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NELSON SENIOR MATHS

FOR THE AUSTRALIAN CURRICULUM | **GENERAL 11**

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NELSON
CENGAGE Learning

Nelson Maths General 11
1st Edition
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CONTENTS

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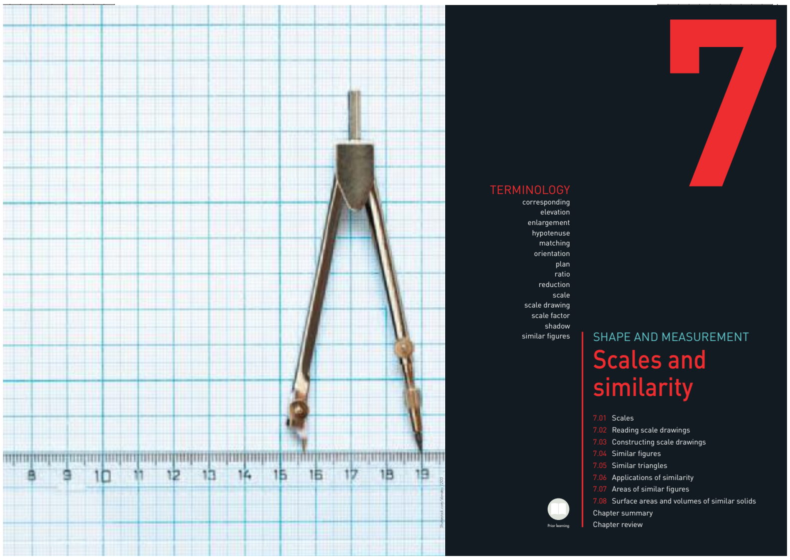
ABOUT THIS BOOK



7

TERMINOLOGY
corresponding
elevation
enlargement
hypotenuse
matching
orientation
plan
ratio
reduction
scale
scale drawing
scale factor
shadow
similar figures

SHAPE AND MEASUREMENT

Scales and similarity

7.01 Scales
7.02 Reading scale drawings
7.03 Constructing scale drawings
7.04 Similar figures
7.05 Similar triangles
7.06 Applications of similarity
7.07 Areas of similar figures
7.08 Surface areas and volumes of similar solids
Chapter summary
Chapter review

Prior learning

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A clear outline of chapter contents is provided.

Links to curriculum content descriptions are included.

A Prior learning worksheet is provided, enabling teachers to assess prerequisite understanding.

Example 1

Yun borrowed $25 000 at a simple interest rate of 8.4% p.a. for 6 years. How much interest did he have to pay?

p.a. means per annum, which is 'per year'

Solution

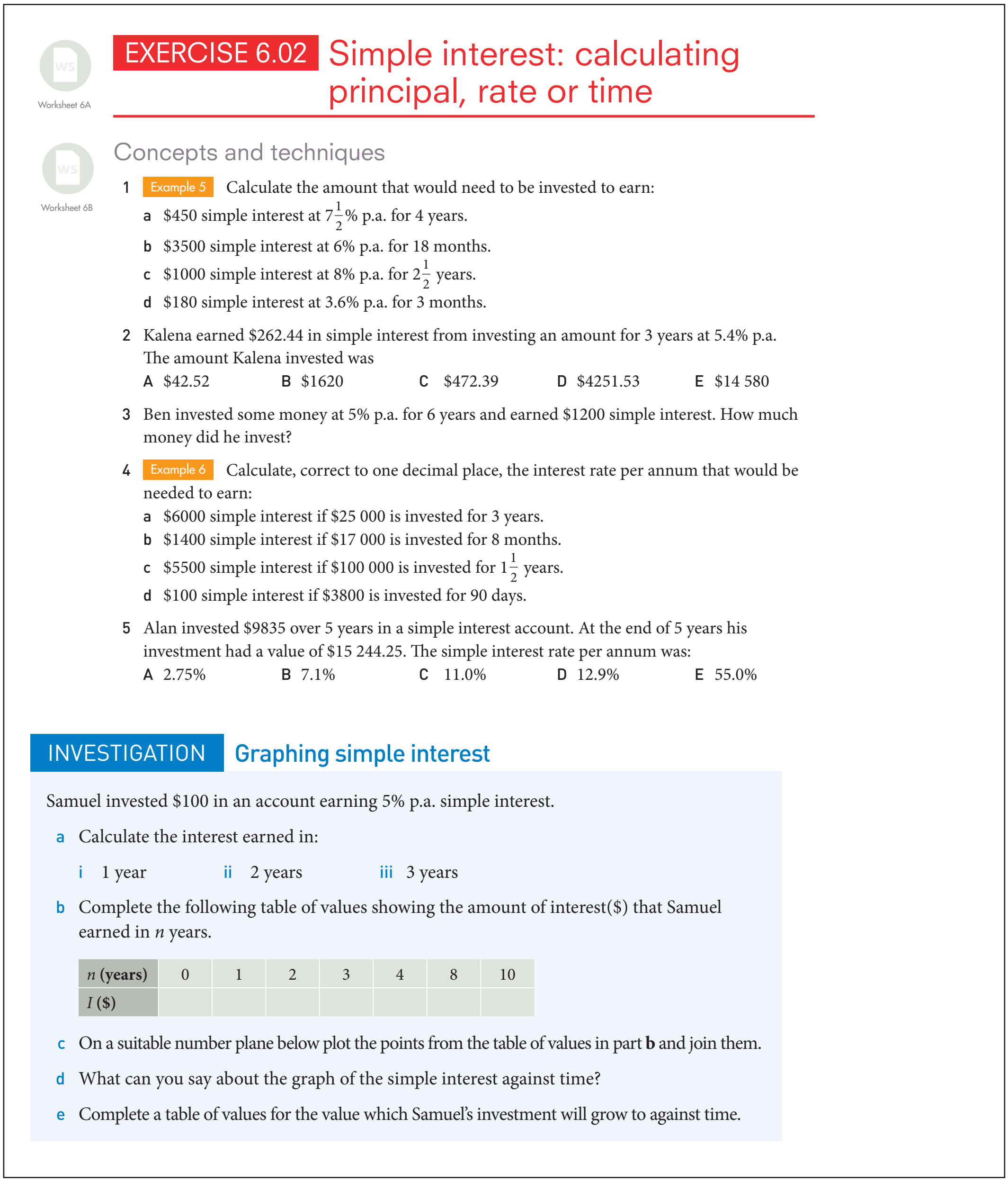
Write the values of the known variables.	$P = 25\,000 \quad r = 8.4 \quad n = 6$
Write the simple interest formula.	$I = \dfrac{Prn}{100}$
Substitute into the formula.	$I = \dfrac{25\,000 \times 8.4 \times 6}{100}$
Evaluate.	$= 12\,600$
Write the answer.	Yun had to pay $12 600 interest.

Examples are clearly set out with reasoning (in black) and writing (in blue) in separate columns.

Examples are sequenced in a logical order.

There are generally three, and no more than four, examples leading to an exercise.

Examples show solutions and steps which guide students through the use of the TI-Nspire CAS CX and the CASIO ClassPad calculators.



Worksheet 6A

Worksheet 6B

EXERCISE 6.02 Simple interest: calculating principal, rate or time

Concepts and techniques

1 Example 5 Calculate the amount that would need to be invested to earn:
   - a $450 simple interest at $7\frac{1}{2}$% p.a. for 4 years.
   - b $3500 simple interest at 6% p.a. for 18 months.
   - c $1000 simple interest at 8% p.a. for $2\frac{1}{2}$ years.
   - d $180 simple interest at 3.6% p.a. for 3 months.

2 Kalena earned $262.44 in simple interest from investing an amount for 3 years at 5.4% p.a. The amount Kalena invested was

A $42.52 B $1620 C $472.39 D $4251.53 E $14 580

3 Ben invested some money at 5% p.a. for 6 years and earned $1200 simple interest. How much money did he invest?

4 Example 6 Calculate, correct to one decimal place, the interest rate per annum that would be needed to earn:
   - a $6000 simple interest if $25 000 is invested for 3 years.
   - b $1400 simple interest if $17 000 is invested for 8 months.
   - c $5500 simple interest if $100 000 is invested for $1\frac{1}{2}$ years.
   - d $100 simple interest if $3800 is invested for 90 days.

5 Alan invested $9835 over 5 years in a simple interest account. At the end of 5 years his investment had a value of $15 244.25. The simple interest rate per annum was:

A 2.75% B 7.1% C 11.0% D 12.9% E 55.0%

INVESTIGATION Graphing simple interest

Samuel invested $100 in an account earning 5% p.a. simple interest.

a Calculate the interest earned in:

i 1 year ii 2 years iii 3 years

b Complete the following table of values showing the amount of interest($) that Samuel earned in *n* years.

n (years)	0	1	2	3	4	8	10
I ($)							

c On a suitable number plane below plot the points from the table of values in part **b** and join them.

d What can you say about the graph of the simple interest against time?

e Complete a table of values for the value which Samuel's investment will grow to against time.

Exercise questions are clearly divided into two categories: concepts and techniques and reasoning and communication.

Where appropriate, worksheets are provided for additional practice and consolidation of key concepts.

Each chapter contains at least one investigation, providing students with the opportunity to apply their understanding to a practical application.

CHAPTER SUMMARY
SIMPLE AND COMPOUND INTEREST 6

- **Simple interest** (or **flat rate interest**) is interest earned or charged only on the original amount of money (**principal**) invested or borrowed.
- $I = \frac{Prn}{100}$ where I = simple interest, P = principal, r = rate of interest per period, n = number of periods
- The **amount** of a loan or investment can be found by adding the interest to the principal. $A = P + I$
- The simple interest formula can also be used to calculate the principle, rate or time by substituting in the given information and solving the resulting equation for the unknown.
- When you make a **deposit** into a bank account, the bank **credits** your account and the amount of the deposit is added to the balance. When you make a **withdrawal** from a bank account, the bank **debits** your account and the amount of the withdrawal is subtracted from the balance.
- Using a **credit card** to purchase goods is like taking out a short-term loan. Some include an interest free period but for some cards the interest is charged daily on each item from the date of purchase.
- For **compound interest** the interest is added to the principal and this is then reinvested. The **compounding periods** say how often the interest is added on. For example yearly, six-monthly, quarterly, monthly or even daily.
- For compound interest, the future value or the amount of the investment can be calculated using $A = P\left(1 + \frac{r}{100}\right)^n$

where: A (amount) = future value or final balance

P = principal or initial quantity

n = number of compounding periods

r = interest rate per compounding period

- The compound interest (I) can be found using $I = A - P$
- The compound interest formula can also be used to calculate the principal, rate or time by substituting in the given information and solving the resulting equation for the unknown.

For a calculation relating to a compound interest investment, the fields for the Financial Solver or Financial Application (Compound Interest) are as follows.

N is the number of time periods.

I% is the interest rate as a percentage per annum.

PV is the present value (for an investment, this is amount invested or the principal and is negative since this amount is being given to the bank).

Pmt or **PMT** is the value of the regular payments being made (for a compound interest investment this is 0).

FV is the future value (this it is positive as it is money that is paid back by the bank).

Pp/Y or **P/Y** is the number of payments per year.

Cp/Y or **C/Y** is the number of times in a year interest is compounded.

Pp/Y or **P/Y** and **CpY** or **C/Y** take the same value for all compound interest calculations.

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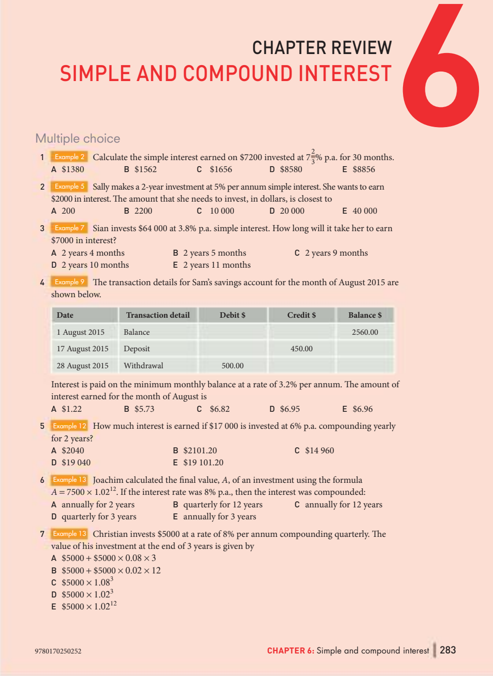
CHAPTER 9: Simple and compound interest | 279



CHAPTER REVIEW
SIMPLE AND COMPOUND INTEREST 6

Multiple choice

1 Example 2 Calculate the simple interest earned on \$7200 invested at $7\frac{2}{3}$% p.a. for 30 months.
A \$1380 B \$1562 C \$1656 D \$8580 E \$8856

2 Example 5 Sally makes a 2-year investment at 5% per annum simple interest. She wants to earn \$2000 in interest. The amount that she needs to invest, in dollars, is closest to
A 200 B 2200 C 10 000 D 20 000 E 40 000

3 Example 7 Sian invests \$64 000 at 3.8% p.a. simple interest. How long will it take her to earn \$7000 in interest?
A 2 years 4 months B 2 years 5 months C 2 years 9 months
D 2 years 10 months E 2 years 11 months

4 Example 9 The transaction details for Sam's savings account for the month of August 2015 are shown below.

Date	Transaction detail	Debit \$	Credit \$	Balance \$
1 August 2015	Balance			2560.00
17 August 2015	Deposit		450.00	
28 August 2015	Withdrawal	500.00		

Interest is paid on the minimum monthly balance at a rate of 3.2% per annum. The amount of interest earned for the month of August is
A \$1.22 B \$5.73 C \$6.82 D \$6.95 E \$6.96

5 Example 12 How much interest is earned if \$17 000 is invested at 6% p.a. compounding yearly for 2 years?
A \$2040 B \$2101.20 C \$14 960
D \$19 040 E \$19 101.20

6 Example 13 Joachim calculated the final value, A, of an investment using the formula $A = 7500 \times 1.02^{12}$. If the interest rate was 8% p.a., then the interest was compounded:
A annually for 2 years B quarterly for 12 years C annually for 12 years
D quarterly for 3 years E annually for 3 years

7 Example 13 Christian invests \$5000 at a rate of 8% per annum compounding quarterly. The value of his investment at the end of 3 years is given by
A $\$5000 + \$5000 \times 0.08 \times 3$
B $\$5000 + \$5000 \times 0.02 \times 12$
C $\$5000 \times 1.08^3$
D $\$5000 \times 1.02^3$
E $\$5000 \times 1.02^{12}$

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CHAPTER 6: Simple and compound interest | 283

Each chapter concludes with a detailed chapter summary and a comprehensive review of concepts.

At the end of every three chapters a comprehensive mixed revision set is provided.

ICONS IN THE TEXTBOOK

Example 1	Link from question to worked example
Prior learning	Prior learning
Worksheets	Worksheet
Spreadsheet	Interactive spreadsheet
Practice quiz	Practice quiz

ABOUT THIS SERIES

There are eight books for this series currently. These cover the subjects Essential Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics.

These books have all been written specifically, from scratch, by a truly national author team for the Senior Australian Curriculum.

There are worked solutions available for purchase, which accompany each of the General, Methods and Specialist Student Books.

There are also ExamView question banks which can be purchased separately. These will contain a large number of multiple choice questions covering each course: General, Methods and Specialist. An ExamView test generator is also provided as a part of this package, so that teachers can construct their own tests for each topic.

Accompanying each printed textbook is a digital textbook called the NelsonNetBook and a NelsonNet website.

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For each chapter the resources are listed. Simply click on the required resource.

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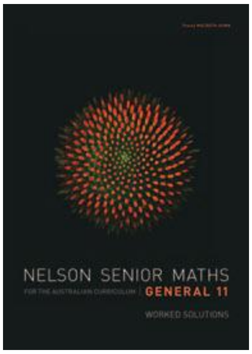

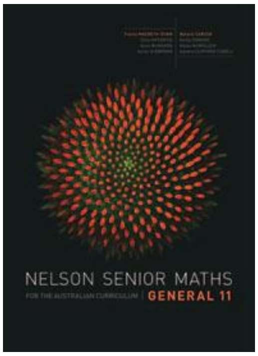

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The **Digital Resources Team** includes Issam Kanaan, Angelique Phillip-Kemp, Kate Sharp and Roger Walter.

ABBREVIATIONS AND SYMBOLS

=	is equal to
≠	is not equal to
≈	is approximately equal to
√	square root
<	is less than
>	is greater than
≤	is less than or equal to
≥	is greater than or equal to
±	plus or minus
∴	therefore
1 : 650 000	at a scale of 1 to 650 000
5 : 8	ratio of 5 to 8
%	percent
/	per
LHS	left-hand side
RHS	right-hand side
Q_3	3rd quartile
$\bar{x}$	mean of x values
s_x or σ_x	standard deviation
π	pi (approximately 3.14159…)
|||	is similar to
∠	angle
△	triangle
36° 27′ 15″	36 degrees 27 minutes and 15 seconds
053°T	a true bearing
N 53° E	a compass bearing

GREEK ALPHABET

Α,	α	alpha
Β,	β	beta
Γ,	γ	gamma
Δ,	δ	delta
Ε,	ε	epsilon
Ζ,	ζ	zeta
Η,	η	eta
Θ,	θ	theta
Ι,	ι	iota
Κ,	κ	kappa
Λ,	λ	lambda
Μ,	μ	mu
Ν,	ν	nu
Ξ,	ξ	xi
Ο,	ο	omicron
Π,	π	pi
Ρ,	ρ	rho
Σ,	σ	sigma
Τ,	τ	tau
Υ,	υ	upsilon
Φ,	ϕ	phi
Χ,	χ	chi
Ψ,	ψ	psi
Ω,	ω	omega


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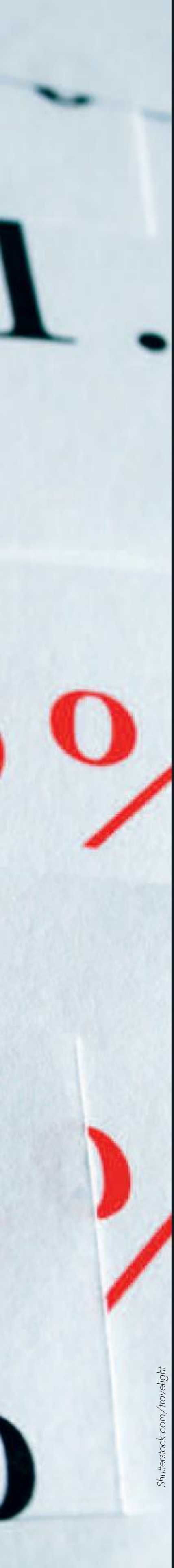

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1

CONSUMER ARITHMETIC

PERCENTAGES AND RATES

TERMINOLOGY

cost price
currency
currency exchange rate
discount
goods and services tax (GST)
inflation
loss
mark-up
percent
profit
rate
retail price
selling price
unit cost
unitary method

Prior learning

APPLICATIONS OF RATES AND PERCENTAGES

- **review rates and percentages (ACMGM001)**
- **compare prices and values using the unit cost method (ACMGM005)**
- **apply percentage increase or decrease in various contexts; for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest (ACMGM006)**
- **use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars (ACMGM007)**

1.01 REVIEW OF PERCENTAGES

The word **percent** means ‘per hundred’ or ‘out of 100’ and is written using the symbol %.

For example, 37% means ‘37 out of 100’ or $\frac{37}{100}$ when written as a fraction.

A fraction or decimal can be converted to a percentage by multiplying by 100%.

IMPORTANT

When converting to a percentage you multiply by 100%.

Example 1

Convert the following to percentages.

a $\frac{3}{4}$ **b** $2\frac{1}{7}$ **c** 0.624 **d** 1.37

Solution

a Multiply by 100%.

$$\frac{3}{4} \times 100\%$$

Rewrite 100% as a fraction.

$$= \frac{3}{4} \times \frac{100}{1}\%$$

Cancel to simplify.

$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{100}^{25}}{1}\%$$

Complete the multiplication and write your answer.

$$= \frac{75}{1}\%$$

$$= 75\%$$

b Convert into an improper fraction.

$$2\frac{1}{7} = \frac{15}{7}$$

Multiply by 100%.

$$\frac{15}{7} \times 100\%$$

Rewrite 100% as a fraction.

$$= \frac{15}{7} \times \frac{100}{1}\%$$

No cancelling is possible so complete the multiplication.

$$= \frac{1500}{7}\%$$

Rewrite the fraction as a mixed number for your final answer. $= 214\frac{2}{7}\%$

c Multiply by 100%. $0.624 \times 100\%$

$= 62.4\%$

d Multiply by 100%. $1.37 \times 100\%$

$= 137\%$

Percentages can be converted to fractions or decimals.

IMPORTANT

When converting a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and simplify.

When converting a percentage to a decimal, divide by 100.

Example 2

Convert the following percentages to

i fractions **ii** decimals.

a 35% **b** 109% **c** 9.4%

Solution

a i Write 35% as a fraction. $35\% = \frac{35}{100}$

Express the fraction in simplest form by cancelling. Divide both the numerator and the denominator by the highest common factor (HCF) of 5. $= \frac{\cancel{35}^{7}}{\cancel{100}_{20}}$

Write your answer. $= \frac{7}{20}$

ii Divide 35 by 100. $35 \div 100$

Write your answer. $= 0.35$

Move the decimal point two places to the left, when converting a percentage to a decimal.

b i Write 109% as a fraction. $109\% = \frac{109}{100}$

Rewrite the fraction as a mixed number for your final answer. Remember to check that the fraction is in simplest form. $= 1\frac{9}{100}$

ii Divide 109 by 100. $109 \div 100$

Write your answer. $= 1.09$

c **i** Write 9.4% as a fraction. $\frac{9.4}{100}$

Multiply both the numerator and the denominator by 10. $= \frac{9.4}{100} \times \frac{10}{10}$

Simplify the fraction and write your answer. $= \frac{\cancel{94}^{47}}{\cancel{1000}_{500}}$

$= \frac{47}{500}$

ii Divide 9.4 by 100. $9.4 \div 100$

Write your answer. $= 0.094$

When solving practical problems involving percentages the same rules apply.

Example 3

What percentage is:

a 70 out of 280? **b** 60 cents out of \$1.25? **c** 36 minutes out of 1 hour?

Solution

a Write as a fraction and multiply by 100%. $\frac{70}{280} \times 100\%$

Rewrite 100% as a fraction.
Cancel to simplify. $= \frac{\cancel{70}^{1}}{\cancel{280}_{\cancel{4}1}} \times \frac{\cancel{100}^{25}}{1}\%$

Complete the multiplication and write your answer. $= \frac{25}{1}\%$

$= 25\%$

b Write as a fraction and multiply by 100%. $\frac{60\text{c}}{\$1.25} \times 100\%$

Convert both to cents. $= \frac{60}{125} \times 100\%$

When comparing two quantities, make sure that they have the same units.

Rewrite 100% as a fraction.
Cancel to simplify. $= \frac{\cancel{60}^{12}}{\cancel{125}_{\cancel{5}1}} \times \frac{\cancel{100}^{4}}{1}\%$

Complete the multiplication and write your answer. $= 48\%$

c Write as a fraction and multiply by 100%. $\frac{36\text{ min}}{1\text{ h}} \times 100\%$

Convert both to minutes. $= \frac{36}{60} \times 100\%$

Rewrite 100% as a fraction.
Cancel to simplify. $= \frac{\cancel{36}^{6}}{\cancel{60}_{\cancel{6}1}} \times \frac{\cancel{100}^{10}}{1}\%$

Complete the multiplication and write your answer. $= 60\%$

A CAS calculator can be used to complete these calculations.

Example 4

a Convert $1\frac{5}{9}$ to a percentage.

b Convert 0.085 to a percentage.

c Convert $24\frac{5}{8}\%$ to a fraction, in simplest form, and a decimal.

d Convert 6500 g out of 8 kg to a percentage.

Solution

TI-Nspire CAS

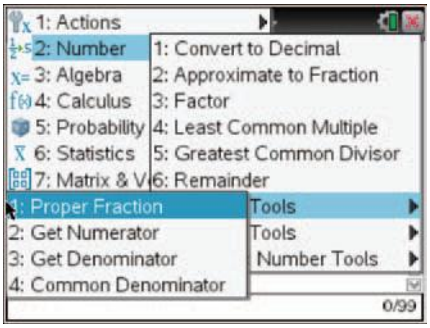
a Key in the whole number component plus the fraction component. Press EXE.

1 + ctrl ÷ 5 ▼ 9 enter

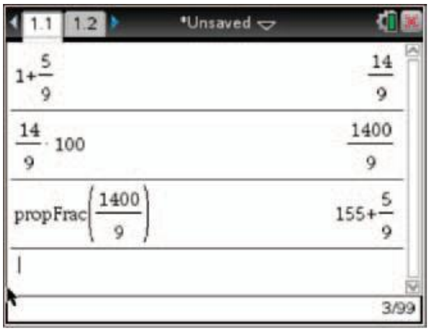
Multiply the improper fraction by 100.

$\times 100$ enter

Press menu
2 (Number)
7(Fraction Tools) then
1(Proper fraction).



Press ▲ enter then enter.



$1+\frac{5}{9}$ $\frac{14}{9}$

$\frac{14}{9}\cdot 100$ $\frac{1400}{9}$

$\text{propFrac}\left(\frac{1400}{9}\right)$ $155+\frac{5}{9}$

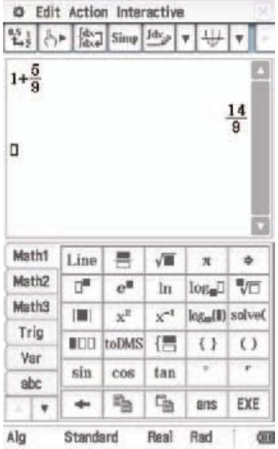
Write your answer.

$1\frac{5}{9} = 155\frac{5}{9}\%$

ClassPad

a Key in the whole number component plus the fraction component. Press EXE.

1 + Keyboard ▤ 5 ▼ 9 EXE



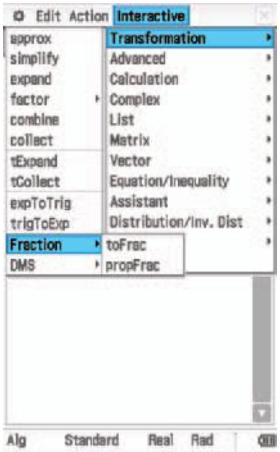
$1+\frac{5}{9}$

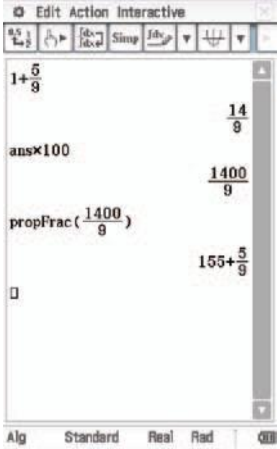
$\frac{14}{9}$

Multiply the improper fraction by 100.

× 100 EXE

Highlight this result and drag to the next line. Highlight, then tap **Interactive**, then **Transformation**, **Fraction**, then **propFrac**.





$1+\frac{5}{9}$

$\frac{14}{9}$

$\text{ans}\times 100$

$\frac{1400}{9}$

$\text{propFrac}\left(\frac{1400}{9}\right)$

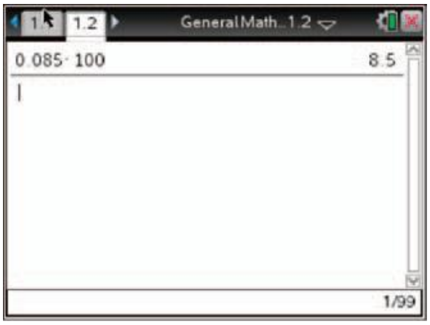
$155+\frac{5}{9}$

Write your answer.

$1\frac{5}{9}=155\frac{5}{9}\%$

TI-Nspire CAS

b Key in the decimal and multiply by 100.
Press enter.

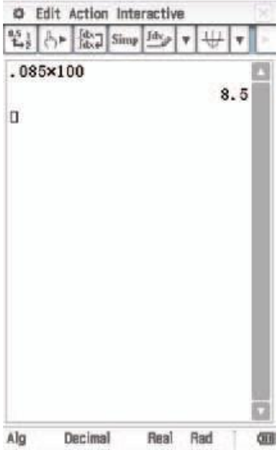


```
0.085· 100                8.5
```

Write your answer. 0.085 = 8.5%

ClassPad

b Use **Decimal** mode by tapping **Standard** on the bottom toolbar.
Key in the decimal and multiply by 100.
Press EXE.



```
.085×100
                8.5
```

Write your answer. 0.085 = 8.5%

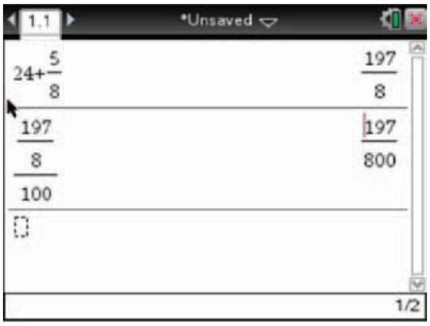
TI-Nspire CAS

c Key in the whole number component and the fraction component. Press EXE.

24 + ctrl ÷ 5 ▼ 8 enter

Then divide by 100.

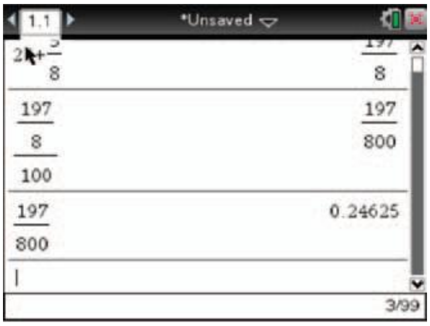
÷ 100 enter



$$24+\frac{5}{8} \qquad \frac{197}{8}$$

$$\frac{\frac{197}{8}}{100} \qquad \frac{197}{800}$$

Press ▲ enter then ctrl enter to express this answer as a decimal.



$24+\frac{5}{8}$ $\frac{197}{8}$

$\frac{\frac{197}{8}}{100}$ $\frac{197}{800}$

$\frac{197}{800}$ 0.24625

Write your answer.

$24\frac{5}{8}\% = \frac{197}{800} = 0.24625$

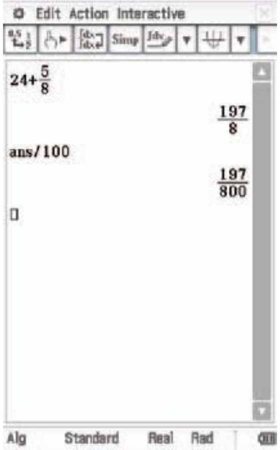
ClassPad

c Using **Standard** mode, key in the whole number component and the fraction component. Press EXE.

24 + Keyboard ▭ 5 ▼ 8 EXE

Divide the improper fraction by 100.

÷ 100 EXE



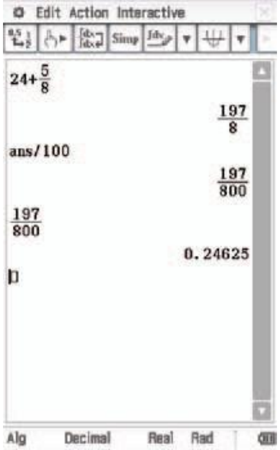
$24+\frac{5}{8}$ $\frac{197}{8}$

ans/100 $\frac{197}{800}$

Change the mode to **Decimal**.

Highlight $\frac{197}{800}$ and drag into the entry line.

Press EXE to express this answer as a decimal.



$24+\frac{5}{8}$ $\frac{197}{8}$

ans/100 $\frac{197}{800}$

$\frac{197}{800}$ 0.24625

Write your answer.

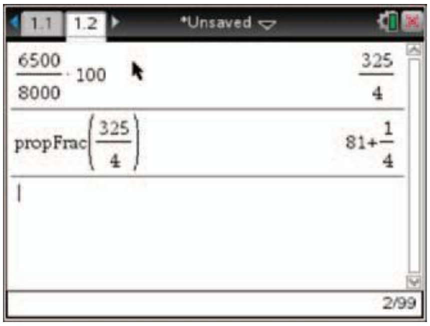
$24\frac{5}{8}\% = \frac{197}{800} = 0.24625$

TI-Nspire CAS

d Write as a fraction in the same units.

$$\frac{6500\text{ g}}{8\text{ kg}}=\frac{6500}{8000}$$

Key in the fraction and multiply by 100. Complete the conversion of the improper fraction into a mixed number.



$\frac{6500}{8000}\cdot 100$ $\frac{325}{4}$

$\text{propFrac}\left(\frac{325}{4}\right)$ $81+\frac{1}{4}$

Write your answer.

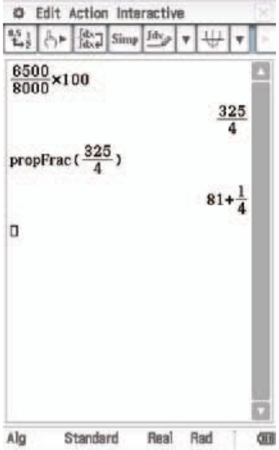
6500 g out of 8 kg $=81\frac{1}{4}\%$

ClassPad

d Write as a fraction in the same units.

$$\frac{6500\text{ g}}{8\text{ kg}}=\frac{6500}{8000}$$

Key in the fraction and multiply by 100. Complete the conversion of the improper fraction into a mixed number.



$\frac{6500}{8000}\times 100$ $\frac{325}{4}$

$\text{propFrac}\left(\frac{325}{4}\right)$ $81+\frac{1}{4}$

Write your answer.

6500 g out of 8 kg $=81\frac{1}{4}\%$

EXERCISE 1.01 Review of percentages

Concepts and techniques

1 Example 1 Convert the following fractions to percentages.

a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{4}{5}$ d $\frac{3}{8}$

e $\frac{2}{3}$ f $\frac{4}{9}$ g $\frac{4}{7}$ h $1\frac{3}{4}$

i $2\frac{1}{5}$ j $3\frac{1}{3}$ k $4\frac{5}{6}$ l $3\frac{2}{7}$

2 Convert the following decimals to percentages.

a 0.75 b 0.076 c 0.05 d 0.8673

e 1.8 f 3.01 g 3.10 h 10.705

i $0.\overline{3}$ j $2.\overline{1}$ k 4.375 l $6.\overline{6}$

3 Example 2 Convert the following percentages to

i simplest fraction form ii decimal form.

a 21% b 84% c 49% d 120%

e 300% f 12.5% g 36% h 110%

i $33\frac{1}{3}\%$ j 0.5% k 180% l 244.4%

4 Copy and complete the following table.

Fraction	Decimal	Percentage
$\frac{7}{10}$		
	0.96	
		14%
	63.125	
$1\frac{7}{8}$		
		225%

5 Example 3 Convert the following to percentages.

a 14 out of 20 b 80 out of 120 c 42 minutes out of 60 minutes

d 60 pages out of 300 pages e 400 m out of 8 km f \$3.50 out of \$15

g 300 L out of 900 kL h 200 mm out of 3 m i 25 seconds out of 2 minutes

j 450 mL out of $1\frac{1}{2}$ L k 200 g out of 7 kg l $26\frac{1}{2}$ mm out of 4 cm

6 Example 4 Convert the following to percentages.

a $1\frac{2}{9}$ b 0.099 c $2\frac{3}{7}$

d 0.128 e 2.789 f $5\frac{5}{11}$

g 49 h $0.\overline{5}$ i 32 out of 48

j \$7.80 out of \$12 k 30 cm out of $2\frac{1}{2}$ m l 16 minutes out of 2 hours

9780170250252

Reasoning and communication

7 A Year 11 student completed the following working to convert a fraction to a percentage. His answer is incorrect.

a Identify where the error was first made in the working shown below.

$$\begin{aligned} 2\frac{5}{12}\times 100\% &= \frac{17}{12}\times\frac{100}{1}\% \\ &= \frac{17}{3}\times\frac{25}{1}\% \\ &= \frac{425}{3}\% \\ &= 141\frac{2}{3}\% \end{aligned}$$

b Complete the conversion correctly.

8 A Year 11 student achieved a score of 11 out of 15 on their Mathematics quiz. Convert this to a percentage.

9 A post office employee noticed that the watermark stamp covered the actual postage stamp on 42 out of 50 occasions. Express this as a percentage.

10 A recent survey indicated that 127 out of 200 Year 11 students watched *Big Sister*. What percentage of students surveyed watched *Big Sister?*

11 Anna saved $36 out of her $80 pay.

a What percentage did she save?

b What percentage did she spend?

12 In a bag of 40 jellybeans, there are 15 red, 5 yellow and 10 black ones.

a What is the percentage of:

i red jellybeans?

ii yellow jellybeans?

iii black jellybeans?

b What percentage of jellybeans are not red, yellow or black?

Getty Images/David Bishop Inc.

13 There were 17 girls in a class of 25 students.

a What was the percentage of girls in the class?

b What percentage of the class was boys?

c A girl left the class, what is the new percentage of boys in the class?

14 John obtained the following results for his examinations:

English: 33 out of 40
Mathematics: 64 out of 75
Science: 17 out of 20

a Convert each of the results into percentages.

b In which subject did he receive the best result?

15 Paul used 270 cm of his 4 m rope. The percentage that he had left was:

A 10.8%
B 32.5%
C 57.5%
D 67.5%
E 130%

16 The Jones family took a full 10 L esky on a hike. When they came home they measured that there was 650 mL left in the esky. What percentage had they consumed?

1.02 PERCENTAGE OF A QUANTITY

To find a percentage of a quantity, multiply the quantity by the percentage. The percentage needs to be expressed as either a fraction or a decimal when multiplying.

You may find it easiest to convert the percentage to a decimal.

Example 5

Find:

a 35% of 360

b 72% of $890

c 15.6% of 78 kg, correct to one decimal place.

Remember that 'of' means multiply.

Solution

a Write the percentage as a fraction and then multiply by the given quantity.

35% of 360

$$= \frac{35}{100} \times 360$$

Cancel to simplify.

$$= \frac{\cancel{35}^{7}}{\cancel{100}_{\cancel{5}1}} \times \frac{\cancel{360}^{18}}{1}$$

Complete the multiplication and write your answer.

$$= 7 \times 18$$
$$= 126$$

b Write the percentage as a decimal and then multiply by the given quantity.

72% of $890

$= 0.72 \times 890$

Write your answer.

$= \$640.80$

c Write the percentage as a decimal and then multiply by the given quantity.

15.6% of 78 kg

$= 0.156 \times 78$

Evaluate

$= 12.168$

Write your answer correct to one decimal place.

≈ 12.2 kg

The following examples demonstrate the correct use of a calculator for solving these types of problems.

Example 6

Calculate:

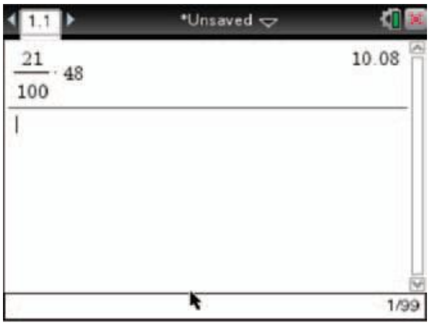
a 21% of $48

b $74\frac{1}{2}$% of 3.5 km, correct to one decimal place.

Solution

TI-Nspire CAS

a Enter 21% as a fraction then multiply by the given quantity. Press ctrl enter to obtain the answer as a decimal.

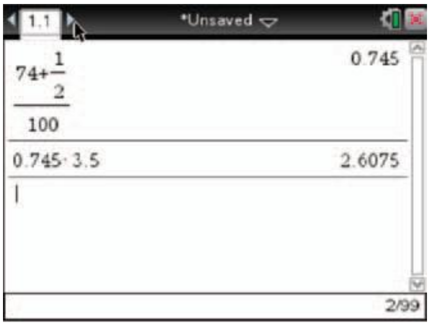


$\frac{21}{100} \cdot 48$ 10.08

Write your answer. 21% of \$48 = \$10.08

b Convert $74\frac{1}{2}$% to a decimal. $74\frac{1}{2} \div 100 = 0.745$

Complete the multiplication, i.e. 0.745 × 3.5.



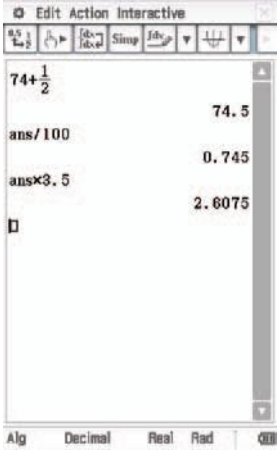
$\frac{74+\frac{1}{2}}{100}$ 0.745

$0.745 \cdot 3.5$ 2.6075

Write your answer, correct to one decimal place. Make sure that you include the correct units. $74\frac{1}{2}$% of 3.5 km = 2.6 km

ClassPad

a With the CAS in **Decimal** mode, enter 21% as a fraction then multiply by the given quantity. Press EXE



$74+\frac{1}{2}$

74.5

ans/100

0.745

ans×3.5

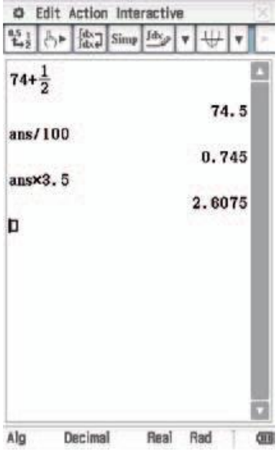
2.6075

Write your answer. 21% of \$48 = \$10.08

b Convert $74\frac{1}{2}$% to a decimal.

$74\frac{1}{2}$ $\boxed{\div}$ $100 = 0.745$

Complete the multiplication, i.e. 0.745×3.5.



```
74+1/2
                74.5
ans/100
               0.745
ans×3.5
              2.6075
```

Write your answer, correct to one decimal place. Make sure that you include the correct units.

$74\frac{1}{2}$% of 3.5 km = 2.6 km

EXERCISE 1.02 Percentage of a quantity

Concepts and techniques

1 Example 5 Find:

- **a** 25% of \$240
- **b** 60% of 80 m
- **c** 15% of 60 L
- **d** 75% of 5 hours
- **e** 40% of 35 cm^2
- **f** 110% of \$44
- **g** $33\frac{1}{3}$% of 960 kg
- **h** $12\frac{1}{2}$% of 720 students

2 Find:

- **a** 41% of \$600
- **b** 11% of 270 L
- **c** 12.5% of \$500
- **d** 6.1% of 420 g
- **e** $5\frac{1}{2}$% of 8 t
- **f** 175% of 7 mm
- **g** $44\frac{3}{5}$% of 90 s
- **h** 15.8% of 28.5 m

3 Example 6 Calculate the following (correct to two decimal places):

- **a** 42% of \$55
- **b** 33% of \$150
- **c** 16.4% of 48.2 kg
- **d** 18.7% of \$185
- **e** 53% of \$286
- **f** 123% of 45 g
- **g** 6% of 42 L
- **h** 205% of \$950

Reasoning and communication

4 A computer has downloaded 62% of a 184 megabyte file. How many megabytes has it downloaded?

5 Susan sees a \$70 jacket at 20% off during a sale. How much will she save?

A \$3.50 **B** \$14 **C** \$20 **D** \$50 **E** \$56

6 Paul completes 75% of a 25 km marathon before having to pull out. How many kilometres did he run?

7 Dave lost 28% of his body weight. If he weighed 120 kg, how much did he lose?

8 Lead contains 23% of the isotope Pb 207. How much of this isotope is there in 50 kg of lead?

9 9 carat gold contains 37.5% pure gold. How much pure gold would be in a 9 carat gold bracelet that weighs 25 g?

10 43% of the students at a school study Japanese. If 839 students attend the school, how many students study Japanese?

11 Sandy works on a commission of 15% of sales plus a weekly retainer of $240. How much does she earn in a week in which she sells $5000 worth of goods?
A $255 B $573 C $750 D $786 E $990

12 Sam invests $6000 in an account that pays interest of 4.5% per year.
- a How much interest would he earn each year?
- b How much would he have in his account at the end of the first year?

13 A school is given a 15% bulk discount if it orders more than $1500 worth of textbooks. The school orders 75 Year 11 books at $64.80 each and 60 Year 12 books at $61.50 each.
- a Calculate the discount the school receives.
- b Calculate the total paid by the school.

1.03 FINDING THE WHOLE FROM A GIVEN PERCENTAGE

If a percentage of an amount is known, then the whole amount can be found using the **unitary method**. This involves dividing to find 1% and then multiplying by 100 to find 100%.

Example 7

If 15% of an unknown amount is $9, find the whole amount.

Solution

Write the equivalence statement.	15% is $9
Divide by 15 to find 1%.	1% is $\$\frac{9}{15}$
Multiply by 100, to find 100%.	100% is $\$\frac{9}{15} \times 100 = \$60$
Write the answer.	The whole amount is $60.

Example 8

A retailer is selling an item for \$59.50. If this is 170% of what he paid for the item, calculate the price he paid.

Solution

Write the equivalence statement.	170% is \$59.50
Divide by 170 to find 1%.	1% is $\$\dfrac{59.50}{170}$
Multiply by 100 to find 100%.	100% is $\$\dfrac{59.5}{170} \times 100 = \$35$
Answer the question.	He paid \$35.

An alternative method can be used to complete these calculations.

170% of price paid = \$59.50

Let x represent the price he paid.

$$1.7x = 59.50$$
$$x = \frac{59.50}{1.7}$$
$$= 35$$

He paid \$35.

The unitary method is useful for finding any percentage that is required, not just 100%.

Example 9

A town has 4800 adults and its population is 65% adults. If 45% of the population are female, how many females are there?

Solution

Write the equivalence statement.	65% is 4800
Divide by 65 to find 1%.	1% is $\dfrac{4800}{65}$
Multiply by 45 to find 45%.	45% is $\dfrac{4800}{65} \times 45 = 3323.076...$
Round to the nearest whole number.	$\approx 3323.$
Answer the question.	There are approximately 3323 females in the town.

EXERCISE 1.03 Finding the whole from a given percentage

Concepts and techniques

1 Example 7 Find the whole quantity, if:

a 4% is \$48
b 75% is 150 km
c 120% is 40 L
d 45% is 54 minutes
e 80% is 320 g
f $33\frac{1}{3}$% is 90 hectares

2 Find 100%, correct to one decimal place if necessary, if:

a 12% is 11.82 m
b 110% is $71.50
c 2.1% is 75 mL
d 70% is 252 L
e 64.5% is 2500 people
f $122\frac{1}{2}$% is 450 g

3 Find the whole quantity, correct to two decimal places if necessary, if:

a 23% is 42.5 kg
b 71% is 400 mL
c 35.2% is 14 mg
d $16\frac{1}{2}$% is 2937 girls
e 126% is 45.35 mm
f 97.5% is $16 980

4 Example 8 245 students in a school voted for a new uniform. If this is 35% of the school population, what is the school population?

A 14
B 280
C 455
D 700
E 8575

5 Shaylee has lost 28.8 kg. If this is 32% of her starting weight, calculate her starting weight.

6 The profit on an iPod is 65% of the cost price. If the profit is $106, find the cost price correct to the nearest dollar.

A $163
B $171
C $175
D $269
E $303

7 A television was bought for $1444.15 at a 15% off storewide sale. What was the original price of the television?

8 Example 9 A computer file was being downloaded. The computer showed that it had downloaded 28% and this was 112 megabytes. How many megabytes would it be when 72% had been downloaded?

9 When booking her holiday Susan found that she would receive a discount of 18% if she paid it off on time. This would save her $4500 on the total cost of her holiday.

a Calculate the deposit that Susan had to pay if it was 10% of the total cost of her holiday before her discount was calculated.
b How much did Susan end up paying in total if she paid it off in time?

10 Rochelle is competing in a marathon. She knows that once she runs 14.7 km she will have completed 35% of the marathon.

a How far will she have run in total when she has completed 65% of the marathon?
b How long is the marathon?

11 The Thomson reservoir is the largest of all of Melbourne's reservoirs. It has a capacity of 1 068 000 ML, and it contributes approximately 60% of Melbourne's total reservoir storage capacity. What is the size of Melbourne's total reservoir storage capacity?

Fairfax Photos/Vince Caligiuri, The Age Melbourne

12 The average attendance at a school is 95% of the enrolment. If the average attendance is 931 students, what is the enrolment at the school?

13 A factory was doing quality control on light globes. It found 36 faulty globes in a batch. If this was 2.3% of the batch, what was the total number of light globes in the batch?

14 All Year 11 students had to choose between three sports to play in at a sports day. 59% chose soccer, 12% chose hockey and 24% chose touch football. If 92 students chose soccer,

- **a** calculate, to the nearest whole number, the number of students who chose:
  - **i** Hockey
  - **ii** Touch football.
- **b** How many students did not make a choice?

15 Running costs of a car include petrol, servicing, registration and insurance. The cost of petrol is about 40% of total running costs. Bradley has bought a second-hand car and found he spent $35 a week on petrol. How much would you expect to spend on the car altogether in a year?

1.04 PERCENTAGE INCREASE AND DECREASE

A whole quantity is 100%. When increasing by a percentage, add the percentage onto 100%, i.e. increasing by 20% means calculate 100% + 20% = 120% of the original amount.

When decreasing by a percentage you subtract the percentage from 100%, i.e. decreasing by 20% means calculate100% – 20% = 80% of the original amount.

Example 10

a Increase 325 mg by 15%.
b Decrease 325 mg by 15%.

Solution

a Increase means add to 100%.	$100\% + 15\% = 115\%$
Write calculation.	115% of 325
Convert the percentage to a decimal.	$= 1.15 \times 325$
Evaluate and write the answer.	$= 373.75$ mg
b Decrease means subtract from 100%.	$100\% - 15\% = 85\%$
Write calculation.	85% of 325
Convert the percentage to a decimal.	$= 0.85 \times 325$
Evaluate and write the answer.	$= 276.25$ mg

Example 11

a Jamie's weekly wage of \$720 increases by 6%. What is his new wage?

b A trampoline with a market price of \$480 is discounted by 45%. What is the sale price?

Solution

a Increase means add to 100%.	$100\% + 6\% = 106\%$
Write the calculation.	106% of \$720
Convert the percentage to a decimal.	$= 1.06 \times 720$ $= 763.2$
Write the answer.	His new weekly wage is \$763.20.
b Discount means decrease, so subtract from 100%.	$100\% - 45\% = 55\%$
Write the calculation.	55% of \$480
Convert the percentage to a decimal.	$= 0.55 \times 480$ $= 264$
Write the answer.	The sale price is \$264.

EXERCISE 1.04 Percentage increase and decrease

Concepts and techniques

1 Example 10 Answer the following correct to one decimal place where necessary.
- a Increase \$700 by 25%
- b Decrease \$700 by 25%
- c Increase 480 kg by 30%
- d Decrease 60 L by 15%
- e Increase 624 hectares by $33\frac{1}{3}\%$
- f Decrease 32 mg by 5%

2 Answer the following correct to one decimal place where necessary.
- a Increase 340 km by 18%
- b Increase \$14 000 by 23%
- c Decrease 320 hectares by 72%
- d Increase 890 L by $6\frac{1}{2}\%$
- e Decrease 7 hours by 42%
- f Decrease 80 cm by 64.5%

3 Answer the following correct to one decimal place where necessary.
- a Increase 156 cm^3 by 2.6%
- b Decrease 18.4 L by 0.5%
- c Increase 78.9 mm by 125%
- d Increase 87.6 kg by 14.3%
- e Decrease 520 tonne by $13\frac{1}{2}\%$
- f Decrease 8.25 mg by 1.5%

4 Example 11 The population of a country town decreased by 13% in the last 5 years. If it started at 2685, what is it now?

A 349 B 940 C 1745 D 2336 E 2672

Alamy/Stephanie Jackson-People collection

5 Paula has a market stall. She sets her selling price by increasing her cost price by 85%. What is her selling price for an item that she bought for $16?

Reasoning and communication

6 A school purchased 150 calculators at $19 each and was given a discount of 12%. Find the total paid for the calculators.

7 Australia's population at the start of last year was 21 435 680. What is the population this year if it has increased by 1.2%?

8 Phuong usually earns $680.50 per week but, this week, she received an 8% bonus. How much did she receive altogether this week?

A $54.44 B $626.06 C $688.50 D $734.94 E $1224.90

9 Hardly Normal Discounts is having a '15% off' sale on all items. How much will you pay for a television set priced at $460?

10 The value of Lisa's computer depreciated (decreased) by 9% this year. If its value last year was $2410, calculate its current value.

11 A restaurant adds a 12% surcharge to all bills on public holidays. Calculate the total to be paid if the bill for a dinner for two on Australia Day comes to $74.50.

12 Laura earns $56 000 a year. What is her new salary if she receives a 7% pay rise?

13 Due to the closure of a large business in the area, a school is expecting an 11% drop in its student numbers for the next year. If its current enrolment is 684 students, what is the anticipated enrolment for the next year?

14 A large workshop normally services an average of 240 cars in a week. Due to an outbreak of the flu there is a 35% reduction in the number of cars serviced in a particular week. How many cars were serviced in this week?

15 With the downturn in the property market, a house that was priced at $4.6 million has been reduced by $8\frac{1}{2}$%. What is the new price?

Alamy/CulturalEyes-AusGS2

1.05 REPEATED PERCENTAGE CHANGE

A retail store increases its prices by 10% at the start of the year, but has a '10% off' clearance sale at the end of the year. Do the increase and decrease cancel each other out? If not, what will be the overall percentage change?

Suppose the original price was \$100.

Increase \$100 by 10%: $110\% \times \$100 = 1.1 \times \$100 = \$110$

Decrease \$110 by 10%: $90\% \times \$110 = 0.9 \times \$110 = \$99$

So, the increase and decrease do not cancel each other out.

The overall change is a decrease of \$1, or 1%.

IMPORTANT

An increase of 10%, followed by a decrease of 10% is equivalent to multiplying the original quantity by 110% and then by 90%.

$$\begin{aligned}110\% \times 90\% &= 1.1 \times 0.9\\ &= 0.99\end{aligned}$$

This is an overall decrease of 1%.

Example 12

What percentage change is equivalent to a decrease of 15%, followed by an increase of 13%?

Solution

Decrease by 15% means subtract from 100%.	$100\% - 15\% = 85\%$
Increase by 13% means add to 100%.	$100\% + 13\% = 113\%$
Calculate the new quantity as a percentage of the original quantity.	New quantity $= 0.85 \times 1.13 \times$ original quantity $= 0.9605 \times$ original quantity New quantity $= 96.05\%$ of original quantity
Calculate the equivalent percentage change.	$100\% - 96.05\% = 3.95\%$
Write the answer.	A decrease of 3.95% is equivalent.

Example 13

J-mart is having a '15% off' sale this week. Sarah works there, so she receives a further 5% staff discount on the sale price. What percentage discount is equivalent to the combined discounts Sarah receives?

Solution

Decrease by 15% means subtract from 100%.	$100\% - 15\% = 85\%$
Decrease by 5% means subtract from 100%.	$100\% - 5\% = 95\%$
Calculate the new price as a percentage of the original price.	$\text{New price} = 0.85 \times 0.95 \times \text{original price}$ $= 0.8075 \times \text{original price}$ $\text{New price} = 80.75\% \text{ of original price}$
Calculate the equivalent percentage change.	$100\% - 80.75\% = 19.25\%$
Write the answer.	A percentage discount of 19.25% is equivalent to the combined discounts Sarah receives.

EXERCISE 1.05 Repeated percentage change

Concepts and techniques

Repeated percentage change

1 Example 12 What percentage change is equivalent to:
- a an increase of 20%, followed by a decrease of 20%?
- b a decrease of 20%, followed by an increase of 20%?
- c an increase of 8%, followed by another increase of 8%?
- d an increase of 10%, followed by a decrease of 15%?
- e a decrease of 4%, followed by a decrease of 6%?
- f a decrease of 5%, followed by an increase of 2%?

Reasoning and communication

2 Example 13 The value of Grant's shares decreased by 8% one year and then increased by 20% the following year. What percentage change is equivalent to a decrease of 8% followed by an increase of 20%?

3 Clara's toy shop increased the price of all toys by 8% due to a manufacturer's price rise. Clara had a sale later that year in which she offered a discount of 5% on all items. For a dolls' house originally marked at $285, calculate:
- a the price after the 8% rise
- b the sale price after the discount
- c the overall change in price and the percentage change on the original price.

4 A town's population of 2000 increased by 10% one year, then decreased by 4% the following year. Which of the following was the town's new population figure?

A 1728 B 1872 C 2006 D 2112 E 2288

5 Which of the following overall percentage increases is equivalent to a percentage increase of 5% followed by another increase of 3%?

A 1.5% B 1.85% C 8% D 8.15% E 15%

6 Dean's investment of \$1000 increased by 8%, 8% and 7% over three years.
  a How much, correct to the nearest cent, was Dean's investment worth at the end of three years?
  b What was the overall percentage change in Dean's investment, to the nearest percentage?

7 An iceberg of mass 3.5 t lost 5% of its mass in the first year, 4% of its mass in the second year and 2% of its mass in the third year.
  a What was the mass of the iceberg after 3 years? Answer correct to two decimal places.
  b What is the percentage change, correct to one decimal place, of the iceberg from its original mass?

1.06 MARK-UPS AND DISCOUNTS

When a retailer buys goods from a wholesaler or manufacturer the price paid is known as the **cost price**. A **mark-up** is usually added to this price in order to cover costs and make money. The price shown on an item is known as the marked price or the **retail price**. The amount the item is sold for is known as the **selling price**. The amount added by the seller depends on many factors, including the turnover, competition, market share, manufacturer's recommendation, freshness and storage life.

A mark-down or **discount** is the amount that is taken off the price of goods so that they sell faster when they are in abundance, selling slowly, damaged or shopsoiled.

Example 14

An electrical store uses a 40% mark-up on large items.
a What is the retail price for a fridge that cost the store \$840?
b What was the cost price for a dishwasher that had a marked price of \$1295?

Solution

a 40% mark-up means that the retail price is 140% of the cost price.	100% + 40% = 140%
Write the calculation.	140% of \$840 = 1.4 × 840 = 1176
Write the answer.	The retail price is \$1176.
b 40% mark-up means that the marked price is 140% of the cost price.	140% is \$1295
Divide both sides by 140 to find 1%.	1% is \$ $\frac{1295}{140}$
Multiply both sides by 100 to find 100%.	100% is \$ $\frac{1295}{140} \times 100 = \$925$
Answer the question.	The cost price was \$925.

An alternative method can be used to complete these calculations.
140% of cost price = 1295
Let x represent the cost price.
$1.4x = 1295$
$x = \frac{1295}{1.4}$
$x = 925$
The cost price is \$925.

Example 15

A store was having a clearance sale where all floor stock was being discounted by 15%.

a How much would a bedroom suite with a retail price of $3500 now cost?

b Angela was entitled to a staff discount of 7% off the already reduced price.

  i What was her total percentage discount?

  ii How much did she pay?

Solution

a	15% off means decrease by 15%.	$100\% - 15\% = 85\%$
	Write the calculation.	85% of \$3500 $= 0.85 \times 3500$ $= 2975$
	Write the answer.	It would cost \$2975.
b i	The initial discount was 15%.	$100\% - 15\% = 85\%$
	Follow this with a further discount of 7%.	$100\% - 7\% = 93\%$
	Combined discount.	New price $= 0.85 \times 0.93 \times$ original price $= 0.7905 \times$ original price New price $= 79.05\%$ of original price
	Calculate the equivalent percentage change.	$100\% - 79.05\% = 20.95\%$
	Write the answer.	A percentage discount of 20.95% is equivalent to the combined discounts Angela receives.
ii	A discount of 20.95% means that Angela pays 79.05% of the original price.	Amount paid = 79.05% of \$3500 $= 0.7905 \times 3500$ $= 2766.75$
	Write the answer.	Angela paid \$2766.75.

IMPORTANT

$$\text{Percentage mark-up} = \frac{\text{mark-up}}{\text{cost price}} \times 100\%$$

$$\text{Percentage discount} = \frac{\text{discount}}{\text{retail price}} \times 100\%$$

Example 16

The retail price of a tyre is \$125 and the cost price was \$75. Calculate the percentage mark-up.

Solution

The mark-up is the difference between the retail price and the cost price.

$$\text{Mark-up} = \$125 - \$75 = \$50$$

Calculate the percentage mark-up.

$$\text{Percentage mark-up} = \frac{50}{75} \times 100\% = 66\frac{2}{3}\%$$

Write the answer.

The percentage mark-up was $66\frac{2}{3}\%$.

EXERCISE 1.06 Mark-ups and discounts

Concepts and techniques

1 Example 14 A hardware shop works on a 65% mark-up. What would be the shop charge for:
  - a a chainsaw with a cost price of \$280?
  - b drills that cost the shop \$140?
  - c packs of ceiling insulation that cost \$60 each?
  - d tubes of silicon costing \$3 each?

2 A takeaway food store works on a mark-up of 120%. What was the cost price of each of the following?
  - a A spring roll sold for \$2.20.
  - b Cod sold for \$4.20 a piece.
  - c Mini dim sims sold for \$1.10 each.
  - d Seafood snacks sold for \$2.85 each.

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3 Example 15 Rachel's ballet fees are \$320 per term, but she gets a sibling discount of 5%. The amount she pays each term is
  - A \$16
  - B \$256
  - C \$304
  - D \$315
  - E \$336

4 The price of a \$2000 television is discounted by 8%.
  - a How much is the discount worth?
  - b What is the discounted price?

5 Example 16 The following table provides cost prices, marked prices, mark-ups and discounts of different items.

a Copy and complete the table.

	Item	Original (cost) price	Marked price	Mark-up or discount?	Amount of mark-up/ discount
i	DVD player	\$240	\$199		
ii	Software bundle	\$235		Mark-up	\$110
iii	Magazine	\$4.05	\$7.50		
iv	Travel bag	\$28	\$62		

b What is the percentage mark-up or discount on each of the items in part **a**? Answer correct to one decimal place.

6 A pool shop sells pumps for \$540. If they cost the shop \$400, what is the percentage mark-up?

7 One day before the use-by date, packets of biscuits are marked down to \$1.50 each. If the original price was \$2.10, what was the percentage discount?

A 28.6% B 40% C 60% D 71.4% E 140%

Reasoning and communication

8 After a fire, smoke-damaged goods are sold at a mark-down of 20% to clear stock.

a What is the marked price of a tracksuit with an original price of \$175?

b What would a dress that originally cost \$240 sell for?

c A jacket is marked down to \$238. What was the original price?

d What was the original price of a top sold for \$68?

9 Mirrors costing \$140 are sold at a mark-up of 60%.

a What is the mark-up? b What is the retail price?

10 A tennis racquet is on special at an 8% discount. If it normally costs \$150, how much does Maryanne save?

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11 A painting is marked up by 125% to \$350. What was the cost price?

A \$155.56 B \$225

C \$280 D \$437.50

E \$787.50

12 An electronics shop buys small resistors for \$30 for each 100 and sells them for 75 cents each. What is the percentage mark-up?

13 A hardware store is having a 15% off sale.

a How much would you pay for a router worth \$282?

b Bob also gets 10% off the total as a builder's discount. How much would he pay for the router?

c The cashier was new and didn't know how to ring up two discounts so she just gave Bob a 25% discount. Was Bob happy with this? Explain.

14 An aluminium dinghy that cost a boatyard \$1200 is sold for \$3160. What is the percentage mark-up?

15 Geoff buys a spa bath. It is on special at 9% discount, and Geoff also gets a 10% builder's discount.

- a What is the total percentage discount that Geoff gets?
- b If the bath was \$1500, how much did Geoff pay?

1.07 PROFIT AND LOSS

A **profit** is made when goods are sold for more than the cost price. A **loss** is made when goods are sold for less than the cost price.

Example 17

Steffi runs a market jewellery stall. She spends \$450 on supplies to make 36 necklaces. She works on a profit margin of 85%. How much should she sell each necklace for?

Solution

Determine the cost for one necklace.

$$\text{Cost per necklace} = \$450 \div 36$$
$$= \$12.50$$

A profit of 85% means increase cost price by 85%.

$$185\% \text{ of } \$12.50 = \frac{185}{100} \times 12.5$$
$$= 23.125$$
$$\approx 23.13$$

With calculations involving money, round to the nearest cent, i.e. two decimal places, unless told otherwise.

Write the answer.

She should sell each necklace for \$23.13.

It is usual to use percentage profit or percentage loss to compare returns for different items.

IMPORTANT

When calculating the percentage profit or loss, cost price is used unless otherwise stated.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\% \qquad \text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

Example 18

Hailey bought a car for \$45 000 and sold it for \$36 000. Brendan bought a car for \$28 000 and sold it for \$23 000. Who had the better percentage return on the sale of their car?

Solution

Calculate Hailey's loss in dollars.

$$\begin{aligned}\text{Loss} &= \$45\,000 - \$36\,000\\ &= \$9000\end{aligned}$$

Calculate Hailey's percentage loss using the formula.

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{cost price}} \times 100\%\\ &= \frac{9000}{45\,000} \times 100\%\\ &= 20\%\end{aligned}$$

Calculate Brendan's loss in dollars.

$$\begin{aligned}\text{Loss} &= \$28\,000 - \$23\,000\\ &= \$5000\end{aligned}$$

Calculate Brendan's percentage loss using the formula.

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{cost price}} \times 100\%\\ &= \frac{5000}{28\,000} \times 100\%\\ &= 17\tfrac{6}{7}\%\end{aligned}$$

Compare the percentage losses and write the answer.

Brendan had a better percentage return as he had a smaller percentage loss.

EXERCISE 1.07 Profit and loss

Profit and loss

Concepts and techniques


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1 Example 17 A local tradesman works on a profit margin of 140%. Calculate the selling price for each of the following.
   - a A dining set that cost him \$2350 to build.
   - b A desk that cost him \$180 to build.
   - c An outdoor table that cost him \$1500 to build.
   - d A rocking horse that cost him \$120 to build.

2 Example 18 A car dealer values a trade-in at \$8000 to help clinch the sale of a new car, but can only get \$6900 for it from a wholesale dealer. What percentage loss does the car dealer make on the trade-in?

3 A newsagent makes a profit of 30% on stationery. What was the cost price of each of these?
   - a Graph pad sold for \$1.95
   - b A4 lined paper sold for \$2.60
   - c Exercise book sold for \$3.25
   - d Pencil sold for 85c

4 A second-hand dealer bought the following items: video camera ($220), jewellery box ($35), filing cabinet ($80) and printer ($340). The video camera sold for $290 and the filing cabinet sold for $130. The jewellery box and printer were in the window for several weeks, and eventually sold for $25 and $255 respectively. Complete the table below to show the percentage profit or loss on each item, correct to one decimal place.

	Item	Profit or loss	Amount of profit/ loss	% profit/ loss
a	video camera			
b	jewellery box			
c	filing cabinet			
d	printer			

5 A computer bought for $2500 was sold 2 years later for $1000. Calculate the percentage loss.

6 An electrical store sells fridges for $850. It buys them for $600. What percentage profit does the store make?

7 Sharni bought a house for $349 000 and spent $80 000 renovating it. She then sold it for $362 000.
- a Did she make a profit or loss?
- b What was the value of the profit or loss?
- c Calculate the percentage profit or loss, correct to one decimal place.

8 A used car dealer, working on a profit margin of 35%, bought a car for $10 300. What price did the dealer sell the car for?

A $3605 B $6695 C $7630 D $10 335 E $13 905

Reasoning and communication

9 Mead Murphy, an antique dealer, made a profit of $68 when he sold an antique bisque doll. His percentage profit was 17%.
- a What did he pay for the doll?
- b What was the sale price?

10 John sells potatoes at a roadside stall. He buys them in bulk for $600/ tonne. He then sells them in 10 kg bags for $9.80. What is his percentage profit?

11 Peter sold a pine bookcase which he had made. When considering only the cost of materials, he made a 55% profit. The sale price was $372. How much did the materials for the bookcase cost?

12 Maxine bought an old washstand at a deceased estate auction. She cleaned it up and sold it to an antiques dealer for a profit of 15%. The dealer sold the washstand to a customer for $2159, including a $20 delivery charge. If the dealer made 55% profit, how much did Maxine pay for the washstand at auction?

13 Hugh bought a used car for $4800 and spent several weekends detailing the car and tuning it up. He spent about $250 on materials. Eventually he sold the car and made a profit of 14%. Hugh's wife told him that he would have been better off spending the 30 hours he worked on the car at his job where he earns $25 an hour.
- a Is she correct?
- b Explain your reasoning.

1.08 INFLATION AND GST

Inflation is a measure of how the cost of living is changing. It gives an indication of the increase in the prices of all goods and services. The yearly percentage increase is called the annual inflation rate and this is usually accompanied by increases in workers' wages and salaries to 'keep up with inflation'. The annual inflation rate in Australia is generally around 3%.

Example 19

A new Holden car costs \$32 460 today. If inflation is 3.4% p.a., what will be the price of a new Holden car next year?

Solution

If the inflation is 3.4% this means that the price increases by 3.4%.	$100\% + 3.4\% = 103.4\%$
Write the calculation.	103.4% of \$32 460
Convert the percentage to a decimal.	$= 1.034 \times 32\,460$ $= 33\,563.64$
Write the answer.	The price of a new Holden next year will be \$33 563.64.

Example 20

Amity receives a pay rise from \$53 000 per year to \$54 800 per year.

a Calculate the percentage increase correct to one decimal place.

b If inflation is at 3.2%, has her salary 'kept up with inflation'?

Solution

a Calculate Amity's pay increase in dollars.	$\text{Pay increase} = \$54\,800 - \$53\,000$ $= \$1800$
Calculate the percentage increase.	$\text{Percentage increase} = \dfrac{1800}{53\,000} \times 100\%$ $= 3.396...\%$
Round to one decimal place.	$\approx 3.4\%$
Write the answer	Her percentage increase was 3.4%.
b Write the answer.	Her salary increase is above the inflation rate of 3.2% so it can be said that her salary has 'kept up with inflation'.

The **goods and services tax (GST)** is a broad sales tax of 10% on most goods and services transactions in Australia. The amount of GST is usually included in the price of a supplied item or service or it will be stated if it is not. GST is not charged for essential items such as basic foods, most health and educational services and local government rates and charges.

Example 21

A camera costs \$568 plus 10% GST. What is its selling price?

Solution

Add the value of the GST to 100% of the price.	100% + 10% = 110%
Write the calculation.	Selling price = 110% of \$568 $= 1.1 \times 568$ $= 624.8$
Write the answer.	The selling price is \$624.80.

Example 22

A television costs \$1995, inclusive of GST. What was the original price of the television, to the nearest five cents, before GST was added?

Solution

Write an equivalence statement.	110% = \$1995
Divide by 110 to find 1%.	$1\% = \$\frac{1995}{110}$
Multiply by 100 to find 100%.	$100\% = \$\frac{1995}{110} \times 100$ $= 1813.636...$
Round to the nearest 5 cents.	≈ 1813.65
Write the answer.	The price before GST was \$1813.65.

EXERCISE 1.08 Inflation and GST

Concepts and techniques

1 Example 19 A store increases their prices so that they stay in line with the rate of inflation. If the rate of inflation is 3.8% p.a., calculate the new price of each of the following. Answer correct to the nearest five cents.

a television priced at \$850

b bedroom suite priced at \$3400

c lamp priced at \$49.95

d dining set priced at \$1298

2 **Example 20** Calculate the percentage increase in the following prices. Answer correct to one decimal place.

a $48 to $56
b $179.50 to $196
c $45 000 to $48 500
d $6.50 to $6.76

3 Calculate the amount of GST that needs to be added to the pre-GST price of each of the following items.

a new car: $15 000
b camera: $320
c novel: $45
d chainsaw: $350

4 **Example 21** Calculate the selling price of each of the following items, given these are pre-GST prices. Answer to the nearest cent where required.

a dinner set: $220
b suitcase: $180
c boat: $880
d Lego set: $29.15

5 A hamburger in Sydney costs $5.50 plus 10% GST. The selling price of a hamburger is:

A $5.00
B $5.60
C $6.05
D $6.50
E $6.60

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6 **Example 22** The Maxwell family is reviewing some of the bills paid for various services in the last 6 months. How much GST did they pay in each case? Answer correct to the nearest cent where required.

a new stormwater drain: $1364
b tax return preparation: $598
c car service: $352
d telephone account: $462.25

7 A car costs $42 000 including 10% GST. Which of the following is the car's price before GST, correct to the nearest dollar?

A $37 800
B $38 181
C $38 182
D $41 990
E $46 200

Reasoning and communication

8 Sally's wage is rising 3.1% to keep pace with inflation. If her wage is $756 per week, what will her new wage be?

9 Pam pays $3300 for a new computer at a major retail store. How much GST is included in the price?

10 A train ticket from Sydney to Nowra costs $18.50 including 10% GST. What is the price of the ticket before tax?

11 Peter's salary rose to $78 000 to keep pace with inflation. If inflation was 4.3%, what was Peter's salary before the rise? Answer correct to the nearest dollar.

12 If an Australian Akubra hat has a 10% GST of $2.83 included in its selling price, find its selling price.

13 The price of a football membership rises from $178 to $185 in one year. If inflation is 4.5% p.a. for that year, have the membership prices kept up with the rate of inflation?

14 A retailer calculates the pre-GST price of a watch to be $420. Find the GST that must be added and the price paid for the watch by the customer.

15 If $37.80 was added to a gas bill for GST, what was the total price of the gas bill, including the GST?

16 Inflation in a certain country is 18% p.a. If a loaf of bread cost $3.45 now:

a what will the price be next year?
b what was the price last year?

17 Josh is a plumber. He charges a call out fee of $70 plus $60 for any amount of time up to half an hour he is at a job. He has to charge GST on top of his bills. How much will Josh charge for a job:

a that did not get done because the clients were not home?
b that takes 2 hours?
c that takes 75 minutes?
d that takes $3\frac{1}{2}$ hours?

18 Suki has an annual salary of $45 000. Inflation is rising at an average of 2.7% p.a. Her salary is rising at the same rate as inflation. She performs the following calculation to work out her salary in 2 years:

$$\begin{aligned} 2.7\% \text{ of } \$45\,000 &= 0.027 \times 45\,000 \\ &= \$1215 \\ \text{New salary} &= \$45\,000 + \$1215 + \$1215 \\ &= \$47\,430 \end{aligned}$$

a Explain why this is incorrect.
b Calculate her salary in 2 years.

19 Electricity is rising at the same rate as inflation. If inflation is rising by an average of 5.6% p.a., calculate the charges on an electricity bill in 2 years if the charges are currently $645.

20 Shaun buys T-shirts from a wholesaler for $7.50. To calculate his selling price he works on a profit margin of 150% and then adds on GST. Calculate:

a the selling price (not including GST)
b the selling price (including GST).

21 A store increases the price of milk to stay in line with inflation. The inflation rates are 2.5% p.a, 3.2% p.a. and 3.3% p.a. for three years respectively. If milk is currently $1.70 per litre, calculate the price of milk in 3 years.

22 A furniture shop buys lounge suites from a distributor for $1420. The shop works on a mark-up of 75%. Calculate the retail price of the lounge suites including GST.

INVESTIGATION GST

a If an advertised price includes GST, why can we find the GST amount by simply dividing the advertised price by 11?

b The Great Gals electrical store adds 10% GST to the prices of its goods. If a washing machine had an original price of $1200, what is its advertised price with GST included?

c At the Great Gals end-of-year sale, customers get a 10% discount on all goods.
  i Will the discounted price be more than, less than or the same as the original price before GST was added?
  ii What is the discounted price of the washing machine in part **b** at the end-of-year sale?
  iii How does the discounted price of the washing machine compare with the original price?
  iv Would it make a difference to the discounted price of the washing machine if the 10% discount was applied first before the 10% GST was added?

d What can you say generally about an item that has 10% added, then 10% subtracted?

e Calculate the overall percentage change when 10% is added to an amount, then 10% is subtracted.

1.09 REVIEW OF RATES

A **rate** is a measurement that compares two different quantities. As rates compare different quantities it is important to include the units. For example, a heart rate compares the number of heartbeats to a period of time, and is measured in beats per minute, written as beats/min.

Example 23

a A hose delivers 3840 litres of water in one hour. What is the flow rate in litres/minute?

b The cost of 35 L of petrol is $44.45. Express this cost as a rate in cents/litre.

Solution

a The flow rate is in litres/minute so write litres divided by time.

$$\text{Flow rate} = \frac{3840\text{ L}}{1\text{ hour}}$$

Convert 1 hour to 60 minutes.

$$= \frac{3840\text{ L}}{60\text{ min}}$$

Divide the number of litres by the number of minutes to calculate the rate.

$$= 64\text{ L/min}$$

Write the answer.

The flow rate is 64 litres/minute.

b Write the rate as cost divided by litres.

$$\text{Petrol cost} = \frac{\$44.45}{35\text{ L}}$$

Convert $44.45 to 4445 cents.

$$= \frac{4445\text{c}}{35\text{ L}}$$

Divide the number of cents by the number of litres to calculate the rate.

$$= 127\text{c/L}$$

Write the answer.

The petrol cost is 127 cents/litre.

Example 24

Craig's heart beats 72 times per minute.

- a How long will Craig's heart take to beat 2052 times?
- b How many times will Craig's heart beat in one hour?

Solution

a Craig's heart beats 72 times in 1 minute. Divide the total number of beats by 72.

$$\text{Time taken for 2052 beats} = 2052 \div 72 = 28.5 \text{ min}$$

Write the answer.

It will take Craig's heart 28.5 minutes.

b There are 60 minutes in 1 hour. Multiply the number of beats by 60.

$$\text{Beats in one hour} = 60 \times 72 = 4320$$

Write the answer.

Craig's heart will beat 4320 times in one hour.

IMPORTANT

Rate problems are solved by either multiplying or dividing. One simple rule is to write the units of the rate as a fraction.

EXERCISE 1.09 Review of rates

Concepts and techniques

1 What units are used to express each of the following rates?
- a wage rate
- b typing speed
- c the cost of potatoes
- d the speed of an athlete

2 Name a rate that uses each of the following units.
- a litres/100 km
- b megabytes/second
- c births/1000 persons
- d $\$/m^2$

3 Example 23 A tree grew 31.2 cm in height in a quarter of a year. Calculate its growth rate in centimetres/week.

4 Troy scored 42 runs in 8 overs of a cricket match. Calculate his run rate in runs/over.

5 A town's population grew by 16 500 over 6 years. What was its growth rate in persons/year?

6 Steak costs $4.55 for 160 g. Calculate the cost as a rate in $/kg.

7 A computer downloaded a 6.5 megabyte file in 20 seconds. Calculate the download rate in MB/s.

8 Australia's population is 23 194 000 and its area is 7 682 300 km^2. Calculate its population density in persons/km^2, correct to two decimal places.

Reasoning and communication

9 Example 24 A bus travels at a speed of 96 km/h.

- a How far will the bus travel in $3\frac{1}{2}$ hours?
- b How long will the bus take to travel 480 km?

10 Jake types 64 words per minute. How long will it take him to type a 1500-word essay? Give your answer to the nearest minute.

11 A breakfast cereal contains 3 mg of fat in every 60 g serve.

- a Express the fat content as a rate in mg/g.
- b How much fat would be contained in a 100 g serving?

12 Hayden's swimming pool is being filled at a rate of 50 L/min. How long will it take to fill if it has a capacity of 38 kL? Give your answer in hours and minutes.

13 A cricket team had a run rate of 4.9 runs/over. How many runs did the team score in 30 overs?

14 The recommended daily dose for a child's medicine is 80 mg for every kilogram of body weight. Which of the following is Elizabeth's daily dose in grams, if she weighs 30 kg?

A 2.4 B 3.75 C 2400 D 375 E 240

15 A tap is dripping water at a rate of 40 drops per minute. Each drop contains 1.3 mL of water. How many litres will drip from the tap in a day?

16 Australia's death rate last year was 5.2 per 1000. If Australia's population was 21 435 680 at the start of the year, how many people died during the year? Give your answer to the nearest whole number.

17 A lawn fertiliser is spread at the rate of 5 kg per 100 m^2 of lawn.

- a How much fertiliser is needed for a rectangular lawn measuring 11 m by 7.5 m? Give your answer to the nearest 0.1 kg.
- b How many times can the whole lawn described in part **a** be treated if a 25 kg bag of fertiliser is purchased?

1.10 CONVERTING RATES

It is important to be able to convert between rates. For example, if it takes an athlete 10.22 seconds to run a 100 m race, we can find the athlete's speed in metres/second. We may want to know the speed in kilometres/hour to get a feel for how fast this is compared with other speeds.

Example 25

Convert:

a 3 L/day to mL/h.

b 9.78 m/s to km/h.

Solution

a Convert both units using known conversion factors.
24 hours in a day and 1000 mL = 1L.

$$3\text{ L/day} = \frac{3\text{ L}}{1\text{ day}}$$

$$= \frac{3000\text{ mL}}{24\text{ h}}$$

Complete the division and simplify.

$$= 125\text{ mL/h}$$

b To change from per second to per hour, we multiply by the number of seconds in 1 hour.

$$9.78\text{ m/s} \times 3600 = 35\,208\text{ m/h}$$

Complete the calculation by converting to km/h. 1000 m = 1 km.

$$= 35.208\text{ km/h}$$

Example 26

A peregrine falcon swoops down towards its prey at 180 km/h.

a What is this speed in m/s?

b If the falcon is flying 180 m above the ground, how many seconds will it take to reach the prey if it flies straight down?

Solution

a Complete the required conversions.
1 km = 1000 m and 1 h = 3600 s.

$$180\text{ km/h} = \frac{180\text{ km}}{1\text{ h}}$$

$$= \frac{180\,000\text{ m}}{3600\text{ s}}$$

Complete the division and simplify.
Write the answer.

$$= 50\text{ m/s}$$

The speed is 50 m/s.

b Calculate the time taken to cover 180 m.

50 m in 1 second.
$180\text{ m} \div 50\text{ m/s} = 3.6\text{ s}$

Write the answer.

It will take 3.6 seconds for the falcon to reach its prey.

EXERCISE 1.10 Converting rates

Concepts and techniques

Rate skills

1 Example 25 Convert:
a $46 212/year to $/month
b 75 km/h to km/min
c 5 L/h to L/day
d 3 m/s to m/min
e 0.8 cm/min to cm/h
f 78 words/min to words/s
g 8.2 L/100 km to L/km
h 1750 mL/s to mL/min.

2 Convert:
a 24 c/min to $/h
b 8 m/s to km/h
c 15 mL/s to L/h
d $10.80/h to c/min
e 60 km/h to m/s
f 3 t/day to kg/min
g 40 m/s to km/min
h 100 mg/s to g/h.

3 The world's fastest cricket bowler is Pakistan's Shoaib Akhtar. In 2003, he bowled a ball 20.12 m in 0.45 seconds. Calculate the speed of the ball he bowled, to the nearest km/h.

4 The speed of sound, called Mach 1, is 1224 km/h. Which of the following is this speed in m/s?
A 34 B 44 C 340 D 441 E 0.34

5 It takes 20 seconds to fill a 10 L bucket with water. What is the rate for filling the bucket, in litres per hour?

6 In 2008, Jamaican athlete Usain Bolt became the world's fastest man when he sprinted 100 metres in 9.72 seconds. What was his speed in km/h, correct to one decimal place?

7 The lift in Sydney Tower travels 298 m in 42 seconds. What is its average speed in km/h, correct to two decimal places?

Reasoning and communication

8 Example 26 A plane is travelling at an average speed of 950 km/h.
a What is this speed in m/s, correct to one decimal place?
b The plane flies over a property 1500 m long. How many seconds does it take to fly over the property (to the nearest second)?

9 The volume of air breathed in through the lungs is measured in litres. The average breathing rate when resting is 8 L/min.
a What is the breathing rate in mL/s?
b How much air is breathed in during an 8-hour sleep?

10 A swimmer won the 100 m butterfly in 1 min 7.75 s. Correct to two decimal places:
a what was her average speed in m/s?
b what was her average speed in km/h?

11 Rental on a telephone is $42 for 12 weeks.
a What is this rate in dollars per week?
b What is the rate in dollars per year?

12 A box contains 30 cups of soap powder. The manufaturer recommends $1\frac{1}{2}$ cups of soap powder per wash.
   - a What is the number of washes that can be done per box?
   - b How many boxes will give 100 washes?

13 A car travels 452 km on 59.8 litres of petrol.
   - a What is the fuel consumption, correct to three decimal places:
     - i in km/L
     - ii in L/100 km?
   - b How far will the car go on 100 L of petrol, correct to the nearest kilometre?
   - c How much petrol will it use for a 1500 km journey, correct to the nearest litre?

1.11 CURRENCY EXCHANGE RATES

If you travel overseas you must change your Australian money to the money of other countries. The money of a country is called its **currency**. The price of a currency is called its **currency exchange rate**. Before 1983, the government fixed the exchange rate of the Australian dollar, but now it 'floats' according to supply and demand.

Banks and other financial institutions do not change your money free of charge. They charge a commission, which is usually included in the exchange rate. They give different rates, depending on whether you are buying or selling Australian dollars. Different banks charge different commissions, so the buying and selling rates will be different even though the exchange rate is the same. The actual exchange rate is sometimes called the midrate to distinguish it from the buying and selling rates. It is close to the average of the buying and selling rates.

The exchange rate of the Australian dollar with important currencies such as the Japanese Yen, American dollar and the European Euro are published almost every day by newspapers.

The International Organisation for Standardisation (ISO) has assigned each currency a three-letter code. The table below shows the currency unit and the ISO code that represents the currency for various countries and regions. Also shown are examples of the buying and selling rates in Australian dollars (AUD) on a certain day.

Country/Region	Currency unit	Code	Buying	Selling
Canada	Dollar ($)	CAD	1.1153	0.9831
China	Renminbi	CNY	6.8290	6.1069
European Union	Euro (€)	EUR	0.8496	0.7559
Fiji	Dollar ($)	FJD	2.0357	1.7333
Hong Kong	Dollar ($)	HKD	8.9430	7.7528
India	Rupee (Rp)	INR	67.7210	49.0576
Japan	Yen (¥)	JPY	100.8400	86.6427
New Zealand	Dollar ($)	NZD	1.3510	1.2063
Thailand	Baht (฿)	THB	35.3250	29.1450
Great Britain (UK)	Pound (£)	GBP	0.6788	0.6143
USA	Dollar ($)	USD	1.0920	1.0077

Example 27

a From the table calculate the exchange rate (midrate) for the Australian dollar (AUD) and American dollar (USD). Answer correct to four decimal places.

b How much, to the nearest dollar, is:

i 5000 AUD worth in USD? ii 5000 USD worth in AUD?

Solution

a Calculate the average exchange rate.

Exchange rate = Average of the buying and selling rate.

$$\begin{aligned}\text{Exchange rate} &= \frac{1.0920 + 1.0077}{2}\\ &= \frac{2.0997}{2}\\ &= 1.04985\\ &\approx 1.0499\end{aligned}$$

Write the answer.

The exchange rate is 1 AUD = 1.0499 USD.

b i The exchange rate is stated in AUD, so we can just multiply by the exchange rate.

Write the AUD to USD exchange rate.

1 AUD = 1.0499 USD

Multiply by 5000 and complete the calculation.

$$\begin{aligned}5000\text{ AUD} &= 1.0499\text{ USD} \times 5000\\ &= 5249.5\text{ USD}\\ &\approx 5250\text{ USD}\end{aligned}$$

Write the answer.

5000 AUD is worth 5250 USD.

ii To change from USD to AUD, we can reverse the method (divide) or work out the reverse exchange.

Write the exchange rate.

1 AUD = 1.0499 USD

Multiply by 5000.

5000 AUD = 1.0499 USD × 5000

Divide by 1.0499.

$$\frac{5000\text{AUD}}{1.0499} = \frac{1.0499\text{USD} \times 5000}{1.0499}$$

This can be done simply using:

$$\begin{aligned}5000\text{ USD} &= 5000 \div 1.0499\\ &= 4762.3\\ &\approx 4762\text{ AUD}\end{aligned}$$

Complete the calculation and round to the nearest dollar.

4762 AUD ≈ 5000 USD

Write the answer.

5000 USD is worth 4762 AUD.

Example 28

Basanti's father is in India. He bought her wedding dress in India for 64 000 rupees. Using the table of exchange rates, calculate the equivalent price in Australian currency.

Solution

She is changing into Australian currency so she is **buying** currency.

Each Australian dollar is worth 67.7210 rupees.

A\$1 = 67.7210 Rupee

Divide to convert to Australian dollars.

$$\begin{aligned}\text{A\$} &= 64\,000 \div 67.7210\\ &= \text{A\$}945.053\ldots\end{aligned}$$

Round to the nearest cent.

≈ A\$945.05

Write the answer.

The equivalent price in Australian currency is \$945.05.

IMPORTANT

To convert:

- Australian dollars into foreign currency MULTIPLY by the rate
- Foreign currency into Australian dollars DIVIDE by the rate.

EXERCISE 1.11 Currency exchange rates

Concepts and techniques

WS Currency rate exchange

For this exercise use the exchange rate table on page 40.

1 Example 27 Assuming that the midrate is the average of the buying and selling rates, calculate the midrate (exchange rate) for each currency in the table.

2 Using the relevant midrate calculated in question **1**, determine how much:

a 3000 AUD is worth in NZD
b 1670 AUD is worth in GBP
c 580 AUD is worth in JPY
d 4900 AUD is worth in HKD
e 4000 EUR is worth in AUD
f 2300 USD is worth in AUD
g 610 NZD is worth in AUD
h 50 000 JPY is worth in AUD.

All of these currencies can be rounded to two decimal places except for Japanese Yen. They are always written to the nearest whole number.

3 Example 28 Convert the following into Australian dollars.

a ฿720
b Rp35 000
c €120 000
d NZ$400

This is buying Australian dollars.

4 Susan is booking her holiday accommodation for one week in Thailand. It is 1645 baht a night. Calculate the cost for the week in A$.

A A$46.57 B A$56.44 C A$325.97 D A$394.09 E A$58 109.63

Remember that Australian dollars are rounded correct to the nearest cent.

5 Calculate the value if you convert $750 Australian to:

a euros
b yen
c rupee
d NZ dollars.

This is selling Australian dollars.

6 Sarah is travelling from Australia to the UK. She has A$3000 spending money that she exchanges into pounds. How much did she get?

A £1842.90 B £2036.40 C £3000 D £4419.56 E £4883.61

Reasoning and communication

7 If Lauren was changing 400 AUD to Fiji dollars, how much would she receive?

8 When Yoshi came back from Japan he had 400 000 JPY left. How much would he get for it in AUD?

9 When Hannah was travelling overseas, she wanted to get some travellers' cheques in US dollars, some in euros and some in Thai baht. If she split 6000 AUD evenly between the currencies, how much of each did she get?

10 Alin was on exchange in Australia, but was paid in USD through electronic funds transfer. His salary was 1200 USD a fortnight after tax. How much in AUD did he get when he drew this out at an automatic teller machine in Australia?

11 Cara thought that a sewing machine advertised in a British magazine was especially cheap at £800. What would be the equivalent price in Australian currency?

12 Juan is selling his band's CD on the internet for A$19.95. What is the equivalent price in Hong Kong dollars?

13 Sienna has booked a tour for 2050 euros on her holiday in Europe. What is the equivalent price in Australian dollars?

14 a Luis is travelling to New Zealand and converts $1500 Australian to New Zealand dollars. How much did he receive?
   b Luis had to cancel his plans at the last minute. He went to convert his New Zealand dollars back to Australian dollars. How much would he receive?
   c Luis was upset he did not get back $1500 Australian dollars. Explain why he didn't.

15 Patrick imports electrical goods from China. The total of one shipment of 500 televisions is 965 000 renminbi.
   a If the exchange rate was A$1 = 6.4595 renminbi, how much did Patrick pay for the shipment in Australian dollars? Answer correct to the nearest dollar.
   b It cost Patrick A$25 000 in import fees. How much did each television cost to import?

16 While travelling in the USA, Mikayla exchanged A$750 for US$804.50.
   a How much was A$1 worth in US dollars? Answer correct to 4 decimal places.
   b Convert US$1500 to Australian dollars, correct to the nearest dollar.

17 Stefani exchanged A$1600 for £1032.48 when travelling in the UK.
   a How much was A$1 worth in UK pounds?
   b Convert £690 to Australian dollars, correct to the nearest dollar.

18 The exchange rate for one Australian dollar was 78.5 Yen. Shaun buys an item over the Internet from Japan. It was priced at 850 Yen. How much was this in Australian dollars?

1.12 BEST BUYS

To ensure that you get the best buy it is important to ensure that the items that you are comparing are equivalent. You need to ask yourself questions such as, are the products of the same quality, do I have to buy more than I need to get a good price, am I making the best purchase from a budget perspective, am I saving money?

When looking at the price of equivalent items it is important to ensure that the prices that are being compared are for the same amount.

To do this, it is important to calculate the **unit cost**. The unit cost is what the item costs for one unit; it could be for 1 item or for 100 mL or for 1 kg, etc. The unitary method is used to calculate the unit cost. The unit cost is an example of a rate as it is the cost/unit.

Many stores now have the unit cost displayed, which makes comparison much easier.

IMPORTANT

To compare prices find the unit cost of each item, then compare this to find the 'best buy'.

Example 29

Paul is buying washing powder. His brand comes in three different sizes, 350 g for \$1.86, 800 g for \$3.28 and 2 kg for \$8.70. Which is the best buy?

Solution

Write equivalence statements for all three sizes.

350 g = \$1.86   800 g = \$3.28   2 kg = \$8.70

To compare the price of 100 g, divide by 3.5, 8 and 20 respectively.

350 g = 3.5 × 100 g
800 g = 8 × 100 g
2 kg = 2000 g
= 20 × 100 g

$$100\text{ g} = \frac{\$1.86}{3.5} = \$0.531\ldots \qquad 100\text{ g} = \frac{\$3.28}{8} = \$0.41 \qquad 100\text{ g} = \frac{\$8.70}{20} = \$0.435$$

The best buy is the one with the lowest price per 100 g. Write the answer.

The 800 g for \$3.28 is the best buy as it is the cheapest at 41c per 100 g.

Example 30

Peta needs to buy soft drink. The brand that she likes has a box of 15 × 375 mL cans of soft drink on sale for \$12 or the 2 L bottles are 15% off their normal price of \$3.79.

a Which is the best buy?
b Justify Peta's reasoning.

Solution

a Calculate the total volume for the cans.

Cans: 15 × 375 = 5625 mL
= 5.625 L

Calculate the cost for the bottles.
15% off means decrease by 15%.
As you are working with money, it is appropriate to round to nearest cent.

Cost per bottle = 85% of \$3.79
= 0.85 × 3.79
= \$3.2215
≈ \$3.22

Calculate the cost per litre for the cans and then the bottles.

Cans: 5.625 L costs \$12

$$1\text{ L costs } \frac{\$12}{5.625} = \$2.133\ldots\ (\$2.13)$$

Bottles: 2 L costs \$3.22

$$1\text{ L costs } \frac{\$3.22}{2} = \$1.61$$

Compare and write the answer.

The 2 L bottles are the best buy as they are cheaper at \$1.61/L.

b Write the answer.

If not a lot of people are drinking the soft drink the 2 L bottle may go flat before it's finished. If taking the drinks somewhere it is often more convenient to take cans in an esky.

There is often a discount given for buying in bulk. It needs to be considered whether the deal is worthwhile for you before you can say whether it is a good buy. For example buying a box of 24 mangos for \$10 is great if you will use them but a waste of money if they are just going to go rotten before they can be used. You may have been better off buying 8 mangos at '4 for \$5', choosing the ones you wanted, and getting better quality mangoes for the same amount of money.

IMPORTANT

When deciding on the 'best buy' make sure factors other than price are considered as well.

Example 31

Fiona needs a new mattress for her bed. She has found the mattress she wants at two different stores for approximately the same retail price. The stores are both having sales. Store A is having a '2 for 1' sale whilst Store B is having a '$\frac{1}{3}$ off' sale. Which should Fiona choose? Explain your answer.

Solution

Consider the cost per mattress at each store and any conditions.	Store A: paying half price but needing to buy two mattresses. Store B: paying $\frac{2}{3}$ of the price.
Write the answer.	Fiona can get a lower price per mattress at Store A but she would need to buy two mattresses. If she knows someone who also wants one then they could buy them together from Store A and this would be the best deal. However if Fiona only wants one mattress then Store B is the best option as she doesn't need to do anything else to get the discount.

EXERCISE 1.12 Best buys

Concepts and techniques

1 Example 29 By determining the unit cost, decide which is the best buy.

- a 150 mL for \$2.36 or 1 L for \$14.80
- b 36 cans for \$20 or 15 cans for \$8
- c \$8.40 for 500 g or \$20.99 for 1.2 kg
- d box of 8 for \$2.88 or a box of 45 for \$15.50

2 Which is the best buy?

- A 2 kg for \$17.80
- B 250 g for \$2.40
- C 1.2 kg for \$10.85
- D 700 g for \$6.30
- E 400 g for \$3.65

3 Example 30 Which is the best buy?

- a 2 kg for \$14.80 or 300 g for \$3.50, which is on a 10% off sale
- b 450 mL for \$7.80, which is reduced by 25% or 1.5 L for \$18.20
- c packet of 4 for \$10.50 with a 15% off discount or packet of 10 for \$23.50
- d 10 L for \$62.70 or 1 L for \$8.80, which is on a '$\frac{1}{3}$ off' sale

4 A book is being sold at five different stores at the recommended retail price of \$47.90. Leading up to Mother's Day all five stores are having a sale. Which discount will give the lowest price?

A 20% off the retail price
B Pay 75% of the retail price
C $\frac{1}{5}$ off the retail price
D Pay $\frac{2}{3}$ of the retail price
E \$13 off the retail price

Reasoning and communication

5 Which cookies are the best buy: chocolate chip at \$4.50 for 300 g, nutty clusters at \$3.70 for 250 g or jam drops at \$10.40 for 700 g?


Shutterstock.com/homydesign


Shutterstock.com/Vydrin

6 **Example 31** Gustavo is buying an electric drill. He has found the drill he wants at two different stores for \$39.95. The stores are both having sales. The first store is having a 'buy 2 get 1 free' sale whilst the second store is having a '10% off' sale. Which should Gustavo choose? Explain your answer.

7 Lucas goes shopping for ice cream. The same brand is available in two different tubs. The 2 L tub is \$7.80. The 5 L tub is \$17.50.

a Which is the best value?
b The next time he goes to buy ice cream he finds it on sale. The 2 L tub is on a 35% off sale and the 5 L one is reduced by \$4.50. Which is now the best buy? Show your working and justify your reasoning.
c What other factors could influence his decision?

8 Susanna works at 'Whitegoods superstore' and can get a 5% staff discount on the final price of any goods. She needs to buy a washing machine and has decided on the one that she wants. 'Whitegoods superstore' has it for \$785 and is currently having a 15% off sale. At 'Bargain whitegoods' they have it for \$699 and are currently having a 10% off sale.

a How much will it cost her at 'Whitegoods superstore'?
b Which is the best buy? Justify your reasoning.
c What other factors could influence her decision?

1

CHAPTER SUMMARY

PERCENTAGES AND RATES

- **Percent** means 'out of 100'.
- When converting a value to a percentage you multiply by 100%.
- The **unitary method** is useful in finding any percentage that is required. Divide to find 1% and multiply to find percentage required.
- When increasing by a percentage, add the percentage onto 100%.
- When decreasing by a percentage, subtract the percentage from 100%.
- For repeated percentage change, multiply the required percentage changes to find the overall percentage change.
- **Mark-up** is usually added to the **cost price** to cover costs and make a profit. The price on an item is called the **marked price** or **retail price**.

  Mark-up = marked price – cost price
- **Discount** (or mark-down) is the amount that is taken off the price of goods to make them sell faster.

  Discount = original marked price – final marked price
- % mark-up usually uses the cost price as the base.
- Unless otherwise stated, **percentage profit** and **percentage loss** are calculated using the cost price as the denominator.
- $\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$

  $\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$
- **Inflation** shows the increase in the prices of all goods and services. The annual inflation rate is given as a yearly percentage increase.
- The **goods and services tax (GST)** is a broad sales tax of 10% on most goods and services transactions in Australia. The amount of GST is usually included in the price of a supplied item or service or it will be stated if it is not.
- A **rate** is a measurement that compares two different quantities. It is important to be able to convert rates.
- **Currency** is the system of money used in a country. Different countries have different currencies.
- **The currency exchange rate** is simply the cost of one currency in terms of another.
- To compare prices find the **unit cost** of each item, then compare this to find the 'best buy'. The unit cost is what the item costs for one unit, i.e. could be for 1 item or for 100 mL or for 1 kg, etc.

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CHAPTER REVIEW
PERCENTAGES AND RATES

1

Multiple choice

1 **Example 2** $7\frac{1}{2}$% written as a decimal is:
A 0.0712 B 0.075 C 0.705 D 0.712 E 0.75

2 **Example 9** In an election one of the candidates received 6.65 million votes, which was 35% of the total votes. Another candidate received 24% of the total votes. How many votes did he receive? Answer correct to the nearest one hundred thousand.
A 1.60 million B 2.33 million C 4.56 million D 9.98 million E 19.00 million

3 **Example 10** A dosage of 172 mg was decreased by $8\frac{1}{2}$%. Calculate the new dosage.
A 14.62 mg B 157.38 mg C 163.50 mg D 180.50 mg E 195.30 mg

4 **Example 15** A store marked down its bakery goods by 70% half an hour before closing. Calculate the price of a loaf of bread that is normally priced at $3.50.
A $1.05 B $2.20 C $2.28 D $2.45 E $4.20

5 **Example 17** Shaun has a second-hand boat business. He bought a boat for $15 000. He spent $3500 on repairs and later sold it. If he has a profit margin of 80%, what was the selling price?
A $18 580 B $20 700 C $27 000 D $30 500 E $33 300

6 **Example 26** A machine digs the ground at a rate of 5 t per hour. How long will it take to dig 200 kg?
A 1.5 minutes B 2.4 minutes C 25 minutes D 40 minutes E 144 minutes

7 **Example 28** Sheldon buys an NBA jersey from America for US$259.99. If the exchange rate was 1.0425 for each Australian dollar, how much did he pay in Australian dollars?
A A$249.39 B A$258.95 C A$259.99 D A$261.03 E A$271.04

8 **Example 29** A certain brand of olive oil comes in different sizes. Which is the best buy?
A 200 mL for $5.60 B 450 mL for $7.85 C 800 mL for $24
D 2.4 L for $49 E 5 L for $89

Short answer

9 **Example 1** Convert the following to percentages.
a $\frac{2}{5}$ b $1\frac{2}{9}$ c 0.3 d 4.05

10 **Example 3** At a family reunion of 300 people, 105 were under 18 years of age. What was the percentage of people who were under 18 years of age?

11 **Example 5** Find 36% of $84.

12 **Example 6** 72.5% of students at a school study a second language. If the school has 912 students, how many study a second language?

13 **Example 7** If 80% of an unknown amount is \$208, find the whole amount.

14 **Example 8** Sam has driven for 858 km. Calculate the total length of his journey if this distance represents 65% of his journey.

15 **Example 11** Peter's renovations will increase the floor area of his house by 45%. If the current floor area is 150 m^2, what will the new floor area be?

16 **Example 13** The value of Debbie's superannuation increased by 8.5% one year and decreased by 6% the next year. What was the overall percentage change in her superannuation?

17 **Example 14** A store sells a treadmill for \$1195. If the mark-up was 65%, what did the store pay for the treadmill? Answer correct to the nearest dollar.

18 **Example 16** The price of all Christmas decorations were reduced in the two days before Christmas. A Christmas tree that was priced at \$76.95 sold for \$50. What was the percentage discount? Answer correct to the nearest percent.

19 **Example 18** Sally made and decorated a cake at a cost of \$12. What was the percentage profit if she sold it for \$54?

20 **Example 19** Last year Louise had a salary of \$106 000. This year her salary has increased to keep in line with inflation. If inflation was 3.9%, what is her new salary?

21 **Example 20** A packet of biscuits rose from \$2.80 to \$2.90 in keeping with inflation. What was the annual inflation rate? Answer correct to one decimal place.

22 **Example 21** An electrician does work worth \$768. Calculate:

a the GST he needs to add to the bill

b the total of the bill.

23 **Example 22** Sun paid \$4700 for a new bedroom suite. How much GST did she pay?

24 **Example 22** Shaun sold a piano for \$5995 including GST. How much was the piano without GST?

25 **Example 23** A bird population is growing at a constant rate. If it increases by 5000 birds in 8 years, what is the growth rate per year?

26 **Example 24** A car's rental rate is \$78/day.

a What is the cost of hiring it for a two-week holiday?

b Dayna has a budget of \$600 for car rental on her holiday. How many days can she hire the car?

27 **Example 25** Convert 16 mL/s to L/h.

28 **Example 30** Xavier is buying coffee. The brand he likes is sold in a 300 g jar for \$16.45 and in a box of 280 sachets, which are each 1.7 g, for \$42.70. He finds that the sachets are on sale at 35% off.

a Which is the better buy? Justify your reasoning.

b What other factors could influence Xavier's decision?

Application

29 Karana bought two investment properties for \$345 000 and \$583 000. She later sold them for \$415 000 and \$689 000 respectively. Which property gave her the better percentage return? Explain.

30 The manufacturing cost of a motor is \$260. The manufacturer applies a mark-up of 30%. The wholesaler has to pay \$20 transport to get the motor from the manufacturer and then adds a mark-up of 20%. It costs the retailer another \$15 to take the motor to his shop. The retailer adds a mark-up of 35%. Finally, GST of 10% is added.

a What will the customer pay for the motor?

b What is the total percentage increase from the manufactured cost? Answer correct to one decimal place.

31 Peter imports Balinese furniture from Indonesia. He buys a shipment of 75 outdoor table settings for A\$18 000. The exchange rate with the Australian dollar was 10 031 rupiah.

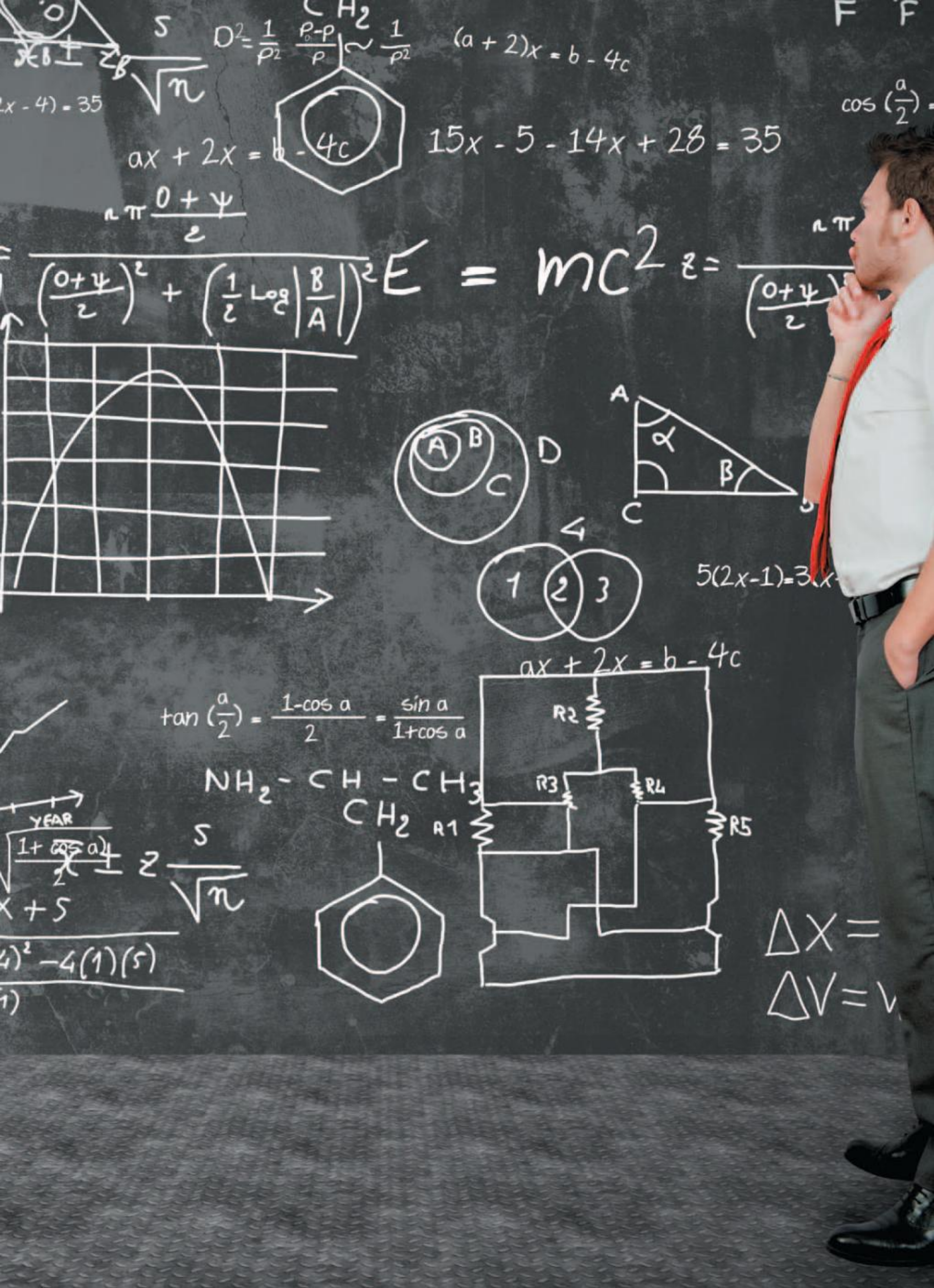
a How much did it cost him in rupiah?

b The shipping costs were 35 108 500 rupiah. How much is this in Australian dollars?

c Calculate the total cost of importing the table settings, in Australian dollars.

d Peter has a 140% mark-up for his profit margin. How much will he need to sell each table setting for?

Practice quiz


CH2
D2= 1/ρ2 ρ0-ρ/ρ ~ 1/ρ2
(a + 2)x = b - 4c
F F
x - 4) = 35
cos (a/2) =
15x - 5 - 14x + 28 = 35
ax + 2x = b - 4c
E = mc2
A B C D
α β
1 2 3
4
5(2x-1)=3
ax + 2x = b - 4c
tan (a/2) = 1-cos a/2 = sin a/1+cos a
NH2 - CH - CH3
CH2
R1 R2 R3 R4 R5
YEAR
x + 5
-4(1)(5)
ΔX=
ΔV=V

Shutterstock.com/alphaspirit

2

ALGEBRA AND MATRICES

ALGEBRA AND EQUATIONS

TERMINOLOGY

algebra
algebraic expression
balanced
coefficient
constant
equation
expanding
formula
highest common factor (HCF)
like terms
linear equation
pronumeral
simplify
solve
subject of a formula
substitution
term
variable

Prior learning

LINEAR AND NON-LINEAR EXPRESSIONS

- **substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate (ACMGM010)**
- **find the value of the subject of the formula, given the values of the other pronumerals in the formula (ACMGM011)**
- **use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights. (ACMGM012)**

LINEAR EQUATIONS

- **identify and solve linear equations (ACMGM038)**
- **develop a linear formula from a word description (ACMGM039)**

2.01 THE FOUR OPERATIONS IN ALGEBRA

Definitions used in algebra

Algebra involves the use of symbols or pronumerals to simplify expressions and solve problems. A **pronumeral** is a symbol or letter that takes the place of a number or a variable. A **variable** is a quantity that can represent different values. A **term** can be a number and one or more pronumerals multiplied together, or it can be just a number. A **coefficient** is the number that appears before a pronumeral within the same term. A **constant** is a term that contains no pronumerals. An **algebraic expression** is a collection of two or more terms. These terms can be added, subtracted, multiplied or divided. An algebraic expression does not include an equals sign.

Examples of algebraic expressions are: $2m - 3n$, $4a + 7b$, $6uv - 2w + 4v$ and $-9ahw \div 3klm$.

Example 1

In the algebraic expression $p - 5qr + 7$ identify the following:

a the number of terms.

b the coefficient of the term containing the variable p.

c the constant term.

Solution

a A term can be a number or a number and one or more pronumerals multiplied together. There are three terms in this expression.

b A coefficient is the number that appears before a pronumeral within the same term. There is no number written in front of the pronumeral p. The coefficient is 1.

c A constant is a term that contains a number only, there is no pronumeral. The constant term is 7.

Adding and subtracting algebraic expressions

Collecting like terms

Like terms are composed of the same pronumerals. The pronumerals may be in a different order. Examples of like terms include the pairs: $4a$ and $2a$; $3xy$ and $-6yx$ and $-2abc^2$ and $5c^2ab$.

Only like terms can be added or subtracted.

To **simplify** an expression means to collect the like terms and then write as simply as possible.

Example 2

Simplify:

a $8x + 2y - 5x - 3y$ **b** $4ab - 5a + 3ba$ **c** $3my - 7m + 2 + 3m$

Solution

a Rearrange the expression to group the like terms. $8x + 2y - 5x - 3y = 8x - 5x + 2y - 3y$

Collect the like terms. $= 3x - y$

TI-Nspire CAS

8·x+2·y−5·x−3·y 3·x−y

ClassPad

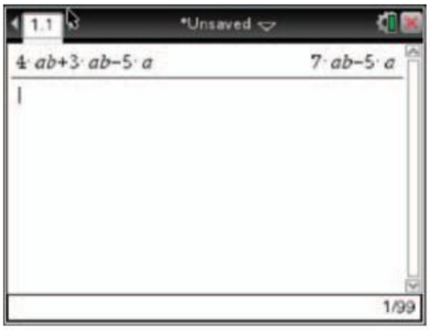
8x+2y−5x−3y

3·x−y

b Rearrange the expression to group the like terms. $4ab - 5a + 3ba = 4ab + 3ba - 5a$

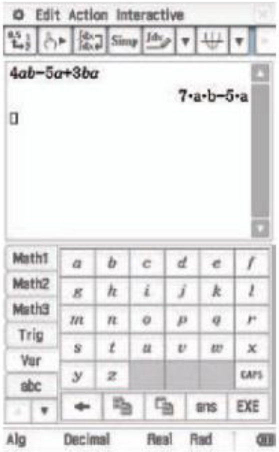
Collect the like terms. $= 7ab - 5a$

TI-Nspire CAS



4·ab+3·ab−5·a 7·ab−5·a

ClassPad



4ab−5a+3ba

7·a·b−5·a

c Rearrange the expression to group the like terms.
Collect the like terms.

$$3my - 7m + 2 + 3m = 3my - 7m + 3m + 2$$
$$= 3my - 4m + 2$$

TI-Nspire CAS

The order of the terms on the TI-Nspire CAS is different, but the expressions are equivalent.

3· my−7· m+3· m+2

-4· m+3· my+2

ClassPad

3my−7m+2+3m

3·m·y−4·m+2

Multiplying and dividing algebraic expressions

Multiplying and dividing

Remember your rules for multiplication and division of integers:

When two integers of the same sign are multiplied or divided, the result is positive.

When two integers of different signs are multiplied or divided, the result is negative.

When multiplying and dividing algebraic expressions, the terms do **not** have to be **like**.

Multiplying algebraic expressions

With multiplication, simplify by multiplying the numbers and then the pronumerals respectively.

Example 3

Simplify:

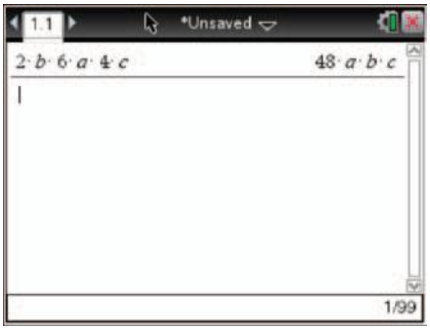
a $2b \times 6a \times 4c$ **b** $5pq \times -3pr$

Solution

a Multiply the numbers and pronumerals respectively.
Write in simplest form.

$$2b \times 6a \times 4c$$
$$= 48 \times b \times a \times c$$
$$= 48abc$$

TI-Nspire CAS



```
2·b·6·a·4·c                          48·a·b·c
```

ClassPad

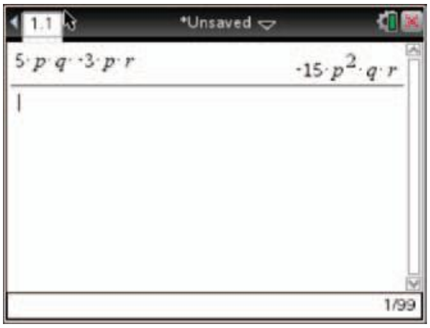
```
2b×6a×4c
                        48·a·b·c
```

b Multiply the numbers and pronumerals respectively.
Write in simplest form.

$$5pq \times -3pr$$
$$= -15pq \times pr$$
$$= -15p^2qr$$

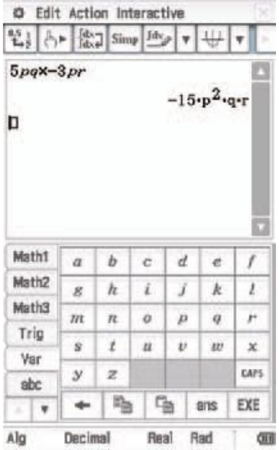
TI-Nspire CAS

Note that you need to include a multiplication sign between each variable in a term when using the TI-Nspire CAS.



```
5·p·q·-3·p·r                        -15·p^2·q·r
```

ClassPad



```
5pq×−3pr
                      −15·p^2·q·r
```

Dividing algebraic expressions

With division, write the expression as a fraction and simplify by cancelling the numbers, and then the pronumerals, using the **highest common factor** (HCF). The answer is usually written as a fraction.

Example 4

Simplify:

a $28xy \div 21yz$ **b** $-\dfrac{36m^2n}{9mnp}$

Solution

a Write the question as a fraction.

$$\frac{28xy}{21yz}$$

Identify the HCF.

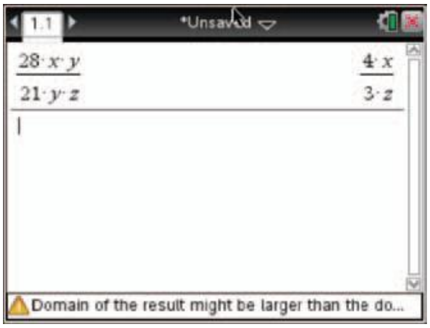
$$\text{HCF} = 7y$$

Divide the numerator and the denominator by the HCF. Write the answer in simplest form.

$$\frac{28xy}{21yz} = \frac{4x}{3z}$$

TI-Nspire CAS

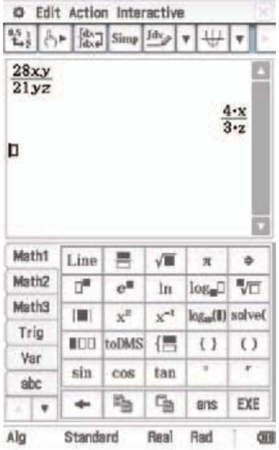
Remember that you need to include a multiplication sign between each variable in a term when using the TI-Nspire CAS.



$\frac{28\cdot x\cdot y}{21\cdot y\cdot z}$ $\frac{4\cdot x}{3\cdot z}$

Domain of the result might be larger than the do...

ClassPad



$\frac{28xy}{21yz}$

$\frac{4\cdot x}{3\cdot z}$

b Write the question as a fraction.

$$-\frac{36m^2n}{9mnp}$$

Identify the HCF.

$$\text{HCF} = 9mn$$

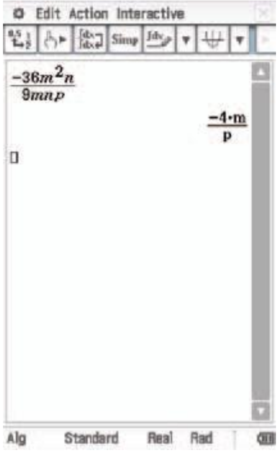
Divide the numerator and the denominator by the HCF. Write the answer in simplest form.

$$-\frac{36m^2n}{9mnp} = -\frac{4m}{p}$$

TI-Nspire CAS

$\frac{-36\cdot m^2\cdot n}{9\cdot m\cdot n\cdot p}$ $\frac{-4\cdot m}{p}$

ClassPad



$\frac{-36m^2n}{9mnp}$

$\frac{-4\cdot m}{p}$

EXERCISE 2.01 The four operations in algebra

1 Example 1 In each of the algebraic expressions identify:

i the number of terms

ii the coefficient of the term with the variable m

iii the constant term

a $3m - 4n + 6$

b $2mn + 7n - 4m - 5$

2 Example 2 Simplify:

a $3x - 2y + 6x - y$

b $4ab - 6a - 9ba$

c $7b - 9a + 3b - 4$

d $8xy + 3x - 4yx - 6$

e $2ab + a + 5ab + b$

f $3r + 4ar - 2ar - 7r$

g $3de - 6ed + 2d$

h $-5 + 7u - 10 - 3u$

i $15x - 5 + 3x + 5$

3 Example 3 Simplify:

a $3x \times 5y$

b $-4a \times 2bc$

c $12mn \times -3n$

d $-5xy \times 3yz$

e $4ak \times 4am$

f $-3d \times 8cd$

g $-6m \times (-6m)$

h $10ab \times (-2ab)$

i $-4bc \times (-3cd)$

4 Example 4 Simplify:

a $\frac{12xy}{3y}$

b $\frac{-30mn}{6np}$

c $\frac{24uv^2}{16vw}$

d $\frac{-48st}{16rt^2}$

e $\frac{4m}{2m}$

f $\frac{14x^2y}{2xy}$

g $\frac{3x}{x^3}$

h $\frac{27pr}{-9pq}$

i $\frac{-4de}{20e}$

5 Simplify:

a $5m - 2n + 3m + n$ b $2ab - 4b + 3b - 2ba$

c $5x - 4y + 2xy - 3x$ d $8pq - 6q + 3p - 4qp$

6 Simplify:

a $2m \times -5n$ b $-4pq \times 2q$

c $8xy \times -7xz$ d $12ab \times 5cd$

7 Simplify:

a $\frac{4xy}{8yz}$ b $\frac{-24ab}{6bc}$ c $\frac{-45x^2y}{9x}$ d $\frac{-32abc}{-4bc^2}$

Reasoning and communication

8 a State two different algebraic expressions involving addition which simplify to$12a - 7b$.

b State two different algebraic expressions involving subtraction which simplify to $2p + 7q$.

c State two different algebraic expressions involving addition which simplify to $12x^2 - 8xy$.

d State two different algebraic expressions involving multiplication which simplify to $6cd$.

e State two different algebraic expressions involving multiplication which simplify to $24a^2b^3$.

f State two different algebraic expressions involving division which simplify to $\frac{2x}{3y}$.

g State two different algebraic expressions involving division which simplify to $\frac{4a^2}{7b^6}$.

9 Select the simplified expression which matches $\frac{24ab}{16b^2m}$.

A $\frac{6a}{12m}$ B $\frac{12a}{8bm}$ C $\frac{3a}{2bm}$ D $\frac{3a}{2m}$ E $\frac{4b}{8m}$

10 Which of the following is the simplified expression for $\frac{2xy}{5} \times \frac{-3}{10y}$?

A $-\frac{3x}{25}$ B $-\frac{3x}{25y^2}$ C $-\frac{6x}{50}$ D $-\frac{3}{25y}$ E $\frac{3x^2}{2y}$

11 Choose a month from any calendar and draw a box around any block of nine numbers. See the boxed numbers for example.

10	11	12
17	18	19
24	25	26

a Add together the numbers in any line of three numbers going through the centre of the box (row, column, or diagonal). For example: 10 + 18 + 26.

b Add another line of three numbers that goes through the centre of the box.

c There are two more lines that go through the centre of the box. Find their sum also.

d What do you notice? Why does this pattern work? *Hint:* Let the number in the top left-hand corner be x.

12 If $5x^2 - 10x - 3x^2 - 12x = px^2 + qx$, state the values of p and q.

2.02 EXPANDING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

Expanding an algebraic expression means that the grouping symbols (brackets) are removed. The distributive law is used and every term in the bracket is multiplied by the term outside the bracket. Remember to take care with multiplication of positive and negative terms.

$$a(b + c) = a \times b + a \times c$$
$$= ab + ac$$

Remember that multiplication signs are omitted in algebra. $a \times b$ means ab

Example 5

Expand and simplify:

a $3(a - 4)$ **b** $x(2x + 5)$ **c** $-4m(2m - n + 3)$

Solution

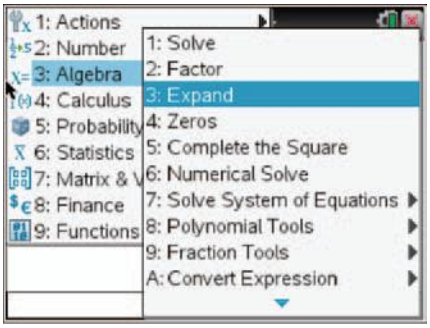
a Write the expression. $3(a - 4)$

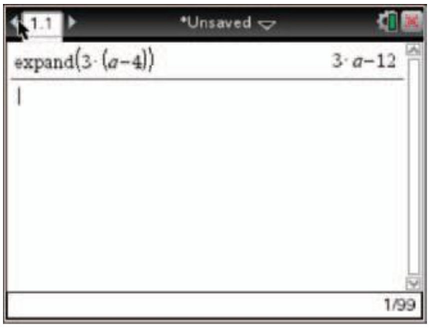
Multiply each term inside the brackets by the term outside the brackets. $= 3 \times a - 3 \times 4$

Simplify the expression. $= 3a - 12$

TI-Nspire CAS

1 Open a New Document with a Calculator page.
2 Press menu.
3 Select 3. Algebra.
4 Select 3. Expand.
5 Type in the expression.
6 Press enter.





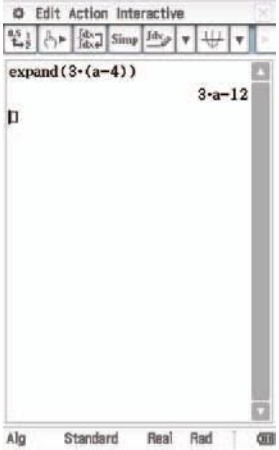
```
expand(3·(a−4))                3·a−12
```

ClassPad

1 In the Main $\sqrt{\alpha}$ application, type the expression.

2 Highlight.

3 Tap Interactive then Transformation then Expand. Make sure Expression is checked then tap $\boxed{\text{OK}}$.



expand(3·(a−4))

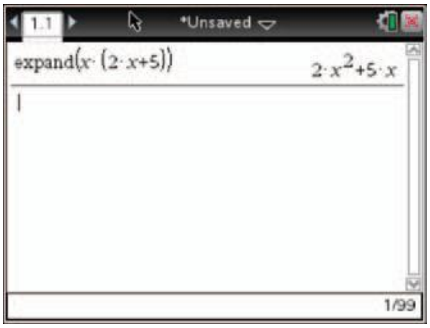
3·a−12

b Write the expression. $x(2x + 5)$

Multiply each term inside the brackets by the term outside the brackets. $= x \times 2x + x \times 5$

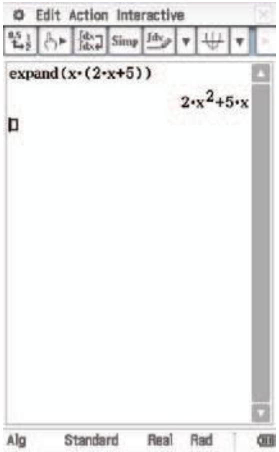
Simplify the expression. $= 2x^2 + 5x$

TI-Nspire CAS



expand(x·(2·x+5)) 2·x²+5·x

ClassPad



expand(x·(2·x+5))

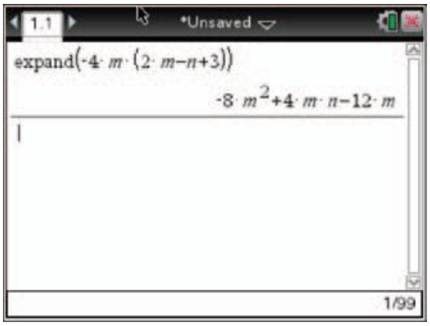
2·x²+5·x

c Write the expression. $-4m(2m - n + 3)$

Multiply each term inside the brackets by the term outside the brackets. $= -4m \times 2m + -4m \times -n + -4m \times 3$

Simplify the expression. $= -8m^2 + 4mn - 12m$

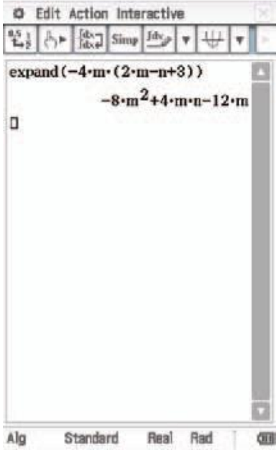
TI-Nspire CAS



expand(-4·m·(2·m−n+3))

$-8\cdot m^2+4\cdot m\cdot n-12\cdot m$

ClassPad



expand(−4·m·(2·m−n+3))

$-8\cdot m^2+4\cdot m\cdot n-12\cdot m$

If there is more than one bracket in the expression expand each bracket separately and then simplify by collecting the like terms.

Note that $-(3x-4) = -1 \times (3x-4)$

Example 6

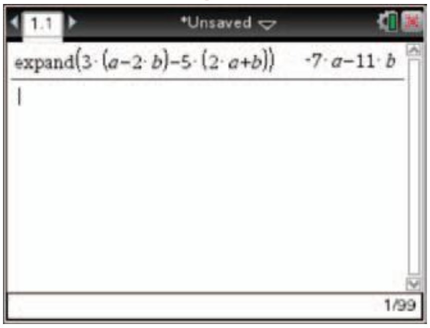
Expand and simplify:

a $3(a-2b)-5(2a+b)$ **b** $2x(5y-3)-(4x+y)$

Solution

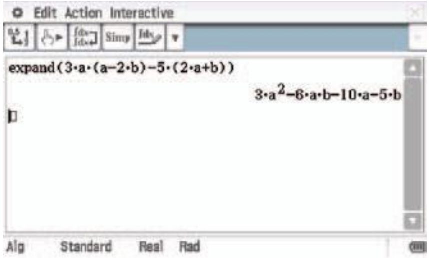
a Write the expression.	$3(a-2b)-5(2a+b)$
Expand each bracket.	$= 3a-6b-10a-5b$
Rearrange the expression grouping like terms.	$= 3a-10a-6b-5b$
Write the expression in simplest form.	$= -7a-11b$

TI-Nspire CAS



expand(3·(a−2·b)−5·(2·a+b)) -7·a−11·b

ClassPad

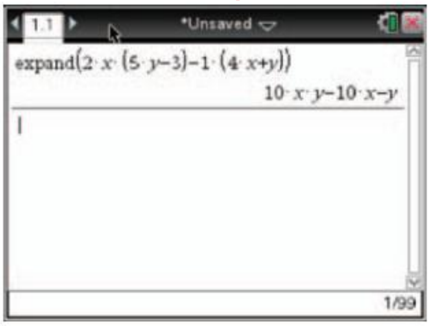


expand(3·a·(a−2·b)−5·(2·a+b))

$3\cdot a^2-6\cdot a\cdot b-10\cdot a-5\cdot b$

b Write the expression.	$2x(5y-3)-(4x+y)$
Expand each bracket separately.	$= 10xy-6x-4x-y$
Write the expression in simplest form by collecting like terms.	$= 10xy-10x-y$

TI-Nspire CAS



```
expand(2·x·(5·y−3)−1·(4·x+y))
                      10·x·y−10·x−y
```

ClassPad

```
expand(2·x·(5·y−3)−(4·x+y))
                      10·x·y−10·x−y
```

Expanding algebra

EXERCISE 2.02 Expanding and simplifying algebraic expressions

Concepts and techniques

1 **Example 5** Expand each of the following expressions:

a $3(a+2)$ b $5(3-2b)$ c $-2(2a+1)$

d $-6(b-2)$ e $3x(x-2)$ f $3p(p-a)$

g $-4(2k+4)$ h $2t(3-4t)$ i $-d(d-5)$

j $k(7-5k)$ k $-9b(b-1)$ l $2y(7x+4y)$

2 Expand:

a $4(x-3)$ b $-3(a+4)$ c $5(2a-1)$ d $3x(x-8)$

3 Expand each of the following expressions:

a $-6n(4-n)$ b $5x(rx+2r)$ c $-(2a^2-4)$

d $5b(a^2+3b-7)$ e $-(x^2-4x+10)$ f $3h(h-7e-4eh)$

g $y(2y+3-y^2)$ h $de(d^2-2+e^2)$ i $-3v(-3av+v-2a)$

4 **Example 6** Expand and simplify each of the following expressions:

a $5(x+4)-2(x+3)$ b $3(d-4)-2(d+5)$ c $6(r+10)-4(r-5)$

d $8(f+2)-(f+7)$ e $(2x-4)-5(3x+4)$ f $6x(x+4)-3x(x-1)$

g $3b(b+5)-b(b-8)$ h $4w(w-7)-w(w+1)$ i $6(k+p)+3(k+2p)$

j $2(a-b)+2(b+a)$ k $x(2v+4)-x(v+1)$ l $-3(t+w)-2(2t-w)$

m $e(3e+5)-(2e-e^2)$ n $-2(a+3)+4(a-3)$ o $p(p-q)-q(q-p)$

5 Which of the following represents the expansion of : $-(2m-3n)$?

A $2m+3n$ B $-2m-3n$ C $3n+2m$

D $3n-2m$ E none of these

6 $5a(b-2a)+3b(4a-b)$ can also be written as:

A $5ab-10a+12ba-3b$ B $17ab-10a^2-3b^2$ C $17ab-10a-3b$

D $5ab-10a^2+12ba-3b^2$ E $10a^2-12ab+3b^2$

7 $6x(x-3) - x(4x+3)$ can also be written as:

A $2x^2 - 3 - 3x$	B $10x^2 - 15x$	C $2x^2 - 15x$
D $10x^2 - 21x$	E $2x^2 - 21x$	

Reasoning and communication

8 a Evaluate 7 – 5 and 5 – 7. How are the two answers related?

b Evaluate 4 – 10 and 10 – 4.

c Is $a - b$ always the same as $-(b - a)$? Can you prove it algebraically?

9 Expanding is useful for multiplying numbers mentally without using a calculator, especially if one of the numbers is close to 10, 100 or 1000.

Study the following examples.

A $35 \times 11 = 35 \times (10 + 1)$	Think of 11 as 10 + 1.
$= 35 \times 10 + 35 \times 1$	Expand.
$= 350 + 35$	Simplify.
$= 385$	
B $43 \times 102 = 43 \times (100 + 2)$	Think of 102 as 100 + 2.
$= 43 \times 100 + 43 \times 2$	Expand.
$= 4300 + 86$	Simplify.
$= 4386$	
C $16 \times 8 = 16 \times (10 - 2)$	Think of 8 as 10 – 2.
$= 16 \times 10 - 16 \times 2$	Expand.
$= 160 - 32$	Simplify.
$= 128$	

Use the strategies shown in the examples above to complete these multiplications without using a calculator.

a 25×12	b 18×9	c 6×105
d 87×11	e 50×99	f 45×8

10 a State two possible algebraic expressions, with brackets, which expand to $12a - 6b$.

b State two possible algebraic expressions, with brackets, which expand to$16cd + 12de$.

c State two possible algebraic expressions, with brackets, which expand to$24mn + 36n^2$.

d State two possible algebraic expressions, with brackets, which expand to $32ab + 16b^2 - 4a^2$.

2.03 SUBSTITUTION

Substitution is where a pronumeral is replaced by a numerical value in order to evaluate an expression. Remember to use the correct order of operations when evaluating.

Example 7

If $p = 2$, $q = -3$ and $r = 8$; evaluate each expression:

a $2pq - r$	b $r^2 + 2q$	c $5p(3q - qr)$

Solution

a Substitute the appropriate values into the expression.	$2pq - r$ $= 2 \times 2 \times -3 - 8$
Evaluate, using the correct order of operations.	$= -12 - 8$ $= -20$

b Substitute the appropriate values into the expression.
Evaluate, using the correct order of operations.

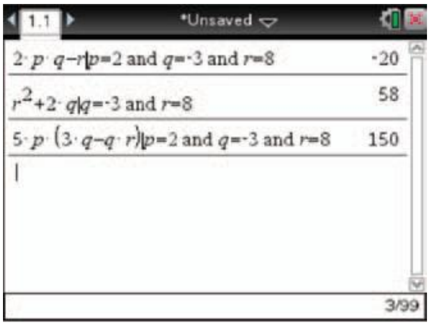
$$\begin{aligned}&r^2 + 2q\\ &= 8^2 + 2 \times - 3\\ &= 64 - 6\\ &= 58\end{aligned}$$

c Substitute the appropriate values into the expression.
Evaluate the bracket first.
Evaluate, using the correct order of operations.

$$\begin{aligned}&5p(3q - qr)\\ &= 5 \times 2 \times (3 \times -3 - -3 \times 8)\\ &= 10 \times (-9 + 24)\\ &= 10 \times 15\\ &= 150\end{aligned}$$

TI-Nspire CAS

The | symbol is used for substitution.
Press `ctrl` `=` then select |. Separate the values of the pronumerals by typing `␣` `A` `N` `D` `␣`.



$2 \cdot p \cdot q - r | p=2$ and $q=-3$ and $r=8$ -20

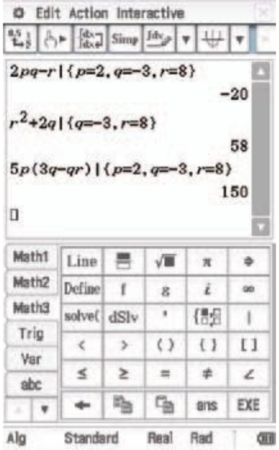
$r^2+2 \cdot q | q=-3$ and $r=8$ 58

$5 \cdot p \cdot (3 \cdot q - q \cdot r) | p=2$ and $q=-3$ and $r=8$ 150

3/99

ClassPad

The | symbol is used for substitution.
Press `Keyboard` then tap `Math3` then select | .
If substituting for more than one pronumeral the set brackets {} need to be used with the values of the pronumerals separated by commas.



$2pq-r | \{p=2, q=-3, r=8\}$ -20

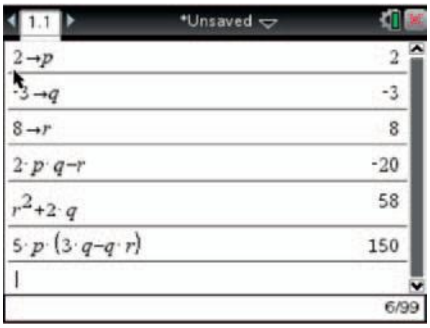
$r^2+2q | \{q=-3, r=8\}$ 58

$5p(3q-qr) | \{p=2, q=-3, r=8\}$ 150

Alternatively, the values of the pronumerals can be stored first. They can then be substituted by typing the required expression into the CAS.

TI-Nspire CAS

Open a New Document with a Calculator page.
Press `2` `ctrl` `var` `P` `enter`
Press `(–)` `3` `ctrl` `var` `Q` `enter`
Press `enter` `ctrl` `var` `R` `enter`
Type an expression and press `enter`
Open a New Document to clear the stored variables.



$2 \to p$ 2

$-3 \to q$ -3

$8 \to r$ 8

$2 \cdot p \cdot q - r$ -20

$r^2+2 \cdot q$ 58

$5 \cdot p \cdot (3 \cdot q - q \cdot r)$ 150

6/99

ClassPad

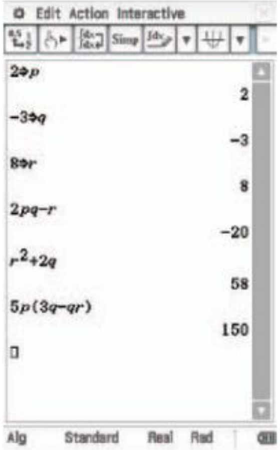
Use the application $\overset{\text{Main}}{\sqrt{\alpha}}$

Press 2 $\boxed{\text{Keyboard}}$ then tap $\boxed{\text{Math2}}$ $\boxed{\Rightarrow}$ $\boxed{\text{Var}}$ p and press $\boxed{\text{EXE}}$

Press $\boxed{(-)}$ 3 tap $\boxed{\Rightarrow}$ q and press $\boxed{\text{EXE}}$

Press 8 tap $\boxed{\Rightarrow}$ r and press $\boxed{\text{EXE}}$

To clear stored variables tap **Edit** then **Clear All Variables**.



$2 \Rightarrow p$
2
$-3 \Rightarrow q$
−3
$8 \Rightarrow r$
8
$2pq-r$
−20
r^2+2q
58
$5p(3q-qr)$
150

A **formula** is an algebraic rule which describes a mathematical relationship between pronumerals or variables. For example: $A = lw$ is the formula used to calculate the area of a rectangle. We say that A is the subject in this formula. You will also notice that a formula contains an equal sign. The value of the subject of a formula can be found by substitution.

You use the same method when substituting into algebraic expressions or formulae.

Substitution code puzzle

Example 8

From a height h m above sea level, an observer can see a distance of d km to the horizon, where $d = 8\sqrt{\frac{h}{5}}$.

What distance, correct to the nearest kilometre, can be seen from a tower 128 m above sea level?

Shutterstock.com/Rob Byron

Solution

Write the formula. $\quad d = 8\sqrt{\frac{h}{5}}$

Substitute $h = 128$ into the formula to find d. $\quad = 8 \times \sqrt{\frac{128}{5}}$

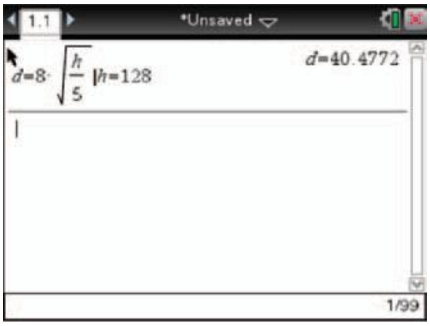
Use your calculator's $\sqrt{}$ button to evaluate. $\quad = 8 \times 5.0596\ldots$

Show each step of the calculation. $\quad = 40.4771\ldots$

Round your answer to match the question. $\quad \approx 40$ km (correct to the nearest km)

TI-Nspire CAS

Press ctrl enter for a decimal answer.



$d=8\cdot\sqrt{\frac{h}{5}}|h=128$

$d=40.4772$

1/99

ClassPad

Use **Decimal** mode to obtain a decimal answer.

$d=8\sqrt{\frac{h}{5}}|h=128$

$d=40.47715405$

IMPORTANT

The **subject of a formula** is the pronumeral by itself, usually on the left of the equals sign.

EXERCISE 2.03 Substitution

Concepts and techniques

1 Example 7 If $m = 6$, $n = 3$ and $p = -4$ evaluate the algebraic expressions:

a $4nm$	b $2np - m$	c $p^2 + mn$	d $24mp - n$
e $3mn - 2p$	f $2n^2 + 3mp$	g $-5p + 2mn$	h $p^2 - n^2 + m^2$

2 The value of the expression $3y^2 - (2y + 8)$ if $y = -5$ is:

A 73 B 77 C −73 D −77 E 27

3 The value of the expression $3a(8 - 2b)$ if $a = 3$ and $b = -12$ is:

A −144 B 144 C 288 D −288 E 96

4 Example 8 Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and perpendicular height of 11.58 cm. Use the formula $V = \pi r^2 h$.

5 The temperature T (in °C) of the water in a kettle t minutes after it is switched on is given by the formula $T = 18t + 28$. Find the temperature of the water:

a 4 minutes after it is turned on.
b $1\frac{1}{2}$ minutes after it is turned on.
c When the kettle is first turned on.

6 The formula for converting Australian dollars (A\$) to US dollars (US\$) is US = 0.835A. Convert the following A\$ amounts to US\$, correct to the nearest cent.

a \$20.00
b \$89.50
c \$4800

7 The angle sum $A°$ of a polygon with n sides is $A = 180(n - 2)$. Find the angle sum of:

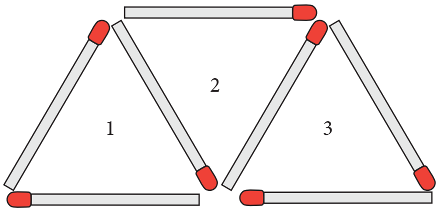
a a quadrilateral
b a hexagon
c a decagon.

8 If an object is moving with speed u m/s and acceleration a m/s^2, then its speed v m/s after t seconds is $v = u + at$.

a Which variable is the subject of the formula?
b Calculate the speed of a car after 5 seconds if its speed now is 6 m/s and it is accelerating at 2 m/s^2.

9 The number of matches, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern. How many matches are required to make:

a 8 triangles?
b 40 triangles?
c 150 triangles?



10 The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of a sphere with radius 14.5 cm.

11 The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$, where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35$ and $g = 10$.


Shutterstock.com/Ball LunlA

12 Use the compound interest $A = P(1+\frac{r}{100})^n$ formula to calculate the amount to which a principal of \$5600 will grow if invested at 9.4% p.a. for 5 years.

13 The maximum distance, d m, a ball travels if thrown with speed v m/s, is $d = \frac{v^2}{g}$. Find the maximum distance if a ball is thrown at a speed of 11.5 m/s and $g = 9.8$, correct to one decimal place.

Reasoning and communication

14 The formula for converting a speed of k km/h to metres per second (m/s) is $M = \frac{5k}{18}$. A speed of 80 km/h is closest to which of the following speeds?

A 22 m/s B 32 m/s C 35 m/s D 47 m/s E 23 m/s

15 The speed, V m/s, required for a spacecraft to escape the Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of the Earth (6 378 000 m). Which of the following is closest to the escape speed of a spacecraft leaving the Earth's atmosphere?

A 8840 m/s B 11 180 m/s

C 12 500 m/s D 35 000 m/s

E 1 180 m/s

Shutterstock.com/Fer Gregory

16 Listed below are commonly-used formulas. As a group or individual activity, select eight formulas and, for each one:

a Describe what the formula is used for.

b State the subject of the formula, and describe what it represents.

c Describe what the other variables in the formula represent.

$$V = \frac{1}{3}Ah \qquad A = 180(n-2) \qquad c^2 = a^2 + b^2$$

$$A = \pi r^2 \qquad S = 2\pi r^2 + 2\pi rh \qquad I = \frac{Prn}{100}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad C = 2\pi r = \pi d \qquad A = \frac{1}{2}xy$$

$$S = \frac{d}{t} \qquad A = \frac{1}{2}(a+b)h \qquad A = P(1+\frac{r}{100})^n$$

$$A = s^2 \qquad V = \pi r^2 h \qquad V = \frac{4}{3}\pi r^3$$

INVESTIGATION The body mass index

The body mass index (BMI) has been used by the World Health Organisation since 1997 as an international standard for health and fitness for adults. The formula for BMI is $B = \frac{m}{h^2}$, where m is a person's mass in kilograms and h is the person's height in metres. This single value, applicable to both males and females, is a convenient indicator of whether a person is overweight or underweight, without the need for reading height-weight charts. The table below shows how BMI, when calculated correct to one decimal place, is interpreted.

BMI	Weight status
Under 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

9780170250252

Because the BMI is an algebraic model, the WHO acknowledges that it has some limitations. It is useful for classifying the general population but does not take into account a person's frame size, muscle mass, bone density or distribution of body fat. For this reason, the BMI should not be used for body builders, athletes, children under 19, pregnant women, the frail and sedentary elderly, and Aboriginal, Pacific Island and Asian people.

Activity

a Jandi is 1.7 m tall and her mass is 60.4 kg. Jay is 1.8 m tall and his mass is 72.2 kg. Use a spreadsheet to calculate Jandi's BMI and Jay's BMI correct to one decimal place.

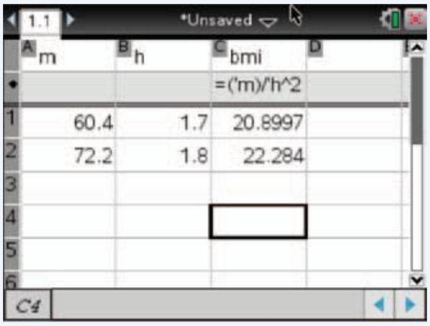
TI-Nspire CAS

Open a new document with a Lists & Spreadsheet page.

In the cell labelled A, type m, in the cell labelled B, type h and in the cell labelled C, type bmi.

Enter Jandi's mass and height in cells A1 and B1 respectively then enter Jay's mass and height in cells A2 and B2 respectively.

In the row immediately below bmi, type the formula ‘$= m/h^2$’. For each pop up window that appears, select Variable Reference then OK.



	A m	B h	C bmi	D
•			=('m)/'h^2	
1	60.4	1.7	20.8997	
2	72.2	1.8	22.284	
3				
4				
5				

Jandi's BMI is 20.9 and Jay's BMI is 22.3.

ClassPad

Use the application.

Tap cell A1. To enter the letter m in this cell, press Keyboard tap abc then m. Similarly, in cell B1 type h and in cell C1 type BMI. (Remember that to see capital letters on the Keyboard it is necessary to first tap CAPS.)

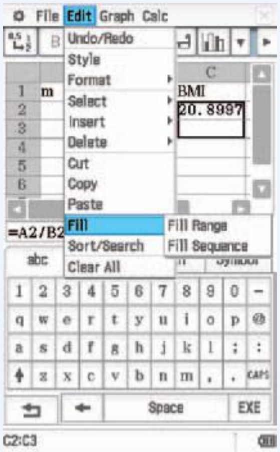
Type Jandi's mass in cell A2 and her height in cell B2. In cell C2 use the buttons and screen keyboard to enter the formula = A2/ B2 ^ 2 then press EXE.

(Note that / can be entered either by pressing ÷ or by tapping Math and selecting it from the screen.)

Type Jay's mass in cell A3 and his height in cell B3.

Highlight cells C2 and C3 then tap **Edit** and **Fill Range**.

On the screen that follows, tap OK



This copies the formula in cell C2 into cell C3 and automatically adjusts this to use the values in cells A3 and B3.

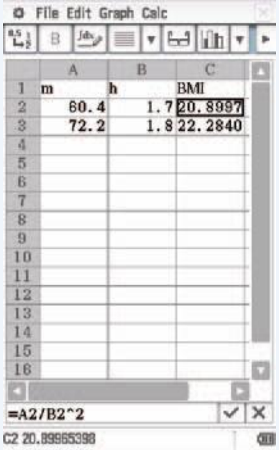
Formula =A2/B2^2

Range C2:C3

=A2/B2^2

C2:C3

Jandi's BMI is 20.9 and Jay's BMI is 22.3.



	A	B	C
1	m	h	BMI
2	60.4	1.7	20.8997
3	72.2	1.8	22.2840

=A2/B2^2

C2 20.89965398

b A doctor sees 15 patients one morning for general medical check-ups and records their masses and heights as part of her routine procedure.

Use a spreadsheet to calculate the BMI, correct to one decimal place, for each patient then classify each one as; underweight, normal, overweight or obese according to the table given in the introduction.

Patient	Mass (kg)	Height (m)	BMI	Interpretation
A	57.4	1.62		
B	63.5	1.58		
C	45.8	1.63		
D	89.6	1.72		
E	104.3	1.85		
F	96.7	1.77		
G	77.2	1.64		

Patient	Mass (kg)	Height (m)	BMI	Interpretation
H	69.5	1.60		
I	47.6	1.53		
J	124.6	1.74		
K	87.9	1.62		
L	82.1	1.71		
M	65.8	1.59		
N	53.5	1.55		
O	52.6	1.63		

2.04 SOLVING EQUATIONS

An **equation** is a mathematical sentence composed of algebraic expressions and numerals which can be on either side of the equals sign. Equations are different from expressions due to the inclusion of an equals sign.

An algebraic expression can be **simplified** but an equation can be **solved**.

For example, $3w - 4$ is an algebraic expression, whereas $3w - 4 = 6$ is an equation.

An equation is solved when the value of the variable, making the equation true (for example, x) is found.

To solve an equation you must keep it **balanced**, that is, you must perform the same operation on both sides of the equation.

Example 9

Solve the equations below:

a $2x - 6 = 14$ **b** $8 - 4b = - 16$

Solution

a Write the equation. $2x - 6 = 14$

Add 6 to both sides. $2x - 6 + 6 = 14 + 6$

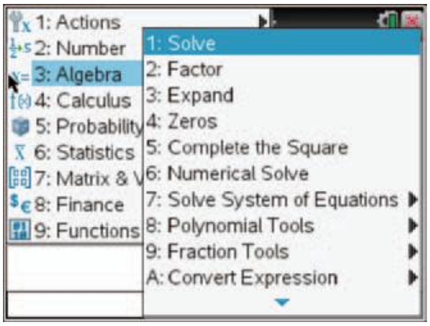
Simplify both sides. $2x = 20$

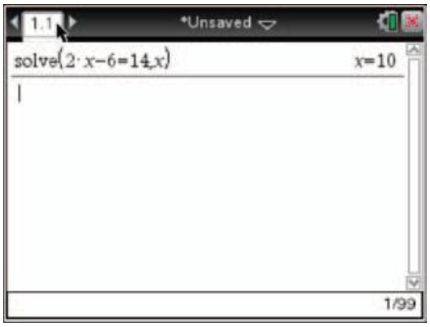
Divide both sides by 2. $\frac{2x}{2} = \frac{20}{2}$

Simplify to find the value of x. $x = 10$

TI-Nspire CAS

1 Use a New Document with a Calculator page.
2 Press menu.
3 Select 3. Algebra
4 Select 1. Solve
5 Type in the equation, pronumeral to be solved for.
6 Press enter.

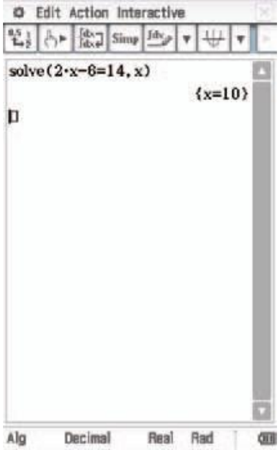




```
solve(2·x−6=14,x)                x=10
```

ClassPad

1 Using the Main $\sqrt{\alpha}$ application, type the equation.
2 Highlight then tap **Interactive** then **Equation/Inequality** then **solve**.
3 Make sure that Solve is checked and the correct variable is in the variable box then tap OK.



```
solve(2·x−6=14, x)
                         {x=10}
```

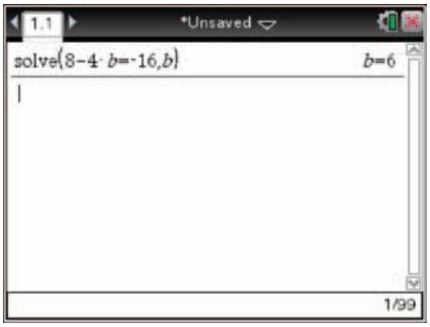
b Write the equation.
Subtract 8 from both sides.
Simplify both sides.

Divide both sides by –4.

Simplify to find the value of *b*.

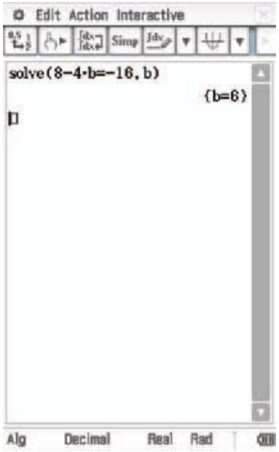
$$8 - 4b = -16$$
$$8 - 4b - 8 = -16 - 8$$
$$-4b = -24$$
$$\frac{-4b}{-4} = \frac{-24}{-4}$$
$$b = 6$$

TI-Nspire CAS



solve(8−4·b=−16,b)   b=6

ClassPad



solve(8−4·b=−16, b)

{b=6}

When solving more complex equations you need to carefully consider the order of the operations performed on the variables.

Once you have identified the order of the operations, perform them in reverse order to solve for the variable.

Example 10

Solve the equations below:

a $\frac{x}{3}+2=7$ **b** $5-\frac{3x}{4}=2$ **c** $\frac{2a-6}{5}=4$

a Write the equation.
$$\frac{x}{3}+2=7$$

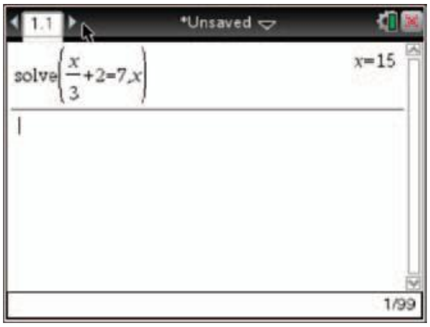
Subtract two from both sides.
$$\frac{x}{3}+2-2=7-2$$

Simplify both sides.
$$\frac{x}{3}=5$$

Multiply both sides by 3.
$$\frac{x}{3}\times 3=5\times 3$$

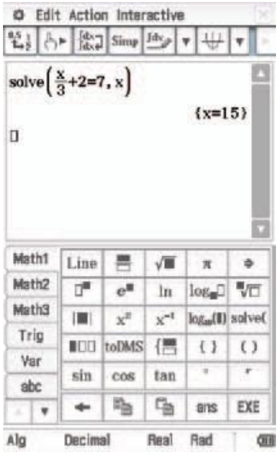
Simplify to find the value of *x*.
$$x=15$$

TI-Nspire CAS



$\text{solve}\left(\frac{x}{3}+2=7,x\right)$ x=15

ClassPad



$\text{solve}\left(\frac{x}{3}+2=7, x\right)$

{x=15}

b Write the equation.
$$5-\frac{3x}{4}=2$$

Subtract 5 from both sides.
$$5-\frac{3x}{4}-5=2-5$$

Simplify both sides.
$$-\frac{3x}{4}=-3$$

Multiply both sides by 4.
$$-\frac{3x}{4}\times 4=-3\times 4$$

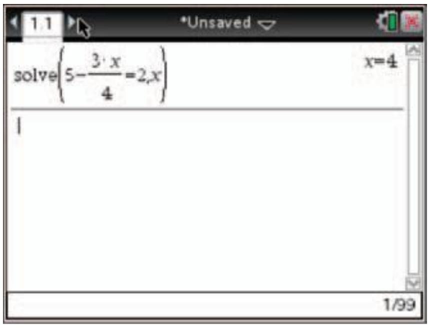
Simplify both sides.
$$-3x=-12$$

Divide both sides by −3.
$$\frac{-3x}{-3}=\frac{-12}{-3}$$

Simplify to find the value of *x*.
$$x=4$$

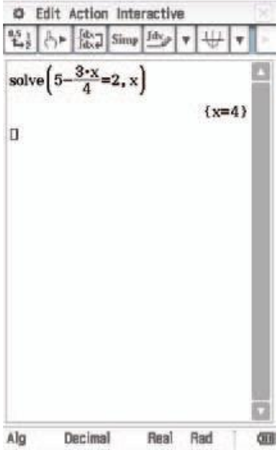
TI-Nspire CAS

TI-Nspire CAS tip: Press `ctrl` `÷` for the fraction template $\frac{\square}{\square}$.



$\text{solve}\left(5-\frac{3\cdot x}{4}=2,x\right)$ x=4

ClassPad



$\text{solve}\left(5-\frac{3\cdot x}{4}=2,x\right)$

{x=4}

c Write the equation.
$$\frac{2a-6}{5}=4$$

Multiply both sides by 5.
$$\frac{2a-6}{5}\times 5=4\times 5$$

Simplify both sides.
$$2a-6=20$$

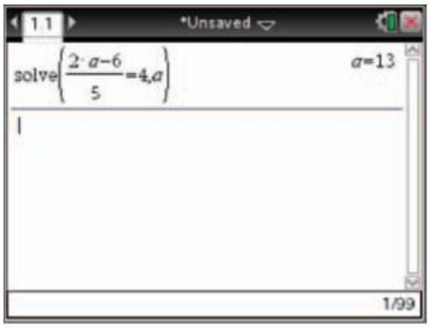
Add 6 to both sides.
$$2a-6+6=20+6$$

Simplify both sides.
$$2a=26$$

Divide both sides by 2.
$$\frac{2a}{2}=\frac{26}{2}$$

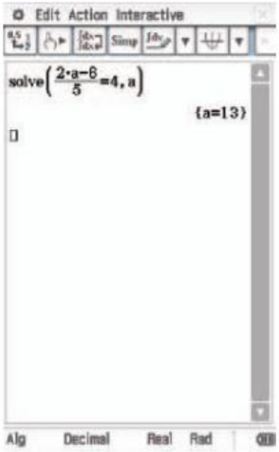
Simplify to find the value of *a*.
$$a=13$$

TI-Nspire CAS



$\text{solve}\left(\frac{2\cdot a-6}{5}=4,a\right)$ $a=13$

ClassPad



$\text{solve}\left(\frac{2\cdot a-6}{5}=4,a\right)$

$\{a=13\}$

EXERCISE 2.04 Solving equations

Concepts and techniques

One- and two-step equations

1 Example 9 Solve each of these equations:

a $3d+2=20$ b $2p-3=2$ c $4u+6=20$

d $5a+3=-12$ e $12b+8=4$ f $3-2a=-6$

g $4x+2=22$ h $5-3y=26$ i $6m-8=16$

j $28-5x=3$ k $22m+12=56$ l $-8-4v=16$

2 Example 10 Solve each of these equations:

a $\frac{3h}{4}=9$ b $\frac{r-1}{6}=2$ c $-\frac{2x}{5}=8$

d $\frac{y+2}{-3}=1$ e $\frac{4c}{10}=3$ f $\frac{z}{3}-11=9$

g $\frac{2a}{3}-4=0$ h $\frac{3x-1}{2}=7$ i $\frac{18-2n}{3}=4$

j $\frac{12a+3}{9}=3$ k $32-\frac{6a}{3}=26$ l $\frac{5x}{4}-12=-7$

3 Solve each of these equations:

a $3m+2=17$ b $2x-4=28$ c $10-3a=16$

d $12+2w=-18$ e $5x-4=26$ f $28-4a=40$

g $\frac{2x}{5}=8$ h $\frac{2f+7}{2}=10$ i $\frac{w}{5}-8=6$

j $\frac{4n+7}{9}=2$ k $\frac{5z+8}{6}=-3$ l $\frac{8-2b}{2}=7$

Reasoning and communication

4 In which line was an error made in solving the following equation?

$\frac{c-4}{8} + 2 = 6$ Line 1

$\frac{c-4}{8} = 8$ Line 2

$c - 4 = 64$ Line 3

$c = 68$ Line 4

A Line 1 B Line 2 C Line 3 D Line 4 E No mistakes

5 Let the number represented in the following be n. For each statement below:
- i Write an equation to match the words.
- ii Solve this equation.
  - a Twice a number less 8 is equal to 18.
  - b Four times the sum of a number and 5 is equal to 28.
  - c Twelve less three times a number equals 39.
  - d Twice the sum of a number and 6 is equal to 46.
  - e One third a number less 4 is equal to 1.
  - f Half the sum of a number and 6 is equal to 8.
  - g Twice a number less 4 is divided by 5 and equals 4.
  - h One third of the sum of a number and 7 is equal to 12.

6 True or false?
- a $w = 5$ is the solution to the equation $3w - 8 = 13$.
- b $x = 4$ is the solution to the equation $12 - 2x = -4$.
- c $m = -3$ is the solution to the equation $\frac{2m-6}{3} = 4$.
- d $n = 7$ is the solution to the equation $\frac{4n}{7} + 8 = 12$.

7 A paddock is 40 m long and x m wide. The perimeter of the paddock is 300 m.
- a Find an expression for the perimeter of the paddock.
- b Write an equation and calculate the width of the paddock.

8 Paula had n marbles and Stephen had 11 less than Paula.
- a Write an expression for the number of marbles Stephen had.
- b Write an expression for the total number of marbles.
- c If they had a total of 61 marbles, write an equation and calculate the number of marbles Paula had.
- d Determine the number of marbles Stephen had.

9 A photo is x cm wide and $x + 6$ cm long. The distance around the outside of the photo is 40 cm.
- a Find an expression for the distance around the outside of the photo.
- b Write an equation and calculate the width of the photo.
- c Determine the length of the photo.

2.05 SOLVING EQUATIONS WITH PRONUMERALS ON BOTH SIDES

Some equations have pronumerals on both sides of the equals sign. To solve this type of equation, collect the pronumerals on one side of the equation then simplify as before to find the value of the pronumeral. To solve an equation with brackets expand the brackets first then solve the equation as before.

Example 11

Solve:

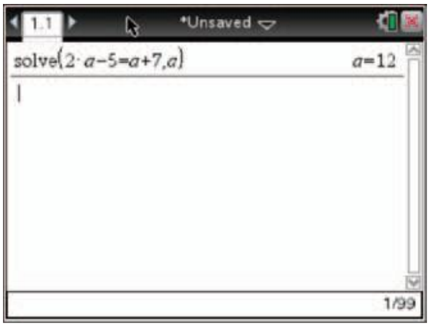
a $2a-5=a+7$ **b** $3(6-2x)=28$ **c** $\frac{2x}{3}-5=3x-19$

Solution

a

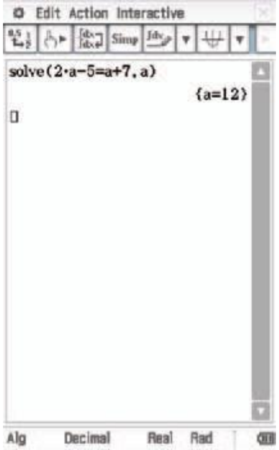
Write the equation.	$2a-5=a+7$
Subtract *a* from both sides.	$2a-5-a=a+7-a$
Simplify both sides.	$a-5=7$
Add 5 to both sides.	$a-5+5=7+5$
Simplify to find the value of *a*.	$a=12$

TI-Nspire CAS



solve(2·a−5=a+7,a) a=12

ClassPad



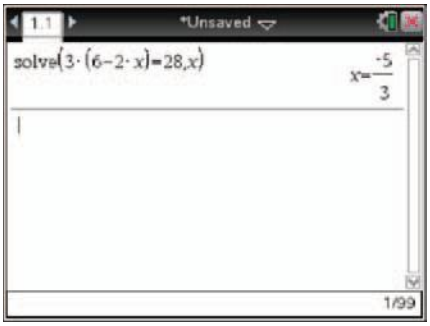
solve(2·a−5=a+7, a)

{a=12}

b

Write the equation.	$3(6-2x)=28$
Expand the brackets.	$18-6x=28$
Subtract 18 from both sides.	$18-6x-18=28-18$
Simplify both sides.	$-6x=10$
Divide both sides by −6.	$\frac{-6x}{-6}=\frac{10}{-6}$
Simplify to find the value of *x*.	$x=\frac{10}{-6}$
	$=-\frac{5}{3}$
	$=-1\frac{2}{3}$

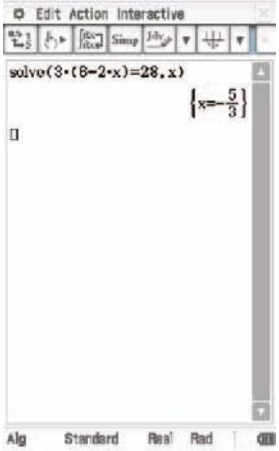
TI-Nspire CAS



solve(3·(6−2·x)=28,x)

$x=\frac{-5}{3}$

ClassPad

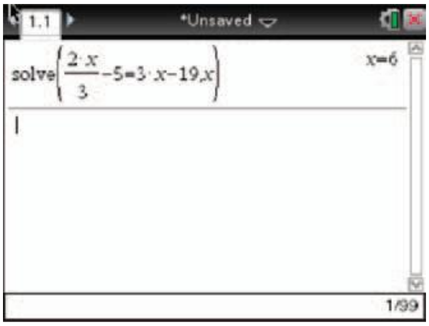


solve(3·(6−2·x)=28,x)

$\left\{x=-\frac{5}{3}\right\}$

c Write the equation.	$\frac{2x}{3}-5=3x-19$
Add 5 to both sides.	$\frac{2x}{3}-5+5=3x-19+5$
Simplify both sides.	$\frac{2x}{3}=3x-14$
Multiply both sides by 3.	$\frac{2x}{3}\times 3=(3x-14)\times 3$
	$2x=3(3x-14)$
Simplify both sides.	$2x=9x-42$
Subtract 9*x* from both sides.	$2x-9x=9x-42-9x$
Simplify both sides	$-7x=-42$
Divide both sides by –7.	$\frac{-7x}{-7}=\frac{-42}{-7}$
Simplify to find the value of *x*.	$x=6$

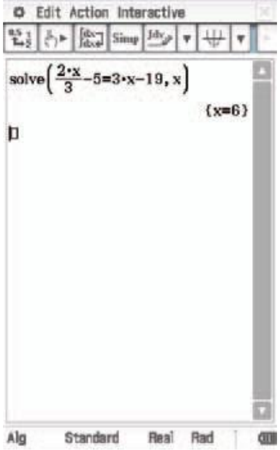
TI-Nspire CAS



$\text{solve}\left(\frac{2\cdot x}{3}-5=3\cdot x-19,x\right)$

$x=6$

ClassPad



$\text{solve}\left(\frac{2\cdot x}{3}-5=3\cdot x-19,x\right)$

{x=6}

Example 12

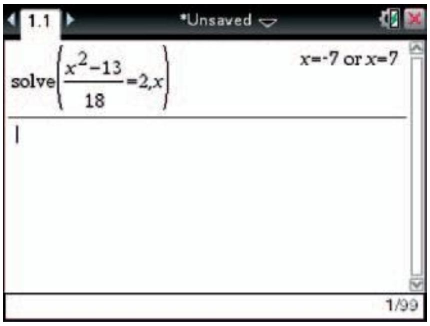
Solve for the pronumeral in each of the following equations.

a $\frac{x^2-13}{18}=2$ **b** $w^3+6=5$ **c** $\sqrt{4d-9}=7$

Reasoning	Working
a Write the equation.	$\frac{x^2-13}{18}=2$
Multiply both sides by 18.	$x^2-13=2\times18$ $x^2-13=36$
Add 13 to both sides.	$x^2=36+13$ $x^2=49$
Take the square root of both sides.	$x=\pm\sqrt{49}$
Write the answer.	$x=\pm7$

$x=\pm7$ means that $x=7$ or $x=-7$. There are two possible solutions to this equation since $7^2=49$ and $(-7)^2=49$.

TI-Nspire CAS



solve$\left(\frac{x^2-13}{18}=2,x\right)$ x=-7 or x=7

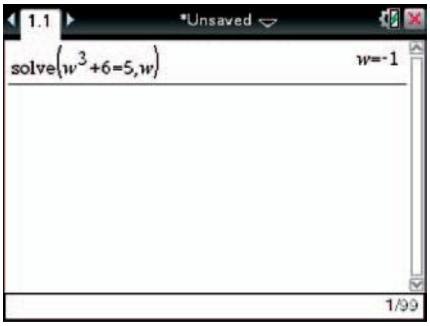
ClassPad

solve$(\frac{x^2-13}{18}=2, x)$

{x=-7, x=7}

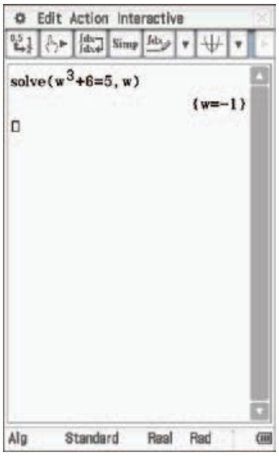
b Write the equation.	$w^3+6=5$
Subtract 6 from both sides.	$w^3=5-6$ $w^3=-1$
Take the cube root of both sides.	$w=\sqrt[3]{-1}$
Write the answer.	$w=-1$

TI-Nspire CAS



solve$(w^3+6=5,w)$ w=-1

ClassPad



solve$(w^3+6=5, w)$

{w=-1}

c Write the equation. $\sqrt{4d-9}=7$

Square both sides. $4d-9=7^2$

$4d-9=49$

Add 9 to both sides. $4d=49+9$

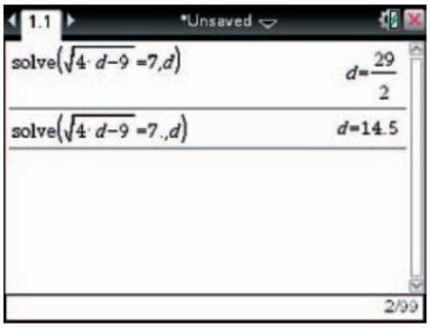
$4d=58$

Divide both sides by 4. $d=\frac{58}{4}$

Write the answer. $d=14.5$

TI-Nspire CAS

Notice that if you include a decimal point in the question, you will get a decimal answer.
If you don't include a decimal point in the question, you will get a fractional answer.

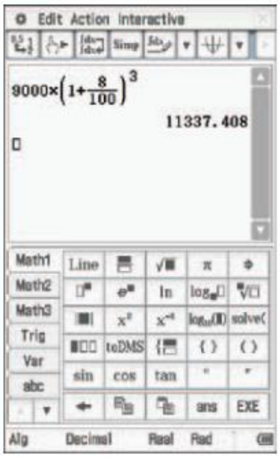


solve($\sqrt{4\cdot d-9}=7$,d) $d=\frac{29}{2}$

solve($\sqrt{4\cdot d-9}=7.$,d) $d=14.5$

ClassPad

To get a fractional answer, set the calculator to Standard.
To get a decimal answer, set the calculator to Decimal.



$9000\times\left(1+\frac{8}{100}\right)^3$

11337.408

Alg Decimal Real Rad

More complex equations

EXERCISE 2.05 Solving equations with pronumerals on both sides

Concepts and techniques

1 Which of the following is the solution for $2t-4=10+t$?

A $t=2$ B $t=3$ C $t=4\frac{2}{3}$ D $t=6$ E $t=14$

2 Example 11 Solve each of these equations:

a $2x-5=x+4$ b $2m+7=m-2$ c $5k-13=3k+9$

d $4x+6=3x-3$ e $7u+7=2u-10$ f $3p+4=4p$

g $8e=2(e-6)$ h $2(m-4)=m+12$ i $3x+6=2(x-8)$

j $\frac{2a}{3}-4=a-7$ k $\frac{2x-1}{3}=\frac{x+4}{2}$ l $\frac{3a}{4}+2=2a-8$

3 Solve each of these equations:

a $4x + 4 = x - 11$ **b** $12d = 8 + 8d$ **c** $3c + 8 = 2(c - 10)$

d $6 - 2y = 3y + 16$ **e** $10x - 11 = 4(2x - 10)$ **f** $10m - 8 = 46 - 2m$

g $5(d + 5) = 2(2d + 18)$ **h** $\frac{2a-8}{5} = 2a$ **i** $\frac{3x}{2} - 5 = 2x + 9$

j $\frac{3a-1}{4} = 2a+5$ **k** $12g - 11 = 4(5g+7)$ **l** $\frac{2m-3}{5} = \frac{3m+1}{2}$

4 The solution to $6(2 - w) = 3w - 15$ is:

A $w = -3$ **B** $w = 3$ **C** $w = 6$ **D** $w = 7$ **E** $w = 9$

5 Example 12 Solve for the pronumeral in each of the following equations.

a $\sqrt{5y+4} = 7$ **b** $3m^2 + 5 = 80$ **c** $5u^3 = -40$

d $\sqrt{y} - 3 = 9$ **e** $\frac{d^2}{8} = 18$ **f** $\sqrt[3]{2e} = -6$

g $\sqrt[3]{2a+1} = 3$ **h** $\frac{x^3 - 7}{5} = 101$ **i** $4k^2 + 11 = 267$

6 Solve for the pronumeral in each of the following equations. Give your answers correct to two decimal places.

a $d^2 = 96$ **b** $5y^2 - 10 = 14$ **c** $3c^3 = 20$

d $\frac{n^2}{4} = 7$ **e** $p^3 + 12 = 6$ **f** $-7w^2 = -10$

g $8h^3 - 12 = 18$ **h** $\frac{r^2+4}{5} = 9$ **i** $\frac{1}{5}p^2 = 6$

Reasoning and communication

7 True or false?

a $x = 2$ is the solution of $4x - 6 = 2x + 2$. **b** $m = -3$ is the solution of $5m - 2 = 3m - 8$.

c $w = 5$ is the solution of $2(3w - 1) = 5w + 4$. **d** $n = -5$ is the solution of $\frac{6n}{5} = n - 1$.

8 In which line was an error made in solving the following equation?

$3x - 4 = 2(x + 8)$
$3x - 4 = 2x + 16$ Line 1
$3x = 2x + 16 - 4$ Line 2
$3x = 2x + 12$ Line 3
$x = 12$ Line 4

A Line 1 **B** Line 2 **C** Line 3 **D** Line 4 **E** No mistakes

9 Let the number represented in the following be x. For each statement below:

i Write an equation to match the words.

ii Solve this equation.

- **a** Twice the number less 6 equals the sum of the number and 4.
- **b** Five times the number plus 2 equals four times the number less 5.
- **c** Six times the number less 8 equals four times the sum of the number and 4.
- **d** Eight times the number plus 7 equals 14 less than 5 times the number.
- **e** Three times the difference between the number and 1 equals twice the number plus 8.
- **f** Five times the number plus 6 equals three times the difference between the number and 2.
- **g** Three times one-fifth of the number equals the sum of the number and 4.
- **h** One-third of the sum of the number and 8 equals one-fifth of 6 less than twice the number.

2.06 PRACTICAL APPLICATIONS OF EQUATION SOLVING

When substituting numerical values into a formula, you may want to find the value of a pronumeral which is *not* the subject of the formula. In this case you must rearrange the formula to make the pronumeral you are solving for the subject of the formula.

Example 13

The formula used to calculate the area of a triangle is $A=\frac{1}{2}bh$. If the area of the triangle is 48 cm^2 and the triangle is 8 cm tall, find the length of the base.

Solution

Write down the formula. $A=\frac{1}{2}bh$

Identify the known variables and state their values. $A=48,\ h=8$

Substitute $A=48$ and $h=8$ into the formula and simplify.

$$48=\frac{1}{2}\times b\times 8$$

$$48=4b$$

Divide both sides by 4.

$$\frac{4b}{4}=\frac{48}{4}$$

Find the value of b. $b=12$

Write the answer. The base of the triangle is 12 cm.

The process of substitution and rearranging the equation to find the value of a pronumeral remains the same, whether or not the formula is familiar.

Example 14

If $A = P(1+\frac{r}{100})^n$ then find the value of:

a P if $A=4500$, $r=8$ and $n=6$.

b r if $A=8650$, $P=6500$ and $n=4$.

Give your answer correct to two decimal places.

Solution

a Write the formula.

$$A = P(1+\frac{r}{100})^n$$

Substitute in $A=4500$, $r=8$ and $n=6$ and simplify.

$$4500=P(1+\frac{8}{100})^6$$

$$4500=P(1.08)^6$$

Solve the equation by dividing both sides by $(1.08)^6$.

$$\frac{4500}{(1.08)^6}=P$$

$$P=2835.763\ldots$$

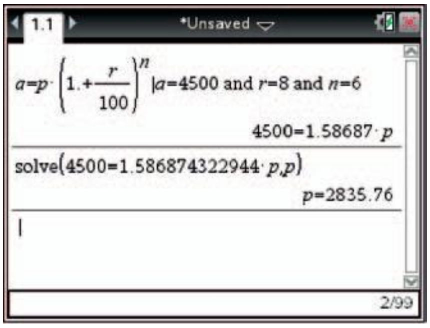
Round your answer to two decimal places.

$$P\simeq 2835.76$$

Using CAS:
First substitute the values of A, r and n.
Then solve for p.

TI-Nspire CAS

In the first line, enter either 1. or 100. so the decimal point will force a decimal answer. This will not work if you put the decimal point in numbers after the vertical bar.
In the second line, enter solve(Ans,p). This will change when you press enter.



$$a=p\cdot\left(1.+\frac{r}{100}\right)^n|a=4500 \text{ and } r=8 \text{ and } n=6$$

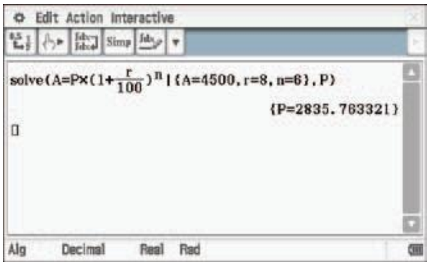
$$4500=1.58687\cdot p$$

$$\text{solve}(4500=1.586874322944\cdot p,p)$$

$$p=2835.76$$

ClassPad

The calculator should be set to Decimal.
The calculator is case sensitive, so be careful that if you use a capital P at the start, you also use it throughout.



$$\text{solve}(A=P\times(1+\frac{r}{100})^n|\{A=4500, r=8, n=6\}, P)$$

$$\{P=2835.763321\}$$

b Write the formula.

$$A = P(1+\frac{r}{100})^n$$

Substitute $A = 8650$, $P = 6500$ and $n = 4$ into the formula.

$$8650 = 6500(1+\frac{r}{100})^4$$

Divide both sides by 6500.

$$\frac{8650}{6500} = (1+\frac{r}{100})^4$$

Simplify.

$$(1+\frac{r}{100})^4 = 1.33076...$$

Find the 4th root of both sides.

$$1+\frac{r}{100} = \sqrt[4]{1.33076...}$$

$$1+\frac{r}{100} = 1.07405...$$

Subtract 1 from both sides.

$$\frac{r}{100} = 0.07405...$$

Multiply by 100.

$$r = 0.07405...\times 100$$

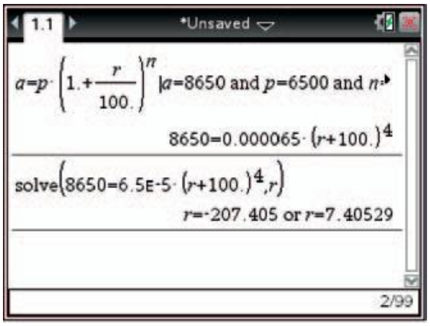
$$r = 7.405...$$

Round your answer to two decimal places.

$$r \approx 7.41\%$$

TI-Nspire CAS

First substitute the values of A, P and n.
Then solve for r.
Choose the positive answer.
For the TI-Nspire, you will need to add a decimal point to both the 1 and the 100.



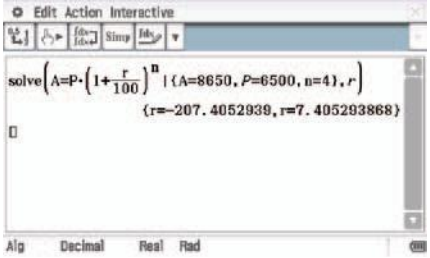
$$a=p\cdot\left(1.+\frac{r}{100.}\right)^n|a=8650 \text{ and } p=6500 \text{ and } n\blacktriangleright$$

$$8650=0.000065\cdot(r+100.)^4$$

$$\text{solve}(8650=6.5\text{E-}5\cdot(r+100.)^4,r)$$

$$r=-207.405 \text{ or } r=7.40529$$

ClassPad



$\text{solve}\left(A=P\cdot\left(1+\frac{r}{100}\right)^n \mid \{A=8650, P=6500, n=4\}, r\right)$

$\{r=-207.4052939, r=7.405293868\}$

Example 15

The formula for finding the volume of a sphere is $V=\frac{4}{3}\pi r^3$. Find the radius, correct to two decimal places, of a sphere which has a volume of 512 cm^3.

Solution

Write the formula.

$$V=\frac{4}{3}\pi r^3$$

Substitute $V=512$ in the formula.

$$512=\frac{4}{3}\pi r^3$$

Divide both sides by $\frac{4}{3}\pi$.

$$\frac{512}{\frac{4}{3}\pi}=r^3$$

Simplify both sides.

$$r^3=122.2309\ldots$$

Take the cube root of both sides.

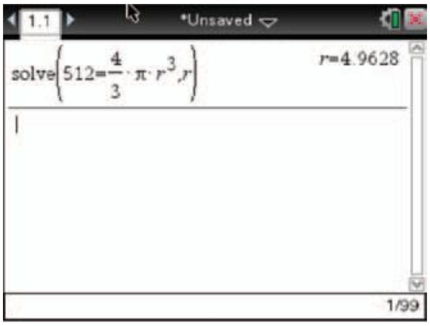
$$r=\sqrt[3]{122.2309\ldots}$$

$$r=4.9628\ldots$$

Round your answer to two decimal places.

$$r\approx 4.96$$

TI-Nspire CAS



$\text{solve}\left(512=\frac{4}{3}\cdot\pi\cdot r^3, r\right)$ $r=4.9628$

ClassPad

$\text{solve}\left(512=\frac{4}{3}\cdot\pi\cdot r^3, r\right)$

$\{r=4.962803927\}$

EXERCISE 2.06 Practical applications of equation solving

Concepts and techniques

Working with formulas

1 Example 13 The formula $A = lw$ can be used to calculate the area, A, of a rectangle with length l and width w. Use this formula to calculate the value of:

- a the length l if w is 8 cm and A is 56 cm^2.
- b the length l if w is 12 cm and A is 132 cm^2.
- c w if the length l is 7 m and A is 63 m^2.
- d w if the length l is 9 m and A is 72 m^2.

2 Find the base length, b, of a triangle with area 90 cm^2 and perpendicular height 15 cm, given the area formula $A = \frac{1}{2}bh$.

3 The average speed of a moving object in metres per second is $s = \frac{d}{t}$, where d is the distance travelled in metres and t is the time taken in seconds. Find the distance travelled by a car in 20 seconds if its speed is 15 m/s.

4 The mean, M, of three numbers x, y and z is calculated using the formula $M = \frac{x+y+z}{3}$. If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

5 The circumference of a circle with radius r is $C = 2\pi r$. If a circle has a circumference of 50.27 cm, find its radius to the nearest centimetre.

6 A kettle is boiled and the temperature, T°C, of the water after t minutes is given by the formula $T = 18t + 28$. After how many minutes is the temperature:

- a 64°C?
- b 92.8°C?

Shutterstock.com/Yarik

7 Example 14 If $A = P(1+\frac{r}{100})^n$ find the value of each of the unknowns, giving your answer correct to two decimal places.

- a Find P if $A = 15\ 000$, $r = 6$ and $n = 8$.
- b Find r if $A = 12\ 350$, $P = 7500$ and $n = 5$.

8 The body mass index (BMI) of an adult is given by $B = \frac{m}{h^2}$ where m is the mass of the adult in kilograms and h is the height of the adult in metres. If Megan is 1.64 m tall and has a BMI of 23.1, determine her mass correct to the nearest 100 grams.

Reasoning and communication

9 Example 15 The distance, d m, a ball travels when thrown with a velocity of v m/s is given by $d = \frac{v^2}{9.8}$. At what velocity was a ball thrown if it travelled a distance of 25.8 m? Answer correct to one decimal place.

10 The volume of a cone with base radius r and perpendicular height h is $V = \frac{1}{3}\pi r^2 h$. If a cone has volume 256 cm^3 and radius 7 cm, find its height correct to two decimal places.

11 The formula for finding the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Find the radius of a sphere which has a volume of 6812 cm^3. Give your answer correct to two decimal places.

12 The formula for finding the area of a trapezium with parallel sides *a* and *b* and distance between them *h*, is: $A = \frac{1}{2}(a+b)h$. If the area of a trapezium is 24 cm^2 and its height is 8cm, then the value of $a + b$ is:

A 12 B 8 C 6 D 4 E 3

13 Another trapezium has an area of 66 cm^2 and a height of 12 cm. The value of *b* when $a = 7$ cm is:

A 11 B 4 C 6 D 5 E 24

14 A sphere with radius *r* cm has total surface area, *A* cm^2, given by $A = 4\pi r^2$ and volume, *V* cm^3, given by $V = \frac{4}{3}\pi r^3$.

- a For a sphere of radius 8 cm determine, correct to one decimal place, the:
  - i total surface area
  - ii volume.
- b A sphere has a total surface area of 50.3 m^2. Determine the radius of the sphere to the nearest 0.1 m.
- c A sphere has a volume of 14 137 mm^3. Determine the radius of the sphere to the nearest mm.

CHAPTER SUMMARY
ALGEBRA AND EQUATIONS

2

- To simplify **algebraic expressions** you can only add and subtract **like terms**.
- Multiplication signs can be omitted and expressions may be multiplied and divided together whether the terms are like or not. With division the answer is written as a fraction.
- To **expand** an algebraic expression involving brackets, use the rule: $a(b + c) = ab + ac$.
- To **substitute** into an algebraic expression or a formula, replace each pronumeral by a given numerical value and simplify.
- An **equation** is a mathematical sentence with an equal sign. To **solve** an equation you find the value of the pronumeral which makes the equation true. You use **inverse operations** to solve an equation.
- To find the value of a pronumeral in a **formula** which is not the subject of the formula, substitute in the known values and solve the resulting equation.
- To solve an equation with pronumerals on both sides, collect the pronumerals on one side of the equation then simplify as before to calculate the value of the pronumeral.

2

CHAPTER REVIEW

ALGEBRA AND EQUATIONS

Multiple choice

1 **Example 4** An expression which simplifies to $\frac{8m}{3n}$ is:

A $\frac{24m}{12n}$ B $\frac{16mn}{6nm}$ C $\frac{32m^2}{12n^2}$ D $\frac{40mn}{15n^2}$ E $\frac{24mn}{12nm}$

2 **Example 6** The coefficient of x in the expansion of $4(2x - 1) - (x + 2)$ is:

A -3 B 3 C 7 D 8 E 11

3 **Example 7** If $x = 3y - yz$ then the value of x when $y = 8$ and $z = -2$ is:

A -32 B -8 C 8 D 14 E 40

4 **Example 11** The solution to $4(23 - 5x) = 12$ is:

A $x = -4$ B $x = -3$ C $x = 3$ D $x = 4$ E $x = 5$

5 **Example 11** If $3(2a + 3) = 2 - a$ then a is equal to:

A -2 B $-\frac{3}{2}$ C -1 D 1 E $\frac{3}{2}$

Short answer

6 **Example 1** For the expression $3p^2 - 5q + 2r - 8$ state:

a the number of terms. b the coefficient of q. c the constant term.

7 **Example 2** Simplify each of the following:

a $6x + 2y - 3x + 4y$ b $4pqr - 5pq + 2rqp - 8qp$

8 **Example 4** Simplify:

a $\frac{12mn}{3mp}$ b $\frac{-5a^2bc}{10abc^2}$

9 **Example 5** Expand and simplify:

a $5(x + 4)$ b $-3(2x - 1)$ c $4m - 2(3m + 4)$

10 **Example 7** If $a = -3$, $b = 5$ and $c = 12$, find the value of:

a $-3ab$ b $\frac{4c}{2a}$ c $4c^2 - bc$

11 **Example 8** The formula $C = \frac{5}{9}(F - 32)$ is used to convert temperatures from degrees Fahrenheit to degrees Celsius. Convert the following to °C. Round to the nearest whole number.

a 22°F b 65° c 100°F

12 **Example 9** Solve:

a $2x - 5 = 13$ b $7 - 2m = -3$

13 **Example 10** Solve:

a $\frac{3a + 4}{2} = 8$ b $4 - \frac{2x}{3} = 5$

14 **Example 11** Solve:

a $3x - 4 = 2x + 18$

b $\frac{5(1-y)}{2} = y + 4$

15 **Example 12** Solve:

a $x^3 - 9 = 1322$

b $\sqrt{4p + 11.2} = 6$

16 **Example 13** The formula to calculate the amount of interest earned, I, when an amount of money, P, is invested at $r\%$ p.a. for n years, is $I = \frac{Prn}{100}$.

Calculate the amount of money that was invested at 7% p.a. for 3 years if it earned $650 interest. Round to the nearest hundred dollars.

17 **Example 14** If $V = \pi r^2 h$ is the formula for the volume, V, of a cylinder of radius r and height h, find its radius if $V = 212$ cm^3 and $h = 8$ cm. Give your answer correct to one decimal place.

Application

18 A rectangular field is x metres wide and $x + 130$ metres long. The perimeter of the field is 520 metres.

a Find an expression for the perimeter of the field.

b Find the width of the field.

c Determine the length of the field.

19 A cylinder with radius r cm and height h cm has total surface area, A cm^2, given by $A = 2\pi r^2 + 2\pi rh$ and volume, V cm^3, given by $V = \pi r^2 h$.

Practice quiz

a For a cylinder of radius 5 cm and height 12 cm determine, correct to one decimal place, the

i total surface area ii volume

b A cylinder of radius 8 cm has a total surface area of 905 cm^2. Determine the height of the cylinder correct to the nearest centimetre.

c The surface area of a cylinder of radius 10 cm is equal in value to its volume. What is the height of this cylinder? Answer correct to one decimal place.

1

MIXED REVISION
CHAPTERS 1 • 2 • 3

Multiple choice

1 Which of the following overall percentage decreases is equivalent to a percentage decrease of 5% followed by another decrease of 3%?

A 7.85%
B 8%
C 15%
D 92%
E 92.15%

2 An expression which simplifies to $5x - 4y$ is:

A $2x + 3y - 3x + y$
B $3x - y + 2x - 3y$
C $6x - y - x - 5y$
D $4x + 2y - 6y - x$
E $3x + y - 2x - 5y$

3 Adding $\begin{bmatrix} 13 & 17 \\ 21 & 19 \\ 6 & 11 \end{bmatrix}$ to $\begin{bmatrix} 15 & 27 \\ 14 & 18 \\ 29 & 33 \end{bmatrix}$ gives a sum of:

A $\begin{bmatrix} 28 & 54 \\ 35 & 37 \\ 35 & 44 \end{bmatrix}$
B $\begin{bmatrix} 28 & 44 \\ 35 & 37 \\ 35 & 44 \end{bmatrix}$
C $\begin{bmatrix} 30 & 42 \\ 40 & 32 \\ 17 & 62 \end{bmatrix}$
D $\begin{bmatrix} 54 & 28 \\ 37 & 35 \\ 44 & 37 \end{bmatrix}$
E $\begin{bmatrix} 2 & 10 \\ 7 & 1 \\ 23 & 22 \end{bmatrix}$

4 Nick's salary rose to \$89 950 when increased with inflation. If inflation was 3.9%, what was Nick's salary before the increase? Answer correct to the nearest dollar.

A \$86 442
B \$86 574
C \$89 946
D \$89 954
E \$93 458

5 The solution to the equation $\dfrac{2x - 3}{3} = 7$ is:

A $x = 3.5$
B $x = 9$
C $x = 12$
D $x = 15$
E $x = 24$

6 Find the correct matrix product **AB** if $\mathbf{A} = \begin{bmatrix} 5 & 4 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 7 & 6 \\ 8 & 5 \\ 9 & 1 \end{bmatrix}$.

A $\begin{bmatrix} 42 & 29 \\ 37 & 25 \end{bmatrix}$
B $\begin{bmatrix} 35 & 18 \\ 32 & 40 \\ 81 & 2 \end{bmatrix}$
C $\begin{bmatrix} 148 & 103 \\ 59 & 60 \end{bmatrix}$
D $\begin{bmatrix} 148 & 59 \\ 103 & 60 \end{bmatrix}$
E $\begin{bmatrix} 35 & 24 \\ 24 & 40 \\ 81 & 2 \end{bmatrix}$

7 Sam can run 100 metres in 13.4 seconds. Convert this to a speed in km/h.

A 0.5 km/h
B 2.1 km/h
C 7.5 km/h
D 26.9 km/h
E 269.7 km/h

8 If $5 - x = 3x + 7$, then $x =$

A $-\frac{1}{2}$ B -2 C 1 D $\frac{1}{2}$ E -1

9 Evaluate $5\mathbf{R} - 3\mathbf{T}$ if $\mathbf{R} = \begin{bmatrix} 7 & 3 \\ 6 & 8 \end{bmatrix}$ and $\mathbf{T} = \begin{bmatrix} 10 & 6 \\ 7 & 9 \end{bmatrix}$.

A $\begin{bmatrix} -3 & -3 \\ -1 & -1 \end{bmatrix}$ B $\begin{bmatrix} 35 & 30 & 5 \\ 30 & 21 & 9 \\ 15 & 18 & -3 \\ 40 & 27 & 13 \end{bmatrix}$ C $\begin{bmatrix} 5 & -3 \\ 9 & 13 \end{bmatrix}$

D $\begin{bmatrix} -29 & -21 \\ -17 & -21 \end{bmatrix}$ E $\begin{bmatrix} -1 & -3 \\ -3 & -1 \end{bmatrix}$

Short answer questions

1 Sammy the cat takes 0.4 g of vitamins in 200 mL of milk to keep his fur shiny.

- a What is this rate in mg/mL?
- b How many mg of vitamins are in 120 mL of milk?

2 Expand and simplify: $a(2a - 1) - 4(3a + 2)$.

3 Given that $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 11 & 8 \\ 7 & 6 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 9 & 12 \\ 10 & 5 & 6 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 3 & 8 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 1 & 7 \\ 3 & 8 \\ 9 & 2 \end{bmatrix}$ answer the following questions:

- a Which pair of matrices can be added?
- b Does the matrix product **AC** exist? Explain.
- c List all of the possible matrix products formed by pairs from the given matrices.
- d State the order of each matrix product that you listed for part **c**.
- e Which matrix can be raised to a power? Why?

4 Gemima was buying chips for her children's lunches. She found 175 g packets for \$2.49 or multipacks, with fifteen 20 g packets, for \$5.68. The multipacks were on sale for 15% off.

- a Which was the best buy?
- b What other factors could influence her decision?

5 Find the value, correct to one decimal place, of A if $A = \pi r^2 + 2\pi rh$, $r = 5$ and $h = 12$.

6 Evaluate:

a $\begin{bmatrix} 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$ b $\begin{bmatrix} 23 \\ 31 \\ 27 \end{bmatrix} - \begin{bmatrix} 18 \\ 40 \\ 26 \end{bmatrix}$

c $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 11 & 5 & 15 \\ 6 & 7 & 12 \end{bmatrix}$ d $\begin{bmatrix} 7 & 6 \\ 12 & 5 \end{bmatrix}^2$

Application questions

1 A musical instrument store bought a saxophone for \$350. They marked it up by 45% and then added the 10% GST.

- a What was the selling price?
- b Sonya bought the saxophone at a 15% off everything sale. How much did she pay?

c Calculate the GST included in the discounted price.

d What was the percentage profit, correct to one decimal place, made on the sale of the saxophone?

2 Sahara imports jewellery from Thailand. She buys a shipment of 300 necklaces for A\$3000. The exchange rate with the Australian dollar was 36.42 baht.

a How much did this equate to in baht?

b The shipping costs were 5463 baht. How much is this equivalent to in Australian dollars?

c Calculate the total cost of importing the necklaces in Australian dollars.

d Sahara has a 220% mark-up for her profit margin. How much will she need to sell each necklace for?

3 The surface area of a sphere can be found using the formula $SA = 4\pi r^2$, where r is the radius of the sphere. Calculate the following, correct to one decimal place.

a The surface area of a sphere with a radius of 5 cm.

b The surface area of a sphere with a diameter of 2 m.

c If the surface area of a sphere is 632.5 mm^2, calculate the radius of the sphere.

4 When hiring a function room there is a set fee of \$250 plus \$5 per person attending.

a Write an equation representing the cost (C) of hiring the function room for n people.

b What will it cost for a function which has 100 people attending?

c If the cost of a function was \$1850, how many people attended the function?

d The owners of the function room decide to add a cleaning fee of \$50. What would be the new equation representing the cost (C) of hiring the function room?

5 Helen, Katie, Grant and Robert play a dice game every night to determine who has to wash the dishes. They take it in turns rolling a pair of dice. Upon a roll of the dice, a player's score is determined by adding the sum of both dice and then bonus points if eligible.

Bonus points are awarded for:

- a double: + 5 bonus points
- the sum of the 2 dice being greater than 8: + 4 bonus points.

So if Katie rolls a pair of 3s, her score will be $(3 + 3) + 5 = 11$.

Each player gets 5 turns.

The results from last night are shown below:

	Sum of dice for all 5 rolls	Doubles	Number of rolls where sum > 8
Helen	29	1	1
Katie	30	0	1
Grant	23	1	2
Robert	27	2	0

a Display this score data in a 4 × 3 matrix called **S**.

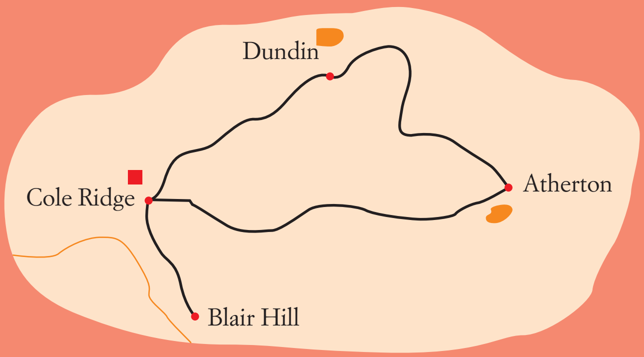
b Grant set up a column matrix $\mathbf{P} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$ which can be multiplied by **S** to find each player's overall score.

i Why is the first element a 1?

ii Find the matrix product **SP**.

iii The player with the lowest score for the game has to wash up. Who had to wash up last night?

6 The map below shows the roads between four properties.



We can see that we can travel directly:

- from Atherton to Dundin or Cole Ridge
- from Blair Hill to Cole Ridge
- from Cole Ridge to Atherton, Blair Hill or Dundin
- from Dundin to Cole Ridge or Atherton.

a Copy and complete the table below showing the number of direct one-step paths from each property to the others.

	Atherton	**Blair Hill**	**Cole Ridge**	**Dundin**
Atherton				
Blair Hill				
Cole Ridge				
Dundin				

b Display the data from the table in a matrix called **M**.

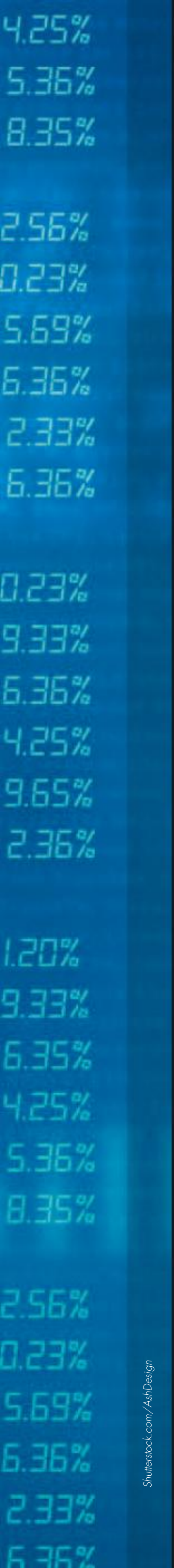
c Find $\mathbf{M}^2$.

d How many 2 step paths are there from:

- i Atherton to Blair Hill?
- ii Blair Hill to Dundin?
- iii Cole Ridge to Blair Hill?

e Without evaluating $\mathbf{M}^4$ write all the 4 step paths that you can find to travel from Blair Hill to Atherton. (Use the map to work this out).


248.36 4.58 65.36 95.36 ▼ 61.4
896.33 3.54 32.23 5.33 ▲ 3.35
896.33 7.63 44.45 2.55 ▼ 6.35
323.24 2.33 42.36 56.35 ▼ 234.6
236.58 6.35 78.96 24.36 ▼ 96.3
596.33 7.98 33.33 72.65 ▲ 25.3
313.63 4.58 65.36 54.36 ▼ 22.3
896.33 3.54 32.23 5.33 ▲ 27.6
245.45 2.33 56.56 7.87 ▼ 45.3
236.58 6.35 78.96 24.36 ▲ 96.3
256.36 2.78 56.39 74.36 ▼ 245.3
313.63 4.58 65.36 54.36 ▼ 22.3
248.36 4.58 65.36 95.36 ▼ 61.4
989.24 6.36 44.55 89.33 ▲ 24.3
545.32 5.36 76.76 7.35 ▼ 88.9
282.80 2.17 83.68 80.25 ▲ 132.10
256.36 2.78 56.39 74.36 ▼ 245.3
375.69 9.56 24.35 45.23 ▼ 82.5
248.36 4.58 65.36 95.36 ▼ 61.4
896.33 3.54 32.23 5.33 ▲ 3.35
896.33 7.63 44.45 2.55 ▼ 6.35
323.24 2.33 42.36 56.35 ▼ 234.6
236.58 6.35 78.96 24.36 ▼ 96.3
596.33 7.98 33.33 72.65 ▲ 25.3
313.63 4.58 65.36 54.36 ▼ 22.3
896.33 3.54 32.23 5.33 ▲ 27.6



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3

MATRICES: MATRIX ARITHMETIC

MATRIX ARITHMETIC

TERMINOLOGY

array
column
column matrix
element
identity matrix
leading diagonal
matrices
matrix
order
row
row matrix
scalar
square matrix
zero matrix

Prior learning

MATRICES AND MATRIX ARITHMETIC

- **use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks (ACMGM013)**
- **recognise different types of matrices (row, column, square, zero, identity) and determine their size (ACMGM014)**
- **perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate (ACMGM015)**
- **use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person. (ACMGM016)**

3.01 TYPES OF MATRICES

A matrix is a rectangular **array** containing numbers which are organised into rows and columns. They are presented in either square brackets or parentheses like so:

$$\begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 4 \end{bmatrix} \text{ or } \begin{pmatrix} 3 & 2 & 4 \\ 1 & -2 & 4 \end{pmatrix}$$

Consider the table below showing how senior students get to their school each day. The data from the table can be presented in the matrix beside it, as shown.

Senior Students' methods of travel to school

	Year 11	**Year 12**
Walk	32	18
Bus	78	38
Car (other driver)	27	12
Car (self-driven)	9	31

$$\begin{bmatrix} 32 & 18 \\ 78 & 38 \\ 27 & 12 \\ 9 & 31 \end{bmatrix}$$

It is customary to name matrices using capital letters. Let's call the matrix representing the Senior Students' methods of travel to school matrix **A**.

Each value in a matrix is called an **element**. Matrix **A** has 8 elements.

In a matrix there are horizontal **rows** and vertical **columns**.

There are four rows in matrix **A**.

The first row in matrix **A** contains the elements 32 and 18.

$$\begin{array}{ll} \text{Row 1} \longrightarrow & \multirow{4}{*}{} \\ \end{array}\begin{array}{l} \text{Row 1} \longrightarrow \\ \text{Row 2} \longrightarrow \\ \text{Row 3} \longrightarrow \\ \text{Row 4} \longrightarrow \end{array} \begin{bmatrix} 32 & 18 \\ 78 & 38 \\ 27 & 12 \\ 9 & 31 \end{bmatrix}$$

There are two columns in matrix **A**. The second column contains the elements 18, 38, 12 and 31.

$$\begin{array}{cc} \text{Column 1} \downarrow & \text{Column 2} \downarrow \\ \left[\begin{matrix} 32 \\ 78 \\ 27 \\ 9 \end{matrix}\right. & \left.\begin{matrix} 18 \\ 38 \\ 12 \\ 31 \end{matrix}\right] \end{array}$$

The **order** of a matrix tells us how many rows and columns it has. The order is expressed as $m \times n$ where m is the number of rows and n is the number of columns.

Example 1

Only one of the following arrays is a matrix. State, with reasons, whether **C** or **D** is the array which is a matrix.

$$\mathbf{C} = \begin{bmatrix} 9 & 2 & 0.5 & 13 \\ 1 & 12 & 3 & 142 \\ -6 & 15 & 0 & -3.5 \end{bmatrix} \qquad \mathbf{D} = \left[\begin{array}{ll} -5 & 30 \\ \ \ 6 & \\ 0.7 & \end{array}\right]$$

Solution

In a matrix, the elements are arranged in rows and columns. Each row has the same number of elements and each column contains the same number of elements.

C is a matrix since each row has the same number of elements and each column has the same number of elements. **D** is not a matrix as one row contains two elements and the other two rows each contain one element. Furthermore, the first column contains three elements and the second column only contains one element.

Example 2

If matrix **B** has 3 rows and 2 columns, how many elements does it contain?

Solution

A matrix with 3 rows and 2 columns will look like this:

$$\begin{bmatrix} * & * \\ * & * \\ * & * \end{bmatrix}$$ where * represents a real number.

Matrix **B** has $3 \times 2 = 6$ elements.

Example 3

State the order of matrix **A**.

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 5 & -9 \\ 1 & 6 & 3 & 1 \\ -3 & 11 & 0 & -3 \end{bmatrix}$$

Solution

Order = number of rows × number of columns

Matrix **A** has 3 rows going across and 4 columns going down.

The order of **A** is 3×4.

A **row matrix** consists of just one row. For example:

$\begin{bmatrix} -2 & 3 & 11 & 5 \end{bmatrix}$ (This is a 1×4 matrix.)

A **column matrix** has just one column. For example:

$\begin{bmatrix} 12 \\ -1 \\ 0 \\ 5 \end{bmatrix}$ (This is a 4×1 matrix.)

A **square matrix** has the same number of rows as it has columns. For example:

$\begin{bmatrix} 0 & 9 \\ 5 & 3 \end{bmatrix}$ (2×2 matrix) or $\begin{bmatrix} 2 & 2 & 6 \\ 7 & 1 & 0 \\ 1 & 0 & 10 \end{bmatrix}$ (3×3 matrix).

