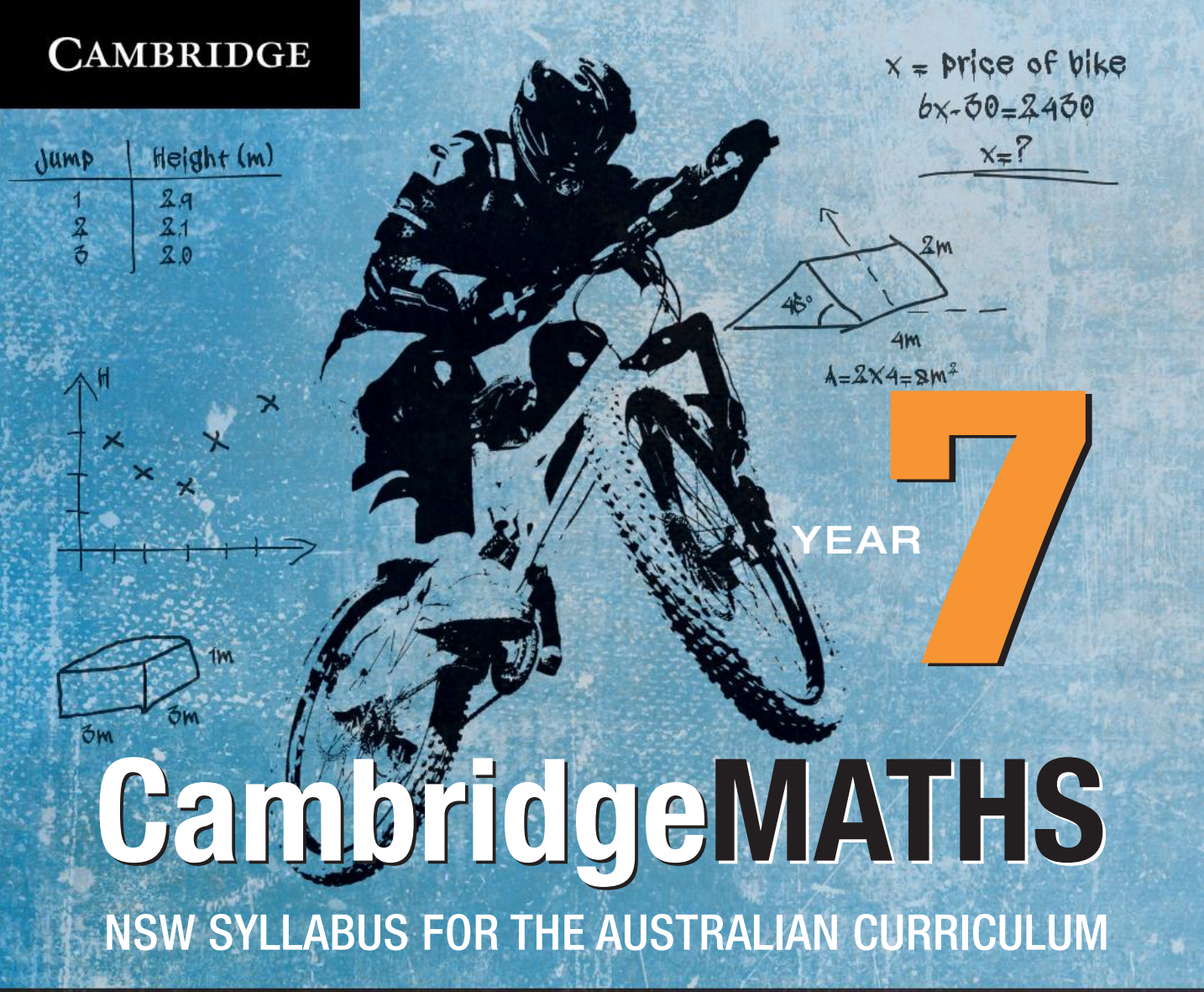
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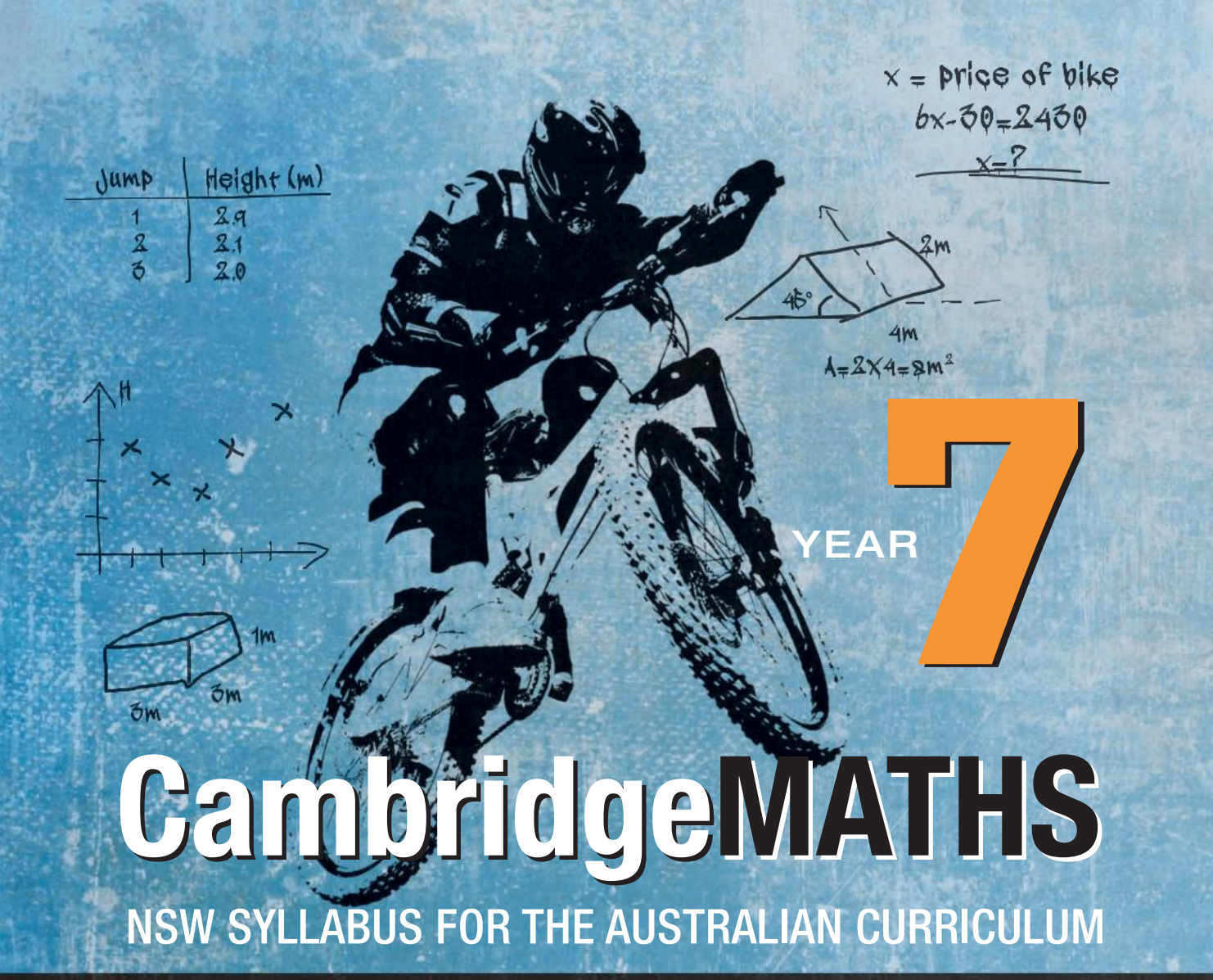
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STAGE 3/4

GOLD

Interactive Textbook included

STUART PALMER | DAVID GREENWOOD
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KAREN McDAID | JENNIFER VAUGHAN | MARGARET POWELL



YEAR 7

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NSW SYLLABUS FOR THE AUSTRALIAN CURRICULUM

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Additional resources online

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Contents

			Strand and Substrand
	About the authors	*vi*	
	Introduction and guide to this book	*viii*	
	Acknowledgements	*xii*	
1	**Computation with positive integers**	2	**Number and Algebra**
1A	Place value in Hindu-Arabic numbers	5	Calculating with Integers
1B	Adding and subtracting positive integers	9	
1C	Algorithms for adding and subtracting	13	
1D	Multiplying small positive integers	17	
1E	Multiplying large positive integers	22	
1F	Dividing positive integers and dealing with remainders	25	
1G	Estimating and rounding positive integers	30	
1H	Order of operations with positive integers	34	
2	**Angle relationships**	44	**Measurement and Geometry**
2A	Points, lines, intervals and angles	47	Angle Relationships
2B	Measuring and classifying angles	52	
2C	Adjacent angles and vertically opposite angles	60	
2D	Transversal lines and parallel lines	66	
3	**Computation with positive and negative integers**	82	**Number and Algebra**
3A	Working with negative integers	85	Calculating with Integers
3B	Adding or subtracting a positive integer	89	
3C	Adding a negative integer	94	
3D	Subtracting a negative integer	98	
3E	Multiplying or dividing by an integer	102	
3F	The Cartesian plane	106	
4	**Understanding fractions, decimals and percentages**	116	**Number and Algebra**
4A	Factors and multiples	119	Fractions, Decimals and Percentages
4B	Highest common factor and lowest common multiple	125	
4C	What are fractions?	131	
4D	Equivalent fractions and simplified fractions	137	

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4E	Mixed numerals and improper fractions	142
4F	Ordering fractions	147
4G	Place value in decimals and ordering decimals	152
4H	Rounding decimals	157
4I	Decimal and fraction conversions	161
4J	Connecting percentages with fractions and decimals	167
4K	Decimal and percentage conversions	173
4L	Fraction and percentage conversions	177
4M	Percentage of a quantity	182
4N	Using fractions and percentages to compare two quantities	185

5 Probability 198 — Statistics and Probability

5A	Describing probability	201
5B	Theoretical probability in single-step experiments	208
5C	Experimental probability in single-step experiments	213
5D	Venn diagrams	217
5E	Two-way tables	223

Probability

6 Computation with decimals and fractions 236 — Number and Algebra

6A	Adding and subtracting decimals	239
6B	Adding fractions	243
6C	Subtracting fractions	249
6D	Multiplying fractions	254
6E	Multiplying and dividing decimals by 10, 100, 1000 etc.	259
6F	Multiplying by a decimal	264
6G	Dividing fractions	267
6H	Dividing decimals	273

Fractions, Decimals and Percentages

Semester review 1 282

7 Time 292 — Measurement and Geometry

7A	Units of time	295
7B	Working with time	300
7C	Using time zones	305

Time

8 Algebraic techniques 1 318 — Number and Algebra

8A	Introduction to formal algebra	321
8B	Substituting positive numbers into algebraic expressions	327

Algebraic Techniques

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8C	Equivalent algebraic expressions	330
8D	Like terms	334
8E	Multiplying and dividing algebraic expressions	338
8F	Applying algebra EXTENSION	342
8G	Substitution involving negative numbers and mixed operations	346
8H	Number patterns EXTENSION	350
8I	Spatial patterns EXTENSION	355
8J	Tables and rules EXTENSION	362
8K	The Cartesian plane and graphs EXTENSION	368

9 Equations 1 380 — Number and Algebra

Equations

9A	Introduction to equations	383
9B	Solving equations by inspection	387
9C	Equivalent equations	390
9D	Solving equations systematically	394
9E	Equations with fractions	401
9F	Formulas and relationships EXTENSION	406
9G	Using equations to solve problems EXTENSION	410

10 Measurement and computation of length, perimeter and area 420 — Measurement and Geometry

Length and Area

10A	Using and converting units of length	423
10B	Perimeter of rectilinear figures	429
10C	Pi and circumference of circles	435
10D	Arc length and perimeter of sectors and composite figures	439
10E	Units of area and area of rectangles	445
10F	Area of triangles	451
10G	Area of parallelograms	458
10H	Mass and temperature	465

11 Introducing indices 476 — Number and Algebra

Indices

11A	Divisibility tests	479
11B	Prime numbers	485
11C	Using indices	489
11D	Prime decomposition	494
11E	Squares, square roots, cubes and cube roots	499

Semester review 2 509

Answers *517*

Index *561*

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About the authors

Stuart Palmer was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. Stuart has taught all the current NSW Mathematics courses in Stages 4, 5 and 6 numerous times. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at The University of Sydney and The University of Western Sydney.



David Greenwood is the head of Mathematics at Trinity Grammar School in Melbourne and has 19 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.



Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, and is currently teaching both junior and senior mathematics in Victoria. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.



Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.



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Karen McDaid is an academic and lecturer in Mathematics Education in the School of Education at the University of Western Sydney. She has taught mathematics to both primary and high school students and is currently teaching undergraduate students on their way to becoming primary school teachers.



Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, QLD and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible, hence facilitating student confidence, achievement and an enjoyment of maths.



Consultant

Margaret Powell has 23 years of experience in teaching special needs students in Sydney and London. She has been head teacher of the support unit at a NSW comprehensive high school for 12 years. She is one of the authors of *Spectrum Maths Gold Year 7* and *Year 8*. Margaret is passionate about ensuring that students with learning difficulties achieve in their academic careers by providing learning materials that are engaging and accessible.



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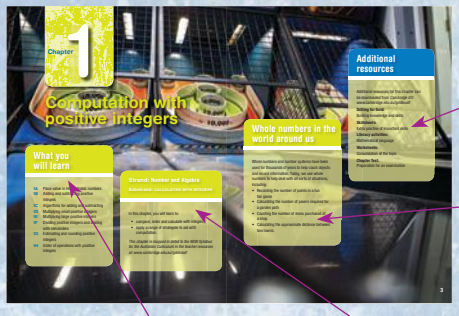
Introduction and guide to this book

Thank you for choosing *CambridgeMATHS Gold*. This book is one component of an integrated package of resources designed for the NSW Syllabus for the Australian Curriculum. *CambridgeMATHS Gold* follows on from the standard CambridgeMATHS series published in 2013–14, and the two series have been structured so that they can be used in the same classroom. Mapping documents showing the relationship between the series can be found on *Cambridge GO*.

Whereas the standard *CambridgeMATHS* books for Years 7 and 8 begin at Stage 4, the *Gold* books for Years 7 and 8 are intended for students who need to consolidate Stage 3 learning prior to progressing to Stage 4. The aim is to develop Understanding and Fluency in core mathematical skills. Clear explanations of concepts, worked examples embedded in each exercise and carefully graded questions contribute to this goal. Problem-solving, Reasoning and Communicating are also developed through carefully-constructed activities, exercises and enrichment.

An important component of *CambridgeMATHS Gold* is a set of worksheets called Drilling for Gold. These are engaging, innovative, skill-and-drill style worksheets that provide the kind of practise and consolidation of the skills required for each topic without adding hundreds of pages to the textbook.

Low literacy can be a barrier for learning mathematics, especially in the transition from primary to secondary school. As such, the relationship between literacy and maths is a major focus of *CambridgeMATHS Gold*. Key words and concepts are defined using student-friendly language; real-world contexts and applications of mathematics help students connect these concepts to everyday life; and a host of literacy activities can be downloaded from the website. In the interactive version of this book, definitions are enhanced by audio pronunciation, visual definitions and examples. More information about the interactive version can be found on page.



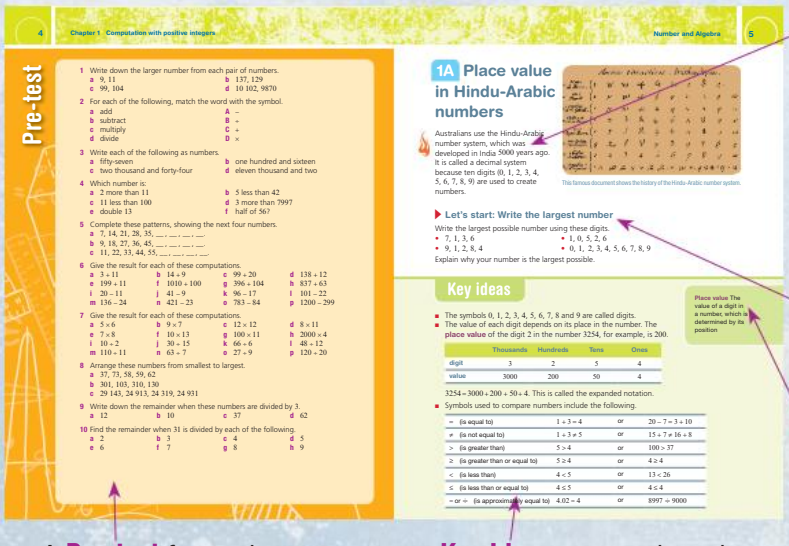
A suite of accompanying resources, including Drilling for Gold worksheets and Literacy activities, can be downloaded from *Cambridge GO.*

Chapter introductions provide real-world context for students.

What you will learn gives an overview of the chapter contents.

A summary of the chapter connects the topic to the NSW Syllabus. Detailed mapping documents to the NSW Syllabus can be found in the teaching program on *Cambridge GO*.

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A **Pre-test** for each chapter establishes prior knowledge.

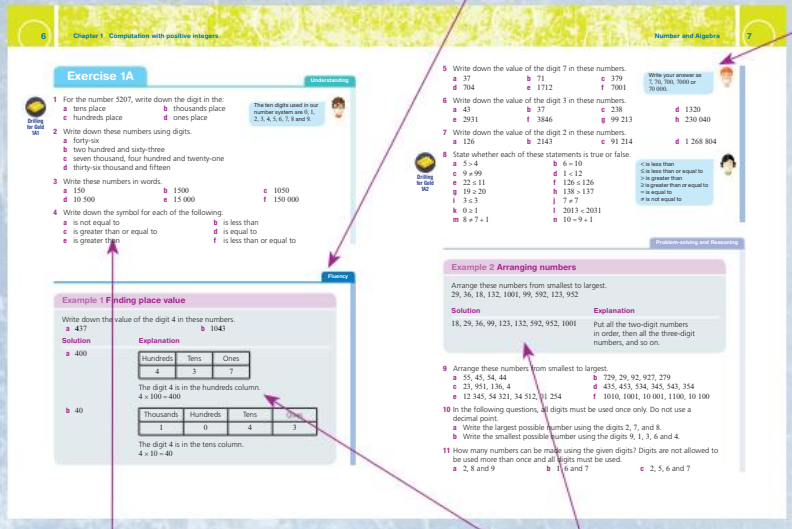
Key Ideas summarises the knowledge and skills for the topic.

Topic introductions relate the topic to mathematics in the wider world.

Let's start activities provide an engaging way to begin thinking about the topic.

Every important term in the **Key Ideas** contains a simple-language definition.

Exercises are structured according to the four Working Mathematically strands: Understanding, Fluency, Problem-solving and Reasoning, with Communicating present in each of the other three. **Enrichment** questions at the end of the exercise challenge students to reach further.



Hint boxes give hints and advice for tackling questions.

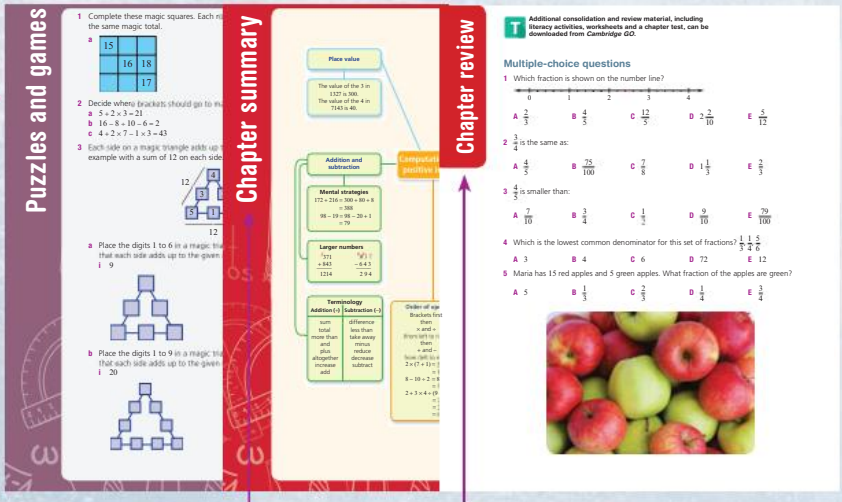
Within each Working Mathematically strand, questions are further carefully graded from easier to challenging.

Examples with worked solutions and explanations are embedded in the exercises immediately before the relevant question/s.

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Puzzles and games allow students to have fun with the mathematics contained in the chapter.

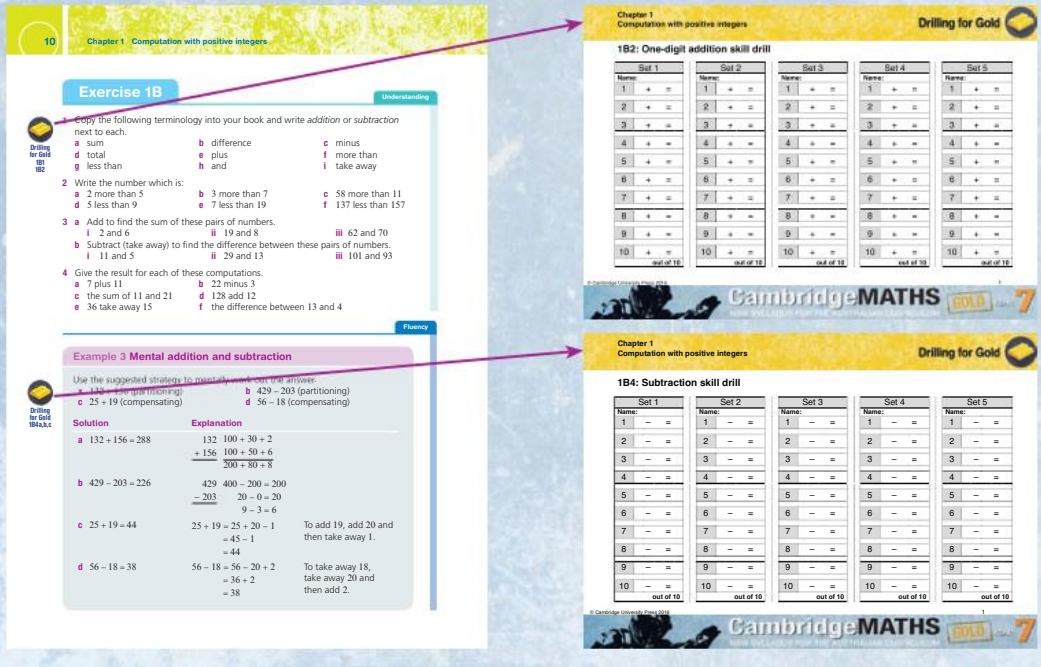
Chapter summaries give mind maps of key concepts and the interconnections between them.



Chapter reviews test comprehension with multiple-choice, short-answer and extended-response questions.

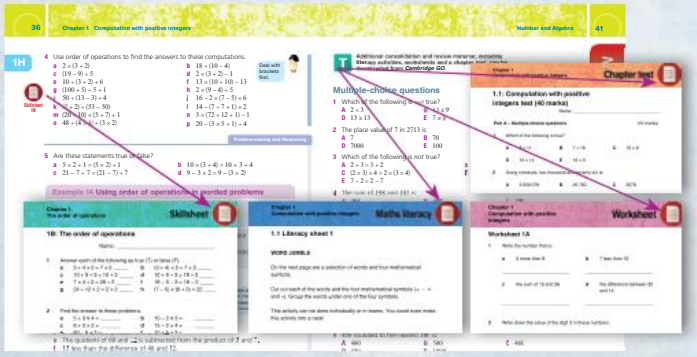
Drilling for Gold

Drilling for Gold is a collection of engaging and motivating learning resources that provide opportunities for students to repeatedly practise routine mathematical skills. Their purpose is to improve automaticity, fluency and understanding through 'hands-on' resources, games, competitions, puzzles, investigations and sets of closed questions. These activities are designed to be used as if they were part of the textbook; each one is referenced in the pages of the textbook via a 'gold' icon and unique reference number. The Drilling for Gold resources can be downloaded via the *Cambridge GO* website.



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Other resources



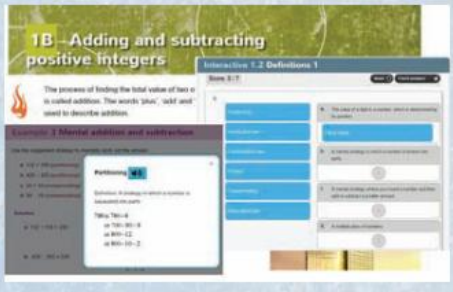
In addition to Drilling for Gold, a host of other resources for each chapter can be downloaded from *Cambridge GO*:

- **Skillsheets** provide practise of the key skills learned across the entirety of the chapter, and are linked to the later sections via their own icon and reference number.
- **Maths literacy** worksheets familiarise students with mathematical English via cloze activities, games, group activities, crosswords and much more.
- A **chapter test** provides exam-style assessment, with multiple-choice, short-answer and extended-response questions.
- **Worksheets** cover multiple topics within a chapter and can be done in class or completed as homework.

About your Interactive Textbook

An interactive digital textbook is included with your print textbook and is an integral part of the *CambridgeMATHS Gold* learning package. As well as being an attractive, easy-to-navigate digital version of the textbook, it contains many features that enhance learning in ways not possible with a print book:

- **Roll-over definitions** give short, simple-language definitions of key terms at the start of a topic
- **Clickable 'Enhanced' definitions** containing diagrams, illustrations, examples and audio pronunciation provide instant assistance and revision within exercises and worked examples
- **Roll-over hints** for selected questions are provided within exercises by rolling your mouse over the cartoon faces
- **Matching HOTmaths lessons** can be accessed by clicking the flame at the start of each topic
- **Additional teacher resources** can be accessed by clicking the 'T' icon in the chapter review
- **Drilling for Gold** and **Skillsheets** can be downloaded by clicking on the respective icons in the margins
- **Fill-the-gap** and **drag-and-drop** activities at the end of each chapter provide a fun way of learning key concepts and consolidating knowledge
- **Answers** for Exercises, Pre-tests, Puzzles and Games and Chapter reviews can be conveniently accessed by clicking the 'Show Answers' button at the bottom of the page
- **Font size** can be increased or decreased as required
- **Annotations** can be added to words, phrases, questions or whole paragraphs to allow critical engagement with the textbook.



A more detailed guide to using the Interactive Textbook can be found on *Cambridge GO*.

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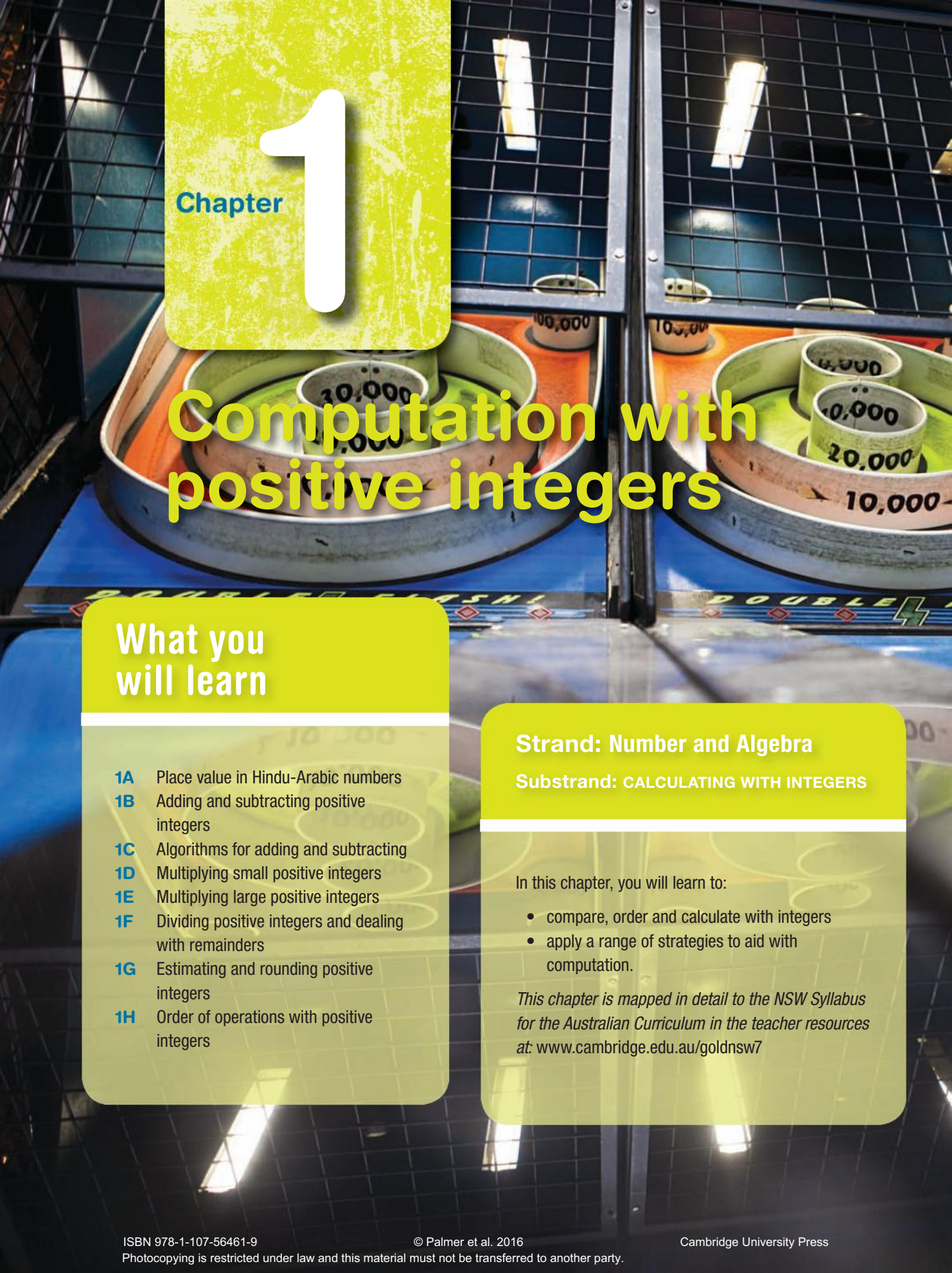
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Chapter 1

Computation with positive integers

What you will learn

- 1A Place value in Hindu-Arabic numbers
- 1B Adding and subtracting positive integers
- 1C Algorithms for adding and subtracting
- 1D Multiplying small positive integers
- 1E Multiplying large positive integers
- 1F Dividing positive integers and dealing with remainders
- 1G Estimating and rounding positive integers
- 1H Order of operations with positive integers

Strand: Number and Algebra

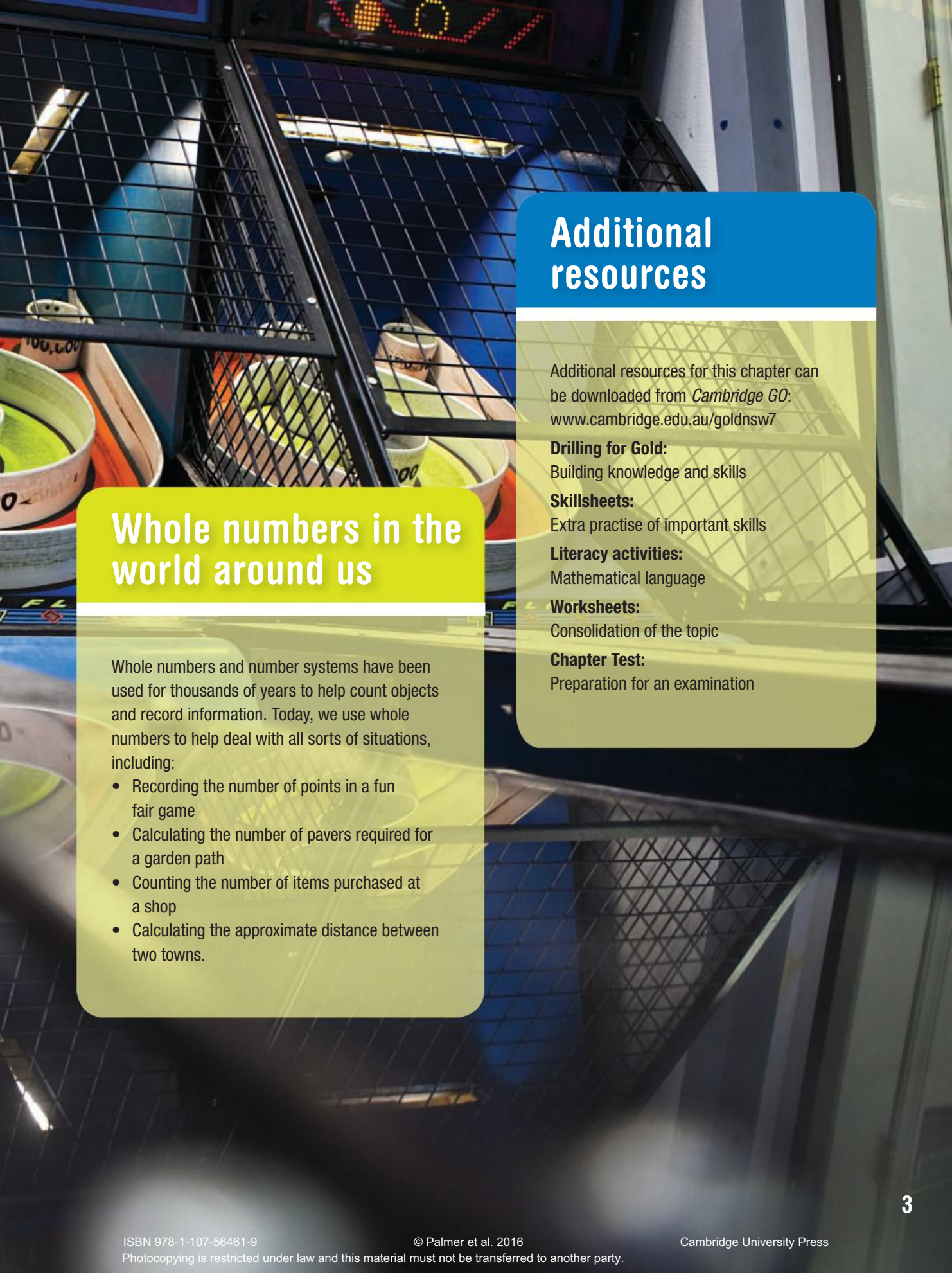
Substrand: CALCULATING WITH INTEGERS

In this chapter, you will learn to:

- compare, order and calculate with integers
- apply a range of strategies to aid with computation.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Whole numbers in the world around us

Whole numbers and number systems have been used for thousands of years to help count objects and record information. Today, we use whole numbers to help deal with all sorts of situations, including:

- Recording the number of points in a fun fair game
- Calculating the number of pavers required for a garden path
- Counting the number of items purchased at a shop
- Calculating the approximate distance between two towns.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Write down the larger number from each pair of numbers.

a 9, 11
b 137, 129
c 99, 104
d 10 102, 9870

2 For each of the following, match the word with the symbol.

a add
b subtract
c multiply
d divide

A –
B ÷
C +
D ×

3 Write each of the following as numbers.

a fifty-seven
b one hundred and sixteen
c two thousand and forty-four
d eleven thousand and two

4 Which number is:

a 2 more than 11
b 5 less than 42
c 11 less than 100
d 3 more than 7997
e double 13
f half of 56?

5 Complete these patterns, showing the next four numbers.

a 7, 14, 21, 28, 35, __ , __ , __ , __.
b 9, 18, 27, 36, 45, __ , __ , __ , __.
c 11, 22, 33, 44, 55, __ , __ , __ , __.

6 Give the result for each of these computations.

a $3 + 11$	**b** $14 + 9$	**c** $99 + 20$	**d** $138 + 12$
e $199 + 11$	**f** $1010 + 100$	**g** $396 + 104$	**h** $837 + 63$
i $20 - 11$	**j** $41 - 9$	**k** $96 - 17$	**l** $101 - 22$
m $136 - 24$	**n** $421 - 23$	**o** $783 - 84$	**p** $1200 - 299$

7 Give the result for each of these computations.

a 5×6	**b** 9×7	**c** 12×12	**d** 8×11
e 7×8	**f** 10×13	**g** 100×11	**h** 2000×4
i $10 \div 2$	**j** $30 \div 15$	**k** $66 \div 6$	**l** $48 \div 12$
m $110 \div 11$	**n** $63 \div 7$	**o** $27 \div 9$	**p** $120 \div 20$

8 Arrange these numbers from smallest to largest.

a 37, 73, 58, 59, 62
b 301, 103, 310, 130
c 29 143, 24 913, 24 319, 24 931

9 Write down the remainder when these numbers are divided by 3.

a 12
b 10
c 37
d 62

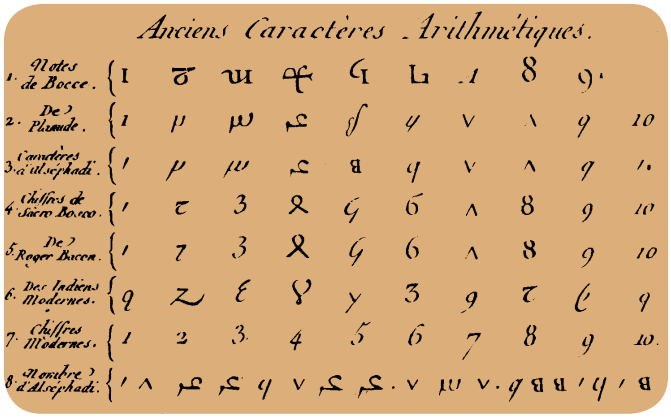
10 Find the remainder when 31 is divided by each of the following.

a 2
b 3
c 4
d 5
e 6
f 7
g 8
h 9

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1A Place value in Hindu-Arabic numbers

Australians use the Hindu-Arabic number system, which was developed in India 5000 years ago. It is called a decimal system because ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are used to create numbers.



