PEARSON

Mathematics

STUDENT BOOK | 3RD EDITION

8

AUSTRALIAN CURRICULUM

Pearson

Pearson Mathematics 8 3rd Edition

Student Book

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Contents

1 Number properties (exponents) 1

Recall 2

1.1 Establish and apply the exponent law for multiplication 3

1.2 Establish and apply the exponent law for division 7

1.3 Establish and apply the exponent law for raising a power to a power 12

1.4 Establish and apply the exponent law for raising to the power of 0 16

Topic review 19

2 Operations with integers, fractions and decimals 22

Recall 23

2.1 Multiply and divide integers 24

2.2 Solve problems by multiplying and dividing integers 29

2.3 Use the four operations with integers 34

2.4 Multiply and divide rational numbers 40

2.5 Apply all four operations with integers, fractions and decimals 44

Topic review 49

3 Rational and irrational numbers 52

Recall 53

3.1 Identify and use irrational numbers 54

3.2 Identify the golden ratio and make the connection between the circumference and diameter of any circle 58

3.3 Identify and use terminating, recurring and non-recurring decimals 64

Topic review 69

4 Percentage increase and decrease 74

Recall 75

4.1 Understand percentage increase and decrease 76

4.2 Understand percentage change in realistic situations 83

Topic review 90

5 Algebra (expand, simplify and factorise) 95

Recall 96

5.1 Use the distributive law to expand simple algebraic expressions 97

5.2 Factorise simple algebraic expressions 100

5.3 Simplify algebraic expressions involving multiple operations 104

Topic review 108

6 Linear graphs 112

Recall 113

6.1 Plot and identify a linear relationship using a set of points 114

6.2 Graph linear relationships with only one axis intercept 120

6.3 Investigate linear graphs using technology 126

6.4 Determine the rule for a linear relationship or pattern 134

6.5 Solve linear equations graphically 140

Topic review 146

7 Linear inequalities 153

Recall 154

7.1 Graph inequalities on number lines 155

7.2 Graph linear inequalities on the Cartesian plane 162

7.3 Solve linear inequalities 170

Topic review 175

Contents

8 Perimeter, area and volume 180

Recall 181
8.1 Estimate length and convert between units of length 182
8.2 Calculate the perimeter of quadrilaterals 188
8.3 Determine unknown side lengths from a given perimeter 196
8.4 Calculate the area of a trapezium 201
8.5 Calculate the area of kites and rhombuses 207
8.6 Calculate the area of composite shapes 212
8.7 Approximate the perimeter and area of irregular shapes 218
8.8 Understand the connection between volume and capacity 229
Topic review 238

9 Circles and sectors 245

Recall 246
9.1 Determine the area of a circle 247
9.2 Determine the area of a sector using common fractions 252
9.3 Determine sector area and arc length 256
9.4 Determine the area of composite shapes involving circles 261
9.5 Solve problems involving circle measurements 266
Topic review 271

10 Time 275

Recall 276
10.1 Understand 24-hour time 277
10.2 Understand the effect of daylight savings and longitude on local time 285
10.3 Understand and work with different time zones 291
Topic review 298

11 Rates 301

Recall 302
11.1 Understand and apply rates 303
11.2 Apply rates to financial situations 307
11.3 Apply rates to tax calculations 312
Topic review 316

12 Pythagoras' theorem in 2D 320

Recall 321
12.1 Measure the side lengths of a right-angled triangle 322
12.2 Understand and use Pythagoras' theorem to identify right-angled triangles 328
12.3 Compare different applications, demonstrations and proofs of Pythagoras' theorem 337
12.4 Use Pythagoras' theorem to determine the length of the hypotenuse 344
12.5 Use Pythagoras' theorem to determine the length of a shorter side in a right-angled triangle 349
Topic review 354

13 Ratios to solve problems 360

Recall 361
13.1 Solve problems involving proportional reasoning 362
13.2 Apply ratios to currency exchange 367
Topic review 371

Contents

14 Triangles and quadrilaterals (similarity and congruence) 375

Recall 376
14.1 Apply transformations to shapes in the plane 377
14.2 Prove congruence in pairs of triangles 384
14.3 Explore the properties of quadrilaterals using congruent triangles 393
14.4 Determine and apply scale factor and angle properties in similar shapes 401
14.5 Prove similarity in pairs of triangles 409
Topic review 416

15 Visualising 3D 427

Recall 428
15.1 Locate and describe the position of objects in 3D space 429
15.2 Use the 3D coordinate system 432
Topic review 439

16 Collecting and analysing data 441

Recall 442
16.1 Compare sampling methods 443
16.2 Understand sampling techniques and data sources 449
16.3 Understand and interpret statistics from different samples in the same population 460
16.4 Plan and conduct a statistical investigation 472
Topic review 484

17 Probability (Venn diagrams and two-way tables) 489

Recall 490
17.1 Understand complementary events 491
17.2 Draw and understand Venn diagrams 499
17.3 Draw and understand two-way tables 511
17.4 Recognise and understand mutually exclusive and non-mutually exclusive events 522
17.5 Use Venn diagrams and two-way tables to solve problems 531
Topic review 539

Glossary 544
Answers 561

How to use Pearson Mathematics

The components of Pearson Mathematics

This *Student Book* is a complementary resource that offers a print medium for corresponding lessons in *Pearson Secondary Teaching Hub* and the *Student Companion*. It is designed on the principles of current best practice and provides students with an offline version to access the theory, worked examples and practice questions from the digital lessons in *Hub*, to give the perfect blend of online and offline learning.

The *Student Book* (also available as an eBook) gives you an offline version of the theory, worked examples and independent practice questions. The *Student Book* has been supplied to minimise digital distraction and screen fatigue, supporting teachers and students to find the perfect balance of online and offline learning.

The *Hub* is a one-of-a-kind digital product designed to simplify teaching and energise learning, with high-quality content created by experienced Australian educators who know how to engage students.

The *Student Companion* is a write-in workbook that has been designed to support the guided practice phase of the lesson, giving students scaffolded instruction as an introduction to the skill. The *Student Companion* also supports the creation of a bound reference or portfolio of learning over the year.

Chunking with learning intentions and success criteria

The structure and relationship between each of the three components in the Pearson Mathematics series (*Hub*, *Student Book* and *Student Companion*) has been designed to manage the cognitive load of our students. Chunking has been used to break down complex concepts and strategies into smaller, more manageable sections, defined by the success criteria.

In the *Hub* and the *Student Companion*, the chunking of content directly supports success criteria, while the *Student Book* is chunked by learning intention, to give both teachers and students autonomy in delivery.

Learning intentions are statements aligned to the relevant curriculum. Each of the lessons in the *Hub* have been developed around a single purpose for learning. The learning intentions are communicated using student-friendly language and share the learning goals or objectives from the corresponding digital lesson in the *Hub* and pages of the *Student Companion*. They describe what learners should know, understand or be able to do by the end of the lesson.

Every lesson starts with a **Lesson warm-up**, a 3- to 5-minute activity designed to activate prior knowledge and prime students for learning. In the *Hub*, teachers are able to access teaching notes that contain sample solutions, different ways to deliver the activity, and enabling and extending prompts to ensure all learners have an entry point and can be extended.

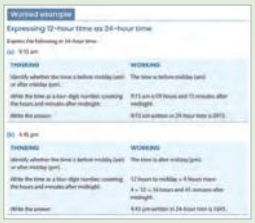
Success criteria clarify expectations and describe what success looks like. They are used to determine how well a student has met the learning intention. The Pearson Mathematics lesson design is based on cognitive load principles, and, as such, each lesson has between one and four success criteria. The success criteria are specific, concrete and measurable, so learners can actively engage with and reflect on their evidence of learning within each lesson. The success criteria form the basis of all three components — the *Hub*, the *Student Book* (and eBook) and the *Student Companion* — to ensure they are referred to throughout the lesson and provide the basis for feedback.

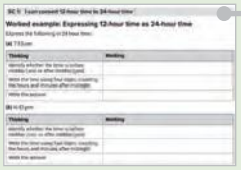
Reducing cognitive load with worked examples

Worked examples provide students with a step-by-step solution to a problem.

The worked examples in the *Student Book* correspond to those in the digital lesson, and are provided for each skill to:

- scaffold learning
- support skill acquisition
- reduce the cognitive load.





In the *Hub*, each worked example is presented as a video walkthrough, and has a 'Check your understanding' section with one to three autocorrecting questions. This gives students and teachers immediate feedback on student understanding of the required skills to succeed, before moving on to the independent practice phase of the lesson.

The *Student Companion* supports the gradual release of responsibility, supporting the guided practice phase of the lesson, by giving students access to a similar problem presented as a 'try yourself' worked example and one to three additional scaffolded practice questions. The *Student Companion* also provides students with the opportunity to develop a learning portfolio, and can be used in a flipped environment, in classroom learning, as homework, or in preparation to create a bound reference or summary.

Gradually releasing responsibility with print and digital

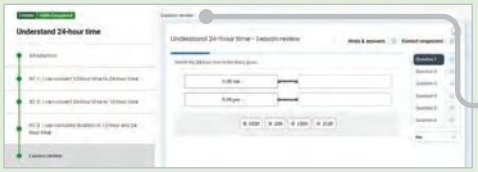
I do: Students and teachers are supported by new information being demonstrated and modelled with worked examples and worked example videos. The 'Check your understanding' questions provide immediate feedback to both students and teachers on the student's level of understanding of each new concept.

We do: The *Student Companion* offers a 'try yourself' version of the worked example and a small set of scaffolded questions designed to facilitate the guided practice phase of the lesson.

You do: The success criteria each have a set of 'Practice' questions available to access in both the *Hub* and the *Student Book*, and are designed to be used in the independent practice phase of the lesson.

Each lesson in the *Student Companion* contains a space for students to reflect on their understanding. The simple and intuitive design of the **lesson reflection tool** allows students to scale their confidence, reflect on their learning, and identify areas in which they need additional support.

The **Practice** section in the *Student Book* contains the independent practice questions for the lesson, still broken down by success criteria. These questions are provided in both the *Hub* and the *Student Book* to give teachers and students the choice of digital or print for the independent practice phase of the lesson. These questions are designed to ensure learners build confidence and demonstrate proficiency.



Each lesson in the *Hub* contains an additional six to eight auto-correcting questions. These questions provide a summative assessment of the lesson and utilise the skills and knowledge obtained throughout the entire lesson. These assessments give students and teachers immediate feedback on student readiness to move forwards in the topic sequence.

Pearson Mathematics 3E

Simplified teaching, energised learning for the new Australian Curriculum Mathematics Year levels 7–10

Powered by the Pearson Secondary Teaching Hub

How have we simplified teaching?

In consultation with Australian teachers and students, we've built a simple-to-use hub of lesson content for the new Australian Curriculum Mathematics.

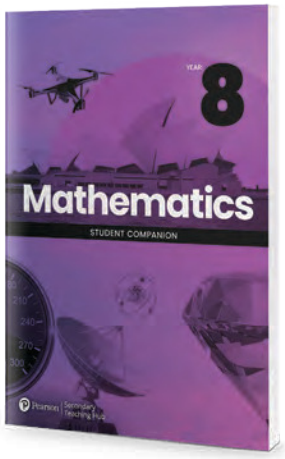
How have we energised learning?

Engaging videos, worksheets, auto-correcting questions, worked examples and more, with the perfect mix of online and offline resources.

Student Books

Teaching Hub with eBooks

Student Companions



Student Books link to the *Hub* for flexible teaching

Giving you the autonomy to teach online or offline to meet the needs of your students.

- Developed using the latest teaching and learning research.
- Access to all 7-10 content in the *Hub* for students that need revision, or those that need extension.
- eBook versions available in the *Hub*.

Student Companions provide write-in practice books

Accompanying the Teaching Hub, with 'try yourself' worked examples and exercise questions to support guided practice.

- 1000s of practice questions across the series
- Use as a homework activity program, guided practice in class, or to support multiple exposures.

Discover Pearson Mathematics at **pearson.com.au/schools/maths-ac**

TOPIC

1

Number properties (exponents)

Recall 2

1.1 Establish and apply the exponent law for multiplication 3

1.2 Establish and apply the exponent law for division 7

1.3 Establish and apply the exponent law for raising a power to a power 12

1.4 Establish and apply the exponent law for raising to the power of 0 16

Topic review 19

Why learn this?

Understanding exponent laws for multiplication is useful for efficiently handling very large and very small numbers in calculations. At the end of this topic students will be able to:

- convert repeated multiplications into a simpler exponent form
- apply the laws of exponents to multiply powers with the same base, as in algebra and higher mathematics.

These skills streamline computations and are widely used in science, economics and finance.

RECALL

ANSWERS Page 561

I can calculate squares and square roots

1 Write the square root as the product of a number multiplied by itself and then simplify.

As an example: $\sqrt{16} = \sqrt{(4 \times 4)} = 4$

(a) $\sqrt{25}$ **(b)** $\sqrt{64}$ **(c)** $\sqrt{81}$ **(d)** $\sqrt{121}$

2 Calculate the value of each of the following square roots.

(a) $\sqrt{100}$ **(b)** $\sqrt{196}$ **(c)** $\sqrt{49}$ **(d)** $\sqrt{169}$

(e) $\sqrt{36}$ **(f)** $\sqrt{256}$ **(g)** $\sqrt{225}$

I can use exponent notation to represent numbers

1 Write each of the following in exponent form.

(a) 4 squared **(b)** 10 cubed **(c)** $3 \times 3 \times 3$ **(d)** two to the power of five

(e) $5 \times 5 \times 5 \times 5$ **(f)** seven squared **(g)** $8 \times 8 \times 8 \times 8 \times 8$

I can use expanded notation to represent numbers

1 Write the following in expanded form.

(a) 5^2 **(b)** 6^4 **(c)** 8^5

2 Write the following in expanded form.

(a) $15^3 =$ _____ **(b)** $9^4 =$ _____

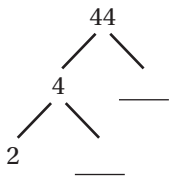
3 Write in expanded form, then evaluate.

(a) 3^4 **(b)** 6^3 **(c)** 10^4

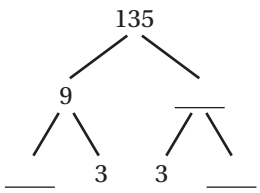
I can represent numbers in prime factor form

1 Complete each factor tree, then express the number as a product of its prime factors in exponent form.

(a)



(b)



Establish and apply the exponent law for multiplication

1.1

Learning intention: To be able to establish and apply the exponent law for multiplication

Success criteria:

SC 1 I can write the expanded form of a multiplication from exponent form and connect the result to the addition of exponents.

SC 2 I can multiply numbers using exponent notation.

Lesson warm-up

Which is bigger?

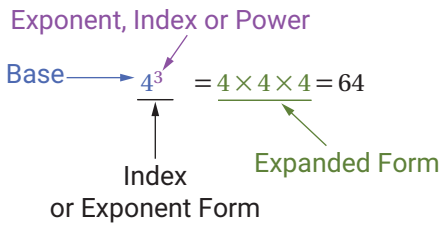
Which is bigger, a^2 or 2^a?

1 Is your answer true for all values of a?

2 Copy and complete the table of values.

a	a^2	2^a
−2		
−1		
−0.5		
0		
0.5		
1		
2		

SC 1 I can write the expanded form of a multiplication from exponent form and connect the result to the addition of exponents



When writing the expanded form in exponent form, the bases must be the same.

For example, $2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$

Worked example

Multiplying in expanded form

Multiply $4^3 \times 4^2$. Write your answer in exponent form.

THINKING	WORKING
Write the terms in expanded form.	$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$
Write the new expression in exponent form.	$= 4 \times 4 \times 4 \times 4 \times 4$ $= 4^5$
Write the answer.	$4^3 \times 4^2 = 4^5$

SC 2 I can multiply numbers using exponent notation

To multiply expressions written in exponent notation with the same base, add the powers.

$x^a \times x^b = x^{a+b}$

Worked example

Applying the first exponent law

Write the following in simplest exponent form.

(a) $7^5 \times 7^2$

THINKING	WORKING
Recall the first exponent law.	$x^a \times x^b = x^{a+b}$
To multiply exponent expressions with the same base, add the powers.	$7^5 \times 7^2 = 7^{(5+2)}$ $= 7^7$
Write the answer.	$7^5 \times 7^2 = 7^7$

(b) $7 \times 7^6 \times 5^2 \times 5^4$

THINKING	WORKING
Recall the first exponent law.	$x^a \times x^b = x^{a+b}$
To multiply exponent expressions with the same base, add the powers. In this case, there are two different bases.	$7 \times 7^6 \times 5^2 \times 5^4 = 7^{(1+6)} \times 5^{(2+4)}$ $= 7^7 \times 5^6$
Write the answer.	$7 \times 7^6 \times 5^2 \times 5^4 = 7^7 \times 5^6$

Practice

ANSWERS Page 561

SC 1 I can write the expanded form of a multiplication from exponent form and connect the result to the addition of exponents

1 Write the following numbers in expanded form.

(a) 3^1 **(b)** 3^2 **(c)** 3^3 **(d)** 3^4

2 Write the following numbers in exponent form.

(a) 5 **(b)** 5×5 **(c)** $5 \times 5 \times 5$ **(d)** $5 \times 5 \times 5 \times 5$

3 Explain how the expanded form and exponent form are linked.

4 Write the following in both expanded form and exponent form.

(a) 3×3^2 **(b)** $3 \times 3^2 \times 3^3$ **(c)** $3 \times 3^2 \times 3^3 \times 3^4$

5 Write the following in both expanded form and exponent form. Use your calculator to check that both forms are the same.

(a) 4×4^2 **(b)** $2 \times 2^3 \times 2^3$ **(c)** $5^2 \times 5^1 \times 5^3$

6 Write the following in both expanded and exponent form. Use your calculator to check that both forms are the same.

(a) $2 \times 3 \times 3^2$ **(b)** $2 \times 2^2 \times 3 \times 3^2$ **(c)** $2 \times 2^2 \times 2^3 \times 3 \times 3^2$

7 Each of the following is incorrect. Explain each mistake that has been made.

(a) $2 \times 2 \times 2 = 6$ **(b)** $2 \times 2^3 = 2^3$

(c) $2 \times 3^2 = 6^2$ **(d)** $2 \times 2^2 \times 3^2 = 2^2 \times 3^2$

8 Consider the following, written in both expanded form and exponent form.

Problem	Expanded form	Exponent form
$5^1 \times 5^2$	$5 \times (5 \times 5)$	5^3
$5^1 \times 5^2 \times 5^3$	$5 \times (5 \times 5) \times (5 \times 5 \times 5)$	5^6
$5^1 \times 5^2 \times 5^3 \times 5^4$	$5 \times (5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5)$	5^{10}

(a) Can you see the pattern of what to do with the powers when you multiply numbers with the same base?

(b) Without writing in expanded form first, express these in simplest exponent form.

(i) $2^2 \times 2^3$ **(ii)** $2^2 \times 2^4$

(iii) $2^2 \times 2^5$ **(iv)** $2^2 \times 2^6$

9 The first exponent law is $x^a \times x^b = x^{(a+b)}$.

(a) Explain the first law in your own words. Include an example.

(b) Does the first law work for $2^2 \times 3^4$? Explain why or why not.

SC 2 I can multiply numbers using exponent notation

1 Use the first exponent law to write the following in simplest exponent form.

(a) 5×5^2 **(b)** 5×5^3 **(c)** $5^2 \times 5^3$ **(d)** $5^2 \times 5^3 \times 5^4$

2 Write the following in simplest exponent form.

(a) $8^3 \times 8^2$ **(b)** $6 \times 6^2 \times 6^3$ **(c)** $3^2 \times 3^3 \times 3^4$

(d) $7^3 \times 7^4 \times 7^5$ **(e)** $9^3 \times 9 \times 9^5$

3 Write the following in simplest exponent form.

(a) $5 \times 5^2 \times 3 \times 3^2$ **(b)** $5^3 \times 3^3 \times 3^4$ **(c)** $2^2 \times 2^3 \times 2^3 \times 3^2 \times 2^3$

(d) $2^4 \times 2^4 \times 2^4 \times 3^4$ **(e)** $2^2 \times 2^3 \times 5^3 \times 5^2 \times 5^4$ **(f)** $3^2 \times 10^3 \times 3^3 \times 10^2$

(g) $3^2 \times 4^3 \times 4^3 \times 5^2 \times 5^3$

4 Each of the following is incorrect. Explain each mistake that has been made.

(a) $2 \times 2^2 \times 2^3 = 2^5$ **(b)** $2 \times 2^2 \times 2^3 = 8^6$

(c) $2 \times 2^2 \times 2^3 = 6^6$ **(d)** $3^3 \times 2^2 \times 2^3 = 3^3 \times 4^5$

5 Use a calculator to determine the value of:

(a) $(-1)^2$ **(b)** $(-1)^3$ **(c)** $(-1)^4$ **(d)** $(-1)^5$

(e) Without calculating, state the value of $(-1)^{99}$. How can you be sure?

6 Express the following in simplest exponent form.

(a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ **(b)** $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

(c) $2.1 \times 2.1 \times 2.1 \times 2.1$ **(d)** $0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8$

7 **(a)** Copy and complete this table of values.

Fraction	Squared	Cubed	Power of 4
$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2 =$	$\left(\frac{1}{2}\right)^3 =$	$\left(\frac{1}{2}\right)^4 =$
$\frac{1}{4}$	$\left(\frac{1}{4}\right)^2 =$	$\left(\frac{1}{4}\right)^3 =$	$\left(\frac{1}{4}\right)^4 =$

(b) Explain why the number decreases as the power increases.

8 Does $\left(\frac{1}{2}\right)^2 = \frac{1^2}{2}$? Explain why or why not.

Establish and apply the exponent law for division

1.2

Learning intention: To establish and apply the exponent law for division

Success criteria:

SC 1 I can write the expanded form of a division from exponent form and connect the result to the subtraction of exponents.

SC 2 I can divide numbers using exponent notation.

SC 3 I can apply the multiplication and division rules, or a combination of both, to simplify an expression.

Lesson warm-up

How do you divide?

How many different ways can you draw $12 \div 3 = 4$?

SC 1 I can write the expanded form of a division from exponent form and connect the result to the subtraction of exponents

Worked example

Dividing numbers written in exponent form

Express $\frac{4^5}{4^2}$ in simplest exponent form.

THINKING	WORKING
Write the numerator and denominator in expanded form.	$\frac{4^5}{4^2} = \frac{4\times4\times4\times4\times4}{4\times4}$
Recall that any number divided by itself is equal to 1. Cancel common factors.	$= \frac{4\times4\times4\times\cancel{4}\times\cancel{4}}{\cancel{4}\times\cancel{4}}$
Express the result using exponential notation.	$= 4^3$
Write the answer.	$\frac{4^5}{4^2} = 4^3$

SC 2 I can divide numbers using exponent notation

To divide expressions written in exponent notation with the same base, subtract the powers.

$x^a \div x^b = x^{a-b}$

Worked example

1

Applying the second exponent law

Write the following in simplest exponent form.

(a) $10^5 \div 10^2$

THINKING	WORKING
Recall the second exponent law.	$\frac{x^a}{x^b} = x^{(a-b)}$
To divide exponent expressions with the same base, subtract the powers.	$10^5 \div 10^2 = 10^{(5-2)}$ $= 10^3$
Write the answer.	$10^5 \div 10^2 = 10^3$

(b) $\frac{2^5 \times 3^8}{2^2 \times 3^4}$

THINKING	WORKING
Recall the second exponent law.	$\frac{x^a}{x^b} = x^{(a-b)}$
To divide exponent expressions with the same base, subtract the powers. In this case, there are two different bases.	$\frac{2^5 \times 3^8}{2^2 \times 3^4} = 2^{(5-2)} \times 3^{(8-4)}$ $= 2^3 \times 3^4$
Write the answer.	$\frac{2^5 \times 3^8}{2^2 \times 3^4} = 2^3 \times 3^4$

SC 3 I can apply the multiplication and division rules, or a combination of both, to simplify an expression

Worked example

Applying the first two exponent laws

Simplify $\frac{8^3 \times 8^8}{8^5}$

THINKING	WORKING
Recall the first two exponent laws.	Law 1: $x^a \times x^b = x^{a+b}$ Law 2: $x^a \div x^b = x^{(a-b)}$
Use the first exponent law to simplify the multiplication of terms with the same base.	$\frac{8^3 \times 8^8}{8^5} = \frac{8^{(3+8)}}{8^5}$ $= \frac{8^{11}}{8^5}$
Use the second exponent law to simplify the division of terms with the same base.	$= 8^{(11-5)}$ $= 8^6$
Write the answer.	$\frac{8^3 \times 8^8}{8^5} = 8^6$

Practice

ANSWERS Page 562

SC 1 I can write the expanded form of a division from exponent form and connect the result to the subtraction of exponents

1 Write the following numbers in expanded notation and calculate the answer.

(a) 4^1 **(b)** 4^2 **(c)** 4^3 **(d)** 4^4 **(e)** 4^5

2 Write the following in expanded notation. Do not simplify.

(a) $\frac{4^2}{4}$ **(b)** $\frac{4^3}{4}$ **(c)** $\frac{4^3}{4^2}$ **(d)** $\frac{4^4}{4^2}$

(e) $\frac{4^5}{4^2}$ **(f)** $\frac{4^4}{4^3}$ **(g)** $\frac{4^5}{4^3}$

3 Write the following in exponent form.

(a) $6 \times 6 \times 6$ **(b)** $6 \times 6 \times 6 \times 6$ **(c)** $6 \times 6 \times 6 \times 6 \times 6$

(d) $8 \times 8 \times 8$ **(e)** $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ **(f)** $106.3 \times 106.3 \times 106.3$

(g) $9 \times 9 \times 9 \times 9$ **(h)** $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ **(i)** $2.8 \times 2.8 \times 2.8 \times 2.8$

4 Write the following in expanded form, then express your answer in simplest exponent form.

(a) $\frac{7^2}{7}$ **(b)** $\frac{7^3}{7}$ **(c)** $\frac{7^3}{7^2}$

(d) $\frac{7^4}{7^2}$ **(e)** $\frac{7^5}{7^2}$ **(f)** $\frac{7^5}{7^3}$

5 Copy and complete the following table.

(a)

Expression	Expanded form	Exponent form	Difference in exponents
$\frac{3^5}{3}$			
$\frac{3^5}{3^2}$			
$\frac{3^5}{3^3}$			
$\frac{3^5}{3^4}$			

(b) What do you notice about the exponent form and the difference in the exponents (powers)?

(c) Complete this statement.
The second law of exponents states that when dividing exponent expressions with the same base, you _____ the powers.

SC 2 I can divide numbers using exponent notation

1 Use the second law of exponents to simplify the following. Leave your answers in exponent form.

(a) $\frac{3^5}{3}$ **(b)** $\frac{3^5}{3^2}$ **(c)** $\frac{3^5}{3^3}$

(d) $\frac{3^5}{3^4}$ **(e)** $3^8 \div 3^3$ **(f)** $3^{10} \div 3^6$

2 Use the second law of exponents to simplify the following.

(a) $\frac{4^5}{4}$ **(b)** $\frac{4^9}{4^2}$ **(c)** $\frac{6^7}{6^3}$ **(d)** $\frac{10^8}{10^4}$

3 Use the second law of exponents to determine the missing exponent in each of the following.

(a) $4^{11} \div 4^x = 4^7$ **(b)** $4^x \div 4^3 = 4^2$ **(c)** $3^{14} \div 3^x = 3^3$

(d) $\frac{2^6}{2^x} = 2^4$ **(e)** $\frac{5^x}{5^3} = 5^5$

4 Which of the following cannot be simplified directly by using the second law of exponents? Why?

(a) $4^{10} \div 2^3$ **(b)** $4^{12} \div 4^3$ **(c)** $30^3 \div 20^3$

(d) $\frac{2^7}{2^2}$ **(e)** $\frac{25^2}{5^2}$ **(f)** $4^{11} \div 5^3$

5 Tim's answer to $8^4 \div 4^2$ is 2^2. Describe the mistake/s Tim has made.

6 Compare the relationship between multiplication and division with the relationship between the first and second exponent laws.

SC 3 I can apply the multiplication and division rules, or a combination of both, to simplify an expression

1 Simplify the following.

(a) $\frac{10^3 \times 10^4}{10^5}$ **(b)** $\frac{10^4 \times 10^4}{10^5}$ **(c)** $\frac{10^3 \times 10^4}{10^2}$

(d) $\frac{10 \times 10^3 \times 10^4}{10^5}$ **(e)** $\frac{10^2 \times 10^3 \times 10^4}{10^5}$

2 Simplify the following.

(a) $\frac{2^3 \times 2^7}{2^5}$ **(b)** $\frac{3^3 \times 3^4}{3^2}$ **(c)** $\frac{4^5 \times 4^4}{4^2}$

(d) $\frac{6 \times 6^3 \times 6^4}{6^5}$ **(e)** $\frac{8 \times 8^2 \times 8^3}{8^4}$

3 Simplify the following.

(a) $\frac{2^3 \times 2^7 \times 5^2}{2^5}$ **(b)** $\frac{3^3 \times 3^4}{3^2 \times 4}$ **(c)** $\frac{4^5 \times 4^4 \times 3^5}{4^2}$

(d) $\frac{6 \times 6^3 \times 6^4}{6^5 \times 7^2}$ **(e)** $\frac{5 \times 8 \times 8^2 \times 8^3}{8^4}$

4 Determine the missing exponent in each of the following.

(a) $4^6 \times 4^3 \div 4^x = 4^7$ **(b)** $4^x \times 4^5 \div 4^3 = 4^7$ **(c)** $3^{14} \div (3^2 \times 3^x) = 3^3$

(d) $\frac{2^6 \times 2^3}{2^x} = 2^4$ **(e)** $\frac{5^x}{5^3 \times 5^4} = 5^5$

5 Simplify the following expressions. Write your answer in exponent form.

(a) $\frac{2^{12}}{2^5} \times \frac{2^3}{2^4}$ **(b)** $\frac{3^3}{3^2} \times \frac{3^2}{3}$ **(c)** $\frac{4^5 \times 4^4}{4^2} \times 4^3$

(d) $\frac{6^3 \times 6^8}{6^5 \times 7^2} \times \frac{7^5}{6^2}$ **(e)** $\frac{5^6 \times 8 \times 8^2 \times 8^3}{8^4} \times \frac{8}{5^2}$

6 Explain how to divide numbers written in exponent notation with the same base. You may include an example.

1.3 Establish and apply the exponent law for raising a power to a power

1

Learning intention: To establish and apply the exponent law for raising a power to a power

Success criteria:

SC 1 I can use expanded form to simplify the power of a power.

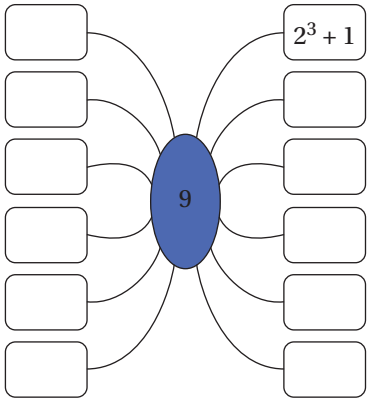
SC 2 I can use exponent laws to simplify a power of a power.

SC 3 I can apply the exponent laws of multiplication, division and raising to a power.

Lesson warm-up

Powers to 9

Copy this diagram.



In each square, write an expression involving exponents (powers) that is equal to 9.

The first one has been filled out for you as an example.

SC 1 I can use expanded form to simplify the power of a power

Worked example

Simplifying an exponential expression raised to a power

Simplify $(5^2)^3$. Leave your answer in exponent form.

THINKING	WORKING
Write the expression in expanded form.	$(5^2)^3 = (5^2) \times (5^2) \times (5^2)$
Use the first exponent law to simplify the multiplication of terms with the same base.	$= 5^{2+2+2}$ $= 5^6$
Write the answer.	$(5^2)^3 = 5^6$

SC 2 I can use exponent laws to simplify a power of a power

To raise a power to a power, you multiply the exponents. Note that the base number stays the same.

$(x^a)^b = x^{a \times b}$

Worked example

Applying the third exponent law

Express $(5^3)^4$ in simplest exponent form.

THINKING	WORKING
Recall the third exponent law.	$(x^a)^b = x^{a \times b}$
To raise a power to a power, you multiply the exponents. Note that the base number stays the same.	$(5^3)^4 = 5^{3 \times 4}$ $= 5^{12}$
Write the answer.	$(5^3)^4 = 5^{12}$

SC 3 I can apply the exponent laws of multiplication, division and raising to a power

Worked example

Applying the first three exponent laws

Express $\dfrac{4^2 \times \left(4^4\right)^3}{4^7}$ in simplest exponent form.

THINKING	WORKING
The order of operations says that raising to a power comes before multiplication and division. To raise a power to a power, you multiply the exponents. Note that the base number stays the same.	$\dfrac{4^2 \times \left(4^4\right)^3}{4^7} = \dfrac{4^2 \times 4^{4\times3}}{4^7}$ $= \dfrac{4^2 \times 4^{12}}{4^7}$
Use the first exponent law to simplify the multiplication of terms with the same base.	$= \dfrac{4^{2+12}}{4^7}$ $= \dfrac{4^{14}}{4^7}$
Use the second exponent law to simplify the division of terms with the same base.	$= 4^{14-7}$ $= 4^7$
Write the answer.	$\dfrac{4^2 \times \left(4^4\right)^3}{4^7} = 4^7$

Practice

ANSWERS Page 562

SC 1 I can use expanded form to simplify the power of a power

1 Simplify the following by expanding, then applying the first exponent law. Leave your answer in exponent form.

(a) $(5^2)^4$ **(b)** $(5^2)^5$ **(c)** $(5^2)^6$

(d) $(3^2)^2$ **(e)** $(3^2)^3$ **(f)** $(3^2)^4$

2 **(a)** Copy and complete the following table.

Expression	Expansion	Simplified form	Multiplying the exponents
$(6^3)^2$	$6^3 \times 6^3$	6^6	$2 \times 3 = 6$
$(7^4)^2$			
$(10^2)^3$			
$(2^4)^3$			

(b) Compare the simplified form and the product of the exponents.

(c) Complete this sentence. When a power is raised to a power, the exponents are _____.

3 Explain how to raise a power to a power. You should include an example.

SC 2 I can use exponent laws to simplify a power of a power

1 Express the following in simplest exponent form.

(a) $(2^3)^4$ **(b)** $(2^3)^5$ **(c)** $(2^3)^6$

(d) $(2^2)^3$ **(e)** $(2^2)^4$ **(f)** $(2^2)^5$

2 Express the following in simplest exponent form.

(a) $(6^3)^4$ **(b)** $(7^2)^5$ **(c)** $(8^3)^3$

(d) $(4^2)^5$ **(e)** $(10^5)^4$ **(f)** $(3^2)^5$

3 Express the following in simplest exponent form.

(a) $\left(\left(\frac{2}{3}\right)^3\right)^4$ **(b)** $\left(\left(\frac{7}{8}\right)^2\right)^5$ **(c)** $(10.5^4)^3$

(d) $(4.4^3)^5$ **(e)** $(1.05^5)^2$ **(f)** $(2.3^3)^4$

4 **(a)** $\left(\frac{1}{3}\right)^3 = \frac{1}{3^3}$, but $\left(\frac{2}{3}\right)^3 \neq \frac{2}{3^3}$. Explain why.

(b) $(4^3)^5 = (4^5)^3$. Explain why.

SC 3 I can apply the exponent laws of multiplication, division and raising to a power

1 Simplify the following. Express your answer in simplest exponent form.

(a) $(2^3)^4 \times 2^2$ **(b)** $(2^3)^4 \times 2^3$ **(c)** $(2^2)^4 \times 2^2$

(d) $(2^3)^4 \times 2^2 \times 2^3$ **(e)** $(2^3)^4 \times 2^2 \times 2^5$

2 Express the following in simplest exponent form.

(a) $(2^3)^4 \div 2^2$ **(b)** $(2^3)^4 \div 2^3$ **(c)** $(2^2)^4 \div 2^2$

(d) $(2^3)^4 \div 2^5$ **(e)** $(2^3)^4 \times 2^2 \div 2^5$

3 Simplify the following. Express your answer in exponent form.

(a) $\dfrac{2^5 \times 2^7}{(2^5)^2}$ **(b)** $\dfrac{3^3 \times 3^4}{(3^2)^3}$ **(c)** $\dfrac{4^5 \times \left(4^4\right)^3}{4^{15}}$

(d) $\dfrac{6^{10} \times (6^3)^2 \times 6^4}{\left(6^5\right)^3}$ **(e)** $\dfrac{8 \times (8^2)^4 \times \left(8^3\right)^2}{8^{14}}$

4 Simplify the following. Write your answer in exponent form.

(a) $\dfrac{2^3 \times \left(2^7\right)^2 \times 5^2}{2^5}$ **(b)** $\dfrac{3^6 \times 4^{15}}{3^2 \times (4^3)^2}$ **(c)** $\dfrac{4^5 \times 4^4 \times 3^5}{\left(4^2\right)^3}$

(d) $\dfrac{6^3 \times (6^2)^4 \times 7^5}{6^5 \times 7^2}$ **(e)** $\dfrac{5^3 \times 8 \times 8^2 \times (8^3)^4}{\left(8^4\right)^2}$

5 Simplify the following. Express your answer in exponent form.

(a) $\dfrac{\left(2^4\right)^5}{2^5} \times \dfrac{2^3}{2^4}$ **(b)** $\dfrac{\left(3^3\right)^2}{3^2} \times \dfrac{3^2}{3^3}$ **(c)** $\dfrac{4^5 \times 4^4}{(4^2)^3} \times 4^3$ **(d)** $\dfrac{6^3 \times 6^8}{6^5 \times 7^2} \times \dfrac{\left(7^5\right)^2}{6^2}$

(e) $\dfrac{5^6 \times 8^2 \times \left(8^3\right)^2}{8^4} \times \dfrac{8}{5^2}$ **(f)** $(3^5 \div 3^2)^2$ **(g)** $(3^5 \times 3^2)^2$

6 Theo's answer to $(8^{10} \div 8^2)^3$ is 8^4. What mistake has Theo made?

7 A cube has a side length of 10 cm.

(a) Express the volume of the cube in exponent and numerical form.

(b) Another cube has sides that are 20 cm in length. Express the volume of the cube in exponent and numerical form.

(c) The dimensions of the second cube have been increased by a factor of 2. Use the third exponent law to explain how the scale factor could be used to calculate the new volume.

8 What power of 4 is equal to 32^2?

9 Determine the values of x and y so that $2^x \times 5^y = 4000$.

1.4 Establish and apply the exponent law for raising to the power of 0

1

Learning intention: To establish and apply the exponent law for raising to the power of 0

Success criteria:

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1.

SC 2 I can simplify and evaluate expressions that require multiple exponent laws to be used.

Lesson warm-up

Dividing a number by itself

When you divide a number by itself, what is the answer?

What does the answer represent?

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1

Any number divided by itself is equal to 1, so $\frac{x^n}{x^n} = 1$.

The second exponent law can be used for a number divided by itself: $\frac{x^n}{x^n} = x^{(n-n)} = x^0$.

This leads to the fourth exponent law: $a^0 = 1, a \neq 0$.

Worked example

Dividing a number written in exponent form by itself

(a) Calculate the value of $\frac{3^5}{3^5}$ by writing the numerator and denominator in expanded form.

THINKING	WORKING
Write the numerator and denominator in expanded form.	$\frac{3^5}{3^5} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3}$
Recall that any number divided by itself is equal to 1. Cancel common factors and write the answer.	$\frac{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}} = 1$

(b) Simplify $\frac{3^5}{3^5}$ using the second exponent law.

THINKING	WORKING
Recall the second exponent law.	$\frac{x^a}{x^b} = x^{(a-b)}$
Use the second law to simplify the division of terms with the same base and write the answer.	$3^5 \div 3^5 = 3^{(5-5)}$ $= 3^0$

(c) What do you conclude from your answers to **(a)** and **(b)**?

THINKING	WORKING
Compare the results from parts **(a)** and **(b)**.	$\frac{3^5}{3^5} = 1$ $\frac{3^5}{3^5} = 3^0$
Write the answer.	$3^0 = 1$

SC 2 I can simplify and evaluate expressions that require multiple exponent laws to be used

Worked example

Applying the first four exponent laws

Express $\frac{\left(2^3\right)^2 \times 4^5}{4^3} \times \frac{4^3}{2^3} \div \frac{4^0}{4^3}$ in simplest exponent form.

THINKING	WORKING
The order of operations says that raising to a power comes before multiplication and division. To raise a power to a power, you multiply the exponents. Note that the base number stays the same.	$\frac{\left(2^3\right)^2 \times 4^5}{4^3} \times \frac{4^3}{2^3} \div \frac{4^0}{4^3}$ $= \frac{2^{3\times2} \times 4^5}{4^3} \times \frac{4^3}{2^3} \div \frac{4^0}{4^3}$ $= \frac{2^6 \times 4^5}{4^3} \times \frac{4^3}{2^3} \div \frac{4^0}{4^3}$
Write the division of a fraction as the multiplication of the inverse fraction.	$= \frac{2^6 \times 4^5}{4^3} \times \frac{4^3}{2^3} \times \frac{4^3}{4^0}$
Use the first exponent law to simplify the multiplication of terms with the same base.	$= \frac{2^6 \times 4^5 \times 4^3 \times 4^3}{4^3 \times 2^3 \times 4^0}$ $= \frac{2^6 \times 4^{(5+3+3)}}{4^{(3+0)} \times 2^3}$ $= \frac{2^6 \times 4^{11}}{4^3 \times 2^3}$
Use the second exponent law to simplify the division of terms with the same base.	$= 2^{(6-3)} \times 4^{(11-3)}$ $= 2^3 \times 4^8$

Practice

ANSWERS Page 563

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1

1 Copy and complete the following table.

Question	Answer	Exponent form	Simplest exponent form answer
$10 \div 10 =$	1	$10^1 \div 10^1 =$	10^0
$4 \div 4 =$		$4^1 \div 4^1 =$	
$4^2 \div 4^2 =$		$4^2 \div 4^2 =$	
$5^2 \div 5^2 =$			
$5^3 \div 5^3 =$			

2 **(a)** Copy and complete the following table.

10^5	10^4	10^3	10^2	10^1	10^0
100 000	10 000	1000			

(b) Describe the pattern in the exponent values.

(c) Describe the pattern in the value of the numbers.

(d) What does this tell you about the value of 10^0?

3 Explain why raising any non-zero number to the power of 0 equals 1 (i.e. $a^0 = 1, a \neq 0$).

4 Determine the value of x in the following.

(a) $3^x = 1$ **(b)** $2^0 + 2^x = 9$ **(c)** $(3^2 \times 3^4)^0 = x$

SC 2 I can simplify and evaluate expressions that require multiple exponent laws to be used

1 Simplify the following. Write your answer in exponent form.

(a) $\dfrac{2^0 \times \left(2^3\right)^2 \times 5^2}{2^5}$ **(b)** $\dfrac{3^6 \times 4^{15}}{3^2 \times (4^3)^0}$ **(c)** $\dfrac{4^5 \times 4^4 \times 3^0}{\left(4^2\right)^3}$ **(d)** $\dfrac{6^0 \times (6^2)^4 \times 7^5}{6^5 \times 7^2}$

2 Simplify the following. Express your answer in simplest exponent form.

(a) $\dfrac{2^{12}}{2^0} \times \dfrac{2^3}{2^4}$ **(b)** $\dfrac{3^3}{3^2} \times \dfrac{3^2}{3^0}$ **(c)** $\dfrac{4^0 \times 4^4}{(4^2)^3} \times 4^3$ **(d)** $\dfrac{6^3 \times 6^8}{6^0 \times 7^2} \div \dfrac{\left(6^3\right)^2}{7^2}$

(e) $\dfrac{5^6 \times 8^2 \times \left(8^3\right)^0}{8^4} \times \dfrac{8^5}{5^2}$ **(f)** $(3^5 \div 3^0)^2$ **(g)** $(3^0 \times 3^2)^2$

3 Determine the missing exponent in each of the following.

(a) $4^9 \times 4^0 \div 4^x = 4^7$ **(b)** $4^x \times 4^5 \div 4^0 = 4^7$ **(c)** $3^{14} \div (3^0 \times 3^x) = 3^3$

(d) $\dfrac{2^6 \times 2^0}{2^x} = 2^4$ **(e)** $\dfrac{5^x}{5^0 \times 5^4} = 5^5$

4 Consider the following: any number divided by itself = 1; 0 divided by any number = 0; any number divided by 0 is undefined.

(a) Using the information above, find a reason why 0^0 might equal 1, 0 or be undefined.

(b) Investigate further. Is there a right or wrong answer?

TOPIC REVIEW

Multiple choice

ANSWERS Page 618

1 3^5 written in expanded form is:

A 3×5 **B** $3 \times 3 \times 3 \times 3 \times 3$ **C** $5 \times 5 \times 5$ **D** $3 \times 3 \times 3 \times 3 \times 3 \times 3$

2 $4 \times 4 \times 4 \times 4 \times 4$ written in exponent form is:

A 4^5 **B** 5^4 **C** 4×5 **D** 4^4

3 In simplest exponent form 3×3^2 is:

A 3^2 **B** 9^2 **C** 6^3 **D** 3^3

4 In simplest exponent form $2^5 \times 2$ is:

A 2^{10} **B** 2^6 **C** 2^3 **D** 4^7

5 In simplest exponent form $2^5 \times 2^3 \times 3^4$ is:

A $2^8 \times 3^4$ **B** $2^{15} \times 3^4$ **C** $2^2 \times 3^4$ **D** $4^8 \times 3^4$

6 In simplest exponent form $3^2 \times 4^2 \times 4^3 \times 5^4 \times 5^3$ is:

A $3^2 \times 16^5 \times 25^7$ **B** $3^2 \times 4^5 \times 5^{12}$ **C** $3^2 \times 4^5 \times 5^7$ **D** 60^{14}

7 $2 \times 2 \times 2 \times 2 \times 10 \times 10 \div 10$ written in exponent notation is:

A 160 **B** $2^4 \times 10^3$ **C** $2^4 \times 10$ **D** 16×10^3

8 In simplest exponent form $6^{10} \div 6^2$ is:

A 6^5 **B** 6^8 **C** 6^{12} **D** 6^{20}

9 In simplest exponent form $\dfrac{2^5 \times 2^2}{2^4}$ is:

A $\dfrac{2^7}{2^4}$ **B** 2^7 **C** 2^3 **D** 2^6

10 In simplest exponent form $(5^3)^2$ is:

A 5^{32} **B** 5^6 **C** 5^5 **D** 5^1

11 In simplest exponent form $(5^2)^0$ is:

A 5^{20} **B** 5^2 **C** 5^1 **D** 1

12 In simplest exponent form $\dfrac{\left(5^0\right)^2 \times 5^6}{5^3}$ is:

A 5^2 **B** 5^3 **C** 5^4 **D** 5^5

Short answer

1 Evaluate:

(a) 3^4 **(b)** 2^7 **(c)** 10^7

(d) 1.2^3 **(e)** $(-2)^3$ **(f)** $\left(\dfrac{1}{2}\right)^3$

2 Express the following in simplest exponent form.

(a) $(3^4)^5$ **(b)** $(3^0)^5$ **(c)** $(3^4)^5 \div 3^8$

3 Express the following in simplest exponent form.

(a) $\dfrac{6^3 \times (6^2)^5 \times 9^5}{6^3 \times 9^2}$ **(b)** $(5^5 \div 5^2)^2$ **(c)** $\dfrac{4^3 \times 4^4 \times 3^5}{(4^0)^3 \times 3^2}$

4 Determine the missing exponent in each of the following.

(a) $3^7 \times 3^3 \div 3x = 3^2$ **(b)** $5x \times 5^4 \div 5^3 = 5^6$ **(c)** $3^{10} \div (3^5 \times 3x) = 3^3$

5 Write these exponent laws in your own words.

(a) First exponent law **(b)** Second exponent law **(c)** Third exponent law

6 Evaluate:

(a) $6^2 + 9^2$ **(b)** $6^0 + (3^2)^2$ **(c)** $(2 + 3^2) + 9^0$ **(d)** $3^2 + (4^5 \div 4^3)^2$

7 **(a)** Demonstrate the first exponent law by first writing the exponents in expanded form: $4^3 \times 4^5$.

(b) Demonstrate the second exponent law by first writing the exponents in expanded form: $3^6 \div 3^2$.

(c) Demonstrate the third exponent law by first writing the exponents in expanded form: $(4^3)^2$.

8 Copy and complete these statements.

(a) In the expression 4^3, 4 is the _____ and 3 is the _____.

(b) The expression $4^3 \times 3^2$ _____ be simplified because the bases are _____.

(c) The simplified form of expression $4^7 \div 4^3$ is _____. This is found by applying the _____ law of exponents and _____ the powers.

(d) The simplified form of expression $4^7 \times 4^3$ is _____. This is found by applying the _____ law of exponents and _____ the powers.

(e) The simplified form of expression $(3.21^3)^5$ is _____. This is found by applying the _____ law of exponents and _____ the powers.

Extended response

1 **(a)** Which is bigger, 2^5 or 5^2?

(b) Which is bigger, 4^3 or 3^4?

(c) Which is bigger, 5^3 or 3^5?

(d) Is there a pattern that indicates whether the larger base or higher power creates the bigger number? Is this true for all combinations? Check your answer with a few examples of your own.

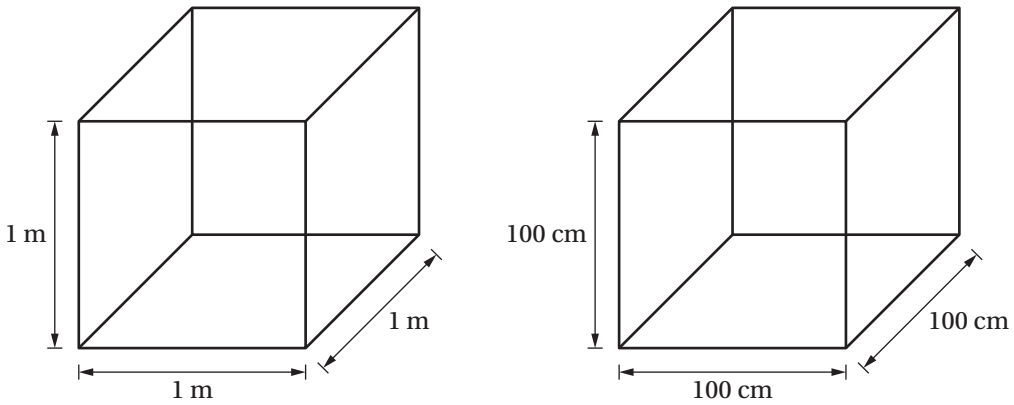
2 An apartment is designed to sit on top of a square concrete base. The sides of the square measure 12 m.

(a) What is the floor area of the apartment? A larger apartment nearby has the same square base, with sides of 24 m in length.

(b) What is the floor area of the bigger apartment?

(c) The bigger apartment has dimensions twice that of the smaller one, but the area is more than double. Explain why.

3 1 m is 100 cm, but 1 m^3 is 1 000 000 cm^3.



(a) Calculate the volume of each cube, showing your calculations.

(b) To convert the length from metres to centimetres, multiply by 100. To convert a volume in cubic metres to cubic centimetres, what do you need to multiply by?

(c) Explain how you would convert an area in square metres to square millimetres.

4 When 2 is raised to a power there is a pattern that forms in the final digit.

(a) Copy and complete the following table to determine the pattern.

$2^1 = 2$
$2^2 =$
$2^3 =$
$2^4 =$
$2^5 =$
$2^6 =$
$2^7 =$
$2^8 =$
$2^9 =$
$2^{10} =$

(b) Describe the pattern in the last digit.

(c) Patterns also exist in the final digits for powers of 3, 4, 6, 9. Investigate these patterns.

(d) Raising 5 to a positive power always ends in 25. Explain why.

(e) Copy and complete these statements using the words 'odd' or 'even'.

(i) An odd number raised to any power will always be an _____ number and an even number raised to any power will always be an _____ number.

(ii) $4^{25} + 5^{25} + 6^{25}$ is an _____ number.

(iii) $1^{15} + 2^{15} + 3^{15}$ is an _____ number.

5 Write your own multiple-choice questions on each of the success criteria in this topic.

TOPIC

2

Operations with integers, fractions and decimals

Recall 23

2.1 Multiply and divide integers 24

2.2 Solve problems by multiplying and dividing integers 29

2.3 Use the four operations with integers 34

2.4 Multiply and divide rational numbers 40

2.5 Apply all four operations with integers, fractions and decimals 44

Topic review 49

Why learn this?

Understanding operations with integers, fractions and decimals is an important step towards developing strong mathematical skills.

These skills are essential in areas such as finance, science and engineering. They lay the groundwork for more advanced mathematical concepts in algebra and calculus, making them invaluable for academic success and practical decision-making in real-life scenarios.

RECALL

ANSWERS Page 564

I can multiply and divide whole numbers

1 Calculate each of the following.

(a) 18×4 **(b)** $84 \div 3$ **(c)** 16×15 **(d)** $204 \div 17$

I can round decimals for different purposes

1 Round each of the following numbers to the number of decimal places stated in the brackets.

(a) 5.2638 (3) **(b)** 11.7745 (2) **(c)** 0.3912 (2) **(d)** 8.007 49 (2)

(e) 20.849 (0) **(f)** 0.9982 (1) **(g)** 3.995 (2)

I can add and subtract decimals

1 Evaluate the following.

(a) $17.34 + 21.2 + 0.43$ **(b)** $8 + 11.23 + 9.1$ **(c)** $9.24 - 6.3$ **(d)** $7.1 - 1.52$

I can multiply decimals

1 Evaluate the following.

(a) 10.5×0.5 **(b)** 0.25×1.5 **(c)** 2.05×1.05

2 Multiply the following decimals by powers of 10.

(a) 1.4537×10 **(b)** 1.4537×100 **(c)** 1.4537×1000 **(d)** $1.4537 \times 10\,000$

3 Multiply the following decimals by integers.

(a) 15.6×3 **(b)** 32.9×7 **(c)** 141.6×12 **(d)** 455.4×23

I can divide decimals

1 Divide the following decimals by powers of 10.

(a) $2943.5 \div 10$ **(b)** $2943.5 \div 100$ **(c)** $2943.5 \div 1000$ **(d)** $2943.5 \div 10\,000$

2 Multiply the following decimals by integers.

(a) $15.6 \div 3$ **(b)** $32.9 \div 7$ **(c)** $141.6 \div 12$ **(d)** $374.6 \div 23$

3 Evaluate the following.

(a) $15 \div 0.3$ **(b)** $4.2 \div 0.07$ **(c)** $8.82 \div 0.6$ **(d)** $82.32 \div 0.02$

I can simplify fractions

1 Simplify the following fractions.

(a) $\frac{12}{15}$ **(b)** $\frac{21}{28}$ **(c)** $\frac{24}{36}$ **(d)** $\frac{10}{18}$

2.1 Multiply and divide integers

Learning intention: To be able to multiply and divide integers

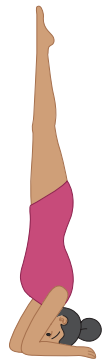
Success criteria:

SC 1 I can multiply two integers.

SC 2 I can divide two integers.

Lesson warm-up

What do multiplication and division look like?



1. Write or draw the multiplication 2×4 in five different ways.
2. Express each calculation below as a multiplication.
   - **(a)** $5 + 5 + 5$
   - **(b)** $(-4) + (-4)$
   - **(c)** $(-3) + (-3) + (-3) + (-3)$
3. Draw or write two representations of $6 \div 2 = 3$.

SC 1 I can multiply two integers

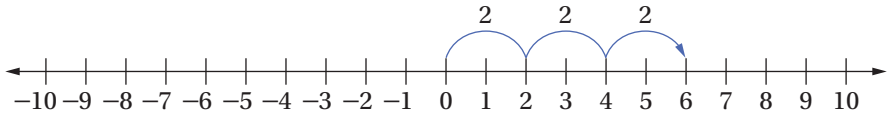
Consider the table of multiplication facts below.

You can see that each calculation results in a change of -3.

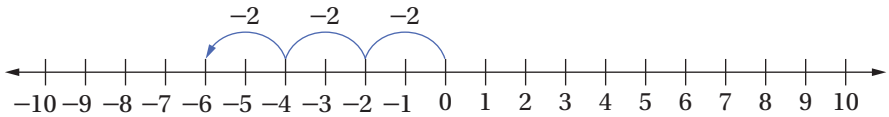
Multiplying a positive integer by another positive integer results in a positive integer. In the examples given, you can read each multiplication as 'three groups of a positive integer'.	$3 \times 4 = 12$ $3 \times 3 = 9$ $3 \times 2 = 6$ $3 \times 1 = 3$
Multiplying any number by zero results in zero. The example given is read as 'three groups of zero'.	$3 \times 0 = 0$
Multiplying a positive integer by a negative integer results in a negative integer. In the examples given, you can read each multiplication as 'three groups of a negative integer'.	$3 \times (-1) = -3$ $3 \times (-2) = -6$ $3 \times (-3) = -9$ $3 \times (-4) = -12$

By reading as 'three groups of ...' you can also use repeated addition to draw the multiplication on a number line.

Three groups of 2 can be drawn as $2 + 2 + 2 = 6$.



Three groups of –2 can be drawn as (–2) + (–2) + (–2) = –6.



You can extend this to show the sequence when multiplying negative integers.

Note that each calculation results in a change of + 3.

As the previous table shows, multiplying a negative integer by a positive integer results in a negative integer.	$(-3) \times 4 = -12$ $(-3) \times 3 = -9$ $(-3) \times 2 = -6$ $(-3) \times 1 = -3$
Multiplying any number by zero results in zero.	$(-3) \times 0 = 0$
Multiplying a negative integer by a negative integer results in a positive integer.	$(-3) \times (-1) = 3$ $(-3) \times (-2) = 6$ $(-3) \times (-3) = 9$ $(-3) \times (-4) = 12$

To remember whether a multiplication expression results in a positive or negative answer:

- For an even number of negative terms in the multiplication, the answer is positive.
- For an odd number of negative terms in the multiplication, the answer is negative.

Worked example

Multiplying integers

Calculate:

(a) $4 \times (-5)$

THINKING	WORKING
Write any negative integers as –1 multiplied by a positive integer.	$4 \times (-5) = 4 \times 5 \times (-1)$
Calculate the product of any positive integers.	$= 20 \times (-1)$ $= -20$
Determine whether the answer is positive or negative.	The answer is negative. There is one negative term in the calculation.
Write the answer.	$4 \times (-5) = -20$

(b) $(-4) \times (-5)$

THINKING	WORKING
Write any negative integers as -1 multiplied by a positive integer.	$(-4) \times (-5) = 4 \times 5 \times (-1) \times (-1)$
Calculate the product of any positive integers.	$= 20 \times (-1) \times (-1)$
Simplify the expression by writing $(-1) \times (-1) = 1$ and determine whether the answer is positive or negative.	$= 20 \times 1$ $= 20$ $(-4) \times (-5)$ has two negative terms, so the answer is positive.
Write the answer.	$(-4) \times (-5) = 20$

SC 2 I can divide two integers

The rules for dividing integers are the same as those for multiplying.

Consider the table of division facts below.

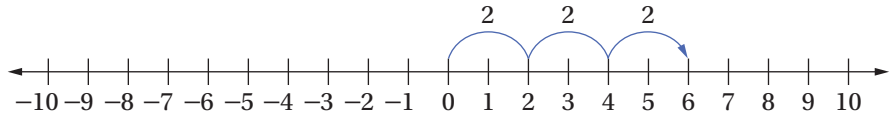
You can see that each calculation results in a change of -1.

Dividing a positive integer by another positive integer results in a positive integer. In the examples given, you can read each division as 'a positive value shared into 3 equal groups'.	$12 \div 3 = 4$ $9 \div 3 = 3$ $6 \div 3 = 2$ $3 \div 3 = 1$
Diving zero into any number results in zero. The example given is read as '0 shared equally into 3 groups'.	$0 \div 3 = 0$
Dividing a negative integer by a positive integer results in a negative integer. In the examples given, you can read each division as 'a negative integer shared into 3 equal groups'.	$(-3) \div 3 = -1$ $(-9) \div 3 = 3$ $(-12) \div 3 = -4$

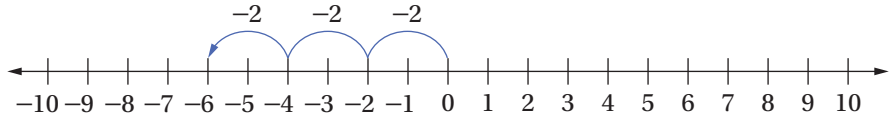
You can also read $\div 3$ as sharing into groups of 3.

Repeated subtraction can be used to draw division on a number line.

6 shared equally into 3 groups results in 2 in each group.



-6 shared equally into 3 groups results in -2 in each group.



You can extend this to show the sequence when dividing by negative integers.

You can see that each calculation results in a change of + 1.

As determined in the table above, dividing a negative integer by a positive integer results in a negative integer.	$12 \div (-3) = -4$ $9 \div (-3) = -3$ $6 \div (-3) = -2$ $3 \div (-3) = -1$
Diving zero into any number results in zero.	$0 \div (-3) = 0$
Dividing a negative integer by a negative integer results in a positive integer. Check using your understanding of multiplication. How many groups of –3 are needed to make –12? 4 groups.	$(-3) \div (-3) = 1$ $(-6) \div (-3) = 2$ $(-9) \div (-3) = 3$ $(-12) \div (-3) = 4$

To remember whether a division expression results in a positive or negative answer:

- For an even number of negative terms in the division, the answer is positive.
- For an odd number of negative terms in the division, the answer is negative.

Worked example

Dividing integers

Calculate and interpret the following expressions.

(a) $(-40) \div 8$

THINKING	WORKING
Write any negative integers as –1 multiplied by a positive integer.	$(-40) \div 8 = \frac{(-1) \times 40}{8}$
Calculate the quotient of the positive integers.	$= (-1) \times 5$
Determine whether the answer is positive or negative.	$(-40) \div 8$ has one negative term, so the answer is negative.
Write the answer.	$(-40) \div 8 = -5$

(b) $(-40) \div (-8)$

THINKING	WORKING
Write any negative integers as –1 multiplied by a positive integer.	$(-40) \div (-8) = \frac{(-1) \times 40}{(-1) \times 8}$
Simplify the expression by writing $(-1) \div (-1) = 1$ and determine whether the answer is positive or negative.	$= 1 \times 5$ $= 5$ $(-40) \div (-8)$ has two negative terms, so the answer is negative.
Write the answer.	$(-40) \div (-8) = 5$

Practice

ANSWERS Page 564

SC 1 I can multiply two integers

1 Calculate:

(a) $(-2) \times 8$ **(b)** $(-2) \times 7$ **(c)** $(-2) \times 6$ **(d)** $(-2) \times 5$

2 Calculate:

(a) $(-4) \times (-1)$ **(b)** $(-4) \times (-2)$ **(c)** $(-4) \times (-3)$

(d) $(-4) \times (-4)$ **(e)** $(-4) \times (-5)$ **(f)** $(-4) \times (-6)$

3 Calculate:

(a) $4 \times (-3)$ **(b)** $4 \times (-2)$ **(c)** $4 \times (-1)$

(d) 4×0 **(e)** 4×1 **(f)** 4×2

4 Calculate:

(a) $5 \times (-3)$ **(b)** $(-4) \times (-2)$ **(c)** $40 \times (-10)$

(d) $(-14) \times 2$ **(e)** $4 \times (-13)$ **(f)** $(-7) \times (-20)$

5 Leah wrote an answer of -6 for the question $(-2) \times (-3)$. Explain the mistake that Leah made.

SC 2 I can divide two integers

1 Calculate:

(a) $(-8) \div 2$ **(b)** $(-6) \div 2$ **(c)** $(-4) \div 2$ **(d)** $(-2) \div 2$

2 Calculate:

(a) $(-4) \div (-1)$ **(b)** $(-4) \div (-2)$ **(c)** $(-4) \div (-4)$

(d) $(-8) \div (-4)$ **(e)** $(-8) \div (-2)$ **(f)** $(-8) \div (-1)$

3 Calculate:

(a) $12 \div (-1)$ **(b)** $12 \div (-2)$ **(c)** $12 \div (-3)$

(d) $12 \div (-4)$ **(e)** $12 \div (-6)$ **(f)** $12 \div (-12)$

4 Calculate:

(a) $14 \div (-1)$ **(b)** $(-12) \div (-2)$ **(c)** $15 \div (-3)$

(d) $(-18) \div (-9)$ **(e)** $26 \div (-13)$ **(f)** $(-120) \div 12$

5 Calculate:

(a) $\frac{-12}{6}$ **(b)** $\frac{15}{-5}$ **(c)** $\frac{-18}{-2}$ **(d)** $-\frac{16}{2}$

6 Evaluate the following.

(a) Divide (-30) by 5. **(b)** Divide (-30) by 6.

(c) Divide 30 by (-10). **(d)** Divide (-30) by (-15).

7 Using the equation $15 \times (-3) = (-45)$, write two equations that divide 45 by negative numbers.

8 Leo gave an answer of $(-12) \div 2 = 6$. Explain the mistake that Leo made.

Solve problems by multiplying and dividing integers

2.2

Learning intention: To be able to solve problems by multiplying and dividing integers

Success criteria:

SC 1 I can determine the sign of problems involving multiple multiplications or divisions.

SC 2 I can multiply and divide integers in word problems.

Lesson warm-up

Strategies for multiplying and dividing

In the multiplication $30 \times 6 \times 2$, which two numbers would you multiply first? Does it make a difference? Explain.

Now, what if the multiplication is changed to division? For the division $30 \div 6 \div 2$, how would you calculate the answer? Explain.

SC 1 I can determine the sign of problems involving multiple multiplications or divisions

Worked example

Determining the sign when multiplying and dividing integers

Calculate:

(a) $(-24) \div 8 \times (-2)$

THINKING	WORKING
Write each of the negative integers as -1 multiplied by a positive integer, and calculate the product or quotient of any positive integers.	$(-24) \div 8 \times (-2) = \frac{(-1) \times 24}{8} \times (-2)$ $= (-1) \times 3 \times (-1) \times 2$ $= (-1) \times (-1) \times 3 \times 2$ $= (-1) \times (-1) \times 6$
Simplify the expression by writing $(-1) \times (-1) = 1$ and determine whether the answer is positive or negative.	$= 1 \times 6$ $= 6$ The answer is positive. There are two negative terms in the calculation.
Write the answer.	$(-24) \div 8 \times (-2) = 6$

(b) $(-18) \div (-3) \div (-6)$

THINKING	WORKING
Write each of the negative integers as -1 multiplied by a positive integer, and calculate the quotient or product of any positive integers.	$(-18) \div (-3) \div (-6)$ $= \frac{(-1) \times 18}{(-1) \times 3} \div (-6)$ $= 6 \div (-6)$ $= \frac{6}{(-1) \times 6}$ $= (-1) \times 1$
Determine whether the answer is positive or negative.	$= -1$ The answer is negative. There are three negative terms in the calculation.
Write the answer.	$(-18) \div (-3) \div (-6) = -1$

Worked example

Determining the sign of a product written in exponent notation

For each of the expressions below, calculate its value and explain why it is either positive or negative.

(a) $(-1)^3$

THINKING	WORKING
Write in expanded notation.	$(-1)^3 = (-1) \times (-1) \times (-1)$
Simplify the expression by writing $(-1) \times (-1) = 1$.	$= 1 \times (-1)$ $= -1$
Write the answer.	The value is negative because there are an odd number of negatives being multiplied.

(b) $(-1)^4$

THINKING	WORKING
Write in expanded notation.	$(-1)^4 = (-1) \times (-1) \times (-1) \times (-1)$
Simplify the expression by writing $(-1) \times (-1) = 1$.	$= 1 \times 1$ $= 1$
Write the answer.	The value is positive because there are an even number of negatives being multiplied.

(c) -1^4

THINKING	WORKING
Write in expanded notation.	$-1^4 = -1 \times 1^4$ $= -1 \times (1 \times 1 \times 1 \times 1)$
Simplify the expression by writing $(-1) \times 1 = -1$.	$= -1 \times 1$ $= -1$
Write the answer.	The value is negative because there is an odd number of negatives.

SC 2 I can multiply and divide integers in word problems

Worked example

Solving integer word problems in context

The water level in a pool lowers by 2 cm every hour on hot days. What will the change in water level be after 8 hours on a hot day?

THINKING	WORKING
Identify whether the values in the question would be positive or negative.	Water level change is: -2 Length of time is 8 hours: $+8$
Write an expression using these values to represent the question.	$8 \times (-2)$
Calculate the value of the expression.	$(-1) \times 8 \times 2 = -1 \times 16$ $= -16$
Check that your answer is reasonable.	$8 \times (-2) = -16$ has one negative term, so the answer is negative. The water level is decreasing each hour, so the answer should be negative.
Write the answer.	The water level will drop by 16 cm.

Worked example

Solving integer word problems

The product of two integers is –14 and their sum is 5. What are the two numbers?

THINKING	WORKING
List all the factors of –14.	Factors of –14 are ±1, ±2, ±7 and ±14.
A negative product value means the two factors are opposite in sign. Write the possible factor pairings.	Factor pairings for –14 are: –1 and 14 or 1 and –14 –2 and 7 or 2 and –7
Identify which pair of factors has the required sum.	–2 + 7 = 5
Write the answer.	The two integers with a product of –14 and sum of 5 are –2 and 7.

Practice

ANSWERS Page 564

SC 1 I can determine the sign of problems involving multiple multiplications or divisions

1 Calculate:

(a) (–1) × (–1) × (–1)

(b) (–1) × (–1) × (–1) × (–1)

(c) (–1) × (–1) × (–1) × (–1) × (–1)

(d) (–1) × (–1) × (–1) × (–1) × (–1) × (–1)

2 Describe how you know whether integers multiply to give a positive or a negative result.

3 Calculate:

(a) (–2) × (–2) × (–2)

(b) (–2) × (–2) × (–2) × (–2)

(c) (–2) × (–2) × (–2) × (–2) × (–2)

(d) (–2) × (–2) × (–2) × (–2) × (–2) × (–2)

4 Identify whether each expression will have a positive or negative result, or 0. You do not need to calculate the result.

(a) (–2) × 2 × (–1)

(b) 2 × (–2) × 2

(c) (–2) × (–1) × (–2)

(d) 2 × (–1) × 0 × (–2)

(e) 1 × (–2) × 1 × (–2)

(f) (–2) × (–1) × 2 × (–1)

5 Calculate:

(a) 12 ÷ (–1) × 2

(b) 12 ÷ (–2) × 4

(c) 12 ÷ (–3) ÷ 2

(d) 12 ÷ 6 × (–5)

(e) 12 ÷ 4 × (–6)

(f) 12 ÷ 2 × (–7)

6 Determine whether each expression has a positive or negative result, or 0. You do not need to calculate the results.

(a) 14 ÷ (–1) × 13 × 4 × (–6)

(b) (–7) × 4 ÷ (–2) × (–18)

(c) (–10) × (–10) × (–10) × (–10) × (–10)

(d) (–10) × 8 ÷ (–6) × 4 ÷ (–2)

(e) 10 × (–2) × 0 × (–2)

(f) (–1) × (–1) × (–1) × (–1) × (–1) × (–1) × (–1) × (–1)

7 **(a)** Calculate:

(i) $(-1)^3$ **(ii)** $(-1)^4$ **(iii)** $(-1)^5$ **(iv)** $(-1)^6$ **(v)** $-(-1)^6$

(b) What is the relationship between the power and the sign of the answers?

8 A student using a calculator to evaluate $(-2)^8$ submits -256 as the answer. Explain the mistake the student made entering figures into the calculator.

SC 2 I can multiply and divide integers in word problems

1 Team A is trailing team B by 8 points after 10 minutes play in a 70-minute game. This scoring trend continues throughout the game. What will team A's final score be in relation to team B's?

2 An aircraft approaching an airport descends by 15 m every second. What is the total change in the aircraft's height after 20 seconds?

3 Over the last 5 days, the average temperature in a region was observed to drop by 15 °C. What was the average daily temperature change in that region?

4 The temperature at sunset in a desert town was 32 °C. After dark, it dropped by an average of 2 °C per hour until dawn 11 hours later. What was the temperature change overnight?

5 The stock price of a commodity fell \$3 in the first hour of trading. Assume that the rate of fall continues.

(a) What will the change in stock price be after an 8-hour day?

(b) The trading price at the beginning of the day is \$245. What is the closing price at the end of the day?

6 The value of some shares fell \$5 in the first hour of trading, rose by \$4 the following hour, then fell by an average of \$2 every hour until the end of the day.

(a) What was the change in the share value after a 9-hour day of trading?

(b) The share price at the beginning of the day was \$43. What was the price at the end of the day?

7 Write each word operation and calculate its value.

(a) Start with 4 and multiply by 2.

(b) Start with -4 and double it.

(c) The product of -4 and -2.

(d) The quotient of -4 and 2.

8 Solve the following.

(a) The product of two numbers is -8 and their sum is -2. What are the two numbers?

(b) The product of two numbers is -8 and their sum is 2. What are the two numbers?

9 Solve the following.

(a) The product of two numbers is 6 and their sum is -5. What are the two numbers?

(b) The product of two numbers is 6 and their sum is -7. What are the two numbers?

10 The mean of two integers is 16. Given that one of the numbers is -2, what is the other?

11 The mean of two integers is -12. Given that one of the numbers is 10, what is the other?

12 The mean of three integers is -5. Given that one of the numbers is 9, what are the other two?

2.3 Use the four operations with integers

Learning intention: To be able to use the four operations with integers

Success criteria:

SC 1 I can use any of the four operations to solve problems.

SC 2 I can solve problems with brackets.

Lesson warm-up

Multiplication table

Complete this table.

×	4	−3	6	−2
−10				
−5				
7				
9				

SC 1 I can use any of the four operations to solve problems

Order of operations

To calculate the value of expressions that contain brackets and a mix of operations, you must follow the order below.

1. Evaluate all expressions inside brackets (B) and other grouping symbols from the innermost ones outwards.
2. Calculate any orders (O). These are powers or roots.
3. Calculate division (D) and multiplication (M) expressions, working from left to right. They are grouped because they are of equal priority.
4. Calculate addition (A) and subtraction (S) expressions last, working from left to right. They are grouped because they are of equal priority.

The letters from the four ordered points create the acronym BODMAS. Use it to remember the calculation order.

When reading expressions, the sign in front of an integer is part of that term. It shows the direction of its value. Forgetting this is a common mistake.

The expression −4 − 3 + 8 can be interpreted as:

(−4) and (−3) and (+8), or (−4) + (−3) + 8.

Addition and subtraction have the same priority, so work from left to right.

$-4 - 3 + 8 = (-4 - 3) + 8$

$= -7 + 8$

$= 1$ ← this is correct

In the common mistake mentioned above, the sign in front of the 3 is ignored and the addition performed first.

$-4 - 3 + 8 = -4 - (3 + 8)$

$= -4 - 11$

$= -15$ ← this is incorrect

Worked example

Working left to right

Calculate $-4 - 18 \div 6 + (-10)$

THINKING	WORKING
Following the order of operations rules, division must be performed first. The negative sign in front of 18 is part of that term.	$-4 - 18 \div 6 + (-10)$ $= -4 - 18 \div 6 + (-10)$
Following the order of operations rules, addition/subtraction is performed next, working left to right.	$= -4 - 3 + (-10)$ $= -7 + (-10)$ $= -17$
Write the answer.	$-4 - 18 \div 6 + (-10) = -17$

Worked example

Regrouping to simplify expressions

(a) Calculate $(-2) \times 19 \times 5$ by working from left to right.

THINKING	WORKING
The only operation listed is multiplication. The order the numbers are multiplied in is not important. Working from left to right, calculate the product of the first two numbers.	$(-2) \times 19 \times 5$
Calculate the product of the two remaining numbers.	$= (-38) \times 5$ $= -190$
Write the answer.	$(-2) \times 19 \times 5 = -190$

(b) Calculate $(-2) \times 19 \times 5$ by regrouping to calculate a simpler product.

THINKING	WORKING
The only operation listed is multiplication. The order the numbers are multiplied in is not important. This order is a simpler calculation.	Multiplying –2 and 5 as the first step means that you will be calculating the product of –10 and in the second step. $\underline{(-2)} \times 19 \times \underline{5}$
Calculate the product of the two numbers identified in the first step.	$= -10 \times 19$
Calculate the product of the two remaining numbers.	$= -190$
Write the answer.	$(-2) \times 19 \times 5 = -190$

(c) Calculate $6 - (-2) \times 19 \times 5 + 100$

THINKING	WORKING
Following the order of operations rules, multiplication must be performed first. A strategy used in part **(a)** or part **(b)** can help determine the product.	$6 - \underline{(-2) \times 19 \times 5} + 100$ $= 6 - (-190) + 100$
Identify the remaining operations and determine the order in which they should be applied.	The two remaining operations are subtraction and addition. Addition and subtraction have the same priority, they will be applied working from left to right.
Calculate the remaining operations.	$= \underline{6 - (-190)} + 100$ $= 196 + 100$ $= 296$
Write the answer.	$6 - (-2) \times 19 \times 5 + 100 = 296$

SC 2 I can solve problems with brackets

Worked example

Simplifying expressions with brackets

Calculate the following.

(a) $4 \times (-5 + 3)$

THINKING	WORKING
Work through the order of operations. Calculate the value inside the brackets.	$4 \times \underline{(-5 + 3)}$ $= 4 \times (-2)$
Complete any multiplication/division, working from left to right.	$= -8$
Write the answer.	$4 \times (-5 + 3) = -8$

(b) $15 - 32 \div ((-6 - 2) \div 2)^2$

THINKING	WORKING
Work through the order of operations. Calculate the value inside the brackets, working from the innermost brackets outwards.	$15 - 32 \div \left(\underline{(-6 - 2)} \div 2\right)^2$ $= 15 - 32 \div \left(\underline{(-8) \div 2}\right)^2$ $= 15 - 32 \div (-4)^2$
Complete any orders (powers or roots). The exponent represents the number of factors in the product.	$= 15 - 32 \div (-4 \times (-4))$ $= 15 - \underline{32 \div 16}$
Complete any multiplication/division, working from left to right.	$= 15 - 2$
Calculate addition/subtraction last, working from left to right.	$= 13$
Write the answer.	$15 - 32 \div ((-6 - 2) \div 2)^2 = 13$

Worked example

Applying the four operations to solve problems

Starting with –11, add 5, divide the result by –3, then multiply by –10. What is the result?

THINKING	WORKING
Write an expression representing the problem. Place brackets around the addition to show that it is performed before the division.	$(-11 + 5) \div (-3) \times (-10)$
Order of operations rules say calculate the value inside the brackets first.	$(-11+5) \div (-3) \times (-10)$ $= (-6) \div (-3) \times (-10)$
Identify the next operation and apply it. For operations that have the same order, work from left to right.	$= (-6) \div (-3) \times (-10)$ $= 2 \times (-10)$
Calculate the result.	$= -20$

Practice

ANSWERS Page 565

SC 1 I can use any of the four operations to solve problems

1 Calculate the following.

(a) $24 \div 4 + (-12)$ **(b)** $(-12) \div 4 + 24$ **(c)** $24 \div (-12) + 4$

2 Calculate the following.

(a) $4 \times (-6) - 24$ **(b)** $24 \div (-4) + 6$ **(c)** $4 \times (-6) \div 24$

3 Josie submitted the answer –44 for the calculation of $-16 - 8 + 20$. Explain the mistake Josie made.

4 Bai and Todd obtained different answers to the expression $20 - 16 - 8 \times 2$.

Bai's answer is –8 and Todd's answer is 20. Explain the mistake each of them made and write the correct value of the expression.

5 Calculate the following, noting that you cannot work from left to right in each problem.

(a) $5 \times 2 - 10 \div 2$ **(b)** $10 \div (-2) \times 2 + 5$

(c) $-2 \times (-5) \div (-10) - 2$ **(d)** $-10 \div 2 + (-5) \times 2$

6 Simplify each expression by regrouping, then calculate its value.

(a) $5 \times 18 \times (-2)$ **(b)** $5 \times (-18) \times (-2)$ **(c)** $-4 \times 17 \times 5$

(d) $-4 \times 17 \times (-5)$ **(e)** $5 \times 15 \times 2 \times (-4)$ **(f)** $5 \times 15 \times (-2) \times (-4)$

7 Simplify each expression by regrouping, then calculate its value.

(a) $13 + (-74) + 87$ **(b)** $32 - (-4) + 8$ **(c)** $360 - 12 + 40$ **(d)** $143 + (-74) - 26$

SC 2 I can solve problems with brackets

1 Calculate the following.

(a) $-2 \times 5 + 3$ **(b)** $-2 + 3 \times 5$ **(c)** $-2 \times 6 - 4$ **(d)** $-2 - 4 \times 6$

(e) $-12 \times 3 + (-4)$ **(f)** $-4 + (-12) \times 3$ **(g)** $-20 \times 6 - 7$ **(h)** $-7 + (-20) \times 6$

2 Calculate the following.

(a) $-40 \div (-10) + 8$ **(b)** $8 + (-40) \div (-10)$ **(c)** $-40 \div (-4) - 8$

(d) $-8 + (-40) \div (-4)$ **(e)** $-30 \div 10 + 8$ **(f)** $8 + (-30) \div 10$

(g) $-30 \div (-5) + (-8)$ **(h)** $-8 + (-30) \div (-5)$

3 Calculate the following.

(a) $4 + (-3 - 8)$ **(b)** $(5 + (-25)) \div (-2)$

(c) $4 \times (-1 + (-6))$ **(d)** $-35 + 15 + (-4) \times 6$

(e) $125 + (-45) + 4 \times (-20)$ **(f)** $-35 \div 7 + (-4) \times (-6)$

4 Calculate the following.

(a) $(2 \times 8 \div (-2)) + 4$ **(b)** $8 \times (-4 \div (2 - (-2)))$

(c) $-8 \div (-2 \times ((-2) + 4))$ **(d)** $(8 \div (-2) \times (-4)) + (-2)$

5 Apply the order of operations to calculate:

(a) $(-2)^2 - (3 - 5)$ **(b)** $(-2)^3 - (3 - (-2))$ **(c)** $(-1)^{15} - (3 - (2^2))$

6 Determine the sign of the answer for these expressions. You do not need to calculate the answers.

(a) $-2 \times (-4) \times (-6) \times (-8) \times (-10)$ **(b)** $2 \times (-4) \times 6 \times (-8) \times 10$

(c) $(-2)^{19}$ **(d)** $(-1)^4 - (-1)^3$

(e) $(-1)^8 - (3 - 2^3)$

7 Write each description below as a mathematical expression, then calculate it.

(a) Start with -40, divide by -4, add 6 divided by -3.

(b) Start with -84, divide by 7, add 3 multiplied by -1.

(c) Start with -40, divide by -4, add 5, then divide by -3.

(d) Start with -24, divide by 6, add 1, then divide by -3.

8 Describe a four-step expression using a mix of operations that starts with -20 and ends with -4.

2.4 Multiply and divide rational numbers

Learning intention: To be able to multiply and divide rational numbers

Success criteria:

SC 1 I can multiply and divide decimals.

SC 2 I can multiply and divide fractions.

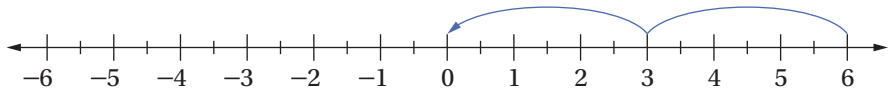
Lesson warm-up

What does division look like?

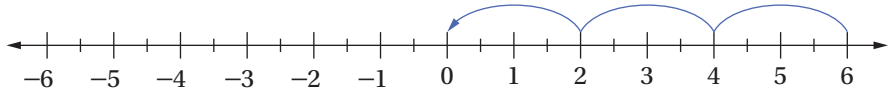
How do you think about division? Does it change depending on the division you are doing?

1 Consider the following representations of $6 \div 2 = 3$ using a number line, and describe what each of the numbers 6, 2 and 3 represent.

(a)

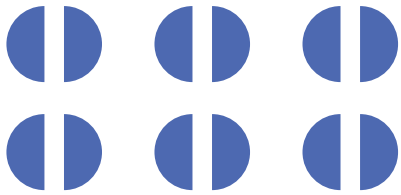


(b)

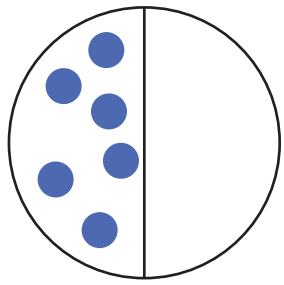


2 Consider the following representations of $6 \div \frac{1}{2} = 12$ using groups. Describe what each of the numbers 6, $\frac{1}{2}$ and 12 represent.

(a)



(b)



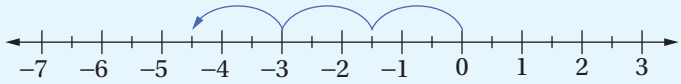
3 Write your own strategies for dividing numbers.

SC 1 I can multiply and divide decimals

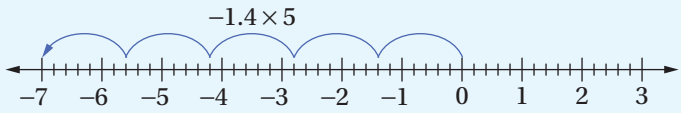
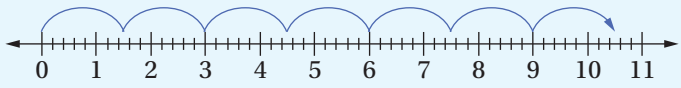
Worked example

Multiplying and dividing positive and negative decimals

(a) Calculate $3 \times (-1.5)$

THINKING	WORKING
Draw the multiplication on a number line. Where possible, read the multiplication as 'groups of'.	The multiplication $3 \times (-1.5)$ can be read as 3 groups of -1.5. 
Write the answer.	$3 \times (-1.5) = -4.5$

(b) Calculate $(-1.4) \times 5 \times (-1.5)$

THINKING	WORKING
Draw the multiplication on a number line. Where possible, read the multiplication as 'groups of'.	The first multiplication $(-1.4) \times 5$ can be read as 5 groups of -1.4. 
Apply the second calculation. Simplify the expression by writing $(-1) \times (-1) = 1$.	$(-1.4) \times 5 \times (-1.5)$ $= -7 \times (-1.5)$ $= (-1) \times (-1) \times 7 \times 1.5$ $= 1 \times 7 \times 1.5$ $= 7 \times 1.5$
Draw the multiplication on a number line. Where possible, read the multiplication as 'groups of'.	7×1.5 can be read as 7 groups of 1.5. 
Write the answer.	$(-1.4) \times 5 \times (-1.5) = 10.5$

SC 2 I can multiply and divide fractions

Common factors and the commutative property can help simplify expressions involving the multiplication of fractions.

An example of commutative property: $a \times (b \times c) = a \times (c \times b)$

After simplifying fractions using common factors, reordering the expression by pairing fractions that share common factors can simplify it further.

Worked example

Multiplying and dividing positive and negative fractions

(a) Simplify, then calculate $-\frac{7}{12} \div \frac{21}{33}$

THINKING	WORKING
Identify fractions in the expression that have common factors in their numerator and denominator.	$\frac{21}{33} = \frac{7 \times 3}{11 \times 3} = \frac{7}{11}$
Use the common factor to simplify the expression.	$-\frac{7}{12} \div \frac{21}{33} = -\frac{7}{12} \div \frac{7}{11}$
When dividing fractions, flip the second fraction and multiply.	$= -\frac{7}{12} \times \frac{11}{7}$
Simplify by dividing by a common factor.	$= -\frac{7 \div 7}{12} \times \frac{11}{7 \div 7}$ $= -\frac{1}{12} \times \frac{11}{1}$
Write the answer.	$-\frac{7}{12} \div \frac{21}{33} = -\frac{11}{12}$

(b) Simplify the calculation $\frac{6}{9} \times \left(-\frac{1}{3}\right) \times \frac{18}{24}$

THINKING	WORKING
Identify fractions in the expression that have common factors in their numerator and denominator.	$\frac{6}{9} = \frac{2 \times 3}{3 \times 3} = \frac{2}{3}$ $\frac{18}{24} = \frac{6 \times 3}{6 \times 4} = \frac{3}{4}$
Use the common factors to simplify each expression.	$\frac{6}{9} \times \left(-\frac{1}{3}\right) \times \frac{18}{24} = \frac{2}{3} \times \left(-\frac{1}{3}\right) \times \frac{3}{4}$ $= \frac{2 \times (-1) \times 3}{3 \times 3 \times 4}$ $= \frac{-6}{36}$
Simplify the fraction by dividing the numerator and denominator by the highest common factor.	$= \frac{-6 \div 6}{36 \div 6}$ $= \frac{-1}{6}$
Write the answer.	$\frac{6}{9} \times \left(-\frac{1}{3}\right) \times \frac{18}{24} = -\frac{1}{6}$

Practice

ANSWERS Page 565

SC 1 I can multiply and divide decimals

1 Calculate:

(a) -4.5×2 **(b)** $-4.5 \times (-3)$ **(c)** $(-1.5) \times (-3)$ **(d)** $2.25 \times (-2)$

2 Calculate:

(a) $2 \times (-4.5) \times 5$ **(b)** $-4.5 \times 2.4 \times (-3)$ **(c)** $-1.25 \times (-4) \times (-1.5)$

3 Calculate:

(a) $-4.5 \div 3$ **(b)** $-3 \div 1.5$ **(c)** $-13.5 \div (-3)$ **(d)** $-4.5 \div 2$

4 Calculate:

(a) $(-7.5) \div (-0.5)$ **(b)** $35.2 \div (-6.4)$ **(c)** $(-22.12) \div 2.8$

5 Risa wrote the answer -12.8 as the value of the expression $-1.6 \times (-8)$.
Explain what mistake was made.

SC 2 I can multiply and divide fractions

1 Simplify and calculate:

(a) $\frac{3}{8} \times \left(-\frac{5}{6}\right)$ **(b)** $-\frac{3}{8} \times \left(-\frac{5}{6}\right)$ **(c)** $\frac{12}{8} \times \left(-\frac{5}{6}\right)$

2 Simplify and calculate:

(a) $\frac{3}{8} \div \left(-\frac{1}{4}\right)$ **(b)** $-\frac{3}{8} \div \left(-\frac{3}{4}\right)$ **(c)** $\frac{9}{8} \div \left(-\frac{3}{5}\right)$

3 Simplify and calculate:

(a) $2 \times \frac{3}{5} \times \left(-\frac{5}{6}\right)$ **(b)** $-\frac{3}{4} \times \left(-\frac{2}{3}\right) \times 4$ **(c)** $-\frac{4}{9} \times (-4) \div \frac{8}{3}$ **(d)** $-\frac{5}{6} \div 2 \times \frac{3}{5}$

4 Simplify and calculate:

(a) $\frac{3}{5} \times \frac{1}{2} \times \left(-\frac{5}{6}\right)$ **(b)** $\frac{5}{6} \times \left(-\frac{3}{4}\right) \times \left(-\frac{2}{3}\right)$ **(c)** $\frac{4}{9} \div (-4) \times \frac{3}{8}$ **(d)** $\frac{3}{5} \div (-2) \times \left(-\frac{5}{6}\right)$

5 Ahmed wrote the answer $-\frac{13}{25}$ as the value of the expression $-\frac{2}{5} \div \frac{13}{10}$.
Explain what mistake was made.

2.5 Apply all four operations with integers, fractions and decimals

Learning intention: To be able to apply all four operations with integers, fractions and decimals

Success criteria:

SC 1 I can use all four operations to simplify and calculate expressions containing rational numbers.

SC 2 I can solve word problems involving rational numbers.

Lesson warm-up

Operations table

1 Copy and complete this table. Write your answer as both a decimal and a fraction.

×	−0.3	$\frac{1}{3}$	$-1\frac{1}{2}$	−0.25	$\frac{3}{4}$
1					
2					
−1					
−2					

2 Provide a real-life example of when you might use fractions.

3 Provide a real-life example of when you might use decimals.

SC 1 I can use all four operations to simplify and calculate expressions containing rational numbers

Recall BODMAS for the order of operations.

B: Evaluate all expressions inside brackets and other grouping symbols from the innermost ones outwards.

O: Calculate any orders (powers or roots).

DM: Calculate division and multiplication expressions.

AS: Calculate addition and subtraction expressions last, working in order from left to right.

When reading expressions, remember that the sign in front of a number is part of that term to show the direction of its value.

Worked example

Applying order of operations

(a) Calculate $\left(-\frac{1}{2}\right) \times 3 + \left(-\frac{3}{4} + \frac{1}{4}\right)$

THINKING	WORKING
Applying the order of operations, calculate the values inside the brackets first.	$-\frac{1}{2} \times 3 + \left(-\frac{3}{4} + \frac{1}{4}\right) = -\frac{1}{2} \times 3 + \left(-\frac{2}{4}\right)$
Simplify the fraction.	$= -\frac{1}{2} \times 3 + \left(-\frac{1}{2}\right)$
Applying the order of operations, calculate the multiplication next. Remember to include the sign in front of each term.	$= -\frac{3}{2} + \left(-\frac{1}{2}\right)$
Applying the order of operations, perform addition/subtraction operations last, from left to right.	$= -\frac{4}{2}$ $= -2$
Write the answer.	$\left(-\frac{1}{2}\right) \times 3 + \left(-\frac{3}{4} + \frac{1}{4}\right) = -2$

(b) Calculate $-1.5 \times 3 + \frac{9}{14} \div \frac{3}{7}$

THINKING	WORKING
Applying the order of operations, calculate the multiplication/division operations first, working from left to right.	$-1.5 \times 3 + \frac{9}{14} \div \frac{3}{7} = -4.5 + \frac{9}{14} \div \frac{3}{7}$ $= -4.5 + \frac{9}{14} \times \frac{7}{3}$
Simplify the fraction.	$= -4.5 + \frac{9 \div 3}{14 \div 7} \times \frac{7 \div 7}{3 \div 3}$ $= -4.5 + \frac{3}{2} \times \frac{1}{1}$ $= -4.5 + \frac{3}{2}$
Change the fraction to an equivalent decimal, then simplify.	$= -4.5 + 1.5$ $= -3$
Write the answer.	$-1.5 \times 3 + \frac{9}{14} \div \frac{3}{7} = -3$

SC 2 I can solve word problems involving rational numbers

Worked example

Solving word problems with rational numbers

(a) Starting with −1.5, add 7.5, divide the result by −3, then multiply by −1.2. What is the result?

THINKING	WORKING
Identify the starting number.	The starting number is −1.5.
Apply the first operation and calculate the result.	$-1.5 + 7.5 = 6$
Apply the next operation and calculate the result.	$6 \div (-3) = -2$
Apply the next operation and calculate the final result.	$-2 \times (-1.2) = 2.4$
Write the answer.	$(-1.5 + 7.5) \div (-3) \times (-1.2) = 2.4$

(b) Starting with $-1\frac{1}{3}$, add $\frac{3}{4}$ and multiply the result by $\frac{4}{7}$. Write the fraction in simplest form.

THINKING	WORKING
Identify the starting number.	The starting number is $-1\frac{1}{3}$.
Apply the first operation, converting any mixed numbers to improper fractions.	$-1\frac{1}{3} + \frac{3}{4} = -\frac{4}{3} + \frac{3}{4}$
Determine the lowest common multiple (LCM) and write the expression with common denominators.	Multiples of 3: 3, 6, 9, 12 Multiples of 4: 4, 8, 12 The lowest common multiple (LCM) of 3 and 4 is 12. $-\frac{4}{3} + \frac{3}{4} = -\frac{4 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3}$ $= -\frac{16}{12} + \frac{9}{12}$
Simplify the expression.	$= -\frac{7}{12}$
Apply the next operation and calculate the result.	$-\frac{7}{12} \times \frac{4}{7} = -\frac{1}{3} \times \frac{1}{1}$ $= -\frac{1}{3}$
Write the answer.	$\left(-1\frac{1}{3} + \frac{3}{4}\right) \times \frac{4}{7} = -\frac{1}{3}$.

Practice

ANSWERS Page 566

SC 1 I can use all four operations to simplify and calculate expressions containing rational numbers

1 Calculate:

(a) $10 - 4.5 \times 2$ **(b)** $10 - 4.5 \div 3$ **(c)** $10 + 4.5 \times (-3)$ **(d)** $10 \div (-2) + 4.5$

2 Calculate:

(a) $12 + (-6) \div 1.5$ **(b)** $1.5 \times (-12 + 6)$ **(c)** $-6 \times 12 \div (-1.5)$ **(d)** $6 \times (10.5 \div (-1.5))$

3 Calculate:

(a) $2 \times (-4.5) \times 5 + 25$ **(b)** $-4.4 + 10.2 \times (-3)$ **(c)** $-1.25 \times (-4) - 2 \times 1.5$

4 Calculate:

(a) $10.2 \div (-3) + 6.4$ **(b)** $-12.8 \div (-4) - 2.8$ **(c)** $7.5 + 12.5 \div (-5)$

5 Margot wrote the answer 6.4 as the value of the expression $-1.6 \times (-8) + 4$.
Explain what mistake was made.

6 Calculate and write your answer as a simplified fraction.

(a) $2 \times \frac{3}{8} + \left(-\frac{1}{8}\right)$ **(b)** $-\frac{3}{4} - \left(-\frac{1}{4}\right) \times 4$ **(c)** $-\frac{1}{5} \times 4 + \frac{2}{5}$

7 Calculate:

(a) $18 \div \left(-\frac{3}{4} \times 12\right)$ **(b)** $\left(\frac{3}{4} + \frac{1}{4}\right) \times (-12)$ **(c)** $\frac{1}{4} \times (-12) + \frac{3}{4}$

(d) $12 \div \left(-\frac{3}{4}\right) - (-18)$

8 Calculate:

(a) $4 \div \left(-\frac{1}{2}\right) + \left(-\frac{2}{5} - \frac{3}{5}\right)$ **(b)** $\left(4 \times \frac{2}{5} - \frac{3}{5}\right) \times \frac{1}{2}$ **(c)** $\frac{4}{5} \div (-4) - \frac{4}{5} \times \frac{1}{2}$

9 Calculate:

(a) $(-3.2) \times \left(3 + \frac{2}{5} \div \frac{1}{5}\right)$ **(b)** $(-0.4) \div \frac{1}{5} + 3 \times \frac{2}{5}$ **(c)** $3 \div \left((-0.4) + 0.2\right) \times \left(-\frac{2}{5}\right)$

SC 2 I can solve word problems involving rational numbers

1 Determine the final value that the following operation steps produce.

(a) Starting with -42, divide by -4, then add 1.5 and divide the result by -3.

(b) Starting with 2.4, divide by -6, then add 24.4 divided by 4.

(c) Starting with $-\frac{1}{2}$, multiply by -4, then add 52 and divide the result by -3.

(d) Starting with $\frac{3}{4}$, divide by $-\frac{7}{8}$, then add $\frac{3}{7}$ multiplied by 3.

2 Calculate:

(a) -50 divided by one quarter of 8

(b) -5 multiplied by one half of -20

(c) -64 divided by one half of $\frac{1}{4} \times 64$

(d) one quarter of 48 divided by $9 \times \left(-\frac{1}{3}\right)$

3 A diver leaps from a 146.5 m high cliff and plunges to a depth of -9.2 m, before swimming back up to the surface.

(a) Write the vertical distance travelled by the diver as a mathematical expression.

(b) What is the total vertical distance travelled by the diver?

4 One day at Uluru, the maximum temperature is 45.5 °C and the minimum temperature is -4.2 °C. Write an expression for the difference between these two temperatures, then calculate the difference.

5 To make 10 choir uniforms, $2\frac{1}{2}$ m of fabric is required for each top half and $2\frac{3}{4}$ m of fabric is required for each bottom half.

(a) The uniform shop only has 10.2 m of fabric. How much extra fabric does the uniform shop need to make all 10 uniforms? Write your answer as a decimal.

(b) Each metre of fabric costs $12.70. How much does the extra fabric cost the uniform shop? Calculate the amount to the nearest cent.

(c) Each choir member pays $60 for a uniform. How much profit does the uniform shop make?

6 A business has a debt of $185 500 and assets worth $250 000. It sells one-fifth of the assets to reduce this debt.

(a) Calculate the amount the debt is reduced by.

(b) What is the new debt for the business after the sale?

(c) Four directors of the business each purchase 0.2 of the remaining assets to further reduce the debt.

(i) How much did each director pay?

(ii) How much is the remaining debt now?

(iii) What does the sign of the answer to **(ii)** mean?

7 A multiple-choice mathematics test with 40 questions awards 2.5 marks for each correct answer. To discourage guessing, -1.5 marks are given for an incorrect answer, and 0 marks for questions not attempted.

(a) What is the total possible score if all 40 questions are answered correctly?

(b) What is the score for a student who correctly answered 34 questions, left two questions unanswered and answered four incorrectly?

(c) Another student scored 84 marks after answering every question. How many questions did the student get correct?

TOPIC REVIEW

Multiple choice

ANSWERS Page 619

1 Choose the correct answer. $-11 \times 2 =$

A 22 **B** 9 **C** −9 **D** −22

2 Choose the correct answer. $-4 \times (-2) =$

A 15 **B** 8 **C** −8 **D** −15

3 Choose the correct answer. $8 \div (-4) =$

A −2 **B** −4 **C** 2 **D** 4

4 Choose the correct answer. $-30 \div 2 =$

A 15 **B** 8 **C** −8 **D** −15

5 Choose the correct answer. $-4 \times 8 \times (-2) =$

A −64 **B** −16 **C** 16 **D** 64

6 Choose the correct answer. $-40 \times (-2) \div 8 =$

A 10 **B** 88 **C** −10 **D** −50

7 Choose the correct answer. $21 \div (-3) \times 4 =$

A −147 **B** −28 **C** 28 **D** 252

8 Choose the correct answer. $8 + (-56) + 92 =$

A 10 **B** 44 **C** −44 **D** −140

9 Choose the correct answer. $(-2)^4 =$

A −64 **B** −16 **C** 16 **D** 64

10 On a cold winter morning the temperature is −3 °C before you get out of bed. By the time you get to school the temperature has risen by 14 °C. The temperature is now:

A −17 °C **B** −11 °C **C** 11 °C **D** 17 °C

11 Choose the correct answer. $0.2 \times (-2.1) =$

A 4.2 **B** −4.2 **C** −0.42 **D** −0.042

12 Choose the correct answer. $-4.2 \div (-3) =$

A 1.4 **B** 12.6 **C** −1.4 **D** −12.6

13 Sal has overdrawn his bank account and has a balance of −$62.50. The bank charges him a fee of $15 for being overdrawn. The next day Sal corrects his mistake by depositing $150 into the account. His new balance is:

A −$72.50 **B** $72.50 **C** $102.50 **D** $135

Short answer

1 Calculate:

(a) $-9 \times 2 =$ **(b)** $9 \times (-3) =$ **(c)** $-9 \times (-4) =$

2 Calculate:

(a) $-4 \times (-20) =$ **(b)** $-5 \times (-10) =$ **(c)** $-9 \times (-6) =$

3 Calculate:

(a) $-9 \div 3 =$ **(b)** $12 \div (-4) =$ **(c)** $-90 \div 10 =$

4 Calculate:

(a) $-10 \div (-2) =$ **(b)** $-12 \div (-4) =$ **(c)** $-40 \div (-8) =$

5 Calculate:

(a) $-10 \times 4 \times (-2) =$ **(b)** $-10 \times -2 \times (-2) =$ **(c)** $6 \times (-2) \times (-3) =$

6 Calculate:

(a) $-16 \div 4 \times (-2) =$ **(b)** $-20 \div 2 \div (-2) =$ **(c)** $-84 \div 7 \div (-6) =$

7 Calculate:

(a) $-0.9 \times 2 =$ **(b)** $9 \times (-0.5) =$ **(c)** $-0.2 \times 1.6 =$

8 Calculate:

(a) $-\frac{4}{5} \times 2 =$ **(b)** $-\frac{5}{8} \times \left(-\frac{1}{4}\right) =$ **(c)** $\frac{9}{10} \times \left(-\frac{1}{3}\right) =$

9 Calculate:

(a) $-6.6 \div 3 =$ **(b)** $12.8 \div (-4) =$ **(c)** $-10 \div 2.5 =$

10 Calculate:

(a) $-10 - (-2) \times 6 =$ **(b)** $-10 - 6 \div 3 =$ **(c)** $-15 \div 3 - (-2) \times 6 =$

11 Calculate:

(a) -20 + half of 16 **(b)** $-6 \times$ half of $16 + 2$ **(c)** $-6 \times$ half of $(16 + 2)$

12 Calculate:

(a) $-\frac{1}{4} \times 12 + (-2)^3 =$ **(b)** $(-3)^3 + (-3)^2 =$ **(c)** $(1.5)^2 + (-2.5)^2 =$

13 Calculate:

(a) $-\frac{1}{2} \div \left(-\frac{3}{4}\right) =$ **(b)** $-\frac{3}{5} \div 3 + \frac{3}{5} =$ **(c)** $\frac{4}{9} \div \left(-\frac{2}{3}\right) =$

14 Min had a balance of $-\$23.50$ on her credit account before she spent another \$51.70 on credit. Determine her new balance.

15 On a clear day on an outback station the maximum daytime temperature was 17.3 °C after an overnight minimum of −2.4°C. By how much did the temperature rise on that day?

16 If you start with −4.2, divide by −2, add 1.5 and then divide by −3, what number do you finish with?

Extended response

1 Cassie is playing a video game where the reward for advancing a level is 1000 points. She starts with 10 lives and for each life she loses, 200 points are deducted.

(a) Determine her score if she:

(i) advances 1 level before losing all 10 lives

(ii) advances 4 levels before losing all 10 lives

(iii) advances 6 levels, losing 5 lives.

(b) She finishes a game with a score of 2200. Determine a possible level she has reached and how many lives she has lost.

2 Solve these word problems:

(a) The school report Alice submitted was initially given 83 points, but 10 points were deducted for each day the report was late. Alice's report was 3 days late. What result did she receive?

(b) Rice is normally sold for a profit of $8 per kg. To make room for new stock the next day, a sale is held at the end of the day, making a loss of $2 per kg. During the day 140 kg of rice was sold at a profit and 24 kg sold at a loss. What was the profit/loss for the day?

(c) A red-hot piece of iron has a temperature of 125 °C. It cools at a rate of 8 °C every 10 minutes. What will the temperature be in 45 minutes?

3 **(a)** Copy the table below and fill in the missing numbers.

Expression	Exponent form	Value	Sign +/–
$-2 \times (-2)$	$(-2)^2$	4	+
$-2 \times (-2) \times (-2)$	$(-2)^3$		
$-2 \times (-2) \times (-2) \times (-2)$			
$-2 \times (-2) \times (-2) \times (-2) \times (-2)$			
$-2 \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$			

(b) Describe the pattern between the power and the sign of the answer.

(c) John thinks $(-2)^{21}$ is 2 097 152. How do we know that he is wrong?

TOPIC

3

Rational and irrational numbers

Recall 53

3.1 Identify and use irrational numbers 54

3.2 Identify the golden ratio and make the connection between the circumference and diameter of any circle 58

3.3 Identify and use terminating, recurring and non-recurring decimals 64

Topic review 69

Why learn this?

Understanding rational and irrational numbers is important for identifying numbers that have an exact value and those that are, at best, approximations. This topic will teach you how to:

- recognise that rational numbers can be written as fractions, while irrational numbers cannot be fully expressed as simple fractions. This deepens the understanding of numbers and related decimal properties
- identify special irrational numbers like the square roots of non-perfect squares and constants like pi (π). These numbers are used in many areas of mathematics and science.

These skills are important for advanced mathematics and help improve problem-solving in various situations where precise and accurate calculations are needed.

RECALL

ANSWERS Page 566

I can recognise and complete patterns

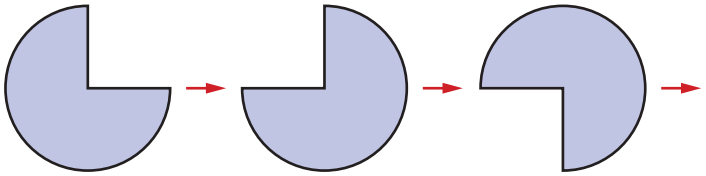
1 Complete the next three values for each number pattern.

(a) 1, 3, 5, 7, __, __, __ **(b)** 4, 8, 12, __, __, __ **(c)** 3, 9, 27, 81, __, __, __

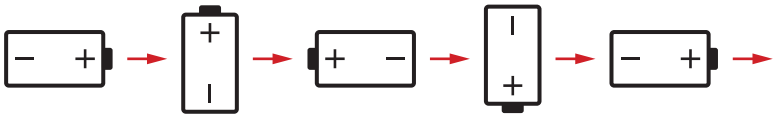
(d) 2, 4, 8, 16, __, __, __ **(e)** 48, 40, 32, 24, __, __, __ **(f)** 1, 1, 2, 3, 5, __, __, __

2 Draw the next shape in these patterns:

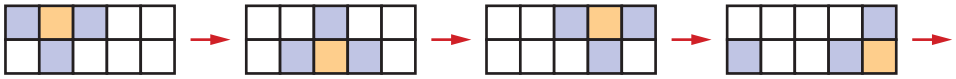
(a)



(b)



(c)



I can classify numbers into sets

1 Using only the numbers 1 to 20, identify the values that would be in each of the following sets.

(a) even numbers **(b)** factors of 24 **(c)** numbers divisible by 5

(d) prime numbers **(e)** composite numbers less than 15

I can calculate with squares and square roots

1 List the square numbers between 1 and 100.

2 Calculate the following.

(a) $\sqrt{9}$ **(b)** $\sqrt{36}$ **(c)** $\sqrt{81}$

(d) $\sqrt{144}$ **(e)** $\sqrt{225}$ **(f)** $\sqrt{400}$

I can simplify fractions

1 List the factors of the following numbers.

(a) 6 **(b)** 10 **(c)** 15 **(d)** 16

2 Simplify the following fractions.

(a) $\frac{6}{10}$ **(b)** $\frac{10}{15}$ **(c)** $\frac{15}{16}$ **(d)** $\frac{6}{16}$

3.1 Identify and use irrational numbers

Learning intention: To be able to identify and use irrational numbers

Success criteria:

SC 1 I can define a rational number, use this to identify irrational numbers and locate irrational numbers on a number line.

SC 2 I can understand situations and solve problems involving irrational numbers.

Lesson warm-up

How many πs?

Pi, π, is the ratio of the circumference divided by the diameter of a circle.

You have 3 minutes to memorise pi to as many places as possible. Be ready to compete with your classmates.

The first 100 digits of pi are **3.14159265358979323846264338327950288419716939937510 58209749445923078164062862089986280348253421170679**

Each year, there is a competition to recite π to the greatest number of decimal places. What do you think the record might be?

SC 1 I can define a rational number, use this to identify irrational numbers and locate irrational numbers on a number line

A rational number is one that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

In the formal world of mathematics, the set of rational numbers is usually referred to by the capital letter $\mathbb{Q}$ and the set of integers by the capital letter $\mathbb{Z}$.

Negative numbers are often written in the form $\frac{-3}{1}$, but they can also be written in the form $-\frac{3}{1}$, or even $\frac{3}{-1}$, and still be correct. The first two forms are the most commonly used.

π is a special number, technically called a transcendental number. There is only one other transcendental number that you will encounter at school, and that is $e \approx 2.718\,281\,8\ldots$

Worked example

Identifying rational numbers

For each of the following numbers, determine whether the number is rational or irrational.

5, 1.9, $\sqrt{9}$, π, –0.006, $\sqrt{3}$

THINKING	WORKING
Recall the definition of a rational number.	A rational number is a number that can be made by dividing an integer, a, by another integer, b.
Classify the examples that are rational numbers. Write the numbers as an integer divided by another to justify your choice.	The rational numbers are: $5 = \frac{5}{1}$ $1.9 = \frac{19}{10}$ $\sqrt{9} = 3 = \frac{3}{1}$ $-0.006 = -\frac{6}{1000}$
Recall the definition of an irrational number.	Irrational numbers are non-terminating, non-recurring decimals.
Classify the examples that are irrational. Use a calculator to show that the numbers are non-terminating and non-recurring decimals.	The irrational numbers are: π = 3.141 592 65... $\sqrt{3}$ = 1.7320508...

As with rational numbers, irrational numbers can be located on a number line, but only an approximation is possible. The irrational number will need to be rounded to a specified degree of accuracy (correct to a stated number of decimal places), then placed on the number line.

Worked example

Locating irrational numbers on a number line

Locate $\sqrt{3}$, correct to 3 decimal places, on a number line.

THINKING	WORKING
Use your calculator to express the irrational number in decimal form.	$\sqrt{3}$ = 1.7320508...
Round the decimal number to the specified number of decimal places.	$\sqrt{3} \approx 1.732$
Draw a number line that shows end values matching the required number of decimal places.	

SC 2 I can understand situations and solve problems involving irrational numbers

Irrational numbers can occur as the solution to some standard questions in mathematics.

Worked example

Using irrational numbers to solve problems

Determine the side length of a square with an area of 30 cm^2.

THINKING	WORKING
Recall the rule for the area of a square.	$A = l^2$
Substitute the known value.	$30 = l^2$
Solve to determine the unknown value.	$\sqrt{30} = \sqrt{l^2}$ $l = \sqrt{30}$ cm
Answer the question.	The length of a square with an area of 30 cm^2 is $\sqrt{30}$ cm.

Practice

ANSWERS Page 567

SC 1 I can define a rational number, use this to identify irrational numbers and locate irrational numbers on a number line

1 Express each of the following in the simplest form, $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, to show that the value is rational.

(a) $\frac{2.07}{3.24}$ **(b)** -2.67 **(c)** 5.0093 **(d)** $\frac{1.077}{30.6}$

2 Express each of the following in the form, $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, to show that the value is rational. If possible, express the fraction in its simplest form.

(a) 1.55 **(b)** -2.006 **(c)** 6.75 **(d)** 19.864

3 Each of the following numbers is written in the form of a whole number inside a square root.

(a) Determine whether the value is rational or irrational.

(i) $\sqrt{121}$ **(ii)** $\sqrt{160}$ **(iii)** $\sqrt{256}$

(iv) $\sqrt{10000}$ **(v)** $\sqrt{8100}$ **(vi)** $\sqrt{1000}$

(b) What type of number needs to be under the square root sign for the square root value to be rational?

4 Each of the following numbers is written in the form of a whole number inside a cube root.

(a) Determine whether the following numbers are rational or irrational. You can use your calculator to help with this question, if necessary.

(i) $\sqrt[3]{8}$ **(ii)** $\sqrt[3]{120}$ **(iii)** $\sqrt[3]{1000}$

(iv) $\sqrt[3]{990}$ **(v)** $\sqrt[3]{4913}$ **(vi)** $\sqrt[3]{2717}$

(b) What type of number needs to be under the cube root sign for the cube root value to be rational?

5 Locate each of the following irrational numbers on number lines. Use a decimal approximation, correct to 3 decimal places in each case.

(a) $\sqrt{47}$ **(b)** $\sqrt{71}$ **(c)** $\frac{\sqrt{29}}{7}$

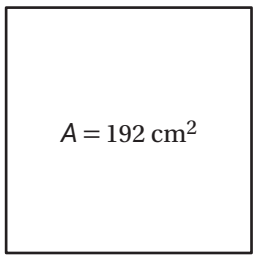
SC 2 I can understand situations and solve problems involving irrational numbers

1 Express, in simplest form, the side length for squares with the following areas.

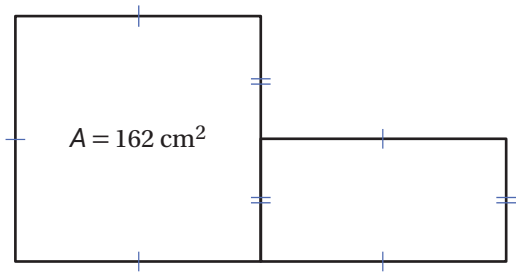
(a) 300 cm^2 **(b)** 405 cm^2 **(c)** 396 cm^2 **(d)** $\sqrt{1225}$ cm^2

2 Determine the perimeter, in exact simplest form, of each of the following shapes.

(a)



(b)



3 A square has a perimeter of 36.7 cm. Determine the area of the square.

4 Substitute the given values and solve to determine the exact value of the unknown in each of the following formulas.

(a) $v^2 = u^2 + 2as$

Calculate the value of v for $u = 10$, $a = 9.8$ and $s = 5$.

(b) $v^2 = \frac{2K}{m}$

Calculate the value of v for $K = 140$ and $m = 30$.

5 Paper comes in various standard sizes. In the metric measurement system, a common set of paper sizes is known as the A series. In this system, the largest paper size is A0, which has the dimensions 841 × 1189 mm. The next size is A1, measuring 594 × 841 mm.

(a) What do you notice about the shorter side of A0 and the longer side of A1?

(b) Determine the value of the ratio $\frac{\text{longer side}}{\text{shorter side}}$ for both A0 and A1 paper sizes. What do you notice?

(c) The exact value of this ratio can be expressed as an irrational number in the form $\sqrt{a}$. What is the value of a?

(d) Correct to the nearest mm, what are the dimensions of A2 paper?

3.2 Identify the golden ratio and make the connection between the circumference and diameter of any circle

Learning intention: To be able to identify the golden ratio and make the connection between the circumference and diameter of any circle

Success criteria:

SC 1 I can identify the golden ratio in various real-world contexts.

SC 2 I can make the connection between the circumference and diameter of any circle.

Lesson warm-up

Fibonacci sequence

What are the next three numbers in this sequence?

1, 1, 2, 3, 5, 8, 13, ...

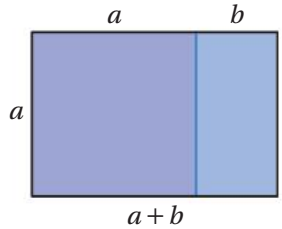
Describe the rule that determines the sequence.

Create your own sequence beginning with any two consecutive single digit numbers. List the first 10 numbers in the sequence.

SC 1 I can identify the golden ratio in various real-world contexts

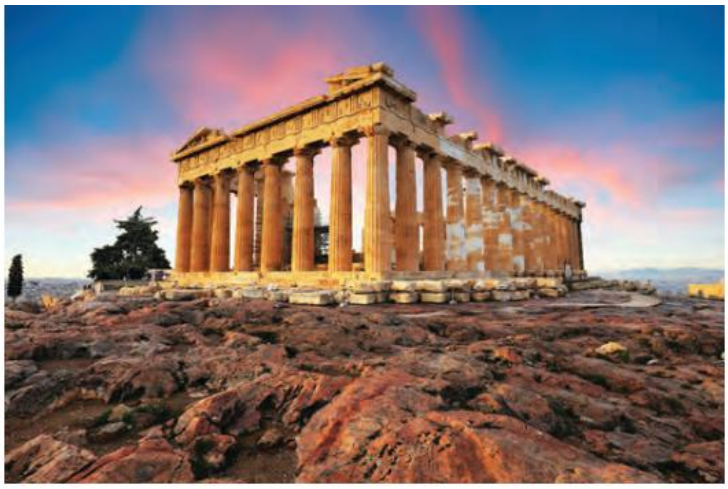
The golden ratio produces rectangles that are the most pleasing to the human eye. The following diagram shows the way in which the sides of a golden rectangle are related to each other. The ratio of the sides in the two rectangles shown are the same:

$(a + b) : a = a : b$



The golden ratio involves an irrational number. It is usually written as $\frac{1+\sqrt{5}}{2} \approx 1.618$. Given one of the side lengths of a rectangle, the other can be found, such that the rectangle will be a golden rectangle.

It is said the golden rectangle can be found in many examples of classical architecture, such as the famous Parthenon in Athens, now partially in ruins. This claim is disputed by many scholars.



Worked example

Determining the length of the second side in a golden rectangle

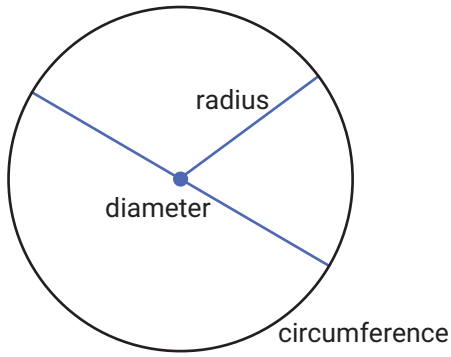
Determine the length of the shorter side of a golden rectangle, given that the longer side is 5 cm.

THINKING	WORKING
Recall the golden ratio.	$\frac{1+\sqrt{5}}{2} \approx 1.618$
Write the ratio equation that needs to be solved.	$\frac{a}{b} = \frac{1+\sqrt{5}}{2}$
Substitute the known value.	$\frac{5}{b} = \frac{1+\sqrt{5}}{2}$
Rearrange to solve for the unknown value.	$10 = b\left(1+\sqrt{5}\right)$ $b = \frac{10}{1+\sqrt{5}}$ ≈ 3.0901699
Check that the ratio obtained is (approximately) correct.	$\frac{a}{b} = \frac{5}{3.0901699}$ $= 1.618034\ldots$ ≈ 1.618

SC 2 I can make the connection between the circumference and diameter of any circle

The circumference of a circle is the distance around its outside. As this is a curved distance, it cannot be measured with a flat ruler like a square or a rectangle. To measure a cylinder, you would instead

wrap a piece of string around the cylinder, then straighten out the string and measure it with a ruler. When working with circles, there are three measurements that are of interest: radius, diameter and circumference.

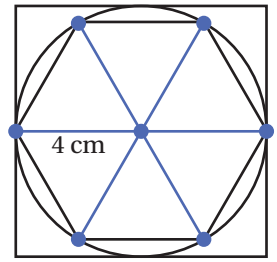


The diameter is twice the radius; the radius is half the diameter.

Worked example

Estimating the circumference of a circle

In the diagram below, the radius of the circle is 4 cm. Determine the perimeter of the square outside the circle and of the hexagon inside the circle. Use these measurements to estimate the circumference of the circle.



THINKING	WORKING
State the side length of the square.	8 cm
Calculate the perimeter of the square.	$P = 4l$ $= 4 \times 8$ $= 32$ cm
State the connection between the radius and the six sides that make up the perimeter of the hexagon.	The radius is equal to the length of the individual sides that make up the perimeter.
Calculate the perimeter of the hexagon.	$P = 6r$ $= 6 \times 4$ $= 24$ cm
Make an estimate of the circumference of the circle, and explain your reasoning.	The circumference of the circle is less than the perimeter of the square, but greater than the perimeter of the hexagon. The circumference is probably closer to the perimeter of the hexagon than the perimeter of the square, so an estimate of 26 cm is reasonable.

Practice

ANSWERS Page 567

SC 1 I can identify the golden ratio in various real-world contexts

1 The Parthenon, a Grecian temple to the goddess Athena, has a width of 30.89 m.

(a) Assuming that the face of the Parthenon is a golden rectangle, what should its height be?

(b) The height from the base to the tip of the triangular top is 18.16 m. How does the actual height compare to the height calculated in part **(a)**?

2 The Fibonacci sequence is a well-known mathematical sequence. It is produced by starting with two 1s, then adding the previous two digits to get the next term.
1, 1, 1 + 1 = 2, 1 + 2 = 3, 3 + 2 = 5, ...
Thus, the initial terms of the sequence are as follows:
1, 1, 2, 3, 5, 8, 13, 21, ...

(a) Determine the value of the ratio $\left(\frac{\text{larger}}{\text{smaller}}\right)$ between the fourth and fifth terms in the sequence. Give your answer correct to 3 decimal places.

(b) Determine the value of the ratio between the seventh and eighth terms in the sequence. Give your answer correct to 3 decimal places.

(c) The twelfth term is 144 and the thirteenth is 233. Determine the ratio between these two values. Give your answer correct to 3 decimal places.

(d) What do you notice about the values of the ratios in parts **(a)**–**(c)**?

(e) The thirtieth value is 2 178 309 and the thirty-first is 3 524 578. Find the ratio between these two values and compare it to the golden ratio.

(f) The nineteenth term is 4181. Use the exact value of the golden ratio to determine the twentieth term.

(g) The twenty-fifth term is 75 025. Use the exact value of the golden ratio to determine the twenty-fourth term.

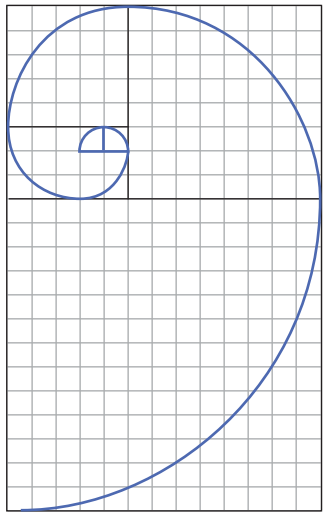
3 There are other sequences that can be referred to as Fibonacci-like sequences that start with two numbers and then add the previous two to produce the sequence.

(a) Create a sequence that starts 3, 3 and follows the same rule as the Fibonacci sequence to see whether it results in the same ratio between terms as the original Fibonacci sequence. Write out the first 12 terms of the sequence and check the ratios of the fourth and fifth, seventh and eighth, and eleventh and twelfth terms.

(b) Create a sequence that starts 2, 7 and follows the same rule as the Fibonacci sequence to see whether it results in the same ratio between terms as the original Fibonacci sequence. Write out the first 12 terms of the sequence and check the fourth and fifth, seventh and eighth, and eleventh and twelfth terms.

(c) Describe your results.

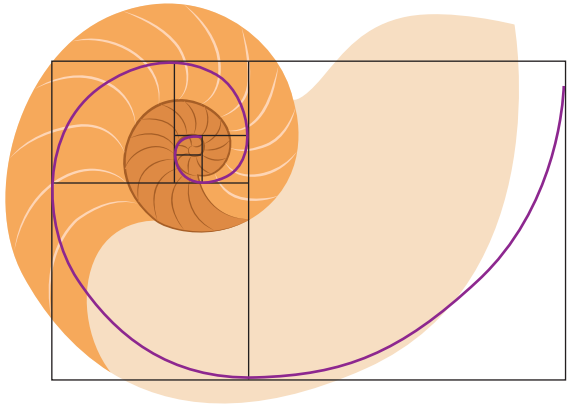
4 Consider the following diagram.



(a) Describe how it has been drawn.

(b) What do the side lengths of the squares remind you of?

(c) Consider this picture of the chambered nautilus.



What has been drawn over the photograph of the shell?

(d) Are you convinced that the golden ratio applies to this shell? Explain your answer.

SC 2 I can make the connection between the circumference and diameter of any circle

1 Use a compass to draw a circle with each radius measurement provided. Use a piece of string to measure the circumference of each circle, correct to the nearest millimetre.

(a) 4 cm **(b)** 5 cm **(c)** 8 cm **(d)** 10 cm

(e) Using your results for parts **(a)**–**(d)**, explain what happens to the length of the circumference when the length of the radius doubles.

(f) Using your results for parts **(a)**–**(d)**, explain what happens when the length of the radius increases by 1 unit and by 3 units.

2 Use your answers from the previous question to complete the following tasks.

(a) Copy and complete the table, writing your answers for the last two columns correct to 3 decimal places.

Radius	Diameter	Circumference	$\frac{C}{d}$	$\frac{C}{r}$
4 cm				
5 cm				
8 cm				
10 cm				

(b) Complete the following statements.

(i) The circumference is about _____ times as long as the diameter.

(ii) The circumference is about _____ times as long as the radius.

3 Use the estimates $\frac{C}{d} = 3.14$ and $\frac{C}{r} = 6.28$ to estimate the circumference of each of the following circles.

(a) The length of the radius is 3 cm.

(b) The length of the diameter is 7 cm.

(c) The length of the radius is 2.5 cm.

(d) The length of the diameter is 3.6 cm.

4 Use the estimate $\frac{C}{d} = 3.14$ to give an approximate diameter length for circles with the following circumferences. Give your answers correct to 2 decimal places.

(a) 20 cm **(b)** 38 cm **(c)** 42.1 cm

5 Use your calculator to answer the following questions.

(a) Press the π button on your calculator. Write down the displayed value correct to 2 decimal places.

(b) How does this connect to the approximate value used for $\frac{C}{d}$?

The value π, which is an irrational number, is the exact value of the relationship between the circumference and the diameter of a circle. Hence, $\frac{C}{d} = \pi$ can be used to calculate the diameter of a circle, given that the circumference is known; or the circumference of a circle, given that the diameter is known.

(c) Complete the following formula: $\frac{C}{d} = \pi \Rightarrow C =$ _____

(d) Use this new value to find the circumference of a circle that has a diameter of 12.6 cm, correct to 3 decimal places.

6 A cylindrical tube has a diameter of 3.8 cm across its circular base.

(a) How many times would a piece of ribbon need to be wrapped tightly around the circumference of this cylinder to get a piece at least 70 cm long? Give your answer as a whole number.

(b) Evelyn has 1.6 m of ribbon. How many complete times will she be able to wrap this tightly around the cylinder?

3.3 Identify and use terminating, recurring and non-recurring decimals

Learning intention: To be able to identify and use terminating, recurring and non-recurring decimals

Success criteria:

SC 1 I can recognise terminating, recurring and non-recurring decimals and choose appropriate representations for each.

SC 2 I can convert recurring decimals to exact fractional form.

Lesson warm-up

Your π day

The most famous irrational number is π. Given that it goes on forever in an unrepeating pattern, eventually there will be a sequence of numbers that match your birthday.

Go to https://mypiday.com/.

Enter your birthdate, and find your π day.

SC 1 I can recognise terminating, recurring and non-recurring decimals and choose appropriate representations for each

Every rational number can be written in an equivalent exact decimal form. In some cases, such as $\frac{1}{4} = 0.25$, it is written as a terminating value. In other cases, such as $\frac{1}{3} = 0.3333333\ldots$, the decimal form goes on infinitely. For such non-terminating numbers, there is a pattern in the decimal form that allows the exact answer to be written. The number of digits in the repeating pattern is called its period. The notation indicating the repeating pattern requires a dot to be placed over the first and last digits in the pattern, or a bar across the top of the pattern. Here are some examples:

Fraction	Decimal form	Repeating pattern (with dots)	Repeating pattern (with bars)
$\frac{1}{3}$	0.333 333 3 ...	$0.\dot{3}$	$0.\overline{3}$
$\frac{1}{15}$	0.066 666 6 ...	$0.0\dot{6}$	$0.0\overline{6}$
$\frac{1}{42}$	0.023 809 523 809 528 ...	$0.0\dot{2}38095\dot{}$	$0.0\overline{238095}$

Worked example

Writing rational numbers in decimal form

Write each of the following rational numbers in exact decimal form.

(a) $\frac{2}{3}$

THINKING	WORKING
Since the number is not in repeating decimal form, use your calculator to find the value.	$\frac{2}{3} = 0.666\,666\,66\ldots$
Identify the period of the pattern.	The period is 1.
Write the exact decimal form using shorthand notation.	$0.666\,666\,66\ldots = 0.\dot{6}$

(b) $\frac{2}{9}$

THINKING	WORKING
Since the number is not in repeating decimal form, use your calculator to find the value.	$\frac{2}{9} = 0.222\,222\,2\ldots$
Identify the period of the pattern.	The period is 1.
Write the exact decimal form using shorthand notation.	$0.222\,222\ldots = 0.\dot{2}$

(c) 0.143 143 143 ...

THINKING	WORKING
Check that the number is in repeating decimal form.	The number is already in repeating decimal form.
Identify the period of the pattern.	The period is 3.
Write the exact decimal form using shorthand notation.	$0.143\,143\,143\ldots = 0.\overline{143}$

(d) 0.016 262 62 ...

THINKING	WORKING
Check that the number is in repeating decimal form.	The number is already in repeating decimal form.
Identify the period of the pattern.	The period is 2.
Write the exact decimal form using shorthand notation.	$0.016\,262\,62\ldots = 0.01\overline{62}$

Irrational numbers cannot be expressed in exact decimal form as they are non-terminating and non-repeating. For example, $\sqrt{2} = 1.414\,213\,56\ldots$ There is no pattern evident in the decimal values.

SC 2 I can convert recurring decimals to exact fractional form

Using your calculator to find the decimal equivalent for $\frac{1}{6}$ will give you a display such as 0.166 666 666 7. The number of 6s before the 7 depends on the number of digits your calculator displays. You should recognise this as being the recurring decimal $0.1\dot{6}$. So, how can you find the exact fractional form for a recurring decimal? The next worked example shows how to do this.

Worked example

Converting a recurring decimal to exact fractional form

Convert the following recurring decimals to exact fractional form:

(a) $0.1\dot{6}$

THINKING	WORKING
Let x represent the repeating decimal.	$x = 0.166\,666\,666\,666\ldots$
Identify the number of digits in the repeating pattern.	There is only one digit in the repeating pattern.
Multiply x by the power of 10 that has the same number of zeros as the number of digits in the repeating pattern.	$10x = 1.666\,666\,666\,66\ldots$
Complete the subtraction, $nx - x$.	$\begin{aligned} 10x &= 1.666\,666\,666\,6 \\ -x &= 0.166\,666\,666\,6 \\ \hline 9x &= 1.5 \end{aligned}$
Now solve the equation and write the fraction in the form $\frac{a}{b}$, where a and b are integers.	$\begin{aligned} x &= \frac{1.5}{9} \\ &= \frac{15}{90} \\ &= \frac{3}{18} \\ &= \frac{1}{6} \end{aligned}$
As a final step, use your calculator to check that the fraction results in the correct recurring decimal.	$\frac{1}{6} = 0.166\,66\ldots$

(b) $0.\overline{123}$

THINKING	WORKING
Identify the number of digits in the repeating pattern.	There are two digits in the repeating pattern.
Multiply x by the power of 10 that has the same number of zeros as the number of digits in the repeating pattern.	$100x = 12.323\ 232\ 32\ ...$
Complete the subtraction, $nx - x$	$\begin{aligned} 100x &= 12.323\,232\,323 \\ -x &= 0.123\,232\,323 \\ \hline 99x &= 12.2 \end{aligned}$
Now solve the equation and write the fraction in the form $\frac{a}{b}$, where a and b are integers.	$\begin{aligned} x &= \frac{12.2}{99} \\ &= \frac{122}{990} \\ &= \frac{61}{495} \end{aligned}$
As a final step, use your calculator to check that the fraction results in the correct recurring decimal.	$\frac{61}{495} = 0.123\,232\,323\ ...$

Practice

ANSWERS Page 568

SC 1 I can recognise terminating, recurring and non-recurring decimals and choose appropriate representations for each

1 Express each of the following rational numbers in exact decimal form.

(a) $\frac{5}{9}$ **(b)** $\frac{4}{11}$ **(c)** $\frac{7}{75}$ **(d)** $\frac{33}{111}$

2 Write the following repeating decimals in shorthand form.

(a) 0.262 626 26 ... **(b)** −2.566 666 66 ... **(c)** 6.897 454 545 ... **(d)** 1.008 080 808 ...

3 For each of the following decimals, state whether the value is rational or irrational, based on the digits you can see.

(a) 0.133 33 ... **(b)** 2.236 067 9 ... **(c)** $1.6\dot{4}$

(d) 0.003 040 404 ... **(e)** $-2.\overline{34}$ **(f)** 3.316 624 7...

4 Using your calculator, express each of the following fractions in exact decimal form, using shorthand notation.

(a) $\frac{1}{7}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{3}{7}$

(d) $\frac{4}{7}$ **(e)** $\frac{5}{7}$ **(f)** $\frac{6}{7}$

(g) What do you notice about the values you have obtained?

(h) Now look at the odd numerators for fourteenths $\left(\frac{1}{14}, \frac{3}{14}, \ldots\right)$. What do you notice here?

Why did you not need to look at the even numerators for fourteenths?

5 $\frac{1}{11} = 0.090\,909\,09\ldots = 0.\overline{09}$. Without using a calculator, write the exact decimal equivalent for each of the following fractions.

(a) $\frac{2}{11}$ **(b)** $\frac{5}{11}$ **(c)** $\frac{7}{11}$ **(d)** $\frac{10}{11}$

6 Audrey wondered whether there was some way of looking at a fraction and knowing if it would lead to a terminating or a repeating decimal. She thought it might have something to do with prime numbers, so she looked at the prime factorisation of the denominators, in simplest form, of many fractions.

(a) For each of the following fractions, find the prime factorisation of the denominator.

(i) $\frac{2}{15}$ **(ii)** $\frac{7}{24}$ **(iii)** $\frac{7}{25}$ **(iv)** $\frac{11}{30}$

(v) $\frac{5}{48}$ **(vi)** $\frac{1}{8}$ **(vii)** $\frac{1}{16}$ **(viii)** $\frac{2}{75}$

(b) Audrey used her calculator to see which of these fractions could be represented as a repeating decimal. Identify the repeating decimals from the list.

(c) Audrey thought she could see a pattern developing. The repeating decimals all had at least one prime factor other than 2 or 5 in the denominator of the fraction. Try at least 10 fractions to check whether Audrey's idea is correct.

SC 2 I can convert recurring decimals to exact fractional form

1 Convert the following recurring decimals to exact fractional form.

(a) $0.1\dot{5}$ **(b)** $0.11\dot{4}$

2 Convert the following recurring decimals to exact fractional form.

(a) $0.1\overline{45}$ **(b)** $0.12\overline{54}$

3 Convert the following recurring decimals to exact fractional form.

(a) $0.1\overline{234}$ **(b)** $0.0\overline{324}$

4 Determine the fractional equivalent of $0.\dot{9}$ and comment on the result.

5 **(a)** Convert each of the following pairs of recurring decimals to exact fractional form.

(i) $0.1\dot{2}$ and $0.\overline{12}$ **(ii)** $0.1\dot{3}$ and $0.\overline{13}$ **(iii)** $0.1\dot{4}$ and $0.\overline{14}$

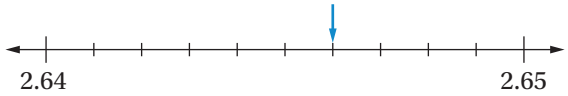
(b) Describe any shortcuts you were able to take when completing this question.

TOPIC REVIEW

Multiple choice

ANSWERS Page 619

1 Choose the irrational number marked on the following number line.



A $\sqrt{5}$ **B** $\frac{2\pi}{3}$ **C** $\sqrt{7}$ **D** $\frac{4\sqrt{8}}{5}$

2 Which of the numbers in the following list are irrational numbers?

$-\frac{3\pi}{2}, \frac{\sqrt{9}}{5}, -\frac{4\sqrt{7}}{3}, \frac{5\pi}{9}, -\frac{7\sqrt{4}}{8}, \frac{\sqrt{1}}{6}, -\frac{5\sqrt{25}}{7}$

A $-\frac{3\pi}{2}, -\frac{4\sqrt{7}}{3}, \frac{5\pi}{9}, -\frac{7\sqrt{4}}{8}$ **B** $-\frac{3\pi}{2}, -\frac{4\sqrt{7}}{3}, \frac{5\pi}{9}$

C $-\frac{3\pi}{2}, -\frac{4\sqrt{7}}{3}, -\frac{7\sqrt{4}}{8}$ **D** All of them

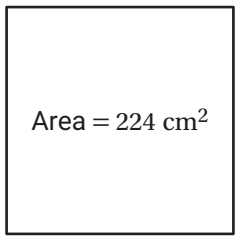
3 When 3.0071 is expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, the result is:

A $\frac{371}{10000}$ **B** $\frac{371}{1000}$ **C** $\frac{30071}{100000}$ **D** $\frac{30071}{10000}$

4 A square with an area of 500 cm^2 has a side length, expressed in simplest form, of:

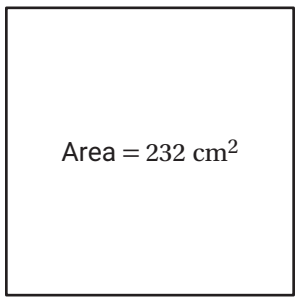
A $10\sqrt{5}$ cm **B** $5\sqrt{10}$ cm **C** 125 cm **D** $\frac{\sqrt{5}}{10}$ cm

5 Determine, in simplest form, the exact side length of the square shown below.



A $2\sqrt{56}$ cm **B** $2\sqrt{112}$ cm **C** $4\sqrt{14}$ cm **D** 14.97 cm

6 Determine the perimeter, in exact simplest form, of the square shown.



A $6\sqrt{58}$ cm^2 **B** $8\sqrt{58}$ cm^2 **C** 60.926 cm^2 **D** $2\sqrt{58}$ cm^2

7 A square with a perimeter of $12\sqrt{5}$ cm has an area of:

A $\sqrt{45}$ cm^2 **B** $3\sqrt{5}$ cm^2 **C** $9\sqrt{5}$ cm^2 **D** 45 cm^2

8 The golden ratio can be written as $\frac{1+\sqrt{5}}{2}$. If one side of a golden rectangle is 7 cm then the longer side is best represented as:

A $\frac{14}{1+\sqrt{5}}$ cm **B** $\frac{7+\sqrt{5}}{2}$ cm **C** $\frac{1+\sqrt{5}}{14}$ cm **D** $\frac{7\left(1+\sqrt{5}\right)}{2}$ cm

9 The 27th term in the standard Fibonacci sequence is 196 418. The 28th term is:

A 314 269 **B** 317 811 **C** 318 197 **D** 514 229

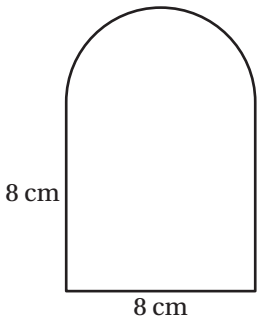
10 A circle has a circumference of 29.8 cm. Which of the following is the best estimate for the radius of the circle?

A 3.08 cm **B** 4.74 cm **C** 6.16 cm **D** 9.48 cm

11 A circle has a diameter of 10.6 cm. The best estimate for its circumference is:

A 33.3 cm **B** 44.12 cm **C** 66.6 cm **D** 88.25 cm

12 What is the perimeter of the following diagram?



A 12.57 cm **B** 36.57 cm **C** 44.57 cm **D** 49.13 cm

13 Written in exact decimal form, $\frac{5}{6}$ is:

A $0.\overline{83}$ **B** $0.\overline{833}$ **C** $0.8\dot{3}$ **D** 0.833...

14 Which of the following is the correct exact decimal representation of $\frac{47}{555}$?

A $0.\overline{0846}$ **B** $0.0\overline{846}$ **C** 0.08468 **D** $0.08\overline{46}$

15 In exact fraction form, $0.\dot{7}$ is:

A $\frac{7}{10}$ **B** $\frac{77}{100}$ **C** $\frac{7}{9}$ **D** $\frac{7}{11}$

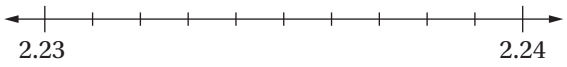
16 In exact fraction form, $0.\overline{18}$ is:

A $\frac{9}{50}$ **B** $\frac{18}{99}$ **C** $\frac{18}{101}$ **D** $\frac{9}{49}$

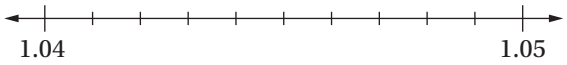
Short answer

1 Locate the following values on the number lines shown. Use an approximation correct to 3 decimal places to do this.

(a) $\sqrt{5}$



(b) $\frac{\pi}{3}$



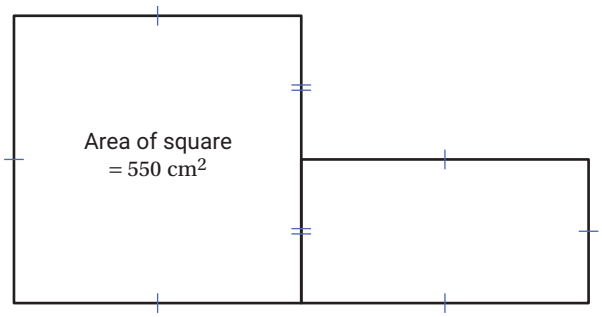
2 Complete the sentences to state the correct decimal values between which each of the following irrational values will lie, when written correct to 3 decimal places.

(a) $\frac{\sqrt{95}}{3}$ will lie between ____ and ____.

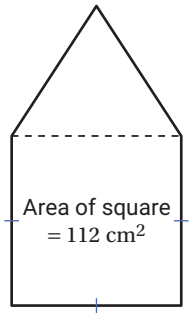
(b) $\frac{5\pi}{9}$ will lie between ____ and ____.

3 Determine the perimeter, in exact simplest form, of each of the following shapes.

(a)



(b)



4 Complete the following statement for a circle with a radius of 15 cm.

A circle with a radius of 15 cm will have a circumference less than 4 × ____ cm, but greater than 6 × ____ cm.

____ < circumference < ____

5 Write each of the following as decimals correct to 3 decimal places.

(a) $\sqrt{11}$ **(b)** $\frac{4\pi}{7}$ **(c)** $\sqrt[3]{15}$

6 Express each of the following in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, to show that the value is rational. Express the result in simplest form.

(a) $\frac{3.06}{21.74}$ **(b)** $-\frac{3.0702}{4.86}$ **(c)** $\frac{12.264}{0.0042}$

7 For each of the following numbers, written in the form of a whole number inside a cube root, state whether the number is rational or irrational.

(a) $\sqrt[3]{1}$ **(b)** $\sqrt[3]{2647}$ **(c)** $\sqrt[3]{4913}$ **(d)** $\sqrt[3]{19\,911}$ **(e)** $\sqrt[3]{24\,389}$

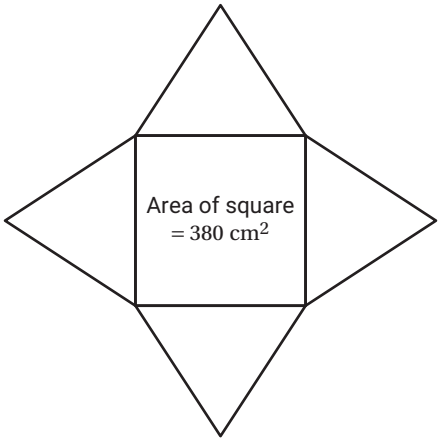
8 Substitute the given values and solve to determine the exact value of the unknown in each of the following formulas.

(a) Calculate the value of v in the formula $v^2 = u^2 + 2as$ when $u = 16$, $a = 9.8$ and $s = 15$.

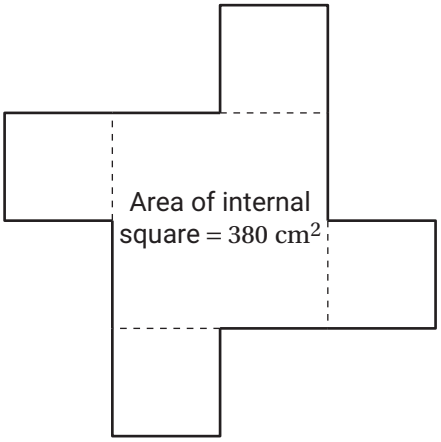
(b) Calculate the value of v in the formula $v^2 = \frac{2K}{m}$ when $K = 240$ and $m = 40$.

9 Determine the perimeter of the following shapes.

(a) In this diagram the triangles are equilateral.



(b)



10 The following questions refer to the Fibonacci-type sequence that starts 4, 4, 8, 12, ...

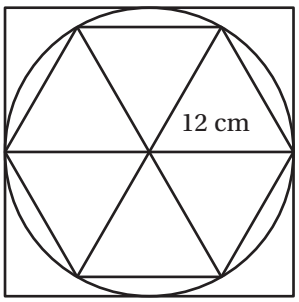
(a) Write out the first 12 terms of this sequence.

(b) Determine the ratio $\left(\frac{\text{larger}}{\text{smaller}}\right)$ between the sixth and fifth terms of this sequence.

(c) Determine the ratio $\left(\frac{\text{larger}}{\text{smaller}}\right)$ between the twelfth and eleventh terms of this sequence.

(d) What do you notice about these ratios?

11 Consider the following diagram.



Without using your calculator, complete the following statements related to between which values the circumference of the circle will fall.

The circumference of the circle is greater than _____ cm, but less than _____ cm. The actual value is likely to be closer to _____ cm.

12 A cylindrical tube has a diameter of 7.5 cm.

(a) Eden has 1.2 m of ribbon. How many complete times will she be able to wrap this tightly around the cylinder?

(b) How much ribbon, correct to 3 decimal places, is left over if only complete rotations are considered?

(c) Express this as a percentage of the original length.

13 Rewrite the following lists in descending (largest to smallest) order.

(a) $\frac{5}{8}, 0.6\dot{1}, \frac{3}{5}, 0.\overline{61}, 0.62$ **(b)** $\frac{1}{5}, \frac{2}{11}, \frac{3}{13}, 0.\dot{2}, 0.2\overline{01}$

14 Write the following repeating decimals in shorthand form.

(a) 0.383183183 ... **(b)** −3.706666 ... **(c)** −2.322323 ... **(d)** 11.991090909 ...

15 Convert the following recurring decimals to exact fractional form.

(a) $0.2\overline{345}$ **(b)** $0.\overline{023}$

Extended response

1 Consider the following decimal equivalents of some related fractions.

$\frac{1}{21} = 0.047619047619\ldots$ $\frac{2}{21} = 0.095238095238\ldots$ $\frac{3}{21} = 0.142857142857\ldots$

$\frac{4}{21} = 0.190476190476\ldots$ $\frac{5}{21} = 0.238095238095\ldots$ $\frac{6}{21} = 0.285714285714\ldots$

$\frac{7}{21} = 0.333333333333\ldots$ $\frac{8}{21} = 0.380952380952\ldots$ $\frac{9}{21} = 0.428571428571\ldots$

$\frac{10}{21} = 0.476190476190\ldots$

(a) Write each of these in exact decimal form.

(b) Compare $\frac{1}{21}$ and $\frac{4}{21}$. What do you notice? Are there any other fractions listed that share this characteristic?

(c) Compare $\frac{2}{21}$ and $\frac{5}{21}$. What do you notice? Are there any other fractions listed that share this characteristic?

(d) Compare $\frac{3}{21}$ and $\frac{6}{21}$. What do you notice? Are there any other fractions listed that share this characteristic?

(e) Can you predict the next related fraction that will be like $\frac{1}{21}$ and $\frac{4}{21}$?

2 Complete the following questions regarding the Fibonacci-type sequence that starts a, a, $a + a$, $a + a + a$, ...

(a) In a vertical list, write the first 10 terms in the sequence, and then write the simplified form for each term.

(b) If $a = 1$, what is the sequence obtained?

(c) If $a = 5$, write the first 10 terms of the sequence.

(d) Calculate the ratio $\frac{\text{term 6}}{\text{term 5}}$ for both of the sequences above. What do you notice?

(e) If $a = 7$, what will be the value of the ratio $\frac{\text{term 6}}{\text{term 5}}$?

(f) What if $a = 44$? Why is this the case?

TOPIC

4

Percentage increase and decrease

Recall 75

4.1 Understand percentage increase and decrease 76

4.2 Understand percentage change in realistic situations 83

Topic review 90

Why learn this?

Calculating the percentage change in values is an important tool for managing and interpreting changes within various contexts.

Managing budgets or resources, making informed consumer decisions, studying subjects that involve data analysis and economic reasoning, are some of the core uses of these skills. Understanding percentage calculations fosters a deeper understanding of how numbers are used in everyday life and professional settings, improving financial literacy and analytical thinking.

RECALL

ANSWERS Page 569

I can identify the change in values as representing an increase or decrease

1 Calculate the difference between the first and second value for the following, stating whether the change represents an increase or a decrease.

(a) 45 to 51 **(b)** 43 to 34 **(c)** $\frac{3}{5}$ to $\frac{2}{5}$ **(d)** 7.8 to 8.0

I can identify the relationship between decimals, percentages and fractions

1 Write the following percentages as fractions.

(a) 17% **(b)** 29% **(c)** 31% **(d)** 77% **(e)** 3%

2 Convert the following fractions to percentages.

(a) $\frac{1}{4}$ **(b)** $\frac{2}{5}$ **(c)** $\frac{3}{25}$ **(d)** $\frac{5}{8}$

I can identify and create equivalent fractions

1 List the factors of each of the following numbers.

(a) 32 **(b)** 56 **(c)** 88 **(d)** 144

2 Determine the highest common factors of the following pairs of numbers.

(a) 32 and 56 **(b)** 56 and 88

3 Determine the missing number in each of the following equivalent fractions.

(a) $\frac{\square}{4} = \frac{33}{44}$ **(b)** $\frac{4}{\square} = \frac{32}{56}$ **(c)** $\frac{25}{45} = \frac{\square}{9}$

4 Simplify the following fractions.

(a) $\frac{65}{91}$ **(b)** $\frac{84}{96}$ **(c)** $\frac{94}{144}$ **(d)** $\frac{56}{88}$

I can calculate with percentages

1 Write each number in the following pairs as a percentage of the other.

(a) 12, 24 **(b)** 12, 60 **(c)** 10, 40 **(d)** 9, 16

2 Calculate the following percentages.

(a) 1% of 360 **(b)** 10% of 360 **(c)** 5% of 360 **(d)** 20% of 360

(e) 25% of 360 **(f)** 50% of 360 **(g)** 75% of 360

4.1 Understand percentage increase and decrease

Learning intention: To be able to understand percentage increase and decrease

Success criteria:

SC 1 I can calculate the percentage increase or decrease between different values.

SC 2 I can calculate the amount of increase or decrease given a value and a percentage change.

SC 3 I can model situations and solve problems involving positive and negative rational numbers and percentages.

Lesson warm-up

Different methods

Use at least two different methods to find a correct answer to 7 + 8.

Is there more than one correct answer?

SC 1 I can calculate the percentage increase or decrease between different values

A percentage is best thought of as a value out of 100. To calculate the percentage increase or decrease between two values, determine the value of the increase or decrease and write it in fraction form, with the change written as the numerator and the original value as the denominator. Then convert it to a percentage by multiplying by 100.

Worked example

Calculating a percentage increase

In each example, calculate the percentage change.

(a) Calculate the percentage increase when 250 is increased to 300.

THINKING	WORKING
Calculate the change.	$300 - 250 = 50$
Recall the percentage value formula.	$\%\text{ change} = \frac{\text{change}}{\text{original}} \times \frac{100}{1}\%$
Substitute the amount of change and original value into the formula.	$= \frac{50}{250} \times \frac{100}{1}\%$
Calculate the value of the percentage change, writing your answer in simplest form.	$= \frac{500}{25}\%$ $= 20\%$
Write the answer.	The percentage increase is 20%.

(b) Calculate the percentage decrease when 300 is decreased to 250.

THINKING	WORKING
Calculate the change.	$300 - 250 = 50$
Recall the percentage value formula.	$\%\ \text{change} = \frac{\text{change}}{\text{original}} \times \frac{100}{1}\%$
Substitute the amount of change and original value into the formula.	$= \frac{50}{300} \times \frac{100}{1}\%$
Calculate the value of the percentage change, writing your answer in simplest form.	$= \frac{500}{30}\%$ $= 16.7\%$ (1 d.p.)
Write the answer.	The percentage decrease is approximately 16.7%.

SC 2 I can calculate the amount of increase or decrease given a value and a percentage change

In this section, you need to recall that to calculate the percentage of a number; for example, 12% of 80, the calculation is:

$$\frac{12}{100} \times \frac{80}{1} = \frac{12 \times 8}{10}$$
$$= \frac{96}{10}$$
$$= 9.6$$

Worked example

Calculating a percentage and a percentage change

A house was originally valued at $850 000.

(a) Determine the new value of the house given an increase of 2.3%.

THINKING	WORKING
Express the question in symbolic form.	2.3% of 850 000
Calculate the value of the change.	$\frac{2.3}{100} \times \frac{850\ 000}{1} = 2.3 \times 8500$ $= 19\,550$
State the change in dollar value.	The value has increased by $19 550.
Write the answer.	The house is now valued at $(850 000 + 19 550) = $869 550

(b) Determine the new value of the house given a decrease of 2.3%.

THINKING	WORKING
Express the question in symbolic form.	2.3% of 850 000
Calculate the value of the change.	$\frac{2.3}{100} \times \frac{850\ 000}{1} = 2.3 \times 8500$ $= 19\,550$
State the change in dollar value.	The value has decreased by $19 550.
Write the answer.	The house is now valued at $(850 000 – 19 550) = $830 450

SC 3 I can model situations and solve problems involving positive and negative rational numbers and percentages

4

Worked example

Determining an original price

A price increases by 8% and becomes $1.42. Determine the original price.

THINKING	WORKING
State the percentage of the original price represented by the price given.	$1.42 represents 108% of the original price.
Write the percentage as a decimal.	108% can be written as 1.08.
Determine the original price, 100%, by dividing the final price by the final percentage.	$\text{Original} = \frac{1.42}{1.08}$ $= 1.314\ 81 \ldots$
Write the answer.	The original price was $1.31.

Practice

ANSWERS Page 569

SC 1 I can calculate the percentage increase or decrease between different values

1 For each of the following number pairs, calculate the percentage increase from the first value to the second. Write your answers in exact mixed number form, if necessary.

(a) 25, 90 **(b)** 40, 110 **(c)** 100, 156 **(d)** 123, 230

2 For each of the following number pairs, calculate the percentage decrease from the first value to the second. Write your answers in exact mixed number form, if necessary.

(a) 75, 45 **(b)** 120, 95 **(c)** 88, 44 **(d)** 190, 125

3 For each of the following situations, calculate the percentage increase or decrease. Write your answers in exact mixed number form, if necessary.

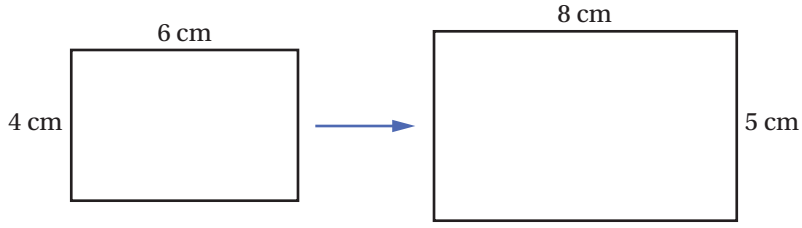
(a) Erica's hourly rate of pay increased from $17.50 to $18.25.

(b) At the end of December 2022, Nathan's house was valued at $750 500. At the end of September 2023, it was valued at $695 000.

(c) At the end of December 2022, Jillian had 12 450 followers on social media. At the end of December 2023, she had 14 230 followers.

(d) Ray was driving home from an interstate holiday. When he started, he was 1350 km from home. After two days of travelling, he was still 420 km from home.

4 Consider the following diagram, showing two rectangles.



(a) Express the change in perimeter going from the rectangle on the left to the rectangle on the right.

(b) Express the change in area going from the rectangle on the left to the rectangle on the right.

5 Calculate the percentage increase or decrease in each of the following situations. Write your answers in decimal form, correct to 2 decimal places, if necessary.

(a) 14, 22 **(b)** 18, 16 **(c)** 150, 130 **(d)** 60, 72

6 Complete the following tasks.

(a) Calculate the percentage increase from 50 to 100.

(b) Calculate the percentage decrease going from 100 to 50.

(c) Why are these two answers different when the change in value is the same?

(d) Now complete both calculations for the numbers 20 and 60.

(e) Do these calculations show the same percentage connection between the values?

SC 2 I can calculate the amount of increase or decrease given a value and a percentage change

1 Complete the following calculations.

(a) Esther earns $20.50 per hour in her part-time job. She is entitled to a 3.25% pay rise. To the nearest cent, what will her new hourly rate be?

(b) Jin-Lee is employed on a casual basis. One week, her wage was $305.90. The next week, she worked 15% less hours. Assuming the same rate of pay, what will be her wage this week?

(c) Diners in some states of the United States are expected to add a tip of 7.5% to the total bill. If you were given a bill stating a total of $62.50, what would you expect to pay?

2 Evelyn thought there had to be an easier way to determine the new value after a percentage increase than finding the increase and adding it to the original value.

(a) Evelyn was trying to calculate her new hourly rate after she received a 2.5% increase on her $18.25 per hour rate. Determine her new rate, correct to the nearest cent.

(b) Evelyn reasoned that her old rate was 100%, so her new rate must be 102.5% of her old rate. What is that percentage expressed as a decimal?

(c) Use the decimal value from part **(b)** and multiply it by the old rate. What do you get?

(d) Was Evelyn's reasoning correct? Explain.

(e) How could Evelyn extend her thinking to cover situations where there is a percentage decrease?

(f) Using both Evelyn's new method, as well as the original method, determine the new value of a car with an initial value of $42 500 that loses 15% of its value.

3 Students were asked to write a calculation to increase $75 000 by 8%. Describe what each of the following students has calculated and identify the student who did it correctly.

Student A	Student B	Student C	Student D
$1.8 \times 75\,000$	$0.92 \times 75\,000$	$1.08 \times 75\,000$	$\frac{8}{100} \times \frac{75\,000}{1}$

4 The annual inflation rate is tracked on a quarterly basis. In the first quarter there was a rise of 1.2%; in the second quarter there was a rise of 2.3%; in the third quarter there was an increase of 0.9%; and in the final quarter the inflation rate fell by 0.75%.

(a) Given that the price of milk follows the inflation rate, determine the final price of a 2 litre carton that started at $4.75.

(b) Which of the following is a correct calculation for the final price of a loaf of bread with an initial price of $2.55?

A 2.55×1.0365

B 2.55×1.0515

C $2.55 \times 1.012 \times 1.023 \times 1.009 \times 1.0075$

D $2.55 \times 1.012 \times 1.023 \times 1.09 \times 1.075$

(c) What is the annual inflation rate, expressed as a percentage, correct to 2 decimal places?

5 Complete the following calculations using whichever process you prefer.

(a) The price of standard unleaded petrol changed from $1.78 per litre to $1.59 per litre. Express this as a percentage change, correct to 2 decimal places.

(b) Determine the new price, given that the cost of petrol had increased by 7%.

(c) Determine the new price, given that the cost of petrol had decreased by 4%.

(d) The price of petrol fluctuates almost daily. On Monday, the price was $1.63 per litre. On Tuesday, it fell by 5%, on Thursday it fell another 7% and on Saturday it rose by 6%. What was the price of petrol after the rise on Saturday?

6 The local scout troop is holding a sausage sizzle as a fundraiser. The total cost of the 120 sausages and bread is $150.

(a) Given that each sausage in bread is sold at a 20% profit, determine the price charged.

(b) Given that each sausage in bread is sold at $2, and everything is sold, determine the profit.

SC 3 I can model situations and solve problems involving positive and negative rational numbers and percentages

1 Trevor said that a 5% increase in a price, followed by a 5% decrease in the price, would lead to the original value. Greg said Trevor was wrong.

(a) Assume the original price was $100. Determine who was correct about the final value, Greg or Trevor, given a 5% increase followed by a 5% decrease.

(b) Repeat this calculation given the original price of $624.

(c) For the $100 calculation, what percentage of the original value is the final value?

(d) For the $624 calculation, what percentage of the original value is the final value?

(e) Explain why this is the case.

2 The following questions are based on data calculated by the Workplace Gender Equality Agency using Australian Bureau of Statistics data (as published in *The Age*, 19/8/2022). Give answers to the following, correct to the nearest dollar.

(a) In May 2022, there was a gender pay gap of 16% in the mining industry. This represented a difference in full-time earnings of $444. Based on this figure, determine the average weekly full-time earnings for males and females in the mining industries.

(b) In the construction industries, the wage gap figures were 19% and $448. Determine the average weekly earnings for males and females in these industries.

(c) On a state basis, the biggest difference was in Western Australia, where the wage gap figures were 22% and $472. Determine the average weekly earnings for males and females in WA.

(d) The smallest dollar difference was in Tasmania, where the wage gap figures were 8% and $122. Determine the average weekly earnings for males and females in Tasmania.

3 Calculate the single percentage change represented by each of the following sequences of change.

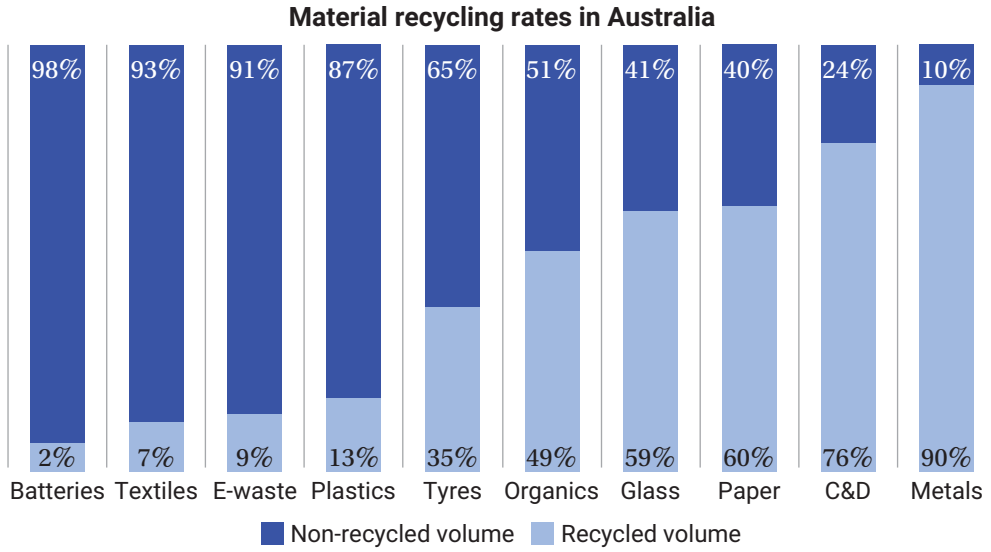
(a) An increase of 3%, followed by a decrease of 6%, followed by an increase of 5%.

(b) An increase of 5%, followed by a decrease of 6%, followed by an increase of 3%.

(c) A decrease of 6%, followed by an increase of 5%, followed by an increase of 3%.

(d) What point does this illustrate?

4 The following graphic gives information about recycling rates in Australia in 2021.



(a) The total mass of batteries recycled was 3.3 kilotonnes (kt). What mass of batteries were not recycled?

(b) The total mass of tyres recycled was 4490 kt. What mass of tyres were not recycled?

(c) The total mass of paper recycled was 5920 kt. What is the total mass of paper in the recycling/waste market?

(d) Why do you think the recycling rate for metals is so high?

5 Complete the following calculations.

(a) The temperature at a ski resort rose 3% to −6.8 °C. Determine the original temperature, correct to 1 decimal place.

(b) Jacques was diving at −110 m before reducing his depth by 18%. Determine his new depth.

(c) Determine the percentage change, correct to 1 decimal place, for a temperature that rises from −10.8 °C to −1.7 °C.

(d) Determine the percentage change, correct to 1 decimal place, if a temperature falls from 2.9 °C to −5.7 °C.

6 The Antarctic is much colder than the Arctic. In Antarctica during winter, the mean temperatures range from −20 °C to −30 °C on the coast and −40 °C to −70 °C in the interior. In Antarctica during the summer, the mean temperatures are about 0 °C on the coast and −20 °C to −30 °C in the interior. In the Arctic, the means range from about 0 °C in summer to about −35 °C in the winter.

(a) Using the warmer end of the range in each case, determine the percentage change in Antarctic coast temperatures from winter to summer.

(b) Using the cooler end of the range in each case, determine the percentage change in Antarctic interior temperatures from summer to winter.

(c) Over the past 50 years, mean temperatures in the Antarctic have risen by 3 °C. Determine what the percentage change this is for the coast in summer.

(d) Using the warmer end of the scale, determine the percentage difference between the interior winter temperature in the Antarctic compared to the Arctic.

Understand percentage change in realistic situations

4.2

Learning intention: To be able to understand percentage changes in realistic situations

Success criteria:

SC 1 I can explain and calculate mark-ups, discounts and GST.

SC 2 I can determine changes as percentage increase or decrease in realistic situations.

SC 3 I can explain income tax and calculate taxation.

Lesson warm-up

Will they fit?

Estimate the percentage of the world's population that could stand on Tasmania at the same time. Be prepared to share your answer and your reasoning.

SC 1 I can explain and calculate mark-ups, discounts and GST

Mark-ups, discounts and the Goods and Services Tax (GST) are all examples of percentages. While mark-ups and discounts can vary in size, the GST is applied at a fixed rate of 10%.

A mark-up or a discount can be expressed in percentage form or as a dollar amount, which can then be converted to a percentage.

Worked example

Calculating the selling price after a discount or mark-up

Determine the selling price of a television marked as $2399 after:

(a) a discount of 15% is applied.

THINKING	WORKING
Calculate the selling price percentage by deducting the discount from 100%, the original price.	(100 – 15)% = 85%
Convert the percentage to a decimal.	85% = 0.85
Multiply the decimal by the marked price.	0.85 × 2399 = 2039.15
Write the answer.	The television will sell for $2039.

(b) a mark-up of 15% is applied.

THINKING	WORKING
Calculate the selling price percentage by adding the mark-up to 100%, the original price.	(100 + 15)% = 115%
Convert the percentage to a decimal.	115% = 1.15
Multiply the decimal by the marked price.	$1.15 \times 2399 = 2758.85$
Write the answer.	The television will sell for $2758.85.

SC 2 I can determine changes as percentage increase or decrease in realistic situations

Percentages are applicable to everyday life. The techniques covered so far can be used in these situations as well.

4

Worked example

Applying percentage increase

According to real estate data, the median house price in a particular suburb of Melbourne in June 2022 was $750 000, which represented an 8.54% growth over a 12-month period. What was the median house price in the same suburb in June 2021?

THINKING	WORKING
Describe the known percentage change.	The 2022 median price is (100 + 8.54)% = 108.54% of the 2021 median price.
Express the percentage as a decimal.	108.54% = 1.0854
Determine the original price by dividing the known price by the decimal.	$\frac{750\,000}{1.0854} = 690\,989.90$
Write the answer.	The median price one year ago was about $691 000.

SC 3 I can explain income tax and calculate taxation

Income tax is the most important source of income for the federal government. The money raised is used to provide funding for national concerns such as defence. Some of the money is given to the state governments to provide for hospitals and schools.

The Australian taxation system is a progressive system. This means that the more money you earn, the greater the tax rate applied to your earnings.

The following table shows the applicable tax rates for the financial year 2023–2024.

Income	Rate	Tax payable
$0–$18 200	0%	Nil
$18 201–$45 000	19%	19 c for each $1 over $18 200
$45 001–$120 000	32.5%	$5092 plus 32.5 c for each $1 over $45 000
$120 001–$180 000	37%	$29 467 plus 37 c for each $1 over $120 000
$180 000 and over	45%	$51 667 plus 45 c for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia

The rates shown are known as the marginal tax rates. So a person earning $95 000 has a marginal tax rate of 32.5%.

Worked example

Calculating income tax using a tax table

Calculate the income tax payable on an annual income of $75 600.

THINKING	WORKING
Identify the marginal tax rate.	The tax bracket is $45 001–$120 000. The marginal tax rate is 32.5%.
Identify the starting amount of tax for this marginal tax rate.	For 32.5%, the starting amount of tax is $5092.
Subtract the 'over' number of the bracket from the income stated.	$(75 600 – 45 000) = $30 600
Multiply this value by the tax rate to calculate the additional amount of tax.	32.5% of $30 600 = 0.325 × 30 600 = $9945
Add the additional amount of tax to the starting amount of tax to calculate the full tax payable.	$(5092 + 9945) = $15 037

Practice

ANSWERS Page 570

SC 1 I can explain and calculate mark-ups, discounts and GST

1 In Australia, a Goods and Services Tax (GST) is applied at a fixed rate of 10% to a wide range of goods and services.
In each of the following situations, calculate the final amount due, given that the value stated does not include GST.

(a) A plumber charges $430 for work on a bathroom renovation.

(b) A book has a price of $35.

(c) A caravan has a price of $23 900.

(d) A room at the holiday resort is $315 per night.

2 In reality, many prices already include the GST. In each of the following cases, calculate:

(i) the pre-GST price

(ii) the amount of GST that has been included in the stated price.

(a) A family meal at a restaurant costs $260.

(b) A block of chocolate costs $4.90.

(c) A jumper costs $135.99.

(d) A tank of petrol costs $112.50.

3 Complete the following statements.

(a) When calculating the price, including GST, multiply/divide the non-GST price by ____.

(b) When calculating the price, excluding GST, multiply/divide the GST-inclusive price by ____.

4 Paul wants to buy a new television. He has seen the model he wants advertised in sales at three different retailers. The offers he has seen are:

- Retailer A: List price of $4850, with a 22% discount
- Retailer B: A fixed price of $4150
- Retailer C: A pre-GST price of $4900, with a 25% discount

Calculate the price at each retailer and place the retailers in order from the least to the most expensive.

5 A clothing retailer has a policy of marking up the cost price of items by 35%. The GST is not applied until after the mark-up has been completed. Calculate the selling price for the following items, with cost price as indicated. Give your answers correct to the nearest dollar.

(a) A pair of shoes, $145

(b) A suit, $450

(c) An umbrella, $23.50

(d) A hat, $49

6 A dining table set has a GST-inclusive price of $2640.

(a) How much of the price is represented by the GST?

(b) After a few weeks the table has not been sold, so it is offered at a discount of 10%. At what price is it now advertised?

(c) After a couple more weeks, the price is reduced by a further 12.5%. What is the new advertised price?

(d) Given that the table is sold at the reduced price, and GST is calculated on this price, calculate the percentage profit, correct to 2 decimal places, made by the retailer if it had a cost price of $1850.

SC 2 I can determine changes as percentage increase or decrease in realistic situations

1 The following table shows the Australian population recorded at 10-year intervals.

Year	Population
1980	14 706 322
1990	17 048 003
2000	19 017 963
2010	22 019 168
2020	25 670 051

(a) In which 10-year period did the Australian population increase by the most people? What was this number?

(b) In which 10-year period did the Australian population increase by the greatest percentage? What was this percentage, correct to 2 decimal places?

(c) Assuming that the population from 1980 until 2020 increased at this maximum 10-year percentage rate, what would the population have been in 2020?

(d) The minimum 10-year percentage rate in this time period was 11.57%. Given that the population from 1980 until 2020 increased at this minimum 10-year percentage rate, what would the population have been in 2020?

2 The following table shows the life expectancy for Australians in the time period 1950–2020.

Year	Life expectancy (years)
1950	68.77
1960	70.61
1970	71.21
1980	74.19
1990	76.79
2000	79.44
2010	81.83
2020	83.50

(a) In which 10-year period did Australian life expectancy increase the most? By how many years did it increase?

(b) In which 10-year period did Australian life expectancy increase by the greatest percentage? What was this percentage, correct to 2 decimal places?

(c) Assuming that life expectancy from 1980 until 2020 increased at this maximum 10-year percentage rate, what would the life expectancy have been in 2020?

(d) The minimum 10-year percentage rate in this time period was 0.85%. Given that life expectancy from 1980 until 2020 increased at this minimum 10-year percentage rate, what would the life expectancy have been in 2020?

3 The following table shows the quantity of CO_2 (carbon dioxide) released into the atmosphere in Australia, recorded on a per capita basis. This means that the total amount is divided by the population at the time to give a more realistic figure about the quantity involved.

Year	Metric tons of CO_2 (per capita)
1994	15.69
1999	17.63
2004	18.17
2009	18.22
2014	15.83
2019	15.24

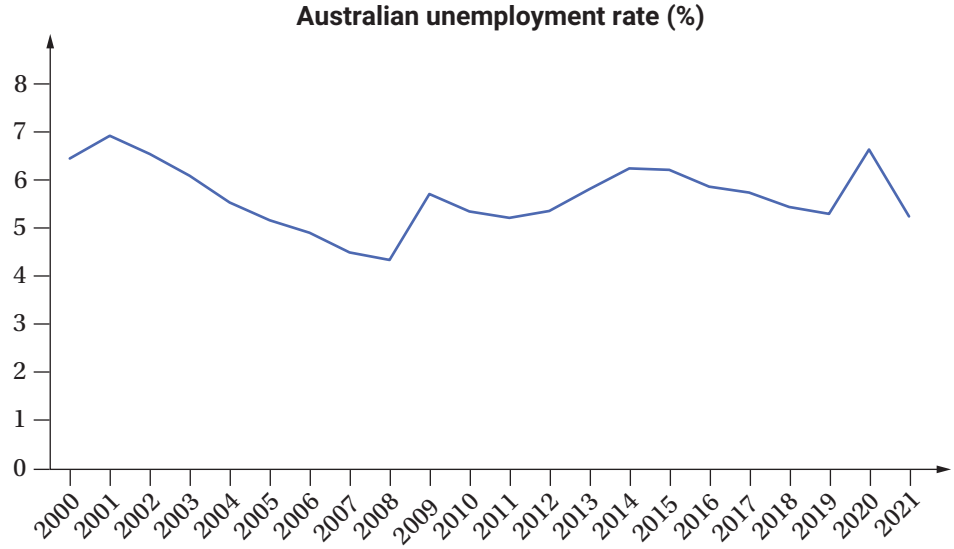
(a) What is different about these figures compared to the previous two tables?

(b) Given that the percentage change from 2014 to 2019 is repeated in the next five-year period, what would the predicted figure be for 2024?

(c) If the change in CO_2 emissions that occurred from 1994 to 1999 had been repeated each five years, what would the figure have been in 2019?

(d) What is the percentage difference between this predicted figure and the actual figure recorded?

4 The following graph shows the Australian unemployment rate for the period 2000–2021.



(a) Describe the trends you can see in this graph.

(b) When a graph or a table of data represents percentages, care needs to be taken in describing any changes that are seen. In 2000, the unemployment rate was 6.28%, and in 2001, it was 6.74%. This can be described as a rise of 0.46 percentage points, but the actual percentage change is different. What is the actual percentage change between the two figures?

(c) When did the biggest upturn in unemployment occur?

(d) When did the biggest downturn in unemployment occur?

SC 3 I can explain income tax and calculate taxation

1 Use the following tax table to determine the tax payable on each of the following annual incomes.

Income	Rate	Tax payable
$0–$18 200	0%	Nil
$18 201–$45 000	19%	19 c for each $1 over $18 200
$45 001–$120 000	32.5%	$5092 plus 32.5 c for each $1 over $45 000
$120 001–$180 000	37%	$29 467 plus 37 c for each $1 over $120 000
$180 000 and over	45%	$51 667 plus 45 c for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia

(a) $19 850 **(b)** $42 750 **(c)** $82 900 **(d)** $128 000

2 Determine the percentage of total income paid in income tax by a person who earns each of the following annual incomes. Give your answers correct to 1 decimal place.

(a) $33 280 **(b)** $88 765 **(c)** $156 745 **(d)** $220 345

3 Some people think that a flat rate of taxation for everyone would be fairer. Determine the amount of tax that would need to be paid on each of the following annual incomes if there was a 17.5% flat rate of taxation.

(a) $33 280 **(b)** $88 765 **(c)** $156 745 **(d)** $220 345

(e) Which end of the income scale benefits from such a scheme?

4 A family has two working-age members.

(a) Determine the income tax payable if only one of the members works and has an annual income of $160 000.

(b) Determine the income tax payable if both members work and each earns $80 000.

(c) Why is there a difference in these two amounts of tax when the annual income is the same?

TOPIC REVIEW

Multiple choice

ANSWERS Page 620

1 The percentage increase when 75 is increased to 100 is:

A 25% **B** 30% **C** $33\frac{1}{3}\%$ **D** 133%

2 The percentage decrease when 480 falls to 360 is:

A −133% **B** −120% **C** 25% **D** 75%

3 Audrey's hourly rate of pay increases by $1.20 per hour on her birthday. If her original hourly rate was $18.25 then the increase is closest to:

A 1.2% **B** 6.58% **C** 12% **D** 17.05%

4 A soft drink was previously sold in a can with capacity 375 mL. The can is downsized by 5%. The new can has capacity:

A 18.75 mL **B** 356.25 mL **C** 370 mL **D** 393.75 mL

5 Which of the following is a correct calculation for determining the new value of a house with an initial value of $955 000 that increases in value by 7.5%?

A $955\,000 \times 1.075$ **B** $955000 \times \frac{7.5}{100}$ **C** $955000 \times \frac{100}{7.5}$ **D** $\frac{955000}{0.75} \times 100$

6 The price of petrol is known to fluctuate wildly. The price per litre on the Monday of each week over one month was:

Week 1: $1.79/L Week 2: $2.25/L Week 3: $1.99/L Week 4: $1.83/L

The average change from week to week is closest to:

A 1% increase **B** 2% increase **C** 3% increase **D** 4% increase

7 A price rose $17\frac{2}{3}\%$ and became $2150. To the nearest dollar, the original price was:

A $38 **B** $1770 **C** $1827 **D** $2530

8 A shirt has a full price of $99.99. In the first week of a sale the price is reduced by 10%. In the second week it is reduced by a further 10%. The marked price of the shirt is now:

A $19.99 **B** $79.99 **C** $80.99 **D** $89.99

9 A plumber charges $616 for a job. The price includes GST, which is charged at a rate of 10%. The amount of GST included is:

A $56 **B** $61.60 **C** $554.40 **D** $560

10 A retail store pays $500 for an article, applies a mark-up of 35%, then adds the 10% GST and advertises it for sale. The price set is:

A $550 **B** $675 **C** $725 **D** $742.50

11 An item advertised for $750 is reduced by 15%. The reduced price includes the GST payable (10%). How much GST is payable on this reduced price?

A $56.25 **B** $63.75 **C** $68.18 **D** $75

12 According to real estate data, the median house price in a Victorian regional city in June 2022 was $752 500, representing 13.92% growth over a 12-month period. The median house price in the same city 12 months ago was closest to:

A $647 752 **B** $660 100 **C** $676 800 **D** $857 250

13 A school has an enrolment of 950. Two years ago it was 770. The annual increase is closest to:

A 9% **B** 12% **C** 18% **D** 23%

14 The population of a small town experienced growth rates over a three-year period of 2%, 1.5% and 2.3%. At the start of this period the population was 10 950. At the end of this time, the population is closest to:

A 11 337 **B** 11 421 **C** 11 585 **D** 11 597

15 Jackson has an annual income of $147 250. This puts him in the tax bracket where he must pay $29 467 plus 37 c for each $1 over $120 000. His tax liability is closest to:

A $10 082.50 **B** $25 015.50 **C** $39 549.50 **D** $54 482.50

16 In a particular country, tax is payable at a flat rate of 17.5% for the first $75 000 and then a flat rate of 22.5% for any additional income. Which of the following best represents the tax payable on an income of $120 000?