A matrix in which all the elements are equal to zero is called a **zero matrix**. A zero matrix can be a row, column, square or rectangular matrix. For example:

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

An **identity matrix** is a square matrix where all the elements in the **leading diagonal** are equal to one and all other elements are zero. The leading diagonal is the diagonal that starts from the top-left element and finishes at the bottom-right element. Here are two examples of identity matrices:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The identity matrix is named using a capital I. The 2×2 identity matrix can be denoted by $\mathbf{I}_2$, the 3×3 identity matrix can be denoted by $\mathbf{I}_3$, and so on.

Example 4

What special type of matrix is **D**?

$$\mathbf{D} = \begin{bmatrix} 3 & 12 & 8 \\ 2 & -5 & 3 \\ 0 & 1 & 9 \end{bmatrix}$$

Solution

D is a 3 × 3 matrix. It has the same number of rows and columns.

D is a square matrix.

IMPORTANT

An $m \times n$ matrix has m rows and n columns.

A matrix with m rows and n columns has $m \times n$ elements.

All elements in a matrix are real numbers.

Rows are horizontal.

Columns are vertical.

A row matrix has only 1 row.

A column matrix has only 1 column.

A square matrix has the same number of rows as it has columns.

An identity matrix is a square matrix with ones on the leading diagonal and zeroes elsewhere.

EXERCISE 3.01 Types of matrices

1 Example 1 Which of the following arrays are matrices?

a $\begin{bmatrix} 4 & 0 & 3 \end{bmatrix}$

b $\begin{bmatrix} 2 & -5 \\ 1 & 0 \\ 3 & -4 \end{bmatrix}$

c $\begin{bmatrix} 1 & 0 & \\ 0 & 1 & 0 \end{bmatrix}$

d $\begin{bmatrix} 2 & 7 \\ 3 & \\ 4 & \end{bmatrix}$

e $\begin{bmatrix} 3 \\ 9 \\ 4 \end{bmatrix}$

f $\begin{bmatrix} -2 & 3.3 & \frac{1}{4} \\ 0 & -0.1 & 8 \\ 2.6 & -11 & 5 \end{bmatrix}$

2 Decide whether the following statements are true or false.

a A matrix of order 2 × 6 has 12 elements.

b A matrix of order 5 × 4 has 5 more elements than a matrix of order 3 × 5.

c The leading diagonal of a 5 × 5 matrix has 10 elements in it.

d The last column of a 6 × 4 matrix contains 4 elements.

e The first column in a 7 × 9 matrix would contain 7 elements.

3 Example 2

a Matrix **A** has 2 rows and 5 columns. How many elements does it contain?

b How many elements would there be in a matrix of order 3 × 3 ?

c A row matrix is known to contain 10 elements. How many columns are in the matrix?

4 **Example 3** State the order of the following matrices.

a $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

b $\begin{bmatrix} 5 & -2 \\ -3 & 6 \end{bmatrix}$

c $\begin{bmatrix} -3 \\ 6 \\ 8 \end{bmatrix}$

d $\begin{bmatrix} 0 & 1 & 1 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$

e $\begin{bmatrix} 2.3 & 3 \\ 5 & 5.1 \\ 0 & 1 \\ 6 & 12 \end{bmatrix}$

f $\begin{bmatrix} 5 & -3 & 10 \\ -6 & 2 & 2 \\ 0 & 1 & 0 \\ -4 & 15 & 38 \\ -47 & 10 & 1 \end{bmatrix}$

g $\begin{bmatrix} 41 & 56 \end{bmatrix}$

h $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

i $\begin{bmatrix} 10 & 13 & 20 & 11 & 27 & 18 \\ 15 & 18 & 32 & 19 & 7 & 32 \\ 16 & 12 & 5 & 44 & 14 & 13 \end{bmatrix}$

j $\begin{bmatrix} 42 & 21 & 67 \\ 23 & 56 & 89 \\ 27 & 34 & 29 \\ 31 & 48 & 87 \end{bmatrix}$

5 Which one of the following is a row matrix?

A $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

B $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

C $\begin{bmatrix} 3 & 5 \end{bmatrix}$

D $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

E $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

6 **Example 4** Each of the matrices below can be classified as one or more of the types listed in the box. State which type each matrix is.

row	column	square	identity	zero

a $\begin{bmatrix} 4 & 7 & 9 \\ 8 & 5 & 9 \\ 2 & 7 & 4 \end{bmatrix}$

b $\begin{bmatrix} 3 & -9 \\ -2 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 8 \\ 5 \end{bmatrix}$

e $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f $\begin{bmatrix} 17 & 22 & 14 & 21 & 15 & 18 \end{bmatrix}$

g $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

h $[0\ 0\ 0\ 0]$

7 If an identity matrix is named I_5:

a how many rows does it have?

b how many columns does it have?

c how is it represented in matrix form?

8 The order of the matrix $\begin{bmatrix} 3 & 5 & 0 \\ -2 & 1 & 0 \end{bmatrix}$ is:

A 2×2

B 2×3

C 3×2

D 2×1

E 3×1

Reasoning and communication

9 Matrix **X** has 24 elements. The order of **X** could be 2×12. What other orders are possible for this matrix?

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10 Matrix **K** has 52 elements. List the six possible orders of matrix **K**.

11 Matrix **W** has a square number of elements less than 50. It is not a square matrix though. Instead it is a rectangular matrix with 5 less rows than columns. State the order of matrix **W**.

3.02 CONSTRUCTING MATRICES

The elements in matrices usually represent real data. The position of each element is important. We can indicate the position of an element by referring to which row and column it is situated in.

IMPORTANT

a_{ij} is an element of matrix **A**

where i is the row number,
j is the column number.

For example, in the matrix $\mathbf{A} = \begin{bmatrix} 5 & 2 \\ 8 & 9 \\ 3 & 7 \end{bmatrix}$:

a_{11} is the element in row 1 and column 1 $(a_{11} = 5)$

a_{11} is read as a 1, 1 not a 11.

a_{12} is the element in row 1 and column 2 $(a_{12} = 2)$

$a_{21} = 8$

$a_{22} = 9$

$a_{31} = 3$

$a_{32} = 7$

Example 5

Given matrix $\mathbf{B} = \begin{bmatrix} -1 & 4 & 16 & -10 & 0 \\ 8 & -2 & 3 & 38 & 9 \end{bmatrix}$, find the value of the following.

a b_{24} **b** b_{15} **c** $b_{22} \times b_{14}$ **d** $b_{11} - b_{12}$

Solution

a b_{24} is the element in the 2nd row and 4th column.

$b_{24} = 38$

b b_{15} is the element in the 1st row and 5th column.

$b_{15} = 0$

c Element b_{22} is the element in the 2nd row and 2nd column, $b_{22} = -2$. Element b_{14} is in the 1st row and the 4th column, $b_{14} = -10$.
Multiply these values.

$$\begin{aligned} b_{22} \times b_{14} &= -2 \times -10 \\ &= 20 \end{aligned}$$

d Element b_{11} is the element that is in the 1st row and 1st column, $b_{11} = -1$ and element b_{12} from the element that is in the 1st row and 2nd column, $b_{12} = 4$.
Subtract b_{12} from b_{11}.

$$\begin{aligned} b_{11} - b_{12} &= -1 - 4 \\ &= -5 \end{aligned}$$

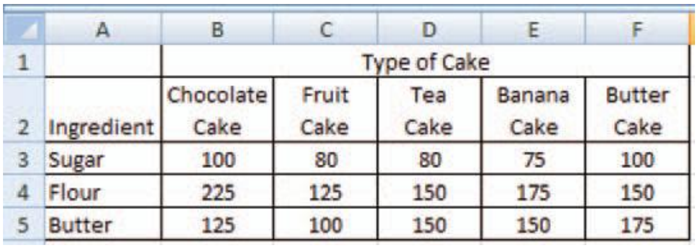
Matrices can be used to represent data displayed in a table.

Example 6

The spreadsheet below shows the quantity (in grams) of the main ingredients used to bake various cakes.

Display this information in a suitable matrix.


Shutterstock.com/ElenaGaak



	A	B	C	D	E	F
1		Type of Cake				
2	Ingredient	Chocolate Cake	Fruit Cake	Tea Cake	Banana Cake	Butter Cake
3	Sugar	100	80	80	75	100
4	Flour	225	125	150	175	150
5	Butter	125	100	150	150	175

Solution

Ignore the headings in the spreadsheet.

The quantities are arranged into 3 rows and 5 columns. So we can display it in a 3 × 5 matrix.

This leaves the ingredients in rows and the cake types in columns.

$$\begin{bmatrix} 100 & 80 & 80 & 75 & 100 \\ 225 & 125 & 150 & 175 & 150 \\ 125 & 100 & 150 & 150 & 175 \end{bmatrix}$$

Alternatively, we could use a 5 × 3 matrix.

But we must keep the data organised into the correct categories.

This alternative formation now has the ingredients in columns and the cake types in rows.

$$\begin{bmatrix} 100 & 225 & 125 \\ 80 & 125 & 100 \\ 80 & 150 & 150 \\ 75 & 175 & 150 \\ 100 & 150 & 175 \end{bmatrix}$$

EXERCISE 3.02 Constructing matrices

Concepts and techniques

1 Example 5 If $\mathbf{K} = \begin{bmatrix} 9 & 12 & 0 \\ 3 & 15 & 2 \end{bmatrix}$ write down the value of:

a k_{12}
b k_{21}
c k_{23}
d k_{31}
e $k_{13} + k_{11}$
f $k_{21} \times k_{12}$

2 Given that $B = \begin{bmatrix} -\frac{2}{5} & 5 & \frac{4}{5} & 17 \\ 0 & 12 & 3 & 2 \\ -3 & 3 & -5 & 0 \end{bmatrix}$ use the notation b_{ij} to show the position of the following elements.

a 2
b $\frac{4}{5}$
c 17
d -3
e $-\frac{2}{5}$
f -5

3 Using the matrices $\mathbf{P} = \begin{bmatrix} 2 & -5 & 22 \\ 24 & 3 & -1 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 8 & -2 & 3 \\ -2.2 & 5 & 11 \\ 16 & 5 & -7 \end{bmatrix}$ and $\mathbf{R} = \begin{bmatrix} 2.1 & 15 \\ -3 & 1.1 \\ 7 & 12 \\ 13 & -5 \\ 21 & 5.1 \end{bmatrix}$ state the value of each of the following.

a $p_{13} + r_{12}$
b $p_{22} \div r_{21}$
c $q_{33} - r_{52}$
d $r_{31} \times p_{21}$
e $p_{13} + q_{31} - r_{12}$
f $\dfrac{p_{11} - r_{42}}{q_{22}}$

4 The following statements relate to matrices **P**, **Q** and **R** from question **3**. Decide whether each statement is true or false.

a $r_{41} = 21$
b $p_{23} \times 7 = q_{33}$
c $r_{31} > q_{31}$
d $q_{23} \div 10 = r_{22}$
e $q_{11} - p_{12} = r_{32}$
f $r_{51} < p_{13} + q_{21}$

5 Example 6 The table below shows the marks achieved by two students, Mike and Kim, in their end of year examinations for Mathematics, English, Science and History.

	Mathematics	English	Science	History
Mike	91	87	94	76
Kim	94	92	67	87

Express this information in a:

a 2×4 matrix
b 4×2 matrix.

6 To make a round fruit cake a baker needs 200 g flour, 500 g dried fruit and 80 g sugar. To make a rectangular slab of fruit cake, the baker needs 500 g flour, 850 g dried fruit and 120 g sugar.

If this information is represented in a 2×3 matrix

a what does the information in the rows represent?
b what does the information in the columns represent?
c write down the matrix.

Reasoning and communication

7 In a group of four, construct a matrix to show the age, number of siblings and number of pets for each group member.

8 This table shows some data about Amy and Ben.

	Age	Hair colour	Weight	Height
Amy	15	brown	59 kg	146 cm
Ben	16	red	65 kg	155 cm

Explain any difficulties you might have constructing a matrix for this data.

3.03 ADDITION AND SUBTRACTION OF MATRICES

Addition and subtraction of matrices can only be performed with matrices of the same order. When adding two matrices, the result is a matrix of the same order as those being added.

Addition of matrices involves adding the elements which are in corresponding positions in both matrices.

That is, if we have matrix $\mathbf{A} = \begin{bmatrix} 2 & 7 \\ 11 & 3 \end{bmatrix}$ and matrix $\mathbf{B} = \begin{bmatrix} 1 & 3 \\ 5 & 12 \end{bmatrix}$, then

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 2+1 & 7+3 \\ 11+5 & 3+12 \end{bmatrix} = \begin{bmatrix} 3 & 10 \\ 16 & 15 \end{bmatrix}.$$

Example 7

Given $\mathbf{T} = \begin{bmatrix} 6 \\ -2 \\ 0.1 \end{bmatrix}$ and $\mathbf{W} = \begin{bmatrix} 0.2 \\ 5 \\ 3.1 \end{bmatrix}$ find $\mathbf{T} + \mathbf{W}$.

Solution

Add corresponding elements from **T** and **W**.

$$\mathbf{T} + \mathbf{W} = \begin{bmatrix} 6 + 0.2 \\ -2 + 5 \\ 0.1 + 3.1 \end{bmatrix}$$

$$= \begin{bmatrix} 6.2 \\ 3 \\ 3.2 \end{bmatrix}$$

If we need to subtract matrix **B** from matrix **A**, we simply subtract each element in matrix **B** from its corresponding element in matrix **A**.

If $\mathbf{A} = \begin{bmatrix} 10 & 3 \\ 16 & 5 \\ 2 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 4 & 11 \\ 4 & -3 \end{bmatrix}$, then $\mathbf{A} - \mathbf{B} = \begin{bmatrix} 10-2 & 3-5 \\ 16-4 & 5-11 \\ 2-4 & 4-(-3) \end{bmatrix} = \begin{bmatrix} 8 & -2 \\ 12 & -6 \\ -2 & 7 \end{bmatrix}$.

Example 8

Find $\mathbf{D} - \mathbf{E}$ if $\mathbf{D} = \begin{bmatrix} 5 & 6 \\ 14 & -7 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} -3 & 0 \\ 11 & 5 \end{bmatrix}$.

Solution

Subtract corresponding elements.

$$\mathbf{D} - \mathbf{E} = \begin{bmatrix} 5-(-3) & 6-0 \\ 14-11 & -7-5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 6 \\ 3 & -12 \end{bmatrix}$$

We can write equations, using our knowledge of addition and subtraction operations on matrices to find the value of unknown elements.

Example 9

Find the values of x, y and z below.

$$\begin{bmatrix} 6 & 12 \\ y & 7 \end{bmatrix} - \begin{bmatrix} 6 & -5 \\ 1 & 2z \end{bmatrix} = \begin{bmatrix} 0 & x \\ 4 & -3 \end{bmatrix}$$

Using the corresponding elements in row 1 and column 2 of each matrix, write down an equation. $12 - (-5) = x$

Simplify. $x = 17$

Using the corresponding elements in row 2 and column 1 of each matrix, write down an equation. $y - 1 = 4$

Solve for y. $y = 5$

Using the corresponding elements in row 2 and column 2 of each matrix, write down an equation. $7 - 2z = -3$

Solve for z.

$$2z = 10$$
$$z = 5$$

A CAS calculator can be used to perform operations on matrices.

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Example 10

Simplify

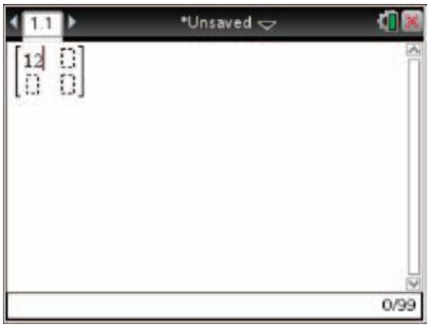
$\begin{bmatrix} 12 & 3 \\ 18 & 5 \end{bmatrix} + \begin{bmatrix} 13 & -9 \\ 6 & 22 \end{bmatrix}$.

TI-Nspire CAS

Use a New Document with a Calculator page.

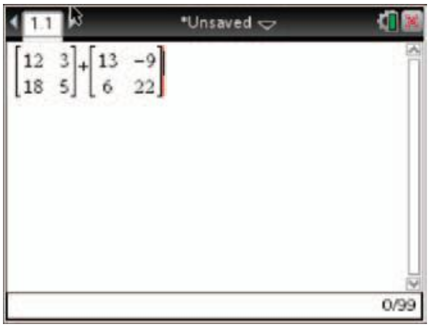
Press ⊞.

Select the 2 × 2 matrix template $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ and press enter.



$\begin{bmatrix} 12 & \square \\ \square & \square \end{bmatrix}$

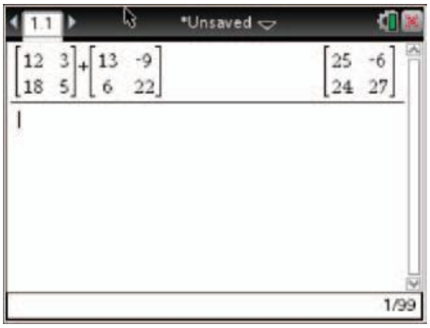
Enter the elements, pressing tab after each entry.



$\begin{bmatrix} 12 & 3 \\ 18 & 5 \end{bmatrix} + \begin{bmatrix} 13 & -9 \\ 6 & 22 \end{bmatrix}$

Press + then create a second 2 × 2 matrix using the method outlined above.

Enter the elements of the second matrix.



$\begin{bmatrix} 12 & 3 \\ 18 & 5 \end{bmatrix} + \begin{bmatrix} 13 & -9 \\ 6 & 22 \end{bmatrix}$ $\begin{bmatrix} 25 & -6 \\ 24 & 27 \end{bmatrix}$

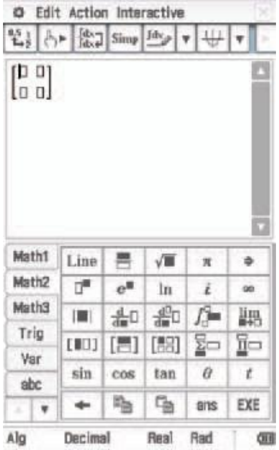
Press enter.

ClassPad

Use the $\overset{\text{Main}}{\sqrt{\alpha}}$ application.

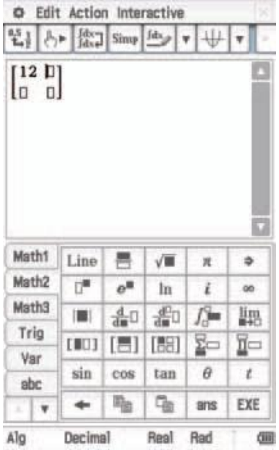
Press [Keyboard].

Tap [Math2] then tap the 2 × 2 matrix template [⊞].



$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

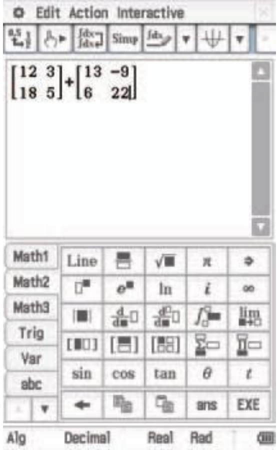
Enter the elements, use the arrow keys [▶] [▼] [◀] to position the cursor in the correct position for each entry.



$\begin{bmatrix} 12 & \square \\ \square & \square \end{bmatrix}$

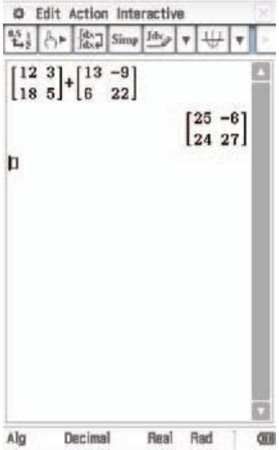
Press [▶] as necessary to move the cursor outside of the first matrix. Press [+] then create a second 2 × 2 matrix using the method outlined above.

Enter the elements of the second matrix.



$\begin{bmatrix} 12 & 3 \\ 18 & 5 \end{bmatrix}+\begin{bmatrix} 13 & -9 \\ 6 & 22 \end{bmatrix}$

Press EXE.



$\begin{bmatrix} 12 & 3 \\ 18 & 5 \end{bmatrix} + \begin{bmatrix} 13 & -9 \\ 6 & 22 \end{bmatrix}$

$\begin{bmatrix} 25 & -6 \\ 24 & 27 \end{bmatrix}$

EXERCISE 3.03 Addition and subtraction of matrices

Concepts and techniques

1 Example 7 Use the matrices shown to perform the following additions.

$$\mathbf{N} = \begin{bmatrix} 16 & 5 \\ 2 & 12 \end{bmatrix} \quad \mathbf{O} = \begin{bmatrix} 10 & 3 \\ 7 & 22 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} 15 & 1 \\ 8 & 34 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 11 & -5 \\ 29 & 36 \end{bmatrix}$$

a $\mathbf{N} + \mathbf{O}$
b $\mathbf{N} + \mathbf{P}$
c $\mathbf{O} + \mathbf{P}$
d $\mathbf{O} + \mathbf{Q}$
e $\mathbf{P} + \mathbf{Q}$
f $\mathbf{N} + \mathbf{Q}$

2 Evaluate $\begin{bmatrix} 0 & 5 & -1.2 \\ 3.6 & -2 & 10 \\ 14 & 5.1 & -6 \end{bmatrix} + \begin{bmatrix} 3.1 & 8 & 6.2 \\ 5 & 12 & -16 \\ 3 & -3.2 & -5 \end{bmatrix}$.

3 Find $\mathbf{X} + \mathbf{Y}$ if $\mathbf{X} = \begin{bmatrix} 121 & 251 & 512 & 108 \\ 305 & 168 & 250 & 366 \\ 299 & 378 & 372 & 451 \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} 450 & 23 & 55 & 216 \\ 207 & 365 & 594 & 333 \\ 45 & 783 & 78 & 96 \end{bmatrix}$.

4 Given $\mathbf{G} = \begin{bmatrix} 2 & 5 \\ 4 & -3 \\ \frac{1}{4} & 7 \end{bmatrix}$ and $\mathbf{H} = \begin{bmatrix} 3 & 8 \\ 0.5 & 6 \\ \frac{1}{4} & 4 \end{bmatrix}$ find $\mathbf{G} + \mathbf{H}$.

5 Example 8 Using matrices **N**, **O**, **P** and **Q** from question **1**, find the following.

a $\mathbf{N} - \mathbf{O}$
b $\mathbf{P} - \mathbf{O}$
c $\mathbf{O} - \mathbf{Q}$
d $\mathbf{Q} - \mathbf{N}$
e $\mathbf{N} - \mathbf{Q}$
f $\mathbf{P} - \mathbf{P}$

6 **Example 9** Find the value of the pronumeral in each of the following.

a $\begin{bmatrix} 21 \\ 35 \\ 16 \\ 15 \end{bmatrix} + \begin{bmatrix} 17 \\ 9 \\ 38 \\ 14 \end{bmatrix} = \begin{bmatrix} 38 \\ m \\ 54 \\ 29 \end{bmatrix}$

b $\begin{bmatrix} 27 & 13 \\ 9 & 44 \end{bmatrix} - \begin{bmatrix} 11 & 14 \\ 28 & 39 \end{bmatrix} = \begin{bmatrix} 16 & -1 \\ -19 & d \end{bmatrix}$

c $\begin{bmatrix} 12 & 3 & 16 \\ 4 & 9 & 27 \end{bmatrix} + \begin{bmatrix} 17 & x & 31 \\ 2 & 13 & 40 \end{bmatrix} = \begin{bmatrix} 29 & 28 & 47 \\ 6 & 22 & 67 \end{bmatrix}$

d $\begin{bmatrix} 31 & 19 \\ 28 & t \end{bmatrix} + \begin{bmatrix} 56 & 47 \\ 37 & 29 \end{bmatrix} = \begin{bmatrix} 87 & 66 \\ 65 & 77 \end{bmatrix}$

e $\begin{bmatrix} 51 & 36 & 48 \end{bmatrix} - \begin{bmatrix} 15 & m & 60 \end{bmatrix} = \begin{bmatrix} 36 & 7 & -12 \end{bmatrix}$

f $\begin{bmatrix} 14 & 3 & 27 \\ 9 & 11 & -5 \\ -12 & 6 & -11 \end{bmatrix} - \begin{bmatrix} -12 & 6 & 13 \\ -10 & 14 & n \\ 21 & 17 & -11 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 14 \\ 19 & -3 & -15 \\ -33 & -11 & 0 \end{bmatrix}$

7 Work out the values of a, b and c in each of the following.

a $\begin{bmatrix} 27 & 18 & a \\ 17 & 22 & 19 \end{bmatrix} + \begin{bmatrix} b & 14 & 9 \\ 18 & 13 & c \end{bmatrix} = \begin{bmatrix} 30 & 32 & 36 \\ 35 & 35 & 37 \end{bmatrix}$

b $\begin{bmatrix} a & 13 \\ 9 & 40 \end{bmatrix} - \begin{bmatrix} 11 & 15 \\ c & 39 \end{bmatrix} = \begin{bmatrix} 16 & b \\ -19 & 1 \end{bmatrix}$

c $\begin{bmatrix} 29 & 17 \\ a & 65 \end{bmatrix} + \begin{bmatrix} b & -71 \\ -27 & 3c \end{bmatrix} = \begin{bmatrix} 87 & -54 \\ 66 & 74 \end{bmatrix}$

d $\begin{bmatrix} 12 & 32 & 25 \\ 9 & 17 & -6 \\ -14 & 2b & -19 \end{bmatrix} - \begin{bmatrix} -21 & 7 & 15 \\ -12 & 18 & 6a \\ 37 & -11 & -35 \end{bmatrix} = \begin{bmatrix} 33 & 25 & 10 \\ 21 & -1 & 24 \\ 3c & 17 & 16 \end{bmatrix}$

8 **Example 10**

a Find $\begin{bmatrix} 8 & 5.2 & 17 \\ 5.8 & 12 & 16 \\ 18 & 9.2 & -11 \end{bmatrix} - \begin{bmatrix} 0 & 3.1 & 9 \\ 5 & 12 & -16 \\ 3 & -3.2 & -5 \end{bmatrix}$.

b Find $\mathbf{R} - \mathbf{Z}$ if $\mathbf{R} = \begin{bmatrix} 220 & 302 & 501 & 662 \\ 723 & 278 & 98 & 519 \\ 561 & 789 & 274 & 368 \end{bmatrix}$ and $\mathbf{Z} = \begin{bmatrix} 250 & 45 & 44 & 599 \\ 207 & 223 & 628 & 189 \\ 45 & 783 & 98 & 87 \end{bmatrix}$.

Reasoning and communication

9 Explain why you can't add matrices of different order.

10 Two car sales people, Henry (H) and Lucy (L) sell sedans (s), station wagons (w) and four wheel drives (f). Their sales for two months, May and June, are shown in the matrices **M** and **J** below.

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$$\mathbf{M} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} & s & w & f \\ H & 12 & 8 & 5 \\ L & 10 & 9 & 6 \end{array}, \quad \mathbf{J} = \begin{array}{cccc} & s & w & f \\ H & 9 & 6 & 4 \\ L & 8 & 8 & 7 \end{array}$$

Find **M** + **J** and explain what this sum represents.

11 Given three matrices $\mathbf{A} = \begin{bmatrix} 5 & 2 & 6 & 1 \\ 0 & 9 & 3 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 7 & 2 & 9 \\ 1 & 0 & 0 & 8 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 12 & 0 & 5 & 3 \\ 7 & 0 & 1 & 10 \end{bmatrix}$ it can be shown that $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = \begin{bmatrix} 5 & 2 & 6 & 1 \\ 0 & 9 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 15 & 7 & 7 & 12 \\ 8 & 0 & 1 & 18 \end{bmatrix} = \begin{bmatrix} 20 & 9 & 13 & 13 \\ 8 & 9 & 4 & 22 \end{bmatrix}$.

Show that $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$.

3.04 SCALAR MULTIPLICATION

The process of scalar multiplication involves multiplying a whole matrix by a scalar (number).

To multiply matrix **A** by the number 4, we would simply multiply each of the elements in matrix **A** by 4.

If $\mathbf{A} = \begin{bmatrix} 5 & 2 \\ 7 & -3 \\ 12 & 6 \end{bmatrix}$, then $4\mathbf{A} = 4\begin{bmatrix} 5 & 2 \\ 7 & -3 \\ 12 & 6 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 28 & -12 \\ 48 & 24 \end{bmatrix}$.

Example 11

Find 5**B** if $\mathbf{B} = \begin{bmatrix} 6 \\ -5 \\ 14 \end{bmatrix}$.

Multiply each element of **B** by 5.

$$5\mathbf{B} = 5\begin{bmatrix} 6 \\ -5 \\ 14 \end{bmatrix}$$

$$= \begin{bmatrix} 30 \\ -25 \\ 70 \end{bmatrix}$$

Example 12

Given that $\mathbf{Q} = \begin{bmatrix} 3 & -5 & 10 \\ 11 & 6 & 17 \\ -2 & 15 & 0 \end{bmatrix}$ and $\mathbf{R} = \begin{bmatrix} 12 & 2 & 1 \\ -3 & 4 & 7 \\ 22 & 14 & -7 \end{bmatrix}$, find

a $4\mathbf{Q} - 2\mathbf{R}$ **b** $3\mathbf{Q} + 7\mathbf{R}$

Solution

a Multiply each element in **Q** by 4 and each element in **R** by **2**.

$$4\mathbf{Q} - 2\mathbf{R} = 4\begin{bmatrix} 3 & -5 & 10 \\ 11 & 6 & 17 \\ -2 & 15 & 0 \end{bmatrix} - 2\begin{bmatrix} 12 & 2 & 1 \\ -3 & 4 & 7 \\ 22 & 14 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -20 & 40 \\ 44 & 24 & 68 \\ -8 & 60 & 0 \end{bmatrix} - \begin{bmatrix} 24 & 4 & 2 \\ -6 & 8 & 14 \\ 44 & 28 & -14 \end{bmatrix}$$

Then perform the subtraction.

$$= \begin{bmatrix} -12 & -24 & 38 \\ 50 & 16 & 54 \\ -52 & 32 & 14 \end{bmatrix}$$

b Multiply each element in **Q** by 3 and each element in **R** by 7.

$$3\mathbf{Q} + 7\mathbf{R} = 3\begin{bmatrix} 3 & -5 & 10 \\ 11 & 6 & 17 \\ -2 & 15 & 0 \end{bmatrix} + 7\begin{bmatrix} 12 & 2 & 1 \\ -3 & 4 & 7 \\ 22 & 14 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -15 & 30 \\ 33 & 18 & 51 \\ -6 & 45 & 0 \end{bmatrix} + \begin{bmatrix} 84 & 14 & 7 \\ -21 & 28 & 49 \\ 154 & 98 & -49 \end{bmatrix}$$

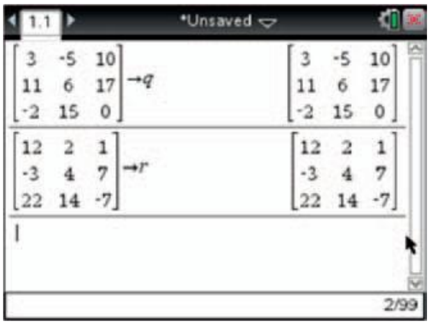
Then perform the addition.

$$= \begin{bmatrix} 93 & -1 & 37 \\ 12 & 46 & 100 \\ 148 & 143 & -49 \end{bmatrix}$$

For repeated calculations using the same matrices, it is useful to store the matrices first.

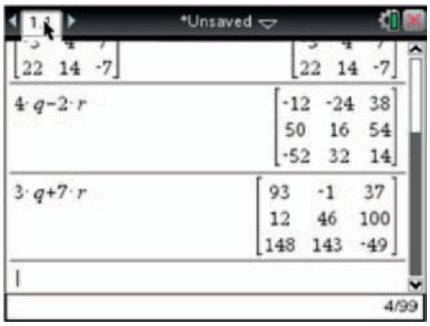
TI-Nspire CAS

Use a new document with a calculator page.
Press ⊞, then select the 3 × 3 matrix template. Set the number of rows to 3 and the number of columns to 3.
Create the first 3 × 3 matrix, then press ctrl var Q enter to store this matrix as *q*.
Create the second 3 × 3 matrix, then press ctrl var R to store this matrix as *r*.
The necessary calculations can then be performed.
Open a new document to clear the stored matrices.



$$\begin{bmatrix} 3 & -5 & 10 \\ 11 & 6 & 17 \\ -2 & 15 & 0 \end{bmatrix} \to q \qquad \begin{bmatrix} 3 & -5 & 10 \\ 11 & 6 & 17 \\ -2 & 15 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 2 & 1 \\ -3 & 4 & 7 \\ 22 & 14 & -7 \end{bmatrix} \to r \qquad \begin{bmatrix} 12 & 2 & 1 \\ -3 & 4 & 7 \\ 22 & 14 & -7 \end{bmatrix}$$



$$4\cdot q-2\cdot r \qquad \begin{bmatrix} -12 & -24 & 38 \\ 50 & 16 & 54 \\ -52 & 32 & 14 \end{bmatrix}$$

$$3\cdot q+7\cdot r \qquad \begin{bmatrix} 93 & -1 & 37 \\ 12 & 46 & 100 \\ 148 & 143 & -49 \end{bmatrix}$$

ClassPad

Use the Main $\sqrt{\alpha}$ application.

Press $\boxed{\text{Keyboard}}$.

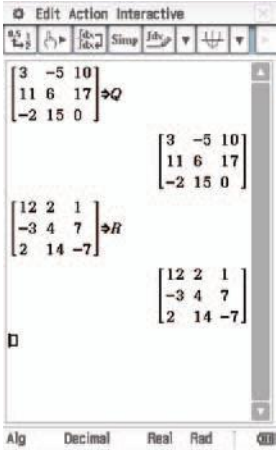
Tap $\boxed{\text{Math2}}$ then tab the 2 × 2 matrix template ▦ twice for a 3 × 3 matrix.

Create the first 3 × 3 matrix, then tap $\boxed{\Rightarrow}$ $\boxed{\text{Var}}$ $\boxed{\text{CAPS}}$ Q and press $\boxed{\text{EXE}}$ to store this matrix as Q.

Create the second 3 × 3 matrix, then tap $\boxed{\Rightarrow}$ $\boxed{\text{Var}}$ R and press $\boxed{\text{EXE}}$ to store this matrix as R.

The necessary calculations can then be performed.

To clear stored matrices, tap **Edit**, then **Clear All Variables**, then **OK**.

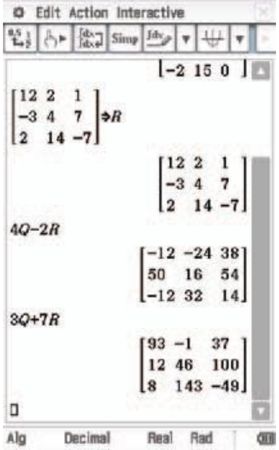


$$\begin{bmatrix}3 & -5 & 10\\ 11 & 6 & 17\\ -2 & 15 & 0\end{bmatrix}\Rightarrow Q$$

$$\begin{bmatrix}3 & -5 & 10\\ 11 & 6 & 17\\ -2 & 15 & 0\end{bmatrix}$$

$$\begin{bmatrix}12 & 2 & 1\\ -3 & 4 & 7\\ 2 & 14 & -7\end{bmatrix}\Rightarrow R$$

$$\begin{bmatrix}12 & 2 & 1\\ -3 & 4 & 7\\ 2 & 14 & -7\end{bmatrix}$$



$$\begin{bmatrix}-2 & 15 & 0\end{bmatrix}$$

$$\begin{bmatrix}12 & 2 & 1\\ -3 & 4 & 7\\ 2 & 14 & -7\end{bmatrix}\Rightarrow R$$

$$\begin{bmatrix}12 & 2 & 1\\ -3 & 4 & 7\\ 2 & 14 & -7\end{bmatrix}$$

$4Q-2R$

$$\begin{bmatrix}-12 & -24 & 38\\ 50 & 16 & 54\\ -12 & 32 & 14\end{bmatrix}$$

$3Q+7R$

$$\begin{bmatrix}93 & -1 & 37\\ 12 & 46 & 100\\ 8 & 143 & -49\end{bmatrix}$$

Example 13

Find the values of the pronumerals in the following matrix equation.

$$3\begin{bmatrix}x & 2 & -1\end{bmatrix}+2\begin{bmatrix}4 & 5 & y\end{bmatrix}=\begin{bmatrix}14 & 16 & 11\end{bmatrix}$$

Solution

Perform the scalar multiplications on the left-hand side.

$$\begin{aligned}&3\begin{bmatrix}x & 2 & -1\end{bmatrix}+2\begin{bmatrix}4 & 5 & y\end{bmatrix}\\ &=\begin{bmatrix}3x & 6 & -3\end{bmatrix}+\begin{bmatrix}8 & 10 & 2y\end{bmatrix}\end{aligned}$$

Add corresponding elements.

$$=\begin{bmatrix}3x+8 & 16 & -3+2y\end{bmatrix}$$

Now equate to the right-hand side of the original equation.

$$\begin{bmatrix}3x+8 & 16 & -3+2y\end{bmatrix}=\begin{bmatrix}14 & 16 & 11\end{bmatrix}$$

Equate corresponding elements and solve for the unknown pronumerals.

$$\begin{aligned}3x+8&=14\\ 3x&=6\\ x&=2\\ -3+2y&=11\\ 2y&=14\\ y&=7\end{aligned}$$

EXERCISE 3.04 Scalar multiplication

Concepts and techniques

Addition and subtraction of matrices

1 **Example 11** Perform the following scalar multiplications.

a $3\begin{bmatrix} 5 \\ 11 \end{bmatrix}$

b $2\begin{bmatrix} -1 & 3 \\ 4 & 4 \\ 7 & 2 \\ 0 & 5 \end{bmatrix}$

c $7\begin{bmatrix} 23 & 4 & 11 & 27 \\ 31 & 18 & 24 & 32 \end{bmatrix}$

d $4\begin{bmatrix} 22 & 17 & 25 \\ 26 & 19 & 22 \\ 13 & 25 & 20 \end{bmatrix}$

e $12\begin{bmatrix} 19 & 37 & 21 \\ 41 & 57 & 33 \end{bmatrix}$

f $5\begin{bmatrix} 13 & 28 \\ 16 & 27 \\ 39 & 40 \\ 15 & 32 \\ 29 & 33 \end{bmatrix}$

2 If matrix $\mathbf{P} = \begin{bmatrix} 57 & 61 \\ 54 & 51 \\ 62 & 78 \end{bmatrix}$, find each of the following.

a $2\mathbf{P}$

b $6\mathbf{P}$

c $23\mathbf{P}$

d $102\mathbf{P}$

3 Given that $\mathbf{M} = \begin{bmatrix} 5 & 9 \\ 3 & 8 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$, decide whether the following statements are true or false.

a $6\mathbf{M} = \begin{bmatrix} 30 & 54 \\ 24 & 32 \end{bmatrix}$

b $4\mathbf{N} = \begin{bmatrix} 12 & 8 \\ 20 & 16 \end{bmatrix}$

c $5\mathbf{M} = \begin{bmatrix} 25 & 45 \\ 15 & 40 \end{bmatrix}$

d $12\mathbf{N} = \begin{bmatrix} 36 & -24 \\ 60 & 48 \end{bmatrix}$

e $5\mathbf{N} = \begin{bmatrix} 15 & -10 \\ 15 & 20 \end{bmatrix}$

f $23\mathbf{M} = \begin{bmatrix} 115 & 207 \\ 69 & 184 \end{bmatrix}$

4 **Example 12** Use the matrices **M** and **N** from question **3** to evaluate the following.

a $2\mathbf{M} + 2\mathbf{N}$

b $3\mathbf{N} - 2\mathbf{M}$

c $5\mathbf{M} + 4\mathbf{N}$

d $6\mathbf{M} - 4\mathbf{N}$

e $12\mathbf{N} - 3\mathbf{M}$

f $\mathbf{M} + 2\mathbf{M}$

g $5\mathbf{N} - \mathbf{M}$

h $10\mathbf{N}–5\mathbf{M}$

5 Evaluate $4\begin{bmatrix} 12 & 10 & 13 \\ 14 & 13 & 9 \end{bmatrix} + 2\begin{bmatrix} 8 & 11 & 12 \\ 12 & 9 & 7 \end{bmatrix} - 3\begin{bmatrix} 12 & 7 & 15 \\ 22 & 11 & 7 \end{bmatrix}$.

6 Complete each of the following by filling in the missing values.

a $5\begin{bmatrix} 3 & 5 \\ 7 & 2 \\ 8 & 5 \\ 1 & 9 \end{bmatrix} + 4\begin{bmatrix} 5 & 1 \\ 6 & 2 \\ 2 & 0 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 35 & \square \\ \square & 18 \\ 48 & 25 \\ \square & \square \end{bmatrix}$

b $7\begin{bmatrix} 3 & -2 & 6 \\ 5 & 11 & 7 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 2 \\ -1 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 17 & -14 & \square \\ 39 & \square & 53 \end{bmatrix}$

7 **Example 13** Find the value of the pronumerals in the following matrix problems.

a $5\begin{bmatrix} a & 3 \\ 7 & -2 \end{bmatrix} + 2\begin{bmatrix} 5 & b \\ 11 & 8 \end{bmatrix} = \begin{bmatrix} 25 & 37 \\ 57 & c \end{bmatrix}$

b $3\begin{bmatrix} 10 & 5 \\ 4 & x \\ 8 & 7 \end{bmatrix} + 4\begin{bmatrix} 1 & 9 \\ 8 & 6 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 34 & 51 \\ z & 45 \\ 60 & 29 \end{bmatrix}$

c $4\begin{bmatrix} 5 & p \\ 7 & 9 \end{bmatrix} - 2\begin{bmatrix} 11 & 10 \\ q & 8 \end{bmatrix} = \begin{bmatrix} 2f & 32 \\ -10 & 20 \end{bmatrix}$

d $4\begin{bmatrix} a \\ -3 \\ b \\ 10 \end{bmatrix} - 7\begin{bmatrix} 12 \\ c \\ -5 \\ 6 \end{bmatrix} = \begin{bmatrix} -52 \\ 23 \\ 19 \\ d \end{bmatrix}$

8 $2\begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 \\ 1 & 6 \end{bmatrix}$ is equal to:

A $5\begin{bmatrix} 2 & 2 \\ 1 & 10 \end{bmatrix}$

B $\begin{bmatrix} 3 & 7 \\ 3 & 26 \end{bmatrix}$

C $6\begin{bmatrix} 2 & 2 \\ 1 & 10 \end{bmatrix}$

D $\begin{bmatrix} 3 & 4 \\ 3 & 26 \end{bmatrix}$

E $\begin{bmatrix} 5 & 4 \\ 3 & 8 \end{bmatrix}$

Reasoning and communication

9 In algebra we know that $5\mathbf{T} - 3\mathbf{T} = 2\mathbf{T}$. Let $\mathbf{T} = \begin{bmatrix} 12 & 7 \\ 21 & 15 \end{bmatrix}$ and use scalar multiplication to determine whether $5\mathbf{T} - 3\mathbf{T} = 2\mathbf{T}$ when **T** is a matrix.

10 If $\mathbf{C} = \begin{bmatrix} 3 & 4 \\ 7 & 9 \\ 2 & 6 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 2 & 8 \\ 1 & 3 \\ 5 & 7 \end{bmatrix}$, find the 3×2 matrix **E** such that $2\mathbf{C} + 3\mathbf{D} - \mathbf{E} = 0$.

11 Find the values of x, y and z if $2\mathbf{K} + 3\mathbf{L} = \mathbf{M}$, given that

$$\mathbf{K} = \begin{bmatrix} 5 & 2 \\ 3 & 5 \\ 7 & 8 \\ 5 & 0 \end{bmatrix}, \mathbf{L} = \begin{bmatrix} x & 3 \\ 12 & 32 \\ 14 & y \\ z & 57 \end{bmatrix} \text{ and } \mathbf{M} = \begin{bmatrix} 16 & 13 \\ 42 & 102 \\ 56 & 31 \\ 55 & 171 \end{bmatrix}.$$

3.05 MATRIX MULTIPLICATION: ROW MATRIX × COLUMN MATRIX

The product of a row matrix and a column matrix can only be found if the number of columns in the row matrix is the same as the number of rows in the column matrix.

Consider a 1 × 4 matrix being multiplied by a 4 × 1 matrix. These can be multiplied because the number of rows in the column matrix is equal to the number of columns in the row matrix.

That is: $(1 \times \underbrace{4) \times (4}_{\text{same}} \times 1)$.

IMPORTANT

The simplest way to check if a row matrix can be multiplied by a column matrix is to check that both matrices have the same number of elements. The product of a row matrix and a column matrix will always be a 1 × 1 matrix.

Example 14

Which of the following matrix products can be calculated?

We don't need to place a × operator between matrices. This is similar to multiplication in algebra where mn is actually $m \times n$ but the × symbol is not required.

a $\begin{bmatrix} 5 & 9 \end{bmatrix}\begin{bmatrix} 3 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 7 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 11 \\ 6 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & 8 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 2 \\ 1 \end{bmatrix}$

Solution

a Row and column matrices can only be multiplied if the number of elements are equal.
There are 2 elements in first matrix but only 1 element in the next one. The product cannot be found.

$\begin{bmatrix} 5 & 9 \end{bmatrix}\begin{bmatrix} 3 \end{bmatrix}$ cannot be found.

b Both matrices have 3 elements so the product can be found.

$\begin{bmatrix} 1 & 7 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 11 \\ 6 \end{bmatrix}$ can be found.

c There are 3 elements in the first matrix and 4 elements in the second one. The product cannot be found.

$\begin{bmatrix} 3 & 8 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 2 \\ 1 \end{bmatrix}$ cannot be found.

When multiplying two matrices, multiply the elements in the first matrix by those in the second using a systematic procedure.

To multiply a row matrix by a column matrix, start with the first element in the row matrix and the top element in the column and multiply these two numbers, then multiply second element in the row with the second element in the column matrix, then multiply the third elements and fourth elements and so on until the end of the row and the end of the column are reached. Add together the products as you go.

IMPORTANT

If $\mathbf{A} = \begin{bmatrix} a & b & c \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then $\mathbf{AB} = \begin{bmatrix} ax + by + cz \end{bmatrix}$

Example 15

Find **AB** if $\mathbf{A} = \begin{bmatrix} 3 & 7 & 2 & 12 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 \\ 9 \\ 7 \\ 2 \end{bmatrix}$

Solution

Multiply the corresponding elements. Then add them together.

$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} 3 \times 5 + 7 \times 9 + 2 \times 7 + 12 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} 116 \end{bmatrix} \end{aligned}$$

When you multiply, as you move across the row, move down the column.

Example 16

Find the value of m if $\begin{bmatrix} 5 & m \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 36 \end{bmatrix}$

Solution

Multiply the corresponding elements. Then add them together. Equate to 36.

$$5 \times 3 + m \times 7 = 36$$

Solve the equation.

$$\begin{aligned} 15 + 7m &= 36 \\ 7m &= 36 - 15 \\ 7m &= 21 \\ m &= \frac{21}{7} \\ m &= 3 \end{aligned}$$

Example 17

A confectionery company manufactures 4 different types of bagged confectionery – Sour Sweeties, Radical Ropes, Pop Frenzy and Berri Chews.

Production data for one month is shown in the two tables below.

Shutterstock.com/Roberaten

Production cost (per carton)			
Sour Sweeties	**Radical Ropes**	**Pop Frenzy**	**Berri Chews**
\$27.50	\$23.90	\$14.70	\$19.75

Quantity manufactured (cartons)	
Sour Sweeties	5000
Radical Ropes	6150
Pop Frenzy	6800
Berri Chews	3600

a Write the production costs as a row matrix **P**.

b Write the quantities manufactured as a column matrix **Q**.

c Find the matrix product **PQ**.

d Explain the information that **PQ** provides.

9780170250252

Solution

a Write the information contained in the first table as a 1 × 4 matrix **P**.

$$\mathbf{P} = \begin{bmatrix} 27.50 & 23.90 & 14.70 & 19.75 \end{bmatrix}$$

b Write the information contained in the second table as a 4 × 1 matrix **Q**.

$$\mathbf{Q} = \begin{bmatrix} 5000 \\ 6150 \\ 6800 \\ 3600 \end{bmatrix}$$

c Multiply each element in **P** by the corresponding element in **Q**. Move across the row and down the column.

$$\begin{aligned}\mathbf{PQ} &= \begin{bmatrix} 27.50 & 23.90 & 14.70 & 19.75 \end{bmatrix}\begin{bmatrix} 5000 \\ 6150 \\ 6800 \\ 3600 \end{bmatrix} \\ &= \begin{bmatrix} 27.50 \times 5000 + 23.90 \times 6150 \\ + 14.70 \times 6800 + 19.75 \times 3600 \end{bmatrix} \\ &= \begin{bmatrix} 455\,545 \end{bmatrix}\end{aligned}$$

A CAS can be used to find the matrix product.

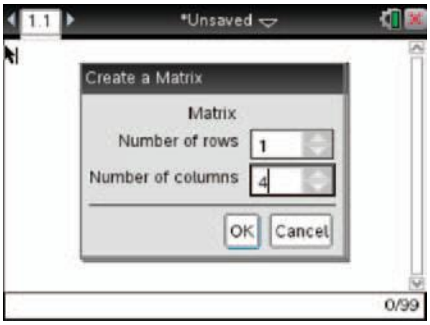
TI-Nspire CAS

Use a New Document with a Calculator page. Press ⊞.

Select the 3 × 3 matrix template $\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$

and press $\boxed{\text{enter}}$.

In the pop up screen that appears change the number of rows to 1 and the number of columns to 4 then press $\boxed{\text{enter}}$.



Create a Matrix

Matrix

Number of rows: 1

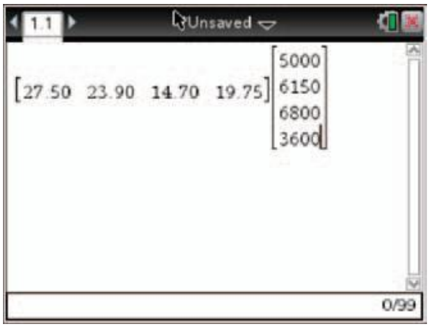
Number of columns: 4

Enter the elements into the first matrix.

[27.50 23.90 14.70 19.75]

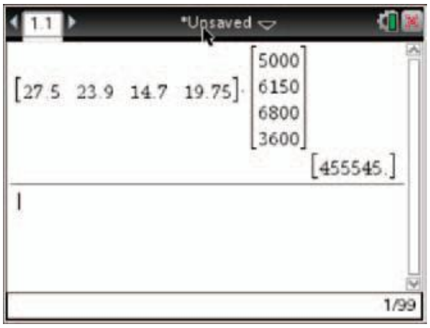
Follow the steps above to enter the second matrix with 4 rows and 1 column.

It is not necessary to press × between the matrices.



$$\begin{bmatrix} 27.50 & 23.90 & 14.70 & 19.75 \end{bmatrix} \begin{bmatrix} 5000 \\ 6150 \\ 6800 \\ 3600 \end{bmatrix}$$

Press enter.

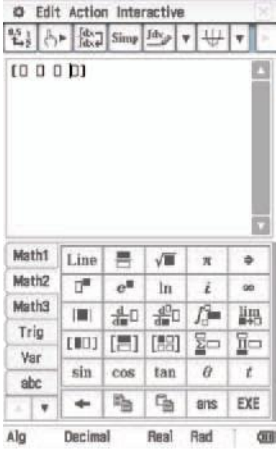


$$\begin{bmatrix} 27.5 & 23.9 & 14.7 & 19.75 \end{bmatrix} \cdot \begin{bmatrix} 5000 \\ 6150 \\ 6800 \\ 3600 \end{bmatrix}$$

$$\begin{bmatrix} 455545. \end{bmatrix}$$

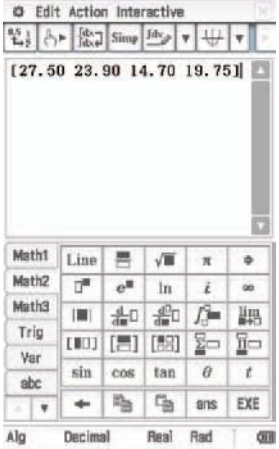
ClassPad

Use the Main √α application.
Press Keyboard.
Tap Math2 then tap the 1 × 2 matrix template, [■□] three times to create a 1 × 4 matrix.



[0 0 0 0]

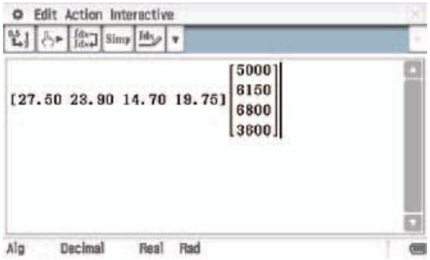
Enter the elements into the first matrix.



[27.50 23.90 14.70 19.75]

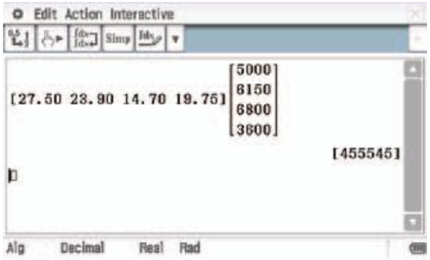
Tap the 2 × 1 matrix template, ▤, three times to enter the second matrix with 4 rows and 1 column.

It is not necessary to press $\boxed{\times}$ between the matrices.



```
[27.50 23.90 14.70 19.75] [5000]
                          [6150]
                          [6800]
                          [3600]
```

Press $\boxed{\text{EXE}}$.



```
[27.50 23.90 14.70 19.75] [5000]
                          [6150]
                          [6800]
                          [3600]
                                    [455545]
```

d Interpret the result.

The information contained in the matrix represents the total production costs for the month which were $455 545.

EXERCISE 3.05 Matrix multiplication: row matrix × column matrix

Concepts and techniques

1 Example 14 For the following pairs of matrices, state whether it is possible to multiply them. Write P if it is possible and N if it is not possible.

a $\begin{bmatrix} 5 & 7 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 9 \end{bmatrix}$ **b** $\begin{bmatrix} 5 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ **c** $\begin{bmatrix} 15 \\ 11 \end{bmatrix}\begin{bmatrix} 7 & 6 & -2 \end{bmatrix}$

d $\begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 6 & 9 \end{bmatrix}$ **e** $\begin{bmatrix} 7 \\ 19 \end{bmatrix}\begin{bmatrix} 5 & 12 & -1 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 12 \end{bmatrix}\begin{bmatrix} 15 \end{bmatrix}$

2 Example 15 Select the correct answer for the following questions.

a $\begin{bmatrix} 3 \\ 4 \end{bmatrix}\begin{bmatrix} 7 & 1 \end{bmatrix}$

A [56] **B** [31] **C** [15] **D** [25] **E** [26]

b $\begin{bmatrix} 20 & 1 & 14 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$

A [50] **B** [157] **C** [175] **D** [185] **E** [525]

3 Evaluate the following matrix products.

a $\begin{bmatrix} 3 & 5 & 22 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \\ 6 \\ 3 \end{bmatrix}$

b $\begin{bmatrix} 16 & 18 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \end{bmatrix}$

c $\begin{bmatrix} 12 & 1 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ 12 \\ 4 \end{bmatrix}$

d $\begin{bmatrix} 6 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

e $\begin{bmatrix} -2 & 5 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

f $\begin{bmatrix} 12 & -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 15 \\ 19 \\ 20 \end{bmatrix}$

4 Example 16 Select the correct response for each question.

a When $\begin{bmatrix} 7 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ d \end{bmatrix} = \begin{bmatrix} 22 \end{bmatrix}$, d must equal:

A 5 **B** 9 **C** 3 **D** 0 **E** 10

b If $\begin{bmatrix} a & 4 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 9 \end{bmatrix} = \begin{bmatrix} 50 \end{bmatrix}$, then a has the value:

A 1 **B** 2 **C** 3 **D** 4 **E** 5

5 Find the value of x if $\begin{bmatrix} 7 & x \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 39 \end{bmatrix}$.

6 $\mathbf{T} = \begin{bmatrix} 6 & 10 \end{bmatrix}$ and $\mathbf{U} = \begin{bmatrix} u \\ 12 \end{bmatrix}$. It is known that $\mathbf{TU} = [156]$. What is the value of u?

7 Evaluate:

a $\begin{bmatrix} 5 & 3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

b $\begin{bmatrix} 9 & 10 & 27 & 5 \end{bmatrix}\begin{bmatrix} 3.2 \\ 2.8 \\ 9 \\ 4.5 \end{bmatrix}$

c $\begin{bmatrix} 16.75 & 32.15 & 14.85 \end{bmatrix}\begin{bmatrix} 120 \\ 200 \\ 150 \end{bmatrix}$

d $\begin{bmatrix} 147.6 & 52.67 \end{bmatrix}\begin{bmatrix} 1300 \\ 1526 \end{bmatrix}$

e $\begin{bmatrix} 120 & 300 & 150 & 290 \end{bmatrix}\begin{bmatrix} 28.75 \\ 31.05 \\ 29.60 \\ 38.35 \end{bmatrix}$

f $\begin{bmatrix} 1520 & 1347 & 2016 \end{bmatrix}\begin{bmatrix} 13.5 \\ 21.6 \\ 37.4 \end{bmatrix}$

Reasoning and communication

8 If $\mathbf{M} = \begin{bmatrix} 2 & 7 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$, then

a find the value of:

i $3\mathbf{M}$ ii $2\mathbf{N}$ iii $\mathbf{MN}$ iv $3\mathbf{M} \times 2\mathbf{N}$ v $6 \times \mathbf{MN}$

b What do you notice about your answers in **iv** and **v**?

9 **Example 17** In a netball season, a team named the “Torpedos” had 7 wins, 3 draws and 2 losses. This can be represented in the row matrix $\begin{bmatrix} 7 & 3 & 2 \end{bmatrix}$.

Points are calculated using the table on the right.

Use matrix multiplication to work out the Torpedos total points for the season.

Win	3
Draw	1
Loss	0

10 A school canteen sells 4 different types of drink in bottles. In February, they sold 15 boxes of flavoured milk, 32 boxes of water, 11 boxes of fruit juice and 9 boxes of flavoured mineral water.

The number of bottles per box of each type of drink is different. (See the table at right.)

Type of drink	**Number of bottles per box**
Flavoured milk	12
Water	30
Fruit juice	24
Flavoured mineral water	18

Using matrix multiplication, find the total number of bottles of drink sold in February.

Shutterstock.com/Monkey Business Images

3.06 MATRIX MULTIPLICATION: MATRIX × COLUMN MATRIX

If we have a matrix of order $m \times n$ and a column matrix of order $n \times 1$, then we can multiply them. The product will be a matrix of order $m \times 1$. The product will have the same number of rows as the first matrix and the same number of columns as the second matrix.

IMPORTANT

$(m \times n) \times (n \times 1)$ gives a product of order $(m \times 1)$

same

order of product

IMPORTANT

It is possible to multiply two matrices provided the number of columns in the first matrix is equal to the number of rows in the second matrix.

Example 18

Determine if it is possible to perform each of the following multiplications and if so, state the order of the product.

a $\begin{bmatrix} 3 & 5 & 1 \\ 2 & 7 & 10 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 9 & 1 \\ 0 & 8 & 11 \\ 3 & 7 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ **c** $\begin{bmatrix} 6 & 4 & 9 \\ 3 & 5 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ 10 \end{bmatrix}$

Solution

a We have $(2 \times \mathbf{3}) \times (\mathbf{3} \times 1)$. Since the number of columns in the first matrix and the number of rows in the second matrix are both 3, we can multiply these matrices.

Yes it is possible.

The order of the product will be the outside numbers (2×1).

The order of the product will be (2×1).

b We have $(3 \times \mathbf{3}) \times (\mathbf{3} \times 1)$. Since the number of columns in the first matrix and the number of rows in the second matrix are both 3 we can multiply these matrices.

Yes it is possible.

The order of the product will be the outside numbers (3×1).

The order of the product will be (3×1).

c We have $(2 \times \mathbf{3}) \times (\mathbf{2} \times 1)$. Since the number of columns in the first matrix is not equal to the number of rows in the second matrix, this multiplication cannot be performed.

No it is not possible.

When multiplying a matrix by a column matrix, every row in the first matrix needs to be multiplied by the column matrix.

When the first row is multiplied by the column matrix the result goes in the first row of the answer matrix, then when the second row is multiplied by the column matrix the result goes in the second row of the answer matrix and so on.

Example 19

Find the products of parts **a** and **b** from Example **18**.

Solution

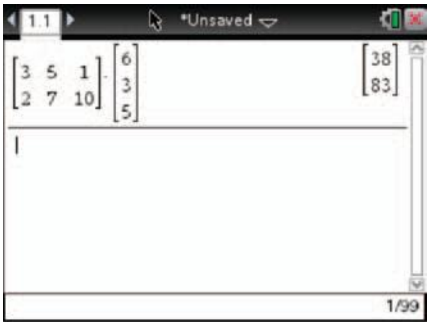
a Multiply each row in the first matrix by the column matrix.

$$\begin{bmatrix} 3 & 5 & 1 \\ 2 & 7 & 10 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3\times6+5\times3+1\times5 \\ 2\times6+7\times3+10\times5 \end{bmatrix}$$

Simplify.

$$= \begin{bmatrix} 38 \\ 83 \end{bmatrix}$$

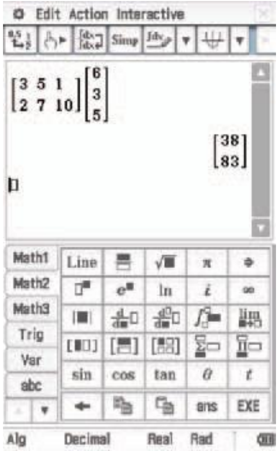
TI-Nspire CAS



$\begin{bmatrix}3&5&1\\2&7&10\end{bmatrix}\cdot\begin{bmatrix}6\\3\\5\end{bmatrix}$ $\begin{bmatrix}38\\83\end{bmatrix}$

When using the ClassPad, tapping the matrix templates multiple times enables you to create matrices with different numbers of rows and columns. For example, for a 2 × 3 matrix, tap [▪□], [▪□], then [▤]. For a 3 × 3 matrix tap [▦] twice.

ClassPad



$\begin{bmatrix}3&5&1\\2&7&10\end{bmatrix}\begin{bmatrix}6\\3\\5\end{bmatrix}$ $\begin{bmatrix}38\\83\end{bmatrix}$

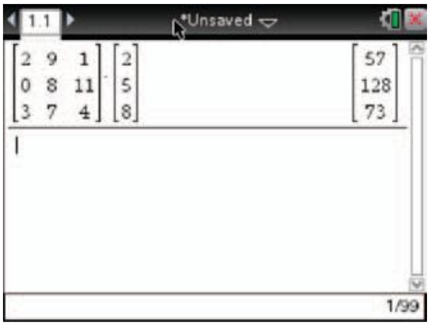
b Multiply each row in the first matrix by the column matrix.

$$\begin{bmatrix}2&9&1\\0&8&11\\3&7&4\end{bmatrix}\begin{bmatrix}2\\5\\8\end{bmatrix}=\begin{bmatrix}2\times2+9\times5+1\times8\\0\times2+8\times5+11\times8\\3\times2+7\times5+4\times8\end{bmatrix}$$

Simplify.

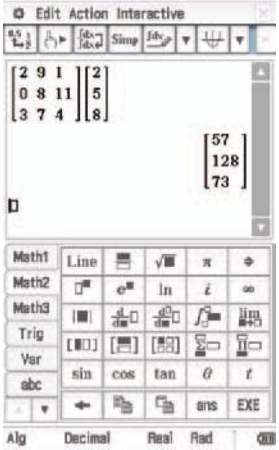
$$=\begin{bmatrix}57\\128\\73\end{bmatrix}$$

TI-Nspire CAS



$\begin{bmatrix}2&9&1\\0&8&11\\3&7&4\end{bmatrix}\cdot\begin{bmatrix}2\\5\\8\end{bmatrix}$ $\begin{bmatrix}57\\128\\73\end{bmatrix}$

ClassPad



$\begin{bmatrix}2&9&1\\0&8&11\\3&7&4\end{bmatrix}\begin{bmatrix}2\\5\\8\end{bmatrix}$ $\begin{bmatrix}57\\128\\73\end{bmatrix}$

Example 20

Find the value of y if $\begin{bmatrix} 3 & y & 8 \\ 5 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 34 \\ 37 \end{bmatrix}$

Solution

The unknown, y, is in the top row so multiply the top row by the column matrix and equate to the element in the top of the matrix on the right-hand side of the equation.

$$(3 \times 4) + (y \times 7) + (8 \times 1) = 34$$

Simplify.

$$\begin{aligned} 12 + 7y + 8 &= 34 \\ 7y + 20 &= 34 \end{aligned}$$

Solve for y.

$$\begin{aligned} 7y &= 34 - 20 \\ 7y &= 14 \\ y &= 2 \end{aligned}$$

EXERCISE 3.06 Matrix multiplication: matrix × column matrix

Concepts and techniques

1 **Example 18** Determine whether or not the following matrix products are possible. If so, state the order of the product.

a $\begin{bmatrix} 5 & 7 & 5 \\ 3 & 2 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix}$

b $\begin{bmatrix} 11 & 3 \\ 16 & 8 \end{bmatrix}\begin{bmatrix} 2 \\ 12 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 9 & -1 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

d $\begin{bmatrix} 17 & 3 & 12 \\ 15 & 22 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 13 \end{bmatrix}$

e $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \end{bmatrix}$

f $\begin{bmatrix} 13 & 5 \\ 1 & 19 \\ 0 & 61 \\ 35 & 23 \end{bmatrix}\begin{bmatrix} 7 \\ 22 \end{bmatrix}$

2 If the matrix product **AB** exists where $\mathbf{A} = \begin{bmatrix} 6 & 12 \\ 8 & 19 \\ 21 & 4 \\ 17 & 32 \\ 5 & 29 \end{bmatrix}$, then which one of the following could be the matrix **B**?

A $\begin{bmatrix} 5 & 6 & 3 & 1 \end{bmatrix}$

B $\begin{bmatrix} 2 \\ 9 \\ 11 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} 3 \\ 2 \\ 8 \\ 9 \\ 4 \end{bmatrix}$

D $\begin{bmatrix} 2 \\ 13 \end{bmatrix}$

E $\begin{bmatrix} 4 & 11 \end{bmatrix}$

3 If $\mathbf{P} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, then the order of **QP** is

A 1×1 **B** 3×1 **C** 1×3 **D** 3×3 **E** 2×3

4 If the matrix $\begin{bmatrix}12 & 5 & 11\\ -3 & 0 & 34\\ 60 & 42 & 9\end{bmatrix}$ was to be multiplied by a column matrix, what would the order of the column matrix need to be?

5 Example 19 Select the correct answer for the following matrix multiplications:

a $\begin{bmatrix}6 & 2 & 3\\ 5 & 2 & 1\end{bmatrix}\begin{bmatrix}5\\ 7\\ 4\end{bmatrix}$

A $\begin{bmatrix}56 & 43\end{bmatrix}$　B $\begin{bmatrix}11\\ 9\\ 7\end{bmatrix}$　C $\begin{bmatrix}56\\ 43\end{bmatrix}$　D $\begin{bmatrix}11 & 9 & 7\\ 10 & 9 & 5\end{bmatrix}$　E $\begin{bmatrix}11 & 10\\ 9 & 9\\ 7 & 5\end{bmatrix}$

b $\begin{bmatrix}1 & 7 & 12\\ 15 & 2 & 26\\ 38 & 0 & 3\end{bmatrix}\begin{bmatrix}5\\ 11\\ 7\end{bmatrix}$

A $\begin{bmatrix}166\\ 279\\ 211\end{bmatrix}$　B $\begin{bmatrix}6 & 18 & 19\\ 20 & 13 & 33\\ 43 & 11 & 10\end{bmatrix}$　C $\begin{bmatrix}43\\ 66\\ 64\end{bmatrix}$

D $\begin{bmatrix}100\\ 473\\ 287\end{bmatrix}$　E $\begin{bmatrix}43 & 66 & 64\end{bmatrix}$

6 Find the matrix product **AB** for each of the following if $\mathbf{B}=\begin{bmatrix}5\\ 8\end{bmatrix}$ and **A** =

a $\begin{bmatrix}6 & 11\end{bmatrix}$　b $\begin{bmatrix}3 & 7\\ 5 & 6\end{bmatrix}$　c $\begin{bmatrix}10 & 15\\ 3 & 17\\ 2 & 8\end{bmatrix}$　d $\begin{bmatrix}2 & -1\\ 3 & 5\\ 7 & 14\\ 6 & 8\\ 9 & 5\end{bmatrix}$

7 Perform the following matrix multiplications.

a $\begin{bmatrix}12 & 13\\ 11 & 21\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}$　b $\begin{bmatrix}11 & 6 & 13\\ 10 & 4 & 20\end{bmatrix}\begin{bmatrix}1\\ 11\\ 3\end{bmatrix}$

c $\begin{bmatrix}3 & 10 & 8 & 1\\ 2 & 7 & 2 & 4\end{bmatrix}\begin{bmatrix}5\\ -1\\ 3\\ 9\end{bmatrix}$　d $\begin{bmatrix}5 & 8 & 2 & 7\\ 6 & 3 & 0 & 1\\ 2 & 1 & 5 & 3\end{bmatrix}\begin{bmatrix}2\\ 5\\ 2\\ 3\end{bmatrix}$

e $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}5\\ 12\end{bmatrix}$　f $\begin{bmatrix}10 & 6 & 9\\ 3 & 5 & 11\\ 12 & 7 & 2\end{bmatrix}\begin{bmatrix}3\\ 8\\ 2\end{bmatrix}$

8 Example 20 Find the value of x if $\begin{bmatrix}1 & 6\\ x & 9\end{bmatrix}\begin{bmatrix}6\\ 7\end{bmatrix}=\begin{bmatrix}48\\ 81\end{bmatrix}$.

9 What is the value of d when $\begin{bmatrix}2 & 5\\ 3 & 2\\ 4 & 6\end{bmatrix}\begin{bmatrix}d\\ 3\end{bmatrix}=\begin{bmatrix}23\\ 18\\ 34\end{bmatrix}$

Reasoning and communication

10 A cinema charges the following ticket prices.

$9.00 for a child under 12 years of age

$13.00 for an adult

$6.50 each for seniors

Their attendance figures for last weekend are shown in the table.

Day	Children	Adults	Seniors
Saturday	62	105	32
Sunday	50	62	45

- a Represent the ticket prices as a column matrix.
- b Represent the attendance figures as a 2×3 matrix.
- c Use matrix multiplication to work out the total amount that the cinema took for tickets at the weekend.

11 If we have $\mathbf{A} = \begin{bmatrix} 3 & 5 & 2 \\ 1 & 9 & 7 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$, we can work out **AB** using the methods above.

Could we also calculate **BA**? In other words, could we perform the calculation in the reverse order $\begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 5 & 2 \\ 1 & 9 & 7 \end{bmatrix}$?

3.07 MATRIX MULTIPLICATION: MATRICES OF VARIOUS ORDERS

It is possible to multiply matrices when the number of columns in the first matrix is the same as the number of rows in the second matrix.

IMPORTANT

If matrix **A** is of order $m \times n$ and matrix **B** is of order $n \times p$ then we can find the product **AB** .

$$m \times \underbrace{n \quad \times \quad n}_{\text{same}} \times p$$

AB will be of order $m \times p$.

Example 21

If $\mathbf{X} = \begin{bmatrix} 5 & 3 \\ 2 & 6 \\ 9 & 10 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 9 & 11 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 7 \\ 1 & 12 \\ 5 & 11 \end{bmatrix}$:

i state whether each of the following products could be found

ii state the order of the product matrix where possible.

a **XA** b **XB** c **AX**

Solution

X is of order 3×2, **A** is of order 2×2 and **B** is of order 3×2.

a i Can we multiply $(3 \times \mathbf{2})$ and $(\mathbf{2} \times 2)$? The number of columns in the first matrix is equal to the number of rows in the second matrix, so we can.

Yes. The product **XA** exists.

ii The 2 outside numbers will determine the order of the product matrix.

The product matrix will have an order of 3×2.

b i We have $(3 \times \mathbf{2}) \times (\mathbf{3} \times 2)$. This multiplication cannot be performed, since the number of columns in the first matrix does not equal the number of rows in the second matrix.

No. The product **XB** does not exist.

c i **AX** would give $(2 \times \mathbf{2}) \times (\mathbf{3} \times 2)$. The number of columns in the first matrix is not equal to the number of rows in the second matrix.

No. The product **AX** does not exist.

To multiply two matrices, we multiply pairs of elements working across the rows in the first matrix and down the columns in the second matrix.

If we have $\mathbf{A} = \begin{bmatrix} 2 & 6 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 5 & 1 \\ 2 & 4 & 7 \end{bmatrix}$, and we need to calculate **AB**, we perform the following calculations by working across the rows in **A** and down the columns in **B**:

$$\begin{bmatrix} (2\times 3 \quad + \quad 6\times 2) & (2\times 5 \quad + \quad 6\times 4) & (2\times 1 \quad + \quad 6\times 7) \\ \mathbf{A}\text{ row } 1\times\mathbf{B}\text{ column } 1 & \mathbf{A}\text{ row } 1\times\mathbf{B}\text{ column } 2 & \mathbf{A}\text{ row } 1\times\mathbf{B}\text{ column } 3 \\ (0\times 3 \quad + \quad 1\times 2) & (0\times 5 \quad + \quad 1\times 4) & (0\times 1 \quad + \quad 1\times 7) \\ \mathbf{A}\text{ row } 2\times\mathbf{B}\text{ column } 1 & \mathbf{A}\text{ row } 2\times\mathbf{B}\text{ column } 2 & \mathbf{A}\text{ row } 2\times\mathbf{B}\text{ column } 3 \end{bmatrix}$$

When we multiply **A** row 1 by **B** column 3, the result goes in row 1, column 3 of the answer matrix.

So $\mathbf{AB} = \begin{bmatrix} 18 & 34 & 44 \\ 2 & 4 & 7 \end{bmatrix}$.

Example 22

Calculate **AB** and **BA** if $\mathbf{A} = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$

Solution

Multiply each row in matrix **A** by each column in matrix **B** then simplify.

$$\mathbf{AB} = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times1+3\times3 & 5\times5+3\times7 \\ 2\times1+6\times3 & 2\times5+6\times7 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 46 \\ 20 & 52 \end{bmatrix}$$

Since **A** and **B** are both 2 × 2 matrices, **AB** and **BA** both exist.

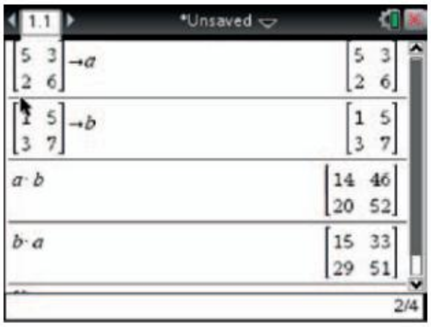
Mulitply each row in matrix **B** by each column in matrix **A** then simplify.

$$\mathbf{BA} = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times5+5\times2 & 1\times3+5\times6 \\ 3\times5+7\times2 & 3\times3+7\times6 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 33 \\ 29 & 51 \end{bmatrix}$$

TI-Nspire CAS



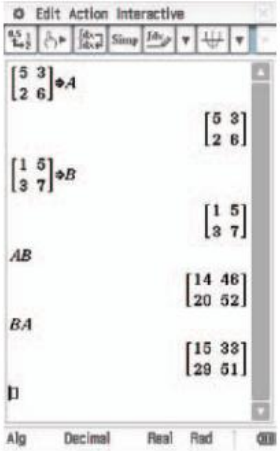
$\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix} \to a$ $\quad \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} \to b$ $\quad \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$

$a \cdot b$ $\quad \begin{bmatrix} 14 & 46 \\ 20 & 52 \end{bmatrix}$

$b \cdot a$ $\quad \begin{bmatrix} 15 & 33 \\ 29 & 51 \end{bmatrix}$

ClassPad



$\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix} \Rightarrow A$ $\quad \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} \Rightarrow B$ $\quad \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$

AB $\quad \begin{bmatrix} 14 & 46 \\ 20 & 52 \end{bmatrix}$

BA $\quad \begin{bmatrix} 15 & 33 \\ 29 & 51 \end{bmatrix}$

EXERCISE 3.07 Matrix multiplication: matrices of various orders

Concepts and techniques

Multiplying matrices

1 Example 21 Let $\mathbf{A} = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 6 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 2 & 3 \\ 5 & 2 \\ 1 & 6 \\ 4 & 7 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 4 & 3 \\ 5 & 1 \\ 2 & 6 \end{bmatrix}$.

i State whether each of the following matrix products exist.

ii For those products that exist, state their order.

a **AB** b **BA** c **AD** d **AC** e **DA**

f **BC** g **CB** h **DB** i **CD** j **BB**

2 Example 22 Given $\mathbf{X} = \begin{bmatrix} 5 & 8 & 11 \\ 3 & 6 & 4 \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} 1 & 5 \\ 6 & 4 \\ 3 & 7 \end{bmatrix}$:

a find **XY**

b find **YX**

c is the order of **XY** the same as the order of **YX**?

d does **XY** = **YX**?

3 Find the following matrix products.

a $\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 5 & 7 & 6 \end{bmatrix}$

b $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 16 & 5 \\ 2 & 11 \end{bmatrix}$

c $\begin{bmatrix} 4 & 2 \\ 5 & 8 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

d $\begin{bmatrix} 1 & 1 & 4 \\ 3 & 2 & 6 \\ 5 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 5 \end{bmatrix}$

e $\begin{bmatrix} 2 & 6 \\ 8 & 7 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 2 & 5 & 9 \end{bmatrix}$

f $\begin{bmatrix} 5 & -2 & 4 \\ -1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ -2 & 5 \\ 1 & 2 \end{bmatrix}$

4 For the matrices $\mathbf{D} = \begin{bmatrix} 2 & 4 \\ 6 & 5 \\ 3 & 4 \end{bmatrix}$, $\mathbf{E} = \begin{bmatrix} 3 & 5 \\ 1 & 4 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}$, $\mathbf{G} = \begin{bmatrix} 7 & 1 & 3 & 4 \\ 5 & 6 & 1 & 3 \\ 2 & 4 & 6 & 9 \end{bmatrix}$ and $\mathbf{H} = \begin{bmatrix} 1 & 5 & 1 & 7 \\ 2 & 0 & 1 & 3 \end{bmatrix}$

calculate the following products:

a **DF**

b **EF**

c **GE**

d **HE**

e **FH**

f **DH**

5 Evaluate the following matrix products.

a $\begin{bmatrix} 4 & 6 \\ 9 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & 8 \\ 6 & 7 \end{bmatrix}$

b $\begin{bmatrix} 11 & 8 & 17 \\ 9 & 12 & 15 \end{bmatrix}\begin{bmatrix} 13 & 5 & 7 \\ 6 & 22 & 23 \\ 14 & 41 & 2 \end{bmatrix}$

c $\begin{bmatrix} 21 & 35 & 26 & 44 \\ 18 & 19 & 20 & 12 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 11 \\ 3 \end{bmatrix}$

d $\begin{bmatrix} 10.2 & 11.5 & 13.6 \\ 19.1 & 13.1 & 14.2 \\ 13.5 & 16.2 & 18.9 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 7 & 9 \\ 11 & 3 \end{bmatrix}$

e $\begin{bmatrix} 201 & 51 \\ 105 & 122 \end{bmatrix}\begin{bmatrix} 12 & 6 & 17 & 13 \\ 18 & 19 & 5 & 7 \end{bmatrix}$

f $\begin{bmatrix} 78 & 56 & 25 \\ 36 & 49 & 62 \end{bmatrix}\begin{bmatrix} 2.1 & 3.5 \\ 4.6 & 7.3 \\ 2.8 & 1.9 \end{bmatrix}$

6 **R** is a 4 × 3 matrix and **S** is a 3 × 6 matrix. The order of matrix **RS** is:

A 4 × 3

B 3 × 6

C 12 × 18

D 4 × 6

E 3 × 4

7 Matrix **T** has 3 rows and 5 columns. For which one of the following matrices **P** would the matrix product **TP** exist?

A $\mathbf{P} = \begin{bmatrix} 2 & 5 & 7 \end{bmatrix}$

B $\mathbf{P} = \begin{bmatrix} 5 & 6 \\ 11 & 7 \\ 9 & 8 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$

C $\mathbf{P} = \begin{bmatrix} 1 & 5 & 6 & 8 & 1 \\ 3 & 2 & 8 & 9 & 4 \\ 7 & 6 & 5 & 8 & 2 \end{bmatrix}$

D $\mathbf{P} = \begin{bmatrix} 7 \\ 2 \\ 5 \\ 9 \end{bmatrix}$

E $\mathbf{P} = \begin{bmatrix} 6 & 6 & 9 \\ 5 & 2 & 4 \end{bmatrix}$

8 Matrix **L** has 2 rows and 4 columns. Matrix **L** was multiplied by matrix **K** and the product **KL** was of order 3×4.

The order of **K** must be

A 3×2 B 2×4 C 2×3 D 4×2 E 12×1

Reasoning and communication

9 A stationery retailer sells various packs of stationery that cater to his various customers' requirements. The names of the packs and details of their contents are in the table below.

Pack	Lead pencils	Coloured pencils	Erasers	Blue pens	Red pens
School	5	24	3	10	3
University	2	0	1	20	5
Home office	2	0	1	30	10

In January he sold 12 School packs, 10 University packs and 3 Home office packs.

In February he sold 8 School packs, 32 University packs and no Home office packs.

Shutterstock.com/erashov

a Represent the above table as a 3×5 matrix. We will call this matrix **P**.

b Organise the sales data for January and February into a 2×3 matrix. The first row will represent January sales. The second row will show February sales. We will call this matrix **S**.

c Perform matrix multiplication to work out **SP**.

d Using matrix **SP**, answer the following questions.

   i How many lead pencils were sold in January?

   ii How many blue pens were sold in February?

   iii For the two months what was the total number of erasers sold?

INVESTIGATION Multiplying matrices: identity matrices

We will investigate the impact of multiplying a matrix by an identity matrix.

Consider the following matrices.

$$\mathbf{A} = \begin{bmatrix} 3 & 5 \\ 6 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 6 \\ 7 \\ 11 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 7 & 5 & 11 \\ 2 & -5 & 4 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} -7 & 4 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 6 & 3 & 8 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 2 \\ 5 \\ 3 \\ 7 \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} 7 & 5 & 11 & 7 \\ 6 & 3 & 9 & 4 \end{bmatrix}$$

$$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{I}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_4$ are identity matrices.

a List all of the products possible when multiplying matrices **A** to **H** by an identity matrix. (For example: The product of **A** and $\mathbf{I}_2$ is called $\mathbf{AI}_2$. It exists because $\mathbf{A} \times \mathbf{I}_2$ gives: $(2 \times 2) \times (2 \times 2) = (2 \times 2)$.)

b Choose 5 of the products that you named in part **a** and evaluate the product matrix.

c What happens when an identity matrix is multiplied by another matrix?

3.08 POWERS OF MATRICES

When a number is raised to a power, this indicates repeated multiplication. For example:

5^2 means 5×5

5^3 means $5 \times 5 \times 5$

5^7 means $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ and so on.

The same concept applies to matrices. So if we have a matrix named **A**, then

$\mathbf{A}^2$ means $\mathbf{A} \times \mathbf{A}$.

We can evaluate powers of matrices using matrix multiplication.

IMPORTANT

Only square matrices can be raised to a power.

Consider a 2 × 3 matrix named **A**. If we tried to evaluate $\mathbf{A}^2$, we would have

$$(2 \times \underbrace{3) \times (2}_{\text{different}} \times 3).$$

Example 23

Find the value of $\mathbf{A}^2$ if $\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix}$.

Solution

Multiply $\mathbf{A} \times \mathbf{A}$.

$$\begin{aligned} \mathbf{A}^2 &= \begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 2 + 5 \times 1 & 2 \times 5 + 5 \times 7 \\ 1 \times 2 + 7 \times 1 & 1 \times 5 + 7 \times 7 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 45 \\ 9 & 54 \end{bmatrix} \end{aligned}$$

Example 24

Find the value of $\mathbf{B}^3$ if $\mathbf{B} = \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}$.

Solution

To evaluate $\mathbf{B}^3$, we can multiply $\mathbf{B} \times \mathbf{B}^2$.

Firstly evaluate $\mathbf{B}^2$ using $\mathbf{B} \times \mathbf{B}$.

$$\begin{aligned} \mathbf{B}^2 &= \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 3 + 6 \times -1 & 3 \times 6 + 6 \times 2 \\ -1 \times 3 + 2 \times -1 & -1 \times 6 + 2 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 30 \\ -5 & -2 \end{bmatrix} \end{aligned}$$

Now multiply $\mathbf{B} \times \mathbf{B}^2$.

$$\begin{aligned} \mathbf{B}^3 &= \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 30 \\ -5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -21 & 78 \\ -13 & -34 \end{bmatrix} \end{aligned}$$

Example 25

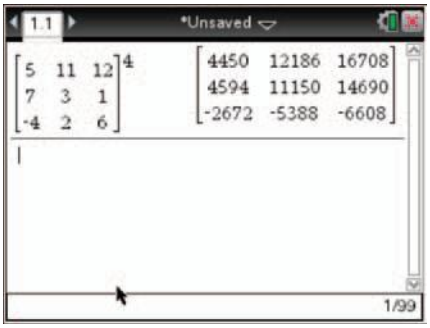
Find the $\mathbf{X}^4$ if $\mathbf{X} = \begin{bmatrix} 5 & 11 & 12 \\ 7 & 3 & 1 \\ -4 & 2 & 6 \end{bmatrix}$

Solution

Since this is a large matrix raised to a relatively large power, it is sensible to use a CAS.

TI-Nspire CAS

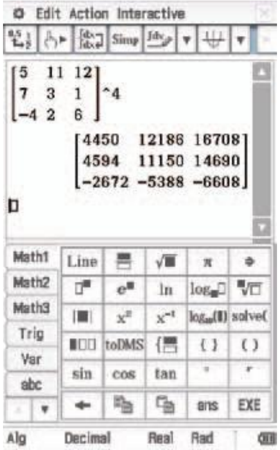
Press ^ 4 then enter to raise the matrix to the power 4.



$$\begin{bmatrix} 5 & 11 & 12 \\ 7 & 3 & 1 \\ -4 & 2 & 6 \end{bmatrix}^4 \qquad \begin{bmatrix} 4450 & 12186 & 16708 \\ 4594 & 11150 & 14690 \\ -2672 & -5388 & -6608 \end{bmatrix}$$

ClassPad

Press $\boxed{\wedge}$4 then $\boxed{\text{EXE}}$ to raise the matrix to the power 4.



$$\begin{bmatrix} 5 & 11 & 12 \\ 7 & 3 & 1 \\ -4 & 2 & 6 \end{bmatrix}^{\wedge}4$$

$$\begin{bmatrix} 4450 & 12186 & 16708 \\ 4594 & 11150 & 14690 \\ -2672 & -5388 & -6608 \end{bmatrix}$$

Write the answer.

$$\mathbf{X}^4 = \begin{bmatrix} 4450 & 12186 & 16708 \\ 4594 & 11150 & 14690 \\ -2672 & -5388 & -6608 \end{bmatrix}$$

EXERCISE 3.08 Powers of matrices

Concepts and techniques

1 Example 23 a For the following matrices state whether they can be raised to a power.

$\mathbf{A} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 10 \\ 4 & 5 & 6 \end{bmatrix}$

$\mathbf{D} = \begin{bmatrix} 2 & 5 \\ -4 & 9 \end{bmatrix}$ $\mathbf{E} = \begin{bmatrix} 1 & 7 \\ 6 & 3 \\ 2 & 11 \end{bmatrix}$ $\mathbf{F} = \begin{bmatrix} 2 & 5 & 1 & 6 \\ 3 & 7 & 2 & 1 \\ 4 & 8 & 3 & 6 \\ 1 & 7 & 5 & 2 \end{bmatrix}$

b For those which you identified in part **a**, find the value when the matrix is raised to the power 2 (squared).

2 Evaluate the following.

a $\begin{bmatrix} 10 & 6 \\ 7 & 3 \end{bmatrix}^2$ b $\begin{bmatrix} 3 & 7 & 5 \\ 1 & 3 & 2 \\ 9 & 4 & 6 \end{bmatrix}^2$ c $\begin{bmatrix} 12 & -2 \\ 5 & -7 \end{bmatrix}^2$

d $\begin{bmatrix} -4 & 3 \\ 6 & -2 \end{bmatrix}^2$ e $\begin{bmatrix} 12 & 9 & 15 \\ 7 & -3 & 6 \\ 13 & 11 & 0 \end{bmatrix}^2$ f $\begin{bmatrix} 20 & 13 \\ 11 & 24 \end{bmatrix}^2$

3 Which of the following numbers is the missing element?

$$\begin{bmatrix} 12 & 8 \\ 10 & 5 \end{bmatrix}^2 = \begin{bmatrix} 224 & 136 \\ 170 & \square \end{bmatrix}$$

A 25 B 89 C 125 D 105 E 169

4 **Example 24** Which of the following is $\mathbf{A}^3$ given that $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}$?

A $\begin{bmatrix} 4 & 1 \\ 25 & 49 \end{bmatrix}$

B $\begin{bmatrix} 8 & 1 \\ 125 & 343 \end{bmatrix}$

C $\begin{bmatrix} 63 & 72 \\ 360 & 423 \end{bmatrix}$

D $\begin{bmatrix} 182 & 255 \\ 348 & 298 \end{bmatrix}$

E $\begin{bmatrix} 63 & 73 \\ 360 & 420 \end{bmatrix}$

5 Find the missing elements in each of the following matrix problems.

a $\begin{bmatrix} 5 & 6 \\ 1 & 7 \end{bmatrix}^3 = \begin{bmatrix} \square & 690 \\ 115 & 457 \end{bmatrix}$

b $\begin{bmatrix} 2 & 9 \\ 6 & 4 \end{bmatrix}^3 = \begin{bmatrix} 440 & 738 \\ \square & 604 \end{bmatrix}$

c $\begin{bmatrix} 3 & 10 \\ 6 & 1 \end{bmatrix}^3 = \begin{bmatrix} 447 & 730 \\ 438 & \square \end{bmatrix}$

d $\begin{bmatrix} 0 & 5 \\ -1 & 4 \end{bmatrix}^3 = \begin{bmatrix} -20 & 55 \\ \square & 24 \end{bmatrix}$

6 **Example 25** Evaluate the following given that

$$\mathbf{X} = \begin{bmatrix} 3 & 7 \\ 9 & 4 \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} 2.5 & 6.1 \\ 3.9 & 2.7 \end{bmatrix} \text{ and } \mathbf{Z} = \begin{bmatrix} 12 & 15 & 21 \\ 17 & 19 & 23 \\ 31 & 11 & 24 \end{bmatrix}$$

a $\mathbf{X}^2$

b $\mathbf{X}^3$

c $\mathbf{Y}^2$

d $\mathbf{X}^2 + \mathbf{Y}^2$

e $\mathbf{X}^5$

f $\mathbf{X}^5 - \mathbf{X}^2$

g $\mathbf{Z}^2$

h $\mathbf{Z}^4$

Reasoning and communication

7 Tim made a mistake when he was asked to square matrix $\mathbf{M} = \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$. His calculations are shown below. Tim has not written the final answer yet.

$$\begin{aligned} M^2 &= \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 2 + 1 \times 2 & 2 \times 1 + 1 \times 1 \\ 2 \times 2 + 0 \times 2 & 2 \times 1 + 0 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} \; & \; \\ \; & \; \end{bmatrix} \end{aligned}$$

a Identify which element has the miscalculation.

b Write what Tim's final answer would have been.

c Write the correct answer for $\mathbf{M}^2$.

8 We know that in matrix multiplication that it cannot be assumed that $\mathbf{AB} = \mathbf{BA}$.
We know that $\mathbf{C}^3 = \mathbf{C} \times \mathbf{C}^2$.

Let matrix $\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$.

a Find the value of $\mathbf{C}^3$ by calculating $\mathbf{C} \times \mathbf{C}^2$.

b Using matrix multiplication, evaluate $\mathbf{C}^2 \times \mathbf{C}$.

c Is the following statement true or false?
"$\mathbf{C}^3 = \mathbf{C}^2 \times \mathbf{C}$ which is the same as $\mathbf{C} \times \mathbf{C}^2$."

3.09 APPLICATIONS OF MATRICES

Matrices are used to represent information so that relevant calculations can be made. Even the basic operations that we have learnt in this chapter can be used in applications of matrices.

Example 26

The manager of a local hardware store sells goods at a 55% mark-up. He recorded his purchase costs for the last 2 weeks in the table below.

	Week 1	Week 2
Paint	\$1060	\$1555
Timber	\$3029	\$1124
Metal fasteners	\$923	\$705
Gardening tools	\$896	\$1230

Shutterstock.com/bikeriderlondon

a Represent these cost sales in a 4 × 2 cost matrix, **C**.

b Selling price is calculated by finding 155% of the cost price. Convert 155% to a decimal.

c Using scalar multiplication, represent the selling prices of these goods in a 4 × 2 matrix, **S**.

d Create a profit matrix.

e Calculate the total profit to be made if all of the goods purchased over these 2 weeks are sold.

Solution

a The table already has 4 rows and 2 columns.

$$\mathbf{C}=\begin{bmatrix}1060 & 1555\\3029 & 1124\\923 & 705\\896 & 1230\end{bmatrix}$$

b To convert a percentage to a decimal, divide by 100.

$$155\% = 155 \div 100$$
$$= 1.55$$

c Selling price = 1.55 × cost price.
Multiplying the cost matrix by the scalar 1.55 will multiply each element by 1.55.

$$\mathbf{S}=1.55\mathbf{C}$$
$$=1.55\begin{bmatrix}1060 & 1555\\3029 & 1124\\923 & 705\\896 & 1230\end{bmatrix}$$
$$=\begin{bmatrix}1643.00 & 2410.25\\4694.95 & 1742.20\\1430.65 & 1092.75\\1388.80 & 1906.50\end{bmatrix}$$

d To create a profit matrix:
Profit = Selling price – Cost price

$$\text{Profit } = \mathbf{S}-\mathbf{C}$$
$$=\begin{bmatrix}1643.00 & 2410.25\\4694.95 & 1742.20\\1430.65 & 1092.75\\1388.80 & 1906.50\end{bmatrix}-\begin{bmatrix}1060 & 1555\\3029 & 1124\\923 & 705\\896 & 1230\end{bmatrix}$$
$$=\begin{bmatrix}583.00 & 855.25\\1665.95 & 618.20\\507.65 & 387.75\\492.80 & 676.50\end{bmatrix}$$

e Total profit can be found by adding all of the elements in the profit matrix.

$$\begin{aligned}\text{Total profit} &= 583.00 + 855.25 + 1665.95 \\ &\quad + 618.20 + 507.65 + 387.75 \\ &\quad + 492.80 + 676.50 \\ &= \$5787.10\end{aligned}$$

Example 27

Each week the coach of the "Little Diggers" basketball team awards the "Best and Fairest Player Award" to the player who scores the most game points.

The results of their last game were as follows.

	3 pointers	2 pointers	1 pointer
Steve	0	5	0
Sarah	1	3	2
Lucy	0	7	0
Matt	2	5	0
Kelly	1	7	0
Jack	0	2	2
Sophie	1	8	0

Corbis/Hero Images

a Write a column matrix, **P**, to represent the 3 different scores possible.

b Write a matrix, **G**, to represent the data from the table.

c Use matrix multiplication to find a score matrix named **S** that represents the total scored by each of the players.

d Which player won the "Best and Fairest" award?

e The opposition team scored a total of 87 points. Did the Little Diggers win the game?

Solution

a When a player scores, they can either get 3 points, 2 points or 1 point.

$$\mathbf{P} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

b Matrix **G** will have 7 rows and 3 columns.

$$\mathbf{G} = \begin{bmatrix} 0 & 5 & 0 \\ 1 & 3 & 2 \\ 0 & 7 & 0 \\ 2 & 5 & 0 \\ 1 & 7 & 0 \\ 0 & 2 & 2 \\ 1 & 8 & 0 \end{bmatrix}$$

9780170250252

c To find **S**, we must find **GP**.

This will be (7 × 3) × (3 × 1), which will result in a 7 × 1 matrix that represents the personal total for each of the seven players.

$$\mathbf{S} = \begin{bmatrix} 0 & 5 & 0 \\ 1 & 3 & 2 \\ 0 & 7 & 0 \\ 2 & 5 & 0 \\ 1 & 7 & 0 \\ 0 & 2 & 2 \\ 1 & 8 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 11 \\ 14 \\ 16 \\ 17 \\ 6 \\ 19 \end{bmatrix}$$

d The largest element in **S** is the highest personal score.

Sophie won with 19 points.

e The total score for Little Diggers can be found by adding all of the elements in **S**.

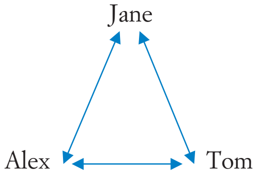
Little Diggers total score
= 10 + 11 + 14 + 16 + 17 + 6 + 19
= 93

Write the answer.

93 is greater than 87, so Little Diggers won.

Matrices can be used to model pathways. These might be social networks, location maps or others. Consider a social network involving 3 people named Alex, Jane and Tom.

We could draw a diagram showing the ways that each person could communicate with each of the others.



We can represent the number of ways that each person can communicate directly with another person in a table.

	Jane	**Alex**	**Tom**
Jane	0	1	1
Alex	1	0	1
Tom	1	1	0

This could then be displayed as a matrix. Let's call it matrix **A**.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The matrix must contain a row for each person and a column for each person. So for three people, we need a 3 × 3 matrix.

We see that Jane can communicate directly with Alex in only 1 way.

What if Jane wanted to communicate with Alex via Tom?

This communication path would involve two steps: Jane → Tom → Alex.

If we needed to know the number of ways that each person could communicate with another via a third person, we can simply square matrix **A**.

$$\mathbf{A}^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Matrix $\mathbf{A}^2$ tells us that there are 2 paths that allow Jane to communicate with herself via a third person.

Jane → Tom → Jane or Jane → Alex → Jane

These are both two-step paths from Jane to Jane.

$\mathbf{A}^2$ tells us that there is only 1 two-step path that allows Jane to speak to Alex.

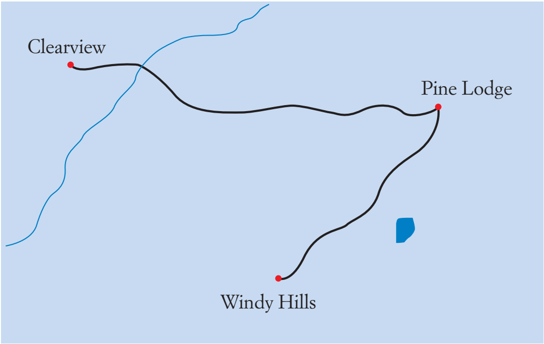
i.e. Jane → Tom → Alex.

To find the number of three-step paths from each person to another, we could simply find $\mathbf{A}^3$.

To find the number of seven-step paths from each person to the others, we would use $\mathbf{A}^7$.

Example 28

The map below shows the roadways connecting three rural properties.



There is a road connecting Clearview to Pine Lodge. There is a road connecting Pine Lodge to Windy Hills. However, there is no road connecting Clearview and Windy Hills.

a Draw a matrix named **P** representing the number of 1-step paths from each of the three properties to the others.

b Calculate $\mathbf{P}^2$.

c How many two-step paths are there from Clearview to Windy Hills? Give details.

d Why are there no two-step paths from Clearview to Pine Lodge?

e How many two-step paths are there from Pine Lodge to Pine Lodge? Give details.

Solution

a Since there are 3 locations, we will need a 3×3 matrix. Use a 1 to represent a path and a 0 to represent no path.

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

In table form we have:

	Clearview	Pine Lodge	Windy Hills
Clearview	0	1	0
Pine Lodge	1	0	1
Windy Hills	0	1	0

b $\mathbf{P}^2 = \mathbf{P} \times \mathbf{P}$

$$\mathbf{P}^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

c The element $(\mathbf{P}^2)_{13}$ shows the number of two-step paths from Clearview to Windy Hills.

There is 1 two-step path from Clearview to Windy Hills: Clearview → Pine Lodge → Windy Hills

d Examine the map.

From Clearview, the first step takes us to Pine Lodge.
A second step would take us back to Clearview or on to Windy Hills.

e The element $(\mathbf{P}^2)_{22}$ shows the number of two-step paths from Pine Lodge to Pine Lodge.

There are 2 two-step paths from Pine Lodge to Pine Lodge: Pine Lodge → Clearview → Pine Lodge or Pine Lodge → Windy Hills → Pine Lodge

Example 29

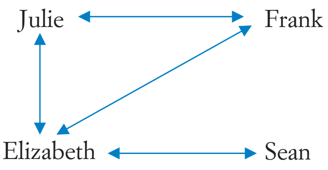
Julie, Frank, Elizabeth and Sean are all members of a social messaging network called "Friends Online".

Julie is friends with Frank and Elizabeth.

Frank is friends with Julie and Elizabeth.

Elizabeth is friends with Julie, Frank and Sean.

Sean is only able to send messages to Elizabeth.



a Draw a 4 × 4 matrix called **C**, showing the number of one-step paths for messages from each of these people to the others.

b Find $\mathbf{C}^2$.

c Find $\mathbf{C}^3$.

d How many two-step paths will allow Sean to message Frank? Give details.

e i How many three-step paths will allow Elizabeth to message Frank? Give details.

  ii Explain why Elizabeth would probably not use any of these three-step paths to send a message to Frank.

Solution

a We can set up a table using the information from the diagram.

	Julie	**Frank**	**Elizabeth**	**Sean**
Julie	0	1	1	0
Frank	1	0	1	0
Elizabeth	1	1	0	1
Sean	0	0	1	0

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

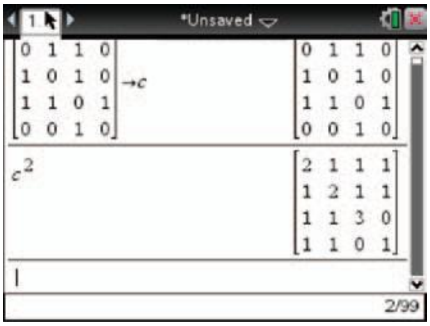
b Use a CAS to find $\mathbf{C}^2$.

TI-Nspire CAS

Since matrix **C** is being used more than once in this question it is advisable to store the matrix as **C**.

Enter the elements in the matrix and press ctrl var for sto→ then type C enter.

For $\mathbf{C}^2$ simply type C x² then enter.



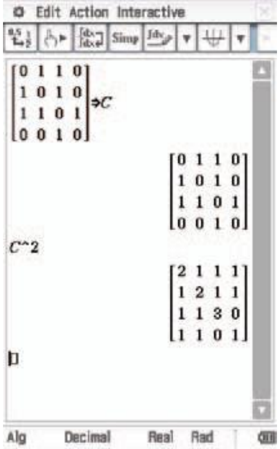
```
[0 1 1 0]         [0 1 1 0]
[1 0 1 0]  →c     [1 0 1 0]
[1 1 0 1]         [1 1 0 1]
[0 0 1 0]         [0 0 1 0]

c^2               [2 1 1 1]
                  [1 2 1 1]
                  [1 1 3 0]
                  [1 1 0 1]
```

ClassPad

Since matrix **C** is being used more than once in this question, it is advisable to store the matrix as **C**. Enter the elements in the matrix, then tap ⇒ Var CAPS C then press EXE.

For $\mathbf{C}^2$ simply tap **C**, then press ^ 2 EXE.



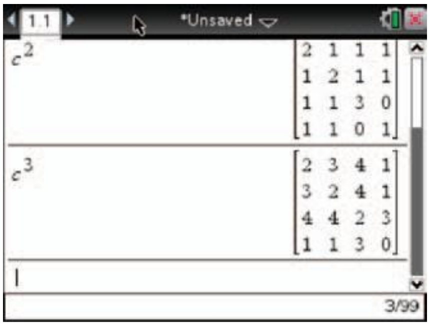
```
[0 1 1 0]
[1 0 1 0]
[1 1 0 1]⇒C
[0 0 1 0]
                  [0 1 1 0]
                  [1 0 1 0]
                  [1 1 0 1]
                  [0 0 1 0]
C^2
                  [2 1 1 1]
                  [1 2 1 1]
                  [1 1 3 0]
                  [1 1 0 1]
```

$$\mathbf{C}^2 = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

c Use a CAS to find $\mathbf{C}^3$.

TI-Nspire CAS

Since matrix **C** is already stored, for $\mathbf{C}^3$ simply type $\boxed{c}\boxed{\wedge}\boxed{3}$ then $\boxed{\text{enter}}$.



c^2

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

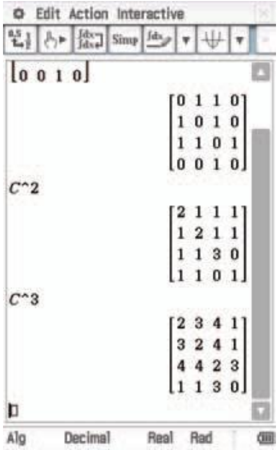
c^3

$$\begin{bmatrix} 2 & 3 & 4 & 1 \\ 3 & 2 & 4 & 1 \\ 4 & 4 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

3/99

ClassPad

Since matrix **C** is already stored, for $\mathbf{C}^3$ simply tap **C**, then press $\boxed{\wedge}$3$\boxed{\text{EXE}}$.



$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

C^2

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

C^3

$$\begin{bmatrix} 2 & 3 & 4 & 1 \\ 3 & 2 & 4 & 1 \\ 4 & 4 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

Alg Decimal Real Rad

$$\mathbf{C}^3 = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 3 & 2 & 4 & 1 \\ 4 & 4 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

d The element $(\mathbf{C}^2)_{42}$ will show the number of paths from Sean to Frank via a third party.

$(C^2)_{42} = 1$

There is 1 two-step path from Sean to Frank.

Sean → Elizabeth → Frank

e i In $\mathbf{C}^3$, element 3,2 shows the number of three-step paths from Elizabeth to Frank.

$(C^3)_{32} = 4$

There are 4 three-step paths that will allow Elizabeth to send a message to Frank.

Elizabeth → Sean → Elizabeth → Frank
Elizabeth → Julie → Elizabeth → Frank
Elizabeth → Frank → Julie → Frank
Elizabeth → Frank → Elizabeth → Frank

ii Look back at matrix **C**.

Elizabeth is able to send Frank a message directly, so there is no need to use extra steps.

EXERCISE 3.09 Applications of matrices

Concepts and techniques

1 Example 26 Each of the elements in matrix **X** are to be increased in size by 60%. Which of the following scalar multiplications will achieve this?

A 0.6**X** B 1.6**X** C 60**X** D 160**X** E 1.06**X**

2 The prices in matrix **Y** need to be reduced by 15% for a sale.

If $\mathbf{Y} = \begin{bmatrix} 2.75 & 11.55 & 3.95 \\ 10.99 & 22.35 & 15.49 \end{bmatrix}$, which matrix below shows the correct sale prices?

A $\begin{bmatrix} 2.60 & 11.40 & 3.80 \\ 10.84 & 22.20 & 15.34 \end{bmatrix}$

B $\begin{bmatrix} 2.90 & 11.70 & 4.10 \\ 11.14 & 22.50 & 15.64 \end{bmatrix}$

C $\begin{bmatrix} 2.33 & 9.81 & 3.35 \\ 9.34 & 18.99 & 13.16 \end{bmatrix}$

D $\begin{bmatrix} 2.34 & 9.82 & 3.36 \\ 9.34 & 19.00 & 13.17 \end{bmatrix}$

E $\begin{bmatrix} 3.16 & 13.28 & 4.54 \\ 12.64 & 25.70 & 17.81 \end{bmatrix}$

3 The manager of a company with three factories was asked to decrease electricity costs. Each factory has its electricity charges broken down into two categories: office electricity and production line electricity.

The March electricity costs are shown in matrix **M**.

The April electricity costs are shown in matrix **A**.

$$\begin{array}{c} \text{March} \\ \begin{bmatrix} 325 & 455 & 450 \\ 1870 & 1960 & 1950 \end{bmatrix} \end{array} \quad \begin{array}{c} \text{Office} \\ \text{Production Line} \end{array} \quad \begin{array}{c} \text{April} \\ \begin{bmatrix} 320 & 445 & 430 \\ 1830 & 1895 & 1880 \end{bmatrix} \end{array}$$

a Find matrix **D**, which shows the difference in electricity costs from March to April.

b What was the total saving in electricity costs that was achieved?

c i Calculate the overall savings made by the three offices.

  ii Express the overall office savings as a percentage of the initial office costs, correct to two decimal places.

d i Calculate the total savings achieved by the three production lines.

  ii Express the total production line savings as a percentage of the initial production line costs, correct to two decimal places.

e A reduction of 3% or more is considered significant. Comment on the manager's success or failure in making a significant cost reduction.

4 Example 27 Mark, Ian and Tina are all sales people. Their personal sales figures for the last two weeks are shown in matrix **S**.

Mark is paid a commission of 7.5 % of his sales.
Ian earns 6.7 % commission on his sales.
Tina's commission is 7 %.

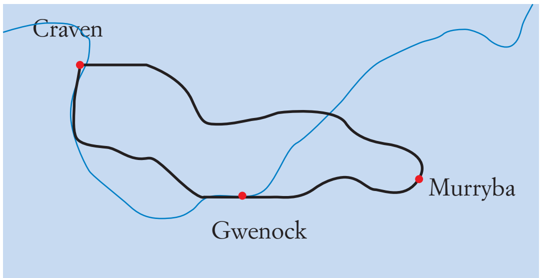
$$\mathbf{S} = \begin{array}{c} \begin{array}{cc} \text{Week 1} & \text{Week 2} \end{array} \\ \begin{bmatrix} 16\,500 & 22\,390 \\ 11\,350 & 14\,275 \\ 17\,100 & 15\,680 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Mark} \\ \text{Ian} \\ \text{Tina} \end{array}$$

a Convert the percentage commissions to decimals and present them in a row matrix named **C**.

b Use matrix multiplication to evaluate **CS**.

c How much was the total of their combined commissions for the fortnight?

5 Example 28 This map shows the linking roads between three villages.



Number of direct paths between towns

	C	M	G
C	0	1	1
M	1	0	1
G	1	1	0

a Display the number of direct paths between towns in a matrix.

b Square the matrix from part **a** to find the number of two-step paths between the towns.

c Name the pairs of villages that have 2 two-step paths between them.

6 Example 29 The number of ways that four people can directly communicate in pairs is shown in matrix **W**.

$$\mathbf{W}=\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$$

The matrix that shows the number of ways in which these four people can communicate via a third party is:

A $\begin{bmatrix}2&0&2&0\\0&2&0&2\\2&0&2&0\\0&2&0&2\end{bmatrix}$

B $\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$

C $\begin{bmatrix}0&2&0&2\\2&0&2&0\\0&2&0&2\\2&0&2&0\end{bmatrix}$

D $\begin{bmatrix}8&0&8&0\\0&8&0&8\\8&0&8&0\\0&8&0&8\end{bmatrix}$

E $\begin{bmatrix}0&1&0&2\\2&0&1&0\\0&1&0&2\\1&0&2&0\end{bmatrix}$

Reasoning and communication

7 Three department stores ordered the same four products from a manufacturer. Depending on the size of their orders, the department stores are given price reductions by the manufacturer.

The prices paid for the products are shown in matrix **P**.

$$\mathbf{P}=\begin{array}{c}\begin{array}{cccc}\text{Product W}&\text{Product X}&\text{Product Y}&\text{Product Z}\end{array}\\\left[\begin{array}{cccc}1.64&9.07&3.26&99.37\\1.62&9.04&3.24&98.99\\1.69&9.21&3.38&99.79\end{array}\right]\begin{array}{l}\text{Store 1}\\\text{Store 2}\\\text{Store 3}\end{array}\end{array}$$

Matrix **M** contains the percentage mark-up for stores 1, 2 and 3 respectively, expressed as decimals.

$\mathbf{M} = \begin{bmatrix} 1.99 & 2.01 & 1.7 \end{bmatrix}$

a Multiply matrix **P** by matrix **M**.
b Explain the meaning of the elements in **MP**.
c Calculate the combined profit across the 3 stores on Product X.

8 A school swimming carnival is being planned as a two day event. On day 1, the whole school will attend. On day 2, the only people attending will be finalists for each event and supervising teachers.

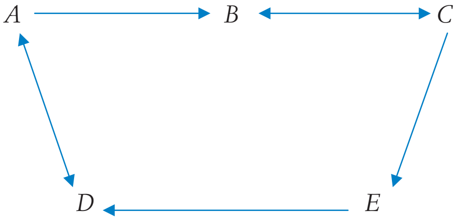
The cost of entry to the local swimming complex is \$5.60 per teacher and \$3.20 per student. Matrix **N** shows the number of students and teachers who will be in attendance over the two days.

$$\mathbf{N} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \text{Students} & \text{Teachers} \\ \left[\begin{matrix} 860 & 75 \\ 60 & 3 \end{matrix}\right. & \left.\begin{matrix} \\ \\ \end{matrix}\right] \end{array}\begin{array}{l} \\ \text{Day 1} \\ \text{Day 2} \end{array}$$


Newsphoto/Phil Rogers

a Display the entry prices in a matrix named **C** such that it can be multiplied by matrix **N**.
b Find the product **NC**.
c What will the total cost be for the 2 days?

9 The points *A*, *B*, *C*, *D* and *E* have links as shown in the diagram below.



It is possible to travel from *A* to *D* and from *D* to *A*.

However, while we can travel from *A* to *B*, we cannot travel from *B* to *A* as there is only a single arrow head on the connection between *A* and *B*.

a Draw a 5 × 5 matrix named **P** showing the number of single-step paths between these five points.
b Evaluate $\mathbf{P}^2$.
c Evaluate $\mathbf{P}^5$.
d List all of the pairs of points that can be linked using a two-step pathway.
e i State how many five-step paths can get us from point *B* to point *D*.
  ii Give details of those paths.
  iii Write a story showing why someone might follow one of the five-step pathways that you detailed in part **ii**. This only needs to be a few sentences.

CHAPTER SUMMARY
MATRIX ARITHMETIC

3

- A matrix holds data displayed in **rows** (horizontal) and **columns** (vertical).
- The order of a matrix is expressed as number of rows × number of columns.
- A matrix of order $n \times m$ has nm **elements**.
- The elements in a matrix must be real numbers.
- A **row matrix** has just 1 row.
- A **column matrix** has just 1 column.
- A **square matrix** has the same number of rows as it has columns.
- A **zero matrix** has every element equal to zero.
- **Identity matrices** are square matrices.
- The elements in the **leading diagonal** of an identity matrix are ones. The rest of the elements are zeros.
- In matrix A, a_{ij} is the element in the ith row and jth column.
- To add or subtract matrices, we simply add or subtract elements that are in corresponding positions.
- Matrices can only be added or subtracted if they are of the same order. The resulting matrix will maintain that order.
- When a matrix is multiplied by a scalar (number), each element in the matrix is multiplied by that same number.
- A row matrix can only be multiplied by a column matrix if they both contain the same number of elements. The product will be a 1×1 matrix.
- Two matrices can only be multiplied if the number of columns in the first matrix is equal to the number of rows in the second matrix.

3

CHAPTER REVIEW

MATRIX ARITHMETIC

Multiple choice

1 **Example 1** Consider the following arrays.

$$\mathbf{J} = \begin{bmatrix} 3 & 5 \\ 6 & 2 \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} -2 \\ 5 \\ 1.1 \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} 2 & 1.6 & 3 \\ 4 & 1 & -2 \\ 5 & & \end{bmatrix} \qquad \mathbf{M} = \begin{bmatrix} 12 \end{bmatrix}$$

Which statement is true?

A None of these arrays are matrices.

B **J** and **K** are matrices, but **L** and **M** are not.

C All of these arrays are matrices.

D **M** is the only array that is not a matrix.

E **L** is the only array that is not a matrix.

2 **Example 2** Matrix **T** has 4 rows and 3 columns. How many elements does it have?

A 3 B 4 C 7 D 12 E 16

3 **Example 5** Given that matrix $\mathbf{P} = \begin{bmatrix} 5 & 6 & 3 & 7 \\ 2 & 10 & 2 & 1 \\ 5 & 0 & 9 & 7 \end{bmatrix}$, $p_{23} + p_{31}$ equals

A 3 B 5 C 7 D 9 E 12

4 **Example 7** $\begin{bmatrix} 5 & 12 \\ 15 & 7 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 25 \\ 18 & 9 \\ 14 & 13 \end{bmatrix}$ gives an answer of

A $\begin{bmatrix} 139 \end{bmatrix}$

B $\begin{bmatrix} 49 \\ 49 \\ 41 \end{bmatrix}$

C $\begin{bmatrix} 12 & 37 \\ 33 & 16 \\ 22 & 20 \end{bmatrix}$

D $\begin{bmatrix} 12 & 37 \\ 33 & 16 \\ 22 & 19 \end{bmatrix}$

E $\begin{bmatrix} 12 & 33 \\ 37 & 16 \\ 22 & 19 \end{bmatrix}$

5 **Example 8** $\begin{bmatrix} 27 & 32 & 8 & 17 \end{bmatrix} - \begin{bmatrix} 18 & 12 & 20 & 5 \end{bmatrix}$ is equal to

A 29

B $\begin{bmatrix} 9 & 20 & -12 & 12 \end{bmatrix}$

C 9 20 −12 12

D $\begin{bmatrix} 9 \\ 20 \\ -12 \\ 12 \end{bmatrix}$

E $\begin{bmatrix} 27 & 32 & 8 & 17 \\ 18 & 12 & 20 & 5 \end{bmatrix}$

6 **Example 9** Consider the problem $\begin{bmatrix} 12 & x \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 15 & 11 \\ 7 & 20 \end{bmatrix} = \begin{bmatrix} 27 & 29 \\ 13 & 27 \end{bmatrix}$. The value of x is

A $\begin{bmatrix} 18 \end{bmatrix}$ B $x + 15$ C 18 D $\begin{bmatrix} 14 \end{bmatrix}$ E $18 - x$

7 Example 11 If $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then 5**C** equals

A 5 **B** $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ **C** $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$

D $\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$ **E** $\begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}$

8 Example 14 Which of the following products cannot be found?

A $\begin{bmatrix} 2 & 7 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$ **B** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 20 & 12 \end{bmatrix}$ **C** $\begin{bmatrix} 6 \\ 3 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \\ 7 & 6 & 8 \\ 3 & 5 & 9 \end{bmatrix}$

D $\begin{bmatrix} 7 & 15 \\ 3 & 6 \\ 72 & 12 \\ 11 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 8 & 13 \end{bmatrix}$ **E** $\begin{bmatrix} 0.5 & 16 \\ 3.8 & 2 \\ 14 & 2.1 \end{bmatrix}\begin{bmatrix} 7 & 6 & 12 & 44 \\ 8 & 52 & 1 & 6 \end{bmatrix}$

9 Example 21 If $\mathbf{G} = \begin{bmatrix} 5 & 4 & 9 \\ 2 & 3 & 7 \\ 6 & 8 & 5 \\ 0 & 1 & 5 \end{bmatrix}$ and $\mathbf{H} = \begin{bmatrix} 7 & 2 \\ 5 & 8 \\ 1 & 9 \end{bmatrix}$, then **GH** will be of order

A 4 × 2 **B** 12 × 6 **C** 4 × 3 **D** 2 × 4 **E** 6 × 12

10 Example 22 What is the value of y if $\begin{bmatrix} 3 & y \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 38 \\ 56 \end{bmatrix}$?

A 1 **B** 4 **C** 5 **D** 6 **E** 8

Short answer

11 Example 3 Matrix **B** has 5 columns and 4 rows. What is the order of this matrix?

12 Example 4 What type of matrix is $\begin{bmatrix} 2 & 5 & 3 \\ -10 & 7 & 1 \\ 12 & 0 & 8 \end{bmatrix}$?

13 Example 4 If a matrix has 3 rows and 1 column, what type of matrix is it?

14 Example 6 The table shows how many students use each type of transport to get to school.

	Walk	Bus	Car
Year 7	91	53	34
Year 8	86	62	27
Year 9	78	57	37
Year 10	79	54	35
Year 11	42	48	40
Year 12	34	41	53


Newspix/Angelo Soulas

a Arrange the data into a 6 × 3 matrix.

b Arrange the data into a 3 × 6 matrix.

15 Example 7 Perform the following matrix additions.

a $\begin{bmatrix} 12 & 7 \\ 19 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 13 \\ 12 & 14 \end{bmatrix}$

b $\begin{bmatrix} 23 & 14 & 52 & 16 \\ 12 & 9 & 21 & 33 \end{bmatrix} + \begin{bmatrix} 15 & 27 & 12 & 11 \\ 72 & 45 & 16 & 28 \end{bmatrix}$

c $\begin{bmatrix} 9 \\ 1 \\ 11 \end{bmatrix} + \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$

d $\begin{bmatrix} 31 & 15 & 44 \\ 27 & 32 & 51 \\ 63 & 21 & 14 \end{bmatrix} + \begin{bmatrix} 17 & 25 & 16 \\ 16 & 45 & 19 \\ 33 & 47 & 29 \end{bmatrix}$

16 Example 8 Perform the following matrix subtractions.

a $\begin{bmatrix} 13 & 18 \\ 21 & 19 \end{bmatrix} - \begin{bmatrix} 8 & 5 \\ 10 & 7 \end{bmatrix}$

b $\begin{bmatrix} 32 & 14 \\ 47 & 56 \\ 28 & 16 \end{bmatrix} - \begin{bmatrix} 10 & 12 \\ 32 & 41 \\ 7 & 9 \end{bmatrix}$

c $\begin{bmatrix} 27 & 39 & 17 \\ 52 & 67 & 84 \end{bmatrix} - \begin{bmatrix} 22 & 2 & 5 \\ 31 & 62 & 43 \end{bmatrix}$

17 Example 9 Find the values of the pronumerals in the following.

a $\begin{bmatrix} 6 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & t \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 5 & 17 \end{bmatrix}$

b $\begin{bmatrix} 12 & a & 8 \end{bmatrix} + \begin{bmatrix} 13 & 5 & 15 \end{bmatrix} = \begin{bmatrix} 25 & 9 & 23 \end{bmatrix}$

c $\begin{bmatrix} p & 19 \\ 22 & 13 \\ 41 & 52 \end{bmatrix} + \begin{bmatrix} 28 & 40 \\ 57 & 34 \\ 27 & 36 \end{bmatrix} = \begin{bmatrix} 60 & 59 \\ 79 & 47 \\ 68 & 88 \end{bmatrix}$

d $\begin{bmatrix} 16 & 22 & 14 \\ 37 & 53 & 49 \\ 55 & 12 & 17 \end{bmatrix} + \begin{bmatrix} -5 & 13 & 26 \\ -12 & -21 & 17 \\ 22 & x & 46 \end{bmatrix} = \begin{bmatrix} 11 & 35 & 40 \\ 25 & 32 & 66 \\ 77 & 1 & 63 \end{bmatrix}$

18 Example 11 Evaluate the following.

a $5\begin{bmatrix} 3 & 7 \\ 5 & 8 \end{bmatrix}$

b $3\begin{bmatrix} 6 & 9 & 11 \\ 4 & 7 & 3 \\ 2 & 10 & 6 \end{bmatrix}$

c $7\begin{bmatrix} 12 & 8 \\ 4 & 9 \\ 3 & 7 \end{bmatrix}$

d $-2\begin{bmatrix} 12 & 24 \\ 15 & 8 \end{bmatrix}$

19 Example 12 Given that $\mathbf{C} = \begin{bmatrix} 4 & 6 & 5 \\ 3 & 7 & 10 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 2 & 5 & 1 \\ 8 & 12 & 9 \end{bmatrix}$, find $3\mathbf{C} - 2\mathbf{D}$.

20 Example 15 Find **JK** if $\mathbf{J} = \begin{bmatrix} 2 & 5 & 9 \end{bmatrix}$ and $\mathbf{K} = \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$.

21 Example 16 What is the value of y if $\begin{bmatrix} 6 & 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ y \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} 98 \end{bmatrix}$?

22 Example 19 Given that $\mathbf{B} = \begin{bmatrix} 6 & 11 \\ 13 & 7 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, find **BC**.

23 Example 20 Determine the value of x if $\begin{bmatrix} 6 & 2 & 9 \\ x & 7 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \\ 11 \end{bmatrix} = \begin{bmatrix} 135 \\ 100 \end{bmatrix}$.

24 Example 22 Find the following matrix products.

a $\begin{bmatrix} 3 & 9 & 8 \\ 6 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 4 & 3 \\ 7 & 6 \end{bmatrix}$

b $\begin{bmatrix} 12 & 6 \\ 4 & 11 \end{bmatrix}\begin{bmatrix} 7 & 1 & 3 \\ 0 & 2 & 5 \end{bmatrix}$

c $\begin{bmatrix} 2 & 7 & 4 & 6 \\ 3 & 2 & 1 & 8 \\ 5 & 7 & 1 & 9 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 6 & 2 \\ 7 & 5 \end{bmatrix}$

25 Example 23 Find the value of $\mathbf{A}^2$ if $\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 4 & 5 \\ 7 & 0 & -2 \end{bmatrix}$.

26 Example 25 Find the value of $\mathbf{M}^4$ if $\mathbf{M} = \begin{bmatrix} 2 & 1 \\ -3 & 7 \end{bmatrix}$.

Application

27 A local bus company charges the following fares for passengers.

Adults – \$2.50

Concession – \$1.20

The table below shows the number of tickets sold in each category for five days.

	Monday	Tuesday	Wednesday	Thursday	Friday
Adult	123	85	112	156	139
Concession	102	115	156	108	121

a Represent the ticket prices in a 1 × 2 matrix. Let that be matrix **A**.

b Represent the data from the table in a 2 × 5 matrix. This will be called matrix **B**.

c Find the product matrix **AB**.

d How much were total ticket sales on Wednesday?

e On which day were the most tickets sold?

f On which day did the bus company receive the most money from ticket sales?

Practice quiz

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1
DOLLAR


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4

TERMINOLOGY

allowance
annual leave loading
bonus
brokerage
budget
commission
dividend
dividend yield
expenses
face value
government allowances
income
market price
overtime
piecework
price-to-earnings ratio
retainer
salary
share market
shareholder
shares or stock
stock exchange
stock market
stockbroker
wage

Prior learning

CONSUMER ARITHMETIC

EARNING MONEY

APPLICATIONS OF RATES AND PERCENTAGES

- **calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework (ACMGM002)**
- **calculate payments based on government allowances and pensions (ACMGM003)**
- **prepare a personal budget for a given income taking into account fixed and discretionary spending (ACMGM004)**
- **calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio. (ACMGM008)**

USE OF SPREADSHEETS

- **use a spreadsheet to display examples of the above computations when multiple or repeated computations are required; for example, preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year. (ACMGM009)**

4.01 WAGES, SALARIES AND OVERTIME

A **wage** is income that is paid by the week for the hours that are worked. Permanent employees may work on a full-time or part-time basis. A full-time employee usually works a 38-hour week and part-time employees work for fewer hours. Casual employees are engaged to work by the hour. Casual workers are not entitled to any benefits, such as superannuation or sick leave, and are paid a higher rate to compensate for this.

A person's wage is based on the number of hours they have worked in a given period. Hours worked in excess of ordinary working hours, outside the usual starting and finishing times, public holidays or weekends, are paid at a higher rate. Special names are given to the higher rates as summarised in the table. **Overtime** is the name given to hours worked outside the normal starting and finishing times.

Rate	Meaning
time-and-a-half	1.5 × normal rate
double time	2 × normal rate
double time-and-a-half	2.5 × normal rate
triple time	3 × normal rate

Example 1

This week, Christy worked at a childcare centre for 37 hours at \$16.40 per hour plus 6 hours overtime at time-and-a-half. Calculate her earnings for the week.

Solution

Multiply her pay rate by the number of normal hours worked.

$$\begin{aligned}\text{Normal pay} &= 37 \times \$16.40\\ &= \$606.80\end{aligned}$$

Time-and-a-half means 1.5 times the normal hourly rate. Multiply the number of overtime hours by 1.5 and the pay rate.

$$\begin{aligned}\text{Overtime pay} &= 6 \times 1.5 \times \$16.40\\ &= \$147.60\end{aligned}$$

Calculate the total earnings.

$$\begin{aligned}\text{Total earnings} &= \$606.80 + \$147.60\\ &= \$754.40\end{aligned}$$

Write the answer.

Christy earned \$754.40 for the week.

Example 2

Ian works as a bartender at an RSL club. His timesheet for the three days he worked in April is shown below.

Day	Start time	Finish time
Tuesday	3:00 p.m.	10:30 p.m.
Wednesday (Anzac Day public holiday)	9:00 a.m.	2:00 p.m.
Thursday	12:00 noon	7:00 p.m.

The rate for bartenders is \$18.17 per hour between 8:30 a.m. and 6:00 p.m., time-and-a-half between 6:00 p.m. and 11:30 p.m., and double time on public holidays. Calculate Ian's pay for the three days he worked.

Solution

Calculate the number of hours that Ian worked.

Day	Normal hours	Time-and-a-half	Double time
Tuesday	3	$4\frac{1}{2}$	0
Anzac Day	0	0	5
Thursday	6	1	0
Total hours	**9**	$5\frac{1}{2}$	**5**

Multiply his pay rate by the number of normal hours worked.

$$\begin{aligned}\text{Normal pay} &= 9 \times \$18.17\\ &= \$163.53\end{aligned}$$

Time-and-a-half means 1.5 times the normal hourly rate.
Multiply Ian's overtime hours by 1.5 and the pay rate.

$$\begin{aligned}\text{Time-and-a-half pay} &= 5\tfrac{1}{2} \times 1.5 \times \$18.17\\ &= \$149.9025\\ &\approx \$149.90\end{aligned}$$

Double-time means twice the normal hourly rate. Multiply the number of overtime hours by 2 and the pay rate.

$$\begin{aligned}\text{Double-time pay} &= 5 \times 2 \times \$18.17\\ &= \$181.70\end{aligned}$$

Calculate the total pay.

$$\begin{aligned}\text{Total pay} &= \$163.53 + \$149.9025 + \$181.70\\ &= \$495.1325\end{aligned}$$

Round to the nearest cent.

$$\approx \$495.13 \longleftarrow$$

As you are working with money, remember that you will need to round your answers to two decimal places where appropriate.

Write the answer.

Ian's pay for the three days was \$495.13.

A **salary** is a fixed amount of annual pay (yearly pay), that is paid weekly, fortnightly or monthly. The hours of work are more flexible.

IMPORTANT

Remember:

1 year = 12 months

1 fortnight = 2 weeks

1 year ≈ 52 weeks

Note: 1 month ≠ 4 weeks. To convert between weeks and months, convert to years first.

Although 52 weeks is not exactly a year, that is, $52 \times 7 = 364$ days, we use this value as an approximation when completing calculations.

Example 3

Siobhan earns a salary of $98 025 as an IT consultant. How much is she paid, to the nearest cent:

a monthly? **b** fortnightly?

Solution

a There are 12 months in a year. Divide her salary by 12.	Monthly pay = \$98 025 ÷ 12 = \$8168.75
Write the answer.	Siobhan is paid \$8168.75 a month.
b There are 26 fortnights in a year. Divide her salary by 26.	Fortnightly pay = \$98 025 ÷ 26 = \$3770.192...
Round to the nearest cent.	≈ \$3770.19
Write the answer.	Siobhan is paid \$3770.19 per fortnight.

Answers to money problems should be rounded to the nearest cent, and in some instances to the nearest 5 cents. For cash payments, amounts are rounded to the nearest 5 cents. When paying by credit, cheque or EFTPOS, amounts to the nearest cent are charged.

EXERCISE 4.01 Wages, salaries and overtime

Concepts and techniques

WS Wage sheet

WS Earning money

1 A gardener works from 6:30 a.m. to 5:00 p.m., Monday to Friday, and earns $18.45 per hour. Calculate the gardener's weekly pay, correct to the nearest cent.

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2 Example 1 A computer operator is paid \$20.50 per hour. She works 9 hours per day, Monday to Friday, and 5 hours on Saturday, for which she is paid time-and-a-half. Calculate her weekly wage.

3 An Internet website designer works a 38-hour week and earns \$2115.35 every fortnight. Calculate her earnings:

a per year   b per week   c per hour.

4 A trainee nurse works 36 hours at an ordinary rate of \$17.44 and 10 hours at time-and-a-half. Calculate her total pay.

5 A childcare worker earns \$682.40 per week. How much will she earn in:

a a year?   b a month?

6 On a long weekend, a waitress is paid time-and-a-half on Sunday and double time on Monday. How much does she earn for working 9 hours on each of the days if her normal wage rate is \$17.54 per hour?

7 Example 2 A chef earns \$23.10 per hour, but is paid time-and-a-half for working after 8:00 p.m. on weekdays and any time on weekends. The timesheet below shows the times he worked last week. Copy and complete the table, then calculate the chef's total pay for the week.

Day	Start time	Finish time	Normal hours	Overtime hours
Monday	12:00 noon	6:00 p.m.		
Tuesday	12:00 noon	6:00 p.m.		
Wednesday	12:00 noon	7:00 p.m.		
Thursday	12:00 noon	10:00 p.m.		
Friday	12:00 noon	12:00 midnight		
Saturday	4:00 p.m.	12:00 midnight		
Sunday	4:00 p.m.	10:00 p.m.		
		Total hours		

8 A personal assistant earns \$18.30 per hour for a 35-hour week, time-and-a-half for the first 4 hours of overtime, and double time thereafter. Calculate her total earnings for a week in which she worked from 8:30 a.m. to 5:30 p.m. from Tuesday to Saturday.

9 Paulini works 7.30 a.m. to 2.30 p.m. from Tuesday to Sunday. She earns time-and-a-half on Saturday and double time on Sunday. If she earns \$26.30 an hour, which expression will give her week's pay?

A $7 \times \$26.30 + 7 \times 2 \times \$26.30$

B $7 \times \$26.30 + 7 \times 1\frac{1}{2} \times \$26.30 + 7 \times 2 \times \$26.30$

C $7 \times 4 \times \$26.30 + 7 \times 1\frac{1}{2} \times \$26.30 + 7 \times 2 \times \$26.30$

D $7 \times 7 \times 1\frac{1}{2} \times 2 \times \$26.30$

E $14 \times 2 \times \$26.30$

10 Example 3 A Member of Parliament earns a salary of \$145 239. How much would he be paid:

a fortnightly?   b monthly?   c weekly?

11 A law clerk earns a salary of \$83 215. How much does he earn:

a per month? b per fortnight? c per day, if he works a 5 day week?

12 If John earns \$10 000 per month, which of the following is the correct expression to find the weekly amount?

A \$10 000 × 12 ÷ 52
B \$10 000 ÷ 4
C \$10 000 ÷ 30 × 7
D \$10 000 × 12
E \$10 000 ÷ 12

13 The table below shows the hours worked by a hotel porter. If he earns \$16.28 per hour from 0600 hours to 1900 hours and time-and-a-half otherwise, calculate his weekly pay.

Day	Start time	Finish time
Monday		
Tuesday	0630 hours	1400 hours
Wednesday	0800 hours	1530 hours
Thursday		
Friday	1500 hours	2330 hours
Saturday		
Sunday	0500 hours	1330 hours

14 If a labourer earns \$728.50 in a week where he works 38 hours at normal pay and 6 hours at time-and-a-half. His hourly rate is:

A \$11.56 B \$14.57 C \$15.50 D \$16.56 E \$19.17

15 A newspaper printer worked for 33 hours at \$16.72 per hour during the week before Good Friday, then for 6 hours at double-time-and-a-half on Good Friday. Calculate the printer's total pay.

16 A survey field hand earns \$22.18 per hour, working from 7:00 a.m. to 4:30 p.m. every weekday. Calculate how much he is paid:

a per day? b per week? c per month?

17 Which one of the following people earns the most?

A Ali: \$18.70 per hour for a 35-hour week
B Boun: \$580 per week
C Carlos: \$3460 per month
D Dimitri: \$31 600 salary
E Emily: \$1100 per fortnight

18 A baker is paid \$163.22 each day for working from 4:30 a.m. to 1:00 p.m. If he works 6 days per week, how much does he earn:

a per hour? b per week? c per year?

19 Each day, a bricklayer earns \$28.10 per hour for 9 hours and time-and-a-half for any extra hours. Last week, he worked the following number of hours each day: 9, 8, 10, 14 and 12.5. Calculate his total pay for the week.

Reasoning and communication

20 Alex worked for 32 hours at the normal rate and 4 hours at time-and-a-half, earning a total of \$720.10. Find her normal rate of pay per hour.

21 The table below shows the weekly pay of four employees at Charlie's sewing centre. Overtime is paid at time-and-a-half. Complete the table by calculating the missing values at **i**, **ii**, **iii** and **iv**.

Name	Pay rate (per hour)	Normal hours	Overtime hours	Weekly pay
Sienna	\$27.00	37	4	i
Meah	\$18.50	17	ii	\$564.25
Zac	iii	40	5	\$1662.50
Caleb	\$23.00	iv	6	\$1012

4.02 COMMISSION AND PIECEWORK

Income – commission and piecework

Some workers earn money on **commission**, which means that they get a percentage of the value of sales that they make, or a percentage based upon a service provided. Most people working on a commission or part-commission basis are involved in direct sales. For example, used car salesmen and real estate agents usually work on a commission basis.

Sometimes a **retainer** is paid as well as a commission. This is a small payment made regardless of sales to act as a 'safety net' for times when the commission payment is small.

Example 4

Ally earns a retainer of \$415 per week plus a commission of 8.5% on the value of Wonder mops sold.

a How much, to the nearest cent, does Ally earn for selling \$1595 worth of mops in one week?

b If Ally earned \$700 in a week, what was the value of mops sold that week, to the nearest dollar?

Solution

a Ally's income includes a retainer plus commission.	Earnings = retainer + commission = \$415 + 8.5% × \$1595
Convert the percentage to a decimal.	= \$415 + 0.085 × \$1595
Evaluate.	= \$550.575
Round to the nearest cent.	≈ \$550.58
Write the answer.	Ally earns \$550.58 for the week.

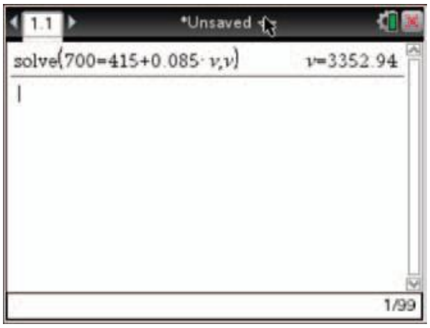
b Define a variable for the value of mops sold. Let \$$V$ = value of mops sold.

Write an equation to represent the earnings. Substitute the known values into the equation.

$$\text{Earnings} = \text{retainer} + 8.5\% \times V$$
$$700 = 415 + 0.085V$$

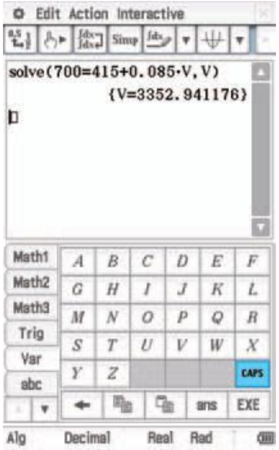
Solve for V.

TI-Nspire CAS



solve(700=415+0.085·v,v) v=3352.94

ClassPad



solve(700=415+0.085·V,V)
{V=3352.941176}

Round to the nearest dollar. $V \approx 3353$

Write the answer. The value of the mops sold is \$3353.

Some people are paid according to the number of items that they make or handle. This is called **piecework**. Fruit pickers and garment makers may earn an income in this way.

Example 5

Josh is paid 45c for each shirt folded and boxed.

- **a** Last week he processed 1440 shirts. How much did he earn?
- **b** How many shirts must he process to earn over \$1000 in a week?

Solution

a Calculate the total amount.

$$\text{Earnings} = 1440 \times \$0.45$$
$$= \$648$$

Write the answer. Josh earned \$648.

b Define a variable for the number of shirts that he needs to process. Let n = number of shirts.

Write an equation to represent the earnings. The earnings are \$1000.

$$\text{Earnings} = n \times \$0.45$$
$$1000 = 0.45n$$

Solve the equation for n.

$$n = \frac{1000}{0.45}$$
$$= 2222.222...$$

As Josh needs at least \$1000, round up to the nearest shirt. 2222 shirts would not be enough to earn \$1000.

$$\approx 2223$$

Write the answer. Josh must process at least 2223 shirts.

EXERCISE 4.02 Commission and piecework

Concepts and techniques

1 **Example 4** A taxi driver works on 45% commission. After working a normal 12-hour shift he has taken $260 in fares. How much does he earn for the shift?

2 Libby earns 28% commission for selling cosmetics door-to-door.
  a How much did she earn if she sold $1350 worth of cosmetics in a week?
  b What value of cosmetics did she sell today if she earned $62.72?

3 A car salesman works on a retainer of $250 a week plus 25% of the profit of cars sold. How much did he earn in a week where he sold:
  a no cars?
  b cars with a profit value of $7000?

4 If a taxi driver working on 45% commission makes $427.50, what is the total taken in fares?
  A $192.38
  B $382.50
  C $472.50
  D $619.88
  E $950.00

5 **Example 5** Grape pickers are paid $2.90 a tin.
  a How much would a picker earn if she picked 80 tins of grapes in a 12-hour day?
  b What is her hourly rate?

6 A computer manufacturer pays $18.50 for each motherboard that is assembled and tested. How much would you earn for doing 23 motherboards?

7 Leila is paid 75c for each garment she irons. If a basket contains 18 items and she irons five baskets, the amount she earns is:
  A $13.50
  B $17.25
  C $37.50
  D $67.50
  E $90

8 At Christmas time, Shane earns $1.05 for every item he giftwraps.
  a Calculate Shane's earnings for wrapping 114 gifts in one day.
  b How many gifts did Shane wrap on the next day if his earnings were $223.65?

9 Truong earns 11 cents for every advertising brochure he delivers to letterboxes.
  a How much does Truong earn in a day in which he delivers 1329 brochures?
  b If Truong earned $90.31 one day, how many brochures did he deliver?

10 Grant is paid 15.5 cents for every newspaper he delivers. How many newspapers did he deliver if he was paid $214.21?

Newspix/Kelly Barnes

11 A life insurance salesperson works on a retainer of \$150 a week plus a commission of 0.2% of the value of policies sold. One week she sells policies worth \$120 000, \$85 000 and \$240 000. What is her gross income for that week?

Reasoning and communication

12 A stockbroker receives a commission of 2% of the selling price of shares.

a If he sold 350 shares at \$4.30 each, calculate his commission.

b What was the value of the shares he sold if he earned \$135 commission?

13 Debbie earns 5% commission on all the plastic containers she sells. If she earned \$83.90 this week, what was the total value of the plastic containers she sold?

14 A real estate agent charges the following commission for selling properties, based on this sliding scale of property prices:

- 5% for the first \$85 000
- 3% for the next \$60 000
- 2.5% for the remaining amount.

What commission will the real estate agent earn for selling:

a an apartment for \$140 000? b a house for \$310 000?

15 Salma earns \$8.40 each time she taste tests a new brand of food. How many tastings must she complete to earn over \$500?

16 Travis is paid the following commission rates for selling mobile phone plans:

- 4.8% for the first \$3000
- 6% for the next \$3000
- 7.5% for the remaining amount.

Calculate Travis' commission for selling plans amounting to \$8758 in value.

17 An advertising material delivery firm pays its delivery people 6.5 cents per leaflet.

a How much would you earn if you delivered leaflets to 850 houses?

b How many leaflets would you have to deliver to earn at least \$180?

c If you are delivering a second leaflet as well, you get an extra 2.45 cents a leaflet for the second one. How many deliveries would you need to make in this case to get at least \$180?

18 A pizza parlour pays \$5.20 per delivery. One night, Peter delivered 24 pizzas between 5:30 p.m. and 11:30 p.m., but also covered 210 km in his car.

a How much was he paid by the pizza parlour?

b If his car uses 11 L/100 km and petrol costs him \$1.23/L, how much did he pay for petrol?

c What were Peter's real earnings and hourly rate correct to the nearest ten cents? Explain your reasoning.

19 Aziz sells music CDs that retail for \$28 each. He has the option of working for either a straight commission of 15% or a commission of 8% and a retainer of \$150 a week. How many CDs would he need to sell in a week to be better off on straight commission? Justify your decision.

20 An agent earned, as his commission, \$2024 of the actor's fee in a commercial. What percentage commission does this represent if the actor's fee was \$23 000?

4.03 BONUSES, ALLOWANCES AND ANNUAL LEAVE LOADING

Bonuses, allowances and annual leave loading are examples of extra pay.

A **bonus** is paid to employees who produce work of a high standard or volume, or for meeting an important quota, goal or deadline. The owners of some businesses give their employees a Christmas bonus to reward them for a productive year's work.

Example 6

A construction company offers a bonus to the project manager for early completion of a job. For every week he finishes ahead of schedule they offer him 20% of his monthly pay. Calculate his bonus if his salary is $230 000 and he finishes 4 weeks ahead of schedule.

Solution

Calculate his monthly pay. $\text{Monthly pay} = \$230\,000 \div 12$

$= \$19\,166.666...$

Calculate his total percentage bonus. $\text{Bonus} = 4 \times 20\%$

$= 80\%$

Calculate the bonus. $\text{Bonus} = 80\% \text{ of } \$19\,166.666...$

Do not use a rounded answer in your calculation as it may cause a rounding error later.

Evaluate. $= 0.80 \times \$19\,166.666...$

$= \$15\,333.333...$

Round to the nearest cent. $\approx \$15\,333.33$

Write the answer. The project manager would earn a bonus of $15 333.33.

An **allowance** is paid to employees for work-related expenses such as travelling, tools or laundry. An allowance can also be paid to employees who work under difficult or dangerous conditions such as in wet weather, in isolated areas, or at great heights.

Example 7

A construction worker in an isolated area earns a weekly wage of $1080.60 plus a site allowance of $104.80 per week, a meal allowance of $16.20 per day and a height allowance of $5.60 per hour. If she works 9 hours a day for 5 days, calculate her total weekly pay.

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Solution

Write all incomes as weekly amounts. $\text{Wage} = \$1080.60$

$\text{Site allowance} = \$104.80$

The meal allowance is per day so multiply this value by 5 to calculate the weekly allowance. $\text{Meal allowance} = 5 \times \$16.20$

$= \$81.00$

The height allowance is per hour, multiply this value by 9 to calculate the allowance payable per day. Then multiply by 5 to find the weekly allowance.

$$\begin{aligned}\text{Height allowance} &= 5 \times 9 \times \$5.60 \\ &= \$252.00\end{aligned}$$

Add all of the components together to calculate the total income.

$$\begin{aligned}\text{Total weekly pay} &= \$1080.60 + \$104.80 \\ &\quad + \$81.00 + \$252.00 \\ &= \$1518.40\end{aligned}$$

Write the answer.

Her total weekly pay is $1518.40.

Annual leave loading, also called holiday loading, is extra pay given during annual leave. It is usually paid at a rate of 17.5% of 4 weeks normal pay.

Example 8

Hayden earns a yearly salary of $46 598. Calculate, correct to the nearest cent, his holiday loading, which is 17.5% of the normal pay for 4 weeks.

Solution

Divide Hayden's salary by 52 to calculate his income for one week. Multiply this value by 4 to calculate 4 weeks' pay.

$$\begin{aligned}\text{4 weeks' pay} &= \$46\,598 \div 52 \times 4 \\ &= \$3584.461\ldots\end{aligned}$$

Calculate the holiday loading.

$$\text{Holiday loading} = 17.5\% \times \$3572.096\ldots$$

Evaluate.

$$\begin{aligned}&= 0.175 \times \$3572.096\ldots \\ &= \$627.280\ldots\end{aligned}$$

Round to the nearest cent.

$$\approx \$627.28$$

Write the answer.

Hayden's holiday loading is $627.28.

EXERCISE 4.03 Bonuses, allowances and annual leave loading

1 **Example 6** For meeting a target one week ahead of schedule, the owners of a firm gave their staff a bonus equal to 15% of the normal pay for one month. What is Amanda's bonus if her salary is $53 045?

A $153.01 B $612.06 C $663.06 D $5083.48 E $7956.75

2 At the end of the year, the owners of a small business gave their employees a Christmas bonus of 60% of a normal week's pay. Calculate Lisa's total pay for that week if her usual pay is $419.70.

3 **Example 7** A painter on the Sydney Harbour Bridge earns $24.15 per hour, as well as a height allowance of $25.10 per day. If he works 10 hours per day for 5 days, calculate his total earnings.

4 An oil rig worker earns a wage of \$32.56 per hour, plus a dirt allowance of \$7.49 per hour, plus an isolation allowance of \$32.50 per day. If she works $9\frac{1}{2}$ hours a day, calculate her total weekly earnings for a 5 day week.

A $9.5 \times (32.56 + 7.49)$
B $9.5 \times (32.56 + 7.49) \times 5 + 32.50$
C $9.5 \times (32.56 + 7.49) \times 5 + 32.50 \times 5$
D $9.5 \times (32.56 + 7.49 + 32.50)$
E $9.5 \times (32.56 + 7.49 + 32.50) \times 5$

5 Example 8 A police officer earns a salary of \$68 275. Calculate her annual leave loading if it is 17.5% of 4 weeks' normal pay.

6 Rebecca earns \$26.80 per hour for a 38 hour week. Calculate:

a the amount that she earns in 4 weeks.
b her holiday loading if she receives 17.5% of her 4 weeks' pay.
c her total holiday pay if she takes her 4 weeks' holiday.

7 Scientists working at the South Pole earn a salary of \$83 204 plus a weekly allowance of \$177.35 for working under extreme and isolated conditions. Calculate their fortnightly pay.

8 A car dealership pays an incentive bonus to any salesperson who can sell over \$500 000 in cars in one month. The bonus is $\frac{1}{4}$% of the salesperson's total sales in the month. Calculate Matt's bonus if he sold \$720 000 worth of cars in a month.

9 Byron works in a mine and earns an hourly rate of \$39.65. He also gets a below ground allowance of \$9.84 per hour, double his hourly rate on weekends and a meal allowance of \$27.40 per day if he works six or more hours.
Calculate Byron's total income in a week he worked the following shifts.

- Monday: 12:00 noon–5:00 p.m. above ground
- Wednesday: 2:00 p.m.–7:00 p.m. below ground
- Friday: 9:00 a.m.–12:00 noon above ground, and 2:00 p.m.–6:00 p.m. below ground
- Saturday: 10:00 a.m.–2:00 p.m. below ground, and 3:00 p.m.–5:00 p.m. above ground

10 Shaun earns a salary of \$87 500. Calculate his holiday pay if he receives 4 weeks' pay plus 17.5% holiday leave loading.

A $0.175 \times 87\,500$
B $0.175 \times 87\,500 \div 4$
C $0.175 \times 87\,500 \div 52 \times 4$
D $0.175 \times 87\,500 \div 4 + 87\,500 \div 4$
E $0.175 \times 87\,500 \div 52 \times 4 + 87\,500 \div 52 \times 4$

11 Brock works at a chemical factory and is paid \$18.14 per hour, plus an additional allowance of \$5.10 per hour for working with toxic substances. What is his total weekly pay if he works from 7:00 a.m. to 3:00 p.m. for 5 days a week?

12 When it rains, road workers are paid an extra \$4.60 per hour. Last week, Paul worked 9 hours a day for 5 days at \$15.23 per hour. If it rained continuously on two of those days, calculate Paul's total earnings for the week.

13 A telephone interviewer employed by a market research company earns \$18.20 per hour for a 35 hour week. Calculate her 4 weeks' holiday pay if her loading is 17.5% of the normal pay for 4 weeks.

Reasoning and communication

14 A miner earns a wage of $28.42 per hour, plus an allowance of $9.97 per hour for working underground in confined spaces. If the miner works from 8:30 a.m. to 4:00 p.m. each day from Monday to Friday, calculate:
- a his total weekly income.
- b how long he has to work to earn at least $2000. Round up to the nearest hour.

15 The owners of a biscuit factory gave their casual staff a bonus of $272 at Christmas, while the permanent staff received a holiday loading of 17.5% of their pay for 4 weeks. If the normal weekly pay for the permanent staff was $632.58, what was the difference between their holiday loading and the bonus given to the casual staff?

16 Each day, an electrician is paid $124.30 plus the following allowances: tool allowance $12.25, meal allowance $8.40 and laundry allowance $3.21.

Shutterstock.com/Lurin

- a If the electrician works for 6 days per week, calculate his total weekly earnings.
- b Calculate the electrician's annual leave loading if it is 17.5% of his normal pay for 4 weeks, without allowances.

17 Each week, a real estate agent earns a retainer of $510, a commission of 3.5% on the value of properties sold, and a car allowance of $128.70. Calculate her total earnings for a week in which she sold $943 658 worth of properties.

18 One year, Kirsten received a holiday loading of $403.12. What was her normal weekly wage?

4.04 GOVERNMENT ALLOWANCES AND PENSIONS

Centrelink is an Australian government agency that supports people in need and assists them to become self-sufficient. Various **government allowances** and benefits are available for the aged, the unemployed, students, parents and the disabled. All figures in this section were correct in 2013.

Age pension

The age pension is a fortnightly payment to seniors, aged 65 or over, who are unable to support themselves in their retirement. To be eligible, a person needs to have low income and limited assets. The payment and income test tables are below.

Payment per fortnight

Marital status	Full age pension payment
Single	\$712.00
A couple	\$536.70 each

Income test

Marital status	To get full payment, earnings per fortnight* can be up to:
Single	\$152
A couple	\$268

*Income over these amounts reduces the rate of pension payable by 50 cents in the dollar.

Example 9

Use the tables above to calculate the fortnightly age pension payment for each of the following seniors.

- a Beryl, aged 72, who receives \$400 per year dividend from her shares.
- b Dulcie (aged 65) and Bert (aged 69), who have a combined income of \$312 per fortnight from doing odd jobs.
- c Stanley, aged 63, who has no income and lives alone.

Solution

a Calculate Beryl's weekly income.

$$\text{Weekly income} = \$400 \div 52 = \$7.692\ldots$$

Calculate her fortnightly income.

$$\text{Fortnightly income} = \$7.692\ldots \times 2 = \$15.384\ldots$$

Answer the question by looking at the table.

Beryl is single and earns less than \$152 per fortnight, so she receives the full payment of \$712.

b Check the income test table.

To receive the full payment, the couple must earn less than \$268 per fortnight. So Dulcie and Bert need to reduce the payment by 50c for each dollar earned over \$268.

Calculate their combined pension.

$$\begin{aligned}\text{Combined pension} &= (2 \times \$536.70) \\ &\quad - 0.5 \times (\$312 - \$268) \\ &= \$1073.40 - \$22 \\ &= \$1051.40\end{aligned}$$

Write your answer.

Dulcie and Bert receive a combined pension of \$1051.40 per fortnight.

c Write your answer.

Stanley is not eligible for the age pension because he is under 65 years old.

Youth and study allowances

There are several allowances available for young people or students. Some examples are shown in the table below.

Allowance	Conditions for eligibility
Youth allowance	Aged 16–20 years, studying full-time, undertaking an apprenticeship or looking for work
	Aged 21–24 years, studying full-time or undertaking an apprenticeship
Austudy	Aged over 25 years and studying full-time or undertaking an apprenticeship
ABSTUDY	Indigenous students or apprentices

Example 10

The tables below show the maximum fortnightly amounts for youth allowance and Austudy.

Youth allowance

Conditions for eligibility	Maximum fortnightly payment
Single with no children, under 18 years and living at home	$223
Single with no children, under 18 years and not living at home	$407.50
Single with no children, 18 years and over and living at home	$268.20
Single with no children, 18 years and over and not living at home	$407.50
Single with children	$533.80
Partnered with no children	$407.50
Partnered with children	$447.40

Austudy (for people aged over 25, studying full-time in an approved course, or undertaking a full-time apprenticeship.)

Conditions for eligibility	Maximum fortnightly payment
Single	$407.50
Single, with children	$533.80
Partnered, no children	$407.50
Partnered, with children	$447.40

Calculate the fortnightly youth allowance or Austudy payment for each of the following people.

a Pablo, aged 17, a first-year apprentice plumber living with his parents

b Cameron, aged 27, studying his HSC full-time at TAFE while raising his son alone

c Sally, aged 32, studying part-time and living with her husband and two children

d Greg, aged 16, has left school, is homeless, and is looking for full-time employment.

Solution

a Check Pablo's eligibility.	Pablo is under 18 and is living at home.
Read the amount paid.	He receives a youth allowance of $223.
b Check Cameron's eligibility.	Cameron is over 25, studying full time and a single parent.
Read the amount paid.	He receives an Austudy payment of $533.80.
c Check Sally's eligibility. Write your answer.	Sally is not eligible for youth allowance as she is over 24. Sally is not eligible for Austudy, because she is not studying full-time.

d	Check Greg's eligibility.	Greg is under 18, is not living at home and is looking for work.
	Read the amount paid.	He receives a youth allowance of $407.50.

Example 11

The table below shows fortnightly ABSTUDY allowances for independent Indigenous students who are studying or undertaking an Australian apprenticeship.

ABSTUDY

Conditions for eligibility	Allowance per fortnight
Single, no children	
16–21 years, not at home	$407.50
Under 16, at home	$223
16–17 years, at home	$223
18–21 years, at home	$268.20
22 years and over	$492.60
Partnered, no children	
16–21 years	$407.50
22 years and over	$444.70
Single with dependent child	
16–21 years	$533.80
22 years and over	$533.00
Partnered with dependent child	
16–21 years	$447.40
22 years and over	$444.70

Calculate the fortnightly ABSTUDY payment for each of the following Indigenous students.

a Joshua, aged 18, who is living with his partner and her eight-year-old son.
b Alanah, aged 14, who is living at home with her parents.
c Susanna, aged 27, who is living with her husband and has no children.

Solution

a	Check Joshua's eligibility.	Joshua is in the category of 16–21 and is partnered, with a dependent child.
	Read the amount paid.	He receives $447.40.
b	Check Alanah's eligibility.	Alanah is under 16 and is living at home.
	Read the amount paid.	She receives $223.
c	Check Susanna's eligibility.	Susanna is 22 years and over and is partnered, with no children.
	Read the amount paid.	She receives $444.70.

EXERCISE 4.04 Government allowances and pensions

Use the tables from Examples **9, 10** and **11** to answer the questions in this exercise.

Concepts and techniques

1 Example 9 If the following people all satisfy the Centrelink assets test, calculate their fortnightly age pension payments.

a Elsie, aged 89, who receives \$850 per year from her investments

b George, aged 71, and Beryl, aged 75, who care for their disabled daughter full-time and have a combined income of \$144 per week from doing odd jobs

c Dell, aged 62 years, who has no savings, no job and lives alone.

2 Why do you think that age pensioners receive less if they are partnered rather than single?

3 The Newstart allowance is paid to unemployed people aged 21 years or over. The maximum fortnightly rates are shown in the table.

Status	**Dependants**	**Newstart allowance**
Single	No children	\$492.60
Single	Children	\$533.00
Partnered		\$444.70

a Hani is unemployed, and is single with no children. How much Newstart allowance would she receive if it was paid:

i fortnightly? ii yearly?

iii monthly?

b Jay is married, unemployed and has two children. How much does he receive over 6 months?

c Does a single unemployed person receive more, less or the same as one with a partner? Is this fair? Explain.

4 Example 10 Carrie is 18 years old and is studying full-time at a senior high school. She lives with her parents and has a two-year-old daughter. Which of the following is her youth allowance?

A \$223 B \$268.20 C \$407.50 D \$447.40 E \$533.80

5 The parenting payment is paid to parents of children aged 18 years or younger and is not dependent on the number of children they have. The maximum fortnightly rates are shown in the table on the right.

Status	**Parenting payment**
Partnered	\$444.70
Partnered but separated due to illness, respite or prison	\$533.00
Single	\$663.70

a Ron and Vicky Taylor have three children: Daniel 15, Julianne 12 and Liam 8. How much does the Taylor family receive as a parenting payment per fortnight?

b The Lovejoys have ten children under 18 at home and two children living away from home. Mr Lovejoy is currently in gaol. What parenting payment does Mrs Lovejoy receive per fortnight?

c Karen is divorced and has a boy aged 9 years and a girl aged 7 years. What is her fortnightly parenting payment?

d Kell's wife is in hospital for the next 12 months. He is looking after their twin 11-year-old boys. How much parenting payment will he receive for:

i 1 year? ii 1 week?

6 Example 11 Dean is a 19-year-old Indigenous student living at home with his parents. Which of the following is his fortnightly ABSTUDY allowance?

A $223 B $268.20 C $407.50 D $447.40 E $533.80

7 Calculate the fortnightly ABSTUDY payment for each Indigenous student described below.
- a Jill is 17 years old, and is living with her partner.
- b Brett is 28 years old, and is living at home with his wife and two children under 10 years.
- c Molly is 21 years old, and is living away from her parents' home with her boyfriend.
- d Dee is 15 years old, and is living at home with her grandparents.

4.05 BUDGETING

A **budget** is a plan for managing your income wisely. Whether you are running a household, a business or just managing your pocket money, it is essential that you do not spend more than you earn. A budget is divided into **income** and **expenses**. A balanced budget is one in which the total of expenses is equal to the total income. When looking at expenses there are some which are fixed expenses and some that can vary, these are called discretionary expenses. Fixed expenses are things like rent and loan repayments. Discretionary expenses are things like entertainment and clothing.

Example 12

Lucy was saving for an overseas trip and accepted some extra casual work. She set up a weekly budget, as shown in the table below, and aimed to save $150 per week.

Income		**Expenses**	
Wages	$ 681	Rent	$ 265
Casual wage	$____	Groceries	$ 143
		Fares	$ 42
		Car loan	$ 90
		Petrol/car maintenance	$ 63
		Entertainment	$ 185
		Savings	$____
Total:	$ 884	**Total:**	$____

- a How much did Lucy earn from her casual job?
- b To balance her budget, what should be the total of Lucy's expenses?
- c How much can Lucy save per week?
- d Suggest a way Lucy could increase her savings to $150 per week.

Solution

a Subtract Lucy's wages from her total income.

Casual wage = $884 – $681
= $203

Write your answer.

Lucy earned $203 from her casual job.

b Lucy's total expenses should be the same as her total income.

$884

c Add all of her expenses.

$$\text{Expenses} = \$265 + \$143 + \$42 + \$90 + \$63 + \$185 = \$788$$

Subtract the total expenses from the total income.

$$\text{Savings} = \$884 - \$788 = \$96$$

d Items like rent and car loan are fixed expenses and could only vary by selling the car or moving house. Lucy needs to consider how she could cut back on her discretionary expenses.

The easiest options would be for Lucy to cut down on her entertainment and look for more specials when grocery shopping. She could also look to rent with a friend or sell her car.

When you first start working out a personal budget, try to do it on a weekly, fortnightly or monthly basis, depending on how often you are paid. A budget can be used to help work out how much money must be set aside for expenses that come less frequently, such as power and phone bills, clothes and insurance.

Example 13

Karen earns $610 a week after tax. She rents a furnished unit with a friend and her share of the rent is $350 a fortnight. She spends about $210 a week on food, household items and toiletries. She catches the train to work each weekday and pays $4.90 for a return ticket. The power bills total about $280 a quarter and she pays half of this. Karen estimates that she spends $310 a month on clothes.

Draw up a weekly budget and work out how much she can save in a year if she sticks with this budget.

Solution

Convert each expense to a weekly cost.

$$\begin{aligned}\text{Rent} &= \$350 \div 2\\ &= \$175 \text{ per week}\end{aligned}$$

Karen travels 5 days a week.

$$\begin{aligned}\text{Fares} &= \$4.90 \times 5\\ &= \$24.50 \text{ per week}\end{aligned}$$

Convert the power bill to a yearly value by multiplying by 4. Remember that Karen only pays half of it.

$$\begin{aligned}\text{Power} &= \frac{1}{2} \text{ of } \$280 \times 4\\ &= \frac{1}{2} \times \$1120\\ &= \$560 \text{ per year}\end{aligned}$$

Convert the power bill to a weekly cost by dividing by 52.

$$\begin{aligned}\text{Power} &= \$560 \div 52\\ &= \$10.769\ldots\\ &\approx \$11 \text{ per week}\end{aligned}$$

Convert the clothing cost to a yearly cost by multiplying by 12.

$$\begin{aligned}\text{Clothes} &= \$310 \times 12\\ &= \$3720 \text{ per year}\end{aligned}$$

Convert this total to a weekly cost by dividing by 52.

$$\begin{aligned}\text{Clothes} &= \$3720 \div 52\\ &= \$71.538\ldots\\ &\approx \$72 \text{ per week}\end{aligned}$$

Draw up a table with income on the left and expenses on the right.

Income		Expenses	
Wages	$610	Rent	$175
		Groceries	$210
		Fares	$24.50
		Power	$11
		Clothes	$72
		Savings ____	$
Total:	$610	**Total:**	$610

Calculate Karen's weekly savings by subtracting her total expenses from her total income.

$$\begin{aligned}\text{Weekly savings} &= \$610 - (\$175 + \$210 \\ &\quad + \$24.50 + \$11 + \$72) \\ &= \$117.50\end{aligned}$$

Calculate Karen's yearly savings.

$$\begin{aligned}\text{Yearly savings} &= \$117.50 \times 52 \\ &= \$6110\end{aligned}$$

Write your answer.

Using this weekly budget Karen can save approximately $6110 in one year.

EXERCISE 4.05 Budgeting

Concepts and techniques

Budget grid

Budgeting scenarios

1 Categorise each of the following items as either income (I) or expense (E).

- a a car insurance payment
- b an inheritance from your grandmother
- c a birthday present for a friend
- d the cost of a plane ticket to London
- e a second prize win in the lottery
- f the monthly rental of a computer
- g a rent payment to a real estate agent
- h the admission price to the cinema
- i the rent from an investment unit
- j pay to an electrician for an installation
- k your weekly wage
- l the cost of a gym class
- m pocket money
- n a Christmas present from your aunt

2 Example 12 Mariella's budget allowed her to save $60 each week. Her budget is shown below.

Income		Expenses	
Youth allowance	$185.70	Rent	$160
Casual job	_____	Fares	$56
		Groceries	$124
		Clothes	$25
		Entertainment	___
		Bills	$58
		Savings	$60
Total:	$537.00	**Total:**	$___

- a How much did Mariella earn for her casual job?
- b What amount did Mariella spend on entertainment?
- c Suggest two ways that Mariella could increase her weekly savings to $80.

9780170250252

3 Jess wants to save \$2200 for an end of year trip. She earns \$615.40 after tax each week and has the following weekly expenses: meals \$40, rent \$138, groceries \$112, fares \$36, tennis \$20, entertainment \$60, car loan \$85, bills \$65 and magazines \$8.

   a Calculate the amount Jess has left for savings each week.
   b Will Jess have enough saved to pay for her trip in one year?
   c If the trip price increases by 20%, would Jess have enough saved in one year to pay the new price?

4 The Rogers family has the following weekly incomes: wages \$1901.77, part-time job wages \$289.53, and parenting allowance \$197.20. The family also has the following weekly expenses: bills \$105.30, school fees \$85.80, entertainment costs \$295, health fund payment \$53.40, clothes \$132, home maintenance \$184, groceries \$210.50, petrol \$85.40, Friday night takeaway \$46, newspapers and magazines \$26, home loan repayments \$545.60, and car loan repayment \$184.22.

   a Use a table or spreadsheet to display this data as a budget, adding an entry for savings.
   b How much does the Rogers family save per week?

Reasoning and communication

5 Example 13 Zac shares a flat and pays \$110 a week towards the rent. Each week he spends \$94 on food and \$46 on public transport. His contribution to the quarterly electricity bill and two-monthly phone bill are \$105.10 and \$42.80 respectively. He earns \$551.40 per week, spends about \$45 per week on entertainment and visits a chiropractor once a month at a cost of \$42 per visit.

   a Convert Zac's expenses to weekly amounts and draw up a weekly budget, including an amount for savings.
   b What is Zac's weekly income?
   c What is the total of Zac's weekly expenses?
   d How much can Zac save per week?
   e Has Zac left out anything obvious in his budget? If so, what?

6 Jack earns \$1072 a fortnight after tax. He shares a house with three other people. The house costs \$350 a week and the shared power bill is usually about \$420 for a quarter. Jack buys lots of takeaways and spends about \$220 a week on food and other household items. He is paying off his car at \$160 a month and spends about \$75 a fortnight to run it. Jack spends about \$1000 a year on clothes. He finds it impossible to save any money, but does go to the football on Saturday and usually goes out on Saturday night.

   a Draw up a budget and work out how much he spends going to the football and going out on Saturday night.
   b What could he save in a year if he gave up his Saturday entertainment?

7 Aya is sharing a small furnished flat in a big house with her best friend. She earns \$720 a week. The flat costs them \$820 a fortnight, but that includes power. Aya hardly ever eats out, so she spends only \$115 a week on food and household necessities. She walks to the train station and pays \$45 for a weekly train ticket to get to and from work. She has a mobile phone that is on an \$89 a month plan. Aya spends about \$2900 a year on clothes and has a large family and spends approximately \$1400 a year on gifts. She gives herself \$50 a week for entertainment and is trying to save up for an overseas trip. Draw up a budget and work out how much she could save in a year.

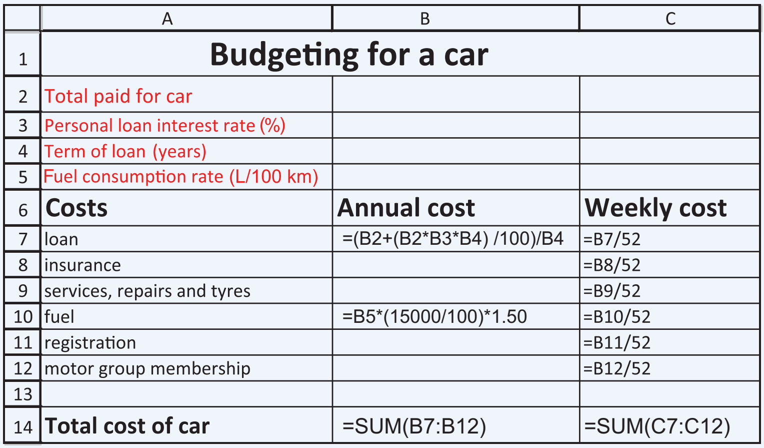
INVESTIGATION Budgeting for a car

Excel spreadsheet: Budgeting for a car

Kim is 18 years old and she is investigating the real cost of owning and running a second-hand car.

- She buys a 2010 Holden Barina for $15 000.
- She takes out a personal loan to cover the cost of the car. The personal loan has an interest rate of 7% p.a. simple interest and she wants to repay the loan in 3 years.
- She wants to get comprehensive insurance. She gets a quote for $1150 a year.
- She budgets that it will cost approximately $600 a year for services. She also allows $400 a year for repairs and tyres.
- Her car has a fuel consumption of 7 L/100 km. She estimates that she will travel around 15 000 km in a year. She is budgeting on the average price of fuel being $1.50 per litre.
- She contacted the transport authority and found that the cost of her registration (including third-party insurance) will cost $800 a year.
- She decides to get a basic membership in the motoring group in her state so that she is covered for roadside assistance. Her motoring club has a membership deal for 18-year-olds that is $44.

a Set up a spreadsheet, like the one below, called ‘Budgeting for a car’.



	A	B	C
1	**Budgeting for a car**		
2	Total paid for car		
3	Personal loan interest rate (%)		
4	Term of loan (years)		
5	Fuel consumption rate (L/100 km)		
6	**Costs**	**Annual cost**	**Weekly cost**
7	loan	=(B2+(B2*B3*B4) /100)/B4	=B7/52
8	insurance		=B8/52
9	services, repairs and tyres		=B9/52
10	fuel	=B5*(15000/100)*1.50	=B10/52
11	registration		=B11/52
12	motor group membership		=B12/52
13			
14	**Total cost of car**	=SUM(B7:B12)	=SUM(C7:C12)

b Collate the given information onto your spreadsheet.

c From the spreadsheet, what is the cost of Kim’s car per week?

d Kim earns $324 per week from her apprenticeship. She decides that she really can’t afford that much of her pay going towards a car. What is the difference in her cost per week if:

- **i** she decides to get 3rd party insurance for $240 a year, instead of comprehensive?
- **ii** she decides to go with a different car that costs $8000 and has a fuel consumption of 8 L/100 km. She stays with the 3rd party insurance on this car and all the other costs are approximately the same.

e Now investigate the cost of owning and running a second-hand car for yourself.

- Choose the type of car.
  (You can obtain car prices from a website such as:
  www. redbookasiapacific.com.au/, www. carsguide.news.com.au/, www.carprices.com/)
- Find the interest rate on a personal loan (you can obtain these from bank websites).
- Contact an insurer to find out how much car insurance would cost for an 18-year-old.
- Check with a local service station to find the cost of services and ask how much you are likely to spend on repairs (you could also check with other car owners).
- Find the typical fuel consumption for the car you have chosen and estimate the number of kilometres you are likely to travel in a year.
- Contact the transport authority in your state to find how much registration (including third-party insurance) will cost a year.
- Ask your state motoring club (i.e. RACV, NRMA, RACQ, etc) about the costs and benefits of being a member. Consider whether you would join.
- Collate the information you have gathered into the spreadsheet you created earlier. Use it to find the real cost of owning and running a second-hand car for an 18-year-old.
- Compare this with the likely earnings of someone of this age.
- What proportion of earnings would be spent on the car you selected?

f How could someone save on these expenses? List at least two options.

4.06 INVESTING IN SHARES

The **stock market or share market** is another form of investment for many Australians. A privately owned company can be floated on the stock market to become a publicly owned entity. It can then sell **shares** at a nominated price per share. This price is known as its **face value**. Investors can buy shares in the company. Each share means that the **shareholder** owns part of the company and may receive part of the profits. Investors can then choose to sell some or all of their shares on the **stock exchange**. If investors think a company is doing well, the price will rise as investors buy more shares and if they think it is doing badly, the price will fall as they sell. The price that shares are traded for on the stock exchange is called the **market price**.

Each year, the directors of a company calculate how much profit has been made. Some of the profit is distributed to shareholders as a **dividend** for every share held. A dividend is usually expressed as cents per share; for example: 38.1c per share.

When trading shares on the stock exchange a **stockbroker** does the trading for you. A stock broker charges a commission, which is called the **brokerage**. This cost should be taken into account when calculating your earnings. The rates for brokerage are different for different stockbrokers.

Example 14

Alyssa bought 8500 shares in Symbio, a listed biotechnology company for \$7.80 each. The company paid a full year dividend of 47.3c a share.

- **a** How much did the shares cost her?
- **b** If her stockbroker charges brokerage of 1.8%, calculate her brokerage fees.
- **c** Calculate her dividend payment.
- **d** At the end of the year she sells her shares for \$8.10. Calculate Alyssa's total earnings over the whole year, after costs.

Solution

a Calculate the cost of Alyssa's shares.

$$\begin{aligned}\text{Cost of shares} &= 8500 \times \$7.80\\ &= \$66\,300\end{aligned}$$

b Calculate the brokerage fee.

$$\begin{aligned}\text{Brokerage} &= 1.8\% \text{ of } \$66\,300\\ &= 0.018 \times 66\,300\\ &= \$1193.40\end{aligned}$$

c Multiply the dividend by the number of shares.

$$\begin{aligned}\text{Dividend} &= 8500 \times 47.3\text{c}\\ &= 402\,050\text{c}\\ &= \$4020.50\end{aligned}$$

d Calculate the selling price of the shares.

$$\begin{aligned}\text{Selling price} &= 8500 \times \$8.10\\ &= \$68\,850\end{aligned}$$

Calculate the brokerage for selling.

$$\begin{aligned}\text{Brokerage} &= 1.8\% \text{ of } \$68\,850\\ &= 0.018 \times 68\,850\\ &= \$1239.30\end{aligned}$$

Calculate the total costs.

$$\begin{aligned}\text{Total costs} &= \text{cost of shares} + \text{brokerage (buying)}\\ &\quad + \text{brokerage (selling)}\\ &= \$66\,300 + \$1193.40 + \$1239.30\\ &= \$68\,732.70\end{aligned}$$

Calculate the earnings.

$$\begin{aligned}\text{Earnings} &= \text{selling price} + \text{dividend}\\ &= \$68\,850 + \$4020.50\\ &= \$72\,870.50\end{aligned}$$

Calculate the total earnings after costs.

$$\begin{aligned}\text{Total earnings} &= \text{earnings} - \text{total costs}\\ &= \$72\,870.50 - \$68\,732.70\\ &= \$4137.80\end{aligned}$$

Write the answer.

Alyssa's total earnings over the whole year, after costs, were \$4137.80.

Example 15

A company has 5.6 million shares and makes an after tax profit of $13.1 million. The company directors decide to pay a dividend of 15% of their profit. What is the dividend paid per share, rounded to the nearest cent?

Solution

Calculate 15% of the total profit.

$$\begin{aligned} &15\% \text{ of } \$13.1 \text{ million} \\ &= 15\% \text{ of } \$13\,100\,000 \\ &= 0.15 \times \$13\,100\,000 \\ &= \$1\,965\,000 \end{aligned}$$

The dividend is 15% of total profit divided by the number of shares.

$$\begin{aligned} \text{Dividend} &= \frac{15\% \text{ of the profit}}{\text{number of shares}} \\ &= \frac{\$1\,965\,000}{5\,600\,000} \end{aligned}$$

Evaluate.

$$\begin{aligned} &= \$0.3508... \\ &= 35.08...\text{c} \end{aligned}$$

Round to the nearest cent.

$$\approx 35\text{c}$$

Write the answer.

The dividend paid is 35 cents per share.

Dividend yield

When investors are looking at their options they need a way of comparing normal bank deposits with the expected returns of stock. As the dividend is determined annually, the **dividend yield** can be compared with the effective interest rate of a bank deposit.

The dividend yield compares the dividend with the market price and is usually written as a percentage. It gives the annual percentage return on the shares. For example, a company with a 5% dividend yield means that the investor has the potential to earn back $5 for every $100 invested.

IMPORTANT

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

Example 16

Jessica had 400 bank shares valued at $7740 and received a dividend of 32c per share.

a What was the dividend yield, correct to two decimal places?

b Jessica could have invested her money in a long term deposit earning 4.3% p.a. Which would have been the best option? Explain.

Solution

a Calculate the market price of each share.

$$\begin{aligned} \text{Market price} &= \$7740 \div 400 \\ &= \$19.35 \end{aligned}$$

Write the dividend in dollars.

$$\text{Dividend} = \$0.32$$

Write the formula.

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

Substitute the known values into the formula and evaluate.

$$\text{Dividend yield} = \frac{0.32}{19.35} \times 100\%$$
$$= 1.653\ldots\%$$

Round to two decimal places.

$$\approx 1.65\%$$

Write your answer.

The dividend yield is 1.65%.

b Compare the dividend yield with the percentage interest.

The dividend yield is lower than the interest rate.

Write the answer.

Jessica would have been better off to invest the money with the bank as the interest rate is better than the dividend yield.

Comparing price to earnings

IMPORTANT

$$P/E = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

The **price-to-earnings ratio (P/E)** is a common measure used by investors to compare the value of stocks. It compares the market value with the company's earnings per share. For example, if Company A has a P/E of 12 and Company B has a P/E of 15, this means that for every $1 of current earnings the investor is paying $12 a share to invest in Company A and $15 a share to invest in Company B. So, all being equal between the two companies, Company A is a better investment.

It is important to remember that the P/E is best used to compare stocks in similar industries.

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Example 17

Martin has decided that he wants to invest in some technology stock. He has narrowed it down to two companies Digimatt and EmBec. He collected the following information on both.

Digimatt: A total of 3 million shares. A market price of \$21.50 per share and annual earnings of \$3.4 million.

EmBec: A market price of \$7.29 per share. A dividend of 11.2c per share and 20% of the company's earnings was paid as a dividend.

a Calculate the price-to-earnings ratio, correct to one decimal place, for Digimatt.
b Calculate the price-to-earnings ratio, correct to one decimal place, for EmBec.
c Based on the price-to-earnings ratio, what would you recommend to Martin? Explain.

Solution

a Divide the annual earnings by the number of shares.

$$\text{Annual earnings per share} = \frac{\$3.4\text{ million}}{\$3\text{ million}} = \$1.13...$$

Write the formula.

$$\text{P/E} = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

Substitute the known values into the equation.

$$\text{P/E} = \frac{\$21.50}{\$1.13...}$$

Evaluate.

$$= 18.97...$$

Round to one decimal place.

$$\approx 19.0$$

Write the answer.

The price-to-earnings ratio for Digimatt is 19.0.

b Write an equation to find the total earnings per share. Let x represent the total earnings per share.

$$20\% \text{ of } x = 11.2$$
$$0.2x = 11.2$$

Solve for x.

$$x = \frac{11.2}{0.2}$$
$$= 56 \text{ cents per share}$$

Write in dollars.

$$= \$0.56$$

Write the formula.

$$\text{P/E} = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

Substitute the known values into the formula.

$$\text{P/E} = \frac{\$7.29}{\$0.56}$$

Evaluate.

$$= 13.01...$$

Round to one decimal place.

$$\approx 13.0$$

Write the answer.

The price-to-earnings ratio for EmBec is 13.0.

c Martin can then make a decision based on comparison of price-to-earnings ratios.

The price-to-earnings ratio is lower for EmBec than for Digimatt. This means that for every dollar of current earnings the investor is paying less to invest in EmBec. So, all other things being equal, it looks like EmBec is the better option.

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EXERCISE 4.06 Investing in shares

Concepts and techniques

WS Share tables

WS Share graphs

1 Susan had $10 000 to invest. How many shares could she buy if the market price is $16.50?

2 Caltex shares have a market value of $7.63. If the dividend is 35% of the market value, calculate the dividend earned from 4600 Caltex shares.

3 A broker charges 1.8% commission on all sales. How much brokerage was charged if 25 500 shares were bought for 89c per share?

4 Example 14 Paul bought 1500 bank shares for $29.29.
- a How much did they cost him?
- b His broker charges commission of 1.4% of the total sales. Calculate the brokerage Paul paid.
- c In the first year the bank paid a dividend of 84c per share and in the second year they paid a dividend of 95c per share. Calculate Paul's total dividend payment for the two years.
- d After two years Paul sells his shares for $24.25. His broker charged him 1.5% brokerage. Calculate his total profit or loss over the two years.

5 Example 15 A company decides to pay a dividend of 12% of its after tax profit of $750 000.
- a What is the total amount paid as a dividend?
- b If it has a total of 100 000 shares, what is the dividend paid per share?

6 A company that has 2.5 million shares makes an after tax profit of $17.1 million. The directors decide to pay a dividend of 24% of the total profit. Calculate the dividend paid per share.

7 Example 16 Sharon had 1600 EHD shares that had a market price of $7.56 per share. She received a dividend payment of $720.
- a Calculate the dividend paid per share.
- b Calculate the dividend yield, correct to one decimal place.

8 Kell bought 100 shares at $4 each and received a dividend of 5c per share. Which of the following is the dividend yield?

A 1.25% B $5 C 12.5% D $20 E 80%

9 Adrian wants to purchase 2500 shares in BOC Banking. The market price of the shares is $14.30.
- a Calculate the total cost of the shares.
- b Adrian's stockbroker charges a commission of 2% of the cost of the shares. Calculate the total brokerage.
- c A dividend of 47c was paid when the market price of the share was $16.50. Calculate the dividend yield, correct to two decimal places.

10 Terry bought 4000 shares for $8920, with a dividend yield of 12%. Calculate:
- a the market price of one share
- b the total dividend earned.

11 Zeli bought 40 000 shares in Austar at $0.75 each. For buying or selling, her stockbroker charged 2.5% brokerage for the first $15 000 of shares and 0.75% thereafter.
- a Calculate the total purchase cost of Zeli's shares.
- b If Zeli collects a dividend yield of 4.8%, calculate the dividend paid.
- c She then sells all her shares for $1.09 each. Calculate her total earnings after costs.

12 Vicky bought 8200 shares in the Ten Network at $1.02 each.

a Calculate the value of her shares.

b Additional costs were:

- broker's service fee: $5.50
- brokerage: 2.5% of the share value up to $5000
  2% of the next $10 000
  1.5% of the next $35 000
  1% of the remainder.

Calculate the total cost of buying the shares.

c If the company pays a dividend of 28c per share, calculate Vicki's dividend.

d Calculate the dividend yield, correct to two decimal places.

13 Karl manages the share portfolio described in the table below.

	Karl's portfolio		
Number of shares	**Company**	**Market value**	**Dividend yield**
1000	Pacifc Mining	$0.25	3.3%
600	QBE Insurance	$23.20	6%
800	Tabcorp	$6.80	5.1%
1700	C-C Amatil	$9.64	5.2%

a Calculate the total value of Karl's shares.

b Calculate the total dividend earned.

14 Example 17 Lachlan bought 1500 RMD shares for $1680. They had annual earnings of 75c per share.

a Calculate the market price per share.

b Calculate the price-to-earnings ratio, correct to one decimal place.

15 SDD shares have a market value of $7.85. They paid a dividend of 12.5c per share and 15% of the company's earnings were paid as a dividend. Calculate:

a the total earnings per share

b the price-to-earnings ratio.

16 Olivia bought 17 000 Timbo shares for $6460. They had annual earnings of 3c per share. Calculate the price-to-earnings ratio of Timbo shares, correct to the nearest whole number.

17 Charlie bought shares for $6.75 each. The directors of the company decided to pay 70% of the company profits to shareholders as a dividend. The dividend was 28 cents per share. Calculate:

a the dividend yield

b the price-to-earnings ratio, correct to one decimal place.

18 Paul bought shares with a dividend yield of 8%. The company returned 55% of its profits to shareholders as a dividend. He paid $1550 for 200 shares. Calculate:

a the market price

b the dividend per share

c the total earnings per share

d the price-to-earnings ratio, correct to one decimal place.

Reasoning and communication

19 George owns 2500 AGL energy shares. The dividend per share is 42c and the market price is $14.05.

- a What dividend will George receive?
- b Calculate the dividend yield, correct to one decimal place.
- c How many extra shares could George purchase if he reinvested his dividend?

20 Evan is looking to invest some money. He is looking at two options:

A: Buying 2000 shares in an oil exploration company with a market value of $3.40 each. They are paying a dividend of 18 cents per share.

B: Putting the same amount of money in his credit union where he can get interest of 5% p.a.

- a Calculate the dividend yield of the shares, correct to one decimal place.
- b Would he do better to buy the shares or invest the money with the credit union?
- c What is the benefit of the credit union over the shares?

21 Kayla was investing money and wanted to choose the option that gave her the best return. Which of the following would be the best investment?

- A Shares paying a dividend of 30c per share with a market price of $3.20.
- B Shares paying a dividend of $3.28 per share with a market price of $34.50.
- C Shares paying a dividend of 87c per share with a market price of $8.90.
- D A bank offering interest of 9.1% p.a.
- E Shares paying a dividend of $2.50 per share with a market price of $26.

22 Scott had 2900 JAD shares worth $39 150. He received a dividend payment of $3327.75. He can get interest of 8.7% p.a. from his bank. Would he do better to keep the shares or sell them and put his money in the bank?

23 Ella is looking to invest in some retail industry stock. She has narrowed it down to two companies Bullseye and Gmart. She collected the following information on both:

Bullseye: A total of 4.2 million shares. A market price of $15.89 per share and annual earnings of $4.8 million.

Gmart: A market price of $31.88 per share. A dividend of 26.1c per share and 15% of the company's earnings was paid as a dividend.

- a Calculate the price-to-earnings ratio for Bullseye. Answer correct to one decimal place.
- b Calculate the price-to-earnings ratio for Gmart. Answer correct to one decimal place.
- c Based on the price-to-earnings ratio what would you recommend to Ella? Explain.

24 Rhani is looking to invest in some auto industry stock.

BMG: 7000 shares cost $472 500 and the annual earnings are $8.90 per share

Vorx: Shares are selling for $8.36 each. The company pays 30% of the total earnings as a dividend. The dividend was 25.8 cents per share.

Which would be her best option? Explain.

25 Chris is looking to invest in some music industry stock.

Shakra: Dividend yield of 4.2% and the company returned 60% of its total earnings as a dividend. The market price is $7.65 per share.

YFT: Dividend yield of 7.2% and the company returned 45% of its total earnings as a dividend. It costs $1550 for 200 shares.

Which would be his best option? Explain.

4

CHAPTER SUMMARY
EARNING MONEY

- A **wage** is income that is paid by the week for the hours that are worked.
- Hours worked in excess of ordinary working hours, outside usual starting and ceasing times, public holidays or weekends, are paid at a higher rate called **overtime**.
- A **salary** is a fixed amount of annual pay (yearly pay), paid weekly, fortnightly or monthly.
- Some workers earn money on **commission**, which means that they get a percentage of the value of sales or of a service provided.
- Sometimes a **retainer** is paid as well as a commission. This is a small payment made regardless of sales.
- **Piecework** is when you are paid according to the number of items that you make or handle.
- A **bonus** is paid to employees who produce work of a high standard or volume, or for meeting an important quota, goal or deadline.
- An **allowance** is paid to employees for work-related expenses (such as travelling, tools or laundry), or who work under difficult or dangerous conditions (such as in wet weather, in isolated areas, or great heights).
- **Annual leave loading** (also called holiday loading) is extra pay given during annual leave. It is usually paid at a rate of 17.5% of 4 weeks normal pay.
- Centrelink is an Australian government agency that supports people in need and helps them to become self-sufficient. Various **government allowances** and benefits are available for the aged, the unemployed, students, parents and the disabled.
- A **budget** is a plan for managing your income wisely.
- A budget is divided into **income** and **expenses.** When looking at the expenses, there are some that are fixed expenses and some that can vary, these are called discretionary expenses.
- The **stock market or share market** is another form of investment for many Australians. Investors can buy **shares** in the company. The price that shares are traded on the stock exchange for is called the **market price**.
- Some of the profit may be distributed to shareholders as a **dividend** for every share held. A dividend is usually expressed as cents per share; 38.1c per share.
- When trading shares on the stock exchange, a stock broker does the trading for you. A stock broker charges a commission, which is called the **brokerage**.
- The **dividend yield** compares the dividend with the market price and is usually written as a percentage.

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

- The **price-to-earnings ratio** is a common measure used by investors to compare the value of stocks. It compares the market value with the company's earnings per share.

$$\text{P/E} = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

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CHAPTER REVIEW
EARNING MONEY
4

Multiple choice

1 **Example 3** Convert $8300 a month to a weekly pay.
A $1915.38 B $1937 C $2075 D $24 900 E $99 600

2 **Example 4** George is a real estate agent. He earns a retainer of $250 a week plus commission. He charges commission for selling properties, based on this sliding scale of property prices:

- 2% for the first $85 000
- 1% for the next $160 000
- 0.5% for the remaining amount.

How much will he earn in a week where he sold a property worth $260 000?
A $1550 B $3375 C $3625 D $4600 E $4850

3 **Example 6** A clothing manufacturer offers each worker a bonus of an extra 5% of their weekly pay for every week where they beat the performance goals he has set. Calculate Maria's pay for a week where she beat her performance goal, if she normally earns $30 160 a year.
A $29 B $580 C $609 D $1508 E $31 668

4 **Example 9** David and May are both eligible for the couple's age pension of $536.70 per fortnight each. They run a stall at the local market and have a combined income of $8580 per year. They are allowed to earn up to $268 a fortnight. For any income over this they lose 50 cents in the dollar. Calculate their fortnightly pension payment.
A $474.70 B $505.70 C $1011.40 D $1042.40 E $1372.40

5 **Example 16** Shaun buys 750 shares for $9.80 each. He is paid a total dividend of $477.75. The dividend yield is:
A 0.065% B 6.5% C 15.4% D 48.75% E 63.7%

Short answer

6 **Example 1** Samantha is a waitress and her normal hourly rate is $23.50. She worked Monday to Thursday from 8 a.m. to 3 p.m. and Friday (which was Christmas day) from 11 a.m. to 8 p.m.
a Calculate her pay for Christmas day if she was paid triple time.
b Calculate her total pay for the week.

7 **Example 2** Marley works three days a week as a part-time carer. She is paid $25.70 per hour up to 5:00 p.m. and time-and-a-half after that. Use the roster to calculate how much she earned this week.

Marley's weekly roster

Day	Start time	Finish time
Monday	10 a.m.	7 p.m.
Wednesday	9 a.m.	1 p.m.
Friday	4 p.m.	10 p.m.

8 **Example 3** For each of the following annual salaries state the amount paid for the pay period given, correct to the nearest cent.
a $98 503 paid fortnightly
b $263 764 paid monthly

9 Example 4 Jarryd earns a 2.5% commission from each car he sells. How much would he earn in a month in which he sells three cars for $57 000, $35 990 and $40 850?

10 Example 4 Christine is a bookseller. She earns a $250 a week retainer plus a 4% commission on all sales. What value of books would she have to sell to earn at least $500 in a week?

11 Example 5 Huon picked 28 buckets of grapes at $3.65 per bucket.

- a How much did Huon earn?
- b How many buckets would Huon need to pick in order to earn $450?

12 Example 7 A window washer earns $17.90 per hour and receives a multistory allowance of $23.40 per day. If he works 10 hours each weekday on a multistory building, calculate his total weekly earnings.

13 Example 8 The Great Gals electrical store pays a holiday loading of 17.5% of four weeks' pay to each employee.

- a How much holiday loading is paid to Richard, the manager, who earns a salary of $86 430?
- b Calculate his holiday pay for 4 weeks.

14 Examples 9–11 Use the tables from Examples **9**, **10** and **11** (on pages 167–9) to help you find the type and amount of the allowance to which each of the following people are entitled.

- a Richard, aged 18, who is Indigenous and is a second-year apprentice living with his parents.
- b Peter and Wendy, aged 68 and 73, who have no other income.
- c Ben, aged 24, who is Indigenous, looking for work and living away from home.
- d Alison, aged 29, who is studying full-time at college while raising her daughter alone.
- e Sandra, aged 67, who is single and earns $100 a fortnight from doing ironing.
- f Sean and Andrea, aged 19 and 22, who are a couple and are both full-time apprentices.
- g Paul, aged 19, who lives out of home and is looking for work.

15 Example 12 Chloe is a manager at a clothing store. She recently bought her own home and has a friend who pays rent to live with her and to share the expenses. Below is Chloe's weekly budget.

Wages	$1130	Home loan repayment	$480
Rent	$200	Bills	$65
		Loan repayments (furniture)	$75
		Loan repayments (white goods)	$40
		Groceries	$125
		Rates & home maintenance	$70
		Public transport	$50
		Health fund	$40
		Clothes	$100
		Entertainment	$80
		Savings	$_____
Total:	$____	**Total:**	$_____

- a Fill in the total for income and expenses in the budget.
- b How much does Chloe save each week?
- c Chloe's friend tells her that she needs to move out. Can Chloe afford to live by herself or does she need to get another flat mate? Justify your answer.

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16 Example 14 RYS Insurance shares have a market value of \$22.60 and pay a dividend of \$3.16.

- a Parker bought 350 RYS shares. How much did they cost him, if he paid 0.5% brokerage?
- b How much dividend was Parker paid?
- c Parker sold his shares a year later for \$24.50. He had to pay a 0.8% brokerage. Calculate Parker's total earnings over the whole year, after costs.

17 Example 15 A company has 250 000 shares and makes an after tax profit of \$1.5 million. The company directors decide to pay a dividend of 45% of their profit. What is the dividend paid per share?

18 Example 16 Kate is looking to invest some money. She is considering two options.

A: Buying 1500 shares in a mining company with a market value of \$32.20 each. They are paying a dividend of \$2.58 per share.

B: Investing the same amount of money in a credit union paying 7.5% p.a.

- a Calculate the dividend yield of the shares, correct to one decimal place.
- b Would she do better to buy the shares or put the money in the credit union?

Application

19 Ingrid earns \$1660 a fortnight after tax and collects \$1270 monthly rent from an investment flat she owns. She pays \$480 per fortnight in rent for her flat and her power costs approximately \$350 per quarter. Her food and other day-to-day expenses amount to \$320 a week, and transport costs her \$70 a week. She is paying off a lounge suite at \$112 a month and also spends about \$380 a month on clothes. Her contents insurance costs her \$390 a year and her mobile phone plan is \$99 per month. Her aerobics class costs \$20 a week.

- a Calculate her weekly income, correct to the nearest \$5.
- b Convert the following to weekly payments, correct to the nearest \$5.
  - i rent
  - ii power
  - iii lounge suite payments
  - iv clothes
  - v contents insurance
  - vi mobile phone plan
- c Draw up a budget. Be sure to include savings.
- d How much she could save in one year?

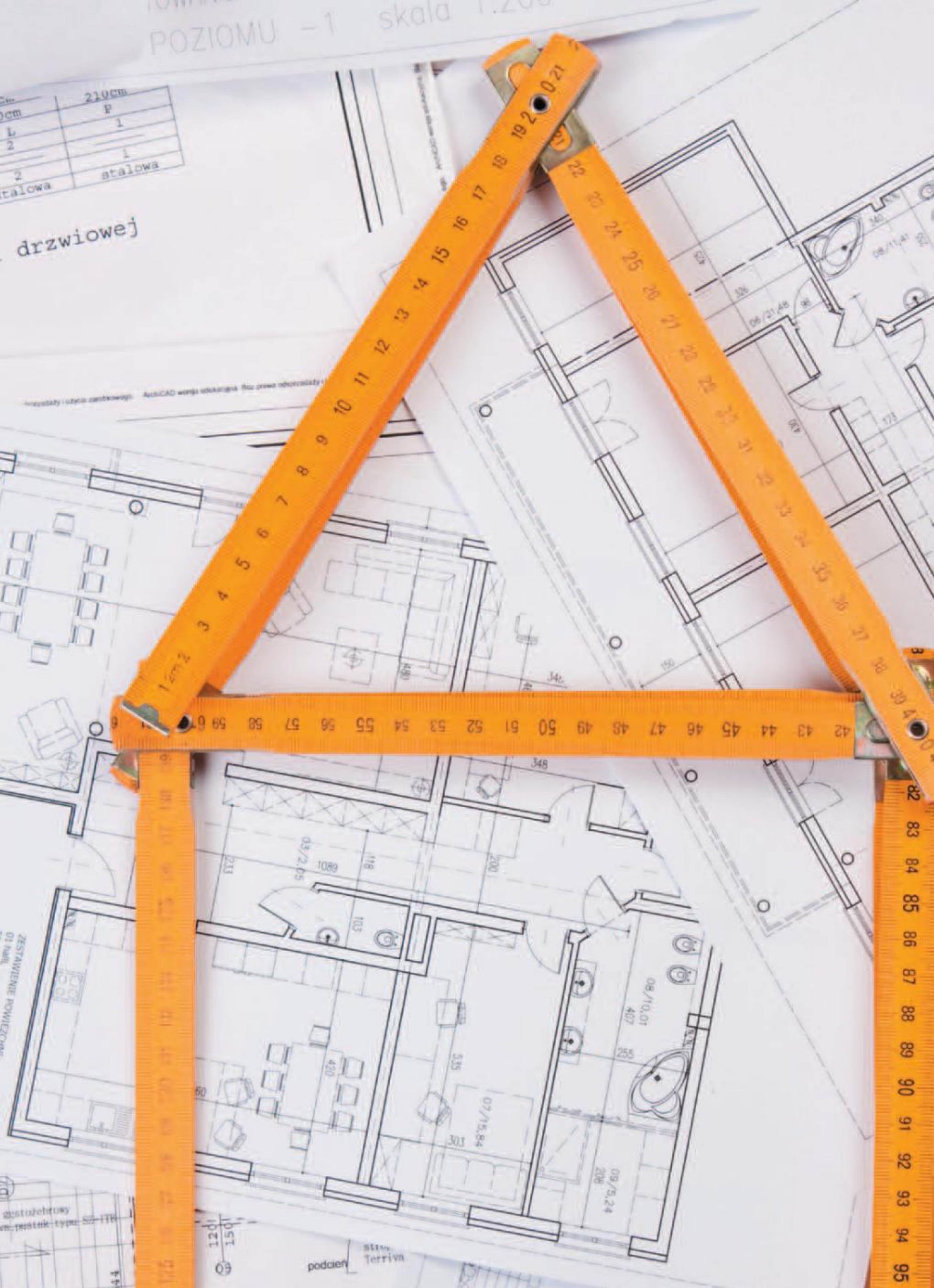
Practice quiz

20 Bob is looking to invest in some technology stock.

STX: 1500 shares cost \$24 450 and the annual earnings are 63 cents per share.

Electro: Shares are selling for \$7.34 each. The company pays 12% of the total profits as a dividend. The dividend was 4 cents per share.

Which would be his best option? Explain.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5

MEASUREMENT

MEASUREMENT CALCULATIONS

TERMINOLOGY

apex
arc
arc length
capacity
circumference
cone
cross-section
cubic units
cylinder
hectare
megalitre (ML)
net
perpendicular height
polygon
prism
pyramid
Pythagoras' theorem
sector
slant height
solid
sphere
surface area

Prior learning

PYTHAGORAS’ THEOREM

- review Pythagoras’ Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions. (ACMGM017)

MENSURATION

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites (ACMGM018)
- calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the volume of water contained in a swimming pool (ACMGM019)
- calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the surface area of a cylindrical food container. (ACMGM020)

5.01 PYTHAGORAS’ THEOREM

The Greek mathematician Pythagoras (570–496 BCE) is said to have discovered this rule.

Pythagoras’ theorem states that:

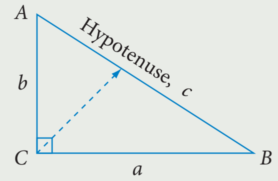
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This theorem can be used to calculate the lengths of sides when working with right-angled triangles.

For any right-angled triangle, we can write a Pythagorean equation in the form: “hypotenuse squared = one short side squared + the other short side squared”.

IMPORTANT

In $\triangle ABC$: $c^2 = a^2 + b^2$

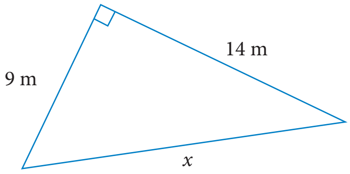


The **hypotenuse** is the longest side of a right-angled triangle.

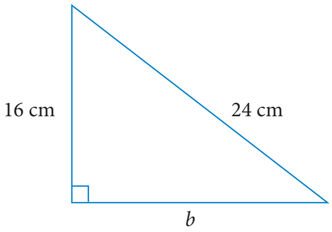
Example 1

Calculate the value of the pronumeral in each triangle. Give your answers correct to one decimal place.

a



b



Solution

a Write a Pythagorean equation where x represents the hypotenuse.

$x^2 = 9^2 + 14^2$

Solve for x.

$x^2 = 81 + 196$

$x^2 = 277$

Evaluate the square root.

$x = \sqrt{277}$

$= 16.64...$

Round to one decimal place.

≈ 16.6 m

Write the answer. The hypotenuse is 16.6 metres long.

b Write a Pythagorean equation where 24 represents the hypotenuse.

$$24^2 = 16^2 + b^2$$

Solve for b.

$$576 = 256 + b^2$$

$$576 - 256 = b^2$$

$$b^2 = 320$$

Evaluate the square root.

$$b = \sqrt{320}$$

$$= 17.88...$$

Round to one decimal place.

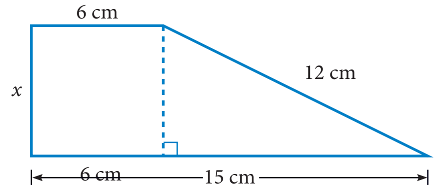
$$\approx 17.9 \text{ cm}$$

Write the answer. Side b has a length of 17.9 cm.

Right-angled triangles can often be found in more complex figures.

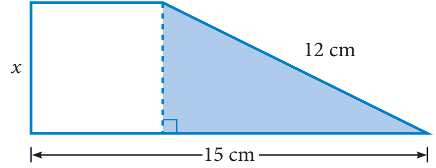
Example 2

Calculate the value of the pronumeral in this composite figure. Give your answer correct to two decimal places.



Solution

Identify the right-angled triangle.

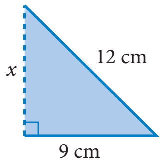


Calculate the length of the horizontal side using the measurements given.

$$15 - 6 = 9 \text{ cm}$$

The hypotenuse is 12 cm.

The third side of the triangle is the same length as x.



Write a Pythagorean equation.

$$12^2 = x^2 + 9^2$$

Solve for x.

$$144 = x^2 + 81$$

$$144 - 81 = x^2$$

$$x^2 = 63$$

Evaluate the square root.

$x = \sqrt{63}$

$= 7.937...$

Round to two decimal places.

≈ 7.94 cm

Write the answer.

The side is 7.94 cm long.

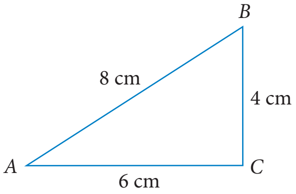
Pythagoras' theorem can be used to determine if a triangle is right-angled. If the three side lengths of a triangle are known, a Pythagorean equation for the triangle can be written.

It is possible to check if squaring the longest side gives the same answer as squaring the other two sides and adding them together. If the same answer is calculated for each, the triangle must be right-angled.

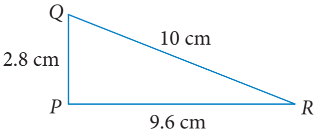
Example 3

Determine whether each of these triangles is right-angled.

a



b



Solution

a Square the longest side.

$8^2 = 64$

Square the other two sides and add them.

$4^2 + 6^2 = 52$

Compare the results.

$64 \neq 52$

$8^2 \neq 4^2 + 6^2$

Write the answer.

Pythagoras' theorem does not work. This triangle is not right-angled.

b Square the longest side.

$10^2 = 100$

Square the other two sides and add them.

$2.8^2 + 9.6^2 = 100$

Compare the results.

$10^2 = 2.8^2 + 9.6^2$

Write the answer.

Pythagoras' theorem works. This triangle is right-angled.

($\angle P$ is the right angle.)

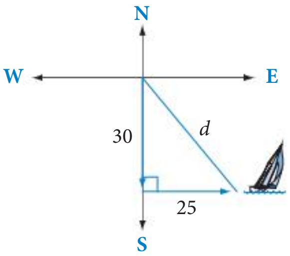
Pythagoras' theorem can be used to calculate lengths in real-life applications.

Example 4

A boat sailed 30 nautical miles due south, then 25 nautical miles due east. How far, correct to the nearest nautical mile, is it from its starting point?

Solution

Let the boat be d nautical miles from its starting point and draw a diagram.



Write the Pythagorean equation.

$$d^2 = 30^2 + 25^2$$

Solve for d.

$$d^2 = 1525$$
$$d = \sqrt{1525}$$
$$= 39.05...$$

Round to nearest nautical mile.

$$\approx 39$$

From the diagram, an answer of 39 nautical miles seems reasonable.

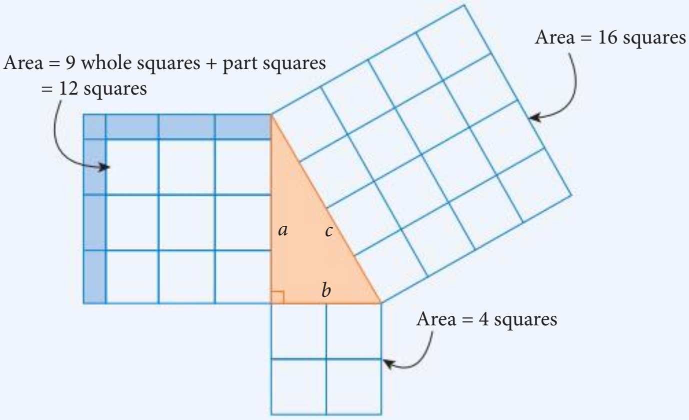
Write the answer.

The boat is 39 nautical miles from its starting point.

INVESTIGATION Pythagorean triples

Geometrically, the area of a square is equal to x^2, where x is equal to the length of the side.

This means that we can calculate the square of a number by counting unit squares in a square of that side length.



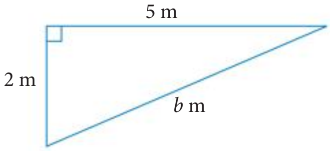
We can show that Pythagoras' theorem is true by counting the area in the squares drawn on the sides of the triangle. The area is counted by dividing the squares into unit squares.