This famous document shows the history of the Hindu-Arabic number system.

Let's start: Write the largest number

Write the largest possible number using these digits.

- 7, 1, 3, 6
- 9, 1, 2, 8, 4
- 1, 0, 5, 2, 6
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Explain why your number is the largest possible.

Key ideas

Place value The value of a digit in a number, which is determined by its position

- The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called digits.
- The value of each digit depends on its place in the number. The **place value** of the digit 2 in the number 3254, for example, is 200.

	Thousands	Hundreds	Tens	Ones
digit	3	2	5	4
value	3000	200	50	4

$3254 = 3000 + 200 + 50 + 4$. This is called the expanded notation.

- Symbols used to compare numbers include the following.

$=$	(is equal to)	$1 + 3 = 4$	or	$20 - 7 = 3 + 10$
$\neq$	(is not equal to)	$1 + 3 \neq 5$	or	$15 + 7 \neq 16 + 8$
$>$	(is greater than)	$5 > 4$	or	$100 > 37$
$\geq$	(is greater than or equal to)	$5 \geq 4$	or	$4 \geq 4$
$<$	(is less than)	$4 < 5$	or	$13 < 26$
$\leq$	(is less than or equal to)	$4 \leq 5$	or	$4 \leq 4$
$\approx$ or $\div$	(is approximately equal to)	$4.02 \approx 4$	or	$8997 \div 9000$

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Exercise 1A

Understanding


Drilling for Gold 1A1

1 For the number 5207, write down the digit in the:
- **a** tens place
- **b** thousands place
- **c** hundreds place
- **d** ones place


The ten digits used in our number system are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

2 Write down these numbers using digits.
- **a** forty-six
- **b** two hundred and sixty-three
- **c** seven thousand, four hundred and twenty-one
- **d** thirty-six thousand and fifteen

3 Write these numbers in words.
- **a** 150
- **b** 1500
- **c** 1050
- **d** 10 500
- **e** 15 000
- **f** 150 000

4 Write down the symbol for each of the following.
- **a** is not equal to
- **b** is less than
- **c** is greater than or equal to
- **d** is equal to
- **e** is greater than
- **f** is less than or equal to

Fluency

Example 1 Finding place value

Write down the value of the digit 4 in these numbers.

a 437 **b** 1043

Solution **Explanation**

a 400

Hundreds	Tens	Ones
4	3	7

The digit 4 is in the hundreds column.
$4 \times 100 = 400$

b 40

Thousands	Hundreds	Tens	Ones
1	0	4	3

The digit 4 is in the tens column.
$4 \times 10 = 40$

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5 Write down the value of the digit 7 in these numbers.

a 37 **b** 71 **c** 379
d 704 **e** 1712 **f** 7001



Write your answer as 7, 70, 700, 7000 or 70 000.

6 Write down the value of the digit 3 in these numbers.

a 43 **b** 37 **c** 238 **d** 1320
e 2931 **f** 3846 **g** 99 213 **h** 230 040

7 Write down the value of the digit 2 in these numbers.

a 126 **b** 2143 **c** 91 214 **d** 1 268 804



Drilling for Gold 1A2

8 State whether each of these statements is true or false.

a $5 > 4$ **b** $6 = 10$
c $9 \neq 99$ **d** $1 < 12$
e $22 \leq 11$ **f** $126 \leq 126$
g $19 \geq 20$ **h** $138 > 137$
i $3 \leq 3$ **j** $7 \neq 7$
k $0 \geq 1$ **l** $2013 < 2031$
m $8 \neq 7 + 1$ **n** $10 = 9 + 1$



$<$ is less than
$\leq$ is less than or equal to
$>$ is greater than
$\geq$ is greater than or equal to
$=$ is equal to
$\neq$ is not equal to

Problem-solving and Reasoning

Example 2 Arranging numbers

Arrange these numbers from smallest to largest.
29, 36, 18, 132, 1001, 99, 592, 123, 952

Solution	Explanation
18, 29, 36, 99, 123, 132, 592, 952, 1001	Put all the two-digit numbers in order, then all the three-digit numbers, and so on.

9 Arrange these numbers from smallest to largest.

a 55, 45, 54, 44 **b** 729, 29, 92, 927, 279
c 23, 951, 136, 4 **d** 435, 453, 534, 345, 543, 354
e 12 345, 54 321, 34 512, 31 254 **f** 1010, 1001, 10 001, 1100, 10 100

10 In the following questions, all digits must be used once only. Do not use a decimal point.

a Write the largest possible number using the digits 2, 7, and 8.
b Write the smallest possible number using the digits 9, 1, 3, 6 and 4.

11 How many numbers can be made using the given digits? Digits are not allowed to be used more than once and all digits must be used.

a 2, 8 and 9 **b** 1, 6 and 7 **c** 2, 5, 6 and 7

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1A

Drilling for Gold 1A3

Enrichment: Large numbers

12 The names of large numbers depend on the number of digits. For example, 1000 is 1 thousand, 1 000 000 is 1 million and 1 000 000 000 is 1 billion.

a Write these numbers using digits.

- **i** 7 thousand
- **ii** 46 thousand
- **iii** 712 thousand
- **iv** 5 million
- **v** 44 million
- **vi** 6 billion
- **vii** 437 billion
- **viii** 15 trillion

b Research the number 1 googol.



In 2008 in Zimbabwe, bank notes were issued in trillions of dollars.





A $1 coin is 3 mm thick. So if you stack up 1 million $1 coins, the total height will be 3 000 000 mm, which is 3 km. That is almost 10 times taller than the Sydney Tower!

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1B Adding and subtracting positive integers



What's the difference in height?

The process of finding the total value of two or more numbers is called addition. The words 'plus', 'add' and 'sum' are also used to describe addition.

The process for finding the difference between two numbers is called subtraction. The words 'minus', 'subtract' and 'take away' are also used to describe subtraction.

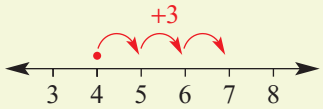
Let's start: Your mental strategy

How could you do the following computations without using a pen or a calculator?

- $132 + 240$
- $99 + 35$
- $73 - 39$

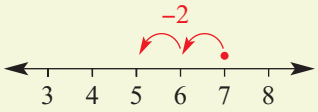
Key ideas

- The symbol + is used to show addition or find a sum; e.g. $4 + 3 = 7$.



  - Note that the order does *not* matter with addition.
    e.g. $5 + 2 = 2 + 5$ and $21 + 12 = 12 + 21$

- The symbol – is used to show subtraction or find a difference.
  e.g. $7 - 2 = 5$



  - Note that the order *does* matter with subtraction.
    e.g. $5 - 2 \neq 2 - 5$ and $21 - 12 \neq 12 - 21$

- Mental addition and subtraction can be done using different strategies.
  - **Partitioning** (Grouping digits in the same position)



$171 + 23 = 194$



$428 - 114 = 314$

  - **Compensating** (Making a 10, 100 etc. and then adjusting or compensating by adding or subtracting)

$46 + 9 = 46 + 10 - 1$
$= 55$

$138 - 99 = 138 - 100 + 1$
$= 39$

Partitioning A mental strategy in which a number is broken into parts
e.g. $123 = 100 + 20 + 3$

Compensating
A mental strategy in which you round a number and then add or subtract a smaller amount

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Exercise 1B

Understanding


Drilling for Gold 1B1 1B2

1 Copy the following terminology into your book and write *addition* or *subtraction* next to each.

a sum	**b** difference	**c** minus
d total	**e** plus	**f** more than
g less than	**h** and	**i** take away

2 Write the number which is:

a 2 more than 5	**b** 3 more than 7	**c** 58 more than 11
d 5 less than 9	**e** 7 less than 19	**f** 137 less than 157

3 **a** Add to find the sum of these pairs of numbers.

i 2 and 6 **ii** 19 and 8 **iii** 62 and 70

b Subtract (take away) to find the difference between these pairs of numbers.

i 11 and 5 **ii** 29 and 13 **iii** 101 and 93

4 Give the result for each of these computations.

a 7 plus 11	**b** 22 minus 3
c the sum of 11 and 21	**d** 128 add 12
e 36 take away 15	**f** the difference between 13 and 4

Fluency

Example 3 Mental addition and subtraction


Drilling for Gold 1B4a,b,c

Use the suggested strategy to mentally work out the answer.

a 132 + 156 (partitioning) **b** 429 – 203 (partitioning)

c 25 + 19 (compensating) **d** 56 – 18 (compensating)

Solution	Explanation	
a 132 + 156 = 288	$\begin{array}{r} 132 \\ +\ 156 \\ \hline \end{array}$ $\begin{array}{l} 100 + 30 + 2 \\ 100 + 50 + 6 \\ \hline 200 + 80 + 8 \end{array}$	
b 429 – 203 = 226	$\begin{array}{r} 429 \\ -\ 203 \\ \hline \end{array}$ $\begin{array}{l} 400 - 200 = 200 \\ 20 - 0 = 20 \\ 9 - 3 = 6 \end{array}$	
c 25 + 19 = 44	25 + 19 = 25 + 20 – 1 = 45 – 1 = 44	To add 19, add 20 and then take away 1.
d 56 – 18 = 38	56 – 18 = 56 – 20 + 2 = 36 + 2 = 38	To take away 18, take away 20 and then add 2.

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5 Mentally find the answers to these sums. Hint: Use the partitioning strategy.

a $11 + 23$ **b** $14 + 32$ **c** $43 + 16$
d $23 + 41$ **e** $71 + 26$ **f** $138 + 441$


Work out the answer by adding the ones, then the tens, and so on.

6 Mentally find the answers to these differences. Hint: Use the partitioning strategy.

a $29 - 18$ **b** $57 - 21$ **c** $94 - 43$
d $249 - 137$ **e** $357 - 124$ **f** $836 - 704$

7 Mentally find the answers to these sums. Hint: Use the compensating strategy.

a $15 + 9$ **b** $64 + 11$
c $19 + 76$ **d** $18 + 115$


Round one of the numbers to the nearest ten, then compensate by adding or subtracting the difference.

8 Mentally find the answers to these differences. Hint: Use the compensating strategy.

a $35 - 11$ **b** $45 - 19$ **c** $156 - 48$
d $244 - 22$ **e** $376 - 59$ **f** $5216 - 199$

9 Mentally find the answers to these sums.

a $3 + 4 + 6 + 7$ **b** $14 + 16$ **c** $6 + 7 + 7$
d $12 + 7 + 8$ **e** $9 + 9 + 1 + 1$ **f** $5 + 6 + 4 + 5 + 3 + 7$


Look for pairs that add to ten; e.g. $3 + 7$.

Problem-solving and Reasoning

10 Mary has \$101 in her piggy bank. She takes out \$22 to buy a top. How much money remains in her piggy bank?

11 Gary worked 7 hours on Monday, 5 hours on Tuesday, 13 hours on Wednesday, 11 hours on Thursday and 2 hours on Friday. What is the total number of hours that Gary worked during the week?

12 In a batting innings, Phil hit 126 runs and Mario hit 19 runs. How many more runs did Phil hit compared to Mario?



13 Mentally find the answers to these computations.

a $11 + 18 - 17$ **b** $37 - 19 + 9$ **c** $101 - 15 + 21$
d $136 + 12 - 15$ **e** $28 - 10 - 9 + 5$ **f** $39 + 71 - 10 - 10$
g $100 - 11 + 21 - 1$ **h** $5 - 7 + 2$ **i** $10 - 25 + 18$

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1B

14 Matt has 36 cards and Andy has 35 more cards than Matt. How many cards does Andy have? If they combine their cards, how many do they have in total?

15 Are these statements true or false?

a $4 + 3 > 6$ **b** $11 + 19 \geq 30$ **c** $13 - 9 < 8$

d $26 - 15 \leq 10$ **e** $1 + 7 - 4 \geq 4$ **f** $50 - 21 + 6 < 35$

g $4 + 11 > 5 + 10$ **h** $4 + 7 = 5 + 6$ **i** $91 + 15 = 90 + 16$

Enrichment: Magic squares



Drilling for Gold 1B3

16 A magic square has every row, column and main diagonal adding to the same number, called the magic sum. For example, this magic square has a magic sum of 15.

4	9	2	15
3	5	7	15
8	1	6	15
15	15	15	15

Find the magic sums for these squares, then fill in the missing numbers.

a

6		
7	5	
2		

b

10		
	11	13
		12

c

15	20	
14		
19		

d

1	15		4
	6		9
		11	
13		2	16

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1C Algorithms for adding and subtracting

To add or subtract larger numbers we can use a step-by-step process called an algorithm.

Adding can involve trading a 'one' **to** the next column, whereas subtracting can involve trading a 'one' **from** the next column.



Let's start: The missing digits

Discuss what digits should go in the empty boxes. Give reasons for your answers.

$$\begin{array}{r} \square\,2 \\ +\ 9\,\square \\ \hline 1\,2\,6 \end{array} \qquad \begin{array}{r} 4\,\square \\ +\ 3\,8 \\ \hline 8\,1 \end{array}$$

$$\begin{array}{r} 1\,\square\,6 \\ -\ 8\,\square \\ \hline 5\,2 \end{array} \qquad \begin{array}{r} 1\,6\,\square \\ -\ \square\,5 \\ \hline 1\,0\,6 \end{array}$$

Key ideas

- An algorithm is a procedure involving a number of steps.
- Algorithm for adding large numbers:
  - Arrange the numbers vertically (i.e. above each other) to line up units with units, tens with tens etc.
  - Add digits in the same column, starting on the right with the units column.
  - If the digits add to more than 9, trade the 1 to the next column on the left.
- Algorithm for subtracting large numbers:
  - Arrange the numbers vertically to line up units with units, tens with tens etc.
  - Starting with the units column, subtract the bottom digit from the top digit.
  - If the bottom digit is greater than the top digit, trade a 1 from the next column to form an extra 10.
- Calculators may be used to check your answers.

Trade the 1

$$\begin{array}{r} {}^{1}2\,3\,4 \\ +\,1\,9\,2 \\ \hline 4\,2\,6 \end{array}$$

$1 + 2 + 1 = 4$ $\quad 4 + 2 = 6$ $\quad 3 + 9 = 12$

Trade 1

$$\begin{array}{r} {}^{1}\,{}^{1}2\,5\,9 \\ -\,1\,8\,2 \\ \hline 7\,7 \end{array}$$

$1 - 1 = 0$ $\quad 9 - 2 = 7$ $\quad 15 - 8 = 7$

Algorithm A step-by-step procedure

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Exercise 1C

Understanding

1 Mentally find the results for these sums.

a $7+6$	**b** $9+4$	**c** $11+9$	**d** $19+3$
e $8+9$	**f** $87+14$	**g** $138+6$	**h** $99+11$
i $998+7$	**j** $19+124$	**k** $102+99$	**l** $52+1053$

2 Mentally find the results for these differences.

a $13-6$	**b** $11-9$	**c** $16-11$	**d** $14-8$
e $13-5$	**f** $36-9$	**g** $75-8$	**h** $100-16$
i $37-22$	**j** $104-12$	**k** $46-17$	**l** $1001-22$

3 What is the missing digit in these computations?

a
$$\begin{array}{r} 2\ 7 \\ +3\ 1 \\ \hline 5\square \end{array}$$

b
$$\begin{array}{r} 3\ 6 \\ +1\ 5 \\ \hline 5\square \end{array}$$

c
$$\begin{array}{r} 1\ 2\ 3 \\ +\ \ 9\ 1 \\ \hline 2\square 4 \end{array}$$

d
$$\begin{array}{r} \ \ 4\ 6 \\ +\ \square\ 5 \\ \hline 1\ 1\ 1 \end{array}$$

e
$$\begin{array}{r} 2\ 4 \\ -1\ \square \\ \hline 1\ 2 \end{array}$$

f
$$\begin{array}{r} 6\ 7 \\ -4\ 8 \\ \hline \square 9 \end{array}$$

g
$$\begin{array}{r} 1\ 6\ 2 \\ -\ \square\ 1 \\ \hline 8\ 1 \end{array}$$

h
$$\begin{array}{r} 1\ 4\ \square\ 2 \\ -\ 6\ 2\ 3 \\ \hline 8\ 0\ 9 \end{array}$$

Fluency

Example 4 Adding larger numbers

Give the result for each of these additions.

a
$$\begin{array}{r} 26 \\ +66 \\ \hline \end{array}$$

b
$$\begin{array}{r} 439 \\ +172 \\ \hline \end{array}$$

Solution	Explanation
a $\begin{array}{r} {}^{1}26 \\ +66 \\ \hline 92 \end{array}$	Add the digits vertically, starting with the 'ones' column on the right. $6 + 6 = 12$, so trade the 1 to the tens column.
b $\begin{array}{r} {}^{1}4{}^{1}39 \\ +1\ 72 \\ \hline 6\ 11 \end{array}$	$9 + 2 = 11$, so trade a 1 to the tens column. $1 + 3 + 7 = 11$, so trade a 1 to the hundreds column.

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4 Give the results for these additions. Check your answers with a calculator.

a $\begin{array}{r} 36 \\ +51 \\ \hline \end{array}$ **b** $\begin{array}{r} 74 \\ +25 \\ \hline \end{array}$ **c** $\begin{array}{r} 17 \\ +24 \\ \hline \end{array}$ **d** $\begin{array}{r} 47 \\ +39 \\ \hline \end{array}$

e $\begin{array}{r} 54 \\ +27 \\ \hline \end{array}$ **f** $\begin{array}{r} 36 \\ +15 \\ \hline \end{array}$ **g** $\begin{array}{r} 64 \\ +28 \\ \hline \end{array}$ **h** $\begin{array}{r} 29 \\ +52 \\ \hline \end{array}$



For parts **c** to **h**, don't forget to trade the 'one'.

5 Give the results for these additions. Check your answers with a calculator.

a $\begin{array}{r} 138 \\ +\ 84 \\ \hline \end{array}$ **b** $\begin{array}{r} 257 \\ +\ 65 \\ \hline \end{array}$ **c** $\begin{array}{r} 449 \\ +\ 72 \\ \hline \end{array}$ **d** $\begin{array}{r} 871 \\ +\ 49 \\ \hline \end{array}$

e $\begin{array}{r} 129 \\ +\ 97 \\ \hline \end{array}$ **f** $\begin{array}{r} 458 \\ +287 \\ \hline \end{array}$ **g** $\begin{array}{r} 1041 \\ +\ 882 \\ \hline \end{array}$ **h** $\begin{array}{r} 3092 \\ +1988 \\ \hline \end{array}$



You will need to trade the 'one' twice in these questions.

Example 5 Subtracting larger numbers

Give the result for each of these subtractions.

a $\begin{array}{r} 74 \\ -15 \\ \hline \end{array}$ **b** $\begin{array}{r} 526 \\ -138 \\ \hline \end{array}$

Solution	Explanation
a $\begin{array}{r} {}^{6}\cancel{7}^{1}\,4 \\ -1\ 5 \\ \hline 5\ 9 \end{array}$	Trade 1 from 7 to make $14 - 5 = 9$. Then subtract 1 from 6 (not 7).
b $\begin{array}{r} {}^{4}\cancel{5}^{11}{}^{1}\cancel{2}^{1}6 \\ -1\ 3\ 8 \\ \hline 3\ 8\ 8 \end{array}$	Trade 1 from 2 to make $16 - 8 = 8$. Trade 1 from 5 to make $11 - 3 = 8$. $4 - 1 = 3$



Drilling for Gold 1C1

6 Give the results for these sums.

a $\begin{array}{r} 17 \\ 26 \\ +34 \\ \hline \end{array}$ **b** $\begin{array}{r} 126 \\ 47 \\ +\ 19 \\ \hline \end{array}$ **c** $\begin{array}{r} 152 \\ 247 \\ +\ 19 \\ \hline \end{array}$ **d** $\begin{array}{r} 2197 \\ 1204 \\ +\ 807 \\ \hline \end{array}$

7 Find the results for these subtractions.

a $\begin{array}{r} 54 \\ -26 \\ \hline \end{array}$ **b** $\begin{array}{r} 85 \\ -66 \\ \hline \end{array}$ **c** $\begin{array}{r} 46 \\ -27 \\ \hline \end{array}$ **d** $\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$

e $\begin{array}{r} 85 \\ -27 \\ \hline \end{array}$ **f** $\begin{array}{r} 43 \\ -14 \\ \hline \end{array}$ **g** $\begin{array}{r} 82 \\ -56 \\ \hline \end{array}$ **h** $\begin{array}{r} 66 \\ -27 \\ \hline \end{array}$



You will need to trade a 'one' from the tens column.

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1C

8 Find the results for these subtractions.

a $\begin{array}{r} 235 \\ -\ 86 \\ \hline \end{array}$ **b** $\begin{array}{r} 352 \\ -\ 79 \\ \hline \end{array}$ **c** $\begin{array}{r} 714 \\ -\ 58 \\ \hline \end{array}$ **d** $\begin{array}{r} 932 \\ -\ 44 \\ \hline \end{array}$

e $\begin{array}{r} 125 \\ -\ 89 \\ \hline \end{array}$ **f** $\begin{array}{r} 241 \\ -129 \\ \hline \end{array}$ **g** $\begin{array}{r} 358 \\ -279 \\ \hline \end{array}$ **h** $\begin{array}{r} 491 \\ -419 \\ \hline \end{array}$



You will need to trade a 'one' twice in each question.

Problem-solving and Reasoning

9 Farmer Green owns 287 sheep, Farmer Brown owns 526 sheep and Farmer Grey owns 1041 sheep. How many sheep are there in total?



10 A car's odometer shows 12 138 kilometres at the start of a journey and 12 714 kilometres at the end of the journey. How far was the journey?



11 Find the missing digits in these sums and differences.

a $\begin{array}{r} 3\,\square \\ +\ 5\ 3 \\ \hline \square\ 1 \end{array}$ **b** $\begin{array}{r} 1\,\square\ 4 \\ +\ \ 7\,\square \\ \hline \square\ 9\ 1 \end{array}$ **c** $\begin{array}{r} 6\,\square \\ -\ 2\ 8 \\ \hline \square\ 4 \end{array}$ **d** $\begin{array}{r} 2\,\square\ 5 \\ -\,\square\ 8\,\square \\ \hline 8\ 1 \end{array}$

12 a What are the missing digits in this sum?

$$\begin{array}{r} 2\,\square\,3 \\ +\,\square\,\square\,\square \\ \hline 4\ 2\ 1 \end{array}$$

b Explain why there is more than one possible set of missing digits in the sum above. Give some examples.

Enrichment: Number problems

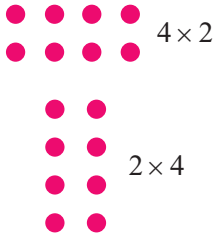
13 The sum of two numbers is 978 and their difference is 74. What are the two numbers?

14 Make up some of your own problems like Question **13** and test them on a friend.

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1D Multiplying small positive integers

The multiplication of two numbers represents a repeated addition. For example, 4×2 could be thought of as 4 groups of 2 or $2 + 2 + 2 + 2$. 4×2 could be also thought of as 2 groups of 4 or 2×4 or $4 + 4$.



Let's start: Class excursion

Your teacher purchases 21 tickets at \$9 each for a class excursion. You need to work out the total cost.

Look at the following strategies. Do any of them give the correct answer?

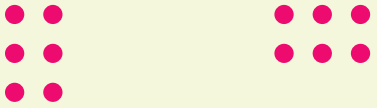
- 21×9 is the same as 20×10, so the answer is \$200.
- 21×9 is the same as $20 \times 9 + 1$, so the answer is $180 + 1 = \$181$.
- 21×9 is the same as $20 \times 9 + 9$, so the answer is $180 + 9 = \$189$.



Key ideas

- Finding the **product** of two numbers involves multiplication. We say 'the product of 2 and 3 is 6'.
  - The order does *not* matter when you multiply numbers.

$3 \times 2 = 6$ and $2 \times 3 = 6$



This example shows the **commutative law**.

$2 \times 3 \times 4 = 6 \times 4 = 24$ $\quad 2 \times 3 \times 4 = 2 \times 12 = 24$

This example shows the **associative law**.

- To multiply by a single digit:
  - Multiply the single digit by each digit in the other number, starting from the right.
  - Trade and add any digits with a higher place value to the total in the next column.

$$\begin{array}{r} {}^{1}23 \\ \times\ \ 4 \\ \hline 92 \end{array} \quad \begin{array}{l} 4 \times 3 = 12 \\ 4 \times 2 + 1 = 9 \end{array}$$

- Mental strategies for multiplication include:
  - Knowing your multiplication tables; e.g. $9 \times 7 = 63$.
  - Changing the order.

e.g. $15 \times 3 = 3 \times 15$ (3 lots of 15)
$= 45$

$5 \times 13 \times 2 = 5 \times 2 \times 13$
$= 10 \times 13$
$= 130$

Product
A multiplication of numbers

Commutative law
When adding and multiplying, the order in which two numbers are combined does not matter

Associative law The result of adding or multiplying three or more numbers does not depend on how they are grouped

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– Using the doubling and halving strategy by doubling one number and halving the other.
– Using the **distributive law** by making a 10, 100 etc. and then adjusting by adding or subtracting.

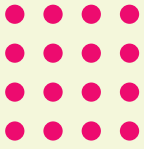
e.g. $6 \times 21 = 6 \times (20 + 1)$
$= 6 \times 20 + 6 \times 1$
$= 120 + 6$
$= 126$

Double the 5 Halve the 4

$5 \times 7 \times 4 = 10 \times 7 \times 2$
$= 70 \times 2$
$= 140$

Distributive law
Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

- When a number is multiplied by itself it is said to be 'squared'.



'4 squared' can be written as 4×4 or 4^2.
$4 \times 4 = 4^2 = 16$

Exercise 1D

Understanding



Drilling for Gold
1D1
1D2
1D3a,b,c
1D4

1 Write the next three numbers in these multiplication patterns.

a 2, 4, 6, 8, __, __, __
b 3, 6, 9, 12, __, __, __
c 7, 14, 21, 28, __, __, __
d 4, 8, 12, 16, __, __, __
e 11, 22, 33, __, __, __
f 9, 18, 27, __, __, __

2 Write the missing number.

a $4 \times 5 = 5 \times$ __
b $2 \times 7 = 7 \times$ __
c $15 \times 11 =$ __ $\times 15$
d $3 \times 2 \times 6 = 6 \times$ __ $\times 3$
e $12 \times 2 \times 4 = 2 \times 12 \times$ __
f $7 \times 3 \times 9 = 9 \times 3 \times$ __

3 Use your knowledge of the multiplication tables to write the answer. Check your answers with your calculator.

a 11×2
b 3×9
c 8×4
d 7×8
e 7×4
f 12×5
g 4×11
h 11×7
i 12×9
j 9×8
k 3×7
l 6×9
m 6×5
n 10×11
o 12×12
p 8×5
q 7×7
r 9×7
s 11×12
t 12×6
u 5×11
v 2×11
w 4×6
x 12×8
y 6×6

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Fluency

Example 6 Using mental strategies for multiplication

Use a mental strategy to find:

a 3×13 **b** 2×19 **c** 5×24 **d** 6^2

Solution	Explanation
a $3 \times 13 = 30 + 9$ $= 39$	$3 \times 13 = 3 \times (10 + 3) = (3 \times 10) + (3 \times 3)$ Break up 13 into $10 + 3$.
b $4 \times 19 = 80 - 4$ $= 76$	$4 \times 19 = 4 \times (20 - 1) = (4 \times 20) - (4 \times 1)$ Break up 19 into $20 - 1$.
c $5 \times 24 = 10 \times 12$ $= 120$	The doubling and halving strategy is being used. Double the 5 and halve the 24.
d $6^2 = 6 \times 6$ $= 36$	The number 6 is multiplied by itself. The number 36 is a square number.

4 Find the results to these products mentally. Check your answers with your calculator.

a 5×21 **b** 4×31 **c** 3×31
d 6×22 **e** 5×23 **f** 7×31
g 9×22 **h** 6×42 **i** 8×42



For part **a**, work out 5×20 and then add 5.

5 Find the answers to these products mentally.

a 3×19 **b** 2×19 **c** 2×29
d 4×29 **e** 5×18 **f** 7×18
g 3×39 **h** 4×49 **i** 6×39



For part **a**, work out 3×20 and then subtract 3.

6 Find the answers to these products mentally.

a 5×14 **b** 5×18 **c** 22×5
d 36×5 **e** 4×24 **f** 3×18
g 6×16 **h** 24×3 **i** 18×4



Double one number and halve the other.
So $5 \times 14 = 10 \times 7$
$= 70$

7 Write down the value of:

a 5^2 **b** 3^2 **c** 9^2

Example 7 Multiplication showing working

Give the result for each of these products.

a 31×4 **b** 197×7

Solution	Explanation
a $\begin{array}{r} 31 \\ \times\ 4 \\ \hline 124 \end{array}$	In the ones column: $4 \times 1 = 4$. In the tens column: $4 \times 3 = 12$, so the 2 goes in the tens column and the 1 goes in the hundreds column.

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1D

Solution	Explanation
b $^{6}1^{4}97 \times 7 = 1379$	In the ones column: $7 \times 7 = 49$. In the tens column: $7 \times 9 + 4 = 67$. In the hundreds column: $7 \times 1 + 6 = 13$.

8 Give the result of each of these products, showing working. Check your answers using a calculator.

a 33×2 **b** 43×3 **c** 72×6 **d** 55×3

e 37×4 **f** 51×9 **g** 48×7 **h** 59×8

i 129×2 **j** 407×7 **k** 526×5 **l** 3509×9

Problem-solving and Reasoning

9 What is the missing digit in these products?

a $21 \times 3 = 6\square$ **b** $36 \times 5 = 18\square$ **c** $76 \times 2 = 1\square2$ **d** $402 \times 3 = 1\square06$

10 A circular race track is 240 metres long and Rory runs seven laps. How far does Rory run in total?



11 Eight tickets costing $33 each are purchased for a concert. What is the total cost of the tickets?

12 Reggie and Angelo combine their packs of cards. Reggie has five sets of 13 cards and Angelo has three sets of 17 cards. How many cards are there in total?

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13 Are these statements true or false? Check your answers using a calculator.

a $4 \times 3 = 3 \times 4$
b $2 \times 5 \times 6 = 6 \times 5 \times 2$
c $5^2 = 10$
d $3 \times 32 = 3 \times 30 + 3 \times 2$
e $5 \times 18 = 10 \times 9$
f $21 \times 4 = 2 \times 42$
g $19 \times 7 = 20 \times 7 - 19$
h $39 \times 4 = 40 \times 4 - 1 \times 4$
i $64 \times 4 = 128 \times 8$

14 Find the missing digits in these products.

a
$$\begin{array}{r} 3\ 9 \\ \times\quad 7 \\ \hline 2\ \square\ 3 \end{array}$$

b
$$\begin{array}{r} 2\ 5 \\ \times\quad \square \\ \hline 1\ 2\ 5 \end{array}$$

c
$$\begin{array}{r} 7\ 9 \\ \times\quad \square \\ \hline \square\ 3\ 7 \end{array}$$

d
$$\begin{array}{r} 1\ 3\ 2 \\ \times\quad \square \\ \hline 10\ \square\ 6 \end{array}$$

e
$$\begin{array}{r} 2\ \square \\ \times\quad 7 \\ \hline \square\ 8\ 9 \end{array}$$

f
$$\begin{array}{r} \square\square \\ \times\quad 9 \\ \hline 3\ 5\ 1 \end{array}$$

15 How many different ways can the two spaces be filled? Explain why.

$$\begin{array}{r} 2\ \square\ 3 \\ \times\quad 4 \\ \hline 8\ \square\ 2 \end{array}$$

Enrichment: Choose your own 'Times Tables Bingo Numbers'

Drilling for Gold 1D5

16 The table below contains all the counting numbers from 1 to 100.
Ask your teacher to give all the students one of these Times Tables Bingo cards.
How to play Times Tables Bingo

- Circle any ten numbers on your Times Tables Bingo card.
- Your teacher will randomly choose some products. If your teacher calls out '8 times 4' (which is 32) and you circled 32, then highlight 32.
- Keep doing this until you have highlighted all your numbers, then call out "Times Table Bingo!".
- For games 2, 3 and 4, start with a new card each game and choose your numbers more carefully each time.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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1E Multiplying large positive integers

There are many situations that require the multiplication of large numbers. For example, finding the total revenue from selling 40 000 tickets at $23 each. Another example is finding the area of a rectangular park with length and breadth dimensions of 65 metres by 122 metres. Doing such calculations by hand requires a number of steps.



How much revenue came from selling tickets to this game?

Let's start: Spot the errors

There are three errors in this computation. Find them and discuss them. What is the correct answer?

$$\begin{array}{r} 82 \\ \times\ 16 \\ \hline 482 \\ 82 \\ \hline 464 \end{array}$$

Key ideas

- When multiplying by 10, 100, 1000, 10 000 etc. each digit moves to the left by the number of zeros.

  $2 \times 100 = 200$ $41 \times 10 = 410$ $279 \times 1000 = 279\,000$

- A strategy for multiplying by multiples of 10, 100 etc. is to first multiply by the number without the zeros and then include the zeros to the answer later.

  For example: $21 \times 3000 = 21 \times 3 \times 1000 = 63 \times 1000 = 63\,000$

- To multiply large numbers, use an algorithm such as:

$$\begin{array}{rl} 37 & \\ \times\ 12 & \\ \hline 74 & \leftarrow 37 \times 2 \\ 370 & \leftarrow 37 \times 10 \\ \hline 444 & \leftarrow 370 + 74 \end{array} \qquad \begin{array}{rl} 143 & \\ \times\ 14 & \\ \hline 572 & \leftarrow 143 \times 4 \\ 1430 & \leftarrow 143 \times 10 \\ \hline 2002 & \leftarrow 1430 + 572 \end{array}$$

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Exercise 1E

Understanding

1 Write the missing number: 10, 100 or 1000.

a $35 \times __ = 350$ **b** $21 \times __ = 2100$

c $49 \times __ = 49\,000$ **d** $213 \times __ = 2130$

2 Copy and complete these products.

a 27×3 **b** 39×2 **c** 92×5 **d** 121×6

3 Write the answers.

a 2×60 **b** 3×40 **c** 5×50 **d** 9×80

e 30×6 **f** 90×2 **g** 70×5 **h** 60×9

i 5×70 **j** 40×5 **k** 20×50 **l** 50×60

Fluency

Example 8 Multiplying large numbers

Give the result for each of these products.

a 37×100 **b** 21×50 **c** 87×13

Solution

a $37 \times 100 = 3700$

b $21 \times 50 = 21 \times 5 \times 10$
$= 105 \times 10$
$= 1050$

c
$$\begin{array}{r} 87 \\ \times\ 13 \\ \hline 261 \\ 870 \\ \hline 1131 \end{array}$$

Explanation

Move the 3 and the 7 two places to the left and add two zeros.

First multiply by 5, then multiply the answer by 10.
$21 \times 5 = 105$
$105 \times 10 = 1050$

First multiply $87 \times 3 = 261$.
Then multiply $87 \times 10 = 870$.
Add the results to give the answer.
$261 + 870 = 1131$

4 Give the result of each of these products.

a 4×100 **b** 29×10 **c** 183×10 **d** 46×100

e 37×1000 **f** 192×10 **g** 3010×100 **h** 248×1000

i 50×1000 **j** 630×100 **k** 1441×10 **l** $2910 \times 10\,000$

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1E

5 Find these products.

a 12×20	**b** 18×30	**c** 26×20
d 21×30	**e** 17×20	**f** 36×40
g 92×70	**h** 45×500	**i** 138×300
j 92×5000	**k** 317×200	**l** 1043×9000



First multiply by the single non-zero digit, then write the zeros:
$12 \times 20 = 12 \times 2 \times 10$

6 Find these products, then check your answers with a calculator.

a 21×12	**b** 26×11	**c** 31×14	**d** 43×15
e 37×11	**f** 72×19	**g** 88×14	**h** 57×22
i 92×23	**j** 84×27	**k** 46^2	**l** 72^2

Problem-solving and Reasoning

7 Mandy buys 28 tickets at \$15 each. What is the total cost of the tickets?

8 A pool area includes 68 square metres of paving at \$32 per square metre. What is the total cost of paving?



9 What is the largest square number less than 100?

10 The product of two numbers is 36. What could the two numbers be? Write all five pairs of numbers.

11 Waldo buys 215 metres of pipe at \$28 per metre. What is the total cost of piping?

12 How many seconds are there in one day? Check your answer using a calculator.



There are 60 seconds in 1 minute.