A $10 125 **B** $13 125 **C** $23 250 **D** $27 000

17 Based on a flat tax rate of 18% Erica paid $23 140 in tax. How much had she earned?

A $27 305 **B** $64 792 **C** $128 556 **D** $416 520

Short answer

1 A house was purchased for $785 600 and sold some years later for $905 400.

(a) What was the profit in dollar terms?

(b) The real estate agent charged a fee of 1.9% of the sale price. How many dollars did this amount to?

(c) Taking into account this fee, what percentage profit was made on the sale? Give your answer correct to 2 decimal places.

2 Julian needs to purchase a new laptop computer. The model he wants has a sale price of $1599. This includes the GST of 10%. What is the amount (exclusive of GST) that the retailer earns for this purchase?

3 A 12-pack of 300 mL of soft drink is purchased for $10.

(a) How much GST will the retailer pay the government for this purchase?

(b) The pack is broken up into individual bottles and sold by the local sporting club at a mark-up of 60%. What is the price charged, to the nearest 5 cents, for each bottle?

(c) If instead of this price the club actually charges $1.50 per bottle, what percentage mark-up would this represent?

4 Calculate the discount percentage, correct to 2 decimal places, on the following sale items.

(a) A TV discounted from $3999 to $3569

(b) A shirt discounted by $17.50 from the original price of $124.99

5 The local golf club indicates that all membership categories will have a 3.5% increase for the following year. The table indicates the current fees.

Category	Adult	Junior	Sub-Junior	5-day Adult
Annual fee	$2590	$1420	$250	$2000

Rewrite the table showing the new fees, rounding to the nearest $5 if necessary.

6 The following prices do not include the GST (10%). In each case, state, to the nearest cent, the price after GST is included.

(a) A motor car purchased for $45 990

(b) A TV purchased for $3549

(c) A bar of chocolate purchased for $3.50

7 Calculate the percentage change in each of the following situations. Give your answers correct to 2 decimal places.

(a) 290 is increased to 310

(b) 92.3 is increased to 96.5

(c) 410 is decreased to 375

(d) 116.7 is decreased to 101.8

8 Over the course of 3 days the price of 1 litre of petrol rose from its starting price of $1.86 per litre by 5 cents per litre on the second day and by 7 cents per litre on the third day.

(a) After the rise on the third day what was the price of petrol?

(b) By what percentage has the price risen in this time period? State your answer correct to 2 decimal places.

(c) On day four the price is reduced to its starting point, $1.86 per litre. What percentage reduction does this represent? State your answer correct to 2 decimal places.

(d) Why aren't the two percentage changes the same?

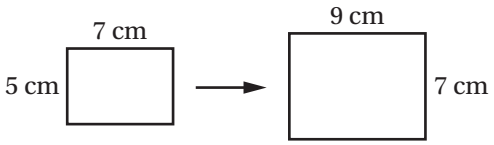
9 The following prices already include the GST cost. Determine the pre-GST cost.

(a) A book with a sale price of $47.85

(b) A drink with a sale price of $4.07

(c) A camper trailer with a sale price of $27 995

10 Consider the following diagram, showing two rectangles.



(a) Starting from the left-hand rectangle, express the change in perimeter going to the right-hand rectangle.

(b) Starting from the left-hand rectangle, express the change in area going to the right-hand rectangle.

11 A painting purchased for $14 500 is sold later for $22 300.

(a) Determine the dollar value of the increase in value.

(b) State the percentage increase in value. Give your answer correct to 2 decimal places.

(c) The person who sold the painting had to pay a $7\frac{1}{2}$% sales commission, which is taken out of the sale price. How much did the seller receive?

(d) What was the percentage profit made on the sale? Give your answer correct to 2 decimal places.

12 Complete the following calculations.

(a) Zac earns $21.70 per hour in his part-time job. He is entitled to a 2.95% pay rise. To the nearest cent, what will his new hourly rate be?

(b) Min-Woo is employed on a casual basis. His wage one week was $425.75. The next week he worked 11% fewer hours. Assuming the same rate of pay, what will be his wage this week?

(c) Diners in some states of the United States are expected to add a tip of 9% to the total of the bill. If the bill states a total of $74.90, what is the expected payment?

13 Calculate the single percentage change represented by each of the following sequences of changes.

(a) An increase of 7%, followed by a decrease of 4%, followed by an increase of 3%.

(b) An increase of 3%, followed by a decrease of 4%, followed by an increase of 7%.

(c) A decrease of 4%, followed by an increase of 7%, followed by an increase of 3%.

(d) What point does this illustrate?

14 Zander wants to buy a new television. He has seen the model he wants advertised in sales at three different retailers. The offers he has seen are:

- Retailer A: List price of $3990 with a 12% discount
- Retailer B: A fixed price of $3450
- Retailer C: A pre-GST price of $4100 with a 25% discount applied before GST is added

Calculate the price in each retail store and place the retailers in order from the cheapest to the most expensive.

Extended response

1 The following table shows the applicable tax rates for the financial year 2023–2024.

Income	Rate	Tax payable
$0–18 200	0%	Nil
$18 201–$45 000	19%	19 c for each $1 over $18 200
$45 000–$120 000	32.5%	$5092 plus 32.5 c for each $1 over $45 000
$120 001–$180 000	37%	$29 467 plus 37 c for each $1 over $120 000
$180 000 and over	45%	$51 667 plus 45 c for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia

(a) Calculate the tax payable on an income of exactly $45 000.

(b) Explain how you can identify the tax payable on an income of $120 000 from the table.

(c) Calculate the percentage of tax paid on an annual income of $100 000, correct to 2 decimal places.

(d) What is the weekly income of Erica, who earns $118 000 annually? Round this to the nearest dollar.

(e) Compare the tax payable on Erica's current salary to that payable after she receives a pay increase of $75 per week.

(f) Why do you think a taxation system like this is called a 'progressive' tax system?

2 The following table shows the tax rates that apply to foreign residents for the tax year 2023–2024.

Income	Tax payable
$0–$120 000	32.5 c for each $1
$120 001–$180 000	$39 000 plus 37 c for each $1 over $120 000
$180 001 and over	$61 200 plus 45 c for each $1 over $180 000

(a) Johsi is a foreign resident who earns $65 450 in the financial year. How much tax will he pay?

(b) Christiaan earns $132 150 in his job, but has paid $40 000 in instalment payments to the tax office. How much more does he owe?

(c) Ciaran earns $193 000. What percentage of his earnings does he end up paying in tax?

3 The following table shows the life expectancy from birth for New Zealand males in the time period 1950–1952 to 2017–2019.

(a) In which time period did the New Zealand male life expectancy increase by the greatest number of years? What was this number?

(b) In which time period did the male life expectancy fall?

(c) In which time period did the New Zealand male life expectancy increase by the greatest percentage? What was this percentage, correct to 2 decimal places?

(d) If the life expectancy from 1950–1952 until 2017–2019 increased at this maximum percentage rate, what would the life expectancy have been in 2017–2019?

Year	Male life expectancy (years)
1950–1952	67.2
1955–1957	68
1960–1962	68.4
1965–1967	68.2
1970–1972	68.5
1975–1977	69
1980–1982	70.4
1985–1987	71.1
1990–1992	72.9
1995–1997	74.4
2000–2002	76.3
2005–2007	78
2012–2014	79.5
2017–2019	80

Based on data from © Stats NZ Tatauranga Aotearoa

TOPIC

5

Algebra (expand, simplify and factorise)

Recall 96

5.1 Use the distributive law to expand simple algebraic expressions 97

5.2 Factorise simple algebraic expressions 100

5.3 Simplify algebraic expressions involving multiple operations 104

Topic review 108

Why learn this?

Understanding how to expand, simplify and factorise algebraic expressions using the distributive law is a fundamental skill in algebra. These skills are an important foundation for further studies in more complex areas of mathematics, engineering, economics, the sciences, and where algebra is used to model and solve real-world problems.

RECALL

ANSWERS Page 570

I can write expressions from words

1 Rewrite each of the following sentences using mathematical symbols.

(a) Eleven plus n is equal to sixteen.

(b) Fourteen is subtracted from m and the result is nine.

(c) x is equal to the difference between y and ten.

(d) The product of five and n is equal to forty-five.

(e) The sum of three and nineteen is divided by q to give a result of seven and one-third.

I can apply the order of operations to calculate the value of an expression

1 Evaluate the following.

(a) $(104) + (2 + 13)$ **(b)** $4 \times 83 \times 9$ **(c)** $2113 - 18 \div 6$

(d) $12 \div (15 - 9) \times 5$ **(e)** $22 - 28 \div 4 \times 3 + 4$

5

I can identify terms and coefficients in expressions and equations

1 Identify the number of terms for each expression.

(a) $6x - 3y$ has ____ terms.

(b) $9x - 3x + 2y - 20$ has ____ terms.

(c) There are ____ terms in the expression $a - b + 4$.

2 Choose the correct answer.

(a) $5a - 3b + 4$ is:

A a term **B** an expression **C** an equation **D** a constant

(b) The coefficient of y in the expression $10 + 5x - 4y$ is:

A -4 **B** 4 **C** 5 **D** 10

I can identify the highest common numerical factor

1 Determine the HCF of each of the following pairs of numbers.

(a) 20 and 45 **(b)** 36 and 72 **(c)** 24 and 36

I can identify all the prime factors of numbers

1 Complete a factor tree, then express the number as a product of its prime factors in exponent form.

(a) 168 **(b)** 216

Use the distributive law to expand simple algebraic expressions

5.1

Learning intention: To be able to use the distributive law to expand simple algebraic expressions

Success criteria:

SC 1 I can describe and illustrate the distributive law using an area model.

SC 2 I can use the distributive law to expand algebraic expressions.

Lesson warm-up

Mental maths

List the different strategies that you can use to solve the following problems.

(a) 25×32

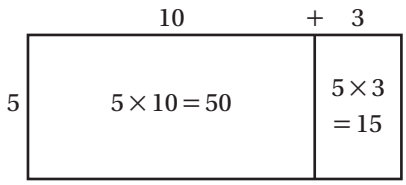
(b) 9×17

(c) 23×46

SC 1 I can describe and illustrate the distributive law using an area model

The distributive law states that in an expression such as $3(4 + 2)$, you can either add the numbers in brackets first, then multiply by 3, or you can multiply 4 and 2 separately by 3, then add the results. The result is the same. You can understand this by using this example and the area model for multiplication.

$5(10 + 3)$ is the same as 5×13.

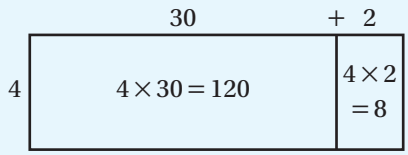


$$\begin{aligned} 5 \times 13 &= 5 \times (10 + 3) \\ &= 5 \times 10 + 5 \times 3 \\ &= 65 \end{aligned}$$

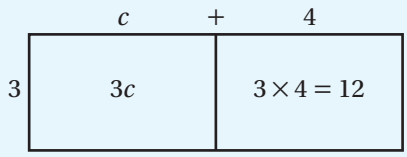
Worked example

Describing the distributive law and illustrating using the area model

(a) Use the area model to show the application of the distributive law to 4×32.

THINKING	WORKING
Break 32 into tens and ones. Draw an area diagram to show the multiplication.	
Write the answer.	$4 \times 32 = 4(30 + 2)$ $= 4 \times 30 + 4 \times 2$ $= 128$

(b) Use the area model to show the application of the distributive law to $3(c + 4)$.

THINKING	WORKING
Break the brackets into its components. Draw an area diagram to show the multiplication.	
Write the answer.	$3(c + 4) = 3 \times c + 3 \times 4$ $= 3c + 12$

SC 2 I can use the distributive law to expand algebraic expressions

The distributive law can be described as: $a(b + c) = ab + ac$

Or, if there are more terms: $a(b + c + d) = ab + ac + ad$

When you expand, the coefficient is placed first and the variables are arranged in alphabetical order.

For example:

$4x(3y + d) = 4x \times 3y + 4x \times d$
$= 12xy + 4dx$

Worked example

Using the distributive law to expand algebraic expressions

Expand each expression using the distributive law.

(a) $y(2a - 6)$

THINKING	WORKING
Multiply each of the terms inside the brackets by the factor outside.	$y(2a - 6) = y \times 2a - 6y$ $= 2ay - 6y$

(b) $-d(5d - 6)$

THINKING	WORKING
Multiply each of the terms inside the brackets by the factor outside.	$-d(5d - 6) = -d \times 5d + (-d) \times (-6)$ $= -5d^2 + 6d$

Practice

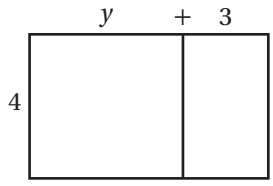
ANSWERS Page 571

Use the distributive law to expand simple algebraic expressions

SC 1 I can describe and illustrate the distributive law using an area model

1 Draw a diagram to show how you calculate 22×7 (if you can, draw a second method). Discuss your preferred model with a partner or small group and decide which model your group thinks is the best.

2 Write the expression represented in this area model diagram.



3 In a new housing estate, each rectangular house block is 32 metres deep. Two blocks next to each other are 24 m and 27 m wide respectively. Write an expression that could be used to find the total area of the two blocks.

SC 2 I can use the distributive law to expand algebraic expressions

1 Expand each expression using the distributive law.

(a) $2(x + 4)$ **(b)** $9(x + 1)$ **(c)** $-3(x - 4)$

(d) $-3(2.1 - b)$ **(e)** $3a(2a - 5)$ **(f)** $-12x(8 - 5x^2)$

2 Expand the expression $m(n + o)$.

3 Expand each expression. Simplify if possible.

(a) $7(g + 5) + 3g$ **(b)** $3(b - 5) + 7b$ **(c)** $6(x + 2) + 3(x + 7)$

(d) $4(b + 2) + 3b(a + 1)$ **(e)** $4(x - 2) - 3x(x - 1)$ **(f)** $-4(y + 2) - 3y(5 - y)$

5.2 Factorise simple algebraic expressions

Learning intention: To be able to factorise simple algebraic expressions

Success criteria:

SC 1 I can factorise algebraic expressions by finding a numerical common factor.

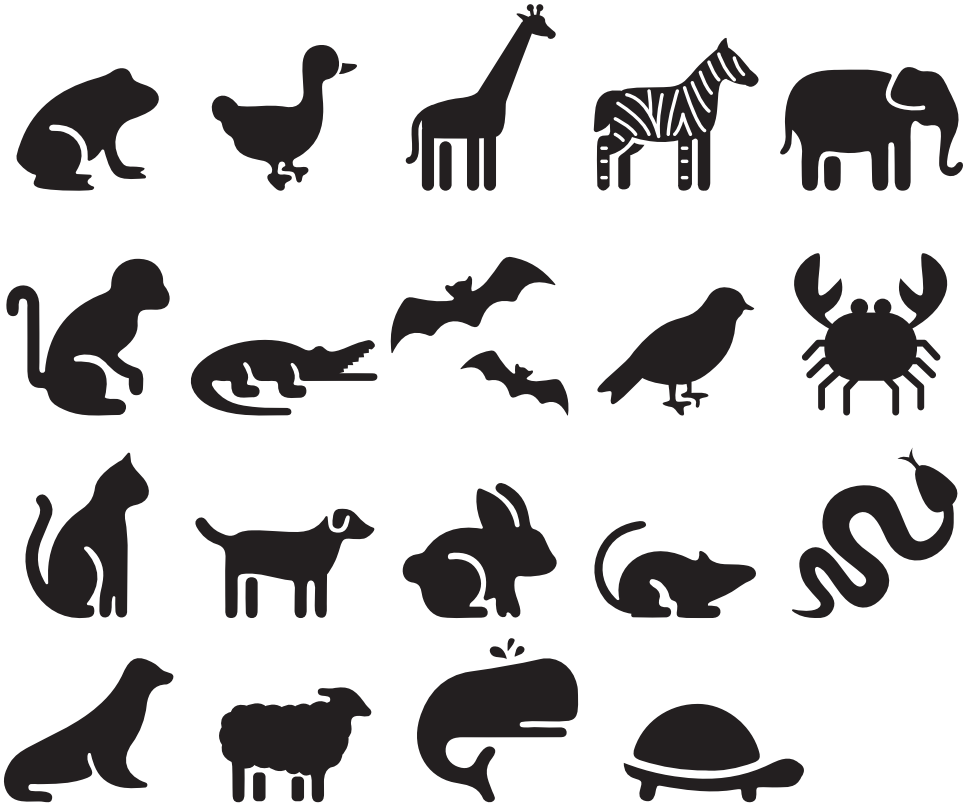
SC 2 I can factorise expressions by finding an algebraic common factor.

SC 3 I can factorise expressions in which the highest common factor is made up of a number and a variable.

Lesson warm-up

What is common?

Sort each of the following pictures into groups and justify your groupings.



SC 1 I can factorise algebraic expressions by finding a numerical common factor

Factorising is the opposite of expanding. An expression is factorised when any common factors have been taken outside a set of brackets.

Worked example

Factorising using a numerical common factor

Factorise the expression $6x + 12$.

THINKING	WORKING
Write each term as a product of its prime factors.	$6x = 2 \times 3 \times x$ $12 = 2 \times 2 \times 3$
Rewrite the expression in expanded form using prime factors.	$6x + 12 = 2 \times 3 \times x + 2 \times 2 \times 3$
Identify all common prime and variable factors. State the highest common factor (HCF).	$2 \times 3 \times x + 2 \times 2 \times 3$ The HCF is $2 \times 3 = 6$
Rewrite the expression, taking the HCF outside a pair of brackets.	$2 \times 3 \times x + 2 \times 2 \times 3$ $= 2 \times 3 \times (x + 2)$ $= 6(x + 2)$
Write your answer.	$6x + 12 = 6(x + 2)$

SC 2 I can factorise expressions by finding an algebraic common factor

A common factor does not need to be restricted to a numerical value. It can also be a variable.

Worked example

Factorising using an algebraic common factor

Factorise the expression $5ab + 3a$.

THINKING	WORKING
Write each term as a product of its prime factors.	$5ab = 5 \times a \times b$ $3a = 3 \times a$
Rewrite the expression in expanded form using prime factors.	$5ab + 3a = 5 \times a \times b + 3 \times a$
Identify all common prime and variable factors. State the HCF.	$5 \times a \times b + 3 \times a$ The HCF is a
Rewrite the expression, taking the HCF outside a pair of brackets.	$5 \times a \times b + 3 \times a$ $= a \times (5 \times b + 3)$ $= a(5b + 3)$
Write the answer.	$5ab + 3a = a(5b + 3)$

SC 3 I can factorise expressions in which the highest common factor is made up of a number and a variable

The highest common factor is not restricted to being just a numerical value or a variable by itself. It can also be a combination of the two. For example, the highest common factor for $12x - 24x^2$ is $12x$, which is $12x - 24x^2 = 12x(1 - 2x)$ in factorised form.

Worked example

Factorising using an algebraic term as the common factor

Factorise the expression $36a^2b - 27ab^2$.

THINKING	WORKING
Write each term as a product of its prime factors.	$36a^2b = 2 \times 2 \times 3 \times 3 \times a \times a \times b$ $27ab^2 = 3 \times 3 \times 3 \times a \times b \times b$
Rewrite the expression in expanded form using prime factors.	$36a^2b - 27ab^2$ $= 2 \times 2 \times 3 \times 3 \times a \times a \times b - 3 \times 3 \times 3 \times a \times b \times b$
Identify all common prime and variable factors. State the HCF.	The HCF is $3 \times 3 \times a \times b = 9ab$
Rewrite the expression, taking the HCF outside a pair of brackets.	$2 \times 2 \times 3 \times 3 \times a \times a \times b - 3 \times 3 \times 3 \times a \times b \times b$ $= 3 \times 3 \times a \times b \times (2 \times 2 \times a - 3 \times b)$ $= 9ab(4a - 3b)$
Write your answer.	$36a^2b - 27ab^2 = 9ab(4a - 3b)$

Practice

ANSWERS Page 571

Factorise simple algebraic expressions

SC 1 I can factorise algebraic expressions by finding a numerical common factor

1 Factorise each expression.

(a) $9x + 30$ **(b)** $8x - 14$ **(c)** $6x - 36$ **(d)** $15x - 3$

2 Factorise each expression.

(a) $24 - 8x$ **(b)** $32 - 12x$ **(c)** $18x + 4y + 10$ **(d)** $26x - 13y - 39$

3 Complete the following sentences.

(a) In factorised form, $48 - 6x$ is best represented as _____.

(b) In factorised form, $26b - 13a + 39$ is best represented as _____.

4 State the highest common factor for each group of terms.

(a) $12a$, 24, $90a$, 36 **(b)** $9b$, $12b$, 72, 81

(c) $10c$, $100c$, $1000c$, 50 **(d)** $11d$, $121d$, 99, 2200

SC 2 I can factorise expressions by finding an algebraic common factor

1 Factorise each expression.

(a) $3a + 7ab$ **(b)** $26a - 15ab$ **(c)** $15bc - 8c$ **(d)** $19cd + 11d$

2 Factorise each expression.

(a) $12b + 13b^2$ **(b)** $13b^2 - 21b$

(c) $15a^2 - 7ab + 2a$ **(d)** $27a^2 - 7ab + 11ab^2$

3 Complete the following sentences.

(a) In factorised form, $15b - 13ab + 7ab^2$ is best represented as _____.

(b) In factorised form, $26a^2b^2 + 15ab - 13b^2$ is best represented by _____.

4 Express the answers to the following in factorised form.

(a) A regular hexagon has a side length of $(5x - 2)$ cm. Calculate its perimeter.

(b) A rectangle has side lengths $(5x + 2)$ cm and $(7 - 3x)$ cm. Calculate its perimeter.

(c) A rectangle has a width of x cm and a length three times the width, less 1 cm. Calculate its perimeter.

SC 3 I can factorise expressions in which the highest common factor is made up of a number and a variable

1 Factorise each expression.

(a) $27ab - 3b$ **(b)** $24a^2b - 16a$ **(c)** $16abc + 22bc$ **(d)** $15abc^2 + 10a^2bc$

2 Factorise each expression.

(a) $14a^2b + 7a^2b^2 - 21a^2b^3$ **(b)** $24ab^2c^3 - 16a^2b^2c^2 + 30a^3b^3c^3$

3 State the highest common factor for each group of terms.

(a) $7abc$, $14abc^2$, $21ab^2c$, $28a^2bc$ **(b)** $26a^2bc$, $4a^2b^2c$, $50abc^2$, $52a^2b^2c^2$

(c) $2(a - b)$, $3a(a - b)$, $5b(a - b)$ **(d)** $-2(2a + 5)$, $3(2a + 5)$, $5(2a + 5)$

4 Factorise each expression.

(a) $7a(3a + 2) - 4(3a + 2)$ **(b)** $6a(4b - 2) - 5(4b - 2)$

5 On a recent algebra topic test, Esther and Mei-Lin were asked to factorise the expression $-4a - 12$.

(a) Esther wrote $-2(2a + 6)$ as her answer. What do you think about her answer?

(b) Mei-Lin wrote $4(a - 3)$ as her answer. What do you think about her answer?

(c) In your opinion, which student should receive more marks?

5.3 Simplify algebraic expressions involving multiple operations

Learning intention: To be able to simplify algebraic expressions involving multiple operations

Success criteria:

SC 1 I can simplify algebraic expressions involving addition, subtraction and multiplication.

SC 2 I can simplify algebraic expressions involving division and fractions.

SC 3 I can apply the order of operations to simplify algebraic expressions involving mixed operations and brackets.

Lesson warm-up

What are like terms?

Decide whether each pair of terms are like or unlike.

$2x$	$2xy$	abc	$2x^2$	a
$-2x$	$5x$	$4bca$	x^2y	$15a$

Write a definition of *like terms*.

SC 1 I can simplify algebraic expressions involving addition, subtraction and multiplication

Many algebraic expressions can be simplified by adding and subtracting like terms, which are terms that have the same variable parts.

Worked example

Adding and subtracting like terms after expanding brackets

Expand and simplify the expression $5(a + 4) - 3(a + 2)$.

THINKING	WORKING
Expand both sets of brackets, taking care with subtraction.	$5(a + 4) - 3(a + 2) = 5a + 20 - 3a - 6$
Now complete the working by collecting like terms.	$= 5a - 3a + 20 - 6$ $= 2a + 14$

You should notice that the final expression could have been factorised to $2(a + 7)$, but the question did not ask you to give the answer in factorised form.

SC 2 I can simplify algebraic expressions involving division and fractions

Algebraic fractions are dealt with in the same way as numerical fractions. Addition and subtraction require the use of a common denominator, while multiplication is the simplest of the four operations. Division involves multiplication by the inverse of the second term.

Worked example

Adding algebraic fractions

Simplify $\frac{5a}{4}+\frac{3a}{7}$

THINKING	WORKING
Identify the lowest common multiple of the two denominators.	The lowest common denominator is 28.
Rewrite the expression so that each term uses this common denominator.	$\frac{5a}{4}+\frac{3a}{7}=\frac{5a\times7}{4\times7}+\frac{3a\times4}{7\times4}$ $=\frac{35a}{28}+\frac{12a}{28}$
Now simplify the expression.	$\frac{5a}{4}+\frac{3a}{7}=\frac{47a}{28}$

Worked example

Multiplying algebraic fractions

Simplify $\frac{3a}{b}\times\frac{8ab}{9}$

THINKING	WORKING
Write the expression, looking for any factors that can be cancelled.	$\frac{\cancel{3}a}{\cancel{b}}\times\frac{8a\cancel{b}}{\cancel{9}_3}$
Then multiply the numerators together and multiply the denominators together.	$\frac{a}{1}\times\frac{8a}{3}=\frac{8a^2}{3}$

SC 3 I can apply the order of operations to simplify algebraic expressions involving mixed operations and brackets

The order of operations is a term that describes the order in which operations should be completed in a complicated expression. It can be expressed in many ways. One such way can be seen in the following steps.

1. Brackets
2. Exponents
3. Multiplications and divisions, as you come to them
4. Additions and subtractions, as you come to them

You are familiar with numerical examples. The same process applies to algebraic expressions.

The process for division of algebraic fractions follows the same process as for numerical calculations. Change the division to a multiply and flip the second fraction to give its inverse.

Worked example

Using the order of operations to simplify algebraic expressions

Simplify the expression $3(x - 4) - 6x \times 3 \div 9$

THINKING	WORKING
Identify the first operation to do and complete it.	The first operation is brackets. $\underline{3(x - 4)} - 6x \times 3 \div 9 = 3x - 12 - 6x \times 3 \div 9$
Identify the next operation to do and complete it.	The next operation is multiplication. $3x - 12 - \underline{6x \times 3} \div 9 = 3x - 12 - 18x \div 9$
Identify the next operation to do and complete it.	The next operation is division. $3x - 12 - \underline{18x \div 9} = 3x - 12 - 2x$
Simplify the expression by collecting like terms as a final step.	$3x - 12 - 2x = x - 12$

5

Practice

ANSWERS Page 572

Simplify algebraic expressions involving multiple operations

SC 1 I can simplify algebraic expressions involving addition, subtraction and multiplication

1 Expand and simplify each expression.

(a) $7(a + 2) + 4(a - 3)$

(b) $8(a - 3) + 5(a + 2)$

(c) $7(2a + 1) + 5(3a - 4)$

(d) $6(1 - 4a) - 3(2 - 5a)$

2 Expand and simplify each expression.

(a) $2a(a + 2) + a(3a - 1)$

(b) $6a(a + b) + 5b(3a + 2b)$

(c) $7a(2a - 3b) - 4b(3a - 2b)$

(d) $9ab(3a - 2b) - 3ab(5a + 6b)$

3 Complete the following sentences.

(a) When expanded and simplified, $-2y(3y - a) + 3y(5a - 2y)$ gives _____.

(b) When expanded and simplified, $6ab(b - 3a) - 4b(3a - 2ab)$ gives _____.

4 Complete the following.

(a) A rectangle has side lengths $(a + 2b)$ m and $(3a - b)$ m. Write an expression for this area and expand and simplify it.

(b) A circle has radius $(2a - b)$ m. Write an expression for its area and expand and simplify it.

SC 2 I can simplify algebraic expressions involving division and fractions

1 Simplify each expression.

(a) $\frac{4a}{3} + \frac{5a}{7}$

(b) $\frac{6a}{5} - \frac{3a}{2}$

(c) $\frac{11a}{6} + \frac{4a}{5}$

(d) $\frac{9a}{8} - \frac{5a}{3}$

2 Simplify each expression.

(a) $\frac{6a}{5b} \times \frac{15ab}{4}$ **(b)** $\frac{14ab}{9} \times \frac{3}{7ba^2}$ **(c)** $\frac{25ab}{9c} \times \frac{18cd}{15b^2}$ **(d)** $\frac{36a^2b^2c}{35d} \times \frac{7d^2}{72abc}$

3 Simplify each expression.

(a) $\frac{6a^2b}{7cd^2} \div \frac{12ab}{49c^2d}$ **(b)** $\frac{81a^3d}{26bc^2} \div \frac{45a^2d^2}{13bc}$

(c) $\frac{121a^3b^2c}{49d} \div \frac{11a^2b^2}{42d^3}$ **(d)** $\frac{110abc^2}{27d^3} \div \frac{66a^2bc}{36d^2e}$

4 Simplify each expression.

(a) $\frac{3a}{2b} + \frac{5a}{3b}$ **(b)** $\frac{4a}{3b} - \frac{2a}{5b}$ **(c)** $\frac{4}{a} + \frac{6}{b}$ **(d)** $\frac{7b}{3a} - \frac{3a}{b}$

SC 3 I can apply the order of operations to simplify algebraic expressions involving mixed operations and brackets

1 Simplify each expression.

(a) $5(x + 2) - 3x \times 2 \div 6$ **(b)** $6x - 5(x - 1) + 6x^2 \div x$

(c) $3a(a - 3) + 5a(a - 3) - 5a^2 \div 5a$ **(d)** $24a^2 \div 6a + 8(a - 2 \times 3a \div a)$

2 Simplify each expression.

(a) $\frac{3x-3}{2x} \times 10x(1-b) + 3b(x-2)$ **(b)** $15x \div 3x - 4(3 - 2x) + 3(5 - 4x)$

(c) $\frac{5x(3-2x)}{6b} \times \frac{12b^2(1-5x)}{5(3-2x)} \div \frac{6(1-5x)}{2x-3}$

3 Simplify the expression $\frac{6-3x}{5} \times \frac{15x}{2(x-2)}$

4 The following terms are defined:

A: $5(x - 3)$

B: $(x - 3)^2$

C: $\frac{5x}{x-3}$

D: $\frac{-3(x-3)}{2x}$

Simplify each of the following expressions using the stated definitions.

(a) $\frac{AC}{B}$ **(b)** $\frac{BC}{D}$ **(c)** $\frac{B}{A} - CD$

5

TOPIC REVIEW

Multiple choice

ANSWERS Page 621

1 Simplifying the expression $\frac{2x}{3(x+1)} \times \frac{(x+1)}{4}$ gives which of the following?

A $\frac{2x}{12}$ **B** $\frac{2x(x+1)}{12}$ **C** $\frac{2x^2+2}{12(x+1)}$ **D** $\frac{x}{6}$

2 Simplifying the expression $\frac{1}{4y} \div \frac{3}{y-2}$ gives which of the following?

A $\frac{3y-2}{4y(y-2)}$ **B** $\frac{y-2}{12y}$ **C** $-\frac{2}{12}$ **D** $\frac{3}{4y^2-8y}$

3 The expansion of $4(x - 7)$ is:

A $4x - 7$ **B** $4x - 28$ **C** $4x + 28$ **D** $32x$

4 The most complete factorisation of $24x - 6$ is:

A $18x$ **B** $2(12x - 3)$ **C** $3(8x - 2)$ **D** $6(4x - 1)$

5 Simplifying the expression $\frac{x}{3} + \frac{3x}{4}$ gives:

A $\frac{4x}{7}$ **B** $\frac{4x}{12}$ **C** $\frac{7x}{12}$ **D** $\frac{13x}{12}$

6 Simplifying the expression $\frac{1}{4y} + \frac{3}{y-2}$ gives:

A $\frac{13y-2}{4y(y-2)}$ **B** $\frac{4}{4y(y-2)}$ **C** $\frac{12y-2}{4y(y-2)}$ **D** $\frac{1(y-2)+3(4y)}{5y-2}$

7 In simplest form, $-8a \times 3b \times (-2c) + 5a \times 2ab$ is:

A $-48abc + 10a^2b$ **B** $-13abc + 10a^2b$ **C** $-7abc + 10ab$ **D** $48abc + 10a^2b$

8 In simplest form, $63xy^2 \div (-9xy) \times 4yz$ is:

A $-28y^2z$ **B** $-28xyz$ **C** $-28xy^2z$ **D** $-\frac{28x}{yz}$

9 Fully factorised, the expression $-25y + 60xy + 35x^2y$ is:

A $-5y(5 - 12x - 7x^2)$ **B** $-5y(5y + 12x + 7x^2)$

C $-5y(5 + 12x - 7x^2)$ **D** $5y(5 + 12x + 7x^2)$

10 When the expression $-2x(4 + r) - 5(2r - 8x)$ is expanded and simplified it becomes:

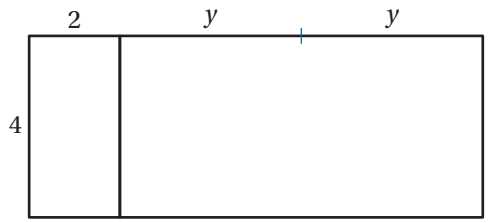
A $-32x - 2xr - 10r$ **B** $-32x + 2xr - 10r$ **C** $32x + 2xr - 10r$ **D** $32x - 2xr - 10r$

Short answer

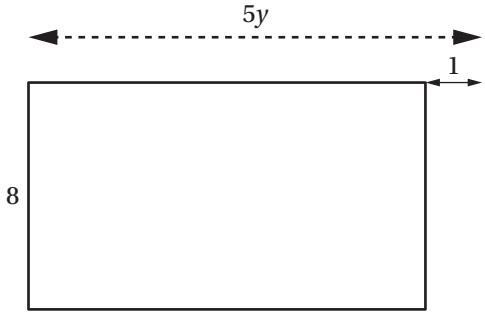
1 Draw an area model to illustrate the expression $3 \times (5 + a)$.

2 Write the expression represented by each of the following area models.

(a)



(b)



3 Expand the following expressions.

(a) $6(2y - 3)$ **(b)** $8(a + b + 2c)$

4 Factorise the following expressions.

(a) $3a + 6$ **(b)** $15b - 10a$

5 Simplify the following expressions.

(a) $a + b + 2a + 2b$ **(b)** $a \times b + 2a \times 2b$ **(c)** $a \times 5a - a^2 \div 3$

6 Simplify the following expressions.

(a) $3(2 + 5a) - 2a$ **(b)** $2(2 - 5a) + 3(2 + 4a)$

7 Simplify the following expressions.

(a) $\frac{a}{2} + \frac{a}{3}$ **(b)** $\frac{2}{a} + \frac{7}{b}$ **(c)** $\frac{x}{2} - \frac{y}{9}$

8 Simplify the following expressions.

(a) $\frac{4}{x} + \frac{5}{y} + \frac{6}{xy}$ **(b)** $\frac{4(f+1)}{2} - \frac{3(f+1)}{5}$

9 Simplify the following expressions.

(a) $\frac{x-2}{5} \times \frac{10}{x+2}$ **(b)** $\frac{5}{x} \div \frac{3}{x+6}$

10 Simplify the expression $\frac{n+3}{3(n+5)} \times \frac{n+5}{3n+9}$.

11 Simplify the following expressions.

(a) $\frac{2a}{3} \times \frac{1}{4}$ **(b)** $\frac{2a}{3} \div \frac{1}{4}$ **(c)** $\frac{5x}{8} \times \frac{2y}{3}$

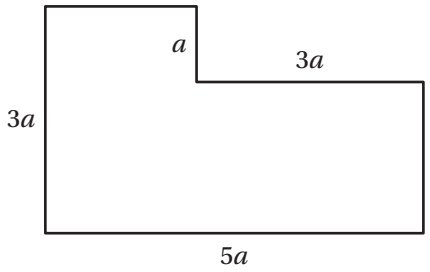
12 Simplify the following expressions.

(a) $-5 \times 3m \times 5 - m$ **(b)** $-\frac{9a}{35d} \times 5ac \times (-2d)$ **(c)** $\frac{2x^2}{3} \times \frac{15xz^3}{4} \div \frac{8x^4yz}{6}$

13 Expand each of the following expressions. Give the answer in simplest form.

(a) $-3p(7r - 5)$ **(b)** $6x(y + 2) - 4y(x + 2)$ **(c)** $\frac{1}{2}\left[16x - \frac{2}{3}(6x - 12)\right]$

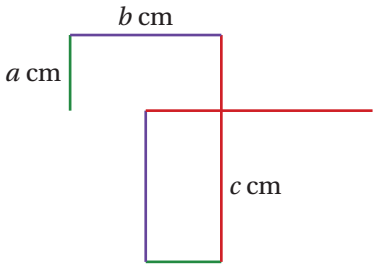
14 Consider the following shape.



(a) Write an expression for the perimeter of the shape and simplify it.

(b) Write and expression for the area of the shape and simplify it.

15 Laika draws the following pattern.



(a) Write an expression for the total length of the lines after the first three lines have been drawn.

(b) Now write an expression for the total length of lines shown in the diagram.

(c) Finally, write an expression for the total length if a total of 30 straight lines are drawn.

16 The area of a triangle is found by multiplying the base length by the perpendicular height and dividing the result by 2.

(a) A triangle has a base length of $8x$ cm and a perpendicular height of $(3x - 7)$ cm. Write an unsimplified expression for the area of this triangle.

(b) Expand and simplify this expression.

Extended response

1 The area of a rectangle is found by multiplying the length by the width. A particular rectangle is 12 cm longer than it is wide.

(a) Write an expression for the area of the rectangle if its width is x cm.

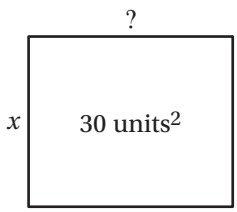
(b) Write an expression in terms of x to calculate the area of the enlarged rectangle if the length is tripled and the width is halved. Write the answer in expanded form.

(c) Write an expanded expression in terms of x to represent the change in area.

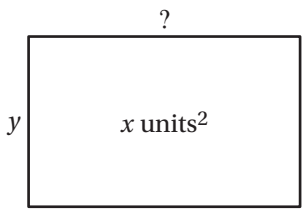
(d) Factorise this answer given in part **(c)**.

2 Determine the missing dimensions in parts **(a)**–**(c)** and then use the diagrams to answer the questions that follow.

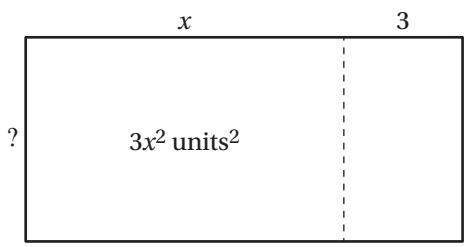
(a)



(b)



(c)



(d) What is the missing area in the diagram for part **(c)**?

(e) Write two expressions for the total area of the diagram in part **(c)**, one in factorised form and one in expanded form.

(f) What law of mathematics does this demonstrate?

3 Complete the following.

(a) Draw an area model showing a square measuring $x + 5$ by $x + 5$ and break the square into four smaller areas. Show the area of each of the four parts on the diagram.

(b) Express the total area of the large square in terms of x in expanded form.

(c) Repeat part **(a)** and part **(b)** with a square measuring $x - 4$ by $x - 4$.

(d) What do you notice about the coefficient of the x term?

(e) Without drawing diagrams or expanding brackets, write the expansion of each of the following:

(i) $(x + 6)^2$ **(ii)** $(x - 7)^2$

TOPIC 6

Linear graphs

Recall 113

6.1 Plot and identify a linear relationship using a set of points 114

6.2 Graph linear relationships with only one axis intercept 120

6.3 Investigate linear graphs using technology 126

6.4 Determine the rule for a linear relationship or pattern 134

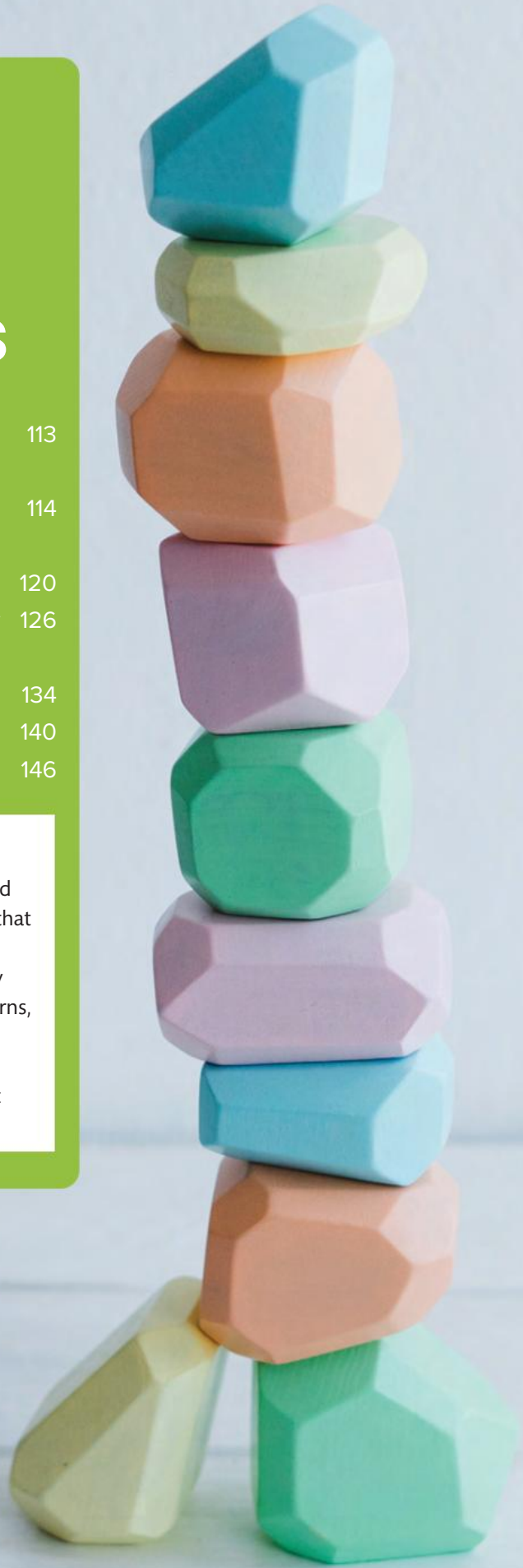
6.5 Solve linear equations graphically 140

Topic review 146

Why learn this?

Linear equations and graphs occur everywhere around us and help make sense of many of the relationships that occur throughout the world.

These skills are not just useful in mathematics. They can be used by scientists for predicting weather patterns, data trends and drawing conclusions; by engineers to design many different projects from buildings to electronic circuitry; and by finance experts to forecast portfolio and sales growth.



RECALL

ANSWERS Page 572

I can plot points on a Cartesian plane

1 Plot the following points on a Cartesian plane.

(a) A(2, 7) **(b)** B(–5, 4) **(c)** C(3, –4)

(d) D(–6, –1) **(e)** E(5, 0) **(f)** F(0, –8)

I can use a rule to fill in a table of values

1 Using the given rule, complete the table of values.

(a) $y = x + 5$

x	–1	0	1	2	3
y					

(b) $y = x - 4$

x	–1	0	1	2	3
y					

(c) $y = 8 - x$

x	–1	0	1	2	3
y					

(d) $y = 3x$

x	–1	0	1	2	3
y					

I can substitute values into equations to check the correct solution

1 For each of the following equations, check whether the solution given in the brackets is correct and write T (true) or F (false).

(a) $x + 5 = 1 \quad [x = 9]$ **(b)** $2y = 8 \quad [y = 4]$ **(c)** $2a + 1 = 4 \quad [a = 1]$

(d) $b - 9 = 12 \quad [b = 21]$ **(e)** $x + 5 = 2x \quad [x = 6]$ **(f)** $\frac{x+1}{5} = 3 \quad [x = 14]$

2 Use substitution to identify the correct answer for each of the following:

(a) If $x + 10 = 23$, then

$x = 3 \quad x = 13 \quad x = 23$

(b) If $2x + 1 = 9$, then

$x = 2 \quad x = 3 \quad x = 4$

(c) If $\frac{x+8}{10} = 1$, then

$x = 2 \quad x = 12 \quad x = 22$

I can solve two–step equations

1 Solve each of these equations for the given variable.

(a) $3a + 3 = 15$ **(b)** $2b - 8 = 12$ **(c)** $\frac{x}{2} - 8 = 4$ **(d)** $\frac{p+1}{5} = 3$

I can write expressions from a sentence

1 Rewrite each of the following sentences using mathematical symbols.

(a) Six is subtracted from one hundred and the result is ninety–four.

(b) Three times four is equal to twelve.

(c) The sum of eight and y is equal to ten.

6.1 Plot and identify a linear relationship using a set of points

Learning intention: To be able to plot and identify a linear relationship using a set of points

Success criteria:

SC 1 I can use two points to plot a linear relationship.

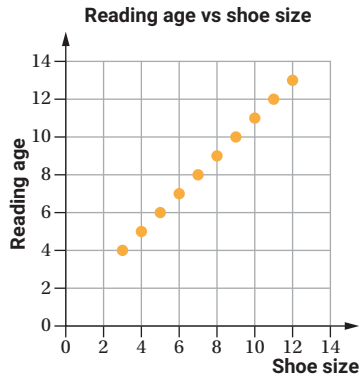
SC 2 I can use a linear relationship to generate coordinate pairs.

SC 3 I can draw a graph of a linear relationship from a practical context.

Lesson warm-up

Reading age and shoe size

The reading age and shoe size of a group of 10 students was measured. A graph of the results seems to show that people with larger feet are better readers. What do you think?



SC 1 I can use two points to plot a linear relationship

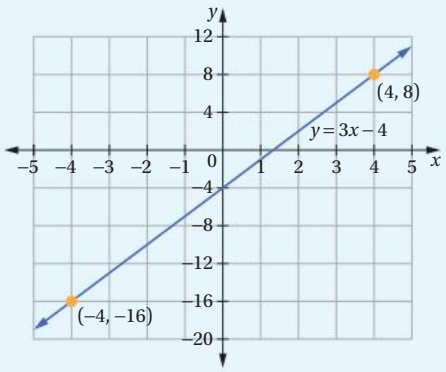
A straight line is uniquely defined by any two points that lie on it. Given the rule, substitution can be used to identify coordinate pairs, which can then be plotted and joined by a straight line.

Worked example

Plotting a linear relationship

Draw a graph for the linear relationship $y = 3x - 4$ using x-values of -4 and 4.

THINKING	WORKING	
Substitute each of the given x-values into the rule to calculate the y-value, then write the coordinate pair.	$y = 3x - 4$	
	For $x = -4$:	For $x = 4$:
	$y = 3 \times (-4) - 4$	$y = 3 \times 4 - 4$
	$= -12 - 4$	$= 12 - 4$
	$= -16$	$= 8$
	$(-4, -16)$	$(4, 8)$

Draw up a set of axes and plot the points on the axes. Plot the points and join them with a line segment. If no domain is given, extend the segment at each end and attach arrows.	
Check the graph, and determine the value for the y-intercept by substituting $x = 0$.	For $x = 0$: $y = 3x - 4$ $= 3 \times (0) - 4$ $= -4$ The coordinates of the y-intercept are $(0, -4)$. The intercept is confirmed on the graph.

SC 2 I can use a linear relationship to generate coordinate pairs

Substitution can be used to set up a table of values if the rule for the linear relationship is known.

6

Worked example

Creating a table of values for a linear relationship

For the linear relationship $y = \dfrac{3x - 4}{2}$ complete the table of values shown.

THINKING	WORKING
Substitute each x-value into the equation to calculate each of the y-values.	$y = \dfrac{3x - 4}{2}$

x	−3	−1	0	4	6
y	$y = \dfrac{3\times(-3)-4}{2}$ $= \dfrac{-9-4}{2}$ $= -\dfrac{13}{2}$ $= -6\dfrac{1}{2}$	$y = \dfrac{3\times(-1)-4}{2}$ $= \dfrac{-3-4}{2}$ $= -\dfrac{7}{2}$ $= -3\dfrac{1}{2}$	$y = \dfrac{3\times0-4}{2}$ $= -\dfrac{4}{2}$ $= -2$	$y = \dfrac{3\times4-4}{2}$ $= \dfrac{12-4}{2}$ $= \dfrac{8}{2}$ $= 4$	$y = \dfrac{3\times6-4}{2}$ $= \dfrac{18-4}{2}$ $= \dfrac{14}{2}$ $= 7$

Complete the table of values.

x	−3	−1	0	4	6
y	$-6\dfrac{1}{2}$	$-3\dfrac{1}{2}$	−2	4	7

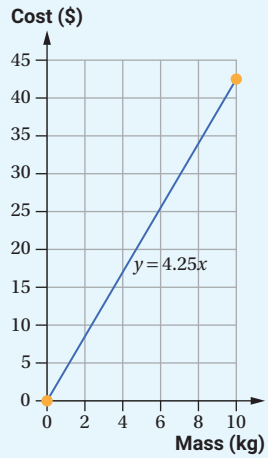
SC 3 I can draw a graph of a linear relationship from a practical context

Many practical situations can be represented by a linear relationship. It is good to be mindful of any constraints in the relationship, such as whether negative values are valid and how far it makes sense to extend the graph to be useful in the given context.

Worked example

Graphing in a practical context

Apples cost $4.25/kg. Draw a graph to show the cost of apples up to and including 10 kg.

THINKING	WORKING
Calculate endpoints for the graph.	0 kg will cost $0: (0, 0). $10 \times 4.25 = 42.5$ 10 kg will cost $42.50: (10, 42.5).
Choose the mass of apples as the independent variable for the horizontal axis and the cost as the dependent variable for the vertical axis. Use the endpoints to determine the values required for each axis. Draw the axes, plot the two calculated points and join with a straight line segment.	

Practice

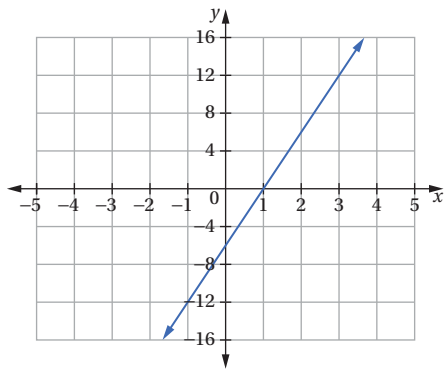
ANSWERS Page 573

SC 1 I can use two points to plot a linear relationship

1 Plot two points for each of the following linear relationships using x-values of -3 and 3, then complete the graph for the relationship.

(a) $y = 2x + 4$ **(b)** $y = 4x - 3$ **(c)** $y = -3x + 2$ **(d)** $y = -5x - 1$

2 Determine the pair of coordinates used to draw this graph, given that $x = -1$ and $x = 3$ were used.



3 For each of the following linear relationships, copy and complete the coordinate pairs shown. If necessary, give your answers in proper fraction or mixed number form.

(a) $y = \frac{3x-2}{4}$: (–3, __) and (5, __)

(b) $y = \frac{3x}{5} - 2$: (–2, __) and (3, __)

(c) $y = 2 - \frac{x}{3}$: (–6, __) and (7, __)

(d) $y = \frac{5-2x}{4}$: (–4, __) and (10, __)

4 For each of the following linear relationships, complete the coordinate pairs shown.

(a) $y = x - 7$: (__, –2) and (__, –10)

(b) $y = 2x + 4$: (__, 10) and (__, –4)

(c) $y = 2 - x$: (__, 8) and (__, –2)

(d) $y = 4 - 3x$: (__, 13) and (__, –11)

SC 2 I can use a linear relationship to generate coordinate pairs

1 For each of the following linear relationships, copy and complete the table of values provided. If necessary, leave the values as proper fractions or mixed numbers.

(a) $y = 3x - 6$

x	–3	–1	0	4	6
y					

(b) $y = 6 - 5x$

x	–5	–2	1	2	5
y					

(c) $y = \frac{2x-1}{3}$

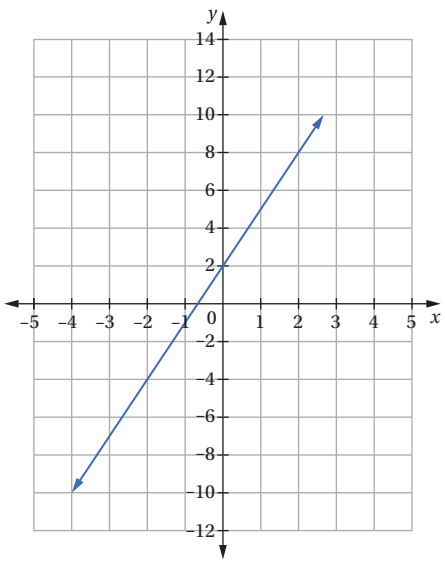
x	–6	–1	0	2	5
y					

(d) $y = \frac{5-4x}{2}$

x	–4	–2	0	1	6
y					

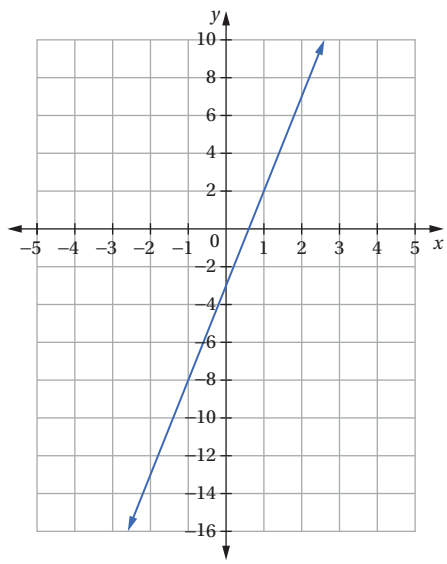
2 For each of the following linear graphs, copy and complete the table of values given.

(a)



x	–4	–2	0	2	4
y					

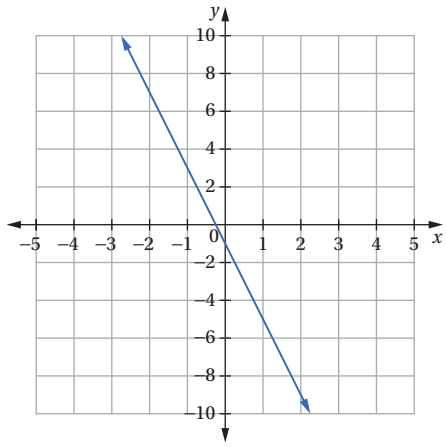
(b)



x	–2	–1	0	1	2
y					

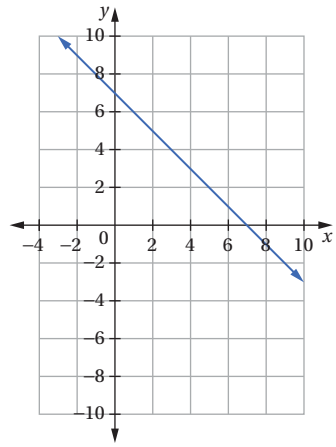
6

(c)



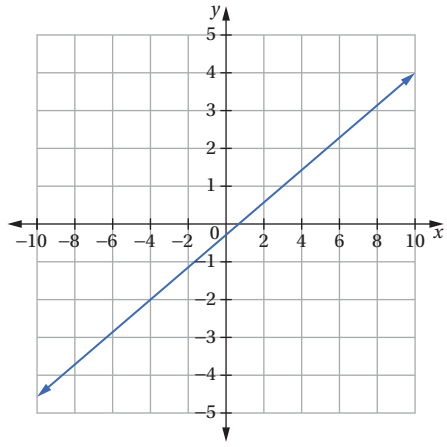
x	−2	−1	0	1	2
y					

(d)



x	−2	0	2	6	10
y					

3 Consider the following linear graph.



(a) You are asked to complete the following table of values. Explain what makes this a difficult task.

x	−8	−2	0	5	9
y					

(b) Determine the value of the y-intercept, correct to 1 decimal place.

(c) You are now told that the rule for the relationship is $y = \frac{3x-2}{7}$. Copy and complete the table of values, giving each value correct to 1 decimal place.

4 Describe the change in the y-values for each increase of 1 in the x-values in each of the following linear relationships.

(a)

x	−2	−1	0	1	2
y	10	5	0	−5	−10

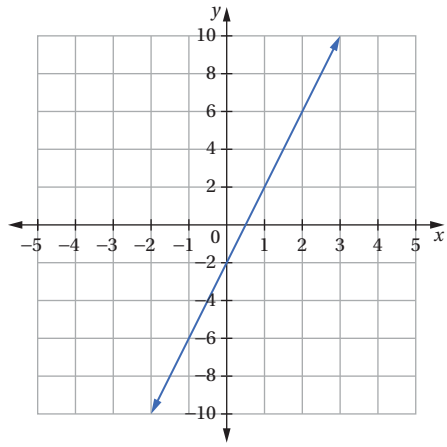
(b)

x	−6	−3	0	3	6
y	−14	−8	−2	4	10

(c)

x	−6	−4	−1	3	8
y	−23	−17	−8	4	19

(d)



SC 3 I can draw a graph of a linear relationship from a practical context

1 Material costs $8.50 per metre.

(a) Draw a graph showing the cost up to 5 m in length.

(b) Use the graph to find the approximate cost for 3 m.

(c) Use the graph to find the approximate cost for 4.5 m.

(d) Describe a more accurate way to find the answers to **(b)** and **(c)** than using the graph.

2 Water is running out of a large water tank that is feeding an irrigation system. The tank initially held 50 000 litres and the irrigation system uses 1500 litres per hour.

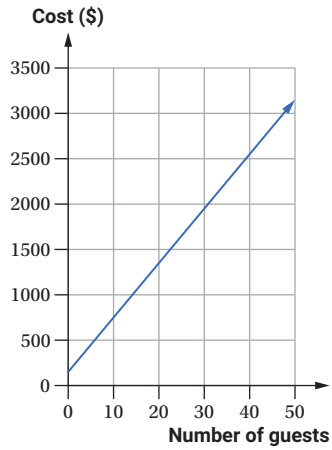
(a) Determine the time taken for the tank to be empty.

(b) Draw a graph showing this linear relationship.

(c) Describe a meaning for each of the axis intercepts of the graph.

(d) Explain why it makes sense to draw this graph only in the first quadrant of the Cartesian axes.

3 Freya is organising a party at the local hall. There are two parts to the cost: a fixed fee for the hire and cleaning of the hall, and a cost per person for food and drinks. The graph below shows the total cost for the party.



(a) Determine the fixed cost for hall hire and cleaning, and explain where the information features on the graph.

(b) Determine the total cost for 10 guests.

(c) Determine the total cost for 30 guests.

(d) Determine the cost per person for food and drinks.

(e) Write a rule to describe the total cost for the party.

(f) The graph starts at $x = 0$ and has an arrow on the right-hand end. Describe the information needed to put appropriate endpoints on the graph.

4 Thongchai needed to call a plumber twice in the last couple of months. He used the same plumber both times. The first job took 3 hours and cost $270, while the second job took 5 hours and cost $410.

(a) Graph the cost for the plumber against time.

(b) From the graph, estimate the call-out fee charged by the plumber.

(c) Determine the plumber's hourly rate.

(d) Write a rule that represents the cost for this plumber.

6.2 Graph linear relationships with only one axis intercept

Learning intention: To be able to graph linear relationships with only one axis intercept

Success criteria:

SC 1 I can recognise and draw vertical and horizontal graphs.

SC 2 I can recognise and draw graphs that pass through the origin.

Lesson warm-up

How often do you eat pizza each month?

Let's look at the different graph types using a spreadsheet.

SC 1 I can recognise and draw vertical and horizontal graphs

Linear relationships involving both x and y lead to oblique graph lines. If just one variable is involved, the line will be either a horizontal or a vertical line.

Rules of the form $y = a$ graph as horizontal lines passing through the y-intercept $(0, a)$.

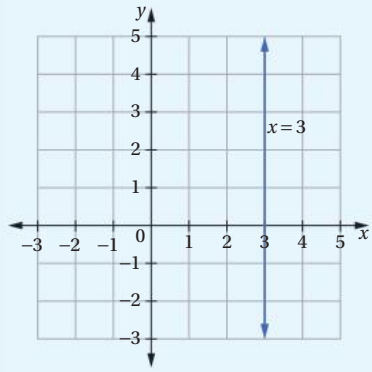
Rules of the form $x = a$ graph as vertical lines passing through the x-intercept $(a, 0)$.

Worked example

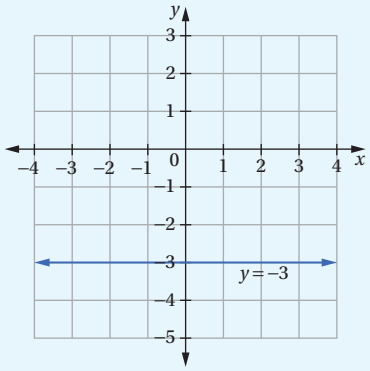
Sketching horizontal and vertical linear graphs

Sketch graphs for each of the rules on a set of Cartesian axes.

(a) $x = 3$

THINKING	WORKING
Draw a set of Cartesian axes. Locate $x = 3$ on the x-axis. Draw a vertical line through the axis point, adding arrows at each end.	

(b) $y = -3$

THINKING	WORKING
Draw a set of Cartesian axes. Locate $y = -3$ on the y-axis. Draw a horizontal line through the axis point, adding arrows at each end.	

SC 2 I can recognise and draw graphs that pass through the origin

With the exception of vertical lines, linear graphs that pass through the origin (0, 0) can be written in the form $y = mx$. The origin is both the x-intercept and the y-intercept.

The value of m gives the gradient, or slope of the line.

- If m is positive, the graph slopes upwards from left to right.
- If m is negative, the graph slopes downwards from left to right.
- If the sign of m is ignored, the larger the size of m, the steeper the graph.

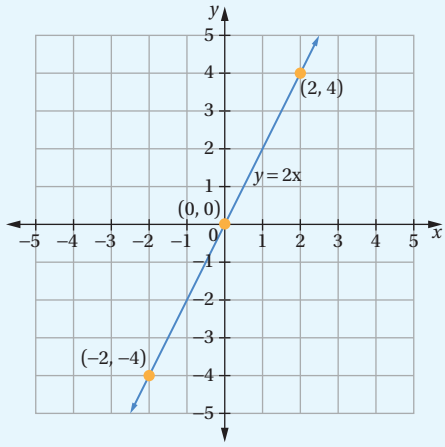
Worked example

Plotting graphs that pass through the origin

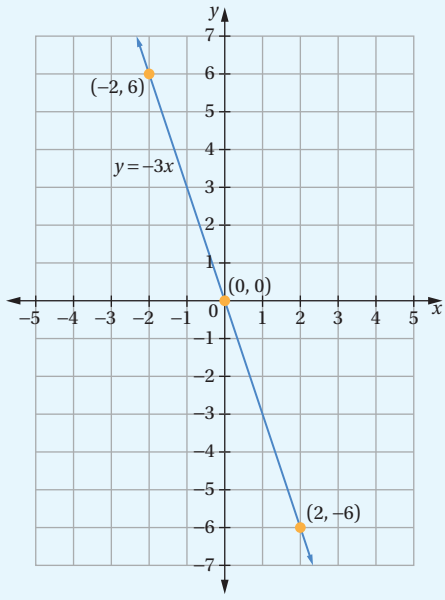
Plot each of the following graphs.

(a) $y = 2x$

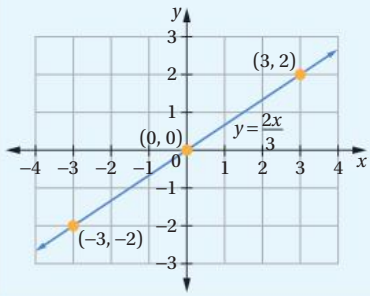
THINKING	WORKING
Determine three coordinate pairs, including the origin. For balance, choose an x-value from each side of the y-axis.	The coordinates of the origin are (0, 0). For $x = -2$: $y = 2 \times (-2)$ $= -4$ The coordinates of another point are (−2, −4). For $x = 2$: $y = 2 \times 2$ $= 4$ The coordinates of another point are (2, 4).

Draw a set of Cartesian axes. Plot the points. Draw a line through the points. Label the graph.	

(b) $y = -3x$

THINKING	WORKING
Determine three coordinate pairs, including the origin. For balance, choose an x-value from each side of the y-axis.	The coordinates of the origin are (0, 0). For $x = -2$: $y = -3 \times (-2)$ $= 6$ The coordinates of another point are (−2, 6). For $x = 2$: $y = -3 \times 2$ $= -6$ The coordinates of another point are (2, −6).
Draw a set of Cartesian axes. Plot the points. Draw a line through the points. Label the graph.	

(c) $y = \frac{2x}{3}$

THINKING	WORKING
Determine three coordinate pairs, including the origin. Choose x-values that give integer y-values; in this case, multiples of 3. For balance, choose an x-value from each side of the y-axis.	The coordinates of the origin are (0, 0). For $x = -3$: $y = \frac{2 \times -3}{3} = -2$ The coordinates of another point are (–3, –2). For $x = 3$: $y = \frac{2 \times 3}{3} = 2$ The coordinates of another point are (3, 2).
Draw a set of Cartesian axes. Plot the points. Draw a line through the points. Label the graph.	

Practice

ANSWERS Page 575

SC 1 I can recognise and draw vertical and horizontal graphs

1 For rules of the form $x = a$:

(a) sketch and label each of the following on the same set of Cartesian axes:

(i) $x = 6$ **(ii)** $x = 1$ **(iii)** $x = -3$ **(iv)** $x = -6$

(b) decide whether the line representing $x = 0$ is the x-axis or the y-axis.

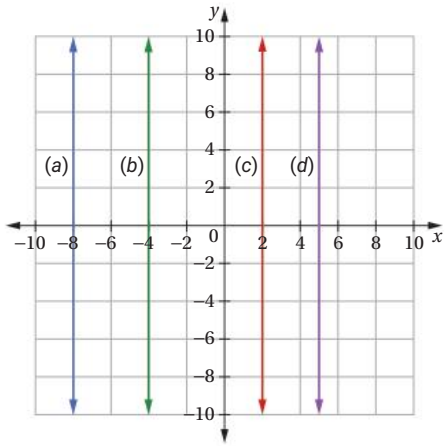
2 For rules of the form $y = a$:

(a) sketch and label each of the following on the same set of Cartesian axes:

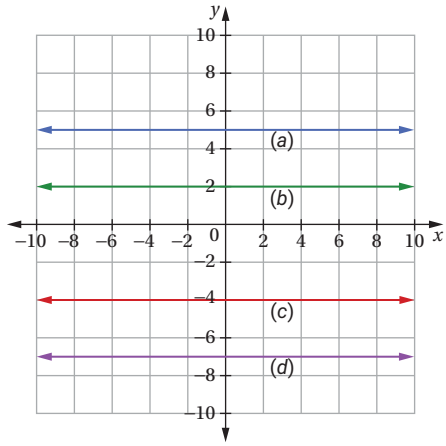
(i) $y = 6$ **(ii)** $y = 1$ **(iii)** $y = -5$ **(iv)** $y = -8$

(b) decide whether the line representing $y = 0$ is the x-axis or the y-axis.

3 Determine the equation for each of the linear relationships shown on the following set of Cartesian axes.



4 Determine the equation for each of the linear relationships shown on the following set of Cartesian axes.



5 Determine the equation for each of the following lines.

(a) A vertical line through (5, 6)

(b) A vertical line through (–9, –12)

(c) A horizontal line through (7, 2)

(d) A horizontal line through (–6, –10)

(e) A vertical line through the origin (0, 0)

(f) A horizontal line through the origin (0, 0)

6 Determine the shapes, with dimensions, that are formed by the graphs of the following linear relationships. Assume a common scale for the two axes.

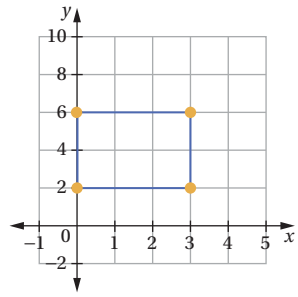
(a) $y = 5, y = -5, x = 5, x = -5$

(b) $y = 0, y = 4, x = 0, x = 8$

7 Consider the following graph.

(a) Determine the set of linear relationships that could be used to draw the shape shown.

(b) Determine the restrictions needed to leave just the line segments shown.



SC 2 I can recognise and draw graphs that pass through the origin

1 Plot and label the following linear relationships on the same set of Cartesian axes. Use the values –4 to 4 on the x-axis.

(a) $y = 3x$ **(b)** $y = -4x$ **(c)** $y = 5x$ **(d)** $y = -x$

2 For each of the following linear relationships:

(a) determine the coordinates for the lowest positive whole number x-value that has an integer for its y-value

(b) draw and label the graphs of the relationships on the same set of axes.

(i) $y = \frac{x}{4}$ **(ii)** $y = \frac{3x}{5}$ **(iii)** $y = -\frac{x}{2}$ **(iv)** $y = -\frac{5x}{4}$

3 Use digital software to graph each of the four linear relationships.

(a) $y = 2x, y = -2x, -y = 2x, -y = -2x$ **(b)** $y = 5x, y = -5x, -y = 5x, -y = -5x$

For each set:

(i) determine the pairs of relationships that give the same graph

(ii) describe the transformation needed to fit one pair of equal graphs to the other.

4 Determine any five coordinate pairs, with integer values, that lie on each of the following linear graphs.

(a) $y = 7x$ **(b)** $y = -4x$ **(c)** $y = 10x$ **(d)** $y = -100x$

5 For each of the following conditions, write an example of a linear relationship with a graph that passes through the origin. Describe what makes the solution correct, so that another student could assess whether their solution is correct.

(a) A graph steeper than $y = -2x$, but also with a negative slope

(b) A graph that has no x-intercept

(c) A graph that has no y-intercept

(d) A graph that is steeper than the graph of $y = 5x$, but also with a positive slope

(e) A graph that is less steep than $y = -2x$, but also with a negative slope

(f) A graph that is less steep than $y = \frac{x}{2}$, but also with a positive slope

6.3 Investigate linear graphs using technology

Learning intention: To be able to detemine the rule for a linear relationship using technology

Success criteria:

SC 1 I can determine the rule for a linear relationship using technology.

SC 2 I understand the connection between the y-intercept and the value of b in $y = mx + b$.

SC 3 I understand the connection between linear relationships in the form $y = mx + b$ and the slope of the line.

Lesson warm-up

Number sequences

Determine the next number in these sequences.

1 2, 4, 6, 8, ...

2 1, 2, 4, 8, 16, ...

And harder sequences?

3 3, 7, 15, 31, ...

4 2, 2, 4, 6, 10, 16, 26, ...

And the hardest of all, determine the next diagram in this sequence.

Hint: it has something to do with numbers and will make sense when you know the answer.

SC 1 I can determine the rule for a linear relationship using technology

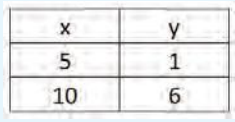
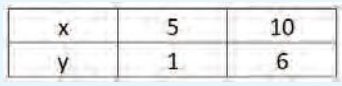
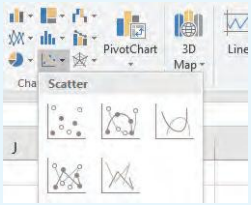
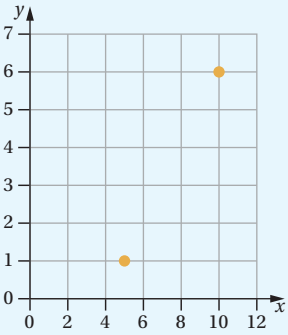
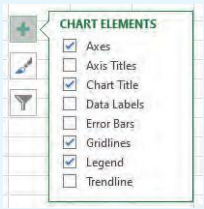
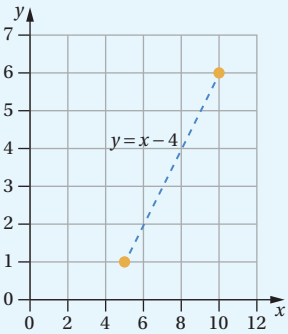
A spreadsheet, such as Microsoft Excel, can be used to determine the rule for a linear relationship, given two points. If more than two points are used, and they all line up, the program will write the rule for the linear relationship. If they don't line up, the software calculates a line of best fit through the points, then provides the equation.

Since it only takes two coordinate pairs to uniquely define a linear relationship, Excel provides the rule for the line through the two points.

Worked example

Using a spreadsheet to determine a linear relationship

Use a spreadsheet to determine the rule for the line that passes through (5, 1) and (10, 6).

THINKING	WORKING
Open a new spreadsheet and set up a table of values for the two points.	 or 
Highlight the values and insert a scatterplot with just the dots from the Charts menu.	
A graph will appear as shown, with the points plotted. If your display has the coordinates the wrong way around, click the 'Switch Row/Column' icon in 'Data'.	
Add a Trendline.	
Hover the mouse pointer over the line drawn on your graph and right-click. Choose 'Format Trendline'. Scroll down to check 'Display Equation on chart'. The equation will automatically appear on your graph.	 The equation is $y = x - 4$.

SC 2 I understand the connection between the y-intercept and the value of b in $y = mx + b$

The general equation for a straight line (excluding vertical lines) is $y = mx + c$ or $y = mx + b$.

The y-intercept is c, with coordinates $(0, c)$.

In some cases, a little work is needed to identify the values of m and c.

For $y = \frac{3x - 4}{5}$:

$$y = \frac{3x - 4}{5}$$

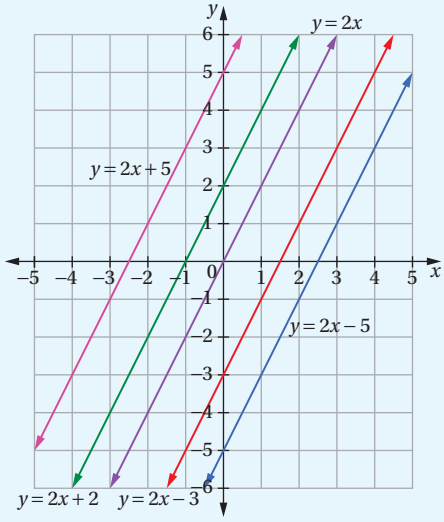
$$= \frac{3}{5}x - \frac{4}{5}$$

So $m = \frac{3}{5}$ or 0.6 and $c = -\frac{4}{5}$ or −0.8.

Worked example

Understanding the role of the constant in $y = mx + c$

Use technology to draw the following lines on the same set of Cartesian axes: $y = 2x - 5$, $y = 2x - 3$, $y = 2x$, $y = 2x + 2$, $y = 2x + 5$. Explain the effect of the constant term on the graphs.

THINKING	WORKING
Use Desmos or a graphing calculator to draw and label the graphs.	
Compare the constant term with the y-intercept for each graph.	The constant term becomes the y-intercept for each graph. $y = 2x - 5$ has y-intercept: $c = -5$ at $(0, -5)$. $y = 2x - 3$ has y-intercept: $c = -3$ at $(0, -3)$. $y = 2x$ has y-intercept: $c = 0$ at $(0, 0)$. $y = 2x + 2$ has y-intercept: $c = 2$ at $(0, 2)$. $y = 2x + 5$ has y-intercept: $c = 5$ at $(0, 5)$.

SC 3 I understand the connection between lines in the form $y = mx + b$ and the slope of the line

For graphs of equations in the form $y = mx + c$:

- for $m < 0$, the line has a negative gradient
- for $m = 0$, the line has a zero gradient
- for $m > 0$, the line has a positive gradient.

Worked example

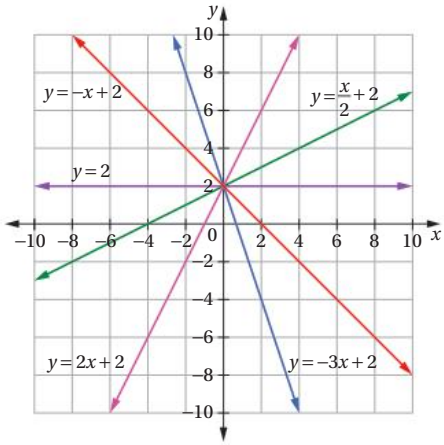
Confirming the function of m in the rule $y = mx + b$

Technology can be used to draw the following lines on the same set of axes:

$y = -3x + 2, y = -x + 2, y = 2, y = \frac{x}{2} + 2, y = 2x + 2$

Discuss the features of the graphs, cover the following points.

- Describe what is common to each graph.
- Determine which graph has a zero gradient. Explain.
- Determine which graphs have positive gradients. Which graph is steepest? Explain.
- Determine which graphs have negative gradients. Which graph is steepest? Explain.



THINKING	WORKING
Describe the point of intersection of the graphs.	All graphs have a y-intercept of 2 at (0, 2).
Identify the horizontal graph line and explain how the equation caused the zero gradient.	The line $y = 2$ has zero gradient because there is no x-term, and hence the y-value does not change.
Identify the lines that increase from left to right and explain the relative steepness in terms of the coefficient of x.	The lines $y = \frac{x}{2} + 2$ and $y = 2x + 2$ have positive gradients because the coefficients of x are positive. $2 > \frac{1}{2}$, so $y = 2x + 2$ is steeper than $y = \frac{x}{2} + 2$ because the y-value increases more for each unit of increase in the x-value.
Identify the lines that decrease from left to right and explain the relative steepness in terms of the coefficient of x.	The lines $y = -3x + 2$ and $y = -x + 2$ have negative gradients because the coefficients of x are negative. $-3 < -1$, so $y = -3x + 2$ is steeper than $y = -x + 2$ because the y-value decreases more for each unit of increase in the x-value.

Practice

ANSWERS Page 576

SC 1 I can determine the rule for a linear relationship using technology

1 Use technology to determine the rule for the line that passes through each of the following pairs of coordinates.

(a) (–1, –3) and (3, 5)

(b) (–2, –4) and (5, 17)

(c) (–5, 26) and (1, 2)

(d) (–4, 5) and (4, –11)

2 Use technology to determine the rule for the line that passes through each of the following pairs of coordinates. Write the answers to the degree of accuracy shown on the technology.

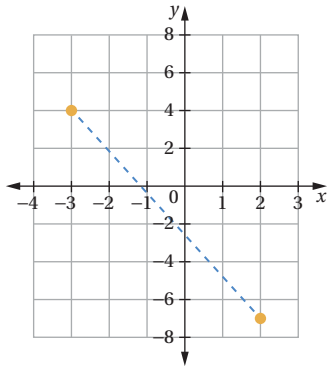
(a) (–1, –15) and (3, 6)

(b) (–2, 8) and (4, –2)

(c) (–3, 7) and (6, 12)

(d) (–6, 7) and (3, –5)

3 Determine the equation of the line shown.

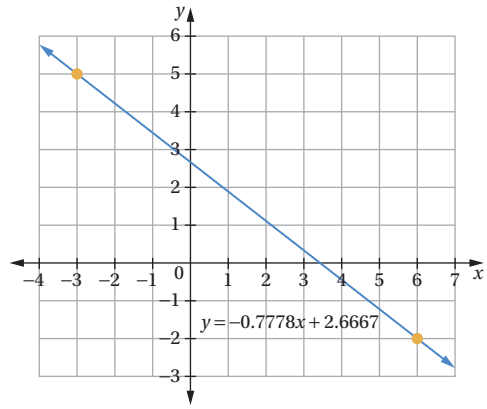


4 The following graph was generated using technology.

Using the rule stated on the graph, complete the following coordinate pairs. Give your answers correct to 4 decimal places.

(a) (–2, _____)

(b) (4, _____)



SC 2 I understand the connection between the y-intercept and the value of b in $y = mx + b$

1 For each of the following linear relationships, state the value of the y-intercept in terms of c from the general equation $y = mx + c$, and then as a coordinate pair, for each of the following linear relationships.

(a) $y = 2x + 7$ **(b)** $y = -3x - 4$ **(c)** $y = -5x + 6$ **(d)** $y = 9x - 10$

2 Use a spreadsheet for this question. Place x-values in the first column, starting with -3, -2, then select these two cells and use the Fill Handle (small green square in the bottom-right of the selected cells) to drag down and automatically fill in the other values for the next few rows. Write the formulas for the linear relationships shown below across the top of the other columns.

	A	B	C	D	E	F
1	x	y=x+3	y=2x+3	y=−2x+3	y=x+5	y=3x−5
2	−3		−3			
3	−2					
4	−1					
5	0					
6	1					
7	2					
8	3					
9	4					
10	5					

In cell C2, the formula =2*A2+3 has been entered for $y = 2x + 3$, using a click on the x-value in the row to achieve the same purpose as typing A2. Type formulas for the first x-value of each equation. Highlight the y-values across the entire row, then click and drag to fill the table. Highlight the row of values representing the y-intercepts for the relationships.

3 For each of the linear relationships:

(i) write the rule in the form $y = mx + c$

(ii) state the values of m and c.

(a) $y = \frac{2x+1}{3}$ **(b)** $y = \frac{x-5}{4}$ **(c)** $y = \frac{4-2x}{6}$ **(d)** $y = \frac{7-x}{2}$

4 For the linear relationship $y = \frac{-5(6-2x)}{7}$, complete the following.

(a) Use technology to draw the graph of the rule.

(b) Read off the y-intercept as a coordinate pair, from the graph.

(c) Use algebra to write the rule in the form $y = mx + c$.

(d) Compare the value of c in the equation with the y-intercept seen on the graph.

5 Determine the exact value for the y-intercept in each of the following linear relationships. Confirm each answer from a digital graph.

(a) $y = 4(3x - 2)$ **(b)** $y = -3(2x + 1)$ **(c)** $y = \frac{2(4-3x)}{5}$

(d) $y = \frac{-6(1-2x)}{7}$ **(e)** $y = 2\left(\frac{3x}{5} - \frac{2}{3}\right)$

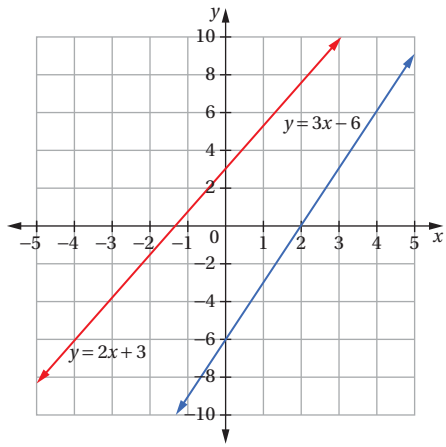
6 Write the rules for the following lines.

(a) The line parallel to $y = 3x - 2$ that passes through (0, 5)

(b) The line parallel to $y = -2x + 7$ that passes through (0, −2)

SC 3 I understand the connection between lines in the form $y = mx + c$ and the slope of the line

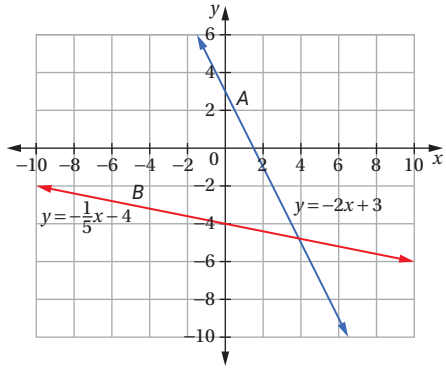
1 Consider the following pairs of linear relationships.



(a) Describe the way in which the gradients are similar.

(b) Which graph is steeper? Explain.

2 Consider the following pairs of linear relationships.



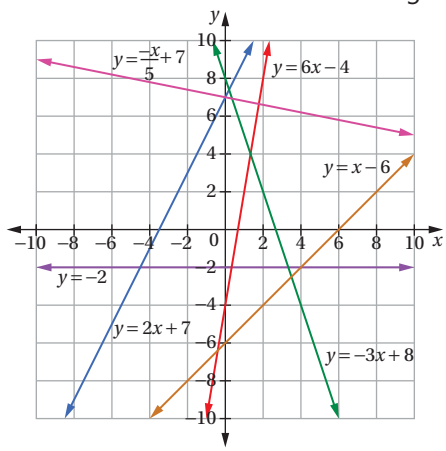
(a) Describe the way in which the gradients are similar.

(b) Which graph is steeper? Explain.

3 State the value of the gradient in each of the following linear relationships.

(a) $y = 3x - 2$ **(b)** $y = -2x + 6$ **(c)** $y = 4 - 3x$ **(d)** $y = -3 - 5x$

4 Write the equation of the graph and the value of the gradient, m, for the lines of the graph:
$y = 2x + 7$, $y = 6x - 4$, $y = -2$, $y = -3x + 8$, $y = x - 6$, $y = 7 - \frac{x}{5}$



(a) with the steepest positive slope

(b) with the steepest negative slope

(c) with zero slope

(d) with the shallowest slope that is not zero.

5 Use an Excel spreadsheet for this question. Place the for x-values in the first column, starting with -3, -2, then highlight these two cells to click and drag down to $x = 5$.
Write the formulas for the linear relationships shown below across the top of the other columns.

	A	B	C	D	E	F
1	x	y=x+3	y=2x+3	y=−2x+3	y=x+5	y=3x−5
2	−3		−3			
3	−2					
4	−1					
5	0					
6	1					
7	2					
8	3					
9	4					
10	5					

In cell C2, the formula =2*A2+3 has been entered for $y = 2x + 3$, using a click on the x-value in the row to achieve the same purpose as typing A2. Type formulas for the first x-value of each equation. Highlight the y-values across the entire row, then click and drag to fill the table.

(a) Highlight the column where the values go down by 2 each time.

(b) Write the equation for the column where the values are going up in threes.

(c) Write the two equations for the columns where the values are going up in ones.

6 State the exact value of the gradient in each of the following linear relationships.

(a) $y = \frac{2x - 5}{7}$ **(b)** $y = \frac{3(4 - 2x)}{5}$ **(c)** $y = \frac{-2(3 - 5x)}{7}$ **(d)** $y = 4\left(\frac{3x}{5} - \frac{2}{7}\right)$

6.4 Determine the rule for a linear relationship or pattern

Learning intention: To be able to determine the rule for a linear relationship or pattern

Success criteria:

SC 1 I can determine the rule for a linear relationship using two points.

SC 2 I can determine the rule for a linear relationship where the y-intercept is known.

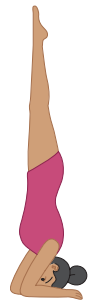
Lesson warm-up

Human graphs

To create a linear model, choose a starting point and then identify constant changes in a horizontal and vertical direction.

Model a linear pattern using a Cartesian plane.

Explain how you could arrange people or classroom objects in a linear pattern.



SC 1 I can determine the rule for a linear relationship using two points

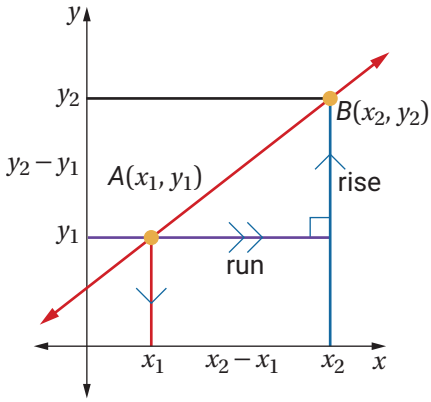
Apart from vertical lines in the form $x = a$, all other linear relationships can be written in the form $y = mx + c$, where m represents the gradient and c is the y-intercept.

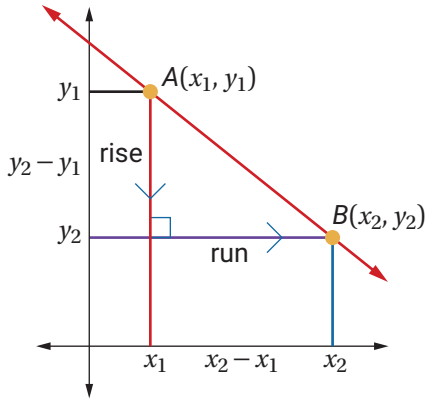
The gradient can be calculated from any two points on the line by comparing the change in the y-values with the change in the x-values. If the two points are taken from left to right, the change in the x-values, the run, will be positive. Then the change in the y-values, the rise, will be positive if the line is increasing and negative if the line is decreasing.

For two points $A(x_1, y_1)$ and $B(x_2, y_2)$: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Graphically this is: gradient = $\dfrac{\text{rise}}{\text{run}}$

The formula works for both positive and negative gradients. Working from left to right, the value of $y_2 - y_1$ gives a negative rise when the value of y_1 is greater than y_2.



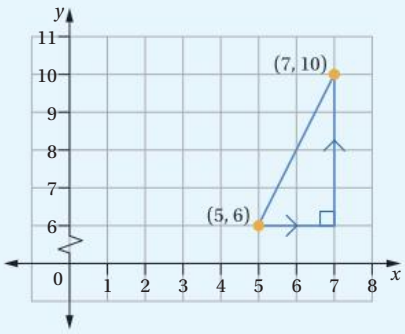


Worked example

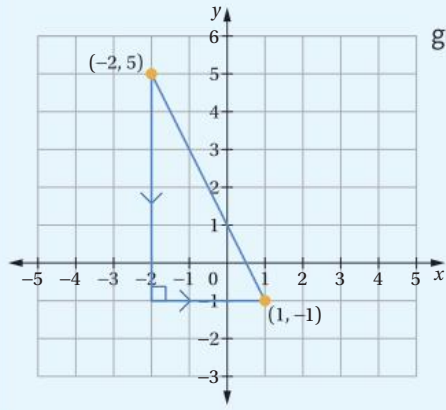
Determining the rule of a graph from two points

Determine the rule for the lines joining:

(a) (5, 6) and (7, 10)

THINKING	WORKING
Assign the given points to (x_1, y_1) and (x_2, y_2).	$(5, 6) \rightarrow (x_1, y_1)$ $(7, 10) \rightarrow (x_2, y_2)$
Recall the rule for the gradient of a straight line.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute and then calculate the gradient.	$m = \frac{10 - 6}{7 - 5}$ $= \frac{4}{2}$ $= 2$
Alternatively, sketch the points on grid paper and use gradient = $\frac{\text{rise}}{\text{run}}$.	 gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{4}{2}$ $= 2$
Write the rule in the form $y = mx + c$.	$y = 2x + c$
Now substitute one of the coordinate pairs to determine the value of c.	Using (5, 6): $x = 5, y = 6$ $6 = 2 \times 5 + c$ $6 = 10 + c$ $-4 = c$ $c = -4$
Write the rule in the form $y = mx + c$, using the solved value for c.	$y = 2x - 4$
You can check this by substituting the other coordinate pair.	Check using (7, 10): $10 = 2 \times 7 - 4$ $10 = 14 - 4$ $10 = 10$ LHS = RHS

(b) (–2, 5) and (1, –1)

THINKING	WORKING
Assign the given points to (x_1, y_1) and (x_2, y_2).	$(-2, 5) \rightarrow (x_1, y_1)$ $(1, -1) \rightarrow (x_2, y_2)$
Recall the rule for the gradient of a straight line.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute and then calculate the gradient.	$m = \frac{-1-5}{1-(-2)}$ $= \frac{-6}{3}$ $= -2$
Alternatively, sketch the points on grid paper and use gradient = $\frac{\text{rise}}{\text{run}}$.	 gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{-6}{3}$ $= -2$
Write the rule in the form $y = mx + c$.	$y = -2x + c$
Now substitute one of the coordinate pairs to find the value of c.	Using (1, –1): $x = 1, y = -1$ $-1 = -2 \times 1 + c$ $-1 = -2 + c$ $-1 + 2 = c$ $c = 1$
Write the rule in the form $y = mx + c$, using the solved value for c.	$y = -2x + 1$
You can check this by substituting the other coordinate pair.	Check using (–2, 5): $5 = -2 \times (-2) + 1$ $5 = 4 + 1$ $5 = 5$ LHS = RHS

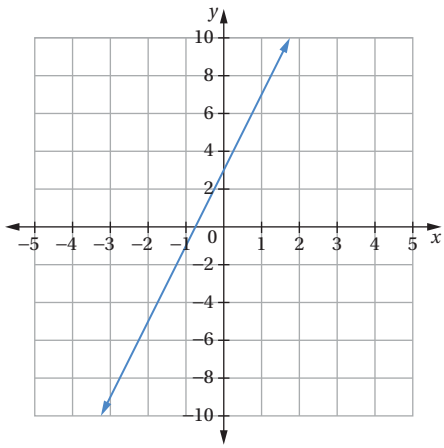
SC 2 I can determine the rule for a linear relationship where the y-intercept is known

If the y-intercept is known, then calculating the gradient will enable the rule to be found in the form $y = mx + c$.

Worked example

Calculating the rule for a line using the y-intercept

Determine the rule for the line shown in the following graph.



THINKING	WORKING
Identify the value of the y-intercept and substitute the value into the rule.	$y = mx + c$, where $c = 3$: $y = mx + 3$
Identify another point on the graph. A whole number pair is best.	Use the point (1, 7).
Calculate the change in x and the change in y between the two points.	Using (0, 3) and (1, 7): Rise: $y_2 - y_1 = 7 - 3$ $= 4$ Run: $x_2 - x_1 = 1 - 0$ $= 1$
Calculate the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{4}{1}$ $= 4$
Substitute the value of the gradient into the rule.	$y = 4x + 3$
Use another whole number pair to check that the rule is correct.	Check using (−2, −5): $-5 = 4 \times (-2) + 3$ $-5 = -8 + 3$ $-5 = -5$ LHS = RHS

Practice

ANSWERS Page 577

SC 1 I can determine the rule for a linear relationship using two points

1 Determine the rule for the linear relationship that connects each of the following pairs of points.

(a) (1, 5) and (4, 14)

(b) (1, –1) and (4, 5)

(c) (2, 13) and (7, 43)

(d) (2, 9) and (7, 44)

2 Determine the rule for the linear relationship that connects each of the following pairs of points.

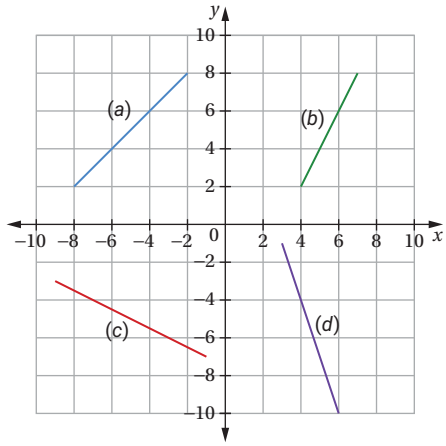
(a) (–1, 5) and (2, 2)

(b) (–3, 3) and (3, –9)

(c) (–2, 11) and (1, –1)

(d) (–1, 0) and (1, –10)

3 The following graph shows some line segments. Determine the rule for each of the line segments.



4 Determine the rule for the linear relationship that connects each of the following pairs of points.

(a) (1, 3) and (6, 4)

(b) (–4, –2) and (2, 3)

(c) (2, 5) and (7, 3)

(d) (–2, 0) and (9, –6)

5 Determine the rule for the line described in each of the following situations.

(a) A line parallel to $y = 2x + 4$ that passes through (2, 1)

(b) A line parallel to $y = 5x - 4$ that passes though (3, –2)

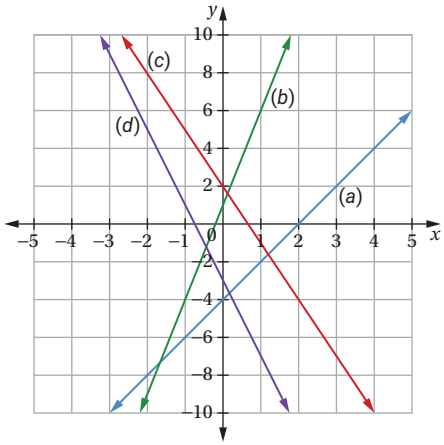
(c) A line parallel to $y = -3x + 4$ that passes through (–4, 6)

(d) A line parallel to $y = -6x - 3$ that passes through (–3, –4)

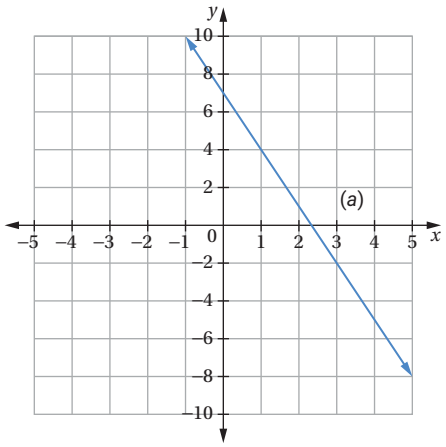
6 A line parallel to $y = -2x + 4$ passes through (3, 5). Determine the coordinates of the x-intercept of the line.

SC 2 I can determine the rule for a linear relationship where the y-intercept is known

1 Determine the rule for each of the linear relationships shown in the graph.



2 One of the questions on a linear graphs test asked the students to determine the rule for the line shown in the graph below.



(a) Alexa wrote that the rule was $y = -2x + 6$. Explain how you know that her answer is incorrect.

(b) Bashir wrote that the rule was $y = 3x + 7$. Explain how you know that his answer is incorrect.

(c) State the correct value of the y-intercept.

(d) Calculate the gradient.

(e) Determine the correct rule for the linear relationship shown.

3 The rule for a linear relationship has a negative gradient and a y-intercept between -6 and -3.

(a) Determine two different rules that fit this description.

(b) Determine the coordinates for the x-intercept for each of the relationships.

4 Determine the rule for each of the following linear relationships.

(a) The y-intercept at (0, 4) and the x-intercept at (−3, 0)

(b) The y-intercept at (0, −2) and the x-intercept at (6, 0)

(c) The y-intercept at (0, 6) and the x-intercept at (4, 0)

(d) The y-intercept at (0, −3) and the x-intercept at (−7, 0)

6.5 Solve linear equations graphically

Learning intention: To be able to solve linear equations graphically

Success criteria:

SC 1 I can solve linear equations of the type $ax + b = c$ using a pair of linear graphs.

SC 2 I can solve linear equations of the type $ax + b = cx + d$ using a pair of linear graphs.

SC 3 I can use digital tools to investigate integer solutions to $ax + by = c$.

Lesson warm-up

Buying a car

The red car is twice the price of the blue car.

The blue car costs $6000 more than the yellow car

The red car is three times the price of the yellow car.

How much is the blue car?

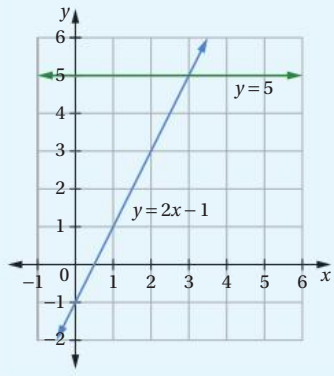
SC 1 I can solve linear equations of the type $ax + b = c$ using a pair of linear graphs

The solution to a linear equation in the form $ax + b = c$, where a, b and c have numerical values, can be determined by the intersection of the graphs of $y = ax + b$ and $y = c$.

Worked example

Solving a linear equation from the intersection of an oblique and a horizontal graph

Use a pair of graphs to solve the equation $2x - 1 = 5$.

THINKING	WORKING
Set the expression on either side of the equals sign to equal y.	$y = 2x - 1$ and $y = 5$
Graph both lines on the same set of axes. Determine the coordinates of the point of intersection.	 The coordinates of the point of intersection are (3, 5).
Write the answer.	The solution is $x = 3$, when $x = 3$, $2x - 1 = 5$

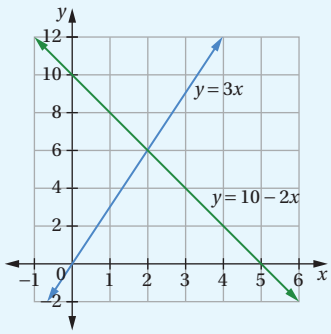
SC 2 I can solve linear equations of the type $ax + b = cx + d$ using a pair of linear graphs

The solution to a linear equation in the form $ax + b = cx + d$, where a, b, c and d have numerical values, can be determined by the intersection of the graphs of $y = ax + b$ and $y = cx + d$.

Worked example

Solving a linear equation from the intersection of two oblique graphs

Use a pair of graphs to solve the equation $3x = 10 - 2x$.

THINKING	WORKING
Set the expression on either side of the equals to equal y.	$y = 3x$ and $y = 10 - 2x$
Graph both lines on the same set of axes.	
Determine the coordinates of the point of intersection.	The coordinates of the point of intersection are (2, 6).
Write the answer.	When $x = 2$, $3x = 6$ and $10 - 2x = 6$. The solution to $3x = 10 - 2x$ is $x = 2$.

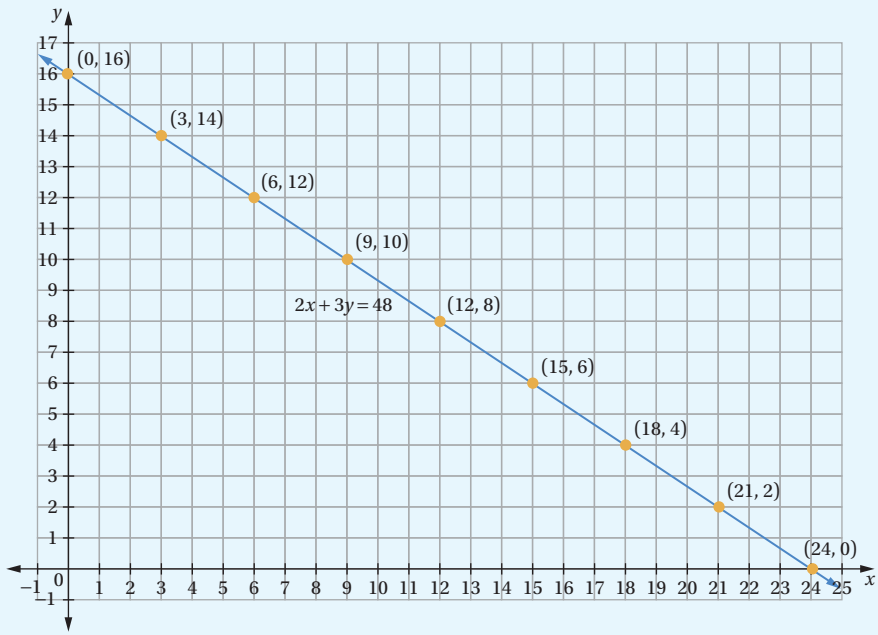
SC 3 I can use digital tools to investigate integer solutions to $ax + by = c$

A linear equation in the form $ax + by = c$, where a, b and c have numerical values, has an infinite number of points, each pair giving a solution to the equation. Drawing the graph on a grid helps reveal coordinate pairs where both values are integers.

Worked example

Using a graph to determine integer solutions to an equation in the form $ax + by = c$

From the graph of $2x + 3y = 48$, determine coordinate pairs representing integer solutions to the equation.

THINKING	WORKING
Determine the integer (whole number) coordinates, for $x \geq 0$.	Pairs of solutions: (0, 16), (3, 14), (6, 12), (9, 10), (12, 8), (15, 6), (18, 4), (21, 2) and (24, 0) 
Describe the pattern in words.	The x-value can be any multiple of 3. The matching y-values are multiples of 2, starting at $y = 16$ when $x = 0$ and going down by 2 for each increase in 3 by the x-value.
Write the answer as an infinite set of solutions, in the form of coordinate pairs.	<table><tr><td>x</td><td>−6</td><td>−3</td><td>0</td><td>24</td><td>27</td><td>30</td></tr><tr><td>y</td><td>20</td><td>18</td><td>16</td><td>0</td><td>−2</td><td>−4</td></tr></table> The set of solutions is: ..., (−6, 20), (−3, 18), (0, 16), (3, 14), (6, 12), ...

Practice

ANSWERS Page 577

SC 1 I can solve linear equations of the type $ax + b = c$ using a pair of linear graphs

1 For each of the following equations:

(i) sketch the graph

(ii) determine the coordinates of the point of intersection

(iii) determine the solution.

(a) $3x - 1 = 11$ **(b)** $4x + 1 = 9$ **(c)** $5 - 2x = 7$ **(d)** $4x + 9 = 1$

2 For each of the following equations:

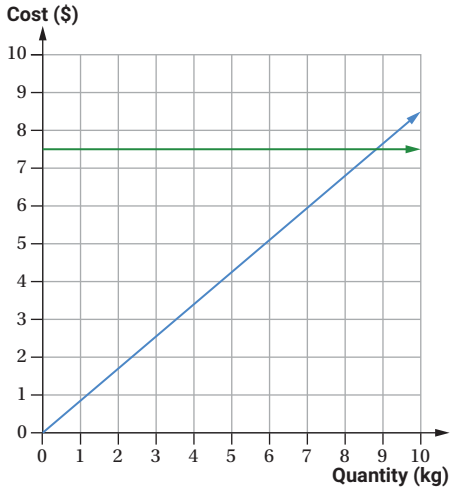
(i) sketch the graph

(ii) determine the coordinates of the point of intersection

(iii) determine the solution.

(a) $\frac{x+5}{4} = 4$ **(b)** $\frac{3x+7}{2} = -4$ **(c)** $\frac{x}{6} + 7 = 9$ **(d)** $3(x - 2) = 15$

3 The following graph shows the cost of potatoes at the greengrocers for up to 10 kg, and a horizontal graph.

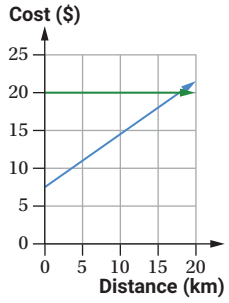


(a) Determine the equation to the oblique line, and hence state the per kg cost of potatoes.

(b) State the equation for the horizontal line and describe a meaning for the line in the context of purchasing potatoes.

(c) Write the coordinates of the point of intersection of the graphs and explain what it means in this context.

4 The following graph shows the cost for delivery by a food courier, $y = 0.8x + 7.5$, for up to 20 km and a horizontal line, $y = 20$.

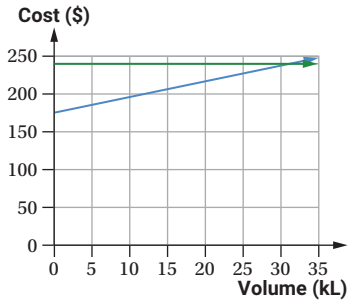


(a) Assign meanings to the numerical values in the equation to the oblique line.

(b) Assign a meaning to the horizontal line in the context of food delivery.

(c) Write the coordinates of the point of intersection of the graphs and explain what this means in this context.

5 The oblique graph line can be used to calculate the annual cost of water at a household for up to 35 kL.



(a) Write the formula for the oblique line, $y = 2.1x + 175$, in words.

(b) Assign a meaning to the horizontal line in the context of annual water costs.

(c) Write the coordinates of the point of intersection of the graphs and explain what it means in this context.

SC 2 I can solve linear equations of the type $ax + b = cx + d$ using a pair of linear graphs

1 For each of the following equations:

(i) sketch the graph

(ii) determine the coordinates of the point of intersection

(iii) determine the solution.

(a) $4x = x + 6$ **(b)** $-5x = 2x + 7$ **(c)** $x = 3x - 6$ **(d)** $2x = 5x + 6$

2 For each of the following equations:

(i) sketch the graph

(ii) determine the coordinates of the point of intersection

(iii) determine the solution.

(a) $2x - 7 = 5 - x$ **(b)** $x + 4 = 2x + 6$ **(c)** $2x - 5 = 3x - 6$ **(d)** $x + 3 = 3x + 5$

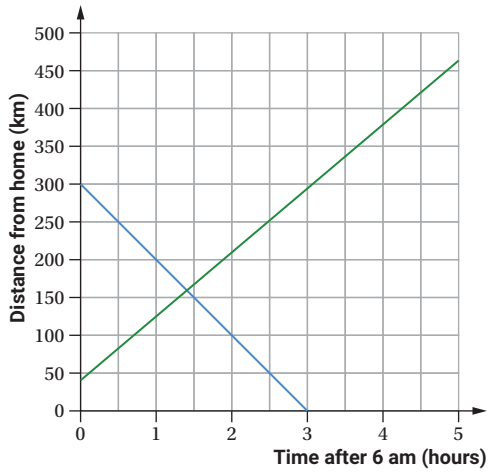
3 Consider the equation $20x + 140 = 50x - 100$.

(a) Sketch the graphs $y = 20x + 140$ and $y = 50x - 100$.

(b) Determine the coordinates of the point of intersection.

(c) Determine the solution to the equation $20x + 140 = 50x - 100$.

4 The following graph shows the distance from home of a man travelling away from home as $y = 85x + 40$, and a second line for his sister's journey along the same road.



(a) Describe the man's journey in terms of speed, direction, place and time.

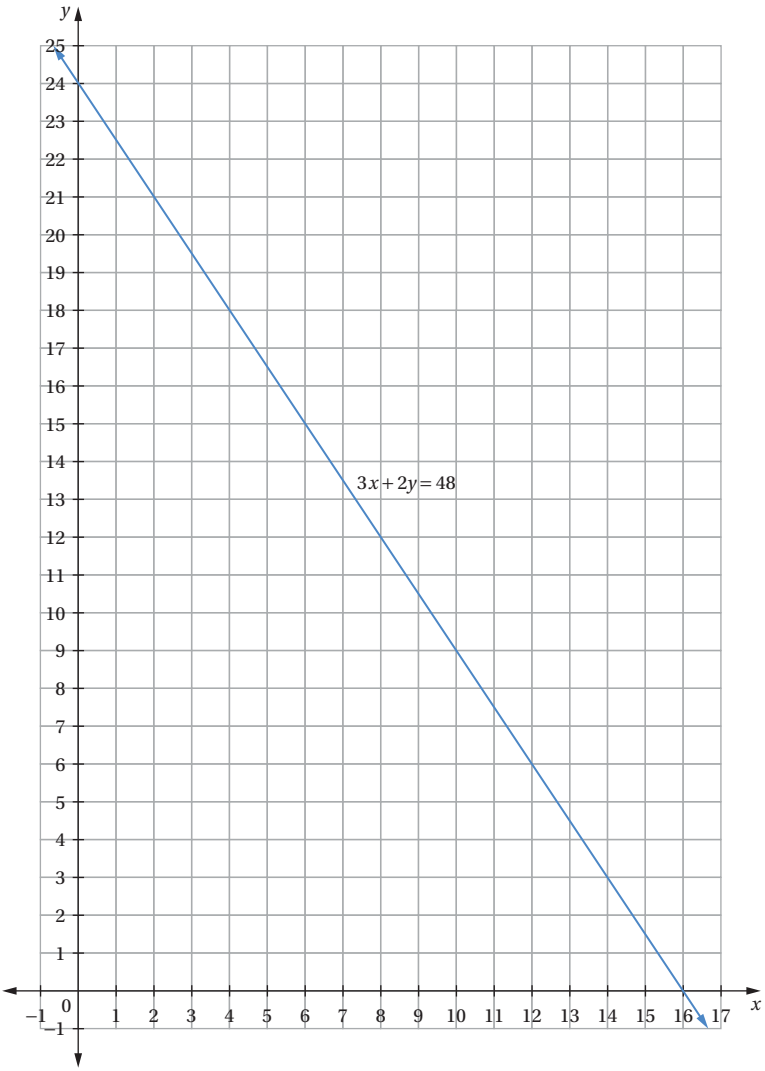
(b) Describe the sister's journey in terms of speed, direction, place and time.

(c) Write an equation for the sister's journey.

(d) Read off the point of intersection and explain what this means in the context of the two journeys.

SC 3 I can use digital tools to investigate integer solutions to $ax + by = c$

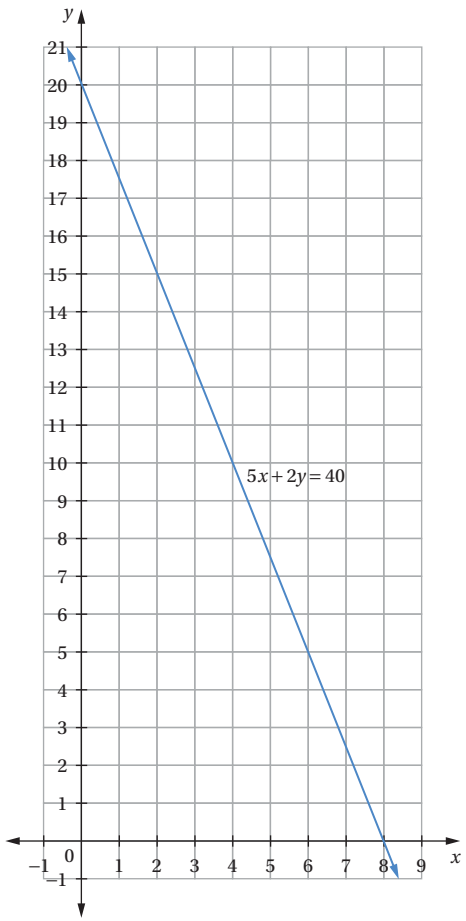
1 From the graph of $3x + 2y = 48$:



(a) determine the first three pairs of integer (whole number) solutions to the equation for $x \geq 0$

(b) list the infinite set of integer solutions as coordinate pairs.

2 From the graph of $5x + 2y = 40$:



(a) determine the first three coordinate pairs of integer (whole number) solutions to the equation, for $x \geq 0$

(b) list the infinite set of integer solutions, listed as coordinate pairs.

3 For each of the following graphs:

(i) sketch the graph using digital technology (for example, Desmos)

(ii) determine the coordinate pairs of integer (whole number) solutions to the equation for $x \geq 0$ and $y \geq 0$.

(a) $x + 2y = 10$ **(b)** $3x + y = 10$ **(c)** $x + 4y = 20$ **(d)** $6x + y = 20$

4 Use graphs to write any four pairs of integer solutions to each of the following equations:

(a) $3x + 4y = 12$ **(b)** $4x + 3y = 12$

5 From the graph of $2x + 2y = 15$:

(a) write all of the pairs of integer solutions to the equation

(b) explain why this result occurred

(c) write three other equations, in the same form, that give the same result.

TOPIC REVIEW

Multiple choice questions

ANSWERS Page 622

1 Which of these linear graphs will have the steepest slope?

A $y = 5x$ **B** $y = -4x + 3$ **C** $y = 6x - 3$ **D** $y = \frac{x}{8} + 16$

2 The coordinates of a point that lies on the graph of $y = 2x + 3$ are:

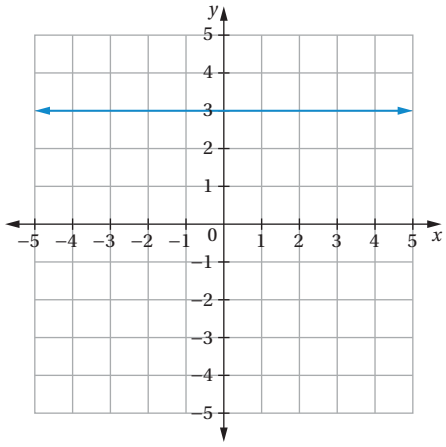
A (2, 6) **B** (1, 6) **C** (1, 5) **D** (3, 2)

3 Which of the following equations matches this table of values?

x	−2	0	2	4
y	−11	−3	5	13

A $y = -11x$ **B** $y = 4x - 3$ **C** $y = 3x - 4$ **D** $y = x - 3$

4 Which of the following equations matches the graph shown below?



A $x = 3$ **B** $x = -3$ **C** $y = 3$ **D** $y = -3$

5 The x-intercept of the line with equation $y = 1 - \frac{5x}{2}$ is:

A $\left(-\frac{5}{2}, 0\right)$ **B** $\left(\frac{5}{2}, 0\right)$ **C** $\left(-\frac{2}{5}, 0\right)$ **D** $\left(\frac{2}{5}, 0\right)$

6 The line given by the equation $y = 0.01x - 5$:

A has a gradient of zero

B is nearly flat

C is very steep

D is vertical

7 An ordered pair representing a point that lies on the x-axis is:

A (−1, −4) **B** (0, 4) **C** (3, 0) **D** (5, 6)

8 If you graph the equation $y = 3x - 7$, the intercept with the y-axis is:

A 3 **B** −7 **C** 0 **D** $\frac{7}{3}$

9 The y-intercept of the line with equation $y = \frac{3x+1}{2}$ is:

A 3 **B** 1 **C** -1 **D** $\frac{1}{2}$

10 The ordered pair $(-4, 1)$ lies on which of the following lines?

A $x + y = 4$ **B** $2x + y = 4$ **C** $x + 3y = -1$ **D** $3x + y = -1$

11 The graphs of $x = 4$ and $y = -2$ are drawn on the same Cartesian axes. The only point the two lines have in common is:

A $(0, 0)$ **B** $(4, 0)$ **C** $(4, -2)$ **D** $(0, -2)$

12 Which line has a gradient equal to 0?

A $x = 5$ **B** $x = y$ **C** $x + y = 1$ **D** $y = 2$

Short answer

1 A shop assistant earns $17.50 per hour. This linear relationship is to be graphed using the general rule $y = mx + b$.

(a) What is the value of b?

(b) What is the value of m?

(c) How much is earnt for a 6-hour workday?

2 Write the equation of the following lines:

(a) Parallel to the y-axis, passing through the point $(3, 5)$

(b) Parallel to the x-axis, passing through the point $(-7, -2)$

(c) Passing through the points $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$

3 State the equation of the horizontal line and the equation of the vertical line passing through the point $\left(-\frac{1}{2}, -2\right)$.

4 Jacob is saving to buy a new touch screen computer. He has $280 already saved and can save another $50/month. He needs a total of $1050 to purchase the computer.

(a) Write Jacob's current savings as a coordinate pair, where the x-value represents time.

(b) Write Jacob's total savings at the end of 2 months as a coordinate pair.

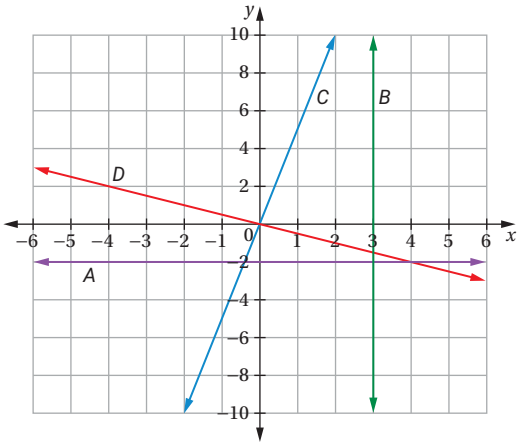
(c) Use these two points to plot a graph of his total savings over 20 months.

(d) How many months will it take Jacob to save enough money to purchase the computer?

5 Complete the following ordered pairs, given the rule $y = 2x + 6$.

(a) $(2, \quad)$ **(b)** $(-1, \quad)$ **(c)** $(\quad, 12)$ **(d)** $(\quad, -8)$

6 State the equation for each of the lines shown on the following set of Cartesian axes.



7 Plot the following points on a Cartesian plane. Join the points with a straight line and determine the:

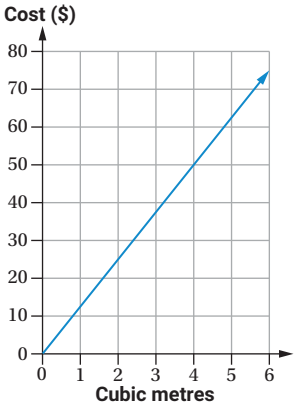
(i) gradient **(ii)** y-intercept **(iii)** rule relating x and y

(a) (2, –4) and (0, 1) **(b)** (4, 5) and (–2, 2)

8 Complete the following table of values for the linear relationship $y = \frac{3(5-x)}{2}$. Give your answers in exact form.

x	–6	–2	0	1	7
y					

9 The following graph shows the cost of various amounts of garden soil.



(a) Estelle has a small project in her garden and requires 2 m^3 of soil. How much will this cost her?

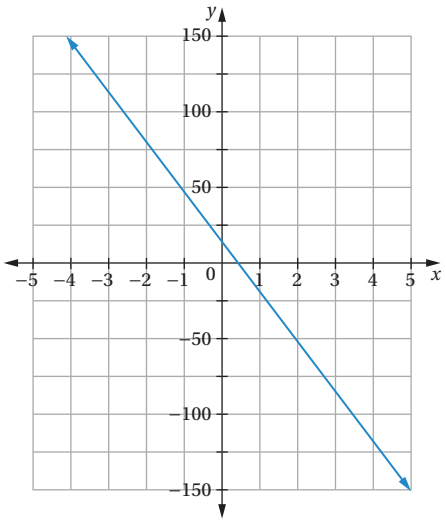
(b) Nigel has a larger project and needs 6 m^3 of soil. How much will this cost him?

(c) Narelle has $60 to spend on soil. What quantity of soil can she buy?

(d) Determine a rule to calculate the cost of this soil.

(e) Use this rule to state the cost of 9.5 m^3 of soil.

10 Consider the following graph.



(a) Which of the following is the best choice for the rule of the linear relationship?

A $y = 30x + 14$

B $y = -33x + 14$

C $y = 0.5x + 20$

D $y = -14x + 0.5$

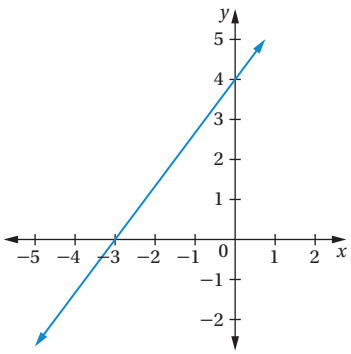
(b) From the graph determine the approximate value of y when $x = 2$.

(c) From the graph determine the approximate value of x when $y = 150$.

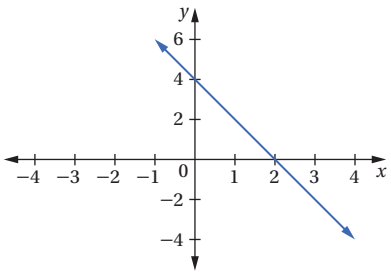
(d) Use these coordinate pairs to determine an approximation for the rule of the linear relationship.

11 Write the equation of each of the following linear relationships.

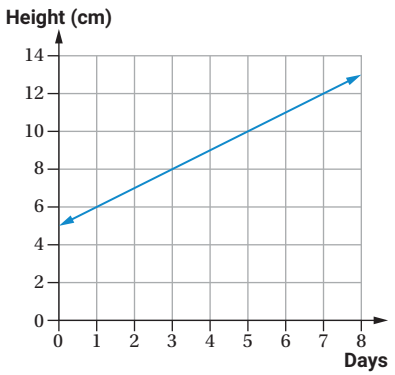
(a)



(b)



12 This graph shows the predicted height, y cm, of a plant as it grows.



(a) What does the y-intercept represent in this situation?

(b) Write a rule to describe this linear relationship.

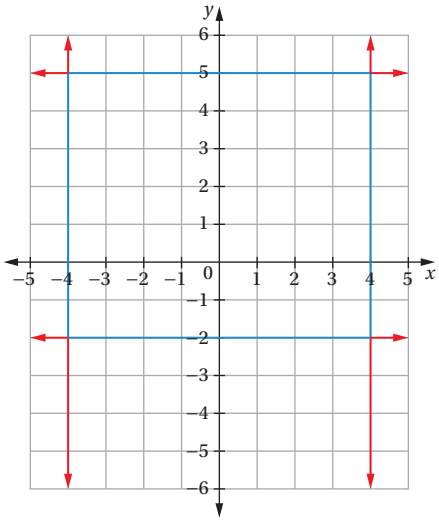
(c) What is the predicted height on day 5?

13 Determine the equations of the lines joining the following pairs of points.

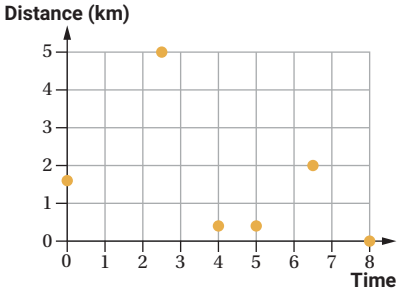
(a) (3, 2) and (7, 10)

(b) (–2, 8) and (3, –7)

14 State the equations of the linear relationships that make up the sides of the rectangle below.



15 The dots on the Cartesian axes, right, represent Lily's position (distance from home) at various times during the day, starting at noon.



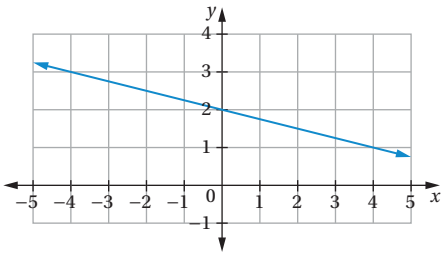
(a) How far from home was Lily at midday?

(b) When can you be certain Lily was back at home?

(c) Name another time that Lily was probably at home?

(d) What is the minimum distance that Lily travelled during the time period shown on the graph?

16 Consider the graph below.



(a) State the coordinates of any three points that lie on this line.

(b) State the rule for the linear relationship.

(c) State the coordinates of the x-intercept.

17 State the coordinates of the x-intercept and the y-intercept for the line with equation $x = -\frac{2y}{5} + \frac{1}{5}$.

18 A linear graph passes through the point (–2, 5).

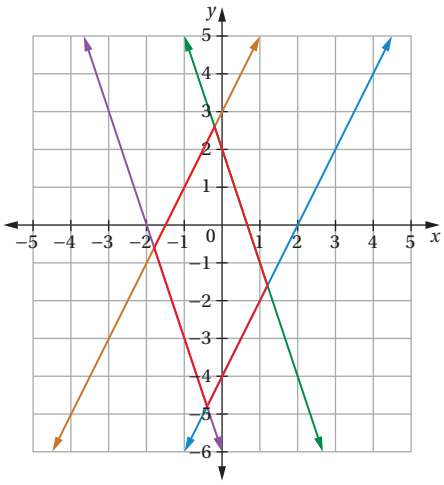
(a) How many rules are possible at this stage for the line?

(b) You are now told the line has a negative gradient. How does this change the number of possible solutions?

(c) You are now told that the line has an x-intercept at $x = 8$. Determine the equation of the line.

(d) What is the y-intercept for this line?

19 Consider the graph below.



(a) State the equations of the linear relationships that make up the shape highlighted.

(b) How can you be sure the shape is a parallelogram?

6

Extended response

1 A mobile car repair company charges a $100 fee to attend for the repair and then $55 per hour for the time spent on the repair.

(a) Determine the rule for calculating the cost, $\$C$, in terms of the time spent on the repair, t hours.

(b) Draw a linear graph of C against t.

(c) Use the graph to estimate how much will you pay if your car is repaired in 3.5 hours.

(d) Now use the rule to determine the cost.

(e) Which of these methods is better, and why?

2 The following table shows the average height of girls at yearly intervals. Use a program such as Excel to plot the points on a set of Cartesian axes.

Age	8	9	10	11	12	13	14	15	16	17	18
Height	127.5	133	138	144	151	157	160.5	162	162.5	163	163

(a) Between which 2 years was the growth rate the highest?

(b) If this growth rate had applied for the whole of the time period shown, how tall would an average 18-year-old be?

(c) Between which 2 years was the growth rate the lowest?

(d) Use the program to determine the equation of the regression line for these points.

(e) Use the rule to determine the predicted height values for 9- and 15-year-old girls.

(f) Do you think the rule for the regression line does a good job in predicting the median height?

3 A household uses 410 kWh of electricity in August 2020 and 380 kWh in June 2021.

(a) Write these two values as coordinate pairs, giving August 2020 a horizontal coordinate of 0.

(b) Use these points to determine a rule for predicted electricity usage.

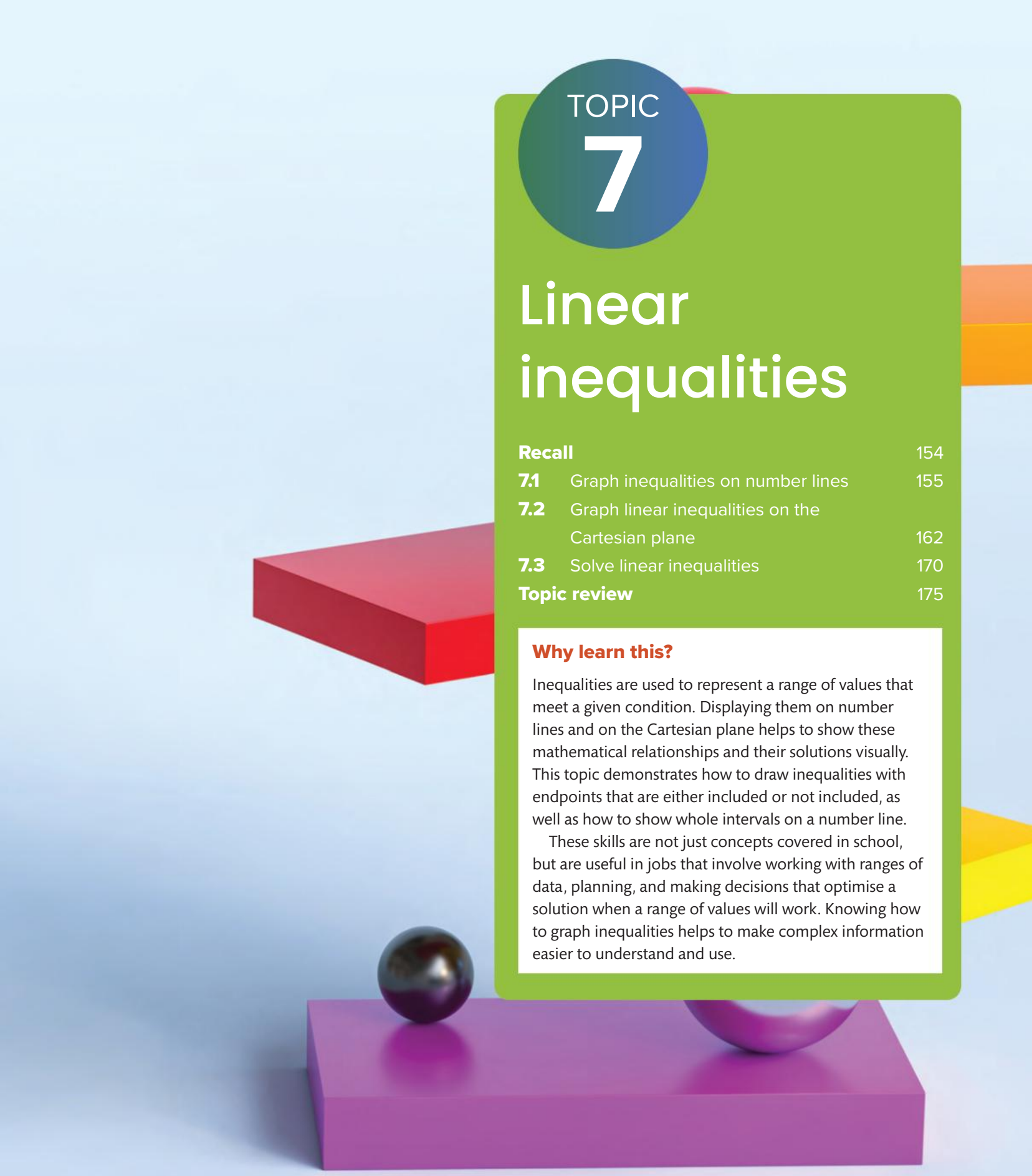
(c) Use this rule to predict the usage for:

(i) October 2020

(ii) July 2021

(d) Do you think it is reasonable to use a linear relationship to predict electricity usage in this context? Explain your answer.

(e) The household actually used 405 kWh in July 2021. Comment on this in the context of your answer to part **(d)**.



TOPIC

7

Linear inequalities

Recall 154

7.1 Graph inequalities on number lines 155

7.2 Graph linear inequalities on the Cartesian plane 162

7.3 Solve linear inequalities 170

Topic review 175

Why learn this?

Inequalities are used to represent a range of values that meet a given condition. Displaying them on number lines and on the Cartesian plane helps to show these mathematical relationships and their solutions visually. This topic demonstrates how to draw inequalities with endpoints that are either included or not included, as well as how to show whole intervals on a number line.

These skills are not just concepts covered in school, but are useful in jobs that involve working with ranges of data, planning, and making decisions that optimise a solution when a range of values will work. Knowing how to graph inequalities helps to make complex information easier to understand and use.

RECALL

ANSWERS Page 581

I can use the <, > and = symbols between pairs of numbers

1 Write < , > or = between the given numbers to make a correct number statement.

(a) 11 ____ 14 **(b)** 21 ____ 12 **(c)** −4 ____ 4

(d) $\frac{1}{2}$ ____ $\frac{1}{5}$ **(e)** $\frac{1}{4}$ ____ 0.25 **(f)** −4.73 ____ 4.37

(g) −4.736 ____ −4.727 **(h)** $-\frac{1}{2}$ ____ $-\frac{1}{5}$ **(i)** 0.02 ____ 0.18

I can plot points on a Cartesian plane

1 Plot and label the following points on a Cartesian plane.

(a) $A(4, -5)$ **(b)** $B(-3, 1)$ **(c)** $C(0, -5)$

(d) $D(-3, 5)$ **(e)** $E(-2, 0)$ **(f)** $F(0, 4)$

I can use substitution to confirm the solution of an equation

1 For each of the following equations, check whether the solution given in the brackets is correct by writing true (T) or false (F).

(a) $2a + 8 = 20 \quad [a = 14]$ **(b)** $2x - 6 = 32 \quad [x = 19]$

(c) $\frac{b}{5} + 3 = 6 \quad [b = 15]$ **(d)** $6 + \frac{y}{11} = 60 \quad [y = 6]$

(e) $5p - 5 = -20 \quad [p = -3]$

I can solve two-step equations

1 Solve these equations.

(a) $3x - 4 = 8$ **(b)** $2b + 14 = -4$ **(c)** $3q + 7 = 28$

(d) $\frac{y}{4} - 5 = 15$ **(e)** $12 + \frac{d}{2} = 4$

Graph inequalities on number lines

7.1

Learning intention: To be able to graph inequalities on number lines

Success criteria:

SC 1 I can graph inequalities with non-inclusive endpoints on number lines.

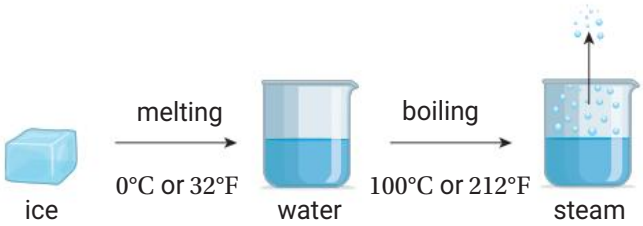
SC 2 I can graph inequalities where the endpoint is included on number lines.

SC 3 I can graph an interval on a number line.

Lesson warm-up

Fahrenheit and Celsius

In Australia, prior to July 1974, Australians measured the temperature in degrees Fahrenheit (°F). In Australia, temperatures are now measured in degrees Celsius (°C).



The freezing point of water is 0 °C or 32 °F.

The boiling point of water is 100 °C or 212 °F.

The relationships are linear.

Is there a temperature that is the same on both scales?

Consider the statement: 'The number value in the temperature written in Fahrenheit is larger than the number of degrees Celsius.' Is the statement sometimes true, always true or never true?

SC 1 I can graph inequalities with non-inclusive endpoints on number lines

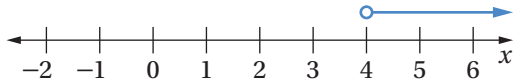
Inequalities are used to compare numbers.

Symbol	Meaning	Example
$<$	is less than	$5 < 10$
$>$	is greater than	$10 > 5$

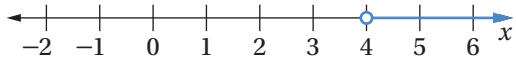
To graph the numbers that are greater than 4, choose either a horizontal or a vertical number line.

For a horizontal number line, the conventional variable is x.

The graph of $x > 4$ is shown.

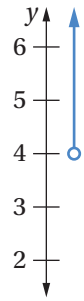


The line can be drawn directly on top of the number line.



The open circle is used to indicate that the starting point is not included.

For a vertical number line, the conventional variable is y. The graph of $y > 4$ is shown on the right.

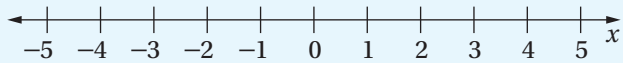
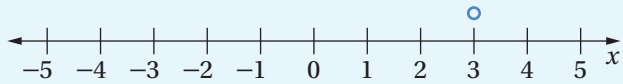
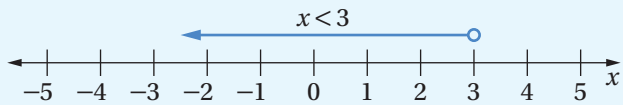


Worked example

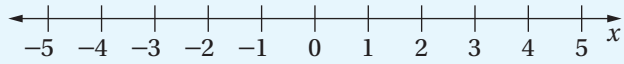
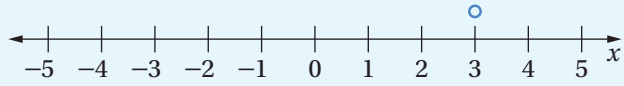
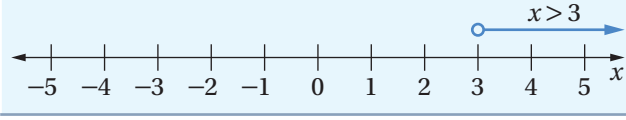
Sketching an inequality with a non-inclusive endpoint on a number line

Graph the following inequalities on a number line.

(a) $x < 3$

THINKING	WORKING
Sketch a number line that includes the number in the inequality (3) and several numbers on either side.	
Plot an open circle above the stated value on the number line.	
Sketch an arrow from the circle in the appropriate direction and label the line (to the left for < and to the right for >).	

(b) $x > 3$

THINKING	WORKING
Sketch a number line that includes the number in the inequality (3) and several numbers on either side.	
Plot an open circle above the stated value on the number line.	
Sketch an arrow from the circle in the appropriate direction and label the line (to the left for < and to the right for >).	

SC 2 I can graph inequalities where the endpoint is included on number lines

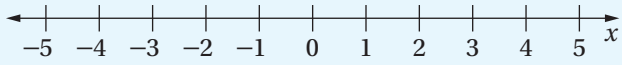
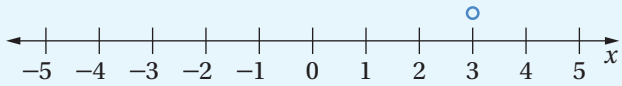
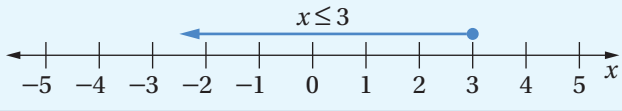
The only difference between $x < 3$ and $x \leq 3$ is that 3 is included. For the graph on a number line, this is shown by a coloured-in or solid circle.

Worked example

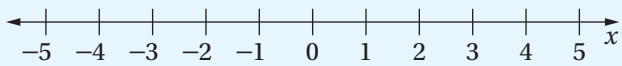
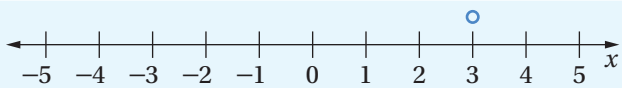
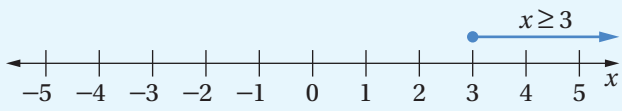
Graphing an inequality with inclusive endpoints on number lines

Graph the inequalities on a number line.

(a) $x \leq 3$

THINKING	WORKING
Sketch a number line that includes the number in the inequality (3) and several numbers on either side.	
Plot a closed circle above the stated value on the number line.	
Sketch an arrow from the circle in the appropriate direction and label the line (to the left for ≤ and to the right for ≥).	

(b) $x \geq 3$

THINKING	WORKING
Sketch a number line that includes the number in the inequality (3) and several numbers on either side.	
Plot a closed circle above the stated value on the number line.	
Sketch an arrow from the circle in the appropriate direction and label the line (to the left for ≤ and to the right for ≥).	

SC 3 I can graph an interval on a number line

An interval is bounded at both ends. A number that lies anywhere from 2 to 10 could be written as $x \geq 2$ and $x \leq 10$ at the same time. Since $2 \leq x$ is the same as $x \geq 2$, we can combine the two inequalities as follows: $2 \leq x \leq 10$.

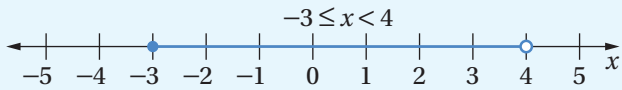
By convention, the interval is written from small to large, i.e. from left to right on a horizontal number line. For a vertical number line, $2 \leq y \leq 10$ is written from bottom to top.

Worked example

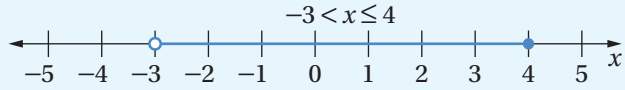
Graphing an interval on a number line

Graph the following intervals on separate number lines.

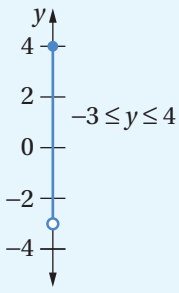
(a) $-3 \leq x < 4$

THINKING	WORKING
Sketch the number line to include the endpoints of the interval and more. Put a solid circle for the included endpoint and an open circle for the non-included endpoint. Join the circles with a line segment.	

(b) $-3 < x \leq 4$

THINKING	WORKING
Sketch the number line to include the endpoints of the interval and more. Put a solid circle for the included endpoint and an open circle for the non-included endpoint. Join the circles with a line segment.	

(c) $-3 \leq y \leq 4$

THINKING	WORKING
Sketch the vertical number line to include the endpoints of the interval and more. Put a solid circle for both of the included endpoints. Join the circles with a line segment.	

Practice

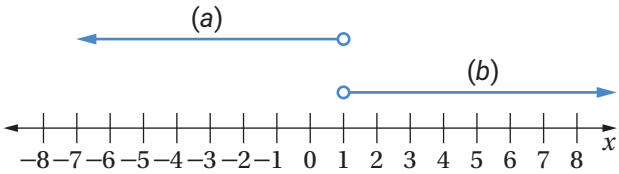
ANSWERS Page 581

SC 1 I can graph inequalities with non-inclusive endpoints on number lines

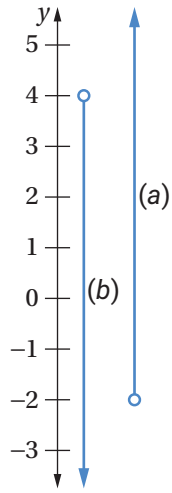
1 Graph the following inequalities on a number line. (You can stack them up on a single number line, but make sure each one is labelled.)

(a) $x < 5$ **(b)** $x > 5$ **(c)** $x < -5$ **(d)** $x > -5$

2 Write the inequalities shown on the number line.

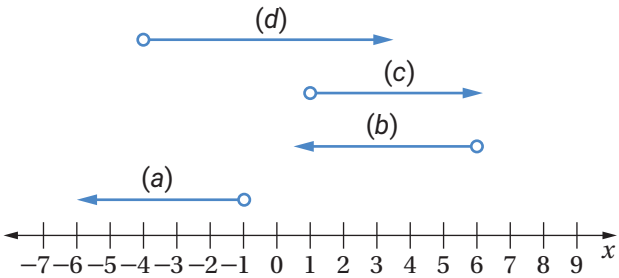


3 Write the inequalities shown on the vertical number line on the right.

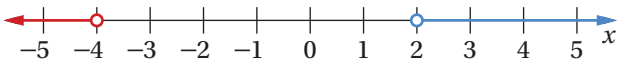


4 Explain how to graph inequalities that use the 'less than' or 'greater than' symbols.

5 The number line shown has several inequalities drawn on it.
Write the inequality for each.



6 The following number line has the inequalities drawn directly on the line.



(a) Write an inequality to match the red line.

(b) Write an inequality to match the blue line.

(c) What would make this presentation style difficult if the direction of each inequality was reversed?

(d) Use this style of presentation to show $x < -3$ and $x > -1$ on the same number line.

7 List any three values of x or y belonging to each of the following inequalities.

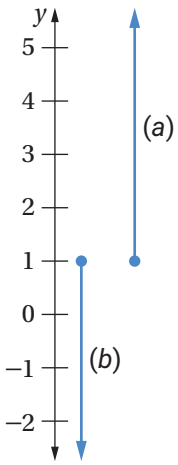
(a) $x < 4$ **(b)** $x > -1$ **(c)** $y < -2$ **(d)** $y > -5$

SC 2 I can graph inequalities where the endpoint is included on number lines

1 Graph the following inequalities on the same number line.

(a) $x \leq 5$ **(b)** $x \geq 5$ **(c)** $x \leq -5$ **(d)** $x \geq -5$

2 Write the inequalities shown on the number line.

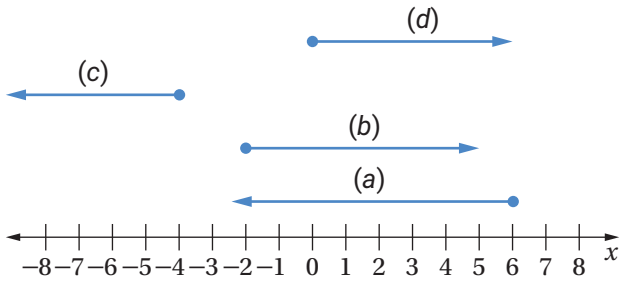


3 Explain how to graph inequalities that use the 'less than or equal to' or the 'greater than or equal to' symbols.

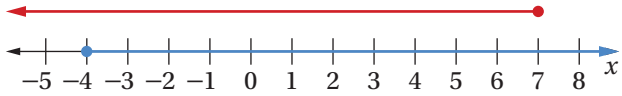
4 Graph the following inequalities on the same number line.

(a) $x \geq 3$ **(b)** $x \geq -4$ **(c)** $x \leq 0$ **(d)** $x \leq 5$

5 The number line shown has several inequalities drawn on it. Write the inequality for each.



6 The following number line has one of the inequalities drawn directly on the line.



(a) Write an inequality to match the red line.

(b) Write an inequality to match the blue line.

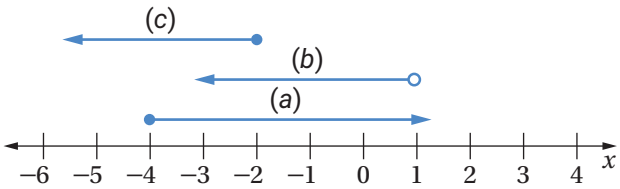
(c) Why did the red line have to be away from the number line, unlike the blue line?

(d) Use this style of presentation to show $x \geq -3$ and $x \leq -1$ on the same number line.

7 For each inequality, list three values for x or y, the end value and any two others that could be included.

(a) $x \leq 6$ **(b)** $x \geq 5$ **(c)** $y \leq -3$ **(d)** $y \geq -6$

8 Consider the number line shown below.



Choose the correct inequality symbol for each graph: $<$, $>$, $\leq$ or $\geq$.

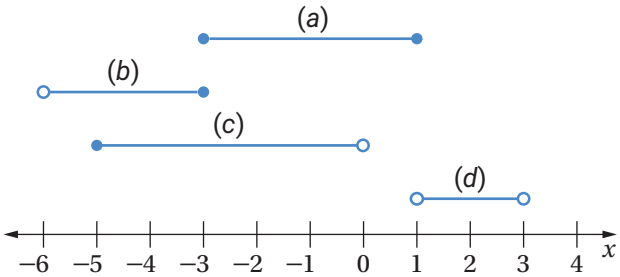
(a) x ___ -4 **(b)** x ___ 1 **(c)** x ___ -2

SC 3 I can graph an interval on a number line

1 Graph the following intervals on separate number lines.

(a) $-1 < x < 2$ **(b)** $-1 < y \leq 2$ **(c)** $-1 \leq x < 2$ **(d)** $-1 \leq y \leq 2$

2 Write the inequalities for the intervals shown on the number line.

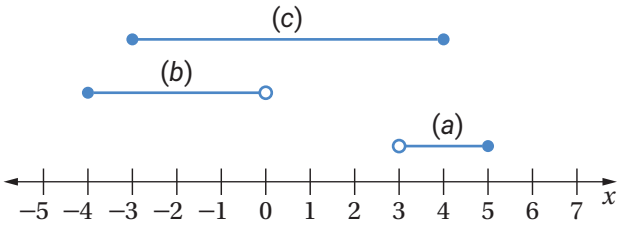


3 Explain how to graph intervals on a number line.

4 Graph the following intervals on the same number line.

(a) $-4 \leq x < 5$ **(b)** $-2 < x < 2$ **(c)** $0 \leq x \leq 6$ **(d)** $3 < x \leq 5$

5 Consider the number line shown below.



Insert the correct two inequality symbols for each interval, where each is either $<$ or $\leq$.

(a) 3 ___ x ___ 5 **(b)** -4 ___ x ___ 0 **(c)** -3 ___ x ___ 4

6 List any three values for x or y that belong in each of the following intervals. For included endpoints, write them in your list.

(a) $-2 \leq x \leq 4$ **(b)** $0 < y \leq 6$ **(c)** $-3 < y \leq 0$ **(d)** $-3 \leq x < 4$

7 In answer to a test question, Andrea wrote that the interval was $-2 < x > 4$. Explain why this is not a possible interval, using some numbers as examples.

7.2 Graph linear inequalities on the Cartesian plane

Learning intention: To be able to graph linear inequalities on the Cartesian plane

Success criteria:

SC 1 I can graph inequalities involving single variables on the Cartesian plane.

SC 2 I can graph intervals involving single variables on the Cartesian plane.

SC 3 I can graph inequalities involving two variables on the Cartesian plane.

Lesson warm-up

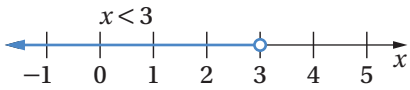
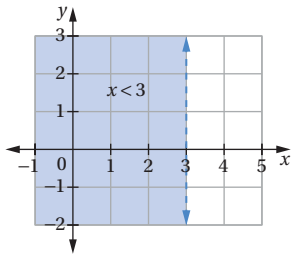
Is it true?

Determine whether the following inequality is sometimes true, always true or never true.

$x + y < xy + 1$

SC 1 I can graph inequalities involving single variables on the Cartesian plane

The inequality $x < 3$ appears differently on the Cartesian plane than its graph on a number line.

Number line	Cartesian plane
	
A ray is drawn to the left of $x = 3$ on the horizontal axis. An empty circle at $x = 3$ shows that the end value is not included.	The area to the left of the $x = 3$ grid line is shaded. The dotted (broken) line $x = 3$ shows that points on the line are not included. Any y-value is allowable.

For $x \leq 3$, a solid dot is used at $x = 3$ on the number line, and a solid (unbroken) line is used through $x = 3$ on the graph.

For $x > 3$ and $x \geq 3$ on the Cartesian plane, the shading is to the left.

For $y < 3$ and $y \leq 3$ on the Cartesian plane, the shading is below the $y = 3$ gridline.

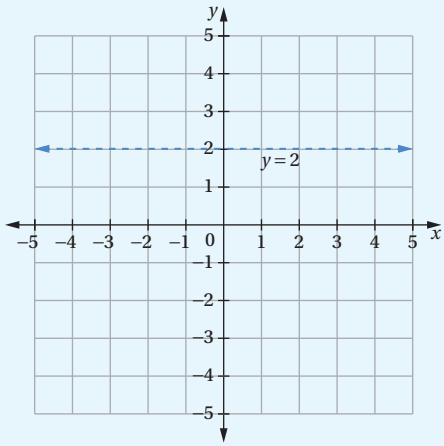
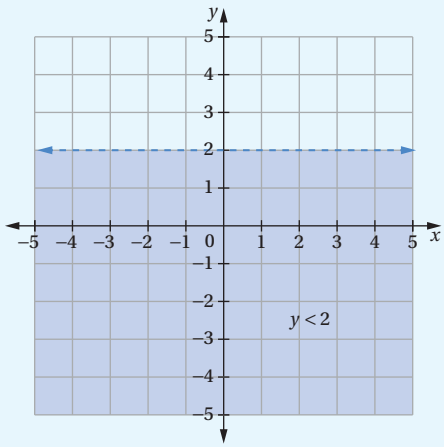
For $y > 3$ and $y \geq 3$ on the Cartesian plane, the shading is above the $y = 3$ gridline.

Worked example

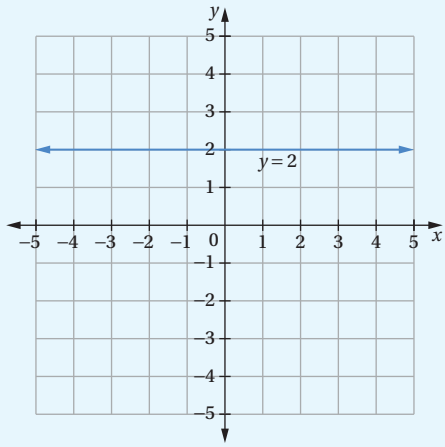
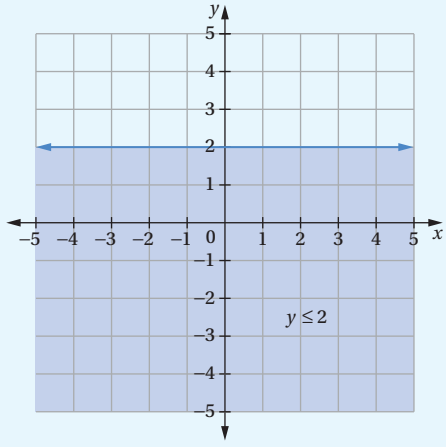
Graphing an inequality of a single variable on the Cartesian plane

Graph each inequality on the Cartesian plane.

(a) $y < 2$

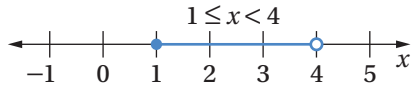
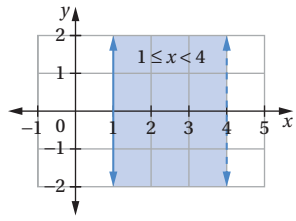
THINKING	WORKING
Sketch the graph, determining whether the line is solid (included) or broken (not included).	$y = 2$ is a horizontal line. The region required is 'less than', so a broken or dashed line is used. 
Shade the region that matches the inequality.	The region is below, as it represents the values less than $y = 2$. 

(b) $y \leq 2$

THINKING	WORKING
Sketch the graph, determining whether the line is solid (included) or broken (not included).	$y = 2$ is a horizontal line. The region required is 'less than or equal to', so a solid line is used. 
Shade the region that matches the inequality.	The region is below, as it represents the values less than or equal to $y = 2$. 

SC 2 I can graph intervals involving single variables on the Cartesian plane

The interval $1 \leq x < 4$ appears differently on the Cartesian plane than its graph on a number line.

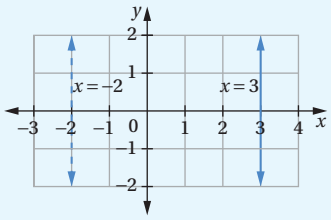
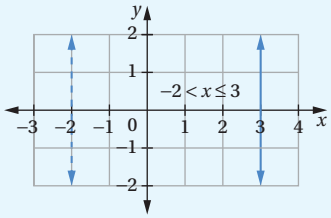
Number line	Cartesian plane
	
A line segment is drawn between $x = 1$ and $x = 4$ on the horizontal axis. A solid circle at $x = 1$ and an empty circle at $x = 4$ shows that $x = 1$ is included and $x = 4$ is not.	The area between the $x = 1$ and $x = 4$ grid lines is shaded. The solid line at $x = 1$ and the dotted (broken) line at $x = 4$ shows whether points on the line are not included. Any y-value is allowable.

Worked example

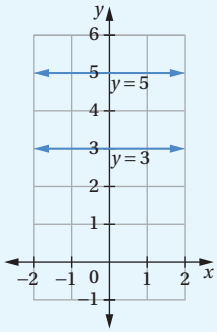
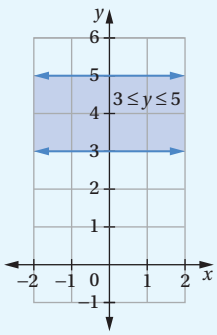
Graphing an interval of a single variable on the Cartesian plane

Graph each interval on the Cartesian plane.

(a) $-2 < x \leq 3$

THINKING	WORKING
Sketch the graphs of the ends of the intervals, determining whether the lines are solid (included) or broken (not included).	Draw the graphs of $x = -2$ (broken line) and $x = 3$ (solid line). 
Shade the graph between the lines and label the shaded area with the inequality.	

(b) $3 \leq y \leq 5$

THINKING	WORKING
Sketch the graphs of the ends of the intervals, determining whether the lines are solid (included) or broken (not included).	Draw the graphs of $y = 3$ (solid line) and $y = 5$ (solid line). 
Shade the graph between the lines and label the shaded area with the inequality.	

SC 3 I can graph inequalities involving two variables on the Cartesian plane

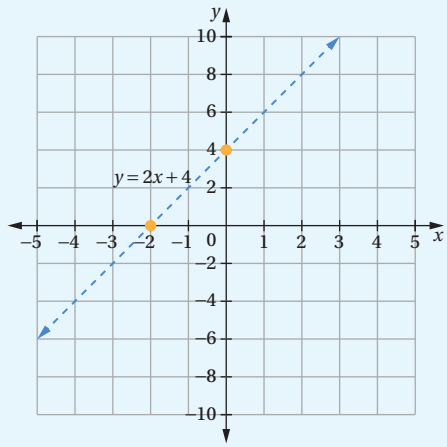
More interesting inequalities are of the form $y > ax + b$ (and the other inequality signs as well).

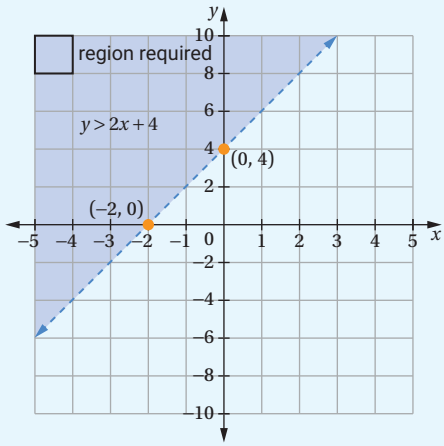
Just as the equation $y = 2x + 4$ can be drawn on the Cartesian plane, so can the inequality $y > 2x + 4$.

Worked example

Sketching an inequality in the form $y > ax + b$

Sketch the inequality $y > 2x + 4$ on the Cartesian plane.

THINKING	WORKING
Determine the intercepts for the straight line.	$y = 2x + 4$ For $x = 0$: $y = 2 \times 0 + 4$ $= 4$ The coordinates of the y-intercept are (0, 4). For $y = 0$: $0 = 2x + 4$ $-4 = 2x$ $x = -2$ The coordinates of the x-intercept are (–2, 0).
Decide whether the straight line should be dotted or solid.	The inequality sign is >, so a dotted line.
Sketch a set of Cartesian axes, plot the two intercepts and join with the appropriate type of line.	
Determine which side of the line should be shaded.	$y > 2x + 4$ The region required is the y-values greater than $y = 2x + 4$. Shade the region above the line of $y = 2x + 4$.

THINKING	WORKING
Check the reasonableness of the region.	Check whether the point (0, 0) is in the inequality: $y > 2x + 4$ $0 > 2 \times 0 + 4$ $0 > 4$ False. As the point (0, 0) returns a false answer, this is the side that needs to be left unshaded.
Shade the graph, making sure a key is placed on the graph to indicate where the required region lies, or write the inequality inside the required region.	

Practice

ANSWERS Page 583

SC 1 I can graph inequalities involving single variables on the Cartesian plane

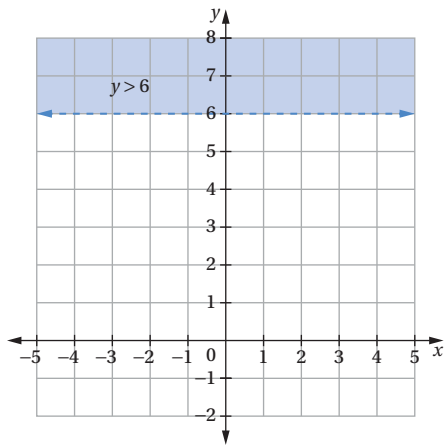
1 Graph each of the following inequalities on separate Cartesian planes.

(a) $y > -3$ **(b)** $y \leq 4$ **(c)** $y > 3$ **(d)** $y \leq -4$

2 Sketch each of the following inequalities on separate sets of Cartesian axes.

(a) $x > 3$ **(b)** $x \geq -2$ **(c)** $x \leq -4$

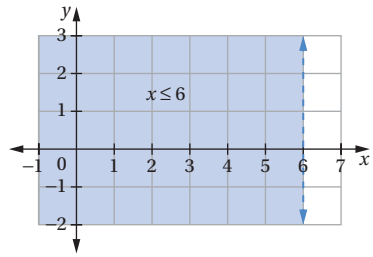
3 Consider the following graph.



(a) Write the inequality shown.

(b) Select the points that belong to the inequality from the following list: (2, 6), (−1, 7), (10, 10), (5, 5), (−3, 7), (6, 8), (−4, 2), (3, 5), (0, 8)

4 Consider the following graph.



(a) Write the inequality shown.

(b) Select the points that belong to the inequality from the following list: (6, 4), (–1, 7), (7, –1), (8, 1), (3, –7), (–4, 2), (–3, 6), (0, 0)

5 Explain how to graph inequalities with one variable that use the 'less than' or 'less than or equal to' symbols, on a Cartesian plane.

6 Graph the regions where both of the following pairs of inequalities apply, on separate Cartesian planes.

(a) $x > 2$ and $y > 4$ **(b)** $x \leq 3$ and $y \geq 2$ **(c)** $x \geq -4$ and $y < 5$ **(d)** $x < -1$ and $y \leq 0$

SC 2 I can graph intervals involving single variables on the Cartesian plane

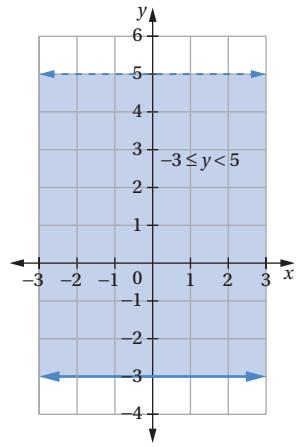
1 Graph each of the following intervals on separate Cartesian planes.

(a) $-4 < x < 2$ **(b)** $1 \leq x \leq 7$ **(c)** $-6 \leq x < -2$

2 Sketch each of the following intervals on separate sets of Cartesian axes.

(a) $-1 < y \leq 7$ **(b)** $5 \leq y \leq 10$

3 Consider the following graph.



(a) Write the interval shown.

(b) Select the points that belong to the interval from the following list: (2, –3), (2, 5), (–1, 0), (6, 4), (5, –3), (–3, 5)

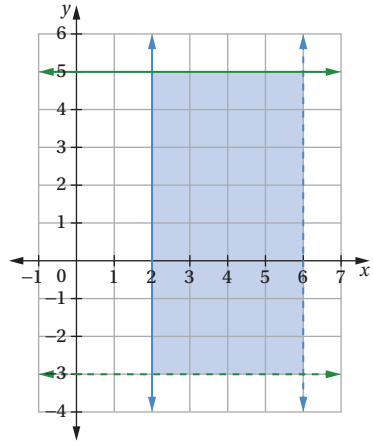
4 Explain how to graph intervals with one variable on a Cartesian plane.

5 Graph the regions where both of the following pairs of intervals apply (the overlap), on separate Cartesian planes.

(a) $2 \leq x \leq 6$ and $3 \leq y \leq 5$ **(b)** $-1 < x < 4$ and $-3 < y < 2$

(c) $-5 < x \leq 5$ and $-4 < y \leq 4$

6 Consider the following graph.



(a) Write the pair of overlapping intervals shown.

(b) Write the coordinates of the four corner points of the graphed region.

(c) List the corner point(s) that belong to the graphed region.

SC 3 I can graph inequalities involving two variables on the Cartesian plane

1 For each of the following, state whether the coordinate pair belongs in the area required for the given inequality.

(a) $y \leq \frac{2x-4}{5}; (1,-1)$ **(b)** $y > \frac{3(x-4)}{2}; (-2,3)$

(c) $y \geq \frac{5(7-3x)}{3}; (-4,-3)$ **(d)** $y < -3\left(\frac{5x+6}{4}\right); (3,4)$

2 You have been asked to draw the inequality $y > \frac{35-5x}{7}$.

(a) State the coordinates of the y-intercept.

(b) State the coordinates of the x-intercept.

(c) State whether the line will be dotted or solid.

(d) Check whether the origin, (0, 0), is in the required area.

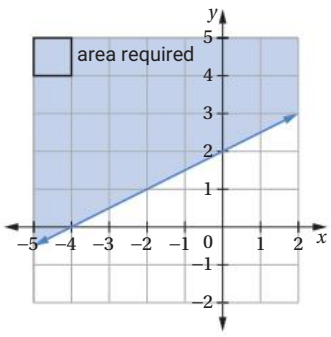
(e) Sketch the inequality.

3 Sketch each of the following on separate sets of Cartesian axes.

(a) $y \leq 2x - 4$ **(b)** $y > 5 - x$ **(c)** $y < 3x + 6$ **(d)** $y \geq \frac{x}{2} + 6$

4 Consider the following graphs of inequalities and choose the option that best represents the inequality shown.

(a)



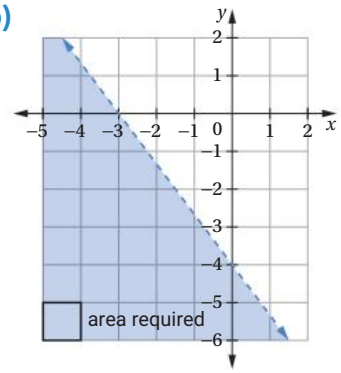
A $y > 2 - \frac{x}{2}$

B $y \geq \frac{x}{2} + 2$

C $y \geq -4x + 2$

D $y \geq 2x - 4$

(b)



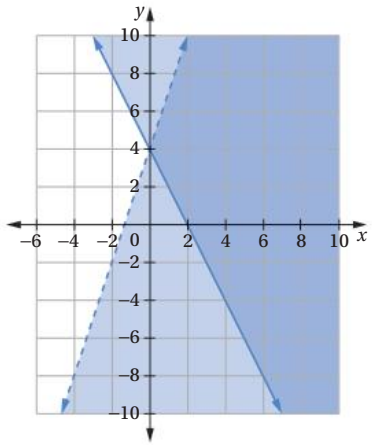
A $y < -3x - 4$

B $y \leq -\frac{3x}{4} - 4$

C $y < -\frac{4x}{3} - 4$

D $y < -\frac{3x}{4} - 4$

5 Consider the following graph.



(a) Determine the rule for each inequality.

(b) Describe what type of coordinate pairs would be found in the unshaded section of the graph.

(c) Choose any coordinate pair in the unshaded region and show that your statement in **(b)** is correct.

(d) Describe what type of coordinate pairs are found in the heavily shaded region of the graph.

(e) Choose any coordinate pair in the heavily shaded region and show that your statement in **(d)** is correct.

(f) Describe what type of coordinate pairs are found in the lightly shaded regions on the graph.

(g) Choose any coordinate pair in a lightly shaded region and show that your statement in **(f)** is correct.

7.3 Solve linear inequalities

Learning intention: To be able to solve linear inequalities

Success criteria:

SC 1 I can solve linear inequalities.

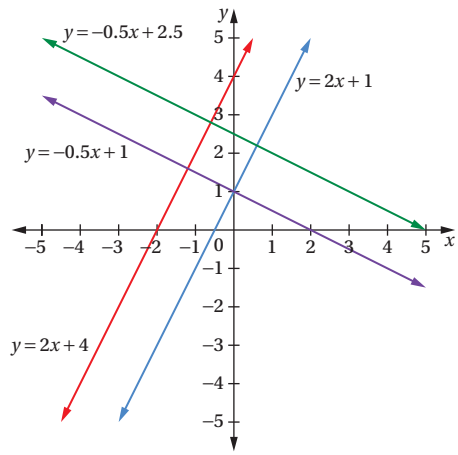
SC 2 I can solve linear inequalities where the symbol must be reversed.

Lesson warm-up

Showing the square

These four graphs enclose a square.

Determine another set of equations that would also enclose a square. What are the characteristics of the rules?



SC 1 I can solve linear inequalities

The solution to a linear equation is calculated by manipulating the equation until the unknown is by itself on one side of the equals sign. The solution to $2x - 5 = 13$ is $x = 9$.

The solution can then be checked by substituting it back into the equation.

$2 \times 9 - 5 = 13$ is true, so the solution is correct.

Linear inequalities can be solved in the same way.

$2x - 5 < 13$ yields the solution $x < 9$, whereas $2x - 5 \leq 13$ yields the solution $x \leq 9$; in both cases there is an infinite number of solutions.

Worked example

Solving an inequality where the symbol stays the same

Solve the following inequalities.

(a) $x + 2 \leq 21$

THINKING	WORKING
Undo the operation that has been performed on the unknown.	$x + 2 \leq 21$ $x + 2 - 2 \leq 21 - 2$ $x \leq 19$
Substitute the end value into the matching equation to check that it is correct.	Substituting $x = 19$ into $x + 2 = 21$: $19 + 2 = 21$ The statement is true. The end value is correct.
Check the reasonableness by substituting any value from the solution (other than the end value).	Substituting $x = 10$ into $x + 2 \leq 21$: $10 + 2 \leq 21$ $12 \leq 21$ The statement is true.
Write the answer.	The solution is $x \leq 19$.

(b) $3x > 15$

THINKING	WORKING
Undo the operation that has been performed on the unknown.	$3x > 15$ $\frac{3x}{3} > \frac{15}{3}$ $x > 5$
Substitute the end value into the matching equation to check that it is correct.	Substituting $x = 5$ into $3x = 15$: $3 \times 5 = 15$ The statement is true. The end value is correct.
Check the reasonableness by substituting any value from the solution (other than the end value).	Substituting $x = 6$ into $3x > 15$: $3 \times 6 > 15$ $18 > 15$ The statement is true.
Write the answer.	The solution is $x > 5$.

(c) $3x - 6 > 12$

THINKING	WORKING
Undo each operation that has been performed on the unknown until the unknown is isolated on the left-hand side.	$3x - 6 > 12$ $3x - 6 + 6 > 12 + 6$ $3x > 18$ $\frac{3x}{3} > \frac{18}{3}$ $x > 6$

Substitute the end value into the matching equation to check that it is correct.	Substituting $x = 6$ into $3x - 6 = 12$: $3 \times 6 - 6 = 12$ The statement is true. The end value is correct.
Check the reasonableness by substituting any value from the solution (other than the end value).	Substituting $x = 7$ into $3x - 6 > 12$: $3 \times 7 - 6 > 12$ $21 - 6 > 12$ $15 > 12$ The statement is true.
Write the answer.	The solution is $x > 6$.

SC 2 I can solve linear inequalities where the symbol must be reversed

There are two instances where the solution to a linear inequality reverses the inequality symbol.

1. When the variable is to the right of the inequality, swap sides and reverse the inequality.
2. When the solving process involves negative quantities of the variable, multiply or divide both sides by a negative number and reverse the inequality.

Worked example

Solving an inequality where the symbol must be reversed

Solve the following inequalities.

(a) $15 > 3x$

THINKING	WORKING
Rewrite the inequality so that the variable is on the left-hand side. The inequality symbol will be reversed.	$15 > 3x$ is the same as $3x < 15$.
Undo the operation that has been performed on the unknown.	$3x < 15$ $\frac{3x}{3} < \frac{15}{3}$ $x < 5$
Substitute the end value into the matching equation to check that it is correct.	Substituting $x = 5$ into $15 = 3x$: $15 = 3 \times 5$ The statement is true. The end value is correct.
Check the reasonableness by substituting any value from the solution (other than the end value).	Substituting $x = 4$ into $15 > 3x$: $15 > 3 \times 4$ $15 > 12$ The statement is true.
Write the answer.	The solution is $x < 5$.

(b) $15 - 2x \geq 5$

THINKING	WORKING
Undo each operation that has been performed on the unknown until the unknown is isolated on the left-hand side. At the step where both sides are divided by −1, the inequality is reversed.	$15-2x \geq 5$ $15-2x-15 \geq 5-15$ $-2x \geq -10$ $\frac{-2x}{2} \geq \frac{-10}{2}$ $-x \geq -5$ $x \leq 5$
Alternatively, add the term with the variable to both sides, then swap sides, reversing the inequality.	$15-2x \geq 5$ $15-2x+2x \geq 5+2x$ $15 \geq 2x+5$ $2x+5 \leq 15$ $2x+5-5 \leq 15-5$ $2x \leq 10$ $\frac{2x}{2} \leq \frac{10}{2}$ $x \leq 5$
Substitute the end value into the matching equation to check that it is correct.	Substituting $x = 5$ into $15 - 2x = 5$: $15 - 2 \times 5 = 5$ The statement is true. The end value is correct.
Check the reasonableness by substituting any value from the solution (other than the end value).	Substituting $x = 4$ into $15 - 2x \geq 5$: $15 - 2 \times 4 \geq 5$ $15 - 8 \geq 5$ $7 \geq 5$ The statement is true.
Write the answer.	The solution is $x \leq 5$.

Practice

ANSWERS Page 586

SC 1 I can solve linear inequalities

1 Solve the following inequalities.

(a) $x + 2 < 8$ **(b)** $x - 2 < 8$ **(c)** $2x < 8$ **(d)** $\frac{x}{2} < 8$

2 Solve the following inequalities and check that your solution is correct.

(a) $x - 1 \geq 6$ **(b)** $x - 2 > 7$ **(c)** $x + 9 \leq 21$ **(d)** $x + 5 < 2$

3 Solve the following inequalities and check that your solution is correct.

(a) $5x < 35$ **(b)** $3x \leq -60$ **(c)** $2x > 14$ **(d)** $8x \geq 12$

4 Determine whether the inequality $2x - 4 \leq -1$ is true for each of the following values of x.

(a) $x = 0$ **(b)** $x = 2$ **(c)** $x = -1$ **(d)** $x = \frac{3}{2}$

5 Solve the following inequalities and check that your solution is correct.

(a) $4x + 2 < 10$ **(b)** $5x - 3 > 12$ **(c)** $7x + 6 \leq 6$ **(d)** $3x - 7 \geq 5$

6 Solve the following inequalities and check that your solution is correct. Write your answers as fractions, where necessary.

(a) $5x - 3 > 4$ **(b)** $2(x - 4) \leq -5$ **(c)** $\frac{4x}{3} - 2 < 5$ **(d)** $\frac{5x}{4} + 6 \geq -4$

7 A corporate logo includes a rectangle, where the height must be less than half of the base.

(a) Allowing the height and base, in centimetres, to be h and b respectively, write the condition as an algebraic inequality.

(b) Simone draws a rectangle with the height 4 cm and the base 7.5 cm. Determine whether the condition is satisfied.

(c) Determine the heights of three satisfactory rectangles that have a base of 12 cm, where the height is a whole number of centimetres.

(d) Determine the base of three satisfactory rectangles that have a height of 7 cm, where the base is a whole number of centimetres.

SC 2 I can solve linear inequalities where the symbol must be reversed

1 Solve the following inequalities.

(a) $-x + 2 < 8$ **(b)** $-x - 2 < 8$ **(c)** $-2x < 8$ **(d)** $-\frac{x}{2} < 8$

2 Solve the following inequalities and check that your solution is correct.

(a) $4 - x \leq 2$ **(b)** $4 < x - 10$ **(c)** $13 \geq x + 8$ **(d)** $12 - x > 15$

3 Solve the following inequalities and check that your solution is correct.

(a) $-5x > 45$ **(b)** $-7x \geq -14$ **(c)** $-4x < 13$ **(d)** $-4x \leq -20$

4 Solve the following inequalities and check that your solution is correct.

(a) $5 - 3x > -10$ **(b)** $2 - 8x < 10$ **(c)** $1 - 9x \geq -17$ **(d)** $29 - 7x \leq 1$

5 Solve the following inequalities and check that your solution is correct.

(a) $4 - 6x > 10$ **(b)** $-3(4x + 5) \leq 9$ **(c)** $6 - \frac{2x}{3} < -6$ **(d)** $\frac{3 - 5x}{4} \geq 2$

6 Solve the following inequalities and check that your solution is correct. If necessary, state your answers in fraction form.

(a) $\frac{5 - 2x}{7} \geq -2$ **(b)** $4 - \frac{3x}{2} < 5$ **(c)** $\frac{-3(2x + 4)}{5} > 2$

(d) $\frac{6(4 - 3x)}{3} \leq -4$ **(e)** $\frac{-7x + 6}{2} \geq -4$

7 Determine whether the inequality $\frac{3(2 - 5x)}{4} < -1$ is true for each of the following values of x.

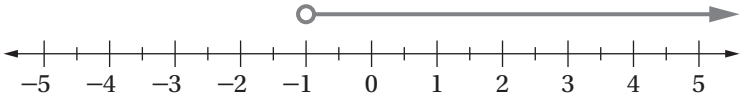
(a) $x = 1$ **(b)** $x = 0$ **(c)** $x = -3$ **(d)** $x = \frac{2}{3}$

TOPIC REVIEW

Multiple choice

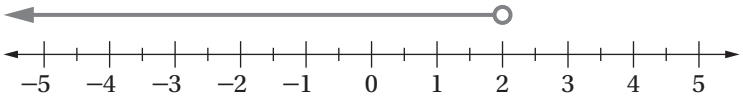
ANSWERS Page 624

1 Which of the following best represents the interval shown on the number line?



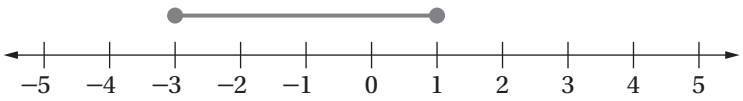
A $x < -1$ **B** $x > -1$ **C** $x \leq -1$ **D** $x \geq -1$

2 Which of the following best represents the interval shown on the number line?



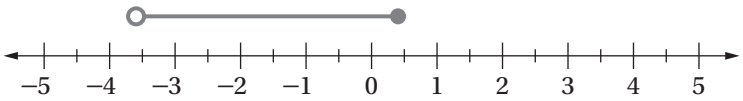
A $x < 2$ **B** $x > 2$ **C** $x \leq 2$ **D** $x \geq 2$

3 Which of the following best represents the interval shown on the number line?



A $-3 < x < 1$ **B** $-3 < x \leq 1$ **C** $-3 \leq x < 1$ **D** $-3 \leq x \leq 1$

4 Which of the following best represents the interval shown on the number line?



A $-4 < x < 0$ **B** $-4 < x \leq 0$ **C** $-4 \leq x < 0$ **D** $-4 \leq x \leq 0$

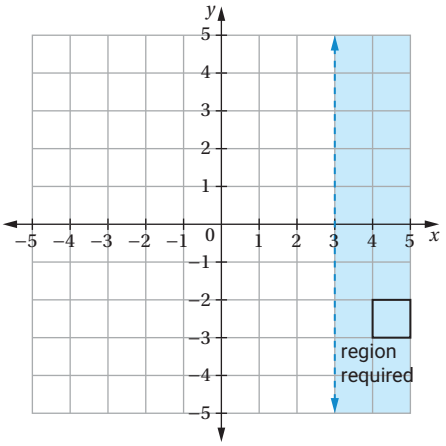
5 Which of the following best represents the inequality shown on the graph?

A $x < 3$

B $x > 3$

C $x \leq 3$

D $x \geq 3$



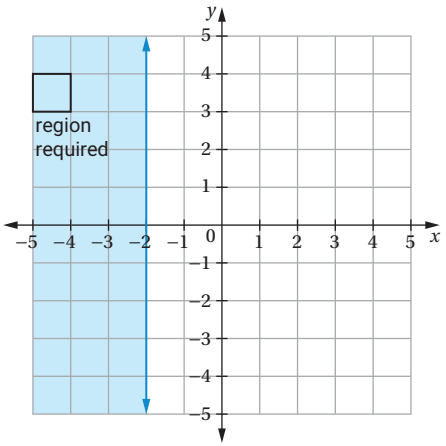
6 Which of the following best represents the inequality shown on the graph?

A $x < -2$

B $x > -2$

C $x \leq -2$

D $x \geq -2$



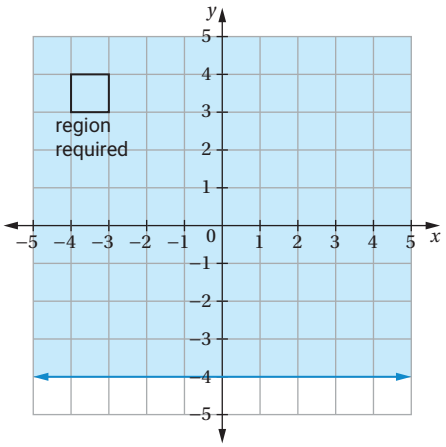
7 Which of the following best represents the inequality shown on the graph?