Work in groups of two or three to prove Pythagoras' theorem using this method. Each group should have different-sized triangles and compare the areas counted in the squares. You may have to count squares by adding smaller parts together.

EXERCISE 5.01 Pythagoras' theorem

Concepts and techniques

1 **Example 1** Calculate the value of the pronumeral in each of the following triangles, correct to two decimal places.

a


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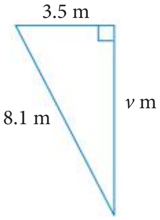

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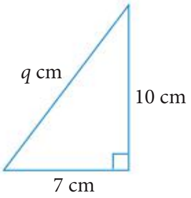

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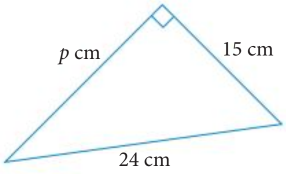

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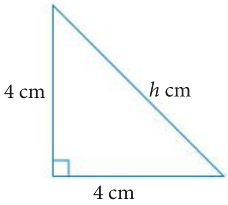

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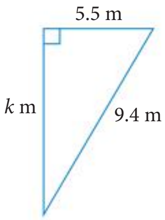

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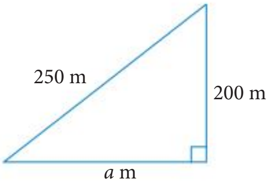

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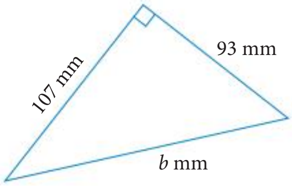

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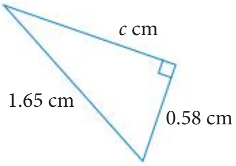

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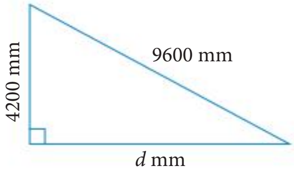

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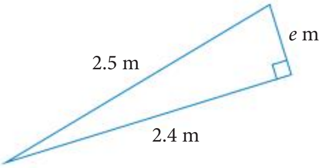

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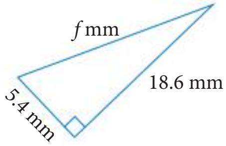
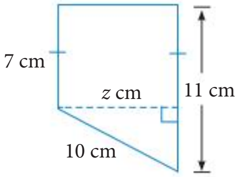


2 **Example 2** Calculate the value of the pronumeral in each of these figures, correct to one decimal place.

a


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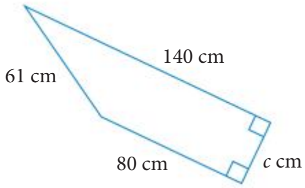

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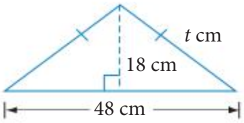

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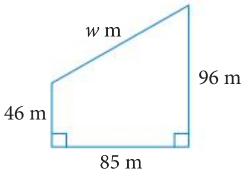

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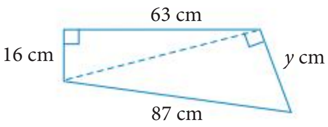

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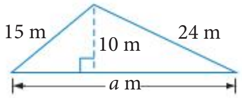

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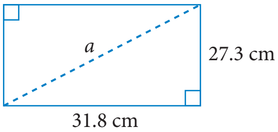

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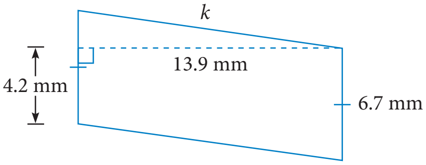

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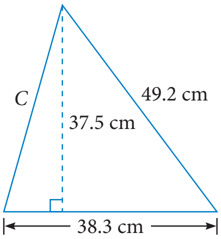
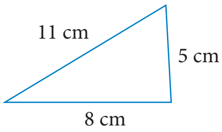


3 Example 3 Determine whether each of the following triangles is right-angled.

a



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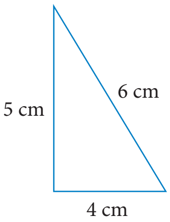

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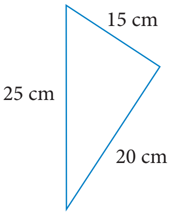

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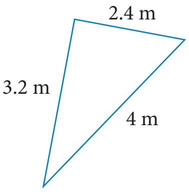

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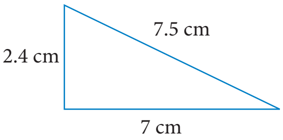

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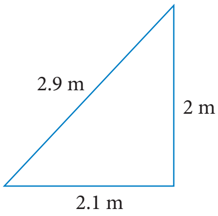
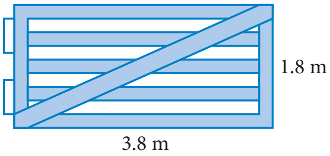
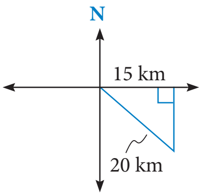


Reasoning and communication

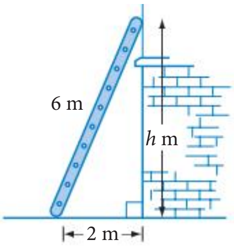
4 Example 4 A rectangular gate has dimensions 3.8 m by 1.8 m. What is the length of its diagonal brace (to the nearest centimetre)?



5 A ship sails in a south-easterly direction for 20 km until it is 15 km east of its starting point. How far south is it from its starting point? Give your answer correct to the nearest metre.

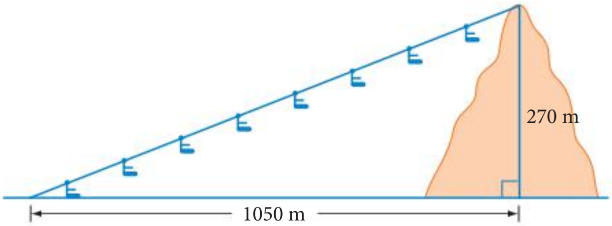


6 A 6 m ladder leans against a house, so that its base is 2 m from the bottom of the house. How far up the wall of the house does the ladder reach, to the nearest metre?



7 A yacht leaves Newcastle and sails 160 nautical miles due north. It turns and sails due east until it is directly 200 nautical miles from Newcastle. How far east did it sail?

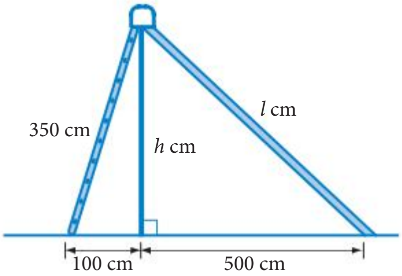
8 A chairlift takes skiers up to the top of a hill 270 m high and 1050 m away. How long is the chairlift's cable, correct to two decimal places?



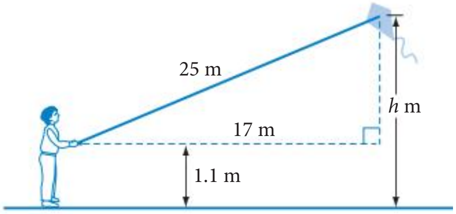
9 A playground slide is made up of two right-angled triangles. Calculate, correct to the nearest centimetre:

a h, the height of the slide.

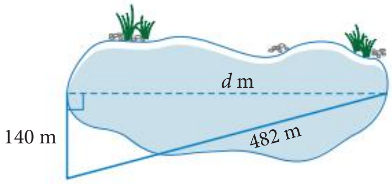
b l, the length of the slide.



10 This diagram shows a boy flying a kite. How high is the kite above the ground, correct to one decimal place?

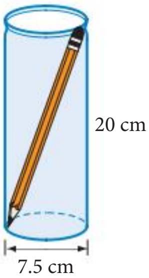


11 Vanessa calculated the distance across the lake by taking the measurements shown on the right. What was the distance across the lake, to the nearest metre?

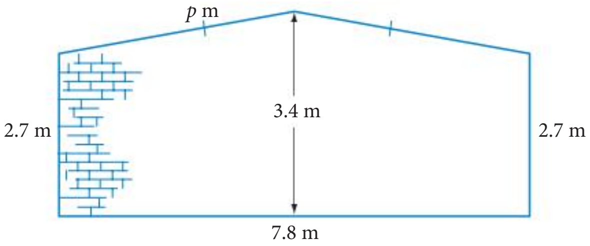


12 A plane flies NW from the airport for 6 km, then turns and flies 4.2 km due east until it is due north of the airport. How far is it from the airport then, correct to two decimal places?

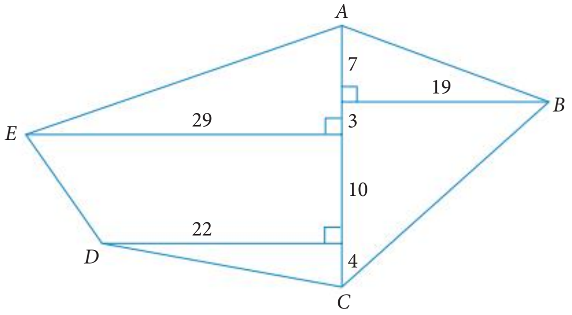
13 Jackie wants to use an old tennis ball container as a pencil case. If the container is a cylinder with diameter of 7.5 cm and a height of 20 cm, what is the length of the longest pencil that will fit inside the container (to the nearest centimetre)?



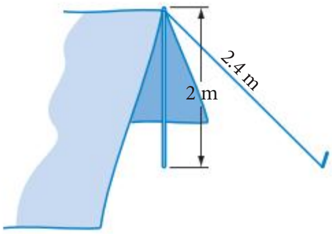
14 Calculate the length of the pitch line, p, of the roof of this house, correct to one decimal place.



15 Calculate the perimeter of the field in this diagram, correct to two decimal places. All measurements are in metres.



16 If a vertical tent pole is 2 m high and the rope attaching it to the ground is 2.4 m long, how far from the base of the pole should the rope be pegged (correct to one decimal place)?



5.02 PERIMETER AND CIRCUMFERENCE

Metric units

IMPORTANT

Metric units

Unit	Relationships
millimetre (mm)	
centimetre (cm)	1 cm = 10 mm
metre (m)	1 m = 100 cm = 1000 mm
kilometre (km)	1 km = 1000 m

To convert a larger unit to a smaller one, *multiply*. There will be more of the smaller units.

To convert a smaller unit to a larger one, *divide*. There will be fewer larger units.

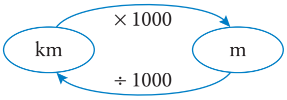
Example 5

Convert 23.5 km to metres.

Solution

To convert from a larger unit of measure to a smaller one we multiply.

$$\begin{aligned}23.5\text{ km} &= 23.5 \times 1000\\ &= 23\,500\text{ m}\end{aligned}$$



Perimeter

Perimeter is a measure of the distance around the outside of a two dimensional shape. It is calculated by adding together all of the side lengths of the shape.

The perimeter of a circle is called its **circumference**. If we know the length of the diameter of a circle, we can use a formula to calculate its circumference.

π is the ratio of a circle's circumference to its diameter. Use the π button on your calculator when calculating circumference.

IMPORTANT

Circumference of a circle

The circumference of a circle can be calculated by using one of the following formulas.

$C = \pi D$

Circumference = $\pi \times$ diameter

The formula $C = 2 \times \pi \times r$ can also be used.

The diameter of a circle is twice the radius.

Example 6

Calculate the circumference of a circle with a radius of 3.4 m, correct to one decimal place.

Solution

Remember that the diameter is twice the radius.

$$\begin{aligned}D &= 2 \times 3.4\\ &= 6.8\text{ m}\end{aligned}$$

Write the appropriate formula.

$C = \pi D$

Substitute the known value for *D*.

$= \pi \times 6.8$

Use π on your calculator.

$= 21.3628\ldots$

Round to one decimal place.

≈ 21.4 m

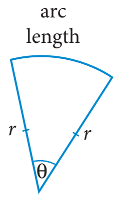
Write the answer.

The circumference is 21.4 m.

In solving practical perimeter problems, sometimes it is necessary to calculate the perimeter of a **sector** of a circle.

A sector has 3 lengths around its boundary:

- two straight lengths that are each equal to the radius of the circle
- a curved length that is called the **arc length** and is a fraction of the circle's circumference.



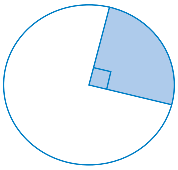
In order to calculate the arc length we need to know what fraction of the circle's circumference it is. The total angle inside a circle is 360°. Dividing the angle in the sector (θ) by 360 tells us what fraction of a circle we have.

Consider these sectors:

The angle inside this sector is 90°.

$$\frac{\theta}{360} = \frac{90}{360} = \frac{1}{4}$$

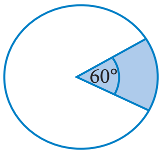
This sector is $\frac{1}{4}$ of the circle.



The angle inside this sector is 60°.

$$\frac{\theta}{360} = \frac{60}{360} = \frac{1}{6}$$

This sector represents $\frac{1}{6}$ of a circle.



We then multiply the fraction by the circumference (πD) to calculate the arc length.

IMPORTANT

Arc length of a sector

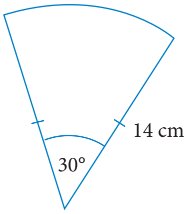
$$\text{Arc length} = \frac{\theta}{360}\pi D$$

The perimeter of the sector can be found by adding the arc length and the two straight sides.

Example 7

Calculate the perimeters of these sectors correct to one decimal place.

a



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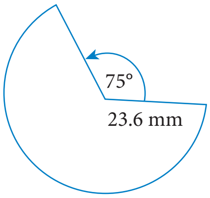


Solution

a Write the formula.	$P = \text{arc length} + 2 \times \text{radius}$
Substitute the angle, diameter and radius (remember that the diameter is twice the radius).	$= \frac{30}{360}\pi \times 28 + 2 \times 14$
Evaluate.	$= 35.330\ldots$
Round to one decimal place.	≈ 35.3 cm
Write the answer.	The perimeter is 35.3 cm

b Calculate the value of the angle within the sector.

$$\theta = 360 - 75$$
$$= 285°$$

Write the formula.

$$P = \text{arc length} + 2 \times \text{radius}$$

Substitute the angle, diameter and radius.

$$P = \frac{285}{360}\pi \times 47.2 + 2 \times 23.6$$

Evaluate.

$$= 164.59\ldots$$

Round to one decimal place.

$$\approx 164.6 \text{ mm}$$

Write the answer.

The perimeter is 164.6 mm.

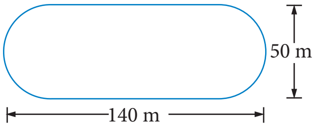
Perimeter calculations are used in real-life scenarios, for example, you would need to know the perimeter of a space when calculating fencing costs.

Example 8

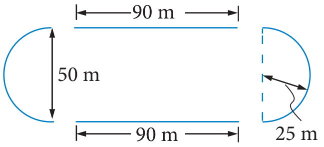
A sportsground 140 m long and 50 m wide has semicircular ends. A fence is to be erected all the way around, with four gates, each 2 m wide. The gates cost $150 each and the fencing costs $45 per metre. Calculate the cost of fencing the ground, correct to the nearest dollar.

Solution

Sketch the sportsground. Show the measurements.



Divide the perimeter into sections.



Calculate the length of the round section at each end of the sportsground.

$$\text{End} = \frac{1}{2}\pi D$$
$$= \frac{1}{2}\pi \times 50$$
$$= 78.5398\ldots \text{ m}$$

Calculate the total distance around the sportsground.

$$\text{Perimeter} = 90 + 78.5398\ldots + 90 + 78.5398\ldots$$
$$= 337.0796\ldots \text{ m}$$

Subtract the gate lengths (4 × 2 m).

$$\text{Fence length} = 337.0796\ldots - 4 \times 2$$
$$= 329.0796\ldots \text{ m}$$

Calculate the total cost.

$$\text{Total cost} = 329.0796\ldots \times \$45 + 4 \times \$150$$
$$= \$14\,808.58 \ldots + \$600$$
$$= \$15\,408.58 \ldots$$

Round to the nearest dollar.

$$\approx \$15\,409$$

Write the answer.

The cost of fencing the ground is $15 409.

EXERCISE 5.02 Perimeter and circumference

WS Units of length and perimeter

Concepts and techniques

1 **Example 5** Change each of the following measurements to the unit indicated.

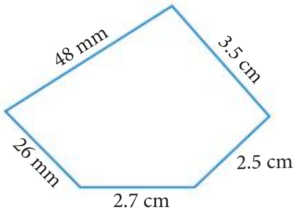
a 320 cm → m
b 5.23 km → m
c 1460 mm → cm
d 58 cm → m
e 537 m → km
f 32.7 m → cm
g 0.054 m → mm
h 47 350 cm → km
i 0.000 64 km → mm

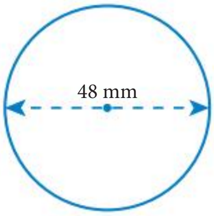
2 Blake's height is 169 cm. Convert this to:

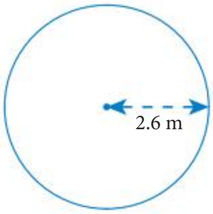
a millimetres
b metres.

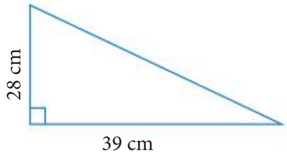
3 Rhys swam in a 1500 m race. How many kilometres did he swim?

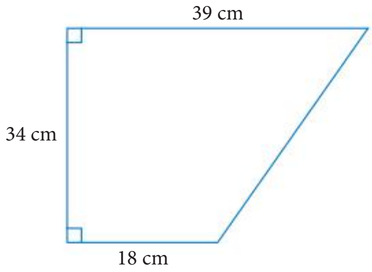
4 **Example 6** Calculate the perimeters of the following shapes. Where rounding is necessary, please round to one decimal place.

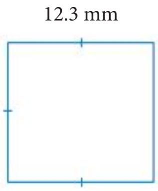
a


b


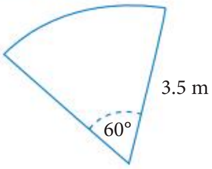
c


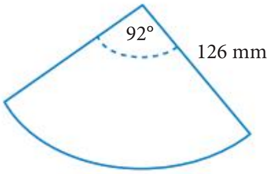
d


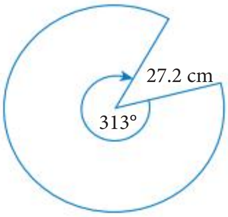
e


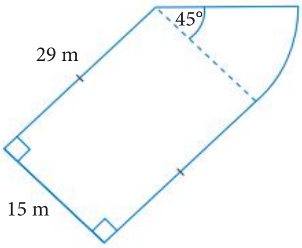
f


5 **Example 7** Calculate the perimeters of these figures, correct to one decimal place.

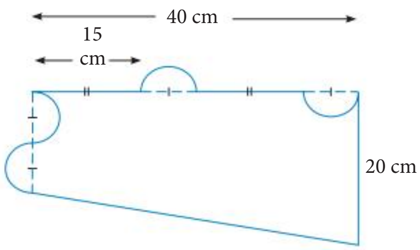
a


b


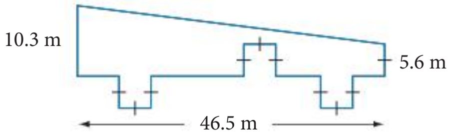
c


d


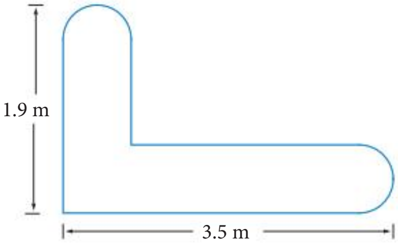
e



f



Reasoning and communication



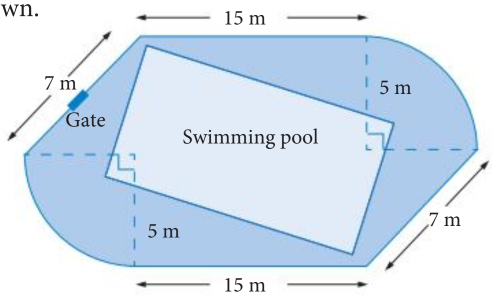
6 **Example 8** The L-shaped kitchen bench with semicircular ends shown here has been made from chipboard veneer. The bench is 600 mm wide. It must have an edging strip of veneer attached all the way along the edge to finish the job. What length of edging veneer, correct to one decimal place, is required?

7 A fence is to be erected around a block of land that has a slanting front boundary and parallel side boundaries of 27 m and 42 m. The rear boundary is 20 m long and is at right angles to the side boundaries. The posts are to be a maximum of 2 m apart, there are two railings, and 100 mm palings are to be spaced 20 mm apart. The posts cost \$12.50 each, the railings cost \$5.40/m and the palings cost \$1.40 each. A 2.5 m gate costing \$130 is to be placed at the front. Calculate the cost of materials to complete the job.

8 If the circumference of a circle is 28 cm, find its radius correct to one decimal place.

9 A swimming pool is set in a wooden deck as shown. A pool fence is to be erected around the border, with a 1200 mm wide gate. The cost of materials for a 1200 mm high fence is \$42/m, and the gate costs \$120. Installation costs are \$10/m for the fence and gate.

- a Calculate the perimeter to the nearest metre.
- b Calculate the cost of materials to complete the job.
- c Calculate the cost of installing the fence and gate.



10 A circular horseracing track is 700 m across the inside and is 15 m wide. The track has to be re-fenced on both sides with post-and-rail fencing at a cost of \$34/m for materials and labour. Calculate the cost of erecting the fences.

Shutterstock.com/Cheryl ann Quigley

11 On a 2.5 km walk, John covers 80 cm with each step, while his daughter Anna covers 55 cm with each of her steps. How many more steps does Anna take than her father during the walk?

5.03 REVIEW OF AREAS OF PLANE SHAPES

The **area** of a two dimensional shape is the number of **square units** it covers.

When converting units of area, we need to multiply or divide by square units.

For example if we want to convert measurements from m^2 into cm^2, we use the fact that
1 m = 100 cm. We then multiply by 100^2.

1 m^2 = 1 × 100^2 cm^2 = 10 000 cm^2.
5 m^2 = 5 × 100^2 cm^2 = 50 000 cm^2.

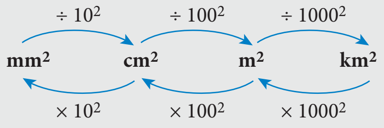
If we need to convert measurements from mm^2 to cm^2, we use the fact that

1 cm = 10 mm. We then divide by 10^2.

1500 mm^2 = 1500 ÷ 10^2 cm^2 = 15 cm^2.

IMPORTANT

Converting units of area



1 hectare =100 m × 100 m = 10 000 m^2
100 hectares = 1 km^2

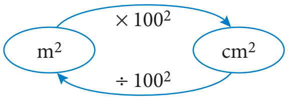
Example 9

Convert:

a 55 m^2 to cm^2 **b** 2350 mm^2 to cm^2 **c** 0.8 km^2 to ha.

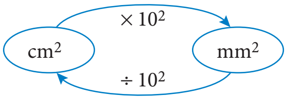
Solution

a



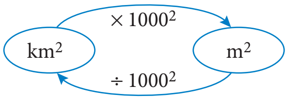
55 m^2 = 55 × 100^2
= 550 000 cm^2

b



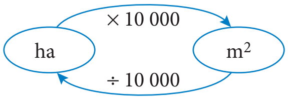
2350 mm^2 = 2350 ÷ 10^2
= 23.5 cm^2

c Convert to m^2 first.



0.8 km^2 = 0.8 × 1000^2
= 800 000 m^2

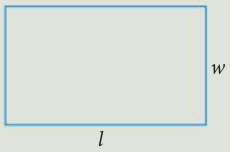
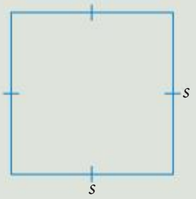
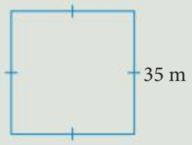
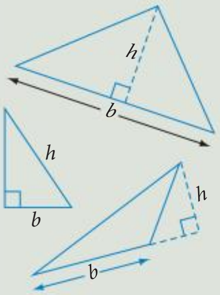
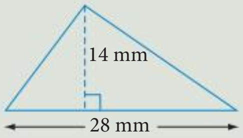
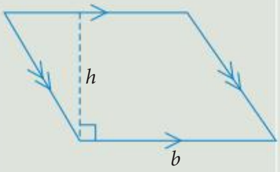
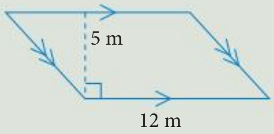
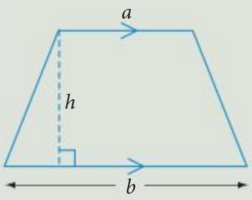
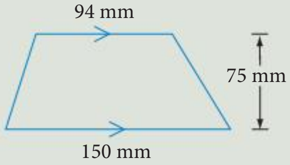
Now convert to ha.

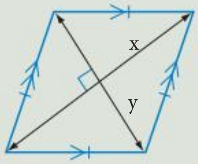
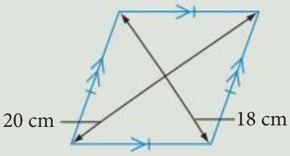
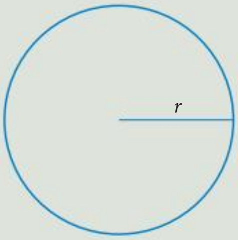
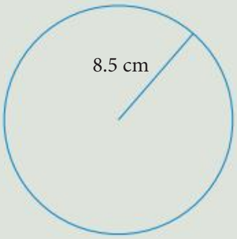


800 000 m^2 = 800 000 ÷ 10 000
= 80 ha

Area formulas

The table below lists the formulas for the areas of plane shapes that you should be familiar with.

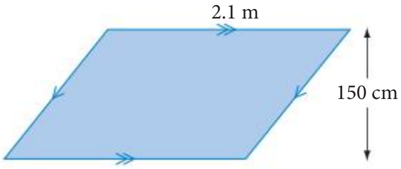
Shape	Area formula	Example
Rectangle  	$A = \text{length} \times \text{width}$ $A = lw$	 $A = 8 \times 6$ $= 48\ \text{cm}^2$
Square  	$A = \text{side} \times \text{side}$ $A = (\text{side})^2$ $A = s^2$	 $A = 35^2$ $= 1225\ \text{m}^2$
Triangle  	$A = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$ $A = \frac{1}{2}\ bh$	 $A = \frac{1}{2} \times 28 \times 14$ $= 196\ \text{mm}^2$
Parallelogram  	$A = \text{base} \times \text{perpendicular height}$ $A = bh$	 $A = 12 \times 5$ $= 60\ \text{m}^2$
Trapezium  	$A = \frac{1}{2} \times \text{sum of parallel sides} \times$ perpendicular distance between sides $A = \frac{1}{2}(a + b)h$	 $A = \frac{1}{2} \times (94 + 150) \times 75$ $= 9150\ \text{mm}^2$

Rhombus or kite  	$A = \frac{1}{2} \times \text{product of diagonals}$ $A = \frac{1}{2}xy$	 $A = \frac{1}{2} \times 20 \times 18$ $= 180\ \text{cm}^2$
Circle  	$A = \pi \times (\text{radius})^2$ $A = \pi r^2$	 $A = \pi \times 8.5^2$ $\approx 227\ \text{cm}^2$

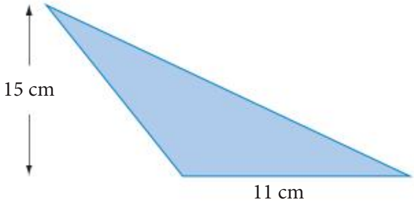
Example 10

Calculate the areas of the following shapes, giving your answers correct to the nearest square centimetre.

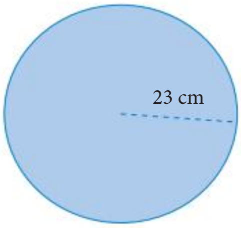
a



b



c



Since the answer is to be given correct to the nearest square centimetre, it is logical to work with side lengths in centimetres.

Solution

a Covert one measurement, so that all lengths are in the same units.	$2.1\ \text{m} = 210\ \text{cm}$
Write the formula for the area of a parallelogram.	$A = bh$
Substitute the known values into the equation.	$= 210 \times 150$
Evaluate and write the answer.	$= 31\ 500\ \text{cm}^2$

b	Write the formula for the area of a triangle.	$A = \frac{1}{2}bh$
	Substitute the known values into the equation.	$= \frac{1}{2} \times 11 \times 15$
	Evaluate.	$= 82.5$
	Write the answer, correct to the nearest square centimetre.	$\approx 83 \text{ cm}^2$
c	Write the formula for the area of a circle.	$A = \pi r^2$
	Substitute the known values into the equation.	$= \pi \times 23^2$
	Evaluate.	$= 1661.90\ldots$
	Write the answer, correct to the nearest square centimetre.	$\approx 1662 \text{ cm}^2$

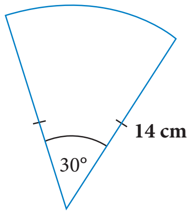
In order to calculate the area of a sector, as a first step, calculate what fraction of a circle the sector represents. As shown in Section 5.02, the fraction of the sector is found using the formula: Fraction $= \frac{\theta}{360}$, where θ is the angle inside the sector. Then multiply that fraction by the area of the whole circle (πr^2) to calculate the area of the sector.

IMPORTANT

Area of a sector $= \frac{\theta}{360}\pi r^2$

Example 11

Calculate the area of the sector shown, correct to one decimal place.



Solution

Write the formula for the area of a sector.	$A = \frac{\theta}{360}\pi r^2$
Substitute the known values into the equation.	$= \frac{30}{360} \times \pi \times 14^2$
Evaluate.	$= 51.31\ldots$
Write the answer, remembering to round to one decimal place.	$\approx 51.3 \text{ cm}^2$

EXERCISE 5.03 Review of areas of plane shapes

Concepts and techniques

1 **Example 9** Which of the following is the number of square centimetres in 300 m^2?

A 3 **B** 300 **C** 3000

D 3 000 000 **E** 30 000 000

2 Which of the following is the number of hectares in 5 km^2?

A 0.5 **B** 50 **C** 500 **D** 5000 **E** 50 000

3 Convert:

a 5 m^2 to cm^2 **b** 2500 cm^2 to mm^2

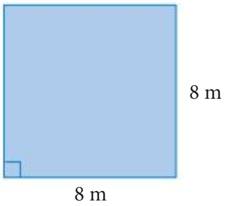
c 72 000 m^2 to hectares **d** 6800 cm^2 to m^2

e 3.09 km^2 to m^2 **f** 3.6 km^2 to hectares

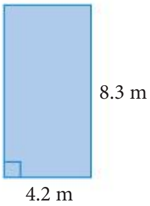
g 4.73 m^2 to mm^2 **h** 540 ha to km^2

4 **Example 10** Calculate the area of each of the following shapes, correct to one decimal place where needed. (Note: Some questions require the use of Pythagoras' theorem first.)

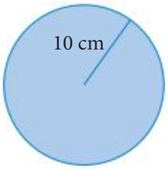
a



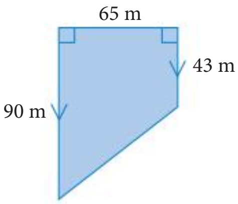
b



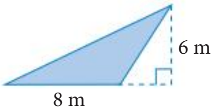
c



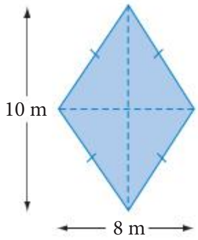
d



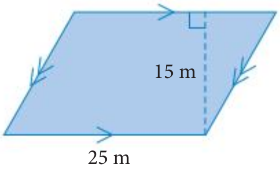
e



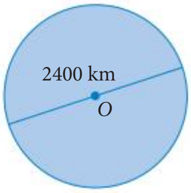
f



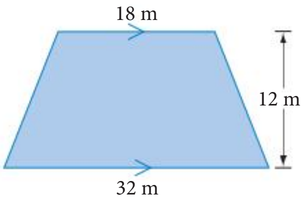
g



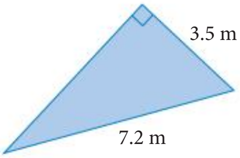
h



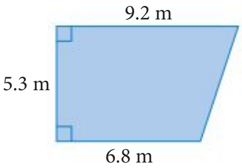
i



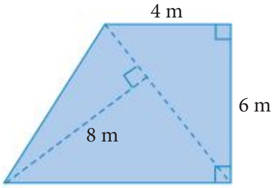
j



k

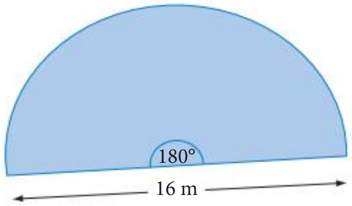


l

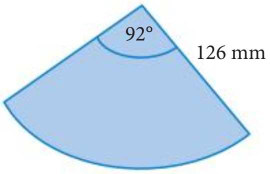


5 **Example 11** Calculate the areas of the sectors below. Give your answers correct to one decimal place.

a



b



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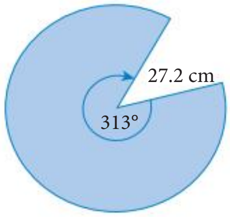

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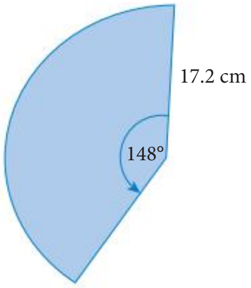

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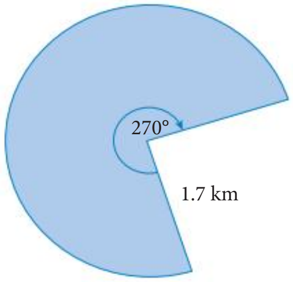

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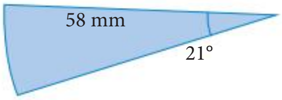


Reasoning and communication

6 If a triangle has an area of 45.6 mm^2 and its base length is 7.1 mm, what would its height be? Give your answer correct to one decimal place.

7 Builders measure lengths in millimetres, and carpet layers quote per square metre. How many square metres of carpet are needed in a room that measures 2300 mm by 1830 mm? Answer this question using the following two methods.

  a Calculate the carpet area in mm^2, and then convert to m^2.

  b Convert the lengths to metres first and then calculate the carpet area in m^2.

8 A farmer owns a large rectangular piece of land, measuring 2 km by 450 m. How many hectares does he own?

9 The area of NSW is approximately 801 600 km^2. How many hectares is this?

10 A DVD has a diameter of 12 cm. Calculate the area of the DVD:

Shutterstock.com/yang na

a to the nearest cm^2

b to the nearest mm^2

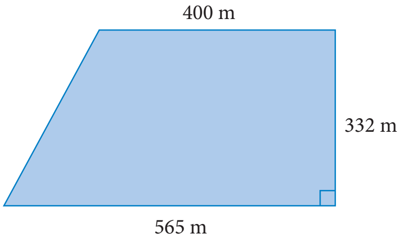
11 A water sprinkler is positioned at the corner of a lawn. It is set so that it rotates through an angle of 85°, moving back and forth as it sprays a stream of water.
The stream of water from the sprinkler reaches 4.5 metres.
What area of lawn is watered in 1 full sweep of the sprinkler? Round your answer to two decimal places.

Shutterstock.com/Alison Hancock

12 The paddock shown below is going to be used to grow wheat. The farmer plans on planting 43 kg of wheat seeds per hectare.

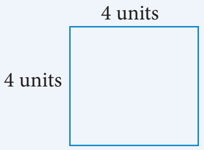
a Calculate the area of the paddock correct to the nearest hectare.

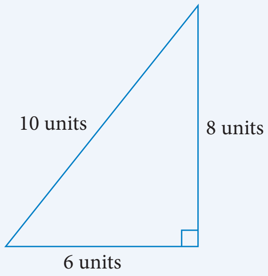
b If wheat seeds are sold in 25 kg bags, how many bags will the farmer need to buy?



INVESTIGATION Perimeter and area

The shapes below have equal areas and perimeters.





For the square: $P = 4 + 4 + 4 + 4$ and $A = 4^2$

$= 16 \text{ units}$ $\quad = 16 \text{ units}^2$

For the triangle: $P = 6 + 8 + 10$ and $A = \frac{1}{2} \times 6 \times 8$

$= 24 \text{ units}$ $\quad = 24 \text{ units}^2$

Try to calculate the dimensions of:

- **a** a rectangle
- **b** a circle
- **c** another right-angled triangle

which also have the same perimeter as they have area.

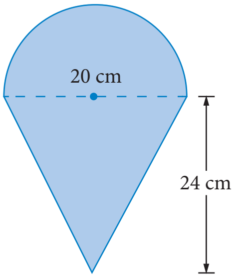
5.04 AREAS OF COMPOSITE FIGURES

A **composite** shape is one that is formed by combining several regular shapes. To calculate the area of a composite shape, calculate and then add together the areas of all of the shapes that comprise the larger shape.

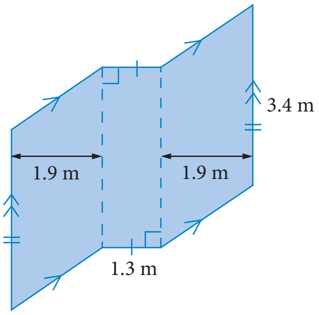
Example 12

Calculate the composite areas shown. Give your answers to the nearest whole number.

a



b



a The composite figure is made up of a semicircle and a triangle.

Calculate the area of the semicircle. (Radius = half of diameter.)

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi \times 10^2$$
$$= 157.0796\ldots \text{ cm}^2$$

Calculate the area of the triangle.

$$\text{Area of triangle} = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 20 \times 24$$
$$= 240 \text{ cm}^2$$

Add the two areas together.

$$\text{Total area} = 157.0796\ldots + 240$$
$$= 397.0796\ldots$$

Round to the nearest whole number.

Write the answer.

$$\approx 397 \text{ cm}^2$$

The total area is 397 cm^2.

b The figure is composed of two equal parallelograms and a rectangle.

Calculate the area of one parallelogram.

$$\text{Area of parallelogram} = bh$$
$$= 3.4 \times 1.9$$
$$= 6.46 \text{ m}^2$$

Calculate the area of the rectangle.

$$\text{Area of rectangle} = l \times w$$
$$= 1.3 \times 3.4$$
$$= 4.42 \text{ m}^2$$

Add the areas together; remember to multiply the area of the parallelogram by two.

$$\text{Total area} = 4.42 + 2 \times 6.46$$
$$= 17.34$$

Round to the nearest whole number.

Write the answer.

$$\approx 17 \text{ m}^2$$

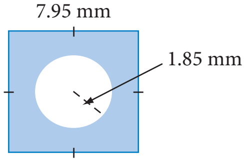
The total area is 17 m^2.

Some composite shapes are made by taking a piece away from a shape.

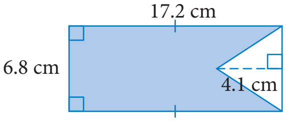
Example 13

Calculate the shaded areas below, correct to one decimal place.

a



b



Solution

a The shaded area is a square with a circle cut out.

Calculate the area of the square.

$$\begin{aligned}\text{Area of square} &= s^2\\ &= 7.95^2\\ &= 63.2025\text{ mm}^2\end{aligned}$$

Calculate the area of the circle.

$$\begin{aligned}\text{Area of circle} &= \pi r^2\\ &= \pi \times 1.85^2\\ &= 10.752\ldots\text{ mm}^2\end{aligned}$$

Subtract the circle's area from the square's area.

$$\begin{aligned}\text{Shaded area} &= 63.2025 - 10.752\ldots\\ &= 52.450\ldots\end{aligned}$$

Round correct to one decimal place.

$$\approx 52.5\text{ mm}^2$$

Write the answer.

The shaded area is 52.5 mm^2.

b The shaded area consists of a rectangle with a triangle cut out.

Calculate the area of the rectangle.

$$\begin{aligned}\text{Area of rectangle} &= l \times w\\ &= 17.2 \times 6.8\\ &= 116.96\text{ cm}^2\end{aligned}$$

Calculate the area of the triangle.

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2}bh\\ &= \frac{1}{2} \times 6.8 \times 4.1\\ &= 13.94\text{ cm}^2\end{aligned}$$

Subtract the triangle's area from the rectangle's area.

$$\begin{aligned}\text{Shaded area} &= 116.96 - 13.94\\ &= 103.02\end{aligned}$$

Round correct to one decimal place.

$$\approx 103.0\text{ cm}^2$$

Write the answer.

The shaded area is 103.0 cm^2.

EXERCISE 5.04 Areas of composite figures

Concepts and techniques

Areas of composite shapes

1 Example 12 The area of the composite figure shown here would be found using which calculation?

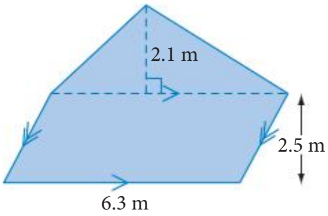
A $A = 6.3 \times (2.1 + 2.5)$

B $A = 6.3 \times 2.5 - 6.1 \times 2.5$

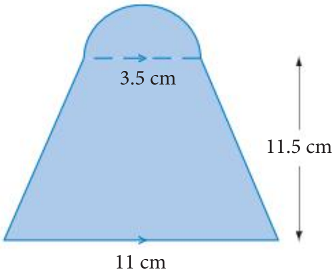
C $A = 6.3 \times 2.1 + \frac{1}{2} \times 6.3 \times 2.5$

D $A = 6.3 \times 2.5 + \frac{1}{2} \times 6.3 \times 2.1$

E $A = \frac{1}{2} \times 2.5 \times 6.3 \times 2.1$



2 Mitchell wrote the following calculations when working out this composite area. He made one mistake which caused his final answer to be incorrect. Determine which line contains the error.



Area of Semicircle $= \frac{1}{2}\pi r^2$ Line 1

$= \frac{1}{2}\pi \times 3.5^2$ Line 2

$= 19.242255 \text{ cm}^2$ Line 3

Area of trapezium $= \frac{1}{2}(a + b)h$ Line 4

$= \frac{1}{2}(11 + 3.5) \times 11.5$ Line 5

$= 83.375 \text{ cm}^2$ Line 6

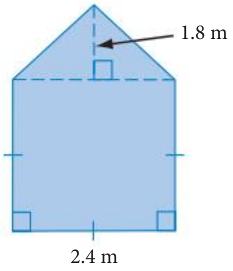
Total Area $= 19.242255 + 83.375$ Line 7

$\approx 102.62 \text{ cm}^2$ Line 8

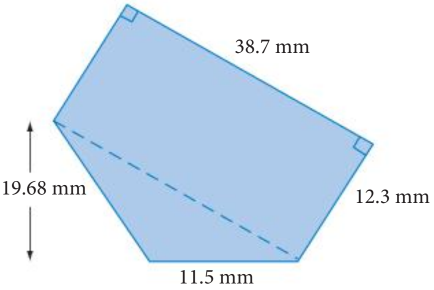
A Line 1 B Line 2 C Line 3 D Line 5 E Line 6

3 Calculate the following composite areas, correct to one decimal place.

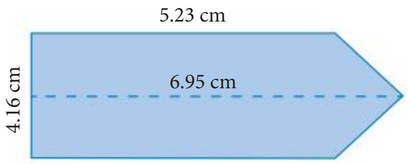
a



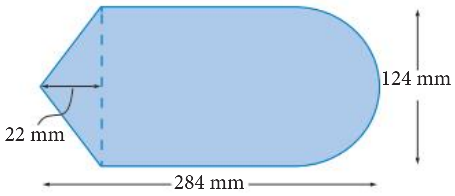
b



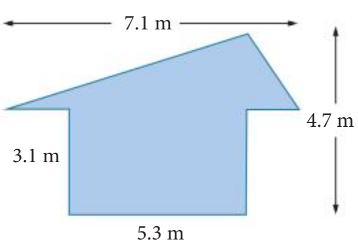
c



d



e



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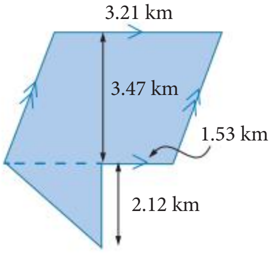
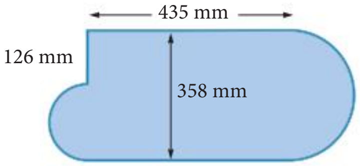
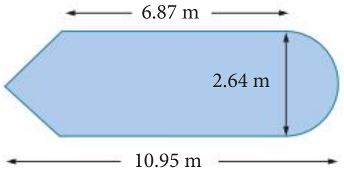


g

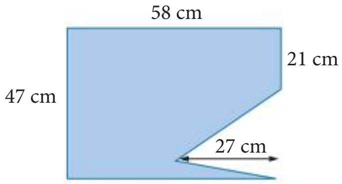


h

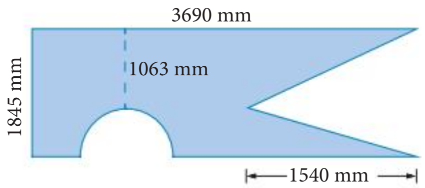


4 **Example 13** Use subtraction to evaluate the following composite areas correct to the nearest whole square unit.

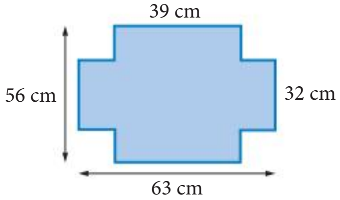
a



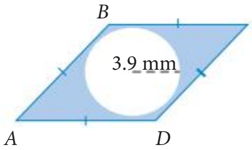
b



c

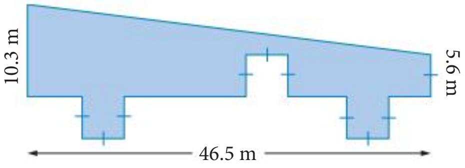


d



ABCD is a rhombus
AC = 22.7 mm
BD = 18.3 mm

e



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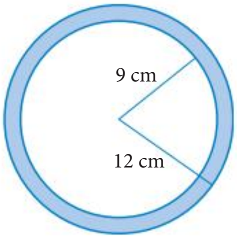
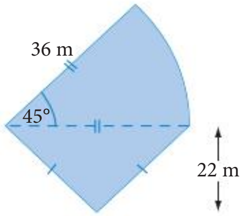
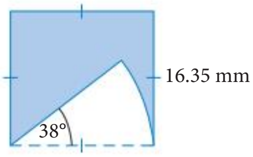


5 The composite areas shown here include sectors. Calculate the areas correct to one decimal place.

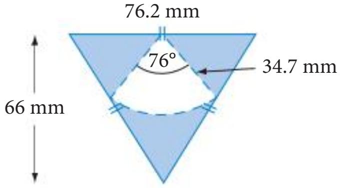
a



b

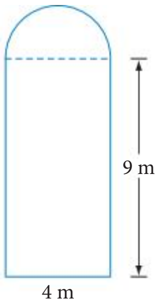


c



Reasoning and communication

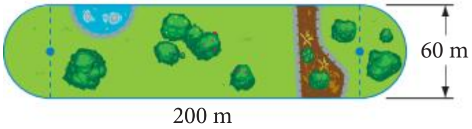
6 Calculate, correct to the nearest 0.1 m^2, the area of the glass in the window at right.



7 A factory worker cuts circular discs from square sheets of metal.

- a What is the area of the largest disc that can be cut from a square of side 20 cm? Give your answer correct to two decimal places.
- b What is the area of the remaining metal, correct to two decimal places, after the disc is cut from the square?

8 Calculate the area of this garden, correct to the nearest square metre.



9 A rectangular photo frame has dimensions 12.7 cm × 17.8 cm. The border of the frame is 1.5 cm wide. If a photo only just fits into the frame:

- a what area of the photo can be seen?
- b what area of the photo will be covered by the frame's border?

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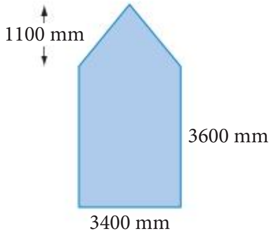
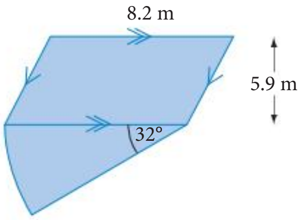
10 The owners of a house that is being built have instructed the builder to make the end wall from glass. They want to be able to take advantage of their ocean view.
The shape of this wall is shown on the right.
How many square metres of glass will be used to make this wall? (Ignore any framework in the glass).



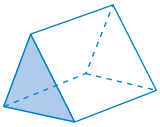
11 A patio floor is to be tiled. The diagram below shows the shape and measurements of the floor. The tiles that will be used are rectangular and have dimensions 15 cm × 12 cm. They cost \$32.50 per square metre. Due to the unusual shape of the floor, the tiler will order 5% more tiles than needed so that he can cut tiles to fit into difficult spots.

- a Calculate the area of the floor correct to 2 decimal places.
- b How many tiles will the tiler order?
- c What will the cost of the tiles be, correct to the nearest dollar?

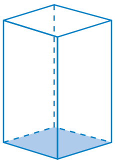


5.05 SURFACE AREAS OF PRISMS AND CYLINDERS

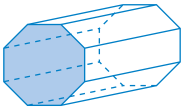
A **prism** is a **solid** with two end faces that are identical. All of the faces joining the two ends are rectangular. The cross-section of a prism is the same shape as its end faces. If a prism has a triangular face at each end, we say its **base** is a triangle and we call the prism a triangular prism. The name of any prism is determined by the shape of its base.



Triangular prism



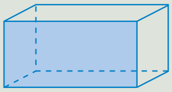
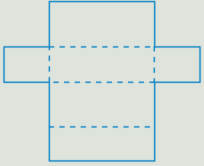
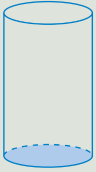
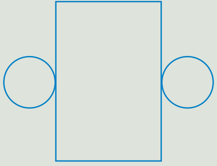
Square prism



Octagonal prism

Nets of solids

If you cut along the edges of a solid and unfold it, its net can be seen. The net shows the shapes of all of the faces of the solid.

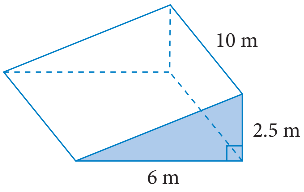
Solid	Net
Rectangular prism 	6 faces (all rectangles) 
Cylinder 	3 faces (one rectangle and two circles) 

Surface area

The **surface area** of a solid is the total area of all of its faces. It is measured in square units. When calculating the surface area of a three-dimensional object, calculate the area of each face and then total all of the areas. It helps to visualise the net of the solid and see the shapes of its faces.

Example 14

Calculate the surface area of this triangular prism.

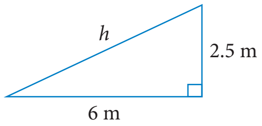


Solution

The triangular prism has five faces: two identical triangles and three different-sized rectangles.
Find the area of the two triangular faces first.

$$\text{Area of two triangles} = 2 \times \frac{1}{2} \times 6 \times 2.5$$
$$= 15\text{ m}^2$$

To calculate the area of the top rectangular face, the length of the hypotenuse, *h*, of the triangle, must be calculated first. This can be done using Pythagoras' theorem.



$$\begin{aligned} h^2 &= 2.5^2 + 6^2 \\ &= 42.25 \\ h &= \sqrt{42.25} \\ &= 6.5\text{ m} \end{aligned}$$

Calculate the areas of the rectangular faces.

$$\begin{aligned} \text{Area of top face} &= 6.5 \times 10 \\ &= 65\text{ m}^2 \end{aligned}$$
$$\begin{aligned} \text{Area of right face} &= 2.5 \times 10 \\ &= 25\text{ m}^2 \end{aligned}$$
$$\begin{aligned} \text{Area of bottom face} &= 6 \times 10 \\ &= 60\text{ m}^2 \end{aligned}$$

Add all of the areas to calculate the total surface area.

$$\begin{aligned} \text{Total surface area} &= 15 + 65 + 25 + 60 \\ &= 165\text{ m}^2 \end{aligned}$$

Write the answer.

The surface area is 165 m^2.

Surface area of cylinders

A cylinder is like a prism because it has two identical ends. These are both circles. The circular ends are joined by a curved surface which is in fact rectangular. The area of the curved surface is equal to $2\pi rh$. To calculate the surface area of a cylinder, add the areas of the two circles to the area of the curved surface.

Surface area formulas for cylinders

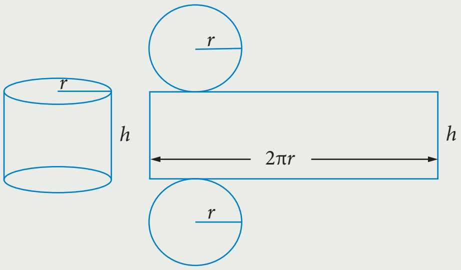
A cylinder can be closed or open at one end or open at both ends.

If a cylinder is open at both ends, its surface area is $2\pi rh$.

If a cylinder is open at one end, its surface area is $2\pi rh + \pi r^2$.

If a cylinder is closed, its surface area is $2\pi rh + 2\pi r^2$.

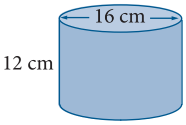
IMPORTANT



$$\text{Surface area} = \pi r^2 + \pi r^2 + 2\pi rh$$
$$= 2\,\pi r^2 + 2\pi rh$$

Example 15

Calculate the surface area of this open-top cylinder, give your answer to the nearest whole unit.



Solution

This solid has two faces. One is a circle. The other is a rectangle. $SA = \pi r^2 + 2\pi rh$

Remember that the radius is half of the diameter. $r = 8$ cm

Substitute the known values into the equation. Evaluate.
$$SA = \pi \times 8^2 + 2\pi \times 8 \times 12$$
$$= 804.247\ldots$$

Round to the nearest whole unit. $\approx 804\text{ cm}^2$

Write the answer. The surface area is 804 cm^2.

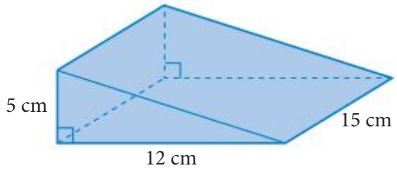
EXERCISE 5.05 Surface areas of prisms and cylinders

Surface area of solids

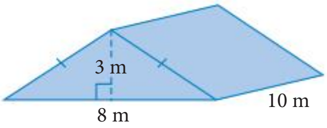
Concepts and techniques

1 **Example 14** Calculate the surface area of each triangular prism below. You will need to use Pythagoras' theorem to calculate an unknown side first.

a

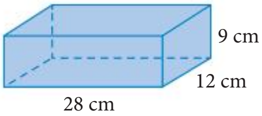


b

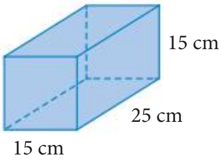


2 Calculate the surface area of each of these solids.

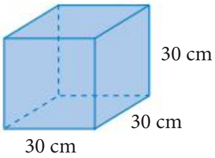
a



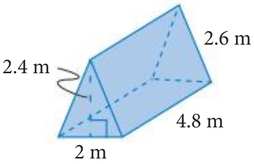
b



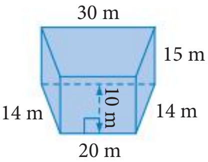
c



d



e



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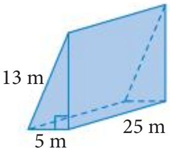
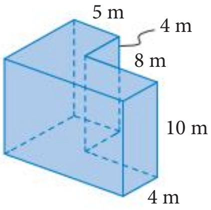
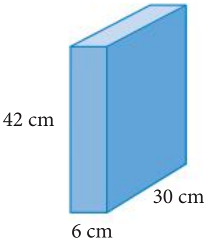


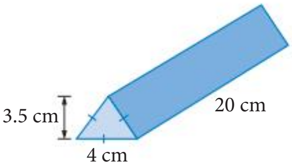
g



h

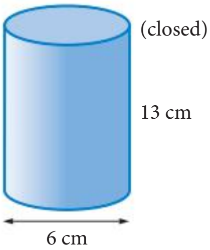


i

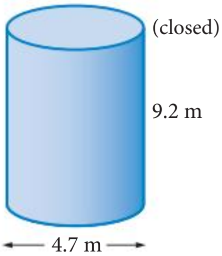


3 **Example 15** Calculate the surface areas of these cylinders correct to two decimal places.

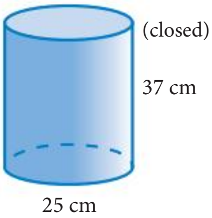
a



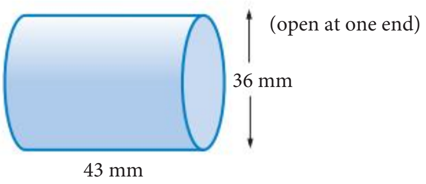
b



c

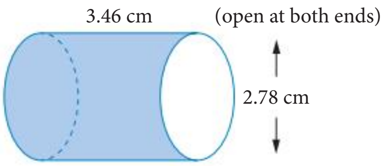


d



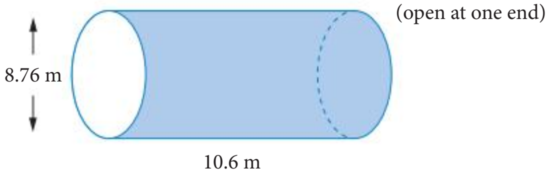
(open at one end)

e



(open at both ends)

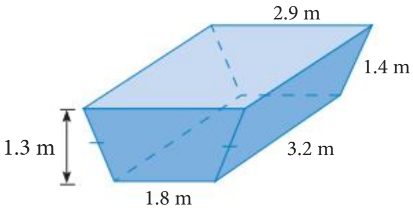
f



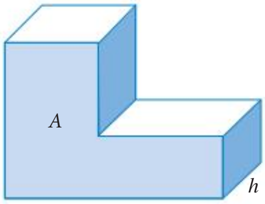
(open at one end)

Reasoning and communication

4 This rubbish skip has the shape of an 'open' trapezoidal prism with no lid. Calculate its external surface area and estimate how many square metres of metal were used to make it.



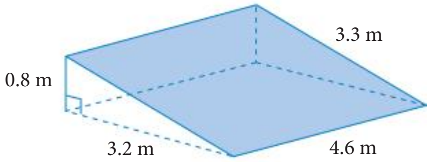
5 If the cross-section of a prism has area A and perimeter P, and the length of the prism is h, as shown in the diagram, then its surface area can be calculated using the formula $SA = 2A + Ph$. Explain why this formula works.



6 Calculate the surface area of this skateboard ramp:

a by adding the areas of its five faces

b using the formula $SA = 2A + Ph$.



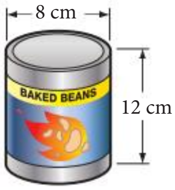
7 This can of baked beans is the shape of a cylinder. It has three faces: two circles and one rectangle. For this cylinder, calculate:

a the area of one circle, correct to two decimal places.

b the height of the rectangle.

c the length of the rectangle, correct to two decimal places.

d the surface area of the can, correct to two decimal places.



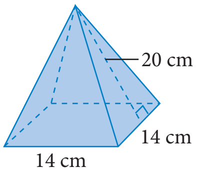
8 A cube has surface area of 864 cm^2. What is the side length of each of its square faces?

5.06 SURFACE AREAS OF PYRAMIDS, CONES AND SPHERES

A **pyramid** is a solid with a **polygon** for its base and triangular side faces meeting at a point called the **apex**. The pyramid takes its name from the shape of its base. For example, a square based pyramid has a square base.

Example 16

Calculate the surface area of this square pyramid.



Solution

The square pyramid has five faces: four identical triangles and one square.

Surface area = 4 × area of triangle + area of square.

Calculate the area of triangle.

$$\text{Area of triangle} = \frac{1}{2} \times 14 \times 20$$
$$= 140\ \text{cm}^2$$

Calculate the area of square.

$$\text{Area of square} = 14 \times 14$$
$$= 196\ \text{cm}^2$$

Add the areas to calculate the total surface area.

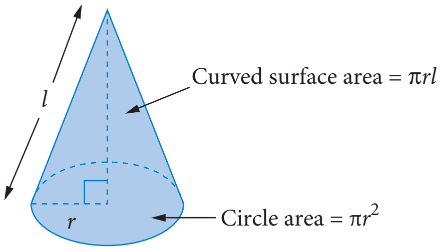
$$SA = 4 \times 140 + 196$$
$$= 756\ \text{cm}^2$$

Write the answer.

The surface area is 756 cm^2.

Surface area of cones

A cone is similar to a pyramid because it has a base and an apex. A cone has two surfaces: a circular base and a curved surface. The area of the curved surface can be found using the formula $A = \pi rl$, where r is the radius of the circular base and l is the **slant height** of the cone.



The slant height is not to be confused with the perpendicular height. While the perpendicular height measures the distance from the centre of the base to the apex, the slant height measures the distance from a point on the circumference of the base to the apex.

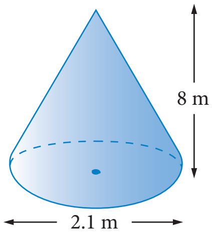
IMPORTANT

Surface area of a cone = $\pi r^2 + \pi rl$

An open cone has no circular face, just a curved surface. When calculating the surface area of an open cone the formula becomes $SA = \pi rl$.

Example 17

Calculate the surface area of the cone. Round your answer to two decimal places.



Solution

The diameter and perpendicular height are known. The radius and slant height are unknown. Calculate these first.

Remember that the radius is half of the diameter.	$r = 1.05$ m
Use Pythagoras' theorem to calculate the slant height.	$l^2 = 1.05^2 + 8^2$ $= 65.1025$ $l = \sqrt{65.1025}$ $= 8.068...$ m
Write the formula for the surface area of a cone.	$SA = \pi r^2 + \pi rl$
Substitute the known values into the formula.	$= \pi \times 1.05^2 + \pi \times 1.05 \times 8.068...$
Evaluate.	$= 30.079...$
Round to two decimal places.	$\approx 30.08 \text{ m}^2$
Write the answer.	The surface area of this cone is 30.08 m^2.

Surface area of spheres

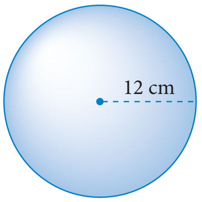
A **sphere** is a solid with only circular cross-sections. It looks like a ball. To calculate the surface area of a sphere use the formula: Surface area = $4\,\pi r^2$.

IMPORTANT

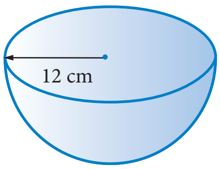
Surface area of a sphere = $4\,\pi r^2$

Example 18

a Calculate the surface area of this sphere correct to the nearest square centimetre.



b The sphere above is cut in half to make a bowl. A lid is added. Calculate the bowl's surface area including the lid. Give your answer correct to one decimal place.



Solution

a Write the formula for the surface area of a sphere.	$SA = 4\pi r^2$
Substitute the known values into the equation.	$= 4 \times \pi \times 12^2$
Evaluate.	$= 1809.557...$
Round to the nearest whole centimetre.	$\approx 1810 \text{ cm}^2$
Write the answer.	The surface area is 1810 cm^2.

b This shape represents half of the surface area of the sphere. It can also be called a hemisphere. To calculate the surface area of a hemisphere, calculate the surface area as if it is a full sphere and divide by two.

$$\text{Hemisphere surface area} = 1809.557\ldots \div 2 = 904.778\ldots \text{ cm}^2$$

Calculate the surface area of the circular lid.

$$A = \pi r^2 = \pi \times 12^2 = 1357.168\ldots \text{ cm}^2$$

Add the areas.

Total surface area

$$= 904.778\ldots + 1357.168\ldots$$
$$= 1357.168\ldots$$

Round to one decimal place.

$$\approx 1357.2 \text{ cm}^2$$

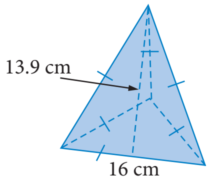
Write the answer.

The surface area of the bowl and lid is 1357.2 cm^2.

EXERCISE 5.06 Surface areas of pyramids, cones and spheres

Concepts and techniques

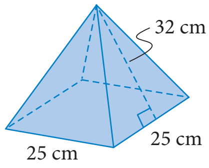
1 Example 16 The surface area of this triangular pyramid is:



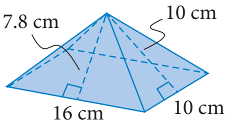
A 443 cm^2 **B** 443 cm **C** 445 cm^2 **D** 462 cm^2 **E** 923 cm^2

2 Calculate the surface areas of the following solids, correct to two decimal places where necessary.

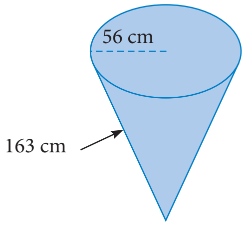
a



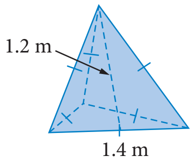
b



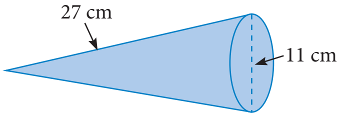
c



d



e



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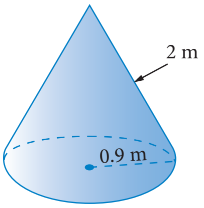
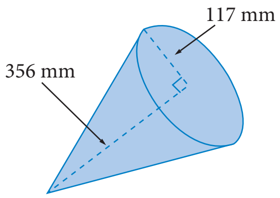


3 **Example 17** The surface area of this closed cone is:



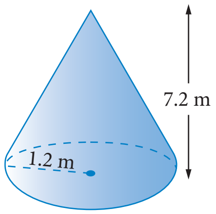
A 84 657.3 mm^2
B 170 087.6 mm^2
C 173 858.9 mm^2
D 180 744.6 mm^2
E 347 717.8 mm^2

4 For the solids shown here you will need to:

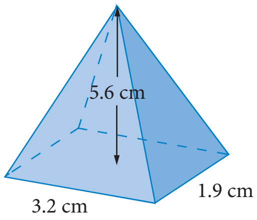
i use Pythagoras' theorem to calculate any necessary slant heights correct to one decimal place

ii calculate the surface area correct to the nearest whole square unit.

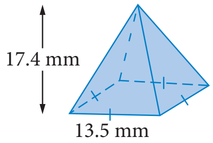
a



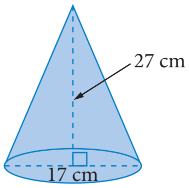
b



c



d



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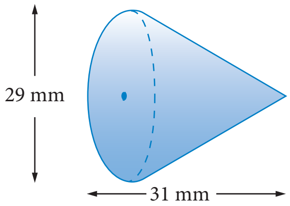

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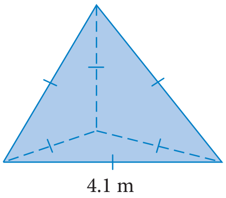
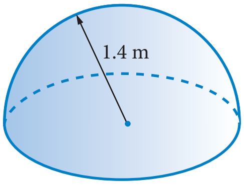


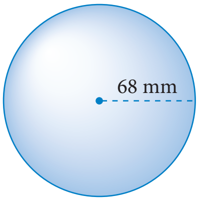
5 **Example 18** To calculate the surface area of this hemisphere you would use the calculation:



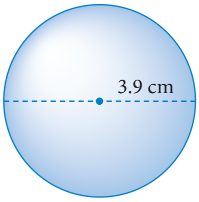
A $\frac{1}{2} \times 4\pi \times 1.4^2$
B $4\pi \times 1.4^2$
C $\frac{1}{2} \times 4\pi \times 2.8^2$
D $\frac{1}{2} \times 4\pi \times 1.4^2 + \pi \times 1.4^2$
E $4\pi \times 1.4^2 + \pi \times 1.4^2$

6 Calculate the surface area of the following solids, correct to two decimal places.

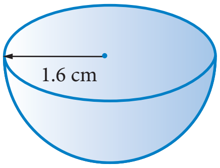
a



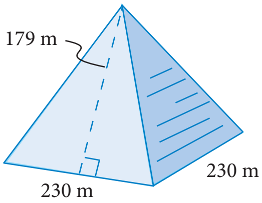
b



c

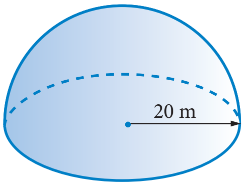


Reasoning and communication



7 The square base of the Great Pyramid of Cheops (or Khufu) at Giza in Egypt has sides of length 230 m, and the slant height of each triangular face is 179 m. Calculate the total surface area of the pyramid, including the base.

8 Craig has to paint the outer surface of this hemispherical dome. The paint that Craig uses comes in 10 litre tins which cost $59 each.

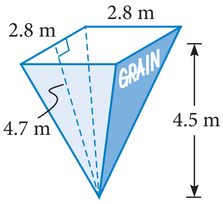


- a Calculate the surface area of the dome correct to one decimal place.
- b One litre of paint covers 6 square metres of wall. How many litres of paint will Craig need to apply one coat of paint over the dome?
- c How many tins of paint will Craig have to buy?
- d What will be the total cost of the paint?

9 A square pyramid has surface area of 40 cm^2. The area of each triangular face is 6 cm^2.

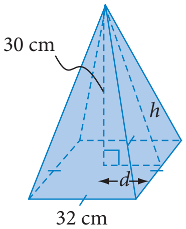
- a Find the area of the square base.
- b Find the length of the sides of the square base.

10 This grain container is in the shape of an open square pyramid. If stainless steel costs $38.60 per square metre, what would be the cost of the steel needed to make this container?



11 This square pyramid has a base of length 32 cm and a perpendicular height of 30 cm.

- a What is the length of d?
- b Calculate the slant height, h, of the pyramid.
- c Calculate the surface area of the pyramid.



12 Tim and Andrew are working together on their history assignment. They have decided to build a model of an Indian tent which is known as a teepee. They need to calculate the number of square metres of material required.

They want their model teepee to be 1 metre tall. The diameter of the base is to be 1.2 metres. There will not be a floor in the tepee.

- a Draw a diagram of the model teepee showing measurements.
- b Calculate the area of fabric needed, ignoring any openings and extra allowances. Give your answer correct to one decimal place.

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5.07 SURFACE AREAS OF COMPOSITE SOLIDS

Composite solids are formed by combining two or more regular solids. Consider the shape of a grain silo. It is made up of a cylinder and a cone.

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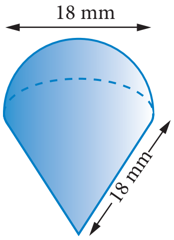
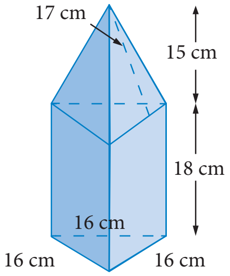
Example 19

Calculate the surface area of these composite solids. Give your answers correct to one decimal place.

a



b



Solution

a We have a cone with no base surface and a hemisphere with no base surface. The radius of both is half of the diameter.

$r = 9$ mm

Calculate the curved surface area of the hemisphere.

$$SA = \frac{1}{2} \times 4\pi r^2$$
$$= 2\pi \times 9^2$$
$$= 508.93\ldots \text{ mm}^2$$

Calculate the curved surface area of the cone.

$$SA = \pi rl$$
$$= \pi \times 9 \times 18$$
$$= 508.93\ldots \text{ mm}^2$$

Add the areas to evaluate the total surface area.

$$\text{Total surface area} = 508.93\ldots + 508.93\ldots$$
$$= 1017.87602\ldots$$

Round to one decimal place.

$$\approx 1017.9 \text{ mm}^2$$

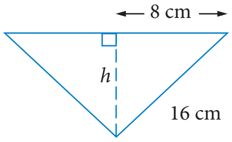
Write the answer.

The total surface area is 1017.9 mm^2.

b We have a triangular pyramid and a triangular prism. Calculate the surface area of the pyramid. Don't include the base.

$$SA = 3 \times \frac{1}{2} \times 16 \times 17$$
$$= 408 \text{ cm}^2$$

Use Pythagoras' theorem to calculate the perpendicular height of the triangle which is the base of the prism.



$$16^2 = h^2 + 8^2$$
$$256 = h^2 + 64$$
$$h^2 = 192$$
$$h = \sqrt{192}$$
$$= 13.85\ldots \text{ cm}$$

Calculate the surface area of the prism. Remember that only 4 faces of the prism are used.

$$SA = 3 \times 16 \times 18 + \frac{1}{2} \times 16 \times 13.85\ldots$$
$$= 974.85\ldots \text{ cm}^2$$

Calculate the total surface area.

$$\text{Total surface area} = 408 + 974.85\ldots$$
$$= 1382.85\ldots$$

Round to one decimal place.

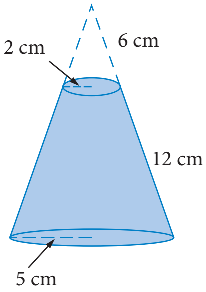
$$\approx 1382.9 \text{ cm}^2$$

Write the answer.

The total surface area is 1382.9 cm^2

Example 20

This solid has been formed by cutting a small cone off the end of a large cone. Calculate the surface area of this composite solid. Give your answer correct to the nearest square centimetre.



Solution

There are three surfaces forming this solid.
Calculate the areas of the two circular surfaces.

$$\text{Area of small circle} = \pi \times 2^2$$
$$= 12.566\ldots \text{ cm}^2$$
$$\text{Area of big circle} = \pi \times 5^2$$
$$= 78.539\ldots \text{ cm}^2$$

The curved surface is made by taking the small cone's curved surface area away from the large cone's curved surface area.
Calculate the curved surface area of both cones using the formula SA = πrl.

$$\text{SA of small cone} = \pi \times 2 \times 6$$
$$= 37.69\ldots \text{ cm}^2$$
$$\text{SA of large cone} = \pi \times 5 \times 18$$
$$= 282.74\ldots \text{ cm}^2$$

Subtract to calculate the composite solid's curved surface area.

$$\text{Curved SA} = 282.74\ldots - 37.69\ldots$$
$$= 245.04\ldots \text{ cm}^2$$

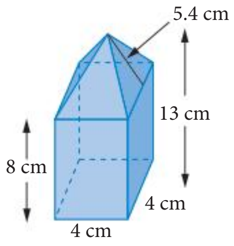
Add the surface areas to calculate the total surface area.	Total SA = 12.566… + 78.539… + 245.04…
	= 336.1...
Round to the nearest square centimetre.	≈ 336 cm^2
Write the answer.	The surface area is 336 cm^2.

EXERCISE 5.07 Surface areas of composite solids

Concepts and techniques

Surface area

1 **Example 19** For the composite solid shown, which calculation would correctly calculate its surface area?



A $4\times\frac{1}{2}\times4\times4+4\times8\times8+4^2$

B $4\times4\times8+4^2-4\times\frac{1}{2}\times4\times5.4$

C $4\times\frac{1}{2}\times4\times5.4+4\times4\times8+4^2$

D $4\times4\times5.4+4\times4\times8+4^2$

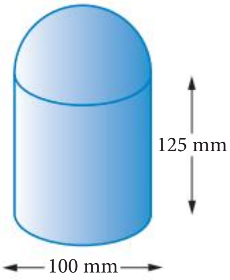
E $4\times\frac{1}{2}\times8\times4+4\times5.4\times8+4^2$

2 How many surfaces does this closed solid have?

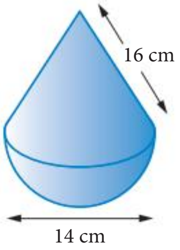
A 2 B 3 C 4 D 6 E 10

3 For each of the solids shown, calculate the total surface area. Round answers to 1 decimal place where necessary.

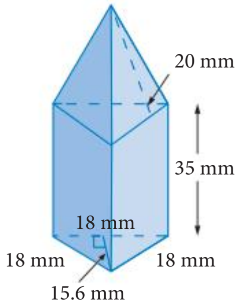
a



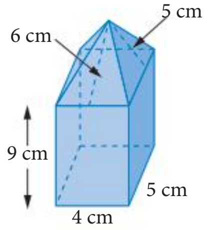
b



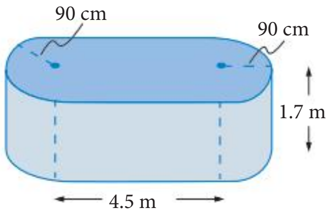
c



d



e



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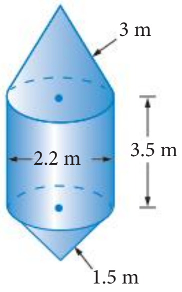
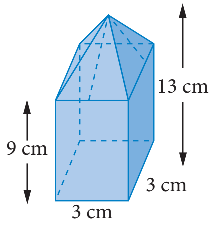


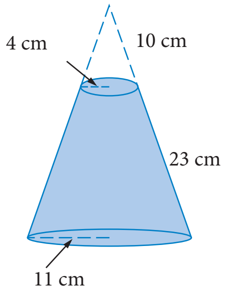
4 For this composite solid:

- a use Pythagoras' theorem to calculate the perpendicular heights of the triangular faces correct to two decimal places
- b calculate the total surface area, correct to two decimal places.

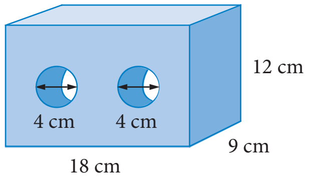


5 **Example 20** Calculate the surface areas of the following composite solids, rounding to two decimal places when necessary.

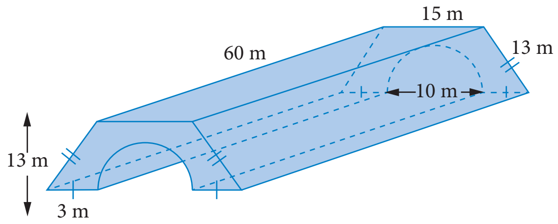
a



b



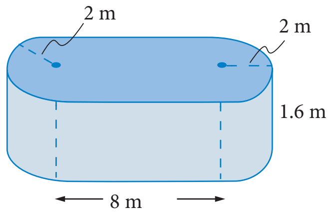
c



Reasoning and communication

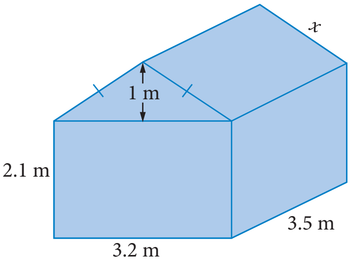
6 The swimming pool pictured here is to have a layer of tiles laid across the floor and up the internal walls.

- a Calculate the total surface area that is to be tiled. Round your answer to two decimal places.
- b Calculate the cost of the tiles if they are priced at $26.70 per square metre.

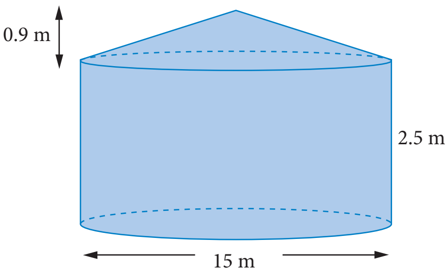


7 A garden shed is to be built in the shape shown on the right. The floor will be a concrete slab. The walls and roof will be made of aluminium sheets.

- a Calculate the value of x correct to one decimal place.
- b Calculate the surface area of aluminium required, ignoring allowances for joins, doors or windows. Give your answer correct to one decimal place.



8 The external surface of this concrete rainwater tank needs to be painted. It will require 2 coats. The base of the water tank will not be painted as the structure is too heavy to lift.



- a Calculate the surface area of the tank, correct to two decimal places..
- b If one tin of paint covers 120 square metres and the tank requires two coats of paint, how many tins will need to be purchased?
- c Given that the tins of paint cost $97.90, calculate the cost of the tins purchased.
- d What is the dollar value of leftover paint (correct to the nearest dollar)?

5.08 VOLUME AND CAPACITY: PRISMS AND CYLINDERS

Metric units for volume

The **volume** of a solid is the amount of space it occupies, measured in **cubic units**. It can be thought of as the number of unit cubes that the solid can hold.

When converting units of volume, we need to multiply or divide by cubed units.

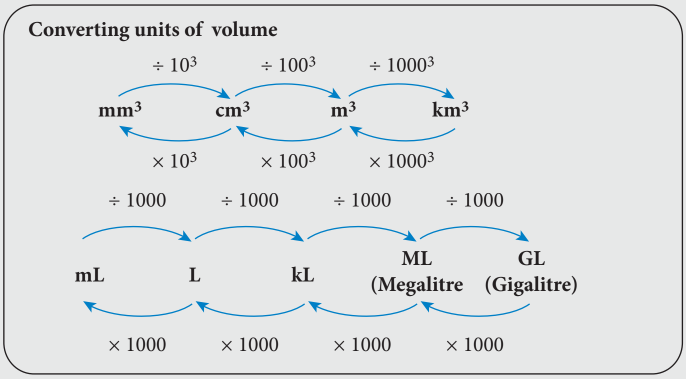
For example if: 1 m = 100 cm

then: $1 \text{ m}^3 = 100^3 \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$

and if: 1 cm = 10 mm

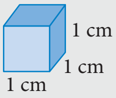
then: $500 \text{ mm}^3 = 500 \div 10^3 \text{ cm}^3 = 0.5 \text{ cm}^3$.

IMPORTANT



1 cm^3 is equivalent to 1 mL

1000 cm^3 is equivalent to 1 L



Example 21

Convert:

a 74 m^3 to cm^3 **b** 4600 mm^3 to cm^3.

Solution

a $1\text{ m}^3 = 100^3\text{ cm}^3$

When converting from a larger unit of measurement to a smaller one, multiply.

$$74\text{ m}^3 = 74 \times 100^3$$
$$= 74\ 000\ 000\text{ cm}^3$$

b $1\text{ cm}^3 = 10^3\text{ mm}^3$

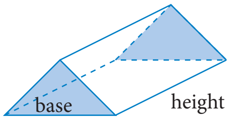
When converting from a smaller unit of measurement to a larger one, divide.

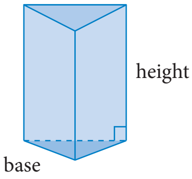
$$4600\text{ mm}^3 = 4600 \div 10^3$$
$$= 4.6\text{ cm}^3$$

Volume formulas

To calculate the volume of a prism, calculate the area of its base and multiply that by its perpendicular height.

For this triangular prism, the base is the triangular face. If you tip the prism over so that its base is on the ground, its height becomes more obvious.





IMPORTANT

Volume of prism = Area of base × perpendicular height
$= Ah$

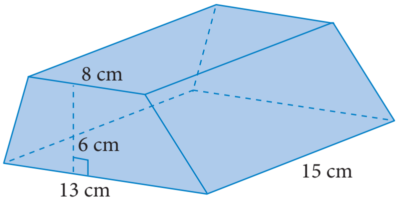
To calculate the volume of a cylinder, we follow the same rule.
Since the base of a cylinder is a circle, its area is πr^2.

IMPORTANT

Volume of cylinder $= Ah$
$= \pi r^2 h$

Example 22

Calculate the volume of this trapezoidal prism.



Solution

Write the formula for the volume of a prism.

$$\text{Volume} = Ah$$

The base of this prism is a trapezium.
Include the formula for the area of a trapezium.

$$= \left[\frac{1}{2}(a+b)h\right] \times h$$

Substitute the known values into the equation.
In the trapezium: $a = 8$ cm, $b = 13$ cm and $h = 6$ cm.
For the prism: $h = 15$ cm.

$$= \left[\frac{1}{2}(8+13) \times 6\right] \times 15$$

Evaluate.

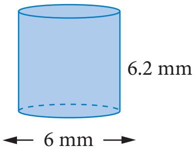
$$= 945 \text{ cm}^3$$

Write the answer.

The volume of the prism is 945 cm^3.

Example 23

Calculate the volume of this cylinder correct to one decimal place.



Solution

Write the formula.
Note that A represents the area of the base circle.

$$V = Ah$$
$$V = \pi r^2 h$$

Remember that the radius is half of the diameter.
Substitute the known values into the equation.

$$= \pi \times 3^2 \times 6.2$$

Evaluate.

$$= 175.30\ldots$$

Round to one decimal place.

$$\approx 175.3 \text{ mm}^3$$

Write the answer.

The volume of the cylinder is 175.3 mm^3.

Capacity

The **capacity** of a solid is a measure of how much liquid it can contain. This is usually measured in millilitres and litres. If the volume of a solid is known, it can be converted into units of capacity.

IMPORTANT

$1\ \text{cm}^3 = 1$ millilitre (mL)

$1\ \text{m}^3 = 1000$ litres (L) $= 1$ kilolitre (kL)

$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

Example 24

Express the following volumes as capacities.

a $683\ \text{cm}^3 =$ _____ mL. **b** $4600\ \text{cm}^3 =$ ______ L. **c** $3500\ \text{m}^3 =$ _______ ML.

Solution

a $1\ \text{cm}^3 = 1\ \text{mL}$

$683\ \text{cm}^3 = 683\ \text{mL}$

b $1\ \text{cm}^3 = 1\ \text{mL}$

$$\begin{tikzcd}
\text{L} \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

$4600\ \text{cm}^3 = 4600\ \text{mL}$

$4600\ \text{mL} = 4600 \div 1000$

$= 4.6\ \text{L}$

c $1\ \text{m}^3 = 1\ \text{kL}$

$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

$3500\ \text{m}^3 = 3500\ \text{kL}$

$3500\ \text{kL} = 3500 \div 1000$

$= 3.5\ \text{ML}$

EXERCISE 5.08 Volume and capacity: prisms and cylinders

Concepts and techniques

1 Example 21 Convert:

a $7\ \text{m}^3$ to cm^3

b $50\ \text{cm}^3$ to mm^3

c $89\ 000\ \text{cm}^3$ to m^3

d $0.468\ \text{m}^3$ to cm^3

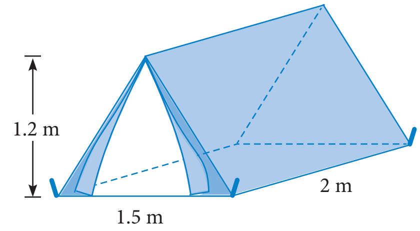
e $2400\ \text{mm}^3$ to cm^3

f $5\ 600\ 000\ \text{cm}^3$ to m^3

g $9\ 100\ 000\ \text{mm}^3$ to cm^3

h $12\ \text{m}^3$ to cm^3.

2 Example 22 Calculate the volume of this tent.



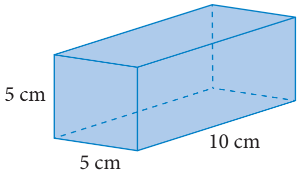
3 Which of the following is the volume of a cube of length 2.5 m?

A 6.25 mm^3 B 8.5 m^3 C 15.625 m^3 D 30 m^3 E 37.5 m^3

4 Example 23 A cylinder with a diameter of 30 cm and perpendicular height of 12 cm would have a volume closest to:

A 8482 cm^3 B 8483 cm^3 C 13 572 cm^3 D 33 929 cm^3 E 33 930 cm^3

5 Calculate the volumes of the following solids. Round your answers correct to one decimal place when necessary.

a


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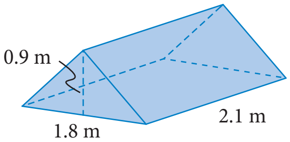
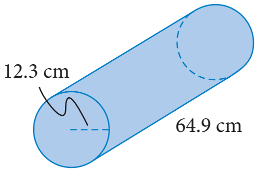
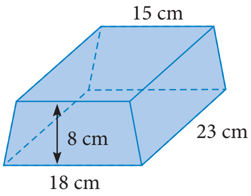
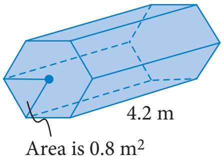


c


d


e


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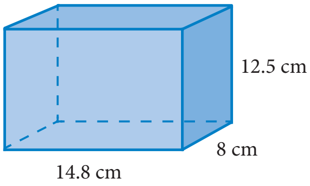

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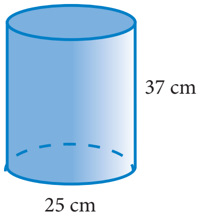

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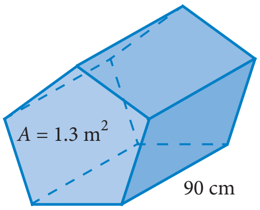
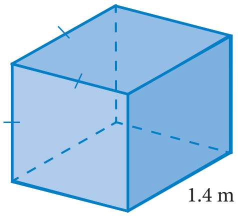


i


6 Example 24 Convert:

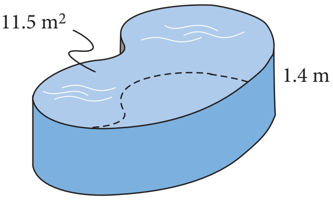
a 680 cm^3 to millilitres
b 8500 cm^3 to litres
c 22 m^3 to litres
d 8000 L to m^3
e 3.5 m^3 to millilitres
f 690 L to cm^3
g 55 m^3 to litres
h 4300 m^3 to kilolitres
i 9500 L to m^3
j 8.5×10^4 cm^3 to litres
k 4.3×10^{-3} kL to cm^3
l 10^6 m^3 to megalitres.

Reasoning and communication

7 The cross-section of a kidney-shaped swimming pool has an area of 11.5 m^2. The pool has a constant depth of 1.4 m.

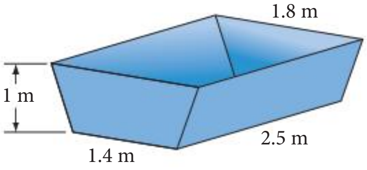
a What is the volume of the pool?

b What is the pool's capacity in litres?



8 a This rubbish skip in the shape of a trapezoidal prism is delivered to a building site. What volume of rubbish will it hold?

b If building rubbish costs $16.50 per cubic metre to dump at the local tip, how much will it cost to dump four full skips?

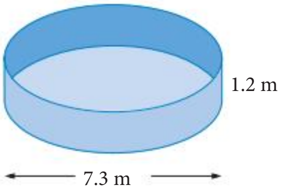


9 A circular swimming pool has a diameter of 7.3 m. It is 1.2 metres deep.

a Calculate the volume of the pool in cubic metres, correct to three decimal places.

b Calculate the capacity of the pool correct to the nearest litre.

c If water is pumped into the empty pool at a rate of 8 L/min, how long will it take to fill?

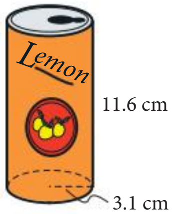


10 a Calculate the volume of this can, correct to the nearest cubic centimetre.

b What is the can's capacity in mL?

c If the radius of the can doubled, would its capacity double? Explain your reasoning.

11 A cylindrical can has a capacity of 850 mL. The can has a height of 15 cm. Calculate the radius of the can correct to two decimal places.



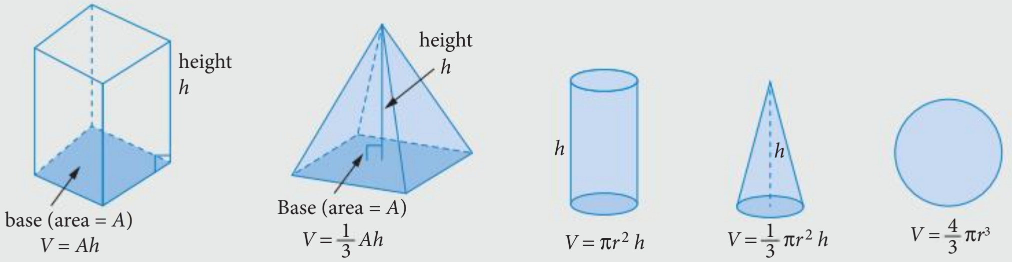
5.09 VOLUME AND CAPACITY: PYRAMIDS, CONES AND SPHERES

Pyramids and cones

The volumes of pyramids and cones are directly related to the volumes of prisms and cylinders. Mathematicians have determined that the volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height. Similarly, the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same base and height.

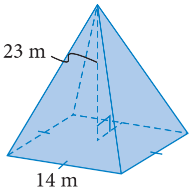
The volume of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$.

IMPORTANT



Example 25

Calculate the volume of this square pyramid, correct to the nearest cubic metre.



Solution

Write the formula for the volume of a pyramid. $\text{Volume} = \frac{1}{3}Ah$

Substitute the known values into the equation:
A = area of square base and h = 23 m. $= \frac{1}{3} \times 14^2 \times 23$

Evaluate. $= 1502.666\ldots$

Round to the nearest whole cubic metre. $\approx 1503 \text{ m}^3$

Write the answer. The volume of the pyramid is 1503 m^3.

Example 26

Calculate the volume of this ball correct to the nearest cm^3.

Shutterstock.com/Boykung

Solution

Remember that the radius is half of the diameter. $r = 15 \text{ cm}$

Write the formula. $\text{Volume} = \frac{4}{3}\pi r^3$

Substitute the known values into the equation. $= \frac{4}{3}\pi \times 15^3$

Evaluate. $= 14\ 137.1669\ldots$

Round to the nearest whole cubic centimetre. $\approx 14\ 137 \text{ cm}^3$

Write the answer. The volume of the ball is 14 137 cm^3.

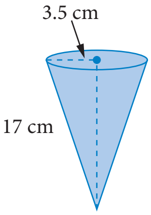
Example 27

An ice-cream cone has a diameter of 7 cm and a slant height of 17 cm.

a Calculate the volume of the cone.

b How many millilitres of ice-cream can the cone hold if the ice-cream is only filled level with the top of the cone?

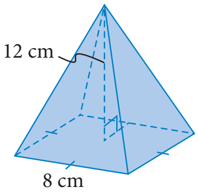
Solution



a Remember that the radius is half of the diameter.	$r = 3.5$ cm
Use Pythagoras' theorem to calculate the perpendicular height of the cone.	$17^2 = 3.5^2 + h^2$ $289 = 12.25 + h^2$ $h^2 = 276.75$ $h = \sqrt{276.75}$ $= 16.63...$ cm
Write the appropriate volume formula.	$V = \frac{1}{3}\pi r^2 h$
Substitute the known values into the equation.	$= \frac{1}{3}\pi \times 3.5^2 \times 16.63...$
Evaluate.	$= 213.40...$
Round your answer to the nearest whole cubic centimetre.	$\approx 213\text{ cm}^3$
Write the answer.	The volume of the cone is 213 cm^3.
b Complete the conversion into a measure of capacity.	$1\text{ cm}^3 = 1\text{ mL}$ $213\text{ cm}^3 = 213\text{ mL}$
Write the answer.	The cone can hold 213 mL of ice-cream.

EXERCISE 5.09 Volume and capacity: pyramids, cones and spheres

Concepts and techniques

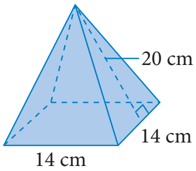


1 Example 25 Which of the following calculations would give the correct volume for the pyramid shown?

A $V = \frac{1}{3} \times 8^2 \times 12$
B $V = \frac{1}{3} \times 8 \times 12^2$
C $V = \frac{1}{3} \times 8^2 \times 12.6$
D $V = \frac{1}{3} \times 4^2 \times 12$
E $V = \frac{1}{3} \times 4^2 \times 12.6$

2 Which of the following is closest to the correct volume for this pyramid?

A 560 cm^3 **B** 980 cm^3 **C** 1224 cm^3 **D** 3920 cm^3 **E** 3921 cm^3



3 **Example 26** The volume of a sphere is calculated using which formula?

A $V = 4\pi r^2$
B $V = \frac{4}{3}\pi r^2$
C $V = \pi r^2 h$
D $V = 4\pi r^3$
E $V = \frac{4}{3}\pi r^3$

4 Which of the following is closest to the capacity of a square pyramid with base length 35 cm and perpendicular height 67 cm?

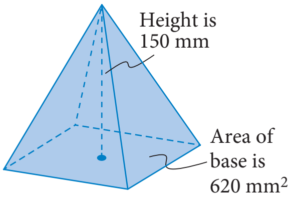
A 782 mL
B 1173 mL
C 2.345 L
D 27 L
E 82 L

5 **Example 27** A cone has a radius of 16 mm and slant height of 25 mm.

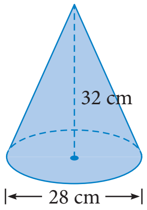
a Use Pythagoras' theorem to calculate its perpendicular height, correct to two decimal places.
b Calculate the volume of the cone correct to one decimal place.
c State the capacity of the cone to the nearest millilitre.

6 Evaluate the volumes of the following solids. Give your answer correct to two decimal places where rounding is necessary.

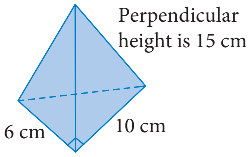
a



b



c



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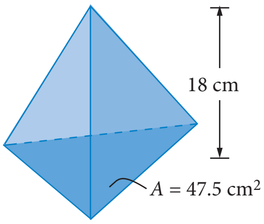

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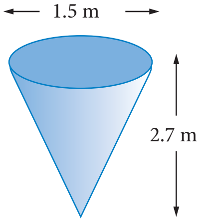

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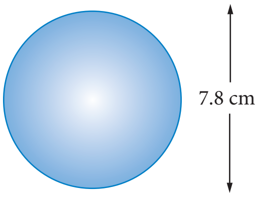

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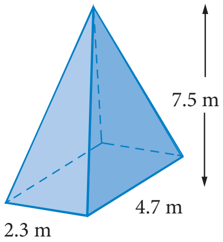
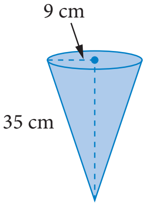
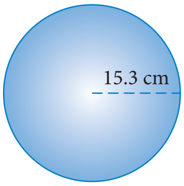


h



i



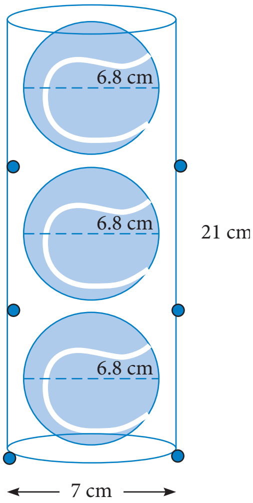
7 A tennis ball is a sphere with diameter 6.8 cm. Calculate its volume, correct to two decimal places.

8 The largest pyramid in Egypt is the Great Pyramid of Cheops (or Khufu). It has a square base of side length 230 m and a perpendicular height of 137 m. What is its volume correct to the nearest 100 cubic metres?

9 Calculate the volume of the Moon given that it has a diameter of 3600 km. Give your answer correct to the nearest million cubic kilometres.

Reasoning and communication

10 A cylindrical container holds three tennis balls. Each tennis ball has a diameter of 6.8 cm.
   a Calculate the volume of the cylinder. Round your answer to two decimal places.
   b Calculate the volume of the three tennis balls. Give your answer correct to two decimal places.
   c What percentage of the cylinder's volume is taken up by the tennis balls? Answer correct to the nearest percent.

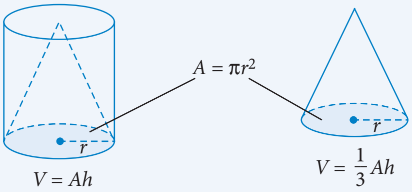


11 A pyramid that has a base area of 16.4 cm^2 and volume of 216.48 cm^3 would be how tall?

12 For a closed cone that has a radius of 11 mm and a volume of 2027 mm^3:
   a calculate its perpendicular height correct to the nearest mm
   b calculate its slant height correct to the nearest mm
   c calculate its total surface area

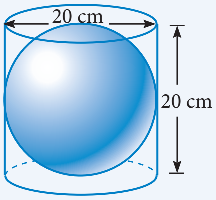
INVESTIGATION Cylinders, cones and spheres

a The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.



Take a cylindrical can and remove the top. Make a paper cone with the same base and height as your can. Fill the cone with sand or water and empty it into the can. Repeat this two more times. The can should now be full of sand (or water). Write a brief report detailing the results of this investigation into your workbook.

b The sphere shown below just fits into a cylinder of diameter and height 20 cm.
   i Calculate the volume of the sphere and the cylinder. Do not round your answers.
   ii What fraction of the cylinder's volume is taken up by the sphere?
   iii Is this fraction the same for any sphere and cylinder with equal diameters? Experiment with other values.
   iv Is there any way of proving this result?

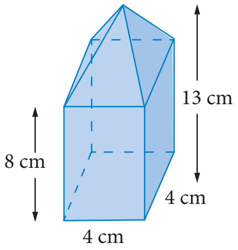


5.10 VOLUME AND CAPACITY: COMPOSITE SOLIDS

Composite solids are made when two or more solids are combined. Use the volume formulas from previous sections in order to calculate the volumes of the following composite solids which include prisms, cylinders, pyramids, cones and spheres.

Example 28

Calculate the volume of this composite solid. Round your answer to the nearest whole cubic centimetre.



Solution

Calculate the volume of the square based pyramid.

Calculate the height of the pyramid.

Height of pyramid = 13 − 8 = 5 cm

Write the appropriate volume formula.

$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3} \times 4^2 \times 5$$
$$= 26.6\ldots \text{ cm}^3$$

Calculate the volume of the rectangular prism.
Write the appropriate volume formula for a rectangular prism.

$$V = Ah$$
$$= 4^2 \times 8$$
$$= 128 \text{ cm}^3$$

Add the volumes to calculate the total volume.

$$\text{Total volume} = 26.6\ldots + 128$$
$$= 154.6\ldots$$

Round to the nearest cubic centimetre.

$$\approx 155 \text{ cm}^3$$

Write the answer.

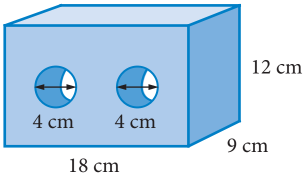
The volume of the solid is 155 cm^3.

Example 29

For the solid shown:

a calculate its volume correct to the nearest cubic centimetre

b state its capacity in litres correct to three decimal places.

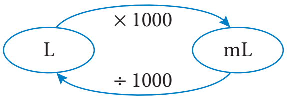


Solution

a Calculate the volume of the rectangular prism.

$$V = Ah$$
$$= 18 \times 9 \times 12$$
$$= 1944 \text{ cm}^3$$

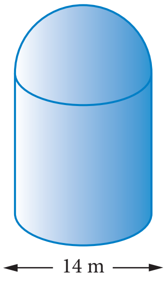
Calculate the volume of one of the cylinders.	$V = \pi r^2 h$ $= \pi \times 2^2 \times 9$ $= 113.09...\ \text{cm}^3$
Subtract to calculate composite volume. (Rectangular prism – 2 × cylinders)	$V = 1944 - 2 \times 113.09...$ $= 1717.80...$
Round to the nearest cubic centimetre. Write the answer.	$\approx 1718\ \text{cm}^3$ The volume is 1718 cm^3.
b Complete the conversion from volume to capacity. 1 cm^3 = 1 mL	1718 cm^3 = 1718 mL 1718 mL = 1718 ÷ 1000 L = 1.718 L
Write the answer.	The volume is 1.718 L.



EXERCISE 5.10 Volume and capacity: composite solids

Concepts and techniques

1 Example 28 Gina was trying to calculate the volume of the figure shown. Determine which line of her working contains an error.



Volume of hemisphere $= \frac{1}{2} \times 4/3\ \pi r^3$ Line 1

$= \frac{1}{2} \times \frac{4}{3} \pi \times 7^3$ Line 2

$= \frac{4}{6} \pi \times 7^3$ Line 3

$= \frac{2}{3} \pi \times 348\ m^3$ Line 4

$= 232\pi\ m^3$ Line 5

Volume of cylinder $= \pi r^2 h$ Line 6

$= \pi \times 7^2 \times 16$ Line 7

$= 784\pi\ m^3$ Line 8

Total Volume $= 784\pi + 232\pi$ Line 9

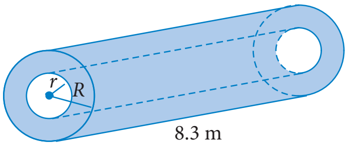
$= 1016\pi$ Line 10

$= 3191.9\ m^3$ Line 11

2 Which of the following answers is the correct volume for the solid shown in question **1**?

A 3191 cm^3 **B** 2808 m^3 **C** 3181 m^3

D 3191 m^3 **E** 3192 m^3



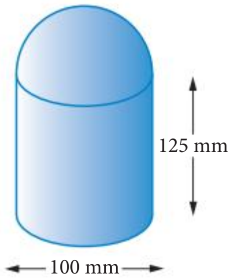
3 **Example 29** What is the volume of this piece of pipe if the outer radius, R, is 54 cm and the inner radius, r, is 32 cm? Give your answer in cubic metres, correct to two decimal places.

4 For each of the solids shown below:

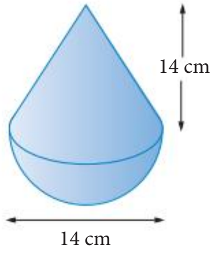
i calculate its volume correct to one decimal place

ii state its capacity correct to the nearest millilitre.

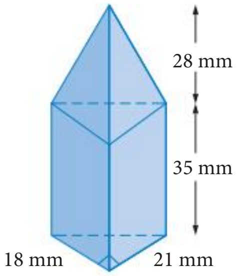
a



b



c



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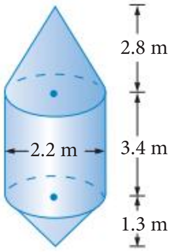

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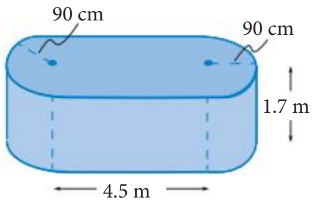

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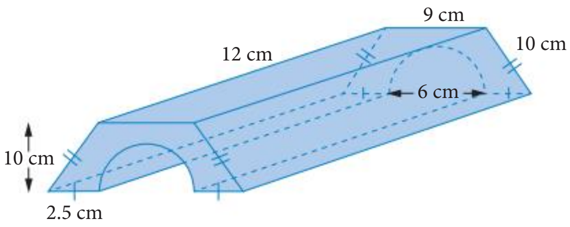
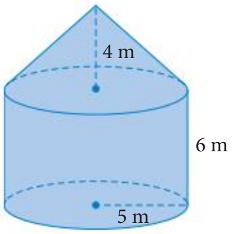
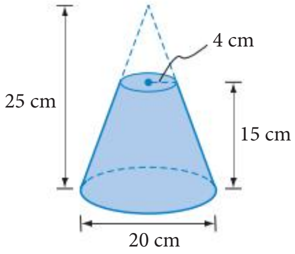


5 A wheat silo is made up of a cylinder with a cone on top. What volume of wheat will it hold when full? Give your answer to the nearest cubic metre.

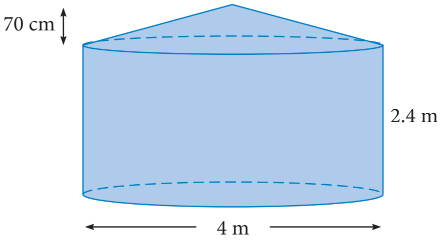


6 A cone of height 25 cm has its top section removed to leave a frustum of height 15 cm. What is the volume of this frustum if the base diameter is 20 cm? Give your answer correct to the nearest cubic centimetre.



Reasoning and communication

7 The James family have a rainwater tank on their property. It currently has only 900 litres of water left in it.

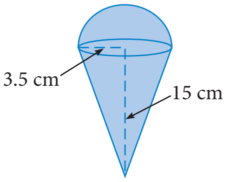


a Calculate the volume of the tank in cubic metres, correct to three decimal places.

b For the tank to be filled to capacity, how many kilolitres of water need to be added? Round up to the nearest kilolitre.

c To purchase water, the James' will have to pay a $160 delivery fee plus $2.20 per kilolitre, or part thereof, of water. How much will it cost them to have their tank filled?

8 The Icy Creams company promise "no gaps" ice-cream cones. This means that they will completely fill each ice cream cone, and then place a hemispherical scoop of ice-cream on top.



a How many millilitres of ice-cream will be in each cone? Give your answer correct to the nearest millilitre.

b If a tray of ice-cream contains 7 litres of ice-cream, how many cones can be filled from one tray?

5

CHAPTER SUMMARY

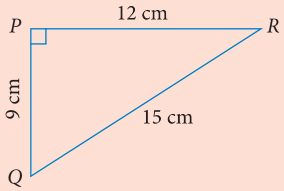
MEASUREMENT CALCULATIONS

- Pythagoras' theorem states that in any right-angled triangle, "hypotenuse squared = short side squared + other short side squared": $c^2 = a^2 + b^2$
- To check if a triangle is right-angled we can see if its side lengths satisfy Pythagoras' theorem.
- To convert a larger unit to a smaller one, we *multiply* to get more of the smaller units.
- To convert a smaller unit to a larger one, we *divide* to get fewer larger units.
- The **perimeter** of a plane (flat) shape is the entire length around the outside of the shape.
- The perimeter of a circle is called the **circumference**. It is calculated using the formula $C = \pi D$.
- A **sector** is a piece of a circle that is enclosed by two radii and an arc.
- An **arc** is a piece of a circle's circumference.
- **Arc length** is found using the formula $l = \frac{\theta}{360}\pi D$.
- The **area** of a plane shape is the amount of surface it occupies.
- Area is measured in square units.
- Area of rectangle $= lw$
- Area of square $= s^2$
- Area of triangle $= \frac{1}{2}bh$
- Area of parallelogram $= bh$
- Area of trapezium $= \frac{1}{2}(a+b)h$
- Area of rhombus or kite $= \frac{1}{2}xy$
- Area of circle $= \pi r^2$
- Area of sector $= \frac{\theta}{360}\pi r^2$
- A **composite** figure is formed when two or more regular shapes are combined.
- A **prism** is a solid with end faces and cross-sections that are identical polygons.
- The **net** of a solid is a flat, two-dimensional (2D) figure, showing the shapes of the faces of the solid. It can be folded to make the solid.
- The **surface area** of a solid is the sum of the areas of its faces, measured in square units.
- Surface area of a closed cylinder $= 2\pi r^2 + 2\pi rh$.
- A **pyramid** is a solid with a polygonal base and triangular side faces meeting at a point called the **apex**.
- A cone is a 3D figure with a circular base and a curved surface that tapers up to its apex.
- The **slant height** of a pyramid or cone is the length from the middle of an outer edge of the base to the apex.
- Surface area of a closed cone $= \pi r^2 + \pi rl$
- A sphere is a solid with circular cross-sections. All points on its surface are the same distance from its centre.
- Surface area of a sphere $= 4\pi r^2$
- The **volume** of a solid is the amount of space it occupies. This is measured in cubic units.
- Volume of prism $= Ah$
- Volume of cylinder $= \pi r^2 h$
- Volume of pyramid $= \frac{1}{3}Ah$
- Volume of cone $= \frac{1}{3}\pi r^2 h$
- Volume of sphere $= \frac{4}{3}\pi r^3$
- **Capacity** is the amount of liquid that a solid can hold. Capacity is measured in metric units.

CHAPTER REVIEW
MEASUREMENT CALCULATIONS
5

Multiple choice

1 **Example 1** Which of the following equations is correct for $\triangle PQR$?



A $q^2 = p^2 + r^2$
B $q^2 = p^2 + p^2$
C $15^2 = 9^2 - 12^2$
D $12^2 = p^2 + r^2$
E $p^2 = q^2 + r^2$

2 **Example 6** The circumference of a circle with a radius of 18 cm is closest to:

A 57 cm
B 108 cm
C 113 cm
D 1018 cm
E 24 429 cm

3 **Example 9** $2.49\ \text{m}^2 =$ _____ cm^2?

A 2.5
B 249
C 2490
D 24 900
E 249 000

4 **Example 17** The formula used to calculate the surface area of a cone is:

A $2\pi r^2 + 2\,\pi rl$
B $\pi r^2 + \pi rl$
C $4\pi r^2$
D $\frac{4}{3}\pi r^2$
E $\pi r^2 + 2\pi rh$

5 **Example 18** The surface area of a sphere with diameter of 22 mm is:

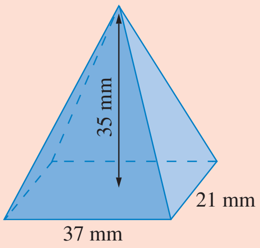
A 434.3 mm^2
B 760.3 mm^2
C 1520.5 mm^2
D 5575.3 mm^2
E 6082.1 mm^2

6 **Example 21** $2.36\ \text{m}^3 =$ ____ mm^3

A 236
B 2360
C 2 360 000
D 2 360 000 000
E 236 000 000 000

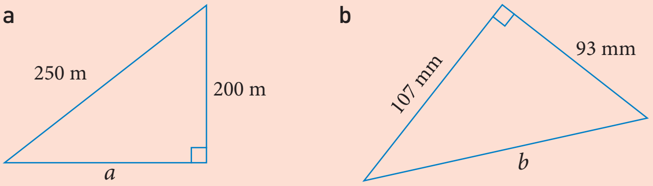
7 **Example 25** The volume of the pyramid shown at right can be found using which calculation?

A $\frac{1}{3} \times 37 \times 21 \times 35$
B $37 \times 21 \times 35$
C $\frac{1}{3} \times 37^2 \times 21 \times 35$
D $\frac{1}{2}(37 + 21) \times 35$
E $\frac{1}{3}(37 + 21) \times 35$

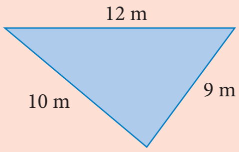


Short answer

8 **Example 1** Calculate the missing side lengths in these right-angled triangles, correct to the nearest whole number.



9 **Example 3** Determine whether this triangle is right-angled or not by using Pythagoras' theorem.



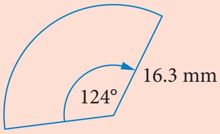
10 **Example 4** A plane flies due west for 15 km. It then travels 32 km north. How far is it from its starting position? Answer correct to one decimal place.

11 **Example 5** Convert:

a 28.5 km to centimetres
b 6.4 m to millimetres
c 340 mm to metres
d 23 m to kilometres
e 678 cm to millimetres
f 38 km to millimetres

12 **Example 6** Calculate the circumference of a circle with a diameter of 24.5 cm. Give your answer correct to the nearest centimetre.

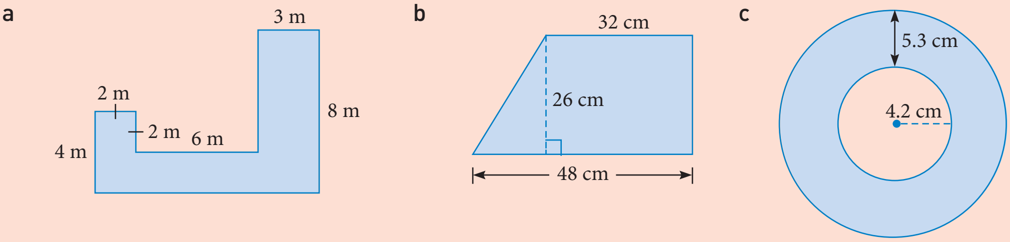
13 **Example 7** Calculate the perimeter of this sector correct to one decimal place.



14 **Example 9** Convert:

a 16 m^2 to cm^2
b 4500 cm^2 to mm^2
c 4.09 km^2 to m^2
d 3.6 km^2 to hectares
e 4735 mm^2 to cm^2
f 685 530 m^2 to km^2

15 **Example 12** Calculate the area of each of the following shapes, correct to the nearest whole number.

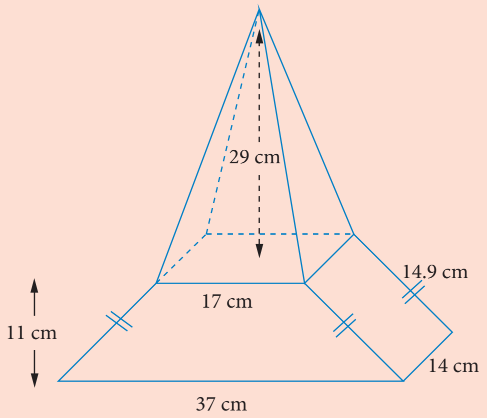


16 **Example 15** A cylinder has a radius of 16 cm and perpendicular height of 27 cm. Calculate the surface area, correct to one decimal place, if:

a the cylinder is closed
b the cylinder is open at one end
c the cylinder is open at both ends.

17 Example 18 Calculate the surface area of a closed hemisphere with a diameter of 47 mm. Give your answer rounded to the nearest square millimetre.

18 Example 19 This composite solid consists of a rectangular pyramid atop a trapezoidal prism. Calculate its total surface area. Round your answer to the nearest 10 square centimetres.



19 Example 21 Convert:

a 20.7 cm^3 to mm^3

b 1 650 000 cm^3 to m^3

c 6.8 km^3 to m^3

d 7 560 000 000 mm^3 to m^3

e 3 782 568 000 cm^3 to m^3

f 45 790 mm^3 to cm^3

20 Example 23 What is the volume of a cylinder if it has a radius of 39.7 m and a perpendicular height of 56.8 m? Give your answer correct to the nearest cubic metre.

21 Example 24 Convert:

a 15 m^3 to L

b 250 mL to L

c 570 cm^3 to mL

d 3680 mm^3 to mL

e 7 320 500 cm^3 to ML

f 3.7 m^3 to mL

22 Example 26 Calculate the volume of a sphere if it has a radius of 3.1 km. Round your answer to one decimal place.

Application

23 Wayne is putting a new roof on his garage. The roof is flat and will be covered with metal sheets that are 2400 mm long and 750 mm wide. The roof is rectangular and is 7400 mm long and 6700 mm wide. Wayne will be laying the sheets so that the length of them runs along the width of the roof. To reduce the chance of leaks, Wayne intends to use whole sheets wherever possible, minimising the use of partial sheets. How many metal sheets will Wayne need to buy?

24 Breanna's office has the following dimensions: width 5300 mm, length 9200 mm and height 3000 mm. Each cubic metre of office space will require approximately 60 watts of air-conditioning power to keep the temperature comfortable.

a Calculate the volume of the office, expressing your answer in cubic metres.

b Determine the size (in watts) of the air-conditioning unit Breanna will require for her office.

Practice quiz

2

MIXED REVISION

CHAPTERS 4 • 5 • 6

Multiple choice

1 Which of the following people earns the most?

A Peter: \$23 per hour for a 37 hour week

B Jessica: commission of 1.5% of \$65 000

C Maddy: salary of \$45 000

D Mitchell: retainer of \$450 per week plus a commission of 3.5% of \$12 000

E Katie: \$4150 a month

2 152 m^2 is equivalent to:

A 1.52 cm^2

B 15 200 cm^2

C 1.52 ha

D 1 520 000 cm^2

E 15 200 000 cm^2

3 Calculate how much Brad will have in 15 months if he invests \$5000 at 8.3% p.a. compounded quarterly.

A \$540.73

B \$5540.73

C \$12 144.84

D \$16 534.72

E \$17 144.84

4 In December Shannon is paid a bonus of 15% of one month's pay as a Christmas bonus. If she earns a salary of \$72 000, calculate her total pay for the month of December.

A \$900

B \$6000

C \$6900

D \$10 800

E \$11 700

5 Calculate the volume of the following cylinder?

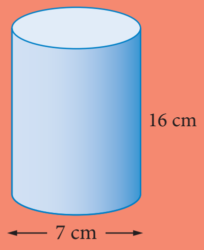
A 352 cm^3

B 616 cm^3

C 704 cm^3

D 1232 cm^3

E 2463 cm^3



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6 Xavier invests \$40 000 at 7.5% p.a. simple interest. How long will it be before he has \$44 500? Answer correct to the nearest month.

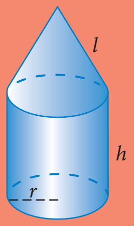
A 2 months
B 11 months
C 15 months
D 18 months
E 178 months

7 Raj works in a convenience store. He is paid normal time if he works Monday to Friday: from 7 a.m. to 5 p.m.; time-and-a-half outside these times and Saturdays; and double time on Sundays. Calculate his pay if he works the following hours and earns \$17.50 per hour.

Day	Start time	Finish time
Wed	5 a.m.	1 p.m.
Thurs	8 a.m.	5 p.m.
Fri	10 a.m.	7 p.m.
Sat	10 a.m.	7 p.m.
Sun	10 a.m.	10 p.m.

A \$822.50
B \$831.25
C \$1111.25
D \$1146.25
E \$1233.75

8 Which of the following formulas would correctly calculate the surface area of the solid shown?



A $2\pi r^2 + 2\pi rh + 2\pi rl$
B $\pi r^2 + 2\pi rh + \pi rl$
C $\pi r^2 + 2\pi r^2h + \pi rl$
D $\pi r^2 + 2\pi h + \pi rl$
E $\pi r^2 + 2\pi rh + \frac{1}{3}\pi rl$

9 Write $8\frac{1}{4}\%$ p.a. as a decimal per six-months.

A $4\frac{1}{8}\%$
B 0.0418
C 0.165
D $16\frac{1}{2}\%$
E 0.04125

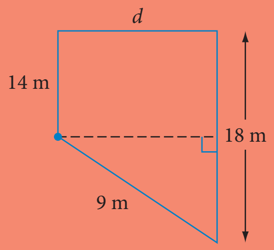
MIXED REVISION • 4 • 5 • 6

Short answer questions

1 A company has a total of 1.2 million shares. Its market price is \$3.65 per share and the annual earnings were \$115 000.

  a Calculate the annual earnings per share, correct to four decimal places.
  b Calculate the price-to-earnings ratio, correct to nearest whole number.

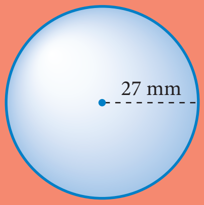
2 Calculate d, correct to one decimal place.



3 How much would you need to put in an investment account to end up with \$15 000 after $2\frac{1}{2}$ years at 5.6% p.a. compounded monthly? Answer correct to the nearest dollar.

4 Steven works a 37 hour week and earns \$34.50 an hour. He is doing up a weekly budget and includes: tax \$320 per week, rent \$350 per week, power \$520 a quarter, groceries \$115 a week, health \$150 a month, fares \$45 a week, phone \$69 per month, clothes \$2400 a year and entertainment \$200 a fortnight. Calculate how much he could save in a year if he sticks to this budget.

5 For the sphere shown:



  a Calculate its surface area, correct to one decimal place.
  b Calculate its volume, correct to one decimal place.
  c State its capacity to the nearest millilitre.

6 Chloe invests \$3200 at 6.24% p.a. simple interest. How much will she have after 8 months?

Application questions

1 A building site manager earns a salary of \$83 561 and a weekly accommodation allowance of \$470.

  a How much is he paid per fortnight?
  b He is taking his 4 weeks holiday.
    i Calculate his annual leave loading if it is 17.5% of 4 weeks of normal pay (with no allowances).
    ii Calculate his total holiday pay.

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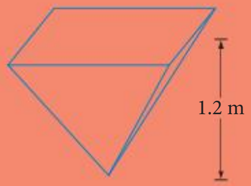
2 Sandy bought 3500 shares in Fosters at \$6.10 with a dividend yield of 7%. Her stockbroker charges 2.5% brokerage for buying shares and 1% brokerage for selling shares.

a Calculate the total cost to Sandy of buying the shares.

b Calculate the dividend paid to Sandy.

c If Sandy sells her shares at \$8.54 per share, one year after buying them, calculate her total profit after costs.

3 This grain hopper is in the shape of a square pyramid. It has a volume of 3.5 m^3 and a depth of 1.2 m.



a What is the area of its square top?

b What is the side length of the square top (correct to three decimal places)?

c What is the slant height of the hopper (correct to three decimal places)?

d How many square metres of stainless steel would have been used to create the surface of the hopper, assuming that there is no lid on the hopper? Round your answer to one decimal place.

4 Tony's fish tank is 1.2 m long, 80 cm wide and 90 cm high. Its four walls and base are made from glass. The top is made of Perspex.

a Calculate the total amount of glass surface in this fish tank to the nearest square centimetre.

b What is the volume of the tank in cubic metres?

c What is the capacity of the tank in litres?

d Tony has chosen fish which are approximately 3 cm in length. He has been advised that each of these fish will require 6 litres of water. How many fish will he put in his fish tank if he follows the recommendation?

5 Olivia borrowed \$15 000 at 6.5% p.a. flat rate interest.

a If the loan was for 3 years, how much interest did she have to pay?

b What was the total amount she had to repay?

c If she paid the loan back in monthly instalments, what was the amount of each instalment? Answer correct to the nearest cent.

6 Samuel is at university when he inherits \$75 000. He wants to look into which account at his bank is best. He has three options:

Option A: 7.65% p.a. simple interest
Option B: 7.1% p.a. compounded monthly
Option C: 6.95% p.a. compounded daily

a He initially wants to invest it for 2 years until he finishes university. Calculate the amount of interest he will earn for each option and hence decide the best option.

b He then changes his mind and decides that he will invest it for 3 years to give himself time to get sorted. Does this change his decision on which account? Explain your reasoning.

109'998.10
04'886.74
1'182.20
052.52
08.36

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6

CONSUMER ARITHMETIC

SIMPLE AND COMPOUND INTEREST

TERMINOLOGY

balance
compound interest
compounding periods
credit
credit card
debit
deposit
future value
flat-rate interest
flat-rate loan
instalments
interest
minimum monthly balance
per annum (p.a.)
present value
principal
simple interest
transaction
withdrawal

Prior learning

APPLICATIONS OF RATES AND PERCENTAGES

- **apply percentage increase or decrease in various contexts: for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest (ACMGM006)**

6.01 SIMPLE INTEREST

When money is borrowed or invested, **interest** is paid.
The amount of money that is borrowed or invested is called the **principal**. When the interest is calculated as a percentage of the principal it is called **simple interest**.

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IMPORTANT

Simple interest can be calculated using the formula:

$$I = \frac{Prn}{100}$$

where

I = simple interest
P = principal
r = rate of interest per time period
n = number of time periods

This formula can also be written as $I = nP\frac{i}{100}$ where i is the interest rate per time period.

Example 1

Yun borrowed \$25 000 at a simple interest rate of 8.4% p.a. for 6 years. How much interest did he have to pay?

p.a. means per annum, which is 'per year'

Solution

Write the values of the known variables.	$P = 25\ 000 \qquad r = 8.4 \qquad n = 6$
Write the simple interest formula.	$I = \frac{Prn}{100}$
Substitute into the formula.	$I = \frac{25\ 000 \times 8.4 \times 6}{100}$
Evaluate.	$= 12\ 600$
Write the answer.	Yun had to pay \$12 600 interest.

It is important that the units for r and n are consistent. If r is the interest rate per annum or per year, then n must be in years. If r is the interest rate per day, then n must be in days.

Example 2

Julia invested her inheritance of $15 520 at 6.35 % p.a. for 20 months. Calculate the amount of simple interest Julia earned.

Solution

Write the values of the variables which can be substituted directly into the formula.

$P = 15\,520 \qquad r = 6.35$

n must be years given that r is per annum, so divide the number of months by 12.

$n = \frac{20}{12}$

It is important to use the exact value of n and not a decimal approximation.

Write the simple interest formula.

$$I = \frac{Prn}{100}$$

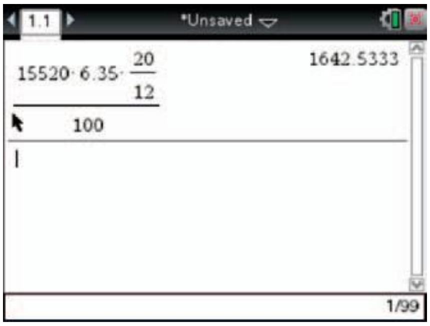
Substitute into the formula.

$$I = \frac{15\,520 \times 6.35 \times \frac{20}{12}}{100}$$

Evaluate.

$$= 1642.533\ldots$$
$$\approx 1642.53$$

TI-Nspire CAS



$\frac{15520 \cdot 6.35 \cdot \frac{20}{12}}{100}$ 1642.5333

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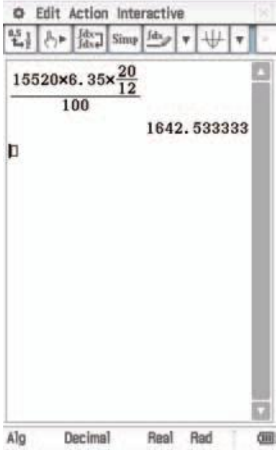

$\frac{15520 \times 6.35 \times \frac{20}{12}}{100}$ 1642.533333

Write the answer.

Julia earned $1642.53 interest.

The amount of an investment or the total amount owed on a loan can be found by adding the interest to the principal.

IMPORTANT

$$A = P + I$$
$$= P + \frac{Prn}{100}$$

○ Example 3

Joseph invests \$7500 at 4.6% p.a. simple interest for 4 years.

a Calculate the amount of interest Joseph earned.

b Find the value of his investment at the end of 4 years.

Solution

a Identify the information given in the question.	$P = 7500 \quad r = 4.6 \quad n = 4$
Write the simple interest formula.	$I = \dfrac{Prn}{100}$
Substitute into the formula.	$I = \dfrac{7500 \times 4.6 \times 4}{100}$ $= 1380$
Write the answer.	Joseph earned \$1380 interest.
b The value of his investment is the sum of the principal and the interest.	$A = P + I$ $= 7500 + 1380$ $= 8880$
Write the answer.	The value of Joseph's investment will be \$8880.

Financial institutions usually use the term **flat-rate interest** when referring to simple interest. In the case of a **flat-rate loan**, interest is calculated on the full amount borrowed for the term of the loan. The loan is repaid by **instalments** or repayments spread evenly over the term of the loan. The amount of each instalment is calculated by dividing the total to be paid by the number of instalments.

○ Example 4

Amina borrowed \$10 000 at 7.25% p.a. flat-rate interest.

a If the loan was for 5 years, how much interest did she have to pay?

b What was the total amount she had to repay?

c If she paid the loan back in monthly instalments, what was the amount of each instalment? Answer correct to the nearest cent.

Solution

a Write the values of the known variables.	$P = 10\,000 \quad r = 7.25 \quad n = 5$
Flat-rate interest is calculated using the simple interest formula.	$I = \dfrac{Prn}{100}$
Substitute the known variables into the formula.	$I = \dfrac{10\,000 \times 7.25 \times 5}{100}$ $= 3625$
Write the answer.	Amina had to pay \$3625 interest.

b The total amount to repay is the sum of the principal and the interest.	$A = P + I$ $= 10\ 000 + 3625$ $= 13\ 625$
Write the answer.	The total amount that she has to repay is \$13 625.
c Calculate the number of monthly instalments.	5 years $= 5 \times 12$ $= 60$ months
Divide total amount by the number of monthly instalments.	Monthly instalment $= 13\ 625 \div 60$ $= 227.0833\ldots$ ≈ 227.09
Write the answer.	Each instalment was \$227.09.

Always round **up** for repayments so there is enough to cover the loan.

EXERCISE 6.01 Simple interest

Concepts and techniques

Simple interest – Buying on terms

1 Example 1 Terrence invested \$4000 at 8% p.a. Calculate the simple interest that he would earn in:
   a 1 year **b** 2 years **c** 5 years **d** 10 years.

2 Calculate the simple interest earned or owed for each of the following.
   a \$1500 borrowed at 4% p.a. for 7 years.
   b \$28 000 invested at $7\frac{1}{2}$% p.a. for 10 years.
   c \$2136 borrowed at 6% p.a. for $2\frac{1}{2}$ years.
   d \$16 200 invested at 7.65% p.a. for 3 years.

3 Example 2 Calculate the simple interest earned or owed for each of the following.
   a \$12 874 invested at 0.61% per month for 6 months.
   b \$4510 borrowed at 0.0301% per day for 31 days.
   c \$20 016 borrowed at $7\frac{1}{2}$% p.a. for 5 months.
   d \$8250 invested at 10.5% p.a. for 240 days.

Assume 365 days in a year

4 Which of the following is the simple interest earned when \$7400 is invested at 5% p.a. for 4 years?
   A \$148 **B** \$14.80 **C** \$1480 **D** \$14 800 **E** \$148 000

5 Example 3 Calculate the total amount owed for the following loans.
   a \$3000 borrowed at a flat rate of $5\frac{1}{2}$% p.a. for 2 years.
   b \$17 000 borrowed at a flat rate of 6.2% p.a. for 18 months.
   c \$11 360 borrowed at a flat rate of 10% p.a. for 2 months.
   d \$4500 borrowed at a flat rate of 3.5% p.a. for 30 days.

6 Amelia invested \$1700 in an account at 4.8% p.a. simple interest. Interest is paid into the account at the end of each month. What is the account balance at the end of 9 months?

7 Masyn borrows \$4000 at a simple interest rate of 3.6% p.a. for 3 years. How much does he have to pay back in total?

Reasoning and communication

8 **Example 4** Jake borrowed $12 000 at 10% p.a. flat-rate interest over 5 years to buy a second-hand car. The loan is to be repaid in equal monthly instalments.

- a How much interest will he be charged?
- b What total amount must he repay?
- c What is his monthly repayment?

9 Joseph wins the lottery and deposits $1 000 000 in a bank account. He quits his job and lives off the interest from his investment. How much does he receive each year if the simple interest rate is 7.5% p.a.?

10 Georgia took out a holiday loan of $13 500 at 15% p.a. flat-rate interest to be repaid in fortnightly instalments over 3 years.

- a How much will her holiday cost her altogether?
- b How much is each fortnightly instalment?
- c How much would she save if the interest rate was reduced to 14% p.a.?


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11 OzExpress Credit Union has the term deposit accounts shown in the table on the right, in which the principal must be invested for a fixed period. Calculate the simple interest earned on the following.

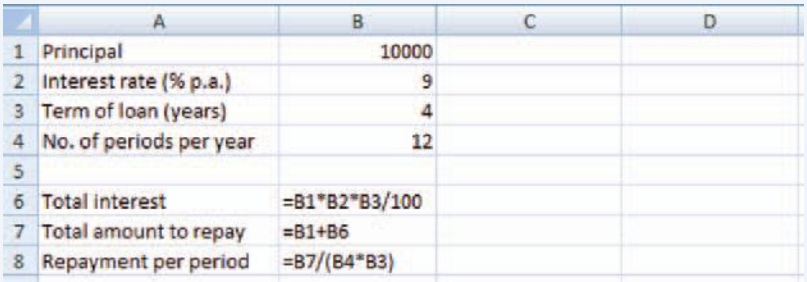
- a $6300 invested for 3 years.
- b $13 750 invested for 8 months.
- c $7800 invested for 5 years.
- d $14 240 invested for 2 months.

Term	Interest rate
1–6 months	8.75% p.a.
7–11 months	9.25% p.a.
1–3 years	10.5% p.a.
4–5 years	12% p.a.
Minimum investment:	$1000

12 Harry invests $5000 in an account that earns simple interest of 6.3% p.a. After 2 years, he deposits another $1000 in this account. How much will he have at the end of 7 years?

INVESTIGATION Flat-rate loan repayments

a Create a spreadsheet to calculate repayments per period for a flat-rate loan by entering amounts shown in cells B1, B2, B3 and B4 and appropriate formulas in cells B6, B7 and B8.



	A	B	C	D
1	Principal	10000		
2	Interest rate (% p.a.)	9		
3	Term of loan (years)	4		
4	No. of periods per year	12		
5				
6	Total interest	=B1*B2*B3/100		
7	Total amount to repay	=B1+B6		
8	Repayment per period	=B7/(B4*B3)		

b Show that the monthly repayment required to pay off a loan of \$10 000 in 4 years at 9% p.a. interest is \$283.34.

c Calculate the monthly repayment needed to pay off a \$6500 loan in three years at 6.5% p.a.

d What fortnightly repayment is required to pay off a loan of \$4750 in 18 months at 8% p.a. interest?

e Suppose you want to buy a car for \$25 000. Find the current interest rate for a car loan plus any fees and charges. If you can afford to repay \$200 per month, would you be able to pay the car off in 5 years?

6.02 SIMPLE INTEREST: CALCULATING PRINCIPAL, RATE OR TIME

The simple interest formula can also be used to calculate the principal, rate or time by substituting values for three of the variables into the formula and solving the resulting equation for the unknown.

Example 5

How much would need to be invested at a simple interest rate of 6.55% p.a. in order to earn \$2500 interest in 2 years? Answer correct to the nearest hundred dollars.

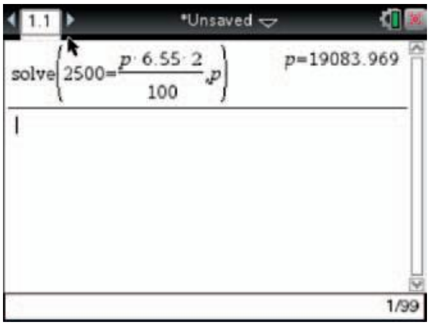
Solution

Identify the information given in the question.	$I = 2500 \quad r = 6.55 \quad n = 2$
Write the simple interest formula.	$I = \frac{Prn}{100}$
Substitute the values of *I*, *r* and *n* into the formula.	$2500 = \frac{P \times 6.55 \times 2}{100}$
Solve for *P*.	$P = \frac{2500 \times 100}{6.55 \times 2}$ $= 19\,083.969\ldots$
Round to the nearest hundred dollars.	$\approx \$19\,100$

Alternatively, solve for P using a CAS.

TI-Nspire CAS

For the TI-Nspire CAS, to view more decimal places press ctrl (–) for ans then press enter.



$\text{solve}\left(2500=\frac{p\cdot 6.55\cdot 2}{100},p\right)$ $p=19083.969$

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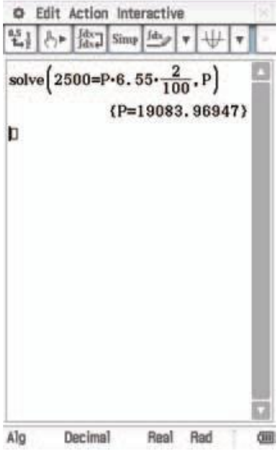


$\text{solve}\left(2500=P\cdot 6.55\cdot\frac{2}{100},P\right)$

$\{P=19083.96947\}$

Write the answer. $19 100 would need to be invested.

Example 6

Paula invests $37 000 in a term deposit account. What percentage interest rate is required for her investment to grow to $50 000 in 4 years? Answer correct to two decimal places.

Solution

Calculate the interest earned.

$$\begin{aligned}\text{Interest} &= 50\,000 - 37\,000\\ &= 13\,000\end{aligned}$$

Write the known values of the variables.

$I = 13\,000 \qquad P = 37\,000 \qquad n = 4$

Write the simple interest formula.

$$I = \frac{Prn}{100}$$

Substitute the values of I, P and n into the formula.

$$13\,000 = \frac{37\,000 \times r \times 4}{100}$$

Solve for r.

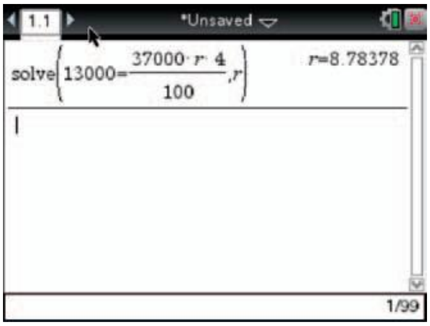
$$\begin{aligned}r &= \frac{13\,000 \times 100}{37\,000 \times 4}\\ &= 8.7837\ldots\end{aligned}$$

Round to two decimal places.

$$\approx 8.78\%$$

Alternatively, solve for r using a CAS.

TI-Nspire CAS



$\text{solve}\left(13000=\frac{37000\cdot r\cdot 4}{100},r\right)$ $r=8.78378$

1/99

ClassPad

$\text{solve}\left(13000=37000\cdot r\cdot\frac{4}{100},r\right)$

$\{r=8.783783784\}$

Write the answer.

Paula needs an interest rate of 8.78% p.a.

Example 7

Jack invests \$5000 at 4% p.a. How long will it take him to earn \$550 interest? Answer in years and months.

Solution

Identify the information given.

$I = 550 \qquad P = 5000 \qquad r = 4$

Write the simple interest formula.

$$I = \frac{Prn}{100}$$

Substitute the values of I, P and r into the formula.

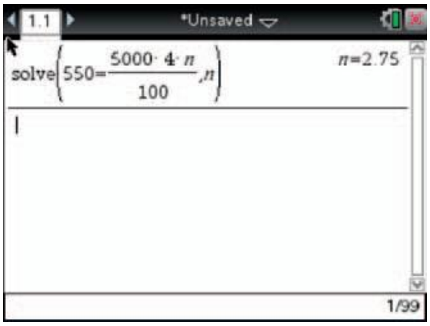
$$550 = \frac{5000 \times 4 \times n}{100}$$

Solve for n.

$$n = \frac{550 \times 100}{5000 \times 4}$$
$$= 2.75 \text{ years}$$

Alternatively, solve for n using a CAS.

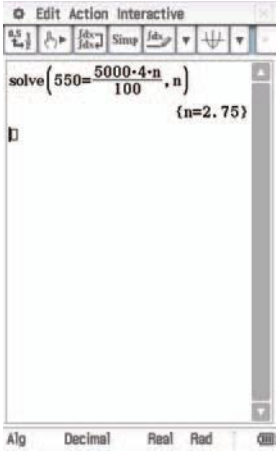
TI-Nspire CAS



$\text{solve}\left(550=\frac{5000\cdot 4\cdot n}{100},n\right)$ $n=2.75$

1/99

ClassPad



$\text{solve}\left(550=\frac{5000\cdot 4\cdot n}{100},n\right)$

$\{n=2.75\}$

Multiply the decimal part of the answer by 12 to find the number of months.	$0.75 \times 12 = 9$ months Time required = 2 years 9 months
Write the answer.	It will take Jack 2 years and 9 months to earn \$550 in interest.

Simple interest

EXERCISE 6.02 Simple interest: calculating principal, rate or time

Concepts and techniques

1 **Example 5** Calculate the amount that would need to be invested to earn:

  a \$450 simple interest at $7\frac{1}{2}\%$ p.a. for 4 years.

  b \$3500 simple interest at 6% p.a. for 18 months.

  c \$1000 simple interest at 8% p.a. for $2\frac{1}{2}$ years.

  d \$180 simple interest at 3.6% p.a. for 3 months.

2 Kalena earned \$262.44 in simple interest from investing an amount for 3 years at 5.4% p.a. The amount Kalena invested was

  A \$42.52 B \$472.39 C \$1620 D \$4251.53 E \$14 580

3 Ben invested some money at 5% p.a. for 6 years and earned \$1200 simple interest. How much money did he invest?

4 **Example 6** Calculate, correct to one decimal place, the interest rate per annum that would be needed to earn:

  a \$6000 simple interest if \$25 000 is invested for 3 years.

  b \$1400 simple interest if \$17 000 is invested for 8 months.

  c \$5500 simple interest if \$100 000 is invested for $1\frac{1}{2}$ years.

  d \$100 simple interest if \$3800 is invested for 90 days.

5 Alan invested \$9835 over 5 years in a simple interest account. At the end of 5 years his investment had a value of \$15 244.25. The simple interest rate per annum was:

  A 2.75% B 7.1% C 11.0% D 12.9% E 55.0%

6 What annual interest rate will earn \$4500 simple interest when \$30 000 is invested for 30 months?

7 **Example 7** Calculate, rounded up to the nearest month, how long it would take for:

  a \$7000 to earn \$300 simple interest at $5\frac{1}{2}\%$ p.a.

  b \$20 000 to earn \$1650 simple interest at 7.6% p.a.

  c \$3500 to grow to \$4500 at 6% p.a. simple interest.

  d \$32 000 to grow to \$38 900 at 8.1% p.a. simple interest.

8 For how long must a principal of \$15 750 be invested at 9.8% p.a. simple interest for it to grow to \$18 837?

A 1.7 years B 2.0 years C 8.5 years D 11.7 years E 12.2 years

9 Zoran earned \$675 interest from \$7500 invested for 9 months. What was the simple interest rate per annum?

10 What principal would earn \$3729.60 in simple interest if invested for 3 years at 16.8% p.a.?

Reasoning and communication

11 Sally has \$210 000 in an investment account where it earns simple interest at a rate of 6% p.a. She uses the interest to help pay her bills each year. How much interest does she receive over 5 years from this investment?

12 Siobhan earned \$80.58 interest from an investment of \$2530 over 91 days. What was the simple interest rate per day, correct to five decimal places?

13 For how long would \$4720 need to be invested at 0.67% per month to earn \$474.36 in simple interest?

14 An amount of \$9020 was invested in an account where simple interest is paid monthly. It grew to \$10 751.84 over a two-year period. Calculate the monthly interest rate.

15 A new guitar costs \$360. Holly's mum pays for it using her special store card and Holly agrees to pay it back plus any interest charged in 4 weeks. Interest is charged at 16% p.a. simple interest from the date of purchase. How much will she owe her mum altogether? Answer correct to the nearest ten cents.

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16 Kylie is planning a 10-day island holiday with her friends at the end of school in 20 months. She has an inheritance from her grandparents of \$3500. She knows she needs \$2900 for the flights and accommodation and she estimates \$1500 for food and spending money. If she invests her savings in an account earning simple interest for the 20 months, what interest rate, correct to one decimal place, will she need in order to have enough for the holiday?

6.03 SAVINGS ACCOUNTS AND CREDIT CARDS

Most people invest money in a bank account. Salaries, pensions and other income are often paid straight into an account at any given time. The **balance** of an account is the amount of money in the account. A **transaction** is made when money is put into or taken out of a bank account. When money is put into this account it is referred to as making a **deposit** and a **credit** is added to the balance. When money is taken out of this account it is referred to as a **withdrawal** and a **debit** is subtracted from the balance.

With savings accounts simple interest is calculated on the balance of the account at a specified time. Interest can be calculated on a yearly, six monthly, quarterly, monthly or daily basis depending on the type of account.

Assume 365 days in a year.

Example 8

Convert 10% p.a. to a

a six monthly interest rate.

b quarterly interest rate.

c monthly interest rate.

d daily interest rate.

Leave answers as an unsimplified fraction where necessary.

Solution

a Six monthly interest is calculated twice per year. Divide the annual interest rate by 2.

$10\% \div 2 = 5\%$ per six months

b Quarterly interest is calculated four times per year. Divide the annual interest rate by 4.

$10\% \div 4 = 2.5\%$ per quarter

c Monthly interest is calculated twelve times per year. Divide the annual interest rate by 12.

$$10\% \div 12 = \frac{10}{12}\%$$

d Daily interest is calculated 365 times per year. Divide the annual interest rate by 365.

$$10\% \div 365 = \frac{10}{365}\%$$

9780170250252

Many everyday bank accounts pay interest on the **minimum monthly balance** meaning that interest is calculated on the smallest amount in the account over the whole month.

Example 9

A section of an account statement is shown below.

Date	Debit $	Credit $	Balance $
1 Mar			80.00
5 Mar		50.00	130.00
22 Mar	10.00		120.00
9 Apr		100.00	220.00
23 Apr		20.00	240.00
2 May	100.00		140.00
18 May	50.00		90.00
20 May		60.00	150.00

The account earns 6% p.a. interest.

a Calculate the daily interest rate for this account.

b Determine the minimum monthly balance for March.

c Calculate the interest for March.

Solution

a There are 365 days in a year. Divide the annual interest rate by 365.

$6\% \div 365 = \frac{6}{365}\%$ per day

b Determine the minimum monthly balance for March.

Balance at 1 Mar is $80.

Write the answer.

The minimum monthly balance for March is $80.

c Write the values of the known variables. There are 31 days in March.

$P = 80,\ r = \frac{6}{365},\ n = 31$

Write the simple interest formula.

$$I = \frac{Prn}{100}$$

Substitute the known variables into the formula.

$$I = \frac{80 \times \frac{6}{365} \times 31}{100}$$

Evaluate.

$$= 0.407\ \ldots$$
$$\approx \$0.41$$

Write the answer.

The interest for March is 41 cents.

Using a **credit card** to purchase goods is like taking out a short-term loan. You use the card to purchase items and pay for them later when you receive a monthly statement. Different cards have different rates and conditions. Many include an interest free period but for some cards the interest is charged daily on each item from the date of purchase.

Example 10

Mitchell's credit card has a flat interest rate of 14% p.a. and no interest free period. Interest is charged from the date of purchase. He uses the card to make the following purchases for the period 1 August to 31 August.

2 August	Xbox games	\$85
16 August	Jeans	\$99
29 August	Movies	\$22

If Mitchell pays his account in full on 3 September, how much does he pay?

Solution

Calculate daily interest rate.

$$14\% \div 365 = \frac{14}{365}\% \text{ per day}$$

Find the number of days for which interest must be paid on the \$85 purchase.

2 August until 3 September inclusive

$$\begin{aligned}\text{Number of days} &= 30 + 3\\ &= 33\end{aligned}$$

Calculate the interest.

$$\begin{aligned}I &= \frac{85 \times \frac{14}{365} \times 33}{100}\\ &= 1.0758\ldots\end{aligned}$$

Find the number of days for which interest must be paid on the \$99 purchase.

16 August until 3 September inclusive

$$\begin{aligned}\text{Number of days} &= 16 + 3\\ &= 19\end{aligned}$$

Calculate the interest.

$$\begin{aligned}I &= \frac{99 \times \frac{14}{365} \times 19}{100}\\ &= 0.7214\ldots\end{aligned}$$

Find the number of days for which interest must be paid on the \$22 purchase.

29 August until 3 September inclusive

$$\begin{aligned}\text{Number of days} &= 3 + 3\\ &= 6\end{aligned}$$

Calculate the interest.

$$\begin{aligned}I &= \frac{22 \times \frac{14}{365} \times 6}{100}\\ &= 0.0506\ldots\end{aligned}$$

Calculate the total interest.

$$\begin{aligned}\text{Total interest} &= 1.0758\ldots + 0.7214\ldots + 0.0506\ldots\\ &= 1.848\ldots\\ &\approx \$1.85\end{aligned}$$

Calculate the total of purchases and interest.

$$\begin{aligned}\text{Total to pay} &= \$85 + \$99 + \$22 + \$1.85\\ &= \$207.85\end{aligned}$$

Write the answer.

He pays \$207.85.

EXERCISE 6.03 Savings accounts and credit cards

Concepts and techniques


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1 **Example 8** Convert 8% p.a. to a

- a six monthly interest rate.
- b quarterly interest rate.
- c monthly interest rate.
- d daily interest rate.

Leave answers as an unsimplified fraction where necessary.

2 Convert $7\frac{1}{2}$% p.a. to a

- a six monthly interest rate.
- b quarterly interest rate.
- c monthly interest rate.
- d daily interest rate.

Give exact answers.

3 The monthly interest rate on a bank account is 0.7%. What is the equivalent interest rate per annum?

4 Which is the highest interest rate?

A 0.049% per day
B 0.69% per fortnight
C 1.33% per month
D 4.25% per quarter
E 7% per six months

5 **Example 9** For the account statement in Example **9**, calculate

- a the minimum monthly balance for April.
- b the interest for April.

6 Nabil opens a new bank account and deposits $130 into this account on 1 March. He then makes another deposit of $50 on 8 March. He withdraws $80 on 15 March.

- a What is the minimum monthly balance for March?
- b If he earns 5.6% p.a., with interest calculated on the minimum monthly balance, what is the interest for March?

7 Karin has a credit card that charges simple interest at a rate of 22.5% p.a. with no interest-free period. She bought shoes for $365 on 21 December and paid her account in full on 30 December. The amount she paid was:

A $2.03
B $365
C $367.03
D $367.25
E $447.13

8 **Example 10** Eamon has a credit card with no interest-free period with interest charged from the date of purchase at a rate of 0.0453% per day. The purchases shown at right were made in October. If Eamon pays his account in full on 1 November, how much does he pay?

4 October	Swimmers	$38.50
18 October	Swim goggles	$120
25 October	Wetsuit	$235.50

Reasoning and communication

9 Zoe is deciding between two credit cards. The first one offers a rate of 3.88% per quarter and the second one a rate of 0.042% per day. Which should she choose? Explain.

10 The following is Eliza's bank account statement from September to November.

a Complete the balance column.

b Find the minimum monthly balance for
  i September.
  ii October.
  iii November.

c Calculate the total interest earned over these 3 months if the interest rate for this account is 3.6% p.a. and interest is calculated on the minimum monthly balance.

Date	Debit $	Credit $	Balance $
Brought forward			870
9 Sept		300	
18 Sept		120	
6 Oct	50		
15 Oct	200		
16 Nov		400	
28 Nov	20		

11 George has a credit card with an interest rate of 17% p.a. and interest free period of up to 55 days. If the account is not paid by the end of the interest free period, interest is calculated from the date of purchase on each item.

In June he went on holiday and used the card as follows:

10 June	Accommodation	$245
12 June	Day trip fares	$48
15 June	Accommodation	$108
20 June	Fares	$55
22 June	Meal	$36
24 June	Accommodation	$130

George arrived back home on 25 June and received his account for June on 3 July. The statement was dated 30 June.

a How much is due if he pays the account immediately?

b What is the last date on which he can pay so as to avoid interest?

c How much is due if he doesn't pay the account until 31 July?

12 NCM Bank credit cards have no interest-free period. A credit card statement for March is given below. If the account is paid in full on 1 April, find:

a the number of days interest charged on the football tickets.

b the total amount due to be paid including any interest.

Due date: 1 April		**NCM BANK** **Credit Card statement**	
Previous balance	**Payment**	**Total purchases**	**Interest**
$876.15	$876.15	…………	……….
Date	**Purchases**	**Amount**	
15 March	Coffee maker	$298.45	
20 March	Football tickets	$139.37	
28 March	Car tyres	$446.79	
Annual simple interest rate: 17.49%			

6.04 COMPOUND INTEREST

Compound interest is interest which is added to the principal and then the total amount is reinvested. The principal plus interest becomes the new principal on which interest is calculated for the next investment period. In other words, 'interest is earned on interest' as well as on the original amount invested.

Example 11

Sumi invests $9000 at 8% p.a. compounded yearly. Calculate:

- **a** the amount of the investment at the end of 3 years.
- **b** the compound interest earned over the 3 years.

Compounded yearly means that the interest is added on each year and then the interest is calculated on the new total for the next year.

Solution

a Interest is compounded yearly, so calculate the interest each year and add it to the principal to get the new principal for the following year.

Write the known values given for the first year.

$P = 9000 \qquad r = 8 \qquad n = 1$

Calculate the interest for the first year using the simple interest formula.

$$I = \frac{Prn}{100}$$

$$= \frac{9000 \times 8 \times 1}{100}$$

$$= 720$$

Calculate the value of the principal at the beginning of the second year.

$$P = 9000 + 720$$
$$= 9720$$

Write the known values for the second year.

$P = 9720 \qquad r = 8 \qquad n = 1$

Calculate the interest for the second year.

$$I = \frac{9720 \times 8 \times 1}{100}$$
$$= 777.60$$

Calculate the principal at the beginning of the third year.

$$P = 9720 + 777.60$$
$$= 10\ 497.60$$

Write the known values for the third year.

$P = 10\ 497.60 \qquad r = 8 \qquad n = 1$

Calculate the interest for the third year.

$$I = \frac{10\,497.60 \times 8 \times 1}{100}$$
$$= 839.81$$

Calculate the value of the investment at the end of the third year.

$$P = 10\ 497.60 + 839.81$$
$$= \$11\ 337.41$$

Write the answer.

The amount of the investment at the end of 3 years is $11 337.41.

b Interest = final amount of investment − original amount invested	$P = 11\ 337.41 - 9000$ $= \$2337.41$
Write the answer.	The compound interest earned over 3 years is \$2337.41.

Instead of repeatedly applying the simple interest formula, we can use the compound interest formula.

IMPORTANT

The compound interest formula is $A = P\left(1+\frac{r}{100}\right)^n$

where: A (amount) = future value or final balance

P = principal or initial investment

n = number of **compounding periods**

r = interest rate per compounding period

The compound interest (I) earned can be found using the formula $I = A - P$.

The following example shows how the compound interest formula can be used to achieve the answers contained in Example **11**.

Example 12

Sumi invests \$9000 at 8% p.a. compounded yearly. Calculate:

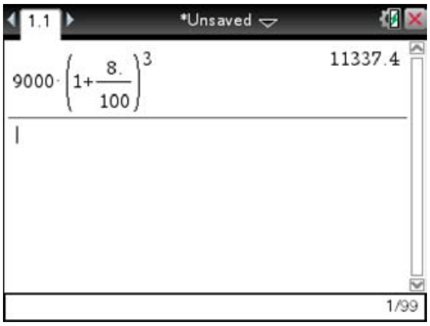
a the amount of the investment at the end of 3 years

b the compound interest earned over the 3 years.

Solution

a Write the known values of the variables.	$P = 9000 \quad r = 8 \quad n = 3$
Write the compound interest formula.	$A = P\left(1+\frac{r}{100}\right)^n$
Substitute the values of P, r and n into the formula.	$A = 9000\left(1+\frac{8}{100}\right)^3$ $= 11\ 337.408$ $\approx 11\ 337.41$
Write the answer.	The amount of the investment at the end of 3 years is \$11 337.41.

TI-Nspire CAS



$9000\cdot\left(1+\frac{8.}{100}\right)^3$ 11337.4

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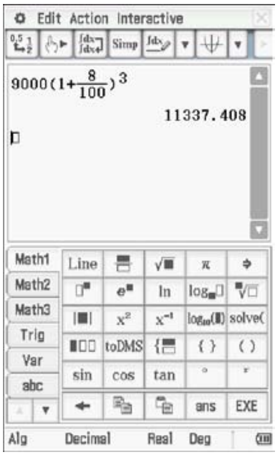


$9000(1+\frac{8}{100})^3$

11337.408

b Interest is calculated by subtracting the principal from the final amount.

$$I = A - P$$
$$= 11\ 337.41 - 9000$$
$$= 2337.41$$

Write the answer. The interest earned would be \$2337.41.

When interest is not compounded yearly, the values of r and n need to be adjusted accordingly. For example, if interest is compounding monthly, then r represents the interest rate per month and n represents the number of months.

Example 13

Calculate the interest earned after 5 years if \$25 500 is invested at 7.2% p.a. compounding quarterly.

Solution

Interest is compounded quarterly so find the quarterly interest rate and the number of quarters in 5 years.

$r = 7.2 \div 4 = 1.8$ per quarter
$n = 5$ years $\times\ 4 = 20$ quarters

Write the values of the known variables.

$P = 25\ 500 \qquad r = 1.8 \qquad n = 20$

Write the compound interest formula.

$$A = P\left(1+\frac{r}{100}\right)^n$$

Substitute the values of P, r and n into the formula.

$$A = 25\,500\left(1+\frac{1.8}{100}\right)^{20}$$
$$= 36\,433.067\ldots$$
$$\approx 36\,433.07$$

Interest is calculated by subtracting the principal from the final amount.

$$I = A - P$$
$$= 36\ 433.07 - 25\ 500$$
$$= 10\ 933.07$$

Write the answer. The interest earned would be \$10 933.07.

EXERCISE 6.04 Compound interest

Concepts and techniques

Spreadsheets – Simple and compound interest

1 **Example 11** Kate invests $5000 in an account earning 4% p.a. compounded annually for 2 years. Calculate:
  - a the interest earned in the first year
  - b the value of the investment at the beginning of the second year
  - c the interest earned in the second year
  - d the value of Kate's investment at the end of the two years
  - e the total interest Kate earned.

2 **Example 12** Cara invests $5000 in an account earning 4% p.a. compounded annually for 2 years.
  - a Use the compound interest formula to calculate the value of Cara's investment at the end of 2 years.
  - b Find the compound interest earned over the two years.

3 Calculate the final amount of each of the following investments.
  - a $7400 invested at 5% p.a., compounded yearly, for 4 years.
  - b $2840 invested at 6.5% p.a., compounded yearly, for 5 years.
  - c $4500 invested at 4.9% p.a., compounded yearly, for 2 years.
  - d $17 000 invested at 0.5% per month, compounded monthly for 10 months.
  - e $9250 invested at 0.82% per month, compounded monthly, for 6 months.
  - f $9000 invested at 0.023% per day, compounded daily, for 240 days.

4 **Example 13** For each of the following investments calculate:
  - i the final amount of the investments
  - ii the amount of compound interest earned.
  - a $12 900 invested at 10.8% p.a. for 1 year, compounded monthly.
  - b $13 800 invested at 7.5% p.a. for 2 years, compounded six-monthly.
  - c $13 800 invested at 7.5% p.a. for 2 years, compounded quarterly.
  - d $6920 invested at 9% p.a. for 4 weeks, compounded daily.
  - e $42 000 invested at $6\frac{1}{2}$% p.a. for 18 months, compounded six-monthly.
  - f $1400 invested at 8.6% p.a. for $2\frac{1}{2}$ years, compounded quarterly.
  - g $24 700 invested at 5.9% p.a. for 3 years, compounded yearly.
  - h $123 000 invested at 9.6% p.a. for 10 years, compounded six-monthly.
  - i $1500 invested at 8.4% p.a. for 2 years, compounded monthly.
  - j $37 500 invested at $8\frac{1}{2}$% p.a. for 6 months, compounded quarterly.
  - k $56 000 invested at 7.9% p.a. for 18 months, compounded daily.
  - l $2700 invested at 6.8% p.a. for $3\frac{1}{2}$ years, compounded monthly.

Reasoning and communication

5 The Bank of Victoria offers compound interest at the rate of 6% p.a. paid annually. The Australia Bank offers 5.5% p.a. paid quarterly.
   - a If $8000 is deposited into an account at each bank, what will be its value at the end of 5 years?
   - b Which bank pays the most interest and by how much?

6 Michael has an account that earns compound interest at 2.1% per quarter. Calculate the value of a $28 000 investment in three years.

7 Lauren inherits $140 000. Whilst she is waiting to buy a house she invests this sum in an account paying 0.0115% per day compounded daily. If it takes her 22 weeks to buy a house, how much will her investment be worth?

8 If a principal of $67 000 earns interest at a rate of 4.5% p.a. compounded monthly, find its value in 5 years time.

9 Susan deposits $5000 into an account for her baby when he is born. The account earns 5.6% p.a. compounded quarterly. If she gives him the account on his 21st birthday, how much will he receive?

10 A sum of $8500 is invested at 7% p.a. for 5 years.
   - a Calculate the total interest earned if the interest is:
     - i calculated at a flat rate (simple interest)
     - ii compounded yearly.
   - b i Why does one type of interest result in a greater amount than the other?
     - ii By how much is it greater?

11 A principal of $21 000 is invested at 5% p.a. Calculate the value of the investment after 2 years if the interest is compounded:
   - a yearly
   - b half-yearly
   - c quarterly
   - d monthly
   - e daily.

12 Judging from your results for question **11**, what happens to the amount of interest earned as the frequency of compounding increases? Why?

13 A principal of $10 000 is to be invested for 3 years. Which of the following is the best investment option?
   - A 6% p.a. simple interest
   - B 5.9% p.a. compounded annually
   - C 5.85% p.a. compounded half-yearly
   - D 5.6% p.a. compounded quarterly
   - E 5% p.a. compounded monthly

6.05 COMPOUND INTEREST: CALCULATING PRINCIPAL, RATE OR TIME

The compound interest formula can also be used to calculate the principal, interest rate or time by substituting the values of three known variables and solving the resulting equation for the unknown.

Example 14

Areti has the opportunity to invest some money at 9% p.a. compounded monthly. How much should she invest if she wants to have \$10 000 available in 5 years? Answer correct to the nearest dollar.

Solution

Interest is compounded monthly, so find the monthly interest rate and the number of months in 5 years.

$r = 9 \div 12 = 0.75$ per month

$n = 5$ years $\times\ 12 = 60$ months

Write the values of known variables.

$A = 10\,000 \qquad r = 0.75 \qquad n = 60$

Write the compound interest formula.

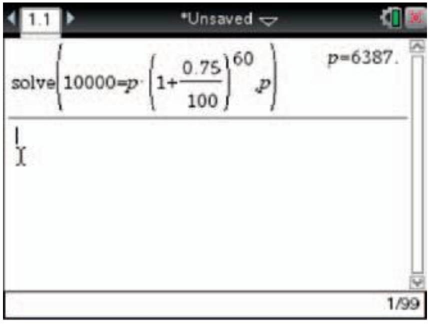
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Substitute the values of A, r and n into the formula.

$$10\ 000 = P\left(1 + \frac{0.75}{100}\right)^{60} \longleftarrow \boxed{P = \frac{10\,000}{(1 + \frac{0.75}{100})^{60}}}$$

Solve for P.

TI-Nspire CAS



$\text{solve}\left(10000=p\cdot\left(1+\frac{0.75}{100}\right)^{60},p\right)$ p=6387.

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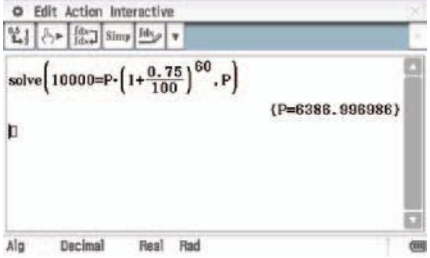


$\text{solve}\left(10000=P\cdot\left(1+\frac{0.75}{100}\right)^{60},P\right)$

{P=6386.996986}

Round to the nearest dollar.

$P = 6386.99\ldots$

$\approx \$6387$

Write the answer.

Areti should invest \$6387.

Example 15

Two years ago, Tanh deposited \$8300 in an investment account paying interest compounded quarterly. If he has earned \$1200 interest, what is the annual interest rate? Answer as a percentage correct to one decimal place.

Solution

Calculate n as interest is compounded quarterly.

$n = 2$ years $\times 4 = 8$ quarters

A is the sum of the principal and the interest.

$$\begin{aligned} A &= P + I \\ &= 8300 + 1200 \\ &= 9500 \end{aligned}$$

Write the values of the known variables.

$A = 9500 \qquad P = 8300 \qquad n = 8$

Write the compound interest formula.

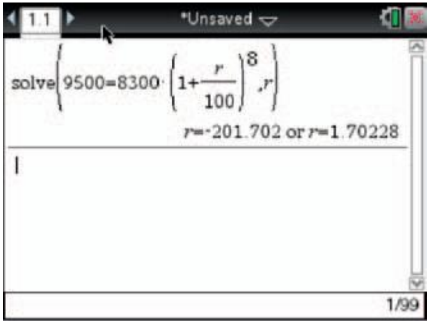
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Substitute the values of A, P and n into the formula.

$$9500 = 8300\left(1 + \frac{r}{100}\right)^8$$

Solve for r.

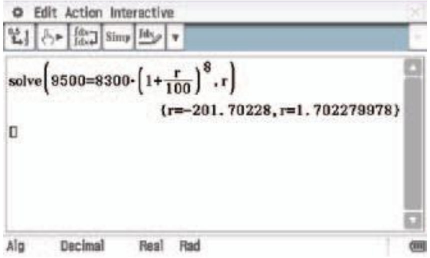
TI-Nspire CAS



$\text{solve}\left(9500=8300\cdot\left(1+\frac{r}{100}\right)^8,r\right)$

$r=-201.702$ or $r=1.70228$

ClassPad



$\text{solve}\left(9500=8300\cdot\left(1+\frac{r}{100}\right)^8,r\right)$

$\{r=-201.70228, r=1.702279978\}$

Select the positive solution.

$r = 1.070\ 22$ ... % per quarter

Multiply the value of r by 4 to calculate the annual percentage interest rate. Round to one decimal place.

$1.070\ 22 \ldots \times 4 = 6.8\%$ per annum

Write the answer.

Tanh has been earning 6.8% p.a. interest on his investment.

Example 16

Steve has \$5000 invested at 8.4% p.a. compounded monthly. How long will it take to accumulate to \$8000?

Solution

Calculate the monthly interest rate, as interest is compounded monthly.

$r = 8.4 \div 12 = 0.7$ per month

Write the values of the known variables.

$A = 8000 \qquad P = 5000 \qquad r = 0.7$

Write the compound interest formula.

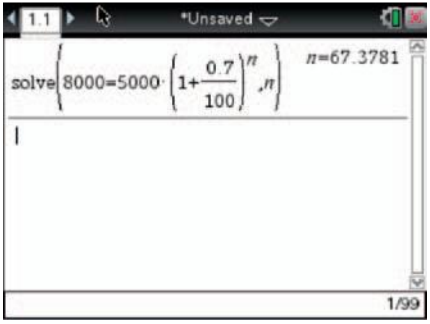
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Substitute the values of A, P and r into the formula.

$$8000 = 5000\left(1 + \frac{0.7}{100}\right)^n$$

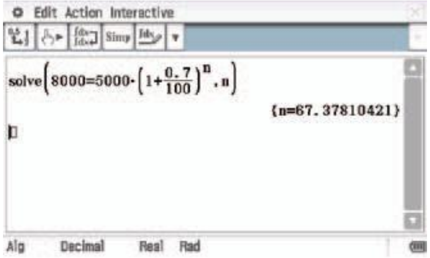
Solve for n.

TI-Nspire CAS



$\text{solve}\left(8000=5000\cdot\left(1+\frac{0.7}{100}\right)^n,n\right)$ $n=67.3781$

ClassPad



$\text{solve}\left(8000=5000\cdot\left(1+\frac{0.7}{100}\right)^n,n\right)$

$\{n=67.37810421\}$

You will need to round up as there is not quite enough money in the account at 67 months.

Round up to the nearest month.

$n = 67.37\ldots$
≈ 68 months

Write the answer.

It will take 68 months (5 years and 8 months) for Steve's investment to accumulate to \$8000.

EXERCISE 6.05 Compound interest: calculating principal, rate or time

Concepts and techniques

Simple and compound interest

Compound interest table

1 Example 14 Paul wants to invest some money so that it will grow to $24 000 in 5 years, when he will travel through Europe. If the interest rate is 5.5% p.a. compounded yearly, what amount should Paul invest, correct to the nearest dollar?

Shutterstock.com/ Vitaly Titov & Maria Sidelnikova

2 After 8 years, the value of Corrina's investment was $6260.14. What was the initial amount invested, correct to the nearest dollar, if the interest rate was 8.8% p.a. compounded quarterly?

A $2632
B $3120
C $5615
D $9544
E $12 561

3 Example 15 If it takes 4 years for $2000 to accumulate to $4000, find the compound interest rate when interest is compounded annually (correct to one decimal place).

4 After 8 years $1000 will accumulate to $3000. If interest is compounded monthly, the interest rate per annum is closest to:

A 1.15%
B 11.4%
C 12.8%
D 13.8%
E 14.7%

5 Example 16 Zara has $4000 in an account earning 4% p.a. interest, compounded yearly. Determine how long it will take Zara to double her money. Give your answer to the nearest year.

6 How long, to the nearest year, will it take an investment of $2400 to grow to $3265 at 9.6% p.a. interest, compounded six-monthly?

7 How much will Judy need to invest if she needs $35 000 in 4 years and she has invested her money at 9.5% p.a. compounding monthly? Answer correct to the nearest hundred dollars.

8 James invested $45 000 in an account that was adjusted quarterly. His investment was worth $61 500 when he withdrew it 5 years later. What was the annual interest rate? Answer as a percentage correct to two decimal places.

Reasoning and communication

9 How much does John need to invest now in order to have $800 in the bank in 2 years to pay for a musical instrument? The bank pays interest at a rate of 5% p.a. compounded quarterly.

10 Nikita had $5000 in an investment and after 4 years with interest compounded monthly it had grown to $6300. What was the annual interest rate? Answer as a percentage correct to two decimal places.

11 Magnus invested a certain amount of money 7 years ago at 5.8% p.a. compounded monthly. He now has $70 000 for a deposit on a house. How much did he invest? Answer correct to the nearest hundred dollars.

12 How much should Renee invest in an account paying 6% p.a. interest, compounded six-monthly, to have $12 500 in 7 years?

Shutterstock.com/Lev Kropotov

13 Tania's credit union pays 5% p.a. interest, compounded daily. How long would it take $10 000 to increase to $10 800 in this account? Round your answer up to the nearest month.

14 Jarrod invests $15 000 in an account paying interest that is compounded fortnightly for 10 years. What interest rate is required if he wishes to triple his investment? Answer as a percentage correct to one decimal place.

15 Jack won $1 000 000 in the lottery. He invests all of his winnings in a term deposit earning interest of 7.4% p.a. compounded six-monthly. How many years will it take for him to double his money?

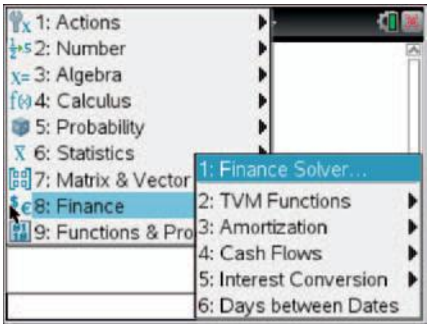
16 How long will it take to double your money if it is invested at 4% compound interest, compounded annually? Answer in years correct to one decimal place.

6.06 USING FINANCE SOLVERS FOR COMPOUND INTEREST PROBLEMS

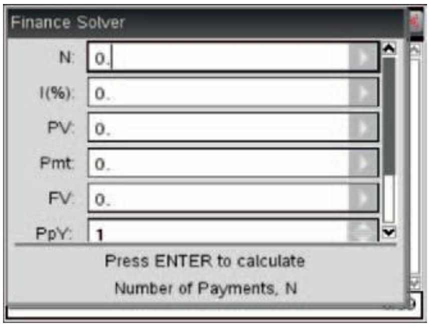
CAS calculators have inbuilt applications called the 'Finance Solver' on the TI-Nspire CAS or 'Financial Application' on the ClassPad. These financial applications can be used to solve compound interest problems.

TI-Nspire CAS

Open a New Document with a Calculator Page
Press menu, 8 (Finance) and 1 (Finance Solver…).



The fields for the Finance Solver are defined as follows:
N is the number of time periods.
I% is the interest rate as a percentage per annum.
PV is the **present value** (when investing this is negative as you are giving the bank this amount).
PMT is the value of any payments being made (for compound interest investments where there is only the initial deposit this is zero).


Finance Solver

N: 0.
I(%): 0.
PV: 0.
Pmt: 0.
FV: 0.
PpY: 1

Press ENTER to calculate
Number of Payments, N

FV is the **future value** (this will be positive as this is the money you get back).
PpY is the number of payments per year.
CpY is the number of times in a year interest is compounded.
PpY and CpY take the same value for all compound interest calculations.
PmtAt is when the payment is made, at the 'beginning' or 'end' of the time period. Compound interest is paid at the end of the time period to leave PmtAt set to END.

When using the Finance solver, press tab to move between fields when entering data.

Once the data is entered press tab to move the cursor to the field representing the unknown quantity and press enter.

ClassPad

Tap Menu followed by the Financial application.

Tap **Compound Interest**. (or **Calc(1)** then **Compound Interest** if you have been using some other function in the **Financial** Application).

The fields for the Compound Interest application are defined as follows:

N is the number of time periods.

I% is the interest rate as a percentage per annum.

PV is the present value (when investing this is negative as you are giving the bank this amount).

PMT is the value of any payments being made (for compound interest investments where there is only the initial deposit this is zero).

FV is the future value (this will be positive as this is the money you get back).

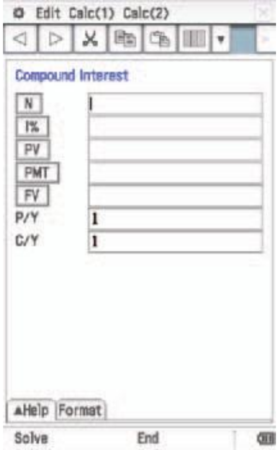
P/Y is the number of payments per year.

C/Y is the number of times in a year interest is compounded.

P/Y and C/Y take the same value for all compound interest calculations.

Tapping Help at the bottom left of the screen gives the definition for a selected field.

Once the data is entered tap the unknown quantity to find its value.



Example 17

Susie invested $54 000 in an account earning 7.2% p.a. compounding yearly.

a What is the value of her investment at the end of 3 years?

b How much interest did she earn?

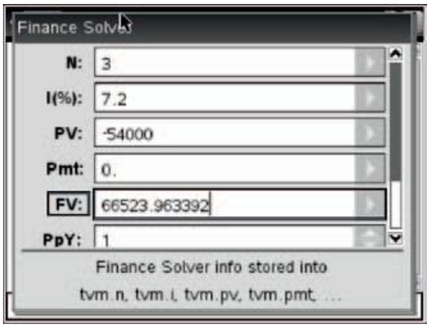
Solution

a Write down values of the known fields.
Interest is compounded yearly, so N is 3 and both PpY (P/Y) and CpY(C/Y) are 1.
The interest rate per annum is 7.2% so I% is 7.2.
$54 000 is being invested so PV is –54 000.

N = 3
I% = 7.2
PV = –54 000
PMT = 0
FV = ?
PpY or P/Y = 1
CpY or C/Y= 1

When solving problems using a finance solver it is usual to write down the values entered in each field.

Enter the data in the finance solver or financial application.



N: 3
I(%): 7.2
PV: -54000
Pmt: 0.
FV: 66523.963392
PpY: 1

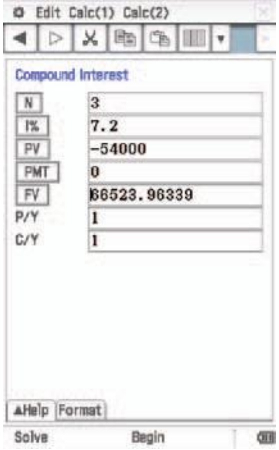
Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

TI-Nspire CAS

For the TI-Nspire move the cursor to the data entry line for FV and press enter.

ClassPad

For the ClassPad, tap FV.



Compound Interest

N 3
I% 7.2
PV -54000
PMT 0
FV 66523.96339
P/Y 1
C/Y 1

Write the answer.

Susie will have $66 523.96 in 3 years.

b To calculate the interest earned use subtract the principal from the final value of the investment.

$$
\begin{aligned}
I &= A - P \\
&= 66\ 523.96 - 54\ 000 \\
&= 12\ 523.96
\end{aligned}
$$

Write the answer.

Susie earned $12 523.96 interest.

Example 18

How long will it take for $7800 to grow to $10 000 if it is invested at 9% p.a. compounded monthly?

Solution

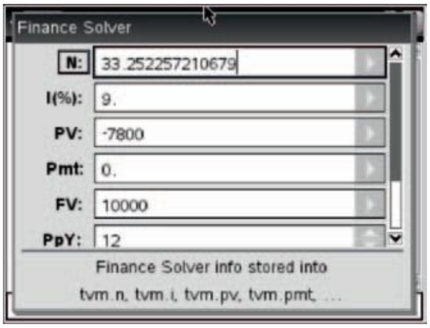
Write down the values of the known quantities.
FV will be positive as that is the amount paid back so FV is 10 000.
The interest rate per annum is 9% so I% is 9.
Interest is compounded monthly so it is paid twelve times in a year. PpY(P/Y) and CpY(C/Y) are both 12.
$7800 is being invested so PV is –7800.

N = ?
I% = 9
PV = –7800
PMT = 0
FV = 10 000
PpY or P/Y = 12
CpY or C/Y = 12

Enter the known values.

TI-Nspire CAS

For the TI-Nspire CAS move the cursor to the data entry line for N and press enter.



Finance Solver
N: 33.252257210679
I(%): 9.
PV: -7800
Pmt: 0.
FV: 10000
PpY: 12
Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

ClassPad

For the ClassPad, tap N.

Compound Interest
N 33.25225721
I% 9
PV −7800
PMT 0
FV 10000
P/Y 12
C/Y 12

Interpret the value of N.
Interest is compounded monthly so N is a number of months.

N = 33.252 … months

The value of the investment needs to reach $10 000 so investing for 33 months will not be quite long enough. Therefore, round answer up to nearest month.

It will take 34 months, which is 2 years and 10 months, for the money to grow to $10 000.

Example 19

Lachlan is looking to invest \$30 000 for 5 years. He needs to decide between:
Bank A: 6.8% p.a. compounded six-monthly, or
Bank B: 6.7% p.a. compounded quarterly.
Which bank should Lachlan use? Justify your answer.

Solution

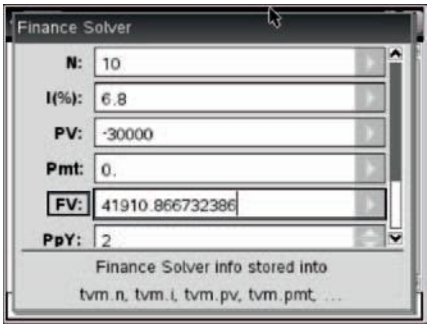
Consider Bank A: Write down the values of the known fields.
Interest compounded six-monthly means that it is paid twice per year. So, N is $5 \times 2 = 10$ and both PpY(P/Y) and CpY(C/Y) are 2.
The interest rate per annum is 6.8% so I% is 6.8.
PV is –30 000 since \$30 000 is being invested.

N = 10
I% = 6.8
PV = –30 000
PMT = 0
FV = ?
PpY or P/Y = 2
CpY or C/Y = 2

Enter the data values.

TI-Nspire CAS

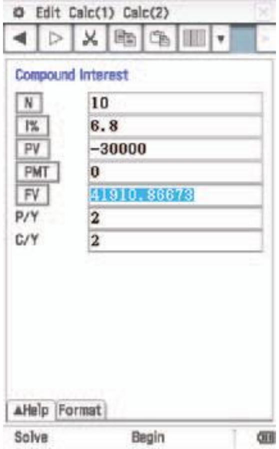
For the TI-Nspire CAS move the cursor to the data entry line for FV and press [enter].



Finance Solver
N: 10
I(%): 6.8
PV: -30000
Pmt: 0.
FV: 41910.866732386
PpY: 2
Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

ClassPad

For the ClassPad, tap [FV].



Compound Interest
N 10
I% 6.8
PV −30000
PMT 0
FV 41910.86673
P/Y 2
C/Y 2

State the value of Lachlan's investment if he invests with Bank A.

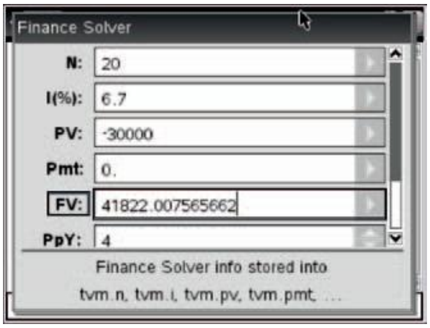
With Bank A, the value of Lachlan's investment at the end of five years is \$41 910.87.

Consider Bank B: Write down the values of the known fields.
Interest compounded quarterly means that it is paid four times in a year. So, N is 5 × 4 = 20 and PpY(P/Y) and CpY(C/Y) both are 4.
The interest rate per annum is 6.7% so I% is 6.7.
PV is –30 000 since $30 000 is being invested.

N = 5 × 4 = 20
I% = 6.7
PV = –30 000
PMT = 0
FV = ?
PpY or P/Y = 4
CpY or C/Y = 4

TI-Nspire CAS

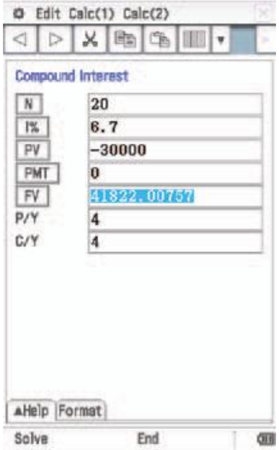
For the TI-Nspire CAS move the cursor to the data entry line for FV and press enter.



Finance Solver
N: 20
I(%): 6.7
PV: -30000
Pmt: 0.
FV: 41822.007565662
PpY: 4
Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

ClassPad

For the ClassPad, tap FV.



Compound Interest
N 20
I% 6.7
PV −30000
PMT 0
FV 41822.00757
P/Y 4
C/Y 4

State the value of Lachlan's investment if he invests with Bank B.

With Bank B, the value of Lachlan's investment at the end of five years is $41 822.01.

Find the difference in the final values of his investment with each back.

$41 910.87 – $41 822.01
= $88.86

Write the answer.

Lachlan should invest in Bank A as he will earn $88.86 in extra interest.

EXERCISE 6.06 Using finance solvers for compound interest problems

Concepts and techniques

1 Example 17 For each of the following investments calculate:

i the total value of the investment

ii the amount of interest earned.

a $36 000 invested at 5% p.a. compounded yearly for 3 years.

b $3700 invested at 7.6% p.a. compounded yearly for 18 years.

c $73 800 invested at 8.88% p.a. compounded yearly for 7 years.

d $100 000 invested at $6\frac{1}{4}$% p.a. compounded yearly for 13 years.

2 Calculate the amount received at the end of each of the following investments.

a $51 000 invested at 6.8% p.a. compounded monthly for 15 years.

b $26 600 invested at 8% p.a. compounded quarterly for 9 years.

c $171 000 invested at $8\frac{1}{2}$% p.a. compounded monthly for $3\frac{1}{2}$ years.

d $1800 invested at 9.25% p.a. compounded daily for 4 years.

Assume 365 days in a year

3 Example 18

a $28 600 is invested at 7% p.a. compounded monthly and grows to $33 400. How long does it take, correct to the nearest month?

b $7200 is invested for 10 years and grows to $11 000. If it was compounded six-monthly, what was the interest rate per annum? Answer as a percentage correct to one decimal place.

c An amount of money was invested at 5.8% p.a. compounded weekly. It grows to $43 250 in 4 years. How much was invested, correct to the nearest hundred dollars?

d $97 000 is invested at $9\frac{1}{2}$% p.a. compounded yearly and grows to $145 000. How long does this take, correct to the nearest year?

4 On her 18th birthday Rebecca was given the money in a savings account from her grandparents. They had invested money on her 10th birthday at 11.5% p.a. paid monthly and it had grown to $21 236. How much had they invested? Answer correct to the nearest dollar.

5 $65 000 was invested at 9% p.a. compounded monthly. The amount of interest that was earned in six months is:

A $243.29 B $2861.99 C $2980.40 D $44 011.51 E $67 980.40

6 Amina buys government bonds worth $16 500. They pay 7.2% p.a. compounded daily. How much will they be worth after 7 years?

7 Stefanie invested $1700 in a savings account 4 years ago. She made no additional deposits and no withdrawals and the balance is now $3146.70. What interest rate did she receive if interest was paid six-monthly? Answer as a percentage p.a. correct to one decimal place.

8 An investment of $45 000 was invested at 6.9% p.a. compounded monthly. How long until the investment was worth $60 000?

A 50 years B 51 years C 4 years 2 months

D 4 years 3 months E 4 years 4 months

Reasoning and communication

9 \$16 000 was invested for 8 years and earned \$5600 interest. If interest was compounded quarterly, what was the interest rate per annum? Answer as a percentage correct to one decimal place.

10 Example 19 Sally inherits \$67 000 from her grandparents. She decides to invest it for 4 years until she finishes university. She is considering two different investment options.

Option A: 8.4% p.a. compounded monthly
Option B: 8.64 % p.a. compounded quarterly
Which option should Sally choose? Justify your answer.

11 Peter deposits \$45 000 in an account that pays 1.8% per quarter, compounded monthly.

a What is the interest rate per annum?
b What is the value of this investment after 5 years?
c How much interest did Peter earn?

12 Joel's credit card charges interest at 0.053% per day, compounded daily with no interest free period. He pays for a \$350 leather jacket using his credit card for a friend. How much interest will he have to charge his friend if he pays it off 45 days later?

13 Sienna has \$45 000 to invest for 3 years. Which of the following will give her the greatest return?

A 0.65% per month, compounded yearly.
B 7.4% p.a. compounded six-monthly.
C 3.8% per six months, compounded quarterly.
D 7.5% p.a. compounded monthly.
E 7.2% p.a. compounded daily.

14 If a principal took approximately 9 years to double in value, at what interest rate was it invested if interest was compounded monthly? Answer as a percentage correct to one decimal place.

15 Carla invests \$20 000 for three years at 5.7% per annum compounded annually. At the end of this time she reinvests the amount returned from her three year investment plus an additional \$4000 that she has saved at 5.8% per annum compounded annually for a further two years. How much does she receive from her investment at the end of the five years?

CHAPTER SUMMARY

SIMPLE AND COMPOUND INTEREST

6

- **Simple interest** (or **flat-rate interest**) is interest earned or charged only on the original amount of money (**principal**) invested or borrowed.
- $I = \frac{Prn}{100}$ where I = simple interest, P = principal, r = rate of interest per period, n = number of periods
- The **amount** of a loan or investment can be found by adding the interest to the principal. $A = P + I$
- The simple interest formula can also be used to calculate the principal, rate or time by substituting in the given information and solving the resulting equation for the unknown.
- When you make a **deposit** into a bank account, the bank **credits** your account and the amount of the deposit is added to the balance. When you make a **withdrawal** from a bank account, the bank **debits** your account and the amount of the withdrawal is subtracted from the balance.
- Using a **credit card** to purchase goods is like taking out a short-term loan. Some include an interest free period but for some cards the interest is charged daily on each item from the date of purchase.
- For **compound interest** the interest is added to the principal and this is then reinvested. The **compounding periods** say how often the interest is added on. For example yearly, six-monthly, quarterly, monthly or even daily.
- For compound interest, the future value or the amount of the investment can be calculated using $A = P\left(1 + \frac{r}{100}\right)^n$

  where: A (amount) = future value or final balance

  P = principal or initial quantity

  n = number of compounding periods

  r = interest rate per compounding period
- The compound interest (I) can be found using $I = A - P$
- The compound interest formula can also be used to calculate the principal, rate or time by substituting in the given information and solving the resulting equation for the unknown.
- For a calculation relating to a compound interest investment, the fields for the Financial Solver or Financial Application (Compound Interest) are as follows.

  N is the number of time periods.

  I% is the interest rate as a percentage per annum.

  PV is the present value (for an investment, this is amount invested or the principal and is negative since this amount is being given to the bank).

  Pmt or **PMT** is the value of the regular payments being made (for a compound interest investment this is 0).

  FV is the future value (this it is positive as it is money that is paid back by the bank).

  Pp/Y or **P/Y** is the number of payments per year.

  Cp/Y or **C/Y** is the number of times in a year interest is compounded.

  Pp/Y or **P/Y** and **CpY** or **C/Y** take the same value for all compound interest calculations.

6

CHAPTER REVIEW
SIMPLE AND COMPOUND INTEREST

Multiple choice

1 **Example 2** Calculate the simple interest earned on \$7200 invested at $7\frac{2}{3}$% p.a. for 30 months.

A \$1380 B \$1562 C \$1656 D \$8580 E \$8856

2 **Example 5** Sally makes a 2-year investment at 5% per annum simple interest. She wants to earn \$2000 in interest. The amount that she needs to invest, in dollars, is closest to

A 200 B 2200 C 10 000 D 20 000 E 40 000

3 **Example 7** Sian invests \$64 000 at 3.8% p.a. simple interest. How long will it take her to earn \$7000 in interest?

A 2 years 4 months B 2 years 5 months C 2 years 9 months
D 2 years 10 months E 2 years 11 months

4 **Example 9** The transaction details for Sam's savings account for the month of August 2015 are shown below.

Date	Transaction detail	Debit \$	Credit \$	Balance \$
1 August 2015	Balance			2560.00
17 August 2015	Deposit		450.00	
28 August 2015	Withdrawal	500.00		

Interest is paid on the minimum monthly balance at a rate of 3.2% per annum. The amount of interest earned for the month of August is

A \$1.22 B \$5.73 C \$6.82 D \$6.95 E \$6.96

5 **Example 12** How much interest is earned if \$17 000 is invested at 6% p.a. compounding yearly for 2 years?

A \$2040 B \$2101.20 C \$14 960
D \$19 040 E \$19 101.20

6 **Example 13** Joachim calculated the final value, A, of an investment using the formula $A = 7500 \times 1.02^{12}$. If the interest rate was 8% p.a., then the interest was compounded:

A annually for 2 years B quarterly for 12 years C annually for 12 years
D quarterly for 3 years E annually for 3 years

7 **Example 13** Christian invests \$5000 at a rate of 8% per annum compounding quarterly. The value of his investment at the end of 3 years is given by

A $\$5000 + \$5000 \times 0.08 \times 3$
B $\$5000 + \$5000 \times 0.02 \times 12$
C $\$5000 \times 1.08^3$
D $\$5000 \times 1.02^3$
E $\$5000 \times 1.02^{12}$

8 Example 16 Kalista has \$7500 invested at 8.4% p.a. compounding quarterly. How long will it take to grow to \$10 500?

A 4 years
B $4\frac{1}{4}$ years
C $4\frac{3}{4}$ years
D 16 years
E 17 years

Short answer

9 Example 1 Trish borrows \$10 300 at 6.25% simple interest for 3 years. How much interest did she have to pay?

10 Example 2 Calculate the simple interest earned or owed for each of the following.

a \$7200 invested at 6.25% p.a. for 3 years
b \$4050 borrowed at 3% p.a. for 16 months
c \$10 300 invested at 0.75% per month for 2 years
d \$40 000 borrowed at $8\frac{1}{2}$% p.a. for 120 days

11 Example 3 Holly invested \$45 000 at 7.45% p.a. simple interest. How much would she have at the end of 3 years?

12 Example 4 Casey bought a motorcycle for \$15 000 and paid 10% deposit. He took a flat rate loan at 12% p.a. for the remainder. If he repaid monthly instalments over 4 years, find:

a the total amount borrowed
b the amount of interest charged
c the monthly instalment.

iStockphoto/Oleksii Kondratiev

13 Example 5 What amount must be invested at 8.8% p.a. simple interest to earn \$15 000 in interest over 8 years? Answer correct to the nearest hundred dollars.

14 Example 6 Finn invests \$8900 for 5 years. If he earned \$2002.50 interest, then what was the annual simple interest rate?

15 Example 7 For how long must a principal of \$4000 be invested at 6.6% p.a. simple interest to grow to \$6376?

16 Example 9 Harmony earns interest of 6.5% p.a. on her bank account. Interest is calculated daily on the minimum monthly balance. On the 1 August the balance was \$769. She then withdrew \$400 on 5 August, and \$312.50 on 18 August. Her pay of \$236 went in on 8 August and 22 August.

a Calculate the minimum monthly balance.
b Calculate the interest for August.

17 **Example 10** Evan's credit card has a flat interest rate of 12.55% p.a. and no interest free period. He had paid off his card in September. He used his card for the following in October.

5 Oct	Lunch	\$36
11 Oct	Shoes	\$79.95
17 Oct	Phone bill	\$124.50
27 Oct	Hotel	\$229

a Evan didn't pay his bill in full until 15 November, how much does he pay?
b How much interest does he pay?

18 **Example 11** Isaac invests \$10 000 in an account earning 7.1% p.a. compounded annually for 3 years. Calculate:
a the interest earned in the first year
b the value of the investment at the beginning of the second year
c the interest earned in the second year
d the value of the investment at the beginning of the third year
e the interest earned in the third year
f the value of Isaac's investment at the end of the three years.

19 **Example 12** For the following calculate:
i the total value of the investment at the end of the period
ii the amount of compound interest earned.
a \$4050 at 3% p.a. for 16 months, compounded monthly
b \$7200 at 6.25% p.a. compounded quarterly for 3 years
c \$10 300 at 0.75% per month for 2 years, compounded monthly
d \$85 000 at 6.2 % p.a. compounded six-monthly for 18 months.

20 **Example 14** What principal must be invested at 5.8% p.a., compounded six-monthly, for it to grow to \$15 000 in 8 years? (Give your answer to the nearest cent.)

21 **Example 15** Sam invests \$26 000 in an account that has its interest compounded annually. If it grows to \$32 500 in 6 years, what was the interest rate? Answer as a percentage p.a. correct to one decimal place.

22 **Example 18** How long will it take \$25 000 to grow to \$30 000 if it is invested in an account that earns 7.2% p.a. compounded monthly?

Application

23 David invested \$8000 in a term deposit on 1 July 2014. The term deposit offered a total of \$1000 interest after two years.
a Determine the annual simple interest rate for this term deposit.
b Alternatively, David had the option of investing with a bank at a rate of 6.9% p.a. compounded annually.
i The total amount in this account after two years is given by: Total amount = $8000 \times k^2$. What is the value of k?
ii How much interest does David earn in two years with this investment option?
c Determine the annual compound interest rate, correct to two decimal places, required for David to earn \$1000 interest on his \$8000 investment in two years.

24 Jessica borrowed $6000 from the bank to purchase a home entertainment system. She was offered a flat-rate loan with interest charged at a rate of 13% p.a. for 30 months.

- **a** Find the total amount of interest that she will pay on this loan.
- **b** How much does she repay overall?
- **c** If the loan is repaid in equal monthly instalments, determine her monthly repayment.

After making repayments for 12 months, Jessica renegotiated her loan to have extra money to buy a new computer. The bank loaned her another $1000 at the same rate as her initial $6000 loan to be repaid in the same time.

- **d** After 12 months, how much is still owed on her original loan?
- **e** How long will the $1000 be charged interest on?
- **f** Determine the amount of interest she will pay on the $1000 balance.
- **g** Calculate the new monthly instalment.
- **h** How much does she repay altogether on the $7000 borrowed?

25 Molly wanted to go on a package holiday that cost $10 000. She had to pay a 15% deposit when she booked her holiday. Molly's grandmother had left her $8500 and she decided to use that to pay her deposit and then invest the rest to pay for her holiday. She investigated rates and the best that she could find was 8.4% p.a. simple interest.

- **a** How much did she have left to pay on the holiday after paying her deposit?
- **b** How much money did she have left to invest after paying the deposit?
- **c** How long will she need to invest the money to have enough to pay for her holiday? Round the answer up to the nearest month.

Once Molly knew how long it would take for her investment to grow to cover the holiday costs she booked her holiday.

After one year Molly's car broke down and she withdrew $3000 to fix it.

- **d** How much had her investment grown to at this point?
- **e** How much did she have left to reinvest?
- **f** How long would she be investing it for? Answer correct to the nearest month.

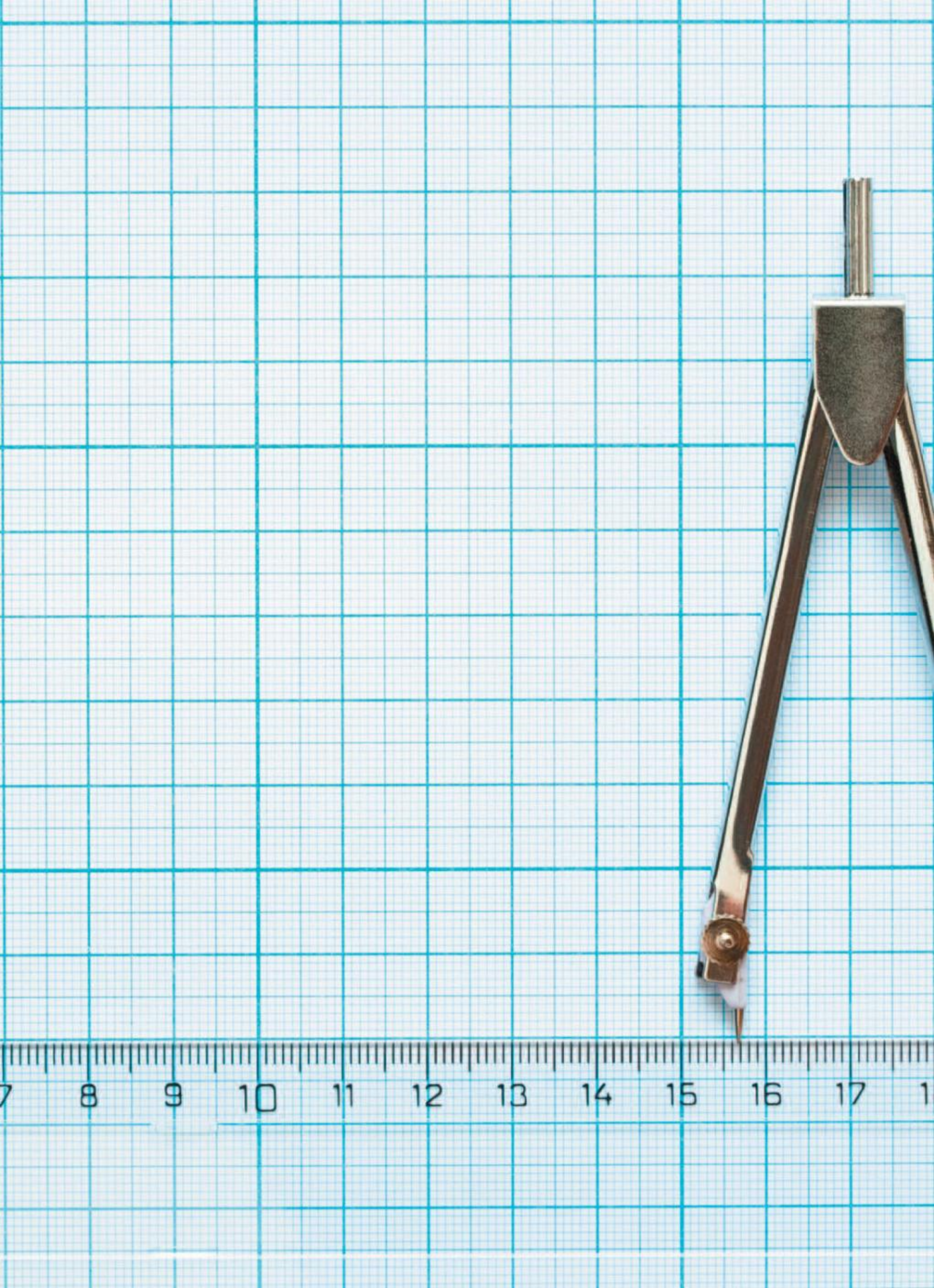
She looked again at rates but this time the best that she could get was 8.05% p.a. compounded monthly.

- **g** How much would she have for her holiday?

Molly did not have enough for her holiday and so she borrowed the balance of the money at 9.6% p.a. compounded quarterly over 18 months.

- **h** How much did she borrow? Answer correct to the nearest hundred dollars.
- **i** How much interest did she have to pay?
- **j** How much did her holiday cost altogether?

Practice quiz



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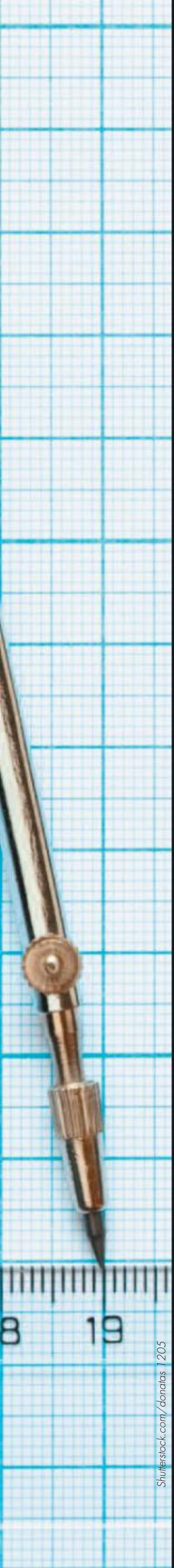

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7

TERMINOLOGY

corresponding
elevation
enlargement
hypotenuse
matching
orientation
plan
ratio
reduction
scale
scale drawing
scale factor
shadow
similar figures

SHAPE AND MEASUREMENT

Scales and similarity

Prior learning

SIMILAR FIGURES AND SCALE FACTORS

- **review the conditions for similarity of two-dimensional figures including similar triangles (ACMGM021)**
- **use the scale factor for two similar figures to solve linear scaling problems (ACMGM022)**
- **obtain measurements from scale drawings, such as maps or building plans, to solve problems (ACMGM023)**
- **obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures (ACMGM024)**
- **obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids. (ACMGM025)**

7.01 SCALES

Scale drawings are used in many areas of everyday life. A scale drawing of an object is the same shape as the object but a different size. To read plans or maps or photographs, we need to be able to understand scales. The **scale** of a drawing is a comparison of the length used on a drawing to the length it represents in real life. So a scale is written as a **ratio**.

IMPORTANT

Scale = plan length : real length

The scale determines how close the drawing is to the size of the real life object. A scale of 1 : 100 means the real length is one hundred times the length on the drawing.

A scale can be written with units of measure included, for example 1 cm : 1 km. This same scale could be written without units of measure included as 1 : 100 000 since there are 100 000 centimetres in one kilometre. A scale is usually written without units in simplest form. This form shows the equivalence for one unit on the plan. A scale of 1 : 1000 indicates that, whatever unit you use to measure a length on the drawing, the real object length is 1000 times larger. So 1 mm represents 1000 mm, 1 cm represents 1000 cm and so on.

Example 1

Copy and complete the following.

a 1 : 1 000 = 1 mm : _____ m **b** 1 : 100 = 1 cm : ______ m

Solution

a Write the question using the same units of measure for both parts.	1 : 1 000 = 1 mm : 1 000 mm
Convert 1000 mm to 1 metre.	1 : 1 000 = 1 mm : 1 m
b Write the question using the same units of measure for both parts.	1 : 100 = 1 cm : 100 cm
Convert 100 cm to 1 m.	1 : 100 = 1 cm : 1 m

Example 2

Write each of the scales below in simplest form without units.

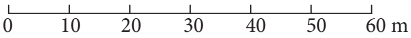
a

b 10 mm to 1 m

c 25 : 1

d 1 cm = 2 m

e



Solution

a Use a ruler to measure the scale given: 1 cm represents 5 km. Write the scale.

Scale = 1 cm : 5 km

Convert both to the same units. Change km to cm by multiplying by 100 000.

= 1 cm : 5 × 100 000 cm
= 1 cm : 500 000 cm

Write the scale without units.

= 1 : 500 000

b Write the scale as a ratio.

Scale = 10 mm : 1 m

Convert both to the same units.
Change m to mm by multiplying by 1000.

= 10 mm : 1000 mm

Simplify by dividing both parts of the ratio by 10.
Then write the scale without units.

= 1 : 100

c This scale is already in ratio form.

Scale = 25 : 1

d Write the scale as a ratio.

Scale = 1 cm : 2 m

Convert to the same units.

= 1 cm : 200 cm

Write your answer without units.

= 1 : 200

e Using a ruler to measure the scale gives 8 mm represents 10 m. Write the ratio.

Scale = 8 mm : 10 m

Convert to the same units.

= 8 mm : 10 000 mm

Simplify by division and write without the units.

= 1 : 1250

A scale can also be used to find unknown lengths on a scale drawing or on a real object.

Corbis/MAPS.com

Example 3

The scale on a map was 1 : 100 000.

a What was the real distance between 2 shops which were 8 cm apart on the map?

b How long was the line on the map indicating a 6.8 km straight stretch of road?

Solution

a Write the scale.	Scale = 1 : 100 000
Write an equivalent ratio to the scale with a pronumeral representing the unknown length.	$1 : 100\,000 = 8 \text{ cm} : x \text{ cm}$
Rewrite the ratios as fractions. Ensure that the pronumeral is on top.	$\frac{100\,000}{1} = \frac{x}{8}$
Multiply both sides by 8.	$x = \frac{100\,000}{1} \times 8$
Simplify. The measurement was in cm so the answer is in cm.	$x = 800\,000 \text{ cm}$
Change to km by dividing by 100 000.	$x = 8 \text{ km}$
Interpret your answer.	The shops are 8 km apart.

b Write the scale.	Scale = 1 : 100 000
Write an equivalent ratio to the scale with a pronumeral representing the unknown length.	1 : 100 000 = x : 6.8 km
Change 6.8 km to centimetres by multiplying by 100 000.	1 : 100 000 = x : 680 000 cm
Rewrite the ratios as fractions.	$\frac{1}{100\,000} = \frac{x}{680\,000}$
Multiply by 680 000 to solve for x.	$x = 6.8$
Interpret the result and write the answer.	The line on the map is 6.8 cm long.

Once you have written an equation using the scale, a CAS can be used to solve for the unknown.

Example 4

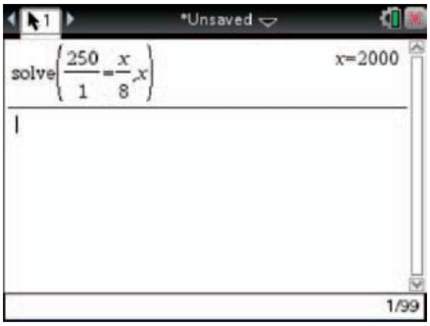
Find the real height of a building represented by an 8 cm high drawing with a scale of 1 : 250.

Solution

Write the scale.	Scale = 1 : 250
Write an equivalent ratio to the scale with a pronumeral representing the unknown length.	1 : 250 = 8 cm : x
Rewrite the ratios as fractions.	$\frac{250}{1} = \frac{x}{8}$

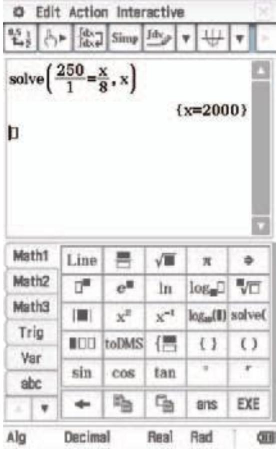
Solve for x.