Enrichment: Multiplication puzzles

13 a What is the largest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

b What is the smallest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

$$\square\square\square \times \square\square$$

14 $8^2 = 8 \times 8 = 64$
64 is a two-digit square number.
Find all the three-digit square numbers in which the 'hundreds' digit is 1 or 2.

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1F Dividing positive integers and dealing with remainders



Division is often used when handling money.

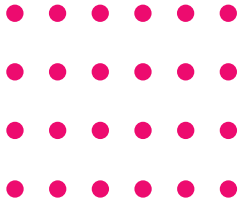
Division is used to split a quantity into equal groups.
Examples include:

- 20 apples shared by five people
- $10 000 shared equally between four people

Let's start: Arranging counters in an array

A total of 24 counters sit on a table. Using whole numbers, in how many ways can the counters be divided into an array?

- Is it also possible to divide the counters into equal-sized groups but with two counters left over?
- If five counters are to remain, how many equal-sized groups can be formed and why?



Key ideas



Drilling for Gold 1F1a,b

- To divide 7 by 3, we mean 7 divided into groups of 3.
  This could be written as:
  7 divided by 3

  or

  3 divided into 7.

  or

  $7 \div 3$

  Start with 7 dots.
  Circle groups of 3.

  There are 2 groups of 3 with 1 remainder.
- Any amount remaining after division into equal-sized groups is called the **remainder**.

$7 \div 3 = 2$ and 1 remainder

dividend, divisor, quotient

This is written $7 \div 3 = 2\frac{1}{3}$. ← remainder, ← divisor

Quotient A number that is the result of division

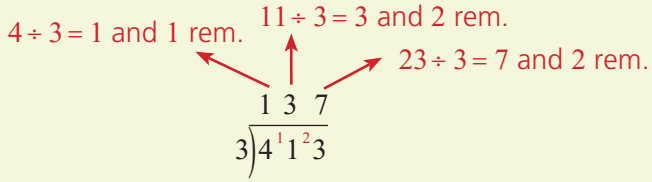
Dividend The number being divided

Divisor The number you are dividing by

Remainder The leftover amount after one number has been divided by another

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- The order *does* matter when you divide numbers. You cannot swap numbers in a division to make the calculation easier.
  e.g. $6 \div 3 = 2$ but $3 \div 6 \neq 2$ $\quad (10 \div 5) \div 2 \neq 10 \div (5 \div 2)$
- Use short division to work with larger numbers.
  – Start by dividing the divisor into the first (i.e. left) digit, then trade any remainder. This example shows that $413 \div 3 = 137$ and 2 remainder.



This is written $413 \div 3 = 137\frac{2}{3}$.

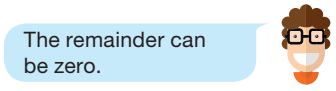
Exercise 1F

Understanding



1 Find the remainder when 24 is divided by:

a 1	**b** 2	**c** 3	**d** 4
e 5	**f** 6	**g** 7	**h** 8



2 **a** If $4 \times 6 = 24$, then $24 \div 6 =$ ☐ and $24 \div 4 =$ ☐.
b If $19 \times 27 = 513$, then $513 \div 27 =$ ☐ and $513 \div 19 =$ ☐.

3 What is the remainder when:

a 2 is divided into 7?	**b** 5 is divided into 37?
c 42 is divided by 8?	**d** 50 is divided by 9?

Fluency

Example 9 Using mental strategies for division

Use a mental strategy to find the answer.

a $56 \div 8$ **b** $93 \div 3$ **c** $57 \div 3$ **d** $128 \div 8$

Solution		Explanation
a	$56 \div 8 = 7$	$8 \times ? = 56$ (Use your knowledge from multiplication tables.)
b	$93 \div 3 = 31$	$90 \div 3 = 30$ so $93 \div 3 = 31$
c	$57 \div 3 = 19$	$60 \div 3 = 20$ so $57 \div 3 = 19$
d	$128 \div 8 = 16$	$128 \div 8 = 64 \div 4 = 32 \div 2 = 16$ (Halve both numbers repeatedly.)

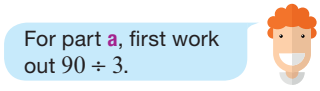
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4 Use your knowledge of multiplication tables to find the quotient. Check with a calculator.

a $28 \div 7$	**b** $36 \div 12$	**c** $48 \div 8$
d $45 \div 9$	**e** $56 \div 8$	**f** $63 \div 7$
g $96 \div 12$	**h** $121 \div 11$	**i** $110 \div 11$
j $36 \div 9$	**k** $77 \div 7$	**l** $48 \div 4$
m $72 \div 9$	**n** $72 \div 6$	**o** $55 \div 5$
p $96 \div 8$		

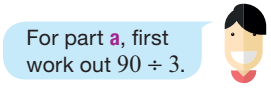
5 Find the answer to these using a mental strategy.

a $93 \div 3$	**b** $42 \div 2$	**c** $84 \div 4$
d $63 \div 3$	**e** $210 \div 10$	**f** $220 \div 20$
g $96 \div 3$	**h** $64 \div 4$	



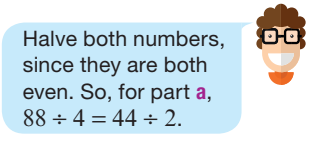
6 Find the answer to these using a mental strategy.

a $87 \div 3$	**b** $76 \div 4$	**c** $96 \div 4$
d $63 \div 7$	**e** $117 \div 3$	**f** $56 \div 4$
g $116 \div 4$	**h** $180 \div 20$	



7 Find the value of:

a $88 \div 4$	**b** $124 \div 4$
c $136 \div 8$	**d** $112 \div 16$



8 Write the answers to these divisions, which involve 0s and 1s.

a $26 \div 1$	**b** $1094 \div 1$
c $0 \div 7$	**d** $0 \div 458$

Example 10 Using short division

Use short division to find the quotient. Express the remainder as a fraction.

a $3\overline{)37}$ **b** $7\overline{)195}$

Solution	Explanation
a $\begin{array}{r} 12 \\ 3\overline{)37} \end{array}$ rem. 1 $37 \div 3 = 12\frac{1}{3}$	$3 \div 3 = 1$ with no remainder. $7 \div 3 = 2$ with 1 remainder.
b $\begin{array}{r} 27 \\ 7\overline{)19^{5}5} \end{array}$ rem. 6 $195 \div 7 = 27\frac{6}{7}$	7 does not divide into 1. $19 \div 7 = 2$ with 5 remainder. $55 \div 7 = 7$ with 6 remainder.

9 Use the short division algorithm and express the remainder as a fraction.

Skillsheet 1A

a $3\overline{)71}$	**b** $7\overline{)92}$	**c** $2\overline{)139}$	**d** $6\overline{)247}$
e $5\overline{)217}$	**f** $4\overline{)506}$	**g** $3\overline{)794}$	**h** $9\overline{)814}$
i $4\overline{)2173}$	**j** $3\overline{)61\,001}$	**k** $5\overline{)4093}$	**l** $9\overline{)90\,009}$

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1F

Problem-solving and Reasoning

10 Write the missing digit in each of these divisions.

a $3\overline{)5\,1}$ $\square 7$ **b** $7\overline{)8\,4}$ $\square 2$ **c** $5\overline{)1\,2\,5}$ $2\square$ **d** $9\overline{)1\,3\,5}$ $1\square$

11 If 117 food packs are divided equally among nine families, how many packs does each family receive?

12 Spring Fresh Company sells mineral water in packs of six bottles. How many packs are there in a truck containing 744 bottles?





13 A straight fence has two end posts. It also has other posts that are divided evenly along the fence 4 metres apart. If the fence is to be 264 metres long, how many posts are needed, including the end posts?

14 Friendly Taxis can take up to four passengers each. How many taxis are required to transport 59 people?



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15 Pies are purchased wholesale at three for $4. How much will it cost to purchase 153 pies?



Enrichment: Long, short division

16 What number am I?

- I am less than 60.
- When I am divided by 3 the remainder is 1.
- When I am divided by 4 the remainder is 2.
- When I am divided by 5 the remainder is 3.

17 Short division can also be used to divide by numbers with more than one digit.

e.g. $215 \div 12 = 17$ and $\frac{11}{12}$

$$12\overline{)21^{9}5} \quad 17 \text{ rem. } 11$$

Use the short division algorithm and express the remainder as a fraction.

a $371 \div 11$ **b** $926 \div 17$ **c** $404 \div 13$

d $1621 \div 15$ **e** $2109 \div 23$ **f** $6914 \div 56$

18 The magic product for this square is 6720. Find the missing numbers.

1	6		56
40		2	3
14			
			10

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1G Estimating and rounding positive integers

Before attempting any computation it is a good idea to estimate the result. In such cases we use rounding to help. For example, the approximate total cost of 18 truckloads of soil at \$54 per load could be estimated by $20 \times 50 = 1000$.

Let's start: Counting crowds

Here is a photo of a crowd at a sporting event. Describe how you might estimate the number of people in the photo. What is your answer? How different is your answer from those of others in your class?



Key ideas

- **Estimates** or **approximations** can be found by **rounding** numbers to the nearest $10, 100, 1000$ etc.
  - If the next digit is $0, 1, 2, 3$ or 4, then round down.
  - If the next digit is $5, 6, 7, 8$ or 9, then round up.
- First digit approximation rounds the first digit to the nearest 10 or 100 or 1000 etc.
  e.g. For 932 use 900 For 968 use 1000
- The symbols $\approx$ and $\fallingdotseq$ mean 'is approximately equal to'.

Estimate
An informed approximation

Approximation A value that is close to the real value

Rounding
Approximating a number

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Exercise 1G

Understanding

1 Have these numbers been rounded up or down?

a $59 \div 60$ **b** $14 \div 10$ **c** $137 \div 140$
d $255 \div 260$ **e** $924 \div 900$ **f** $1413 \div 1000$

2 The following numbers are to be rounded to the nearest 10. Should they be rounded up or down?

a 19 **b** 37 **c** 21 **d** 14
e 72 **f** 33 **g** 45 **h** 95
i 132 **j** 176 **k** 288 **l** 304

3 The following numbers are to be rounded to the nearest 100. Decide if they would be rounded up or down.

a 103 **b** 201 **c** 195 **d** 186
e 172 **f** 131 **g** 427 **h** 552
i 956 **j** 349 **k** 198 **l** 359

Fluency

Example 11 Rounding

Round these numbers as indicated.

a 86 (to the nearest 10) **b** 4142 (to the nearest 100)

Solution	Explanation
a $86 \approx 90$ or $86 \div 100$	The digit after the 8 is greater than or equal to 5, so round up.
b $4142 \approx 4100$ or $4142 \div 4100$	The digit after the 1 is less than or equal to 4, so round down. Keep the 1 and make the digits that follow zero.

4 Round these numbers as indicated.

a 59 (nearest 10) **b** 32 (nearest 10)
c 124 (nearest 10) **d** 185 (nearest 10)
e 231 (nearest 100) **f** 894 (nearest 100)
g 96 (nearest 10) **h** 584 (nearest 100)
i 1512 (nearest 1000) **j** 1492 (nearest 1000)
k 7810 (nearest 1000) **l** 10 200 (nearest 1000)



If the next digit is 0, 1, 2, 3 or 4, round down. Otherwise, round up.

5 Round these numbers using the first digit.

a 21 **b** 29 **c** 136
d 857 **e** 241 **f** 729
g 98 **h** 962 **i** 5600
j 92 104 **k** 9999 **l** 14

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1G

Example 12 Using leading digit approximation

Estimate the answers to these problems by rounding each number to the first digit.

a 42×7 **b** 95×326 **c** $302 \div 29$

Solution	Explanation
a $42 \times 7 \fallingdotseq 40 \times 7 \fallingdotseq 280$	The first digit in 42 is the 4 in the 'tens' column.
b $95 \times 326 \fallingdotseq 100 \times 300 \fallingdotseq 30\,000$	The nearest 'ten' to 95 is 100, and the first digit in 326 is in the 'hundreds' column.
c $302 \div 29 \fallingdotseq 300 \div 30 \fallingdotseq 10$	302 rounds to 300 and 29 rounds to 30.

6 Estimate the value of:

a 29×4 **b** $124 + 58$ **c** $232 - 106$
d $61 \div 5$ **e** $103 \div 11$ **f** 32×99
g 59×21 **h** $279 \div 95$ **i** $394 \div 10$
j 97×21 **k** $1390 + 3244$ **l** $999 - 888$



Round each number to the first digit before making the calculation.

Problem-solving and Reasoning



For questions **7** to **10**, use first digit approximation to make your estimate.

7 You purchase 59 tickets at \$21 each. Give an estimate for the total cost of the tickets.

8 A digger can dig 29 scoops of soil per hour and work 7 hours per day. Approximately how many scoops can be dug over 10 days?

9 Most of the pens at a stockyard are full of sheep. There are 55 pens and one of the pens has 22 sheep. Give an estimate for the total number of sheep at the stockyard.

10 A whole year group of 159 students is roughly divided into 19 groups. Estimate the number in each group.

11 For the given estimates, decide if the approximate answer is going to give a larger or smaller result compared to the true answer.

a $58 + 97 \fallingdotseq 60 + 100$ **b** $24 \times 31 \fallingdotseq 20 \times 30$
c $130 - 79 \fallingdotseq 130 - 80$ **d** $267 - 110 \fallingdotseq 270 - 110$

12 It is sensible sometimes to round one number up if the other number is going to be rounded down. Use first digit approximation to estimate the answers to these computations.

a 11×19 **b** 129×954
c 25×36 **d** 1500×2500

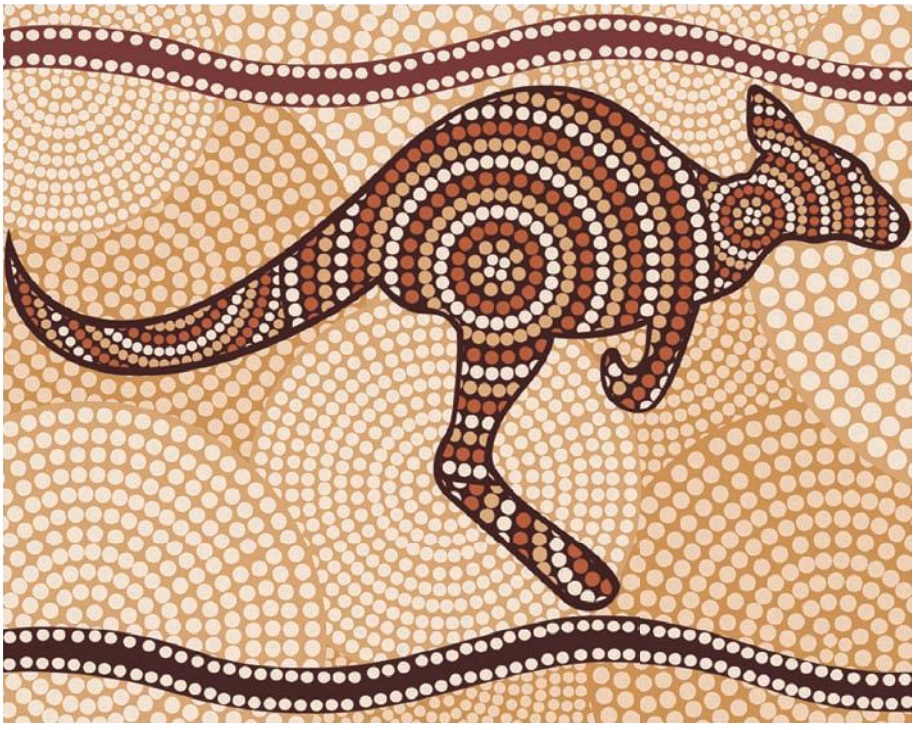
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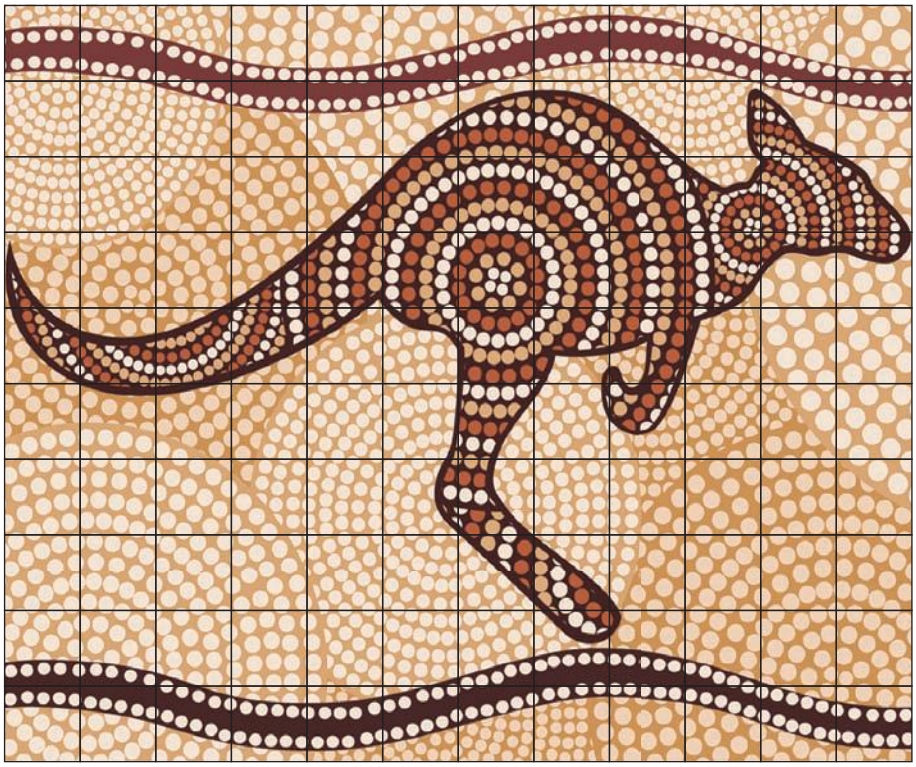
Enrichment: Aboriginal dot painting

13 Many examples of Aboriginal art include dot paintings. Estimate the number of dots in the painting.



Hint: Use the grid in the second photo to help you.





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1H Order of operations with positive integers

When combining the operations of addition, subtraction, multiplication and division, a special order needs to be followed. Multiplication and division sit higher in the order than addition and subtraction.

Look at the colours of the chairs in the photo and then consider these two statements.

- 2 groups of 3 chairs plus 5 chairs
- 5 chairs plus 2 groups of 3 chairs

In both cases, there are $2 \times 3 + 5 = 11$ chairs.

This means that $2 \times 3 + 5 = 5 + 2 \times 3$.

This also suggests that for $5 + 2 \times 3$, the multiplication should be done first.



Let's start: Make it true!

Can you insert a pair of brackets to make the following true?

- $2 + 3 \times 5 = 25$
- $3 \times 2 + 6 = 24$
- $20 \div 8 - 3 = 4$
- $7 - 1 \div 3 = 2$

Discuss whether you think the following need brackets to make them true.

- $2 + 6 \times 1 = 8$
- $10 \div 2 + 3 = 2$

Key ideas

- When working with more than one operation:
  - First, deal with operations inside the brackets.
  - Then do multiplication and division, working from left to right.
  - Finally, do addition and subtraction, working from left to right.

$7 \div (4 + 3)$

1st: $4 + 3 = 7$

2nd: $7 \div 7 = 1$

This is written as:

$7 \div (4 + 3) = 7 \div 7$

$= 1$

$4 \times (2 + 3) - 12 \div 6$

1st: $2 + 3 = 5$

2nd: $4 \times 5 = 20$

3rd: $12 \div 6 = 2$

last: $20 - 2 = 18$

This is written as:

$4 \times (2 + 3) - 12 \div 6 = 4 \times 5 - 2$

$= 20 - 2$

$= 18$

- Note that some calculators apply the **order of operations** and some do not.
  For example: $5 + 2 \times 3 = 11$ (not 21).
  Try this on a variety of calculators and mobile phones.

Order of operations
The sequence in which computations are done

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Exercise 1H

Understanding

1 Which goes first?

a addition or multiplication
b brackets or division
c subtraction or brackets
d multiplication or subtraction

2 Which operation (i.e. +, –, × or ÷) must be done first?

a $2 + 5 - 3$
b $5 \div 5 \times 2$
c $2 \times 3 \div 6$
d $5 \times 2 + 3$
e $7 \div 7 - 1$
f $(6 + 2) \times 3$
g $(8 \div 4) - 1$
h $4 + 7 \times 2$
i $8 - 10 \div 5$
j $10 - 2 + 3$
k $6 + 2 \times 3 - 1$
l $5 \times (2 + 3 \div 3) - 1$

Fluency

Example 13 Using order of operations

Use order of operations to find the answers to these computations.

a $5 + 10 \div 2$
b $3 \times (2 + 4)$
c $5 \times 2 - 8 \div 4$
d $6 \times (2 + 10) - 24$
e $18 - 2 \times (4 + 6) \div 5$

Solution	Explanation
a $5 + 10 \div 2 = 5 + 5$ $= 10$	Do the division before the addition.
b $3 \times (2 + 4) = 3 \times 6$ $= 18$	Deal with brackets before multiplication.
c $5 \times 2 - 8 \div 4 = 10 - 2$ $= 8$	Do the multiplication and division before the subtraction.
d $6 \times (2 + 10) - 24 = 6 \times 12 - 24$ $= 72 - 24$ $= 48$	Deal with brackets first. Do the multiplication before subtraction. Do the subtraction last.
e $18 - 2 \times (4 + 6) \div 5 = 18 - 2 \times 10 \div 5$ $= 18 - 20 \div 5$ $= 18 - 4$ $= 14$	Deal with brackets first. Do the multiplication and division next, working from left to right. Do the subtraction last.


Drilling for Gold 1H1

3 Use order of operations to answer these computations.

a $1 + 2 \times 3$
b $5 + 7 \times 2$
c $9 - 10 \div 5$
d $4 \times (3 + 2)$
e $21 \div (3 + 4)$
f $18 \div (10 - 1)$
g $(7 + 2) \div 3$
h $(10 - 4) \times 4$
i $(6 - 5) \div 1$
j $2 + 3 \times 7$
k $5 + 8 \times 2$
l $10 - 20 \div 2$
m $22 - 16 \div 4$
n $6 \times 3 + 2 \times 7$
o $1 \times 8 - 2 \times 3$
p $18 \div 9 + 60 \div 3$
q $2 + 3 \times 7 - 1$
r $40 - 25 \div 5 + 3$
s $63 \div 3 \times 7 + 2 \times 3$
t $78 - 14 \times 4 + 6$
u $300 - 100 \times 4 \div 4$


Remember that × and ÷ go before + and –. Work from left to right after you have chosen which operation goes first.

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1H

Skillsheet 1B

4 Use order of operations to find the answers to these computations.

a $2 \times (3 + 2)$
b $18 \div (10 - 4)$
c $(19 - 9) \div 5$
d $2 \times (3 + 2) - 1$
e $10 \div (3 + 2) + 6$
f $13 \times (10 \div 10) - 13$
g $(100 + 5) \div 5 + 1$
h $2 \times (9 - 4) \div 5$
i $50 \div (13 - 3) + 4$
j $16 - 2 \times (7 - 5) + 6$
k $(7 + 2) \div (53 - 50)$
l $14 - (7 \div 7 + 1) \times 2$
m $(20 - 10) \times (5 + 7) + 1$
n $3 \times (72 \div 12 + 1) - 1$
o $48 \div (4 + 4) \div (3 \times 2)$
p $20 - (3 \times 5 + 1) \div 4$


Deal with brackets first.

Problem-solving and Reasoning

5 Are these statements true or false?

a $5 \times 2 + 1 = (5 \times 2) + 1$
b $10 \times (3 + 4) = 10 \times 3 + 4$
c $21 - 7 \div 7 = (21 - 7) \div 7$
d $9 - 3 \times 2 = 9 - (3 \times 2)$

Example 14 Using order of operations in worded problems

Find the result when 6 is multiplied by the sum of 2 and 7.

Solution	Explanation
$6 \times (2 + 7) = 6 \times 9$ $= 54$	First, write the problem using symbols and numbers. Use brackets for the sum since this operation is to be completed first.

6 Find the answer to these worded problems by first writing the sentence using numbers and symbols. Check your answers with a calculator.


Sum means add.
Difference means subtract.
Product means multiply.
Quotient means divide.

a Triple the sum of 3 and 6.
b Double the quotient of 20 and 4.
c The quotient of 44 and 11 plus 4.
d 5 more than the product of 6 and 12.
e The quotient of 60 and 12 is subtracted from the product of 5 and 7.
f 15 less than the difference of 48 and 12.
g The product of 9 and 12 is subtracted from double the product of 10 and 15.

7 A delivery of 15 boxes of books arrives. Each box contains eight books. The bookstore owner removes three books from each box. How many books still remain in total?




Hint: Draw a diagram, then write the number sentence.

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8 In a class, eight students have three TV screens at home, four have two TV screens, 13 have one TV screen and two students have no TV screens. How many TV screens are there in total?





9 Insert brackets into these statements to make them true. Check your answers with a calculator.

a $4 + 2 \times 3 = 18$ **b** $9 \div 12 - 9 = 3$ **c** $2 \times 3 + 4 - 5 = 9$

d $3 + 2 \times 7 - 3 = 20$ **e** $10 - 7 \div 21 - 18 = 1$ **f** $4 + 10 \div 21 \div 3 = 2$

10 Use a calculator to decide if the brackets given in each statement are actually necessary.

a $2 + (3 \times 6) = 20$ **b** $(2 + 3) \times 6 = 30$ **c** $(20 \times 2) \times 3 = 120$

d $10 - (5 + 2) = 3$ **e** $22 - (11 - 7) = 18$ **f** $19 - (10 \div 2) = 14$

Enrichment: Brackets within brackets

11 These computations involve brackets within brackets. Make sure you work with the inner brackets first. (The first one has already been done for you.) Check your answers with a calculator.

a $2 \times [(2 + 3) \times 6 - 1]$

$= 2 \times [5 \times 6 - 1]$

$= 2 \times [30 - 1]$

$= 2 \times 29$

$= 58$

b $[10 \div (2 + 3) + 1] \times 6$

c $26 \div [10 - (17 - 9)]$

d $[6 - (5 - 3)] \times 7$

e $2 + [103 - (21 + 52)] - (9 + 11) \times 6 \div 12$

12 Insert brackets to make the following true. (You may need to use more than one pair.)

a $20 - 31 - 19 \times 2 = 16$

b $50 \div 2 \times 5 - 4 = 1$

c $25 - 19 \times 3 + 7 \div 12 + 1 = 6$

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Puzzles and games

1 Complete these magic squares. Each row, column and main diagonal add up to the same magic total.

a

15		
	16	18
		17

b

		9
	12	14
		13

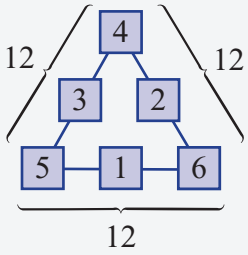
2 Decide where brackets should go to make each statement true.

a $5 + 2 \times 3 = 21$

b $16 - 8 \div 10 - 6 = 2$

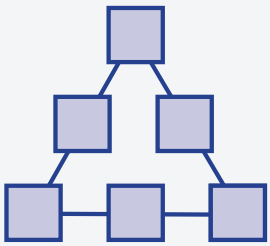
c $4 + 2 \times 7 - 1 \times 3 = 43$

3 Each side on a magic triangle adds up to the same number, as shown in this example with a sum of 12 on each side.

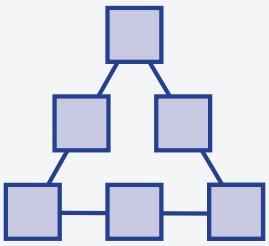


a Place the digits 1 to 6 in a magic triangle with three digits along each side so that each side adds up to the given number.

i 9

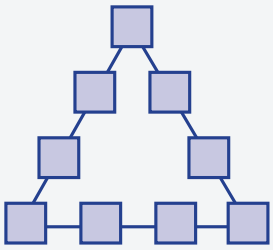


ii 10

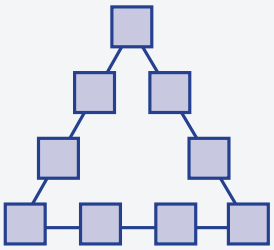


b Place the digits 1 to 9 in a magic triangle with four digits along each side so that each side adds up to the given number.

i 20



ii 23



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4 Sudoku is a popular logic number puzzle. These sudoku puzzles are made up of a 4 by 4 square, where each column and row can use the digits 1, 2, 3 and 4 only once. Also, each digit is to be used only once in each 2 by 2 square. Solve these puzzles.

Starting out **Getting better** **Superstar**

a

		2	1
2		4	3
3			2
	2		

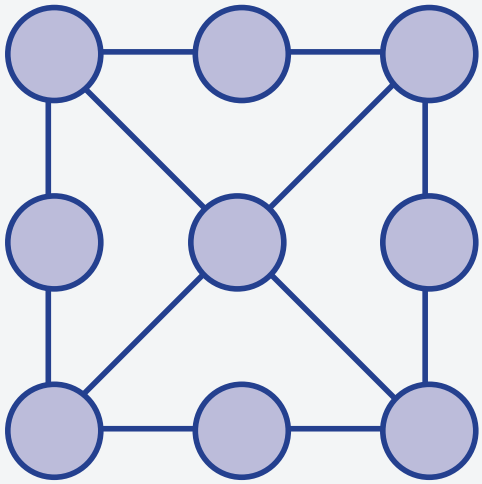
b

			2
2	3		
3	2		4
	4	2	3

c

	3	2	
4			1
2	4		

5 The sum along each line is 15. Can you place each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make this true?



6 Find all the missing digits in these products.

a
$$\begin{array}{r} \square 1 \square \\ \times \quad 7 \\ \hline \square 5 1 \square \end{array}$$

b
$$\begin{array}{r} 2\,9\,\square \\ \times \quad 3 \\ \hline 8 \square \square \end{array}$$

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Chapter summary

Computation with positive integers

Place value

The value of the 3 in 1327 is 300.
The value of the 4 in 7143 is 40.

Estimation

955 to the nearest 10 is 960.
850 to the nearest 100 is 900.

First digit approximation
$39 \times 326 \approx 40 \times 300 = 12\,000$

Addition and subtraction

Mental strategies

$172 + 216 = 300 + 80 + 8$
$= 388$
$98 - 19 = 98 - 20 + 1$
$= 79$

Larger numbers

$$\begin{array}{r} {}^{1}371 \\ +\,843 \\ \hline 1214 \end{array} \qquad \begin{array}{r} {}^{8}\not{9}{}^{1}37 \\ -\,643 \\ \hline 294 \end{array}$$

Terminology

Addition (+)	Subtraction (–)
sum	difference
total	less than
more than	take away
and	minus
plus	reduce
altogether	decrease
increase	subtract
add	

Order of operations

Brackets first
then
× and ÷
(from left to right)
then
+ and –
from (left to right)

$2 \times (7 + 1) = 2 \times 8$
$= 16$
$8 - 10 \div 2 = 8 - 5$
$= 3$
$2 + 3 \times 4 \div (9 \div 3) = 2 + 12 \div 3$
$= 2 + 4$
$= 6$

Multiplication and division

Mental strategies

$7 \times 31 = 7 \times 30 + 7 \times 1 = 217$
$5 \times 14 = 10 \times 7 = 70$
$64 \div 8 = 32 \div 4 = 16 \div 2 = 8$
$156 \div 4 = 160 \div 4 - 4 \div 4$
$= 40 - 1$
$= 39$

Larger numbers

$$\begin{array}{r} {}^{2}29 \\ \times\,13 \\ \hline 87 \\ 290 \\ \hline 377 \end{array} \qquad 3\overline{)205} \rightarrow 68 \text{ with 1 remainder}$$

$205 \div 3 = 68\frac{1}{3}$

Multiplying by 10, 100, …

$38 \times 100 = 3800$
$38 \times 700 = 38 \times 7 \times 100$
$= 26\,600$

Terminology

Multiplication (×)	Division (÷)
product	quotient
times	share
lots of	divide
multiples	
multiply	

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 Which of the following is *not* true?
A $2 < 3$ **B** $12 \leq 9$ **C** $15 > 2$
D $13 \geq 13$ **E** $7 \neq 8$

2 The place value of 7 in 2713 is:
A 7 **B** 70 **C** 700
D 7000 **E** 100

3 Which of the following is *not* true?
A $2 + 3 = 3 + 2$ **B** $2 \times 3 = 3 \times 2$
C $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ **D** $5 \div 2 \neq 2 \div 5$
E $7 - 2 = 2 - 7$

4 The sum of 198 and 103 is:
A 301 **B** 304 **C** 299
D 199 **E** 95

5 The difference between 126 and 29 is:
A 102 **B** 97 **C** 103
D 98 **E** 99

6 The product of 7 and 21 is:
A 147 **B** 141 **C** 21
D 140 **E** 207

7 The missing digit in this division is: $3\overline{)4^{1}1^{2}1}$ $\square\,3\,7$
A 2 **B** 0 **C** 4
D 1 **E** 3

8 The remainder when 317 is divided by 9 is:
A 7 **B** 5 **C** 2
D 1 **E** 0

9 458 rounded to the nearest 100 is:
A 400 **B** 500 **C** 460
D 450 **E** 1000

10 The value of $4 \times 3 - 26 \div 13$ is:
A 10 **B** 25 **C** 6
D 12 **E** 14

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Chapter review

Short-answer questions

1 Arrange these numbers from smallest to largest.
a 317, 713, 731, 371, 173, 137
b 1001, 1010, 199, 999, 1000, 1900, 1090

2 Write down the place value of the digit 5 in these numbers.
a 357 **b** 5249 **c** 356 612

3 Use a mental strategy to find these sums and differences.
a $124 + 335$ **b** $687 - 324$
c $59 + 36$ **d** $256 - 39$

4 Find these sums and differences.

a
$$\begin{array}{r} 76 \\ +52 \\ \hline \end{array}$$

b
$$\begin{array}{r} 137 \\ +218 \\ \hline \end{array}$$

c
$$\begin{array}{r} 329 \\ -138 \\ \hline \end{array}$$

d
$$\begin{array}{r} 926 \\ -187 \\ \hline \end{array}$$

5 Use a mental strategy to perform these computations.
a 5×19 **b** 22×6 **c** 5×44
d $123 \div 3$ **e** $264 \div 8$ **f** $96 \div 4$
g 29×1000 **h** 36×300 **i** $14\,678 \div 1$

6 Show your working to find each answer.

a
$$\begin{array}{r} 39 \\ \times\ 4 \\ \hline \end{array}$$

b
$$\begin{array}{r} 21 \\ \times 40 \\ \hline \end{array}$$

c
$$\begin{array}{r} 157 \\ \times\ 9 \\ \hline \end{array}$$

d
$$\begin{array}{r} 27 \\ \times 13 \\ \hline \end{array}$$

e $3\overline{)135}$ **f** $9\overline{)912}$ **g** $7\overline{)327}$ **h** $4\overline{)30\,162}$

7 Find the missing digits in these computations.

a
$$\begin{array}{r} 2\square 3 \\ +7\ 3\square \\ \hline 9\ 6\ 1 \end{array}$$

b
$$\begin{array}{r} \square\ 2\square \\ -\ 4\ \square\ 3 \\ \hline 2\ 5\ 6 \end{array}$$

c
$$\begin{array}{r} \square 1 \\ \times\ \ 7 \\ \hline 2\ 8\ \square \end{array}$$

d $\begin{array}{r} 4\ \square\ 8 \\ 2\overline{)\square^{1}1^{1}\square} \end{array}$ with no remainder

8 Round these numbers as indicated.
a 72 (nearest 10) **b** 3268 (nearest 100) **c** 951 (nearest 100)

9 Use *first digit approximation* to estimate the answers to these computations.
a $289 + 532$ **b** 22×19 **c** 452×11 **d** $99 \div 11$

10 Use *order of operations* to find the value of these computations.
a $3 \times (2 + 6)$ **b** $6 - 8 \div 4$ **c** $(7 - 4) \div 3$
d $20 \div 10 + 9 \times 10$ **e** $2 \times 8 - 12 \div 6$ **f** $40 \div (5 + 3) - 2$
g $(5 + 2) \times 3 - (8 - 7)$ **h** $0 \times (988\,234 \div 3)$ **i** $1 \times (3 + 2 \times 5)$

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Extended-response questions

1 A city tower construction uses 450 tonnes of concrete. Each mixer can carry 5 tonnes of concrete. The concrete costs $350 per truck load for the first 10 loads and $300 per load after that.



a How many loads of concrete are needed?
b Find the total cost of concrete needed for the tower construction.
c If the price of concrete was always $350 regardless of the number of loads, how much more would it cost for the concrete?