A $y < -4$

B $y > -4$

C $y \leq -4$

D $y \geq -4$



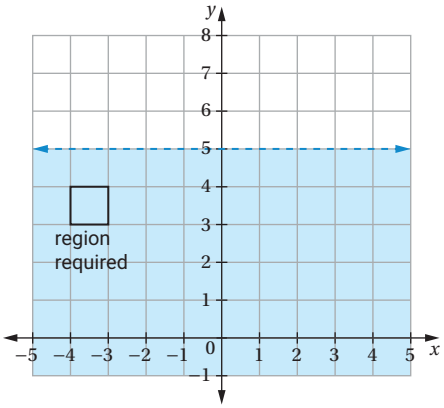
8 Which of the following best represents the inequality shown on the graph?

A $y < 5$

B $y > 5$

C $y \leq 5$

D $y \geq 5$



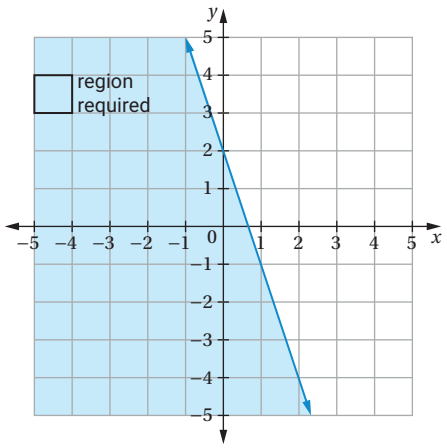
9 Which of the following best represents the inequality shown on the graph?

A $y \geq 2 - 3x$

B $y \geq -2x + \frac{2}{3}$

C $y \leq -3x + 2$

D $y \leq -2x + \frac{2}{3}$



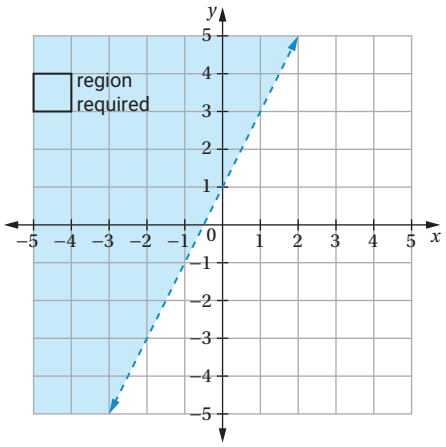
10 Which of the following best represents the inequality shown on the graph?

A $y > 0.5x + 1$

B $y > 2x + 1$

C $y < 0.5x + 1$

D $y < 2x + 1$



11 The solution to $\frac{5x-2}{3} \geq 1$ is:

A $x > 1$ **B** $x \geq 5$ **C** $x > 5$ **D** $x \geq 1$

12 The solution to $\frac{3(2-4x)}{2} < 1$ is:

A $x > \frac{1}{3}$ **B** $x > \frac{16}{3}$ **C** $x < \frac{16}{3}$ **D** $x < \frac{1}{3}$

Short answer

1 Convert the following word descriptions to symbolic form. In each case, use x to represent the variable.

(a) The variable is greater than 4.

(b) The variable is less than or equal to 6.

(c) The variable is between –3 and 5, including both endpoints.

(d) The variable is greater than 5 and less than or equal to 10.

2 Substitute each of the given points into the inequality $y < \frac{3x-2}{4}$ and state whether the point lies in the required region of the graph.

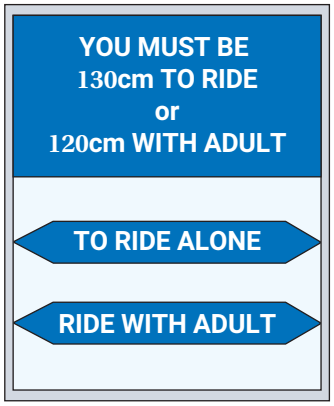
(a) (0, 5) **(b)** (–1, 3) **(c)** (–2, –3)

3 Write an inequality based on each of the following images.

(a)

(b)

(c)



(d)

4 Draw each of the following inequalities on a number line.

(a) $x > -2$ **(b)** $x \leq 5$ **(c)** $x \geq 1$

5 Draw each of the following intervals on a number line.

(a) $-3 < x \leq 4$ **(b)** $2 \leq x < 5$ **(c)** $-5 \leq x \leq 0$

6 For each of the following inequalities, complete the coordinate pair that lies on the boundary line of the region.

(a) $y \geq \frac{2x-5}{3}; (4, \square)$ **(b)** $y \leq \frac{-2(x-7)}{3}; (\square, 2)$

7 Solve each of the following inequalities.

(a) $6 - 4x < -2$ **(b)** $\frac{3x}{4} + 6 \leq 3$ **(c)** $\frac{5x-4}{2} > 2$

8 Draw each of the following inequalities on separate Cartesian axes.

(a) $y \geq 2x - 1$ **(b)** $y < 3 - 4x$

9 **(a)** Write the following intervals in words.

(i) $3 \leq x < 4$

(ii) $-2 < x < 7$

(iii) $-7 \leq x \leq 5$

(b) In each case, state how many whole numbers are contained in the interval.

Extended response

1 Complete the following.

(a) Draw each of the following inequations on the same set of Cartesian axes: $y \geq x + 4$; $y \leq x - 4$; $y \geq -x + 4$; $y \leq -x - 4$. In each case, shade the area required.

(b) Substitute the point (0, 0) into each inequality to check that the shading is correct.

(c) Describe the shape that has been left unshaded. Justify any claims you make about parallel lines with evidence.

(d) What is the area of the unshaded section of the graph?

TOPIC

8

Perimeter, area and volume

Recall 181

8.1 Estimate length and convert between units of length 182

8.2 Calculate the perimeter of quadrilaterals 188

8.3 Determine unknown side lengths from a given perimeter 196

8.4 Calculate the area of a trapezium 201

8.5 Calculate the area of kites and rhombuses 207

8.6 Calculate the area of composite shapes 212

8.7 Approximate the perimeter and area of irregular shapes 218

8.8 Understand the connection between volume and capacity 229

Topic review 238

Why learn this?

Calculating the perimeter and area of shapes and the volume of objects builds the spatial awareness of the real world within which we live.

The skills covered here further spatial awareness and problem-solving skills, applicable in various fields, such as engineering, architecture, interior design and logistics.

RECALL

ANSWERS Page 587

I understand how to multiply and divide decimals by powers of 10

1 Copy and calculate the following.

(a) $2.8 \times 100 =$ **(b)** $3.71 \times 100 =$ **(c)** $0.4 \times 100 =$

(d) $0.067 \times 1000 =$ **(e)** $8.205 \times 10 =$ **(f)** $9.27 \times 1000 =$

2 Copy and calculate the following.

(a) $72 \div 10 =$ **(b)** $24 \div 100 =$ **(c)** $14 \div 1000 =$

(d) $127 \div 1000 =$ **(e)** $0.4 \div 10 =$ **(f)** $0.03 \div 100 =$

I can add and subtract decimals

1 Evaluate the following.

(a) $8.5 + 3.2 + 4.8$ **(b)** $23.78 + 15.21 + 9.03$ **(c)** $8.25 + 11.94 + 14.8 + 20.05$

(d) $25 - 15.2$ **(e)** $13.57 - 5.36$ **(f)** $28.42 - 17.59$

I can multiply and divide decimals

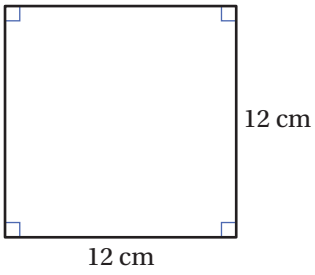
1 Evaluate the following.

(a) 4.8×6.5 **(b)** 15.05×0.2 **(c)** $4.5 \div 0.5$ **(d)** $11.6 \div 0.02$

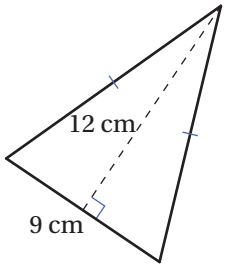
I can calculate the area of plane shapes

1 Calculate the area of each of the following shapes.

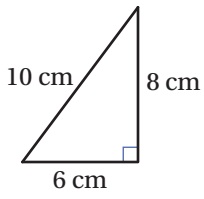
(a)



(b)



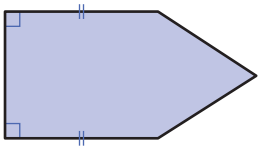
(c)



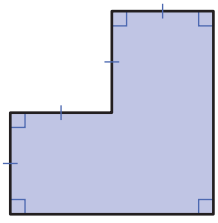
I can identify the plane shapes used to form a composite shape

1 Identify the two plane shapes used to form each composite shape.

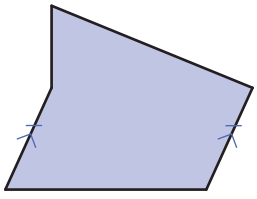
(a)



(b)



(c)



8.1 Estimate length and convert between units of length

Learning intention: To be able to estimate length and convert between units of length

Success criteria:

SC 1 I can estimate lengths by comparing with known lengths.

SC 2 I can convert between units of length using metric prefixes.

Lesson warm-up

Estimating length in different units

Your teacher will walk around holding up some objects. Estimate their length in centimetres, then millimetres. Record your results and compare them with your classmates.

SC 1 I can estimate lengths by comparing with known lengths

In Australia, the metric system is used for length. In this system, the basic unit of length is the metre (m).

- A tall adult may have a striding step length of a metre. A child may have a step length of half a metre.
- A person can walk 1 kilometre (km) in about 10 to 15 minutes.

For smaller lengths:

- 1 millimetre (mm) is the approximate thickness of a paperclip or a ruler.

- 1 centimetre (cm) is the approximate width of a finger.

Worked example

Estimating lengths from known lengths

The distance between the stumps at each end of a cricket pitch is 20.12 m.

To mark out the distance between stumps on a field, describe:

(a) what a teacher might do.

THINKING	WORKING
Describe a known length.	A teacher might be able to comfortably stride 1 metre.
Calculate the number of steps.	$\frac{20.12\,\text{m}}{1\,\text{m}} \approx 20$
Answer the question.	The tall teacher might mark the position of one end, then stride out 20 steps to mark the other end.

(b) what a primary school-aged student might do.

THINKING	WORKING
Describe a known length.	A primary student might be able to comfortably stride half a metre.
Calculate the number of steps.	$\frac{20.12\,\text{m}}{0.5\,\text{m}} \approx 40$
Answer the question.	A primary school student might mark the position of one end, then stride out 40 steps to mark the other end.

SC 2 I can convert between units of length using metric prefixes

Some of the common prefixes used in the metric system are:

- milli, as in millimetre or mm, where 1 mm $= \frac{1}{1000}$ m or 0.001 m
- centi, as in centimetre or cm, where 1 cm $= \frac{1}{100}$ m or 0.01 m and
- kilo, as in kilometre or km, where 1 km = 1000 m.

Prefix	Standard abbreviation	Numerical meaning
milli	m	$\frac{1}{1000}$ or 0.001
centi	c	$\frac{1}{100}$ or 0.01
deci	d	$\frac{1}{10}$ or 0.1
deca	D	10
hecto	h	100
kilo	k	1000
mega	M	1 000 000

Worked example

Converting units of length

Convert each of the following quantities.

(a) 16 metres to centimetres

THINKING	WORKING
Identify the prefix.	cm is centimetres, centi: $\frac{1}{100}$
Write the conversion as a scale factor.	1 m = 100 cm
Convert the length to the base unit, metres, by multiplying by the scale factor.	16 m = 16 × 100 cm = 1600 cm
Write the answer.	16 m = 1600 cm

(b) 160 centimetres to metres

THINKING	WORKING
Identify the prefix.	cm is centimetres, centi: $\frac{1}{100}$
Write the conversion as a scale factor.	$1\text{ m} = 100\text{ cm}$ $1\text{ cm} = \frac{1}{100}\text{ m}$
Convert the length to the base unit, metres, by multiplying by the scale factor.	$160\text{ cm} = 160 \times \frac{1}{100}\text{ m}$ $= 1.6\text{ m}$
Write the answer.	160 cm = 1.6 m

(c) 1.6 centimetres to millimetres

THINKING	WORKING
Identify the prefix.	cm is centimetres, centi: $\frac{1}{100}$ mm is millimetres, milli: $\frac{1}{1000}$
Write the conversion as a scale factor.	1 cm = 10 mm
Convert the length by multiplying by the scale factor.	1.6 cm = 1.6 × 10 mm = 16 mm
Write the answer.	1.6 cm = 16 mm

(d) 160 centimetres to kilometres

THINKING	WORKING
Identify the prefix.	cm is centimetres, centi: $\frac{1}{100}$
Write the conversion as a scale factor.	$1\text{ cm} = \frac{1}{100}\text{ m}$
Convert the length to the base unit, metres, by multiplying by the scale factor.	$160\text{ cm} = 160 \times \frac{1}{100}\text{ m}$ $= 1.6\text{ m}$
Identify the prefix.	km is kilometres, kilo: 1000
Write the conversion as a scale factor.	$1\text{ m} = \frac{1}{1000}\text{ km}$
Convert the length in metres to the larger unit by multiplying by the scale factor.	$1.6\text{ m} \times \frac{1}{1000} = 0.0016\text{ km}$
Write the answer.	160 cm = 0.0016 km

Practice

ANSWERS Page 587

SC 1 I can estimate lengths by comparing with known lengths

1 Describe how to estimate each of the following lengths.

(a) The width of the palm of your hand, given the width of a fingernail is about 1 cm.

(b) The width of a grain of rice, given the width of a paperclip is about 1 mm.

(c) The length of a grain of rice.

(d) The distance you have walked in two and a half hours, given an estimated walking pace of 15 minutes for each kilometre.

2 Estimate the length of the unmarked blue line by working out how many of the reference lines are needed to make the same length.

(a) 3 mm

(b) 2 cm

(c) 6 m

(d) 8 km

3 Estimate the length of the unmarked line by comparing its length with the reference line.

(a) 12 mm

(b) 8 cm

(c) 9 m

(d) 24 km

4 The following locations have been plotted, based on a map with the scale line shown. Use the scale line to estimate the distances between the points indicated.

School • • Sportsground

• Grandparents' house

Home • • Shops

1 km

(a) home and school

(b) home and shops

(c) home and sportsground

(d) home and grandparents' house

(e) school and sportsground

(f) shops and grandparents' house

5 The photographs show a mother with her daughter and a father with his son.

(a) The mother is 165 cm tall. Estimate the height of the daughter and explain how you did this.

(b) Given that the son is 120 cm tall, estimate the height of the father and explain how you did this.

6 Estimate the heights of the following objects you might see while travelling.

(a) The height of a grain silo

(b) The height of a road sign

SC 2 I can convert between units of length using metric prefixes

1 Convert each of the following lengths to centimetres.

(a) Height of a 2-storey building: 5.8 m

(b) Length of A4 paper: 297 mm

(c) Distance walked to school: 1.4 km

(d) Distance from Sydney to Canberra: 235 km

2 Convert 832 cm to each of the following units.

(a) metres **(b)** kilometres **(c)** millimetres **(d)** decimetres

3 Place the following quantities in ascending order (from smallest to largest).

1 m, 1 hm, 1 mm, 1 dm, 1 Dm

Remember: h (hector): 100; m (milli): 0.001; d (deci): 0.1; D (deca): 10

4 Insert a greater than symbol (>) or less than symbol (<) in each of the following statements to make them true.

(a) 0.5 km ☐ 5500 m **(b)** 15 m ☐ 11000 mm

(c) 250 mm ☐ 0.026 km **(d)** 3700 mm ☐ 340 cm

5 Change the units to the largest quantity, then perform the following additions.

(a) 3 cm + 6 mm + 45 mm **(b)** 569 m + 3.3 km + 0.7 km

(c) 85 mm + 23 cm + 0.92 m **(d)** 1.3 km + 400 m + 600 cm

6 Change the units to the smaller quantity, then perform the following subtractions.

(a) 12 cm – 15 mm **(b)** 1 km – 450 m

(c) 0.7 m – 6.5 cm **(d)** 0.004 km – 25 cm

8.2 Calculate the perimeter of quadrilaterals

Learning intention: To be able to calculate the perimeter of quadrilaterals

Success criteria:

SC 1 I can calculate the perimeter of rectangles.

SC 2 I can calculate the perimeter of any quadrilateral.

SC 3 I can extend my understanding of perimeter to any polygon.

Lesson warm-up

Quadrilaterals made from string

Your teacher will give each student a length of string.

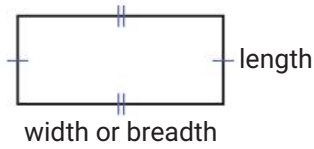
1. Cut your piece of string into four pieces that can be used to form a quadrilateral, so don't make any piece too long.
2. Arrange the pieces into a quadrilateral.
3. Place the lengths of string along a ruler to determine the length of the perimeter of your quadrilateral.

SC 1 I can calculate the perimeter of rectangles

The perimeter of a two-dimensional shape is the length of its boundary.

A rectangle is a quadrilateral, a shape with four sides.

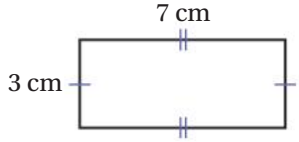
The opposite side lengths in a rectangle are equal in length.



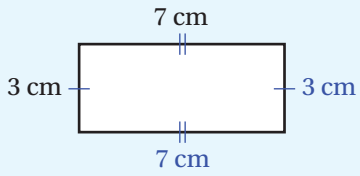
Worked example

Calculating the perimeter of a rectangle

Calculate the perimeter of the following rectangle using the given strategy.



(a) Determine the unknown side lengths, then add all four side lengths together.

THINKING	WORKING
Identify and label the side lengths that are equal in length.	
Calculate the perimeter by adding the four side lengths.	$P = 3 + 7 + 3 + 7$ $= 20$
Answer the question.	The perimeter of the rectangle is 20 cm.

(b) Double the length of the two given side lengths and add them together.

THINKING	WORKING
Identify the known side lengths of the rectangle.	The length l is 7 cm. The breadth or width b is 3 cm.
Calculate the perimeter by doubling the given side lengths and adding.	$P = 2l + 2b$ $= 2 \times 7 + 2 \times 3$ $= 14 + 6$ $= 20$
Answer the question.	The perimeter of the rectangle is 20 cm.

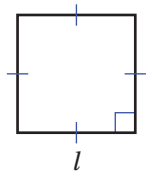
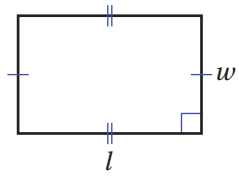
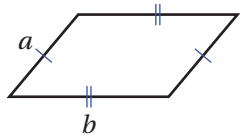
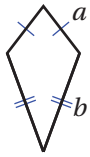
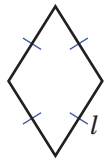
(c) Determine the total of the length and breadth, then double the result.

THINKING	WORKING
Identify the known side lengths of the rectangle.	The length l is 7 cm. The width w is 3 cm.
Calculate the total of the length and breadth.	$l + w = 7 + 3$ $= 10$
Calculate the perimeter by doubling the result.	$P = 2 \times 10$ $= 20$
Answer the question.	The perimeter of the rectangle is 20 cm.

SC 2 I can calculate the perimeter of any quadrilateral

The perimeter of regular quadrilaterals

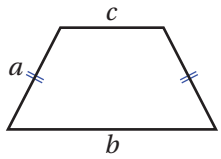
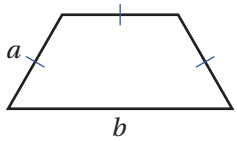
For regular quadrilaterals, you can use the properties of side lengths to write general perimeter formulas. Some examples are given in the table below.

Name of quadrilateral	Diagram	Perimeter formula
Square		$P = 4l$
Rectangle		$P = 2l + 2w$ or $P = 2(l + w)$
Parallelogram		$P = 2a + 2b$ or $P = 2(a + b)$
Kite		$P = 2a + 2b$ or $P = 2(a + b)$
Rhombus		$P = 4l$

The perimeter of other quadrilaterals

For a quadrilateral with side lengths a, b, c and d, the perimeter is $P = a + b + c + d$.

For shapes with two or three side lengths that are the same, you can use a version of the general formula with only two or three variables. Some examples are given in the table below.

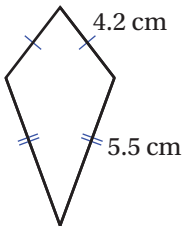
Type of quadrilateral	Diagram	Perimeter formula
Isosceles trapezium		$P = 2a + b + c$
Irregular quadrilateral with three sides the same length		$P = 3a + b$

Worked example

Calculating the perimeter of a quadrilateral

Calculate the perimeter of each quadrilateral in centimetres.

(a)

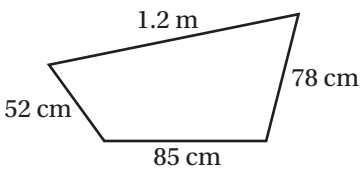


THINKING	WORKING
Calculate the perimeter by adding the length of each side. You can be efficient by multiplying repeated lengths by 2.	$P = 4.2 \times 2 + 5.5 \times 2$ $= 19.4$
Write the answer.	The perimeter of the kite is 19.4 cm.

(b) Repeat the calculation for **(a)** using a formula.

THINKING	WORKING
Identify the shape.	The shape is a kite.
Write a formula, defining the variables.	$P = 2a + 2b$ where $a = 4.2$ cm and $b = 5.5$ cm.
Calculate the perimeter by substituting the known values.	$P = 2 \times 4.2 + 2 \times 5.5$ $= 19.4$
Write the answer.	The perimeter of the kite is 19.4 cm.

(c)



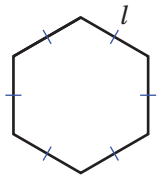
THINKING	WORKING
Convert the measurements so all lengths have the same unit.	1.2 m = 120 cm
The perimeter is the total of all of the lengths.	$P = 120 + 78 + 85 + 52$ $= 335$
Write the answer.	The perimeter of the quadrilateral is 335 cm.

SC 3 I can extend my understanding of perimeter to any polygon

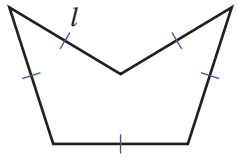
For any polygon (a two-dimensional shape whose boundary is made up of line segments), the perimeter is the total of the individual lengths of the line segments.

Regular polygons, or any polygon, use perimeter formulas similar to those for the perimeters of squares or rhombuses.

Regular hexagon: $P = 6l$



Pentagon with all sides equal: $P = 5l$



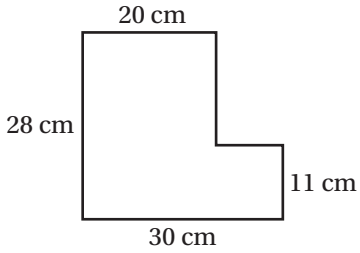
For the perimeter of polygons, in general, calculate any unknown side lengths, change all measurements to the same unit, then calculate the total of the side lengths.

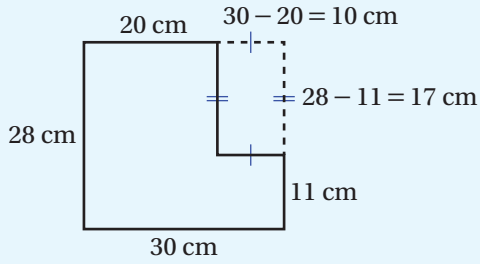
Worked example

Calculating the perimeter of a polygon

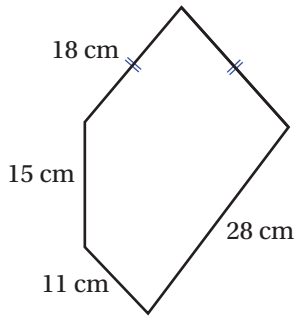
Calculate the perimeter of each polygon.

(a)



THINKING	WORKING
Determine the length of unlabelled sides, assuming all corners are right-angled.	
The perimeter is the total of all of the lengths.	$P = 30 + 28 + 20 + 17 + 10 + 11$ $= 116$ cm
Write the answer.	The perimeter of the polygon is 116 cm.

(b)



THINKING	WORKING
Calculate the perimeter by adding the length of each side. You can multiply the length of the equal sides by 2.	$P = 18 \times 2 + 28 + 11 + 15$ $= 90$ cm
Write the answer.	The perimeter of the pentagon is 90 cm.

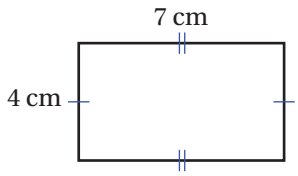
Practice

ANSWERS Page 587

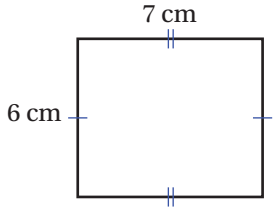
SC 1 I can calculate the perimeter of rectangles

1 Choose a strategy to calculate the perimeter of each rectangle.

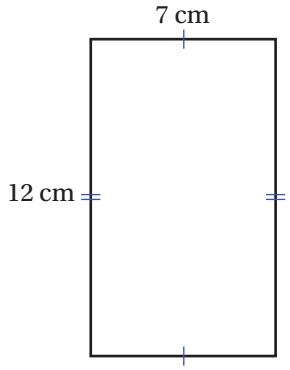
(a)



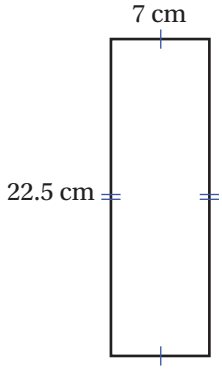
(b)



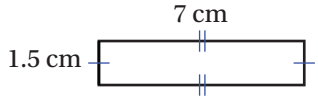
(c)



(d)



(e)



2 Calculate the perimeter of the following rectangles. Make sure all measurements are written in the same units before calculating. Draw the rectangle first if needed.

(a) Length: 3.8 cm; width: 27 mm

(b) Length: 1.3 cm; width: 4 mm

(c) Length: 2.13 cm; width: 7.7 mm

(d) Length: 3.6 m; width: 85 cm

(e) Length: 1.41 m; width: 30 cm

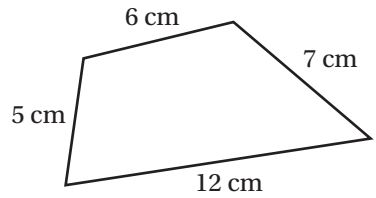
3 Determine the perimeter of fencing required to enclose a rectangular chicken pen 5 m in length by 3.5 m in width.

4 Draw three rectangles with a perimeter of 16 cm.

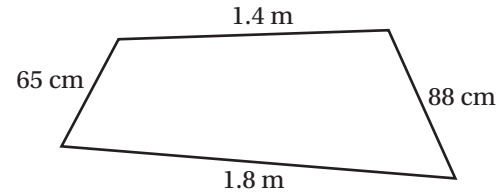
SC 2 I can calculate the perimeter of any quadrilateral

1 Calculate the perimeter of each quadrilateral in centimetres.

(a)



(b)

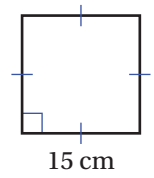


(c) A quadrilateral with side lengths 14 cm, 6.5 cm, 17.2 cm and 9 cm

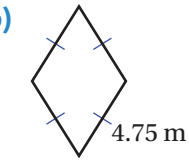
(d) A quadrilateral with side lengths 4 mm, 5 mm, 1.6 cm and 1.1 cm

2 Calculate the perimeter of each special quadrilateral.

(a)



(b)

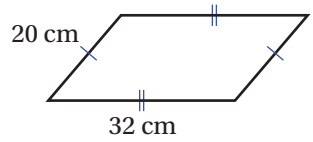


(c) A square with side lengths 6.5 mm

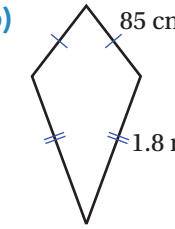
(d) A rhombus with side lengths 1.7 m

3 Calculate the perimeter of each special quadrilateral in centimetres.

(a)



(b)

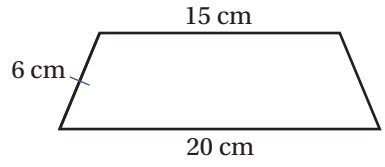


(c) A kite with side lengths 56 mm and 82 mm

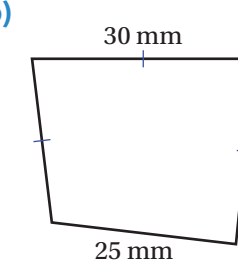
(d) A parallelogram with side lengths 0.43 m and 0.2 m

4 Calculate the perimeter of each quadrilateral in centimetres.

(a)



(b)



(c) A quadrilateral with two sides of 18 cm and one each of 10 cm and 20 cm.

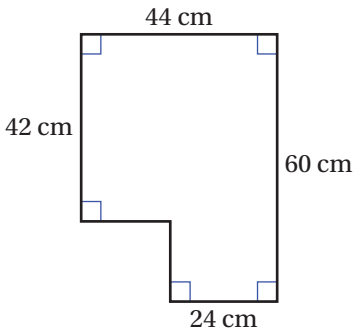
(d) A quadrilateral with three sides of 75 mm and one of 126 mm.

5 A property has four sides, two each of 420 m, one of 1.7 km and one of 2.2 km. How many metres of fencing are needed to enclose the property?

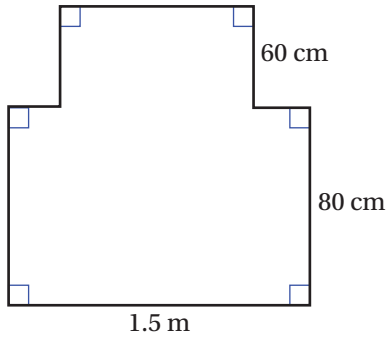
SC 3 I can extend my understanding of perimeter to any polygon

1 Calculate the perimeter of each shape in centimetres. Assume that all corners are right-angled.

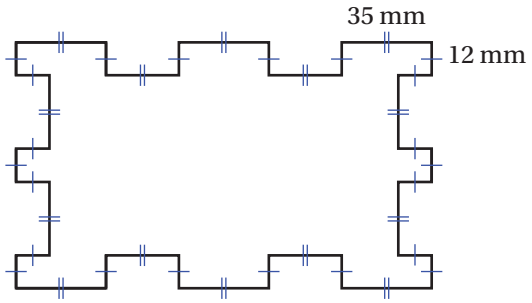
(a)



(b)

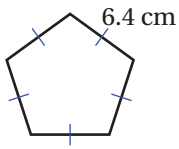


(c) Assume that there are only two different side lengths.

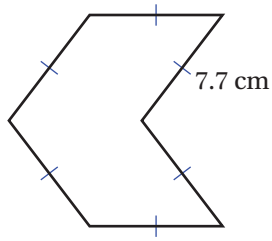


2 Calculate the perimeter of each shape.

(a)



(b)

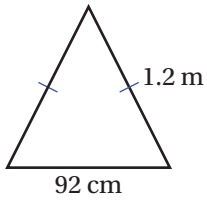


(c) An equilateral triangle with side length 2.5 m

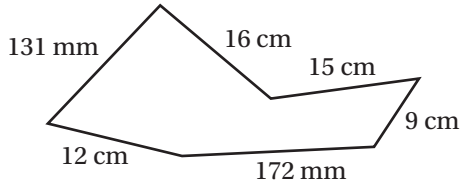
(d) A decagon (10-sided shape) with each side length 12 mm

3 Calculate the perimeter of each shape.

(a)



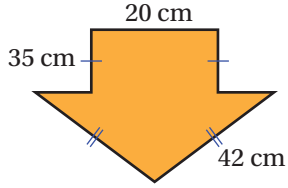
(b)



(c) A pentagon with two sides of 11 cm and three sides of 15 cm

(d) A hexagon with three sides of 2.4 cm and one side each of 8 mm, 6 mm and 7 mm

4 Determine the painted perimeter of the following symbol on a footpath.



8.3 Determine unknown side lengths from a given perimeter

Learning intention: To be able to determine unknown side lengths from a given perimeter

Success criteria:

SC 1 I can determine unknown side lengths in a rectangle and other quadrilaterals, given the perimeter.

SC 2 I can determine unknown side lengths in any polygon, given the perimeter.

Lesson warm-up

Unknown side lengths

Use your fingers to stretch 32 cm of string to make straight sides of the following polygons, commenting on the side lengths of the shape. Accuracy of length is not important – the string is just to help you visualise.

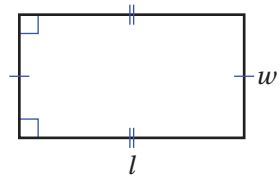
- Rhombus
- Rectangle with one side length equal to 10 cm
- Quadrilateral of known side lengths 4 cm, 11 cm and 12 cm
- Isosceles triangle with a side length of 10 cm

SC 1 I can determine unknown side lengths in a rectangle and other quadrilaterals, given the perimeter

Every rectangle has two different side lengths, which is shown in this formula:

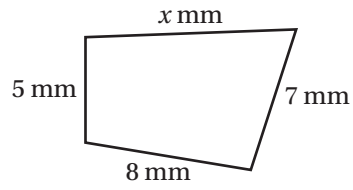
$P = 2l + 2w$ or $P = 2(l + w)$

If the perimeter is known, then the length of an unknown side length can be determined.



Worked example

Calculating the unknown side length of a quadrilateral given the perimeter

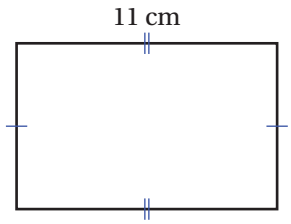


(a) Calculate the unknown side length in the shape, given its perimeter is 30 mm.

THINKING	WORKING
The perimeter is the total of all the side lengths. Write an equation for the given perimeter.	$P = a + b + c + d$
Substitute the known values.	$30 = 5 + 8 + 7 + x$ $= 20 + x$

Solve the equation.	$30 - 20 = 20 + x - 20$ $10 = x$
Write the answer.	The unknown side length is 10 mm.

(b) Calculate the unknown side length in the rectangle below, given its perimeter is 36 cm, by using the methods described.



(i) Backtrack using the formula $P = 2l + 2b$.

THINKING	WORKING
Assign a pronumeral to the unknown length.	Let the unknown side length be b.
Write an equation for the given perimeter.	Use $P = 2l + 2b$ $36 = 2 \times 11 + 2b$ $36 = 22 + 2b$
Solve the equation.	$36 - 22 = 22 + 2b - 22$ $14 = 2b$ $\frac{14}{2} = \frac{2b}{2}$ $7 = b$
Write the answer.	The unknown side length is 7 cm.

(ii) Backtrack using the formula $P = 2(l + b)$.

THINKING	WORKING
Write an equation for the given perimeter.	Use $P = 2(l + b)$ $36 = 2(11 + b)$
Solve the equation.	$\frac{36}{2} = \frac{2(11+b)}{2}$ $18 = 11 + b$ $18 - 11 = 11 + b - 11$ $7 = b$
Write the answer.	The unknown side length is 7 cm.

SC 2 I can determine unknown side lengths in any polygon, given the perimeter

For polygons with equal side lengths, the side length can be calculated from the perimeter by dividing by the number of sides.

To calculate the unknown side length in any polygon, first subtract the total of the known side lengths from the perimeter. For an unknown side length that appears more than once, divide by the number of times it appears.

Polygons with equal side lengths and interior angles are called 'regular' polygons.

Worked example

Calculating the unknown side length of a polygon, given the perimeter

Calculate the unknown side length of each polygon, given the perimeter.

(a) A regular pentagon with $P = 33$ cm

THINKING	WORKING
Write a suitable formula for the perimeter.	$P = 5l$, where l is the side length in centimetres.
Substitute the given value, then calculate the unknown.	$33 = 5l$ $l = \frac{33}{5}$ $= 6.6$
Write the answer.	The side length is 6.6 cm.

(b) A hexagon with two sides of 5 m, one of 8 m and three others of the same length, given $P = 46.5$ m

THINKING	WORKING
Write a suitable formula for the perimeter.	$P = 2 \times 5 + 8 + 3x$, where x is the unknown side length in metres.
Substitute the given values, then calculate the unknown.	$46.5 = 2 \times 5 + 8 + 3x$ $46.5 = 18 + 3x$ $3x = 46.5 - 18$ $= 28.5$ $x = \frac{28.5}{3}$ $= 9.5$
Write the answer.	The unknown side length is 9.5 m.

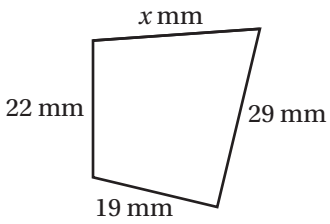
Practice

ANSWERS Page 588

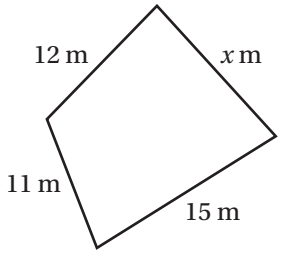
SC 1 I can determine unknown side lengths in a rectangle and other quadrilaterals, given the perimeter

1 Determine the unknown side length for each of the quadrilaterals, given the perimeter.

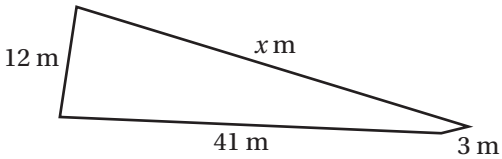
(a) $P = 95$ mm



(b) $P = 51$ m

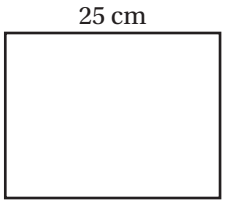


(c) $P = 100$ m



2 Determine the unknown side length for each of the rectangles, given the perimeter.

(a) $P = 70$ cm



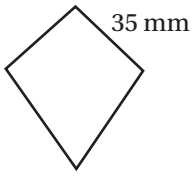
(b) $P = 28$ cm

(c) One pair of sides is 6.1 cm and $P = 21.8$ cm

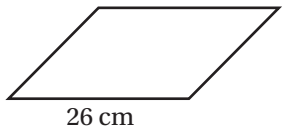
(d) One pair of sides is 82 cm and $P = 4.7$ m

3 Determine the unknown side length for each of the special quadrilaterals, given the perimeter.

(a) Kite: $P = 180$ mm



(b) Parallelogram: $P = 80$ cm



(c) A kite with one pair of sides 72 cm and $P = 4.04$ m

(d) A parallelogram with one pair of sides 6.3 cm and $P = 32$ cm

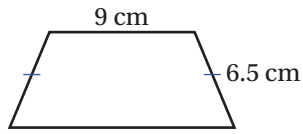
4 Determine the unknown side length for each of the quadrilaterals, given the perimeter.

(a) Three sides of 11.5 cm and $P = 50$ cm

(b) Three sides of the same length with the fourth side 1.6 m and $P = 5.8$ m

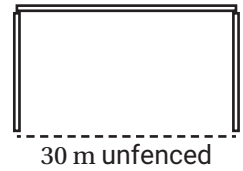
5 Determine the unknown side length for each of the quadrilaterals, given the perimeter.

(a) $P = 34.3$ cm

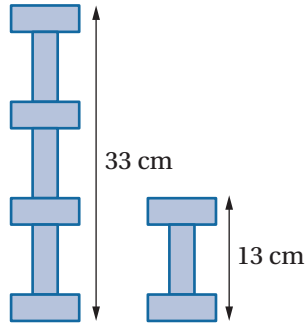


(b) Two sides of the same length, the other sides 21 cm and 26 cm respectively, and $P = 1.13$ m.

6 A rectangular house block requires just 70 m of fencing, since the 30 m of frontage is to be left unfenced. Determine the depth of the block.



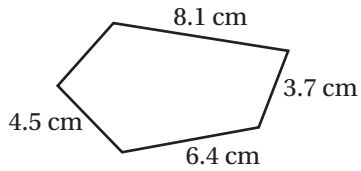
7 Find the perimeter of a single rectangle, given that all dimensions are whole numbers. Explain how you found your answer.



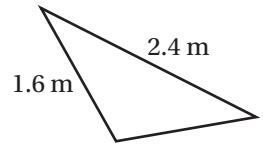
SC 2 I can determine unknown side lengths in any polygon, given the perimeter

1 Calculate the unknown side length of each polygon, given the perimeter.

(a) $P = 26.1$ cm



(b) $P = 5.4$ m



(c) A hexagon with one side each of 8.3 cm, 4.2 cm, 5.9 cm, 7.5 cm and 8.8 cm, and $P = 40.2$ cm

(d) A decagon with nine sides each of 22 cm and $P = 2.13$ m

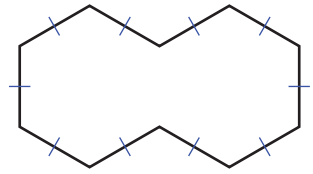
2 Calculate the unknown side length of each polygon, given the perimeter.

(a) A square with $P = 10$ m

(b) A regular hexagon with $P = 96$ cm

(c) A rhombus with $P = 58.4$ cm

(d) $P = 213$ cm



3 A pentagon has three sides each of 15.6 cm and a perimeter of 87.2 cm. Determine the lengths of the other two sides, given that they are both the same.

4 An octagon with a perimeter of 49 m has two sides of 4.1 m and 5.3 m respectively.

(a) Determine the total of the side lengths of the other four sides.

(b) Write possible lengths for the other four sides, given that they are all different from each other and the given sides.

Calculate the area of a trapezium

8.4

Learning intention: To be able to calculate the area of a trapezium

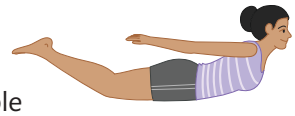
Success criteria:

SC 1 I can determine the area of a trapezium by drawing a rectangle with the same area.

SC 2 I can calculate the area of a trapezium using a formula.

Lesson warm-up

Comparing trapeziums to related parallelograms

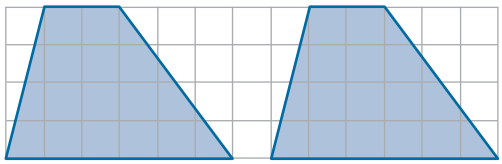


Use grid paper to draw two copies of the same trapezium of any dimensions you choose. Make your dimensions different from the example and different from the students sitting near you.

Cut out the second trapezium, rotate it 180°, then slide it so that the matching sides fit together.

Copy down the two sentences and fill in the missing words.

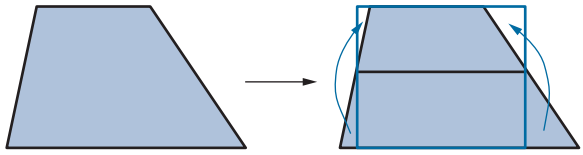
The new shape is a parallelogram with its base as the ____ of the ____ sides of the trapezium. The height of the parallelogram is the ____ as the height of the trapezium.



SC 1 I can determine the area of a trapezium by drawing a rectangle with the same area

A trapezium is a quadrilateral with just one pair of parallel sides.

As with parallelograms, the midpoint method can be used to construct a rectangle with the same area as any trapezium.



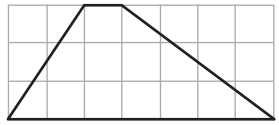
The height of the trapezium gives the height of the rectangle.

The average of the parallel sides in the trapezium becomes the length of the rectangle.

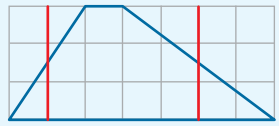
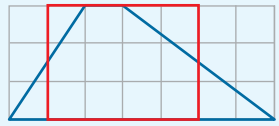
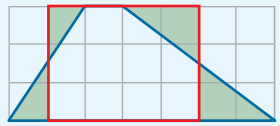
Worked example

Deducing the area of a trapezium from a rectangle with the same area

Consider the trapezium below.



(a) Use the midpoint method to form a rectangle of equal area to the trapezium, shading triangles so that equal areas are easily seen.

THINKING	WORKING
Draw vertical lines through the midpoints of the oblique sides.	
Use the vertical sides to construct the rectangle.	
Shade the four triangles to show that equal areas have been added and subtracted.	

(b) Relate the lengths of the parallel sides to the distance between the midpoints.

THINKING	WORKING
Describe the length of the rectangle in terms of the top and bottom lengths of the original shape.	Length of the rectangle: 4 units Parallel sides of the trapezium: 1 unit, 7 units The length of the rectangle is the average of the parallel sides of the trapezium. $\frac{1+7}{2} = 4$

(c) Calculate the area of the rectangle.

THINKING	WORKING
Calculate the area of the rectangle using the grid.	$4 \times 3 = 12$
Write the answer.	The area of the rectangle is 12 square units.

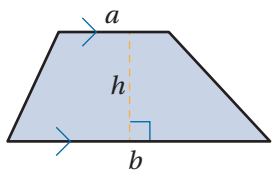
(d) Using the results of part **(c)**, state the area of the trapezium.

THINKING	WORKING
Since equal triangular areas have been subtracted from the trapezium, then added to complete the rectangle, you can conclude that the areas are the same.	The area of the trapezium is 12 square units.

SC 2 I can calculate the area of a trapezium using a formula

The area of a rectangle is length × breadth or base × height, where the two dimensions are perpendicular.

The area of a trapezium can be found by multiplying the average of the parallel sides by the distance between them.



For parallel sides a and b, their average is given by $\frac{a+b}{2}$ or $\frac{1}{2}(a+b)$.

The parallel sides are perpendicular to the height, h.

So the area is given by:

$$A = \frac{a+b}{2} \times h \text{ or } A = \frac{1}{2}(a+b)h$$

which is known as the trapezoid formula.

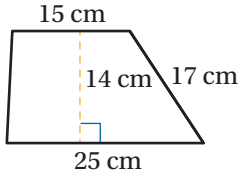
Ensure that the dimensions are in the same units before using any formula for area.

8

Worked example

Calculating the area of a trapezium from a formula

Use the trapezoid formula to determine the area of the trapezium.



THINKING	WORKING
Recall the formula for the area of a trapezium.	$A = \frac{1}{2}(a+b)h$
Identify the dimensions to be used in the formula.	Parallel sides: a = 15 cm, b = 25 cm Perpendicular height: h = 14 cm

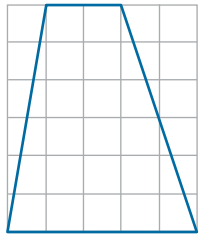
Calculate the area of the trapezium.	$A = \frac{1}{2}(15+25)\times 14$ $= 20\times 14$ $= 280$
Write the answer.	The area of the trapezium is 280 cm^2.

Practice

ANSWERS Page 588

SC 1 I can determine the area of a trapezium by drawing a rectangle with the same area

1 Consider the trapezium below.



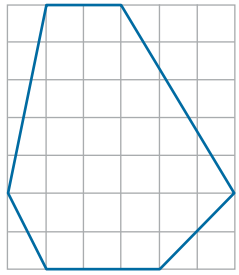
(a) Use the midpoint method to form a rectangle of equal area to the trapezium, shading triangles so that equal areas are easily seen.

(b) Show that the distance between the midpoints is the average of the parallel sides.

(c) Calculate the area of the rectangle.

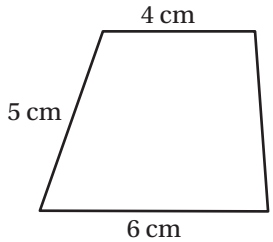
(d) State the area of the trapezium.

2 Give the dimensions of two rectangles whose areas could be added together to calculate the area of the hexagon.

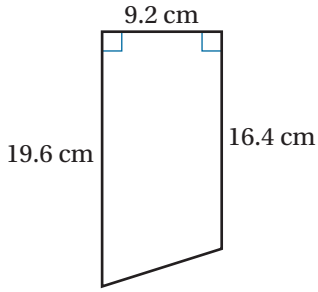


3 Calculate the average of the parallel sides for each trapezium.

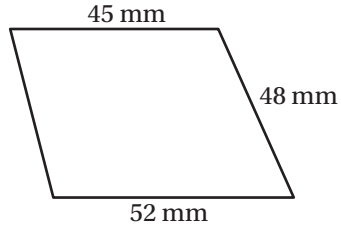
(a)



(b)

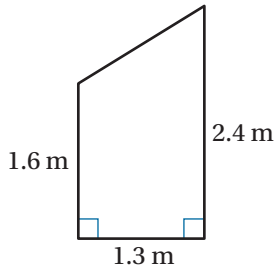


(c)

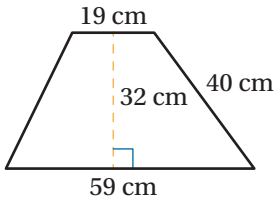


4 Determine the dimensions of a rectangle, the same height as each trapezium, which will have the same area as the trapezium. Express your answers in the form length × height.

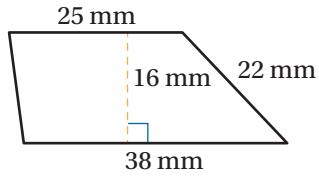
(a)



(b)



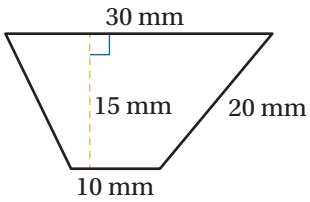
(c)



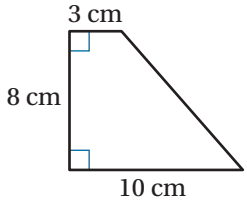
SC 2 I can calculate the area of a trapezium using a formula

1 Use the trapezoid formula to calculate the area of each trapezium.

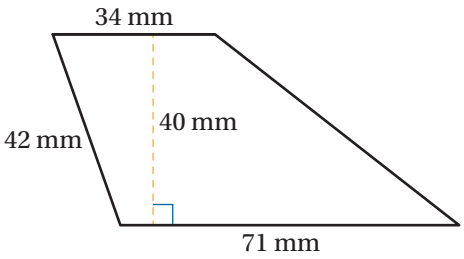
(a)



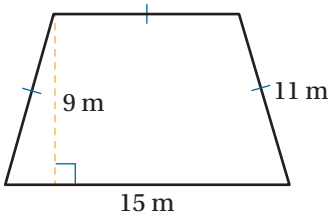
(b)



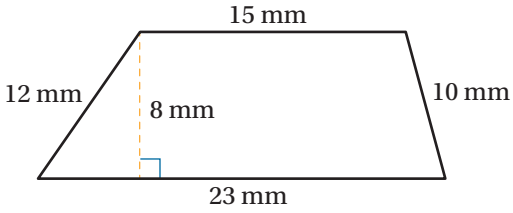
(c)



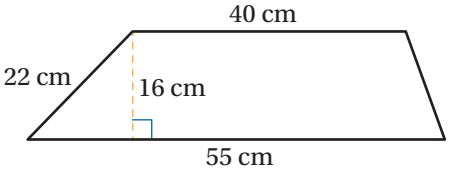
(d)



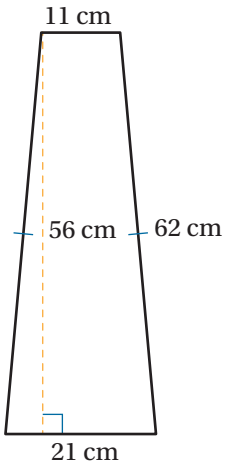
(e)



(f)



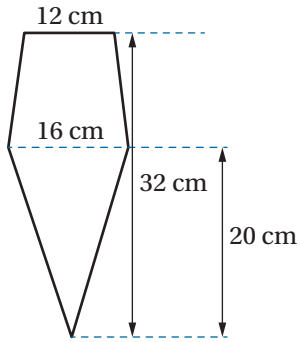
(g)



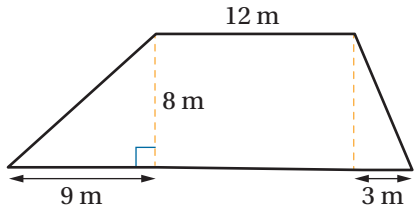
2 Explain the common error in the calculation below, then write the corrected final answer.

$$A = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(60+90)\times 40$$
$$= 3000\ \text{mm}^2$$
$$= 3\ \text{m}^2$$

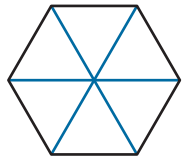
3 Determine the area of the pentagon in cm^2.



4 Determine the cost of concreting the patio area, given a rate of $25.80 per square metre.



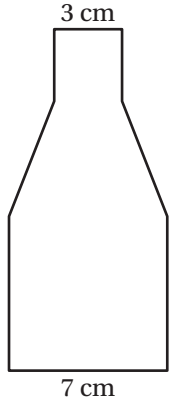
5 A regular hexagon has a side length of 20 cm and an overall height of 34.6 cm.



(a) Determine the area of the hexagon.

(b) Determine the dimensions of a rectangle with the same height and area of the regular hexagon.

6 The compound shape below is made up of a trapezium between two squares. The shape has an overall height of 15 cm. Determine the area of the compound shape.



Calculate the area of kites and rhombuses

8.5

Learning intention: To be able to calculate the areas of kites and rhombuses

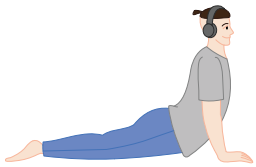
Success criteria:

SC 1 I can determine the area of kites and rhombuses from the areas of rectangles and triangles.

SC 2 I can calculate the area of kites and rhombuses using a formula.

Lesson warm-up

Making rectangles from kites



Use grid paper to draw two copies of the same kite of any dimensions you choose. Make sure your dimensions are different from the example and different from the students sitting near you.

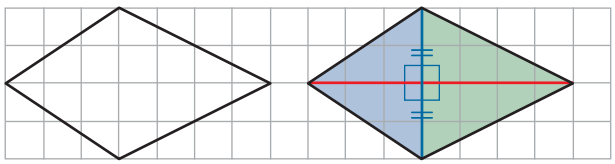
Mark in the diagonals, the right angles where the diagonals cross and the equal parts of one of the diagonals.

Cut out the second kite and cut along each of its diagonals. Form a rectangle from the four triangles.

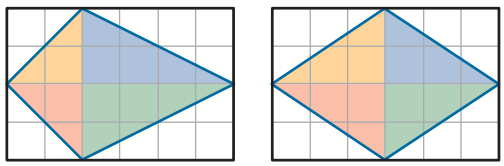
Compare the dimensions of the rectangle with the diagonals of the kite.

Copy down the following sentence and fill in the missing words.

The new shape is a rectangle with one side as the unbisected _____ of the kite and the other side as _____ of the bisected diagonal.



Kites (and the special case of a rhombus) have diagonals at right angles and at least one line of symmetry. The diagonals break a kite into two pairs of congruent (identical) right-angled triangles, and any rhombus into four congruent right-angled triangles.



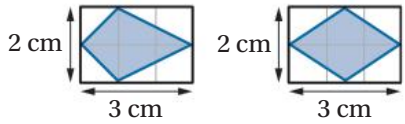
Kite Rhombus

Given the lengths of the diagonals, the area of a kite can be determined by first calculating the area of a rectangle or a variety of triangles.

Worked example

Determining the areas of a kite and a rhombus from a rectangle and various triangles

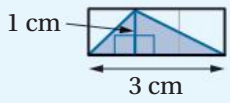
The kite and rhombus below are drawn within identical rectangles.



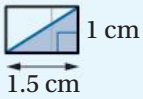
(a) Calculate the area of the rectangle, and from this, calculate the area of both the kite and the rhombus.

THINKING	WORKING
Recall the formula to calculate the area of a rectangle.	$A = lb$
Use the grid squares to calculate the area of the rectangles.	$A = 3 \times 2$ $= 6$ The area of each rectangle is 6 cm^2.
In each of the four sections formed by the diagonals, the kite and rhombus take up half of the area. So, the areas of both the kite and rhombus are half the area of the rectangle overall.	$\frac{6}{2} = 3$ Hence, the areas of both the kite and the rhombus are 3 cm^2.

(b) Calculate the area of the triangle that forms the top half of the kite, and from this, calculate the area of the kite.

THINKING	WORKING
Use the base and perpendicular height to calculate the area of the top triangle.	 $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 3 \times 1$ $= 1.5$ The area of the top triangle of the kite is 1.5 cm^2.
Two congruent triangles form the kite, so multiply the area of the triangle by 2.	$1.5 \times 2 = 3$ Hence, the area of the kite is 3 cm^2.

(c) Calculate the area of the right-angled triangle that forms a quarter of the rhombus, and from this, calculate the area of the rhombus.

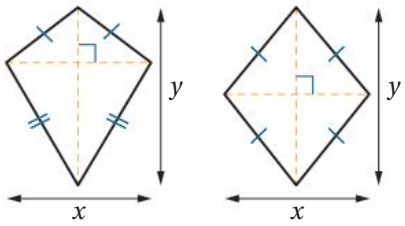
THINKING	WORKING
Use the perpendicular sides of the right-angled triangles to calculate the area.	 $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 1.5 \times 1$ $= 0.75$ The area of the right-angled triangle that is a quarter of the rhombus is 0.75 cm^2.
Four congruent right-angled triangles form the rhombus, so multiply the area of the triangle by 4.	$0.75 \times 4 = 3$ Hence, the area of the rhombus is 3 cm^2.

SC 2 I can calculate the area of kites and rhombuses using a formula

Because of the symmetry of kites, and the special case of the rhombus, a formula involving the lengths of the diagonals can be derived.

$A = \frac{1}{2}xy$ or $A = \frac{xy}{2}$, where x and y are the lengths of the diagonals.

Note: The values of x and y are interchangeable.

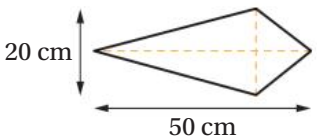


Kite Rhombus

Worked example

Calculating the area of a kite from a formula

Use a formula to calculate the area of the kite.



THINKING	WORKING
Recall the formula for the area of a kite.	$A = \frac{1}{2}xy$
Write the formula and identify the variables.	Diagonals: $x = 20$ cm, $y = 50$ cm
Substitute the values and calculate.	$A = \frac{1}{2} \times 20 \times 50$ $= 500$
Write the answer, including the units of area.	The area of the kite is 500 cm^2.

Practice

ANSWERS Page 588

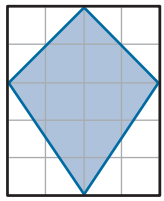
SC 1 I can determine the area of kites and rhombuses from the areas of rectangles and triangles

1 For each diagram, assume the grid units are in centimetres.

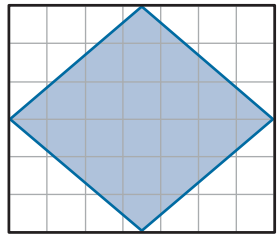
(i) Calculate the area of the rectangle containing each kite or rhombus.

(ii) Then, calculate the area of the kite or rhombus.

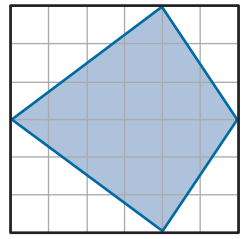
(a)



(b)



(c)

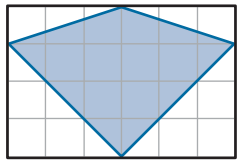


2 For each diagram, assume the grid units are in centimetres.

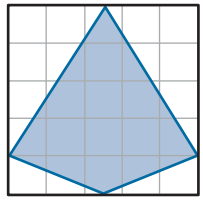
(i) Calculate the area of one of the congruent triangles that forms half of the kite.

(ii) Then, calculate the area of the kite.

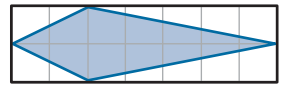
(a)



(b)



(c)

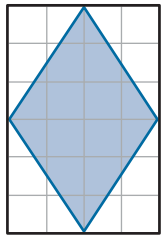


3 For each diagram, assume the grid units are in centimetres.

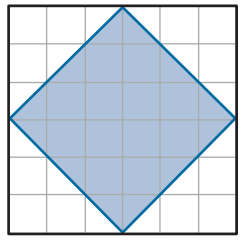
(i) Calculate the area of one of the congruent right-angled triangles that forms a quarter of each shaded rhombus (or square).

(ii) Then, calculate the area of the rhombus (or shaded square).

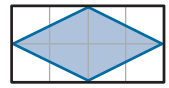
(a)



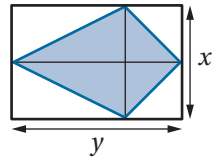
(b)



(c)



4 The diagonal lengths of the kite are x and y, respectively.

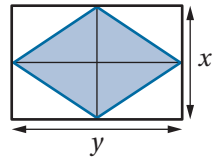


(a) Write the area for the rectangle in terms of x and y.

(b) For any one of the congruent triangles that form half of the kite, write its area in terms of x and y.

(c) From your answer to **(a)** or **(b)**, write the area of the kite in terms of x and y.

5 The diagonal lengths of the rhombus are x and y, respectively.

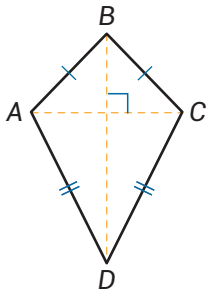


(a) For any one of the congruent right-angled triangles that form a quarter of the rhombus, write its area in terms of x and y.

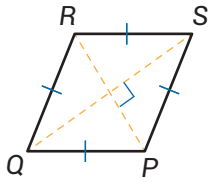
(b) From your answer to **(a)**, write the area of the rhombus in terms of x and y.

6 For each quadrilateral, use the end letters to name any diagonals that divide the quadrilateral into two congruent (identical) triangles.

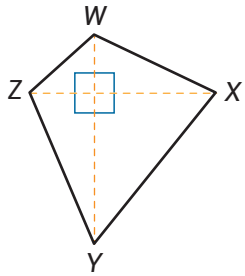
(a) a kite



(b) a rhombus



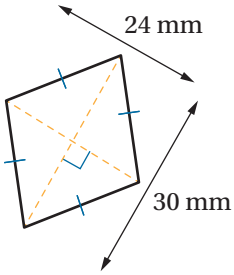
(c) a quadrilateral with perpendicular diagonals



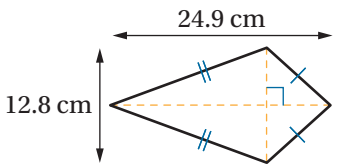
SC 2 I can calculate the area of kites and rhombuses using a formula

1 Use the formula to calculate the area of each kite or rhombus.

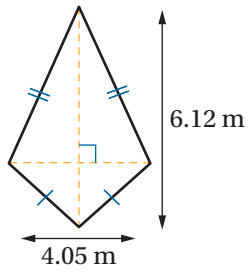
(a)



(b)



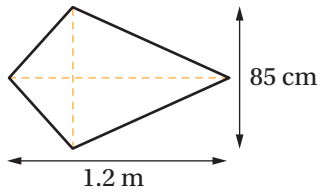
(c)



2 Explain the error in the following calculation, then write the corrected final answer.

$$A = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 1.3 \times 1.4$$
$$= 0.91\,\text{m}^2$$
$$= 91\,\text{cm}^2$$

3 Determine the area of the kite from each of the given units, then check that the answers convert to each other correctly.



4 A rhombus is a special kite, so the formula $A = \frac{1}{2}xy$ can be used to find its area, when the lengths of the diagonals are known.

A rhombus is also a special parallelogram, so the formula $A = bh$ can be used to find its area, when the base and perpendicular height are known.

(a) Given that the diagonals of a rhombus are 32.8 cm and 18.7 cm, and it has a side length of 20 cm, determine its height. Hint: Construct a diagram first.

(b) Given that the side length of a rhombus is 2.47 m and its perpendicular height is 2.1 m, determine the length of the longer diagonal, given that the shorter diagonal is 2.5 m.

8.6 Calculate the area of composite shapes

Learning intention: To be able to calculate the area of composite shapes

Success criteria:

SC 1 I can calculate the area of composite shapes.

Lesson warm-up

What shapes make up this shape?

Consider this house.

The owner of the house wants to paint the end wall (outlined in red).

They remove the plants in front of the wall first.

Describe how you could work out the area to be painted.

SC 1 I can calculate the area of composite shapes

You already know formulas to calculate the area of several different shapes.

Composite shapes are made up of other shapes. You divide a composite shape into known shapes, then use the area formulas you know to calculate the area of the whole shape.

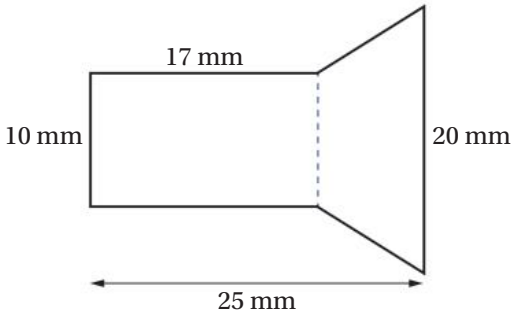
For some composite shapes, you will need to add the areas of two or more known shapes. For other composite shapes, where a known shape has been removed, you will need to subtract.

Worked example

Calculating the area of composite shapes

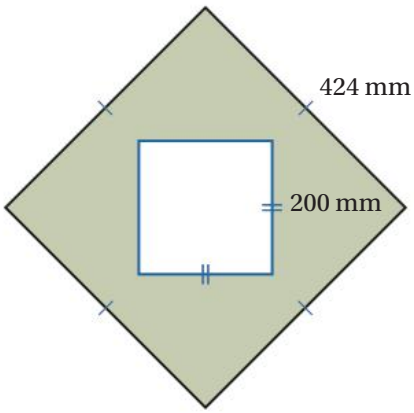
Determine the area of each composite shape.

(a)



THINKING	WORKING
Identify the known shapes.	The composite shape is made from a rectangle and a trapezium.
State the dimensions of the shapes.	Rectangle: $l = 17$ mm, $w = 10$ mm Trapezium: $a = 10$ mm, $b = 20$ mm $h = 25 - 17$ $= 8$ mm
Write a method, including formulas, to calculate the total area.	$A = lw + \left(\frac{a+b}{2}\right) \times h$
Substitute the required dimensions and calculate the area of the composite shape.	$A = 17 \times 10 + \left(\frac{10+20}{2}\right) \times 8$ $= 290 \text{ mm}^2$
Write the answer.	The area of the shape is 290 mm^2.

(b)



THINKING	WORKING
Identify the known shapes.	The composite shape is made from a larger square with a smaller square subtracted from it.
State the dimensions of the shapes.	Larger square: $L = 424$ mm Smaller square: $l = 200$ mm
Write a method, including formulas, to calculate the total area.	$A = L^2 - l^2$
Substitute the required dimensions and calculate the area of the composite shape.	$A = 424^2 - 200^2$ $= 139\,776$ mm^2
Write the answer.	The area of the shaded shape is 139 776 mm^2.

Practice

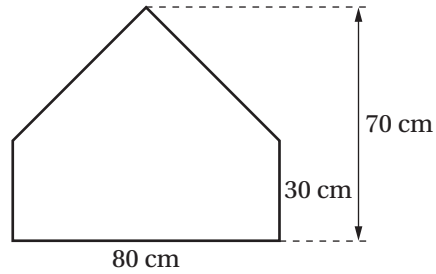
ANSWERS Page 588

SC 1 I can calculate the area of composite shapes

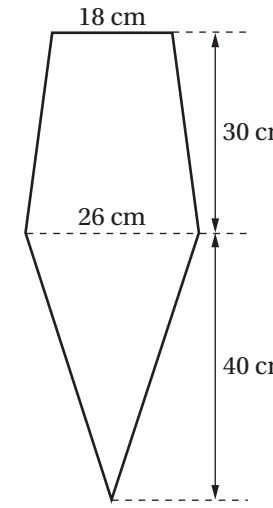
1 Calculate the area of each composite shape. Round your answers to 2 decimal places where necessary.

Hint: You will need to add shapes together.

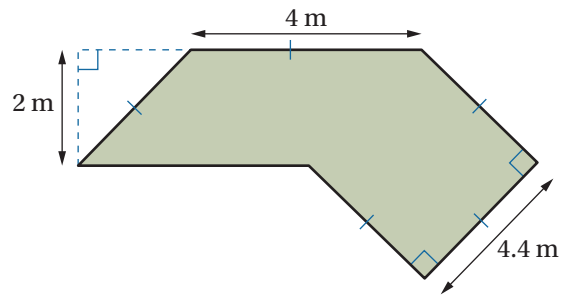
(a)



(b)

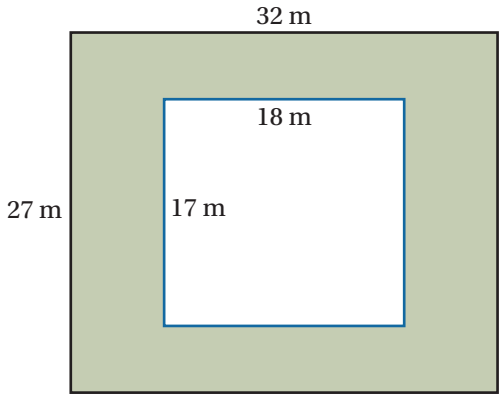


(c)

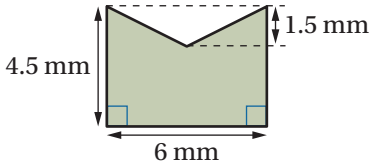


2 Calculate the area of each composite shape by subtracting the area of one shape from another.

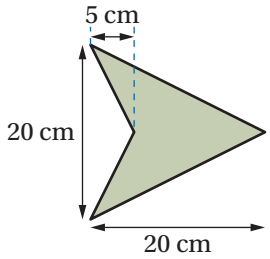
(a)



(b)

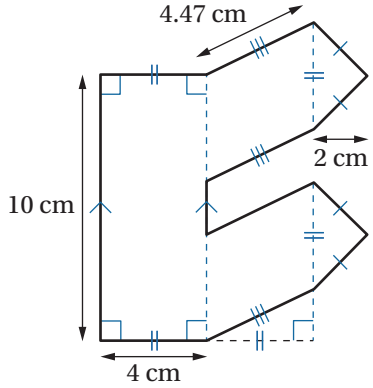


(c)

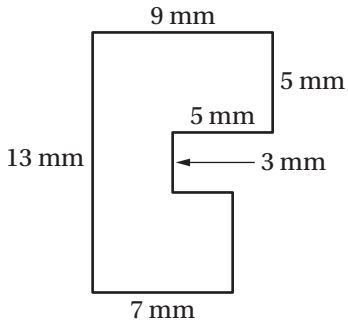


3 Calculate the area of each composite shape.

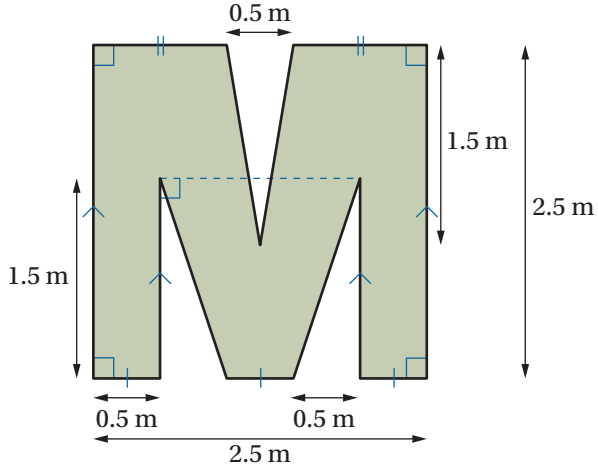
(a)



(b)

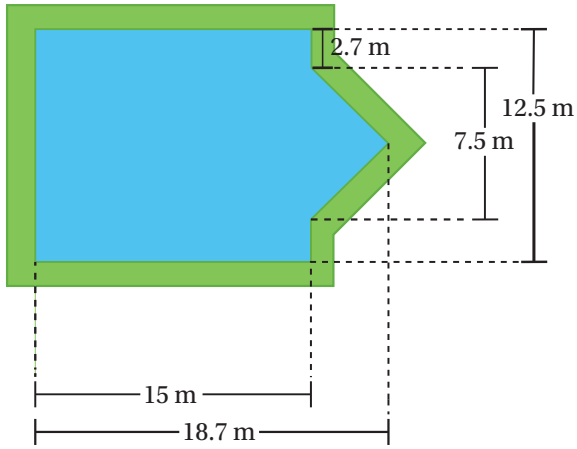


(c)

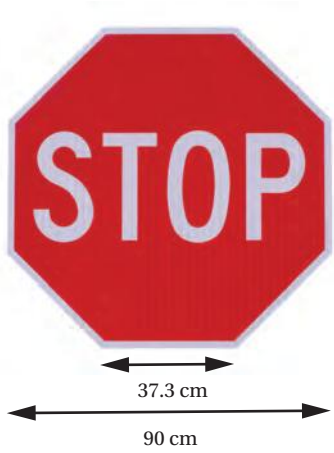


4 Miriam wants to know the rate of water evaporation from her swimming pool. This will depend on the surface area.

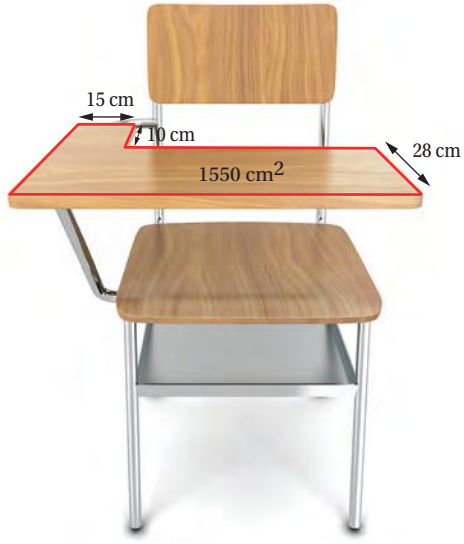
Calculate the surface area of the water in the pool. Round your answer to the nearest square metre.



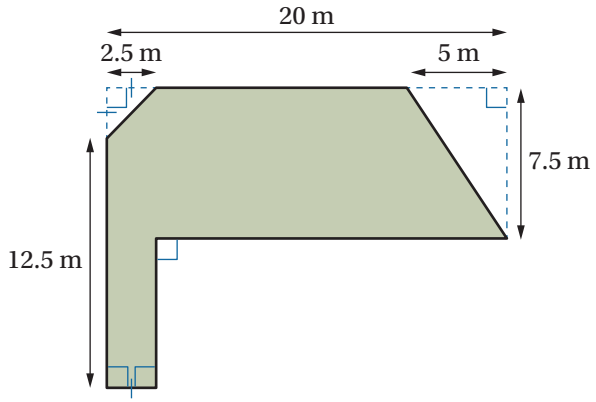
5 A stop sign is a regular octagon, with eight sides of equal length and equal angles. Determine the area of the stop sign to the nearest square centimetre.



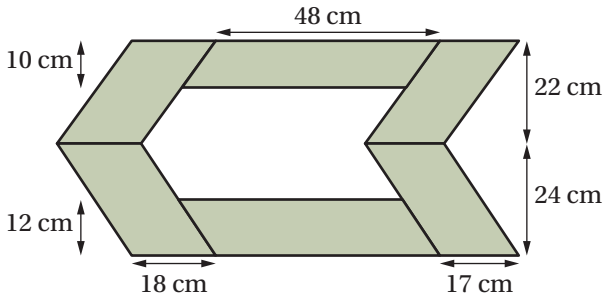
6 An office supplier sells a chair with a built-in desk, with lengths as shown.
The desk is advertised as having an area of 1550 cm^2. Determine the width of the desk.



7 A home owner wants to lay grass in their backyard. The grass costs \$14/m^2, with area rounded up to the nearest square metre. Calculate the cost of the grass for the backyard shown.



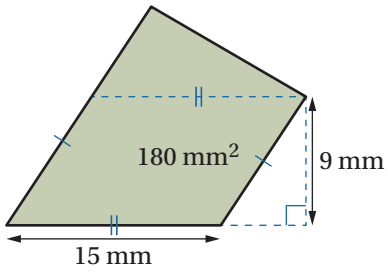
8 A model arrow to be used as a road sign is shown below.



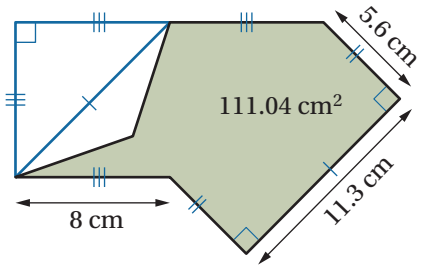
(a) The shaded area is made up of six parallelograms with dimensions as shown. What area of material is needed to construct this section of the arrow?

(b) The inner section of the arrow is made up of two other parallelograms. What area of material is needed to construct this part of the arrow?

9 This composite shape has a total area of 180 mm^2. It can be separated into a parallelogram and a triangle as shown. Determine the height of the triangle.



10 This composite shape can be separated into a rectangle and a parallelogram, with a triangle cut out, as shown. The shaded area is 111.04 cm^2.
Determine the height of the triangle that is cut out. Give your answer correct to 1 decimal place.



8.7 Approximate the perimeter and area of irregular shapes

Learning intention: To be able to approximate the perimeter and area of irregular shapes

Success criteria:

SC 1 I can approximate the perimeter of irregular shapes.

SC 2 I can approximate the area of irregular shapes.

SC 3 I can solve practical problems involving irregular shapes.

Lesson warm-up

Swimming around Australia

As part of a fundraising event, a swimming team will swim the length of the entire perimeter of Australia.

They need to estimate the total distance around Australia so they can plan how long the trip will take.

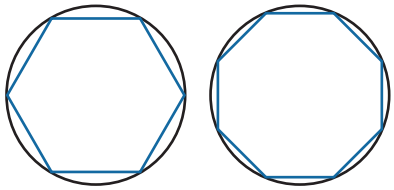
The Australian coastline has been formed by natural processes over millions of years, so it is not a regular shape. The swimmers will need a method to approximate the distance they will swim.

Suggest how they might do this.

SC 1 I can approximate the perimeter of irregular shapes

You can estimate the length of curved parts of a perimeter by using many small straight pieces.

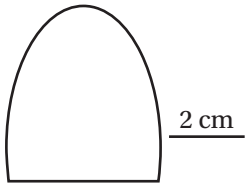
For a circle, the perimeter of the inscribed hexagon gives a fairly good estimate, but the perimeter of the inscribed octagon gives an even better estimate.



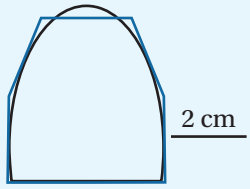
Worked example

Approximating the perimeter

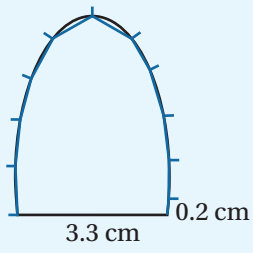
Approximate the perimeter of this shape.



(a) Use a straight-edged ruler and 2 cm segments.

THINKING	WORKING
Measure the 2 cm segment. Starting at the bottom left corner of the shape, mark off 2 cm segments around the edge.	
Write a calculation to find the total of the measured distances.	$P \approx 7 \times 2$ $= 14$ cm

(b) Refine your estimate using 1 cm segments.

THINKING	WORKING
Measure the straight edge of the shape. Then, starting at the bottom left corner of the shape, mark off 1 cm segments around the curve. When the last part is less than 1 cm, measure the distance to the end of the curve.	
Write a calculation to find the total of the measured distances.	$P \approx 3.3 + 10 \times 1 + 0.2$ $= 13.5$ cm

SC 2 I can approximate the area of irregular shapes

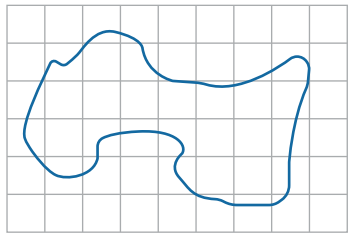
Laying a grid over an irregular shape can help with estimating the area of the shape. Within the grid, squares can be counted if they are more than half-shaded, while squares that are less than half-shaded are not counted. This balances the inaccuracies in the method to some extent.

The accuracy of a grid estimate can be improved by increasing the number of grid lines, so the irregular shape is divided into a larger number of smaller squares.

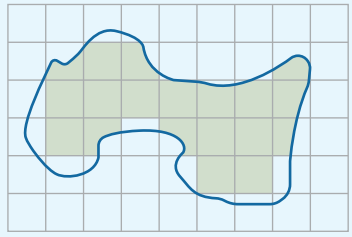
Worked example

Approximating area from a grid

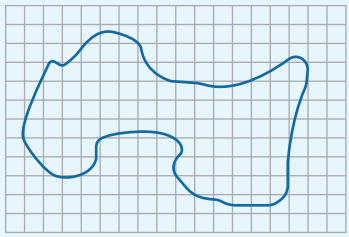
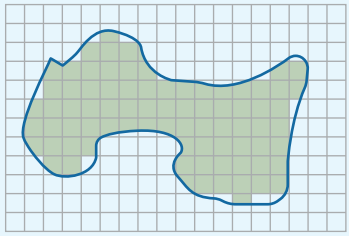
Approximate the area of this irregular shape.



(a) Use the 1 cm^2 grid.

THINKING	WORKING
Shade the grid squares that are entirely within the shape. Then shade pieces of the shape that fill more than half a grid square.	
Count the shaded squares.	19 squares are fully or partly shaded.
Calculate the area of the shape by multiplying the number of squares by the area of each square.	$A \approx 19 \times 1\ cm^2$ $= 19\ cm^2$
Write the answer.	The area is approximately 19 cm^2.

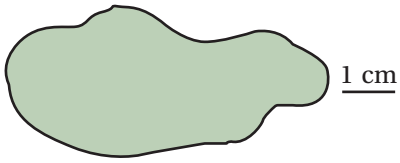
(b) Refine the approximation by doubling the number of grid lines.

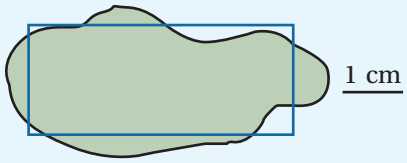
THINKING	WORKING
Draw grid lines midway between the existing grid lines.	
Shade all grid squares that are fully within the shape. Then shade pieces of the shape that fill more than half a grid square.	
Count the shaded squares.	79 squares are fully or partly shaded.
Calculate the area of the shape by multiplying the number of squares by the area of each square.	$A \approx 79 \times \frac{1}{4}$ cm^2 ≈ 19.75 cm^2
Write the answer.	The area is approximately 19.75 cm^2.

Worked example

Approximating area using standard shapes

Approximate the area of this irregular shape using an overlaid rectangle.



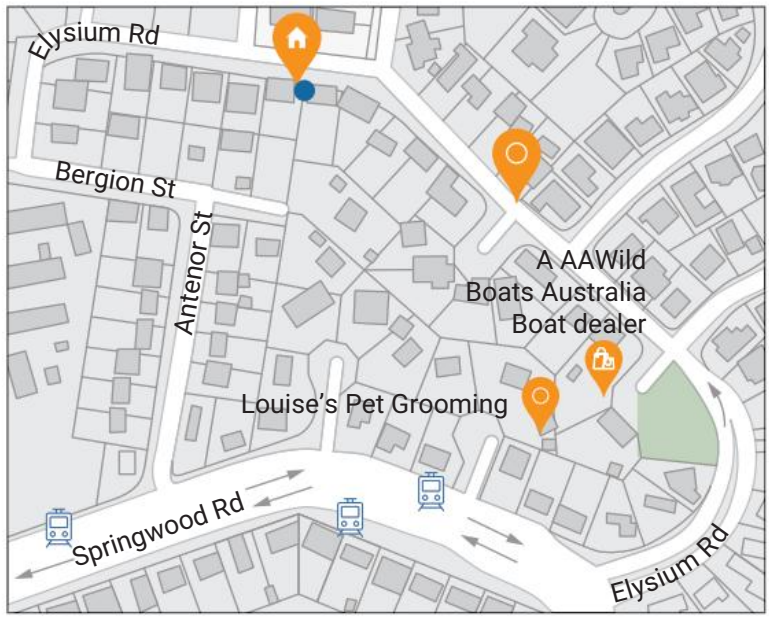
THINKING	WORKING
Draw a rectangle over the shape. Make sure the shaded area outside the rectangle is roughly equal to the unshaded area inside the rectangle.	
Measure the length and width of the rectangle.	$l = 7.0$ cm, $w = 2.9$ cm
Calculate the area of the rectangle.	$A = lw$ $= 7.0 \times 2.9$ $= 20.3$ cm^2
Write the answer.	The shape has an area of about 20 cm^2.

SC 3 I can solve practical problems involving irregular shapes

Worked example

Using estimation to solve practical problems

Elysium Road goes from Bergion Street to Springwood Road. There is a divided section at the Springwood Road end.



(a) Antenor Street is about 160 m long. Use this fact to estimate the length of Elysium Road, counting both parts of the divided section.

THINKING	WORKING
Measure the map length of the given street.	Antenor St: 3 cm ↔ 160 m
Estimate the map length for the required street length, including both parts of the divided road.	Elysium Road map length ≈ 17 cm
Use proportion to calculate the actual length of Elysium Road. Round appropriately.	Elysium Road scale: $\frac{17}{3} \approx 5.7$ times the length of Antenor Street Elysium Road actual length: $\frac{17\text{cm}}{3\text{cm}} \times 160\,\text{m} \approx 910\,\text{m}$
Write the answer.	Elysium Road is approximately 910 m long.

(b) Elysium Road is approximately 8 m wide. Calculate the cost of resurfacing it at $\$50/m^2$.

THINKING	WORKING
Estimate the area, treating the road as a long rectangle.	Area of road surface: $A = lw$ $= 910 \times 8$ $= 7280\ m^2$
Calculate the cost by multiplying the area by the cost per m^2.	Cost: $7280\ m^2 \times \$50/m^2 = \$364\ 000$
Write the answer.	It will cost around $364 000 to resurface Elysium Road

Practice

ANSWERS Page 589

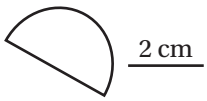
SC 1 I can approximate the perimeter of irregular shapes

1 Approximate the perimeter of each shape.

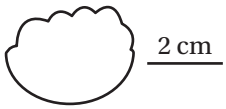
(i) Use a straight-edged ruler and 2 cm segments.

(ii) Refine your estimate using 1 cm segments.

(a)



(b)



2 A lake has a path going all the way around it.
Use this path to estimate the perimeter of the lake, given the scale marking of 100 m.

3 The straight-line distance from Gladstone to Rockhampton is 92 km.

(a) Estimate the distance by plane from Bundaberg to Gladstone.

(b) Estimate the distance by car along the main road from Bundaberg to Rockhampton.

4 The land border of Western Australia is 1874 km long. Use this information and the map to estimate:

(a) the perimeter of Western Australia

(b) the perimeter of New South Wales

(c) the length of the land border of Queensland

(d) the length of the land border of Victoria

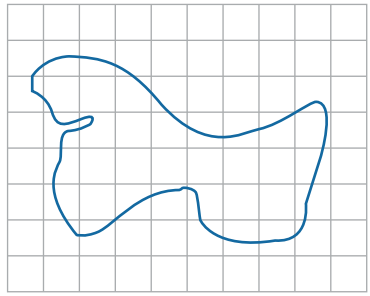
(e) the length of the land border of South Australia

(f) the length of the land border of the Northern Territory

(g) the length of the main coastline of Tasmania.

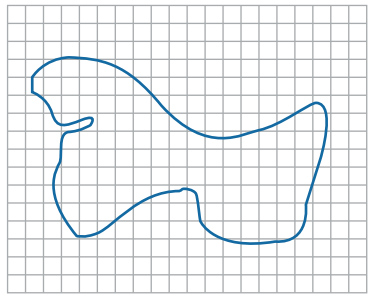
SC 2 I can approximate the area of irregular shapes

1 Approximate the area of this irregular shape.



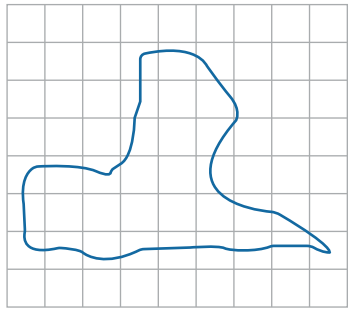
(a) Use the 1 cm^2 grid.

(b) Refine the approximation by doubling the number of grid lines.

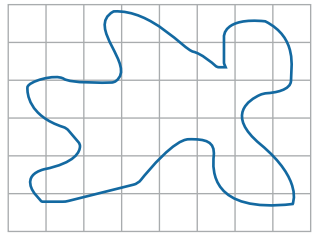


2 Approximate the area of each shape using the 1 cm^2 grid.

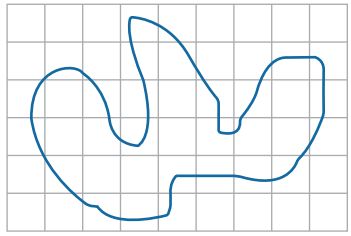
(a)



(b)



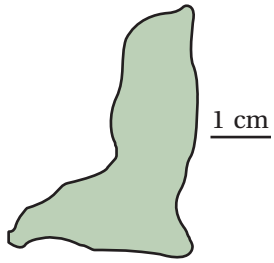
(c)



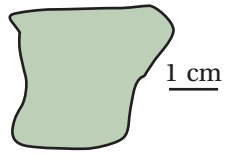
3 Approximate the area of the irregular shape using the suggested overlaid shape.

(a) Parallelogram

(b) Triangle



(c) Trapezium



4 Use the map of Australia on page 225 to answer the following questions.

(a) Use a rectangular overlay to estimate the area of the Northern Territory on the map.

(b) Estimate the actual area of the Northern Territory.

(c) Use a triangle overlay to estimate the area of Queensland.

(d) Use a trapezium overlay to estimate the area of Tasmania.

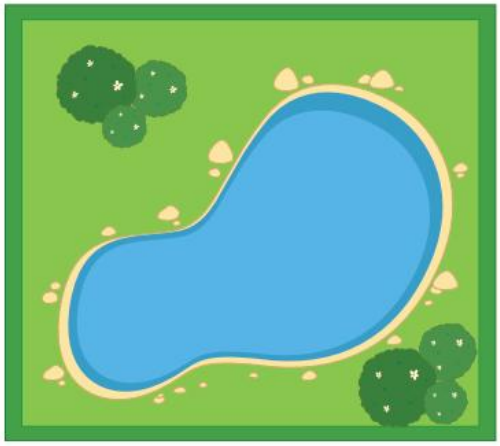
(e) Estimate the actual areas of:

(i) Western Australia **(ii)** South Australia

(iii) New South Wales **(iv)** Victoria

SC 3 I can solve practical problems involving irregular shapes

1 The pool in the picture is 10 m long.



(a) Estimate the perimeter of the pool.

(b) A slate path is to be laid around the edge of the pool. The path will be 80 cm wide. Estimate the cost at \$60/m^2.

2 The pool in this diagram is 15 m long.

(a) Estimate the area of lawn.

(b) Calculate the cost of:

(i) new grass seed, at \$70/100 m^2

(ii) laying turf, at \$1045/100 m^2 plus \$100 for delivery.

3 The front of this house block is estimated to be 40 m wide. The fence line marks the boundary.

(a) Estimate the total cost of guttering for the house and garage at $120/m.

(b) Estimate the cost of installing aluminium-slat fencing at $600/m^2, given that the fence is 1.5 m high.

4 A developer is dividing a block of land into 30 lots. Half of the lots will be 620 m^2, one-third of them will be 500 m^2 and the rest will be 440 m^2.

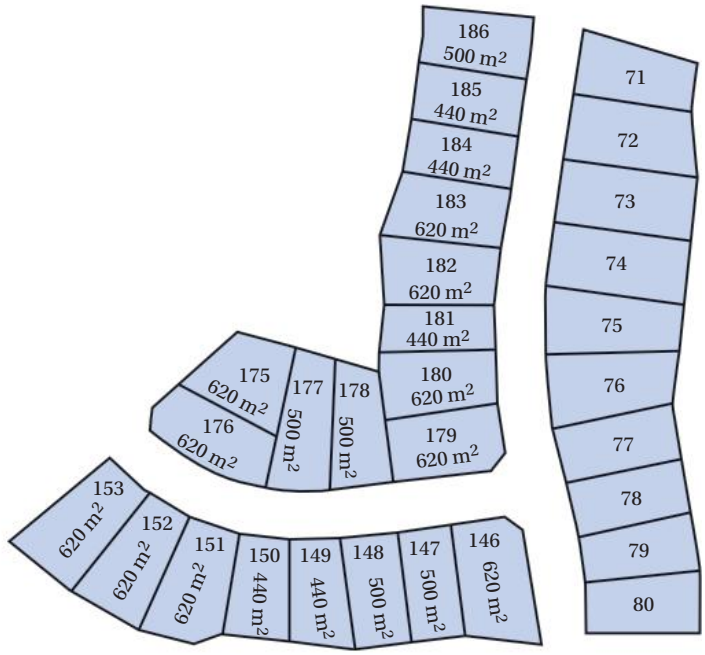
(a) Identify the 500 m^2 lots out of lots 71 to 80.

(b) Work out the size of the other lots from 71 to 80.

(c) Estimate the length of new kerb-and-channelling (road edge) needed for the T-shaped road shown.

(d) The developer must contribute money to public parks at the rates shown below. The charge is for each additional lot after the first, so assume there is no charge for one of the largest lots.

Calculate the developer's contribution.



Residential lot size	Contribution
≥ 900 m^2	$3800
750 m^2–899 m^2	$3500
600 m^2–749 m^2	$3100
450 m^2–599 m^2	$2900
300 m^2–449 m^2	$2800
< 300 m^2	$2000

Understand the connection between volume and capacity

8.8

Learning intention: To understand the connection between volume and capacity

Success criteria:

SC 1 I can convert between metric units of volume.

SC 2 I can calculate volume to determine the capacity of prism-shaped containers.

SC 3 I can solve a variety of practical problems involving the volumes and capacities of prisms.

Lesson warm-up

How much soft drink is needed to fill a bathtub?

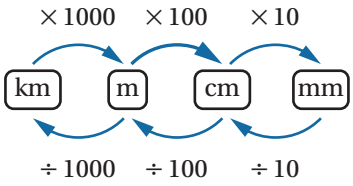
If you add Mentos to a bottle of soft drink, there is a spectacular reaction that causes the liquid to squirt out of the bottle like a fountain.

An influencer wants to create a dramatic video by filling a rectangular bathtub with soft drink and then throwing 1000 Mentos into it.

Work out what supplies they will need and calculate the cost of the experiment.

SC 1 I can convert between metric units of volume

Here are some simple rules for converting between units of length:

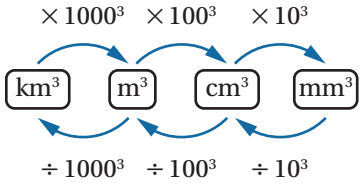


1 km = 1000 m

1 m = 100 cm

1 cm = 10 mm

Here are similar rules for converting between units of volume:



$1\ km^3 = 1000^3\ m^3$

$1\ m^3 = 100^3\ cm^3$

$1\ cm^3 = 10^3\ mm^3$

Worked example

Converting between metric units of volume

Describe how to convert:

(a) cm^3 to mm^3

THINKING	WORKING
Determine whether the number of cubic units will increase or decrease.	There are 10 mm in every 1 cm so the number of cubic units will increase.
Identify the conversion required.	Length: 1 cm = 10 mm Area: 1 cm^2 = 10^2 mm^2 Volume: 1 cm^3 = 10^3 mm^3
Convert the volume from cubic centimetres to cubic millimetres.	1 cm^3 = 10 × 10 × 10 mm^3 = 1000 mm^3
Write the answer.	To convert cm^3 to mm^3, multiply the number of cm^3 by 1000.

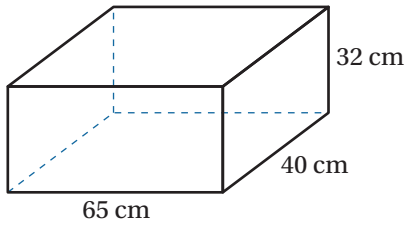
(b) cm^3 to m^3

THINKING	WORKING
Determine whether the number of cubic units will increase or decrease.	There are 100 cm in every 1 m. The number of cubic units will decrease.
Identify the conversion required.	Length: $1\text{ cm} = \frac{1}{100}\text{ m}$ Area: $1\text{ cm}^2 = \frac{1}{100^2}\text{ m}^2$ Volume: $1\text{ cm}^3 = \frac{1}{100^3}\text{ m}^3$
Convert the volume from cubic centimetres to cubic metres.	$1\text{ cm}^3 = \frac{1}{100 \times 100 \times 100}\text{ m}^3$ $= \frac{1}{1\,000\,000}\text{ m}^3$
Write the answer.	To convert cm^3 to m^3, divide the number of cm^3 by 1 000 000.

Worked example

Calculating the volume of prisms

Consider the rectangular prism shown.



(a) Calculate the volume in cubic centimetres.

THINKING	WORKING
Identify given information in centimetres.	$l = 65$ cm, $w = 40$ cm, $h = 32$ cm
Recall the formula to calculate the volume of a prism.	$V = lwh$
Substitute the lengths and calculate the volume.	$V = 65 \times 40 \times 32$ $= 83\,200$ cm^3
Write the answer.	The volume is 83 200 cm^3.

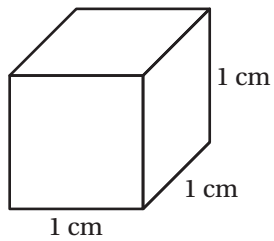
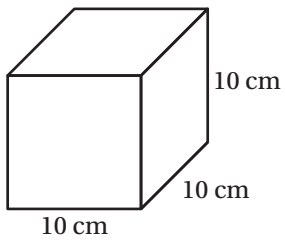
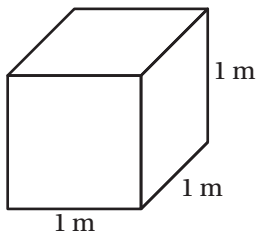
(b) Convert the lengths to metres and calculate the volume in cubic metres.

THINKING	WORKING
Determine the dimensions in metres.	$l = 0.65$ m, $w = 0.4$ m, $h = 0.32$ m
Substitute the lengths into the volume formula and calculate the volume.	$V = lwh$ $= 0.65 \times 0.4 \times 0.32$ $= 0.0832$ m^3
Write the answer.	The volume is 0.0832 m^3.

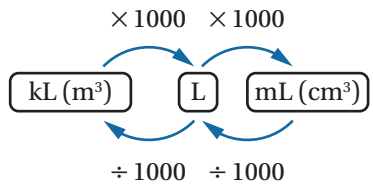
SC 2 I can calculate volume to determine the capacity of prism-shaped containers

The term capacity is used to describe the volume of a substance that can be held within a container. Units for volume and capacity are often interchangeable.

The main units of capacity are millilitres (mL) and litres (L). You may also use the larger unit kilolitres (kL).

		
1 cm^3 = 1 mL	1000 cm^3 = 1 L, where 1 L = 1000 mL	1 m^3 = 1 kL, where 1 kL = 1000 L

There are some simple rules for converting between units of capacity:



Worked example

Converting units of volume and capacity

Convert:

(a) 3.5 L to mL

THINKING	WORKING
Determine whether the conversion will result in a bigger number or a smaller number.	1 L = 1000 mL The number of millilitres will be greater than the number of litres.
To convert from a larger unit to a smaller unit, multiply by the conversion factor.	3.5 L = 3.5 × 1000 mL = 3500 mL
Write the answer.	3.5 L = 3500 mL

(b) 495 cm^3 to L.

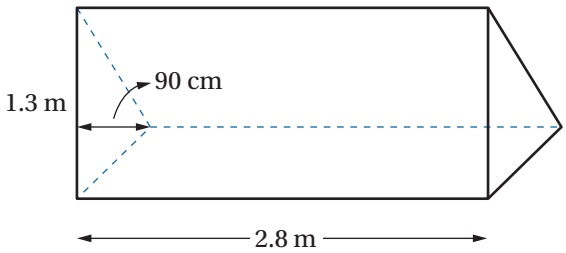
THINKING	WORKING
Identify the equivalent units of volume and capacity.	1 cm^3 = 1 mL
Convert to the equivalent capacity unit.	495 cm^3 = 495 mL
Determine whether the conversion will result in a bigger number or a smaller number.	1000 mL = 1 L The number of litres will be smaller than the number of millilitres.
To convert from a smaller unit to a larger unit, divide by the conversion factor.	$495\,\text{mL} = \frac{495}{1000}\,\text{L}$ $= 0.495\,\text{L}$
Write the answer.	495 cm^3 = 0.495 L

Worked example

Calculating capacity given mixed units

A triangular prism has the interior dimensions shown.

Calculate the capacity of the prism in litres.



THINKING	WORKING
Write all the dimensions in centimetres.	Triangle: $b = 130$ cm, $h = 90$ cm Prism: $H = 280$ cm
Recall the formula for the volume of a prism.	$V = AH$
Determine the area of the triangular cross-section.	$A = \frac{bh}{2}$ $= \frac{130 \times 90}{2}$ $= 5850 \text{ cm}^2$
Substitute the area and height to calculate the volume.	$V = AH$ $= 5850 \times 280$ $= 1\,638\,000 \text{ cm}^3$
Recall the conversions.	$1 \text{ cm}^3 = 1 \text{ mL}$ $1000 \text{ mL} = 1 \text{ L}$
Convert volume units to the equivalent capacity units.	$1\,638\,000 \text{ cm}^3 = 1\,638\,000 \text{ mL}$
Convert from millilitres to litres.	$1\,638\,000 \text{ mL} = 1638 \text{ L}$
Write the answer.	The volume of the triangular prism is 1638 L.

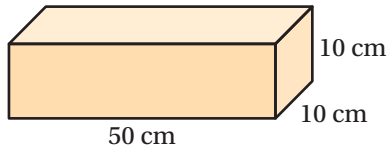
SC 3 I can solve a variety of practical problems involving the volumes and capacities of prisms

Worked example

Applying volume and capacity to solve practical problems

A fish tank has the interior dimensions shown.

Calculate the number of 375 mL jugs of water needed to fill the fish tank.



THINKING	WORKING
Identify the given information.	$l = 50$ cm, $w = 10$ cm, $h = 10$ cm
Calculate the volume of the tank in cubic units.	$V = lwh$ $= 50 \times 10 \times 10$ $= 5000$ cm^3
Convert to capacity units.	5000 cm^3 = 5000 mL
Divide to determine the number of jugs needed.	Number of jugs: $\frac{5000\text{ mL}}{375\text{ mL}} = 13.3\ldots$
Write the answer.	It will take between 13 and 14 jugs to fill the fish tank.

Practice

ANSWERS Page 589

SC 1 I can convert between metric units of volume

1 Describe how to convert from:

(a) mm^3 to cm^3 **(b)** m^3 to cm^3 **(c)** m^3 to km^3

2 Convert the following volumes.

(a) 7 m^3 to cm^3 **(b)** 20 000 cm^3 to m^3 **(c)** 34 000 mm^3 to cm^3

(d) 25 m^3 to mm^3 **(e)** 56.7 cm^3 to m^3 **(f)** 0.03 m^3 to cm^3

(g) 4.01 cm^3 to mm^3 **(h)** 1034.8 mm^3 to m^3

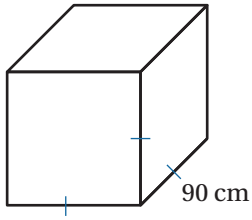
3 Determine which volume is greater in each pair.

(a) 56 700 cm^3 or 56.7 m^3 **(b)** 0.027 m^3 or 27 cm^3

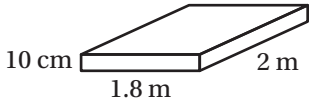
(c) 6 cm^3 or 60 mm^3 **(d)** 1 000 000 mm^{3}or 10 m^3

4 For each prism, calculate the volume in each of the units required.

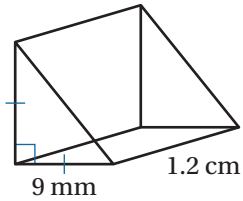
(a) cm^3 and m^3



(b) cm^3 and m^3



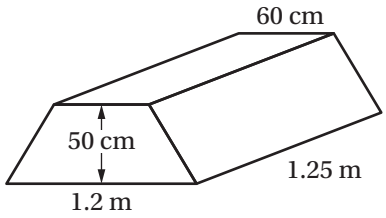
(c) mm^3 and cm^3



5 Calculate the volume of this trapezium-based prism.

(a) Give the volume in cm^3 and m^3.

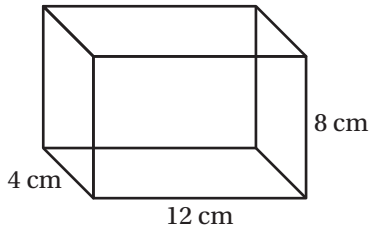
(b) Increase each dimension by a factor of 10 and calculate the new volume in both cm^3 and m^3.



6 Calculate the volume of this rectangular prism.

(a) Give the volume in cm^3.

(b) If the dimensions are doubled, does the volume also double? Explain.

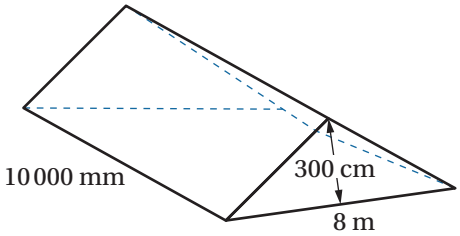


7 Calculate the volume of this triangular prism in the units required.

(a) cm^3

(b) m^3

(c) km^3



SC 2 I can calculate volume to determine the capacity of prism-shaped containers

1 Convert:

(a) 7200 mL to L **(b)** 0.85 L to mL

(c) 15 kL to L **(d)** 400 kL to ML (where 1 ML = 1 000 000 L)

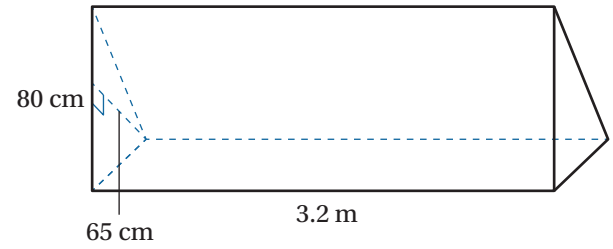
(e) 0.052 kL to mL **(f)** 750 L to kL

2 Convert:

(a) 85 cm^3 to mL **(b)** 150 mL to cm^3 **(c)** 6.7 m^3 to kL

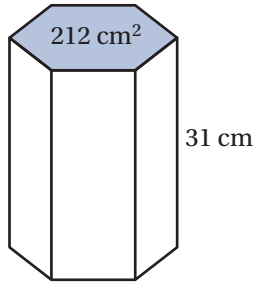
(d) 0.096 kL to m^3 **(e)** 800 cm^3 to L **(f)** 5.2 L to cm^3

3 A triangular prism has the interior dimensions shown.
Calculate the capacity of the prism in litres.

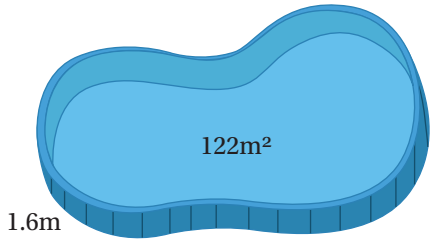


SC 3 I can solve a variety of practical problems involving the volumes and capacities of prisms

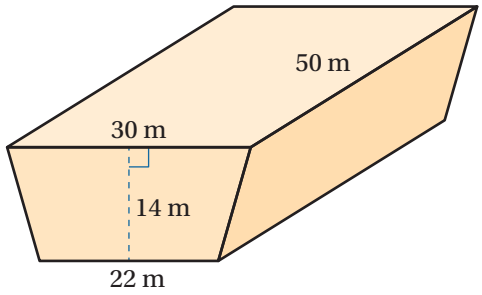
1 A hexagonal prism has the interior dimensions shown.
Calculate the capacity of the prism in litres, correct to 1 decimal place.



2 Calculate the capacity of this swimming pool, giving your answer in kilolitres.



3 Calculate the capacity of this tank, giving your answer in megalitres.



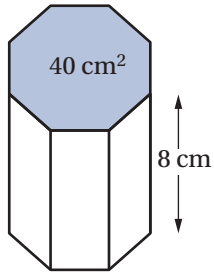
4 Three prism-shaped containers each have a capacity of 5 L.
Use the dimensions given below to calculate the height of each container.
Give your answers correct to 1 decimal place.

(a) The base is a rectangle: $l = 25$ cm and $w = 33$ cm.

(b) The base is a triangle: $b = 30$ cm and $h = 22$ cm.

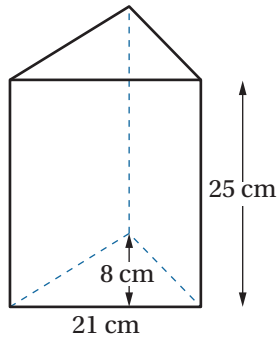
(c) The base is a polygon with area 300 cm^2.

5 A spice container has the dimensions shown. Calculate how many 5 mL teaspoons can be filled with spice.

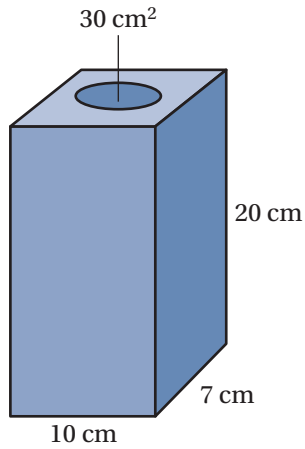


6 A cube of aluminium of side 4 cm has a mass of 172.8 g. Calculate the density of aluminium, in g/cm^3.

7 A company makes containers in the shape of triangular prisms. One of these containers is shown.
The company needs to change the dimensions of the container so that the proportions remain the same, but the new container holds 1 L. Calculate the new dimensions of the container, giving your answers to the nearest millimetre.



8 A vase is shaped as a rectangular prism (with dimensions shown). Within the prism, there is a hollow with an oval cross-section, down to a base that is 15 mm thick.



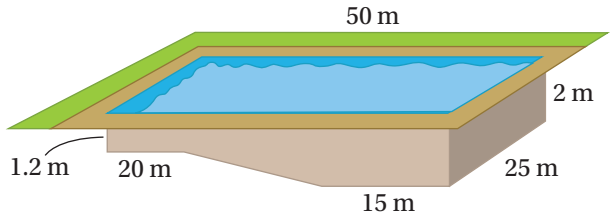
(a) Calculate the volume of glass in the vase.

(b) Determine the capacity of the vase.

(c) Given that the density of glass is 2.7 g/cm^3, determine the mass of the empty vase. Give your answer in kg.

(d) Determine the mass of the filled vase when filled with water up to 2 cm from the top (the density of water is 1 g/mL). Give your answer in kg.

9 A 50 m pool has the interior dimensions shown.



(a) Determine the capacity of the pool.

(b) A hose is used to fill the pool. The hose can fill a 20 L bucket in 8 seconds. The pool begins filling at 7:30 pm on 1 September. When will the pool be full?

(c) The local council charges $4/kL for water. Calculate the cost of the water to fill the pool.

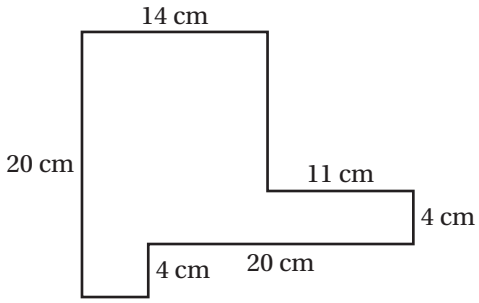
(d) The pool requires a 5 cm top-up 20 times per year. Calculate the annual cost of water to top up the pool.

TOPIC REVIEW

Multiple choice

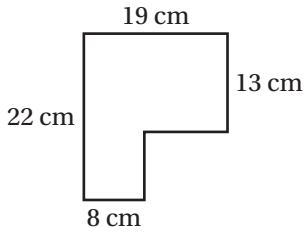
ANSWERS Page 625

1 The perimeter of the shape is:



A 73 cm **B** 85 cm **C** 90 cm **D** 288 cm

2 The perimeter of the shape is:



A 82 cm **B** 75 cm **C** 73 cm **D** 62 cm

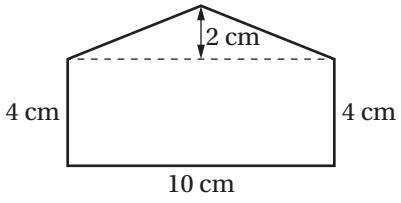
3 A trapezium has an area of 518 cm^2. If its parallel sides are 17 cm and 20 cm long, the height from one side could be approximately:

A 12 cm **B** 24 cm **C** 28 cm **D** 30 cm

4 The formula to calculate the area of a trapezium is:

A $A = lw$ **B** $A = \frac{h}{2}(a+b)$ **C** $A = bh$ **D** $A = \frac{xy}{2}$

5 Choose the correct calculation for area of the composite shape in m^2.



A $2\times 2+10\times 2$ **B** $10\times 4+\frac{1}{2}\times 10\times 2$ **C** $10\times 2+10\times 2$ **D** $10\times 4-\frac{1}{2}\times 10\times 2$

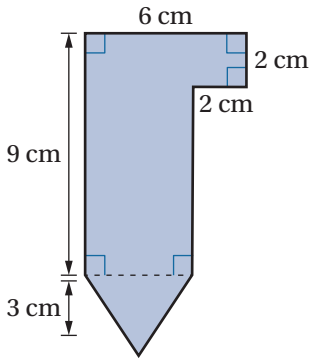
6 Choose the correct calculation for area of the composite shape in cm^2.

A $6\times9+2\times2+\frac{1}{2}\times4\times3$

B $6\times2+9\times4+\frac{1}{2}\times4\times3$

C $2\times2+\frac{1}{2}\times4\times12$

D $4\times9+2\times2+\frac{1}{2}\times4\times3$



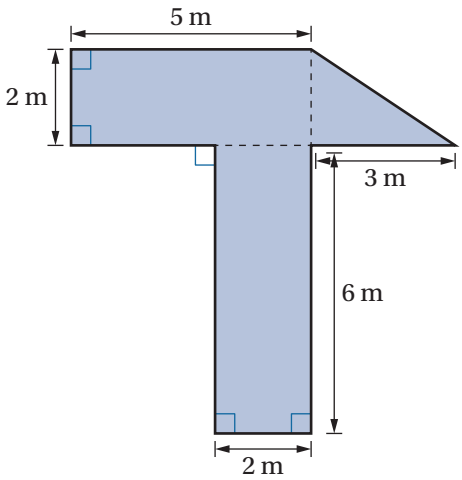
7 Choose the correct calculation for area of the composite shape in m^2.

A $5\times6-3\times6+\frac{1}{2}\times2\times3$

B $2\times6+\frac{1}{2}\times2\times8$

C $2\times8+2\times6+\frac{1}{2}\times2\times3$

D $2\times5+2\times6+\frac{1}{2}\times2\times3$



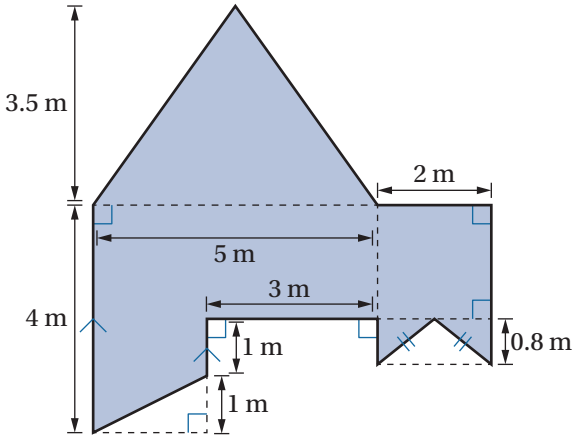
8 Choose the correct calculation for area of the composite shape in m^2.

A $\frac{1}{2}\times5\times3.5+7\times2+4\times2+\frac{1}{2}\times2\times0.8$

B $\frac{1}{2}\times5\times3.5+7\times4-\frac{1}{2}\times2\times1-3\times1-\frac{1}{2}\times2\times0.8$

C $\frac{1}{2}\times5\times3.5+7\times2+2\times2+2\times\left(\frac{1}{2}\times2\times0.8\right)$

D $\frac{1}{2}\times5\times3.5+7\times2+2\times2-\frac{1}{2}\times2\times1+\frac{1}{2}\times2\times0.8$



9 Which of the following is the same as 85 cm?

A 850 m **B** 8.5 km **C** 8.5 mm **D** 850 mm

10 Choose the equivalent area for 35 500 mm^2.

A 35.5 m^2 **B** 0.355 m^2 **C** 0.0355 m^2 **D** 0.00355 m^2

11 Choose the equivalent area for 200 mm^2.

A 2 m^2 **B** 20 cm^2 **C** 200 cm^2 **D** 2 cm^2

12 Choose the equivalent area for 7.85 m^2.

A 785 cm^2 **B** 78 500 cm^2 **C** 7850 mm^2 **D** 0.007 85 mm^2

13 Choose the equivalent volume or capacity for 5.2 L.

A 520 cm^3 **B** 5200 cm^3 **C** 0.52 cm^3 **D** 0.0052 cm^3

14 Choose the equivalent volume or capacity for 74 400 cm^3.

A 7.44 m^3 **B** 0.007 44 m^3 **C** 744 m^3 **D** 74.4 L

15 Choose the equivalent volume or capacity for 1 m^3.

A 1 cm^3 **B** 100 cm^3 **C** 10 000 cm^3 **D** 1 000 000 cm^3

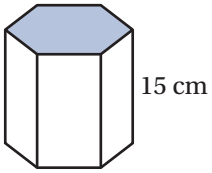
16 Choose the equivalent volume or capacity for 95 500 mm^3.

A 0.955 m^3 **B** 95.5 cm^3 **C** 9.55 cm^3 **D** 955 cm^3

17 The volume of a rectangular prism with height 0.6 m, width 0.5 m and length 80 cm is:

A 24 m^3 **B** 2.4 m^3 **C** 0.24 m^3 **D** 240 000 m^3

18 The volume of the hexagonal prism is 500 cm^3. The area of cross-section is closest to:

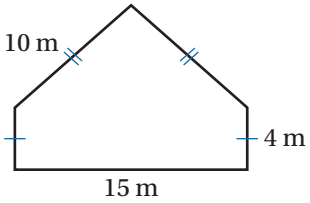


A 2 cm^2 **B** 17 cm^2 **C** 20 cm^2 **D** 33 cm^2

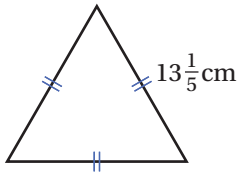
Short answer

1 Calculate the perimeter of each shape.

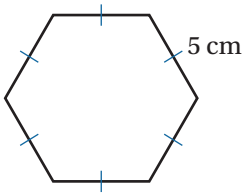
(a)



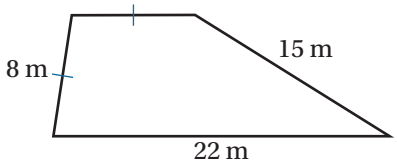
(b)



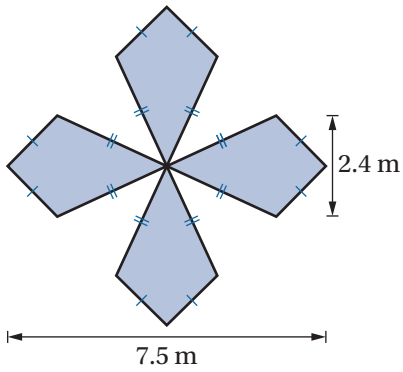
(c)



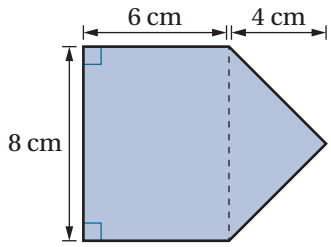
(d)



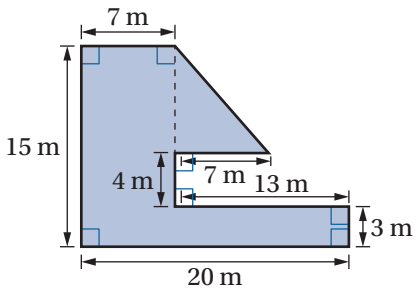
2 Calculate the total shaded area of the shape.



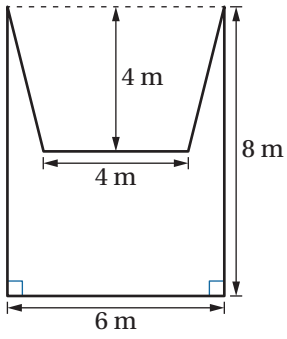
3 Calculate the area of the composite shape.



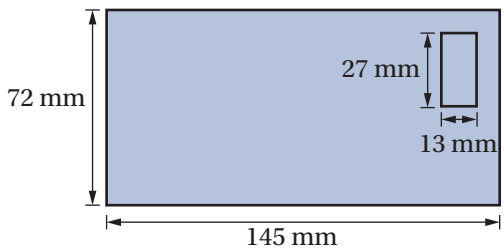
4 Determine the total area shaded.



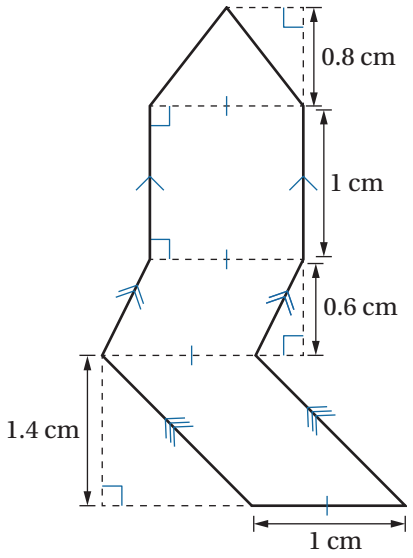
5 Determine the area of the composite shape.



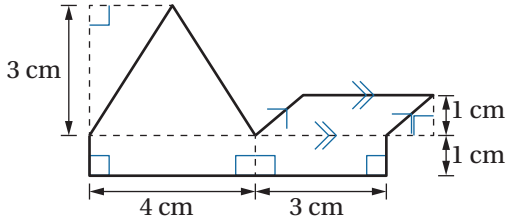
6 An artist is designing a new phone case, which must be a rectangle with a smaller rectangular hole cut out for the camera lens. Calculate the area of material needed for the phone case shown.



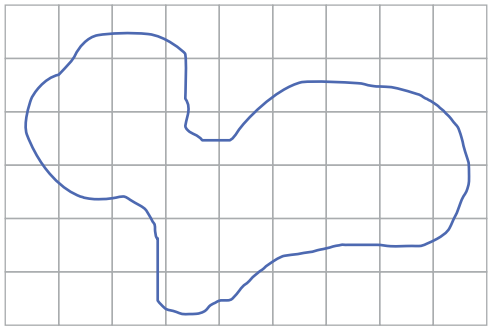
7 Determine the area of this composite shape.



8 Calculate the area of the composite shape.

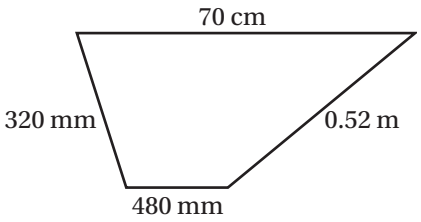


9 Estimate the perimeter and area of the shape on the 1 cm grid to the nearest whole number.

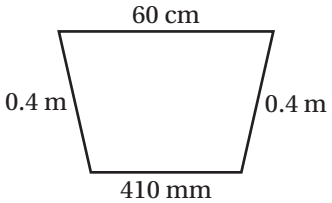


10 Determine the perimeter of each of the following shapes.

(a)



(b)

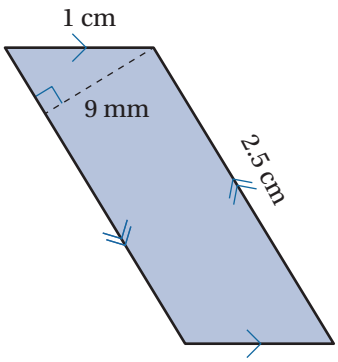


11 Perform each of the following area conversions.

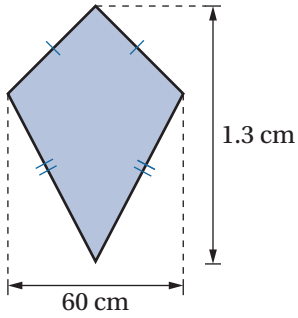
(a) 6.5 m^2 to mm^2 **(b)** 0.25 ha to m^2 **(c)** 10 cm^2 to mm^2 **(d)** 0.5 km^2 to ha

12 Determine the area of each of the following shapes in cm^2.

(a)

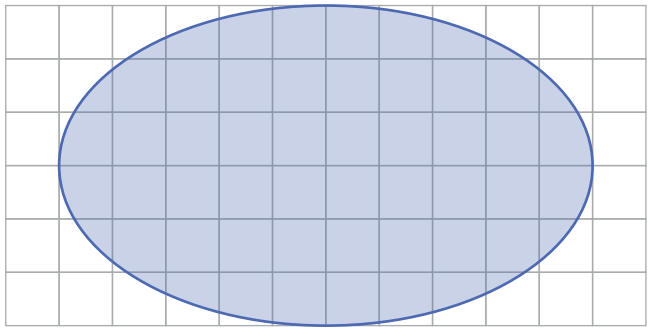


(b)



Extended response

1 An ellipse is drawn on a 1 cm grid.

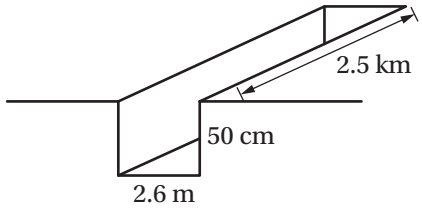


(a) Estimate the area of the ellipse to the nearest whole cm^2.

(b) If the ellipse represents the surface of a basin of depth 29 cm and approximately 10 cm wide, calculate the capacity of the basin.

(c) If the grid squares represented a 1 m grid and the ellipse represents the surface of a pond of average depth 2.9 m, calculate the capacity of the pond.

2 A trench is dug along a silted-up creek bed in the suburbs.



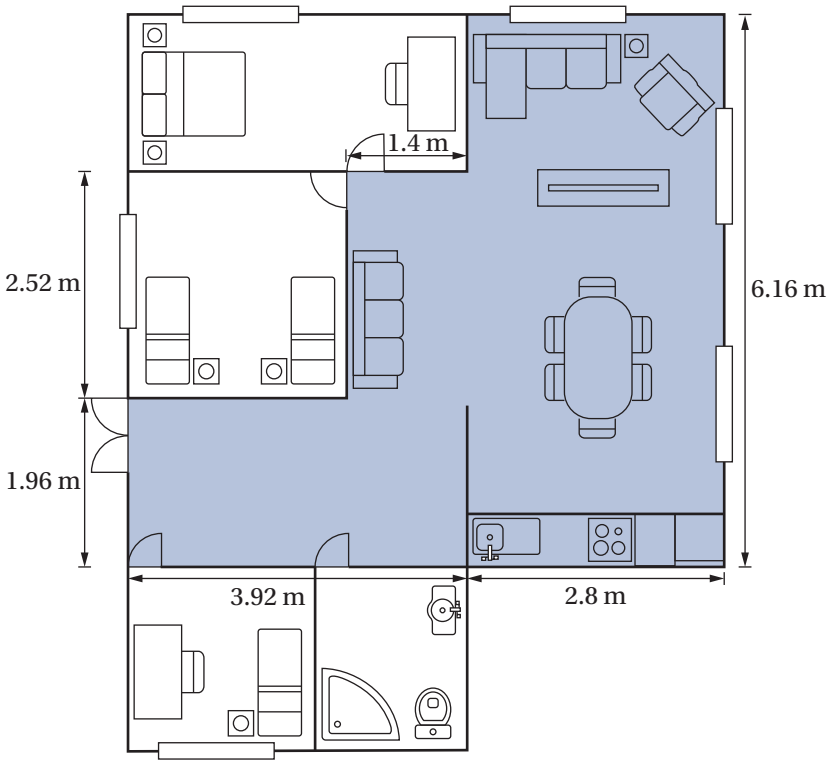
(a) What volume of soil is removed?

(b) If the trench is to be lined with concrete 10 cm thick on all three sides, what volume of concrete is needed.

(c) If a concrete truck can carry 16 m^3, how many truck loads are needed to complete the task?

(d) After heavy rain, how many kilolitres of water could there be within the drain?

3 Sophia wants new carpet for some rooms in her house.



(a) Calculate the area of carpet needed for the rooms highlighted on the floorplan.

(b) Calculate the cost of the carpet at \$45/m^2, where area is rounded up to the nearest m^2.

(c) Rolls of carpet are usually 3.66 m wide. Calculate the lineal amount of carpet needed from the roll. Give an answer for each of the two possible directions.

(d) Is there any benefit in considering a more expensive carpet of width 4 m? Explain your answer.

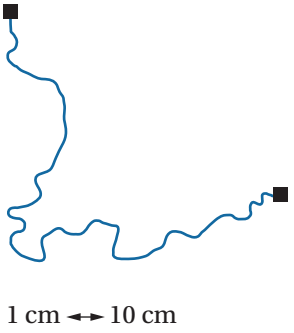
4 Hobart's Seven Mile Beach is about 11 km long and 20 m wide.

(a) What is the area of the beach?

(b) On a king tide the water covers the entire beach. If the average depth of water 20 m from the edge is 1.6 m, estimate the volume of water that is above the 11 km strip.

(c) Salty water is about 3.5% salt, so 3.5 g/mL or 3.5 kg/kL. How much salt could be harvested from above the 11 km strip?

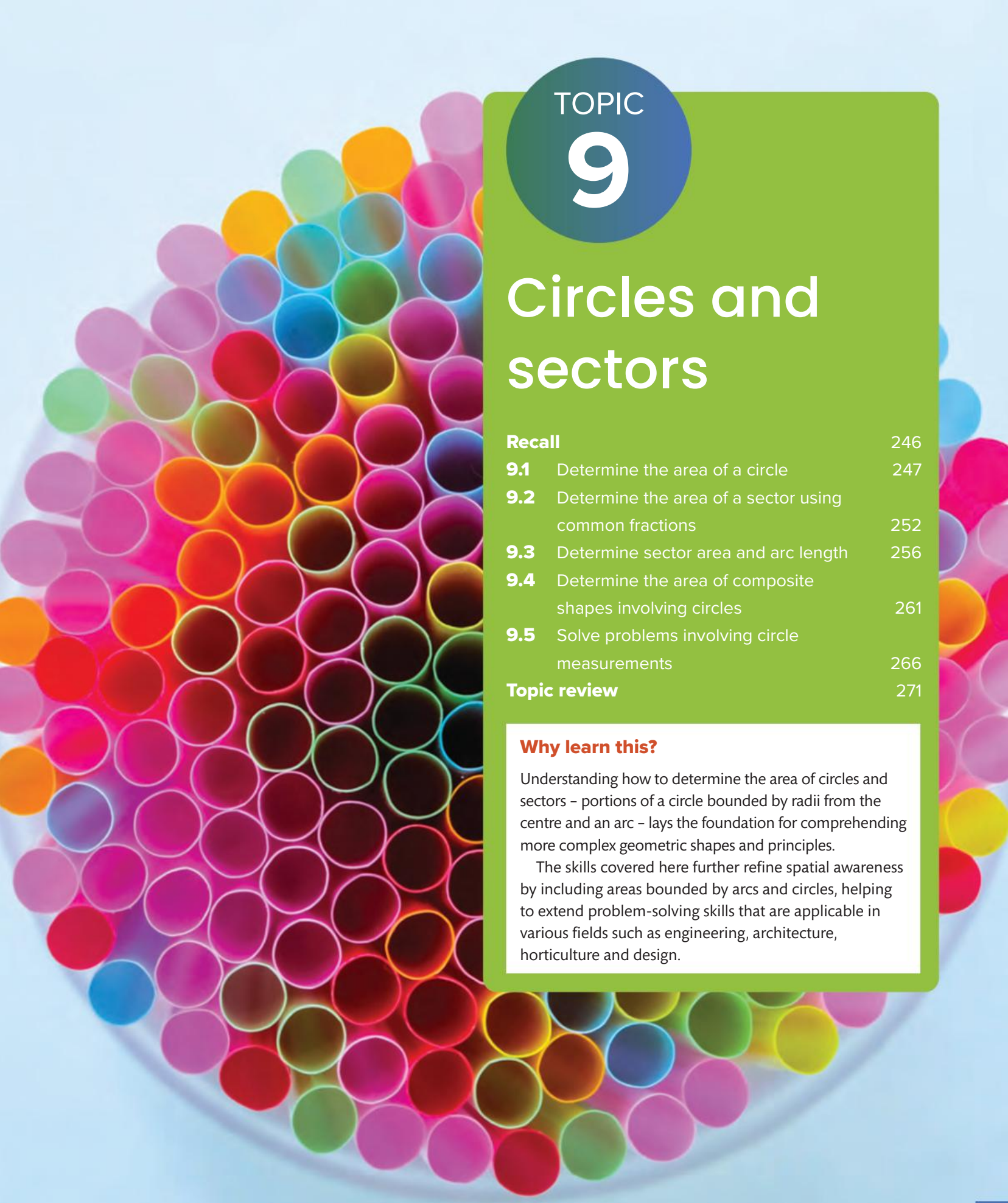
5 The road between two country towns is to be resurfaced.



(a) Estimate the distance along the road between the two towns.

(b) The road is to be 20 m wide within 10 km of each town and 25 m wide for 3 km for passing lanes. Five sections of passing lanes is anticipated. Calculate the area of road to be surfaced if other parts of the road are only 10 m wide.

(c) What volume of bitumen is needed if the surface is to be 8 cm thick?



TOPIC 9

Circles and sectors

Recall 246

9.1 Determine the area of a circle 247

9.2 Determine the area of a sector using common fractions 252

9.3 Determine sector area and arc length 256

9.4 Determine the area of composite shapes involving circles 261

9.5 Solve problems involving circle measurements 266

Topic review 271

Why learn this?

Understanding how to determine the area of circles and sectors – portions of a circle bounded by radii from the centre and an arc – lays the foundation for comprehending more complex geometric shapes and principles.

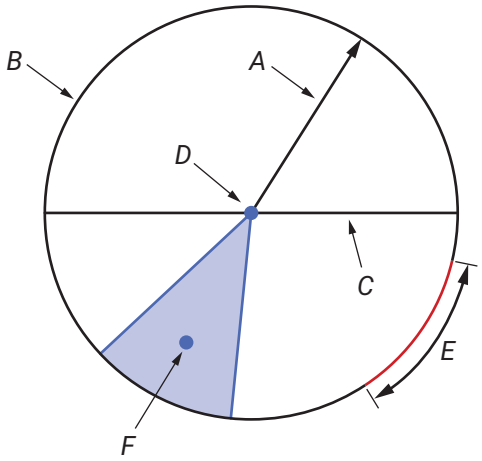
The skills covered here further refine spatial awareness by including areas bounded by arcs and circles, helping to extend problem-solving skills that are applicable in various fields such as engineering, architecture, horticulture and design.

RECALL

ANSWERS Page 589

I can identify features of a circle

1 Name the circle part represented by each label.



I can identify the size of special angles

1 Write the size of the angle described by the following names.

(a) right angle **(b)** straight angle **(c)** revolution

I can simplify fractions

1 Write the following fractions in simplest form.

(a) $\frac{30}{90}$ **(b)** $\frac{45}{120}$ **(c)** $\frac{52}{180}$ **(d)** $\frac{36}{270}$

I can write angles as a fraction of a revolution

1 Write the following angles as simplified fractions of a circle (360°).

(a) 180° **(b)** 90° **(c)** 270°

(d) 45° **(e)** 120° **(f)** 60°

I can round decimals for different purposes

1 Round each of the following numbers to the number of decimal places stated in the brackets.

(a) 8.3625 (3) **(b)** 5.477 (2) **(c)** 2.193 (2) **(d)** 9.407 (2)

(e) 94.82 (0) **(f)** 9.954 (1) **(g)** 5.981 (2)

I can round calculations to the required number of decimal places

1 Calculate each of the following, correct to 2 decimal places.

(a) 3.14×4 **(b)** $(1.6)^2$ **(c)** $3.14 \times (1.5)^2$ **(d)** $3.142 \times (2.5)^2$

Determine the area of a circle

9.1

Learning intention: To be able to determine the area of a circle

Success criteria:

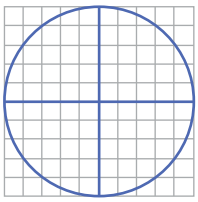
SC 1 I can establish the formula for the area of a circle using approximations.

SC 2 I can calculate the area of a circle.

Lesson warm-up

Area of a circle, using squares

Use a pair of compasses and 1 cm grid paper to draw a circle of radius 4 cm. By counting whole grid squares and approximating partial ones, estimate the area of the circle below, correct to the nearest square centimetre.



SC 1 I can establish the formula for the area of a circle using approximations

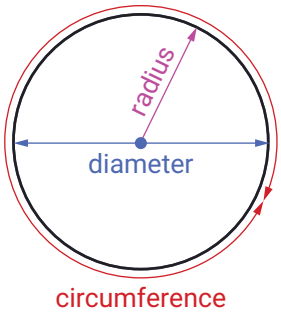
The circumference of a circle of known radius or diameter can be calculated using π (pi).

π is an irrational number, so it is not expressed as an exact decimal, but $\pi \approx 3.14159$.

The diameter is the distance from one edge of the circle to the opposite edge of the circle.

The radius is the distance from the centre to the edge of the circle.

The perimeter of a circle is called the circumference.



The circumference can be calculated using the formula

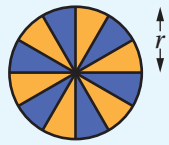
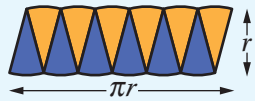
$C = \pi d$ or $C = 2\pi r$.

The area of a circle can also be calculated using π.

Worked example

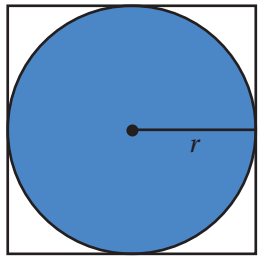
Establishing the formula for the area of a circle from the circumference of a circle

Establish a formula for the area of a circle using the circumference formula $C = 2\pi r$ and a circle divided into 12 equal sectors.

THINKING	WORKING
Divide a circle into twelfths.	
Cut out each segment and place half of them with curves placed along a line, and the other half inverted in the gaps.	
Describe the approximate shape.	The shape approximates a parallelogram.
Determine the dimensions, in terms of the radius, using the wavy length as the base. Note: the more sectors used, the more the shape becomes like a rectangle.	The base is the length of half the circumference. $b = \frac{2\pi r}{2}$ $= \pi r$ The height is the radius of the circle. $h = r$
Determine the area in terms of the radius.	$A = bh$ $= \pi r \times r$ $= \pi r^2$
Write the answer.	The area of a circle is $A = \pi r^2$.

SC 2 I can calculate the area of a circle

The formula for the area of a circle is $A = \pi r^2$.



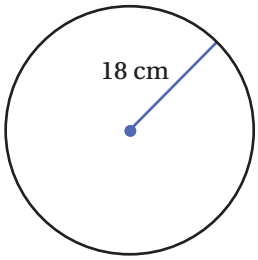
Since π is slightly more than 3, this means that a circle takes up a little more than $\frac{3}{4}$ of the area of the square that contains it; $\frac{\pi}{4}$ of the area of the square, to be exact.

Worked example

Calculating the area of a circle

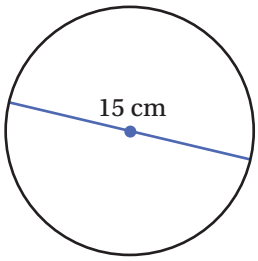
Calculate the area of each circle, correct to 2 decimal places.

(a)



THINKING	WORKING
Identify the radius.	Radius: $r = 18$ cm
Recall the formula for the area of a circle.	$A = \pi r^2$
Substitute the value of r into the formula and calculate the answer.	$A = \pi \times 18^2$ $= 1017.876 \ldots$
Write the area using squared units.	The area is 1017.88 cm^2 (2 d.p.).

(b)



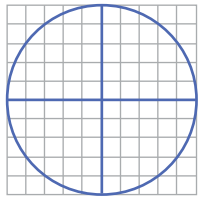
THINKING	WORKING
Identify the diameter.	Diameter: $d = 15$ cm
Determine the length of the radius.	Radius: $r = \frac{15}{2}$ $= 7.5$ cm
Recall the formula for the area of a circle.	$A = \pi r^2$
Substitute the value of r into the formula and calculate the answer.	$A = \pi \times 7.5^2$ $= 176.714 \ldots$
Write the area using squared units.	The area is 176.71 cm^2 (2 d.p.).

Practice

ANSWERS Page 590

SC 1 I can establish the formula for the area of a circle using approximations

1 A circle is drawn on a grid of unit squares.



(a) Calculate the area of the entire grid in square units.

(b) Calculate the area of one-quarter of the grid.

(c) By counting whole grid squares and approximating with partial ones, estimate the area of the circle.

(d) Describe how the area of the circle compares with the entire grid.

2 Establish a formula for the area of a circle using the circumference formula $C = 2\pi r$ and a circle divided into eight equal sectors. Cut up and paste the sectors into your exercise book to accompany your work.

3 A circle with a circumference of 30 cm is divided into 10 equal sectors.

(a) What is the length of the arc of each sector?

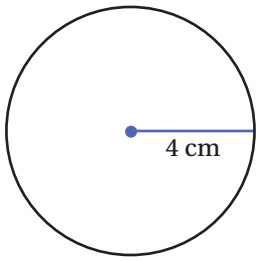
(b) What is the length of the radius, correct to 2 decimal places?

(c) The sectors are arranged to form a parallelogram. Determine the area of the parallelogram, correct to 2 decimal places.

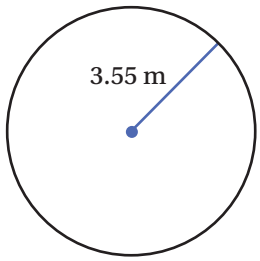
SC 2 I can calculate the area of a circle

1 Calculate the area of each circle, correct to 2 decimal places.

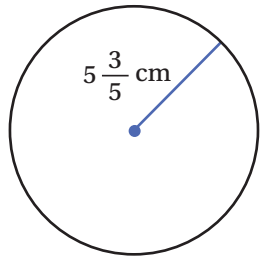
(a)



(b)

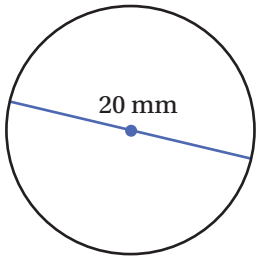


(c)

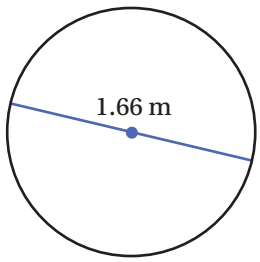


2 Calculate the area of each circle, correct to 2 decimal places.

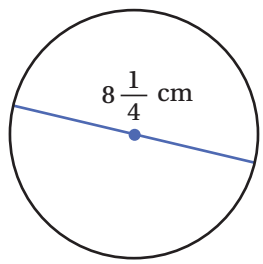
(a)



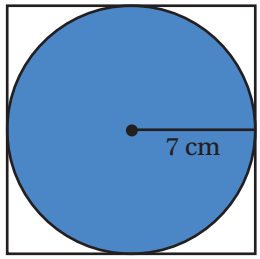
(b)



(c)



3 A circle of radius 7 cm is contained within a square. Answer the following questions, rounding answers to the nearest whole number, where necessary.



(a) Determine each of the following.

(i) The perimeter of the square

(ii) The circumference of the circle

(iii) The area of the square

(iv) The area of the circle

(b) The diagram is now doubled in size so that the radius is 14 cm. Determine each of the following for the new diagram.

(i) The perimeter of the square **(ii)** The circumference of the circle

(iii) The area of the square **(iv)** The area of the circle

(c) Describe what happened to both perimeters after the radius was doubled.

(d) Describe what happened to both areas after the radius was doubled.

4 An area in the shape of a circle, diameter 5 m, is to be covered with mosaic tiles.

(a) Calculate the area, rounded up to the nearest m^2.

(b) Determine the cost of tiling the area, given that tiles cost \$85/m^2.

(c) The border area is to be increased by 10 cm all the way around. What will be the increase in the cost? Round up the new area to the nearest m^2 first.

5 Three circles have diameters of 6 cm, 8 cm and 10 cm respectively.

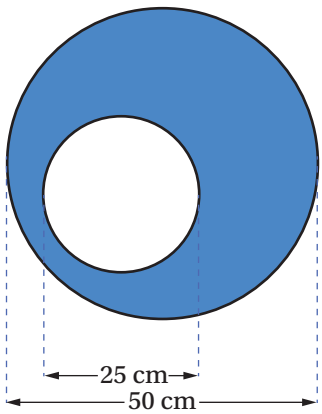
(a) Calculate their circumferences to the nearest cm.

(b) Compare the size of the circumferences.

(c) Calculate the areas of the circles to the nearest cm^2.

(d) Is there a similar increase in area from one circle to the next? Explain.

6 A circle of diameter 25 cm is drawn inside a circle of diameter 50 cm.



(a) Calculate the shaded area, to the nearest cm^2.

(b) Is the shaded area the same as the area of the smaller circle? Explain.

9.2 Determine the area of a sector using common fractions

Learning intention: To be able to determine the area of a sector using common fractions

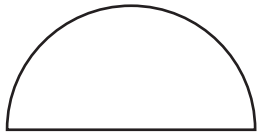
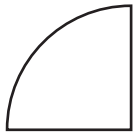
Success criteria:

SC 1 I can determine the area of a sector using common fractions.

Lesson warm-up

Parts of a circle

Copy and complete the table below showing the equivalent sector drawing, its central angle and fraction of a whole circle. The first one has been completed for you.

Sector	Angle	Fraction of a whole circle
	180°	$\frac{180}{360}=\frac{1}{2}$
		
	60°	
		$\frac{3}{4}$
	260°	

SC 1 I can determine the area of a sector using common fractions

The area of any sector of a circle is a fraction of the area of a circle: $A = \pi r^2$.

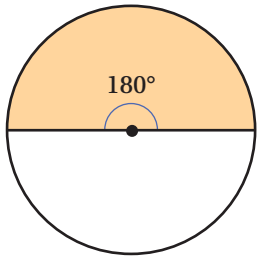
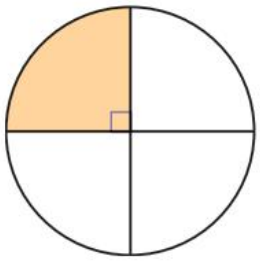
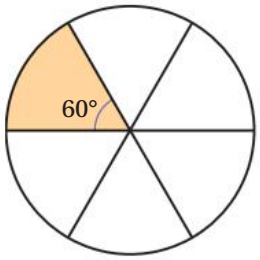
Diagram	Properties
	Name: Semicircle Fraction of circle: $\frac{180°}{360°}=\frac{1}{2}$ Area formula: $A=\pi r^2 \times \frac{1}{2}=\frac{\pi r^2}{2}$

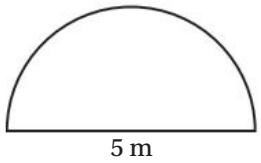
Diagram	Properties
	Name: Quarter circle Fraction of circle: $\frac{90°}{360°}=\frac{1}{4}$ Area formula: $A=\pi r^2\times\frac{1}{4}=\frac{\pi r^2}{4}$
	Description: Sector with 60° angle at the centre Fraction of circle: $\frac{60°}{360°}=\frac{1}{6}$ Area formula: $A=\pi r^2\times\frac{1}{6}=\frac{\pi r^2}{6}$

Worked example

Determining the sector area using common fractions

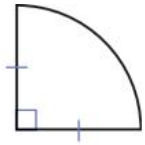
Determine the area of the sector below. Round your answer correct to 1 decimal place.

(a)



THINKING	WORKING
Determine the length of the radius.	$r=\frac{5}{2}$ $=2.5\,\text{m}$
Recognise the fraction of a circle.	Semicircle: $\frac{1}{2}$ of a circle
Recall the formula for the area of a circle.	$A=\pi r^2$
Write a formula. Substitute the radius value. Calculate the answer.	$A=\pi r^2\times\frac{1}{2}$ $=\frac{\pi r^2}{2}$ $=\frac{\pi\times2.5^2}{2}$ $=9.817...$
Write the answer.	The area of the sector is 9.8 m^2 (1 d.p.).

(b)

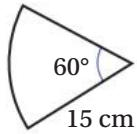


THINKING	WORKING
Identify the length of the radius.	$r = 5$ cm
Recognise the fraction of a circle.	Quarter circle: $\frac{1}{4}$ of a circle
Recall the formula for the area of a circle.	$A = \pi r^2$
Write a formula. Substitute the radius value. Calculate the answer.	$A = \frac{\pi r^2}{4}$ $= \frac{\pi \times 5^2}{4}$ $= 19.6349...$
Write the answer.	The area of the sector is 19.6 cm^2 (1 d.p.).

Worked example

Determining the sector area using an angle

Calculate the area of the sector below. Round your answer correct to 1 decimal place.



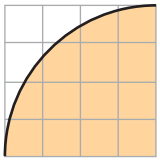
THINKING	WORKING
Identify the length of the radius.	$r = 15$ cm
Determine the fraction of a circle.	$\frac{60°}{360°} = \frac{1}{6}$ of a circle
Recall the area of a circle.	$A = \pi r^2$
Write a formula. Substitute the radius value. Calculate the answer.	$A = \frac{\pi r^2}{6}$ $= \frac{\pi \times 15^2}{6}$ $= 117.809...$
Write the answer.	The area of the sector is 117.8 cm^2 (1 d.p.).

Practice

ANSWERS Page 590

SC 1 I can determine the area of a sector using common fractions

1 A quarter circle is drawn on a grid of unit squares.

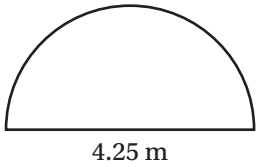


(a) By counting whole shaded squares and approximating with the partly shaded squares, estimate the area of the quarter circle to the nearest square unit.

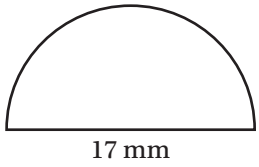
(b) Calculate the area as a quarter of the area of the circle using the formula. Round your answer correct to 1 decimal place.

2 Calculate the area of each sector below. Round your answers correct to 1 decimal place.

(a)



(b)



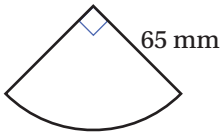
3 Calculate the area of a semicircle (half circle) correct to 2 decimal places, given the semicircle has a diameter of:

(a) 1 cm **(b)** 2 cm **(c)** 5 cm **(d)** 10 cm **(e)** 100 cm

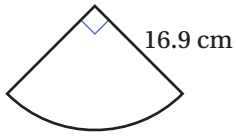
4 Explain how to calculate the area of a semicircle.

5 Calculate the area of each sector below using a formula. Round your answers correct to 1 decimal place.

(a)



(b)



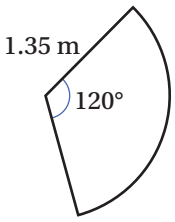
6 Calculate the area of a quarter circle correct to 2 decimal places, given the circle has a radius of:

(a) 1 cm **(b)** 2 cm **(c)** 5 cm **(d)** 10 cm **(e)** 100 cm

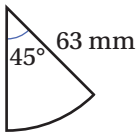
7 Explain how to calculate the area of a quarter circle.

8 Calculate the area of each sector below. Round your answers correct to 1 decimal place.

(a)



(b)



9 Calculate the area of the sectors taken from a circle with a radius of 5 cm. Write your answer correct to 2 decimal places.

(a) 36° **(b)** 45° **(c)** 60° **(d)** 180° **(e)** 270°

10 Explain how to calculate the area of a sector using an angle.

9.3 Determine sector area and arc length

Learning intention: To be able to determine sector area and arc length

Success criteria:

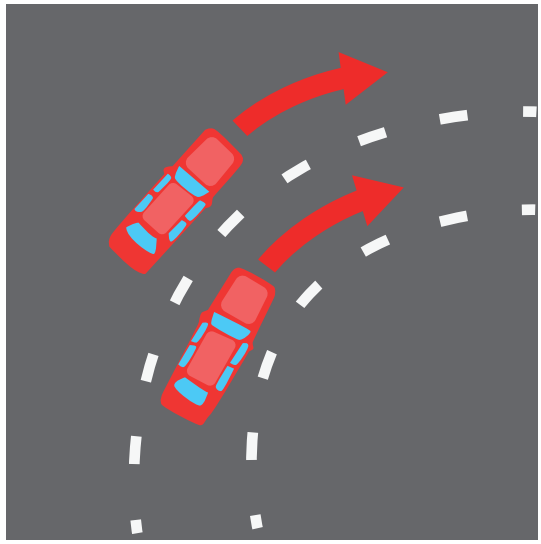
SC 1 I can determine the area of a sector with an angle of any size.

SC 2 I can determine the arc length of a sector.

Lesson warm-up

Arc length and sectors

These two cars are turning at the same time and travelling at the same safe speed, changing direction from going straight to heading right. Explain what will happen during the turn and why.



SC 1 I can determine the area of a sector with an angle of any size

To determine the area of a sector with central angle 90°:

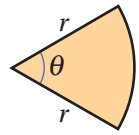
$\frac{90°}{360°} = \frac{1}{4}$, so divide the area of the circle by 4. This is easier than multiplying by $\frac{1}{4}$.

For sectors of circles in which the angle at the centre is not a factor of 360°, it is not possible to divide the area of the circle by a whole number. Instead, the fraction of the circle is multiplied by the area of the circle.

To determine the area of a sector with central angle 55°:

$\frac{55°}{360°} = \frac{11}{72}$, so multiply the area of the circle by $\frac{11}{72}$.

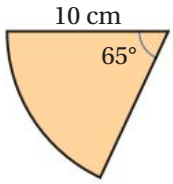
Formula for the area of a sector with radius r and central angle θ.



$$A = \frac{\theta}{360} \times \pi r^2$$

Worked example

Area of a sector formula

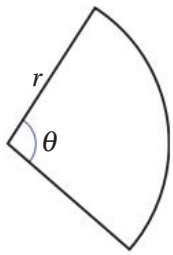


Use the formula to calculate the area of the sector. Round your answer to 1 decimal place.

THINKING	WORKING
List the given information.	$r = 10$ cm, $\theta = 65°$
Recall the formula for the area of a sector.	$A = \frac{\theta}{360} \times \pi r^2$
Substitute the known values into the formula. Calculate the answer.	$A = \frac{65}{360} \times \pi \times (10)^2$ $= 56.7232...$
Write the area, correctly rounded, in square units.	The area is 56.7 cm^2 (1 d.p.).

SC 2 I can determine the arc length of a sector

The boundary, or perimeter, of a sector is made up of two radii and an arc that is a fraction of the circumference of a circle.



The formula for the perimeter of a sector with radius, r and central angle θ is:

$P = l + 2r$, where $l = \frac{\theta}{360} \times 2\pi r = \frac{\pi r \theta}{180}$ is the arc length.

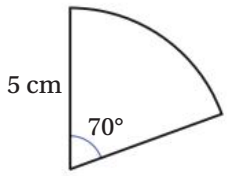
Worked example

Perimeter of a sector

For the sectors below, calculate the following, correct to 1 decimal place.

(i) arc length **(ii)** perimeter.

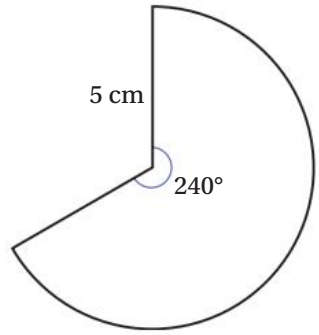
(a)



THINKING	WORKING
(i) List the given information.	Radius: $r = 5$ cm Central angle: $\theta = 70°$
Recall the formula for arc length.	$l = \frac{\theta}{360} \times 2\pi r$

Substitute in the angle and radius values and calculate the arc length.	$l = \frac{70}{360} \times 2 \times \pi \times 5$ $= 6.10\ldots$
Write the arc length, correctly rounded, and include units of length.	The arc length is 6.1 cm (1 d.p.).
(ii) Write the formula for perimeter. Substitute in the length and radius values. Calculate and show units of length.	$P = l + 2r$ $= 6.10\ldots + 2 \times 5$ $= 16.1$ cm (1 d.p.)
Write the perimeter, correctly rounded, and include units of length.	The perimeter is 16.1 cm (1 d.p.).

(b)



THINKING	WORKING
(i) List the given information.	Radius: $r = 5$ cm Central angle: $\theta = 240°$
Recall the formula for arc length.	$l = \frac{\theta}{360} \times 2\pi r$
Substitute in the angle and radius values and calculate the arc length.	$l = \frac{240}{360} \times 2 \times \pi \times 5$ $= 20.94\ldots$
Write the arc length, correctly rounded, and include units of length.	The arc length is 20.9 cm (1 d.p.).
(ii) Write the formula for perimeter. Substitute in the length and radius values. Calculate and show units of length.	$P = l + 2r$ $= 20.94\ldots + 2 \times 5$ $= 30.9$ cm (1 d.p.)
Write the perimeter, correctly rounded, and include units of length.	The perimeter is 30.9 cm (1 d.p.).

Practice

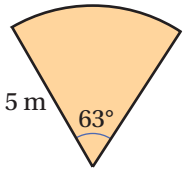
ANSWERS Page 590

Determine sector area and arc length

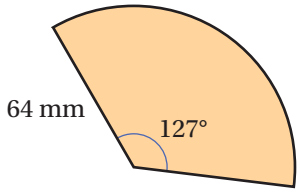
SC 1 I can determine the area of a sector with an angle of any size

1 Calculate the area of each sector below. Round your answers correct to 1 decimal place.

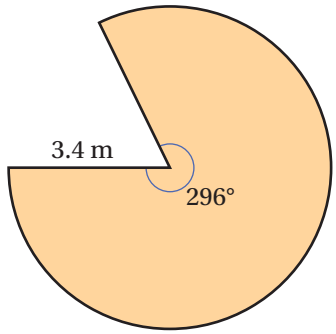
(a)



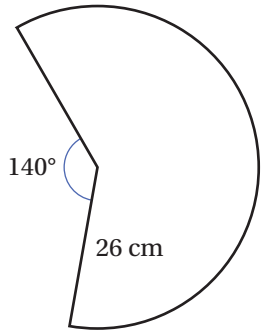
(b)



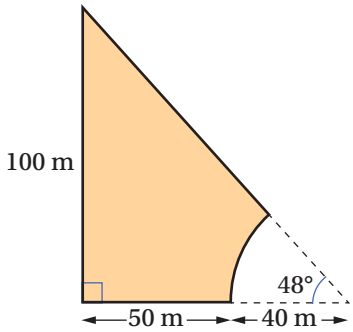
(c)



2 Calculate the area and perimeter of the sector below. Round your answers correct to 1 decimal place.



3 Calculate the area shaded in the shape, correct to 1 decimal place.



4 The hour hand of a clock is 5.5 cm long and the minute hand is 8.5 cm long. Write your answers to the nearest square centimetre.

(a) Calculate the area swept by the minute hand between 8.45 am and 9.05 am.

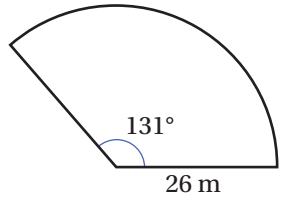
(b) Calculate the area swept by the hour hand between 9.05 am and 2.15 pm.

SC 2 I can determine the arc length of a sector

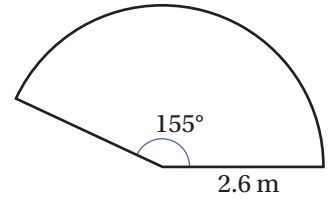
1 For the sectors below, calculate the following, correct to 1 decimal place.

(i) arc length **(ii)** perimeter

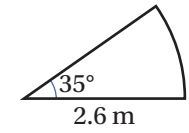
(a)



(b)

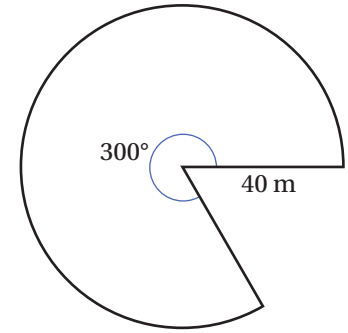


(c)

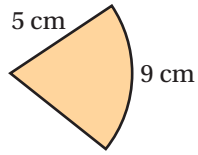


2 For the shape below, calculate the following, to the nearest whole number.

(a) arc length **(b)** perimeter



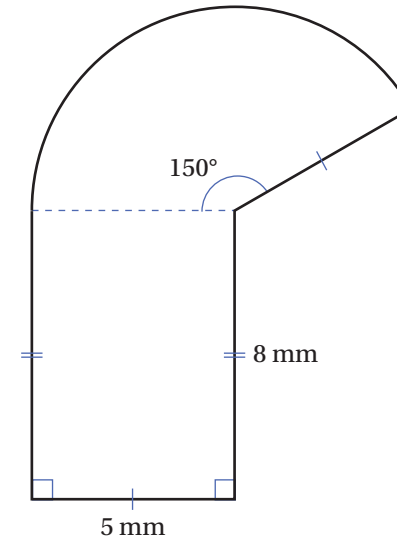
3 For the given sector, calculate:



(a) the central angle, to the nearest degree

(b) the area, correct to 1 decimal place.

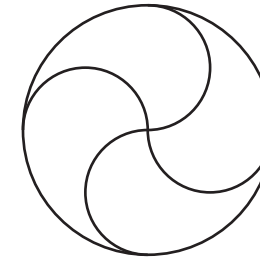
4 Calculate the area and perimeter of the shape below. Round your answers to 1 decimal place.



5 The following circle has a diameter of 10 cm.

(a) Calculate the perimeter of one of the pieces.

(b) Calculate the area of one of the pieces.



Determine the area of composite shapes involving circles

9.4

Learning intention: To be able to determine the area of composite shapes involving circles

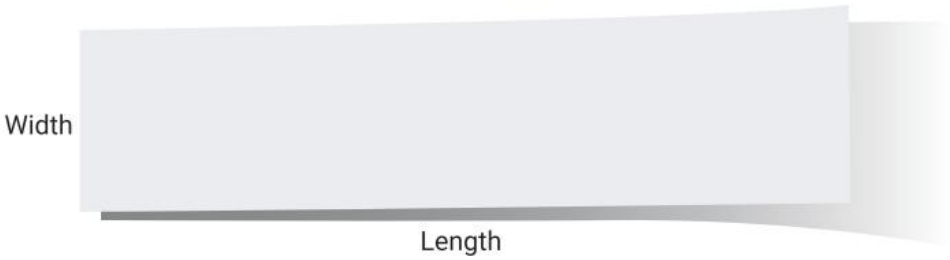
Success criteria:

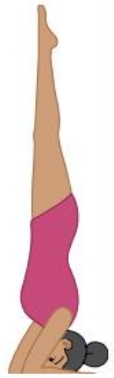
SC 1 I can determine the area of composite shapes involving circles.

Lesson warm-up

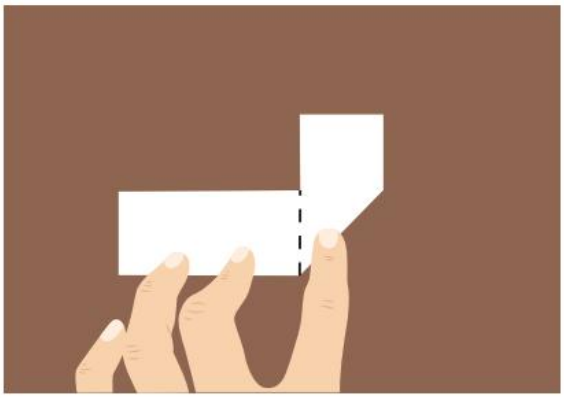
Composite shapes

The area of a rectangular piece of paper can be calculated by multiplying its length by the width.





Fold the rectangular strip of paper as shown.



1. Has the area covered by the paper changed?
2. If so, has it increased or decreased?
3. Describe how the shape has changed.

SC 1 I can determine the area of composite shapes involving circles

Calculating the area of odd shapes involves identifying standard shapes within the shape.

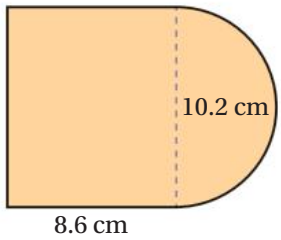
Determining the area of a composite shape involves adding together individual areas, but sometimes a shape removes an area, so subtraction is required as well.

Worked example

Composite areas involving circles

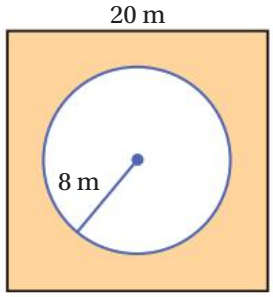
Calculate the area of each shape, correct to 1 decimal place.

(a)



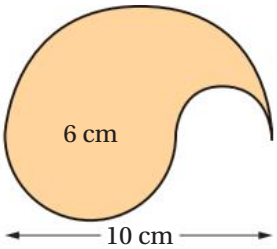
THINKING	WORKING
Describe the shapes to be used in the calculation, including the dimensions.	Rectangle: $l = 8.6$ cm, $w = 10.2$ cm Semicircle: $d = 10.2$ cm, so $r = 5.1$ cm
Write a formula for the calculation.	$A = lw + \frac{\pi r^2}{2}$
Substitute in the values for length, width, and radius. Calculate.	$A = 8.6 \times 10.2 + \frac{\pi \times (5.1)^2}{2}$ $= 128.57...$
Write the area, correctly rounded, in square units.	The area is 128.6 cm^2 (1 d.p.).

(b)



THINKING	WORKING
Describe the shapes to be used in the calculation, including the dimensions.	Square: $l = 20$ m Circle: $r = 8$ m
Write a formula for the calculation.	$A = l^2 - \pi r^2$
Substitute in the values for length and radius. Calculate.	$A = (20)^2 - \pi \times (8)^2$ $= 198.938...$
Write the area, correctly rounded, in square units.	The area is 198.9 m^2 (1 d.p.).

(c)



THINKING	WORKING
Describe the shapes to be used in the calculation, including the dimensions.	There are three semicircles that make up the diagram. Large semicircle: $d_1 = 10$ cm, so $r_1 = 5$ cm Medium semicircle: $d_2 = 6$ cm, so $r_2 = 3$ cm Small semicircle: $d_3 = 4$ cm so, $r_3 = 2$ cm
Write a formula for the calculation.	$A = \frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} - \frac{\pi r_3^2}{2}$
Substitute in the radius values. Calculate.	$A = \frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} - \frac{\pi r_3^2}{2}$ $= \frac{\pi \times (5)^2}{2} + \frac{\pi \times (3)^2}{2} - \frac{\pi \times (2)^2}{2}$ $= 47.12...$
Write the area, correctly rounded, in square units.	The area is 47.1 cm^2 (1 d.p.).

Practice

ANSWERS Page 591

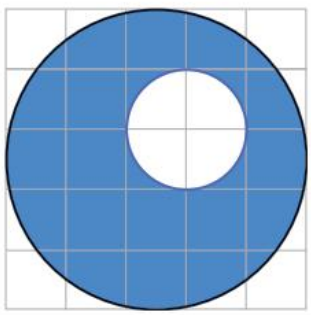
SC 1 I can determine the area of composite shapes involving circles

1 Shapes involving circles have been drawn on grids of unit squares.

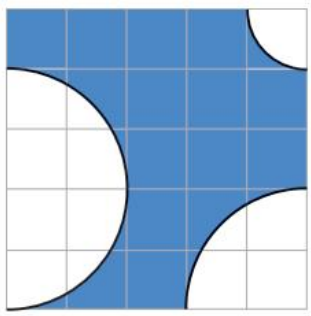
(a) (i) By counting whole shaded squares and approximating with the partly shaded squares, estimate the area of each shape to the nearest square unit.

(ii) Calculate the area of each shape by adding or subtracting individual areas. Round your final answers to 1 decimal place.

(b) (i)

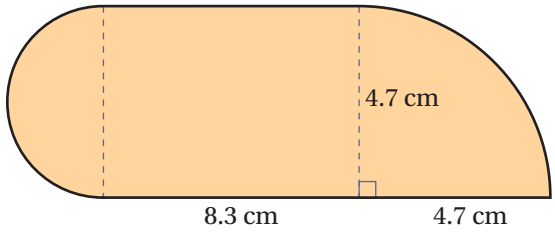


(ii)

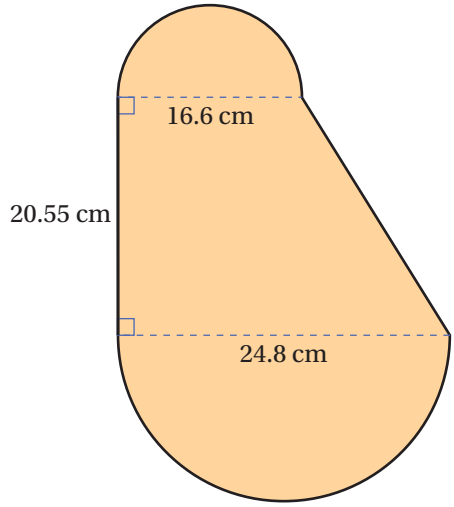


2 Calculate the area of each shape, correct to 1 decimal place.

(a)

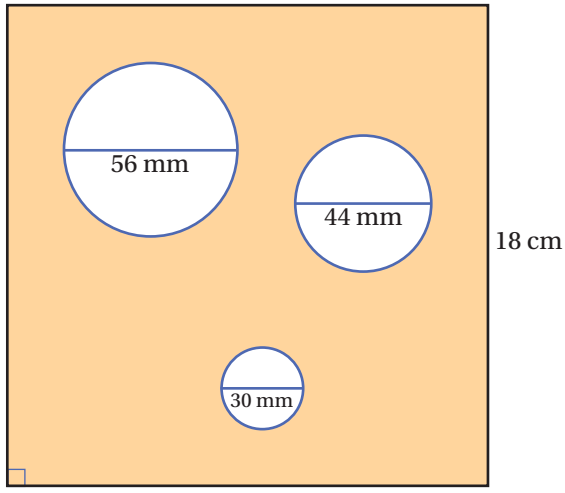


(b)

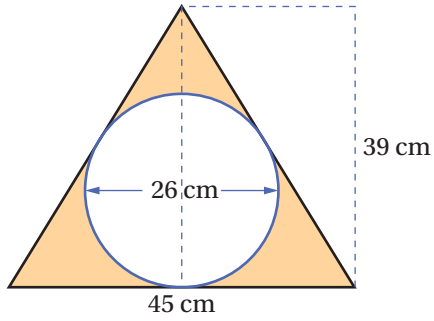


3 Calculate the shaded area of each shape, correct to 1 decimal place.

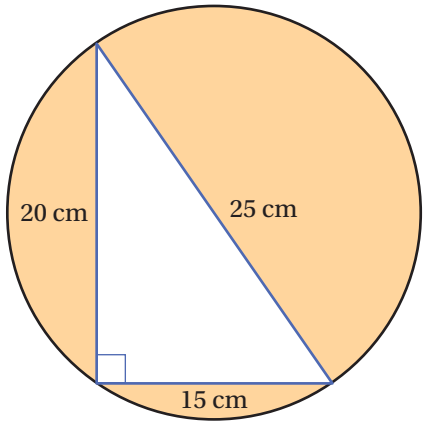
(a)



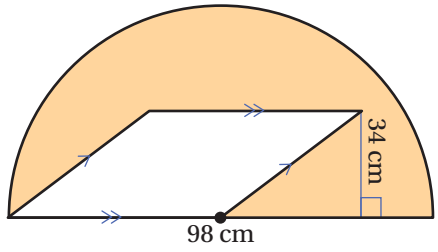
(b)



(c)

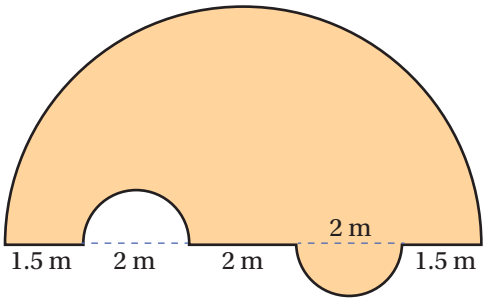


(d)

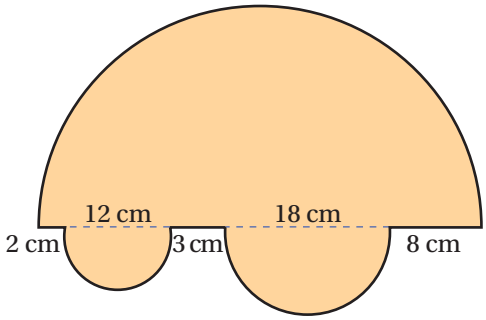


4 Calculate the area of each shape, correct to 1 decimal place.

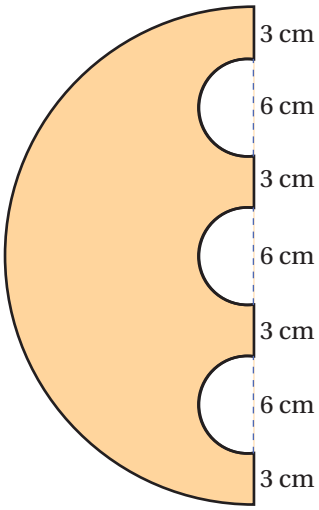
(a)



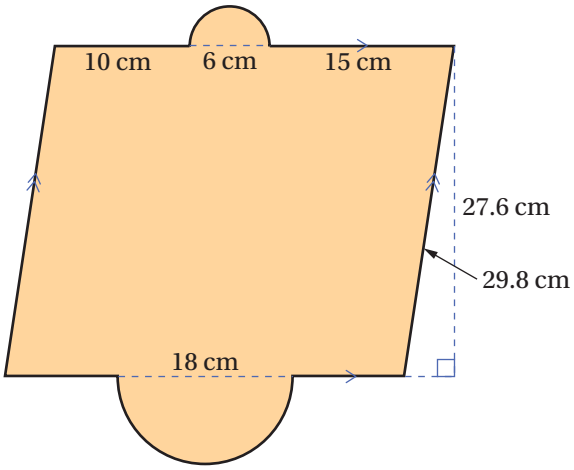
(b)



(c)



5 Calculate the area and perimeter of the shape, correct to 2 decimal places.

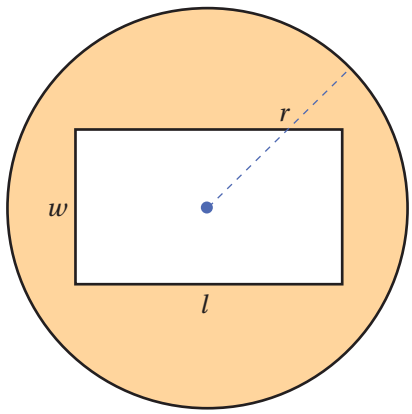


6 A rectangle is removed from a circle, as shown, with the remainder shaded.

The shaded area is between 500 cm^2 and 600 cm^2.

(a) Given that the circle has a radius of 20 cm, determine possible dimensions for the rectangle.

(b) Given that the length of the rectangle is 20 cm, determine possible pairs of values for the width of the rectangle and the radius of the circle.



9.5 Solve problems involving circle measurements

Learning intention: To be able to solve problems involving circle measurements

Success criteria:

SC 1 I can solve practical problems involving circle measurements.

Lesson warm-up

Bicycle wheels and circumference

Bike wheels come in different sizes. The size is measured by the height of the wheel (diameter). Using the wheel shown, find its circumference and then check your calculation by measuring the distance it covers after one full revolution.

Exploration activity

ANSWERS Page 591

The Penny Farthing Bike

One of the early bicycles in England was called the Penny Farthing. This was because the front and back wheels were different sizes and reminded people of two coins – a penny, which was a coin slightly larger than a 20 cent piece, and a farthing, a very small coin, close to the size of a 5 cent coin.

1. The larger wheel is 130 cm in diameter. How far does it travel in one revolution?
2. If the bike was ridden a distance of 1 km, how many turns of the wheel have there been?
3. The rear wheel is only 46 cm in diameter. How many turns has it made during the same 1 km trip?
4. Which tyre will wear out the fastest?
5. The pedals are situated on the large wheel. Why do you think this is this case?

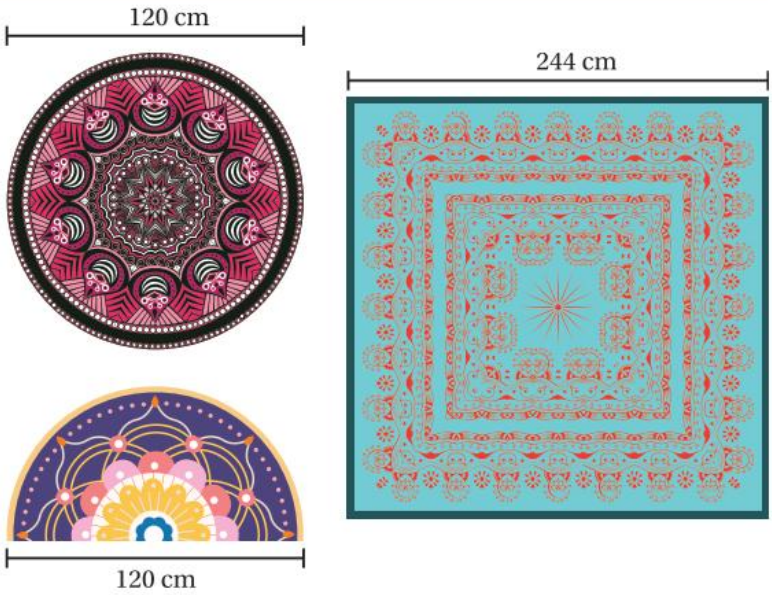
SC 1 I can solve practical problems involving circle measurements

Worked example

Practical problems with circles

Mathew has a choice of buying a square rug of side length 244 cm, or a circular and semicircular rug, each with a diameter of 120 cm.

How much more area, to the nearest cm^2, does the square rug cover compared with the total area covered by the smaller rugs?



THINKING	WORKING
Write the task in terms of the shapes.	Difference in area: Area of square – areas of circle and semicircle
Identify the given information.	Square: $l = 244$ cm Circle and semicircle: $d = 120$ cm so, $r = 60$ cm
Recall the required area formulas.	Square: $A = l^2$ Circle: $A = \pi r^2$
Substitute in the values for length and radius. Calculate.	Square: $A = l^2$ $= 244^2$ $= 59\,536$ cm^2 Circle and semicircle: $A = \pi r^2 + \frac{\pi r^2}{2}$ $= \pi \times 60^2 + \frac{\pi \times 60^2}{2}$ $= 16\,964.6\ldots$ cm^2
Calculate the difference.	$59\,536 - 16\,964.6 = 42\,571.4$
Write the answer.	The difference in area is approximately $42\,571$ cm^2 (to the nearest square centimetre).

Worked example

Practical problem involving circumference

The radius of the Earth is 6400 km, to the nearest 100 km. Give all answers to the nearest 100 km.

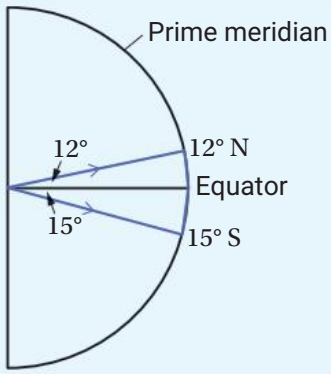
(a) Determine the distance around the Equator.

THINKING	WORKING
Identify the given information.	Radius of Earth: $r = 6400$ km
Write the task in terms of a circle.	Determine the circumference of Earth.
Recall the formula for the circumference.	$C = 2\pi r$
Determine the circumference by substituting in the value for radius.	$C = 2\pi r$ $= 2 \times \pi \times 6400$ $= 40\,212.3859\ldots$
Write the answer.	The distance around the equator is approximately 40 200 km (to the nearest 100 km).

(b) Calculate the distance from the North Pole to the South Pole.

THINKING	WORKING
Write the task in terms of a circle.	Determine half of the circumference of Earth.
Write the formula. Substitute the distance from part **(a)** and calculate.	$\frac{C}{2} = \frac{40\,212.3859\ldots}{2}$ $= 20\,106.1929\ldots$
Write the answer in words, correctly rounded to the nearest 100 km, with units of length.	The distance from the North Pole to the South Pole is approximately 20 100 km.

(c) How far would a ship travel going from 12°N to 15°S along the prime meridian?

THINKING	WORKING
Draw the situation.	
Write the task in terms of a circle.	Determine the arc length of a sector of central angle 12° + 15° = 27°.
Recall the formula for arc length.	$l = \frac{\theta}{360} \times C$
Write the formula and substitute in the arc length and circumference values. Calculate.	$l = \frac{27}{360} \times 40\,212.3859\ldots$ $= 3015.9289\ldots$
Write the answer in words, correctly rounded to the nearest 100 km, with units of length.	The distance from 12°S to 15°N is approximately 3000 km.

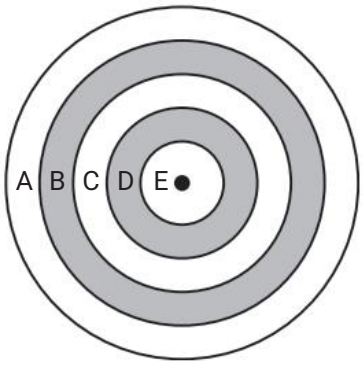
Practice

ANSWERS Page 591

SC 1 I can solve practical problems involving circle measurements

1 Determine the area, to the nearest m^2, of a model race track, 25 cm wide, whose inner shape is a rectangle 10 m by 8 m, with semicircles on each of the shorter sides.

2 The dartboard has an inner ring of diameter 5 cm, with the rings 2.5 cm wide.

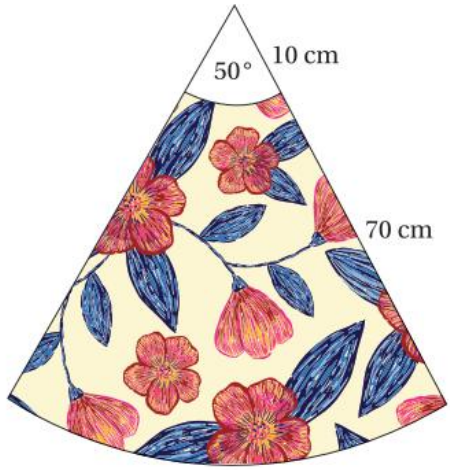


(a) Determine the total area of the shaded rings to the nearest cm^2.