TI-Nspire CAS



$\text{solve}\left(\frac{250}{1}=\frac{x}{8},x\right)$ $x=2000$

ClassPad



$\text{solve}\left(\frac{250}{1}=\frac{x}{8},x\right)$

$\{x=2000\}$

Give an answer with units.	$x = 2000$ cm
Convert to an appropriate unit of measurement.	$x = 2000 \div 100$ m $= 20$ m
Interpret your answer.	The building is 20 m high.

EXERCISE 7.01 Scales

Concepts and techniques

1 **Example 1** Copy and complete each of the following.

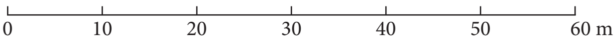
a 1 : 10 = 1 mm : _____ cm
b 1 : 1 000 000 = 1 mm : _____ km
c 1 : 1000 = 1 m : _____ km
d 1 : 1000 = 1 cm : _____ m
e 1 : 100 000 = 1 m : _____ km
f 1 : 100 = 1 mm : _____ cm

2 **Example 2** Write each of these scales as a ratio in simplest form, without units.

a

b 2 mm : 1 cm

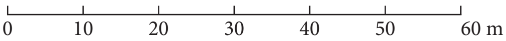
c



d 10 cm to 1 m

e 5 m : 2 cm

f



3 Write as a ratio in simplest form, without units.

a 1 mm : 1 m
b 1 mm : 10 m
c 1 cm : 10 km
d 1 cm : 4 m
e 1 m : 7 km
f 4 cm : 1 m
g 200 mm : 1 m
h 50 cm : 1 km
i 1 m : 0.68 km

4 **Example 3** Find the real length of objects represented on a drawing, with a scale of 1 : 300.

a 1 cm
b 1 mm
c 3 cm
d 28 mm
e 9.7 cm
f 20 cm

5 **Example 4** Find the real length of objects represented on a plan, with a scale of 1 cm to 5 m.

a 8 cm
b 2.5 cm
c 3.1 cm
d 1 mm
e 15 mm
f 0.6 cm

6 What length of line would be used on a plan, with a scale of 1 cm : 2.5 m, to represent a real distance of:

a 10 m
b 5 m
c 35 m
d 7.5 m
e 1.25 m
f 13.75 m

7 What length of line would be used on a map, with a scale of 1 : 10 000, to represent a real distance of:

a 1 km
b 3.5 km
c 4.9 km
d 500 m
e 730 m
f 60 m

8 The distance from Sydney to Penrith is 55 km. What scaled distance would this be on a map with a scale of 1 : 2 000 000?

Reasoning and communication

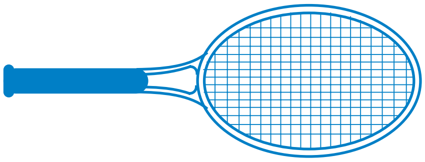
9 The first part of the ratio given for a scale is always the length on the drawing. The scale used for a drawing was given as 10 : 1. Is it possible to have a scale drawing with this scale? Explain.

7.02 READING SCALE DRAWINGS

A scale drawing is usually a **reduction** of a real object, but can be an **enlargement** of a very small object, such as a computer chip. When using a scale drawing to find measurements, it is important to measure lengths accurately.

Example 5

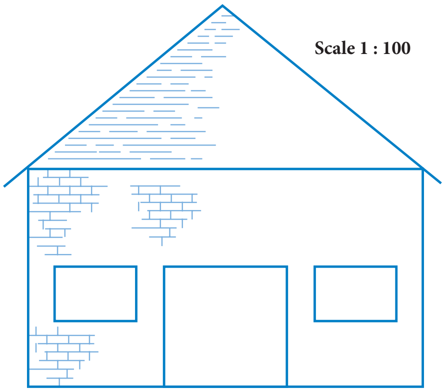
a By measurement and calculation, find the real length (in centimetres) of this tennis racquet.



Tennis racquet 1 : 12

b This diagram of the front of a house is drawn to a scale of 1 : 100.

- **i** Find the actual height of the door in the drawing, in metres.
- **ii** What is the scaled length, in millimetres, of the house if it is actually 9.5 m long?



Solution

a	Measure the length of the racquet.	Drawing length = 5.5 cm
	The scale is 1 : 12, so the real racquet is 12 times larger than the scale drawing. So we multiply by 12.	Real length = 12 × 5.5 cm = 66 cm
b i	Measure the height of the door.	Drawing length = 16 mm
	The scale is 1 : 100, so the real door is 100 times larger than the scale drawing. So we multiply by 100.	Real length = 100 × 16 mm = 1600 mm
	Convert to an appropriate unit.	= 1.6 m
ii	Write the real length.	Real length = 9.5 m
	The scale is 1 : 100 so the length in the scale drawing is 100 times smaller than the real length. So we divide by 100.	Drawing length = 9.5 m ÷ 100
	Convert to a smaller unit of measure.	= 9500 mm ÷ 100
	Write the answer.	= 95 mm

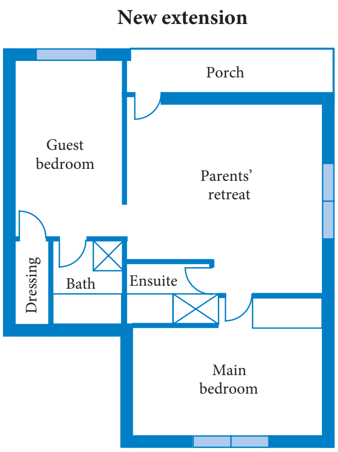
When working with scale drawings, you should check that answers seem reasonable.

Example 6

An architect drew this scale drawing of an extension to Natalie's house.

a What is the length of the porch, in metres?

b What is the cost of tiling the porch if tiles are \$62.50 per m^2 laid?



Scale 1 : 200

Solution

a Measure the length of the porch.	Drawing length = 27 mm
The scale is 1 : 200 so the real length is 200 times larger.	Real length = 200 × 27 mm = 5400 mm
Convert to metres.	= 5.4 m
b Measure the width of the porch.	Drawing width = 6 mm
Multiply by 200 to find the real width.	Real width = 200 × 6 mm = 1200 mm
Convert to metres.	= 1.2 m
Calculate the area of the porch using the area of a rectangle formula, $A = lw$.	Area = 5.4 × 1.2 = 6.48 m^2
Multiply by the cost per m^2, to find the total cost.	Cost = 6.48 × \$62.50 = \$405

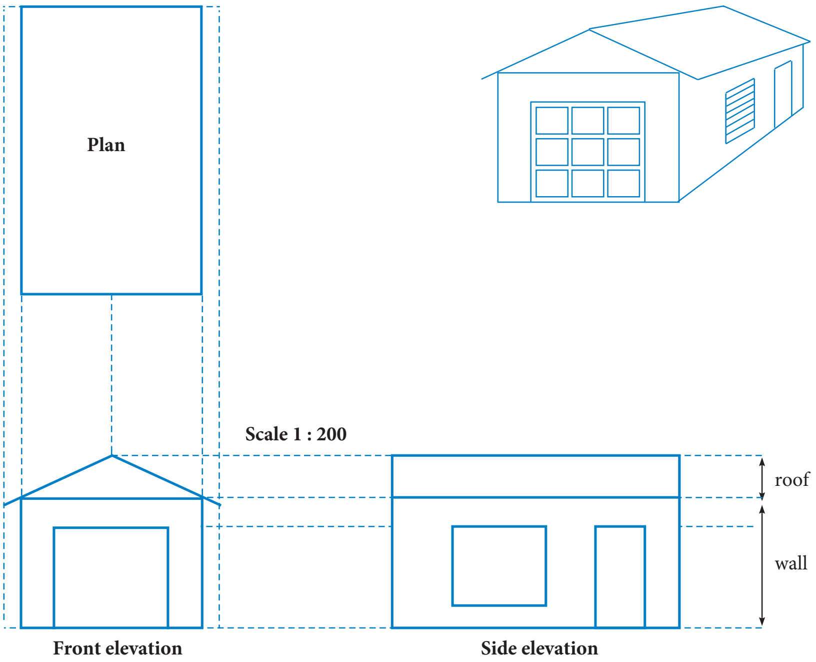
An architect constructing diagrams for a new building includes

- a **plan**: a diagram of the floor
- the **elevations**: the views of the front, back and sides.

9780170250252

Example 7

Here is a drawing of a garage, showing its plan, front and side elevations.



a What is the floor area of the garage in m^2?

b What is the height of the highest point of the garage's roof?

c The four inside walls of the garage need to be painted, including the doors but not the window (of dimensions 2.4 m by 2 m). How many whole litres of paint would you need to buy to put two coats of paint on the walls if 1 L covers $10m^2$?

Solution

a Measure the length of the garage. Drawing length = 3.8 cm

The scale is 1 : 200. Multiply by 200 to find the actual length. Then convert to metres.

Real length = 200 × 3.8 cm
= 760 cm
= 7.6 m

Measure the width of the garage. Drawing width = 2.4 cm

Multiply by 200. Then convert to metres.

Real width = 200 × 2.4 cm
= 480 cm
= 4.8 m

Calculate the area of the garage floor.

$$\text{Area} = 7.6 \times 4.8$$
$$= 36.48 \text{ m}^2$$

b Measure the height from the floor to the peak of the roof. Drawing height of highest point = 2.3 cm

Multiply by 200. Then convert to metres.

Real height of highest point = 200 × 2.3 cm
= 460 cm
= 4.6 m

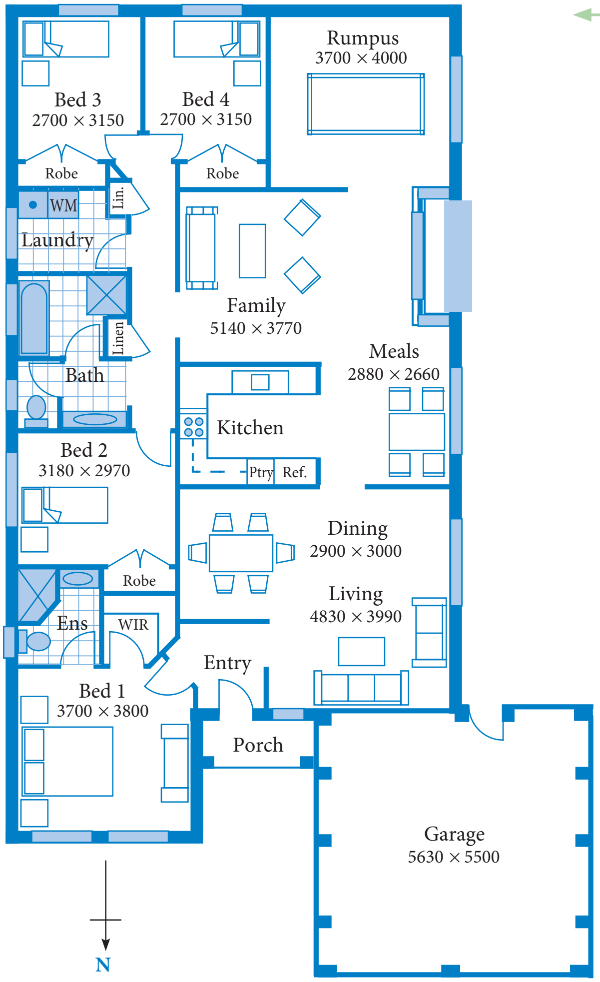
c Measure the height of the wall.	Drawing height of wall = 1.7 cm
Multiply by 200. Then convert to metres.	Real height of wall = 200 × 1.7 cm = 3.4 m
Calculate the area of the front and back walls using the width of the garage found in part **a**.	Area of front and back walls = 2 × 4.8 × 3.4 = 32.64 m^2
Calculate the area of the side walls using the length of the garage found in part **a**.	Area of side walls = 2 × 7.6 × 3.4 = 51.68 m^2
Calculate the area of the window from the dimensions given.	Area of window = 2.4 × 2 = 4.8 m^2
Calculate the total wall area to be painted.	Area to be painted = 32.64 + 51.68 – 4.8 = 79.52 m^2
Calculate the total area to be painted for 2 coats.	Total area of paint = 2 × 79.52 = 159.04 m^2
Calculate the amount of paint needed, given that 1 L covers 10 m^2.	Amount of paint needed = 159.04 ÷ 10 L = 15.904 L
Answer the question. Remember that you can only buy whole litres of paint so you need to round up the number of litres needed.	16 L of paint would need to be bought.

Plans usually contain symbols and abbreviations showing features, such as doors and toilets. Some plan symbols are shown here.

	Hinged door	Ens	Ensuite (bathroom)
 or WC	Toilet (water closet)		Bath
	Shower		Vanity
	Stove		Kitchen sink
Ptry	Pantry		Laundry tub
WM	Washing machine		Linen cupboard
Robe	Wardrobe	WIR	Walk-in (ward)robe
FL	Floor level	GL	Ground level
CL	Ceiling level		Window

Example 8

This house plan, not to scale, shows measurements in millimetres.



Measurements on plans are always shown in millimetres because this avoids the use of decimal points, which can lead to errors in printing and reading.

a Write the symbol for a window and state the number of windows on the plan.

b State the number of hinged doors on the plan, not including those on wardrobes or cupboards.

c How many showers are there in this house?

d What is used to indicate the wall cupboards in the kitchen on this plan?

e What is the area of the family room, correct to the nearest m^2?

Solution

a From the table of plan symbols, find the window symbol. Count how many are on the plan.	▭ There are 16 windows.
b Count the number of ⌒ symbols in the house. Be careful not to count wardrobes or cupboards.	There are 10 hinged doors.

c Count the number of ⊠ symbols in the house.	There are two showers.
d Wall cupboards in a kitchen are not as wide as the floor units. Look at the plan to see how this is indicated.	The wall units are indicated by dotted lines.
e Read off the measurements for the family room given by 5140 × 3770. Convert to metres before multiplying.	Area of family room $= 5.14 \text{ m} \times 3.77 \text{ m}$ $= 19.3778 \text{ m}^2$
Round to the nearest m^2.	$\approx 19 \text{ m}^2$
Write the answer.	The area of the family room is 19 m^2.

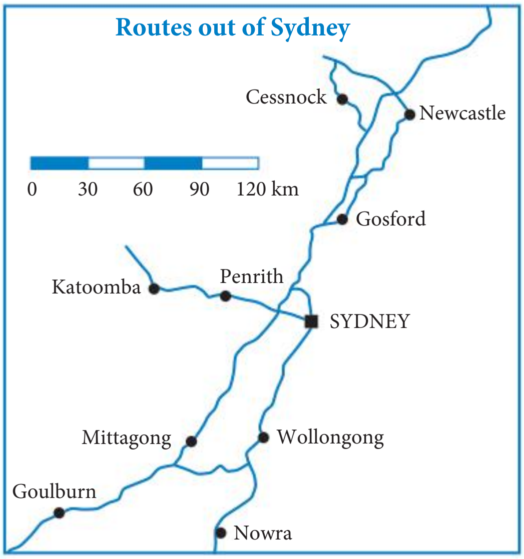
WS Interpreting an office plan

EXERCISE 7.02 Reading scale drawings

Concepts and techniques

1 Example 5 The map 'Routes out of Sydney' is a scale drawing. Find:

- a the scale factor
- b the distance (in a straight line) from:
  - i Sydney to Katoomba
  - ii Newcastle to Nowra
  - iii Goulburn to Gosford
  - iv Sydney to Cessnock
- c the town that is 92 km from Sydney
- d the towns that are exactly 40 km apart.



2 By measurement and calculation, find the real lengths of these objects:

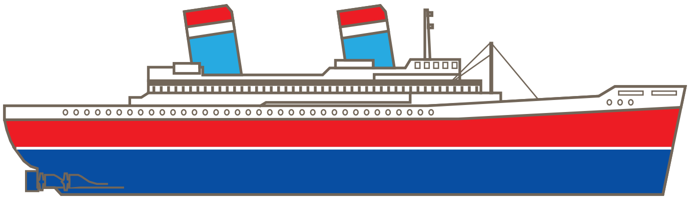
a

Car 1 : 100

b

Ant 4 : 1

c



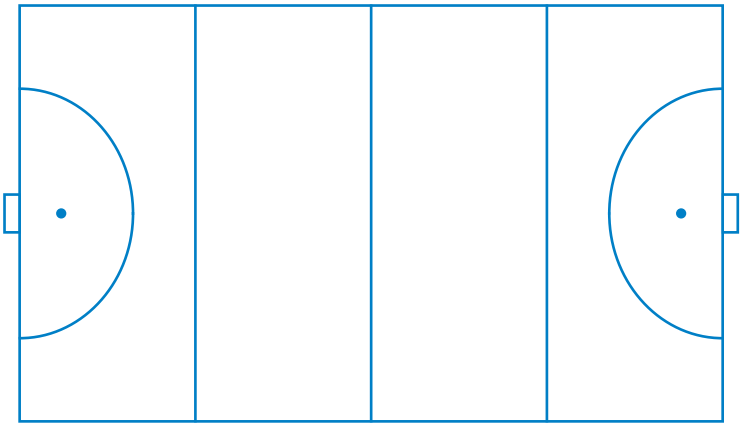
Ship 1 : 3000

3 Below is a scale drawing of a hockey field.

a Copy the diagram carefully, then calculate and write the actual dimensions on your drawing.

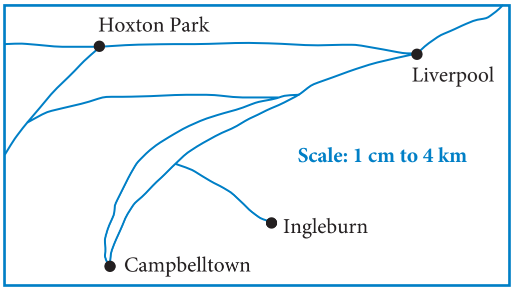
b What is the actual width of the hockey field?

c What is the actual length of the hockey field?



Scale: 1 cm = 10 m

4 What is the distance (in a direct line) between:

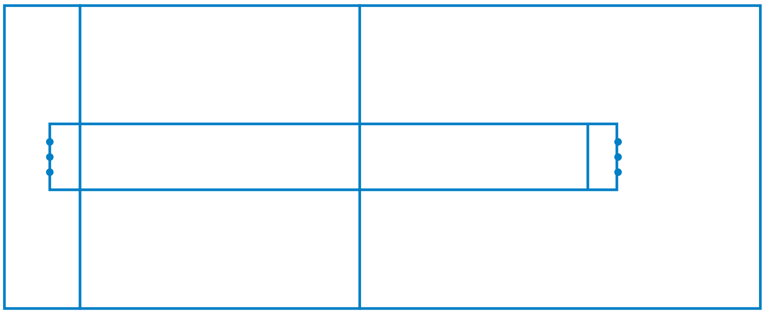


a Liverpool and Campbelltown?

b Hoxton Park and Ingleburn?

c Campbelltown and Hoxton Park?

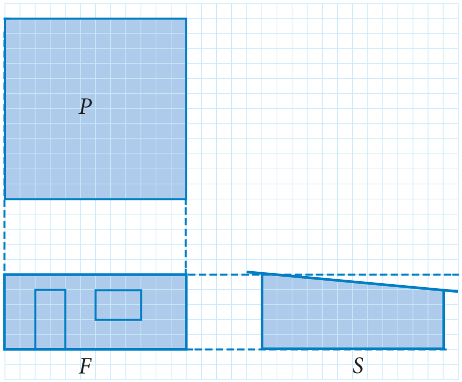
5 Below is a scale drawing of an indoor cricket pitch.



Scale: 1 cm to 3 m

a What is the actual length of the total playing area?
b What is the actual width of the cricket pitch?
c What is the actual length of the cricket pitch?

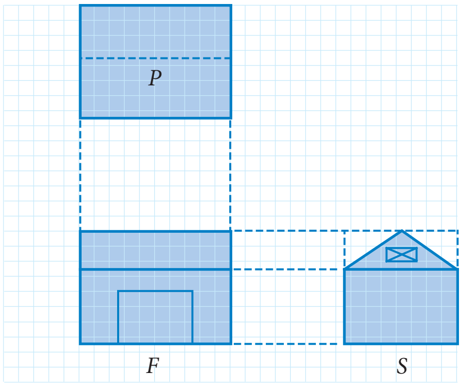
Reasoning and communication



Scale 1 cm : 2.5 m

6 Example 6 The scaled plan for a fishing shack is shown on the right. The floor plan is labelled *P*, the front elevation is labelled *F* and the side elevation is labelled *S*.

a What is the height of the back wall?
b What is the width of the shack?
c What is the floor area?
d How many whole litres of paint are required to paint the outside walls of the shack, including the door but not the window, if 1 L of paint covers 10 m^2?



Scale 1 : 400

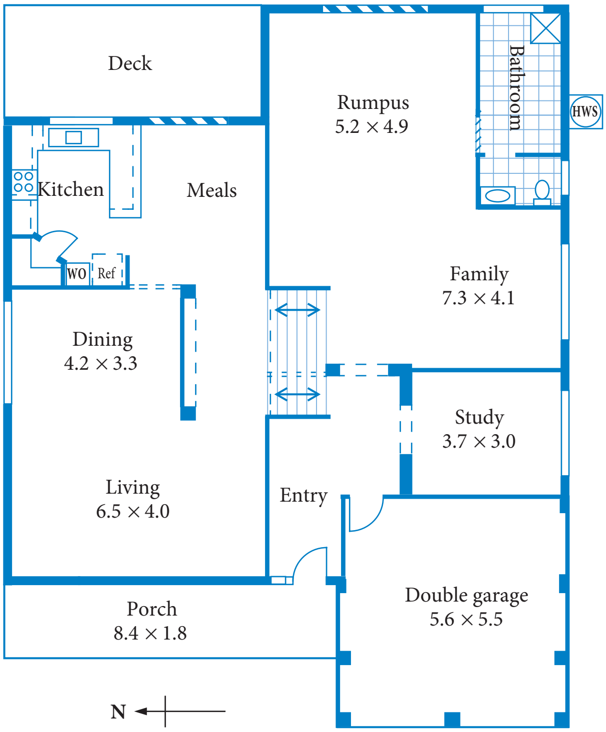
7 Example 7 The scaled plans for a barn are shown on the right. The floor plan is labelled *P*, the front elevation is labelled *F* and the side elevation is labelled *S*.

a What is the height of the highest point of the roof from the ground?
b What is the length of the barn?
c What is the floor area?
d What is the cost of tiling the floor if tiles are $84.50 per square metre laid?

8 Example 8 Refer to the house plan in Example **8** to answer these questions.

a How many bedrooms are there in the house?
b What does the symbol WIR stand for?
c How many bathtubs are there in the house?
d In which room is the pantry?
e What is the area of the combined living and dining room, to the nearest m^2?

9 This is the floor plan of a split-level home.



a How many steps do we walk down from the family room to the dining and living level?

b Draw the symbols used on the plan to represent:
  - i the hot water service
  - ii the hotplates
  - iii the wall oven
  - iv the front door
  - v the sliding door onto the deck

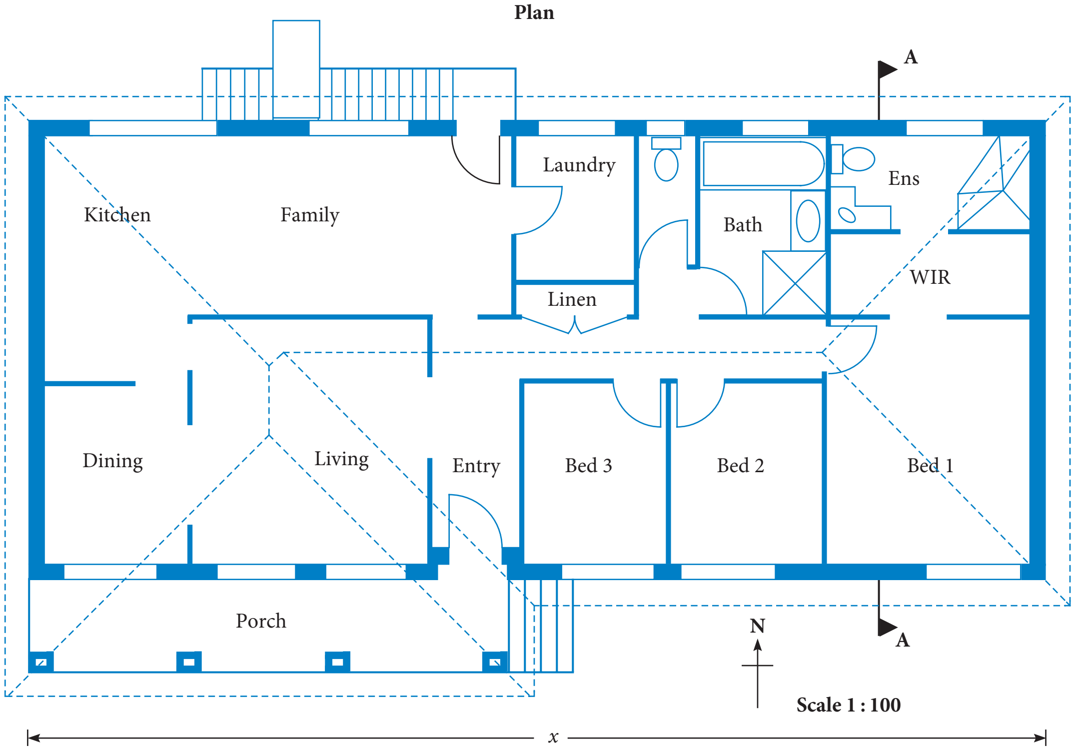
c The porch is to be tiled with terracotta tiles. How many square metres of tiles are needed (rounded to the nearest 0.5 m^2)?

d If each tile measures 30 cm by 30 cm, how many whole tiles are needed to tile the porch?

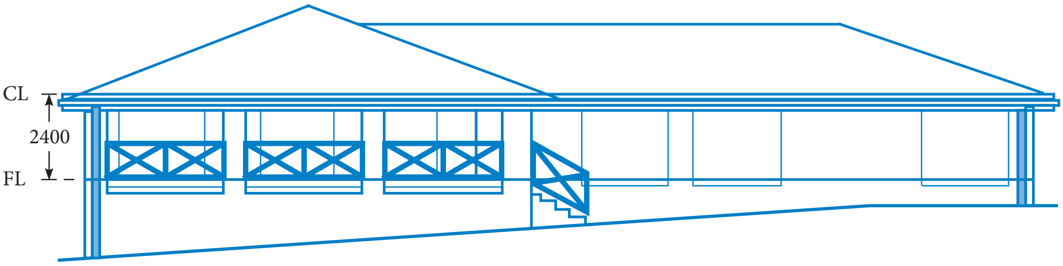
e The dining and living areas are to be carpeted. How much will it cost to carpet the rooms if carpet costs \$68 per m^2 laid?

10 The floor plan and south elevation of a home built on a sloping block of land are shown below.

Plan



South elevation



a What scale is shown on the plan?

b What is the actual length of the house (denoted by x)?

c How many windows are there in the house?

d How many external doors are there in the house?

e What is the area of the porch? Give your answer to the nearest m^2.

f What is the ceiling height?

g The entry and hallway are to have slate tiles. These cost \$105 per square metre laid. How much will it cost to have the entry and hallway tiled? Give your answer correct to the nearest \$100.

9780170250252

7.03 CONSTRUCTING SCALE DRAWINGS

A scale drawing is drawn on a piece of paper (or computer screen), so the drawing can fit on the paper. When choosing what scale to use, we try to make the scale fit the metric system, so we use scales such as 1 : 10, 1 : 20, 1 : 25, 1 : 40, 1 : 50, 1 : 100, 1 : 200 . . . 1 : 100 00, 1 : 500 000, etc. This makes the conversion from the real lengths to the drawn lengths easier.

Shutterstock.com/Van Zoeren Designs

IMPORTANT

How to determine the scale for a drawing.

1. Change the measurements of the real object to the same units being used on the drawing paper.
2. Determine the size of the space available. Allow for margins.
3. Divide the size of the real object by the size of the space for the drawing in both directions to obtain a **scale factor** for each direction.
4. Choose the larger scale factor from the two directions.
5. Round up to a simple number.

Example 9

A house that is 16 m long and 9 m wide is to be drawn on A4 paper. What scale should be used?

Solution

Change the units of the house measurements to mm.	16 m = 16 000 mm 9 m = 9000 mm
Measure the paper.	A4 paper size = 297 mm × 210 mm
Allow for margins.	Margins should be 25 mm.
Find the amount of drawing space.	Drawing space = 247 mm × 160 mm
Divide longer dimensions.	Length scale factor = 16 000 mm ÷ 247 mm ≈ 64.8
Divide shorter dimensions.	Width scale factor = 9000 mm ÷ 160 mm ≈ 56.3
Choose the larger scale factor.	Use 64.8.
Round the larger scale factor up.	Make the scale 1 : 100.
Write the answer.	A scale of 1 : 100 should be used.

Example 10

A desk 120 cm by 50 cm is shown on a scale drawing at a scale of 1 : 50. What are the dimensions of the drawing?

Solution

Divide the width, in mm, by the scale factor.	Width = 1200 mm ÷ 50 = 24 mm
Divide the depth, in mm, by the scale factor.	Depth = 500 mm ÷ 50 = 10 mm
Write the answer.	The dimensions of the drawing are 24 mm by 10 mm.

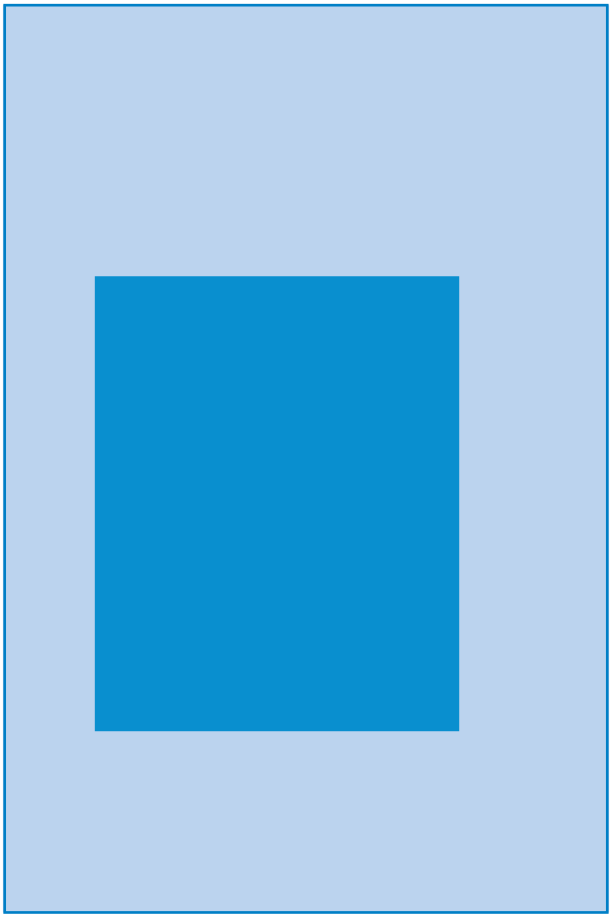
Example 11

A scale drawing of a house on a block of land is to be made. The block is 20 m wide and 30 m deep, and the house is 15 m wide and 12 m deep. The house is 3 m from the left boundary and 6 m from the front boundary. The drawing must fit into a space 10 cm wide by 12 cm deep. Choose a scale and make the drawing.

Solution

Change the units from metres to centimetres.	20 m = 2000 cm, 30 m = 3000 cm
Divide the dimensions of the block by those of the paper.	2000 cm ÷ 10 cm = 200 3000 cm ÷ 12 cm = 250
Choose an appropriate scale.	Use a scale of 1 : 250.
Calculate the drawing dimensions in mm.	Block width = 20 000 mm ÷ 250 = 80 mm Block depth = 30 000 mm ÷ 250 = 120 mm House width = 15 000 mm ÷ 250 = 60 mm House depth = 12 000 mm ÷ 250 = 48 mm
Calculate the house position in mm.	Distance from left side = 3000 mm ÷ 250 = 12 mm Distance from front = 6000 mm ÷ 250 = 24 mm

Make the drawing.
Write the scale on the drawing



Scale 1 : 250

INVESTIGATION Crowded classrooms

1 Determine the scale required to plot your classroom so that it fits on an A4 sheet with some room left on the edges as margins.

2 Now work in groups to make a scale drawing of the classroom showing door(s), windows and any furniture.

Shutterstock.com/Monica Butnaru

3 Calculate the floor space left when the furniture is taken into account. Find the ratio of the free floor space to total floor space. Find the amount of floor space per person, both in terms of total space and free space.

4 Discuss the amount of floor space needed to avoid feeling ‘crowded’ in a classroom or work environment. Does this depend on the person, or is there general agreement?

EXERCISE 7.03 Constructing scale drawings

Concepts and techniques

1 Example 9 What scale should be used for a drawing on A4 paper (allowing for 25 mm margins) of:

a a house block 70 m by 28 m? **b** a park 2 km by 1.8 km?

c a room 6 m by 3.5 m? **d** a house 18 m by 10 m?

e a subdivision 3 km by 5 km? **f** a cattle station 20 km by 16 km?

2 Example 10 Objects of the following sizes are drawn to scale. Find the dimensions of each drawing.

a 30 m by 40 m at 1 : 100. **b** 16 m by 11 m at 1 : 200.

c 3.0 m by 4.5 m at 1 : 50. **d** 900 mm by 2100 mm at 1 : 50.

e 28 km by 60 km at 1 : 50 000. **f** 450 m by 800 m at 1 : 500.

Reasoning and communication

3 Example 11 A house 16 m wide by 19 m deep is on a block of land 20 m wide by 35 m deep. The house is set back 7 m from the front boundary and 2 m from either side. Make a scale drawing of the block and house on A4 paper.

4 A 16-year-old girl wants to rearrange the furniture in her bedroom. She has measured the room and finds that it is 3.00 m by 2.40 m, with the 820 mm door in one corner on the 2.40 m wall and a 1200 mm-wide window in the centre of the opposite wall. Her bed is 1950 mm by 900 mm, her wardrobe is 1200 mm by 500 mm, her desk is 900 mm by 500 mm and her dressing table is 900 mm by 450 mm. Make a scale drawing on A4 paper of her bedroom, and use cut-outs of furniture to work out an arrangement of her furniture. Justify any decisions you make.

5 An office has to accommodate 6 staff in a room 10 m long and 6 m wide with two 900 mm-wide doors. Each staff member needs a desk 1200 mm by 600 mm, a filing cabinet 450 mm wide and 600 mm deep, and space for a chair. In addition, space is needed for a photocopier, shared fax and shared printer in the office. These are 1200 mm by 600 mm, 600 mm by 450 mm and 600 mm by 600 mm respectively. Dividers may be placed in the room to ensure privacy. Choose a scale and design a layout to fit:

a A4 paper **b** A3 paper (297 × 420 mm).

Shutterstock.com/06photo

7.04 SIMILAR FIGURES

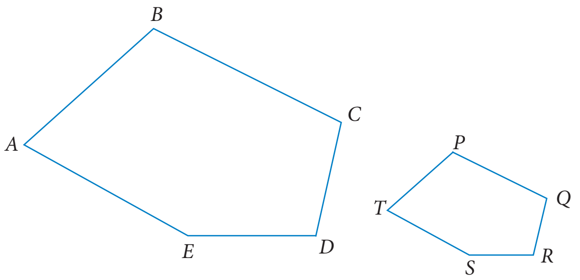
The symbol for similarity is ||| or ~. Both mean “is similar to”.

When we name similar figures, it is important to name them in matching angle order.

IMPORTANT

Similar figures have the same shape but not necessarily the same size. They have:

- corresponding angles equal.
- corresponding sides in the same ratio.



If you are told that figure *ABCDE* is similar to figure *TPQRS*, then we say *ABCDE* ||| *TPQRS* and we know that:

$\angle A = \angle T, \angle B = \angle P, \angle C = \angle Q, \angle D = \angle R$ and $\angle E = \angle S$ since corresponding angles are equal

and $\frac{AB}{TP} = \frac{BC}{PQ} = \frac{CD}{QR} = \frac{DE}{RS} = \frac{EA}{ST}$ since corresponding sides are in the same ratio.

The ratio of two corresponding sides in similar figures is called the scale factor.

The scale factor measures how much the object would be enlarged or reduced by to produce the similar figure known as the image.

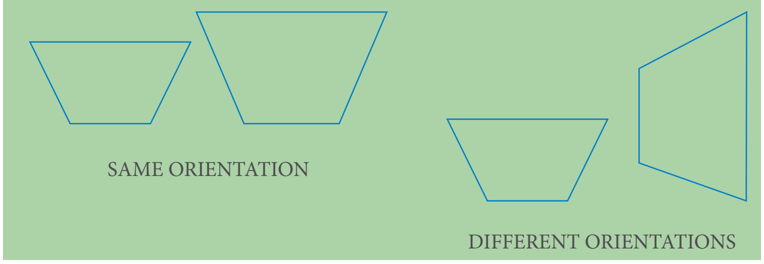
IMPORTANT

$$\text{Scale factor} = \frac{\text{length in image}}{\text{corresponding length in object}}$$

If the scale factor is greater than 1, then the image is an enlargement of the object and if the scale factor is less than one, the image is smaller than the object.

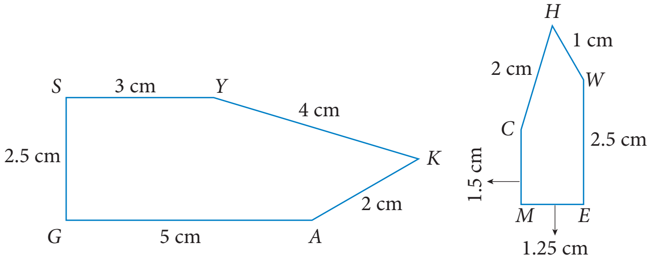
To make it easier to match **corresponding** sides and angles in similar shapes, you can sketch the figures in the same **orientation**.

Shapes in the same orientation, are around the same way.



Example 12

For the following two shapes:



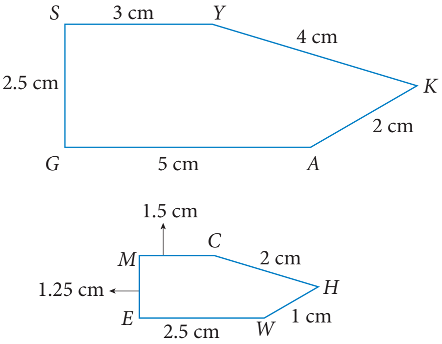
a prove that the two shapes are similar

b name the similar shapes in matching angle order

c state the scale factor.

Solution

a Sketch the two shapes with the same orientation.



Measure matching angles.

$\angle S = \angle M = 90°$

$\angle G = \angle E = 90°$

$\angle A = \angle W = 150°$

$\angle K = \angle H = 52°$

$\angle Y = \angle C = 158°$

Write the answer.

The figures are similar as corresponding angles are equal.

b State that the figures are similar using matching angle order.

$SYKAG \,|||\, MCHWE$

c Scale factor

$$= \frac{\text{length in image}}{\text{corresponding length in object}}$$

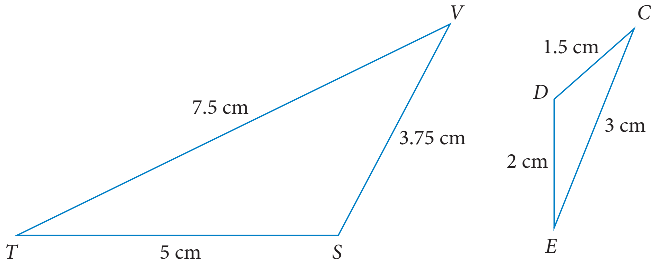
$$\text{Scale factor} = \frac{ME}{SG}$$

$$= \frac{1.25}{2.5}$$

$$= \frac{1}{2}$$

Example 13

The following shapes are similar.



a Name the side corresponding to *DE* in ΔSTV.

b Calculate the scale factor used to obtain ΔDEC from ΔSTV.

Solution

a *DE* is the second longest side in ΔDEC and so corresponds to the second longest side of ΔSTV.

ST corresponds to *DE*.

b Scale factor $= \dfrac{\text{length in image}}{\text{corresponding length in object}}$

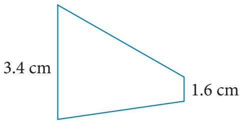
$$\begin{aligned}\text{Scale factor} &= \frac{DE}{ST}\\ &= \frac{2}{5}\end{aligned}$$

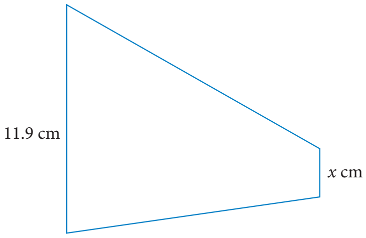
A scale factor can be a whole number or a fraction. A scale factor is usually left as an improper fraction, not a mixed number, where necessary.

Example 14

For these similar figures, find the:

a scale factor **b** value of x





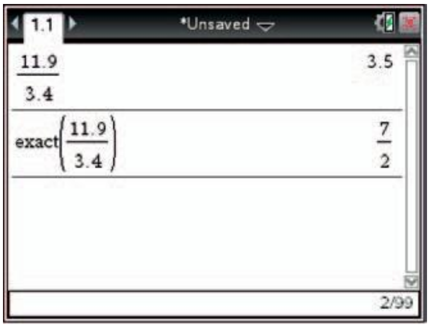
Solution

a Scale factor $= \dfrac{\text{length in image}}{\text{corresponding length in object}}$

$$\begin{aligned}\text{Scale factor} &= \frac{11.9}{3.4}\\ &= 3.5\\ &= \frac{7}{2}\end{aligned}$$

TI-Nspire CAS

In AUTO mode, if decimals are used in the input then the CAS will automatically return a decimal answer. To obtain a fractional answer type exact $\left(\frac{11.9}{3.4}\right)$ then press enter.



$\frac{11.9}{3.4}$ 3.5

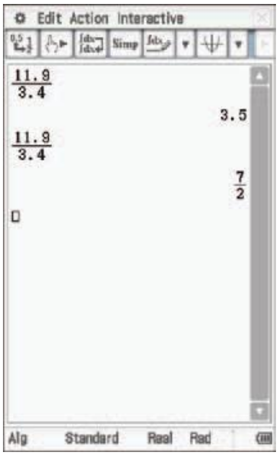
exact$\left(\frac{11.9}{3.4}\right)$ $\frac{7}{2}$

ClassPad

With the calculator in **Decimal** mode input then the CAS will automatically return a decimal answer.

$\frac{11.9}{3.4}$ 3.5

To obtain a fractional answer tap **Decimal** on the bottom toolbar to change the mode to **Standard**.



$\frac{11.9}{3.4}$ 3.5

$\frac{11.9}{3.4}$ $\frac{7}{2}$

b The scale factor compares corresponding sides. Identify the corresponding side to the unknown. Write an equation using the ratio of corresponding sides.

Solve for x.

$7 \leftrightarrow 2, \quad x \text{ cm} \leftrightarrow 1.6 \text{ cm}$

$$\frac{7}{2} = \frac{x}{1.6}$$

$$x = \frac{7}{2} \times 1.6$$

$$= 5.6$$

INVESTIGATION Scale factors

1 Investigate the scale factor of enlargements obtained using an overhead projector or data projector. Is there a relationship between the distance of the projector from the screen and the scale factor of the resulting projection?

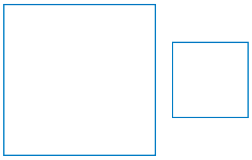
2 Use a photocopier to investigate enlarging and reducing a figure. Select, for example, 120% and 70%, then measure the sides and write the scale factors.

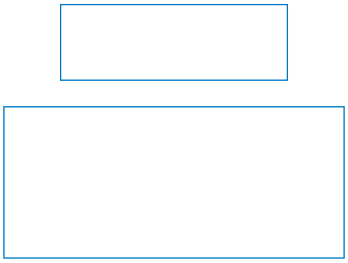

Alamy/aberCPC

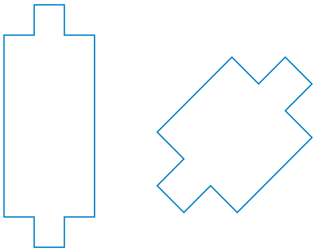
EXERCISE 7.04 Similar figures

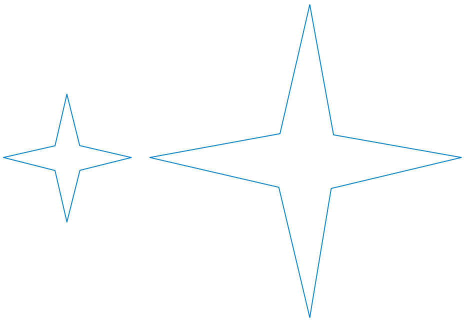
Concepts and techniques

1 By measurement, state whether each pair of shapes is similar or not.

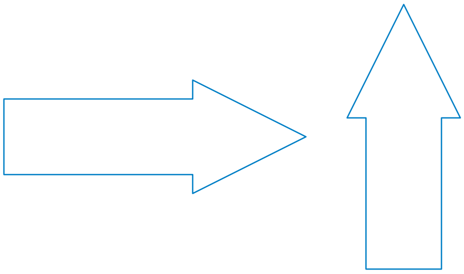
a


b


c


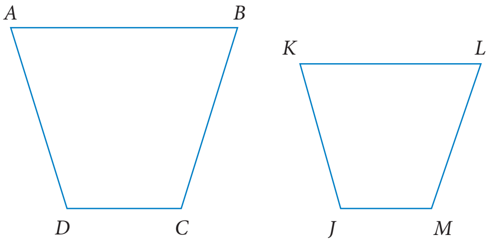
d


e

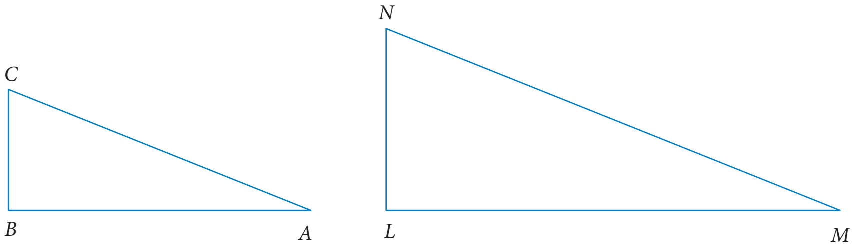

f


2 **Example 12** Using measurement, state which of the following pairs of figures are similar. For similar figures, name the shapes in matching angle order and calculate the scale factor.

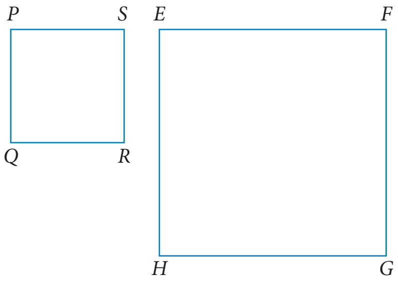
a



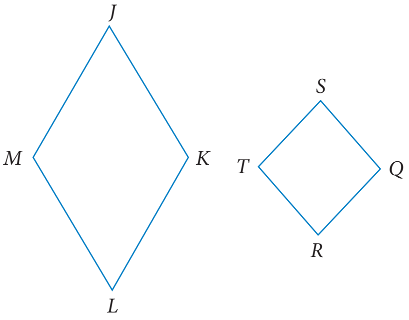
b



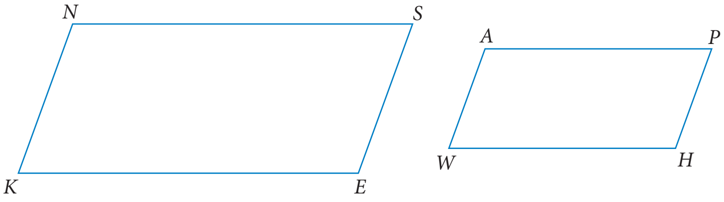
c



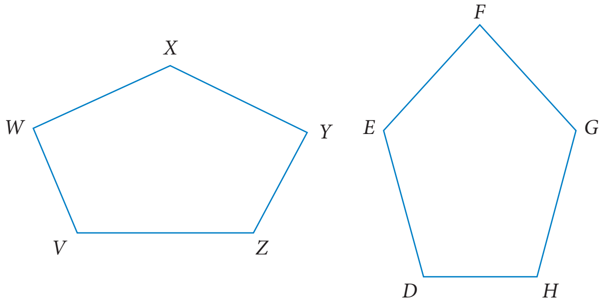
d



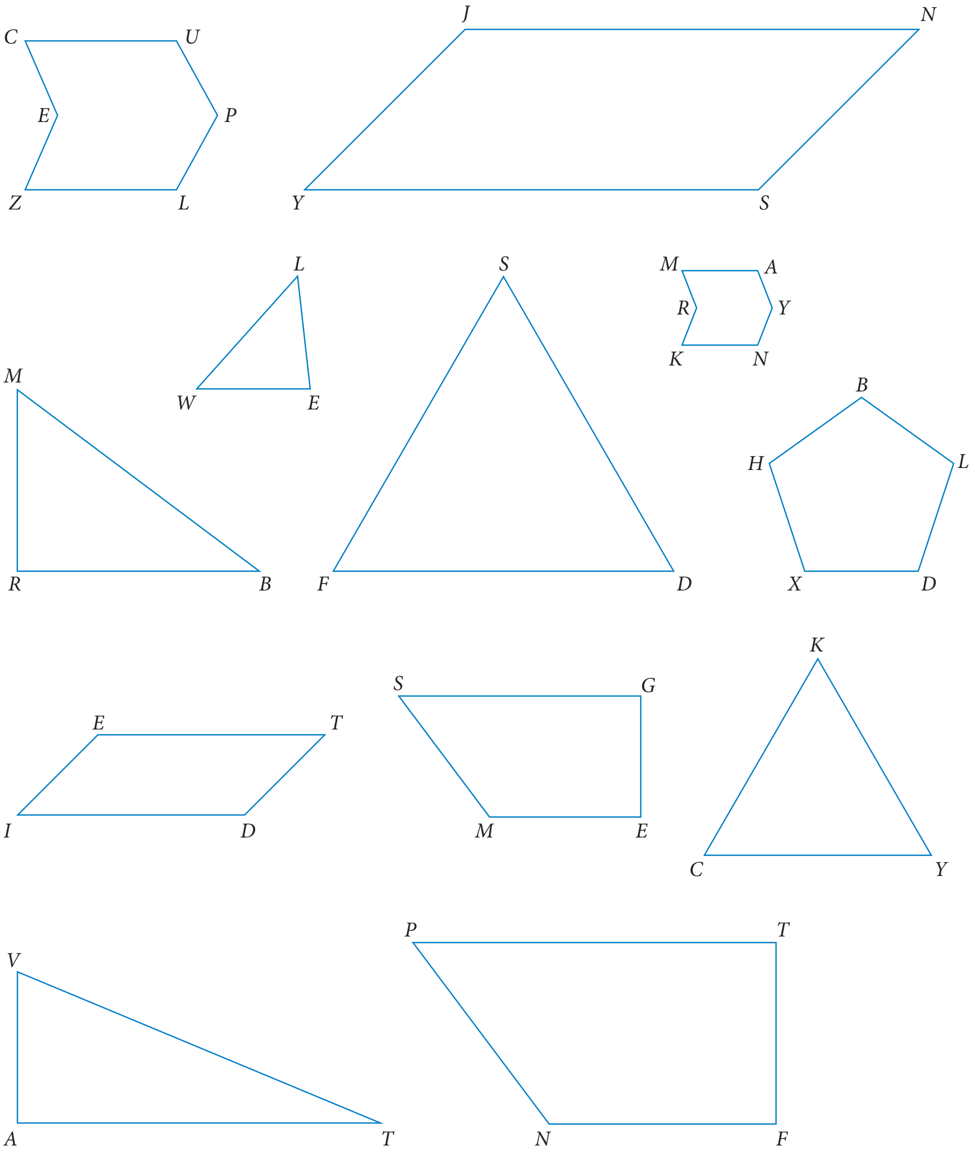
e



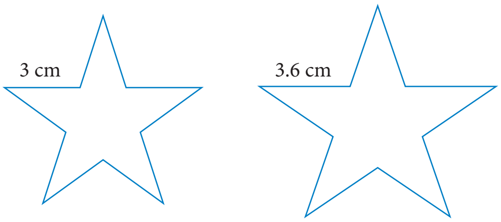
f



3 **a** Name any similar figures in matching angle order and calculate their scale factor.



4 Determine the scale factor of these similar figures.



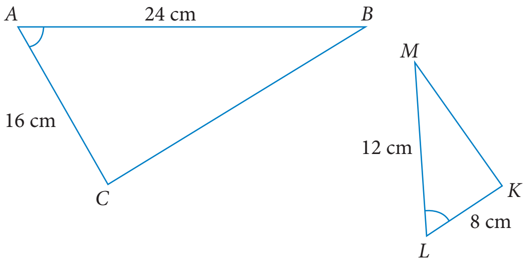
5 **Example 13** Each of the following pairs of shapes are similar. They are not drawn to scale.

i Name the similar shapes in matching angle order.

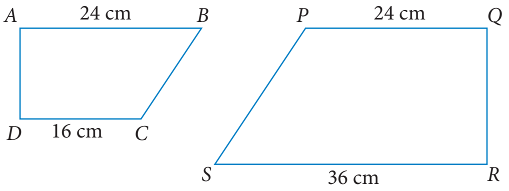
ii Name the side corresponding to AB.

iii Calculate the scale factor.

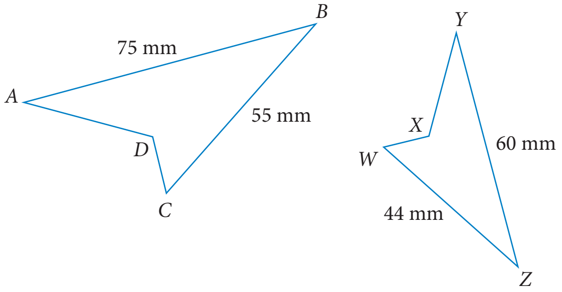
a



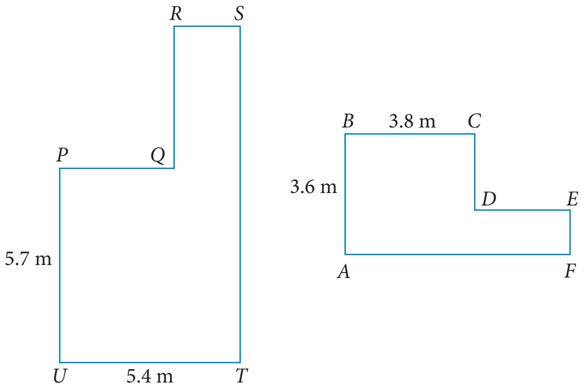
b



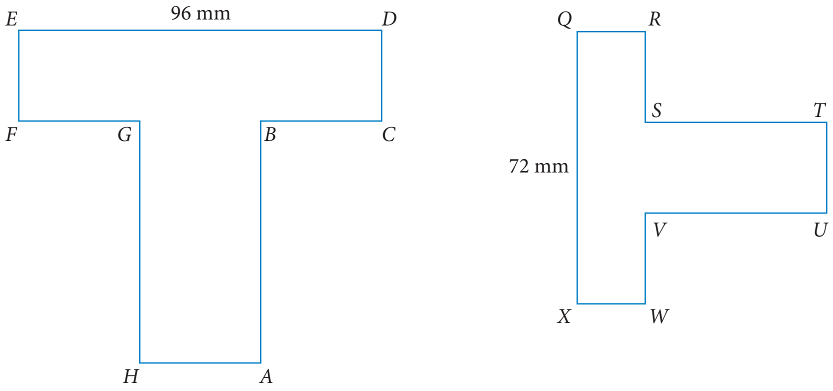
c



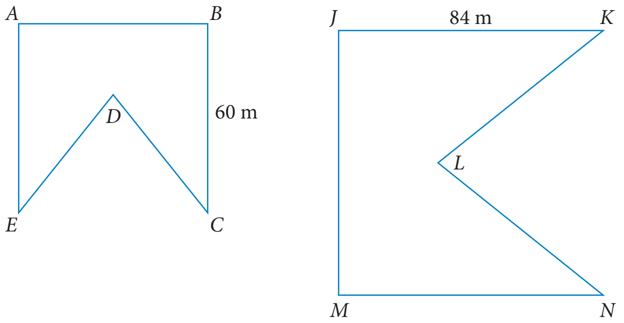
d



e

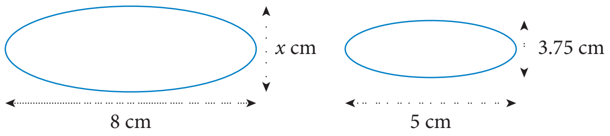


f

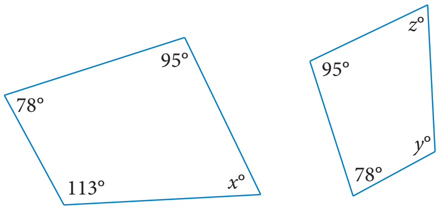


6 **Example 14** Each of the following pairs of figures are similar. Find the value of each pronumeral.

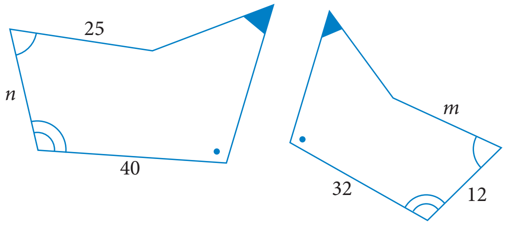
a



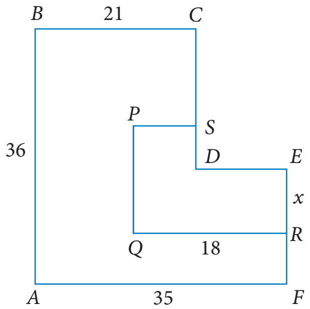
b



c



d



Reasoning and communication

7 Answer True (T) or False (F) to each of the following statements.
  - a All circles are similar.
  - b The corresponding angles of similar figures are equal.
  - c All rectangles are similar.
  - d The corresponding sides in similar figures are always equal.
  - e All regular hexagons are similar.
  - f All right-angled triangles are similar.
  - g The corresponding sides of similar figures are in the same ratio.
  - h All pentagons are similar.
  - i If two triangles have two pairs of corresponding angles equal then they are similar.
  - j Two hexagons that have all of their corresponding angles equal are similar.
  - k If two quadrilaterals have two pairs of corresponding angles equal, then they are similar.
  - l Similar figures must have at least one pair of corresponding sides equal.

8 a Explain why all parallelograms are not similar.
  b Draw two parallelograms that are not similar.

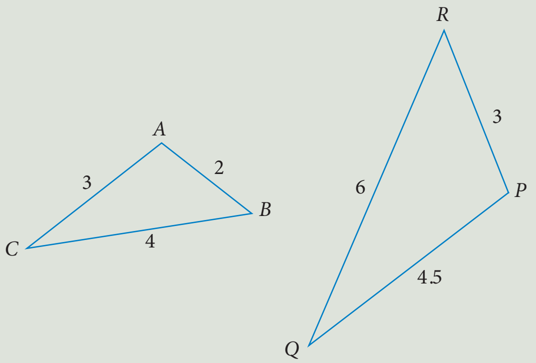
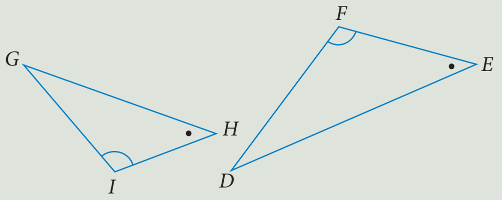
9 Are all squares similar? Explain.

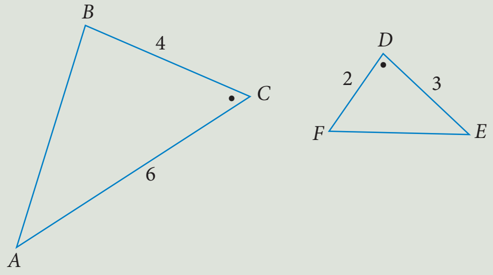
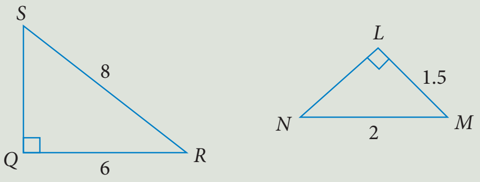
7.05 SIMILAR TRIANGLES

Similar triangles are figures that are frequently used in areas such as surveying where it is difficult to measure lengths or heights. All of the properties that apply for the similar figures studied previously also apply to similar triangles.

IMPORTANT

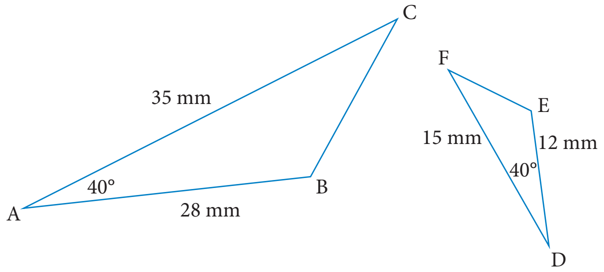
There are four tests for triangle similarity.

Similarity test	**Example**
All ratios of corresponding side lengths are equal. We abbreviate this test to **SSS**.	 $\frac{QR}{CB}=\frac{6}{4}$ $=\frac{3}{2}$ $\frac{PR}{AB}=\frac{3}{2}$ $\frac{PQ}{AC}=\frac{4.5}{3}$ $=\frac{3}{2}$ So $\triangle PRQ \mid\mid\mid \triangle ABC$ by SSS.
The angles of each of the triangles are the same. It is sufficient to show that two angles of each triangle are equal because the third angles must therefore be the same, as the angle sum is 180°. We abbreviate this test to **AAA** or we say the triangles are **equiangular**.	 $\angle H=\angle E$ $\angle I=\angle F$ $\angle G=\angle D$ So $\triangle EFD \mid\mid\mid \triangle HIG$ by AAA.

Similarity test	Example
Two pairs of corresponding sides in the triangles are in the same ratio and the angles between them are equal. We abbreviate this test to **SAS**.	 $\frac{CB}{DF} = \frac{4}{2}$ $= 2$ $\frac{CA}{DE} = \frac{6}{3}$ $= 2$ $\angle C = \angle D$ So $\triangle CAB \parallel\!| \triangle DEF$ by SAS.
The **hypotenuses** and a pair of corresponding sides are in the same ratio in two right-angled triangles. We abbreviate this test to **RHS**.	 $\angle Q = \angle L = 90°$ $\frac{SR}{NM} = \frac{8}{2}$ $= 4$ $\frac{QR}{LM} = \frac{6}{1.5}$ $= 4$ So $\triangle QRS \parallel\!| \triangle LMN$ by RHS.

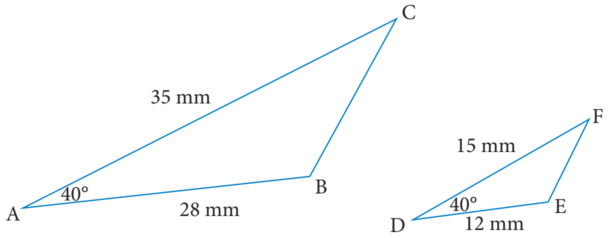
Example 15

Determine whether or not $\triangle DEF$ is similar to $\triangle ABC$.



Solution

Redraw $\triangle DEF$ with the same orientation as $\triangle ABC$.



In each triangle, the lengths of two sides are given together with the size of the angle between them so the most appropriate test to use is SAS. The side *DE* corresponds to the side *AB*.

$$\frac{DE}{AB} = \frac{12}{28}$$
$$= \frac{3}{7}$$

The side *DF* corresponds to the side *AC*.

$$\frac{DF}{AC} = \frac{15}{35}$$
$$= \frac{3}{7}$$

$\angle CAB$ corresponds to $\angle FDE$.

$$\angle CAB = \angle \text{FDE}$$
$$= 40°$$

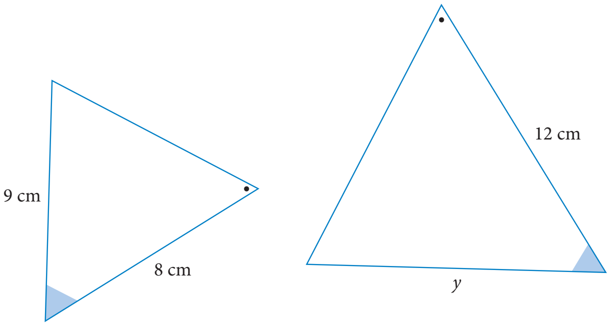
Write the answer stating the test used.

$\triangle ABC \,|||\, \triangle DEF$ by SAS.

Since we know that the sides of similar figures are in the same ratio, we can use this to find the lengths of unknown sides.

Example 16

Find the value of the pronumeral in the pair of similar triangles.



Solution

The triangles are similar, so you know that matching sides will be in the same ratio.

Identify corresponding sides.	9 cm ↔ y cm, 8 cm ↔ 12 cm.
Write an equation using the ratio of corresponding sides.	$\frac{y}{9}=\frac{12}{8}$
Solve for y.	$y=\frac{12}{8}\times 9$
Simplify.	$=13.5$

It is important to keep the sides from the same triangle in the same position in each ratio: y and 12 are both in the numerator, they are both side lengths from the same triangle.

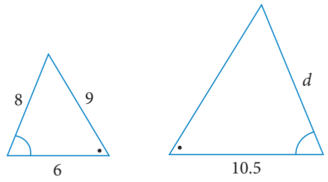
Scale factors can also be used to find side lengths in triangles.

Example 17

a Show that the pair of triangles below are similar.

b State the scale factor.

c Find the value of the pronumeral.

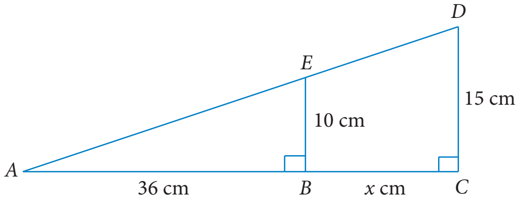


Solution

a AAA is the test to use in this case because we know that the triangles have two pairs of equal angles.	The angles labelled ∠ in each triangle are equal. The angles labelled ∡ in each triangle are equal. Therefore the remaining angles in each triangle must also be equal since the angle sum in a triangle is 180°.
State the test.	The two triangles are similar by AAA.
b Identify a pair of corresponding sides where both their measurements are given.	$6 \leftrightarrow 10.5$
Write the scale factor using matched sides. Put the second diagram side over the first diagram side. Simplify.	$\text{Scale factor} = \frac{10.5}{6}$ $= \frac{7}{4}$
c The scale factor compares corresponding sides.	$7 \leftrightarrow 4, \quad d \leftrightarrow 8$
Identify the corresponding side to the unknown. Write an equation using the ratio of corresponding sides.	$\frac{7}{4} = \frac{d}{8}$
Solve for *d*.	$d = \frac{7}{4} \times 8$ $= 14$

Example 18

Triangle *ABE* is similar to triangle *ACD*.



a Explain why the length of *AC* is $(x + 36)$ cm.

b Find the value of the pronumeral.

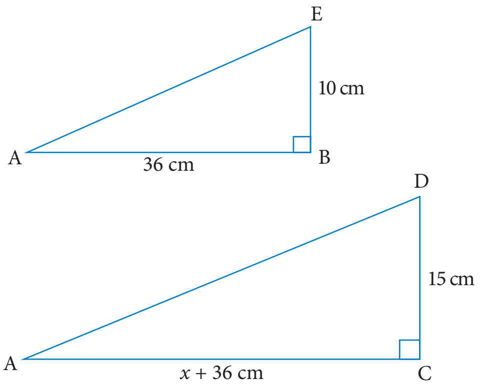
Solution

a From the diagram the side *AC* consists of two parts *AB* and *BC*.

$AB = 36$ cm and $BC = x$ cm

$$\begin{aligned} AC &= AB + BC \\ &= (36 + x) \text{ cm} \\ &= (x + 36) \text{ cm as required.} \end{aligned}$$

b Redraw the diagram as two separate triangles so that corresponding sides can be easily identified.



Determine corresponding sides.

$AB \leftrightarrow AC,\quad BE \leftrightarrow CD$

$36 \leftrightarrow x + 36,\quad 10 \leftrightarrow 15$

Write an equation using the ratios of corresponding sides.

$$\frac{x+36}{36} = \frac{15}{10}$$

Solve for *x*.

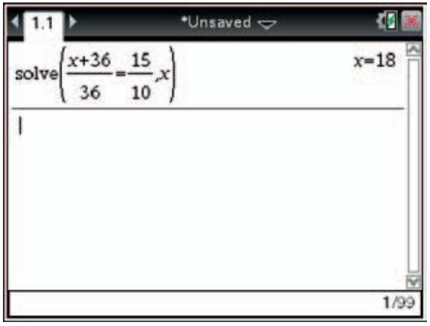
$$x + 36 = \frac{15}{10} \times 36$$

$$x + 36 = 54$$

$$x = 18$$

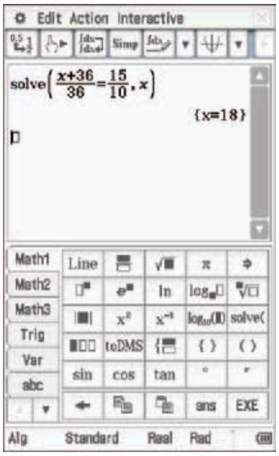
Alternatively, solve for *x* using a CAS.

TI-Nspire CAS



$\text{solve}\left(\frac{x+36}{36}=\frac{15}{10},x\right)$ $x=18$

ClassPad



$\text{solve}\left(\frac{x+36}{36}=\frac{15}{10},x\right)$

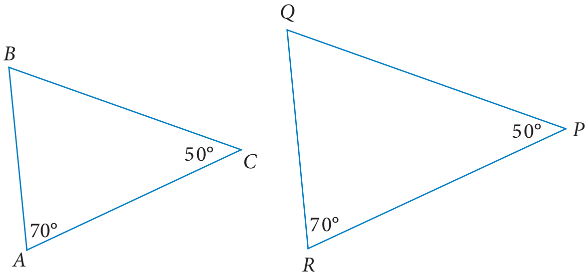
$\{x=18\}$

EXERCISE 7.05 Similar triangles

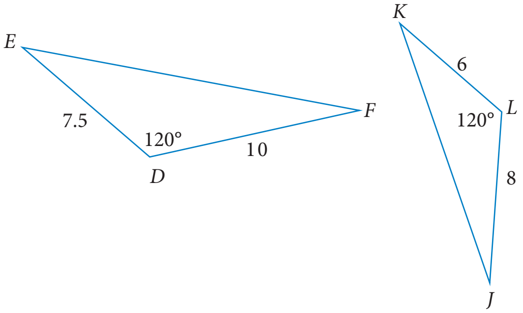
Concepts and techniques

1 Example 15 Determine which of the following pairs of triangles are similar. For each pair of similar triangles, state the test for similarity that was used. The diagrams are not drawn to scale.

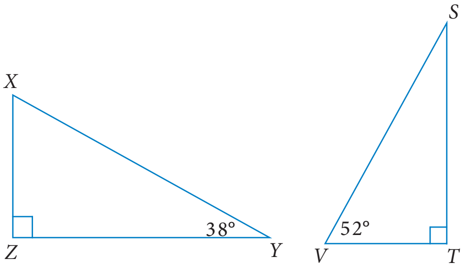
a



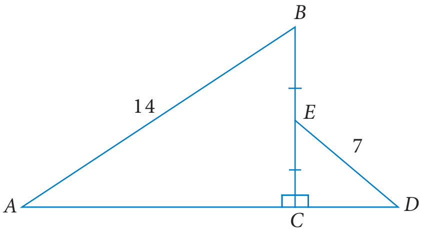
b



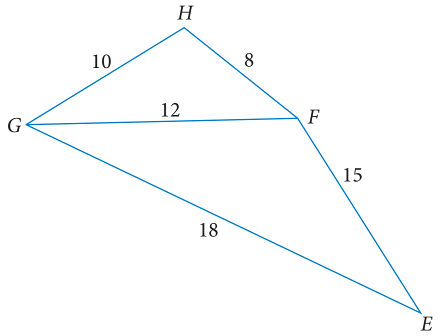
c



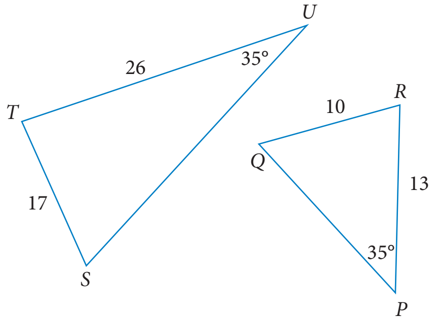
d



e



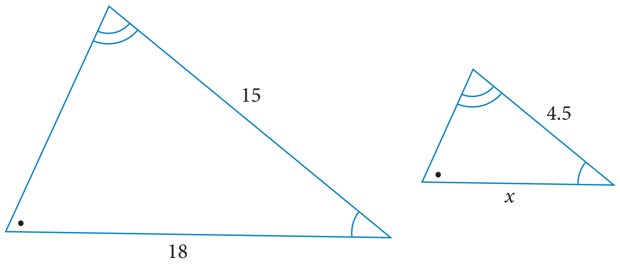
f



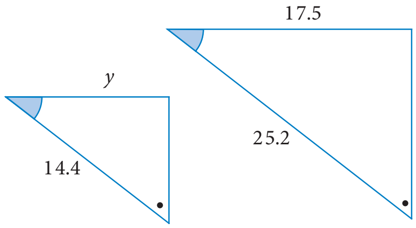
2 **Example 16** Find the scale factor used for each pair of similar triangles in question **1**, where possible.

3 **Example 17** The following pairs of triangles are similar. State the scale factor used and find the value of the pronumeral.

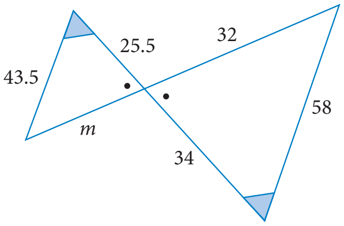
a



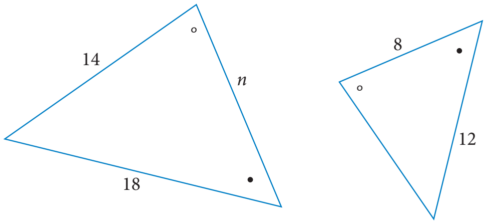
b



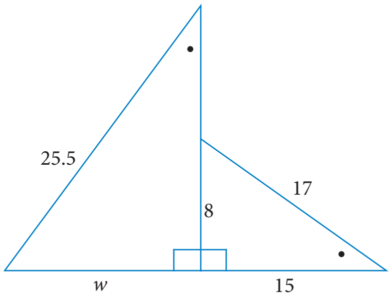
c



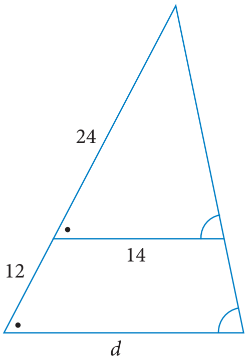
d



e

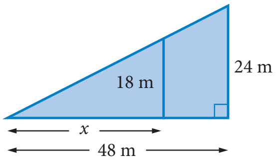


f

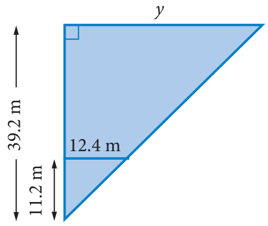


4 These diagrams show similar triangles. Use scale factors to find the unknown sides.

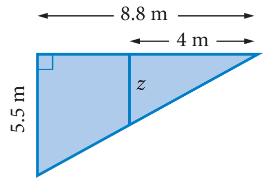
a



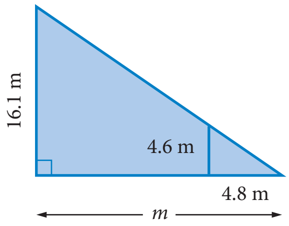
b



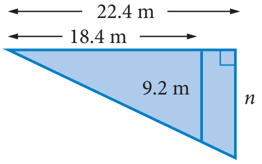
c



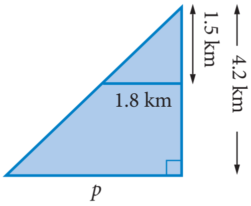
d



e



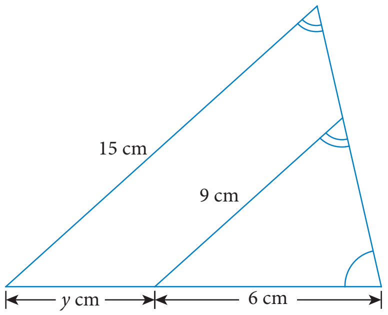
f



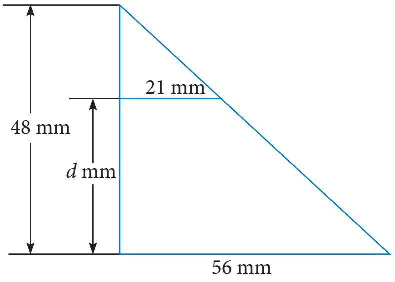
Reasoning and communication

5 Example 18 The triangles in the following diagrams are similar. Find the value of the pronumeral.

a



b



(*Hint:* Find an expression for the vertical side of the small triangle first.)

7.06 APPLICATIONS OF SIMILARITY

The similarity principle can be applied to solve real-life problems. Similar triangles are frequently used to find lengths or heights that are too difficult to measure by other means.

If the height of an object such as a building or a flagpole is too difficult to measure, we can use shadows to create similar triangles. Firstly measure the length of the shadow that the tall object casts. Then measure the length of the shadow cast by a stick at the same time of day. Since the Sun's rays make the same angle with the ground, two similar triangles can be drawn and so the unknown height can be determined.

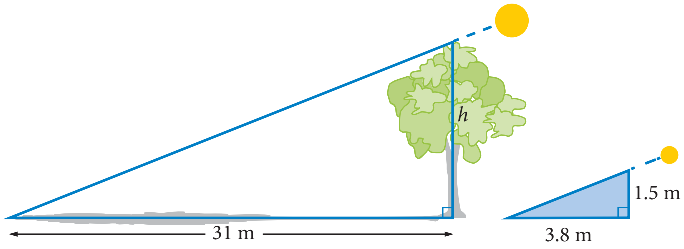
Example 19

Shutterstock.com/Nico Traut

A stick of height 1.5 m casts a shadow of length 3.8 m. At the same time, a tree casts a shadow 31 m long. What is the height of the tree?

Solution

Draw a diagram to show the information.



Determine the corresponding sides. $h \leftrightarrow 1.5$ and $31 \leftrightarrow 3.8$

Write the ratio equation using corresponding sides.

$$\frac{h}{1.5} = \frac{31}{3.8}$$

Solve for h.

$$h = \frac{31}{3.8} \times 1.5$$

$$h = 12.23684211...$$

Round the answer correct to one decimal place.

$$\approx 12.2 \text{ m}$$

Write the answer to the question. The height of the tree is 12.2 m.

INVESTIGATION Using shadow sticks

A metre ruler makes an excellent shadow stick because the calculations are simplified by having the stick exactly 1 m high. Use a metre ruler and a long tape in groups of two or three to measure the heights of buildings and trees around the school.

You will need to make sure the metre ruler is vertical when you are measuring its shadow. This can be done using a spirit level. If you don't have a spirit level, you can check that the stick is vertical by line of sight against something known to be vertical, such as the corner of a building or a telephone or light pole.

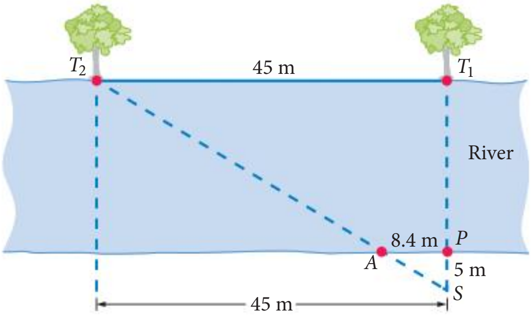
Example 20

A surveyor needs to measure the distance across a river. There are two trees on the opposite bank that are 45 m apart. She stands 5 m from the bank, directly opposite the first tree. Her assistant has to move 8.4 m along the bank to place a stick directly in her line of sight to the second tree. Find the width of the river, correct to the nearest centimetre.

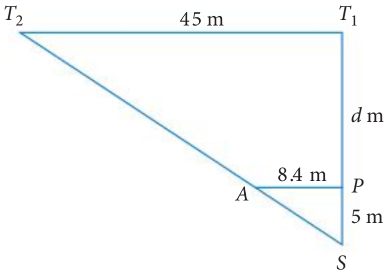
Finding sides in similar triangles

Solution

Draw a diagram to show the information.
Label points for reference and write in lengths.



Identify corresponding sides.



$T_2T_1 \leftrightarrow AP$ and $T_1S \leftrightarrow PS$

$45 \leftrightarrow 8.4$ and $d + 5 \leftrightarrow 5$

Write the ratio equation using corresponding sides.

$$\frac{45}{8.4} = \frac{d+5}{5}$$

Solve for d.

$$\frac{45}{8.4} \times 5 = d + 5$$

$$d + 5 = 26.785...$$

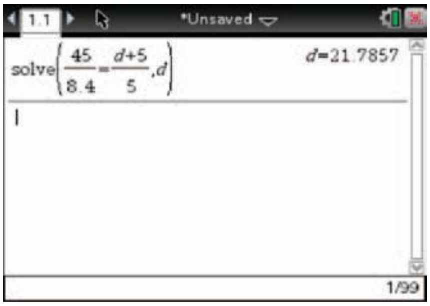
$$d = 26.785... - 5$$

$$= 21.785...$$

$$\approx 21.79 \text{ m}$$

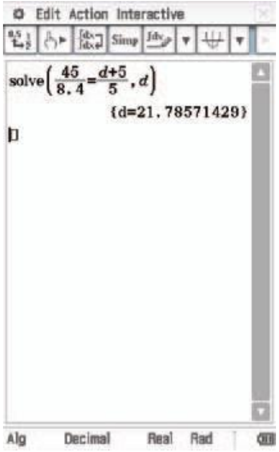
Alternatively, solve for d using a CAS.

TI-Nspire CAS



$\text{solve}\left(\frac{45}{8.4}=\frac{d+5}{5},d\right)$ $d=21.7857$

ClassPad



$\text{solve}\left(\frac{45}{8.4}=\frac{d+5}{5},d\right)$

{d=21.78571429}

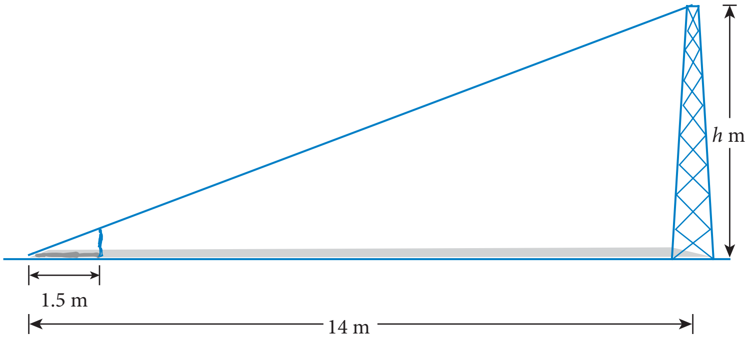
Correct to the nearest centimetre means rounding off to two decimal places when working in metres.

Answer the question. The river is approximately 21.79 m wide.

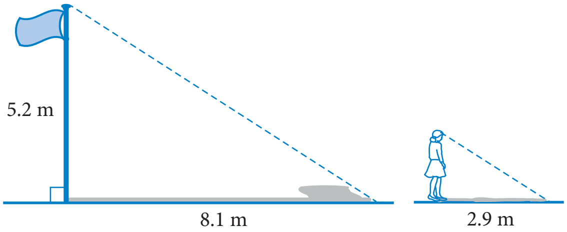
EXERCISE 7.06 Applications of similarity

Concepts and techniques

1 Example 19 Robert is 2 m tall and casts a shadow 1.5 m long. At the same time, a tower casts a shadow 14 m long. How high is the tower? Answer correct to one decimal place.



2 A 5.2 m flagpole casts a shadow of 8.1 m, while Arlene casts a shadow of 2.9 m. How tall is Arlene? Answer correct to one decimal place.



3 A school building throws a shadow of 26 m while a 3 m high tree throws a shadow of 5 m. What is the height of the building?

4 At sunset, a mountain of height 1500 m casts a shadow of length 6 km, while a television tower throws a shadow of 15 m.

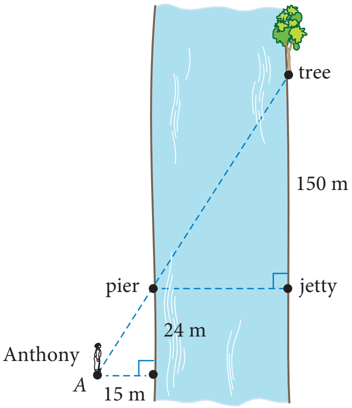
a Why are shadows longer at sunset?

b How tall is the tower?

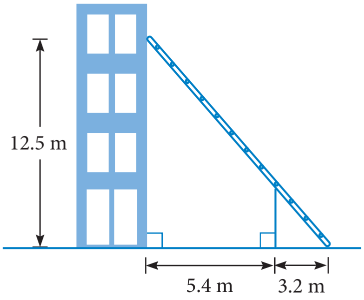
5 A 1.5 m high fence has a shadow of length 1.8 m. At the same time, how long is the shadow of a building that is 4.8 m high?

6 Jaani holds a 30 cm ruler and measures its shadow to be 24 cm. At the same time, Costa measures the shadow of a lamp post to be 10.5 m long. How tall is the lamp post?

7 Example 20 Anthony stands at point *A*, shown on the diagram, so that he is in line with the pier and the tree on the other side of the canal. He then takes measurements of 15 m and 24 m, as shown. The distance of the tree from the jetty is known to be 150 m. Find the width of the canal.



8 A fireman's ladder reaches 12.5 m up a building. The ladder is propped up by a support, placed 5.4 m from the building and 3.2 m from the base of the ladder. What is the length of the support, correct to one decimal place?

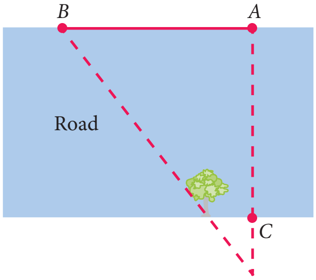


Reasoning and communication

9 A student used a metre ruler as a shadow stick to measure the heights of some school buildings. At 2 p.m. the length of the shadow cast by the ruler was 60 cm and the length of the shadow of A-block was 4.86 m. The student then moved to the oval and measured the length of the shadow of the gym, which was 3.5 m long. After that the student moved to C-block and measured its shadow as 7.2 m. It was then 2:45 p.m. and the student made a last check of the stick's shadow length before going inside. It was now 90 cm long. The students were to calculate the heights of the buildings for homework.

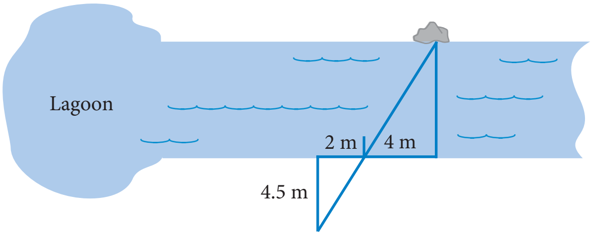
a What did this student do incorrectly?
b What was the height of A-block?
c What was the height of C-block?
d What do you estimate the height of the gym to be?

10 The distance across a road cutting is to be calculated using triangulation. Sightings are taken of two points *A* and *B* on the opposite side of the road that are known to be 120 m apart. The sides of the road cutting are parallel, and point *C* on the surveyor's side is directly opposite point *A*. From a point 8 m away from the road edge at *C*, the surveyor notices that a small tree on the edge of the road just obscures point *B*. The tree is found to be 6.7 m from *C* on the surveyor's side of the road. What is the width of the road cutting?

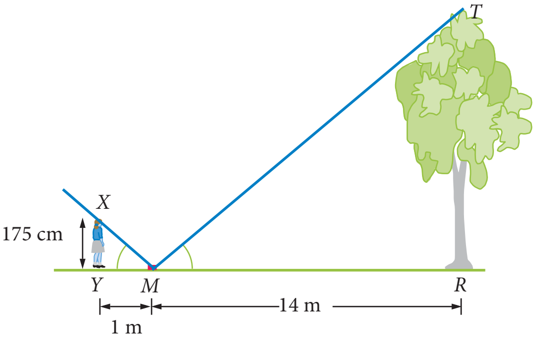



Alamy/Mireille Vautier

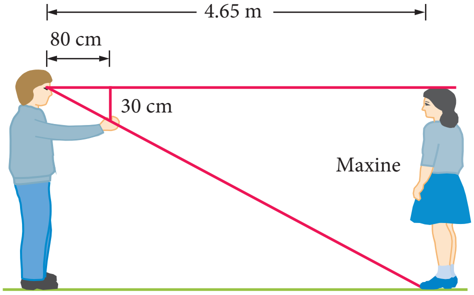
11 Water is flowing out the long, narrow entrance to a lagoon at low tide. There is a large rock on the opposite bank of the entrance. A fisherman is directly opposite the rock. He walks 4 m along the bank of the entrance and puts a stick in the sand. After walking a further 2 m along the bank, he then has to move 4.5 m back from the bank in order to line up the stick and rock. How wide is the lagoon entrance?



12 You can estimate the height of a tree (*TR*), as shown in the diagram, by placing a mirror (*M*) on level ground and moving backwards in line with the tree to a point *Y* where you can see the image of the top of the tree in the mirror. Sally is standing at *Y* and her eyes are 175 cm above the ground. When the mirror is placed 1 m from her and 14 m away from the tree, she can see the top of the tree. How high is the tree?



13 If you hold a ruler in front of yourself, you can line it up so that the head and feet of a friend standing some distance from you coincide with the top and bottom of the ruler, as shown in the diagram. Steve is holding a 30 cm ruler at a distance of 80 cm from his eyes as shown. He finds that when Maxine is standing 4.65 m away from him, the top of her head lines up with the top of the ruler and her feet line up with the bottom. How tall is Maxine?

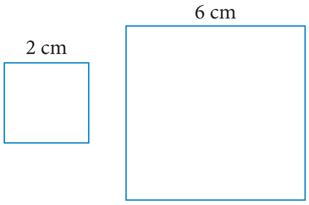


7.07 AREAS OF SIMILAR FIGURES

We know that if two figures are similar, then the lengths of their corresponding sides are in the same ratio. The areas of similar figures are also in proportion.

Consider two squares of side lengths of 2 cm and 6 cm respectively.

Square 2 is an enlargement of Square 1.



Area = 4 cm^2
Square 1

Area = 36 cm^2
Square 2

The length scale factor gives the factor by which side lengths in Square 1 are multiplied to obtain the side lengths in Square 2.

$$\begin{aligned}\text{Length scale factor} &= \frac{\text{side length of Square 2}}{\text{side length of Square 1}}\\ &= \frac{6}{2}\\ &= 3\end{aligned}$$

So the side lengths in Square 2 are 3 times those in Square 1.

The area scale factor gives the factor by which the area of Square 1 is multiplied to obtain the area of Square 2.

$$\begin{aligned}\text{Area scale factor} &= \frac{\text{area of Square 2}}{\text{area of Square 1}}\\ &= \frac{36}{4}\\ &= 9\end{aligned}$$

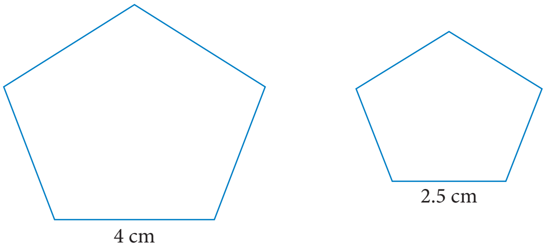
So the area of Square 2 is 9 (or 3^2) times the area of Square 1.

IMPORTANT

In general, if the length scale factor is k, then the area scale factor is k^2.

Example 21

Two regular pentagons have sides of length 4 cm and 2.5 cm respectively.



Find the

a length scale factor

b area scale factor.

Solution

a Write down the ratio of the two corresponding sides. Write the side length from the second figure on top.

$$\text{Length scale factor} = \frac{2.5}{4}$$

Simplify.

$$= \frac{5}{8}$$

b Area scale factor = (length scale factor)^2

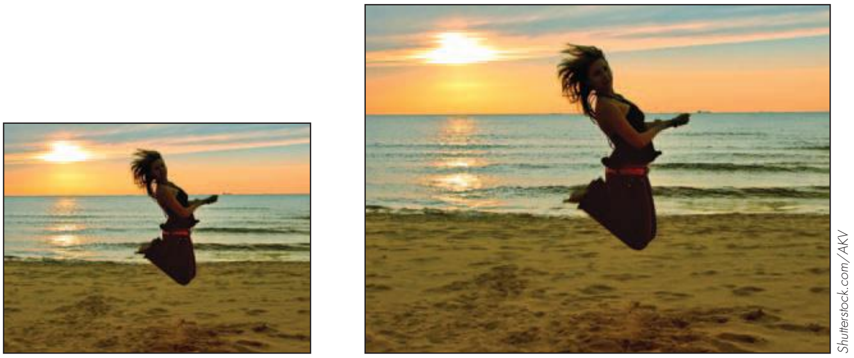
$$\text{Area scale factor} = \left(\frac{5}{8}\right)^2$$

Simplify.

$$= \frac{25}{64}$$

Example 22

A photograph is 8 cm long. Its enlargement is 12 cm long. If the area of the small photo is 48 cm^2, what is the area of the enlargement?



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Solution

Write down the ratio of the two corresponding sides.

$$\text{Length scale factor} = \frac{12}{8}$$

Simplify.

$$= \frac{3}{2}$$

The area scale factor is the square of the length scale factor.

$$\text{Area scale factor} = \left(\frac{3}{2}\right)^2$$

Simplify.

$$= \frac{9}{4}$$

The area scale factor compares areas.
Let the area of the enlargement be x.

$$9 \leftrightarrow 4, \qquad x \leftrightarrow 48$$

Write an equation using the ratio of corresponding areas.

$$\frac{9}{4} = \frac{x}{48}$$

Solve for x.

$$x = \frac{9}{4} \times 48$$
$$= 108$$

Answer the question.

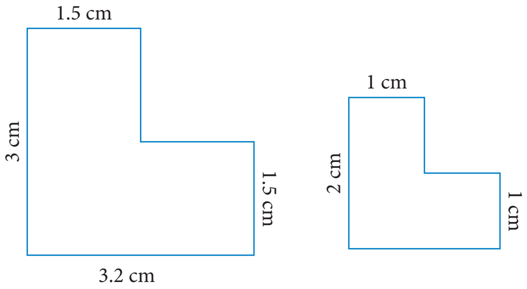
The area of the enlargement is 108 cm^2.

EXERCISE 7.07 Areas of similar figures

Concepts and techniques

1 **Example 21** The corresponding sides of two similar figures are in the ratio 3 : 2.

- a What is the length scale factor?
- b What is the area scale factor?

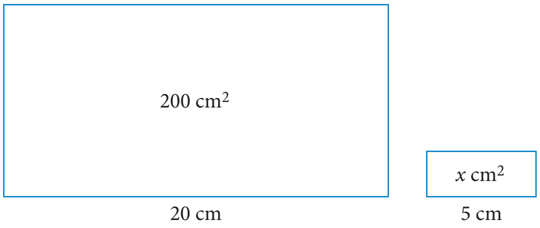


2 Find the area scale factor for each of the following linear scale factors.

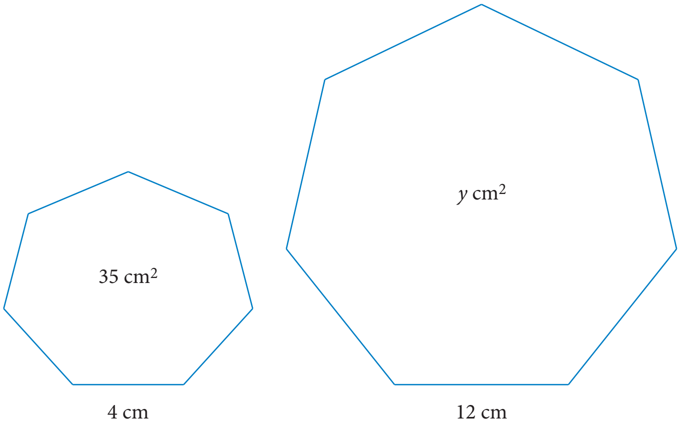
a 6 b $\frac{5}{4}$ c $\frac{2}{7}$

3 Find the value of the pronumeral in each of the following pairs of similar figures.

a



b



4 Example 22 The corresponding sides of two similar triangles are 9 cm and 15 cm. If the area of the smaller triangle is 63 cm^2, find the area of the larger triangle.

5 A square of side length 5 cm is enlarged using a linear scale factor of $k = 3$. What is the area of the image?

Reasoning and communication

6 **a** The diameter of a circle is trebled. What happens to its area?

b The height of a triangle is double the height of a similar triangle. What do you know about their areas?

c Two octagons are regular. If the area of the larger octagon is sixteen times bigger than the area of the smaller octagon, what is the relationship between the lengths of their sides?

d The lengths of the sides of a quadrilateral are halved. What relationship is there between the area of the original quadrilateral and the area of the new quadrilateral?

7 The ratio of matching sides of similar parallelograms is 4 : 7. The area of the smaller parallelogram is 80 cm^2.

a Calculate the area of the larger parallelogram.

b The length of the base of the longer parallelogram is 35 cm. What is the length of the base of the smaller parallelogram?

8 A triangle has side lengths of 5 cm, 12 cm, and 13 cm. A similar triangle to it has nine times the area of the first triangle.

a What is the ratio of the sides of the two triangles?

b What are the lengths of the side of the second triangle?

c What is the relationship between the perimeters of the two triangles?

9 The ceiling of a 30 m by 15 m hall is to be painted. How much paint is needed, if it took 1.5 L of paint to paint a similar ceiling that was 6 m by 3 m?

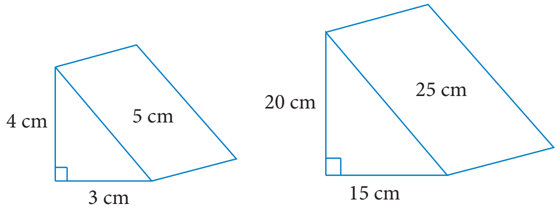
10 A rectangular billboard has length 12 metres. On a computer screen the corresponding length of the billboard is 21 centimetres. If the area of the actual billboard is 96m^2, what is the area of the image of the billboard on the computer screen?

7.08 SURFACE AREAS AND VOLUMES OF SIMILAR SOLIDS

An area scale factor can be used to find the surface area of a three dimensional figure if the area of another similar three dimensional figure is known.

Example 23

Two similar right triangular prisms are shown below.



a What is the length scale factor?
b What is the area scale factor?
c The surface area of the larger prism is 2400 cm^2, what is the surface area of the smaller prism?

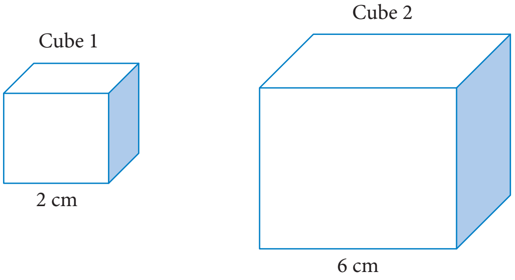
Solution

a Write down the ratio of any two corresponding sides. Write the side length from the second figure on top.	Length scale factor $= \dfrac{25}{5}$
Simplify.	$= 5$
b Area scale factor = (length scale factor)^2	Area scale factor $= 5^2$ $= 25$
c The area scale factor compares areas. Let the surface area of the smaller prism be x.	$25 \leftrightarrow 1, \qquad 2400 \leftrightarrow x$
Write an equation using the ratio of corresponding areas.	$\dfrac{1}{25} = \dfrac{x}{2400}$
Solve for x.	$x = \dfrac{1}{25} \times 2400$ $= 96$
Write the answer with units.	The surface area of the smaller prism is 96 cm^2.

The volumes of two similar figures are in proportion. The following shows how the volume scale factor can be determined from a length scale factor.

Consider two cubes of side length 2 cm and 6 cm respectively.

Cube 2 is an enlargement of Cube 1.



$$\text{Length scale factor} = \frac{\text{side length of Cube 2}}{\text{side length of Cube 1}}$$
$$= \frac{6}{2}$$
$$= 3$$

So the side lengths in Cube 2 are 3 times those in Cube 1.

The volume scale factor gives the factor by which the volume of Cube 1 is multiplied to obtain the volume of Cube 2.

$$\text{Volume scale factor} = \frac{\text{volume of Cube 2}}{\text{volume of Cube1}}$$
$$= \frac{216}{8}$$
$$= 27$$

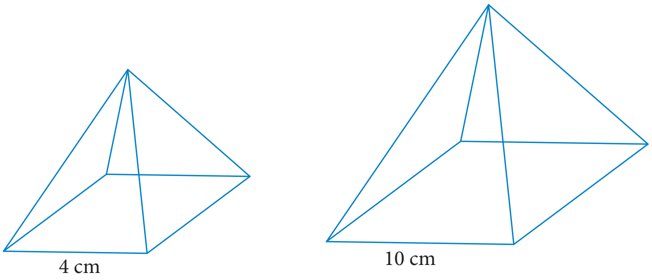
So the volume of Cube 2 is 27 (or 3^3) times the volume of Cube 1.

IMPORTANT

In general, if the length scale factor is k, then the volume scale factor is k^3.

Example 24

A square based pyramid with base of length 4 cm is enlarged to produce a similar pyramid with base length 10 cm.



a What is the length scale factor?
b What is the volume scale factor?
c If the volume of the smaller pyramid is 32 cm^3, what is the volume of the larger pyramid?

Solution

a Write down the ratio of corresponding sides.

$$\text{Length scale factor} = \frac{10}{4}$$
$$= \frac{5}{2}$$

b Volume scale factor = (length scale factor)^3

$$\text{Volume scale factor} = \left(\frac{5}{2}\right)^3$$
$$= \frac{125}{8}$$

c The volume scale factor compares volumes.
Let the volume of the larger pyramid be x.

$125 \leftrightarrow 8, \qquad x \leftrightarrow 32$

Write an equation using the ratio of corresponding volumes.

$$\frac{125}{8} = \frac{x}{32}$$

Solve for x.

$$x = \frac{125}{8} \times 32$$
$$= 500$$

Write the answer with units.

The volume of the larger pyramid is 500 cm^3.

Example 25

A chocolate bar is in the shape of rectangular prism. The manufacturer wants to increase profits by reducing the size of the bar.

a If the dimensions of the bar are reduced by one quarter, what is the ratio of the volume of the new bar compared to the volume of the original?

b If the original bar was 320 g, what is the weight of the new bar?

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Solution

a If the dimensions are reduced by $\frac{1}{4}$, then the dimensions of the new bar are $\frac{3}{4}$ of the original.

Dimensions of new bar $= \frac{3}{4} \times$ original dimensions

Write down the length scale factor.

Length scale factor $= \frac{3}{4}$

Find the volume scale factor.
Volume scale factor = (length scale factor)^3.

$$\text{Volume scale factor} = \left(\frac{3}{4}\right)^3$$
$$= \frac{27}{64}$$

Interpret the volume scale factor.

$$\text{Volume scale factor} = \frac{\text{Volume of new chocolate bar}}{\text{Volume of original chocolate bar}}$$
$$= \frac{27}{64}$$

State your answer.

Volume of new bar : volume of original bar = 27 : 64

b The volume scale factor compares volumes.

Let the volume of the new bar be x. $27 \leftrightarrow 64, \quad x \leftrightarrow 320$

Write an equation using the ratio of corresponding volumes.

$$\frac{27}{64} = \frac{x}{320}$$

Solve for x.

$$x = \frac{27}{64} \times 320$$
$$= 135$$

Write the answer with units. The weight of the new bar is 135 g.

EXERCISE 7.08 Surface areas and volumes of similar solids

Concepts and techniques

1 Example 23 The bases of two similar rectangular prisms have side lengths of 28 cm by 21 cm and 8 cm by 6 cm respectively.

- a What is the length scale factor?
- b What is the surface area scale factor?
- c If the surface area of the larger prism is 2450 cm^2, what is the surface area of the smaller prism?

2 Two pentagonal prisms are similar. The areas of their cross-sectional faces are given.

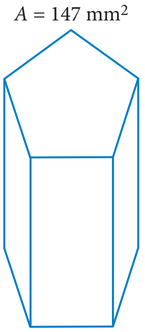


Figure I

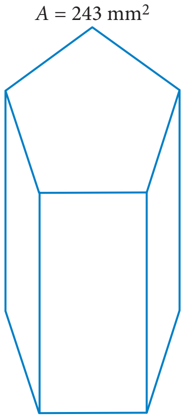


Figure II

- a What is the scale factor of the surface areas?
- b What is the length scale factor?
- c If Figure I is 14 mm high, what is the height of Figure II?

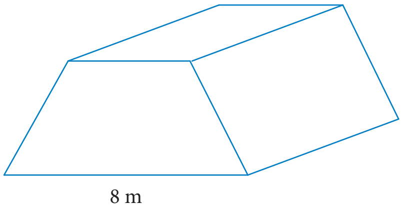
3 Example 24 Two similar cones have bases with radius 20 cm and 24 cm respectively.

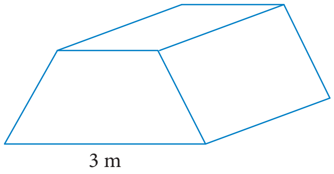
a What is the scale factor of the heights of the cones?

b What is the volume scale factor of the cones?

c If the volume of the smaller cone is 937.5 cm^3, what is the volume of the large cone?

4 Given that these two trapezoidal prisms are similar, what is the volume scale factor?





5 Example 25 If the dimensions of a cement slab are reduced to one third of the original, what happens to the surface area? What happens to the volume?

6 If the dimensions of a cement slab are trebled, what happens to the surface area? What happens to the volume?

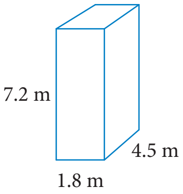
7 a Show that these two rectangular prisms are similar.

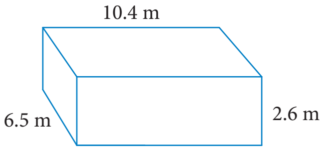
b What is the scale factor of the lengths of their corresponding sides?

c What is the scale factor of their surface areas?

d What is the scale factor of their volumes?

e If the measurements of the smaller prism are doubled, what would the scale factor of the volumes be?





8 The radii of two spherical balloons are 24 cm and 30 cm respectively.

a What is the ratio of their radii?

b What is the ratio of their surface areas?

c What is the ratio of their volumes?

d Half of the air is released from the smaller balloon. What now is the ratio of their volumes?

9 The surface areas of two similar triangular prisms are in the ratio 81 : 64.

a What is the scale factor of their sides?

b What is the scale factor of their volumes?

10 The volumes of two similar crates are in the ratio 343 : 729.

a What is the ratio of their sides?

b What is the ratio of their surface areas?

Reasoning and communication

11 Nicole wanted to build a 3.4 m long yacht. She modelled it on the Sydney-Hobart winner, Wild Oats XI, which is 5.1 m in length. Wild Oat's mainsail is 382 m^2. If Nicole wanted to rig the yacht the same way, what area would Nicole have to make her mainsail? Give your answer correct to the nearest m^2.

12 Ken visited the Eiffel Tower when he was in Paris. He bought a replica of the tower as a souvenir. The model tower was made in the ratio 1 : 1800.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Shutterstock.com/Valerie Potapova

The real tower is 324 m high.

a What was the height of the souvenir?

b What is the ratio of the surface area of the replica to the real tower?

13 A length of a model car is in the ratio of 1 : 20 with the real car.

a If it takes 15 mL of paint to cover the model, how much paint is needed to paint the real car? Give your answer in litres.

b The fuel tank of the real car holds 60 L. What would be the capacity of the fuel tank on the model?

14 A beehive consists of hexagonal cells with sides of length 2.4 mm and a depth of 2.4 mm.

Joy wanted to build a storage unit modelled on the beehive. The side length of each hexagon in her unit is 15 cm.

a What is the ratio of the lengths of the sides, in simplest form?

b What is the ratio of the surface area of the honeycomb cell to the surface area of the materials used to construct each storage unit cell?

c What is the ratio of the volumes of 10 real honeycomb cells compared to 10 storage unit cells?


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15 Jodie was making a trial birthday cake for her son. After cooking up one batch of cake mix she found that the cake was only half as long and half as wide and half as high as she needed. To make the correct sized cake, does Jodie need to double the mixture? Justify your answer.

16 Natalie collected teddy bears. Two of her collection were identical in shape, but one was four times as tall as the other one.

If 350 cm^2 of 'fake fur' was used to make the smaller bear, how much material was needed to make the larger bear?

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17 Two similar octagonal gazebos are to be tiled. The scale factor of the tiled areas is $\frac{81}{100}$.

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a If 405 tiles are needed to cover the smaller gazebo, how many tiles are needed to cover the larger one?

b Ornamental fencing is to be placed around each gazebo. If it costs $264 to fence the larger gazebo, how much would it cost to fence the smaller gazebo?

7

CHAPTER SUMMARY
SCALES AND SIMILARITY

- A **scale drawing** of an object is the same shape as the object but a different size.
- The **scale** of a drawing is a comparison of the length used on a drawing to the length it represents in real life.
- A scale is written as a **ratio**. The first part of the ratio given for a scale is always the length on the drawing. Scale = plan length : real length
- A scale is usually written without units in simplest form.
- A scale can also be used to find unknown lengths on a scale drawing or on a real object.
- A scale drawing is usually a **reduction** of a real object, but can be an **enlargement** of a very small object.
- A **plan** is a detailed drawing or diagram of an object. E.g. house plan.
- An **elevation** of a building is a view of it from the front, side or back.
- Measurements on plans are always shown in millimetres.
- A **scale factor** is the ratio of any two matching lengths in two similar figures.
- **Similar figures** have the same shape but not necessarily the same size.
- **Matching sides** or angles (also called **corresponding sides** or angles) refers to sides or angles in similar shapes. The sides or angles are in the same relative position in each figure.
- The symbol for **similarity** is ||| or ~.
- Similar figures are named in matching angle order.
- Similar figures have their matching angles equal and their matching sides are in the same ratio.
- There are four tests for similar triangles:
  - SSS: The three matching sides of the triangles are in the same ratio.
  - AAA or equiangular: The angles of each of the triangles are the same. It is good enough to prove that two angles of each triangle are equal because the third must be the same, since the angle sum is 180°.
  - SAS: Two matching sides of the triangles are in the same ratio and the included angles are equal.
  - RHS: The **hypotenuse** and a matching side are in the same ratio in two right-angled triangles.
- Ratio can be used to find unknown side lengths in similar figures.
- In general, if the length scale factor is k, then the area scale factor is k^2.
- In general, if the length scale factor is k, then the volume scale factor is k^3.

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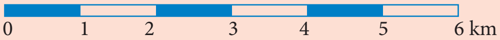
7

CHAPTER REVIEW

SCALES AND SIMILARITY

Multiple choice

1 **Example 2** Write this scale in ratio form.



A 1 : 6 B 1 : 100 000 C 1 : 600 000

D 1 : 1 000 000 E 1 : 6 000 000

2 **Example 3** The width of a terrace house is 7.4 m. On a drawing with a scale of 1 : 200, which of the following would be the width of the house?

A 14.8 mm B 1.48 cm C 370 mm D 3.7 cm E 14.8 cm

3 **Example 5** By measurement and calculation, find the real length of this ladybird.

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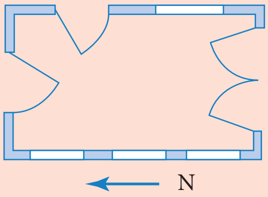
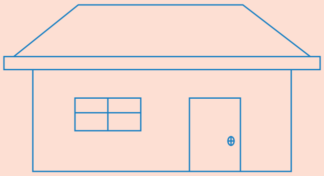
Scale = 10 : 1

A 3.9 mm B 10 mm C 3.9 cm D 10 cm E 39 cm

4 **Example 7** A floor plan and one elevation of a warehouse are shown here.



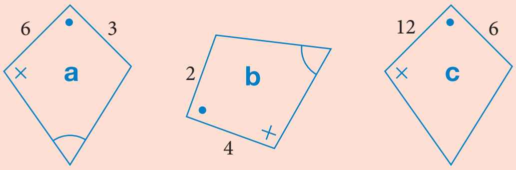


Which elevation is shown?

A Northern elevation B Eastern elevation C Southern elevation

D Western elevation E Left side elevation

5 **Example 13** If quadrilateral **a** below is the original, which of the following statements is true?



A **a** and **c** are similar, with scale factor $\frac{1}{2}$.

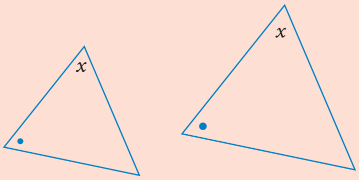
B **a** and **c** are similar, with scale factor 2.

C **a** and **b** are similar, with scale factor $\frac{2}{3}$.

D **a** and **b** are similar, with scale factor $\frac{3}{4}$.

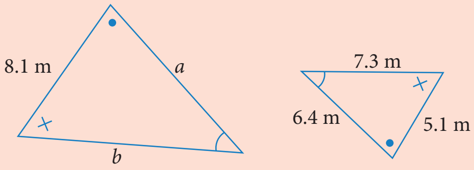
E **a** and **b** are similar, with scale factor $\frac{1}{2}$.

6 **Example 15** For the triangles shown, which one of the following statements is correct?



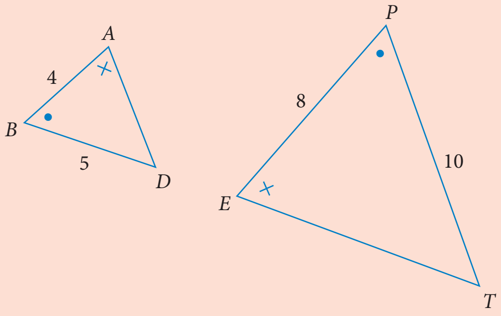
A They are not similar.
B They are similar by SSS.
C They are similar by SAS.
D They are similar by RHS.
E They are similar by AAA.

7 **Example 16** These two triangles are similar. The values of the pronumerals (correct to one decimal place) are:



A $a = 9.2, b = 6.5$
B $a = 4.0, b = 4.6$
C $a = 10.2, b = 11.6$
D $a = 4.6, b = 4.0$
E $a = 11.6, b = 10.2$

8 **Example 17** If $\triangle BAD$ and $\triangle PET$ are similar, what scale factor has been used to change $\triangle BAD$ into $\triangle PET$?

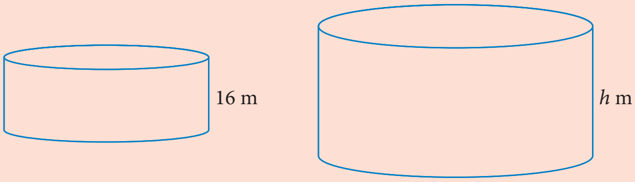


A $\frac{1}{2}$
B $\frac{5}{4}$
C $1\frac{1}{2}$
D 2
E 3

9 Example 19 A 1.5 m high fence has a shadow of length 1.8 m. At the same time, how long is the shadow of a building that is 4.8 m high?

A 2.7 m B 4 m C 5.76 m D 7.2 m E 8.64 m

10 Example 24 These two cylinders are similar.



The scale factor of their volumes is $\frac{64}{729}$. If the height of the smaller cylinder is 16 m, what is the height of the larger cylinder?

A 36 m B 54 m C 182.25 m D 9 m E 18 m

Short answer

11 Example 1 Copy and complete the following.

a 1 : 10 000 = 1 mm : ______ m

b 1 : 500 = 1 cm : ______ m

c 1 : 6 000 000 = 1 mm : ______ km

d 25 : 1 = ______ cm : 1 mm

12 Example 2 Write each of the scales given below in simplest form.

a 1 cm : 10 km

b 1 mm : 3.5 m

c 40 cm : 2 m

d 125 mm : 5 km

13 Example 5 By measurement and calculation find: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Scale = 1 : 10

a the real length of the skateboard

b the real width of the skateboard

c the distance apart the central bolts are, that hold the wheels on. Give your answer to the nearest 5 cm.

14 Example 7 This is the plan for a three-bedroom house. The dimensions are in millimetres.

a What are the internal dimensions of bedroom 1?

b A wardrobe, 600 mm wide, is to be built along the eastern wall of bedroom 1.

i What is the maximum length that the wardrobe can be?

ii What floor area will be left in the bedroom to carpet?

c What is the area of the deck (to the nearest square metre)?

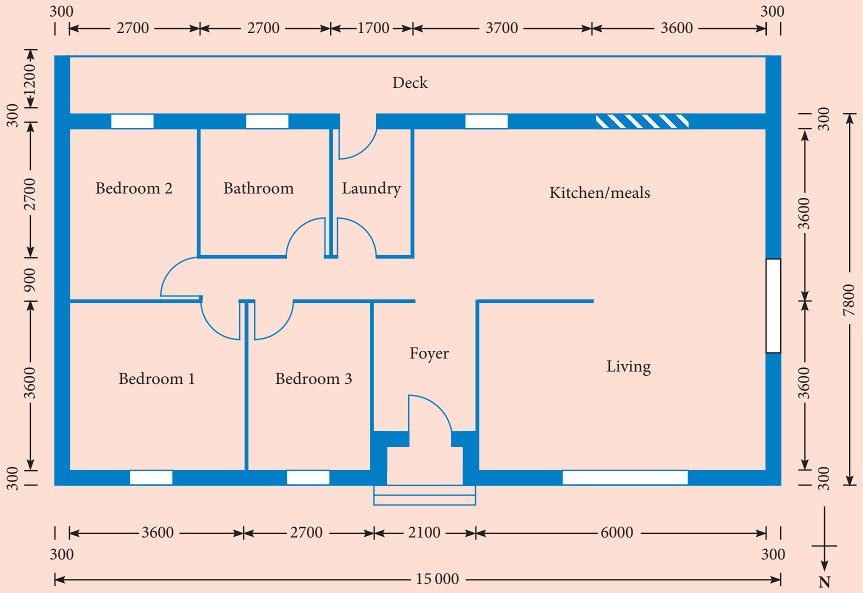
d What is the overall area of the house (to the nearest square metre)?

e How much will it cost to build this house (including the deck) if the current building rate is \$1540 per square metre?

f The bathroom floor is to be tiled with 20 cm square tiles. Find:

i the number of tiles (rounded to the nearest whole number) that will fit along the length of the bathroom floor.

ii the number of tiles required.



15 Example 8 Refer to the house plan in question **14** to answer the following questions.

a How many hinged doors are in the house?

b How many windows are in the house?

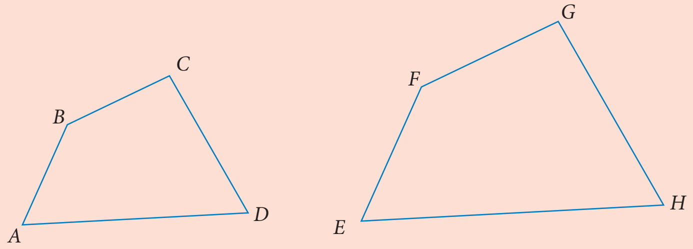
c In which room is there a sliding door?

16 Example 9 What scale should be used for a drawing (with margins) on A4 paper of a reserve 3.4 km by 2.1 km? (A4 paper is 297 mm × 210 mm)

17 Example 10 The reserve in question **16** is drawn on a larger sheet of paper using a scale of 1 : 10 000. What are its dimensions in the drawing?

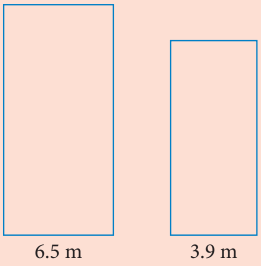
18 **Example 11** A house 18 m wide by 24 m deep is to be shown on a piece of A4 paper. The house is set back 4 m from the front boundary and 2 m from the left boundary of the block, which is 23 m wide by 35 m deep. Make a scale drawing of the house and block.

19 **Example 12** For the following two shapes:

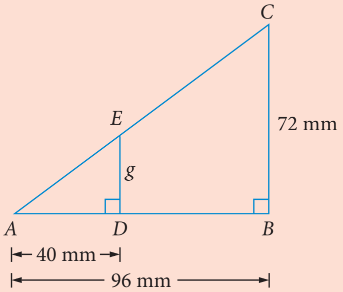


a Prove they are similar.
b Name the similar shapes in matching angle order.
c State the scale factor.

20 **Example 14** Find the scale factor used to transform the figure on the left into the similar figure on the right.

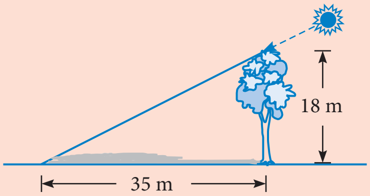


21 **Example 18** In the diagram, $\triangle AED \,|||\, \triangle ACB$.

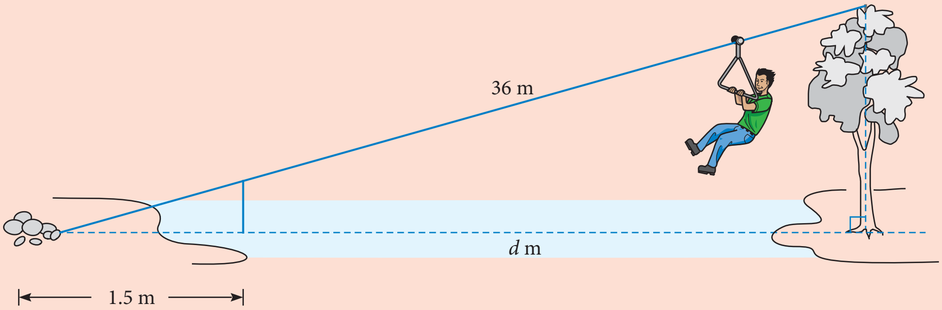


a What similarity test could be used to prove that these triangles are similar?
b Find the value of g.
c Given $AE = 50$ mm, find the length of EC.

22 **Example 19** A tree 18 m high casts a shadow of length 35 m. How long would the shadow of a 1.5 m fence post be at exactly the same time? Give your answer correct to 1 decimal place.



23 **Example 20** A flying fox is constructed between the top of a tree and a large rock. It has a length of 36 m. A post is put into the ground 1.5 m from the rock. The length of cable from the rock to the top of the post is 2 m. What is the (horizontal) distance between the tree and the rock?



24 **Example 22** A club's logo was circular with a diameter of 6 cm. The club President wanted a banner made with an enlarged logo on it. They found that they needed 144 times the area of material that was used for the original logo. What was the diameter of the logo on the banner?

25 **Example 23** A model airplane had dimensions: length 24 cm, wing span 18 cm and wing surface area 40 cm^2. A scale of 1 : 200 had been used to make it.

a What was the length of the real plane?

b What was the wing span of the real plane?

c What was the wing surface area of the real plane?

26 **Example 24** Two similar prisms had sides with a scale factor of $\frac{3}{7}$.

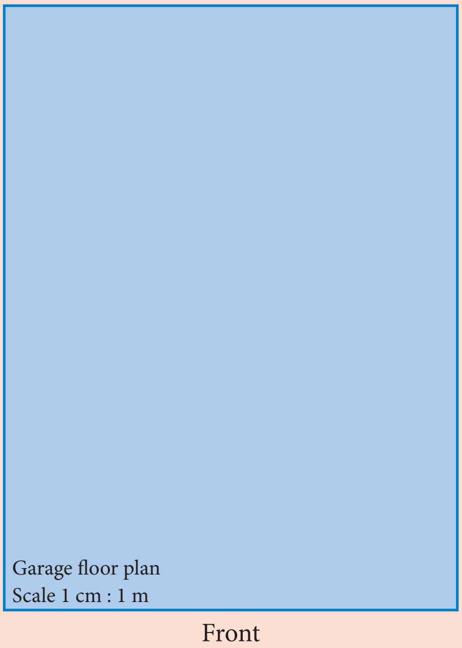
a What is the volume scale factor?

b If the smaller prism had a volume of 121.5 m^3, what is the volume of the larger prism?

27 **Example 25** Two cylindrical tanks have identical dimensions. The first tank then has its height and radius doubled, while the second tank has its height and radius halved. How many times would you have to empty the second tank into the first tank to now fill it?

Application

28 The floor plan of a garage is shown. With allowance for waste and mortar, it is usual to assume that it requires 53 bricks to complete a square metre of a single wall. The two side and back walls of the garage are to be brick and will be 2.5 m high. How many bricks are needed, given that the side entry door is 820 mm by 2030 mm and the two windows are 1800 mm by 1200 mm? Round your answer up to the nearest ten bricks.



Practice quiz

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8

UNIVARIATE DATA ANALYSIS AND THE STATISTICAL INVESTIGATION PROCESS

STATISTICAL MEASURES

TERMINOLOGY

bimodal
box-and-whisker plot
categorical, nominal data
categorical, ordinal data
class interval
continuous numerical data
discrete numerical data
five-number summary
frequency table
interquartile range (IQR)
mean
median
mode
outlier
population
quartile
random sample
range
sample
skew
standard deviation
stem-and-leaf plot
summary statistics

Prior learning

THE STATISTICAL INVESTIGATION PROCESS

- **review the statistical investigation process ; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results (ACMGM026)**

MAKING SENSE OF DATA RELATING TO A SINGLE STATISTICAL VARIABLE

- **classify a categorical variable as ordinal, such as income level (high, medium, low), or nominal, such as place of birth (Australia, overseas), and use tables and bar charts to organise and display the data (ACMGM027)**
- **classify a numerical variable as discrete, such as the number of rooms in a house, or continuous, such as the temperature in degrees Celsius (ACMGM028)**
- **with the aid of an appropriate graphical display (chosen from dot plot, stem plot, bar chart or histogram), describe the distribution of a numerical dataset in terms of modality (uni or multimodal), shape (symmetric versus positively or negatively skewed), location and spread and outliers, and interpret this information in the context of the data (ACMGM029)**
- **determine the mean and standard deviation of a dataset and use these statistics as measures of location and spread of a data distribution, being aware of their limitations. (ACMGM030)** AC

8.01 COLLECTION OF DATA

Statistics is the study of collecting, organising, presenting, analysing and interpreting data.

Study of data	How is this done?
Collecting	By observations, interviews, existing records or surveys.
Organising	Collected data is organised using tables or lists.
Presenting	Organised data is represented by appropriate graphical displays.
Analysing	Summary statistics such as mean, median and mode are calculated.
Interpreting	Use graphs and summary statistics to draw conclusions about the data.

When conducting a survey, you need to think carefully about the types of questions that you will use. Survey questions need to be clear, have a limited number of answers or a choice of possible answers to make data analysis easier and unbiased.

Example 1

Explain whether the following questions would be appropriate to use on a survey. If not, state how they could be made appropriate.

a How old are you?

b How many pieces of fruit did you eat today?

c How do you think the current government is performing?

d It is important to eat vegetables every day. How many vegetables did you eat yesterday?

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Solution

a	The question has a large range of answers.	Not appropriate due to large range of answers, therefore provide groupings to select from: such as 0 to 9 years, 10 to 19 years, etc.
b	The question has a fairly small range of answers.	Appropriate due to small range of answers.
c	The question is open-ended.	Not appropriate, as data is difficult to analyse. Provide responses. ☐ Very well ☐ Well ☐ Neither well nor poorly ☐ Poorly ☐ Very poorly
d	The question is biased.	Not appropriate, since the first sentence may lead a person into answering the question dishonestly. Just ask the question: 'How many vegetables did you eat today?'

Data can be collected from a population or sample.

A **population** includes all items in the group being studied. For example: investigating the height of Year 11 students in Victoria, the population would be every Year 11 student in Victoria.

A **sample** represents a portion of the population and is used when a population is large. It would be very time consuming and costly to measure every Year 11 student in Victoria.

A **random sample** is the fairest way of sampling, as everything or everyone in the population has an equal chance of being chosen. Random sampling helps to eliminate bias in the sample.

Random numbers can be generated on a CAS calculator to help in selecting a sample. Each member of the population is assigned a number starting at 1. If the numbers 2, 32, 54 are generated, it means select person 2, 32 and 54 from the population.

Example 2

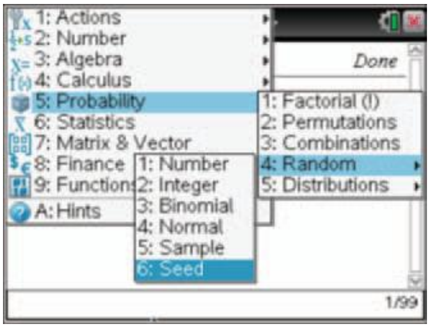
Generate 15 random numbers ranging from 1 to 200.

Solution

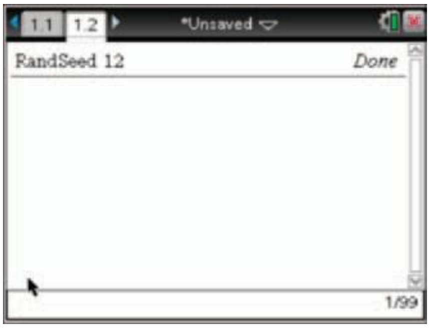
TI-Nspire CAS

Set seed. (This only needs to be done once.) Each student should enter a different value or they will all get the same numbers.

Press menu 5: Probability, 4: Random, 6: Seed



Then enter the chosen value; we have chosen 12.



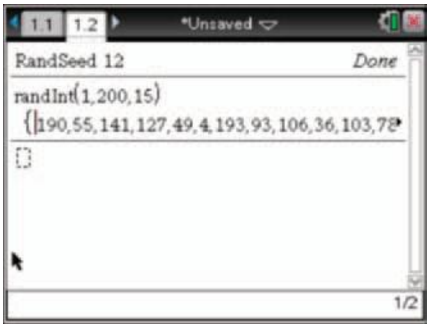
RandSeed 12 Done

Generate random integers.
Press `menu` 5: Probability, 4: Random, 2: Integer

randint(starting number, end number, amount of numbers to generate)

`enter`

Depending on how many numbers there are, you may need to Press → to see them all.



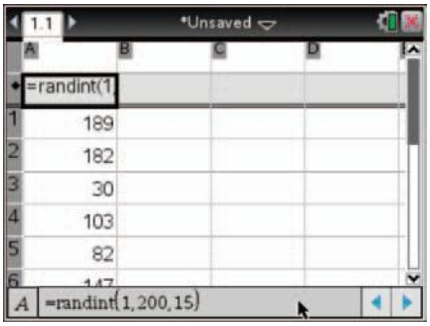
RandSeed 12 Done

randInt(1,200,15)

{190,55,141,127,49,4,193,93,106,36,103,78▸

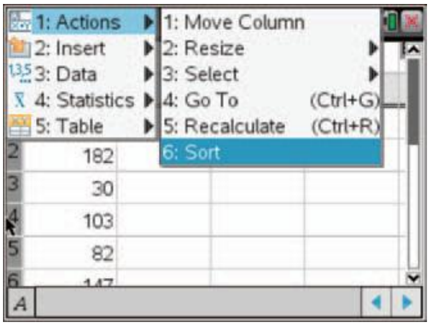
Therefore the random sample contains person 190, 55, 141, etc.

This same method can be used in a Lists & Spreadsheet page.
Press `menu` 3: Data, 5: Random, 2: Integer then **randint(starting number, end number, amount of numbers to generate)** `enter` into dark formula cell (row 2) of column A. The benefit of using a Lists & Spreadsheets page is that you can then sort the data from smallest to largest. Press `menu` 1: Actions, 6: Sort



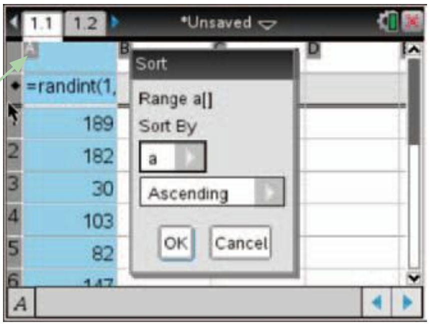
	A	B	C	D
◆	=randint(1			
1	189			
2	182			
3	30			
4	103			
5	82			
6	147			

A =randint(1,200,15)



Select the column to be sorted and Ascending for smallest to largest. Click OK.

The cursor must be in the top cell of column A to sort the data.

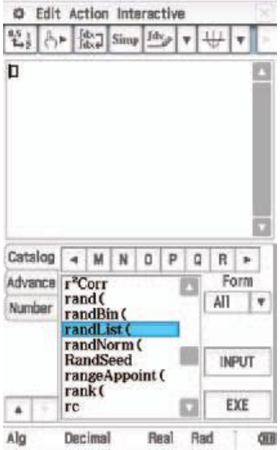


The random numbers are now sorted.

ClassPad

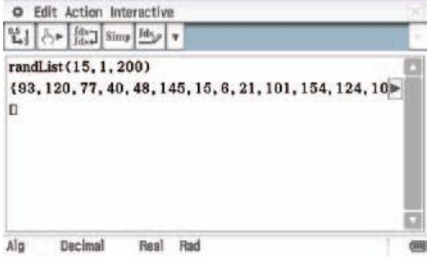
Tap Menu ⊞. If necessary, use the **Edit** menu to **Clear All**.

Press Keyboard and tap ▼ **to access** the Catalog of functions. Use ▶ to scroll along as necessary to see R. Tap R then **randList(** on the list of functions.



Add to **randList(** displayed on the screen by typing **15** , **1** , **200**) and press EXE.

The first number states how many numbers to generate (15); the next two are the lower and upper limits (1 to 200).


```
randList(15,1,200)
{93,120,77,40,48,145,15,6,21,101,154,124,10▸
```

Sort the list of numbers.

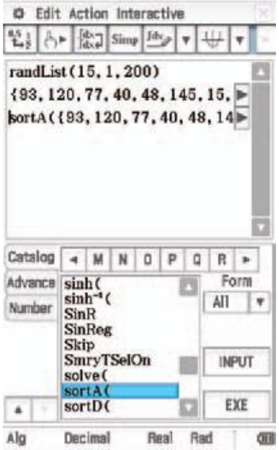
Select **sortA(** from the Catalog.

Tap the list of numbers. They should all be highlighted.

Copy and paste the numbers into the **sortA** field.

(Tap **Edit** then **Copy**, tap next to **sortA**, tap **Edit** then **Paste**.)

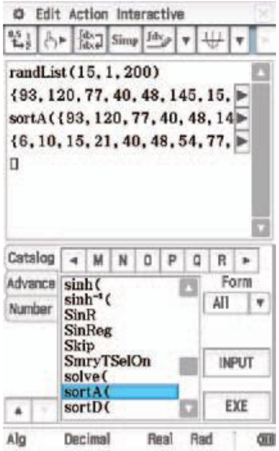
To see the numbers not displayed on the screen, tap the arrows at the end of the list, ▶ to scroll right or ◀ to scroll left as necessary.



```
randList(15,1,200)
{93,120,77,40,48,145,15,▶
sortA({93,120,77,40,48,14▶
```

Pressing EXE will result in the list being sorted in ascending order.

Write down the numbers in order.

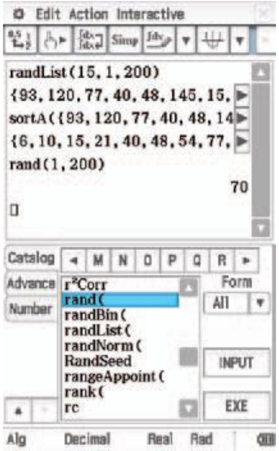


```
randList(15,1,200)
{93,120,77,40,48,145,15,▶
sortA({93,120,77,40,48,14▶
{6,10,15,21,40,48,54,77,▶
```

Sometimes you will notice that the same number appears twice. Simply use **rand(** to select a new random number from 1 to 200.

rand(1, 200) generates a random number from 1 to 200.

Repeat this as often as required.



```
randList(15,1,200)
{93,120,77,40,48,145,15,▶
sortA({93,120,77,40,48,14▶
{6,10,15,21,40,48,54,77,▶
rand(1,200)
                       70
```

EXERCISE 8.01 Collection of data

Concepts and techniques

1 Example 1 Explain whether the following questions would be appropriate to use on a survey. If not, state how they should be altered.
   - a Which AFL football team do you support?
   - b How many songs do you have on your iPod?
   - c How many siblings do you have?
   - d What is your favourite food?
   - e Do you think teenagers are lazy?

2 Match each of the survey items **a** to **e** with an appropriate set of responses from **A** to **C**.
   - a How many siblings do you have?
   - b Will you travel this year?
   - c Is Queensland larger than the Northern Territory?
   - d All school students should have a laptop computer.
   - e Do cats have more chromosomes than dogs?

   A ☐ Yes ☐ No ☐ Don't know
   B ☐ 0 ☐ 1 ☐ 2 ☐ 3 or more
   C ☐ Strongly disagree ☐ Disagree ☐ Neither agree nor disagree
   ☐ Agree ☐ Strongly agree

3 Classify each of the following questions as either biased or unbiased.
   - a You do not like apricots, do you?
     ☐ Yes ☐ No ☐ Don't know
   - b How many people live at your house?
   - c Exercising is part of a healthy lifestyle. How many times do you exercise per week?
     ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 or more
   - d It is important to have a health check every six months. How many times per year do you have a health check?
     ☐ 0 ☐ 1 ☐ 2 ☐ 3 or more
   - e What is your favourite ice-cream flavour?
   - f The new pool is a great success. Do you agree?

4 Example 2 Generate the following lists of random numbers and then sort them from smallest to largest.
   - a 5 different numbers ranging from 10 to 50
   - b 15 different numbers ranging from 0 to 150
   - c 30 different numbers ranging from 1 to 250

Reasoning and communication

5 A confectionary company wishes to determine the favourite lolly in a population of 15 000 five-year-olds.
   - a Discuss how a sample of 150 should be selected.
   - b Write two appropriate questions that could be asked.

8.02 TYPES OF DATA

Data is information that is collected about a certain group. A **variable** is a particular characteristic that can be observed, counted or measured, such as eye colour or height. Data can be classified as categorical data or numerical data.

Categorical data

Variables that represent qualities/characteristics described with words.

They can be classified further as

Nominal: characteristics have no order, for example eye colour.

Ordinal: characteristics for which there is an order, for example star movie rating.

Numerical data

Variables that represent quantities or measurements represented with numbers.

They can be classified further as

Discrete: can take only particular values, in most cases whole numbers, for example number of runs in cricket.

Continuous: can take all values usually within a particular range, for example the height of a tree.

IMPORTANT

When variables are classified as **categorical data** we should also state if they are **nominal** or **ordinal**.

When variables are classified as **numerical data** we should also state if they are **discrete** or **continuous**.

Example 3

Classify each of the following variables.

a Gender

b Number of children

c Finishing position in a 100 metre race

d Height

e Height to the nearest cm

Solution

a	Gender is a category (male or female) with the order not being important.	Categorical and nominal
b	Number of children is represented by numbers and can only be whole numbers.	Numerical and discrete
c	Categories can be numbers. Each finishing position in a race is a category (1st place, 2nd place, etc., or 1, 2, 3, etc.) and the order is important.	Categorical and ordinal
d	Height is a measureable numerical value and can take all values within a particular range.	Numerical and continuous
e	Height to the nearest cm is a measured variable but can only take particular values, multiples of 1 cm, within a reasonable range.	Numerical and discrete

EXERCISE 8.02 Types of data

Concepts and techniques

1 Example 3 What is the difference between categorical data and numerical data? Give an example of each type of data.

2 State what type of data each of the following variables represents.
   a A person's religion
   b Number of computers in homes
   c Populations of cities
   d Marital status
   e Hotel star-rating
   f The amount of time spent on the Internet
   g The amount of rainfall per month
   h Letter grades on a test
   i Olympic swim times expressed correct to two decimal places

3 Classify each of the following types of numerical data as discrete (D) or continuous (C).
   a weights of athletes
   b shirt sizes
   c running times for a 100 m race
   d speeds of cyclists
   e number of seats in a bus
   f number of goals scored in netball
   g heights of the world's tallest buildings to the nearest metre.
   h running times for a 100 m race recorded to the nearest hundredth of a second.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Reasoning and communication

4 Jan said that the distance for a javelin throw is a discrete variable because it is always measured to the nearest 2 cm. Deirdre disagreed with Jan, saying that the distance for a throw is continuous. Explain who you think is correct, carefully justifying your answer.

5 The table below shows the ages and average heights of a group of students.

Age	15	16	17	18
Number of students	4	15	3	1
Average height	1.62 m	1.68 m	1.73 m	1.79 m

   a What kind of variable is student's age? Give reasons for your answer.
   b What kind of variable is the number of students? Give reasons for your answer.
   c What kind of variable is average height? Give reasons for your answer.

8.03 DISPLAYING CATEGORICAL DATA

When a large amount of data is collected it can be organised into a **frequency table** so that we can see how many times each data value occurs. An appropriate graphical display can then be more easily constructed to visualise the data. A frequency table shows each response and how often this response occurred. For example,

Variable name	Tally	Frequency (f)
Response 1		
Response 2		
Response 3		
Response 4		
	Total	

Place a line in the tally column each time a response occurs. This helps us to keep track. Remember that 𝍸 means 5.

Example 4

A coffee shop owner decided to record the first 20 types of hot drinks he sold. They were

cappuccino latte tea latte latte
cappuccino tea latte latte cappuccino
cappuccino tea cappuccino cappuccino latte
tea latte latte cappuccino cappuccino


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Construct a frequency table to display this information.

Solution

Create a table with the correct variable name and possibilities.

Hot drink	Tally	Frequency (f)
Cappuccino		
Latte		
Tea		

Place a line in the tally column for each response.

Hot drink	Tally	Frequency (f)
Cappuccino	𝍸 |||	
Latte	𝍸 |||	
Tea	||||	

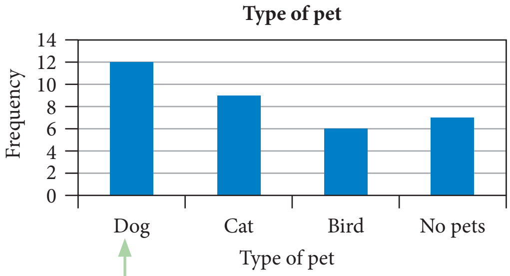
Count each tally and record the total as a number in the frequency column. Check that the frequency column adds up to 20.

Hot drink	Tally	Frequency (f)
Cappuccino	𝍸 |||	8
Latte	𝍸 |||	8
Tea	||||	4
Total		20

Always add up the frequency column to make sure that you have included all the data.

For categorical data displayed in a frequency table the most efficient graphical display is a column or bar graph.

Column graph



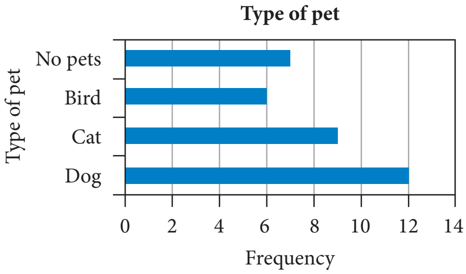
There must be gaps between each column or bar and labelling is in the middle of the column or bar.

Label each axis.

Frequency is displayed on the vertical axis.

Draw in columns to match the frequency of each type of pet.

Bar graph



Label each axis.

Frequency is displayed on the horizontal axis.

Draw in bars to match the frequency of each type of pet.

Shutterstock.com/Eric Isselee

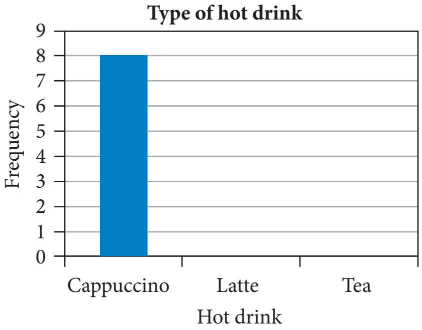
Example 5

Construct a column graph for the frequency table in Example **4**. Write an explanation of the data displayed on the graph.

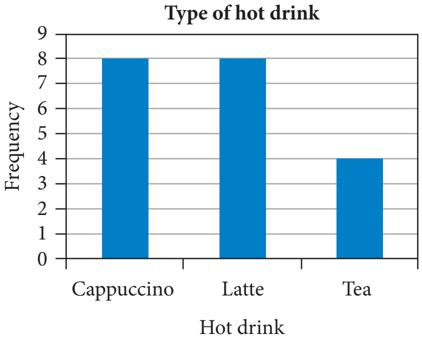
Hot drink	**Tally**	**Frequency (*f*)**
Cappuccino	𝍸 |||	8
Latte	𝍸 |||	8
Tea	||||	4
	Total	20

Solution

Construct axes with frequency on the vertical axis. Determine an appropriate scale for the vertical axis; look at the highest frequency to decide this. Add title and labels. Draw the cappuccino column up to 8.



Draw in the remaining columns.



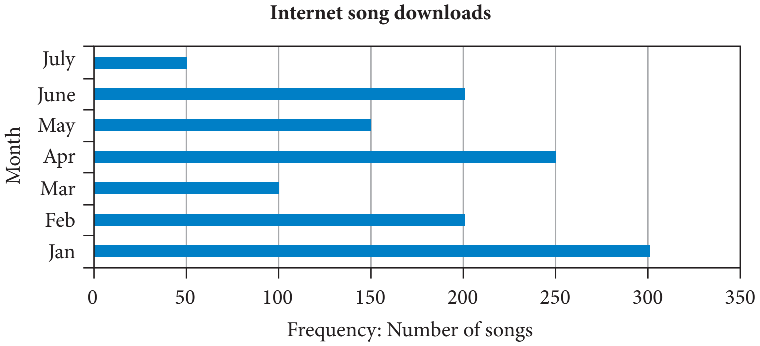
Summarise what the graph shows.

The graph shows that cappuccinos and lattes were the most popular hot drinks sold, with tea sales being the lowest.

When given a bar or column graph representing information, we can use careful interpretation to describe the key findings.

Example 6

This bar graph shows the number of songs downloaded from the Internet over 7 months by a group of teenagers.



July	50
June	200
May	150
Apr	250
Mar	100
Feb	200
Jan	300

Use the graph to answer the following questions.

a In which month were most songs downloaded from the Internet?

b How many songs were downloaded over the 7-month period?

c What percentage of songs were downloaded in March?

Solution

a The longest bar is January with a frequency of 300.

Most songs were downloaded in January.

b Add up the frequency for each month.

$300 + 200 + 100 + 250 + 150 + 200 + 50 = 1250$
Therefore 1250 songs were downloaded over the 7-month period.

c Use the formula $\frac{\text{downloads in March}}{\text{total downloads}} \times 100\%$

$\frac{100}{1250} \times 100\% = 8\%$, therefore 8% of the downloads occurred in March.

EXERCISE 8.03 Displaying categorical data

Concepts and techniques

1 Example 4 A survey of eye colour was conducted in a Year 11 class containing 30 students. The results were as follows.

blue, brown, green, hazel, hazel, blue, blue, brown, green, brown, green, hazel, green, brown, brown, blue, hazel, green, green, blue, blue, green, hazel, green, brown, hazel, blue, brown, blue, blue.

Construct a frequency table to display this information.

2 25 boys were asked to select their favourite sport from football, soccer, cricket or other. The results are given below. Construct a frequency table to display this information.

football, soccer, cricket, other, other, soccer, cricket, cricket, football, football, soccer, soccer, other, football, soccer, cricket, football, other, soccer, soccer, cricket, other, football, football, soccer.

Shutterstock.com/Sonya Etchison

3 50 parents were asked the following question: 'Are teenagers lazy?' They had to select from yes, no or undecided. The results are given below. Construct a frequency table to display this information.

yes, yes, no, yes, undecided, undecided, undecided, no, no, yes, yes, yes, yes, undecided, undecided, no, no, yes, yes, undecided, no, yes, no, no, no, undecided, undecided, yes, yes, no, no, no, no, undecided, undecided, yes, yes, no, yes, undecided, undecided, undecided, no, no, yes, yes, undecided, no, yes, yes.

4 Example 5 Construct a column graph for the frequency table in question **1**. Explain what the graph tells you about the data.

5 Construct a bar graph for the frequency table in question **2**. Explain what the graph tells you about the data.

6 Construct a column graph for the frequency table in question **3**. Explain what the graph tells you about the data.

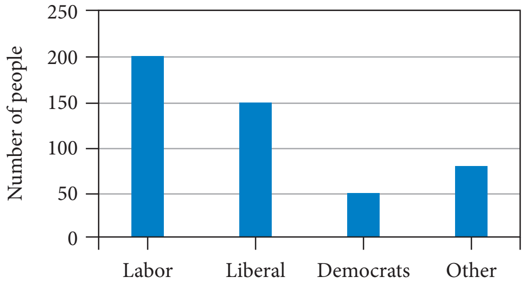
7 A survey of people in a supermarket to find out their meat preferences gave the following results.

Type of meat	Beef	Lamb	Pork	Chicken	Veal	None
Frequency	42	38	13	17	3	5

Draw a bar chart showing these results.

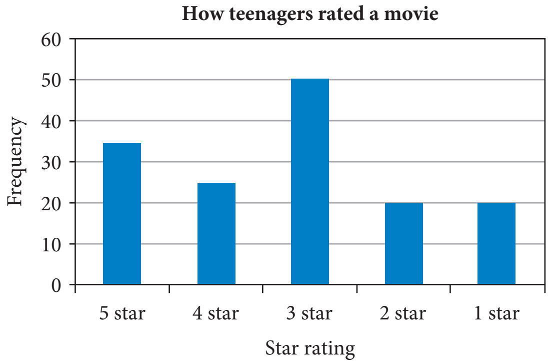
Reasoning and communication

8 **Example 6** This graph shows the results of a poll before an election.



a How many people favour the Democrats?
b How many people favour the Liberal Party?
c Which is the most popular party in the poll?
d How many people favour parties other than the Democrats?
e How many people favour the Labor Party above the Liberal Party?
f How many people were polled altogether?
g What percentage of those polled favoured the Liberal Party?

The following graph relates to questions **9**, **10** and **11**.



9 The most common star rating for the movie is:
A 1 star B 2 star C 3 star D 4 star E 5 star

10 The number of teenagers who were asked to rate the movie is:
A 75 B 100 C 150 D 175 E 200

11 The percentage of teenagers who rated the movie as 2 star is:
A 9% B 13% C 25% D 33% E 50%

8.04 DISPLAYING NUMERICAL DATA: STEM-AND-LEAF PLOTS AND DOT PLOTS

When the amount of numerical data collected is not too large, a **stem-and-leaf plot**, also known as a stemplot, or a **dot plot** can be used. Both plots enable the entire set of data to be observed.

Stem-and-leaf plot

Data is split into a stem and leaf. For example:

- two digit numbers like 15 have a stem of 1 and a leaf of 5
- three digit numbers like 126 have a stem of 12 and a leaf of 6
- decimal numbers like 12.5 have a stem of 12 and a leaf of 5

A stem-and-leaf plot should always have a key.

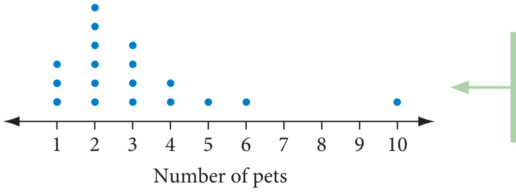
1|2 means 12 is very different to 1|2 means 1.2.

Key: 7|9 means 79

Stem	Leaf							
7	1	5	6	8	9	9		
8	0	0	2	3	4	6	8	9
9	0	2	5	7	9			
10	2	3	5	6	7	7	8	

Dot plot

Each dot represents an occurrence of a score



The data point at 10 is very different from the rest of the data. This is classed as an outlier.

A dot plot can be used for categorical or numerical data that has a small range of responses or values.

IMPORTANT

An **outlier** is an extreme (high or low) value in the data. When interpreting a graph an outlier can be seen as a score or point that is away from the main cluster of data as in the dot plot above.

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Example 7

The following data represents heights in centimetres.

167	189	170	179	160	159	172	183	191	159
154	170	177	168	174	174	192	175	157	190
123	162	168	177	175	163	159	177		

a Explain why a stem-and-leaf plot is more appropriate than a dot plot for displaying this data.

b Arrange the data in a stem-and-leaf plot.

c Describe what the plot shows, identifying any outliers if appropriate.

Solution

a The heights vary from 123 to 192 so for a dot plot the scale would be too long.

As the range of heights given is large, a stem-and-leaf plot is more appropriate than a dot plot.

b Set up the stem-and-leaf plot. There are three digit numbers, therefore the key is 12|3 means 123 cm.

Place every third digit against its stem.

Key: 12|3 means 123 cm

Stem	Leaf
12	3
13	
14	
15	9 9 4 7 9
16	7 0 8 2 8 3
17	0 9 2 0 7 4 4 5 7 5 7
18	9 3
19	1 2 0

Order the leaves from smallest to largest.

Key: 12|3 means 123 cm

Stem	Leaf
12	3
13	
14	
15	4 7 9 9 9
16	0 2 3 7 8 8
17	0 0 2 4 4 5 5 7 7 7 9
18	3 9
19	0 1 2

c Look for the stem with the most leaves, the most common number and any outliers.

The plot shows that the 170–179 cm height bracket was the most common and that 159 cm and 177 cm were the most common individual heights.
123 cm would be considered an outlier as it is away from the main cluster of data.

Example 8

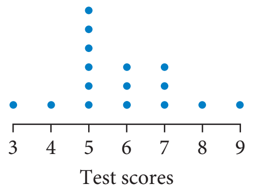
Sixteen students completed a spelling test with ten words. Their scores were as follows.

7, 3, 5, 6, 6, 9, 7, 5, 6, 5, 5, 8, 5, 7, 5, 4.

Display the test scores as a dot plot and interpret.

Solution

Set up the horizontal scale from 3 to 9 with a label, then place a dot for each time a score appears.
There are 16 values so make sure that you have 16 dots.



Interpret the data including the highest, lowest and most common score and any outliers.

The lowest score was 3 and the highest was 9.
The most common score was 5 correct words.
No one got all the words correct. All values are clustered together, therefore there are no outliers.

Example 9

This stem-and-leaf plot shows the examination marks of a class of students.

Key: 4|3 means 43%

Stem	Leaf
4	3 7
5	0 2 2 4 8
6	1 3 3 3 6 6 9
7	5 7 8
8	4 6
9	2

a What was the highest mark?

b How many students are in the class?

c What was the most common mark?

d What percentage of students scored over 60%?

Solution

a The last value in the plot is 92%. The highest mark is 92%.

b Count how many numbers there are in the leaf column. There are 20 students in the class.

c The mark that appears the most. The most common mark is 63%.

d Use the formula:

$$\frac{\text{number of students with a score over 60\%}}{\text{total students}} \times 100\%$$

$$\frac{13}{20} \times 100\% = 65\%,$$

therefore 65% of students achieved a score over 60%.

EXERCISE 8.04 Displaying numerical data: stem-and-leaf plots and dot plots

Stem-and-leaf-plots

Concepts and techniques

1 Example 7 Construct a stem-and-leaf plot for each set of data.

a 11, 15, 29, 30, 16, 18, 9, 12, 26, 29, 25, 17, 15, 15, 7, 27, 24, 28, 29, 18, 17, 15, 21, 20, 30, 25, 14, 28, 27, 30, 22, 21, 30

b 64, 39, 50, 38, 55, 89, 76, 66, 65, 59, 50, 47, 82, 81, 60, 62, 55, 41, 67, 60, 78, 71, 74, 55, 57

c 48, 76, 75, 83, 55, 60, 67, 63, 81, 90, 49, 50, 61, 62, 76, 68, 64, 55, 51

d 354, 371, 388, 323, 336, 376, 399, 347, 341, 338, 376, 366, 369, 370, 345, 331, 338, 365, 360, 370

e 82, 91, 120, 113, 78, 96, 87, 112, 110, 99, 87, 99, 101, 115, 76, 106

2 **Example 8** Construct a dot plot for each set of data.

- a 5, 8, 5, 4, 7, 9, 5, 6, 8, 7, 6, 5, 5, 6, 7
- b 103, 105, 102, 102, 105, 102, 103, 104
- c 58, 59, 58, 55, 57, 58, 56, 54, 58
- d 19, 20, 12, 17, 15, 17, 19, 20, 12, 16, 19, 18, 20
- e 3.5, 3.3, 3.7, 3.5, 3.7, 3.5, 3.6, 3.5, 3.9

3 Draw either a dot plot or a stem-and-leaf plot for the following data. Choose the most appropriate display for each set.

- a 45, 79, 61, 63, 55, 80, 69, 89, 71, 56, 23
- b 46, 49, 49, 47, 41, 49, 48, 49, 40, 45, 46

Reasoning and communication

4 **Example 9** A class was surveyed to find out how many hours each student spent on maths homework each week. The results are shown below.

7	6	8	9	5	10	6	9	9	0	9	8
18	7	5	3	4	9	6	7	8	10	7	8

- a Draw a dot plot for this data.
- b How many students are in the class?
- c List any outliers. Suggest possible reasons for these.

5 Jeremy sells sausages in bread outside the hardware store on Sundays. The number of sales each day over 30 Sundays was recorded as follows.

66	64	28	93	47	110	53	68	117	43
72	68	84	103	59	82	78	61	104	79
51	63	112	81	79	94	42	57	83	100

- a Draw a stem-and-leaf plot to represent this data.
- b On what percentage of days did Jeremy have more than 50 sales?
- c What is the outlier? Give one possible reason for this outlier.

6 A survey was conducted to determine the number of people in each car in a sample of cars on a road. The results are as follows.

6	3	1	2	3	1	1	3	4	2
5	3	1	1	2	3	4	5	6	4

- a Display the data using a dot plot.
- b How many cars were surveyed?
- c What was the highest number of people recorded in a car?
- d Comment on any outliers.

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The following stem-and-leaf plot relates to questions **7**, **8** and **9**.

This plot shows the times (in seconds) in a Slalom ski race.

Key: 9|1 means 91 seconds

Stem	**Leaf**
9	1 5 7 9
10	2 4 5 6 6 8
11	0 2 2 3 4 4 5
12	1 2 3 3 3 7 9
13	2 3 4 5 7 7
14	3 6 9
15	0 1 2

7 The number of skiers who participated in the race is:

A 7 **B** 15 **C** 23 **D** 36 **E** 152

8 The winning time is:

A 91 seconds **B** 110 seconds **C** 132 seconds
D 143 seconds **E** 152 seconds

9 If skiers with times under 110 seconds were of Olympic standard, the percentage of skiers of this standard is?

A 12% **B** 22% **C** 25% **D** 27% **E** 28%

10 A nurse at Greenacres Hospital took the pulse rates, in heartbeats per minute, of 40 patients. The results were as follows.

71	81	63	55	93	52	69	78	84	65
72	80	68	74	85	79	90	84	76	68
58	64	60	97	83	69	74	56	64	89
94	81	63	60	76	72	110	83	90	64

a Construct a stem-and-leaf plot for this data.

b If the typical human pulse rate is from 65 to 75 beats per minute inclusive, how many patients had a typical pulse rate?

c Identify the outlier and interpret this data value.

d A very fit person has a low resting pulse rate (say, less than 60). How many patients were in this category?

11 At the school athletics carnival, runners recorded the following times (in seconds) for the 100 metres sprint.

12.1	13.6	11.8	18.1	12.0	15.6	13.9	17.2
14.5	18.7	15.7	14.6	16.3	11.6	17.7	15.6
14.6	15.4	12.4	16.5	17.4	14.6	16.8	14.3

a Represent this data using a stem-and-leaf plot using stems of 11, 12, etc.

b How many runners were there?

c What was

i the best time recorded? ii the worst time recorded?

d What percentage of students took more than 15 seconds to run the distance? Answer correct to one decimal place.

e To qualify for the regional carnival, a runner's time must be less than 13.5 seconds. What fraction of students qualified for the regional carnival?

8.05 DISPLAYING NUMERICAL DATA: HISTOGRAMS

Dot plots and stem-and-leaf plots represent small amounts of numerical data effectively but are not practical for large amounts of data.

A frequency table can be used to organise large amounts of numerical data following the same process used for categorical data.

IMPORTANT

When the data is discrete and has a small range of values, each value is listed individually in the frequency table.

Example 10

Construct a frequency table for Jack's golf scores for the year.

62	67	65	68	65	68	65	63	65	63	69
65	62	68	67	64	65	68	69	64	65	63

Solution

Construct a table with the correct variable name and numbers. Look for the smallest number to start your table and the largest number to finish it.

Golf score	Tally	Frequency (f)
62		
63		
64		
65		
66		
67		
68		
69		
	Total	

Place a line in the tally column for each golf score.
Count each tally and record the number in the frequency column.
Make sure the frequency column adds up to the total number of golf scores recorded (22).

Golf score	Tally	Frequency (f)
62	||	2
63	|||	3
64	||	2
65	𝍸 ||	7
66		0
67	||	2
68	||||	4
69	||	2
	Total	22

When data is discrete with a large range of values or is continuous, it is not practical to list each individual number in a frequency table. For example, if you have discrete values ranging between 0 and 99 you would need 100 rows in a frequency table.

A class interval describes a range of values such as 0–9, therefore any number from 0 to 9 is assigned to that class interval. For discrete values ranging between 0 and 99, using class intervals of size 10 (0–9, 10–19 etc.) requires a frequency table with 10 rows.

IMPORTANT

When numerical data has a large range of values, numbers need to be grouped into **class intervals** for the frequency table.

Example 11

Group the following scores into a frequency table using class intervals of size 20 (0–19, 20–39 ...).

45 78 67 68 59 32 12 99 45 58 56 69 78 16 67 65 51
50 43 22 70 35 66 43 19 21 77 80 89 56 54 61 68 74

Solution

Construct a table with correct variable name and class intervals. The first class interval needs to contain the lowest score in it but does not have to start with that lowest score.

Class intervals must all be the same size.

In general there should be between 5 to 12 class intervals.

Score	Tally	Frequency (f)
0–19		
20–39		
40–59		
60–79		
80–99		
	Total	

The first score is 45, so place a line in the tally column for class interval 40–59.
Place a tally line for all scores. Count the tallies and record their number in the frequency column.
Make sure the frequency column adds up to the total number of scores (34).

Score	Tally	Frequency (f)
0–19	|||	3
20–39	||||	4
40–59	𝍸 𝍸 |	11
60–79	𝍸 𝍸 |||	13
80–99	|||	3
	Total	34

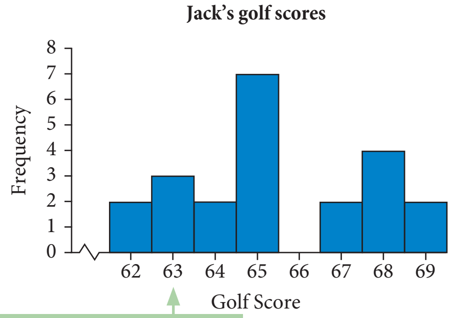
For numerical data organised into a frequency table, the most efficient graphical representation is a histogram. For a histogram the horizontal axis represents the data values and the vertical axis represents frequency.

Unless the first class interval starts at zero, leave a gap between the first column and the vertical axis. All other columns have no gaps.

The height of each column matches the frequency listed in the frequency table.

Ungrouped data

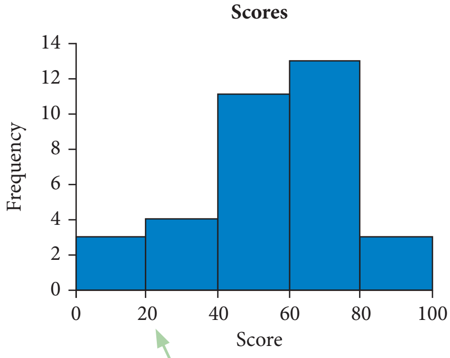
Using the frequency table from Example **10**.



Column labelling is in the middle of each column.

Data grouped into class intervals

Using the frequency table from Example **11**.



Column labelling is on the left edge of each column using the lower limit of the class interval.

Example 12

The following table shows the number of hours of sleep had by a group of 16-year-olds.

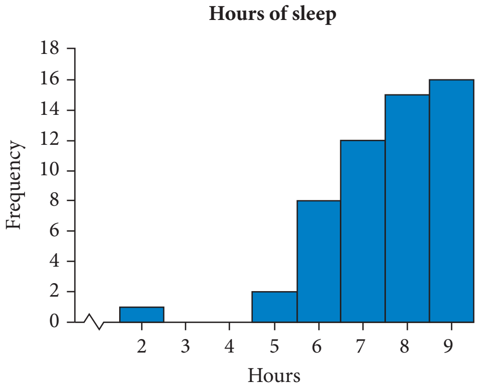
Hours	2	3	4	5	6	7	8	9
Frequency	1	0	0	2	8	12	15	16

Note that the table is drawn in two rows rather than two columns. Always remember that frequency is assigned to the vertical axis of a histogram regardless of how the table is presented.

Display this information on a histogram. Describe the graph.

Solution

Construct the axes using an appropriate scale. Look at the highest frequency to decide this. Add titles and labels in the centre of each column. Draw each column to match the frequency, remembering that there are no gaps.

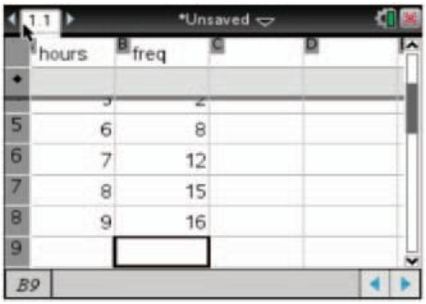


Comment on the most common score, the general pattern and the presence of any outliers.

9 hours sleep was the most common. Most 16-year-olds have between 6 to 9 hours sleep, with an increasing number having more hours of sleep, up to 9 hours. There is one outlier present in the data (1 person with 2 hours sleep) indicated by the large gap between the first and second columns.

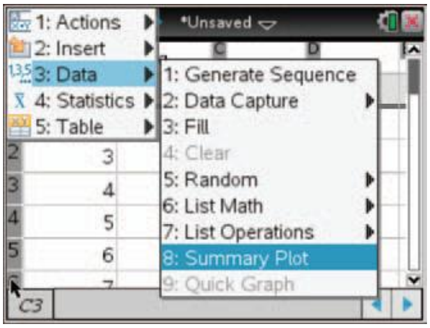
TI-Nspire CAS

Use a Lists & Spreadsheet page.
Name column A hours and column B freq, then enter the data.

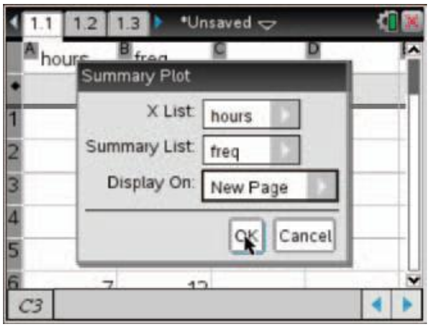


	hours	freq
5	6	8
6	7	12
7	8	15
8	9	16
9		

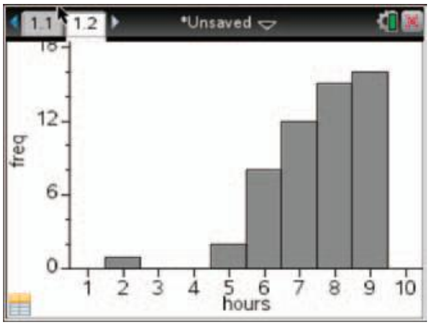
Press menu 3: Data, 8: Summary Plot



Fill in the summary plot details as per the third screen and click OK.



X List: hours
Summary List: freq
Display On: New Page

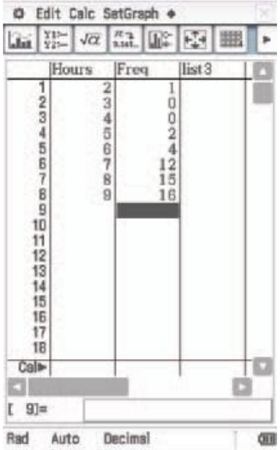


ClassPad

Tap Menu ⊞ and the Statistics application.

Rename **list1** as **Hours** and **list2** as **Freq**.

Enter the data.



	Hours	Freq	list3
1	2	1	
2	3	0	
3	4	0	
4	5	2	
5	6	4	
6	7	12	
7	8	15	
8	9	16	

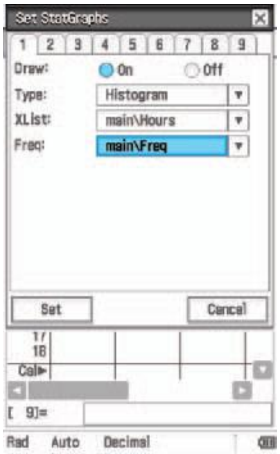
[9]=

Tap **SetGraph**.

Make sure **StatGraph1** only is ticked. If necessary, tap **Stat Window Auto** (under the ◆ menu) to make sure it is on.

When everything is correct, tap **SetGraph**, then **Setting...**

Set the screen as shown and tap Set.



Draw: ● On ○ Off

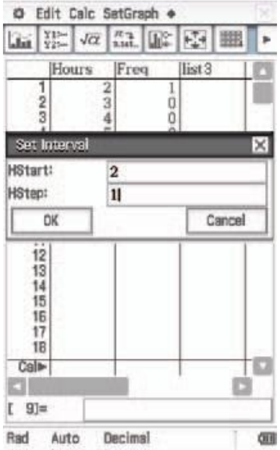
Type: Histogram

XList: main\Hours

Freq: main\Freq

Tap the top left graph icon ▥.
Change the HStep to 1.

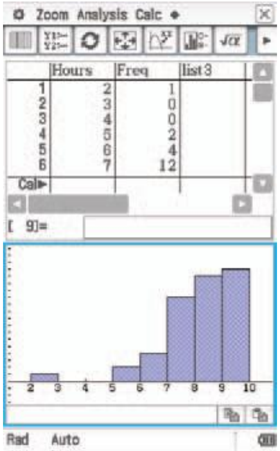
Tap OK.



HStart: 2

HStep: 1

The histogram is displayed.



	Hours	Freq	list3
1	2	1	
2	3	0	
3	4	0	
4	5	2	
5	6	4	
6	7	12	
Cal▶			

[9]=

Example 13

The following table shows the exam scores, out of 100, achieved by a class of Year 11 students on their end-of-year mathematics examination.

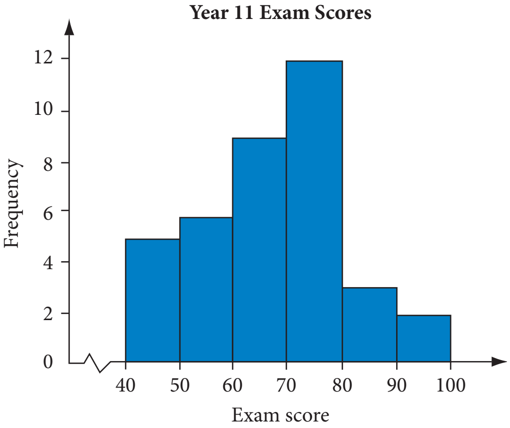
Exam score	Frequency (*f*)
40–<50	5
50–<60	6
60–<70	9
70–<80	12
80–<90	3
90–<100	2

The class interval 60–<70 includes all scores starting at 60 up to but not including 70.

Display this information on a histogram.

Solution

Construct axes using an appropriate scale. Look at your highest frequency to determine the scale for the vertical axis. Add a title and labels on the edges of each column. Draw each column to match its frequency, remembering that there should be no gaps between columns.



TI-Nspire CAS

Name column A score and column B freq then enter the data. As class intervals are used, enter the midpoint of each interval. Press `menu` 3: Data, 8: Summary Plot

The midpoint of the first class interval is $\frac{40+50}{2} = 45$

	score	freq
1	45	5
2	55	6
3	65	9
4	75	12
5	85	3

B3 9

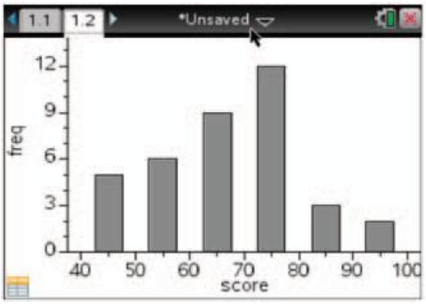
Fill in the summary plot details as shown on the second screen, then click **OK**.

Summary Plot

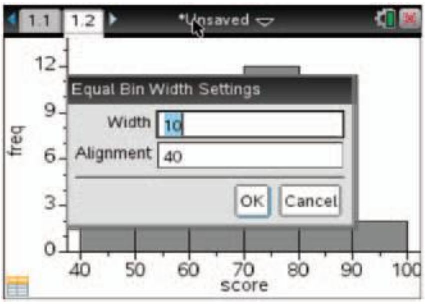
X List: score

Summary List: freq

Display On: New Page



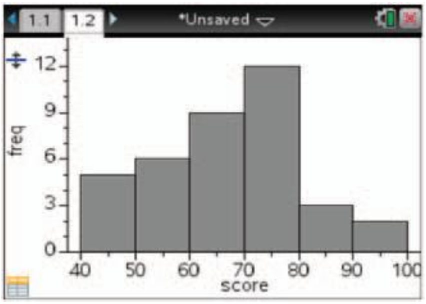
The histogram that appears is not in class intervals of 10. Change this by pressing `menu` 2: Plot Properties, 2: Histogram Properites, 2: Bin Settings, 1: Equal Bin Width
Fill in the width of 10 and Alignment 40.
Click **OK**.



Equal Bin Width Settings

Width 10

Alignment 40

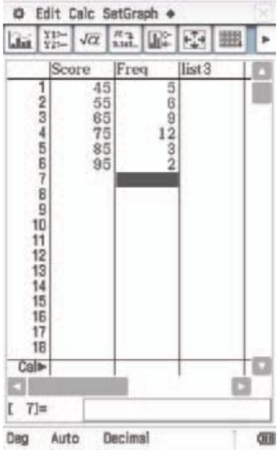


ClassPad

Tap and the **Statistics** application.

Rename **list1** as **Score** and **list2** as **Freq**.

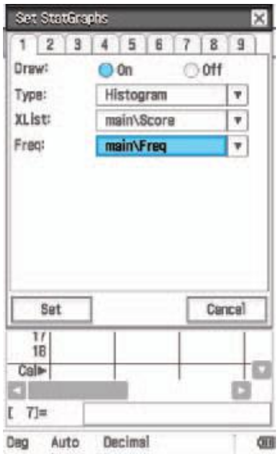
As class intervals are used, enter the midpoints of the class intervals and the frequencies as before.



	Score	Freq	list3
1	45	5	
2	55	6	
3	65	9	
4	75	12	
5	85	3	
6	95	2	

Use **SetGraph** as before, making sure **StatGraph1** only is ticked and **Stat Window Auto** is on

Tap **Setting...**, Set the screen as shown and tap Set.



Set StatGraphs

Draw: ● On ○ Off

Type: Histogram

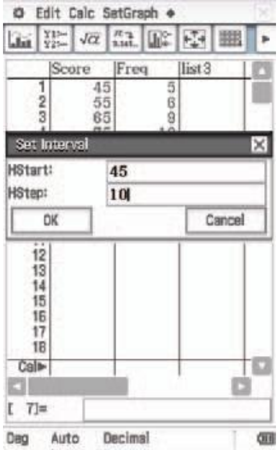
XList: main\Score

Freq: main\Freq

Tap the top left graph icon .

Set **Hstart** to 45 and the columns must be ten units wide, so set **HStep** to 10.

Tap OK.

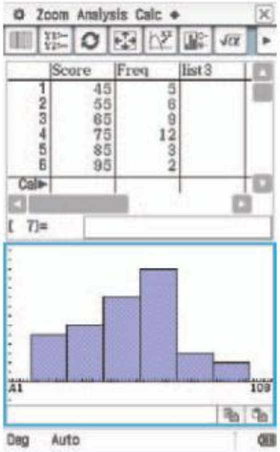


Set Interval

HStart: 45

HStep: 10

The histogram is displayed.



	Score	Freq	list 3
1	45	5	
2	55	6	
3	65	9	
4	75	12	
5	85	3	
6	95	2	

The table below describes the most appropriate graphical display when given a data set.

Graph	Type of data	Important points
Bar graph or column graph	Categorical	
Histogram	Numerical, discrete and continuous	Medium/ large data sets
Dot plot	Categorical and discrete numerical	Small data sets
Stem-and-leaf plot	Discrete and continuous numerical	Small data sets

EXERCISE 8.05 Displaying numerical data: histograms

Frequency distribution tables

Histograms

Concepts and techniques

1 Example 10 Construct a frequency table for each set of scores.

- a 8, 7, 9, 5, 7, 3, 5, 6, 1, 5, 6, 5, 8, 5, 6, 3, 5, 7, 4, 7, 5, 9, 4, 4, 8
- b 12, 16, 10, 16, 14, 16, 12, 11, 14, 15, 17, 18, 15, 16, 15, 14, 11, 10, 15, 12, 18, 19, 15, 14, 15
- c 103, 105, 104, 102, 103, 101, 100, 105, 104, 103, 103, 103, 104, 101, 107, 102, 101, 104, 106, 104, 102, 102, 101
- d 78, 76, 72, 74, 78, 75, 78, 78, 75, 76, 77, 71, 78, 72, 78, 77, 75, 76, 75, 78, 73, 76, 71, 78, 77, 75, 76, 77, 76, 72
- e 4, 9, 8, 4, 3, 2, 6, 6, 9, 8, 5, 4, 2, 5, 5, 7, 4, 4, 5, 4, 6, 6, 4, 4, 4, 4, 7, 2

2 Example 11 Construct a grouped frequency table for each set of scores.

- a 18, 73, 95, 56, 71, 33, 50, 65, 19, 54, 61, 53, 87, 52, 61, 37, 58, 74, 45, 76, 55, 92, 48, 49, 81, 57, 81, 44, 50, 36, 71, 55, 30, 20, 11, 77, 91 (use groups of 10–19, 20–29 and so on)
- b 102, 116, 100, 136, 154, 176, 112, 141, 134, 175, 167, 108, 115, 176, 115, 124, 161, 160, 155, 162, 188, 139, 125, 154, 165 (use groups of 100–119, 120–139, 140–159 and so on)
- c 103, 115, 107, 132, 123, 122, 109, 106, 123, 131, 109, 113, 116, 107, 118, 130, 103, 116, 130, 128, 122, 104, 101, 122, 103, 134, 122, 108, 111 (use groups of 100–104, 105–109, 110–114 and so on)

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3 In the frequency table below, the missing information is

Score	Frequency (f)
0–4	5
5–9	
10–14	15
	17
20–24	10
Total	50

A a frequency of 10 and the class interval 15–19
B a frequency of 3 and the class interval 15–19
C a frequency of 3 and the class interval 10–19
D a frequency of 5 and the class interval 15–19
E a frequency of 47 and the class interval 15–19

4 **Example 12** Construct histograms for the frequency tables in question **1**. Interpret each histogram.

Reasoning and communication

5 The masses in grams of a sample of 48 eggs were measured. The results were as follows.

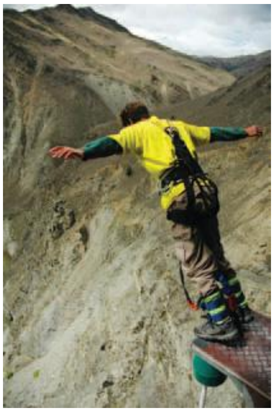
Shutterstock.com/gillmar

57	58	61	59	62	59	59	56	60	64
58	58	56	59	64	57	60	62	58	60
64	57	61	58	59	57	64	58	59	57
58	64	60	58	60	57	61	64	58	60
61	62	62	58	60	61	57	58		

a Construct a frequency table for this data.
b Construct a histogram to display this data.
c If these eggs are labelled as being 60 grams, would you say that the label is misleading? Give reasons for your answer using percentages.

6 **Example 13** Bungee jumpers must be weighed before they 'take the plunge'. The weights in kg of 40 jumpers were recorded as follows.

41	58	63	37	49
58	71	33	85	58
60	73	81	46	55
38	80	48	50	62
61	59	63	44	77
62	58	73	62	75
52	60	69	61	55
47	76	42	66	70


Shutterstock.com/Henrique Daniel Araujo

- a Use class intervals of 30–<40, 40–<50 and so on, to construct a frequency table for the data.
- b Construct a histogram to display this data.
- c From 200 bungee jumpers, how many would you expect to weigh 70 kg or more? Justify your answer.

7 The numbers of litres of oil used in servicing cars each day by a mobile mechanic were as follows.

172	164	115	142	98	104	157
149	267	163	124	201	187	79
133	152	221	184	226	205	98
162	171	213	188	173	156	224
184	166	152	210	172	181	207
178	224	167	185	110	108	

- a Construct a frequency table for the data using class intervals of size 20, starting at 70.
- b Construct a histogram to display this data.
- c Describe what the graph shows.

8 The age, in years, of employees of Burger Heaven were recorded as follows.

18	19	18	17	20	20	24	15	24	19
15	40	21	17	20	22	23	21	24	23
34	19	45	20	15	21	24	27	19	33
34	24	16	18	30	21	26	31	16	25
49	21	21	35	16	22	15	25	44	23.

- a Organise the data into a frequency table using class intervals of 15–<20, 20–<25 and so on.
- b How many people work at Burger Heaven?
- c Construct a histogram to display this data.
- d What percentage of employees are 40 or over?
- e Describe what the graph shows.

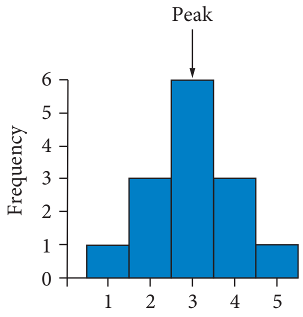
8.06 DESCRIBING THE DISTRIBUTION OF NUMERICAL DATA

When numerical data is displayed using a histogram, the shape of the distribution of the data can be identified, together with the centre of the data and its spread. Any outliers within the data can be identified.

Shape and outliers

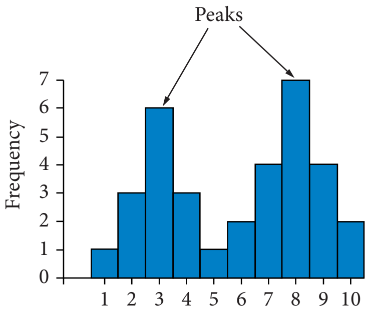
The shape of a histogram can be described as symmetrical or skewed; of course it may be neither. Any outliers must be identified.

Symmetrical



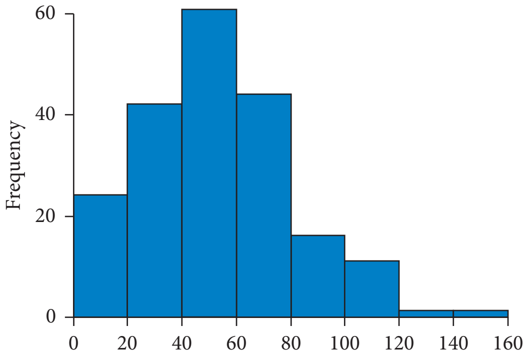
The tallest column (one peak) is in the middle. Either side of the peak is approximately the same.

Bimodal



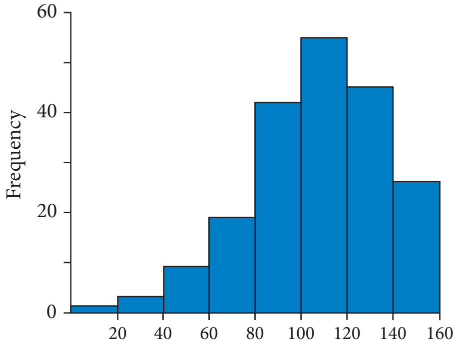
The histogram has two distinct peaks. If there are more than two peaks, then the histogram can be described as multimodal.

Postively skewed

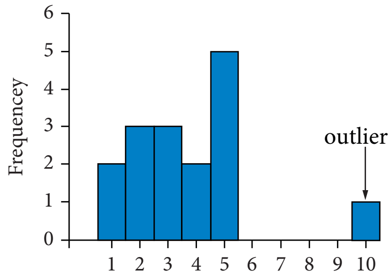


The taller columns are near the start then columns become smaller as you move to the right.

Negatively skewed



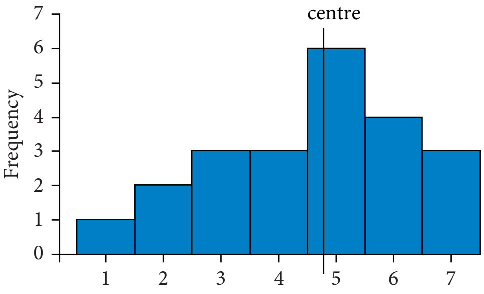
Smaller columns at the start then columns become taller as you move to the right.



Outliers are data values that stand out from the main body of data. When describing other features of a histogram outliers may need to be ignored.

Centre

When the data is symmetrical the centre can be located by sight. It will correspond to the highest peak. When the data is not symmetrical, you need to find the centre.



The centre position can be found using $\frac{\text{total frequency} + 1}{2}$.

For this histogram the frequencies total 22 therefore the centre position $= \frac{23}{2} = 11.5$.

Add up the frequencies starting at the first column and working right until you get to the column that has the 11th and 12th values in it. The centre is in the column representing data values of 5.

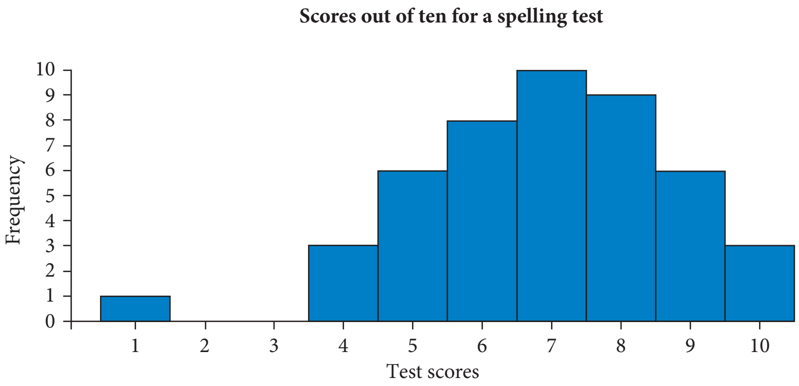
If a histogram has class intervals, state which interval the centre value is in.

Spread

For a histogram where each column represents one score, the spread is from the lowest data value to the highest data value. If the histogram represents grouped data, then the approximate spread of the data is from the lower end of the first class interval to the upper end of the last class interval.

Example 14

The following histogram displays the scores out of ten for a spelling test taken by forty six students. Write a brief report describing the data displayed by this graph.



Solution

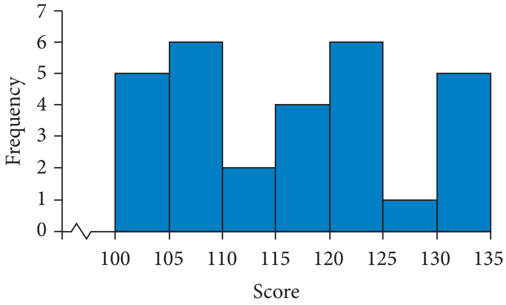
Find the centre.	The centre position = $\frac{46+1}{2} = 23.5$.
Add up the frequencies until you reach the column with the 23rd and 24th data values in it.	$1 + 3 = 4$, $4 + 6 = 10$, $10 + 8 = 18$, $18 + 10 = 28$, therefore the centre data value is a score of 7.
Write a report commenting on shape and outliers, centre and spread, relating them to what the data represents.	The histogram shows the test scores ranged from 1 to 10 with 1 being an outlier. Ignoring the outlier, the shape of the data is approximately symmetrical with the centre score and the most common score being 7.

EXERCISE 8.06 Describing the distribution of numerical data

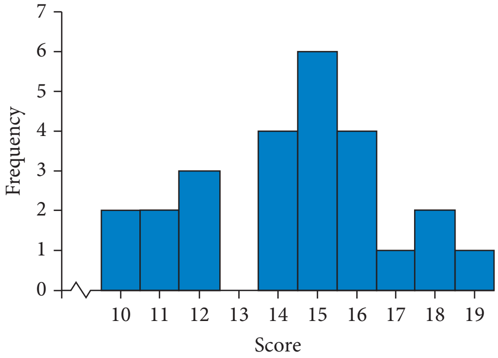
Concepts and techniques

1 Example 14 Find the centre value or class interval for the following histograms.

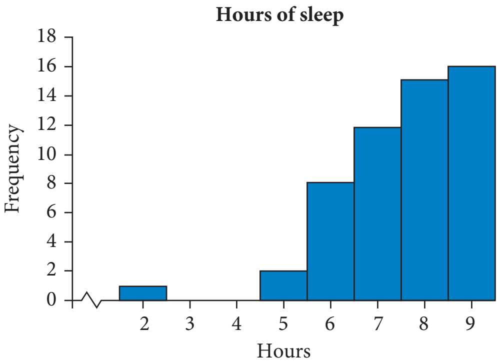
a



b



The following histogram relates to questions **2**, **3** and **4**.




Shutterstock.com/Andywak

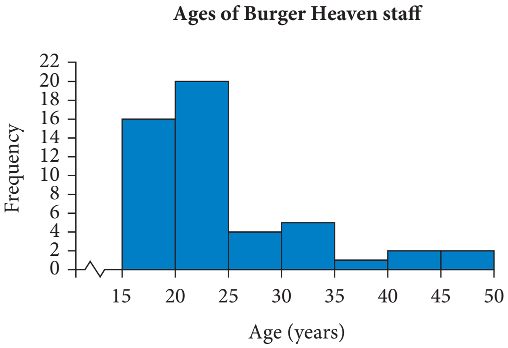
2 The most appropriate description for the shape of the histogram is
- A symmetrical
- B bimodal
- C positively skewed with an outlier
- D negatively skewed with an outlier
- E none of the above

3 The centre value for the number of hours sleep is
A 6 B 7 C 8 D 15 E 27

4 The spread of the data could be described as:
- A Hours of sleep varies from 0 to 9 hours
- B Frequency varies from 0 to 9 hours
- C Hours of sleep varies from 5 to 9 hours
- D Frequency varies from 5 to 9 hours
- E Hours of sleep varies from 2 to 9 hours

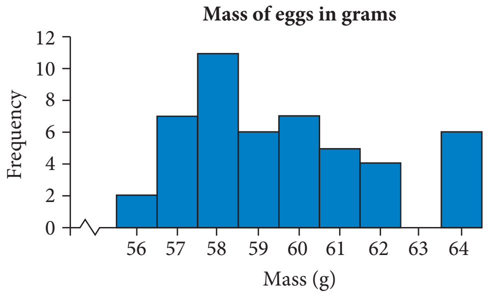
Reasoning and communication

5 The following histogram displays the ages of 50 Burger Heaven staff members. Write a brief report describing the data displayed in this graph.



9780170250252

6 The following histogram displays the masses of eggs in grams. Write a brief report describing the data displayed in this graph.



8.07 MEASURES OF CENTRAL TENDENCY: MEAN

Summary statistics for numerical data relating to the measure of centre and spread can be calculated to help further analyse data.

The measures of centre are **mean**, **median** and **mode**.

The mean

The mean, or average, is a measure of centre.

IMPORTANT

When given a list of values,

$$\text{Mean, } \bar{x} = \frac{\text{sum of all the values}}{\text{number of values}} = \frac{\sum x}{n}$$

Σ means 'the sum of'
x represents a value
n is the total number of values

Example 15

Calculate, correct to one decimal place, the mean of the following data set.
The maximum daily temperatures in °C in Mudgee for the first two weeks in January.

30	28	26	31	34	35	32
33	21	25	28	32	32	35

Solution

Count how many values there are in the list then use the formula.

$$\bar{x} = \frac{\text{sum of all the values}}{\text{number of values}}$$

$$= \frac{\sum x}{n}$$

$$\bar{x} = \frac{30 + 28 + 26 + 31 + 34 + 35 + 32 + 33 + 21 + 25 + 28 + 32 + 32 + 35}{14}$$

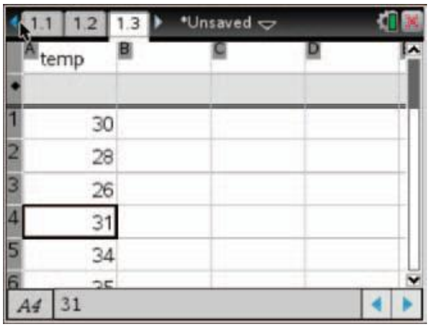
$$= \frac{422}{14}$$

$$= 30.14285...$$

$$= 30.1 \text{ correct to 1 decimal place}$$

TI-Nspire CAS

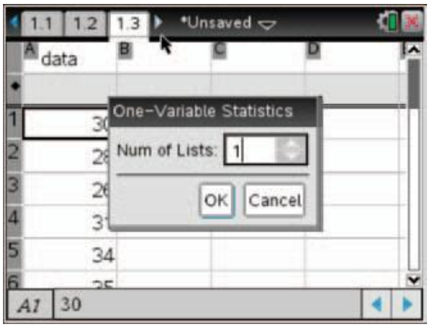
Use New Document with a Lists & Spreadsheet page.
Name column A temp then enter data.



	temp
1	30
2	28
3	26
4	31
5	34

A4 31

Press menu 4: Statistics, 1: Stat Calculations, 1: One-variable Statistics. Set Number of lists to 1 then click OK on the pop-up screen.

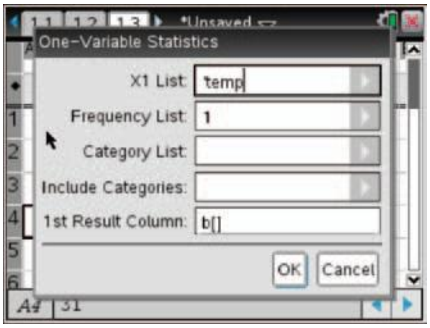


One-Variable Statistics

Num of Lists: 1

A1 30

On the next pop-up screen set the X1 list to temp, Frequency List should be 1, then click OK.



One-Variable Statistics

X1 List: temp

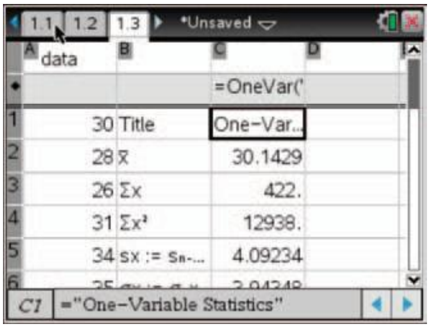
Frequency List: 1

Category List:

Include Categories:

1st Result Column: b[]

The mean is the answer in column C row 2, $\overline{x} = 30.142857 \approx 30.1$.



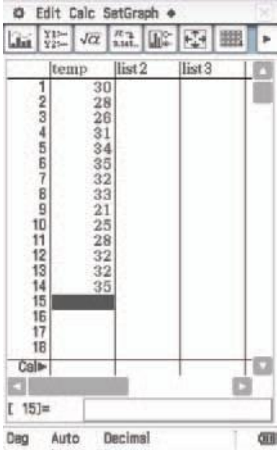
	A data	B	C	D
◆			=OneVar('	
1	30	Title	One-Var...	
2	28	x̄	30.1429	
3	26	Σx	422.	
4	31	Σx²	12938.	
5	34	sx := sn-...	4.09234	

C1 ="One-Variable Statistics"

ClassPad

Use the **Statistics** application.

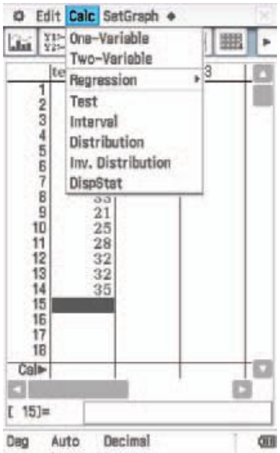
Rename list1 as temp then enter the data into this list.



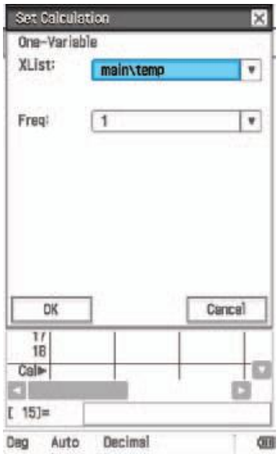
	temp	list 2	list 3
1	30		
2	28		
3	26		
4	31		
5	34		
6	35		
7	32		
8	33		
9	21		
10	25		
11	28		
12	32		
13	32		
14	35		
15			
16			
17			
18			

[15]=

Tap **Calc** then **One-Variable**.

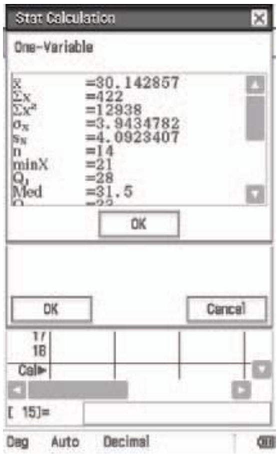


Set the XList as main\temp and ensure that Freq is 1 then tap **OK**.



```
Set Calculation
One-Variable
XList:  main\temp
Freq:   1
```

The mean is the first result, $\overline{x} = 30.142857 \approx 30.1$.



```
Stat Calculation
One-Variable
x̄      =30.142857
Σx     =422
Σx²    =12938
σx     =3.9434782
sx     =4.0923407
n      =14
minX   =21
Q1     =28
Med    =31.5
```

IMPORTANT

When calculating the mean of data which is represented in a frequency table the following formula applies.

$$\text{Mean, } \overline{x} = \frac{\sum fx}{\sum f}$$

The use of this formula requires an extra column to be added to the frequency table, $f \times x$.

Data (x)	Frequency (f)	$f \times x$
1	2	$1 \times 2 = 2$
2	7	14
3	11	33
Total	20	49

Example 16

The scores for the players in a nine-hole golf competition are represented in the following frequency table.

Score (x)	Frequency (f)
37	2
38	4
39	7
40	4
41	1

Calculate the mean score correct to one decimal place.

Solution

Add an extra column to the table and an extra row for totals. Fill in the $f \times x$ column and totals.

Score (x)	Frequency (f)	$f \times x$
37	2	74
38	4	152
39	7	273
40	4	160
41	1	41
Total	18	700

Use the formula.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{700}{18}$$

$$= 38.888.....$$

$$= 38.9$$

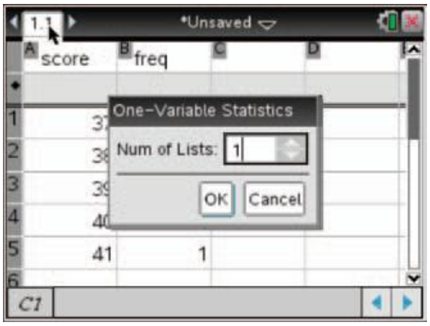
The mean score for the nine-hole golf competition was 38.9.

TI-Nspire CAS

Use a New Document with a Lists & Spreadsheet page.
Name column A score and column B freq then enter the data.

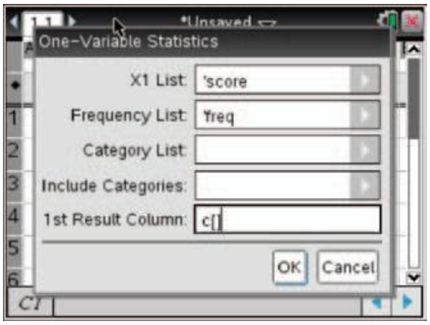
score	freq
37	2
38	4
39	7
40	4
41	1

Press menu 4: Statistics, 1: Stat Calculations, 1: One-variable Statistics. Set Number of lists to 1 then click **OK** on the pop-up screen.



Num of Lists: 1

On the next pop-up screen select variable name score for the X1 List and select freq for Frequency then click **OK**.



X1 List: 'score

Frequency List: 'freq

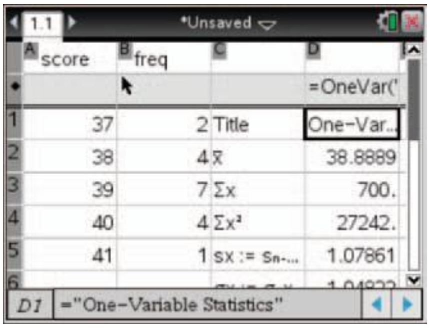
Category List:

Include Categories:

1st Result Column: c[]

The mean is located in row 2 and column D.

$\bar{x} = 38.888889 \approx 38.9$.



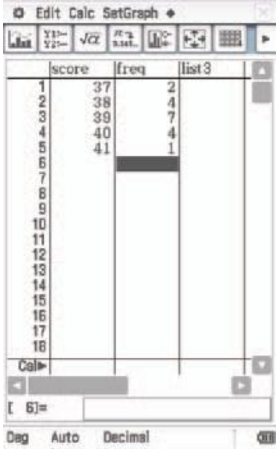
	A score	B freq	C	D
◆				=OneVar('
1	37	2	Title	One-Var...
2	38	4	$\bar{x}$	38.8889
3	39	7	Σx	700.
4	40	4	Σx^2	27242.
5	41	1	sx := Sn-...	1.07861

D1 ="One-Variable Statistics"

ClassPad

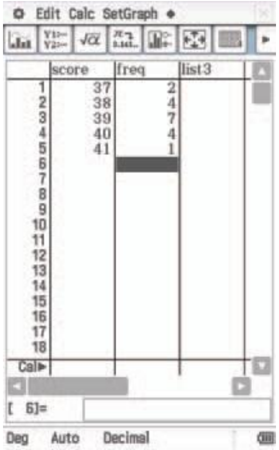
Use the **Statistics** application.

Rename **list1** as score and **list2** as freq then enter the data into the lists. To clear variables from previous calculations tap ⚙ then **Variable Manager** then select **main** and tap **Edit** then **Delete** then **OK**.



	score	freq	list3
1	37	2	
2	38	4	
3	39	7	
4	40	4	
5	41	1	

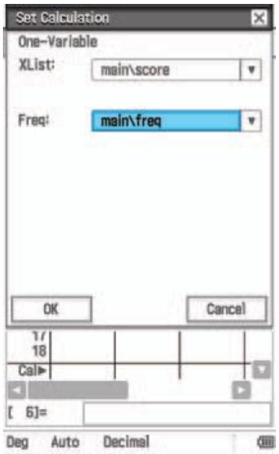
Tap **Calc** then **One-Variable**.



	score	freq	list3
1	37	2	
2	38	4	
3	39	7	
4	40	4	
5	41	1	

Set XList to main\score and Freq to main\freq.

Tap OK.

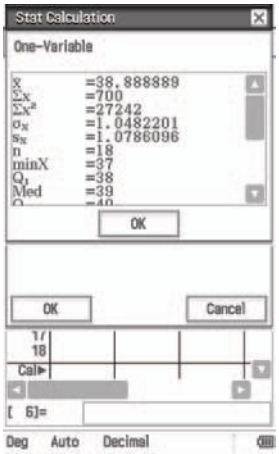


One-Variable

XList: main\score

Freq: main\freq

The mean is the first result,
$\bar{x} = 38.888889 \approx 38.9$.



One-Variable

```
x̄       =38.888889
Σx      =700
Σx²     =27242
σx      =1.0482201
sx      =1.0786096
n       =18
minX    =37
Q₁      =38
Med     =39
```

If the data in the frequency table is grouped into class intervals, only an estimate of the mean can be calculated. This is due to the fact that x is taken to be the midpoint of the class interval, which may not accurately reflect the original data. If the original data is provided it should be used to calculate the mean rather than the grouped frequency table.

Class interval	***x* midpoint**	**Frequency (*f*)**	$f \times x$
0–4	2	2	$2 \times 2 = 4$
5–9	7	3	21
10–14	12	5	60
	Total	10	85

As the first class interval is 0–4, the midpoint x is found by adding the 2 end numbers and dividing by 2, $\frac{0+4}{2} = 2$.

Example 17

The ages of patients at a medical centre in one afternoon were recorded and are displayed in the following frequency table.

Age	**Frequency (*f*)**
0–9	8
10–19	7
20–29	6
30–39	8
40–49	5
50–59	4
60–69	3
70–79	1

Calculate the mean age of patients that visited the medical centre.

Solution

Add and fill in the two extra columns (x and $f \times x$) and the 'Total' row.

Age	**Midpoint (x)**	**Frequency (*f*)**	$f \times x$
0–9	4.5	8	36.0
10–19	14.5	7	101.5
20–29	24.5	6	147.0
30–39	34.5	8	276.0
40–49	44.5	5	222.5
50–59	54.5	4	218.0
60–69	64.5	3	193.5
70–79	74.5	1	74.5
	Total	42	1269.0

Use the formula.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

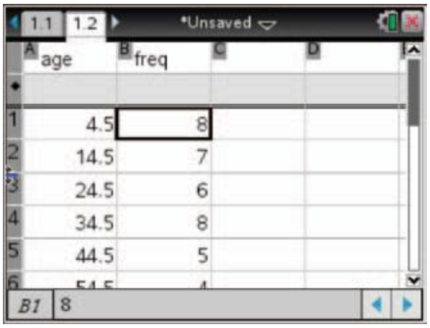
$$\begin{aligned} \bar{x} &= \frac{1269}{42} \\ &= 30.2142..... \\ &\approx 30.2 \end{aligned}$$

The estimated mean age of patients to visit the medical centre is
30.2 years old.

TI-Nspire CAS

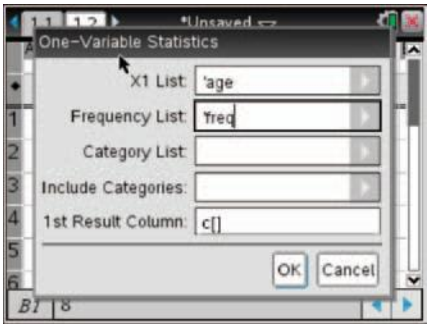
Use a New Document with a Lists & Spreadsheet page.

This is the same as Example **16** but you must enter the midpoint for each age class interval in the age column.



	A age	B freq	C	D
◆				
1	4.5	8		
2	14.5	7		
3	24.5	6		
4	34.5	8		
5	44.5	5		

B1 8



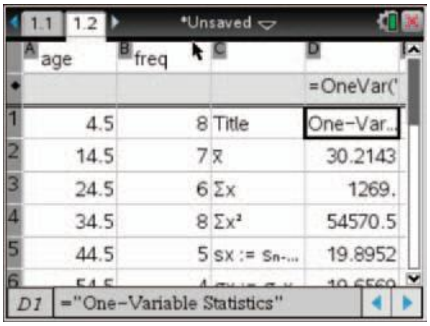
X1 List: 'age

Frequency List: 'freq

Category List:

Include Categories:

1st Result Column: c[]

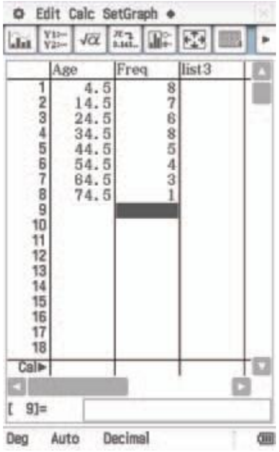


	A age	B freq	C	D
◆				=OneVar('
1	4.5	8	Title	One-Var..
2	14.5	7	$\bar{x}$	30.2143
3	24.5	6	Σx	1269.
4	34.5	8	Σx^2	54570.5
5	44.5	5	sx := sn-...	19.8952

D1 ="One-Variable Statistics"

ClassPad

Use the **Statistics** application. This is the same as Example **16** except that you enter the midpoints into the Age list. Enter the frequencies into the Freq list as before.



	Age	Freq	list3
1	4.5	8	
2	14.5	7	
3	24.5	6	
4	34.5	8	
5	44.5	5	
6	54.5	4	
7	64.5	3	
8	74.5	1	

[9]=

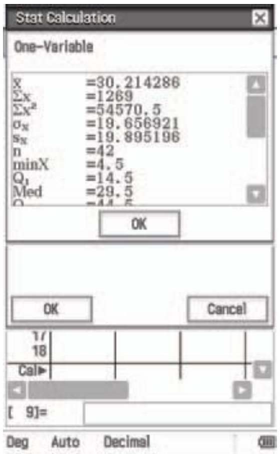
Tap **Calc** then **One-Variable.**

Make sure the screen looks like the one on the right.

Tap $\boxed{\text{OK}}$.

```
Set Calculation
One-Variable
XList:   main\Age
Freq:    main\Freq
```

The mean is the first result,
$\bar{x} = 30.214286 \approx 30.2$.



```
Stat Calculation
One-Variable
x̄        =30.214286
Σx       =1269
Σx²      =54570.5
σx       =19.656921
sx       =19.895196
n        =42
minX     =4.5
Q1       =14.5
Med      =29.5
```

The advantage of using the mean as a measure of centre is that it uses all of the data and is reliable if there are no outliers.