2 Ricky and her brother Micky went into a sweets shop. In the shop they collected 3 tins of 25 jelly beans, 4 packets of 32 choc buds, 5 boxes of 10 smarties and 12 packets of 5 liquorice sticks.



a Find the total number of sweets.
b Find the difference between the number of choc buds and the number of smarties.
c Ricky and Micky decide to divide each type of sweet into groups of 7 and then eat any remainder. Which type of sweet will they eat the most of and how many?

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Chapter 2

Angle relationships

What you will learn

2A Points, lines, intervals and angles
2B Measuring and classifying angles
2C Adjacent angles and vertically opposite angles
2D Transversal lines and parallel lines

Strand: Measurement and Geometry

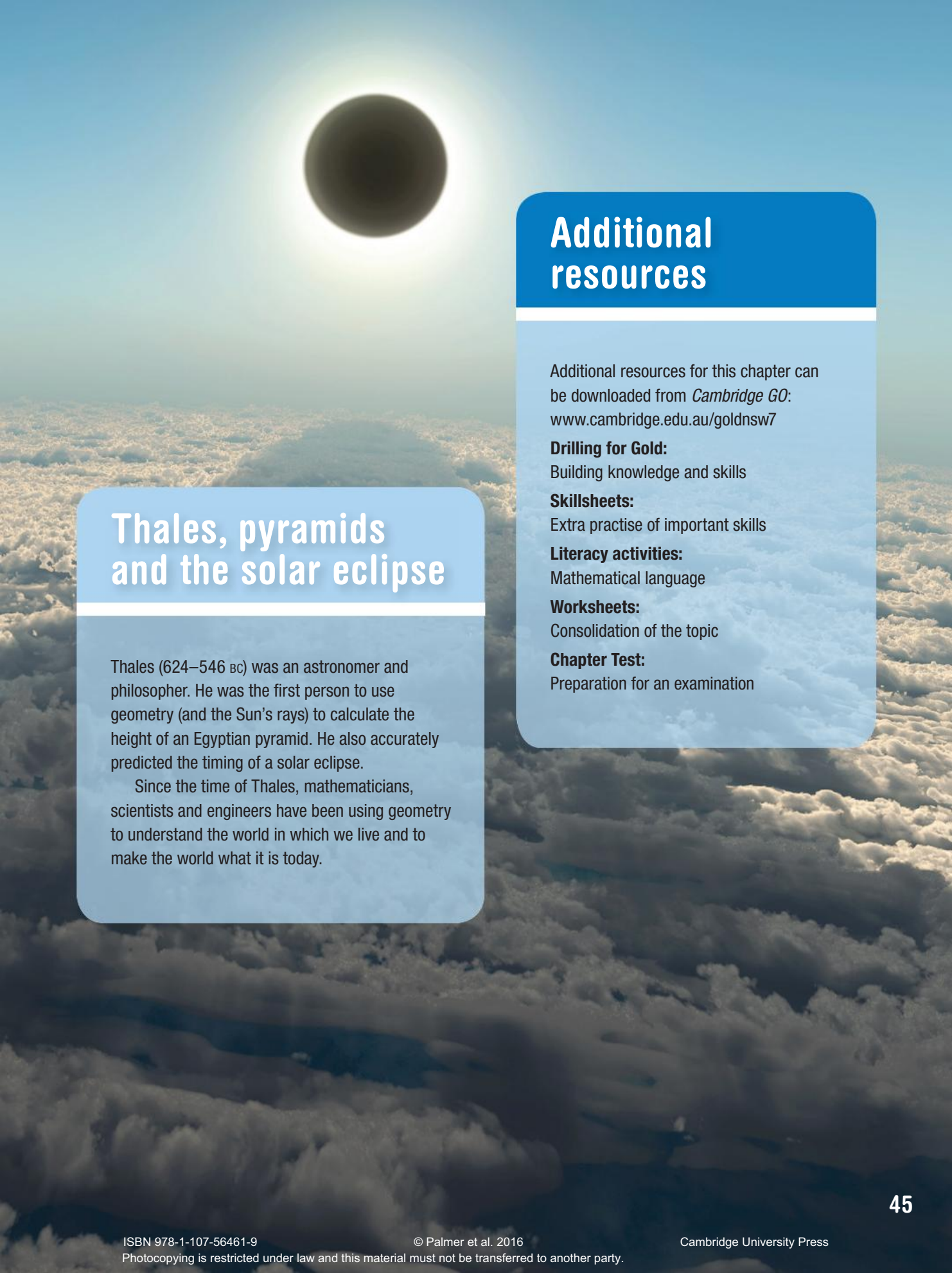
Substrand: ANGLE RELATIONSHIPS

In this chapter, you will learn to:

- identify and use angle relationships, including those relating to transversals on sets of parallel lines.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Thales, pyramids and the solar eclipse

Thales (624–546 BC) was an astronomer and philosopher. He was the first person to use geometry (and the Sun's rays) to calculate the height of an Egyptian pyramid. He also accurately predicted the timing of a solar eclipse.

Since the time of Thales, mathematicians, scientists and engineers have been using geometry to understand the world in which we live and to make the world what it is today.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

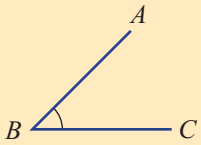
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Pre-test

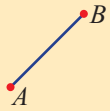
1 Match the description (**a** to **f**) to the object (**A** to **F**).

a a point
b a line
c a circle
d a segment AB (or interval AB)
e an angle ABC
f an angle DEF

A



B



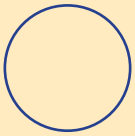
C



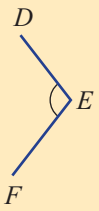
D



E

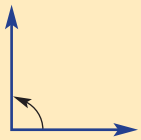


F

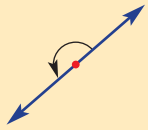


2 Estimate the size of these angles. Remember there are 360° in a full circle.

a



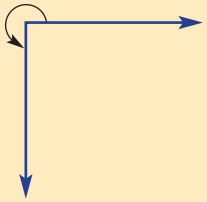
b



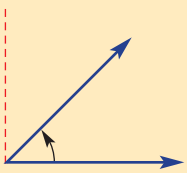
c



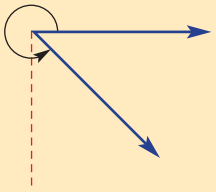
d



e

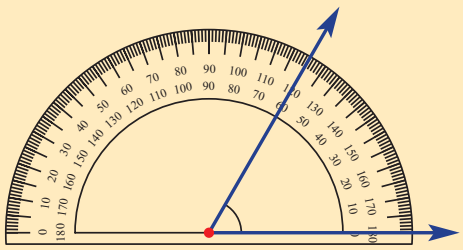


f

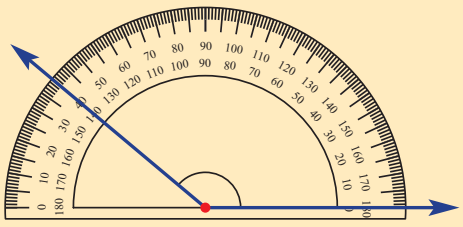


3 What angle measurements are shown on these protractors?

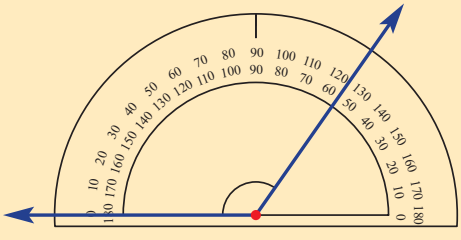
a



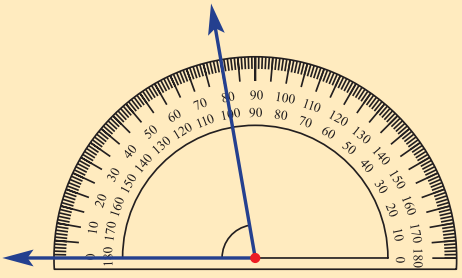
b



c



d



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2A Points, lines, intervals and angles

The basic building blocks of geometry are the point, line and plane. They are the objects used to construct angles, triangles and other more complex shapes and objects.



The Sun emits light in rays.

Let's start: Can you see these in your school?

Before you start learning about the geometrical objects in this topic, look around your school for objects that contain:

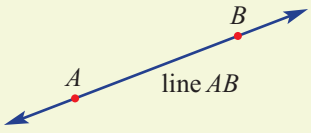
- a right angle
- an acute angle
- an obtuse angle
- lines that intersect

Key ideas

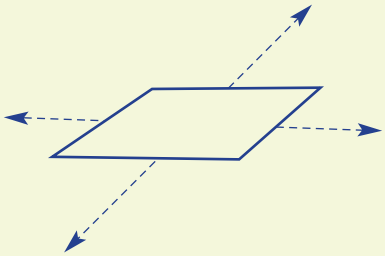
- A **point** is usually labelled with a capital letter.



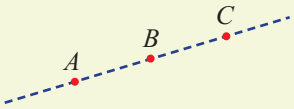
- A **line** can be named using two points.



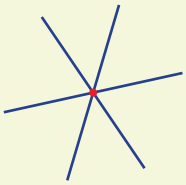
- A **plane** is a flat surface and extends indefinitely.



- Points that all lie on a single line are **collinear**.



- More than two lines that meet at the same intersection point are **concurrent**.



Point A position in space, marked with a dot and named with a capital letter

Line A set of points forming a straight path that extends forever in opposite directions

Plane A flat surface that extends forever in all directions

Collinear Points on the same straight line

Concurrent Lines that share exactly one point

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Segment or **Interval** A section of a line with two end points

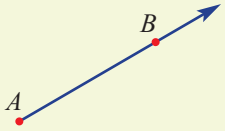
Ray A section of a line with one end point

Vertex A point from which two rays or segments extend in different directions

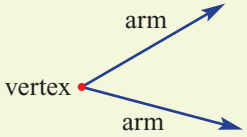
- A line **segment** (or **interval**) is part of a line with a fixed length and end points.



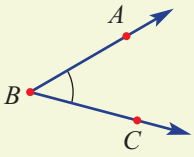
- A **ray** AB is part of a line, with an end point A and passing through point B.



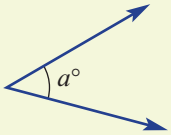
- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called the arms of the angle.



- An angle is named using three points, with the vertex as the middle point.



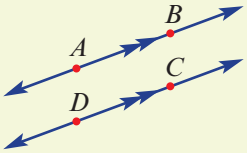
or



$\angle ABC$ or $\angle CBA$

Lower-case letters are often used to represent the number of degrees in an unknown angle.

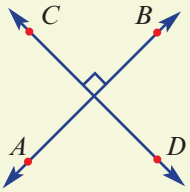
- These two lines are parallel. This is written $AB \parallel DC$.



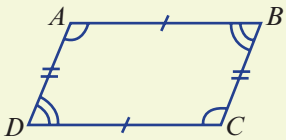


Drilling for Gold 2A1

- These two lines are perpendicular. This is written $AB \perp CD$.



- The markings on this diagram show that $AB = CD$, $AD = BC$, $\angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$.



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Exercise 2A

Understanding

1 Match the description (**a** to **f**) to the object (**A** to **F**).

a ray **b** point **c** line segment
d line **e** angle **f** plane

Look back at the Key ideas if required.

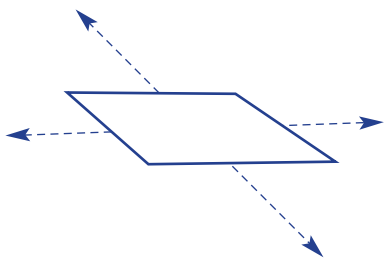


A

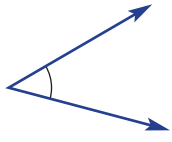


B

C



D



E



F



2 Draw the following objects.

a a point P **b** a line AN **c** an angle $\angle ABC$
d a ray ST **e** a plane **f** a line segment ST

3 Match the words *line*, *segment* or *ray* to the correct description.

a Starts from a point and extends forever in one direction.
b Extends forever in both directions, passing through two points.
c Starts and ends at two points.

Fluency

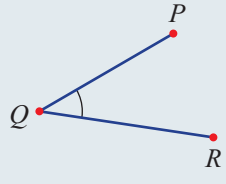
Example 1 Naming objects

Name this line segment and angle, using the given letters.

a



b



Solution	Explanation
a segment AB	Segment BA, interval AB or interval BA are also acceptable.
b $\angle PQR$ or $\angle RQP$	Point Q is the vertex and sits in between P and R. Either order ($\angle PQR$ or $\angle RQP$) is correct.

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2A

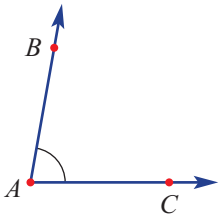
4 Name the following objects, using the given letters.

a $\bullet T$

b



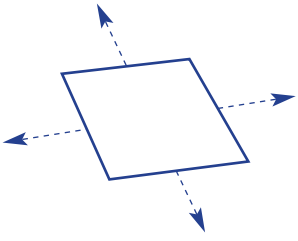
c



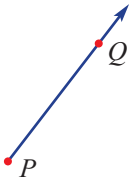


For parts **b**, **c**, **e** and **f**, you can reverse the letters and still be correct.

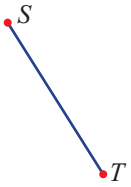
d



e

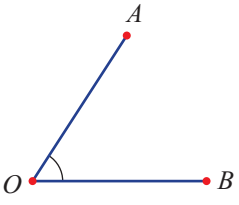


f

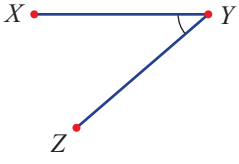


5 Use three letters to name the angle marked in these diagrams.

a



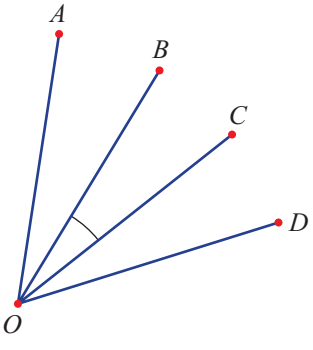
b



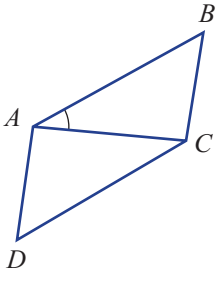


Name the angle that is marked by a small arc: ∠

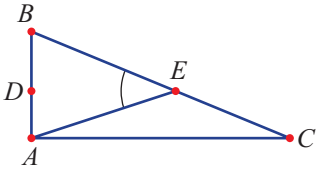
c



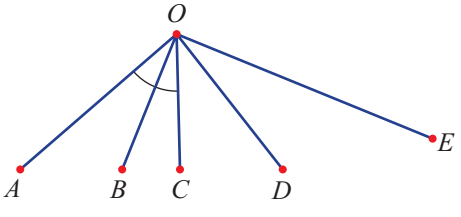
d



e



f



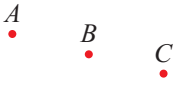
6 Write the missing word.

a Points that sit in a straight line are called _____.

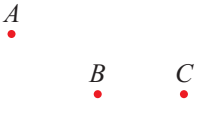
b Lines that meet at the same point are called _____.

7 Do the following sets of points look collinear?

a



b



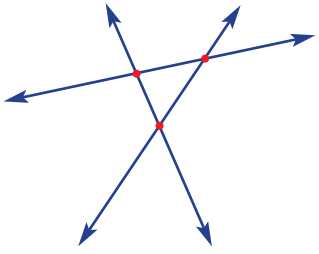


Collinear points lie in a straight line.

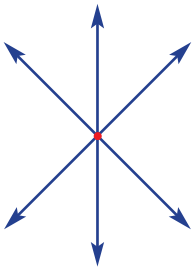
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8 Are the following sets of lines concurrent?

a



b

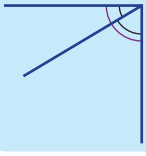


Problem-solving and Reasoning

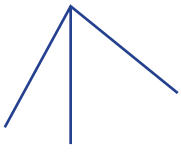
9 Count the number of angles formed inside these shapes. Count all angles, including those that may be the same size. You should also count those that are divided by another segment.



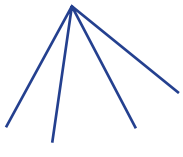
There are three angles in a corner like this:



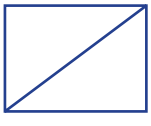
a



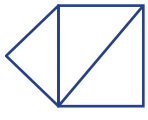
b



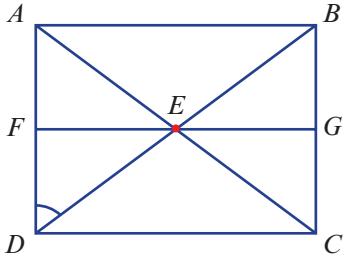
c



d

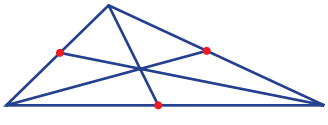


10 Write down six different names for the marked angle. For example, $\angle ADB$.



Enrichment: Are they concurrent?

11 The lines joining each vertex (corner) of a triangle with the midpoint (middle point) of the opposite side are drawn here.



a Draw any triangle and use a ruler to measure and mark the midpoints of each side.

b Join each vertex with the midpoint of the opposite side.

c Are your segments from part **b** concurrent?

d Do you think your answer to part **c** will always be true for any triangle? Try one other triangle of a different size to check.

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2B Measuring and classifying angles

An angle is a measure of rotation. The amount of rotation is measured using a protractor. The unit of measurement is degrees.



On this Ferris wheel, what angle is between each spoke and the next?

Let's start: Estimating angles

How good are you at estimating the size of angles? Estimate the size of these angles and then check with a protractor.

If your protractor only goes to 180°, discuss how you might measure the second angle.



Key ideas

- Angles are classified according to their size.



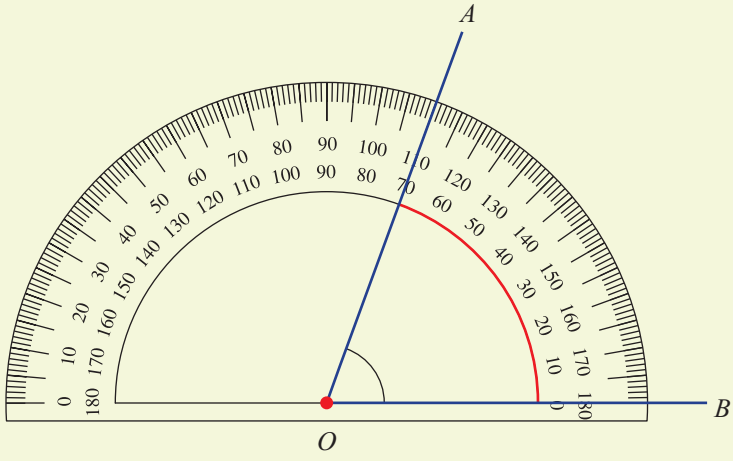
Drilling for Gold 2B1

Angle type	Size	Examples
acute angle	between 0° and 90°	
right angle	90°	
obtuse angle	between 90° and 180°	
straight angle	180°	
reflex angle	between 180° and 360°	
revolution	360°	

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- A **protractor** can be used to measure angles.
  To use a protractor:
  - Place the centre of the protractor on the vertex of the angle.
  - Align the base line of the protractor along one arm of the angle.
  - Measure the angle using the other arm and the scale on the protractor.
  - Be sure to use the scale that starts from 0° on the base line of the protractor. The angle shown in the diagram below is 70°, not 110°.

Protractor
A semicircular or circular tool for measuring or drawing angles



  - A reflex angle can be measured by subtracting a measured angle from 360°.

Exercise 2B

Understanding

Example 2 Classifying angles

Classify the following angles as acute, obtuse, straight or reflex.

a 47° **b** 172° **c** 180° **d** 282°

Solution	Explanation
a acute	Angles between 0° and 90° are acute.
b obtuse	Obtuse angles are between 90° and 180°.
c straight	A straight angle is 180°.
d reflex	A reflex angle is between 180° and 360°.

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2B

Skillsheet 2A

1 Classify the following angles as acute, right, obtuse, straight, reflex or revolution.

a $31°$	**b** $127°$	**c** $90°$
d $180°$	**e** $360°$	**f** $83°$
g $291°$	**h** $320°$	**i** $93°$



acute: $0°$ to $90°$
right: $90°$
obtuse: $90°$ to $180°$
straight: $180°$
reflex: $180°$ to $360°$
revolution: $360°$

2 Without using a protractor, draw an example of the following types of angles.

a acute	**b** right	**c** obtuse
d straight	**e** reflex	**f** revolution

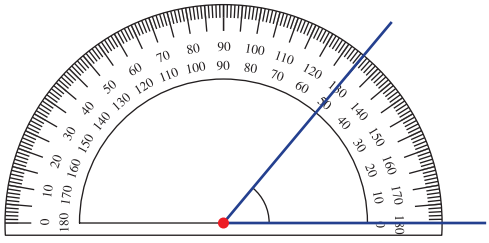
Drilling for Gold 2B2

3 What is the size of the angle measured with these protractors?

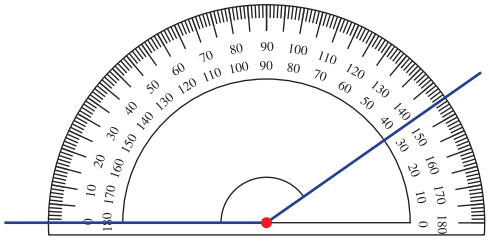


Carefully align your protractor with one arm of the given angle. Place the centre of the protractor on the vertex of the angle. Be sure to start from $0°$!

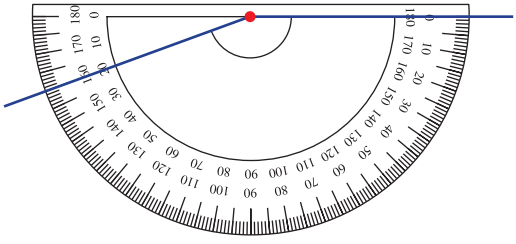
a



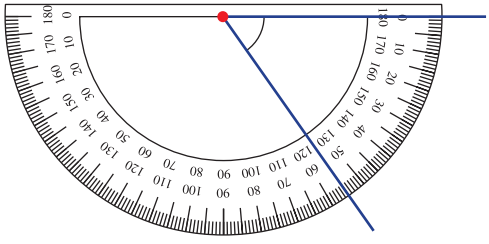
b



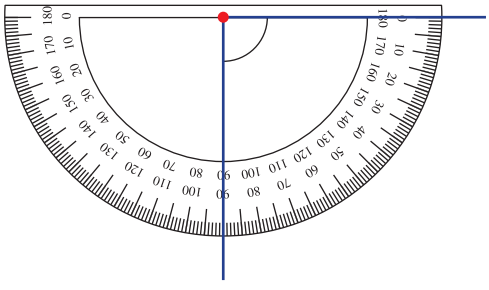
c



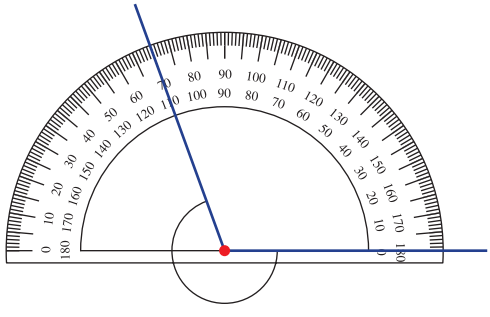
d



e



f



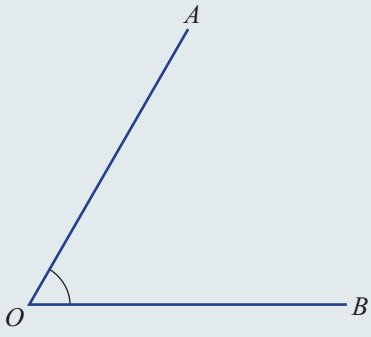
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Fluency

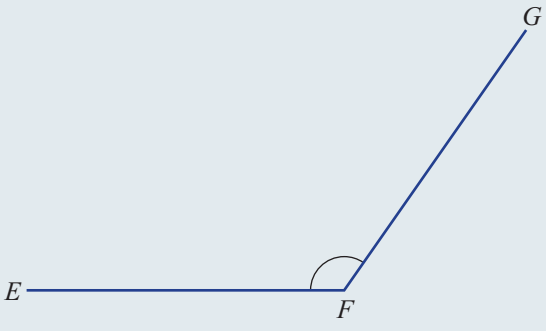
Example 3 Measuring with a protractor

Measure the size of each angle.

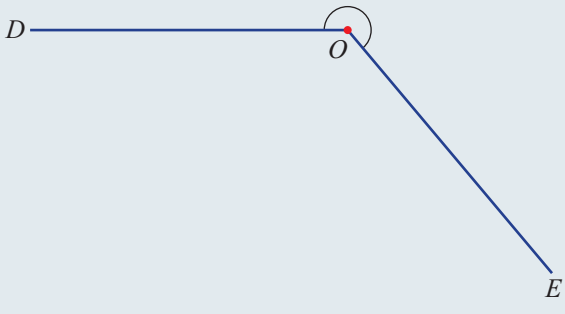
a



b



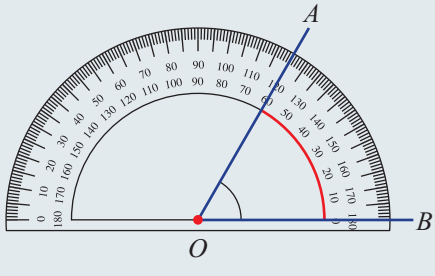
c



Solution

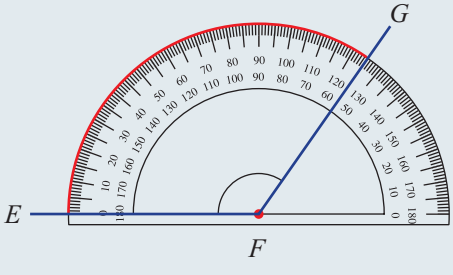
a $\angle AOB = 60°$

Explanation



Start from $0°$ on the inner scale.

b $\angle EFG = 125°$



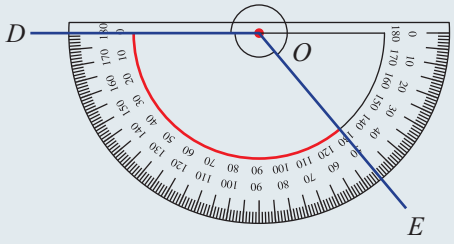
Start from $0°$ on the outer scale.

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2B

c obtuse $\angle DOE = 130°$

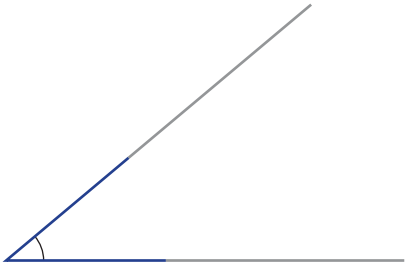
reflex $\angle DOE = 360° - 130°$
$= 230°$



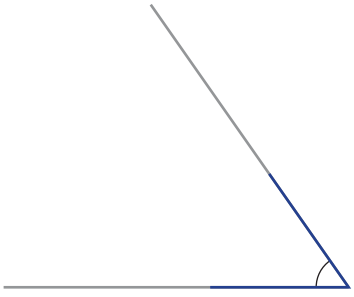
Start from $0°$ on the inner scale.

4 Use a protractor to measure the size of each angle.

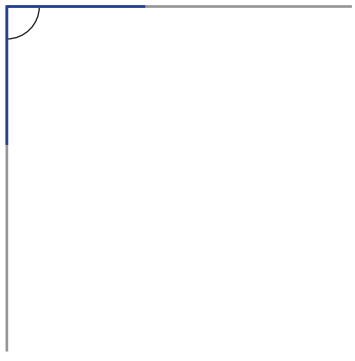
a



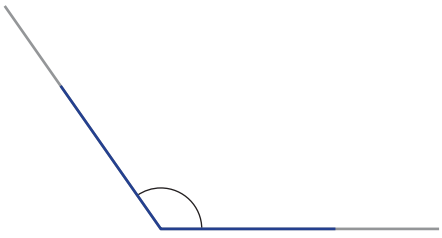
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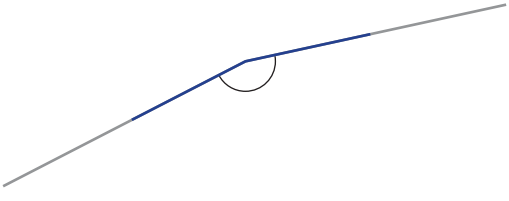
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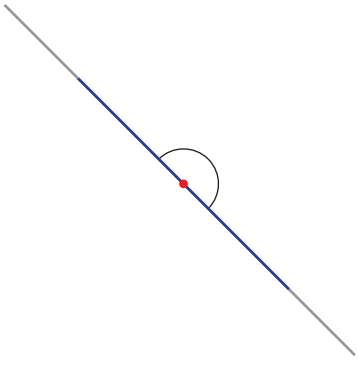
d



e

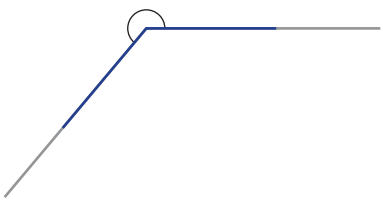


f

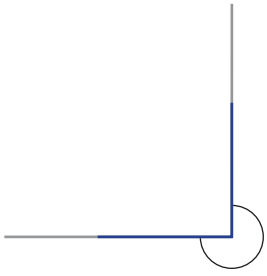


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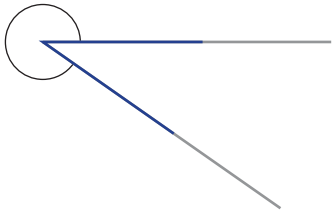
g



h



i



5 Classify each of the angles in Question **4** as acute, right, obtuse, straight, reflex or revolution.

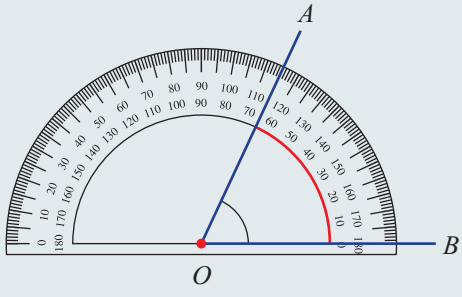
Example 4 Drawing angles

Use a protractor to draw each of the following angles.

a $\angle AOB = 65°$ **b** $\angle WXY = 130°$ **c** $\angle MNO = 260°$

Solution

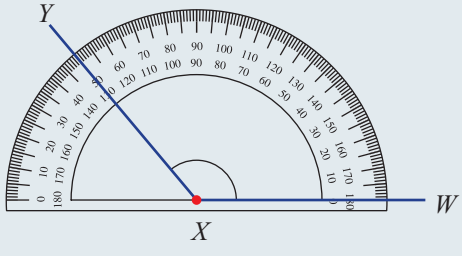
a



Explanation

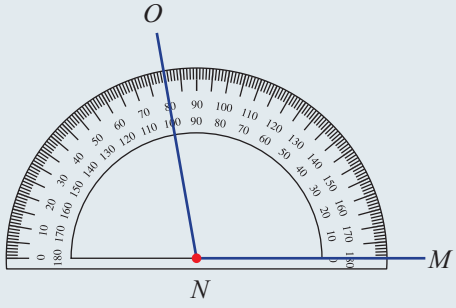
Step 1: Draw a base line OB.
Step 2: Align the protractor along the base line with the centre at point O.
Step 3: Starting from $0°$ on the base line, measure $65°$ and mark a point, A.
Step 4: Draw the arm OA.

b



Step 1: Draw a base line XW.
Step 2: Align the protractor along the base line with the centre at point X.
Step 3: Starting at $0°$ on the base line measure $130°$ and mark a point, Y.
Step 4: Draw the arm XY.

c



Step 1: Draw an angle of $360° - 260° = 100°$.
Step 2: Mark the reflex angle on the opposite side to the obtuse angle of $100°$.
Alternatively, draw a $180°$ angle and measure an $80°$ angle to add to the $180°$ angle.

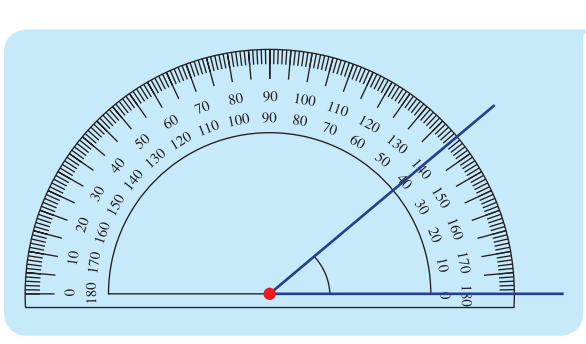
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2B

Skillsheet 2B

6 Use a protractor to draw each of the following angles.

a 40° **b** 75°
c 90° **d** 135°
e 175° **f** 205°
g 260° **h** 270°
i 295° **j** 352°





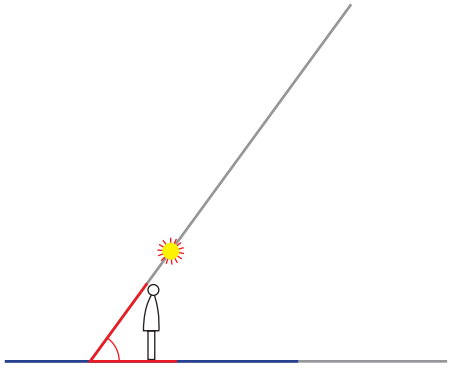
Problem-solving and Reasoning

7 Use a protractor to measure:

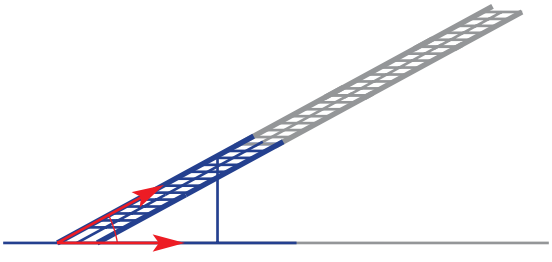
a the angle the Sun's rays make with the ground



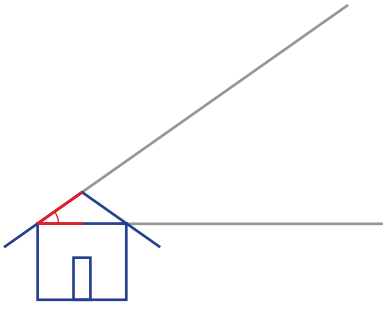
Place the centre of the protractor at the vertex of the marked angle.



b the angle that this ramp makes with the ground

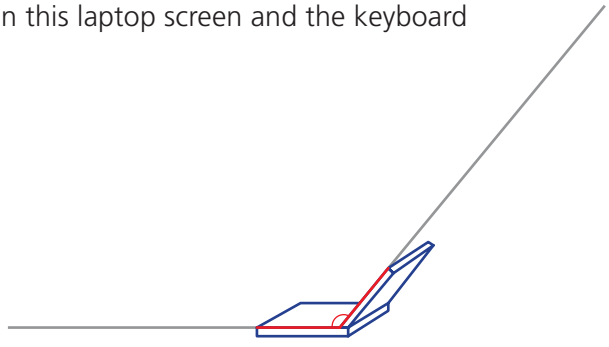


c the angle or pitch of this roof



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d the angle between this laptop screen and the keyboard

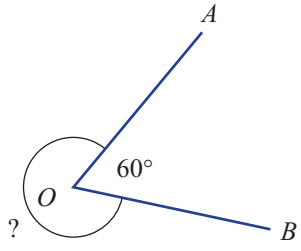


8 How many right angles (i.e. angles of 90°) make up:
a a straight angle **b** 270° **c** a revolution?

9 A clock face is numbered 1 to 12. Find the angle the minute hand turns in:
a 30 minutes **b** 1 hour **c** 15 minutes
d 45 minutes **e** 5 minutes **f** 20 minutes
g 55 minutes **h** 1 minute


There are 360° in a full circle.

10 An acute angle $\angle AOB$ is equal to 60°. Do you need to use a protractor to work out the size of the reflex angle $\angle AOB$? Why not?



11 Without using a protractor, estimate and draw the following angles. (Check your accuracy using a protractor.)
a 45° **b** 135° **c** 75° **d** 160°

Enrichment: Hour-hand and minute-hand angles

12 A clockface is numbered 1 to 12. Find the angle between the hour hand and the minute hand at:
a 6 p.m. **b** 3 p.m.
c 4 p.m. **d** 11 a.m.


Draw a clockface for each time. Each 5-minute turn of the minute hand is 360° ÷ 12 = 30°.