(b) Describe the relationship between the area of the whole dartboard and:

(i) E **(ii)** the outermost ring (labelled A)

3 A flared skirt is to be made from panels as shown.



(a) Calculate the area of fabric in each panel, to the nearest cm^2.

(b) Determine the perimeter of the panel, to the nearest cm.

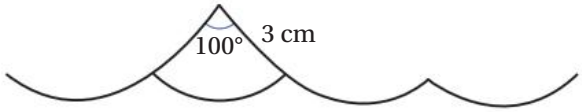
4 The large back tyre of a tractor has a diameter of 108 cm, while the small front tyre has a diameter of 72 cm. Given that the tractor is moving at 12 km/h, how many more revolutions, to the nearest whole number, does the front tyre experience:

(a) in 1 minute? **(b)** in 10 minutes? **(c)** in 2 hours?

5 The Wheel of Brisbane on South Bank is 60 m high.

Each ride completes 4 revolutions and costs $15.75. How much does it cost to travel at least 10 km on the ride?

6 A shawl has a scalloped edge, a section of which is shown below, featuring 4 scallops. What length of edging is needed to trim the scallops, given that there are 80 scallops along the edge of the material? Round your answer to the nearest centimetre.



7 The radius of the Earth is approximately 6400 km. Aircraft travel at a height of 10 975 m. For an aircraft that travels halfway around the world, what is the extra distance travelled compared to the distance measured on the surface of Earth? Give your answer in kilometres, correct to 1 decimal place.

TOPIC REVIEW

Multiple choice

ANSWERS Page 626

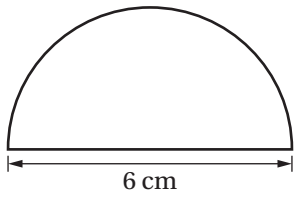
1 The area of a circle of with a diameter of 0.2 m is approximately:

A 3140 cm^2 **B** 3.14 cm^2 **C** 314 cm^2 **D** 31.4 cm^2

2 A circle with an area of 48 cm^2 has a radius correct to 2 decimal places, of:

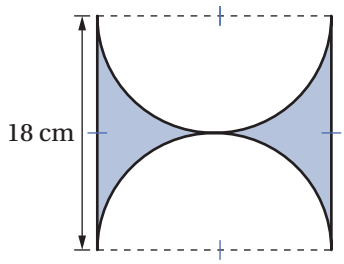
A 3.91 cm **B** 5.53 cm **C** 10.92 cm **D** 12.28 cm

3 What is the area of the semicircle below (correct to 2 decimal places)?



A 113.10 cm^2 **B** 18.85 cm^2 **C** 12.00 cm^2 **D** 14.14 cm^2

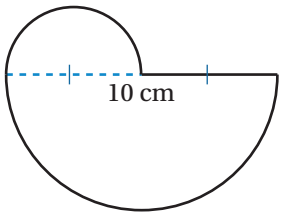
4 The area in cm^2 of the shaded region is:



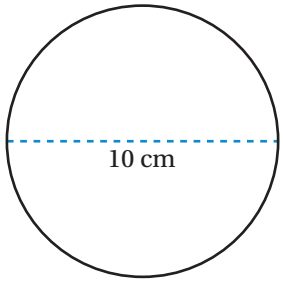
A $18^2 - \pi \times 18^2$ **B** $18^2 - \pi \times 9^2$ **C** $18^2 + \pi \times 18^2$ **D** $18^2 - \frac{1}{2} \times \pi \times 9^2$

5 The perimeters of the shapes below, in order from smallest to largest, are:

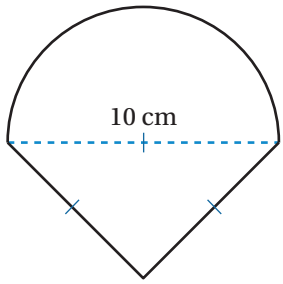
I



II



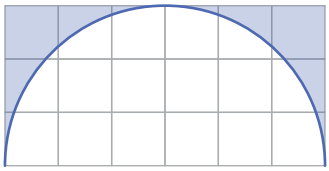
III



A II, I, III **B** II, III, I **C** I, III, II **D** III, II, I

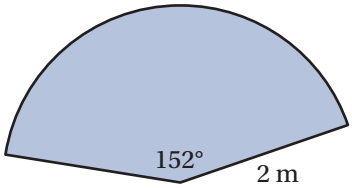
Short answer

1 Write a unit fraction to approximate the portion of the total area that is shaded, so $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ etc.

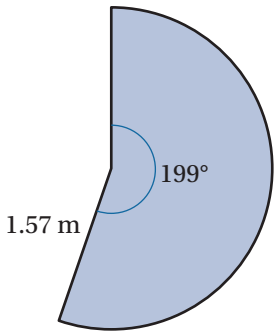


2 Calculate the area of each sector correct to 1 decimal place.

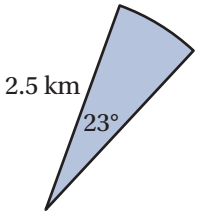
(a)



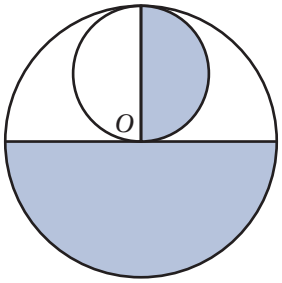
(b)



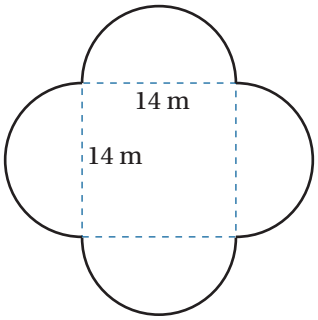
(c)



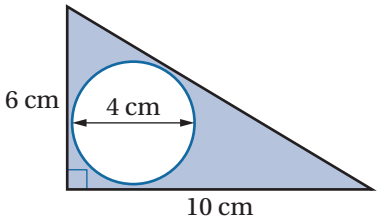
3 The area of the larger circle, centre O, is 144 cm^2. Determine the area shaded.



4 Calculate the area of the shape, correct to the nearest m^2.

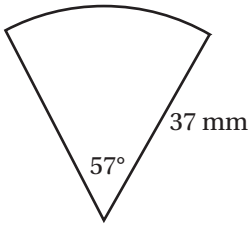


5 Calculate the area shaded, correct to 1 decimal place.

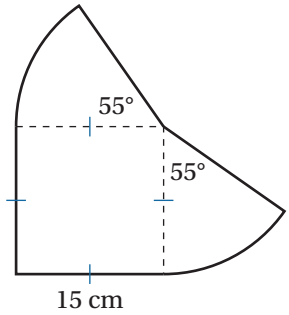


6 Calculate the perimeter of each shape, correct to the nearest whole number.

(a)

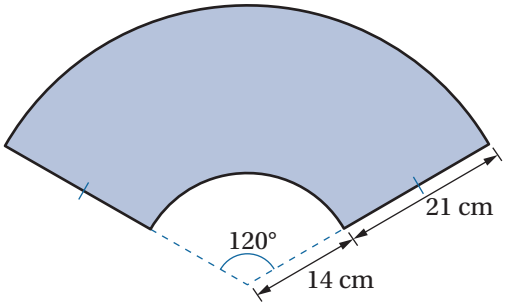


(b)

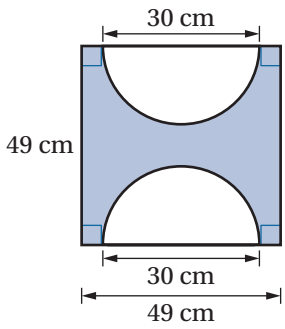


7 Calculate the perimeter and the area of each shape, correct to 2 decimal places.

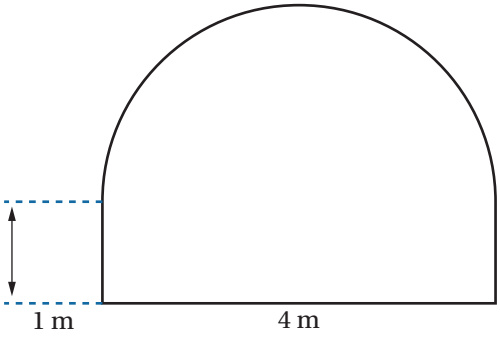
(a)



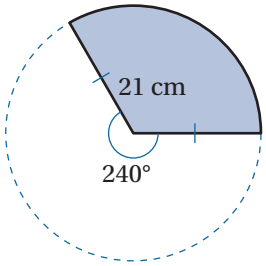
(b)



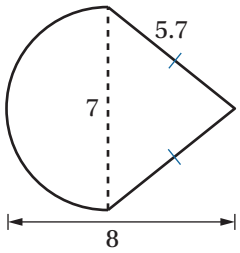
(c)



8 Calculate the area and perimeter of the shaded sector, correct to 1 decimal place.

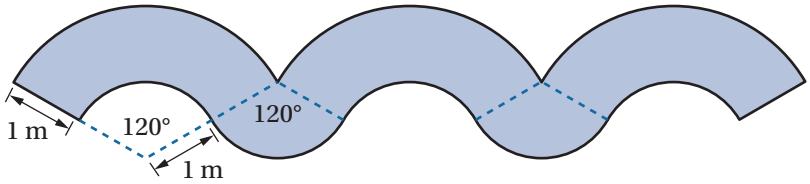


9 The dimensions on the shape below are given in centimetres.



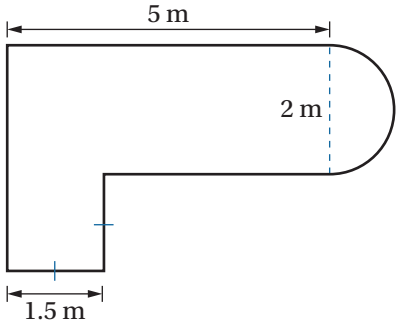
Determine the perimeter of the shape, correct to 1 decimal place.

10 Part of an awning is shaped as shown. Calculate the area of the given part, correct to 1 decimal place.



Extended response

1 Tim has decided to retile his backyard swimming pool, which has the dimensions outlined in the diagram below.



(a) Calculate the area of the base of the pool, rounded up to the nearest m^2.

(b) Determine the perimeter of the pool, correct to 2 decimal places.

(c) If the whole pool is 1.2 m deep, determine the area of the vertical surface around the pool, rounded up to the nearest m^2.

(d) If the tiles cost \$45 per m^2, how much will they cost Tim for the internal area of the pool? (Base the cost on the rounded up areas.)

TOPIC 10

Time

Recall 276

10.1 Understand 24-hour time 277

10.2 Understand the effect of daylight savings and longitude on local time 285

10.3 Understand and work with different time zones 291

Topic review 298

Why learn this?

Understanding 24-hour time and its conversion from 12-hour time format offers several practical advantages. It eliminates confusion between am and pm designations, making scheduling clearer in contexts such as travel, military operations and emergency services, where precision is vital.

The 24-hour time format aids in developing mathematical skills, such as calculating time durations, which are applicable in everyday scenarios such as cooking, transport timetables, parking stations or tracking work hours.

Learning this system enhances time management skills and equips learners with a universally recognised method of telling time, beneficial for personal organisation and professional contexts.

RECALL

ANSWERS Page 591

I can convert between different units of time

1 How many minutes are there in each of the following time durations?

(a) 1 hour **(b)** 60 seconds **(c)** $\frac{3}{4}$ of an hour

(d) $1\frac{1}{4}$ hours **(e)** 360 seconds

2 How many seconds are there in each of the following time durations?

(a) 1 minute **(b)** 20 minutes **(c)** $\frac{1}{4}$ of an hour

(d) 1 hour **(e)** 45 minutes

I can solve problems involving time duration

1 How many minutes are there between each pair of times on the same day?

(a) 7:00 am and 10:30 pm **(b)** 1:30 pm and 5:45 pm **(c)** 9:15 am and 12:00 pm

(d) 11:30 am and 2:45 pm **(e)** 5:15 am and 11:45 pm

2 What is the new time after each current time and duration?

(a) Current time: 7:00 am, duration: 4 hours and 30 minutes

(b) Current time: 4:20 pm, duration: 2 hours and 10 minutes

(c) Current time: 11:50 am, duration: 5 hours and 25 minutes

(d) Current time: 9:30 pm, duration: 6 hours and 45 minutes

3 What was the time in each of these cities 3.5 hours ago (ignore am/pm)?

New York

London

Moscow

Tokyo

Sydney

Understand 24-hour time

10.1

Learning intention: To be able to understand 24-hour time

Success criteria:

SC 1 I can convert 12-hour time to 24-hour time.

SC 2 I can convert 24-hour time to 12-hour time.

SC 3 I can calculate duration with 12-hour and 24-hour time.

Lesson warm-up

Ancient timing at night

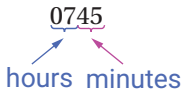
Sundials were used by some ancient civilisations to track the passage of time throughout each day.

What were some of the problems they faced using such a timing device?

SC 1 I can convert 12-hour time to 24-hour time

24-hour times represent the duration of time from midnight each day, written as a four-digit number.

The first two digits represent hours, the second two digits represent minutes.



Times in this format can be communicated in a number of ways.

On-the-hour times before midday

For example:
0400 is four hours after midnight and read as 'oh-four-hundred hours' or 'zero-four-hundred hours'.

Other times before midday

For example:
0420 is four hours and twenty minutes after midnight and read as 'Oh-four-twenty hours' or 'zero-four-twenty hours'.

Times after midday

For example:
1215 is fifteen minutes after midday and read as 'twelve-fifteen hours.'

2350 is 10 minutes before midnight and read as 'twenty-three-fifty hours'.

Times between midnight and 1:00 am

0010 is 10 minutes after midnight and read as 'oh-ten hours', or 'zero-ten hours'.

0000 is used for midnight, although 2400 is sometimes used for time duration calculations, 0000 is read as 'zero hundred hours'.

When converting to 12-hour time, note that 1800 is 6 pm (not 8 pm) because it is 6 hours after midday (1200).

Worked example

Expressing 12-hour time as 24-hour time

Express the following in 24-hour time.

(a) 9:15 am

THINKING	WORKING
Identify whether the time is before midday (am) or after midday (pm).	The time is before midday (am).
Write the time as a four-digit number, counting the hours and minutes after midnight.	9:15 am is 09 hours and 15 minutes after midnight.
Write the answer.	9:15 am written in 24-hour time is 0915.

(b) 4:45 pm

THINKING	WORKING
Identify whether the time is before midday (am) or after midday (pm).	The time is after midday (pm).
Write the time as a four-digit number, counting the hours and minutes after midnight.	12 hours to midday + 4 hours more 4 + 12 = 16 hours and 45 minutes after midnight.
Write the answer.	4:45 pm written in 24-hour time is 1645.

SC 2 I can convert 24-hour time to 12-hour time

Worked example

Expressing 24-hour time as 12-hour time

Express the following in 12-hour time.

(a) 0615

THINKING	WORKING
Determine the hours and minutes after midnight.	0615 is 6 hours and 15 minutes after midnight.
Determine whether the time will be written as am or pm. If the time is less than 12 hours after midnight, it is morning (am). If the time is 12 hours or more after midnight, it is afternoon (pm).	6:15 am
Write the answer.	0615 written in 12-hour time is 6:15 am.

(b) 1737

THINKING	WORKING
Determine the hours and minutes after midnight. For times more than 12 hours after midnight, determine the time after midday.	17 – 12 = 5 hours after midday 37 minutes
Determine whether the time will be written as am or pm. If the time is less than 12 hours after midnight, it is morning (am). If the time is 12 hours or more after midnight, it is afternoon (pm).	5:37 pm
Write the answer.	1737 written in 12-hour time is 5:37 pm.

SC 3 I can calculate duration in 12-hour and 24-hour time

Worked example

Duration between two 12-hour times on the same day

Calculate the duration between these 12-hour times.

(a) 2:15 pm and 11:25 pm

THINKING	WORKING
Determine whether the times are in the same half of the day (before or after midday).	2:15 pm and 11:25 pm are both pm, therefore they are in the same half of the day (after midday).
Subtract the hours and minutes for the earlier time from the hours and minutes of the later time.	11:25 pm – 2:15 pm = (11 – 2):(25 – 15) = (9):(10) This represents 9 hours and 10 minutes.
Write the answer.	The duration between 2:15 pm and 11:25 pm is 9 hours and 10 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	2:15 pm + 10 mins = 2:25 pm 2:25 pm + 9 hours = 11:25 pm The total duration is 9 hours and 10 minutes.

(b) 8:45 am and 10:15 am

THINKING	WORKING
Determine whether the times are in the same half of the day (before or after midday).	8:45 am and 10:15 am are both am, therefore they are in the same half of the day (before midday).
Subtract the hours and minutes for the earlier time from the hours and minutes of the later time.	10:15 am – 8:45 am = (10 – 8):(15 – 45) = (2):(–30) This represents 2 hours and (–30) minutes.
Adjust due to the negative minutes result. The minutes value is subtracted from the hours value.	2 hours – 30 minutes = 1 hour and 30 minutes.
Write the answer.	The duration between 8:45 am and 10:15 am is 1 hour and 30 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	8:45 am + 30 mins = 9:15 am 9:15 am + 1 hour = 10:15 am The total duration is 1 hour and 30 minutes.

(c) 10:20 am and 4:50 pm

THINKING	WORKING
Determine whether the times are in the same half of the day (before or after midday).	10:20 am and 4:50 pm are not in the same half of the day.
Calculate the time duration between each time and midday (12:00 pm).	12:00 pm – 10:20 am = (12 – 10):(00 – 20) = (2):(–20) This represents 2 hours and (–20) minutes or 1 hour and 40 minutes before midday. 4:50 pm is 4 hours and 50 minutes after midday.
Add the hours and minutes of the two durations together.	1 hour and 40 minutes + 4 hours and 50 minutes = (1 + 4):(40 + 50) = 5:90 This represents 5 hours and 90 minutes, or 6 hours and 30 minutes.
Write the answer.	The duration between 10:20 am and 4:50 pm is 6 hours and 30 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	10:20 am + 30 mins = 10:50 am 10:50 am + 6 hours = 4:50 pm The total duration is 6 hours and 30 minutes.

Worked example

Duration between two 24-hour times on the same day

Calculate the duration between these 24-hour times.

(a) 1415 and 2325

THINKING	WORKING
Subtract the hours and minutes of the earlier time from the later time separately.	2325 – 1415 = (23 – 14):(25 – 15) = (09):(10) This represents 9 hours and 10 minutes.
Write the answer.	The duration between 1415 and 2325 is 9 hours and 10 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	1415 + 10 mins = 1425 1425 + 9 hours = 2325 The total duration is 9 hours and 10 minutes.

(b) 1050 and 1620

THINKING	WORKING
Subtract the hours and minutes of the earlier time from the later time separately.	1620 – 1050 = (16 – 10):(20 – 50) = (06):(–30) This represents 6 hours and (–30) minutes.
Adjust due to the negative minutes result. The minutes value is subtracted from the hours value.	6 hours – 30 minutes = 5 hours and 30 minutes
Write the answer.	The duration between 1020 and 1650 is 5 hours and 30 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	1050 + 30 mins = 1120 1120 + 5 hours = 1620 The total duration is 5 hours and 30 minutes.

(c) 1042 and 1810

THINKING	WORKING
Subtract the hours and minutes of the earlier time from the later time separately.	1810 – 1042 = (18 – 10):(10 – 42) = (08):(–32) This represents 8 hours and –32 minutes. 8 hours – 32 minutes = 7 hours and (60 – 32) minutes = 7 hours and 28 minutes
Write the answer.	The duration between 10:42 am and 1810 is 7 hours and 28 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	1042 + 18 mins = 1100 1100 + 10 mins = 1110 1110 + 7 hours = 1810 The total duration is 7 hours and 28 minutes.

Worked example

Duration between two times on different days

Calculate the duration between these times and days.

(a) 5:20 pm Monday and 3:20 am Tuesday.

THINKING	WORKING
Calculate the time duration between each time and midnight (12:00 am).	12:00 am – 5:20 pm = (12 – 5):(00 – 20) = (7):(–20) This represents 7 hours and (–20) minutes or 6 hours and 40 minutes before midnight. 3:20 am is 3 hours and 20 minutes after midnight (12:00 am).
Add the hours and minutes of the two durations together.	6 hours and 20 minutes + 3 hours and 20 minutes = (6 + 3):(20 + 20) = 9:40 This represents 9 hours and 40 minutes.
Write the answer.	The duration between 5:20 pm Monday and 3:20 am Tuesday is 9 hours and 40 minutes.

(b) 1050 Wednesday and 1620 Friday

THINKING	WORKING
Calculate the time duration between each time and midnight (2400).	2400 – 1050 = (24 – 10):(00 – 50) = (14):(–50) 1050 Wednesday is 10 hours and (–50) minutes or 13 hours and 10 minutes before midnight. Thursday is 24 hours and 0 minutes before midnight. Friday 1620 is 16 hours and 20 minutes after midnight.
Add the hours and minutes of the three durations together.	(13 + 24 + 16):(10 + 0 + 20) = (53):(30) This represents 53 hours and 30 minutes.
Write the answer.	The duration between 1050 Wednesday and 1620 Friday is 53 hours and 30 minutes.
Note: Another strategy is to count up to align the minutes, then the hours.	1050 Wednesday + 48 hours = 1050 Friday 1050 + 30 mins = 1120 1120 + 5 hours = 1620 The total duration is 48 + 5 = 53 hours and 30 minutes.

Practice

ANSWERS Page 591

SC 1 I can convert 12-hour time to 24-hour time

1 Express the following am times in 24-hour time.

(a) 5:30 am **(b)** 11:20 am **(c)** 10 am **(d)** 7 am

2 Express the following pm times in 24-hour time.

(a) 12:40 pm **(b)** 1:20 pm **(c)** 7:10 pm **(d)** 10 pm

SC 2 I can convert 24-hour time to 12-hour time

1 Express the following times before midday in 12-hour time.

(a) 0428 **(b)** 1040 **(c)** 1100

2 Express the following times after midday in 12-hour time.

(a) 1235 **(b)** 1600 **(c)** 1852 **(d)** 2035 **(e)** 2210

3 Write the 12-hour time that each 24-hour time description represents.

(a) Oh-seven-hundred hours

(b) Fourteen-twenty hours

(c) Twenty-twenty-two hours

(d) Oh-nine-fifty hours

(e) Thirteen-hundred hours

(f) Zero-forty-five hours

SC 3 I can calculate duration in 12-hour and 24-hour time

1 Determine the amount of time that has passed between the given times.

(a) 0740 and 1025

(b) 1050 and 2212

(c) 0615 and 2030

(d) 1623 and 0425 the next day

2 Determine the time of day.

(a) 3 h 30 min after 0915

(b) 10 h 50 min after 1430

(c) 5 h 20 min before 1455

(d) 6 h 25 min before 0215

Understand the effect of daylight savings and longitude on local time

10.2

Learning intention: To understand the effect of daylight savings and longitude on local time

Success criteria:

SC 1 I can determine the effect of DST on local time.

SC 2 I can determine the effect of longitude on local time.

Lesson warm-up

Meeting times

With advancements in technology and increased connectivity, companies are able to allow employees to work remotely with flexible hours. This has led to companies being able to employ people from anywhere across the country, not just those who live within travelling distance of an office.

This presents some unique challenges, such as organising team meetings.

List the sorts of challenges that would be faced when organising such meetings between employees located in different states.

SC 1 I can determine the effect of DST on local time

Closer to the Equator, the duration of daylight hours is less affected by the seasons of the year.

Further from the Equator, winter daylight hours become considerably shorter, while summer daylight hours become considerably longer. This is due to the tilt of Earth's axis. The North and South Poles are located furthest away from the Equator and experience 6 months of daylight followed by 6 months of darkness.

Daylight savings was introduced in some countries that are further away from the Equator in an attempt to conserve energy usage and ensure an extra hour of daylight near the end of the working day.

Countries or states whose governments choose to have Daylight Saving Time (DST) across summer put their clocks forward by one hour in the spring months ('spring forward'), and change them back by one hour to Standard Time (ST) in the autumn months ('fall back').

Within Australia, states choosing to observe DST use the first Sunday in October, when 2 am jumps to 3 am to begin DST, and then the first Sunday in April, when 3 am goes back to 2 am, to end DST.

In Australia, Queensland, the Northern Territory and Western Australia currently choose not to observe DST in the summer.

Worked example

Calculating the effect of Daylight Saving Time (DST) on local time

Queensland and NSW are positioned in the same standard time zone. NSW observes DST throughout summer, while Queensland does not.

During the summer months, determine the time in:

(a) Tweed Heads, NSW, when it is 9:35 am in Coolangatta, Qld.

THINKING	WORKING
Determine the effect of Daylight Saving Time (DST).	Throughout summer, the clocks in DST zones are advanced forward by 1 hour.
Identify the location affected by DST.	The clocks in NSW observe DST. Therefore, the time in NSW is one hour ahead of the time in Qld.
Write the answer.	When it is 9:35 am in Coolangatta, Qld, the time in Tweed Heads, NSW (DST) is 10:35 am.

(b) Rockhampton, Qld, when it is 3:40 pm in Newcastle, NSW.

THINKING	WORKING
Determine the effect of Daylight Saving Time (DST).	Throughout summer, the clocks in DST zones are advanced forward by 1 hour.
Identify the location affected by DST.	The clocks in NSW observe DST. Therefore, the time in Qld is 1 hour behind the time in NSW.
Write the answer.	When it is 3:40 pm in Newcastle, NSW (DST), the time in Rockhampton, Qld is 2:40 pm.

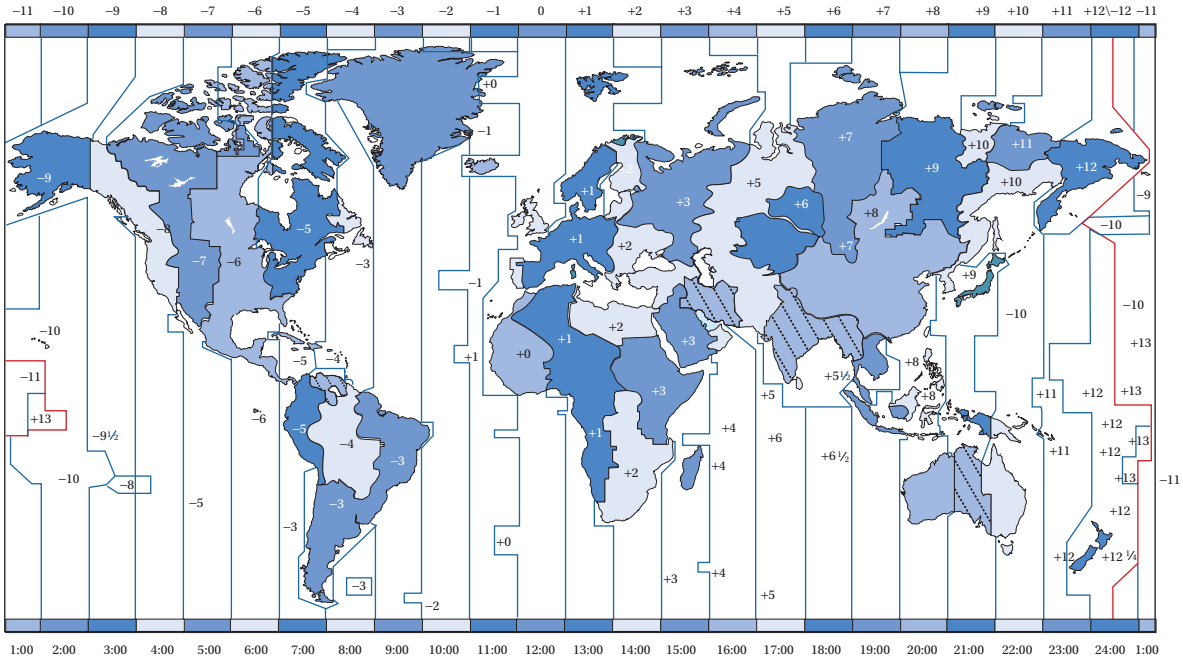
SC 2 I can determine the effect of longitude on local time

Locations to the east of any position on Earth experience sunrise (and sunset) each day before locations to the west.

When there is no adjustment for daylight savings, the east coast of Australia experiences sunrise approximately 2 hours ahead of (before) the west coast.

Apart from regional adjustments by governments, Earth locations that are close to/on the same meridian (line of longitude) experience the same time of day and will be in the same (or similar) time zone.

Worldwide map of local time zones



The global map of Earth is divided into imaginary vertical lines called meridians. Every one of these lines meets at the North and South Poles, and they are furthest away from each other at the Equator.

Longitude is the measurement in degrees of how far east or west a location on Earth is away from the prime meridian, which is the starting point (0°).

As the world rotates on its axis, it turns 360° of longitude every 24 hours, which is equivalent to 15°/h. The Sun reaches its zenith (highest position during the middle of the day) an hour later for every 15° towards the west.

At midday along the prime meridian, the whole world is on the same day, with the Eastern Hemisphere experiencing afternoon and the Western Hemisphere experiencing morning.

When it is midday Friday at the prime meridian, it is midnight Friday at 180°E and midnight Thursday (about to become Friday) at 180°W.

The imaginary line where the Eastern and Western hemispheres meet is called the International Date Line (IDL). Although the clock times immediately on either side of the IDL are identical, the western side is actually a whole day (24 hours) behind the eastern side. Therefore, when it is 3 pm Wednesday on the eastern side of the IDL, it is 3 pm Tuesday on the western side.

Worked example

Determining time from longitude

Eastern Australia is assigned its time zone based on the 150°E meridian (just west of Brisbane). Determine the time in each of these places when it is 0830 on Thursday in Brisbane (assume no DST or unusual adjustments by governments).

(a) London, prime meridian (or 0°)

THINKING	WORKING
Determine the difference between the meridians.	150°E to 0° is 150° difference.
Determine the time difference using 15° per hour.	$\frac{150^\circ}{15^\circ} = 10$ h
Determine whether the location for the given time is east (ahead) or west (behind) of the required location.	Brisbane (150°E) is further east of London (0°). London is west of Brisbane and therefore 10 h behind.
Subtract the time difference because London is west of Brisbane.	10 h before 0830 is 2230 the previous day.
Write the answer.	It is 2230 on Wednesday in London when it is 0830 on Thursday in Brisbane.

(b) Los Angeles (LA), 120°W meridian

THINKING	WORKING
Determine the difference between the meridians. Note: The longitudes are added here because the locations are on opposite sides of the prime meridian.	Brisbane (150°E) to Los Angeles (120°W) is 150° + 120° = 270° difference.
Determine the time difference using 15°per hour.	$\frac{270^\circ}{15^\circ} = 18$ h
Determine whether the location for the given time is east (ahead) or west (behind) of the required location.	Brisbane (150°E) is further east of Los Angeles (120°W). LA is west of Brisbane and therefore 18 h behind.
Subtract the time difference because LA is west of Brisbane.	18 h before 0830 is 1430 the previous day.
Write the answer.	It is 1430 on Wednesday in Los Angeles when it is 0830 on Thursday in Brisbane.

(c) Auckland, 180°E meridian

THINKING	WORKING
Determine the meridian difference.	Brisbane (150°E) to Auckland (180°E) is $180 - 150 = 30^\circ$ difference.
Determine the time difference using 15° per hour.	$\frac{30^\circ}{15^\circ} = 2$ h
Determine whether the location for the given time is east (ahead) or west (behind) of the required location.	Brisbane (150°E) is further west of Auckland (180°E). Auckland is east of Brisbane and therefore 2 h ahead.
Add the time difference because Auckland is east of Brisbane.	2 h after 0830 is 1030.
Write the answer.	It is 1030 on Thursday in Auckland when it is 0830 on Thursday in Brisbane.

Practice

ANSWERS Page 591

SC 1 I can determine the effect of DST on local time

1 Central Africa and the Baltic States of Europe share the same standard time zone. The Baltic States observe DST throughout summer, while Central Africa does not. During summer months, determine:

(a) the DST in Finland, Europe when it is 8:05 am in Egypt, Central Africa

(b) the DST in Latvia, Europe when it is 4:12 pm in South Sudan, Central Africa

(c) the time in Sudan, Central Africa when the DST in Estonia, Europe is 6:55 pm

(d) the time in Zimbabwe, Central Africa when the DST in Lithuania, Europe is 10:35 am.

2 Western Australia (WA), Queensland (Qld) and the Northern Territory (NT) do not observe Daylight Saving Time (DST). In winter, when it is 6 am in all of the eastern Australian states, it is 4 am in WA. During summer, determine:

(a) the time in WA when it is 3:25 pm in Qld

(b) the time in WA when the DST in NSW is 10:20 am

(c) the DST in NSW when it is 6:55 am in WA

(d) the time in Qld when it is 1:15 pm WA.

3 Western Australia, Queensland and the Northern Territory do not observe Daylight Saving Time (DST).

(a) In summer, when the DST is 4 pm in NSW, it is 3:30 pm in South Australia (SA). During winter, determine the time in:

(i) SA when it is 2:10 pm in Qld

(ii) Victoria when it is 8:50 am in SA

(iii) SA when it is 10:10 am in Tasmania

(iv) ACT when it is 9:45 pm in SA.

(b) In summer, when it is 1 pm in Victoria, it is 11:30 am in NT. During winter, determine the time in:

(i) NT when it is 4:45 pm in Qld

(ii) NSW when it is 7:40 am in NT

(iii) NT when it is 11:30 am in ACT

(iv) Tasmania when it is 10:15 pm in NT.

SC 2 I can determine the effect of longitude on local time

1 NSW is assigned its time zone based on the 150°E meridian. Determine the time in each of the listed places when it is 1140 on Saturday in Sydney (assume DST and/or unusual adjustments by governments are not being observed).

(a) Glasgow, prime meridian 0°

(b) Christchurch, 180°E meridian

(c) Perth, 120°E meridian

(d) Mumbai, 75°E meridian

(e) Tokyo, 135°E meridian

(f) Kyiv, 30°E meridian

(g) New York, 75°W meridian

(h) Vancouver, 120°W meridian

2 It is 5 pm on Thursday in Hobart. Assuming standard times are being observed (no DST), use the meridian values to estimate the time in each of these locations, correct to the nearest hour.

(a) Beijing (China) 40°N, 116°E

(b) Capetown (South Africa) 34°S, 18°E

(c) Anchorage (Alaska) 61°N, 150°W

(d) Frankfurt (Germany) 50°N, 9°E

3 It is 7 am on a winter Sunday in Rockhampton, Queensland. Use meridian values to estimate the time in the listed locations, correct to the nearest hour.

(a) Edmonton (Canada), 54°N, 113°W observing DST

(b) La Paz (Bolivia), 16°S, 68°W observing standard time

(c) Cardiff (Wales), 51°N, 3°W observing DST

(d) Islamabad (Pakistan), 34°N, 73°E observing standard time

4 It is 8 pm on a summer Wednesday in Ballarat, Victoria. Use meridian values to estimate the time in the listed locations, correct to the nearest hour.

(a) Monrovia (Liberia), 6°N, 11°W

(b) Sau Paulo (Brazil), 24°S, 47°W observing standard time

(c) Lagos (Nigeria), 7°N, 3°E

(d) Kolkata (India), 23°N, 88°E

Understand and work with different time zones

10.3

Learning intention: To understand and be able to work with different time zones

Success criteria:

SC 1 I can determine local time throughout Australia.

SC 2 I can determine local time around the world.

SC 3 I can make travel plans across different time zones.

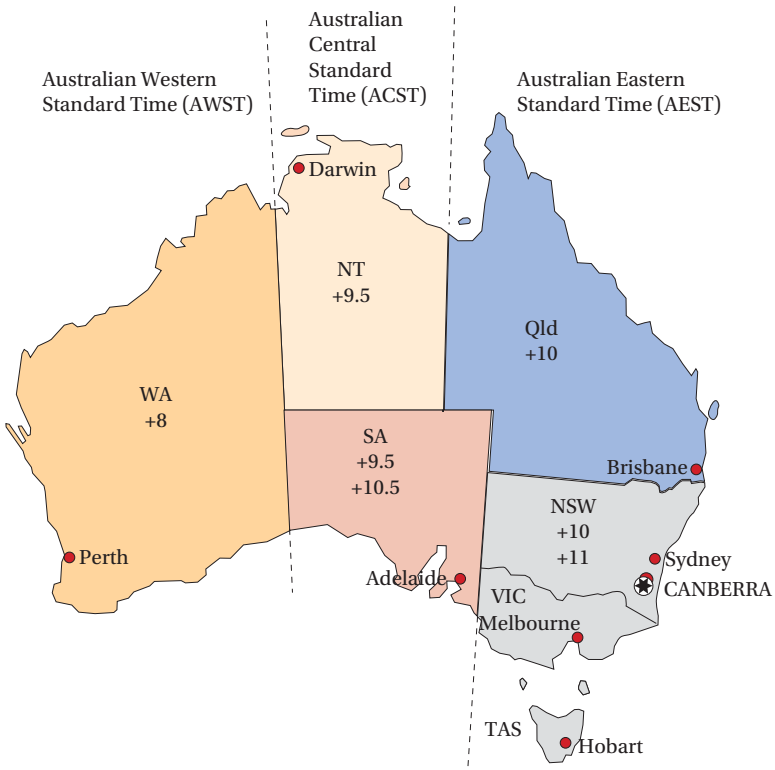
Lesson warm-up

Planning a travel holiday

When planning a holiday to another country, apart from passports and travel insurance, what needs to be considered in advance?

Write a quick list of what you would want to have booked prior to travelling.

SC 1 I can determine local time throughout Australia



Australia uses three standard time zones.

Location	Time zone
The eastern states and territory (Queensland, New South Wales, ACT, Victoria and Tasmania).	AEST 10 hours ahead of Greenwich Mean Time (at the prime meridian) GMT+10 UTC+10 (Coordinated Universal Time)
The central state and territory (Northern Territory and South Australia).	ACST 9.5 hours ahead of GMT GMT+9.5 UTC+9.5
The western state (Western Australia).	AWST 8 hours ahead of UTC UTC+8 GMT+8

From the first Sunday in October until the first Sunday in April, some states observe Daylight Saving Time (DST), during which time all clocks are set forward one hour. This means that Australia has five different time zones at certain times of the year.

During Daylight Saving Time, DST:

- NSW, ACT, Vic and Tas use Australian Eastern Daylight Time (AEDT), which is UTC+11.
- SA uses ACDT, which is UTC+10.5.
- Qld, NT and WA do not follow DST, so they do not change their clocks (UTC+10, UTC+9.5 and UTC+8 respectively).

Worked example

Determining local time within Australia

It is 6:20 pm in Sydney. What time is it in Darwin during:

(a) winter?

THINKING	WORKING
During the winter months, all Australian states observe standard time. Determine the time zones for the two locations in terms of UTC.	Sydney (AEST): UTC+10 Darwin (ACST): UTC+9.5
Calculate and interpret the UTC time difference.	(+10) – (+9.5) = (+0.5) Sydney is half an hour (30 min) ahead of Darwin.
Calculate the time using the difference.	Darwin time is 30 min behind. 6:20 pm – 0:30 = 5:50 pm
Write the answer.	The time in Darwin is 5:50 pm when it is 6:20 pm in Sydney during winter.

(b) summer?

THINKING	WORKING
During the summer months, certain Australian states observe Daylight Saving Time. Determine the time zones for the two locations in terms of UTC.	Sydney (AEDT): UTC+11 Darwin (ACST): UTC+9.5
Calculate and interpret the UTC time difference.	(+11) – (+9.5) = (+1.5) Sydney is one-and-a-half hours (1 hour, 30 minutes) ahead of Darwin.
Calculate the time using the difference.	Darwin is 1 hour, 30 minutes behind. 6:20 pm – 1:30 = 4:50 pm
Write the answer.	The time in Darwin is 4:50 pm when it is 6:20 pm in Sydney during summer.

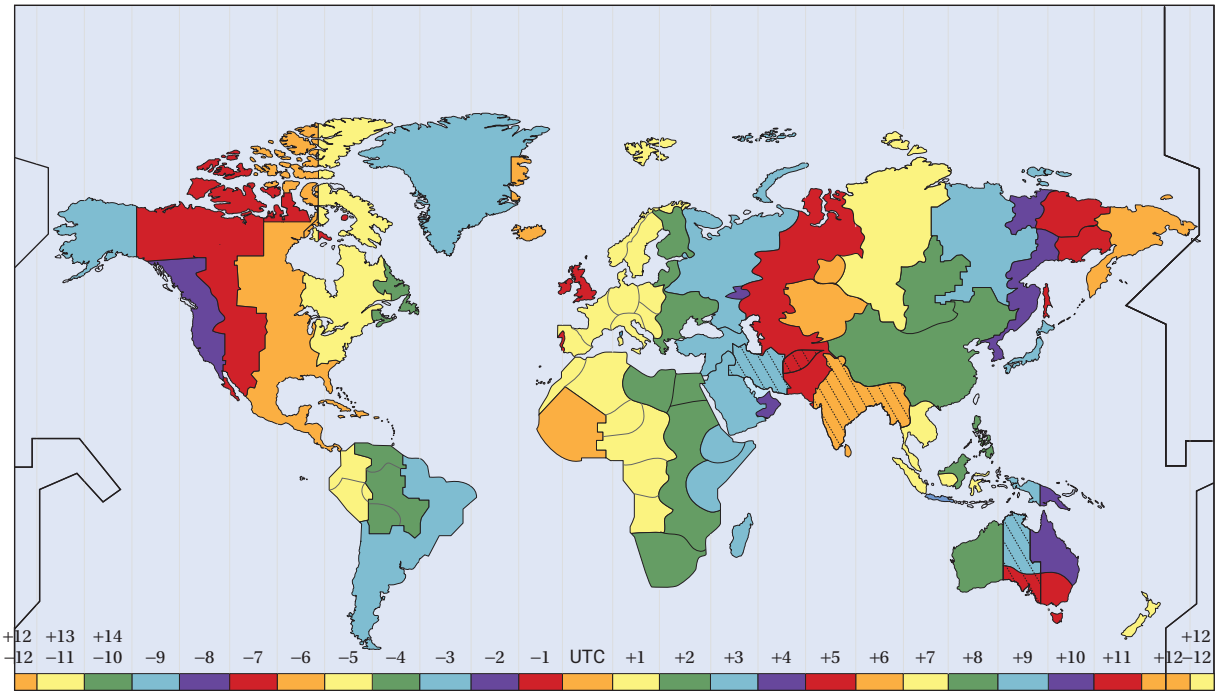
SC 2 I can determine local time around the world

As Earth rotates on its axis, the sun will be in different positions above various locations as daytime moves around the globe. Earth is divided into time zones to accommodate for this.

The world map shows time zones during the Northern Hemisphere summer (and Southern Hemisphere winter).

The Southern Hemisphere colour shadings roughly match the shadings along the bottom (eastern Australia UTC+10, central Australia UTC+9.5).

The Northern Hemisphere lines up nicely with regions nearer the Equator; however, further north where DST is observed, the shadings match one hour to the right.



This shows why sports fans in Australia prefer major sporting events to be held in the Asian regions rather than in Europe or the Americas. The live broadcasts occur at times that are more convenient to watch.

Some countries, such as Australia, Brazil, the USA and Canada, use several times zones, whereas all of China uses just one.

The International Date Line (IDL) is halfway around the world from the prime meridian, where 180°E meets 180°W. This imaginary line stretches along an area consisting mainly of ocean.

Where the meridian passes through countries with a very large land mass (such as Russia) or through island groups, the IDL deviates in certain areas so those nations/communities are on the same day as each other.

Worked example

Local time around the world

At 2:50 pm in Brisbane, what time is it in:

(a) Beijing, UTC+8?

THINKING	WORKING
Write both time zones in terms of UTC.	Brisbane: UTC+10 Beijing: UTC+8
Calculate and interpret the UTC time difference.	(+10) – (+8) = (+2) Brisbane is 2 hours ahead of Beijing.
Calculate the time using the difference.	Beijing is 2 hours behind Brisbane. 2:50 pm – 2:00 = 12:50 pm
Write the answer.	The time in Beijing is 12:50 pm when it is 2:50 pm in Brisbane.

(b) London, UTC?

THINKING	WORKING
Write both time zones in terms of UTC.	Brisbane: UTC+10 London: UTC
Calculate and interpret the UTC time difference.	(+10) – (0) = (+10) Brisbane is 10 hours ahead of London.
Calculate the time using the difference.	London is 10 hours behind Brisbane. 2:50 pm – 10:00 = 4:50 am
Write the answer.	The time in London is 4:50 am when it is 2:50 pm in Brisbane.

(c) Panama City, UTC−5?

THINKING	WORKING
Write both time zones in terms of UTC.	Brisbane: UTC+10 Panama City: UTC−5
Calculate and interpret the UTC time difference.	(+10) − (−5) = (+15) Brisbane is 15 hours ahead of Panama City.
Calculate the time.	2:50 pm − 15:00 = 11:50 pm the previous night
Write the answer.	The time in Panama City is 11:50 pm the previous night when it is 2:50 pm in Brisbane.

SC 3 I can make travel plans across different time zones

When planning international travel for holidays or business trips, both the duration of the travel and difference in time zones require careful consideration. Arriving in a country and finding out that an accommodation or travel connection booking has been made for the wrong date can get holidays off to a bad start.

It is always important to determine the local arrival time and date before making further holiday/business arrangements.

On long trips out of Australia, you could travel for 20 hours and arrive in the new location on the same date you left on. When travelling home from such distant locations, you can actually skip a day.

Worked example

Calculating travel times across international time zones

Anne is flying from Brisbane to Vancouver via Sydney on 14 May. The departure times from Brisbane and Sydney are 0635 and 0945 respectively, on the same day.

(a) Calculate the duration of the stopover in Sydney, given that the Brisbane–Sydney flight takes 1 h 35 min to complete.

THINKING	WORKING
Establish any time zone difference.	There is no AEDT in Australia during May, so Brisbane and Sydney are both UTC+10.
Calculate the difference in departure times.	0945 − 0635 = 0310 0635 to 0945 is 3 h 10 min
Calculate the time remaining for the stopover after the flight duration is subtracted.	3 h 10 min − 1 h 35 min = 0310 − 0135 = 0135 Stopover time: 1 h 35 min

(b) Determine the time in Sydney when Anne arrives in Vancouver, given the duration of the Sydney–Vancouver flight is 14 h 10 min.

THINKING	WORKING
Add the flight duration to the Sydney departure time.	0945 + 14h 10 min = 0945 + 1410 = 2355 14 h 10 min after 0945 is 2355.
Write the answer.	It will be 2355 on 14 May in Sydney when Anne arrives in Vancouver.

(c) Determine Anne's arrival time in Vancouver (UTC−7).

THINKING	WORKING
Establish the time zones.	Sydney: UTC+10 Vancouver: UTC−7
Calculate and interpret the UTC time difference.	(+10) − (−7) = (+17) Sydney is 17 hours ahead of Vancouver. (Vancouver is 17 hours behind Sydney).
Calculate the difference between the time in Sydney and the flight duration to determine when the plane arrives in Vancouver.	2355 − 1700 = 0655 17 hours before 2355 is 0655.
Write the answer.	After the long journey, Anne arrives at a local time in Vancouver that is earlier than the local departure time in Sydney! Anne's arrival time in Vancouver is 0655 on 14 May.

Practice

ANSWERS Page 592

SC 1 I can determine local time throughout Australia

1 It is 10:30 am in Brisbane, Qld. Determine the time in Adelaide, SA, during:

(a) winter **(b)** summer.

2 It is 5:40 pm in Perth, WA. Determine the time in Hobart, Tas, during:

(a) winter **(b)** summer.

3 A game of football is being played in Brisbane, Qld, starting at 3 pm on the third Sunday in April. Determine the local time of the live broadcast of the game:

(a) in Adelaide (SA) **(b)** in Perth (WA)

4 A plane departs Brisbane, Qld, at 0720 and arrives in Alice Springs, NT, at 1010 local time. Determine the duration of the flight.

5 An office in Perth, WA hosted an online meeting starting at 0930 local time during summer. The meeting duration was 45 minutes. At what local time did the meeting end for employees based in Sydney, NSW?

SC 2 I can determine local time around the world

1 At 7:15 am on a Tuesday in Perth, determine the time and day in:

(a) Tokyo, Japan, UTC+9 **(b)** Lisbon, Portugal, UTC **(c)** Minneapolis, USA, UTC−6

2 What time (AEST) and date is it in Sydney at these listed 23 June times?

(a) 4:20 am in Dunedin, NZ, UTC+12 **(b)** 2:55 pm in London, England, UTC+1

(c) 9:40 am in Las Vegas, USA, UTC−7

3 An online meeting is scheduled to begin anytime between 9 am and 4 pm, AEDT in Canberra. Determine the range of starting times for each location, assuming they would also prefer it to be between 9 am and 4 pm locally. Write the possible start times for the locations that can meet both requirements.

(a) Wellington, NZ, UTC+13 **(b)** Dublin, Ireland, UTC **(c)** Dallas, Texas, UTC−6

4 The French Open (tennis) final in Paris is scheduled for a 3 pm start on a Sunday in June. France is UTC+2 during summer. What time will the live broadcast start in:

(a) Adelaide, AUS, UTC+9.5 **(b)** Christchurch, NZ, UTC+12

(c) Mumbai, India, UTC+5.5 **(d)** Washington DC, UTC−4?

SC 3 I can make travel plans across different time zones

1 Calculate the local arrival time at each destination for flights leaving Brisbane at 8 am during summer.

(a) Singapore, UTC+8 with a flight duration of 8 h 5 min

(b) Auckland, UTC+13 with a flight duration of 3 h 5 min

(c) New York, UTC−5 with a flight duration of 20 h 55 min

2 An airline passenger departs Rome (UTC+2) at 8:05 am, arriving in New York at 7:45 pm the same day, following a stopover in London. The flight from Rome to London is 2 h 45 min, and from London to New York is 7 h 45 min. What is the duration of the stopover in London, given that New York is UTC−4?

3 Callum is flying from Perth to Los Angeles via Melbourne on 6 June. The departure times from Perth and Melbourne are 0940 and 1825, respectively, on the same day.

(a) Calculate the duration of the stopover in Melbourne, given that the Perth–Melbourne flight took 3 h 25 min.

(b) What is the time in Melbourne when Callum arrives in Los Angeles, given that the Melbourne to Los Angeles flight took 14 h 20 min to complete?

(c) Calculate Callum's arrival time in Los Angeles (UTC−7).

TOPIC REVIEW

Multiple choice

ANSWERS Page 626

1 If it is 1630 on New Year's Day in Sydney, what is the time in Mackay, Queensland?

A 4:30 am **B** 3:30 pm **C** 4:30 pm **D** 5:30 pm

2 If it is 1630 on New Year's Day in Sydney, what is the time in Adelaide, South Australia?

A 3:30 pm **B** 4 pm **C** 4:30 pm **D** 5 pm

3 If it is 1630 on New Year's Day in Sydney, what is the time in Broome, WA?

A 1:30 pm **B** 2:30 pm **C** 3:30 pm **D** 4:30 pm

4 If it is 1630 on New Year's Day in Sydney, what is the time in Hobart, Tasmania?

A 4:30 am **B** 3:30 pm **C** 4:30 pm **D** 5:30 pm

5 If it is 0945 in October in Perth, what is the time in each place, given the meridian used to determine the time: Suva, Fiji, 180°E (no DST)?

A 10:15 am **B** 9:45 am **C** 10:45 am **D** 11:45 am

6 If it is 0945 in October in Perth, what is the time in each place, given the meridian used to determine the time: Cambodia, 105°E?

A 8:45 am **B** 9:45 am **C** 10:45 am **D** 11:45 am

7 If it is 0945 in October in Perth, what is the time in each place, given the meridian used to determine the time: Sweden, 15°E?

A 12:45 am **B** 1:45 am **C** 2:45 am **D** 3:45 am

8 If it is 0945 in October in Perth, what is the time in each place, given the meridian used to determine the time: New Orleans, USA, 90°W?

A 7:45 am previous day **B** 7:45 pm previous day

C 7:45 am same day **D** 7:45 pm same day

9 Convert each 24-hour time to am or pm time: 1342.

A 1:42 pm **B** 3:42 pm **C** 3:42 am **D** 1:42 am

10 Convert each 24-hour time to am or pm time: 0620.

A 4:20 pm **B** 6:20 pm **C** 4:20 am **D** 6:20 am

11 Convert each 24-hour time to am or pm time: 1255.

A 2:55 pm **B** 10:55 pm **C** 12:55 pm **D** 12:55 am

12 Convert each 24-hour time to am or pm time: 2215.

A 2:15 pm **B** 12:15 am **C** 10:15 pm **D** 12:15 pm

13 Convert each am or pm time to 24-hour time: 8:25 am.

A 1825 **B** 0825 **C** 2025 **D** 825

14 Convert each am or pm time to 24-hour time: 5:57 pm.

A 0557 **B** 1557 **C** 1757 **D** 2557

15 Convert each am or pm time to 24-hour time: 12:40 am.

A 1240 **B** 1040 **C** 0240 **D** 0040

16 Convert each am or pm time to 24-hour time: 12:50 pm.

A 1250 **B** 0050 **C** 2250 **D** 2450

Short answer

1 During summer, a plane leaves Perth at 10:50 pm arriving in Melbourne at 5:15 am the next day. Determine the duration of the flight.

2 Assuming no DST, what time is it in Port Augusta, SA, when it is:

(a) 3:25 pm in Mount Isa, Queensland

(b) 7:40 am in Bunbury, WA?

3 A winter flight from Canberra to Perth lists departure and arrival times as 7 pm and 9:30 pm respectively. What is the duration of the flight?

4 A winter flight from Hobart to Adelaide lists departure and arrival times as 1110 and 1235 respectively. What is the duration of the flight?

5 The 9:45 am flight from Brisbane to Alice Springs takes 3 h 25 min. What is ETA (estimated time of arrival) in Alice Springs?

6 A plane leaves London, GMT+1, during summer at 4:55 pm arriving in Rome, GMT+2, at 8:35 pm. Determine the duration of the flight.

7 What time is it in Cloncurry, Qld, when it is:

(a) 6:55 pm in Houston, Texas, GMT−6

(b) 10:10 am in Beirut, Lebanon, GMT+2?

8 A flight from Salt Lake City, Utah, GMT−7, to Atlanta, Georgia, GMT−5, lists departure and arrival times as 6:55 am and 12:34 pm respectively. What is the duration of the flight?

9 A winter flight from Sao Paulo, Brazil, GMT−3, to La Paz, Bolivia, GMT−4, takes about 3 h 35 min. If the arrival time in La Paz is 1440, determine the departure time from Sao Paulo.

10 The 6:30 am flight from Hobart to Queenstown, NZ, with stops in Sydney and Auckland, takes 11 h 15 min. What is ETA (estimated time of arrival) in Queenstown, GMT+12, assuming it is a skiing trip?

Extended response

1 **(a)** Complete the table below.

State/territory	Standard time	Summer time
Queensland		
NSW (except Broken Hill)		
ACT	1200	

Victoria		
Tasmania		
NT		
SA (and Broken Hill)		
WA		

(b) Describe the time difference between Queensland and SA throughout the year.

(c) In Perth at 4 pm in February, what time is it in Melbourne?

(d) At 0920 in Hobart in August, what time is it in Adelaide?

(e) A row of clock faces near a domestic airport shows analogue and digital time for capital cities in Australia during summer. Complete the table.

Perth	Darwin	Adelaide	Sydney	Melbourne	Brisbane

2 Denpasar is the capital of Bali, an island of Indonesia. Bali shares its time zone with WA all year round. Details of Jesse's trip to Bali are as follows:

Departure: Sunday 1 May 2025			
From	To	Departure	Arrival
Brisbane	Darwin	2050	0035 2 May
Darwin	Denpasar, Bali	0930 2 May	1045 2 May

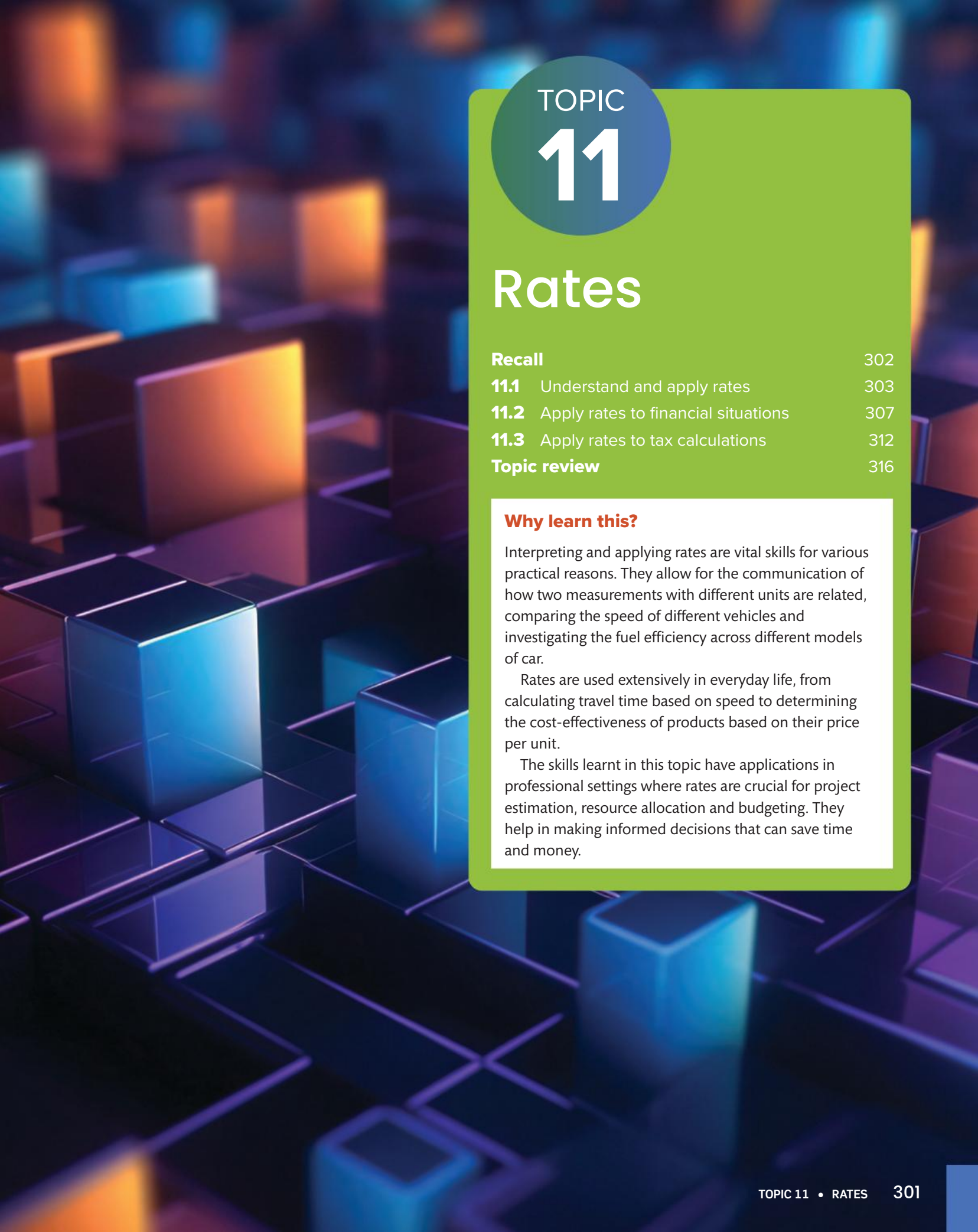
Departure: Sunday 11 May 2025			
From	To	Departure	Arrival
Denpasar, Bali	Brisbane	1450	2220

(a) Calculate the duration of each flight.

(b) Calculate the duration of the stopover in Darwin.

(c) Calculate the duration of the stay in Bali.

(d) Calculate the saving in flying time due to not splitting the trip home.



TOPIC 11

Rates

Recall 302
11.1 Understand and apply rates 303
11.2 Apply rates to financial situations 307
11.3 Apply rates to tax calculations 312
Topic review 316

Why learn this?

Interpreting and applying rates are vital skills for various practical reasons. They allow for the communication of how two measurements with different units are related, comparing the speed of different vehicles and investigating the fuel efficiency across different models of car.

Rates are used extensively in everyday life, from calculating travel time based on speed to determining the cost-effectiveness of products based on their price per unit.

The skills learnt in this topic have applications in professional settings where rates are crucial for project estimation, resource allocation and budgeting. They help in making informed decisions that can save time and money.

RECALL

ANSWERS Page 592

I can convert between different units of length, mass and money

1 Change each measurement into the units given.

(a) 20 m to cm **(b)** 3000 mL to L **(c)** 827c to $ **(d)** 357 g to kg

(e) 5.2 km to m **(f)** 3.4 L to mL **(g)** 22.51 kg to g **(h)** $0.78 to cents

I can identify and write equivalent fractions

1 Calculate the missing number in each of the following fractions.

(a) $\frac{12}{28} = \frac{\square}{7}$ **(b)** $\frac{8}{9} = \frac{24}{\square}$ **(c)** $\frac{\square}{42} = \frac{5}{7}$ **(d)** $\frac{3}{\square} = \frac{54}{72}$

2 Write true (T) or false (F) to identify whether each pair of fractions is equivalent or not.

(a) $\frac{4}{5} = \frac{24}{30}$ **(b)** $\frac{48}{54} = \frac{2}{3}$ **(c)** $\frac{18}{24} = \frac{45}{64}$ **(d)** $\frac{45}{72} = \frac{15}{24}$

I can write fractions in their simplest form

1 Simplify these fractions.

(a) $\frac{22}{55}$ **(b)** $\frac{28}{49}$ **(c)** $\frac{32}{48}$ **(d)** $\frac{36}{96}$

I can convert between mixed numerals and improper fraction forms

1 Write these mixed numerals as equivalent improper fractions.

(a) $3\frac{7}{8}$ **(b)** $6\frac{1}{7}$ **(c)** $12\frac{2}{9}$ **(d)** $8\frac{3}{5}$

2 Write these improper fractions as mixed numerals in simplest form.

(a) $\frac{9}{2}$ **(b)** $\frac{27}{5}$ **(c)** $\frac{20}{6}$ **(d)** $\frac{52}{8}$

I can interpret percentages in sentences

1 Write the percentage amount represented by these sentences.

(a) 5 out of 20 TV channels are sport channels. **(b)** 3 out of 5 students have brown hair.

(c) 6 out of 16 giraffes in the zoo were fully grown. **(d)** 9 out of 16 people have a gym membership.

I can solve one-step equations

1 Solve the following equations.

(a) $\frac{a}{2} = 7$ **(b)** $\frac{x}{5} = 4.5$ **(c)** $6y = 24$

(d) $8m = 20$ **(e)** $4n = -36$ **(f)** $\frac{p}{12} = -3.5$

Understand and apply rates

11.1

Learning intention: To be able to understand and apply rates

Success criteria:

SC 1 I understand the difference between a rate and a ratio and can convert a speed to a related rate.

SC 2 I can use rates to determine unknown values.

Lesson warm-up

Drawing stars

How quickly can you draw stars?

- Set a timer for 10 seconds and see how many stars you can draw.
- Multiply this number by 6 to find the number of stars you can draw in a minute.

What is the highest number of 'stars per minute' in the class?
Who was fastest?

Estimate how many stars your whole class could draw in 1 hour.

SC 1 I understand the difference between a rate and a ratio and can convert a speed to a related rate

Both rates and ratios compare quantities.

A ratio compares the same sorts of things, such as comparing one volume to another volume. If you are mixing cordial, you might use a water to cordial ratio of 5 : 1. This means that for every 5 cups of water, you use 1 cup of cordial, or for every 5 L of water, you use 1 L of cordial.

A rate compares different things, such as distance and time. Your heart rate is the number of times your heart beats in one minute. It is measured in beats per minute.

Another rate is speed. For example, the speed of a car is usually measured in kilometres per hour (km/h). This is the number of kilometres the car travels in 1 hour.

A rate must include units, so that it is clear what is being compared. For example, 20 km/h (the speed of a cyclist) is a lot faster than 20 metres per hour (m/h) (the speed of a very fast snail).

A ratio does not need units because it compares numbers. In effect, a ratio has the same units on both sides.

Worked example

Calculating and converting rates

(a) A bullet train in Japan travels 610 km in 2 hours. Calculate the average speed of the train in kilometres per hour (km/h).

THINKING	WORKING
Write the information as a fraction, following the order of the rate in the question. Include the units.	For the rate measured in km/h, use the distance measured in km as the numerator, and time measured in hours as the denominator. $\frac{610\text{ km}}{2\text{ hours}}$
Divide to change the fraction into a single number.	$\frac{610\text{ km}}{2\text{ hours}} = 305\text{ km/h}$
Write the answer.	The average speed is 305 km/h.

(b) Convert 72 km/h to m/s (metres per second).

THINKING	WORKING
Write the rate as a fraction, including the units.	$\frac{72\text{ km}}{1\text{ h}}$
Convert the numerator to the new unit by multiplying (for a smaller unit) or dividing (for a larger unit).	1 km = 1000 m, so $\frac{72\text{ km}}{1\text{ h}} = \frac{72\times 1000\text{ m}}{1\text{ h}} = \frac{72000}{1}\text{ m/h}$
Convert the denominator to the new unit by multiplying (for a smaller unit) or dividing (for a larger unit).	1 h = 3600 s, so $\frac{72\,000\text{ m}}{1\text{ h}} = \frac{72\,000\text{ m}}{1\times 60\times 60\text{ s}} = \frac{72\,000}{3600}\text{ m/s}$
Cancel or divide to change the fraction into a single number.	$= \frac{720}{36}$ $= 20\text{ m/s}$
Write the answer.	72 km/h = 20 m/s

SC 2 I can use rates to determine unknown values

Worked example

Using rates to calculate unknown values

How long will it take to travel 95 km at a speed of 60 km/h?

Give your answer correct to 2 decimal places.

THINKING	WORKING
Define a variable for the unknown.	Let x be the time taken to travel 95 km.
Write the rate as a fraction and write an equivalent fraction with the unknown.	$\frac{60\text{ km}}{1\text{ h}} = \frac{95\text{ km}}{x\text{ h}}$
Make sure the units are the same, then remove the units.	$\frac{60}{1} = \frac{95}{x}$
Solve for the unknown.	$\frac{x}{95} = \frac{1}{60}$ $x = 95 \times \frac{1}{60}$ $= 1.58333\ldots$ $= 1.58\text{ h (to 2 d.p.)}$
Write the answer.	It will take 1.58 hours to travel 95 km.
Check that the answer is reasonable.	60 km/h means 60 km in 1 hour, which is 120 km in 2 hours. The answer is between these values, so it is reasonable.

Practice

ANSWERS Page 593

SC 1 I understand the difference between a rate and a ratio and can convert a speed to a related rate

1 For each of the following journeys, write the average speed in km/h. Write your answers to the nearest whole number.

(a) 500 km in 8 hours

(b) 1000 km in 15 hours

(c) 300 km in $3\frac{1}{4}$ hours

(d) 420 km in $2\frac{1}{2}$ hours

2 Complete the following unit conversions. Write your answers correct to 2 decimal places where necessary.

(a) 50 km/h to m/h

(b) 50 km/h to km/s

(c) 50 km/h to m/s

(d) 3.6 km/h to m/s

3 Complete the following unit conversions.

(a) 16 m/s to km/h **(b)** 40 m/s to km/h **(c)** 100 m/s to km/h

4 Copy and complete these sentences by crossing out the incorrect options in brackets.

(a) The wind was blowing at 15 km/h from the north-east. This is an example of a rate/ratio where it is/is not important to include units.

(b) When mixing concrete, for every 1 bucket of cement you should use 2 buckets of sand and 3 buckets of stones. This is an example of a rate/ratio where units are/are not required.

5 A spaceship takes 3 days to travel the 386 400 km from Earth to the Moon. Calculate, correct to the nearest whole number, the average speed in:

(a) kilometres per hour (km/h) **(b)** metres per second (m/s).

6 90 km/h converts to exactly 25 m/s. Use this fact to write exact answers for the following conversions.

(a) Convert 1 km/h to m/s. **(b)** Convert 1 m/s to km/h.

SC 2 I can use rates to determine unknown values

1 A school excursion requires adult supervision at a rate of 0.2 adults/student.

(a) How many adults are needed for an excursion with 60 students?

(b) How many adults are needed for an excursion with 74 students?

2 At a shop, a particular sweet can be bought at a rate of 6 sweets per dollar.

(a) How many sweets can be purchased for $5?

(b) How much does it cost to buy 54 sweets?

3 Petrol costs $1.73/L.

(a) How much does it cost to buy 95 L of petrol? Give your answer to the nearest cent.

(b) How much petrol can be bought for $100? Give your answer correct to 2 decimal places.

4 On a long journey, David's car uses petrol at a rate of 5.8 L/100 km.

(a) Why do you think the rate is expressed per 100 km instead of per km?

(b) On a journey, David used 32.54 L of petrol. How far did he travel? Give your answer to the nearest kilometre.

(c) How much petrol would David use for a trip of 295 km? Give your answer to the nearest litre.

(d) A full tank holds 60 L of petrol. How far can the car travel on one tank? Give your answer to the nearest kilometre.

Apply rates to financial situations

11.2

Learning intention: To be able to apply rates to financial situations

Success criteria:

SC 1 I can apply rates to earning and spending money.

SC 2 I understand and can apply simple interest.

Lesson warm-up

Part-time job

Angelo needs a part-time job. He is trying to choose between two jobs.

He wants to know which job will pay him more money.

- Job A pays $110 for 5 hours work.
- Job B pays $140 for 7 hours work.

Which job pays more? How can you work it out?

What other information do you think Angelo will use to help him choose?

SC 1 I can apply rates to earning and spending money

People use rates to describe financial situations, such as earning money and buying items.

Knowing a rate helps you to make predictions about how much you will earn or spend. For example,

- if you know that your rate of pay is $22/h, you can predict how many hours you need to work to earn the money you need
- if you know that somethings costs $4.99/kg, you can predict the total cost of different weights of the product
- if you know that one brand of a product costs $12/100 g and another brand costs $11/100 g, you can see which brand is cheaper.

Worked example

Calculating money earnt and spent

Pip has a part-time job at a cafe. Her rate of pay is $17.50/h.

(a) One week, Pip works for 12.5 hours. Calculate how much money she earns in the week.

THINKING	WORKING
Write the meaning of the rate.	$17.50/h means that Pip is paid $17.50 for each hour she works.
Multiply the rate by the number of hours.	$17.50 \times 12.5 = 218.75$
Write the answer.	Pip earns $218.75

(b) How many hours does Pip need to work, to the nearest half-hour, to earn over $150?

THINKING	WORKING
Define a variable for the unknown.	Let the number of hours be n.
Write the rate as a fraction and write an equivalent fraction that includes the unknown.	$\frac{\$17.50}{1\text{ hour}} = \frac{\$150}{n\text{ hours}}$
Make sure the units are the same, then remove the units.	$\frac{17.50}{1} = \frac{150}{n}$
Solve for the unknown.	$17.5 \times n = 150 \times 1$ $n = \frac{150}{17.5}$ $= 8.57\ldots$
Read the question again and decide how to round the answer.	Round up the number of hours so that the amount earnt is larger than $150.
Write the answer.	To the nearest half-hour, Pip needs to work for 9 hours to earn over $150.

(c) Pip orders 30 kg of coffee beans. The total cost is $714.60. What is the cost per kilogram?

THINKING	WORKING
Write the information as a fraction, following the order of the rate in the question. Include the units.	Cost per kilogram: $\frac{\$714.60}{30\text{ kg}}$
Divide to change the fraction into a single number.	$\frac{\$714.60}{30\text{ kg}} = \$23.82/\text{kg}$
Write the answer.	The coffee beans cost $23.82/kg.

SC 2 I understand and can apply simple interest

When you keep money in a bank, the bank pays you a small amount. This amount is called interest. The longer your money is in the bank, and the more money you have, the more interest you are paid.

The interest rate is usually expressed as a percentage per year. For example, for an interest rate of 4% per year, the bank pays you 4% of your money every year. Another way or writing 'per year' is 'per annum' or 'p.a.'.

The simplest type of interest is called 'simple interest'. You can calculate simple interest using what you have learnt about rates, or by using a formula.

A formula for simple interest is:

Simple interest = principal × rate × time

$$I = PRT$$

- The principal is the amount of money you have in the bank.
- The rate is usually a percentage per year, written as a decimal.
- The time is the length of time your money is in the bank. The units you use for the time must match the rate: for example, for a rate of 4% per year, the time must be in years.

Worked example

Calculating simple interest

(a) Camryn has $2500 in a bank account that pays simple interest at a rate of 3.15% p.a. Calculate the amount of interest she earns in a year.

THINKING	WORKING
Calculate the percentage of the amount in the account.	$3.15\% \text{ of } \$2500 = \frac{3.15}{100} \times \frac{2500}{1}$
Calculate the value of the expression.	$= \frac{3.15 \times 25}{1}$ $= 78.75$
Write the answer.	Camryn earns $78.75 interest in a year.

(b) Ravi has $7640 in a bank account that pays 4.05% p.a. Use the simple interest formula to calculate the amount of interest he earns in 200 days. Give your answer to the nearest cent.

THINKING	WORKING
Compare the units in the question.	The rate is 'per annum', which means per year. The time is in days, so it needs to be converted to years.
Recall the simple interest formula.	$I = PRT$
Identify the values to be used in the formula.	$P = \$7640$ $R = 4.05\% = 0.0405$ $T = \frac{200}{365}$ years
Substitute the values in and evaluate.	$I = 7640 \times 0.0405 \times \frac{200}{365}$ $= 169.55 \text{ (2 d.p.)}$
Write the answer.	Ravi earns $169.55 interest in 200 days.

Practice

ANSWERS Page 593

SC 1 I can apply rates to earning and spending money

1 For each of the following, calculate the total earnings. Write your answers correct to the nearest cent.

(a) 12.5 hours at $16.85/h **(b)** 22.75 hours at $19.22/h

(c) 18.3 hours at $21.97/h **(d)** 33 hours at $26.45/h

2 For each of the following pay rates, calculate the time that must be worked to earn at least $350. State your answers to the nearest half hour.

(a) $16.75/h **(b)** $22.22/h **(c)** $26.50/h **(d)** $47/h

3 For each of the following items, calculate the price per unit. State your answers to the nearest cent.

(a) A 292 g piece of fish sells for $13.50. What is the price per kilogram?

(b) A 1.5 L bottle of orange juice sells for $5. What is the price per litre?

(c) A 300 g portion of cheese sells for $8.50. What is the price per kilogram?

4 A supermarket labels products with the cost per 100 g, so customers can compare different brands and packet sizes. Calculate the cost per 100 g for each of the following items.

(a) A 5 kg bag of rice costs $11.50.

(b) A 375 g box of cereal costs $3.25.

(c) A 1 kg bag of muesli costs $4.

5 People are not always paid per hour. In some industries, people are paid per item completed. This is called a 'piece rate'.

(a) Stella is paid $0.50 per shirt sewn. How much does she earn for sewing 99 shirts?

(b) Joost is paid $0.95 per tray of strawberries picked. How much does he earn for picking 310 trays?

SC 2 I understand and can apply simple interest

1 Jermaine has $7000 in a bank account that pays simple interest at a rate of 2% p.a. Calculate the interest earnt in each of the following time periods. Give your answers correct to the nearest cent.

(a) 1 year **(b)** 3 years **(c)** 6 months **(d)** 100 days

2 Sara has $15 000 in a bank account. Calculate how much interest she earns in 2 years at each of the following interest rates.

(a) 1% p.a. **(b)** 3% p.a. **(c)** 6% p.a. **(d)** 0.1% p.a.

3 Calculate the time it would take to earn the amount of interest stated. Give answers to the nearest day.

(a) A deposit of $3000 at 2% p.a. earns $55 interest.

(b) A deposit of $990 at 1.1% p.a. earns $15 interest.

4 Calculate the interest rate for each of the following situations. Give each answer as a percentage, correct to 2 decimal places.

(a) A deposit of $1000 earns $100 interest in 5 years.

(b) A deposit of $650 earns $10 interest in 200 days.

(c) A deposit of $3600 earns $65 interest in 150 days.

5 Michelle has $2500 in an account that pays simple interest at a rate of 1.3% p.a.

(a) How much interest would Michelle earn in a full year?

(b) After 150 days, Michelle withdraws the money and the interest she has earnt in that time. She deposits the combined amount in a new account that pays simple interest at a rate of 1.55% p.a. How much interest does Michelle earn on this deposit in 215 days?

(c) In total, how much interest does Michelle earn over the 365 days?

(d) What simple interest rate would give Michelle this amount of interest in 1 year? Give your answer as a percentage, correct to 2 decimal places.

Exploration activity

ANSWERS Page 593

Get rich quick?

The value of shares of Company X is predicted to grow at a rate of 2.5% per annum over the next year. This 'growth' is paid to shareholders in the form of dividends. A dividend is a distribution of profits by a corporation to its shareholders.

Shareholders choose between the following two options.

- Option 1: Get the dividends paid once a year.
- Option 2: Get the dividends paid every quarter.

Scenario 1

Sam claims that the second option is better because you get paid more money in total. Do you agree?

Scenario 2

The company now provides the option of reinvesting the dividends, either every quarter or once per year. Sam still claims that the quarterly option is better because you got more money. Do you agree?

11.3 Apply rates to tax calculations

Learning intention: To be able to apply rates to tax calculations

Success criteria:

SC 1 I can calculate tax payable from taxation tables.

SC 2 I can solve problems involving taxation.

Lesson warm-up

Why do we have tax?

List five things in your community that people can use without paying. (The local library is an example.)

How does the community pay for these things?

SC 1 I can calculate tax payable from taxation tables

Tax is money collected by local, state and federal governments to pay for services such as schools, roads and hospitals. There are lots of different types of tax.

Income tax is paid to the government by people who earn money. The amount of tax a person pays depends on how much they earn in the year (their income).

We can use a table such as the one below to calculate how much tax a person needs to pay.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	19 cents for each $1 over $18 200
$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia

- The table has five rows, showing five ranges of income. Each range is called a 'tax bracket'.
- People in the lowest tax bracket pay no income tax (nil).
- Each bracket has a different calculation that includes a rate.
- A person's 'taxable income' is their earnings minus any 'deductions', such as work-related expenses and donations.

Worked example

Calculating tax from tax tables

Use the tax table to calculate the tax to be paid on each taxable income.

(a) $15 482

THINKING	WORKING
Determine the tax bracket from the tax table.	$15 482 is in the range 0–$18 200.
Determine the tax on this income.	Tax payable is nil, so $0.

(b) $25 631

THINKING	WORKING
Determine the tax bracket from the tax table.	$25 631 is in the range $18 201–$45 000.
Identify the rule for this tax bracket.	19 cents for each $1 over $18 200.
Subtract to determine the amount the tax rate applies to.	Amount over $18 200 is: $25 631 − $18 200 = $7431
Write the tax rate in dollars.	19 cents = $0.19
Multiply the amount that the tax rate applies to by the tax rate per dollar.	0.19 × 7431 = $1411.89
Write the answer.	The tax payable on $25 631 is $1411.89.

(c) $103 528

THINKING	WORKING
Determine the tax bracket from the tax table.	$103 528 is in the range $45 001–$120 000.
Identify the rule for this tax bracket.	$5092 plus 32.5 cents for each $1 over $45 000.
Subtract to determine the amount the tax rate applies to.	$103 528 − $45 000 = $58 528
Write the tax rate in dollars.	32.5 cents = $0.325
Multiply the amount that the tax rate applies to by the tax rate per dollar.	0.325 × 58 528 = $19 021.60
Follow the rule and add the two amounts.	$5092 + $19 021.60 = $24 113.60
Write the answer.	The tax payable on $103 528 is $24 113.60.

SC 2 I can solve problems involving taxation

So that people do not have to pay all their tax at once, pay as you go (PAYG) tax is collected throughout the year.

This tax is collected every time a person gets paid – for example, weekly, fortnightly or monthly. The amount collected is an estimate based on their predicted yearly income.

At the end of the financial year, people fill in a form called a tax return. The Australian Tax Office (ATO) then calculates whether a person has paid too much tax or not enough. If they have paid too much tax, the person receives a tax refund. If they have not paid enough tax, the person must pay the difference to the ATO.

An extra payment, called the Medicare levy, helps to pay for the cost of medical treatment in the community. This is 2% of taxable income for people who earn at least $29 207 a year.

Worked example

Calculating tax collected on fortnightly earnings

Alex is paid $1850 each fortnight.

(a) Calculate the tax collected from Alex's earnings of $1850 each fortnight.

THINKING	WORKING
Calculate the predicted annual earnings, assuming there are 26 fortnights in a year.	$1850 × 26 = $48 100
Identify the tax bracket.	Tax bracket: $45 001–$120 000 Tax on this income: $5092 plus 32.5 cents for each $1 over $45 000.
Calculate the tax to be paid on the predicted annual earnings.	$48 100 – $45 000 = $3100 Tax: 0.325 × $3100 = $1007.50 Tax to be paid: $5092 + $1007.50 = $6099.50
Calculate the tax per fortnight, assuming there are 26 fortnights in a year.	$6099.50 ÷ 26 = $234.60
Write the answer.	The tax collected will be $234.60 each fortnight.

(b) Calculate the amount Alex takes home (his take-home pay) after tax and the Medicare levy have been removed.

THINKING	WORKING
Calculate the Medicare levy as 2% of the fortnightly wage.	Medicare levy: $1850 × 0.02 = $37
Subtract the fortnightly tax and Medicare levy from the fortnightly earnings.	Take-home pay: $1850 – $234.60 – $37 = $1578.40
Write the answer.	Alex takes home $1578.40 each fortnight.

Practice

ANSWERS Page 593

SC 1 I can calculate tax payable from taxation tables

1 Use the tax table to calculate the tax payable on a taxable income of $38 491.

2 Use the tax table to calculate the tax payable on each of the following taxable incomes.

(a) $45 000 **(b)** $45 001

3 Use the tax table to calculate the tax payable on a taxable income of $70 280.

4 Determine how much more tax is paid by a person with a taxable income of $80 000 than by a person with a taxable income of $75 000.

5 A person working as a casual driver earns $15 000. They also have a second job. How much tax do they pay on this part of their income, given that they already have a taxable income of:

(a) $25 000 **(b)** $50 000?

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$45 000	19 cents for each $1 over $18 200
$45 001–$120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001–$180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia

SC 2 I can solve problems involving taxation

1 Calculate the tax collected from a worker who earns $4230 per fortnight.

2 A worker earns $1210 per week.

(a) Calculate the tax collected each week.

(b) Calculate the amount the worker takes home each week, after tax and the 2% Medicare levy have been removed.

3 A worker has an annual income of $50 000. Determine the overall percentage of their earnings paid in tax, including the Medicare levy.

4 Caron used to earn $5800 each month. She now earns $1000 less each month. Calculate the reduction in her monthly take-home pay (after tax and the Medicare levy have been removed).

TOPIC REVIEW

Multiple choice

ANSWERS Page 627

1 290 km travelled in 3 hours 25 minutes is an average speed closest to:

A 82.86 km/h **B** 84.88 km/h **C** 89.23 km/h **D** 96.67 km/h

2 42 km travelled in 40 minutes is an average speed closest to:

A 0.95 km/min **B** 1.05 km/min **C** 63 km/min **D** 1680 km/min

3 560 m travelled in 0.05 hours is an average speed closest to:

A 3.11 m/s **B** 28 m/s **C** 155.56 m/s **D** 186.67 m/s

4 The time taken to travel 350 km at an average speed of 110 km/h is closest to:

A 19 minutes **B** 31 minutes **C** 3 h 11 min **D** 3 h 18 min

5 Which of the following is the closest to the distance travelled in 5 h 45 min at an average speed of 92.5 km/h?

A 504.1 km **B** 531.9 km **C** 1609 km **D** 1697 km

6 Petrol costs $1.93/L. Correct to 2 decimal places, the amount of petrol that can be purchased for $125 is:

A 64.77 L **B** 123.07 L **C** 126.93 L **D** 241.25 L

7 Carmella earns $32.55/h. Her wage in a week when she works 36 hours is:

A $110.60 **B** $904.17 **C** $1105.99 **D** $1171.80

8 The local butcher's shop is selling chuck steak at $11.99/kg. Stella can afford $10 for some of this meat. Which of the following best represents the amount of steak Stella can purchase?

A 0.834 kg **B** 0.99 kg **C** 1.199 kg **D** 1.99 kg

9 Angelo earns $21.45/h. One week he takes home $664.95. This means he has worked:

A 14.26 h **B** 31 h **C** 32.25 h **D** 64.35 h

10 Graham deposits $1500 into an account paying 2.75% p.a. simple interest. The amount in the account after the interest is added at the end of one year is:

A $41.25 **B** $412.50 **C** $1541.25 **D** $1912.50

11 Peta deposits $4750 into an account paying 1.95% p.a. simple interest. She withdraws her money, plus interest, after 275 days. The amount of interest Peta receives is:

A $69.79 **B** $92.63 **C** $4430.21 **D** $4819.79

12 Charlotte earnt $55.65 in interest on a deposit of $3000 for 400 days. The annual rate of simple interest she receives is closest to:

A 1.69% p.a. **B** 1.86% p.a. **C** 2.03% p.a. **D** 2.40% p.a.

13 On a particular day the exchange rate is A$1 = US$0.645. At this rate, US$550 is equivalent to:

A A$354.75 **B** A$485.50 **C** A$614.50 **D** A$852.71

14 On a particular day the exchange rate is A$1 = NZ$1.11. Bernice is on holiday in New Zealand and is trying to decide whether to buy a jumper for NZ$130. The equivalent price in A$ is:

A $117.12 **B** $128.89 **C** $131.11 **D** $144.30

15 Given 1 km/h = 0.621371 miles/h, then 75 km/h is closest to:

A 46.6 miles/h **B** 68.8 miles/h **C** 120.7 miles/h **D** 137.1 miles/h

Short answer

1 In September 2022, the following were the world records for various distances in freestyle swimming.

	50 m	100 m	200 m	400 m
Men	20.91 s	46.86 s	1 min 42.00 s	3 min 40.07 s
Women	23.67 s	51.71 s	1 min 52.98 s	3 min 56.40 s

(a) Rewrite the table showing the speed in m/s, correct to 2 decimal places.

(b) Rewrite the table showing the speed in km/h, correct to 2 decimal places.

2 On a long journey Jessica's car uses 7.1 L/100 km.

(a) To the nearest kilometre, how far did Jessica travel if she used 52.54 L of petrol?

(b) Correct to 2 decimal places, how much petrol would be used for a trip of 595 km?

(c) If a full tank is 95 L what is the theoretical range, correct to the nearest kilometre, for the vehicle?

3 Monica earns $22.50/h for the first 36 hours she works in a week. For any hours after that she is paid time-and-a-half, which means for each extra hour she works she is paid as if it was 1.5 hours. Determine Monica's wage for each of the weeks where she works the following hours.

(a) 35 hours **(b)** 39 hours **(c)** 41 hours

4 Ray deposits $5000 into an account paying 2.15% p.a. simple interest.

(a) How much interest is earnt in the first year?

At the end of the first year, Ray has a further $5000 to invest.

(b) If he withdrew the interest and added the new deposit to the existing principal, how much interest would he earn in the next year, assuming the interest rate remains the same?

(c) If instead, Ray keeps kept the interest in the account and just adds the new deposit, how much interest would he earn in the second year?

5 On a particular day, the following exchange rates apply to various currencies compared to the Australian dollar. Assume these rates apply when answering the following questions.

Country	Currency	A$1 equivalent
Japan	Yen	88.28
Europe	Euro	0.6428
UK	Pound sterling	0.5745
Thailand	Baht	23.29

(a) How many Australian dollars would you receive for 65 000 baht?

(b) How many euros would you receive for A$12 500?

(c) How many Australian dollars would you receive for £555 pounds sterling?

(d) How many yen would you receive for A$6540?

6 In September 2022, the NASA DART spacecraft deliberately crashed into the asteroid Dimorphos at a reported speed of 6.44 km/s.

(a) Express this speed in km/h.

(b) Express this speed in m/s.

(c) At this speed, how far did the spacecraft travel in its final 5 minutes?

(d) A newspaper headline reported the speed at impact as 22 500 km/h. What is this in km/s, correct to 2 decimal places?

7 Convert the following annual interest rates to the time period shown. Where necessary, write the answers as percentages correct to 4 decimal places.

(a) 3.05% p.a. as a monthly rate

(b) 2.37% p.a. as a daily rate

(c) 1.99% p.a. as a weekly rate

8 A department store is having a sale. The full price of a shirt is $79.99, but there are several different offers available. Work out the cost per shirt in each of the following circumstances, stating your answers correct to the nearest cent.

(a) Buy three shirts, get a fourth shirt free.

(b) Buy two shirts, get a third shirt at half-price.

(c) Buy one shirt and get 25% off a second shirt.

9 The fuel excise tax on petrol and diesel is 46 c/L.

(a) The local petrol station sells 23 500 L of petrol and diesel one day. How much fuel excise tax will they need to pay for this?

(b) Another petrol station owes $28 560 in fuel excise tax. How many litres, to the nearest litre, of fuel does this represent?

10 At the local bakery a single Portuguese custard tart costs $3.50. However, a 4-pack of the same tarts costs $13.

(a) What is the discount in dollars and cents for the 4-pack?

(b) What is the percentage discount, based on the selling price?

(c) What is the price per tart for the 4-pack?

11 Calculate the average speed in km/h for each of the following journeys. Where necessary, state the answers correct to 2 decimal places.

(a) 250 km in 3.25 h

(b) 620 km in 9.15 h

(c) 830 km in 6 h 25 min

12 One of the questions on a topic test asked students to convert US$650 into A$, where the conversion rate was A$1 = US$0.6434. Janelle gave the answer as A$418.21.

(a) Give a brief explanation, without doing a calculation, of how you can tell this answer is incorrect.

(b) What is the correct answer?

(c) What do you think Janelle did wrong?

Extended response

1 In 2022 the law was changed regarding piecework rates. Although workers could still be paid by the piece or item, it was now required that the hourly wage could not be less than the national minimum adult wage of $25.41/h.

- **(a)** Alphonse was picking strawberries and could fill 20 punnets in an hour. What is the minimum piece rate he could be working at, rounded to the nearest cent?
- **(b)** Celeste was picking apricots and was paid by the bucket. If the rate per bucket is $2.55, what is the minimum number of buckets expected to be filled each hour?
- **(c)** Kyah works in a laundry, where her job is to iron the towels for the accommodation venues that use the laundry. Her piece rate is $0.78/towel. How many towels must she iron to make the minimum wage?
- **(d)** The legislation also requires that some workers must be able to earn 15% above the minimum hourly rate. At the orchard where Celeste works, how many buckets per hour would this represent?

2 A particular soft drink is available in a variety of sizes. On one day at the local supermarket the following options were noted. In each case calculate the cost per litre.

- **(a)** 300 mL × 4-pack for $6.15
- **(b)** 1.25 L bottle for $2.45
- **(c)** 2 L bottle for $3.65
- **(d)** 300 mL × 12-pack for $10
- **(e)** 250 mL mini-can chilled for $2.25
- **(f)** Discuss which option is the most cost-effective, taking into account the temperature and packaging of the drinks.

Exploration activity

ANSWERS Page 628

Fastest on Earth

What are the fastest animals in the world? How fast can they run, fly or swir

In this activity you will compare the maximum speeds of different animals v their speeds are given in different units. Many of the data sources are from t United States and give the speed of an animal in miles per hour (mph). Other sources give the speed in km/h or m/s.

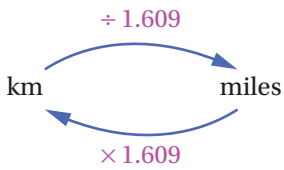
It is easier to compare the speeds of the different animals when the values are in the same units. To convert between mph and km/h, only the distance part (miles) needs to be changed (to kilometres), or vice versa. This can be done using the conversion diagram below (given to 3 decimal places).

1 Create and complete a table to determine the maximum speeds recorded for each type of animal. Give your answers as mph, km/h and m/s, rounding to 1 decimal place (where necessary).

2 Which of these is the fastest animal?

3 Which of these is the slowest animal?

4 Why is it important to convert rates to the same units?



TOPIC
12

Pythagoras' theorem in 2D

Recall 321

12.1 Measure the side lengths of a right-angled triangle 322

12.2 Understand and use Pythagoras' theorem to identify right-angled triangles 328

12.3 Compare different applications, demonstrations and proofs of Pythagoras' theorem 337

12.4 Use Pythagoras' theorem to determine the length of the hypotenuse 344

12.5 Use Pythagoras' theorem to determine the length of a shorter side in a right-angled triangle 349

Topic review 354

Why learn this?

Pythagoras' theorem is one of the most recognisable and fundamental theorems in mathematics.

Learning Pythagoras' theorem provides an additional tool to explore and understand the world in a more structured and quantitative way, making it easier to engage with tasks that require precise calculations and spatial reasoning.

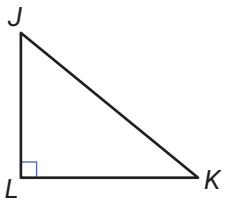
RECALL

ANSWERS Page 594

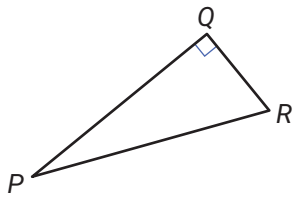
I can identify the hypotenuse of a right-angled triangle

1 Name the hypotenuse in each of these triangles.

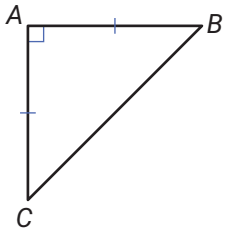
(a)



(b)



(c)



I can round decimals numbers

1 Round each of the following numbers to 2 decimal places.

(a) 2.018 **(b)** 6.794 **(c)** 1500.156 **(d)** 239.800 923

I can perform calculations with square numbers

1 Calculate the following squares.

(a) 6^2 **(b)** 1.2^2 **(c)** $\left(\frac{1}{2}\right)^2$ **(d)** $\left(\sqrt{11}\right)^2$

2 Evaluate each of the following, correct to 2 decimal places.

(a) 1.25^2 **(b)** 6.05^2 **(c)** 0.07^2 **(d)** 1.203^2

3 Evaluate each of the following, rounding the result correct to 2 decimal places.

(a) $0.25^2 + 1.2^2$ **(b)** $2.4^2 + 3.85^2$ **(c)** $544.73^2 - 470.56^2$

I can use a calculator to determine exact and approximate square roots

1 Use your calculator to find the exact value of each of the following.

(a) $\sqrt{441}$ **(b)** $\sqrt{2704}$ **(c)** $\sqrt{0.0225}$ **(d)** $\sqrt{0.1296}$

2 Use your calculator to find the value of each of the following, rounded to 2 decimal places.

(a) $\sqrt{7}$ **(b)** $\sqrt{11}$ **(c)** $\sqrt{31}$ **(d)** $\sqrt{123}$

I can solve linear equations

1 Solve each of the following equations for a.

(a) $a + 11 = 81$ **(b)** $36 + a = 64$ **(c)** $4 - a = 16$

I can solve equations including squared terms

1 Solve each of the following equations for positive values of x, rounding to 2 decimal places, if required.

(a) $x^2 = 121$ **(b)** $x^2 = 1764$ **(c)** $x^2 = 37$ **(d)** $x^2 = 185$

12.1 Measure the side lengths of a right-angled triangle

Learning intention: To be able to measure the sides of right-angled triangles

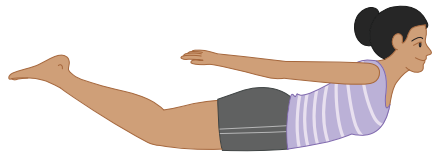
Success criteria:

SC 1 I can identify the hypotenuse in a right-angled triangle.

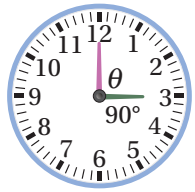
SC 2 I can construct and measure the side lengths of a right-angled triangle.

Lesson warm-up

When is the time 'right'?



In the clock below, the hands form a right angle when displaying the time 3:00.



Write down three other times where the hands will form a right angle.

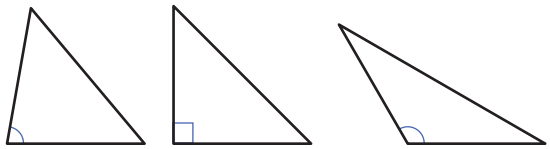
SC 1 I can identify the hypotenuse in a right-angled triangle

Triangle properties

All triangles contain at least two internal acute angles.

The longest side of any triangle is always opposite the largest internal angle.

The largest angle of a triangle can be an acute angle, a right angle or an obtuse angle.



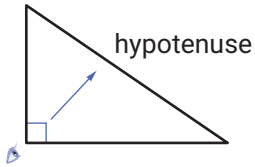
The longest side of a right-angled triangle is called the hypotenuse.

The hypotenuse is the side opposite the right angle.

Right-angled triangles

The longest side of a right-angled triangle is called the hypotenuse.

The hypotenuse is located opposite the largest angle, the right angle.

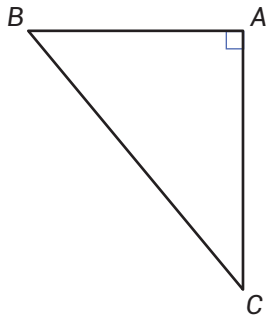


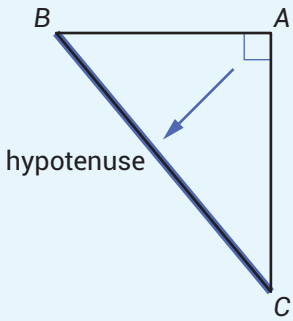
Worked example

Identifying the right angle and the hypotenuse

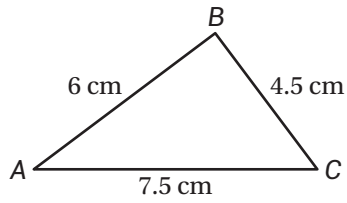
Identify and name the right angle and hypotenuse for each of the right-angled triangles.

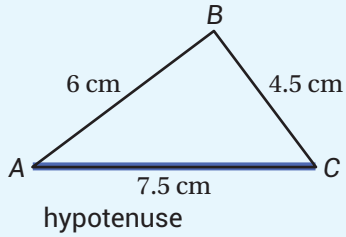
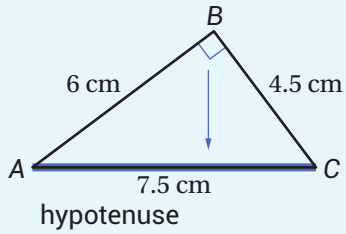
(a)



THINKING	WORKING
Identify the known information.	The symbol for a right angle is ∟. On the diagram, it looks like a square.
Locate the right angle.	The right angle is at vertex A.
Name the angle. One way to name angles is to write the angle symbol and the label of the vertex (the point where two sides meet).	The right angle can be labelled as $\angle A$.
Another way to name the angle is to identify the two edges (arms) that form the angle. This is written with the angle symbol, followed by the three vertex labels in the order they are joined to form the angle.	The right angle can be labelled as $\angle BAC$ (or $\angle CAB$).
Locate the hypotenuse.	The hypotenuse is the longest side length and lies opposite the right angle in a right-angled triangle. 
Name the hypotenuse using the vertex labels at either end of the side. The vertices are usually written in alphabetical order.	The hypotenuse is BC.

(b)



THINKING	WORKING
Identify the known information.	The side lengths are marked.
Locate the hypotenuse.	The hypotenuse is the longest side length in a right-angled triangle. The side length marked 7.5 cm is the hypotenuse.  hypotenuse
Name the hypotenuse using the vertex labels at either end of the side. The vertices are usually written in alphabetical order.	The hypotenuse is AC.
Locate the right angle.	The right angle is the largest angle; it is opposite the hypotenuse.  hypotenuse
Name the angle.	Written as a single vertex, the right angle is $\angle B$. The right angle can also be written as $\angle ABC$ (or $\angle CBA$).

SC 2 I can construct and measure the side lengths of a right-angled triangle

A right-angled triangle can be constructed from the given lengths of any two sides.

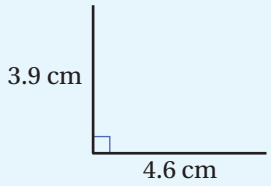
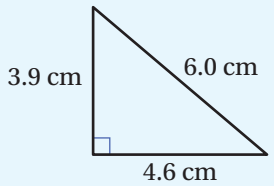
Determine whether the two side lengths given are the two shorter sides (drawn at a right angle) or the hypotenuse and one of the shorter side lengths, then use a ruler and a protractor to draw the triangle.

Worked example

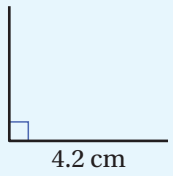
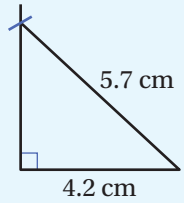
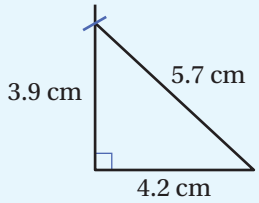
Constructing right-angled triangles given the lengths of two sides

Use a ruler, a protractor and a compass to construct a right-angled triangle as directed. Measure the length of the other side to the nearest millimetre.

(a) The length of the two shorter sides are 4.6 cm and 3.9 cm.

THINKING	WORKING
Draw a horizontal line using the first given length. Form a right angle at one end and extend the vertical line to the second given length.	
Join the ends of the horizontal and vertical lines to form the hypotenuse. Measure the length of the hypotenuse and write it onto the diagram.	
State the length of the hypotenuse to the nearest centimetre.	The hypotenuse is 6.0 cm.

(b) The hypotenuse is 5.7 cm and a shorter side is 4.2 cm.

THINKING	WORKING
Draw a horizontal line using the given shorter side length. Form a right angle at one end and draw an extended vertical line.	
From the other end of the horizontal line, draw the hypotenuse the required length to just meet the vertical line (extended, if necessary).	
Measure the length of the vertical side and write it on the diagram.	
State the length of the third shorter side to the nearest centimetre.	The other short side is 3.9 cm.

Practice

ANSWERS Page 594

SC 1 I can identify the hypotenuse in a right-angled triangle

1 Given the lengths of sides in each of the following right-angled triangles, identify the length of the hypotenuse.

(a) 3 mm, 4 mm, 5 mm **(b)** 5 mm, 12 mm, 13 mm **(c)** 9 mm, 12 mm, 15 mm

(d) Explain how you determined which side length represented the hypotenuse in each case.

2 Use the side lengths in **(a)** to **(e)** below to determine whether they form a right-angled triangle.

First choose an appropriate method. Will you:

(i) draw the two shorter side lengths at a right angle, then measure the length of the hypotenuse to see whether it matches the third side given?

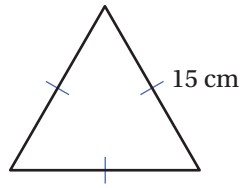
(ii) draw one side length and use a compass to mark the arcs onto which the other two edges meet, then measure the largest angle to see whether it is a right angle?

(iii) use a different method? Please describe.

(a) 6 cm, 14.4 cm, 15.6 cm **(b)** 6 cm, 7.5 cm, 11 cm **(c)** 6 cm, 4.5 cm, 11 cm

(d) 4.2 cm, 14.4 cm, 15 cm **(e)** 3.2 cm, 4.1 cm, 4.8 cm

3 An equilateral triangle has equal side lengths of 15 cm. If you were to cut the equilateral triangle into two right-angled triangles, what would be the length of each of the hypotenuse?



4 Sketch a triangle with side lengths 10 cm and 8 cm. Use the line of symmetry to cut the isosceles triangle into two right-angled triangles. State the length of each hypotenuse in the right-angled triangles.

5 Sketch a triangle with side lengths 10 cm and 14 cm. Draw a line through the isosceles triangle to form two right-angled triangles.

(a) Describe how to draw the line to create two right-angled triangles with the same length hypotenuse.

(b) Determine the shortest possible length of the hypotenuse.

(c) Determine the longest possible length of the hypotenuse.

SC 2 I can construct and measure the side lengths of a right-angled triangle

1 Draw a right-angled triangle using the two shorter side lengths listed. In each case, label and measure the hypotenuse.

(a) $AB = 4$ cm and $AC = 5$ cm **(b)** $AB = 4$ cm and $BC = 6$ cm

2 A right-angled triangle has side lengths of 5.8 cm and 7.8 cm. By drawing and measuring, determine the length of the third side, correct to the nearest millimetre, given:

(a) neither given side is the hypotenuse

(b) one of the given sides is the hypotenuse.

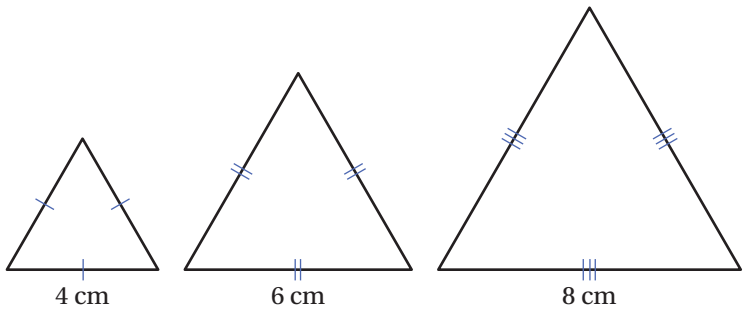
3 An isosceles triangle has side lengths of 5.8 cm and 7.8 cm. By drawing and measuring, determine the length of the line of symmetry, correct to the nearest millimetre, given:

(a) the equal sides are shorter than the third side

(b) the equal sides are longer than the third side.

4 An equilateral triangle has side lengths of 6.6 cm. By drawing and measuring, determine the length of the line of symmetry inside the triangle.

5 Draw three equilateral triangles of side lengths as shown in the diagram.



(a) Describe the side lengths of the second and third triangles, in terms of the side lengths of the first triangle.

(b) Measure the length of the line of symmetry for each triangle to the nearest millimetre.

(c) Describe the lengths of the lines of symmetry of the second and third triangles, in terms of the length of the line of symmetry of the first triangle (allowing for rounding errors).

12.2 Understand and use Pythagoras' theorem to identify right-angled triangles

Learning intention: To be able to understand and use Pythagoras' theorem to identify right-angled triangles

Success criteria:

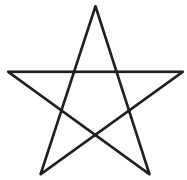
SC 1 I can establish the relationship between the side lengths in a right-angled triangle.

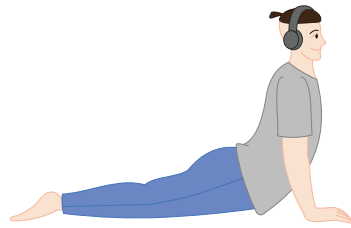
SC 2 I can identify and use Pythagorean triples.

SC 3 I can recognise the relationship between the squares of lengths of sides for different types of triangles.

Lesson warm-up

Identify this shape

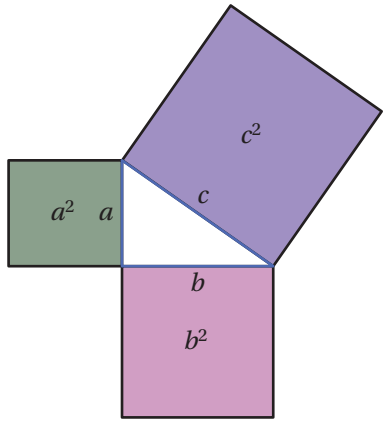




SC 1 I can establish the relationship between the side lengths in a right-angled triangle

An introduction to Pythagoras' theorem

Pythagoras' theorem states that for any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



For the triangle shown, the hypotenuse is labelled c and the two shorter side lengths are labelled a and b. Using these variables, Pythagoras' theorem is $c^2 = a^2 + b^2$.

The converse of Pythagoras' theorem

The converse of a theorem is when the conclusion is switched with the hypothesis.

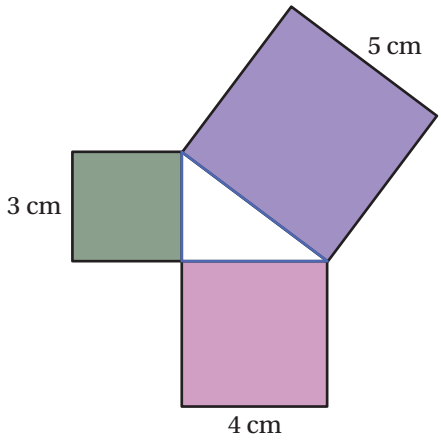
The converse of Pythagoras' theorem is that if the square of the longest side is equal to the sum of the squares of the other two sides, then the triangle will be right-angled.

Worked example

Using the converse of Pythagoras' theorem

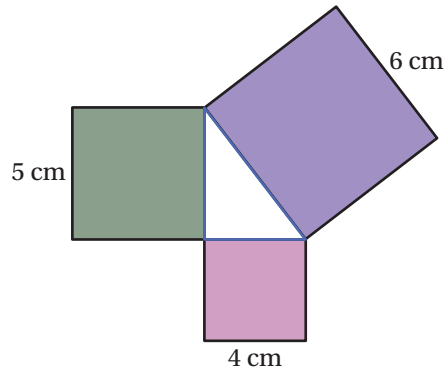
Determine whether each of the following triangles is right-angled.

(a)



THINKING	WORKING
Recall Pythagoras' theorem.	The triangle is a right-angled triangle if the sum of the squares of the shorter lengths is equal to the square of the hypotenuse (longest side length).
Identify the two shorter side lengths.	The two shorter side lengths are 3 cm and 4 cm.
Determine the sum of the squares of the two shorter side lengths.	$3^2 + 4^2 = 3 \times 3 + 4 \times 4$ $= 9 + 16$ $= 25$ The sum of the areas of the squares of the two shorter lengths is 25 cm^2.
Identify the longest side length.	The longest side length is 5 cm.
Determine the square of the longest side length.	$5^2 = 5 \times 5$ $= 25$ The area of the square of the longest side is 25 cm^2.
Compare the sum of the squares of the two shorter sides with the square of the longest side length.	The square of the longest side is equal to the sum of the squares of the shorter lengths.
Write a conclusion.	The triangle is a right-angled triangle. The hypotenuse is 5 cm, and the right angle is opposite the hypotenuse.

(b)



12

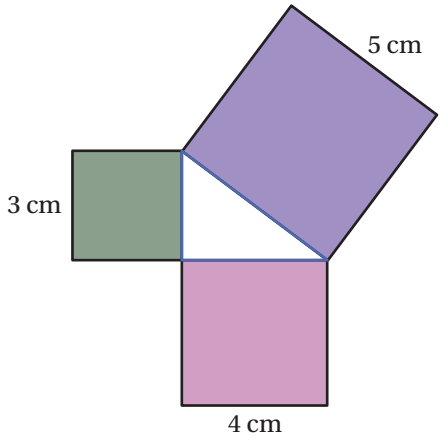
THINKING	WORKING
Recall the converse of Pythagoras' theorem.	If the square of the longest side is equal to the sum of the squares of the other two sides, then the triangle will be right-angled.
Identify the two shorter side lengths.	The two shorter side lengths are 4 cm and 5 cm.
Determine the sum of the squares of the two shorter side lengths.	$4^2 + 5^2 = 4 \times 4 + 5 \times 5$ $= 16 + 25$ $= 41$ The sum of the areas of the squares of the two shorter lengths is 41 cm^2.
Identify the longest side length.	The longest side length is 6 cm.
Determine the square of the longest side length.	$6^2 = 6 \times 6$ $= 36$ The area of the square of the longest side is 36 cm^2.
Compare the sum of the squares of the two shorter sides with the square of the longest side length.	The square of the longest side is not equal to the sum of the squares of the shorter lengths.
Write a conclusion.	The triangle is not a right-angled triangle.

SC 2 I can identify and use Pythagorean triples

Pythagorean triples

A Pythagorean triple is an ordered set of three positive integers, such as (3, 4, 5), where the numbers all satisfy Pythagoras' theorem, with the hypotenuse as the last value.

For (3, 4, 5): $3^2 + 4^2 = 5^2$ since $9 + 16 = 25$



Some other Pythagorean triples include:

(5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (12, 35, 37), (20, 21, 29)

If each value in a Pythagorean triple is multiplied by the same positive integer, a similar (proportional) right-angled triangle will be formed.

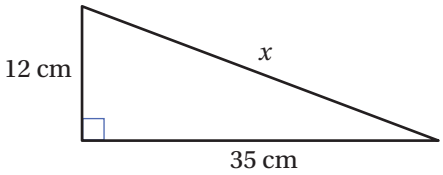
The triple (3, 4, 5) leads to other Pythagorean triples such as (6, 8, 10), (9, 12, 15), (30, 40, 50) and so on.

Worked example

Using Pythagorean triples to solve unknown sides

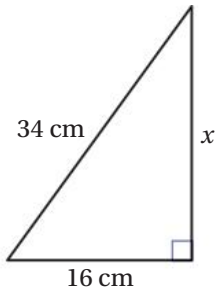
Use Pythagorean triples to determine the lengths of the unknown side in each right-angled triangle.

(a)



THINKING	WORKING
Interpret the information given in the right-angled triangle as a Pythagorean triple.	The sides of the right-angled triangle represent the Pythagorean triple (12, 35, x).
Recall the relationship between the three values in a Pythagorean triple.	The sum of the squares of the two shorter side lengths is equal to the square of the hypotenuse.
Determine the relationship between the values in the Pythagorean triple.	The length of the hypotenuse is x. $12^2 + 35^2 = x^2$ $144 + 1225 = x^2$ $1369 = x^2$
Determine the value of the variable.	$x^2 = 37^2$ $x^2 = 1369$ $x = \sqrt{1369}$ $x = 37$
Interpret the answer.	The length of the hypotenuse is 37 cm.

(b)



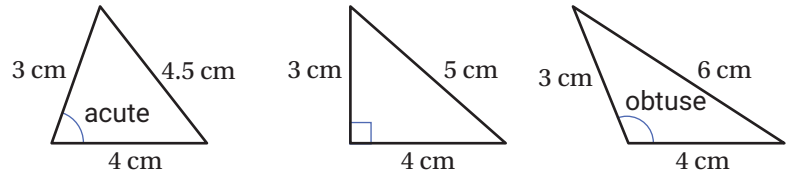
THINKING	WORKING
Interpret the information given in the right-angled triangle as a Pythagorean triple.	No Pythagorean triple is found, so determine whether the two known side lengths have a common factor. Factors: 16 = {1, 2, 4, 8, 16} 34 = {1, 2, 3, 10, 15, 30} The highest common factor is 2. The Pythagorean triple is $2\times\left(8,\frac{1}{2}x,17\right)$ or (16, x, 34).
Recall the relationship between the three values in a Pythagorean triple.	The sum of the squares of the two shorter side lengths is equal to the square of the hypotenuse.
Determine the relationship between the values in the Pythagorean triple.	One of the short sides is labelled x. $16^2 + x^2 = 34^2$ $256 + x^2 = 1156$ $x^2 = 900$
Determine the value of the variable.	$x^2 = 30^2$ $x^2 = 900$ $x = \sqrt{900}$ $x = 30$
Interpret the answer.	The length of the unknown side is 30 cm.

SC 3 I can recognise the relationship between the squares of lengths of sides for different types of triangles

Any triple with values proportional to a Pythagorean triple – such as (3, 4, 5) – will form a right-angled triangle, with the right angle opposite the longest side.

If the longest side is shortened, the right angle will be reduced to an acute angle.

If the longest side is lengthened, the right angle will be enlarged to an obtuse angle.



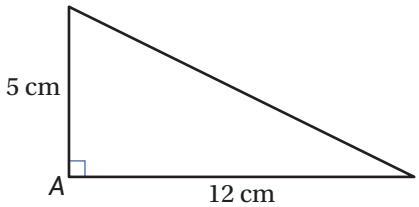
In each triangle, at least two of the internal angles must be acute. A triangle is classified as acute, right-angled or obtuse, depending on the size of its largest angle.

Worked example

Classifying triangles with angle names

Two shorter sides of a triangle are 5 cm and 12 cm.

Match the length of the longest side (12.5 cm, 13 cm, 14 cm) to the triangle it represents (acute, right-angled, obtuse).



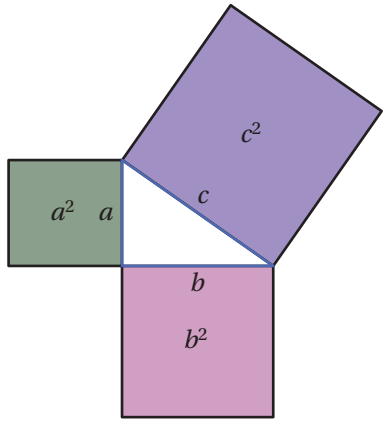
THINKING	WORKING
Determine the sum of the squares of the two shorter sides.	$5^2 + 12^2 = 25 + 144$ $= 169$
Determine the length of the hypotenuse of a right-angled triangle.	The square of the hypotenuse is $13^2 = 169$. For the triangle to be a right-angled triangle, the length of the longest side (hypotenuse) is equal to 13 cm.
Use the length of the hypotenuse to classify the other side lengths.	If the unknown side length is shorter than 13 cm, then angle A is smaller than 90° and the triangle is classified as an acute triangle. If the longest side is longer than 13 cm, then angle A is larger than 90° and the triangle is classified as an obtuse triangle.
Draw a conclusion.	(5 cm, 12 cm, 12.5 cm) is an acute triangle. (5 cm, 12 cm, 13 cm) is a right-angled triangle. (5 cm, 12 cm, 14 cm) is an obtuse triangle.

Practice

ANSWERS Page 595

SC 1 I can establish the relationship between the side lengths in a right-angled triangle

1 Consider the right-angled triangle shown.



(a) Copy and complete the table for each of the right-angled triangles with the side lengths given.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
30 mm	40 mm	50 mm				
6 cm	8 cm	10 cm				
60 mm	80 mm	100 mm				
5 cm	12 cm	13 cm				

(b) Which variable is used for the hypotenuse in each case?

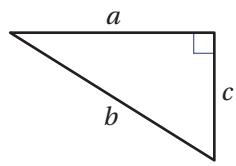
(c) What sort of units do each of the values of a^2, b^2 and c^2 have?

(d) What is the relationship between $a^2 + b^2$ and c^2?

2 Copy and complete the table to determine whether each of the following triangles is a right-angled triangle or not.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2	**Right-angled?**
4.1 cm	8 cm	10.5 cm					
1.4 cm	4.8 cm	5 cm					
18 mm	24 mm	30 mm					
7 cm	7 cm	14 cm					

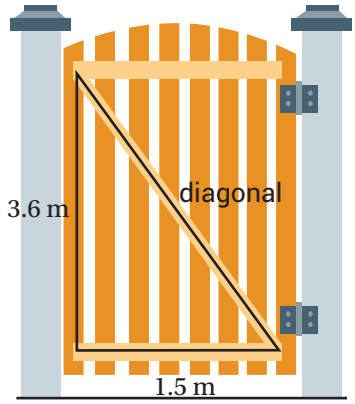
3 Consider the triangle below.



(a) Explain why Pythagoras' theorem, written as $c^2 = a^2 + b^2$, does not accurately describe the triangle.

(b) Using Pythagoras' theorem, write a true equation based on the triangle.

4 A rectangular gate has a height of 1.5 m and a width of 3.6 m.



Three students have calculated the length of the diagonal brace.

Student A	Student B	Student C
The length of the diagonal brace is the sum of the two shorter sides. The length of the diagonal is $1.5 + 3.6 = 5.1$ m.	The length of the diagonal brace is the sum of the squares of the two shorter sides. The length of the diagonal is $1.5^2 + 3.6^2 = 15.21$ m.	The square of the length of the diagonal, is equal to the sum of the squares of the two shorter sides. $1.5^2 + 3.6^2 = 15.21$ $3.9^2 = 15.21$ The length of the diagonal is 3.9 m.

(a) Which student is correct?

(b) Explain the common mistakes made by the other two students.

SC 2 I can identify and use Pythagorean triples

This list of Pythagorean triples will be helpful for the following questions.

(3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (12, 35, 37), (20, 21, 29)

1 Use the Pythagorean triple (3, 4, 5) to determine the length of the hypotenuse for similar right-angled triangles, given the lengths of the two short sides of a right-angled triangle.

(a) 6 mm, 8 mm **(b)** 9 mm, 12 mm **(c)** 12 mm, 16 mm **(d)** 15 mm, 20 mm

(e) Describe how you can use a Pythagorean triple to determine the length of an unknown hypotenuse.

2 Use the Pythagorean triples from the list to determine the length of the hypotenuse, given the lengths of the two short sides.

(a) 10 cm, 24 cm **(b)** 27 m, 36 m **(c)** 18 km, 80 km

3 Use the Pythagorean triple (5, 12, 13) to determine the length of the unknown side length, given the length of the hypotenuse and the other side.

(a) 10 m, 26 m **(b)** 24 m, 26 m **(c)** 36 m, 39 m **(d)** 15 m, 39 m

(e) Describe how the process of determining the length of a shorter side is different from determining the length of the hypotenuse.

4 Use Pythagorean triples from the list to determine the length of a shorter side, given the length of the hypotenuse and the other shorter side.

(a) 50 mm, 130 mm **(b)** 84 cm, 116 cm **(c)** 25 km, 65 km

5 Use the definition to decide whether each of the following ordered triples is a Pythagorean triple. Give a reason for each decision.

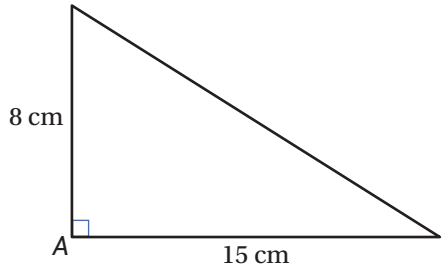
(a) (6, 8, 10) **(b)** (10, 18, 42) **(c)** (20, 48, 52) **(d)** (3.3, 4.4, 5.5)

6 Use each of the given Pythagorean triples to create all the similar right-angled triangles with a side length of 30 mm. Express each triangle as a Pythagorean triple.

(a) (8, 15, 17) **(b)** (3, 4, 5)

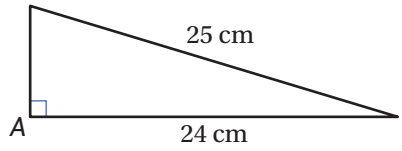
SC 3 I can recognise the relationship between the squares of lengths of sides for different types of triangles

1 The two shorter sides of a triangle are 8 cm and 15 cm. Give a third possible side length that would form:



(a) an acute triangle **(b)** a right-angled triangle **(c)** an obtuse triangle.

2 The length of the longest side of a triangle is 25 cm, and the length of one of the shorter sides is 24 cm. Give a third possible side length that would form:



(a) an acute triangle **(b)** a right-angled triangle **(c)** an obtuse triangle.

3 Use the Pythagorean triple (3, 4, 5) to classify the triangles with the side lengths listed as either acute-angled or obtuse-angled triangles.

(a) (3, 4, 4) **(b)** (3, 4, 6) **(c)** (6, 8, 9)

(d) (5, 8, 10) **(e)** (6, 8, 11)

(f) Describe how Pythagorean triples can help you classify triangles.

4 Identify whether each triple represents a Pythagorean triple, an acute-angled triangle, or an obtuse-angled triangle.

(a) (8, 15, 19) **(b)** (8, 15, 16) **(c)** (6, 12, 13)

(d) (20, 21, 30) **(e)** (9, 40, 41)

5 A parallelogram $ABCD$ has $AB = 8$ cm and $BC = 4$ cm. The diagonal, DB, is 10 cm long. Determine which angles in the parallelogram are acute.
HINT: A diagram should be helpful.

Compare different applications, demonstrations and proofs of Pythagoras' theorem

12.3

Learning intention: To be able to compare different applications, demonstrations and proofs of Pythagoras' theorem

Success criteria:

SC 1 I can demonstrate and prove Pythagoras' theorem using a square shape.

SC 2 I can use Pythagoras' theorem for similar shapes on the sides of right-angled triangles.

Lesson warm-up

What is a proof?

Can you think of a way to prove that 1 + 1 = 2?

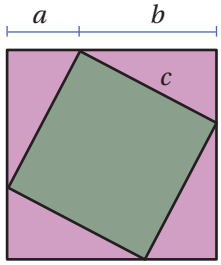
SC 1 I can demonstrate and prove Pythagoras' theorem using a square shape

History

The credit for Pythagoras' theorem goes to the mathematician and philosopher who lived in Greece around 2500 years ago; however, clay tablets show that the Babylonians used the theorem thousands of years earlier. Early evidence of the theorem has been found in Mesopotamia, India and China, sometimes as Pythagorean triples (or triads).

Pythagoras' theorem proof with squares

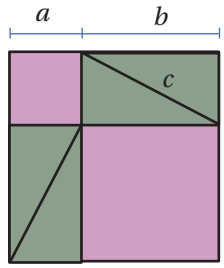
One well-known geometric proof of Pythagoras' theorem begins with a large square in which four identical right-angled triangles have been drawn at the corners.



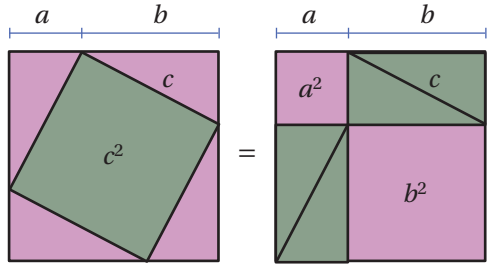
The shape remaining in the middle is a square of side length c, where c represents the length of the hypotenuse of each of the right-angled triangles.

The angles of the middle square are formed by the acute angles of each triangle. These acute angles are complementary (add up to 90°) because their third angle is a right angle (90°). Because all triangles have an angle total of 180°, you can conclude that the angles of the middle square are also right angles.

If the triangles are moved around inside the square, the resulting diagram shows that the removal of four triangles will leave two squares of side lengths a and b, where a and b represent the lengths of the other two sides.



The remaining areas must be equal; therefore, $c^2 = a^2 + b^2$

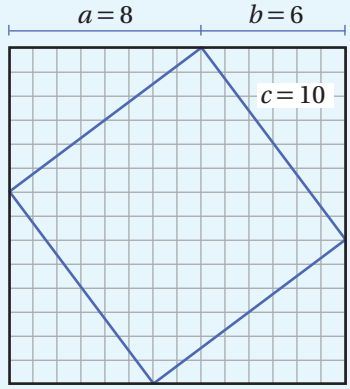


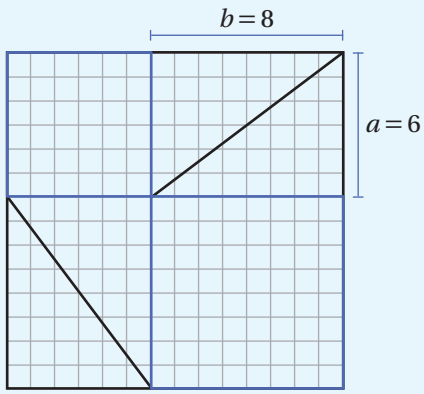
The overall square for both diagrams has a side length of $a + b$.

Worked example

Illustrating the geometric proof using a grid

Demonstrate Pythagoras' theorem using squares on grid paper and the Pythagorean triple (6, 8, 10).

THINKING	WORKING
On grid paper, draw the large square to the required size. Draw the right-angled triangles at each corner so that a square is left in the middle.	
Determine the dimensions of the large square.	$a + b = 8 + 6$ $= 14$ The length of each side of the square is 14 units.

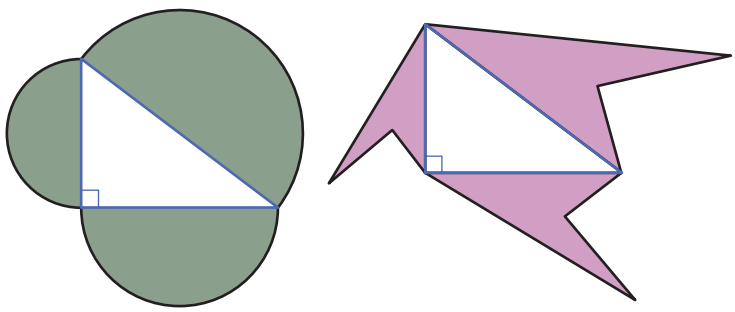
Determine the area of the interior square.	$c^2 = 10^2$ $= 100$ The area of the interior square is 100 square units.
On grid paper, draw the overall square to the required size. Draw the right-angled triangles as two pairs of rectangles, so that two squares are left at opposite corners.	
Determine the areas of the remaining squares.	$a^2 + b^2 = 6^2 + 8^2$ $= 36 + 64$ $= 100$ The sum of the areas of the corner squares is 100 square units.
Show that the area of the square of the hypotenuse is equal to the sum of the squares on the other two sides.	$c^2 = 10^2$ $= 100$ $= a^2 + b^2$ The area of the square of the hypotenuse is equal to the sum of the squares on the other two sides.

SC 2 I can use Pythagoras' theorem for similar shapes on the sides of right-angled triangles

For a right-angled triangle with hypotenuse c and short sides a and b, the Pythagorean equation $c^2 = a^2 + b^2$ has a direct connection with the area of the three squares built onto the sides.

Other shapes besides squares can be built onto the sides of right-angled triangles. In such cases, the areas of any three similar shapes built onto the sides are in proportion to the area of the squares. The formula for those shapes is in proportion to Pythagoras' theorem,
so, $c^2 = a^2 + b^2$ becomes $kc^2 = ka^2 + kb^2$, where the area of the shape on the hypotenuse is kc^2 and the areas of the other shapes are ka^2 and kb^2, and k is a constant number.

The value of k gives the relationship between the area of the shape on the side compared to the area of a square built on the same side.

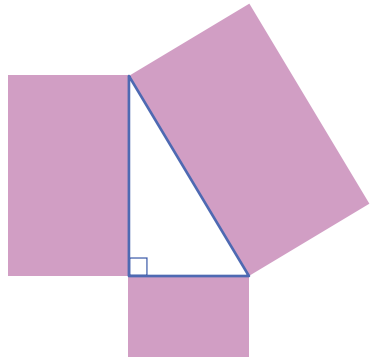


Worked example

Using Pythagoras' theorem with similar shapes built on the side of right-angled triangles

The right-angled triangle below has a shape based on the (3, 4, 5) Pythagorean triple.

The hypotenuse is 10 cm long, and the rectangle on the hypotenuse has an area of 60 cm^2.



Given that the three rectangles are similar, and all have a longer side along the side of the triangle, determine:

(a) the value of k in $kc^2 = ka^2 + kb^2$

THINKING	WORKING
Use the length of the hypotenuse as c and the area of the rectangle on the hypotenuse as kc^2.	$c = 10$ cm, so $c^2 = 100$ cm^2 $kc^2 = 60$ cm^2, therefore: $k = \frac{60}{100}$ $= 0.6$
Explain the meaning of the value of k.	Each rectangle has 0.6 of the area of a square built on the same side.

(b) the lengths of the short sides of the triangle

THINKING	WORKING
Use the ratio 3 : 4 : 5 to determine scale factor.	3 : 4 : 5 = 6 : 8 : 10
Interpret the answer.	The lengths of the short sides are 6 cm and 8 cm.

(c) the areas of the two smaller rectangles

THINKING	WORKING
Use ka^2, kb^2 and kc^2 as the areas.	Areas of smaller rectangles: $ka^2 = 0.6 \times 6^2$ $= 21.6$ cm^2 $kb^2 = 0.6 \times 8^2$ $= 38.4$ cm^2

(d) the width of each rectangle.

THINKING	WORKING
Use $A = lw$ for each rectangle.	$21.6 = 6 \times w_1$ $\quad$ $38.4 = 8 \times w_2$ $\quad$ $60 = 10 \times w_3$ $w_1 = 3.6$ cm $\quad$ $w_2 = 4.8$ cm $\quad$ $w_3 = 6$ cm The widths of the rectangles, in ascending order are: 3.6 cm, 4.8 cm, 6 cm

Practice

ANSWERS Page 596

SC 1 I can demonstrate and prove Pythagoras' theorem using a square shape

1 Use a square with side lengths of 7 units drawn on grid paper to complete the proof of Pythagoras' theorem with a (3, 4, 5) triangle.

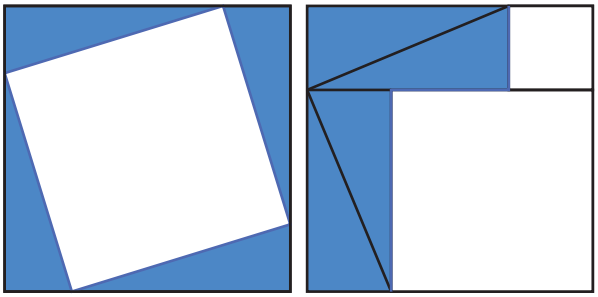
(a) Use grid paper to draw right-angled triangles with side length (3, 4, 5).

(b) Calculate the area of the centre square.

(c) Rearrange the right-angled triangles to form two squares.

(d) Use your answer to part **(c)** to demonstrate that the sum of the squares of the two shorter sides is equal to the square of the hypotenuse.

2 Another pair of square diagrams that illustrate Pythagoras' theorem is given below.



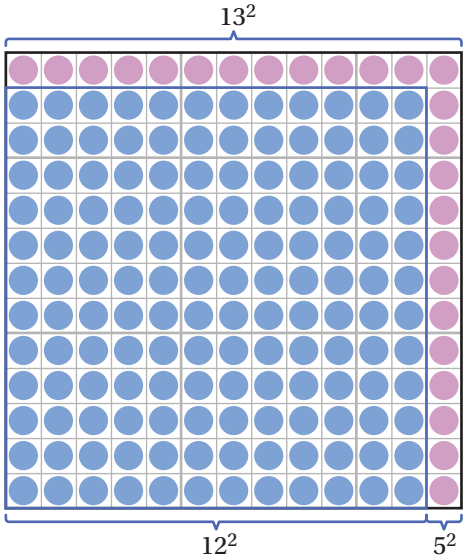
(a) For a right-angled triangle with hypotenuse c and short sides a and b, what is the side length in both of the two larger squares?

(b) On grid paper, illustrate this method of proof using the Pythagorean triple (5, 12, 13).

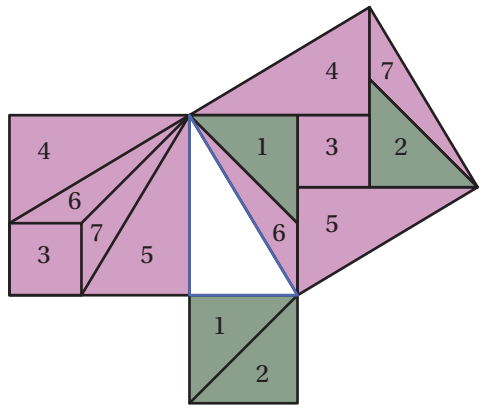
3 A dot array diagram shows that the Pythagorean triple (5, 12, 13) satisfies Pythagoras' theorem in the following way: A 12 × 12 array of dots is formed, then a 5 × 5 array is sectioned off in one corner.

It is easy to check that the total remaining dots, 25 in this case, could form a 5 × 5 array.

Draw a dot array diagram in the same style to demonstrate that the Pythagorean triple (3, 4, 5) satisfies Pythagoras' theorem.



4 Chinese mathematician Liu Hui used the seven pieces of a tangram to illustrate Pythagoras' theorem.



(a) On grid paper, draw a (6, 8, 10) right-angled triangle with 6 × 6 and 8 × 8 squares.

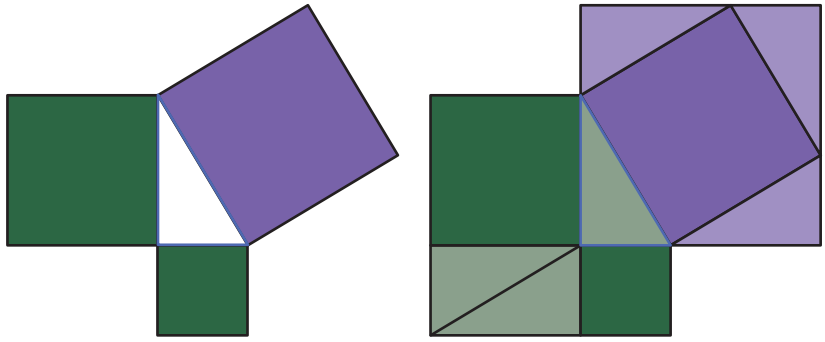
(b) Copy the shape of the lines in Liu Hui's example, noting that the triangles marked '4' is identical to the original triangle.

(c) Make a second copy for cutting out.

(d) Shade the original squares in contrasting colours and number the shapes.

(e) Cut out all shapes of the two small squares and arrange them on the hypotenuse to form a 10 × 10 square.

5 The diagram below shows yet another geometric proof of Pythagoras' theorem.



Using the total areas of each colour, explain how this diagram shows $c^2 = a^2 + b^2$ for a right-angled triangle with hypotenuse c and short sides a and b.

SC 2 I can use Pythagoras' theorem for similar shapes on the sides of right-angled triangles

1 A right-angled triangle has short side lengths of 60 mm and 80 mm.

Rectangles with lengths of double their widths are built onto each side, with the width along the side of the triangle in each case. Determine:

(a) the length of the hypotenuse

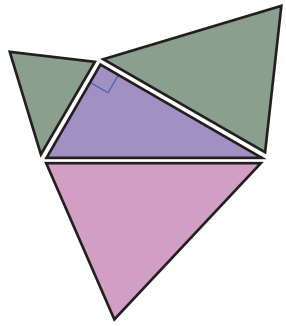
(b) the length of each rectangle

(c) the area of each rectangle

(d) the value of k in $kc^2 = ka^2 + kb^2$.

2 A right-angled triangle's shape is based on the (3, 4, 5) Pythagorean triple, with a hypotenuse of 20 cm.

Three similar triangles are built onto the sides, with the longest side of each triangle attached to the right-angled triangle in each case. The height of the triangle on the hypotenuse is 14 cm.



Determine:

(a) the lengths of the two short sides of the right-angled triangle

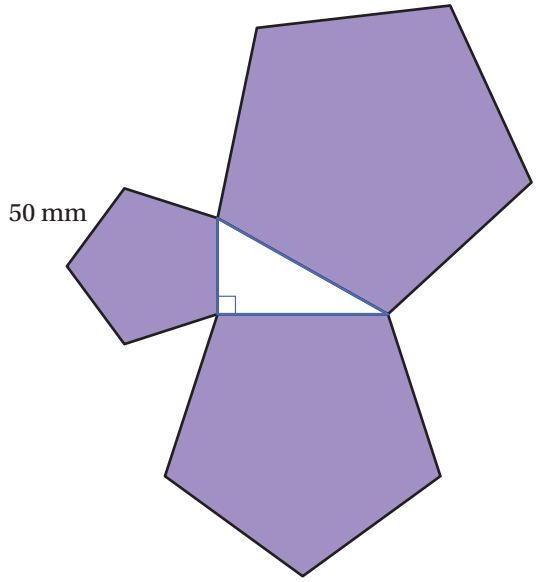
(b) the height of the other similar triangles

(c) the area of each similar triangle

(d) the value of k in $kc^2 = ka^2 + kb^2$.

3 A right-angled triangle's shape is based on the (5, 12, 13) Pythagorean triple, with the shortest side measuring 50 mm.

Regular pentagons are built onto the sides. The area of a regular pentagon is approximated by the formula $A = 1.72l^2$, where l is the side length of the regular pentagon.



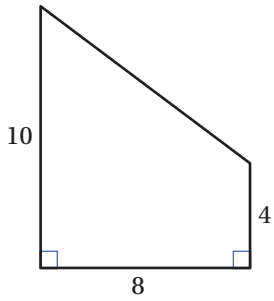
Determine:

(a) the lengths of the other sides of the right-angled triangle

(b) the area of each regular pentagon

(c) the value of k in $kc^2 = ka^2 + kb^2$.

4 Shapes similar to the one below are to be built onto the sides of a right-angled triangle whose shape is based on the Pythagorean triple (8, 15, 17).



The shortest side length of the right-angled triangle is 16 cm. The base of each of the similar shapes attaches to the side of the triangle in every case.

Determine:

(a) the lengths of the other sides of the right-angled triangle

(b) the left and right heights of each similar figure

(c) the area of each similar figure

(d) the value of k in $kc^2 = ka^2 + kb^2$.

12.4 Use Pythagoras' theorem to determine the length of the hypotenuse

Learning intention: To be able to use Pythagoras' theorem to determine the length of the hypotenuse

Success criteria:

SC 1 I can use Pythagoras' theorem to determine the length of a hypotenuse.

SC 2 I can solve problems involving determining the length of a hypotenuse.

Lesson warm-up

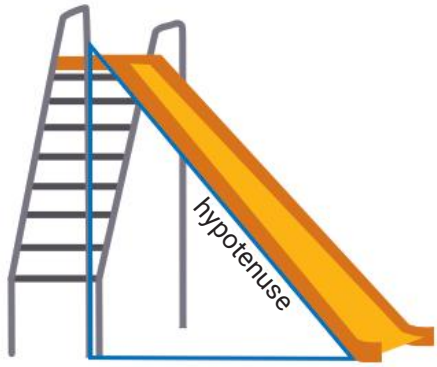
Which side of a right-angled triangle is the hypotenuse?

What do you think of this poem to help you remember which side is the hypotenuse?

Oh no! The hypotenuse
has broken loose.
The slide is no fun to ride,
because it is missing its longest side.

See if you can do better.

Design a method for remembering which side of a right-angled triangle is the hypotenuse.



SC 1 I can use Pythagoras' theorem to determine the length of a hypotenuse

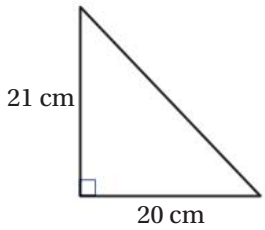
For a right-angled triangle with side lengths a, b and c, where c represents the length of the hypotenuse, $c^2 = a^2 + b^2$ can be used to determine the length of the hypotenuse, given the lengths of the other two sides.

Worked example

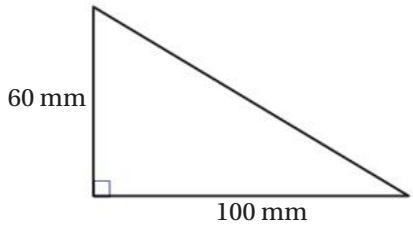
Calculating the length of the hypotenuse

Calculate the length of the hypotenuse in each triangle, correct to 2 decimal places, where necessary.

(a)



(b)



(a)

THINKING	WORKING
Use a variable to represent the length of the unknown side.	Let the length of the hypotenuse be x.
Write an equation for Pythagoras' theorem in terms of the given values.	$x^2 = 20^2 + 21^2$
Solve for the unknown.	$x^2 = 400 + 441$ $x^2 = 841$ $x = \sqrt{841}$ $x = 29$
Write the answer in words.	The length of the hypotenuse is 29 cm.
Check that the answer is reasonable.	The answer is reasonable because this length is greater than the lengths of the other two sides, and less than their sum ($29 < 20 + 21$).

(b)

THINKING	WORKING
Use a variable to represent the length of the unknown side.	Let the length of the hypotenuse be x.
Write an equation for Pythagoras' theorem in terms of the given values.	$x^2 = 60^2 + 100^2$
Solve for the unknown.	$x^2 = 3600 + 10000$ $x^2 = 13600$ $x = \sqrt{13600}$ $x = 116.619...$
Write the answer in words.	The length of the hypotenuse is 116.62 mm.
Check that the answer is reasonable.	The answer is reasonable because this length is greater than the lengths of the other two sides, and less than their sum ($116.62 < 60 + 100$).

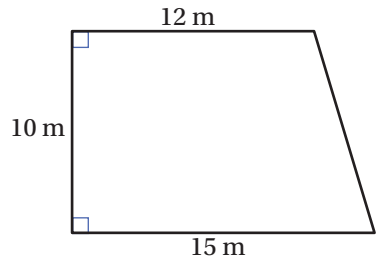
SC 2 I can solve problems involving determining the length of a hypotenuse

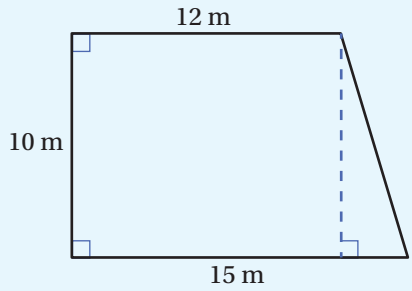
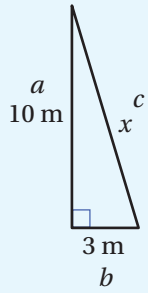
A construction line can sometimes produce a right-angled triangle in a diagram where there wasn't one before. Pythagoras' theorem may then be applied to solve unknown lengths.

Worked example

Solving the length of the hypotenuse by first forming a right-angled triangle

Solve the unknown side length, correct to the nearest centimetre.



THINKING	WORKING
Draw a vertical line to form a right-angled triangle.	
Determine the length of the two shorter sides in the right-angled triangle by matching with known lengths, or by simple subtraction.	The height of the triangle is the same height as the trapezium, $a = 10$ m. The base of the triangle is $b = 15 - 12 = 3$ m.
	Let the unknown side length be x. 
Substitute the known lengths into Pythagoras' theorem, $c^2 = a^2 + b^2$.	$x^2 = 10^2 + 3^2$ $= 100 + 9$ $= 109$
Solve for the unknown side length.	$x = \sqrt{109}$ $= 10.440\ldots$
Write the answer.	The length of the unknown side is 10.44 m.

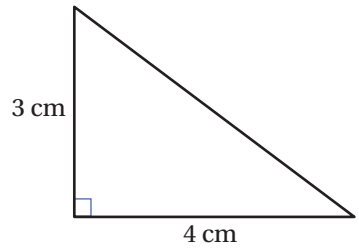
Practice

ANSWERS Page 596

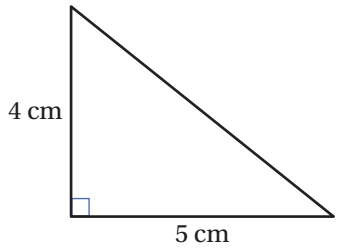
SC 1 I can use Pythagoras' theorem to determine the length of a hypotenuse

1 Calculate the length of the hypotenuse in each right-angled triangle. Round your answers to 2 decimal places where necessary.

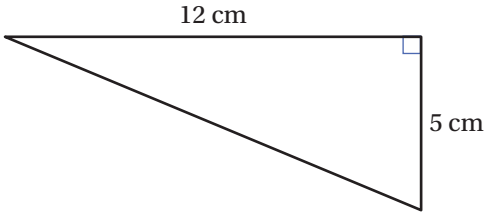
(a)



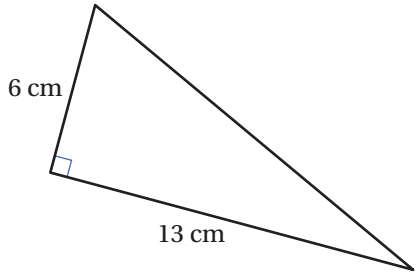
(b)



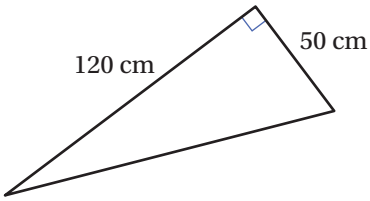
(c)



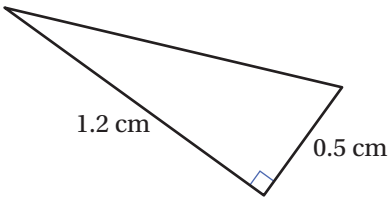
(d)



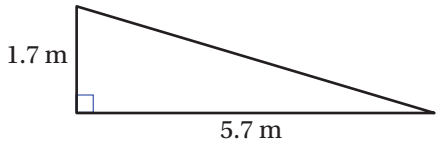
(e)



(f)



2 A ramp is built with a horizontal distance of 5.7 m and vertical distance of 1.7 m.



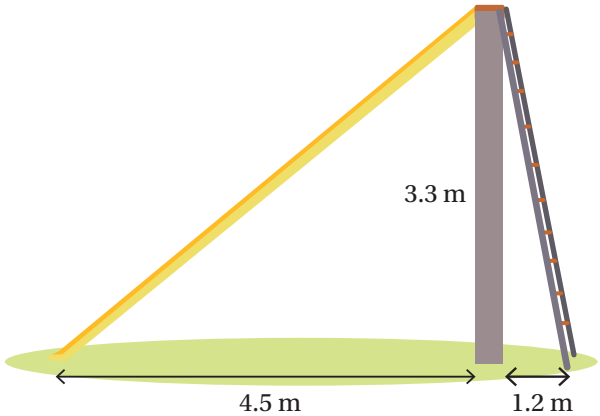
Determine the length of the ramp to the nearest millimetre.

3 A playground slide's base is 4.5 m, measured horizontally. The ladder's base is 1.2 m, measured horizontally.

Given that the height of the top seat is 3.3 m above the ground, determine, to the nearest centimetre:

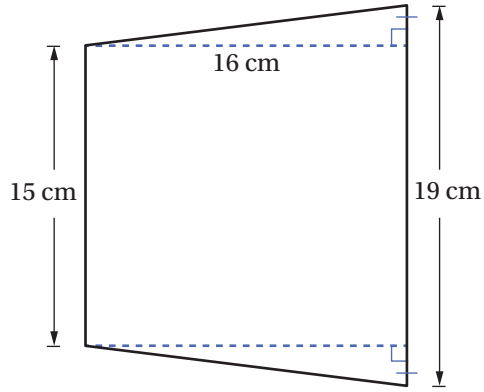
(a) the length of the slide

(b) the length of the ladder.

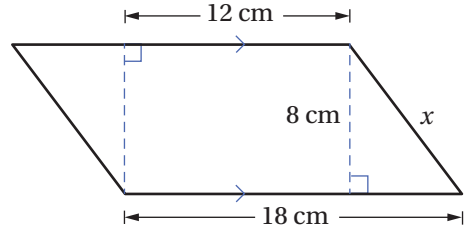


SC 2 I can solve problems involving determining the length of a hypotenuse

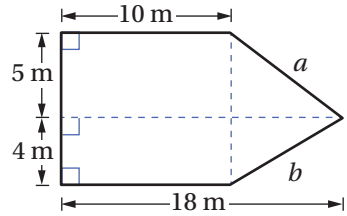
1 Determine the perimeter of the symmetrical shape, correct to the nearest millimetre.



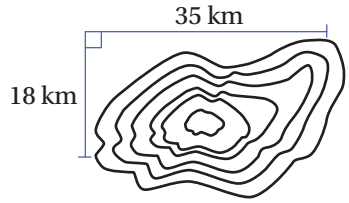
2 Determine the length of the side marked with a pronumeral.



3 Determine the lengths of the sides marked with pronumerals, correct to the nearest centimetre.

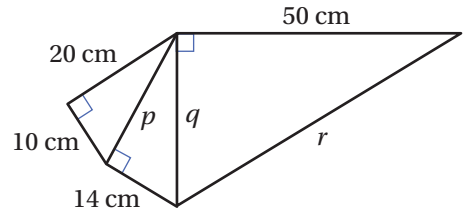


4 Two roads at right angles can be built, as shown, at a cost of $5000 per kilometre. Alternatively, a single diagonal road can be built at a cost of $7500 per kilometre by cutting through the hill.



Determine which proposal is cheaper and by how much. Give your answer to the nearest thousand dollars.

5 Determine the lengths of the sides marked with pronumerals, correct to the nearest millimetre.



Use Pythagoras' theorem to determine the length of a shorter side in a right-angled triangle

12.5

Learning intention: To be able to use Pythagoras' theorem to determine the length of a shorter side in a right-angled triangle

Success criteria:

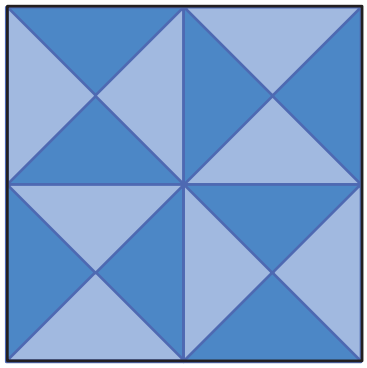
SC 1 I can rearrange Pythagoras' theorem to determine the length of a shorter side of a right-angled triangle.

SC 2 I can solve problems using Pythagoras' theorem.

Lesson warm-up

How many right-angled triangles?

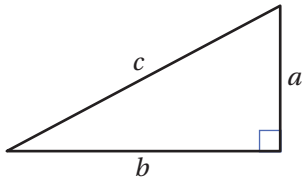
How many right-angled triangles can you see in this picture?



SC 1 I can rearrange Pythagoras' theorem to determine the length of a shorter side of a right-angled triangle

For right-angled triangles with side lengths a, b and c, such that $a^2 + b^2 = c^2$, c represents the length of the hypotenuse.

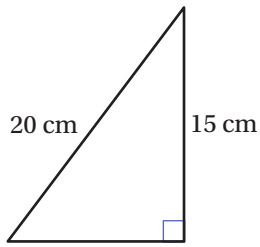
Transforming the formula to $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$ allows you to determine the length of a shorter side when the hypotenuse and the other short side is known.

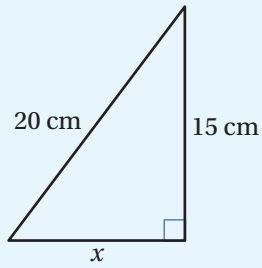
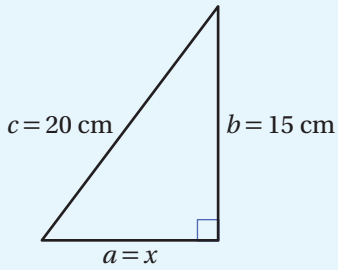


Worked example

Calculating the length of a shorter side in a right-angled triangle

Determine the length of the unknown side, correct to the nearest millimetre.



THINKING	WORKING
Label the unknown length with a variable.	
Label the two shorter side lengths a and b. Label the hypotenuse c.	
Write an equation for Pythagoras' theorem in terms of the given values. Note: Subtraction is used because the unknown side must be shorter than the hypotenuse.	$a^2 = c^2 - b^2$ $x^2 = 20^2 - 15^2$
Solve for the unknown.	$x^2 = 400 - 225$ $x^2 = 175$ $x = \sqrt{175}$ $x = 13.22\ldots$
Write the answer in words with appropriate units, rounding as directed.	The length of the unknown side is 13.2 cm.

SC 2 I can solve problems using Pythagoras' theorem

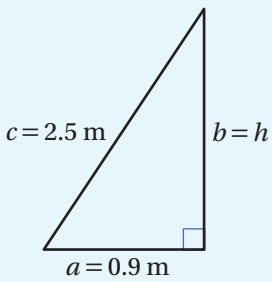
Problems requiring the application of Pythagoras' theorem may not have a diagram. To solve these problems, it is always good practice to use the following steps.

1. Draw and label a right-angled triangle with the given information.
2. Identify the unknown side length.
3. Substitute the values into Pythagoras' theorem.
4. Solve for the unknown.
5. Interpret and present the answer.
6. Always check that the answer is reasonable.

Worked example

Solving a described problem by first drawing a right-angled triangle

A ladder measuring 2.5 m is leaning against a wall with its base 90 cm from the wall. How high up the wall does the ladder reach? Give your answer in metres, correct to the nearest centimetre.

THINKING	WORKING
Step 1: Draw a right-angled triangle using the wall and ground as the natural right angle. Ensure that units are the same. Step 2: Identify and mark the unknown length with a suitable pronumeral.	
Step 3: Substitute the values into Pythagoras' theorem.	The unknown side to be found is a shorter side of the right-angled triangle. Therefore we use: $b^2 = c^2 - a^2$ $h^2 = 2.5^2 - 0.9^2$
Step 4: Solve for the unknown.	$h^2 = 6.25 - 0.81$ $= 5.44$ $h = \sqrt{5.44}$ $= 2.332 \ldots$
Step 5: Interpret and present the answer in words with appropriate units.	The ladder reaches 2.33 m up the wall.
Step 6: Check that the answer is reasonable.	The answer is reasonable because this length is shorter than the length of the hypotenuse (ladder), and the hypotenuse is less than the sum of the two shorter sides ($2.5 < 0.9 + 2.33$).

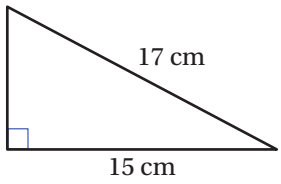
Practice

ANSWERS Page 597

SC 1 I can rearrange Pythagoras' theorem to determine the length of a shorter side of a right-angled triangle

1 Calculate the lengths of the unknown sides. Round your answers to the nearest millimetre where required.

(a)



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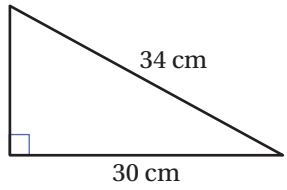

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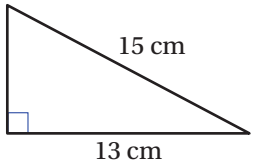
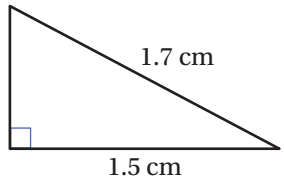
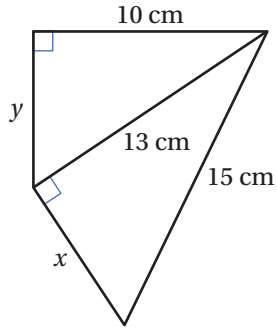


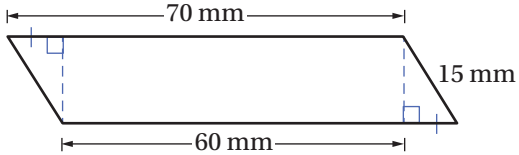
(d)



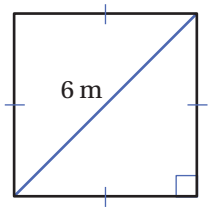
2 Determine the values of the pronumerals, correct to 2 decimal places.



3 Determine the height of the quadrilateral below, correct to 1 decimal place.

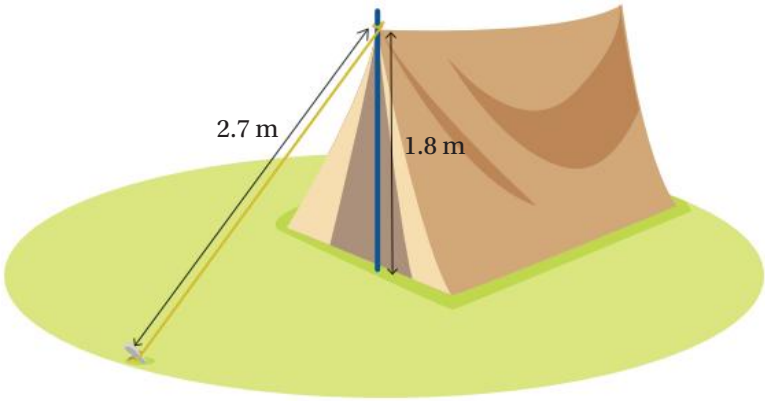


4 A square has a diagonal length of 6 m. Determine the side length of the square to the nearest centimetre.

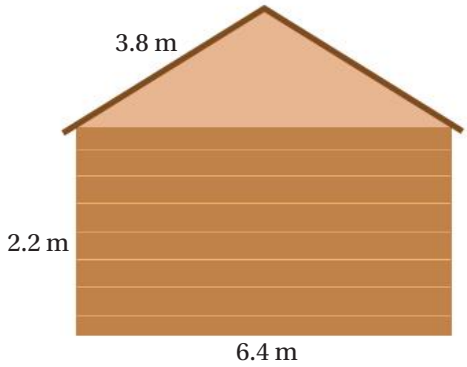


SC 2 I can solve problems using Pythagoras' theorem

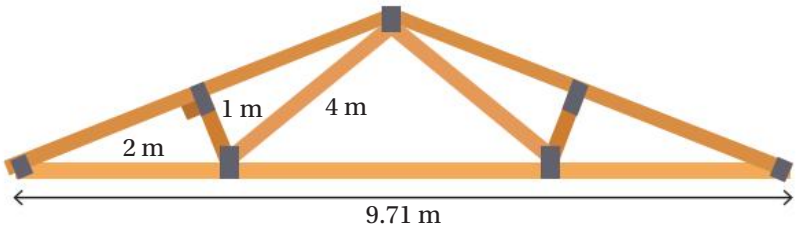
1 A taut 2.7 m guy rope stretches from a peg on the ground to the top of a 1.8 m tall vertical tent pole. How far is the peg from the base of the tent pole, correct to the nearest centimetre?



2 A symmetrical garden shed is 6.4 m wide, and its walls are 2.2 m high. The roof measures 3.8 m along each sloping side. Calculate the overall height of the shed, including the roof, to the nearest centimetre.



3 A symmetrical timber roof frame is constructed as shown. Calculate the total length of timber used to construct this symmetrical roof frame, accurate to the nearest centimetre.



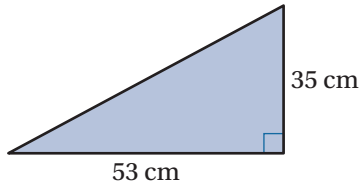
4 The perimeter of an equilateral triangle is 42 cm. Determine the perpendicular height of the triangle to the nearest millimetre.

TOPIC REVIEW

Multiple choice

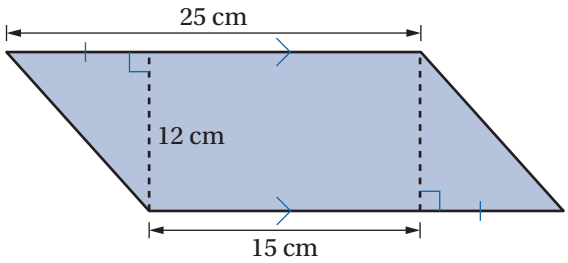
ANSWERS Page 628

1 The length of the hypotenuse is closest to:



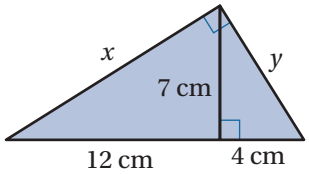
A 40 cm **B** 64 cm **C** 88 cm **D** 4034 cm

2 The value of x is closest to:



A 16 cm **B** 17 cm **C** 19 cm **D** 28 cm

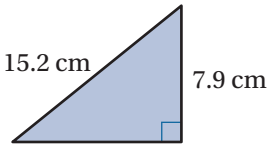
3 The values of x and y are closest to:



A $x = 10$ cm, $y = 8$ cm **B** $x = 10$ cm, $y = 6$ cm

C $x = 14$ cm, $y = 8$ cm **D** $x = 14$ cm, $y = 6$ cm

4 The length of the unknown side, to the nearest millimetre, is:



A 7.3 cm **B** 13.0 cm **C** 17.1 cm **D** 23.1 cm

5 The longer two sides of a right-angled triangle are 55 cm and 53 cm. The length of the shortest side is closest to:

A 2 cm **B** 15 cm **C** 23 cm **D** 51 cm

6 The top of a 6.5 m ladder is resting against a wall at a point 4.2 m above the ground. Which angle and length represent the right angle and the hypotenuse?

A between the ladder and the wall; the height up the wall

B between the ground and the wall; the length of the ladder

C between the ladder and the ground; the distance along the ground

D between the ladder and the wall; the length of the ladder

7 The top of a 6.5 m ladder is resting against a wall at a point 4.2 m above the ground. Which of the following best approximates the distance of the bottom of the ladder from the wall?

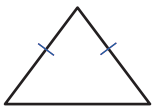
A 2.5 m **B** 3.0 m **C** 3.2 m **D** 5.0 m

8 A 17 m wire is attached to a tower 2 m below the top of the tower with the other end pegged to the ground 7.5 m from the base of the tower. The total height of the tower is closest to:

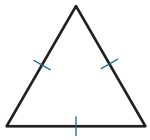
A 13.3 m **B** 15.0 m **C** 15.3 m **D** 17.3 m

9 Which of the triangles has sides that satisfy Pythagoras' theorem?

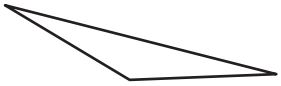
A



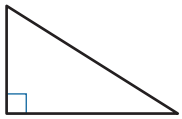
B



C

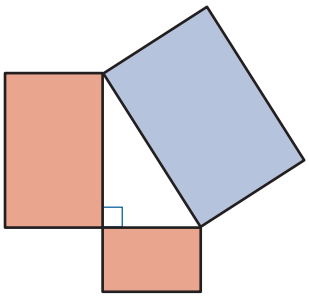


D

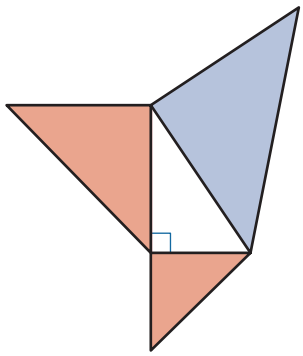


10 Which of the diagrams below could not support a proof of Pythagoras' theorem?

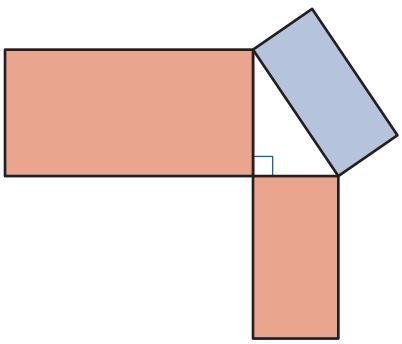
A



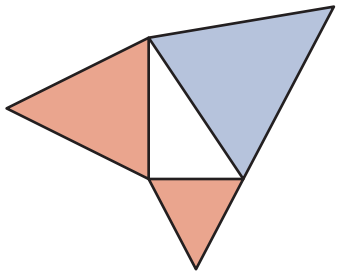
B



C



D



11 Choose the Pythagorean triple from the ordered triples.

A (20, 21, 23) **B** (20, 48, 52) **C** (6, 7, 8) **D** (5, 7, 10)

12 Which Pythagorean triple can be deduced from (36, 105, 111)?

A (12, 35, 37) **B** (36, 110, 333) **C** (24, 52.5, 55.5) **D** (72, 110, 222)

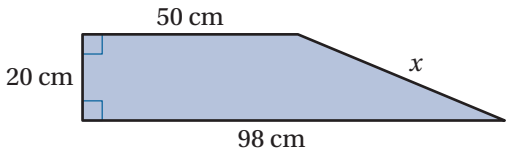
13 The two shorter sides of a right-angled triangle are 14 cm and 48 cm. The length of the hypotenuse is:

A 25 cm **B** 34 cm **C** 46 cm **D** 50 cm

14 In a right-angled triangle, the hypotenuse is 87 mm long and one of the other sides is 63 mm. The third side has a length of:

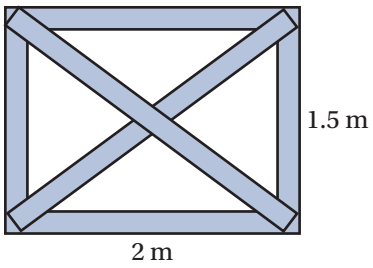
A 60 mm **B** 65.2 mm **C** 71.4 mm **D** 89.6 mm

15 The value of x is:



A 42 cm **B** 48 cm **C** 52 cm **D** 64 cm

16 A rectangular gate with diagonal supports is to be built from lengths of timber. What total length of timber will be used?

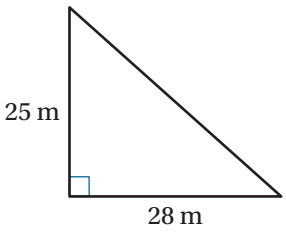


A 6 m **B** 7 m **C** 9.5 m **D** 12 m

Short answer

1 Calculate the length of each hypotenuse, correct to the nearest centimetre.

(a)



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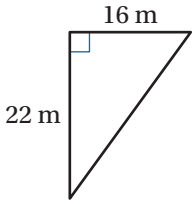

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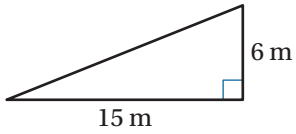
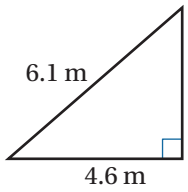


2 In each of the following right-angled triangles, calculate the length of the unknown side, correct to the nearest centimetre.

(a)



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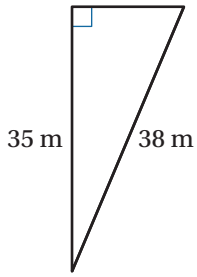

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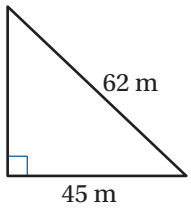


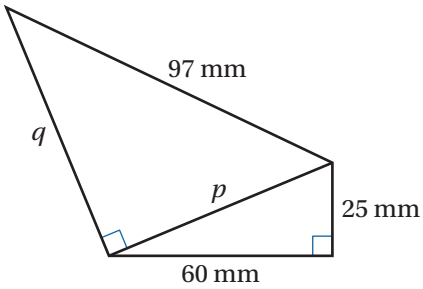
3 Use Pythagoras' theorem to show that a triangle with side lengths 10 cm, 24 cm and 26 cm must be right-angled.

4 A ball 20 m south of a particular spot is thrown and caught 16 m east of the same spot. Calculate, to the nearest centimetre, how far the ball is from its initial position.

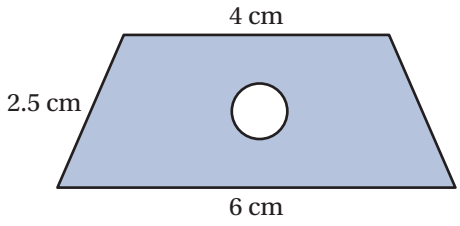
5 For each triple, determine whether sides proportional to the given values would form a right-angled triangle. If not, say whether the triangle is acute-angled or obtuse-angled.

(a) (16, 63, 65) **(b)** (48, 55, 73) **(c)** (13, 84, 86)

6 Use Pythagorean triples to calculate the values of p and q.

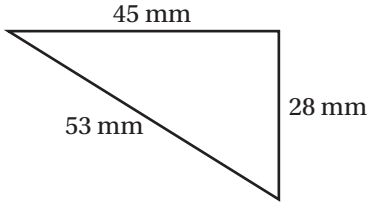


7 A box-cutter blade is in the shape of a symmetrical trapezium. Calculate the height of the blade to the nearest millimetre.

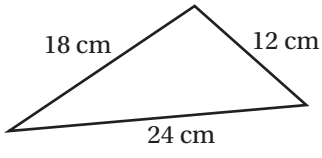


8 Determine whether or not each of the following triangles is right-angled. If not, say whether the triangle is acute-angled or obtuse-angled.

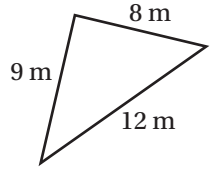
(a)



(b)

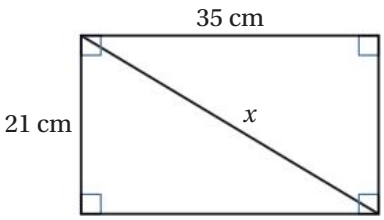


(c)

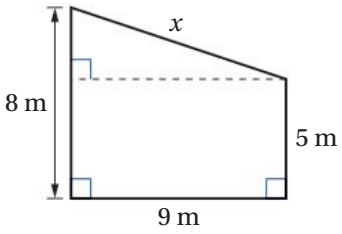


9 Determine the value of x in each diagram, rounded correct to one decimal place, where necessary.

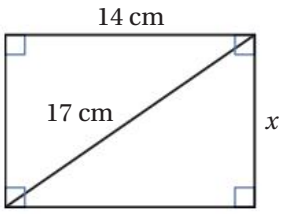
(a)



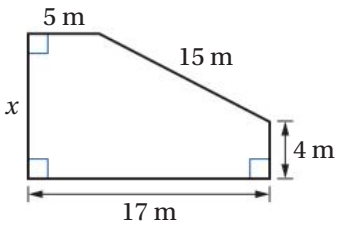
(b)



(c)



(d)

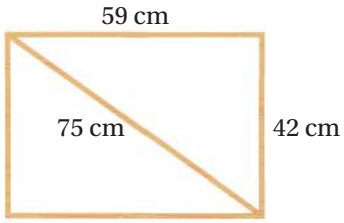


10 Use Pythagorean triples to determine the length of the hypotenuse given the lengths of the other two sides.

(a) 12 m, 16 m

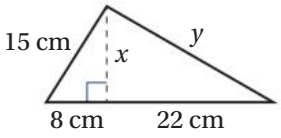
(b) 45 cm, 200 cm

11 A wooden rectangular frame is to be built with a diagonal brace. Say whether the brace needs to be lengthened or shortened and by how many millimetres to ensure the frame is 'square' (with right-angled corners).

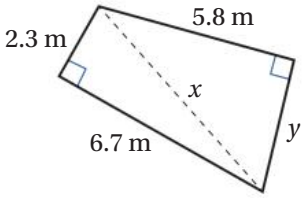


12 For each diagram determine the values of x and y correct to 2 decimal places.

(a)

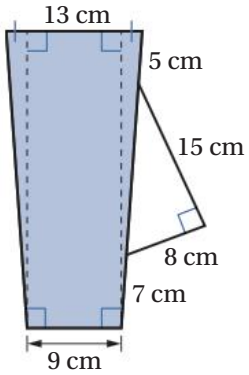


(b)



Extended response

1 A water jug has the cross-section shown below.

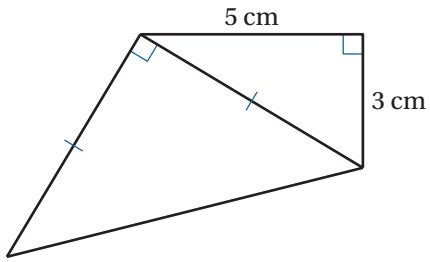


(a) Calculate the length of the hypotenuse of the triangle formed by the handle.

(b) Calculate the length of the hypotenuse and the length of the shortest side of the narrow right-angled triangle next to the rectangle.

(c) Calculate the height of the water jug, correct to 2 decimal places.

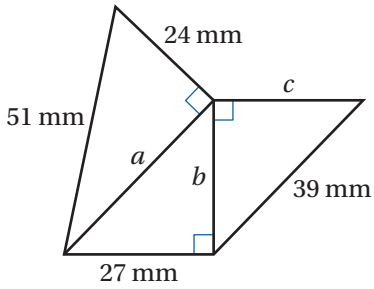
2 Calculate the following lengths, correct to 2 decimal places.



(a) The hypotenuse of the smaller triangle

(b) The hypotenuse of the larger triangle

3 Use Pythagorean triples to determine the lengths of a, b and c.

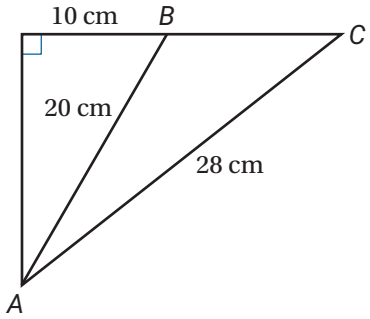


4 For the diagram below, calculate:

(a) the height

(b) the width

(c) hence, calculate the perimeter of ΔABC, rounding to 2 decimal places where necessary.



TOPIC
13

Ratios to solve problems

Recall 361

13.1 Solve problems involving proportional reasoning 362

13.2 Apply ratios to currency exchange 367

Topic review 371

Why learn this?

Understanding proportional reasoning and applying ratios to solve problems is a crucial skillset as ratios provide a method for comparing quantities, enabling effective decision-making and problem-solving in situations such as adjusting paint tones to produce variants in colour and amount required.

In professional and academic settings, ratios are essential for interpreting data, understanding scale models, and working with scaled representations like maps and technical drawings.

Learning to manipulate and apply ratios effectively prepares you for challenges in diverse fields such as science, statistics, engineering, finance and everyday practical tasks.

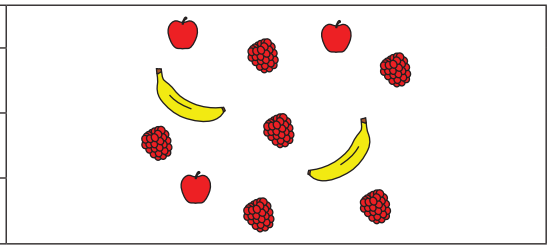
RECALL

ANSWERS Page 597

I can group and count items by common attributes

1 Complete the table to identify the number of pieces of each type of fruit shown.

Fruit	Number
Bananas	
Raspberries	
Apples	



2 For the numbers listed, identify how many prime numbers there are compared to composite numbers.
14, 2, 5, 13, 8, 7, 6, 3, 12, 4, 9, 16

I can write ratios to represent grouped values

1 Write the totals of each group in the following as ratios.

(a) Twenty adults and thirteen children **(b)** Three bananas to ten dragon fruits

I can convert between different units of length, mass and money

1 Change each measurement into the units given.

(a) 35 cm to mm **(b)** 4.9 L to mL **(c)** $12.50 to cents

I can write fractions in their simplest form

1 Simplify these fractions.

(a) $\frac{24}{28}$ **(b)** $\frac{36}{54}$ **(c)** $\frac{72}{96}$ **(d)** $\frac{35}{125}$

I can convert between mixed numerals and improper fraction forms

1 Convert the following between mixed numerals and improper fractions.

(a) $4\frac{2}{5}$ **(b)** $5\frac{3}{7}$ **(c)** $\frac{67}{4}$ **(d)** $\frac{73}{8}$

I can identify and write equivalent ratios

1 Which pair of ratios below are equivalent? Write true (T) or false (F) to identify them.

(a) 2 : 3 and 16 : 28 **(b)** 4 : 5 and 36 : 45

2 Identify the missing value in each of these pairs of equivalent ratios.

(a) 3 : 4 and $\square$: 28 **(b)** 75 : 60 and 15 : $\square$

13.1 Solve problems involving proportional reasoning

Learning intention: To be able to solve problems involving proportional reasoning

Success criteria:

SC 1 I can use ratios to solve practical problems.

SC 2 I can work with scales on maps and plans.

SC 3 I can calculate magnification factors.

Lesson warm-up

A recipe for rainbow pancakes

Cooper and Willow have a recipe for blue pancakes.

The recipe says to use three drops of blue food colouring in the pancake batter.

Cooper and Willow want to make rainbow pancakes, so they split the batter into six small bowls, and put three drops of food colouring in each bowl.

Discuss the following.

- Do you think they followed the recipe correctly?
- How bright do you think their pancakes will be?
- How many drops of food colouring should they have used?

SC 1 I can use ratios to solve practical problems

Algebra can be used to solve ratio problems. Choose a variable to represent the unknown in the ratio, then write the equivalent ratios as fractions and solve.

Always check that your answer makes sense by predicting whether it should be larger or smaller than the amount given in the question. You can also use estimation or substitution to check.

Worked example

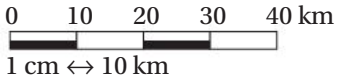
Solving ratio problems algebraically

A pharmacist is making a nutritional supplement by mixing a concentrate with water in the ratio 4 : 9. If the pharmacist has 275 mL of concentrate, how much water should they add?

THINKING	WORKING
Define a variable for the unknown quantity.	Let the volume of water be w.
Write the ratios in the same order.	Concentrate to water $4 : 9 = 275 \text{ mL} : w$
Write the ratios as fractions.	$\frac{4}{9} = \frac{275}{w}$
Invert both fractions, so that the unknown is in the numerator.	$\frac{9}{4} = \frac{w}{275}$
Swap sides, so that the unknown is on the left.	$\frac{w}{275} = \frac{9}{4}$
Solve for the unknown.	$w = 275 \times \frac{9}{4}$ $= 618.75$
Write the answer in words, checking that the calculation is reasonable.	618.75 mL of water should be added. Check: Water is the larger part of the ratio 4 : 9, so the volume of water should be greater than the volume of concentrate. 618.75 mL > 275 mL, so the calculation makes sense.

SC 2 I can work with scales on maps and plans

The scale on a map links distances on the map to the distances on the ground. The scale can be written (e.g. 1 : 1 000 000 or 1 cm ↔ 10 km) or drawn:



1 cm ↔ 10 km

This scale shows that every 1 cm on the map represents 10 km on the ground.

Worked example

Using scales on maps to calculate distances

For the scale 1 cm ↔ 10 km:

(a) calculate the real distance shown as 6.7 cm on the map

THINKING	WORKING
Multiply both parts of the scale by the same number.	1 cm ↔ 10 km, so 6.7 cm ↔ 6.7 × 10 km
Write the answer.	The real distance is 67 km.

(b) calculate the map distance that represents 183 km on the ground.

THINKING	WORKING
Determine the number to multiply the scale by.	$\frac{183\text{ km}}{10\text{ km}} = 18.3$
Multiply both parts of the scale by the same number.	1 cm ↔ 10 km, so 18.3 × 1 cm ↔ 18.3 × 10 km 18.3 cm ↔ 183 km
Write the answer.	The map distance is 18.3 cm.

SC 3 I can calculate magnification factors

Scales on maps and technical drawings are often unit ratios, such as 1 : 10 000.

A scale of 1 : 10 000 means that 1 cm on the map represents 10 000 cm (100 m) in real life.

The number 10 000 is called the magnification factor. You multiply a length on the drawing by the magnification factor to determine the actual length. You divide an actual length by the magnification factor to determine the length on the drawing.