The mean is affected by outliers in the data set and thus can be misleading. Think about the average age of students in your class. You are all basically the same age. Now add in your 40-year-old teacher. Is the mean still an accurate measure of the average age of the class?

IMPORTANT

The mean is not a good representation for the measure of centre when there are outliers present in the data.

EXERCISE 8.07 Measures of central tendency: mean

Concepts and techniques

1 Example 15 Find the mean for each set of data below.

a	1	1	2	5	5	7	9	10			
b	37	31	35	39	31	32	34	32	35	38	
c	28	40	38	42	45	29	31	41	30		
d	5	8	14	9	10	7	11	15	8	7	5

9780170250252

2 The stem-and-leaf plot below represents the number of points scored by the Sharks in every match of the football season.
Key: 4|3 means 43

Stem	Leaf					
4	3	5	9			
5	0	2	7	8		
6	1	2	4	5	7	8
7	0	2	3	9		
8	2	4	9			

a How many matches were played in the season?
b Calculate the mean score.

3 Joel is training for a triathlon. He swam the following times, in minutes, in his last ten races.

28 34 22 24 25 24 26 26 24 27

a Joel's mean swim time is?
A 24 min. B 25 min. C 25.5 min. D 26 min. E 27 min.
b If the outlier score of 34 was removed from the set of data, would the mean increase or decrease? Give a reason for your answer.

4 The stem-and-leaf plot below shows the times, in seconds, for competitors in a slalom ski race.
Key: 9|1 means 91 seconds

Stem	Leaf					
9	1	7	9			
10	4	5	6	6	8	
11	0	2	3	4	4	5
12	1	2	3	3	3	9
13	2	3	4	7	7	
14	6	9				
15	0	1				

a How many skiers participated in the race?
b Calculate the mean time for the race, correct to one decimal place.

5 Example 16 Students were surveyed about the number of DVDs they had purchased in the last six months. The results are shown in the frequency table below.

DVDs (x)	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5


Alamy/PhotoEdit

a How many students were surveyed?
b Calculate the mean number of DVDs purchased, correct to one decimal place.

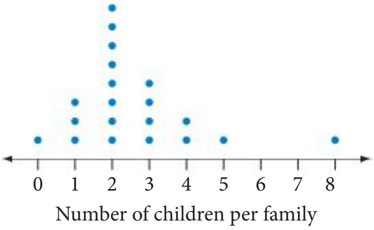
Reasoning and communication

6 Meg's Matches has 'average contents 50' printed on each box. A quality controller counted the contents of a sample of 160 matchboxes. The results are displayed in the frequency table below.

Number of matches (x)	Frequency (f)
48	10
49	45
50	52
51	39
52	9
53	5

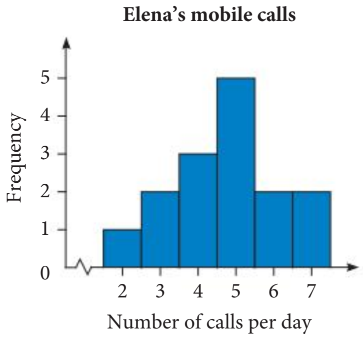
- **a** Calculate the average number of matches for the sample, correct to one decimal place.
- **b** Is the claim 'average contents 50' justified? Give a reason for your answer.

7 This dot plot shows the number of children in each family living on Willard Crescent.



- **a** How many families live on Willard Crescent?
- **b** Calculate the mean number of children per family.
- **c** What is the outlier?
- **d** If the outlier is removed from the data set, how is the mean affected?
- **e** Give two reasons why the mean is not a good measure of centre for this data.
- **f** An extra family moved into Willard Crescent, increasing the mean number of children to 3. How many children did the new family have?

8 This frequency histogram shows the number of mobile phone calls made by Elena each day over a number of days.



- **a** Construct a frequency table for this data, including an 'fx' column.
- **b** Over how many days was the number of calls Elena made recorded?
- **c** Calculate the mean number of phone calls made by Elena per day, correct to one decimal place.

9780170250252

9 Example 17 The police used radar to check the speeds of motor vehicles driving in a 40 km/h zone outside a local primary school one morning. They recorded the results in the table below.

Speed (km/h) (*x*)	Frequency (*f*)
36–40	64
41–45	36
46–50	18
51–55	15
56–60	11
61–65	5

Newspix/Tim Carrafa

a How many motor vehicles had their speeds checked?

b Calculate the mean speed of the vehicles, correct to two decimal places. Explain why this answer can only be considered as an estimate.

10 The weekly wages at Yen's restaurant are displayed in the frequency table below.

Wage (\$) (*x*)	Frequency (*f*)
100–<200	5
200–<300	11
300–<400	20
400–<500	4
500–<600	3
600–<700	1

a How many employees are there?

b Explain what 100–<200 represents.

c Calculate the estimated mean weekly wage, correct to the nearest dollar.

d What could account for the large number of employees earning under \$400 per week?

8.08 MEASURES OF CENTRAL TENDENCY: MEDIAN AND MODE

Two other measures of centre are the median and mode.

IMPORTANT

The **median** represents the middle score when the data values are ordered from smallest to largest, therefore equal numbers of the data values lie below the median and above it.

The **mode** is the most common data value or the data value with the highest frequency. There can be more than one mode but if every data value occurs the same number of times then there is no mode.

When locating the median for a set of data values it is helpful to know its position, for example the 11th data value.

IMPORTANT

The position of the median is the $\frac{n+1}{2}$th data value, where n is the total number of data values.

When using this formula, interpreting the answer is slightly different, depending on whether there is an even or odd number of data values.

ODD number of data values

If you have 21 data values then the median is the $\frac{21+1}{2}$th = the 11th data value.

For an odd number of data values the median is always one of the data values.

EVEN number of data values

If you have 20 data values, then the position of the median is the $\frac{20+1}{2}$th = the 10.5th data value. This indicates that the median is positioned halfway between the 10th and 11th data values. Median = $\frac{\text{value 10} + \text{value 11}}{2}$

For an even number of data values the median is always positioned in the middle of two data values. If these two data values are the same, then the median is equal to this value.

Example 18

For the following set of data, find

30	28	26	31	34	35	32
33	21	25	28	32	32	35

a the median

b the mode.

Solution

a Order the data from the smallest to the largest.

21 25 26 28 28 30 31 32 32 32 33 34 35 35

Locate the position of the median.

The median is the $\frac{14+1}{2}$th = the 7.5th data value, therefore halfway between the 7th and 8th data values.

Locate the 7th and 8th data values and state the median.

21 25 26 28 28 30 31 32 32 32 33 34 35 35

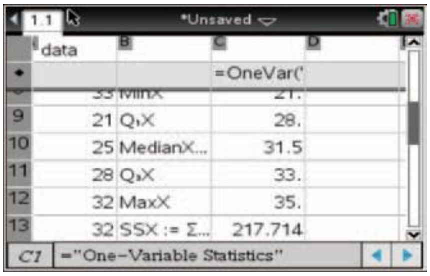
Median = $\frac{31+32}{2} = 31.5$

TI-Nspire CAS

Use a New Document with a Lists & Spreadsheet page.

Use the same steps as for finding the mean in the previous exercise.

Scroll down the page until you find MedianX.



	data	B	C	D
◆			=OneVar('	
	33	MinX	21.	
9	21	Q₁X	28.	
10	25	MedianX...	31.5	
11	28	Q₃X	33.	
12	32	MaxX	35.	
13	32	SSX := Σ...	217.714	

C1 ="One–Variable Statistics"

b State the number with the highest frequency.

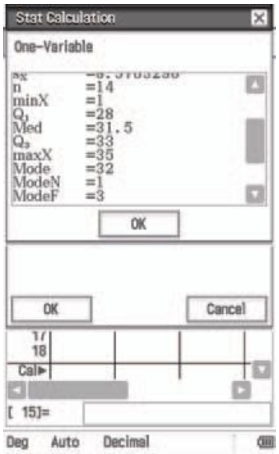
Mode = 32 as it appears 3 times.

ClassPad

Use the same steps as for the mean in the previous exercise.

Scroll down to Med = 31.5 and Mode = 32. Ignore ModeN and ModeF.

If there is more than one mode, both or all will be listed as Mode.



```
Stat Calculation
One-Variable
n       =14
minX    =1
Q1      =28
Med     =31.5
Q3      =33
maxX    =35
Mode    =32
ModeN   =1
ModeF   =3
```

Remember that for the median, data values must be in order from smallest to largest. Stem-and-leaf plots, dot plots and frequency tables all represent a list of ordered data values and thus can be used to easily locate the median.

Example 19

The following table shows the temperatures recorded in Sydney over a time period.

Temperature (°C)	Frequency
28	5
29	6
30	5
31	3
32	4

a For how many days was the temperature measured?

b Find

i the mode

ii the median temperature.

Solution

a Add up the frequency column.

5 + 6 + 5 + 3 + 4 = 23

The temperature was measured on 23 days.

b i The temperature with the highest frequency.

The modal temperature is 29°C.

ii Find the position of the median.

The median position = $\frac{23+1}{2}$th = 12th data value.

Add up the frequencies in the table until you reach the 12th data value.

Temp	Frequency
28	5
29	6
30	5
31	3
32	4

The first five temperatures are 28. 5 + 6 = 11, therefore the 6th to the 11th temperatures are 29.

11 + 5 = 16, therefore the 12th to the 16th temperatures are 30.

Therefore the median temperature is 30°C.

If data is presented in a frequency table with class intervals, then you will be asked to find the median class (the class interval with the median in it) and the modal class (the class interval with the highest frequency).

Example 20

The following data relates to phone call costs.

Call cost (cents)	Frequency
0–<20	10
20–<40	16
40–<60	21
60–<80	11
80–<100	10

The class interval 20–<40 means calls that cost 20 cents but less than 40 cents. 40 cents is not included in this group of values.

Find

a the median class

b the modal class.

Solution

a Find the position of the median.

The median position is the $\frac{68+1}{2}$th = 34.5th data value, therefore it is between the 34th and 35th data values.

Add up the frequencies in the table until you reach the class that has the 34th and 35th data values.

Call cost (cents)	Frequency	
0–<20	10	First 10 values
20–<40	16	11th to the 26th value
40–<60	21	27th to the 47th value
60–<80	11	
80–<100	10	

As both the 34th and 35th data values are in the same class interval, the median class is 40–<60.

b State the class interval with highest frequency.

The modal class is 40–<60.

Deciding which measure of centre is the most appropriate to use

Use the mean when

- **i** the data is numerical
- **ii** the question asks for the average
- **iii** the data has no outliers and is not skewed.

Use the median when

- **i** the data is numerical
- **ii** the question asks for a middle value
- **iii** the data has outliers or is skewed.

Use the mode when

- **i** the data is categorical or numerical
- **ii** the question asks for the most common feature
- **iii** the data has outliers or is skewed.

EXERCISE 8.08 Measures of central tendency: median and mode

Mode, median, mean

Measures of central tendency

Concepts and techniques

1 Example 18 For each data set below, find

i the median ii the mode(s).

a 1 1 2 5 5 7 9 10
b 37 31 35 39 31 32 34 32 35 38
c 28 40 38 42 45 29 31 41 30
d 5 8 14 9 10 7 11 15 8 7 5

2 The maths test marks obtained by a class of students are shown in the following ordered stem-and-leaf plot.

Key: 4|7 means 47%

Stem	Leaf					
4	4	7	7	8		
5	2	6	8	9	9	
6	1	3	5	5	7	8
7	0	2	3	4	5	
8	3	7	8			
9	2	8				

For this data, find

a the median **b** the mode(s).

Shutterstock.com/Ddana E Fry

The following information relates to questions **3** and **4**.

Ebony swam the following times, in minutes, during ten races.

28 34 22 24 25 24 26 26 24 27

3 Her median swim time, in minutes, was:

A 24 **B** 25 **C** 25.5 **D** 26 **E** 24.5

4 Her modal swim time, in minutes, was:

A 24 **B** 25 **C** 25.5 **D** 26 **E** 24.5

5 Example 19 Some students were surveyed about the number of DVDs they had purchased in the last six months. The results are shown in the following frequency table.

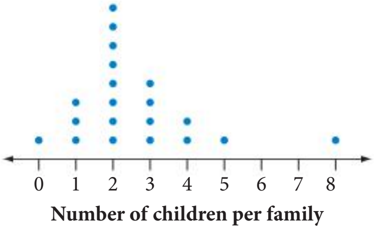
Number of DVDs	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5

a How many students were surveyed?

b Find the mode.

c Find the median.

6 This dot plot shows the number of children in each family living on Willard Crescent.



a What is the median?

b What is the mode?

c State any outliers.

d If the outlier is removed from the data set, explain why there is no effect on

i the median

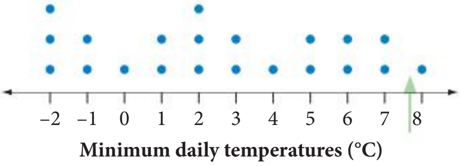
ii the mode

7 This stem-and-leaf plot shows the weights in kilograms of the competitors in an open water swimming race.
Key: 5|1 means 51 kilograms

Stem	Leaf					
5	2	8	9			
6	4	5	7	7	9	
7	0	1	2	4	4	7
8	1	2	3	3	3	9
9	1	2	5	8	9	
10	5	8				
11	0	2				

a Find the median weight of the swimmers in the race.
b Find the modal weight.
c Find the average weight of the competitors, correct to two decimal places.

8 This dot plot shows the minimum daily temperatures (°C) in Hobart over a 3-week period.



a What is the mode?
b What position is the median in? What is the median?
c Calculate the mean correct to one decimal place.

9 Example 20 The following frequency table shows the heights of young trees in a section of a national park before they were planted.

Height (cm)	Frequency (f)
20–29	28
30–39	45
40–49	74
50–59	63
60–69	24

For this data, find
a the position of the median, and hence state the median class
b the modal class.

10 The heights of a group of junior tennis players are shown in the frequency table below.

Height (cm)	Frequency (*f*)
140–<145	3
145–<150	5
155–<160	6
160–<165	9
165–<170	5
170–<175	2
175–<180	2

a What is the modal class for heights?

b What is the median class for the heights?

Reasoning and communication

11 Which measure of centre is most appropriate for describing each of the following sets of data? Justify your answer.

a Exam marks for a class.

b The most common shirt size purchased by teenage girls.

c Rent paid for houses in Melbourne.

d Weights of football players in a team.

e Brands of mobile phones.

AAP Images/Matt Grayson/PA Wire

12 Ten houses were sold this week in Darwin for the following prices.

\$376 000	\$1 200 000	\$270 000	\$308 000	\$372 000
\$409 000	\$387 000	\$582 000	\$460 000	\$238 000

a Calculate the mean house price.

b Calculate the median house price.

c Which measure of centre is higher, the mean or the median?

d Which measure of centre is more appropriate for describing the average house price? Explain your reasoning.

8.09 MEASURES OF SPREAD: RANGE AND INTERQUARTILE RANGE

The mean, median and mode are measures of centre for a data set. There are three summary statistics that are measures of spread: the **range**, the **interquartile range (IQR)** and the **standard deviation**.

IMPORTANT

The range is the simplest measure of spread.

Range = highest data value – lowest data value

Example 21

Find the range for each of the following data sets.

a 3 4 5 6 7

b

Score	Frequency (f)
11	3
12	6
13	7
14	11
15	12
16	6

Solution

a Use the formula for range.

Range = highest data value – lowest data value
= 7 – 3
= 4

b The frequency column can be ignored for the range. Use the formula.

Range = highest data value – lowest data value
= 16 – 11
= 5

A data set can be split into quarters using the median (Q_2), the first quartile (Q_1) and the third quartile (Q_3).

Q_2 is the median of all the values and splits the data set into two equal groups – the lower group and the upper group.

Q_1 is the median of the lower group of the data.

Q_3 is the median of the upper group of the data.

Always calculate the median first, then Q_1 and Q_3

For example

Lower group Upper group

		Q_1			Q_2			Q_3		
21	22	**23**	23	24	**25**	26	27	**27**	28	28

2 values 2 values 2 values 2 values

IMPORTANT

The middle 50% of the data lies between Q_1 and Q_3 .The interquartile range (IQR) measures the spread of the middle 50% of data

$$\text{IQR} = Q_3 - Q_1$$

As calculating Q_3 and Q_1 is basically finding a median, the working is slightly different for odd and even data values.

Odd number of data values

12 13 13 14 $\boxed{15}$ 15 16 16 17

Lower group Upper group

The median is one of the data values, in the case above 15. This data value is not included in either the lower group or upper group to calculate Q_1 and Q_3.

Even number of data values

21 21 22 23 24 | 25 25 27 28 29

Lower group Upper group

The median is not one of the data values, it is between two, in the case above it is 24.5. The median splits the data directly in half, therefore all data values are included in the lower group or upper group to calculate Q_1 and Q_3.

Example 22

Find the IQR for each of the following data sets.

a 1 1 2 2 3 4 5 6
6 7 7 8 8 8 9 9

b Key: 4|7 means 47 out of 100

Stem	**Leaf**					
4	4	7	7	8		
5	2	6	8	9	9	
6	1	3	5	5	7	8
7	0	2	3	4	5	
8	3	7	8			
9	2	8				

Solution

a Find the median by locating its position.

1 1 2 2 3 4 5 6 |
6 7 7 8 8 8 9 9

Median is the $\frac{16+1}{2}$th = 8.5th data value, so between the 8th and the 9th values.

Median = 6

Remember that the data values must be in order from smallest to largest.

Locate Q_1 and Q_3.
There are 8 values in each half, therefore the position of the quartiles is $\frac{8+1}{2}$th = 4.5th data value, so between the 4th and the 5th values in both groups.

Lower group
1 1 2 2 | 3 4 5 6
6 7 7 8 | 8 8 9 9
Upper group

$Q_1 = 2.5$ and $Q_3 = 8$.

Calculate the IQR using the formula.

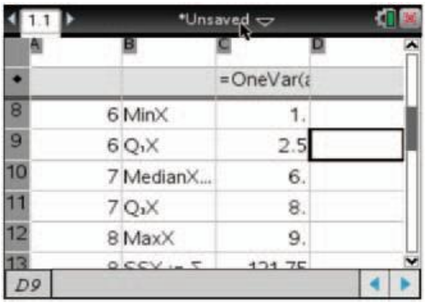
$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 8 - 2.5 \\ &= 5.5 \end{aligned}$$

TI-Nspire CAS

Calculate the one variable statistics.

The CAS calculator will not find the IQR but it finds Q_1 and Q_3.

Notice that the maximum and minimum values are also listed. These can be used to calculate the range.



1.1 *Unsaved

	A	B	C	D
◆			=OneVar(	
8		6 MinX	1.	
9		6 Q₁X	2.5	
10		7 MedianX...	6.	
11		7 Q₃X	8.	
12		8 MaxX	9.	
13		9 SSX := Σ...	121.75	

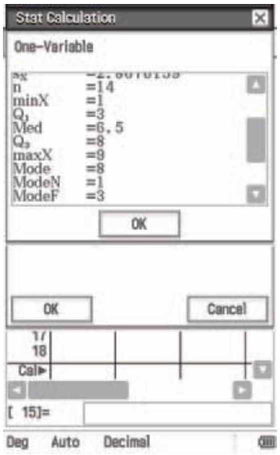
D9

ClassPad

Calculate the one variable statistics.

The CAS calculator will not find the IQR but it finds Q_1 and Q_3.

Notice that the maximum and minimum values are also listed. These can be used to calculate the range.



Stat Calculation

One-Variable

```
sx     =2.8010139
n      =14
minX   =1
Q1     =3
Med    =6.5
Q3     =8
maxX   =9
Mode   =8
ModeN  =1
ModeF  =3
```

b Find the median by locating it's position.

The median is the $\frac{25+1}{2}$th = the 13th value.

Key: 4|7 means 47 out of 100

Stem	Leaf					
4	4	7	7	8		
5	2	6 ∣	8	9	9	
6	1	3	5	$\boxed{5}$	7	8
7	0	2	3	4 ∣	5	
8	3	7	8			
9	2	8				

Locate Q_1 and Q_3.

There are 12 values in each half, therefore the position of the quartiles is $\frac{12+1}{2}$th = 6.5th data value, so it is between the 6th and 7th values in both groups.

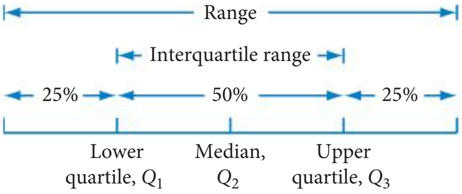
Median = 65

$Q_1 = 57$ and $Q_3 = 74.5$

Calculate the IQR using the formula.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 74.5 - 57 \\ &= 17.5 \end{aligned}$$

The **range** represents the total spread of scores but it is not a good measure of spread if there are outliers. The **interquartile range** is not affected by outliers, because it measures the range of half of the data.



Mean, median, mode and range

EXERCISE 8.09 Measures of spread: range and interquartile range

Concepts and techniques

1 Example 21 Calculate the range of each data set below.

a The number of accidents per month in a factory.

3 0 0 1 2 1 6 0 0 2 1 0

b A golfer's scores for the first nine holes of a golf course.

4 3 5 6 4 3 8 6 6

c Weekly mortgage repayments in dollars.

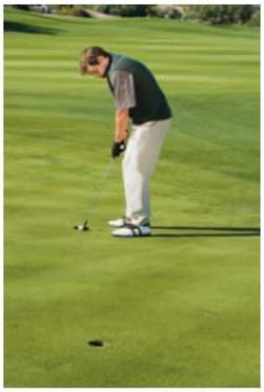
370 628 299 417 354 1027 585 435 509 652 481

d Times in minutes for the swim leg of a triathlon.

28 34 22 24 25 24 26 26 24 27

2 The following frequency table shows Peter's golf scores for 22 games. Find the range of Peter's golf scores.

Golf score	Frequency (*f*)
62	2
63	3
64	2
65	7
66	0
67	2
68	4
69	2
Total	22

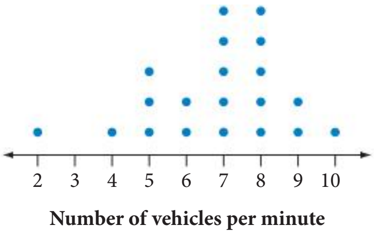


Shutterstock.com/bikeriderlondon

3 Calculate the interquartile range of each data set in question **1**.

The following information relates to questions **4** and **5**.

The dot plot below shows the number of vehicles driving past Westvale High School per minute in a 20-minute period.



4 Example 22 The interquartile range of this data set is:

A 2.5 B 3 C 5 D 8 E 10

5 The range of this data set is:

A 2.5 B 3 C 5 D 8 E 10

6 For the data displayed in the stem-and-leaf plot below, find

a the range

b the interquartile range.

Key: 3|2 means 3.2

Stem	**Leaf**					
3	2	3	3	5	8	
4	0	1	3	5	6	9
5	0	1	7	9		
6	1	2	4	6	7	7
7	0	3	4	5	5	8
8	2	4	4	6	7	

7 Find the range and interquartile range of each of the following sets of data.

a 4 2 5 2 7 5 8 8 3 5 8 3 5 9

b 18 15 14 12 13 17 18 16 14 16 17 19 12 16 16 19

c 24 26 29 33 39 42 41 27 36 34 28 27 24 27 44

d

Score	0	1	2	3	4	5	6	7	8	9
Frequency	2	5	5	7	7	6	6	3	2	1

e

Score	8	9	10	11	12	13	14	15	16	17	18
Frequency	3	7	6	10	12	13	6	5	3	0	2

Reasoning and communication

8 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their completion times, in seconds, for the test were:

45	37	46	34	26	15	35	61
43	48	52	38	30	44	37	

a What was the median time?

b What was the mean time?

c What was the interquartile range of the times?

d What was the range of the times?

e Which measure of centre and spread is more appropriate to use? Justify your answer.

f Based on your answer to part **e** write a short report relating to the data.

8.10 MEASURES OF SPREAD: STANDARD DEVIATION

Another measure of spread is the standard deviation. The range is the difference between the highest and lowest data values, and the interquartile range gives the range of the middle 50% of the data and utilises the upper and lower quartiles. We will now look at the standard deviation which measures the spread of data relative to the mean.

Sometimes data is collected from the whole population, in which case the population standard deviation, σ_x, can be calculated. As the majority of our data sets are from samples, we will look at how to calculate the standard deviation for a sample rather than a population.

The sample standard deviation is denoted by s_x or just s. $s_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$ where x is a data value, $\bar{x}$ is the mean of the data values and n is the number of data values

Using a CAS calculator is the preferred method for calculating the standard deviation. Both the sample and population standard deviations can be calculated. The standard deviation using a CAS calculator is found when the one variable statistics are calculated.

Example 23

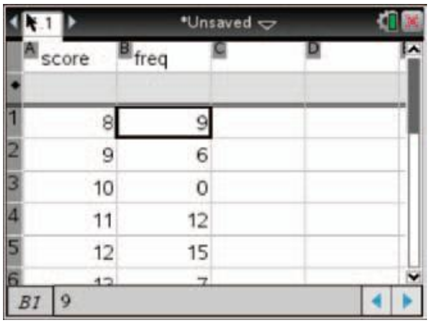
Calculate the standard deviation for the following frequency table. Round your answer correct to two decimal places.

Score (x)	Frequency (f)
8	9
9	6
10	0
11	12
12	15
13	7

Solution

TI-Nspire CAS

Use a New Document with a Lists & Spreadsheet page and enter the data.

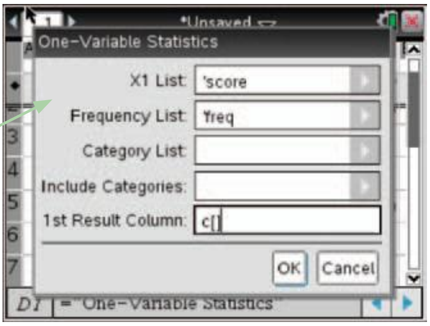


	A score	B freq	C	D
1	8	9		
2	9	6		
3	10	0		
4	11	12		
5	12	15		
6	13	7		

B1 9

Press menu 4: Statistics, 1: Stat Calculations, 1: One-variable Statistics. Click OK on the first pop-up screen, then fill in the second pop-up screen as shown.

If the data is in a single list rather than a frequency table, remember to have Frequency List set to 1.


One-Variable Statistics

X1 List: 'score

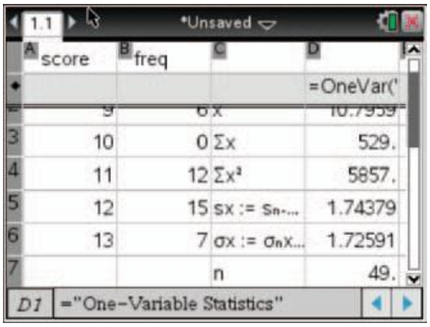
Frequency List: 'freq

Category List:

Include Categories:

1st Result Column: c[]

State the answer.

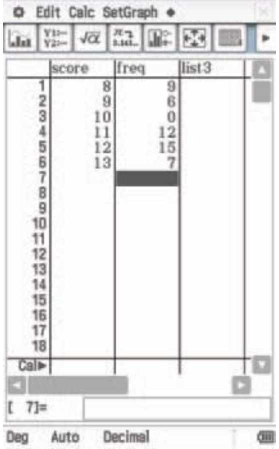


	A score	B freq	C	D
				=OneVar('
3	10	0	Σx	529.
4	11	12	Σx²	5857.
5	12	15	sx := sn-...	1.74379
6	13	7	σx := σnx...	1.72591
7			n	49.

$s_x = 1.74$

ClassPad

Use the **Statistics** application. Enter the scores and frequencies.



	score	freq	list3
1	8	9	
2	9	6	
3	10	0	
4	11	12	
5	12	15	
6	13	7	

Use **Calc** and tap **One-Variable**.

Make sure the screen looks like the one on the right.

Tap OK.

If the data is just a single list and not a frequency table, make **Freq:** 1.

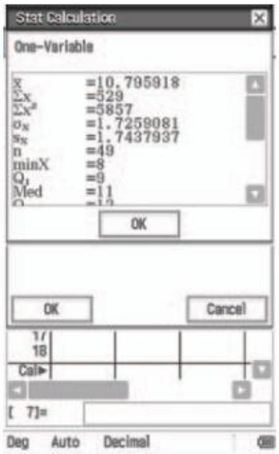

Set Calculation

One-Variable

XList: main\score

Freq: main\freq

The standard deviation
$s = 1.7437937 \approx 1.74$.



```
x̄      =10.795918
Σx     =529
Σx²    =5857
σx     =1.7259081
sx     =1.7437937
n      =49
minX   =8
Q1     =9
Med    =11
```

Summary statistics allow us to describe a data set but their true worth will be shown in Chapter **11** where sets of data will be compared using their summary statistics.

Standard deviation

EXERCISE 8.10 Measures of spread: standard deviation

Concepts and techniques

1 Example 23 The number of monthly accidents at a construction site over 8 months was as follows.

3	0	4	2	3	0	2	2

a Calculate the mean number of accidents per month.

b Find the standard deviation for the data, correct to one decimal place.

2 A sample of mobile phone batteries was tested for charge life in hours. The results are shown below.

60	73	65	84	77	64	66	73	88	90	79	81

Find, correct to two decimal places,

a the mean

b the standard deviation.

3 Blake's weekly commissions for selling Internet plans were as follows.

\$540	\$510	\$1100
\$1350	\$780	\$650
\$920	\$590	\$1080

Calculate for this data, correct to the nearest dollar,

a the mean

b the standard deviation.

Shutterstock.com/Ashwin

4 Find the standard deviation for each of the following frequency tables, correct to two decimal places.

a

Score	Frequency (f)
62	2
63	3
64	2
65	7
66	0
67	2
68	4
69	2
Total	22

b

Score	25	26	27	28	29	30	31	32	33
Frequency	3	6	10	16	22	20	17	6	4

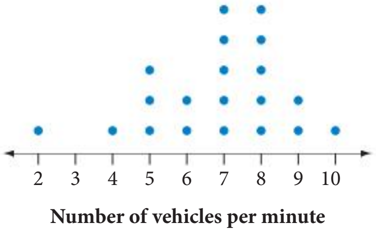
5 Find the range, interquartile range and standard deviation for each of the following sets of scores. Give answers correct to two decimal places.

a 10 11 12 13 9 14 12 11 9 12 8 16 14 12 13 9

b 4 3 2 8 4 0 1 7 3 4 2 6 5 4 8

Reasoning and communication

6 This dot plot shows the number of vehicles driving past Westvale High School every minute for a 20-minute period.



a Find the mean.

b Calculate the standard deviation correct to two decimal places.

c How many scores were within one standard deviation of the mean?

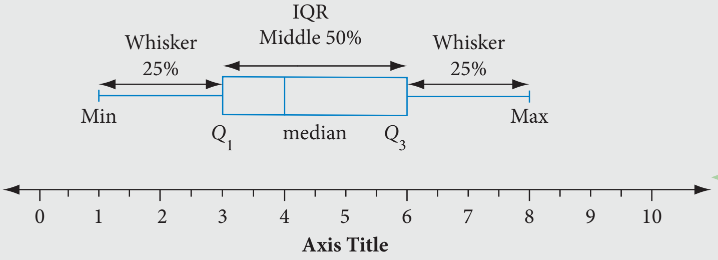
d Write a report about the number of vehicles driving past Westvale High School every minute referring to as many of the summary statistics as possible.

8.11 BOXPLOTS

A boxplot, also known as a box-and-whisker plot, is another way to graphically display numerical data.

IMPORTANT

To construct a boxplot you need to calculate the five-number summary, which consists of the minimum value, Q_1, the median, Q_3 and the maximum value



It is important to use a consistent scale, for example, going up by twos.

DO NOT just use the five-number summary values as the scale.

A boxplot can be used to describe the spread (range and IQR) and the centre (median) of a data set. As the boxplot splits the data into quarters it can also be used to display the bottom 25% of data, the middle 50% of data and so on.

Example 24

The ages of 10 people at a park were as follows.

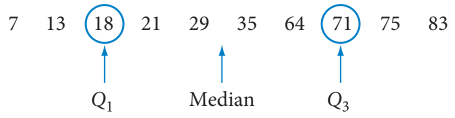
21	13	64	75	35	83	7	71	18	29

a Find the five-number summary for this data.

b Construct a boxplot.

Solution

a The data must be in order to calculate the values needed.



Minimum = 7

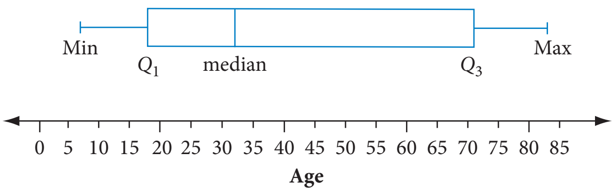
$Q_1 = 18$

$$\text{Median} = \frac{29 + 35}{2} = 32$$

$Q_3 = 71$

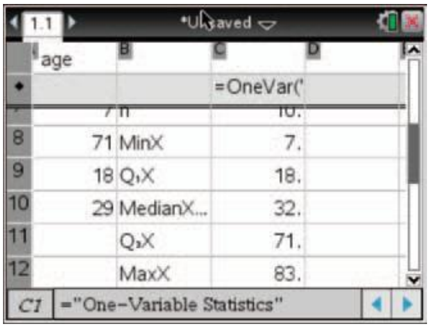
Maximum = 83

b Set up a consistent scale that goes up by fives.
Draw the boxplot.



TI-Nspire CAS

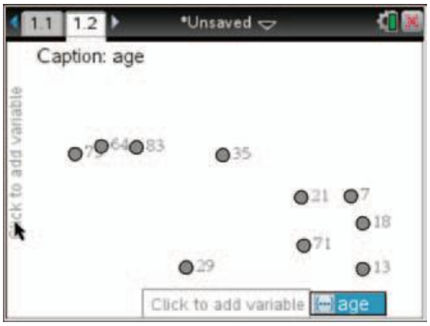
a Using a New Document enter data into the Lists & Spreadsheet page and calculate 1 variable statistics.

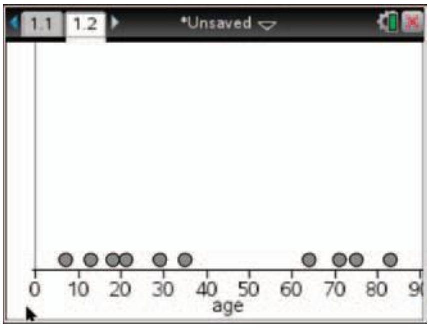


	age	B	C	D
◆			=OneVar('	
7		n	10.	
8	71	MinX	7.	
9	18	Q_1X	18.	
10	29	MedianX...	32.	
11		Q_3X	71.	
12		MaxX	83.	

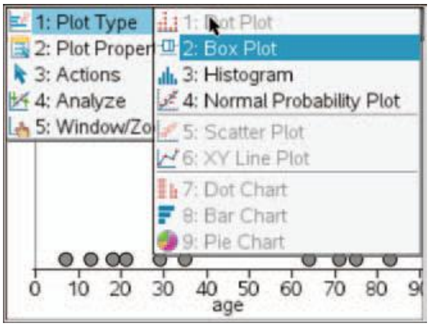
C1 ="One-Variable Statistics"

b Add a Data & Statistics page then press tab and select age as the x variable.

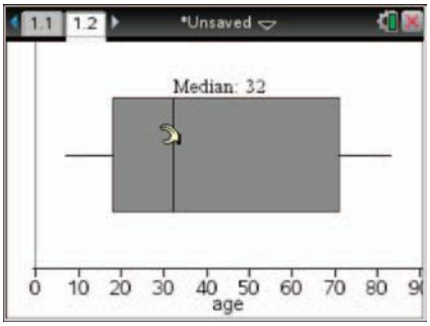




A dot plot will appear. To change it to a boxplot, press menu 1: Plot Type, 2: Box Plot.



As you move the cursor over the graph, the important statistics will appear.



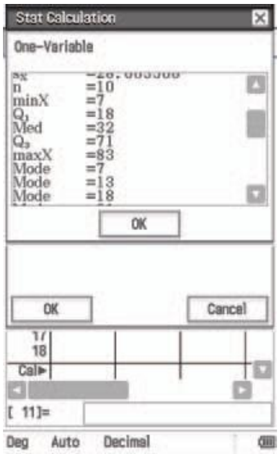
ClassPad

a Calculate the one-variable statistics.

Scroll down to find the five number summary.

Min = 7, Q_1 = 18 and Med (Q_2) = 32, Q_3 = 71, Max = 83.

Tap the top **OK** when you've finished.

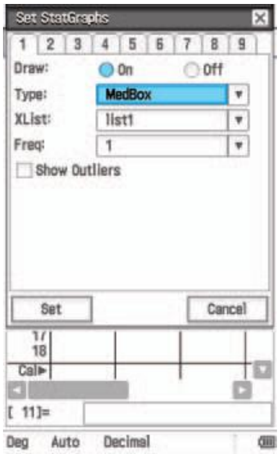


One-Variable

```
n         =10
minX      =7
Q1        =18
Med       =32
Q3        =71
maxX      =83
Mode      =7
Mode      =13
Mode      =18
```

b Use **SetGraph** as before, making sure that only **StatGraph1** is ticked and **Stat Window Auto** is on.

Tap **Setting...** and choose MedBox. The screen should be like the one on the right. Tap **Set**.



Draw: ● On ○ Off

Type: MedBox

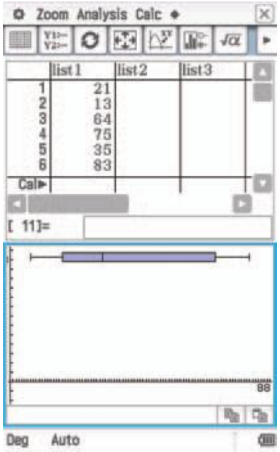
XList: list1

Freq: 1

☐ Show Outliers

9780170250252

Tap the graph icon 📊 to obtain the boxplot shown on the right.



	list1	list2	list3
1	21		
2	13		
3	64		
4	75		
5	35		
6	83		

When constructing a boxplot, outliers in the data need to be considered. In previous exercises we visually identified outliers but there is a formula that can be used to identify them. When using a CAS calculator to construct a boxplot an outlier will automatically be identified.

IMPORTANT

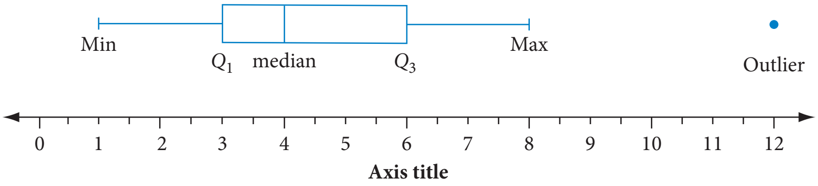
An outlier is a data value, *x*, which lies outside the interval:

$Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$

There can be more than one outlier and they can be at either end.

Any value smaller than $Q_1 - 1.5 \times \text{IQR}$ or any value greater than $Q_3 + 1.5 \times \text{IQR}$ is considered to be an outlier.

Graphically an outlier is represented as a dot. This will affect the minimum or maximum value for the boxplot, depending at which end the outlier is, but it does not affect the median or quartiles.



The boxplot above represents the data

$1 \quad 1 \quad 2 \quad 2 \quad 2 \quad \boxed{3} \quad 3 \quad 3 \quad 3 \quad 4 \quad 4 \quad \boxed{4} \quad 5 \quad 5 \quad 6 \quad 6 \quad 6 \quad \boxed{6} \quad 7 \quad 7 \quad 7 \quad 8 \quad 12$

It can be seen that $Q_1 = 3$ and $Q_3 = 6$, therefore the IQR = 3.

Outlier calculation: $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$

$$3 - 1.5 \times 3 \leq x \leq 6 + 1.5 \times 3$$

$$-1.5 \leq x \leq 10.5$$

The only purpose of these values is to determine the outliers. They will not become points on the boxplot.

Any value smaller than –1.5 is considered to be an outlier, but from the data it can be seen that the minimum value is 1, so there is no outlier at this end.

Any value greater than 10.5 is considered to be an outlier, therefore 12 is an outlier. As 12 is an outlier, the maximum value is taken to be 8, which is the next highest value in the list of data values.

Example 25

The following data represents maximum temperatures in degrees over a two-week period.

28	29	30	27	28	30	32
28	29	30	29	23	27	28

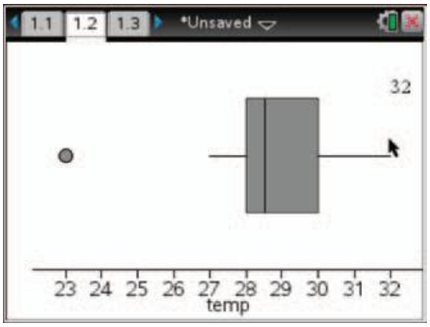
a Construct a boxplot.

b Is there an outlier present? If yes, prove why.

Solution

TI-Nspire CAS

a Construct a boxplot.



b Move the cursor over the plot to identify the value of the outlier. Record the value of the outlier and the five-number summary.

Minimum = 27 (because of outlier)

$Q_1 = 28$

Median = 28.5

$Q_3 = 30$

Maximum = 32

IQR = 2

23 is an outlier.

Use the formula to prove 23 is an outlier.

$$Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$$

$$28 - 1.5 \times 2 \le x \le 30 + 1.5 \times 2$$

$$25 \le x \le 33$$

As 23 is smaller than 25 it is considered to be an outlier.

ClassPad

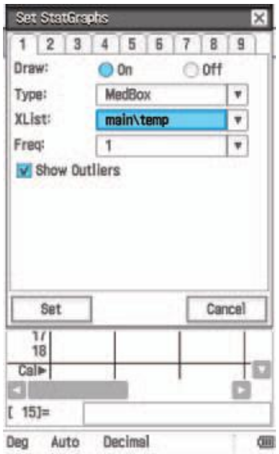
a Enter the data.

Use **SetGraph** as before, making sure **StatGraph1** only is ticked and **Stat Window Auto** is on.

Tap **Setting...** and choose MedBox. The screen should be like the one on the right.

Make sure the box for **Show outliers** is ticked. Tap it if it isn't.

Tap **Set**.



Draw: ● On ○ Off

Type: MedBox

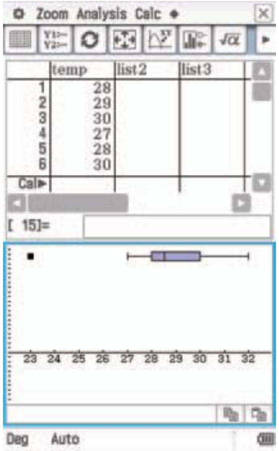
XList: main\temp

Freq: 1

☑ Show Outliers

Tap the graph icon to get the boxplot shown on the right.

The outlier is shown on the left and the next smallest temperature which is not an outlier becomes the new minimum.



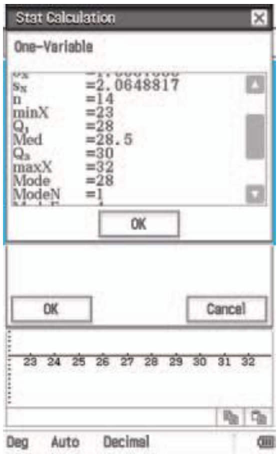
	temp	list2	list3
1	28		
2	29		
3	30		
4	27		
5	28		
6	30		

b Tap the lists icon

Tap **Calc** and **One-Variable**, follow the same steps used previously to obtain the five number summary.

Min = 23, Q_1 = 28 and Med = 28.5, Q_3 = 30, Max = 32.

Note that the outlier 23 is listed as the minimum.



One-Variable

```
sx     =2.0648817
n      =14
minX   =23
Q1     =28
Med    =28.5
Q3     =30
maxX   =32
Mode   =28
ModeN  =1
```

Use Q_1 and Q_3 in the formula to show that 23 is an outlier.

$$Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$$

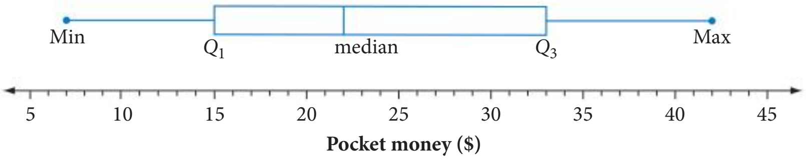
$$28 - 1.5 \times 2 \le x \le 30 + 1.5 \times 2$$

$$25 \le x \le 33$$

As 23 is smaller than 25 it is considered to be an outlier.

Example 26

This boxplot represents the amount of pocket money in dollars earned by a sample of 48 children.



a Find the median.
b Find the range.
c Find the interquartile range.
d What percentage of children earned between \$33 and \$42?
e Estimate how many children earned less than \$15.

Solution

a Visually inspect the graph. Median = \$22

b Find the largest and smallest data values and use the formula to calculate the range.

$$\begin{aligned}\text{Range} &= \text{highest data value} - \text{lowest data value}\\ &= \$42 - \$7\\ &= \$35\end{aligned}$$

c Find the Q_1 and Q_3 and use the formula to calculate the IQR.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= \$33 - \$15\\ &= \$18\end{aligned}$$

d Remember that each section of the boxplot represents 25% of the data.

\$33 to \$42 is the right whisker, therefore 25% of the data lies between \$33 and \$42.

e Under \$15 represents 25% of the children.

$$\begin{aligned}25\% \text{ of } 48 \text{ children} &= \frac{25}{100} \times 48 \text{ children}\\ &= 12 \text{ children}\end{aligned}$$

EXERCISE 8.11 Boxplots

Boxplots 1

Boxplots 2

Concepts and techniques

1 Example 24 Tom's scores for the 18 holes of a golf course were as follows.

3	4	5	8	7	9	5	9	11
5	7	4	5	8	6	9	10	5

a Find the five-number summary for this data.
b Represent this data on a box-and-whisker plot.

9780170250252

2 An archaeologist discovered 24 coins that had been produced in the Middle Ages. The masses of the coins, in grams, were as follows. Construct a boxplot to show these masses.

8.12	8.21	8.33	8.12	8.26	8.14	8.21	8.56	7.78	8.24	8.34	8.62
8.45	7.89	8.25	8.65	8.19	8.77	7.91	7.80	8.32	8.56	8.64	8.73

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3 The numbers of mistakes on a page of typing were recorded as follows for 24 business college students. Construct a boxplot for this data.

4	1	0	12	9	4	5	7	2	0	1	9
5	9	10	12	4	7	6	3	5	4	8	0

4 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their completion times in seconds for the test were as shown below. Represent this data using a box-and-whisker plot.

45	37	46	34	26
20	35	61	43	48
52	38	30	44	37

5 Find the five-number summary for the data in this stem-and-leaf plot of the ages of people at the cinema, then display the data using a boxplot.
Key: 1|4 means 14 years old

Stem	Leaf					
1	4	7	7	8		
2	6	8	9	9		
3	1	3	5	5	7	8
4	0	2	2	4	5	
5	3	7	8			
6	2	3				

6 **Example 25** The heights in centimetres of fifteen Year 11 students were recorded as follows.

155	156	153	158	161	155	154	157
158	159	160	161	170	154	155	

a Construct a boxplot to display this data.

b Are there any outliers present? If there are, use the formula to prove why they are outliers.

7 The following data represents the number of hours of television a group of Year 11 students watch in a week.

12	10	13	21	14	15	13	11	10
2	3	10	15	12	13	14	15	12

a List any outliers present in the data.

b Construct a boxplot.

Reasoning and communication

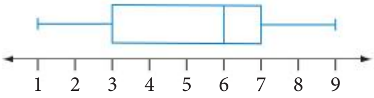
8 **Example 26** For each box-and-whisker plot, find

i the median,

ii the interquartile range,

iii the range.

a



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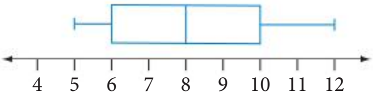

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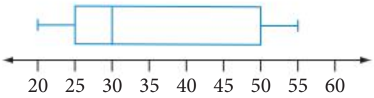

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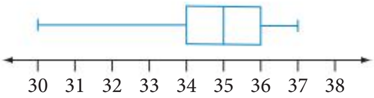

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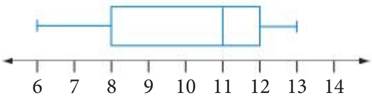
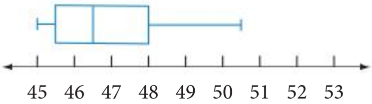


f



9 This box-and-whisker plot shows the number of cigarettes smoked per day by a sample of 60 smokers who were trying to quit.

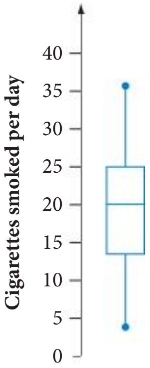
a What is the median number of cigarettes smoked per day?

b What is the interquartile range?

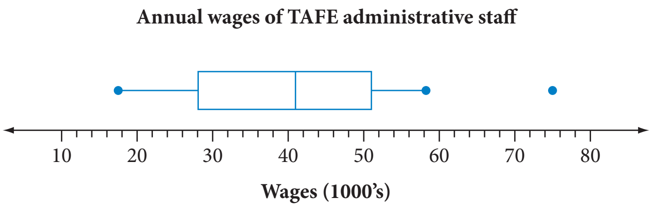
c What is the lower extreme?

d What percentage of people smoked between 20 and 25 cigarettes per day?

e Estimate how many people smoked fewer than 20 cigarettes per day.



10 This boxplot represents the annual wages (× \$1000) of the administration staff at a TAFE college.



a By inspecting the boxplot, one of the wages is shown as an outlier. Prove why the graph is incorrect.

b The median wage is

A \$18 **B** \$41 **C** \$18 000 **D** \$41 000 **E** \$52 000

c The range of wages is

A \$58 **B** \$41 **C** \$58 000 **D** \$41 000 **E** \$52 000

d Between what two amounts are the middle 50% of staff wages?

e What percentage of the staff earn less than \$28 000?

INVESTIGATION Trees

The table below lists the age in years and circumference in centimetres of trees in a local park.

Age (years)	Circumference (cm)	Species
5	6.3	Oak
33	65.9	Elm
4	6.5	Elm
9	7.7	Elm
41	59.8	Oak
8	16.3	Oak
10	24.2	Elm
32	48.1	Oak
11	27.6	Oak
13	39.3	Elm
17	20.1	Elm
21	36.8	Elm
22	43.9	Oak
26	46.2	Elm
29	50.5	Elm
30	47.8	Oak
14	27.6	Elm
33	55.9	Elm
10	27.6	Elm
37	51.9	Oak
40	55.7	Elm
10	15.9	Elm
43	59.8	Oak
17	34.8	Oak

a For each of the three variables state the type of data represented.

b State the most appropriate graphical display for each of the three variables.

c Construct a stem-and-leaf plot for the ages of the 24 trees.

d For the variable age calculate
  i the mean
  ii the five-number summary
  iii the standard deviation

e Write a brief report about the ages of trees in the local neighbourhood.

f Construct a boxplot representing the circumferences of the elm trees and state any other relevant summary statistics.

g On the same set of axes, construct a boxplot for the circumferences of the oak trees and state any other relevant summary statistics.

h Justify with working why neither boxplot has any outliers.

i Summarise the findings about the circumferences of elm and oak trees.

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CHAPTER SUMMARY
STATISTICAL MEASURES
8

- Statistics is the study of collecting, organising, presenting, analysing and interpreting data.
- A population includes all items in a group whereas a sample represents a portion of the population.
- A random sample eliminates bias in the selection process.
- Data can be classified as categorical or numerical. Categorical data can be further classified as nominal or ordinal and numerical data can be further classified as discrete or continuous data.
- A frequency table is used to organise data and lists the frequencies of the different scores.
- A column graph or bar chart is a graphical display used for categorical data. It displays the frequency of each different category using a vertical column or a horizontal bar.
- A stem-and leaf plot is a graphical display for discrete numerical data where each number is split into a stem and a leaf.

  Key: 7|9 means 79

Stem	**Leaf**							
7	1	5	6	8	9	9		
8	0	0	2	3	4	6	8	9
9	0	2	5	7	9			
10	2	3	5	6	7	7	8	

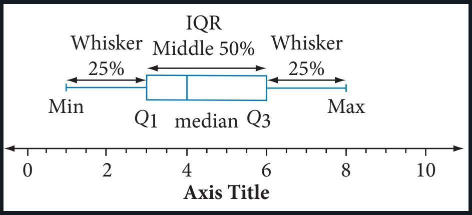
- A dot plot is a graphical display for numerical discrete data where each occurrence of a data value is represented by a dot.
- A histogram is a graphical display for large amounts of numerical data that is discrete or continuous. The columns are drawn side-by-side with no gaps between them.
- When describing the distribution of numerical data from a graphical display the following should be discussed.
  - Shape (symmetrical, bimodal, positively skewed and negatively skewed) and outliers (data values that stand out from the main body)
  - Centre
  - Spread
- Measures of centre are
  - mean or average

    $$\bar{x} = \frac{\text{sum of all the values}}{\text{number of values}} = \frac{\sum x}{n}$$

  - median, the middle value when the data is in order. The position of the median is the $\frac{n+1}{2}$th data value.
  - mode, the most common number or highest frequency
- The measures of spread are:
  - range = highest data value – lowest data value
  - interquartile range or IQR = $Q_3 - Q_1$
  - standard deviation can be calculated using a CAS calculator.

    $$s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

- A boxplot is a graphical display for numerical data and requires the five-number summary (minimum value, Q_1, median, Q_3 and maximum value) to be calculated



- Outliers can be formally identified using $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$

CHAPTER REVIEW
STATISTICAL MEASURES
8

Multiple choice

The following information relates to questions **1** and **5**.

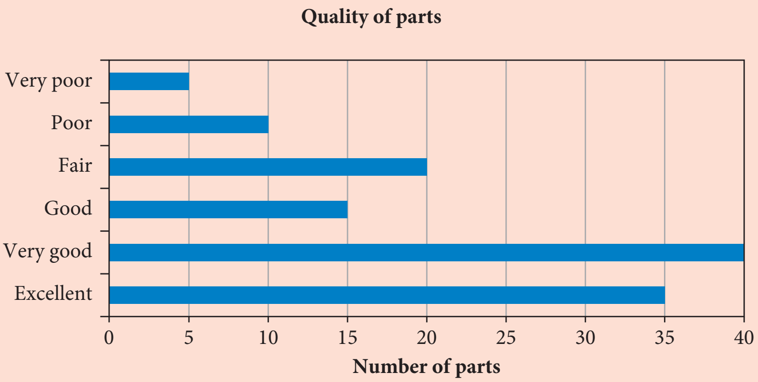
Kylie measured the heights of 50 Year 11 students for her statistics assignment.

1 Example 3 The type of data collected is best described as

A categorical and nominal data
B categorical and continuous data
C categorical and ordinal data
D numerical and discrete data
E numerical and continuous data.

The following information relates to questions **2**, **3** and **4**.

The bar graph shows the results of a quality control survey of parts manufactured at a factory.



2 Example 3 The type of data collected is best described as

A categorical and nominal data
B numerical and ordinal data
C categorical and ordinal data
D numerical and discrete data
E numerical and continuous data.

3 Example 6 The total number of parts surveyed was

A 20
B 40
C 120
D 125
E 132

4 Example 6 The percentage of parts rated as fair is

A 4%
B 10%
C 16%
D 28%
E 32%

5 Example 13 The most appropriate graphical display for the data in question **1** is

A a bar graph
B a dot plot
C a column graph
D a histogram
E a stem-and-leaf plot

The following information relates to questions **6** and **7**.

The marks for a spelling test of ten words were recorded as follows

8 6 8 4 5 6 8 5
7 4 7 8 6 8 9

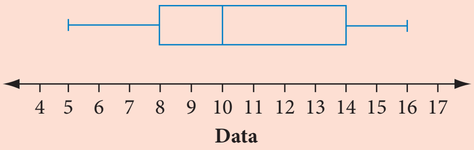
6 Example 14, 17 For the spelling test marks the mean, median and mode respectively are

A 7, 6 and 8
B 8, 7 and 6.6
C 6.6, 8 and 7
D 6.6, 7 and 8
E 7, 7 and 7

7 Example 20, 21 For the spelling test marks the range and IQR respectively are

A 5 and 3
B 9 and 5
C 3 and 5
D 5 and 9
E 7 and 7

The following information relates to questions **8** and **9**.



8 Example 26 The percentage of data below 10 is

A 25%
B 50%
C 75%
D 100%
E None of the above

9 Example 26 The value of Q_1 is

A 5
B 8
C 10
D 14
E 16

Short answer

10 Example 1 Explain what is wrong with each of the following survey questions and suggest how each one can be improved.

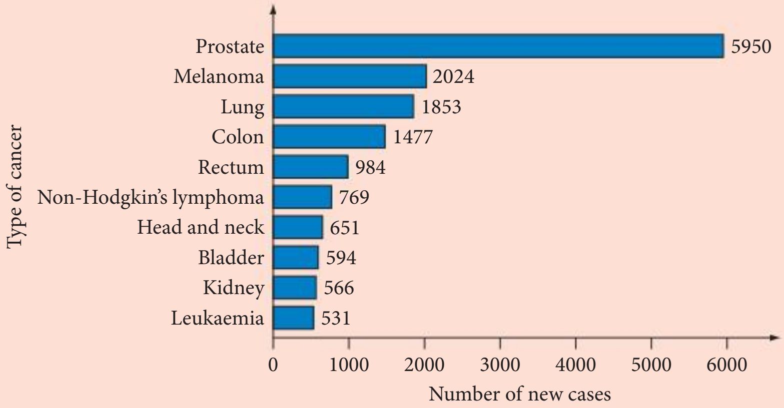
a How often do you visit the doctor?
b What did you like or dislike about the film?

11 Example 3 Classify each of the following types of data.

a The number of people watching a State of Origin football match.
b The different brands of surfing magazines.
c The heights of swimmers in a squad.
d The finishing places of 11 horses in a race.
e The number of levels in an office building.
f The state of birth of each person born in Australia.

12 Example 6 The bar graph shows the ten most common types of cancer for males in 2005 as registered by the Cancer Council.

Most common sites of cancer in males (2005)



- a How many cases of colon cancer were registered?
- b What was the sixth most common type of cancer?
- c How many types of cancer recorded fewer than 1000 cases each?
- d Find the percentage of males that had melanoma. Give your answer correct to the nearest percent.

13 Example 8 The ages of a sample of children at a Wiggles concert are listed below.

6	0	4	1	5	6	4	2	8	6	4	4
3	6	5	5	2	5	3	1	5	6	3	3

- a Construct a dot plot for this data.
- b How many children were in the sample?
- c What fraction of children were over 5 years old?
- d By visual inspection are there any outliers? If so, state what they are. Check by using the rule.

14 Example 9 This stem-and-leaf plot shows the ages of visitors entering the Royal Melbourne Show in a 5-minute period.

Key: 0|3 means 3 years old

Stem	Leaf							
0	3	8	9					
1	0	2	2	2	5	6	7	9
2	0	2	3	4	6	7		
3	1	3	3	4	9			
4	3	4	7	8				
5	5	5	8					

- a What was the age of the youngest visitor?
- b How many visitors entered the show during the 5-minute period?
- c What was the most common age of the visitors?
- d What percentage (correct to the nearest whole number) of visitors were over 30 years old?

15 **Example 11, 13** The masses (correct to the nearest kilogram) of forty skydivers were recorded.

58	63	77	82	53	69	65	80	96	105
79	63	52	90	104	85	65	87	68	105
65	87	109	84	62	75	102	78	93	84
68	105	74	59	68	74	88	66	70	62

a Are the masses discrete or continuous data?
b Organise this data into a frequency table with class intervals of 50–<60, 60–<70, etc.
c Construct a histogram to display the data.
d Which class interval had the highest frequency?
e What fraction of skydivers were in the 80–<90 kg class?

16 **Example 14** The heights in centimetres of a group of ballet dancers are listed below. Calculate the mean height correct to one decimal place.

165	183	170	168	175	179	168	170
181	168	172	177	171	170	175	179

17 **Example 14, 15, 16** Find the mean, median, mode, range, interquartile range and standard deviation of the following data sets. Where necessary, round answers to one decimal place.

a 23, 28, 29, 25, 26, 25, 29, 28, 22, 24, 21, 31, 32, 24, 27, 24, 26

b

x	Frequency (f)
12	3
13	4
14	7
15	9
16	6
17	5
18	4
19	3
20	1

c

Score	Frequency (f)
5–9	3
10–14	6
15–19	12
20–24	18
25–29	19
30–34	23
35–39	21
40–44	17
45–49	14
50–54	8
55–59	4

18 **Example 14, 17** The house prices realised at auction one Saturday in Vincentia were:

\$342 000	\$264 000	\$268 000	\$517 000
\$1 044 000	\$420 000	\$348 000	\$297 000

a Calculate the mean price.
b Calculate the median price.
c Is the mean or the median the better measure of centre? Why?

19 Example 11, 16, 19 Police radar measured the speeds of motor vehicles in km/h. The results are listed below.

78	95	64	77	81	84	77	89	90	78
79	80	82	84	80	79	95	86	84	70
78	65	82	91	89	60	85	81	78	68
90	84	69	70	80	91	85	84	80	76

- **a** Organise the data into a frequency table using classes of 60–< 70, 70–< 80, and so on, and include a column of class midpoints.
- **b** Use the frequency table to calculate an estimate for the mean speed.
- **c** State the median class and the modal class.

20 Example 21, 23 The stem-and-leaf plot below represents the number of points per match scored by the Cats in an AFL football season.

Key: 7|2 means 72 points

Stem	Leaf				
7	2	4	5	5	9
8	0	4			
9	2	5	7	7	9
10	1	2	7		
11	0	2	4	8	9
12	3	6			
13	4				

For this data, find:

- **a** the median.
- **b** the range.
- **c** the interquartile range.
- **d** Construct a boxplot to represent the data.

21 Example 14, 22 A manufacturer tests a random sample of 10 screws, each designed to have a length of 2 cm. The actual lengths of the screws, in centimetres, are as follows.

2.00 1.99 1.98 2.01 2.01 1.97 2.03 1.98 2.01 2.00

- **a** Find the mean screw length.
- **b** Calculate the sample standard deviation, correct to two decimal places.

22 Example 7, 24 The following information represents the response times (in seconds) for emergency calls by 000 telephone operators.

12	75	45	52	61	49	53	45	64	22	54
15	23	45	68	51	40	39	60	61	55	58
41	69	66	52	38	79	18	54	22	31	47

a Construct a stem-and-leaf plot to display the data.
b Find the five-number summary.
c Use your answers to part **b** to determine whether there are any outliers in the data.
d Construct a boxplot to represent the data.

Application

23 The ages in years of a sample of patients at a hospital are shown in the stem-and-leaf plot below.
Key: 1|2 means 12 years old

Stem	Leaf
1	2 2 3 4 6
2	1 2
3	0 0 0 3
4	4 7 8
5	1 1
6	
7	5 7 8
8	1

a Calculate the mean age of the patients.
b Find the median age of the patients.
c Is the mean or median more appropriate for describing the average age of the patients? Give a reason for your answer.
d Find the interquartile range of the patients' ages.
e Represent this data set using a box-and-whisker plot.
f Using the summary statistics already calculated, write a brief report about the age of patients at the hospital.

24 A test out of 20 produced the following scores.

15	19	12	2	19	16	13	18	11	15
17	11	18	14	14	16	18	14	12	

a Calculate the mean, mode and the five-number summary.
b One score is an outlier. Find this outlier and use the formula to verify this.
c Ignoring the outlier, recalculate the mean, mode and the five-number summary.
d Explain which calculations have been affected by the outlier and how they have been affected.

9780170250252

25 Two food-packaging machines are used to pack 2 kg containers of flour. A sample of 20 packs from each machine was taken and weighed. The results in kilograms were as follows.

Practice quiz

Machine 1:	2.088	2.076	2.099	2.070	2.095	2.068
2.008	2.099	2.016	2.087	2.012	2.021	2.091
2.037	2.074	2.085	2.002	2.070	2.068	2.006
Machine 2:	2.070	2.068	2.061	2.076	2.077	2.070
2.081	2.071	2.062	2.069	2.066	2.079	2.064
2.072	2.068	2.072	2.077	2.071	2.068	2.066

Use the mean and standard deviation, correct to 2 decimal places, to compare the outputs of the two machines, and comment on the effectiveness of each machine.

3

MIXED REVISION
CHAPTERS 7 • 8 • 9

Multiple choice

1 Convert 2 cm : 0.5 km to an equivalent ratio in simplest form.

A 1 : 50 **B** 1 : 250 **C** 1 : 500
D 1 : 25 000 **E** 1 : 250 000

2 A survey was completed which collected the information about the heights of students within a cohort of Year 11 students. The type of data collected is best described as:

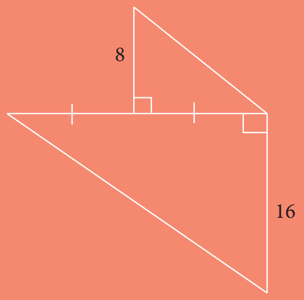
A categorical and nominal data.
B numerical and ordinal data.
C categorical and ordinal data.
D numerical and discrete data.
E numerical and continuous data.

3 The y-intercept of the line $3x - 2y + 8 = 0$ is:

A -2 **B** $\frac{3}{2}$ **C** 3 **D** 4 **E** 8

4 Which similarity test, if any, could be used to show that the two triangles are similar?

A They are not similar. **B** SSS **C** SAS
D AAA **E** RHS



5 For the spelling test results given below; the range, mode, mean (rounded to one decimal place) and median respectively are:

6, 5, 6, 7, 9, 8, 10, 7, 7, 7, 8, 6, 8, 9, 9, 7, 7

A 5, 7, 7.4, 7 **B** 7.4, 7, 7, 5 **C** 5, 7, 7, 7.4
D 1, 7, 7.4, 7 **E** 7, 7, 7, 7

6 Calculate the gradient of the line connecting (–5, 2) and (3, 1).

A -8 **B** $-\frac{1}{8}$ **C** $\frac{1}{8}$ **D** $\frac{1}{6}$ **E** 6

7 A street directory has a scale of 1 : 16 000. What distance is represented by 35 mm?

A 56 m **B** 560 m **C** 5.6 km **D** 56 km **E** 560 km

8 For the spelling test results in Question **5**, the IQR is:

A 2 **B** 3.5 **C** 5 **D** 7 **E** 15

9 The equation of the graph at right is:

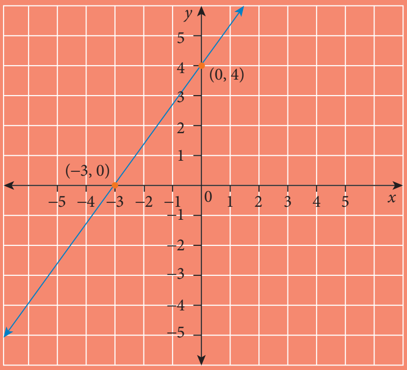
A $y = 4 - 3x$

B $y = 3 - 4x$

C $y = 4 + \frac{4}{3}x$

D $y = 4 - \frac{4}{3}x$

E $y = -3 + 4x$



Short answer questions

1 The picture of the whale has a scale of 1 : 300.

a If measuring from the tip of the nose to the tip of the tail, what is the length of the real whale?

b What is the tail width of the real whale?

c A fully grown male gray whale can grow up to 15 m long. How long would its picture be using the same scale?

Shutterstock.com/Jan Kratochvila

2 Motor vehicles were clocked, by police radar, travelling at the following speeds (in km/h).

78	95	64	77	81	84	77	89	90	78	79	80	82	84	80	79	95	86	84	70
78	65	82	91	89	60	85	81	78	68	90	84	69	70	80	91	85	84	80	76

a Sort the data in a frequency table using classes of 60 – < 70, 70 – < 80, and so on.

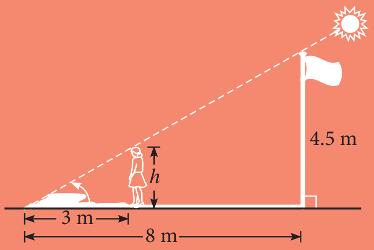
b Use the frequency table to calculate an estimate for the mean speed rounded to one decimal place.

c State both the median class and modal class.

3 a What is the gradient and y-intercept of $2x - 4y = 12$?

b Sketch the line.

4 Ashleigh casts a 3 m shadow at the same time as a 4.5 m flagpole throws an 8 m shadow. Calculate h, Ashleigh's height in metres, to the nearest cm.



5 The following information represents the response times (in seconds) for emergency calls by 000 telephone operators:

12	75	45	52	61	49	53	45	64	22	54	15	23	45	68	51	40
39	60	61	55	58	41	69	66	52	38	79	18	54	22	31	47	

a Construct a stem-and-leaf plot to display the data.

b Calculate the five-number summary.

c Use your answers to part **b** verify if there are any outliers in the data.

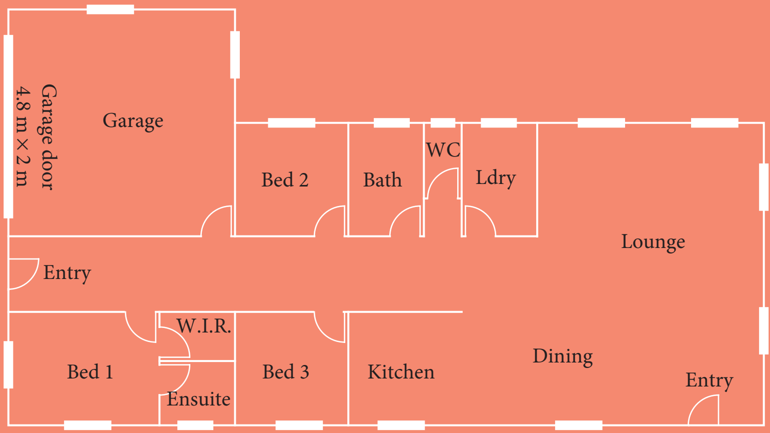
6 Sketch $x = 4$ and $y = -2$ on the same number plane.

MIXED REVISION • 7 • 8 • 9

Application questions

1 One albatross with body length 135 cm had a wing span of 3 m. A similar albatross had a wing span of 2.5 m.

- a What was the body length of the smaller bird?
- b What was the scale factor of the surface areas of their wings?
- c What was the scale factor of their volumes?
- d If the larger albatross weighed 11.88 kg, what would you expect the smaller bird to weigh?



Doors: 820 × 2030

Windows:
1200 × 1200,
except Bath/Ldry/Ens:
900 × 600,
WC: 600 × 600

Ceilings: 2400

Scale 1 : 200

2

- a Find the length of 3.66 m wide carpet needed to carpet the three bedrooms and the walk-in robe.
- b Find the number of rolls of wallpaper needed for the walls of Bed 2, given that the chosen wallpaper comes in 50 cm wide rolls of length 10 m, with no pattern-matching required.
- c Find the cost of painting the ceilings of the house and garage with two coats, given that the paint covers 11 m^2/L and only comes in 4 L tins costing $78.60 each. Fully explain your reasoning.

3 The ages, in years, of a sample of patients at a hospital are shown in the stem-and-leaf plot on the right.

Key: 1|2 means 12 years old

- a Calculate the mean age of the patients.
- b Find the median age of the patients.
- c Is the mean or median more appropriate for describing the average age of the patients? Justify your reasoning.
- d Calculate the interquartile range of the patients' ages.
- e Represent this data set on a box-and-whisker plot.

Key: 1|2 means 12 years old

Stem	Leaf
1	2 2 3 4 6
2	1 2
3	0 0 0 3
4	4 7 8
5	1 1
6	
7	5 7 8
8	1

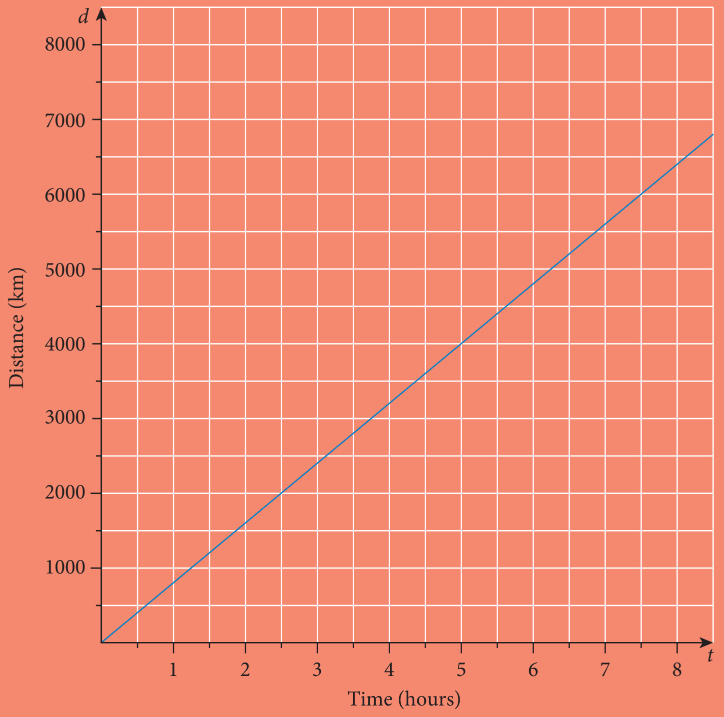
4 The cost, C, of hiring a function room includes a set fee of \$250 and \$5 per head for the number of people attending. The table below shows the total cost \$$C$ for a different number of people, n, attending the function.

No of students (n)	20	30	40	50	100
Total cost (\$$C$)	350	400	450	500	750

- a Which variable is the dependent variable?
- b Explain how you can determine if the relationship shown is linear.
- c Calculate the gradient of the linear relationship.
- d What is the vertical intercept?
- e Write the linear function of C in terms of n.
- f What does the vertical intercept represent?
- g What would be the cost of the function if 135 people attended?

5 A plane is travelling at a constant speed. The graph below shows the distance travelled, d, after time, t, hours.

- a How far will the plane travel in 2 hours?
- b How long will it take the plane to travel 4000 km?
- c Calculate the gradient of the linear relationship between d and t.
- d What is the vertical intercept?
- e Write the linear function of d in terms of t.
- f What does the vertical intercept represent?
- g What does the gradient represent?



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9

ALGEBRA REVIEW AND LINEAR GRAPHS

LINEAR EQUATIONS AND GRAPHS

TERMINOLOGY

balanced
coordinates
dependent variable
equation
general equation
gradient
horizontal line
independent variable
linear equation
linear modelling
negative gradient
positive gradient
solve
vertical line
x-intercept
y-intercept

Prior learning

LINEAR EQUATIONS AND THEIR GRAPHS

- **identify and solve linear equations (ACMGM038)**
- **develop a linear formula from a word description (ACMGM039)**

STRAIGHT-LINE GRAPHS AND THEIR APPLICATIONS

- **construct straight-line graphs both with and without the aid of technology (ACMGM040)**
- **determine the slope and intercepts of a straight-line graph from both its equation and its plot (ACMGM041)**
- **interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation (ACMGM042)**
- **construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required. (ACMGM043)**

9.01 REVIEW OF LINEAR EQUATIONS

Recall that an equation contains an algebraic expression and an equals (=) sign. A linear equation is one in which the variable is raised to the power 1. For example: $-7f + 11$ is a linear expression, while $-7f + 11 = 15$ is a **linear equation**. An equation is **solved** when a value of the variable (for example, f) is found and when substituted into the equation, the equation is true; that is, both the left-hand side and the right-hand side are equal.

IMPORTANT

To solve an equation:

- use inverse operations
- keep the equation balanced by performing the same operation on both sides
- aim to have any terms containing the unknown variable on one side of the equation and any numbers on the other side
- work in a logical progression through the steps, undoing each of the operations that have been performed on the variable in the reverse order to which they were performed

Example 1

Solve each of these equations.

a $3x - 4 = -13$ **b** $\frac{b}{2} + 7 = 1$ **c** $\frac{h - 10}{4} = 3$

Solution

a Write the equation.

$$3x - 4 = -13$$

Add 4 to both sides.

$$3x - 4 + 4 = -13 + 4$$

$$3x = -9$$

Divide both sides by 3.

$$\frac{3x}{3} = \frac{-9}{3}$$

Write the answer.

$$x = -3$$

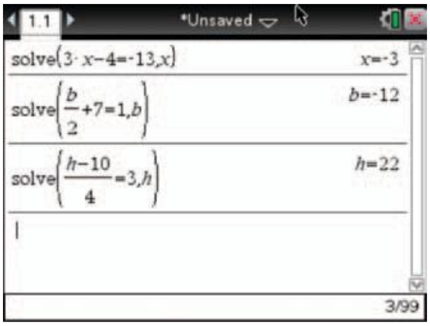
b	Write the equation.	$\frac{b}{2}+7=1$
	Subtract 7 from both sides.	$\frac{b}{2}+7-7=1-7$
		$\frac{b}{2}=-6$
	Multiply both sides by 2.	$\frac{b}{2}\times 2=-6\times 2$
	Write the answer.	$b=-12$
c	Write the equation.	$\frac{h-10}{4}=3$
	Multiply both sides by 4.	$\frac{h-10}{4}\times 4=3\times 4$
		$h-10=12$
	Add 10 to both sides.	$h-10+10=12+10$
	Write your answer.	$h=22$

Remember that you can check your solution by substituting the value represented by the variable back into the original equation and checking if the right-hand side and the left-hand side are equal when each side is evaluated.

$$\frac{22-10}{4}=\frac{12}{4}$$
$$3=3$$

Solutions can also be found or checked using a CAS calculator.

TI-Nspire CAS



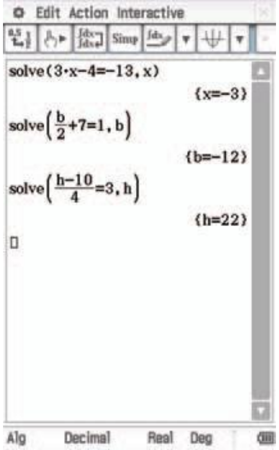
solve(3·x−4=−13,x) x=−3

$\text{solve}\left(\frac{b}{2}+7=1,b\right)$ b=−12

$\text{solve}\left(\frac{h-10}{4}=3,h\right)$ h=22

3/99

ClassPad



solve(3·x−4=−13, x)
{x=−3}

$\text{solve}\left(\frac{b}{2}+7=1, b\right)$
{b=−12}

$\text{solve}\left(\frac{h-10}{4}=3, h\right)$
{h=22}

Solving more complex equations

Some equations will require more than one or two steps to solve them. Ensure that all terms containing the variable are on one side of the equation. Work in a logical progression through the steps, undoing each of the operations that have been performed on the variable in the reverse order to which they were performed.

Example 2

Solve each of these equations.

a $5y + 5 = 2y + 17$ **b** $\dfrac{2m-7}{3} = 6$ **c** $4(1 - 2t) = 16$

Solution

a Write the equation.

$$5y + 5 = 2y + 17$$

Subtract $2y$ from both sides so all terms containing y are on one side of the equation.

$$5y + 5 - 2y = 2y + 17 - 2y$$

Simplify.

$$3y + 5 = 17$$

Subtract 5 from both sides and simplify.

$$3y + 5 - 5 = 17 - 5$$
$$3y = 12$$

Divide both sides by 3.

$$\frac{3y}{3} = \frac{12}{3}$$

Write the answer.

$$y = 4$$

b Write the equation.

$$\frac{2m-7}{3} = 6$$

Multiply both sides by 3.

$$\frac{2m-7}{3} \times 3 = 6 \times 3$$
$$2m - 7 = 18$$

Add 7 to both sides and simplify.

$$2m - 7 + 7 = 18 + 7$$
$$2m = 25$$

Divide both sides by 2.

$$\frac{2m}{2} = \frac{25}{2}$$

Write the answer.

$$m = \frac{25}{2} \text{ or } 12\frac{1}{2}$$

c Write the equation.

$$4(1 - 2t) = 16$$

This equation could also be solved by expanding the brackets first. The answer will be the same.

Divide both sides by 4.

$$\frac{4(1-2t)}{4} = \frac{16}{4}$$
$$1 - 2t = 4$$

Subtract 1 from both sides and simplify.

$$1 - 2t - 1 = 4 - 1$$
$$-2t = 3$$

Divide both sides by −2.

$$\frac{-2t}{-2} = \frac{3}{-2}$$

Write the answer.

$$t = -\frac{3}{2} \text{ or } -1\frac{1}{2}$$

EXERCISE 9.01 Review of linear equations

Concepts and techniques

1 **Example 1** Solve each of the following linear equations.

a $\frac{b-3}{2}=2$

b $3(f+3)=-18$

c $10-3i=-17$

d $2(a+6)=10$

e $\frac{l}{2}-7=-11$

f $8s+6=-18$

g $\frac{u+1}{-2}=2$

h $3x-7=1$

i $21-5m=31$

2 **Example 2** Solve each of the following linear equations.

a $4n+12=3n+6$

b $5(2d+1)=35$

c $r+4=8-r$

d $5k-2=2k+10$

e $-(2x-4)=2$

f $8-t=56+11t$

g $3e-4=e+16$

h $\frac{2r-12}{4}=5$

i $\frac{40-2a}{4}=4$

Reasoning and communication

3 When solving the equation $3q+4=4(q-1)$, which of the following statements is **false**?

A The brackets on the RHS can be expanded.

B The qs can only be collected on the LHS of the equation.

C Expanding the brackets on the RHS gives $3q+4=4q-4$.

D The solution to this equation is $q=8$.

E $-q=-8$ is obtained if the qs are collected on the LHS of the equation.

4 The first error in the solution of the following equation is in:

Line A: $3(2h-14)=4(h-5)$
Line B: $6h-42=4h-20$
Line C: $6h-42-4h=4h-20-4h$
Line D: $2h-42=20$
Line E: $2h-42+42=20+42$
Line F: $2h=62$
Line G: $\frac{2h}{2}=\frac{62}{2}$
Line H: $h=31$

A Line B.
B Line C.
C Line D.
D Line E.
E Line F.

5 When solving the equation: $3(e+5)-4(e-1)=26$, which of the following statements is **false**?

A Expanding the brackets on the LHS gives $3e+15-4e-4=26$.

B The solution to the equation is $e=-7$.

C The LHS of the equation simplifies to $-e+19=26$.

D Keeping e on the LHS of the equation gives $-e=7$.

E The equation simplifies to $19=26+e$.

6 The error in the solution of the following equation is in:

Line A: $7(t-2) - 3(4t + 2) = 30$
Line B: $7t - 14 - 12t - 6 = 30$
Line C: $-5t - 20 = 30$
Line D: $-5t - 20 + 20 = 30 + 20$
Line E: $-5t = 50$
Line F: $\frac{5t}{5} = \frac{50}{5}$
Line G: $t = 10$

A Line B. **B** Line C. **C** Line D. **D** Line E. **E** Line F.

9.02 GRAPHING STRAIGHT LINES

The formula or equation $y = 4x + 3$ is called a **linear equation** and its graph is a straight line. The **coordinates** of points on this line are found by substituting x-values into the equation to calculate corresponding y-values. While a straight line can be plotted using as few as two points, sometimes a table of values is constructed and each point is plotted in turn to form the straight line.

IMPORTANT

Rearranging the equation to make y the subject makes it easier to calculate the table of values.

Example 3

Graph the straight lines represented by the linear equations:

a $y = 2x - 1$ **b** $3x + y = 5$

Solution

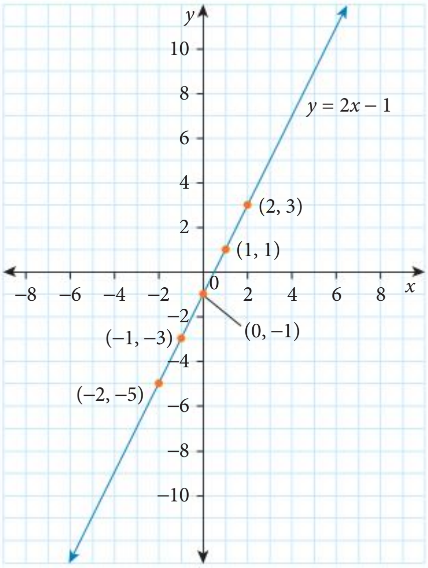
a Select integer values of x, say between -2 and 2 to construct a table of values.
When $x = -2$, $y = 2 \times (-2) - 1 = -5$.
When $x = -1$, $y = 2 \times (-1) - 1 = -3$.
When $x = 0$, $y = 2 \times 0 - 1 = -1$.
When $x = 1$, $y = 2 \times 1 - 1 = 1$.
When $x = 2$, $y = 2 \times 2 - 1 = 3$.

$y = 2x - 1$

x	-2	-1	0	1	2
y	-5	-3	-1	1	3
(x, y)	$(-2, -5)$	$(-1, -3)$	$(0, -1)$	$(1, 1)$	$(2, 3)$

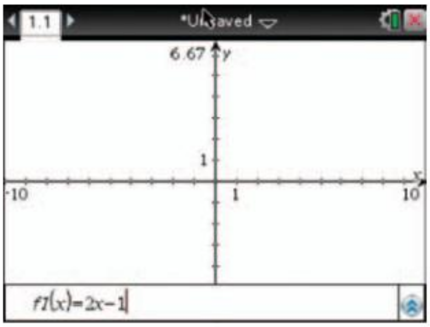
Creating a table using any two or more values of x and finding the corresponding values of y will yield the same straight line when the points are plotted.

Plot the points on a Cartesian plane.
Join the points. Extend and place arrows on both ends of the line.
Label the graph with its equation.

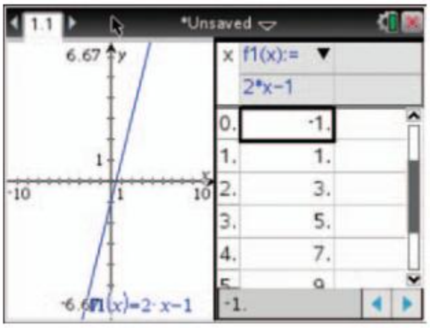


TI-Nspire CAS

Tables of values can be generated using a CAS calculator.
Open a New Document with a Graphs page.
At the bottom of the screen there is an entry line with the prompt $f1(x) =$.
Type $2x - 1$ and press enter.
Press ctrl T for a table of values.
Use the arrow keys to scroll up and down the table.



f1(x)=2x−1



f1(x)=2·x−1

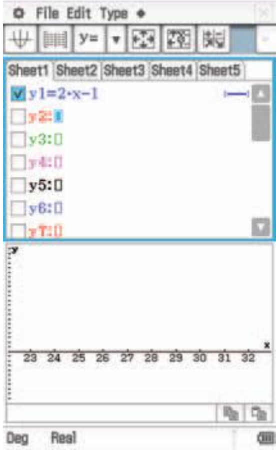
x	f1(x):= 2*x−1
0.	-1.
1.	1.
2.	3.
3.	5.
4.	7.
5.	9.

-1.

ClassPad

Tap Menu then the **Graph**&Table application.

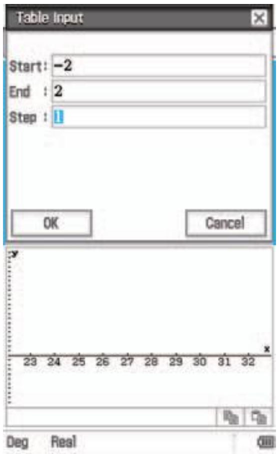
In the entry line next to $y1$ = type 2$\boxed{x}$ $\boxed{-}$1 then press $\boxed{\text{EXE}}$.



☑ y1=2·x−1
☐ y2:□
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

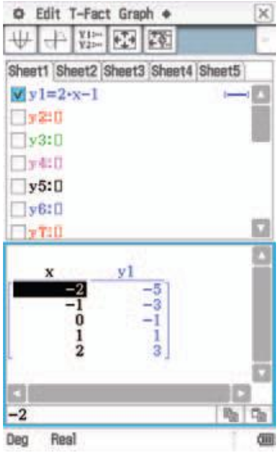
Tap to create a table of values.

In the pop up screen that appears type −2 for Start, 2 for End and 1 for Step then tap $\boxed{\text{OK}}$.



Start: −2
End : 2
Step : 1

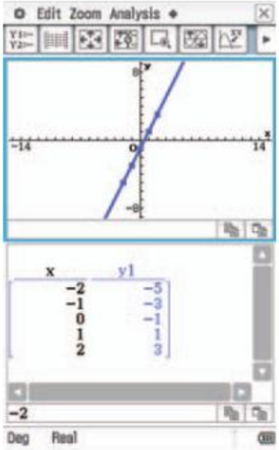
To view the table of values, tap .



☑ y1=2·x−1
☐ y2:□
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

x	y1
−2	−5
−1	−3
0	−1
1	1
2	3

To plot the graph, tap ⊞.
If necessary, tap **Zoom** then **Zoom Out** to show more of the graph.



x	y1
−2	−5
−1	−3
0	−1
1	1
2	3

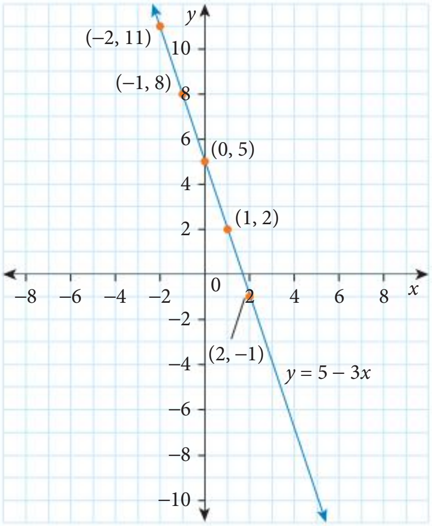
−2

b Make y the subject of the equation.
Construct a table of values.
The values of x are not specified so use the values of $-2 \le x \le 2$ as before.
When $x = -2$, $y = 5 - 3 \times (-2) = 5 + 6 = 11$.
When $x = -1$, $y = 5 - 3 \times (-1) = 5 + 3 = 8$.
When $x = 0$, $y = 5 - 3 \times 0 = 5 - 0 = 5$.
When $x = 1$, $y = 5 - 3 \times 1 = 2$.
When $x = 2$, $y = 5 - 3 \times 2 = -1$.

Plot the points on a Cartesian plane.
Join the points. Extend and place arrows on both ends of the line.
Label the graph with its equation.

$y = 5 - 3x$

x	−2	−1	0	1	2
y	11	8	5	2	−1
(x, y)	(−2, 11)	(−1, 8)	(0, 5)	(1, 2)	(2, −1)



Sketching linear graphs

A sketch of a linear function can be made by finding and plotting any two points. The two special points that are usually found are the ***x*-intercept** and the ***y*-intercept**. The x-intercept is the point where the line crosses the x-axis and occurs at a point where $y = 0$. The y-intercept is the point where the line crosses the y-axis and occurs at a point where $x = 0$.

Example 4

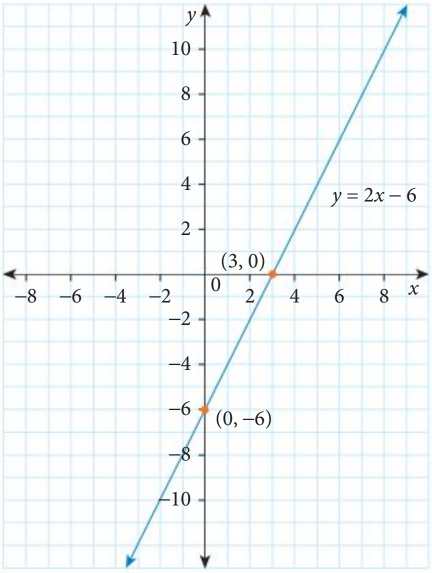
Sketch the linear functions below by first finding the x- and y-intercepts.

a $y = 2x - 6$ **b** $4x + 3y = 8$

Solution

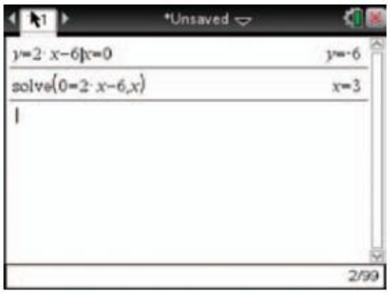
a Write the equation.	$y = 2x - 6$
Find the x-intercept by letting $y = 0$.	When $y = 0$, $2x - 6 = 0$
Solve the equation.	$2x = 6$ $x = 3$ The coordinates of the x-intercept are $(3, 0)$.
Find the y-intercept by letting $x = 0$.	When $x = 0$, $y = 2 \times 0 - 6$
Solve the equation.	$y = -6$ The coordinates of the y-intercept are $(0, -6)$.

Plot the intercepts.
Join them to draw the line.
Extend the line and place arrows on both ends of the line.
Label the graph with its equation.



TI-Nspire CAS

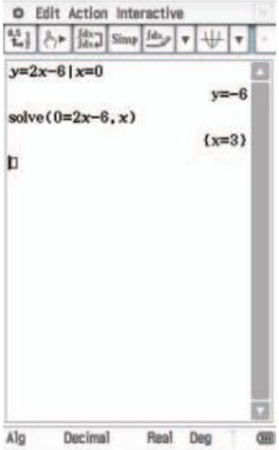
Rather than solving algebraically, CAS can be used to complete the calculations required to find the intercepts.
For the y-intercept, substitute $x = 0$ into the rule using the | symbol.
For the x-intercept, let $y = 0$ and then solve for x.



```
y=2·x−6|x=0                 y=-6
solve(0=2·x−6,x)             x=3
```

9780170250252

ClassPad



```
y=2x−6 | x=0
                    y=−6
solve(0=2x−6, x)
                   {x=3}
```

b Write the equation.

$4x + 3y = 8$

Find the x-intercept by letting $y = 0$.

Solve the equation.

When $y = 0$, $4x + 0 = 8$

$4x = 8$

$x = 2$

Coordinates of the x-intercept are $(2, 0)$.

Find the y-intercept by letting $x = 0$.

Solve the equation.

When $x = 0$, $0 + 3y = 8$

$3y = 8$

$$y = \frac{8}{3}$$

$$= 2\frac{2}{3}$$

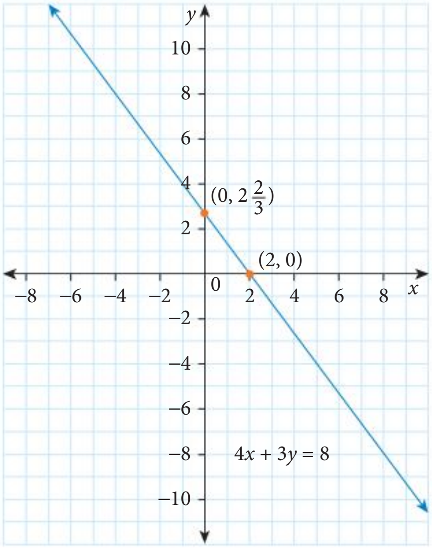
Coordinates of the y-intercept are $(0, 2\frac{2}{3})$.

Plot the intercepts.

Join them to draw the line.

Extend the line and place arrows on both ends of the line.

Label the graph with its equation.



Example 5

Sketch the lines below using a CAS calculator.

a $y = -\frac{3}{4}x + 3$ **b** $y = 4 - 3x$ **c** $2x - y + 3 = 0$

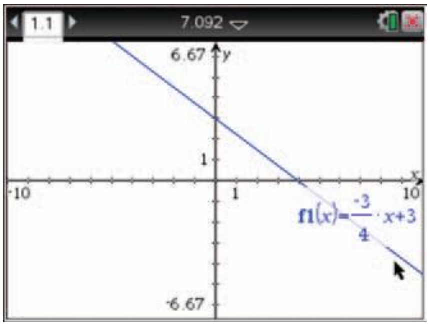
Solution

a **TI-Nspire CAS**

Open a New Document with a Graphs page.
In the entry line with the prompt f1(x) =, type $-\frac{3}{4}x + 3$.

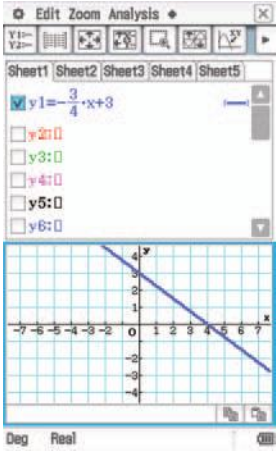
Press enter.

The y-intercept is (0, 3) and the x-intercept is (4, 0).

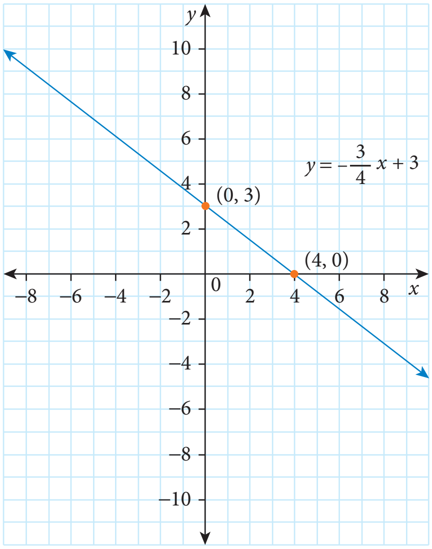


ClassPad

Use the **Graph&Table** application.
In the entry line next to $y1$ =, type $-\frac{3}{4}x + 3$ then press EXE.
Tap .
The y-intercept is (0, 3) and the x-intercept is (4, 0).
Sketch a graph.

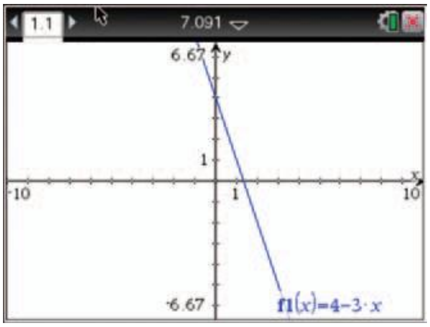


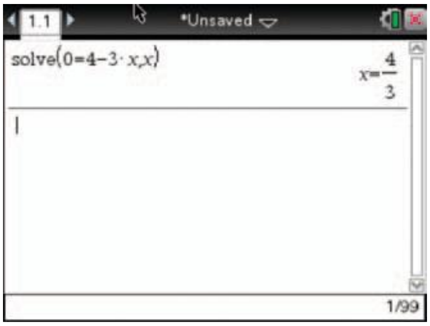
Sketch a graph.



b **TI-Nspire CAS**

Open a New Document with a Graphs page.
In the entry line with the prompt f1(x) =, type $4 - 3x$.
Press enter.
The y-intercept can be read directly from the graph, (0, 4).
Find the x-intercept.



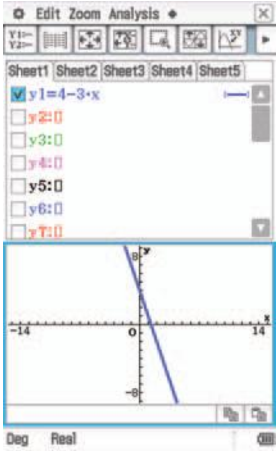


solve(0=4-3·x,x) $x=\frac{4}{3}$

1/99

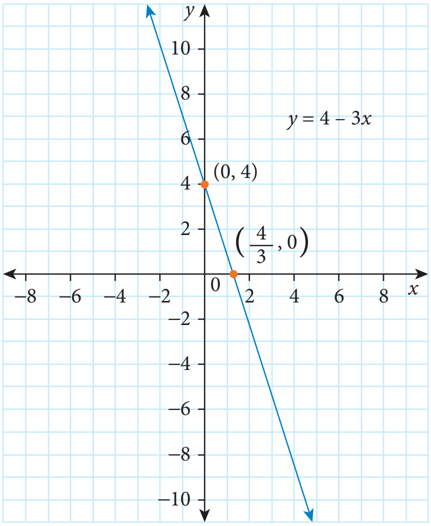
ClassPad

Use the **Graph&Table** application.
In the entry line next to $y1$ = type $4 – 3x$ then press EXE.
Tap .
The y-intercept can be read directly from the graph, (0,4).
Find the x-intercept.
Sketch a graph.



☑ y1=4-3·x
☐ y2:□
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

Sketch a graph.



c **TI-Nspire CAS**

Rearrange the equation to make *y* the subject.

Graph using a CAS calculator.

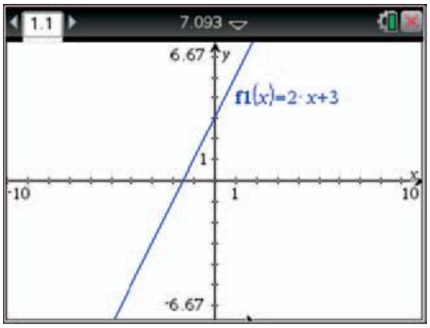
The *y*-intercept can be read directly from the graph, (0, 3).

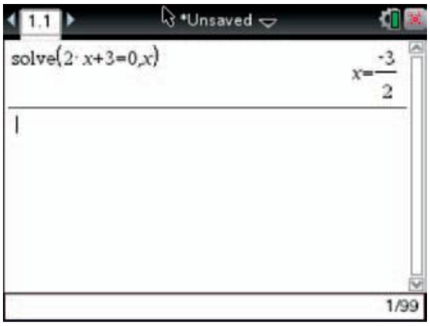
Find the *x*-intercept.

$2x - y + 3 = 0$

$2x + 3 = y$ or $y = 2x + 3$

It is important to note that *y* must be the subject of the equation in order to graph using a CAS calculator.

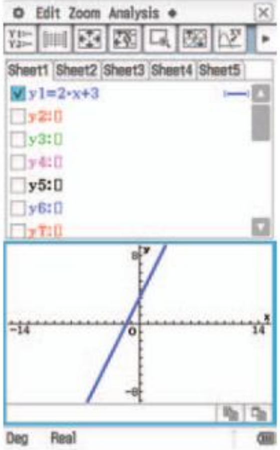




solve(2·x+3=0,x)　$x=\frac{-3}{2}$

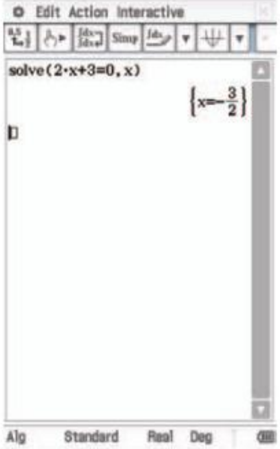
ClassPad

Rearrange the equation to make y the subject. Graph using a CAS calculator.	$2x - y + 3 = 0$ $2x + 3 = y$ or $y = 2x + 3$



☑ y1=2·x+3
☐ y2:□
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

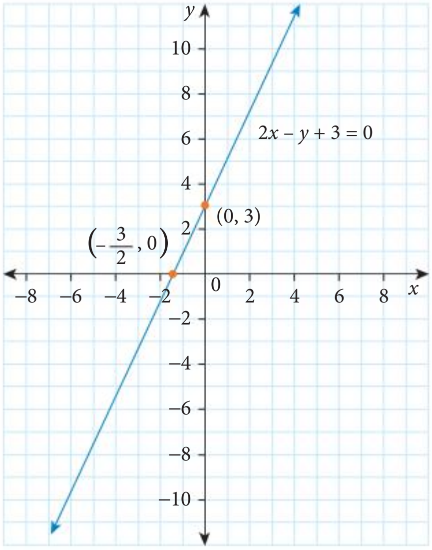
Graph using a CAS.



solve(2·x+3=0, x)

$\left\{x=-\frac{3}{2}\right\}$

Sketch a graph.



To determine whether a point lies on a line, substitute the x-coordinate and y-coordinate, in turn, into the equation of the line and check if the resulting mathematical statement is true or false. If it is true, then the point lies on the line, and if it is false, then the point does not lie on the line.

Example 6

Determine if the point (–2, 3) lies on each of the lines below.

a $y = 2x + 7$ **b** $3x - y = 8$

Solution

Identify the x-coordinate and the y-coordinate of the point (–2, 3).	$x = -2$ and $y = 3$.
Substitute the values into the equation of the line and evaluate.	$y = 2x + 7$ $3 = 2 \times (-2) + 7$ $3 = -4 + 7$; true.
State whether the equation is true or false and write the answer.	(–2, 3) lies on the line $y = 2x + 7$.
Identify the x-coordinate and the y-coordinate of the point (–2, 3).	$x = -2$ and $y = 3$.
Substitute the values into the equation of the line and evaluate.	$3x - y = 8$ $3 \times (-2) - 3 = 8$ $-6 - 3 = 8$; false.
State whether the equation is true or false and write the answer.	(–2, 3) does not lie on the line $3x - y = 8$.

Linear graphs

EXERCISE 9.02 Graphing straight lines

Concepts and techniques

1 **Example 3**

i Complete a table of values for $-2 \le x \le 2$ for each of the following equations.

ii Plot the points to construct a linear graph representing each equation.

a $y = 3x + 1$

b $y = 4 - 2x$

c $y = -2x + 5$

d $y = x - 1$

e $y = \frac{1}{2}x + 2$

f $y = 3x - \frac{1}{2}$

g $4x + y = 3$

h $2x - y = 6$

2 **Example 4** For the line $y = 2x + 4$:

a what is the x-intercept?

b what is the y-intercept?

c sketch the line using these points.

3 Sketch the lines below by first finding the x- and y-intercepts.

a $y = 3x - 5$

b $y = 2x + 1$

c $y = 4 - 3x$

d $y = -x - 3$

e $2x + y = 5$

f $3x - y = 4$

g $\frac{1}{3}x + y = 6$

h $\frac{3}{4}x - 2y = 8$

4 **Example 5** Sketch the lines below using a CAS calculator.

a $y = 2x + 3$	b $y = 3x - 2$	c $y = -x - 4$
d $y = 5 - 3x$	e $3x - y = 5$	f $2x + y = 8$
g $\frac{1}{2}x + 4y = 8$	h $3x - \frac{2}{3}y = 6$	

5 **Example 6** Does the point (5, 20) lie on the line $y = 15x - 55$? Justify your answer.

6 Does the point (3, –4) lie on the line $2x + 3y + 6 = 0$? Justify your answer.

7 Does the point (–10, 15) lie on the line $5x - 3y = -5$? Justify your answer.

Reasoning and communication

8 For the linear function $y = 2x - 4$ the x-intercept is:

A –4 B –2 C 2 D 4 E none of these

9 For the linear function $4x - 2y = 12$ the y-intercept is:

A –6 B –3 C 3 D 4 E 6

10 i State if the following equations are linear. Justify your answer.

ii Find the coordinates of the x- and y-intercepts of the linear equations.

a $y = 4x + 2$	b $y = 4x^2 + 2$	c $y = 4x + 2x^2$
d $y = 8 - 3x$	e $y = 8 - 3x^3$	f $y = 8 \times 3^x$

11 Find a point that satisfies the equation $C = 45 - 6.5n$.

12 Find a point that satisfies the equation $5m - 4.8n = 38.4$.

13 a What strategy could you use to check if the points below form a straight line when graphed?
(–1, 3), (0, 6), (1, 9), (–2, 3)

b Do these points form a straight line? Explain your reasoning.

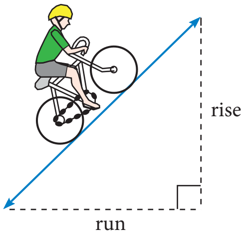
9.03 CALCULATING THE GRADIENT OF A LINE

The slope of a line is also known as the **gradient**.

The gradient of a line can be calculated using the formula:

$$\text{Gradient} = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)},$$

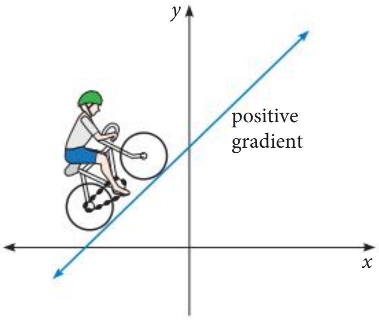
where the rise is the vertical distance between two points and the run is the horizontal distance between the same two points.



The gradient of a line may be **positive** or **negative**.

Positive gradient

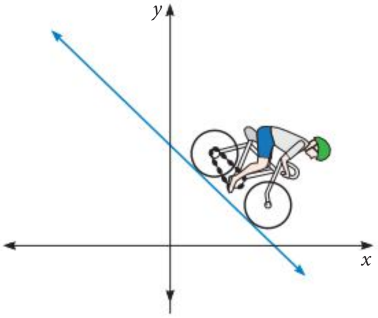
A line with a positive gradient slopes upwards, from left to right, because its y-values are increasing as x increases. A gradient of 3 means that, for each 1 unit we move across, the line goes up 3 units or, as the x values increase by 1, the y values *increase* by 3.



Graph sloping upwards

Negative gradient

A line with a negative gradient slopes downwards, from left to right, because its y-values are decreasing as x increases. A negative gradient has a 'negative rise', or a 'drop'. A gradient of –2 means that, for each unit we move across, the line goes down 2 units, or as the x values increase by 1, the y values *decrease* by 2.



Graph sloping downwards

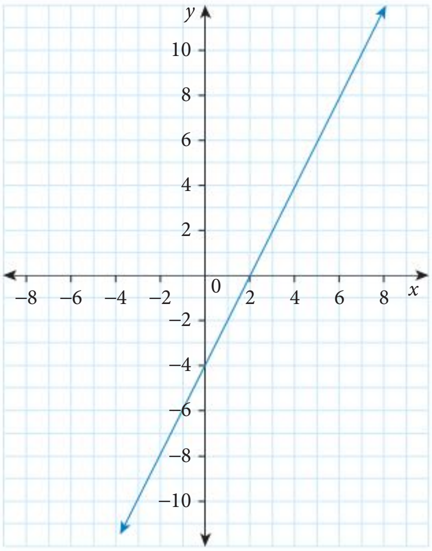
Example 7

For each of the lines below:

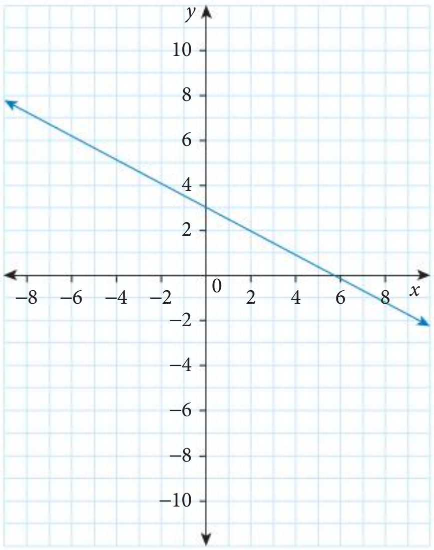
i state whether the gradient is positive or negative

ii calculate the gradient of the line.

a



b

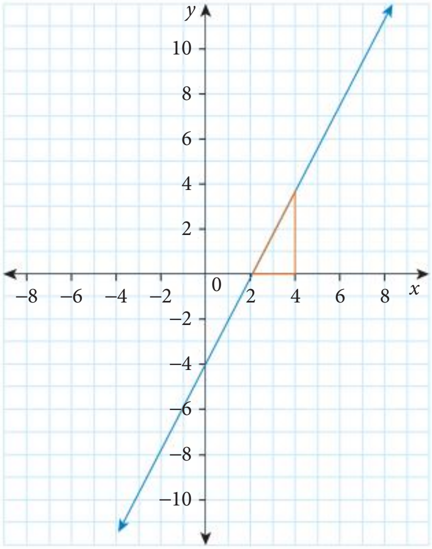


9780170250252

Solution

a i The line is going up from left to right. — Positive gradient

ii Draw in a right-angled triangle.



Write the formula.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

Substitute in rise and run from the triangle that you have drawn onto the graph.

$$= \frac{4}{2}$$

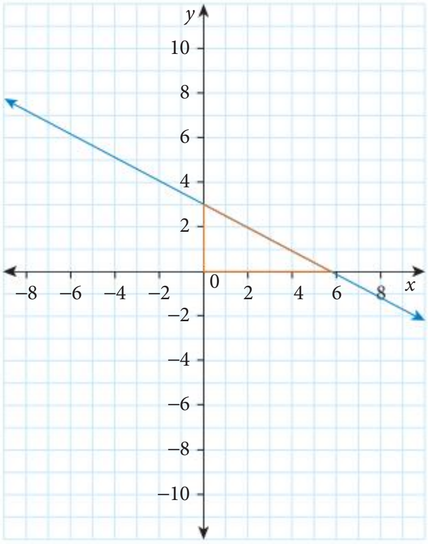
$$= 2$$

Simplify and write the answer.

The gradient of this line is 2.

b i The line is going down from left to right. — Negative gradient

ii Draw in a right-angled triangle.



Write the formula.	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$
Substitute in rise and run from the triangle that you have drawn onto the graph.	$= -\frac{3}{6}$
Simplify and write the answer.	$= -\frac{1}{2}$ The gradient of this line is $-\frac{1}{2}$.

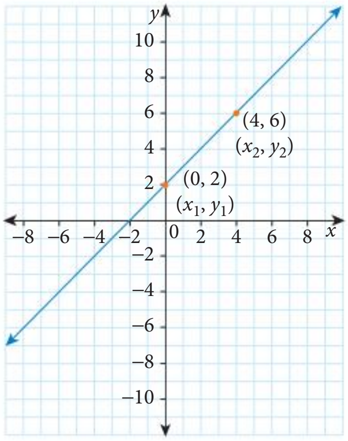
An alternative method to calculate the gradient.

To find the gradient of a line which is plotted on a Cartesian plane, select two points on the line. The two points can be named as (x_1, y_1) and (x_2, y_2).

To calculate the gradient of the straight line joining (x_1, y_1) and (x_2, y_2) use the formula

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

This is a way of calculating gradient without the need for a diagram and also represents the change in y over change in x. The same formula can be used if given a table of values. Any two points may be used.



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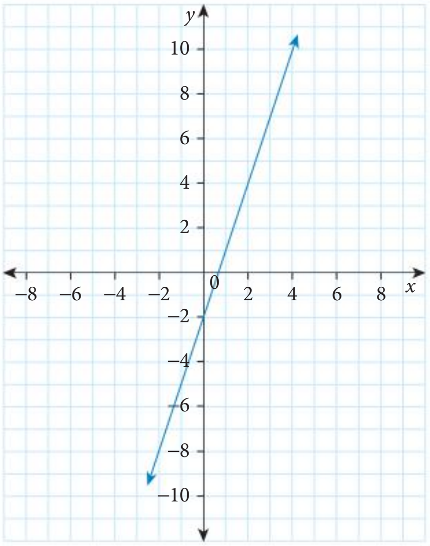
Example 8

Calculate the gradient of the following linear functions.

a



b

x	−2	0	2
y	16	6	−4

Solution

a Select two points on the line, say (0, −2) and (2, 4).

Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (2, 4)$.

Write the formula.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute these points into the formula and simplify.

$$\text{Gradient} = \frac{4-(-2)}{2-0}$$
$$= \frac{6}{2}$$
$$= 3$$

Write the answer.

The gradient is 3.

b Select two points from the table, say (0, 6) and (2, −4).

Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (2, -4)$.

Write the formula.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute these points into the formula and simplify.

$$\text{Gradient} = \frac{-4-6}{2-0}$$
$$= \frac{-10}{2}$$
$$= -5$$

Write the answer.

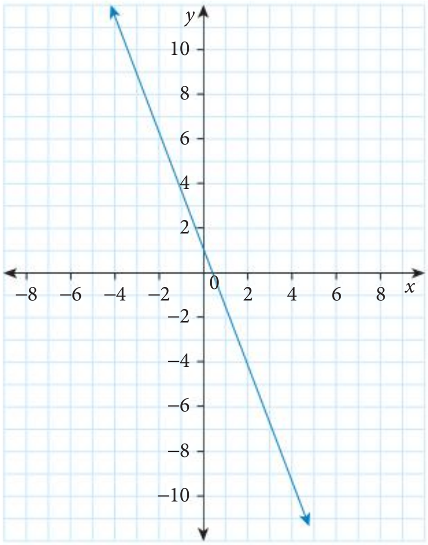
The gradient is −5.

EXERCISE 9.03 Calculating the gradient of a line

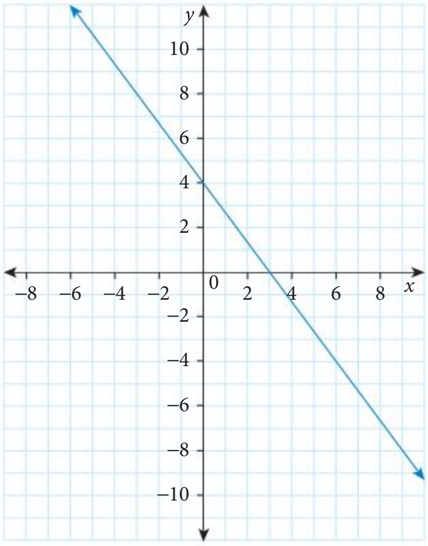
Concepts and techniques

1 **Example 7** State if the following lines have a positive or negative gradient.

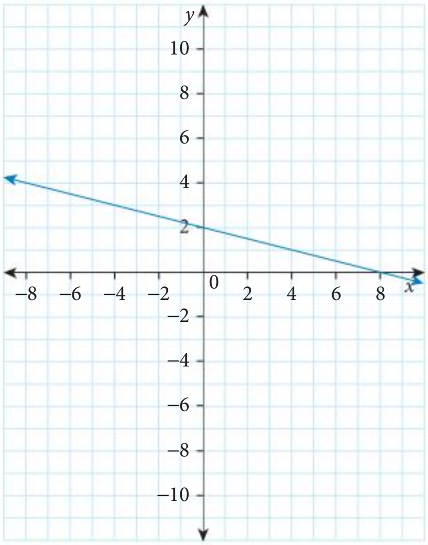
a



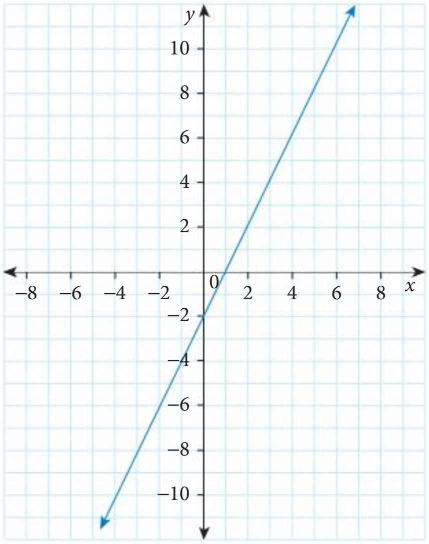
b



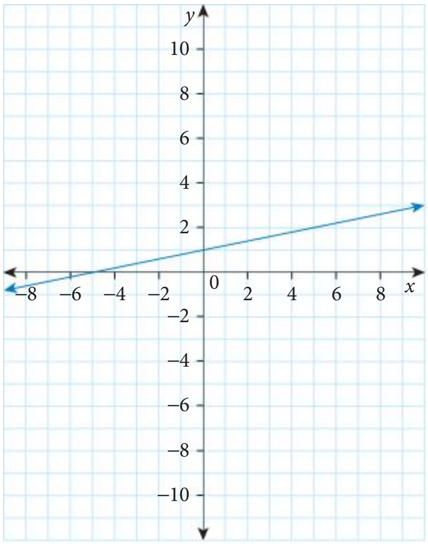
c



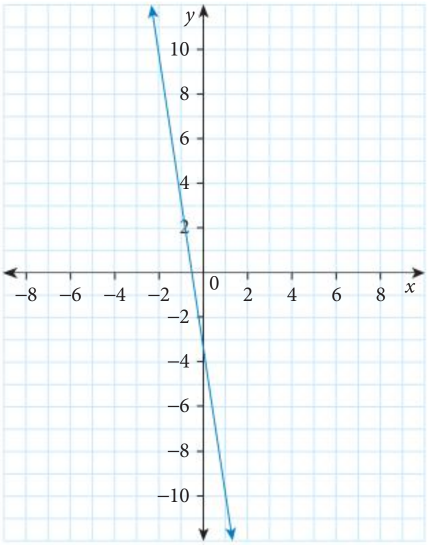
d



e


y
10
8
6
4
2
0
−2
−4
−6
−8
−10
x
−8
−6
−4
−2
2
4
6
8

f


y
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x
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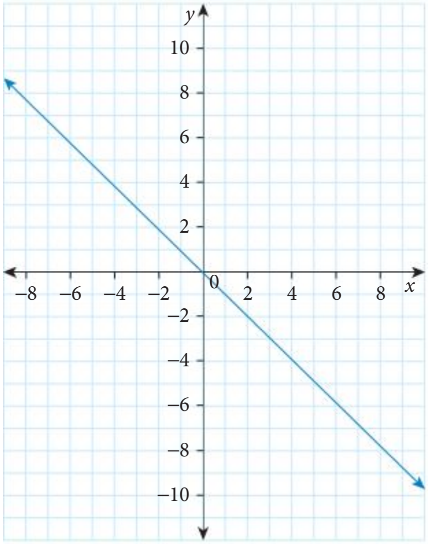
4
6
8

g


y
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8
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−10
x
−8
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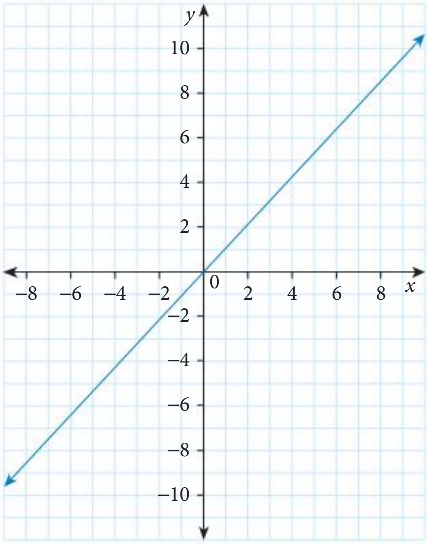
−4
−2
2
4
6
8

h


y
10
8
6
4
2
0
−2
−4
−6
−8
−10
x
−8
−6
−4
−2
2
4
6
8

2 Calculate the gradient of each line above by drawing in right-angled triangles and using the formula:

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}.$$

3 **Example 8** The gradient of the line joining point $A(-1, 3)$ to $B(5, -5)$ is:

A -2 B $-\frac{4}{3}$ C $-\frac{3}{4}$ D $-\frac{1}{3}$ E $\frac{3}{4}$

4 Find the gradient of the lines joining the points below using the formula: Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$.

a (2, 1) and (4, 5)
b (0, 2) and (2, 0)
c (5, 3) and (11, 15)
d (6, 8) and (10, 6)
e (−1, 2) and (3, −4)
f (2, −8) and (5, −12)
g (−2, −5) and (4, −6)
h (6, 7)and (−8, −10)

5 Find the gradient of the line represented by each of the following tables of values.

a

x	0	4	7	10
y	−1	11	20	29

b

x	1	5	7	12
y	1	21	31	56

c

x	0	8	12	20
y	3	5	6	8

d

x	1	3	6	8
y	7	3	−3	−7

e

x	0	10	25	40
y	−4	2	11	20

f

x	2	8	14	16
y	4	1	−2	−3

Reasoning and communication

6 Find two points which when joined result in a straight line with a gradient of 3.

7 Find two points which when joined result in a straight line a gradient of −3.

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9.04 THE GENERAL EQUATION OF A STRAIGHT LINE

The general equation of a straight line is $y = a + bx$ where a represents the y-intercept and b represents the gradient of the line.

IMPORTANT

Note that the value representing the gradient is the coefficient of the term containing the pronumeral x. In some instances you may need to rearrange the equation into the general form before identifying the gradient and the y-intercept. It is important to note that no matter what the order of the terms in the equation; the coefficient of x will always represent the gradient of the line and the constant will always represent the y-intercept of the line.

Example 9

State the gradient and y-intercept of the lines below.

a $y = 8 + 3x$ **b** $y = -2x - 5$ **c** $2x - 3y = 8$

Solution

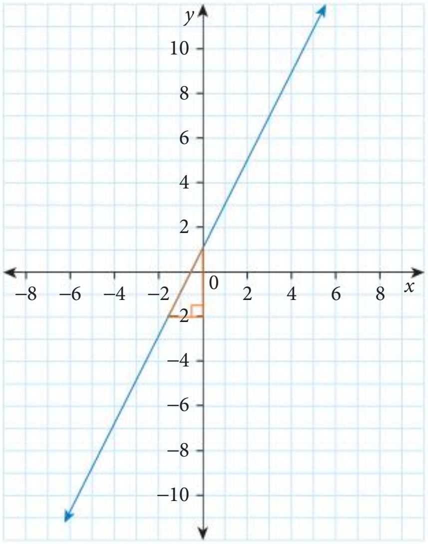
a	The general equation is $y = a + bx$ where b = gradient and a = y-intercept. State the values of the gradient and y-intercept.	$y = 8 + 3x$ $b = 3, a = 8$ The gradient is 3 and the y-intercept is 8.
b	The general equation is $y = a + bx$ where b = gradient and a = y-intercept. State the values of the gradient and y-intercept.	$y = -2x - 5$ $b = -2, a = -5$ The gradient is −2 and the y-intercept is −5.
c	Rewrite the equation in the form $y = a + bx$, that is, rearrange the equation to make y the subject.	$2x - 3y = 8$ $2x = 8 + 3y$ $2x - 8 = 3y$ $y = \frac{2}{3}x - \frac{8}{3}$
	The general equation is $y = a + bx$ where b = gradient and a = y-intercept. State the values of the gradient and y-intercept.	$b = \frac{2}{3}, a = -\frac{8}{3}$ or $-2\frac{2}{3}$ The gradient is $\frac{2}{3}$ and the y-intercept is $-2\frac{2}{3}$.

If the gradient and y-intercept are given, you can write the equation of the line. Additionally, if you are provided with the graph of a line you can calculate the gradient and identify the value of the y-intercept. When you have this information you can also write the equation of the line.

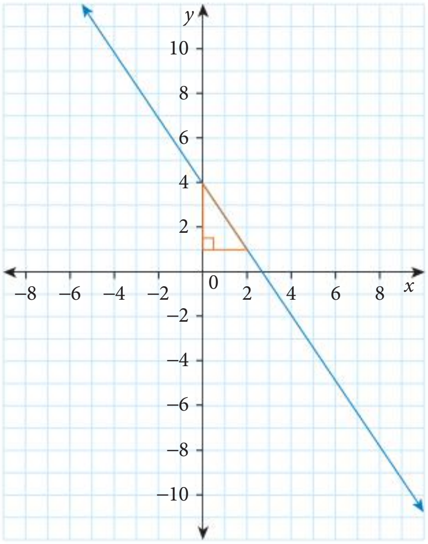
Example 10

Find the gradient and y-intercept of each of the lines below and hence find the equation of the line.

a



b

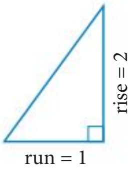


Solution

a Identify the y-intercept. $a = 1$

Calculate the gradient. $b = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

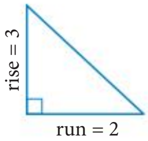
Substitute the values of a and b into the general equation: $y = a + bx$. The equation is $y = 1 + 2x$.



b Identify the y-intercept. $a = 4$

Calculate the gradient. $b = \frac{\text{rise}}{\text{run}} = -\frac{3}{2}$

Substitute the values of a and b into the general equation: $y = a + bx$. The equation is $y = 4 - \frac{3}{2}x$.



Sometimes the equation has a negative term first. It can be rearranged so that the positive term is written first. For example: $y = -8 + 4x$ can also be written as $y = 4x - 8$. The equations are the same.

EXERCISE 9.04 The general equation of a straight line

Gradient and y-intercept

Concepts and techniques

1 **Example 9** State the gradient and y-intercept of the lines below.

a $y = 4 + 3x$ b $y = -5x + 2$

c $y = \frac{3}{4} - 2x$ d $y = -6x - 8$

e $y = 5 - \frac{2}{3}x$ f $y = 10 - 3x$

g $y = \frac{2}{5} + x$ h $3x + y = 6$

i $2x - y = 8$ j $y - 4x + \frac{1}{3} = 0$

2 The gradient of the line $y = 6 - 3x$ is:

A -6 B -3 C 2 D 3 E 6

3 The y-intercept of the line $y = \frac{2}{5}x - 3$ is:

A -3 B $-\frac{2}{5}$ C $\frac{2}{5}$ D 2 E 3

4 Find the equation of the line with:

a gradient $= 3$, y-intercept $= 7$ b gradient $= -2$, y-intercept $= 1$

c gradient $= 1$, y-intercept $= -1$ d gradient $= \frac{1}{3}$, y-intercept $= -\frac{1}{2}$

e gradient $= -\frac{5}{4}$, y-intercept $= 0$ f gradient $= 0$, y-intercept $= 5$

5 a Write the equation $2x - 4y = 6$ in the form $y = a + bx$.

b State the gradient.

c State the y-intercept.

6 For each of the following equations:

i rearrange the equation into the general form of a straight line

ii state the gradient and y-intercept.

a $3x - 6y = 12$ b $2x + y - 4 = 0$

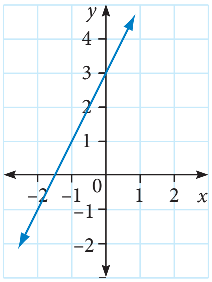
c $4x + 3y = 16$ d $x - 2y - 8 = 0$

e $5x + y = 4$ f $3x - 2y + 7 = 0$

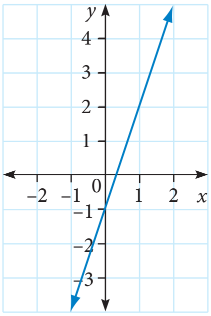
g $2x - 6y = 12$ h $4x + 5y - 8 = 0$

7 Example 10 Find the gradient and y-intercept of each of the lines shown below and hence write the equation of the line.

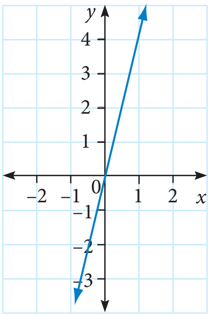
a



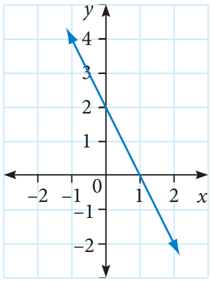
b



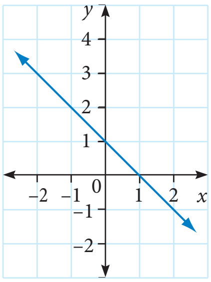
c



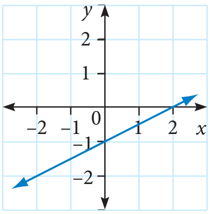
d



e



f



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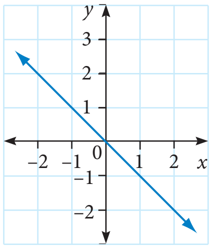

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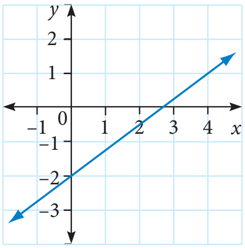

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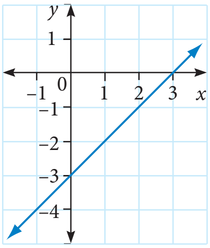


Reasoning and communication

8 State the gradient and vertical axis intercept for the formulas below.

a $C = 5N + 12$

b $S = 12.8 - 6.35n$

9 The gradient of the linear function $C = 1.8k + 5.2$ is:

A -1.8

B $\frac{1.8}{5.2}$

C 1.8

D $\frac{5.2}{1.8}$

E 5.2

9.05 GRAPHING USING GRADIENT AND y-INTERCEPT

It is quick and easy to draw linear graphs using your knowledge of gradient and y-intercept.

Plot the y-intercept on the y-axis. Use the gradient to identify the second point. Plot the second point and join the points to complete the sketch of the line.

For example: if the gradient of a line was known to be 2, to plot the next point, to the right of the y-intercept, you would move one unit across followed by two units up. If the gradient was −2, to plot the next point, to the right of the y-intercept, you would move one unit across and two units down.

Example 11

Sketch the lines below using the values of the gradient and y-intercept.

a $y = 2x + 3$ **b** $y = -2 + \frac{1}{2}x$ **c** $y = 4 - \frac{2}{3}x$

Solution

a Identify the gradient and the y-intercept from the equation.

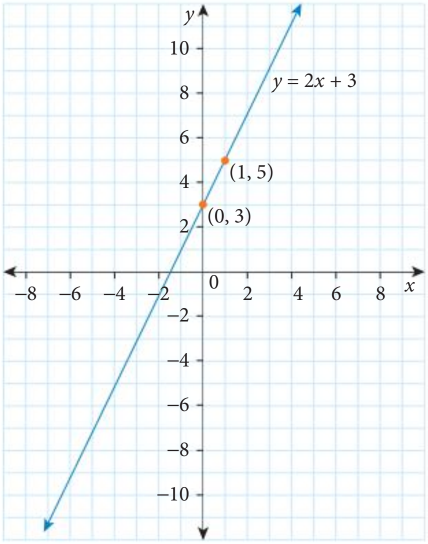
Gradient: $b = 2$
y-intercept: $a = 3$

Plot the coordinates of the y-intercept: (0, 3).

Move 1 unit across to the right followed by 2 units up, plotting a point at (1, 5).

Join these points, extending the line past them, to sketch the line.

Label the graph with its equation.



b Identify the gradient and the y-intercept from the equation.
Plot the coordinates of the y-intercept: (0, −2).

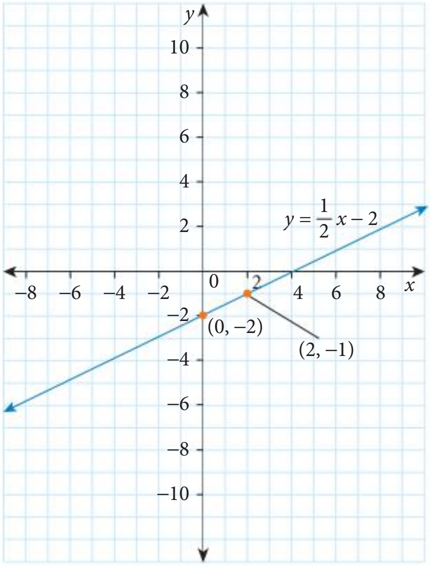
Gradient: $b = \frac{1}{2}$
y-intercept: $a = -2$

A gradient of $\frac{1}{2}$ means a rise of 1 and a run of 2.

Move 2 units across to the right followed by 1 unit up, plotting a point at (2, −1).

Join these points, extending the line past them, to sketch the line.

Label the graph with its equation.



c Identify the gradient and the y-intercept from the equation.

Gradient: $b = -\frac{2}{3}$

y-intercept: $a = 4$

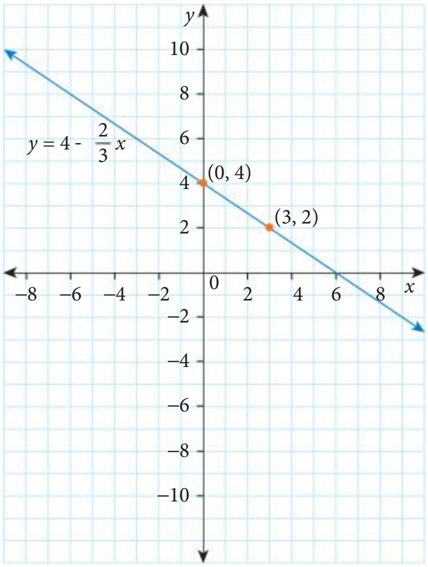
Plot the coordinates of the y-intercept: (0, 4).

A gradient of $-\frac{2}{3}$ means a rise of -2 and a run of 3.

Move 3 units across to the right followed by 2 units down, plotting a point at (3, 2).

Join these points, extending the line past them, to sketch the line.

Label the graph with its equation.



Sometimes the equation must be rearranged into the form $y = a + bx$ before this method can be used.

Example 12

Graph the lines below using the gradient and y-intercept method.

a $4x + 2y = 8$ **b** $3x - y + 5 = 0$

Solution

a Rearrange the equation to make y the subject.

$$4x + 2y = 8$$
$$2y = 8 - 4x$$
$$y = 4 - 2x$$

Identify the gradient and the y-intercept.

Gradient: $b = -2$

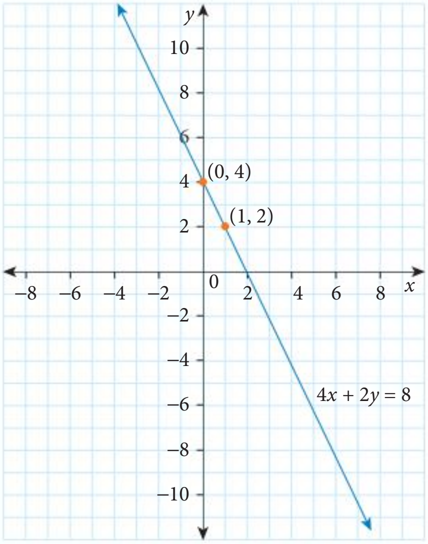
y-intercept: $a = 4$

Plot the coordinates of the y-intercept: (0, 4).

Move 1 unit across to the right followed by 2 units down, plotting a point at (1, 2).

Join these points to sketch the line.

Label the graph with its equation.



b Rearrange the equation to make y the subject.

$$3x - y + 5 = 0$$
$$-y + 5 = -3x$$
$$-y = -3x - 5$$
$$y = 3x + 5$$

Identify the gradient and the y-intercept.

Gradient: $b = 3$

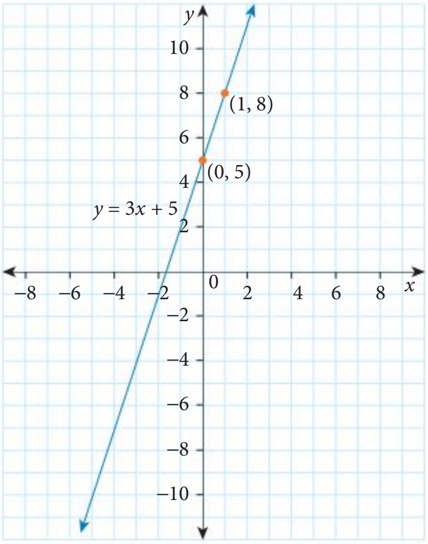
y-intercept: $a = 5$

Plot the coordinates of the y-intercept: (0, 5).

Move 1 unit across to the right followed by 3 units up, plotting a point at (1, 8).

Join these points to sketch the line.

Label the graph with its equation.



If two points lie on the same line, then we can use the given points to find the gradient of the line by using the formula: gradient $= \frac{y_2 - y_1}{x_2 - x_1}$.

Then use the given points to find the value of the y-intercept.

And finally substitute the values of the gradient (b) and the y-intercept (a) into the equation: $y = a + bx$.

Example 13

The points (0, –3) and (2, 7) lie on the same line.

a Find the gradient of the line.

b Find the equation of the line and hence sketch its graph.

Solution

a Write down the formula for finding the gradient of a line when given two points.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Identify the values of (x_1, y_1) and (x_2, y_2).

Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (2, 7)$.

Substitute these points into the formula and simplify.

$$\text{Gradient} = \frac{7+3}{2-0} = \frac{10}{2} = 5$$

b Write the general equation of the straight line, where a = y-intercept and b = gradient.

$y = a + bx$

Identify the values of a and b.

The point (0, –3) lies on the line and has the x-coordinate of zero so the y-intercept is –3.

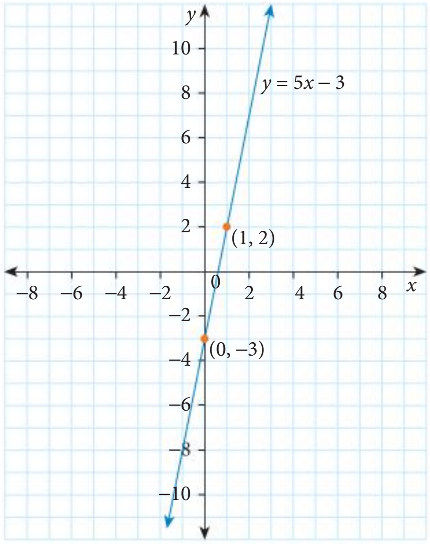
$a = -3, b = 5$

Substitute these values into the equation.

$y = -3 + 5x$ or $y = 5x - 3$

Sketch the graph using the y-intercept and gradient method.

Label the graph with its equation.



EXERCISE 9.05 Graphing using gradient and y-intercept

WS General equation of a straight line

Concepts and techniques

1 Example 11 Graph each of the following using the gradient and y-intercept method.

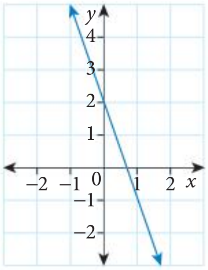
a $y = x - 2$
b $y = \frac{1}{2}x + 2$
c $y = 4x - 3$
d $y = -2x$
e $y = 2x - 5$
f $y = \frac{3}{5}x - 1$
g $y = -x + 4$
h $y = \frac{4}{3}x + 1$
i $y = -\frac{1}{4}x + 3$

2 Example 12 Graph the lines below using the gradient and y-intercept method.

a $5x + y = 6$
b $4x - y = 3$
c $6x + 2y = -8$
d $3x - y + 5 = 0$
e $4x + 2y = 6$
f $2x - 5y + 10 = 0$
g $x + 2y = 8$
h $4x - 2y - 12 = 0$

3 Which of the following is the gradient of this line?

A -3
B -2
C $-\frac{1}{3}$
D $\frac{1}{3}$
E 3



4 What is the y-intercept of the line in question **3**?

A -3
B -2
C -0.7
D 0.7
E 2

5 a Using the gradient and y-intercept method, sketch the following lines on the same Cartesian plane.

i $y = x$
ii $y = 2x$
iii $y = -\frac{1}{5}x$
iv $y = -3x$
v $y = \frac{1}{4}x$

b List any similarities and differences between the lines.

6 The y-intercept for the graph with equation $4x - 2y = 10$ is:

A -10
B -5
C 2
D 4
E 10

7 Example 13 For each of the given pairs of points find the gradient of the line connecting them, and hence the equation of the line.

a $(0, 1)$ and $(2, 7)$
b $(-1, 5)$ and $(0, 4)$
c $(0, 3)$ and $(-1, 0)$
d $(0, -10)$ and $(5, -2)$
e $(3, 8.5)$ and $(0, -4.5)$
f $(-4, -6)$ and $(0, -11)$

Reasoning and communication

8 a What is the gradient and vertical axis intercept of the line given by $C = 2.6 - 4.5n$?

b Sketch a graph of the line given by $C = 2.6 - 4.5n$.

9 a State the gradient and vertical axis intercept of $P = -2.8t + 3.95$.

b Sketch a graph of the line with rule $P = -2.8t + 3.95$.

10 For each set of points below:

i plot the points and draw the line that passes through both

ii find the gradient of the line connecting the two points

iii find the coordinates of the point where the line passes through the y-axis

iv find the equation of the line connecting the two points.

a (3, 5) and (1, 1)   b (4, −2) and (10, −14)   c (−8, −4) and (−6, −3)

11 a If the cost, C, of running a taxi is represented by the equation $C = 3.5 + 2.8k$, where k is the number of kilometres travelled, what does the 3.5 represent?

b Find the cost of travelling 10 kilometres in the taxi.

c Draw a graph of the line with rule $C = 3.5 + 2.8k$.

INVESTIGATION Stairway to heaven

Alamy/Forget Patrick/SAGAPHOTO.COM

Chris was on holidays in Europe when he entered a huge cathedral on top of a hill 50 m above sea level. It had beautiful stained glass windows and a huge flight of stairs leading all the way up through the steeple.

He decided to climb the stairs to see where they led.

He climbed 25 stairs and came to a landing.

He walked 2 m across the level landing and came to the next flight of stairs.

He climbed this flight of stairs, which also had 25 steps, and noticed each step was 20 cm high. There was another landing 2 m wide before the next flight of stairs.

The guide for the cathedral found Chris and explained to him that these stairs and landings continued on in the same pattern for 30 floors. It was only after going up 30 floors that he would be at the top and see the panoramic view of the city below.

a If H is Chris's height above sea level in metres and s is the number of stairs he climbs, write an equation to illustrate his height above sea level whilst climbing the first 25 stairs.

b What is his height above sea level when he has finished climbing the first 25 stairs?

c What is Chris's height above sea level after climbing the next flight of stairs?

d Write this equation as a linear function using H for Chris's height above sea level and n for the number of flights of stairs climbed, stating its gradient and H-intercept.

e What does the H-intercept mean in this situation?

f What is the meaning of the gradient in this case?

g Find Chris's height above sea level after climbing 5 flights of stairs.

h How many stairs had he climbed when he was 95 m above sea level?

i What is his height above sea level when he reaches the top of the steeple?

j If it takes Chris 1 minute 10 seconds to walk up 25 stairs and another 25 seconds to cross each landing, how long will it take him to climb to the top of the steeple if he rests for 5 minutes halfway up?

k With a friend, find a set of stairs close by and do the same investigation as above.

l Outline any differences you find between your flights of stairs and the cathedral's stairs.

9.06 SPECIAL LINES

If an equation contains both x and y, then the line matching that equation will be sloped or oblique. There are some special lines that contain only one of the pronumerals.

Such a line is $x = 3$, where all of the points on the line have an x value of 3. If you plot these points, such as (3, 2), (3, 1) and (3, –1) you will notice that they form a **vertical** line (or a line parallel to the y-axis).

Another such line is $y = 2$, in this case, all of the points on the line have a y value of 2. If you plot these points, such as (3, 2), (5, 2) and (–2, 2) you will notice that they form a **horizontal** line (or a line parallel to the x-axis).

IMPORTANT

Lines with equations in the form $x = a$, where a is a constant, are vertical lines through the number represented by the constant. For example, $x = 3$ is a vertical line through 3 on the x-axis.

Lines with equations in the form $y = a$, where a is a constant, are horizontal lines through the number represented by the constant. For example, $y = –7$ is a horizontal line through –7 on the y-axis.

The gradient of a horizontal line is zero whereas the gradient of a vertical line is undefined.

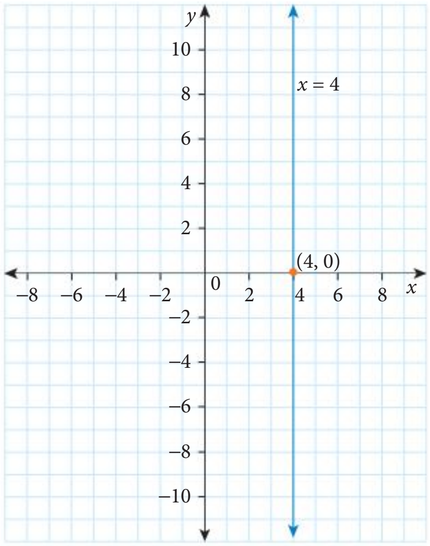
Example 14

Sketch the graphs of the following equations.

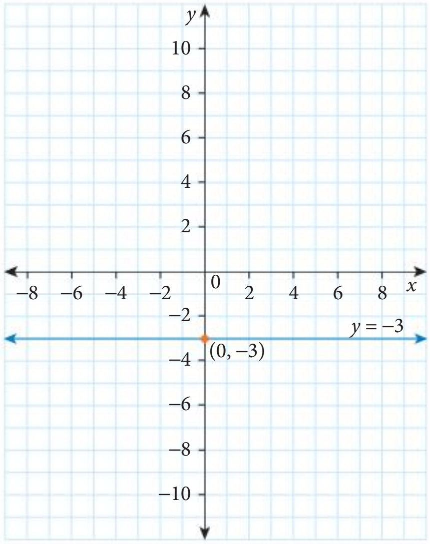
a $x = 4$ **b** $y = -3$

Solution

a The equation is in the form $x = a$. The graph is a line parallel to the y-axis.
Draw a vertical line passing through the point (4, 0).
Label the graph with its equation.



b The equation is in the form $y = a$. The graph is a line parallel to the x-axis.
Draw a horizontal line passing through the point (0, −3).
Label the graph with its equation.



If an equation is written in the form $y = bx$, the constant term, a, is equal to 0. The graph of this equation is a line passing through the origin where the coordinates of the x- and y-intercepts are both equal to zero. The gradient–intercept method can be used to draw this line or a second point can be found to plot. The second point is usually $x = 1$ and then the corresponding y-value is found.

Example 15

Sketch the graphs of the following equations.

a $y = \frac{1}{2}x$ **b** $y = -4x$

Solution

a Write the equation.

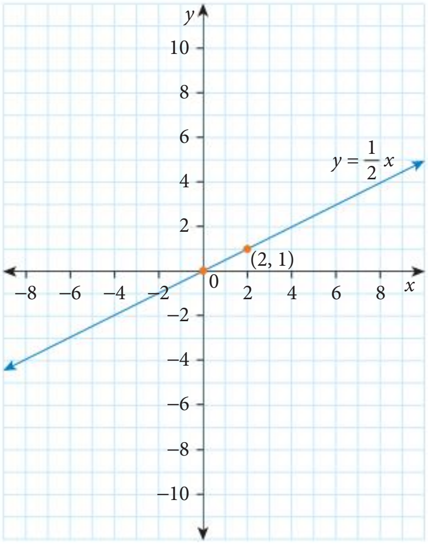
$y = \frac{1}{2}x$

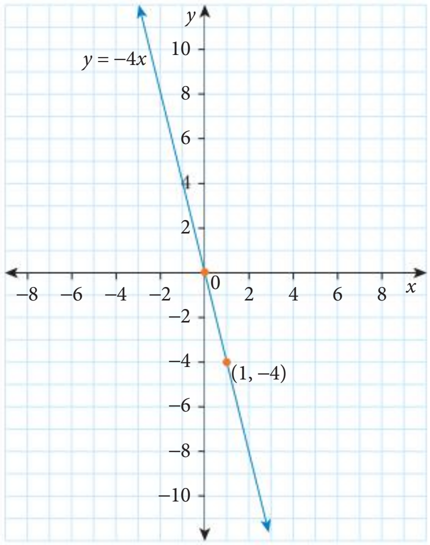
The equation is written in the form $y = bx$, hence the line passes through the point (0, 0). In this case the coefficient of x is $\frac{1}{2}$, so find another point by letting $x = 2$ (the number in the denominator). This will make our calculations easier.

When $x = 2$, $y = \frac{1}{2} \times 2 = 1$

The second point is (2, 1).

Sketch the graph.
Label the graph with its equation.



b Write the equation.	$y = -4x$
The equation is written in the form $y = bx$, hence the line passes through the point (0, 0). Find another point by letting $x = 1$.	When $x = 1$, $y = -4 \times 1 = -4$ The second point is (1, −4).
Sketch the graph. Label the graph with its equation.	

EXERCISE 9.06 Special lines

Concepts and techniques

1 Example 14 Sketch the following lines.

- **a** $x = -2$
- **b** $y = 4$
- **c** $y = 5$
- **d** $x = -4$
- **e** $x = 1.5$
- **f** $y = -3.5$
- **g** $x = 0$
- **h** $y = 0$

2 The graph of the equation $x = -3$ is:

- **A** a horizontal line through $x = -3$.
- **B** a vertical line through $x = -3$.
- **C** a horizontal line through $y = -3$.
- **D** a vertical line through $y = -3$.
- **E** a diagonal line through $x = -3$.

3 The graph of the equation $y = 5$ is:

- **A** a horizontal line through $y = 5$.
- **B** a horizontal line through $x = 5$.
- **C** a vertical line through $x = 5$.
- **D** a diagonal line through $y = 5$.
- **E** a vertical line through $y = 5$.

9780170250252

4 a Sketch the lines $x = 4$ and $y = -3$ on the same number plane.
  b Identify the point of intersection.

5 Example 15 Sketch the following lines.

a $y = x$ b $y = \frac{1}{3}x$ c $y = -\frac{1}{2}x$ d $y = 10x$ e $y = -5x$

Reasoning and communication

6 Without graphing the lines below, write down the point of intersection for each pair.

a $x = 2, y = -4$
b $x = -5, y = 3$
c $y = 4, x = -1$
d $y = -6, x = 8$
e $x = 14, y = -10$
f $y = 8, x = -12$
g $x = -1.5, y = 6.2$
h $y = -9.8, x = -3.6$

7 Sketch each of the special lines below on the same number plane and state the figure enclosed by each set of lines.

a $x = 2, y = -2, x = 6, y = 2$
b $x = -1, y = -5, x = 2, y = 3$
c $x = -2, y = -4, x = 6, y = 0$
d $x = 2, y = -1, x = 0, y = 1$

8 State another name for the line with equation $x = 0$.

9 State another name for the line with equation $y = 0$.

9.07 DEVELOPING A LINEAR FORMULA

In Mathematics you can sometimes represent patterns in nature and real life situations using a rule or model. When the rule is linear we call it **linear modelling**. In this case the rule will be of the form: $y = a + bx$.

It can be said that the value of y depends on the value of x, that is, y is the **dependent variable** and x is the **independent variable**.

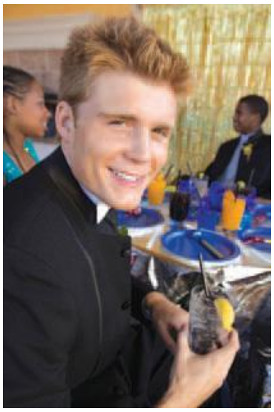
When using examples in linear modelling, the variables are not always x and y.

Often other pronumerals are used, such as C for cost or d for distance travelled.

When a function is graphed, the independent variable is represented on the horizontal axis and the dependent variable is on the vertical axis.

Example 16

Jules is organising the annual Year 11 dance. The total cost for the event will include \$500 room hire, \$280 for the DJ and \$25 per head for food. The table below shows the total cost, $\$C$, for a different number of students, n, attending the dance.


Alamy/moodboard

No. of students (n)	80	100	120	140	160
Total cost ($\$C$)	2780	3280	3780	4280	4780

A linear model of the form $C = a + bn$ can be used to find the total cost of the event, C, for n students.

- a Which variable is the dependent variable?
- b Find the value of b.
- c In the rule $C = a + bn$, what does a represent in this context?
- d Find the value of a.
- e Write down the linear rule for C in terms of n.
- f Sketch a graph of C against n.
- g What would be the total cost of the dance if 105 students attended?

Solution

a The dependent variable depends on the other variable.	C is the dependent variable, as cost depends on the number of students, n, attending the dance.
b b is the gradient. Choose 2 points on the line. Use the formula: Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$, to find the gradient	(80, 2780) and (100, 3280) $b = \frac{3280 - 2780}{100 - 80}$ $= \frac{500}{20}$ $= 25$
c a is the vertical axis intercept and is therefore the cost when $n = 0$.	a represents the cost of the dance if no students attend. It is the initial set up cost.
d Initial set up cost = Room hire plus DJ.	$a = 500 + 280$ $= 780$
e Substitute the values of a and b into the equation $y = a + bx$. Remember to use C for y and n for x.	$C = a + bn$ $a = 780$, $b = 25$ $C = 780 + 25n$

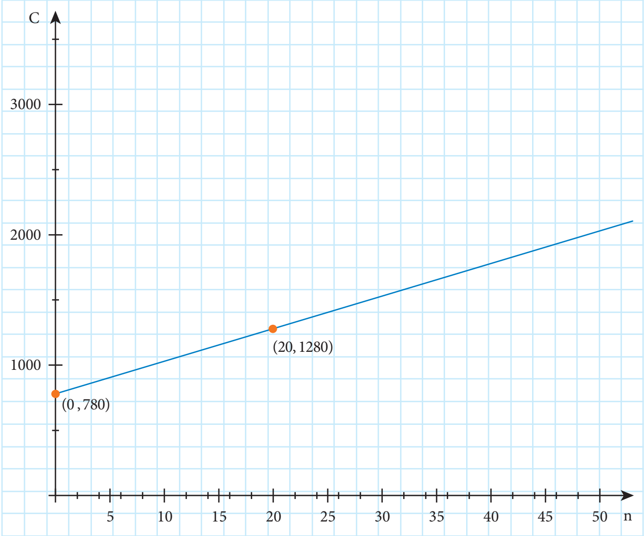
f It is not appropriate to consider negative values of C or n in this situation. Find the coordinates of two points on the graph.

The C-axis intercept is already known.
To sketch the graph, another point is required.
Find the coordinates of the point on the graph where, say, $n = 20$.
Sketch the graph.
Label the graph with its equation.

The C-axis intercept has coordinates (0, 780).

$n = 20$: $C = 780 + 25 \times 20$
$= 1280$

Another point on the line has coordinates (20, 1280).



g Write the equation for the total cost.
Substitute $n = 105$ into the equation.

$C = 780 + 25n$
$= 780 + 25 \times 105$
$= 3405$

The cost of the dance will be \$3405 if 105 students attend.

Example 17

A criminologist discovered that the number of crimes committed per month, C, in a big city decreased as the number of police officers, P, patrolling the city increased. After graphing her data on a number plane, she found the linear relationship to be $C = -3P + 3250$.

a What is the independent variable in this relationship?

b Copy and complete this table for the equation $C = -3P + 3250$.

Number of police, *P*	50	150	200	250	300
Crimes per month, *C*					

c Graph $C = -3P + 3250$ on the number plane.

d What is the gradient of the graph you drew in part **c** and what does it represent?

e What is the vertical axis intercept of the graph you drew in part **c** and what does it represent?

f Calculate how many crimes are committed when 100 police officers are on patrol.

g Calculate how many police officers are required to reduce the number of crimes to 1900.

Solution

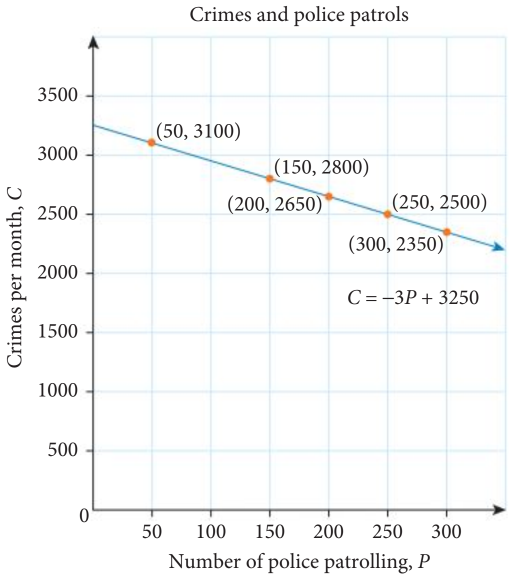
a The independent variable does not depend on the other variable.

P is the independent variable; as the number of police does not depend on the number of crimes committed.

b Complete the table by substituting values into: $C = -3P + 3250$.

Number of police, *P*	50	150	200	250	300
Crimes/month, *C*	3100	2800	2650	2500	2350

c Plot the points and draw the graph. Label the graph with its equation.



d Find the gradient of $C = -3P + 3250$.

The gradient is −3 and it represents the rate of change in crime as the number of police increases. As the number of police increases by 1, the number of crimes decreases by 3.

e Find the vertical axis intercept of $C = -3P + 3250$.

The vertical axis intercept is 3250, and this represents the number of crimes if no police officers were on patrol. It is the value of C when $P = 0$.

f Substitute $P = 100$ into $C = -3P + 3250$.

$$C = -3P + 3250$$
$$C = -3(100) + 3250$$
$$= 2950$$

2950 crimes are committed when 100 police are on patrol.

g Substitute $C = 1900$ into $C = -3P + 3250$.

$$C = -3P + 3250$$
$$1900 = -3P + 3250$$
$$-3P = -1350$$
$$P = \frac{-1350}{-3}$$
$$= 450$$

450 police officers are required to reduce the number of crimes to 1900.

EXERCISE 9.07 Developing a linear formula

Practical applications

Concepts and techniques

1 For the linear rule $C = 4.5n + 22$, which variable is the independent variable?

2 For the linear rule $M = 280 - 6.2p$, which variable is the dependent variable?

3 Write down the gradient for the linear rule $V = 22.6x + 280$.

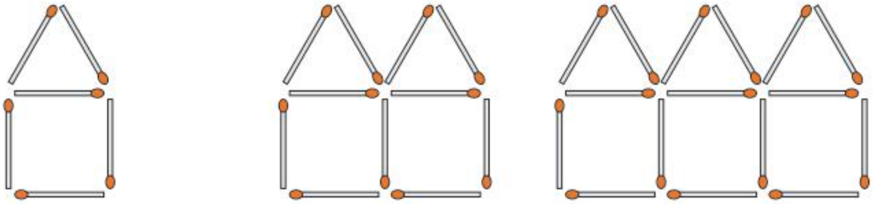
4 What is the vertical axis intercept for the linear rule in question **3**?

5 The gradient for the line with rule $C = 290 - 6.5n$ is:

A -290 B -6.5 C 0 D 6.5 E 290

Reasoning and communication

6 Example 16 This is a matchstick pattern of houses.



One house uses 6 matches.

Two houses use 11 matches.

Three houses use 16 matches.

a Copy and complete this table for the pattern above.

Number of houses, *h*	1	2	3	4	5	6
Number of matches, *N*						

b What is the dependent variable?

c Find the linear relationship in the form $N = a + bh$.

d How many matches are required to make 20 houses?

e How many houses can be made from 81 matches?

f Sketch a graph of N against h on a number plane.

g State the gradient and the N-intercept of the line.

7 This table shows the cost, C cents, of mobile phone calls under the Oz-Zone Budget Plan, for calls of different lengths, in t minutes.

Length of call, *t* (min)	1	2	5	10	15
Cost, *C* (cents)	102	182	422	822	1222

a Find the linear relationship in the form $C = a + bt$.

b If this rule was graphed on a number plane, which variable would be shown on the vertical axis?

c Use the relationship you found in part **a** to calculate the cost of an 18-minute call.

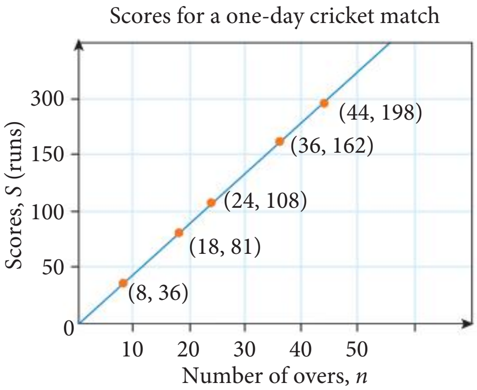
d What is the vertical axis intercept of the graph and what does it represent?

e If a phone call is extended by 3 minutes, by how much would its cost increase?

f How long is a phone call under this plan if it cost \$5.82?

8 A cricket team's progressive score during a one-day cricket match can be approximated by the linear rule graphed below.

A one-day match has 50 overs, where an over is a set of 6 balls bowled by the same bowler. The variable, n, represents the number of overs bowled, while S represents the total number of runs scored by the batting team.



- a Is S the dependent variable or the independent variable?
- b Find the formula for S in terms of n.
- c The gradient of this graph is also the team's run rate. What is the gradient of the graph and in what units is this **run rate** measured?
- d What is the S-intercept and what does it represent?
  - i the 21st over? ii the 50th over?
- e What was the score after:
  - i 54 runs? ii 180 runs?
- f At the end of which over had the score reached:
- g The graph for a real cricket match would not be a straight line but would normally flatten out later in the innings. Explain your reasoning.

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9 Example 17 The value of a notebook computer depreciates according to the formula $V = -420t + 1900$, where V is the value in dollars and t is the time in years.

- a Copy and complete this table for the formula $V = -420t + 1900$.

Time, t (years)	1	2	3	4
Value of computer, V (\$)				

- b Graph this linear relationship on a number plane.
- c What does the gradient of this line represent?
- d What was the value of the computer after $2\frac{1}{2}$ years?
- e What was the original value of the computer?
- f This linear model does not work when $t = 5$ and beyond. Why not?
- g Find, correct to one decimal place, the time when the computer has zero value.

10 Jessica works for a pizza shop and each day she earns a base pay of \$50, plus \$3 for every pizza she delivers.

- a If n is the number of pizzas Jessica delivers in a day and P is her total pay in dollars, write a formula for P in terms of n.
- b If this line was graphed, which variable would be represented on the horizontal axis?
- c What is the vertical axis intercept of this line and what does it represent?
- d How much will Jessica earn for delivering 28 pizzas in a day?
- e If Jessica earned \$98 today, how many pizzas did she deliver?

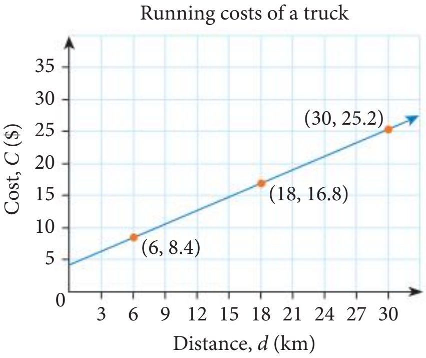
11 This table shows the linear relationship between distances measured in miles and distances measured in kilometres.

Miles, *M*	15	25	30	45
Kilometres, *K*	24	40	48	72

- a Graph this linear relationship on a number plane.
- b Find the equation of the line.
- c What can you say about the value of the vertical axis intercept? Why?
- d What is the gradient and what does it represent?
- e Use your equation from part **b** to convert:
  - i 100 miles to kilometres.
  - ii 100 km to miles.
- f Use your graph from part **a** to convert:
  - i 12 miles to kilometres.
  - ii 20 km to miles.

12 This graph shows the linear relationship between the distance, d km, travelled by a truck and the running costs, \$$C$, for the trip.

- a What is the dependent variable?
- b Find the gradient and the vertical intercept of this linear relationship.
- c Write the formula for this linear relationship.
- d If the length of a trip is extended by 5 km, by how much will the charge increase?
- e Calculate the running costs for a trip of length:
  - i 20 km.
  - ii 0 km.
- f Calculate the distance travelled if the running costs were $37.80.



13 During summer, crickets chirp faster at night if the temperature is higher. There is a linear relationship between the temperature and a cricket's chirping rate, as shown in the table below.

Temperature, *T* (°C)	12	15	19	22	28
Chirp rate, *n* (chirps/min)	72	96	128	152	200

- a Is T the dependent variable or the independent variable?
- b Find the linear rule for n in terms of T.
- c Graph the linear rule you found in part **b**.
- d If the temperature increases by 2°C, what happens to the crickets' chirp rate?
- e Find the chirp rate of a cricket when the temperature is 26°C.
- f At what temperature does a cricket chirp 144 times per minute?
- g What is the vertical axis intercept of this line? Why doesn't the linear model work for this value?

14 George works in a burger shop and each day he earns a base pay of $75, plus $2 for every burger he makes.

Alamy/David Levenson

- a If n is the number of burgers George makes in a day and P is his total pay in dollars, write a formula for P in terms of n.
- b If this linear rule was graphed, which variable would be represented on the horizontal axis?
- c What is the vertical axis intercept of this function and what does it represent?
- d How much will George earn for making 50 burgers in a day?
- e If George earned $315 today, how many burgers did he make?

9

CHAPTER SUMMARY

LINEAR EQUATIONS AND GRAPHS

- An **equation** is a mathematical sentence with an equals sign. To **solve** an equation you find the value of the pronumeral which makes the equation true. You use **inverse operations** to solve an equation.
- The graph of a **linear rule** is a **straight line**.
- The **gradient–intercept form** of a straight line is: $y = a + bx$ where $a = y$-intercept and $b =$ gradient of the line.
- The **gradient of a line** is $\frac{\text{rise}}{\text{run}}$.
- The **gradient of a line given two points** (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.
- There are many ways to **graph** a straight line:
  - by **plotting points** in a table
  - by finding ***x* and *y*-intercepts**
  - by using the **gradient** (slope) and ***y*-intercept**
  - by using a **CAS** calculator.
- **Special lines** are vertical ($x = a$) or horizontal ($y = a$).
- When using practical applications with linear functions you place the **independent variable** on the horizontal axis and the **dependent variable** on the vertical axis.

9780170250252

CHAPTER REVIEW
LINEAR EQUATIONS AND GRAPHS

9

Multiple choice

1 **Example 2** The solution to the equation $4(23 - 5x) = 12$ is

A $x = -4$ B $x = -3$ C $x = 3$ D $x = 4$ E $x = 5$

2 **Example 4** The straight line with equation $3x - 2y = 6$ has a y-intercept with the coordinates

A $(0, 6)$ B $(2, 0)$ C $(0, 2)$ D $(-3, 0)$ E $(0, -3)$

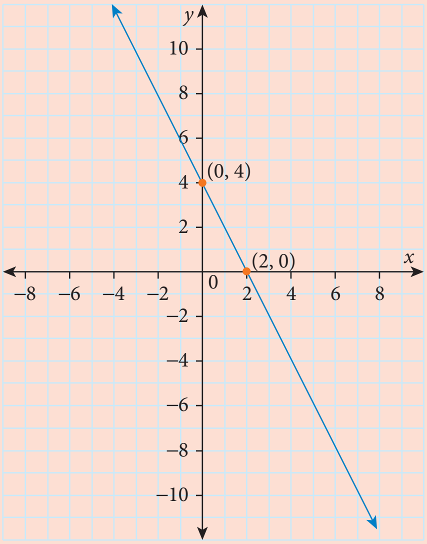
3 **Example 6** Which one of the following points does **not** lie on the straight line with equation $y = -3x + 1$?

A $(0, 1)$ B $(2, -5)$ C $(-1, 4)$ D $(-4, -11)$ E $(-10, 31)$

4 **Example 8** A line passes through the origin and the point $(3, -1)$. The gradient of this line is

A -3 B -1 C $-\frac{1}{3}$ D $\frac{1}{3}$ E 3

5 **Example 10** The graph of $y = a + bx$ is shown below.



The values of a and b are

A $a = -2, b = 4$ B $a = 4, b = -2$ C $m = 4, b = 2$

D $a = 4, b = -\frac{1}{2}$ E $a = -\frac{1}{2}, b = 4$

6 **Example 12** The gradient of the line $5x - 4y + 6 = 0$ is:

A -4 B $-\frac{5}{4}$ C $\frac{5}{4}$

D 5 E none of the above

Short answer

7 Example 2 Solve:

a $2x - 5 = 13$

b $\frac{3a+4}{2} = 8$

c $3x - 4 = 2x + 18$

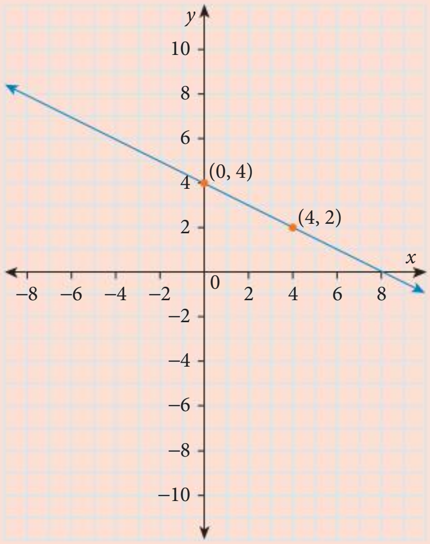
8 Examples 3, 11 Sketch the line $y = 3x - 2$ by:

a plotting 2 points.

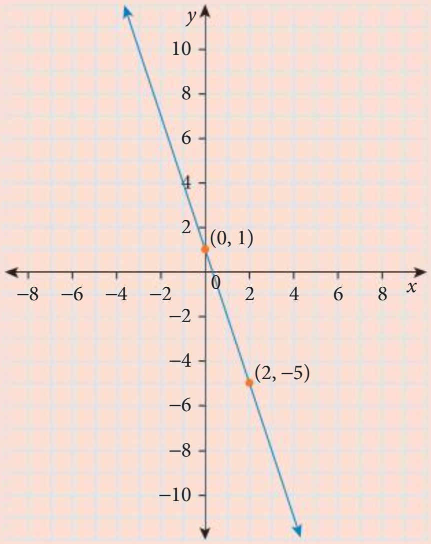
b using the gradient and y-intercept method.

9 Example 10 Write down the equation of the lines sketched below.

a



b



10 Example 9 State the gradient and y-intercept of each of the following straight lines.

a $y = 6 - 2x$

b $y = -3x - 7$

c $3x - 2y + 10 = 0$

11 Example 13 Find the equation of the straight line that contains the points (0, 7) and (2, 13).

12 Example 14 Sketch graphs of the straight lines with equations:

a $x = -3$.

b $y = 2$.

13 Example 15 Sketch graphs of the straight lines with equations:

a $y = 3x$.

b $y = -\frac{1}{5}x$.

Application

14 The cost of a taxi fare is represented by the equation $C = 4.5 + 2k$, where C is the total cost of the fare and k is the number of kilometres travelled.

- **a** What is the cost to the passenger before the taxi even starts off?
- **b** What is the gradient of the linear function?
- **c** Sketch a graph of the linear function.
- **d** How much would it cost to travel a distance of 11.2 km?

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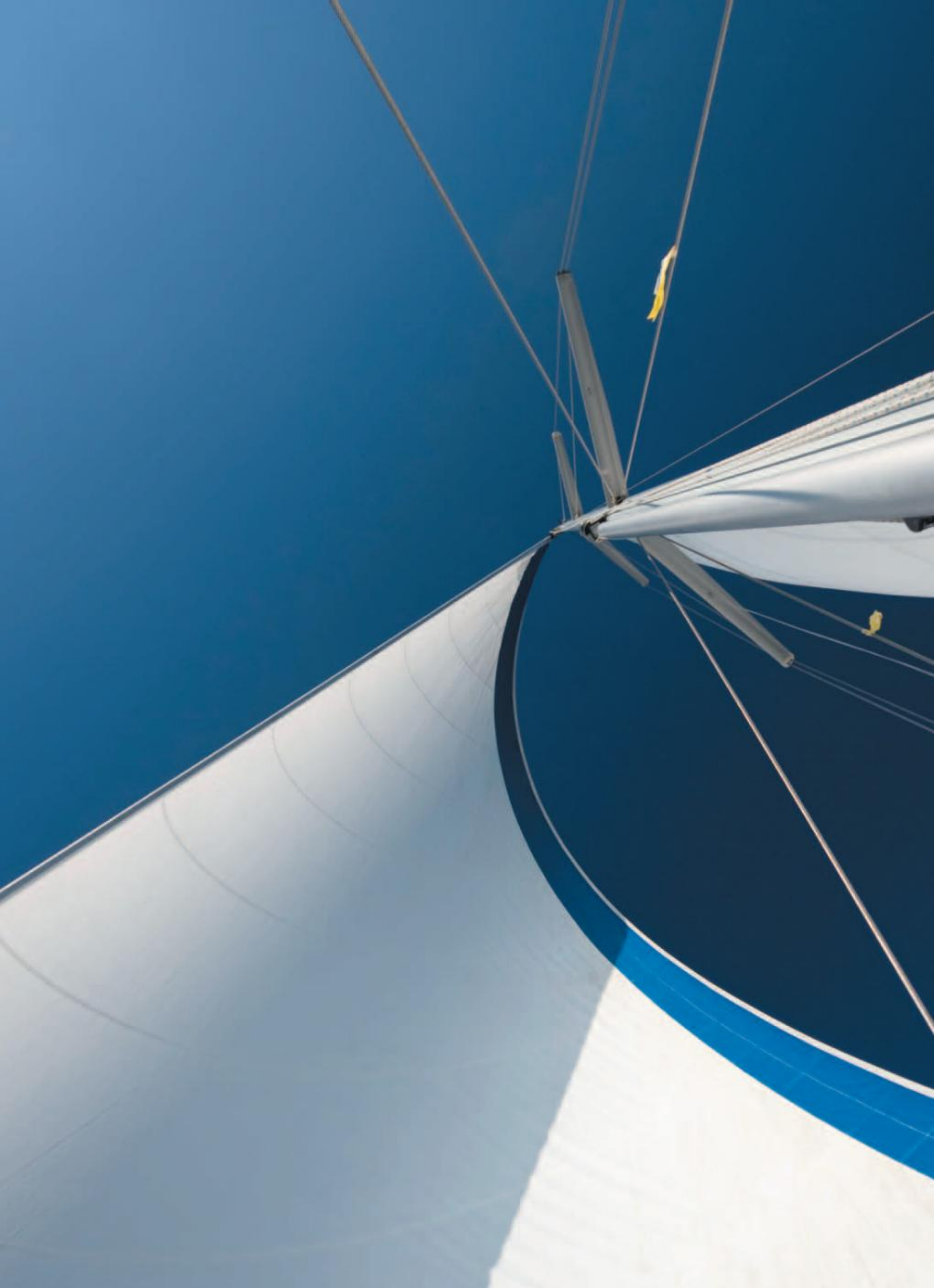
15 The value of an iPad depreciates according to the formula $V = -240t + 1100$, where V is the value in dollars and t is the time in years.

- **a** Copy and complete this table for the formula $V = -240t + 1100$.

Time *t* (years)	0.5	1	1.5	2
Value of iPad, *V* ($)				

- **b** Graph this linear relationship on a number plane.
- **c** What does the gradient of this line represent?
- **d** What was the value of the iPad after 1.5 years?
- **e** What was the original value of the iPad?
- **f** This linear model does not work when $t = 5$ and beyond. Why not?
- **g** Find, correct to one decimal place, the time when the iPad has zero value.

Practice quiz

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10

TRIGONOMETRY

APPLICATIONS OF TRIGONOMETRY

TERMINOLOGY

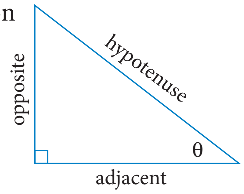
adjacent
angle of depression
angle of elevation
area of a triangle
compass bearing
cosine rule
Heron's formula
hypotenuse
included angle
non-included angle
non-right-angled triangle
opposite
right-angled triangle
sine rule
true bearing

Prior learning

APPLICATIONS OF TRIGONOMETRY

- **review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle (ACMGM034)**
- **determine the area of a triangle given two sides and an included angle by using the rule Area = $\frac{1}{2}$ *ab* sin *C*, or given three sides by using Heron's rule, and solve related practical problems (ACMGM035)**
- **solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule (ACMGM036)**
- **solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation (ACMGM037).**

10.01 FINDING AN UNKNOWN SIDE



Trigonometry has many real-life applications, some of which are construction and navigation. It allows us to measure unknown lengths and angles that may be too difficult to measure directly, for example, measuring the height of a building.

When we are working with **right-angled triangles** we need to be able to name the sides of the triangle. The convention is to look at the given angle, θ (named 'theta'), then label the sides according to their position in relation to θ, as the **hypotenuse**, **adjacent** side or **opposite** side.

There are three **trigonometric functions,** each defined by a specific ratio, which can be used to find the lengths of unknown sides and the size of unknown angles in right-angled triangles.

IMPORTANT

For angle θ in the diagram above:

Function	Abbreviation	Ratio	Initials
sine	sin	$\sin(\theta)=\frac{\text{opposite}}{\text{hypotenuse}}$	SOH
cosine	cos	$\cos(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$	CAH
tangent	tan	$\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}$	TOA

Some students memorise the initials of these ratios, SOH-CAH-TOA (pronounced 'so car toe ah'), while others use a phrase such as '**S**ome **O**ld **H**ens **C**an **A**lways **H**ide **T**heir **O**ld **A**ge' to remember them. Find your own mnemonic (memory aid) to remember this.

A trigonometric ratio can be used to calculate the length of an unknown side in a right-angled triangle if one angle and one side are known. Select the ratio that links the angle to both the unknown side and the known side.

IMPORTANT

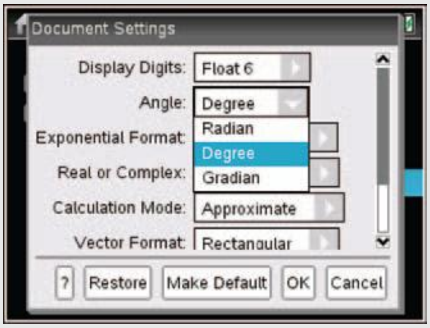
Before you begin your calculations you must have your calculator set to 'degree' mode. Your CAS will then identify that you are working in degrees and you will not have to use the degrees symbol when working with angles.

TI-Nspire CAS

On your home screen select 5: Settings.

From the menu select: Document Settings.

Open the menu within the Angle option. Highlight Degree using the ▼ key. Press enter. Press tab until Make Default is highlighted then press enter.


Document Settings

Display Digits: Float 6

Angle: Degree

Exponential Format:

Real or Complex:

Calculation Mode: Approximate

Vector Format: Rectangular

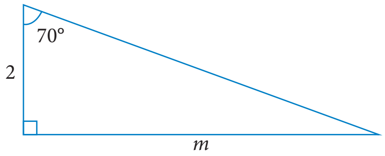
ClassPad

Using the $\overset{\text{Main}}{\sqrt{\alpha}}$ application, check **Deg** is showing on the bottom toolbar. If the mode is either **Rad** or **Gra**, tap until **Deg** appears.

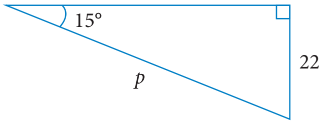
Example 1

Calculate the value of the pronumerals, correct to two decimal places.

a

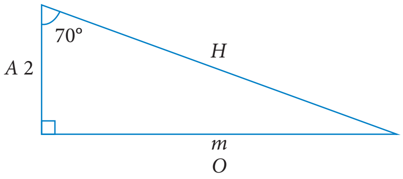


b



Solution

a Draw and label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the opposite and the adjacent side, the ratio will be tangent.

$$\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}$$

Substitute known values into the equation.

$\theta = 70°$, adjacent $= 2$

$$\tan(70°)=\frac{m}{2}$$

Solve the equation.

$$2\times\tan(70°)=\frac{m}{\cancel{2}}\times\cancel{2}$$

$$m = 2\tan(70°)$$

$$m = 5.49$$

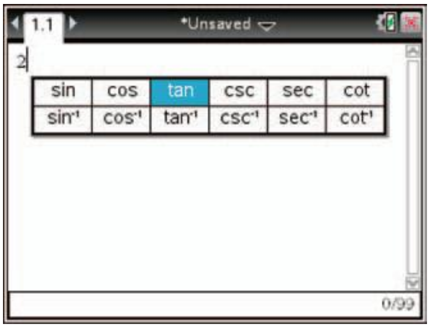
TI-Nspire CAS

Press 2.

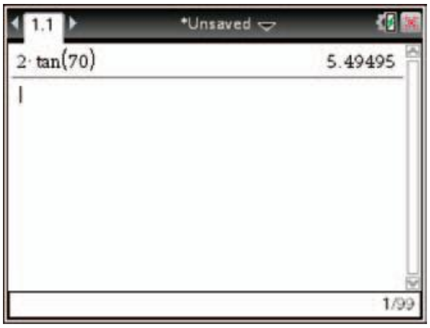
Press trig.

Select ‘tan’.

Key in 70. Press ctrl enter.



The side opposite the angle, represented by *m*, equals approximately 5.49 units.



```
2· tan(70)                5.49495
```

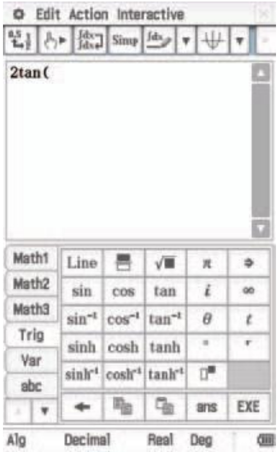
ClassPad

Use the Main $\sqrt{\alpha}$ Application in **Decimal** mode.

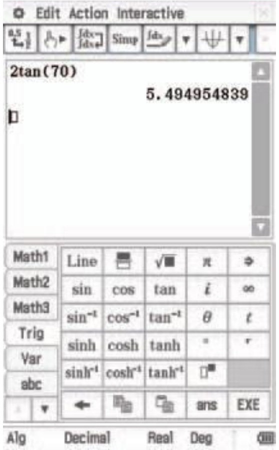
Press 2.

Press Keyboard.

Tap Trig and tan.

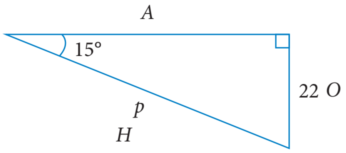


Type 70). Press EXE.



```
2tan(70)
            5.494954839
```

b Draw and label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the opposite and the hypotenuse, the ratio will be sin.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute known values into the equation.

$\theta = 15°$, opposite $= 22$, hypotenuse $= p$

$$\sin(15°) = \frac{22}{p}$$

Solve the equation.

Multiply both sides by p.

$$p \times \sin(15°) = \frac{22}{\cancel{p}} \times \cancel{p}$$

$$p \sin(15°) = 22$$

Divide both sides by sin (15°).

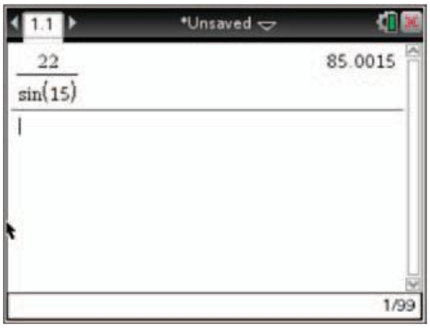
$$\frac{p \sin(15°)}{\sin(15°)} = \frac{22}{\sin(15°)}$$

$$p = \frac{22}{\sin(15°)}$$

$$p = 85$$

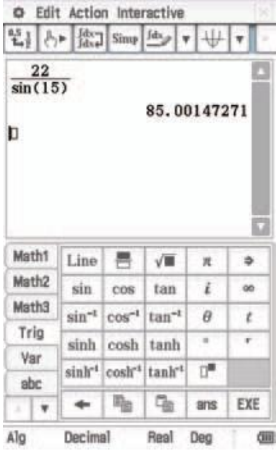
The hypotenuse, represented by p, equals approximately 85 units.

TI-Nspire CAS



$\frac{22}{\sin(15)}$ 85.0015

ClassPad

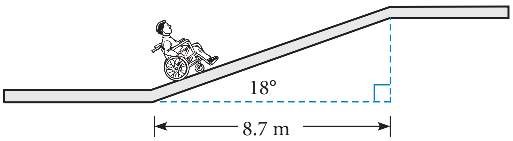


$\frac{22}{\sin(15)}$

85.00147271

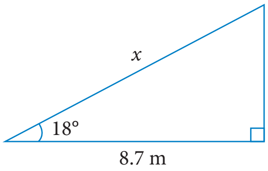
Example 2

A wheelchair ramp is inclined at 18° to the horizontal. How long is the ramp if it links two levels 8.7 m apart horizontally? Round your answer correct to one decimal place.

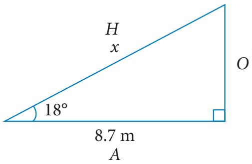


Solution

Redraw the triangle using the given values. Represent the length of the ramp by x.



Label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the adjacent side and the hypotenuse, the ratio will be cosine.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute the known values into the equation.

$\theta = 18°$, adjacent $= 8.7$

$$\cos(18°) = \frac{8.7}{x}$$

Solve the equation.

Multiply both sides by x.

$$x\cos(18°) = \frac{8.7}{\cancel{x}} \times \cancel{x}$$

Divide both sides by $\cos(18°)$.

$$\frac{x\cos(18°)}{\cos(18°)} = \frac{8.7}{\cos(18°)}$$

$$x = \frac{8.7}{\cos(18°)}$$

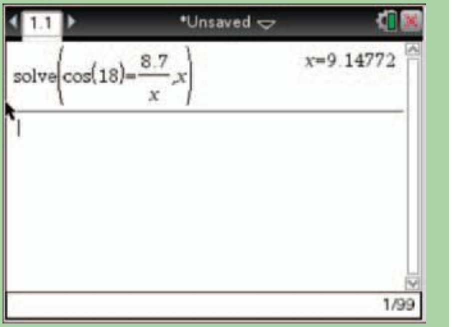
Evaluate and round correct to one decimal place.

$$x = 9.1477...$$
$$\approx 9.1$$

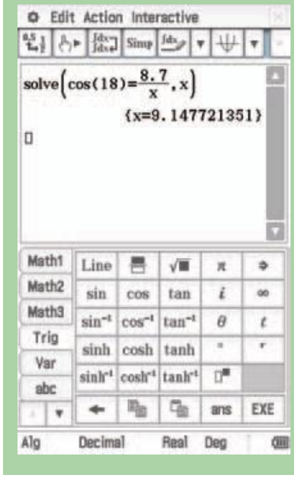
Write the answer.

The length of the ramp is 9.1 metres.

A CAS can be used to solve for an unknown side length.



$\text{solve}\left(\cos(18)=\frac{8.7}{x},x\right)$ $x=9.14772$

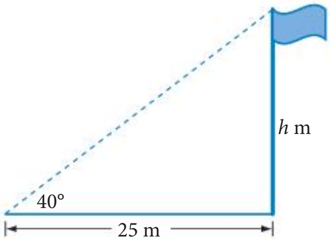


$\text{solve}\left(\cos(18)=\frac{8.7}{x},x\right)$

$\{x=9.147721351\}$

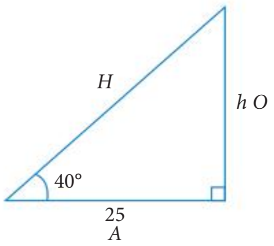
Example 3

Find the height, h m, of this flagpole correct to one decimal place.



Solution

Redraw the triangle using the given values.
Represent the height of the flagpole by h.
Label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation and solve for h.

$$\tan(40°) = \frac{h}{25}$$

$$\begin{aligned} h &= 25\tan(40°) \\ &= 20.9774\ldots \\ &\approx 20.98 \end{aligned}$$

Write the answer including units.

The height of the flagpole is 20.98 m.

Identifying the correct trigonometric ratio

EXERCISE 10.01 Finding an unknown side

Concepts and techniques

1 Evaluate each of these expressions correct to two decimal places.

a $7\tan(28°)$

b $\dfrac{25}{\sin(10°)}$

c $\dfrac{7}{\tan(63°)}$

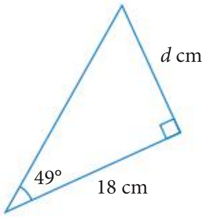
d $8\cos(12°)$

e $\dfrac{11.2}{\tan(77°)}$

f $7\sin(25°)$

2 Example 1a Calculate the value of the pronumerals, correct to two decimal places.

a



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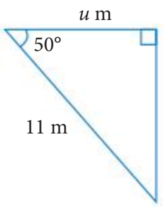
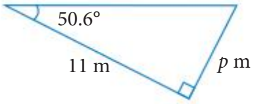
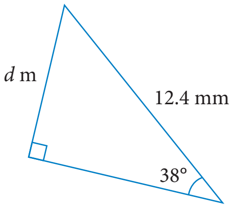


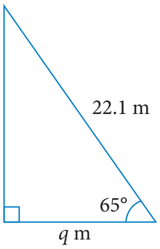
c



d



e



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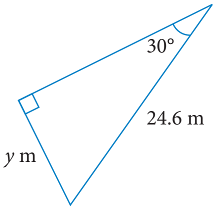
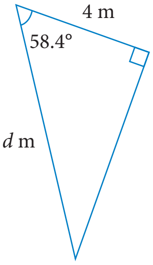
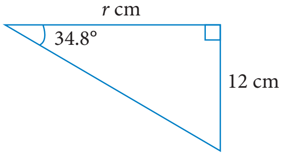


3 **Example 1b** Find the value of the pronumerals, correct to two decimal places.

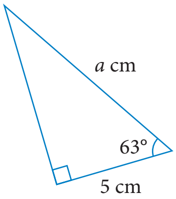
a



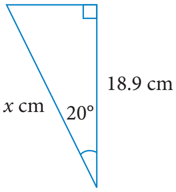
b



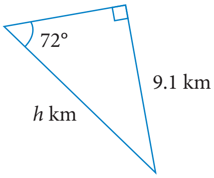
c



d



e



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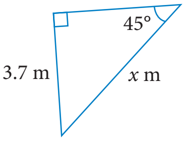
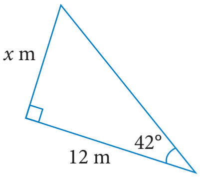


4 The expression used to find the value of x in the following triangle is:

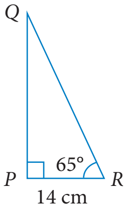
A $12\cos(42°)$ **B** $12\tan(42°)$

C $\dfrac{12}{\tan(42°)}$ **D** $\dfrac{12}{\cos(42°)}$

E $12\sin(42°)$



5 A triangle PQR has the following dimensions: $\angle P = 90°$, $\angle R = 65°$ and $PR = 14$ cm. Find the length of side PQ, correct to one decimal place.



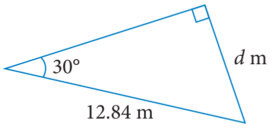
6 In ΔHIJ, $\angle J = 90°$, $\angle I = 26°$ and $HJ = 5.1$ m. Find the length of HI, correct to one decimal place.

7 Which of the following is not equal to the unknown side in the following triangle?

A $12.84\cos(60°)$ **B** $12.84\tan(60°)$

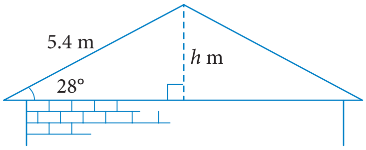
C $\dfrac{12.84}{2}$ **D** 6.42

E $12.81\sin(30°)$

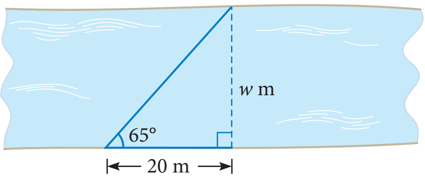


Reasoning and communication

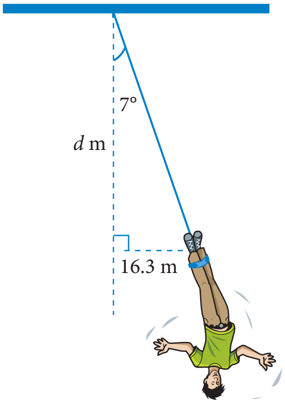
8 Example 2 The pitch of a roof is 28°. If the peak of the roof is 5.4 m from the gutter, how much higher is the peak than the gutter? Give your answer correct to one decimal place.



9 Example 3 Find the width, w metres, of the river shown in the diagram, correct to one decimal place.



10 A bungy jumper leaps off a bridge at an angle of 7° to the vertical. If he is swinging 16.3 m off-centre, calculate the vertical distance that he has dropped correct to one decimal place.

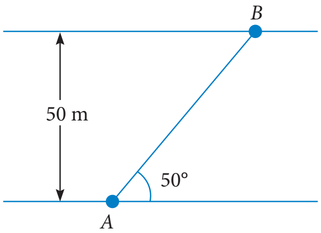


11 A rectangular field of width 38 m has a diagonal path that makes an angle of 40° with the 38 m side. Find the length of the path correct to the nearest 0.1 m.

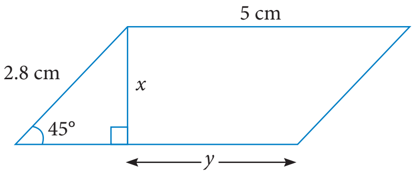
12 A ladder leaning against a building makes an angle of 22° with the building. If the foot of the ladder is 2.8 m from the building, calculate, correct to two decimal places.

- a the length of the ladder.
- b the height it reaches above the ground.

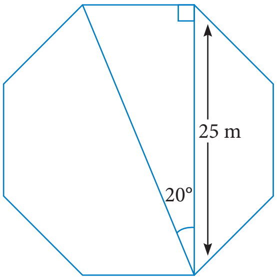
13 A swimmer swims across a river 50 m wide. She starts at point A and finishes at point B. What is the actual distance that she swam (to the nearest metre)?



14 In the parallelogram on the right calculate the values of x and y correct to one decimal place.



15 A right-angled triangle is drawn inside a regular octagon as shown on the right. Calculate the perimeter of the octagon correct to one decimal place.



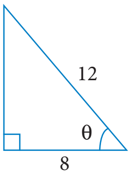
10.02 FINDING AN UNKNOWN ANGLE

Trigonometric ratios can also be used to find unknown angles in right-angled triangles if two side lengths are known. Select the appropriate function linking the unknown angle with the two known sides. Solve to find the angle by using the inverse trigonometric function.

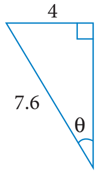
Example 4

Calculate the values of the unknown angles, correct to the nearest degree.

a

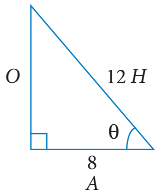


b



Solution

a Draw and label the sides of the triangle.



Select and write the appropriate ratio. $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$

Substitute the known values into the equation. $\cos(\theta) = \frac{8}{12}$

Solve the equation by using $\cos^{-1}$ to evaluate θ.

$$\theta = \cos^{-1}\left(\frac{8}{12}\right)$$

$$\theta = 48.1897...$$

$$\approx 48°$$

The inverse of a trigonometric function is represented in the following way.

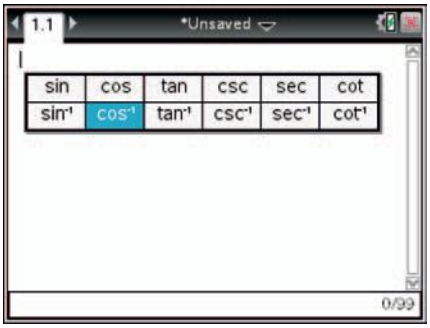
The inverse of sine = $\sin^{-1}$.
The inverse of cosine = $\cos^{-1}$.
The inverse of tangent = $\tan^{-1}$.

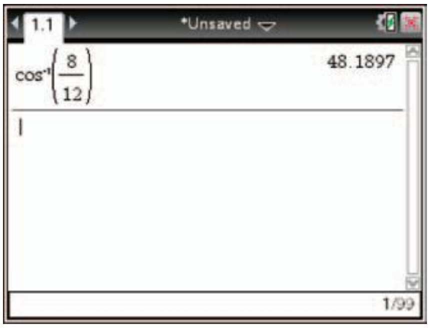
TI-Nspire CAS

Press trig and select $\cos^{-1}$.

Key in $\left(\frac{8}{12}\right)$. Press ctrl enter.

Calculate and round to the nearest degree.



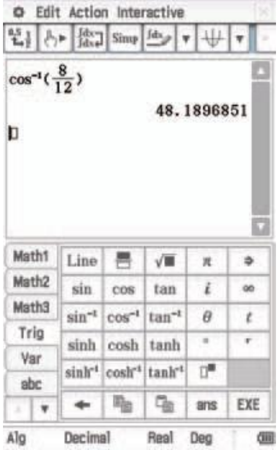


$\cos^{-1}\left(\frac{8}{12}\right)$ 48.1897

ClassPad

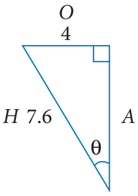
Use the Main $\sqrt{\alpha}$
Application in **Decimal** mode.
Press Keyboard then tap Trig and $\cos^{-1}$.

Key in $\left(\frac{8}{12}\right)$. Press EXE.



$\cos^{-1}\left(\frac{8}{12}\right)$
48.1896851

b Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute known values into the equation.

$$\sin(\theta) = \frac{4}{7.6}$$

Solve the equation by using $\sin^{-1}$ to evaluate θ.

$$\theta = \sin^{-1}\left(\frac{4}{7.6}\right)$$

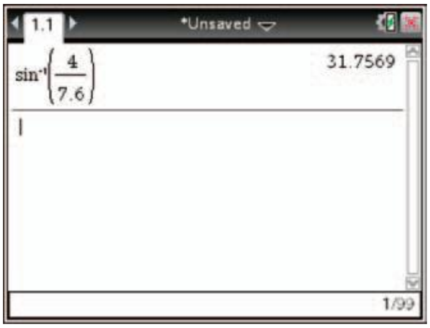
$$\theta = 31.7569...$$
$$\approx 32°$$

TI-Nspire CAS

Press trig and select $\sin^{-1}$.

Key in $\left(\frac{4}{7.6}\right)$. Press ctrl enter.

Calculate and round to the nearest degree.



$\sin^{-1}\left(\frac{4}{7.6}\right)$ 31.7569

9780170250252

ClassPad

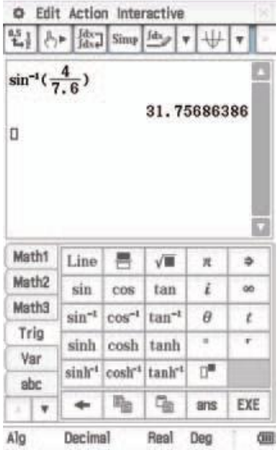
Use the Main $\sqrt{\alpha}$

Application in **Decimal** mode.

Press `Keyboard` then tap `Trig` and `sin⁻¹`.

Key in $\left(\frac{4}{7.6}\right)$.

Press `EXE`. Calculate and round to the nearest degree.



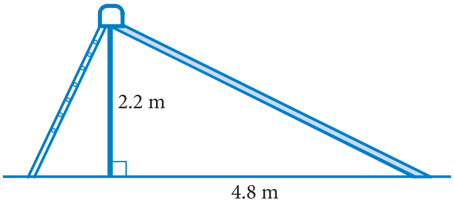
$\sin^{-1}\left(\frac{4}{7.6}\right)$

31.75686386

Example 5

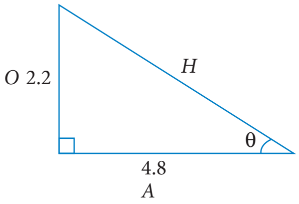
A playground slide is made up of a ladder and a metal slide. The top of the metal slide is 2.2 m from the ground and the bottom of the slide is 4.8 m from a point directly under the top of the slide.

What angle does the slide make with the ground, correct to the nearest degree?



Solution

Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(\theta) = \frac{2.2}{4.8}$$

Solve the equation by using $\tan^{-1}$ to evaluate θ.

$$\theta = \tan^{-1}\left(\frac{2.2}{4.8}\right)$$

Calculate and write the answer, remembering to round to the nearest degree.

$$\theta = 24.6236\ldots$$
$$\approx 25°$$

The metal slide makes an angle of 25° with the ground.

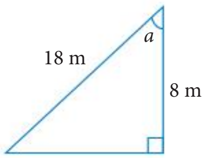
EXERCISE 10.02 Finding an unknown angle

Calculating lengths and angles

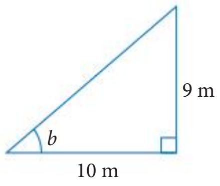
Concepts and techniques

1 **Example 4** Find the value of the pronumeral in each of the following, correct to the nearest degree.

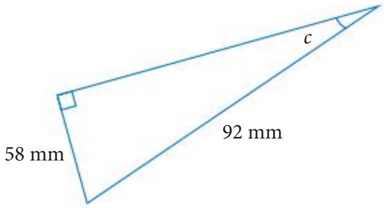
a



b



c



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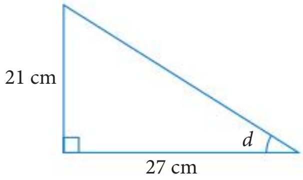
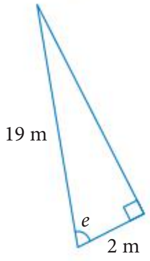
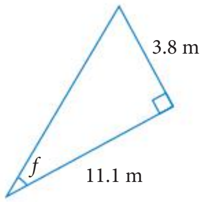


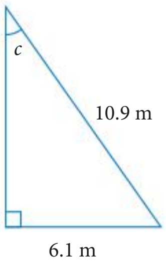
e



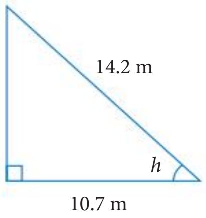
f



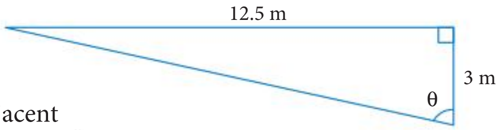
g



h



2 To calculate the value of θ in the following triangle, which trigonometric ratio would you use?



A $\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}$

B $\sin(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$

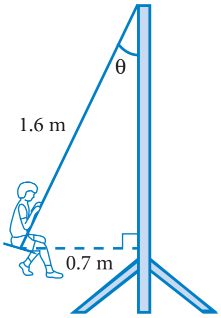
C $\cos(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$

D $\tan(\theta)=\frac{\text{adjacent}}{\text{opposite}}$

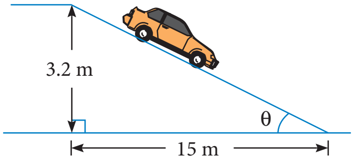
E $\sin(\theta)=\frac{\text{opposite}}{\text{hypotenuse}}$

Reasoning and communication

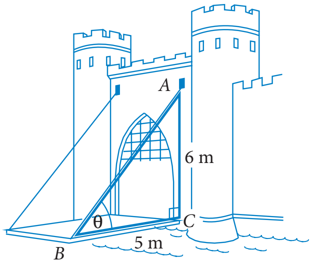
3 **Example 5** Caitlin is on a swing of length 1.6 m. When she is 0.7 m away from the swing's frame, what angle does the swing make with the vertical? Give your answer to the nearest degree.



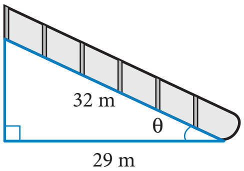
4 Find the angle of inclination, θ, of this ramp in a multistorey car park, correct to the nearest degree.



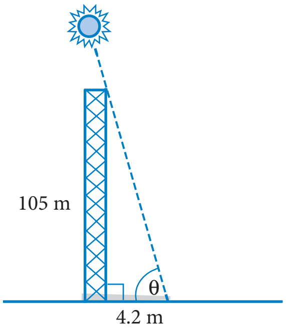
5 The drawbridge in front of this castle is 5 m long and is pulled by chains from 6 m above ground level. What angle do the chains make with the ground, to the nearest degree?



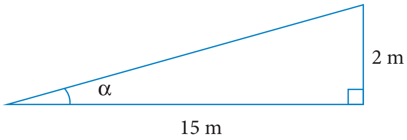
6 An escalator ramp has a length of 32 m and covers a horizontal distance of 29 m. What is its angle of inclination, θ, to the nearest degree?



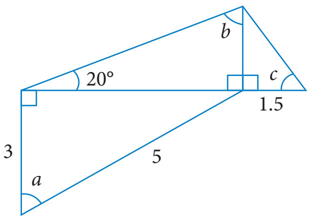
7 What angle do the Sun's rays make with the horizontal when a tower that is 105 m tall has a shadow of 4.2 m? Give your answer correct to the nearest degree.



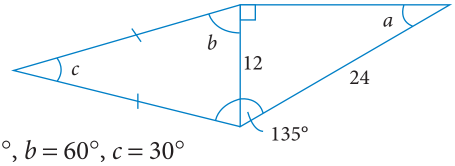
8 A hill has a gradient of $\frac{2}{15}$, which means that it rises 2 m for every 15 m horizontally. Find the angle that the hill makes with the horizontal, correct to the nearest degree.



9 In the diagram below, find the values of a, b, and c to the nearest degree.



10 An isosceles triangle is placed next to a right-angled triangle as shown in the diagram. The magnitudes of angles a, b, and c to the nearest degree are:



A $a = 30°, b = 30°, c = 30°$

B $a = 30°, b = 60°, c = 30°$

C $a = 30°, b = 75°, c = 30°$

D $a = 60°, b = 75°, c = 30°$

E $a = 30°, b = 30°, c = 60°$

INVESTIGATION Investigating the trigonometric ratios

a i Use your calculator to find the ratios of the angles in this table (correct to four decimal places).

θ	1°	10°	30°	45°	60°	80°	89°
sin (θ)							
cos (θ)							
tan (θ)							

ii Look at the sine and cosine ratios for 30° and 60°. Why do we get these values?

iii What happens to the sine and cosine ratios as θ increases in size from 0° to 90°?

iv What happens to the tan ratio as θ increases in size from 0° to 90°?

b i Using your ruler and a protractor accurately draw a right-angled triangle that has one angle measuring 30°.

ii Use your protractor to measure the size of the third angle.

iii Is there only one possible triangle with the angle measurements described? Explain your reasoning.

iv Measure the sides of your triangle using your ruler and then calculate the sine, cosine and tangent ratios for 30°. Compare them with those in the completed table from question **a** part **i**.

v Repeat part **iv** for the third angle of the triangle (60°).

c i Using your ruler and a protractor accurately draw a right-angled triangle that has one angle measuring 45°.

ii Use your protractor to measure the size of the third angle.

iii Is there only one possible triangle with the angle measurements described? Explain your reasoning.

iv Measure the sides of your triangle using your ruler and then calculate the sine, cosine and tangent ratios for 45°. Compare them with those in the completed table from question **a** part **i**.

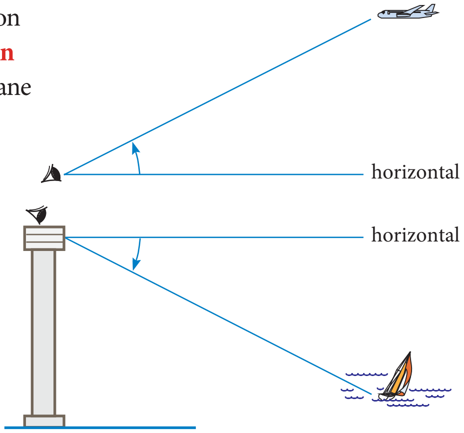
10.03 ANGLES OF ELEVATION AND DEPRESSION

One application of trigonometry is the use and calculation of angles of elevation and depression. **Angles of elevation** and **angles of depression** are taken from a horizontal plane or reference line as shown on the right.