13 Find the angle between the hour hand and the minute hand of a clock at these times.
a 10:10 a.m.
b 4:45 a.m.
c 11:10 p.m.
d 2:25 a.m.




Think carefully! The answer to part **a** is *not* 120°.

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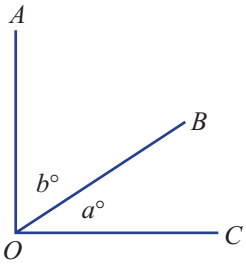
2C Adjacent angles and vertically opposite angles

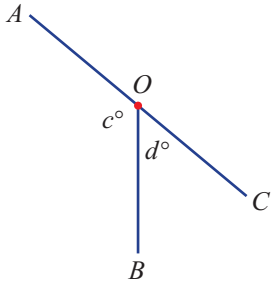


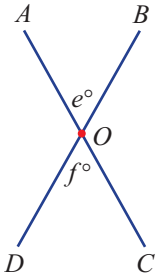
When lines, segments or rays meet at a point, special pairs of angles are sometimes formed.

Let's start: Special pairs of angles

By making a drawing or using computer geometry, construct the diagrams below. Measure the two marked angles. What do you notice about the two marked angles?

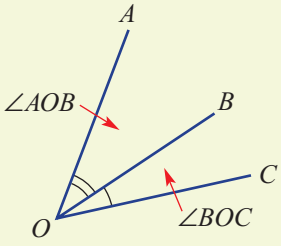




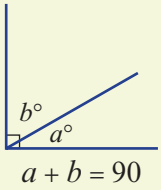


Key ideas

- **Adjacent** angles are side by side and share a vertex and an arm.

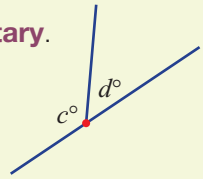


- Two adjacent angles in a right angle are **complementary**.
  They add to 90°.
  If the value of a is 30, then the value of b is 60 because $30° + 60° = 90°$.
  30° is the complement of 60°.



$a + b = 90$

- Two adjacent angles on a straight line are **supplementary**.
  They add to 180°.
  If the value of $c = 130$, then the value of d is 50 because $130° + 50° = 180°$.
  130° is the supplement of 50°.



$c + d = 180$


Drilling for Gold 2C1

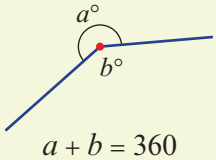
Adjacent Next to each other

Complementary Two angles having a sum of 90°

Supplementary Two angles having a sum of 180°

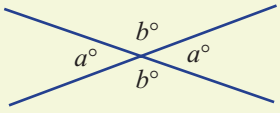
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- Angles in a **revolution** have a sum of 360°.

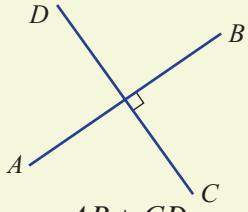


$a + b = 360$

- **Vertically opposite** angles are formed when two lines intersect. The opposite angles are equal.



- **Perpendicular** lines meet at right angles (90°).



$AB \perp CD$

Revolution A full 360 degree turn or rotation

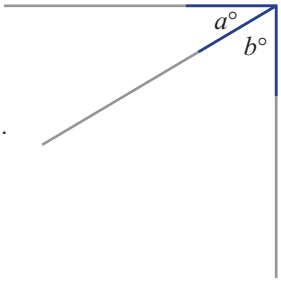
Vertically opposite On either side of a common vertex

Perpendicular Two lines that meet at right angles

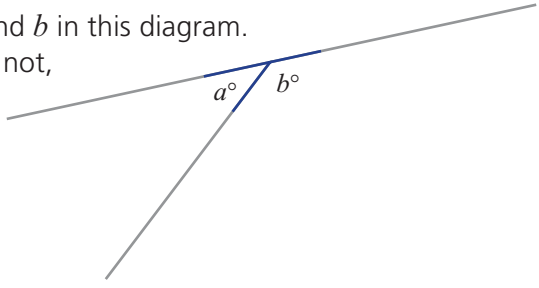
Exercise 2C

Understanding

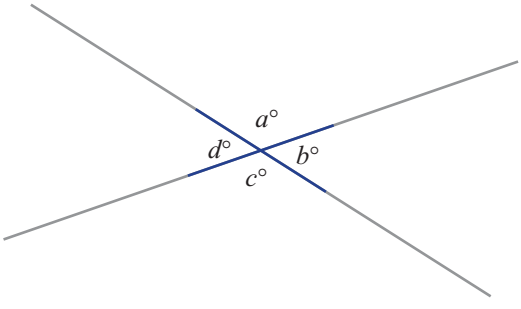
1 **a** Using a protractor give a value for a and b in this diagram.
b Calculate $a + b$. Is your answer 90? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are _____ angles.



2 **a** Using a protractor give a value for a and b in this diagram.
b Calculate $a + b$. Is your answer 180? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are _____ angles.



3 **a** Using a protractor give a value for a, b, c and d in this diagram.
b What do you notice about the sum of the four angles?
c Write the missing words: $b°$ and $d°$ are _____ angles.



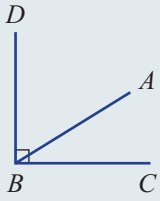
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2C

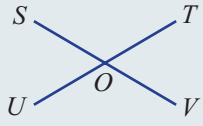
Example 5 Naming a special angle

Name the angle that is:

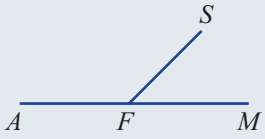
a complementary to $\angle ABC$



b vertically opposite to $\angle SOT$



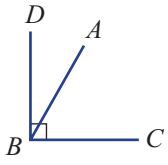
c supplementary to $\angle AFS$



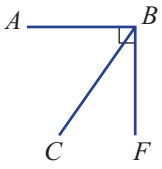
Solution	Explanation
a $\angle ABD$	$\angle ABD$ and $\angle ABC$ make a right angle, so they are complementary.
b $\angle UOV$	$\angle SOT$ is vertically opposite to $\angle UOV$.
c $\angle SFM$	$\angle AFS$ and $\angle SFM$ make a straight line, so they are supplementary.

4 Name the angle that is complementary to $\angle ABC$.

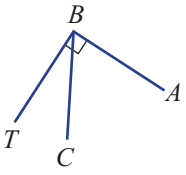
a



b



c

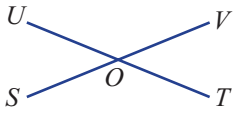




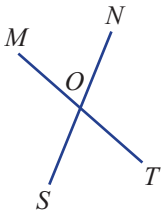
Choose the angle next to $\angle ABC$ that makes up 90°.

5 Name the angle that is vertically opposite to $\angle SOT$.

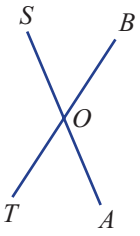
a



b



c

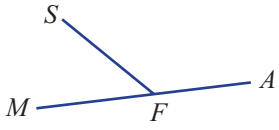




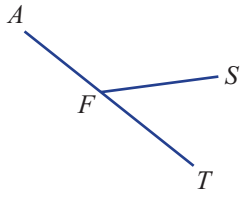
Choose the angle opposite to $\angle SOT$.

6 Name the angle that is supplementary to $\angle AFS$.

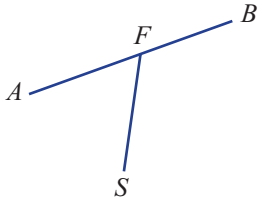
a



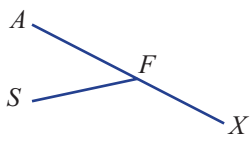
b



c



d





Choose the angle next to $\angle AFS$ that makes up 180°.

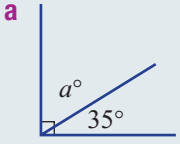
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Fluency

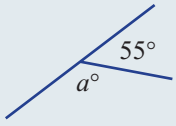
Example 6 Finding angles at a point

Without using a protractor, find the value of a in these diagrams.

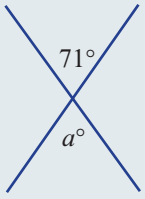
a



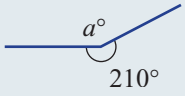
b



c



d



Solution	Explanation
a $a + 35 = 90$ $a = 55$	Angles in a right angle add to 90°. $90 - 35 = 55$
b $a + 55 = 180$ $a = 125$	Angles on a straight line add to 180°. $180 - 55 = 125$
c $a = 71$	Vertically opposite angles are equal.
d $a + 210 = 360$ $a = 150$	The sum of angles in a revolution is 360°. a is the difference between 210 and 360.

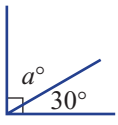
Skillsheet 2C

7 Without using a protractor, find the size of each angle marked with the letter a. (The diagrams shown may not be drawn to scale.)

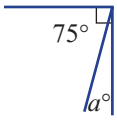


Complementary angles add to 90°. Supplementary angles add to 180°. Vertically opposite angles are equal. Angles in a revolution add to 360°.

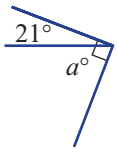
a



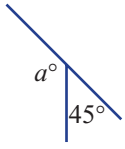
b



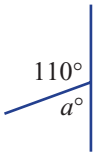
c



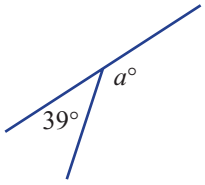
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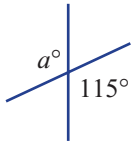
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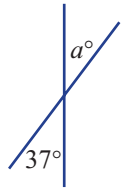
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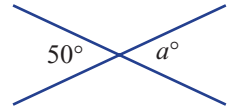
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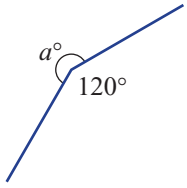
h



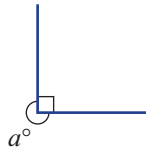
i



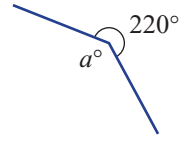
j



k



l



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2C

8 For each of the given pairs of angles, write C if they are complementary (i.e. add to 90°), S if they are supplementary (i.e. add to 180°) or N if they are neither.

a 21°, 79° **b** 130°, 60° **c** 98°, 82° **d** 180°, 90°
e 17°, 73° **f** 31°, 59° **g** 68°, 22° **h** 93°, 87°

9 Write down the complement of:

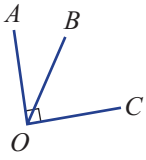
a 10° **b** 40° **c** 70° **d** 5°

10 Write down the supplement of:

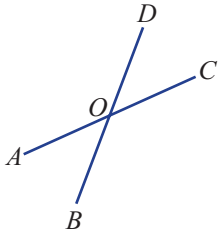
a 10° **b** 40° **c** 70° **d** 130°

Problem-solving and Reasoning

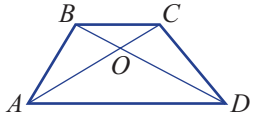
11 a Name the angle that is the complement of $\angle AOB$ in this diagram.



b Name the two angles that are supplementary to $\angle AOB$ in this diagram.

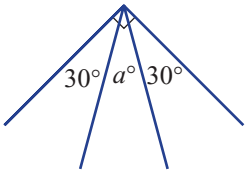


c Name the angle that is vertically opposite to $\angle AOB$ in this diagram.

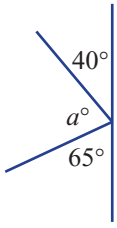


12 Without using a protractor, find the value of a in these diagrams.

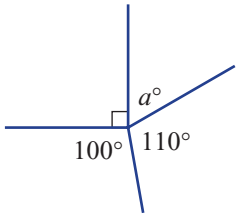
a



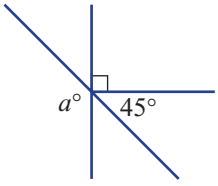
b



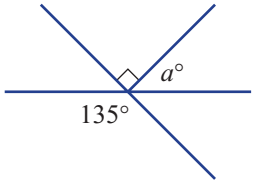
c



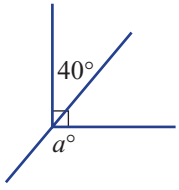
d



e



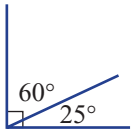
f



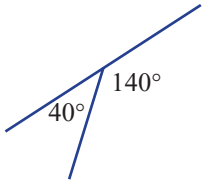
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13 Do these diagrams have the correct information? Give reasons.

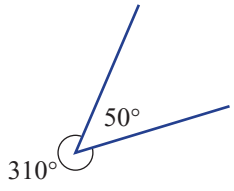
a



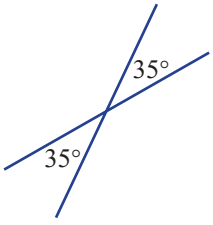
b



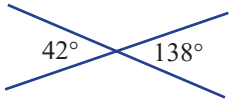
c



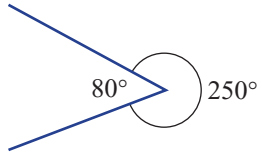
d



e



f



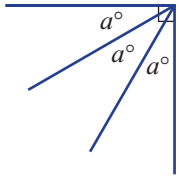
14 A pizza is divided between four people. Bella is to get twice as much as Dom. Dom gets twice as much as Rick. Rick gets twice as much as Marie. Find the angle at the centre of the pizza for Marie's piece. (Hint: Begin by thinking, "Marie will get one piece.")



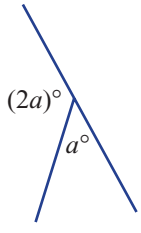
Enrichment: More than one $a°$

15 Find the value of a in these diagrams.

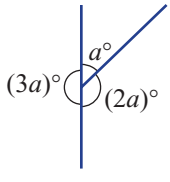
a



b



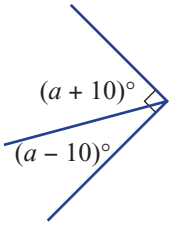
c



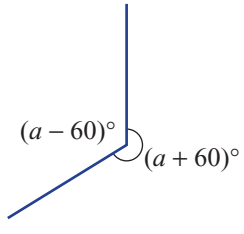


You may need to use 'guess and check'.

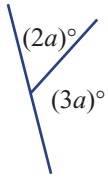
d



e



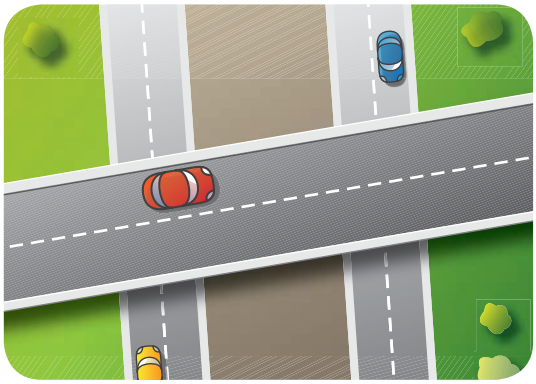
f



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2D Transversal lines and parallel lines

When a line cuts two other lines it is called a transversal and forms eight angles. If the two other lines are parallel, then special pairs of angles are formed.



Multiple angels are formed when this road crosses over parallel lanes of a freeway.

Let's start: What's formed by a transversal?



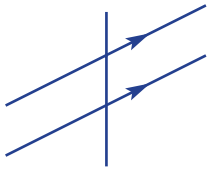
Drilling for Gold 2D1

Draw a pair of parallel lines using either:

- two sides of a ruler; or
- computer geometry (parallel line tool).

Then cross the two lines with a third line (transversal) at any angle.

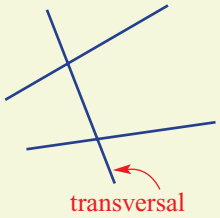
Measure each of the eight angles formed and discuss what you find. If computer geometry is used, drag the transversal and see if your observations apply to all the cases that you observe.



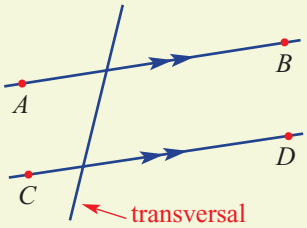
Key ideas

- A **transversal** is a line that intersects two or more lines.

Two **lines** crossed by a transversal.

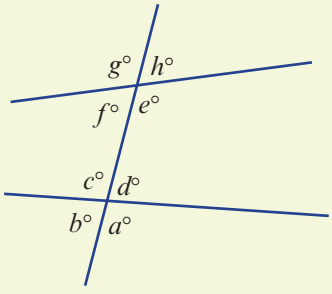


Two **parallel lines** crossed by a transversal.



The arrows on the lines indicate that AB is parallel to CD. This is written $AB \parallel CD$.

- When a transversal crosses two lines eight angles are formed.



Transversal A line that cuts two or more lines

Parallel lines Lines in the same plane that are a fixed distance apart and never intersect

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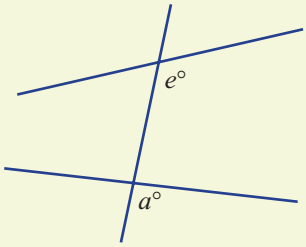
- Some angles can be paired with others and given a name, such as:
  - **Corresponding angles** (matching positions)
  - **Alternate angles** (inside the lines but on opposite sides of the transversal)
  - **Cointerior angles** (inside the lines and on the same side of the transversal)

Corresponding angles
Pairs of angles formed by two lines cut by a transversal

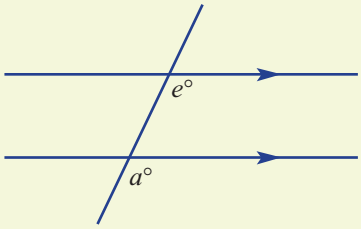
Alternate angles
Two angles that lie between two lines on either side of a transversal

Cointerior angles
A pair of angles lying between two lines on the same side of a transversal

A pair of corresponding angles

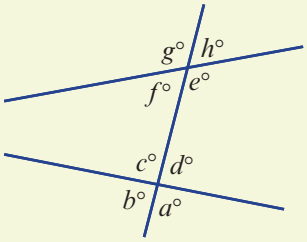


A pair of corresponding angles on parallel lines



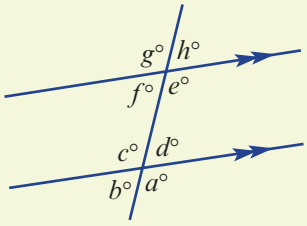
In parallel lines, corresponding angles are equal, so $a = e$.

In the following diagram:



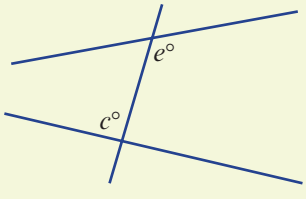
$a°$ and $e°$ are corresponding angles.
$b°$ and $f°$ are corresponding angles.
$c°$ and $g°$ are corresponding angles.
$d°$ and $h°$ are corresponding angles.

In the following diagram:

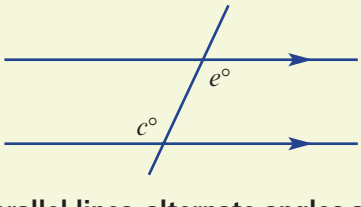


$a°$ and $e°$ are corresponding angles on parallel lines, so $a = e$.
$b°$ and $f°$ are corresponding angles on parallel lines, so $b = f$.
$c°$ and $g°$ are corresponding angles on parallel lines, so $c = g$.
$d°$ and $h°$ are corresponding angles on parallel lines, so $d = h$.

A pair of alternate angles



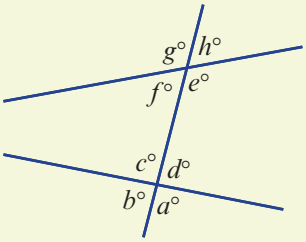
A pair of alternate angles on parallel lines



In parallel lines, alternate angles are equal, so $c = e$.

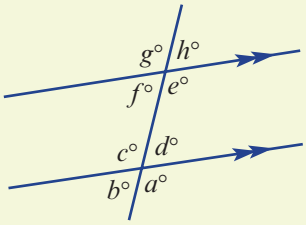
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In the following diagram:



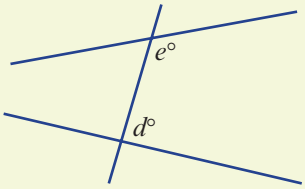
$c°$ and $e°$ are alternate angles.
$d°$ and $f°$ are alternate angles.

In the following diagram:

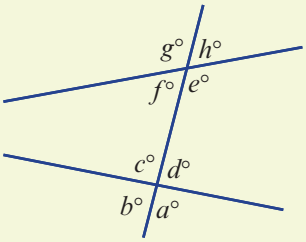


$c°$ and $e°$ are alternate angles on parallel lines, so $c = e$.
$d°$ and $f°$ are alternate angles on parallel lines, so $d = f$.

A pair of cointerior angles

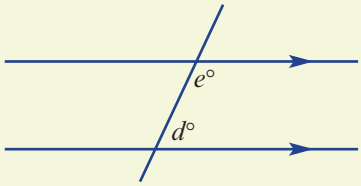


In the following diagram:



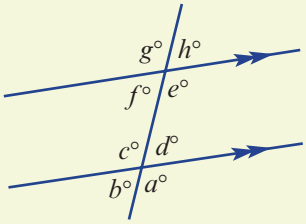
$d°$ and $e°$ are cointerior angles.
$c°$ and $f°$ are cointerior angles.

A pair of cointerior angles on parallel lines



In parallel lines, cointerior angles are supplementary (i.e. add to 180°), so $d + e = 180$.

In the following diagram:

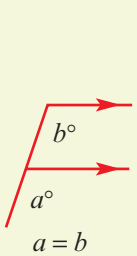


$d°$ and $e°$ are cointerior angles on parallel lines, so $d + e = 180$.
$c°$ and $f°$ are cointerior angles on parallel lines, so $c + f = 180$.

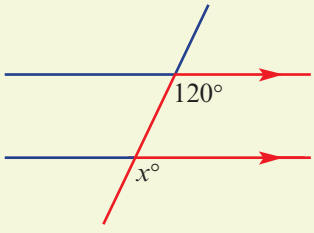
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- In a complex diagram it can be difficult to decide which angle pair may unlock the problem. Some students use the letters F, Z and C, as shown below.
  - To find a pair of corresponding angles, look for the letter F.

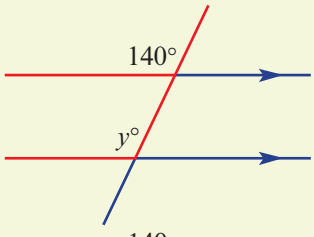




$a = b$

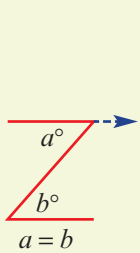


$x = 120$

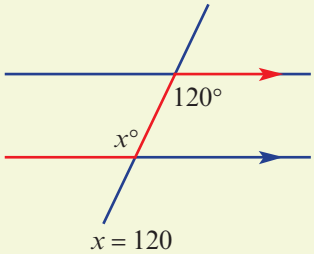


$y = 140$

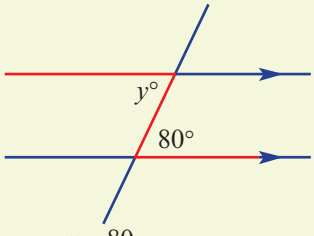
  - To find a pair of alternate angles, look for the letter Z.



$a = b$

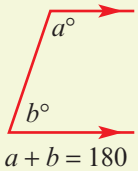


$x = 120$

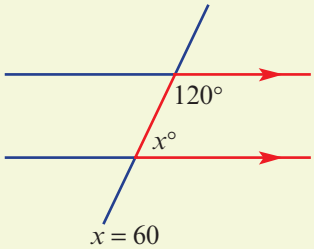


$y = 80$

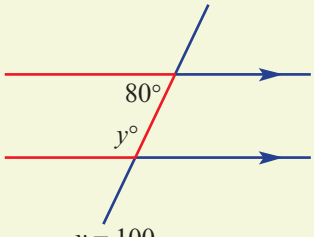
  - To find a pair of cointerior angles, look for the letter C.



$a + b = 180$



$x = 60$



$y = 100$

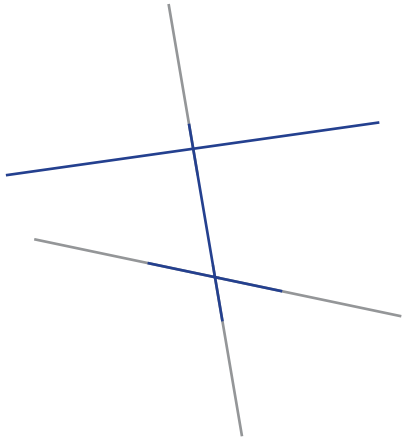
Exercise 2D

Understanding

1 Use a protractor to measure each of the eight angles in this diagram.

a How many *different* angle measurements did you find?

b Do you think that the two lines cut by the transversal are parallel?



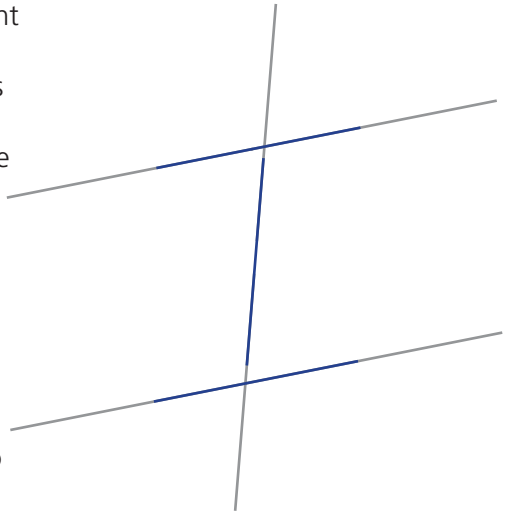
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2D

2 Use a protractor to measure each of the eight angles in this diagram.

a How many *different* angle measurements did you find?

b Do you think that the two lines cut by the transversal are parallel?



3 Choose the word *equal* or *supplementary* to complete these sentences.
When a transversal cuts two parallel lines, then:

a alternate angles are _____.

b cointerior angles are _____.

c corresponding angles are _____.

d vertically opposite angles are _____.



When angles add to 180° they are called supplementary.

Fluency

Example 7 Naming pairs of angles

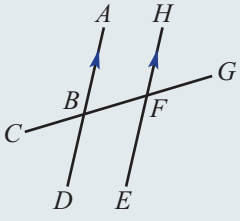
Name the angle that is:

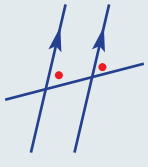
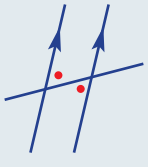
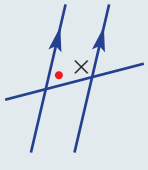
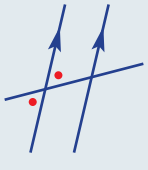
a corresponding to $\angle ABF$

b alternate to $\angle ABF$

c cointerior to $\angle ABF$

d vertically opposite to $\angle ABF$



Solution	Explanation	Solution	Explanation
a $\angle HFG$		**b** $\angle EFB$	
c $\angle HFB$		**d** $\angle CBD$	

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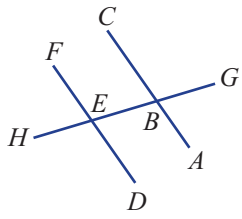
4 Name the angle that is:

a corresponding to $\angle ABE$

b alternate to $\angle ABE$

c cointerior to $\angle ABE$

d vertically opposite to $\angle ABE$



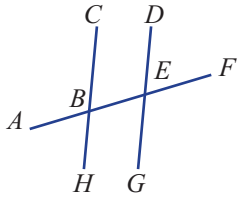
5 Name the angle that is:

a corresponding to $\angle EBH$

b alternate to $\angle EBH$

c cointerior to $\angle EBH$

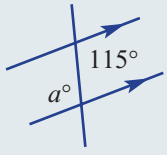
d vertically opposite to $\angle EBH$



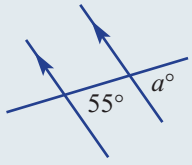
Example 8 Finding angles in parallel lines

Find the value of a in these diagrams and give a reason for each answer.

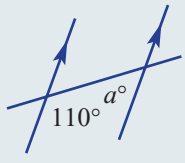
a



b



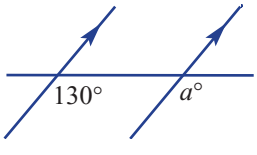
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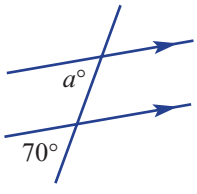
Solution	Explanation
a $a = 115$ (alternate angles in parallel lines)	Alternate angles in parallel lines are equal. (The arrows show that the lines are parallel.)
b $a = 55$ (corresponding angles in parallel lines)	Corresponding angles in parallel lines are equal.
c $a = 180 - 70$ $= 70$ (cointerior angles in parallel lines)	Cointerior angles in parallel lines sum to $180°$.

6 Find the value of a in these diagrams, giving a reason.

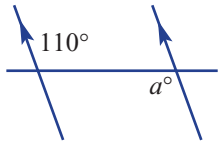
a



b



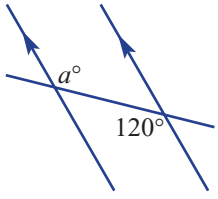
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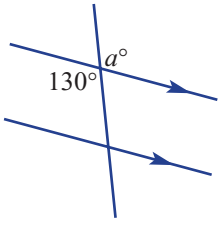
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2D

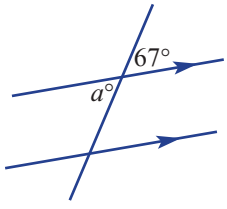
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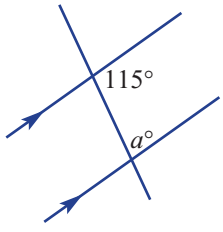
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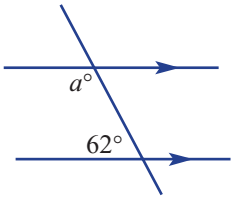
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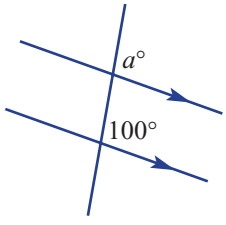
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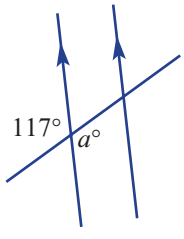
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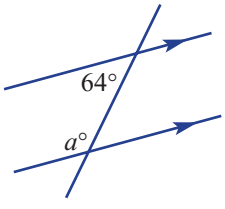
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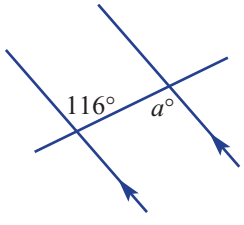
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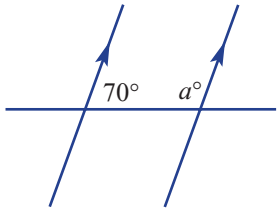
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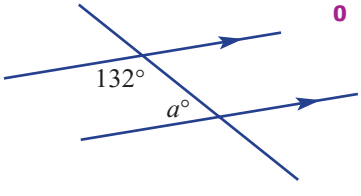
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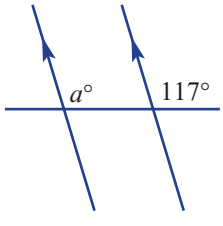
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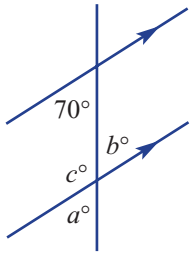


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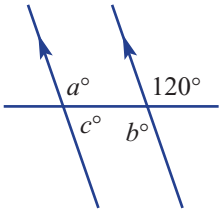


7 Find the value of each unknown pronumeral in the following diagrams.

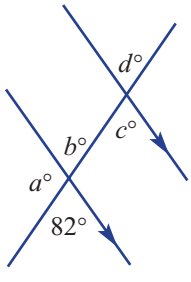
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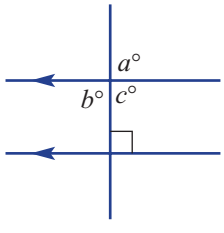
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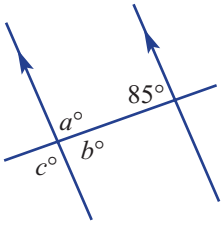
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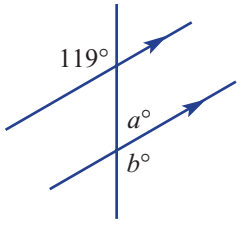
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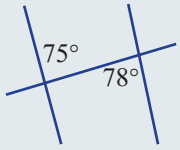


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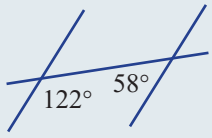
Example 9 Proving lines are parallel

Giving reasons, state whether the two lines cut by the transversal are parallel.

a



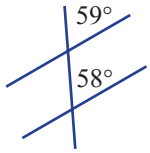
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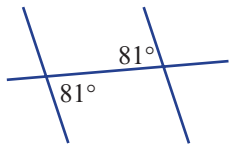
Solution	Explanation
a not parallel Alternate angles are not equal.	Parallel lines have equal alternate angles.
b parallel The cointerior angles sum to 180°.	$122° + 58° = 180°$ Cointerior angles inside parallel lines are supplementary (i.e. sum to 180°).

8 Giving reasons, state whether the two lines cut by the transversal are parallel.

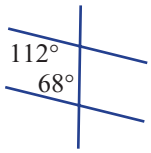
a



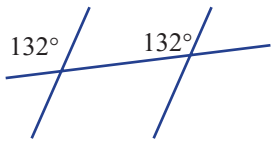
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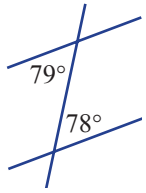
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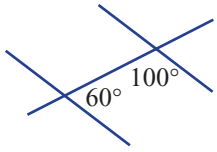
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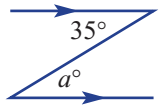


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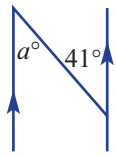


9 Find the value of a in these diagrams.

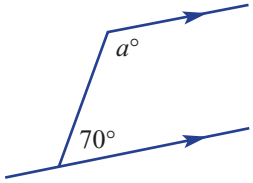
a



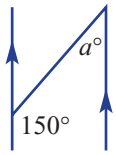
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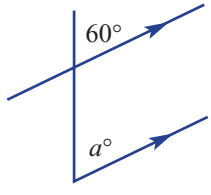
c



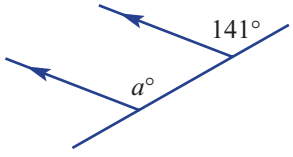
d



e



f

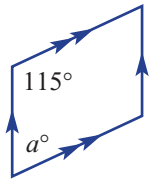


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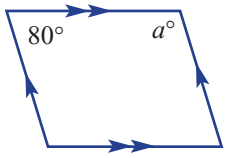
2D

10 Find the value of a in these diagrams.

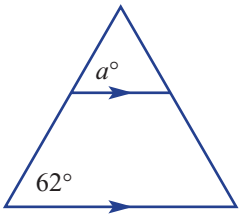
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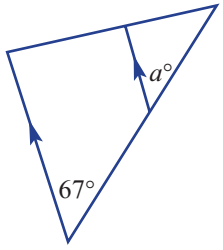
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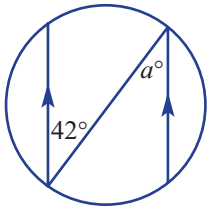
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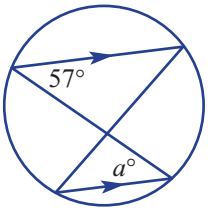
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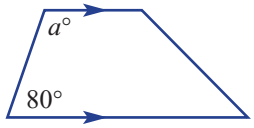
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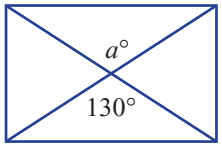
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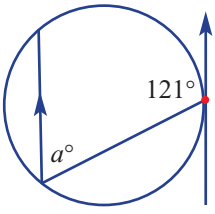
g



h



i

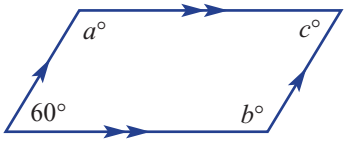


11 This shape is a parallelogram with two pairs of parallel sides.

a Use the $60°$ angle to find the value of a and b.

b Find the value of c.

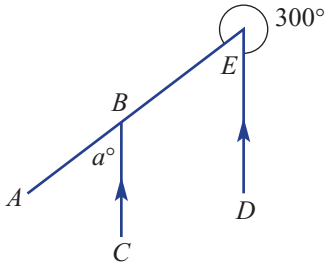
c What do you notice about the angles inside a parallelogram?