Worked example

Using scales on maps to calculate magnification factors

For the scale 1 cm ↔ 10 km:

(a) convert the scale to a unit ratio in simplest form

THINKING	WORKING
Write the scale as a ratio, and change the larger amount to the same units as the smaller amount.	1 cm : 10 km = 1 cm : 10 000 m = 1 cm : 1 000 000 cm
Cancel out the units.	1 : 1 000 000

(b) express the meaning of the unit ratio in words using the magnification factor.

THINKING	WORKING
State the magnification factor (the part of the unit ratio that is not 1).	The magnification factor is 1 000 000.
Write the answer.	The real distances are 1 000 000 (1 million) times the size of the map distances.

Practice

ANSWERS Page 597

SC 1 I can use ratios to solve practical problems

1 Calculate the value of each unknown.

(a) $a : 180 = 5 : 3$ **(b)** $210 : b = 7 : 3$ **(c)** $11 : 4 = c : 100$

(d) $9 : 8 = d : 1200$ **(e)** $e : 3 = 7 : 4$ **(f)** $2 : f = 5 : 11$

2 The ratio of rainbow lorikeets to little wattlebirds visiting a banksia tree each day tends to be about 6 : 5. How many lorikeets would you expect there to be when there are 43 little wattlebirds?

3 The width to height ratio of the Australian flag is 19 : 10. To the nearest centimetre, how high would a flag be, given its width is 1.8 m?

4 Two students put in money to buy a bag of sweets. They agree to share the sweets in the same proportion as the money they put in. If one student put in $2.50 and the other $1.80, what is the larger share of sweets, to the nearest gram, given that the smaller share is 350 g?

5 Freshly squeezed juice is being sold in 200 mL and 450 mL cups at $3 and $5 respectively. What is the discount for purchasing the larger size?

SC 2 I can work with scales on maps and plans

1 A plan of a room uses a scale of 1 : 100. Calculate the actual lengths in metres for each of the distances on the plan.

(a) 3 cm **(b)** 6 mm **(c)** 42 mm **(d)** 12.6 cm

2 Alberto is making a 1 : 400 scale model of a house. Determine the model lengths in millimetres for each of the actual lengths below.

(a) 4 m **(b)** 80 cm **(c)** 20 m **(d)** 42 m **(e)** 2.4 m

3 A map of a town has the scale 1 : 2000. Calculate the actual distances in metres for each of the following lengths measured on the map.

(a) 5 cm **(b)** 12 cm **(c)** 3.2 cm

(d) 25.6 cm **(e)** 43 mm **(f)** 7 mm

4 A map of Australia is drawn to the scale 1 cm ↔ 500 km. Calculate the map distances between each pair of capital cities listed below. Use the actual distances given in the question. State your answers in centimetres, to the nearest millimetre.

(a) Brisbane to Perth, 3607 km **(b)** Sydney to Hobart, 1150 km

(c) Adelaide to Darwin, 1623 km **(d)** Canberra to Melbourne, 467 km

5 A map of Japan is drawn to the scale 1 cm ↔ 50 km. Calculate the real distances in kilometres between each pair of cities listed below. Use the map distances given in the question.

(a) Tokyo to Sapporo, 16.4 cm **(b)** Fukuoka to Nagasaki, 2.3 cm

(c) Kyoto to Osaka, 0.9 cm

SC 3 I can calculate magnification factors

1 By first converting each scale to a unit ratio, determine the magnification factor.

(a) 1 mm ↔ 10 cm **(b)** 1 mm ↔ 40 cm **(c)** 1 cm ↔ 40 cm **(d)** 1 cm ↔ 400 m

2 For each scale in Question **1**, what real distance is represented by 5 cm on a map with that scale?

3 Determine the magnification factor for each scale.

(a) 2 mm ↔ 1 m **(b)** 2 mm ↔ 15 m **(c)** 5 cm ↔ 3 m **(d)** 5 cm ↔ 3 km

4 For each scale in Question **3**, what distance on a plan would you need to use to represent 1 km in real life?

5 The magnification factor for a housing estate plan is 900. A rectangular block measures 3 cm by 2.4 cm on the plan.

(a) Determine the actual dimensions of the block.

(b) Calculate the area of the rectangle on the plan.

(c) Calculate the actual area of the rectangular block.

(d) Determine the factor connecting the plan area and the actual area.

Exploration activity

ANSWERS Page 597

Big battles, scaled down!

Popular ratios (scale : actual) used for miniature figures are 1 : 100, 1 : 60 and 1 : 56. Scenery in model railways is usually 1 : 87 scale.

You can convert between model size and real life using the following equation:

Model size = actual size ÷ scale factor

Actual size = model size × scale factor

Some common miniature figure characters used in role-playing games are humans, goblins, trolls and werewolves (as shown).

Human Goblin Troll Werewolf

1 **(a)** What is the magnification factor for the ratio 1 : 56?

(b) The goblin figure is 28 mm tall. How tall would this goblin be in real life if it is 1 : 56 scale? Give your answer in metres to 2 decimal places.

(c) The werewolf figure is also 28 mm tall, but it is 1 : 100 scale. How tall would it be in real life? Give your answer in metres to 2 decimal places.

(d) The troll figure is 42 mm tall in 1 : 60 scale. How tall would it really be? Give your answer in metres to 2 decimal places.

Apply ratios to currency exchange

13.2

Learning intention: To be able to apply ratios to currency exchange

Success criteria:

SC 1 I can determine the value of amounts of money in foreign currencies.

SC 2 I can manage a budget involving foreign currency.

Lesson warm-up

Cheaper overseas?

A game costs $109.99 in Australia, but the same game only costs $69.99 in the United States. Eric says that this is because everything is cheaper overseas. Lanna says that you can't compare Australian dollars to overseas dollars.

Discuss.

- How much more does the game cost in Australia?
- Is an Australian dollar the same as a US dollar?

SC 1 I can determine the value of amounts of money in foreign currencies

Different countries use different money. Foreign currency means money from another country. Several countries call their money 'dollars', but a US dollar is different from an Australian dollar, so every currency has a three-letter code. Some codes are in the table below.

Country	Currency name	Code
Australia	dollar	AUD
United States of America	dollar	USD
New Zealand	dollar	NZD
Japan	yen	JPY
China	yuan	CNY
European Union	euro	EUR

People travelling or buying items overseas have to swap their money for the local currency. This is called currency exchange. The swap is not 1 : 1; for example, you don't receive 1 euro for $1. The amount of money you receive for $1 is called the exchange rate, which is like a ratio.

Exchange rates change by small amounts all the time.

Worked example

Converting currency to and from Australian dollars (AUD)

Convert the following currencies. Use exchange rates 1 AUD ↔ 0.66 EUR and 1 AUD ↔ 0.64 USD.

(a) Convert $500 of Australian currency (500 AUD) to US dollars (USD).

THINKING	WORKING
Write the exchange rate.	1 AUD ↔ 0.64 USD
Multiply both sides by the same number.	500 AUD ↔ 0.64 × 500 USD 500 AUD ↔ 320 USD
Write the answer.	500 AUD converts to 320 USD. Check: the number of US dollars should be less than the number of Australian dollars to match the exchange rate.

(b) Convert 500 euros (EUR) to Australian dollars (AUD).

THINKING	WORKING
Define a variable for the unknown.	Let x be the number of Australian dollars.
Write the exchange rate as a ratio, and write an equivalent ratio using the variable.	1 AUD ↔ 0.66 EUR, so $1 : 0.66 = x : 500$
Solve for the variable, and round to the nearest cent.	$\frac{x}{500} = \frac{1}{0.66}$ $x = 500 \times \frac{1}{0.66}$ $x = 757.58$
Write your answer.	500 EUR converts to 757.58 AUD. Check: the number of euros should be less than the number of Australian dollars to match the exchange rate

SC 2 I can manage a budget involving foreign currency

Worked example

Making a budget involving foreign currency

An Australian planning a trip to Japan wants to spend no more than 150 AUD a night on accommodation. They also want to be able to compare the Japanese food prices with Australian prices. The exchange rate is 1 AUD ↔ 92.81 JPY.

(a) How many yen is equivalent to 150 AUD? Round your answer to the nearest thousand.

THINKING	WORKING
Write the exchange rate.	1 AUD ↔ 92.81 JPY
Multiply both sides by the same number.	150 AUD ↔ 150 × 92.81 JPY 150 AUD ↔ 13 921.50 JPY
Write the answer, rounding as required.	150 AUD is equivalent to approximately 14 000 JPY.

(b) A lunch costs 1100 JPY. How much is this in Australian dollars?

THINKING	WORKING
Define a variable for the unknown.	Let d be the number of Australian dollars.
Write the exchange rate as a ratio, and write an equivalent ratio using the unknown.	1 AUD ↔ 92.81 JPY, so $1 : 92.81 = d : 1100$
Solve for the unknown, and round to the nearest cent.	$\frac{d}{1100} = \frac{1}{92.81}$ $d = 1100 \times \frac{1}{92.81}$ $d = 11.85$
Write the answer.	A lunch costing 1100 JPY costs $11.85 in Australian dollars.

Practice

ANSWERS Page 597

SC 1 I can determine the value of amounts of money in foreign currencies

1 Change 800 AUD to each of the following currencies, given the exchange rate.

(a) Polish zloty, 1 AUD ↔ 3.18 PLN

(b) New Zealand dollars, 1 AUD ↔ 1.14 NZD

(c) Chinese yuan, 1 AUD ↔ 4.55 CNY

(d) British pounds sterling, 1 AUD ↔ 0.58 GBP

2 Change each of the following currencies to Australian dollars, to the nearest 5 cents, using the given exchange rate.

(a) 640 Singapore dollars at 1 AUD ↔ 0.91 SGD

(b) 4000 Mexican pesos at 1 AUD ↔ 12.81 MXN

(c) 200 Canadian dollars at 1 AUD ↔ 0.87 CAD

(d) 5000 Japanese yen at 1 AUD ↔ 92.81 JPY

3 Jaryn has 2000 AUD saved for his holiday to New Zealand. He changes it to NZD at an exchange rate of 1 AUD ↔ 1.1586 NZD. At the end of his holiday, he has 300 NZD left over.

(a) How many NZD does Jaryn receive for 2000 AUD?

(b) The exchange rate at the end of his holiday has changed to 1 AUD ↔ 1.0976 NZD. What amount in AUD does he receive?

4 Travellers often swap money at a bank or a currency exchange. Usually there are two rates: one for buying and one for selling. This makes a profit for the bank. For example, the 'buy' rate for euros might be 1 AUD ↔ 0.65 EUR, while the 'sell' rate might be 1 AUD ↔ 0.68 EUR.

(a) Start with 1000 AUD and use it to 'buy' euros at the rate 1 AUD ↔ 0.65 EUR. How many euros will you receive?

(b) Sell the euros received in part **(a)** at the rate 1 AUD ↔ 0.68 EUR. How many Australian dollars will you receive?

(c) How much has been kept by the bank?

SC 2 I can manage a budget involving foreign currency

1 Two Australian friends are visiting Germany for a week. They have budgeted 30 AUD each for lunch each day. They spend 25 EUR each on lunch the first day, and 32 EUR each on the second day. The exchange rate is 1 AUD ↔ 0.66 EUR.

(a) How much have they spent so far in Australian dollars?

(b) To stick to their budget, how much should they spend on lunch for the next five days?

2 An Australian planning a trip to the United States has paid for most of the big items, such as accommodation and day trips, ahead of time. For the rest of their expenses, they have a mixture of Australian and US dollars, depending on where the information came from.

	USD	AUD
Meals		$500
Tips @ 15% of meals		
Souvenirs	$150	
Bottled water	$20	
Total		

Using the exchange rate 1 AUD ↔ 0.64 USD, complete columns for both currencies to the nearest dollar.

3 An Australian planning a trip to Vietnam and Thailand has paid for some accommodation ahead of time and researched prices for other expenses. A 1.5 L bottle of water costs 16.10 Thai baht and 12 000 Vietnamese dong. Tourists generally spend 480 baht per day on food in Thailand and 276 000 dong per day in Vietnam. She has two days booked in Thailand and three in Vietnam. She expects to drink a bottle of water per day and plans to spend about $100 on souvenirs in each country. Using the exchange rates 1 AUD ↔ 24.00 THB and 1 AUD ↔ 17 600 VND, complete the table for all currencies, rounding to the nearest whole number.

	THB	VND	AUD
Food			
Souvenirs			$200
Bottled water			
Total			

4 An Australian travelling to Canada pays for most items using her credit card. The credit card company uses an exchange rate of 1 AUD ↔ 0.87 CAD and charges a fee of 2 AUD per transaction. Determine the cost in Australian dollars of:

(a) coffee, 5 CAD **(b)** jeans, 70 CAD **(c)** dinner, 45 CAD **(d)** hotel, 215 CAD

TOPIC REVIEW

Multiple choice

ANSWERS Page 629

1 Solve for a. $800 : a = 40 : 9$

A $a = 40$ **B** $a = 180$ **C** $a = 450$ **D** $a = 900$

2 Solve for b. $5 : 4 = b : 16$

A $b = 20$ **B** $b = 16$ **C** $b = 4$ **D** $b = 1$

3 Choose the correct equation that could be used to solve for a. $a : 4 = 9 : 7$

A $a = 4 \times \frac{7}{9}$ **B** $a = \frac{1}{4} \times \frac{9}{7}$ **C** $a = 4 \times \frac{9}{7}$ **D** $a = \frac{1}{4} \times \frac{7}{9}$

4 Choose the correct equation that could be used to solve for b. $2 : 5 = 19 : b$

A $b = \frac{1}{19} \times \frac{5}{2}$ **B** $b = 19 \times \frac{5}{2}$ **C** $b = \frac{1}{19} \times \frac{2}{5}$ **D** $b = 19 \times \frac{2}{5}$

5 In the ratio 9 : 8, the magnification factor is:

A 9 **B** 8 **C** $\frac{9}{8}$ **D** $\frac{8}{9}$

6 A map uses the scale 1 : 5000. If the distance between two positions on the map is 20 cm, the actual distance is:

A 1 km **B** 10 000 cm **C** 250 m **D** 20 m

7 The tax payable on $74 280 is closest to:

A $24 141 **B** $14 608 **C** $9516 **D** $5092

8 The 2% Medicare levy is collected from the pay of any person expected to earn more than $29 207 in a year. Which of the following wages will not have to pay the levy?

A Weekly wage: $563 **B** Fortnightly wage: $1230

C Fortnightly wage: $1342 **D** Monthly wage: $1900

9 Which of the following wages do not put a person in the tax bracket $45 001–$120 000 for annual income?

A Weekly wage: $2450 **B** Fortnightly wage: $2861

C Fortnightly wage: $3947 **D** Monthly wage: $4500

10 If a person has a taxable income of $20 000, the overall percentage of earnings collected in tax is closest to:

A 1% **B** 2% **C** 9% **D** 19%

11 If 1 Australian dollar is equivalent to 1.14 New Zealand dollars, which amount is closest to A$200?

A NZ$175 **B** NZ$186 **C** NZ$228 **D** NZ$328

12 If 1 Australian dollar is equivalent to 0.66 euros, which amount is closest to €500?

A A$330 **B** A$434 **C** A$566 **D** A$758

Short answer

1 Calculate the value of the unknown in each of the following.

(a) $a : 42 = 11 : 7$ **(b)** $12 : b = 1 : 5$

(c) $9 : 21 = 24 : c$ **(d)** $12 : 10 = d : 15$

2 Solve for x in each of the following:

(a) $x : y = 1.6$, where $y = 5$ **(b)** $y : x = 5.2$, where $y = 26$

3 If the adult to student ratio for an excursion needs to be at least 1 : 5, determine the number of parents needed to assist if 2 teachers and 29 students are attending.

4 A thick-shake is made of ice-cream whipped up in milk in the ratio 2 : 5. Calculate the amount of ice-cream to be added to 240 mL of milk.

5 The planets closer to the Sun take a shorter time for each revolution, with Earth taking 365.25 days or 1 year.

(a) Calculate each of the following ratios, to 2 decimal places where necessary, given the revolution times around the Sun.

(i) Venus to Mercury; Mercury, 88 days and Venus, 224.7 days

(ii) Saturn to Earth; Saturn, 29.5 years

(iii) Neptune to Jupiter; Neptune, 164.8 years and Jupiter, 11.9 years

(b) Calculate each of the following revolution times around the Sun, given the ratios of times.

(i) Mars, to the nearest day; Mars to Earth ratio is 1.88.

(ii) Uranus, to the nearest year; Neptune to Uranus ratio is 1.96.

6 1.2 L of orange juice is mixed with 800 mL of pineapple juice and 1 L of water. Determine the fraction of the mixture that is orange juice.

7 Determine the magnification factor suggested by each of the following scales.

(a) 1 cm ↔ 20 m **(b)** 5 mm ↔ 12 km **(c)** 4 cm ↔ 1 km

8 Determine the real lengths represented by each of the diagram lengths, given the scale.

(a) 4 cm, 1 : 1000 **(b)** 16 mm, 1 : 200 000 **(c)** 3.6 cm, 1 : 50 000

9 Determine the map lengths, in millimetres, corresponding to each distance, given the scale.

(a) 600 m, 1 : 1500 **(b)** 42 km, 1 : 400 000 **(c)** 256 km, 1 : 500 000

10 A carpenter earns an annual income of $55 432 and is paid fortnightly. Calculate the carpenter's:

(a) annual tax payable

(b) fortnightly take-home pay after tax and the 2% Medicare levy have been removed.

11 An accountant's monthly salary, before tax and other deductions is $8502. Calculate the accountant's:

(a) annual income

(b) annual tax payable, assuming no deductions

(c) monthly tax payable

(d) take-home pay after tax and the 2% Medicare levy have been removed.

12 Change A$850 to each of the following currencies, given the exchange rate.

(a) British pounds sterling, 1 AUD ↔ 0.58 GBP **(b)** Chinese yuan, 1 AUD ↔ 4.55 CNY

13 Change each of the following currencies to Australian dollars (AUD), to the nearest 5 cents, given the exchange rate.

(a) ¥3000 Japanese yen, 1 AUD ↔ 92.81 JPY **(b)** C$200 Canadian dollars, 1 AUD ↔ 0.87 CAD

Extended response

1 The floor plan of a house uses a scale of 1 : 50. Determine:

(a) the magnification factor

(b) the real dimensions of a room that is 7 cm × 6.5 cm on the plan

(c) the area of the room on the plan

(d) the actual area of the room

(e) the real area compared to the plan area as a single number ratio.

(f) Write the answer to part **(e)** as a power of the magnification factor.

2 At the beginning of the financial year, a man is on an annual salary of $85 384, paid weekly.

(a) What is the tax payable on the anticipated annual taxable income?

(b) What is the anticipated annual Medicare levy, at 2% of taxable income?

(c) How much, in total, is collected from each pay for tax and Medicare?

(d) The man quits his job 12 weeks before the end of the financial year to go travelling, and doesn't work again that year. How much tax and Medicare were collected altogether?

(e) What was the man's taxable income over the 52 weeks, assuming he has no deductions?

(f) How much tax should have been paid?

(g) How much Medicare levy should have been paid?

(h) What refund will the man receive after the end of the financial year?

3 Tourists in St Petersburg, Russia, need cash for lunches, ice-cream and street-vendor items, like Fabergé-style eggs and Matryoshka dolls. There are warnings about Giardia in the water, so bottled water for drinking and teeth cleaning is essential.

If the exchange rate from Australian dollars to Russian rubles is 1 AUD ↔ 39.85 RUB, determine the amount of money, in Australian dollars, a person will spend for:

(a) Three bottles of water at 53 rubles each

(b) Two lunches at 350 rubles each and three dinners at 700 rubles each

(c) Six ice-creams at 350 rubles each, two apples at 350 rubles each and five coffees at 172 rubles each

(d) Two souvenirs at 1200 rubles each

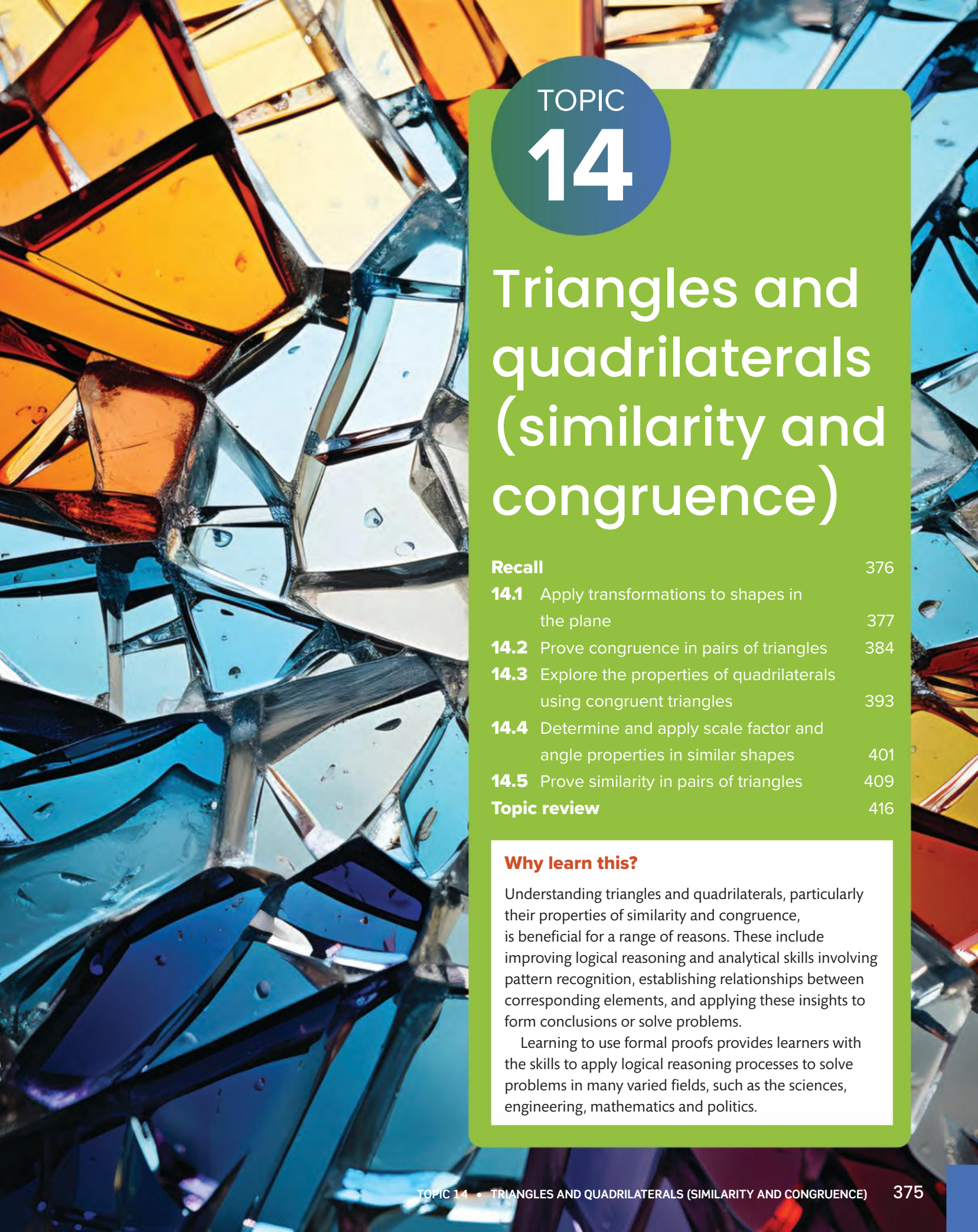
(e) Entry to the Hermitage, 800 rubles

(f) A taxi ride from the airport to the motel 23 km away at 20 rubles/km

(g) Russian ATMs allow only 2000 rubles to be withdrawn at one time, at a cost of A$2 per withdrawal, taken from the client's bank account.

- **(i)** How many rubles need to be withdrawn to cover the expenses in parts **(a)** to **(f)**?
- **(ii)** What is the cost of the withdrawals?

(h) If the unspent rubles cannot be exchanged for Australian dollars, what is the actual cost of the stay in St Petersburg, in Australian dollars, to the nearest dollar?



TOPIC
14

Triangles and quadrilaterals (similarity and congruence)

Recall 376

14.1 Apply transformations to shapes in the plane 377

14.2 Prove congruence in pairs of triangles 384

14.3 Explore the properties of quadrilaterals using congruent triangles 393

14.4 Determine and apply scale factor and angle properties in similar shapes 401

14.5 Prove similarity in pairs of triangles 409

Topic review 416

Why learn this?

Understanding triangles and quadrilaterals, particularly their properties of similarity and congruence, is beneficial for a range of reasons. These include improving logical reasoning and analytical skills involving pattern recognition, establishing relationships between corresponding elements, and applying these insights to form conclusions or solve problems.

Learning to use formal proofs provides learners with the skills to apply logical reasoning processes to solve problems in many varied fields, such as the sciences, engineering, mathematics and politics.

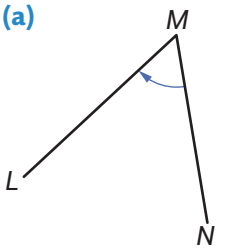
RECALL

ANSWERS Page 598

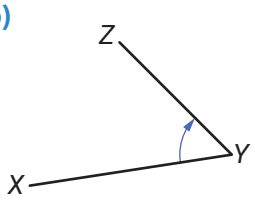
I can name angles

1 Name these angles.

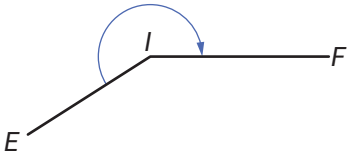
(a)



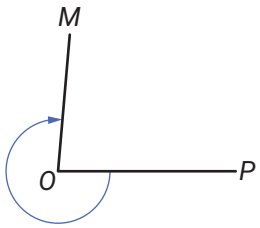
(b)



(c)



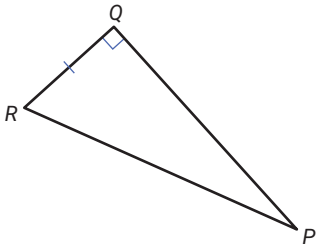
(d)

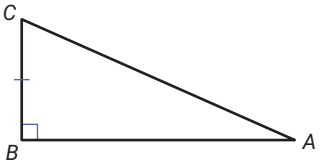


I can identify matching sides and angles in shapes

1 Name the sides and angles that correspond to each of these in the pair of identical triangles below.

(a) PQ **(b)** $\angle RQP$ **(c)** BC **(d)** $\angle BAC$

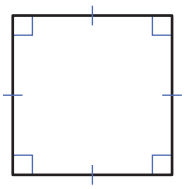




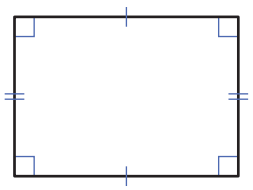
I can identify and name different types of quadrilaterals

1 Name the following quadrilaterals.

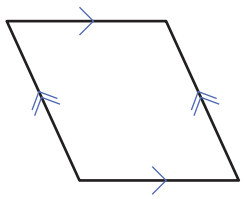
(a)



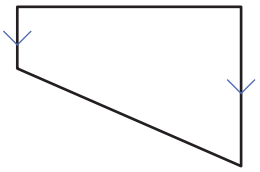
(b)



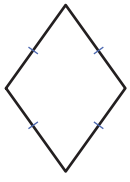
(c)



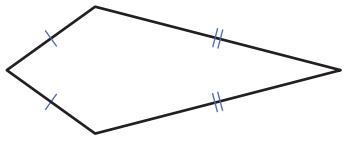
(d)



(e)



(f)



I can simplify ratios to unit ratios

1 Simplify to find the value of m for each of these ratios.

(a) $36 : 6 = m : 1$ **(b)** $72 : 24 = m : 1$ **(c)** $120 : 15 = m : 1$

(d) $35 : 14 = m : 1$ **(e)** $153 : 36 = m : 1$

Apply transformations to shapes in the plane

14.1

Learning intention: To be able to apply transformations to shapes in the plane

Success criteria:

SC 1 I can describe and apply a translation.

SC 2 I can apply a reflection and draw lines of symmetry.

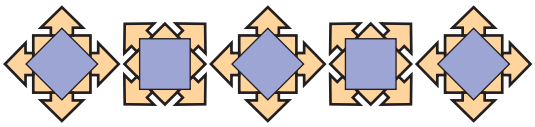
SC 3 I can apply a rotation and determine the order of rotational symmetry.

Lesson warm-up

Designing a border

This decorative design uses a repeated motif.

Rotating (turning) and translating (sliding) the motif makes the next part of the pattern.



Describe how you could use a stencil of the motif to continue the pattern.

This design uses reflection (flipping), as well as rotation and translation.

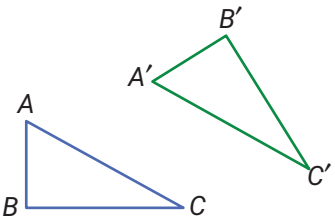
Describe how you could use a stencil of the motif to continue the pattern.

Draw your own shape and use it to make a border design. Describe the rotations, reflections and translations you have used.

SC 1 I can describe and apply a translation

A transformation changes the position, orientation or size of a shape, according to a rule. Types of transformations include translation (sliding), rotation (turning), reflection (flipping) and enlargement (increasing or decreasing the size).

After a shape ABC is transformed, the new shape (the image) is called $A'B'C'$, with the vertices named in the matching order.



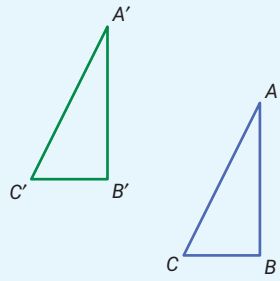
When a shape is translated, rotated or reflected, the transformed shape is the same shape and size. The two shapes are said to be congruent.

For two shapes to be congruent, one shape could fit exactly over the other shape if you translate it, rotate it or reflect it (or a combination of these).

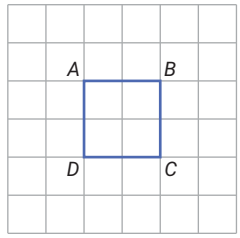
Worked example

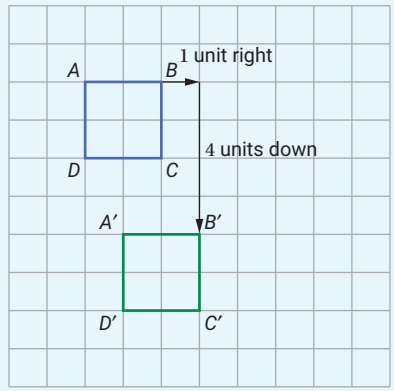
Describing and performing translations

(a) Describe the translation shown.

THINKING	WORKING
Choose a vertex (e.g. A) and locate its image (A'). Count how many units to the right (or left) the point has moved, then count the units up (or down).	
Write the answer.	The translation is 4 units left, 2 units up.

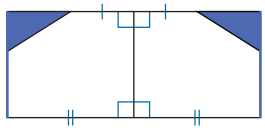
(b) Copy the square onto grid paper, then translate it one unit right and four units down.



THINKING	WORKING
Choose a vertex (e.g. B) and translate it to determine where to draw its image (B'). Complete the image, keeping the same orientation, size and shape.	

SC 2 I can apply a reflection and draw lines of symmetry

A reflection flips the original shape across a mirror line, changing its position and reversing its orientation. Each point on the image is the same distance behind the mirror as the original point, where distance is measured perpendicular to the mirror.



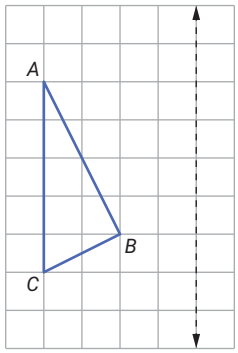
A symmetrical shape is one that has at least one line of symmetry.

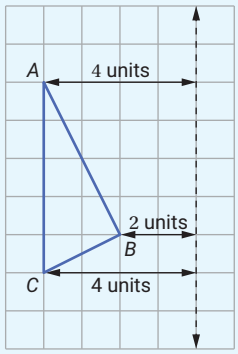
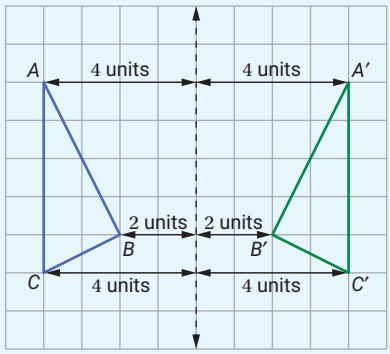
If you fold a shape on a line of symmetry, it is folded in half and each half fits exactly over the other. One half is a reflection of the other half.

Worked example

Reflecting a shape

Reflect each vertex of ΔABC in the mirror. Draw and label the image.



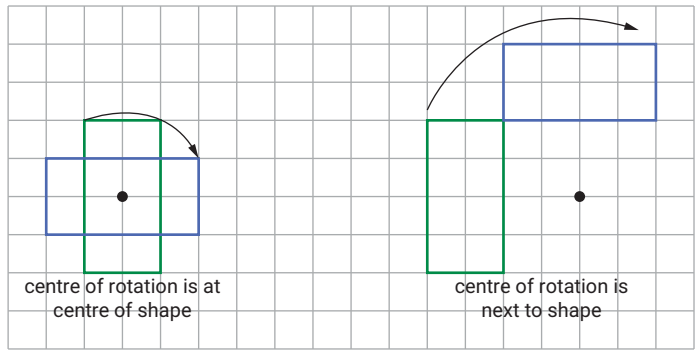
THINKING	WORKING
Measure the perpendicular distance from each vertex of ΔABC to the mirror line.	
Draw the image of each vertex the same distance on the other side of the mirror. Draw the shape.	

SC 3 I can apply a rotation and determine the order of rotational symmetry

To rotate a shape, you need to know three things:

- the centre of rotation
- the size of the angle of rotation
- the direction of rotation, either clockwise or anticlockwise.

The centre of rotation can be anywhere inside or outside the shape. It does not have to be at the centre of the shape.

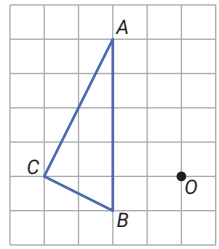


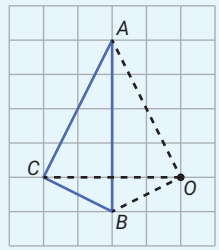
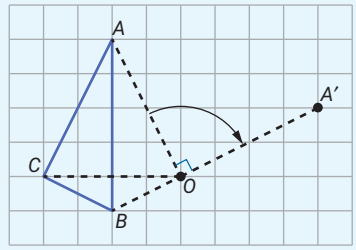
A shape that has rotational symmetry can be rotated around its own centre, with the image matching the original shape before a full rotation of 360° has been completed. The order of rotational symmetry is the number of positions during the complete rotation that the image matches the original shape. The order of rotational symmetry of the pinwheel shape below is 4.

Worked example

Rotating a shape

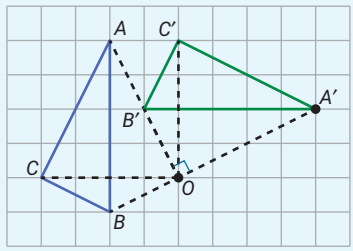
Copy the shape onto grid paper. Rotate the shape 90° clockwise around the centre of rotation, O. Label the image.



THINKING	WORKING
Join the centre of rotation to each vertex with a straight line segment.	
Measure 90° clockwise from a line segment. Draw the image vertex the same distance from the centre of rotation as the original vertex.	

Repeat the process with the other vertices.

Join the image vertices to complete the image of the rotated shape.



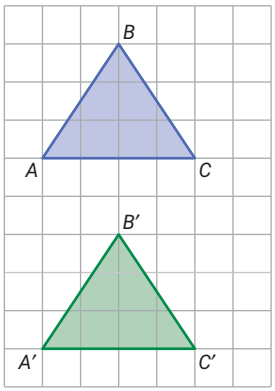
Practice

ANSWERS Page 598

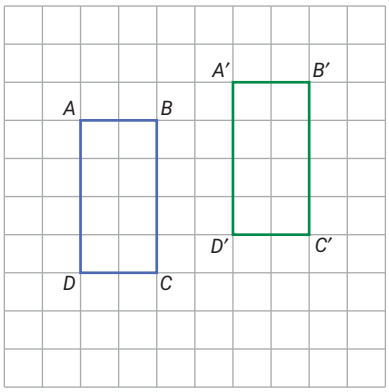
SC 1 I can describe and apply a translation

1 Describe the translation.

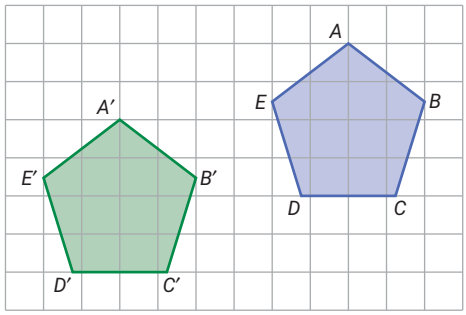
(a)



(b)

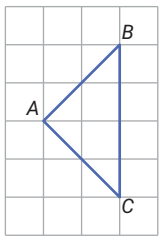


(c)

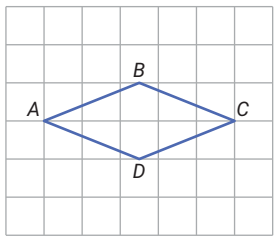


2 Copy each shape onto grid paper, then translate.

(a) 2 units right, 1 unit down



(b) 1 unit left, 6 units down



3 When a shape is translated, which of the following things change and which stay the same?

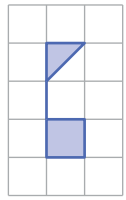
(a) The side lengths

(b) The angles

(c) The position

(d) The orientation (turning)

4 Copy the shape onto grid paper, then translate.



(a) 4 units left, 1 unit up

(b) 3 units right, 2 units down

5 Write a single translation that would replace each of the following multiple translations.

(a) 4 units left, 3 units down, then 6 units left, 7 units up

(b) 2 units right, 3 units up, then 2 units left, 6 units up

(c) 12 units down, 5 units right, 7 units left, 8 units up, 2 units right

(d) 2 units right, repeated for a total of 100 times

6 Copy the shape onto paper, then use a ruler and protractor to translate.

(a) 40 mm left

(b) 42 mm up and to the right, along a line at 35° to the vertical

(c) 50 mm up and to the right, along a line at 50° to the horizontal

SC 2 I can apply a reflection and draw lines of symmetry

1 For each shape, draw all possible lines of symmetry.

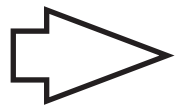
(a)

(b)

2 How many lines of reflectional symmetry do each of the following shapes have?

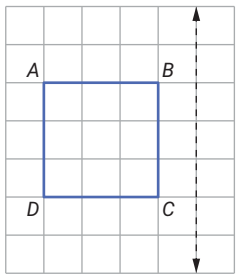
(a)

(b)

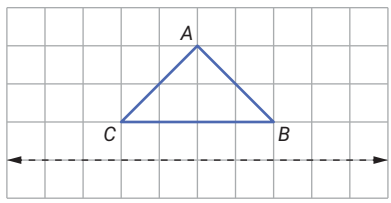
(c) 

3 Copy each shape and mirror line onto grid paper. Reflect each shape in the mirror and label the image.

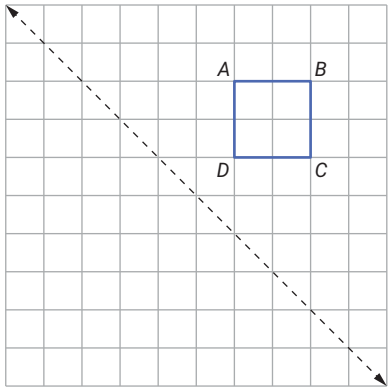
(a)



(b)



(c)



4 When a shape is reflected, which of the following things change and which stay the same?

(a) The side lengths **(b)** The angles

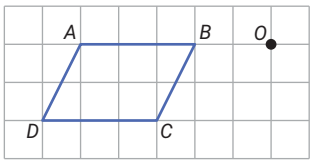
(c) The position **(d)** The order of the vertices

5 Create four equal sections with a vertical mirror line and a horizontal mirror line running through the middle of a piece of grid paper 6 squares wide and 6 squares high. Shade at least one square and one triangle in the top left section. Then reflect the shading in the horizontal mirror line. After this, reflect the image in the vertical mirror line.

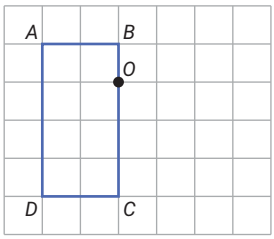
SC 3 I can apply a rotation and determine the order of rotational symmetry

1 Copy each shape onto grid paper. Rotate as directed. Label the image.

(a) Rotate 180° clockwise around O.

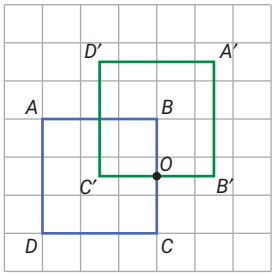


(b) Rotate 90° anticlockwise around O.

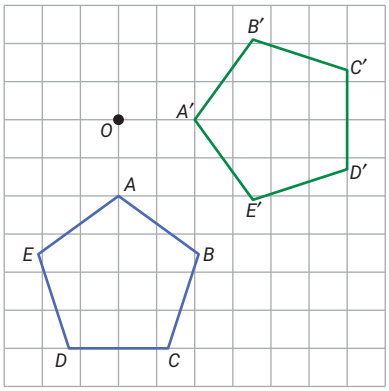


2 Determine the angle size and direction of the rotation around O that has taken place in each of the following. Use the smallest angle possible.

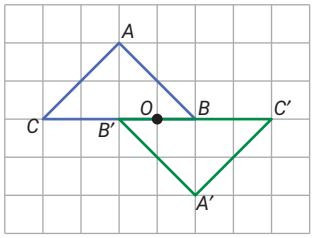
(a)



(b)



(c)

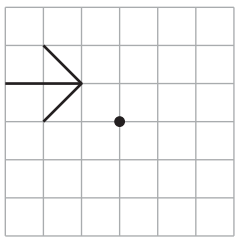


3 When a shape is rotated, which of the following things change and which stay the same?

(a) The side lengths **(b)** The angles **(c)** The position

(d) The orientation **(e)** The order of the vertices

4 Rotate this shape 90° clockwise about the dot, then repeat this procedure with the image until the final image is back in the original position.



14.2 Prove congruence in pairs of triangles

Learning intention: To be able to prove congruence in pairs of triangles

Success criteria:

SC 1 I can draw triangles given side lengths and angles.

SC 2 I can use each of the four tests for congruent triangles.

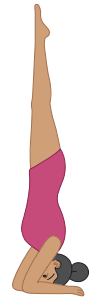
SC 3 I can prove congruence in pairs of triangles to determine unknown side lengths or angles.

Lesson warm-up

What does a maths proof need to do?

'I have seen hundreds of sheep and they are all white. I can therefore conclude that all sheep are white.'

Does this statement prove what it sets out to prove? Why or why not?

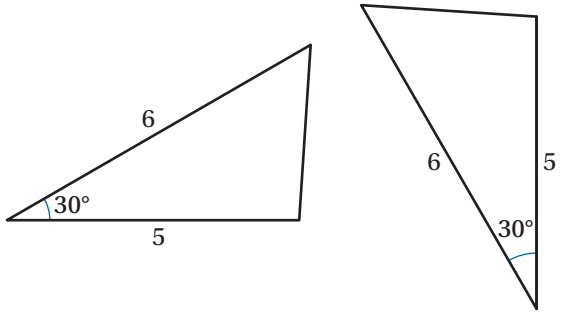


SC 1 I can draw triangles given side lengths and angles

Two triangles are congruent if they have the same size and shape.

To decide whether two triangles are congruent, measure the sides and angles and check that they are the same. You don't have to measure all sides and angles. There are six values (three sides and three angles) for each triangle. If you know which sides and angles to measure, you can determine congruency from just three values on each triangle.

Drawing triangles can help you understand which sides and angles are important for congruency. For example, try to draw a triangle with side lengths of 5 cm and 6 cm, with an angle of 30° in between those sides:



There is only one way to draw the final side.

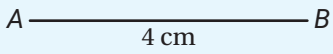
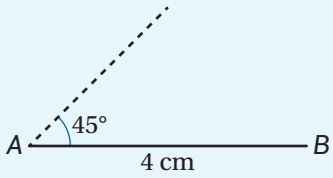
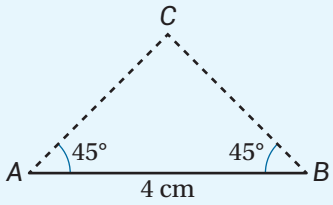
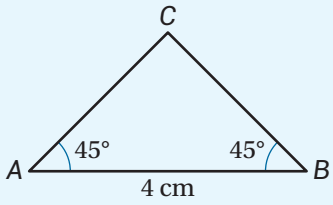
If there is only one triangle you can draw from a set of instructions, then any triangle that matches those instructions is congruent.

Worked example

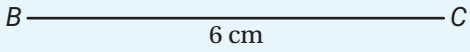
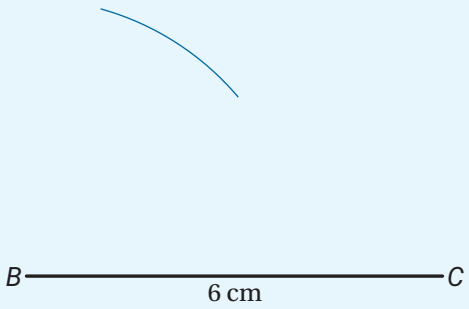
Constructing triangles from three measurements

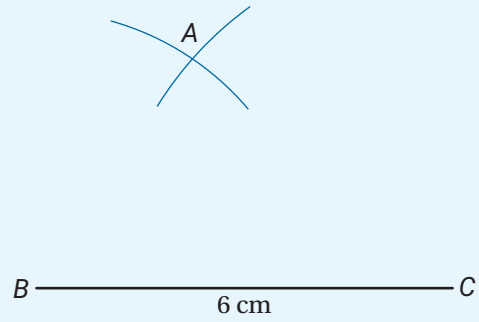
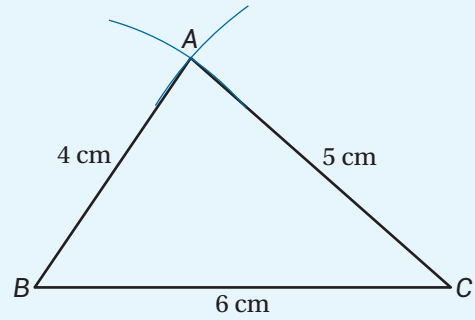
Construct a triangle ΔABC with:

(a) $AB = 4$ cm, $A = 45°$ and $B = 45°$ (one side and two angles, ASA)

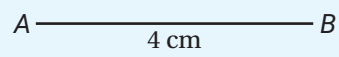
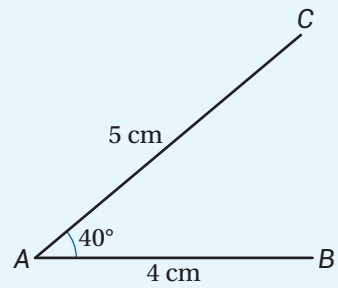
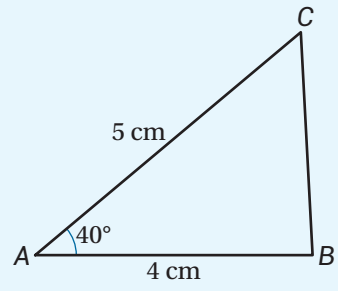
THINKING	WORKING
Use a ruler to draw and label the given side.	
Use a protractor and ruler to construct and label one of the given angles at one end of the segment.	
Use a protractor and ruler to construct the other given angle at the other end of the segment. Label the point where they meet with the name of the vertex.	
Complete the triangle using the intersection of the arms of the angles.	

(b) $AB = 4$ cm, $AC = 5$ cm and $BC = 6$ cm (three side lengths, SSS)

THINKING	WORKING
Use a ruler to draw and label a given side.	
Use a compass, open to the length of one of the sides and draw an arc.	

Use a compass, open to the length of the other side and draw an arc. Mark the point where the two arcs meet with the name of the vertex.	
Complete the triangle using the intersection of the arcs as the third vertex.	

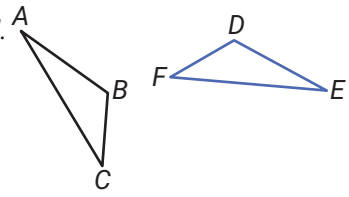
(c) $AB = 4$ cm, $AC = 5$ cm and $A = 40°$ (two side lengths and the angle between them, SAS)

THINKING	WORKING
Use a ruler to draw and label a given side.	
Use a protractor to measure the angle, and rule through the known side length.	
Use a ruler to join the other two vertices.	

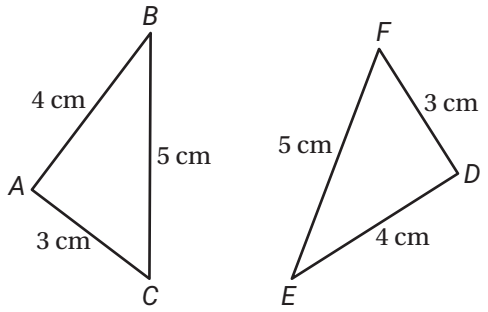
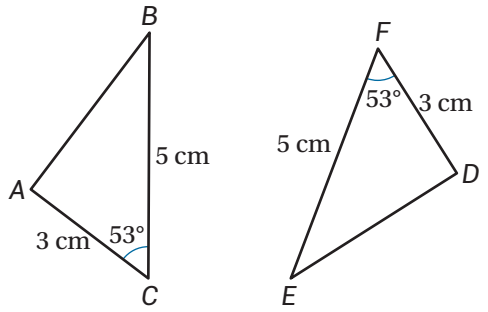
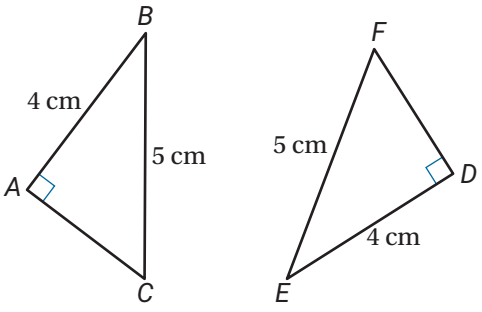
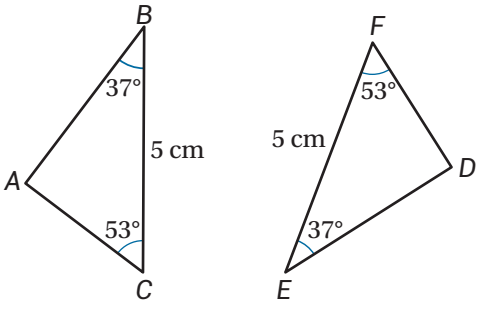
SC 2 I can use each of the four tests for congruent triangles

$\Delta ABC \cong \Delta EDF$ means that triangle ABC 'is congruent to' triangle EDF.

Name the triangles so that their matching vertices are in the same order: E is the matching vertex of A, D is the matching vertex of B, and so on.



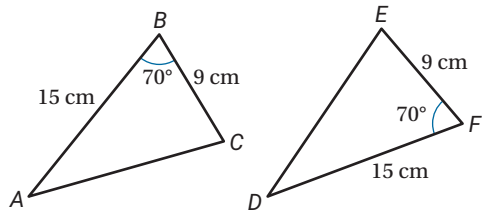
In the last section, you drew triangles from three pieces of information (angles and sides). Sometimes only one triangle could be drawn with those three pieces of information. Congruence tests are based on this idea.

Congruence test	Explanation	Example
SSS (side, side, side)	If three sides of one triangle are the same lengths as three sides of another, then the two triangles are congruent.	 $\Delta ABC \cong \Delta DEF$
SAS (side, angle, side)	If two sides and the included angle of one triangle are the same as two sides and the included angle of another triangle, then the two triangles are congruent.	 $\Delta ABC \cong \Delta DEF$
RHS (right angle, hypotenuse, side)	If the hypotenuse and one other side of two right-angled triangles are equal, the two triangles are congruent.	 $\Delta ABC \cong \Delta DEF$
AAS (angle, angle, side)	If two angles and one side of a triangle are the same as two angles and the matching side of another triangle, the two triangles are congruent.	 $\Delta ABC \cong \Delta DEF$

Worked example

Proving congruence in triangles

Prove that these triangles are congruent.



THINKING	WORKING
Name the triangles, matching the vertices.	ΔABC and ΔDFE
Identify the equal sides and angles, giving reasons.	$AB = DF$ (given as 15 cm) ... S $\angle ABC = \angle DFE$ (given as 70°) ... A $BC = FE$ (given as 9 cm) ... S
Write the congruence relationship and justify with the congruence test.	$\Delta ABC \cong \Delta DFE$ (SAS congruence test)

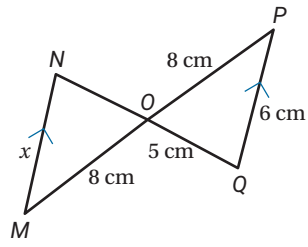
SC 3 I can prove congruence in pairs of triangles to determine unknown side lengths or angles

If two triangles are congruent, it means that their matching sides and angles are equal. After proving congruence, you can determine unknown side lengths or angles.

Worked example

Proving congruence to solve for unknowns

Prove that $\Delta MNO \cong \Delta PQO$, and hence determine the value of x.



THINKING	WORKING
For the pair of triangles, identify the equal pairs of sides and angles, giving reasons.	In triangles MNO and PQO: $\angle MNO = \angle PQO$ (alternate angles, $NM \parallel PQ$) ... A $\angle NMO = \angle QPO$ (alternate angles, $NM \parallel PQ$) ... A (Or: $\angle NOM = \angle QOP$ (vertically opposite angles)) $MO = PO$ (given 8 cm) ... S
Write the congruence statement and the congruence test.	$\therefore \Delta MNO \cong \Delta PQO$ (AAS congruence test)
Determine the side corresponding to x and write the answer.	Hence $MN = PQ$ (corresponding sides) $x = 6$ cm

Practice

ANSWERS Page 599

SC 1 I can draw triangles given side lengths and angles.

1 Use a protractor, ruler and compass to construct the following triangles.

(a) Sides 7 cm and 4 cm, with 30° angle between the sides

(b) Angles of 20° and 100°, with the side between these angles 8 cm

(c) Right-angled triangle with hypotenuse 6 cm and another side 4 cm

(d) Sides 3 cm, 6 cm and 8 cm

2 For each triangle in Question **1**, consider whether any other triangles can be drawn using the same instructions.

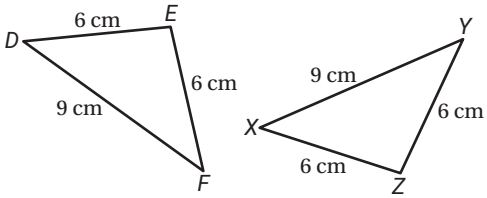
3 Explain why two triangles with three angles the same may not be congruent. Use triangles with angles of 35°, 65° and 80° in your explanation, supported with a diagram.

4 Explain why a pair of triangles with two pairs of matching sides does not mean that the triangles will be congruent. Use triangles with side lengths of 2 cm and 5 cm in your explanation, supported with a diagram.

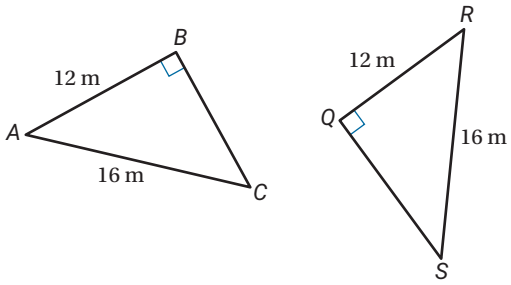
SC 2 I can use each of the four tests for congruent triangles

1 For each pair of triangles, write the congruence relationship and justify with the congruence test.

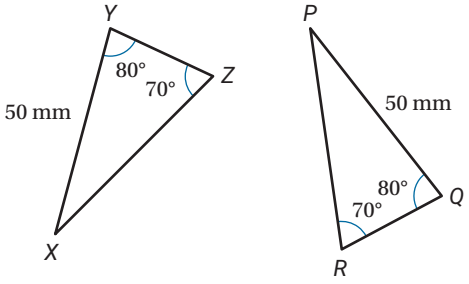
(a)



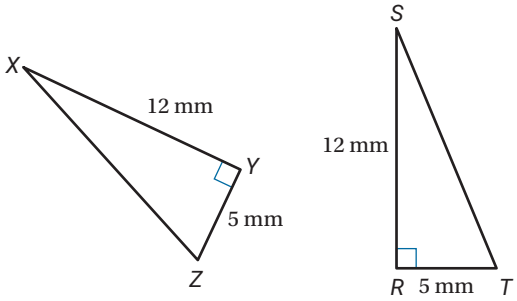
(b)



(c)

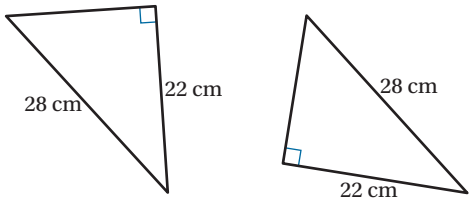


(d)



2 Write the congruence test that justifies why each pair of triangles is congruent.

(a)



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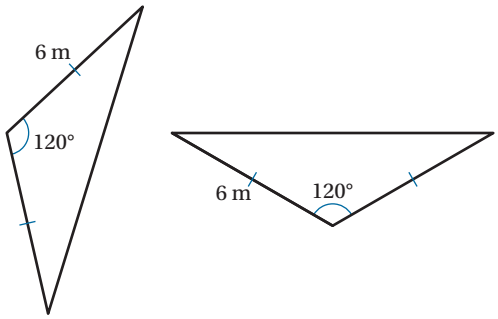
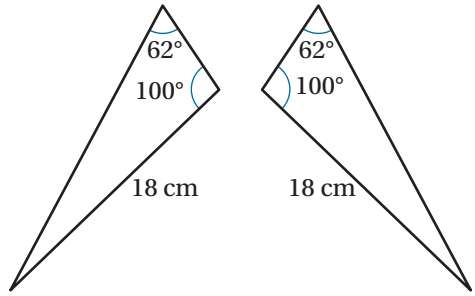
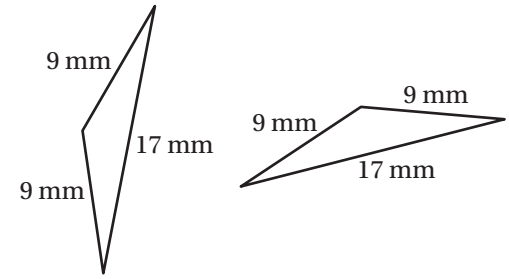


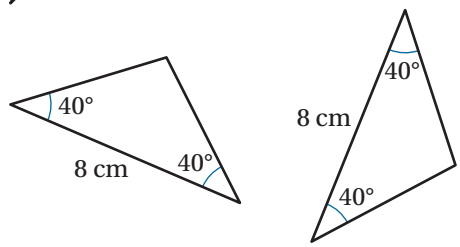
(c)



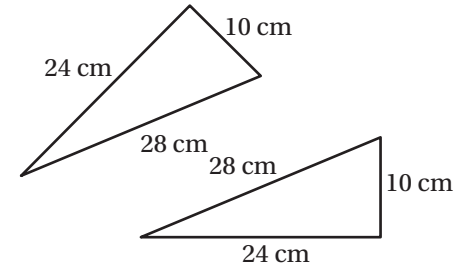
(d)



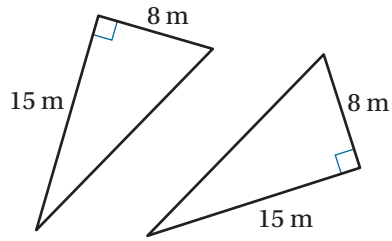
(e)



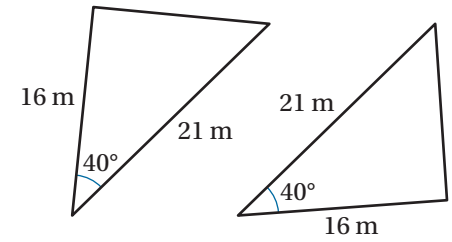
(f)



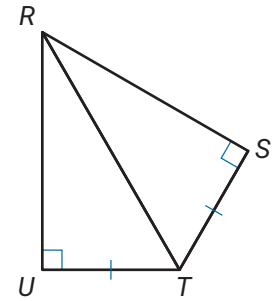
(g)



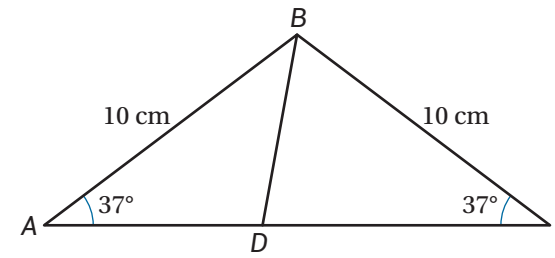
(h)



3 Prove congruence in the pair of triangles. Write a congruence statement and a congruence test to justify your answer.



4 Isosceles triangle ABC has $BA = BC = 10$ cm and $\angle A = \angle C = 37°$. The line segment BD cuts it into two smaller triangles, ΔABD and ΔCBD.



(a) In the two smaller triangles, list three pairs of sides or angles that are equal.

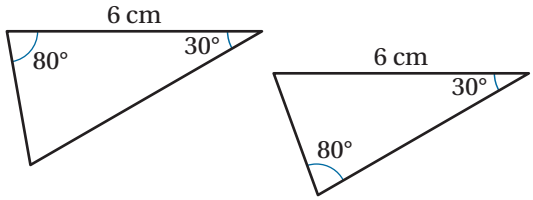
(b) Are the three equal values in matching positions?

(c) Is ΔABD congruent to ΔCBD? Justify your answer.

(d) Why does the SAS congruency test use only the angle between the sides (the included angle)?

(e) How could you move line segment BD to make congruent triangles? What tests could you use to prove congruency?

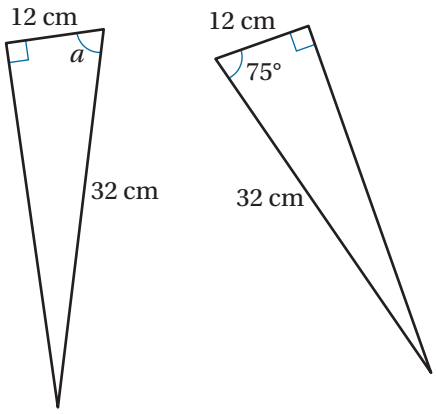
5 This pair of triangles both have angles of 80° and 30°, and one side 6 cm. Are these triangles congruent? Justify your answer.



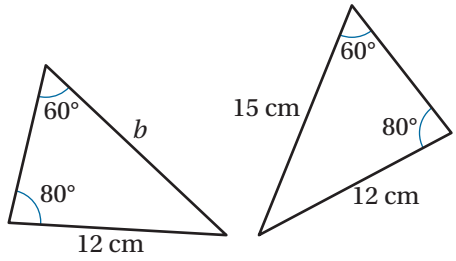
SC 3 I can prove congruence in pairs of triangles to determine unknown side lengths or angles

1 The following pairs of triangles are congruent. Determine the value of the variable in each.

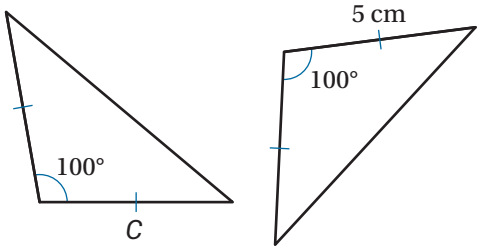
(a)



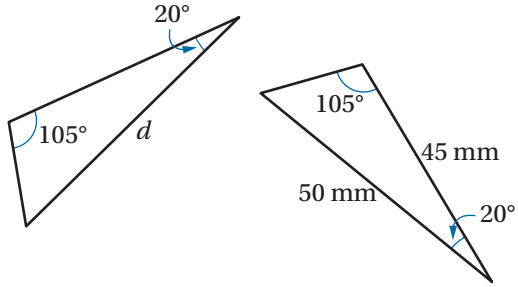
(b)



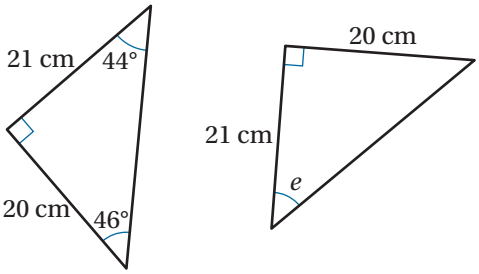
(c)



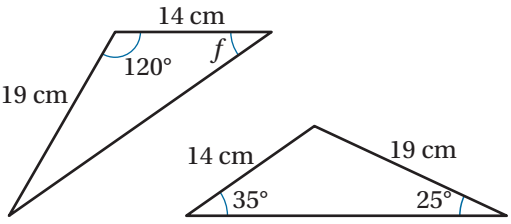
(d)



(e)

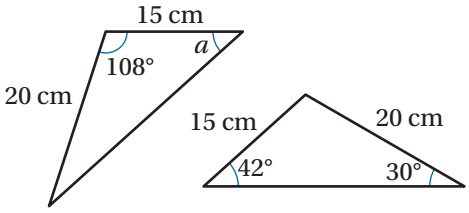


(f)

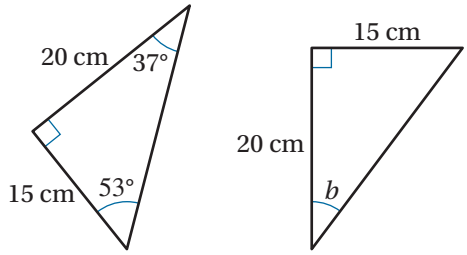


2 In each of the following pairs of triangles, state the test that proves congruence, then use corresponding sides and angles to determine the value of the variable.

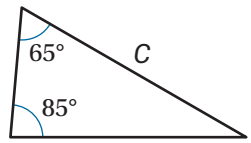
(a)

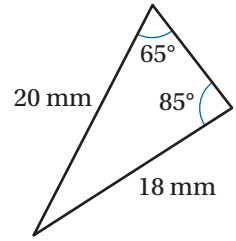


(b)

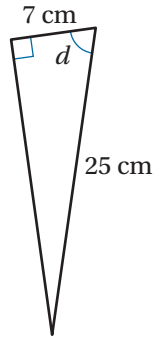


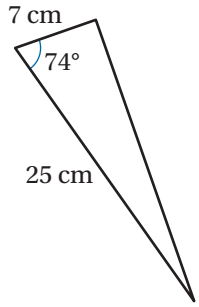
(c)



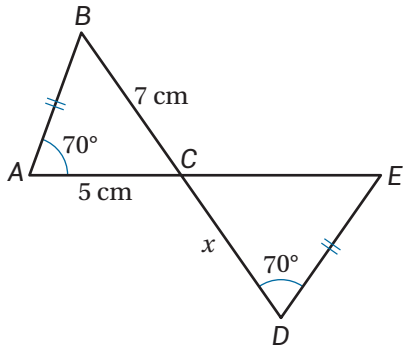


(d)

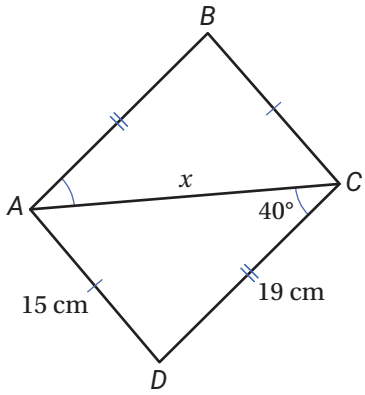




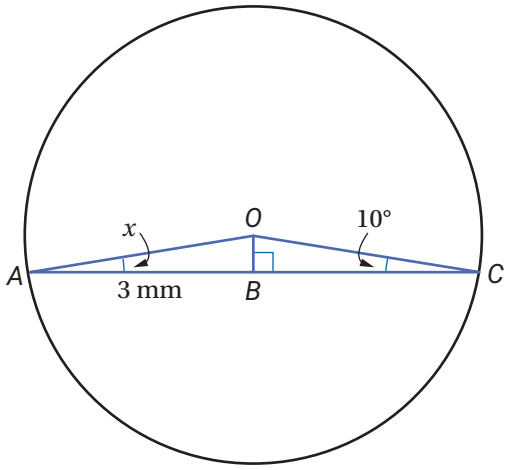
3 Prove that these triangles are congruent. Write the congruence statement with the congruence test. Then solve for the variable.



4 Prove that these triangles are congruent, then solve for the variable.



5 Prove that these triangles are congruent, then solve for the pronumeral.



Explore the properties of quadrilaterals using congruent triangles

14.3

Learning intention: To be able to explore the properties of quadrilaterals using congruent triangles

Success criteria:

SC 1 I can establish properties of quadrilaterals using congruent triangles.

SC 2 I can prove congruence in pairs of triangles to determine unknown side lengths or angles in symmetrical quadrilaterals.

SC 3 I can draw diagrams identifying the main properties of quadrilaterals, including side lengths, parallel sides, angles, diagonals and symmetry.

14

Lesson warm-up

Who did it?

In Shape world, there were two famous detectives, Captain Geo and Captain Tree. One day, they were both called to the scene of a crime, a large rectangular prism. All the suspects had been gathered and were waiting to be questioned. Captain Geo parallel-parked while Captain Tree parked at 90°, being acutely aware of the need to get the right angle to the curb.

Geo met Tree at the vertex of the prism before each suspect was asked the same question, 'What did you see?' After hearing all the answers, the two detectives reached congruence on who was guilty. See if you can find the guilty shape.

The Isosceles Triangle said, 'It was a bit dark, but I saw the guilty shape had at least one pair of parallel sides.'

The Rhombus said, 'I think the shape was a quadrilateral.'

The Kite said, 'I saw at least two sides of the same length.'

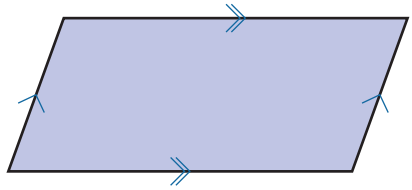
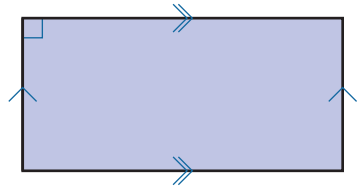
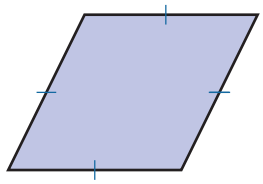
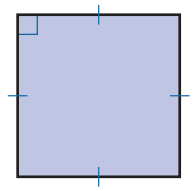
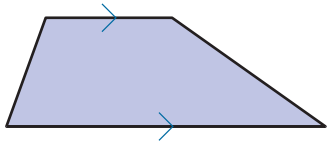
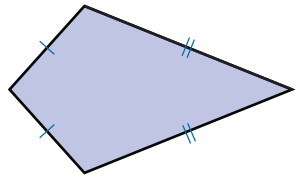
The Trapezium said, 'The guilty shape had all equal sides.'

The Square said, 'I am sure the guilty shape was a triangle.'

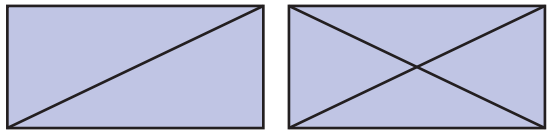
The Equilateral Triangle said, 'I saw two pairs of equal angles.'

SC 1 I can establish properties of quadrilaterals using congruent triangles

Quadrilaterals are four-sided plane figures. The special quadrilaterals and their definitions are shown in the table below. Their properties can be proven from their definitions, using congruent triangles.

 A parallelogram has both pairs of opposite sides parallel.	 A rectangle has both pairs of opposite sides parallel and a right angle.
 A rhombus has all sides equal.	 A square has all sides equal and a right angle.
 A trapezium has just one pair of opposite sides parallel.	 A kite has two pairs of adjacent sides equal.

Quadrilaterals can be divided by a diagonal into two triangles. A second diagonal gives four triangles.

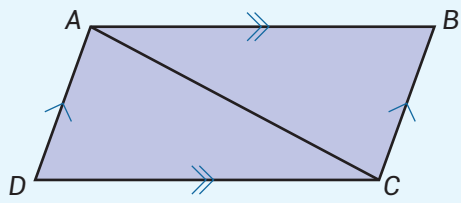


In the special quadrilaterals, apart from the trapezium, at least one pair of the triangles is congruent. These congruent triangles are then used to establish the properties of the special quadrilaterals.

Worked example

Establishing side properties of parallelograms

Using the definition of a parallelogram only, prove congruence in a pair of triangles to conclude that both pairs of opposite sides are equal.

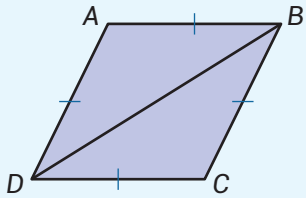
THINKING	WORKING
Draw and label the parallelogram, marking both pairs of parallel sides. Draw one diagonal.	

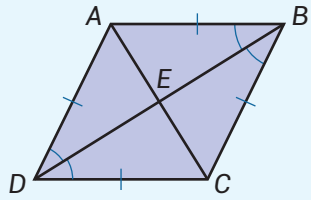
Prove congruence in the pair of triangles.	In triangles ABC, CDA: $\angle BCA = \angle DAC$ (alternate angles, $AD \parallel BC$) ... A $\angle BAC = \angle DCA$ (alternate angles, $AB \parallel DC$) ... A AC is shared ... S $\therefore \Delta ABC \cong \Delta CDA$ (AAS)
Write the pairs of corresponding sides in the triangles. These are also the opposite sides of the parallelogram.	$AB = CD$ (corresponding sides) $BC = DA$ (corresponding sides)
Write the conclusion.	Hence, both pairs of opposite sides of the parallelogram are equal.

Worked example

Establishing properties of the diagonals of a rhombus

Using just the definition of a rhombus, prove congruence in a pair of triangles to conclude that the diagonals are at right angles.

THINKING	WORKING
Draw and label the rhombus, marking all sides equal. Draw one diagonal.	
Prove congruence in the pair of triangles.	In triangles ABD, CBD: $AB = CB$ (definition) ... S $AD = CD$ (definition) ... S DB is shared ... S $\therefore \Delta ABD \cong \Delta CBD$ (SSS)
Identify the alternative congruence due to the isosceles triangles.	Alternatively, $\Delta ABD \cong \Delta CDB$
Determine corresponding pairs of angles from the triangles that are needed in the next stage of the proof.	$\angle ABD = \angle CBD$ (corresponding angles) $\angle ADB = \angle CDB$ (corresponding angles) Alternatively, $\angle ABD = \angle CDB$ Hence all four angles next to the diagonal are equal.

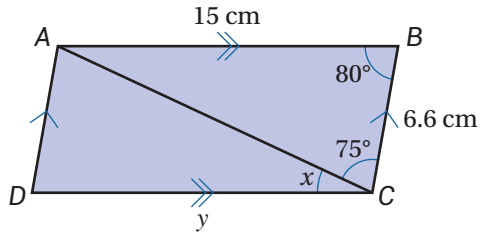
Mark the equal angles and draw the second diagonal, labelling the intersection.	
Prove congruence in a pair of triangles such that a pair of adjacent angles at the intersection of diagonals is involved.	In triangles ABE, CBE: $AB = CB$ (definition) ... S $\angle ABE = \angle CBE$ (proven) ... A EB is shared ... S $\therefore \Delta ABE \cong \Delta CBE$ (SAS)
Determine equivalence of corresponding angles at the intersection of diagonals.	Hence $\angle BEA = \angle BEC$ (corresponding angles)
Write the conclusion.	The adjacent angles where the diagonals meet are equal and supplementary, hence they are both 90°. Hence the diagonals of a rhombus intersect at right angles.

SC 2 I can prove congruence in pairs of triangles to determine unknown side lengths or angles in symmetrical quadrilaterals

Worked example

Proving congruence to solve unknown sides and angles in special quadrilaterals

Using just the information given on the diagram, prove that the pair of triangles are congruent, then solve for the variables.



THINKING	WORKING
Prove congruence in the pair of triangles.	In triangles ABC, CDA: $\angle BAC = \angle DCA$ (alternate angles, $AB \parallel DC$) ... A $\angle BCA = \angle DAC$ (alternate angles, $AD \parallel BC$) ... A AC is shared ... S $\therefore \Delta ABC \cong \Delta CDA$ (AAS)

Determine corresponding sides from the triangles that match the variable.	$AB = CD$ (corresponding sides) So, $y = 15$ cm
Determine corresponding angles from the triangles that match the variable.	$\angle BAC = \angle DCA$ (corresponding angles) From the angle sum of a triangle: $\angle BAC = 180° - 80° - 75°$ $= 25°$ So, $x = 25°$
Write the answer.	$x = 25°$ and $y = 15$ cm

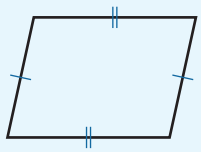
SC 3 I can draw diagrams identifying the main properties of quadrilaterals, including side lengths, parallel sides, angles, diagonals and symmetry

Worked example

Marking up quadrilaterals to show properties

Draw three parallelograms to show each of the following properties.

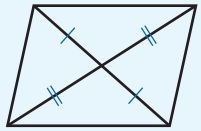
(a) Equal sides

THINKING	WORKING
Mark both pairs of opposite sides equal.	

(b) Equal angles

THINKING	WORKING
Mark both pairs of opposite angles equal.	

(c) Properties of the diagonals

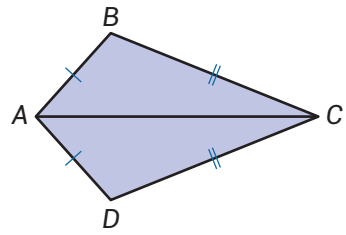
THINKING	WORKING
Show that the diagonals bisect each other.	

Practice

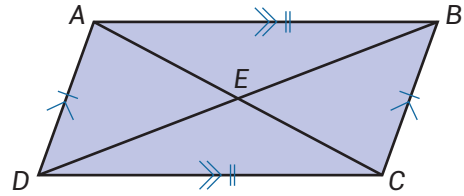
ANSWERS Page 600

SC 1 I can establish properties of quadrilaterals using congruent triangles

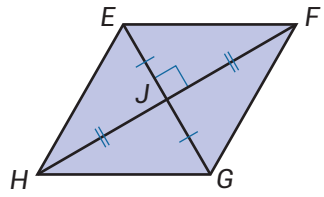
1 Using only the definition of a kite, prove congruence in a pair of triangles to conclude that one pair of opposite angles is equal.



2 Using the definition of a parallelogram, and assuming both pairs of opposite sides have been proven equal, prove congruence in a pair of triangles to conclude that the diagonals bisect each other.



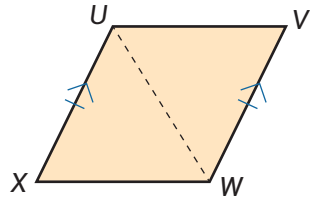
3 The diagonals of quadrilateral $EFGH$ bisect each other at right angles at J.



Prove that $EFGH$ is a rhombus.

4 From only the definition of a kite, prove that the diagonals intersect at right angles.

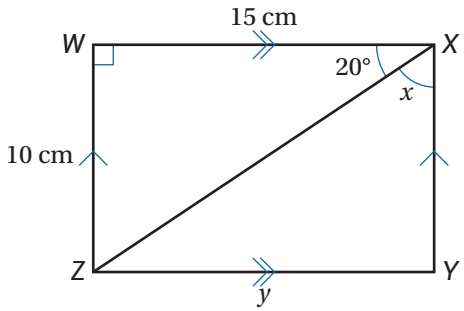
5 A quadrilateral has one pair of opposite sides equal and parallel.



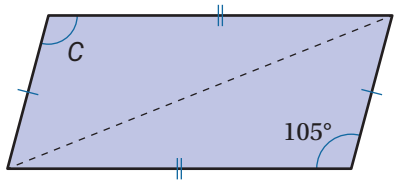
Prove that the quadrilateral is a parallelogram.

SC 2 I can prove congruence in pairs of triangles to determine unknown side lengths or angles in symmetrical quadrilaterals

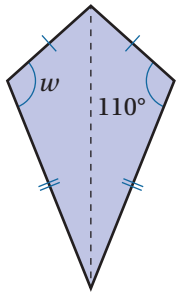
1 Using only the information given on the diagram, prove that the pair of triangles are congruent, then solve for the variables.



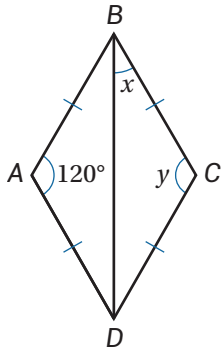
2 Use congruent triangles to determine the value of c.



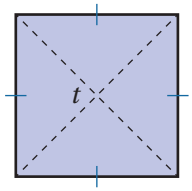
3 Use congruent triangles to determine the value of w.



4 Use congruent triangles to determine the value of the variables x and y.



5 Use congruent triangles to determine the value of angle t.



SC 3 I can draw diagrams identifying the main properties of quadrilaterals including side lengths, parallel sides, angles, diagonals and symmetry

1 Draw a kite to show each of the following properties.

(a) Equal sides **(b)** Equal angles **(c)** Properties of the diagonals

2 Identify which of the special quadrilaterals have lines of symmetry:

(a) through both pairs of sides and both pairs of vertices

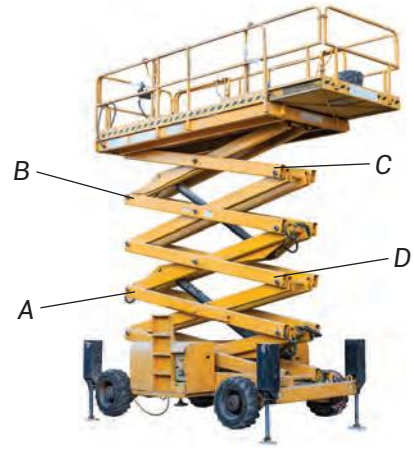
(b) only through both pairs of sides

(c) only through both pairs of vertices.

3 Mark the equal angles created by the pair of diagonals of each of the following.

(a) Parallelogram **(b)** Rhombus

4 The illustration below shows a scissor lift with a horizontal platform on top.



(a) Identify the shape of the quadrilateral $ABCD$.

(b) Explain what happens to the shape of $ABCD$ as the platform is elevated.

(c) Identify the properties of the diagonals of $ABCD$ that can be seen in the illustration.

(d) Explain the benefits of the shape of $ABCD$ in this kind of lift.

Determine and apply scale factor and angle properties in similar shapes

14.4

Learning intention: To be able to determine and apply scale factor and angle properties in similar shapes

Success criteria:

SC 1 I can draw the result of an enlargement by a given scale factor.

SC 2 I can determine the scale factor used to enlarge a figure.

SC 3 I can determine lengths and angles in similar shapes.

Lesson warm-up

Zooming in

When you zoom in on a photo, what changes and what stays the same?

The photo below on the right has been zoomed in.

1 Choose one thing that has changed, and one thing that has stayed the same.

2 Measure the wingspan of the flying birds in both photos. How much larger is the wingspan in the zoomed-in photo?

3 Measure the angle of the wings of the flying birds in both photos. How much larger is the angle in the zoomed-in photo?

4 Measure a vertical distance in both photos (e.g. the distance from the horizon to the foreground bird's beak). How much larger is it in the zoomed-in photo?

5 Measure the length of the foreground bird's front leg. How long do you think it would be in the zoomed-in photo?

SC 1 I can draw the result of an enlargement by a given scale factor

Similar figures are the same shape but different sizes. All of the corresponding angles are equal and pairs of corresponding lengths are in the same ratio.

An enlargement changes the size of an object to produce a similar figure in which the angles are the same as the original and the lengths are in proportion.

If the scale factor is 2, the image is an enlargement, with all lengths double the original lengths.

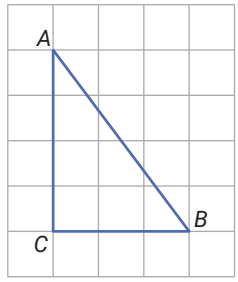
If the scale factor is $\frac{1}{2}$, the image is a reduction, with all lengths half the original lengths.

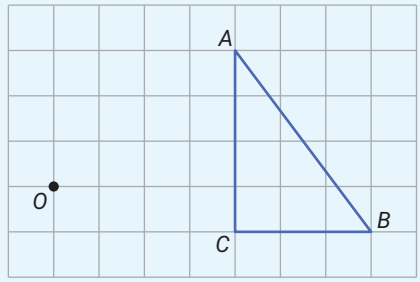
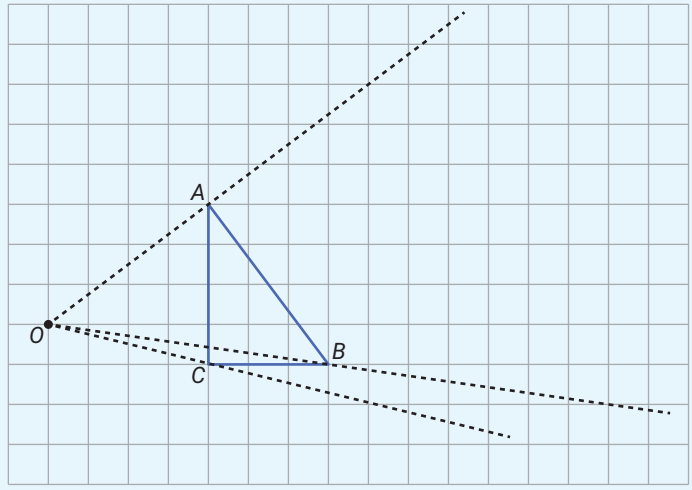
One way of enlarging a shape is to use a centre of enlargement.

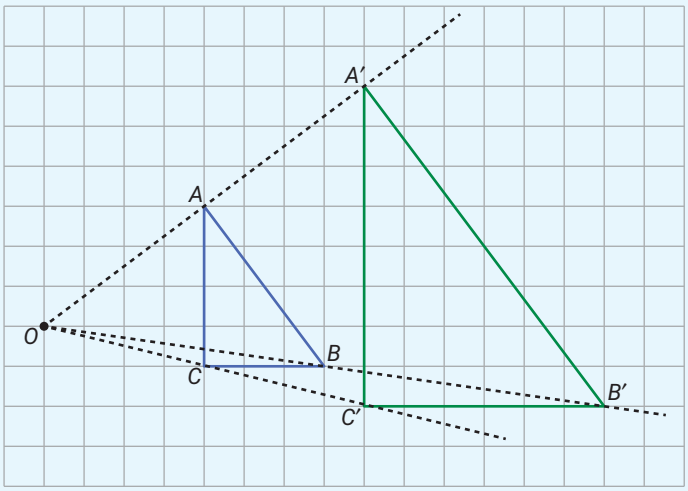
Worked example

Enlarging a shape using a centre of enlargement

Enlarge the triangle by a factor of 2.



THINKING	WORKING
Select a centre of enlargement, anywhere outside the shape.	
Rule lines from the centre of enlargement through each vertex and out the other side.	

Measure the distances from the centre to each vertex (or use the grid), multiply by the scale factor and draw the image vertices. Draw the image.	
Check that the lengths of the sides have been increased by the scale factor.	The side lengths have doubled as intended.

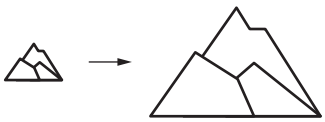
SC 2 I can determine the scale factor used to enlarge a figure

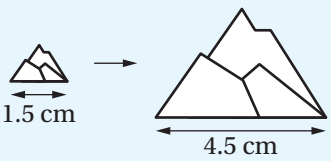
For pairs of similar figures, any matching pair of dimensions can be used to determine the scale factor.

Worked example

Determining scale factor for an enlarged figure

Determine the scale factor that has been used in the enlargement.



THINKING	WORKING
Choose corresponding dimensions on each shape and measure them.	
Determine the ratio of the new length to the original. Simplify the ratio.	new length : old length 4.5 cm : 1.5 cm 3 : 1
Convert the ratio to a number.	The scale factor is 3.

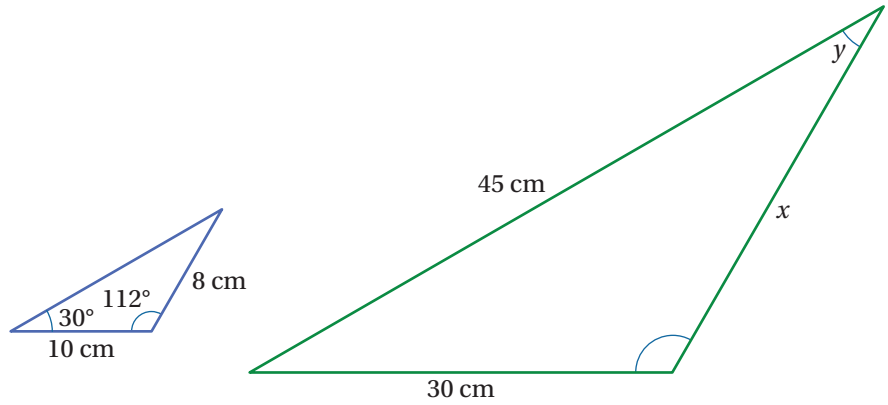
SC 3 I can determine lengths and angles in similar shapes

Similar shapes are the same shape, but different sizes. If two shapes are similar, they have all pairs of corresponding angles equal and all pairs of corresponding sides in proportion.

Worked example

Calculating lengths and angles in similar shapes

For this pair of similar triangles, calculate the values of the pronumerals representing the sizes of angles and the lengths of sides.



THINKING	WORKING
Match any pairs of known sides. Calculate the scale factor.	10 cm ↔ 30 cm $\frac{30}{10} = 3$ The scale factor is 3.
Match the side with the pronumeral to a known side. Multiply (or divide) by the scale factor to calculate the value of the pronumeral.	8 cm ↔ x $x = 8 \times 3$ $= 24$ cm
Match the angle with the pronumeral to a known angle. Calculate the value if necessary.	Third angle in the small triangle: $180° - 30° - 112° = 38°$ Hence $y = 38°$
Write the answer.	The length of the missing side, $x = 24$ cm. The size of the missing angle, $y = 38°$.

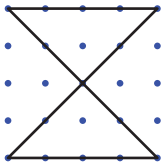
Practice

ANSWERS Page 602

SC 1 I can draw the result of an enlargement by a given scale factor

1 Copy each figure onto grid or dot paper, then enlarge them using a scale factor of 2.

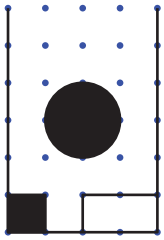
(a)



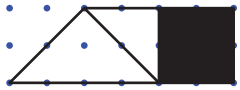
(b)

2 Copy each figure onto grid or dot paper, then reduce them using a scale factor of $\frac{1}{2}$.

(a)



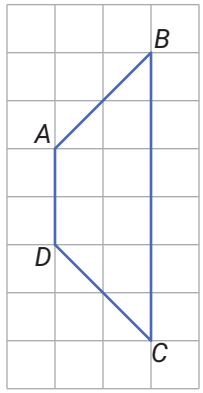
(b)



3 Copy the quadrilateral, choose a centre of enlargement, then enlarge by the given scale factors.

(a) scale factor 3

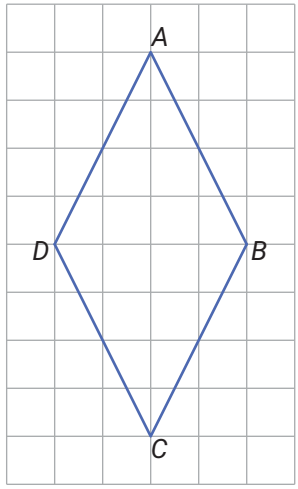
(b) scale factor 1.5



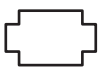
4 Copy the shape, choose a centre of reduction, then reduce by the given scale factors. (Leave room between the centre of reduction and the original shape to draw the image.)

(a) scale factor $\frac{1}{3}$

(b) scale factor 0.8



5 Use any method to enlarge this shape by a scale factor of 2.5.

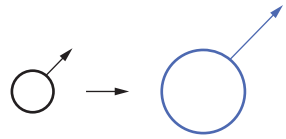


SC 2 I can determine the scale factor used to enlarge a figure

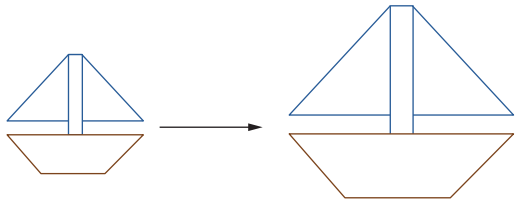
1 Determine the scale factor that has been used in each enlargement.

(a)

(b)

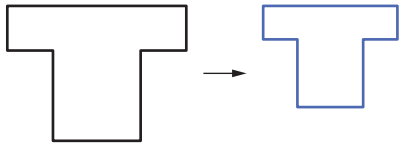


(c)

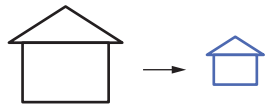


2 Determine the scale factor that has been used in each reduction.

(a)



(b)

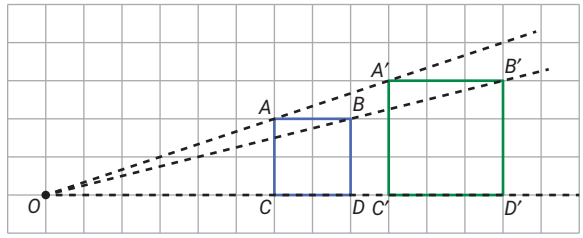


3 A square has been enlarged as shown.

(a) Calculate values for $\frac{A'B'}{AB}$ and $\frac{B'D'}{BD}$ as fractions in simplest form.

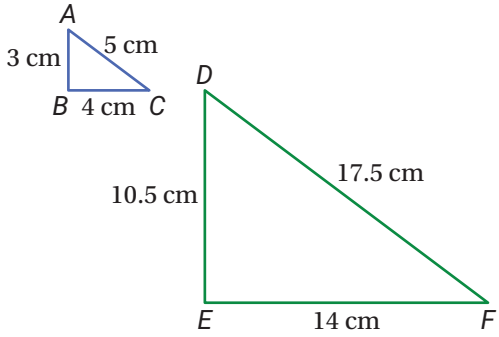
(b) Write the scale factor as a decimal.

(c) What would be the value of $\frac{OA'}{OA}$?

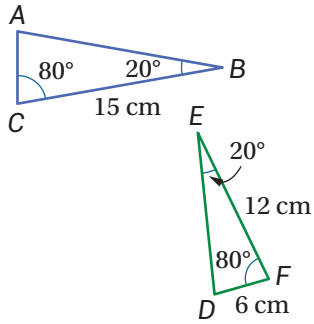


4 Determine the scale factor, as a decimal number, for the following pairs of similar triangles. Assume the original to be the triangle on the left.

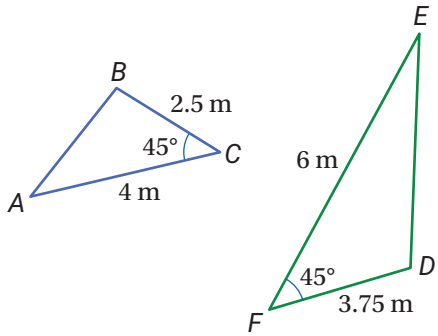
(a)



(b)

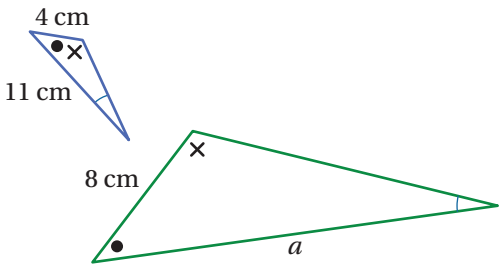


(c)

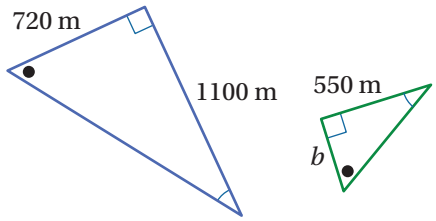


5 In each pair of similar triangles, calculate the scale factor and use it to determine the value of the variable. Assume the original to be the triangle on the left.

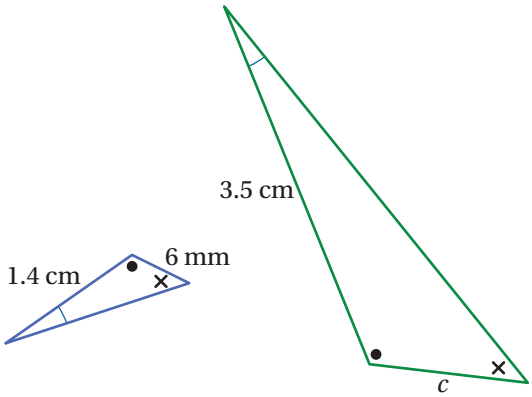
(a)



(b)



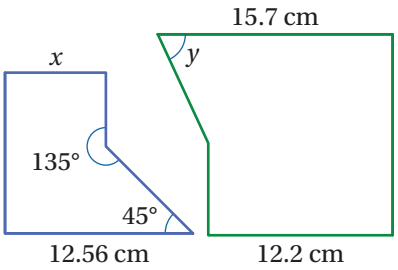
(c)



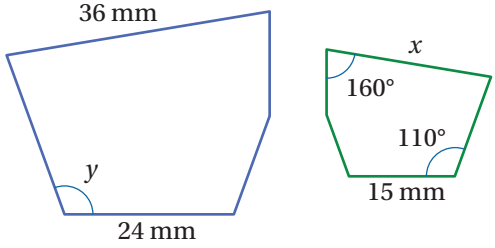
SC 3 I can determine lengths and angles in similar shapes

1 For each pair of similar figures, determine the values of the pronumerals representing the sizes of angles and the lengths of sides.

(a)

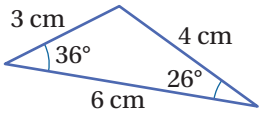


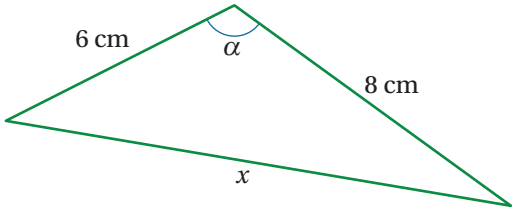
(b)



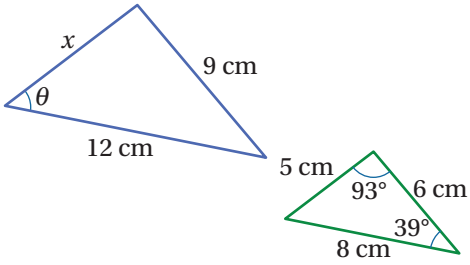
2 For each pair of similar triangles, calculate the values of the pronumerals representing the sizes of angles and the lengths of sides.

(a)

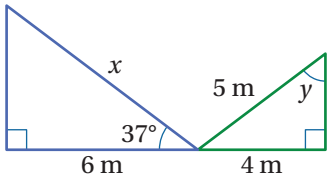




(b)

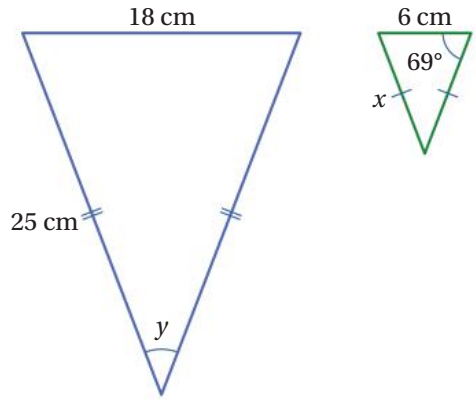


(c)

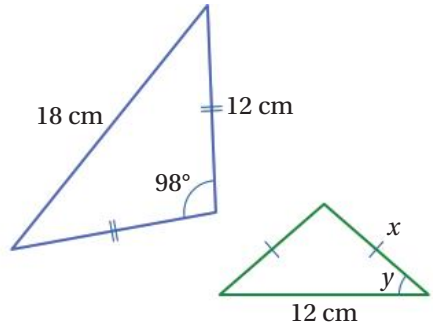


3 For each pair of similar isosceles triangles, calculate the values of the pronumerals representing the sizes of angles and the lengths of sides.

(a)

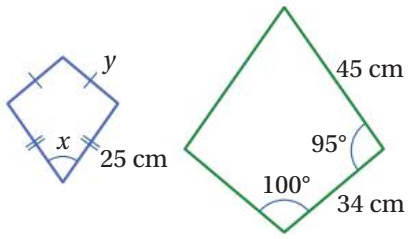


(b)

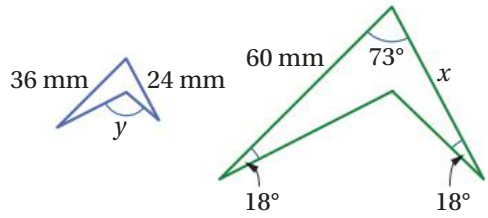


4 For each pair of similar quadrilaterals, determine the values of the pronumerals representing the sizes of angles and the length of sides.

(a)



(b)

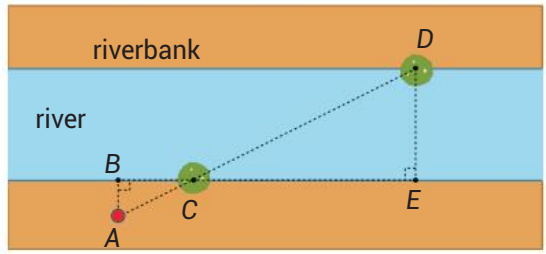


5 This symmetrical five-pointed star has side lengths of 15 cm and acute angles all 36°.

An enlargement of the star is made using a scale factor of 2.2.

For the enlarged star, calculate the lengths of the sides and the sizes of the angles.

6 Similar triangles can be used to determine lengths that are hard to measure in real life. The following diagram shows a river and some trees. A person stands at point A, so that trees C and D are lined up when they look across the river. They have measured distances on their side of the river as follows: $AB = 7$ m, $BC = 16$ m, $CE = 49$ m. Determine the width of the river, DE, to the nearest metre.



Prove similarity in pairs of triangles

14.5

Learning intention: To be able to prove similarity in pairs of triangles

Success criteria:

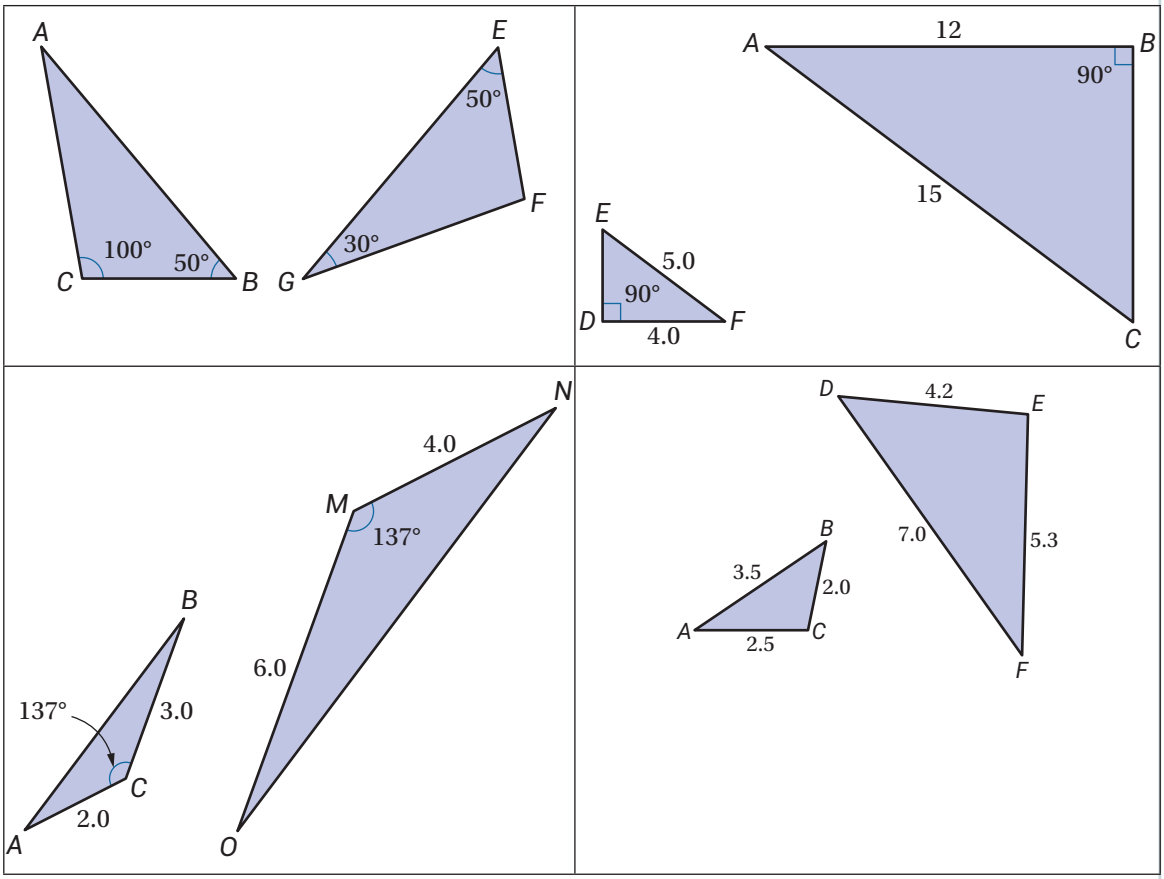
SC 1 I can use each of the four tests for similar triangles.

SC 2 I can apply logical reasoning to determine side lengths and angles.

Lesson warm-up

What makes two triangles similar to each other?

State whether each of the pairs of triangles given below are similar, congruent or neither.
Justify your answers.

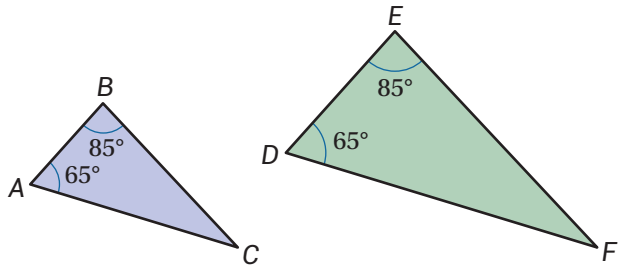
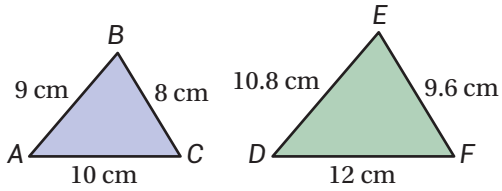
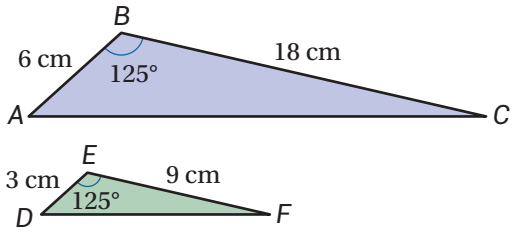
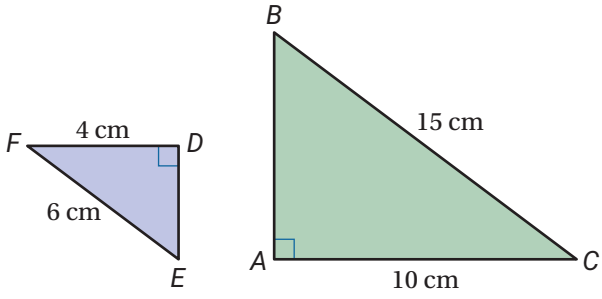


SC 1 I can use each of the four tests for similar triangles

When two triangles are similar, there will be three pairs of corresponding angles that are equal and three pairs of corresponding sides in the same proportion.

$\Delta ABC \sim \Delta PQR$ means 'ΔABC is similar to ΔPQR.' This can also be written as $\Delta ABC \parallel\!\parallel\!\parallel \Delta PQR$. Remember to write the vertices in matching order.

As for congruency, similarity can be proved with just three sets of information. These are known as the tests for similar triangles.

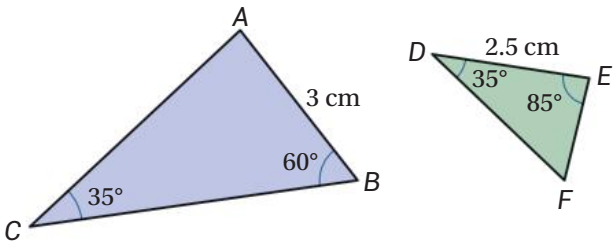
Test	Explanation	Diagram
AAA	Three pairs of equal angles. (If there are two pairs of equal angles, the third pair will also be equal because the angle sum of a triangle is a constant 180°.)	 $\Delta ABC \sim \Delta DEF$ (AAA similarity test)
SSS	Three pairs of sides in the same proportion, e.g. $\frac{12}{10} = \frac{10.8}{9} = \frac{9.6}{8} = 1.2$ (Match the longest sides with each other, and the shortest sides with each other, etc.)	 $\Delta ABC \sim \Delta DEF$ (SSS similarity test)
SAS	Two pairs of sides in the same proportion and the angle between them equal, e.g. $\frac{6}{3} = \frac{18}{9} = 2$.	 $\Delta ABC \sim \Delta DEF$ (SAS similarity test)
RHS	Right-angled triangles where the hypotenuses and another pair of sides are in the same proportion, e.g. $\frac{15}{6} = \frac{10}{4} = 2.5$.	 $\Delta ABC \sim \Delta DEF$ (RHS similarity test)

Worked example

Proving triangles are similar

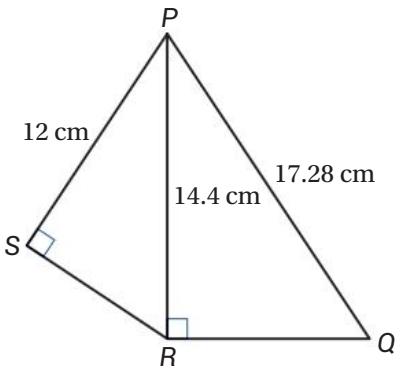
Prove each pair of triangles is similar. Write a similarity statement justified with the appropriate test.

(a)



THINKING	WORKING
Solve angles where necessary.	$\angle A = 180° - 35° - 60°$ $= 85°$ Alternatively, $\angle F = 180° - 35° - 85°$ $= 60°$
Match equal angles.	$\angle A = \angle E\,(85°)$ $\angle C = \angle D\,(35°)$ $\angle B = \angle F$ (angle sum of a triangle is constant)
Write the similarity statement, matching the vertices. Justify the statement with the relevant similarity test.	$\Delta ABC \sim \Delta EFD$ (AAA similarity test)

(b)



THINKING	WORKING
Match the right angles.	$\angle PSR = \angle PRQ$ (right angles)
Determine the proportions for the pair of hypotenuses.	$\frac{PQ}{PR} = \frac{17.28}{14.4}$ $= 1.2$
Determine the proportions for the other pair of sides, ensuring the same order for the triangles.	$\frac{PR}{PS} = \frac{14.4}{12}$ $= 1.2$
Write the similarity statement, matching the vertices. Justify the statement with the relevant similarity test.	$\Delta PSR \sim \Delta PRQ$ (RHS similarity test)

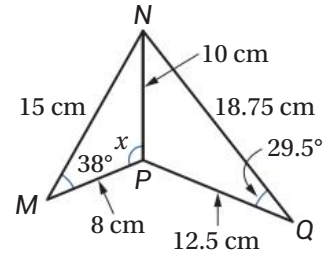
14

SC 2 I can apply logical reasoning to determine side lengths and angles

Worked example

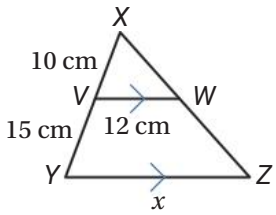
Proving similarity to determine angles and side lengths

(a) Prove similarity in a pair of triangles and determine the value of x.



THINKING	WORKING
Prove similarity by ratios of matching sides.	$\frac{NQ}{NM} = \frac{18.75}{15} = 1.25$ $\frac{PQ}{NP} = \frac{12.5}{10} = 1.25$ $\frac{NP}{MP} = \frac{10}{8} = 1.25$ $\therefore \Delta MPN \sim \Delta NPQ$ (SSS similarity)
Use angle sum of a triangle to determine the unknown angle value.	$180° - 38° - 29.5° = 112.5°$
Determine the value of x using corresponding angles in similar triangles.	$\angle MPN = \angle NPQ$ $x = 112.5°$

(b) Prove similarity in a pair of triangles and determine the value of x.



THINKING	WORKING
Prove similarity using equal angles.	$\angle X$ is the same in both triangles. $\angle XVW = \angle XYZ$ (corresponding angles, $VW \parallel YZ$) Alternatively: $\angle XWV = \angle XZY$ (corresponding angles, $VW \parallel YZ$) $\therefore \Delta XVW \sim \Delta XYZ$ (AAA similarity)
Write equal ratios involving known lengths and x.	$\frac{YZ}{VW} = \frac{XY}{XV}$
Calculate the value of x.	$\frac{x}{12} = \frac{10+15}{10}$ $x = \frac{25}{10} \times 12$ $= 30$ cm

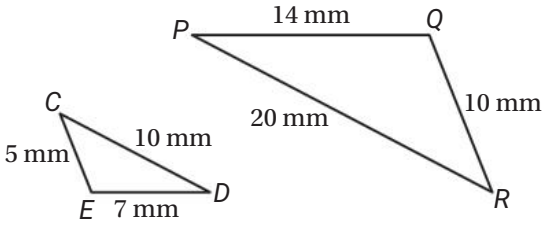
Practice

ANSWERS Page 603

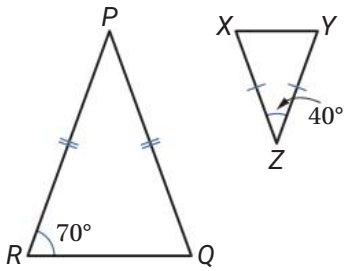
SC 1 I can use each of the four tests for similar triangles

1 Prove that each pair of triangles are similar. Write a similarity statement justified with the appropriate test.

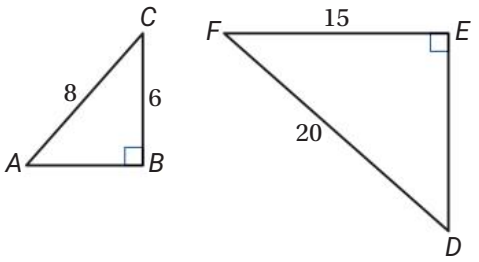
(a)



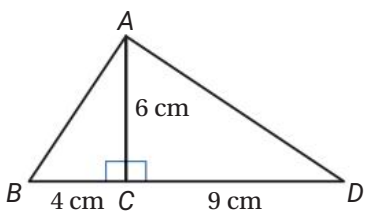
(b)



(c)

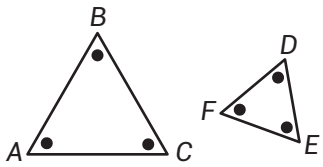


(d)

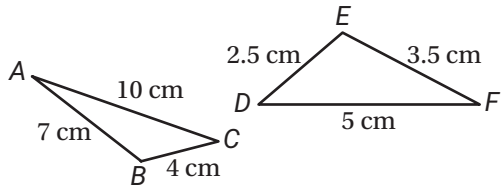


2 Determine whether each pair of triangles is similar. Justify your answers.

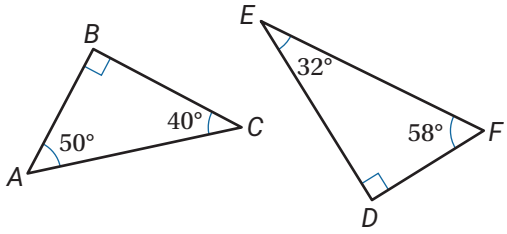
(a)



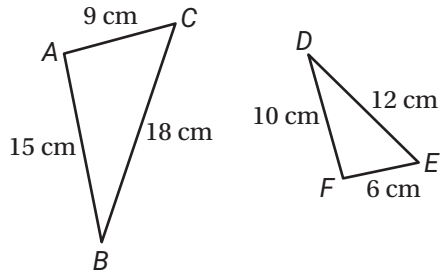
(b)



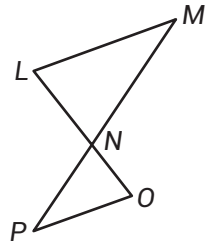
(c)



(d)



3 The diagram shows a pair of triangles drawn from three lines.

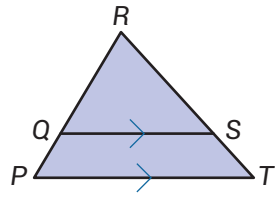


(a) State the only pair of angles known to be equal, giving a reason.

(b) What additional piece of information involving LM would ensure similarity? Explain why, writing the similarity statement and test used.

(c) If the condition from **(b)** is not true, name any pair of angles, if equal, would ensure similarity. Write the similarity statement.

4 The diagram below contains two triangles.

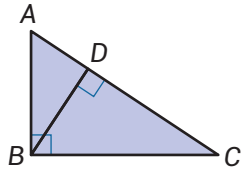


(a) Mark in angles that are equal due to the parallel sides.

(b) Redraw as two separate triangles, marking the equal angles as before. Also mark the shared angle, not appearing twice, as equal.

(c) Write the similarity statement for the triangles, with the appropriate test.

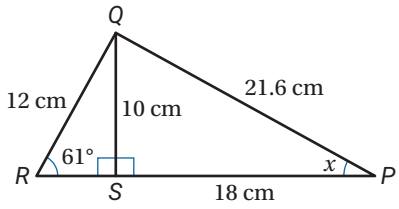
5 The diagram shows a right-angled triangle that has been divided into two smaller right-angled triangles. Use diagrams to help establish that all three triangles are similar. Write the similarity statements with the similarity test that applies.



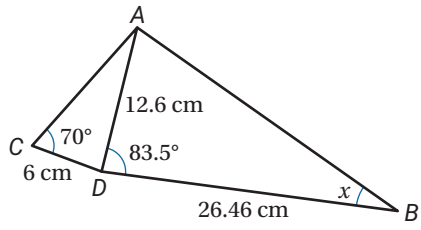
SC 2 I can apply logical reasoning to determine side lengths and angles

1 Prove similarity in a pair of triangles and hence determine the value of x.

(a)

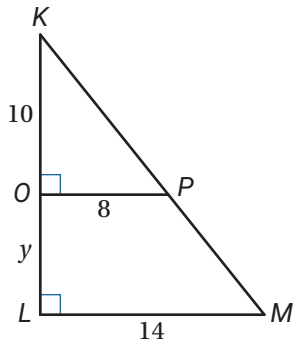


(b)

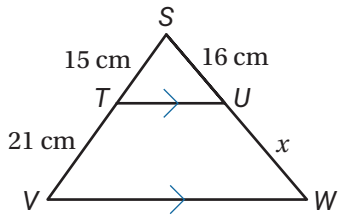


2 Prove similarity in a pair of triangles and hence determine the value of the variable.

(a)

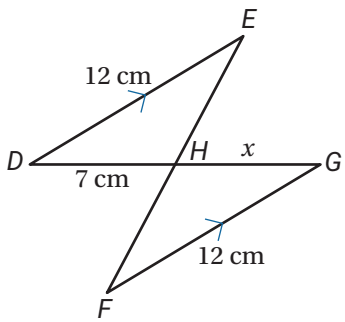


(b)

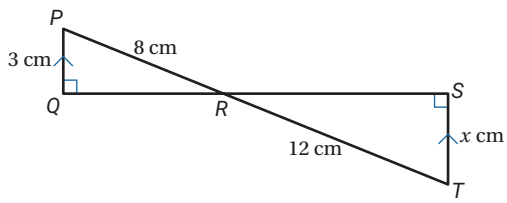


3 Prove similarity in a pair of triangles and hence determine the value of x.

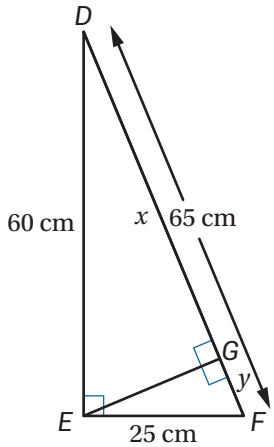
(a)



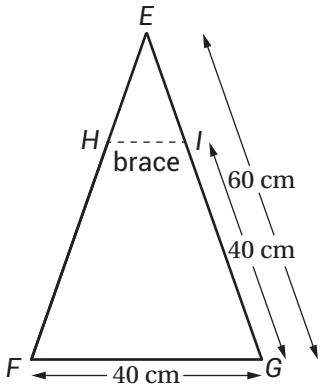
(b)



4 Use similar triangles to determine the values of x and y.



5 Sophia designed a window frame for her new house. Determine the length of the brace needed.

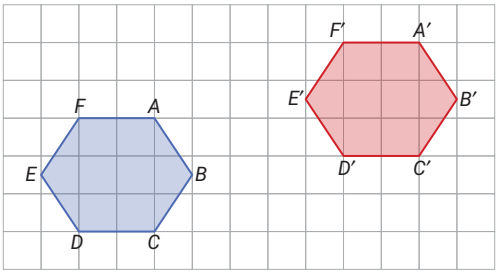


TOPIC REVIEW

Multiple choice

ANSWERS Page 629

1 How has the hexagon been translated?



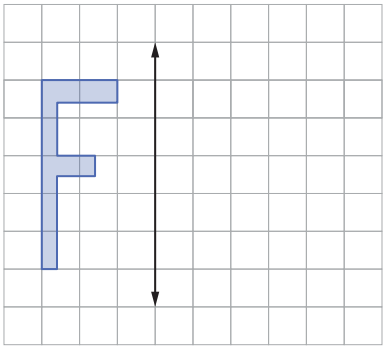
A 7 units up and 2 units right

B 7 units right and 2 units up

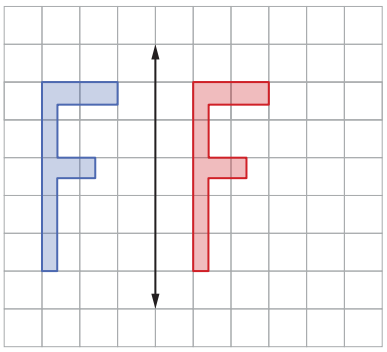
C 7 units left and 2 units down

D 6 units right and 2 units up

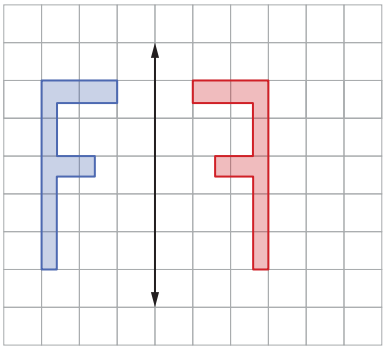
2 Which of the following options shows the reflection of F in the mirror line?



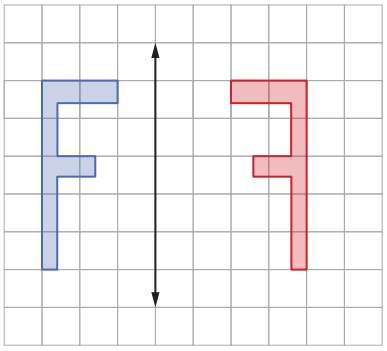
A



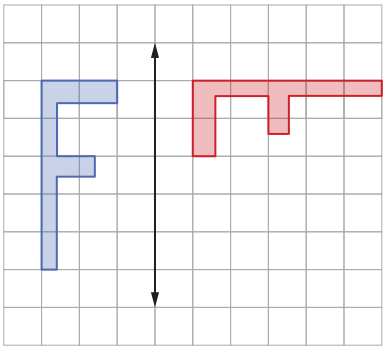
B



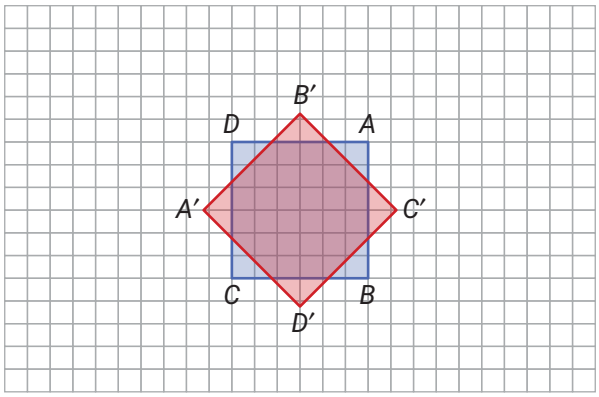
C



D



3 The rotation needed to transform $ABCD$ to $A'B'C'D'$ is:



A 45° clockwise **B** 135° anticlockwise **C** 90° clockwise **D** 90° anticlockwise

4 For the letters of the word M A T H S, which letter does not have an axis of symmetry?

A M **B** A **C** H **D** S

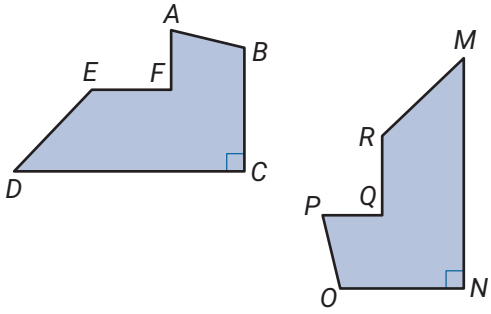
5 For the letters of the word M A T H S, how many of the letters have just one axis of symmetry?

A 5 **B** 2 **C** 3 **D** 4

6 For the letters of the word M A T H S, which letter has two axes of symmetry?

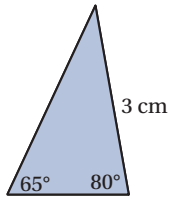
A H **B** M **C** A **D** S

7 For the pair of congruent hexagons, which pairing is not correct?

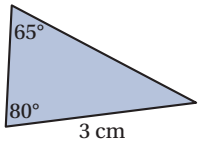


A $BC \leftrightarrow ON$ **B** $\angle NOP \leftrightarrow \angle CBA$ **C** $\angle DEF \leftrightarrow \angle RMN$ **D** $P \leftrightarrow A$

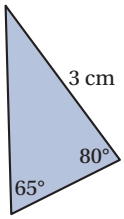
8 Which of the triangles is not congruent to the triangle shown?



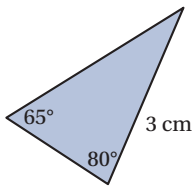
A



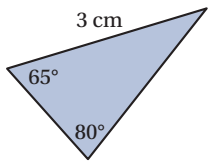
B



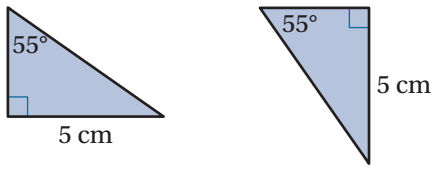
C



D

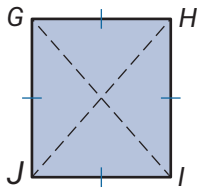


9 Which test can be used to show that these two triangles are congruent?

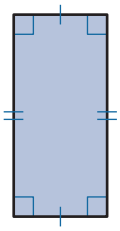


A SSS **B** SAS **C** AAS **D** RHS

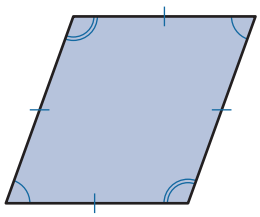
10 A square GHIJ is divided into four congruent triangles by its diagonals. Which of the following quadrilaterals are divided into four congruent triangles by their diagonals?



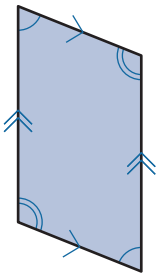
A



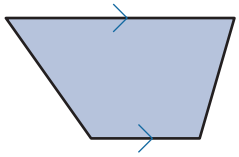
B



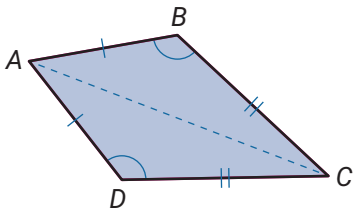
C



D

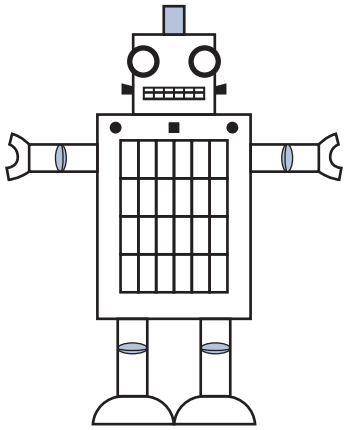


11 A kite $ABCD$ has been divided into two congruent triangles. Which of the following is true?



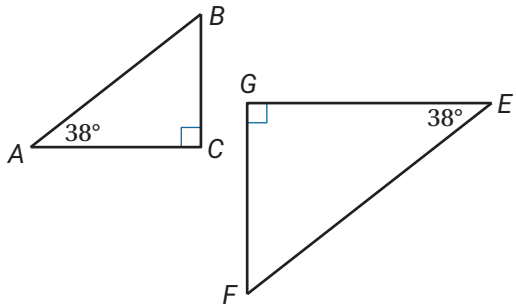
A $AD \parallel BC$ **B** $\angle ABC = \angle ADC$ **C** $\Delta BCD \cong \Delta DAB$ **D** $CD = AB$

12 A scale factor of $\frac{1}{4}$ has been used to draw this toy robot. The actual height of the toy is:



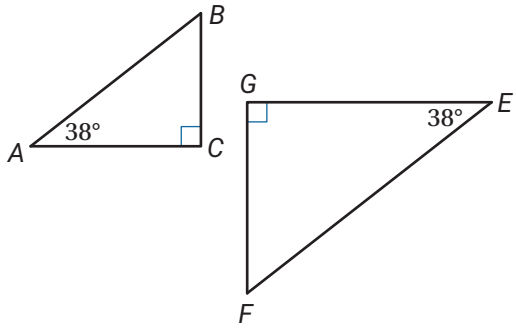
A 1.2 cm **B** 18 cm **C** 19.2 cm **D** 20 cm

13 For the pair of similar triangles, the side corresponding to AB is:



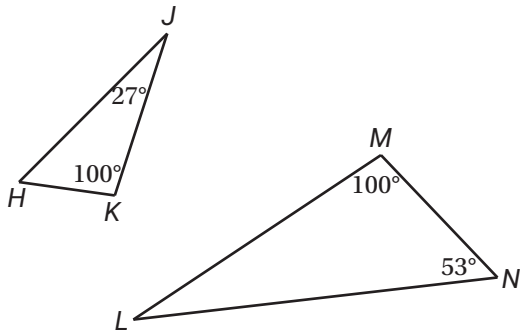
A FG **B** EF **C** BC **D** EG

14 For the pair of similar triangles, the angle corresponding to $\angle EFG$ is:



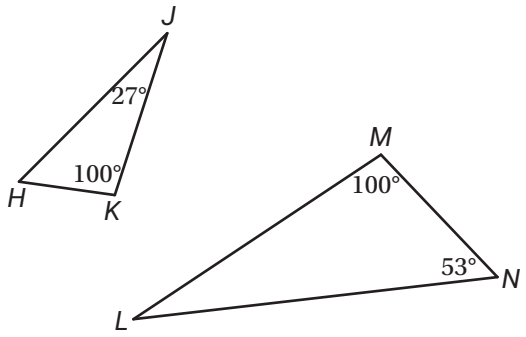
A $\angle CAB$ **B** $\angle ACB$ **C** $\angle ABC$ **D** $\angle BAC$

15 For the pair of similar triangles, the side corresponding to LM is:



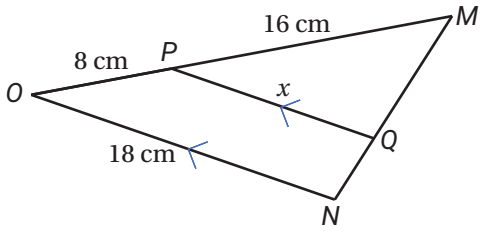
A HJ **B** MN **C** JK **D** LN

16 For the pair of similar triangles, the angle corresponding to $\angle JHK$ is:



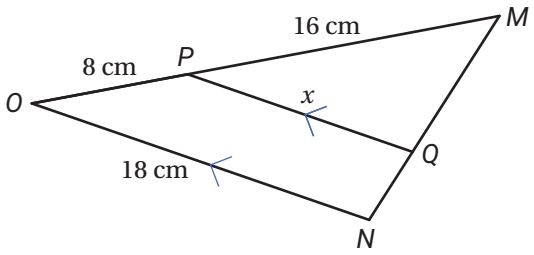
A $\angle LMN$ **B** $\angle NLM$ **C** $\angle LNM$ **D** $\angle MLN$

17 The diagram contains a pair of similar triangles. Which equation could be used to calculate the length of PQ?



A $\frac{8}{18} = \frac{x}{16}$ **B** $\frac{x}{18} = \frac{8}{16}$ **C** $\frac{x}{18} = \frac{16}{24}$ **D** $\frac{x}{16} = \frac{8}{24}$

18 The diagram contains a pair of similar triangles. The scale factor that produces the larger triangle from the smaller is:

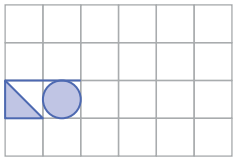


A 0.5 **B** $\frac{2}{3}$ **C** 1.5 **D** 2

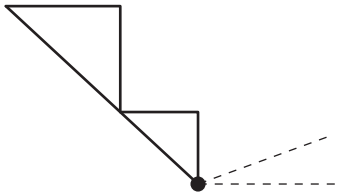
Short answer

1 Translate each shape, as instructed.

(a) 2 up, 3 right

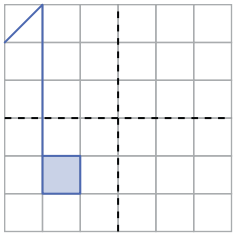


(b) 40 mm along a line at 20° to the horizontal, as shown.

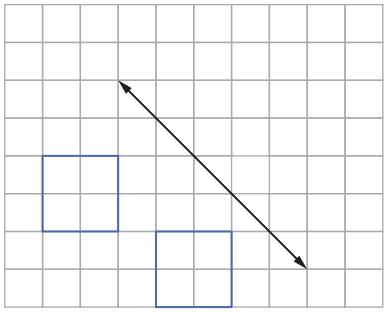


2 Reflect each shape, as instructed.

(a) Reflect first in the vertical mirror line, then in the horizontal mirror line.

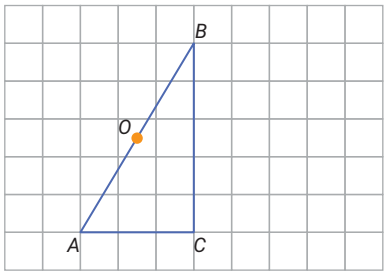


(b) Reflect in the oblique mirror line.

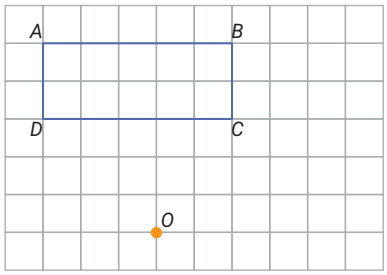


3 Show the image after rotating as instructed. Label the image using dash notation.

(a) Rotate 180° about O.

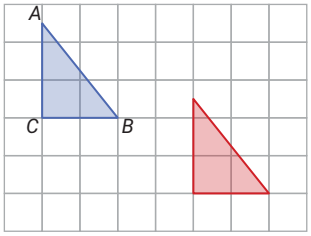


(b) Rotate 90° clockwise about O.

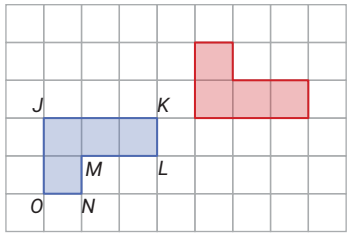


4 Copy the pairs of congruent figures onto grid paper. Describe each transformation to move the blue diagram to the pink diagram. Label the image using dash notation.

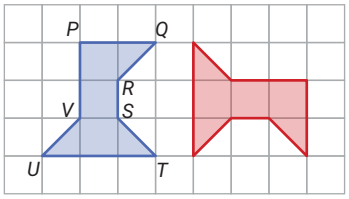
(a)



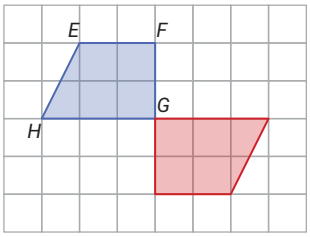
(b)



(c)

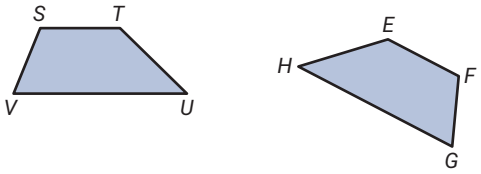


(d)



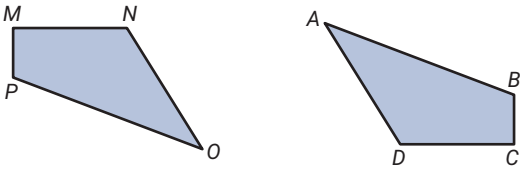
5 For each pair of congruent figures, complete the pairings.

(a)



$H \leftrightarrow _$, $TU = ___$, $\angle EFG = \angle ___$

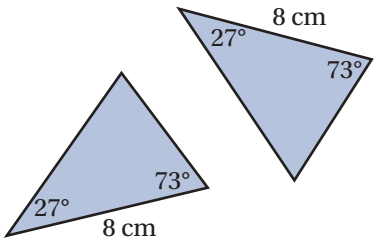
(b)



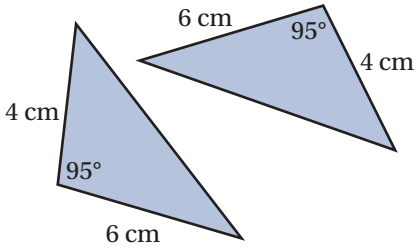
$N \leftrightarrow _$, $AD = ___$, $\angle CDA = \angle ___$

6 Write the congruence test used to prove each pair of triangles congruent.

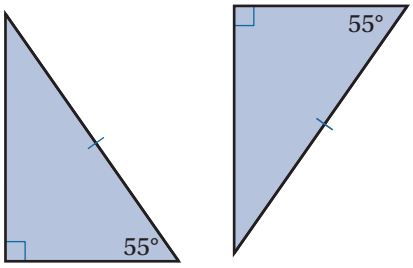
(a)



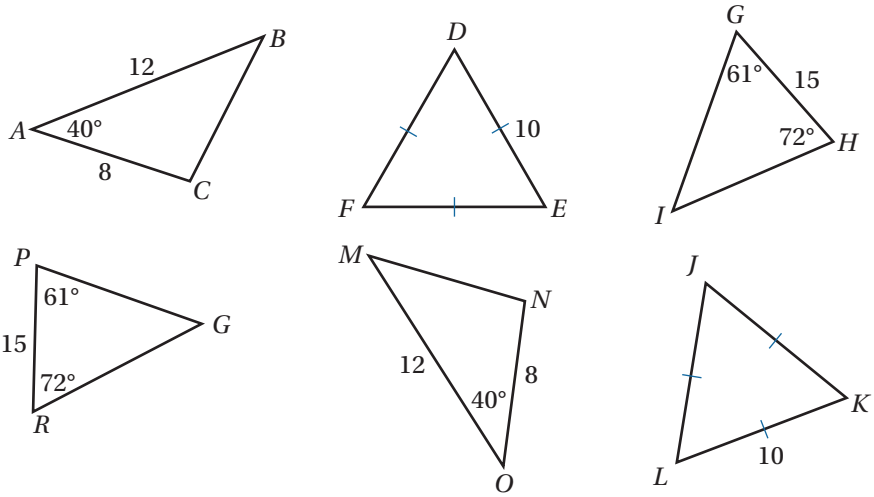
(b)



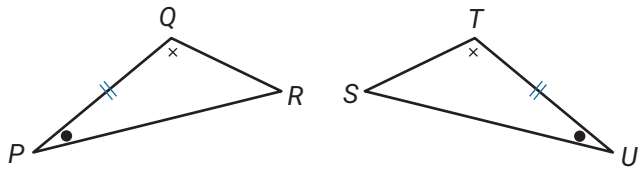
(c)



7 Locate three pairs of congruent triangles. Write the congruence statement for each pair, as well as the test used to prove congruence.

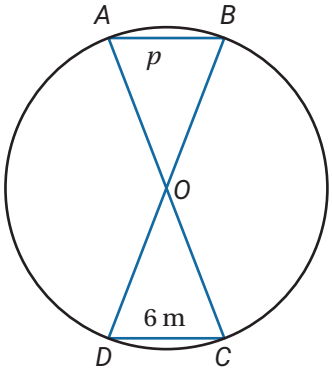


8 Prove that $\Delta PQR \cong \Delta UTS$.

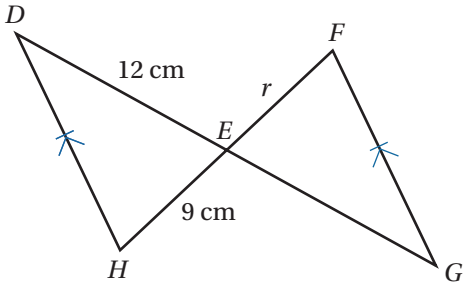


9 Prove each pair of triangles congruent and hence determine the values of the variables.

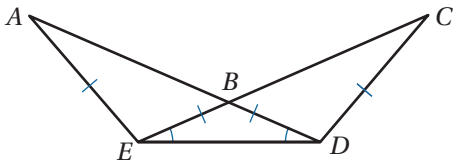
(a)



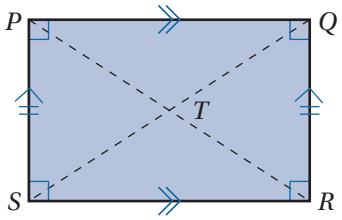
(b)



10 ΔAED and ΔCDE are placed such that an isosceles triangle forms. Can you be sure that $\Delta AED \cong \Delta CDE$? Explain your answer.



11 For the diagram below:



(a) Copy and complete the proof.

In triangles PRS and QSR:

$\angle$___ = $\angle$___ (given 90°)

$\angle$___ = $\angle$___ (alternate angle, $PS \parallel QR$)

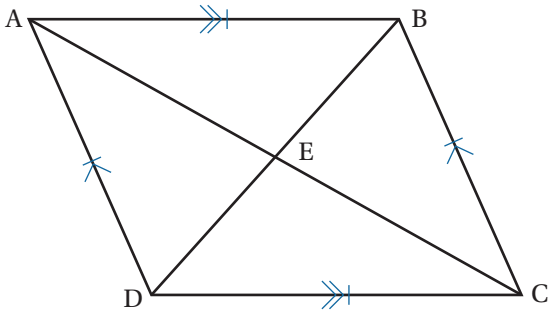
___ = __ (given)

$\therefore \Delta PRS \cong \Delta$___ ($AAS$ congruence test)

(b) Justify that $PR = QS$.

(c) What property of a special quadrilateral have you proved?

12 For the diagram below:



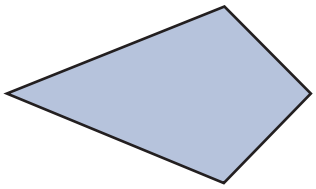
(a) Prove that $BD \perp AC$.

(b) What property of a special quadrilateral have you proved?

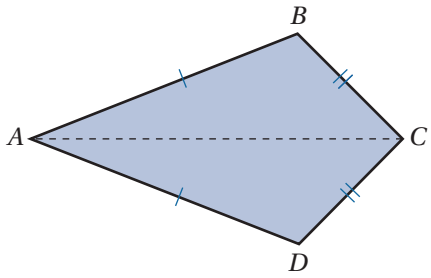
13 Copy each special quadrilateral. Write the special name for each. Mark any equal sides, equal angles and parallel sides.

(a)

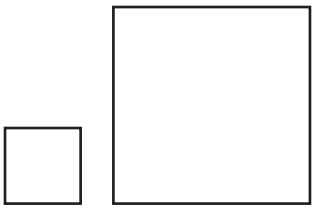
(b)

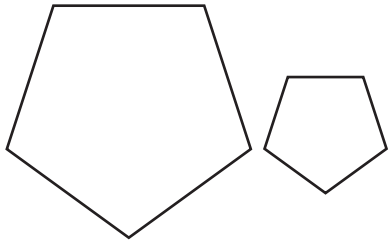
(c) 

14 Use congruent triangles to prove that a pair of opposite angles in the kite are equal.

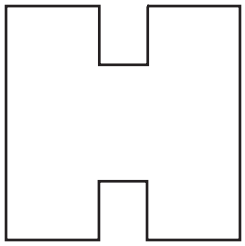


15 Calculate the scale factor for each pair of similar figures, assuming the original figure to be on the left each time.

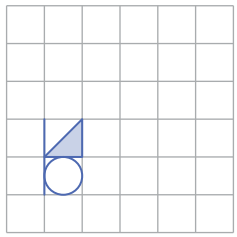
(a) 

(b) 

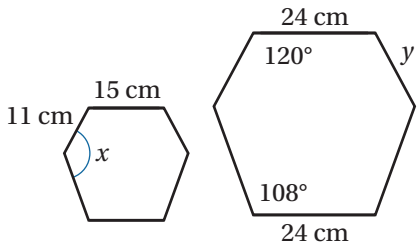
16 Reduce the H shape using the scale ratio of $\frac{1}{4}$.



17 Dilate the shape using a scale factor of 2. Make sure you allow space to draw it.



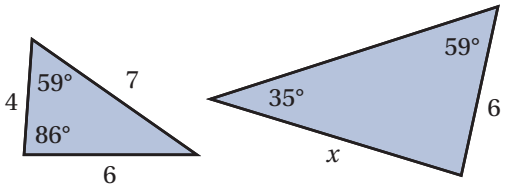
18 The pair of similar hexagons are oriented the same way, each having a vertical line of symmetry.



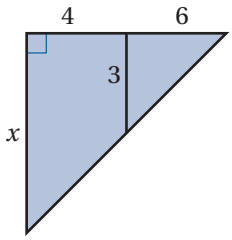
Calculate the values of the variables.

19 State a similarity test to prove a pair of triangles similar in each diagram, then solve for the variables. (All dimensions are in centimetres.)

(a)



(b)



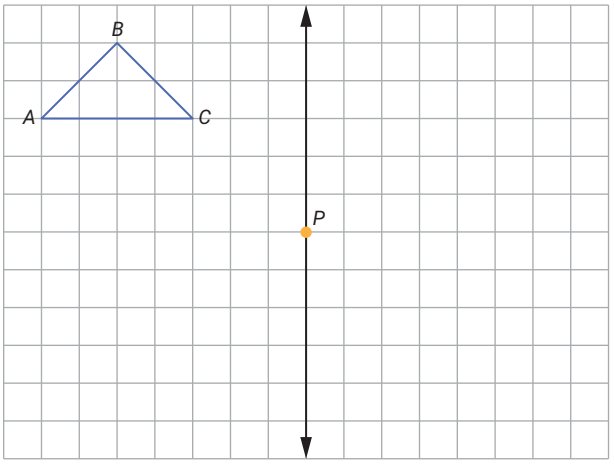
Extended response

1 The following transformations are performed, in sequence, on triangle ABC:

- a reflection in the mirror line (to get A'B'C')
- a rotation of 180° about P (to get A''B''C'')
- a reflection in the mirror line (to get A'''B'''C''')

(a) Draw a single diagram to show the labelled images.

(b) Determine a single transformation to move the image $A'''B'''C'''$ back to the original position.



2 In some fonts many upper-case letters of the alphabet have symmetry.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

(a) Which of the letters are asymmetrical (no line symmetry)?

(b) Which letters have just one axis of symmetry?

(c) Which letters have two axes of symmetry?

(d) Which letters have rotational symmetry?

3 ΔABC has $AB = 8$ cm, $\angle B = 55°$ and $AC = 7.5$ cm.

(a) Use a protractor, ruler and compass to draw two possible triangles with these measurements.

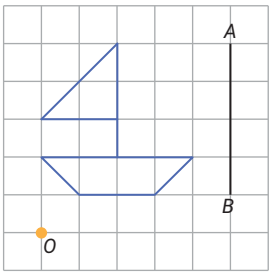
(b) Measure the sides and angles not given for each triangle.

(c) Give one extra piece of information for ΔABC to ensure that it is unique.

4 A sail boat is drawn on a grid. For each of the following transformations:

(i) Copy the original onto a grid then perform the transformation.

(ii) Describe the transformation that will map the image back to the original.



(a) Translate 3 units right, 2 units down.

(b) Reflect in AB.

(c) Rotate 90° clockwise about B.

(d) Enlarge with O as the centre of projection using a scale factor of 1.5.

TOPIC 15

Visualising 3D

Recall 428

15.1 Locate and describe the position of objects in 3D space 429

15.2 Use the 3D coordinate system 432

Topic review 439

Why learn this?

Understanding how to visualise and describe the position of objects in 3D space is essential for numerous practical and theoretical reasons. It enhances spatial awareness and the ability to think in three dimensions, which is crucial in fields such as architecture, engineering and design, where one must visualise and manipulate objects and structures in space. This knowledge is vital in navigation and geography for pinpointing locations precisely using 3D coordinates, extending positioning on a flat plane to include vertical positioning above and below.

In science and technology, understanding 3D space is necessary for disciplines such as physics (for studying motion and forces), computer graphics (for creating and manipulating digital models), and robotics (for programming robots to interact with their environments).

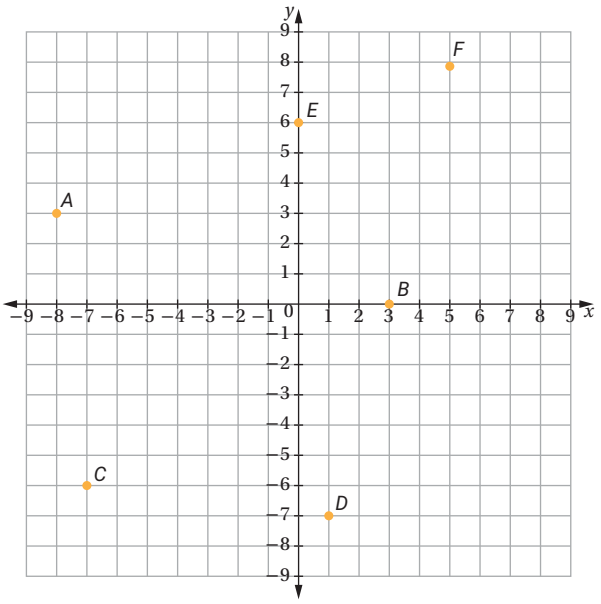
By mastering the ability to read and apply coordinates in three dimensional spaces, you will be better equipped to handle advanced technological tools and contribute to innovations in science, technology and digital media.

RECALL

ANSWERS Page 603

I can describe the position of plotted points on a Cartesian plane from the origin

1 Complete the sentences for each plotted point.



(a) A is ___ units to the ___, and up ___ units from the origin.

(b) B is ___ units to the ___ of the origin.

(c) C is ___ units to the ___, and 6 units ___ the origin.

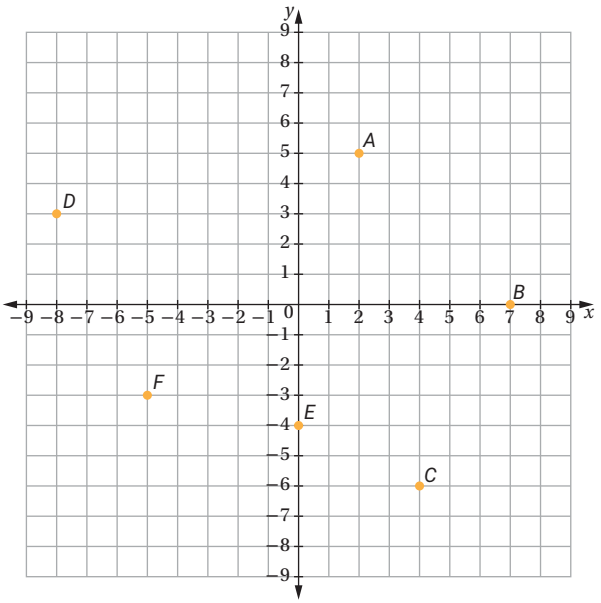
(d) D is ___ unit ___ of the origin and ___ units below it.

(e) E is ___ units directly ___ the origin.

(f) F is ___ units to the right and ___ units above the origin.

I can read and plot points on a Cartesian plane

1 Write the coordinates for each point plotted on the number plane shown.



2 Plot and label the following points on a Cartesian plane.

(a) $A(-2, 2)$ **(b)** $B(5, -6)$ **(c)** $C(-3, -8)$

(d) $D(8, 5)$ **(e)** $E(0, 4)$ **(f)** $F(-6, 0)$

Locate and describe the position of objects in 3D space

15.1

Learning intention: To be able to locate and describe the position of objects in 3D space

Success criteria:

SC 1 I can locate objects in space using three dimensions.

Lesson warm-up

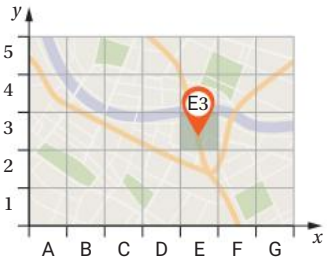
Locating cars

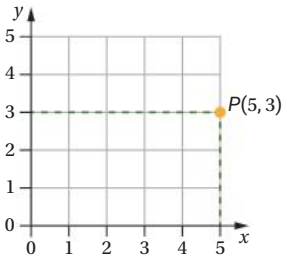
A driver hired the yellow car circled in the diagram and needs to collect it. List the directions you would give to the driver to find the exact location of the hired car in the parking station.

Assume the stairs are right next to the drive-in entrance, and the pedestrian pathway is around the edge of the building on each floor.

SC 1 I can locate objects in space using three dimensions

Locating positions in two-dimensional (2D) space, such as maps, can been achieved by using a coordinate system that combines letters and numbers on a grid, a pair of coordinates on a Cartesian plane or by using longitude and latitude.



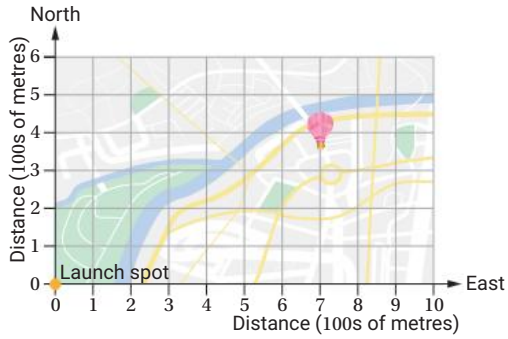


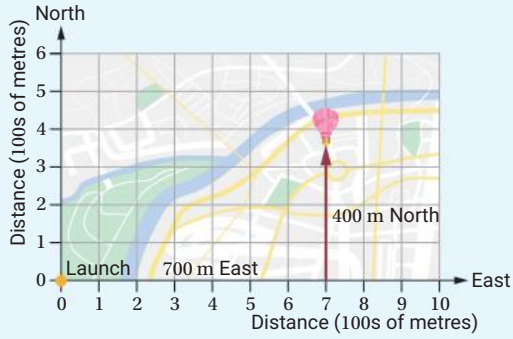
Most locations or objects in the real world are not on the same flat 2D plane. A system is needed to reference the positions of objects in three dimensions, such as those in the air, above or below ground in buildings, or in the ocean depths. The altitude of an object is the distance above (positive) or below (negative) the point of reference.

Worked example

Locating objects in 3D space

A hot air balloon at an altitude of 250 m is tracked using a global positioning system (GPS). Describe the exact position of the hot air balloon from the launch spot.



THINKING	WORKING
Locate the position of the balloon on the GPS map. Start with the horizontal axis.	The hot air balloon is: 700 m east of the launch spot, and 400 m north of the launch spot. 
Identify the altitude of the hot air balloon. The altitude is a positive value, so it is above the launch spot on the ground.	The hot air balloon is 250 m above the ground.
Describe the exact position of the hot air balloon from the launch position.	The hot air balloon is 700 m east and 400 m north of the launch spot, at an altitude of 250 m.

Practice

ANSWERS Page 604

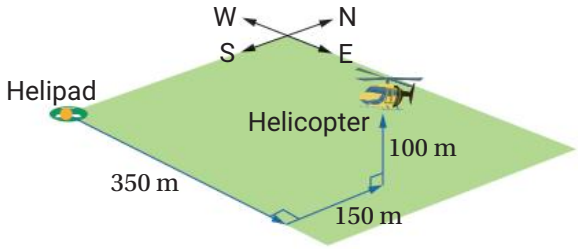
SC 1 I can locate objects in space using three dimensions

1 The diagram shows the current position of a helicopter from ground level at a helipad.

(a) How far north of the helipad is the helicopter?

(b) How far east of the helipad is the helicopter?

(c) What is the altitude of the helicopter compared to the ground?

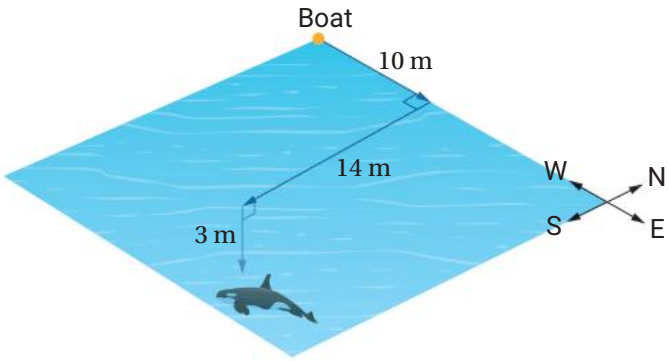


2 The diagram shows the current position of a whale observed from a boat.

(a) How far south of the boat is the whale?

(b) How far east of the boat is the whale?

(c) What is the altitude of the whale compared to the boat?



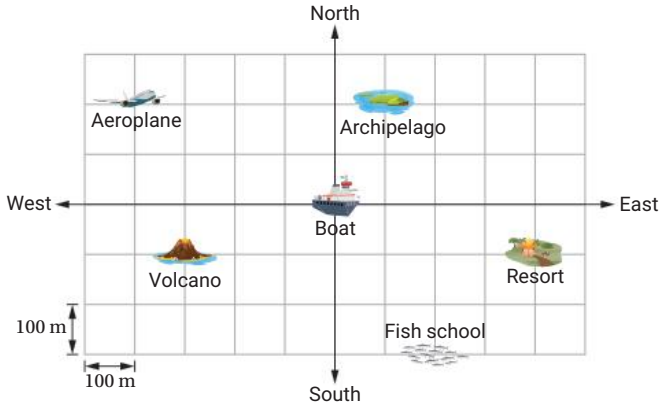
3 Use compass distances and the word 'altitude' to describe the location of each listed observation from the boat located on the ocean surface. Start with the east/west distances. Each grid line on the map is 100 m apart.

(a) The aeroplane flying 450 m above the ocean

(b) The peak of the 200 m-high volcano

(c) The school of fish swimming at a depth of 15 m

(d) The resort on the beach that is level with the ocean



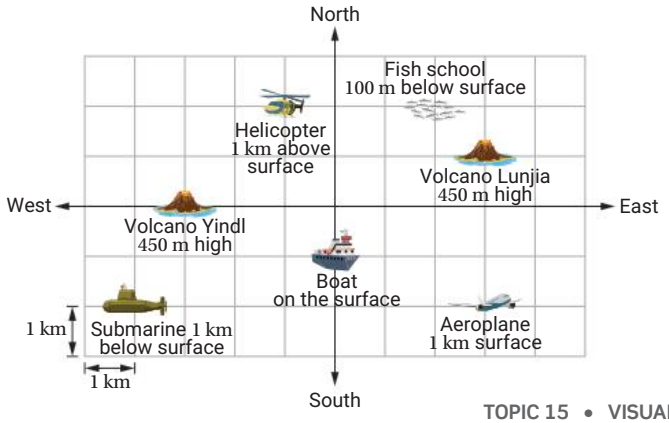
4 Use the 1 km grid map of the ocean below to identify what is located at each given description.

(a) 6 km east and 2 km south of Volcano Yindl, with an altitude of 1 km

(b) 4 km west and 1 km south of the boat, with an altitude of –1 km

(c) 3 km east and 2 km north of the boat, with an altitude of 450 m

(d) 2 km east and 3 km north of the boat, with an altitude of –100 m



15.2 Use the 3D coordinate system

Learning intention: To be able to use 3D coordinate systems

Success criteria:

SC 1 I can describe and compare 2D and 3D coordinate systems.

Lesson warm-up

Make your own 3D plane

Cut the largest square possible out of a piece of A4 grid paper.

Fold the square in half and half again.

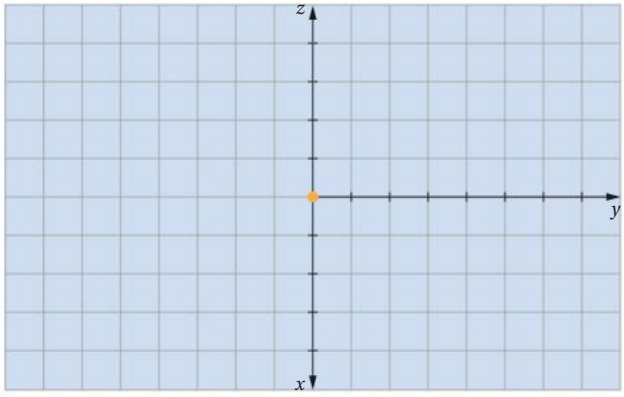
Open the square.

Draw a point in the centre.

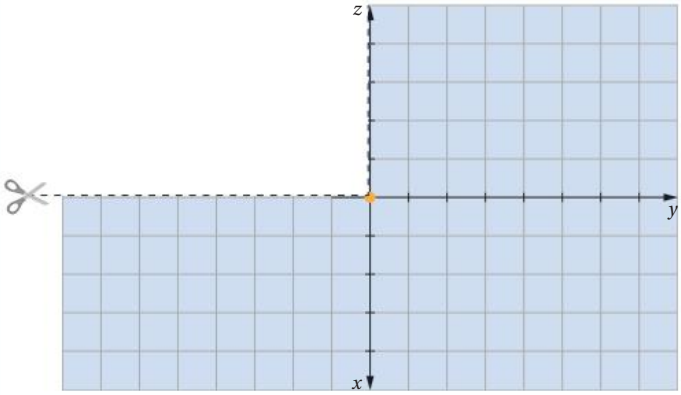
Draw a vertical axis from the centre to the top and label it z.

Draw a vertical axis from the centre to the bottom of the square and label it x.

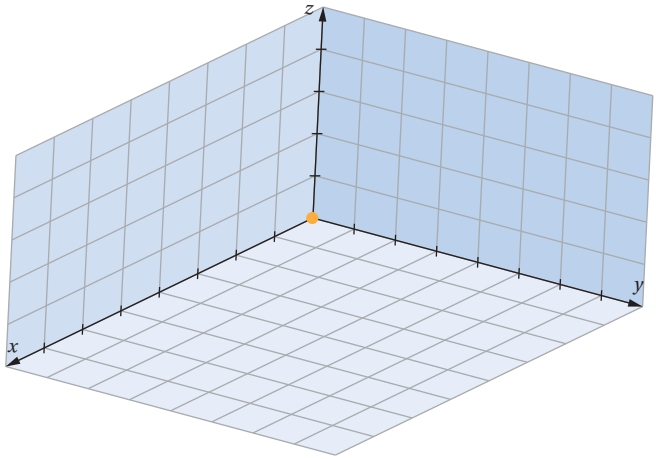
Draw a horizontal axis from the centre to the right-hand side of the square and label it y.



Next, cut out the top left-hand quarter.



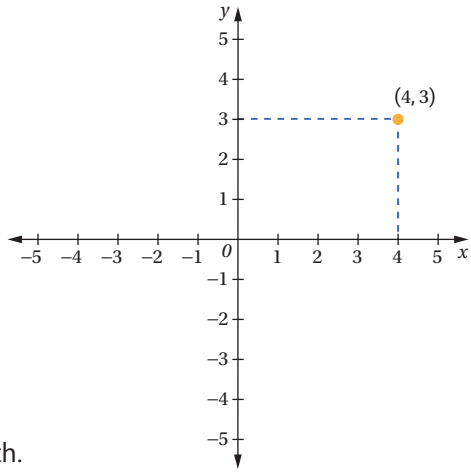
Fold the quarters to form the 3D plane.



You can use the plane to practise plotting and locating points, as well as describing points on each plane.

SC 1 I can describe and compare 2D and 3D coordinate systems

Coordinates are used to describe the exact position of a point or object in space. For flat plane shapes in 2D space, the Cartesian coordinate system is used, consisting of an x-axis and a y-axis. Any location can be identified by its x- and y-coordinates.



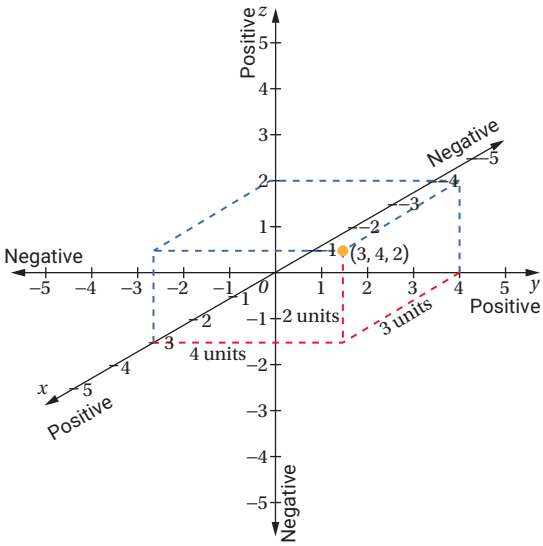
The point (4, 3) is located moving 4 units in the positive x-direction, then 3 units in the positive y-direction.

To represent points in three-dimensional (3D) space using coordinates, an additional z-axis is needed. This axis forms a plane that is perpendicular to the 2D Cartesian plane. It allows 3D objects to be represented in terms of horizontal and vertical positions, as well as the height or depth.

Points or locations in the 3D system have three values.

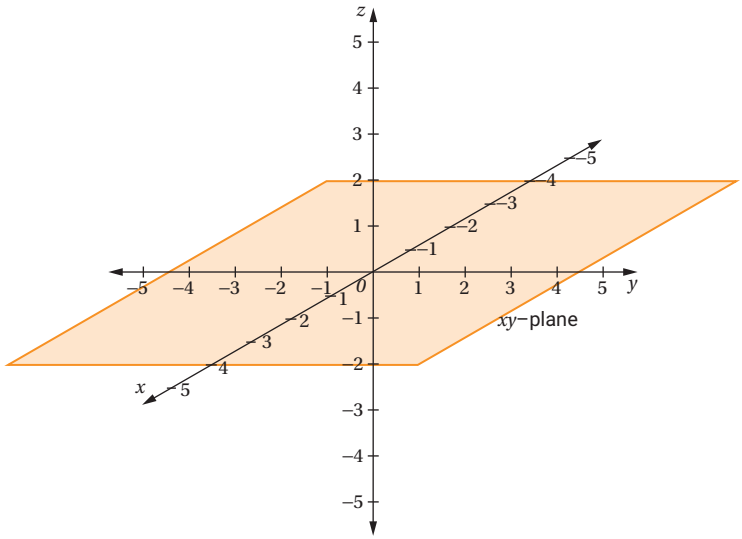
A point with the coordinates (3, 4, 2) means the x-value is 3, the y-value is 4 and the z-value is 2.

This point is located by moving 3 units in the positive x-direction, then 4 units in the positive y-direction and finally 2 units in the positive z-direction.

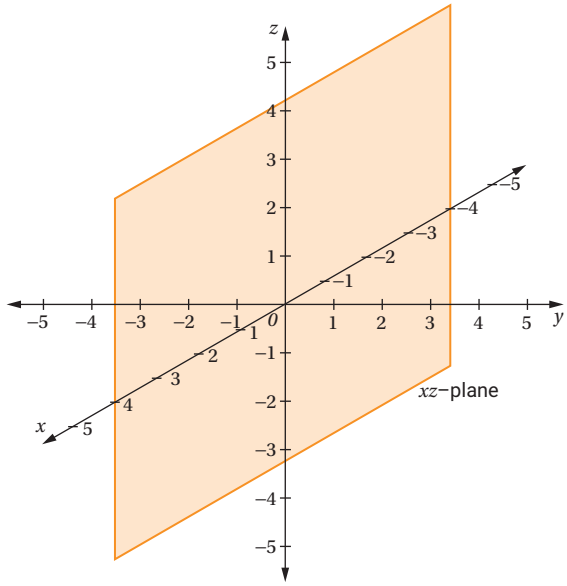


Positive z-values are above the xy-plane, and negative z-values are below the xy-plane.

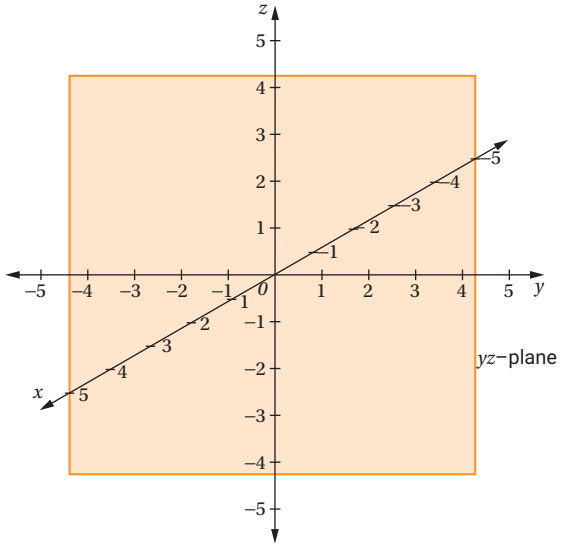
Points sitting on the flat xy-plane, always have a z-value equal to 0, $(x, y, 0)$.



Points sitting on the flat xz-plane always have a y-value equal to 0, $(x, 0, z)$.

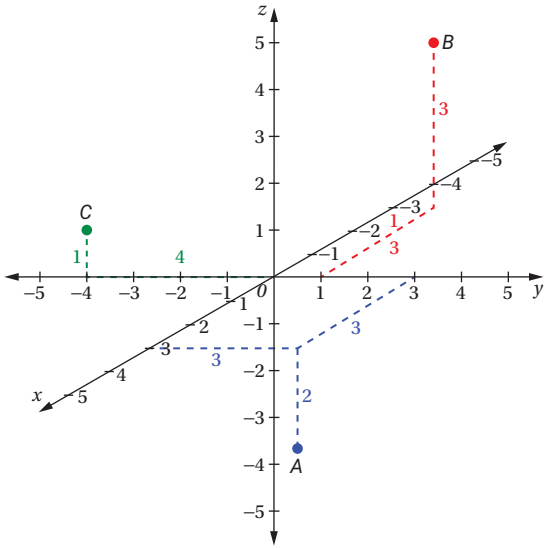


Points sitting on the flat yz-plane always have an x-value equal to 0, $(0, y, z)$.



Worked example

Identifying points plotted on a 3D coordinate plane

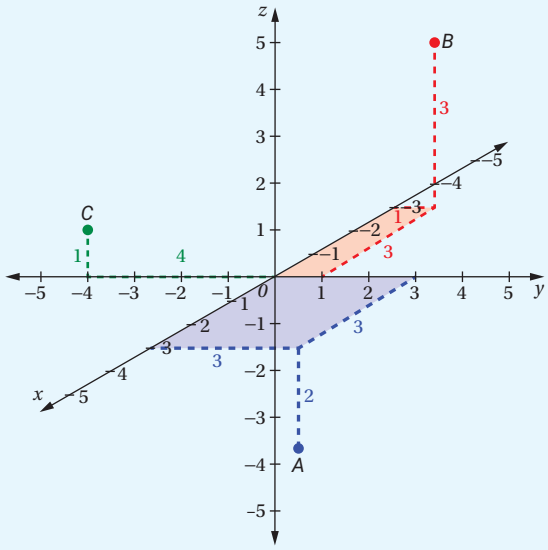


(a) Write the coordinates for each plotted point, A, B and C.

THINKING

Identify the coordinates of each point in the flat xy-plane first.

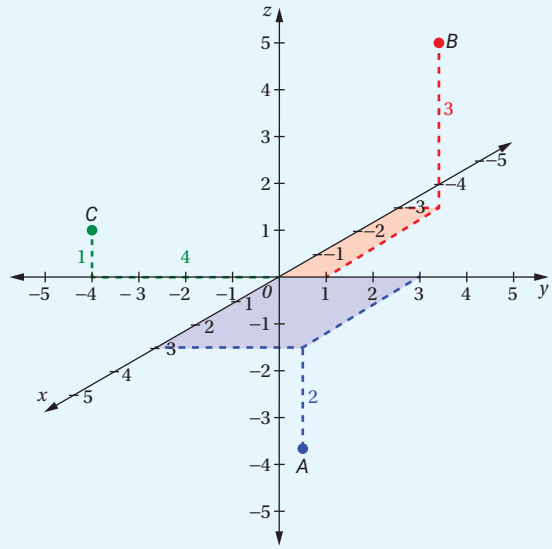
WORKING



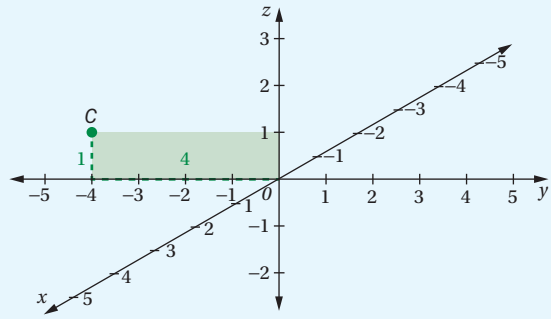
A: the x-value is 3 and the y-value is 3

B: the x-value is -3 and the y-value is 1

C: the x-value is 0 and the y-value is -4

Identify the coordinates of each point on the z-plane. Recall that points below the xy-plane have a negative z-value and points above the xy-plane have a positive z-value.	 A: The point is below the xy-plane, therefore the z-value is negative. $z = -2$ B: The point is above the xy-plane, therefore the z-value is positive. $z = 3$ C: The point is above the xy-plane, therefore the z-value is positive. $z = 1$
Write the answer in the form (x, y, z).	Point A is located at $(3, 3, -2)$. Point B is located at $(-3, 1, 3)$. Point C is located at $(0, -4, 1)$.

(b) Which 2D plane does point C lie on?

THINKING	WORKING
Identify the value in the coordinates for point C that is equal to 0.	Point C is located at $(0, -4, 1)$. Therefore, $x = 0$.
Identify the 2D plane on which coordinates with an x-value of 0 lie.	
Write the answer.	Point C is on the yz-plane because its x-value is 0.

Practice

ANSWERS Page 604

SC 1 I can describe and compare 2D and 3D coordinate systems

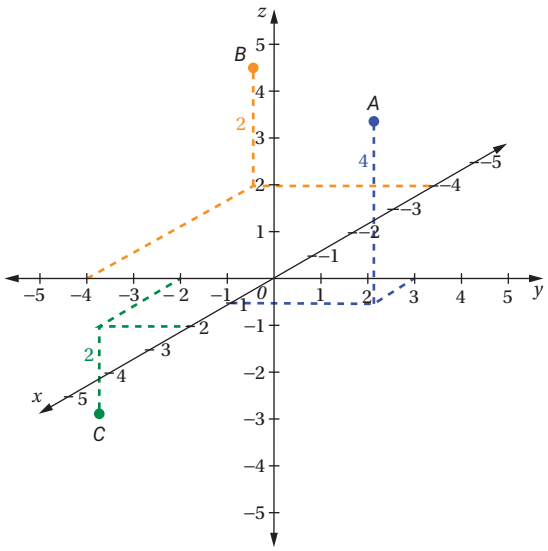
1 Identify the related value for each of these points in the 3D coordinate system.

(a) x-value for the point (3, 3, –2)

(b) y-value for the point (–3, 1, 3)

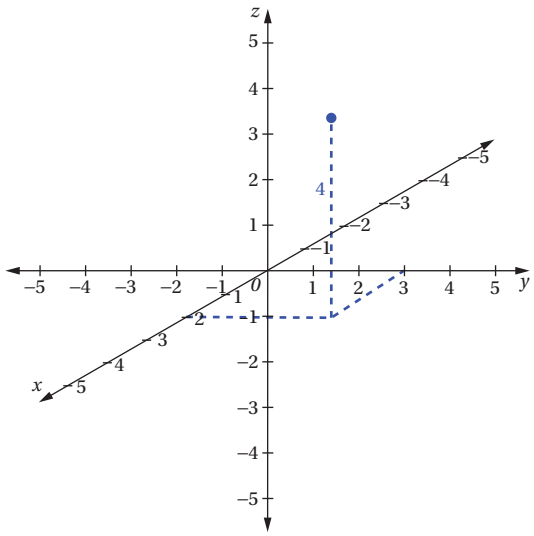
(c) z-value for the point (0, –4, 1)

2 Identify the letter plotted at each given set of coordinates.

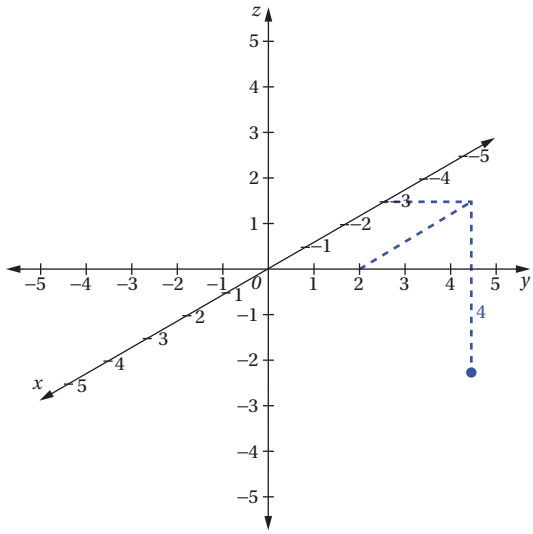


(a) (1, 3, 4) **(b)** (–4, –4, 2) **(c)** (2, –2, –2)

3 Write the coordinates for this point plotted on the 3D plane.



4 Write the coordinates for this point plotted on the 3D plane.



5 Write the coordinates for the following descriptions in the 3D plane.

(a) 2 units along the positive x-axis, 3 units along the positive y-axis, and 4 units above the xy-plane.

(b) 2 units along the y-axis, 3 units in the negative direction along the x-axis, and 4 units below the xy-plane.

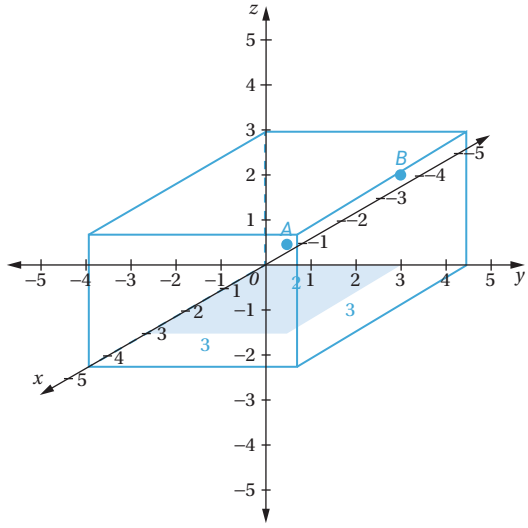
(c) 0 units along the x-axis, 3 units in the negative direction along the y-axis, and on the xy-plane.

6 Identify the flat plane each of the following 3D coordinates lie on and explain your reasoning.

(a) (2, 3, 0) **(b)** (2, 0, 3) **(c)** (0, 2, 3)

7 **(a)** A prism with a square base is drawn on the 3D plane. What are the coordinates of the vertices marked A and B?

(b) Which two coordinate values remain the same for the two points?



TOPIC REVIEW

Multiple choice

ANSWERS Page 632

1 To locate a position in 2D space, this can be achieved by which of the following?

- **A** Using a coordinate system by combining letters and numbers on a grid
- **B** Locating a pair of coordinates on a Cartesian plane
- **C** Using longitude and latitude
- **D** All the above

2 What is the altitude of an object?

- **A** The distance above or to the west of the point of reference
- **B** The distance above or below the point of reference
- **C** The distance above or to the east of the point of reference
- **D** The distance below or to the east of the point of reference

3 Which of the following examples best describes an object positioned in a 3D space?

- **A** Solar panels on the roof of a two-storey house 50 metres away
- **B** A car in the driveway of a house
- **C** A pebble on a beach
- **D** A dog walking on a footpath

4 In which of the examples below is the object positioned in a 3D space?

- **A** Your shoes on the floor
- **B** A chair at your dining table
- **C** A bird sitting on top of a tree
- **D** The carpet in your house

5 Which of the following axes would you determine on a Cartesian plane?

A x and y **B** x and z **C** y and z **D** None of the above

6 Which of the following axes would you determine on a 3D plane?

A c, x and z **B** x, y and z **C** c, y and z **D** None of the above

7 Which of the following points represents an x-coordinate of 2 and a y-coordinate of 5?

A (5, 2) **B** (2, 0) **C** (2, 5) **D** (5, 0)

8 Which of the following points represents an x-coordinate of -5 and a y-coordinate of 3?

A (5, 3) **B** (3, -5) **C** (-5, -3) **D** (-5, 3)

9 Which of the following points represents an x-coordinate of 4, a y-coordinate of -3 and a z-coordinate of 2?

A (4, 3, 2) **B** (4, 2, -3) **C** (-3, 4, 2) **D** (4, -3, 2)

10 Which of the following points represents an x-coordinate of 10, a y-coordinate of 13 and a z-coordinate of 12?

A (10, 12, 13) **B** (10, 13, 12) **C** (12, 13, 10) **D** None of the above

11 If $y = 2$ and $z = 1$ on the flat yz-plane, which of the coordinates below is the correct location?

A (0, 2, 1) **B** (2, 1, 0) **C** (2, 0, 1) **D** (2, 1)

12 If $x = 7$ and $z = 1$ on the flat xz-plane, which of the coordinates below is the correct location?

A (0, 7, 1) **B** (1, 0, 7) **C** (7, 1) **D** (7, 0, 1)

Short answer

1 Define the Cartesian plane, explaining also what the horizontal and vertical lines are called.

2 Define the three-dimensional plane, explaining also what the three axes are called and where they are positioned.

3 For all points below, identify which is the x-coordinate, which is the y-coordinate and which is the z-coordinate.

(a) (5, 6, 2) **(b)** (3, 7) **(c)** (6, 8, 1) **(d)** (1, 4)

4 If a point is sitting on the x-plane what value is the z-coordinate? Explain your answer and show the coordinates.

5 On a Cartesian plane, plot the point (3, 5). Explain the route you would take to get to the coordinate using compass points north, south, east and west. For example, for coordinate (2, –2) you would write 'two points east and two points south'.