The angle of elevation is the angle looking up.

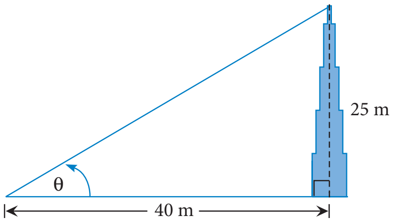
The angle of depression is the angle looking down.

Remember that the angle is measured from the horizontal. To distinguish between the two types of angles you could remember that when we feel 'elevated' things are 'looking up'. When we feel 'depressed' things are 'looking down'.



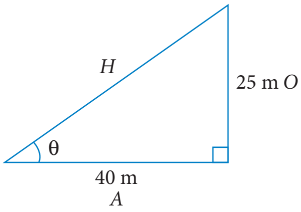
Example 6

Imran stands 40 m from the base of a 25 m tower. What is the angle of elevation, to the nearest degree, of the tower from Imran?



Solution

Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(\theta) = \frac{25}{40}$$

Solve the equation and write the answer, remembering to round to the nearest whole degree.

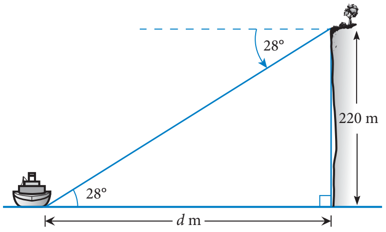
$$\theta = \tan^{-1}\left(\frac{25}{40}\right)$$

$$= 32.0053...$$

$$\approx 32°$$

The angle of elevation is 32°.

Example 7

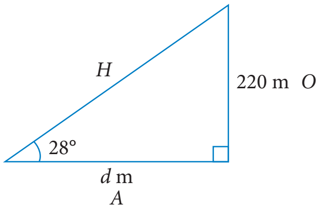


The angle of depression of a ship from the top of a 220 m vertical cliff is 28°. To the nearest metre, how far is the ship from the base of the cliff?

Solution

Draw and label the sides of the triangle.

The angle of depression of the ship from the cliff top is 28°, so the angle of elevation from the ship to the cliff top must also be 28°.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(28°) = \frac{220}{d}$$

Solve the equation and write the answer, remembering to round to the nearest whole metre.

$$d \times \tan(28°) = \frac{220}{\cancel{d}} \times \cancel{d}$$

$$d\tan(28°) = 220$$

$$\frac{d\tan(28°)}{\tan(28°)} = \frac{220}{\tan(28°)}$$

$$d = \frac{220}{\tan(28°)}$$

$$= 413.76...$$

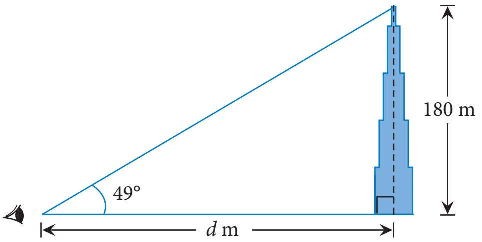
$$\approx 414 \text{ m}$$

The ship is 414 m from the base of the cliff.

EXERCISE 10.03 Angles of elevation and depression

Reasoning and communication

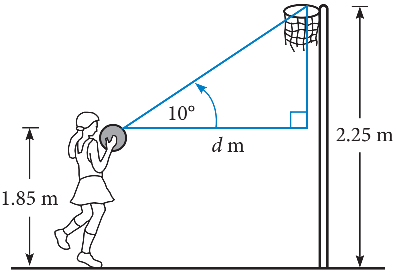
1 Example 6 From street level, Sonja sees the top of a 180 m tower at an angle of elevation of 49°. How far, to the nearest metre, is she from the tower?



2 Jaz is shooting for goal towards a netball ring at an angle of 10°. Find, correct to two decimal places:

a the distance of the ball from the ring

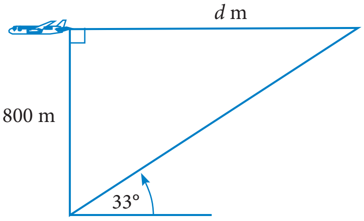
b the distance, d, from Jaz to the goalpost.



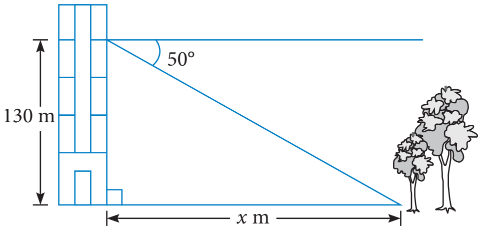
3 Ben observed a plane at an angle of elevation of 33°. It flew at a constant height of 800 m until it was directly above him.

a How far was the plane from Ben when he first saw it (correct to two decimal places)?

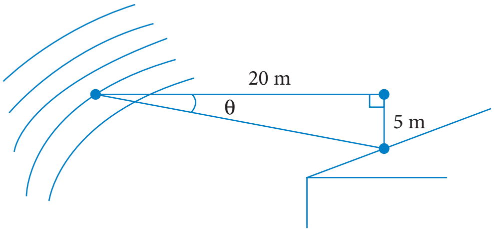
b What was the distance, d, travelled by the plane (correct to two decimal places)?



4 **Example 7** From her apartment, 130 m above ground level, Maddy sights the park at an angle of depression of 50°. How far, to the nearest metre, is the park from the base of Maddy's building?



5 In a concert hall, Liam's seat is 20 m from the stage and 5 m above it. What is his angle of depression to the stage, correct to the nearest degree?



6 Alexis is on a swing of length 1.4 m. She pulls back 0.6 m from the frame to commence swinging. Which of the following is the expression that will give you the angle that the swing makes with the vertical?

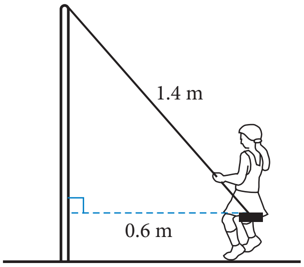
A $\cos^{-1}\left(\frac{0.6}{1.4}\right)$

B $\sin^{-1}\left(\frac{1.4}{0.6}\right)$

C $\tan^{-1}\left(\frac{0.6}{1.4}\right)$

D $\cos^{-1}\left(\frac{1.4}{0.6}\right)$

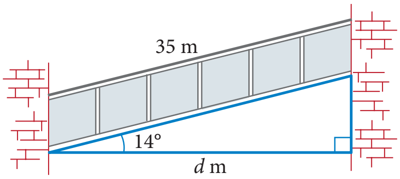
E $\sin^{-1}\left(\frac{0.6}{1.4}\right)$



7 A pedestrian walkway, linking two buildings, is 35 m long and is inclined at 14° to the horizontal. Calculate, correct to two decimal places:

a the distance between the two buildings

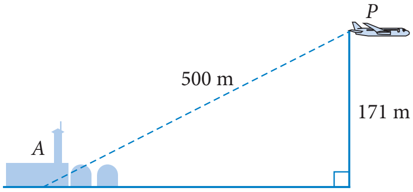
b the vertical rise of the skyway.



8 A plane, P, leaves an airport, A, and flies 500 m to reach a height of 171 m.

a What is the angle of elevation of the plane from the airport (to the nearest degree)?

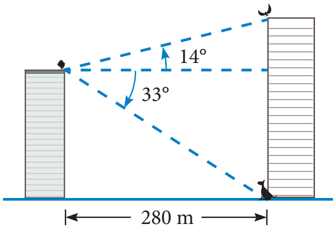
b What is the horizontal distance between the plane and the airport (correct to two decimal places)?



9 Amien stands on the roof of his apartment block and sees a bird on the roof of a neighbouring office building at an angle of elevation of 14°. At the same time, he sees a dog at the bottom of the office building at an angle of depression of 33°. If the two buildings are 280 m apart:

a how far, correct to two decimal places, is the bird from Amien?

b how far, correct to two decimal places, is the bird from the dog?

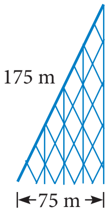


10 A 2.5 m totem pole casts a shadow of 2 m.

a What is the length of the shadow of a 6.4 m tree at the same time, correct to two decimal places?

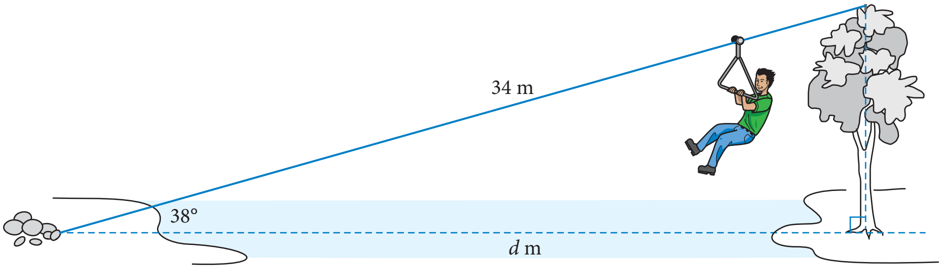
b What is the angle of elevation of the Sun, correct to the nearest degree?

11 A section of a rollercoaster is shown in the diagram. What is the angle of elevation of this section to the nearest degree?



12 A flying fox is constructed between the top of a tree and a large rock. It has a length of 34 m and an angle of elevation of 38°. Which of the following is the (horizontal) distance between the tree and the rock, correct to one decimal place?

A 20.9 m B 26.0 m C 26.6 m D 26.7 m E 26.8 m



INVESTIGATION Using a clinometer to measure heights

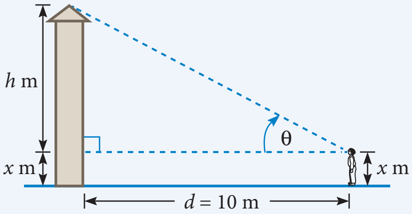
You will need: A clinometer, a trundle wheel, a metre ruler or height chart and a calculator. Choose a tall object outside, such as a tree, a flagpole, a football goalpost, a tower or a building. You will use a clinometer and a trundle wheel to measure the object's height.

a Copy the following table.

Distance, d	**Angle of elevation, θ**	**Height, $h = d\tan(\theta)$**	**Revised height, $h + x$**
10 m			
15 m			
20 m			
30 m			
50 m			

b Use the trundle wheel to measure 10 m from the base of the object and stand at that point.

c Use the clinometer to view and measure the angle of elevation, θ, to the top of the object. Write this value in the 'Angle of elevation, θ' column in the table.



d $\tan(\theta) = \frac{h}{d}$, so $h = d\tan(\theta)$. Calculate h correct to two decimal places. Write that value in the 'Height, $h = d\tan(\theta)$' column of the table.

e Measure x, the height of the observer's eye above the ground in metres, correct to two decimal places.

f Calculate $h + x$ and write the answer in the 'Revised height, $h + x$' column of the table. This is the height of the object.

g Repeat this experiment for other values of d, at different distances from the base of your object, using the values given in the table, or some of your own.

h How do your calculated results compare with each other? They are all meant to be the same, but why might the answers be different?

i How could we come up with a single accurate value for the height of the object, using these results?

j Use the clinometer to measure and calculate the heights of other objects around your school.

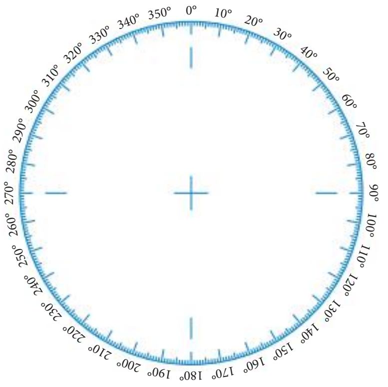
10.04 BEARINGS

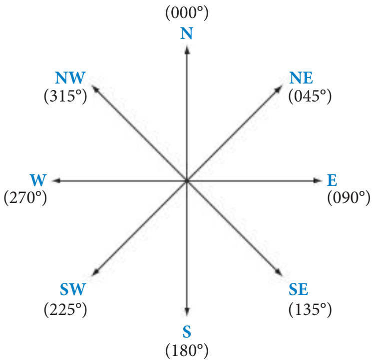
Bearings use angles to show the direction of one location from a given point. Bearings or directions are found by connecting the two points with a straight line on a horizontal plane.

There are two types of bearings: **true bearings** and **compass bearings**.

True bearings

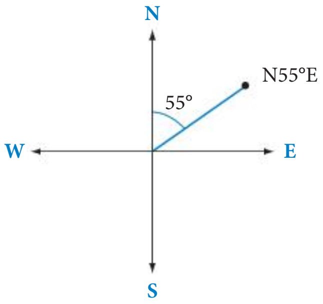
True bearings (also known as three-figure bearings) are angles measured in a clockwise direction from north, which is at 000°T. The diagram below allows you to see how one full revolution of a circle (360°) compares to the compass rose.

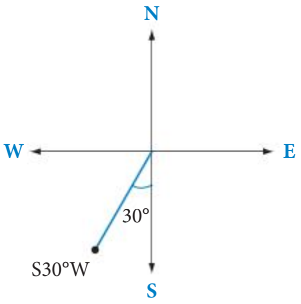




Compass bearings

Compass bearings differ to true bearings although they are still referenced to a compass rose. Compass bearings are referenced from either the north or the south; and are measured in either an east or west direction.

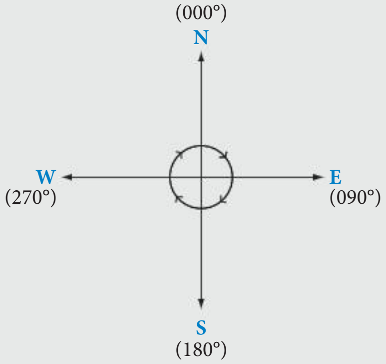




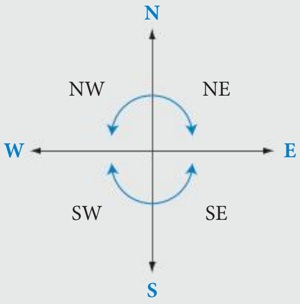
In the diagram above, you can see how the bearing is taken from either north or south. With compass bearings, north (N) or south (S) is written first. The angle, which is between 0° and 90°, is written next and this is followed by the reference to east (E) or west (W).

IMPORTANT

True bearings always start from north and go in a clockwise direction. True bearings will be between 000°T and 360°T (with north being at both 000°T and 360°T).



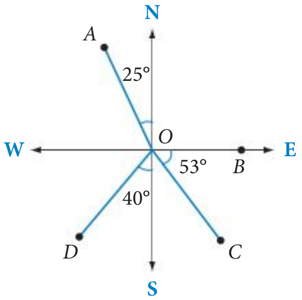
Compass bearings depend on which quadrant is being referenced. Compass bearings are referenced from either north or south.



Example 8

a Write the true bearings of locations *A*, *B*, *C* and *D* from point *O*.

b Write the compass bearings of locations *A*, *B*, *C* and *D* from point *O*.



Solution

a True bearings are measured clockwise from north.

The first bearing from north is *B*. *B* is due east, so it is 90° clockwise from north.

The bearing of *B* is 090°T.

C is 53° past east.

The bearing of *C* is (90° + 53°)T = 143°T.

D is 40° past south (180°).

The bearing of *D* is (180° + 40°)T = 220°T.

A is 25° before north (360°).

The bearing of *A* is (360° – 25°)T = 335°T.

b Compass bearings depend on which quadrant the point is in. Point B is on east.

B = east

C is in the SE quadrant and forms an angle of 37° with south.

90° – 53° = 37°

The bearing of C is S 37° E.

D is in the SW quadrant and forms an angle of 40° with south.

The bearing of D is S 40° W.

A is in the NW quadrant and forms an angle of 25° with north.

The bearing of A is N 25° W.

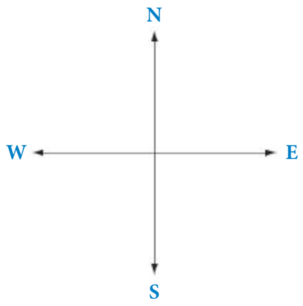
Example 9

Sketch each of the following bearings on a compass rose:

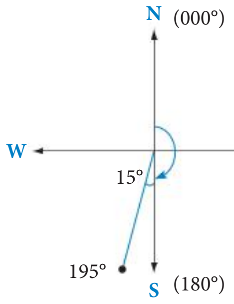
a 195°T. **b** N 25° W.

Solution

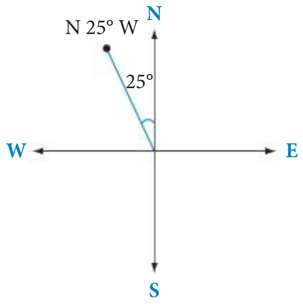
Draw a compass rose.



a 195° is 15° greater than 180°, therefore draw the bearing at 15° past south.



b This compass bearing is in the NW quadrant. It is 25° from north.



EXERCISE 10.04 Bearings

Bearings

Identifying bearings

Concepts and techniques

1 Write the true bearings of each of the following.

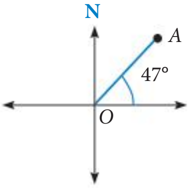
a south
b north-east
c west
d south-west
e north-west
f east

2 In which quadrant (NE, SE, SW, NW) will each of these bearings lie?

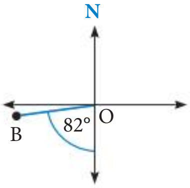
a 260°
b 145°
c 073°
d 102°
e 340°
f 237°

3 Example 8 Write the true bearing of each of the following points from *O*.

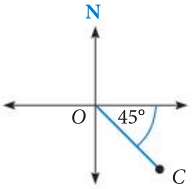
a



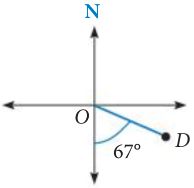
b



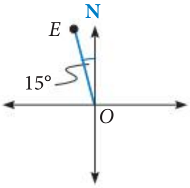
c



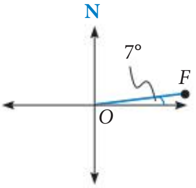
d



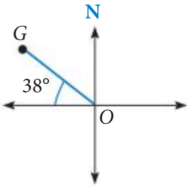
e



f



g



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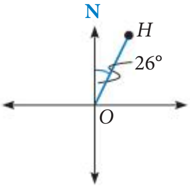

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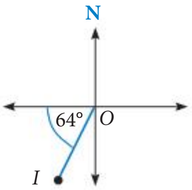
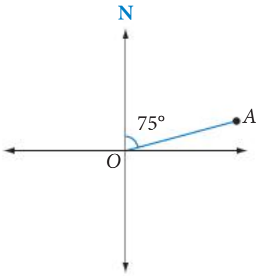
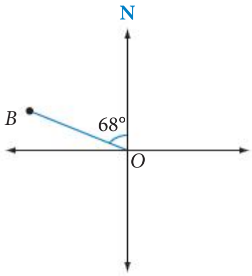


4 Write the compass bearing of each of the following points from *O*.

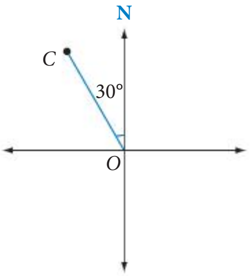
a



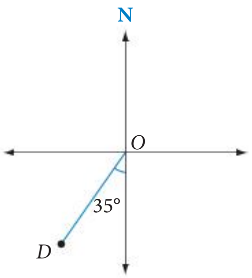
b



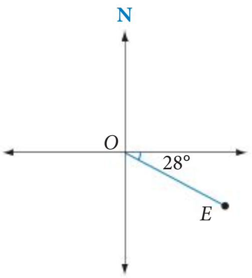
c



d



e



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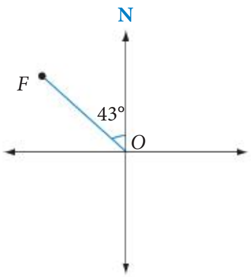

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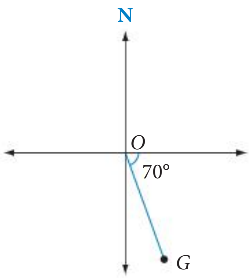

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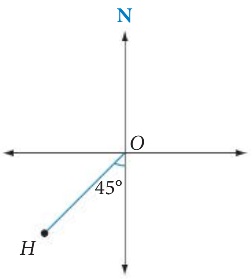

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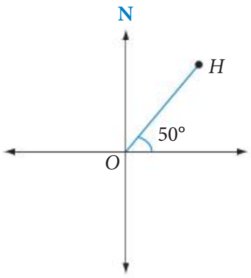


5 147° is in which quadrant?

A NW

B NE

C SW

D SE

E NS

6 The compass bearing displayed at right is:

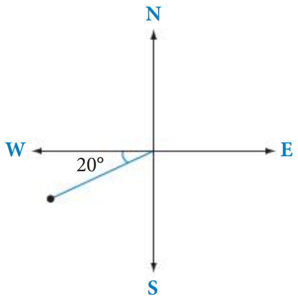
A S 70° W

B S 20° W

C W 70° S

D N 20° W

E S 250° W



7 Example 8 Sketch each of the following bearings on a compass rose.

a 352°T

b N 40° W

c 166°T

d 078°T

e S 22° W

f 303°T

g 147°T

h 270°T

i N 10° E

j S 70° E

k 065°T

l S 58° W

10.05 BEARING APPLICATIONS

For problems involving bearings, take note where the bearing is taken from. This is the point where we draw the centre of our compass rose and from where we take the bearing.

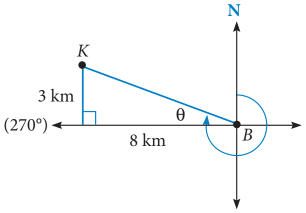
It is important to always work from an accurate diagram and to have north pointing up. To find the lengths and angles in problems involving bearings, you will need to apply all of your trigonometry skills.

Example 10

Kerr's Creek, *K*, is 8 km west and 3 km north of Bobtown, *B*.

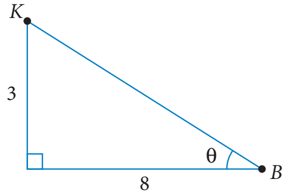
a Find the true bearing, correct to the nearest degree, of Kerr's Creek from Bobtown.

b Find the direct distance between Kerr's Creek and Bobtown correct to one decimal place.



Solution

a Find the right-angled triangle in the diagram and draw it, including all of the relevant information.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{3}{8}$$

Calculate θ.

$$\theta = \tan^{-1}\left(\frac{3}{8}\right)$$
$$= 20.5560\ldots$$
$$\approx 21°$$

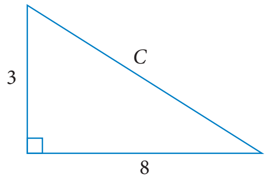
The true bearing is taken from North in a clockwise direction. Add θ to 270° (the angle between north and west).

$$21° + 270° = 291°$$

Write the answer.

The true bearing of Kerr's Creek from Bobtown is 291°T.

b The direct distance can be found using Pythagoras' theorem.

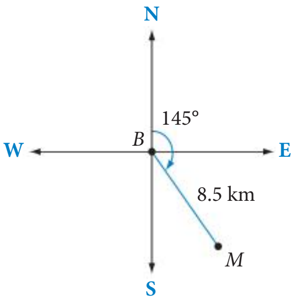


$$a^2 + b^2 = C^2$$
$$3^2 + 8^2 = C^2$$
$$9 + 64 = C^2$$
$$73 = C^2$$
$$C = \sqrt{73}$$
$$= 8.5440\ldots$$
$$\approx 8.5$$

The direct distance between Kerr's Creek and Bobtown is 8.5 km.

Example 11

Matthew walked for 8.5 km on a true bearing of 145° from base camp. His position is shown by *M* on the diagram on the right.



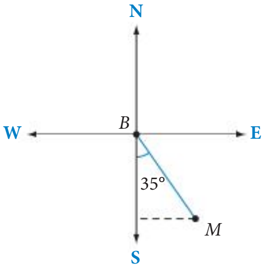
a How far south is he from base camp (correct to one decimal place)?

b How far east is he from base camp (correct to one decimal place)?

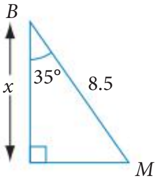
c What is the true bearing of base camp from Matthew?

Solution

a Find the right-angled triangle in the diagram.



Draw it, including all of the relevant information.



Select and write the appropriate ratio.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute the values into the equation and solve for the unknown length.

$$\cos(35°) = \frac{x}{8.5}$$

$$x = 8.5\cos(35°)$$

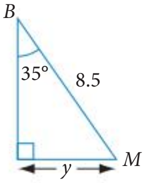
$$= 6.9627...$$

$$\approx 7.0 \text{ km}$$

Write the answer.

Matthew is 7.0 km south of base camp.

b Using the same right-angled triangle, calculate the distance east (*y*) using the sine ratio.



You could use either Pythagoras or the tan ratio to find the length of *y*, but it is always better to use the values given in the original question.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(35°) = \frac{y}{8.5}$$

$$y = 8.5\sin(35°)$$

$$= 4.8754...$$

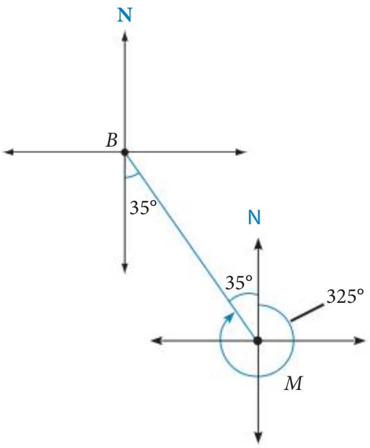
$$\approx 4.9 \text{ km}$$

Write the answer.

Matthew is 4.9 km east of base camp.

c To find the true bearing of the base from Matthew we need to sketch his journey using two compass roses.

Using alternate angles, it can be seen that the angle formed between north and Matthew's journey is 35°.



The true bearing must go in a clockwise direction from north.

$360^\circ - 35^\circ = 325^\circ$

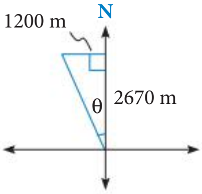
Write the answer.

The bearing of the base camp from Matthew is 325°T.

EXERCISE 10.05 Bearing applications

Reasoning and communication

1 Example 10 Selena walks 2670 m north, then turns and walks 1200 m west, as shown in the diagram.

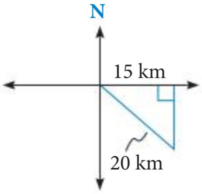


a Find the value of θ, to the nearest degree.

b Calculate Selena's true bearing from her starting point.

c How far is Selena from her starting point (to the nearest 10 m)?

2 A ship sails in a south-easterly direction for 20 km until it is 15 km east of its starting point.



a What is the bearing of the starting position from the ship's current position?

b How far south has the boat sailed, to the nearest km?

3 A triangular orienteering course starts at Alpha and passes through the checkpoints Bravo and Charlie before finishing back at Alpha. Bravo is 8.5 km due east of Alpha and Charlie is 10.5 km due south of Bravo.

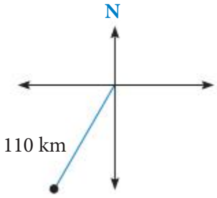
a Draw a diagram showing this information.

b Calculate the length of the circuit to the nearest 0.1 km.

c Find the compass bearing of Charlie from Alpha to the nearest degree.

d Find the compass bearing of Alpha from Charlie to the nearest degree.

4 Example 11 Amy drives on a true bearing of 210° for 110 km. Find:

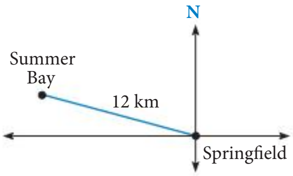


a how far west Amy is from her starting point, correct to the nearest kilometre

b how far south Amy is from her starting point, correct to the nearest kilometre

c the true bearing of Amy's starting point from her finishing point.

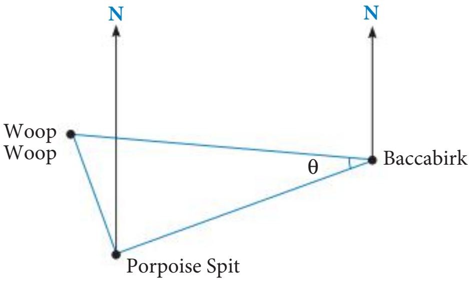
5 Springfield and Summer Bay are 12 km apart and the compass bearing of Summer Bay from Springfield is N 65° W. How far is Summer Bay west of Springfield, correct to two decimal places?



6 A hot air balloon travelled 15 km due east but drifted 1.8 km south. What is its true bearing from its starting point, to the nearest degree?

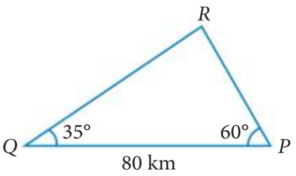
7 Two cars leave from the same place: the first heads due east, the other on a true bearing of 165°. After travelling 8 km, the first car is due north of the second car. How far has the second car travelled, to the nearest kilometre?

8 The town of Woop Woop is 22 km from the town of Porpoise Spit, on a true bearing of 340°. The town of Baccabirk is 48 km from Porpoise Spit, on a true bearing of 070°.
- a Find θ to the nearest degree.
- b Find the true bearing of Woop Woop from Baccabirk. (*Hint*: Use alternate angles.)

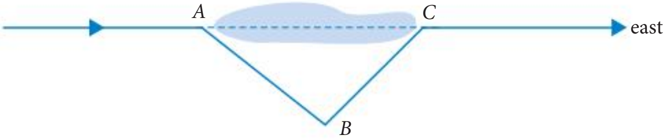


9 Farmer Fred needs to complete the fencing for his triangular pig enclosure. The enclosure currently has two fences. From point O, one fence line is on a bearing of N 30° E for 200 m and the other is on a bearing of S 60° E for 400 m. Find the length of fencing (to the nearest metre) required to complete the pig enclosure.

10 Petersville (P) is 80 km due east of Queensfield (Q). Rossmore (R) is on a true bearing of 055° from Queensfield and 300° from Petersville as shown in the diagram.
- a What is the true bearing of Queensfield from Petersville?
- b What is the true bearing of Queensfield from Rossmore?

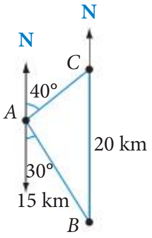


11 A surveyor travelling due east needs to make a detour on a true bearing of 128° to avoid a flooded area. After detouring 8 km, he turns and travels NE until he rejoins the original line of travel.



Copy the diagram above and mark on it all the information given and then find the size of each angle in $\triangle ABC$.

12 Emily is going on a three-day bushwalking adventure. She starts at camp site A and walks for 15 km on a compass bearing of S 30° E to camp site B. On the second day, she walks 20 km due north to campsite C. On the third day, Emily plans to return to campsite A. She knows campsite C is on a compass bearing of N 40° E from campsite A.
What compass bearing does Emily need to travel on in order to return to camp site A?



10.06

THE SINE RULE: FINDING AN UNKNOWN SIDE

Pythagoras' theorem and trigonometric ratios can be used when working with right-angled triangles. When working with **non-right-angled triangles**, the **sine rule** can be used to calculate side lengths and angles. We will consider finding unknown side lengths first.

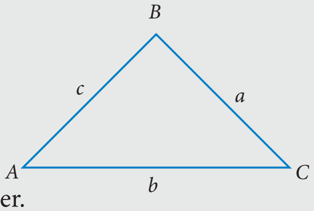
IMPORTANT

The sine rule for the triangle ABC shown on the right is

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

A, B and C represent the angles and a, b and c represent the side lengths.

The angle is opposite the side length represented by the same letter.



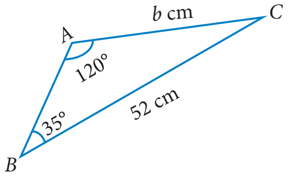
The sine rule can be used if given a non-right-angled triangle and either of the following conditions exist:

– one side and two angles are given

– two sides and a **non-included angle** are given.

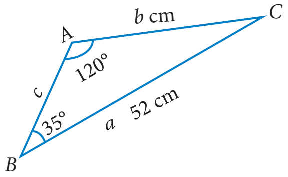
Example 12

Calculate the side length b, correct to two decimal places.



Solution

Label the sides of the triangle using a, b, and c; ensuring that the sides are opposite the angles represented by the same letter.



Select and write the appropriate section of the sine rule.

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$

Substitute the known values into the equation.

$$\frac{b}{\sin(35°)} = \frac{52}{\sin(120°)}$$

Solve the equation.

$$\sin(35°)\times\frac{b}{\sin(35°)}=\frac{52\sin(35°)}{\sin(120°)}$$

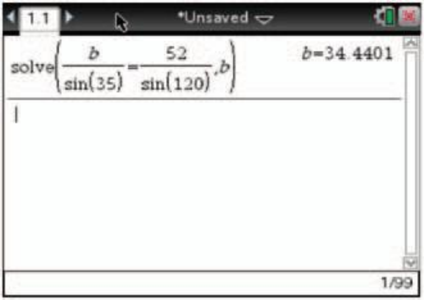
$$b=\frac{52\sin\left(35°\right)}{\sin\left(120°\right)}$$

$$=34.440...$$

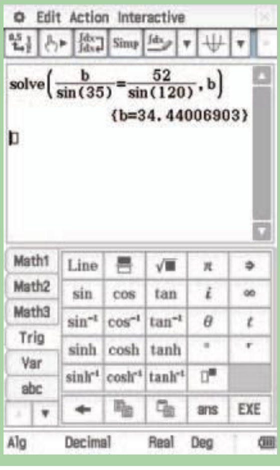
$$\approx 34.44\text{ cm}$$

Write the answer with units.

A CAS can be used to solve for the unknown side.



$\text{solve}\left(\frac{b}{\sin(35)}=\frac{52}{\sin(120)},b\right)$ $b=34.4401$

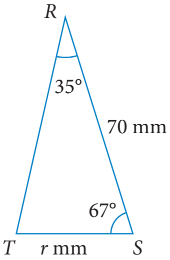


$\text{solve}\left(\frac{b}{\sin(35)}=\frac{52}{\sin(120)},b\right)$

{b=34.44006903}

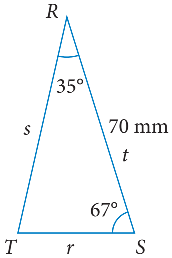
Example 13

In $\triangle RST$, $\angle R=35°$, $\angle S=67°$, and $t=70$ mm.
Find r, correct to one decimal place.



Solution

Label the sides of the triangle.



Note that letters other than *a*, *b* and *c* can be used. It is best to match the letters used to label the side lengths to the letters used to label the angles.

To solve for *r*, angle *T* is needed since its corresponding side is given.

$$T=180-35°-70°$$
$$=75°$$

Write the sine rule using the appropriate pronumerals.

$$\frac{r}{\sin(R)}=\frac{t}{\sin(T)}$$

Substitute the known angles and side length into the rule.

$$\frac{r}{\sin(35^\circ)}=\frac{70}{\sin(75^\circ)}$$

Solve the equation.

$$\sin 35^\circ \times \frac{r}{\sin 35^\circ}=\frac{70\sin 35^\circ}{\sin 75^\circ}$$

$$r=\frac{70\sin 35^\circ}{\sin 75^\circ}$$

$$=41.5667...$$

Write the answer with the correct units.

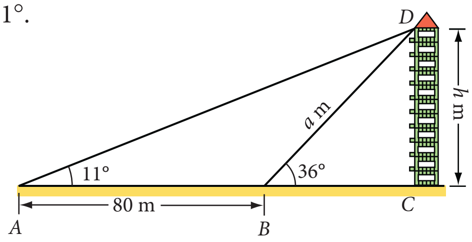
$$\approx 41.57 \text{ mm}$$

Example 14

Kasun observes a tower at an angle of elevation of 11°. Walking 80 m towards the tower, he finds that the angle of elevation increases to 36°.

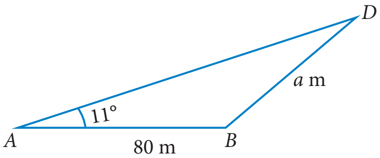
a Show that $a=\frac{80\sin(11^\circ)}{\sin(25^\circ)}$.

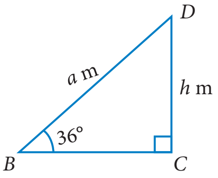
b Hence evaluate the height, h metres, of the tower correct to two decimal places.

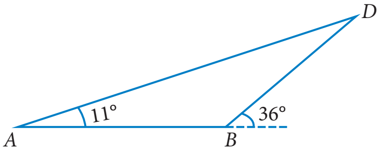


Solution

a Redraw the two triangles separately.



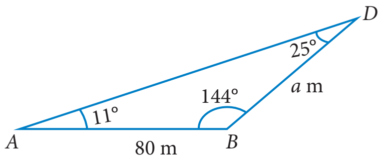




The non-right-angled triangle has more information, therefore use it first. With the given information we can find ∠ABD and ∠ADB.

$$\angle ABD = 180^\circ - 36^\circ$$
$$= 144^\circ$$
$$\angle ADB = 180^\circ - 11^\circ - 144^\circ$$
$$= 25^\circ$$

Label the non-right-angled triangle.



Show that $a = \dfrac{80\sin(11°)}{\sin(25°)}$ using the sine rule.

$$\frac{a}{\sin(A)} = \frac{d}{\sin(D)}$$

$$\frac{a}{\sin(11°)} = \frac{80}{\sin(25°)}$$

$$\sin(11°) \times \frac{a}{\sin(11°)} = \frac{80\sin(11°)}{\sin(25°)}$$

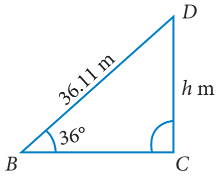
$$a = \frac{80\sin(11°)}{\sin(25°)} \text{ as required.}$$

b Evaluate *BD* from part **a**.

$$BD = \frac{80\sin(11°)}{\sin(25°)}$$

$$= 36.11940...$$

Now that length *BD* is known, it is possible to work with the right-angled triangle to find *h*.



IMPORTANT

Use the unrounded answer 36.11940… in the calculations rather than rounding to a smaller number of decimal places so that rounding errors do not occur.

Use the sine ratio to solve for *h*.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(36°) = \frac{h}{36.11940\ldots}$$

$$h = 36.11940\ldots \sin(36°)$$

$$= 21.2304\ldots$$

$$\approx 21.23$$

Write the answer.

The height of the tower is 21.23 m.

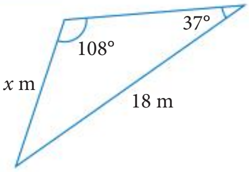
EXERCISE 10.06 The sine rule: finding an unknown side

The sine rule – Finding lengths of sides

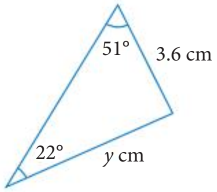
Concepts and techniques

1 **Example 12** Find the value of the pronumeral, correct to two decimal places.

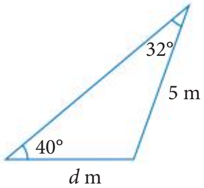
a



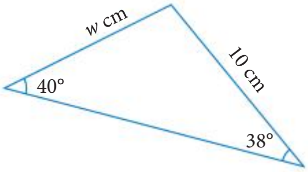
b



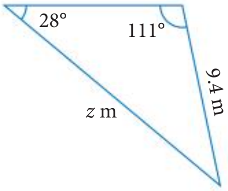
c



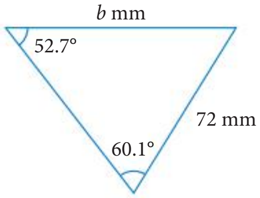
d



e



f



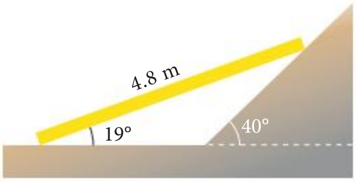
2 **Example 13** In $\triangle MTV$, $\angle M = 71°$, $\angle T = 46°$ and $TV = 18.3$ cm. Calculate the length of MV, correct to one decimal place.

3 In $\triangle JKL$, $\angle J = 25°$, $\angle K = 87°$ and $KL = 7.8$ m. The length of JL, correct to one decimal place, is:

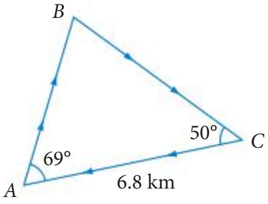
A 3.3 m **B** 16.7 m **C** 17.1 m **D** 18.4 m **E** 148.8 m

Reasoning and communication

4 A 4.8 m plank is placed between the ground and a ramp that is inclined at 40° to the ground.
If the plank is inclined at 19° to the ground, how far up the ramp is the other end of the plank (give your answer correct to two decimal places)?



5 Which of the following expressions will give you the total distance of the orienteering course outlined on the right?



A $\dfrac{6.8\sin(69°)}{\sin(61°)} + \dfrac{6.8\sin(50°)}{\sin(61°)} + 6.8$

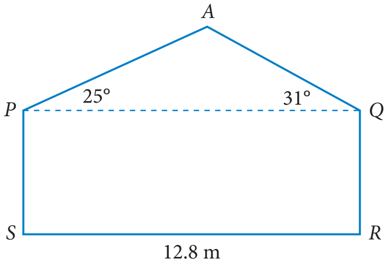
B $\dfrac{6.8\sin(61°)}{\sin(69°)} + \dfrac{6.8\sin(61°)}{\sin(50°)} + 6.8$

C $\dfrac{\sin(69°)}{6.8\sin(61°)} + \dfrac{\sin(50°)}{6.8\sin(61°)} + 6.8$

D $\dfrac{6.8\sin(69°)}{\sin(50°)} + \dfrac{6.8\sin(61°)}{\sin(69°)} + 6.8$

E $\dfrac{\sin(50°)}{6.8\sin(69°)} + \dfrac{\sin(69°)}{6.8\sin(61°)} + 6.8$

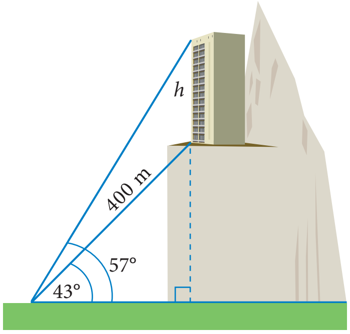
6 A possum walks across the two sides, PA and AQ, of the roof of this house. Find the total distance travelled across the roof, correct to one decimal place.



7 Example 14 Rhys observes the top and bottom of a mountain resort to be at angles of elevation of 57° and 43° respectively. He is 400 m from the bottom of the resort.

a Copy this diagram and find the sizes of all angles.

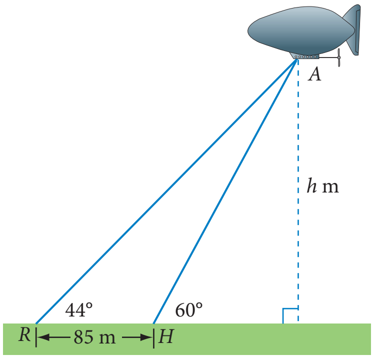
b Find the height of the resort, h, correct to two decimal places.



8 Hayley (H) and Rebecca (R), standing 85 m apart, observe an airship (A) at angles of elevation of 60° and 44° respectively.

a Find the distance between Hayley and the airship, correct to two decimal places.

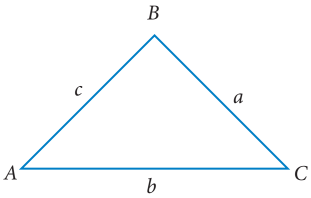
b Calculate the height of the airship, h metres, correct to two decimal places.



10.07 THE SINE RULE: FINDING AN UNKNOWN ANGLE

The sine rule, in the form as shown below, can also be used to find unknown angles in non-right-angled triangles. This form of the rule keeps the unknown value in the numerator, making the equation easier to solve.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

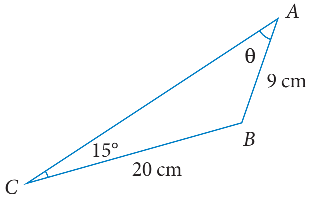


where A, B and C are the angles and a, b and c are the side lengths.

It doesn't matter which form of the sine rule you begin with as long as you label your triangle carefully and substitute the values correctly.

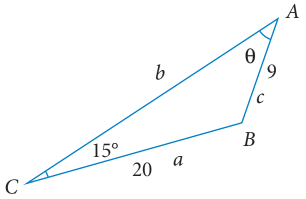
Example 15

Find the size of the angle θ, correct to the nearest degree.



Solution

Label the sides of the triangle.



Since θ = A, we need to use the corresponding side a, and angle C and side c are also known.

$$\frac{\sin(A)}{a}=\frac{\sin(C)}{c}$$

Substitute in the known values.

$$\frac{\sin(\theta)}{20}=\frac{\sin(15°)}{9}$$

Solve the equation for sin (θ).

$$20\times\frac{\sin(\theta)}{20}=\frac{20\sin(15°)}{9}$$

$$\sin(\theta)=\frac{20\sin(15°)}{9}$$

Use the inverse sine function to calculate the size of the angle. Calculate and write your answer, rounding to the nearest whole degree.

$$\theta=\sin^{-1}\left[\frac{20\sin(15°)}{9}\right]$$

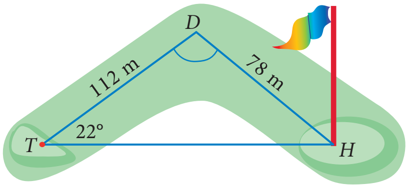
$$=35.1103...$$

$$\approx 35°$$

Example 16

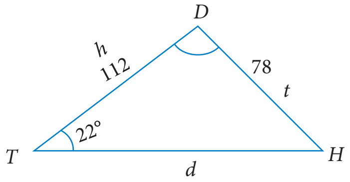
The 18th hole at a golf course has a tee at T, a hole at H and a dogleg at D.

If $TD = 112$ m, $DH = 78$ m and $\angle HTD = 22°$, find $\angle TDH$ correct to the nearest degree.



Solution

Label the sides of the triangle.



For the purpose of using the sine rule:

$\angle HTD = T$

$\angle TDH = D$

$\angle DHT = H$

Since *d* is not known, *D* cannot be calculated using the sine rule. However, *H* can be calculated and this can be used to find *D*.

$$\frac{\sin(H)}{h} = \frac{\sin(T)}{t}$$

Substitute the known values into the sine rule to calculate *H*.

$$\frac{\sin(H)}{112} = \frac{\sin(22°)}{78}$$

$$\sin(H) = \frac{122\sin(22°)}{78}$$

$$H = \sin^{-1}\left[\frac{122\sin(22°)}{78}\right]$$

$$\approx 35.87°$$

Since all the internal angles of a triangle must add to 180°, *D* can now be calculated.

$$D + T + H = 180°$$

$$D + 22° + 35.87° = 180°$$

$$D = 180° - 22° - 35.87°$$

$$= 122.13°$$

Write your answer, rounding to the nearest whole degree.

$$\angle TDH = 122°$$

WS

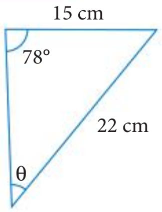
The sine rule – Finding angles

EXERCISE 10.07 The sine rule: finding an unknown angle

Concepts and techniques

1 Example 15 Find θ in each triangle. Give your answer correct to the nearest degree.

a



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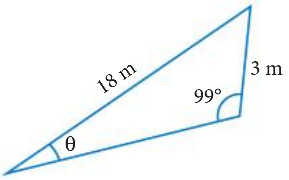
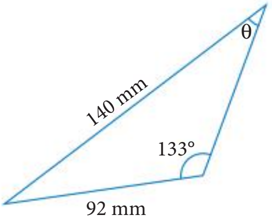


c



2 The expression for finding θ in the triangle at right is:

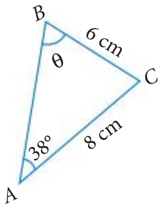
A $\frac{8\sin(38°)}{6} \times \frac{8\sin(38°)}{6}$

B $\sin^{-1}\left[\frac{6\sin(38°)}{8}\right]$

C $\sin^{-1}\left[\frac{8\sin(38°)}{6}\right]$

D $\frac{8\sin[\sin(38°)]}{6}$

E $\frac{6\sin(38°)}{8}$



9780170250252

3 Triangle *PQR* has sides $PQ = 15$ mm, $QR = 14.7$ mm and $\angle PRQ = 62°$. The values of $\angle QPR$ and $\angle PQR$ respectively are:

A 59°, 59° **B** 60°, 58° **C** 60°, 30° **D** 62°, 56° **E** 30°, 60°

4 In $\triangle RCN$, $NR = 7.8$ m, $NC = 4.5$ m and $\angle C = 120°$.

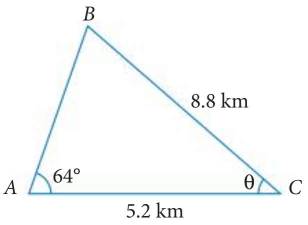
- **a** Calculate the size of $\angle R$, to the nearest degree.
- **b** What is the size of $\angle N$, to the nearest degree?
- **c** What type of triangle is $\triangle RCN$?
- **d** What is the length of *CR* correct to two decimal places?

5 Example 16 A non-right-angled triangle *PQR* has $\angle RPQ = 72°$, side $PR = 26.1$ m and side $QR = 32.8$ m.

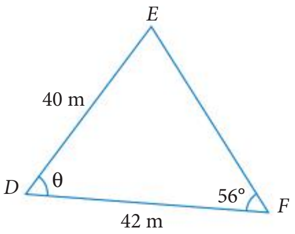
- **a** Draw a diagram showing the information given.
- **b** Find the value of $\angle PQR$, correct to the nearest degree.
- **c** Hence find the value of $\angle PRQ$, correct to the nearest degree.

6 Find θ in each triangle. Give your answer correct to the nearest degree.

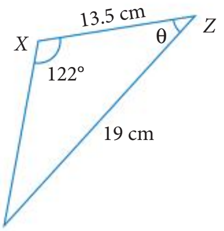
a



b



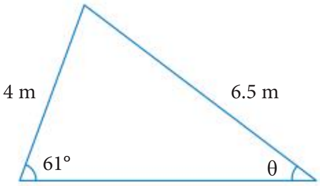
c



7 A surveyor positions three checkpoints *M*, *N* and *P* around a lake at a picnic ground. $MN = 728$ m, $MP = 638$ m and $\angle N = 57°$. Find the size of $\angle M$, correct to the nearest degree.

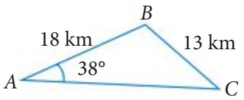
A 47° **B** 50° **C** 73° **D** 76° **E** 107°

8 The gable end of a roof is shown here. Find the value of θ to the nearest degree.

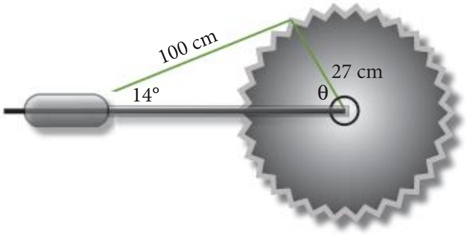


Reasoning and communication

9 Deanna canoes down a river from point *A* to point *B*, then on to point *C*. If $AB = 18$ km, $BC = 13$ km, and $\angle CAB = 38°$, calculate $\angle ABC$ to the nearest degree.

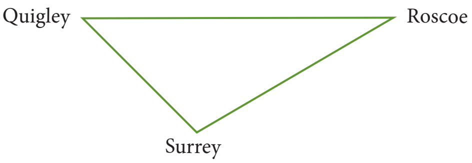


10 The crank and connecting rod of an engine are 27 cm and 100 cm long respectively. What angle does the crank make with the horizontal when the angle made by the connecting rod is 14°? Answer to the nearest degree.



11 Litza is taking a scenic helicopter flight over Melbourne. She leaves Melbourne and flies 20 km due south over the bay. The helicopter then turns on an angle of 120° to fly back over land. When Litza realises that she is running out of petrol she flies 36 km back to the airport where she started. Determine the angle (to the nearest degree) that Litza must turn to return to the airport.

12 Surrey is 21 km on a bearing of 140° from Quigley. Roscoe is 31 km from Surrey and due east of Quigley. Find the bearing of Surrey from Roscoe to the nearest degree.



10.08 THE COSINE RULE

The **cosine rule** can be used to calculate unknown side lengths and angles in non-right-angled triangles when there is not enough information to use the sine rule.

IMPORTANT

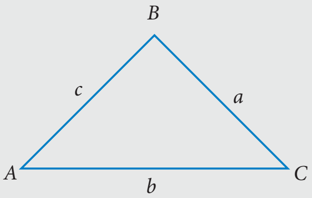
The cosine rule for the triangle ABC can be written as $a^2 = b^2 + c^2 - 2bc\cos(A)$.

It can also be written in other forms:

$b^2 = a^2 + c^2 - 2ac\cos(B)$

or

$c^2 = a^2 + b^2 - 2ab\cos(C)$.



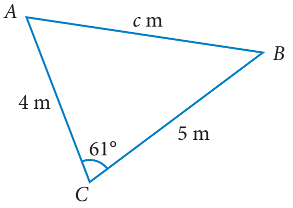
The cosine rule can be used if given a non-right-angled triangle under either of the following conditions exist:

– two side lengths and an **included angle** are given

– three side lengths are given.

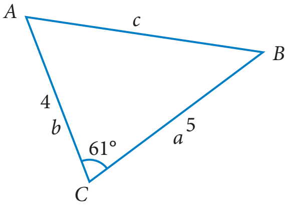
Example 17

Find the value of the pronumeral in the following triangle, correct to two decimal places.



Solution

Label the sides of the triangle using a, b, and c; ensuring that the sides are opposite the angles represented by the same letter.



Write the appropriate variation of the cosine rule. $c^2 = a^2 + b^2 - 2ab\cos(C)$

Substitute the known values into the equation. $c^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos(61°)$

Calculate.

$$\begin{aligned} c^2 &= 25 + 16 - 40\cos(61°) \\ &= 21.6076... \end{aligned}$$

Find c by taking the square root of both sides.

$$\begin{aligned} c &= \sqrt{21.6076...} \\ &= 4.6483... \end{aligned}$$

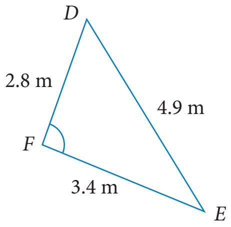
Write your answer, rounding to two decimal places and including units. $c \approx 4.65$ m

If given all three side lengths and no angles, the cosine rule can be transposed in order to calculate the size of an angle.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} \quad \text{or} \quad \cos(B) = \frac{a^2 + c^2 - b^2}{2ac} \quad \text{or} \quad \cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

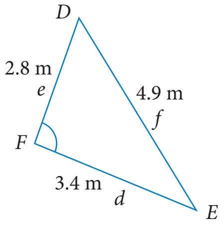
Example 18

Triangle DEF has $DE = 4.9$ m, $EF = 3.4$ m and $DF = 2.8$ m. Find angle F, correct to the nearest degree.



Solution

Label the sides of the triangle.



Note that letters other than a, b and c can be used. It is essential to match the letters used to label the side lengths to the letters used to label the opposite angles.

Write the appropriate variation of the cosine rule.

$$\cos(F) = \frac{d^2 + e^2 - f^2}{2de}$$

Substitute the known values into the equation and solve for cos (F).

$$\begin{aligned} \cos(F) &= \frac{3.4^2 + 2.8^2 - 4.9^2}{2 \times 3.4 \times 2.8} \\ &= -\frac{4.61}{19.04} \end{aligned}$$

Solve for F and write your answer. Remember to round to the nearest whole degree.

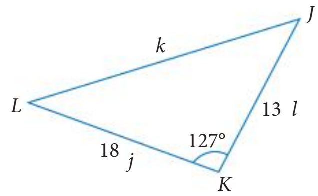
$$\begin{aligned} F &= \cos^{-1}\left(-\frac{4.61}{19.04}\right) \\ &= 104.0118... \\ &\approx 104° \end{aligned}$$

Example 19

Kelly, Jessica and Lena are testing their new walkie-talkies. Kelly is 18 m from Lena and 13 m from Jessica. The angle formed between Lena, Kelly and Jessica is 127°. Calculate the distance between Lena and Jessica, correct to the nearest metre.

Solution

Label the sides and angles of the triangle. The distance between (*L*) Lena and (*J*) Jessica is represented by *k* as it is the side opposite *K* (Kelly).



Write the cosine rule.

$$k^2 = j^2 + l^2 - 2jl\cos(K)$$

Substitute known values into the equation.

$$k^2 = 18^2 + 13^2 - 2 \times 18 \times 13 \times \cos(127^\circ)$$

Calculate k^2.

$$\begin{aligned} k^2 &= 324 + 169 - 468 \times \cos(127^\circ) \\ &= 774.6494... \end{aligned}$$

Find *k* by taking the square root of both sides.

$$\begin{aligned} k &= \sqrt{774.6494...} \\ &= 27.8325... \\ &\approx 27.83 \end{aligned}$$

Write the answer.

The distance between Lena and Jessica is 28 metres.

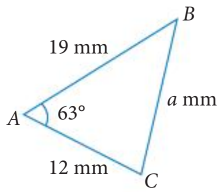
The cosine rule – Angles and sides

EXERCISE 10.08 The cosine rule

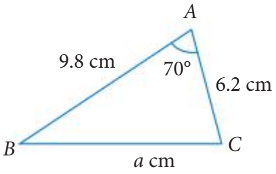
Concepts and techniques

1 Example 17 Find the length of the unknown side in each triangle, correct to two decimal places.

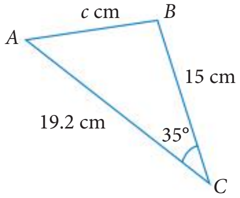
a



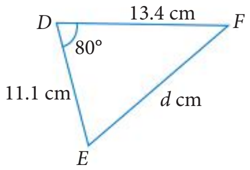
b



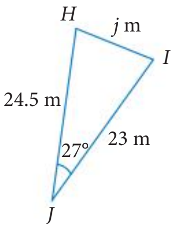
c



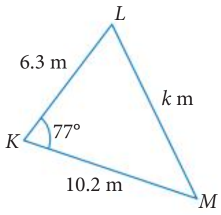
d



e



f

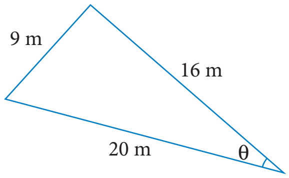


2 In $\triangle ABC$, $\angle B = 38°$, $AB = 12$ m and $BC = 17.5$ m. The length of AC to the nearest metre is:

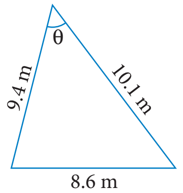
A 9 m **B** 10 m **C** 11 m **D** 14 m **E** 119 m

3 **Example 18** Find θ, to the nearest degree, in each triangle.

a



b



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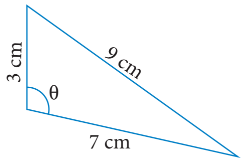
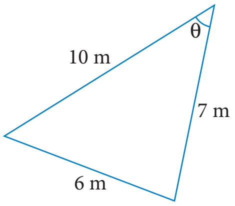
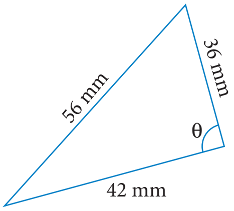


d



e



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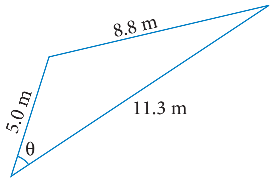
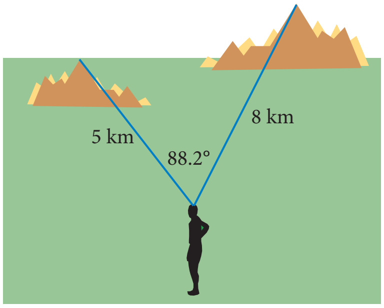


4 In $\triangle GST$, $GS = ST = 8.4$ cm and $GT = 12.7$ cm.

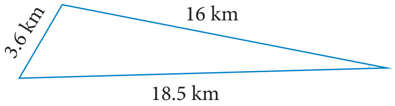
- **a** Calculate the size of $\angle S$, to the nearest degree.
- **b** Calculate the size of $\angle G$, to the nearest degree.
- **c** What type of triangle is $\triangle GST$?
- **d** What is the size of $\angle T$?

Reasoning and communication

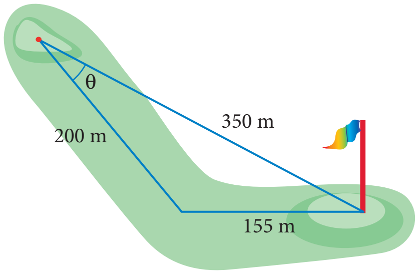
5 **Example 19** Angus observes the peaks of two hills from a plain. One is at a distance of 5 km and the other is at a distance of 8 km. If the angle between the lines of sight is 88.2°, calculate the distance between the peaks, correct to the nearest kilometre.



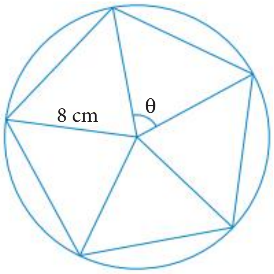
6 A triathlon has a triangular circuit with three legs: swimming 3.6 km, cycling 18.5 km and running 16 km. What is the size of the angle between the cycling and running legs? Answer correct to the nearest degree.



7 A golf hole is 350 m in a straight line from the tee. A ball is driven 200 m from the tee but it is off-line. The ball lands 155 m to the right of the hole. By what angle, to the nearest degree, is the shot off-line?

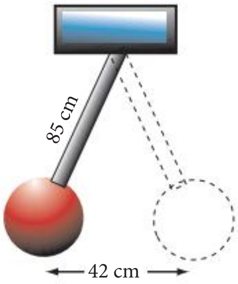


8 A regular pentagon is inscribed in a circle of radius 8 cm.
   a How many degrees in a revolution?
   b What is the size of angle θ?
   c Calculate the perimeter of the pentagon, giving the answer correct to two decimal places.



9 The two legs of a ladder are 2.1 m long and stand on the ground 0.9 m apart. Calculate, correct to the nearest degree:
   a the angle between the legs
   b the angle that each leg makes with the ground.

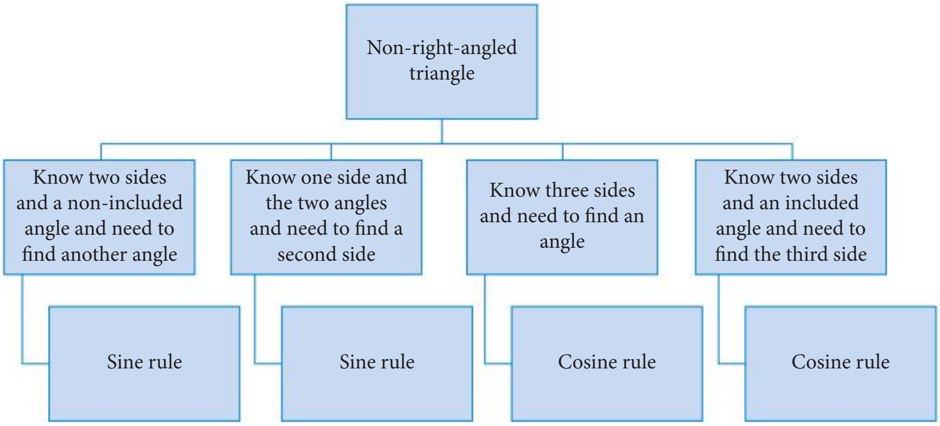
10 A pendulum of length 85 cm swings through a horizontal distance of 42 cm. What is the angle swept by the pendulum? Answer correct to the nearest degree.



10.09

MIXED APPLICATIONS INVOLVING NON-RIGHT-ANGLED TRIANGLES

When working with non-right-angled triangles either the sine rule or cosine rule can be used to calculate an unknown angle or side length. Each rule has its own purpose and it is important to be familiar with what each rule is used for. The following flow chart will assist you in choosing the appropriate rule.

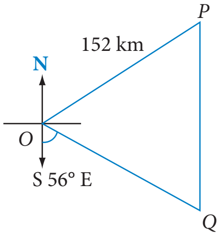


Example 20

Two ships, P and Q, leave port O at the same time. Ship P sails on a course NE, while ship Q sails on a compass bearing of S 56° E. After P sails 152 km, it is due north of Q. Calculate, correct to two decimal places:

a the distance between P and Q

b how far Q has sailed.

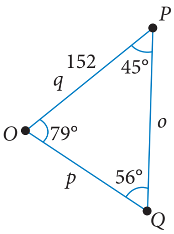


Solution

a Redraw the triangle and calculate the internal angles.

$$\angle POQ = 180° - 45° - 56°$$
$$= 79°$$
$$\angle OPQ = 45°$$
$$\angle OQP = 56°$$

Label angles O, P and Q, and side lengths o, p and q.



Use the sine rule to calculate the distance between P and Q (labelled as o on the triangle).

$$\frac{o}{\sin(O)} = \frac{q}{\sin(Q)}$$
$$\frac{o}{\sin(79°)} = \frac{152}{\sin(56°)}$$
$$o = \frac{152\sin(79°)}{\sin(56°)}$$
$$= 179.9765...$$
$$\approx 180$$

Write your answer, rounding to the nearest whole kilometre, including units.

The distance between P and Q is 180 km.

b Use the sine rule to calculate the distance that Q has travelled (labelled as p on the triangle).

$$\frac{p}{\sin(P)} = \frac{q}{\sin(Q)}$$
$$\frac{p}{\sin(45°)} = \frac{152}{\sin(56°)}$$
$$p = \frac{152\sin(45°)}{\sin(56°)}$$
$$= 129.6445...$$
$$\approx 130$$

Write your answer, rounding to the nearest whole kilometre, including units.

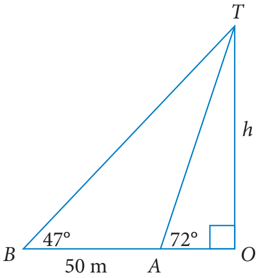
Q has sailed 130 km.

Example 21

The angle of elevation of a tower from point A is 72°. From point B, 50 m further away from the tower than A, the angle of elevation is 47°.

a Find the length of AT correct to two decimal places.

b Find the height h of the tower correct to two decimal places.



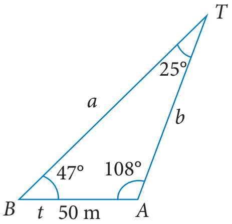
Solution

a To find AT, use $\triangle BAT$ as there is not enough information to use $\triangle BOT$. $\angle BAT$ and $\angle BTA$ can also be found.

$$\angle BAT = 180° - 72° = 108°$$

$$\angle BTA = 180° - 108° - 47° = 25°$$

Draw and label the diagram carefully.



Solve for AT using the sine rule. Let $AT = b$.

$$\frac{b}{\sin(47°)} = \frac{50}{\sin(25°)}$$

$$b = \frac{50\sin(47°)}{\sin(25°)}$$

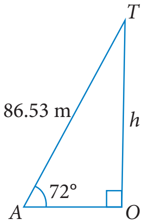
$$= 86.5265...$$

$$\approx 86.53$$

Write your answer, rounding to two decimal places, and include the unit.

The length of AT is 86.53 m.

b To find h, we use $\triangle AOT$ with the length of AT found in part **a**.



Use the sine ratio to solve for h.

$$\sin(\theta) = \frac{O}{H}$$

$$\sin(72°) = \frac{h}{86.53}$$

$$h = 86.53\sin(72°)$$

$$= 82.2949...$$

$$\approx 82.29$$

Write your answer, rounding to two decimal places, and include the unit.

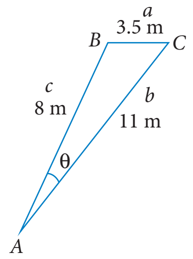
The height of the tower is 82.29 m.

Example 22

Football posts are 3.5 m apart. If a footballer is standing 8 m from one post and 11 m from the other, find the angle within which the ball must be kicked to score a goal, correct to the nearest degree.

Solution

Sketch a diagram illustrating the above situation.



As three side lengths are known and an angle needs to be calculated, the cosine rule is used.

Select and write the appropriate variation of the cosine rule.

$$\cos(A)=\frac{b^2+c^2-a^2}{2bc}$$

Substitute in the known values and solve.

$$\cos(\mathrm{A})=\frac{11^2+8^2-3.5^2}{2\times 11\times 8}$$

$$=\frac{172.75}{176}$$

$$\mathrm{A}=\cos^{-1}\left(\frac{172.75}{176}\right)$$

$$=11.0279...$$

$$\approx 11°$$

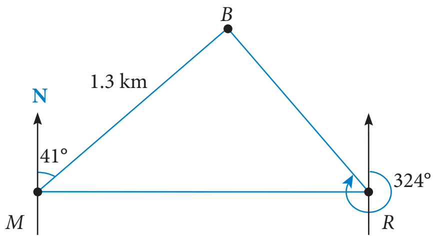
Write your answer, remembering to round to the nearest whole degree.

The angle within which the ball must be kicked is 11°.

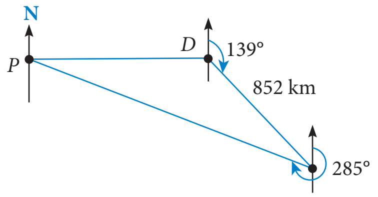
EXERCISE 10.09 Mixed applications involving non-right-angled triangles

Reasoning and communication

1 Example 20 A boat is sinking 1.3 km out to sea from a marina. Its bearing is 041° from the marina and 324° from a rescue boat. The rescue boat is due east of the marina. How far, correct to 2 decimal places, is the rescue boat from the sinking boat?



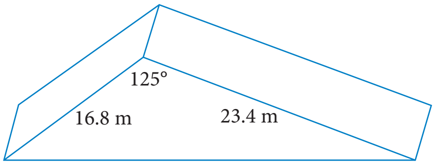
2 A plane flies from Dubbo on a bearing of 139° for 852 km, then turns and flies on a bearing of 285° until it is due west of Dubbo. How far from Dubbo is the plane, to the nearest km?



3 Example 21 The angle of elevation of the top of a flagpole from a point a certain distance away from its base is 20°. After walking 80 m towards the flagpole, the angle of elevation is 75°. Find the height of the flagpole, correct to the nearest metre.

4 Example 22 A triangular field *ABC* has sides $AB = 85$ m and $AC = 50$ m. If *B* is on a bearing of 065° from *A* and *C* is on a bearing of 166° from *A*, find the length of *BC*, correct to the nearest metre.

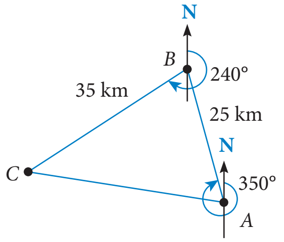
5 A triangular roof is 16.8 m up to its peak, then 23.4 m on the other side with a 125° angle at the peak as shown. Find the horizontal length of the roof correct to one decimal place.



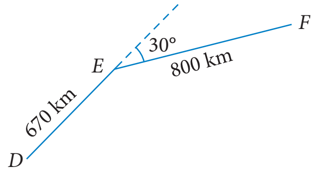
6 Claire just returned from a three-day cycling trip. On the first day she left point *A* on a true bearing of 350° and cycled for 25 km to point *B*. On the second day she cycled for 35 km on a true bearing of 240° to reach point *C*. On the third day she returned to point *A* but she didn't record the distance or bearing.

a What distance did she travel on the third day (correct to two decimal places)?

b What is the true bearing of point *A* from point *C* (to the nearest degree)?



7 A plane flies from Dunstan to Emmett, a distance of 670 km. It then turns 30° and flies another 800 km to Frankston. Calculate the distance between Dunstan and Frankston, correct to the nearest kilometre.



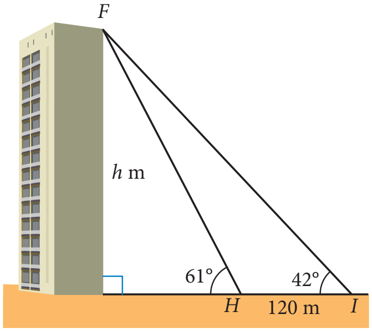
8 At point *I*, Justine observes the top, *F*, of an office tower at an angle of elevation of 42°. She walks 120 m towards the tower to *H*, where the angle of elevation increases to 61°.

a Explain why $\angle IFH = 19°$.

b Show that $FH = \dfrac{120\sin(42°)}{\sin(19°)}$.

c Hence show that $h = \dfrac{120\sin(42°)\sin(61°)}{\sin(19°)}$.

d Evaluate *h* to the nearest metre.



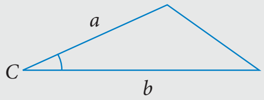
10.10 AREA OF A TRIANGLE: TRIGONOMETRY

The **area of a triangle** can be found using the formula $A = \frac{1}{2}bh$, where b is the base length and h is the perpendicular height. There is another formula that can be used to calculate the area of a triangle if given the length of two sides and the angle between them.

IMPORTANT

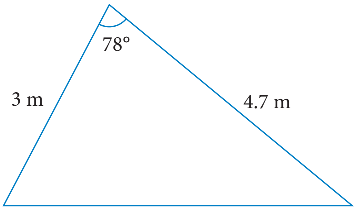
When two side lengths and the included angle are known:

$A_{\text{triangle}} = \frac{1}{2}ab\sin(C)$



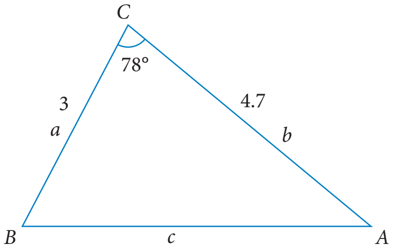
Example 23

Calculate the area of the following triangle correct to two decimal places.



Solution

Label the side lengths a, b and c and the corresponding angles A, B and C.



Identify the values of a, b and C.

$a = 3$
$b = 4.7$
$C = 78°$

Substitute the known values into the formula.

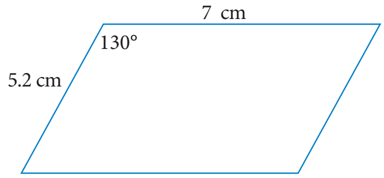
$$A = \frac{1}{2}ab\sin(C)$$
$$= \frac{1}{2} \times 3 \times 4.7 \times \sin(78°)$$

Evaluate, round correct to two decimal places and write your answer.

$$A = 6.8959\ldots$$
$$\approx 6.90 \text{ m}^2$$

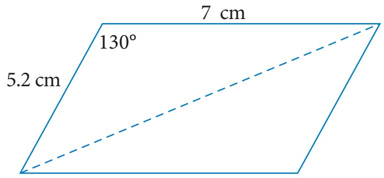
Example 24

A parallelogram has sides of length 7 cm and 5.2 cm, and an included angle of 130°. Calculate its area, correct to one decimal place.



Solution

The question specifies that the angle is included, so the appropriate formula to calculate half the area of the parallelogram is $A = \frac{1}{2}\, ab \sin(C)$.



Identify the values of a and b: that is, the side lengths. Identify the value of the included angle.

$a = 7$ cm, $b = 5.2$ cm
$C = 130°$

Substitute the known values into the formula.

$$\begin{aligned} A &= \frac{1}{2} ab \sin(C) \\ &= \frac{1}{2} \times 7 \times 5.2 \times \sin(130°) \\ &= 13.94 \text{ cm}^2 \end{aligned}$$

Remember that a parallelogram is constructed of two non-right-angled triangles. Multiply your answer by 2, round correct to one decimal place and write your answer.

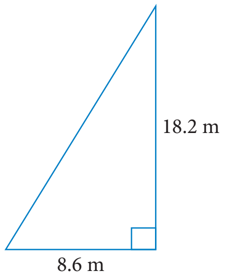
$$\begin{aligned} \text{Area of parallelogram} &= 2 \times 13.94 \text{ cm}^2 \\ &= 27.9 \text{ cm}^2 \end{aligned}$$

EXERCISE 10.10 Area of a triangle: trigonometry

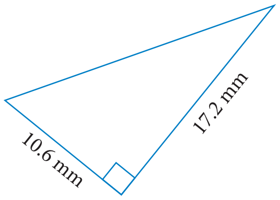
Concepts and techniques

1 Calculate the area of the following triangles using $A = \frac{1}{2} bh$.

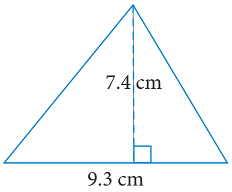
a



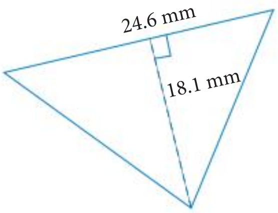
b



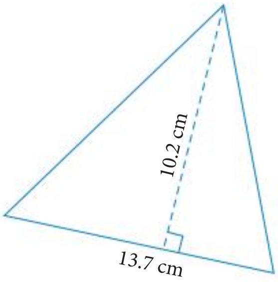
c



d



e



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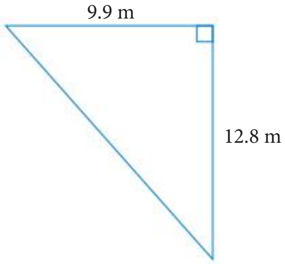
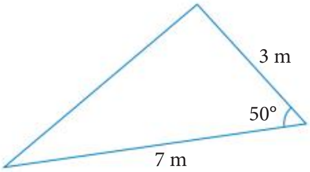
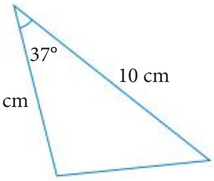


2 Example 23 Calculate the area of the following triangles using $A = \frac{1}{2}ab\sin(C)$, correct to two decimal places.

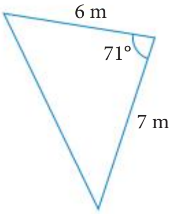
a



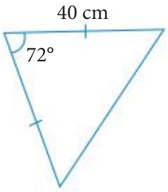
b



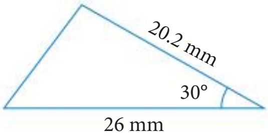
c



d



e



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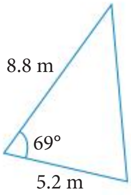
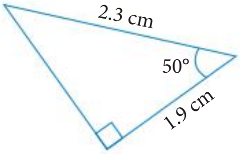
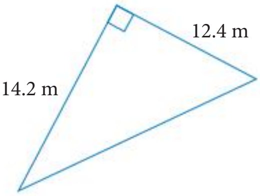


3 Calculate the area of the following triangles correct to one decimal place.

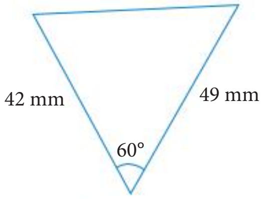
a



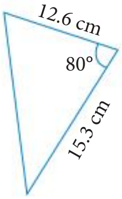
b



c



d



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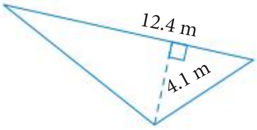
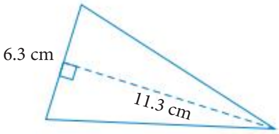


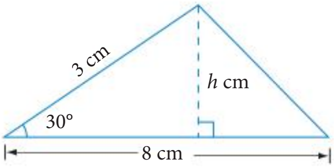
f



4 Calculate the area of this triangle:

a using $A = \frac{1}{2}ab\sin(C)$.

b using $A = \frac{1}{2}bh$ (find h using trigonometry first).

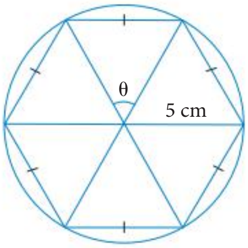


5 Calculate, correct to two decimal places, the area of an equilateral triangle with side lengths of 7 cm:

- a using the formula $A = \frac{1}{2}ab \sin(C)$.
- b using the formula $A = \frac{1}{2}bh$ (after first finding the value of h).

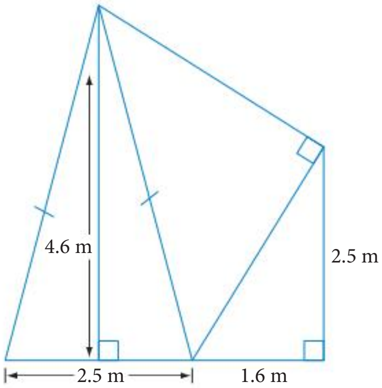
Reasoning and communication

6 **Example 24** A regular hexagon is inscribed in a circle of radius 5 cm. The area of the hexagon is:



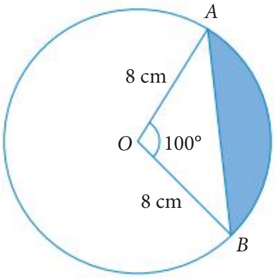
A 10.83 cm^2
B 54.13 cm^2
C 64.95 cm^2
D 75.00 cm^2
E 78.54 cm^2

7 Jim has designed the following garden bed as a feature in his backyard. He needs to fill the garden bed with top soil. Calculate the total area that requires top soil.



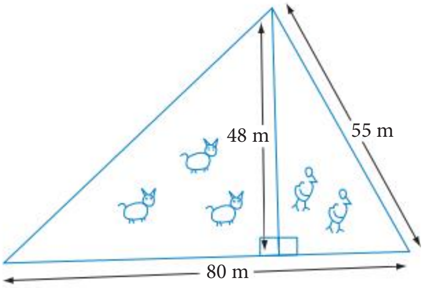
8 O is the centre of a circle with radius 8 cm shown at right.

- a Calculate, correct to two decimal places, the area of sector OAB using the formula $A = \frac{\theta}{360}\pi r^2$.
- b Calculate the area of ΔOAB, correct to two decimal places.
- c Hence, find the area of the shaded segment.



9 Farmer Mac has divided his farm into two sections as illustrated. On one side he has chickens and on the other he has pigs.

- a Calculate the area of each section, correct to two decimal places.
- b Hence calculate the total area of the farm, correct to the nearest square metre.



10.11 AREA OF A TRIANGLE: HERON'S FORMULA

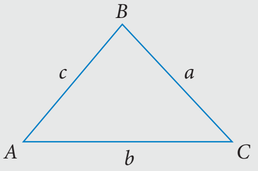
The area of a triangle can also be found using **Heron's formula**. Heron's formula is used when all three side lengths are known.

IMPORTANT

Heron's formula:

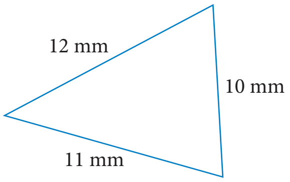
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ (the semi-perimeter).



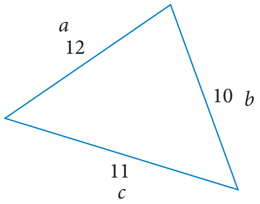
Example 25

Calculate the area, correct to two decimal places, of the following triangle.



Solution

Label the sides of the triangle *a*, *b* and *c*.



To calculate the semi-perimeter, write the formula for the semi-perimeter and substitute in the known values.

$$s = \frac{a+b+c}{2}$$
$$= \frac{12+10+11}{2}$$
$$= 16.5$$

To calculate the area, write Heron's formula and substitute in the known side lengths and semi-perimeter.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16.5(16.5-12)(16.5-10)(16.5-11)}$$

Evaluate.

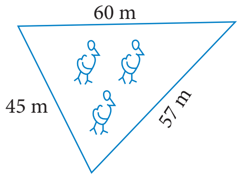
$$= \sqrt{2654.4375}$$
$$= 51.5212...$$

Round to two decimal places and write the answer with the appropriate unit.

$$\text{Area} \approx 51.52 \text{ mm}^2$$

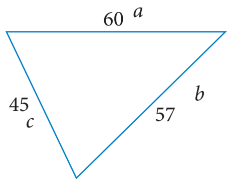
Example 26

Local law states that owners require a certain amount of area for each chicken kept on a property. Mike has the following space allocated for keeping chickens. Calculate this area to the nearest square metre.



Solution

Label the sides of the triangle *a*, *b* and *c*.



Write the formula for the semi-perimeter and substitute in the known values.

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{60+57+45}{2} \\ &= 81 \end{aligned}$$

Substitute the known side lengths and semi-perimeter into Heron's formula.

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{81(81-60)(81-57)(81-45)} \end{aligned}$$

Evaluate.

$$= \sqrt{1\,469\,664}$$

Round to two decimal places and write the answer with the appropriate unit.

$$= 1212.2969...$$

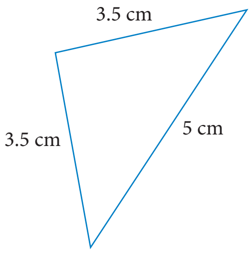
$$\text{Area} \approx 1212.30\ \text{m}^2$$

EXERCISE 10.11 Area of a triangle: Heron's formula

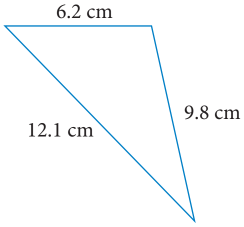
Concepts and techniques

1 Example 1 Calculate the area of the following triangles, correct to two decimal places.

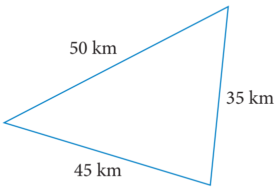
a



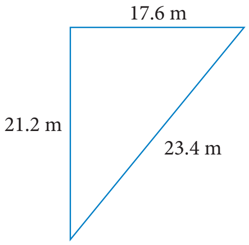
b



c



d



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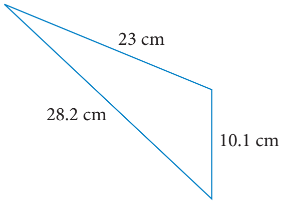

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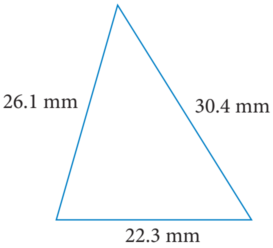


2 In triangle *ABC*, *AB* = 12 cm, *AC* = 7 cm and *BC* = 9 cm. To calculate the area, which of the following equations would be used?

A $A=\sqrt{14(14-12)(14-7)(14-9)}$

B $A=\sqrt{28(28-12)(28-7)(28-9)}$

C $A=\frac{12+7+9}{2}$

D $A=\frac{1}{2}\times 12\times 7$

E $A=\frac{1}{2}\times 12\times 7\times 9$

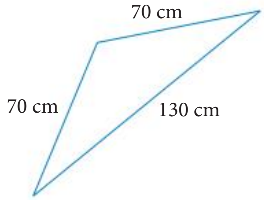
3 Find the area (correct to two decimal places) of an equilateral triangle with a perimeter of 18 cm using:

a Heron's formula

b $A=\frac{1}{2}ab\sin(C)$

Reasoning and communication

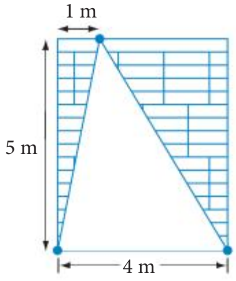
4 Theo is designing a television cabinet to fit in the corner of his lounge room. He needs to know the area of the top of the cabinet so he knows how much paint is required. Calculate the area of the top of the cabinet to the nearest cm^2.



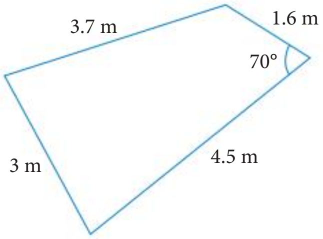
5 A shade sail is to be tied to three posts on a rectangular patio, as shown in the diagram on the right.

a Calculate the dimensions of this shade sail , correct to two decimal places.

b Calculate the area of material required to make the shade sail (to the nearest square metre).



6 Eric has designed a fishpond in the shape of a quadrilateral for a local park, as shown in the diagram below. Calculate the total area of the fishpond, correct to the nearest square metre.



10

CHAPTER SUMMARY

APPLICATIONS OF TRIGONOMETRY

- The **trigonometric ratios** allow us to solve unknown side lengths and unknown angles in right-angled triangles.
- We use the mnemonic (memory aid) **SOH-CAH-TOA** to remember the trigonometric ratios:

Ratio	Meaning	Initials
sine	$\sin(\theta)=\frac{\text{opposite}}{\text{hypotenuse}}$	SOH
cosine	$\cos(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$	CAH
tangent	$\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}$	TOA

- **Angles of elevation** are taken from the horizontal looking up.
- **Angles of depression** are taken from the horizontal looking down.
- There are two types of **bearings.**
    - **True bearings** are taken from north (0°) in a clockwise direction. True bearings will always be between 0° and 360° and are generally written with three digits (e.g. 060°)
    - **Compass bearings** depend on which quadrant the bearing is in. Compass bearings either go from north or south and in an east or west direction. Compass bearings will always be between 0° and 90°. An example of a compass bearing is N 60° E.
- The **sine rule**: $\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$ is used to find an unknown side or angle in a non-right-angled triangle under the following conditions:
    - two sides and a non-included angle are known
    - one side and two angles are known.
- The **cosine rule**: $a^2 = b^2 + c^2 - 2bc\cos(A)$ is used to find an unknown side.
- A variation of the cosine rule,

$$\cos(A)=\frac{b^2+c^2-a^2}{2bc}$$

  is used to find an unknown angle in a non-right-angled triangle under the following conditions:
    - two sides and an included angle are known
    - three sides are known.
- There are three different formulas which can be used to calculate the **area of a triangle**:
    - $A=\frac{1}{2}bh$
    - $A=\frac{1}{2}ab\sin(C)$
    - $A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{a+b+c}{2}$ (the semi-perimeter)

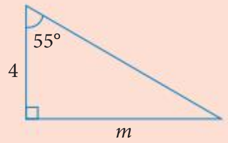
9780170250252

CHAPTER REVIEW
APPLICATIONS OF TRIGONOMETRY
10

Multiple choice

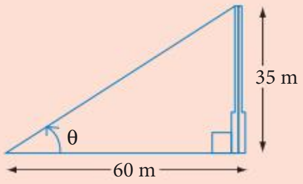
1 **Example 1** The value of the pronumeral, correct to two decimal places, in the diagram on the right is:

A 2.29 B 2.80 C 3.28
D 5.71 E 6.97



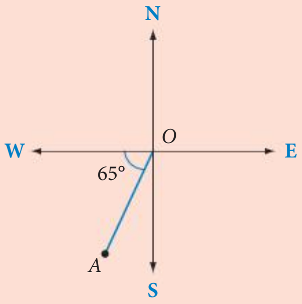
2 **Example 6** Kevin stands 60 m from the base of a 35 m tower. The angle of elevation, to the nearest degree, of the tower from Kevin is:

A 30° B 31° C 36°
D 54° E 60°



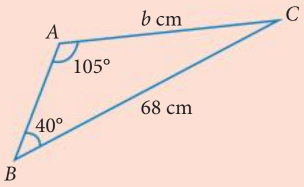
3 **Example 8** Respectively, the true bearing and compass bearing of *A* from *O* is:

A 215°, S 65° W
B 065°, W 65° S
C 205°, W 65° S
D 215°, S 25° W
E 205°, S 25° W



4 **Example 12** The value of *b*, correct to two decimal places, is:

A 42.22 cm
B 43.63 cm
C 45.25 cm
D 67.88 cm
E 102.18 cm



5 **Example 17** Which of the following expressions is equal to *c*?

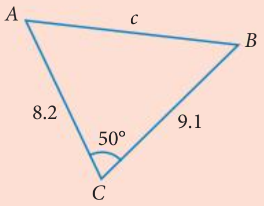
A $8.2^2+9.1^2$

B $\sqrt{8.2^2+9.1^2-2\times 8.2\times 9.1\cos(50°)}$

C $8.2^2+9.1^2-2\times 8.2\times 9.1\cos(50°)$

D $\sqrt{8.2^2+9.1^2}$

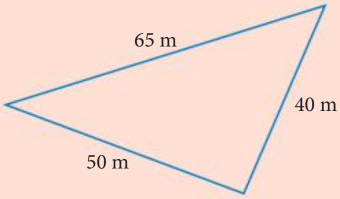
E $\dfrac{8.2\sin(50°)}{9.1}$



6 **Example 25** The area of the triangle (correct to two decimal places) is:

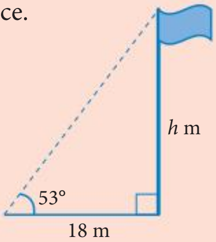
A 77.50 m^2 **B** 155.00 m^2 **C** 999.51 m^2

D 1000.00 m^2 **E** 1300.00 m^2

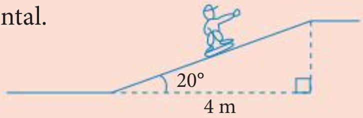


Short answer

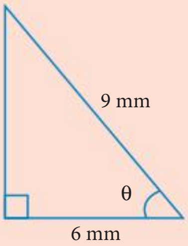
7 **Example 3** Find the height, *h* m, of this flagpole correct to one decimal place.



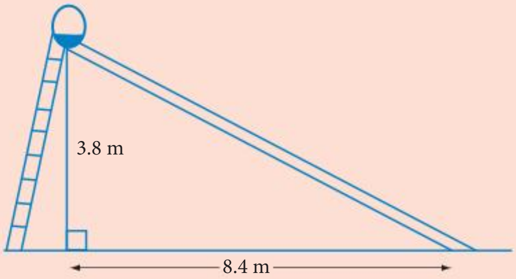
8 **Example 2** A skateboard ramp is inclined at 20° to the horizontal. How long is the ramp if it links two levels that are 4 m apart? Give your answer correct to one decimal place.



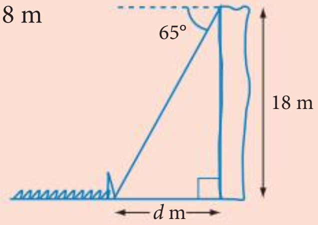
9 **Example 4** Find the unknown angle, correct to the nearest degree.



10 **Example 5** A pool slide is made up of a ladder and a plastic slide. The top of the plastic slide is 3.8 m from the ground and the bottom of the slide is 8.4 m from a point directly under the top of the slide. To the nearest degree, what angle does the slide make with the ground?



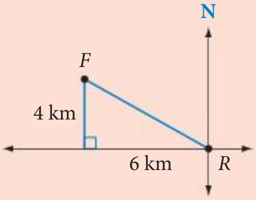
11 **Example 7** The angle of depression of a buoy from the top of a 18 m vertical cliff is 65°. To the nearest metre, how far is the buoy from the base of the cliff?



12 **Example 10** Flannigan's Primary School, F, is 6 km west and 4 km north of Riverside Mall, R.

a Find the true bearing, correct to the nearest degree, of Flannigan's Primary School from Riverside Mall.

b Find the direct distance between Flannigan's Primary School and Riverside Mall, correct to one decimal place.

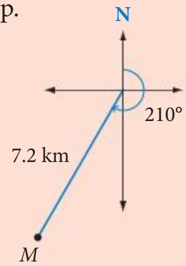


13 **Example 11** Max walked for 7.2 km on a true bearing of 210° from base camp. His position is shown by M on the diagram on the right.

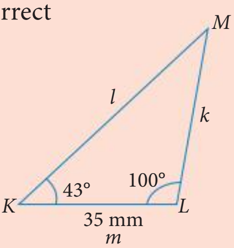
a How far south is he from base camp (correct to one decimal place)?

b How far west is he from base camp (correct to one decimal place)?

c What is the true bearing of base camp from Max?



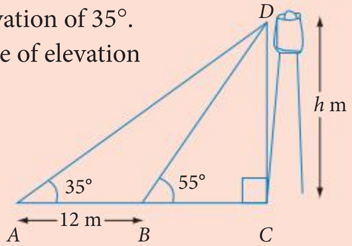
14 **Example 13** In $\triangle KLM$, $\angle K = 43°$, $\angle L = 100°$, and $m = 35$ mm. Find k, correct to one decimal place.



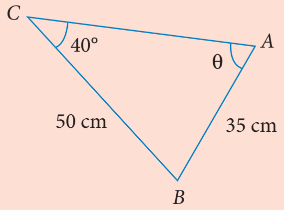
15 **Example 14** Monique observes a lighthouse at an angle of elevation of 35°. Walking 12 m towards the lighthouse, she finds that the angle of elevation increases to 55°.

a Calculate the distance between B and D, correct to two decimal places.

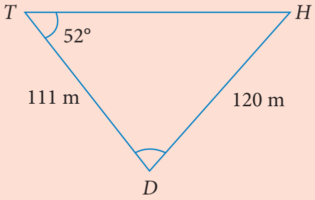
b Hence evaluate the height, h metres, of the lighthouse, correct to two decimal places.



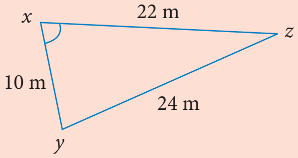
16 **Example 15** Find the value of θ, correct to the nearest degree, in the following triangle.



17 **Example 16** The 12th hole at a golf course has a tee at T, a hole at H and a dogleg at D. If $TD = 111$ m, $DH = 120$ m and $\angle HTD = 52°$, find $\angle TDH$ correct to the nearest degree.



18 **Example 18** Triangle XYZ has $XY = 10$ m, $YZ = 24$ m and $XZ = 22$ m. Find angle X, correct to the nearest degree.

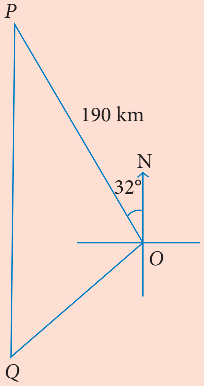


19 **Example 19** James, Hannah and Caitlin are playing hide-and-seek. James is 12 m from Hannah and 19 m from Caitlin. The angle formed between Hannah, James and Caitlin is 105°. Calculate the distance between Hannah and Caitlin, correct to the nearest metre.

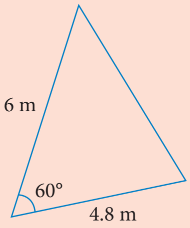
20 **Example 20** Two ships, P and Q, leave port O at the same time. Ship P sails on a course N 32° W, while ship Q sails SW. After P sails 190 km, it is due north of Q. Calculate, correct to two decimal places:

a the distance between P and Q.

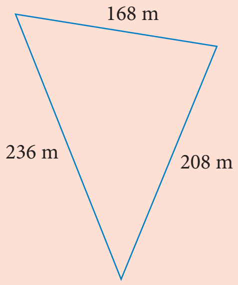
b how far Q has sailed.



21 **Example 24** Calculate the area of the following triangle using $A = \frac{1}{2} ab \sin (C)$, correct to two decimal places.



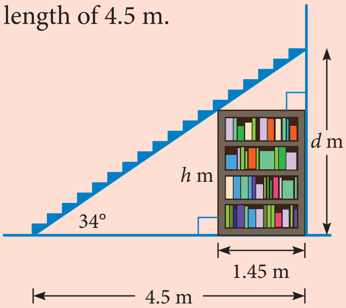
22 **Example 27** Michelle has measured the dimensions of a triangular dam on her property, as shown in the diagram on the right. Calculate the area of the dam, correct to the nearest square metre.



Application

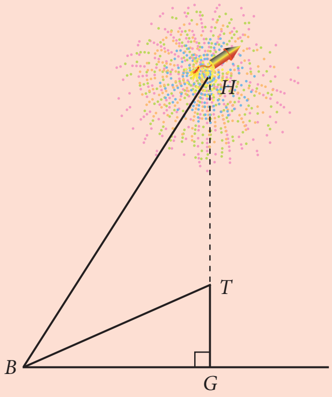
23 A staircase is inclined at an angle of 34° and has a horizontal length of 4.5 m. A bookcase, 1.45 m long, is placed under the stairs. Find, correct to the nearest metre,

- **a** the vertical height, h, of the tallest bookcase that will fit under the stairs
- **b** the distance, d, of the top step of the staircase from the ground
- **c** the length from the bottom of the staircase to the top.



24 Boun (B) sees fireworks being launched from the top of a tower, TG, at an angle of elevation of 38°. When the fireworks are at their highest point, H, 800 m above the tower, the angle of elevation is 80°.

- **a** Copy the diagram and mark on it all the information given.
- **b** Show that $\angle BTH = 128°$.
- **c** Calculate the distance between Boun and the fireworks at H, correct to the nearest metre.
- **d** Calculate the distance between Boun and the top, T, of the tower, correct to the nearest metre.
- **e** Calculate the height of the tower, correct to the nearest metre.



Practice quiz

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11

TERMINOLOGY

back-to-back stem-and-leaf plot
interquartile range
mean
median
mode
negatively skewed
parallel boxplots
positively skewed
range
side-by-side column graphs
standard deviation
summary statistics

STATISTICS: TWO VARIABLES

MAKING COMPARISONS

Prior learning

THE STATISTICAL INVESTIGATION PROCESS

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM026)

COMPARING DATA FOR A NUMERICAL VARIABLE ACROSS TWO OR MORE GROUPS

- construct and use parallel box plots (including the use of the 'Q1 − 1.5 × IQR' and 'Q3 + 1.5 × IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and to interpret and communicate the differences observed in the context of the data (ACMGM031)
- compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate, interpret the differences observed in the context of the data and report the findings in a systematic and concise manner (ACMGM032)
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups; for example, are Year 11 students the fittest in the school? (ACMGM033)

11.01 USE OF SUMMARY STATISTICS

This chapter will investigate many of the summary statistics and graphical displays introduced in Chapter 8 and how they can be utilised to compare sets of numerical data.

When interpreting summary statistics, it is important to understand what each statistic tells you about the data.

Refer to Chapter 8 (Sections 8.07 to 8.09) to revise how to calculate each individual summary statistic by hand and using a CAS if required.

- Mean: the average of the data being investigated.
- Median: the middle value of the data set when in order. This means that 50% of the data lies below the median and 50% lies above it.
- Mode: the most frequent or popular data value.
- Range: the difference between the highest data value and the lowest data value, therefore describing the total spread of scores.
- Interquartile range: the spread of the middle 50% of the data.
- Standard deviation: how data deviates from the mean.

Example 1

The following data are the English examination results (%) for a Year 11 class.

50	45	60	76	78	90	58	79	85	92	56
34	56	93	84	76	65	78	89	95	81	54

a Find the mean, correct to one decimal place.
b Find the median.
c Find the mode.
d Find the range.
e Find the interquartile range.
f Explain what each statistic tells you about the data set.

Solution

a Calculate the mean using the formula.

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{1573}{22}$$

$= 71.5$ correct to one decimal place

The mean mark is 71.5%.

b Order the data.

34 45 50 54 56 56 58 60 65 76 76
78 78 78 81 84 85 89 90 92 93 95

Find the median position.

Median position is the $\frac{22+1}{2}$th $= 11.5$th.

Find the median.

The median is half way between the 11th and 12th data values.

Median = 77%

Interpret.

The middle examination result was 77%, which indicates that 50% of the students scored below 77% and 50% scored above it.

c The mode is the most commonly occurring value.

Mode = 78%

d Calculate the range using the formula.

Range = highest data value – lowest data value
Range = 95 – 34
= 61%

e Find the IQR using $Q_3 - Q_1$.

$34 \quad 45 \quad 50 \quad 54 \quad 56 \quad \boxed{56} \quad 58 \quad 60 \quad 65 \quad 76 \quad 76$

$Q_1 = 56$

$78 \quad 78 \quad 78 \quad 81 \quad 84 \quad \boxed{85} \quad 89 \quad 90 \quad 92 \quad 93 \quad 95$

$Q_3 = 85$

$IQR = Q_3 - Q_1$
$= 85 - 56$
$= 29\%$

f The mean tells you the average score.

The average English examination mark for the Year 11 class was 71.5%.

The median tells you the middle score.

The middle mark was 77% which indicates that 50% of students scored below 77% and 50% scored above it.

The mode is the most common score.

The most common examination score was 78%.

The range is the difference between the highest score and the lowest score.

There was a difference of 61% between the highest and lowest scores on the examination.

The interquartile range gives the range of the middle 50% of scores.

The examination scores have a IQR of 29%, which tells us that the middle 50% of scores on the examination had a range of 29%, or that the middle 50% of data lies between 85% and 56%.

Example 2

A small business employs staff with the following salaries.

• General Manager	\$158 300
• 3 factory hands	\$64 300 each
• supervisor	\$85 000
• 2 clerical staff	\$68 500 each

a How many people are on staff?

b Calculate the mean salary of the staff, correct to the nearest \$100.

c Calculate the median salary of the staff.

d Which measure of centre is the highest, mean or median? Calculate the difference.

e Can you suggest a possible reason for the difference?

f Which measure of centre best describes the salaries at this business?

Solution

a Count the number of staff.

$1 + 3 + 1 + 2 = 7$ workers

b Calculate the mean using the formula.

$$\bar{x} = \frac{\sum x}{n}$$

Round to the nearest \$100.

$$\bar{x} = \frac{573\ 200}{7}$$
$$= 81\ 885.714...$$
$$\approx \$81\ 900$$

c List the data in order and find the median.

64 300 64 300 64 300 **68 500**
68 500 85 600 158 300

Median = \$68 500

d Calculate the difference between the mean and the median.
Interpret the result.

Mean – Median = 81 900 – 68 500
= 13 400

The mean is higher by \$13 400.

e Suggest a possible reason.

The average is higher as the last wages of the general manager and supervisor are significantly higher than the other five staff. This results in the average salary being higher than the middle salary.

f The median is unaffected by extreme high or low data values.

The median of \$68 500 best represents the salaries at this business as five out of the seven salaries are close to this figure.

Summary statistics are very important and informative when comparing 2 or more sets of data. Comparisons can be made, such as:

1 Group A has a higher average than Group B
2 Group A has a higher median value than Group B
3 Group A has a smaller range than Group B.

Example 3

The number of kilometres travelled per week (Monday to Friday) by two families was recorded over ten weeks and the results are shown below. Calculate the:

Thom family: 125 156 136 152 129 145 154 143 132 126
Mills family: 116 135 145 165 159 137 149 154 137 148

a mean for each family and state which family travels the most, on average
b median for each family and make a comparison
c range for each data set and state which one has the greater spread.

Solution

a Calculate the mean for each family.

Thom family

$$\bar{x} = \frac{1398}{10}$$
$$= 139.8 \text{ km}$$

Mills Family

$$\bar{x} = \frac{1445}{10}$$
$$= 144.5 \text{ km}$$

Compare the means.

On average, the Mills family travels more kilometres than the Thom family.

b Calculate the median for each family.

Thom family:
125 126 129 132 136 | 143 145 152 154 156

$$\text{Median} = \frac{136+143}{2} = 139.5 \text{ km}$$

Mills family:
116 135 137 137 145 | 148 149 154 159 165

$$\text{median} = \frac{145+148}{2} = 146.5 \text{ km}$$

Compare the medians.

The median number of kilometres travelled is higher for the Mills family than the Thom family. The Mills family travelled more kilometres than the Thom family.

c Find each range using:
Range = highest data value – lowest data value.

Thom family:

$$\text{Range} = 156 - 125$$
$$= 31 \text{ km}$$

Mills family:

$$\text{Range} = 165 - 116$$
$$= 49 \text{ km}$$

Compare the ranges.

The Mills family has a higher range, indicating they have more variation in the number of kilometres travelled each week.

EXERCISE 11.01 Use of summary statistics

Concepts and techniques

1 Example 1 The following data represents the number of runs made by a cricket team throughout the 2012/13 season.

145 167 185 123 146 167 178 156 128 192 188

a Find the mean.
b Find the median.
c Find the mode.
d Find the range.
e Find the interquartile range.
f Explain what each statistic tells you about the data set.

2 The following data represents the heights (cm) of students in a Year 11 class.

167 182 175 156 168 159 160 175 180 168 154 159
175 179 163 169 178 171 161 170 157 165 162

a Find the mean.
b Find the median.
c Find the mode.
d Find the range.
e Find the interquartile range.
f Explain what each statistic tells you about the data set.

The following information relates to questions **3** and **4**.

The times, in seconds, of competitors in a slalom ski race are displayed in the stem-and-leaf plot below.

Key: 9|1 means 91 seconds

Stem	Leaf
9	1 7 9
10	4 5 6 6 8
11	0 2 3 4 4 5
12	1 2 3 3 3 9
13	2 3 7 7 7
14	6 9
15	0 1

Shutterstock.com/Mitch Gunn

3 Which one of the following statements is **false**?
A The range of the data is 60 seconds.
B The mean of the data is 120.9 seconds.
C The IQR of the data is 28 seconds.
D The median of the data is 121 seconds.
E The mode of the data is 123 seconds.

4 Which one of the following statements is **true?**
A There are 30 competitors in the slalom race.
B The middle 50% of the data has a range of 30 seconds.
C 50% of the data lies above 121 seconds.
D 50% of the data lies below 116 seconds.
E None of the above.

5 **Example 2** The makes and prices of cars sold at a second-hand dealership in one week are shown below.

Fairfax Photos/Vince Caligiuri, The Age Melbourne

- Toyota: Two at $12 000 and one at $14 000
- Holden: Three at $11 000 and two at $13 000
- Ford: One at $40 000 and four at $14 500
- Nissan: One at $11 000

a How many cars were sold over the week?
b What was the most popular make of car sold during the week?
c Calculate the mean price of the cars sold, correct to the nearest dollar.
d Calculate the median price of the cars sold.
e Which measure of centre is the highest, mean or median? Calculate the difference.
f Can you suggest a possible reason for the difference?
g Which measure of centre best describes the selling price of the cars for the week?

6 The heights, in centimetres, of a team of basketball players are:

204 195 184 189 193 178 199

a Find the mean height of the team, correct to one decimal place.
b Find the median height of the team and interpret.

The 184 cm tall player is replaced by a 200 cm tall player.

c Recalculate the mean.
d Recalculate the median.
e Describe how and why the mean and median have been affected by the change of players.

7 **Example 3** The amount spent (in dollars) by a group of boys and girls attending the Royal Melbourne Show was recorded and the results are shown below.

Boys: 11 12 20 23 30 32 34 34 35 36 36 38 43 45 45 46 46 46
Girls: 11 12 13 18 19 20 22 24 24 26 27 28 34 49

a What gender was the person who spent the most?
b Calculate the mean for each gender and state which gender has spent the most on average.
c Find the median for each gender and make a comparison.
d Calculate the range for each gender and state which one is greater.

8 Twelve people took part in the QUIT smoking program. The number of cigarettes smoked per day was recorded in week 1 of the program and in week 7.

Week 1: 21	10	36	16	32	42	9	14	21	18	34	45
Week 7: 6	24	31	21	16	19	16	18	28	32	8	13

a Calculate the mean number of cigarettes per day (to the nearest whole number) for each week and state which week has the lower average.

b Find the median for each week and make a comparison.

c Calculate the range for each week and state which one has the greater spread.

d Using your answers from part **a–c** discuss if the QUIT program has been successful or not.

9 Liz and George deliver pamphlets to letterboxes. The number of pamphlets delivered over 12 hours is shown below.

Liz:	24	25	26	27	28	28	31	32	32	32	35	35
George:	15	18	21	24	25	29	31	31	32	38	38	45

Which one of the following statements is **true**?

A On average, George delivers more pamphlets than Liz.

B The median for George is lower than that of Liz's.

C The range for George is smaller than for Liz.

D Liz's middle 50% of data has a smaller range than George's.

E None of the above.

Reasoning and communication

10 The colours of the new cars sold last week at Huxley Motors were recorded. The results are shown in the table below.

Colour	Black	Blue	Red	Silver	White
Frequency	4	7	7	9	12

a How many new cars were sold?

b What is the mode for this data?

c Why isn't it possible to find the median or mean colour?

11 The weights (in kilograms) of two teams of players were recorded.

Netball players: 77	72	80	77	62	72	82	79	58	75		
Hockey players: 81	86	64	74	92	75	73	81	64	52	82	79

a Which team has the most players?

b Which team has the lightest player?

c Find the median weight for each team and make a comparison.

d Calculate the mean weight for each team and make a comparison.

e Which summary statistic would you use to see which team has the largest total spread of weights? Calculate this for each team.

f The hockey coach states that hockey players weigh less than netball players. Do you agree or disagree with the hockey coach? Support your answer with the summary statistics already calculated.

11.02 WORKING WITH OUTLIERS

In some situations summary statistics are used when data is skewed or there are outliers.

IMPORTANT

Outliers are extreme (high or low) values in a data set and can be identified using the following calculation.

An outlier is a data value, x, which lies outside the interval:

$$Q_1 - 1.5 \times IQR \leq x \leq Q_3 + 1.5 \times IQR$$

There can be more than one outlier and they can be at one end or both ends.

To calculate Q_1 and Q_3, data must be arranged in ascending order.

Calculate the interquartile range using $IQR = Q_3 - Q_1$.

Example 4

The temperatures (°C) of a sample of hospital patients are shown below.

36 37 37 38 38 38 38 39 39 43

Identify if there are any outliers present in the data.

Solution

Find the median, Q_1 and Q_3.

36 37 37 38 38 | 38 38 39 39 43

Median = 38

$Q_1 = 37$ and $Q_3 = 39$

Calculate the IQR.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 39 - 37 \\ &= 2 \end{aligned}$$

Identify any outliers.

$$\begin{gathered} Q_1 - 1.5 \times IQR \leq x \leq Q_3 + 1.5 \times IQR \\ 37 - 1.5 \times 2 \leq x \leq 39 + 1.5 \times 2 \\ 34 \leq x \leq 42 \end{gathered}$$

43°C is an outlier as it is larger than 42°C.

IMPORTANT

Outliers need to be identified using $Q_1 - 1.5 \times IQR \leq x \leq Q_3 + 1.5 \times IQR$ and not just by visual inspection. A sentence must be written stating, for example, 43 is an outlier as it is larger than 42.

Depending on the shape of the data distribution and if outliers are present, summary statistics are more appropriate to use as a measure of centre and spread.

This is summarised in the following table.

Summary statistics	What it measures	Conditions of use
Mean	Centre	Is affected by skewed data and outliers. When investigating discrete data it may give an unrealistic answer (i.e. 2.7 children).
Median	Centre	Is not affected by skewed data and outliers.
Mode	Centre	Use when asked to find the most popular/frequent data value. (The mode is seldom used in statistics as a reliable measure of centre.)
Range	Spread	Is affected by outliers.
Interquartile range	Spread	Is not affected by outliers.
Standard deviation	Spread	Is affected by skewed data and outliers.

If the data is not skewed and no outliers are present then all measures of centre and spread are appropriate.

Example 5

The weekly mortgage repayments (in dollars) of 11 home owners are:

370 628 299 417 354 1027 585 435 509 652 481

- **a** Calculate the mean and median for the data set.
- **b** Find the difference between the mean and median.
- **c** Identify if any outliers are present in the data.
- **d** Which measure of centre is more appropriate to use and why?
- **e** Recalculate the mean and median ignoring any outliers and calculate their difference.
- **f** Using your answers to part **b** and **e**, discuss the effect of ignoring the outlier.

Solution

a Calculate the mean using the formula. Then arrange the data in ascending order to find the median.

$$\text{Mean} = \frac{5757}{11} = \$523.36$$

$299 \quad 354 \quad 370 \quad 417 \quad 435 \quad \boxed{481} \quad 509 \quad 585 \quad 628 \quad 652 \quad 1027$

Median = $481

b Difference = mean – median.

Difference = $523.36 – $481 = $42.36

c Identify the outliers.

$Q_1 = \$370$, $Q_3 = \$628$ and IQR = $258

$Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$

$370 - 1.5 \times 258 \le x \le 628 + 1.5 \times 258$

$\${-17} \le x \le \$1015$

$1027 is an outlier as it is larger than $1015.

As the data represents money repayments, it cannot be negative so the result of the outlier calculation can be written as

$\$\,0 < x \le \$1015$

d Refer to the table above.

As there is an outlier present, the median is a better measure of centre.

e Recalculate the mean, median and their difference, ignoring the outlier ($1027).

$$\text{Mean} = \frac{4730}{10} = \$473$$

$299 \quad 354 \quad 370 \quad 417 \quad 435 \,|\, 481 \quad 509 \quad 585 \quad 628 \quad 652$

Median = $458

Difference = $473 – $458 = $15

f Compare the answers and interpret the result.

The mean and median are much closer together when the outlier is ignored and therefore each statistic is a good measure of centre.

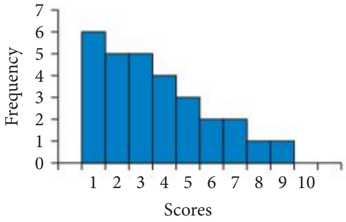
Example 6

For each of the following data sets, state the most appropriate measure of:

i centre

ii spread

a

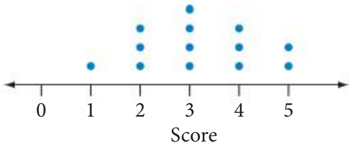


b

Stem	Leaf
1	4
2	
3	
4	
5	2
6	0 0 3
7	1 2 2 4
8	0 1 1 3 3 4 4 5
9	0 0 3 4 7 7 7 8 9 9
10	0 1 1 2 2 3 4 5 6 7 7 8

Key: 5|2 means 52 heartbeats per minute

c



Solution

a Determine the shape of the data distribution and whether outliers are present.

The data displayed in the histogram is positively skewed and has no outliers.

i The median is the most appropriate measure of centre, due to the data being skewed.

ii Either the range or the interquartile range are the most appropriate measures of spread, as there are no outliers.

b Determine the shape of the data distribution and whether outliers are present.

The data displayed in the stem-and-leaf plot is negatively skewed with one outlier.

i The median is the most appropriate measure of centre as the data is skewed and there is one outlier in the data set.

ii The interquartile range is the most appropriate measure of spread due to the presence of an outlier in the data set.

c Determine the shape of the data distribution and whether outliers are present.

The data displayed in the dot plot is approximately symmetrical and has no outliers.

i Either the mean or the median can be used as the measure of centre, as the data is approximately symmetrical and there are no outliers in the data set.

ii The range, the interquartile range or the standard deviation can be used to describe the spread of this approximately symmetrical data set.

EXERCISE 11.02 Working with outliers

Concepts and techniques

1 Example 4 For each of the following data sets identify if any outliers exist.

a Shoe sizes of a group of Year 11 students.

6 6 7 7 7 8 8 9 9 13

b Points scored per match for a football team.

20 45 57 64 72 89 43 52 67 68 79 84 49 50 68
58 65 73 82 61 70

c The number of kilograms lost by a group of people.

1.8 2.5 4.3 6.5 2.7 4.6 11.0 10.8 0.3 8.2 2.0 3.8 4.4 5.8 1.6
5.9 7.6 9.3 4.8 3.4 12.5 4.6 2.5 6.9 7.5 3.5 4.8 12.2 4.3 3.7
0.0 0.9 2.6 7.8 4.9 7.4 9.8 10.4 2.6 8.2

d Maths test scores (%) in a Year 11 class.

12 43 56 51 65 53 55 49 91 63 60 55 54 48 61
54 56 52 60 61 52 98

2 Example 5 The number of cups of coffee drunk by a sample of external examination markers in one night were:

3 3 3 3 4 4 4 4 4 4 5 5 5 5 5
5 5 5 5 8

a How many markers were surveyed?

b Identify if there are any outliers in the data.

c What is the mean if the outlier:

i is included? **ii** is not included?

d If the outlier is included, what effect does this have on the mean number of cups of coffee that were drunk?

3 The dot plot below shows the shoe sizes of a sample of Year 11 students.

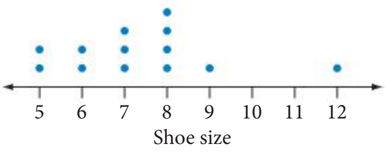
a For this data, find:

i the mean **ii** the median

iii the mode.

b Identify if any outliers are present in the data.

Shoe sizes of Year 11 students



c If the outlier is removed, state what will happen to:

i the mean   ii the median   iii the mode.

d A shoe store needs to buy more shoes for a back-to-school sale. Should they use the mean, median or mode as an indication of what to order? Justify your answer.

4 The employees at the Bread and Butter Café earned the following wages in a week.

$450 $520 $570 $230 $800 $420 $560

a What is the mean and median wage?

b Which of the above is the more accurate reflection of the data? Justify your answer.

c What is the range and interquartile range for the set of wages?

d Which of the above is the more accurate reflection of the data? Justify your answer.

e The manager of the store earns the most and his wage is considered to be an outlier. State how this supports your answers to parts **b** and **d** and prove why the manager's wage is an outlier.

f If the manager's wage is not included, how does this affect the mean and median wage?

5 A group of friends go to the movies. The ages of the friends are:

13   12   11   14   12   15   14   13

If Sally brings her 5-year-old sister as well:

A the median age will increase

B the median age will decrease

C the mean age will increase

D the mean age will decrease

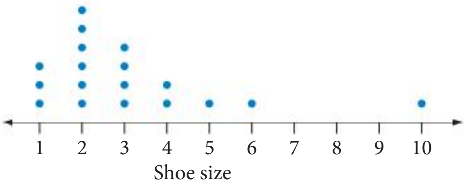
E none of the above

6 Example 6 For each of the following data sets, state the most appropriate measure of:

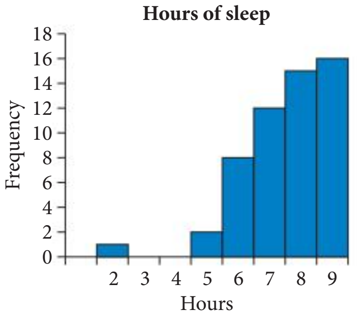
i centre   ii spread.

Justify each of your selections.

a



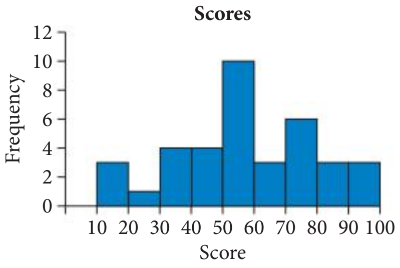
b



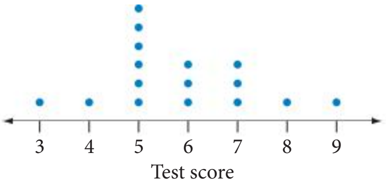
c Key: 4|3 means 43%

Stem	Leaf										
4	0	1	2	3	5	6	7	8	9	9	9
5	0	2	2	4	8	8	9				
6	1	3	3	3	6						
7	5	7	8								
8	4	6									
9	2										

d



e



f Key: 9|1 means 91 seconds

Stem	Leaf
9	1 5 7 9
10	2 4 5 6 6 8
11	0 2 2 3 4 4 5
12	1 2 3 3 3 7 9
13	2 3 4 5 7 7
14	3 6 9
15	0 1 2

Reasoning and communication

7 The following data represents the scores (out of 10) achieved by students competing in a spelling bee.

3 6 8 10 10 9 7 4 6 10 9 9 8 4 6
8 7 9 9 10 10 7 7 8 8 9 9 10 10 9

- **a** What percentage of students scored more than 8?
- **b** Construct a dot plot to represent this data.
- **c** Use the dot plot to determine the shape of the data distribution.
- **d** Based on your answer to part **c**, calculate the most appropriate measure of centre.
- **e** Determine if any outliers are present in the data.
- **f** Based on your answer to part **e**, what is the most appropriate measure of spread?

8 Rupert's bookstore employs the following people with annual wages as shown.

1 store manager	$74 300
2 cashiers	$34 200 each
2 part-time clerical staff	$28 500 each
3 salespeople	$46 500 each
2 part-time cleaners	$13 500 each

- **a** How many employees work for Rupert's bookstore?
- **b** Find the following statistical measure and explain what it tells you about the data:
  - **i** mean **ii** median **iii** mode.
- **c** Which of the above measures would Rupert use to make the salaries appear higher? Why?
- **d** Which of the above measures best represents the annual wages for an employee at Rupert's bookstore? Why?

9 Pam and Percy sell copiers. The numbers of copiers each person sold per week are sorted in ascending order.

Pam:	1	2	3	3	5	6	7	8	12	25
Percy:	3	3	3	14	16	18	18	24	32	35

- **a** Construct two stem-and-leaf plots, one for Pam and one for Percy.
- **b** What is the modal number of copiers sold by each person?
- **c** What would you conclude about each person if you only knew the modes?
- **d** Determine the shape of the data distribution and find the outliers for each person.
- **e** Find the most appropriate measure of centre for each person.
- **f** Find the most appropriate measure of spread for each person.
- **g** Do these findings support your conclusion from part **c**? Justify your answer.

11.03 USING THE MEAN AND STANDARD DEVIATION TO DESCRIBE AND COMPARE DATA

When data is not skewed and has no outliers, the mean and standard deviation are both appropriate statistical measures for making comparisons.

IMPORTANT

For two related sets of data, a report can be written comparing the:

mean or average: this needs to be interpreted in relation to what the data represents. For example, if the data is test scores, then the set with the larger average has achieved higher marks, whereas if the data is time to complete a race, then the set with the lower average are the better runners.

standard deviation: the set with the higher standard deviation is considered to have a greater spread about the mean, therefore it is more variable. The set with the smaller standard deviation has its values more closely clustered around the mean, therefore the data is more consistent.

Example 7

The following scores were obtained by students from the same class in Maths and English.

English: 13 14 16 12 8 6 15 18 12 14 13 11 10
9 7 9 12 8 9 7 10 10 9 11 13

Maths: 5 2 9 7 9 12 8 9 7 10 10 9 11
18 11 14 16 17 8 6 20 18 12 4 6

a Calculate the mean and standard deviation for each subject.

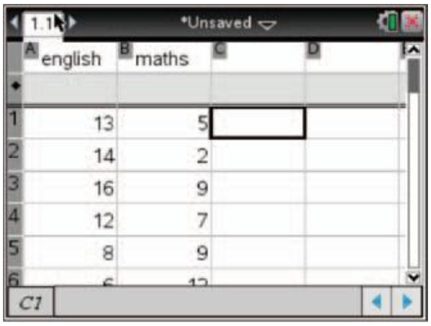
b Interpret and compare the mean for each subject.

c Interpret and compare the standard deviation for each subject.

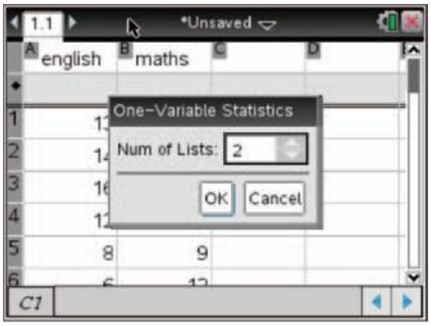
Solution

a Calculate the mean and standard deviation for both subjects.

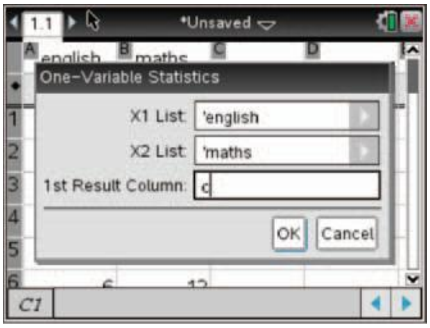
TI-Nspire CAS



	A english	B maths	C	D
◆				
1	13	5		
2	14	2		
3	16	9		
4	12	7		
5	8	9		



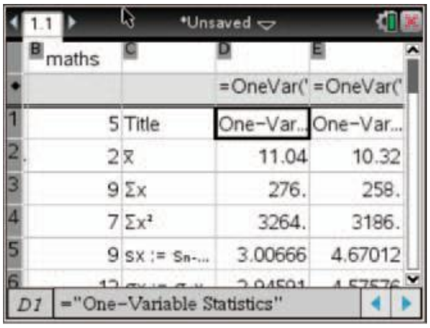
Num of Lists: 2



X1 List: 'english

X2 List: 'maths

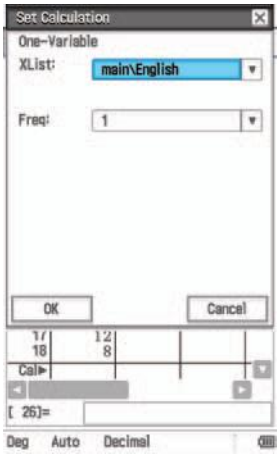
1st Result Column: c



B maths	C	D	E
		=OneVar('	=OneVar('
5	Title	One-Var...	One-Var...
2	$\bar{x}$	11.04	10.32
9	Σx	276.	258.
7	Σx²	3264.	3186.
9	sx := Sn-...	3.00666	4.67012

D1 ="One-Variable Statistics"

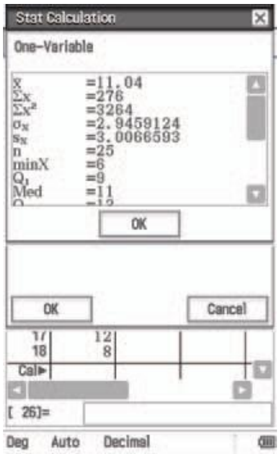
ClassPad



One-Variable

XList: main\English

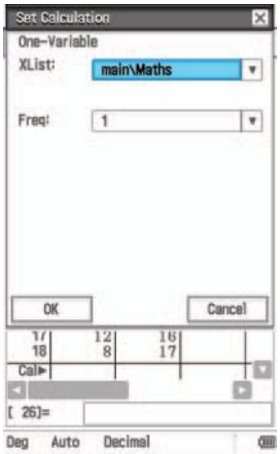
Freq: 1



Stat Calculation

One-Variable

```
x̄      =11.04
Σx     =276
Σx²    =3264
σx     =2.9459124
sx     =3.0066593
n      =25
minX   =6
Q1     =9
Med    =11
```

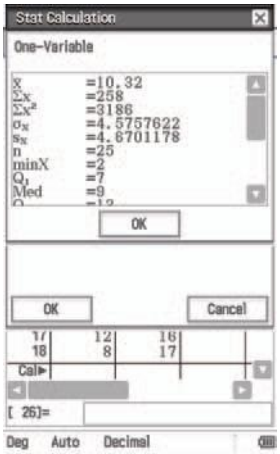


Set Calculation

One-Variable

XList: main\Maths

Freq: 1



Stat Calculation

One-Variable

```
x̄      =10.32
Σx     =258
Σx²    =3186
σx     =4.5757622
sx     =4.6701178
n      =25
minX   =2
Q1     =7
Med    =9
```

Means: English = 11.04 and Maths = 10.32

Standard deviations: English = 3.01 and Maths = 4.67

b Interpret the means.

English has a higher average, therefore the class has performed better in English.

c Interpret the standard deviations.

Maths has a higher standard deviation, which means that the scores in Maths had a greater spread than in English. The English results were more consistent.

Calculating and interpreting summary statistics

EXERCISE 11.03 Using the mean and standard deviation to describe and compare data

Concepts and techniques

1 Example 7 Select the data set (A or B) that has the higher mean.

a A: 101 99 80 98 90 100 114 93 109 100 80 92 91 120 103
B: 98 104 89 93 118 80 120 120 101 112 106 119 117 80 117

b A: 56 30 30 60 35 38 40 54 45 48 41 53 39 45 48 55 46 37
B: 40 48 30 60 45 44 45 41 51 33 37 41 38 42 38 33 32 49

c A: 110 113 127 108 130 106 139 69 95 70 85 119 106 99 84
B: 53 106 90 112 109 65 88 100 91 170 39 28 103 59 88

d A: 9.1 11.8 9 12 9.8 9.7 9.4 10.8 10.5 9.4
9.9 12 10.5 11.5 9.8 11.2 9.5 9 10.7 9
B: 11.9 10.6 9 12 11.6 9 11.4 11.7 10.5 11.3
12 10.9 10.6 12 11.8 10.7 12 11.5 11.1 12

2 Select the data set (A or B) that has the greater spread.

a A: 20.7 20.6 22 19.2 21.1 19.5 20.8 20.6 18 19 20.1 19.3
B: 19.2 18.1 18.7 18 20.9 22 22 19.9 19.9 20 19.1 18.7

b A: 36.1 51.2 56 33.7 51.6 53.9 59.8 36.4
50.3 44.2 40.6 55.9 79.9 52.5 50.8
B: 37.1 35.7 80.9 33.5 58.9 54.1 34.9 48.5
9 23 47.1 41.4 31.5 54.2 36.6

3 Select the data (A or B) that is the most consistent.

a A: 188 213 185 255 209 238 214 191 249 237
228 243 231 216 240 226 227 238 212 227
B: 185 236 185 255 211 204 226 190 185 192
219 254 215 203 243 247 239 255 210 227

b A: 54 43 71 67 31 32 50 26 54 56 31 42 21 49 53 35
B: 38 21 48 40 42 48 44 41 58 60 71 38 39 54 59 59

The following information relates to questions **4** and **5**.
Two Year 11 PE classes had a competition to see how many push-ups each student could complete. The results are shown below.

Class A: 1 3 3 6 8 9 10 11 11 12 12
16 17 19 20 20 21 22 25 29 30 35

Class B: 1 5 5 8 9 10 11 11 12 13 14
16 18 20 21 21 21 25 25 30 30 37

4 The average number of push-ups for Classes A and B respectively are:

A 16.5 and 15.46
B 15.46 and 16.5
C 16.5 and 15.45
D 15.45 and 16.5
E none of the above

9780170250252

5 Which one of the following statements is **false**?

A Class A on average does less push ups than Class B.

B Class A's results are more clustered around the mean than Class B.

C The mean for Class B is larger than Class A.

D Class B's results are more clustered around the mean than Class A.

E Class A's results have a slightly larger spread than Class B.

Reasoning and communication

6 The results of fifteen students who do both Art and Music are shown below.

Art:	38	28	56	35	40	17	53	29	80	60	40	32	53	41	47
Music:	63	52	64	58	48	56	34	60	48	55	61	46	49	53	43

a Calculate the mean and standard deviation for each subject, correct to one decimal place.

b Discuss which subject achieved the better results.

c Discuss which subject achieved the most consistent results.

7 Sue and Michelle recorded the number of glasses of water they drank over a two-week period. The results are shown below.

Sue:	4	9	8	5	6	4	6	7	5	8	6	7	5	6
Michelle:	9	3	6	4	5	7	5	8	6	8	5	9	6	5

a Calculate the mean number of glasses of water drunk by each person, correct to one decimal place.

b What would you conclude about the amount of water drunk by each person if only given the mean?

c Construct a dot plot to represent the data for each person.

d Does the spread of each dot plot support your conclusion from part **b**?

e Calculate the standard deviation for each data set.

f Conclude who drinks the most consistent amount of water per day.

8 A large catering company that specialises in supplying nursing homes and other residential hostels with hot meals buys its vegetables directly from producers. It has samples of brown onions from two farmers. The masses (grams) of the onions for each farmer are shown below.

Farmer Moe:	201	212	263	253	73	292	190	198	200	289
	321	122	348	374	129	143	96	208	187	206
Farmer Joe:	189	282	162	165	152	218	161	312	122	192
	199	190	127	204	195	140	156	205	133	198

a Calculate and compare the mean weight of onions for each farmer, correct to one decimal place.

b Calculate and compare the standard deviation of onions for each farmer, correct to one decimal place.

c Which farmer has the most consistent size of onion? Justify your answer.

d The catering company will buy the onions in 5 kg bags. Justify which farmer the company should select.

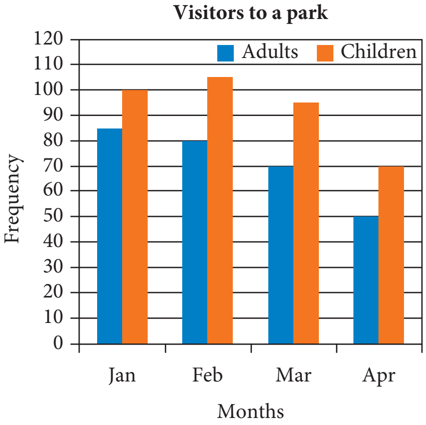
11.04 SIDE-BY-SIDE COLUMN GRAPHS

Side-by-side column graphs are used to represent two sets of categorical data so that the frequency of each can be compared. For example, the number of adult visitors compared to child visitors at a park per month.

The following table displays the number of adult visitors and child visitors to a park over the first four months of the year.

Month	Adults	Children
Jan	85	100
Feb	80	105
Mar	70	95
Apr	50	70

The corresponding side-by-side column graph is:



The key features of a side-by-side column graph are:

1 sets of two joined columns, but with gaps between each set
2 each column's height is determined by the frequency stated in the table
3 columns for each data set must be shaded differently with a matching key.

A side-by-side column graph can be analysed by visual inspection and a report can be written to describe similarities and differences between the two sets of data. For the example above, the following could be stated:

- more children visited the park every month than adults
- January and February had equal numbers of visitors and were the most popular months to visit the park.
- In April, the park had the least number of visitors.

Example 8

The data in the following table represents the final scores for the AFL Anzac Day clash over the last four years.

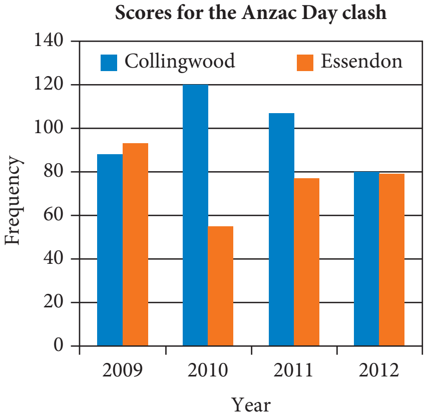
Year	Collingwood	Essendon
2009	88	93
2010	120	55
2011	107	77
2012	80	79

a Display this data using a side-by-side column graph.

b Write a report comparing the final scores for the AFL Anzac Day clashes over the last four years.

Solution

a Set up axes allowing for two columns for each year on the horizontal axis. The vertical axis must extend past the highest score of 120. Construct a column for each score.



b Write a report which describes the information presented in the graph.

Overall, Collingwood has won more, 3 out of 4, of the matches and have also won by the biggest margin. The clash in 2012 was the closest match.

EXERCISE 11.04 Side-by-side column graphs

Concepts and techniques

1 Example 8 The number of days that it rained was recorded for each season of a year. The results are displayed in the table below.

Season	Days of rain	Days of no rain
Summer	30	60
Autumn	50	42
Winter	45	47
Spring	30	61

Shutterstock.com/Alexander Shadrin

a Display this data using a side-by-side column graph.

b Write a report which describes the information presented in the graph.

2 The number of spectators at Rugby League and Rugby Union matches is recorded in the following table.

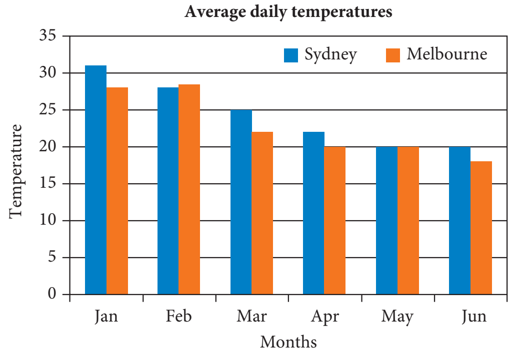
- a Display this data using a side-by-side column graph.
- b Write a report which describes the information presented in the graph.

Match number	Rugby Union	Rugby League
1	7000	12 000
2	6500	18 500
3	9000	23 000
4	11 000	17 500
5	7500	25 500
6	8000	31 000

3 The graph on the right shows the average daily temperatures in Sydney and Melbourne in a certain year.

Which one of the following statements is **true**?

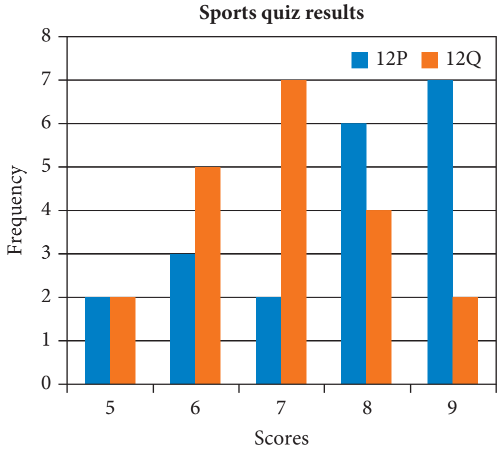
- A In general, Melbourne has higher average temperatures than Sydney.
- B Melbourne had the same average temperature in April and May.
- C Sydney's hottest month was March.
- D Sydney and Melbourne had the same average temperature in May and June.
- E Melbourne's highest average temperature was 31°C.



Reasoning and communication

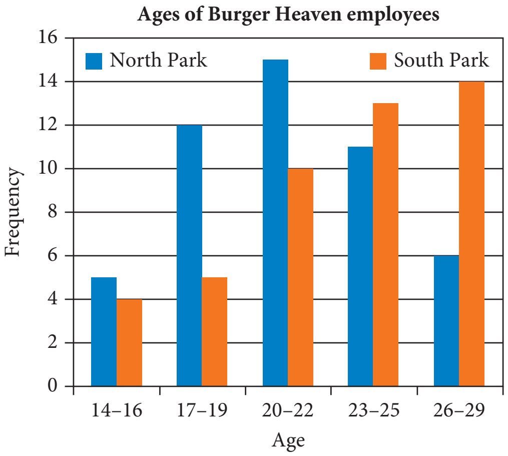
4 Two Year 12 classes, 12P and 12Q, were given a sports quiz, and the student's scores are shown in the graph on the right.

- a How many students were there in 12P?
- b How many students were there in 12Q?
- c What was the average score for each class?
- d What percentage of students in 12Q got a score of 7?
- e 12P had more students with a score of 9 than in 12Q. Find how many more, expressing your answer as a percentage.
- f Write a report which describes the information presented in the graph.



5 The ages of employees at Burger Heaven were compared across two stores: North Park and South Park. The results are shown in the graph on the right.

- a Which store has the most employees?
- b What was the modal age group at each store?
- c Which store employs more young people? Justify your answer.
- d Write a report which describes the information presented in the graph.



11.05 BACK-TO-BACK STEM-AND-LEAF PLOTS

A back-to-back stem-and-leaf plot is a stem-and-leaf plot with a shared stem but two sets of leaves, one to the right of the stem and one to the left of the stem, each representing a different set of data. They are used to display numerical data for two different groups (categories). For example, test scores for class A and class B.

Key: 5|6 means 56%

Class A	Stem	Class B
4 3 2	5	6
8 5 3 4 1	6	0 0 2 4
9 6 4 4 3 1 0	7	3 5 6 7 8 8 8 9
8 8 7 5 3	8	0 1 4 6
8 6 3	9	2 3 5 6 7 8

Each row of leaves must be ordered from the centre out.

There must be a title for each set of leaves indicating the group it represents.

Don't forget the key.

Example 9

Mr Mathis gave his class two tests and the results were:

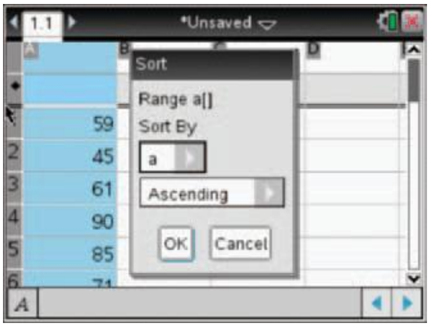
Test 1:	59	45	61	90	85	71	63	72	82
	85	87	81	51	53	70	54	78	62
Test 2:	41	52	50	67	51	74	55	67	80
	51	84	49	60	70	51	64	72	49

Display this information in a back-to-back stem-and-leaf plot.

Solution

Start with the Test 1 results creating a stem-and-leaf plot. Remember that leaves must be ordered. A CAS can be used to order the data.

TI-Nspire CAS



Sort

Range a[]

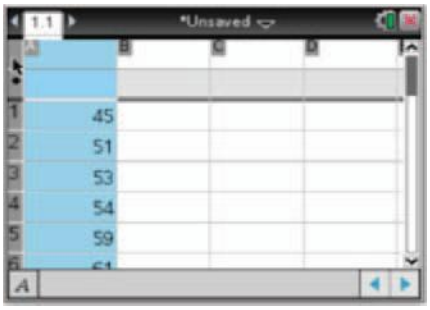
Sort By a

Ascending

A
59
45
61
90
85

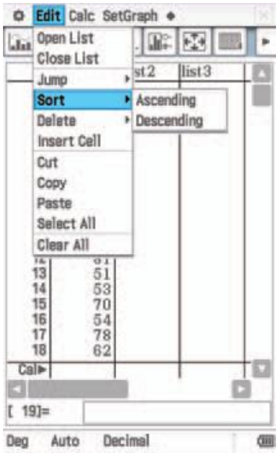
Key: 4|5 means 45%

Stem	Test 1
4	5
5	1 3 4 9
6	1 2 3
7	0 1 2 8
8	1 2 5 5 7
9	0



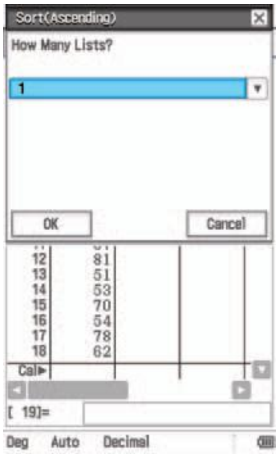
1	45
2	51
3	53
4	54
5	59

ClassPad



13	51
14	53
15	70
16	54
17	78
18	62

[19]=

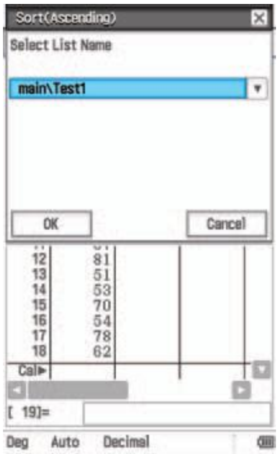


How Many Lists?

1

12	81
13	51
14	53
15	70
16	54
17	78
18	62

[19]=

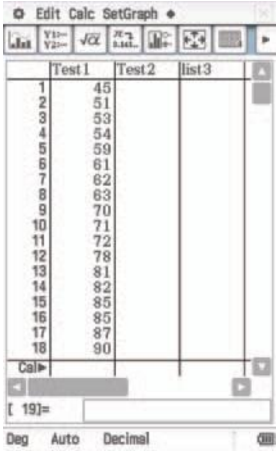


Select List Name

main\Test1

12	81
13	51
14	53
15	70
16	54
17	78
18	62

[19]=



	Test1	Test2	list3
1	45		
2	51		
3	53		
4	54		
5	59		
6	61		
7	62		
8	63		
9	70		
10	71		
11	72		
12	78		
13	81		
14	82		
15	85		
16	85		
17	87		
18	90		

[19]=

Add an additional leaf column onto the left side of the stem and add the ordered data for Test 2.

Key: 4|5 means 45%

Test 2	Stem	Test 1
9 9 1	4	5
5 2 1 1 1 0	5	1 3 4 9
7 7 4 0	6	1 2 3
4 2 0	7	0 1 2 8
4 0	8	1 2 5 5 7
	9	0

When writing a report which compares two sets of data:

1. use a graphical display of the data to describe the shape of the distribution and determine if any outliers exist
2. based on your findings from step 1, select and calculate the appropriate measures of centre and spread
3. compare each summary statistic calculated in step 2 in relation to the data

If all measures of centre and spread are appropriate, make sure that you calculate the same measure for each set of data. There is no point comparing the mean of one set to the median of the other.

4 compile a report by making a general conclusion about the data set (i.e., boys are generally taller than girls), then support this conclusion by summarising the findings from all of the steps above.

A report that compares data should not just state values; it should discuss what is higher, lower, similar or different between the two data sets.

Example 10

The following back-to-back stem-and-leaf plot shows the number of mobile phones sold in January across various stores in two different Australian states.

Key: 2|2 means 22 mobile phones sold

Victoria	Stem	New South Wales
9 9 2 2 1	2	2 6
5 2 1 1 1 0	3	0 1
9 8 7 7 4 0	4	4 8
9 4 2 0	5	2 6 9
4 4 2 1 1 0	6	1 3 4 5
	7	0 2 3 4 4 5 5 7 7 7 8 9
	8	3 5 7 7 8 8 8 8 9
9	9	2 8

a Describe the shape of the distribution of each of the data sets.

b Determine if outliers exist for either data set.

c Hence determine and calculate the most appropriate summary statistics to represent the data.

d Compare each of the statistics calculated in part **c** in the context of the question.

e Using all the previous answers, compile a report comparing mobile phone sales in Victoria and New South Wales.

Solution

a The stem-and-leaf plot is like a histogram on its side. Consider Victoria's data first.

Victoria's data appears to be approximately symmetrical with the value of 99 being a possible outlier.

Now consider the data for NSW.

The data for New South Wales is negatively skewed.

Stem	New South Wales
2	2 6
3	0 1
4	4 8
5	2 6 9
6	1 3 4 5
7	0 2 3 4 4 5 5 7 7 7 8 9
8	3 5 7 7 8 8 8 8 9
9	2 8

For NSW, the tail on the left extends further than the tail on the right.

b Determine whether there are any outliers by using the formula.

Victoria

$Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$

$31 - 1.5 \times 28.5 \leq x \leq 59.5 + 1.5 \times 28.5$

$-11.75 \leq x \leq 102.25$

99 is not an outlier as it is smaller than 102.25.

NSW

$Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$

$60 - 1.5 \times 26 \leq x \leq 86 + 1.5 \times 26$

$21 \leq x \leq 125$

There are no outliers.

c Determine the most appropriate summary statistics and calculate them.

NSW has skewed data therefore the median will be used as the measure of centre and the IQR will be used for the measure of spread. The mode is also useful in making comparisons.

	Victoria	NSW
Median	47	74.5
IQR	28.5	26
Mode	31	88

d Compare the medians.

Median: NSW have a higher median than Victoria, indicating that they sold more phones in January.

Compare the IQRs.

IQR: The results are very similar, indicating that the middle 50% of sales for both states was spread nearly the same; although the NSW middle 50% ranged from 60 to 86 whereas Victoria's ranged from 31 to 59.5.

Compare the modes.

Mode: The most common number of phones sold was much higher for NSW (88) than Victoria (31).

e Write a report comparing the data sets for NSW and Victoria.

In general, NSW sold more mobile phones in the month of January than Victoria. This is supported by NSW having a median that is 27.5 phones higher than Victoria and a mode that is much higher. Even though the IQR was of a similar spread for both states, the middle 50% for NSW was located at higher values than Victoria. This is due to the fact that the NSW data is negatively skewed, indicating that most of its values are at the higher end.

EXERCISE 11.05 Back-to-back stem-and-leaf plots

Concepts and techniques

1 Example 9 The heights (in cm) of students were taken from 2 classes, Class A and Class B. Display this information using a back-to-back stem-and-leaf plot.

Class A:	158	169	177	147	160	153	167	164	171	159	162	166	172
	148	157	176	155	170	149	150	161	163	177	168	157	159
Class B:	181	166	179	181	164	173	168	165	176	181	157	162	178
	182	157	172	180	173	159	151	168	165	175	169	167	158

2 Two radar cameras positioned on different roads recorded car speeds (in km/h) as follows. Display this information in a back-to-back stem-and-leaf plot.

Camera 1:	85	66	75	69	72	83	80	69	74	77	73	74
	90	84	65	73	69	89	76	103	83	78	69	70
Camera 2:	122	142	120	118	116	135	140	123	135	124	120	119
	138	131	122	119	125	130	130	133	123	148	169	130

3 The parents of some students in Year 11 have the following ages. Display this information in a back-to-back stem-and-leaf plot.

Mothers' ages:	38	43	35	52	55	57	47	49	39	44	46	43
	48	44	40	51	53	36	42	49	52	39	44	
Fathers' ages:	41	46	38	55	58	57	48	48	40	49	45	49
	52	48	43	57	61	38	42	51	54	41	44	

4 Example 10 Using the back-to-back stem-and-leaf plot constructed in question **1**, complete the following questions.

a Describe the shape of the data sets for each class.

b Determine if any outliers exist for each class.

c Determine and calculate the most appropriate summary statistics to represent the data.

d Compare the statistics calculated in part **c** in context of the question.

e Using all the previous answers, compile a report comparing the heights of students in Class A and Class B.

5 Using the back-to-back stem-and-leaf plot constructed in question **2**, complete the following questions.

a Describe the shape of each of the data sets for each radar camera.

b Determine if any outliers exist for each radar camera.

c Determine and calculate the most appropriate summary statistics to represent the data.

d Compare each of the statistics calculated in part **c** in context of the question.

e Using all the previous answers, compile a report comparing car speeds recorded by the two cameras.

6 Write a report comparing the parents' ages of the Year 11 students from question **3**.

The following information relates to questions **7** and **8**.

The back-to-back stem-and-leaf plot shows the relative humidity at 3 p.m. in two towns over a 2-week period.

Key: 4|3 means 43%

Town A	Stem	Town B
5 3	3	
8 8 5 5	4	3 5 8
8 7 6 6	5	8 9
8 7 7	6	5 7 9 9
	7	8 8 9
4	8	4 9

7 Which one of the following statements is **true**?

A Town B's median humidity is 67%.

B Town B has a larger range than Town A.

C Town A experienced the highest percentage humidity.

D Town A has more rain than Town B.

E Town B has a larger IQR than Town A.

8 Which one of the following statements is **false**?

A The average humidity for Town B is higher than Town A.

B Ignoring the possible outlier, Town A's data is approximately symmetrical.

C The median humidity for Town A and B are 68 and 56 respectively.

D Town B experienced the highest percentage humidity.

E Town B had a smaller percentage of humidities in the 60s than Town A.

Reasoning and communication

9 The numbers of words per sentence in a computer magazine article were:

10 28 31 17 23 27 18 15 26 24 20 19
36 27 14 25 15 22 11 21 24 27 17 29

The numbers of words per sentence in a newspaper article were:

27 39 33 24 28 19 32 41 33 27 35 12
38 41 27 13 22 23 18 46 32 22 18 32

a Construct a back-to-back stem-and-leaf plot showing both articles.

b Copy and complete the following table.

	Computer magazine article	**Newspaper article**
Range		
Median		
IQR		
Mean		

c Describe the shape of the data for each article and select the appropriate measures of centre.

d Write a report comparing the number of words in each article.

10 The following back-to-back stem-and-leaf plot displays the numbers of mammals found in two separate surveys of random areas of forest.
Key: 2|5 means 25 mammals

Survey 2	Stem	Survey 1
	2	2 5 6
5 4 2	3	4 8 9
8 7 6 3 3	4	0 0 1 3 7
6 5 4 4 2 2	5	1 1 2 3 4 4 7 8
5 5 5 5 2 0	6	7 8 8 9 9
9 8 5 2 1 1 0	7	0 1 8 8 8
6 1	8	2 5 6
8 7 4 0	9	1

a Describe the distribution shape of the data collected in Survey 1 and Survey 2.

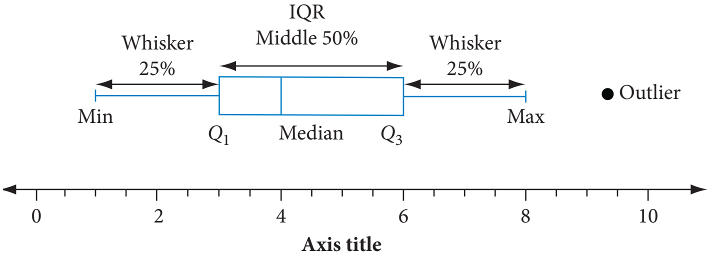
b Calculate all the relevant summary statistics for each survey and display the information in a table.

c Write a report about the number of mammals found in the forest for each of the two surveys.

Oxygen Group Photography/David M. Barron

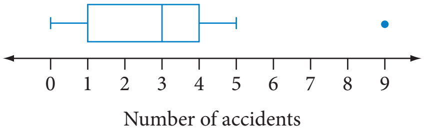
11.06 CONSTRUCTING AND INTERPRETING PARALLEL BOXPLOTS

From a boxplot, various summary statistics can be read and interpreted. They are the median, maximum and minimum values, from which the range can be determined, and Q_1 and Q_3, from which the interquartile range can be found. Outliers are easily identified from a boxplot. Boxplots split the data into quarters, which means that each section of the boxplot represents 25% of the data. The rectangular box section of the plot represents the IQR or the middle 50% of the data.



Example 11

The boxplot below represents the number of accidents per month at a factory over one year.



a State the median. What does this tell you about the data?

b Calculate the range, excluding the outlier, and interpret this value.

c Calculate the IQR and interpret this value.

d What is the range of the bottom 25% of the data?

e What percentage of accidents per month were between 3 and 4?

f What percentage of the accidents per month were 4 or less?

Solution

a The median corresponds to the vertical line in the box.

Median = 3 accidents

Half of the values lie below the median and half lie above the median.

For half of the months there were less than 3 accidents and for the other half there were more than 3 accidents.

b Read the minimum and maximum values from the ends of the whiskers.

Range = 5 – 0 = 5 accidents

The range is not an appropriate measure of spread when outliers are present, which is why the question has asked for the outlier to be excluded from the calculation.

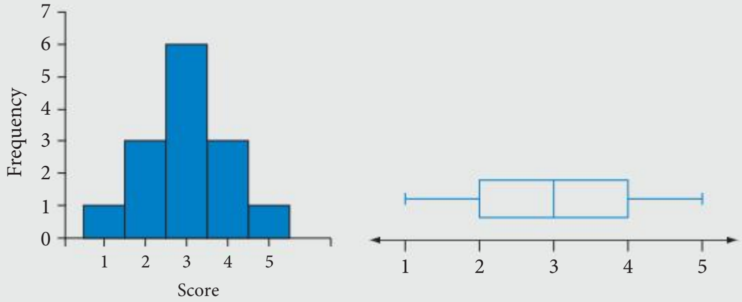
	The range is the difference between the highest value of the data and the lowest value of the data.	When the outlier is excluded, the difference between the maximum and minimum accidents per month over the year is 5.
c	Read Q_1 and Q_3 values from boxplot. Q_1 corresponds to the left end of the box and Q_3 corresponds to the right end of the box.	IQR = 4 − 1 = 3 accidents
	Interpret the results. The IQR represents the spread of the middle half of data values.	The middle 50% of the data has a range of 3 accidents.
d	The bottom 25% of data is represented by the whisker to the left of the box. Read off the values, from left to right, at the ends of this whisker.	Between 0 and 1 accident per month.
e	3 corresponds to the vertical line within the box and is therefore the median. 4 corresponds to the vertical line at the right end of the box and is therefore Q_3. 25% of the data lies between the median and Q_3.	Median = 3 $Q_3 = 4$ 25% of the accidents were between 3 and 4 per month.
f	75% of the data lies below Q_3.	On 75% of months in the year there were 4 or less accidents.

As with histograms, dot plots and stem-and-leaf plots, a boxplot gives a visual display of a data set and therefore can be used to describe the shape of the distribution of the data. It can be categorised as positively skewed, negatively skewed or symmetrical. Based on the shape of the boxplot and the presence of outliers, the appropriate measure of centre and spread can be selected.

Since a boxplot gives a summary of the data, the median is the only measure of centre available if the actual data is not given. The range and interquartile range are appropriate measures of spread for data that is symmetrical with no outliers. If outliers are present or the data is skewed, then the interquartile range is a more appropriate measure of spread.

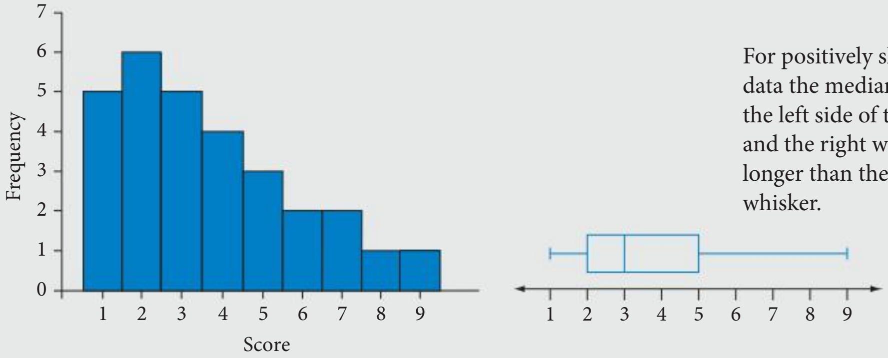
IMPORTANT

Symmetrical



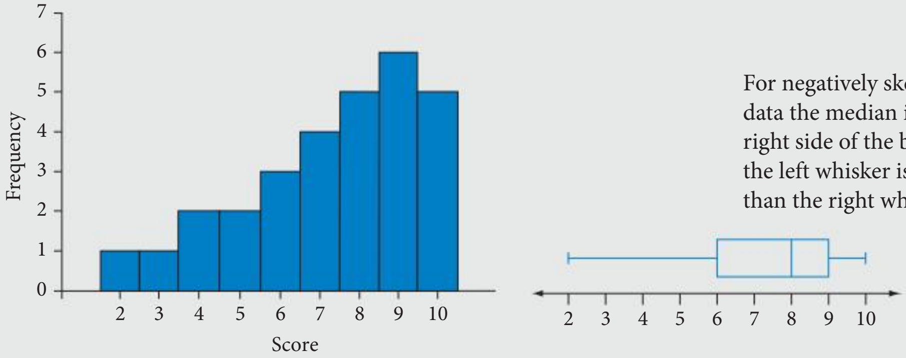
For symmetrical data the median is in the centre of the box and the whiskers are the same length.

Positively skewed



For positively skewed data the median is to the left side of the box and the right whisker is longer than the left whisker.

Negatively skewed



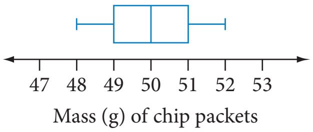
For negatively skewed data the median is to the right side of the box and the left whisker is longer than the right whisker.

Some boxplots may not exactly match the shapes above. If this is the case, we choose the shape that the boxplot most closely resembles and use the word approximately when describing the shape of the boxplot, for example approximately symmetrical.

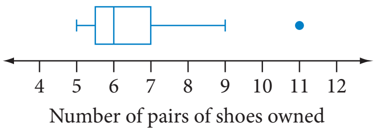
Example 12

For each of the following boxplots state the shape of the distribution of the data and hence state the most appropriate measures of centre and spread.

a



b



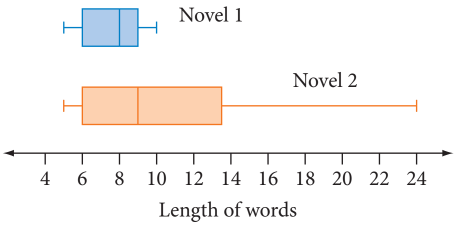
Solution

a Determine the distribution shape from the boxplot. The median is in centre of the box and each whisker is the same length.	The distribution of the data is symmetrical.
Determine the most suitable measures of centre and spread based on the shape of the distribution.	The median is the only measure of centre that can be obtained from the boxplot. Since the data is symmetrical, without outliers, both the range and interquartile range are appropriate measures of spread.
b Determine the distribution shape from the boxplot. The median is to the left side of the box and the right whisker is longer than the left whisker.	The boxplot is positively skewed with an outlier.
State the appropriate measures of centre and spread.	The median is the most appropriate measure of centre and the IQR is the most appropriate measure of spread.

Often multiple boxplots will appear on the one scale. They are called parallel boxplots and are used to display numerical data across different categories. Each individual boxplot can be interpreted but comparisons between each boxplot can also be made. This is usually done by comparing the centre or median of each data set and the spread of the data sets by comparing interquartile ranges.

Example 13

The following parallel boxplots show the number of words counted in the sentences of two English novels.



- **a** What is the median word length for Novel 1 and what does it mean?
- **b** What is the maximum word length for Novel 2?
- **c** Without performing any calculations, which novel has the largest IQR?
- **d** What is the same for each novel?
- **e** Describe the shape of the data distribution for Novel 2.

Solution

a Use the top boxplot labelled Novel 1 and read the value from the horizontal axis that corresponds to the vertical line within the box.

The median is 8, which means that 50% of the sentences are shorter than 8 words and 50% are longer.

b Read from the bottom boxplot labelled Novel 2. Read the value from the horizontal axis corresponding to the end of the right whisker.

The maximum sentence length is 24 words.

c The boxplot with the longer box has the larger interquartile range.

Novel 2 has the larger IQR.

d The left-hand whisker for each boxplot starts at the same value and ends at the same value.

The minimum number of words per sentence in each novel was 5. $Q_1 = 6$ for both novels so in each novel 25% of sentences had 5 or 6 words.

e Describe the distribution shape. The median is to the left side of the box and the right whisker is longer than the left whisker.

The data for sentence length in Novel 2 is positively skewed.

Example 14

The data below shows the times (seconds) taken by a group of 14-year-old boys and girls to run 50 m.

Boys: 8.3 8.1 8.1 7.8 8.8 8.1 7.4 8.0 7.8 9.1 8.2 8.6 8.1
8.2 7.6 8.0 7.6 7.9 8.8 8.5 8.3 8.3 8.3 8.5 8.2 8.5
8.0 8.0 8.6 7.9

Girls: 8.4 8.3 8.8 8.2 8.9 9.7 8.5 8.4 8.8 9.0 8.7 8.3 8.1
8.4 9.0 8.6 8.9 8.4 9.3 9.1 8.5 8.4 9.5 8.5 8.5 8.5
9.7 8.9 8.6 8.4

a Construct parallel boxplots to represent the data.

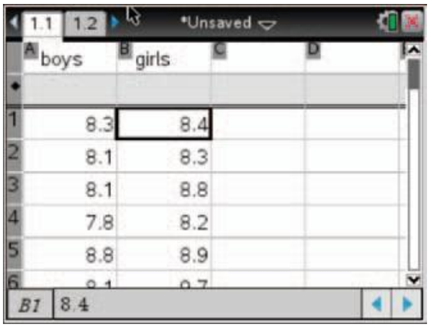
b Compare the performances of the boys and girls.

Solution

a Use CAS to create the boxplots.

TI-Nspire CAS

Open a New Document with a Lists & Spreadsheet page.
Type each set of data into a column.



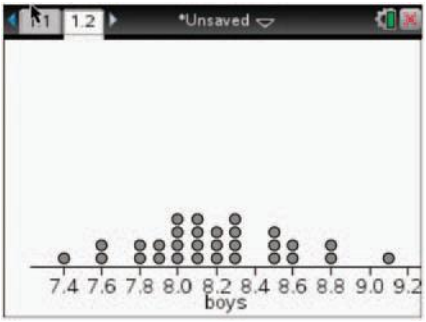
	A boys	B girls	C	D
1	8.3	8.4		
2	8.1	8.3		
3	8.1	8.8		
4	7.8	8.2		
5	8.8	8.9		

B1 8.4

Add a Data & Statistics page.

Click to add variable space at the bottom of the page.

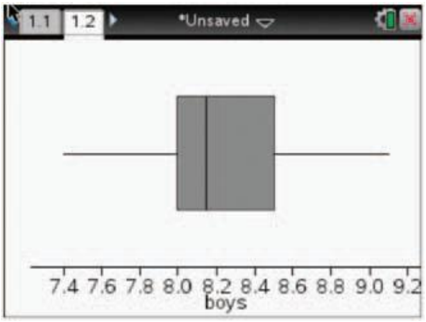
Select boys.



Press menu

1: Plot Type

2: Box Plot

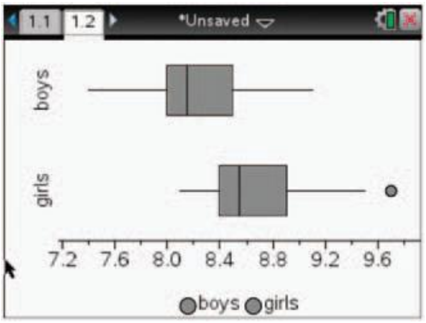


Press menu

2: Plot Properties

5: Add X Variable

Select girls.



Press menu

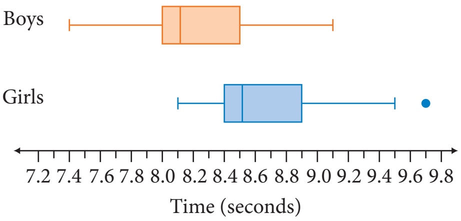
4: Analyze

A: Graph Trace

Press ▶ to read the five-number summary from each boxplot. Identify the value of any outliers.

	Boys	Girls
Minimum	7.4	8.1
Q_1	8.0	8.4
Median	8.15	8.55
Q_3	8.5	8.9
Maximum	9.1	9.5
Outliers		9.7

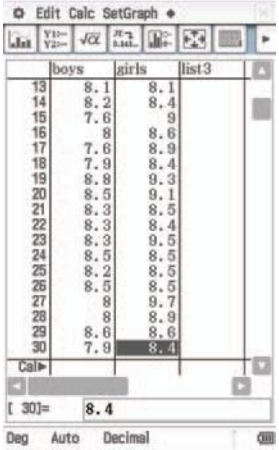
Reproduce the boxplots accurately on paper.



ClassPad

Use the Statistics application.

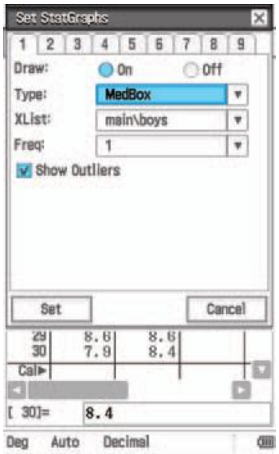
Type each set of data into a column.



	boys	girls	list3
13	8.1	8.1	
14	8.2	8.4	
15	7.6	9	
16	8	8.6	
17	7.6	8.9	
18	7.9	8.4	
19	8.8	9.3	
20	8.5	9.1	
21	8.3	8.5	
22	8.3	8.4	
23	8.3	9.5	
24	8.5	8.5	
25	8.2	8.5	
26	8.5	8.5	
27	8	9.7	
28	8	8.9	
29	8.6	8.6	
30	7.9	8.4	

[30]= 8.4

Tap **SetGraph**.
Make sure **StatGraph1** is checked and **StatGraph2** is checked.
Tap **Setting**.
Complete the window as shown, then tap the tab numbered $\boxed{2}$.

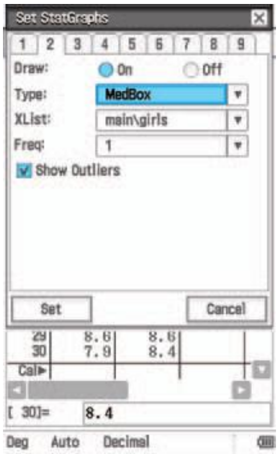


Draw: ● On ○ Off
Type: MedBox
XList: main\boys
Freq: 1
☑ Show Outliers

29	8.6	8.6
30	7.9	8.4

[30]= 8.4

Complete the window as shown, then tap $\boxed{\text{Set}}$.

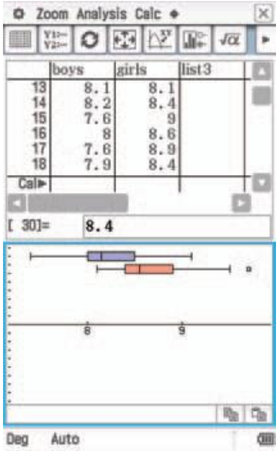


Draw: ● On ○ Off
Type: MedBox
XList: main\girls
Freq: 1
☑ Show Outliers

29	8.6	8.6
30	7.9	8.4

[30]= 8.4

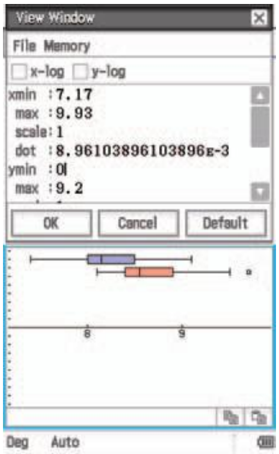
Tap 📊



	boys	girls	list3
13	8.1	8.1	
14	8.2	8.4	
15	7.6	9	
16	8	8.6	
17	7.6	8.9	
18	7.9	8.4	

[30]= 8.4

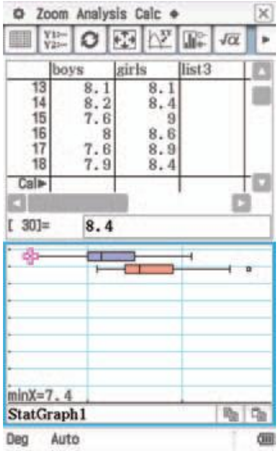
Tap ⛶ and set *y*min to be 0. Tap OK.



xmin :7.17
max :9.93
scale: 1
dot :8.96103896103896E-3
ymin :0
max :9.2

Tap **Analysis** then **Trace**.
Use the arrow keys ▼ or ▲ to move from one plot to the other then ▶ or ◀ to read the five-number summary from each boxplot.

	Boys	Girls
Minimum	7.4	8.1
Q_1	8.0	8.4
Median	8.15	8.55
Q_3	8.5	8.9
Maximum	9.1	9.5
Outliers		9.7



	boys	girls	list3
13	8.1	8.1	
14	8.2	8.4	
15	7.6	9	
16	8	8.6	
17	7.6	8.9	
18	7.9	8.4	

[30]= 8.4

b Compare the medians and interquartile ranges.
Due to the presence of the outlier in the girls' data, the IQR is the better measure of spread to use for making comparisons.

In general, the boys were faster than the girls as evidenced by the lower median for the boys. The boys and girls had a similar range of times since each group had approximately the same interquartile range.

Parallel boxplots are useful for making comparisons between the variables in relation to their centre (median), spread (IQR and range) and outliers. A comparison report can be written using the following steps as a guide.

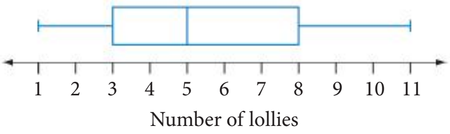
1. Use the boxplots to describe the shape of the distribution of each data set and determine if any outliers exist.
2. The median, IQR and range for each data set can all be determined from the boxplots.
3. Compare each summary statistic calculated in step 2 in relation to the data.
4. Compile a report by making a general conclusion about the data sets, then support this conclusion by summarising the findings from all the above steps.

Comparing group measures

EXERCISE 11.06 Constructing and interpreting parallel boxplots

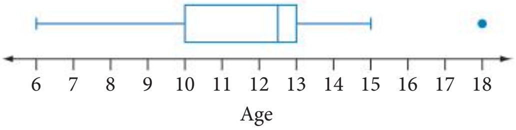
Concepts and techniques

1 **Example 11** The boxplot represents the number of lollies eaten by a group of 13-year-old girls at a birthday party.



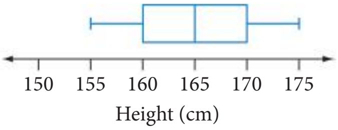
- **a** State the median and explain what this tells you about the data.
- **b** Calculate the range and interpret this value.
- **c** Calculate the IQR and interpret this value.
- **d** What is the range of the top 25% of data?
- **e** What percentage of the girls ate between 1 and 5 lollies?
- **f** What percentage of the girls ate 11 or less lollies?

2 The boxplot represents the ages of cars owned by a group of people.



- **a** State the median and explain what this tells you about the data.
- **b** Calculate the range, excluding the outlier, and interpret this value.
- **c** Calculate the IQR and interpret this value.
- **d** Show why the 18-year-old car is an outlier.
- **e** What range of values does the bottom 25% of the data represent?
- **f** What percentage of the cars are below 13 years old?

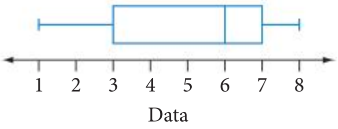
3 The boxplot represents the heights of a group of adults.



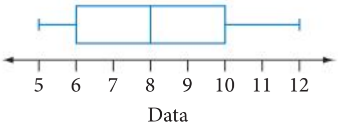
- **a** State the median. What information does this provide?
- **b** Calculate the range and interpret this value.
- **c** Calculate the IQR and interpret this value.
- **d** What range of values does the third quarter of the data represent?
- **e** What percentage of the heights are above 160 cm?
- **f** What percentage of the heights are below 160 cm or above 170 cm?

4 **Example 12** Describe the shape of the following boxplots, commenting on the presence of outliers.

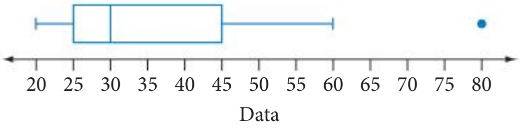
a



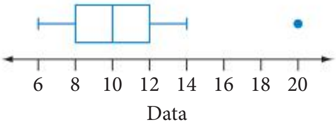
b



c



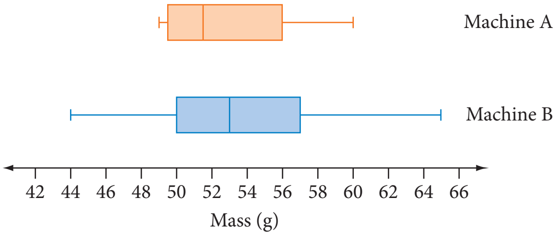
d



The following information relates to questions **5** and **6**.

Shutterstock.com/ArtThailand

The boxplots below show the number of faulty tennis balls produced per week by each of two machines in a factory over a one-year period.



5 Which one of the following statements is **true**?

A The data for Machine A is negatively skewed.

B The data for Machine B has an outlier.

C The data for Machine A is positively skewed.

D The data for Machine B is negatively skewed.

E The data for Machine A is symmetrical.

6 Which one of the following statements is **false**?

A The minimum data value for Machine A was 49.

B 50% of the data for Machine B lies above 53.

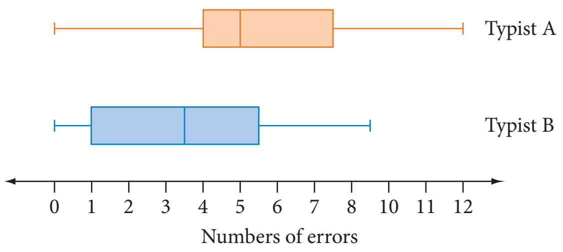
C 25% of the data for Machine B is less than 50.

D 25% of the data for Machine B is between 53 and 65.

E The range of the data for Machine A is less than that for Machine B.

The following information relates to questions **7** and **8**.

The number of errors made per page by two typists was recorded and is displayed in the parallel boxplots on the right.



7 Which one of the following statements is **true**?
- A Typist B has a larger range of errors per page than Typist A.
- B Typist A is the better typist.
- C 50% of Typist A's mistakes per page were below 4.
- D Typist B is the better typist.
- E Typist B made the most errors.

8 Which one of the following statements is **false**?
- A The median number of errors for Typist A is 5.
- B The range for Typist B is 12.
- C The middle 50% for number of errors per page for Typist B is 4.5.
- D Both typists made no errors at some time.
- E The value of quartile 1 for Typist A is 4.

9 Two golf ball manufacturers produce balls with a nominal diameter of 4.3 cm. A sample of twenty balls was taken from those produced by each company and measured. The results (in centimetres) were:

Beaut Ball Company:	4.30	4.29	4.30	4.34	4.30	4.32	4.29	4.29	4.28	4.31
	4.31	4.27	4.28	4.30	4.29	4.30	4.28	4.28	4.29	4.30
E-Zee Ball Company:	4.27	4.36	4.31	4.30	4.30	4.29	4.28	4.30	4.37	4.31
	4.30	4.37	4.28	4.30	4.26	4.34	4.31	4.35	4.34	4.39

- a If any outliers exist, show why these data values are outliers.
- b Construct parallel boxplots to represent the data.

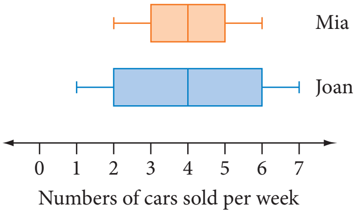
Reasoning and communication

10 The number of matches in boxes produced by 2 different companies were recorded as follows.

Company 1:	49	50	52	48	50	51	49	50	52	51	50	50
Company 2:	47	48	50	53	49	50	51	52	49	50	48	49

- a Construct parallel boxplots to represent this data.
- b Describe the distribution shape of each company's data.
- c Hence calculate the appropriate measures of centre and spread.
- d Which company had the largest median?
- e Which company had the largest range of matches per box?
- f Which company has the most consistent number of matches per box?
- g What percentage of Company 1's matches per box was above 50 matches?

11 Example 13 The following parallel boxplots represent the number of cars sold by two salespeople, Mia and Joan, per week over a six-month period.



a What is the median number of cars sold by Mia?

b What was the least number of cars sold in a week and who was the salesperson?

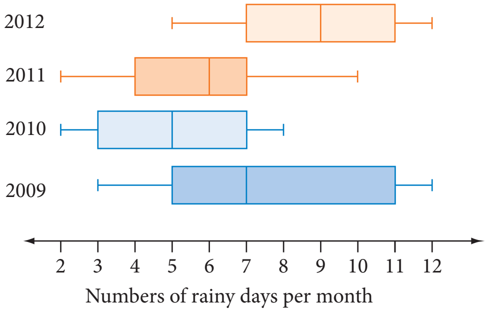
c What is the IQR for Mia's data?

d What percentage of the cars sold by Joan per week was above 6?

e What is the data shape for each of the salespeople?

f List things that Mia and Joan have in common.

12 The following parallel boxplots show the number of rainy days per month for 2012 to 2009.



a Which year has the lowest median number of rainy days per month?

b Which year has the highest median number of rainy days per month?

c In 2010, what percentage of the rainy days per month were between two days and five days?

d What year has 50% of its data above nine rainy days per month?

e Which year has the largest range of rainy days per month?

f For which year/s would you describe the data distribution shape as symmetrical?

g What can be said about seven rainy days per month in terms of percentages for each of the years?

13 Example 14 The relative humidities (%) at 3 p.m. in two towns over a period of two weeks were:

Town A: 33 35 67 45 48 67 84 56 58 57 45 48 68 56

Town B: 45 48 67 78 79 84 65 58 43 59 69 89 78 69

a Construct parallel boxplots to represent the data.

b Compare the relative humidity in each town.

14 A local cricket club has narrowed the choice of the last batsman for a match down to two candidates. Their scores in their last twelve matches are as follows.

Basher Smith: 6 10 5 0 14 12 36 16 15 3 10 13

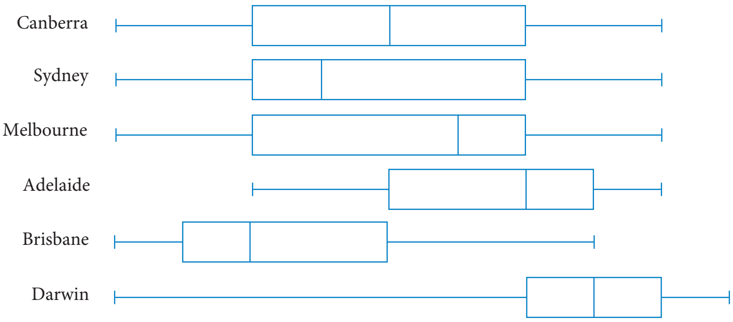
Slogger Jones: 12 15 9 7 15 20 5 8 7 15 17 19

a Construct parallel boxplots to represent the data.

b If any outliers exist, show why these data values are outliers.

c Which batsman should the club select for the match if they want a consistent performer? Justify your answer using appropriate summary statistics.

15 The box-and-whisker plots on the right show the distribution of petrol prices in six Australian capital cities.



a Which city has:
  i the greatest range of prices?
  ii the lowest median price?
  iii the smallest interquartile range of prices?
  iv a symmetrical distribution of prices?

b Is petrol cheaper in Sydney than Melbourne? How can you tell?

16 The lengths of words in typical definitions in two biology books were counted and found to be as follows.

Book 1: 5 8 10 6 9 6 10 8 5 6 8 10 8 7 9 7 8 10
Book 2: 12 8 5 24 7 9 5 5 22 5 13 9 10 9 5 17 14 8 14 7

a Construct parallel boxplots to represent the data.

b Compare the difficulty of the texts, assuming that longer words make a text harder to read. Justify your answer.

c Compare the consistency of the length of words used in each text. Which book has the most consistent word length? Justify your answer.

17 The maximum temperatures (°C) in two Queensland towns over a fortnight were:

Town A: 22 24 28 32 35 27 24 29 28 29 27 31 23 30
Town B: 25 26 26 24 26 28 28 26 29 30 27 24 29 31

a Construct parallel boxplots to represent the data.

b Describe the data distribution shape for both of the towns.

c Are any outliers present for either town?

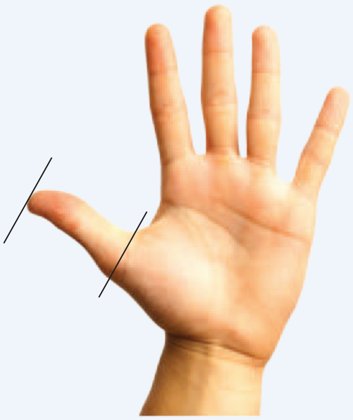
d Use the boxplots to calculate any summary statistics that describe the data.

e Using all the previous answers, compile a report comparing the temperatures in Town A with those in Town B.

INVESTIGATION Does gender determine body measurements?

a Collect the following data for at least 30 people of your own age, ensuring that you have a similar spread of males and females.

- Gender
- Thumb length, measured to the nearest mm
- Height, measured to the nearest cm
- Arm length, measured to the nearest cm



Shutterstock.com/Aaron Amat

Shutterstock.com/Andresr

Shutterstock.com/Andresr

Record the gathered information in a table, for example:

	Gender	Thumb length	Height	Arm length
1				
2				

b By constructing a back-to-back stem-and-leaf plot, investigate if there is a relationship between Gender and Height.

c Using appropriate summary statistics (remember to look for shape and outliers), discuss if there is a relationship between Gender and Height.

d By constructing parallel boxplots, investigate if there is a relationship between:

- **i** Gender and Thumb length
- **ii** Gender and Arm length.

e Answer the title question ‘Does gender determine body measurements?’, ensuring you support your answer with findings from questions **a–d**.

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11

CHAPTER SUMMARY
MAKING COMPARISONS

- Each summary statistic has its own unique interpretation.
  Mean: the average of the data being investigated.
  Median: the middle value of the data set when in order. This means that 50% of the data lies below the median and 50% lies above it.
  Mode: the most frequent or popular data value.
  Range: the difference between the highest data value and the lowest data value, therefore describing the total spread of scores.
  Interquartile range: the spread of the middle 50% of the data.
  Standard deviation: how data deviates from the mean.
- When two sets of data are being investigated, comparisons between the same summary statistic can be made. For example, the mean is higher for data set 1 than set 2.
- An outlier is a data value, x, which lies outside the interval

  $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$
- When data is skewed, the mean and standard deviation are not appropriate measures of centre or spread.
- When the data has outliers, the mean, standard deviation and range are not appropriate measures of centre or spread.
- The mean and standard deviation can be used to compare sets of data when the data is not skewed and there are no outliers present.
- In particular situations a higher mean is the better result (test scores), but in other circumstances a lower mean is the desired result (time to complete a race).
- A higher standard deviation means the data is more variable. Alternatively, a lower standard deviation means that the data is more consistent.
- Side-by-side column graphs are used to represent and compare two sets of categorical data.
- Back-to-back stem-and-leaf plots are used to represent and compare two related sets of numerical data.
- A comparison report can be written when comparing two sets of data using the following steps.
  1. Using a graphical display of the data describe the shape and determine if any outliers exist.
  2. Based on the findings from Step 1, select and calculate the appropriate measures of centre and spread.
  3. Compare each summary statistic calculated in Step 2 in relation to the data.
  4. Compile a report by making a general conclusion about the data set, then support this conclusion by summarising findings from all the above steps.
- Interpreting boxplots – the five-number summary can be read off a boxplot and a boxplot splits the data into quarters, therefore every section of the plot represents 25%.

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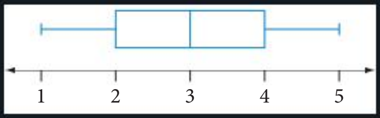
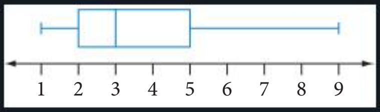
- The shape of a boxplot can be described as:

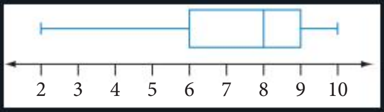
Symmetrical



Positively skewed



Negatively skewed



- Parallel boxplots are used to represent and compare the same numerical variable in two or more categories.

11

CHAPTER REVIEW
MAKING COMPARISONS

Multiple choice

The following information relates to questions **1** and **2**.

The goals scored by a netball team for a season are displayed in the stem-and-leaf plot below.

Key: 1|2 means 12 goals

Stem	Leaf
1	2 3 7
2	1 4 5 7
3	0 3
4	5
5	1 5
6	0

1 **Example 1** The median and interquartile range respectively are:

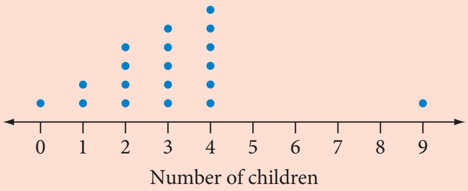
- A 29 goals and 24 goals
- B 27 goals and 29 goals
- C 17 goals and 21 goals
- D 21 goals and 17 goals
- E 24 goals and 30 goals

2 **Example 1** Which one of the following statements is **true**?

- A The average number of goals scored was 24.
- B The middle 50% of data has a range of 24 goals.
- C The netball team played 15 games for the season.
- D 50% of the goals scored were below 27.
- E None of the above.

3 **Example 6** The number of children in a sample of families is displayed in the dot plot on the right. Which one of the following best describes the shape of the data?

- A Positively skewed with an outlier.
- B Approximately symmetrical.
- C Positively skewed.
- D Negatively skewed.
- E Negatively skewed with an outlier.



4 **Example 7** Data set A and data set B have the same mean but data set B has a larger standard deviation. Which one of the following statements is **true**?

- A Set A has a higher average.
- B Set B has a higher average.
- C Set A's data is more clustered about the mean than set B.
- D Set B is more clustered about the mean than set A.
- E Set A's data has a wider spread than set B.

5 **Example 8** The graph on the right shows what employment sectors a sample of Australians worked in.

Which one of the following statements is **true**?

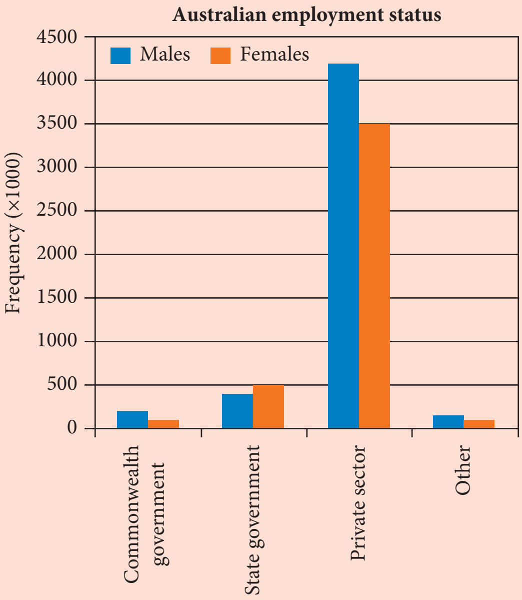
A More men worked in all employment sectors.

B Less females worked in State government than in the Commonwealth government.

C An equal number of males and females are employed by the employment sector 'other'.

D The private sector employs the most people from the sample of Australians.

E More males than females worked in every employment sector.



The following stem-and-leaf plot relates to questions **6** and **7**.

Key: 1|3 means 13

Set 2	Stem	Set 1
4	1	3 3 4
7 7 6 3	2	1 2 5 6 8 9 9
8 7 5 5 2 0	3	0 2 2 5 6
8 8 6	4	3 6 6
9 8 8 7 1 1	5	0 1
4 3 0	6	0 1
1 1	7	

6 **Example 10** The interquartile ranges of Set 1 and 2 respectively are:

A 48 and 57

B 21 and 27.5

C 47 and 48

D 27.5 and 21

E 30 and 30

7 **Example 10** Which one of the following statements is **false**.

A Set 1 has a lower mean than Set 2.

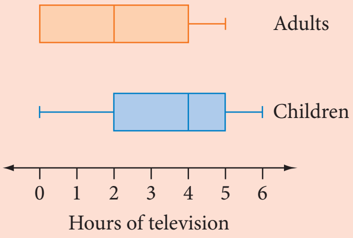
B The median of Set 2 is larger than Set 1.

C The interquartile range of Set 2 is larger than Set 1.

D The range of Set 1 is larger than Set 2.

E Set 2 has more modes than Set 1.

8 Example 13 The parallel boxplots on the right show the hours of television watched per day for adults and children.



Which one of the following statements is **true**?

A 25% of adults and children watch between 0–2 hours of television a day.

B The median number of hours of watching television is the same for adults and children.

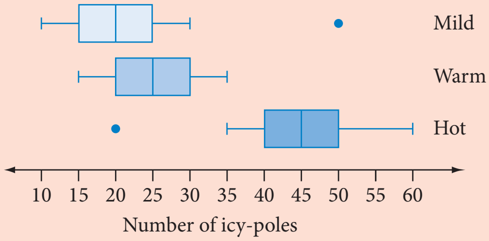
C The spread of hours of watching television is larger for the adults than the children.

D The interquartile range for adults and children is 4 hours.

E None of the above.

The following information relates to questions **9** and **10**.

The parallel boxplots on the right show the number of icy-poles sold at a school canteen for three different weather conditions: mild, warm and hot.



9 Example 13 If outliers are ignored, which one of the following statements is **false**?

A Each weather condition has the same range for the middle 50% of data.

B The number of icy-poles sold in mild weather conditions is symmetrical.

C The largest range of icy-poles sold occurs in mild weather conditions.

D The maximum number of icy-poles sold increases as the weather conditions get hotter.

E None of the above.

10 Example 14 From the parallel boxplots it can be concluded that as the weather conditions change from mild to warm to hot, the number of icy poles sold:

A decreases and becomes less variable

B decreases and becomes more variable

C increases and becomes less variable

D increases and becomes more variable

E stays the same.

Short answer

11 Example 2 The house prices realised at auction one Saturday in Vincentia were:

$342 000 $264 000 $268 000 $517 000 $1 044 000 $420 000 $348 000 $297 000

a Calculate the mean price.

b Calculate the median price.

c Which measure of centre is the highest: mean or median? Calculate the difference.

d Can you suggest a possible reason for the difference?

e Which measure of centre best describes the prices of the houses sold at auction?

12 Example 8 The following table shows the caffeine and herbal contents of some popular "energy drinks".

Caffeine level	Herbs	No herbs
None	3	0
High	4	4
Medium	1	1
Low	2	4

a Display this information in a side-by-side column graph.
b Discuss what the side-by-side column graph shows.

13 Example 9 The following data shows the weekly sales claimed by 2 door-to-door salespeople selling sets of reference books.

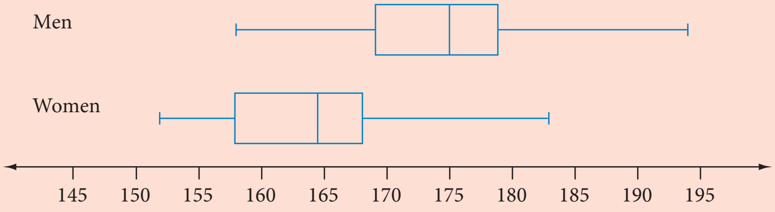
Max: 5 8 10 7 15 21 32 41 15 40
25 14 6 31 12 34 15 13 4 30
Jane: 2 15 23 17 31 43 48 12 14 2
17 24 17 41 30 12 16 15 9 10

a Display this information in a back-to-back stem-and-leaf plot.
b Calculate the median for each salesperson and make a comparison.
c Calculate the range for each salesperson and state which one has the greater spread.
d Using the appropriate summary statistics, state who is the most consistent salesperson.

14 Examples 11, 12 The heights, in centimetres, of a group of men and women are displayed in the two plots below.

Key: 15 | 2 means 152 cm

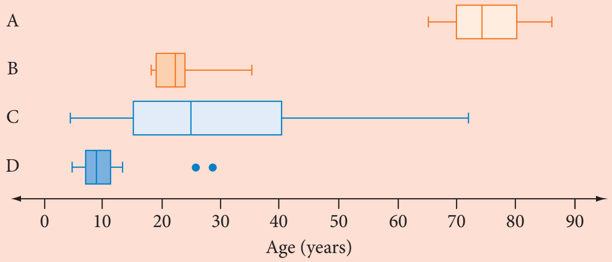
Men		Women
8	15	2 4 4 5 6 8 8 9
9 7 7 5 2	16	0 2 3 3 4 5 5 5 5 6 7 8 8
9 9 8 8 6 5 5 4 4 4 2 1	17	2 3 4 4
8 6 3 2 1	18	3
4	19	



a Which group had the higher median?
b Which measure of spread was equal to 10 for both groups?
c Which group had the wider spread of heights?
d Comment on the skewness of each data set.
e Calculate the mean heights of men and women (correct to one decimal place).
f Which is the better measure of centre for these data sets, the mean or the median? Why?

15 **Example 13** Match each sample of people to the correct box-and-whisker plot.

- **a** Spectators at a football match.
- **b** Senior citizens on a bus trip.
- **c** Visitors to a dance party.
- **d** Primary school zoo excursion attendees.



16 **Examples 4, 12, 14** A golfer had the following scores for her last twenty rounds of golf:

86 102 81 90 105 78 89 86 80 91 82 79 81 88 90 101 94 82 86 88

A new member of the club has the following scores for her first ten rounds of golf:

132 118 128 141 120 92 138 90 147 144

- **a** Use parallel boxplots to display the performances of the two golfers.
- **b** If any outliers exist, show why these data values are outliers.
- **c** By describing the shape of each data set, state the appropriate measure of spread that should be used to compare the two golfers' scores.
- **d** Who is the better player? Explain.

Application

17 A factory produces small metal rods designed to have a mass of 50 g. Samples were taken from two different machines and a back-to-back stem-and-leaf plot drawn.

Key: 4|8 means 48 grams

Machine A	Stem	Machine B
	4	4
9 9 9 9 9	4	8 9
3 2 2 1 1 0 0 0	5	0 0 0 0 0 1 1 1 2 3 3 3 4 4
8 8 7 6 6 5	5	5 6 7 7 8 9
0	6	1 2 3
	6	5

- **a** Find the modal mass for each machine.
- **b** Find the median and interquartile range for each machine.
- **c** Which machine produces smaller rods on average?
- **d** Find the five-number summary for each data set and construct parallel boxplots for the data.
- **e** ‘Machine B produces rods of a more consistent mass than Machine A’. Do you agree with this statement? Justify your answer using an appropriate summary statistic.

18 The following information is the final score for each game in the 2012 season for three AFL teams.

Brisbane:	119	63	35	111	41	62	58	132	96	102	70
	114	122	63	92	59	77	84	77	95	104	128
GWS:	37	54	69	50	62	43	94	40	53	61	74
	57	38	31	59	37	54	107	79	59	35	77
Melbourne:	78	58	74	67	66	76	49	37	49	58	87
	135	61	78	83	56	73	108	82	84	81	40

a Construct parallel boxplots to represent the data.

b Why is the mean not a good measure of centre for Melbourne?

c What range of values does the bottom 25% of the data have for each team?

d State the shape of the data distribution for each team.

e Compile a report comparing the final scores for the season for all three teams.

Practice quiz

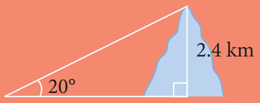
MIXED REVISION

CHAPTERS 10 • 11 • 12

4

Multiple choice

1 Danielle walked up a mountain with an incline of 20° to reach a height of 2.4 km above sea level. The total distance walked by Danielle (to the nearest kilometre is) is:



A 2 km   B 3 km   C 5 km   D 6 km   E 7 km

2 The side-by-side column graph displays the number of days that it rained for each season of a year.

Which one of the following statements is **true**?

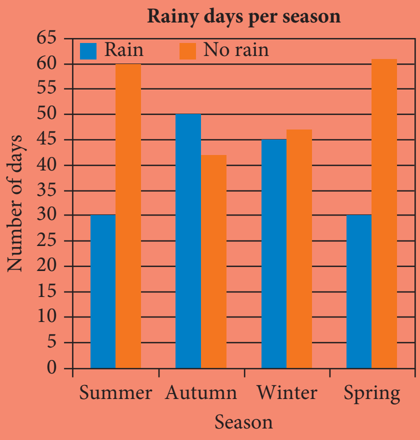
A There were more rainy days than dry days in all the seasons.

B Autumn had the most rainy days and the least dry days.

C Summer had more rainy days than dry days.

D Winter had the same number of rainy and dry days.

E None of the above.



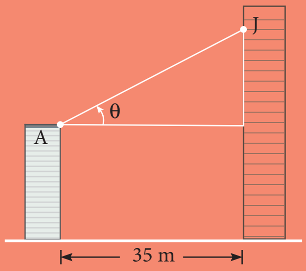
3 The gradient and y-intercept respectively of the equation $3x - 4y = 20$ are

A $-3, 5$ B $\frac{3}{4}, -5$ C $\frac{4}{3}, -5$ D $\frac{3}{4}, 5$ E $5, \frac{3}{4}$

4 Angus is standing on his balcony which is 30 m above street level. He spots his friend, Josephine, on her balcony which is 60 m above street level. If the two buildings are 35 m apart, the angle of elevation of Josephine from Angus (to the nearest degree) is:

A 30°   B 41°   C 49°

D 60°   E 90°



5 The following boxplot shows the wages of employees at a fashion outlet.

The middle 50% of staff wages is between:

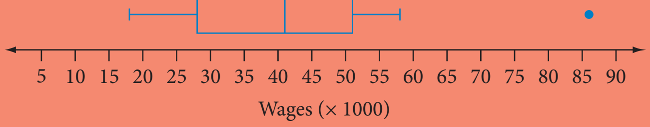
A \$51 and \$28

B \$41 and \$18

C \$41 000 and \$18 000

D \$51 000 and \$28 000

E none of the above



6 The solution to the simultaneous equations $y = 5x - 1$ and $2y = 3x - 9$ is:

A $x = -1, y = -4$ B $x = 1, y = 4$ C $x = -4, y = -21$

D $x = -1, y = -6$ E $x = 4, y = 1$

7 The length of *FH* can be found using:

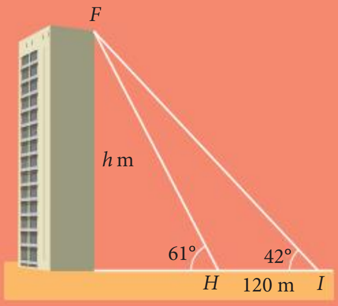
A $FH = 120 \tan(42°)$

B $FH = \dfrac{120\sin(42^\circ)}{\sin(61^\circ)}$

C $FH = \dfrac{120\sin(42^\circ)}{\sin(19^\circ)}$

D $FH = 120 \tan(61°)$

E $FH = \dfrac{120\sin(42^\circ)}{\sin(119^\circ)}$



8 The dot plot represents the amount of pocket money given to a group of 5 year old children.

Select the measure of centre and spread that best represents the data.

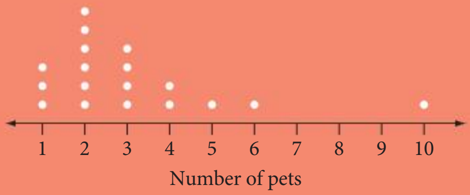
A Mode and range

B Median and interquartile range

C Mean and range

D Mean and interquartile range

E Mean and standard deviation



9 The equation which best describes the step graph is:

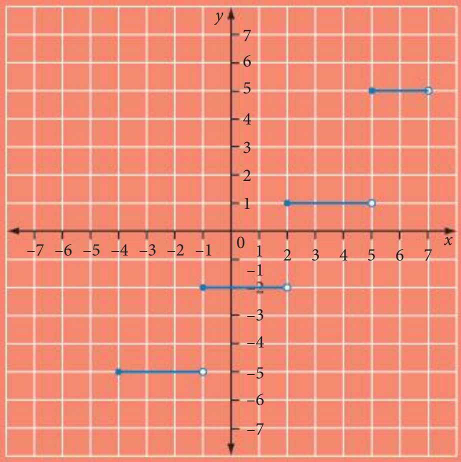
A $y = \begin{cases} -5, -4 \le x < -1 \\ -2, -1 \le x < 2 \\ 1, 2 \le x < 5 \\ 5, 5 \le x < 7 \end{cases}$

B $y = \begin{cases} -5, -4 < x \le -1 \\ -2, -1 < x \le 2 \\ 1, 2 < x \le 5 \\ 5, 5 < x \le 7 \end{cases}$

C $y = \begin{cases} -5, -4 \le x \le -1 \\ -2, -1 \le x \le 2 \\ 1, 2 \le x \le 5 \\ 5, 5 \le x \le 7 \end{cases}$

D $y = \begin{cases} -5, -4 \le x < -1 \\ -2, -1 \le x < 2 \\ 1, 2 \le x < 5 \\ 5, 6 \le x \le 7 \end{cases}$

E $y = \begin{cases} -5, -4 < x < -1 \\ -2, -1 < x < 2 \\ 1, 2 < x < 5 \\ 5, 6 < x < 7 \end{cases}$



Short answer questions

1 A triangular shade sail has sides of length 8 m, 11 m and 14 m. Calculate the area of material, correct to the nearest square metre.

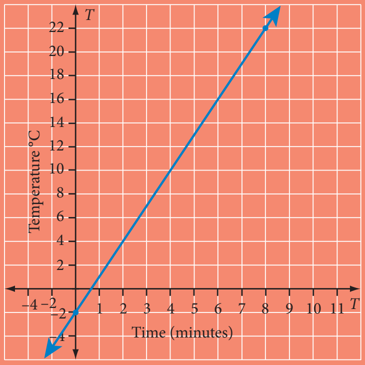
2 The ages of members of a yoga class at Husky Body Shop one morning are shown below.

Women: 24 32 43 48 50 50 54 59 63 63 64 65 68

Men: 22 26 32 32 35 36 38 42 44 45 46 53 63 63 64

a Find the interquartile range of the ages for the women in the class and interpret.
b Compare the mean age for men and women.
c Justify why no outliers exist for each gender.

3 The temperature in a room t minutes after the heating is turned on is given by the graph.
a What was the initial temperature of the room?
b By how many degrees did the temperature rise each minute?
c Use your answers to parts **a** and **b** to write a rule showing the relationship between the temperature, T, to the time, t.



4 A hang-glider pilot takes off from the top of a 100 m cliff that is at an angle of 103° to the beach. He flies 135 m to land on the beach. What is the angle of elevation from where he lands to the top of the cliff, correct to the nearest degree?

5 The following stem-and leaf plot shows the results (%) of an algebra test for Mr A's Year 10 class.

Key: 3|1 means 31%

Stem	Leaf
3	1
4	0 1 2
5	0 0 1 2 3
6	2 3 4 5 6 7 8
7	3 4 5 6 7
8	0 1 2
9	9

a Calculate the mean and standard deviation of the test results for Mr A's class.
b Why are the mean and standard deviation appropriate measures of centre and spread?

Mr A decides to compare his class' results to Mr B's class. They had an average mark of 72.61% and a standard deviation of 11.1%.

c Which teacher's class achieved higher results? Explain your reasoning.
d Which teacher's class achieved the most consistent results? Justify your thinking.

6 The equations $3x + 2y = -8$ and $7x - 3y = 35$ are to be solved simultaneously.
a Sketch the graphs of each equation on the same set of axes.
b Find their point of intersection using the graph.
c Solve the equations algebraically using substitution or elimination. Compare your answers.

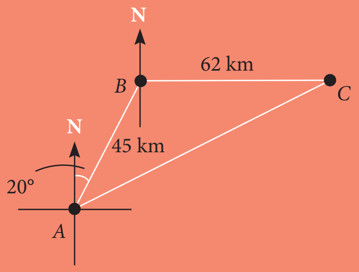
Application questions

1 A helicopter is delivering supplies to three islands off the Australian coast as shown in the diagram below. The helicopter leaves island *A* on a bearing of 020° and flies for 45 km to reach island *B*. It then heads due east to reach island *C*, which is 62 km away.

Calculate:

a The distance between island *C* and island *A*, correct to two decimal places.

b The true bearing of island *A* from island *C*, correct to the nearest degree.



2 The parallel boxplots show the diameters (cm) of a sample of elm and oak trees.

a Describe the shape of the distributions.

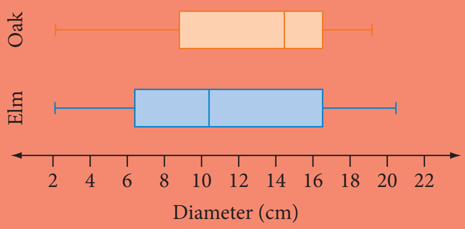
b Which measures of centre and spread will best represent the data?

c Explain why the mean diameter for each type of tree cannot be calculated.

d What percentage of both trees had a diameter greater than 16.5 cm?

e Which feature of the boxplot tells us that elm trees have a larger range for the middle 50% of the data?

f Compare the diameters of the two types of trees.