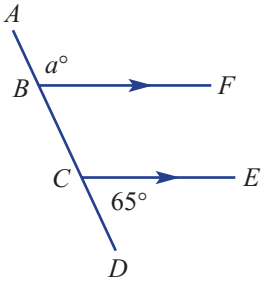
Enrichment: Parallel challenge

12 Find the value of a in these diagrams.

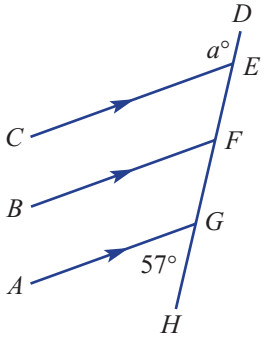
a



b



c



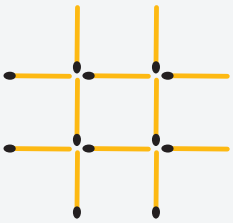
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1 Move three matchsticks to turn the fish to face the opposite direction.

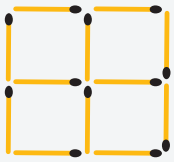


2 Move three matchsticks to get three squares of the same size.

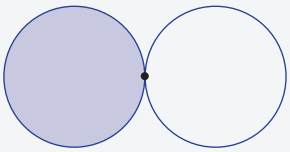
a



b



3 Two circles are the same size. The shaded circle rolls around the other circle. How many degrees will it have turned after returning to its starting position?

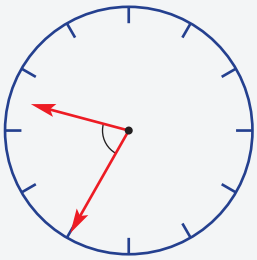


4 Measure the angle between the hour hand and minute hand of a clock at:

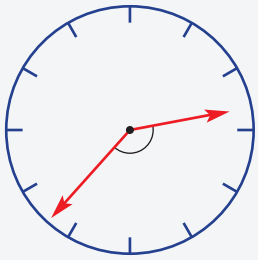
a 1:30 pm



b 9:35 am



c 2:37 pm



5 The picture shown is a tangram puzzle. It contains seven shapes within a square.

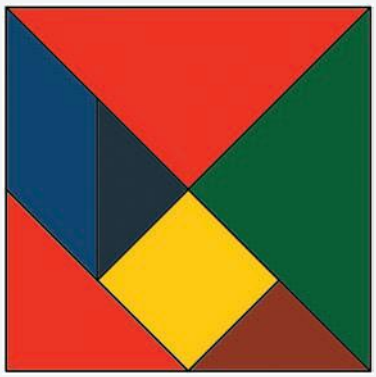
a Find the size of all the angles in each shape.

b Search the internet for a tangram template, print it and cut it out. Create a hexagon (6 sides) using all seven pieces.

i Find the size of each interior angle.

ii Find the sum of all six angles.

c Create a different hexagon using all seven pieces. Find the sum of the interior angles. What do you notice?



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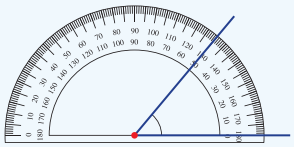
Chapter summary



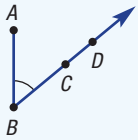
Drilling for Gold 2S1

Angle relationships

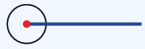
Measuring angles



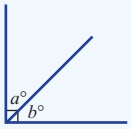
Geometrical objects



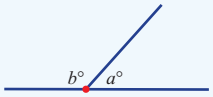
- $\angle ABC$ or $\angle CBA$
- ray BD
- segment AB
- collinear points B, C, D
- vertex B

Angle type	Size	Examples
acute angle	between 0° and 90°	
right angle	90°	
obtuse angle	between 90° and 180°	
straight angle	180°	
reflex angle	between 180° and 360°	
revolution	360°	

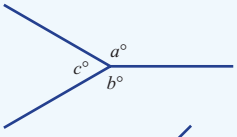
Angles at a point



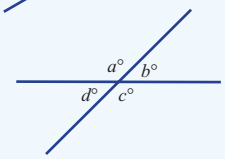
Angles in a right angle
$a + b = 90$



Angles on a straight line
$a + b = 180$



Angles in a revolution
$a + b + c = 360$

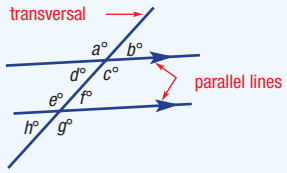


Vertically opposite angles
$a = c$ and $b = d$



Perpendicular lines meet at 90°
$AC \perp BD$

Parallel lines



$a = e$ (corresponding)
$b = f$ (corresponding)
$c = g$ (corresponding)
$d = h$ (corresponding)
$d = f$ (alternate)
$c = e$ (alternate)
$d + e = 180$ (cointerior)
$c + f = 180$ (cointerior)

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

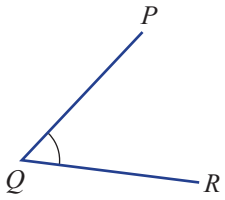
Multiple-choice questions

1 The object shown here would be called:



A ray AB **B** angle AB
C segment AB **D** plane AB
E line AB

2 The angle shown here can be named:



A $\angle QRP$ **B** $\angle PQR$ **C** $\angle QPR$
D $\angle QRR$ **E** $\angle PQP$

3 Supplementary angles:

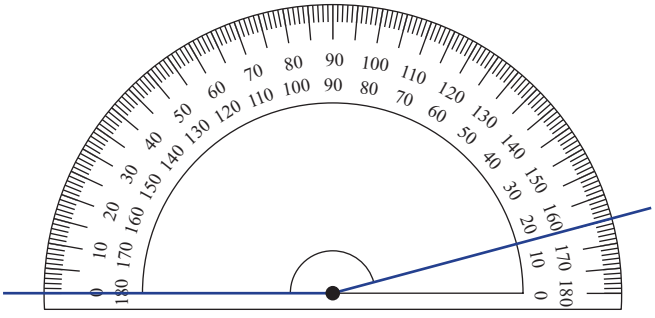
A sum to 90° **B** sum to 270° **C** sum to 360°
D sum to 180° **E** sum to 45°

4 An acute angle is:

A 90° **B** 180° **C** between 180° and 360°
D between 90° and 180° **E** between 0° and 90°

5 What is the angle measured on this protractor?

A 15° **B** 30° **C** 105° **D** 165° **E** 195°



6 The angle in a revolution is:

A 45° **B** 270° **C** 180° **D** 90° **E** 360°

7 If a transversal cuts two parallel lines, then:

A cointerior angles are equal
B alternate angles are supplementary (i.e. sum to 180°)
C corresponding angles are equal
D vertically opposite angles are supplementary
E supplementary angles add to 90°

8 The angle between two perpendicular lines is:

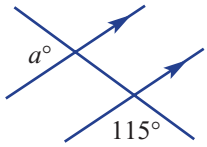
A 90° **B** 360° **C** 10 cm **D** 0° **E** 180°

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Chapter review

9 The value of a in this diagram is:

A 115 **B** 75 **C** 60 **D** 55 **E** 65



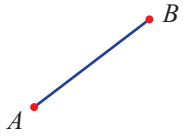
10 The complement of 50° is:

A 130° **B** 40° **C** 310° **D** 150° **E** 50°

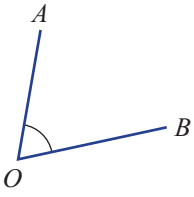
Short-answer questions

1 Name each of these objects.

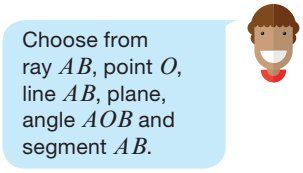
a



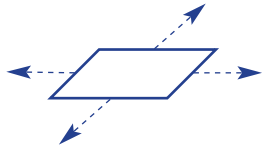
b



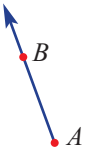
c $\bullet O$



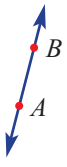
d



e

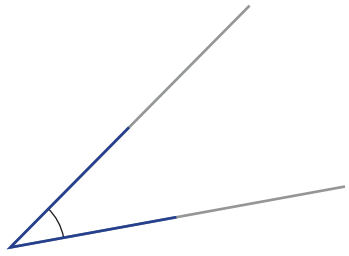


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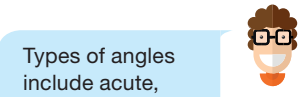
2 For the angles shown, state the type of angle and measure its size using a protractor.

a

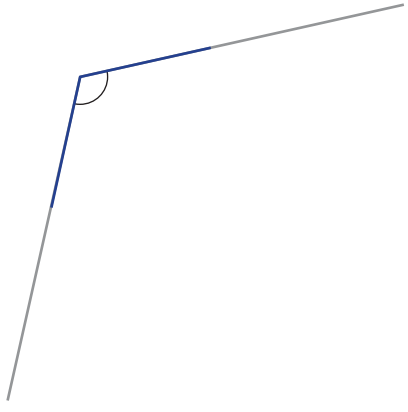


b

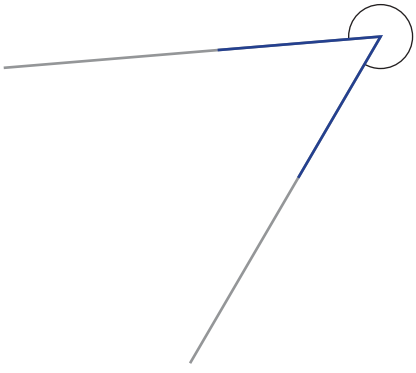




c



d



3 Draw each of the following angles using a protractor.

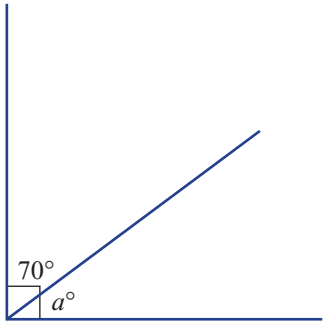
a 60° **b** a right angle **c** 170° **d** 265°

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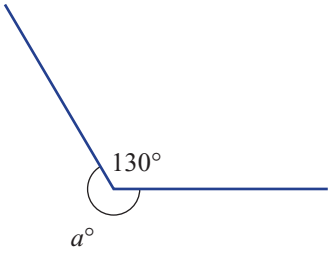
Chapter review

4 Without using a protractor, find the value of a in these diagrams.

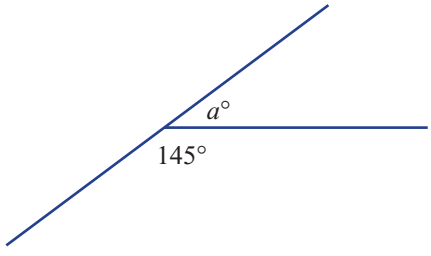
a



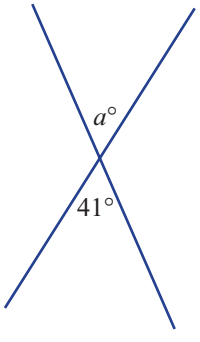
b



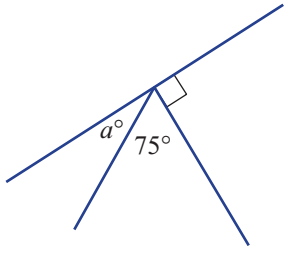
c



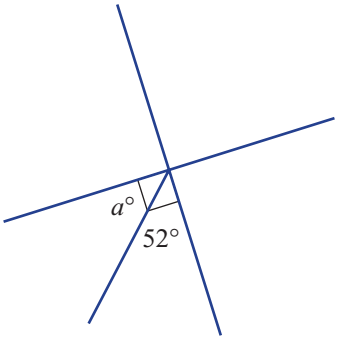
d



e

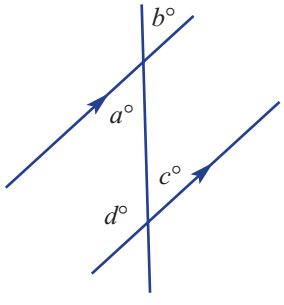


f



5 Using the letters a, b, c or d given in the diagram, write down a pair of angles that are:

a vertically opposite
b cointerior
c alternate
d corresponding
e supplementary but not cointerior

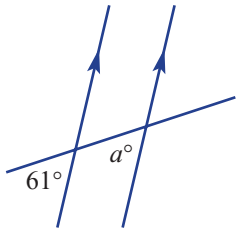


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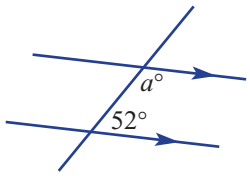
Chapter review

6 Find the value of a in these diagrams, which include parallel lines.

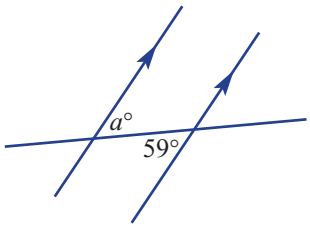
a



b

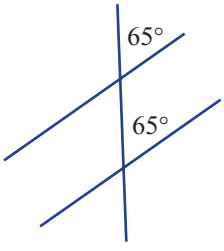


c

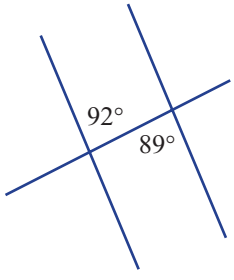


7 For each of the following, state whether the two lines cut by the transversal are parallel. Give reasons for each answer.

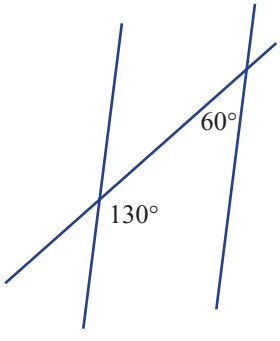
a



b

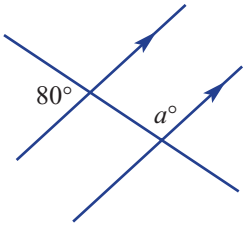


c

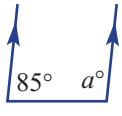


8 Find the value of a in these diagrams.

a



b



c



Extended-response questions

1 A clockface is numbered from 1 to 12.



a Find the angle the minute hand turns in:

i 5 minutes **ii** $\frac{1}{2}$ hour **iii** 45 minutes

b Find the angle the hour hand turns in:

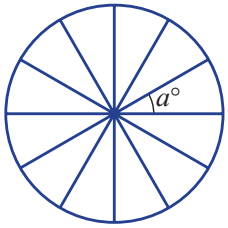
i 1 hour **ii** 7 hours **iii** $\frac{1}{2}$ hour

c Find the angle between the hour and minute hands at these times.

i 6 p.m. **ii** 3 p.m. **iii** 10 p.m.

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2 A circular birthday cake is cut into pieces of equal size, cutting from the centre outwards. Each cut has an angle of $a°$ at the centre.



Kayla's family takes four pieces. George's family takes three pieces. Lavinia's family takes two pieces. Brittany's family takes two pieces. Yi-Ming takes one piece.

a How many pieces were taken all together?

b If there is no cake left after all the pieces are taken, find the value of a.

c Find the value of a if:

i half of the cake still remains

ii one-quarter of the cake still remains

iii one-third of the cake still remains

iv one-fifth of the cake still remains



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Chapter 3

Computation with positive and negative integers

What you will learn

3A Working with negative integers
3B Adding or subtracting a positive integer
3C Adding a negative integer
3D Subtracting a negative integer
3E Multiplying or dividing by an integer
3F The Cartesian plane

Strand: Number and Algebra

Substrand: CALCULATING WITH INTEGERS

In this chapter, you will learn to:

- compare, order and calculate with integers
- apply a range of strategies to aid with computation.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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The coldest place on Earth

The maximum daily temperature in Antarctica is below freezing (0° Celsius) for much of the year. To record temperatures below 0° Celsius, we use negative numbers. For example, the coldest temperature ever recorded on Earth was about –89°C in 1983 at the Russian Vostok Station in Antarctica.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Evaluate the following.

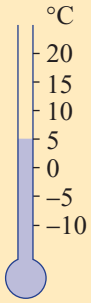
a $7 + 4$	**b** $0 + 5$	**c** $7 + 0$	**d** $13 + 9$
e $10 - 10$	**f** $14 - 8$	**g** $9 - 8$	**h** $26 - 14$
i $2 + 3 - 4$	**j** $7 - 4 + 5$	**k** $21 - 16 + 4$	**l** $36 + 24 - 47$

2 Insert the symbols < (is less than) or > (is greater than) to make each statement true.

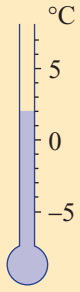
a $5 \square 7$	**b** $0 \square 10$	**c** $9 \square 11$	**d** $3 \square 0$
e $10 \square 12$	**f** $13 \square 26$	**g** $2 \square 1$	**h** $101 \square 99$

3 Read the temperature on these thermometers, correct to the nearest degree.

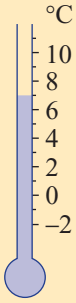
a



b



c



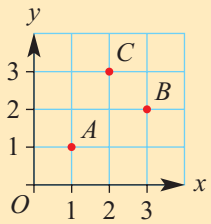
4 Evaluate these products.

a 2×15	**b** 11×7	**c** 9×8	**d** 3×13

5 Evaluate these quotients.

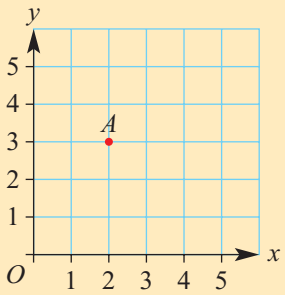
a $35 \div 7$	**b** $121 \div 11$	**c** $63 \div 7$	**d** $84 \div 12$

6 Write down the coordinates (x, y) of A, B and C for this Cartesian plane. Choose from (1, 1), (2, 3) and (3, 2).



7 Plot these points on the given Cartesian plane. The first one is done for you.

a A (2, 3)
b B (4, 1)
c C (5, 4)
d D (0, 2)
e E (3, 0)



8 If the temperature rises from –3°C to 3°C, by how much has it increased?

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3A Working with negative integers



John Wallis invented the number line.

Positive numbers are greater than zero. Negative numbers are less than zero. All the positive whole numbers (1, 2, 3, …) and the negative whole numbers (–1, –2, –3, …), together with zero (0), are called integers.

An English mathematician named John Wallis (1616–1703) invented the number line. He also invented the idea that numbers have a direction.

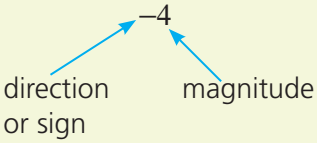
Let's start: Who uses negative numbers?

How might negative numbers be used:

- on the weather report?
- on a bank statement?
- at a golf tournament?
- at the launch of a rocket?

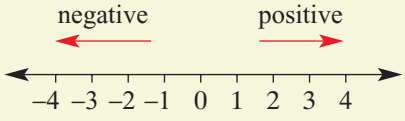
Key ideas

- **Negative** numbers are numbers less than zero.
- **Integers** are whole numbers that can be negative, zero or positive.
  … –4, –3, –2, –1, 0, 1, 2, 3, 4, …
- The number –4 is read as 'negative 4'.
- The number 4 is sometimes written as +4 and is sometimes read as 'positive 4'.

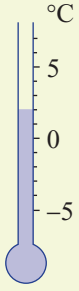


- Every number has *direction* and *magnitude*.
- A **number line** shows:
  – positive numbers to the right of zero
  – negative numbers to the left of zero.

Drilling for Gold 3A1



- A thermometer shows:
  – positive temperatures above zero
  – negative temperatures below zero.
- Each number other than zero has an **opposite**. 3 and –3 are examples of opposite numbers. They are equal in **magnitude** but opposite in sign.



Negative number A number less than zero

Integers The set of positive and negative whole numbers and zero

Number line A line on which numbers are represented by points

Magnitude The size of something

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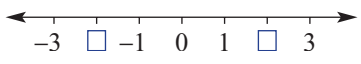
Exercise 3A

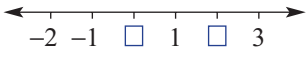
Understanding

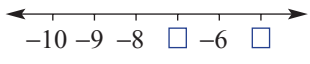
1 State the missing number.

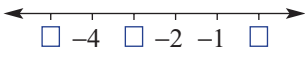
a –3, –2, –1, 0, ☐, 2
b –1, ☐, 1, 2, 3, 4
c –6, –5, ☐, –3, –2, –1, 0
d –4, –3, –2, ☐, 0, 1
e –21, –20, –19, ☐, –17, –16
f –37, ☐, –35, –34, –33, –32

2 What are the missing numbers on these number lines?

a


b


c


d


3 Fill in each blank using the word *greater* or *less*.

a 5 is _____ than 0
b –3 is _____ than 0
c 0 is _____ than –6
d 0 is _____ than 1
e –2 is _____ than –3
f –6 is _____ than –2



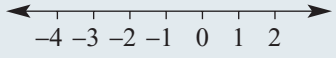
On the number line, numbers to the *right* are greater than numbers to their *left*.

Fluency

Example 1 Drawing a number line

Draw a number line, showing all integers from –4 to 2.

Solution



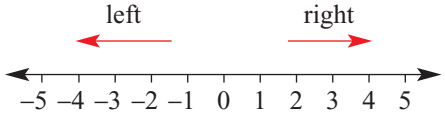
Explanation

Use equally spaced markings and put –4 on the left and 2 on the right.

4 Draw a number line showing all integers:

a from –2 to 2
b from –5 to 1
c from –10 to –4
d from –16 to –12

5 Add the word *right* or *left* to make the following statements true. Use this number line to help.



a 2 is to the _____ of 0
b 1 is to the _____ of 3
c –1 is to the _____ of 2
d –4 is to the _____ of –5
e –4 is to the _____ of –1
f 2 is to the _____ of –4

6 –5 is the opposite number of 5, and 5 is the opposite number of –5. Write down the opposite of these numbers.

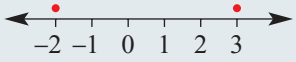
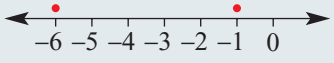
a 2
b 6
c –3
d –7

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Example 2 Is less than or is greater than

Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.

a $-2 \square 3$ **b** $-1 \square -6$

Solution	Explanation
a $-2 < 3$	-2 is to the left of 3 on a number line. 
b $-1 > -6$	-1 is to the right of -6 on a number line. 

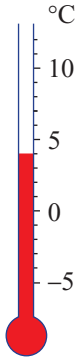


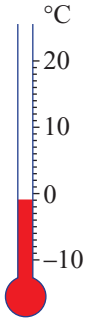
On the number line, numbers to the *left* are less than numbers to their *right*.

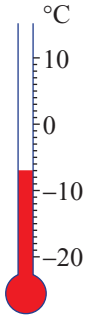
7 Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.

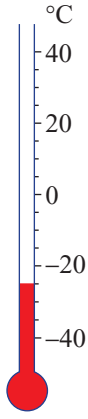
a $7 \square 9$	**b** $3 \square 2$	**c** $0 \square -2$	**d** $-4 \square 0$
e $-1 \square -5$	**f** $-7 \square -6$	**g** $-11 \square -2$	**h** $-9 \square -13$
i $-3 \square 3$	**j** $3 \square -3$	**k** $-130 \square 1$	**l** $-2 \square -147$
m $-7 \square -6$	**n** $-1 \square -21$	**o** $-116 \square -118$	**p** $-231 \square -162$

8 Give the temperature for these thermometers, correct to the nearest degree. Hint: A thermometer is like a vertical number line.

a 

b 

c 

d 

Problem-solving and Reasoning

9 True or false?

a -3 is the opposite of 6	**b** 0 is a negative number
c 0 is a positive number	**d** -5 is less than -2
e -8 is greater than -1	**f** -2 is equal to 2
g $-11 < -6$	**h** $-7 > -2$

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3A

10 List all the integers that fit the given description.

a from –2 up to 4
b from –7 up to 0
c is greater than –3 and is less than 2
d is greater than –5 and is less than 1
e is less than 4 and is greater than –4
f is less than –3 and is greater than –10

11 Arrange these numbers in *ascending* order.

a –3, –6, 0, 2, –10, 4, –1
b –304, 126, –142, –2, 1, 71, 0



Ascending means 'to rise', so arrange numbers from lowest to highest (i.e. *left* to *right* on a number line).

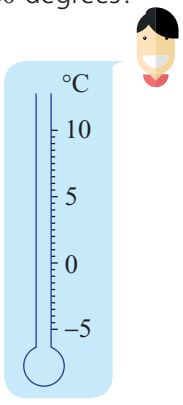
12 Write the next three numbers in these simple patterns.

a 3, 2, 1, ___, ___, ___
b –8, –6, –4, ___, ___, ___
c 10, 5, 0, ___, ___, ___
d –38, –40, –42, ___, ___, ___
e –91, –87, –83, ___, ___, ___
f 199, 99, –1, ___, ___, ___

13 A thermometer shows a temperature of 10 degrees Celsius. What would be the new temperature if it drops by:

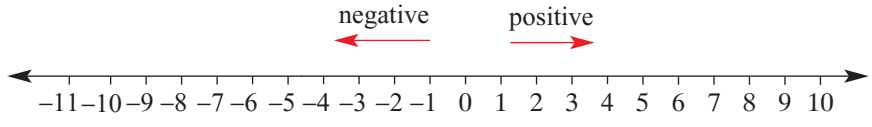
a 6 degrees? **b** 10 degrees? **c** 12 degrees? **d** 20 degrees?





Enrichment: The final position

14 For these sets of numbers, a positive number means to move right and a negative number means to move left. Start at zero each time and find the final position.



a Move –1, then 4, then –5
b Move 3, –5, –1, 4
c Move –5, –1, 3, 1, –2, –1, 4
d Move –10, 20, –7, –14, 8, –4

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3B Adding or subtracting a positive integer

When we add a positive number such as 3, we move 3 places to the right on the number line. $2 + 3$ means start at 2 and move 3 places to the right. $-5 + 3$ means start at -5 and move 3 places to the right.

To subtract the positive number 4, we move 4 places to the left on the number line. For $7 - 4$, we start at 7 and move 4 places left. For $-2 - 4$, we start at -2 and move four places left.

So, to add or subtract a positive number, we need to pick a starting point and then move right for addition or left for subtraction.

Let's start: Walking the number line

Imagine a number line running from one side of the room to the other, large enough to walk along. Decide which half is positive (the right) and which is negative (the left). Each step moves you one whole number along the line.

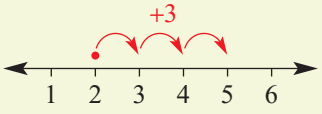
- Stand at zero and move to the right for addition and left for subtraction.
- Another student calls out a command such as 'add 3' or 'subtract 5'. Move the appropriate number of steps on the number line and tell the class your new position.

Key ideas

- If a positive number is added to an integer, you move right on a number line.

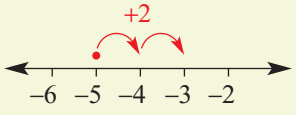
$2 + 3 = 5$

Start here. Move right 3.



$-5 + 2 = -3$

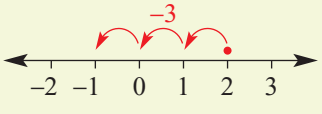
Start here. Move right 2.



- If a positive number is subtracted from an integer, you move left on a number line.

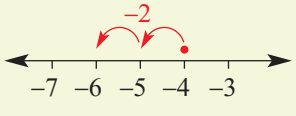
$2 - 3 = -1$

Start here. Move left 3.



$-4 - 2 = -6$

Start here. Move left 2.



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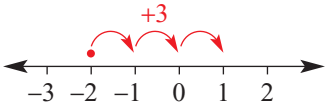
Exercise 3B

Understanding

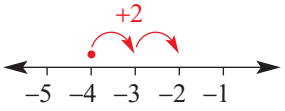

Drilling for Gold 3B1

1 Use the number line to help find the answer.

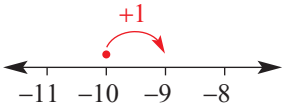
a $-2 + 3$



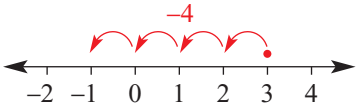
b $-4 + 2$



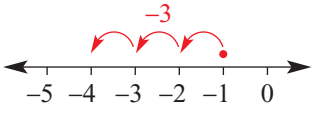
c $-10 + 1$



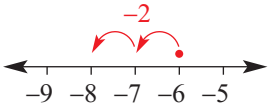
d $3 - 4$



e $-1 - 3$



f $-6 - 2$



2 In which direction (i.e. right or left) on a number line do you move for the following calculations?

a Start at –5, add 2

b Start at –4, add 6

c Start at 2, subtract 4

d Start at –4, subtract 3

3 Match up the problems (**a** to **d**) with the number lines (**A** to **D**).

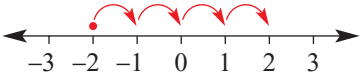
a $5 - 6 = -1$

b $-2 + 4 = 2$

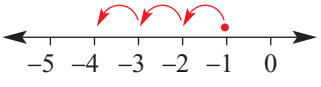
c $-1 - 3 = -4$

d $-6 + 3 = -3$

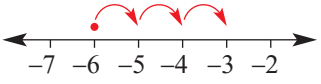
A



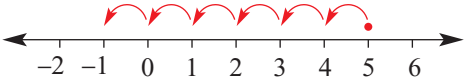
B



C



D



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Fluency

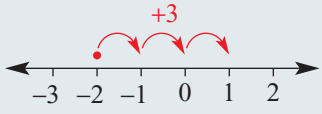
Example 3 Adding a positive integer

Add the following.

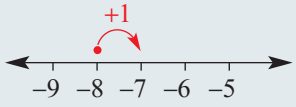
a $-2 + 3$ **b** $-8 + 1$ **c** $-11 + 4$

Solution **Explanation**

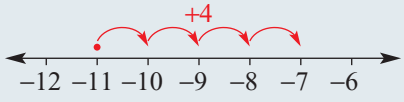
a $-2 + 3 = 1$



b $-8 + 1 = -7$



c $-11 + 4 = -7$



4 Add the following.

a $-1 + 2$	**b** $-1 + 4$	**c** $-3 + 5$	**d** $-10 + 11$
e $-7 + 2$	**f** $-10 + 7$	**g** $-13 + 14$	**h** $-9 + 13$
i $-4 + 3$	**j** $-5 + 2$	**k** $-11 + 9$	**l** $-20 + 18$
m $-4 + 0$	**n** $-8 + 0$	**o** $-30 + 29$	**p** $-39 + 41$



Remember to move *right* on the number line when *adding* a *positive* integer.

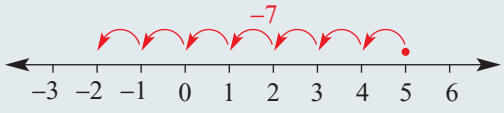
Example 4 Subtracting a positive integer

Subtract the following.

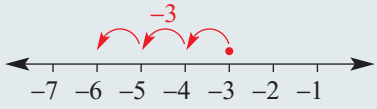
a $5 - 7$ **b** $-3 - 3$ **c** $-12 - 4$

Solution **Explanation**

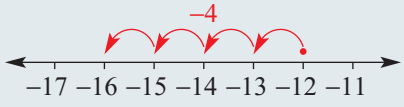
a $5 - 7 = -2$



b $-3 - 3 = -6$



c $-12 - 4 = -16$



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3B

5 Subtract the following.

a 4 – 6	**b** 7 – 8	**c** 3 – 11	**d** 1 – 20
e –1 – 2	**f** –3 – 4	**g** –5 – 10	**h** –11 – 2
i –3 – 1	**j** –5 – 5	**k** –2 – 13	**l** –7 – 0
m –37 – 4	**n** 39 – 51	**o** 62 – 84	**p** –21 – 26



Remember to move left on the number line when subtracting a positive integer.

6 Evaluate:

a –4 + 6	**b** –4 – 6	**c** 4 – 6	**d** 6 – 4
e –8 + 12	**f** –8 – 12	**g** 8 – 12	**h** 12 – 8

Example 5 Working from left to right

Evaluate the following, working from left to right.

a 2 + 3 – 6 **b** –6 + 3 – 5

Solution	Explanation
a 2 + 3 – 6 = 5 – 6 = –1	First work out 2 + 3 = 5, then subtract 6.
b –6 + 3 – 5 = –3 – 5 = –8	First work out –6 + 3 = –3, then subtract 5.



7 Evaluate the following. Check your answers using a calculator.

a 3 – 4 + 6	**b** 2 – 7 – 4	**c** –1 – 4 + 6	**d** –5 – 7 – 1
e –2 + 7 – 4	**f** –3 + 1 – 6	**g** –4 – 5 + 2	**h** –16 + 4 – 1
i –3 + 5 + 4	**j** –3 – 5 + 4	**k** –3 – 5 – 4	**l** 3 – 5 + 4



Remember to work from left to right.

Problem-solving and Reasoning

8 Find the missing number.

a 2 + □ = 7	**b** –2 + □ = 7	**c** –2 + □ = 3	**d** –4 + □ = –2
e 5 – □ = 0	**f** 3 – □ = –4	**g** –9 – □ = –12	**h** –20 – □ = –30
i –6 + □ = –1	**j** –8 – □ = –24	**k** □ + 1 = –3	**l** □ + 7 = 2
m □ – 4 = –10	**n** □ – 7 = –20	**o** □ + 6 = –24	**p** □ – 100 = –213

9 Determine how much debt remains in these bank accounts.

a Owes $300 and pays back $155
b Owes $20 and borrows another $35
c Owes $21 500 and pays back $16 250

10 a The reading on a thermometer rises 18°C from –15°C. What is the final temperature?
b The reading on a thermometer falls 7°C from 4°C. What is the final temperature?
c The reading on a thermometer falls 32°C from –14°C. What is the final temperature?

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11 For an experiment, a chemical solution starts at a temperature of 25°C. It falls to –3°C, rises to 15°C and then falls again to –8°C. What is the total change in temperature? Add all the changes together for each rise and fall.

12 An ocean sensor is raised and lowered to different depths in the sea. Note that –100 m means 100 m below sea level.



a If the sensor is initially at –100 m and then raised to –41 m, how far does the sensor rise?

b If the sensor is initially at –37 m and then lowered to –93 m, how far is the sensor lowered?

13 Insert + or – signs into these statements to make them true.

a $3 \square 4 \square 5 = 4$

b $1 \square 7 \square 9 \square 4 = -5$

c $-4 \square 2 \square 1 \square 3 \square 4 = 0$

d $-20 \square 10 \square 7 \square 36 \square 1 \square 18 = -4$

Enrichment: Positive and negative possibilities

14 Decide if it is possible to find an example of the following. If so, give a specific example. It might help to draw a number line.

a A positive number added to a positive number gives a positive number.

b A positive number added to a positive number gives a negative number.

c A positive number added to a negative number gives a positive number.

d A positive number added to a negative number gives a negative number.

e A positive number subtracted from a positive number gives a positive number.

f A positive number subtracted from a positive number gives a negative number.

g A positive number subtracted from a negative number gives a positive number.

h A positive number subtracted from a negative number gives a negative number.

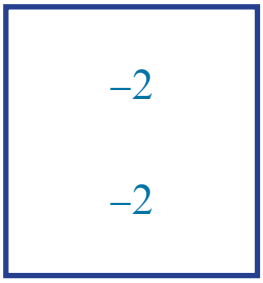
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3C Adding a negative integer

So far, we have added or subtracted a positive integer to other integers. For example: $-4 + 2$ or $-4 - 2$

We will now look at the case where we add a negative integer to another integer. For example: $-4 + (-2)$

Look at the diagram on the right. At the moment, the sum of the numbers inside the box is -4. If we add -2, what is the sum?



Let's start: What does the pattern tell us?

Look at this addition pattern.

$2 + 3 = 5$ -1
$2 + 2 = 4$ -1
$2 + 1 = 3$ -1
$2 + 0 = 2$ -1
$2 + (-1) = 1$ -1
$2 + (-2) = 0$ -1
$2 + (-3) = -1$

Note that $2 + (-3) = -1$ and $2 - 3 = -1$.

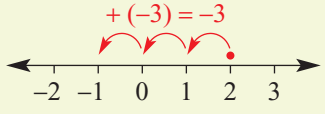


- Describe the vertical patterns that you see.
- What do the patterns tell you about adding a negative integer?

Key ideas

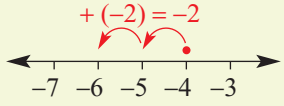
- The opposite of 3 is -3, and the opposite of -2 is 2.
- To add a negative number, subtract its opposite.
  For example, to add (-3), simply subtract 3.

$2 + (-3) = 2 - 3 = -1$



To add -3, subtract 3.

$-4 + (-2) = -4 - 2 = -6$



To add -2, subtract 2.

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Exercise 3C

Understanding

1 Copy and complete.

a To add (–5), subtract ___.
b To add (–8), _____ ___.
c To add ___, subtract 6.
d To ___ (–1), subtract ___.