6 On a Cartesian plane, plot the point (–2, 4). Explain the route you would take to get to the coordinate using compass points north, south, east and west.

7 On a Cartesian plane plot the point (–1, 0). Explain the route you would take to get to the coordinate using compass points north, south, east and west.

8 On a Cartesian plane mark the coordinates (4, –5). Explain the route you would take to get to the coordinate using compass points north, south, east and west.

9 On a 3D plane each grid represents a kilometre (km). Explain the route you would take to get to the coordinate (4, 5, 7) using the x, y and z axes.

10 Draw the position of a drone hovering at coordinate (8, 7, 2). Explain the route you would take to get to the drone using the x, y and z axes.

Extended response

1 Design a scavenger hunt in your classroom using the 3D coordinate plane. As an example, the plan might use the floor as the xy-plane and two of the adjacent walls to form the xz- and yz- planes (assuming they meet at a right-angle). Use coordinates to show where clues are to be placed and the order in which your classmates would need to locate them.

TOPIC
16

Collecting and analysing data

Recall 442
16.1 Compare sampling methods 443
16.2 Understand sampling techniques and data sources 449
16.3 Understand and interpret statistics from different samples in the same population 460
16.4 Plan and conduct a statistical investigation 472
Topic review 484

Why learn this?

Learning how to collect and analyse data helps people to make smart choices based on information gathered from different sources.

- Know when to use a complete survey of everyone (census) or just a few selected people (sample), depending on how big the group is.
- Learn different ways to select people for a sample, like choosing randomly or picking specific people with common traits. This helps obtain a clear and fair view of what those surveyed think or do as a representative of others similar to them.
- Spot mistakes in how data are collected to make sure the information is accurate and trustworthy.

These skills are useful in school for projects and in everyday life when using feedback from a target audience to make better decisions. They are valuable in many jobs that rely on understanding data to improve products or services.

RECALL

ANSWERS Page 604

I can complete a frequency distribution table

1 Complete a frequency table for this data, showing the number of wi-fi signals recorded in 15 different locations in a shopping centre.
2, 4, 3, 3, 0, 3, 2, 4, 3, 3, 2, 4, 1, 2, 4.

I can calculate the mean, mode and median for a set of scores

1 Calculate the mean (correct to 2 decimal places) of each of the following sets of numbers.

(a) 2, 3, 7, 6, 5, 4, 3 **(b)** 8, 11, 12, 9, 6, 3 **(c)** 1.1, 1.5, 1.4, 1.8, 1.7, 2.0

2 Determine the mode of each of the following sets of numbers.

(a) 1, 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 6, 6, 7, 8 **(b)** 3, 4, 2, 1, 3, 6, 7, 3, 3, 2, 1, 4, 3, 5, 8, 9, 8, 8, 8

(c) 3, 2, 4, 6, 8, 5, 3, 2, 8, 6, 5, 4, 8, 5

3 Determine the median of each of the following sets of numbers.

(a) 7, 9, 11, 8, 15, 14, 12 **(b)** 2, 1, 21, 31, 4, 52 **(c)** 101, 112, 117, 118, 119, 104

I can calculate the mean, mode and median from a frequency distribution table

1 Copy and complete the xf column in this frequency table, then determine the mean, median and mode.

Minutes (x)	Frequency (f)	$x \times f$
30	2	$30 \times 2 =$ .....
35	4	$35 \times 4 =$ .....
40	5	$40 \times$ ..... $=$ .....
45	12	..... $\times$ ..... $=$ .....
50	13	..... $\times$ ..... $=$ .....
55	4	..... $\times$ ..... $=$ .....
Total	**40**	.....

I can calculate the mean, mode and median from a stem-and-leaf plot

1 Find the mean, mode and median for the following stem-and-leaf plot.

Stem	Leaf
1	0 1 3 4
1	6 8 9 9 9
2	2 2 3 4
2	5 7 7
3	1 1 3 3 4
3	6 8
4	0 1 4
4	8 8

Compare sampling methods

16.1

Learning intention: To be able to compare sampling methods

Success criteria:

SC 1 I can identify situations where a census or sample has been used to collect data.

SC 2 I can collect samples using a variety of techniques, including random and non-random sampling.

SC 3 I can recognise where bias may occur in a sampling method.

Lesson warm-up

School memories

Many high schools take a photograph of their Year 12 classes each year. Then they hang these photos in a prominent place for visitors and other school students to see.

What information could you gain by studying the photos from many different years? What could you learn about what has changed over the years for Year 12 students at your school?

SC 1 I can identify situations where a census or sample has been used to collect data

Statistics are all around us. For example:

- daily weather reports provide information about the temperature and whether it is above or below the average temperature for the time of year
- before elections, statistics can provide information about the predicted result
- consumer confidence is measured on a regular basis
- the Australian Bureau of Statistics (ABS) conducts a census of the Australian population every five years.

A census occurs when every member of a population is involved in the data gathering process. Note that 'population' does not necessarily mean everybody in Australia. For example, gathering data about every student in your school is also a census, because every member of the school population is involved.

In a sample, only part of the population is involved in the data-gathering process. A sample of the population can be selected in several different ways, some of which provide more reliable results than others.

It is possible to collect data by observation. To do this, the data collector observes people in a particular location and makes notes about their actions or behaviours.

Worked example

Identifying a census or a sample

Read the following scenarios. State whether a census is conducted or a sample is taken in each case.

(a) A school is considering some uniform changes. They ask every Year 7 student to comment on the proposed changes.

THINKING	WORKING
Identify the population in this case.	The population is every member of the school community.
Determine whether the whole population is involved in the process.	No; only Year 7 students are involved.
Write the answer.	This is a sample.

(b) The owners of a block of units want to employ a contractor to look after all the common areas. They prepare a short survey about the proposal and send it to every unit owner.

THINKING	WORKING
Identify the population in this case.	The population is every unit owner in the block.
Determine whether the whole population is involved in the process.	Yes.
Write the answer.	This is a census.

(c) A workers' union has been involved in negotiations with an employer about a new workplace agreement. A meeting is held to find out whether the union members support the proposed agreement.

THINKING	WORKING
Identify the population in this case.	The population is every member of the union employed at the workplace.
Determine whether the whole population is involved in the process.	The whole population is invited to the meeting, but some people may not attend.
Write the answer.	This is more likely to be a sample than a census.

SC 2 I can collect samples using a variety of techniques, including random and non-random sampling

In a random sample, every member of the population has an equal chance of being selected. This is not the case for a non-random sample.

Consider a school with a population of 2600 students.

- To obtain a random sample of 100 students, you could give each student a unique number from 1 to 2600, then use a random number generator to select 100 different numbers.
- To obtain a non-random sample of 100 students, you could simply select the first 100 students who pass through the gates one morning.

When sampling from a large population, some statisticians think $\sqrt{n}$, where n is the population size, is a reasonable sample size.

Worked example

Deciding whether a sample is random or non-random

For each of the following methods, decide whether the sampling process is random or non-random.

(a) The 26 letters of the alphabet are written on separate pieces of paper. All the pieces of paper are put into a hat and one letter is chosen from the hat. The letter chosen is C. All students with a surname starting with C are included in the sample.

THINKING	WORKING
Determine whether every person in the population has an equal chance of being in the survey.	Yes.
Write the answer.	A random sample.

(b) The first 50 people leaving a train station are surveyed about their attitudes to public transport.

THINKING	WORKING
Determine whether every person in the population has an equal chance of being in the survey.	No.
Write the answer.	A non-random sample.

(c) A large sporting club has 500 members. Each person is allocated a number from 1 to 500 and a random number generator is used to choose the 50 members to be included in the sample.

THINKING	WORKING
Determine whether every person in the population has an equal chance of being in the survey.	Yes.
Write the answer.	A random sample.

SC 3 I can recognise where bias may occur in a sampling method

A sample may be biased if it is not chosen carefully to reflect the population. For example, a survey about the annual costs of running a car will almost certainly be biased if the sample only includes owners of electric vehicles.

Worked example

Understanding bias

Explain why the following sampling method may lead to a biased sample.

The research question relates to the use of public transport. The sample of 200 people is obtained by stopping people inside a busy metropolitan railway station.

THINKING	WORKING
Identify possible sources of bias and write an explanation based on these sources.	Public transport involves more than trains, so this survey does not include all users of public transport. For example, bus users will not be included in the survey unless they also happen to use the train. A busy metropolitan station is likely to have a better service than a smaller rural station, so the survey will not reflect the opinions of all train users, let alone all public transport users. People who do not use public transport at all are unlikely to be at the station, so they will have no chance to explain why they do not use public transport.

Practice

ANSWERS Page 605

SC 1 I can identify situations where a census or sample has been used to collect data

1 Read the following scenarios. For each one, state whether the process uses a census approach or a sample approach.

(a) After each car service, a car dealership sends a survey to the customer seeking feedback about their service experience.

(b) At a shopping centre, two mobile telephone service providers have shops side-by-side. Anthea counts the number of people who enter each shop over a five-hour period.

(c) Every person entering Gate 5 at a large sporting arena is asked to complete a survey about prices for the venue.

2 Identify the population in each of the following situations.

(a) A researcher wants to find out about the cost of housing in a particular suburb.

(b) A researcher wants to find the unemployment level for each state.

(c) A researcher wants to know the average number of people in Australian households.

3 In each of the following cases, state whether a census or sample would be used. Explain your reasoning.

(a) A researcher wants to find the most popular car colour in Australia.

(b) A researcher wants to find the average height of 14-year-old students in Australia.

(c) A researcher wants to determine the likely effect of increasing the tax on nicotine products on usage rates of these products.

(d) A researcher wants to find out how 100 members of a tennis club feel about extending the clubhouse.

4 The official Australian Census is conducted by the Australian Bureau of Statistics every five years.

(a) Why do you think the census is not conducted annually?

(b) How long do you think it takes to collect the information?

(c) How long do you think it takes to collate and interpret the information collected?

(d) Who do you think pays for the census to be conducted?

SC 2 I can collect samples using a variety of techniques, including random and non-random sampling

1 For each of the following scenarios, decide whether the sampling process is random or non-random.

(a) When investigating the most popular car colour, you record the colour of every car in the school car park.

(b) When investigating the average height of 14-year-old students, you measure and record the height of every 14-year-old at your school.

(c) When investigating the views of the 500 members of a sporting club, you allocate a number to each member, then use a random number generator to pick 35 members to be surveyed.

2 A company that manufactures batteries wants to find out the average life of its batteries, to use in an advertising campaign.

(a) Why would the company not conduct a census to find out the average life of the batteries?

(b) If the factory produces a batch of 20 000 batteries, how many should be included in the sample?

(c) If the factory produces a batch of 200 000 batteries, how many should be included in the sample?

3 In each of the following cases, say whether a census or a sample should be undertaken. For each sample, explain how you would obtain a random sample of the population.

(a) A school wants to find out the opinions of Year 9 students before changing the elective subjects offered.

(b) A local council needs to repair a road with 50 houses on it and wants a financial contribution from property owners.

(c) The manager of a concert hall wants to gather feedback about a concert held one night. The venue holds 1500 people.

4 A State election is to be held soon. A large metropolitan newspaper has commissioned surveys on various aspects of policy relevant to the election. One survey relates to public transport.

(a) State whether each of the following processes would lead to a random sample or a non-random sample of the state's population.

(i) The survey is conducted outside metropolitan train stations only.

(ii) The survey is conducted outside metropolitan train stations and rural bus stops.

(iii) The survey is conducted in the central business district of the capital city.

(iv) The survey is conducted in five regional towns only.

(b) What do your answers to part **(a)** tell you about obtaining a random sample?

SC 3 I can recognise where bias may occur in a sampling method

1 Explain why the following sampling method may lead to a biased sample.

The question relates to the average weekly cost of petrol in a family car.

A survey is conducted at three different petrol stations, on three different days, in three different suburbs of the capital city.

2 Before the introduction of mobile telephones, many houses had a landline telephone connections. The number for this telephone was usually listed in a telephone directory, so if you knew the name of a person, you could look up their telephone number in this directory. Many surveys were conducted by choosing people 'at random' from the telephone directory. Explain your thoughts about this survey method and whether it would produce a biased sample.

3 For each of the following methods, indicate why a biased sample is likely.

(a) A school is seeking comments on some uniform changes and surveys every student in Year 7.

(b) A sporting club that has cricket, football, and netball teams is considering making changes to the clubrooms. They conduct a survey during the summer, by speaking to a sample of the cricket-playing members.

(c) A confectionary company is planning to introduce a new chocolate bar. They need to choose between three different bars. The company employs people to stop shoppers at a local shopping centre and ask them to choose their favourite bar from the three.

4 A company has 450 employees, of which 10 are managers, 25 are middle managers and the rest are workers. Each employee is given a unique number from 1 to 450 for the purposes of selecting a sample of 35 employees for the annual worker satisfaction survey. Explain why this sample will probably meet the definition of being random, but will also probably be biased.

Understand sampling techniques and data sources

16.2

Learning intention: To understand sampling techniques and data sources

Success criteria:

SC 1 I can distinguish between many different sampling methods.

SC 2 I can explain the benefits and limitations of various sampling techniques.

SC 3 I can explain the difference between primary and secondary data.

SC 4 I understand and can explain various statistical terms.

Lesson warm-up

The interview

After applying for a job, you are invited for an interview.

However, you are not sure what to wear for the interview.

1. You have identified two options: Go to the place where the job will be and see what people wear while working in similar roles. Then you can dress appropriately to match.
2. Conduct some online research about what to wear to interviews for similar positions. Then you can find an outfit that matches this information.

Which approach would you take and why?

SC 1 I can distinguish between many different sampling methods

There are many different sampling techniques that can be used in different situations. Some of the most common techniques are outlined below.

- A convenience sample involves surveying people in a way that is convenient to the survey taker. For example, a researcher might stand in a shopping centre and stop people as they walk past.
- A random sample involves surveying people who have been selected at random – for example, by allocating a unique number to every member of a population, then using a random number generator to choose the sample.

- A stratified sample may be used when there are clearly identifiable layers, or strata, in the target population. For example, in a school, each year level may be regarded as a separate layer. A sample should include balanced representation from each layer.
- Purposeful sampling is used when a specific subset of the population is the target. For example, a survey on the effect of being a twin should only include twins.

Worked example

Choosing a sampling technique

What type of data collection method could be used in each of the following situations? Explain your reasoning.

(a) Identify the most popular make of car in Australia.

THINKING	WORKING
Decide whether you need a census or a sample.	It is not practical to find out the make of every car in the country, so you will need to take a sample.
Decide the simplest non-biased way of taking a sample.	Any random sampling technique would be suitable. In this situation, convenience sampling would be a good choice – for example, stand on the corner of a street and note the make of every car that passes.

(b) Determine the average arm span of 14-year-old students in Australia.

THINKING	WORKING
Decide whether you need a census or a sample.	It is not practical to conduct a census, so you will need to take a sample.
Decide the simplest non-biased way of taking a sample.	It would be best to take a stratified random sample, including students from every state and territory in Australia.

(c) Determine the average time taken to get to school for students in your class.

THINKING	WORKING
Decide whether you need a census or a sample.	The population is small, so conduct a census.
Decide the simplest non-biased way of generating the data.	Collect the information directly from each student.

SC 2 I can explain the benefits and limitations of various sampling techniques

The cost of conducting surveys is an important consideration. This is why, in most cases, a sample is used rather than a census.

Unfortunately, cheaper and easier survey methods usually give less reliable results.

Worked example

Describing the limitations of a sampling method

A school was interested in finding the average time taken by students to get to school each day. They stationed staff members at the two main entrances to the school for 20 minutes before school started one Monday and asked as many students as possible to estimate how long it had taken them to get to school that day.

Write a few sentences about this sampling method and the problems it may contain.

THINKING	WORKING
Identify the type of sampling used.	This is an example of convenience sampling. A convenience sample involves surveying people in some manner convenient to the survey taker.
Write a few sentences about any problems you can see with the process.	The scenario mentions 'the two main entrances', which implies there are other entrances. Students who use these other entrances are excluded from the sample. Students who arrive more than 20 minutes before the start of school, or who arrive late, are also excluded. The sample only considers one Monday, but it may take students more or less time to get to school on different days.

SC 3 I can explain the difference between primary and secondary data

Data sets can be categorised in different ways. One of these ways is based on how the data was collected.

Data that is collected by the person doing the analysis is known as primary data.

Data that is collected by some other person or organisation is known as secondary data.

If you stand in front of your school and note each type of vehicle that passes, you are collecting primary data. If you refer to a database to find the average height of the players in an Australian professional football code, you are using secondary data.

Worked example

Categorising data as primary or secondary

Every five years, the Australian Bureau of Statistics (ABS) conducts a census of the Australian population. Over the following five years, statisticians working for the ABS analyse the data collected and release their findings. In many cases, the data collected is also published on the ABS website, where it can be accessed by anybody who is interested in doing some data analysis.

(a) Is the data primary or secondary when it is analysed by the ABS statisticians?

THINKING	WORKING
Recall the definitions of primary and secondary data.	Primary data is collected and analysed by the same researcher or organisation. Secondary data is collected by one researcher or organisation and analysed by another.
Identify the organisation that collected the data.	The ABS collected the data.
Identify the organisation that analysed the data.	The data is analysed by ABS statisticians.
Write the answer.	The data is collected and analysed by the same organisation, so it is primary data.

(b) Is the data primary data or secondary data when it is analysed by you?

THINKING	WORKING
Recall the definitions of primary and secondary data.	Primary data is collected and analysed by the same researcher or organisation. Secondary data is collected by one researcher or organisation and analysed by another.
Identify the organisation that collected the data.	The ABS collected the data.
Identify the organisation that analyses the data.	The data is analysed by you.
Write the answer.	The data is collected and analysed by different researchers or organisations, so it is secondary data.

SC 4 I understand and can explain various statistical terms

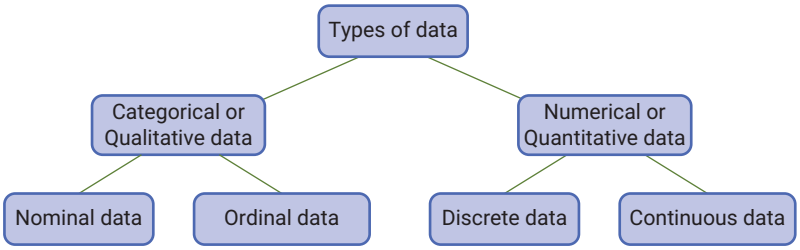
In the study of statistics, you will need to use some technical terms to distinguish between similar things. You have already come across some of these terms, such as census and sample, and primary data and secondary data.

Numerical data is data where the values are numbers, such as height or number of children. Data of this type is sometimes also referred to as quantitative data.

There are two distinct types of numerical data:

- Discrete data can only take particular values – for example, number of children.
- Continuous data can take any value (within a range), but is typically rounded to a predetermined number of decimal places – for example, height may be rounded to two decimal places for better viewing.

Data that is not numerical is called categorical data. Categorical data is also sometimes referred to as qualitative data. Nominal data is a type of categorical data. With nominal data, values fall into set categories, such as the mode of transport used to get to school. In some cases, there is a sense of order to the categorical data. For example, a survey question might ask you to choose an answer on a scale from strongly agree to strongly disagree. This type of categorical data is known as ordinal data.



A random number sequence is a series of numbers that have no relation to each other. Random number sequences are often used to choose individuals from a population to form a sample. You can produce random numbers using a scientific calculator or a computer application such as Excel. For example, in Excel, the formula =INT(RAND()*20+1) will produce random whole numbers in the range 1–20.

The sample space is the set of values that is produced.

The sampling distribution allows the researcher to draw conclusions about the data – for example, the mean value of the samples taken from a population.

Worked example

Distinguishing between discrete and continuous data

In each of the following cases, decide whether the data set contains discrete or continuous data.

(a) The arm span of each player on a team in the Super Netball League.

THINKING	WORKING
Recall the definitions of discrete and continuous data.	Discrete data can take a finite number of values. Continuous data can take an infinite number of values (within a range).
Determine whether the number of possible values is finite or infinite.	The results are only dependent on the accuracy of the measuring device. There is an infinite number of possible results.
Write the answer.	The arm span of each player on a team in the Super Netball League is continuous data.

(b) The foot length of each player on a team in the Super Netball League.

THINKING	WORKING
Recall the definitions of discrete and continuous data.	Discrete data can take a finite number of values. Continuous data can take an infinite number of values (within a range).
Determine whether the number of possible values is finite or infinite.	The results are only dependent on the accuracy of the measuring device. There is an infinite number of possible results.
Write the answer.	The foot length of each player on a team in the Super Netball League is continuous data.

(c) The number of goals scored by each player on a team in the Super Netball League.

THINKING	WORKING
Recall the definitions of discrete and continuous data.	Discrete data can take a finite number of values. Continuous data can take an infinite number of values (within a range).
Determine whether the number of possible values is finite or infinite.	The number of goals scored by any player is limited to whole numbers (0, 1, 2, etc.). There is a finite number of possible results.
Write the answer.	The number of goals scored by each player on a team in the Super Netball League is discrete data.

Practice

ANSWERS Page 606

SC 1 I can distinguish between many different sampling methods

1 What type of data collection process should be used in each of the following situations? Briefly explain your reasoning.

(a) A market research firm has been commissioned to predict the outcome of the next federal election.

(b) A school is thinking about making changes to the food items available for purchase in the school canteen. They want to find out what students think about the proposed changes.

(c) A car dealership wants to find out whether customers who have their vehicle serviced are satisfied with the service they receive.

2 In each of the following situations, explain what strata you might use to conduct a stratified random sample.

(a) A school is thinking about making changes to the school uniform.

(b) A sporting club is planning to renovate the clubrooms.

(c) A researcher is investigating the time spent using mobile devices.

3 When conducting a stratified random sample, the number of people in each stratum of the sample should be in proportion to the number of people in that stratum of the population. Use this table to answer the following questions:

Stratum	Number of people
A	290
B	510
C	615
D	220

(a) What is the total population across the four strata?

(b) What percentage of the population does stratum A represent? Give your answer to 1 decimal place.

(c) In a sample of 75, how many people should come from stratum A?

(d) Now calculate the number for each of the other strata. Round your answers to the nearest whole number.

(e) What do these numbers add to?

(f) What should you do if these numbers don't add up to 75? Why might they not add up to 75?

4 For each of the following populations, copy and complete the table, showing the number of people to be chosen from each stratum.

(a) A stratified random sample of 100 students from a school population.

Year level	Number of students	Number to be surveyed
7	300	
8	290	
9	350	
10	420	
11	375	
12	350	

(b) A stratified random sample of 150 people from a country town. Only the age groups shown were considered relevant to the survey.

Age group	Number of people	Number to be surveyed
10–<15	450	
15–<20	600	
20–<25	295	
25–<30	315	
30–<35	460	
35–<40	270	

5 Write answers to the following questions.

(a) A telephone survey is usually regarded as quite unreliable. Explain why this is the case.

(b) Internet-based surveys are sometimes described as 'self-selecting'. Explain what you think this means and how it might affect the reliability of the survey.

(c) A group of students stands at the front of the school and records the colour of cars passing by. What type of sampling is this, and what issues might arise during the survey?

SC 2 I can explain the benefits and limitations of various sampling techniques

1 Local council elections are to be held soon. The council is divided into seven wards and each ward elects two councillors. Voting is compulsory for all landowners in the council area.

A political party is running candidates in all wards. They want to know what level of support they can expect at election time.

There is a large business district within the council area, so the political party sends volunteers to various locations within this business district. Each volunteer is asked to interview 30 people each day.

(a) What type of sampling technique is this?

(b) What are the limitations of this process?

(c) Suggest a better process that could be used in this situation.

2 A school is interested in students' views on a range of issues, such as House sports and community service. The school has 2540 students spread across Years 7–12. The sample size will be 150 students. The school allocates each student a number and then uses a random number generator to select the 150 students to be surveyed.

(a) What type of sampling technique is this?

(b) What are the limitations of this process?

(c) Suggest a better sampling technique that would produce more reliable results.

3 A company has 270 employees; 5 are senior managers, 30 are middle managers, and the rest are workers. The company is interested in employees' attitudes to management and conducts a properly structured, stratified random sample.

(a) Is this a reasonable sampling technique in this scenario?

(b) Suggest a more appropriate sampling process and explain your choice.

4 The Victorian State Government has spent a large amount of money removing level crossings where a rail line crosses a road. This has been achieved by building tunnels (so the road can pass under the rail line) or bridges (so the rail line can pass over the road).

(a) Why do you think the Victorian State Government invested in this process?

(b) There is a wide range of opinions about this use of public funds. Who would you consider to be the major stakeholders who needed to be consulted about this initiative?

(c) How would you construct a sampling process with the aim of identifying the opinion of the 'general public' regarding this issue?

SC 3 I can explain the difference between primary and secondary data

1 Categorise each of the following data sets as primary data or secondary data.

(a) At the start of each cricket season, a booklet is produced giving various data values, such as height, age and number of games played for each player in each team in the competition. You use this data to find the average height of the players at your favourite club.

(b) The Bureau of Meteorology (BOM) publishes data related to the weather, such as daily maximum and minimum temperatures, for hundreds of locations in Australia. You access this data to find information about your nearest weather station. Then you use the data to write a report about the normal winter weather conditions for your location.

(c) As part of your study of statistics, each student in your class measures their own hand span and records it on the board. Then the class analyses the data together.

2 The following table shows data collected at the Albury Airport weather station during May 2022.

Date	Day	Temps		Rain	Evap	Sun	Max Wind gust			9 am						3 pm					
		Min	Max				Dir	Spd	Time	Temp	RH	Cld	Dir	Spd	MSLP	Temp	RH	Cld	Dir	Spd	MSLP
		°C	°C	mm	mm	hours		km/h	local	°C	%	8th		km/h	hPa	°C	%	8th		km/h	hPa
1	Su	4.7	19.2	0			WSW	24	15:54	10.1	96			Calm	1025.7	17.4	61	6	WSW	15	1022.6
2	Mo	5.0	20.4	0			WSW	17	13:13	10.3	99	8	ENE	7	1025.1	19.7	62	8	S	6	1021.6
3	Tu	5.8	22.4	0			N	24	13:28	11.8	98			Calm	1022.1	22.4	51	3	NNE	9	1017.3
4	We	9.5	21.8	0			NNW	24	12:41	13.4	94	8		Calm	1015.7	20.0	69	8	SE	6	1012.3
5	Th	6.7	16.5	0			WNW	30	13:58	11.4	78		NW	9	1016.2	15.9	40		WNW	19	1014.6
6	Fr	1.8	13.8	0			NW	31	16:34	8.4	89	8	E	4	1020.3	12.1	63	8	NNW	20	1018.7
7	Sa	3.5	16.3	0.6			W	35	14:37	8.1	88	7	NNW	9	1022.7	14.7	50	5	WNW	22	1020.9
8	Su	7.4	15.1	0.4			NNE	15	12:00	11.0	92	8		Calm	1026.6	14.5	71	8	W	9	1025.7
9	Mo	3.3	20.3	0			ENE	20	14:29	9.3	98			Calm	1029.8	20.3	49	1		Calm	1027.4
10	Tu	5.5	20.5	0			SE	20	13:39	12.3	88	2	SSW	6	1031.0	19.5	58	1	SE	11	1027.7
11	We	10.0	19.2	0			SE	20	12:45	13.0	94			Calm	1027.7	18.9	61	8	SE	9	1024.4
12	Th	13.0	17.3	8.2			SE	24	08:11	14.3	99	8	SE	13	1020.0	16.1	100	8	SSE	7	1015.7
13	Fr	14.3	20.1	18.0			ENE	17	09:07	17.2	100	8	E	9	1017.5	19.6	99	8	ENE	7	1014.1
14	Sa	15.1	20.2	1.8			NW	24	11:38	17.3	100	8	NE	4	1015.3	18.7	99	8	WSW	13	1013.4
15	Su	14.1	20.6	1.6			NNW	28	14:35	16.0	100	2	NNE	7	1015.8	18.8	74	1	WNW	15	1012.5
16	Mo	12.1	18.1	0			W	35	13:01	14.2	89	8	W	13	1016.8	17.3	65	2	WNW	17	1014.5
17	Tu	7.5	15.5	0.8			W	30	13:15	9.3	99	8	WSW	6	1021.8	14.7	60	4	WNW	13	1020.0
18	We	9.3	15.8	0			W	41	11:47	11.3	91	7	W	20	1021.6	15.3	57	8	WSW	20	1020.8
19	Th	1.2	16.2	0			SSE	17	01:49	5.0	99	8	SSW	4	1028.9	15.7	54		W	6	1027.0
20	Fr	1.6	16.3	0			SE	24	13:59	7.1	92		NNW	2	1034.3	15.5	54		SE	13	1031.4
21	Sa	3.3	19.1	0			ESE	15	10:08	9.8	88		SE	2	1034.0	18.2	48		ESE	7	1029.9
22	Su	3.4	21.0	0			E	13	10:11	8.8	90			Calm	1030.8	20.0	47		NE	2	1026.8
23	Mo	3.0	19.9	0			ESE	13	14:32	8.8	91			Calm	1030.8	18.9	52		SSE	4	1027.1
24	Tu	3.5	19.4	0			ESE	15	12:05	9.0	90			Calm	1030.1	18.8	44		ESE	7	1024.8
25	We	3.6	18.5	0			SSE	17	13:37	8.7	89		W	6	1025.3	17.3	60		ESE	11	1020.5
26	Th	6.8	16.4	0.2			ESE	15	12:30	11.0	96	8		Calm	1020.3	15.5	87	8	SE	7	1017.4
27	Fr	11.0	17.1	1.4			NNW	13	16:12	13.3	99	8	SE	7	1019.2	15.8	93	8	SW	7	1015.8
28	Sa	9.8	15.9	6.6			WNW	30	12:50	12.9	99	7	WNW	9	1015.0	15.0	77		WNW	22	1012.2
29	Su	5.5	14.3	0			WNW	26	13:46	8.2	99	8	SE	4	1012.2	12.4	62	1	W	19	1008.3
30	Mo	3.5	9.3	0			W	31	20:28	8.0	99	8	ESE	7	997.0	9.0	99	8	NW	7	987.7
31	Tu	6.9	11.0	29.2			WNW	50	12:50	8.3	95	6	W	24	996.7	9.8	78	1	W	30	997.2
Statistics for May 2022																					
Mean		6.8	17.7							10.9	94	7		5	1021.5	16.7	65	5		11	1018.4
Lowest		1.2	9.3	0						5.0	78	2		Calm	996.7	9.0	40	1		Calm	987.7
Highest		15.1	22.4	29.2			WNW	50		17.3	100	8	W	24	1034.3	22.4	100	8	W	30	1031.4
Total				68.8																	

Data based on © Copyright Commonwealth of Australia, Bureau of Meteorology

Estelle wants to look at a sample of this data. She uses a random number generator to give her five numbers in the range 1–31.

Using Excel, she obtains:

5
9
22
1
9

(a) What type of data is Estelle using?

(b) What would Estelle need to do if her list of five random numbers included one or more repeated values?

(c) Use the random numbers Estelle generated to find the average maximum and minimum temperatures at Albury Airport in May 2022.

(d) Compare the values Estelle calculated with the average values recorded in the official table.

3 While preparing for an annual athletics carnival, every student at your school completes the 100 m sprint and has their time recorded. The results for your class, in seconds to 1 decimal place, were:

17.1	18.2	16.5	19.2	15.1
13.8	18.7	21.1	17.7	18.6
17.8	20.0	16.4	15.5	16.9
13.2	20.6	18.8	15.3	14.9

(a) How many students are in your class?

(b) Is this primary data or secondary data? Give a brief justification for your answer.

(c) What is the average time taken to run 100 m for the students in your class?

4 On 7 June 2023, the following information was at the top of the ABS website.

Population	Consumer price index	Gross domestic product	Average weekly earnings	Unemployment rate
26,124,814	**7.0%**	**0.2%**	**$1,807.70**	**3.7%**
30 September 2022	Annual change Mar 2023 quarter	Quarterly change Mar 2023	November 2022	April 2023

(a) Would the data used to produce these figures be considered primary or secondary data?

(b) The population figure is dated 30 September, 2022. Why do you think this figure is not more current?

(c) The consumer price exponent and gross domestic product figures refer to the March 2023 quarter. Is it possible for the ABS to provide a more current figure in either of these cases?

(d) Why do you think the average weekly earnings figure is dated November 2022?

SC 4 I understand and can explain various statistical terms

1 Classify each of the following data sets as numerical or categorical.

(a) The number of students studying advanced mathematics each year.

(b) The average speed of Formula 1 cars.

(c) The colour of cars in the school car park.

2 Complete the following statements.

(a) Nominal data is a type of _____ data.

(b) The two types of numerical data are _____ and _____.

(c) A survey that includes every member of the group under investigation is called a _____.

(d) A small group that is randomly selected from a larger population is called a _____.

(e) A sample where everyone in the population has an equal chance of being selected is a _____ sample.

3 Every night for the month of July, Virat watched the weather report on the news and recorded the maximum daily temperature.

(a) Is the data Virat recorded discrete or continuous?

(b) Is the data Virat recorded primary or secondary?

(c) Virat writes in his report that the number of days where the maximum temperature was greater than 15°C was five. Is this value discrete or continuous?

4 Choose the best response for each of the following.

(a) The time taken for each member of a Year 8 class to complete the Statistics Topic Test.

A discrete **B** continuous **C** nominal **D** ordinal

(b) The number of students served each day in the school canteen.

A discrete **B** continuous **C** nominal **D** ordinal

(c) Oranges are graded as very small, small, large or extra large.

A discrete **B** continuous **C** nominal **D** ordinal

(d) The vehicles in the school car park are classified as car, SUV, 4WD or other.

A discrete **B** continuous **C** nominal **D** ordinal

5 There are 263 students in Year 8 at Survey Secondary College. They are placed in alphabetical order and each student is allocated a number from 1–263 based on their position in the alphabetical listing. The first student on the list is 1 and the last is 263.

(a) Write an Excel formula that you could use to select a random sample of these students.

(b) The following sample of size 20 is obtained using your formula.

What should you do if one or more of the numbers are repeated?

237	69	121	234	204
9	209	195	97	208
90	19	18	188	90
125	167	146	100	172

(c) If another student used your formula, would you expect them to get the same 20 values? Give a brief explanation for your answer.

(d) What could you do to check that your formula is correct?

16.3 Understand and interpret statistics from different samples in the same population

Learning intention: To be able to understand and interpret statistics from different samples in the same population

Success criteria:

SC 1 I can compare variations obtained from random samples drawn from the same population.

SC 2 I can explain the effect of individual values, including outliers, on statistical values.

SC 3 I can conduct a simulation using a digital tool.

Lesson warm-up

Draw bag

Each student has a bag and all the bags contain the same tokens.

Every student shakes their bag exactly two times, then draws one token and records the result.

If every student performs the same action, with a bag containing the same tokens, would you expect all the outcomes to be the same?

From your class results, you can see that this is not the case. Briefly discuss why.

SC 1 I can compare variations obtained from random samples drawn from the same population

When the word statistics is used in this context, it refers to values such as the mean or the median of the data values produced. For quantitative data, it would refer to things such as the most popular response.

Worked example

Calculating the mean of a sample

A sample of 15 students in Year 8 have their height measured, in centimetres, correct to 1 decimal place. The results are as follows:

141.8	152.3	146.4	161.5	130.8
153.6	141.8	140.2	138.6	131.3
151.5	161.0	149.2	157.8	142.4

(a) Calculate the mean of all 15 values.

THINKING	WORKING
To calculate the mean, use the formula: $\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$	The sum of the values is 2200.2. $\text{Mean} = \frac{2200.2}{15}$ $= 146.68$

(b) Calculate the mean of the first row of five values.

THINKING	WORKING
To calculate the mean, use the formula: $\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$	The sum of the first five values is 732.8. $\text{Mean} = \frac{732.8}{5}$ $= 146.56$

(c) Make a comment about your results.

THINKING	WORKING
Compare the two values obtained.	The two values are close, but if rounded to 1 decimal place, they differ by 0.1. The smaller sample has a smaller mean value.

SC 2 I can explain the effect of individual values, including outliers, on statistical values

In a data set, an outlier is a value that seems quite different to the rest of the data values. It may be the result of an error in recording, or it may be a true value that is very different to the rest.

For example, imagine your class is collecting data about the number of pets at home. Common values are likely to be small, such as 0, 1, 2, 3, 4 and 5. But what if one of the students breeds small birds, such as budgerigars? They could have 50 or more birds that they consider to be pets. The value 50 is very different to all the other values, so it is an outlier.

Three of the most important statistics calculated for numerical data sets are the mean, the median and the range. The mean and the median are measures of central tendency, while the range is a measure of spread. The range is simply the difference between the lowest and highest values.

These statistics are affected by outliers in different ways, so you will need to think carefully about how to deal with any outliers in your data set.

Worked example

Understanding the effect of an outlier

The following data set shows the number of pets for each person in a class.

2	3	1	4	0	1	1	1	0	0	7	0
33	2	5	0	0	2	1	1	2	4	2	0

(a) Calculate the mean for the 24 values.

THINKING	WORKING
Recall the formula for the mean and determine the number of values and the sum of the values.	$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$ $\sum x = 72$ $n = 24$
Calculate the mean by substituting the values into the formula.	$\text{Mean} = \frac{72}{24}$ $= 3$

(b) Determine the median for the 24 values.

THINKING	WORKING
To determine the median, first put the values in order from smallest to largest. Then find the middle value.	In order, the values are: 0 0 0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 3 4 4 5 7 33 The median is the 12.5th value, which is between the last value on the first line and the first value on the second line. These values are the same, so the median is 1.

(c) Calculate the range for the 24 values.

THINKING	WORKING
Calculate the range using the formula: Range = max value – min value	Range = 33 – 0 = 33

(d) Which value, or values, are outliers?

THINKING	WORKING
Which value, or values, are outliers?	33 is definitely an outlier. Some people may consider 7 to be an outlier as well, but in these calculations, only 33 will be considered.

(e) Calculate the mean for the values not considered outliers.

THINKING	WORKING
Calculate the mean, excluding any outliers from the data set.	Excluding the value 33: $\sum x = 39$ $n = 23$ $\text{Mean} = \frac{39}{23}$ $= 1.6956\ldots$ Rounding to 2 decimal places, the mean is 1.70.

(f) Calculate the median for the values not considered outliers.

THINKING	WORKING
Calculate the median, excluding any outliers from the data set.	0 0 0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 3 4 4 5 7 The median is the 12th value in the ordered list, which is 1.

(g) Calculate the range for the values that are not considered to be outliers.

THINKING	WORKING
Calculate the range, excluding any outliers from the data set.	Range = 7 – 0 = 7

(h) Comment on the statistics you have calculated.

THINKING	WORKING
Compare the statistics with and without the outlier and comment on any differences.	The two values for the mean are quite different. When the outlier is excluded, the mean is significantly smaller and seems to represent the data values more accurately. The value of the median is the same with and without the outlier. The two values for the range are very different; this is the value that has changed the most.

SC 3 I can conduct a simulation using a digital tool

Scientific calculators and spreadsheet programs such as Excel can produce lists of random numbers. These random numbers can be used to simulate an actual experiment.

A die (a single dice) is often used in probability and statistics experiments. However, some students find it difficult to roll a die properly, it can be noisy, and it takes a long time to roll 100 times or more.

It can be much easier – and more reliable – to use a list of random numbers, often generated using technology.

Worked example

Using a random number generator

In some games involving a die, players must roll a 6 to start.

Use random numbers to estimate the number of times a player will need to roll the die before a 6 appears. Use a random list of 100 numbers in the range 1–6 to make this estimate.

THINKING	WORKING
Generate a list of 100 numbers in the range 1–6. (In Excel, the formula to use is =INT(RAND()*6+1).)	4 5 5 3 3 1 1 5 1 2 1 4 4 6 5 3 1 4 2 6 2 3 2 6 6 5 4 5 1 1 1 6 4 1 2 4 4 6 5 5 6 5 2 3 2 1 5 3 4 4 3 2 2 6 5 4 6 6 6 1 2 4 6 2 5 2 6 6 5 4 6 2 2 3 5 5 4 1 5 4 4 3 4 4 6 5 4 5 2 4 3 6 1 4 2 6 4 5 5 6
Start at the top left-hand corner of the data list and count how many numbers there are before the first 6 appears. At the end of the first row, go to the start of the second row, and so on.	The first 6 is the 14th value in the list.
Now count how many numbers there are before the second 6.	The second 6 appears after 6 more rolls.
Keep going until you reach the end of the number list.	The third 6 takes 4 rolls. The fourth 6 takes 1 roll. The fifth 6 takes 7 rolls. The complete list of numbers of rolls is: 14, 6, 4, 1, 7, 6, 3, 13, 3., 1, 1, 4, 4, 1, 3, 14, 7, 4, 4
Now find the mean number of rolls, rounding to 2 decimal places.	$\text{Mean} = \frac{100}{19}$ $= 5.263\,15\ldots$ The mean number of rolls before a 6 appears is 5.26 rolls.

Practice

ANSWERS Page 608

SC 1 I can compare variations obtained from random samples drawn from the same population

1 Consider the following set of randomly generated whole numbers in the range 1–10.

5	7	4	9	5	4	7	7	6	1
9	4	4	9	6	2	6	10	8	5
10	9	8	4	10	7	10	5	4	7
7	2	5	3	1	1	10	1	9	4
4	6	6	4	4	3	6	8	5	4

(a) Copy and complete the frequency table for these values.

Score (x)	Frequency (f)	$x \times f$
1	4	
2	2	
3	2	
4	11	
5	6	
6	6	
7	6	
8	3	
9	5	
10	5	
	$\sum f =$	$\sum xf =$

(b) What is the mean of this data set? State your answer correct to 1 decimal place, if necessary.

(c) What is the median of this data set? State your answer correct to 1 decimal place, if necessary.

(d) If the 50 values were equally spread between the 10 possible outcomes, what would be the mean and the median? State your answers correct to 1 decimal place, if necessary.

2 Consider the following set of values. Each value is a randomly generated whole number in the range 1–20.

6	2	15	16	13	20	5	12	4	14
12	11	1	2	17	14	11	17	11	18
4	20	1	15	8	20	8	19	16	20
13	10	19	16	4	18	17	20	18	1
14	13	4	10	7	18	9	20	10	17
19	7	14	9	8	16	2	17	20	4
5	20	16	15	8	14	20	9	12	20
20	3	12	5	19	1	19	1	12	13

(a) Construct and complete a frequency table showing these results.

(b) What is the mean of this data set? State your answer correct to 1 decimal place, if necessary.

(c) What is the median of this data set? State your answer correct to 1 decimal place, if necessary.

(d) If the 80 values were equally spread between the 10 possible outcomes, what would be the mean and the median? State your answers correct to 1 decimal place, if necessary.

3 Consider the following data set of 50 randomly generated whole numbers in the range 1–10. Some statistics associated with the values have been calculated.

5	6	1	5	2	4	6	7	8	6
7	8	9	4	9	5	4	8	4	3
2	9	1	6	7	2	2	4	5	2
5	2	6	6	3	2	2	7	10	7
2	4	4	9	6	10	4	9	9	10

Mean	5.33	Mean 1	5	Median 1	5.5
Median	5	Mean 2	6.1	Median 2	6
		Mean 3	4	Median 3	3
		Mean 4	5	Median 4	5.5
		Mean 5	6.7	Median 5	7.5

(a) Mean 1 is the mean of the first row of 10 values. What is the mean of the five individual row means?

(b) What do you notice about this value?

(c) What is the median of the five individual median values?

(d) What do you notice about this value?

(e) What is the mean of the five individual median values?

(f) What do you notice about this value?

(g) What conclusion can you draw from this?

4 The following table gives some statistics related to five different random samples. Each sample is a set of 50 whole numbers in the range 1–10.

Sample #	1	2	3	4	5
Mean	5.68	4.9	5.42	5.9	5.54
Median	6	4	5	6	5

(a) What is the mean of all 250 values?

(b) What can you say about the overall median for the 250 values?

(c) What sort of values are possible for the median in this case?

5 The following table gives some statistics related to five different random samples. Each sample is a set of 80 whole numbers in the range 1–20.

Sample #	1	2	3	4	5
Mean	10.7625	11.8625	10.8	11.55	10.7625

(a) What is the mean of all 400 values?

(b) If the 400 values perfectly matched the predicted distribution of values, what value would the mean have?

(c) What seems slightly unusual about these five random samples?

Another set of five samples give the following statistics.

Sample #	1	2	3	4	5
Mean	11.25	10.9125	10.525	10.3875	10.6375

(d) What is the overall mean for this set of 400 values?

(e) How does this set of samples compare with the first set of samples?

6 Complete the following statements by circling the correct words or filling in the blanks.

(a) Each random sample taken from a large population is likely to have the same/different values for the mean and the median.

(b) It is/is not possible to calculate the expected value for both the mean and the median of a sample.

(c) As the sample size increases, it is expected that the sample mean and sample median will be closer to/be further away from/stay the same as the expected values.

(d) For a data set that contains the whole number values 1–30, the expected value for both the mean and the median is _____.

(e) For a data set that contains the whole number values 1–n, the expected value for both the mean and the median is _____.

SC 2 I can explain the effect of individual values, including outliers, on statistical values

1 Consider the following data set

2 10 10 11 11 12 12 12 12 13

13 13 14 14 15 15 16 16 17 20

(a) Calculate the mean for the full data set.

(b) Calculate the median for the full data set.

(c) Calculate the range for the full data set.

(d) List any values that could be considered outliers in this data set.

(e) Calculate the mean for the data set without the outlier(s). State your answer correct to 2 decimal places.

(f) Calculate the median for the data set without the outlier(s).

(g) Calculate the range for the data set without the outlier(s).

(h) Comment on your results.

2 Consider the following data set.

Score (x)	Frequency (f)
1	6
3	5
5	4
6	4
7	5
10	6
20	1

(a) Calculate the mean for the full data set. State your answer correct to 2 decimal places.

(b) Calculate the median for the full data set.

(c) Calculate the range for the full data set.

(d) List any values that could be considered outliers in this data set.

(e) Calculate the mean for the data set without the outlier(s). State your answer correct to 2 decimal places.

(f) Calculate the median for the data set without the outlier(s).

(g) Calculate the range for the data set without the outlier(s).

(h) Comment on your results.

3 Consider the following data set.

15	20	21	18	17	14	13	15
21	18	17	2	21	20	15	14
16	19	41	21	19	20	18	15

(a) List the data in order, from smallest to largest.

(b) Calculate the mean, median and range of the full data set. If necessary, state answers correct to 2 decimal places.

(c) List any values that could be considered outliers in this data set.

(d) Calculate the mean, median and range of the data set without the outlier(s). If necessary, state answers correct to 2 decimal places.

(e) Comment on your results.

4 Consider the following data set.

Score (x)	Frequency (f)
1	2
13	5
14	6
16	7
17	3
19	4
20	2

(a) Calculate the mean, median and range of the full data set. If necessary, state answers correct to 2 decimal places.

(b) List any values that could be considered outliers in this data set.

(c) Calculate the mean, median and range of the data set without the outlier(s). If necessary, state answers correct to 2 decimal places.

(d) Comment on your results.

5 Consider the following data set.

1	3	5	4	6	2	8	5	5	7
2	4	21	5	7	9	23	6	2	8
7	6	2	1	4	8	7	6	4	5

(a) List the data set in order, from smallest to largest.

(b) Calculate the mean, median and range of the full data set.

(c) List any values that could be considered outliers in this data set.

(d) Calculate the mean, median and range of the data set without the outlier(s). If necessary, state answers correct to 2 decimal places.

(e) Comment on your results.

6 For each of the following data sets, calculate the mean and the median. If necessary, round your answers correct to 2 decimal places.

(a)	1	1	2	3	4	5	6	6	6	7	8	15
(b)	1	1	2	3	4	5	6	6	6	7	8	32
(c)	1	1	2	3	4	5	6	6	6	7	8	55

(d) What do you notice from these calculations?

7 You can see from your answers to the previous questions that the range is most affected by outliers. For this reason, statisticians prefer to use a different measure of spread that is not usually affected by outliers. This statistic is the interquartile range.

Work through the following steps to understand how this statistic is calculated.

(a) Copy this list of values and draw a circle around the median.

1	1	2	3	4	4	4	4	5	5
6	6	7	7	7	7	8	8	9	

(b) Now find the median of the lower half of the data set. This value is called the lower quartile.

(c) Now find the median of the upper half part of the data set. This value is called the upper quartile.

(d) Now find the interquartile range (IQR) using IQR = upper quartile – lower quartile

8 If a data set has an even number of data values, the median is midway between two of the values. When calculating the quartiles, the left-hand of these values is part of the lower half of the data, and the right-hand value is part of the upper half of the data.

Use this information to calculate the mean, median, range and interquartile range for each of the following data sets. If necessary, round your answers correct to 2 decimal places.

(a)	2	2	3	4	4	5	6	7	8	9	9	16
(b)	2	2	3	4	4	5	6	7	8	9	9	36
(c)	2	2	3	4	4	5	6	7	8	9	9	56

(d) Complete the following to make a true statement:
As the outlier increases, the mean and the range both decrease/increase/remain the same while the median and the interquartile range decrease/increase/remain the same.

SC 3 I can conduct a simulation using a digital tool

1 A certain brand of cereal runs a promotion where one of 8 small superhero figures is included in each box. To simulate this, Erica produced the following list of random numbers.

3	4	7	1	2	4	7	8	5	6
2	4	1	1	8	4	6	3	3	3
6	8	2	5	5	2	6	6	5	4
1	5	1	7	4	5	6	2	3	5
5	8	3	2	6	1	1	3	6	4
4	8	5	8	8	5	2	4	6	2
1	4	8	8	8	5	6	1	7	1
5	4	5	1	8	6	2	6	6	8
6	8	7	4	5	6	1	5	5	2
3	1	6	7	5	7	7	3	6	7

(a) What needs to occur in the number list to show that one of every figure has been collected?

(b) How many boxes need to be purchased to collect all 8 figures for the first time?

(c) Start the count again after the last cereal box purchased. How many boxes need to be purchased to collect all 8 figures again?

(d) In total, how many times can the collection of a full set of 8 figures be simulated by this table of values?

(e) What is the mean number of boxes of cereal that must be purchased to complete the collection?

2 A box contains 3 green marbles and 7 blue marbles. Use the following table of random values to simulate choosing marbles, with replacement, from the box.

1	5	7	4	6
10	9	10	1	9
2	10	9	1	10
8	4	6	10	10
5	10	9	3	6
4	9	4	1	5
6	8	4	2	2
4	6	4	1	9
7	5	7	4	2
2	3	3	5	2

(a) Let the digits 1–3 represent green marbles and the digits 4–10 represent blue marbles. How many blue marbles appear in the first 5 draws? (This is the top row of values.)

(b) How many blue marbles appear in the first 50 draws?

3 William is an archer. He has a 60% chance of hitting the bullseye from a distance of 50 metres.

(a) Explain how you could use a list of random numbers generated using technology to simulate William's success with 5 arrows from 50 metres.

Use the following list of random numbers to answer the following questions.

9	8	6	6	5
4	8	7	6	7
7	2	5	2	9
3	3	6	2	9
5	10	5	9	5
3	6	2	1	7
3	5	9	2	4
4	10	10	3	9
3	4	7	8	9
5	6	7	7	6

(b) Using the numbers 1–6 to represent a bullseye, state the number of 'hits' in William's first set of 5 arrows.

(c) Based on this simulation, what is William's mean number of hits per set of 5 arrows?

(d) If you produced another set of random numbers, do you think the results would be the same?

4 A normal pack of playing cards consists of 52 different cards. There are four different suits: Hearts (H), Diamonds (D), Clubs (C) and Spades (S).

In each suit, there are 13 cards: Ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q) and King (K).

Each card is allocated a number from 1–52, as shown in the following table.

Number range	Card allocation
1–13	Ace H, 2 H, 3 H, ..., JH, QH, KH
14–26	Ace D, 2 D, 3 D, ..., JD, QD, KD
27–39	Ace C, 2 C, 3 C, ..., JC, QC, KC
40–52	Ace S, 2 S, 3 S, ..., JS, QS, KS

(a) What cards are in a hand represented by the numbers 13, 28, 19, 39, 50?

Use this set of random numbers to answer the following questions.

37	23	30	28	8	9	47	46	4	31
24	49	6	46	21	22	20	5	30	18
41	43	33	25	17	50	20	45	29	48
14	52	40	21	51	20	1	24	22	27
52	4	32	1	45	7	49	42	51	14
5	4	26	27	51	12	25	41	4	3
17	36	52	52	4	1	2	21	50	1
39	27	47	45	35	1	47	35	32	36
25	21	39	23	17	5	45	7	4	48
18	25	41	40	36	51	26	34	43	7

(b) 10 cards are selected at random, with replacement, from the pack. How many cards are there of each suit? Use the first row of the table to answer this question.

(c) Of the 100 cards represented in the table of random numbers, how many are there of each suit?

(d) Do you think this table of values is a reasonable simulation of an actual pack of cards? Explain your answer in a sentence or two.

16.4 Plan and conduct a statistical investigation

Learning intention: To be able to plan and conduct a statistical investigation

Success criteria:

SC 1 I can recognise ethical and fair methods in an investigation.

SC 2 I can draw conclusions from the data collected.

SC 3 I can explain why sampling is necessary in many situations, yet still allows for reliable inferences about a population.

Lesson warm-up

Light the lighthouse!

The lighthouse light has gone out!

You need to find the lighthouse so you can turn it back on before it gets dark.

The lighthouse is located on a desk in the classroom. As a class, you have 3 minutes to write five survey questions. The answers to the questions should help you to identify which desk the lighthouse is on.

No questions will be answered until the 3 minutes are up.

Once you have collected the answers to the five questions, you will have one more minute to draw a conclusion from your survey and locate the lighthouse.

Questions cannot involve any student information and the teacher can only answer yes or no to each question. Good luck!

SC 1 I can recognise ethical and fair methods in an investigation

Researchers must conduct investigations in an ethical manner. This means, for example, that they cannot trick people into participating or conduct an investigation in a way that might cause harm to participants.

In the medical industry, new drugs are often tested on animals before humans. This is to ensure that the new drug does not have any serious side effects and that the animal responds to the drug as expected. The new drug is then tested on humans, with some participants given the real drug and others given a placebo (a fake medicine that does not do anything), to test if the two groups of participants have a statistically different outcome to each other.

Researchers must also think carefully about the methods they use. For example, the sampling method chosen can lead to bias in an investigation. If the participants are chosen in a way that means they are likely to share the same opinion, then the conclusions drawn from the analysis of the data are likely to be invalid.

Writing survey questions is also a difficult task. Even official, professionally written surveys sometimes contain poorly worded questions that will not lead to valid conclusions.

Worked example

Analysing a survey method

RediData is a market research company that conducts surveys on behalf of its clients. Its survey method involves recruiting people via the internet to become paid survey takers. The payment is very small, but an individual could complete several surveys every day.

(a) Do you think this sample method would produce a random sample of the community? Give a reason for your answer.

THINKING	WORKING
Some questions to think about include: Are all age groups likely to be equally represented? Are people from non-metropolitan areas likely to be equally represented? What is the target population for the research?	If the target population is the whole Australian community, then this method is unlikely to produce valid results. However, the target population may be much narrower and more likely to be represented by internet users who have the free time to complete such surveys.

(b) Do you think the results obtained using these surveys would be valid? Give a reason for your answer.

THINKING	WORKING
A valid survey is one where the results obtained give a true reflection of the target population. Is this likely to be the case here?	It depends on the target population. The companies who employ RediData to complete these surveys must have some confidence that the data collected will be of use to them.

(c) Most surveys include a question about the age of the person taking the survey. Why do you think this is the case?

THINKING	WORKING
What relevance does age have on these surveys?	Age is one of the most common strata used when sampling. If the first question in a survey asks people about their age, the researchers can filter the participants, only providing the rest of the survey to people who are in the target age range.

SC 2 I can draw conclusions from the data collected

Once the data have been collected, it is time to carry out statistical analysis before drawing any conclusions. The data that need to be analysed may be in raw form, in table form or in graphical form.

Worked example

Analysing data in a table

The following table was published in Bureau of Infrastructure and Transport Research Economics (BITRE), 2023, *Road Trauma Australia 2022 statistical summary*, BITRE, Canberra ACT. It shows the number of deaths on roads in each jurisdiction, from 2013 to 2022.

Table 1.1 Deaths by jurisdiction

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT	Australia
2013	333	243	271	98	161	36	37	7	1,186
2014	307	248	223	108	182	33	39	10	1,150
2015	350	252	243	102	161	34	49	15	1,206
2016	380	290	251	86	195	37	45	10	1,294
2017	389	259	247	100	160	32	31	5	1,223
2018	347	213	245	80	159	32	50	9	1,135
2019	353	266	219	114	163	29	36	6	1,186
2020	284	211	278	93	155	38	31	7	1,097
2021	275	231	277	99	166	35	35	11	1,129
2022	293	242	297	71	175	51	47	18	1,194
% change 2021-2022	6.5	4.8	7.2	-28.3	5.4	45.7	34.3	63.6	5.8
Ave. trend change p.a. (%)	-2.0	-1.1	1.6	-2.0	-0.4	2.1	-0.7	2.7	-0.5

Augustus analysed the data in the table. He concluded that in 2022, ACT was the most dangerous jurisdiction in Australia for drivers, because it showed the highest percentage increase in road deaths, while South Australia was the safest, because it had the greatest percentage decrease.

What do you think about the conclusions Augustus has drawn?

THINKING	WORKING
Is there any statistical backing for the conclusions Augustus has drawn?	The figures quoted by Augustus are correct: ACT showed the highest percentage increase in road deaths and South Australia showed the greatest percentage decrease.

Are the conclusions reasonable, and based on the whole body of statistics presented?	It is not certain that the conclusions are correct. Augustus would need to provide more detail about what he means by 'most dangerous' in this context. Also, the numbers for ACT are so low that any change is a significant percentage change.
What other information would be useful?	The number of road deaths per 10 000 people would be useful. This would provide a 'rate' of road deaths for each jurisdiction, which would be more appropriate as a source for comparison.

SC 3 I can explain why sampling is necessary in many situations, yet still allows for reliable inferences about a population

If you survey three people and find that two of them support higher taxes, you would be unwise to infer (draw the conclusion) that two-thirds of the population support higher taxes. However, if you survey 1000 people and find that 667 of them support higher taxes, then the inference that two-thirds of the population support higher taxes is more likely to be correct.

Collecting a sample is quick, and, if the sample is chosen carefully, it is usually a reliable way of predicting population attributes. The size of the sample required depends on the following factors:

- the size of the population
- the level of confidence you require (usually 95%)
- the margin of error in the mean (usually set at 3%).

For example, in the run-up to an election, opinion polls are produced almost weekly. Each poll uses a random sample of the population.

A researcher might survey a sample of the population about their voting intentions. If the researcher reports that Party A is leading Party B 52% to 48%, this means the pollster is 95% confident that the vote for Party A will be (52 ± 3)% and the vote for Party B will be (48 ± 3)%. In other words, the vote for Party A will be between 49% and 55%, and the vote for Party B will be between 45% and 51%.

If Party B wins 51% to 49%, the pollster can still claim that their survey conclusions were accurate.

The following table shows the sample size required for various combinations of confidence level, margin of error and population size. You will see there are some interesting results.

	Confidence level: 95% Margin of error			**Confidence level: 99% Margin of error**		
Population size	**5%**	**2.5%**	**1%**	**5%**	**2.5%**	**1%**
100	80	94	99	87	96	99
1000	278	606	906	399	727	943
10 000	370	1332	4899	622	2098	6239
100 000	383	1513	8762	659	2585	14 227
1 000 000	384	1534	9512	663	2647	16 317

For a large population, the sample size does not increase much as the population size increases.

The greater the level of confidence and the smaller the margin of error you need, the larger the sample size required.

Before a federal election, the major polling companies run many nationwide polls. These typically have around 1600 respondents and the results are reported with a 3% margin of error.

Worked example

Choosing a sample size

A sample is to be chosen from a population of 900. The researcher requires a 95% confidence level and a 2.5% margin of error. What size should the sample be? Use the table above to answer this question.

THINKING	WORKING
Choose the correct column of the table based on the confidence level and margin of error. Then look for the nearest value for the population.	The closest population size is 1000. The sample size for the stated confidence level and margin of error is 606.
Make an estimate based on the actual size of the population.	For a population size of 100 with the same confidence level and margin of error, the sample size is 94. Assuming a linear progression, the sample size increases by approximately 50 for each 100. So, for 900 the sample size should be approximately $606 - 1 \times 50 = 556$. Since this is an estimate, a sample size around 550–560 would seem appropriate.

Practice

ANSWERS Page 609

SC 1 I can recognise ethical and fair methods in an investigation

1 A new drug is being tested. The researchers find that more than 60% of the trial subjects show improvement in their condition and 35% of the trial subjects show no difference in their condition. Among the remaining 5%, some subjects report serious side-effects. The researchers decide to ignore this 5%, reasoning that they are outliers.

(a) Explain whether you think this approach is ethical or not.

(b) What problems could arise for the researchers as a result of this decision?

(c) What course of action would you recommend?

2 A researcher was testing a new drug to see whether it reduced the time taken to recover from the common cold. They planned to test 300 people in the first stage of the research. However, they found that 45 of the first 50 test subjects recovered in less than the commonly accepted time. Based on these results, the researcher stopped testing and the company started marketing the drug.

(a) Was it an ethical decision to stop the trial?

(b) Explain why the trial should have been continued.

3 The Australian Government is considering a new tax. A market research firm has produced two different questions related to this proposed tax.

Question 1: Do you support the introduction of a new tax that will cost you about $110 per year?

Question 2: Do you support the introduction of a new tax that will allow the government to allocate more money to the provision of social housing?

(a) What differences would you expect to find in the survey results for these two questions?

(b) Which question is sympathetic to the introduction of the new tax?

(c) Do you think either of these questions is unethical?

(d) Do you think both questions would be included in the same survey?

4 In its annual report, XYZ Manufacturing reports that the average income of its workers is more than $95 000 per year.

Elsewhere in the report, the following table shows the actual income values for the employees.

Employee type	Number	Annual income
Management	3	$300 000
Middle management	10	$175 000
Assembly workers	40	$60 000

(a) Is the claim about the average income correct?

(b) Is the claim in the report justified?

(c) Can you write a more ethical statement about 'average' income?

5 Researchers hope that 70% of people treated by a new drug will be cured.

The following table of random numbers simulates a drug trial.

Assign the digits 0–6 to represent 'cure' and 7–9 to represent 'no cure'.

6	0	4	3	9	2	8	6	2	9
1	7	5	1	6	8	3	7	6	4
5	8	1	1	5	4	0	9	8	5
5	1	5	2	7	6	7	1	1	4
4	2	8	8	8	8	3	3	1	1
9	2	6	1	2	8	4	4	9	1
9	1	3	1	7	9	0	2	7	3
7	1	7	1	6	7	0	3	2	8
3	5	2	4	7	8	4	3	6	8
0	1	8	6	5	9	3	8	7	9

(a) For the 100 people represented in this table, what is the cure rate?

(b) The researcher decided to not use the 10 subjects from Row 5 and the 10 subjects from Row 10, arguing that 80 was a big enough sample. Why do you think the researcher chose to exclude these two rows?

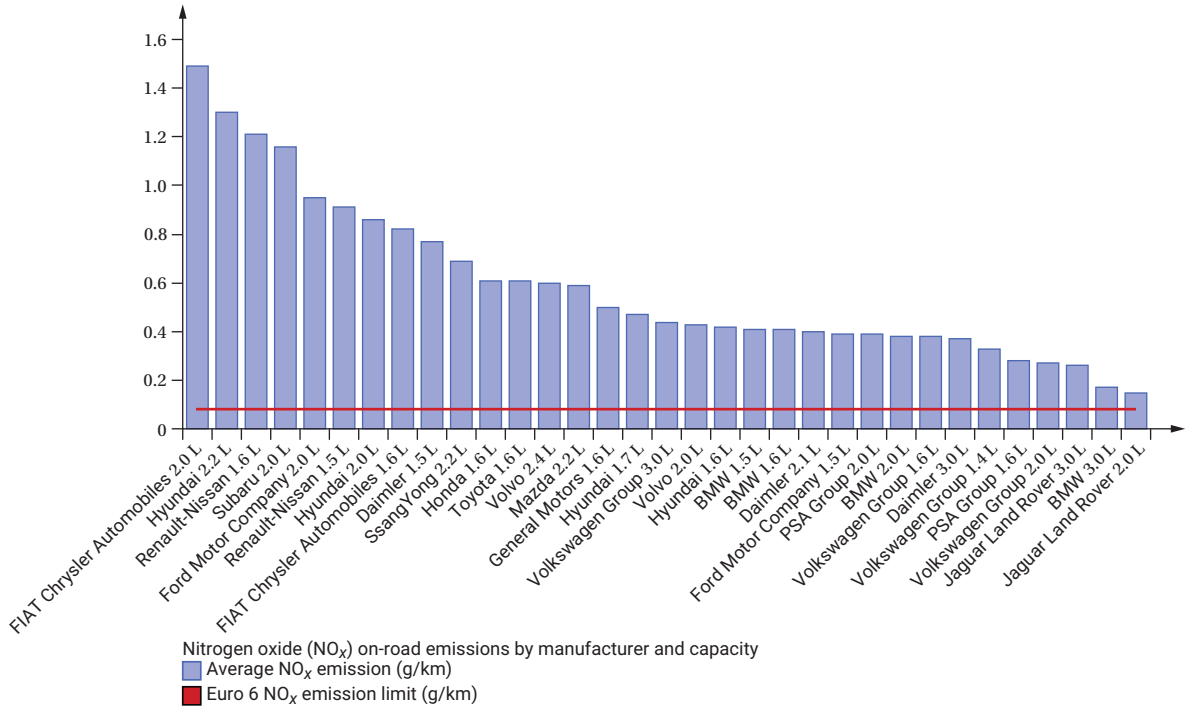
(c) Re-calculate the success rate for the trial, excluding rows 5 and 10.

(d) Were the researcher's actions ethical?

6 Many countries have legislation to limit the fuel emissions that cars are allowed to produce. In 2015, the Volkswagen group was sued by the United States Environmental Protection Agency for falsifying the results of emissions tests for their diesel-fuelled vehicles. The EPA found that the vehicle manufacturer had installed software that could recognise when a vehicle was being tested. During tests, various emission control systems were turned on, allowing the vehicles to pass the tests. Under normal running conditions, however, these controls were disabled. As a result, the real emissions of the vehicles were much higher than the company claimed.

It has since been found that other car manufacturers have been manipulating emissions tests in the same way. This scandal, sometimes known as 'Dieselgate', is still ongoing.

The following table shows the actual test results for some vehicles that were supposed to meet emissions requirements.



(a) Why do you think the companies manipulated the test results in this way?

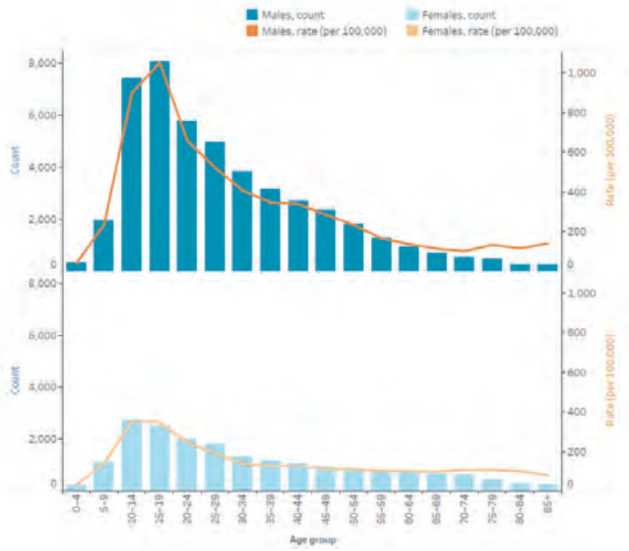
(b) Was this an ethical approach to marketing?

(c) The scandal has cost the Volkswagen group more than $33 billion dollars and some executives have been jailed. What do you think this tells you about the profits to be made in the automobile business?

(d) What does the table tell you about the scope of the scandal?

SC 2 I can draw conclusions from the data collected

1 Consider the following graph, which shows sports injury hospitalisations, by age and sex.



Based on data from © Commonwealth of Australia, Australian Institute of Health and Welfare.

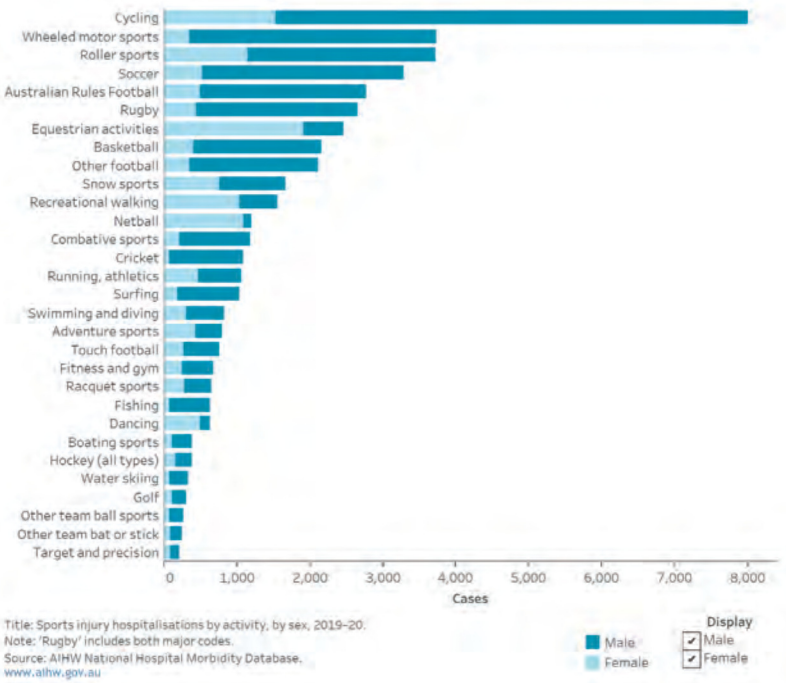
(a) What appears to be the most dangerous age group for males?

(b) What appears to be the most dangerous age group for females?

(c) There is an increase in the rate per 100 000 people for males in the age group 85+. Suggest what this indicates.

(d) Why do you think the values are so much lower for females than for males?

2 Consider the following graph, which shows data about sports injury hospitalisations, by activity and sex.



Based on data from © Commonwealth of Australia, Australian Institute of Health and Welfare.

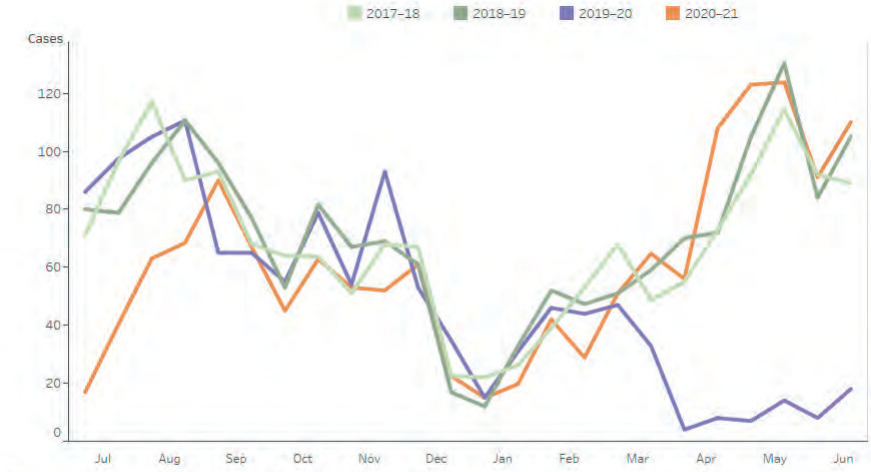
(a) Which sporting activity results in the most hospitalisations for males?

(b) Which sporting activity results in the most hospitalisations for females?

(c) In which activities are more females hospitalised than males?

(d) Approximately how many males were hospitalised for injuries sustained while cycling?

3 Consider the following graph, which shows data about netball injury hospitalisations.



Title: Netball injury hospitalisations, by month of admission, 2017–18 to 2020–21.
Notes
1. Admission counts have been standardised into two 15-day periods per month.
2. A scale up factor was applied to the data for June admissions to account for cases not yet separated. See the data tables for details.
Source: AIHW National Hospital Morbidity Database.
www.aihw.gov.au

Based on data from © Commonwealth of Australia, Australian Institute of Health and Welfare.

(a) What appears unusual about the figures for the later part of 2019–2020 and the first part of 2021–2021? Why do you think this is the case?

(b) Which months seem to be the worst for hospitalisations from netball? Why do you think this is the case?

(c) Approximately how many cases per month is the worst value seen in this graph?

4 The following table was published in June 2023.

Hitting the plastic

Total credit card spending (year ending)		Average credit card balance	Total number of credit card accounts
April 2020	$333.47 billion	$2893	14.35 million
April 2021	$306.61 billion	$2870	13.37 million
April 2022	$332.27 billion	$2862	13.15 million
April 2023	$394.05 billion	$2988	13.36 million

(a) If you multiply the average credit card balance by the total number of credit card accounts for April 2020, the value obtained is $41 514.55 million. Write this number in billions of dollars.

(b) Why does this number not match the number in the 'Total credit card spending' column for that year?

(c) What has happened to the number of credit card accounts during the period shown by this data?

(d) What has happened to the average credit card balance during the period shown by this data?

(e) The first line of the accompanying report reads: 'Annual spending on credit cards is almost 20% higher than at the same time last year as households resort to short-term borrowing to make ends meet amid Australia's cost of living crunch'. Is the figure quoted correct? What is the exact figure, given the rounding in the table?

(f) The first line of the report suggests that the figures are a cause for concern. Do you agree with this assessment?

5 The following table was published in the media in June 2023.

Domestic cats and injuries to wildlife

	Australia	Sydney	Melbourne
Pet cat population	5 333 000	1 081 324	1 027 045
Roaming pet cat population	2 953 415	767 740	729 202
Animals killed per year*	428 481 506	111 383 675	105 792 632
Native animals killed†	253 403 041	65 872 066	62 565 535
Animals killed per day	1 173 922	305 161	289 843
Native animals killed per day	694 255	180 471	171 412

*Annual death toll for vertebrates. †Annual death toll for native vertebrates.

Source: Based on data from ANU Professor Sarah Legge in: Laura Chung, The staggering death toll of roaming pet cats finally revealed, *The Age*, 9 June 2023.

(a) What is your first reaction when reading these figures?

(b) Do you think these figures have been compiled using a census approach or a sampling approach?

(c) What percentage, correct to 2 decimal places, of the pet cats in Australia is listed as being able to roam?

(d) What percentage of the pet cats are listed as being able to roam in:

(i) Sydney **(ii)** Melbourne?

(e) What do these figures suggest?

(f) Using the data for Australia, how many animals per day are killed by each roaming domestic cat? Does this figure seem reasonable to you?

SC 3 I can explain why sampling is necessary in many situations, yet still allows for reliable inferences about a population

1 A sample is to be chosen from a population of 10 000. Due to time and financial constraints, the sample can be no larger than 500. Use the table provided on page 475 to answer the following questions.

(a) What confidence level and margin of error will be provided by a sample no larger than 500?

(b) The calculated sample mean is 221. Within what range would you expect to find other sample means from this population?

2 A sample mean of 11.5 is recorded at a 95% confidence level and a margin of error of 5%.

(a) Within what range would you expect 95% of the sample means to fall?

(b) A sample mean of 12.2 is recorded. Should this sample be excluded from further analysis?

3 The following table shows a selection of random numbers in the range 1–10. The mean has been calculated for each subset of 10 values.

3	4	2	2	4	3	3	10	2	3	Mean:	3.6
3	8	7	2	3	6	4	10	3	9	Mean:	5.5
7	7	2	10	10	7	4	2	8	2	Mean:	5.9
10	1	10	10	8	7	7	9	4	10	Mean:	7.6
6	10	1	5	2	5	6	8	8	8	Mean:	5.9
9	7	4	4	3	7	10	1	8	1	Mean:	5.4
4	10	3	8	9	10	4	8	7	9	Mean:	7.2
4	1	2	10	10	7	9	2	6	7	Mean:	5.8
8	2	6	9	8	9	4	5	3	7	Mean:	6.1
5	6	2	3	7	6	10	2	6	10	Mean:	5.7
3	9	5	5	3	5	1	8	1	7	Mean:	4.7
3	1	7	2	3	2	3	7	5	7	Mean:	4
8	5	4	3	4	10	7	9	1	5	Mean:	5.6
6	10	10	2	8	2	2	1	8	8	Mean:	5.7
7	2	1	3	2	7	7	3	10	10	Mean:	5.2
10	6	6	10	6	3	1	2	2	1	Mean:	4.7
7	10	6	3	2	10	10	7	2	3	Mean:	6
8	10	4	7	4	1	7	8	1	6	Mean:	5.6
5	6	6	1	6	6	10	6	7	4	Mean:	5.7
3	7	4	9	7	8	6	2	5	5	Mean:	5.6
9	1	10	1	1	4	6	9	4	8	Mean:	5.3
8	3	8	7	5	1	8	2	1	5	Mean:	4.8
4	3	4	7	5	3	9	4	1	3	Mean:	4.3
5	3	3	4	10	3	10	4	5	6	Mean:	5.3
6	3	9	4	1	10	8	5	3	3	Mean:	5.2
2	3	4	8	2	6	9	5	9	7	Mean:	5.5
6	8	9	7	8	9	6	2	1	4	Mean:	6
3	10	3	4	3	7	6	9	1	7	Mean:	5.3
1	1	1	1	4	10	10	4	9	5	Mean:	4.6
7	9	1	9	8	10	3	10	9	7	Mean:	7.3
9	5	2	8	7	5	7	9	5	2	Mean:	5.9
10	9	9	5	10	2	7	7	7	5	Mean:	7.1
6	2	7	3	6	2	1	8	8	6	Mean:	4.9
4	5	2	3	9	4	8	5	6	10	Mean:	5.6
2	3	8	2	8	6	3	10	6	8	Mean:	5.6
4	7	7	2	7	3	6	1	10	2	Mean:	4.9
4	9	3	1	2	3	4	6	6	6	Mean:	4.4
8	1	8	7	4	9	4	3	4	4	Mean:	5.2
7	5	10	6	9	6	2	4	5	9	Mean:	6.3
7	3	10	4	5	2	10	2	10	4	Mean:	5.7
7	1	5	1	3	5	1	2	2	4	Mean:	3.1
2	9	2	8	4	4	4	4	10	4	Mean:	5.1
9	7	9	10	8	10	5	8	8	8	Mean:	8.2
											5.513953

(a) What is the theoretical mean value for numbers in the range 1–10?

(b) Complete the following frequency table for the mean values for each line of 10 values.

Mean	Frequency
3.0–<3.5	
3.5–<4.0	
4.0–<4.5	
4.5–<5.0	
5.0–<5.5	
5.5–<6.0	
6.0–<6.5	
6.5–<7.0	
7.0–<7.5	
7.5–<8.0	
8.0–<8.5	

(c) The mean for the 430 values shown is 5.513 95 ..., which rounds to 5.5. What is significant about this value?

(d) What is the median for this data set (the row means)?

(e) What is the mode for this data set (the row means)?

(f) Make a comment about how well, or otherwise, this data set matches what is expected.

4 Complete the following statements by circling the correct words.

(a) The smaller the sample, the more/less likely it is to give a true indication of the population.

(b) The size of the sample needed to produce results at a certain level of confidence and within a specified margin of error rises/does not rise in proportion with the population.

(c) Sampling is sometimes/never necessary due to the nature of the population.

(d) When a sensible sample is taken from a population, the conclusions drawn will/will not be valid for the population.

TOPIC REVIEW

Multiple choice

ANSWERS Page 633

1 Which one of the following is an example of a census?

- **A** All students in Years 9–12 are surveyed to see who they would vote for school captain.
- **B** 10 students from all classes are surveyed to see who they would vote for school captain.
- **C** All students in all classes are surveyed to see who they would vote for school captain.
- **D** The first 100 students who arrive to school are surveyed to see who they would vote for school captain.

2 Which of the following is an example of a sample?

- **A** You survey all students enrolled at a university.
- **B** You survey all customers who have used the services of an accountant.
- **C** You survey all students from Year 7 at your school.
- **D** You survey all teachers at your school.

3 A restaurant that seats 200 people is changing its menu and has decided to survey its customers for their opinions. Which one of the following is an example of a random sample?

- **A** All diners are surveyed over one month.
- **B** Diners who arrive between 6 pm and 7 pm are surveyed over one month.
- **C** Numbers are put on the bottom of chairs. All diners are surveyed over one month, and a random number generator is used to choose 50 diners to be included in the survey.
- **D** The first 50 diners who arrive for lunch each day are surveyed over one month.

4 A research topic relates to how much a person is spending on groceries each week. Which of the following is an example of a biased sample?

- **A** A sample is taken from an inner-city apartment complex of 200 one-bedroom apartments.
- **B** A sample is taken from the parents at a high school.
- **C** A sample is taken from all employees and students who were on campus at a university on a Monday.
- **D** All the above.

5 Which of the following is most likely to be a non-random data set?

- **A** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- **B** 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9
- **C** 0, 0, 0, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9
- **D** 0, 0, 1, 3, 4, 4, 5, 6, 7, 9

6 Your school has 100 students spread across six classes in years 7–12. You have been asked to put together a school council of 6 members to represent the school. Which of the following would represent a stratified sample?

- **A** You stand at the tuck shop and ask students who approach the tuck shop if they would like to be included.
- **B** You give all students a number from 1 to 100 and use a random number generator to select 6 students.

C You select the first 6 students who nominate themselves.

D One student from each of the 6 classes is nominated by their class.

7 A librarian wants to do a survey in their lunch break, so they decide to conduct the survey between 12:00 and 12:30 pm outside the building. What sampling technique is the survey-taker using?

A Random sampling **B** Convenience sampling

C Stratified sampling **D** Non-random sampling

8 Which of the following could be classified as purposeful sampling?

A A survey on the number of cars in each household

B A survey on the quality of the food at a restaurant

C A survey on people with red hair at your school

D A survey on all people who have a mobile phone

9 Your research relates to the amount of rain Sydney receives in the month of March. Which of the following would be referred to as primary data?

A You place a rain gauge in your garden and count the amount of rain it gathers each day in March.

B Your brother places a rain gauge in his garden and counts the amount of rain it gathers each day in March and gives you the results.

C You go onto the Bureau of Meteorology website and look up how much rain fell in March for the past 5 years.

D You watch the weather report on the television and record the rain stated each day for the month of March.

10 Which of the following is known as discrete numerical data?

A The data represents students' eye colour in class 3A.

B The data represents the heights of all students in class 3A.

C The data represents the results of a maths test in class 3A.

D The data represents the number of children in each family in class 3A.

11 If 5 random samples of 10 numbers each are taken from a large population, the mean of each sample is most likely to be:

A different to each other **B** within 0.5 as each other

C 5 **D** same as each other

12 The following data represents the arm span in metres of all 10 students in a class. What is the mean arm span of the class to 2 decimal places?

1.2, 1.25, 1.05, 1.22, 1.21, 1.24, 1.2, 1.09, 1.13, 1.08

A 1.20 m **B** 1.19 m **C** 1.18 m **D** 1.17 m

13 Which of the following represent measures of central tendency?

A Mean and mode **B** Mean and median **C** Mean and range **D** Median and range

14 Which of the following data sets includes an outlier?

A 10, 11, 12, 13, 14, 15, 16, 38 **B** 0, 0, 2, 2, 3, 3, 5, 5

C 21, 22, 24, 25, 26, 26, 27, 27 **D** 3, 3, 3, 4, 4, 6, 6, 7

15 Which of the following could represent an unethical investigation in a medical investigation trialling a new drug?

A The researcher discloses all tests, components being used and known risks to the participants.

B The participants are all male and between the ages of 24 and 39.

C Not all risks have been disclosed to the participants.

D The participants are all aged between 24 and 39.

16 When data is being analysed it must be in which format?

A Raw format **B** Table format **C** Graphical format **D** Any of the above

Short answer

1 Explain the difference between a census and a sample, giving an example of each.

2 Sandy is conducting a survey on who is wearing a football jumper at an AFL match. She positions herself at the front entrance to the oval.

(a) Is this a census or sample approach?

(b) If Sandy counts the first 100 people to enter the oval, would this data collection method be random or non-random?

3 Jerome is conducting a survey on how many people talk on their phones while walking. He positions himself opposite a walking track for 5 hours and counts the number of people he sees walking while talking on their phone. What type of data-gathering process is Jerome using in this instance? Explain your answer.

4 Ganna is conducting a survey on how many people use taxis. They have positioned themselves at a taxi rank outside a busy metropolitan train station. Does this data gathering method have the opportunity to be biased? Explain your answer.

5 State a similarity and a difference between a random sample and a purposeful sample.

6 The table below represents the number of students which attend four different schools A, B, C and D. If you need a total random sample of 60 students representing the four schools, how many students will come from school C?

School	Number of students
A	1590
B	710
C	600
D	1100

7 Miriam has completed a survey using convenience sampling. List a benefit and a limitation of this data-gathering technique.

8 Explain the difference between primary and secondary data and give an example of each.

9 What is categorical data? Give two different examples.

10 What are the two subtypes of numerical data? Give an example of each.

11 Explain the following terms:

(a) Mean **(b)** Median **(c)** Range

12 The following table shows the summary statistics for five different random samples of size 50 for whole numbers in the range 1–10.

Sample #	1	2	3	4	5
Mean	5.68	4.9	5.42	5.9	5.54
Median	6	4	5	6	5

(a) Calculate the overall mean for the total 250 values.

(b) Comment on what the overall mean for the whole population could be.

13 Explain the term outlier and explain how it can affect the range of your data set.

14 Which of the following methods would be most effective in producing a list of random numbers which could then be used to simulate an actual experiment?

(a) Rolling a die 100 times

(b) Tossing a coin 100 times

(c) Using a spreadsheet program such as Excel

(d) Giving all students a number and selecting the numbers of those students who have blue eyes.

15 Explain the difference between an ethical and a biased investigation.

Extended response

1 A university is gathering information to see if the staff would like a working from home arrangement, where they can work 2 days at home and spend 3 days in the office per week. There are 2340 staff across three campuses and a sample is to be collected. Each staff member is invited by email to complete an online survey on a certain date and a random sample generator will be used to select a number of surveys from which a decision will be made.

(a) Is the university conducting a census or a sample in its data collection method?

(b) Make two suggestions which would improve the quality of the data being gathered.

(c) When deciding how big your sample should be, list three factors that would influence your decision.

(d) What level of confidence is usually required to allow for a reliable inference about a population?

2 The federal government is conducting market research to see how the public would respond if a fast (bullet) train was introduced on some routes. Travel time could be reduced by over 50%. They send survey takers to stand outside the 5 main train stations for one week in both Sydney and Melbourne and ask 100 people per day to complete the survey.

(a) What type of sampling technique is being used?

(b) List two limitations that could arise from implementing this sampling technique.

(c) List two benefits that could arise from implementing this sampling technique.

(d) Suggest how you could improve the quality of the data being gathered.

3 The following data set represents the number of people living in 20 different houses:

2, 3, 4, 5, 6, 5, 1, 2, 1, 5, 6, 7, 4, 7, 12, 2, 1, 4, 6, 5

(a) Calculate the mean for the full data set correct to 2 decimal places.

(b) Calculate the median for the full data set correct to 2 decimal places.

(c) Calculate the range for the full data set.

(d) List if any values could be considered an outlier.

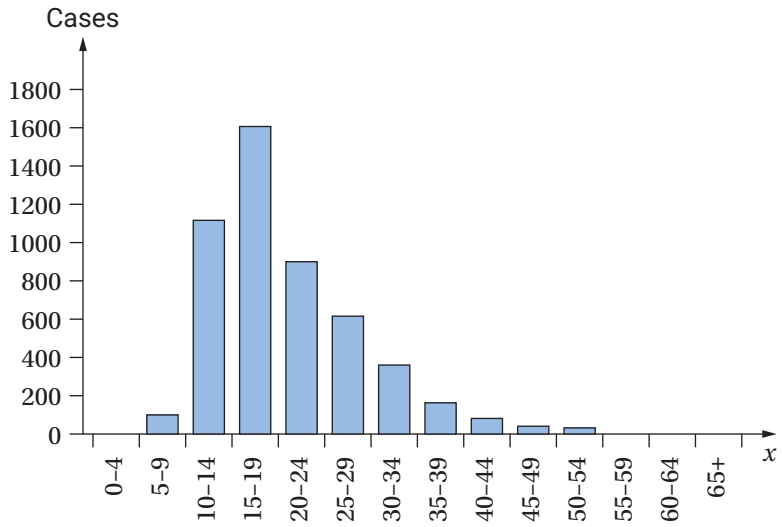
(e) If this outlier is removed, what is your mean value?

(f) If this outlier is removed, what is your median value?

(g) State if the data collected is categorical or numerical. Explain your answer.

4 Consider the following bar chart.

Age distribution of rugby injury hospitalisations 2020–2021



Based on data from © Australian Institute of Health and Welfare.

(a) What age category has the highest number of injuries? Why do you think this is the case?

(b) Why are injuries so low in the 50–54 age category?

(c) If there is a total of 5050 hospitalisations in the 2020–2021 period, what percentage of injuries does the 25–29 age group represent? Give your answer to the nearest round number.

(d) If the average age of a professional rugby player is 27 years, why are the number of injuries in this age category so much lower than for the 15–19 age category?

TOPIC
17

Probability (Venn diagrams and two-way tables)

Recall 490

17.1 Understand complementary events 491

17.2 Draw and understand Venn diagrams 499

17.3 Draw and understand two-way tables 511

17.4 Recognise and understand mutually exclusive and non-mutually exclusive events 522

17.5 Use Venn diagrams and two-way tables to solve problems 531

Topic review 539

Why learn this?

Learning about complementary events and displaying data using Venn diagrams or two-way tables leads to an understanding of how to classify and group data in a meaningful way.

These skills are not only useful in subjects like mathematics, but also in everyday life when making choices based on chances. Knowing these basics prepares students for more complex studies in statistics and data analysis, which can be useful across many professions including risk analysis and safety.

RECALL

ANSWERS Page 611

I can write fractions in their simplest form

1 Simplify these fractions.

(a) $\frac{4}{10}$ **(b)** $\frac{25}{100}$ **(c)** $\frac{25}{75}$ **(d)** $\frac{15}{50}$

I can subtract fractions from whole numbers

1 Calculate these subtractions.

(a) $1-\frac{2}{5}$ **(b)** $1-\frac{1}{9}$ **(c)** $2-\frac{3}{7}$ **(d)** $2-\frac{5}{8}$

I can write fractions in their equivalent decimal form

1 Write these fractions as equivalent decimals.

(a) $\frac{55}{100}$ **(b)** $\frac{8}{40}$ **(c)** $\frac{36}{90}$ **(d)** $\frac{141}{188}$

I can list the sample space for all possible outcomes of an event

1 List the sample space for each of the following cases.

(a) A standard coin is tossed.

(b) A marble is drawn from a bag containing red, blue and green marbles.

I can identify elements that are common to two different sets

1 The even numbers from 1 to 20 are divided into two sets:

- numbers divisible by 4
- numbers divisible by 2.

Write the values that could be placed into both sets.

I can compare probabilities

1 For each set of probability values below, state which value corresponds to the greatest probability.

(a) 0.3, 0.1, 0.7, 0.6 **(b)** $\frac{7}{12}, \frac{5}{6}, \frac{2}{3}, \frac{1}{4}, \frac{5}{8}$

I can write probabilities

1 A standard six-sided die is rolled. Write the following probabilities as fractions in simplest form.

(a) Rolling a 5 **(b)** Rolling a 1 or a 4 **(c)** Rolling an even number

(d) Rolling a multiple of 3 **(e)** Rolling a 0

Understand complementary events

17.1

Learning intention: To understand complementary events

Success criteria:

SC 1 I can identify events and complementary events.

SC 2 I can solve problems by knowing that the probabilities of an event and its complement add to 1.

SC 3 I can use digital tools to conduct probability simulations to determine if long-run events are complementary.

Lesson warm-up

How many matches?

128 players start the Australian Open tournament.

How many matches are played before the winner is crowned?

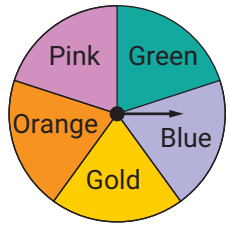
SC 1 I can identify events and complementary events

Term	Definition
Probability	The chance of an event occurring The probability of an event is given by a numerical value between 0 and 1. Probability is a scale, ranging from 0 being impossible to 1 being certain. The probability of an event occurring can be found using the formula: $\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
Event	An event is the result of a probability experiment.
Complementary events	Events are complementary when there are only two outcomes and if, and only if, when one event occurs the other cannot. For example, flipping a head on a coin and flipping a tail are complementary events.
Complement	The complement of A is written as A'. $\Pr(A + A') = 1$

Worked example

Identifying a complementary event

The spinner shown is spun once. For each pair of events shown, determine whether the events are *complementary.*



(a) Spinning blue and pink

THINKING	WORKING
Recall the definition of complementary events.	Events are complementary when: • the outcomes of the two events are the only possible outcomes • the two events cannot happen at the same time.
Check the given pair against the criteria given.	Spinning blue or spinning pink are not the only possible outcomes. You can also land on green, orange or gold.
Write the answer.	Spinning pink and spinning blue are not complementary events.

(b) Spinning blue and not-blue

THINKING	WORKING
Recall the definition of complementary events.	Events are complementary when: • the outcomes of the two events are the only possible outcomes • the two events cannot happen at the same time.
Check the given pair against the criteria for complementary events.	Spinning blue and not spinning blue are the only possible outcomes. Not spinning blue would mean that the spinner lands on green, orange, pink or gold. The two events cannot happen at the same time, so the result after one spin is either blue or not blue.
Write the answer.	Spinning blue and not spinning blue are complementary events.

SC 2 I can solve problems by knowing that the probabilities of an event and its complement add to 1

The sum of the probabilities of an event and its complement is always equal to 1.

For example, when rolling a 6-sided die, the probability of the event rolling a 5 is one out of six, or $\frac{1}{6}$.

The probability of the complementary event, not rolling a 5, is the probability of all remaining outcomes not including the probability of rolling a 5, which can be calculated by $1-\frac{1}{6}=\frac{5}{6}$.

Worked example

Calculating the probability of complementary events

Consider the following situations and calculate the probabilities using the fact that the sum of the probabilities of an event and its complement is always equal to 1.

(a) A coin is flipped 5 times. What is the probability of flipping *at least one* head?

THINKING	WORKING
Recognise a complementary event.	The complement of flipping at least one head is flipping no (zero) heads. Pr(≥ 1 head) and Pr(0 heads) are complementary events because Pr(≥ 1 head) + Pr(0 heads) = 1
Calculate the total number of outcomes.	For each flip of the coin, there are two possible outcomes, so the total number of outcomes would be $2 \times 2 \times 2 \times 2 \times 2 = 32$
Determine the probability of the complementary event.	There is only one way to obtain 0 heads after 5 flips of a coin, which would be the outcome $TTTTT$. The probability of this one outcome out of the possible 32 is $\frac{1}{32}$.
Determine the probability using the sum of an event and its complement.	Since the sum of the probabilities of an event and its complement are always equal to 1, this means: Pr(≥ 1 head) + Pr($TTTTT$) = 1 Because $\Pr(TTTTT)=\frac{1}{32}$, $\Pr(\geq 1 \text{ head})+\frac{1}{32}=1$ $\Pr(\geq 1 \text{ head})=1-\frac{1}{32}$ $=\frac{31}{32}$
Write the answer.	The probability of flipping at least one head is $\frac{31}{32}$.

(b) Jane has a 50-50 chance of scoring every time she shoots a free throw in basketball. After the siren she has two free throws and her team need her to score with *at least one* of them to win the game. What is the probability of Jane's team winning?

THINKING	WORKING
List the sample space.	S = score M = miss The sample space for Jane's two throws is: $\{SS, SM, MS, MM\}$ Scoring and missing are equally likely.
Determine the total number of outcomes and the number of favourable outcomes.	Jane needs at least one scoring shot. From the sample space, $\Pr(\text{miss both shots}) = \frac{1}{4}$. The probability of at least one scoring shot is the probability of all outcomes, except missing both shots: $\Pr(\text{at least one scoring shot})$ $= 1 - \Pr(\text{miss both shots})$ $= 1 - \frac{1}{4}$ $= \frac{3}{4}$
Write the answer.	The probability of Jane getting at least one shot and her team winning is $\frac{3}{4}$.

17

SC 3 I can use digital tools to conduct probability simulations to determine if long-run events are complementary

A simulation is an experiment that is used to estimate the probability of a real-world event.

For very large numbers of trials, it is easier to run these simulations using technology.

Given the probability of an outcome, we can simulate outcomes and occurrences of events over the long term.

Worked example

Making observations from simulation data

After 1000 trials of a probability experiment using a spinner with two colours, these results were obtained.

Colour	Frequency
Yellow	248
Blue	752

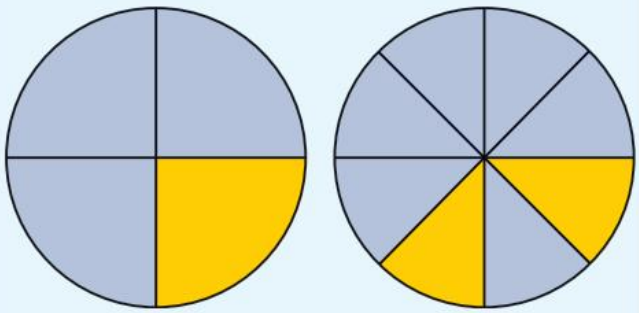
(a) Are the events complementary?

THINKING	WORKING
Are there only two possible outcomes?	Yes: yellow or blue.
Does the total frequency add to the number of trials?	Yes. 248 + 752 = 1000 This confirms that only one of the events can occur at a time and that no other event is possible.
Write your conclusion.	If the answer to both questions is yes, then the events are complementary. The events are complementary.

(b) Estimate the fraction of the area covered by each colour.

THINKING	WORKING
Write each event as a relative frequency.	Round if necessary: $\frac{248}{1000} \approx \frac{250}{1000} = \frac{1}{4}$ $\frac{752}{1000} \approx \frac{750}{1000} = \frac{3}{4}$
Write the answer.	From the simulation, the probability of yellow being spun is roughly $\frac{1}{4}$ of the simulated outcomes, while blue corresponds to roughly $\frac{3}{4}$ of the outcomes.

(c) Draw two different spinners that could have been used for this simulation.

THINKING	WORKING
Draw a spinner corresponding to the simulated probabilities.	The spinners should reflect the probabilities found in the simulation. Draw a spinner where yellow occupies $\frac{1}{4}$ of the area and blue occupies $\frac{3}{4}$ of the area. For example: 

Practice

ANSWERS Page 611

SC 1 I can identify events and complementary events

1 Describe the complementary event of:

(a) flipping heads on a coin

(b) rolling an odd number on a die

(c) rolling a 4 on a die

(d) spinning red on a spinner.

2 A single 6-sided die is rolled. Identify the following pairs of events as complementary or not.

(a) Rolling a 5 or a 6

(b) Rolling an odd number or an even number

(c) Rolling a 3 or not a 3

(d) Rolling a number less than 5 or a 6

(e) Rolling a multiple of 3 or an even number

3 A single 6-sided die with the numbers written in words rather than numerals is rolled. Identify the following pairs of events as complementary or not.

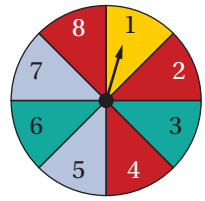
(a) A 'five' or a 'six'

(b) A number containing the letter e or one not containing the letter e

(c) A number containing the letter o or being a number greater than 4

(d) A number containing the letter o or a number containing the letter e

4 The following spinner is to be spun once:



Which of these pairs of events are complementary?

(a) A yellow or a green

(b) A red or an even number

(c) An even number or multiple of 3

(d) An odd number or an even number

(e) A blue or not a blue

5 The letters of the word 'complementary' are placed in a bag and one is drawn out. Identify the following pairs of events as complementary or not.

(a) A consonant or a vowel **(b)** A consonant or a p **(c)** A p or not a p

(d) A letter that is repeated or a letter that is not repeated

SC 2 I can solve problems by knowing that the probabilities of an event and its complement add to 1

1 Calculate the following.

(a) The probability of rolling a 5 is $\frac{1}{6}$. What is the probability of *not* rolling a 5?

(b) The probability of rolling a 4 is $\frac{1}{6}$. What is the probability of *not* rolling a 4?

(c) The probability of flipping a head is $\frac{1}{2}$. What is the probability of *not* flipping a head?

(d) The probability of flipping at least 3 heads in 4 flips is $\frac{5}{16}$. What is the probability of flipping *fewer than* 3 heads?

2 Calculate the following.

(a) The probability of selecting a red card from a standard deck of playing cards is $\frac{1}{2}$. What is the probability of selecting a black card?

(b) The probability of choosing a club from a standard deck of playing cards is $\frac{1}{4}$. What is the probability of *not* selecting a club?

(c) The probability of selecting a King from a standard deck of playing cards is $\frac{1}{13}$. What is the probability of *not* selecting a King?

(d) The probability of selecting a picture card (Jack, Queen, King) from a standard deck of playing cards is $\frac{3}{13}$. What is the probability of not selecting a picture card?

3 The sample space for flipping 3 coins is:
$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(a) Calculate the probability of flipping *exactly* 2 heads.

(b) Calculate the probability of flipping *at least* 2 heads.

(c) Use your knowledge of complementary events to calculate the probability of flipping fewer than 2 heads.

4 When given a penalty kick in soccer, Jai has a 50-50 chance of scoring. In a single game, Jai is awarded 2 penalty kicks. What is the probability of Jai scoring at least once?

5 When given a penalty kick in soccer, Jac has a 50-50 chance of scoring. In a single game, Jac is awarded 3 penalty kicks. What is the probability of Jac scoring at least once?

6 Albi is going to flip a fair coin 5 times. What advice would you give him on calculating the probability of flipping at least one tail?

SC 3 I can use digital tools to conduct probability simulations to determine if long-run events are complementary

1 After 1000 trials of a probability experiment using a spinner with two colours, these results were obtained.

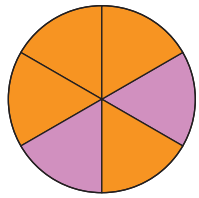
Colour	Frequency
Red	330
Blue	670

(a) Are the events complementary?

(b) Estimate the fraction of the area covered by each colour.

(c) Draw two different spinners that could have been used for this simulation.

2 1000 trials of a probability experiment using the spinner below is to be run.



(a) Determine the fraction of the area covered by each colour.

(b) Estimate how many outcomes of each colour will be spun in 1000 trials.

(c) Draw a different spinner which will produce similar results.

3 After 1000 trials of a probability experiment flipping a 6-sided die, these results were obtained.

Number	Frequency
1	130
2	170
3	180
4	120
5	210
6	190
odd	520
even	480

(a) State the pair of complementary events.

(b) Explain why the frequency adds to more than 1000.

4 An experiment is to be run drawing random numbers from 1 to 10.

(a) What is the expected number of times the number will be divisible by 5, given that an experiment with 50 trials is run?

(b) Are the events *drawing a number divisible by 5* and *not drawing a number divisible by 5* complementary? Discuss.

(c) Use the random number button on your calculator to run 50 trials of this simulation. Draw up the following table and record your results in it.

	Tally	Frequency
Divisible by 5		
Not divisible by 5		
	Total	

(d) Did the results of your simulation support your answers to parts **(a)** and **(b)**?

Draw and understand Venn diagrams

17.2

Learning intention: To be able to draw and understand Venn diagrams

Success criteria:

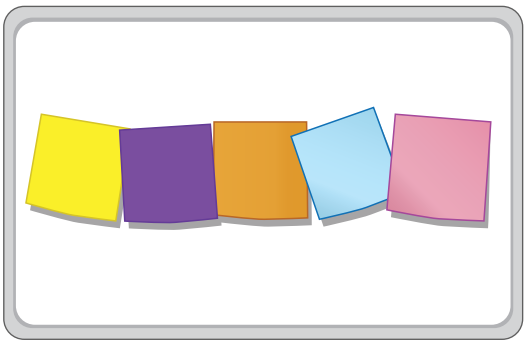
SC 1 I can represent data using set notation and incorporate data into a Venn diagram.

SC 2 I can draw Venn diagrams showing sets, complementary sets, union and intersection of sets and the universal set.

SC 3 I can use given data to calculate probabilities from a Venn diagram.

Lesson warm-up

Post It



Write your name on the sticky note and place it on the board.

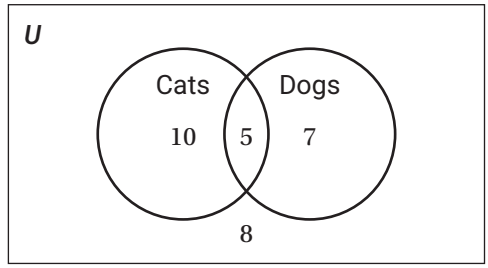
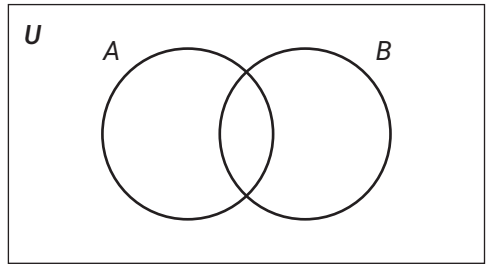
Group together students who are left-handed in one group and those who play organised sport in another group.

What happens if someone is in neither group? What if someone is in both groups?

Try to come to a decision as a class.

SC 1 I can represent data using set notation and incorporate data into a Venn diagram

Term	Definition	Example
Set	A set is a collection of objects or things that share a common property. We use braces around numbers or objects to show they are part of a set and give the set a label (usually a letter) to represent it.	For example, set A of the first four counting numbers could be written $A = \{1, 2, 3, 4\}$.
Element	An element of a set is any one of the objects or things that belong to that set.	For example, in set $A = \{1, 2, 3, 4\}$, the elements are the numbers 1 to 4.

Term	Definition	Example
Venn diagram	A Venn diagram is a useful way of representing relationships between sets of data. Sets could contain numbers, letters, symbols, objects or even other sets! A Venn diagram is usually created using circles to represent the sets. The area inside each circle represents the elements of the set. The area where two or more circles overlap represents the elements that are common to these sets.	An example of a Venn diagram showing how many students have either a dog or cat (or neither) is shown below: 
Universal Set	The universal set (usually labelled U) is a list of all possible elements. In probability, this is the same as the sample space.	The universal set is represented by everything inside the rectangle in the Venn diagram below: 
Cardinal number	The cardinal number, n, of a set is the number of elements in that set.	For example, the cardinal number of set $A = \{1, 2, 3, 4\}$ is $n = 4$.

17

Worked example

Drawing a Venn Diagram

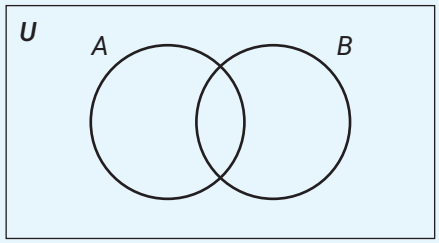
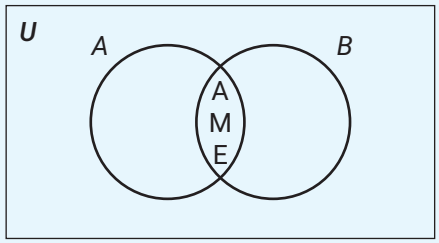
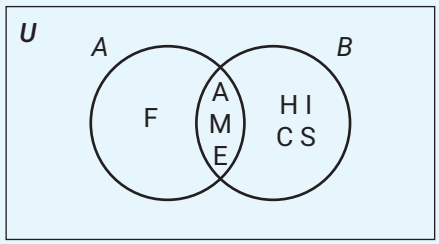
Set A is the letters in the word FAME.

Set B is the letters in the word MATHEMATICS.

(a) List the elements of both sets in set notation.

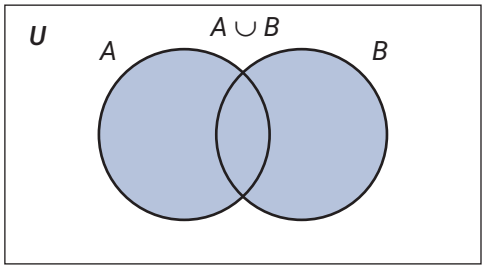
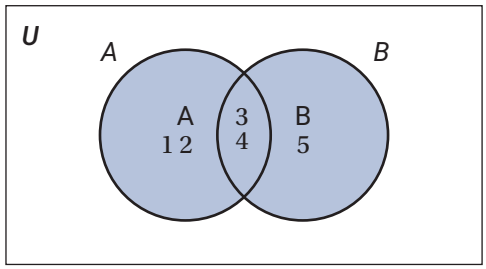
THINKING	WORKING
List the different letters inside curly brackets { }, separating each element of the set by a comma.	Set $A = \{F, A, M, E\}$ Set $B = \{M, A, T, H, E, I, C, S\}$

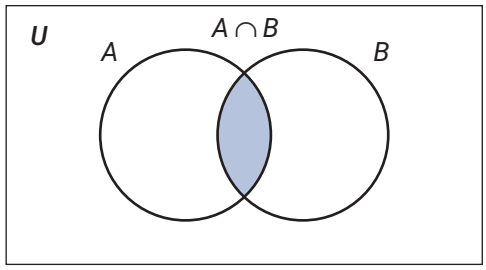
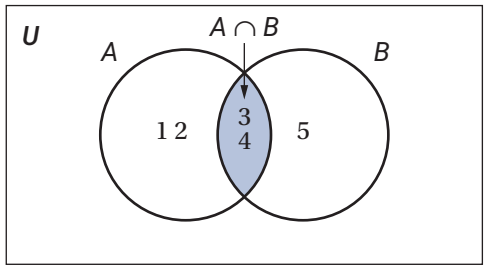
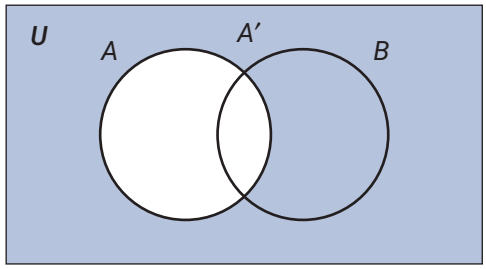
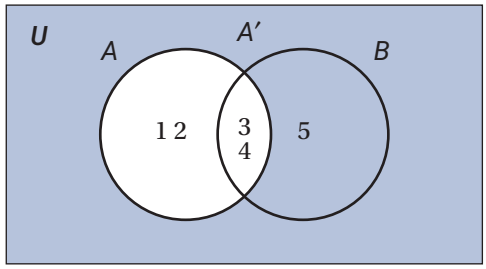
(b) Draw a Venn diagram to show the relationship between sets.

THINKING	WORKING
Draw a circle for each set. Label each circle to represent each of the sets. There are some common elements, so there must be an overlap of the two circles.	
Identify the elements common to both sets. Add these to the overlapping section.	
Add the remaining elements to each circle (so that each set contains all required elements).	

SC 2 I can draw Venn diagrams showing sets, complementary sets, union and intersection of sets and the universal set

The union and intersection of sets can be described visually, using a Venn diagram.

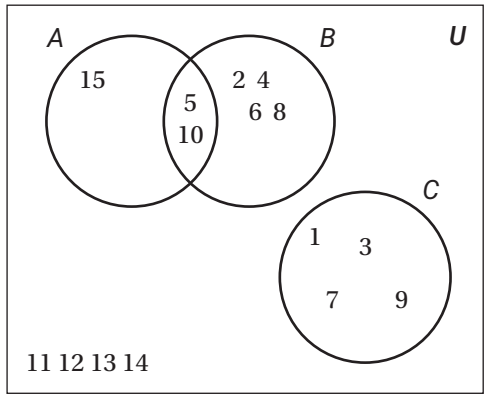
Term	Definition	Example
Union of sets	The union of two sets A and B (written as $A \cup B$) contains all the elements that belong in either A or B, or both sets combined. Any elements that are repeated are recorded only once. In probability, this is when either A or B or both occur, and is represented by the shaded region in the Venn diagram below: 	For example, for two sets $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, the union is $A \cup B = \{1, 2, 3, 4, 5\}$ and corresponds to the shaded region in the following Venn diagram: 

Term	Definition	Example
Intersection of sets	The intersection (written as $A \cap B$) of two sets A and B is the elements that belong to both A and B; that is, the elements both sets have in common. In probability, this is when both A and B occur together, and is represented by the shaded region in the Venn diagram below: 	For example, for the two sets $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, the intersection is $A \cap B = \{3, 4\}$ and is shown in the Venn diagram below: 
Complement of sets	The complement of a set A is all the elements contained in the universal set that don't belong to set A. The complement set to set A is labelled A', and is represented by the shaded region in the Venn diagram below: 	For example, for the two sets A = {1, 2, 3, 4} and B = {3, 4, 5}, A′ is the complement of A and A′ = {5}. 

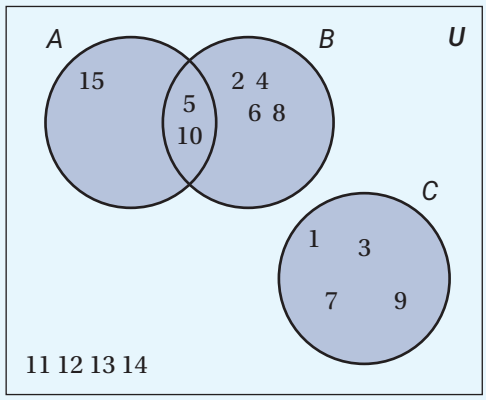
Worked example

Understanding union, intersection and complementary sets

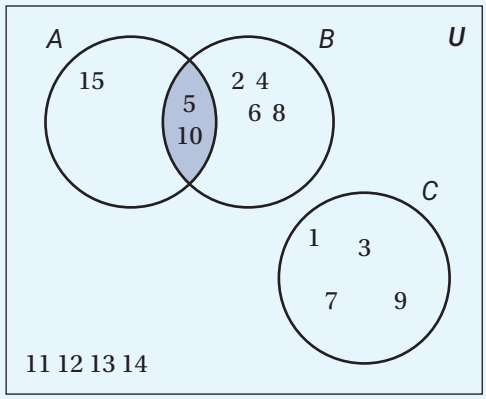
Consider the following Venn diagram.



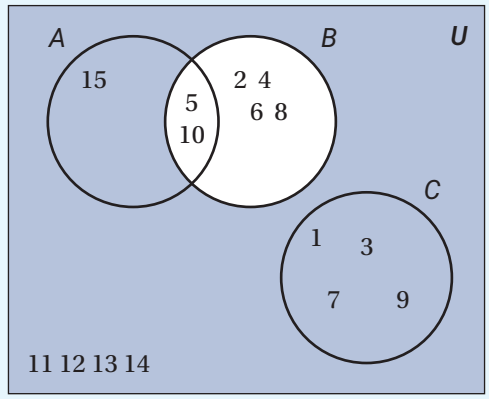
(a) Shade and list the elements in $A \cup B \cup C$.

THINKING	WORKING
Interpret the information given.	$A \cup B \cup C$ represents the union, the set containing all elements that are in each of set A, set B or set C (as well as any overlap).
Shade the set.	
List the elements.	$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15\}$

(b) Shade and list the elements in $A \cap B$.

THINKING	WORKING
Interpret the information given.	$A \cap B$ represents the intersection, the elements that are in both set A and set B.
Shade the set.	
List the elements.	$A \cap B = \{5, 10\}$

(c) Shade and list the elements in B'.

THINKING	WORKING
Interpret the information given.	B' represents the elements not in set B.
Shade the set.	
List the elements.	$B' = \{1, 3, 7, 9, 11, 12, 13, 14, 15\}$

SC 3 I can use given data to calculate probabilities from a Venn diagram

Missing probabilities in a Venn diagram can be found by first adding the given information to the diagram, then using knowledge of the union, intersection and complement of sets to find the remaining values.

Worked example

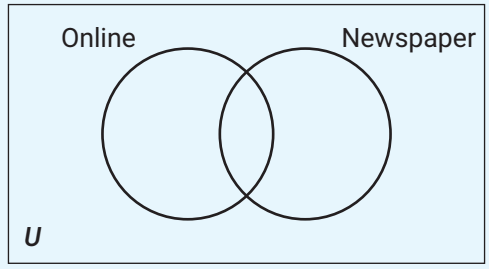
Calculating probabilities from Venn diagrams

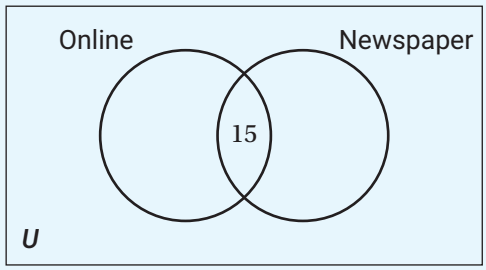
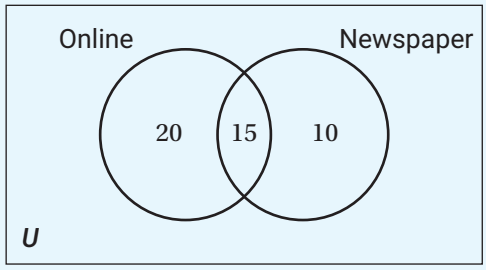
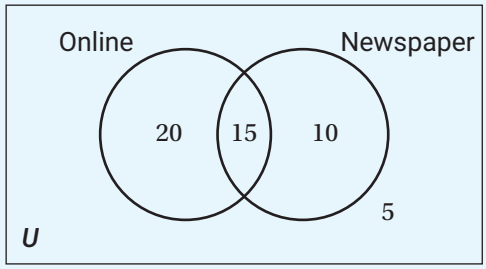
A group of people were surveyed on how they preferred to get their news. The options were 'reading a newspaper' and 'reading online'.

While 35 people were happy to read online, 25 preferred to read a newspaper.

While 15 answered that they were happy with either method, 5 said they did not get their news from a newspaper or online.

(a) Draw a Venn diagram to represent the situation.

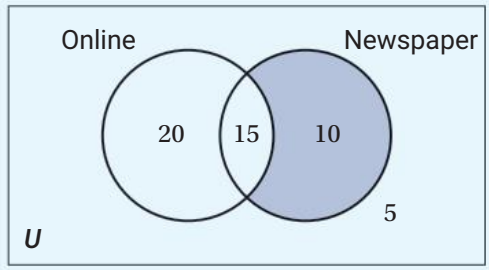
THINKING	WORKING
Construct a Venn diagram with a circle for each event.	

Identify the cardinality or the number in the intersection or overlap of the circles.	
Determine the cardinality for each set by subtracting the intersection value from each total. Add these values to the diagram.	n(online only) = 35 – 15 = 20 n(newspaper only) = 25 – 15 = 10 
Identify how many people chose neither option and add this value outside both circles, but inside the rectangle (the universal set).	

(b) How many people were surveyed?

THINKING	WORKING
Add all the numbers in the Venn diagram for the total.	20 + 15 + 10 + 5 = 50
Write the answer.	There were 50 people surveyed.

(c) Determine the probability that a person chosen at random reads newspapers, but not online news.

THINKING	WORKING
Determine the number of favourable outcomes by identifying the relevant part of the Venn diagram.	The shaded region below shows people who read newspapers, but not online news.  Favourable outcomes: $n(\text{newspaper only}) = 10$
Determine the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{newspaper only}) = \frac{10}{50}$ $= \frac{1}{5}$
Write the answer.	The probability that one of these people chosen at random reads newspapers, but not online news is $\frac{1}{5}$.

Practice

ANSWERS Page 612

SC 1 I can represent data using set notation and incorporate data into a Venn diagram

1 Set A is the letters in the word ROUND.
Set B is the letters in the word DROWN.

(a) List the elements of both sets using set notation.

(b) Draw a Venn diagram to show the relationship between sets.

2 Fun fact: the word 'coroner' originally meant someone sent by the crown (the King or Queen), to collect taxes from his/her subjects.

The coroners were so unpopular they often 'disappeared' during their trips, so the King would need to send out another coroner to investigate what had happened to the original tax collector. Let set A be the letters in the word CROWN and set B be the letters in the word CORONER.

(a) List the elements of both sets using set notation.

(b) Draw a Venn diagram to show the relationship between the sets.

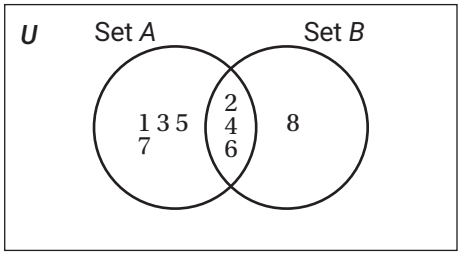
3 Set A is the letters in the word 'anagram'.
Explain why set A is not written as $\{a, n, a, g, r, a, m\}$.

4 Set A is the natural numbers less than 10.
Set B is the multiples of 2 less than 13.

(a) List the elements of both sets using set notation.

(b) Draw a Venn diagram to show the relationship between sets.

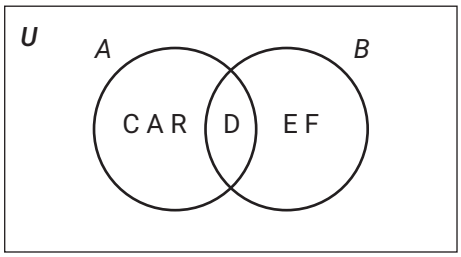
5 Consider the Venn diagram shown.



(a) Write set A in set notation. **(b)** Write set B in set notation.

(c) Describe set A. **(d)** Describe set B.

6 Recall that the cardinal number, n, of a set is the number of elements in that set. For this Venn diagram:



(a) determine the cardinal number of set A, $n(A)$

(b) determine the cardinal number of set B, $n(B)$.

7 Determine the cardinal number of the set of letters in the following words.

(a) Set A is the selection of letters in the word TRAVEL.

(b) Set B is the selection of letters in the word WELCOMING.

(c) Set C is the selection of letters in the word DIDGERIDOO.

8 Construct a Venn diagram of the following situation.
There are 24 students in a class.

- 16 enjoy action movies
- 10 enjoy comedies
- 6 enjoy both

9 In a squad of 15 soccer players, there are:

- 2 goalies
- 8 forwards
- 8 defenders
- 3 who can play as a forward or a defender, but not goalie.

Draw the Venn diagram of this squad.

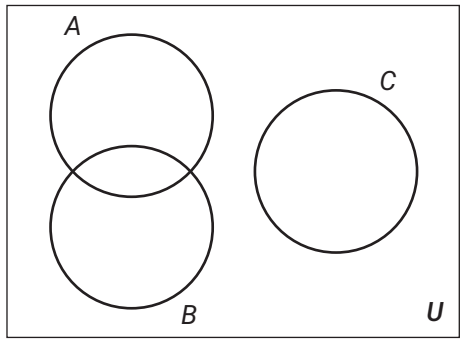
10 Sixteen friends are comparing the elective subjects they have chosen for next year.

- 8 chose art.
- 7 chose PE.
- 9 chose cooking.
- There are 2 students who have chosen all three subjects.
- 5 have chosen both cooking and PE.
- 3 have chosen both cooking and art.
- 2 have chosen both art and PE.

Draw a Venn diagram of this situation.

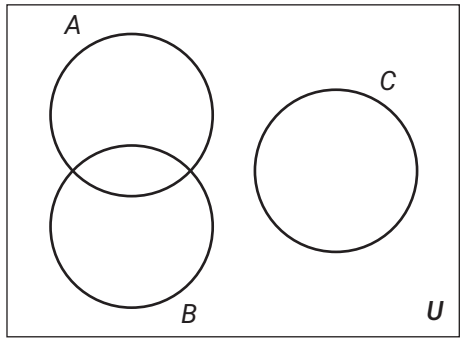
SC 2 I can draw Venn diagrams showing sets, complementary sets, union and intersection of sets and the universal set

1 Copy the Venn diagram shown and shade the given sets.



(a) A **(b)** C **(c)** $A \cup B$ **(d)** $A \cap B$ **(e)** $A \cup B \cup C$

2 Copy the Venn diagram shown and shade the given complements.



(a) A' **(b)** C' **(c)** $A' \cap B'$ **(d)** $C \cup C'$ **(e)** $(A \cup B)'$

3 Set A is the set of positive even numbers.
Set B is the set of positive multiples of 6.
The universal set is natural numbers less than 19.

(a) Draw a Venn diagram to represent this situation.

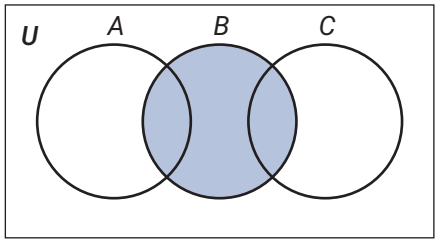
(b) Shade A'.

(c) Determine the cardinal number, $n(A \cap B)$.

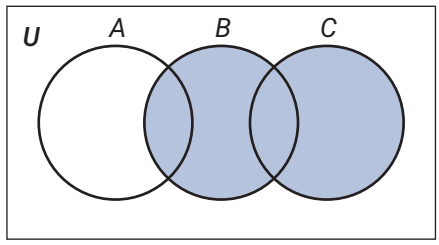
(d) Determine the cardinal number, $n(A)$.

4 Determine the set represented by the shaded region in these Venn diagrams.

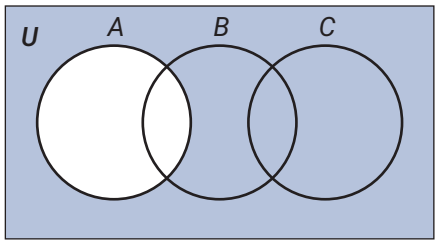
(a)



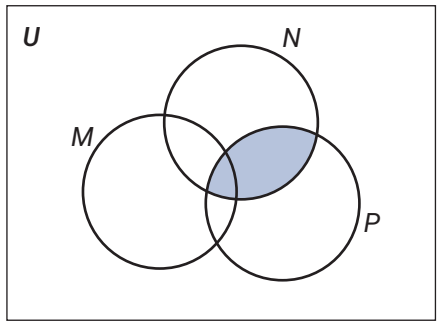
(b)



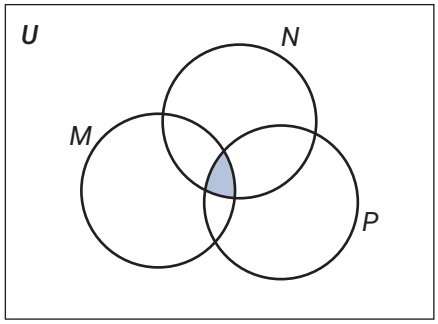
(c)



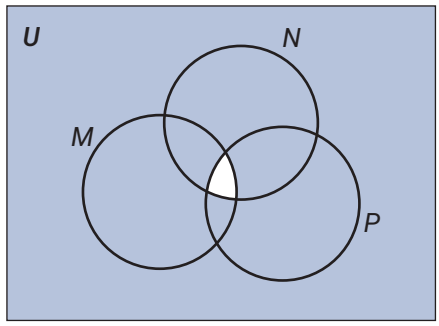
(d)



(e)



(f)



5 Consider the universal set of 52 cards in a standard deck.

Set A is the set of red cards in a standard deck of cards.

Set B is the set of picture cards in a standard deck of cards.

(a) Draw a Venn diagram to represent this situation.

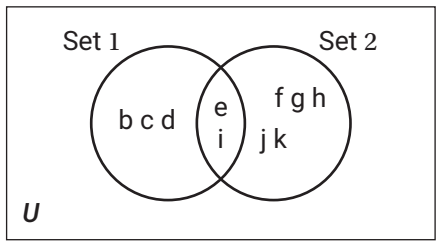
(b) Describe the elements in A'.

(c) Describe the elements in $A \cap B$.

(d) Using set notation, define the section of the Venn diagram where the black 10s belong.

SC 3 I can use given data to calculate probabilities from a Venn diagram

1 Given this Venn diagram, determine the probability that a single letter drawn at random is:



(a) the letter b **(b)** the letter i **(c)** the letter g

(d) from Set 1 **(e)** from Set 2.

2 Ten friends enjoy having coffee together each week. The order for one week was 6 coffees and 5 pieces of cake. Two people ordered both cake and coffee.

(a) Draw the Venn diagram of the situation.

(b) How many, if any, did not have cake or coffee?

(c) One of these people is selected at random. Determine the probability of:

(i) the person having cake only

(ii) the person having coffee only

(iii) the person having both cake and coffee.

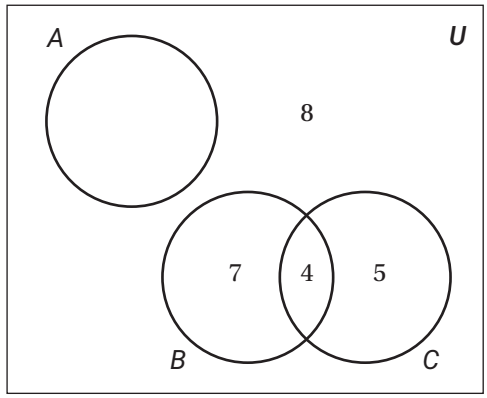
(d) Determine the probability of:

(i) the person having cake also having coffee

(ii) the person having coffee also having cake

(iii) the person having neither cake nor coffee.

3 Thirty people were surveyed and the following partially completed Venn diagram created.



(a) Calculate $n(A)$. Explain how you calculated this.

(b) Determine the probability of a random selected survey response containing the opinion:

(i) A **(ii)** B **(iii)** C **(iv)** B or C.

(c) Determine the probability of a random survey not containing the opinion:

(i) A **(ii)** B **(iii)** C

(iv) B or C **(v)** A, B or C.

4 Ten students take part in the school choir. There are 6 boys, 4 girls, 2 sopranos and 4 basses.

Mr B says there must be some mistake as this adds to more than 10 students. Can you offer some advice to sort out his confusion?

Draw and understand two-way tables

17.3

Learning intention: To be able to draw and understand two-way tables

Success criteria:

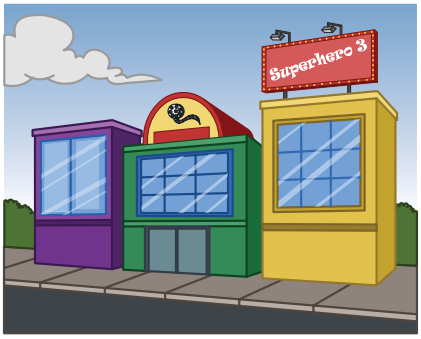
SC 1 I can interpret the data in a two-way table.

SC 2 I can create a two-way table with probabilities.

SC 3 I can use given data to calculate missing probabilities in a two-way table.

Lesson warm-up

Superhero



Sixty people attend a session of this film, with 26 of them sitting near the front of the theatre.

Of the 22 adults, 13 sit near the back.

How many children are sitting at the front?

17

SC 1 I can interpret the data in a two-way table

A two-way table is used to display the **frequencies** of two different categories for the same experiment (how often they occur). One category is displayed as the rows and the other as the columns.

Consider the following example of a two-way table.

The number of tickets sold for one session at a cinema is recorded. The data can be displayed in a two-way table to try to establish patterns in the genre of movie that different age groups prefer.

	Comedy	Action	Total
Child	45	25	70
Adult	28	52	80
Total	73	77	150

Reading *across* the rows shows, for each age group, the number of people who attended each genre of film; while reading *down* the columns shows, for each genre, how many people attended from each age group.

For example:

- reading down the first column: of the 73 people who attended a comedy, 45 of these people were children and 28 were adults
- reading across the first row: of the 70 children at this movie session, 45 of these children went to the comedy movie and 25 went to the action movie
- reading from anywhere in the table: of the 150 people in attendance, 77 people watched an action film.

Each value in a two-way table is contained in a cell. A cell in a table is a rectangular area that is formed by the intersection of a row and a column. Each cell in a table can contain text, numbers, formulas or other types of data.

Probabilities can also be calculated from a two-way table.

If you consider the total of 150 people, the probability of a random person being chosen who is going to the action film is $\Pr(\text{action}) = \frac{77}{150}$

The total you use to calculate the probability (the denominator) can *change*, depending on the group of interest. So, if you wanted to find the probability that, given the person was an adult, they went to a comedy, you would use the total as the number of adults (the denominator) and find out how many of these adults went to a comedy (the numerator):

$\Pr(\text{comedy given they are an adult}) = \frac{28}{80}$

The symbol $|$ can be used to mean *given that*, so this could also be written as:

$\Pr(\text{comedy}|\text{adult}) = \frac{28}{80}$.

Worked example

Calculating probabilities using a two-way table

A survey of golf club members was taken to find their views on increasing the length of the 10th hole of a golf course.

Use the following two-way table to solve the following problems.

Age group	Yes	No	Total
10–30	26	4	30
31–50	46	40	86
> 50	30	64	94
Total	102	108	210

(a) Determine the percentage of the over-50s age group that voted to lengthen the hole.

THINKING	WORKING
Identify the relevant data in the table and establish the largest group you are selecting from (the total).	In this case, we want the over-50s row: Age group | Yes | No | Total > 50 | 30 | 64 | 94 The total will be the number of over-50s, which will be the denominator for the probability calculation. This is a 'given that' probability because we want the probability that given respondents are over 50, they vote 'yes' to lengthen the golf hole.
Determine the number of favourable outcomes.	The number of over-50s who voted yes is 30.
Identify the total.	There are 94 over-50s who voted.
Calculate the probability.	$\Pr(\text{lengthen the hole} \mid \text{over 50}) = \frac{30}{94}$
Convert the probability to a percentage.	$\frac{30}{94} \times 100 = 31.9\%$
Write the answer.	31.9% of the over-50s age group voted to lengthen the golf hole.

(b) Determine the probability that a random member is 50 years of age or younger.

THINKING	WORKING
Identify the relevant data in the table.	50 years of age or younger, is given by the age groups 10–30 and 31–50.
Determine the number of favourable outcomes.	10–30 years old: 30 members 31–50 years old: 86 members 30 + 86 = 116 people who are 50 years old or younger.
Identify the total.	Here, the total is taken as the entire group (210 people).
Calculate the probability.	$\Pr(\text{age} < 50) = \frac{116}{210}$ $= \frac{58}{105}$
Write the answer.	The probability of a random member being less than 50 years old is $\frac{58}{105}$.

SC 2 I can create a two-way table with probabilities

Missing values in two-way tables can be found using known values or totals to fill in the empty cells.

The row, table and column totals is a useful place to start for calculating missing values.

For example, consider the table for a class survey on pet ownership, shown below.

	Dog	Cat	Total
Own	2	4	
Don't own		7	10
Total	5	11	

THINKING	RESULT
The missing value in the first row can be found by adding the two known values to find the total:	<table><tr><th></th><th>Dog</th><th>Cat</th><th>Total</th></tr><tr><td>Own</td><td>2</td><td>4</td><td>2 + 4 = 6</td></tr><tr><td>Don't own</td><td></td><td>7</td><td>10</td></tr><tr><td>Total</td><td>5</td><td>11</td><td></td></tr></table>
The missing value in the second row can be found by looking down the first column, and subtracting the one known value from the total:	<table><tr><th></th><th>Dog</th><th>Cat</th><th>Total</th></tr><tr><td>Own</td><td>2</td><td>4</td><td>6</td></tr><tr><td>Don't own</td><td>5 – 2 = 3</td><td>7</td><td>10</td></tr><tr><td>Total</td><td>5</td><td>11</td><td></td></tr></table>
The table total (bottom right cell) is found by either adding together the values in the last row, or the right column. Using the right column, reading down we have:	<table><tr><th></th><th>Dog</th><th>Cat</th><th>Total</th></tr><tr><td>Own</td><td>2</td><td>4</td><td>6</td></tr><tr><td>Don't own</td><td>3</td><td>7</td><td>10</td></tr><tr><td>Total</td><td>5</td><td>11</td><td>6 + 10 = 16</td></tr></table>

Probabilities and frequencies can then be calculated from the two-way table as normal.

Worked example

Calculating missing frequencies in two-way tables

Mr Smyth asked his maths class whether they prefer to go swimming at the beach or at the pool.

This table shows some of the results.

	Beach	Pool	Total
Year 7		3	
Year 8	6	11	
Total		14	25

(a) Complete the table.

THINKING	WORKING
Use the known values and the column, row or table totals to determine the missing values.	

	Beach	Pool	Total
Year 7	$8 - 3 = 5$	3	$25 - 17 = 8$
Year 8	6	11	$6 + 11 = 17$
Total	$25 - 14 = 11$	14	25

Write the answer.

	Beach	Pool	Total
Year 7	5	3	8
Year 8	6	11	17
Total	11	14	25

(b) Determine the probability that someone who prefers swimming at the beach is in Year 7.

THINKING	WORKING
Identify the relevant total.	Focus on the relevant group. In this case, people who prefer the beach.

	Beach
Year 7	5
Year 8	6
Total	11

There are 11 students who prefer the beach.

Calculate the probability.

Of the 11 students who prefer the beach, 5 of them are Year 7 students.

$$\Pr(\text{Yr7} \mid \text{beach}) = \frac{5}{11}$$

Write the answer.

The probability that someone who prefers swimming at the beach is a Year 7 student is $\frac{5}{11}$.

SC 3 I can use given data to calculate missing probabilities in a two-way table

Missing probabilities in two-way tables can be found using known values or totals to fill in the empty cells, the same way as for frequency two-way tables.

The row, table and column probabilities is a useful place to start to calculate missing probabilities.

Worked example

Calculating missing probabilities in two-way tables

A group of swimmers at the beach were surveyed about whether they liked chocolate or vanilla ice-cream.

This table shows the probability of some of the results:

	Chocolate	Vanilla	Neither	Total
Children	$\frac{8}{25}$	$\frac{3}{25}$		$\frac{11}{25}$
Teens	$\frac{4}{25}$		0	$\frac{6}{25}$
Adults		$\frac{5}{25}$	$\frac{3}{25}$	$\frac{8}{25}$
Total			$\frac{3}{25}$	1

(a) Complete the table.

THINKING

Use the known values to calculate the value of the missing values.

WORKING

	Chocolate	Vanilla	Neither	Total
Children	$\frac{8}{25}$	$\frac{3}{25}$	$\frac{8}{25}+\frac{3}{25}+a=\frac{11}{25}$ $\frac{11}{25}+a=\frac{11}{25}$ $a=0$	$\frac{11}{25}$
Teens	$\frac{4}{25}$	$\frac{4}{25}+b+0=\frac{6}{25}$ $b=\frac{6}{25}-\frac{4}{25}$ $b=\frac{2}{25}$	0	$\frac{6}{25}$
Adults	$c+\frac{5}{25}+\frac{3}{25}=\frac{8}{25}$ $c+\frac{8}{25}=\frac{8}{25}$ $c=0$	$\frac{5}{25}$	$\frac{3}{25}$	$\frac{8}{25}$

Total	$\frac{8}{25}+\frac{4}{25}+c=d$ $\frac{12}{25}+0=d$ $d=\frac{12}{25}$	$\frac{3}{25}+b+\frac{5}{25}=e$ $\frac{8}{25}+\frac{2}{25}=e$ $e=\frac{10}{25}$	$\frac{3}{25}$	1

(b) A selected survey response said that they like vanilla ice-cream. Determine the probability that this response was given by a child.

THINKING	WORKING
Identify the relevant total.	Choose the given column of figures. In this case, $\frac{10}{25}$ of the people surveyed like vanilla ice-cream. **Vanilla** **Children** $\frac{3}{25}$ **Teens** $\frac{2}{25}$ **Adults** $\frac{5}{25}$ **Total** $\frac{10}{25}$
Identify the probability of the favourable outcome.	The probability in this table section of being a child is $\frac{3}{25}$.
Calculate the probability.	The probability that, given the person likes vanilla, they will be a child, is $\Pr(\text{child}\mid\text{vanilla})=\frac{3}{25}\div\frac{10}{25}=\frac{3}{10}$
Write the answer.	The probability of a 'likes vanilla' vote coming from a child is $\frac{3}{10}$.

Practice

ANSWERS Page 614

SC 1 I can interpret the data in a two-way table

1 Students responded to a survey in the following way.

	Like skateboards	Do not like skateboards	Total
Like roller blades	78	27	105
Do not like roller blades	23	22	45
Total	101	49	150

(a) How many students were surveyed?

(b) How many students do not like roller blades?

(c) How many students like skateboards?

(d) How many students who like roller blades also like skateboards?

(e) How many students did not like either roller blades or skateboards?

2 People responded to a survey about their preferred social media, Facebook (FB) or Instagram (IG), in the following way.

	Like IG	Do not like IG	Total
Like FB	41	24	65
Do not like FB	4	16	20
Total	45	40	85

(a) How many were surveyed?

(b) How many students do not like IG?

(c) How many students like FB?

(d) How many students who like FB also like IG?

(e) How many students did not like either FB or IG?

3 Students responded to a survey in the following way.

	Like video games	Do not like video games	Total
Like board games	82	29	111
Do not like board games	19	41	60
Total	101	70	171

What is the probability that a student chosen at random:

(a) likes board games?

(b) likes video games?

(c) does not like either video games or board games?

(d) likes board games or video games, but not both?

4 A group of teenagers responded to a survey about their preferred sport to watch on television, football or tennis, in the following way.

	Like tennis	Do not like tennis	Total
Like football	36	25	61
Do not like football	6	16	22
Total	42	41	83

What is the probability that a student chosen at random:

(a) likes tennis?

(b) likes football?

(c) does not like either tennis or football?

(d) likes tennis or football, but not both?

SC 2 I can create a two-way table with probabilities

1 This partially completed table shows how a class of students responded to a survey about drinks.

	Like juice	Do not like juice	Total
Like cola	78		
Do not like cola	23	22	
Total	101	49	

(a) Complete the table.

(b) How many students were surveyed?

(c) How many students do not like cola?

(d) How many students like cola?

(e) How many students who like cola do not like juice?

(f) How many students do not like either cola or juice?

2 Two hundred Year 12 students were surveyed and responded in the following way.

	Studies science	Does not study science	Total
Studies maths	102	33	145
Does not study maths	34	31	55
Total	136	64	200

(a) What is the probability of a student not studying maths?

(b) What is the probability of a student studying science?

(c) What is the probability of a student studying both science and maths?

3 Students responded to a survey about what genre of movie they would like to watch the next two times they plan to go to the cinema.

		Movie 2		
		Superhero	Cartoon	Total
Movie 1	Superhero	70	30	100
	Cartoon	20	30	50
	Total	90	60	150

(a) Change the frequencies to probabilities. Do not simplify the fractions.

(b) What is the probability of watching two superhero movies?

(c) What is the probability of watching no superhero movies?

(d) What is the probability of watching at least one superhero movie?

4 **(a)** Why is the value in the bottom right-hand corner of a two-way probability table always equal to 1?

(b) Fast Eddie says there is a reason he doesn't simplify the fractions in a two-way probability table. What do you think his reason might be?

SC 3 I can use given data to calculate missing probabilities in a two-way table

1 A group of 40 joggers at the beach were surveyed about whether they liked brand name shoes or generic running shoes. This table shows the probability of some of the results.

	Type of running shoe			
	Brand name	**Generic**	**Both**	**Total**
Distance runners	$\frac{21}{40}$	a	0	$\frac{22}{40}$
Sprinters	b	$\frac{14}{40}$	$\frac{2}{40}$	$\frac{18}{40}$
Total	c	d	e	1

(a) Complete the table of probabilities.

(b) What is the probability of a distance runner liking the generic brand?

(c) What is the probability of a sprinter not liking the generic brand?

(d) What is the probability of a distance runner liking both the brand name and the generic brand?

2 A group of 100 Australians were surveyed about whether they liked Coconut brand sunscreen or Divia brand sunscreen. This table shows the probability of some of the results.

	Type of sunscreen				
	Coconut	**Divia**	**Neither**	**Both**	**Total**
Under 40	$\frac{21}{100}$	a	$\frac{8}{100}$	$\frac{24}{100}$	b
Over 40	$\frac{14}{100}$	$\frac{18}{100}$	c	$\frac{2}{100}$	$\frac{40}{100}$
Total	d	e	f	g	1

(a) Complete the table of probabilities.

(b) What is the probability of a person being both under 40 and liking the Divia brand?

(c) What is the probability of a person being both over 40 and liking neither brand?

(d) What is the probability of a person liking the Divia brand?

(e) Given that a person chosen likes the Coconut brand, what is the probability of them being over 40?

3 At Pearson High School, there is a $\frac{4}{10}$ chance of a student catching the bus to school. There is a $\frac{1}{10}$ chance of a student catching both a bus and a train, and a $\frac{4}{10}$ chance of catching a train, but not a bus. Catching the bus and/or train are the only methods of getting to school.

(a) Construct a completed two-way table using the probability of each event.

(b) What is the probability of a student catching the train?

(c) What is the probability of a student catching the train and not catching the bus?

(d) What is the probability of a student catching the train and the bus?

(e) What is the probability that a student who catches the train also catches the bus?

4 Probabilities can be written as fractions, decimals or percentages. For example, a probability of 45% could also be written as 0.45 or as a fraction $\frac{45}{100}$ or $\frac{9}{20}$.

Complete the following two-way tables and convert the probabilities to fractions.

(a)

	Movie	**Not movie**	**Total**
Dinner		21%	62%
Not dinner	12%		38%
Total		47%	100%

(b)

	Netball	**Not netball**	**Total**
Softball	0.35		0.61
Not softball	0.27		0.39
Total			1

5 In a small town in the 1960s there was an 82% chance of a male teenager playing Australian Rules football during the winter months. There was also a 63% chance of the male teenager playing cricket during the summer. There was a 51% chance of a male teenager playing both football and cricket.

(a) Construct a completed two-way table using the probability of each event.

(b) What is the probability of a teenager not playing football?

(c) What is the probability of a teenager not playing football or cricket?

(d) A teenager who plays football is selected at random. What is the probability that they will also play cricket?

17.4 Recognise and understand mutually exclusive and non-mutually exclusive events

Learning intention: To be able to recognise and understand mutually exclusive and non-mutually exclusive events

Success criteria:

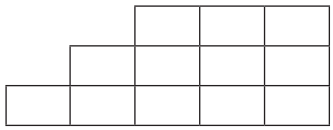
SC 1 I can recognise mutually exclusive and non-mutually exclusive events.

SC 2 I can calculate the probability of mutually exclusive events.

SC 3 I can use Venn diagrams and two-way tables to demonstrate the difference between mutually exclusive and non-mutually exclusive events.

Lesson warm-up

The biggest number



3-digit number
4-digit number
5-digit number

Roll a die 12 times. After each time choose where to place the digit in the table to create a 3-digit, a 4-digit and a 5-digit number.

After all 12 digits are placed, add the numbers together to determine your score. Compare your results with a friend.

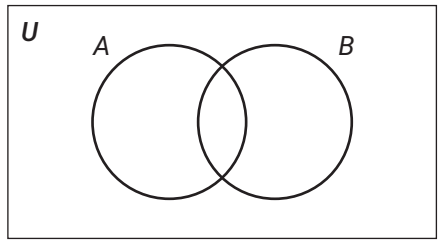
SC 1 I can recognise mutually exclusive events and non-mutually exclusive events

Mutually exclusive events are things that cannot both happen at the same time. For example, rolling a 3 or rolling a 4 on a die are mutually exclusive events – these events cannot both occur on one roll of the die.

This is not the same as complementary events, where if one event does not happen, the other must happen. For example, the complementary event of rolling a 4 on a die is *not* rolling a 4.

With non-mutually exclusive events, there is a chance that *both* events could happen at the same time. There is also a chance that *neither* event may occur.

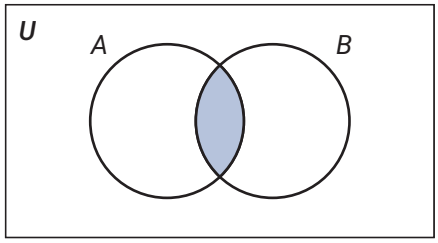
A Venn diagram is a useful way to visualise these situations.



Mutually exclusive events can be represented by a Venn diagram with no intersection (no overlap in the circles).

The probability of either event A or event B happening is the sum of their probabilities:

$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$



Non-mutually exclusive events can be represented by a Venn diagram with an intersection (there *is* an overlap in the circles).

The intersection of two events can also be represented by the symbol ∩. The probability of the intersection of these events A and B would be written as $\Pr(A \cap B)$.

In a two-way table of the frequency of outcomes, you can check whether events are mutually exclusive by looking for a zero in the intersection cell.

The following table shows the number of outcomes for each event (shown along the top of the columns and down the rows on the left). Here the events 'Rolling a 3' and 'Rolling an even number' on a die are mutually exclusive because they cannot occur together.

	Rolling an even number	Not rolling an even number	Total
Rolling a 3	0	1	1
Not rolling a 3	3	2	5
Total	3	3	6

17

Worked example

Identifying mutually exclusive events

For each of the following, determine whether or not the events are mutually exclusive and give your reasons.

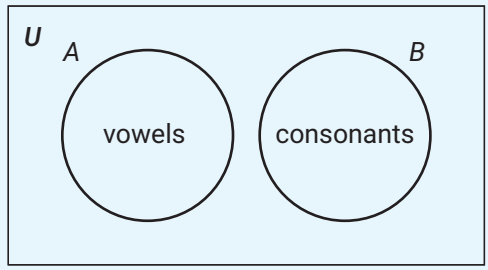
(a) Rolling an even number on a die and rolling a 4 on a die.

THINKING	WORKING
Decide whether the events can occur at the same time.	Can an even number and 4 occur at the same time on a single roll? Since 4 is also an even number, these events can occur together.
Write the answer.	These events are non-mutually exclusive because 4 is also an even number (these events can occur together).

(b) The chance of it raining tomorrow and the temperature being over 25°C.

THINKING	WORKING
Decide whether the events can occur at the same time.	Can it rain and be over 25°C at the same time? Yes.
Write the answer.	These events are non-mutually exclusive because it can rain and be over 25°C at the same time.

(c) A letter chosen at random from a word being a vowel or a consonant.

THINKING	WORKING
Decide whether the events can occur at the same time.	A letter cannot be both a vowel and a consonant. This can be represented by a Venn diagram:  These events are mutually exclusive because there is no intersection.
Write the answer.	These events are mutually exclusive because a letter cannot be both a vowel and a consonant at the same time.

(d) Geelong winning the AFL premiership and Essendon winning the AFL premiership

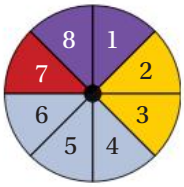
THINKING	WORKING
Decide whether the events can occur at the same time.	Can both teams win the Premiership at the same time? Since only one team can win the Premiership, these events are mutually exclusive.
Write the answer.	Mutually exclusive, because both teams cannot win the Premiership at the same time.

SC 2 I can calculate the probability of mutually exclusive events

If two events are mutually exclusive, the probability of either event happening is the *sum* of the probabilities of each event.

Worked example

Calculating the probability of mutually exclusive events



The following spinner is to be spun once. For each of the following, confirm whether the events are mutually exclusive, and if they are, calculate the probability of the event.

(a) Spinning red or purple

THINKING	WORKING
Recall the definition of mutually exclusive events.	Mutually exclusive events are those which cannot occur at the same time.
Decide whether the events can happen at the same time.	Spinning a red or purple cannot occur at the same time, so the events are mutually exclusive.
Recall the rule to calculate the probability of mutually exclusive events.	$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$
Substitute the values to calculate the probability of the mutually exclusive events.	$\Pr(\text{red or purple}) = \Pr(\text{red}) + \Pr(\text{purple})$ $= \frac{1}{8} + \frac{2}{8}$ $= \frac{3}{8}$
Write the answer.	The probability of spinning red or purple is $\frac{3}{8}$.

(b) Spinning an odd number or an 8

THINKING	WORKING
Recall the definition of mutually exclusive events.	Mutually exclusive events are those which cannot occur at the same time.
Decide whether the events can happen at the same time.	An odd number or an 8 cannot occur at the same time, so the events are mutually exclusive.
Recall the rule to calculate the probability of mutually exclusive events.	$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$
Substitute the values to calculate the probability of the mutually exclusive events.	$\Pr(\text{odd or } 8) = \Pr(\text{odd}) + \Pr(8)$ $= \frac{4}{8} + \frac{1}{8}$ $= \frac{5}{8}$
Write the answer.	The probability of spinning an odd number or an 8 is $\frac{5}{8}$.

(c) Spinning yellow or an even number

THINKING	WORKING
Recall the definition of mutually exclusive events.	Mutually exclusive events are those which cannot occur at the same time.
Decide whether the events can happen at the same time.	The number 2 is both an even number and yellow.
Write the answer.	The events are non-mutually exclusive.

SC 3 I can use Venn diagrams and two-way tables to demonstrate the difference between mutually exclusive and non-mutually exclusive events

Worked example

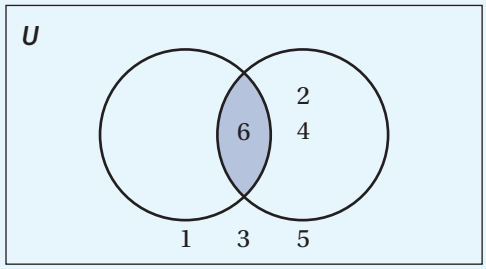
Using Venn diagrams and two-way tables to determine whether events are mutually exclusive

Determine whether the event of rolling a 6 and rolling an even number on a standard 6-sided die are mutually exclusive or not, using:

(a) a two-way table

THINKING	WORKING
List the sample space.	6 and even 6 and odd Not 6 and even Not 6 and odd
Construct a two-way table and state the probabilities.	<table><tr><th></th><th>6</th><th>Not a 6</th><th>Total</th></tr><tr><td>**Even**</td><td>$\frac{1}{6}$</td><td>$\frac{2}{6}$</td><td>$\frac{3}{6}$</td></tr><tr><td>**Odd**</td><td>0</td><td>$\frac{3}{6}$</td><td>$\frac{3}{6}$</td></tr><tr><td>**Total**</td><td>$\frac{1}{6}$</td><td>$\frac{2}{6}$</td><td>$\frac{6}{6}=1$</td></tr></table>
Write the answer.	There is a possibility, $\frac{1}{6}$, of both events occurring at the same time, so the events are non-mutually exclusive.

(b) a Venn diagram

THINKING	WORKING
List the universal set. List the sample space for each outcome.	$U = \{1, 2, 3, 4, 5, 6\}$ Rolling a 6: {6} Rolling an even number: {2, 4, 6}
Check for overlap.	6 is the overlap between the sets.
Draw the Venn diagram.	
Write the answer.	There is overlap, so the events are non-mutually exclusive.

Practice

ANSWERS Page 615

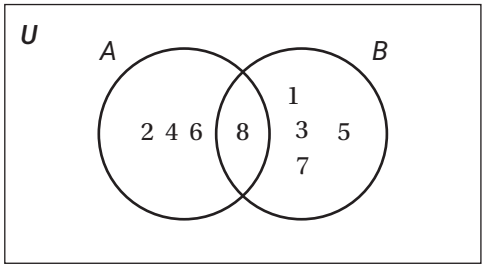
SC 1 I can recognise mutually exclusive events and non-mutually exclusive events

1 For each of the following, identify whether or not the events are mutually exclusive and give your reasons.

(a) Rolling an odd number on a 6-sided die and rolling a 3

(b) Driving to work and arriving late for work

(c) The Venn diagram for the two sets $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7, 8\}$



(d) Melbourne Storm winning the NRL Premiership and Cronulla Sharks winning the NRL Premiership.

2 You are going to roll a 6-sided die once.

(a) List two possible mutually exclusive events. Explain why they are mutually exclusive.

(b) List two possible non-mutually exclusive events. Explain why they are non-mutually exclusive.

3 From a set of cards numbered 1 to 20 you will draw a single card.

(a) List two possible mutually exclusive events. Explain why they are mutually exclusive.

(b) List two possible non-mutually exclusive events. Explain why they are non-mutually exclusive.

(c) Decide whether the following events are mutually exclusive or non-mutually exclusive. Explain your answer.

(i) Drawing a multiple of 2 and drawing a multiple of 3.

(ii) Drawing a multiple of 2 and drawing a multiple of 5.

(iii) Drawing a multiple of 3 and drawing a multiple of 5.

(iv) Drawing a multiple of 6 and drawing a multiple of 8.

4 Write an explanation of what a mutually exclusive event is.

5 Ash or Serena will win the Women's Singles final at the Australian Tennis Open. Jack says that since either player can win, the two events (Ash winning or Serena winning) must be non-mutually exclusive. Is he correct? Explain why or why not.

6 Complementary events are also events that cannot happen at the same time. How are complementary events and mutually exclusive events different?

SC 2 I can calculate the probability of mutually exclusive events

1 A 6-sided die is rolled once. Determine the probability of rolling:

(a) a 5 **(b)** an odd number

(c) a 5 or an odd number **(d)** an odd number and a 5.

2 A 6-sided die is rolled once. Determine the probability of rolling:

(a) a 4 **(b)** an odd number

(c) a 4 or an odd number **(d)** an odd number and a 4.

3 The letters in the word 'teacher' are placed in a bag and one letter is drawn at random. Determine the probability of drawing:

(a) a 't' **b)** an 'e' **(c)** an 'e' or a 't' **(d)** an 'e' and a 't'.

4 The letters in the word 'rapid' are placed in a bag and one drawn at random. Determine the probability of drawing:

(a) a vowel **(b)** a consonant **(c)** a vowel or a consonant

(d) a vowel and a consonant **(e)** neither a vowel nor a consonant.

5 For this spinner, determine the probability of spinning:

(a) a red **(b)** a green

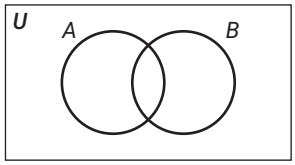
(c) a green or red **(d)** a blue or even number

(e) a yellow or an odd number **(f)** an odd number or a green.

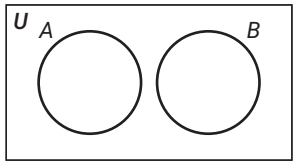
SC 3 I can use Venn diagrams and two-way tables to demonstrate the difference between mutually exclusive and non-mutually exclusive events

1 For each of the following, decide whether the events A and B are mutually exclusive or not and explain why.

(a)



(b)



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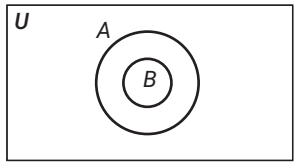


2 A single card is drawn from a set of cards numbered 1 to 10. Draw a Venn diagram to determine whether each of the following events are mutually exclusive or not.

(a) The card is a 3 or an odd number.

(b) The card is a 3 or an even number.

(c) The card is a 6 or a multiple of 3.

(d) The card is a 6 or a multiple of 6.

3 The first 10 letters of the alphabet are placed in a box, and one is drawn at random. Use a two-way table to demonstrate whether the following pairs of events are mutually exclusive or not.

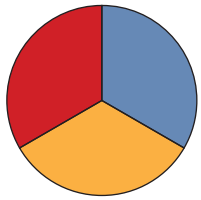
(a) Drawing a vowel and drawing an e.

(b) Drawing a vowel and drawing a consonant.

(c) Drawing a letter from the word 'dig' and drawing a letter from the word 'had'.

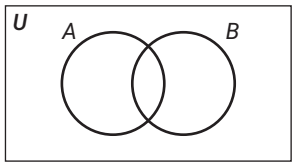
(d) Drawing a letter from the word 'bad' and drawing a letter from the word 'fig'.

4 For the following spinner, draw a Venn diagram to show that the events of spinning a blue, spinning a red and spinning a yellow are all mutually exclusive.

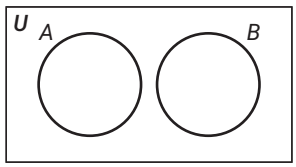


5 For each of the following, give an example of events A and B when drawing cards from a deck.

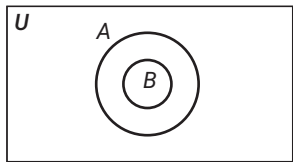
(a)



(b)



(c)



6 The following two-way table shows the results of a survey taken at a local voting station. The survey recorded the age of people voting and their vote as either yes or no.

	Yes	No	Total
Under 40	10	20	30
40 and over	30	40	70
Total	40	60	100

(a) Create the equivalent Venn diagram, where A is the set of people under the age of 40 and B is the set of people who voted yes.

(b) Determine the number of people that were surveyed.

(c) Determine the probability that a random person chosen:

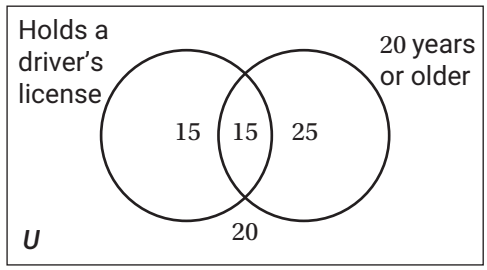
(i) is younger than 40 and voted yes

(ii) is 40 and older

(iii) voted yes

(iv) is 40 and older and voted yes.

7 The following Venn diagram summarises the results of a survey. The survey was conducted on public transport in a major city and recorded the age of the person who responded and whether they had a driver's license.



(a) Create an equivalent two-way table.

(b) Determine the number of people surveyed.

(c) Determine the probability of a random person chosen being:

(i) 20 years or older with a driver's license

(ii) younger than 20 years old and does not have a driver's license

(iii) younger than 20 years old

(iv) a non-driver.

Use Venn diagrams and two-way tables to solve problems

17.5

Learning intention: To be able to use Venn diagrams and two-way tables to solve problems

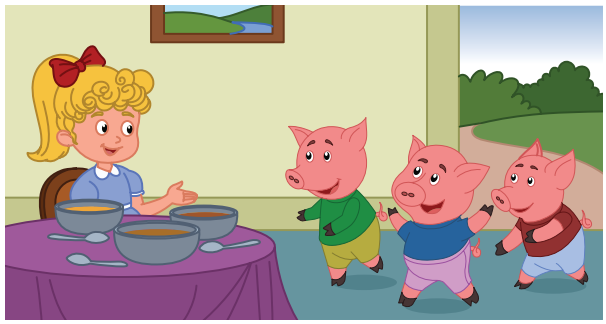
Success criteria:

SC 1 I can use the terminology 'at least', 'exclusive or', 'inclusive or' and 'and'.

SC 2 I can calculate probabilities for events satisfying 'and', 'or' and 'not'.

Lesson warm-up

Goldilocks and the three little pigs



These two stories have elements they share and elements they don't. Draw a blank Venn diagram template with two overlapping circles (one for each story) and see how many items you can add to it in 3 minutes.

SC 1 I can use the terminology 'at least', 'exclusive or', 'inclusive or' and 'and'

Where there are two identified events A and B, there are four possible outcomes for a single trial.

Pr(A and B) + Pr(A and not B) + Pr(not A and B) + Pr(not A and not B) = 1

Terminology	Example
at least	A coin tossed 5 times. The statement 'at least' 3 heads means either 3, 4 or 5 heads.
and	Both things must occur.
exclusive or (in coding this is referred to as xor)	Must be one or the other.

Terminology	Example
inclusive or	One or the other or both.

Worked example

Using the language 'at least' when flipping coins

A coin is flipped three times. The sample space is:

$HHH, HHT, HTH, HTT, THH, THT, TTH, TTT$

Determine the probability of:

(a) flipping at least 1 head

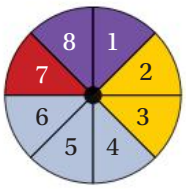
THINKING	WORKING
Interpret the question.	At least 1 head is 1 or more heads. This is every outcome except for TTT.
Count the total number of outcomes and the number of favourable outcomes.	Total number of outcomes = 8 Number of favourable outcomes = 7
Recall the formula to calculate the probability.	$\Pr(x) = \frac{\text{no. favourable outcomes}}{\text{total no. of outcomes}}$
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{at least 1 head}) = \frac{7}{8}$
Write the answer.	The probability of flipping at least 1 head in 3 flips is $\frac{7}{8}$.

(b) flipping at least 2 heads.

THINKING	WORKING
Interpret the question.	At least 2 heads is 2 or 3 heads: $HHH, HHT, HTH, THH,$
Count the total number of outcomes and the number of favourable outcomes.	Total number of outcomes = 8 Number of favourable outcomes = 4
Recall the formula to calculate the probability.	$\Pr(x) = \frac{\text{no. favourable outcomes}}{\text{total no. of outcomes}}$
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{at least 2 heads}) = \frac{4}{8}$
Write the answer.	The probability of flipping at least 2 heads in 3 flips of a coin is $\frac{4}{8}$.

Worked example

Using the language 'at least', 'exclusive or' and 'inclusive or' in a spinner



This spinner is spun once. Determine the probability of:

(a) spinning at least a 5

THINKING	WORKING
Interpret the question.	Success is landing on numbers greater than or equal to 5: 5, 6, 7 or 8.
Count the total number of outcomes and the number of favourable outcomes.	Total number of outcomes = 8 Number of favourable outcomes = 4
Recall the formula to calculate the probability.	$\Pr(x) = \frac{\text{no. favourable outcomes}}{\text{total no. of outcomes}}$
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(5,6,7,8) = \frac{4}{8} = \frac{1}{2}$
Write the answer.	The probability of spinning a 5, 6, 7 or 8 is $\frac{1}{2}$.

(b) spinning yellow or an odd number, but not both

THINKING	WORKING
Interpret the question.	This is an example of 'exclusive or'. Success is landing on yellow, or an odd number, but not both. Yellow = {2, 3} Odd = {1, 3, 5, 7} Yellow or odd, but not both = {1, 2, 5, 7}
Count the total number of outcomes and the number of favourable outcomes.	Total number of outcomes = 8 Number of favourable outcomes = 4
Recall the formula to calculate the probability.	$\Pr(x) = \frac{\text{no. favourable outcomes}}{\text{total no. of outcomes}}$
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{yellow or odd but not both}) = \frac{4}{8} = \frac{1}{2}$
Write the answer.	The probability of spinning a yellow or an odd number, but not both is $\frac{1}{2}$.

(c) spinning yellow or an odd number

THINKING	WORKING
Interpret the question.	This implies an 'inclusive or'. Success is landing on yellow, or an odd number. Yellow = {2, 3} Odd = {1, 3, 5, 7} Yellow or odd = {1, 2, 3, 5, 7}
Count the total number of outcomes and the number of favourable outcomes.	Total number of outcomes = 8 Number of favourable outcomes = 5
Recall the formula to calculate the probability.	$\Pr(x) = \frac{\text{no. favourable outcomes}}{\text{total no. of outcomes}}$
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{yellow or odd}) = \frac{5}{8}$
Write the answer.	The probability of spinning a yellow or an odd number is $\frac{5}{8}$.

SC 2 I can calculate probabilities for events satisfying 'and', 'or' and 'not'

And means that both events must occur at the same time, **not** means something does not occur.

Worked example

Calculating probabilities in two-way tables

Professional and amateur golfers were asked whether a particular golfer should try to hit over the water and onto the green – the aggressive approach – or hit the ball short of the water to ensure an easy next shot – the conservative approach. The results are shown in this two-way table.

	Conservative	Aggressive	Total
Professional	32	14	46
Amateur	16	38	54
Total	48	52	100

(a) Determine the probability that a random golfer is both a professional and recommends the conservative approach.

THINKING	WORKING
Identify the intersection of both conditions.	<table><tr><th></th><th>Conservative</th><th>Aggressive</th><th>Total</th></tr><tr><td>**Professional**</td><td>32</td><td>14</td><td>46</td></tr><tr><td>**Amateur**</td><td>16</td><td>38</td><td>54</td></tr><tr><td>**Total**</td><td>48</td><td>52</td><td>100</td></tr></table>
Calculate the probability by dividing the number of favourable outcomes by the total number of outcomes.	$\Pr(\text{professional and conservative}) = \frac{32}{100} = \frac{8}{25}$
Write the answer.	The probability that a random golfer is a professional who recommends the conservative approach is $\frac{8}{25}$.

(b) Determine the probability that a random golfer is a professional, but does not recommend the conservative approach.

THINKING	WORKING
Identify the intersection of both conditions.	<table><tr><th></th><th>Conservative</th><th>Aggressive</th><th>Total</th></tr><tr><td>**Professional**</td><td>32</td><td>14</td><td>46</td></tr><tr><td>**Amateur**</td><td>16</td><td>38</td><td>54</td></tr><tr><td>**Total**</td><td>48</td><td>52</td><td>100</td></tr></table>
Calculate the probability.	$\Pr(\text{professional and not conservative}) = \frac{14}{100} = \frac{7}{50}$
Write the answer.	The probability that a random golfer is a professional, but does not recommend the conservative approach is $\frac{7}{50}$.

(c) Determine the probability that a random golfer is an amateur or recommends the aggressive approach.

THINKING	WORKING
Identify the union of all conditions.	<table><tr><th></th><th>Conservative</th><th>Aggressive</th><th>Total</th></tr><tr><td>Professional</td><td>32</td><td>14</td><td>46</td></tr><tr><td>Amateur</td><td>16</td><td>38</td><td>54</td></tr><tr><td>Total</td><td>48</td><td>52</td><td>100</td></tr></table>
Determine the number of favourable outcomes.	$14 + 16 + 38 = 68$
Calculate the probability.	$\Pr(\text{amateur and aggressive}) = \frac{68}{100} = \frac{17}{25}$
Write the answer.	The probability that a random golfer is an amateur or recommends the aggressive approach is $\frac{17}{25}$.

(d) Determine the probability that a random golfer is an amateur or recommends the aggressive approach, but not both.

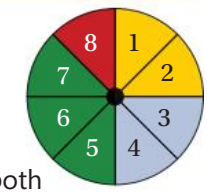
THINKING	WORKING
Identify the intersection of all conditions.	<table><tr><th></th><th>Conservative</th><th>Aggressive</th><th>Total</th></tr><tr><td>Professional</td><td>32</td><td>14</td><td>46</td></tr><tr><td>Amateur</td><td>16</td><td>38</td><td>54</td></tr><tr><td>Total</td><td>48</td><td>52</td><td>100</td></tr></table>
Determine the number of favourable outcomes.	$14 + 16 = 30$
Calculate the probability.	$\Pr(x) = \frac{30}{100} = \frac{3}{10}$
Write the answer.	The probability that a random golfer is an amateur or recommends the aggressive approach, but not both is $\frac{3}{10}$.

Practice

ANSWERS Page 617

SC 1 I can use the terminology 'at least', 'exclusive or', 'inclusive or' and 'and'

1 The spinner is spun once. Determine the probability of:



(a) spinning at least 3

(b) spinning a 3 or a yellow

(c) spinning a 3 or a blue

(d) spinning a 3 or a blue, but not both

(e) spinning a green or an odd number

(f) spinning a green or an odd number, but not both.

2 From a set of 10 cards numbered 1 to 10, a single card is drawn randomly. Determine the probability of drawing:

(a) a multiple of 3

(b) a multiple of 3 or a multiple of 2

(c) a multiple of 3 or a multiple of 2, but not both

(d) a multiple of both 2 and 3

(e) a multiple of 2 or a multiple of 5

(f) a multiple of 2 or a multiple of 5, but not both

(g) a multiple of both 2 and 5.

3 A standard 6-sided die is rolled. Write an event whose favourable outcomes are the elements of these sets:

(a) {2, 3, 4, 5, 6} **(b)** {2, 4, 6} **(c)** {3, 4, 5, 6} **(d)** {2, 3, 4}

4 A family has 3 cars. Each car has an equal chance of being either black or white. Determine the probability of:

(a) having 3 black cars **(b)** having exactly 2 black cars

(c) having at least 2 black cars **(d)** having exactly 1 black car

(e) having at least 1 black car.

5 A roulette wheel is spun 4 times. On each spin, there is an equal chance of either red or black.

(a) List the sample space.

(b) Determine the probability of spinning:

(i) 4 reds **(ii)** 4 blacks **(iii)** at least 1 black

(iv) at least 2 blacks **(v)** at least 3 blacks.

(c) Write an example of a question that includes an 'exclusive or'.

6 Jose says that 'or' means or and there are not two meanings. Use an example to show him he is wrong.

SC 2 I can calculate probabilities for events satisfying 'and', 'or' and 'not'

1 One hundred and fifty people who purchased raffle tickets to win a car were surveyed about their age and whether their raffle preference would be an SUV or a sports car. The results are shown in this table.

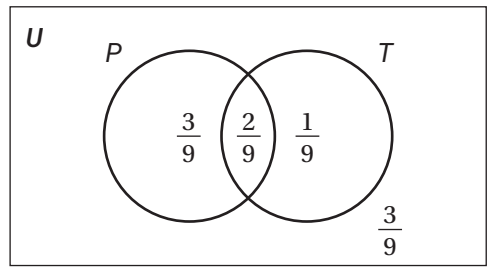
	SUV	Sports car	Total
$\leq$ **30**	21	49	70
$>$ **30**	61	19	80
Total	82	68	150

Determine the probability that a person selected at random is:

(a) over the age of 30 **(b)** over 30 and prefers an SUV

(c) over 30 or prefers an SUV **(d)** over 30 or prefers an SUV, but not both.

2 A group of tennis players were asked what brand of tennis racquet they would purchase next: Pike, P or Tonnex, T. The probability of each response is shown in the Venn diagram below.



Determine the probability of:

(a) buying a Pike racquet or a Tonnex racquet

(b) not buying a Pike racquet

(c) choosing a different brand altogether

(d) being willing to buy only a Tonnex racquet

(e) being willing to buy only one type of racquet.

3 At a live performance there are three classes of seats. The following two-way table shows the distribution of seats on opening night.

	Premium	Elite	Gold Class	Total
Adults	36	39		112
Children	41		31	
Total		60		

(a) Complete the table.

(b) Determine the probability of a random audience member being:

(i) an adult sitting in the Elite section

(ii) a child sitting in the Elite or Gold Class sections

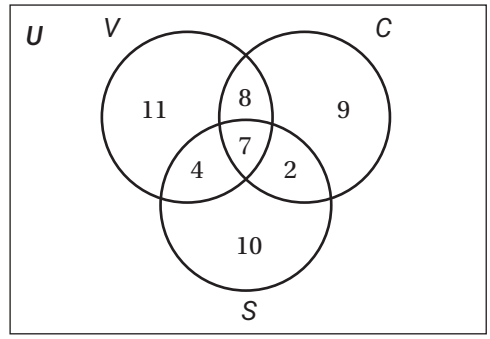
(iii) an adult or child sitting in a Premium seat

(iv) an audience member sitting in either a Premium or Elite seat.

(c) It is known that an audience member is sitting in the Elite section. What is the probability that this member is an adult?

(d) It is known that an audience member is a child. What is the probability that the child is sitting in the Premium section?

4 An ice-cream seller has three flavours available: vanilla, chocolate and strawberry. A single ice-cream may contain 1, 2 or 3 scoops. The Venn diagram shows the sales on one afternoon.



(a) Determine the probability of an order chosen at random containing:

(i) vanilla

(ii) vanilla or strawberry

(iii) vanilla or strawberry, but not both

(iv) vanilla and strawberry

(v) vanilla and strawberry, but not chocolate

(vi) only vanilla.

(b) Write a pair of questions that demonstrate the difference between 'inclusive or' and 'exclusive or'.

(c) Write a pair of questions that demonstrate the difference between 'inclusive or' and 'and'.

5 Which is greater, the probability of A or B 'exclusive or' or 'inclusive or'? Explain.

TOPIC REVIEW

Multiple choice

ANSWERS Page 635

1 A 6-sided die is rolled once. Let A = the set of numbers less than 2. The complement of A is:

A {odd numbers} **B** {1} **C** {2, 3, 4, 5, 6} **D** {3, 4, 5, 6}

2 Which of these pairs of events are complementary?

A rolling a 5 and rolling a 6 on a die

B rolling a 5 and rolling a number < 5 on a die

C rolling a multiple of 2 and rolling a multiple of 3

D rolling a 5 and not rolling a 5

3 A coin is tossed 5 times. The probability of tossing 5 tails is $\frac{1}{32}$. The probability of tossing at least one head is:

A $\frac{1}{32}$ **B** $\frac{16}{32}$ **C** $\frac{26}{32}$ **D** $\frac{31}{32}$

4 If Set $A = \{1, 2, 3, 4, 5\}$ and Set $B = \{2, 4, 6, 8\}$, then $A \cup B$ is:

A {1, 2, 2, 3, 4, 5, 6, 8} **B** {1, 2, 3, 4, 5, 6, 8} **C** {1, 2, 3, 4, 5, 6, 7, 8} **D** {2, 4}

5 If Set $A = \{1, 2, 3, 4, 5, 6\}$ and Set $B = \{2, 4, 6, 8\}$, then $A \cap B$ is:

A {1, 2, 2, 3, 4, 4, 5, 6, 6, 8} **B** {2, 4, 6, 8}

C {1, 2, 3, 4, 5, 6, 7, 8} **D** {2, 4, 6}

6 If Set A = {positive odd numbers less than 12}, Set $B = \{2, 4, 6, 8\}$ and the universal set is the natural numbers less than 14, then A' is:

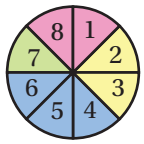
A {1, 3, 5, 7, 9, 11} **B** {2, 4, 6, 8, 10, 12}

C {2, 4, 6, 8, 10, 12, 14} **D** {1, 2, 3, 4, 5, 6, 7, 8, 9, 11}

7 If Set A = {positive odd numbers less than 12}, Set B = {multiples of 3 less than 15}, then $A \cap B$ is:

A {3, 9} **B** {3, 6, 9} **C** {1, 3, 5, 7, 9, 11} **D** {1, 3, 5, 6, 7, 9, 11}

8 The probability of spinning a yellow or green is:



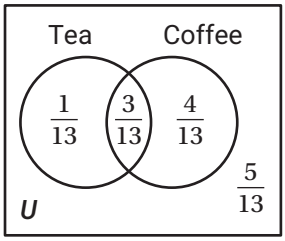
A $\frac{3}{8}$ **B** $\frac{2}{8}$ **C** $\frac{2}{7}$ **D** $\frac{3}{4}$

9 Parents were surveyed about a possible excursion to either Ballarat or Bendigo. Using the results in this two-way table, determine the probability that a random person being selected is a mother.

	Ballarat	Bendigo	Total
Fathers	15	8	23
Mothers	11	13	24
Total	26	21	47

A $\frac{11}{47}$ **B** $\frac{24}{47}$ **C** $\frac{11}{13}$ **D** $\frac{13}{47}$

10 A group of shoppers is asked if they will buy tea or coffee the next time they go shopping. The results are shown in this Venn diagram.



The probability of buying tea or coffee, but not both, is:

A $\frac{4}{13}$ **B** $\frac{5}{13}$ **C** $\frac{8}{13}$ **D** $\frac{3}{13}$

Short answer

1 Max has a long-time record of shooting 50% from the free-throw line in basketball. (This means he scores with 1 out of 2 shots.) When the siren sounds, his team is level with their opponents, but Max has been awarded 2 free throws.

(a) List the sample space for the outcomes of his free throws.

(b) Determine the probability of the game remaining even after his throws.

(c) Determine the probability of Max's team winning the game.

2 Out of 20 friends, 10 drank coffee, 5 ate cake and 3 had both cake and coffee.

(a) Draw a Venn diagram to represent this situation.

(b) Change the figures in the diagram to represent the probability of each event.

(c) Determine the probability of a person chosen at random:

(i) eating cake

(ii) eating cake, but not drinking coffee

(iii) not having cake or coffee

(iv) having coffee or cake, but not both.

(d) Why was it useful to not simplify the fractions in the Venn diagram?

3 Complementary and mutually exclusive events are similar but not the same. Use an example to explain the difference.

4 Complete the following sentences:

(a) The probabilities of complementary events add to _____.

(b) Mutually exclusive events _____ occur at the same time.

(c) $A \cup B$ can be read as A _____ B.

(d) $A \cap B$ can be read as A _____ B.

(e) A' is the _____ of A.

5 A card is drawn randomly from a standard deck of 52 cards. Determine the probability that the card is:

(a) black

(b) a Heart

(c) a Queen

(d) a black Queen

(e) a black Queen or a King

(f) a black card or a Spade, but not both.

6 Two hundred people were surveyed about whether their next car would be an SUV or a sports car. The results are shown in this table. Determine the probability that a person selected at random is:

	SUV	Sports car	Totals
$\leq$ **30**	61	49	110
> 30	188	12	200
Total	249	61	310

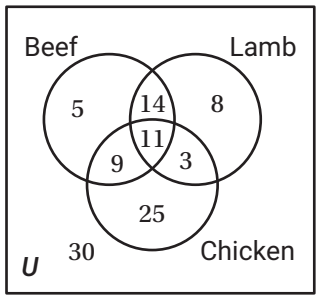
(a) over 30

(b) over 30 and will select an SUV

(c) over 30 or will select an SUV

(d) over 30 or will select an SUV, but not both.