3 The town of WallyWater uses a piecewise linear function to calculate the monthly water charge for its consumers. The piecewise linear function, where C = the cost of the water and l = the number of thousands of litres, is:

$$C = \begin{cases} 5l + 20, & 0 \le l \le 5 \\ 9l, & 5 < l \le 10 \\ 10l - 10, & l > 10 \end{cases}$$

a Sketch the graph of the function to show the relationship between the litres used and the cost of water in WallyWater town.

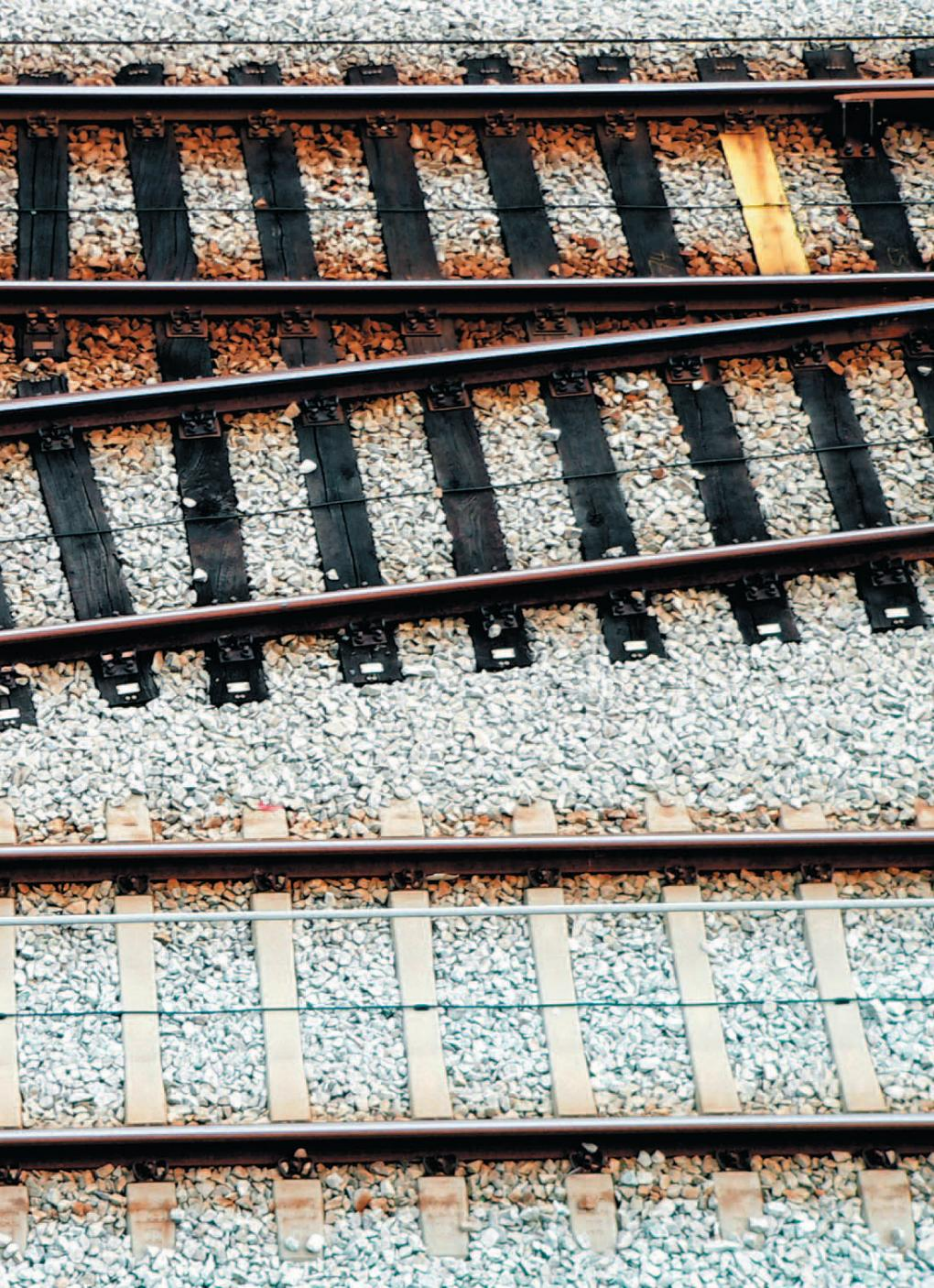
b Describe the graph you have sketched.

c Which points are the critical points on the graph. Explain your reasoning.

d What do you notice about the slope of each line segment? What do you think the changing slope means?

e Use your graph to determine the monthly water charge if a consumer used:

i 3500 litres ii 9000 litres iii 12 500 litres

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12

SIMULTANEOUS LINEAR EQUATIONS AND THEIR APPLICATIONS, PIECEWISE LINEAR AND STEP GRAPHS

APPLICATIONS OF SIMULTANEOUS EQUATIONS, PIECEWISE AND STEP GRAPHS

TERMINOLOGY

elimination method
extrapolating
general form of a linear equation
gradient
interpolating
linear rule
linear graph
parallel lines
piecewise linear graph
point of intersection
simultaneous linear equations
step graph
substitution method
x-intercept
y-intercept

Prior learning

SIMULTANEOUS LINEAR EQUATIONS AND THEIR APPLICATIONS

- **solve a pair of simultaneous linear equations, using technology when appropriate (ACMGM044)**
- **solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations. (ACMGM045)**

PIECEWISE LINEAR GRAPHS AND STEP GRAPHS

- **sketch piecewise linear graphs and step graphs, using technology when appropriate (ACMGM046)**
- **interpret piecewise linear and step graphs used to model practical situations; for example, the tax paid as income increases, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time, the charging scheme for sending parcels of different weights through the post. (ACMGM047)**

12.01 REVIEW OF LINEAR GRAPHS

A **linear equation** connecting y and x is one that results in a straight line when you graph it.

Equations such as $y = 2x + 1$; $y = \frac{1}{2}x$; $3x - 2y = 12$; $y = 8 - x$ are examples of linear equations.

The **general form of a linear equation** is $y = a + bx$, where a represents the y-intercept and b represents the **gradient** of the straight line.

IMPORTANT

Note that the value representing the gradient is the coefficient of the term containing the pronumeral x. In some instances you may need to rearrange the equation into the general form, by making y the subject, before identifying the gradient and the y-intercept.

Remember that the gradient of a line can be calculated using $\frac{\text{rise}}{\text{run}}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

Example 1

State the gradient and y-intercept of the lines below.

a $y = 4x - 3$ **b** $y - 2x = 6$ **c** $4x + 3y = 9$

Solution

a Write the equation in the form $y = a + bx$. $y = -3 + 4x$

State the values of the gradient and y-intercept. $b = 4$, $a = -3$

The gradient is 4 and the y-intercept is −3.

b Write the equation in the form $y = a + bx$.

$$y - 2x = 6$$
$$y = 6 + 2x$$

State the values of the gradient and y-intercept. $b = 2$, $a = 6$

The gradient is 2 and the y-intercept is 6.

c Write the equation in the form $y = a + bx$.

$$4x + 3y = 9$$
$$3y = 9 - 4x$$
$$y = 3 - \frac{4}{3}x$$

State the values of the gradient and y-intercept.

$b = -\frac{4}{3}$, $a = 3$

The gradient is $-\frac{4}{3}$ and the y-intercept is 3.

The gradient and y-intercept can be used to sketch a linear graph.

- Plot the y-intercept on the y-axis.
- Use the gradient to locate another point on the line.
- Rule a line through both marked points.
- Label the graph with its equation.

Remember that:

- a positive gradient slopes up from left to right
- a negative gradient slopes down from left to right.

IMPORTANT

To sketch the graph of a straight line, only two points are required.

Example 2

Sketch the graphs for each of the following linear functions using the gradient and y-intercept method.

a $y = 2x + 1$ **b** $2x + 3y = 12$

Solution

a Write the equation.

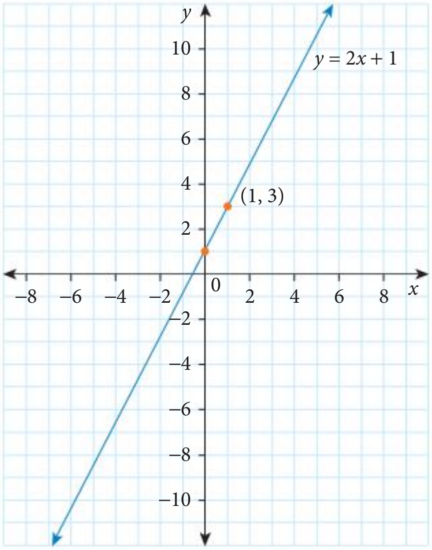
$y = 2x + 1$

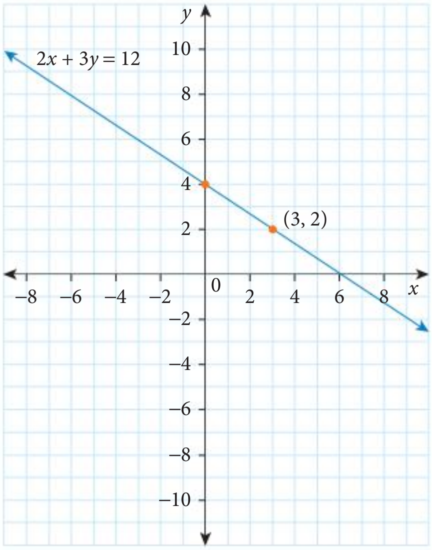
Identify the gradient and the y-intercept from the equation.

Gradient: $b = 2$

y-intercept: $a = 1$

Plot the coordinates of the y-intercept: (0, 1).
Move 1 unit across to the right followed by 2 units up, plotting a point at (1, 3).
Join these points to sketch the line.
Label the line with its equation.



b	Write the equation.	$2x + 3y = 12$
	Rearrange the equation to make y the subject. Subtract $2x$ from both sides.	$3y = 12 - 2x$
	Divide both sides by 3.	$y = \frac{12}{3} - \frac{2}{3}x$ $= 4 - \frac{2}{3}x$
	Identify the gradient and the y-intercept.	Gradient: $b = -\frac{2}{3}$ y-intercept: $a = 4$
	Plot the coordinates of the y-intercept: (0, 4). Move 2 units down and 3 units across, plotting a point at (3, 2). Join these points to sketch the line. Label the line with its equation.	

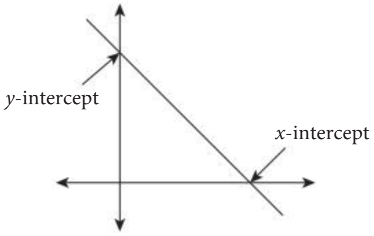
Two points that can be used to sketch the graph of a straight line are the **x-intercept** and the **y-intercept**.

All the points along the x-axis have coordinates with $y = 0$.

To find the x-intercept, let $y = 0$.

All the points along the y-axis have coordinates with $x = 0$.

To find the y-intercept, let $x = 0$.



IMPORTANT

Lines with equations in the form $y = a$, where a is a constant, are horizontal lines (or lines parallel to the x-axis). These lines pass through the number represented by the constant.

Lines with equations in the form $x = a$, where a is a constant, are vertical lines (or lines parallel to the y-axis). These lines pass through the number represented by the constant.

Example 3

Find the x- and y-intercepts for each of the following linear functions, and hence sketch their graphs.

a $y = 3 + 2x$ **b** $2x - 3y = 18$ **c** $y = 3$ **d** $x = -2$

Solution

a Write the equation.

$y = 3 + 2x$

Find the x-intercept by letting $y = 0$.

$0 = 3 + 2x$

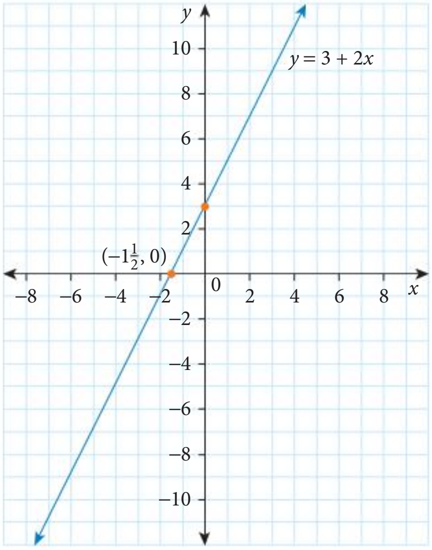
Solve the equation.

$$2x = -3$$
$$x = -\frac{3}{2}$$
$$= -1\frac{1}{2}$$

Find the y-intercept by letting $x = 0$.

$$y = 3 + 2 \times 0$$
$$y = 3$$

Plot the intercepts.
Join these points to sketch the line. Extend the line.
Remember to write the equation of the line next to its graph.



b Write the equation.

$2x - 3y = 18$

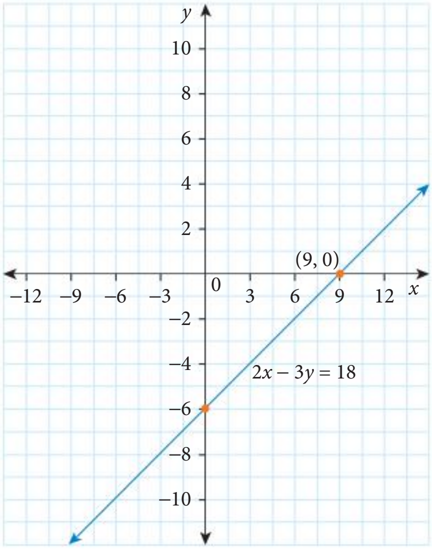
Find the x-intercept by letting $y = 0$.
Solve the equation for x.

$$2x - 3 \times 0 = 18$$
$$2x = 18$$
$$x = 9$$

Find the y-intercept by letting $x = 0$.
Solve the equation.

$$2 \times 0 - 3y = 18$$
$$-3y = 18$$
$$y = -6$$

Plot the intercepts.
Join these points to sketch the line. Extend the line.
Remember to write the equation of the line next to its graph.

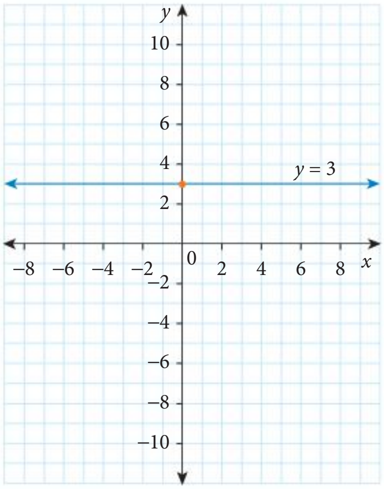


c Write the equation.

The equation is in the form $y = a$, hence the graph is a line parallel to the x-axis. The y-intercept is 3.

$y = 3$

Draw a horizontal line passing through the point (0, 3).
Remember to write the equation of the line next to its graph.

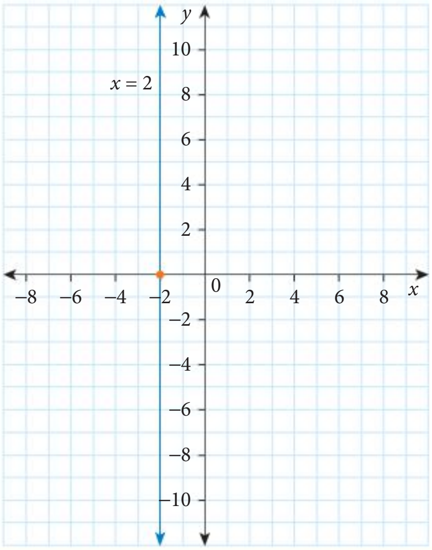


d Write the equation.

$x = -2$

The equation is in the form $x = a$. There is no y value present in the equation; hence the graph is a line parallel to the y-axis. The x-intercept is −2.

Draw a vertical line passing through the point (–2, 0). Remember to write the equation of the line next to its graph.



EXERCISE 12.01 Review of linear graphs

Substitution

Concepts and techniques

1 Example 1 For each of the linear functions given below, find:

i the gradient

ii the y-intercept

- a $y = x - 2$
- b $y = 6 + 3x$
- c $y = 7 - \frac{1}{4}x$
- d $x + 2y = 6$
- e $8x - 4y = -24$
- f $3y - 2x = 8$
- g $3x + 5y - 20 = 0$
- h $3x - 4y - 6 = 0$

2 Find the equation of each linear function.

- a gradient = 1, y-intercept = –2
- b gradient = –2, y-intercept = 2
- c gradient = 5, y-intercept = 3
- d gradient = –10, y-intercept = 0
- e gradient = $\frac{1}{2}$, y-intercept = 1
- f gradient = $-\frac{3}{4}$, y-intercept = –5
- g gradient = $-\frac{3}{2}$, y-intercept = 3
- h gradient = $\frac{2}{3}$, y-intercept = 0

3 Example 2 Sketch the following graphs using the gradient and y-intercept.

- a $y = 3 + x$
- b $y = -2 - x$
- c $y = \frac{1}{3}x + 1$
- d $4x + y = 8$
- e $y = 3x$
- f $4x + 6y = 0$
- g $10y - 6x + 5 = 0$
- h $5x + 4y - 100 = 0$

4 a The y-intercept of the line on the right is:

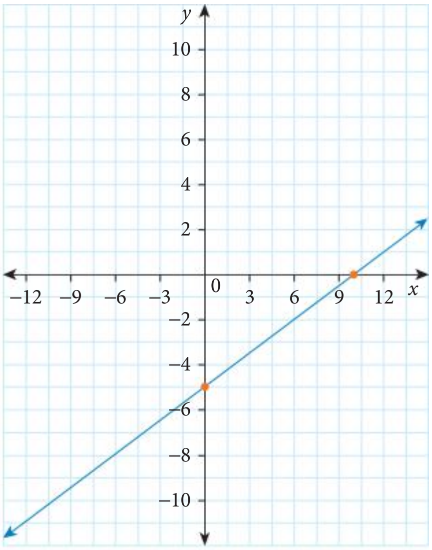
A -5 B -4 C $-\frac{1}{2}$

D $\frac{1}{2}$ E 2

b The gradient of the line on the right is:

A -5 B -2 C $-\frac{1}{2}$

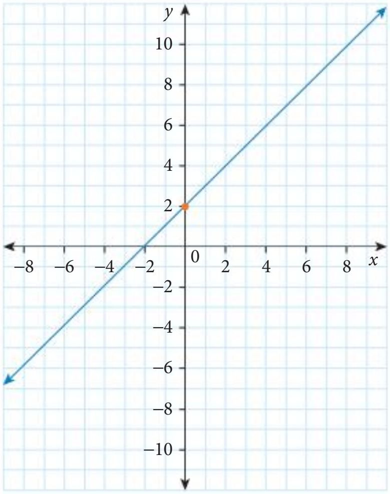
D $\frac{1}{2}$ E 2



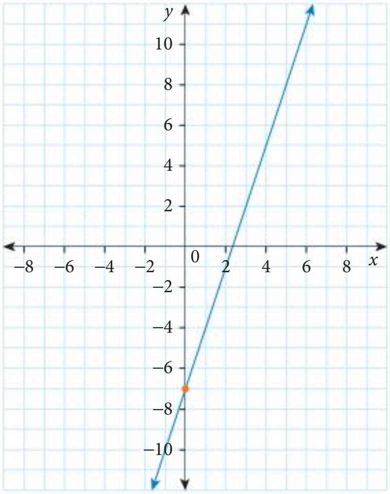
5 For each of the following graphs find:

i the y-intercept ii the gradient iii the equation of the line.

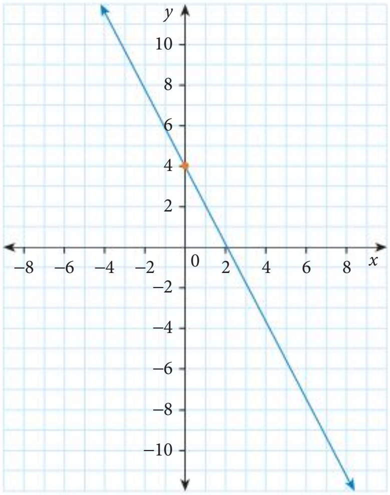
a



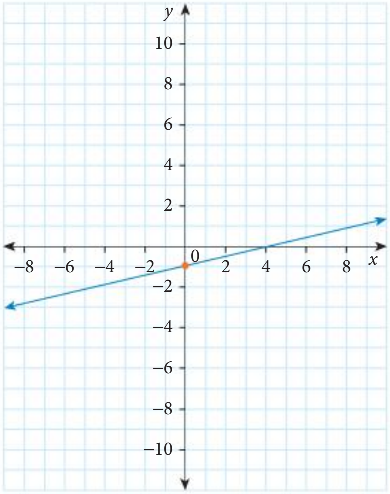
b



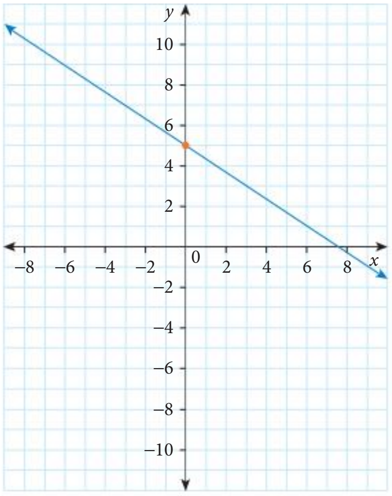
c



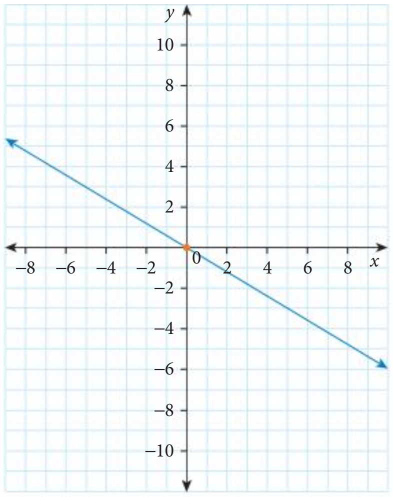
d



e



f



6 **Example 3** The coordinates of the x- and y-intercepts respectively for the graph below are:

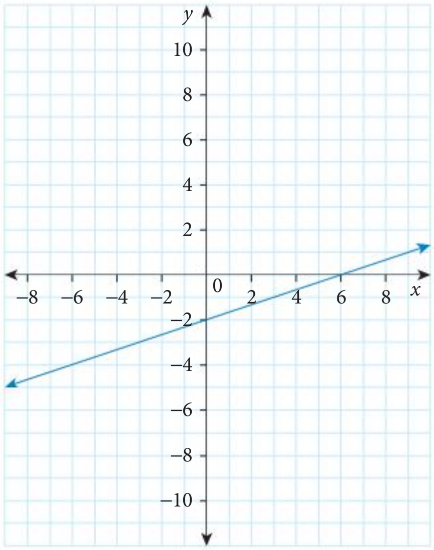
A (6, 0) and (0, −2)

B (−2, 6)

C (−2, 0) and (0, 6)

D (−2, 0) and (6, 0)

E (6, −2)



7 Calculate the x- and y-intercepts for each linear equation. Use the intercepts to sketch their graphs.

a $y = 2x - 2$

b $y = 8 - 2x$

c $4x + 3y = 12$

d $2x + y = 7$

e $y = \frac{2}{3}x + 5$

f $5y - 2x + 15 = 0$

g $x = -5$

h $y = 6$

8 Sketch the following graphs.

a $y = -5x$

b $y = \frac{1}{4}x$

c $y = 3$

d $x = 7$

e $x = -9$

f $y = -5.5$

g $y = 0$

h $x = 0$

Reasoning and communication

9 Write a sentence explaining what is special about the lines $x = 0$ and $y = 0$. (Hint: use your answers from question **8**.)

Finding the gradient between two points on a line

10 Find the equation (in the form $y = a + bx$) of the straight line passing through the points:

a (−1, 3) and (0, 4)
b (5, 1) and (0, 5)
c (−2, 4) and (0, −6)
d (3, 6) and (1, 0)
e (−4, 2) and (0, 0)
f (−3, 7) and (3, 3).

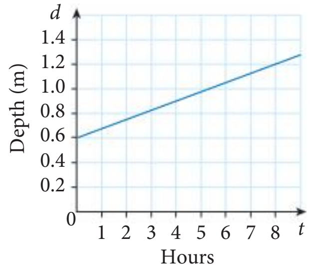
11 Write down the gradient and C-intercept for the equation $C = 125 + 42n$.

12 A taxi fare costs $5 flag fall plus $2 per km.

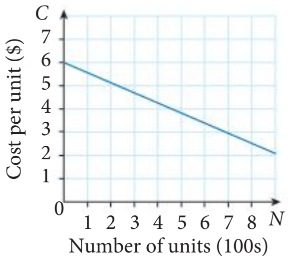
a Write an equation for the cost (C) of the taxi when travelling x km.
b Sketch the graph of this equation on the number plane.
c Find the gradient of this graph. What does the gradient represent?
d What is the vertical axis intercept of this graph? What does this point represent?

13 A swimming pool is being filled and the depth of water is recorded as it rises. The graph shows the changes in depth.

a What is the depth of water after 6.5 hours?
b When is the pool 1.2 m deep?
c Where does the graph intersect the vertical axis and what does this point represent?
d Find the gradient of the line. What does the gradient tell us about the depth of the water in the pool?
e Find the equation of the depth (d) of water over time (t).



14 The cost per unit of an electrical part decreases with more orders. The graph shows the cost.



In part **i** you found an answer within the given values or range of the graph. This is called **interpolating**.

In part **ii** you found an answer that fell outside the range of the graph. This is called **extrapolating**.

a Find the equation for the cost per unit of n electrical parts.
b Use this equation to find:
  i the cost per unit of 300 parts
  ii the number of parts that would each cost $1.50.

15 The table below shows the quarterly cost of water usage for different volumes of water.

Volume, v (kL)	16	25	48	60	81	99
Cost, C ($)	168.24	183	220.72	240.40	274.84	304.36

a What is the independent variable and what does it represent?
b Find a linear function for C in terms of v.
c Sketch a graph of this linear function.
d What does the gradient represent?
e What does the vertical axis intercept represent?
f What is the cost of using 54 kL of water?
g Find the volume of water usage that costs $265.

12.02 SOLVING SIMULTANEOUS EQUATIONS GRAPHICALLY

A pair of linear equations, such as $y = x + 2$ and $y = 2x - 5$, can be solved together to find the values of x and y that satisfy *both* equations. As they are solved at the same time, they are called **simultaneous linear equations**.

If simultaneous linear equations are graphed on the same Cartesian plane, their solution is the coordinates of the point where their graphs intersect.

The point where two or more graphs intersect is called the **point of intersection**.

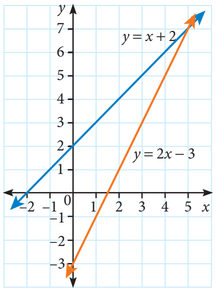
You can check that your solution is correct by substituting the x and y values back into the original equations.

Sometimes the equations represent parallel lines that never intersect and therefore the simultaneous equations have no solution.

Example 4

Graph the equations $y = x + 2$ and $y = 2x - 3$ on the same set of axes and find their point of intersection.

Solution

Both equations are written in the form $y = a + bx$ so it is easiest to use the gradient and y-intercept method.	$y = x + 2$	$y = 2x - 3$
Find the gradient and y-intercept for each equation.	gradient = 1 y-intercept = 2	gradient = 2 y-intercept = −3
Sketch both lines on the same set of axes. Remember to write the equation of each line next to its graph.		
Use the graph to read the coordinates of the point of intersection.	Point of intersection = (5, 7)	
Check the solutions by substituting (5, 7) into the original equations.	$y = x + 2$ $7 = 5 + 2$ ✓ $y = 2x - 3$ $7 = 2 \times 5 - 3$ ✓	
Write the answer.	The point of intersection is (5, 7).	

It may be easier to use the x- and y-intercept method to sketch the graphs of the pair of simultaneous equations.

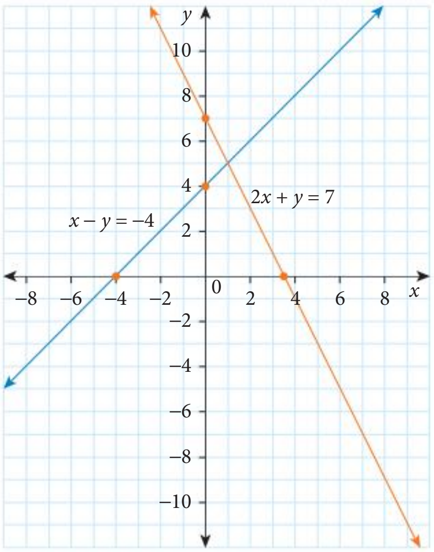
Example 5

Solve the simultaneous equations $2x + y = 7$ and $x - y = -4$ graphically.

Solution

Write down the equation of the first line.	$2x + y = 7$
Find the x-intercept by letting $y = 0$.	$2x + 0 = 7$
Solve the equation.	$2x = 7$ $x = 3\frac{1}{2}$
State the coordinates of the x-intercept.	The coordinates of the x-intercept are $\left(3\frac{1}{2}, 0\right)$.
Find the y-intercept by letting $x = 0$.	$2 \times 0 + y = 7$
Solve the equation.	$y = 7$
State the coordinates of the y-intercept.	The coordinates of the y-intercept are (0, 7).
Write down the equation of the second line.	$x - y = -4$
Find the x-intercept by letting $y = 0$ and solve.	$x - 0 = -4$ $x = -4$
State the coordinates of the x-intercept.	The coordinates of the x-intercept are (–4, 0).
Find the y-intercept by letting $x = 0$ and solve.	$0 - y = -4$ $y = 4$
State the coordinates of the y-intercept.	The coordinates of the y-intercept are (0, 4).

Sketch both lines on the same set of axes. Remember to write the equation of each line next to its graph.



Use the graph to read the coordinates of the point of intersection.

Point of intersection = (1, 5)

Check the solutions by substituting (1, 5) into the original equations.

$2x + y = 7$
$2 + 5 = 7$ ✓
$x - y = -4$
$1 - 5 = -4$ ✓

Write the answer.

The solution to the simultaneous equations is $x = 1$, $y = 5$.

Example 6

Use a CAS calculator to graph and solve the equations $y = 3x - 2$ and $x + 2y = 10$.

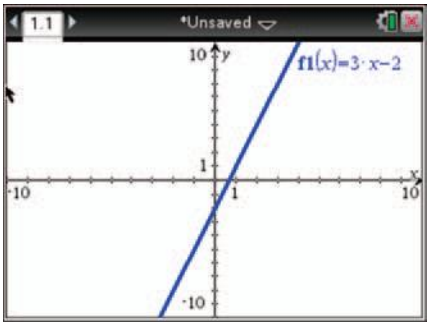
Solution

Write the equation.

$y = 3x - 2$

TI-Nspire CAS

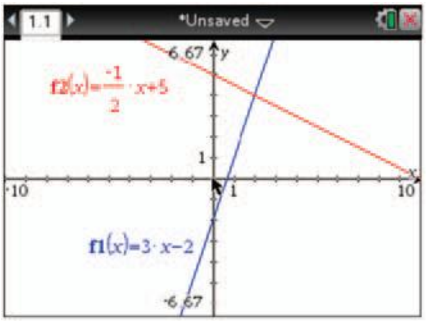
Open a New Document with a Graphs page.
In the entry line with the prompt f1(x) =, type $3x - 2$.
Press enter.



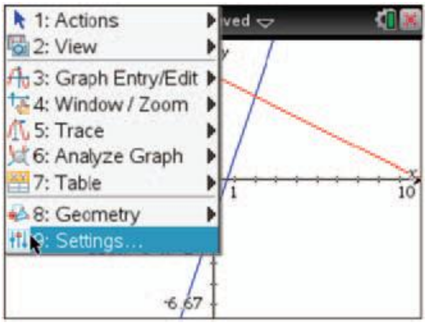
To type in the second equation we need to first write the equation in the form $y = a + bx$.

$x + 2y = 10$
$2y = -x + 10$
$y = -\frac{1}{2}x + 5$

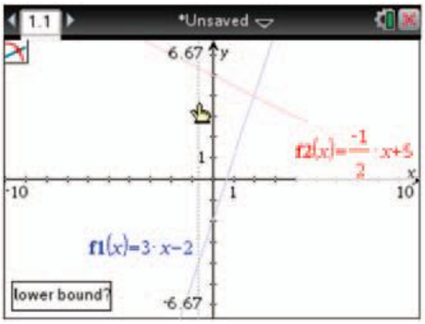
Press ctrl G to view another entry line.
Type in the second equation.
Press enter.
(You can use the arrow ▲▼ keys to move from one entry line to another.)



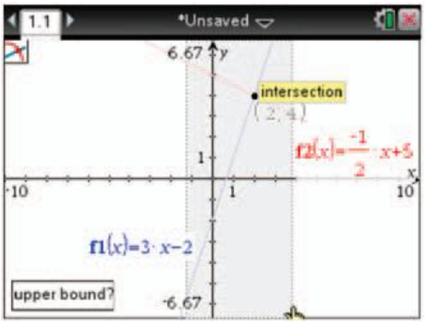
To find the point of intersection:
Press `menu`.
Select: 6: Analyze Graph then
Select: 4: Intersection
Press `enter`.



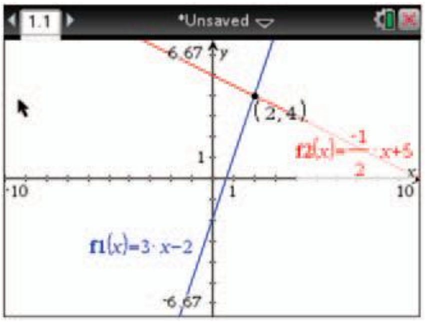
Use the ◀ key to move the lower bound line to the left of the point of intersection and press `enter`.



Use the ▶ key to move the upper bound line to the right of the point of intersection.



Press `enter`.



Use the graph to read the coordinates of the point of intersection.

Point of intersection = (2, 4)

Remember you can check that the solution is correct by substituting back into the original equations.

$y = 3x - 2$
$4 = 6 - 2$ ✓
$x + 2y = 10$
$2 + 8 = 10$ ✓

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Write the answer.

The solution to the simultaneous equations is $x = 2$, $y = 4$.

ClassPad

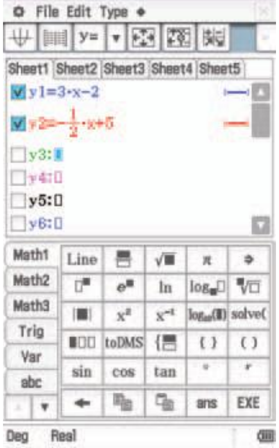
Tap Menu then the **Graph** & Table application.

In the entry line next to $y1$= type 3 $\boxed{x}$ $\boxed{-}$ 2 then press $\boxed{\text{EXE}}$.

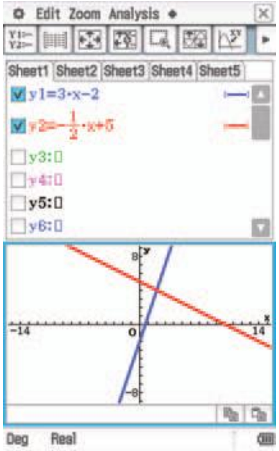
To type in the second equation, we need to first write the equation in the form $y = a + bx$.

$$x + 2y = 10$$
$$2y = -x + 10$$
$$y = -\frac{1}{2}x + 5$$

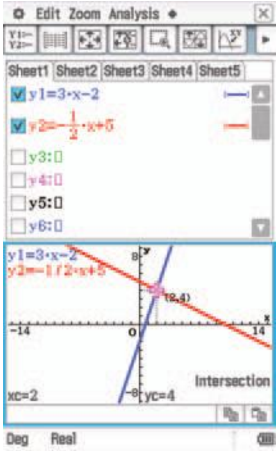
In the entry line next to $y2$= type $-\frac{1}{2}x+5$ then press $\boxed{\text{EXE}}$.



Tap . To ensure that you can see the lines, you may need to tap **Zoom** then **Zoom Out,** or tap to change the maximum and minimum of the x or y values.



Tap the graph screen then tap **Analysis** then **G-Solve** then **Intersection**.



The coordinates of the point of intersection are shown at the bottom of the graph screen.

Point of intersection = (2, 4)

Remember that you can check that the solution is correct by substituting back into the original equations.

$y = 3x - 2$
$4 = 6 - 2$ ✓
$x + 2y = 10$
$2 + 8 = 10$ ✓

Write the answer.

The solution to the simultaneous equations is $x = 2$, $y = 4$.

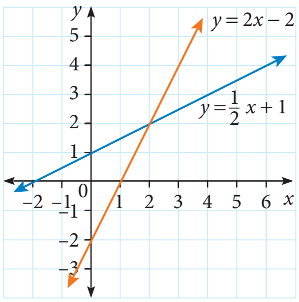
EXERCISE 12.02 Solving simultaneous equations graphically

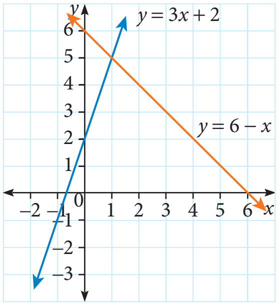
Concepts and techniques

1 Example 4 Find the point of intersection of each pair of simultaneous equations given their graphs below.

a $y = \frac{1}{2}x + 1$
$y = 2x - 2$

b $y = 3x + 2$
$y = 6 - x$





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Sketching simultaneous equations

2 **Example 5** **a** Graph the equations $y = 2x + 2$ and $y = x + 5$ on the same set of axes.

b Use the graphs constructed in part **a** to solve the simultaneous equations $y = 2x + 2$ and $y = x + 5$.

3 Sketch each pair of linear functions to find the point of intersection.

a $y = 2x$
$y = x + 3$

b $y = x + 4$
$y = -x - 6$

c $y = 3x + 4$
$y = x + 1$

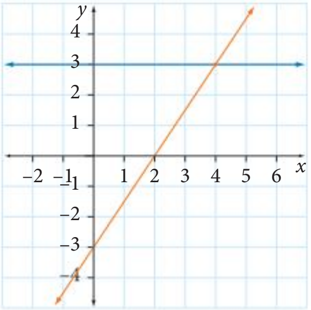
d $y = 2x + 1$
$y = 7 - x$

e $y = -4$
$y = 7x - 4$

f $y = -4x - 1$
$y = 3x + 6$

Remember to check that the solution is correct by substituting back into the original equations.

4 The graph shows the intersection of two linear functions.



a The equations of the lines are:

A $y = \frac{2}{3}x - 3$ and $x = 3$

B $y = \frac{3}{2}x - 3$ and $y = 3$

C $y = -\frac{2}{3}x - 3$ and $y = 3$

D $y = \frac{3}{2}x - 3$ and $x = 3$

E $y = \frac{2}{3}x - 3$ and $y = 3$

b The coordinates of their point of intersection are:

A (0, 3) **B** (2, –3) **C** (4, 3) **D** (2, 0) **E** (3, 4)

5 **Examples 5, 6** Solve the following pairs of simultaneous equations graphically and check the solutions using a CAS.

a $y = x + 3$
$2x - y = -4$

b $x + y = 2$
$x - y = -6$

c $2x + y = 3$
$4x - y = 3$

d $3x - y = 4$
$2x - y = 2$

e $2x - y = -1$
$4x + y + 5 = 0$

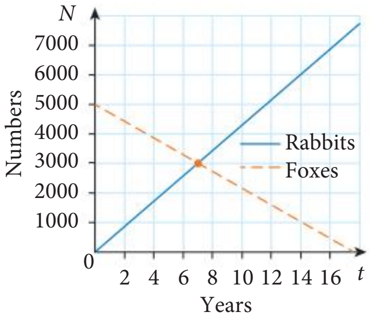
f $5y - 2x = 20$
$2y - 2x = 5$

Reasoning and communication

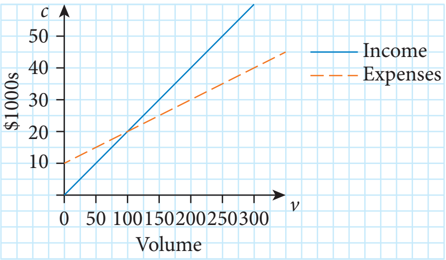
6 The number of rabbits in Blackwood Forest is increasing steadily, while the number of foxes is falling. The graph shows this information.

a When will the numbers of foxes and rabbits be equal?

b How many of each will there be at that time?



7 The graph below shows the income and expenses for the Gosford Petroleum Company.



a What is the point of intersection?

b Write a sentence discussing the value of the expenses and income at this point. What does the point of intersection represent?

8 a Sketch the line: $y=\frac{1}{2}x-4$.

b On the same set of axes, sketch the line $4y = 2x + 12$.

c Comment on the type of lines that have been drawn.

d Will these lines have a point of intersection? Give reasons for your answer.

e How can you determine if two lines will intersect or not intersect without sketching their graphs? (*Hint*: What does the gradient determine when sketching a line?)

9 a Sketch the lines $3x – 2y = 5$ and $6y = 9x – 15$ on the same set of axes.

b What do you notice about the graphs?

c These lines are called **coincident lines**. Using your answers to parts **a** and **b**, write a definition of coincident lines.

12.03 SOLVING SIMULTANEOUS EQUATIONS BY SUBSTITUTION

You have seen how to graphically find solutions for pairs of simultaneous linear equations. This does not always give accurate results and can be time consuming. Algebraic methods can be more accurate and may also provide a solution more quickly.

The first algebraic method is called the **substitution method**.

IMPORTANT

Steps for the substitution method

1 Use one equation to construct a more complex equation that can be used to solve for one variable.

2 Substitute the formula for the variable into the second equation.

3 Solve the second equation.

4 Substitute the solution into one of the original equations.

Check the values in *both* of the original equations.

Example 7

Solve the following simultaneous equations by substitution.

$y = 3 - 2x$

$-5x + 2y = -12$

Solution

Write and label the equations.	$y = 3 - 2x$ $-5x + 2y = -12$	[1] [2]
Substitute equation [1] into equation [2].	$-5x + 2(3 - 2x) = -12$	y from [1] into [2]
Expand the brackets and simplify the terms on the left-hand side.	$-5x + 6 - 4x = -12$ $-9x + 6 = -12$ $-9x = -18$	
Solve for x.	$x = 2$	
Substitute $x = 2$ into equation [1].	$y = 3 - 2 \times 2$	$x = 2$ into [1]
Simplify to solve for y.	$= -1$	
Remember that you can check if the solution is correct by substituting back into the original equations. Substitute (2, −1) into equations [1] and [2].	[1] $-1 = 3 - 2 \times 2$ ✓ [2] $-5x + 2y = -5 \times 2 + 2 \times (-1)$ $= -10 - 2$ $= -12$ ✓	
Write the answer.	The solution to the simultaneous equations is $x = 2$ and $y = -1$.	

In some cases it may be necessary to transpose one equation to write one unknown in terms of the other before using the substitution method.

Example 8

Solve the following simultaneous equations by substitution:

$6x + 3y = 12$

$3x + 2y = -3$

Solution

Write and label the equations.	$6x + 3y = 12$ $3x + 2y = -3$	[1] [2]
Rearrange equation [1] to make y the subject. Label this equation [3].	$3y = 12 - 6x$ $y = 4 - 2x$	Rearrange [1] [3]
Substitute equation [3] into equation [2].	$3x + 2(4 - 2x) = -3$	y from [3] into [2]
Expand the brackets. Simplify the terms on the left-hand side. Solve for x.	$3x + 8 - 4x = -3$ $-x + 8 = -3$ $-x = -11$ $x = 11$	

Substitute $x = 11$ into equation [3].	$y = 4 - 2 \times 11$	$x = 11$ into [3]
Simplify to solve for y.	$y = -18$	
Remember that you can check if the solution is correct by substituting back into the original equations. Substitute (11, −18) into equations [1] and [2].	[1] $6 \times 11 + 3 \times (-18) = 12$ $66 - 54 = 12$ ✓ [2] $3 \times 11 + 2 \times (-18) = -3$ $33 - 36 = -3$ ✓	
Write the answer.	The solution to the simultaneous equations is $x = 11$, $y = -18$.	

EXERCISE 12.03 Solving simultaneous equations by substitution

Concepts and techniques

Solving simultaneous equations

1 Example 7 Find the point of intersection for the lines $y = 3x - 1$ and $5x - 2y = 9$.

2 Solve the following simultaneous equations by substitution.

a $y = 11 - 2x$
$3x - 2y = 6$

b $y = -3 + 2x$
$3x + 2y = 22$

c $y = 2x - 1$
$2x - 3y = 11$

d $y = -4 - x$
$2x - 3y = 7$

e $y = -3x$
$2x + 3y = 14$

f $y = 2x - 12$
$3x + 2y = 4$

g $x = 4 - 5y$
$2y - 3x = 22$

h $x = 17 - 2y$
$3x - y = 2$

3 The equations $y = 3x - 6$ and $y = x + 4$ are solved simultaneously using the substitution method. The solution is:

A $x = -1, y = -9$
B $x = 5, y = 9$
C $x = -1, y = 3$
D $x = 10, y = 14$
E $x = -5, y = -1$

4 Example 8

$$y - x = 2 \quad [1]$$
$$3y - 4x = 12 \quad [2]$$

a Make y the subject in equation [1].

b Use your answer to part **a** to solve the equations using the substitution method.

5 Solve the following simultaneous equations.

a $y - x = 3$
$4x - 3y = 3$

b $2x - y = -10$
$7x - 2y = 1$

c $3x + y = 7$
$5x - 2y = -3$

d $3y - 12x = 9$
$4x - 3y = -1$

e $x + 3y = 23$
$3x - 2y = 3$

f $21x + 7y = 84$
$2x - 5y = -9$

g $2y = 16 - 3x$
$x - 8y = -129$

h $5x - 2y = -3$
$7x + 4y = 40$

Reasoning and communication

6 Using the substitution method, solve the following equations.

$3t - 2d = -12$

$7t - 3d = 48$

7 The following equations represent the cost (C) of running two different dance parties depending on the number of people (n) attending.

$C = 12n$ and $C = 5.8n + 155$

- **a** Solve the equations simultaneously using the substitution method to find the number of people attending when the cost of the parties are the same.
- **b** What is the cost of each party when this occurs?

8 Joseph's solution to the simultaneous equations $y = x + 2$ and $5x - 2y = 5$ is shown below.

$y = x + 2$ [1]

$5x - 2y = 5$ [2]

Substitute [1] into [2].

$5x - 2(x + 2) = 5$

$5x - 2x + 4 = 5$

$3x = 1$

$x = \frac{1}{3}$

Substitute $x = \frac{1}{3}$ into [2].

$y = \frac{1}{3} + 2$

$y = 2\frac{1}{3}$

The solution is $x = \frac{1}{3}$, $y = 2\frac{1}{3}$.

- **a** Identify the step in the working shown where the error was made.
- **b** Solve the equations correctly.

9 The point where the equations $y = 11 - 2x$ and $y = 6$ intersect is:

A $(2\frac{1}{2}, 6)$ **B** $(-5, 6)$ **C** $(6, -1)$ **D** $(-1, 6)$ **E** $(-1, 2\frac{1}{2})$

10 Can you find another line apart from $y = 11 - 2x$, which will give the same point of intersection as your answer to question **9** when passing through the line $y = 6$?

11 **a** Solve the following simultaneous equations.

$y = 11 - 2x$ and

$2y = 22 - 4x$

- **b** Did you find a solution to the simultaneous equations above? If not, why do you think a solution does not exist for this pair of equations?

12.04 SOLVING SIMULTANEOUS EQUATIONS BY ELIMINATION

The next algebraic method of finding simultaneous solutions is called the **elimination method**. In this method we add suitable multiples of the equations to *eliminate* one variable at a time.

IMPORTANT

Steps for the elimination method

1. Choose which variable to eliminate.
2. Make the coefficients of that variable the same size, but opposite in sign, by multiplying by suitable numbers.
3. Add the equations to eliminate the variable.
4. Solve the new equation.
5. Substitute the solution into either of the original equations and solve for the other variable *or* repeat the elimination method for the other variable.
6. Check the values in *both* of the original equations.

Example 9

Solve the following simultaneous equations by elimination.

$2x - y = 0$

$3x + y = 10$

Write down and label the equations.	$2x - y = 0$ $3x + y = 10$	[1] [2]
We will eliminate y first. The coefficients of y are the same and the signs are different. Eliminate y by adding equations [1] and [2].	$5x = 10$	[1] + [2]
Solve for x.	$x = 2$	
Substitute $x = 2$ into equation [1].	$2 \times 2 - y = 0$	$x = 2$ into [1]
Simplify. Subtract 4 from both sides. Solve for y.	$4 - y = 0$ $-y = -4$ $y = 4$	
Remember to check that the solution is correct by substituting back into the original equations. Substitute (2, 4) into equations [1] and [2].	[1] $2 \times 2 - 4 = 0$ $4 - 4 = 0$ ✓ [2] $3 \times 2 + 4 = 10$ $6 + 4 = 10$ ✓	
Write the answer.	The solution to the simultaneous equations is $x = 2$, $y = 4$.	

Sometimes the coefficient of one of the variables needs to be changed to the same size and opposite sign before we can eliminate the variable and then solve the simultaneous equations. This can be achieved by performing an operation on one of the equations as shown in Example **10a**. In some instances you may need to change both equations as shown in Example **10b**. In some instances you may need to subtract equation 2 from equation 1 to eliminate the variable as shown in Example **10c**. You must always examine the equations carefully and select the correct method to use.

Example 10

Solve the following simultaneous equations by elimination.

a $2x + y = 12$ $3x - 4y = 7$	**b** $4x + 3y = -12$ $6x + 2y = 8$	**c** $2x + y = 9$ $2x + 3y = 15$

a Write and label the equations.

$2x + y = 12$ [1]
$3x - 4y = 7$ [2]

We will eliminate y first.
Multiply equation [1] by 4 and label the new equation [3].

$4 \times [1]$
$8x + 4y = 48$ [3]

Eliminate y by adding equations [2] and [3].

$11x = 55$ [2] + [3]

Solve for x.

$x = 5$

Substitute for $x = 5$ into equation [2].

$3 \times 5 - 4y = 7$ $x = 5$ into [2]

Simplify.
Subtract 15 from both sides.
Solve for y.

$15 - 4y = 7$
$-4y = -8$
$y = 2$

Remember to check that the solution is correct by substituting back into the original equations.
Substitute (5, 2) into equations [1] and [2].

[1] $2 \times 5 + 2 = 12$
$10 + 2 = 12$ ✓
[2] $3 \times 5 - 4 \times 2 = 7$
$15 - 8 = 7$ ✓

Write the answer.

The solution to the simultaneous equations is $x = 5$, $y = 2$.

b Write and label the equations.

$4x + 3y = -12$ [1]
$6x + 2y = 8$ [2]

We will eliminate y first.
Multiply equation [1] by 2 and label the new equation [3].

$2 \times [1]$
$8x + 6y = -24$ [3]

Multiply equation [2] by –3 and label the new equation [4].

$-3 \times [2]$
$-18x - 6y = -24$ [4]

Eliminate y by adding equations [3] and [4].

$-10x = -48$ [3] + [4]

Solve for x.

$$\frac{-10}{-10}x = \frac{-48}{-10}$$

$$x = 4\frac{8}{10} \text{ or } 4\frac{4}{5} = 4.8$$

	Substitute for $x = 4.8$ into equation [1]. Simplify and solve for y.	$4x + 3y = -12$ $x = 4.8$ into [1] $4(4.8) + 3y = -12$ $19.2 + 3y = -12$ $3y = -12 - 19.2$ $3y = -31.2$ $\frac{3y}{3} = \frac{-31.2}{3}$ $y = -10.4$
	Remember you can check that the solution is correct by substituting back into the original equations. Substitute (4.8, –10.4) into equations [1] and [2].	[1] $4x + 3y = -12$ $4(4.8) + 3(-10.4) = -12$ $19.2 + (-31.2) = -12$ $-12 = -12$ ✓ [2] $6x + 2y = 8$ $6(4.8) + 2(-10.4) = 8$ $28.8 + (-20.8) = 8$ $8 = 8$ ✓
	Write the answer.	The solution to the simultaneous equations is $x = 4.8$, $y = -10.4$.
c	Write and label the equations.	$2x + y = 9$ [1] $2x + 3y = 15$ [2]
	We will eliminate x first. Subtract equation [2] from equation [1].	$-2y = -6$ [1] – [2]
	Solve for y.	$\frac{-2y}{-2} = \frac{-6}{-2}$ $y = 3$
	Substitute $y = 3$ into equation [2]. Simplify and solve for x.	$2x + 3y = 15$ $y = 3$ into [2] $2x + 3(3) = 15$ $2x + 9 = 15$ $2x = 15 - 9$ $2x = 6$ $\frac{2x}{2} = \frac{6}{2}$ $x = 3$
	Remember that you can check if the solution is correct by substituting back into the original equations. Substitute (3, 3) into equations [1] and [2].	[1] $2x + y = 9$ $2(3) + 3 = 9$ $6 + 3 = 9$ $9 = 9$ ✓ [2] $2x + 3y = 15$ $2(3) + 3(3) = 15$ $6 + 9 = 15$ $15 = 15$ ✓
	Write the answer.	The solution to the simultaneous equations is $x = 3$, $y = 3$.

Example 11

Solve the equations

$y = 2x + 1$

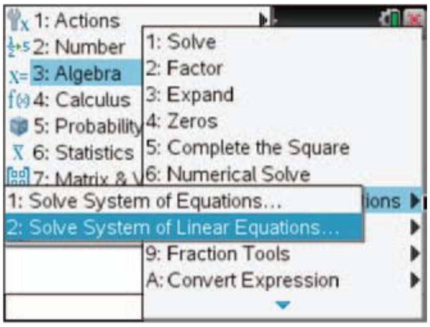
$4x + y + 5 = 0$

using a CAS calculator.

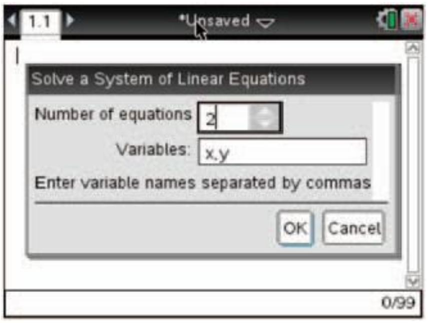
Solution

TI-Nspire CAS

Open a New Document with a Calculator page.
Press menu.
Select 3: Algebra.
Select 7: Solve System of Equations.
Select 2: Solve System of Linear Equations.



Complete the windows shown on the screen.
When OK is highlighted, press enter.



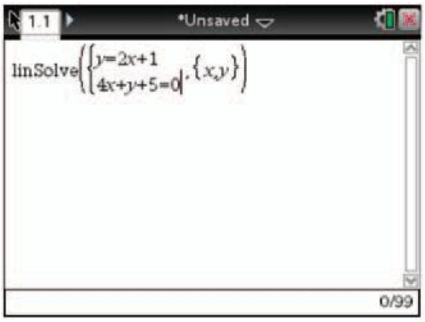
Solve a System of Linear Equations

Number of equations: 2

Variables: x,y

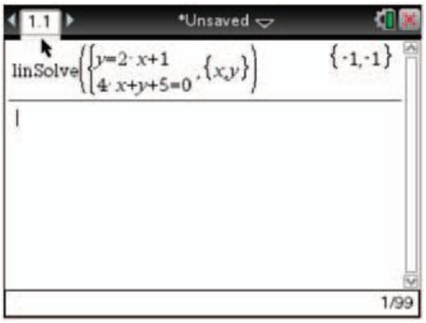
Enter variable names separated by commas

Enter the equations into the boxes as shown.
You can move between boxes by using the tab key.



$$\text{linSolve}\left(\begin{cases} y=2x+1 \\ 4x+y+5=0 \end{cases}, \{x,y\}\right)$$

Press enter to show the solution:
$x = -1, y = -1.$



$$\text{linSolve}\left(\begin{cases} y=2\cdot x+1 \\ 4\cdot x+y+5=0 \end{cases}, \{x,y\}\right) \quad \{-1,-1\}$$

Remember that you can check if the solution is correct by substituting back into the original equations.
Substitute (−1, −1) into equations [1] and [2].

[1] $-1 = 2 \times (-1) + 1$ ✓

[2] $4 \times (-1) - 1 + 5 = 0$

$-4 + 4 = 0$ ✓

Write the answer.

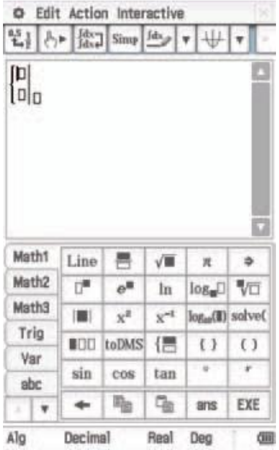
The solution to the simultaneous equations is $x = -1, y = -1$.

ClassPad

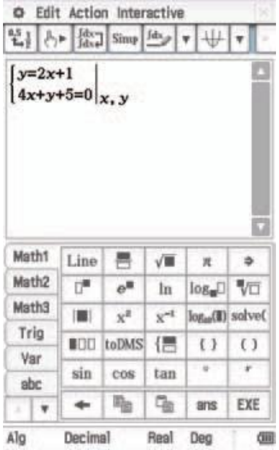
Use the Main √α application.

Press $\boxed{\text{Keyboard}}$

then tap $\boxed{\{▤}$.

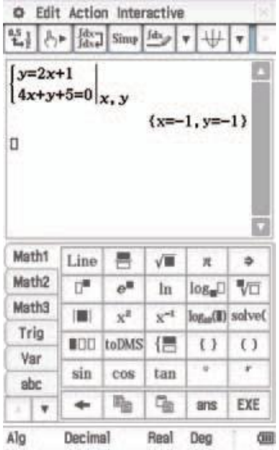


In the top line type $\boxed{y}$ $\boxed{=}$ 2$\boxed{x}$ $\boxed{+}$ 1.
In the bottom line type 4$\boxed{x}$ $\boxed{+}$ $\boxed{y}$ $\boxed{+}$ 5 $\boxed{=}$ 0.
After the vertical line, in the box provided by the template, type $\boxed{x}$ $\boxed{,}$ $\boxed{y}$.



$\begin{cases} y=2x+1 \\ 4x+y+5=0 \end{cases}\Big|_{x,\,y}$

Press $\boxed{\text{EXE}}$.



$\begin{cases} y=2x+1 \\ 4x+y+5=0 \end{cases}\Big|_{x,\,y}$

{x=-1, y=-1}

Remember that you can check if the solution is correct by substituting back into the original equations.
Substitute (−1, −1) into equations [1] and [2].

[1] $-1 = 2 \times (-1) + 1$ ✓
[2] $4 \times (-1) - 1 + 5 = 0$
$-4 + 4 = 0$ ✓

Write the answer.

The solution to the simultaneous equations is $x = -1$, $y = -1$.

EXERCISE 12.04 Solving simultaneous equations by elimination

Concepts and techniques

Solving simultaneous equations

1 **Example 9** Solve the simultaneous equations below using the elimination method.
$2x + y = -5$
$5x - y = 19$

2 The equations $7x - y = -8$ and $-5x - y = 16$ are to be solved simultaneously using the elimination method.

a The equation that results when one variable is eliminated is:
A $2x = 8$
B $12x = -24$
C $12x = 8$
D $2x = -24$
E $2x = -8$

b The point of intersection between the two lines is:
A (4, 36)
B (−12, −44)
C (−2, 11)
D (−4, −36)
E (−2, −6)

3 Solve the following simultaneous equations by elimination.

a $x + y = 3$
$3x - y = -15$

b $3x + y = 3$
$2x - y = 7$

c $3x + 2y = 12$
$4x - 2y = 2$

d $-5x + 5y = 25$
$3x - 5y = -21$

e $-6x - 2y = 22$
$3x + 2y = -16$

f $-3x + 2y = -6$
$3x + y = 18$

g $-5x + 2y = 16$
$5x + 7y = 11$

h $3x - 4y = -13$
$-3x - 4y = -19$

4 **Example 10** For parts **a** and **b** use the simultaneous equations below.

$$-3x + 2y = -3 \quad [1]$$
$$x + 3y = 23 \quad [2]$$

a The equations are to be solved using the elimination method. To eliminate the y variable by adding the equations we must first:
A multiply equation [1] by −3 and multiply equation [2] by −3
B multiply equation [1] by 3 and multiply equation [2] by 2
C multiply equation [1] by 2 and multiply equation [2] by 2
D multiply equation [1] by −3 and multiply equation [2] by 2
E multiply equation [1] by 1 and multiply equation [2] by 3

b The point of intersection is:

A (5, –6) **B** (–6, –5) **C** (5, 6) **D** $\left(5\frac{1}{5}, 6\frac{11}{13}\right)$ **E** (4, 11)

5 Solve the following simultaneous equations by elimination.

a $3x + 4y = -5$
$x - y = 3$

b $3x - 4y = 8$
$2x + y = 9$

c $2x + 3y = 5$
$3x + 5y = 9$

d $2x + 3y = -6$
$3x + y = 5$

e $x + 2y = 3$
$2x + y = 9$

f $2x + 3y = 10$
$4x - 2y = -4$

g $2x - 9y = -7$
$3x + 2y = 36$

h $4x - 3y = 11$
$3x - 4y = 17$

6 Example 11 Solve the following simultaneous equations.

a $3x + 2y = 14$
$4x - 2y = 7$

b $-2x + 5y = 20$
$-2x + 2y = 5$

c $3x - 2y = 10$
$-3x - 2y = -14$

d $2x + 3y = 11$
$5x - 2y = -1$

e $4x + 7y = 10$
$2x - 5y = -46$

f $2x - 3y = 13$
$3y - x = -8$

Reasoning and communication

7 **a** Solve the following cost equations simultaneously by using the elimination method.
$C = 60n$
$C = 20n - 256$

b Is the elimination method the most appropriate method to use when solving these simultaneous equations? Give reasons for your answer.

8 Jack and Jenny solved the simultaneous equations below by elimination, but obtained two different answers.
$4x - 3y = -4$ and
$6x - 4y = 9$
Jack's answer was $x = 21\frac{1}{2}, y = 30$ whilst Jenny insisted that $x = 5\frac{1}{2}, y = 8\frac{2}{3}$
Solve the equations to find who was correct.

9 Solve the following simultaneous equations.

a $5(2x - y) + 4y = 3(3x + 1)$
$5(2x - 1) = 5x - 2y + 38$

b $\frac{x}{2} + y = 5$
$\frac{3x}{4} + \frac{y}{3} = 4$

c $\frac{2x}{3} = 13 + y$
$3x - \frac{3y}{2} = 7$

12.05 PRACTICAL APPLICATIONS OF SIMULTANEOUS EQUATIONS

Simultaneous equations can be used to solve problems involving linear modelling. You may be required to draw the graphs for two equations on the same set of axes and use the graph to solve the problem.

Example 12

The income and expenses equations for the Superman Products Company are given below.

Expenses: $V = 15x$

Income: $V = 2000 + \frac{5x}{3}$

where V is measured in thousands of dollars and x is the number of units sold.

a Graph both equations on the same set of axes. (Use values of x from 0 to 300).

b Find the break-even point for the company.

The break-even point is the point where the expenses equal the income.

Solution

a Write the first equation.
Identify its gradient and vertical intercept.

$V = 15x$

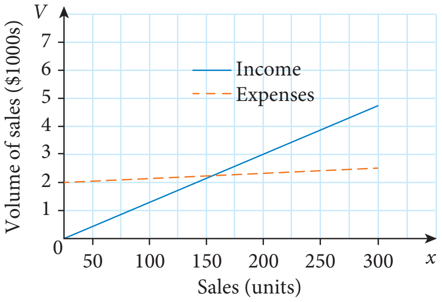
Gradient = 15, vertical intercept = 0

Write the second equation.
Identify its gradient and vertical intercept.

$V = 2000 + \frac{5x}{3}$

Gradient = $\frac{5}{3}$, vertical intercept = 2000

Graph both lines on the same set of axes. Remember to use an appropriate scale on the horizontal and vertical axis.



Alternatively, the graphs could also be drawn by finding two points on each line.

For expenses:

if $x = 0$, $V = 15 \times 0 = 0$, so (0, 0)

if $x = 300$, $V = 15 \times 300 = 4500$, so (300, 4.5)

For Income:

if $x = 0$, $V = 2000 + \frac{5}{3} \times 0 = 2000$, so (0, 2)

if $x = 300$, $V = 2000 + \frac{5}{3} \times 300 = 2500$, so (300, 2.5)

b The break-even point is where the lines intersect.
Use the graph to identify the point of intersection.

Point of intersection = (150, 2250)

Write the answer.

The break-even point is at sales of 150 units, where both the income and expenses are $2250.

Sometimes it is necessary to construct simultaneous linear equations from a worded problem.

Steps for solving worded problems

1. Read the question carefully.
2. Identify the unknowns and assign a pronumeral for the variables.
3. Set up the equations by converting the given information into mathematical sentences.
4. Solve the simultaneous equations using the most appropriate method.
5. Check the values in *both* of the original equations.
6. Answer the original question in sentence form.

Example 13

The length of a rectangle is 5 cm longer than its width. If the perimeter of the rectangle is 38 cm, find its dimensions.

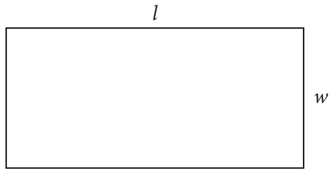
Solution

Define the variables.

Let l = length and w = width.

You can dra

Remember that the formula for the perimeter of a rectangle is: $P = 2l + 2w$.



Use the given information to write two linear equations. Label the equations [1] and [2].

$l = w + 5$ [1]

$38 = 2l + 2w$ [2]

Equation [1] is written in the form $y = a + bx$, so the substitution method is the most appropriate method to use. Substitute equation [1] into equation [2].

$38 = 2(w + 5) + 2w$

l from [1] into [2]

Expand the brackets and solve for w.

$38 = 2w + 10 + 2w$

$38 = 4w + 10$

$28 = 4w$

$7 = w$

$w = 7$

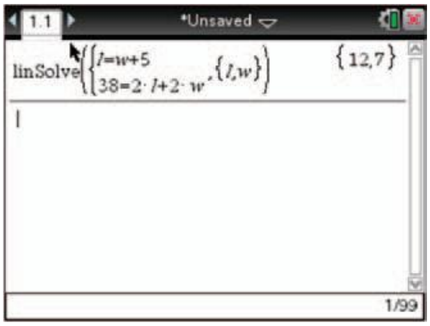
Substitute $w = 7$ into equation [1] to find the value of l.

$l = 7 + 5$ $w = 7$ into [1]

$l = 12$

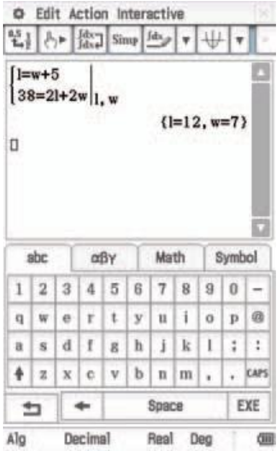
Alternatively, solve using a CAS calculator.

TI-Nspire CAS



$$\text{linSolve}\left(\begin{cases} l=w+5 \\ 38=2\cdot l+2\cdot w \end{cases},\{l,w\}\right) \qquad \{12,7\}$$

ClassPad



$$\begin{cases} l=w+5 \\ 38=2l+2w \end{cases}\Big|_{l,\,w} \qquad \{l=12, w=7\}$$

Remember to check that the solution is correct by substituting back into the original equations. Substitute $l = 12$ and $w = 7$ into equations [1] and [2].

[1] $12 = 7 + 5$ ✓

[2] $38 = 2 \times 12 + 2 \times 7$

$38 = 24 + 14$ ✓

Write the answer.

The length of the rectangle is 12 cm and the width of the rectangle is 7 cm.

Cost equations are common examples of practical applications of simultaneous equations.

Always check that variables are given in the same units. Convert units where necessary.

Example 14

Toni spent $16.40 on pens and pencils. She purchased a total of 8 items. If the pens cost $2.80 each and the pencils cost 80 cents each, how many pens and pencils did Toni purchase?

Solution

Define the variables.

Let p = number of pens

q = number of pencils

Use the given information to write two linear equations. Label the equations [1] and [2]. Remember to convert the 80 cents to $0.80.

$p + q = 8$ [1]

$2.8p + 0.8q = 16.4$ [2]

The coefficients of the variables are decimals, so it is easier to use the elimination method.
We will eliminate q first.
Multiply equation [1] by –0.8 and label the new equation [3].

$-0.8 \times$ [1]

$-0.8p - 0.8q = -6.4$ [3]

Eliminate q by adding equations [2] and [3].

$2p = 10$ [2] + [3]

Solve for p.

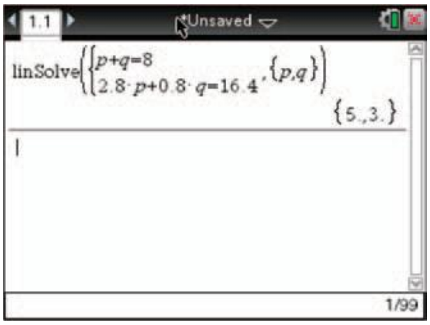
$p = 5$

Substitute $p = 5$ into equation [1].
Solve for q.

$$5 + q = 8 \qquad p = 5 \text{ into } [1]$$
$$q = 3$$

Alternatively, a CAS calculator can be used to solve the equations.

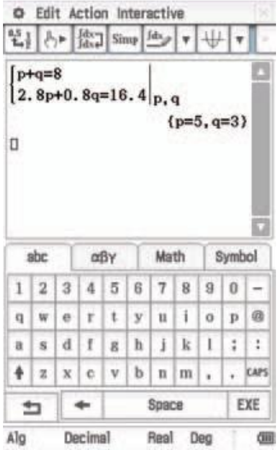
TI-Nspire CAS



$\text{linSolve}\left(\begin{cases} p+q=8 \\ 2.8\cdot p+0.8\cdot q=16.4 \end{cases},\{p,q\}\right)$

$\{5.,3.\}$

ClassPad



$\begin{cases} p+q=8 \\ 2.8p+0.8q=16.4 \end{cases} \Big| p,q$

$\{p=5, q=3\}$

Remember to check that the solution is correct by substituting back into the original equations. Substitute (5, 3) into equations [1] and [2].

[1] $5 + 3 = 8$ ✓

[2] $2.8 \times 5 + 0.8 \times 3 = 16.4$

$14 + 2.4 = 16.4$ ✓

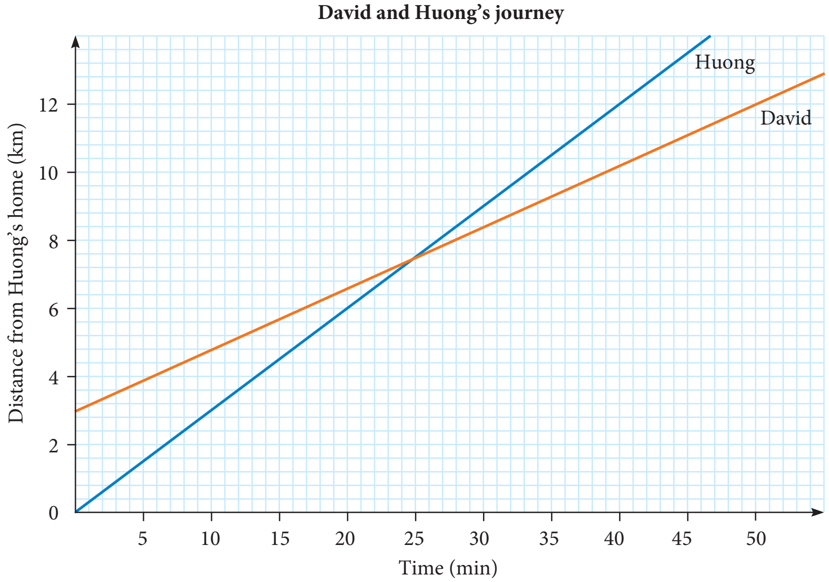
Write the answer.

Toni purchased 5 pens and 3 pencils.

EXERCISE 12.05 Practical applications of simultaneous equations

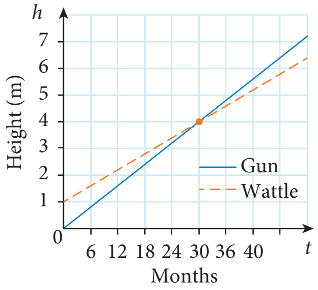
Concepts and techniques

1 Example 12 David jogged to the park while Huong rode her bike. David was already 3 km ahead of Huong when she started from home, and jogged at 10.8 km/h, while Huong rode at a speed of 18 km/h. The travel graph below shows their journeys, with the distances from Huong's home shown.



- **a** Did Huong catch up to David? If so, when?
- **b** If the park is 12 km from Huong's home, who got there first and by how many minutes?

2 The graph shows the growth of 2 different types of trees, measured from the time they were seedlings.



- **a** Which tree was taller as a seedling and what was the difference in height?
- **b** After how many months do the trees reach the same height?
- **c** What is that height?
- **d** Find an equation for the height of the:
  - **i** gum tree
  - **ii** wattle tree.
- **e** Are these equations good models for the height of the trees after 10 years? Why or why not?

Use the following information to answer questions **3** and **4**.

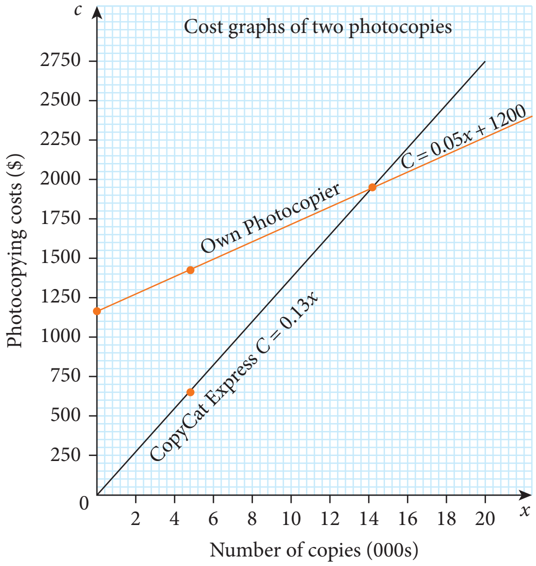
At CopyCat Express, the charge for photocopying is 13 cents per sheet. Klaus is investigating the purchase of his own photocopier for $1200, which brings the cost down to 5 cents per sheet. The two cost functions are:

CopyCat Express: $C = 0.13x$

Own photocopier: $C = 0.05x + 1200$

where C is in dollars and x represents the number of copies made.

The graph of the functions is shown below.



3 Which option is cheaper for making 5000 copies and by what amount?

A Own photocopier by $1450

B CopyCat Express by $650

C Own photocopier by $800

D CopyCat Express by $800

E None of the above

4 What are the coordinates of the break-even point?

A (16 000, 2080)

B (15 000, 1950)

C (0, 1200)

D (21 000, 2730)

E (0, 0)

5 **Example 13** The length of a rectangle is three times as long as its width. If the perimeter of the rectangle is 52 cm, find the dimensions of the rectangle.

6 The sum of two consecutive numbers is 93. Find the numbers.

7 The sum of two numbers is 39. The larger number is 5 more than the smaller number. What are the numbers?

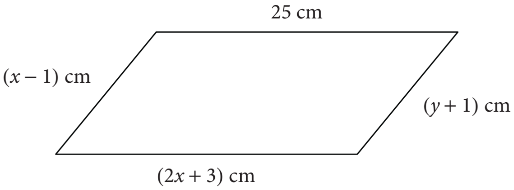
8 The average of two numbers is 18. When 2 times the first number is added to 5 times the second number, the result is 120. Find the two numbers.

9 Petra is four times as old as her daughter Philippa. In 8 years time she will be five times as old as Philippa. How old are Philippa and Petra?

10 a Find the values of x and y.

b State the dimensions of the parallelogram.

c Find the perimeter of the parallelogram.



11 Example 14 Jan saves money regularly, while Tony spends his money. The amount they have in the bank is given by the simultaneous equations below, where a = the amount of money (in \$) and m = number of months.

Jan: $2a - 5m = 20$

Tony: $a + 5m = 40$

Solve the equations simultaneously to find the number of months it will take for them to have the same amount of money in the bank. What is this amount?

12 Beachside Ice-creams sells ice-cream in single cones and double cones. The price of a single cone is \$3.80, whereas the price of a double cone is \$5.20. On a particular day, they sold a total of 450 cones and their takings for the day were \$1990. How many of each type of cone did Beachside Ice-creams sell on the day?

13 Jenni is comparing two Internet cafes. QuickByte has a monthly membership fee of \$8 and charges 60 cents per hour of Internet use. Game Hunters charges \$1 per hour of Internet use but has no monthly access fee. The two costs are represented by the formulas:

QuickByte: $C = 0.6t + 8$

Game Hunters: $C = t$

where C is in dollars and t is the number of hours of Internet use.

Alamy/David R.Frazier Photolibrary, Inc.

a If Jenni uses an Internet cafe for 24 hours per month, which cafe is better for her? Why?

b For what number of hours is the cost the same for both plans? What is this cost?

14 Geoff wants to hire a car over the three days of a long weekend. There are two rental companies he can choose from.

Shifty: \$45 per day plus \$1.85 per kilometre

Megahertz: \$80 per day plus \$1.55 per kilometre

a If d is the distance travelled in kilometres, and C is the cost in dollars, write a formula representing the cost of hiring a car for three days from:

i Shifty ii Megahertz

b After how many kilometres will the cost for hiring a car be the same for both companies? What is the cost at this time?

c If Geoff plans to drive an average of 100 km each day on the long weekend, which plan would be better for him and what would be his cost? Give reasons for your answer.

Reasoning and communication

15 The cost, $\$C$, of making n yo-yos is given by the formula $C = 3n + 90$, while the revenue, $\$R$, from selling n yo-yos is given by the formula $R = 5n$.

a Graph both formulas on the same axes, for values of n from 0 to 50.

b What is the cost of making 20 yo-yos?

c What is the revenue from selling 20 yo-yos?

Revenue means income or money received.

d What is the loss from making 20 yo-yos?

e How much does one yo-yo sell for?

f What is the break-even point? (Hint: For what value of n does the revenue equal the cost?)

16 Megan and Jane both sell kitchenware and earn commissions. Megan's commission is \$240 plus 5% of her sales, while Jane's commission is 20% of her sales (with no retainer). These rates can be expressed by the formulas below.

Megan's commission: $C = 0.05x + 240$

Jane's commission: $C = 0.2x$

where \$$x$ is the total value of sales and \$$C$ is the commission.

- a Graph both commission formulas on the same axes, for values of x from \$0 to \$2400.
- b If each person sells \$1200 worth of kitchenware, who earns more?
- c For what value of sales does Megan earn the same amount as Jane?
- d What is the value of their commissions at this point?
- e For what value of sales does Megan earn more than Jane?

17 Dave claims that a quick 'rule-of-thumb' for converting Celsius temperatures to Fahrenheit is 'double it and add 30'.

- a Graph Dave's rule and the actual conversion formula for values of C from 0 to 100.

  Dave's rule: $F = 2C + 30$

  Actual conversion formula: $F = \frac{9}{5}C + 32$

  where C represents the temperature in °C and F represents the temperature in °F.
- b For what value of C is Dave's rule exactly equal to the actual conversion formula?
- c For what values of C does Dave's rule give answers that are too high?
- d For what values of C is the difference between the two formulas not more than 5°F?

18 Mathew purchases 4 CDs and 2 DVDs and pays \$86. Dion buys 3 CDs and 5 DVDs and pays \$131.

- a Write an equation which represents Mathew's purchases.
- b Write an equation which represents Dion's purchases.
- c Solve the equations above simultaneously to find the price of each CD and each DVD.

INVESTIGATION Taxi charges

Burntrubber Taxis charges \$4.60 flag fall plus 90 cents per kilometre. Their rival company, Whiteknuckle Cabs, charges \$2.50 flag fall plus \$1.20 per kilometre.

- a If C represents the dollars charged and k represents the kilometres travelled, write an equation that represents the trip charges for each company.
- b Which variable is the dependent variable and which one is the independent variable?
- c Graph both formulas for values of k from 0 to 11 on the same set of axes.
- d State the vertical intercept for both lines.
- e How much will each company charge to travel the following distances?

  i 2 km ii 5 km iii 12 km

f There is no value for $k = 12$ on the horizontal axis. How can you determine the cost of travelling 12 kilometres using the graph?

g Use the graph to find their point of intersection.

h What does the point of intersection represent?

i Solve the equations simultaneously using one of the algebraic methods (substitution or elimination). Does your solution agree with your answer to part **g**? Give reasons for your answer.

j When is each company cheaper to use?

k Jake travels to and from work each day by taxi, a distance of 11 km each way. Determine which is the better taxi for him to use, and calculate the saving he would make using it over 5 days.

12.06 DRAWING PIECEWISE GRAPHS

A **piecewise graph** is a combination of two or more straight lines. It is made up of separate line segments (‘pieces’) with different gradients.

For example, we can draw the graph of $y = -3x$ for $-1 \le x < 3$ and then draw the graph of $y = 2x - 1$ for $3 \le x \le 6$. The second line segment starts from where the first line segment finishes. The two line segments have different gradients (or slopes) and form the piecewise linear graph.

The equation of the piecewise linear function above is written as:

$$y = \begin{cases} -3x, & -1 \le x < 3 \\ 2x - 1, & 3 \le x \le 6 \end{cases}$$

Remember:
$<$ means less than
$>$ means greater than
$\le$ means less than or equal to
$\ge$ means greater than or equal to

IMPORTANT

$-1 \le x < 3$ means all the x values between -1 and 3. The -1 is included but 3 is not.

That is, x is greater than or equal to -1 but is less than 3.

Example 15

A piecewise linear equation is given below.

$$y = \begin{cases} -3x, & -1 \leq x < 3 \\ 2x - 1, & 3 \leq x \leq 6 \end{cases}$$

a Complete a table of values for this equation.

b Use the table of values to draw its graph.

Solution

a There are two parts to the piecewise linear function. Hence, we will need to draw two separate line segments.

Identify the first line segment.

$y = 3x$

Complete a table of values for this equation. We only require the x values between −1 and 3.

x	−1	0	1	2	3
y	−3	0	3	6	9

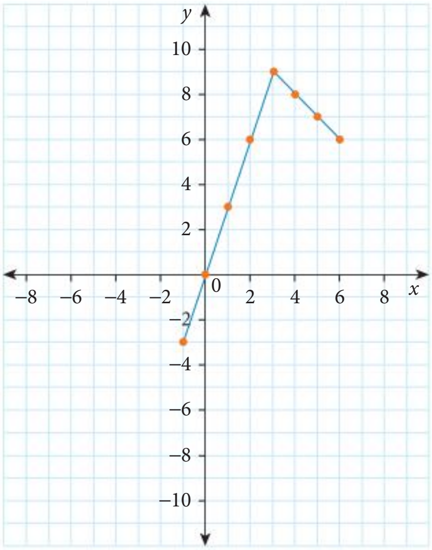
Identify the second line segment.

$y = 12 - x$

Complete a table of values for this equation. We only require the x values between 3 and 6.

x	3	4	5	6
y	9	8	7	6

b Plot the points for $y = 3x$ from the first table of values.
On the same set of axes, plot the points for $y = 12 - x$ from the second table of values.
This line segment begins where the first line segment ended.



Using a table of values to plot a piecewise linear graph can be very time-consuming. You can use your knowledge of sketching linear functions to draw the graph of each line segment in the piecewise linear function. The critical points are the end points of each line segment and therefore we find the coordinates of these points and join them to draw the line segment.

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Example 16

a Sketch the graph of $y = 3x - 2$ for $-3 \leq x < 2$.

b On the same set of axes, sketch the graph of $y = 4$ for $2 \leq x \leq 5$.

c Comment on the graph formed by sketching these two lines on the same set of axes.

Solution

a Write the equation.

$y = 3x - 2$

The graph of $y = 3x - 2$ is only required for x values between -3 and 2. Find the value of y at each of the end points, that is, where $x = -3$ and $x = 2$.

When $x = -3$,

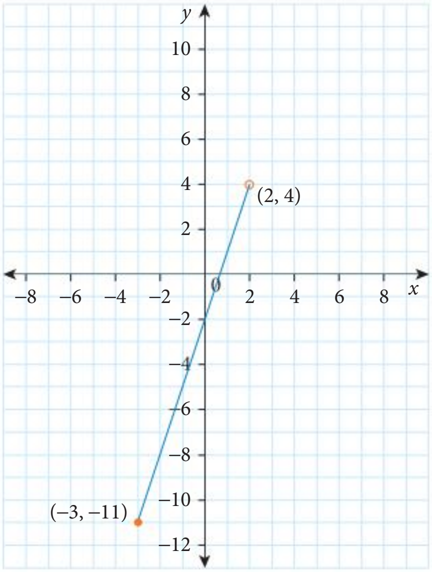
$$\begin{aligned} y &= -9 - 2 \\ &= -11 \end{aligned}$$

When $x = 2$,

$$\begin{aligned} y &= 6 - 2 \\ &= 4 \end{aligned}$$

The coordinates of the end points are $(-3, -11)$ and $(2, 4)$.

Plot the coordinates of the end points: $(-3, -11)$ and $(2, 4)$. Join these points to sketch the line.



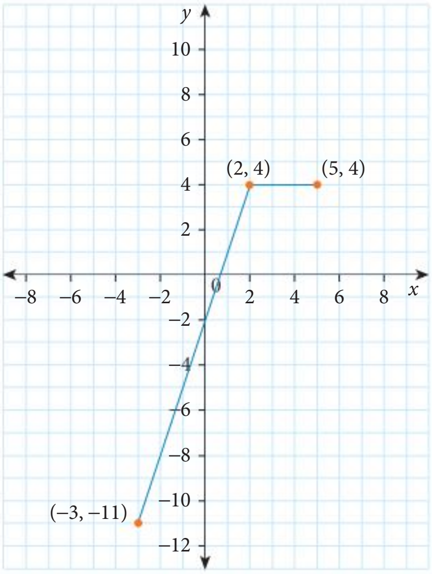
b Write the equation.

$y = 4$

The graph of $y = 4$ is only required for x values between 2 and 5. State the coordinates of the points at $x = 2$ and $x = 5$.

Coordinates of the end points are: $(2, 4)$ and $(5, 4)$.

The graph of $y = 4$ starts where the graph of $y = 3x - 2$ finishes.
Draw a horizontal line from (2, 4) to (5, 4).



c Examine the graphs and write a comment.

The graph is made up of two different line segments.
The first line segment has a positive gradient, hence the graph is increasing.
The coordinates of its end points are (−3, −11) and (2, 4).
The second line segment starts from the point where the first line segment ends, that is, at (2, 4) and finishes at (5, 4). It is a horizontal line.

Example 17

For the piecewise equation

$$y = \begin{cases} 2 - x, & -6 \le x < 1 \\ x, & 1 \le x \le 5 \end{cases}$$

a sketch a graph

b check your answer to part **a** with your CAS calculator.

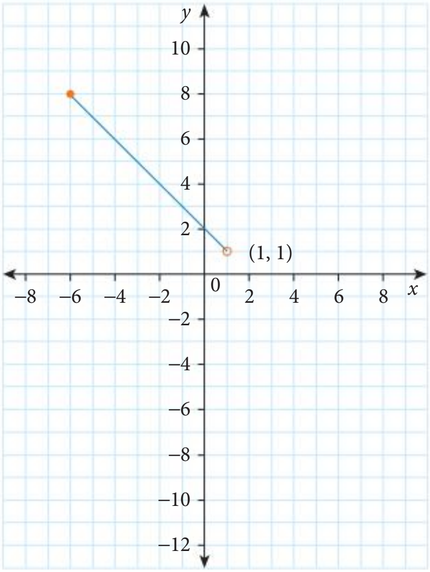
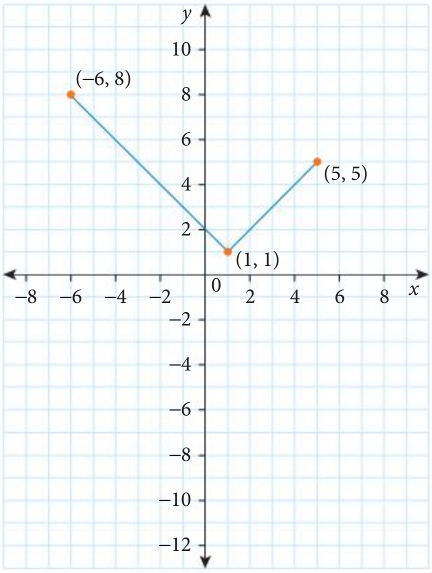
Solution

a Write the equation of the first line.

$y = 2 - x$

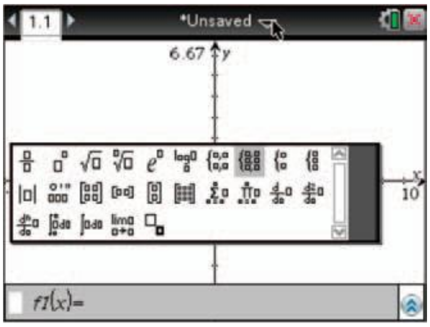
The graph of $y = 2 - x$ is only required for x values between −6 and 1.
Find the value of y at the end points, that is where $x = -6$ and $x = 1$.

When $x = -6$,
$y = 2 + 6$
$= 8$
When $x = 1$,
$y = 2 - 1$
$= 1$
The coordinates of the end points are (−6, 8) and (1, 1).

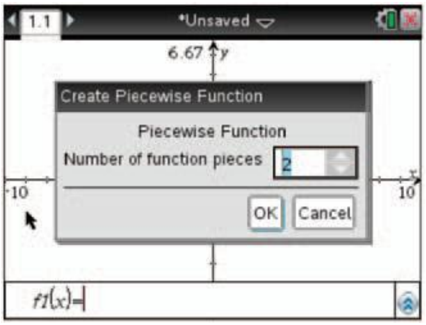
Use the points above to sketch the first line segment.	
Write the equation of the second line segment.	$y = x$
The graph of $y = x$ is only required for x values between 1 and 5. Find the value of y at the end points, that is, where $x = 1$ and $x = 5$.	When $x = 1$, $y = 1$ When $x = 5$, $y = 5$ The coordinates of the end points are (1, 1) and (5, 5).
Use the points above to sketch the second line segment on the same set of axes.	

TI-Nspire CAS

Open a New Document with a Graphs page.
Press `⊞`, then select the piecewise equation template.
Press `enter`.



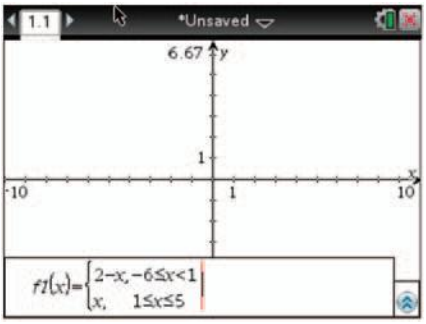
In the pop up screen set the number of function pieces to 2.
Press `tab` so OK is highlighted, then press `enter`.



Piecewise Function

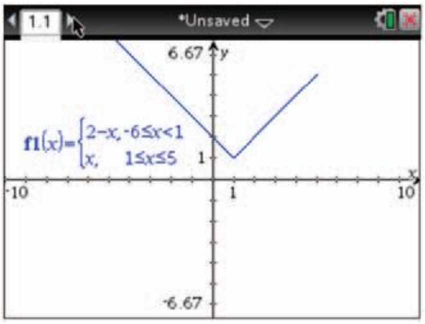
Number of function pieces 2

Type the piecewise equation into the template.
Inequality signs < and ≤ can be found by pressing `ctrl` `=`.

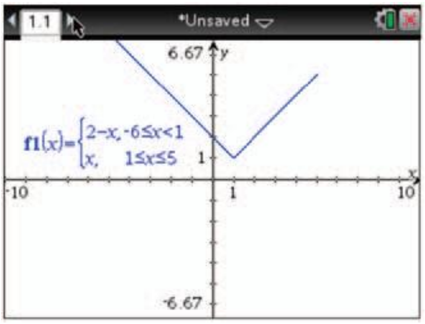


$$f1(x)=\begin{cases}2-x, & -6\le x<1\\ x, & 1\le x\le 5\end{cases}$$

Press `enter`.



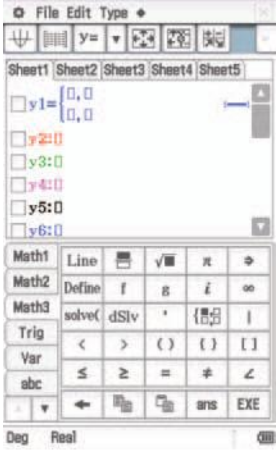
Press menu.
4: Window/Zoom
4: Zoom-Out
Move the cursor (Center?) over the origin and press enter.



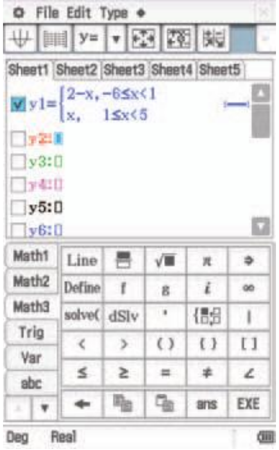
ClassPad

Use the **Graph & Table** application.
With the cursor in the entry line next to *y*1= ,
press Keyboard.
Tap Math3 then the piece-wise equation template.

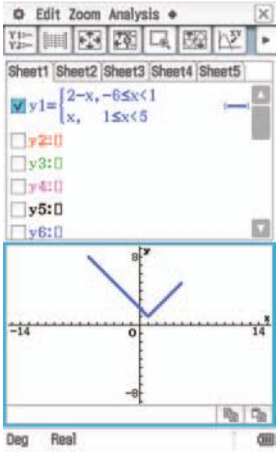
For a piecewise equation with three parts, tap twice to get the correct template.



Type the piecewise equation into the template.
Inequality signs < and ≤ can be found on the screen given by Math3.
Press EXE when you have finished entering the equation.

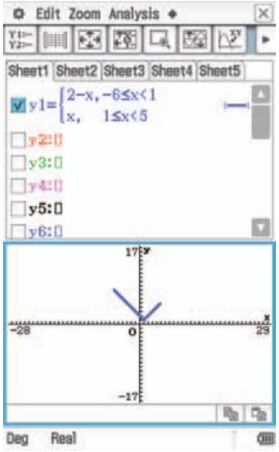


Tap ⊞.



$y1=\begin{cases}2-x, & -6\le x<1\\ x, & 1\le x<5\end{cases}$

Tap **Zoom** then **Zoom Out**.



$y1=\begin{cases}2-x, & -6\le x<1\\ x, & 1\le x<5\end{cases}$

EXERCISE 12.06 Drawing piecewise graphs

Concepts and techniques

1 Example 15 A piecewise linear equation is given below.

$$y=\begin{cases}x+3, & -5\le x<0\\ 3-2x, & 0\le x\le 4\end{cases}$$

a Complete a table of values for the first line segment.

b Complete a table of values for the second line segment.

c Use the tables of values to create a graph.

2 A piecewise graph consists of 3 line segments given by the rule below.

$$y = \begin{cases} -x, & -7 \le x < -3 \\ 3, & -3 \le x < 2 \\ x+1, & 2 \le x \le 7 \end{cases}$$

a Complete a table of values for the rule.

b Create a graph using the points in the table of values.

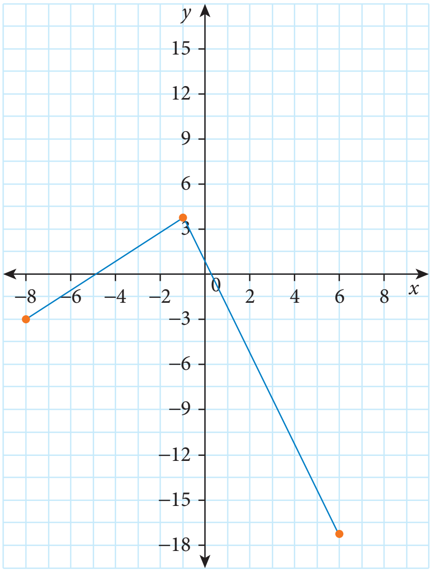
3 **Example 16** **a** Sketch the graph of $y = \frac{1}{2}x$ for $-8 \le x < 2$.

b On the same set of axes, sketch the graph of $y = 1$ for $2 \le x \le 8$.

c Comment on the graph formed by sketching these two lines on the same set of axes.

For questions **4** and **5**, use the graph below.

$$y = \begin{cases} x+5, & -8 \le x < -1 \\ 1-3x, & -1 \le x \le 6 \end{cases}$$



4 The first line segment will be drawn for all x values that fall between −8 and −1 but ...

- **A** does not include the points where $x = -8$ and $x = -1$
- **B** does not include the point where $x = -8$, but includes the point where $x = -1$
- **C** includes the points where $x = -8$ and $x = -1$
- **D** includes the points where $y = -8$ and $y = -1$
- **E** includes the point where $x = -8$, but does not include the point where $x = -1$.

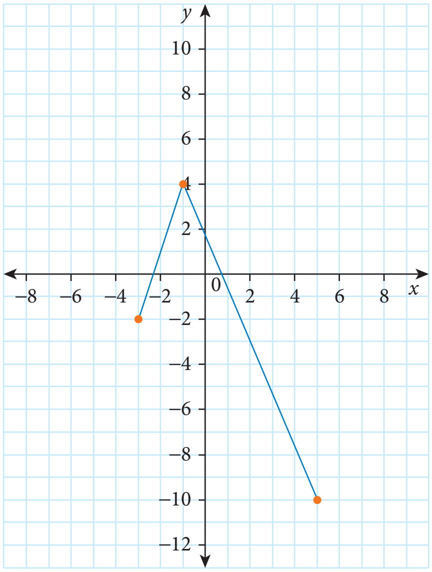
5 The point where the two line segments meet is:

A (−8, −3) **B** (4, −1) **C** (6, −15) **D** (−1, 4) **E** (−1, 10)

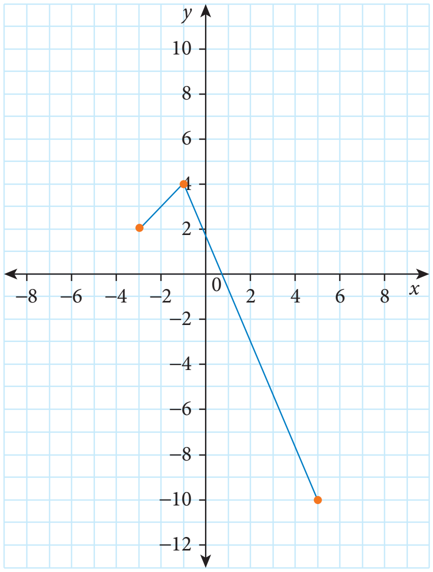
6 Which of the following graphs best represents the piecewise linear equation below?

$$y=\begin{cases}3x+7, & -3\le x<-1\\ x+5, & -1\le x\le 5\end{cases}$$

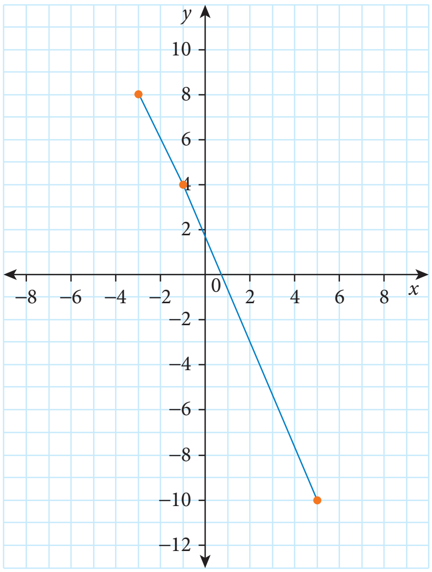
A



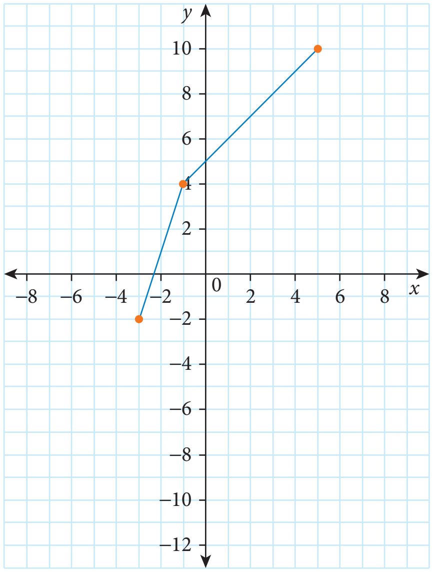
B



C



D



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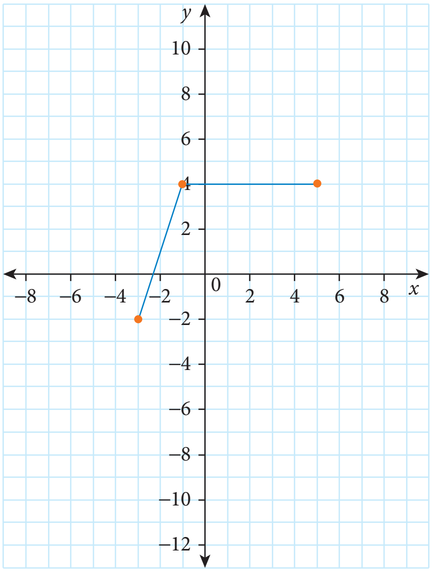
E



7 Example 17 Sketch a graph for each of the following rules.

a $y=\begin{cases}2x-5, & -1\le x<4\\ 3, & 4\le x\le 10\end{cases}$

b $y=\begin{cases}-6, & -6\le x<2\\ -3x, & 2\le x\le 3\end{cases}$

c $y=\begin{cases}-x+5, & -4\le x<2\\ 9-3x, & 2\le x\le 5\end{cases}$

d $y=\begin{cases}7-2x, & -5\le x<1\\ 4x+1, & 1\le x\le 4\end{cases}$

e $y=\begin{cases}x-3, & -7\le x<-1\\ -4, & -1\le x<5\\ 1-x, & 5\le x\le 11\end{cases}$

f $y=\begin{cases}8, & -8\le x<-2\\ -4x, & -2\le x<3\\ 2x-18, & 3\le x<10\end{cases}$

8 Check the accuracy of your graphs in question **7** by using your CAS calculator to draw each piecewise linear graph. Check to see if your answers match.

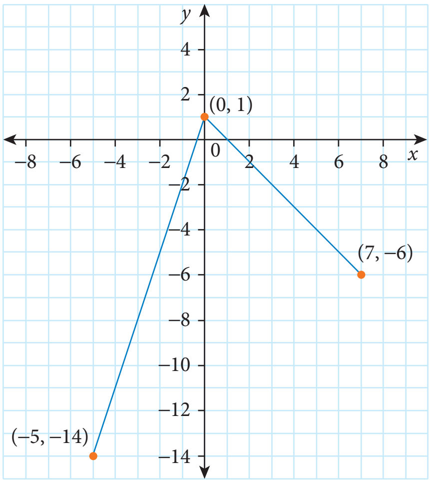
Reasoning and communication

9 **a** The piecewise graph on the right consists of two line segments. Between what values of x does each line segment lie?

b Find the equation of the first line segment.

c Find the equation of the second line segment.

d Using your answers above, write an equation for the piecewise linear graph.



10 The equation below is for a piecewise linear graph

$$y=\begin{cases}2x, & x<1\\ 2, & x\geq 1\end{cases}$$

- a State the equation of the first part of the linear piecewise graph.
- b For what values of x do you need to draw the line segment? How does this differ from the way the previous piecewise linear equations were written?
- c How can you show this when drawing the graph?
- d State the equation of the second part of the linear piecewise graph.
- e For what values of x do you need to draw this line segment?
- f Use your answer from parts **a** to **e** to draw the graph of this equation.

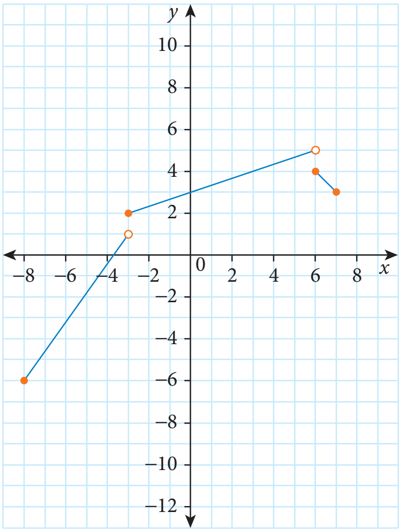
Remember: A line segment has a starting point and an end. A ray has a starting point but no end point.

11 Sketch the piecewise linear graph given by the equation

$$y=\begin{cases}2-x, & x\leq -1\\ 3, & -1<x\leq 2\\ 2x-1, & x\geq 2\end{cases}$$

12 a Examine the graph on the right. How does this graph differ from the previous graphs?

- b How many line segments form this piecewise linear graph?
- c State the values of x used for each line segment.
- d Write a sentence explaining why some of the end points have open circles whilst others have closed circles.



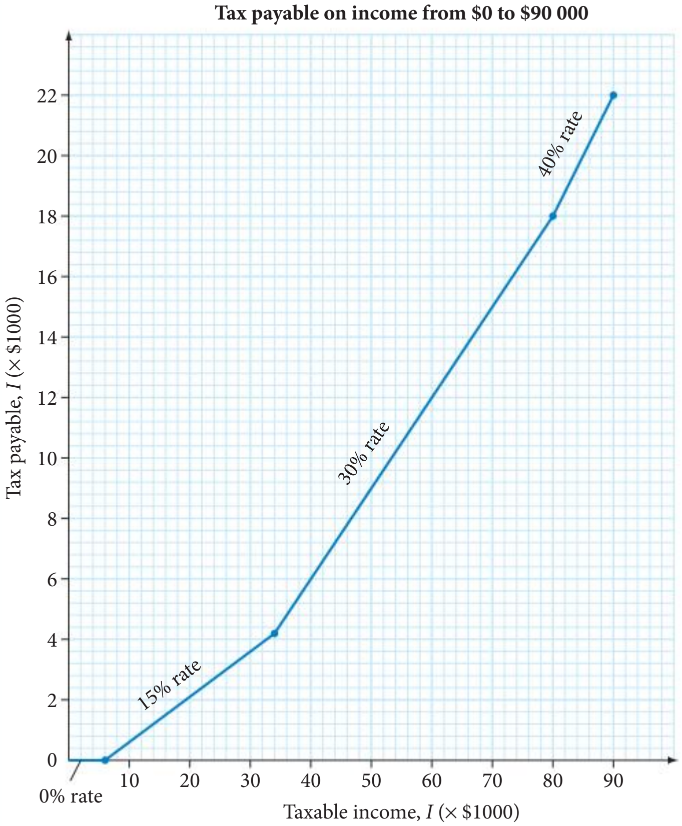
12.07 APPLICATIONS INVOLVING PIECEWISE GRAPHS

Piecewise linear graphs are useful for displaying information that involves different rates of change. Examples of situations where piecewise linear rules and graphs are useful include:

- taxation rates
- commissions
- mobile phone rates, and
- utility rates.

Example 18

This piecewise linear graph illustrates the income tax payable for incomes up to $90 000.



a Describe the graph and what it represents.

b Use the graph to find the tax payable on an income of:

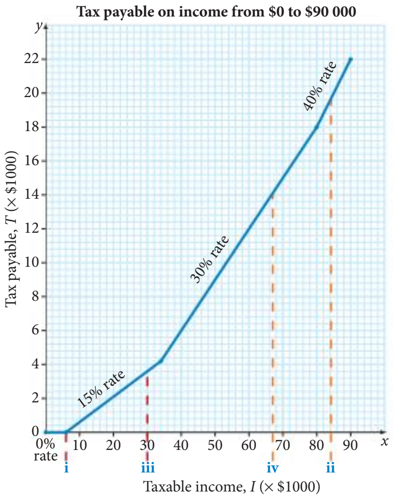
i $6000 ii $84 000 iii $30 000 iv $67 000.

Solution

a Study the graph. There are 4 line segments showing different rates of change.
Comment on your observations.

There are 4 line segments in this piecewise graph. The gradient of the line segments increases as the amount of taxable income increases. No tax is charged for the first $6000.
Between $6000 and $34 000 the tax rate is 15%.
Between $34 000 and $80 000 the tax rate is 30%.
Between $80 000 and $90 000 the tax rate is 40%.

b **i** Locate $6000 on the horizontal axis. Draw a broken line to the graph and then read the y value at this point.



y value = 0

Write your answer. The tax payable on an income of $6000 is $0.

ii Locate $84 000 on the horizontal axis. Draw a broken line to the graph and then read the y value at this point.

y value = $19 600

Write your answer. The tax payable on an income of $84 000 is $19 600.

iii Locate $30 000 on the horizontal axis. Draw a broken line to the graph and then read the y value at this point.

y value = $3600

Write your answer. The tax payable on an income of $30 000 is $3600.

iv Locate $67 000 on the horizontal axis. Draw a broken line to the graph and then read the y value at this point.

y value = $14 100

Write your answer. The tax payable on an income of $67 000 is $14 100.

Sometimes a piecewise graph is not provided but instead we are given data or information from which we can construct a graph.

Example 19

A car salesman’s commission is 5% for the first \$18 000 worth of sales and 3.5% for the remaining amount.

a Copy and complete this table of values using the rates given.

Car price, \$$P$	0	18 000	25 000	50 000
Commission, \$$C$				

b Use the table of values to help you draw a piecewise linear graph that represents the car salesman’s commission for values of P from \$0 to \$50 000.

Solution

a Copy the table.
If there are no sales, the commission will be \$0.

$P = 0, C = 0$

Car sales to the value of \$18 000 earn the salesman a commission of 5%.

$$5\% \text{ of } \$18\,000 = \frac{5}{100} \times 18\,000 = \$900$$

Car sales of \$25 000 give the salesman 5% of the first \$18 000 and 3.5% of the remaining amount. Calculate the remaining amount.

$\$25\,000 - \$18\,000 = \$7000$

$P = 18\,000, C = 900$

Calculate the commission on the remaining amount.

$$3.5\% \text{ of } \$7000 = \frac{3.5}{100} \times 7000 = \$245$$

Add this amount to the commission paid on the first \$18 000.

$\$900 + \$245 = \$1145$

$P = 25000, C = 1145$

Repeat the steps above for a commission of \$50 000.

$\$50\,000 - \$18\,000 = \$32\,000$

$$3.5\% \text{ of } \$32\,000 = \frac{3.5}{100} \times 32\,000 = \$1120$$

$\$900 + \$1120 = \$2020$

$P = 50\,000, C = 2020$

Enter the values into a table. This can be done as each one is calculated.

Car price, \$$P$	0	18 000	25 000	50 000
Commission, \$$C$	0	900	1145	2020

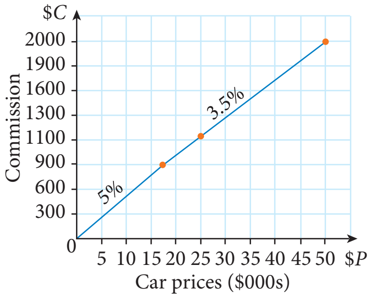
b Draw a set of axes. Label the horizontal axis 'Car price (\$$P$)' and the vertical axis 'Commission (\$$C$)'.

Plot the points from the table.

There are two segments to this graph.

Join the first two points (0, 0) and (18 000, 900) to give the first line segment.

Join the points (18 000, 900), (25 000, 1145) and (50 000, 2020) to give the second line segment.



At times we may be given a piecewise rule to show how an object is moving. We can now build on the work of the previous section to sketch the graph of the given rule.

Example 20

A moving object follows the path given by the piecewise linear function

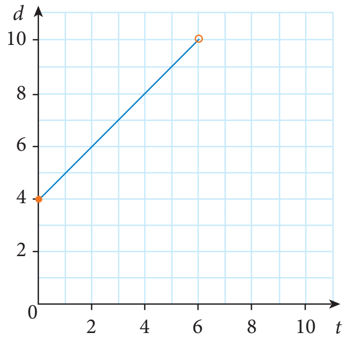
$$d = \begin{cases} t+4, & 0 \leq t < 6 \\ 10, & 6 \leq t < 8 \\ 50-5t, & 8 \leq t \leq 10 \end{cases}$$

where d is the distance travelled in metres and t is the time taken in seconds. Sketch the path followed by the object.

Solution

Write the equation of the first line segment.	$d = t + 4$
The graph of $d = t + 4$ is only required for the x values between 0 and 6. Find the value of d at the end points, that is, where $t = 0$ and $t = 6$.	When $t = 0$, $d = 4$ When $t = 6$, $d = 10$ The end points are (0, 4) and (6, 10).

Use the points (0, 4) and (6, 10) to sketch the first line segment.

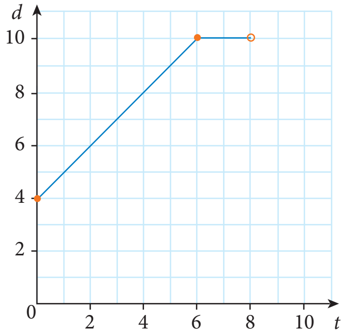


Write the equation of the second line. $d = 10$

The graph of $d = 10$ is only required for t values between 6 and 8.
State the coordinates of the points at $t = 6$ and $t = 8$.

The end points are (6, 10) and (8, 10).

Use the points (6, 10) and (8, 10) to sketch the second line segment on the same set of axes.



Write the equation of the third line segment.

$d = 50 - 5t$

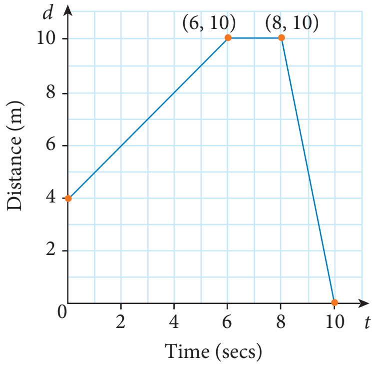
The graph of $d = 50 - 5t$ is only required for t values between 8 and 10.
Find the value of d at the end points, that is, where $t = 8$ and $t = 10$.

When $t = 8$,
$$\begin{aligned} d &= 50 - 40 \\ &= 10 \end{aligned}$$
When $t = 10$,
$$\begin{aligned} d &= 50 - 50 \\ &= 0 \end{aligned}$$

State the coordinates of the points at $t = 8$ and $t = 10$.

The end points are (8, 10) and (10, 0).

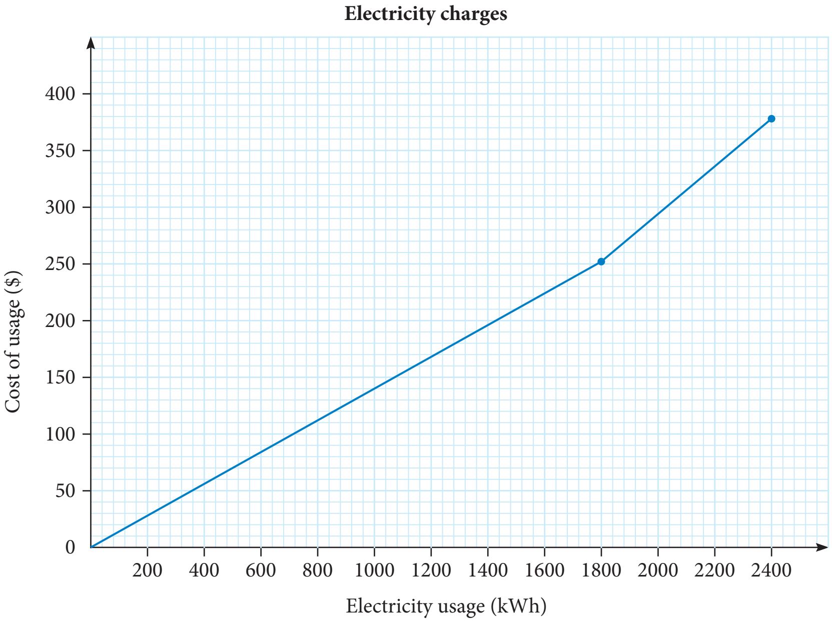
Use the points (8, 10) and (10, 0) to sketch the third line segment on the same set of axes.



EXERCISE 12.07 Applications of piecewise graphs

Reasoning and communication

1 Example 18 This piecewise linear graph shows the cost of electricity for different amounts of usage in kWh (kilowatt-hours).



a How many different cost rates are shown on the graph?

b What is the rate for the first 1800 kWh in c/kWh?

c Is the second rate higher or lower than the first rate?

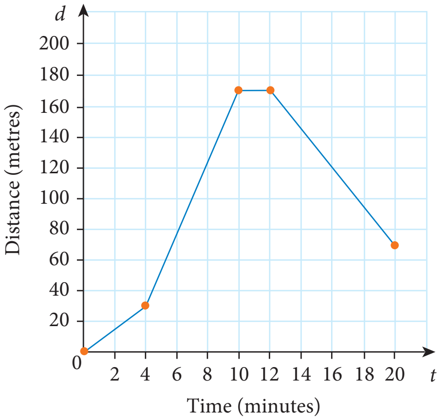
d Use the graph to find the cost of using the following amounts of electricity.

i 880 kWh ii 2100 kWh iii 1920 kWh

e Use the graph to find the amount of electricity used if the cost was:

i $100 ii $220 iii $370

The distance-time graph below shows the different movements of an object over a period of time. Use the graph to answer questions **2**, **3** and **4**.



2 The object changes speed

A 0 times	B 1 time	C 2 times
D 3 times	E 4 times	

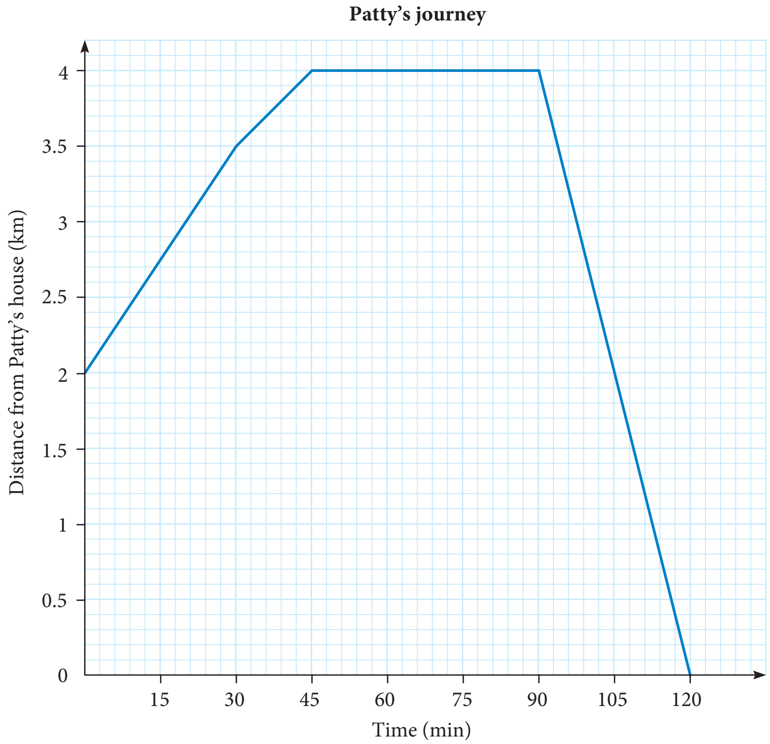
3 The total distance travelled by the object is:

A 70 metres	B 80 metres	C 150 metres
D 170 metres	E 270 metres	

4 For how many minutes is this object falling?

A 2 minutes	B 4 minutes	C 8 minutes
D 10 minutes	E 20 minutes	

5 Patty walked to Susan's house, stayed there for a while, then borrowed Susan's bike and cycled home. This travel graph illustrating Patty's movements has four distinct sections.

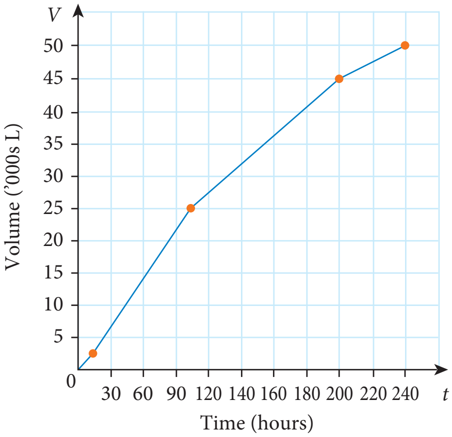

Patty's journey

- a How long did Patty stay at Susan's house?
- b The vertical intercept of this graph is 2 km. What does this mean?
- c What is the distance between Patty's house and Susan's house?
- d Describe briefly what happened at each section of the graph.
- e Calculate Patty's speed in kilometres/hour at each section of her journey.

Remember: speed = distance divided by time.

6 A rain water tank holds 50 000 litres. The piecewise linear graph below shows the rate at which the rain water tank fills with water.

Shutterstock.com/kezza



a Does the rain water tank fill at the same rate until it reaches capacity? Give reasons for your answer.

b How many litres does the rain water tank hold after:

i 10 hours? **ii** 100 hours? **iii** 200 hours?

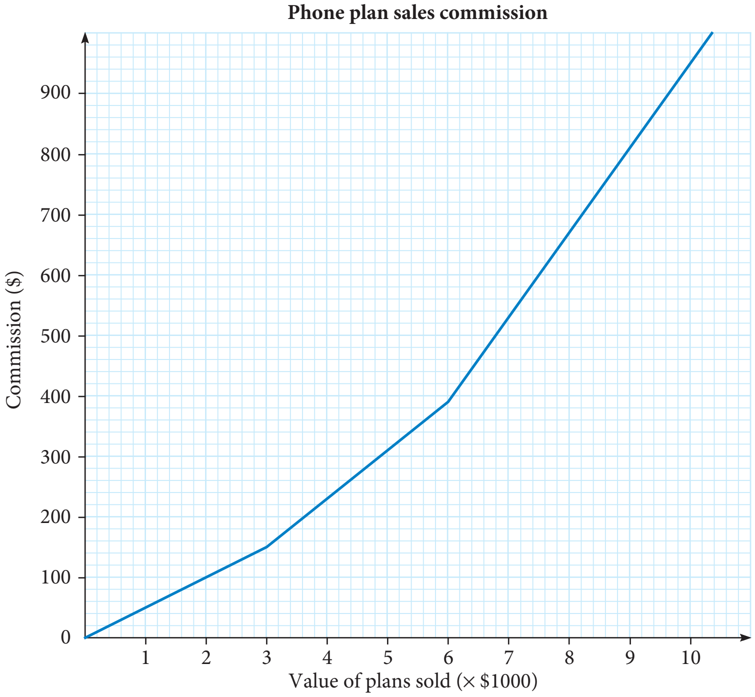
c After how many hours is the rain water tank filled to capacity?

d After how many hours does the rate at which the water is filling the rain water tank change? (Note: The rate changes three times.)

7 Damien is paid the following weekly commission rates for selling mobile phone plans.

- 5% of the first $3000
- 8% of the next $3000
- 14% of the remaining amount.

The piecewise linear graph below shows Damien's commission based on the value of the plans he sells.



a How much commission would Damien earn for selling phone plans at each of the following amounts per week?

i $5000 **ii** $1500 **iii** $8000

b If Damien earned $500 in commission this week, what was the value of the phone plans he sold?

c As Damien sells more plans, his rate of commission increases. How is this shown on the graph?

8 Example 19 A real estate agent charges the following commission rates for selling a property.

- 5% of the first $85 000 of the property's price
- 3% for the next $60 000
- 2.5% of the remaining amount.

a Copy and complete this table of values using the rates given above.

Property price, \$*P*	0	85 000	145 000	500 000
Commission, \$*C*				

b Use the table from part **a** to help you draw a piecewise linear graph that illustrates the real estate agent's commission for values of *P* from $0 to $500 000.

c What happens to the gradient of the graph as the property prices increase? Why?

d Use the graph to find the commission earned for selling:

i a block of land for $210 000

ii an apartment for $330 000

iii a house for $450 000.

9 The new Australian taxation rates are given in the table below.

Taxable income	Tax on this income
0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Source: Australian Taxation Office. Go to http://www.ato.gov.au/content/12333.htm

a Copy and complete the table below.

Taxable income (\$*I*)	0	30 000	50 000	150 000	200 000
Tax payable (\$*T*)					

b Use the table from part **a** to draw a piecewise linear graph that shows the tax payable on income up to \$200 000.

10 Example 20 A yacht race has 2 legs that need to be completed for the yachts to reach the finish line. One yacht takes 5 hours to complete the race. Each leg is defined by the equation:

$$d=\begin{cases}8t+4, & 0\le t<2\\ 30-5t, & 2\le t\le 5\end{cases}$$

where d is the distance travelled by the yacht in kilometres and t is the time taken in hours.

a Draw a piecewise linear graph to represent this information.

b How many kilometres has the yacht sailed over the two legs?

11 Joseph rides his bike from his home to the shops 12 kilometres away. His distance from home is given by the equation

$$d=\begin{cases}6t, & 0\le t<1\\ 6, & 1\le t<1.5\\ 12-4t, & 1.5\le t\le 3\end{cases}$$

where d is the distance Joseph rides on his bike and t is the time taken in hours.

a Draw a piecewise linear graph to represent his journey to the shops.

b During which part of the journey was he travelling the fastest? Explain your answer.

c Between $t = 1$ and $t = 1.5$ the graph is a horizontal line. What is the gradient of this line? Explain what you think happened between $t = 1$ and $t = 1.5$.

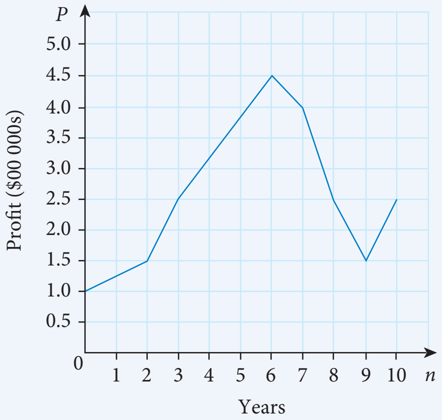
d How many kilometres did Joseph cycle in total?

INVESTIGATION Piecewise linear rules in business

Companies often chart their profit results over a number of years to help them analyse how their company is performing and to help them predict how their company may perform in the future. At times they use line graphs that resemble piecewise linear graphs.

The Mountain Resort Building Company's performance over 10 years is shown in the graph below.

Mountain Resort Building Company's profit



a How many line segments form this piecewise linear graph?

b What profit was the company showing at the start of this 10-year period?

c What profit was the company showing at the end of the 10 years?

d Use the graph to determine the company's profit at:

- **i** 2.5 years **ii** 4 years **iii** 7.5 years

e After how many years did the company show a profit of $420 000?

f For how many years was the company's profit:

- **i** increasing? **ii** decreasing?

g Between which years did the company grow the fastest? Give a reason to explain why there might have been such a growth.

h Between which years did the company show a fall in profits? How much did their profits fall by during this time?

i How can you explain this drop in profits?

j State the gradient of each of the seven line segments that form this piecewise linear graph.

k Using the trend shown in the graph, what do you think the company may do in the next five years?

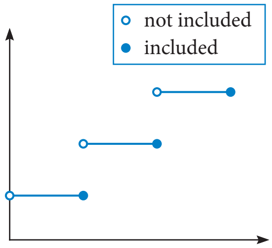
The Australian Stock Exchange lists public companies and provides information to the pubic about these companies. It shows the value of stocks and may represent the value of these stocks in a graph that resembles a piecewise linear graph. Visit the ASX website: http://www.asx.com.au/

Weblink

Research some of the companies listed there to see how their share prices are presented to the public.

12.08 STEP GRAPHS

A **step graph** is made up of horizontal line segments that look like steps. The ends of each line segment in a step graph are labelled with circles. These can be open (hollow) or closed (shaded). An open circle means that the point is not included as part of the graph while a closed circle means that the point is included as part of the graph. A step graph is said to be **discontinuous** because the line segments do not connect.

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IMPORTANT

A step graph is a type of piecewise graph.

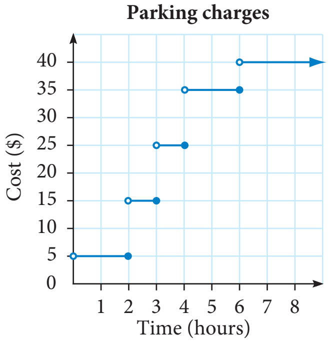
Example 21

This step graph shows the parking charges at a car park.

a Find the charge for parking for:
  - i 3 hours
  - ii 55 minutes
  - iii $3\frac{1}{2}$ hours
  - iv 4 hours 1 minute.

b For what range of times can a driver park for $25?

c What does the arrow on the $40 step mean?



9780170250252

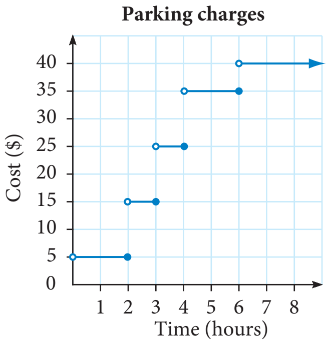
Solution

a The costs will vary according the amount of time a driver parks in the car park.

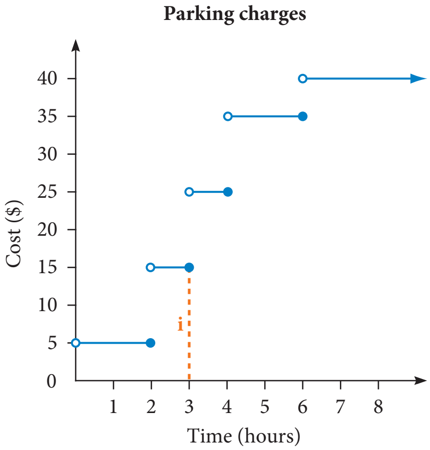
For example, if a driver parks in the car park for more than 0 hours but less than or equal to 2 hours, they will pay $5.

If a driver parks in the car park for more than 2 hours but less than or equal to 3 hours, they will pay $15.

1f a driver parks in the car park for more than 3 hours but less than or equal to 4 hours, they will pay $25, and so on.



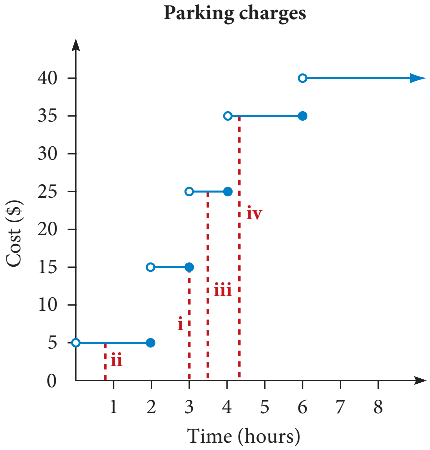
i Find 3 hours on the horizontal axis and draw a vertical line from this point until you reach one of the steps. The vertical line meets the second step at the end of the interval. The circle at the end of the step is closed, so 3 hours is included in this interval.



Read the answer from the graph as indicated and answer the question.

3 hours of parking will cost $15.

Repeat the process from part **i** for each of questions **ii**, **iii** and **iv**.



ii Read the answer from the graph as indicated and answer the question.

55 minutes of parking will cost $5.

iii

$3\frac{1}{2}$ hours of parking will cost $25.

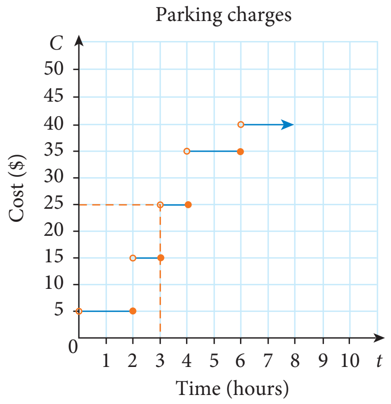
iv

4 hours 1 minute of parking will cost $35.

b Using the graph, find $25 dollars on the vertical axis.

Move across horizontally until you reach the step which is in line with 25.

The step ranges from 3 to 4. The open circle at the start means that 3 is not included. The closed circle at the end means that 4 is included.



Write the answer.

The driver can park for more than 3 hours but less than or equal to 4 hours if he pays $25.

c The $40 step has an arrow at the end of the interval. This means that the parking charge is the same for any time after 6 hours.

Write the answer.

After 6 hours, the parking charge remains constant at $40. The maximum daily charge is $40.

At times we may be given a rule and be required to draw a step graph.

Example 22

Graph the following stepwise linear function.

$$y = \begin{cases} 1, & \text{for } -2 < x \leq 1 \\ 3, & \text{for } 1 < x \leq 4 \\ 5, & \text{for } 4 < x \leq 7 \\ 7, & \text{for } 7 < x \leq 10 \end{cases}$$

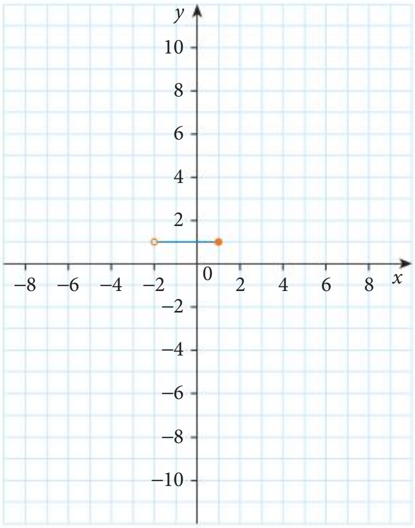
Solution

The graph of this equation will resemble a step graph. There are 4 steps in this graph. Consider each part separately.

The first step or interval has the equation $y = 1$ when x is greater than –2 but less than or equal to 1.

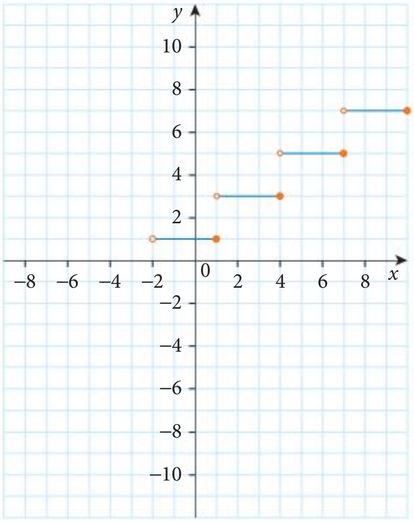
Draw a horizontal line from –2 to 1 at $y = 1$.

–2 is not included, so place an open circle at this end. 1 is included, so place a closed circle at this end.



Repeat this process for the remaining 3 steps to complete the graph.

Each step should have the open circle at the start and the closed circle at the end.



Step graphs are used in real-life situations, such as the cost of hiring items, the cost of sending parcels or car parking costs.

Example 23

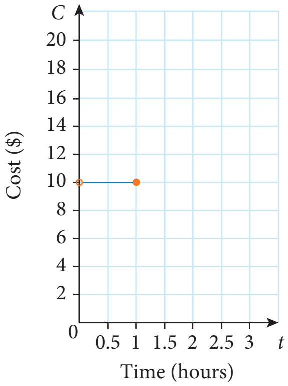
The cost of hiring a bicycle from Happy Cycling Rentals varies according to the number of hours the bicycles are rented for. The rates are as follows:

$10 up to and including 1 hour, then $2.50 for each additional half hour or part thereof.

Draw a step graph to represent the cost of hiring a bicycle for up to 3 hours from Happy Cycling Rentals.

Solution

The graph representing this information will be a step graph. Draw a set of axes. Label the horizontal axis 'Number of hours' (t) and the vertical axis 'Hiring cost' ($$C$).
The bike will cost $10 to hire for the first hour.
Draw a horizontal line from 0 to 1 at $C = 10$.
As this is the cost of hiring the bicycle for up to and including 1 hour, draw a closed circle at the right end of the line interval.
An open circle is required at the left end of this line segment as you don't pay $10 if you don't use the bicycle at all.



It will cost an additional $2.50 for the next half hour. Add this cost to $10.

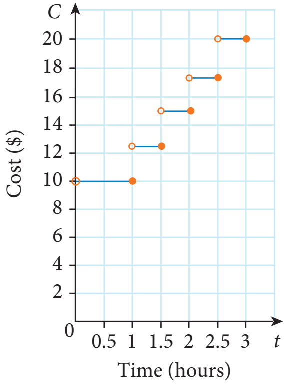
$10 + $2.50 = $12.50

Draw a horizontal line from 1 to 1.5 at $C = 12.5$.
Place an open circle at the start of this interval since the cost for 1 hour is $10, not $12.50.
This is the price for the next half hour, hence we need to place a closed circle at the end of this interval.

Repeat this process until you reach 3 hours on the horizontal axis.
Remember to place an open circle at the start and a closed interval at the end of each line segment.

$12.5 + 2.5 = 15$
$15 + 2.5 = 17.5$
$17.5 + 2.5 = 20$

Happy Cycling Rentals hiring costs



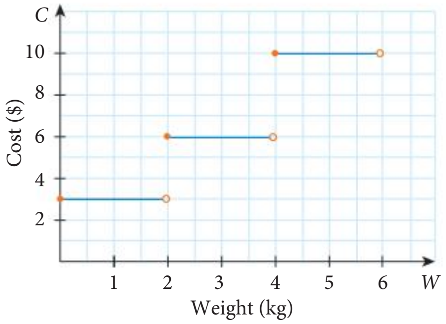
EXERCISE 12.08 Step graphs

Step graphs

Concepts and techniques

1 a In your own words, describe why the graphs above are called 'step graphs'.
  b List 3 features of a step graph.

2 Example 21 A step graph shows the cost of sending parcels of different weights by air freight.



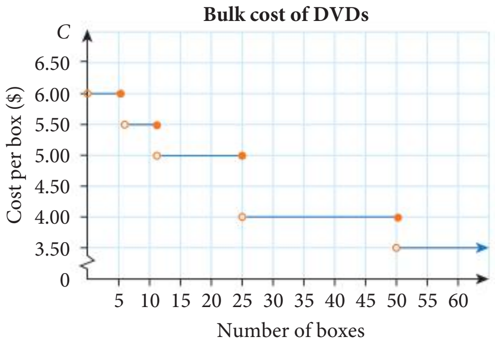
How much does it cost to send:

a one parcel weighing 3.4 kg?

b one parcel weighing 1.7 kg?

c two parcels, weighing 2 kg and 4 kg?

3 This step graph shows the cost per box of blank DVDs when they are bought in bulk.



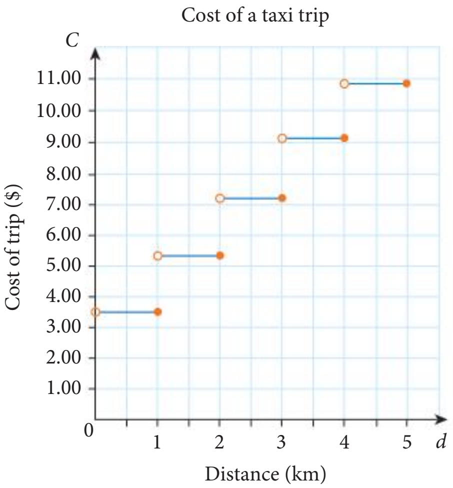
a What happens to the cost per box as the number of boxes increases? Why?

b Find the cost per box when purchasing:
  i 40 boxes ii 8 boxes iii 50 boxes iv 1 box.

c What is the maximum number of boxes that can be bought for $4 each?

d What does the arrow on the $3.50 step mean?

4 The cost of a trip in a taxi depends on the distance travelled, as shown by this step graph.



a Find the cost of travelling a distance of:

i 4 km ii 2.7 km iii 1.1 km.

b For what range of trip lengths is the cost $9.05?

c What is the 'flag fall' (the charge for starting a trip)?

d Which of the following is the amount charged, after flag fall, for every additional kilometre?

A $1.85 B $2.68 C $3.50 D $3.60 E $7.40

5 This step graph shows the annual premiums (costs) of life insurance policies for males of different ages.

a What is life insurance?

b What is the annual premium for a man aged:

i 35 years?

ii 48 years?

iii 22 years?

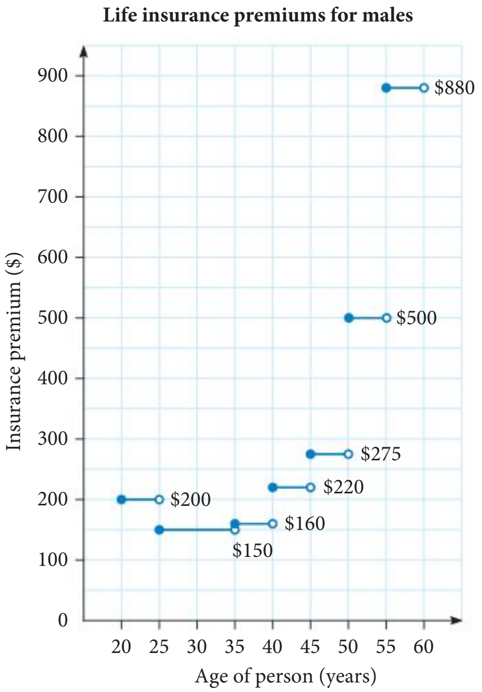
c Why do you think the cost of life insurance is lowest for men aged 25 to 34?

d As the age of a man increases, what generally happens to the insurance premium? Why?

e For what age group is the annual premium $200?

f What is the age of the youngest man who can pay a premium of:

i $220? ii $500?



6 Example 22 Graph the following equations.

a $y = \begin{cases} -3, & \text{for } -5 \le x < -1 \\ 1.5, & \text{for } -1 \le x < 1 \\ 4, & \text{for } 1 \le x < 6 \end{cases}$

b $y = \begin{cases} -3.5, & \text{for } -3 < x \le -1 \\ -1, & \text{for } -1 < x \le 2.5 \\ 3.5, & \text{for } 2.5 < x \le 5.5 \\ 6.5, & \text{for } 5.5 < x \le 8 \end{cases}$

c $C = \begin{cases} \$11, & \text{for } 0 < n \le 3 \\ \$3.80, & \text{for } 3 < n \le 5 \\ \$4.20, & \text{for } 5 < n \le 8 \\ \$6.60, & \text{for } n > 8 \end{cases}$

Reasoning and communication

7 Example 23 A taxi driver charges $5 flag fall and $2 per kilometre per hour or part thereof. Draw a step graph showing the cost of hiring the taxi for the first 5 kilometres.

8 The size of the speeding fine imposed on a driver depends upon the number of km/h by which the speed limit was exceeded. The table below shows the approximate cost of a fine depending on the number of km/h over the speed limit.
Represent this data on a step graph.

Speed in excess of limit (km/h)	**Fine ($)**
Less than 10	176
10 or more but less than 25	282
25 or more but less than 30	387
30 or more but less than 35	458
35 or more but less than 40	528
40 or more but less than 45	599
45 or more	704

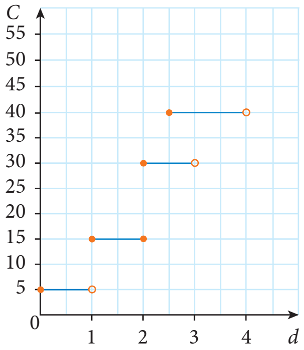
Source: Vicroads

9 The rates for hiring a fishing boat from ‘Hook, Line and Sinker’ are as follows.

- $20 for the first hour
- $5 for each additional half hour or part thereof
- $40 for the day

Represent these costs on a step graph.

10 Find the faults in the step graph below.



11 A step graph is a type of piecewise graph, which you studied in the previous section. List some of the differences between the two types of graphs.

Alamy/

CHAPTER SUMMARY

APPLICATIONS OF SIMULTANEOUS EQUATIONS, PIECEWISE AND STEP GRAPHS

12

- The graph of a **linear equation** is a straight line (or **linear graph**).
- The **general form of a linear equation** is $y = a + bx$, where a represents the y-intercept and b represents the **gradient** of the straight line.
- The gradient of a straight line is the slope of the line and is defined as $\frac{\text{rise}}{\text{run}}$. A linear graph can be sketched using the gradient and y-intercept.
- A positive gradient slopes up to the right. A negative gradient slopes down to the right. The gradient will determine the steepness of the line.
- The gradient between two points (x_1, y_1) and (x_2, y_2) can be found using the formula:

  $$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
- The x- and y-intercepts are the points where a straight line graph cuts the axes. To find the ***x*-intercept** let $y = 0$. To find the ***y*-intercept** let $x = 0$.
- Straight line graphs can be drawn using the gradient and y-intercept method or by using the x- and y-intercepts method.
- Lines of equations in the form $y = a$, where a is a constant, are horizontal lines.
- Lines of equations in the form $x = a$, where a is a constant, are vertical lines.
- When reading graphs we can find answers within the given values or range of the graph. This is called **interpolating**.
- If we predict answers that fall outside the range of the graph we are **extrapolating**.
- **Simultaneous equations** are equations that can be solved at the same time.
- The point where two or more graphs intersect is called the **point of intersection**.
- Simultaneous linear equations can be solved graphically or algebraically. There are two algebraic methods for solving simultaneous linear equations – **substitution method** and **elimination method**. Answers should be checked by substituting the solution into the original linear equations.
- A **piecewise linear equation** is a combination of two or more linear equations.
- The graph of a piecewise linear equation is called a **piecewise graph** and is comprised of separate line segments. The critical points in a piecewise linear graph are the end points of each line segment.
- Piecewise linear graphs are useful for displaying information that has different rates of change.
- A **step graph** is made up of horizontal segments. The ends of each step in a step graph are labelled with circles. An open circle means that the point is not included, while a closed circle means that the point is included.
- A step graph is said to be **discontinuous** because the line segments do not connect.

12

CHAPTER REVIEW

APPLICATIONS OF SIMULTANEOUS EQUATIONS, PIECEWISE AND STEP GRAPHS

Multiple choice

1 **Example 1** The equation $3x + y = 5$ transposed into the form $y = a + bx$ is

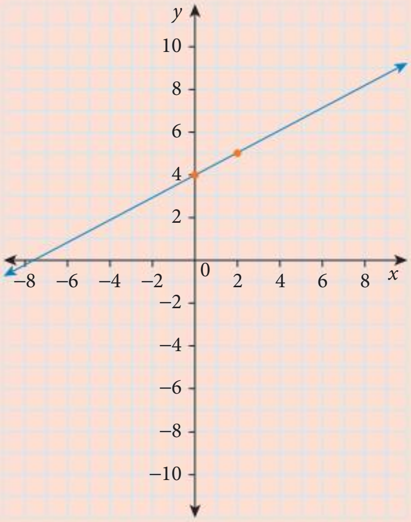
A $x = \frac{y}{3} - \frac{5}{3}$
B $y = -5 + 3x$
C $y = -5 - 3x$
D $y = 5 + 3x$
E $y = 5 - 3x$

2 **Example 1** The gradient of the line $2y - 4x = 6$ is:

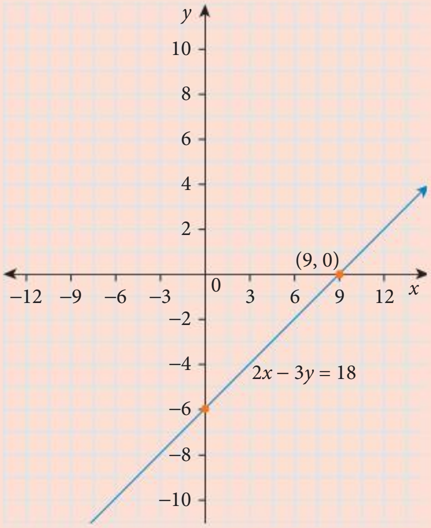
A -6
B -3
C $\frac{1}{2}$
D 2
E 4

3 **Example 2** The graph of the equation $y = -\frac{1}{2}x + 4$ is:

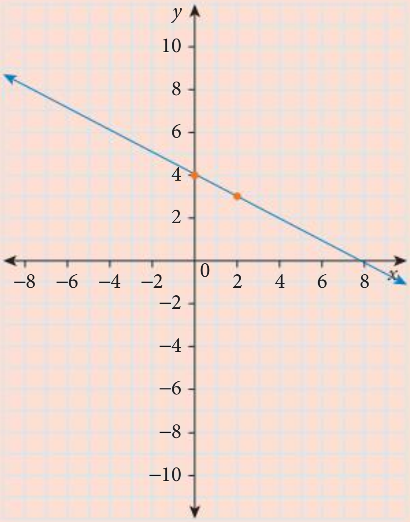
A



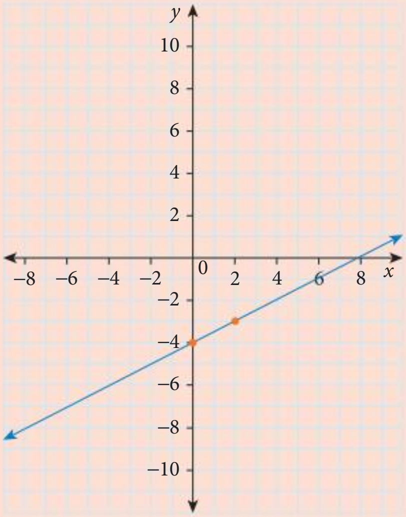
B



C



D



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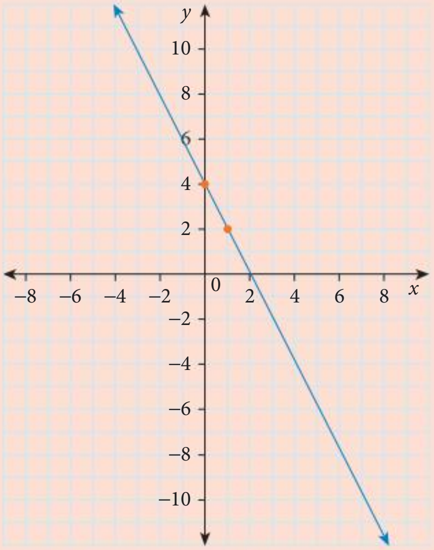
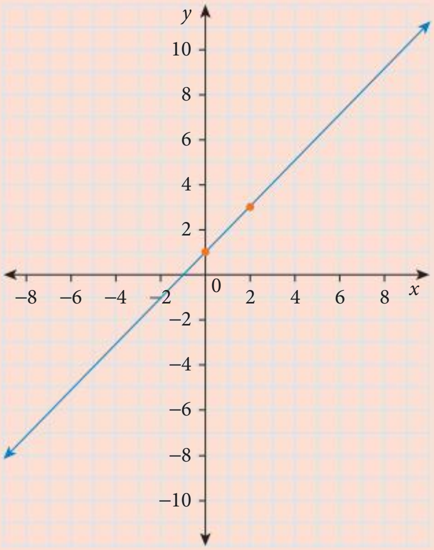
E



4 **Example 2** The equation of the line on the right is:



A $y = 1 + 2x$

B $y = -1 - 2x$

C $y = 1 - x$

D $y = 1 - \frac{1}{2}x$

E $y = 1 + x$

5 **Example 3** The line with the equation $5x - 3y = 30$ intersects the axes at which of the following points?

A (6, 0) and (0, −10)	B (−6, 0) and (0, 10)	C (0, 6) and (−10, 0)
D (−6, 0) and (0, −10)	E (−6, 0) and (10, 0)	

6 **Example 7** The solution to the simultaneous equations, $y = x - 5$ and $3x + 2y = -5$ is:

A $x = -1, y = 4$	B $x = -4, y = 1$	C $x = -1, y = -4$
D $x = 1, y = -4$	E $x = 4, y = -1$	

For questions **7** and **8** use the simultaneous equations

$5x + 3y = 7$

$-2x - 3y = -37$

7 Example 9 The equation which results when one variable is eliminated is:

A $7x = -30$ B $7x = -44$ C $3x = 30$

D $3x = -30$ E $-3x = 30$

8 Example 9 The point of intersection of the two lines is:

A $\left(10, -\frac{17}{3}\right)$ B $(-10, 19)$ C $\left(-10, -\frac{41}{3}\right)$

D $(10, 19)$ E $(10, -19)$

9 Example 10 For the simultaneous equations

$7x - 2y = 25$

$-4x + 3y = -5$

the solution is:

A $x = -5, y = -5$ B $x = 5, y = 5$ C $x = \frac{55}{13}, y = \frac{15}{13}$

D $x = -\frac{55}{13}, y = -\frac{15}{13}$ E $x = \frac{35}{13}, y = \frac{155}{13}$

10 Example 15 The equation which best represents the piecewise linear graph below is:

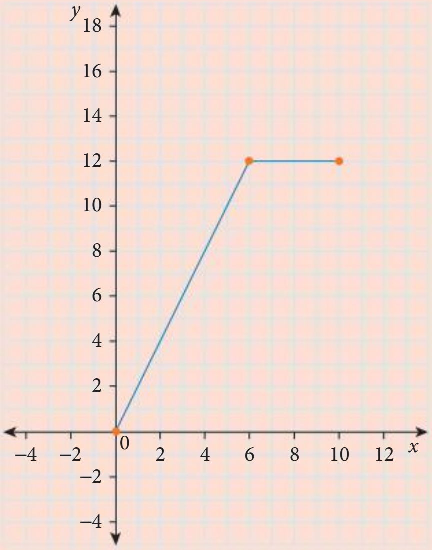
A $y = \begin{cases} -2x, & 0 \le x \le 6 \\ 12, & 6 \le x \le 10 \end{cases}$

B $y = \begin{cases} 2x, & 0 \le x < 6 \\ 12x, & 6 \le x \le 10 \end{cases}$

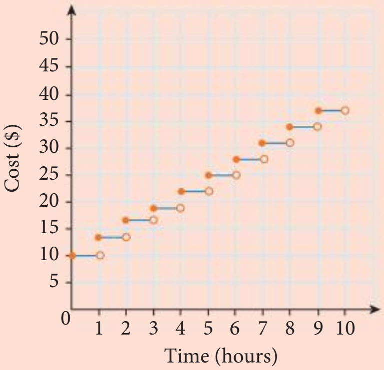
C $y = \begin{cases} -2x, & 0 \le x < 6 \\ 12, & 6 \le x \le 10 \end{cases}$

D $y = \begin{cases} 2x, & 0 < x < 6 \\ 12, & 6 < x < 10 \end{cases}$

E $y = \begin{cases} 2x, & 0 \le x < 6 \\ 12, & 6 \le x \le 10 \end{cases}$



11 Example 21 The step graph below shows the charge for hiring a taxi to travel up to 10 kilometres.



The fare for travelling 6 km is:

A \$3 **B** \$10 **C** \$18 **D** \$25 **E** \$28

Short answer

12 Example 1 Find the gradient and y-intercept of the line with the equation $3x + 2y + 10 = 0$

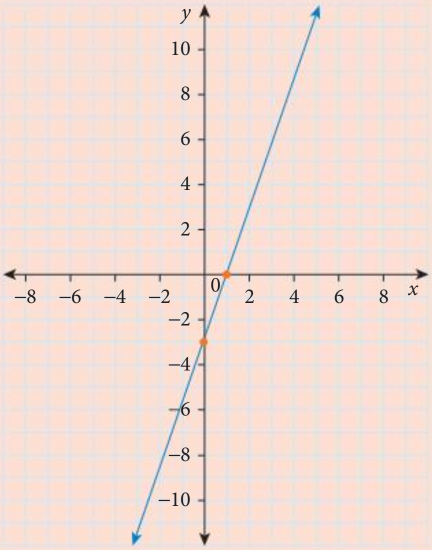
13 Example 1 A promotions company are selling theatre tickets for \$75 each. There is also a flat fee of \$10 for postage and handling for each booking (no matter how many tickets are bought). Write an equation for the cost (C) of n tickets.

14 Example 1 **a** Find the gradient of the line passing through the points (4, 3) and (0, –5).

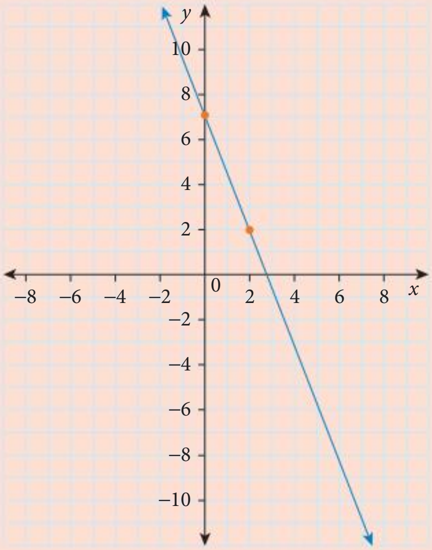
b Find the equation of the line.

15 Example 2 Find the equation of each of these lines.

a



b



16 Example 2 a Use the gradient and y-intercept method to sketch the lines $y = 2x$ and $y = 2x + 3$ on the same set of axes.

b Comment on the slope of the lines.

17 Example 3 a Find the coordinates of the x-and y-intercepts for the line with the equation $2x - y = 4$.

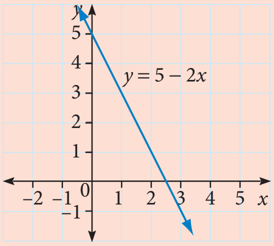
b Sketch the graph of the line.

18 Example 3 Sketch the graphs of $x = -1$ and $y = 5$ on the same set of axes.

19 Example 4 a Graph the equations $3x - y = 11$ and $2x + y = 4$ on the same set of axes.

b Use the graph to find the point of intersection of $3x - y = 11$ and $2x + y = 4$.

20 Example 5 a Copy the graph below and graph the equation $y = x - 1$ on the same axes.



b Use your graph from part **a** to solve the simultaneous equations $y = 5 - 2x$ and $y = x - 1$.

21 Example 6 Using your CAS calculator, sketch the graphs $y = x + 2$ and $2x - y = 1$. State the point of intersection.

22 Example 7 Solve the simultaneous linear equations below using the substitution method.

$y = 2x + 9$
$3x + 2y = 4$

23 Example 8 a Express $x + y + 6 = 0$ in the form $y = a + bx$.

b Use your answer to part **a** to solve the following pair of simultaneous linear equations by substitution.

$x + y + 6 = 0$
$2x - 5y = 9$

24 Example 9 The equations below are to be solved simultaneously using the method of elimination.

$x - 2y = 11$
$x + 2y = 9$

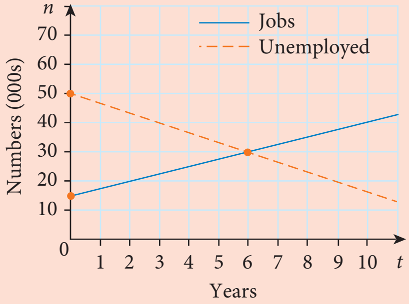
a Which variable would be the most appropriate to eliminate first?

b Solve the simultaneous equations by elimination.

25 Example 10 Using the elimination method, solve the following simultaneous linear equations.

$3x + 4y = 5$
$2x - 3y = -8$

26 Example 12 The number of unemployed people is declining at a steady rate, while the number of jobs available is rising, according to the graph.



a How many people were initially unemployed?

b When will the number of unemployed people equal the number of jobs? How many people were unemployed at this point?

27 Example 13 The perimeter of a rectangle is 72 cm. The length of the rectangle is 8 cm longer than its width.

a Write a set of two of linear equations using the information above.

b Solve the equations simultaneously to find the length and the width of the rectangle.

28 Example 14 A screen printing company has income given by $A = 15p$ and expenses of $A = 54 + 6p$, where p is the number of T-shirts printed. Solve the equations simultaneously to find the break-even point.

29 Example 16 **a** Sketch the graph of $y = 4x + 1$ for $-3 \le x < 1$.

b On the same set of axes, sketch the graph of $y = 7 - 2x$ for $1 \le x \le 5$.

30 Example 17 Sketch the graph of

$$y = \begin{cases} -x-3, & -10 \le x < -5 \\ 2, & -5 \le x < 1 \\ 2x, & 1 \le x \le 8. \end{cases}$$

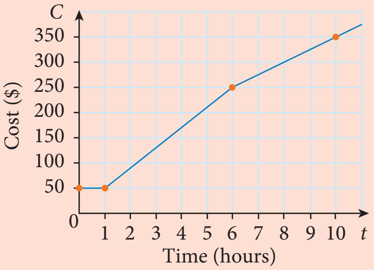
31 Example 18 The graph below shows the cost of servicing a washing machine.

How much will it cost to service a machine for:

a 1 hour?

b 2 hours?

c 8 hours?



32 Example 22 Sketch the step graph given by the equation

$$y = \begin{cases} -5, & -6 \le x < -2 \\ -1, & -2 \le x < 2 \\ 3, & 2 \le x < 6. \end{cases}$$

Application

33 A shipping container holds 5 pallets of washing powder and 3 pallets of sugar. The weight of the contents is 1880 kg. The contents of another shipping container, holding 3 pallets of washing powder and 5 pallets of sugar, weighs 1640 kg.

a Write a pair of equations to represent the contents of each container. Let $w =$ the weight of each washing powder pallet and $s =$ weight of each sugar pallet.

b Find the weight of a pallet of washing powder and a pallet of sugar.

34 Salma is organising an end-of-year outdoor lunch for her staff. The cost of hiring a tent is $133 and the food is $7.50 per guest. To cover these costs, Salma is charging each guest $11. The cost and revenue functions may be represented by the following linear rules.

Cost: $C = 7.5n + 133$
Revenue: $R = 11n$

where C and R are in dollars and n represents the number of guests.

a Graph both the C and R functions on the same axes, for values of n from 0 to 50.

b Use the graph to find the cost and revenue for running the lunch for 32 guests.

c For what value of n does the revenue from the lunch equal the cost?

d Why is the value you found in part **c** called the ‘break-even point’?

e What is the profit made when the lunch caters for 50 guests?

35 The step graph below shows the cost of mobile phone calls, for call durations from 0 minutes to 4 minutes.

a What is the cost of a mobile phone call that lasts:

i 3 min 10 s? **ii** 15 s? **iii** 1 min 59 s?

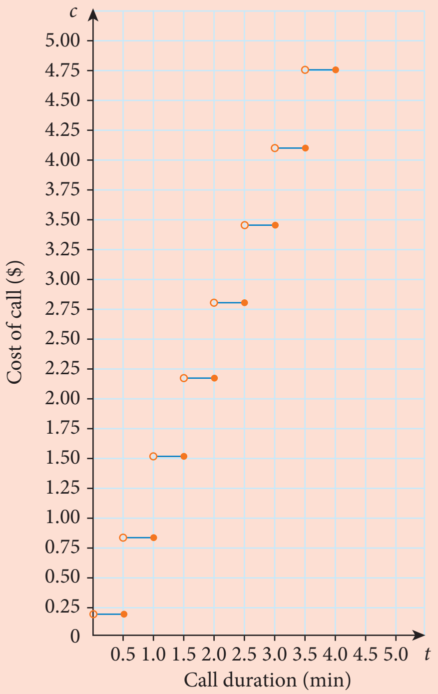
b What is the longest call that can be made for $2.15?

c One particular call cost $4.10. What was the duration of the call?

d By what rate does the call cost increase?

Practice quiz

Mobile phone call charges



ANSWERS

1.01

1 a 50% b 25% c 80%
d $37\frac{1}{2}\%$ e $66\frac{2}{3}\%$ f $44\frac{4}{9}\%$
g $57\frac{1}{7}\%$ h 175% i 220%
j $333\frac{1}{3}\%$ k $483\frac{1}{3}\%$ l $328\frac{4}{7}\%$

2 a 75% b 7.6% c 5%
d 86.73% e 180% f 301%
g 310% h 1070.5% i $33.\overline{3}\%$
j $211.\overline{1}\%$ k 437.5% l $666.\overline{6}\%$

3 a i $\frac{21}{100}$ ii 0.21
b i $\frac{21}{25}$ ii 0.84
c i $\frac{49}{100}$ ii 0.49
d i $1\frac{1}{5}$ ii 1.2
e i 3 ii 3
f i $\frac{1}{8}$ ii 0.125
g i $\frac{9}{25}$ ii 0.36
h i $1\frac{1}{10}$ ii 1.1
i i $\frac{1}{3}$ ii $0.\overline{3}$
j i $\frac{1}{200}$ ii 0.005
k i $1\frac{4}{5}$ ii 1.8
l i $2\frac{111}{250}$ ii 2.444

4

Fraction	Decimal	Percentage
$\frac{7}{10}$	0.7	70%
$\frac{24}{25}$	0.96	96%
$\frac{7}{50}$	0.14	14%
$63\frac{1}{8}$	63.125	6312.5%
$1\frac{7}{8}$	1.875	187.5%
$2\frac{1}{4}$	2.25	225%

5 a 70% b $66\frac{2}{3}\%$ c 70%
d 20% e 5% f $23\frac{1}{3}\%$
g $\frac{1}{30}\%$ h $6\frac{2}{3}\%$ i $20\frac{5}{6}\%$
j 30% k $2\frac{6}{7}\%$ l $66\frac{1}{4}\%$

6 a $122\frac{2}{9}\%$ b 9.9% c $242\frac{6}{7}\%$
d 12.8% e 278.9% f $545\frac{5}{11}\%$
g 4900% h $55.\overline{5}\%$ i $66\frac{2}{3}\%$
j 65% k 12% l $13\frac{1}{3}\%$

7 a The error is in Step 1, where the mixed number is converted to an improper fraction. $2\frac{5}{12} = \frac{29}{12}$
b $241\frac{2}{3}\%$

8 $73\frac{1}{3}\%$
9 84%
10 63.5%
11 a 45% b 55%
12 a i 37.5% ii 12.5% iii 25%
b 25%
13 a 68% b 32% c $33\frac{1}{3}\%$
14 a English: 82.5%, Mathematics: $85\frac{1}{3}\%$, Science: 85%
b Mathematics
15 B
16 93.5%

1.02

1 a \$60 b 48 m c 9 L
d 3.75 hours e 14 cm^2 f \$48.40
g 320 kg h 90 students
2 a \$246 b 29.7 L c \$62.50
d 25.62 g e 0.44 t f 12.25 mm
g 40.14 s h 4.503 m
3 a \$23.10 b \$49.50 c 7.90 kg
d \$34.60 e \$151.58 f 55.35 g
g 2.52 L h \$1947.50
4 114.08 megabytes
5 B
6 18.75 km
7 33.6 kg
8 11.5 kg

9 9.375 g
10 361 students
11 E
12 a $270 b $6270
13 a $1282.50 b $7267.50

1.03

1 a $1200 b 200 km c $33\frac{1}{3}$ L
d 120 minutes = 2 hours
e 400 g
f 270 hectares
2 a 98.5 m b $65 c 3571.4 mL
d 360 L e 3876.0 people f 367.3 g
3 a 184.78 kg b 563.38 mL
c 39.77 mg d 17 800 girls
e 35.99 mm f $17 415.38
4 D
5 90 kg
6 A
7 $1699
8 288 megabytes
9 a $2500 b $20 500
10 a 27.3 km b 42 km
11 1 780 000 ML
12 980 students
13 1565 light globes
14 a i 19 ii 37 b 8
15 $4550

1.04

1 a $875 b $525 c 624 kg
d 51 L e 832 hectares f 30.4 mg
2 a 401.2 km b $17 220 c 89.6 hectares
d 947.9 L e 4.1 hours f 28.4 cm
3 a 160.1 cm^3 b 18.3 L c 177.5 mm
d 100.1 kg e 449.8 tonne f 8.1 mg
4 D
5 $29.60
6 $2508
7 21 692 908
8 D
9 $391
10 $2193.10
11 $83.44
12 $59 920
13 609 students
14 156 cars
15 $4.209 million

1.05

1 a –4% b –4% c 16.64%
d –6.5% e –9.76% f –3.1%
2 10.4%
3 a $307.80 b $292.41 c $7.41 increase; 2.6%
4 D
5 D
6 a $1248.05 b 25%
7 a 3.13 t b –10.6%

1.06

1 a $462 b $231 c $99 d $4.95
2 a $1 b $1.91 c 50c d $1.30
3 C
4 a $160 b $1840
5 a

i	$199	Discount	$41
ii	$345	Mark-up	$110
iii	$7.50	Mark-up	$3.45
iv	$62	Mark-up	$34

b i 17.1% ii 46.8% iii 85.2% iv 121.4%
6 35%
7 A
8 a $140 b $192 c $297.50 d $85
9 a $84 b $224
10 $12
11 A
12 150%
13 a $239.70 b $215.73
c Bob would have been happy with this as it was more than he should have got. A decrease of 15% and then 10% means 0.85 × 0.9 = 0.765. This is 76.5% of the price, so it is equivalent to a discount of 23.5%, not 25%.
14 $163\frac{1}{3}$%
15 a 18.1% b $1228.50

1.07

1 a $5640 b $432 c $3600 d $288
2 13.75% loss
3 a $1.50 b $2 c $2.50 d 65c
4

	Item	Profit or loss	Amount of profit/ loss	% profit/ loss
a	video camera	profit	$70	31.8%
b	jewellery box	loss	$10	28.6%
c	filing cabinet	profit	$50	62.5%
d	printer	loss	$85	25.0%

5 60%
6 $41\frac{2}{3}$%
7 a loss b $67 000 c 15.6%
8 E
9 a $400 b $468
10 $63\frac{1}{3}$%

11 $240
12 $1200
13 a Yes
b The money is almost the same $707 profit for 30 hours = $23.57 per hour, so less than his job.

1.08

1 a $882.30 b $3529.20
c $51.85 d $1347.30
2 a 16.7% b 9.2% c 7.8% d 4.0%
3 a $1500 b $32 c $4.50 d $35
4 a $242 b $198 c $968 d $32.07
5 C
6 a $124 b $54.36 c $32 d $42.02
7 C
8 $779.44
9 $300
10 $16.82
11 $74 784
12 $31.13
13 No. It has only risen by 3.9%.
14 $42, $462
15 $415.80
16 a $4.07 b $2.92
17 a $77 b $341 c $275 d $539
18 a Suki's salary has increased by the same amount for the 2 years but in the second year her salary should have increased by 2.7% of the new salary – not the old one.
b $47 462.81
19 $719.26
20 a $18.75 b $20.63
21 $1.86
22 $2733.50

INVESTIGATION: GST

a Advertised price (incl. GST) = 100% + 10% = 110%
We know 110% ÷ 11 = 10% (which is the GST).
∴ advertised price ÷ 11 = GST
b $1320
c i less than ii $1188
iii It is $12 less iv No
d It will be less than the original price.
e 1% less (i.e. 1.1 × 0.9 = 0.99 ∴ 1% decrease)

1.09

1 a $/hour b words/min
c $/kg d m/s
2 a fuel consumption
b download speed from Internet
c birth rate
d cost of tiling a floor
3 2.4 cm/week
4 5.25 runs/over
5 2750 persons/year
6 $28.44/kg
7 0.325 MB/s
8 3.02 persons/km^2
9 a 336 km b 5 h
10 23 min
11 a 0.05 mg/g b 5 mg
12 12 h 40 min
13 147
14 A
15 74.88 L
16 111 466
17 a 4.1 kg b 6 times

1.10

1 a $3851/month b 1.25 km/min
c 120 L/day d 180 m/min
e 48 cm/h f 1.3 words/s
g 0.082 L/km h 105 000 mL/min
2 a $14.40/h b 28.8 km/h
c 54 L/h d 18c/min
e $16\frac{2}{3}$ m/s f $2\frac{1}{12}$ kg/min
g 2.4 km/min h 360 g/h
3 161 km/h
4 C
5 1800 L/h
6 37.0 km/h
7 25.54 km/h
8 a 263.9 m/s b 6 s
9 a $133\frac{1}{3}$ mL/s b 3840 L
10 a 1.48 m/s b 5.31 km/h
11 a $3.50/week b $182/year
12 a 20 washes/box b 5
13 a i 7.559 km/L ii 13.230 L/100 km
b 756 km c 198 L

1.11

1

Canada	1.0492
China	6.4680
European Union	0.8028
Fiji	1.8845
Hong Kong	8.3479
India	58.3893
Japan	93.7414
New Zealand	1.2787
Thailand	32.2350
Great Britain (UK)	0.6466
USA	1.0499

2 a 3836.10 NZD b 1079.82 GBP
c 54 370 JPY d 40 904.71 HKD
e 4982.56 AUD f 2190.68 AUD
g 477.05 AUD h 533.38 AUD

3 a A$20.38 b A$516.83
c A$141 242.94 d A$296.08
4 C
5 a €566.93 b ¥64 982
c Rp36 793.20 d NZ$904.73
6 A
7 693.32 FJD
8 3966.68 AUD
9 2015.40 USD, 1511.80 EUR, 58 290 THB
10 1098.90 AUD
11 A$1178.55
12 154.67 HKD
13 2412.90 AUD
14 a 1809.45 NZD b 1339.34 AUD
c The buying and selling rate is different and the rates vary every day. When he changed it back it was lower than when he first exchanged it.
15 a A$149 392 b A$348.78
16 a 1.0727 b A$1398
17 a 0.6453 b A$1069
18 A$10.83

1.12

1 a 1 L b 15 cans
c 500 g d box of 45
2 A
3 a 2 kg b 1.5 L
c packet of 4 d 1 L
4 D
5 Nutty clusters
6 Store 1 is the cheapest when considering the cost per drill. For someone who only wants one drill, Store 2 is cheaper.
7 a 5 L tub ($3.50/L)
b 2 L tub ($2.54/L)
c If the 5L tub is too much ice-cream; if the tub will fit into his freezer.
8 a $633.89 b ‘Bargain whitegoods’ at $629.10
c She may consider store loyalty as she works at Whitegoods Superstore; reputation of the two stores on customer service.

CHAPTER 1 REVIEW

1 B
2 C
3 B
4 A
5 E
6 B
7 A
8 B
9 a 40% b $122\frac{2}{9}$% c 30% d 405%
10 35%
11 $30.24
12 661 students
13 $260
14 1320 km
15 217.5 m^2
16 1.99% increase
17 $724
18 35%
19 350%
20 $110 134
21 3.6%
22 a $76.80 b $844.80
23 $427.27
24 $5450
25 625 birds/year
26 a $1092 b 7 days
27 57.6 L/h
28 a 300 g jar. It is $5.48 for 100g, whilst sachets are $5.83.
b As price is close he may like the convenience of sachets.
29 The $345 000 one; as it gave a profit of 20.3% whilst the other only gave 18.2%.
30 a $660.23 b 153.9%
31 a 180 558 000 rupiah b A$3500
c A$21 500 d A$688

2.01

1 a i 3 ii 3 iii 6
b i 4 ii –4 iii –5
2 a $9x - 3y$ b $-5ab - 6a$
c $10b - 9a - 4$ d $4xy + 3x - 6$
e $7ab + a + b$ f $-4r + 2ar$
g $-3de + 2d$ h $4u - 15$ i $18x$
3 a $15xy$ b $-8abc$ c $-36mn^2$
d $-15xy^2z$ e $16a^2km$ f $-24cd^2$
g $36m^2$ h $-20a^2b^2$ i $12bc^2d$
4 a $4x$ b $-\frac{5m}{p}$ c $\frac{3uv}{2w}$
d $-\frac{3s}{rt}$ e 2 f $7x$
g $\frac{3}{x^2}$ h $-\frac{3r}{q}$ i $-\frac{d}{5}$
5 a $8m - n$ b $-b$
c $2x - 4y + 2xy$ d $4pq - 6q + 3p$
6 a $-10mn$ b $-8pq^2$ c $-56x^2yz$ d $60abcd$
7 a $\frac{x}{2z}$ b $-\frac{4a}{c}$ c $-5xy$ d $\frac{8a}{c}$
8 a Answers will vary. For example:
$5a + 3b - 10b + 7a$ or $10a - 5b + 2a - 2b$
b Answers will vary. For example:
$8p + 20q - (6p + 13q)$ or $12p - 5q - (10p - 12q)$
c Answers will vary. For example:
$3x^2 - 5xy + 9x^2 - 3xy$ or $5x^2 + 12xy - 20xy + 7x^2$
d Answers will vary. For example:
$3c \times 2d$ or $-6c \times (-d)$
e Answers will vary. For example:
$8a^2b \times 3b^2$ or $6ab^2 \times 4ab$

f Answers will vary. For example: $\frac{24xy}{36y^2}$ or $\frac{10x^2y}{15xy^2}$
g Answers will vary. For example: $\frac{8a^3b}{14ab^7}$ or $\frac{12a^4b^3}{21a^2b^9}$
9 C
10 A
11 The three numbers will always add up to the same number.
The total will always be $3x + 24$.
12 $p = 2$, $q = -22$

2.02

1 a $3a + 6$ b $15 - 10b$
c $-4a - 2$ d $-6b + 12$
e $3x^2 - 6x$ f $3p^2 - 3ap$
g $-8k - 16$ h $6t - 8t^2$
i $-d^2 + 5d$ j $7k - 5k^2$
k $-9b^2 + 9b$ l $14xy + 8y^2$
2 a $4x - 12$ b $-3a - 12$
c $10a - 5$ d $3x^2 - 24x$
3 a $-24n + 6n^2$ b $5rx^2 + 10rx$
c $-2a^2 + 4$ d $5a^2b + 15b^2 - 35b$
e $-x^2 + 4x - 10$ f $3h^2 - 21eh - 12eh^2$
g $2y^2 + 3y - y^3$ h $d^3e - 2de + de^3$
i $9av^2 - 3v^2 + 6av$
4 a $3x + 14$ b $d - 22$
c $2r + 80$ d $7f + 9$
e $-13x - 24$ f $3x^2 + 27x$
g $2b^2 + 23b$ h $3w^2 - 29w$
i $9k + 12p$ j $4a$
k $vx + 3x$ l $-7t - w$
m $4e^2 + 3e$ n $2a - 18$
o $p^2 - q^2$
5 D
6 B
7 E
8 a 2 and −2; opposites b −6 and 6
c yes; $-(b - a) = -b + a = a - b$
9 a 300 b 162 c 630
d 957 e 4950 f 360
10 a Answers will vary. For example:
$3(4a - 2b)$ or $6(2a - b)$
b Answers will vary. For example:
$4d(4c + 3e)$ or $2d(8c + 6e)$
c Answers will vary. For example:
$12n(2m + 3n)$ or $6n(4m + 6n)$
d Answers will vary. For example:
$4(8ab + 4b^2 - a^2)$ or $2(16ab + 8b^2 - 2a^2)$

2.03

1 a 72 b −30 c 34 d −579
e 62 f −54 g 56 h 43
2 B
3 C
4 602.63 cm^3
5 a 100°C b 55°C c 28°C
6 a \$16.70 b \$74.73 c \$4008.00
7 a 360° b 720° c 1440°
8 a v b 16 m/s
9 a 17 b 81 c 301
10 12 770.1 cm^3
11 3.05 s
12 \$8775.56
13 13.5 m
14 A
15 B
16 Teacher to check, for example:
a Formula for volume of a pyramid, subject of the formula is V, and A is area of the base and h is height of pyramid.
b Formula for angle sum of a polygon, subject of the formula is A, and n is the number of sides in the polygon.

INVESTIGATION: THE BODY MASS INDEX

Patient	Mass (kg)	Height (m)	BMI	Interpretation
A	57.4	1.62	21.9	normal
B	63.5	1.58	25.4	overweight
C	45.8	1.63	17.2	underweight
D	89.6	1.72	30.3	obese
E	104.3	1.85	30.5	obese
F	96.7	1.77	30.9	obese
G	77.2	1.64	28.7	overweight
H	69.5	1.60	27.1	overweight
I	47.6	1.53	20.3	normal
J	124.6	1.74	41.2	obese
K	87.9	1.62	33.5	obese
L	82.1	1.71	28.1	overweight
M	65.8	1.59	26.0	overweight
N	53.5	1.55	22.3	normal
O	52.6	1.63	19.8	normal

2.04

1 a $d = 6$ b $p = 2\frac{1}{2}$ c $u = 3\frac{1}{2}$ d $a = -3$
e $b = -\frac{1}{3}$ f $a = 4\frac{1}{2}$ g $x = 5$ h $y = -7$
i $m = 4$ j $x = 5$ k $m = 2$ l $v = -6$
2 a $h = 12$ b $r = 13$ c $x = -20$
d $y = -5$ e $c = 7\frac{1}{2}$ f $z = 60$
g $a = 6$ h $x = 5$ i $n = 3$
j $a = 2$ k $a = 3$ l $x = 4$
3 a $m = 5$ b $x = 16$ c $a = -2$
d $w = -15$ e $x = 6$ f $a = -3$
g $x = 20$ h $f = 6\frac{1}{2}$ i $w = 70$
j $n = 2\frac{3}{4}$ k $z = -5\frac{1}{5}$ l $b = -3$
4 B

5 a i $2n - 8 = 18$ ii $n = 13$
b i $4(n + 5) = 28$ ii $n = 2$
c i $12 - 3n = 39$ ii $n = -9$
d i $2(n + 6) = 46$ ii $n = 17$
e i $\frac{n}{3} - 4 = 1$ ii $n = 15$
f i $\frac{n+6}{2} = 8$ ii $n = 10$
g i $\frac{2n-4}{5} = 4$ ii $n = 12$
h i $\frac{n+7}{3} = 12$ ii $n = 29$

6 a False b False c False d True

7 a $P = 80 + 2x$ b 110 m

8 a $n - 11$ b $2n - 11$
c 36 marbles d 25 marbles

9 a $4x + 12$ b 7 cm c 13 cm

2.05

1 E

2 a $x = 9$ b $m = -9$ c $k = 11$ d $x = -9$
e $u = -3\frac{2}{5}$ f $p = 4$ g $e = -2$ h $m = 20$
i $x = -22$ j $a = 9$ k $x = 14$ l $a = 8$

3 a $x = -5$ b $d = 2$ c $c = -28$ d $y = -2$
e $x = -14\frac{1}{2}$ f $m = 4\frac{1}{2}$ g $d = 11$ h $a = -1$
i $x = -28$ j $a = -4\frac{1}{5}$ k $g = -4\frac{7}{8}$ l $m = -1$

4 B

5 a $y = 9$ b $m = \pm 5$ c $u = -2$
d $y = 144$ e $d = \pm 12$ f $e = -108$
g $a = 13$ h $x = 8$ i $k = \pm 8$

6 a $d \approx \pm 9.80$ b $y \approx \pm 2.19$ c $c \approx 1.88$
d $n \approx \pm 5.29$ e $p \approx -1.82$ f $w \approx \pm 1.20$
g $h \approx 1.55$ h $r \approx \pm 6.40$ i $p \approx \pm 5.48$

7 a False b True c False d True

8 B

9 a i $2x - 6 = x + 4$ ii $x = 10$
b i $5x + 2 = 4x - 5$ ii $x = -7$
c i $6x - 8 = 4(x + 4)$ ii $x = 12$
d i $8x + 7 = 5x - 14$ ii $x = -7$
e i $3(x - 1) = 2x + 8$ ii $x = 11$
f i $5x + 6 = 3(x - 2)$ ii $x = -6$
g i $\frac{3x}{5} = x + 4$ ii $x = -10$
h i $\frac{x+8}{3} = \frac{2x-6}{5}$ ii $x = 58$

2.06

1 a 7 cm b 11 cm c 9 m d 8 m

2 12 cm

3 300 m

4 21

5 8 cm

6 a 2 min b 3.6 min

7 a 9411.19 b 10.49

8 62.1 kg

9 15.9 m/s

10 4.99 cm

11 11.76 cm

12 C

13 B

14 a i 804.2 cm^2
ii 2144.7 cm^3
b 2.0 m
c 15 mm

CHAPTER 2 REVIEW

1 D

2 C

3 E

4 D

5 C

6 a 4 b -5 c -8

7 a $3x + 6y$ b $6pqr - 13pq$

8 a $\frac{4n}{p}$ b $-\frac{a}{2c}$

9 a $5x + 20$ b $-6x + 3$ c $-2m - 8$

10 a 45 b -8 c 516

11 a -6 b 13 c 38

12 a $x = 9$ b $m = 5$

13 a $a = 4$ b $x = 1.5$

14 a $x = 22$ b $y = -\frac{3}{7}$

15 a $x = 11$ b $p = 6.2$

16 \$3100

17 $r = 2.9$ cm

18 a $4x + 260$ b 65 metres c 195 metres

19 a i 534.1 cm^2 ii 942.5 cm^3
b 10 cm c 2.5 cm

3.01

1 a, b, e and f

2 a True b True c False
d False e True

3 a 10 b 9 c 10

4 a 2×3 b 2×2 c 3×1 d 3×3
e 4×2 f 5×3 g 1×2 h 2×2
i 3×6 j 4×3

5 C

6 a square b square
c zero d column
e identity
f row
g identity
h zero and row

7 a 5 b 5 c $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

8 B

9 Any of the following pairs:
1 × 24, 2 × 12, 3 × 8, 4 × 6, 6 × 4, 8 × 3, 12 × 2, 24 × 1

10 1 × 52, 52 × 1, 2 × 26, 26 × 2, 4 ×13, 13 × 4

11 4 × 9

3.02

1 a 12 b 3 c 2
d Does not exist as there is no 3rd row.
e 9 f 36

2 a b_{24} b b_{13} c b_{14}
d b_{31} e b_{11} f b_{33}

3 a 37 b −1 c −12.1
d 168 e 23 f $\frac{7}{5}=1\frac{2}{5}$

4 a False b True c False
d True e False f False

5 a $\begin{bmatrix}91 & 87 & 94 & 76\\ 94 & 92 & 67 & 87\end{bmatrix}$ b $\begin{bmatrix}91 & 94\\ 87 & 92\\ 94 & 67\\ 76 & 87\end{bmatrix}$

6 a Cake type b Ingredients
c $\begin{bmatrix}200 & 500 & 80\\ 500 & 850 & 120\end{bmatrix}$

7 Teacher to check.

8 Since matrices can only contain elements that are numbers, the hair colour could not be included.

3.03

1 a $\begin{bmatrix}26 & 8\\ 9 & 34\end{bmatrix}$ b $\begin{bmatrix}31 & 6\\ 10 & 46\end{bmatrix}$
c $\begin{bmatrix}25 & 4\\ 15 & 56\end{bmatrix}$ d $\begin{bmatrix}21 & -2\\ 36 & 58\end{bmatrix}$
e $\begin{bmatrix}26 & -4\\ 37 & 70\end{bmatrix}$ f $\begin{bmatrix}27 & 0\\ 31 & 48\end{bmatrix}$

2 $\begin{bmatrix}3.1 & 13 & 5\\ 8.6 & 10 & -6\\ 17 & 1.9 & -11\end{bmatrix}$

3 $\begin{bmatrix}571 & 274 & 567 & 324\\ 512 & 533 & 844 & 699\\ 344 & 1161 & 450 & 547\end{bmatrix}$

4 $\begin{bmatrix}5 & 13\\ 4.5 & 3\\ \frac{1}{2} & 11\end{bmatrix}$

5 a $\begin{bmatrix}6 & 2\\ -5 & -10\end{bmatrix}$ b $\begin{bmatrix}5 & -2\\ 1 & 12\end{bmatrix}$ c $\begin{bmatrix}-1 & 8\\ -22 & -14\end{bmatrix}$
d $\begin{bmatrix}-5 & -10\\ 27 & 24\end{bmatrix}$ e $\begin{bmatrix}5 & 10\\ -27 & -24\end{bmatrix}$ f $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

6 a $m = 44$ b $d = 5$ c $x = 25$
d $t = 48$ e $m = 29$ f $n = 10$

7 a $a = 27, b = 3, c = 18$ b $a = 27, b = -2, c = 28$
c $a = 93, b = 58, c = 3$ d $a = -5, b = 3, c = -17$

8 a $\begin{bmatrix}8 & 2.1 & 8\\ 0.8 & 0 & 32\\ 15 & 12.4 & -6\end{bmatrix}$ b $\begin{bmatrix}-30 & 257 & 457 & 63\\ 516 & 55 & -530 & 330\\ 516 & 6 & 176 & 281\end{bmatrix}$

9 The process of adding matrices involves adding pairs of elements in corresponding positions in both matrices. If we have, for example a 2 × 2 matrix $\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$ and we try to add it to a 2 × 3 matrix $\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\end{bmatrix}$ we can find 4 pairs of corresponding elements to add, but b_{13} and b_{23} have no partners to be added to.

10 $\mathbf{M}+\mathbf{J}=\begin{array}{c} \\ \text{H}\\ \text{L}\end{array}\!\!\begin{array}{c}\begin{array}{ccc}\text{s} & \text{w} & \text{f}\end{array}\\ \begin{bmatrix}21 & 14 & 9\\ 18 & 17 & 13\end{bmatrix}\end{array}$

The sum represents the total sales of each type of vehicle by each person in May and June.

11 $(\mathbf{A}+\mathbf{B})+\mathbf{C}$
$=\begin{bmatrix}8 & 9 & 8 & 10\\ 1 & 9 & 3 & 12\end{bmatrix}+\begin{bmatrix}12 & 0 & 5 & 3\\ 7 & 0 & 1 & 10\end{bmatrix}$
$=\begin{bmatrix}20 & 9 & 13 & 13\\ 8 & 9 & 4 & 22\end{bmatrix}$
$=\mathbf{A}+(\mathbf{B}+\mathbf{C})$

3.04

1 a $\begin{bmatrix}15\\ 33\end{bmatrix}$ b $\begin{bmatrix}-2 & 6\\ 8 & 8\\ 14 & 4\\ 0 & 10\end{bmatrix}$
c $\begin{bmatrix}161 & 28 & 77 & 189\\ 217 & 126 & 168 & 224\end{bmatrix}$ d $\begin{bmatrix}88 & 68 & 100\\ 104 & 76 & 88\\ 52 & 100 & 80\end{bmatrix}$
e $\begin{bmatrix}228 & 444 & 252\\ 492 & 684 & 396\end{bmatrix}$ f $\begin{bmatrix}65 & 140\\ 80 & 135\\ 195 & 200\\ 75 & 160\\ 145 & 165\end{bmatrix}$

2 a $\begin{bmatrix}114 & 122\\ 108 & 102\\ 124 & 156\end{bmatrix}$ b $\begin{bmatrix}342 & 366\\ 324 & 306\\ 372 & 468\end{bmatrix}$
c $\begin{bmatrix}1311 & 1403\\ 1242 & 1173\\ 1426 & 1794\end{bmatrix}$ d $\begin{bmatrix}5814 & 6222\\ 5508 & 5202\\ 6324 & 7956\end{bmatrix}$

3 a False b False c True
d True e False f True

4 a $\begin{bmatrix}16 & 14\\ 16 & 24\end{bmatrix}$ b $\begin{bmatrix}-1 & -24\\ 9 & -4\end{bmatrix}$ c $\begin{bmatrix}37 & 37\\ 35 & 56\end{bmatrix}$
d $\begin{bmatrix}18 & 62\\ -2 & 32\end{bmatrix}$ e $\begin{bmatrix}21 & -51\\ 51 & 24\end{bmatrix}$ f $\begin{bmatrix}15 & 27\\ 9 & 24\end{bmatrix}$
g $\begin{bmatrix}10 & -19\\ 22 & 12\end{bmatrix}$ h $\begin{bmatrix}5 & -65\\ 35 & 0\end{bmatrix}$

5 $\begin{bmatrix}28 & 41 & 31\\ 14 & 37 & 29\end{bmatrix}$

6 a $\begin{bmatrix}35 & 29\\59 & 18\\48 & 25\\25 & 77\end{bmatrix}$ b $\begin{bmatrix}17 & -14 & 34\\39 & 89 & 53\end{bmatrix}$

7 a $a = 3, b = 11, c = 6$ b $x = 7, y = 9, z = 44$
c $p = 13, q = 19, f = -1$
d $a = 8, b = -4, c = -5, d = -2$

8 D

9 $2\mathbf{T} = 2\begin{bmatrix}12 & 7\\21 & 15\end{bmatrix} = \begin{bmatrix}24 & 14\\42 & 30\end{bmatrix}$

So we need to show that $5\mathbf{T} - 3\mathbf{T} = \begin{bmatrix}24 & 14\\42 & 30\end{bmatrix}$

$$5\mathbf{T} - 3\mathbf{T} = 5\begin{bmatrix}12 & 7\\21 & 15\end{bmatrix} - 3\begin{bmatrix}12 & 7\\21 & 15\end{bmatrix}$$
$$= \begin{bmatrix}60 & 35\\105 & 75\end{bmatrix} - \begin{bmatrix}36 & 21\\63 & 45\end{bmatrix} = \begin{bmatrix}24 & 14\\42 & 30\end{bmatrix}$$

Therefore, $5\mathbf{T} - 3\mathbf{T}$ does equal $2\mathbf{T}$ when $\mathbf{T}$ is a matrix.

10 $\begin{bmatrix}12 & 32\\17 & 27\\19 & 33\end{bmatrix}$

11 $x = 2, y = 5, z = 15$

3.05

1 a P b P c N
d N e N f P

2 a D b B

3 a [181] b [242] c [156]
d [16] e [–7] f [108]

4 a C b D

5 $x = 5$

6 $u = 6$

7 a [49] b [322.3] c [10 667.5]
d [272 254.42] e [28 326.5]
f [125 013.6]

8 a i $\begin{bmatrix}6 & 21\end{bmatrix}$ ii $\begin{bmatrix}18\\10\end{bmatrix}$ iii [53] iv [318] v [318]
b The 2 answers are the same.

9 24

10 1566 bottles.

3.06

1 a Not possible b Possible, order 2 × 1
c Not possible d Possible, order 2 × 1
e Possible, order 2 × 1 f Possible, order 4 × 1

2 D

3 D

4 3 × 1

5 a C b A

6 a [118] b $\begin{bmatrix}71\\73\end{bmatrix}$ c $\begin{bmatrix}170\\151\\74\end{bmatrix}$ d $\begin{bmatrix}2\\55\\147\\94\\85\end{bmatrix}$

7 a $\begin{bmatrix}38\\53\end{bmatrix}$ b $\begin{bmatrix}116\\114\end{bmatrix}$ c $\begin{bmatrix}38\\45\end{bmatrix}$ d $\begin{bmatrix}75\\30\\28\end{bmatrix}$
e $\begin{bmatrix}5\\12\end{bmatrix}$ f $\begin{bmatrix}96\\71\\96\end{bmatrix}$

8 $x = 3$

9 $d = 4$

10 a $\begin{bmatrix}9.00\\13.00\\6.50\end{bmatrix}$ b $\begin{bmatrix}62 & 105 & 32\\50 & 62 & 45\end{bmatrix}$
c $\begin{bmatrix}62 & 105 & 32\\50 & 62 & 45\end{bmatrix}\begin{bmatrix}9.00\\13.00\\6.50\end{bmatrix} = \begin{bmatrix}2131.00\\1548.50\end{bmatrix}$

Therefore total weekend ticket sales were $3679.50.

11 No. In multiplying **AB** we have (2 × 3) × (3 × 1), which works because the middle 2 numbers are both 3. In multiplying **BA** we would have (3 × 1) × (2 × 3), which we can't do since the 2 middle numbers are different.

3.07

1 a i Not exist b i Exists ii 2 × 3
c i Exists ii 2 × 2
d i Not exist e i Exists ii 3 × 3
f i Not exist g i Exists ii 4 × 2
h i Exists ii 3 × 2
i i Not exist j i Exists ii 2 × 2

2 a $\begin{bmatrix}86 & 134\\51 & 67\end{bmatrix}$ b $\begin{bmatrix}20 & 38 & 31\\42 & 72 & 82\\36 & 66 & 61\end{bmatrix}$
c No, **XY** is 2 × 2, **YX** is 3 × 3.
d No

3 a $\begin{bmatrix}31 & 37 & 30\\19 & 17 & 12\end{bmatrix}$ b $\begin{bmatrix}16 & 5\\2 & 11\end{bmatrix}$ c $\begin{bmatrix}10 & 12\\29 & 26\\6 & 8\end{bmatrix}$
d $\begin{bmatrix}15 & 27\\25 & 48\\19 & 43\\22 & 43\end{bmatrix}$ e $\begin{bmatrix}14 & 18 & 32 & 62\\22 & 38 & 43 & 95\end{bmatrix}$
f $\begin{bmatrix}43 & 13\\-11 & 16\end{bmatrix}$

4 a $\begin{bmatrix}16 & 26\\27 & 36\\18 & 27\end{bmatrix}$ b $\begin{bmatrix}21 & 33\\14 & 25\\19 & 32\\18 & 27\end{bmatrix}$ c $\begin{bmatrix}40 & 70\\32 & 66\\49 & 92\end{bmatrix}$
d $\begin{bmatrix}31 & 58\\17 & 27\end{bmatrix}$ e $\begin{bmatrix}4 & 10 & 3 & 17\\15 & 15 & 9 & 39\end{bmatrix}$
f $\begin{bmatrix}10 & 10 & 6 & 26\\16 & 30 & 11 & 57\\11 & 15 & 7 & 33\end{bmatrix}$

5 a $\begin{bmatrix}48 & 74\\33 & 79\\36 & 51\end{bmatrix}$ b $\begin{bmatrix}429 & 928 & 295\\399 & 924 & 369\end{bmatrix}$
c $\begin{bmatrix}768\\479\end{bmatrix}$ d $\begin{bmatrix}250.5 & 205.5\\286.1 & 275.1\\348.3 & 283.5\end{bmatrix}$

e $\begin{bmatrix}3330 & 2175 & 3672 & 2970\\ 3456 & 2948 & 2395 & 2219\end{bmatrix}$

f $\begin{bmatrix}491.4 & 729.3\\ 474.6 & 601.5\end{bmatrix}$

6 D

7 B

8 A

9 a $\mathbf{P}=\begin{bmatrix}5 & 24 & 3 & 10 & 3\\ 2 & 0 & 1 & 20 & 5\\ 2 & 0 & 1 & 30 & 10\end{bmatrix}$

b $\mathbf{S}=\begin{bmatrix}12 & 10 & 3\\ 8 & 32 & 0\end{bmatrix}$

c $\mathbf{SP}=\begin{bmatrix}86 & 288 & 49 & 410 & 116\\ 104 & 192 & 56 & 720 & 184\end{bmatrix}$

d i 86 ii 720

iii 49 + 46 = 105 erasers

INVESTIGATION: MULTIPLYING MATRICES: IDENTITY MATRICES

a $\mathbf{AI_2}$, $\mathbf{EI_2}$, $\mathbf{I_2A}$, $\mathbf{I_2B}$, $\mathbf{I_2D}$, $\mathbf{I_2H}$, $\mathbf{DI_3}$, $\mathbf{FI_3}$, $\mathbf{I_3C}$, $\mathbf{HI_4}$, $\mathbf{I_4G}$

b Teacher to check.

c It is like multiplying by 1, you just get the matrix that you started with.

3.08

1 a **A** – no, **B** – yes, **C** – yes, **D** – yes, **E** – no, **F** – yes

b $\mathbf{B}^2=\begin{bmatrix}28 & 3\\ 9 & 7\end{bmatrix}$; $\mathbf{C}^2=\begin{bmatrix}3 & -2 & 23\\ 42 & 56 & 58\\ 38 & 42 & 82\end{bmatrix}$;

$\mathbf{D}^2=\begin{bmatrix}-16 & 55\\ -44 & 61\end{bmatrix}$; $\mathbf{F}^2=\begin{bmatrix}29 & 95 & 45 & 35\\ 36 & 87 & 28 & 39\\ 50 & 142 & 59 & 62\\ 45 & 108 & 40 & 47\end{bmatrix}$

2 a $\begin{bmatrix}142 & 78\\ 91 & 51\end{bmatrix}$ b $\begin{bmatrix}61 & 62 & 59\\ 24 & 24 & 23\\ 85 & 99 & 89\end{bmatrix}$

c $\begin{bmatrix}134 & -10\\ 25 & 39\end{bmatrix}$ d $\begin{bmatrix}34 & -18\\ -36 & 22\end{bmatrix}$

e $\begin{bmatrix}402 & 246 & 234\\ 141 & 138 & 87\\ 233 & 84 & 261\end{bmatrix}$ f $\begin{bmatrix}543 & 572\\ 484 & 719\end{bmatrix}$

3 D

4 C

5 a 227 b 492 c 301 d −11

6 a $\begin{bmatrix}72 & 49\\ 63 & 79\end{bmatrix}$ b $\begin{bmatrix}657 & 700\\ 900 & 757\end{bmatrix}$

c $\begin{bmatrix}30.04 & 31.72\\ 20.28 & 31.08\end{bmatrix}$ d $\begin{bmatrix}102.04 & 80.72\\ 83.28 & 110.08\end{bmatrix}$

e $\begin{bmatrix}91\,404 & 87\,493\\ 112\,491 & 103\,903\end{bmatrix}$ f $\begin{bmatrix}91\,332 & 87\,444\\ 112\,428 & 103\,824\end{bmatrix}$

g $\begin{bmatrix}1050 & 696 & 1101\\ 1240 & 869 & 1346\\ 1303 & 938 & 1480\end{bmatrix}$

h $\begin{bmatrix}3\,400\,143 & 2\,368\,362 & 3\;722\,346\\ 4\,133\,398 & 2\,880\,749 & 4\,526\,994\\ 4\,459\,710 & 3\,110\,250 & 4\,887\,551\end{bmatrix}$

7 a element 1,2 or $(\mathbf{M}^2)_{12}$ i.e. the element in the 1st row and 2nd column.

b $\begin{bmatrix}6 & 3\\ 4 & 2\end{bmatrix}$ c $\begin{bmatrix}6 & 2\\ 4 & 2\end{bmatrix}$

8 a $\mathbf{C}^2=\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}9 & 8\\ 0 & 1\end{bmatrix}$

$\mathbf{C}^3=\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}9 & 8\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}27 & 26\\ 0 & 1\end{bmatrix}$

b $\mathbf{C}^2\times\mathbf{C}=\begin{bmatrix}9 & 8\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}27 & 26\\ 0 & 1\end{bmatrix}$

c True.

3.09

1 B

2 D

3 a $\mathbf{D}=\begin{bmatrix}5 & 10 & 20\\ 40 & 65 & 70\end{bmatrix}$

b \$210

c i \$35

ii 2.85%

d i \$175

ii 3.03%

e Percentage saving $=\dfrac{\$210}{\$7010}\times 100\approx 2.996\%$. This is slightly below the required 3%, however, if we give the answer to 2 decimal places, the manager can say that he has succeeded. He was able to reduce production line costs significantly (3.03%), but not office costs (2.85%).

4 a $\mathbf{C}=\begin{bmatrix}0.075 & 0.067 & 0.07\end{bmatrix}$

b $\mathbf{CS}=\begin{bmatrix}3194.95 & 3733.275\end{bmatrix}$

c \$6928.23

5 a $\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ b $\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$

c Craven to Craven, Murryba to Murryba, Gwenock to Gwenock

6 A

7 a $\mathbf{MP} = \begin{bmatrix} 9.39 & 51.88 & 18.75 & 566.36 \end{bmatrix}$

b The elements in **MP** show the total combined revenue across all 3 stores for each of the four products once the prices have been marked up.

c $24.56

8 a $\mathbf{C} = \begin{bmatrix} 3.20 \\ 5.60 \end{bmatrix}$

b $\mathbf{NC} = \begin{bmatrix} 3172.00 \\ 208.80 \end{bmatrix}$

c $3380.80

9 a $\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ b $\mathbf{P}^2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

c $\mathbf{P}^5 = \begin{bmatrix} 1 & 3 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 & 1 \end{bmatrix}$

d A to A, A to C, B to B, B to E, C to C, C to D, D to B, D to D, E to A

e i 2

ii B to C to E to D to A to D or
B to C to B to C to E to D

iii Check with your teacher as these will vary. For example B to C to B to C to E to D could be something like: John lived at B. He needed to deliver a parcel to his grandmother who lived at D. John headed for C. When he got there he realised he'd forgotten the parcel. He returned to B, then travelled to C, then E and finally to his grandmother's at D.

CHAPTER 3 REVIEW

1 E
2 D
3 C
4 D
5 B
6 C
7 B
8 C
9 A
10 B
11 4 × 5
12 square matrix
13 column matrix

14 a $\begin{bmatrix} 91 & 53 & 34 \\ 86 & 62 & 27 \\ 78 & 57 & 37 \\ 79 & 54 & 35 \\ 42 & 48 & 40 \\ 34 & 41 & 53 \end{bmatrix}$ b $\begin{bmatrix} 91 & 86 & 78 & 79 & 42 & 34 \\ 53 & 62 & 57 & 54 & 48 & 41 \\ 34 & 27 & 37 & 35 & 40 & 53 \end{bmatrix}$

15 a $\begin{bmatrix} 18 & 20 \\ 31 & 19 \end{bmatrix}$ b $\begin{bmatrix} 38 & 41 & 64 & 27 \\ 84 & 54 & 37 & 61 \end{bmatrix}$

c $\begin{bmatrix} 17 \\ 7 \\ 16 \end{bmatrix}$ d $\begin{bmatrix} 48 & 40 & 60 \\ 43 & 77 & 70 \\ 96 & 68 & 43 \end{bmatrix}$

16 a $\begin{bmatrix} 5 & 13 \\ 11 & 12 \end{bmatrix}$ b $\begin{bmatrix} 22 & 2 \\ 15 & 15 \\ 21 & 7 \end{bmatrix}$ c $\begin{bmatrix} 5 & 37 & 12 \\ 21 & 5 & 41 \end{bmatrix}$

17 a $t = 8$ b $a = 4$
c $p = 32$ d $x = -11$

18 a $\begin{bmatrix} 15 & 35 \\ 25 & 40 \end{bmatrix}$ b $\begin{bmatrix} 18 & 27 & 33 \\ 12 & 21 & 9 \\ 6 & 30 & 18 \end{bmatrix}$

c $\begin{bmatrix} 84 & 56 \\ 28 & 63 \\ 21 & 49 \end{bmatrix}$ d $\begin{bmatrix} -24 & -48 \\ -30 & -16 \end{bmatrix}$

19 $\begin{bmatrix} 8 & 8 & 13 \\ -7 & -3 & 12 \end{bmatrix}$

20 $\begin{bmatrix} 61 \end{bmatrix}$

21 $y = 1$

22 $\begin{bmatrix} 74 \\ 93 \end{bmatrix}$

23 $x = 7$

24 a $\begin{bmatrix} 95 & 90 \\ 49 & 66 \end{bmatrix}$ b $\begin{bmatrix} 84 & 24 & 66 \\ 28 & 26 & 67 \end{bmatrix}$

c $\begin{bmatrix} 77 & 72 \\ 70 & 59 \\ 86 & 90 \end{bmatrix}$

25 $\begin{bmatrix} 14 & 14 & 11 \\ 28 & 14 & 9 \\ 7 & 14 & 11 \end{bmatrix}$

26 $\begin{bmatrix} -242 & 423 \\ -1269 & 1873 \end{bmatrix}$

27 a $\begin{bmatrix} 2.50 & 1.20 \end{bmatrix}$ b $\begin{bmatrix} 123 & 85 & 112 & 156 & 139 \\ 102 & 115 & 156 & 108 & 121 \end{bmatrix}$

c $\begin{bmatrix} 429.90 & 350.50 & 467.20 & 519.60 & 492.70 \end{bmatrix}$

d $467.20

e Wednesday

f Thursday

MIXED REVISION 1

Multiple choice

1 A
2 B
3 B
4 B
5 C
6 D
7 D

8 A
9 C

Short answer questions

1 a 2 mg/mL b 240 mg
2 $2a^2 - 13a - 8$
3 a **A** and **D**
b Yes. **C** has the same number of rows as **A** has columns.
c **AC, AB, BA, BD, CB, DC, DB**
d **AC** is of order 3 × 2, **AB** is of order 3 × 3, **BA** is of order 2 × 2, **BD** is of order 2 × 2, **CB** is of order 2 × 3, **DC** is of order 3 × 2, **DB** is of order 3 × 3
e **C** – only square matrices can be raised to a power
4 a The 175 g packet; $1.42 for 100 g whereas the multipack is $1.61 for 100 g.
b Factors such as convenience could influence her decision. Small chip packets can easily be put into children's lunch boxes. Chips in large packets might go stale if they are not all eaten in one day.
5 455.5
6 a $\begin{bmatrix} 83 \end{bmatrix}$ b $\begin{bmatrix} 5 \\ -9 \\ 1 \end{bmatrix}$
c $\begin{bmatrix} 22 & 10 & 30 \\ 23 & 19 & 39 \end{bmatrix}$ d $\begin{bmatrix} 121 & 72 \\ 144 & 97 \end{bmatrix}$

Application questions

1 a $558.25 b $474.51
c $43.14 d 23.2%
2 a 109 260 baht b $150
c $3150 d $33.60
3 a 314.2 cm^2 b 12.6 m^2
c 7.1 mm
4 a $C = 250 + 5n$ b $750
c 320 people d $C = 300 + 5n$
5 a $\mathbf{S} = \begin{bmatrix} 29 & 1 & 1 \\ 30 & 0 & 1 \\ 23 & 1 & 2 \\ 27 & 2 & 0 \end{bmatrix}$
b i Each player's total score will be calculated using:
"sum of dice for all 5 rolls"+ 5 × the number of doubles rolled + 4 × the "Number of rolls where sum = 8".
So the number 1 is used because when we multiply **P** by **S**, that will correctly use 1 lot of the "Sum of dice for all 5 rolls".
ii $\mathbf{SP} = \begin{bmatrix} 38 \\ 34 \\ 36 \\ 37 \end{bmatrix}$
iii Katie had the lowest score of 34, so she had to wash up.