2 Change addition to subtraction.

a $1 + (-3) = 1 - \square$
b $4 + (-3) = 4 - \square$
c $6 + (-2) = 6 - \square$
d $-3 + (-2) = -3 - \square$

3 Match each addition (**a** to **d**) with the correct diagram (**A** to **D**).

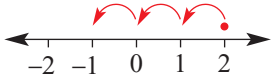
a $4 + (-1)$

b $2 + (-3)$

c $-6 + (-2)$

d $-4 + (-1)$

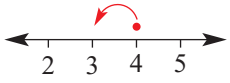
A



B



C



D



Fluency

Example 6 Adding a negative integer

Calculate answers to the following.

a $7 + (-2)$
b $-2 + (-3)$

Solution	Explanation
a $7 + (-2) = 7 - 2$ $= 5$	To add –2, subtract 2.
b $-2 + (-3) = -2 - 3$ $= -5$	To add –3, subtract 3.

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3C



Drilling for Gold 3C1

4 Calculate:

a $3 + (-2)$ **b** $8 + (-3)$ **c** $12 + (-6)$
d $9 + (-7)$ **e** $1 + (-4)$ **f** $6 + (-11)$
g $20 + (-22)$ **h** $0 + (-4)$ **i** $-2 + (-1)$
j $-7 + (-15)$ **k** $-5 + (-30)$ **l** $-28 + (-52)$
m $-7 + (-3)$ **n** $-20 + (-9)$ **o** $-31 + (-19)$
p $-103 + (-9)$ **q** $-99 + (-10)$ **r** $-12 + (-101)$



To add a negative integer, subtract its opposite.
$-3 + (-2) = 3 - 2$

5 Find the answer when:

a -2 is added to 7 **b** -3 is added to 10
c -10 is added to -2 **d** -1 is added to -6
e -13 is added to -2 **f** -31 is added to -11



For part **c**, start at -2 and subtract 10.

Example 7 Working from left to right

Calculate the answers, working from left to right.

a $3 + (-2) + 5$ **b** $-2 + (-3) + (-4)$

Solution	Explanation
a $3 + (-2) + 5 = 3 - 2 + 5$ $= 1 + 5$ $= 6$	To add -2, subtract 2, then add 5 to finish.
b $-2 + (-3) + (-4) = -2 - 3 - 4$ $= -5 - 4$ $= -9$	Starting at -2, add -3 by subtracting 3. Then add -4 by subtracting 4.



6 Work from left to right to evaluate the following. Check your answers using a calculator.

a $3 + (-1) + 2$ **b** $7 + (-2) + 4$ **c** $2 + (-3) + 1$
d $-1 + (-3) + (-4)$ **e** $-4 + (-2) + (-1)$ **f** $-6 + (-3) + (-1)$
g $3 + (-10) + 6$ **h** $-15 + 2 + (-4)$ **i** $-13 + (-17) + (-3)$

Problem-solving and Reasoning

7 Find the missing number.

a $2 + \square = -1$ **b** $3 + \square = -7$ **c** $-2 + \square = -6$
d $\square + (-3) = 1$ **e** $\square + (-10) = -11$ **f** $\square + (-4) = 0$
g $\square + (-3) = 2$ **h** $\square + (-6) = -1$ **i** $-2 + \square = -3$
j $-5 + \square = -7$ **k** $-10 + \square = -12$ **l** $-37 + \square = -51$

8 A person has \$120 of debt. If \$70 of debt is added to this, how much debt is now owed by the person?

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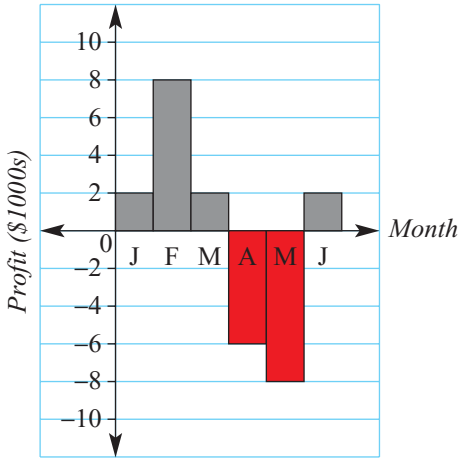
9 Here is a profit graph showing the profit for each month of the first half of the year for a bakery shop.

a What is the profit for:

i February?

ii April?

b What is the overall profit for the 6 months?



10 Are the following always true?

a $\begin{pmatrix}\text{positive}\\\text{number}\end{pmatrix} + \begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix} = \begin{pmatrix}\text{positive}\\\text{number}\end{pmatrix}$

b $\begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix} + \begin{pmatrix}\text{positive}\\\text{number}\end{pmatrix} = \begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix}$

c $\begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix} + \begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix} = \begin{pmatrix}\text{negative}\\\text{number}\end{pmatrix}$



Try substituting some numbers!

Enrichment: Negative magic squares

11 Complete these magic squares, using addition. The sum of each row, column and diagonal should be the same.

a

−2		5
	1	
		4

b

		−6
−3		−17
		−7

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3D Subtracting a negative integer

Look at the diagram on the right. The sum of the numbers in the box is –7.

If the –3 is removed from the box, what happens to the sum? This suggests that subtracting –3 is the same as adding 3.

–2
–2
–3

Let's start: Patterns reveal the trick

Look at this subtraction pattern.



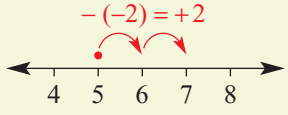
$2 - 3 = -1$ (+1)
$2 - 2 = 0$ (+1)
$2 - 1 = 1$ (+1)
$2 - 0 = 2$ (+1)
$2 - (-1) = 3$ (+1)
$2 - (-2) = 4$ (+1)
$2 - (-3) = 5$

- Describe the vertical patterns that you see.
- What do the patterns tell you about subtracting a negative integer?

Key ideas

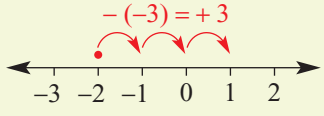
- To subtract a negative number, add its opposite.

$5 - (-2) = 5 + 2 = 7$



To subtract –2, add 2.

$-2 - (-3) = -2 + 3 = 1$



To subtract –3, add 3.

Exercise 3D

Understanding

1 Write the missing number.

a To subtract –3, add ☐.
b To subtract –6, add ☐.
c To subtract –4, add ☐.
d To subtract –11, add ☐.
e To subtract –15, add ☐.
f To subtract –312, add ☐.

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2 Write down the missing numbers in these sentences. Parts **a** and **c** have been started for you.

a 5 – 3 means that 3 is subtracted from ☐.
b –2 – 6 means that ☐ is subtracted from ☐.
c 7 – (–3) means that –3 is subtracted from ☐.
d –7 – (–11) means that ☐ is subtracted from ☐.
e –2 – (–4) means that ☐ is subtracted from ☐.
f –6 – (–1) means that ☐ is subtracted from ☐.
g –11 – (–7) means that ☐ is subtracted from ☐.
h 27 – (–12) means that ☐ is subtracted from ☐.

3 Match each subtraction (**a** to **d**) with the correct diagram (**A** to **D**).

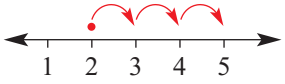
a 5 – (–1)

b 2 – (–3)

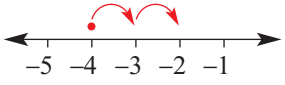
c –4 – (–2)

d –1 – (–1)

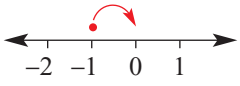
A



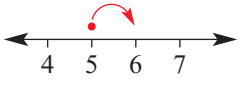
B



C



D



Fluency

Example 8 Subtracting a negative integer

Calculate:
a 1 – (–3) **b** –6 – (–2)

Solution	Explanation
a $1 - (-3) = 1 + 3$ $= 4$	To subtract –3, add 3.
b $-6 - (-2) = -6 + 2$ $= -4$	To subtract –2, add 2.

4 Calculate:

a 2 – (–3) **b** 5 – (–6) **c** 20 – (–30)
d 5 – (–1) **e** –5 – (–1) **f** –7 – (–4)
g –11 – (–6) **h** 4 – (–6) **i** –4 – (–6)
j –9 – (–10) **k** –20 – (–20) **l** 20 – (–20)
m 5 – (–23) **n** 28 – (–6) **o** –28 – (–6)

Drilling for Gold 3D1



To subtract a negative integer, add its opposite.
2 – (–3) = 2 + 3

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3D

5 Find the answer if:

a –2 is subtracted from 6
b –1 is subtracted from 10
c –4 is subtracted from –1
d –7 is subtracted from –3
e –2 is subtracted from –7
f –8 is subtracted from –13



For part **a**, start with 6 and subtract –2.

Example 9 Working from left to right

Work from left to right to evaluate the following.

a $7 - 9 - (-3)$ **b** $-2 - (-5) - 9$

Solution	Explanation
a $7 - 9 - (-3) = -2 - (-3)$	First work out 7 – 9.
$= -2 + 3$	To subtract –3, add 3.
$= 1$	–2 plus 3 is 1.
b $-2 - (-5) - 9 = -2 + 5 - 9$	To subtract –5, add 5.
$= 3 - 9$	–2 plus 5 = 3.
$= -6$	3 minus 9 is –6.



Skillsheet 3A

6 Work from left to right to evaluate the following.

a $6 - 9 - (-1)$
b $2 - 4 - (-3)$
c $-1 - 3 - (-2)$
d $-3 - (-1) - 4$
e $-4 - (-2) - 5$
f $-10 - (-2) - 6$
g $2 - (-1) - (-3)$
h $-10 - (-4) - (-3)$
i $-16 - (-10) - (-7)$



7 Calculate the answer, working from left to right. Check your answers using a calculator.

a $3 + (-2) + (-1)$
b $2 + (-1) + (-6)$
c $3 - (-1) - (-4)$
d $10 - (-6) + (-4)$
e $-7 - (-1) + (-3)$
f $-20 - (-10) - (-15)$
g $-9 - (-19) + (-16)$
h $-15 - (-20) + (-96)$
i $-13 - (-19) + (-21)$
j $-2 - (-3) - (-5)$
k $-18 - (-16) - (-19)$
l $5 + (-20) - (-26)$

Problem-solving and Reasoning

8 Find the missing number.

a $5 - \square = 6$
b $2 - \square = 7$
c $-1 - \square = 3$
d $\square - (-3) = 7$
e $\square - (-10) = 12$
f $\square - (-4) = -20$
g $5 - \square = 11$
h $\square - (-2) = -3$
i $-2 - \square = -4$
j $\square - (-5) = -1$
k $6 - \square = 15$
l $\square - (-2) = -4$

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9 A diver is at a height of –19 m from sea level. During a diving exercise, the diver rises 10 m, falls 18 m and then rises once again by 15 m. What is the diver's final height from sea level?



10 A small business has a bank balance of –$50 000. An amount of $20 000 of extra debt is added to the balance and, later, $35 000 is paid back. What is the final balance?



11 $100 of debt is added to an existing balance of $50 of debt. Later, $120 of debt is removed from the balance. What is the final balance?

12 Rachel said, "If you add two negative numbers together, the result is positive." Make up an example to show that she is incorrect.

Enrichment: Make it true

13 Insert + or – signs to make each statement true.

a $-2 \square 6 \square (-2) = 2$

b $-3 \square (-3) \square 2 = -8$

c $-2 \square (-5) \square 3 = 0$

d $-4 \square (-3) \square (-1) = -6$

e $4 \square (-10) \square (-2) = 12$

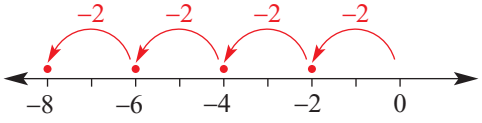
f $-3 \square (-16) \square (-2) = -17$

Make up your own and try it on a friend!

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3E Multiplying or dividing by an integer

Multiplication is repeated addition.
$2 + 2 + 2 + 2$ is '4 lots of 2'.
So $2 + 2 + 2 + 2$ is equal to 4×2, which is 8.



This diagram shows $0 + (-2) + (-2) + (-2) + (-2)$, which gives -8.

Similarly,
$(-2) + (-2) + (-2) + (-2)$ is equal to $4 \times (-2)$, which is -8.
Multiplication is reversible, so
$4 \times (-2) = (-2) \times 4 = -8$
So it looks like the product of a positive number and a negative number will always be negative.

Let's start: Patterns in times tables

- In the table below, use the pattern to fill in the blanks.

The 3 times table	The −3 times table
$3 \times 4 =$	$-3 \times 4 =$
$3 \times 3 = 9$	$-3 \times 3 =$
$3 \times 2 = 6$	$-3 \times 2 = -6$
$3 \times 1 = 3$	$-3 \times 1 = -3$
$3 \times 0 = 0$	$-3 \times 0 = 0$
$3 \times -1 = -3$	$-3 \times -1 =$
$3 \times -2 =$	$-3 \times -2 =$
$3 \times -3 =$	$-3 \times -3 =$
$3 \times -4 =$	$-3 \times -4 =$
$3 \times -5 =$	$-3 \times -5 =$

What do you notice about the numbers and products in the:
- yellow zone?
- orange zone?
- green zone?
- blue zone?

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Key ideas

In the previous table:

- The yellow zone shows that **positive × positive = positive**.
- The green zone shows that **positive × negative = negative**.
- The blue zone shows that **negative × positive = negative**.
- The orange zone shows that **negative × negative = positive**.

In the yellow and orange zones:
When two numbers have the same sign, their product is positive.
For example, $4 \times 3 = 12$ and $(-4) \times (-3) = 12$.

In the green and the blue zones:
When two numbers have opposite signs, their product is negative.
For example, $4 \times (-3) = -12$ and $(-4) \times 3 = -12$.

The same rules apply for division:
positive ÷ positive = positive
positive ÷ negative = negative
negative ÷ negative = positive
negative ÷ positive = negative

Exercise 3E

Understanding

1 Write the missing numbers in these tables. You should create a pattern in the third column.

a

□	△	□ × △
3	5	15
2	5	
1	5	
0	5	
−1	5	
−2	5	
−3	5	

b

□	△	□ × △
3	−5	−15
2	−5	−10
1	−5	
0	−5	
−1	−5	
−2	−5	
−3	−5	

2 Write the missing numbers in these sentences. Use the tables in Question **1** to help.

a $3 \times 5 = \square$ so $15 \div 5 = \square$

b $-3 \times 5 = \square$ so $-15 \div 5 = \square$

c $3 \times (-5) = \square$ so $15 \div (-5) = \square$

d $-3 \times (-5) = \square$ so $15 \div (-5) = \square$

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3E

3 Without finding the answer to these products, decide if the answer would be positive or negative.

a 109×4 **b** -76×5 **c** $15 \times (-9)$ **d** $-6 \times (-13)$
e 89×104 **f** -74×8 **g** $-94 \times (-5)$ **h** $80 \times (-7)$

4 Without finding the answer to these quotients, decide if the answer would be positive or negative.

a $16 \div 2$ **b** $24 \div (-3)$ **c** $78 \div (-2)$ **d** $-56 \div 2$ **e** $-81 \div 9$ **f** $-99 \div (-11)$

Fluency

Example 10 Finding products

Evaluate the following.

a $3 \times (-7)$ **b** $-4 \times (-12)$

Solution	Explanation
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative. $[+] \times [-] = [-]$
b $-4 \times (-12) = 48$	-4 and -12 are both negative and so the product will be positive. $[-] \times [-] = [+]$

Drilling for Gold 3E1

5 Evaluate the following.

a $4 \times (-5)$ **b** $6 \times (-9)$ **c** -4×10 **d** -11×9
e $-2 \times (-3)$ **f** -6×7 **g** -9×8 **h** $-11 \times (-9)$
i $20 \times (-2)$ **j** -16×4 **k** $-5 \times (-7)$ **l** $8 \times (-4)$
m $-10 \times (-6)$ **n** $44 \times (-1)$ **o** $-9 \times (-1)$ **p** -5×12

Example 11 Finding quotients

Evaluate the following.

a $-63 \div 7$ **b** $-121 \div (-11)$

Solution	Explanation
a $-63 \div 7 = -9$	The two numbers are of opposite sign so the answer will be negative. $[-] \div [+] = [-]$
b $-121 \div (-11) = 11$	-121 and -11 are both negative so the quotient will be positive. $[-] \div [-] = [+]$

6 Evaluate the following.

a $-10 \div 2$ **b** $-38 \div 19$ **c** $-60 \div 15$ **d** $-120 \div 4$
e $32 \div (-16)$ **f** $-6 \div 2$ **g** $6 \div (-2)$ **h** $-6 \div (-2)$
i $-12 \div 6$ **j** $-24 \div (-3)$ **k** $-45 \div 5$ **l** $-45 \div (-9)$
m $-66 \div (-6)$ **n** $-5 \div (-5)$ **o** $-8 \div 1$ **p** $-8 \div (-1)$

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Example 12 Order of operations

a $-7 + 6 \times (-5)$ **b** $-4 \times 6 \div (-2)$

Solution	Explanation
a $-7 + 6 \times (-5)$	The order of operations: multiplication first.
$= -7 + (-30)$	$6 \times (-5) = -30$
$= -7 - 30$	Last, addition of a negative = subtraction.
$= -37$	$-7 + (-30) = -7 - 30$
b $-4 \times 6 \div (-2)$	Multiplication and division: work from left to right.
$= -24 \div (-2)$	-4×6 *First* $\boxed{-} \times \boxed{+} = \boxed{-}$
$= 12$	$-24 \div -2$ *Last* $\boxed{-} \div \boxed{-} = \boxed{+}$

7 Follow the order of operations to find the following. Check your answers using a calculator.

a $10 + (-6) \times 5$ **b** $15 - 3 \times (-2)$ **c** $18 \times (-2) \div 3$
d $-9 \times 2 + (-5)$ **e** $45 - 50 \div (-10)$ **f** $9 - 6 \times 3$
g $-10 \div (-2) \times (-3)$ **h** $9 \times 3 - 6 \times (-2)$ **i** $18 \div (-3) + 3 \times (-4)$

8 If $(-2)^2 = -2 \times -2 = 4$, find each value of the following.

a $(-5)^2$ **b** $(-6)^2$ **c** $(-7)^2$ **d** $(-8)^2$ **e** $(-9)^2$

Problem-solving and Reasoning

9 Write the missing number.

a $\square \times 3 = -9$ **b** $\square \times (-7) = 35$ **c** $\square \times (-4) = -28$ **d** $-3 \times \square = -18$
e $-19 \times \square = 57$ **f** $\square \div (-9) = 8$ **g** $\square \div 6 = -42$ **h** $-150 \div \square = 5$

10 Will $(-2)^3$ give a positive or negative answer?

11 Insert a $\times$ sign and/or $\div$ sign to make these equations true.

a $-2 \square 3 \square (-6) = 1$ **b** $10 \square (-5) \square (-2) = 25$
c $6 \square (-6) \square 20 = -20$ **d** $-14 \square (-7) \square (-2) = -1$

12 The product of two numbers is –24 and their sum is –5. What are the two numbers?

Enrichment: Missing brackets

13 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$ **b** $-10 \div 3 - (-2) = -2$
c $-8 \div (-1) + 5 = -2$ **d** $-1 - 4 \times 2 + (-3) = 5$
e $-4 + (-2) \div 10 + (-7) = -2$ **f** $20 + 2 - 8 \times (-3) = 38$
g $1 - (-7) \times 3 \times 2 = 44$ **h** $4 + (-5) \div 5 \times (-2) = -6$

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3F The Cartesian plane

Street maps use a coordinate system. A capital letter and a number, such as D3, are used to locate a plane on the map.

This kind of coordinate system is accurate enough to find a street name or park. In mathematics we need a more precise system so that we can locate points and draw graphs.

The number plane we use today is called the Cartesian plane after the 17th century mathematician, René Descartes.



Let's start: North, south, east and west

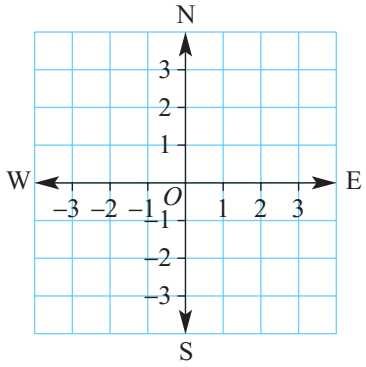
The units for this grid are in metres.
René starts at position O and moves:

- 3 m east
- 4 m west
- 2 m south
- 5 m north.

Pierre starts at position O and moves:

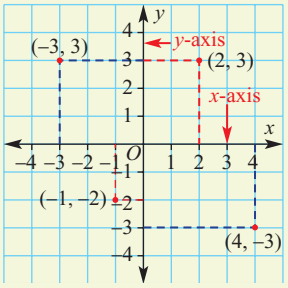
- 1 m west
- 4 m east
- 3 m south
- 5 m north.

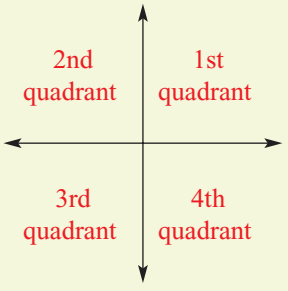
Using the number plane, how would you describe René and Pierre's final positions?



Key Ideas

- The **number plane** is also called the Cartesian plane.
- A point plotted on the plane has an x-coordinate and y-coordinate, which are written as (x, y).
- The point $(0, 0)$ is called the origin and labelled O.
- To plot points, always start at the origin.
  - For (2, 3) move 2 right and 3 up.
  - For (4, –3) move 4 right and 3 down.
  - For (–3, 3) move 3 left and 3 up.
  - For (–1, –2) move 1 left and 2 down.





Number plane A diagram on which two numbers can be used to locate any point

Cartesian plane Another name for the number plane

Origin The point on the number plane with coordinates (0, 0)

x–axis Horizontal axis of the number plane

y–axis Vertical axis of the number plane

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Exercise 3F

Understanding

1 Put the words *left* or *right* and *down* or *up* into these sentences.

a To move from (0, 0) to (3, 4) you would shift 3 units _____ and 4 units _____.

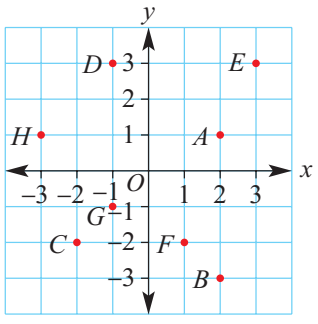
b To move from (0, 0) to (2, –1) you would shift 2 units _____ and 1 unit _____.

c To move from (0, 0) to (–3, 2) you would shift 3 units _____ and 2 units _____.

d To move from (0, 0) to (–1, –4) you would shift 1 unit _____ and 4 units _____.

2 Match the points A, B, C, D, E, F, G and H with the given coordinates.

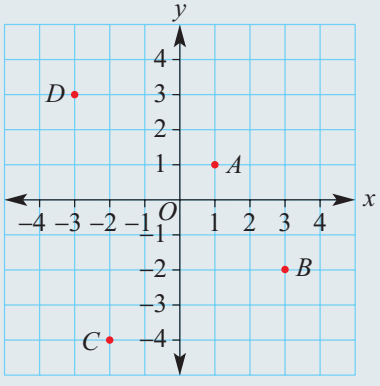
a (–1, 3) **b** (2, –3)
c (2, 1) **d** (–2, –2)
e (3, 3) **f** (–3, 1)
g (1, –2) **h** (–1, –1)



Fluency

Example 13 Finding coordinates

For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C and D.



Solution

$A = (1, 1)$
$B = (3, -2)$
$C = (-2, -4)$
$D = (-3, 3)$

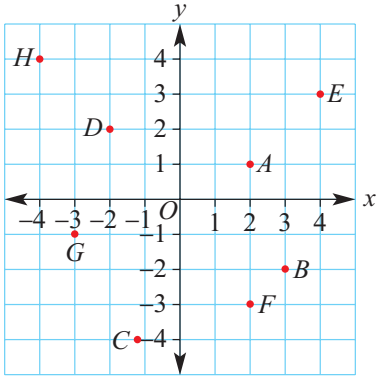
Explanation

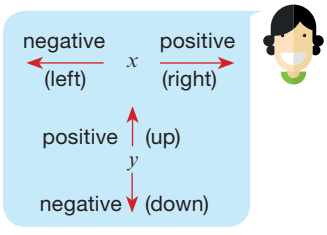
For each point, write the x-coordinate first (from the horizontal axis) followed by the y-coordinate (from the vertical axis).



Drilling for Gold 3F1 3F2

3 For the Cartesian plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.



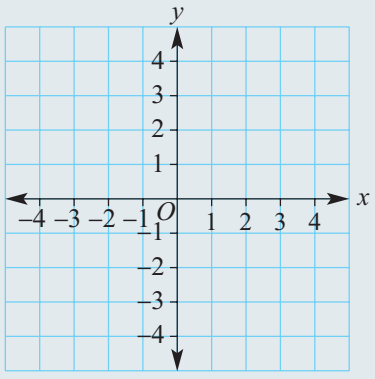


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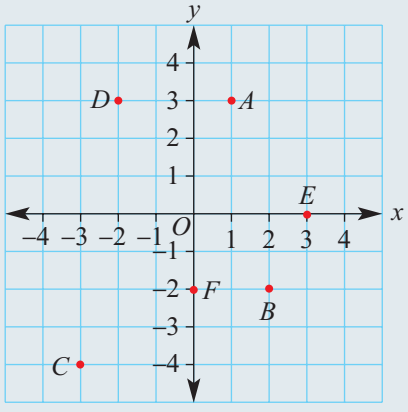
3F

Example 14 Plotting points

Plot these points on the given number planes.
$A(1, 3)$, $B(2, -2)$, $C(-3, -4)$, $D(-2, 3)$, $E(3, 0)$, $F(0, -2)$



Solution



Explanation

For each point, start at the origin $(0, 0)$.
First move right (if x is positive) or left (if x is negative).
Then move up (if y is positive) or down (if y is negative).

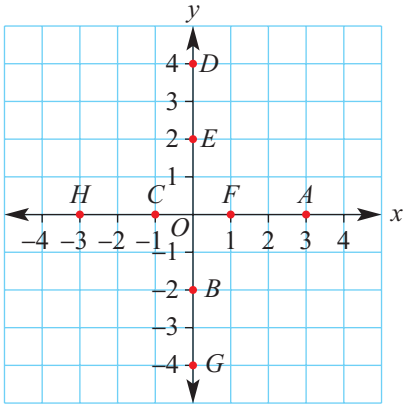
4 **a** Draw a set of axes like those in the example above. (You can use grid paper to help.)

b Now plot these points.

i $A(-3, 2)$ **ii** $B(1, 4)$ **iii** $C(2, -1)$
iv $D(-2, -4)$ **v** $E(2, 2)$ **vi** $F(-1, 4)$
vii $G(-3, -1)$ **viii** $H(1, -2)$ **ix** $I(3, -2)$
x $J(-2, 1)$ **xi** $K(-1, -1)$ **xii** $L(1, 2)$

Skillsheet 3B

5 For the number plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.



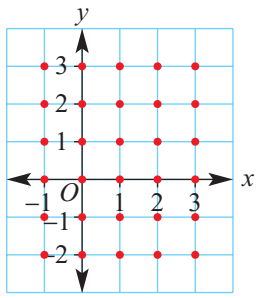


For each point, either the x-coordinate or y-coordinate will be zero.

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Problem-solving and Reasoning

6 Count the number of points (red dots) on this plane that have:



- **a** both x- and y-coordinates as positive numbers
- **b** an x-coordinate as a positive number
- **c** a y-coordinate as a positive number
- **d** an x-coordinate as a negative number
- **e** a y-coordinate as a negative number
- **f** both x- and y-coordinates as negative numbers
- **g** neither x nor y as positive or negative numbers

7 When plotted on the Cartesian plane, what shape does each set of points form?



If you can't 'picture' the points, plot them on a grid.

- **a** $A(-2, 0)$, $B(0, 3)$, $C(2, 0)$
- **b** $A(-3, -1)$, $B(-3, 2)$, $C(1, 2)$, $D(1, -1)$
- **c** $A(-4, -2)$, $B(3, -2)$, $C(1, 2)$, $D(-1, 2)$
- **d** $A(-3, 1)$, $B(-1, 3)$, $C(4, 1)$, $D(-1, -1)$

8 Karen's bushwalk starts at a point (2, 2) on a grid map. Each square on the map represents 1 km. If Karen walks to the points (2, –7), then (–4, –7), then (–4, 0) and then (2, 0), how far has she walked in total?



9 Seven points have the following x- and y-coordinates.

x	–3	–2	–1	0	1	2	3
y	–2	–1	0	1	2	3	4

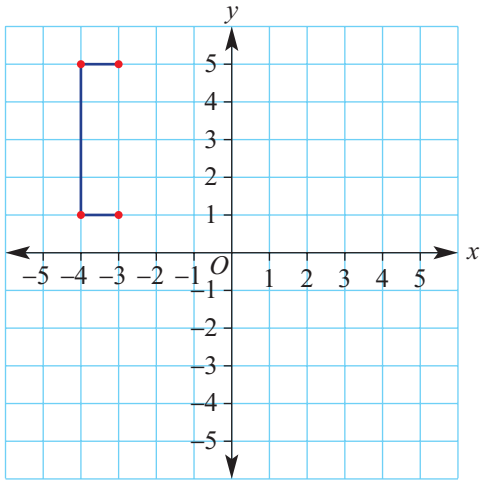
- **a** Plot the seven points on a Cartesian plane. Use –3 to 3 on the x-axis and –2 to 4 on the y-axis.
- **b** What do you notice about these points on the Cartesian plane?

10 The points $A(-2, 0)$, $B(-1, ?)$ and $C(0, 4)$ all lie on a straight line. Find the y-coordinate of point B.

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Enrichment: The secret message

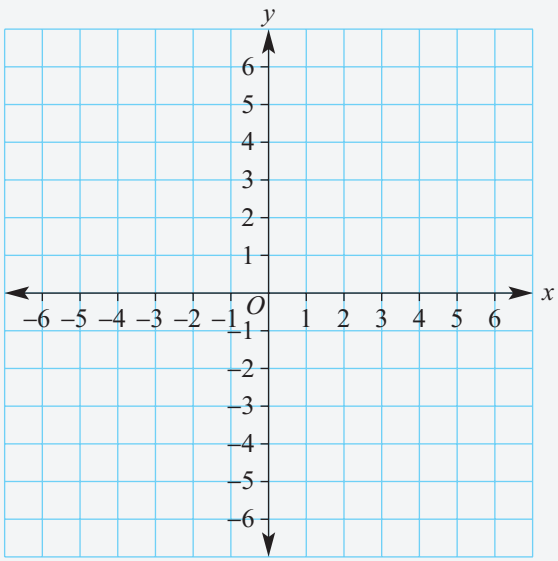
11 Plot these points to decode a secret message. Join points with a line segment if there is a + sign between coordinate pairs. The first one has been done for you.



(–3, 5) + (–4, 5) + (–4, 1) + (–3, 1)
(–2, 2) + (–2, 4) + (–3, 4) + (–3, 2) + (–2, 2)
(0, 2) + (0, 4) + (–1, 4) + (–1, 2) + (0, 2)
(0, 5) + (1, 5) + (1, 1) + (0, 1)
(2, 2) + (2, 4)
(3, 2) + (4, 2) + (4, 3) + (3, 3) + (3, 4) + (4, 4)
(–1, –2) + (–1, 0)
(–2, 0) + (0, 0)
(1, 0) + (1, –2)
(1, –1) + (2, –1)
(2, 0) + (2, –2)
(4, 0) + (3, 0) + (3, –2) + (4, –2)
(3, –1) + (4, –1)
(–4, –3) + (–4, –5) + (–3, –5) + (–3, –3) + (–4, –3)
(–2, –5) + (–2, –3) + (–1, –3) + (–1, –4) + (–2, –4) + (–1, –5)
(0, –3) + (0, –5)
(2, –3) + (1, –3) + (1, –5) + (2, –5) + (2, –4) + $(1\frac{1}{2}, -4)$
(3, –3) + (3, –5)
(4, –5) + (4, –3) + (5, –5) + (5, –3)

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1 Plot the following points to find out what I am. Join points with a line segment if there is a + sign between coordinate pairs.
(–3, 2) + (–3, –4) + (3, –4) + (3, 2)
(–1, –4) + (–1, 0) + (1, 0) + (1, –4)
(–5, 0) + (0, 5) + (5, 0)



2 Insert brackets (if necessary) and symbols (i.e. +, –, × or ÷) into these number sentences to make them true.

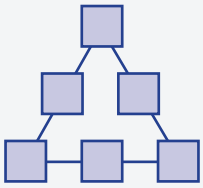
a –3 ☐ 4 ☐ –2 = –6

b –2 ☐ 5 ☐ –1 ☐ 11 = 21

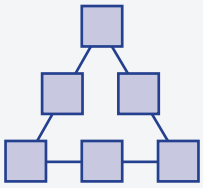
c 1 ☐ 30 ☐ –6 ☐ –2 = –3

3 Place the integers –3, –2, –1, 0, 1 and 2 into the triangle so that the sum of every side is:

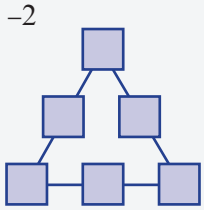
a –3



b 0



c –2



4 Find the next three numbers in these patterns.

a 3, –9, 27, ___, ___, ___

b –32, 16, –8, ___, ___, ___

c 0, –1, –3, –6, ___, ___, ___

d –1, –1, –2, –3, –5, ___, ___, ___

5 **a** The difference between two numbers is 14 and their sum is 8. What are the two numbers?

b The difference between two numbers is 31 and their sum is 11. What are the two numbers?

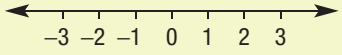
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Chapter summary

Computation with positive and negative integers

Number line

negative positive

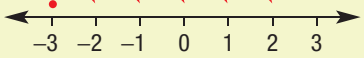


$-2 < 3$ $1 > -1$

$-5 < -1$ $-2 > -6$

Adding or subtracting a positive integer

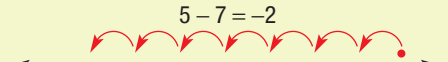
$-3 + 5 = 2$



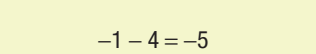
$-4 + 3 = -1$



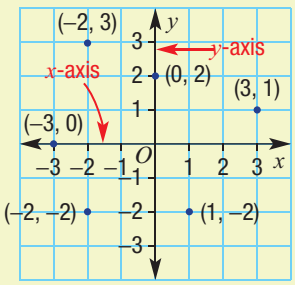
$5 - 7 = -2$



$-1 - 4 = -5$



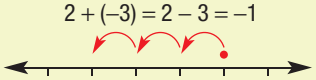
Cartesian plane



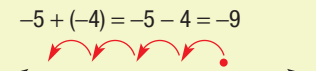
O is the origin.

Adding or subtracting a negative integer

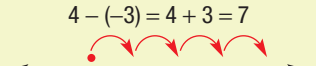
$2 + (-3) = 2 - 3 = -1$



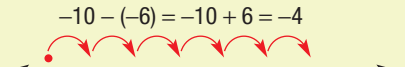
$-5 + (-4) = -5 - 4 = -9$



$4 - (-3) = 4 + 3 = 7$



$-10 - (-6) = -10 + 6 = -4$



Multiplication

(pos.) × (pos.) = (pos.)
(neg.) × (neg.) = (pos.)

(pos.) × (neg.) = (neg.)
(neg.) × (pos.) = (neg.)

The same rules apply for division.