7 Consider the following Venn diagram related to whether a group of people like eating beef, chicken and lamb.



(a) How many people were surveyed?

(b) What is the probability of not liking any of the 3 meats?

(c) What is the probability of liking any of the 3 meats?

(d) What is the probability of liking only 2 of the meats?

8 The spinner below is spun once. Determine the probability of:

(a) spinning at least 2

(b) spinning a 2 or a green

(c) spinning a 2 or a yellow

(d) spinning a 2 or a yellow, but not both

(e) spinning a 2 and a yellow.

9 From set of 8 cards numbered 1, 1, 1, 2, 2, 3, 4 and 5, a single card is drawn randomly. What is the probability of drawing:

(a) a 1?

(b) not a 1?

(c) a 1 or a 2?

(d) a multiple of 3?

(e) a multiple of 3 or a multiple of 2?

10 In probability the word 'or' can have two meanings. Give the name of each meaning and use an example to describe the difference between them.

11 A number is drawn from the first 16 positive integers.

(a) What is the probability it is divisible by 2 or 3?

(b) What is the probability it is divisible by 2 or 3, but not both?

(c) What is the probability it is divisible by 2 and 3?

12 In a raffle with 1000 tickets, you buy 40 tickets. What is the probability that you do not win the first prize?

13 Gai is asked to choose a number between 1 and 15 (inclusive). What is the probability that the number is:

(a) odd?

(b) greater than 10?

(c) odd and greater than 10?

(d) odd and less than 10?

(e) greater than 5 but less than 10?

14 The sample space, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

Set $A = \{2, 4, 6, 8, 10\}$, Set $B = \{1, 2, 3, 5, 7, 9, 11, 13, 15, 16, 17\}$

(a) Draw the Venn diagram.

(b) Determine the probability of A'.

(c) Determine the probability of $A \cup B$.

(d) Determine the probability of $A \cap B$.

15 The eye colour and gender of 390 people is recorded in the table below.

	Brown	Blue	Grey	Green	Total
Male	60	70	35	25	190
Female	70	65	40	25	200
Total	130	135	75	50	390

What is the probability that a person chosen at random is:

(a) blue-eyed?

(b) a male?

(c) a female that does not have blue eyes?

(d) has blue eyes or is a female?

Extended response

1 Cassie's, Diva's and Fashionistas are three fashion stores located in the same shopping centre. Records are kept over an 8-hour period as to the number of shoppers who enter each store.

83 shoppers entered Diva's.

101 entered Cassie's.

22 entered Fashionistas.

31 entered both Cassie's and Diva's.

8 entered both Diva's and Fashionistas.

10 entered both Cassie's and Fashionistas.

6 entered all three stores.

34 did not enter any of the stores.

(a) Draw a Venn diagram to represent this situation.

(b) How many shoppers were in the shopping centre?

(c) Determine the probability of:

(i) a shopper not visiting any of the stores

(ii) a shopper visiting only Diva's

(iii) a shopper visiting Diva's and at least one other store

(iv) a shopper visiting only one of the stores

(v) a shopper visiting any two of the stores, but not all three.

(d) A particular shopper entered Diva's. What is the probability that was the only store she visited?

2 A store keeps T-shirts, jumpers and coats in three sizes, S, M and L. The stock numbers are shown in this partially completed two-way table.

	T-shirts	Jumpers	Coats	Total
Small	2	36	28	
Medium		0	1	10
Large		51		
Total			44	200

(a) Complete the table of values.

(b) What is the probability that a chosen random item of clothing is:

(i) a small size

(ii) a medium-sized jumper

(iii) a large sized T-shirt

(iv) a coat or a large sized item

(v) a coat or a large sized item, but not both.

(c) Given that the item was a large size, what is the probability that it is a jumper?

3 The teacher tells you there is an upcoming exam on what you have learnt in this probability topic. She also says you can bring in one page of handwritten notes to refer to. Prepare a one-page summary that would assist you on the exam. You might consider including some theory, some definitions, some formulas and a few worked examples.

GLOSSARY

acquire data
The process of obtaining data by direct collection or from other sets of data so that it can be sorted, displayed and analysed.

acute angle
An angle that measures between 0 and 90 degrees, i.e. less than one right angle.

addition facts
The results associated with the sums of pairs of natural numbers from 0 to 9. They are foundational to arithmetic.

additive
A situation or relationship that involves addition, subtraction or both, e.g. giving change from a simple money transaction.

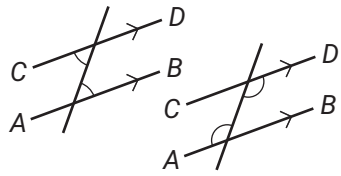
algebra
The process of manipulating variables and constants in a mathematical expression according to laws, properties or rules, e.g. simplifying an expression or solving an equation.

algebraic expression
A mathematical statement formed by combining numbers and algebraic symbols using arithmetic operations, e.g. $a^2 + 3ab - 2b^2$.

algebraically
Carrying out a process symbolically using variables, constants, operations and rules.

algorithm
A process that can be carried out systematically, using a well-defined set of instructions, to perform a particular task or solve a type of problem.

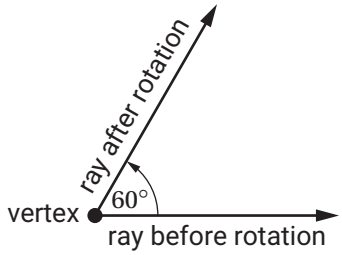
alternate angles
Angles on alternate sides of a transversal of a pair of parallel lines. If the lines AB and CD are parallel, then each pair of alternate angles are equal.



am
Any time before midday when using 12-hour time. An abbreviation of the Latin term *ante meridiem*.

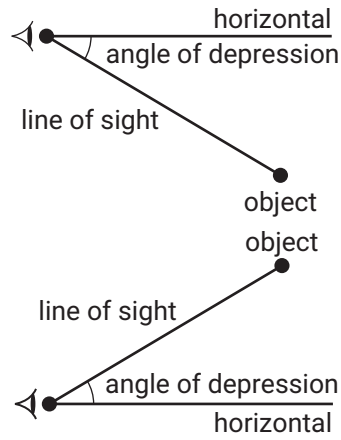
analog clock
A clock with hours marked from 1 to 12 arranged in a circle and moving hands to show you the time.

angle
The figure formed at the intersection of two rays in the plane with a common endpoint, called its vertex. It is typically indicated with an arc.

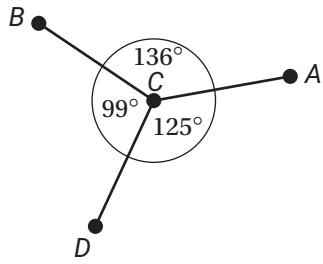


angle of depression
The angle between horizontal and the line of sight from an observer to an object that is lower than the observer.

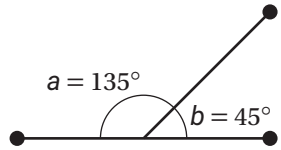
angle of elevation
The angle between horizontal and the line of sight from an observer to an object that is higher than the observer.



angles at a point
The sum of the angles with vertices at a common point in the plane is equivalent to four right angles or 360 degrees. This spatial fact can be applied to determine the size of missing angles.



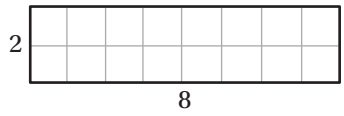
angles on a straight line
The sum of the angles that form a straight line is equivalent to two right angles or 180 degrees. This spatial fact can be applied to determine the size of missing angles.



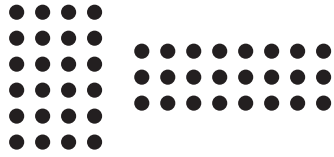
approximate
To obtain or state a value to a particular accuracy.

approximation
A result that is not exact, but is close enough for a given purpose, e.g. giving an approximation of the area of a complex shape by using a combination of basic shapes.

area
The amount of interior space bounded by the perimeter of a shape. It is usually measured in square units. For example, the area of this rectangle is given by $8 \times 2 = 16$ units.



array
An ordered collection of discrete objects or numbers, such as rectangular arrangements of rows and columns. For example, two arrays of dots (4×6 and 8×3) produce two different representations of the number 24.



Asia/Asian

Geographically, the largest continent, bounded by Europe and the Pacific, Arctic and Indian oceans. It can be described in cultural, religious, historical and language boundaries or commonalities.

assign probabilities

The process of providing a qualitative or quantitative value for the likelihood or chance of an event occurring. This may involve judgement based on experience, experiment, counting or measurement.

association(s)

Relationships between pairs of variables for different data types, e.g. in terms of strength, form and direction.

associative

Of or relating to an operation, which, that when applied to any three elements of an expression, is the same regardless of which pair of elements (without changing their order) is combined first.

associative law

In general, the addition or multiplication of a set of real numbers gives the same output, no matter how the numbers are grouped, e.g. $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$.

attribute

A property of an object or event that can be measured and/or directly or indirectly compared.

back-to-back stem-and-leaf-plot

A method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem'.

Pulse rate

Before		After
9 8 8 8	6	
8 6 6 4 1 1 0	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	
	14	6

base-10

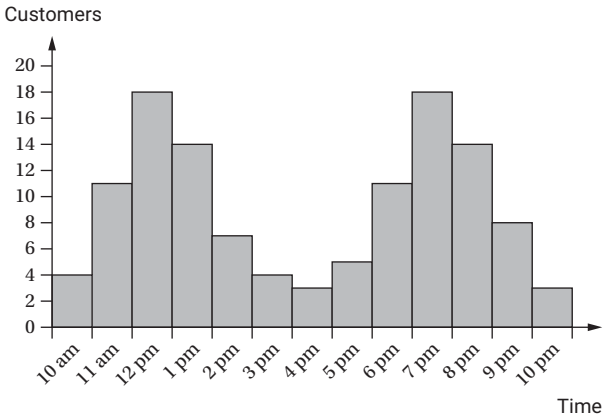
A number system which uses the digits 0–9 and the value of the digit is determined by its face value and its place value, e.g. $283 = 2 \times 100 + 8 \times 10 + 3 \times 1$ and $283 = 200 + 80 + 3$.

bias (statistics)

A measure being systematically moved away from the true value, leading to overestimating or underestimating that value.

bimodal distribution

The distribution of a set of data that has two modes. The term 'bimodal' is also used if the graph of the distribution has two distinct peak values, which may not be equal.

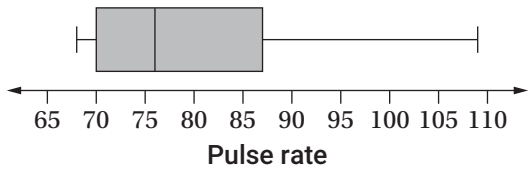


bivariate data

Data relating to measurement of two variables. It can be categorical data, numerical data or a combination of both.

boxplot

A graphical display of a 5-number summary of data. The 'box' covers the middle 50% of scores. The 'whiskers' extend to the maximum and minimum values in the data set.

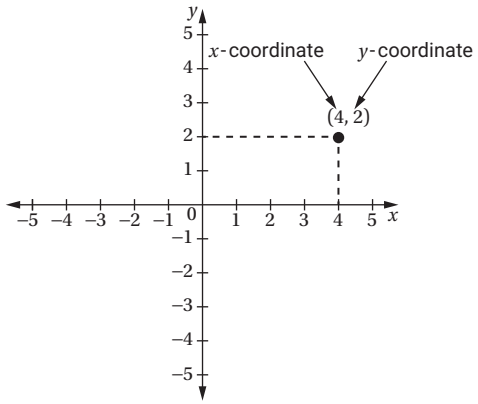


capacity

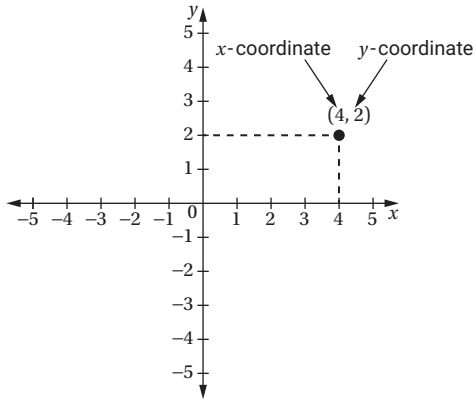
The amount a container will hold. It is often used in relation to the volume of fluids. Units of capacity (volume of fluids or gases) include litres (L) and millilitres (mL).

Cartesian coordinate system

The positioning system for any point on the Cartesian plane, represented by an ordered pair of real numbers (x, y), which are called the coordinates of the point relative to the origin at (0, 0).



Cartesian plane
A plane divided into 4 regions (quadrants) by 2 perpendicular axes, the x-axis and the y-axis, which intersect at a point called the origin.



categorical data
Data in discrete categories, e.g. data on blood groups with values of type A, B, AB or O.

categorical variable
A variable that takes values from sets of categories. There are two types of categorical variables: nominal and ordinal.

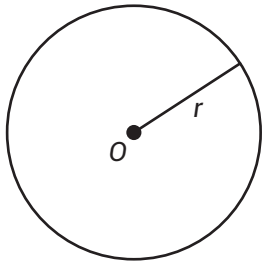
census
Collection of information about the whole population.

centre (space)
The middle point of a shape, object or space.

centre (statistics)
In a set of data, a single value that is used to represent or typify that data. Informally, it is a notion of average.

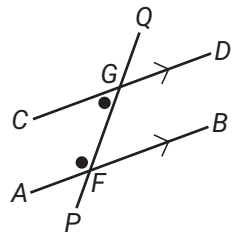
chance experiment (trial)
An investigation process that involves planning and conducting a repeatable experiment involving chance, where more than one possible outcome may occur. Experiments can be single or multi-step and results can vary across trials.

circle
The set of all points in the plane that are a fixed distance (the radius) from a given point (the centre of the circle).



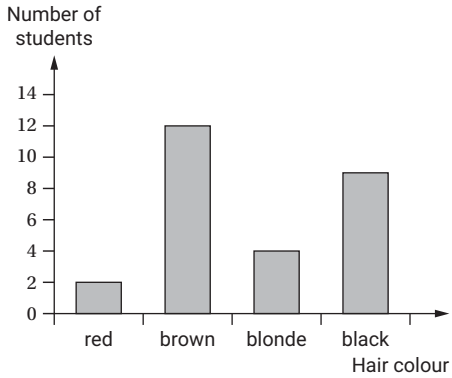
circumference
The boundary (perimeter) of a circle. The length of the circumference C is given by $C = \pi d$, where d is the diameter. Alternatively, it is given by $C = 2\pi r$ where r is the radius.

co-interior angles
A pair of angles that lie between a pair of parallel lines cut by a transversal and has a sum of 180 degrees or two right angles.



colonisation
The act of setting up a colony away from one's place of origin. With humans, it can be seen negatively because it tends to involve an invading culture taking control over an indigenous population.

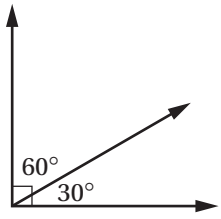
column graph
A graph for organising and displaying categorical data. Equal-width rectangular bars are constructed for each category with height equal to the observed frequency of the category. Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees, is 90°.



commutative
An operation in which the result of applying it to any two elements of a set is the same, regardless of the order of the elements. Addition and multiplication are commutative on the set of natural numbers.

commutative property
In general, the commutative property of addition and multiplication of real numbers is that for all real numbers a and b, $a + b = b + a$ and $a \times b = b \times a$, respectively.

complementary angles
Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees is 90°.



complementary events
Two mutually exclusive outcomes in a probability experiment, such that Pr(A) + Pr(B) = 1, where Pr(A) is the probability of event A and Pr(B) is the probability of event B.

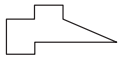
complex shapes and objects

A complex shape is a closed shape comprised of a number of plane shapes. Complex objects are comprised of simpler objects.

composite number

A non-zero natural number that has a factor other than 1 and itself.

composite shape



A shape that is formed by combining other plane shapes. Composite shapes are often described as 'complex' when they are made up of many and different shapes.

compound event

In probability, an event that consists of more than one event occurring at the same time, e.g. tossing a coin and selecting a card.

compound interest

The interest earned by investing a sum of money (the principal) when each successive interest payment is added to the principal for calculating the next interest payment.

compounding error

The increasing level of error that arises when measurements with errors are used to calculate other quantities.

computation

The action of performing a mathematical calculation. It can be performed mentally or by using an algorithm or a digital tool, such as a calculator.

computational thinking

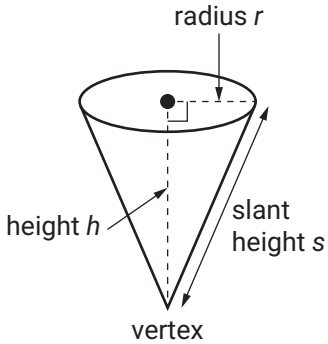
A way of thinking that helps to organise data logically by breaking down problems into parts; defining abstract concepts; and designing and using algorithms, patterns and models.

conditional probability

Consideration of whether the knowledge of the occurrence of one event, A, affects the probability of occurrence of another event, B.

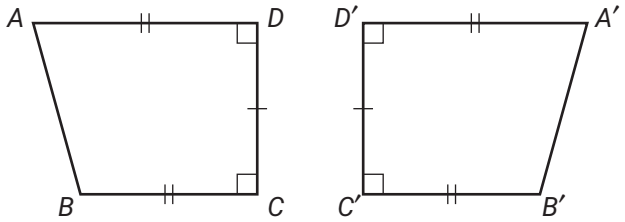
cone

An object that is formed by taking a circular base and a point not in the plane of this circle, called the vertex, and joining the vertex to each point on the circumference of the base.



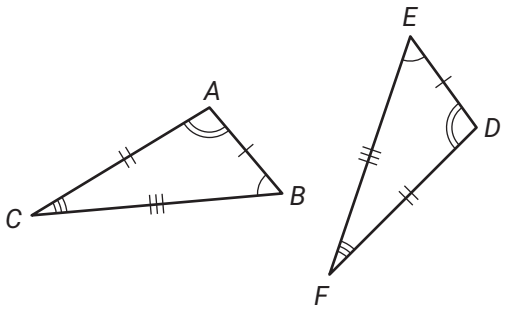
congruent

An exact match between every part of one figure with the corresponding part of the other figure, i.e. congruent figures can be exactly superimposed on each other.



congruent triangles

Triangles that have the same size and shape. When not all measurements are known, the determination can be made using the four standard congruence tests for triangles (SSS, SAS, AAS, RHS).



conjecture

A statement whose truth or otherwise is not yet determined, but is open to further investigation, e.g. Goldbach's conjecture of 1742 has not yet been either proven to be true or shown to be false.

construct

To draw, make or build a mathematical object such as a shape, pattern, algorithm or proof.

context

The situation or setting of an event, problem or application. Contexts can be real world or constructed.

continuous numerical variable

A numerical variable that can take any value that lies within an interval. In practice, the values taken are recorded to the accuracy of measurement of the instrument used to obtain these values.

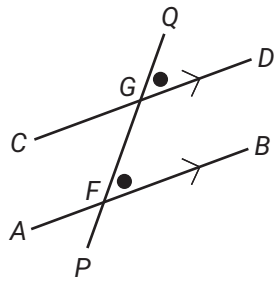
coordinate system

A system for showing a set of values that give an exact position, e.g. Cartesian coordinate system, grid map or a seating plan for a venue.

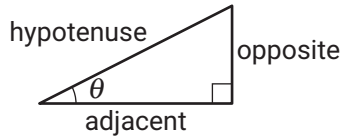
coordinates

A set of values that define the position of a point or an object in a space. The coordinate system being used will govern the nature of the coordinates.

corresponding angles
Angles that lie in the same relative position at the points of intersection, where a pair of parallel lines is cut by a transversal and which are equal.



cosine ratio
The ratio of the length of the base, b, of a right-angled triangle to the length of its hypotenuse, h, for a given base angle theta, θ. In any right-angled triangle,
$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.

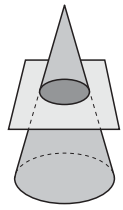


counting
The process of quantifying the number of objects in a set or collection.

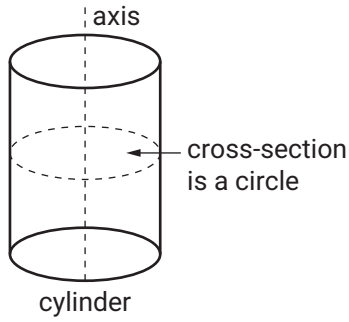
Country
A space mapped out that individuals or groups of First Peoples of Australia occupy and regard as their own and having varying degrees of spirituality. It includes lands, waters and sky.

Country/Place
Spaces mapped out that individuals or groups of Aboriginal and Torres Strait Islander Peoples occupy and regard as their own and that have varying degrees of spirituality. They include lands, waters and sky.

cross-section
The intersection of a solid object with a plane. It may differ for a given solid depending on the orientation of the slicing plane.



cylinder
An object that has parallel circular discs of equal radius at the ends, which are joined by a curved surface.



data
A general term for a set of observations or measurements collected during an investigation. Primary data is generated by the user; secondary data is collected from a secondary source, such as a journal, book or website.

data displays
Visual formats for organising, representing and summarising data. Also called visualisations.

data distributions
Patterns in the values of a set of data. The representation used to show the distribution depends on the type of data and how it is measured.

decay
Decrease in the value of the dependent variable of a function as the value of the independent variable increases.

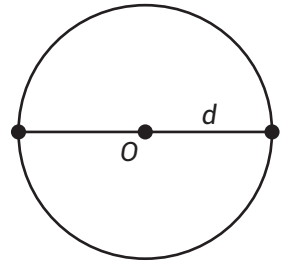
decimal
Used to describe aspects of the base-10 number system. The decimal point (. or ,) separates the whole number part of a number from its decimal part.

degrees
A unit of measurement for measuring an angle. Angles are measured as a proportion of a full turn, which is equivalent to 360 degrees, so that one degree is equal to $\frac{1}{360}$ of a full turn. Written as $\alpha°$.

denominator
In the fraction $\frac{a}{b}$, b is the denominator. It is the number of equal parts into which the whole is divided.

diagonal
A line segment joining two vertices of a polygon that do not share an edge.

diameter
A chord passing through the centre of a circle. Also describes the length of the diameter.



digit
A single symbol that is used to represent a number as a numeral. In the base–10 number system there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

digital tools
Electronic devices, programs, websites or online resources that can be used to support the learning and doing of mathematics.

direct comparison
Comparing objects and events using a particular attribute, e.g. measuring the length of two pencils by placing them side by side or starting two events at the same time to compare duration.

direct proportion
The relationship between corresponding values of two variables in a fixed ratio relative to each other;
i.e. for variables x and y, $\frac{y}{x} = k$ and $y = kx$, where $k \neq 0$.

directional language
Words that communicate position, location and movement within a space.

discrete numerical data
Numerical data that can take only a countable number of values, e.g. the number of people in a car, or clothing sizes.

distributive
An operation that can take priority over the operation used for combination within brackets. Multiplication is distributive over addition, e.g. $8 \times 17 = 136$ as $8 \times 17 = 8 \times (10 + 7)$ and $8 \times (10 + 7) = 8 \times 10 + 8 \times 7$ and $8 \times 10 + 8 \times 7 = 80 + 56$, which equals 136.

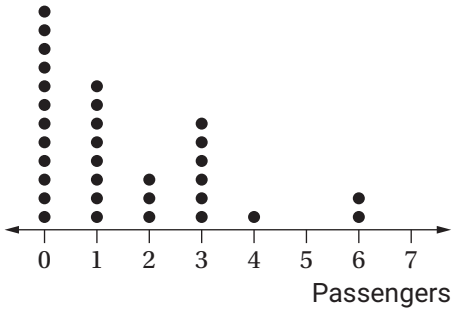
distributive property
In general, the distributive law (property) for multiplication over addition for real numbers states that for all real numbers a, b and c: $a(b + c) = a \times b + a \times c$.

divisible
Able to be divided exactly, i.e. without leaving a remainder.

division
For a finite set, the process of partitioning the set into subsets of equal size. For natural numbers, it expresses a given number as a multiple of a smaller number and any remainder.

division facts
Facts that draw on the inverse relationship between division and multiplication and are directly related to the multiplication facts, e.g. $2 \times 5 = 10$, so $\frac{10}{2} = 5$ and $\frac{10}{5} = 2$.

dot plot
A graph used to display numerical data. Using a number line, it displays a dot for each piece of data and, when there is more than one piece of data with the same value, the dots are stacked vertically.



duration
A measure of time that an event or activity takes to complete.

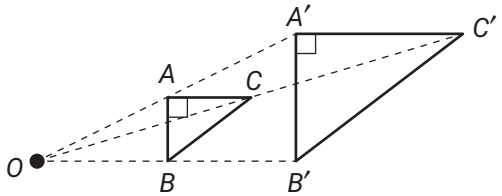
dynamic geometric software
Software that supports the representation, construction and manipulation of geometric shapes and objects, and related measures and other functions in an interactive and dynamic digital environment.

efficient strategies
The choice of suitable approaches (mental, written, using digital tools), and methods within these. The means used for calculation reflect the purpose and context.

element(s)
A member of a set, e.g. 3 is a member of the set of natural numbers $N = \{0, 1, 2, 3, ...\}$. This relation can be written more concisely as $3 \in N$ ('3 is an element of the set N').

enclosed spaces
Space that is enclosed by a boundary, such as the perimeter of a shape or the outer surface area of an object.

enlargement transformation
A scaling of a figure in which the corresponding lengths in the transformed figure are increased or decreased in proportion to the original figure. The two figures are similar.



equal sharing
Dividing a collection, shape or object into equal parts.

equally likely outcomes
An event for which all outcomes have the same probability of occurring.

equation
A statement that includes the '=' symbol. Equations are used to show the equality of two expressions.

equivalent
Equal in value or meaning, such as an expression or statement that is essentially the same, two or more sets that are capable of being mapped in a one-to-one relationship.

equivalent fractions
Fractions that are different representations of the same value, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

error
The difference between an actual value and its measured or estimated value.

estimation
The skill of conceptualising and mentally manipulating numbers or measurements to find an approximate answer. The capacity to make reasonable adjustments to estimates is essential in estimating.

estimation strategies
Strategies used to estimate or find an approximate value for the amount, extent, duration, position or size of something.

Euler's formula
A relation between the number of faces, vertices and edges of any polyhedron. It is written F + V = E + 2 , where F is the number of faces, V is the number of vertices and E is the number of edges.

even number
An integer that is divisible by 2. The even natural numbers are {0, 2, 4, 6, ...}

event
A subset of the sample space of a random experiment.

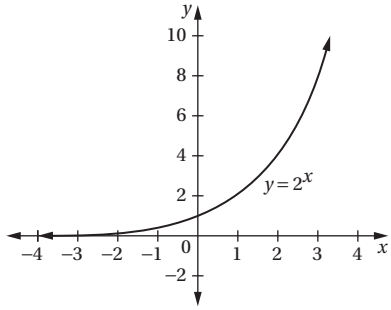
expanded notation
A way of representing a number, separating each digit and representing it in an additive number sentence to show each digit's place value, e.g. $2345 = 2 \times 1000 + 3 \times 100 + 4 \times 10 + 5$.

expected frequencies
The number of times that a particular event or outcome is predicted to occur using theoretical probability.

exponent
The power (or index) to which a number or algebraic expression is raised, shown using superscript, e.g. in the following expressions the exponent is 3: 2^3, $2x^3$, $2(x + 5)^3$.

exponent laws
Rules for manipulating exponents. Also referred to as index laws.

exponential functions
Functions comprising a positive constant raised to the power of the variable; i.e. $f(x) = ax$ where a is a constant and x is the variable.



expression
Two or more numbers or variables connected by operations.

factors
Let a, b and c be natural numbers such that $a \times b = c$, then a and b are factors (or divisors) of c, e.g. $3 \times 4 = 12$, so 3 and 4 are factors (divisors) of 12.

formulating expressions
The process of using mathematical notation, conventions and symbols to represent situations (including informal or worded statements) as mathematical expressions.

fraction
A number of the form $\frac{a}{b}$, where a is an integer and b is a non-zero integer, e.g. $\frac{4}{6}, \frac{11}{5}, \frac{-3}{7}$. A fraction is said to be in simplest form when a and b have no common divisor greater than 1.

frequency
The number of times that a particular value occurs in a data set. For grouped data, it is the number of observations that lie in that group or class interval.

frequency table
A table listing the frequency (number of occurrences) of observations, measurements or outcomes in different ranges, called class intervals.

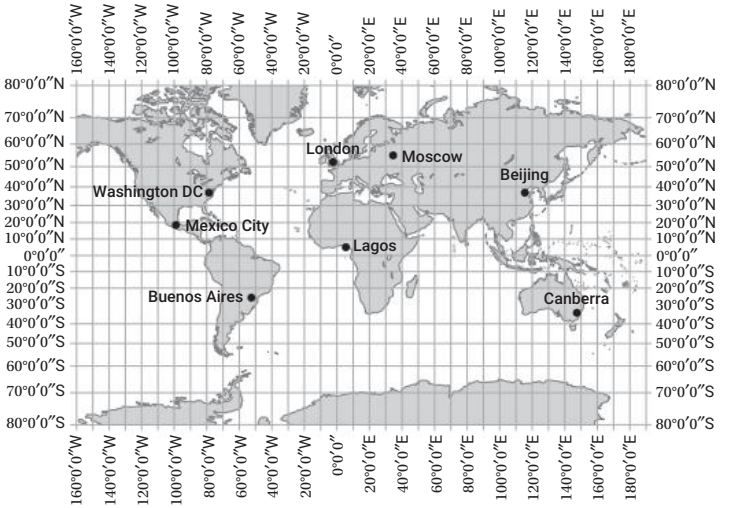
function
A correspondence (map or relation) between the elements of two sets where each element in the first set is mapped to exactly one corresponding element in the second set, either one-to-one or many-to-one.

function machine
An algorithmic process which takes an input, applies an operation (or operations) and results in an output.

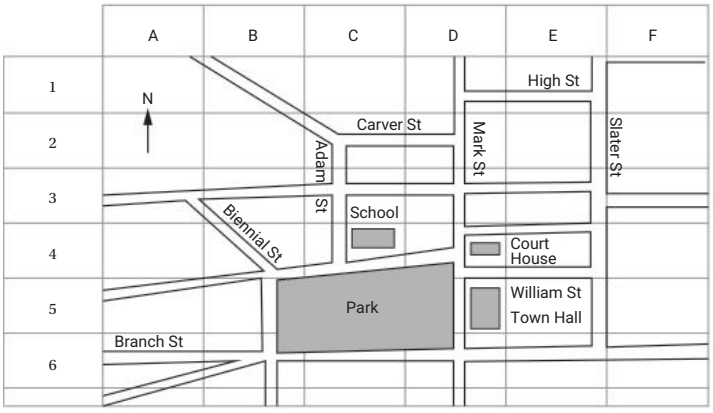
geometric patterns
Patterns formed by a series of shapes. They may be repeating or growing patterns which change in predicted ways that can be generalised. Tessellations are geometric patterns formed by transforming shapes.

gradient
The slope of a line. It is calculated as the gradient of a line segment it contains. The gradient (m) of the line containing $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated as $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

grid coordinate system
A system for referencing location within a space using labelled horizontal and vertical grid lines. Locations are defined using a set of coordinates which indicate points in the plane where two grid lines meet.



grid reference
A way of identifying a region on a map. Coordinates and grid lines are used to refer to specific features or locations, e.g. in the map shown, the school is located at the grid reference C4.



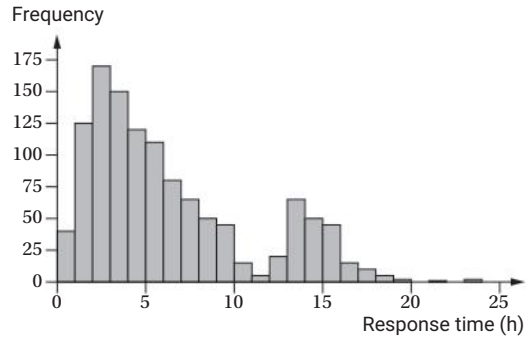
grouping
A process of partitioning a set into subsets according to a given criterion, e.g. a class of students according to eye colour. Equal grouping means each subset has the same number of elements.

growing pattern
A pattern where each term is either an increase or a decrease on the previous term. Change from term to term is predictable if the pattern is growing additively, multiplicatively or exponentially.

growth
A measure of the increase in value of the dependent variable of a function as the value of the independent variable increases, e.g. the length of a spring when weights are added.

guided statistical investigation
A teacher-led, scaffolded approach to the statistical investigation process, including collecting, representing and analysing data, and interpreting and communicating results.

histogram
A statistical graph for displaying the frequency distribution of continuous data. It is also a graphical representation of the information contained in a frequency table.



improper fraction
A fraction where the numerator is greater than or equal to the denominator, e.g. $\frac{7}{5}$ is an improper fraction, whereas $\frac{5}{7}$ is a proper fraction.

inequality
A mathematical expression containing the terms 'less than', 'less than or equal to', 'greater than', or 'greater than or equal to' and their respective symbolic representations $<$, $>$, $\leq$ and $\geq$.

inference (statistics)
The process for determining consequences or conclusions based on assumptions, evidence and reasoning.

informal units
Units whose values are decided on in a given context, e.g. the use of pace to measure distance (non-uniform unit); the use of paperclips to measure length (uniform unit).

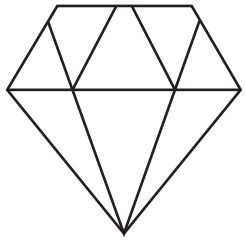
integer
An element of the infinite set of numbers $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$.

interval(s)
A continuous subset of the real number line, e.g. 'the set of all real numbers greater than or equal to 10'.

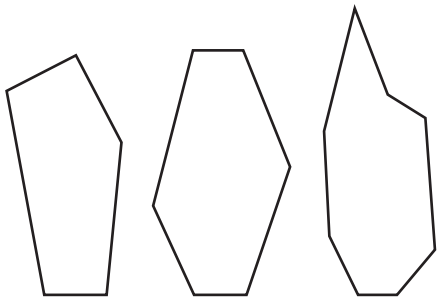
inverse operation
An operation in arithmetic which undoes the effect of another operation. Multiplication and division are inverse operations, as are addition and subtraction.

irrational number
A number that cannot be expressed as a fraction in the form $\frac{m}{n}$, where m and n are integers and n is non-zero. The decimal expansions of such numbers are infinite and non-recurring.

irregular object
An object with faces of varying shapes. Contrast with a regular object, such as a cube.

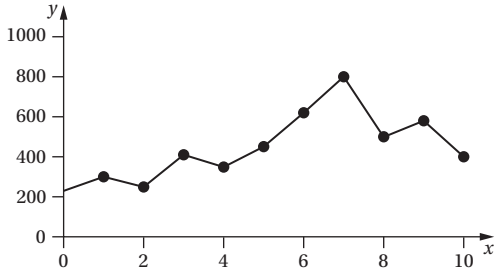

diamond

irregular shape
A shape with sides of varying lengths. Contrast with a regular shape, such as a rhombus or equilateral triangle.



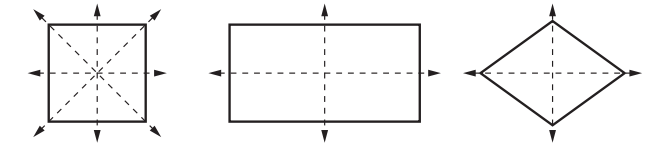
line
A model of what is perceived visually as straight. It has one dimension and extends indefinitely in the plane.

line graphs
Graphs that represent the changing relationship between two variables. They use lines to connect individual data points, and are generally used to show how something changes over time.



line segment (interval)
Let A and B be two distinct points on a line, then the part of the line between and including A and B is a line segment.

line symmetry
A line in which a shape is symmetrical, such that if you were to fold the shape about this line it would map onto itself exactly.


Square
4 lines of symmetry

Rectangle
2 lines of symmetry

Rhombus
2 lines of symmetry

linear equations
Equations involving linear expressions. The general form of a linear equation in one variable is $ax + b = c$, where a, b and c are constants.

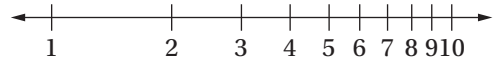
linear expression
An expression involving only variables to the power 1, and constants, e.g. $2x + 3$, $180(n - 2)$, $4x - 5y$.

linear functions
A function which when graphed on a Cartesian plane produces a straight line. It can be reduced to the general form $y = mx + b$.

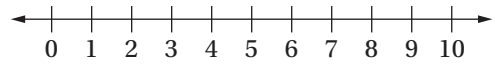
linear inequalities
Inequalities that involve linear expressions and contain one of the inequality signs: either $<$, $>$, $\leq$ or $\geq$.

logarithmic scales
Scales where successive endpoint values of intervals increase by a constant factor (multiplicatively). Contrast with linear scales in which the increase is a constant amount.

Logarithmic scale



Linear scale



mass
The measure for how much matter an object contains. The SI unit for mass is kilogram (kg).

mathematical modelling
A mathematical process involving the identification of a real-world problem, formulation and calculation of a solution to the problem mathematically using knowledge and understanding of mathematical concepts, structures and relationships, and the interpretation and communication of the mathematical solution(s) in terms of the situation. As students', mathematical modelling skills become more sophisticated, they begin to consider assumptions, choose variables, justify decisions and evaluate the effectiveness of the model.

mean
The sum of values in a data set divided by the total number of values in the data set. Also called the average.

measure
A record of the magnitude of an attribute (such as weight, length, duration and likelihood) associated with an object or event.

measure of central tendency
A statistic that is used to summarise a data set. There are three common measures of centre for a data set: mode, median and mean.

measure of turn
A measurement of rotation from an initial starting position. A turn can be a fraction of a circle (quarter turn) or complete rotation (revolution). It can also be directional (clockwise or anti-clockwise).

median
The value in a set of ordered data that divides the data into two equal parts. It is often called the 'middle value'.

mixed numeral
A form of representation of a rational number (also called a mixed fraction or mixed number) composed of an integer and a proper fraction, i.e. a fraction with a smaller numerator than denominator, e.g. $2\frac{3}{5}$.

mode
The most frequently occurring value in a set of data. There can be more than one mode. When there are two modes, the data set is said to be bimodal.

modelling
Using mathematical concepts, structures and relationships to describe and characterise, or model, a situation in a way that captures its essential features.

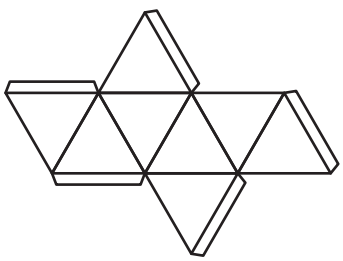
multiples
A multiple of a number is the product of that number and an integer. A multiple of a real number x is any number that is a product of x and an integer.

multiplication facts
The results associated with the products of pairs of natural numbers from 0 to 9, associated with reasons. They are foundational to arithmetic.

multiplicative
Problems or contexts that involve multiplication or division, e.g. calculating the number of seats in a theatre that has 30 rows of 24 seats.

natural numbers
The set $N = \{0, 1, 2, 3, ...\}$ or $N = \{1, 2, 3, ...\}$ depending on whether counting is started at 0 or 1. The elements of N are also called the counting numbers, used to count the number of elements in finite sets.

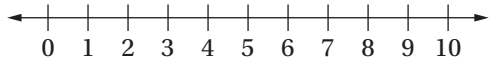
nets
Plane figures that can be folded to form a polyhedron. More specifically, two-dimensional representations comprising joined shapes (the faces) that can be folded (along edges) to form the object.



network

A set of points (vertices or nodes), some of which are joined by lines or curves (edges) and sometimes enclose regions (faces), e.g. road networks, a family tree or the edges lining a tennis court.

number line
A line that gives a pictorial representation of real numbers.



number sentence
A statement of equality or inequality using numbers, operations and common symbols, e.g. $8 + 5 = 13$ and $16 - 6 = 10$.

numeral
The designation of a number in a given language, e.g. the number 'three' is designated by the Hindu-Arabic numeral 3, the Roman numeral III, and the Chinese numeral 三.

numerical data
Data associated with a numerical variable. It can be discrete or continuous, e.g. number of siblings in a family or heights of students in the class.

numerical variables
Variables with values that are numbers, and for which measuring and arithmetic processes, such as adding and subtracting or calculating an average, make sense.

object
A solid or three-dimensional (3D) skeletal representation composed of a number of faces, edges, vertices or closed surfaces, such as a prism, pyramid, cylinder or sphere. The term three-dimensional shape is also used.

obtuse angle
An angle that measures between 90 and 180 degrees, i.e. between one and two right angles.

odd number
An integer that is not divisible by 2. The odd numbers are $\{... -5, -3, -1, 1, 3, 5, ...\}$.

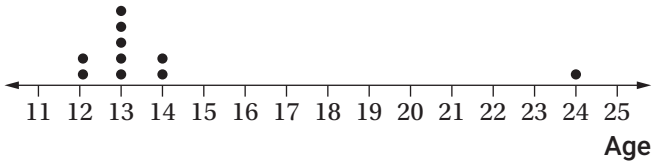
online tools
Digital hardware, software, platforms and resources that are connected and support communications between users. Examples of online tools include websites for online shopping and fitness apps.

operation
The process of combining numbers or expressions. Operations are arithmetic – addition, subtraction, multiplication and division – and also include exponentiation and substitution.

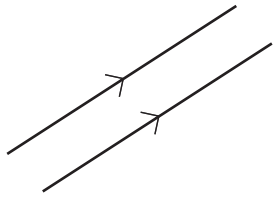
order of operations
A set of conventions for evaluating expressions involving several operations. Operations in brackets are first, followed by exponents, multiplication/division, then addition/subtraction left to right.

ordinal categorical data
Categorical data that has a ranking or order, such as Likert scales, clothing sizes, placegetters in a race.

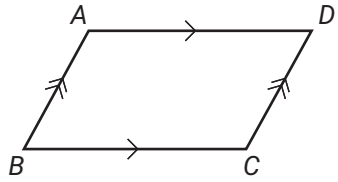
outlier
A data value that appears to stand out from the other members of the data set by being unusually high or low.



parallel lines
Two lines L and M in a plane that have no point of intersection in that plane, written $L \parallel M$.



parallelogram
A quadrilateral in which opposite sides are parallel and equal in length. The quadrilateral $ABCD$ shown is a parallelogram because $BA \parallel CD$ and $AD \parallel BC$.



parameters
A factor for which a specific value defines a condition or constraint for a function or relation.

partition numbers
Separating numbers additively or multiplicatively into two or more parts, e.g. 10 is 8 + 2 and 8 is 3 + 3 + 2. 12 can be divided into six equal parts of 2 (so $12 = 6 \times 2$ or $12 \div 6 = 2$), or into 3 equal parts of 4 (so $12 = 3 \times 4$ or $12 \div 3 = 4$).

partitioning
The ability to think about numbers as made up of two or more parts. Numbers can be partitioned into standard or non-standard place value partitions, such as 248 as $200 + 40 + 8$ or 62 as $50 + 10 + 2$.

part-part-whole reasoning
A model used to recognise relationships between a whole number and its parts, e.g. $7 = 5 + 2$ or $7 = 4 + 3$.

percentage
A ratio to 100 or a fraction whose denominator is 100.

perfect square
A number that is the square of an integer, e.g. 169 is a perfect square as $13^2 = 169$. Perfect squares can also be represented pictorially.

perimeter
The total length of the boundary of a plane figure or space.

perpendicular
Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle) are considered perpendicular.

pi (π)
The ratio of the circumference of any circle to its diameter. It is denoted by the Greek letter π (pi). Approximate values are 3.14 or $\frac{22}{7}$.

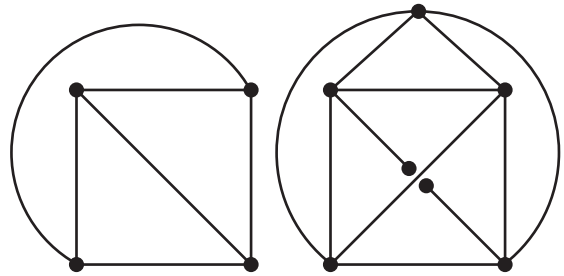
picture graph (pictographs)
A statistical graph for organising and displaying categorical data.

Ball sports played by students in Year 4	
Football	⚽ ⚽ ⚽ ⚽
Basketball	⚽ ⚽ ⚽
Netball	⚽ ⚽ ⚽ ⚽ ⚽
Soccer	⚽ ⚽
Rugby	⚽ ⚽ ⚽
Hockey	⚽ ⚽

Key ⚽ = 10 students

place value
The value of a digit as determined by its position in a number relative to the ones (or units) place. For integers, the ones place is occupied by the rightmost digit in the number.

planar graph
A connected graph that is drawn within a plane whose edges do not cross.



platonic solids
Three-dimensional, convex and regular objects that have polygonal faces. The five platonic solids are the tetrahedron, cube, octahedron, dodecahedron and icosahedron.

point
Representation of a location in space that has zero dimensions. Lines, curves, shapes, surfaces and objects are constructed from sets of points. Points on the Cartesian plane are specified by coordinates.

polyhedra
Three-dimensional objects whose faces are adjacent polygons. A convex polyhedron is a finite region bounded by planes, in the sense that the region lies entirely on one side of the plane.

population
The complete set of individuals, objects, places, etc. that we want information about. A census collects information about the whole population.

prime number
A natural number that is greater than 1 and its only factors are 1 and itself.

prism
A convex polyhedron that has two congruent and parallel polygonal faces; all its other faces are parallelograms. It is named according to these two congruent faces, e.g. a triangular prism.

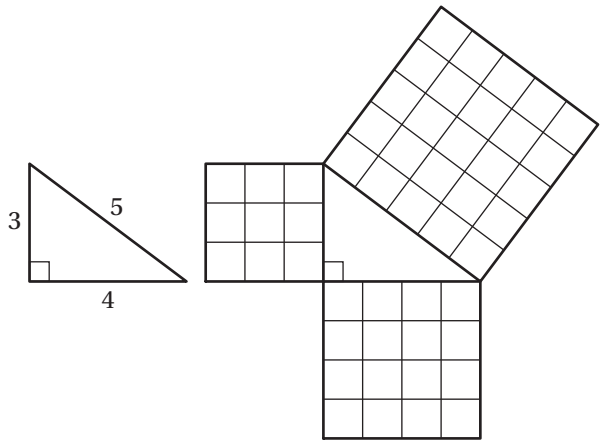
probability
The chance of something happening shown on a scale from 0 to 1 (inclusive), e.g. the probability that a fair coin toss will come up 'heads' is 0.5.

product
The result of multiplying together two or more numbers or algebraic expressions.

proof
A mathematical argument that demonstrates whether a proposition is true. A mathematical statement that has been proved is called a theorem.

proportion
A constant ratio between corresponding elements of two sets, e.g. the circumference and diameter of a circle are in proportion as their ratio is the constant π (pi).

Pythagoras' theorem
The square of the length of the hypotenuse, c, of a right-angled triangle equals the sum of the squares of the lengths of the other two sides, a and b, such that $c^2 = a^2 + b^2$.



quadratic equation
A quadratic expression that has a particular value. The general quadratic equation in one variable is $ax^2 + bx + c = 0$, for constants a, b, c and where $a \neq 0$.

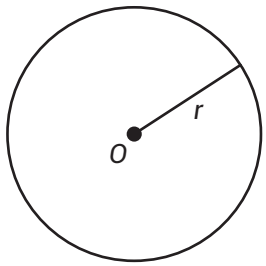
quadratic expression
An expression that contains one or more of the terms in which the variable is raised to the second power, but no variable is raised to a higher power. Its general form is $ax^2 + bx + c$, where a≠0.

quadratic function
A function in which the rule is a quadratic expression. When graphed on the Cartesian plane it produces a parabola and algebraically it can be represented in the form $y = ax^2 + bx + c$, where $a \neq 0$.

quadrilateral
A four-sided polygon. Examples include square (a regular quadrilateral, all sides and angles equal), rectangle, kite and trapezium.

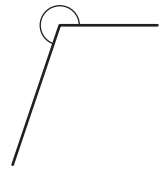
quotient
The result of dividing one number or algebraic expression by another.

radius
The distance from the centre to the circumference of a circle.

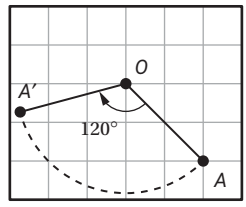


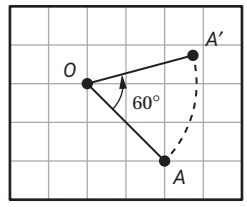
random sample
A subset of the population chosen such that every element of the population has an equal chance of being selected; i.e. their selection is governed by chance, e.g. by using a random number generator.

random variable
A function that assigns real numbers to random events.
range
A set of different things of the same general type, e.g. different representations of the same fraction or a variety of strategies for solving problems involving the basic operations.
range (statistics)
The difference between the largest and smallest numerical values in a data set. It can be used as a measure of spread in a data set. It is sensitive to outliers and should be interpreted with care.
rate
A comparison of two quantities measured in different units, e.g. the rate of distance to time, known as speed, because distance and time are measured in different units (such as kilometres and hours).
ratio
A comparison of magnitudes of sets, quantities of the same kind or algebraic expressions. It is often used as a comparison of the size of two (or more) quantities relative to each other. and is written as $a{:}b$, $a{:}b{:}c$, etc .
rational number
An element of the infinite set of numbers $Q = \frac{m}{n}$, where m and n are integers and $n \neq 0$. It may be expressed in decimal form, e.g. $\frac{1}{8} = 0.125$ and $\frac{4}{9} = 0.444\ldots$
real number line
The representation of real numbers using the infinite set of points on a continuous line.
real numbers
The numbers generally used in mathematics, scientific work and everyday life. They are any of the elements of the infinite set R, which includes all the rational numbers and irrational numbers.
reasonableness (calculation)
Checking the results of calculations or operations to see if they are reasonable in the context, e.g. the sum of two odd natural numbers must be an even number.
rectangle
A quadrilateral in which all angles are right angles.
recurring decimal
A decimal expansion of a number that contains a finite sequence of digits that repeats indefinitely, such as 34.271271271271... The repeating sequence is often designated by a bar over it, such as $34.\overline{271}$.
reflection
A transformation where each point of a shape is mirrored to a position the same distance from an axis of symmetry.
reflex angle
An angle that measures between 180 and 360 degrees, i.e. between two and four right angles.

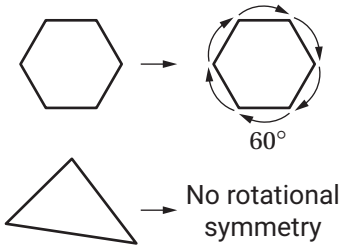


regular shape
A shape that has all sides of equal length, and all interior angles equal.
related denominators
One of the denominators is a multiple of the other denominator of a pair of fractions.
relation
A correspondence (map) between the elements of two sets, e.g. 'favourite colour' related to the set of students in a class (the domain) and the set of colours (the co-domain).
rename
To express a number according to the relationship between the place value powers of 10, e.g. 263 can be renamed 2 hundreds and 63 ones or 1 hundred, 16 tens and 3 ones.
repeated addition
Adding the same number several times, e.g. $3 + 3 + 3 + 3 = 4 \times 3 = 12$. A strategy sometimes used for multiplication.
repeating pattern
A pattern where there is an identifiable repeating unit, such as beads arranged in a sequence of red, green, blue, red, green, blue ...; days of the week or months of the year.
revolution
A complete turn around a point.
right angle
An angle formed by a quarter turn of a revolution. It is equal to 90 degrees. If two lines are at a right angle, they may also be referred to as being perpendicular.
right prism
Convex polyhedra that have two congruent and parallel faces and all remaining faces are congruent rectangles.
rotation
A transformation where each point in the plane is rotated through a given angle about a fixed point in the plane (the point or centre of rotation). For example, point A is rotated 120° clockwise and 60° anti-clockwise.





rotational symmetry
A quality of some shapes and objects to appear the same when turned around their centre for less than a full turn, e.g. a square has quarter-turn rotational symmetry.



rounding
The process for approximating a value that lies between two known values. It is used to specify a number correct to a given accuracy, e.g. 4.027 becomes 4.03 when rounded to 2 decimal places.

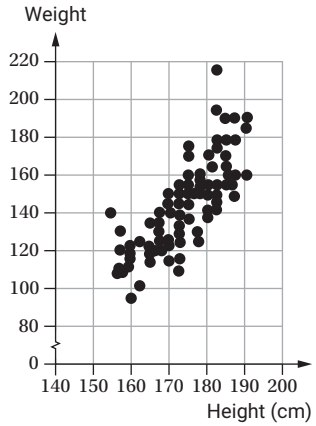
sample
A subset of a population used to estimate characteristics of the population, e.g. a randomly selected group of 8-year-olds (sample) selected to estimate the height of 8-year-olds in Australia (population).

sample space
The set of all possible outcomes of a chance experiment, e.g. the set of outcomes (also called sample points) from tossing two heads is {HH, HT, TH, TT}, where H represents a 'head' and T a 'tail'.

sampling
A process by which a subset of a population is selected for the purposes of data analysis. This subset is called a sample.

scale
Specification of proportion between two measures, e.g. a model of a house may be made on a 1:10 scale of length; on a graph, each interval of the vertical axis corresponds to a given value.

scatter plot
A visual representation of bivariate numerical data using the position of each dot on the horizontal and vertical axis to indicate the numerical values for an individual data point. Scatter plots are used to observe relationships between variables.



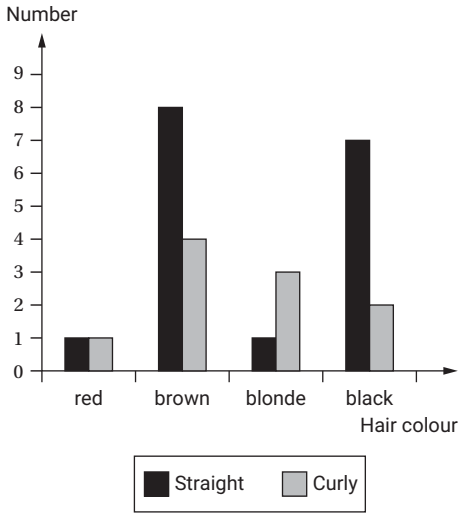
scientific notation
A way of writing numbers that are too big or too small to be written in an accessible way. It uses powers of 10, e.g. the scientific notation for 34 590 is 3.459×104.

set
A collection of objects or elements, usually specified by listing its elements (e.g. {1, 2, 3, 4}), by describing it in words (e.g. 'the set of primes') or by using a rule such as $\{(x, y): y = 2x + 1, x \in N\}$.

shape (space)
A geometric figure in 2-dimensional space, such as a freehand closed curve, a triangle or square. It can be regular or irregular, simple or complex.

shape (statistics)
The form of the visual representation of the distribution of numerical data, which may be symmetrical, skewed or multimodal.

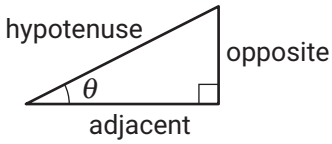
side-by-side column graph
A type of column graph that can be used to organise and display the data that arises when a group of individuals or things are categorised according to two or more criteria.



simulation
The process of simulating an event using technology or a device, e.g. if two players are equally likely to win a game on past performance, then games could be simulated by successive tossing of a fair coin.

simulations
A simulation is a re-creation of a random phenomena used to collect probability data using either physical or virtual manipulatives such as spinners, coins and cards or computer generated simulations, and which can be performed a large number of times.

sine ratio
In any right-angled triangle, $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.



skip counting
Counting from a given starting value using multiples of a fixed natural number, e.g. {2, 4, 6, ...} or {7, 12, 17, ...}.

spread
A statistic that indicates how widely the values of a data set are distributed. Common measures of spread include range, interquartile range, quantiles and percentiles.

square number
The result of multiplying a natural number by itself, e.g. $16 = 4 \times 4 = 42$, and is an element of the set {0, 1, 4, 9, 16, 25}.

square root
A value which, when multiplied by itself gives the original number. The positive square root of a given real number x is the positive real number y, such that $y^2 = x$.

statistical investigation
An investigation process involving a chosen context, area of interest or an identified problem scenario, identifying the subjects and variables to be investigated, planning the approach, collecting data, representing, interpreting and analysing the data, making inferences and communicating the results of the investigation in terms of the context.

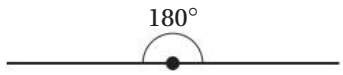
stem-and-leaf plot
A method of organising and displaying numerical data in which each data value is split into two parts: a 'stem' and a 'leaf'. Stem plots give a visual indication of spread.

Pulse rate

Stem	Leaves
6	8 8 8 9
7	0 1 1 4 6 6 8
8	2 6 8 8
9	0 6
10	4
11	0

key 6|8 = 68 bpm

straight angle
The angle formed by two opposite rays on a straight line from a common point. A straight angle is also formed by the half-turn of a ray about its endpoint. It measures 180 degrees.



subitising
The capacity to visually recognise the size of a small set of objects without counting.

supplementary angles
Two adjacent angles that form a straight angle and their sum is 180 degrees (a straight angle).

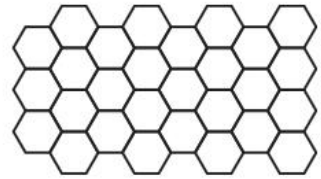
surface area
The measure of the total area of the surface(s) of a three-dimensional shape or object.

survey
A tool for collecting data that comprises a series of questions, which can be constructed to collect quantitative or qualitative data, numerical or categorical responses.

tangent ratio
The ratio of the length of the altitude, a, of a right-angled triangle to the length of its base, b, for a given base angle, theta, θ.

terminating decimal
A decimal expansion of a number which has a finite number of non-zero digits, e.g. 3.125.

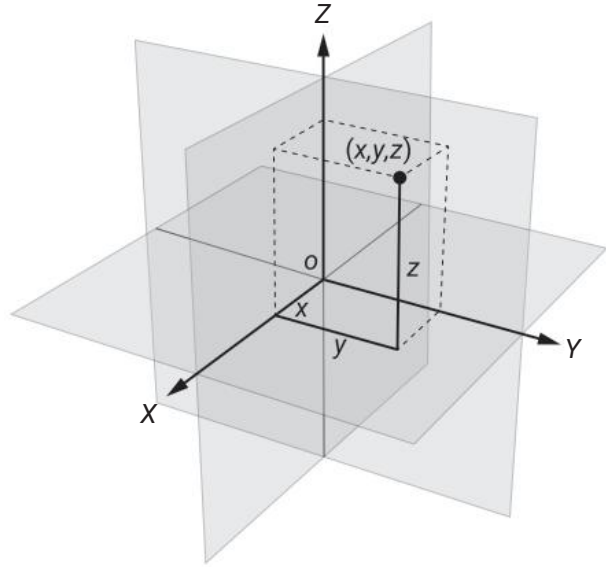
tessellation
A repeated pattern in the plane or on a surface where shapes completely fill all of the space around a given point where their boundaries meet, e.g. a honeycomb is a tessellation using hexagons.



theorem
A statement which has been derived from given statements using agreed rules of deduction (proof).

three-dimensional
An object that has three independent dimensions of length, width (breadth) and depth, or a space in which position is specified by three independent coordinates.

three-dimensional coordinate system
A coordinate system where any point in space is defined using three different dimensions, e.g. longitude, latitude and altitude for locating the position of a drone in the sky.



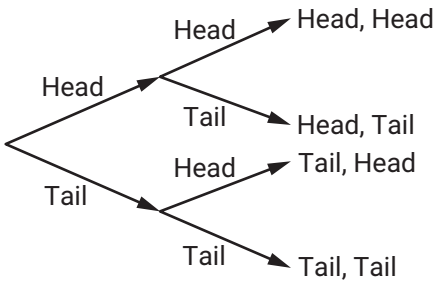
time zones
The 24 divisions of the globe, where each change of 15 degrees longitude corresponds to one hour. As time zones often align with national or regional boundaries, this is an approximate relationship.

transformation
A map of the plane onto itself. Transformations include dilation (enlargements or compressions), reflections, rotations and translations.

translation
Shifting a figure in the plane without turning it. Translations can be specified as a combination of a horizontal shift and a vertical shift.

transversal
A line that intersects two other lines obliquely.

tree diagram
A diagram consisting of line segments (edges) connected to points (vertices) like the branches of a tree. It shows the relationship between sets, events, or the set of outcomes of a multi-step random experiment.



trials
Any repeatable procedures with a well-defined sample space, e.g. the flipping of a coin.

trigonometric ratios
Relationships between the angles and sides of right triangles. The three basic trigonometric ratios are: sine (sin), cosine (cos) and tangent (tan).

two-dimensional
A shape that has two independent dimensions of length and width (breadth) is two-dimensional, as is a space in which position is specified by two independent coordinates, such as the plane.

two-way tables
A common way of displaying the two-way frequency distribution that arises when a group is categorised according to two criteria.

	Curly hair	Straight hair	Total
Red hair	1	1	2
Brown hair	8	4	12
Blonde hair	1	3	4
Black hair	7	2	9
Total	17	10	27

unit fraction
A fraction of the form $\frac{1}{n}$, where n is a non-zero natural number, such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

unit of repeat
In a repeating pattern, the pattern sequence that is repeated within the pattern, e.g. 1234 in the pattern 1234123412341234...

units
The basis for counting or measurement or both, e.g. the number 1 is the unit for counting; the metre is the standard unit for measurement of length in the metric system.

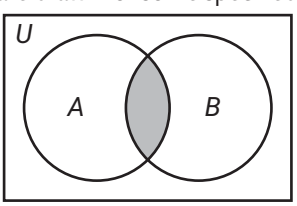
univariate data
Data relating to measurement of a single variable, e.g. shoe size.

validate (data)
The process of checking that data to be analysed is reasonable and fit for purpose. This can involve checking for accuracy, clarity, detail, consistency, correctness and completeness.

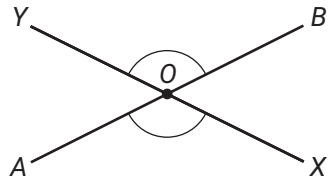
variable
Things that are measurable or observable and are expected to either change over time or between individual observations. They are often designated by symbols, such as n, x and y, to represent members of a set.

variation in data
The characteristic that elements of a data set differ, e.g. the maximum temperature recorded at different suburbs of a town on a given day.

Venn diagrams
Graphical representations, using several typically overlapping circles, showing elements of sets in relation to properties or attributes. They are drawn for some specified universal set.



vertically opposite angles
Two pairs of vertically opposite angles formed at the point of intersection of two lines in the plane. Vertically opposite angles are equal.



virtual materials
Dynamic digital representations of physical materials (sometimes called virtual manipulatives).

visualisations
Visual formats for organising, representing and summarising data. Also called data displays.

volume
The amount of space bounded by the surface of an object. It is usually measured in cubic units.

zero
The number represented by the numeral or the symbol 0, e.g. the origin on a number line. It also functions as an empty place-holder digit in the decimal expansion of numbers, e.g. 205.

Based on Australian Curriculum, Assessment and Reporting Authority (ACARA) materials

ANSWERS

Topic 1: Number properties (exponents)

Recall

I can calculate squares and square roots

1 **(a)** $\sqrt{25} = \sqrt{(5 \times 5)} = 5$
(b) $\sqrt{64} = \sqrt{(8 \times 8)} = 8$
(c) $\sqrt{81} = \sqrt{(9 \times 9)} = 9$
(d) $\sqrt{121} = \sqrt{(11 \times 11)} = 11$

2 **(a)** 10 **(b)** 14 **(c)** 7 **(d)** 13
(e) 6 **(f)** 16 **(g)** 15

I can use exponent notation to represent numbers

1 **(a)** 4^2 **(b)** 10^3 **(c)** 3^3 **(d)** 2^5
(e) 5^4 **(f)** 7^2 **(g)** 8^5

I can use expanded notation to represent numbers

1 **(a)** $5^2 = 5 \times 5$ **(b)** $6^4 = 6 \times 6 \times 6 \times 6$
(c) $8^5 = 8 \times 8 \times 8 \times 8 \times 8$
2 **(a)** $15^3 = 15 \times 15 \times 15$ **(b)** $9^4 = 9 \times 9 \times 9 \times 9$
3 **(a)** $3^4 = 3 \times 3 \times 3$
$= 9 \times 9$
$= 81$
(b) $6^3 = 6 \times 6 \times 6$
$= 36 \times 6$
$= 216$
(c) $10^4 = 10 \times 10 \times 10 \times 10$
$= 100 \times 100$
$= 10\,000$

I can represent numbers in prime factor form

1 **(a)** $44 = 2 \times 2 \times 11$
$= 2^2 \times 11$
(b) $135 = 9 \times 15$
$= 3 \times 3 \times 3 \times 5$
$= 5 \times 3^3$

1.1 Establish and apply the exponent law for multiplication

SC 1

1 **(a)** 3 **(b)** 3×3
(c) $3 \times 3 \times 3$ **(d)** $3 \times 3 \times 3 \times 3$
2 **(a)** 5^1 **(b)** 5^2 **(c)** 5^3 **(d)** 5^4
3 Sample answer: Expanded form shows how many times the base must be multiplied by itself.
4 **(a)** $3 \times (3 \times 3) = 3^3$
(b) $3 \times (3 \times 3) \times (3 \times 3 \times 3) = 3^6$
(c) $3 \times (3 \times 3) \times (3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^{10}$
5 **(a)** $4 \times (4 \times 4) = 4^3$
(b) $2 \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^7$
(c) $(5 \times 5) \times 5 \times (5 \times 5 \times 5) = 5^6$
6 **(a)** $2 \times 3 \times 3 \times 3 = 2 \times 3^3$
(b) $2 \times (2 \times 2) \times 3 \times (3 \times 3) = 2^3 \times 3^3$
(c) $2 \times (2 \times 2) \times (2 \times 2 \times 2) \times 3 \times (3 \times 3) = 2^6 \times 3^3$
7 **(a)** The answer given is 3 times 2, not 2^3.
(b) The question means $2^1 \times 2^3 = 2^4$
(c) The bases must be the same. You cannot multiply different bases in this way.
(d) The question means $2^1 \times 2^2 \times 3^2 = 2^3 \times 3^2$
8 **(a)** Add the powers when multiplying numbers with the same base.
(b) **(i)** 2^5 **(ii)** 2^6 **(iii)** 2^7 **(iv)** 2^8
9 **(a)** When you multiply exponent expressions with the same base, the powers are added.
For example, to find $4^2 \times 4^4$, keep the base and add the exponents. $4^{(2+4)} = 4^6$
(b) No, because the bases are different.

SC 2

1 **(a)** 5^3 **(b)** 5^4 **(c)** 5^5 **(d)** 5^9
2 **(a)** 8^5 **(b)** 6^6 **(c)** 3^9 **(d)** 7^{12}
(e) 9^9
3 **(a)** $5^3 \times 3^3$ **(b)** $5^3 \times 3^7$ **(c)** $2^{11} \times 3^2$
(d) $2^{12} \times 3^4$ **(e)** $2^5 \times 5^9$ **(f)** $3^5 \times 10^5$
(g) $3^2 \times 4^6 \times 5^5$
4 **(a)** $2^1 \times 2^2 \times 2^3 = 2^6$
The answer ignores the fact that $2 = 2^1$.
(b) $2^1 \times 2^2 \times 2^3 = 2^6$
The answer multiplied the bases in addition to adding the powers.
(c) The answer added the bases to get 6.
(d) $3^3 \times 2^2 \times 2^3 = 3^3 \times 2^5$
The answer added (or multiplied) the base 2s.
5 **(a)** 1 **(b)** −1 **(c)** 1 **(d)** −1
(e) −1, −1 raised to an odd power is negative.
A number raised to an even power is positive.
6 **(a)** $\left(\frac{1}{2}\right)^4$ **(b)** $\left(\frac{1}{3}\right)^5$ **(c)** 2.1^4 **(d)** 0.8^6
7 **(a)**

Fraction	Squared	Cubed	Power of 4
$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$	$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$
$\frac{1}{4}$	$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$	$\left(\frac{1}{4}\right)^3 = \frac{1}{64}$	$\left(\frac{1}{4}\right)^4 = \frac{1}{256}$

(b) Multiplying by a fraction decreases the number.
Multiplying by $\frac{1}{2}$ is the same as dividing by 2.

8 No. $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and $\frac{1^2}{2} = \frac{1 \times 1}{2} = \frac{1}{2}$

1.2 Establish and apply the exponent law for division

SC 1

1 **(a)** 4
(b) $4 \times 4 = 16$
(c) $4 \times 4 \times 4 = 64$
(d) $4 \times 4 \times 4 \times 4 = 256$
(e) $4 \times 4 \times 4 \times 4 \times 4 = 1024$

2 **(a)** $\frac{4 \times 4}{4}$ **(b)** $\frac{4 \times 4 \times 4}{4}$
(c) $\frac{4 \times 4 \times 4}{4 \times 4}$ **(d)** $\frac{4 \times 4 \times 4 \times 4}{4 \times 4}$
(e) $\frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4}$ **(f)** $\frac{4 \times 4 \times 4 \times 4}{4 \times 4 \times 4}$
(g) $\frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4}$

3 **(a)** 6^3 **(b)** 6^4 **(c)** 6^5
(d) 8^3 **(e)** $\left(\frac{2}{5}\right)^3$ **(f)** 106.3^3
(g) 9^4 **(h)** $\left(\frac{3}{4}\right)^4$ **(i)** 2.8^4

4 **(a)** $\frac{7 \times 7}{7} = 7^1$ **(b)** $\frac{7 \times 7 \times 7}{7} = 7^2$
(c) $\frac{7 \times 7 \times 7}{7 \times 7} = 7^1$ or 7 **(d)** $\frac{7 \times 7 \times 7 \times 7}{7 \times 7} = 7^2$
(e) $\frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7} = 7^3$ **(f)** $\frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7} = 7^2$

5 **(a)**

Expression	Expanded form	Exponent form	Difference in exponents
$\frac{3^5}{3}$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3}$	3^4	$5 - 1 = 4$
$\frac{3^5}{3^2}$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$	3^3	$5 - 2 = 3$
$\frac{3^5}{3^3}$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3}$	3^2	$5 - 3 = 2$
$\frac{3^5}{3^4}$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$	3^1	$5 - 4 = 1$

(b) The power in the final exponent number is the same as the difference of the exponents.

(c) The second law of exponents states that when dividing exponent expressions with the same base, you subtract the powers.

SC 2

1 **(a)** 3^4 **(b)** 3^3 **(c)** 3^2
(d) 3^1 **(e)** 3^5 **(f)** 3^4
2 **(a)** 4^4 **(b)** 4^7 **(c)** 6^4 **(d)** 10^6
3 **(a)** $x = 4$ **(b)** $x = 5$ **(c)** $x = 11$
(d) $x = 2$ **(e)** $x = 8$
4 **(a)**, **(c)**, **(e)**, **(f)** different bases.
5 The 8 and 4 are not the same base, so he cannot apply the second law of exponents.
6 Multiplication and division are inverse operations, as are addition and subtraction. Multiplying exponent expressions involves adding exponents, while dividing involves subtracting exponents.

SC 3

1 **(a)** 10^2 **(b)** 10^3 **(c)** 10^5 **(d)** 10^3 **(e)** 10^4
2 **(a)** 2^5 **(b)** 3^5 **(c)** 4^7 **(d)** 6^3 **(e)** 8^2
3 **(a)** $2^5 \times 5^2$ **(b)** $\frac{3^5}{4}$ **(c)** $4^7 \times 3^5$
(d) $\frac{6^3}{7^2}$ **(e)** 5×8^2
4 **(a)** $x = 2$ **(b)** $x = 5$ **(c)** $x = 9$
(d) $x = 5$ **(e)** $x = 12$
5 **(a)** 2^6 **(b)** 3^2 **(c)** 4^{10}
(d) $6^4 \times 7^3$ **(e)** $5^4 \times 8^3$
6 Sample answer. When exponent expressions with the same base are divided, the powers or exponents are subtracted.
For example, $4^{10} \div 4^3 = 4^{(10-3)} = 4^7$

1.3 Establish and apply the exponent law for raising a power to a power

SC 1

1 **(a)** 5^8 **(b)** 5^{10} **(c)** 5^{12}
(d) 3^4 **(e)** 3^6 **(f)** 3^8
2 **(a)**

Expression	Expansion	Simplified form	Multiplying the exponents
$(6^3)^2$	$6^3 \times 6^3$	6^6	$2 \times 3 = 6$
$(7^4)^2$	$7^4 \times 7^4$	7^8	$4 \times 2 = 8$
$(10^2)^3$	$10^2 \times 10^2 \times 10^2$	10^6	$2 \times 3 = 6$
$(2^4)^3$	$2^4 \times 2^4 \times 2^4$	2^{12}	$4 \times 3 = 12$

(b) The exponent of the simplified expression is the product of the original exponents.

(c) When a power is raised to a power, the exponents are multiplied.

3 Sample answer: When a power is raised to a power, multiply the exponents.
For example, $(4^3)^2 = 4^3 \times 4^3 = 4^6$ or multiply the exponents. $4^{3\times 2} = 4^6$

SC 2

1 **(a)** 2^{12} **(b)** 2^{15} **(c)** 2^{18}
(d) 2^6 **(e)** 2^8 **(f)** 2^{10}

2 **(a)** 6^{12} **(b)** 7^{10} **(c)** 8^9
(d) 4^{10} **(e)** 10^{20} **(f)** 3^{10}

3 **(a)** $\left(\frac{2}{3}\right)^{12}$ **(b)** $\left(\frac{7}{8}\right)^{10}$ **(c)** 10.5^{12}
(d) 4.4^{15} **(e)** 1.05^{10} **(f)** 2.3^{12}

4 **(a)** $\left(\frac{1}{3}\right)^3 = \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{1^3}{3^3} = \frac{1}{3^3}$, but

$\left(\frac{2}{3}\right)^3 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{2^3}{3^3} \neq \frac{2}{3^3}$

(b) $(4^3)^5 = (4^5)^3$ because the new power is the product of 3 and 5. The order is not important. $3 \times 5 = 5 = 3$

SC 3

1 **(a)** 2^{14} **(b)** 2^{15} **(c)** 2^{10} **(d)** 2^{17} **(e)** 2^{19}

2 **(a)** 2^{10} **(b)** 2^9 **(c)** 2^6 **(d)** 2^7 **(e)** 2^9

3 **(a)** 2^2 **(b)** 3^1 **(c)** 4^2 **(d)** 6^5 **(e)** 8^1

4 **(a)** $2^{12} \times 5^2$ **(b)** $3^4 \times 4^9$ **(c)** $4^3 \times 3^5$
(d) $6^6 \times 7^3$ **(e)** $5^3 \times 8^7$

5 **(a)** 2^{14} **(b)** 3^3 **(c)** 4^6 **(d)** $6^4 \times 7^8$
(e) $5^4 \times 8^5$ **(f)** 3^6 **(g)** 3^{14}

6 The order of operations has been incorrectly applied. Operations inside the brackets must be done first.
$(8^{10} \div 8^2)^3 = (8^8)^3 = 8^{24}$

7 **(a)** $V = 10^3 = 1000 \text{ cm}^3$
(b) $V = 20^3 = 8000 \text{ cm}^3$
(c) $V = (2 \times 10)^3$
$= (2 \times 10) \times (2 \times 10) \times (2 \times 10)$
$= 2^3 \times 10^3$
$= 2^3 \times 1000 \text{ cm}^3$
$= 8000 \text{ cm}^3$

8 5

9 $x = 5$ and $y = 3$

1.4 Establish and apply the exponent law for raising to the power of 0

SC 1

1

Question	Answer	Exponent form	Simplest exponent form answer
$10 \div 10 =$	1	$10^1 \div 10^1 =$	10^0
$4 \div 4 =$	1	$4^1 \div 4^1 =$	4^0
$4^2 \div 4^2 =$	1	$4^2 \div 4^2 =$	4^0
$5^2 \div 5^2 =$	1	$5^2 \div 5^2 =$	5^0
$5^3 \div 5^3 =$	1	$5^3 \div 5^3 =$	5^0

2 **(a)** Complete this table.

10^5	10^4	10^3	10^2	10^1	10^0
100 000	10 000	1000	100	10	1

(b) The exponent values decrease by 1 as we move to the right.

(c) The number of zeroes decreases by 1 as we move to the right.

(d) Based on this pattern, the value of 10^0 is 1.

3 Sample answer: Expressed in exponent form $\frac{x^y}{x^y} = x^0$, and any number divided by itself equals 1, so x^0 must equal 1.

4 **(a)** $x = 0$ **(b)** $x = 3$ **(c)** $x = 0$

SC 2

1 **(a)** 2×5^2 **(b)** $3^4 \times 4^{15}$
(c) 4^3 **(d)** $6^3 \times 7^3$

2 **(a)** 2^{11} **(b)** 3^3 **(c)** 4^1 **(d)** 6^5
(e) $5^4 \times 8^3$ **(f)** 3^{10} **(g)** 3^4

3 **(a)** $x = 2$ **(b)** $x = 2$ **(c)** $x = 11$
(d) $x = 2$ **(e)** $x = 9$

4 **(a)** Sample answer:
$\frac{0^4}{0^4} = 0^0 = 1$? Any number divided by itself is equal to 1.
$\frac{0^1}{0^1} = 0^0 = 0$? Because 0 is divided by something the answer must be 0.
$\frac{0^1}{0^1} = 0^0$ is undefined because any number divided by 0 is undefined.

(b) The accepted answer is that it is undefined.

Topic 2: Operations with integers, fractions and decimals

Recall

I can multiply and divide whole numbers

1 **(a)** 72 **(b)** 28 **(c)** 240 **(d)** 12

I can round decimals for different purposes

1 **(a)** 5.264 **(b)** 11.77 **(c)** 0.39 **(d)** 8.01
(e) 21 **(f)** 1.0 **(g)** 4.00

I can add and subtract decimals

1 **(a)** 38.97 **(b)** 59.3 **(c)** 2.94 **(d)** 5.58

I can multiply decimals

1 **(a)** 5.25 **(b)** 0.375 **(c)** 2.1525
2 **(a)** 14.537 **(b)** 145.37 **(c)** 1453.7 **(d)** 14 537
3 **(a)** 46.8 **(b)** 230.3 **(c)** 1699.2 **(d)** 10 474.2

I can divide decimals

1 **(a)** 294.35 **(b)** 29.435 **(c)** 2.9435 **(d)** 0.29435
2 **(a)** 5.2 **(b)** 4.7 **(c)** 11.8 **(d)** 19.8
3 **(a)** 50 **(b)** 60 **(c)** 14.7 **(d)** 4116

I can simplify fractions

1 **(a)** $\frac{4}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{5}{9}$

2.1 Multiply and divide integers

SC 1

1 **(a)** −16 **(b)** −14 **(c)** −12 **(d)** −10
2 **(a)** 4 **(b)** 8 **(c)** 12
(d) 16 **(e)** 20 **(f)** 24
3 **(a)** −12 **(b)** −8 **(c)** −4
(d) 0 **(e)** 4 **(f)** 8
4 **(a)** −15 **(b)** 8 **(c)** −400
(d) −28 **(e)** −52 **(f)** 140
5 Sample answer: The question has an even number of negative integer terms, so the answer must be positive.

SC 2

1 **(a)** −4 **(b)** −3 **(c)** −2 **(d)** −1
2 **(a)** 4 **(b)** 2 **(c)** 1
(d) 2 **(e)** 4 **(f)** 8
3 **(a)** −12 **(b)** −6 **(c)** −4
(d) −3 **(e)** −2 **(f)** −1
4 **(a)** −14 **(b)** 6 **(c)** −5
(d) 2 **(e)** −2 **(f)** −10
5 **(a)** −2 **(b)** −3 **(c)** 9 **(d)** −8
6 **(a)** −6 **(b)** −5 **(c)** −3 **(d)** 2
7 45 ÷ (−3) = −15, 45 ÷ (−15) = −3
8 Sample answer: There is one negative number in the question, so the answer must be negative.

2.2 Solve problems by multiplying and dividing integers

SC 1

1 **(a)** −1 **(b)** 1 **(c)** −1 **(d)** 1
2 If an even number of negative terms are multiplied, then the result will be positive. If an odd number of negative terms are multiplied, then the result will be negative.
3 **(a)** −8 **(b)** 16 **(c)** −32 **(d)** 64
4 **(a)** positive or + **(b)** negative or −
(c) negative or − **(d)** neither or 0
(e) positive or + **(f)** negative or −
5 **(a)** −24 **(b)** −24 **(c)** −2
(d) −10 **(e)** −18 **(f)** −42
6 **(a)** positive or + **(b)** negative or −
(c) negative or − **(d)** negative or −
(e) neither or 0 **(f)** positive or +
7 **(a)** **(i)** −1 **(ii)** 1 **(iii)** −1
(iv) 1 **(v)** −1
(b) If the power is an odd number, then the answer is negative and if the power is an even number, then the answer is positive.
8 Sample answer: Raising a negative integer to an even power means there are an even number of negative terms in the calculation. Therefore, the answer should be a positive value.

SC 2

1 −56 points
2 −300 m
3 −3°C per day
4 −22°C
5 **(a)** −$24 **(b)** $221
6 **(a)** −$15 **(b)** $28
7 **(a)** 4 × 2 = 8 **(b)** −4 × 2 = −8
(c) −4 × (−2) = 8 **(d)** −4 ÷ 2 = −2
8 **(a)** −4, 2 **(b)** −2, 4
9 **(a)** −3, −2 **(b)** −6, −1
10 34
11 −34
12 −10 and −14 or 6 and −30
Sample answer: Any pair of integers that has a sum of −24 will work.

2.3 Use the four operations with integers

SC 1

1 **(a)** −6 **(b)** 21 **(c)** 2

2 **(a)** −48 **(b)** 0 **(c)** −1

3 Josie did not include the negative sign in front of the 8 when first calculating −8 + 20 and calculated −(8 + 20) instead. Therefore, Josie got −16 − (8 + 20) = −44

4 Bai performed each operation in order from left to right, ignoring the order of operations.
Todd calculated 20 − 16 first, then added 8 × 2 to the result, instead of subtracting it from the result.
The correct value is:
$20 - 16 - 8 \times 2 = 20 - 16 - 16$
$= 4 - 16$
$= -12$

5 **(a)** 10 − 5 = 5 **(b)** −10 + 5 = −5
(c) −1 − 2 = −3 **(d)** −5 − 10 = −15

6 **(a)** −10 × 18 = −180 **(b)** −10 × (−18) = 180
(c) −20 × 17 = −340 **(d)** 20 × 17 = 340
(e) −20 × 30 = −600 or 10 × (−60) = −600
(f) −20 × (−30) = 600 or −10 × (−60) = 600

7 **(a)** 100 + (−74) = 26 **(b)** 40 − (−4) = 44
(c) 400 − 12 = 388 **(d)** 143 − 100 = 43

SC 2

1 **(a)** −7 **(b)** 13 **(c)** −16 **(d)** −26
(e) −40 **(f)** −40 **(g)** −127 **(h)** −127

2 **(a)** 12 **(b)** 12 **(c)** 2 **(d)** 2
(e) 5 **(f)** 5 **(g)** −2 **(h)** −2

3 **(a)** −7 **(b)** 10 **(c)** −28
(d) −44 **(e)** 0 **(f)** 19

4 **(a)** −4 **(b)** −8 **(c)** 2 **(d)** 14

5 **(a)** 6 **(b)** −13 **(c)** 0

6 **(a)** negative **(b)** positive **(c)** negative
(d) positive **(e)** positive

7 **(a)** −40 ÷ (−4) + 6 ÷ (−3) = 8
(b) −84 ÷ 7 + 3 × (−1) = −15
(c) (−40 ÷ (−4) + 5) ÷ (−3) = −5
(d) (−24 ÷ 6 + 1) ÷ (−3) = 1

8 Individual answers.
Sample: Start with −20, divide by 4, subtract 15 and then divide by 5.

2.4 Multiply and divide rational numbers

SC 1

1 **(a)** −9 **(b)** 13.5 **(c)** 4.5 **(d)** −4.5

2 **(a)** −45 **(b)** 32.4 **(c)** −7.5

3 **(a)** −1.5 **(b)** −2 **(c)** 4.5 **(d)** − 2.25

4 **(a)** 15 **(b)** −5.5 **(c)** −7.9

5 Risa did not notice that both the terms had negative signs in the multiplication, which makes the answer positive.

SC 2

1 **(a)** $\frac{1}{8} \times \left(-\frac{5}{2}\right) = -\frac{5}{16}$

(b) $\left(-\frac{1}{8}\right) \times \left(-\frac{5}{2}\right) = \frac{5}{16}$

(c) $\frac{3}{2} \times \left(-\frac{5}{6}\right) = \frac{1}{2} \times \left(-\frac{5}{2}\right)$
$= -\frac{5}{4}$

2 **(a)** $\frac{3}{8} \times \left(-\frac{4}{1}\right) = \frac{3}{2} \times \left(-\frac{1}{1}\right)$
$= -\frac{3}{2}$

(b) $-\frac{3}{8} \times \left(-\frac{4}{3}\right) = -\frac{1}{2} \times \left(-\frac{1}{1}\right)$
$= \frac{1}{2}$

(c) $\frac{9}{8} \times \left(-\frac{5}{3}\right) = \frac{3}{8} \times \left(-\frac{5}{1}\right)$
$= -\frac{15}{8}$

3 **(a)** $2 \times \frac{3 \div 3}{5 \div 5} \times \left(-\frac{5 \div 5}{6 \div 3}\right) = 2 \times \frac{1}{1} \times \left(-\frac{1}{2}\right)$
$= -1$

(b) $-\frac{3 \div 3}{4 \div 2} \times \left(-\frac{2 \div 2}{3 \div 3}\right) \times 4 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{1}\right) \times 4$
$= 2$

(c) $-\frac{4}{9} \times (-4) \div \frac{8}{3} = -\frac{4}{9} \times (-4) \times \frac{3}{8}$
$= -\frac{4 \div 4}{9 \div 3} \times \frac{3 \div 3}{8 \div 4} \times (-4)$
$= -\frac{1}{3} \times \frac{1}{2 \div 2} \times (-4 \div 2)$
$= -\frac{1}{3} \times \frac{1}{1} \times (-2)$
$= \frac{2}{3}$

(d) $-\frac{5}{6} \div 2 \times \frac{3}{5} = -\frac{5}{6} \times \frac{1}{2} \times \frac{3}{5}$
$= -\frac{5 \div 5}{6 \div 3} \times \frac{3 \div 3}{5 \div 5} \times \frac{1}{2}$
$= -\frac{1}{2} \times \frac{1}{1} \times \frac{1}{2}$
$= -\frac{1}{4}$

4 **(a)** $\frac{3}{5} \times \frac{1}{2} \times \left(-\frac{5}{6}\right) = \frac{3 \div 3}{5 \div 5} \times \left(-\frac{5 \div 5}{6 \div 3}\right) \times \frac{1}{2}$
$= \frac{1}{1} \times \left(-\frac{1}{2}\right) \times \frac{1}{2}$
$= -\frac{1}{4}$

(b) $\frac{5}{6}\times\left(-\frac{3}{4}\right)\times\left(-\frac{2}{3}\right)=\frac{5}{6\div3}\times\left(-\frac{3\div3}{4\div2}\right)\times\left(-\frac{2\div2}{3}\right)$

$=\frac{5}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{3}\right)$

$=\frac{5\times(-1)\times(-1)}{2\times2\times3}$

$=\frac{5}{12}$

(c) $\frac{4}{9}\div(-4)\times\frac{3}{8}=\frac{4}{9}\times\left(-\frac{1}{4}\right)\times\frac{3}{8}$

$=\frac{4\div4}{9\div3}\times\frac{3\div3}{8\div4}\times\left(-\frac{1}{4}\right)$

$=\frac{1}{3}\times\frac{1}{2}\times\left(-\frac{1}{4}\right)$

$=\frac{1\times1\times(-1)}{3\times2\times4}$

$=-\frac{1}{24}$

(d) $\frac{3}{5}\div(-2)\times\left(-\frac{5}{6}\right)=\frac{3}{5}\times\left(-\frac{1}{2}\right)\times\left(-\frac{5}{6}\right)$

$=\frac{3\div3}{5\div5}\times\left(-\frac{5\div5}{6\div3}\right)\times\left(-\frac{1}{2}\right)$

$=\frac{1}{1}\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)$

$=\frac{1}{4}$

5 Ahmed did not flip the second fraction before multiplying the two fractions.

$-\frac{2}{5}\div\frac{13}{10}=-\frac{2}{5\div5}\times\frac{10\div5}{13}$

$=-\frac{2}{1}\times\frac{2}{13}$

$=-\frac{4}{13}$

2.5 Apply all four operations with integers, fractions and decimals

SC 1

1 **(a)** 1 **(b)** 8.5 or $8\frac{1}{2}$

(c) −3.5 **(d)** −0.5 or $-\frac{1}{2}$

2 **(a)** 8 **(b)** −9 **(c)** 48 **(d)** −42

3 **(a)** −20 **(b)** −35 **(c)** 2

4 **(a)** 3 **(b)** 0.4 **(c)** 5

5 Margot did not follow the order of operations, calculating the addition before the multiplication.

$-1.6\times(-8)+4=12.8+4$

$=16.8$

6 **(a)** $\frac{5}{8}$ **(b)** $\frac{1}{4}$ **(c)** $-\frac{2}{5}$

7 **(a)** −2 **(b)** −12

(c) $-\frac{9}{4}$ or $-2\frac{1}{4}$ **(d)** 2

8 **(a)** −9 **(b)** $\frac{1}{2}$ **(c)** $-\frac{3}{5}$

9 **(a)** −16 **(b)** −0.8 or $-\frac{4}{5}$ **(c)** 6

SC 2

1 **(a)** −4 **(b)** 5.7 or $5\frac{7}{10}$

(c) −18 **(d)** $\frac{3}{7}$

2 **(a)** −25 **(b)** 50 **(c)** −8 **(d)** −4

3 **(a)** Vertical distance travelled is 146.5 − (−9.2) m

(b) 155.7 m

4 Difference = 45.5°C − (−4.2°C)

= 49.7°C

5 **(a)** Total length of fabric required $=10\times\left(2\frac{1}{2}+2\frac{3}{4}\right)$ m

$=10\times\frac{21}{4}$

$=52.5$ m

Extra fabric required = (52.5 − 10.2) m

= 42.3 m

(b) $12.70 × 42.3 = $537.21

(c) Total paid by choir members = 10 × $60

= $600

Uniform shop profit = $600 − $537.21

= $62.79

6 **(a)** $50 000 **(b)** $135 500

(c) **(i)** $40 000 **(ii)** −$24 500

(iii) The sign of the answer to **(ii)** means the company now has a profit.

7 **(a)** 100 **(b)** 79 **(c)** 36

Topic 3: Rational and irrational numbers

Recall

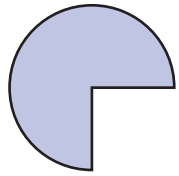
I can recognise and complete patterns

1 **(a)** 9, 11, 13 **(b)** 16, 20, 24

(c) 243, 729, 2187 **(d)** 32, 64, 128

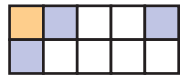
(e) 16, 8, 0 **(f)** 8, 13, 21

2 **(a)**



(b)

(c)



I can classify numbers into sets

1 **(a)** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
(b) 1, 2, 3, 4, 6, 8, 12
(c) 5, 10, 15, 20
(d) 2, 3, 5, 7, 11, 13, 17, 19
(e) 4, 6, 8, 9, 10, 12, 14

I can calculate with squares and square roots

1 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
2 **(a)** 3 **(b)** 6 **(c)** 9
(d) 12 **(e)** 15 **(f)** 20

I can simplify fractions

1 **(a)** 1, 2, 3 , 6 **(b)** 1, 2, 5, 10
(c) 1, 3, 5, 15 **(d)** 1, 2, 4, 8, 16

2 **(a)** $\frac{3}{5}$ **(b)** $\frac{2}{5}$ **(c)** $\frac{15}{16}$ **(d)** $\frac{3}{8}$

3.1 Identify and use irrational numbers

SC 1

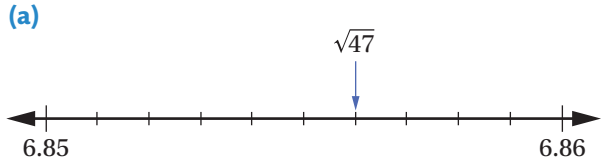
1 **(a)** $\frac{207}{324}$ **(b)** $-\frac{267}{100}$
(c) $\frac{50093}{10000}$ **(d)** $\frac{1077}{30600}$

2 **(a)** $\frac{155}{100} = \frac{31}{20}$ **(b)** $-\frac{2006}{1000} = -\frac{1003}{500}$
(c) $\frac{675}{100} = \frac{27}{4}$ **(d)** $\frac{19864}{1000} = \frac{2483}{125}$

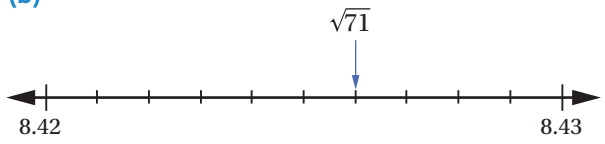
3 **(a)** **(i)** rational **(ii)** irrational
(iii) rational **(iv)** rational
(v) rational **(vi)** irrational
(b) The number needs to be a perfect square.

4 **(a)** **(i)** rational **(ii)** irrational
(iii) rational **(iv)** irrational
(v) rational **(vi)** irrational
(b) The number needs to be a perfect cube.

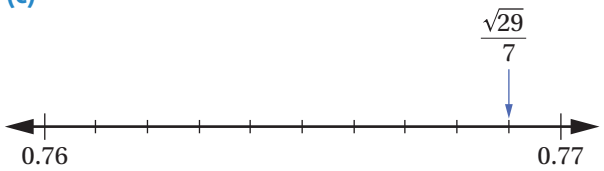
5 **(a)**



(b)



(c)



SC 2

1 **(a)** $10\sqrt{3}$ cm **(b)** $9\sqrt{5}$ cm
(c) $6\sqrt{11}$ cm **(d)** 35 cm

2 **(a)** $32\sqrt{3}$ cm **(b)** $54\sqrt{2}$ cm

3 252 cm^2

4 **(a)** $\sqrt{198}$ **(b)** $\sqrt{\frac{28}{3}} = 2\sqrt{\frac{7}{3}}$

5 **(a)** These measurements are the same.
(b) Within the limitations of rounding, these two values are the same, approximately 1.41, correct to 2 decimal places.
(c) $a = 2$; the exact value of the ratio is $\sqrt{2}$
(d) 420 × 594 mm

3.2 Identify the golden ratio and make the connection between the circumference and diameter of any circle

SC 1

1 **(a)** 19.09 m
(b) The measured value is close to the required value for a golden rectangle. Perhaps this was as close as the ancient builders could get to the golden ratio without reliable measuring devices.

2 **(a)** 1.667 **(b)** 1.615 **(c)** 1.618
(d) The ratios are getting progressively closer to the golden ratio.
(e) The ratios are the same: 1.618 033 989 ...
(f) 6765
(g) 46 368

3 **(a)** 3, 3, 6, 9, 15, 24, 39, 63, 102, 165, 267, 432
1.667, 1.615, 1.618
(b) 2, 7, 9, 16, 25, 41, 66, 107, 173, 280, 453, 733
1.563, 1.621, 1.618
(c) Both appear to approach the golden ratio. The sequence that did not start with the same numbers takes a little bit longer to settle into the golden ratio, but it appears nonetheless.

4 **(a)** A square of side length 1 is drawn and another of the same size is drawn next to it. Then a square of side length 2 is attached to one side of the rectangle formed by the first two squares. Then a square of side length 3 (2 + 1) is drawn, followed by a square of side length 5 (3 + 2) and so on. Arcs are then drawn in each of the squares.

(b) The side lengths are the terms in the Fibonacci sequence.

(c) The Fibonacci spiral has been drawn over the picture.

(d) Students will have their own answer for this, but it would probably be wise to remain sceptical. Many scholars refute the claim that the golden ratio is involved.

SC 2

1 These answers will be approximations, depending on the manual dexterity of the students. The answers provided are the 'exact' answers, correct to 1 decimal place.

(a) 25.1 cm **(b)** 31.4 cm

(c) 50.3 cm **(d)** 62.8 cm

(e) When the radius doubles, the length of the circumference also doubles, within the bounds of measurement errors.

(f) When the radius increases by 1 unit, the circumference increases by a little over 6 units, while an increase of 3 units leads to an increase in the circumference of a little under 19. This might be recognised as about 3 times the increase for 1 unit.

2 (a)

Radius	Diameter	Circumference	$\frac{C}{d}$	$\frac{C}{r}$
4 cm	8 cm	25.1 cm	3.138	6.275
5 cm	10 cm	31.4 cm	3.14	6.28
8 cm	16 cm	50.3 cm	3.144	6.288
10 cm	20 cm	62.8 cm	3.14	6.28

(b) (i) The circumference is about 3.14 times as long as the diameter.

(ii) The circumference is about 6.28 times as long as the radius.

3 (a) 18.84 cm **(b)** 21.98 cm

(c) 15.7 cm **(d)** 11.304 cm

4 (a) 6.37 cm **(b)** 12.10 cm **(c)** 13.41 cm

5 (a) 3.14

(b) The value for π is the same as the approximate value being used for $\frac{C}{d}$.

(c) $\frac{C}{d} = \pi \Rightarrow C = \pi d$

(d) 39.584 cm

6 (a) At least 6 times.

(b) Evelyn will be able to completely wrap it 13 times, with a small piece left over.

3.3 Identify and use terminating, recurring and non-recurring decimals

SC 1

1 (a) $0.\dot{5}$ **(b)** $0.\overline{36}$ **(c)** $0.09\dot{3}$ **(d)** $0.\overline{297}$

2 (a) $0.2\overline{6}$ **(b)** $-2.5\dot{6}$ **(c)** $6.897\,\overline{45}$ **(d)** $1.00\overline{8}$

3 (a) rational **(b)** irrational

(c) rational **(d)** rational

(e) rational **(f)** irrational

4 (a) $0.\overline{142\,857}$ **(b)** $0.\overline{285\,714}$

(c) $0.\overline{428\,571}$ **(d)** $0.\overline{571\,428}$

(e) $0.\overline{714\,285}$ **(f)** $0.\overline{857\,142}$

(g) The repeating digits are all the same. If you think about them as being in a loop, each answer just starts at a different value in the loop.

(h) These also just use the loop values from sevenths, with a digit at the start. As an example, $\frac{1}{14} = 0.0\overline{714285}$. The even fourteenths are just the sevenths that have already been examined as they reduce down to sevenths.

5 (a) $0.\overline{18}$ **(b)** $0.\overline{45}$ **(c)** $0.\overline{63}$ **(d)** $0.\overline{90}$

6 (a) (i) $15: 3 \times 5$ **(ii)** $24: 2^3 \times 3$

(iii) $25: 5^2$ **(iv)** $30: 2 \times 3 \times 5$

(v) $48: 2^4 \times 3$ **(vi)** $8: 2^3$

(vii) $16: 2^4$ **(viii)** $75: 3 \times 5^2$

(b) (i) repeating **(ii)** repeating

(iii) terminating **(iv)** repeating

(v) repeating **(vi)** terminating

(vii) terminating **(viii)** repeating

(c) Students are to choose their own values to check. Audrey's suggestion is correct: All repeating decimals are rational numbers that can be written as reduced fractions with denominators containing at least one prime number factor other than 2 or 5.

SC 2

1 (a) $\frac{7}{45}$ **(b)** $\frac{103}{900}$

2 (a) $\frac{8}{55}$ **(b)** $\frac{69}{550}$

3 (a) $\frac{137}{1110}$ **(b)** $\frac{6}{185}$

4 $0.\dot{9} = 1$; this result may come as a surprise.

5 (a) (i) $\frac{11}{90}$ and $\frac{12}{99} = \frac{4}{33}$

(ii) $\frac{12}{90} = \frac{2}{15}$ and $\frac{13}{99}$

(iii) $\frac{13}{90}$ and $\frac{14}{99}$

(b) Before cancelling to simplest form, the next set just adds 1 to the numerator and 9 to the denominator for each fraction in the pair.

Topic 4: Percentage increase and decrease

Recall

I can identify the change in values as representing an increase or decrease

1 **(a)** Increase of 6 **(b)** Decrease by 9
(c) Decrease by $\frac{1}{5}$ **(d)** Increase by 0.2

I can identify the relationship between decimals, percentages and fractions

1 **(a)** $\frac{17}{100}$ **(b)** $\frac{29}{100}$ **(c)** $\frac{31}{100}$ **(d)** $\frac{77}{100}$
(e) $\frac{3}{100}$
2 **(a)** 25% **(b)** $\frac{2}{5} \times \frac{100}{1} = 40\%$
(c) $\frac{3}{25} \times \frac{100}{1} = 12\%$ **(d)** $\frac{5}{8} \times \frac{100}{1} = 62.5\%$

I can identify and create equivalent fractions

1 **(a)** 1, 2, 4, 8, 16, 32 **(b)** 1, 2, 4, 7, 8, 14, 28, 56
(c) 1, 2, 4, 8, 11, 22, 44, 88
(d) 1, 2, 3, 4, 6, 8, 12, 18, 24, 48, 72, 144
2 **(a)** 8 **(b)** 8
3 **(a)** 3 **(b)** 7 **(c)** 5
4 **(a)** $\frac{5}{7}$ **(b)** $\frac{7}{8}$ **(c)** $\frac{47}{72}$ **(d)** $\frac{7}{11}$

I can calculate with percentages

1 **(a)** 50%, 200% **(b)** 20%, 500%
(c) 25%, 400% **(d)** 56.25%, 178%
2 **(a)** 3.6 **(b)** 36 **(c)** 18 **(d)** 72
(e) 90 **(f)** 180 **(g)** 270

4.1 Understand percentage increase and decrease

SC 1

1 **(a)** 260% **(b)** 175% **(c)** 56%
(d) $86\frac{122}{123}\%$
2 **(a)** 90% **(b)** $20\frac{5}{6}\%$ **(c)** 50%
(d) $34\frac{4}{19}\%$
3 **(a)** $4\frac{2}{7}\%$ increase **(b)** $7\frac{593}{1501}\%$ decrease
(c) $14\frac{64}{249}\%$ increase **(d)** $68\frac{8}{9}\%$ decrease
4 **(a)** 30% increase **(b)** $66\frac{2}{3}\%$ increase
5 **(a)** 57.14% increase **(b)** 11.11% decrease
(c) 13.33% decrease **(d)** 20% increase
6 **(a)** 100% increase **(b)** 50% decrease
(c) The change in values (the numerator) is the same. However, each calculation uses a different original value (the denominator).
(d) 200% increase and a $66\frac{2}{3}\%$ decrease.
(e) Apart from the increase being a much higher value than the decrease, there is no obvious connection between the percentages.

SC 2

1 **(a)** \$21.17 **(b)** \$260.02 **(c)** \$67.19
2 **(a)** \$18.71 **(b)** 1.025 **(c)** \$18.71
(d) Yes, Evelyn's reasoning was correct.
(e) Instead of adding the percentage change to 100%, subtract it.
(f) Both methods result in a value of \$36 125.
3 Student A has calculated an increase of 80%.
Student B has calculated a decrease of 8%.
Student C has shown the correct calculation.
Student D has calculated 8% of \$75 000.
4 **(a)** \$5 per litre **(b)** C **(c)** 5.24%
5 **(a)** 10.67% decrease **(b)** \$1.90
(c) \$1.71 **(d)** \$1.53
6 **(a)** \$1.50 **(b)** 60% profit

SC 3

1 **(a)** The final price is \$99.75, so Greg is correct.
(b) The final price is \$622.44, so Greg is correct again.
(c) 99.75%
(d) 99.75%
(e) $1.05 \times 0.95 = 0.9975 = 99.75\%$ of the original price.
2 **(a)** Male: \$2775, Female: \$2331
(b) Male: \$2358, Female: \$1910
(c) Male: \$2145, Female: \$1673
(d) Male: \$1525, Female: \$1403
3 **(a)** An increase of 1.016 61%.
(b) An increase of 1.016 61%.
(c) An increase of 1.016 61%.
(d) So long as the changes are the same, the order in which they are applied does not make a difference.
4 **(a)** 161.7 kt **(b)** 8338.57 kt
(c) 9866.67 kt
(d) The recycling of metals has been occurring for many years. These products are expensive and relatively easy to repurpose.

5 **(a)** −7.0°C **(b)** −90.2 m
(c) An increase of 84.3%
(d) A decrease of 296.6%
6 **(a)** 100% increase
(b) 133.3% decrease
(c) 300% increase
(d) Compared to the Arctic, the Antarctic is 14.3% colder.

4.2 Understand percentage changes in realistic situations

SC 1

1 **(a)** $473 **(b)** $38.50
(c) $26 290 **(d)** $346.50
2 **(a)** **(i)** $236.36 **(ii)** $23.64
(b) **(i)** $4.45 **(ii)** $0.45
(c) **(i)** $123.63 **(ii)** $12.37
(d) **(i)** $102.27 **(ii)** $10.23
3 **(a)** multiply, 1.1 **(b)** divide, 1.1
4 Retailer A: $4268; Retailer B: $4150; Retailer C: $4042.50
In order from least to most expensive: Retailer C, Retailer B, Retailer A
5 **(a)** $215 **(b)** $668 **(c)** $35 **(d)** $73
6 **(a)** $240 **(b)** $2376 **(c)** $2079 **(d)** 2.16%

SC 2

1 **(a)** 2010 to 2020; 3 650 883
(b) 2010 to 2020; 16.58%
(c) 27 164 410
(d) 22 787 349
2 **(a)** 1970 to 1980; 2.98 years
(b) 1970 to 1980; 4.18%
(c) 91.60 years
(d) 72.97 years
3 **(a)** For the first time there has been an increase, followed by decreases in the later time periods.
(b) 14.67 metric tons
(c) 28.10 metric tons
(d) The actual figure is 54.23%, less than the predicted figure.
4 **(a)** The figure fluctuates, but has values between 4% and 7%. There was a steady decline in the rate in the period 2001 to 2008. This was the longest period over which a consistent movement can be seen.
(b) It is a 7.32% increase.
(c) From 2008 to 2009
(d) From 2020 to 2021

SC 3

1 **(a)** $313.50 **(b)** $4664.50
(c) $17 409.50 **(d)** $32 427
2 **(a)** 8.6% **(b)** 21.8% **(c)** 27.5% **(d)** 31.7%
3 **(a)** $5824 **(b)** $15 533.88
(c) $23 511.75 **(d)** $38 560.38
(e) The more your income increases, the more you save under this taxation arrangement.
4 **(a)** $44 267
(b) $(16 467 × 2) = $32 934
(c) When the income is split, both people are entitled to the $18 200 tax-free component, plus the marginal rate of taxation is lower: 32.5% compared to 37%.

Topic 5: Algebra (expand, simplify and factorise)

Recall

I can write expressions from words

1 **(a)** $11 + n = 16$ **(b)** $m - 14 = 9$
(c) $x = y - 10$ **(d)** $5n = 45$
(e) $\frac{3+19}{q} = \frac{1}{3}$ or $(3+19) \div q = \frac{1}{3}$

I can apply the order of operations to calculate the value of an expression

1 **(a)** 21 **(b)** 5 **(c)** 5 **(d)** 10 **(e)** 5

I can identify terms and coefficients, in expressions and equations

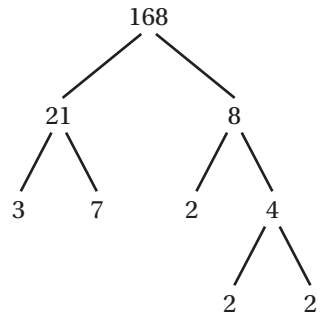
1 **(a)** 2 **(b)** 4 **(c)** 3
2 **(a)** B **(b)** A: 4

I can identify the highest common numerical factor

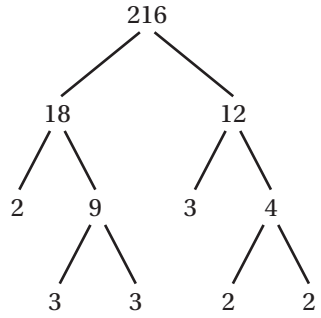
1 **(a)** 5 **(b)** 36 **(c)** 12

I can identify all the prime factors of numbers

1 **(a)** $2^3 \times 3 \times 7 = 168$



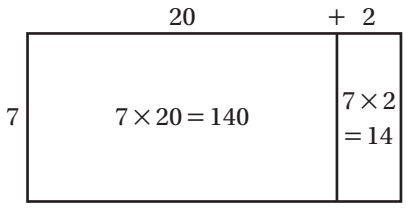
(b) $2^3 \times 3^3 = 216$



5.1 Use the distributive law to expand simple algebraic expressions

SC 1

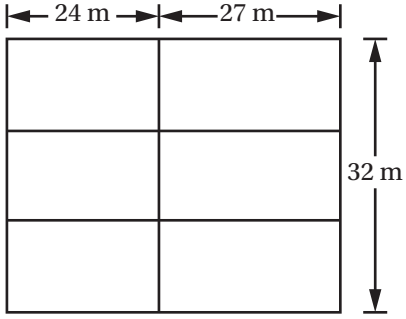
1 Students' answers will vary. Sample answer:



$22 \times 7 \equiv (20 + 2) \times 7$
$\equiv (20 \times 7) + (2 \times 7)$
$= 140 + 14$
$= 154$

2 The dimensions of the rectangle are 4 and $y + 3$
The expression is $4(y + 3)$.

3 $32(24 + 27)$



Total area $= 32 \times 24 + 32 \times 27$
$= 32(24 + 27)$

SC 2

1 **(a)** $2x + 8$ **(b)** $9x + 9$
(c) $-3x + 12$ **(d)** $-6.3 + 3b$
(e) $6a^2 - 15a$ **(f)** $-96x + 60x^3$

2 $mn + mo$

3 **(a)** $7(g + 5) + 3g = 7 \times g + 7 \times 5 + 3g$
$= 7g + 35 + 3g$
$= 10g + 35$

(b) $3(b - 5) + 7b = 3 \times b + 3 \times (-5) + 7b$
$= 3b - 15 + 7b$
$= 10b - 15$

(c) $6(x + 2) + 3(x + 7) = 6 \times x + 6 \times 2 + 3x + 3 \times 7$
$= 6x + 12 + 3x + 21$
$= 9x + 33$

(d) $4(b + 2) + 3b(a + 1) = 4 \times b + 4 \times 2 + 3b \times a$
$+ 3b \times 1$
$= 4b + 8 + 3ab + 3b$
$= 7b + 3ab + 8$

(e) $4(x - 2) - 3x(x - 1) = 4 \times x + 4 \times (-2) - 3x \times x$
$+ (-3x) \times (-1)$
$= 4x - 8 - 3x^2 + 3x$
$= -3x^2 + 7x - 8$

(f) $-4(y + 2) - 3y(5 - y) = (-4) \times y + (-4) \times 2 +$
$(-3y) \times 5 + (-3y) \times (-y)$
$= -4y - 8 - 15y + 3y^2$
$= 3y^2 - 19y - 8$

5.2 Factorise simple algebraic expressions

SC 1

1 **(a)** $3(3x + 10)$ **(b)** $2(4x - 7)$
(c) $6(x - 6)$ **(d)** $3(5x - 1)$

2 **(a)** $8(3 - x)$ **(b)** $4(8 - 3x)$
(c) $2(9x + 2y + 5)$ **(d)** $13(2x - y - 3)$

3 **(a)** $-6(x - 8)$ **(b)** $-13(a - 2b - 3)$

4 **(a)** 6 **(b)** 3 **(c)** 10 **(d)** 11

SC 2

1 **(a)** $a(3 + 7a)$ **(b)** $a(26 - 15b)$
(c) $c(15b - 8)$ **(d)** $d(19c + 11)$

2 **(a)** $b(12 + 13b)$ **(b)** $b(13b - 21)$
(c) $a(15a - 7b + 2)$ **(d)** $a(27a - 7b + 11b^2)$

3 **(a)** $b(15 - 13a + 7ab)$
(b) $b(26a^2b + 15a - 13b)$

4 **(a)** $6(5x - 2)$ cm **(b)** $2(9 + 2x)$ cm
(c) $2(4x - 1)$ cm

SC 3

1 **(a)** $3b(9a - 1)$ **(b)** $8a(3ab - 2)$
(c) $2bc(8a + 11)$ **(d)** $5abc(3c + 2a)$

2 **(a)** $7a^2b(2 + b - 3b^2)$
(b) $2ab^2c^2(12c - 8a + 15a^2bc)$

3 **(a)** $7abc$ **(b)** $2a^2bc$
(c) $(a - b)$ **(d)** $(2a + 5)$

4 **(a)** $(3a + 2)(7a - 4)$ **(b)** $(4b - 2)(6a - 5)$

5 **(a)** This answer is incomplete, but is equivalent to the fully factorised expression $-4(a + 3)$.
(b) This answer is incorrect as it does not expand to the original expression.
(c) Esther deserves some marks for her incomplete answer.

5.3 Simplify algebraic expressions involving multiple operations

SC 1

1 **(a)** $11a + 2$ **(b)** $13a - 14$
(c) $29a - 13$ **(d)** $-9a$

2 **(a)** $5a^2 + 3a$ **(b)** $6a^2 + 21ab + 10b^2$
(c) $14a^2 - 33ab + 8b^2$ **(d)** $12a^2b - 36ab^2$

3 **(a)** $-12y^2 + 17ay$ **(b)** $14ab^2 - 18a^2b - 12ab$

4 **(a)** $3a^2 + 5ab - 2b^2$
(b) $4\pi a^2 - 4\pi ab + \pi b^2$

SC 2

1 **(a)** $\frac{43a}{21}$ **(b)** $-\frac{3a}{10}$ **(c)** $\frac{79a}{30}$ **(d)** $-\frac{13a}{24}$

2 **(a)** $\frac{9a^2}{2}$ **(b)** $\frac{2}{3a}$
(c) $\frac{10ad}{3b}$ **(d)** $\frac{abd}{10}$

3 **(a)** $\frac{7ac}{2d}$ **(b)** $\frac{9a}{10cd}$
(c) $\frac{66acd^2}{7}$ **(d)** $\frac{20ce}{9ad}$

4 **(a)** $\frac{19a}{6b}$ **(b)** $\frac{14a}{15b}$
(c) $\frac{4b+6a}{ab}$ **(d)** $\frac{7b^2-9a^2}{3ab}$

SC 3

1 **(a)** $4x + 10$ **(b)** $7x + 5$
(c) $8a^2 - 25a$ **(d)** $12a - 48$

2 **(a)** $15x - 12bx + 9b - 15$
(b) $8 - 4x$ **(c)** $\frac{2bx^2 - 3xb}{3}$

3 $-\frac{9x}{2}$

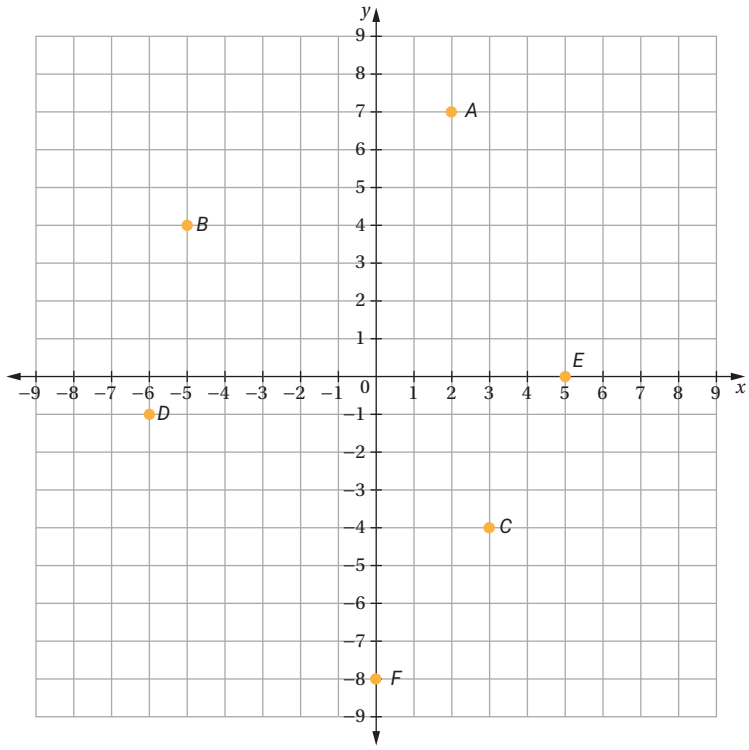
4 **(a)** $\frac{25x}{(x-3)^2}$ **(b)** $-\frac{10x^2}{3}$ **(c)** $\frac{10x+345}{50}$

Topic 6: Linear graphs

Recall

I can plot points on a Cartesian plane

1



I can use a rule to fill in a table of values

1 Using the given rule, complete the table of values.

(a) $y = x + 5$

x	-1	0	1	2	3
y	4	5	6	7	8

(b) $y = x - 4$

x	-1	0	1	2	3
y	-5	-4	-3	-2	-1

(c) $y = 8 - x$

x	-1	0	1	2	3
y	9	8	7	6	5

(d) $y = 3x$

x	-1	0	1	2	3
y	-3	0	3	6	9

I can substitute values into equations to check the correct solution

1 **(a)** F **(b)** T **(c)** F
(d) T **(e)** F **(f)** T

2 **(a)** $x = 13$ **(b)** $x = 4$ **(c)** $x = 2$

I can solve two-step equations

1 **(a)** $a = 4$ **(b)** $b = 10$
(c) $x = 24$ **(d)** $p = 14$

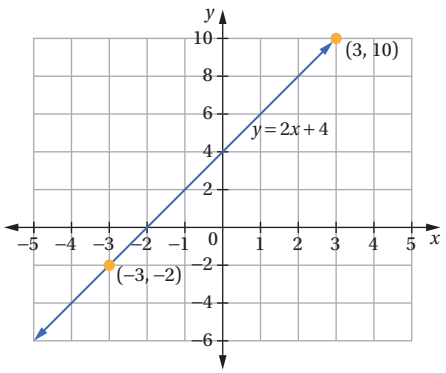
I can write expressions from a sentence

1 **(a)** $100 - 6 = 94$ **(b)** $3 \times 4 = 12$
(c) $8 + y = 10$

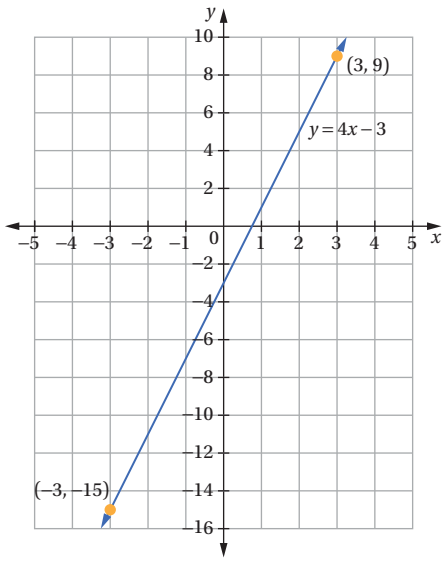
6.1 Plot and identify a linear relationship using a set of points

SC 1

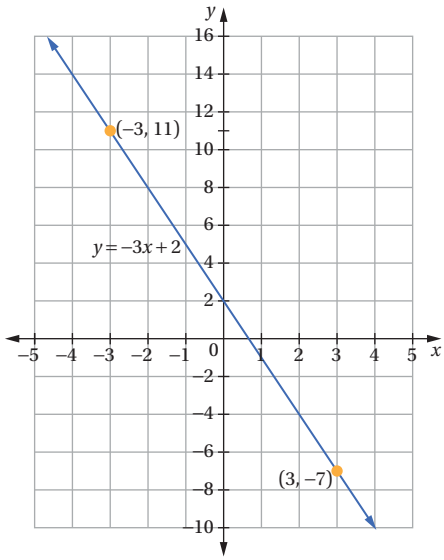
1 **(a)**



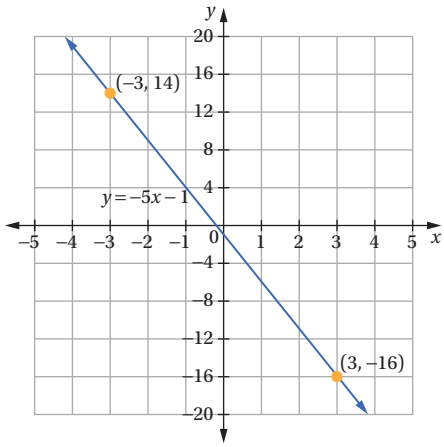
(b)



(c)



(d)



2 $(-1, -12)$ and $(3, 12)$

3 **(a)** $\left(-3, -2\frac{3}{4}\right)$ and $\left(5, 3\frac{1}{4}\right)$

(b) $\left(-2, -3\frac{1}{5}\right)$ and $\left(3, -\frac{1}{5}\right)$

(c) $(-6, 4)$ and $\left(7, -\frac{1}{3}\right)$ **(d)** $\left(-4, 3\frac{1}{4}\right)$ and $\left(10, -3\frac{3}{4}\right)$

4 **(a)** (5, −2) and (−3, −10) **(b)** (3, 10) and (−4, −4)
(c) (−6, 8) and (4, −2) **(d)** (−3, 13) and (5, −11)

SC 2

1 **(a)**

x	−3	−1	0	4	6
y	−15	−9	−6	6	12

(b)

x	−5	−2	1	2	5
y	31	16	1	−4	−19

(c)

x	−6	−1	0	2	5
y	$-4\frac{1}{3}$	−1	$-\frac{1}{3}$	1	3

(d)

x	−4	−2	0	1	6
y	$10\frac{1}{2}$	$6\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{2}$	$-9\frac{1}{2}$

2 **(a)**

x	−4	−2	0	2	4
y	−10	−4	2	8	14

(b)

x	−2	−1	0	1	2
y	−13	−8	−3	2	7

(c)

x	−2	−1	0	1	2
y	7	3	−1	−5	−9

(d)

x	−2	0	2	6	10
y	9	7	5	1	−3

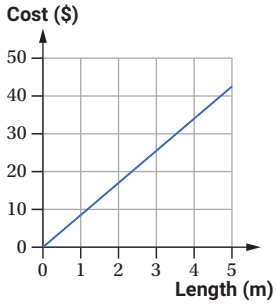
3 **(a)** The given x-values correspond to y-values that are not on the grid lines.
(b) −0.3
(c)

x	−8	−2	0	5	9
y	−3.7	−1.1	−0.3	1.9	3.6

4 **(a)** A decrease of 5 **(b)** An increase of 2
(c) An increase of 3 **(d)** An increase of 4

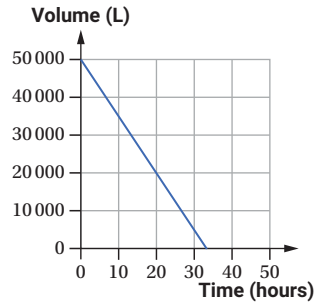
SC 3

1 **(a)**



(b) $26 **(c)** $38
(d) Multiply the number of metres by the cost per metre. This gives the answers in dollars and cents.

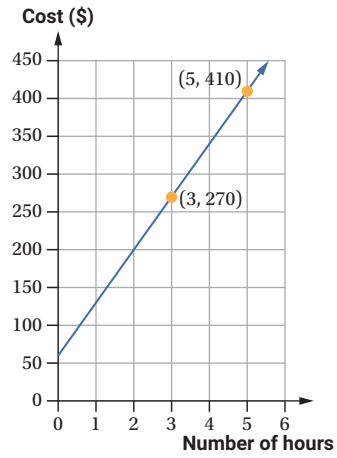
2 **(a)** $33\frac{1}{3}$ h
(b)



(c) The y-intercept gives the initial volume of water in the tank; the x-intercept gives the time taken to empty the tank.
(d) Neither the volume nor the time can be negative, so only the first quadrant makes sense.

3 **(a)** $150: the y-intercept
(b) $750 **(c)** $1950
(d) $\frac{750-150}{10} = \frac{1950-150}{30} = 60$
The cost per person is $60
(e) Total cost ($) = number of guests × 60 + 150
(f) The venue would have to inform organisers of the minimum number of people required before they would agree to hold the event, as well as the maximum number of people possible, depending on the size of the venue or the catering capacity.

4 **(a)**

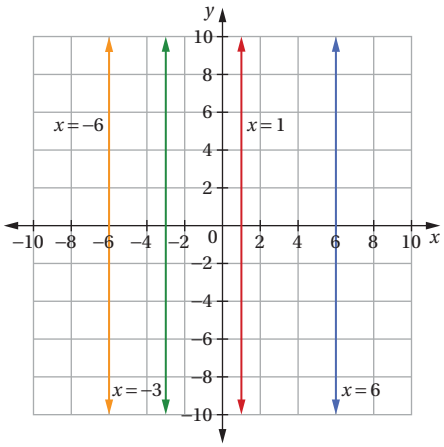


(b) The call-out fee is $60.
(c) $\frac{410-270}{2} = \frac{140}{2} = 70$
$70/h
(d) Cost ($) = number of hours × 70 + 60

6.2 Graph linear relationships with only one axis intercept

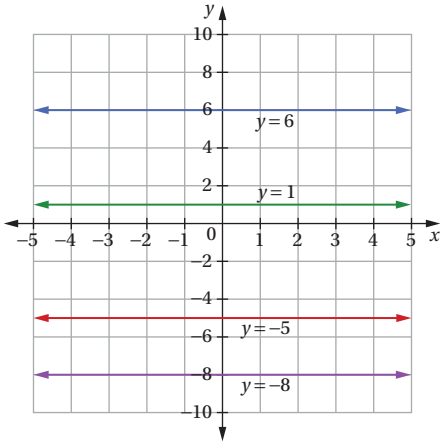
SC 1

1 (a)



(b) y-axis

2 (a)



(b) x-axis

3 (a) $x = -8$ **(b)** $x = -4$ **(c)** $x = 2$ **(d)** $x = 5$

4 (a) $y = 5$ **(b)** $y = 2$ **(c)** $y = -4$ **(d)** $y = -7$

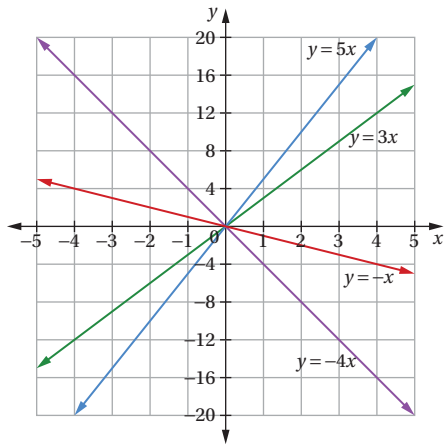
5 (a) $x = 5$ **(b)** $x = -9$ **(c)** $y = 2$
(d) $y = -10$ **(e)** $x = 0$ **(f)** $y = 0$

6 (a) A square of side length 10 units
(b) A rectangle 8 units by 4 units

7 (a) $x = 0, x = 3, y = 2, y = 6$
(b) $x = 0$ and $x = 3$: $2 \leq y \leq 6$; $y = 2$ and $y = 6$: $0 \leq x \leq 3$.

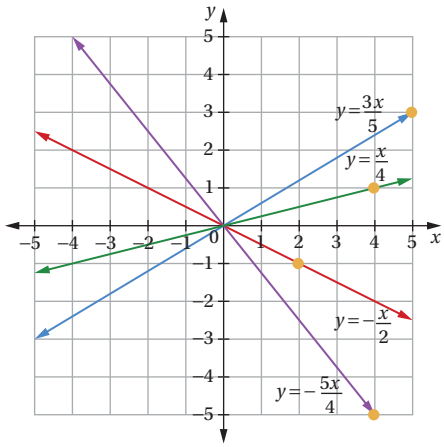
SC 2

1

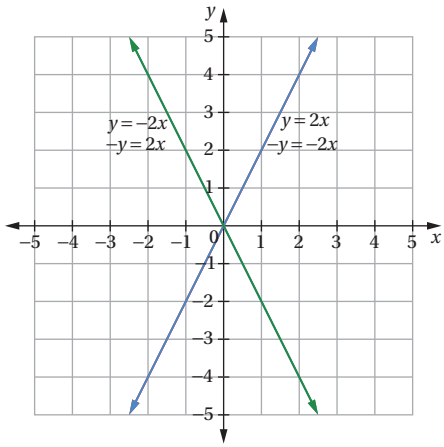


2 (a) (i) (4, 1) **(ii)** (5, 3)
(iii) (2, −1) **(iv)** (4, −5)

(b)



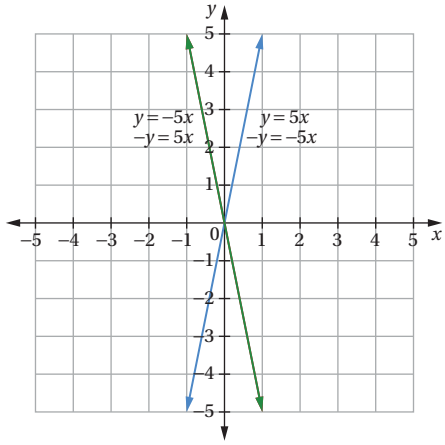
3 (a)



(i) $y = 2x$ has the same graph as $-y = -2x$ and $y = -2x$ has the same graph as $-y = 2x$.

(ii) Either pair of graphs can be reflected in the vertical axis to fit onto the other pair.

(b)



(i) $y = 5x$ has the same graph as $-y = -5x$ and $y = -5x$ has the same graph as $-y = 5x$.

(ii) Either pair of graphs can be reflected in the vertical axis to fit onto the other pair.

4 Sample answers:

(a) (−6, −42), (−3, −21), (0, 0), (3, 21), (6, 42)

(b) (−6, 24), (−3, 12), (0, 0), (3, −12), (6, −24)

(c) (−6, −60), (−3, −30), (0, 0), (3, 30), (6, 60)

(d) (−6, 600), (−3, 300), (0, 0), (3, −300), (6, −600)

5 Sample answers:

(a) $y = -4x$; any rule of the form $y = ax$, where $a < -2$.

(b) $y = 5$; any rule of the form $y = a$, where $a \neq 0$.

(c) $x = 2$; any rule of the form $x = a$, where $a \neq 0$.

(d) $y = 6x$; any rule of the form $y = ax$, where $a > 5$.

(e) $y = -x$; any rule of the form $y = ax$, where $-2 < x < 0$.

(f) $y = \frac{x}{4}$; any rule of the form $y = ax$, where $0 < a < \frac{1}{2}$.

6.3 Investigate linear graphs using technology

SC 1

1 **(a)** $y = 2x - 1$ **(b)** $y = 3x + 2$

(c) $y = -4x + 6$ **(d)** $y = -2x - 3$

2 **(a)** $y = 5.25x - 9.75$

(b) $y = -1.6667x + 4.6667$

(c) $y = 0.5556x + 8.6667$

(d) $y = -1.3333x - 1$

3 $y = -2.2x - 2.6$

4 **(a)** (−2, 4.2223) **(b)** (4, −0.4445)

SC 2

1 **(a)** $c = 7$, (0, 7) **(b)** $c = -4$, (0, −4)

(c) $c = 6$, (0, 6) **(d)** $c = -10$, (0, −10)

2

x	y=x+3	y=2x+3	y=−2x+3	y=x+5	y=3x−5
−3	0	−3	9	2	−14
−2	1	−2	7	3	−11
−1	2	1	5	4	−8
0	3	3	3	5	−5
1	4	5	1	6	−2
2	5	7	−1	7	1
3	6	9	−3	8	4
4	7	11	−5	9	7
5	8	13	−7	10	10

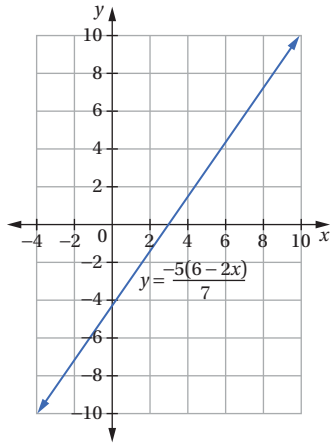
3 **(a)** **(i)** $y = \frac{2}{3}x + \frac{1}{3}$ **(ii)** $m = \frac{2}{3}, c = \frac{1}{3}$

(b) **(i)** $y = \frac{1}{4}x - 1\frac{1}{4}$ **(ii)** $m = \frac{1}{4}, c = -1\frac{1}{4}$

(c) **(i)** $y = -\frac{1}{3}x + \frac{2}{3}$ **(ii)** $m = -\frac{1}{3}, c = \frac{2}{3}$

(d) **(i)** $y = -\frac{1}{2}x + 3\frac{1}{2}$ **(ii)** $m = -\frac{1}{2}, c = 3\frac{1}{2}$

4 **(a)**



(b) (0, −4.286)

(c) $y = \frac{10}{7}x - 4\frac{2}{7}$

(d) $-4\frac{2}{7} \approx -4.286$

5 **(a)** $c = -8$ **(b)** $c = -3$

(c) $c = 1\frac{3}{5}, c = 1.6$ **(d)** $c = -\frac{6}{7}, c = -0.857$

(e) $c = -1\frac{1}{3}, c = -1.333$

6 **(a)** $y = 3x + 5$ **(b)** $y = -2x - 2$

SC 3

1 **(a)** Both gradients are positive.

(b) $y = 3x - 6$ is steeper than $y = 2x + 3$ because its x coefficient is greater (i.e. $3 > 2$), so the y-value increases more for each unit of change in the x-value.

2 **(a)** Both gradients are negative.

(b) $y = -2x + 3$ is steeper than $y = -\frac{1}{5}x - 4$ because the magnitude of the coefficient of x is greater $\left(\text{i.e. } 2 > \frac{1}{5}\right)$. In the graph of $y = -2x + 3$, the y-value decreases by 2 for every unit of increase in the x-value by 1. In the graph of $y = -\frac{1}{5}x - 4$, the y-value only decreases by $\frac{1}{5}$ for every unit of increase in the x-value by 1. Graphically, the graph of $y = -2x + 3$ is closer to vertical than the graph of $y = -\frac{1}{5}x - 4$.

3 **(a)** $m = 3$ **(b)** $m = -2$

(c) $m = -3$ **(d)** $m = -5$

4 **(a)** $y = 6x - 4, m = 6$

(b) $y = -3x + 8, m = -3$

(c) $y = -2, m = 0$

(d) $y = \frac{-x}{5} + 7, m = -\frac{1}{5}$

5 **(a)**

x	y=x+3	y=2x+3	y=−2x+3	y=x+5	y=3x−5
−3	0	−3	9	2	−14
−2	1	−1	7	3	−11
−1	2	1	5	4	−8
0	3	3	3	5	−5
1	4	5	1	6	−2
2	5	7	−1	7	1
3	6	9	−3	8	4
4	7	11	−5	9	7
5	8	13	−7	10	10

(b) $y = 3x - 5$

(c) $y = x + 3$ and $y = x + 5$

6 **(a)** $m = \frac{2}{7}$ **(b)** $m = -\frac{6}{5}$

(c) $m = \frac{10}{7}$ **(d)** $m = \frac{12}{5}$

6.4 Determine the rule for a linear relationship or pattern

SC 1

1 **(a)** $y = 3x + 2$ **(b)** $y = 2x - 3$

(c) $y = 6x + 1$ **(d)** $y = 7x - 5$

2 **(a)** $y = -x + 4$ **(b)** $y = -2x - 3$

(c) $y = -4x + 3$ **(d)** $y = -5x - 5$

3 **(a)** $y = x + 10$ **(b)** $y = 2x - 6$

(c) $y = -\frac{x}{2} - 7\frac{1}{2}$ **(d)** $y = -3x + 8$

4 **(a)** $y = \frac{1}{5}x + 2\frac{4}{5}$ **(b)** $y = \frac{5}{6}x + 1\frac{1}{3}$

(c) $y = -\frac{2}{5}x + 5\frac{4}{5}$ **(d)** $y = -\frac{6}{11}x - 1\frac{1}{11}$

5 **(a)** $y = 2x - 3$ **(b)** $y = 5x - 17$

(c) $y = -3x - 6$ **(d)** $y = -6x - 22$

6 (5.5, 0)

SC 2

1 **(a)** $y = 2x - 4$ **(b)** $y = 5x + 1$

(c) $y = -3x + 2$ **(d)** $y = -4x - 3$

2 **(a)** The y-intercept is not 6.

(b) The graph has a negative gradient, so the coefficient of x cannot be 3.

(c) $c = 7$ **(d)** $m = -3$

(e) $y = -3x + 7$

3 Sample answers:

(a) $y = -2x - 4$ and $y = -x - 5$

(b) (−2, 0) and (−5, 0), respectively

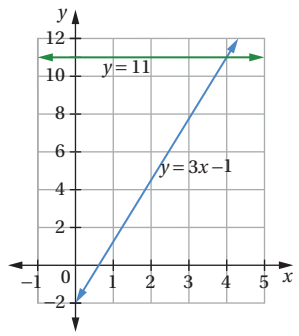
4 **(a)** $y = \frac{4}{3}x + 4$ **(b)** $y = \frac{1}{3}x - 2$

(c) $y = -\frac{3}{2}x + 6$ **(d)** $y = -\frac{3x}{7} - 3$

6.5 Solve linear equations graphically

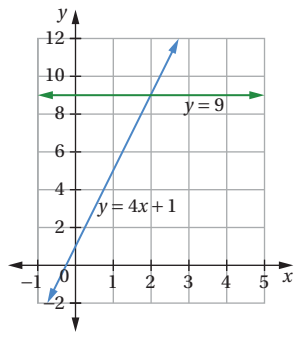
SC 1

1 **(a)** **(i)**



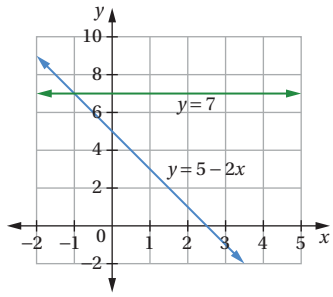
(ii) (4, 11) **(iii)** $x = 4$

(b) **(i)**



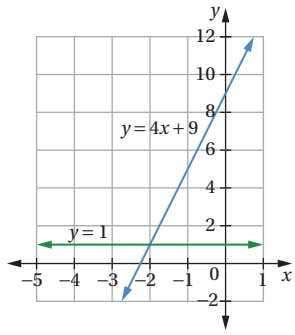
(ii) (2, 9) **(iii)** $x = 2$

(c) **(i)**



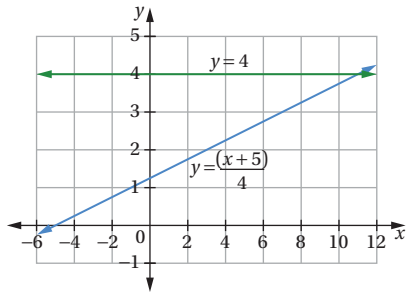
(ii) (−1, 7) **(iii)** $x = -1$

(d) **(i)**



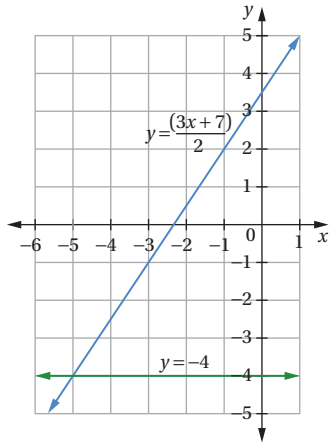
(ii) (−2, 1) **(iii)** $x = -2$

2 **(a)** **(i)**



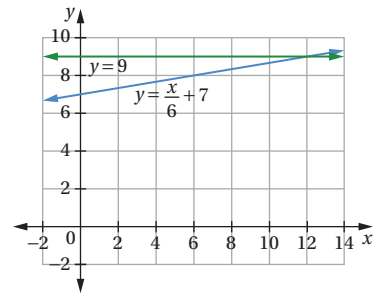
(ii) (11, 4) **(iii)** $x = 11$

(b) **(i)**



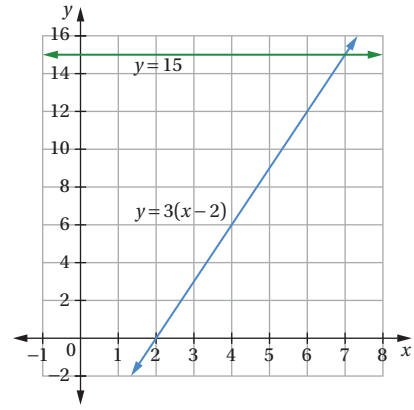
(ii) (−5, −4) **(iii)** $x = -5$

(c) **(i)**



(ii) (12, 9) **(iii)** $x = 12$

(d) **(i)**



(ii) (7, 15) **(iii)** $x = 7$

3 **(a)** $y = 0.85x$; potatoes cost 85 c/kg.

(b) $y = 7.5$; \$7.50 spent on potatoes.

(c) (8.8, 7.5); about 8.8 kg of potatoes can be bought for \$7.50.

4 **(a)** $y = 0.8x + 7.5$; it costs \$7.50 to make a delivery plus 80 c/km for travel.

(b) $y = 20$; \$20 is the total cost of delivery.

(c) (15.5, 20); about 15.5 km of travel for a \$20 delivery charge, including travel.

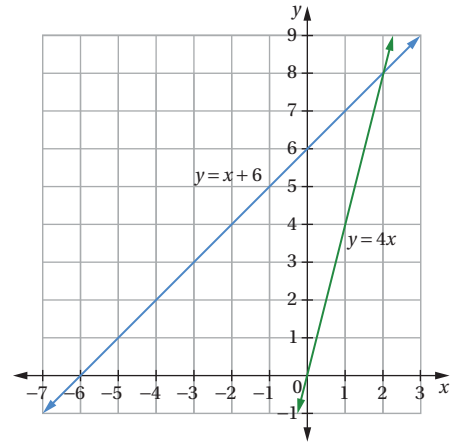
5 **(a)** Total annual cost of water (\$) = 2.1 × number of kilolitres + 175

(b) $y = 240$; \$240 is the total annual cost of water.

(c) (31, 240); about 31 kL of water was used in the year giving a total cost of \$240.

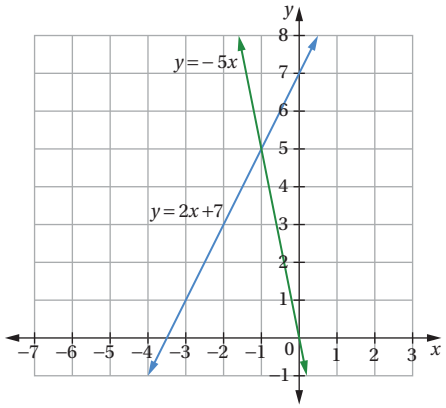
SC 2

1 **(a)** **(i)**



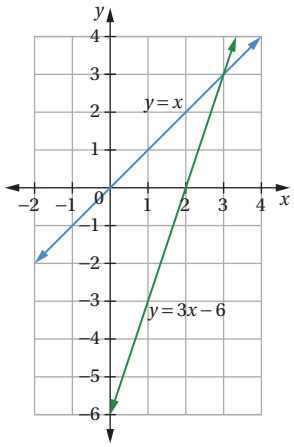
(ii) (2, 8) **(iii)** $x = 2$

(b) **(i)**



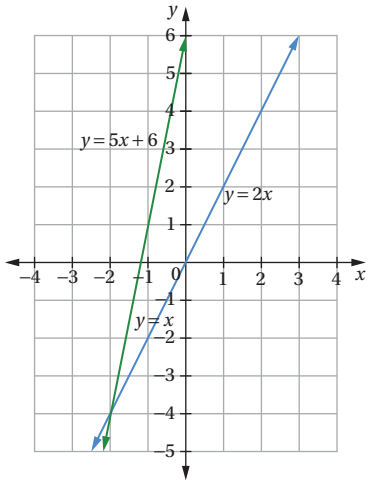
(ii) (−1, 5) **(iii)** $x = -1$

(c) **(i)**



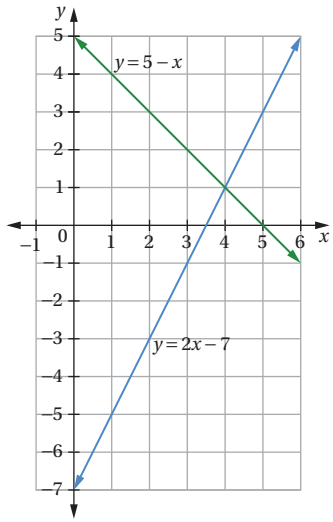
(ii) (3, 3) **(iii)** $x = 3$

(d) **(i)**



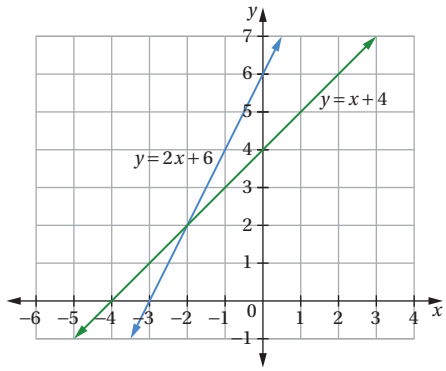
(ii) (−2, −4) **(iii)** $x = -2$

2 **(a)** **(i)**



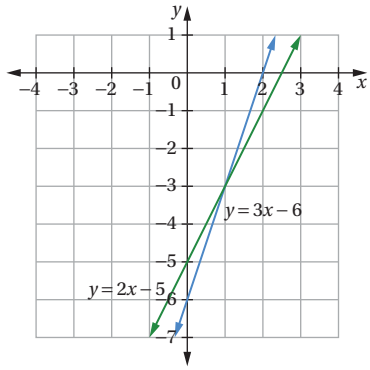
(ii) (4, 1) **(iii)** $x = 4$

(b) **(i)**



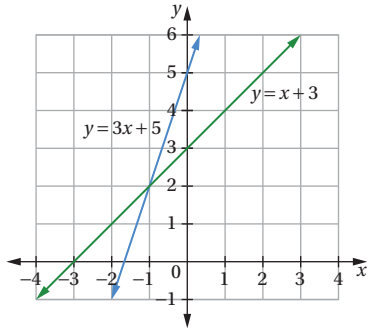
(ii) (−2, 2) **(iii)** $x = -2$

(c) **(i)**



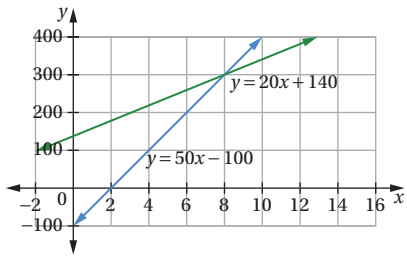
(ii) (1, −3) **(iii)** $x = 1$

(d) **(i)**



(ii) (−1, 2) **(iii)** $x = -1$

3 **(a)**



(b) (8, 300) **(c)** $x = 8$

4 **(a)** The man started his the journey at 6 am, 40 km from home, travelling for 5 h at a speed of 85 km/h, away from home. At the end of the journey he was 465 km from home at 11 am.

(b) The sister starts her journey at 6 am, 300 km from home, heading straight home at 100 km/h. She arrives home after 3 h at 9 am.

(c) $y = 300 - 100x$, for $0 \leq x \leq 3$.

(d) (1.4, 160); The man and his sister pass each other, going in opposite directions, 1.4 h after leaving, so 7:24 am, 160 km from home.

SC 3

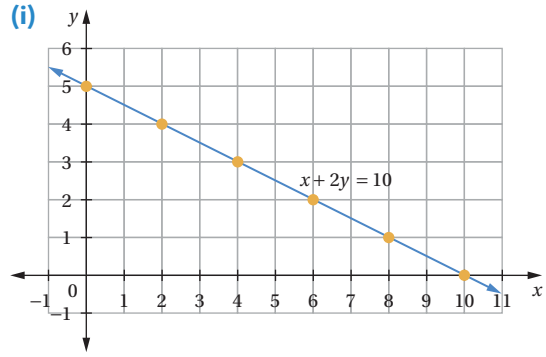
1 **(a)** (0, 24), (2, 21), (4, 18)

(b) ..., (−4, 30), (−2, 27), (0, 24), (2, 21), (4, 18), ...

2 **(a)** (0, 20), (2, 15), (4, 10)

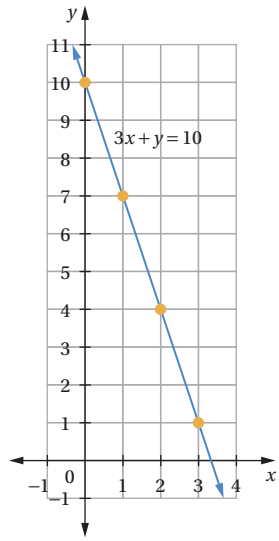
(b) ..., (−4, 30), (−2, 25), (0, 20), (2, 15), (4, 10), (6, 5), (8, 0), (10, −5), (12, −10), ...

3 **(a)** **(i)**



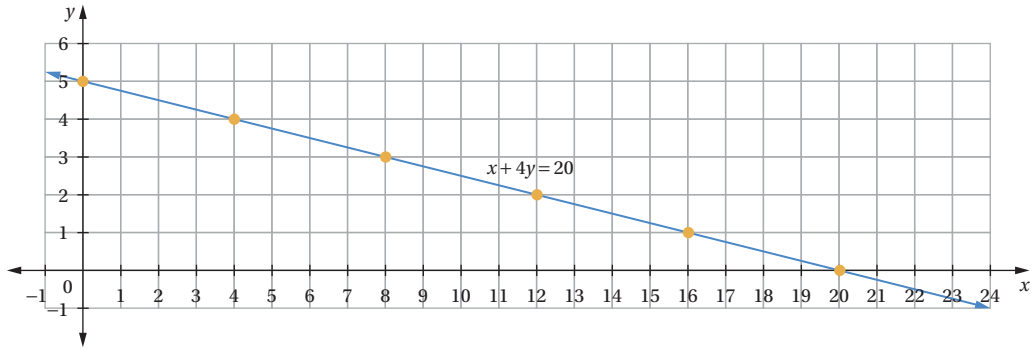
(ii) (0, 5), (2, 4), (4, 3), (6, 2), (8, 1), (10, 0)

(b) **(i)**



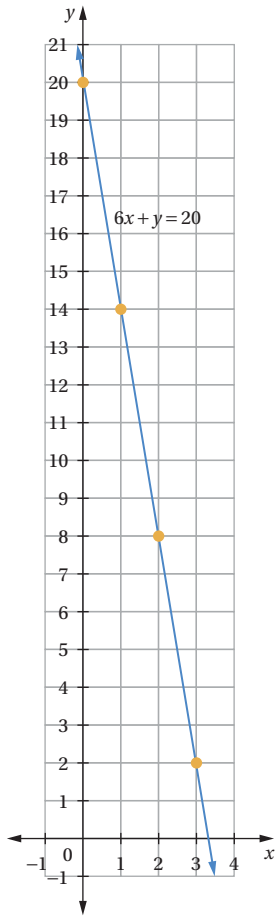
(ii) (0, 10), (1, 7), (2, 4), (3, 1)

(c) **(i)**



(ii) (0, 5), (4, 4), (8, 3), (12, 2), (16, 1), (20, 0)

(d) **(i)**



(ii) (0, 20), (1, 14), (2, 8), (3, 2)

4 Possible answers:

(a) (−4, 6), (0, 3), (4, 0), (8, −3)

(b) (−3, 8), (0, 4), (3, 0), (6, −4)

5 **(a)** No pairs of integer solutions

(b) If 15 was changed to a multiple of 2, the solutions would be many and all lined up. This line is parallel to $2x + 2y = 14$ and $2x + 2y = 16$, so misses all of these solutions.

(c) Possible solutions: $3x + 3y = 10$, $4x + 4y = 18$, $6x + 2y = 15$

Topic 7: Linear inequalities

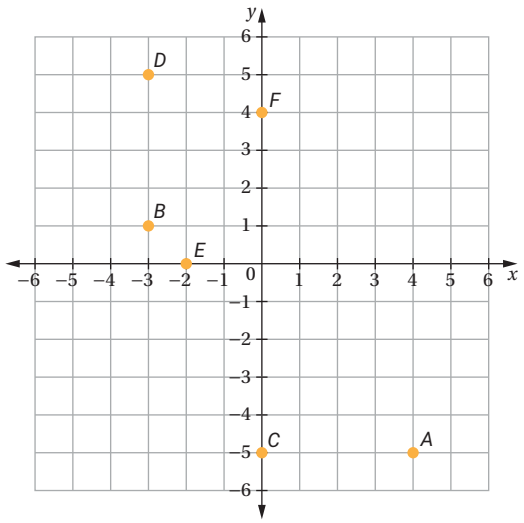
Recall

I can use the <, > and = symbols between pairs of numbers

1 **(a)** < **(b)** > **(c)** < **(d)** > **(e)** =
(f) < **(g)** > **(h)** < **(i)** <

I can plot points on a Cartesian plane

1



I can use substitution to confirm the solution of an equation

1 **(a)** F **(b)** T **(c)** T **(d)** F **(e)** T

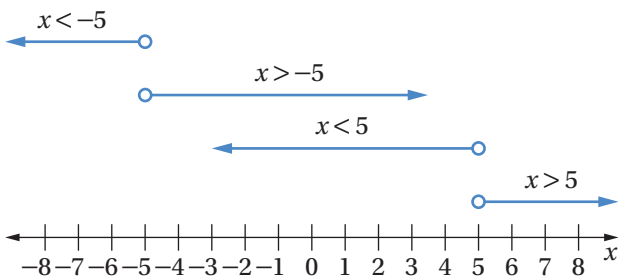
I can solve two-step equations

1 **(a)** $x = 4$ **(b)** $b = -9$ **(c)** $q = 7$
(d) $y = 80$ **(e)** $d = -16$

7.1 Graph inequalities on number lines

SC 1

1



2 **(a)** $x < 1$ **(b)** $x > 1$

3 **(a)** $y > -2$ **(b)** $y < 4$

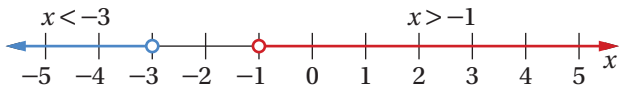
4 Sketch a horizontal or vertical number line, labelled x or y, respectively. Place a circle next to the non-included end value. From the circle, draw an arrowed line parallel to the axis, to the right or up for 'greater than' (>), to the left or down for 'less than' (<).

5 **(a)** $x < -1$ **(b)** $x < 6$ **(c)** $x > 1$ **(d)** $x > -4$

6 **(a)** $x < -4$ **(b)** $x > 2$

(c) If the lines overlap, it may get difficult to determine what is actually shown.

(d)



7 Possible answers:

(a) $x = 3.6, x = 0, x = -2.5$

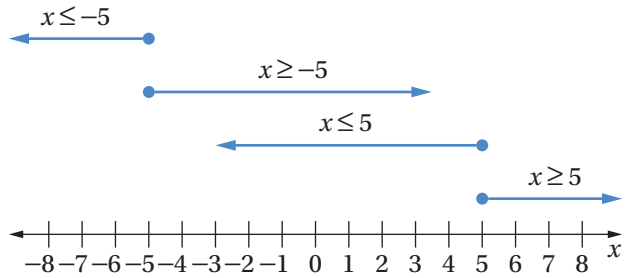
(b) $x = -0.4, x = 0, x = 6$

(c) $y = -2.1, y = -8, y = -8.6$

(d) $y = -4.7, y = 0, y = 2.3$

SC 2

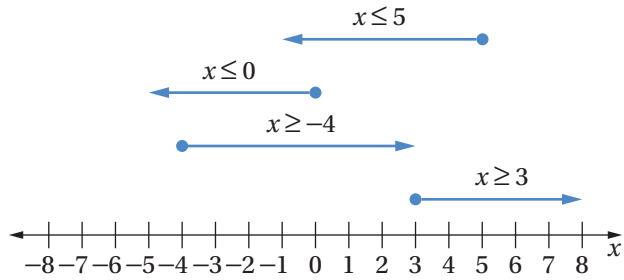
1



2 **(a)** $y \geq 1$ **(b)** $y \leq 1$

3 Sketch a horizontal or vertical number line, labelled x or y, respectively. Place a solid circle next to the end value. From the circle, draw an arrowed line parallel to the axis, to the right or up for 'greater than or equal to' (≥), to the left or down for 'less than or equal to' (≤).

4

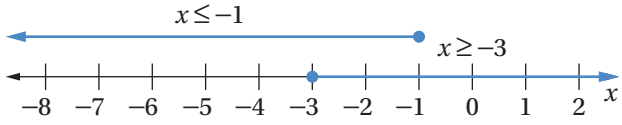


5 **(a)** $x \leq 6$ **(b)** $x \geq -2$ **(c)** $x \leq -4$ **(d)** $x \geq 0$

6 **(a)** $x \leq 7$ **(b)** $x \geq -4$

(c) There is overlap, which would make the number line difficult to interpret.

(d)



7 Possible answers:

(a) $x = 6, x = 0, x = -4.5$

(b) $x = 5, x = 5.1, x = 10$

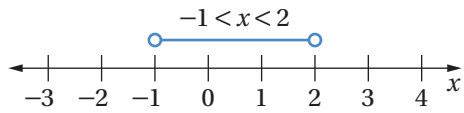
(c) $y = -3, y = -3.8, y = -15$

(d) $y = -6, y = 0, y = 10.8$

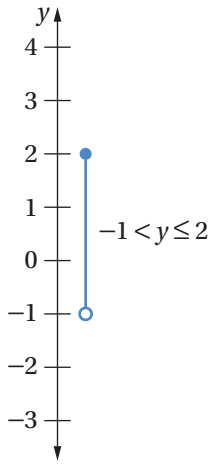
8 **(a)** $x \geq -4$ **(b)** $x < 1$ **(c)** $x \leq -2$

SC 3

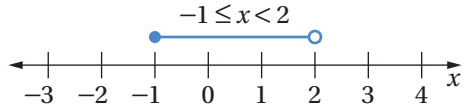
1 **(a)**



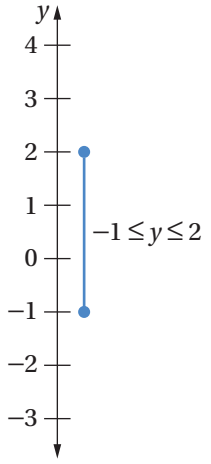
(b)



(c)



(d)

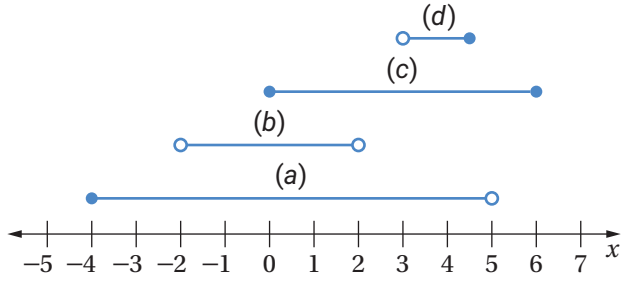


2 **(a)** $-3 \leq x \leq 1$ **(b)** $-6 < x \leq -3$

(c) $-5 \leq x < 0$ **(d)** $1 < x < 3$

3 Sketch a horizontal or vertical number line, labelled x or y, respectively. Place a circle next to each end value, solid if the value is included, open if it is not. Between the circles, draw a line interval parallel to (or on) the axis.

4



5 **(a)** $3 < x \leq 5$

(b) $-4 \leq x < 0$

(c) $-3 \leq x \leq 4$

6 Possible answers:

(a) $x = -2, x = 0.5, x = 4$

(b) $y = 0.1, y = 3.4, y = 6$

(c) $y = -2.6, y = -1.8, y = 0$

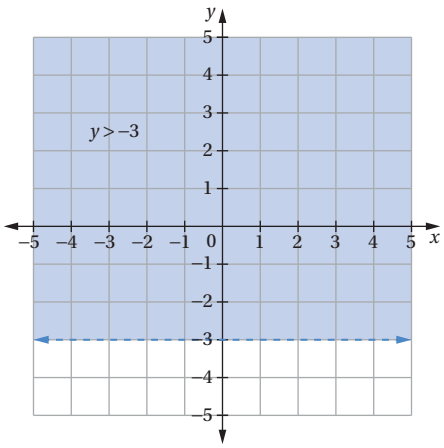
(d) $x = -3, x = -1.1, x = 3.7$

7 The double-ended inequality is made up of two separate inequalities. In this case, they are $-2 < x$ and $x > 4$. The first of these is equivalent to $x > -2$. Putting these together in words reads, 'The number must be greater than -2 and it must be greater than 4'. The only numbers that satisfy both parts are the numbers represented by $x > 4$, an open-ended set of numbers, not an interval.

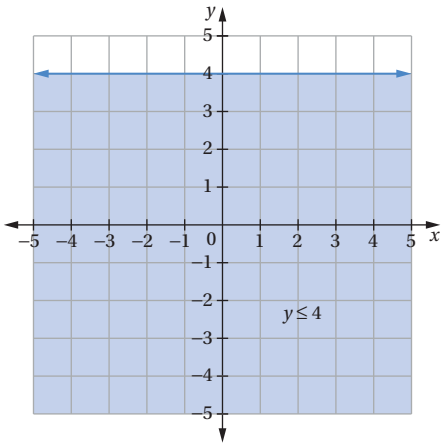
7.2 Graph linear inequalities on the Cartesian plane

SC 1

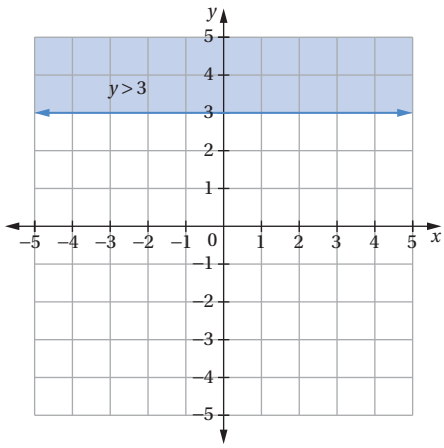
1 **(a)**



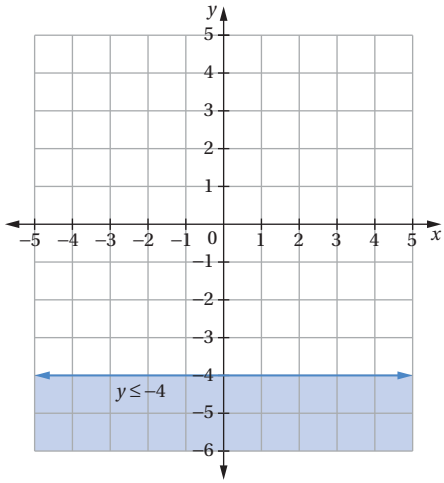
(b)



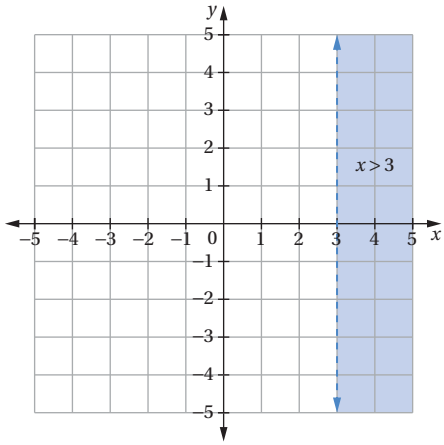
(c)



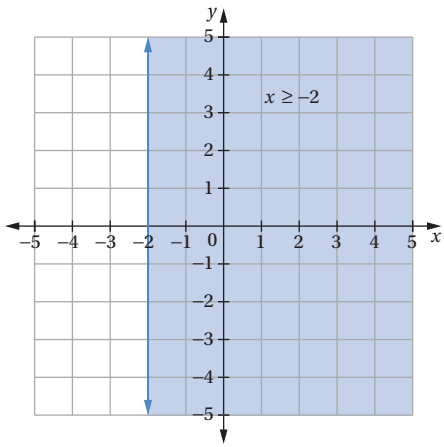
(d)



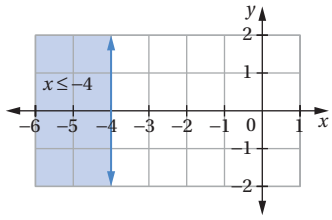
2 **(a)**



(b)



(c)



3 **(a)** $y > 6$

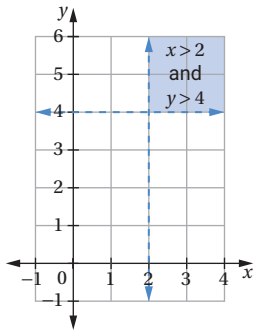
(b) (−1, 7), (10, 10), (−3, 7), (6, 8), (0, 8)

4 **(a)** $x \leq 6$

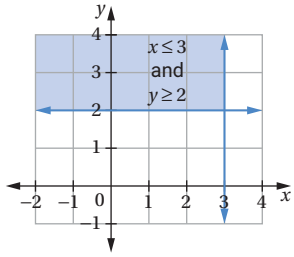
(b) (6, 4), (−1, 7), (3, −7), (−4, 2), (−3, 6), (0, 0)

5 Sketch a Cartesian plane. If x is the variable, draw a vertical grid line through the given value, and if y is the variable, draw a horizontal grid line through the given value. For a 'less than' inequality, make the line dotted to show that points on the line are not included. For a vertical line, shade everything to the left of the line. For a horizontal line, shade everything below the line.

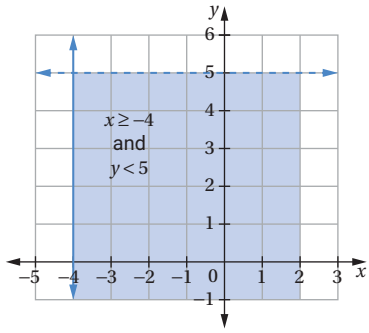
6 **(a)**



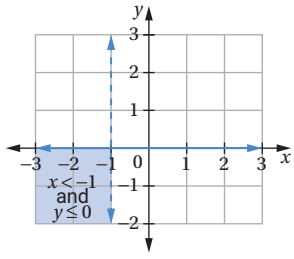
(b)



(c)

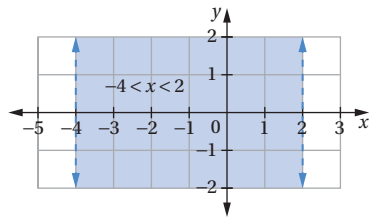


(d)

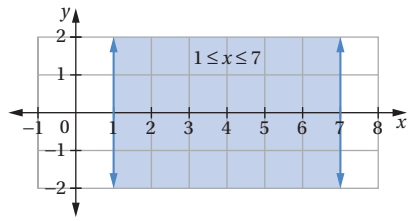


SC 2

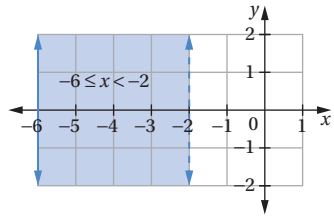
1 **(a)**



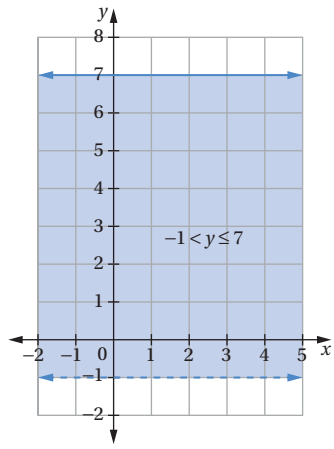
(b)



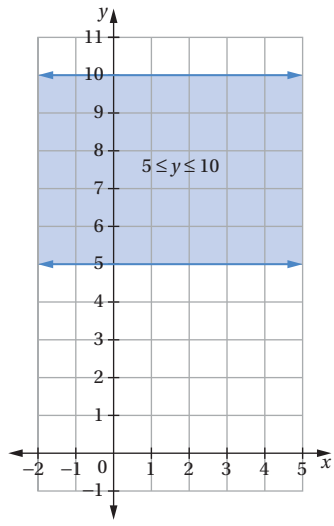
(c)



2 **(a)**



(b)

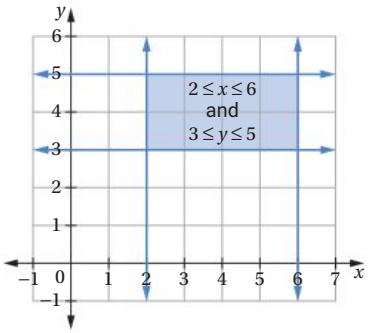


3 **(a)** $-3 \leq y < 5$

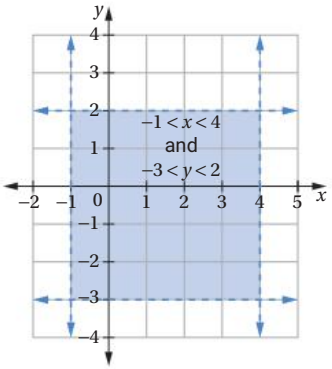
(b) $(2, -3), (-1, 0), (6, 4), (5, -3)$

4 Sketch a Cartesian plane. If x is the variable, draw vertical grid lines through the end values, and if y is the variable, draw horizontal grid lines through the end values. For a 'less than' inequality, use a dotted line to show that points on the line are not included. Shade everything between the lines.

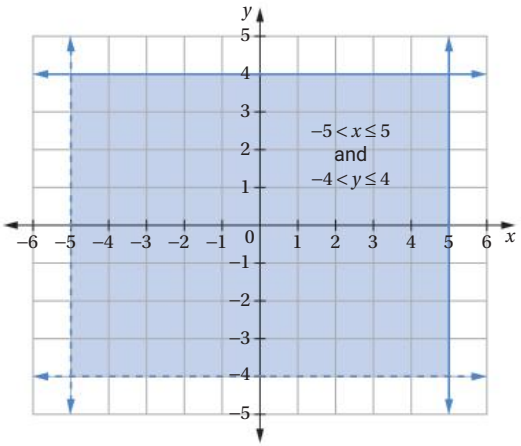
5 **(a)**



(b)



(c)



6 **(a)** $2 \leq x < 6$ and $-3 < y \leq 5$

(b) $(2, -3), (2, 5), (6, -3), (6, 5)$

(c) $(2, 5)$

SC 3

1 **(a)** Yes, it belongs in the required area.

(b) Yes, it belongs in the required area.

(c) No, it does not belong in the required area.

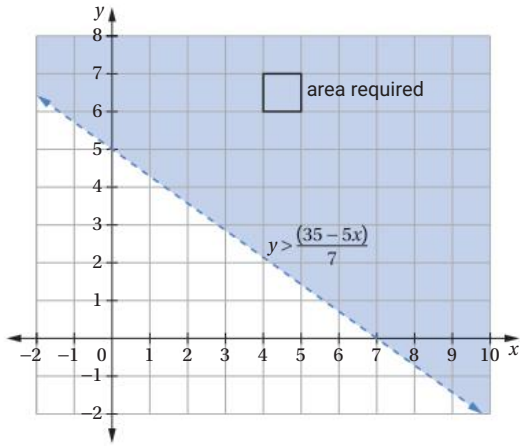
(d) No, it does not belong in the required area.

2 **(a)** $(0, 5)$ **(b)** $(7, 0)$

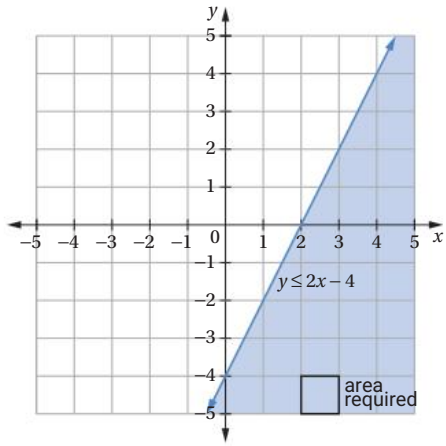
(c) The line will be dotted.

(d) The origin is not in the area required.

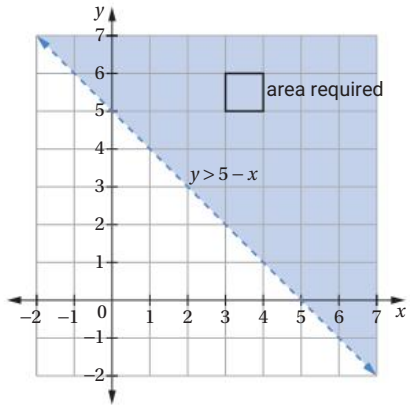
(e)



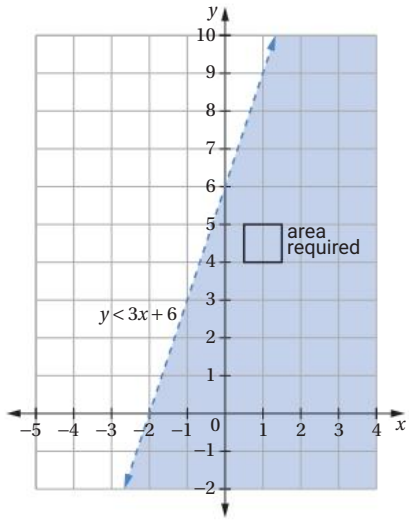
3 **(a)**



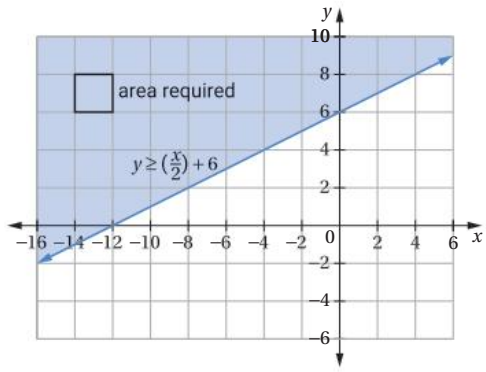
(b)



(c)



(d)



4 **(a)** $y \geq \frac{x}{2} + 2$

(b) $y < -\frac{4x}{3} - 4$

5 **(a)** $y < 3x - 2$ and $y \geq -2x + 4$

(b) These coordinate pairs are in the required region for neither of the inequalities.

(c) Sample answer using (0, 0)

$y < 3x - 2$

$0 < 3 \times 0 - 2$

$0 < -2$

false

$0 \geq -2 \times 0 + 4$

$0 \geq 4$

false

(d) These coordinate pairs are in the required region for both inequalities.

(e) Sample answer using (5, 0)

$0 < 3 \times 5 - 2$

$0 < 15 - 2$

$0 < 13$

true

$0 \geq -2 \times 5 + 4$

$0 \geq -10 + 4$

$0 \geq -6$

true

(f) These coordinate pairs are in the required region for only one of the inequalities.

(g) Sample answer using (0, 5)

$5 < 3 \times 0 - 2$

$5 < -2$

false

$5 \geq -2 \times 0 + 4$

$5 \geq 4$

true

7.3 Solve linear inequalities

SC 1

1 **(a)** $x < 6$ **(b)** $x < 10$ **(c)** $x < 4$

(d) $x < 16$

2 **(a)** $x \geq 7$ **(b)** $x > 9$ **(c)** $x \leq 12$

(d) $x < -3$

3 **(a)** $x < 7$ **(b)** $x \leq -20$ **(c)** $x > 7$

(d) $x \geq 1.5$ or $x \geq \frac{3}{2}$ or $x \geq 1\frac{1}{2}$

4 **(a)** Yes **(b)** No **(c)** Yes **(d)** Yes

5 **(a)** $x < 2$ **(b)** $x > 3$ **(c)** $x \leq 0$ **(d)** $x \geq 4$

6 **(a)** $x > \frac{7}{5}$ **(b)** $x \leq \frac{3}{2}$

(c) $x < \frac{21}{4}$ **(d)** $x \geq -8$

7 **(a)** $h < \frac{b}{2}$ or $2h < b$

(b) $4 < \frac{7.5}{2}$ is not true, so this is not a satisfactory rectangle.

(c) Sample answer: 5 cm, 4 cm, 3 cm

(d) Sample answer: 15 cm, 16 cm, 17 cm

SC 2

1 **(a)** $x > -6$ **(b)** $x > -10$

(c) $x > -4$ **(d)** $x > -16$

2 **(a)** $x \geq 2$ **(b)** $x > 14$

(c) $x \leq 5$ **(d)** $x < -3$

3 **(a)** $x < 9$ **(b)** $x \leq 2$

(c) $x > -3.25$ or $x > -3\frac{1}{4}$ or $x > -\frac{13}{4}$

(d) $x \geq 5$

4 **(a)** $x < 5$ **(b)** $x > -1$

(c) $x \leq 2$ **(d)** $x \geq 4$

5 **(a)** $x < -1$ **(b)** $x \geq -2$

(c) $x < 18$ **(d)** $x \leq -1$

6 **(a)** $x \leq \frac{19}{2}$ **(b)** $x > -\frac{2}{3}$

(c) $x < -\frac{11}{3}$ **(d)** $x \geq 2$

(e) $x \leq 2$

7 **(a)** Yes **(b)** No **(c)** No **(d)** No

Topic 8: Perimeter, area and volume

Recall

I understand how to multiply and divide decimals by powers of 10

1 **(a)** 280 **(b)** 371 **(c)** 40
(d) 67 **(e)** 82.05 **(f)** 9270
2 **(a)** 7.2 **(b)** 0.24 **(c)** 0.014
(d) 0.127 **(e)** 0.04 **(f)** 0.0003

I can add and subtract decimals

1 **(a)** 16.5 **(b)** 48.02 **(c)** 55.04
(d) 9.8 **(e)** 8.21 **(f)** 10.83

I can multiply and divide decimals

1 **(a)** 31.2 **(b)** 3.01 **(c)** 9 **(d)** 580

I can calculate the area of plane shapes

1 **(a)** 144 cm^2 **(b)** 54 cm^2 **(c)** 24 cm^2

I can identify the plane shapes used to form a composite shape

1 **(a)** rectangle and triangle
(b) square and rectangle
(c) parallelogram and triangle.

8.1 Estimate length and convert between units of length

SC 1

1 **(a)** Check that the width of your fingernail is about a centimetre. Count the number of times your fingernail fits across your palm. This estimates the width of your palm in centimetres.
(b) Check that the width of the paperclip is about 1 millimetre. Count the number of times a paperclip width fits across the grain of rice. This estimates the width of the grain in millimetres.
(c) Compare the length of the grain of rice with the width of a paperclip. This estimates the length of the rice grain in millimetres.
(d) Calculate how many blocks of 15 minutes are walked in two and a half hours (10). This gives the distance in kilometres.
2 **(a)** 9 mm **(b)** 8 cm **(c)** 15 m **(d)** 24 km
3 **(a)** 6 mm **(b)** 2 cm **(c)** 3 m **(d)** 6 km
4 **(a)** 2 km **(b)** 1.5 km **(c)** 4 km **(d)** 5 km
(e) 3.75 km **(f)** 3.75 km
5 **(a)** Any value in the range 85–100 cm would be acceptable. The daughter is a bit more than half the height of her mother.
(b) Any value in the range 175–185 cm would be acceptable. The son is about two thirds the height of his father.
6 **(a)** 8–10 m **(b)** 2–2.5 m

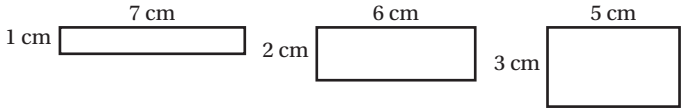
SC 2

1 **(a)** 580 cm **(b)** 29.7 cm
(c) 140 000 cm **(d)** 23 500 000 cm
2 **(a)** 8.32 m **(b)** 0.008 32 km
(c) 8320 mm **(d)** 83.2 dm
3 1 mm, 1 dm, 1 m, 1 Dm, 1 hm
4 **(a)** 0.5 km < 5500 m
(b) 15 m > 11 000 mm
(c) 250 mm < 0.026 km
(d) 3700 mm > 340 cm
5 **(a)** 8.1 cm **(b)** 4.569 km
(c) 1.235 m **(d)** 1.706 km
6 **(a)** 105 mm **(b)** 550 m
(c) 63.5 cm **(d)** 375 cm

8.2 Calculate the perimeter of quadrilaterals

SC 1

1 **(a)** 22 cm **(b)** 26 cm **(c)** 38 cm
(d) 59 cm **(e)** 17 cm
2 **(a)** 13 cm or 130 mm **(b)** 3.4 cm or 34 mm
(c) 5.8 cm or 58 mm **(d)** 8.9 m or 890 cm
(e) 3.42 m or 342 cm
3 17 m
4 Answers will vary.
There are many different rectangles, the sum of the two different side lengths is equal to 8 cm.



SC 2

1 **(a)** 30 cm **(b)** 473 cm **(c)** 46.7 cm **(d)** 3.6 cm
2 **(a)** 60 cm **(b)** 19 cm **(c)** 26 mm **(d)** 6.8 m
3 **(a)** 104 cm **(b)** 530 cm **(c)** 27.6 cm **(d)** 126 cm
4 **(a)** 47 cm **(b)** 11.5 cm **(c)** 66 cm **(d)** 35.1 cm
5 4740 m

SC 3

1 **(a)** 208 cm **(b)** 580 cm **(c)** 75.4 mm
2 **(a)** 32 cm **(b)** 46.2 cm **(c)** 7.5 m **(d)** 120 mm
3 **(a)** 332 cm **(b)** 82.3 cm **(c)** 67 cm **(d)** 93 mm
4 244 cm

8.3 Determine unknown side lengths from a given perimeter

SC 1

1 **(a)** 25 mm **(b)** 13 m **(c)** 44 m

2 **(a)** 10 cm **(b)** 11 cm **(c)** 4.8 cm
(d) 1.53 m or 153 cm

3 **(a)** 55 mm **(b)** 14 cm
(c) 130 cm or 1.3 m **(d)** 9.7 cm

4 **(a)** 15.5 cm **(b)** 1.4 m

5 **(a)** 12.3 cm **(b)** 33 cm

6 20 m

7 $l = 7$, $w = 3$; $P = 20$ cm

SC 2

1 **(a)** 3.4 cm **(b)** 1.4 m **(c)** 5.5 cm **(d)** 15 cm

2 **(a)** 2.5 m **(b)** 16 cm **(c)** 14.6 cm **(d)** 21.3 cm

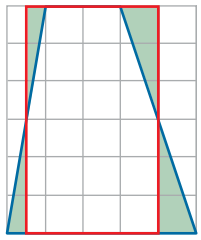
3 20.2 cm

4 **(a)** 30.2 m
(b) Answers will vary. Any four different values, not including 4.1 m or 5.3 m, that total 30.2 m. A possible answer is: 7 m, 7.2 m, 7.6 m and 8.4 m.