6 a

	Atherton	Blair Hill	Cole Ridge	Dundin
Atherton	0	0	1	1
Blair Hill	0	0	1	0
Cole Ridge	1	1	0	1
Dundin	1	0	1	0

b $\mathbf{M} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ c $\mathbf{M}^2 = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$
d i 1 ii 1 iii 0
e Blair Hill → Cole Ridge → Dundin → Cole Ridge → Atherton
Blair Hill → Cole Ridge → Atherton → Cole Ridge → Atherton
Blair Hill → Cole Ridge → Atherton → Dundin → Atherton
Blair Hill → Cole Ridge → Blair Hill → Cole Ridge → Atherton

4.01

1 $968.63
2 $1076.25
3 a $54 999.10 b $1057.68 c $27.83
4 $889.44
5 a $35 484.80 b $2957.07
6 $552.51
7 $1501.50
8 $969.90
9 C
10 a $5586.12 b $12 103.25
c $2793.06
11 a $6934.58 b $3200.58
c $320.06
12 A
13 $565.73
14 C
15 $802.56
16 a $210.71 b $1053.55
c $4565.38
17 C
18 a $19.20 b $979.32
c $50 924.64
19 $1636.83
20 $18.95
21 i $1161 ii 9 hours
iii $35 iv 35 hours

4.02

1 $117
2 a $378 b $224

3 a $250 b $2000
4 E
5 a $232 b $19.33
6 $425.50
7 D
8 a $119.70 b 213
9 a $146.19 b 821
10 1382
11 $1040
12 a $30.10 b $6750
13 $1678
14 a $5900 b $10 175
15 60
16 $530.85
17 a $55.25 b 2770 c 2012
18 a $124.80 b $28.41
c $96.40; $16.10
19 2143 or more
20 8.8%

4.03

1 C
2 $671.52
3 $1333
4 C
5 $919.09
6 a $4073.60 b $712.88
c $4786.48
7 $3554.85
8 $1800
9 $1371.93
10 E
11 $929.60
12 $768.15
13 $2993.90
14 a $1439.63 b 53 hours
15 $170.81
16 a $888.96 b $522.06
17 $33 666.73
18 $575.89

4.04

1 a $712 b $1063.40 c $0 (as <65 years)
2 Because a couple live in the same house and the cost of living is shared.
3 a i $492.60 ii $12 807.60
iii $1067.30
b $5781.10
c More. This is fair as they have no partner to share the costs.
4 E
5 a $444.70 b $533 c $663.70
d i $13 858 ii $266.50
6 B
7 a $407.50 b $444.70 c $407.50 d $223

4.05

1 a E b I c E d E
e I f E g E h E
i I j E k I l E
m I n I
2 a $351.30 b $54
c Spend less on clothes, spend less on groceries by looking for specials.
3 a $51.40 b Yes c Yes
4 a

The Rogers family weekly budget

Income		Expenses	
wages	$1901.77	bills	$105.30
part time job	$289.53	school fees	$85.80
parenting allowance	$197.20	entertainment	$295
		health fund	$53.40
		clothes	$132
		home maintenance	$184
		groceries	$210.50
		petrol	$85.40
		takeaway	$46
		n/papers and mags	$26
		home loan repay.	$545.60
		car loan repay.	$184.22
		savings	$_____
Total:	$2388.50	Total:	$2388.50

b $435.28
5 a

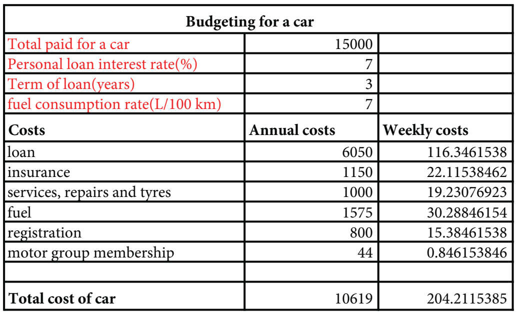
Zac's weekly budget

Income		Expenses	
wage	$551.40	rent	$110
		food	$94
		public transport	$46
		elec bill	$8
		phone bill	$5
		entertainment	$45
		chiropractor	$10
		savings	$_____
Total:	$551.40	Total:	$551.40

b $551.40 c $318 d $233.40
e Yes. He hasn't budgeted for clothes.
6 a $127 b $6604
7 Aya could save approximately $10 452 in a year.

INVESTIGATION: BUDGETING FOR A CAR

a, b



Budgeting for a car		
Total paid for a car	15000	
Personal loan interest rate(%)	7	
Term of loan(years)	3	
fuel consumption rate(L/100 km)	7	
Costs	**Annual costs**	**Weekly costs**
loan	6050	116.3461538
insurance	1150	22.11538462
services, repairs and tyres	1000	19.23076923
fuel	1575	30.28846154
registration	800	15.38461538
motor group membership	44	0.846153846
Total cost of car	10619	204.2115385

c $204.21

d i $186.71. She saves $17.50 per week.

ii $136.74. She saves $67.47 per week.

e Answers will vary

f Before she buys a car she could save the money so she doesn't need a loan. She can also not drive as much so that she uses less petrol and there is less wear and tear on the car.

4.06

1 606 shares
2 $12 284.30
3 $408.51
4 a $43 935 b $615.09 c $2685
d loss of $6035.72
5 a $90 000 b 90c per share
6 $1.64
7 a $0.45 b 6.0%
8 A
9 a $35 750 b $715 c 2.85%
10 a $2.23 b $1070.40
11 a $30 487.50 b $1440
c $13 963
12 a $8364 b $8561.78
c $2296 d 27.45%
13 a $35 998 b $1973.07
14 a $1.12 b 1.5
15 a $0.83 b 9.42
16 13
17 a 4.1% b 16.9
18 a $7.75 b $0.62 c $1.13 d 6.9
19 a $1050 b 3.0% c 74
20 a 5.3%
b Buy shares. The dividend yield is better than the credit union interest.
c A credit union is safer as there is no chance of losing your money like there is if share prices fall.
21 C
22 In the bank. The interest rate is better than the dividend yield.
23 a 13.9 b 18.3
c P/E is lower for Bullseye. Therefore, for every dollar of current earnings, the investor pays less for Bullseye. So Bullseye is the best option.
24 P/E BMG = 7.6; P/E Vorx = 9.7. Therefore P/E is lower for BMG. For every dollar of current earnings, the investor pays less for BMG. So BMG is the best option.
25 P/E Shakra = 14.3; P/E YFT = 6.25. Therefore P/E is lower for YFT. For every dollar of current earnings, the investor pays less for YFT. So YFT is the best option.

CHAPTER 4 REVIEW

1 A
2 C
3 C
4 D
5 B
6 a $634.50 b $1292.50
7 $578.25
8 a $3788.58 b $21 980.33
9 $3346
10 $6250
11 a $102.20
b 124 buckets (round up as 123 buckets would not be enough)
12 $1012
13 a $1163.48 b $7811.94
14 a ABSTUDY, $268.20
b couples age pension, $536.70 each
c youth allowance, $407.50
d Austudy, $533.80
e single age pension, $712
f youth allowance, $407.50 each
g youth allowance, $407.50
15 a both $1330 b $205
c Chloe will need a new flat mate. Without the flatmate's rent, she only has $5 left to save. This is not enough to cover any extra on bills she may need to pay or other contingencies.
16 a $7949.55 b $1106 c $1662.85
17 $2.70
18 a 8.0%
b Buy shares. The dividend yield is better than the credit union interest.
19 a $1125
b i $240 ii $25 iii $25
iv $90 v $10 vi $25

c

Weekly budget			
Income		Expenses	
wages	$830	rent	$240
rent	$295	power	$25
		food	$320
		transport	$70
		lounge repayments	$25
		clothes	$90
		contents insurance	$10
		mobile phone	$25
		aerobics	$20
		savings	$300
Total:	$1125	Total:	$1125

d $15 600

20 P/E STX = 25.9; P/E Electro = 22.0. Therefore P/E is lower for Electro. For every dollar of current earnings the investor pays less for Electro. So Electro is the best option.

5.01

1 a 5.39 m b 7.30 m c 12.21 cm
d 18.73 cm e 5.66 cm f 7.62 m
g 150.00 m h 141.77 mm
i 1.54 cm j 8632.50 mm
k 0.70 m l 19.37 mm
2 a 9.2 cm b 11.0 cm c 30.0 cm
d 98.6 m e 57.8 cm f 33.0 m
g 41.9 cm h 14.1mm i 38.1 cm
3 a N b N c Y
d Y e N f Y
4 4.20 m
5 13 229 m
6 6 m
7 120 nautical miles
8 1084.16 m
9 a 335 cm b 602 cm
10 19.4 m
11 461 m
12 4.28 km
13 21 cm
14 4.0 m
15 110.99 m
16 1.3 m

5.02

1 a 3.2 m b 5230 m c 146 cm
d 0.58 m e 0.537 km f 3270 cm
g 54 mm h 0.4735 km i 640 mm
2 a 1690 mm b 1.69 m
3 1.5 km
4 a 16.1 cm b 150.8 mm
c 16.3 m d 115.0 cm
e 131.0 cm f 49.2 mm
5 a 10.7 m b 454.3 mm
c 203.0 cm d 99.8 m
e 122.6 cm f 142.7 m
6 10.3 m.
7 $3361.20 (for 58 posts, 223 m of railing, 930 full palings and a gate)
8 4.5 cm
9 a 60 m b $2589.60
c $600 (including gate)
10 $152 744.23
11 About 1420

5.03

1 D
2 C
3 a 50 000 cm^2 b 250 000 mm^2 c 7.2 ha
d 0.68 m^2 e 3 090 000 m^2 f 360 ha
g 4 730 000 mm^2 h 5.4 km^2
4 a 64 m^2 b 34.9 m^2
c 314.2 cm^2 d 4322.5 m^2
e 24 m^2 f 40 m^2
g 375 m^2 h 4 523 893.4 km^2
i 300 m^2 j 11.0 m^2
k 42.4 m^2 l 40.8 m^2
5 a 100.5 m^2 b 12 746.1 mm^2
c 2020.8 cm^2 d 382.1 cm^2
e 6.8 km^2 f 616.5 mm^2
6 12.8 mm
7 a 4 209 000 mm^2, 4.209 m^2
b $2.3 \times 1.83 = 4.209$ m^2
8 90
9 80 160 000 ha
10 a 113 cm^2 b 11 310 mm^2
11 15.02 m^2
12 a 16 ha b 28 bags

INVESTIGATION: PERIMETER AND AREA

a 3×6 b radius = 2 c 5, 12, 13

5.04

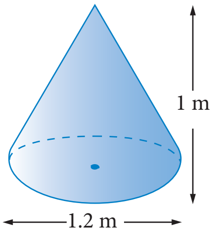
1 D
2 B
3 a 7.9 m^2 b 589.2 mm^2
c 25.3 cm^2 d 32 202.1 mm^2
e 22.1 m^2 f 12.9 km^2
g 227 196.5 mm^2 h 24.5 m^2
4 a 2375 cm^2 b 4 426 820 mm^2
c 2952 cm^2 d 160 mm^2
e 401 m^2 f 198 cm^2
5 a 904.9 m^2 b 178.7 mm^2
c 1716.0 mm^2
6 42.3 m^2
7 a 314.16 cm^2 b 85.84 cm^2
8 14 827 m^2
9 a 143.56 cm^2 b 82.5 cm^2
10 14.11 m^2
11 a 67.16 m^2 b 3918 tiles c $2292

5.05

1 a 510 cm^2 b 204 m^2
2 a 1392 cm^2 b 1950 cm^2
c 5400 cm^2 d 39.36 m^2
e 1670 m^2 f 810 m^2
g 564 m^2 h 3384 cm^2
i 254 cm^2
3 a 301.59 cm^2 b 170.54 m^2
c 3887.72 cm^2 d 5881.06 mm^2
e 30.22 cm^2 f 351.99 m^2
4 20.83 m^2
5 A is the area of the end faces, all the other faces can be combined into a long rectangle with length P and height h.
6 a, b 36.14 m^2
7 a 50.27 cm^2 b 12 cm
c 25.13 cm d 402.12 cm^2
8 12 cm

5.06

1 C
2 a 2225 cm^2 b 384.8 cm^2
c 38 528.49 cm^2 d 3.36 m^2
e 561.56 cm^2 f 8.20 m^2
3 D
4 a i 7.3 m ii 32 m^2
b i 5.7 cm and 5.8 cm ii 36 cm^2
c i 18.7 mm ii 686 mm^2
d i 28.3 cm ii 983 cm^2
e i 34.2 mm ii 2220 mm^2
f i 3.6 m ii 29 m^2
5 D
6 a 58 106.90 mm^2 b 47.78 cm^2
c 24.13 cm^2
7 135 240 m^2
8 a 2513.3 m^2 b 419 L
c 42 tins d $2478
9 a 16 cm^2 b 4 cm
10 $1015.95
11 a 16 cm b $h = 34$ cm c 3200 cm^2
12 a



b 2.2 m^2

5.07

1 C
2 B
3 a 62 831.9 mm^2 b 659.7 cm^2
c 2570.4 mm^2 d 231 cm^2
e 46.2 m^2 f 39.7 m^2
4 a 4.27 cm b 142.63 cm^2
5 a 1445.13 cm^2 b 1147.93 cm^2
c 4086.94 m^2
6 a 90.27 m^2 b $2410.28
7 a 1.9 m b 44.5 m^2
8 a 295.79 m^2 b 5 tins
c $489.50 d $7

5.08

1 a 7 000 000 cm^3 b 50 000 mm^3
c 0.089 m^3 d 468 000 cm^3
e 2.4 cm^3 f 5.6 m^3
g 9100 cm^3 h 12 000 000 cm^3
2 1.8 m^3
3 C
4 A
5 a 250 cm^3 b 1.7 m^3
c 30 846.4 cm^3 d 3036 cm^3
e 20.2 m^3 f 1480 cm^3
g 18 162.3 cm^3 h 1.2 m^3
i 2.7 m^3
6 a 680 mL b 8.5 L
c 22 000 L d 8 m^3
e 3 500 000 mL f 690 000 cm^3
g 55 000 L h 4300 kL
i 9.5 m^3 j 85 L
k 4300 cm^3 l 1000 ML
7 a 16.1 m^3 b 16 100 L
8 a 4 m^3 b $264
9 a 50.225 m^3 b 50 225 L
c 104 hours and 38 minutes
10 a 350 cm^3 b 350 mL
c No. The radius is squared when calculating volume and capacity, so doubling it will more than double the can's capacity.
11 4.25 cm

5.09

1 A
2 C
3 E
4 D
5 a 19.21 mm b 5149.7 mm^3
c 5 mL
6 a 31 000 mm^3 b 6568.02 cm^3
c 150 cm^3 d 285 cm^3
e 1.59 m^3 f 248.47 cm^3
g 27.03 m^3 h 2868.97 cm^3
i 15 002.47 mm^3
7 164.64 cm^3
8 2 415 800 m^3
9 24 429 million km^3
10 a 808.17 cm^3 b 493.91 cm^3 c 61%
11 39.6 cm
12 a 16 mm b 19 mm c 1051 mm^2

INVESTIGATION: CYLINDERS, CONES AND SPHERES

a Teacher to check.

b i Volume of sphere = 4188.79... cm^3,
(or $1333\frac{1}{3}\pi$ cm^3)
volume of cylinder = 6283.18... cm^3 (or 2000π cm^3)

ii $\frac{2}{3}$

iii Yes – it is true for all spheres and cylinders where the radius is equal.

iv Volume of sphere = $\frac{4}{3}\pi r^3$.
Volume of cylinder = $\pi r^2 \times h$ – but the height of the cylinder is equal to 2 × radius.
∴ Volume of cylinder = $\pi r^2 \times 2r = 2\pi r^3$

So $\frac{V_{\text{sphere}}}{V_{\text{cylinder}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3}$.
We can cancel the πr^3 from the numerator and denominator, leaving $\frac{\frac{4}{3}}{2} = \frac{2}{3}$.

5.10

1 Line 4
2 C
3 4.93 m^3
4 a i 1 243 547.1 mm^3 ii 1244 mL
b i 1436.8 cm^3 ii 1437 mL
c i 8379.0 mm^3 ii 8 mL
d i 18.1 m^3 ii 18 119 659 mL
e i 18.1 m^3 ii 18 095 973 mL
f i 1030.4 cm^3 ii 1030 mL
5 576 m^3
6 2450 cm^3
7 a 33.091 m^3 b 34 kL c $234.80
8 a 282 mL
b 24 cones (there will not quite be enough for 25 cones)

CHAPTER 5 REVIEW

1 E
2 C
3 D
4 B
5 C
6 D
7 A
8 a 150 m b 142 mm
9 Not right-angled. $12^2 \neq 10^2 + 9^2$
10 35.3 km
11 a 2 850 000 cm b 6400 mm c 0.34 m
d 0.023 km e 6780 mm
f 38 000 000 mm
12 77 cm
13 67.9 mm
14 a 160 000 cm^2 b 450 000 mm^2
c 4 090 000 m^2 d 360 ha
e 47.35 cm^2 f 0.68553 km^2
15 a 44 m^2 b 1040 cm^2 c 228 cm^2
16 a 4322.8 cm^2 b 3518.6 cm^2
c 2714.3 cm^2
17 5205 mm^2
18 2460 cm^2
19 a 20 700 mm^3 b 1.65 m^3
c 6 800 000 000 m^3 d 7.56 m^3
e 3782.568 m^3 f 45.79 cm^3
20 281 241 m^3
21 a 15 000 L b 0.25 L
c 570 mL d 3.68 mL
e 0.007 3205 ML f 3 700 000 mL
22 124.8 km^3
23 30 sheets
24 a 146.28 m^3 b 8776.8 W

6.01

1 a $320 b $640
c $1600 d $3200
2 a $420 b $21 000
c $320.40 d $3717.90
3 a $471.19 b $42.08
c $625.50 d $569.59
4 C
5 a $3330 b $18 581
c $11 549.33 d $4512.95
6 $1761.20
7 $4432
8 a $6000 b $18 000 c $300
9 $75 000
10 a $19 575 b $250.97 c $405
11 a $1984.50 b $847.92
c $4680 d $207.67
12 $8520

INVESTIGATION: FLAT-RATE LOAN REPAYMENTS

a B1: 10000, B2: 9, B3: 4, B4: 12
b Teacher to check.
c $215.77
d $136.42
e Teacher to check.

6.02

1 a $1500 b $38 888.89
c $5000 d $20 000
2 C
3 $4000
4 a 8.0% b 12.4%
c 3.7% d 10.7%

5 C
6 6% p.a.
7 a 10 months
b 14 months = 1 year 2 months
c 58 months = 4 years 10 months
d 32 months = 2 years 8 months
8 B
9 12%
10 $7400
11 $63 000
12 0.035 00%
13 15 months
14 0.8%
15 $364.40
16

$$\begin{aligned}900 &= 3500 \times \frac{r}{100} \times \frac{20}{12}\\ &= 15.428...\%\\ &\approx 15.4\%\end{aligned}$$

6.03

1 a 4% b 2%
c $\frac{2}{3}\%$ d $\frac{8}{365}\%$
2 a 3.75% b 1.875%
c 0.625% d $\frac{7\frac{1}{2}}{365}\%$
3 8.4%
4 B
5 a $120 b 59c
6 a $100 b 48c
7 D
8 Interest = 0.49 + 0.76 + 0.75 = $2
Total = $396
9 The second one. It is equivalent to 15.33% p.a. whereas the first one is higher at 15.52% p.a.
10 a

Balance	870	1170	1290	1240	1040	1440	1420

b i $870 ii $1040 iii $1040
c $8.83
11 a $622 b 25 July c $635.17
12 a 12 b $888.70

6.04

1 a $200 b $5200 c $208
d $5408 e $408
2 a $5408 b $408
3 a $8994.75 b $3891.05
c $4951.80 d $17 869.38
e $9714.53 f $9510.71
4 a i $14 364.27 ii $1464.27
b i $15 989.38 ii $2189.38
c i $16 011.06 ii $2211.06
d i $6967.94 ii $47.94
e i $46 229.53 ii $4229.53
f i $1731.86 ii $331.86
g i $29 334.91 ii $4634.91
h i $314 145.44 ii $191 145.44
i i $1773.37 ii $273.37
j i $39 110.68 ii $1610.68
k i $63 044 38 ii $7044.38
l i $3423.21 ii $723.21
5 a Bank of Victoria: $10 705.80; Australia bank: $10 512.53
b Bank of Victoria by $193.27.
6 $35 930.80
7 $142 501.34
8 $83 870.32
9 $16 075.27
10 a i $2975 ii $3421.69
b i Compound interest is greater because interest is calculated on accumulated interest as well.
ii $446.69
11 a $23 152.50 b $23 180.07
c $23 194.21 d $23 203.77
e $23 208.43
12 Amount of interest increases because you get interest on your interest more often.
13 C

6.05

1 $18 363
2 B
3 18.9% p.a.
4 D
5 18 years
6 4 years
7 $24 000
8 6.30%
9 $724.32
10 5.79% p.a.
11 $46 700
12 $8263.97
13 19 months
14 11.0% p.a.
15 10 years
16 17.7 years

6.06

1 a i $41 674.50 ii $5674.50
b i $13 830.01 ii $10 130.01
c i $133 873.05 ii $60 073.05
d i $219 925.81 ii $119 925.81
2 a $141 026.32 b $54 261.00
c $230 008.18 d $2605.80
3 a 27 months = 2 years 3 months
b 4.3% c $34 300 d 5 years
4 $8500
5 C
6 $27 311.58
7 16.0% p.a.

8 D
9 3.8% p.a.
10 Option A: $93 646.03; Option B: $94 312.43. She should choose Option B as she ends up with more money.
11 a 7.2% p.a. b $64 430.48
c $19 430.48
12 $8.45
13 C
14 7.7% p.a.
15 $30 915.31

CHAPTER 6 REVIEW

1 A
2 D
3 E
4 C
5 B
6 D
7 E
8 B
9 $1931.25
10 a $1350 b $162
c $1854 d $1117.81
11 $55 057.50
12 a $13 500 b $6480 c $416.25
13 $21 300
14 4.5%
15 9 years
16 a $292.50 b $1.61
17 a $4.21 b $473.66
18 a $710 b $10 710
c $760.41 d $11 470.41
e $814.40 f $12 284.81
19 a i $4215.07 ii $165.07
b i $8672.28 ii $1472.28
c i $12 323.06 ii $2023.06
d i $93 152.59 ii $8152.59
20 $9493.91
21 3.8% p.a.
22 31 months
23 a 6.25%
b i 1.059
ii $971.85
c 6.06%
24 a $1950 b $7950
c $265 d $4770
e 18 months f $195
g $331.39 h $9145.02
25 a $8500 b $7000
c 2 years 7 months d $7588
e $4588 f 1 year 7 months
g $5209.46 h $3300
i $504.64 j $10 504.64

MIXED REVISION 2

Multiple choice

1 B
2 D
3 B
4 C
5 B
6 D
7 D
8 B
9 E

Short answer questions

1 a $0.0958 b 38
2 8.1 m
3 $13 045
4 ≈$10 894
5 a 9160.9 mm^2 b 82 448.0 mm^3 c 82 mL
6 $3333.12

Application questions

1 a $4153.88
b i $1124.86 ii $7552.63
2 a $21 883.75 b $1494.50
c $9201.85
3 a 8.75 m^2 b 2.958 m
c 1.905 m d 11.3 m^2
4 a 45 600 cm^2 b 0.864 m^3
c 864 L d 144 fish
5 a $2925 b $17 925
c $497.92
6 a A: $11 475
B: $11 407.09
C: $11 183.17
∴ Option A is the best option.
b A: $17 212.50
B: $17 745.62
C: $17 385.34
∴ It does change his decision. Option B is now the best option.

7.01

1 a 1 : 10 = 1 mm : 1 cm
b 1 : 1 000 000 = 1 mm : 1 km
c 1 : 1000 = 1 m : 1 km
d 1 : 1000 = 1 cm : 10 m
e 1 : 100 000 = 1 m : 100 km
f 1 : 100 = 1 mm : 10 cm
2 a 1 : 400 b 1 : 5
c 1 : 800 d 1 : 10
e 250 : 1 f 1 : 1000
3 a 1 : 1000 b 1 : 10 000
c 1 : 1 000 000 d 1 : 400
e 1 : 7000 f 1 : 25
g 1 : 5 h 1 : 2000
i 1 : 680

4 a 3 m b 30 cm c 9 m
d 8.4 m e 29.1 m f 60 m
5 a 40 m b 12.5 m c 15.5 m
d 50 cm e 7.5 m f 3 m
6 a 4 cm b 2 cm c 14 cm
d 3 cm e 0.5 cm f 5.5 cm
7 a 10 cm b 35 cm c 49 cm
d 5 cm e 7.3 cm f 0.6 cm
8 2.75 cm
9 Yes, the scale is possible. It means that the drawing is ten times larger than the real object. E.g, the drawing of an insect could use this scale.

7.02

1 a 1 : 4 000 000
b i 84 km ii 244 km
iii 216 km iv 120 km
c Mittagong
d Katoomba and Penrith; Mittagong and Wollongong
2 a 4 m b 5 mm c 270 m
3 a length = 9.3 cm, width = 5.5 cm
b 55 m c 93 m
4 a 20 km b 13.2 km c 11.6 km
5 a 30 m b 2.7 m c 22.5 m
6 a 2 m b 6 m c 36 m^2
d 52.5 m^2 to paint; 6 L needed
7 a 6 m b 8 m c 48 m^2 d $4056
8 a 4 b walk-in robe
c 1 d kitchen
e 28 m^2
9 a 7
b i 

ii 
iii 

iv 
v 
c 15.0 m^2 d 168 e $2710.48
10 a 1 : 100 b 13.5 m
c 12 windows d 2 doors
e 8 m^2 f 2400 mm (2.4 m)
g $700

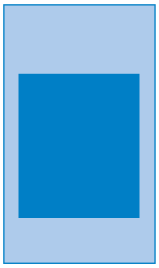
INVESTIGATION: CROWDED CLASSROOMS

Teacher to check results.

7.03

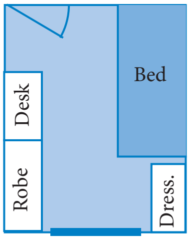
1 a 1 : 300 b 1 : 11 250
c 1 : 25 d 1 : 100
e 1 : 25 000 f 1 : 100 000
2 a 30 cm × 40 cm b 8 cm × 5.5 cm
c 6 cm × 9 cm d 18 mm × 42 mm
e 56 cm × 120 cm f 90 cm × 160 cm

3 Choose the scale of 1 : 200. Your diagram should look like this but a bit bigger.

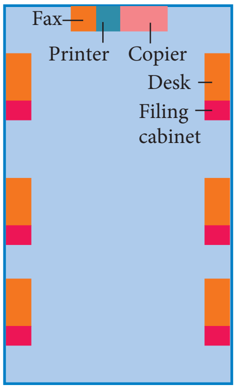


Front

4 Choose the scale of 1 : 20. Your drawing should look like this, but will be bigger. Other arrangements are possible.



5 Your drawing should look like this, but will be bigger. A lot more people could work in the office. Other arrangements are possible
a Choose scale 1 : 50. b Choose scale 1 : 50.



7.04

1 a Yes b No c No
d Yes e Yes f No
2 a $ABCD \,|||\, KLMJ, \frac{4}{5}$ b $CAB \,|||\, NML, \frac{3}{2}$
c $PSRQ \,|||\, EFGH, 2$ d Not similar
e $NSEK \,|||\, APHW, \frac{2}{3}$ f Not similar

3 $CUPLZE \,|||\, MAYNKR, \frac{1}{2}$; $JNSY \,|||\, ETDI, \frac{1}{2}$

$\triangle SDF \,|||\, \triangle KYC, \frac{2}{3}$; $SGEM \,|||\, PTFN, \frac{3}{2}$

4 $\frac{6}{5}$

5 a i $\triangle ABC \,|||\, \triangle LMK$ ii LM iii $\frac{1}{2}$

b i $ABCD \,|||\, RSPQ$ ii RS iii $\frac{3}{2}$

c i $ABCD \,|||\, YZWX$ ii YZ iii $\frac{4}{5}$

d i $PQRSTU \,|||\, CDEFAB$ ii TU iii $\frac{2}{3}$

e i $EDCBAHGF \,|||\, XQRSTUVW$

ii TS iii $\frac{3}{4}$

f i $ABCDE \,|||\, MJKLN$ ii MJ iii $\frac{7}{5}$

6 a $x = 6$ b $x = z = 74, y = 113$

c $m = 20, n = 15$ d $x = 10.5$

7 a T b T c F d F

e T f F g T h F

i T j F k F l F

8 a Similar figures MUST have all corresponding angles equal. Some parallelograms have equal angles and so may be similar, however, not all parallelograms have equal angles and so these cannot be similar.

b

9 All squares are similar. All of their angles are equal to 90° and their sides are in the same ratio.

7.05

1 a Yes, AAA b Yes, SAS

c Yes, AAA d Yes, RHS

e Yes, SSS

f Not necessarily. The angle is not the included angle.

2 b $\frac{4}{5}$ d $\frac{1}{2}$ e $\frac{3}{2}$

3 a $\frac{3}{10}, x = 5.4$ b $\frac{7}{4}, y = 10$

c $\frac{4}{3}, m = 24$ d $\frac{2}{3}, n = 12$

e $\frac{2}{3}, w = 12$ f $\frac{3}{2}, d = 21$

4 a 36 m b 43.4 m c 2.5 m

d 16.8 m e 11.2 m f 5.04 km

5 a $y = 4$ b $d = 30$

7.06

1 18.7 m

2 1.9 m

3 15.6 m

4 a The sun is lower. b 3.75 m

5 5.76 m

6 13.125 m

7 93.75 m

8 4.7 m

9 a They omitted the metre rule shadow measurement at the gym.

b 8.1 m c 8 m d About 5 m

10 135.3 m

11 9 m

12 24.5 m

13 1.75 m

7.07

1 a $\frac{2}{3}$ b $\frac{4}{9}$

2 a 36 b $\frac{25}{16}$ c $\frac{4}{49}$

3 a $x = 12.5$ b $y = 315$

4 175 cm^2

5 225 cm^2

6 a The area is nine times larger.

b The large area is four times the smaller area.

c The sides are in the ratio 4 : 1.

d The area of the new quadrilateral is a quarter of the area of the original quadrilateral.

7 a 245 cm^2 b 20 cm

8 a 1 : 3 b 15 cm, 36 cm, 39 cm

c The perimeter of the larger triangle is three times larger than the perimeter of the smaller triangle.

9 37.5 L

10 294 cm^2

7.08

1 a $\frac{2}{7}$ b $\frac{4}{49}$ c 200 cm^2

2 a $\frac{81}{49}$ b $\frac{9}{7}$ c 18 mm

3 a $\frac{6}{5}$ b $\frac{216}{125}$ c 1620 cm^3

4 $\frac{27}{512}$

5 $\frac{1}{9}$ of the original, $\frac{1}{27}$ of the original

6 9 times larger than the original, 27 times larger than the original

7 a $\frac{7.2}{10.4} = \frac{1.8}{2.6} = \frac{4.5}{6.5} = \frac{9}{13}$, corresponding sides are in the same ratio.

b $\frac{13}{9}$ c $\frac{169}{81}$

d $\frac{2197}{729}$ e $\frac{2197}{5832}$

8 a 4 : 5 b 16 : 25

c 64 : 125 d 32 : 125

9 a $\frac{8}{9}$ b $\frac{512}{729}$

10 a 7 : 9 b 49 : 81

11 170 m^2

12 a 18 cm b 1 : 3 240 000

13 a 6 L b 7.5 ml

14 a 2 : 125 b 4 : 15 625 c 8 : 1 953 125

15 No. To double the ratio of the sides, the volume of mixture needs to be 8 times the original batch.

16 5600 cm^2

17 a 500 b $237.60

CHAPTER 7 REVIEW

1 B

2 D

3 A

4 B

5 C

6 E

7 C

8 D

9 C

10 A

11 a 10 m b 5 m c 6 km d 2.5 cm

12 a 1 : 1 000 000 b 1 : 3500
c 1 : 5 d 1 : 40 000

13 a 63 cm b 16 cm c 30 cm

14 a 3.6 m × 3.6 m
b i 3.6 m ii 10.8 m^2
c 17 m^2
d 135 m^2
e $207 900
f i 14 ii 196

15 a 7 b 7 c Kitchen/meals

16 1 : 20 000

17 34 cm by 21 cm

18 Choose scale 1 : 200. Your drawing should look like the following, but will be bigger.
Block
width 11.5 cm
depth 17.5 cm
House
width 9 cm
depth 12 cm
House is 2 cm from the front boundary.
House is 1 cm from the left boundary.

19 a They are similar because their matching sides are in the same ratio and matching angles are equal.
b *ABCD* ||| *EFGH* c $\frac{4}{3}$

20 $\frac{3}{5}$

21 a AAA b 30 mm c 70 mm

22 2.9 m

23 27 m

24 72 cm

25 a 48 m b 36 m c 160 m^2

26 a $\frac{27}{343}$ b 1543.5 m^3

27 64

28 2600 bricks

8.01

1 a Appropriate, as they will state one of the 18 AFL teams.
b Not appropriate, as there is a large range of possible answers. Provide groupings such as 0 to 50 songs, etc.
c Appropriate, as there is only a small range of possible answers.
d Not appropriate, as the word food could be interpreted in many ways. Therefore you need to provide answers to select from, for example, takeaway, dessert, etc.
e Appropriate, as the only answers are YES, NO or MAYBE.

2 a B b A c A
d C e A

3 a Biased b Unbiased c Biased
d Biased e Unbiased f Biased

4 a, b, c all students will have different answers, show your teacher your calculator screen.

5 a A CAS calculator can be used to generate the random numbers *randint(1,15000,150).*
b Questions should relate to what is being investigated and one question should specifically be 'What is your favourite lolly from the list provided?' snake, banana, etc.

8.02

1 Categorical data are variables represented by qualities or words and numerical data are variables represented by quantities or numbers. An appropriate example of each should be stated.

2 a categorical, nominal
b numerical, discrete
c numerical, discrete
d categorical, nominal
e categorical, ordinal
f numerical, continuous
g numerical, continuous
h categorical, ordinal
i numerical, discrete

3 a C b D c C d C
e D f D g D h D

4 Answers may vary. Measurements are generally considered to be numerical and continuous. However, when stated that they are rounded, they can be considered to be discrete.

5 a Numerical and continuous (although it looks discrete from the table) as age can have any value within a given range.
b Numerical and discrete, as you can only have a whole person.
c Numerical and continuous, as average height can have any value within a given range, including decimal places.

8.03

1

Eye colour	Tally	Frequency (f)
blue	𝍸 ||||	9
brown	𝍸 ||	7
green	𝍸 |||	8
hazel	𝍸 |	6
	Total	30

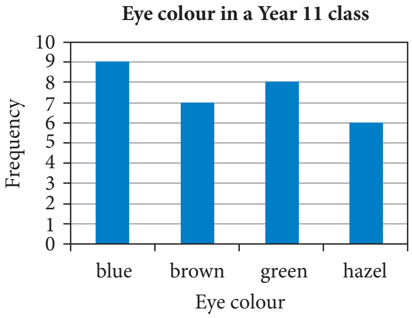
2

Sport	Tally	Frequency (f)
football	𝍸 ||	7
soccer	𝍸 |||	8
cricket	𝍸	5
other	𝍸	5
	Total	25

3

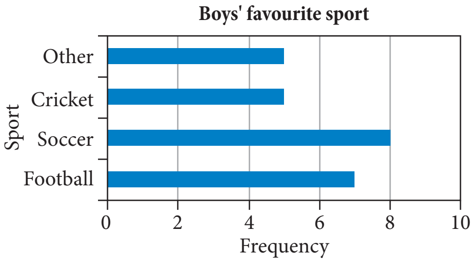
Answer	Tally	Frequency (f)
Yes	𝍸 𝍸 𝍸 ||||	19
No	𝍸 𝍸 𝍸 ||	17
undecided	𝍸 𝍸 ||||	14
	Total	50

4



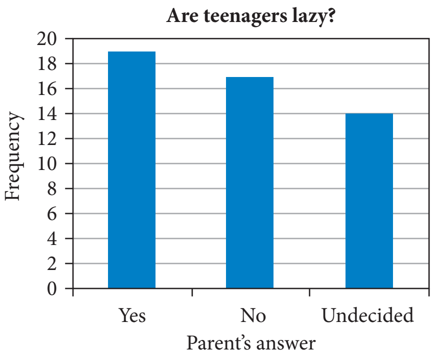
The graph shows that the most common eye colour was blue, closely followed by green. Hazel was the least common eye colour.

5



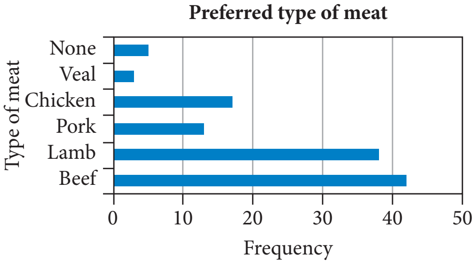
The graph shows that the favourite sport for boys was soccer, closely followed by football. An equal number of boys named cricket or other as their favourite sport.

6



The graph shows that more parents said yes, teenagers are lazy, but only by 2. Another 14 were undecided.

7



8 **a** 50 **b** 150 **c** Labor **d** 430

e 50 **f** 480 **g** 31.25%

9 C

10 C

11 B

8.04

1 **a**

Stem	Leaf
0	7 9
1	1 2 4 5 5 5 5 6 7 7 8 8
2	0 1 1 2 4 5 5 6 7 7 8 8 9 9 9
3	0 0 0 0

Key: 0|7 means 7

b

Stem	Leaf
3	8 9
4	1 7
5	0 0 5 5 5 7 9
6	0 0 2 4 5 6 7
7	1 4 6 8
8	1 2 9

Key: 3|8 means 38

c

Stem	Leaf
4	8 9
5	0 1 5 5
6	0 1 2 3 4 7 8
7	5 6 6
8	1 3
9	0

Key: 4|8 means 48

d

Stem	Leaf
32	3
33	1 6 8 8
34	1 5 7
35	4
36	0 5 6 9
37	0 0 1 6 6
38	8
39	9

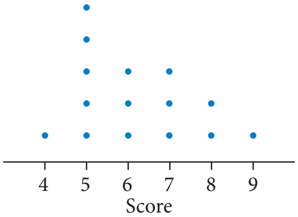
Key: 32|3 means 323

e

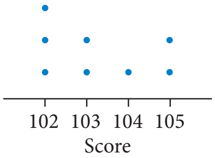
Stem	Leaf
7	6 8
8	2 7 7
9	1 6 9 9
10	1 6
11	0 2 3 5
12	0

Key: 7|6 means 76

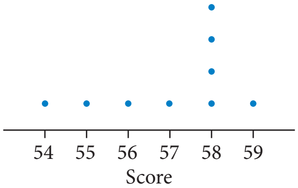
2 **a**



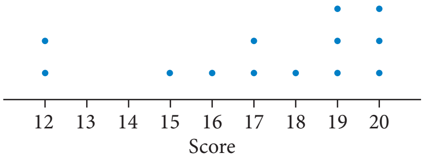
b



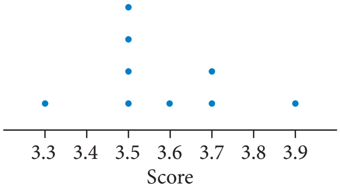
c



d



e

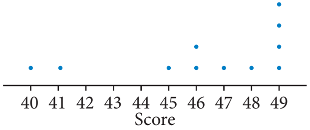


3 **a** A stem-and-leaf plot is more appropriate as there is a large range of values.

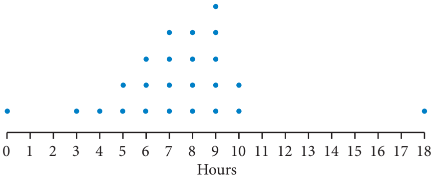
Stem	Leaf
2	3
3	
4	5
5	5 6
6	1 3 9
7	1 9
8	0 9

Key: 2|3 means 23

b A dot plot is more appropriate as there is a small range of values.



4 **a**



b 24 **c** 0 and 18

5 **a**

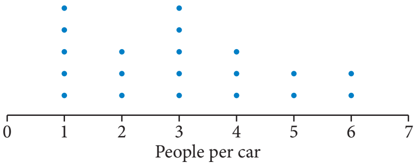
Stem	Leaf
2	8
3	
4	2 3 7
5	1 3 7 9
6	1 3 4 6 8 8
7	2 8 9 9
8	1 2 3 4
9	3 4
10	0 3 4
11	0 2 7

Key: 2|8 means 28 sales

b 87%

c The outlier is 28 sales. This could be due to bad weather so fewer customers are going to the hardware store.

6 **a**



b 20 **c** 6

d No outliers.

7 D

8 A

9 E

10 a

Stem	Leaf
5	2 5 6 8
6	0 0 3 3 4 4 4 5 8 8 9 9
7	1 2 2 4 4 6 6 8 9
8	0 1 1 3 3 4 4 5 9
9	0 0 3 4 7
10	
11	0

Key: 5|2 means 52 heartbeats per minute

b 10

c 110 heartbeats per minute, heart is beating quickly.

d 4

11 a

Stem	Leaf
11	6 8
12	0 1 4
13	6 9
14	3 5 6 6 6
15	4 6 6 7
16	3 5 8
17	2 4 7
18	1 7

Key: 11|6 means 11.6 seconds

b 24

c i 11.6 seconds ii 18.7 seconds

d 50.0%

e $\frac{5}{24}$

8.05

1 a

Score	Frequency
1	1
2	0
3	2
4	3
5	7
6	3
7	4
8	3
9	2
Total	25

b

Score	Frequency
10	2
11	2
12	3
13	0
14	4
15	6
16	4
17	1
18	2
19	1
Total	25

c

Score	Frequency
100	1
101	4
102	4
103	5
104	5
105	2
106	1
107	1
Total	23

d

Score	Frequency
71	2
72	3
73	1
74	1
75	5
76	6
77	4
78	8
Total	30

e

Score	Frequency
2	3
3	1
4	10
5	4
6	4
7	2
8	2
9	2
Total	28

2 a

Score (x)	Frequency (f)
10–19	3
20–29	1
30–39	4
40–49	4
50–59	10
60–69	3
70–79	6
80–89	3
90–99	3
Total	37

b

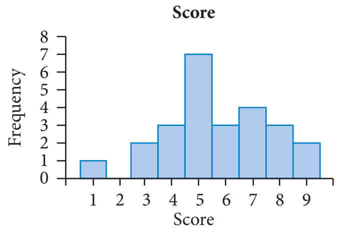
Score (x)	Frequency (f)
100–119	7
120–139	5
140–159	4
160–179	8
180–199	1
Total	25

c

Score (x)	Frequency (f)
100–104	5
105–109	6
110–114	2
115–119	4
120–124	6
125–129	1
130–134	5
Total	29

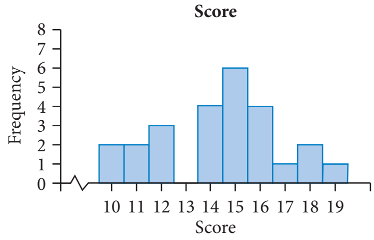
3 B

4 a



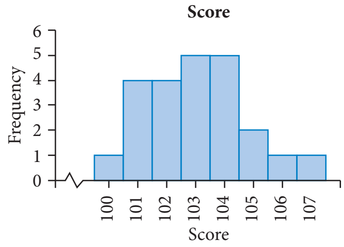
The most common score was 5. Scores ranged from 1 to 9.

b



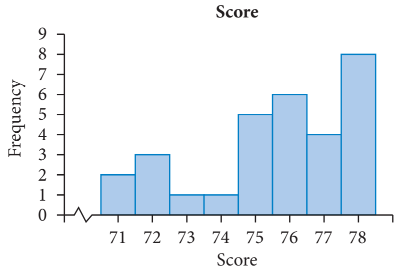
The most common score was 15. Scores ranged from 10 to 19, with a cluster of scores from 14 to 16.

c



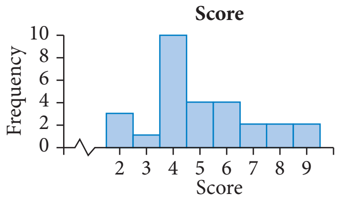
The most common scores were 103 and 104. Scores ranged from 100 to 107, with a cluster of scores from 101 to 104.

d



The most common score was 78. Scores ranged from 71 to 78, with a cluster of scores from 75 to 78.

e

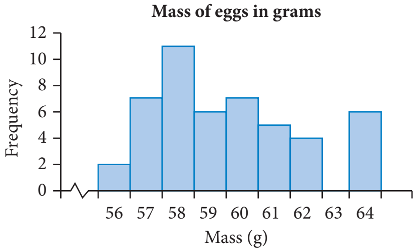


The most common score was 4. Scores ranged from 2 to 9. The frequencies were fairly consistent, except for a score of 4.

5 a

Mass (g)	Frequency (f)
56	2
57	7
58	11
59	6
60	7
61	5
62	4
63	0
64	6
Total	48

b

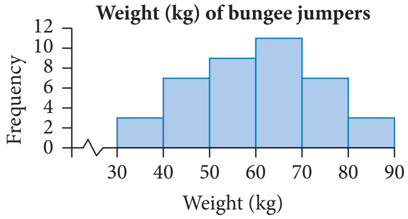


c Yes, the label is misleading as 54% of the eggs weigh less than 60 grams. Only 15% of the eggs actually weigh 60 grams.

6 a

Weight (kg)	Frequency (f)
30–<40	3
40–<50	7
50–<60	9
60–<70	11
70–<80	7
80–<90	3
Total	40

b

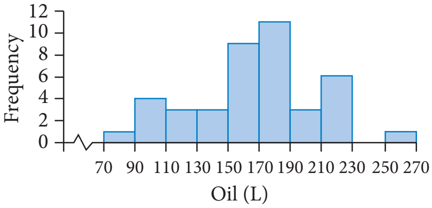


c $\frac{10}{40} \times 200 = 50$ bungee jumpers

7 a

Oil used (L)	Frequency (*f*)
70–<90	1
90–<110	4
110–<130	3
130–<150	3
150–<170	9
170–<190	11
190–<210	3
210–<230	6
230–<250	0
250–<270	1
Total	41

b


Litres of oil used servicing cars

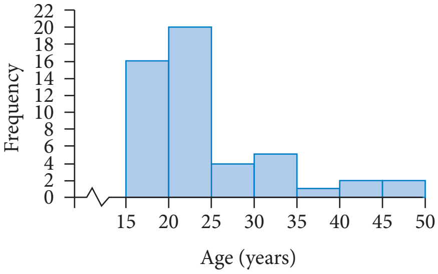
c The graph shows that 20 out of the 41 cars required between 150 L and 189 L of oil. One car required 267 litres, which could be considered an outlier.

8 a

Age (*x*)	Frequency (*f*)
15–<20	16
20–<25	20
25–<30	4
30–<35	5
35–<40	1
40–<45	2
45–<50	2
Total	50

b 50

c


Ages of Burger Heaven staff

d 8%

e The graph shows that Burger Heaven employs a greater proportion of younger people, particularly between the ages of 15–24.

8.06

1 **a** class interval 115–119 **b** 15

2 D

3 C

4 E

5 The shape of the histogram is positively skewed, which means that there is a greater number of younger staff members than older. The ages of staff members range from 15 up to 50 and the centre age is the class interval of 20 to 24 years old, which is also the most common class interval for age.

6 The histograms shape is neither symmetrical nor skewed. The range of egg masses is from 56 grams to 64 grams, with the eggs weighing 64 grams possibly being outliers. The most common egg mass is 58 grams with the centre mass being 59 grams.

8.07

1 a 5 b 34.4 c 36 d 9

2 a 20 b 64.5

3 a D

b Decrease, the high outlier increases the mean so its removal would decrease the mean.

4 a 29 b 120.8 s

5 a 50 b 2.9

6 a 50.0 b Yes, the mean is 50.0.

7 a 20 b 2.6

c 8 d decreases

e An outlier is present and 2.6 children is not realistic, as data is discrete.

f 11

8 a

No. of calls (x)	Frequency (f)	$f \times x$
2	1	2
3	2	6
4	3	12
5	5	25
6	2	12
7	2	14
Total	15	71

b 15 c 4.7

9 a 149

b The mean speed is 44.24. It is only an estimate because the original data is not known and therefore the midpoints of the class intervals were used to calculate the mean.

10 a 44

b Weekly wage from $100 to less than $200.

c $332

d part-time or casual staff

8.08

1 **a** **i** 5 **ii** 1, 5
b **i** 34.5 **ii** 31, 32, 35
c **i** 38 **ii** no mode
d **i** 8 **ii** 5, 7, 8

2 **a** 65% **b** 47%, 59%, 65%

3 C

4 A

5 **a** 50 students **b** 3 DVDs
c 3 DVDs

6 **a** 2 children
b 2 children
c 8 children
d **i** There would only be 19 values if the outlier was removed, so the 10th value is in the position of the median, which is still 2 children.
ii Removing the outlier does not change the frequency of the mode, which is 2 children.

7 **a** 81 kg **b** 83 kg **c** 81.03 kg

8 **a** −2°C, 2°C **b** 11th, 2°C **c** 2.6°C

9 **a** between the 117th and 118th values, 40–49 cm
b 40–49 cm

10 **a** 160–<165 **b** 160–<165

11 **a** mean, to find the class average
b mode, as the question asks for the most common
c median, as there could be outliers in rental prices
d mean, as data is numerical and continuous
e mode, as data is categorical

12 **a** $460 200 **b** $381 500 **c** mean
d median, as $1 200 000 is an outlier

8.09

1 **a** 6 **b** 5 **c** 728 **d** 12

2 7

3 **a** 2 **b** 2.5 **c** 258 **d** 3

4 A

5 D

6 **a** 5.5 **b** 3.05

7 **a** range = 7, IQR = 5
b range = 7, IQR = 3.5
c range = 20, IQR = 12
d range = 9, IQR = 4
e range = 10, IQR = 2

8 **a** 38 **b** 39.4 **c** 12 **d** 46
e The median and IQR are more appropriate to use as the data set has outliers.
f Mention middle score of 38, therefore half above and below, mention IQR of 12 seconds, therefore the middle 50% of the group completed the test within 12 seconds of each other.

8.10

1 **a** 2 **b** 1.4

2 **a** 75 **b** 9.84

3 **a** $836 **b** $294

4 **a** 2.22 **b** 1.87

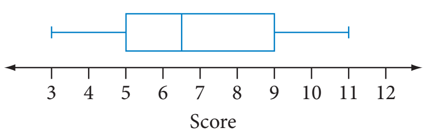
5 **a** range = 8, IQR = 3.5 and $s_x = 2.19$
b range = 8, IQR = 4 and $s_x = 2.40$

6 **a** 6.8 **b** 1.91 **c** 15 values
d mention mean = 6.8, range = 8, IQR = 2.5, median = 7 and $s_x = 2.19$

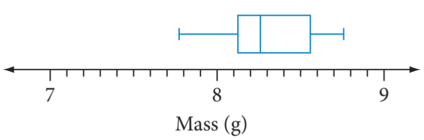
8.11

1 **a** minimum = 3
$Q_1 = 5$
median = 6.5
$Q_3 = 9$
maximum = 11

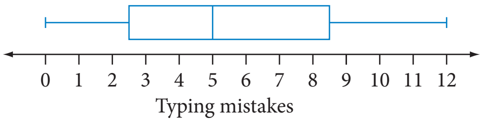
b



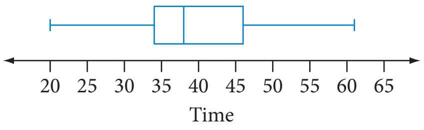
2



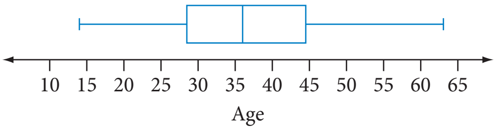
3



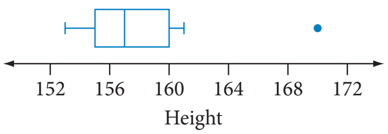
4



5 minimum = 14
$Q_1 = 28.5$
median = 36
$Q_3 = 44.5$
maximum = 63



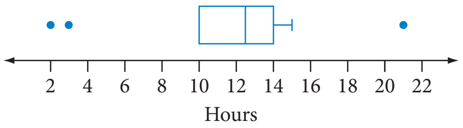
6 **a**



b $147.5 \le x \le 167.5$. As 170 is bigger than 167.5, it is an outlier.

7 **a** $4 \le x \le 20$. As 2 and 3 are less than 4, they are outliers and as 21 is bigger than 20, it is an outlier.

b



8 **a** i 6 ii 4 iii 8
b i 8 ii 4 iii 7
c i 30 ii 25 iii 35
d i 35 ii 2 iii 7
e i 11 ii 4 iii 7
f i 46.5 ii 2.5 iii 5.5

9 **a** 20 **b** 12 **c** 3
d 25% **e** 30

10 **a** $-6.5 \le x \le 85.5$. There are no outliers as all values are inside this range. Thus, the graph is incorrect.
b D **c** C
d \$28 000 and \$51 000 **e** 25%

INVESTIGATION: TREES

a Age: Numerical and Discrete, Circumference: Numerical and continuous, Species: Categorical and nominal

b Age: Stem-and-leaf plot, Circumference: Histogram with class intervals, Species: Bar or column graph

c Construct a stem-and-leaf plot for the age of the 24 trees.

Stem	Leaf
0	4 5 8 9
1	0 0 0 1 3 4 7 7
2	1 2 6 9
3	0 2 3 3 7
4	0 1 3

Key: 0|4 = 4 years

d i Mean $\bar{x} = 21.4583$

ii five-number summary

minimum = 4

$Q_1 = 10$

median = 19

$Q_3 = 32.5$

maximum = 43

iii standard deviation $s_x = 12.4096$

e The youngest tree is 4 years old and the oldest tree is 43 years old, therefore the range of ages is 39 years. The average tree age in the local neighbourhood is approximately 21 years. The median age is 19 and the middle 50% (IQR) of ages has a range of 22.5 years.

f, g

Elm

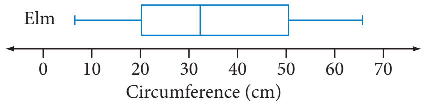
minimum = 6.5

$Q_1 = 20.1$

median = 32.2

$Q_3 = 50.5$

maximum = 65.9



Oak

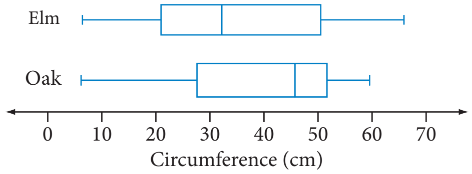
minimum = 6.3

$Q_1 = 27.6$

median = 45.85

$Q_3 = 51.9$

maximum = 59.8



h Justify (with working) why neither boxplot has any outliers.

Oak trees

IQR = 24.3

Outliers lie outside the range of $-8.85 \le x \le 88.35$.

All circumferences for the oak trees fall within this range, therefore no outliers are present.

Elm trees

IQR = 30.4

Outliers lie outside the range of $-25.5 \le x \le 96.1$.

All circumferences for the elm trees fall within this range, therefore no outliers are present.

i Report that discusses all relevant summary statistics such as mean, median, IQR, etc. A discussion comparing the two is also acceptable.

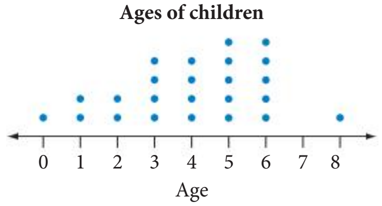
CHAPTER 8 REVIEW

1 E
2 C
3 D
4 C
5 D
6 D
7 A
8 B
9 B
10 **a** Too many options, give answers that may be selected, such as every day, once a week, etc.
b Each individual could give a different response so a checklist needs to be provided for respondents to tick what they liked about the movie.

11 **a** numerical and discrete
b categorical and nominal
c numerical and continuous
d categorical and ordinal
e numerical and discrete
f categorical and nominal

12 **a** 1477 **b** Non-Hodgkin's Lymphoma
c 6 **d** 13%

13 **a**



b 24 **c** $\frac{1}{4}$

d By visual inspection: yes, 8. Using the rule: $-0.75 \le x \le 9.25$, so no outliers.

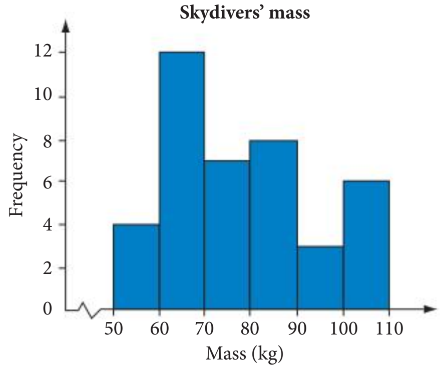
14 **a** 3 **b** 29 **c** 12 **d** 41%

15 **a** numerical and discrete

b

Mass (kg)	Frequency (f)
50–<60	4
60–<70	12
70–<80	7
80–<90	8
90–<100	3
100–<110	6
Total	40

c



d 60–<70 **e** $\frac{1}{5}$

16 173.2 cm

17 **a** $\bar{x} = 26.1$, median = 26, mode = 24, range = 11, IQR = 4.5 and $s_x = 3.1$
b $\bar{x} = 15.5$, median = 15, mode = 15, range = 8, IQR = 3 and $s_x = 2.1$
c $\bar{x} = 32.7$, median = 30–34, mode = 30–34, range = 50, IQR = 20 and $s_x = 11.9$

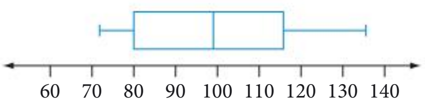
18 **a** $437 500 **b** $345 000
c Median, as $1 044 000 is an outlier.

19 **a**

Speed (km/h)	Midpoint (x)	Frequency (f)
60–<70	65	5
70–<80	75	11
80–<90	85	18
90–<100	95	6
	Total	40

b 81.25 km/h
c Both 80–<90

20 **a** 99
b 62
c 34
d



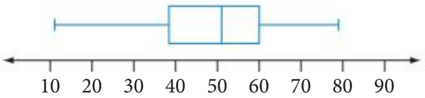
21 **a** 1.998 cm **b** 0.02

22 **a**

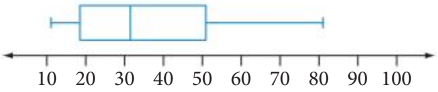
Stem	Leaf
1	2 5 8
2	2 2 3
3	1 8 9
4	0 1 5 5 5 7 9
5	1 2 2 3 4 4 5 8
6	0 1 1 4 6 8 9
7	5 9

Key: 1|2 means 12 seconds

b min = 12, $Q_1 = 38.5$, median = 51, $Q_3 = 60.5$, max = 79
c $5.5 \le x \le 93.5$, all values are inside this range, therefore there are no outliers.
d



23 **a** 39.25 years
b 31.5 years
c median, data skewed
d 32.5 years
e



f Sentence that mentions median and IQR.

24 **a** $\bar{x} = 14.4$, mode = 14, 18, min = 2, $Q_1 = 12$, median = 15, $Q_3 = 18$ and max = 19
b $3 \le x \le 27$. 2 is an outlier as it is smaller than 3.
c $\bar{x} = 15.1$, mode = 14, 18, min = 11, $Q_1 = 13$, median = 15, $Q_3 = 18$ and max = 19
d the mean, minimum and Q_1 all became larger.

25 Machine 1: $\bar{x} = 2.06$ and $s_x = 0.04$

Machine 2: $\bar{x} = 2.07$ and $s_x = 0.01$

The means are very similar, so on average they produce the same weight of flour. But machine 1 has a greater spread of values and so is not as consistent. This is indicated by the larger standard deviation.

9.01

1 a $b = 7$ b $f = -9$ c $i = 9$
d $a = -1$ e $l = -8$ f $s = -3$
g $u = -5$ h $x = \frac{8}{3}$ or $2\frac{2}{3}$ i $m = -2$

2 a $n = -6$ b $d = 3$ c $r = 2$
d $k = 4$ e $x = 1$ f $t = -4$
g $e = 10$ h $r = 16$ i $a = 12$

3 B

4 C

5 A

6 E

9.02

1 i a $y = 3x + 1$

x	−2	−1	0	1	2
y	−5	−2	1	4	7

b $y = 4 - 2x$

x	−2	−1	0	1	2
y	8	6	4	2	0

c $y = -2x + 5$

x	−2	−1	0	1	2
y	9	7	5	3	1

d $y = x - 1$

x	−2	−1	0	1	2
y	−3	−2	−1	0	1

e $y = \frac{1}{2}x + 2$

x	−2	−1	0	1	2
y	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3

f $y = 3x - \frac{1}{2}$

x	−2	−1	0	1	2
y	$-6\frac{1}{2}$	$-3\frac{1}{2}$	$-\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{2}$

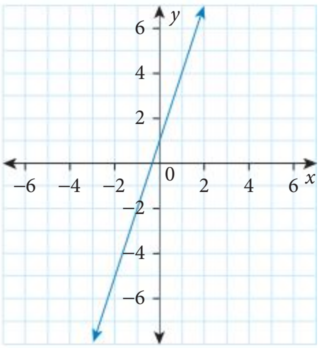
g $y = 3 - 4x$

x	−2	−1	0	1	2
y	11	7	3	−1	−5

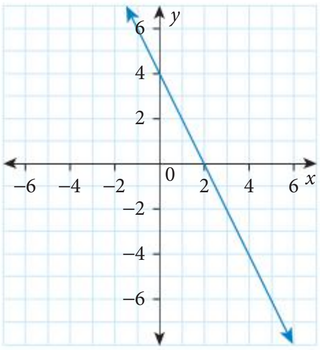
h $y = 2x - 6$

x	−2	−1	0	1	2
y	−10	−8	−6	−4	−2

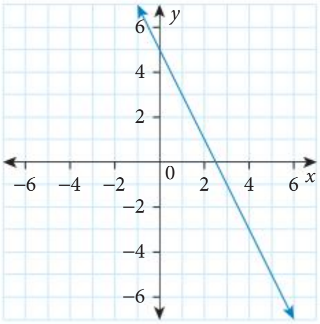
ii a



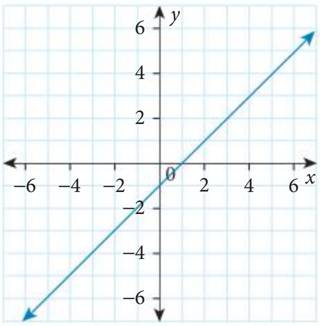
b



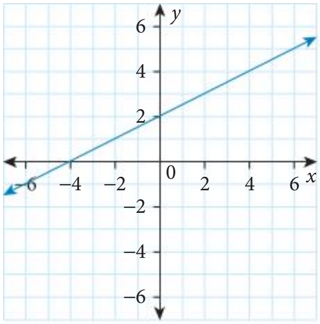
c



d



e



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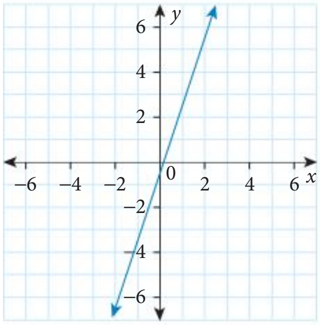

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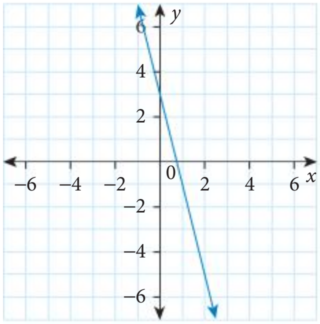

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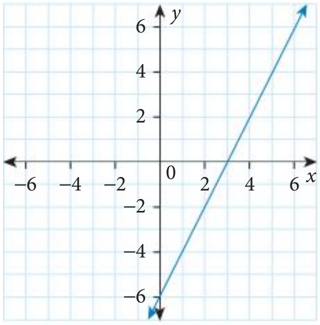
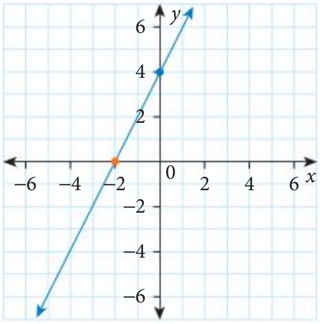
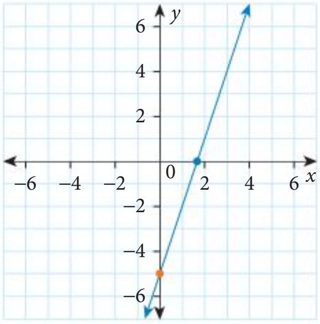


2 **a** (−2, 0) **b** (0, 4)

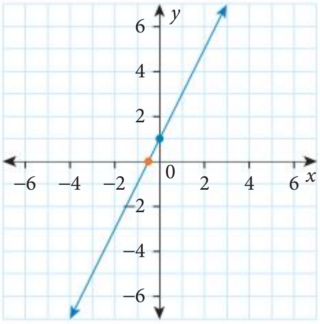
c



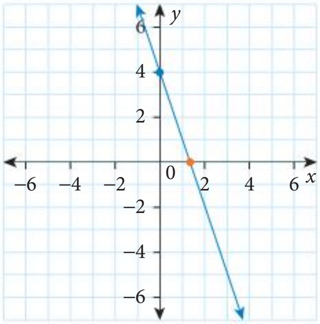
3 **a** $(1\frac{2}{3}, 0)$ (0, −5)



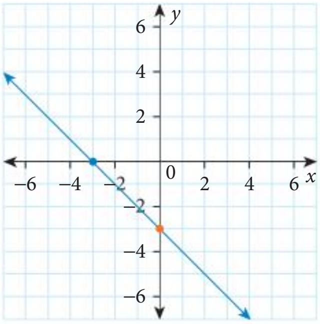
b $(-\frac{1}{2}, 0)$ (0, 1)



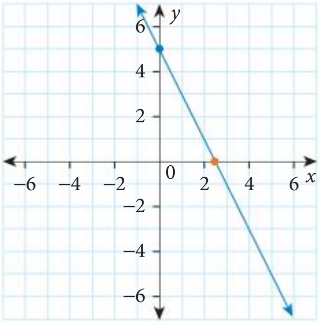
c $(1\frac{1}{3}, 0)$ (0, 4)



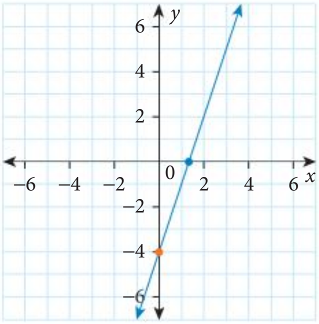
d (−3, 0) (0, −3)



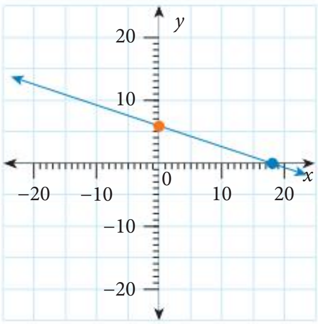
e $(2\frac{1}{2},0)$ (0, 5)



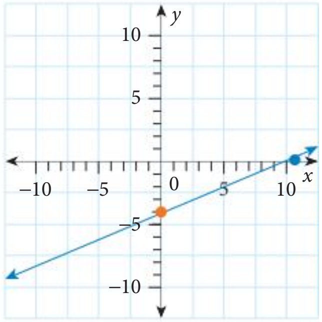
f $(1\frac{1}{3}, 0)$ (0, −4)



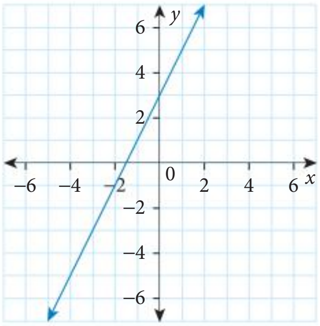
g (18, 0) (0, 6)



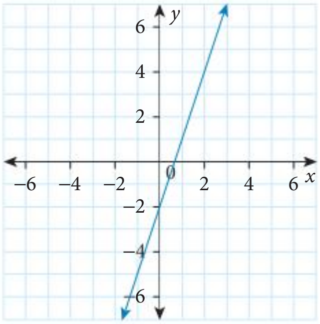
h $(10\frac{2}{3}, 0)$ (0, −4)



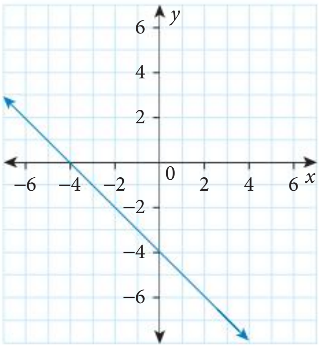
4 a



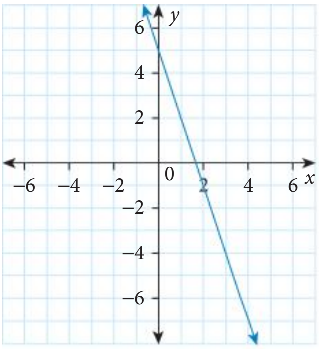
b



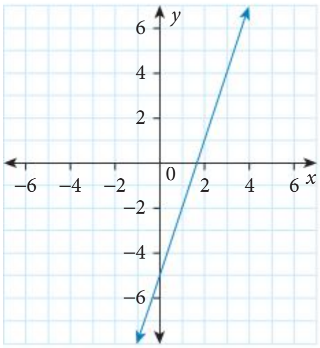
c



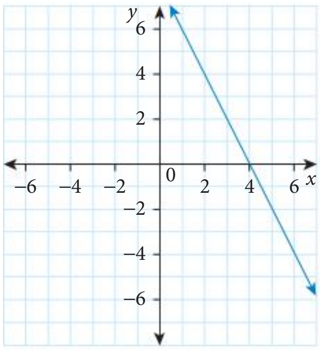
d



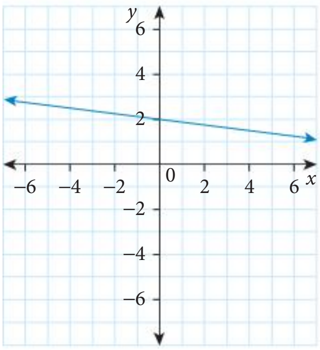
e



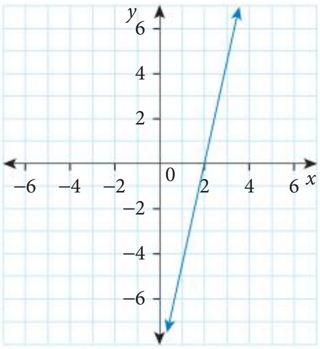
f



g



h



5 Yes: $20 = 75 - 55$

6 Yes: $6 - 12 + 6 = 0$

7 No: $-50 - 45 \neq -5$

8 C

9 A

10 **a** **i** Yes, graph is a straight line. Highest power of x is 1.

ii $\left(-\frac{1}{2}, 0\right)$ and (0, 2)

b **i** No, the highest power of x is 2.

c **i** No, the highest power of x is 2.

d **i** Yes, graph is a straight line. Highest power of x is 1.

ii $\left(2\frac{2}{3}, 0\right)$ and (0, 8)

e **i** No, the highest power of x is 3.

f **i** No, 3^x is not linear.

11 One point that satisfies the equation is $n = 2$, $C = 32$.

12 One point that satisfies the equation is $n = 2$, $m = 9.6$.

13 **a** Plot the points and see if you can draw a straight line through them.

b No. When you join the points the line is not straight. There are 2 different lines.

9.03

1 **a** negative **b** negative **c** negative
d positive **e** positive **f** negative
g negative **h** positive

2 **a** -4 **b** $-\frac{4}{3}$ **c** $-\frac{1}{4}$ **d** 2
e $\frac{1}{5}$ **f** -6 **g** -1 **h** 1

3 B

4 **a** 2 **b** -1 **c** 2 **d** $-\frac{1}{2}$
e $-\frac{3}{2}$ **f** $-\frac{4}{3}$ **g** $-\frac{1}{6}$ **h** $\frac{17}{14}$

5 **a** 3 **b** 5 **c** $\frac{1}{4}$
d -2 **e** $\frac{3}{5}$ **f** $-\frac{1}{2}$

6 Various answers, for example (2, 5) and (3, 8).

7 Various answers, for example (1, 16) and (3, 10).

9.04

1 a gradient = 3 and y-intercept = 4
b gradient = –5 and y-intercept = 2
c gradient = –2 and y-intercept = $\frac{3}{4}$
d gradient = –6 and y-intercept = –8
e gradient = $-\frac{2}{3}$ and y-intercept = 5
f gradient = –3 and y-intercept = 10
g gradient = 1 and y-intercept = $\frac{2}{5}$
h gradient = –3 and y-intercept = 6
i gradient = 2 and y-intercept = –8
j gradient = 4 and y-intercept = $-\frac{1}{3}$

2 B

3 A

4 a $y = 7 + 3x$ b $y = 1 - 2x$
c $y = x - 1$ d $y = \frac{1}{3}x - \frac{1}{2}$
e $y = -\frac{5}{4}x$ f $y = 5$

5 a $y = -\frac{3}{2} + \frac{1}{2}x$ b gradient = $\frac{1}{2}$
c y-intercept = $-1\frac{1}{2}$

6 a i $y = \frac{1}{2}x - 2$
ii gradient = $\frac{1}{2}$ and y-intercept = –2
b i $y = 4 - 2x$
ii gradient = –2 and y-intercept = 4
c i $y = \frac{16}{3} - \frac{4}{3}x$
ii gradient = $-\frac{4}{3}$ and y-intercept = $5\frac{1}{3}$
d i $y = \frac{1}{2}x - 4$
ii gradient = $\frac{1}{2}$ and y-intercept = –4
e i $y = 4 - 5x$
ii gradient = –5 and y-intercept = 4
f i $y = \frac{7}{2} + \frac{3}{2}x$
ii gradient = $\frac{3}{2}$ and y-intercept = $3\frac{1}{2}$
g i $y = \frac{1}{3}x - 2$
ii gradient = $\frac{1}{3}$ and y-intercept = –2
h i $y = \frac{8}{5} - \frac{4}{5}x$
ii gradient = $-\frac{4}{5}$ and y-intercept = $1\frac{3}{5}$

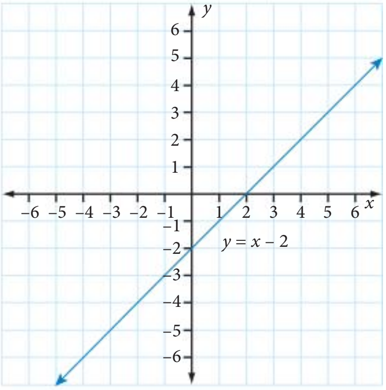
7 a $y = 3 + 2x$ b $y = 3x - 1$ c $y = 4x$
d $y = 2 - 2x$ e $y = 1 - x$ f $y = \frac{1}{2}x - 1$
g $y = -x$ h $y = \frac{3}{4}x - 2$ i $y = x - 3$

8 a gradient = 5, vertical axis intercept = 12
b gradient = –6.35, vertical axis intercept = 12.8

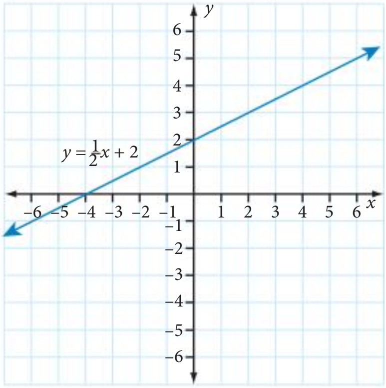
9 C

9.05

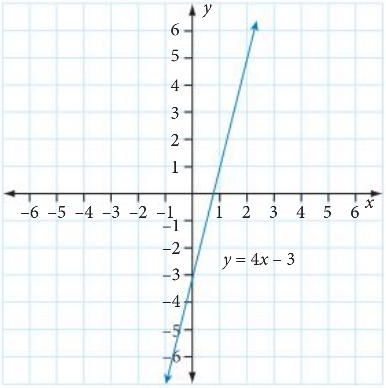
1 a



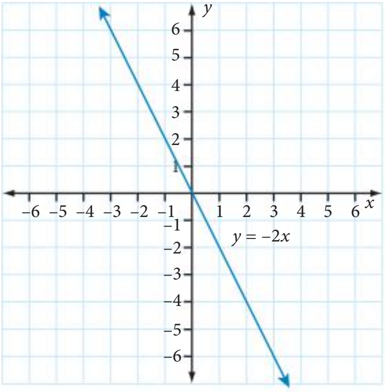
b



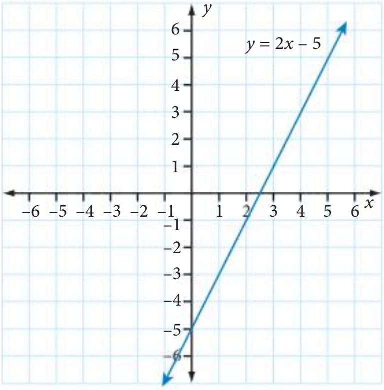
c



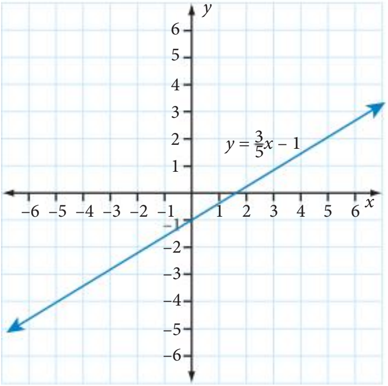
d


y
x
y = −2x
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

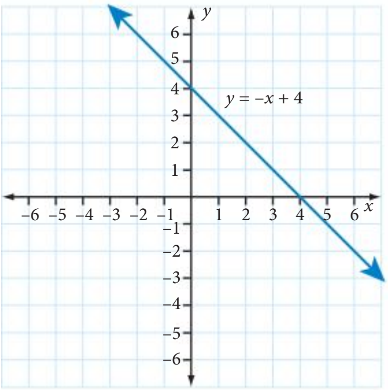
e


y
x
y = 2x − 5
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

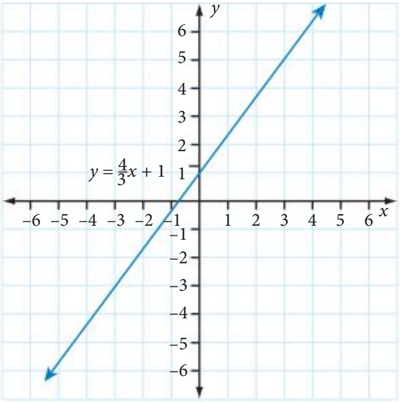
f


y
x
$y = \frac{3}{5}x - 1$
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

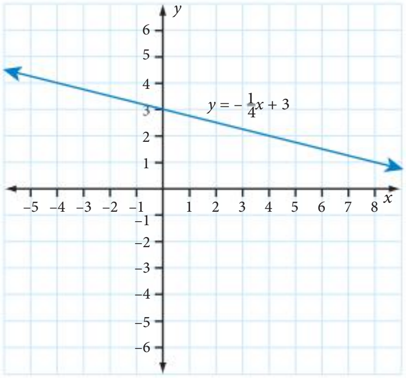
g


y
x
y = −x + 4
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

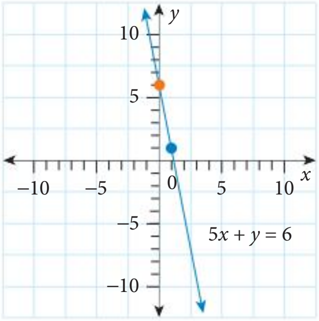
h


y
x
$y = \frac{4}{3}x + 1$
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

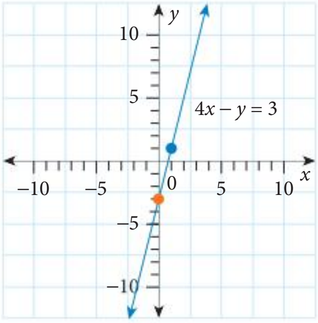
i


y
x
$y = -\frac{1}{4}x + 3$
−5 −4 −3 −2 −1 1 2 3 4 5 6 7 8
6 5 4 3 2 1 −1 −2 −3 −4 −5 −6

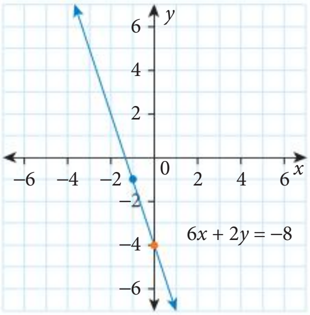
2 a


y
x
5x + y = 6
−10 −5 0 5 10
10 5 −5 −10

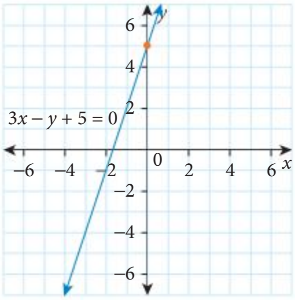
b


y
x
4x − y = 3
−10 −5 0 5 10
10 5 −5 −10

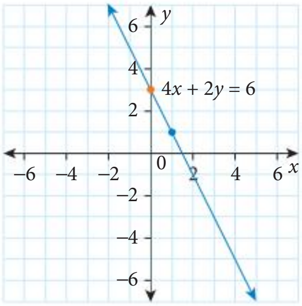
c



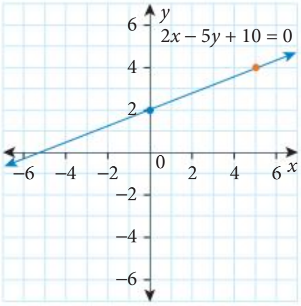
d



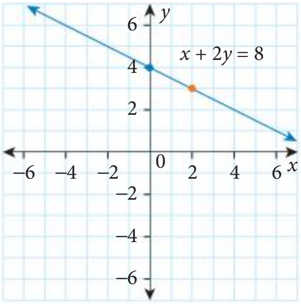
e



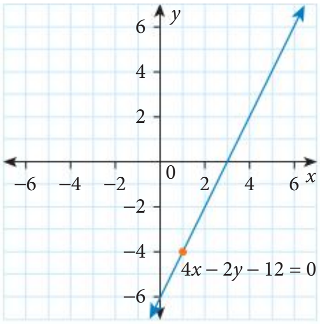
f



g



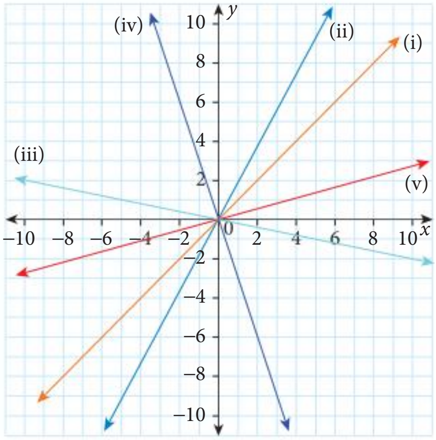
h



3 A

4 E

5 **a**



b Similarities: All graphs are straight lines; all pass through the origin; they all have x- and y-intercepts equal to 0.
Differences: Lines vary in steepness – the greater the coefficient of x, the steeper the line. The graphs of the equations where the coefficients of x are negative, slope in the opposite direction.

6 B

7 **a** gradient = 3; $y = 3x + 1$

b gradient = −1; $y = 4 - x$

c gradient = 3; $y = 3 + 3x$

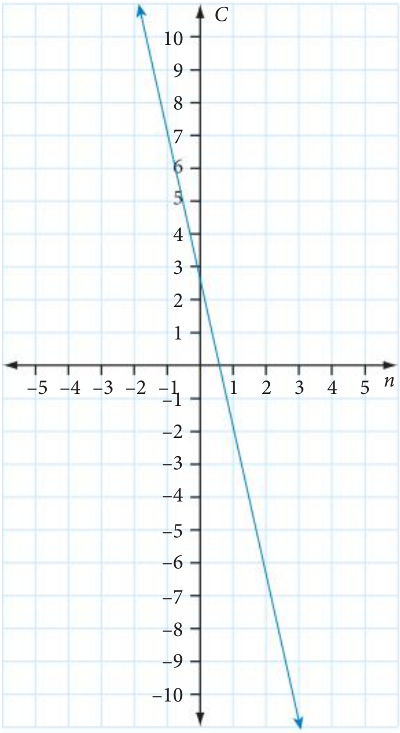
d gradient = $\frac{8}{5}$; $y = \frac{8}{5}x - 10$

e gradient = $\frac{13}{3}$; $y = \frac{13}{3}x - 4.5$

f gradient = $-\frac{5}{4}$; $y = -11 - \frac{5}{4}x$

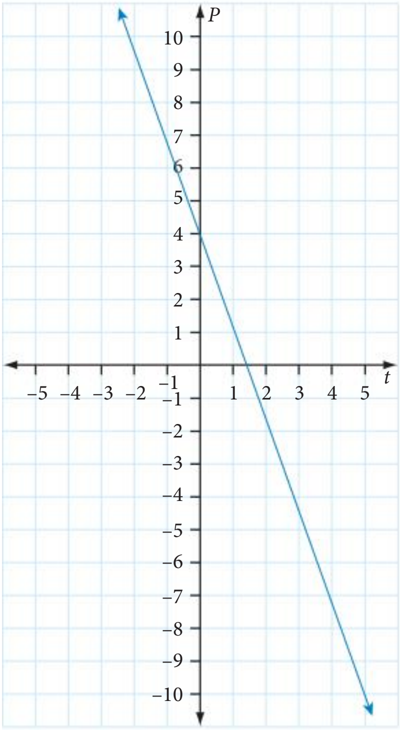
8 **a** Gradient = -4.5 , vertical axis intercept = 2.6

b

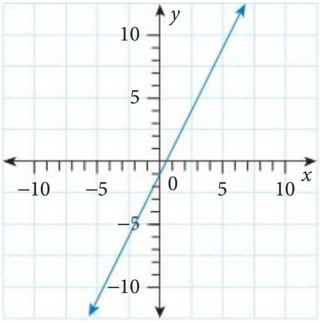


9 **a** Gradient = -2.8, vertical axis intercept = 3.95

b

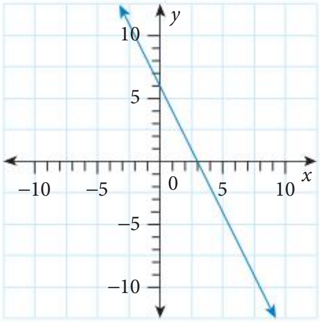


10 **a** **i**



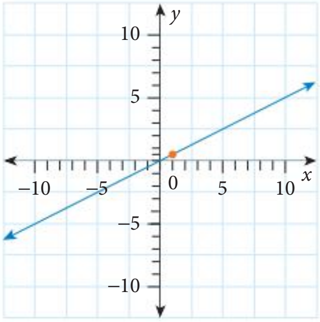
ii 2 **iii** $(0, -1)$ **iv** $y = 2x - 1$

b **i**



ii -2 **iii** $(0, 6)$ **iv** $y = 6 - 2x$

c **i**

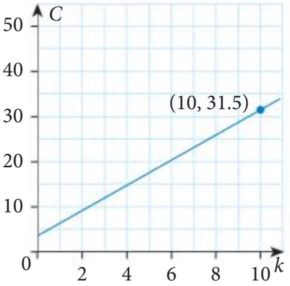


ii $\frac{1}{2}$ **iii** $(0, 0)$ **iv** $y = \frac{1}{2}x$

11 **a** The cost for getting in the taxi or the flag fall, which is \$3.50.

b \$31.50

c



INVESTIGATION: STAIRWAY TO HEAVEN

a $H = 50 + 0.2s$ b 55 m

c 60 m

d $H = 50 + 5n$, gradient = 5, H-intercept = 50.

e His height above sea level when he enters the cathedral.

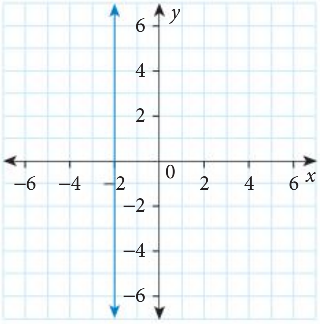
f The increase in vertical height for each flight of stairs climbed.

g 75 m h 225 i 200 m

j 52 minutes 5 seconds

9.06

1 a



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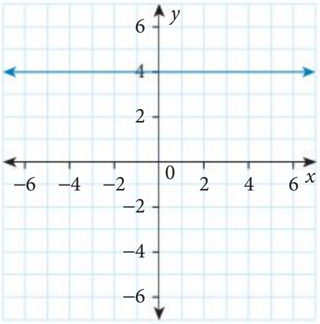

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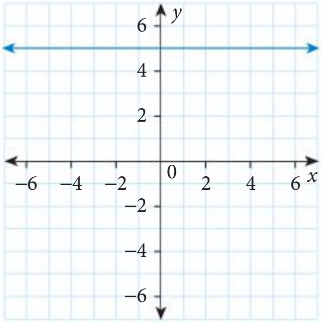

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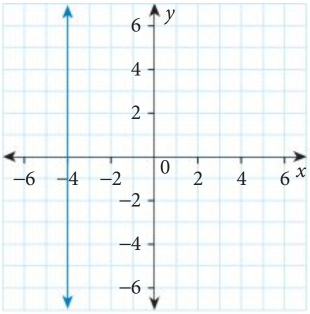

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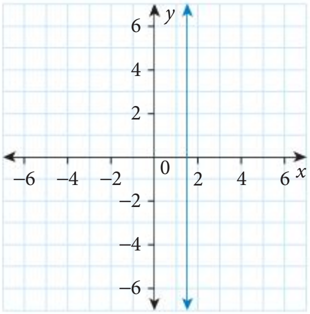

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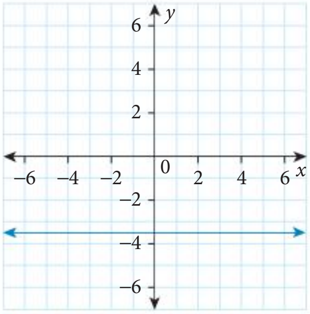

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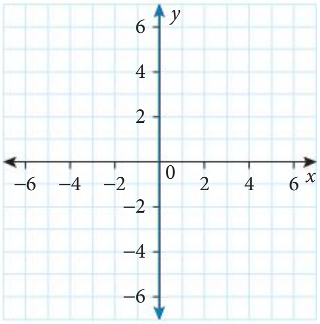

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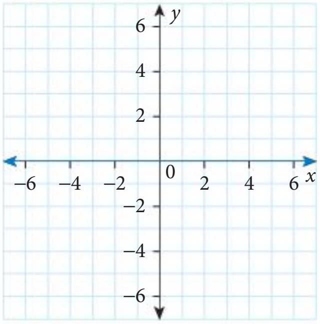
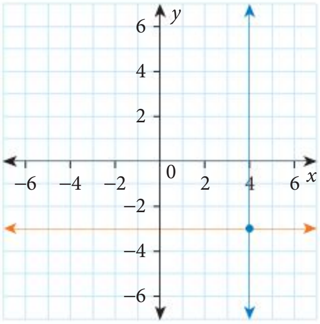


9780170250252

2 B

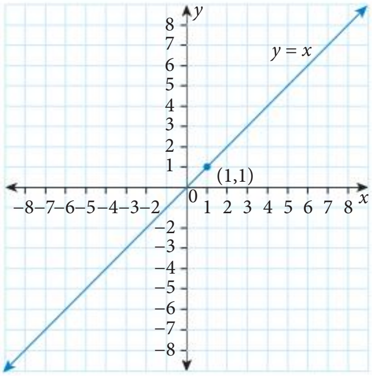
3 A

4 a



b Point of intersection is (4, −3)

5 a



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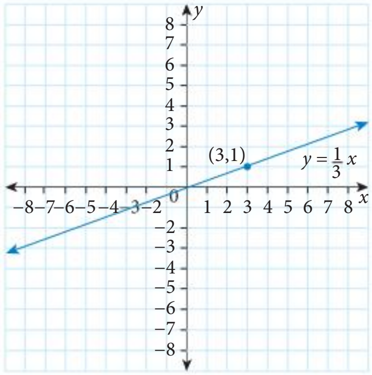

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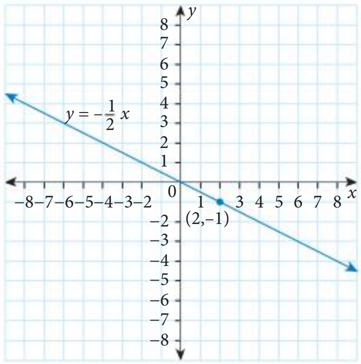
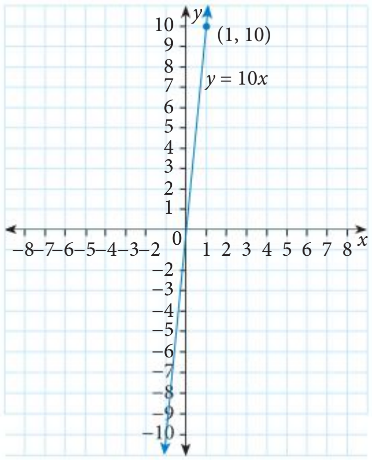
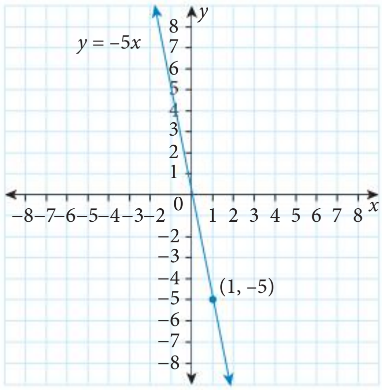


d



e

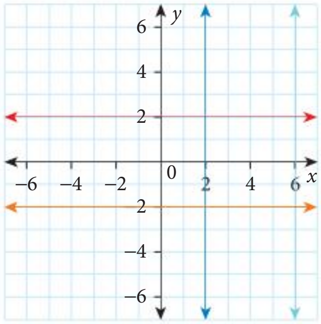


6 a (2, −4) b (−5, 3) c (−1, 4)

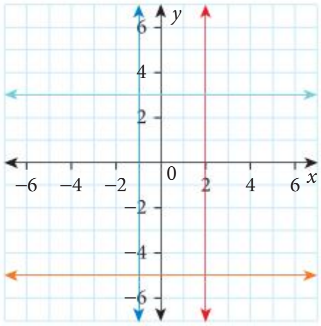
d (8, −6) e (14, −10) f (−12, 8)

g (−1.5, 6.2) h (−3.6, −9.8)

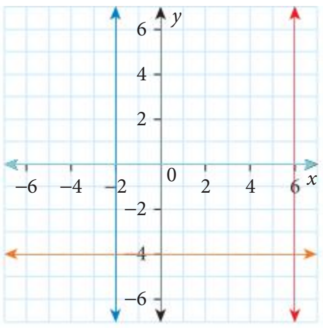
7 a square



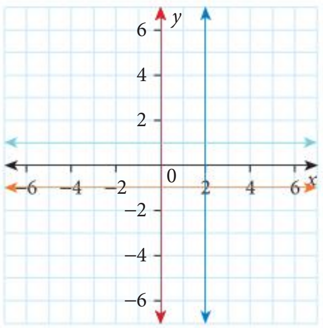
b rectangle



c rectangle



d square



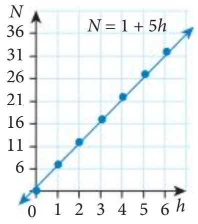
8 y-axis
9 x-axis

9.07

1 n
2 M
3 22.6
4 280
5 B
6 a

h	1	2	3	4	5	6
N	6	11	16	21	26	31

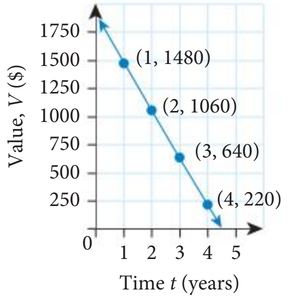
b N c $N = 1 + 5h$
d 101 e 16
f



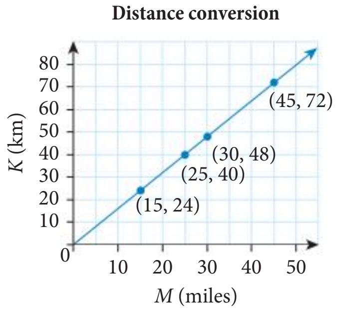
g gradient 5, N-intercept 1
7 a $C = 22 + 80t$
b C
c $14.62
d 22, starting cost of call (at 0 min)
e $2.40
f 7 min
8 a dependent
b $S = 4.5n$
c 4.5 runs/over
d 0, the number of runs scored after 0 overs
e i 95 ii 225
f i 12 ii 40
g Weaker batters bat later and have lower run rates.
9 a

t	1	2	3	4
V	1480	1060	640	220

b



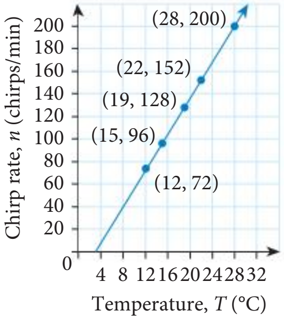
c rate of depreciation in dollars per year
d $850 e $1900
f V becomes negative. g 4.5 years
10 a $P = 50 + 3n$
b n
c 50, the base pay
d $134
e 16
11 a



b $K = 1.6M$ c 0, 0 miles = 0 km
d 1.6, the number of kilometres in 1 mile
e i 160 km ii 62.5 miles
f i 19 km ii 12.5 miles
12 a C
b gradient 0.7, vertical intercept 4.2
c $C = 4.2 + 0.7d$
d $3.50
e i $18.20 ii $4.20
f 48 km

13 **a** independent **b** $n = 8T - 24$

c



d increases by 16 chirps/min

e 184 chirps/min

f 21°C

g −24, the temperature would not be −24°C

14 **a** $P = 75 + 2n$

b n, the number of burgers made

c 75, George's base pay if he makes no burgers

d $175

e 120

CHAPTER 9 REVIEW

1 D

2 E

3 D

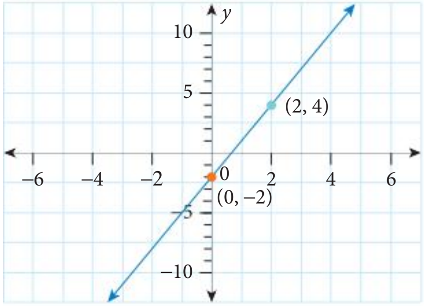
4 C

5 B

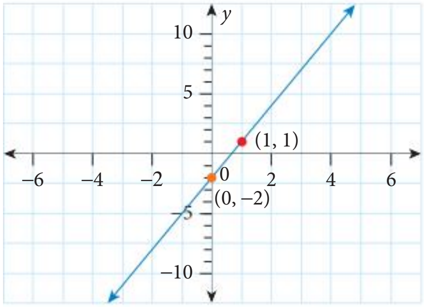
6 C

7 **a** $x = 9$ **b** $a = 4$ **c** $x = 22$

8 **a**



b Use y-intercept = −2 and gradient = 3



9 **a** $y = 4 - \frac{1}{2}x$ **b** $y = 1 - 3x$

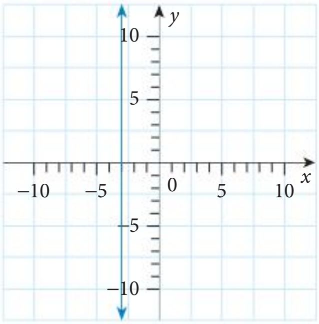
10 **a** gradient = −2, y-intercept = 6

b gradient = −3, y-intercept = −7

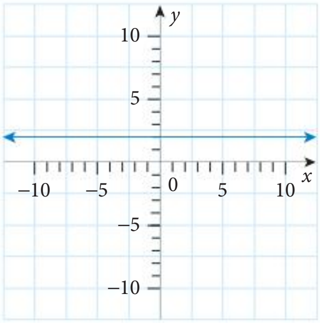
c gradient = $\frac{3}{2}$, y-intercept = 5

11 $y = 7 + 3x$

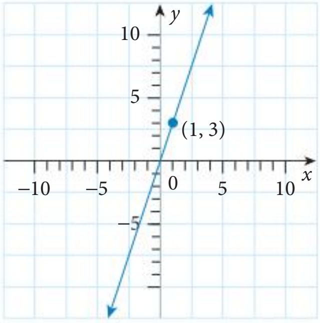
12 **a**



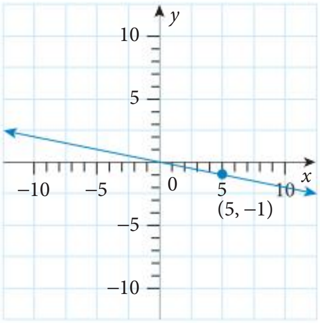
b



13 **a**

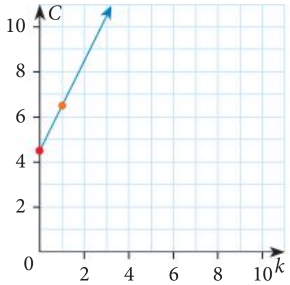


b



14 **a** \$4.50 **b** 2

c

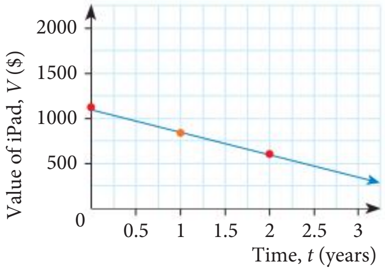


d \$26.90.

15 **a**

Time t in years	0.5	1	1.5	2
Value of iPad, V(\$)	980	860	740	620

b



c The depreciation of the iPad / year.

d \$740

e \$1100

f Value of the iPad will be less than \$0.

g 4.6 years.

MIXED REVISION 3

Multiple choice

1 D
2 E
3 D
4 C
5 A
6 B
7 B
8 A
9 C

Short answer questions

1 **a** 13.5 m **b** 2.7 m **c** 5 cm

2 **a**

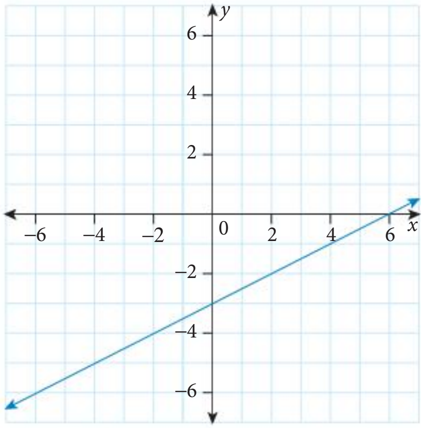
Speed (km/h)	Frequency (f)
60–<70	5
70–<80	11
80–<90	18
90–<100	6
Total	40

b 81.3

c 80–<90 for both

3 **a** gradient = $\frac{1}{2}$; y-intercept = –3

b



4 1.69 m

5 **a**

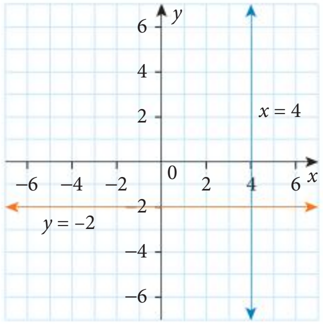
Stem	Leaf
1	2 5 8
2	2 2 3
3	1 8 9
4	0 1 5 5 5 7 9
5	1 2 2 3 4 4 5 8
6	0 1 1 4 6 8 9
7	5 9

Key: 1|2 means 12 seconds

b min = 12, Q_1 = 38.5, median = 51, Q_3 = 60.5, max = 79

c $5.5 \le x \le 93.5$. All values are inside this range, therefore there are no outliers.

6



Application questions

1 **a** 112.5 cm **b** $\frac{25}{36}$

c $\frac{125}{216}$ **d** 6.875 kg

2 **a** 12 m

b 6 rolls

c \$707.40

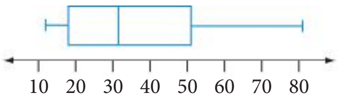
3 **a** $\bar{x} = 39.25$

b 31.5

c The data is skewed, therefore the median is more appropriate.

d 32.5

e



4 a C

b As n goes up by a constant amount (10), so does C (50).

c 5

d 250

e $C = 250 + 5n$

f The cost when there are no people attending the function.

g $925

5 a 1600 km b 5 hours

c 800 d 0

e $d = 800t$

f When the time is zero, the plane has travelled 0 km.

g The plane's constant speed of 800 km/h.

10.01

1 a 3.72 b 143.97 c 3.57

d 7.83 e 2.59 f 2.96

2 a 20.71 cm b 7.07 m c 13.39 m

d 7.63 mm e 9.34 m f 12.30 m

3 a 7.63 m b 17.27 cm c 11.01 cm

d 20.11 cm e 9.57 km f 5.23 m

4 B

5 30.0 cm

6 11.6 m

7 B

8 2.5 m

9 42.9 m

10 132.8 m

11 49.6 m

12 a 7.47 m b 6.93 m

13 65 m

14 $x = 2.0$ cm $y = 3.0$ cm

15 72.8m

10.02

1 a 64° b 42° c 39° d 38°

e 84° f 19° g 34° h 41°

2 A

3 26°

4 12°

5 50°

6 25°

7 88°

8 8°

9 $a = 53°$, $b = 70°$, $c = 44°$

10 C

INVESTIGATION: INVESTIGATING THE TRIGONOMETRIC RATIOS

a i

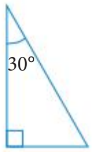
θ	1°	10°	30°	45°	60°	80°	89°
sin (θ)	0.0175	0.1736	0.5000	0.7071	0.8660	0.9848	0.9998
cos (θ)	0.9998	0.9848	0.8660	0.7071	0.5000	0.1736	0.0175
tan (θ)	0.0175	0.1763	0.5774	1.0000	1.7321	5.6713	57.2900

ii They are complementary angles.

iii The values of sine increase and the values of cosine decrease.

iv It increases dramatically.

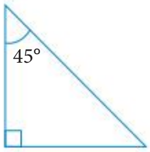
b i



ii 60°

iii Yes. The sum of all internal angles must equal 180°, therefore these angles must stay the same.

c i



ii 45° iii Yes.

10.03

1 156 m

2 a 2.30 m b 2.27 m

3 a 1468.86 m b 1231.89 m

4 109 m

5 14°

6 E

7 a 33.96 m b 8.47 m

8 a 20° b 469.85 m

9 a 288.57 m b 251.65 m

10 a 5.12 m b 51°

11 65°

12 E

INVESTIGATION: USING A CLINOMETER TO MEASURE HEIGHTS

Teacher to check results.

10.04

1 a 180° b 045° c 270°
d 225° e 315° f 090°

2 a SW b SE c NE
d SE e NW f SW

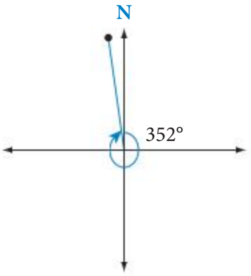
3 a 043° b 262° c 135°
d 113° e 345° f 083°
g 308° h 026° i 206°

4 a N 75° E b N 68° W c N 30° W
d S 35° W e S 62° E f N 43° W
g S 20° E h S 45° W i N 40° E

5 D

6 A

7 a



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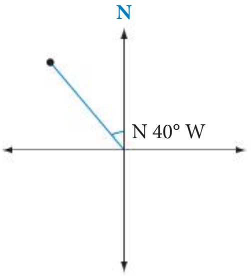

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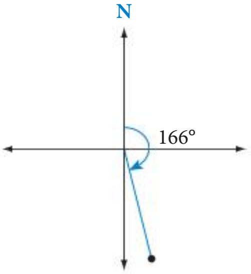

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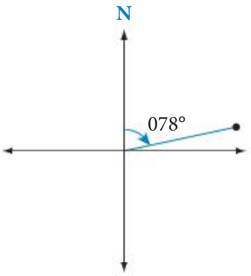

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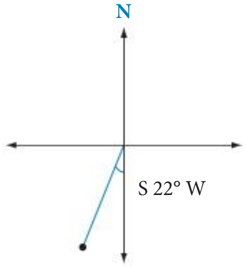

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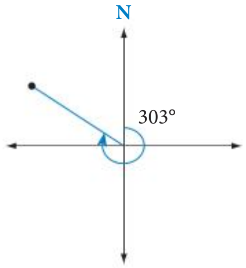

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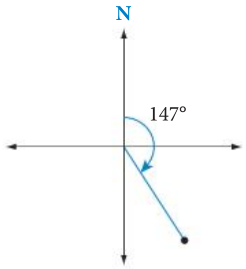

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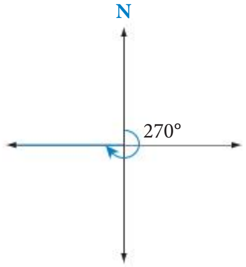

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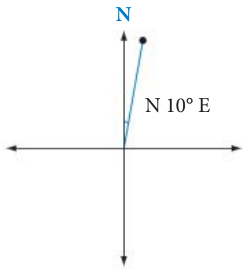

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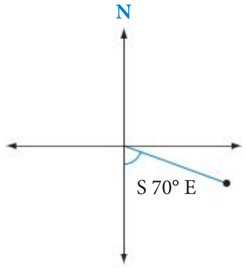

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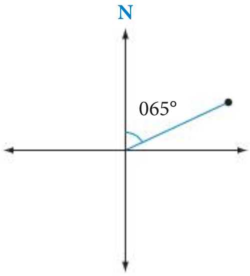
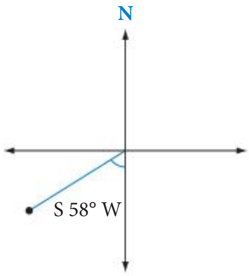
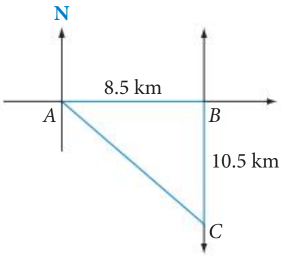


l



10.05

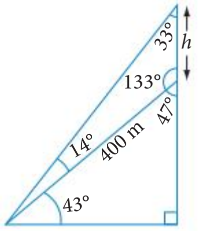
1 a 24° b 336° c 2930 km
2 a N 49° W b 13 km
3 a



b 32.5 km c S 39° E d N 39° W
4 a 55 km b 95 km c 030°
5 10.88 km
6 097°
7 31 km
8 a 25° b 275°
9 447 m
10 a 270° b 235°
11 $\angle A = 38^\circ$, $\angle B = 97^\circ$, $\angle C = 45^\circ$
12 S 40° W

10.06

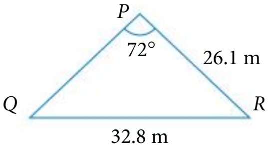
1 a 11.39 m b 7.47 cm c 4.12 m
d 9.58 cm e 18.69 m f 78.46 mm
2 13.9 cm
3 D
4 2.43 m
5 A
6 14.5 m
7 a



b 177.68 m
8 a 214.22 m b 185.52 m

10.07

1 a 42° b 9° c 29°
2 C
3 B
4 a 30° b 30° c isosceles d 4.50 m
5 a



b 49°
c 59°
6 a 84° b 63° c 21°
7 B
8 33°
9 84°
10 64°
11 151°
12 239°

10.08

1 a 17.26 mm b 9.64 cm c 11.04 m
d 15.85 cm e 11.18 m f 10.72 m
2 C
3 a 26° b 52° c 123°
d 36° e 91° f 48°
4 a 98° b 41° c isosceles d 41°
5 9 km
6 9°
7 8°
8 a 360° b 72° c 47.02 cm
9 a 25° b 78°
10 29°

10.09

1 1.21 km
2 1841 km
3 32 m
4 107 m
5 35.8 m

6 a 35.38 km b 102°
7 1420 km
8 a $\angle IFH + 42° = 61°$ (exterior angle $\triangle FIH$)
$\angle IFH = 19°$
b Teacher to check.
c Teacher to check.
d 216 m

10.10

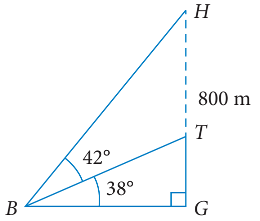
1 a 78.26 m^2 b 91.16 mm^2
c 34.41 cm^2 d 222.63 mm^2
e 69.87 cm^2 f 63.36 m^2
2 a 8.04 m^2 b 24.07 cm^2
c 19.86 m^2 d 760.85 cm^2
e 131.30 mm^2 f 21.36 m^2
3 a 1.7 cm^2 b 88.0 m^2
c 891.1 mm^2 d 94.9 cm^2
e 25.4 m^2 f 35.6 cm^2
4 a 6 cm^2 b 6 cm^2
5 a 21.22 cm^2 b 21.22 cm^2
6 C
7 13.29 m^2
8 a 55.85 cm^2 b 31.51 cm^2
c 24.34 cm^2
9 a Pigs: 1275.57 m^2; Chickens: 644.43 m^2
b 1920 m^2

10.11

1 a 6.12 cm^2 b 30.23 cm^2
c 764.85 km^2 d 178.90 m^2
e 108.65 cm^2 f 283.98 mm^2
2 A
3 a 15.59 cm^2 b 15.59 cm^2
4 1689 cm^2
5 a 4.00 m, 5.10 m, 5.83 m
b 10 m^2
6 9 m^2

CHAPTER 10 REVIEW

1 D
2 A
3 E
4 C
5 B
6 C
7 23.9 m
8 4.3 m
9 48°
10 24°
11 8 m
12 a 304° b 7.2 km
13 a 6.2 km b 3.6 km c 030°
14 39.7 mm
15 a 20.12 m b 16.48 m
16 67°
17 81°
18 89°
19 25 m
20 a 261.81 km b 142.39 km
21 12.47 m^2
22 17 020 m^2
23 a 2 m b 3 m c 5 m
24 a



b $\angle BTG = 180° - 90° - 38°$
$= 52°$ (angle sum of $\triangle TGB$)
$\angle BTH = 180° - 52°$
$= 128°$ (angles on a straight line)
c 942 m
d 208 m e 128 m

11.01

1 a 161.4 b 167 c 167
d 69 e 40
f The average number of runs scored for the season was 161. 50% of the scores were below 167 runs and 50% of the scores were above 167. 167 runs was the most common score. There was a difference of 69 runs between the highest and lowest number of runs scored. The middle 50% of runs scored has a range of 40 runs.
2 a 167.5 b 168 c 175
d 28 e 15
f The average height of a student is 167.5 cm. 50% of the student's heights are below 168 cm and 50% are above 168 cm. The most common height of students is 175 cm. There is a difference of 28 cm between the height of the tallest student and that of the shortest. The middle 50% of heights has a range of 15 cm.
3 E
4 C
5 a 14 b Holden and Ford c $14 714
d $13 000 e Mean, by $1714
f One car is a much higher price than all of the others.
g Median, as the mean is greater than 13 out of the 14 cars sold.
6 a 191.7 cm b 193 cm
c 194 cm d 195 cm
e Both increase as a taller player (top half) is replacing a shorter player (lower half).

7 a Girl
b Boys = \$34, Girls = \$23.36, on average the boys spent more.
c Boys = \$35.50, Girls = \$23, the median for boys is larger than for girls.
d Boys = \$35, Girls = \$38, range for girls is greater.

8 a Week 1 = 25, Week 7 = 19, Week 7
b Week 1 = 21, Week 7 = 18.5, Week 1 has a higher median than Week 7.
c Week 1 = 36, Week 7 = 26, Week 1 has the greater spread.
d The program has been a success as all summary statistics calculated are lower for Week 7 (meaning less smokes per day) than Week 1.

9 D

10 a 39 b White cars
c not numerical data

11 a Hockey b Hockey
c Netball = 76 kg, Hockey = 77 kg, the median weight for hockey players is higher than for netball players.
d Netball = 73.4 kg, Hockey = 75.3 kg, the mean weight for hockey players is higher than for netball players.
e Range: Netball = 24 kg, Hockey = 40 kg
f Disagree, even though hockey has the lightest player they have a wider spread of weights with the average and median weight for hockey players being slightly higher than netball players.

11.02

1 a $4 \le x \le 12$, therefore 13 is an outlier.
b $18.75 \le x \le 104.75$, therefore there are no outliers.
c $-4.925 \le x \le 15.275$, therefore there are no outliers.
d $38.5 \le x \le 74.5$, therefore 12, 91 and 98 are outliers.

2 a 20 b 8 is an outlier ($2.5 \le x \le 6.5$).
c i 4.45 cups ii 4.26 cups
d It increases the mean number of cups.

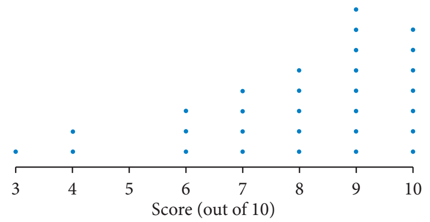
3 a i 7.38 ii 7 iii 8
b 12 is an outlier ($3 \le x \le 11$).
c i decrease ii stay the same
iii stay the same
d There are outliers, so the median or mode should be used. He needs to order the most popular size, so the mode should be used as an indication of what to order.

4 a $\bar{x}$ = \$507.14, median = \$520
b Either seem fairly reasonable as most wages are around in the \$400s and \$500s.
c range = \$570, IQR = \$150
d IQR, as most wages are clustered apart from one.
e For part **b** I would now only select the median and it supports my choice in part **d**. \$800 is an outlier as it is outside the range $195 \le x \le 795$.
f $\bar{x}$ = \$458.3 median = \$485. They are both lower.

5 D

6 a i Median, as data is positively skewed with an outlier.
ii IQR, as data is positively skewed with an outlier.
b i Median, as data is negatively skewed with a possible outlier.
ii IQR, as data is negatively skewed with a possible outlier.
c i Median, as data is positively skewed with a possible outlier.
ii IQR, as data is positively skewed with a possible outlier.
d i Mean or median, as data is not skewed and no outliers are present.
ii Range, IQR or standard deviation, as data is not skewed and no outliers are present.
e i Mean or median, as data is not skewed and no outliers are present.
ii Range, IQR or standard deviation, as data is not skewed and no outliers are present.
f i Mean or median, as data is approximately symmetrical.
ii Range, IQR or standard deviation, as data is approximately symmetrical.

7 a 50%
b



c negatively skewed
d median = 8.5
e 3 is an outlier ($4 \le x \le 12$).
f IQR

8 a 10
b i \$36 620. The average annual wage is \$36 620.
ii \$34 200. 50% of the annual wages are below \$34 200 and 50% are above it.
iii \$46 500. The most common annual wage is \$46 500.
c Mode, as it is the largest of all 3 measures.
d Median, as \$74 300 is an outlier ($1500 \le x \le 73\,500$).

9 a **Pam**

Stem	**Leaf**
0	1 2 3 3 5 6 7 8
1	2
2	5

Key: 0|1 means 1 copier sold

Percy

Stem	Leaf
0	3 3 3
1	4 6 8 8
2	4
3	2 5

b Pam = 3, Percy = 3

c Conclude that Pam and Percy sell the same number of copiers.

d Pam – positively skewed and 25 is an outlier ($0 \le x \le 15.5$).
Percy – not skewed and no outliers ($0 \le x \le 55.5$).

e Pam: median = 5.5;
Percy: median = 17 or mean = 16.6

f Pam: IQR = 5; Percy: IQR = 21 or range = 32

g No, it is clear from the other summary statistics that Percy sells more copiers.

11.03

1 a B ($\bar{x} \approx 104.9$) is higher than A ($\bar{x} \approx 98$).

b A ($\bar{x} \approx 44.4$) is higher than B ($\bar{x} \approx 41.5$).

c A ($\bar{x} \approx 104$) is higher than B ($\bar{x} \approx 86.7$).

d B ($\bar{x} \approx 11.2$) is higher than A ($\bar{x} \approx 10.2$).

2 a B ($s_x \approx 1.4$) is higher, therefore it has a greater spread than A ($s_x \approx 1.1$).

b B ($s_x \approx 16.8$) is higher, therefore it has a greater spread than A ($s_x \approx 11.5$).

3 a A ($s_x \approx 19.7$) is more consistent than B ($s_x \approx 24.8$).

b B ($s_x \approx 12.2$) is more consistent than A ($s_x \approx 14.5$).

4 D

5 B

6 a **Art**
$\bar{x} = 43.3$, $s_x = 15.5$
Music
$\bar{x} = 52.7$, $s_x = 8.2$

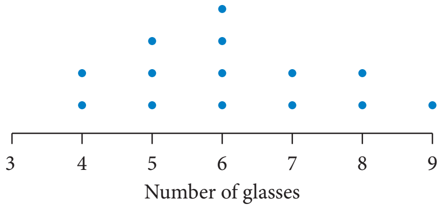
b Music has a higher mean (52.7 compared to 43.3), therefore the students have achieved better in Music than Art.

c Music has a smaller standard deviation (8.2 compared to 15.5), therefore the results for Music are more clustered around the mean.
Note: References to other summary statistics could also be made.

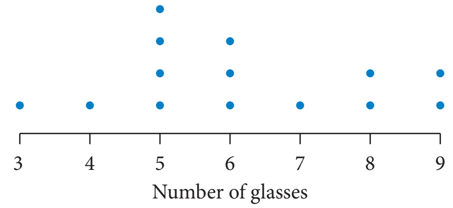
7 a Sue = 6.14 glasses, Michelle = 6.14 glasses

b From the mean you would conclude that they drink the same number of glasses per day.

c Sue



Michelle



d No, Michelle has a wider spread.

e Sue = 1.51, Michelle = 1.83

f Sue is the most consistent consumer of water as her standard deviation is lower.

8 a Farmer Joe = 185.1 g; Farmer Moe = 215.3 g.
Farmer Moe on average has larger onions.

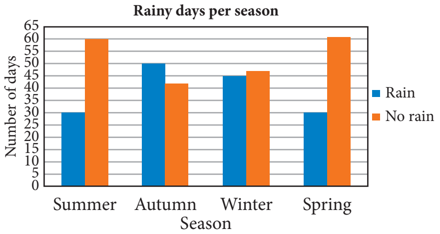
b Farmer Joe = 47.8 g; Farmer Moe = 81.8 g
Farmer Moe's onion size has a greater spread.

c Farmer Joe's onions are more consistent since they have a lower standard deviation.

d Farmer Joe should get the contract as his onions are of a more consistent size. Farmer Moe's onions, on average, are larger but as the onions are sold in 5 kg lots, this is not important.

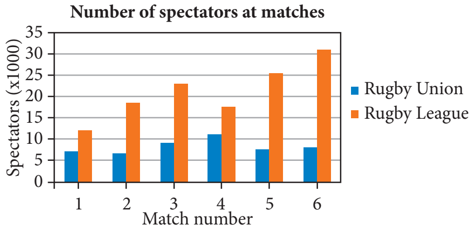
11.04

1 a



b The season with the most days of rain was Autumn followed by Winter. Spring and Summer had the same amount of rainy days, these were far less than their days with no rain.

2 a



b Rugby League has far more spectators than Rugby Union. The most popular match for Rugby League was number 6, whereas for Rugby Union it was number 4.

3 B

4 a 20 **b** 20

c 12P = 7.65 and 12Q = 6.95

d 35% **e** 25%

f 12P scored higher on the sports quiz as they had more students scoring 8 and 9. The most common score for 12Q and 12P were 7 and 9 respectively.

5 a North Park
  b North Park: 20–22, South Park: 26–29.
  c North Park, as it has higher columns for lower age groups compared to South Park.
  d North Park has more employees in the younger age groups whereas South Park has more in the older age groups. The most common age of employees for North and South Park were 20–22 and 26–29 respectively.

11.05

1

Class B	Stem	Class A
	14	7 8 9
9 8 7 7 1	15	0 3 5 7 7 8 9 9
9 8 8 7 6 5 5 4 2	16	0 1 2 3 4 6 7 8 9
9 8 6 5 3 3 2	17	0 1 2 6 7 7
2 1 1 1 0	18	

Key: 14|7 means 147cm

2

Camera 2	Stem	Camera 1
	6	5 6 9 9 9 9
	7	0 2 3 3 4 4 5 6 7 8
	8	0 3 3 4 5 9
	9	0
	10	3
9 9 8 6	11	
5 4 3 3 2 2 0 0	12	
8 5 5 3 1 0 0 0	13	
8 2 0	14	
	15	
9	16	

Key: 6|5 means 65 km/h

3

Father’s age	Stem	Mother’s age
8 8	3	5 6 8 9 9
9 9 8 8 8 6 5 4 3 2 1 1 0	4	0 2 3 3 4 4 4 6 7 8 9 9
8 7 7 5 4 2 1	5	1 2 2 3 5 7
1	6	

Key: 3|5 means 35 years old

4 a Approximately symmetrical
  b Class A – no outliers (139 ≤ x ≤ 187) and Class B – no outliers (143 ≤ x ≤ 199).
  c As data is not skewed and has no outliers, all summary statistics are appropriate.

	Class A	Class B
Mean	162.12 cm	169.5 cm
Median	161.5 cm	168.5 cm
IQR	12 cm	14 cm
Range	30 cm	31 cm
Standard deviation	8.89 cm	8.89 cm

  d Class B has a higher median and mean than Class A. The spread of both classes is very similar, with the measures of spread being the same or very similar.
  e In general, students in Class B are slightly taller than students in Class A. This is supported by Class B having a mean that was 7.38 cm higher and a median that was 7 cm higher than Class A. Class B also had the tallest person. Both classes’ data was approximately symmetrical with a very similar spread.

5 a Camera 1 – positively skewed
  Camera 2 – approximately symmetrical with possible outlier
  b Camera 1 – no outliers (49.25 ≤ x ≤ 103.25)
  Camera 2 – 169 km/h is an outlier (100 ≤ x ≤ 156).
  c As data is skewed and an outlier is present, only the median, mode and IQR are appropriate.

	Camera 1	Camera 2
Median	74.5 km/h	127.5 km/h
Mode	69 km/h	130 km/h
IQR	13.5 km/h	14 km/h

  d Mode and median speed are much higher for Camera 2. The IQR for each camera is virtually the same.
  e In general, the speeds taken by Camera 2 are much higher than the speeds taken by Camera 1. This is supported by Camera 2 having a median that is 53 km/h higher than that of Camera 1. The most common speed recorded by Camera 2 is also much higher than that of Camera 1. Even though Camera 2 has recorded much higher speeds, the spread of the middle 50% of data for each camera is nearly the same. The data shows that each camera was in a different speed zone or everyone recorded on Camera 2 was speeding by a lot.

6 In general, fathers’ ages are higher than mothers’ ages. This is supported by the age of fathers having a median that is 4 years higher and a mean that is 2.5 years higher than that of the ages of mothers. Both sets of data are approximately symmetrical and they both have a similar spread indicated by the similar values for all measures of spread.

7 E

8 C

9 a

Newspaper article	Stem	Computer article
9 8 8 3 2	1	0 1 4 5 5 7 7 8 9
8 7 7 7 4 3 2 2	2	0 1 2 3 4 4 5 6 7 7 7 8 9
9 8 5 3 3 2 2 2	3	1 6
6 1 1	4	

Key: 1|0 means 10 words per sentence

  b

	Newspaper article	Computer article
Range	34	26
Median	27.5	22.5
IQR	12	10
Mean	28.4	21.9

c Newspaper article – approximately symmetrical; Computer article – positively skewed
As one data set is skewed, the median is the best measure of centre.

d In general, newspaper articles have more words per sentence than computer articles. This is supported by newspaper articles having a median that is 5 words higher than that of computer articles. The newspaper articles also use a wider range (by 8) of words per sentence compared to the computer magazine articles.

10 a Approximately symmetrical

b

	Survey 1	Survey 2
Range	69	66
Median	54	65
IQR	30	26.5
Mean	57.1	63.5
s_x	18.9	18.1

c In general, Survey 2 sighted more mammals than Survey 1.This is supported by Survey 2 having a mean that is 6.4 mammals higher and a median that is 11 mammals higher than that of Survey 1. Survey 1 had a slightly higher range and standard deviation than Survey 2, indicating a larger spread of mammals sighted.

11.06

1 a 5, 50% of the girls ate less than 5 lollies and 50% ate more than 5 lollies.

b 10, the number of lollies eaten had a range of 10 lollies.

c 5, the middle 50% of the girls ate between 3 and 8 lollies.

d 8 – 11 lollies e 50% f 100%

2 a 12.5, 50% of the cars were younger than 12.5 years and 50% were older than 12.5 years.

b 9, the age of the cars had a range of 9 years when the outlier was ignored.

c 3, the middle 50% of cars have ages with a range of 3 years.

d 18 is an outlier as it is outside the range $5.5 \le x \le 17.5$.

e 6 – 10 years f 75%

3 a 165, 50% of the people were shorter than 165 cm and 50% were taller than 165 cm.

b 20, the people's heights had a range of 20 cm.

c 10, the middle 50% of heights have a range of 10 cm.

d 165 cm – 170 cm e 75% f 50%

4 a Negatively skewed with no outliers.

b Approximately symmetrical with no outliers.

c Positively skewed with an outlier at 80.

d Symmetrical with an outlier at 20.

5 C

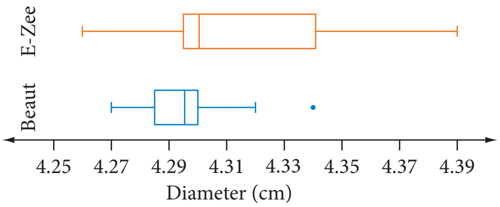
6 D

7 D

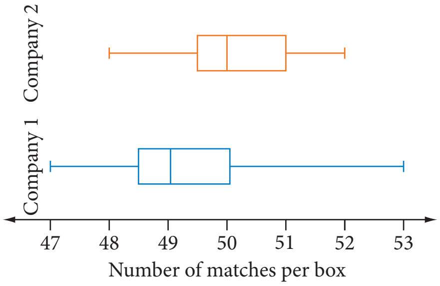
8 B

9 a For Beaut 4.34 cm is an outlier because it is outside the range of $4.2625 \le x \le 4.3225$.

b



10 a



b Company 1 not skewed, no outliers.
Company 2 approximately symmetrical.

c All measures are appropriate.

	Company 1	Company 2
Median	50	49.5
Mean	50.2	49.7
IQR	1.5	2
Range	4	6
Standard deviation	1.2	1.7

d Company 1 e Company 2

f Company 1 g 50%

11 a 4 cars b 1 car, Joan c 2 cars

d 25% e both are symmetrical

f Median, symmetrical

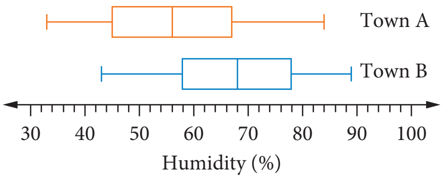
12 a 2010 b 2012 c 50%

d 2012 e 2009

f 2010 (2012 approximately symmetrical)

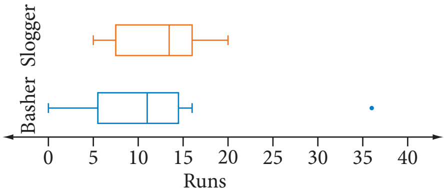
g 2009: 50% is above and below 7
2010 & 2011: 25% is above and 75% is below 7
2012: 25% is below and 75% is above 7.

13 a



b The relative humidity in Town A, median 56, is less than that in Town B, median 68. The towns have similar variation in humidity with the variation in humidity being slightly less in Town B. The IQR for Town A is 22 and for Town B it is 20.

14 **a**



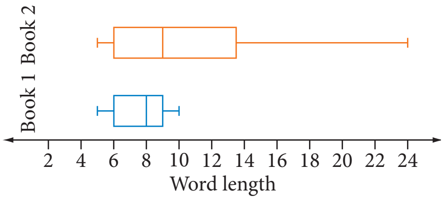
b For Basher, 36 runs is an outlier as it is outside the range $-8 \le x \le 28$ (realistically, the range would be $0 \le x \le 28$ as you cannot have negative runs).

c Ignoring outliers, they should select Slogger as he has been the more consistent batsman. This is supported by the position of his boxplot compared to Basher's as well as the standard deviation (ignoring outlier), which is lower for Slogger.

15 **a** **i** Darwin **ii** Brisbane
iii Darwin **iv** Canberra

b Yes, as Sydney has a lower median even though the spread for both cities is the same.

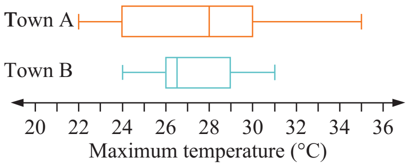
16 **a**



b Book 2 is harder as it has longer words in it, indicated by its higher median (9 compared to 8 and mean (10.4 compared to 7.8).

c Book 1 has a more consistent word length, indicated by its lower standard deviation (1.7 compared to 5.5).

17 **a**



b Town A is not skewed or symmetrical and Town B is approximately positively skewed.

c No outliers

d

	Town A	Town B
Median	28	26.5
IQR	6	3
Range	13	7

e In general, the maximum temperatures are higher for Town A than Town B. This is supported by Town A having a median temperature that is 1.5°C higher than Town B. Town A has a much larger spread of temperatures, indicated by its higher range and IQR.

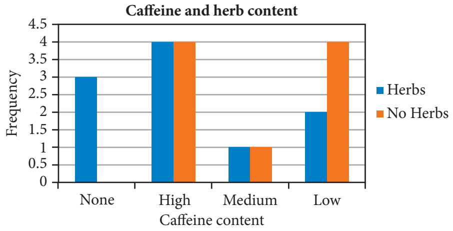
INVESTIGATION: DOES GENDER DETERMINE BODY MEASUREEMNTS?

Teacher to check results.

CHAPTER 11 REVIEW

1 B
2 D
3 E
4 C
5 D
6 B
7 D
8 A
9 C
10 D
11 **a** $437 500 **b** $345 000
c mean, by $92 500
d large value of $1 044 000
e The median of $345 000, as 6 out of the 8 house prices are close to this figure whereas the mean is higher due to the influence of the one very high value.

12 **a**



b Energy drinks that contain a high and low level of caffeine have the same amount with no herbs. Drinks with no caffeine only contain herbs. Energy drinks with high and medium caffeine content have equal amounts with herbs and no herbs.

13 **a**

Max	Stem	Jane
8 7 6 5 4	0	2 2 9
5 5 5 4 3 2 0	1	0 2 2 4 5 5 6 7 7 7
5 1	2	3 4
4 2 1 0	3	0 1
1 0	4	1 3 8

Key: 0|4 means 4 books sold

b Max = 15, Jane = 16.5, therefore Jane's median sales are higher.

c Max = 37, Jane = 46, therefore Jane's sales have a greater spread.

d Standard deviation, Max = 11.96, Jane = 12.83, therefore Max is the most consistent salesperson.

14 a Men b IQR c Men

d Men – not skewed

Women – approximately positively skewed

e Men = 175.3 cm Women = 164.0 cm

f Median, as one data set is skewed.

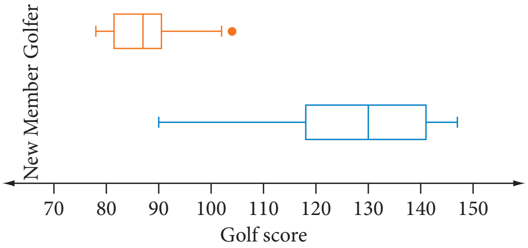
15 Spectators at a football match – boxplot C

Senior citizens on a bus trip – boxplot A

Visitors to a dance party – boxplot B

Primary school zoo excursion attendees – boxplot D

16 a The golfer.



b 105 is an outlier as it is larger than 90.5 + 1.5 × 9 = 104.

c Golfer = approximately positively skewed with an outlier

New member = negatively skewed

Use IQR to compare spread.

d The new member generally has higher scores, therefore he/she is not the better player.

17 a Machine A = 49 g and Machine B = 50 g

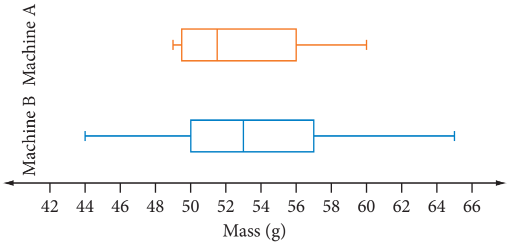
b

	Machine A	Machine B
Median	51.5 g	53 g
IQR	6.5 g	7 g

c Machine A (Machine A average = 52.7 g, Machine B average = 53.9 g)

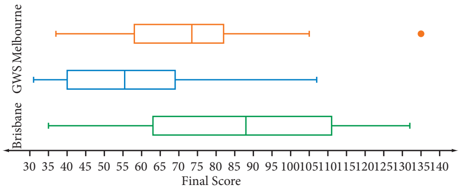
d Machine A: Min = 49, Q_1 = 49.5, median = 51.5, Q_3 = 56, max = 60

Machine B: Min = 44, Q_1 = 50, median = 53, Q_3 = 57, max = 65



e No, Machine B has a larger standard deviation (5 compared to 3.6), therefore it produces a wider spread of rod masses. This is also evident in the boxplots as Machine B has a far greater range.

18 a



(Note: $22 \le x \le 118$, therefore 135 is an outlier for Melbourne.)

b Outlier

c Melbourne = 58 to 37, GWS = 40 to 31, Brisbane = 63 to 35

d Melbourne – negatively skewed with an outlier

GWS – positively skewed

Brisbane – approximately symmetrical

e In general, Brisbane's final scores for the season were the highest, followed by Melbourne then GWS. This is supported by the median of Brisbane, which is the largest. Brisbane also has the widest range of scores due to its total spread being the largest as well as its middle 50% of data. Although there are outliers and some data is skewed, the standard deviation shows that GWS were the most consistent team but not consistently high.

12.01

1 a i 1 ii −2

b i 3 ii 6

c i $-\frac{1}{4}$ ii 7

d i $-\frac{1}{2}$ ii 3

e i 2 ii 6

f i $\frac{2}{3}$ ii $\frac{8}{3} = 2\frac{2}{3}$

g i $-\frac{3}{5}$ ii 4

h i $\frac{3}{4}$ ii $-\frac{3}{2} = -1\frac{1}{2}$

2 a $y = x - 2$ b $y = 2 - 2x$

c $y = 3 + 5x$ d $y = -10x$

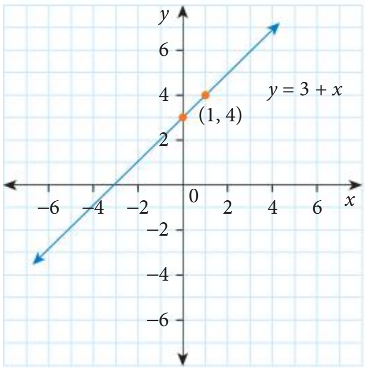
e $y = \frac{1}{2}x + 1$ or $2y = x + 2$

f $y = -5 - \frac{3}{4}x$ or $3x + 4y = 20$

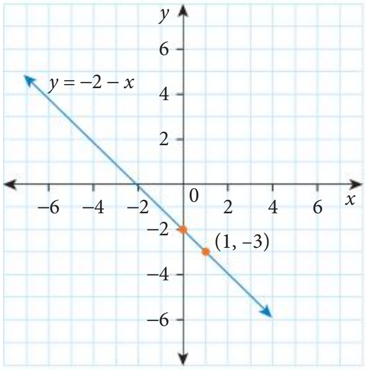
g $y = 3 - \frac{3}{2}x$ or $3x + 2y = 6$

h $y = \frac{2}{3}x$ or $3y = 2x$

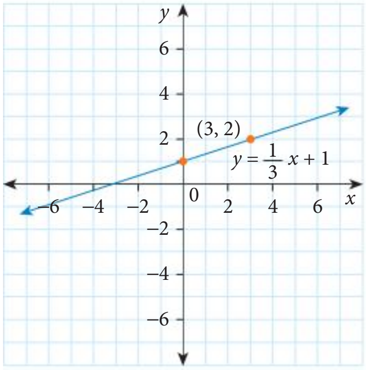
3 **a**


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
y = 3 + x
(1, 4)

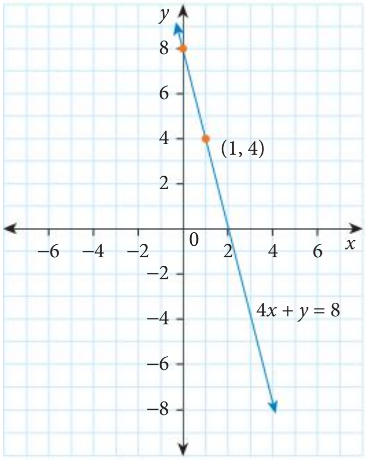
b


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
y = −2 − x
(1, −3)

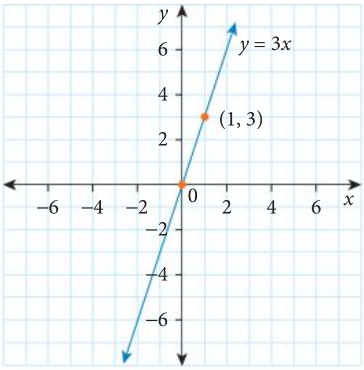
c


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
(3, 2)
y = 1/3 x + 1

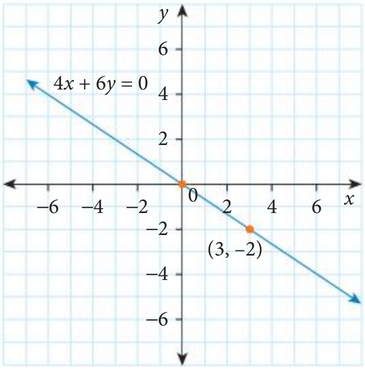
d


y
x
8
6
4
2
0
−2
−4
−6
−8
−6 −4 −2 2 4 6
(1, 4)
4x + y = 8

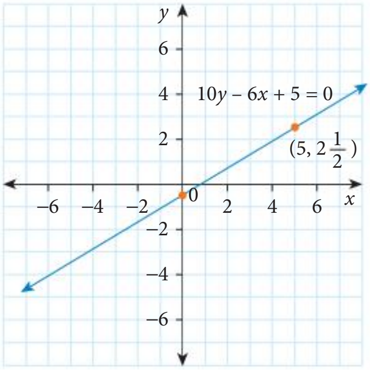
e


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
y = 3x
(1, 3)

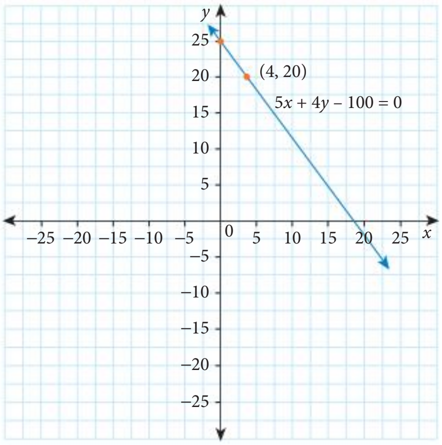
f


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
4x + 6y = 0
(3, −2)

g


y
x
6
4
2
0
−2
−4
−6
−6 −4 −2 2 4 6
10y − 6x + 5 = 0
(5, 2 1/2)

h


y
x
25
20
15
10
5
0
−5
−10
−15
−20
−25
−25 −20 −15 −10 −5 5 10 15 20 25
(4, 20)
5x + 4y − 100 = 0

4 **a** A **b** D

5 **a** i 2 ii 1 iii $y = x + 2$

b i -7 ii 3 iii $y = 3x - 7$

c i 4 ii -2 iii $y = 4 - 2x$

d i -1 ii $\frac{1}{4}$

iii $y = \frac{1}{4}x - 1$ or $4y = x - 4$

e i 5 ii $-\frac{2}{3}$

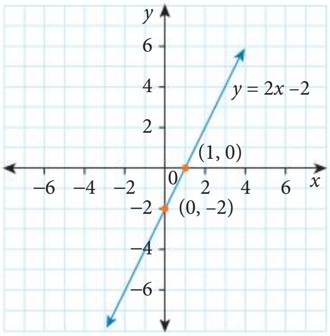
iii $y = 5 - \frac{2}{3}x$ or $2x + 3y = 15$

f i 0 ii $-\frac{3}{5}$

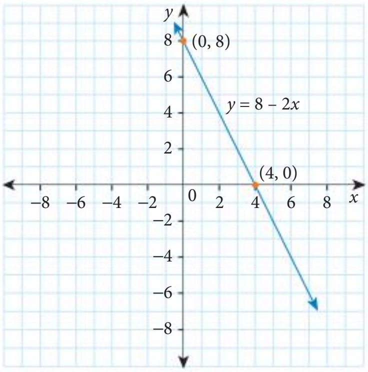
iii $y = -\frac{3}{5}x$ or $3x + 5y = 0$

6 A

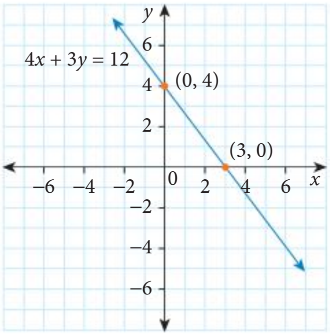
7 **a**



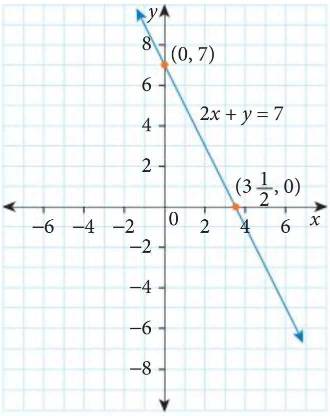
b



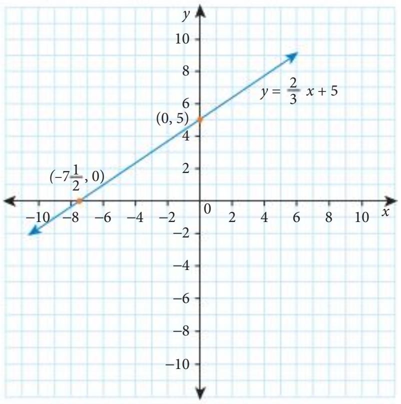
c



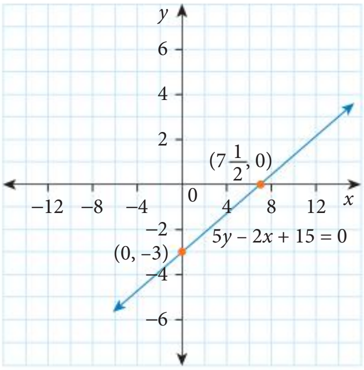
d



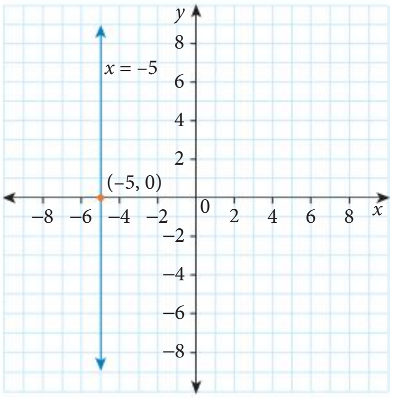
e



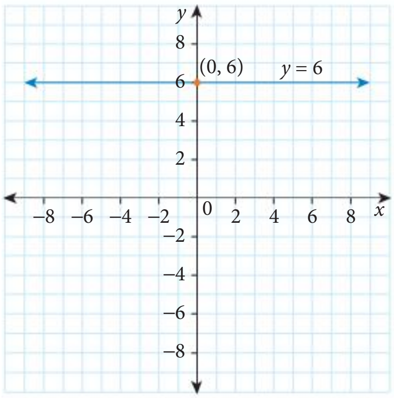
f



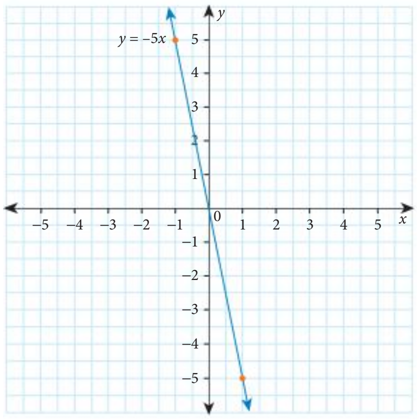
g



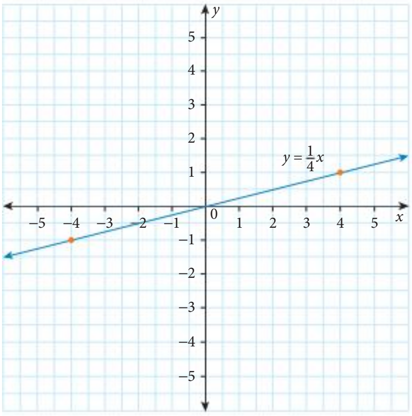
h


y
x
(0, 6)
y = 6
8
6
4
2
0
−2
−4
−6
−8

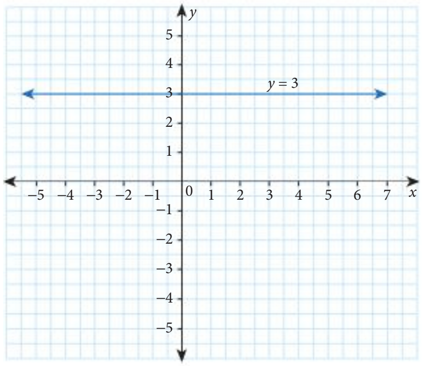
8 a


y
x
y = −5x
0

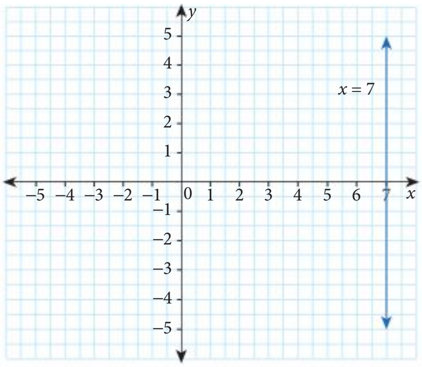
b


y
x
$y = \frac{1}{4}x$
0

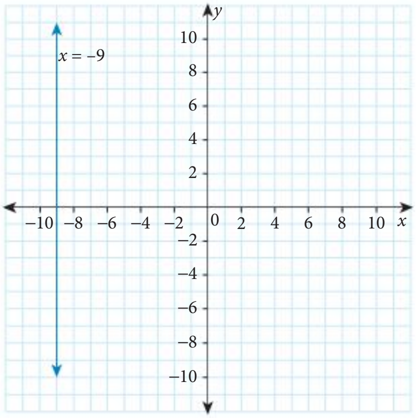
c


y
x
y = 3
0

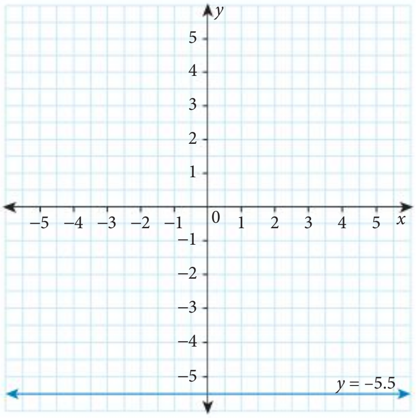
d


y
x
x = 7
0

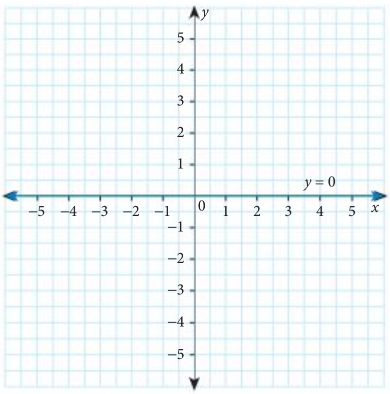
e


y
x
x = −9
0

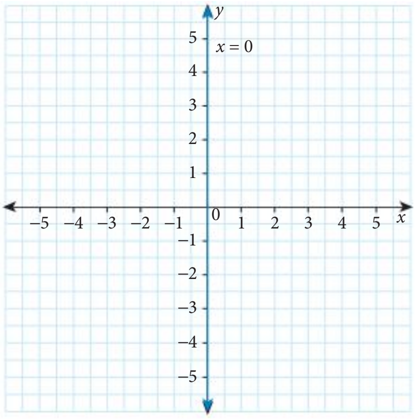
f


y
x
y = −5.5
0

g


y
x
y = 0
0

h

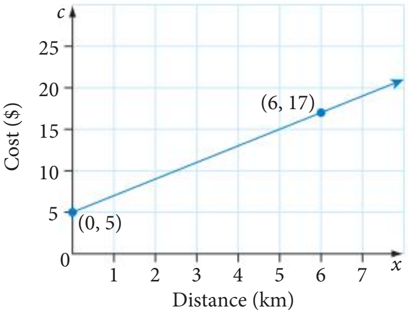


9 The line $x = 0$ is the same as the y-axis.
The line $y = 0$ is the same as the x-axis.

10 a $y = 4 + x$ b $y = 5 - \frac{4}{5}x$
c $y = -6 - 5x$
d $y = -3 + 3x$ e $y = -\frac{1}{2}x$
f $y = 5 - \frac{2}{3}x$

11 gradient = 42, C-intercept = 125

12 a $C = 5 + 2x$
b

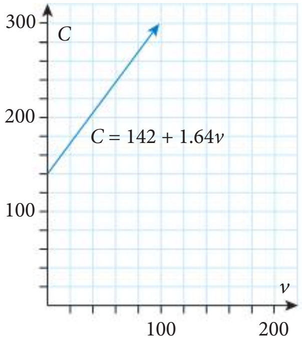


c 2; gradient represents the increase in fares for each kilometre travelled.
d 5; the vertical axis intercept represents the initial cost of the taxi (prior to being charged for any kilometres travelled).

13 a 1.1 m b After 8 hours.
c 0.6; the depth of the water in metres before filling the swimming pool.
d 0.075 m/h; the rate at which the depth is changing in metres per hour.
e $d = 0.6 + 0.075t$

14 a $C = 6 - \frac{1}{250}n$
b i \$4.80 per unit for 300 parts.
ii 1125 units will give a cost of \$1.50 per unit.

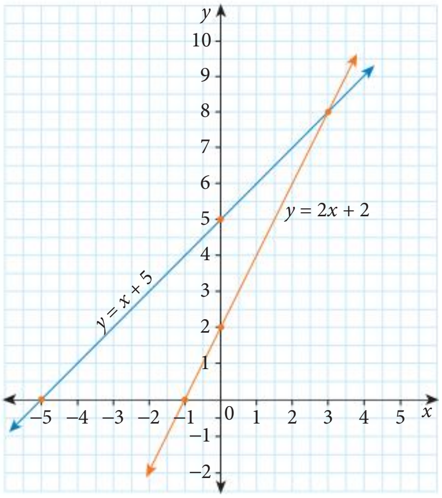
15 a Volume, v; the volume of water usage in kL.
b $C = 142 + 1.64v$
c



d The rate at which the cost of water usage is changing in volume per kL.
e The initial cost of the water usage.
f \$230.56 g 75 kL

12.02

1 a (2, 2) b (1, 5)
2 a



b $x = 3, y = 8$

3 a (3, 6) b (−5, −1) c $\left(-1\frac{1}{2}, -\frac{1}{2}\right)$
d (2, 5) e (0, −4) f (−1, 3)

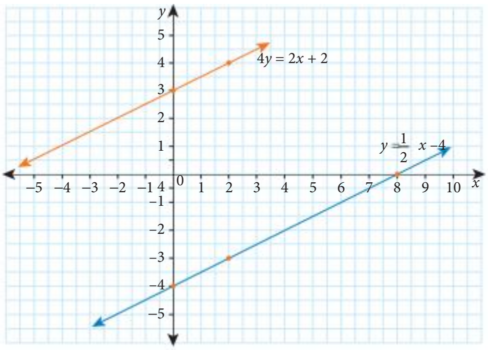
4 a B b C

5 a $x = -1, y = 2$ b $x = -2, y = 4$
c $x = 1, y = 1$ d $x = 2, y = 2$
e $x = -1, y = -1$ f $x = 2\frac{1}{2}, y = 5$

6 a 7 years b 3000

7 a (100, 20 000)
b The expenses and income are of equal value at the point of intersection. It shows the volume of sales needed for the company to break even.

8 **a** and **b**

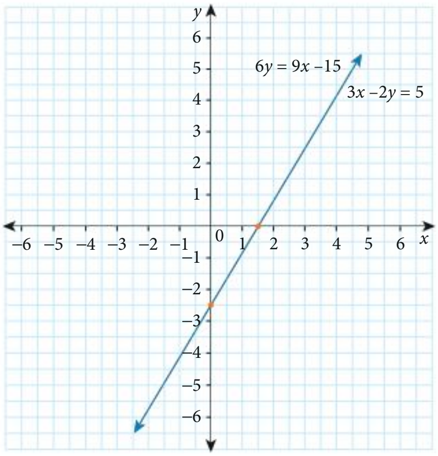


c The lines are parallel.

d There will be no point of intersection as parallel lines never meet.

e Write both equations in the form $y = a + bx$. If two lines have the same gradient (b value), they are parallel and will not intersect.

9 **a**



b Both equations give the same line graph.

c Answers may vary. Possible answers are:
Coincident lines have all points in common.
Coincident lines have an infinite number of solutions.

12.03

1 $(-7, -22)$

2 **a** $x = 4, y = 3$ **b** $x = 4, y = 5$ **c** $x = -2, y = -5$
d $x = -1, y = -3$ **e** $x = -2, y = 6$ **f** $x = 4, y = -4$
g $x = -6, y = 2$ **h** $x = 3, y = 7$

3 B

4 **a** $y = x + 2$ **b** $x = -6, y = -4$

5 **a** $x = 12, y = 15$ **b** $x = 7, y = 24$ **c** $x = 1, y = 4$
d $x = -1, y = -1$ **e** $x = 5, y = 6$ **f** $x = 3, y = 3$
g $x = -5, y = 15\frac{1}{2}$ **h** $x = 2,\ y = 6\frac{1}{2}$

6 $t = 26.4, d = 45.6$

7 **a** 25 people **b** \$300

8 **a** Error occurred when expanding the brackets after substituting equation [1] into equation [2].

b $y = x + 2 \quad [1]$

$5x - 2y = 5 \quad [2]$

Substitute[1]into[2].

$$5x - 2(x + 2) = 5$$

$$5x - 2x - 4 = 5$$

$$3x = 9$$

$$x = 3$$

Substitute $x = 3$ into [2].

$$y = 3 + 2$$

$$y = 5$$

Solution is (3, 5).

Solution is $x = 3, y = 5$.

9 A

10 $y = 1 + 2x$

11 **a** It is not possible to find one point of intersection, so there is no solution.

b The equations represent the same lines, hence they are coincidental and there are an infinite number of solutions.

12.04

1 $x = 2, y = -9$

2 **a** B **b** E

3 **a** $x = -3, y = 6$ **b** $x = 2, y = -3$
c $x = 2, y = 3$ **d** $x = -2, y = 3$
e $x = -2, y = -5$ **f** $x = 4\frac{2}{3}, y = 4$
g $x = -2, y = 3$ **h** $x = 1, y = 4$

4 **a** D **b** C

5 **a** $x = 1, y = -2$ **b** $x = 4, y = 1$
c $x = -2, y = 3$ **d** $x = 3, y = -4$
e $x = 5, y = -1$ **f** $x = \frac{1}{2}, y = 3$
g $x = 10, y = 3$ **h** $x = -1, y = -5$

6 **a** $x = 3, y = 2.5$ **b** $x = 2.5, y = 5$
c $x = 4, y = 1$ **d** $x = 1, y = 3$
e $x = -8, y = 6$ **f** $x = 5, y = -1$

7 **a** $n = -6.4, C = -384$

b The substitution method would be more appropriate when solving these simultaneous linear equations as the equations are written in the form $y = a + bx$.

8 Jack was correct with $x = 21\frac{1}{2}, y = 30$.

9 **a** $x = 7, y = 4$

b $x = 4, y = 3$

c $x = -6\frac{1}{4}, y = -17\frac{1}{6}$

12.05

1 a Yes, after 25 minutes.
  b Huong, by 10 minutes.

2 a Wattle tree; 1 metre b 30 months
  c 4 metres
  d i $h = \frac{2}{15}t$ ii $h = 0.1t + 1$
  e No, the trees may have stopped growing or slowed down before then.

3 D

4 B

5 $l = 19.5$ cm, $w = 6.5$ cm

6 46 and 47

7 22 and 17

8 20 and 16

9 Philippa is 8 years old; Petra is 32 years old.

10 a $x = 11$ cm, $y = 9$ cm
  b 25 cm by 10 cm
  c 70 cm

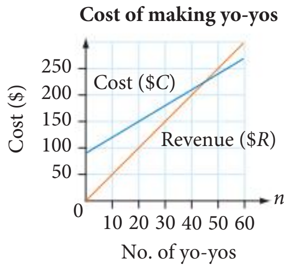
11 4 months; $20

12 250 single cones; 200 double cones

13 a QuickByte is cheaper. QuickByte costs $22.40 and Game Hunters costs $24.
  b 20 hours; $20

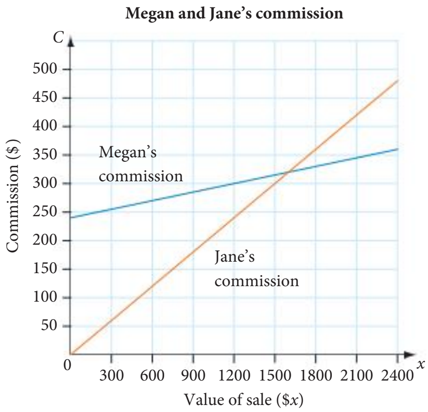
14 a i Shifty: $C = 1.85d + 135$
   ii Megahertz: $C = 1.55d + 240$
  b 350 km, $782.50
  c Shifty, as it is cheaper. Shifty charges $690 and Megahertz charges $705.

15 a



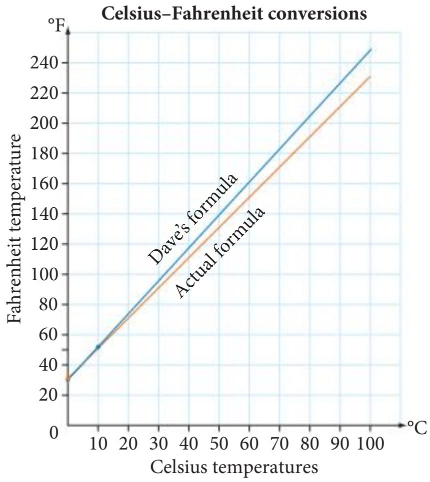
  b $150 c $100 d $50
  e $5 f 45

16 a



  b Megan c $1600 d $320
  e less than $1600

17 a



  b $C = 10$ c $C > 10$
  d From $C = 0$ to $C = 35$.

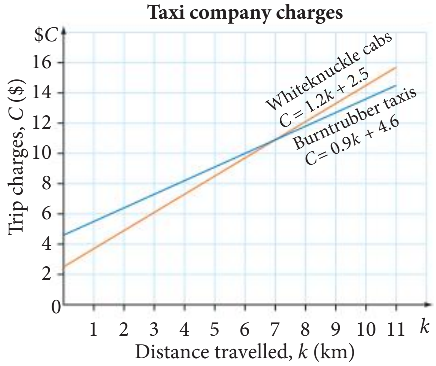
18 a $4c + 2d = 86$
  b $3c + 5d = 131$
  c CD: $12; DVD: $19

INVESTIGATION: TAXI CHARGES

a Burntrubber Taxis: $C = 4.6 + 0.9k$ ($4.60 plus 90 c/km)
 Whiteknuckle Cabs: $C = 2.5 + 1.2k$ ($2.50 plus $1.20/km)

b independent variable: k; dependent variable: C

c



d Burntrubber Taxis: 4.60; Whiteknuckle Cabs: 2.50

e i Burntrubber Taxis: $6.40
  Whiteknuckle Cabs: $4.90
 ii Burntrubber Taxis: $9.10
  Whiteknuckle Cabs: $8.50
 iii Burntrubber Taxis: $15.40
  Whiteknuckle Cabs: $16.90

f Use extrapolation. Extend the horizontal axis to 12, then extend the line to find the value of C when $k = 12$.

g (7, 10.9)

h The point where the cost and the kilometres travelled are the same for the two companies.

i Answers should be the same. Check with teacher.

j Whiteknuckle Cabs is cheaper for trips under 7 km, Burntrubber Taxis is cheaper for trips over 7 km.

k Burntrubber Taxis, $12

12.06

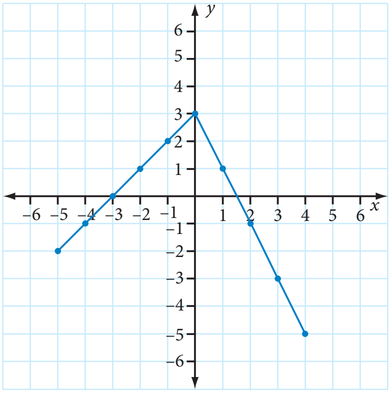
1 a $y = x + 3$

x	–5	–4	–3	–2	–1	0
y	–2	–1	0	1	2	3

b $y = 3 - 2x$

x	0	1	2	3	4
y	3	1	–1	–3	–5

c



2 a $y = -x$

x	–7	–6	–5	–4	–3
y	7	6	5	4	3

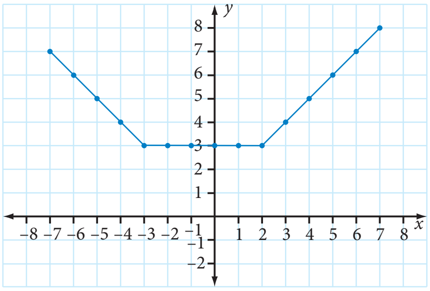
$y = 3$

x	–3	–2	–1	0	1	2
y	3	3	3	3	3	3

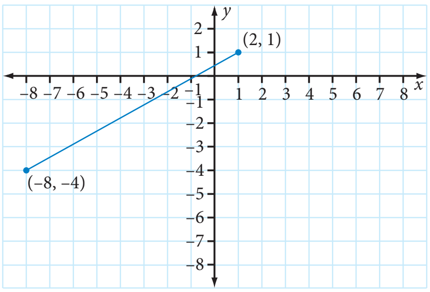
$y = x + 1$

x	2	3	4	5	6	7
y	3	4	5	6	7	8

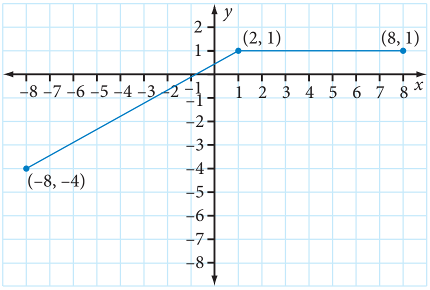
b



3 a



b



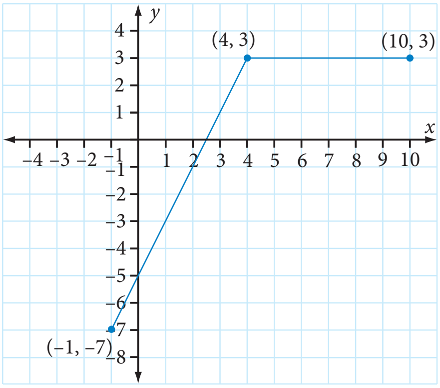
c The graph is made up of two different line segments. The first line segment has a positive gradient, hence the graph is increasing. The coordinates of its end points are (–8, –4) and (2, 1). The second line segment starts from the point where the first line segment ends, that is at (2, 1), and finishes at (8, 1). It is a horizontal line.

4 E

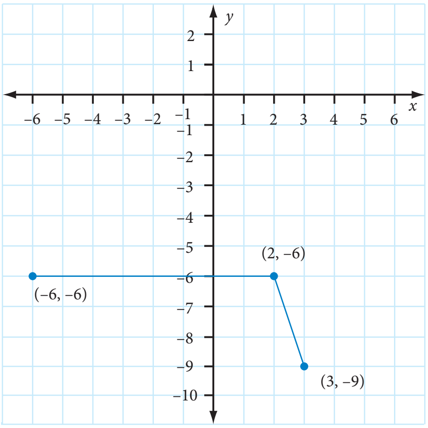
5 D

6 D

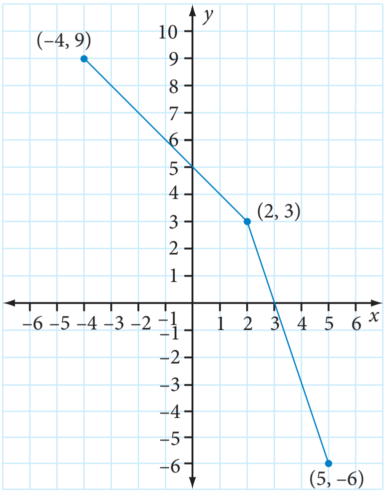
7 a



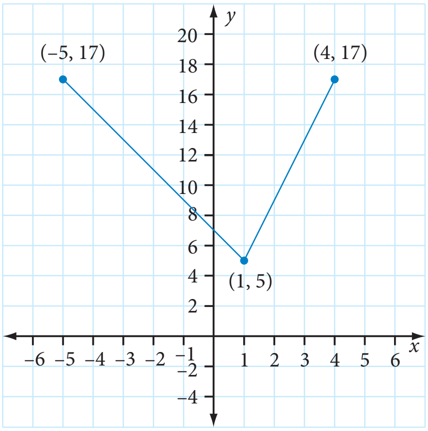
b



c



d



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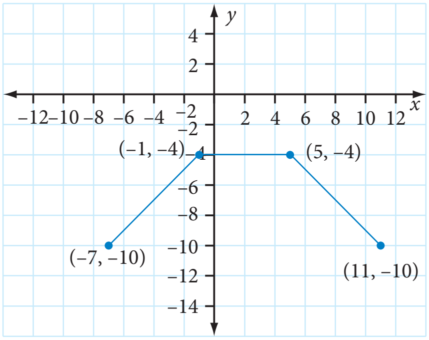

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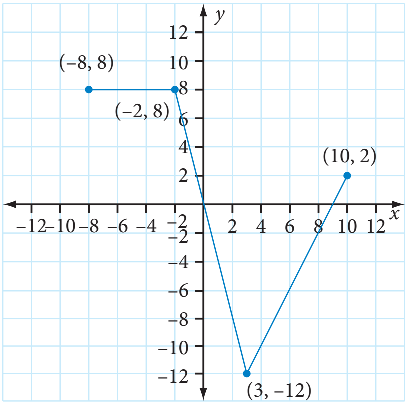


8 Refer to question **7**.

9 **a** line segment 1: $-5 \le x < 0$;
line segment 2: $0 \le x \le 7$

b $y = 1 + 3x$

c $y = 1 - x$

d $$y = \begin{cases} 1 + 3x, & -5 \le x < 0 \\ 1 - x, & 0 \le x \le 7 \end{cases}$$

10 **a** $y = 2x$

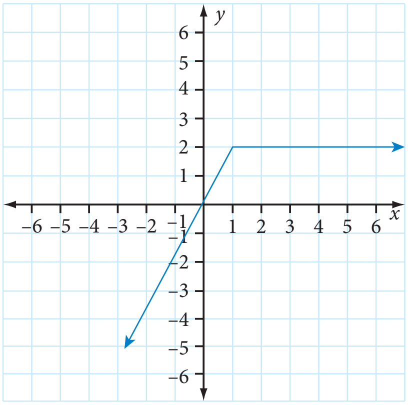
b All the values of x less than 1. It differs as before there were two definite endpoints for each segment.

c Draw a ray with an end point at (1, 2) and an arrow at the other end indicating that the line is moving towards infinity in the negative direction.

d $y = 2$

e All the values of x greater than or equal to 1.

f



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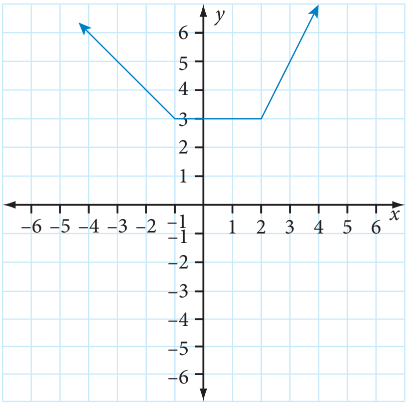


12 **a** The line segments do not meet.
b 3 line segments
c line segment 1: $-8 \le x < -3$;
line segment 2: $-3 \le x < 6$;
line segment 3: $6 \le x \le 7$
d Open circles indicate the points that are not included in the line segment and the closed circles indicate the points that are included in the line segment.

12.07

1 **a** 2 **b** 14c/kWh **c** higher
d **i** $120 **ii** $315 **iii** $280
e **i** 720 kWh **ii** 1560 kWh **iii** 2360 kWh

2 D

3 E

4 C

5 **a** 45 min
b Patty started her journey 2 km from her house.
c 4 km
d Patty began her journey 2 km from home, and spent 30 minutes walking 1.5 km. Then she walked for 15 minutes for another 0.5 km to reach Susan's house. She stayed there for 45 minutes. Then she travelled home on the bicycle, covering 4 km in 30 minutes.
e 3 km/h, 2 km/h, 0 km/h, 8 km/h

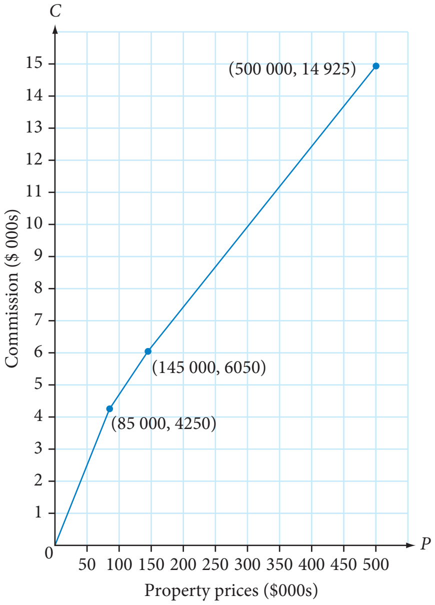
6 **a** No. The graph is made up of four different line segments and each line segment has a different gradient. Hence, the rain water tank is filling at four different rates.
b **i** 2000 L **ii** 24 500 L **iii** 45 500 L
c 240 hours
d After 10 hours and again after 100 and 200 hours.

7 **a** **i** $310 **ii** $75 **iii** $670
b $6800 **c** Gradient of the line increases.

8 **a**

Property price, $P	0	85 000	145 000	500 000
Commission, $C	0	4250	6050	14 925

b

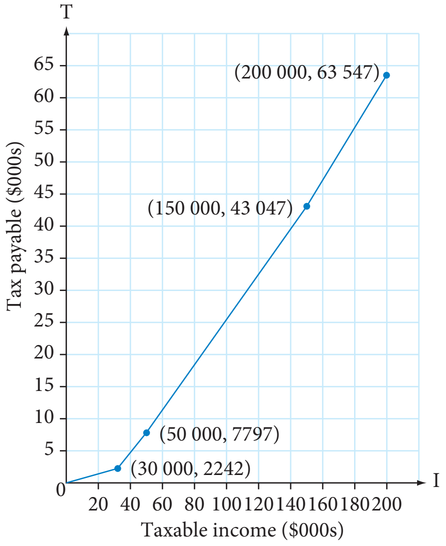


c decreases, rate of commission decreases
d **i** $7700 **ii** $10 700 **iii** $13 700

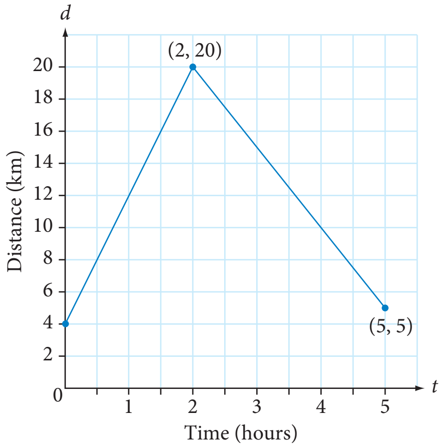
9 **a**

Taxable income ($*I*)	0	30 000	50 000	150 000	200 000
Tax payable ($*T*)	0	2242	7797	43 447	63 547

b

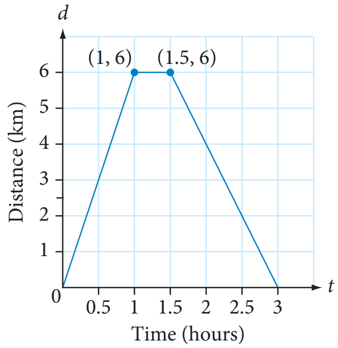


10 a



b 31 km

11 a



b The first line segment is the steepest, therefore Joseph is travelling the fastest during the first part of his journey.

c Gradient = 0. Joseph stopped for a rest.

d 12 km

INVESTIGATION: PIECEWISE LINEAR RULES IN BUSINESS

a seven line segments b $100 000

c $250 000

d i $200 000 ii $320 000

iii $315 000

e 5.5 years and 6.5 years

f i 7 years ii 3 years

g Between the second and third years and ninth and tenth years. There might have been a building boom.

h Between the sixth and ninth years. $300 000.

i Answers may vary. Possible comment is: There might have been a building slump due to an increase in interest rates.

j segment 1: 25 000; segment 2: 100 000;

segment 3: $\frac{200000}{3}$; segment 4: –50 000;

segment 5: –150 000; segment 6: –100 000;

segment 7: 100 000

k Answers may vary. Possible comment is: The company is likely to continue to increase its profits.

12.08

1 a Graph consists of horizontal line segments which look like steps on a staircase.

b Short horizontal line segments; open circle at one end and closed circle at the other end; one line starts where the other line finishes; not continuous.

2 a $6 b $3 c $16

3 a cost decreases, cheaper to buy in bulk, less packaging and handling

b i $4.00 ii $5.50

iii $4.00 iv $6.00

c 50

d Cost stays constant after 50 boxes, minimum cost is $3.50.

4 a i $9.05 ii $7.20 iii $5.35

b 3–<4 km c $3.50 d A

5 a payment to family/spouse if a person dies

b i $160 ii $275 iii $200

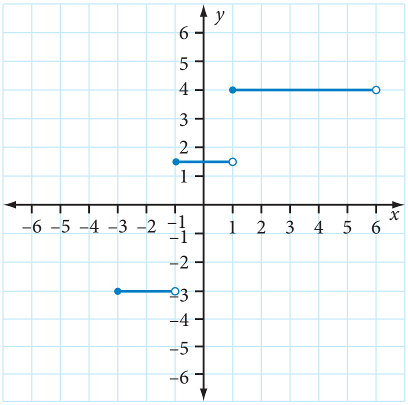
c least likely to die in that age group

d increases, more likely to die

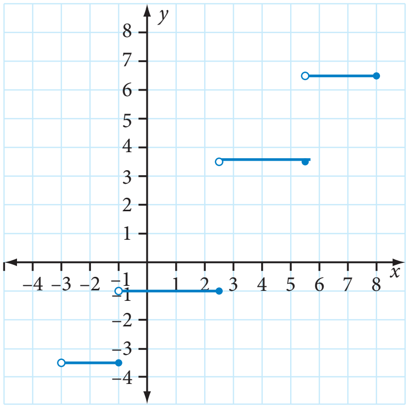
e 20 to <25 years

f i 40 years ii 50 years

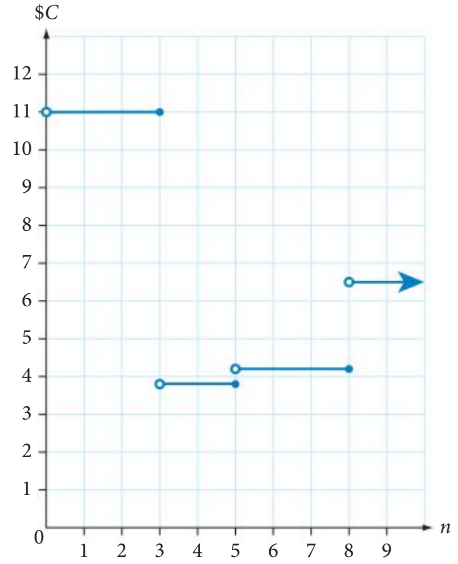
6 a



b



c



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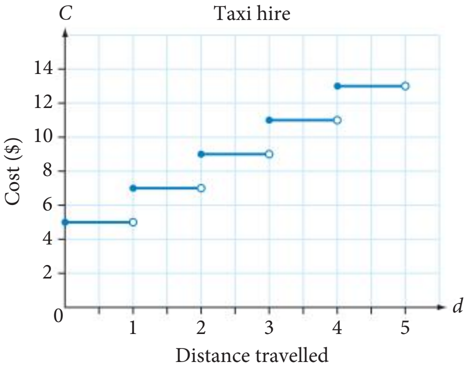

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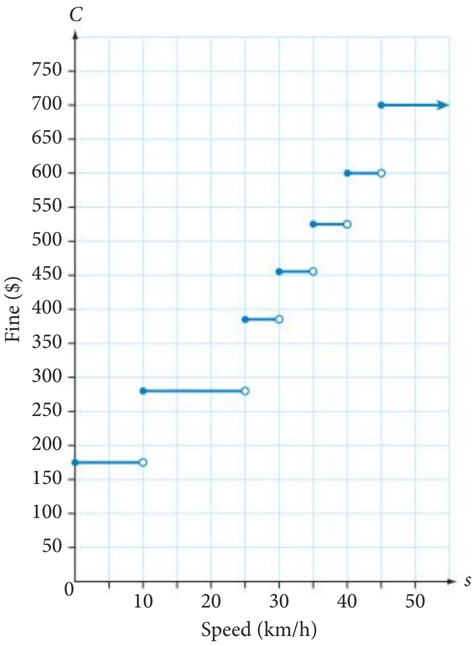

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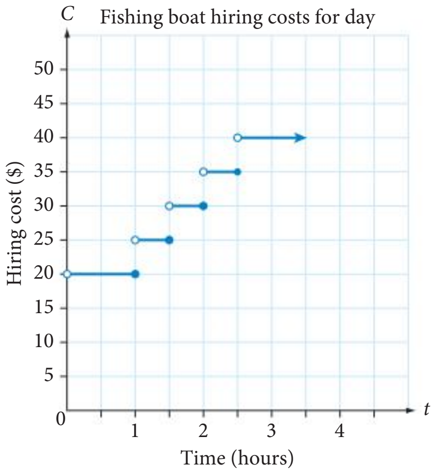
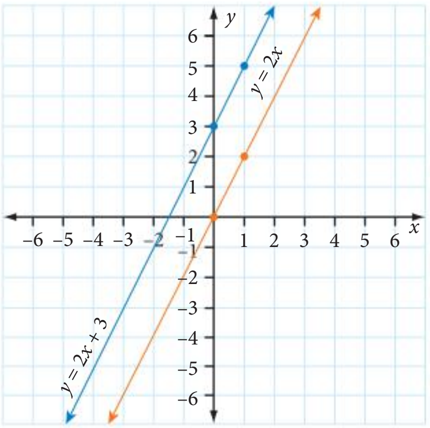


10 $d = 2$ has two values as the second and third line segments both have closed circles above it. The third and fourth line segments are overlapping.

11 Piecewise graphs are made up of line segments that have different gradients (including 0). The segments can also be joined together. Step graphs consist of only horizontal line segments.

CHAPTER 12 REVIEW

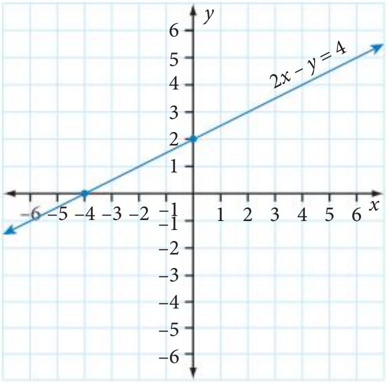
1 E
2 D
3 C
4 E
5 A
6 D
7 D
8 B
9 B
10 E
11 E
12 Gradient = $-\frac{3}{2}$; y-intercept = -5
13 $C = 10 + 75n$
14 a Gradient = 2 b $y = 2x - 5$
15 a $y = 3x - 3$ b $y = 7 - \frac{5}{2}x$
16 a



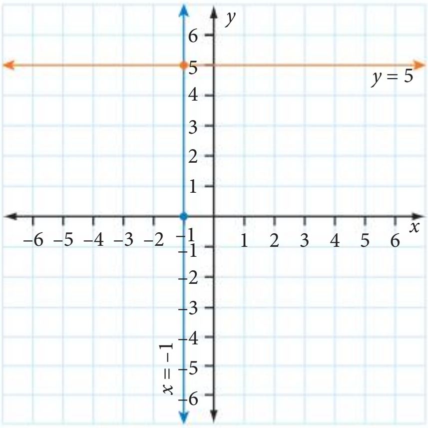
b The lines have the same slope. The lines are parallel.

17 **a** (2, 0) and (0, –4)

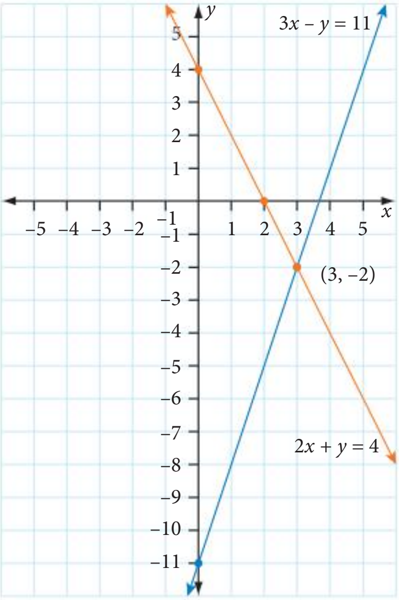
b



18

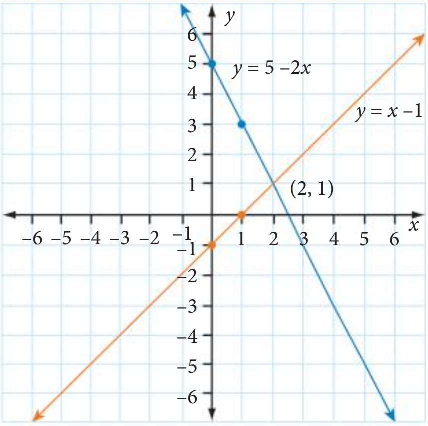


19 **a**



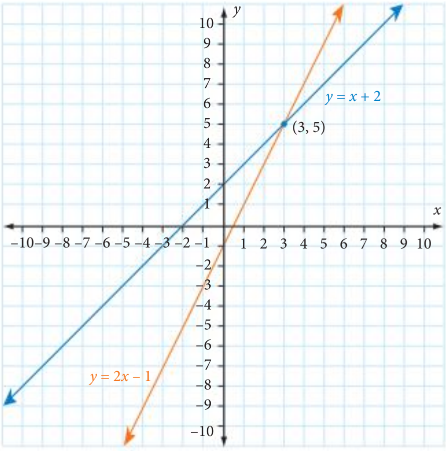
b (3, –2)

20 **a**



b $x = 2, y = 1$

21



Point of intersection = (3, 5)

22 $x = -2, y = 5$

23 **a** $y = -x - 6$ **b** $x = -3, y = -3$

24 **a** y should be eliminated.

b $x = 10, y = -\frac{1}{2}$

25 $x = -1, y = 2$

26 **a** 50 000 people

b 6 years; approximately 30 000 people

27 **a** $2l + 2w = 72$

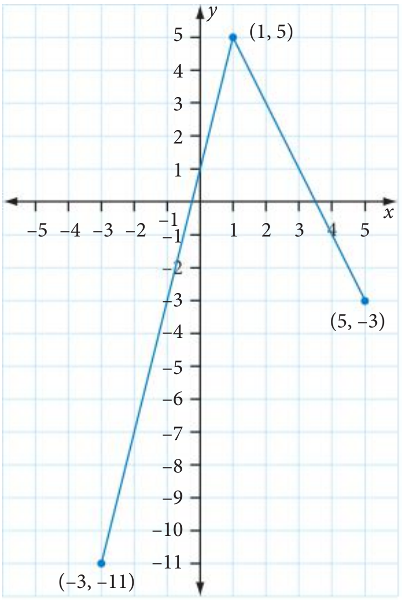
$l = w + 8$

where l = length and w = width of the rectangle

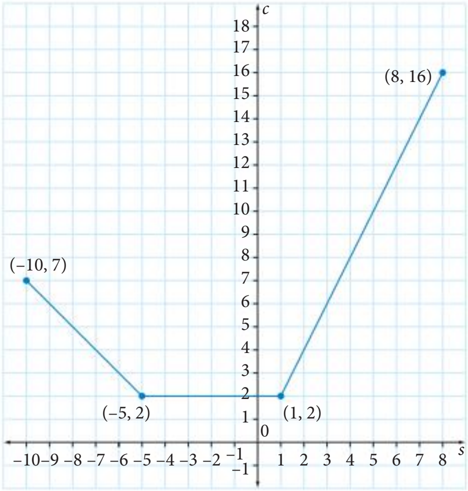
b $l = 22$ cm, $w = 14$ cm

28 $p = 6$ and $A = 90$.

29 **a** and **b**

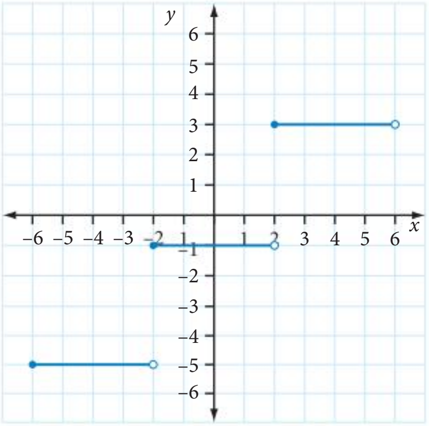


30



31 **a** $50 **b** $90 **c** $300

32



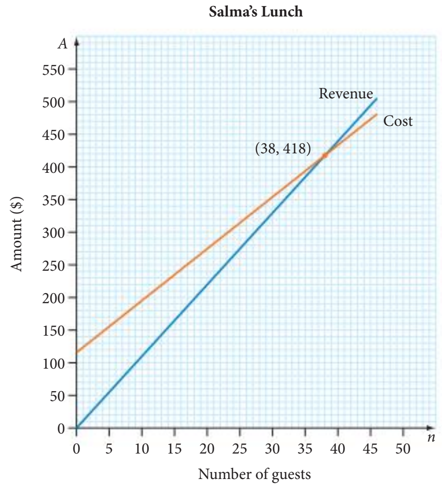
33 **a** $5w + 3s = 1880$

$3w + 5s = 1640$

b w = 280 kg

s = 160 kg

34 **a**



b cost $373; revenue $352 **c** 38

d It is the number of guests (38) at which Salma will stop incurring a loss because the revenue will equal the cost rather than be below it. For higher numbers of guests, Salma will make a profit.

e $42

35 **a** **i** $4.10 **ii** $0.20 **iii** $2.15

b 2 min

c Over 3 minutes and up to and including 3.5 minutes.

d 65c/30 s or 65c/ $\frac{1}{2}$ min

MIXED REVISION 4

Multiple choice

1 E
2 B
3 B
4 B
5 D
6 D
7 C
8 B
9 A

Short answer questions

1 44 m^2

2 **a** 18, the middle 50% of the women's ages has a range of 18 years.

b Women = 52.5 Men = 42.7. The average age of the women who attend the yoga class is higher than that of the men.

c **Women**

$18.5 \le x \le 90.5$, therefore there are no outliers as all values are inside this range.

Men

$0.5 \le x \le 84.5$, therefore there are no outliers as all values are inside this range.

3 a −2°C b 3°

c $T = 3t - 2$

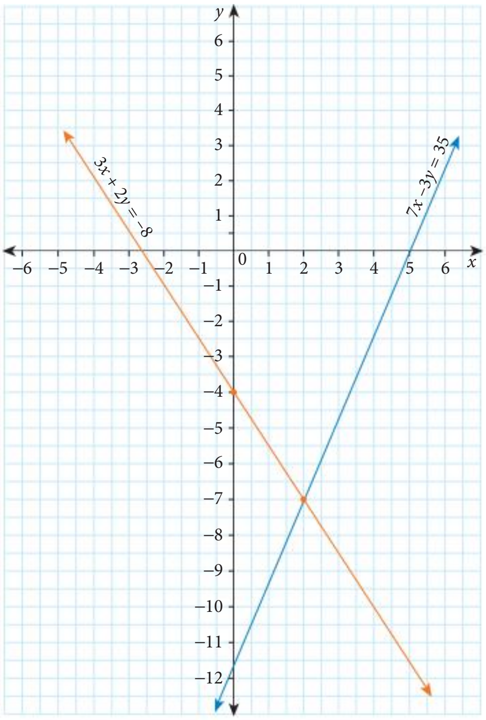
4 46°

5 a 63.28, 16.12 b Symmetrical

c Mr B's class as the average is higher.

d Mr B's class as the standard deviation is lower.

6 a



b (2, −7)

c The answers are the same using both methods.

Application questions

1 a 88.19 km b 241°

2 a Oak: Negatively skewed with no outliers.

Elm: Slightly positively skewed with no outliers.

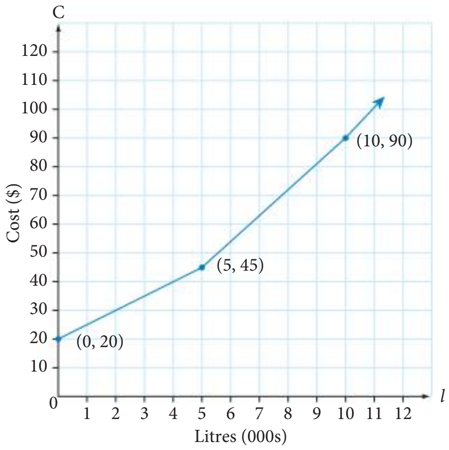
b Median and all measures of spread.

c You would need the original data, which a boxplot does not provide.

d 25% e IQR (the size of the box)

f In general, oak trees have a larger diameter than elm trees. This is supported by the larger median for oak trees. The data for oak trees also has a more consistent range for the middle 50% as shown by the lower IQR. The elm tree diameters have a larger range than that of oak trees.

3 a



b A graph which is comprised of three line segments, each with a different gradient.

c The end points of each line segment: (0, 20), (5, 45), (10, 90). They determine where each line segment starts and ends.

d Each line segment is steeper than the previous one. The greater the water usage, the greater the monthly water charge.

e i $37.50 ii $81 iii $115

GLOSSARY

Chapter 1

cost price The amount paid by someone who intends to sell an item.

currency The system of money used in a country.

currency exchange rate The cost of one currency in terms of another.

discount Amount an item is reduced by.

goods and services tax (GST) The tax a consumer pays on any purchased item or service (for example buying a car, or hiring a painter).

inflation The increase in the prices of goods and services over time.

loss When money is lost. The difference between selling price and cost price if selling price is lower.

mark-up An amount added to the cost price.

percent Out of 100; 29% means 29 out of 100 = $\frac{29}{100}$.

profit When money is made. The difference between selling price and cost price if selling price is higher.

rate A measurement that compares two different quantities.

retail price The amount that is asked for an item.

selling price The amount an item sells for.

unit cost What an item costs for one unit; could be 1 item or for 100 mL or for 1 kg, etc.

unitary method A way of calculating a value by first finding one unit then multiplying.

Chapter 2

algebra Involves the use of symbols or pronumerals to simplify expressions.

algebraic expression A collection of two or more terms involving pronumerals and numerals.

balanced When the same operation is performed on both sides of an equation.

coefficient The number which appears before a pronumeral within the same term.

constant A term which contains a number only, there is no pronumeral.

equation A mathematical sentence composed of algebraic expressions and numerals either side of an equals sign.

expanding To remove grouping symbols.

formula An algebraic rule which describes a mathematical relationship between pronumerals or variables.

highest common factor (HCF) The largest factor which is common to all terms.

like terms Are composed of the same pronumerals.

pronumeral A symbol or letter which takes the place of a number or a variable.

simplify To collect the like terms and then write as simply as possible.

solve To find the value of the variable in an equation.

substitution Where a pronumeral is replaced by a numerical value in order to evaluate an expression.

subject of a formula The pronumeral by itself on the left hand side of the equals sign.

term A number or a number and one or more pronumerals multiplied together.

variable A quantity which can represent different values.

Chapter 3

array A set of numbers arranged in a specific order.

column A vertical list of elements.

column matrix A matrix with only 1 column.

element Any value that is entered in a matrix.

identity matrix A square matrix in which the elements in the leading diagonal are all ones. The other elements are all zeros.

leading diagonal The diagonal in a square matrix which starts at the top left and finishes at the bottom right.

matrices Plural of matrix.

matrix An array of numbers arranged in rows and columns.

order The size of a matrix written as number of rows × number of columns.

row A horizontal list of elements.

row matrix A matrix with only 1 row.

scalar A real number.

square matrix A matrix with the same number of rows as it has columns.

zero matrix A matrix which has all of its elements equal to zero.

Chapter 4

allowance (worker's) Money paid to a worker for expenses incurred as part of his or her job (for example, for travel, for special clothing, or for working in isolated or dangerous areas).

annual leave loading Extra payment to a worker based on a percentage (usually 17.5%) of 4 weeks annual leave.

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

brokerage Commission paid to a stockbroker for buying or selling a parcel of shares for a client.

budget A plan for managing money.

commission The earnings of a sales person or agent; usually a percentage of the value of items sold.

dividend The share of profits in a company paid to each shareholder.

dividend yield The annual percentage return on the shares.

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

expenses The cost of spending.

face value The nominated price for shares when they are originally listed on the stock market.

government allowances Money paid by the government to support individuals for specific purposes (for example, to support the aged, unemployed, disabled, students and parents).

income Money that is earned or gained (usually regularly).

market price Current sale value of a share or item.

overtime Time worked beyond usual working hours, usually paid at a higher rate.

piecework Earnings based on the number of items processed, mode or delivered, paid at a rate per item (rather than an hourly rate).

price-to-earnings ratio Compares the market value with the earnings per share.

$$P/E = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

retainer A fixed amount paid to a salesperson before commission is added.

salary Fixed earnings quoted as a yearly amount, but paid weekly, fortnightly or monthly.

share (or stock) market Where the buying and selling of shares takes place.

shareholder A person who owns shares in a company.

shares (or stock) A portion of a company owned by a shareholder.

stock exchange See share market.

stockbroker An agent who buys and sells shares.

wage The amount earned by an employee for a set number of working hours, usually paid weekly or fortnightly.

Chapter 5

apex The point at which all of the triangular faces of a pyramid meet.

arc A section of a circle's perimeter.

arc length The length of a particular arc.

capacity The amount of cubic units which can be held within a three dimensional figure.

circumference The perimeter of a circle.

cone A three dimensional figure with a circular base and a curved surface which joins the base to an apex.

cross-section The two dimensional shape that we get when we cut a solid object parallel to its base.

cubic units Three dimensional units of measurement which are 1 unit × 1 unit × 1 unit.

cylinder A three dimensional figure which has equal circles at each end, joined by a curved surface.

frustum A three dimensional figure obtained by cutting the top off a cone parallel to its base.

hectare A metric unit of area which is equal to 10 000 square metres.

megalitre (ML) A measure of capacity which is equal to 1 million litres.

net A two dimensional figure which shows all of the faces of a solid and can be folded up to form that solid.

perpendicular height Measures the distance from the centre of the base to the top surface or apex of a solid.

polygon A two dimensional closed shape with three or more straight sides.

prism A solid with end faces and cross-sections that are identical polygons.

pyramid A solid with a polygon base that is joined by triangular side faces meeting at a point called the apex.

Pythagoras' theorem A mathematical theorem which is said to have been developed by a Greek mathematician called Pythagoras. It states that: in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

sector A part of a circle which is enclosed by two radii and the arc that is between them.

slant height Measures the distance from a point on the perimeter of the base to the apex of a pyramid or cone.

solid A three dimensional figure.

sphere A three dimensional figure which is round. It has only circular cross-sections. All of the points on its surface are the same distance from its centre.

surface area The sum of the areas of all of the faces of a solid.

Chapter 6

balance (of account) The amount of money remaining in the account.

compound interest Interest paid on the principal invested as well as any accumulated interest.

compounding periods How often interest is calculated, for example daily, monthly, quarterly and so on.

credit The amount of money put into an account.

credit card A card used to obtain goods and services where you pay for them later.

debit The amount of money taken out of an account.

deposit Putting money into an account.

flat-rate interest See simple interest.

flat-rate loan Loan where simple interest is charged on the amount borrowed for the term of the loan.

future value The total amount you end up with at the end of an investment.

instalment (repayment) Amount paid regularly to pay off a loan.

interest Money earned on an investment or paid on a loan.

minimum monthly balance The smallest amount of money that was in an account in a given month.

per annum (p.a.) Per year.

present value The total amount of the investment now.

principal The original amount of money invested or borrowed.

simple interest Also known as flat-rate interest. Interest earned or charged only on the original amount of money (principal) invested or borrowed.

transaction When money is put into or taken out of an account.

withdrawal When money is taken out of an account.

Chapter 7

corresponding This refers to sides or angles in similar shapes. Matching sides or angles are the same as corresponding sides or angles. The sides or angles are in the same relative position in each figure.

elevation The front, side or back view of a building.

enlargement This is a transformation which makes a figure larger without changing its shape.

hypotenuse The longest side of a right angle triangle.

orientation Shapes in the same orientation, are around the same way. It is the relative position of the shape.

plan A detailed drawing or diagram of an object. E.g. house plan.

ratio A comparison of two like quantities. E.g. the comparison of two lengths.

reduction This is a transformation which makes a figure smaller without changing its shape.

scale The scale of a drawing is a comparison of the length used on a drawing to the length it represents in real life.

scale drawing A drawing of an object showing all parts in the same ratio as their true size.

scale factor The ratio of any two matching lengths in two similar figures.

shadow stick A vertical stick or pole placed in the ground in order to measure its shadow, so that the height of other objects may be calculated.

similar figures Plane figures that are exactly the same shape but not necessarily the same size.

Chapter 8

bar graph Graphical display for categorical data with horizontal columns (not joined).

boxplot Graphical display for numerical data that shows the centre (median) and spread (range and IQR) of a data set.

categorical data Variables represented by qualities or characteristics that can be either nominal or ordinal.

column graph Graphical display for categorical data with vertical columns (not joined).

distribution of data Described as symmetrical, positively skewed or negatively skewed.

dot plot Graphical display for categorical or numerical discrete data.

five-number summary Five values required to construct a boxplot which are the minimum value, Q_1, median, Q_3 and maximum value.

frequency table Table used to organise large amounts of data.

histogram Graphical display for numerical data (discrete or continuous) with vertical joined columns.

interquartile range (IQR) Measure of spread calculated using IQR = $Q_3 - Q_1$.

mean The average of a data set and is calculated using $\bar{x} = \frac{\Sigma x}{n}$, and is considered a measure of centre.

median Middle value when a data set is in order from smallest to largest, and is considered a measure of centre.

mode Most common data value is a set, and is considered a measure of centre.

numerical data Variables represented by quantities or measurements that can be either discrete or continuous.

outlier By visual inspection, an extreme (high or low value) in the data or formally any value outside the range of $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$.

population All items/people of the group being studied.

range Measure of spread calculated using range = highest data value – lowest data value.

sample Portion of the population and is used when the population is large.

standard deviation Measures the spread of data about the mean and for a sample is calculated using $s_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$.

stem-and-leaf plot Graphical display for numerical data that is either discrete or continuous.

Chapter 9

balanced When the same operation is performed on both sides of an equation.

coordinates The x and y values of a point.

dependent variable A variable whose value depends on another variable, usually the y variable.

equation A mathematical sentence composed of algebraic expressions on either side of an equals sign.

general equation An equation of a straight line written in the form $y = a + bx$.

gradient The slope of a line calculated by vertical rise over horizontal run.

horizontal line A line parallel to the x-axis.

independent variable A variable which can take any value, not dependent on another variable, usually the x variable.

linear equation An equation with x to the power of 1 where its graph is a straight line.

linear modelling When a linear equation represents real life situations.

negative gradient Where the line slopes down from left to right.

positive gradient Where the line slopes up from left to right.

solve To find the value of the variable in an equation.

vertical line A line parallel to the y-axis.

x-intercept The point where the line crosses the x-axis.

y-intercept The point where the line crosses the y-axis.

Chapter 10

adjacent In a right-angled triangle, the side ‘next to’ the given angle, leading to the right angle.

angle of depression The angle between the horizontal and the line of sight, looking down.

angle of elevation The angle between the horizontal and the line of sight, looking up.

area of a triangle When the base and height are known: $A = \frac{1}{2}bh$.

When an included angle and two sides are known: $A = \frac{1}{2}ab \sin(C)$

compass bearing A direction measured as an angle depending on the quadrant. Compass bearings go from North or South in either an East or West direction.

cosine rule Used in non-right-angled triangles to find an unknown side or angle given two sides and an included angle or three sides: $a^2 = b^2 + c^2 - 2bc \cos(A)$.

Heron's formula Used to find the area of a triangle when three sides are known: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

hypotenuse The longest side in a right-angled-triangle.

included angle An angle where both of its surrounding sides are known.

non-included angle An angle where one of its surrounding sides is unknown.

opposite In a right-angled triangle, the side directly facing the given angle.

right-angled triangle A triangle which contains a 90° angle.

sine rule Used in non-right-angled triangles to find an unknown side or angle given either two sides and a non-included angle or one side and two angles: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$.

trigonometric ratio Used to find unknown sides and angles in right-angled triangles.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

true bearing A direction measured as an angle clockwise from due north, from 000° to 360°.

Chapter 11

back-to-back stem-and-leaf plot Graphical display used to compare numerical data for two different groups. Displays every data value.

comparing summary statistics When investigating two or more data sets comparisons between summary statistics can be made, for example the median is larger for data set 1.

interquartile range Describes the range of the middle 50% of data.

mean The average of the data being investigated.

median Tells you the value that 50% of the data lies above and below.

mode Describes the most common/frequent data value.

outlier Any value outside the range of $Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$.

parallel boxplots Graphical display used to compare the same numerical variable for two or more data sets.

range Describe the total spread of the data.

standard deviation Describes how the data deviates from the mean. A smaller standard deviation indicates more consistency in the data.

side-by-side column graph Graphical display used to compare two sets of categorical data so that the frequency of each can be compared over different periods.

Chapter 12

discontinuous A graph that is interrupted or has a break in it is said to be discontinuous.

elimination method A method for solving simultaneous linear equations where one variable is eliminated in order to solve the second variable and hence the simultaneous equations.

extrapolating The process of estimating the value of a quantity that is outside the given range of the values.

general form of a linear equation A linear equation which is written in the form $y = a + bx$.

gradient Slope of a line.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

interpolating The process of estimating the value of a quantity that is within the given range of values.

linear equation An equation of the form $y = a + bx$, whose graph is a straight line.

linear graph The graph of a linear function. The graph is in the form of a straight line.

parallel lines Lines that are the same distance apart and never meet. Parallel lines do not have a point of intersection.

piecewise graph The graph of a piecewise linear function. It is made up of separate lines or 'pieces'.

piecewise linear equation An equation comprised of two or more linear functions. Its graph is a piecewise graph.

point of intersection The point where two or more lines meet.

simultaneous linear equations A system of two or more linear equations which are solved at the same time.

step graph The graph of a stepwise linear function.

stepwise linear equation An equation whose graph is comprised of two or more horizontal line segments forming 'steps'.

substitution method A method for solving simultaneous linear equations where one equation is substituted into the other in order to eliminate one variable and solve for the other.

***x*-intercept** The value at which a straight line cuts the horizontal axis.

***y*-intercept** The value at which a straight line cuts the vertical axis.

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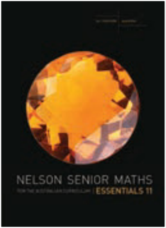

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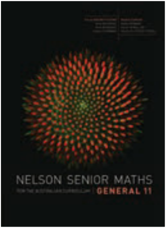

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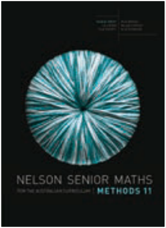

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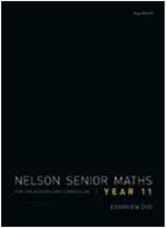

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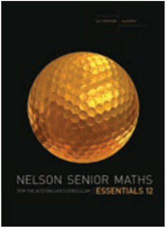

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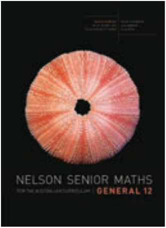

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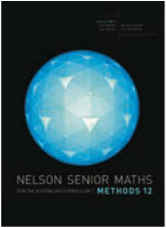

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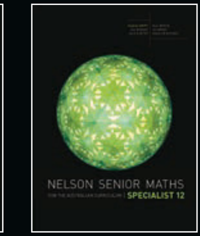

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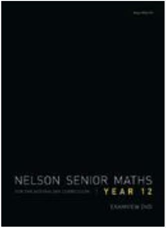

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