8.4 Calculate the area of a trapezium

SC 1

1 **(a)**



(b) Distance between the midpoints: 3.5 units.
The average of the parallel side lengths (2 units and 5 units, $\frac{2+5}{2} = 3.5$. units).

(c) 21 square units

(d) 21 square units

2 Top rectangle: 4 units by 5 units;
Bottom rectangle: 4.5 units by 2 units.

3 **(a)** 5 cm **(b)** 18 cm **(c)** 48.5 mm

4 **(a)** 2 m × 1.3 m
(b) 39 cm × 32 cm
(c) 31.5 mm × 16 mm

SC 2

1 **(a)** 300 mm^2 **(b)** 52 cm^2
(c) 2100 mm^2 **(d)** 117 m^2
(e) 152 mm^2 **(f)** 760 cm^2
(g) 896 cm^2

2 The conversion factor from m to mm is 1000^2, not 1000; 0.003 m^2.

3 328 cm^2

4 $3715.20

5 **(a)** 1038 cm^2 **(b)** 30 cm × 34.6 cm

6 83 cm^2

8.5 Calculate the area of kites and rhombuses

SC 1

1 **(a)** **(i)** 20 cm^2 **(ii)** 10 cm^2
(b) **(i)** 42 cm^2 **(ii)** 21 cm^2
(c) **(i)** 36 cm^2 **(ii)** 18 cm^2

2 **(a)** **(i)** 6 cm^2 **(ii)** 12 cm^2
(b) **(i)** 6.25 cm^2 **(ii)** 12.5 cm^2
(c) **(i)** 3.5 cm^2 **(ii)** 7 cm^2

3 **(a)** **(i)** 3 cm^2 **(ii)** 12 cm^2
(b) **(i)** 4.5 cm^2 **(ii)** 18 cm^2
(c) **(i)** 1 cm^2 **(ii)** 4 cm^2

4 **(a)** $A = xy$ **(b)** $A = \frac{1}{4}xy$ or $A = \frac{xy}{4}$
(c) $A = \frac{1}{2}xy$ or $A = \frac{xy}{2}$

5 **(a)** $A = \frac{1}{8}xy$ or $A = \frac{xy}{8}$
(b) $A = \frac{1}{2}xy$ or $A = \frac{xy}{2}$

6 **(a)** BD **(b)** RP and QS **(c)** none

SC 2

1 **(a)** 360 mm^2 **(b)** 159.36 cm^2 **(c)** 12.393 m^2

2 The conversion factor from m^2 to cm^2 is 100^2, not 100; 9100 cm^2.

3 5100 cm^2, 0.51 m^2

4 **(a)** 15.334 cm **(b)** 4.1496 m

8.6 Calculate the area of composite shapes

SC 1

1 **(a)** 4000 cm^2 **(b)** 1180 cm^2 **(c)** 27.36 m^2

2 **(a)** 558 m^2 **(b)** 22.5 mm^2 **(c)** 150 cm^2

3 **(a)** 80.56 cm^2 **(b)** 92 mm^2 **(c)** 5.125 m^2

4 201 m^2

5 6711 cm^2

6 50 cm

7 $1974

8 **(a)** 2666 cm^2 **(b)** 1152 cm^2

9 6 mm

10 2.9 cm

8.7 Approximate the perimeter and area of irregular shapes

SC 1

1 Answers will vary, but should be approximately:
(a) 14 cm **(b)** 15 cm
2 4.1 km
3 **(a)** 190 km **(b)** 285 km
4 **(a)** 14 700 km **(b)** 6000 km
(c) 3300 km **(d)** 2500 km
(e) 3100 km **(f)** 3100 km
(g) 2800 km

SC 2

1 Answers will vary, but should be approximately:
(a) 25 cm^2 **(b)** 24.5 cm^2
2 Answers will vary, but should be approximately:
(a) 21 cm^2 **(b)** 22 cm^2 **(c)** 20 cm^2
3 Answers will vary, but should be approximately:
(a) 20 cm^2 **(b)** 24 cm^2 **(c)** 10 cm^2
4 Answers will vary, but should be approximately:
(a) 8 cm^2 **(b)** 1 300 000 km^2
(c) 1 900 000 km^2 **(d)** 90 000 km^2
(e) **(i)** 2 520 000 km^2 **(ii)** 1 000 000 km^2
(iii) 800 000 km^2 **(iv)** 200 000 km^2

SC 3

1 Sample answers based on approximation of 10 m long and 3 m wide.
(a) 26 m **(b)** $1260
2 Sample answer, based on the length of lawn being 2.5 × 15 m and width 2 × 15 m.
(a) 1125 m^2
(b) **(i)** $787.50 **(ii)** $11 856.25
3 Sample answer based on the following estimated dimensions: house 12 m × 21 m; garage 12 m × 8 m; yard 40 m × 31 m with 3 m gaps in the fencing at the front.
(a) $12 720 **(b)** $122 400
4 **(a)** 71, 77, 78, 79, 80 **(b)** All 620 m^2
(c) 600 m **(d)** $86 400

8.8 Understand the connection between volume and capacity

SC 1

1 **(a)** Divide the number of cm^3 by 1000 to find the number of mm^3.
(b) Multiply the number of m^3 by 1 000 000 to find the number of cm^3.
(c) Divide the number of m^3 by 1 000 000 000 to find the number of km^3.
2 **(a)** 7 000 000 cm^3 **(b)** 0.02 m^3
(c) 34 cm^3 **(d)** 25 000 000 000 mm^3
(e) 0.000 056 7 m^3 **(f)** 30 000 cm^3
(g) 4010 mm^3 **(h)** 0.000 001 034 8 m^3
3 **(a)** 56.7 m^3 **(b)** 0.027 m^3
(c) 6 cm^3 **(d)** 10 m^3
4 **(a)** 729 000 cm^3, 0.729 m^3
(b) 360 000 cm^3, 0.36 m^3
(c) 486 mm^3, 0.486 cm^3
5 **(a)** 562 500 cm^3, 0.5625 m^3
(b) 562 500 000 cm^3, 562.5 m^3
6 **(a)** 384 cm^3
(b) No. If the dimensions are doubled, the new volume is 3027 cm^3. This is $2^3 = 8$ times the original volume.
7 **(a)** 120 000 000 cm^3 **(b)** 120 m^3
(c) 0.000 000 12 km^3

SC 2

1 **(a)** 7.2 L **(b)** 850 mL **(c)** 15 000 L
(d) 0.4 ML **(e)** 52 000 mL **(f)** 0.75 kL
2 **(a)** 85 mL **(b)** 150 cm^3 **(c)** 6.7 kL
(d) 0.096 m^3 **(e)** 0.8 L **(f)** 5200 cm^3
3 300 cm^3 0.3 L

SC 3

1 6.6 L
2 195.2 kL
3 18.2 ML
4 **(a)** 6.1 cm **(b)** 15.2 cm **(c)** 16.7 cm
5 64
6 2.7 g/cm^3
7 16.4 cm, 6.2 cm, 19.5 cm
8 **(a)** 845 cm^3 **(b)** 555 mL
(c) 2.2815 kg **(d)** 2.7765 kg
9 **(a)** 1950 kL
(b) 8:10 pm on September 10
(c) $7800
(d) $5000

Topic 9: Circles and sectors

Recall

I can identify features of a circle

1 A: radius, B: circumference, C: diameter
D: centre, E: arc, F: sector

I can identify the size of special angles

1 **(a)** 90° **(b)** 180° **(c)** 360°

I can simplify fractions

1 **(a)** $\frac{1}{3}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{13}{45}$ **(d)** $\frac{2}{15}$

I can write angles as a fraction of a revolution

1 **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{3}{4}$
(d) $\frac{1}{8}$ **(e)** $\frac{1}{3}$ **(f)** $\frac{1}{6}$

I can round decimals for different purposes

1 **(a)** 8.363 **(b)** 5.48 **(c)** 2.19 **(d)** 9.41
(e) 95 **(f)** 109 **(g)** 5.98

I can round calculations to the required number of decimal places

1 **(a)** 12.56 **(b)** 2.56 **(c)** 7.07 **(d)** 19.64

9.1 Determine the area of a circle

SC 1

1 **(a)** 100 square units
(b) 25 square units
(c) 79 square units
(d) A bit more than three quarters of the grid.
2 $A = \pi r^2$
3 **(a)** 3 cm **(b)** 4.77 cm **(c)** 71.55 cm^2

SC 2

1 **(a)** 50.27 cm^2 **(b)** 39.59 m^2 **(c)** 98.52 cm^2
2 **(a)** 314.16 cm^2 **(b)** 2.16 m^2 **(c)** 53.46 cm^2
3 **(a)** **(i)** 56 cm **(ii)** 44 cm
(iii) 196 cm^2 **(iv)** 154 cm^2
(b) **(i)** 112 cm **(ii)** 88 cm
(iii) 784 cm^2 **(iv)** 616 cm^2
(c) Both perimeters doubled.
(d) Both areas quadrupled (became 4 times as large).
4 **(a)** 20 m^2 **(b)** \$1700 **(c)** \$170
5 **(a)** 19 cm, 25 cm and 31 cm
(b) There is an increase of about 6 cm for each 2 cm increase of diameter: $2\pi \approx 6$
(c) 28 cm^2, 50 cm^2 and 79 cm^2
(d) No. There is an increase of about 22 cm^2 and then 29 cm^2. Because the radius is squared, a constant increase in radius does not give a constant increase in area.
6 **(a)** 1473 cm^2
(b) No. The smaller circle is only about 491 cm^2, one-quarter of the larger circle's area, so the shaded area would be exactly three times larger than the smaller circle's area.

9.2 Determine the area of a sector using common fractions

SC 1

1 **(a)** 13 square units
(b) 12.6 square units
2 **(a)** 7.1 m^2 **(b)** 113.5 mm^2
3 **(a)** 0.39 cm^2 **(b)** 1.57 cm^2
(c) 9.81 cm^2 **(d)** 39.27 cm^2
(e) 3926.99 cm^2
4 Sample answer. Calculate the length of the radius, then substitute the radius into the formula to calculate the area of a circle, $A = \pi r^2$, then divide by 2 because a semicircle is half a circle.
5 **(a)** 3318.3 mm^2 **(b)** 224.3 cm^2
6 **(a)** 0.78 cm^2 **(b)** 3.14 cm^2
(c) 19.63 cm^2 **(d)** 78.54 cm^2
(e) 7853.98 cm^2
7 Sample answer. Substitute the radius into the formula to calculate the area of a circle, $A = \pi r^2$, then divide by 4.
8 **(a)** 1.9 m^2 **(b)** 1558.6 mm^2
9 **(a)** 7.85 cm^2 **(b)** 9.82 cm^2
(c) 13.09 cm^2 **(d)** 39.27 cm^2
(e) 58.90 cm^2
10 Sample answer. Substitute the radius into the formula to calculate the area of a circle, $A = \pi r^2$, then multiply by the fraction of the circle given. The fraction is the angle divided by 360.

9.3 Determine sector area and arc length

SC 1

1 **(a)** 13.7 m^2 **(b)** 4539.5 mm^2 **(c)** 29.9 m^2
2 $A = 1297.8$ cm^2, $P = 151.8$ cm
3 3829.8 m^2
4 **(a)** 76 cm^2 **(b)** 41 cm^2

SC 2

1 **(a)** **(i)** 5.9 m **(ii)** 11.1 m
(b) **(i)** 7.0 m **(ii)** 12.2 m
(c) **(i)** 1.6 m **(ii)** 6.8 m
2 **(a)** 209 m **(b)** 289 m
3 **(a)** 103° **(b)** 22.5 cm^2
4 $A = 72.7$ cm^2, $P = 39.1$ cm
5 **(a)** A single piece is made of an arc one-quarter the length of the circumference of the 10 cm circle plus 2 × the arc of the circumference of the 5 cm circle.

$$P = \frac{1}{4} \times \pi \times 10 + 2\left(\frac{1}{2} \times \pi \times 5\right)$$

$P = 23.56$ cm

(b) Finding the area may present a challenge for students until they realise the area is:

$A = \frac{\pi 5^2}{4}$

$A = 19.63 \text{ cm}^2$

9.4 Determine the area of composite shapes involving circles

SC 1

1 **(a)** **(i)** 17 square units **(ii)** 16.5 square units
(b) **(i)** 15 square units **(ii)** 14.8 square units
2 **(a)** 65.0 m^2 **(b)** 774.1 cm^2
3 **(a)** 277.1 cm^2 **(b)** 346.6 cm^2
(c) 340.9 cm^2 **(d)** 2105.5 cm^2
4 **(a)** 31.8 m^2 **(b)** 909.9 cm^2 **(c)** 311.0 cm^2
5 $A = 996.97 \text{ cm}^2$, $P = 135.30 \text{ cm}$
6 **(a)** Possible answer: $l = 32$ cm, $w = 23$ cm
(b) Possible answer: $w = 11$ cm, $r = 16$ cm

9.5 Solve problems involving circle measurements

Exploration activity

1 $C = \pi d; C = \pi \times 1.3; C = 4.08 \text{ m}$ (2 d.p.)
2 $\frac{1000}{4.08} = 244.85$ revolutions. $C = \pi d$
3 $C = \pi \times 0.46$
$C = 1.45 \text{ m}$ (2 d.p.)
$\frac{1000}{1.45} = 691.98$ revolutions
4 The smaller tyre
5 The rider only needs to turn the pedals 245 revolutions rather than 692 revolutions.

SC 1

1 11 m^2
2 **(a)** 196 cm^2
(b) **(i)** The whole dartboard is 471 cm^2 larger, or 25 times larger than the middle circle.
(ii) The whole dartboard is 314 cm^2 larger, or $2\frac{7}{9}$ or about 3 times larger than the outermost ring.
3 **(a)** 2749 cm^2 **(b)** 219 cm
4 **(a)** 29 **(b)** 295 **(c)** 3537
5 \$220.50
6 4.19 m or 419 cm
7 34.5 km

Topic 10: Time

Recall

I can convert between different units of time

1 **(a)** 60 **(b)** 1 **(c)** 45 **(d)** 75 **(e)** 6
2 **(a)** 60 **(b)** 1200 **(c)** 900 **(d)** 3600 **(e)** 2700

I can solve problems involving time duration

1 **(a)** 3 hours, 30 minutes **(b)** 4 hours, 15 minutes
(c) 2 hours, 45 minutes **(d)** 3 hours, 15 minutes
(e) 18 hours, 30 minutes
2 **(a)** 11:30 am **(b)** 6:30 pm
(c) 5:15 pm **(d)** 4:15 am
3 New York: 2:40, London: 6:40, Moscow: 9:40, Tokyo: 3:40, Sydney: 5:40

10.1 Understand 24-hour time

SC 1

1 **(a)** 0530 **(b)** 1120 **(c)** 1000 **(d)** 0700
2 **(a)** 1240 **(b)** 1320 **(c)** 1910 **(d)** 2200

SC 2

1 **(a)** 4:28 am **(b)** 10:40 am **(c)** 11 am
2 **(a)** 12:35 pm **(b)** 4 pm **(c)** 6:52 pm
(d) 8:35 pm **(e)** 10:10 pm
3 **(a)** 7:00 am **(b)** 2:20 pm **(c)** 10:22 pm
(d) 9:50 am **(e)** 1:00 pm **(f)** 12:45 am

SC 3

1 **(a)** 2 h 45 min **(b)** 11 h 22 min
(c) 14 h 15 min **(d)** 12 h 2 min
2 **(a)** 1245 **(b)** 0120 next day
(c) 0935 **(d)** 1950 previous day

10.2 Understand the effect of daylight savings and longitude on local time

SC 1

1 **(a)** 9:05 am **(b)** 5:12 pm
(c) 5:55 pm **(d)** 9:35 am
2 **(a)** 1:25 pm **(b)** 7:20 am
(c) 9:55 am **(d)** 3:15 pm
3 **(a)** **(i)** 1:40 pm **(ii)** 9:20 am
(iii) 9:40 am **(iv)** 10:15 pm
(b) **(i)** 4:15 pm **(ii)** 8:10 am
(iii) 11 am **(iv)** 10:45 pm

SC 2

1 **(a)** 0140 on Saturday **(b)** 1340 on Saturday
(c) 0940 on Saturday **(d)** 0640 on Saturday
(e) 1040 on Saturday **(f)** 2340 on Friday
(g) 2040 on Friday **(h)** 1740 on Friday

2 **(a)** 3 pm on Thursday **(b)** 8 am on Thursday
(c) 9 pm on Wednesday **(d)** 5 am on Thursday

3 **(a)** 2 pm on Saturday **(b)** 5 pm on Saturday
(c) 10 pm on Saturday **(d)** 2 am on Sunday

4 It would be 7 pm at 150°E if no DST.
None of the locations has DST, since they are either in the Northern Hemisphere (so experiencing winter) or it is stated that they have no DST.
(a) 8 am on Wednesday **(b)** 6 am on Wednesday
(c) 9 am on Wednesday **(d)** 3 pm on Wednesday

10.3 Understand and work with different time zones

SC 1

1 **(a)** 10 am **(b)** 11 am
2 **(a)** 7:40 pm **(b)** 8:40 pm
3 **(a)** 2:30 pm **(b)** 1 pm
4 Times given are local time, so the arrival time of 1010 in Alice Springs (SA) is 1040 in Brisbane (Qld), because South Australia is UTC+10.5 and Queensland is UTC+10, regardless of the season.
The flight time is (1010 − 0720) + 0030 = 0320, or 3 h 20 min.
5 In Perth, the meeting ended at 1015. During summer, NSW observes daylight saving, so the time there is 3 hours ahead of NSW. Therefore, the time in Sydney when the meeting ended is 1315 or 1:15 pm.

SC 2

1 **(a)** 8:15 am Tuesday **(b)** 9:15 pm Monday
(c) 5:15 pm Monday
2 **(a)** 2:20 am 23 June **(b)** 11:55 pm 23 June
(c) 2:40 am 24 June
3 Canberra while observing AEDT is UTC+11
(a) 9 am to 4 pm for UTC+11 is 11 am to 6 pm for UTC+13
Canberra: 9 am to 2 pm; Wellington: 11 am to 4 pm
(b) 9 am to 4 pm for UTC+11 is 10 pm previous night to 4 am for UTC
There is no suitable time for Canberra and Dublin.
(c) 9 am to 4 pm for UTC+11 is 4 pm to 11 pm (both previous day) for UTC−6.
Canberra: 9 am and Dallas: 4 pm is the only suitable starting time.
4 **(a)** Adelaide is UTC+9.5 in winter. 7.5 h after 3 pm is 10:30 pm (Sunday).
(b) 10 h after 3 pm is 1 am Saturday.
(c) 3.5 h after 3 pm is 6:30 pm (Sunday).
(d) 6 h before is 9 am (Sunday).

SC 3

1 Brisbane is always UTC+10.
(a) 8 am + 8 h 5 min = 4:05 pm in Brisbane, so 2:05 pm in Singapore.
(b) 8 am + 3 h 5 min = 11:05 am in Brisbane, so 2:05 pm in Auckland.
(c) 8 am + 20 h 55 min = 4:55 am next day in Brisbane, so 1:55 pm same day as departure in New York.
2 7:45 pm at UTC−4 is 1:45 am the next day at UTC+2.
8:05 am to 1:45 am the next day is 17 h 40 min.
17 h 40 min − 2 h 45 min − 7 h 45 min = 7 h 10 min
The stopover is 7 h 10 min.
3 **(a)** 0940 UTC+8 to 1825 UTC+10 is 8 h 45 min − 2 h = 6 h 45 min.
Stopover: 6 h 45 min − 3 h 25 min = 3 h 20 min
(b) 1825 + 14 h 20 min = 3245 or 0845, 7th June
(c) 0845 − 17 h = 1545 previous day, so local time in LA is 1545, 6th June.

Topic 11: Rates

Recall

I can convert between different units of length, mass and money

1 **(a)** 2000 cm **(b)** 3 L **(c)** $8.27
(d) 0.357 kg **(e)** 5200 m **(f)** 3400 mL
(g) 22 510 g **(h)** 78 c

I can identify and write equivalent fractions

1 **(a)** 3 **(b)** 27 **(c)** 30 **(d)** 4
2 **(a)** T **(b)** F **(c)** F **(d)** T

I can write fractions in their simplest form

1 **(a)** $\frac{2}{5}$ **(b)** $\frac{4}{7}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{3}{8}$

I can convert between mixed numerals and improper fraction forms

1 **(a)** $\frac{31}{8}$ **(b)** $\frac{43}{7}$ **(c)** $\frac{110}{9}$ **(d)** $\frac{43}{5}$
2 **(a)** $4\frac{1}{2}$ **(b)** $5\frac{2}{5}$ **(c)** $3\frac{1}{3}$ **(d)** $6\frac{1}{2}$

I can interpret percentages in sentences

1 **(a)** 25% **(b)** 60% **(c)** $37\frac{1}{2}\%$ **(d)** $56\frac{1}{4}\%$

I can solve one-step equations

1 **(a)** $a = 14$ **(b)** $x = 22.5$ **(c)** $y = 4$
(d) $m = 2.5$ **(e)** $n = -9$ **(f)** $p = -42$

11.1 Understand and apply rates

SC 1

1 **(a)** 63 km/h **(b)** 67 km/h
(c) 92 km/h **(d)** 168 km/h

2 **(a)** 50 000 m/h **(b)** 0.01 km/s
(c) 13.89 m/s **(d)** 1 m/s

3 **(a)** 57.6 km/h **(b)** 144 km/h
(c) 360 km/h

4 **(a)** The wind was blowing at 15 km/h from the north-east. This is an example of a *rate* where it *is* important to include units.
(b) When mixing concrete, for every 1 bucket of cement, you should use 2 buckets of sand and 3 buckets of stones. This is an example of a *ratio* where units *are not* required.

5 **(a)** 5367 km/h **(b)** 1491 m/s

6 **(a)** $\frac{25}{90} = \frac{5}{18}$ m/s

(b) $\frac{90}{25} = \frac{18}{5} = 3\frac{3}{5}$ km/h

SC 2

1 **(a)** 12 adults
(b) 15 adults (a whole number of adults is needed)

2 **(a)** 30 sweets **(b)** $9

3 **(a)** $164.35 **(b)** 57.80 L

4 **(a)** The rate per km is a very small number (0.058 L/km). This is more difficult to work with.
(b) 561 km **(c)** 17.11 L **(d)** 1034 km

11.2 Apply rates to financial situations

SC 1

1 **(a)** $210.63 **(b)** $437.26
(c) $402.05 **(d)** $872.85

2 **(a)** 21 hours **(b)** 16 hours
(c) 13.5 hours **(d)** 7.5 hours

3 **(a)** $46.23/kg **(b)** $3.33/L
(c) $28.33/kg

4 **(a)** $0.23/100 g **(b)** $0.87/100 g
(c) $0.40/100 g

5 **(a)** $49.50 **(b)** $294.50

SC 2

1 **(a)** $140 **(b)** $420 **(c)** $70
(d) $38.36

2 **(a)** $300 **(b)** $900 **(c)** $1800
(d) $30

3 **(a)** 335 days **(b)** 503 days

4 **(a)** 2% p.a. **(b)** 2.81% p.a.
(c) 4.39% p.a.

5 **(a)** $32.50 **(b)** $22.95
(c) $36.31 **(d)** 1.45% p.a.

Exploration activity

In Scenario 1 (payout), it doesn't matter which option you choose because you get the same amount of money.

In Scenario 2 (reinvest), the quarterly option is best since you do get more money. This is because the interest is calculated on an increasing principal. Note that while there is a minor difference (24 cents) between the two options, this difference would be compounded as the initial principal.

11.3 Apply rates to tax calculations

SC 1

1 $3855.29 (Tax bracket is $18 201–$45 000; tax on extra $20 291 above $18 200 is $3855.29.)

2 **(a)** $5092 (Tax bracket is $18 201–$45 000; tax on extra $26 800 above $18 200 is $5092.)
(b) $5092.33 to the nearest cent. (Tax bracket is $45 001–$120 000; tax on extra $1 above $45 000 is 32.5 cents.)

3 $13 308 (Tax bracket is $45 001–$120 000; tax on extra $25 280 above $45 000 is $8216; tax to be paid is 5092 + 8216 = 13 308.)

4 $1625 (Both people are in the same tax bracket $45 001–$120 000, so they both pay $14 842 on the first $75 000 of their income. The person with a taxable income of $80 000 also pays tax on an extra $5000 at the rate 32.5 cents per dollar. This is $1625 extra.)

5 **(a)** Tax on the extra $15 000 is $2850 (All of the $15 000 is in the tax bracket $18 201–$45 000, so the rate is 19 cents per dollar.)
(b) Tax on the extra $15 000 is $4875 (All of the $15 000 is in the tax bracket $45 001–$120 000, so the rate is 32.5 cents per dollar.)

SC 2

1 $1008.10 each fortnight (Annual earnings: $109 980; annual tax $5092 + $21 118.50 = $26 210.50)

2 **(a)** $209.92 (Annual earnings: $62 920; annual tax: $5092 + $5824 = $10 916)
(b) $975.88 (Medicare levy: $24.20 per week)

3 15.4% (Tax, including Medicare levy, is $6717 + $1000 = $7717)

4 $655 (Annual earnings before and after: $69 600 and $57 600, which are both in the same tax bracket. Annual tax before and after: $13 087 and $9187. Medicare levy before and after: $1392 and $1152. Monthly take-home pay before and after: $4593.42 and $3938.42.)

Topic 12: Pythagoras' theorem in 2D

Recall

I can identify the hypotenuse of a right-angled triangle

1 **(a)** JK **(b)** PR **(c)** BC

I can round decimal numbers

1 **(a)** 2.02 **(b)** 6.79 **(c)** 1500.16 **(d)** 239.80

I can perform calculations with square numbers

1 **(a)** 36 **(b)** 1.44 **(c)** $\frac{1}{4}$ **(d)** 11
2 **(a)** 1.56 **(b)** 36.60 **(c)** 0.04 **(d)** 1.45
3 **(a)** 1.50 **(b)** 20.58 **(c)** 75 304.06

I can use a calculator to determine exact and approximate square roots

1 **(a)** 21 **(b)** 52 **(c)** 0.15 **(d)** 0.36
2 **(a)** 2.65 **(b)** 3.32 **(c)** 5.57 **(d)** 11.09

I can solve linear equations

1 **(a)** $a = 70$ **(b)** $a = 28$ **(c)** $a = -12$

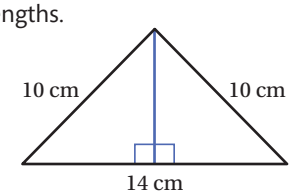
I can solve equations including squared terms

1 **(a)** $x = 11$ **(b)** $x = 42$ **(c)** $x = 6.08$
(d) $x = 13.27$

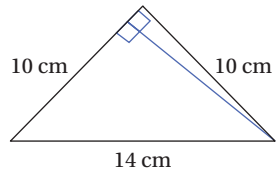
12.1 Measure the side lengths of a right-angled triangle

SC 1

1 **(a)** 5 mm **(b)** 13 mm **(c)** 15 mm
(d) The hypotenuse is the longest side length in a right-angled triangle.
2 Student answers may vary.
(a) Right angle; hypotenuse: 15.6 cm
(b) Not a right-angled triangle.
(c) Not a triangle, the sum of the two shorter sides is shorter than the longest side.
(d) Right angle; hypotenuse: 15 cm
(e) Not a right-angled triangle.
3 The hypotenuse is 15 cm in each right-angled triangle.
4 Both hypotenuses are 10 cm.
5 **(a)** Two right-angled triangles will have the same length hypotenuse if they have two pairs of equal side lengths.



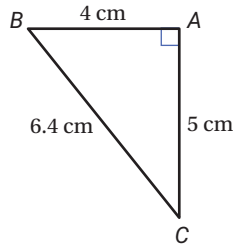
Two right-angled triangles will have different length hypotenuses if the pairs of shorter side lengths are unequal lengths.



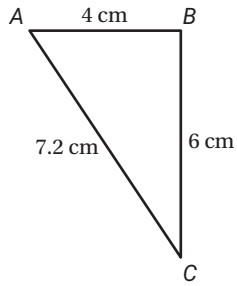
(b) 10 cm **(c)** 14 cm

SC 2

1 **(a)** The hypotenuse, $BC = 6.4$ cm.



(b) The hypotenuse, $AC = 7.2$ cm.



2 **(a)** 9.7 cm **(b)** 5.2 cm
3 **(a)** 4.3 cm **(b)** 7.2 cm
4 5.7 cm
5 **(a)** The second triangle has sides 1.5 times the lengths of the first. The third triangle has sides double the lengths of the first.
(b) 3.5 cm, 5.2 cm and 6.9 cm
(c) The corresponding lengths are in proportion:
The second triangle has a line of symmetry 1.5 times the length of the first. The third triangle has a line of symmetry double the length of the first.

12.2 Understand and use Pythagoras' theorem to identify right-angled triangles

SC 1

1 (a)

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
30 mm	40 mm	50 mm	900 mm^2	1600 mm^2	2500 mm^2	2500 mm^2
6 cm	8 cm	10 cm	36 cm^2	64 cm^2	100 cm^2	100 cm^2
60 mm	80 mm	100 mm	3600 mm^2	6400 mm^2	10 000 mm^2	10 000 mm^2
5 cm	12 cm	13 cm	81 cm^2	144 cm^2	169 cm^2	169 cm^2

(b) c

(c) Units of area, square units.

(d) The expressions are equal: $c^2 = a^2 + b^2$

2

a	b	c	a^2	b^2	$a^2 + b^2$	c^2	**Right-angled?**
4.1 cm	8 cm	10.5 cm	16.81 cm^2	64 cm^2	80.81 cm^2	110.25 cm^2	No
1.4 cm	4.8 cm	5 cm	1.96 cm^2	23.04 cm^2	25 cm^2	25 cm^2	Yes
18 mm	24 mm	30 mm	324 mm^2	576 mm^2	900 mm^2	900 mm^2	Yes
7 cm	7 cm	14 cm	49 cm^2	49 cm^2	98 cm^2	196 cm^2	No

3 (a) The equation assumes that the hypotenuse is c. The triangle shown has a hypotenuse labelled b.

(b) $b^2 = a^2 + c^2$ or $b^2 = c^2 + a^2$

4 (a) Student C is correct.

(b) Student A found the total length of the horizontal and vertical pieces of timber. Student B realised that they could use Pythagoras' theorem, but they worked out the square of the diagonal instead of the length of the diagonal.

SC 2

1 (a) 10 mm **(b)** 15 mm **(c)** 20 mm **(d)** 25 mm

(e) If the two shorter side lengths are in the same proportion as (3, 4, 5), then the hypotenuse will be in the same proportion also. You can multiply by the scale factor.

2 (a) 26 cm **(b)** 45 m **(c)** 82 km

3 (a) 24 m **(b)** 10 m **(c)** 15 m **(d)** 36 m

(e) When determining the length of the hypotenuse, you are always using the longest side. When determining the length of a shorter side, you first need to determine which is the corresponding side length.

4 (a) 120 mm **(b)** 80 cm **(c)** 60 km

5 (a) Yes: (6, 8, 10) is proportional to (3, 4, 5).

(b) No: (10, 18, 42) is proportional to (5, 9, 21).
$5^2 + 9^2 \neq 21^2$ because $25 + 81 \neq 441$

(c) Yes: (20, 48, 52) is proportional to (5, 12, 13).

(d) No: While (3.3, 4.4, 5.5) is proportional to (3, 4, 5), the values are not integers, so the ordered triple fails to meet the definition.

6 (a) Answers may vary, e.g. (16, 30, 34)

(b) Answers may vary, e.g. (30, 40, 50), (18, 24, 30)

SC 3

1 (a) Any length greater than 7 cm, but less than 17 cm.

(b) 17 cm

(c) Any length greater than 17 cm, but less than 23 cm.

2 (a) Any length greater than 7 cm, but less than 25 cm.

(b) 7 cm

(c) Any length greater than 1 cm, but less than 7 cm.

3 (a) Acute-angled

(b) Obtuse-angled

(c) Acute-angled

(d) Obtuse-angled

(e) Obtuse-angled

(f) If the two shorter side lengths match a Pythagorean triple, but the longest side of the triangle is longer than the Pythagorean hypotenuse, an obtuse-angled triangle is formed.
If the two shorter side lengths match a Pythagorean triple, but the longest side of the triangle is shorter than the Pythagorean hypotenuse, an acute-angled triangle is formed.

4 (a) Obtuse-angled triangle

(b) Acute-angled triangle

(c) Acute-angled triangle

(d) Obtuse-angled triangle

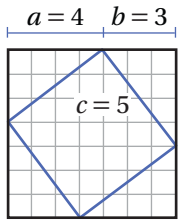
(e) Pythagorean triple

5 $\angle D$ and $\angle B$

12.3 Compare different applications, demonstrations and proofs of Pythagoras' theorem

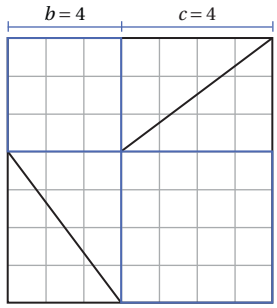
SC 1

1 (a)



(b) $c^2 = 5^2 = 25$

(c)

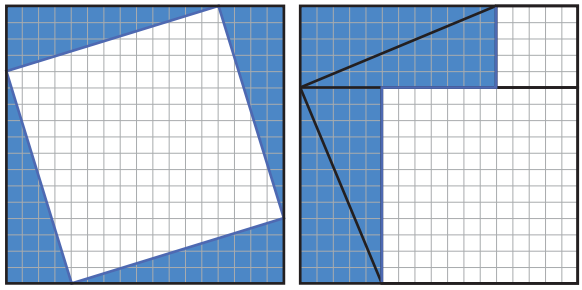


(d) $a^2 + b^2 = 4^2 + 3^2$
$= 16 + 9$
$= 25$
$= c^2$

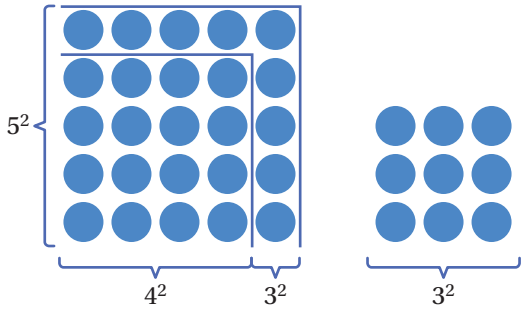
The area of the square of the hypotenuse is equal to the sum of the squares on the other two sides.

2 (a) $a + b$

(b)



3



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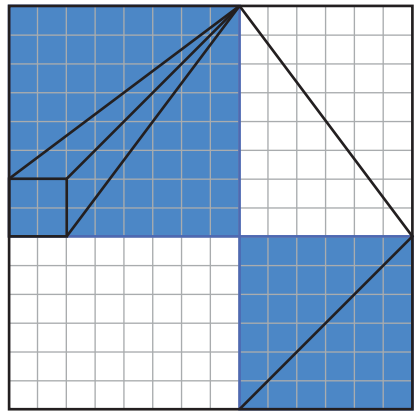


5 The total area in red is $(a + b)^2$ – area of 1 triangle.
The total area in blue is $(a + b)^2$ – area of 1 triangle.
Removing the three triangles from each of the two colour shadings leaves the three squares from the first diagram.
Hence, $c^2 = a^2 + b^2$.

SC 2

1 (a) 100 mm
(b) 120 mm, 160 mm, 200 mm
(c) 7200 mm^2, 12 800 mm^2, 20 000 mm^2
(d) $k = 2$

2 (a) 12 cm, 16 cm
(b) 8.4 cm, 11.2 cm
(c) 50.4 cm^2, 89.6 cm^2, 140 cm^2
(d) $k = 0.35$

3 (a) 120 mm, 130 mm
(b) 4300 mm^2, 24 768 mm^2, 29 068 mm^2
(c) $k = 1.72$

4 (a) 30 cm, 34 cm
(b) 20 cm, 8 cm; 37.5 cm, 15 cm; 42.5 cm, 17 cm
(c) 224 cm^2, 787.5 cm^2, 1011.5 cm^2
(d) $k = 0.875$

12.4 Use Pythagoras' theorem to determine the length of the hypotenuse

SC 1

1 (a) 5 cm **(b)** 6.40 cm **(c)** 13 cm
(d) 13.93 cm **(e)** 130 cm **(f)** 1.3 cm

2 5.948 m

3 (a) 5.58 m **(b)** 3.51 m

SC 2

1 50.1 cm

2 $x = 10$ cm

3 $a = 9.43$ m, $b = 8.94$ m

4 $265 000 for two roads; $295 000 for diagonal road.
It is $30 000 cheaper to build the two roads.

5 $p = 22.4$ cm, $q = 26.4$ cm, $r = 56.5$ cm

12.5 Use Pythagoras' theorem to determine the length of a shorter side in a right-angled triangle

SC 1

1 **(a)** 8 cm **(b)** 16 cm
(c) 19.85 cm **(d)** 0.8 cm
2 $x = 7.48$ cm, $y = 8.31$ cm
3 11.2 mm
4 4.24 m

SC 2

1 2.01 m
2 4.25 m
3 30.92 m
4 12.1 cm

Topic 13: Ratios to solve problems

Recall

I can group and count items by common attributes

1 Bananas: 2, Raspberries: 6, Apples: 3
2 Prime numbers: 5, Composite numbers: 7

I can write ratios to represent grouped values

1 **(a)** 20 : 13 **(b)** 3 : 10

I can convert between different units of length, mass and money

1 **(a)** 350 mm **(b)** 4900 mL **(c)** 1250 cents

I can write fractions in their simplest form

1 **(a)** $\frac{6}{7}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{7}{25}$

I can convert between mixed numerals and improper fraction forms

1 **(a)** $\frac{22}{5}$ **(b)** $\frac{38}{7}$ **(c)** $16\frac{3}{4}$ **(d)** $9\frac{1}{8}$

I can identify and write equivalent ratios

1 **(a)** F **(b)** T
2 **(a)** 21 **(b)** 12

13.1 Solve problems involving proportional reasoning

SC 1

1 **(a)** $a = 300$ **(b)** $b = 90$ **(c)** $c = 275$
(d) $d = 1350$ **(e)** $e = 5.25$ **(f)** $f = 4.4$
2 51.6, so expect 52 little wattle birds
3 95 cm
4 486 g
5 Without a discount, 450 mL should cost \$6.75, so the discount is \$1.75.

SC 2

1 **(a)** 3 m **(b)** 0.6 m
(c) 4.2 m **(d)** 12.6 m
2 **(a)** 10 mm **(b)** 2 mm **(c)** 50 mm
(d) 105 mm **(e)** 6 mm
3 **(a)** 100 m **(b)** 240 m **(c)** 64 m
(d) 512 m **(e)** 86 m **(f)** 14 m
4 **(a)** 7.2 cm **(b)** 2.3 cm **(c)** 3.2 cm
(d) 0.9 cm
5 **(a)** 820 km **(b)** 115 km **(c)** 45 km

SC 3

1 **(a)** 1 : 100; 100 **(b)** 1 : 400; 400
(c) 1 : 40; 40 **(d)** 1 : 40 000; 40 000
2 **(a)** 500 cm or 5 m
(b) 2000 cm or 20 m
(c) 200 cm or 2 m
(d) 200 000 cm or 2 km
3 **(a)** 1 : 500; 500
(b) 1 : 7500; 7500
(c) 1 : 60; 60
(d) 1 : 60 000; 60 000
4 **(a)** 2 m **(b)** 13.3 cm
(c) 16.7 m **(d)** 1.7 cm
5 **(a)** 27 m by 21.6 m **(b)** 7.2 cm^2
(c) 583.2 m^2
(d) $900^2 = 810\,000$, or $\frac{583.2 \times 100^2}{7.2} = 81000$

Exploration activity

1 **(a)** Scale factor = 56 **(b)** 1.57 m
(c) 2.80 m **(d)** 2.52 m

13.2 Apply ratios to currency exchange

SC 1

1 **(a)** 2544 PLN **(b)** 912 NZD
(c) 3640 CNY **(d)** 464 GBP
2 **(a)** 703.30 AUD **(b)** 312.25 AUD
(c) 229.90 AUD **(d)** 53.85 AUD
3 **(a)** 2317.20 NZD **(b)** 273.30 AUD
4 **(a)** 650 EUR **(b)** 955.88 AUD **(c)** 44.12 AUD

SC 2

1 **(a)** 172.73 AUD
(b) 163.2 EUR for 5 days or 16.32 EUR each per day, on average.

2

	USD	AUD
Meals	$320	$500
Tips @ 15% of meals	$48	$75
Souvenirs	$150	$234
Bottled water	$20	$31
Total	$538	$840

3

	THB	VND	AUD
Food	960	828 000	40 + 47 = $87
Souvenirs	2400	1 760 000	$200
Bottled water	32	36 000	1.33 + 2.05 ≈ $3
Total	3392	2 624 000	$290

4 **(a)** 7.75 AUD **(b)** 82.46 AUD
(c) 53.72 AUD **(d)** 249.13 AUD

Topic 14: Triangles and quadrilaterals (similarity and congruence)

Recall

I can name angles

1 **(a)** $\angle LMN$ **(b)** $\angle XYZ$ **(c)** $\angle EIF$
(d) $\angle MOP$

I can identify matching sides and angles in shapes

1 **(a)** AB **(b)** $\angle CBA$ **(c)** QR
(d) $\angle QPR$

I can identify and name different types of quadrilaterals

1 **(a)** square **(b)** rectangle **(c)** parallelogram
(d) trapezium **(e)** rhombus **(f)** kite

I can simplify ratios to unit ratios

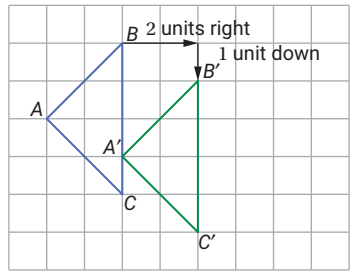
1 **(a)** $m = 6$ **(b)** $m = 3$ **(c)** $m = 8$
(d) $m = 2.5$ or $2\frac{1}{2}$ **(e)** $m = 4.25$ or $4\frac{1}{4}$

14.1 Apply transformations to shapes in the plane

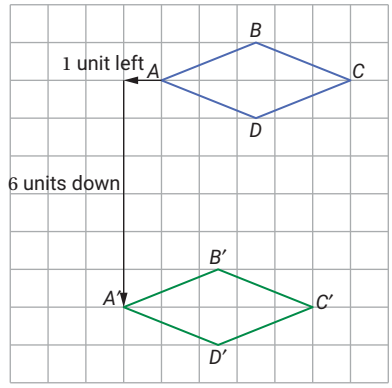
SC 1

1 **(a)** 5 units down
(b) 4 units right, 1 unit down
(c) 6 units left, 2 units up

2 **(a)**

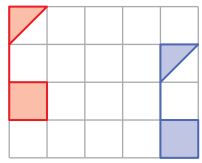


(b)

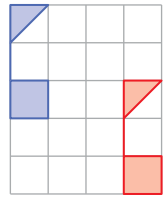


3 The original shape and its translated image are congruent.
(a) The side lengths stay the same
(b) The angles stay the same
(c) The position changes
(d) The orientation stays the same

4 **(a)**

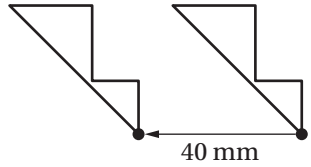


(b)

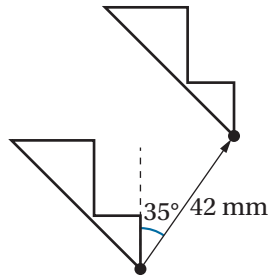


5 **(a)** 10 units left, 4 units up
(b) 9 units up
(c) 4 units down
(d) 200 units right

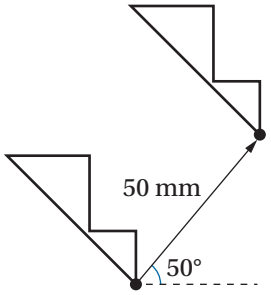
6 **(a)**



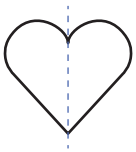
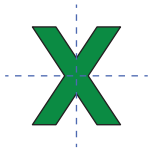
(b)



(c)

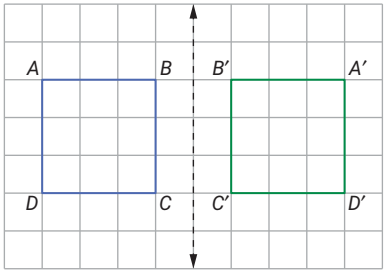


SC 2

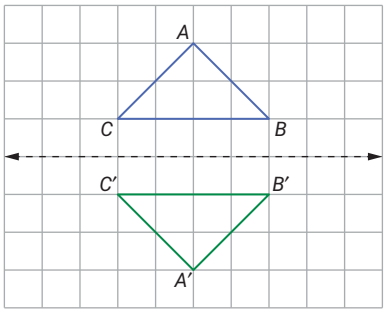
1 **(a)**  **(b)** 

2 **(a)** 4 **(b)** 2 **(c)** 1

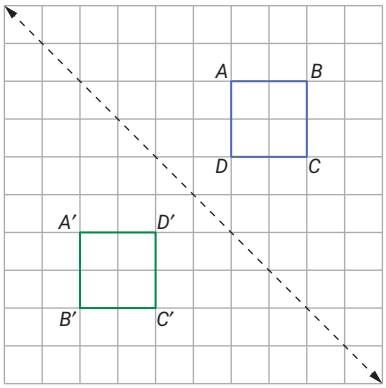
3 **(a)**



(b)

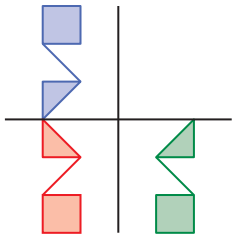


(c)



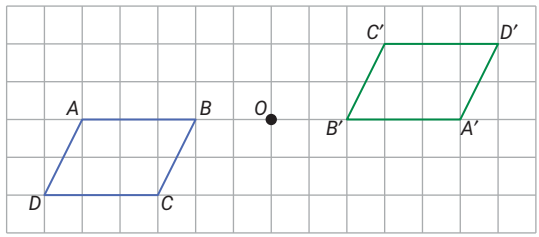
4 The original shape and its reflected image are congruent.
- **(a)** The side lengths stay the same
- **(b)** The angles stay the same
- **(c)** The position changes, unless the shape is reflected around its own axis
- **(d)** The order of the vertices changes

5 Sample answer

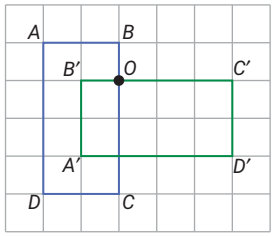


SC 3

1 **(a)**



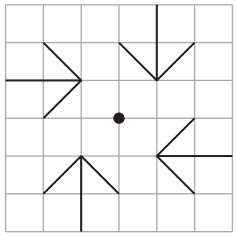
(b)



2 **(a)** 90° clockwise
- **(b)** 90° anticlockwise
- **(c)** 180° clockwise or anticlockwise

3 The original shape and its rotated image are congruent.
- **(a)** The side lengths stay the same.
- **(b)** The angles stay the same.
- **(c)** The position changes, unless the shape is rotated around its own centre.
- **(d)** The orientation changes, unless the rotation is 360°.
- **(e)** The order of the vertices stay the same.

4



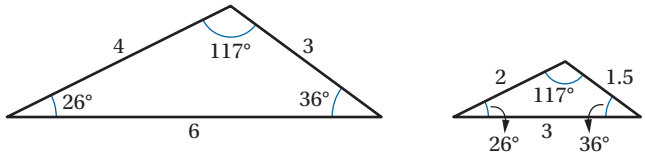
14.2 Prove congruence in pairs of triangles

SC 1

1 **(a)** As a check, the unknown side should be about 4.1 cm.
- **(b)** As a check, the longest side should be about 9.1 cm.
- **(c)** As a check, the unknown side should be about 4.5 cm.
- **(d)** As a check, the largest angle should be about 122°.

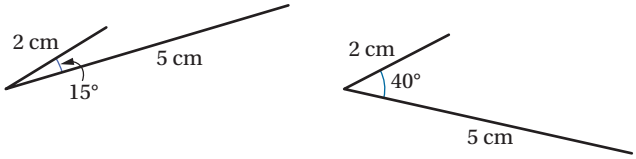
2 For each triangle, its reflection can be drawn using the same instructions. The two triangles are congruent.

3 Three angles the same means that the triangles will be the same shape, but not necessarily the same size.



If one pair of matching sides were equal, the triangles would be the same size and congruent.

4 The given sides can meet at a variety of angles. The size of the angle determines the size of the third side.



SC 2

1 **(a)** $\Delta DEF \cong \Delta XZY$ or $\Delta DEF \cong \Delta YZX$ (SSS)
(b) $\Delta ABC \cong \Delta RQS$ (RHS)
(c) $\Delta XYZ \cong \Delta PQR$ (AAS)
(d) $\Delta XYZ \cong \Delta SRT$ (SAS)

2 **(a)** RHS **(b)** SAS **(c)** AAS **(d)** SSS
(e) AAS **(f)** SSS **(g)** SAS **(h)** SAS

3 $\Delta RST \cong \Delta RUT$ (RHS) Both triangles have a right angle (R). Both triangles share the same hypotenuse RT (H). Side length ST is equal to side length UT (S). Therefore, the two triangles satisfy the congruence rule RHS.

4 **(a)** $BA = BC = 10$ cm, $\angle A = \angle C = 37°$, BD is shared, so the same length in each triangle.
(b) Yes
(c) No. The only tests for two sides and an angle are RHS, which needs a right-angled triangle, and SAS, which needs the angle between the sides (the included angle). If the angles at D were both right angles, the triangles would be congruent.
(d) The diagram shows that many different triangles can be made by moving line segment BD, and all of these have 2 sides and an angle equal. For the test SAS to work, it must be the included angle.
(e) If line segment BD moved to make 2 right angles at D, then congruency could be proven using RHS or AAS.

5 The position of the 6 cm side is different in the two triangles, so the AAS congruence test cannot be used with this side. Looking further, the third angle of the triangle is $180° - 80° - 30° = 70°$, so the triangle is scalene, with all sides different. This means that the unknown sides of each triangle are not equal to 6 cm.

SC 3

1 **(a)** $a = 75°$ **(b)** $b = 15$ cm **(c)** $c = 5$ cm
(d) $d = 50$ mm **(e)** $e = 44°$ **(f)** $f = 35°$

2 **(a)** SAS; $a = 42°$ **(b)** SAS; $b = 37°$
(c) AAS; $c = 20$ mm **(d)** RHS; $d = 74°$

3 $\Delta BAC \cong \Delta EDC$ (AAS); $x = 5$ cm

4 $\Delta ABC \cong \Delta CDA$ (SSS); $x = 40°$

5 $\Delta OAB \cong \Delta OCB$ (RHS); $x = 10°$

14.3 Explore the properties of quadrilaterals using congruent triangles

SC 1

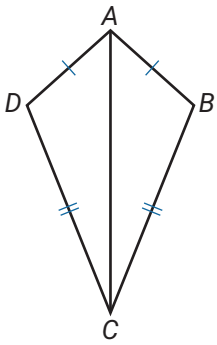
1 In triangles ABC, ADC:
$AB = AD$ (given) ...S
$BC = DC$ (given) ...S
AC is shared ...S
$\therefore \Delta ABC \cong \Delta ADC$ (SSS)
$\angle ABC = \angle ADC$ (corresponding angles)
Hence, one pair of opposite angles of the kite are equal.

2 In triangles AEB, CED:
$\angle BAE = \angle DCE$ (alternate angles, $AB \parallel DC$) ...A
$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel DC$) ....A
$AB = CD$ (opposite sides equal, assumed proved) ...S
$\therefore \Delta AEB \cong \Delta CED$ (AAS)
$AE = CE$ and $BE = DE$ (corresponding sides)
Hence, the diagonals of a parallelogram bisect each other.

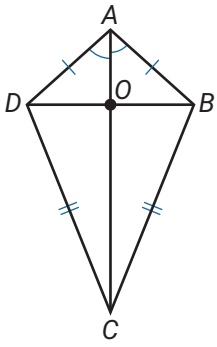
3 In triangles EJF, GJH:
$EJ = GJ$ (given) ...S
$\angle EJF = \angle GJH$ (vertically opposite angles) ...A
$FJ = HJ$ (given) ...S
$\therefore \Delta EJF \cong \Delta GJH$ (SAS)
$EF = GH$ (corresponding sides)

In triangles EJF, GJF:
$EJ = GJ$ (given) ...S
$\angle EJF = \angle GJF$ (Angle adjacent to 90° is supplementary, so also 90°) ...A
FJ is shared ...S
$\therefore \Delta EJF \cong \Delta GJF$ (SAS)
$EF = GF$ (corresponding sides)
Similarly, it can be shown using another pair of adjacent triangles that $EF = EH$.
Therefore, all sides of the quadrilateral are equal, so the quadrilateral is a rhombus by definition.

4



In triangles ADC, ABC:
$AD = AB$ (given) ...S
$CD = CB$ (given) ...S
AC is shared ...S
$\therefore \Delta ADC \cong \Delta ABC$ (SSS)
$\angle DAC = \angle BAC$ (corresponding angles)



In triangles DAO, BAO:
$AD = AB$ (given) ...S
$\angle DAO = \angle BAO$ (Proven) ...A
AO is shared ...S
$\therefore \Delta DAO \cong \Delta BAO$ (SAS)
$\angle DOA = \angle BOA$ (corresponding sides)
Since these two angles are adjacent, they are supplementary, so both angles are right angles.
Hence, the diagonals of a kite intersect at right angles.

5 In triangles XUW, VWU:
$XU = VW$ (given) ...S
$\angle XUW = \angle VWU$ (alternate angles, $XU \parallel WV$) ...A
UW is shared ...S
$\therefore \Delta XUW \cong \Delta VWU$ (SAS)
$\angle XWU = \angle VUW$ (corresponding angles)
Hence $UV \parallel XW$ (alternate angles equal)
Now that both pairs of opposite sides are known to be parallel, the quadrilateral is a parallelogram, by definition.

SC 2

1 In triangles ZWX, XYZ:
$\angle WXZ = \angle YZX$ (alternate angles, $WX \parallel ZY$) ...A
$\angle WZX = \angle YXZ$ (alternate angles, $WZ \parallel XY$) ...A
XZ is shared ...S
$\therefore \Delta ZWX \cong \Delta XYZ$ (AAS)
$WX = YZ$ (corresponding sides)
So, $y = 15$ cm
$\angle WZX = \angle YXZ$ (corresponding angles)
From the angle sum of a triangle:
$\angle WZX = 180° - 90° - 20° = 70°$
So, $x = 70°$

2 SSS congruence test, $c = 105°$

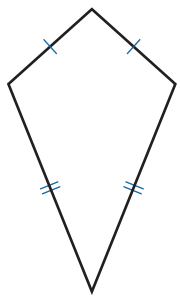
3 SSS congruence test, $w = 110°$

4 In triangles BAD, BCD:
$AB = CB$ (given) ...S
$AD = CD$ (given) ...S
BD is shared ...S
$\therefore \Delta BAD \cong \Delta BCD$ (SSS)
$\angle BAD = \angle BCD$ (corresponding angles)
So, $y = 120°$
$\angle ABD = \angle CBD$ (corresponding angles)
From the angle sum of a triangle and isosceles triangles:
$\angle ABD = \dfrac{180° - 120°}{2} = 30°$
So, $x = 30°$

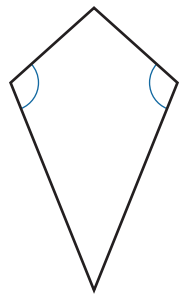
5 Draw one diagonal, marking equal angles from the isosceles triangle as x.
The SSS congruence test gives two more angles to mark as x.
Draw the other diagonal, marking equal angles from the isosceles triangle as y.
The SSS congruence test gives two more angles to mark as y.
Now $4x + 4y = 360°$, so $x + y = 90°$.
In any of the smallest triangles, $x + y + t = 180°$.
So, $t = 90°$.

SC 3

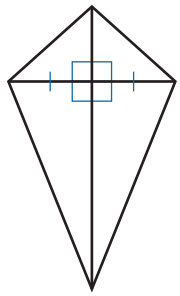
1 **(a)**



(b)

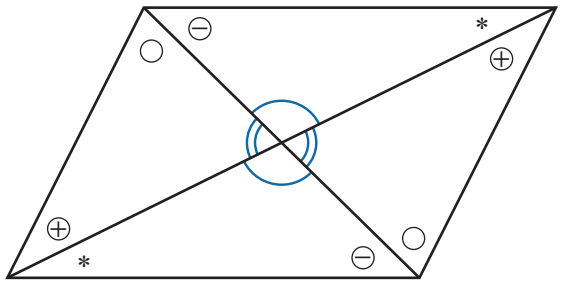


(c)

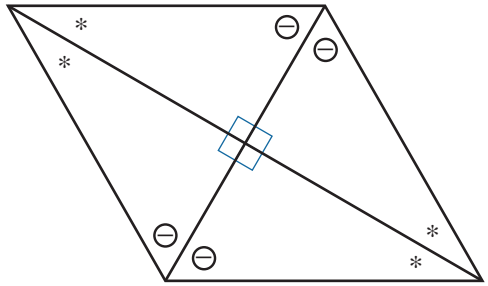


2 **(a)** square **(b)** rectangle **(c)** rhombus

3 **(a)**



(b)



4 **(a)** A rectangle.

(b) Greater height, less width, but still a rectangle.

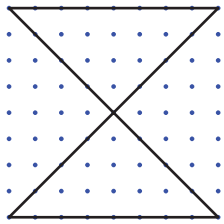
(c) The diagonals are equal length and bisect each other.

(d) The platform remains horizontal as its base rises vertically because the sides of a rectangle are perpendicular. Because no other quadrilateral has diagonals of equal length that bisect each other, the shape remains as a rectangle.

14.4 Determine and apply scale factor and angle properties in similar shapes

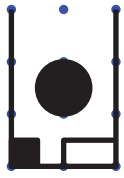
SC 1

1 **(a)**



(b)

2 **(a)**



(b)

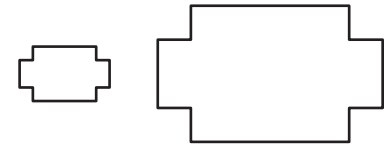
3 **(a)** Image should be the same shape, with sides parallel to the original sides. All sides will be three times the original lengths.

(b) Image should be the same shape, with sides parallel to the original sides. All sides will be 1.5 times the original lengths.

4 **(a)** Image should be the same shape, with sides parallel to the original sides. All sides will be $\frac{1}{3}$ the original lengths.

(b) Image should be the same shape, with sides parallel to the original sides. All sides will be 0.8 times the original lengths.

5



SC 2

1 **(a)** 1.5 or $\frac{3}{2}$ or $1\frac{1}{2}$

(b) 2

(c) 1.6 or $\frac{8}{5}$ or $1\frac{3}{5}$

2 **(a)** $\frac{3}{4}$ or 0.75 **(b)** $\frac{1}{2}$ or 0.5

3 **(a)** Both $\frac{3}{2}$ **(b)** 1.5 **(c)** $\frac{1}{2}$ or 0.5

4 **(a)** 3.5 **(b)** 0.8 **(c)** 1.5

5 **(a)** 2; $a = 22$ cm

(b) 0.5; $b = 360$ m

(c) 2.5; $c = 1.5$ cm

SC 3

1 **(a)** $x = 9.76$ cm, $y = 45°$

(b) $x = 22.5$ mm, $y = 110°$

2 **(a)** $x = 12$ cm, $\alpha = 118°$

(b) $x = 7.5$ cm, $\theta = 48°$

(c) $x = 7.5$ cm, $y = 53°$

3 **(a)** $x = 8\frac{1}{3}$ cm ≈ 8.3 cm, $y = 42°$

(b) $x = 8$ cm, $y = 41°$

4 **(a)** $y = 18\frac{8}{9}$ cm ≈ 18.9 cm, $x = 70°$

(b) $x = 90$ mm, $y = 109°$

5 All side lengths 33 cm; all acute angles 36°; all reflex angles 252°.

6 21 m

14.5 Prove similarity in pairs of triangles

SC 1

1 (a) $\Delta CED \sim \Delta RQP$ (SSS); scale factor 2

(b) $\Delta PQR \sim \Delta ZXY$ or $\Delta PQR \sim \Delta ZYX$ (AAA); $70° \times 2 + 40° = 180°$

(c) $\Delta ABC \sim \Delta DEF$ (RHS); scale factor 2.5

(d) $\Delta ACB \sim \Delta DCA$ (SAS); scale factor 1.5

2 (a) $\Delta ABC \sim \Delta DEF$ or $\Delta ABC \sim \Delta DFE$ or $\Delta ABC \sim \Delta EDF$ (AAA) because all angles are the same.

(b) Not similar. Ratio of largest sides: $\frac{10}{5} = 2$; ratio of smallest sides: $\frac{4}{2.5} \neq 2$

(c) Not similar. Only one pair of angles is the same. The triangles are different shapes.

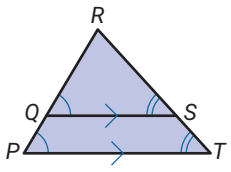
(d) $\Delta ABC \sim \Delta DFE$ (SSS). Ratio of three pairs of sides is $\frac{2}{3}$.

3 (a) $\angle LNM = \angle ONP$ (vertically opposite angles)

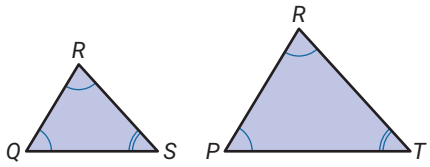
(b) If $LM \parallel PO$, then either $\angle L = \angle O$ or $\angle M = \angle P$ (alternate angles, $LM \parallel PO$).
Hence $\Delta LMN \sim \Delta OPN$ (AAA).

(c) Either $\angle L = \angle P$ or $\angle M = \angle O$; $\Delta LMN \sim \Delta PON$

4 (a)

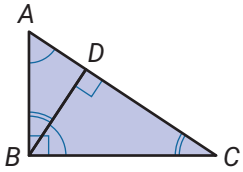


(b)



(c) $\Delta RPT \sim \Delta RQS$ (AAA)

5



ΔABC has a right angle at B and acute angles at A and C.
ΔADC has a right angle at D and shares $\angle A$, so that $\Delta ADC \sim \Delta ABC$.
ΔBDC has a right angle at D and shares $\angle C$, so that $\Delta BDC \sim \Delta ABC$.
The AAA similarity test is applied each time.
It follows that $\angle ABD = \angle C$ and $\angle DBC = \angle A$, as shown on the diagram.

SC 2

1 (a) $\Delta RSQ \sim \Delta QSP$ (RHS); Scale factor: 1.8; $x = 29°$

(b) $\Delta ADC \sim \Delta BDA$ (SAS); Scale factor: 2.1; $x = 26.5°$

2 (a) $\Delta KOP \sim \Delta KLM$ (AAA); $y = 7.5$ cm

(b) $\Delta SVW \sim \Delta STU$ (AAA); $x = 22.4$ cm

3 (a) $\Delta DEH \sim \Delta GFH$ (ASA); $x = 12$ cm

(b) $\Delta PQR \sim \Delta TSR$ (AAA); $x = 4.5$ m

4 $\Delta DGE \sim \Delta DEF$; $\frac{x}{60} = \frac{60}{65}$ so $x = 55\frac{5}{13}$ cm ≈ 55.4 cm

$\Delta EGF \sim \Delta DEF$; $\frac{y}{25} = \frac{25}{65}$ so $y = 9\frac{8}{13}$ cm ≈ 9.6 cm

5 $\Delta EFG \sim \Delta EHI$ (AAA); $\frac{HI}{FG} = \frac{EI}{EG}$; $\frac{x}{40} = \frac{20}{60}$;
$x = 13.3$ cm. The brace needs a length of 13.3 cm.

Topic 15: Visualising 3D

Recall

I can describe the position of plotted points on a Cartesian plane from the origin

1 (a) A is 8 units to the left, and up 3 units from the origin.

(b) B is 3 units to the right of the origin.

(c) C is 7 units to the left, and 6 units below the origin.

(d) D is 1 unit right of the origin and 7 units below it.

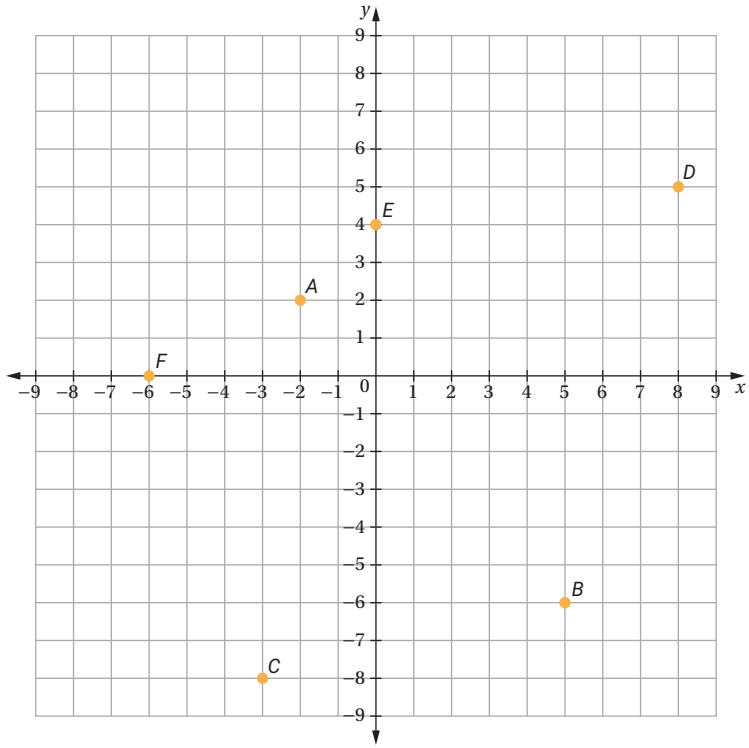
(e) E is 6 units directly above the origin.

(f) F is 5 units to the right and 8 units above the origin.

I can read and plot points on a Cartesian plane

1 $A(2, 5)$, $B(7, 0)$, $C(4, -6)$, $D(-8, 3)$, $E(0, -4)$ and $F(-5, -3)$

2



15.1 Locate and describe the position of objects in 3D space

SC 1

1 **(a)** 150 m **(b)** 350 m **(c)** 100 m

2 **(a)** 14 m **(b)** 10 m

(c) −3 m (The whale is below the boat, not above.)

3 **(a)** 400 m west and 200 m north of the boat at an altitude of 450 m

(b) 300 m west and 100 m south of the boat at an altitude of 200 m

(c) 200 m east and 300 m south of the boat at an altitude of −15 m

(d) 400 m east and 100 m south of the boat at an altitude of 0 m

4 **(a)** aeroplane

(b) submarine

(c) volcano Lunjia

(d) fish school

15.2 Use the 3D coordinate system

SC 1

1 **(a)** 3 **(b)** 1 **(c)** 1

2 **(a)** C **(b)** B **(c)** A

3 (2, 3, 4)

4 (−3, 2, −4)

5 **(a)** (2, 3, 4) **(b)** (−3, 2, −4) **(c)** (0, −3, 0)

6 **(a)** xy-plane because the z-value is equal to 0.

(b) xz-plane because the y-value is equal to 0.

(c) yz-plane because the x-value is equal to 0.

7 **(a)** A: (3, 3, 2), B: (0, 3, 2)

(b) The y-values and z-values remain the same.

Topic 16: Collecting and analysing data

Recall

I can complete a frequency distribution table

1

Wi-Fi signals	Tally	Frequency
0	|	1
1	|	1
2	|||	4
3	||||	5
4	|||	4
Total	**15**	**15**

I can calculate the mean, mode and median for a set of scores

1 **(a)** 4.29 **(b)** 8.17 **(c)** 1.58
2 **(a)** 4 **(b)** 3 **(c)** 5 and 8
3 **(a)** 11 **(b)** 12.5 **(c)** 114.5

I can calculate the mean, mode and median from a frequency distribution table

1

Minutes (x)	Frequency (f)	$x \times f$
30	2	60
35	4	140
40	5	200
45	12	540
50	13	650
55	4	220
Total	**40**	**1810**

Mean: $\frac{1810}{40} = 45.25$
Median: 45
Mode: 50

I can calculate the mean, mode and median from a stem-and-leaf plot

1 Mean: $\frac{766}{28} = 27.36$, Median: 26, Mode: 19

16.1 Compare sampling methods

SC 1

1 **(a)** It is likely that some customers will not reply to the survey. However, this fits the definition of a census because the survey is sent to every customer who has their car serviced, which is the whole population.
(b) This is most likely a sample. We do not know what question Anthea is trying to answer, but the survey period – 5 hours – is not long enough to draw any conclusions.
(c) This is a sample; the population is everyone visiting the sporting arena, but this process only involves people using Gate 5.
2 **(a)** The population is all houses that have been sold in that suburb in a defined time period. (The cost can only be based on actual house sales, not on the estimated value of all the houses that have not been sold recently.)
(b) The population is the people of working age in each state.
(c) The population is every household in Australia.
3 **(a)** Sample; it is not practical to find the colour of every car in Australia. (Although states or territories might record this information as part of the car registration process, these databases usually record only broad colour categories.)
(b) Sample; it is not practical to measure every student of this age in Australia.
(c) Sample; it is not practical to survey every user of nicotine products.
(d) Census; the population (100 club members) is relatively small, so every person can be surveyed.
4 **(a)** It takes too long to organise and conduct the census, and to collate and interpret the information collected.
(b) Much of the information is collected on Census Night via an electronic process. However, households are given the opportunity to complete a paper-based form. It can take around a month for these forms to be distributed and collected. There are many remote communities in Australia and every person has the same right to have their information collected.
(c) This is an ongoing process that effectively fills the time between census dates.
(d) The Australian Government pays, which effectively means the taxpayers of Australia pay.

SC 2

1 **(a)** non-random **(b)** non-random
(c) random
2 **(a)** If every battery is tested to exhaustion, the company will have no batteries left to sell.
(b) Using the $\sqrt{n}$ rule, the sample should be about 140 batteries.
(c) Using the $\sqrt{n}$ rule, the sample should be about 450 batteries.
3 **(a)** census **(b)** census
(c) sample; each seat could be allocated a number and a random number generator used to choose the seats for which the occupants will be surveyed
4 **(a)** **(i)** non-random **(ii)** non-random
(iii) non-random **(iv)** non-random
(b) It is difficult to ensure a sample is random.

SC 3

1 Motorists who live in rural areas have little chance of being included in the survey. Motorists who do not fill their car on that day will not be surveyed. However, some effort has been made to make the survey more robust than, for example, a survey completed at only one petrol station.

2 If the telephone directory included a phone number for every member of the target population, then the method had a slight chance of being fair and unbiased. In most cases, however, more than one directory would have to have been used. Also, people who did not have a landline (or who chose to keep their details private) would not have any chance of being included in the sample.

3 **(a)** Students in other year groups may have very different opinions to those in Year 7.

(b) Those playing sport in the winter, for example, football and netball players, may have different needs in clubrooms to those playing cricket or other summer sports; for example, climate control.

(c) The people surveyed are simply asked to choose their favourite bar from three possible options: the question does not find out whether they would buy any of the chocolate bars. Also, the profile of the target population should be used to inform the sampling process. For example, are children likely to eat the product and if so, do they have any chance of being included in the survey?

4 Assuming the employee numbers are used with a random number generator, the sample will be random. However, it is likely to be biased because there is a high chance that all the employees surveyed will be 'workers', who may have very different views to those of managers and middle managers.

16.2 Understand sampling techniques and data sources

SC 1

1 **(a)** A stratified random sample should be used, because people's voting intentions may vary according to where they live (for example, inner metropolitan area, outer metropolitan area, large country town, rural area), their age, their gender, and so on.

(b) There are probably too many students to conduct a census, so a simple random sample or a stratified random sample should be used because student thinking might vary based on age (for example, Year 7, Year 8).

(c) Depending on the number of customers, this could be a census or a random sample. If you think the type of vehicle may lead to different experiences, you should consider a stratified random sample.

2 **(a)** The strata should include all the key stakeholders, such as students, parents and teachers. You may need to further divide these groups – for example, older students and their parents may have different opinions to those who are just starting their school journey.

(b) The strata should cover the different sports available at the club. It may also be useful to sub divide according to gender.

(c) The strata should probably be age-based as younger people may be more likely to interact with mobile devices than their grandparents, for example.

3 **(a)** 1635 **(b)** 17.7% **(c)** 13 people

(d) Stratum B: 23 people; stratum C: 28 people; stratum D: 10 people

(e) 74 people

(f) The number may be one less or one more than the target number due to rounding errors. It is acceptable to ignore this small difference.

4 **(a)**

Year level	Number of students	Number to be surveyed
7	300	14
8	290	14
9	350	17
10	420	20
11	375	18
12	350	17

(b)

Age group	Number of people	Number to be surveyed
10 – <15	450	28
15 – <20	600	38
20 – <25	295	19
25 – <30	315	20
30 – <35	460	29
35 – <40	270	17

5 **(a)** Many people will not answer calls from unknown numbers, so these people are excluded from the sample. People who do not have a telephone are excluded. People who agree to be surveyed may not take the survey seriously – for example, they might give answers that do not reflect their true opinions.

(b) This type of survey requires a person to click a button to take part, so the person chooses (selects) to be in the sample. It is not usually possible to know whether the person has already completed the survey, so one person might answer multiple times. People are more likely to choose to take part if they have strong views about the research topic. This means people with moderate views are likely to be excluded from the sample.

(c) This is convenience sampling. The students will need to produce a list of colours before they begin and it is unlikely that this will cover every colour seen. Also, some colours are difficult to identify – for example, students may disagree about whether a car is bluish-green or greenish-blue.

SC 2

1 (a) This is convenience sampling.
(b) People in the business district are unlikely to represent a broad cross-section of the council area, so their views will not match those of the population as a whole. The people surveyed may not be landowners, so they would not be eligible to vote. Many people do not know the name of the Ward in which they live, so they will not be able to provide this information to the researchers.
(c) A stratified random sample should be taken across all Wards. Landowners from each Ward could be selected at random from the council rates database.

2 (a) This is a random sample.
(b) The sample group may not include students from all year levels.
(c) The school should use a stratified random sample, to ensure all year levels are represented fairly.

3 (a) Although a stratified random sample is often regarded as the 'gold standard', it is not appropriate here, because the question is self-referential for two of the strata (senior managers and middle managers). This may mean the results are not as reliable as they otherwise might be.
(b) In this scenario, purposeful sampling is more appropriate because only part of the company population is truly relevant to the research question.

4 (a) Safety and efficient use of the roads are major considerations. Level crossings can cause significant disruption to traffic flow, especially at peak periods.
(b) Key stakeholders are drivers and people who live near the intersections, especially where the rail line is to be elevated. However, as the building works will use funds from all Victorians, via taxation, it would be appropriate to survey the opinions of the wider population.
(c) This would be extremely difficult. You will need some input from the wider community, but the opinions of key stakeholders should carry more weight when you draw your final conclusions. Some form of random sample is needed, but it will be complex and costly to implement.

SC 3

1 (a) secondary data
(b) secondary data
(c) primary data

2 (a) Although Estelle is choosing the sample herself, the data was collected by someone else, so it is secondary data.
(b) Estelle would need to keep generating random numbers until she had five different values.
(c) Minimum: 3.9°C; maximum: 18.6°C
(d) The average values in the official table for May 2022 are minimum 6.8°C and maximum 17.7°C. The average maximum temperature for Estelle's sample is close to the official figure, but Estelle's minimum value is quite a bit lower. This can be partially explained by the fact that Estelle's sample includes 19 May, which had the lowest minimum temperature in the whole month.

3 (a) There are 20 students in the class.
(b) This is primary data. The times were collected and recorded at your school and are being analysed at your school. The data collection does not have to be completed by you personally.
(c) 17.3 s

4 (a) The ABS collected the data, so it is primary data.
(b) It takes time for births and deaths to be registered, so the ABS probably only updates this figure once or twice each year.
(c) No, because the next quarter does not end until the end of June.
(d) This figure is based on a survey that the ABS conducts at various fixed times during the year. It is too expensive and time-consuming for the ABS to collect the data more frequently and the statisticians need time to collate and analyse the data collected.

SC 4

1 (a) numerical
(b) numerical
(c) categorical

2 (a) categorical
(b) discrete, continuous
(c) census
(d) sample
(e) random

3 (a) continuous
(b) secondary
(c) discrete

4 (a) B continuous
(b) A discrete
(c) D ordinal
(d) C nominal

5 (a) =INT(RAND()*263+1)
(b) Produce more values and use the first 20 unique numbers.
(c) No: if the formula really produces random numbers, it is extremely unlikely to produce the same list of numbers for another student.
(d) You could produce a very large list of numbers, say 400, and identify the smallest and largest numbers produced. If they are not 1 and 263 (in this case), you might need to check your formula.

16.3 Understand and interpret statistics from different samples of the same population

SC 1

1 (a)

Score (x)	Frequency (f)	$x \times f$
1	4	4
2	2	4
3	2	6
4	11	44
5	6	30
6	6	36
7	6	42
8	3	24
9	5	45
10	5	50
	$\sum f = 50$	$\sum xf = 285$

(b) 5.7 **(c)** 5.5

(d) The mean and the median would both be 5.5.

2 (a)

Score (x)	Frequency (f)	$x \times f$
1	5	5
2	3	6
3	1	3
4	5	20
5	3	15
6	1	6
7	2	14
8	4	32
9	3	27
10	3	30
11	3	33
12	5	60
13	4	52
14	5	70
15	3	45
16	5	80
17	5	85
18	4	72
19	5	95
20	11	220
	$\sum f = 80$	$\sum xf = 970$

(b) 12.1 **(c)** 13

(d) Both the mean and the median would be 10.5.

3 (a) 5.36

(b) This is the same as the overall mean of the 50 values.

(c) 5.5

(d) This is not the same as the overall median of the 50 values.

(e) 5.5

(f) This is not the same as the overall median of the 50 values.

(g) When you have a table of random values (like the one in this question), the overall mean is the mean of the individual row means (or the individual column means). However, there is no simple way of converting the individual row median values into an overall median value.

4 (a) 5.488

(b) There is no simple way of finding the overall median value, but it is likely to be around 5.

(c) All the values in the data sets are whole numbers, so the median must be a whole number or a number ending in .5. All the data values are in the range 1–10, so the median must also be within this range. It is likely to be around 5, for example, 4.5, 5, 5.5.

5 (a) 11.1475 **(b)** 10.5

(c) Every subset has a mean higher than the expected value.

(d) 10.7425

(e) In this set of five samples, one sample mean is lower than the expected value and the overall mean is much closer to the expected value.

6 (a) different **(b)** is

(c) get closer to **(d)** 15.5

(e) $\frac{n+1}{2}$

SC 2

1 (a) 12.9 **(b)** 13 **(c)** 18 **(d)** 2

(e) 13.47 **(f)** 13 **(g)** 10

(h) The median has stayed the same, the mean has increased a little and the range has been halved.

2 (a) 5.81 **(b)** 6 **(c)** 19 **(d)** 20

(e) 5.33 **(f)** 5.5 **(g)** 9

(h) The mean and the median have both changed by relatively small amounts. The range has changed significantly, becoming much smaller.

3 (a) 2 13 14 14 15 15 15 15
16 17 17 18 18 18 19 19
20 20 20 21 21 21 21 41

(b) Mean: 17.92; median: 18; range: 39

(c) 2 and 41

(d) Mean: 17.59; median: 18; range: 8

(e) The mean has changed slightly, the median has remained the same and the range has become significantly smaller.

4 (a) Mean: 14.83; median: 16; range: 19

(b) The two values of 1 are outliers.

(c) Mean: 15.85; median: 16; range: 7

(d) The mean has become slightly larger, the median is unchanged and the range has become significantly smaller.

5 (a) 1 1 2 2 2 2 3 4 4 4
4 5 5 5 5 5 6 6 6 6
7 7 7 7 8 8 8 9 21 23
(b) Mean: 6.1; median: 5; range: 22
(c) 21 and 23
(d) Mean: 4.96; median: 5; range: 8
(e) The mean has become slightly smaller, the median has stayed the same and the range has become significantly smaller.

6 (a) Mean: 5.33; median: 5.5
(b) Mean: 6.75; median: 5.5
(c) Mean: 8.67; median: 5.5
(d) The mean is affected by outliers: the larger the outlier, the larger the mean. The median is not affected by the outlier in each data set. The median is a more stable measure of central tendency.

7 (a) 1 1 2 3 4 4 4 4 5 **5** 6 6 7 7 7 7 8 8 9
(b) 1 1 2 3 **4** 4 4 4 5 **5** 6 6 7 7 7 7 8 8 9
The lower quartile is 4.
(c) 1 1 2 3 **4** 4 4 4 5 **5** 6 6 7 **7** 7 8 8 9
The upper quartile is 7
(d) IQR = 7 – 4 = 3

8 (a) Mean: 6.25; median: 5.5; range: 14; IQR: 5
(b) Mean: 7.92; median: 5.5; range: 34; IQR: 5
(c) Mean: 9.58; median: 5.5; range: 54; IQR: 5
(d) As the outlier increases, the mean and the range both *increase*, while the median and the interquartile range *remain the same*.

SC 3

1 (a) Each of the numbers 1–8 must appear.
(b) 10 boxes need to be purchased.
(c) 24 boxes
(d) 4 times (10, 24, 35, 22)
(e) From this simulation, the mean number of boxes is 22.75.

2 (a) 4 blue marbles
(b) 36 blue marbles

3 (a) Allocate 6 of the digits 1–10 to represent a hit, then produce a set of random numbers in the range 1–10 and count the number of 'hits' in each set of 5 numbers.
(b) 3 hits
(c) 3 hits
(d) No, each set of random numbers is likely to produce slightly different results. However, you can expect the average number of hits per 5 arrows to be close to 3 for each simulation.

4 (a) KH, 2C, 6D, KC, JS
(b) There are three Hearts, one Diamond, four Clubs and two Spades.
(c) There are 23 Hearts, 26 Diamonds, 20 Clubs and 31 Spades.
(d) In this sample, there are fewer Clubs and more Spades than expected, but this would probably even out over a larger number of trials. The structure of the simulation seems sound.

16.4 Plan and conduct a statistical investigation

SC 1

1 (a) This is not an ethical approach. If serious side-effects have been reported, they must not be ignored. The side-effects may not be enough to discount the drug, but, at the very least, further research is required.
(b) If people find out that these side-effects were known and then ignored, the company will be liable to be sued by patients who suffer these side-effects in the future.
(c) The researchers need to share what they have learnt about the potential side-effects and include information about them in product disclosure statements if the drug is eventually released. They should also conduct further research to find out more about the issue.

2 (a) No, the trial should have continued as planned, with 300 test subjects.
(b) The results of a small sample are always less reliable than the results of a larger sample. In a small sample, the results may be skewed by any outliers. In a larger sample, this is less of a problem.

3 (a) Question 1 gives people 'permission' to oppose the tax as it is going to affect them directly. Question 2 tries to persuade people to agree with the tax, by phrasing the outcome as something that would probably be considered positive. People are more like to answer 'No' to Question 1 and 'Yes' to Question 2.
(b) Question 2 is clearly sympathetic to the introduction of the new tax.
(c) It could be argued that both questions are unethical as they both push people, to some extent, towards a particular response.
(d) It is unlikely that both questions would appear in the same survey.

4 (a) Yes, this is correct, because the mean income is $95 283.
(b) No, it is not justified. Only 3 of the 53 employees actually earn more than $95 000, while 50 employees earn less.

(c) It would be more accurate to use the mode ($60 000), but the business probably would not want to use this figure in the annual report.

5 **(a)** The cure rate is 66%.

(b) These two rows have more than three values for 'no cure'.

(c) The cure rate is now 69%.

(d) No, the researcher behaved in an unethical manner.

6 **(a)** The companies did not want to spend the money required to make their cars compliant with the regulations.

(b) No, this was clearly an unethical approach.

(c) The profits must be huge to be able to survive penalties of this size and to motivate the executives to take this risk.

(d) The table shows that many manufacturers have been caught up in the scandal. It also shows that many cars produce significantly more pollution than their manufacturers claim.

SC 2

1 **(a)** 15–19 **(b)** 10–14

(c) As the age groups go up, the raw numbers and the rate per 100 000 fall. However, males aged 85+ are slightly over-represented in hospitalisations. It would be interesting to see which sports are contributing to these hospitalisations.

(d) There are approximately equal numbers of males and females in the general population, so girls may participate in sports that have lower rates of hospitalisation.

2 **(a)** cycling

(b) equestrian activities

(c) equestrian activities; recreational walking; netball; dancing

(d) About 6500 males were hospitalised due to injuries sustained while cycling.

3 **(a)** These figures are much lower than in other years. They were almost certainly affected by the COVID-related lockdowns when competitive sport was restricted.

(b) April–July seem to be the worst months. This is probably because netball is usually a winter sport run in conjunction with the major football codes.

(c) Around 125–130 hospitalisations per month seems to be the highest figure.

4 **(a)** $41.51 billion

(b) If money spent using credit cards is repaid during the interest-free period, it does not appear on the balance at the end of the month. It seems that most credit card spending is paid off during the interest-free period.

(c) The total number of credit card accounts fell by 8.36% from 2020 to 2022, before rising by 1.60% the following year. Overall, the value in 2023 is only 6.90% lower than the value in 2020.

(d) The credit card balance has been relatively stable. The lowest figure was in 2022 and the highest was in 2023. Over the whole period from 2020 to 2023, the figure has risen by 3.28%.

(e) The quoted figure is approximately correct; the actual figure is 18.59%.

(f) Probably not. However, if the upward trend continues, there may be cause for concern.

5 **(a)** It is probably amazement at the size of the numbers! They seem extremely high.

(b) The figures must have been obtained using a sampling approach, and then extrapolated to give the final figures. It would be impossible to count every cat, or every animal killed.

(c) 55.38%

(d) **(i)** Sydney: 71.00%

(ii) Melbourne: 71.00%

(e) The Sydney and Melbourne figures are very similar, and give the same answers in part **(d)**. This suggests that some assumptions have been made, and then applied to the number of cat registrations in each city.

(f) The figure is less than 1, about 0.4. This means that one roaming cat kills about 2 animals every 5 days. This seems like a reasonable figure.

SC 3

1 **(a)** 5% margin of error at 95% confidence level

(b) 209.95–232.05

2 **(a)** 10.925–12.075

(b) No, although this sample mean is outside the expected range for 95% of the sample means, it should not be excluded.

3 **(a)** 5.5

(b)

Mean	Frequency
3.0–<3.5	1
3.5–<4.0	1
4.0–<4.5	3
4.5–<5.0	6
5.0–<5.5	8
5.5–<6.0	15
6.0–<6.5	4
6.5–<7.0	0
7.0–<7.5	3
7.5–<8.0	1
8.0–<8.5	1

(c) It is the same as the theoretical value for the mean.
(d) The median occurs in the range 5.5–<6.0.
(e) The mode is the range of values 5.5–<6.0.
(f) All three measures of central tendency are approximately the same and match the theoretical value.

4 **(a)** less **(b)** does not rise
(c) sometimes **(d)** will

Topic 17: Probability (Venn diagrams and two-way tables)

Recall

I can write fractions in their simplest form

1 **(a)** $\frac{2}{5}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{3}{10}$

I can subtract fractions from whole numbers

1 **(a)** $\frac{3}{5}$ **(b)** $\frac{8}{9}$ **(c)** $\frac{11}{7} = 1\frac{4}{7}$
(d) $\frac{11}{8} = 1\frac{3}{8}$

I can write fractions in their equivalent decimal form

1 **(a)** 0.55 **(b)** 0.2 **(c)** 0.4 **(d)** 0.75

I can list the sample space for all possible outcomes of an event

1 **(a)** heads, tails
(b) red marble, blue marble, green marble

I can identify elements that are common to two different sets

1 4, 8, 12, 16, 20

I can compare probabilities

1 **(a)** 0.7 **(b)** $\frac{5}{6}$

I can calculate probabilities

1 **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{3}$ **(e)** 0

17.1 Understand complementary events

SC 1

1 **(a)** Flipping a tail on a coin
(b) Rolling an even number on a die
(c) Not rolling 4 on a die
(d) Not spinning red on a spinner

2 **(a)** Not complementary
(b) Complementary
(c) Complementary
(d) Not complementary
(e) Not complementary

3 **(a)** Not complementary
(b) Complementary
(c) Not complementary
(d) Not complementary

4 **(a)** Not complementary
(b) Not complementary
(c) Not complementary
(d) Complementary
(e) Complementary

5 **(a)** Complementary
(b) Not complementary
(c) Complementary
(d) Complementary

SC 2

1 **(a)** $\frac{5}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{11}{16}$

2 **(a)** $\frac{1}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{12}{13}$ **(d)** $\frac{10}{13}$

3 **(a)** $\frac{3}{8}$ **(b)** $\frac{4}{8} = \frac{1}{2}$ **(c)** $1 - \frac{1}{2} = \frac{1}{2}$

4 $\frac{3}{4}$

5 $\frac{7}{8}$

6 Calculating the possibilities for 1 tail *or* 2 tails *or* 3 tails *or* 4 tails *or* 5 tails will take too long! Since flipping zero tails is the complementary event to flipping *at least* one tail, and the probability of flipping zero tails is easy to calculate, use the fact that the probability of an event and its complement is equal to 1 to solve the problem. For five flips, there are two possible outcomes each flip, so the total number of outcomes is $2 \times 2 \times 2 \times 2 \times 2 = 32$. There is only one way to flip zero tails, so

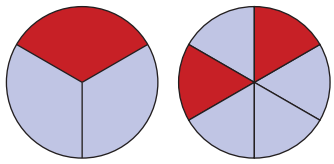
$$\Pr(\text{no tails}) = \frac{1}{32}$$

$$\Pr(\text{at least one tail}) = 1 - \Pr(\text{no tails})$$
$$= 1 - \frac{1}{32}$$
$$= \frac{31}{32}$$

SC 3

1 **(a)** Yes
(b) $\frac{1}{3}$ red, $\frac{2}{3}$ green

(c) Sample answers:

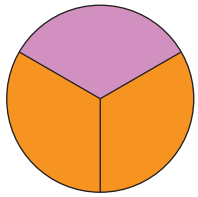


2 **(a)** $\frac{1}{3}$ pink, $\frac{2}{3}$ orange

(b) Sample answers should be around one third for pink and two thirds for orange.

Colour	Frequency
Pink	333
Orange	667

(c) Sample answer:



3 **(a)** Rolling odd or even

(b) Each result is counted twice, because all the individual numbers also fall into the odd and even categories. The total is 2000.

4 **(a)** 10

(b) Yes, because they cover all possibilities and cannot both occur at the same time.

(c) Individual answers

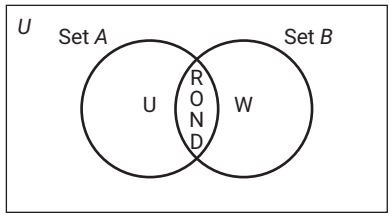
(d) Individual answers

17.2 Draw and understand Venn diagrams

SC 1

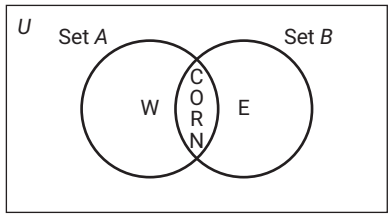
1 **(a)** $A = \{R, O, U, N, D\}, B = \{D, R, O, W, N\}$

(b)



2 **(a)** $A = \{C, R, O, W, N\}, B = \{C, O, R, N, E\}$

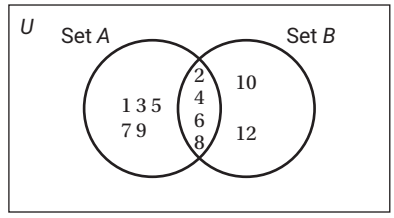
(b)



3 Repeated elements are only included once.

4 **(a)** $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, B = \{2, 4, 6, 8, 10, 12\}$

(b)



5 **(a)** $A = \{1, 2, 3, 4, 5, 6, 7\}$

(b) $B = \{2, 4, 6, 8\}$

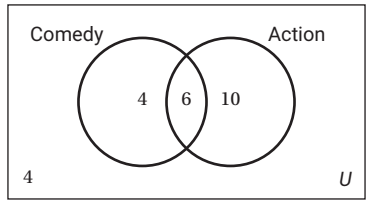
(c) natural numbers less than 8

(d) even numbers less than 9

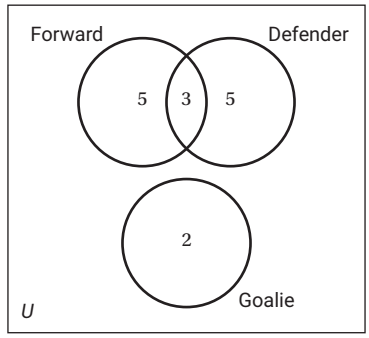
6 **(a)** $n(A) = 4$ **(b)** $n(B) = 3$

7 **(a)** $n(A) = 6$ **(b)** $n(B) = 9$ **(c)** $n(C) = 6$

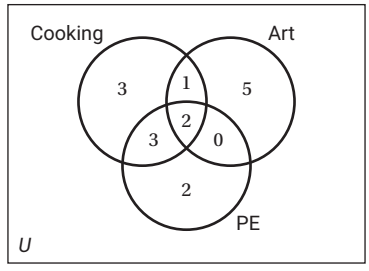
8



9

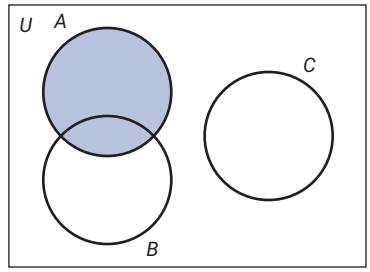


10

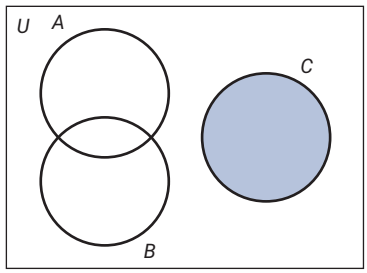


SC 2

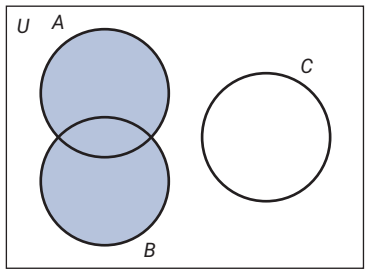
1 **(a)**



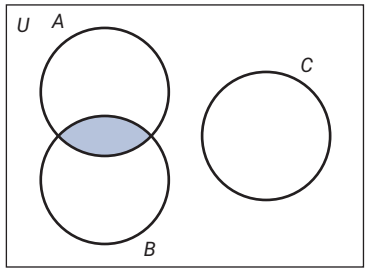
(b)



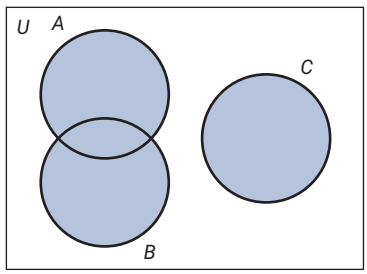
(c)



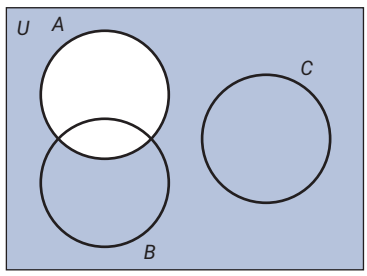
(d)



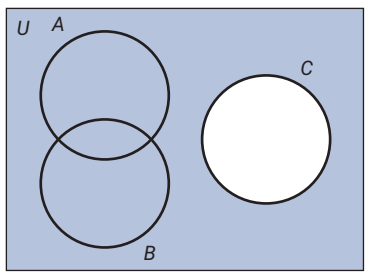
(e)



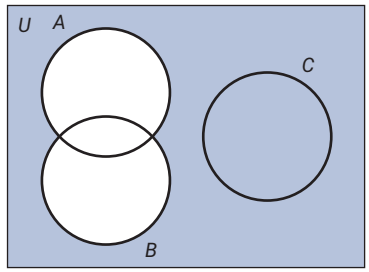
2 (a)



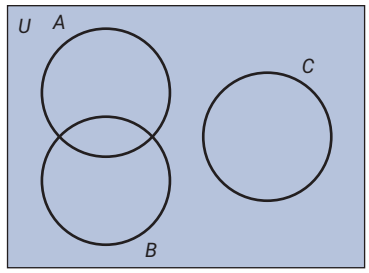
(b)



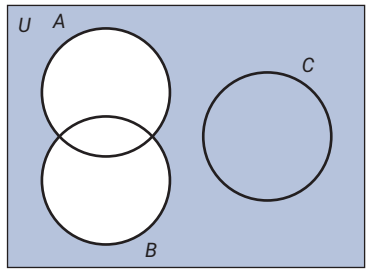
(c)



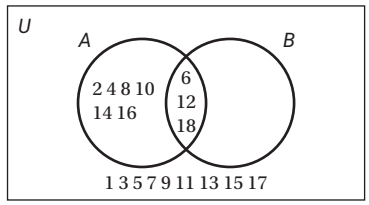
(d)



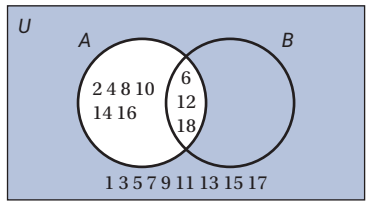
(e)



3 (a)



(b)



(c) $n(A \cap B) = 3$

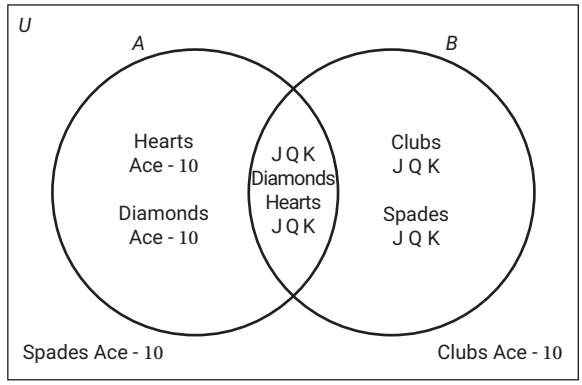
(d) $n(A) = 9$

4 (a) B **(b)** $B \cup C$

(c) A' **(d)** $N \cap P$

(e) $M \cap N \cap P$ **(f)** $(M \cap N \cap P)'$

5 (a)

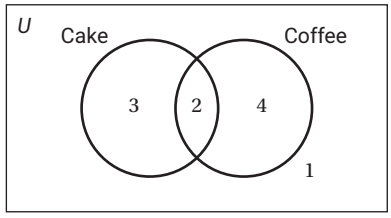


(b) All black cards.
(c) Red picture cards.
(d) $(A \cup B)'$

SC 3

1 **(a)** $\frac{1}{10}$ **(b)** $\frac{1}{10}$ **(c)** $\frac{1}{10}$
(d) $\frac{5}{10} = \frac{1}{2}$ **(e)** $\frac{7}{10}$

2 **(a)**



(b) 1
(c) **(i)** $\frac{3}{10}$ **(ii)** $\frac{4}{10} = \frac{2}{5}$ **(iii)** $\frac{2}{10} = \frac{1}{5}$
(d) **(i)** $\frac{2}{5}$ **(ii)** $\frac{2}{6} = \frac{1}{3}$ **(iii)** $\frac{1}{10}$

3 **(a)** 6. All the figures must add to 30.
(b) **(i)** $\frac{6}{30} = \frac{1}{5}$ **(ii)** $\frac{11}{30}$
(iii) $\frac{9}{30} = \frac{3}{10}$ **(iv)** $\frac{16}{30} = \frac{8}{15}$
(c) **(i)** $\frac{24}{30} = \frac{4}{5}$ **(ii)** $\frac{19}{30}$
(iii) $\frac{21}{30} = \frac{7}{10}$ **(iv)** $\frac{14}{30} = \frac{7}{15}$
(v) $\frac{8}{30} = \frac{4}{15}$

4 The Sopranos are also girls, and the basses are boys. If we drew a Venn diagram, there would be overlap.

17.3 Draw and understand two-way tables

SC 1

1 **(a)** 150 **(b)** 45 **(c)** 101
(d) 78 **(e)** 22
2 **(a)** 85 **(b)** 40 **(c)** 65
(d) 41 **(e)** 16
3 **(a)** $\frac{111}{171}$ **(b)** $\frac{101}{171}$ **(c)** $\frac{41}{171}$ **(d)** $\frac{48}{171}$
4 **(a)** $\frac{42}{83}$ **(b)** $\frac{61}{83}$ **(c)** $\frac{16}{83}$ **(d)** $\frac{31}{83}$

SC 2

1 **(a)**

	Like juice	Do not like juice	Total
Like cola	78	27	105
Do not like cola	23	22	45
Total	101	49	150

(b) 150 **(c)** 45 **(d)** 105 **(e)** 27 **(f)** 22
2 **(a)** $\frac{55}{200} = \frac{11}{40}$ **(b)** $\frac{136}{200} = \frac{17}{25}$ **(c)** $\frac{102}{200} = \frac{51}{100}$
3 **(a)**

		Movie 2		
		Superhero	Cartoon	Total
Movie 1	Superhero	$\frac{70}{150}$	$\frac{30}{150}$	$\frac{100}{150}$
	Cartoon	$\frac{20}{150}$	$\frac{30}{150}$	$\frac{50}{150}$.
Total		$\frac{90}{150}$	$\frac{60}{150}$	1

(b) $\frac{70}{150}$ **(c)** $\frac{30}{150}$ **(d)** $\frac{120}{150}$
4 **(a)** It is the total of all probabilities (the probability of all possible outcomes must be 100% or 1).
(b) If the fractions have the same denominator, probabilities can be more easily compared and checked directly from the original table.

SC 3

1 **(a)**

	Type of running shoe			
	Brand name	Generic	Both	Total
Distance runners	$\frac{21}{40}$	$\frac{1}{40}$	0	$\frac{22}{40}$
Sprinters	$\frac{2}{40}$	$\frac{14}{40}$	$\frac{2}{40}$	$\frac{18}{40}$
Total	$\frac{23}{40}$	$\frac{15}{40}$	$\frac{2}{40}$	1

(b) $\frac{1}{40}$ **(c)** $\frac{2}{40}$ **(d)** 0
2 **(a)**

	Type of sunscreen				
	Coconut	Divia	Neither	Both	Total
Under 40	$\frac{21}{100}$	$\frac{7}{100}$	$\frac{8}{100}$	$\frac{24}{100}$	$\frac{60}{100}$
Over 40	$\frac{14}{100}$	$\frac{18}{100}$	$\frac{6}{100}$	$\frac{2}{100}$	$\frac{40}{100}$
Total	$\frac{35}{100}$	$\frac{25}{100}$	$\frac{14}{100}$	$\frac{26}{100}$	1

(b) $\frac{7}{100}$ **(c)** $\frac{6}{100}$ **(d)** $\frac{25}{100}$ **(e)** $\frac{14}{35}$

3 (a)

	Bus	Not bus	Total
Train	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{5}{10}$
Not train	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{5}{10}$
Total	$\frac{4}{10}$	$\frac{6}{10}$	1

(b) $\frac{5}{10} = \frac{1}{2}$ **(c)** $\frac{4}{10} = \frac{2}{5}$

(d) $\frac{1}{10}$ **(e)** $\frac{1}{10} \div \frac{5}{10} = \frac{1}{5}$

4 (a)

	Movie	Not movie	Total
Dinner	$\frac{41}{100}$	$\frac{21}{100}$	$\frac{62}{100}$
Not dinner	$\frac{12}{100}$	$\frac{26}{100}$	$\frac{38}{100}$
Total	$\frac{53}{100}$	$\frac{47}{100}$	1

(b)

	Netball	Not netball	Total
Softball	$\frac{35}{100}$	$\frac{26}{100}$	$\frac{61}{100}$
Not softball	$\frac{27}{100}$	$\frac{12}{100}$	$\frac{39}{100}$
Total	$\frac{62}{100}$	$\frac{38}{100}$	1

5 (a)

	Football	Not football	Total
Cricket	$\frac{51}{100}$	$\frac{12}{100}$	$\frac{63}{100}$
Not cricket	$\frac{31}{100}$	$\frac{6}{100}$	$\frac{37}{100}$
Total	$\frac{82}{100}$	$\frac{18}{100}$	1

(b) $\frac{18}{100} = \frac{9}{50}$ **(c)** $\frac{6}{100} = \frac{3}{50}$ **(d)** $\frac{51}{82}$

17.4 Recognise and understand mutually exclusive and non-mutually exclusive events

SC 1

1 (a) Non-mutually exclusive because if a 3 is rolled it is also an odd number.

(b) Non-mutually exclusive because you can still be late for work.

(c) Non-mutually exclusive because there is an intersection.

(d) Mutually exclusive because both teams cannot win the Premiership at the same time.

2 (a) Sample answer: rolling a 6 and rolling an odd number. If one event occurs, then the other cannot.

(b) Sample answer: rolling a 6 and rolling an even number. If a 6 is rolled, then both events have occurred at the same time.

3 (a) Sample answer: drawing card number 12 and drawing an odd numbered card. If one event occurs, then the other cannot.

(b) Sample answer: drawing card number 12 and drawing an even numbered card. If the 12 is drawn, both events have occurred at the same time.

(c) (i) Non-mutually exclusive. A 6, 12 or 18 satisfy both events.

(ii) Non-mutually exclusive. A 10 or 20 satisfies both events.

(iii) Non-mutually exclusive. A 15 satisfies both events.

(iv) Mutually exclusive. There are no numbers that satisfy both events.

4 Mutually exclusive events cannot both happen at the same time. For example, rolling a 2 or rolling an odd number on a die.

5 Jack is incorrect. Non-mutually exclusive events are things that can both happen at the same time. It is not possible for both Ash and Serena to win the event.

6 In the case of complementary events, if one event does not occur, then the other must. For mutually exclusive events, there remains the possibility that neither event will occur.

SC 2

1 (a) $\frac{1}{6}$ **(b)** $\frac{3}{6}$ **(c)** $\frac{3}{6}$ **(d)** $\frac{1}{6}$

2 (a) $\frac{1}{6}$ **(b)** $\frac{3}{6}$ **(c)** $\frac{4}{6}$ **(d)** 0

3 (a) $\frac{1}{7}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{3}{7}$ **(d)** 0

4 (a) $\frac{2}{5}$ **(b)** $\frac{3}{5}$ **(c)** 1 **(d)** 0 **(e)** 0

5 **(a)** $\frac{3}{8}$ **(b)** $\frac{2}{8}=\frac{1}{4}$ **(c)** $\frac{5}{8}$

(d) $\frac{6}{8}=\frac{3}{4}$ **(e)** $\frac{4}{8}=\frac{1}{2}$ **(f)** $\frac{5}{8}=\frac{3}{4}$

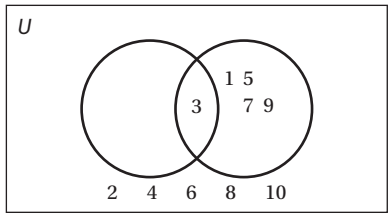
SC 3

1 **(a)** They are only mutually exclusive if the overlap A and B is empty.

(b) They are mutually exclusive because there is no overlap.

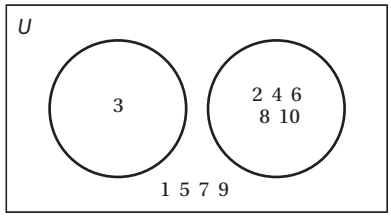
(c) They are not mutually exclusive because the event B is a subset of A (or only occurs when event A occurs).

2 **(a)**



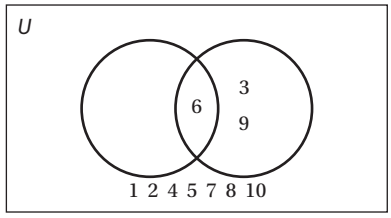
The events are non-mutually exclusive because there is overlap.

(b)



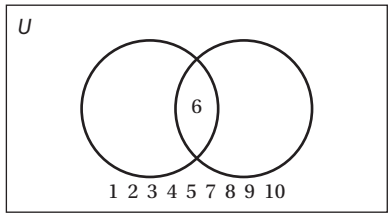
The events are mutually exclusive because there is no overlap.

(c)



The events are non-mutually exclusive because there is overlap.

(d)



The events are non-mutually exclusive because there is overlap.

3 **(a)** They are non-mutually exclusive.

	Vowel	Not a vowel	Total
e	1	0	1
Not an e	2	7	9
Total	3	7	10

The shaded square shows that there can be both an e and a vowel at the same time.

(b) They are mutually exclusive.

	Vowel	Not a vowel	Total
Consonant	0	7	7
Not a consonant	3	0	3
Total	3	7	10

The shaded square shows that there cannot be both a consonant and a vowel at the same time.

(c) They are non-mutually exclusive.

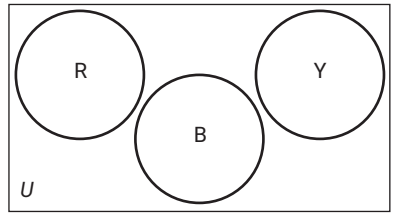
	Dig	Not dig	Total
Had	1	2	3
Not had	2	5	7
Total	3	7	10

The shaded square shows that there can be a letter from both 'dig' and 'had' at the same time: the letter d.

(d) They are mutually exclusive.

	Bad	Not bad	Total
Fig	0	3	3
Not fig	3	4	7
Total	3	7	10

4



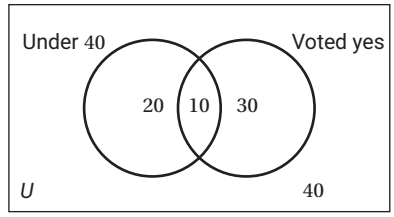
5 Sample answers:

(a) A is selecting an ace and B is selecting a black card.

(b) A is selecting an ace and B is selecting a seven.

(c) A is selecting a red card and B is selecting a Heart.

6 **(a)**



(b) 100

(c) **(i)** $\frac{10}{100}=\frac{1}{10}$ **(ii)** $\frac{70}{100}=\frac{7}{10}$

(iii) $\frac{40}{100}=\frac{4}{10}$ **(iv)** $\frac{30}{100}=\frac{3}{10}$

7 **(a)**

	Holds a driver's license	Does not hold a driver's license	Total
Younger than 20	15	20	35
20 years or older	15	25	40
Total	30	45	75

(b) 75

(c) **(i)** $\frac{15}{75}=\frac{1}{5}$ **(ii)** $\frac{20}{75}=\frac{4}{15}$

(iii) $\frac{35}{75}=\frac{7}{15}$ **(iv)** $\frac{45}{75}=\frac{3}{5}$

17.5 Use Venn diagrams and two-way tables to solve problems

SC 1

1 **(a)** $\frac{6}{8}=\frac{3}{4}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{2}{8}=\frac{1}{4}$

(d) $\frac{1}{8}$ **(e)** $\frac{5}{8}$ **(f)** $\frac{3}{8}$

2 **(a)** $\frac{3}{10}$ **(b)** $\frac{7}{10}$ **(c)** $\frac{6}{10}=\frac{3}{5}$ **(d)** $\frac{1}{10}$

(e) $\frac{6}{10}=\frac{3}{5}$ **(f)** $\frac{5}{10}=\frac{1}{2}$ **(g)** $\frac{1}{10}$

3 Suggested answers:

(a) A number that is at least 2

(b) A number that is divisible by 2

(c) A number greater than 2

(d) A number that is divisible by 2 or 3 but not both

4 **(a)** $\frac{1}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{4}{8}=\frac{1}{2}$

(d) $\frac{3}{8}$ **(e)** $\frac{7}{8}$

5 **(a)** *RRRR*, *RRRB*, *RRBR*, *RRBB*, *RBRR*, *RBRB*, *RBBR*, *RBBB*, *BRRR*, *BRRB*, *BRBR*, *BRBB*, *BBRR*, *BBRB*, *BBBR*, *BBBB*

(b) **(i)** $\frac{1}{16}$ **(ii)** $\frac{1}{16}$ **(iii)** $\frac{15}{16}$

(iv) $\frac{11}{16}$ **(v)** $\frac{5}{16}$

(c) Sample answer: Determine the probability of spinning 1 black or 3 blacks.

6 Sample answer:

'Do you want to play tennis or golf?' This is an example of 'exclusive or': you cannot play both at once. An 'exclusive or' can be either, but not both.

'Do you enjoy tennis or golf?' is an example of 'inclusive or', as it is possible to enjoy both at the same time. An 'inclusive or' can be either, but can also be both.

SC 2

1 **(a)** $\frac{80}{150}=\frac{8}{15}$ **(b)** $\frac{61}{150}$

(c) $\frac{101}{150}$ **(d)** $\frac{40}{150}=\frac{4}{15}$

2 **(a)** $\frac{6}{9}=\frac{2}{3}$ **(b)** $\frac{4}{9}$ **(c)** $\frac{3}{9}=\frac{1}{3}$

(d) $\frac{1}{9}$ **(e)** $\frac{4}{9}$

3 **(a)**

	Premium	Elite	Gold Class	Total
Adults	36	39	37	112
Children	41	21	31	93
Total	77	60	68	205

(b) **(i)** $\frac{39}{205}$ **(ii)** $\frac{52}{205}$

(iii) $\frac{77}{205}$ **(iv)** $\frac{137}{205}$

(c) $\frac{39}{60}=\frac{13}{20}$ **(d)** $\frac{41}{93}$

4 **(a)** **(i)** $\frac{30}{51}$ **(ii)** $\frac{42}{51}$ **(iii)** $\frac{31}{51}$

(iv) $\frac{11}{51}$ **(v)** $\frac{4}{51}$ **(vi)** $\frac{11}{51}$

(b) Sample answer:

Inclusive or: Determine the probability of an order chosen at random containing chocolate or strawberry.

Exclusive or: Determine the probability of an order chosen at random containing chocolate or strawberry, but not both.

(c) Sample answer:

Inclusive or: What is the probability of an order chosen at random containing chocolate or strawberry?

And: Determine the probability of an order chosen at random containing chocolate and strawberry.

5 Inclusive or includes the possibility of A or B or both.

Exclusive or includes A or B but not both.

Inclusive or is larger (or if $n(A \cap B) = 0$, then it is equal to exclusive or). It is never less than exclusive or.

Topic 1: Number properties (exponents)

Topic review

Multiple choice

1 B
2 A
3 D
4 B
5 A
6 C
7 C
8 B
9 C
10 B
11 D
12 B

Short answer

1 (a) 81 (b) 128 (c) 10 000 000
(d) 1.728 (e) -8 (f) $\frac{1}{8}$

2 (a) 3^{20} (b) 1 (c) 3^{12}

3 (a) $6^{10} \times 9^3$ (b) 5^6 (c) $4^7 \times 3^3$

4 (a) $x = 8$ (b) $x = 5$ (c) $x = 2$

5 (a) First exponent law: When multiplying exponent expressions with the same base, add the powers.
(b) Second exponent law: When dividing index numbers with the same base, subtract the powers.
(c) Third exponent law: When raising a power to a power, multiply the exponents.

6 (a) 117 (b) 82 (c) 122 (d) 265

7 (a) $4^3 \times 4^5 = (4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4) = 4^8$
(b) $3^6 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = 3^4$
(c) $(4^3)^2 = 4^3 \times 4^3 = 4^6$

8 (a) base, index (or exponent or power)
(b) cannot, different
(c) 4^4, second, subtracting
(d) 4^{11}, first, adding
(e) 3.21^{15}, third, multiplying

Extended response

1 (a) $2^5 = 32$, $5^2 = 25$, so 2^5 is bigger.
(b) $4^3 = 64$, $3^4 = 81$, so 3^4 is bigger.
(c) $5^3 = 125$, $3^5 = 243$, so 3^5 is bigger.
(d) It is not true that having the bigger power creates the larger number for all numbers (e.g. 1^5 and 5^1, 2^3 and 3^2).

2 (a) 144 m^2 (b) 576 m^2
(c) The scale factor of 2 is applied to both the length and width, so the area increases by a factor of $2^2 = 4$.

3 (a) $V = 1 \times 1 \times 1 = 1\text{ m}^3$ and $V = 100 \times 100 \times 100 = 1\,000\,000\text{ m}^3$
(b) 100^3
(c) $A = 1 \times 1 = 1\text{ m}^2$ and $A = 1000 \times 1000 = 1\,000\,000\text{ mm}^2$

4 (a)

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

(b) 2, 4, 8, 6
(c)

$3^1 = 3$	$4^1 = 4$	$6^1 = 6$	$9^1 = 9$
$3^2 = 9$	$4^2 = 16$	$6^2 = 36$	$9^2 = 81$
$3^3 = 27$	$4^3 = 64$	$6^3 = 216$	$9^3 = 729$
$3^4 = 81$	$4^4 = 256$	$6^4 = 1296$	$9^4 = 6561$
$3^5 = 243$	$4^5 = 1024$		$9^5 = 59\,049$
$3^6 = 729$			
$3^7 = 2187$			
$3^8 = 6561$			
$3^9 = 19\,683$			
$3^{10} = 59\,049$			
3, 9, 7, 1...	4, 6...	6...	9, 1...

(d) Use the example 75^2 and 45^2:

$$\begin{array}{r} 75 \\ \times\ 75 \\ \hline 375 \\ 5250 \\ \hline 5625 \end{array} \quad \text{and} \quad \begin{array}{r} 45 \\ \times\ 45 \\ \hline 225 \\ 1800 \\ \hline 2025 \end{array}$$

The final digit must always be a 5, from the $5 \times 5 = 25$.
If the first digit is odd, the figures in the ten's column will always add to 2 or 12.
If the first digit is even, the figures in the ten's column will always add to 2.

(e) (i) An odd number raised to any power will always be an odd number and an even number raised to any power will always be an even number.

(ii) $4^{25} + 5^{25} + 6^{25}$ is an odd number.

(iii) $1^{15} + 2^{15} + 3^{15}$ is an even number.

5 A variety of answers. The questions should cover each of the laws and raising to the power of 0.

Topic 2: Operations with integers, fractions and decimals

Topic review

Multiple choice

1 D
2 B
3 A
4 D
5 D
6 A
7 B
8 B
9 C
10 C
11 C
12 A
13 B

Short answer

1 **(a)** −18 **(b)** −27 **(c)** 36
2 **(a)** 80 **(b)** 50 **(c)** 54
3 **(a)** −3 **(b)** −3 **(c)** −9
4 **(a)** 5 **(b)** 3 **(c)** 5
5 **(a)** 80 **(b)** −40 **(c)** 36
6 **(a)** 8 **(b)** 5 **(c)** 2
7 **(a)** −1.8 **(b)** −4.5 **(c)** −0.32
8 **(a)** $-\frac{8}{5} = -1\frac{3}{5}$ **(b)** $\frac{5}{32}$ **(c)** $-\frac{3}{10}$
9 **(a)** −2.2 **(b)** −3.2 **(c)** −4
10 **(a)** 2 **(b)** −12 **(c)** 7
11 **(a)** −12 **(b)** −46 **(c)** −54
12 **(a)** −11 **(b)** −18 **(c)** 8.5
13 **(a)** $\frac{2}{3}$ **(b)** $\frac{2}{5}$ **(c)** $-\frac{2}{3}$
14 −$75.20
15 19.7°C
16 −1.2

Extended response

1 **(a)** **(i)** −1000 **(ii)** 2000 **(iii)** 4000
(b) Advances 3 levels losing 4 lives
2 **(a)** 53 **(b)** $1072 **(c)** 89°C

3 **(a)**

Expression	Exponent form	Value	Sign +/−
$-2 \times (-2)$	$(-2)^2$	4	+
$-2 \times (-2) \times (-2)$	$(-2)^3$	−8	−
$-2 \times (-2) \times (-2) \times (-2)$	$(-2)^4$	16	+
$-2 \times (-2) \times (-2) \times (-2) \times (-2)$	$(-2)^5$	−32	−
$-2 \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$	$(-2)^6$	64	+

(b) Odd powers give a negative result. Even powers give a negative result.
(c) −2 is being raised to an odd power, so the answer must be negative.

Topic 3: Rational and irrational numbers

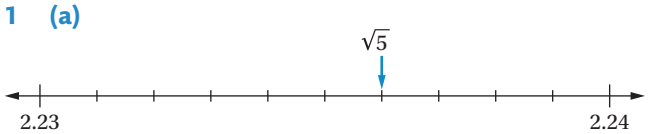
Topic review

Multiple choice

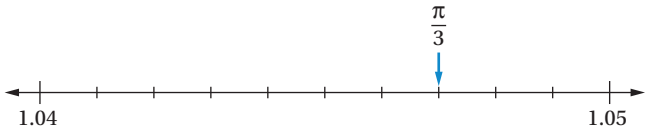
1 C
2 B
3 D
4 A
5 C
6 B
7 D
8 D
9 B
10 D
11 A
12 B
13 C
14 B
15 C
16 B

Short answer

1 **(a)**



(b)



2 **(a)** $\frac{\sqrt{95}}{3}$ will lie between 3.24 and 3.25.
(b) $\frac{5\pi}{9}$ will lie between 1.74 and 1.75.

3 **(a)** $30\sqrt{22}$ cm^2 **(b)** $20\sqrt{7}$ cm^2

4 A circle with a radius of 15 cm will have a circumference less than 4×30 cm, but greater than 6×15 cm.
$90 <$ circumference < 120

5 **(a)** 3.317 **(b)** 1.795 **(c)** 2.466

6 **(a)** $\frac{153}{1087}$ **(b)** $-\frac{30702}{486} = -63\frac{14}{81}$
(c) $\frac{2920}{1} = 2920$

7 **(a)** Rational **(b)** Irrational **(c)** Rational
(d) Irrational **(e)** Rational

8 **(a)** $\sqrt{550}$ **(b)** $2\sqrt{3}$

9 **(a)** $16\sqrt{95}$ cm **(b)** $80\sqrt{3}$ cm

10 **(a)** 4, 4, 8, 12, 20, 32, 52, 84, 136, 220, 356, 576
(b) $\frac{32}{20} = 1.6$ **(c)** $\frac{576}{356} = 1.617977\ldots$
(d) The value of the ratio is approaching the golden ratio.

11 The circumference of the circle is greater than 72 cm but less than 96 cm. The actual value is likely to be closer to 72 cm.

12 **(a)** 5 times **(b)** 2.190 cm **(c)** 1.825%

13 **(a)** $\frac{5}{8}, 0.62, 0.6\overline{1}, 0.6\dot{1}, \frac{3}{5}$ **(b)** $\frac{3}{13}, 0.\dot{2}, 0.2\overline{01}, \frac{1}{5}, \frac{2}{11}$

14 **(a)** $0.38\overline{31}$ **(b)** $-3.70\dot{6}$
(c) $-2.32\overline{23}$ **(d)** $11.99\overline{109}$

15 **(a)** $\frac{781}{3330}$ **(b)** $\frac{23}{999}$

Extended response

1 **(a)** $\frac{1}{21} = 0.\overline{047619}$
$\frac{2}{21} = 0.\overline{095238}$
$\frac{3}{21} = 0.\overline{142857}$
$\frac{4}{21} = 0.\overline{190476}$
$\frac{5}{21} = 0.\overline{238095}$
$\frac{6}{21} = 0.\overline{285714}$
$\frac{7}{21} = 0.\dot{3}$
$\frac{8}{21} = 0.\overline{380952}$
$\frac{9}{21} = 0.\overline{428517}$
$\frac{10}{21} = 0.\overline{476190}$
(b) The repeating digits are the same, just in a different order. From the list $\frac{10}{21}$ also shares these digits.
(c) The repeating digits are the same, just in a different order. From the list $\frac{8}{21}$ also shares these digits.
(d) The repeating digits are the same, just in a different order. From the list $\frac{9}{21}$ also shares these digits.
(e) $\frac{13}{21} = 0.\overline{619047}$

2 **(a)**
a
a
$a + a = 2a$
$2a + a = 3a$
$3a + 2a = 5a$
$5a + 3a = 8a$
$5a + 8a = 13a$
$13a + 8a = 21a$
$21a + 13a = 34a$
$34a + 21a = 55a$
(b) This produces the standard Fibonacci sequence.
(c) 5, 5, 10, 15, 25, 40, 65, 105, 170, 275
(d) $\frac{8}{5} = 1.6; \frac{40}{25} = 1.6$. The value is the same in each case.
(e) It will be 1.6.
(f) It also will be 1.6. This is because the ratio is actually $\frac{8a}{5a} = \frac{8}{5} = 1.6$, where $a = 44$ in this case.

Topic 4: Percentage increase and decrease

Topic review

Multiple choice

1 C
2 C
3 B
4 B
5 A
6 A
7 C
8 C
9 A
10 D
11 B
12 B
13 B
14 D
15 C
16 C
17 C

Short answer

1 **(a)** $119 800 **(b)** $17 202.60
(c) 13.06% profit
2 $1453.64
3 **(a)** $0.90 **(b)** $1.35
(c) 80% mark-up
4 **(a)** 10.75%
(b) 14.00%
5

Category	Annual fee
Adult	$2680
Junior	$1470
Sub-Junior	$260
5-day Adult	$2070

6 **(a)** $50 589 **(b)** $3903.90 **(c)** $3.85
7 **(a)** 6.89% increase **(b)** 4.55% increase
(c) 8.54% decrease **(d)** 12.77% decrease
8 **(a)** $1.98/L **(b)** 6.45% increase
(c) 6.06% decrease
(d) The change in terms of the number of cents is the same, but the base number is different in each case. For the increase, the base number is $1.86/L, whereas for the decrease, the base number is $1.98/L.
9 **(a)** $43.50 **(b)** $3.70 **(c)** $25 450
10 **(a)** $33\frac{1}{3}$% increase **(b)** 80% increase
11 **(a)** $7800 **(b)** 53.79%
(c) $20 627.50 **(d)** 42.23%
12 **(a)** $22.34/h **(b)** $378.92 **(c)** $81.64
13 **(a)** 5.8% increase
(b) 5.8% increase
(c) 5.8% increase
(d) As long as the changes are the same, the order in which they are applied does not make a difference.
14 Retailer A: $3511.20; Retailer B: $3450; Retailer C: $3382.50. In order from cheapest to most expensive: Retailer C, Retailer B, Retailer A.

Extended response

1 **(a)** $5092
(b) $29 467, as it is the base tax amount for the next tax bracket. It is stated in the tax table.
(c) Tax payable: $22 967; Percentage of income: 22.97%
(d) $2269
(e) On $118 000: $28 817, which is 24.42% of their income. On $121 900: $30 170, which is 24.75% of their income.
(f) The word progressive means that as the income increases so too does the percentage of tax paid. The assumption is that the more a person earns the more they can contribute to taxation.
2 **(a)** $21 271.25 **(b)** He still owes $3495.50.
(c) 34.75%
3 **(a)** For the period 2000–2002 the male life expectancy rose by 1.9 years compared to the previous period.
(b) In the period 1965–1967 the male life expectancy fell by 0.2 years compared to the previous period.
(c) In the period 2000–2002 the male life expectancy rose by 2.55% compared to the previous period.
(d) If this rate had been seen for the whole time shown the male life expectancy would be 93.23 years.

Topic 5: Algebra (expand, simplify and factorise)

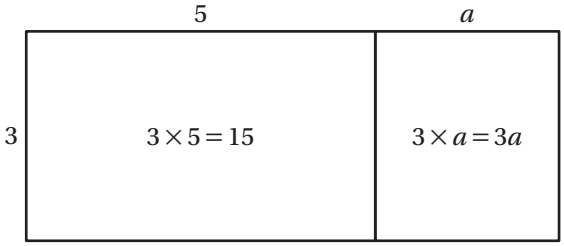
Topic review

Multiple choice

1 D
2 B
3 B
4 D
5 D
6 A
7 D
8 A
9 A
10 D

Short answer

1



2 **(a)** $4(2 + 2y)$ **(b)** $8(5y - 1)$
3 **(a)** $12y - 18$ **(b)** $8a + 8b + 16c$
4 **(a)** $3(a + 2)$ **(b)** $5(3b - 2a)$
5 **(a)** $3a + 3b$ **(b)** $5ab$ **(c)** $\frac{14a^2}{3}$
6 **(a)** $6 + 13a$ **(b)** $10 + 2a$
7 **(a)** $\frac{5a}{6}$ **(b)** $\frac{2b+7a}{ab}$
(c) $\frac{9x-2y}{18}$
8 **(a)** $\frac{4y+5x+6}{xy}$ **(b)** $\frac{7f+7}{5}$
9 **(a)** $\frac{2x-4}{x+2}$ **(b)** $\frac{5(x+6)}{3x}$
10 $\frac{1}{9}$

11 **(a)** $\frac{a}{6}$ **(b)** $\frac{8a}{3}$ **(c)** $\frac{5xy}{12}$

12 **(a)** $-76m$ **(b)** $\frac{18a^2c}{7}$ **(c)** $\frac{15z^2}{xy}$

13 **(a)** $15\pi - 21\pi r$

(b) $12x + 2xy - 8y$

(c) $6x + 4$

14 **(a)** $5a + 3a + 2a + a + 3a + 2a = 16a$

(b) $(3a \times 2a) + (2a \times 3a) = 12a^2$

15 **(a)** $(a + b + c)$ cm

(b) $2(a + b + c)$ cm

(c) $10(a + b + c)$ cm

16 **(a)** $\frac{8x(3x-7)}{2}$ cm^2

(b) $\frac{8x(3x-7)}{2} = 4x(3x-7)$

$= 12x^2 - 28x$ cm^2

Extended response

1 **(a)** $x(x + 12)$ **(b)** $\frac{3x^2}{2} + 18x$

(c) $\frac{x^2}{2} + 6x$ **(d)** $x\left(\frac{x}{2} + 6\right)$

2 **(a)** The missing dimension is $\frac{30}{x}$.

(b) The missing dimension is $\frac{x}{y}$.

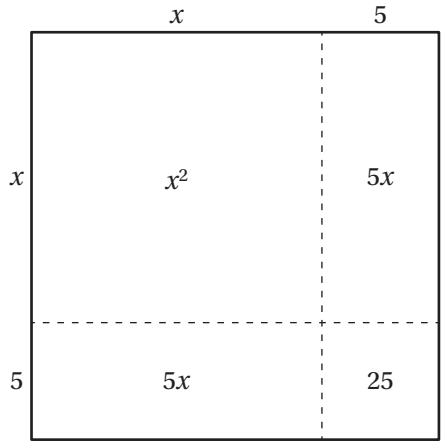
(c) The missing dimension is $3x$.

(d) The missing area is $9x$ units^2.

(e) $3x(x + 3) = 3x^2 + 9x$

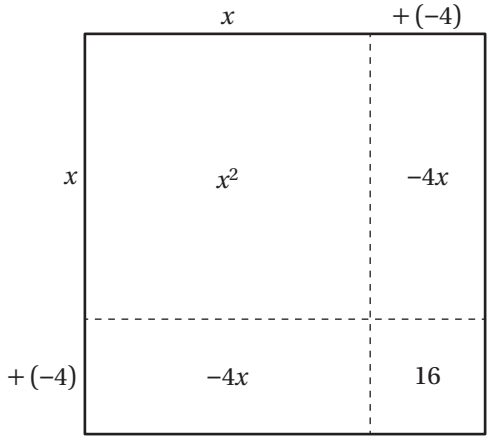
(f) This is a demonstration of the distributive law.

3 **(a)**



(b) The area is $x^2 + 10x + 25$ units^2.

(c)



The area is $x^2 - 8x + 16$.

(d) This term is double the product of the two parts of the side length. Its sign is the same as the operator between the two parts of the side length.

(e) **(i)** $x^2 + 12x + 36$

(ii) $x^2 - 14x + 49$

Topic 6: Linear graphs

Topic review

Multiple choice

1 C

2 C

3 B

4 C

5 D

6 B

7 C

8 B

9 D

10 C

11 C

12 A

Short answer

1 **(a)** $b = 0$ **(b)** $m = 17.5$ **(c)** \$105

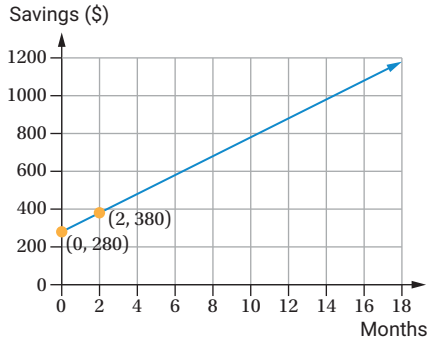
2 **(a)** $x = 3$ **(b)** $y = -2$ **(c)** $y = x$

3 Horizontal: $y = -2$; Vertical: $x = -\frac{1}{2}$

4 **(a)** (0, 280)

(b) (0, 380)

(c)

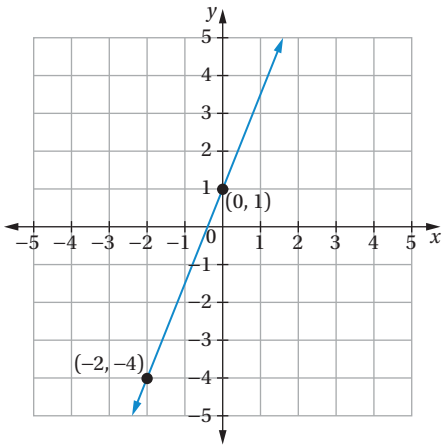


(d) It will take 16 months for Jacob to reach his target.

5 **(a)** (2, 10) **(b)** (−1, 4)
(c) (3, 12) **(d)** (−7, −8)

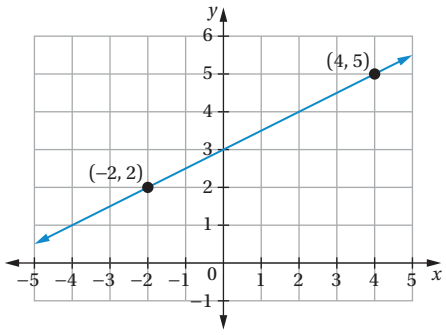
6 **(a)** $y = -2$ **(b)** $x = 3$
(c) $y = 5x$ **(d)** $y = -\frac{x}{2}$

7 **(a)**



(i) $m = \frac{5}{2}$ **(ii)** $b = 1$ **(iii)** $y = \frac{5x}{2} + 1$

(b)



(i) $m = \frac{1}{2}$ **(ii)** $b = 3$ **(iii)** $y = \frac{x}{2} + 3$

8

x	−6	−2	0	1	7
y	$\frac{33}{2}$	9	$\frac{15}{2}$	6	−3

9 **(a)** $25 **(b)** $75
(c) 4.8 m^3; it is unlikely this quantity could be purchased.
(d) Cost = 12.5 × volume in cubic metres
(e) $118.75

10 **(a)** B: $y = -33x + 14$
(b) $y = -50$:(2, −50)
(c) $x = -4$:(−4, 150)
(d) $y = -33\frac{1}{3}x + 16\frac{2}{3}$

11 **(a)** $y = \frac{4x}{3} + 4$
(b) $y = -2x + 4$

12 **(a)** This is the height of the plant when it was planted.
(b) Height = day + 5
(c) 10 cm

13 **(a)** $y = 2x - 4$
(b) $y = -3x + 2$

14 $y = 5$; $y = -2$; $x = -4$; $x = 4$

15 **(a)** 1.6 km from home
(b) At 8 pm
(c) Sometime between 4 pm and 5 pm
(d) The minimum distance is 11.6 km

16 **(a)** Probably a selection from the following, but others are possible: (−4, 3), (−2, 2.5), (0, 2), (2, 1.5), (4, 1)
(b) $y = -\frac{x}{4} + 2$
(c) (8, 0)

17 x-intercept: $\left(\frac{1}{5}, 0\right)$
y-intercept: $\left(0, \frac{1}{2}\right)$

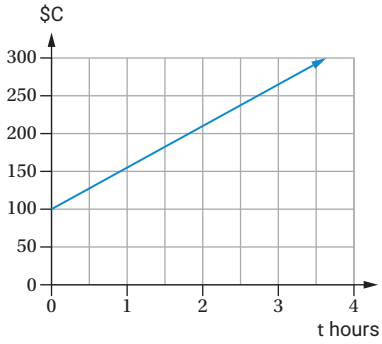
18 **(a)** An infinite number of solutions is possible.
(b) There is still an infinite number of solutions possible.
(c) $y = -\frac{x}{2} + 4$
(d) $b = 4$; (0, 4)

19 **(a)** $y = 2x + 3$; $y = 2x - 4$; $y = -3x + 2$; $y = -3x - 6$
(b) Both pairs of opposite sides are parallel since they have the same gradient.

Extended response

1 **(a)** $C = 55t + 100$

(b)



(c) Somewhere in the region of $290 (or something else reasonable).

(d) $292.50

(e) Using the rule is better as it gives an exact answer.

2 **(a)** Between ages 10 and 11 and between 12 and 13 the increase is cm per year.

(b) $10 \times 6 = 60$; the average girl would be 187.5 cm tall.

(c) Between ages 17 and 18 there was no change. It would appear the average girl has stopped growing by this age.

(d) The equation is height = $3.7864 \times$ age + 101.82.

(e) 9:135.8976 cm
15:158.616 cm

(f) The prediction for age 9 is quite good, but not so good for age 15, although this age is close to the end of the biggest growth spurt.

3 **(a)** (0, 410) and (10, 380)

(b) amount = $-3 \times$ month number + 410

(c) **(i)** 404 kWh **(ii)** 377 kWh

(d) No, the electricity usage is almost certainly dependent on the season. This is especially true if the household has heating based on electricity.

(e) The difference between the actual usage and the value given by the equation is approximately 7%.

Topic 7: Linear inequalities

Topic review

Multiple choice

1 B
2 A
3 C
4 B
5 B
6 C
7 D
8 A
9 C
10 B
11 D
12 A

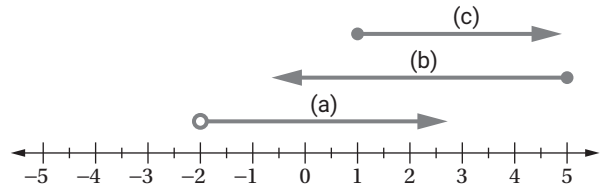
Short answer

1 **(a)** $x > 4$ **(b)** $x \leq 6$
(c) $-3 \leq x \leq 5$ **(d)** $5 < x \leq 10$

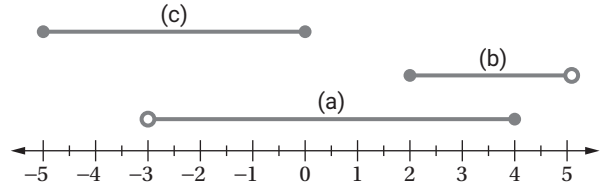
2 **(a)** False; the point does not lie in the required region.
(b) False; the point does not lie in the required region.
(c) True; the point does lie in the required region.

3 **(a)** Speed $\leq$ 80 km/h
(b) Vehicle height $\leq$ 4.4 m
(c) Ride alone height > 130 cm
Ride with adult height > 120 cm
(d) Safe distance $\geq$ 150 cm

4



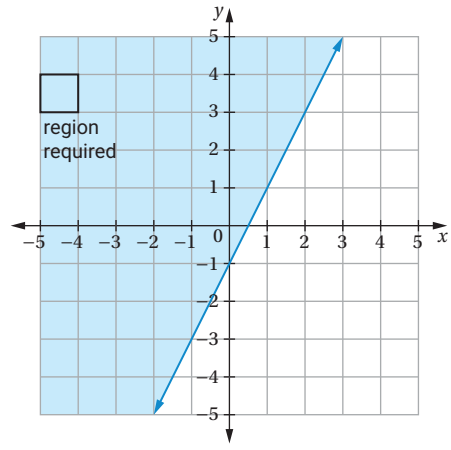
5



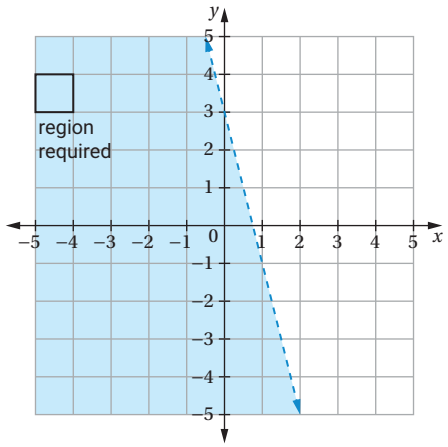
6 **(a)** (4, 1) **(b)** (4,2)

7 **(a)** $x > 2$ **(b)** $x \leq -4$ **(c)** $x > \frac{8}{5}$

8 **(a)**



(b)



9 (a) (i) The variable x is greater than or equal to 3 and less than 4.

(ii) The variable x is greater than -2 and less than 7.

(iii) The variable x is greater than or equal to -7 and less than or equal to 5.

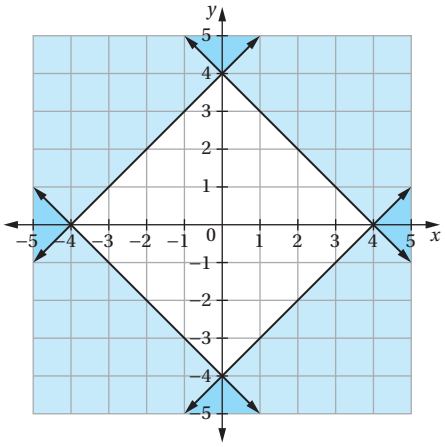
(b) (i) One whole number

(ii) Eight whole numbers

(iii) Thirteen whole numbers

Extended response

1 (a) X



(b) For each inequality, substituting (0, 0) gives a false result, hence each inequality is shaded on the correct side of the boundary line. Additional points would need to be checked to determine if region is correct.

(c) The shape is a square, although it may look like a rhombus. The opposite sides are parallel since the inequations have the same gradient. The product of the gradients of the lines that meet is –1, so the angles are right angles. Hence, the shape is a square. You can imply that the sides are the same length as they join the same values on each axis.

(d) The area is 32 units^2.

Topic 8: Perimeter, area and volume

Topic review

Multiple choice

1 C
2 A
3 C
4 B
5 C
6 D
7 D
8 D
9 D
10 C
11 D
12 B
13 B
14 D
15 D
16 B
17 C
18 D

Short answer

1 (a) 43 m **(b)** 39.6 cm **(c)** 30 cm **(d)** 53 m
2 18 m^2
3 64 cm^2
4 172 m^2
5 28 m^2
6 10 089 mm^2 or 100.89 cm^2
7 3.4 cm^2
8 16 cm^2
9 perimeter 23 cm, area 24 cm^2
10 (a) 2020 mm or 202 cm or 2.02 m
(b) 1810 mm or 181 cm or 1.81 m
11 (a) 6 500 000 mm^2 **(b)** 2500 m^2
(c) 1000 mm^2 **(d)** 50 ha
12 (a) 2.25 cm^2 **(b)** 3900 cm^2

Extended response

1 (a) 48 cm^2 **(b)** 1.392 L **(c)** 139.2 kL
2 (a) 3250 m^3 **(b)** 850 m^3
(c) 54 **(d)** 2400 kL
3 (a) 28.5096 m^2 **(b)** $1305
(c) 10.64 m, 10.92 m
(d) No, the same lineal amount is needed.
4 (a) 220 000 m^2 or 22 ha or 0.22 km^2
(b) 176 000 m^3 **(c)** 6160 kg or 6.16 tonnes
5 (a) 93 km
(b) 1 355 000 m^2 or 135.5 ha or 1.355 km^2
(c) 108 400 m^3

Topic 9: Circles and sectors

Topic review

Multiple choice

1 C
2 A
3 D
4 B
5 A

Short answer

1 $\frac{1}{5}$
2 **(a)** 5.3 m^2 **(b)** 4.3 m^2 **(c)** 1.3 km^2
3 90 cm^2
4 504 m^2
5 17.4 cm^2
6 **(a)** 111 mm **(b)** 89 cm
7 **(a)** $P = 144.63$ cm, $A = 1077.57$ cm^2
(b) $P = 194.25$ cm, $A = 893.14$ cm^2
(c) $P = 12.28$ m, $A = 10.28$ m^2
8 $A = 461.8$ cm^2, $P = 86.0$ cm
9 22.4 cm
10 11.5 m^2

Extended response

1 **(a)** 14 m^2 **(b)** 18.14 m **(c)** 22 m^2 **(d)** $1620

Topic 10: Time

Topic review

Multiple choice

1 B
2 B
3 A
4 C
5 D
6 A
7 C
8 B
9 A
10 D
11 C
12 C
13 B
14 C
15 D
16 A

Short answer

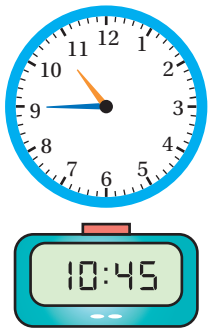
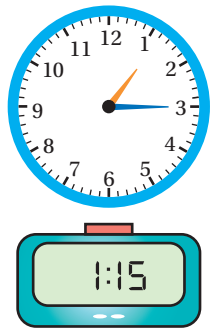
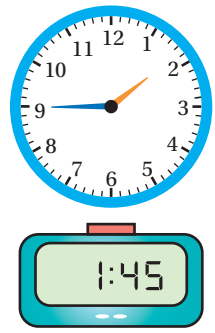
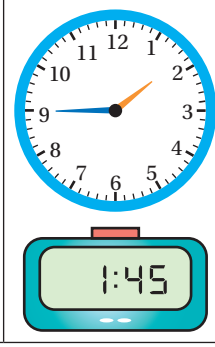
1 3 h 25 min
2 **(a)** 2:55 pm **(b)** 9:10 am
3 4 h 30 min
4 1 h 55 min
5 12:40 pm
6 2 h 40 min
7 **(a)** 10:55 am next day **(b)** 6:10 pm
8 3 h 39 min
9 1205
10 7:45 pm

Extended response

1 **(a)**

State/territory	Standard time	Summer time
Queensland	1200	1200
NSW (except Broken Hill)	1200	1300
ACT	1200	1300
Victoria	1200	1300
Tasmania	1200	1300
NT	1130	1130
SA (and Broken Hill)	1130	1230
WA	1000	1000

(b) Across winter Queensland is 30 minutes ahead of SA, whereas across summer Queensland is 30 minutes behind SA.
(c) 7 pm
(d) 0850
(e)

Perth	Darwin	Adelaide	Sydney	Melbourne	Brisbane
					

2 **(a)** Brisbane to Darwin: time difference 30 minutes. 2050 to 0035 next day is 3 h 45 min + 30 min = 4 h 15 min
Darwin to Denpasar: time difference 1 h 30 min. 0930 to 1045 is 1 h 15 min + 1 h 30 min = 2 h 45 min
Denpasar to Brisbane: time difference 2 hours 1450 to 2220 is 7 h 30 min − 2 h = 5 h 30 min.
(b) Darwin stopover: 0035 to 0930 is 8 h 55 min
(c) Bali stay: 1045, 2 May to 1450, 11 May is 9 days 4 h 5 min
(d) Saving in flying: 4 h 15 min + 2 h 45 min − 5 h 30 min = 1 h 30 min

Topic 11: Rates

Topic review

Multiple choice

1 B
2 C
3 A
4 C
5 B
6 A
7 D
8 A
9 B
10 C
11 A
12 A
13 D
14 A
15 A

Short answer

1 **(a)**

	50 m	100 m	200 m	400 m
Men	2.39 m/s	2.13 m/s	1.96 m/s	1.81 m/s
Women	2.11 m/s	1.93 m/s	1.77 m/s	1.69 m/s

(b)

	50 m	100 m	200 m	400 m
Men	8.61 km/h	7.68 km/h	7.06 km/h	6.52 km/h
Women	7.60 km/h	6.95 km/h	6.37 km/h	6.08 km/h

2 **(a)** 740 km **(b)** 42.25 L **(c)** 1338 km
3 **(a)** $787.50 **(b)** $911.25 **(c)** $978.75
4 **(a)** $107.50 **(b)** $215 **(c)** $217.31
5 **(a)** A$2790.90 **(b)** 8035 euro
(c) $966.06 **(d)** 577 351 yen
6 **(a)** 23 184 km/h **(b)** 6440 m/s
(c) 1932 km **(d)** 6.25 km/s
7 **(a)** 0.2542% **(b)** 0.0065% **(c)** 0.0383%
8 **(a)** $59.99/shirt **(b)** $66.66/shirt **(c)** $69.99/shirt
9 **(a)** $10 810 **(b)** 62 087 L
10 **(a)** $1 **(b)** 7.69% **(c)** $3.25
11 **(a)** 76.92 km/h **(b)** 67.76 km/h **(c)** 129.35 km/h
12 **(a)** One Australian dollar is worth less than one US dollar. So, this means one US dollar is worth more than one Australian dollar. The converted amount must be greater than A$650.
(b) A$1010.26
(c) Janelle multiplied by the exchange rate instead of dividing by it.

Extended response

1 **(a)** $1.27/punnet
(b) 9.96, so, say 10 buckets/h
(c) 32.58, so, say 33 towels/h
(d) 11.46, so, say 11.5 buckets/h
2 **(a)** $5.13/L **(b)** $1.96/L **(c)** $1.83/L
(d) $2.78/L **(e)** $9/L
(f) The smallest can is clearly the most expensive. This is a chilled option, so the price reflects the cost of keeping it cold, but it is really a very high price to pay for convenience. The price per litre is lower as the size of the bottle increases. The packs of cans are interesting as the 12-pack is around half the cost per litre of the 4-pack, yet the convenience level is the same, as the chance of it going off in the bottle is the same.

Exploration activity

Sample answer:

1

Animal	Speed in mph	Speed in km/h	Speed in m/s
Mexican free-tailed bat	99	$99 \times 1.609 = 159.3$	$\frac{159.3 \times 1000}{1 \times 3600} = 44.3$
Kangaroo	$70 \div 1.609 = 43.5$	70	$\frac{70 \times 1000}{1 \times 3600} = 19.4$
Cheetah	$115.2 \div 1.609 = 71.6$	$\frac{32 \div 1000}{1 \div 3600} = 115.2$	32
Lion	$80 \div 1.609 = 49.7$	80	$\frac{80 \times 1000}{1 \times 3600} = 22.2$
House sparrow (flying)	$46.1 \div 1.609 = 28.6$	$\frac{12.8 \div 1000}{1 \div 3600} = 46.1$	12.8
Penguin (swimming)	$9 \div 1.609 = 5.6$	9	$\frac{9 \times 1000}{1 \times 3600} = 2.5$
Elephant	15.5	$15.5 \times 1.609 = 24.9$	$\frac{24.9 \times 1000}{1 \times 3600} = 6.9$
Peregrine falcon (diving through air)	$321.8 \div 1.609 = 200$	$\frac{89.4 \div 1000}{1 \div 3600} = 321.8$	89.4
Hummingbird	$79 \div 1.609 = 49.1$	79	$\frac{79 \times 1000}{1 \times 3600} = 21.9$
Tortoise	0.63	$0.63 \times 1.609 = 1.0$	$\frac{1.0 \times 1000}{1 \times 3600} = 0.28$
Average human (running)	10	$10 \times 1.609 = 16.1$	$\frac{16.1 \times 1000}{1 \times 3600} = 4.5$

2 The fastest animal is the peregrine falcon (diving through air).

3 The slowest animal is the tortoise.

4 Converting rates to the same units makes it easier to compare different speeds. For example, 10 mph and 10 km/h share the same number, but represent very different speeds. Converting both speeds to either mph or km/h makes it easier to make a comparison.

Topic 12: Pythagoras' theorem in 2D

Topic review

Multiple choice

1 B
2 A
3 C
4 B
5 B
6 B
7 D
8 D
9 D
10 C
11 B
12 A
13 D
14 A
15 C
16 D

Short answer

1 **(a)** 37.54 m **(b)** 27.20 m **(c)** 16.16 m

2 **(a)** 4.01 m **(b)** 14.80 m **(c)** 42.65 m

3 $10^2 + 24^2 = 676$ and $26^2 = 676$. The square on the hypotenuse is equal to the sum of the squares on the other two sides. Pythagoras' theorem is satisfied, so the triangle is right-angled.

4 25.61 m

5 **(a)** Yes **(b)** Yes
(c) No, obtuse -angled

6 65 mm, 72 mm

7 2.3 cm
8 **(a)** Right-angled triangle **(b)** Obtuse-angled **(c)** Acute-angled
9 **(a)** $x = 40.8$ cm **(b)** $x = 9.5$ m **(c)** $x = 9.6$ cm **(d)** $x = 13$ m
10 **(a)** 20 m **(b)** 205 cm
11 Shorten by 26 mm
12 **(a)** $x = 12.69$ cm, $y = 25.40$ cm **(b)** $x = 7.08$ m, $y = 4.07$ m

Extended response

1 **(a)** 17 cm **(b)** Hypotenuse: 29 cm; shortest side 2 cm. **(c)** 28.93 cm
2 **(a)** 5.83 cm **(b)** 8.25 cm
3 $a = 45$ mm, $b = 36$ mm, $c = 15$ mm
4 **(a)** 17.32 cm **(b)** 22 cm **(c)** 60 cm

Topic 13: Ratios to solve problems

Topic review

Multiple choice

1 B
2 A
3 C
4 B
5 C
6 A
7 B
8 D
9 A
10 B
11 C
12 D

Short answer

1 **(a)** $a = 66$ **(b)** $b = 60$ **(c)** $c = 56$ **(d)** $d = 18$
2 **(a)** $x = 8$ **(b)** $x = 5$
3 4 parents
4 96 mL
5 **(a)** **(i)** 2.55 **(ii)** 29.5 **(iii)** 13.85
(b) **(i)** Mars: 687 days **(ii)** Uranus: 84 years
6 $\frac{2}{5}$
7 **(a)** 2000 **(b)** 2400000 **(c)** 25000
8 **(a)** 40 m **(b)** 800 m **(c)** 1.8 km
9 **(a)** 400 mm **(b)** 105 mm **(c)** 512 mm
10 **(a)** \$8482.40 **(b)** \$1763.11
11 **(a)** \$102024 **(b)** \$23624.80 **(c)** \$1968.73 **(d)** \$6363.96
12 **(a)** £493 (British pounds sterling) **(b)** 3867.5 Chinese yuan
13 **(a)** A\$32.30 **(b)** A\$229.90

Extended response

1 **(a)** 50 **(b)** 3.5 m × 3.25 m **(c)** 45.5 cm^2 **(d)** 11.375 m^2 **(e)** 2500 **(f)** 50^2
2 **(a)** Anticipated tax payable: \$18 216.80
(b) Anticipated Medicare levy: \$1707.68
(c) Weekly tax plus Medicare: \$383.16
(d) Tax plus Medicare collected: \$15 326.40
(e) Actual taxable income: \$65 680
(f) Tax on taxable income: \$11 813
(g) Medicare levy: \$1313.60
(h) Refund: \$2199.80
3 **(a)** 53 × 3 = 159 rubles: A\$3.99
(b) 350 × 2 + 700 × 3 = 2800 rubles: A\$70.26
(c) 350 × 6 + 350 × 2 + 172 × 5 = 3660 rubles: A\$91.84
(d) 1200 × 2 = 2400 rubles: A\$60.23
(e) 800 rubles: A\$20.08
(f) 20 × 23 = 460 rubles: A\$11.54
(g) **(i)** 10 279 rubles requires 6 withdrawals to get 12 000 rubles
(ii) A\$12
(h) 12 000 rubles is A\$301, so A\$313 altogether.

Topic 14: Triangles and quadrilaterals (similarity and congruence)

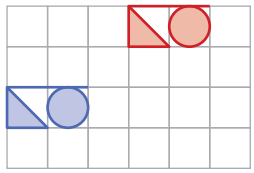
Topic review

Multiple choice

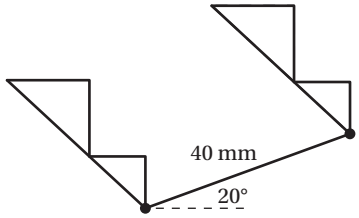
1 B
2 B
3 B
4 D
5 C
6 A
7 C
8 C and D
9 C
10 B
11 B
12 D
13 B
14 C
15 C
16 C
17 C
18 C

Short answer

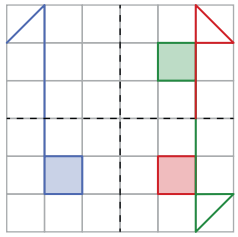
1 (a)



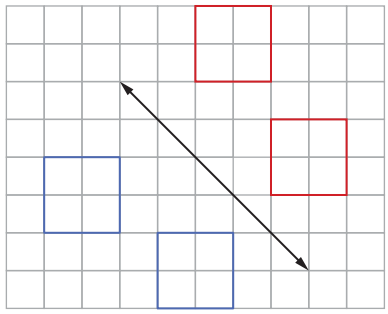
(b)



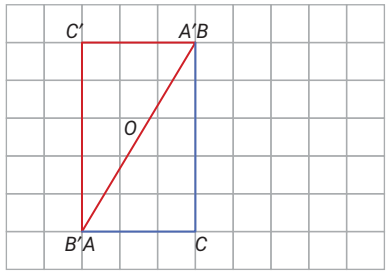
2 (a)



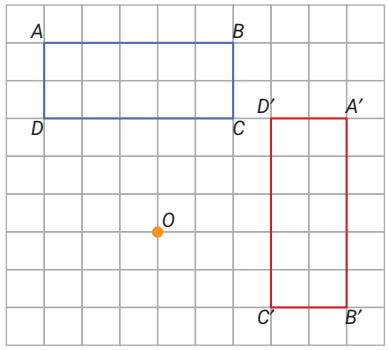
(b)



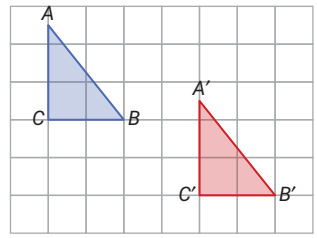
3 (a)



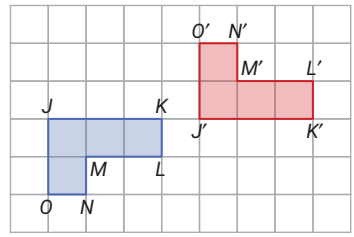
(b)



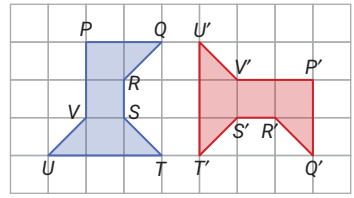
4 (a) Translation 4 units right, 2 units down



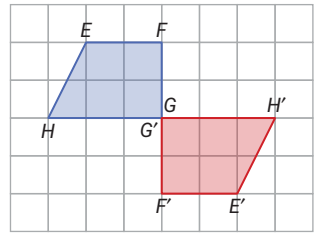
(b) Reflection in a line JK then translation 4 units right



(c) Rotation of 90° clockwise around T then translation 1 unit right



(d) Rotation of 180° around G



5 (a) H ↔ U, TU = EH, ∠EFG = ∠TSV

(b) N ↔ D, AD = ON, ∠CDA = ∠MNO

6 (a) AAS **(b)** SAS **(c)** AAS

7 $\Delta ABC \cong \Delta OMN$ (SAS); $\Delta DEF \cong \Delta JKL$, vertices in any order, (SSS); $\Delta GHI \cong \Delta PRG$ (AAS)

8 In triangles PQR and UTS:
$\angle P = \angle U$ (given)...A
$\angle Q = \angle T$ (given)...A
$PQ = UT$ (given)...S
$\therefore \Delta PQR \cong \Delta UTS$ (AAS congruence test)

9 (a) $\Delta OAB \cong \Delta OCD$ or $\Delta OAB \cong \Delta ODC$ (SAS) due to a pair of vertically opposite angles and equal radii; $p = 6$ m

(b) $\Delta DEH \cong \Delta GEF$ (AAS) due to: any two of the pair of vertically opposite angles and two pairs of alternate angles with parallel lines, as well as the pair of sides given equal; $r = 9$ cm

10 In ΔAED and ΔCDE: $AE = CD$ (given), ED is shared and $\angle ADE = \angle CED$ (given); however, two sides and a non-included angle are not sufficient to prove congruence.

11 **(a)** In triangles PRS and QSR:
$\angle PSR = \angle QRS$ (given 90°)
$\angle SPR = \angle RQS$ (alternate angle, $PS \parallel QR$)
$PS = QR$ (given)
$\therefore \Delta PRS \cong \Delta QSR$ (AAS congruence test)

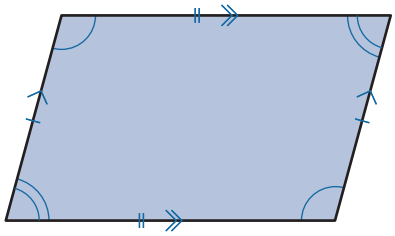
(b) Corresponding sides in congruent triangles.

(c) The diagonals of a rectangle are equal.

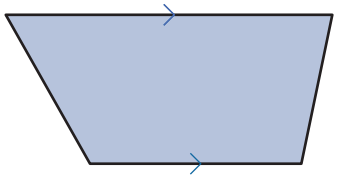
12 **(a)** Possible method: $\Delta BAE \cong \Delta DCE$ (AAS) due to any two of: the pair of vertically opposite angles and two pairs of alternate angles with parallel lines, as well as the pair of sides given equal.
Hence, $AE = CE$ (corresponding sides);
So, $\Delta BAE \cong \Delta BCE$ (SSS) with the pair of sides proven equal, the pair given equal and the shared side.
$\therefore \angle AEB = \angle CEB$ (corresponding angles) and since they are adjacent, both are 90°.
Hence $BD \perp AC$.

(b) The diagonals of a rhombus are perpendicular.

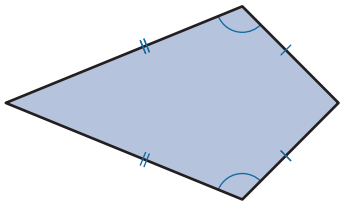
13 **(a)** Parallelogram



(b) Trapezium



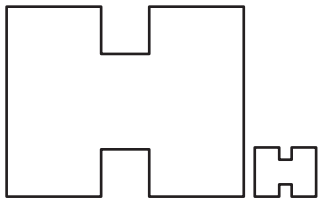
(c) Kite



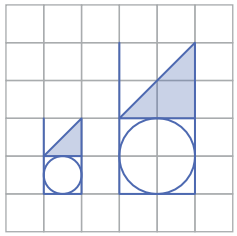
14 $\Delta ABC \cong \Delta ADC$ (SSS) due to two pairs of sides given equal and a shared side. Hence $\angle ABC = \angle ADC$ (corresponding angles). Hence, a pair of opposite angles in the kite are equal.

15 **(a)** 2.6 **(b)** 0.5 or $\frac{1}{2}$

16



17

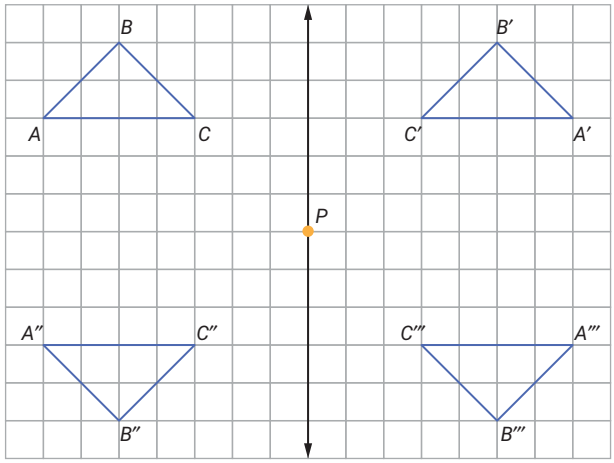


18 $x = 132°$, $y = 17.6$ cm

19 **(a)** AAA; $x = 9$ cm **(b)** AAA; $x = 5$ cm

Extended response

1 **(a)**



(b) A rotation of 180° about P.

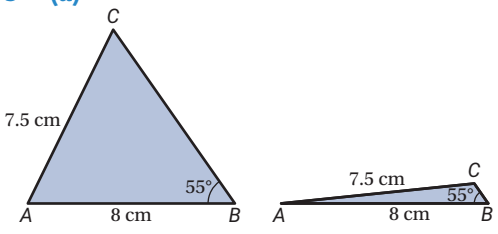
2 **(a)** F, G, J, K, L, N, P, Q, R, S, Z

(b) A, B, C, D, E, M, T, U, V, W, Y

(c) H, I, O, X

(d) H, I, N, O, S, X, Z

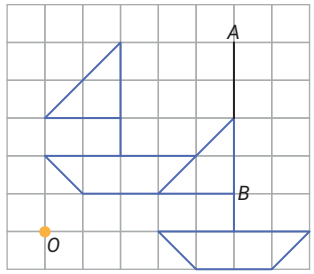
3 **(a)**



(b) Acute-angled triangle: $\angle A = 64°$, $\angle C = 61°$, $CB = 8.2$ cm
Obtuse-angled triangle: $\angle A = 6°$, $\angle C = 119°$, $CB = 0.9$ cm

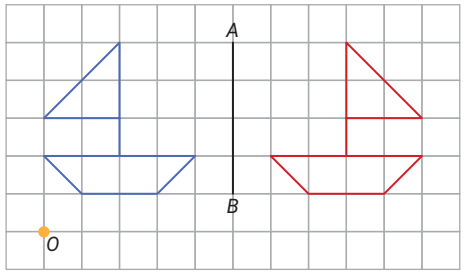
(c) Any one of the three values for either triangle will fix the shape as either the acute-angled or obtuse-angled triangle.

4 (a) (i)



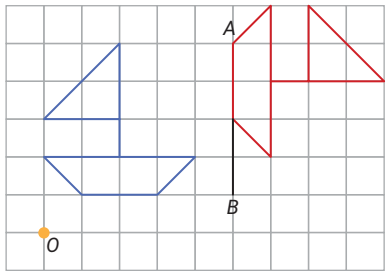
(ii) Translate 3 units left, 2 units up.

(b) (i)



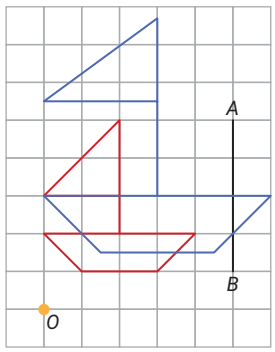
(ii) Reflect in AB.

(c) (i)



(ii) Rotate 90° anticlockwise about B.

(d) (i)



(ii) Reduce with O as the centre of projection using a scale ratio of $\frac{2}{3}$.

Topic 15: Visualising 3D

Topic review

Multiple choice

1 D
2 B
3 A
4 C
5 A
6 B
7 C
8 D
9 D
10 B
11 A
12 D

Short answer

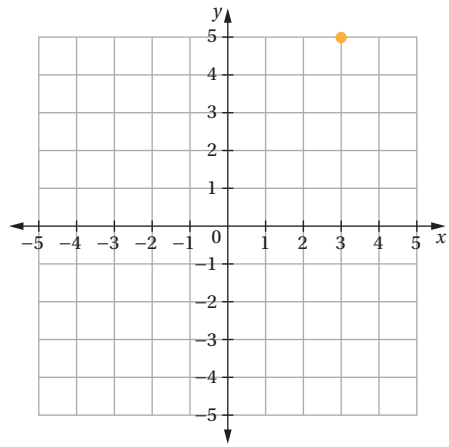
1 The Cartesian plane is a 2-dimensional space or plane. It is created by intersecting a horizontal line (x-axis) with a vertical line (y-axis).

2 The three-dimensional plane contains three axes. The x-axis, y-axis and z-axis are mutually perpendicular, and all meet at the origin.

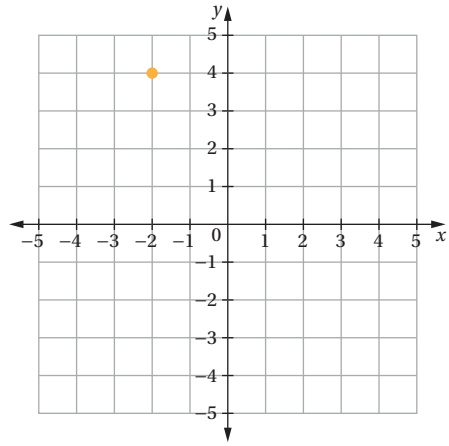
3 **(a)** $x = 5, y = 6, z = 2$
(b) $x = 3, y = 7$
(c) $x = 6, y = 8, z = 1$
(d) $x = 1, y = 4$

4 Points sitting on the flat xy plane will have a z value of 0. Coordinates are: $(x, y, 0)$

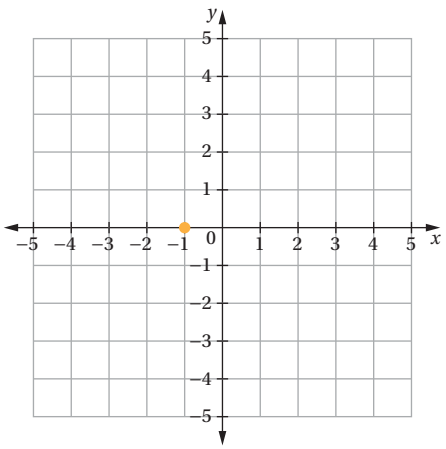
5 For coordinate (3, 5) the location is 3 units east and 5 units north.



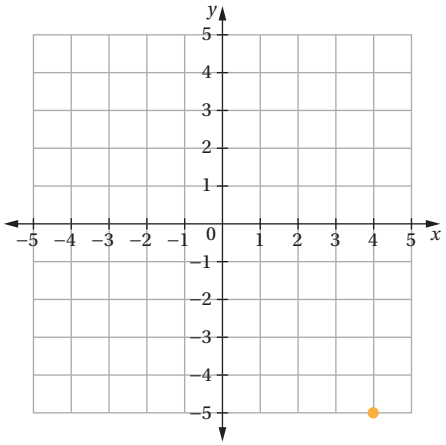
6 For coordinate (−2, 4) the location is 2 units west and 4 units north.



7 For coordinate (−1, 0) the location is 1 unit west.



8 For coordinate the location(4, −5) is 4 units east and 5 units south



9 For coordinate (4, 5, 7) the location is 4 points along the x axis, 5 points along the y axis and 7 points along the z axis

10 For coordinates (8, 7, 2) the location is 8 km along the x axis, 7 km along the y axis and 2 km along the z axis.

Topic 16: Collecting and analysing data

Topic review

Multiple choice

1 C
2 C
3 C
4 D
5 D
6 A
7 B
8 C
9 A
10 D
11 A
12 D
13 B
14 A
15 C
16 D

Short answer

1 A census is where every member of a population is included in the data gathering process, whereas a sample only a part of the population is included.
Census examples:
- All houses in a town receive a survey to complete.
- All students in a school vote on a school excursion destination

Sample examples:
- Every fifth house in a town receives a survey to complete.
- 10 students from each class vote on a school excursion destination.

2 **(a)** This is a sample approach as she will not capture the entire population at only one entrance.
(b) The data collection method would not produce a random sample. Each member of the population does not have an equal chance of being counted as only the people who get to the oval early and can line up first will be included.

3 Jerome is obtaining an observation sample. This involves observing people in a location and noting certain behaviours.

4 Yes, this gathering method could lead to a biased result as it does not fairly represent the population of people who catch taxis. The fact it is a busy train station could mean that more taxis get used here than other locations. The day that Ganna is doing the survey could be an influencing factor, as it could be busier or quieter travel day than normal.

5 Similarity: both are samples, therefore only a portion of the population's data is included in the results
Difference: A random sample uses a lottery-style draw to generate a random data set, whereas a purposeful sample targets a specific subset of the population, such as people with red hair.

6 Nine students. The total number of students in all schools is 4000. School C has 600 students, which represents 15% of the total; 15% of the random sample of 60 is 9 students.

7 Convenience sampling is where the researcher collects the data at a location convenient to themselves; for example, outside a shopping centre. A benefit of this technique is that it is cheaper and quicker to obtain a sample of the population than to survey the entire population. A limitation of this technique is that you do not get a fair representation of all the population, so the result may be biased.

8 Primary data is where the person collecting the data is also analysing the data. An example of primary data could be where you take your dog to a vet when it is unwell. The vet asks you to monitor your dog's weight over a period of time and return with the results. Secondary data is where a third party collects the data and other people use the data. An example is where you use the Bureau of Meteorology to see the temperature of a city; you did not gather the data, but are using BOM's data.

9 Categorical data refers to data that can be categorised, but is not counted, collected or analysed. Examples are eye colour or the model of car your family drives.

10 Numerical data refers to data that can be collected, counted and analysed. The two subtypes are discrete and continuous. An example of discrete data is how many cars each household has, while an example of continuous data would be heights of the children in class.

11 Mean is the sum of the values divided by the number of values; the value if all data was shared equally between the data values.
Median is the middle score when data is ordered from smallest to largest.
Range is the highest subtract the lowest score.

12 **(a)** $\text{Mean} = \frac{\text{sum of all the numbers}}{\text{number of numbers}} = \frac{27.44}{5} = 5.49$

(b) The median will either be a whole number or will include a 0.5 and in this population is likely to be 5.

13 An outlier is a value that is much higher or much lower than the rest of the values in the data set. It may be the result or an error in recording or measurement. The range will reduce dramatically when an outlier is removed, as the highest number or the lowest number is removed, thus reducing the range.

14 All the above methods could produce a random set; however, tossing a coin and rolling a die could be biased towards how the die is thrown or how high the coin is tossed. Giving students a number is time consuming, so the most effective method is **(c)** and results can be used to simulate an experiment

15 A ethical investigation is one where there is transparency in the investigation process and participants are fully aware of the risks. A biased investigation can arise if there is a deliberate attempt to include people who have the same opinion.

Extended response

1 **(a)** The random sampling technique is being used as there is no predictability on whose survey will be included in the sample.

(b) A stratified random sample should be taken to ensure staff from different teams and different campuses are all represented. The staff should be given a date range in which to complete the online survey to reduce the number of people who could miss the survey due to absence.

(c) The three factors that need to be considered are: the size of the population, the level of confidence you require, and the margin of error.

(d) 95%

2 **(a)** The sampling technique being used is convenience sampling, as it is convenient to the survey-takers to stand in an area where there are train commuters.

(b) The sampling technique won't represent all commuters. Some people may currently drive as the trains take too long. The survey-takers are currently targeting train users, so it may be a biased sample.

(c) It costs less than surveying the whole population and the method is quick as the data is gathered within one week.

(d) A stratified random sample should be taken across all regions of the country, so that all of the population is represented.

3 **(a)** $\text{Mean} = \frac{\text{sum of all the numbers}}{\text{number of numbers}} = \frac{87}{20} = 4.35$

(b) Median = middle score from an ordered data set = 4

(c) Range = maximum score − minimum score
$12 - 1 = 11$

(d) 12 is unusually high.

(e) $\text{Mean} = \frac{\text{sum of all the numbers}}{\text{number of numbers}} = \frac{75}{19} = 3.95$

(f) Median = middle score from an ordered data set = 4

(g) The data is numerical as it can be counted.

4 **(a)** 15–19. Many people in this age group play sport.

(b) Most people stop playing rugby as they get older.

(c) 12%

(d) They have high levels of training, which could reduce the number of injuries, or they may have retired due to injuries.

Topic 17: Probability (Venn diagrams and two-way tables)

Topic review

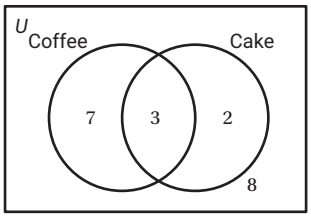
Multiple choice

1 C
2 D
3 D
4 B
5 D
6 B
7 A
8 A
9 B
10 B

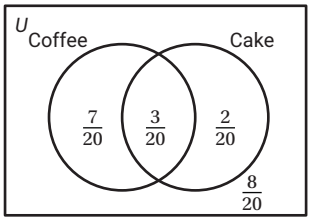
Short answer

1 (a) S = success, F = failure. SS SF FS FF
(b) $\frac{1}{4}$ (c) $\frac{3}{4}$

2 (a)



(b)



(c) (i) $\frac{5}{20} = \frac{1}{4}$ (ii) $\frac{2}{20} = \frac{1}{10}$
(iii) $\frac{8}{20} = \frac{2}{5}$ (iv) $\frac{9}{20}$

(d) It made the fractions easier to add because they had the same denominator.

3 Sample answer:
When we toss a coin, tossing a head and tossing a tail are complementary and mutually exclusive, because if one happens the other cannot.
When we roll a die, rolling a 4 and rolling a 5 are mutually exclusive, because if one happens the other cannot, but they are not complementary.
Complementary events must cover all possibilities, so in this case, rolling a 4 and not rolling a 4 would be complementary.

4 (a) 1 (b) cannot (c) union
(d) intersection (e) complement

5 (a) $\frac{26}{52} = \frac{1}{2}$ (b) $\frac{13}{52} = \frac{1}{4}$ (c) $\frac{4}{52} = \frac{1}{13}$
(d) $\frac{2}{52} = \frac{1}{26}$ (e) $\frac{6}{52} = \frac{3}{26}$ (f) $\frac{13}{52} = \frac{1}{4}$

6 (a) $\frac{200}{310} = \frac{20}{31}$ (b) $\frac{188}{310} = \frac{94}{155}$
(c) $\frac{261}{310}$ (d) $\frac{73}{310}$

7 (a) 105 (b) $\frac{30}{105} = \frac{2}{7}$
(c) $\frac{11}{105}$ (d) $\frac{26}{105}$

8 (a) $\frac{7}{8}$ (b) $\frac{4}{8} = \frac{1}{2}$ (c) $\frac{2}{8} = \frac{1}{4}$
(d) $\frac{1}{8}$ (e) $\frac{1}{8}$

9 (a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{5}{8}$
(d) $\frac{1}{8}$ (e) $\frac{4}{8} = \frac{1}{2}$

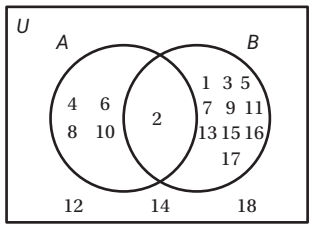
10 Sample answer:
An 'inclusive or' allows for the possibility for either event to take place, but also the possibility of both. An 'exclusive or' does not allow the possibility of both occurring. For example, when rolling a standard die the probability of a 6 or an even number is inclusive and is {2, 4, 6}. The probability of a 6 or an even number but not both is exclusive and does not include the 6 {2, 4}.

11 (a) $\frac{11}{16}$ (b) $\frac{9}{16}$ (c) $\frac{2}{16} = \frac{1}{8}$

12 $\frac{960}{1000} = \frac{24}{25}$

13 (a) $\frac{8}{15}$ (b) $\frac{5}{15} = \frac{1}{3}$ (c) $\frac{3}{15} = \frac{1}{5}$
(d) $\frac{5}{15} = \frac{1}{3}$ (e) $\frac{4}{15}$

14 (a)

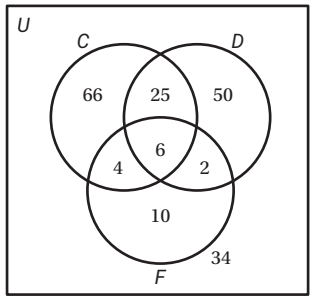


(b) $\frac{13}{18}$ (c) $\frac{15}{18} = \frac{5}{6}$ (d) $\frac{1}{18}$

15 (a) $\frac{135}{390} = \frac{9}{26}$ (b) $\frac{190}{390} = \frac{19}{39}$
(c) $\frac{135}{390} = \frac{9}{26}$ (d) $\frac{270}{390} = \frac{9}{13}$

Extended response

1 **(a)**



(b) 197

(c) **(i)** $\frac{34}{197}$ **(ii)** $\frac{50}{197}$ **(iii)** $\frac{33}{197}$

(iv) $\frac{126}{197}$ **(v)** $\frac{31}{197}$

(d) $\frac{50}{83}$

2 **(a)**

	T-shirts	Jumpers	Coats	Totals
Small	2	36	28	66
Medium	9	0	1	10
Large	58	51	15	124
Total	69	87	44	200

(b) **(i)** $\frac{66}{200} = \frac{33}{100}$ **(ii)** $\frac{10}{200} = \frac{1}{20}$

(iii) $\frac{124}{200} = \frac{31}{50}$ **(iv)** $\frac{153}{200}$

(v) $\frac{69}{100}$

(c) $\frac{51}{124}$

3 $Pr(A) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Inclusive of - can be both
Exclusive of - can not be both.

$Pr(A) + Pr(A') = 1$ — A' — complement

Complementary Events - two events that cannot occur at the same time, and cover all possible outcomes.

Mutually Exclusive Events - events that cannot happen at the same time. Not all possibilities are necessarily covered.

e.g. Tossing an H or T are mutually exclusive, but are complementary.

Rolling a 3 or 4 is mutually exclusive as they cannot happen together, but there are other possibilities, so they are not complementary.

$A \cup B$ → A union B → a list of all the different elements of A and B

$A \cap B$ → A intersection B → where they overlap.

Universal set → A list of all possible outcomes. The same as sample space.

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