Order of operations

First, brackets
then
× or ÷ (left to right)
then
+ or – (left to right)

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 When the numbers –4, 0, –1, 7 and –6 are arranged from lowest to highest, the correct sequence is:
A 0, –1, –4, –6, 7 **B** 0, –4, –6, –1, 7 **C** –6, –4, –1, 0, 7
D –1, –4, –6, 0, 7 **E** –6, –1, 0, –4, 7 **F** 7, –6, –4, –1, 0

2 The difference between –3 and 4 is:
A –12 **B** –1 **C** 1 **D** –7 **E** 7

3 The missing number in $2 - \square = 3$ is:
A 1 **B** –1 **C** 5 **D** –5 **E** 2

4 Which of the following is true?
A $2 < -1$ **B** $-4 > -3$ **C** $0 < -3$ **D** $-4 < -2$ **E** $1 < -4$

5 The temperature inside a mountain hut is –5°C. After burning a fire for 2 hours, the temperature rises to 17°C. What is the rise in temperature?
A –12°C **B** 12°C **C** 22°C **D** –85°C **E** –22°C

6 $-2 + (-3)$ is equal to:
A –5 **B** 1 **C** –2 **D** –1 **E** 5

7 $5 - (-2) + (-7)$ is equal to:
A –4 **B** 10 **C** 7 **D** 0 **E** 14

8 Which operation (i.e. addition, subtraction, multiplication or division) is completed *second* in the calculation of $(-2 + 5) \times 3 + 1$?
A addition **B** subtraction **C** multiplication
D division **E** brackets

9 $5 - (-2)$ is equal to:
A –3 **B** 3 **C** 7 **D** 10 **E** –7

10 The points $A(-2, 3)$, $B(-3, -1)$, $C(1, -1)$ and $D(0, 3)$ are joined on a Cartesian plane. What shape do they make?
A triangle **B** square **C** trapezium
D kite **E** parallelogram

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Chapter review

Short-answer questions

1 Write the missing numbers.

a $-4, \square, -2, -1, \square, 1$ **b** $3, 2, \square, 0, -1, \square$

c $-10, -8, \square, -4, \square, 0$ **d** $9, 4, \square, -6, \square, -16$

2 Insert the symbol < (is less than) or > (is greater than) into each of these statements to make it true.

a $0 \square 7$ **b** $-1 \square 4$ **c** $3 \square -7$ **d** $-11 \square -6$

3 Evaluate:

a $2 - 7$ **b** $-4 + 2$ **c** $0 - 15$ **d** $-36 + 37$

e $-3 + 2$ **f** $-7 + 9$ **g** $-2 - 5$ **h** $-6 - 19$

i $-4 - 7$ **j** $4 - 11$ **k** $-16 - 31$ **l** $-126 - 5$

4 Evaluate:

a $2 + (-1)$ **b** $5 + (-2)$ **c** $-1 + (-2)$ **d** $-3 + (-4)$

e $5 + (-7)$ **f** $-1 + (-4)$ **g** $10 - (-2)$ **h** $-21 - (-3)$

i $4 - (-3)$ **j** $-18 - (-1)$ **k** $-2 - (-5)$ **l** $-15 - (-18)$

5 Evaluate:

a $1 - 5 + (-2)$ **b** $-3 + 7 - (-1)$ **c** $0 + (-1) - 10$

d $-2 - (-3) - (-4)$ **e** $-1 + (-2) - 3$ **f** $-4 - (-1) - 3$

g $-6 - (-4) + 7$ **h** $-17 + (-14) - (-2)$ **i** $-13 + 2 - (-5)$

6 Find the missing number for each of the following.

a $-2 + \square = -3$ **b** $-1 + \square = -10$ **c** $5 - \square = 6$ **d** $-2 - \square = -4$

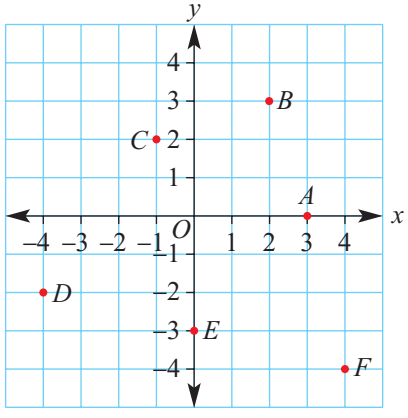
e $-1 - \square = 20$ **f** $-15 - \square = -13$ **g** $7 + \square = -80$ **h** $-15 + \square = 15$

7 Evaluate, using order of operations.

a $-2 + 3 \times 2$ **b** $-4 - 7 \div 7$ **c** $3 \times 4 - 15$ **d** $25 \div 5 - (-2)$

e $(3 - (-2)) \times 6$ **f** $2 \times (4 - (-1))$ **g** $(7 + (-3)) \div 4$ **h** $20 \times (-1 - (-3))$

8 For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C, D, E and F.



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Chapter review

Extended-response questions

1 A scientist is camped on the ice in Greenland. She records the following details in her notepad about the temperature over five days. Note that 'min' stands for minimum and 'max' stands for maximum.

- Monday: min = –18°C, max = –2°C.
- Decreased 29°C from Monday's max to give Tuesday's min.
- Wednesday's min was –23°C.
- Max was only –8°C on Thursday.
- Friday's min is 19°C colder than Thursday's max.

a What is the overall temperature increase on Monday?
b What is Tuesday's minimum temperature?
c What is the difference between the minimum temperatures for Tuesday and Wednesday?
d What is the overall temperature drop from Thursday's maximum to Friday's minimum?
e By how much will the temperature need to rise on Friday if its maximum is 0°C?



2 When joined, these points form a picture on a Cartesian plane. What is the picture?
$A(0, 5)$, $B(1, 3)$, $C(1, 1)$, $D(2, 0)$, $E(1, 0)$, $F(1, -2)$, $G(3, -5)$, $H(-3, -5)$, $I(-1, -2)$, $J(-1, 0)$, $K(-2, 0)$, $L(-1, 1)$, $M(-1, 3)$, $N(0, 5)$

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Chapter 4

Understanding fractions, decimals and percentages

What you will learn

4A Factors and multiples
4B Highest common factor and lowest common multiple
4C What are fractions?
4D Equivalent fractions and simplified fractions
4E Mixed numerals and improper fractions
4F Ordering fractions
4G Place value in decimals and ordering decimals
4H Rounding decimals
4I Decimal and fraction conversions
4J Connecting percentages with fractions and decimals
4K Decimal and percentage conversions
4L Fraction and percentage conversions
4M Percentage of a quantity
4N Using fractions and percentages to compare two quantities

Strand: Number and Algebra

Substrand: FRACTIONS, DECIMALS AND PERCENTAGES

In this chapter, you will learn to:

- operate with fractions, decimals and percentages.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Ancient Egyptian fractions

The ancient Egyptians used fractions over 4000 years ago. They had a system for writing fractions and they also used special symbols for six important fractions. Together, these six symbols made up a design called the Eye of Horus, after the Egyptian sky god. For example, instead of writing $\frac{1}{2}$, Egyptians would draw to represent part of the eye .

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

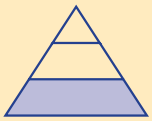
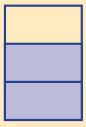
Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

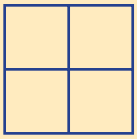
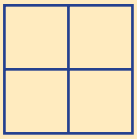
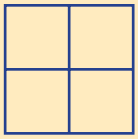
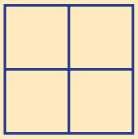
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Pre-test

1 Which of the following indicates that one-third is shaded?

A  **B**  **C**  **D** 

2 Copy each diagram and shade the given fraction.

a  $\frac{1}{2}$ **b**  $\frac{1}{4}$ **c**  $\frac{3}{4}$ **d**  $\frac{4}{4}$

3 Write the following as fractions.

a one-half **b** one-third **c** two-thirds **d** one-tenth **e** three-quarters

4 Which of the following is *not* equivalent to (i.e. *not* the same as) one whole?

A $\frac{2}{2}$ **B** $\frac{6}{6}$ **C** $\frac{1}{4}$ **D** $\frac{12}{12}$

5 Which of the following is *not* equivalent to one-half?

A $\frac{2}{4}$ **B** $\frac{3}{9}$ **C** $\frac{5}{10}$ **D** $\frac{10}{20}$

6 Find:

a $1-\frac{1}{4}$ **b** $1-\frac{1}{2}$ **c** $1-\frac{1}{3}$ **d** $1-\frac{1}{5}$

7 Find:

a $3-\frac{1}{4}$ **b** $2-\frac{1}{2}$ **c** $10-\frac{1}{2}$ **d** $6-\frac{3}{4}$

8 Tom eats half a block of chocolate on Monday and half of the remaining block on Tuesday. What fraction of the block is left for Wednesday?

9 Find the next three terms in these number sequences.

a $0, \frac{1}{2}, 1, 1\frac{1}{2}, __, __, __$ **b** $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, __, __, __$

c $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, __, __, __$ **d** $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, __, __, __$

10 Write 1, 3 or 4 in each box to make a true statement.

a $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\square\times\frac{1}{2}$ **b** $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\square\times\frac{3}{4}$ **c** $\frac{5}{6}\times\square=\frac{5}{6}$ **d** $\frac{6}{8}\div\square=\frac{6}{8}$

11 Find:

a $\frac{1}{2}$ of \$15 **b** $\frac{1}{4}$ of \$160 **c** $\frac{3}{4}$ of \$1 **d** $\frac{1}{3}$ of \$6

12 State whether each of the following is true or false.

a $\frac{1}{2}$ of $16 = 16 \div 2$ **b** $\frac{16}{4}=\frac{1}{4}$ of 16 **c** $\frac{3}{4}$ of $100 = 75$ **d** one-tenth $=\frac{1}{100}$

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4A Factors and multiples

One dozen doughnuts are generally packed into bags or boxes with 3 rows of 4 doughnuts each. Since $3 \times 4 = 12$, we can say that 3 and 4 are factors of 12.

Purchasing 'multiple' packs of 12 doughnuts could result in buying 24, 36, 48 or 60 doughnuts, depending on the number of packs. These numbers are known as multiples of 12.

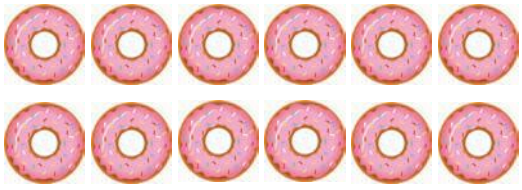


How many factors are there in a set of 12?

Let's start: Using factors

Dinky Doughnuts want to make a new box for 12 doughnuts. Here are three possibilities.

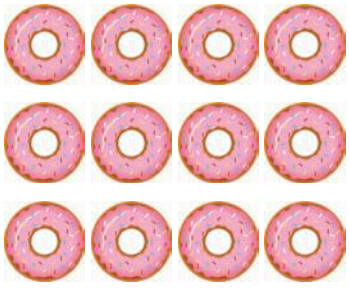
1 A 2 by 6 array



2 A 1 by 12 array



3 A 3 by 4 array



- Are there any other ways to package the 12 doughnuts?
  2, 6, 1, 12, 3 and 4 are all factors of 12. When arranged in ascending order this gives 1, 2, 3, 4, 6, 12.
- Use the arrays to list all the factors of 12.
- Use arrays to find all the ways of packaging 16 doughnuts.
- List all the factors of 16.

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Key ideas

Factor A whole number that will divide into another number without a remainder

Ascending Increasing, from smallest to largest

Multiple A multiple of a number is the product of that number and any positive integer

- **Factors** of a number divide exactly into that number. For example, $20 \div 4 = 5$ exactly, so 4 is a factor of 20.
  - Factors of 20 listed in pairs: $1 \times 20 = 20$, $2 \times 10 = 20$, $4 \times 5 = 20$
  - Factors of 20 in **ascending** (i.e. increasing) order: 1, 2, 4, 5, 10, 20

1 is the smallest factor of any number.

The largest factor of any number is the number itself.

- **Multiples** of a number are made when that number is multiplied by 1, 2, 3 etc.
  - For example, the multiples of 5 (in ascending order) are 5, 10, 15, 20, ...
  - Another way to find multiples of 5 is to start with 5 and keep adding 5. (The same as the 5 times table.)

+5 +5 +5

5, 10, 15, 20, ...

The smallest multiple of a number is the number itself.

Multiples just keep on getting bigger.

Exercise 4A

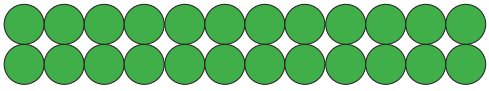
Understanding

1 You and a partner will need 24 counters for this question.

Refer back to 'Let's start'.



a Arrange the 24 counters into an array with 2 rows of 12 counters.



b Copy and complete the factor pairs for this rectangle.
$12 \times \square = 24$ or $\square \times 12 = 24$

c Using all 24 counters each time, make three other rectangles and write down a factor pair for each.

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Example 1 Finding factor pairs

Write the pairs of factors for 18. Then write the factors in ascending order.

Solution	Explanation
$1 \times 18 = 18$, $2 \times 9 = 18$, $3 \times 6 = 18$	Think of all the number pairs that multiply together to equal 18.
Factors of 18 are: 1, 2, 3, 6, 9, 18	Ascending means increasing, so start with the smallest factor.

2 Copy and complete:

a $1 \times \square = 12$, $2 \times \square = 12$, $3 \times \square = 12$
The factors of 12 are 1, __, __, __, __, 12

b $1 \times \square = 5$
The factors of 5 are __, __

c $1 \times \square = 30$, $2 \times \square = 30$, $3 \times \square = 30$, $5 \times \square = 30$
The factors of 30 are __, __, __, __, __, __, __, __



List the factors in ascending order (i.e. from smallest to largest).

Example 2 Finding multiples

Copy and complete these multiples of 8: 8, __, __, 32

Solution	Explanation
8, 16, 24, 32	+8 +8 +8 8, 16, 24, 32

3 Copy and complete:

a The first six multiples of 5 are 5, __, 15, __, 25, __

b The first six multiples of 10 are 10, __, 30, __, __, __

c The first six multiples of 7 are 7, __, __, 28, __, 42



To make multiples of 5, start with 5 and keep adding 5.

4 Copy and complete:

a The first eight even numbers are: 2, __, 6, __, 10, __, 14, __

b An even number is a multiple of __.

c Odd numbers do not have __ as a factor.

d The first eight odd numbers are: 1, __, 5, __, 9, __, 13, __

5 Copy and complete:

a $7 \times 6 = 42$ so 7 and 6 are _____ of 42.

b To make multiples of 6, multiply 6 by whole numbers. For example:
$1 \times 6 = 6$, $\square \times 6 = 12$, $\square \times 6 = 18$, $\square \times 6 = 24$

c Some multiples of 6 are: 6, $\square$, 18, $\square$, $\square$

d Counting in sixes gives _____ of 6.

e If you start with 6 and then keep adding 6, you will produce a list of _____ of 6.

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4A

Fluency

Example 3 Finding factors

Find the complete set of factors for each of these numbers.

a 15 **b** 40

Solution	Explanation
a Factors of 15 are 1, 3, 5, 15.	$1 \times 15 = 15$, $3 \times 5 = 15$
b Factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40.	$1 \times 40 = 40$, $2 \times 20 = 40$ $4 \times 10 = 40$, $5 \times 8 = 40$ The last number you need to check is 7.

Drilling for Gold 4A1

6 List the complete set of factors for each of the following numbers.

a 10 **b** 24 **c** 17
d 36 **e** 60 **f** 42
g 80 **h** 12 **i** 28



List the factors in ascending order (smallest to largest).

Example 4 Listing multiples

Write down the first six multiples for each of these numbers.

a 11 **b** 35

Solution	Explanation
a 11, 22, 33, 44, 55, 66	The first multiple is always the given number. Add on the given number to find the next multiple. Repeat this process to get more multiples.
b 35, 70, 105, 140, 175, 210	Start at 35, the given number, and repeatedly add 35 to continue producing multiples.

Drilling for Gold 4A2

7 Write down the first six multiples for each of the following numbers.

a 5 **b** 8 **c** 12
d 7 **e** 20 **f** 75
g 15 **h** 100 **i** 37



List the multiples in ascending order.

8 Fill in the gaps to complete the set of factors for each of the following numbers.

a 18 1, 2, ___, 6, 9, ___
b 25 1, ___, 25
c 50 ___, 2, ___, 10, ___, ___
d 100 1, ___, 4, ___, ___, 20, ___, 50, ___

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Example 5 Choosing multiples

Which number is the *wrong* multiple for the sequence 7, 14, 20, 28, 35? Write the correct sequence.

Solution	Explanation
20 is incorrect. 7, 14, 21, 28, 35	$14 + 7 = 21$ or $3 \times 7 = 21$ A multiple of 7 must be a whole number times 7.

9 Find the *wrong* multiple in each of the following. Write the correct sequence.

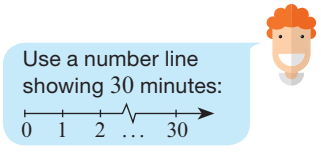
a 3, 6, 9, 12, 15, 18, 22, 24, 27, 30
b 5, 10, 15, 20, 24, 30
c 11, 21, 33, 44, 55, 66, 77, 88, 99, 110
d 12, 24, 36, 49, 60, 72, 84

Problem-solving and Reasoning

10 What is the smallest number which has exactly:

a 3 factors? **b** 4 factors? **c** 5 factors? **d** 6 factors?

11 Zane and Matt are both keen runners. Zane takes 4 minutes to jog around a running track and Matt takes 5 minutes. They start together at the same time and jog 6 laps for training.



a Copy and complete:
  i When Zane jogged 6 laps around the track, he crossed the start/finish line after 4, 8, ___, ___, ___ and ___ minutes.
  ii Matt crossed the line after ___, ___, ___, ___, ___ and ___ minutes.
b How many laps had each boy jogged when they crossed the line together?
c For how long had the boys jogged before they crossed the line together?

12 Anson is preparing for his 12th birthday party. He has invited 5 friends and is making each of them a 'lolly bag' to take home after the party. To be fair, he wants to make sure that each friend has the same number of lollies. Anson has a total of 67 lollies to share among the lolly bags.



a How many lollies does Anson put in each of his friends' lolly bags?
b How many lollies does Anson have left over to eat himself?

Anson then decides that he wants a lolly bag for himself also.

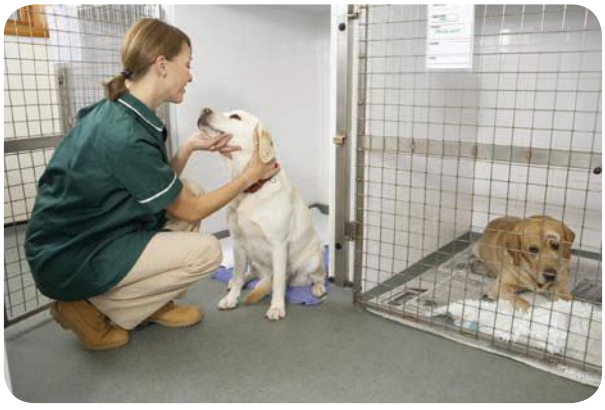
c How many lollies will now go into each of the 6 lolly bags?
d With 6 lolly bags, are there any lollies left over?

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4A

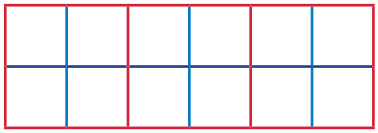
Enrichment: Using factors to make squares or rectangles

13 Joanna is a vet. She wants to buy several pens for separating sick animals. There are square or rectangular pens available, all with sides that are 1, 2, 3 or 4 metres long.

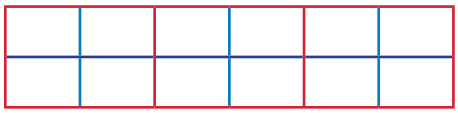


a Joanna wants to buy equal-sized pens to cover an area of 12 m^2. Two possibilities are shown below. Find the other four possibilities Joanna could buy.

Option 1: Buy 3 square pens.
Each pen has: $\ell = 2$ m,
$b = 2$ m, $A = 4$ m^2
Total area $= 3 \times 4 = 12$ m^2



Option 2: Buy 2 rectangular pens.
Each pen has: $b = 2$ m, $\ell = 3$ m, $A = 6$ m^2
Total area $= 2 \times 6 = 12$ m^2





Use grid paper and draw the pens. Note that 2 m × 3 m pens are the same as 3 m × 2 m pens, so don't count them twice.

For the following questions, list all possible answers.

b Work out the number of equal-sized *square* pens and their sizes so all these pens joined side by side cover an area of:
i 32 m^2 **ii** 54 m^2

c Work out the number of equal-sized *rectangular* pens and their sizes so all these pens joined side by side cover an area of:
i 32 m^2 **ii** 54 m^2

d A zoo vet wants to buy several larger, equal-sized pens to cover 150 m^2. Find all the possibilities with whole-number sides from 2 to 25 metres.

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4B Highest common factor and lowest common multiple

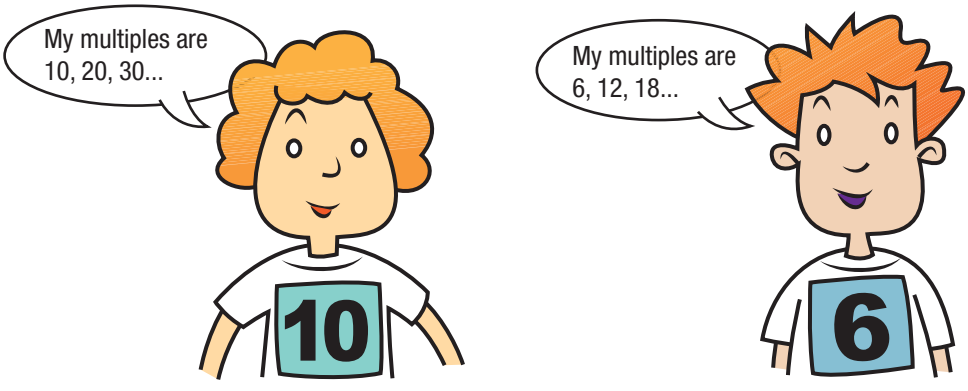
In Section 4B, we worked with factors and multiples. In this section, we look at common factors, or multiples that are shared by two numbers.

It is often useful to be able to find the highest common factor (HCF) or lowest common multiple (LCM).

Let's start: Finding special numbers



We share some common factors. What are they?
Which is the highest (i.e. biggest) common factor we share?



We share some common multiples. What are they?
What is the lowest (i.e. smallest) multiple we share?

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Key ideas

Highest common factor (HCF) The largest number that is a factor of all the given numbers

Lowest common multiple (LCM) The smallest number that two or more numbers divide into without a remainder

- **HCF** stands for **highest common factor**. (Common means shared.)
  - For example: Find the HCF of 24 and 40.
    Factors of 24 are (1), (2), 3, (4), 6, (8), 12 and 24.
    Factors of 40 are (1), (2), (4), 5, (8), 10, 20 and 40.
    ← List the factors. Circle shared factors.
    HCF = 8. ← The biggest shared factor is the HCF.
- **LCM** stands for **lowest common multiple**.
  - For example: Find the LCM of 20 and 12.
    Multiples of 20 are 20, 40, (60), 80, 100, 120, 140, …
    Multiples of 12 are 12, 24, 36, 48, (60), 72, 84, 96, …
    ← Circle the first common multiple.
    LCM = 60. ← LCM is first shared multiple.

Exercise 4B

Understanding

1 Copy and complete the following.

a HCF stands for _____ _____ _____.

b To find the HCF, we first list all the _____ of each number.

c The HCF is the highest _____ (i.e. shared) factor.

d LCM stands for _____ _____ _____.

e To find the LCM, we list _____ of each number until we get a _____ (i.e. shared) multiple.



Look back at the Key ideas.

Example 6 Listing common factors to find the HCF



Drilling for Gold 4B1

The factors of 8 are 1, 2, 4, 8. The factors of 20 are 1, 2, 4, 5, 10, 20.

a What are the common factors of 8 and 20?

b What is the HCF of 8 and 20?

Solution	Explanation	
a 1, 2, 4	(1), (2), (4), 8 (1), (2), (4), 5, 10, 20	Circle common (i.e. shared) factors.
b HCF = 4	Largest circled number is the HCF.	

2 The factors of 12 are 1, 2, 3, 4, 6, 12. The factors of 16 are 1, 2, 4, 8, 16.

a What are the common factors of 12 and 16?

b What is the HCF of 12 and 16?



Common factors are shared factors.

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3 Copy and complete to find the HCF of 18 and 30:
Factors of 18 are 1, __, 3, __, __, 18.
Factors of 30 are 1, __, __, 5, __, 10, __, 30.
HCF = __.



$1 \times 18 = 18$
$_ \times _ = 18$
$3 \times _ = 18$

Example 7 Listing common multiples to find the LCM



The first six multiples of 3 are 3, 6, 9, 12, 15, 18.
The first six multiples of 2 are 2, 4, 6, 8, 10, 12.

a What are two common multiples of 2 and 3?
b What is the LCM of 2 and 3?

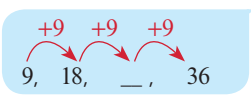
Solution	Explanation
a 6, 12	3, (6), 9, (12), 15, 18 2, 4, (6), 8, 10, (12) Circle common (i.e. shared) multiples.
b LCM = 6	The first circled number is the LCM.

4 The first 10 multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80.
The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60.

a What are two common multiples of 8 and 6?
b What is the LCM of 8 and 6?

Fluency

5 Copy and complete to find the LCM of 9 and 15:
Multiples of 9 are 9, 18, __, 36, __, __, __, __, 81, __.
Multiples of 15 are __, 30, __, 60, 75, __, __, 120.
LCM = __.





Example 8 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 36 and 48.

Solution	Explanation
Factors of 36 are: (1), (2), (3), (4), (6), 9, (12), 18 and 36.	Find the factors of 36 ($1 \times 36 = 36$, $2 \times 18 = 36$ etc.) and list them in order.
Factors of 48 are: (1), (2), (3), (4), (6), 8, (12), 16, 24 and 48.	Do the same thing for 48 ($1 \times 48 = 48$, $2 \times 24 = 48$ etc.).
HCF = 12.	Circle common (i.e. shared) factors. Pick the HCF (biggest shared factor).

6 Find the HCF of the following numbers.

a 4 and 5
b 8 and 13
c 2 and 12
d 3 and 15
e 16 and 20
f 15 and 60
g 20, 40 and 50
h 12, 15 and 30



First list the factors of each number.

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4B

Example 9 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 6 and 10.

Solution	Explanation
6, 12, 18, 24, (30), 36, ...	List multiples of each number.
10, 20, (30), 40, ...	Circle the first common (i.e. shared) multiple.
LCM = 30	This is the LCM.

Skillsheet 4A

7 Find the LCM of the following numbers.

a 4 and 5 **b** 3 and 7 **c** 5 and 6
d 8 and 10 **e** 4 and 6 **f** 5 and 10
g 2, 3 and 5 **h** 3, 4 and 5 **i** 2, 3 and 7



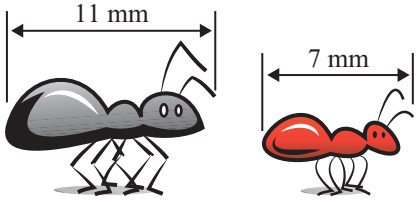
First list some multiples of each number.

Problem-solving and Reasoning

8 A trail (i.e. line) of red ants is next to a trail of black ants.

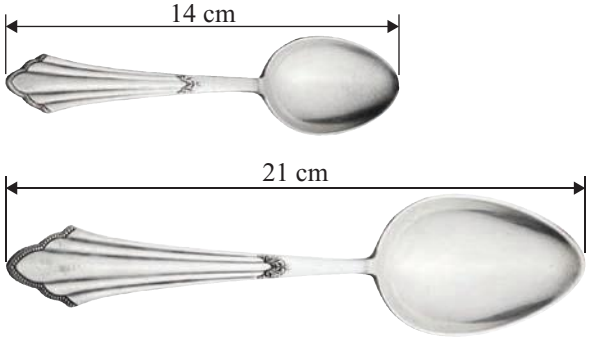


The LCM might be useful.



a What is the smallest number of red ants and the smallest number of black ants that would make the trails equal in length?
b How long would that trail be?

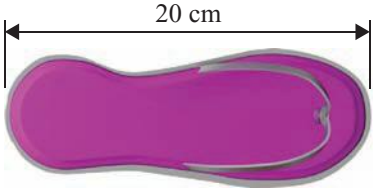
9 A line of spoons is next to a line of teaspoons.

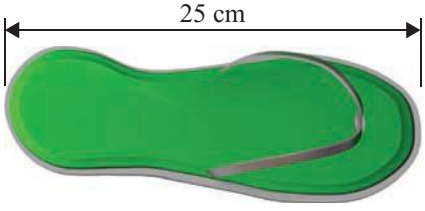


a What is the smallest number of each spoon that would make the lines equal in length?
b How long would that line be?
c The answer to part **b** is called the _____ of 14 and 21.

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10 A line of pink thongs is next to a line of green thongs.







Remember that LCM means lowest common multiple.

a Suppose that the two lines of thongs are the same length. How long could each line be? Find three possible answers.

b Which of the answers in part **a** is the LCM of 20 and 25?

11 Wendy is a florist who is making up small bunches of roses for sale. She has 36 red roses, 42 pink roses and 30 cream roses.







Wendy uses only one colour for each bunch. She wants to use all the roses. Each bunch must have the same total number of roses.

a What is the largest number of roses Wendy can put in each bunch?

b The answer to part **a** is called the _____ of 36, 42 and 30.

c How many bunches of each colour can she make?

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4B

12 Given that the HCF of a pair of different numbers is 8, find the two numbers:

a if both numbers are less than 20

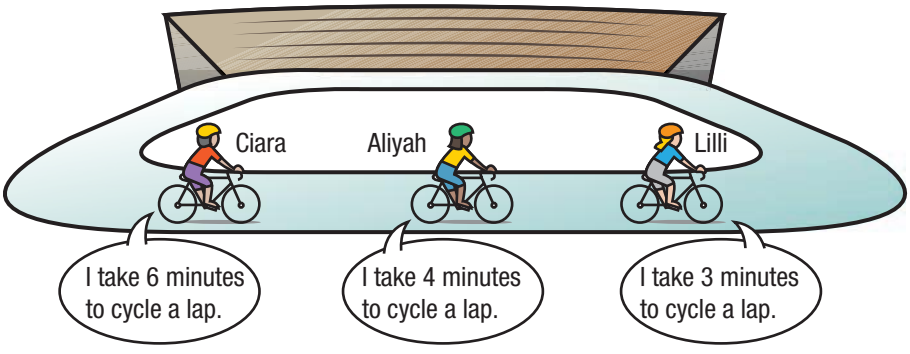
b when one number is in the 20s and the other in the 30s



Remember, HCF means highest common factor.

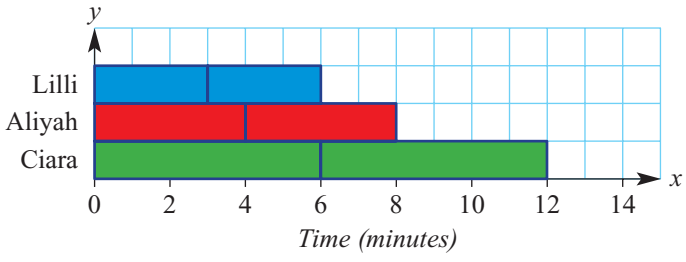
Enrichment: Cycling laps

13 Three girls are riding their bikes around a circular track. They all start together.



Lilli takes 3 minutes to cycle each lap (i.e. she crosses the starting line every 3 minutes). Aliyah takes 4 minutes for each lap and Ciara takes 6 minutes.

This bar graph shows the time it takes each girl to complete the first two laps.



a On grid paper, copy the graph and extend it to show the number of laps each girl completes in 24 minutes. (Draw a rectangle for each lap cycled.)

b When do all three girls first cross the starting line together? (Give your answer as the number of minutes after the start.)

c How many full laps has each girl completed after 24 minutes? (Count whole laps only.)

d Suppose that each girl rides 15 laps. How many minutes after Lilli will:

i Aliyah finish? **ii** Ciara finish?

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4C What are fractions?

The word fraction comes from the Latin word 'frangere', which means 'to break into pieces'. Fractions are parts of a whole.

We all use fractions every day. Examples could include cooking, shopping, sporting, building construction and more.



Every piece is a fraction of the whole block.

Let's start: What strength do you like your cordial?

- Tom uses 40 mL of cordial and 120 mL of water.
- Amelia uses 40 mL of cordial with 200 mL of water.
- Who likes their drink the strongest? How can fractions be used to describe the strengths of cordial?

Key ideas

- Every **fraction** has a **n**umerator (**u**p) and a **d**enominator (**d**own).
  - For example: 3 out of 5 regions are shaded. This circle is divided into five equal **regions**.



$\frac{3}{5}$ ← numerator
$\frac{3}{5}$ ← denominator

  - The **denominator** tells you how many regions the whole is divided into.
  - The **numerator** tells you how many regions are shaded.
  - The horizontal line separating the numerator and the denominator is called the **vinculum**.
- A **proper fraction** is less than a whole (i.e. the numerator must be smaller than the denominator).
  For example: $\frac{2}{7}$ is a proper fraction.
- An **improper fraction** is greater than a whole (i.e. the numerator must be larger than the denominator).
  For example: $\frac{5}{3}$ is an improper fraction.
- Whole numbers can be represented as fractions.
  For example: $1 = \frac{4}{4}$, $1 = \frac{3}{3}$, $2 = \frac{8}{4}$
- Mixed numerals have a whole number and a fraction.
  For example: $1\frac{2}{3}$ is a mixed numeral.

Fraction Part of a whole

Denominator The number in a fraction below the vinculum. It is the number of equal parts into which the whole is divided

Numerator The number in a fraction above the vinculum

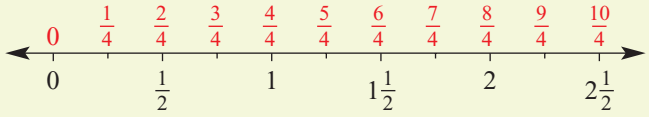
Vinculum The separating line in a fraction

Proper fraction A fraction where the numerator is less than the denominator

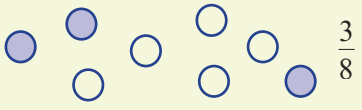
Improper fraction A fraction where the numerator is greater than the denominator

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- All fractions can be placed on the number line.



- Fractions are also used to represent a group of objects within a larger group.
  - For example: In this **set** of 8 circles, 3 out of 8 are shaded.

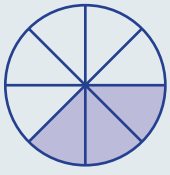


Exercise 4C

Understanding

Example 10 Understanding the numerator and the denominator

A pizza has been divided into equal pieces.



a Into how many pieces has the whole pizza been divided?
b How many pieces have been selected (i.e. shaded)?
c In representing the shaded fraction of the pizza:
  i What must the denominator equal?
  ii What must the numerator equal?
  iii Write the amount of pizza selected (i.e. shaded) as a fraction.
  iv Write the fraction in words.
d Is the fraction proper or improper?

Solution	Explanation
a 8	Pizza cut into 8 equal pieces.
b 3	3 of the 8 pieces are shaded.
c i 8	Denominator shows the number of parts the whole has been divided into.
ii 3	Numerator tells how many of the divided parts you have selected.
iii $\frac{3}{8}$	Shaded fraction is the numerator over the denominator; i.e. 3 out of 8 divided pieces.
iv three-eighths	The denominator tells us we have eighths.
d proper	The fraction is less than 1.

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1 **a** State the denominator of this proper fraction: $\frac{2}{9}$.

b State the numerator of this improper fraction: $\frac{7}{5}$.

2 Group the following list of fractions into proper fractions, improper fractions and whole numbers.

a $\frac{7}{6}$ **b** $\frac{2}{7}$ **c** $\frac{50}{7}$ **d** $\frac{3}{3}$

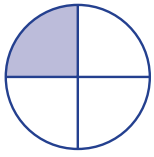
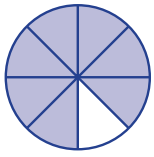
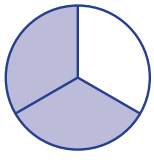
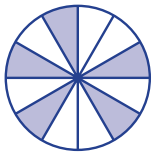
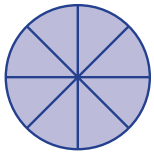
e $\frac{3}{4}$ **f** $\frac{5}{11}$ **g** $\frac{1}{99}$ **h** $\frac{9}{4}$

i $\frac{11}{8}$ **j** $\frac{10}{10}$ **k** $\frac{5}{1}$ **l** $\frac{121}{5}$

m $\frac{12}{12}$ **n** $\frac{12}{6}$ **o** $\frac{9}{4}$ **p** $\frac{100}{3}$

3 Copy and complete the following table.


Drilling for Gold 4C1

The whole (divided into equal parts)	Number of equal parts in the whole (denominator)	Number of shaded equal parts (numerator)	Fraction shaded	Name of fraction
	4			one-quarter
				
		2	$\frac{2}{3}$	
				
		8		one whole

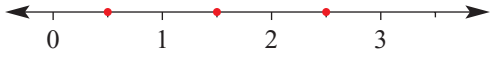
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4C

Drilling for Gold 4C2

4 What fractions are shown on these number lines?

a



b



Write any improper fractions as mixed numerals as well.

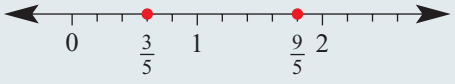


Fluency

Example 11 Representing fractions on a number line

Show the fractions $\frac{3}{5}$ and $\frac{9}{5}$ on a number line.

Solution



Explanation

Draw a number line starting at 0 and mark on it the whole numbers 0, 1 and 2. Divide each whole unit into five segments of equal length. Each of these segments has a length of one-fifth.

5 Represent the following fractions on a number line.

a $\frac{3}{7}$ and $\frac{6}{7}$ **b** $\frac{2}{3}$ and $\frac{5}{3}$ **c** $\frac{1}{6}$ and $\frac{5}{6}$

In part a, to show sevenths, rule a 7 cm line for one whole and mark each cm for one-seventh.



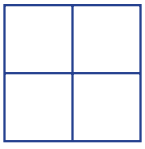
6 Write the following as whole numbers.

a $\frac{5}{5}$ **b** $\frac{6}{6}$ **c** $\frac{8}{8}$ **d** $\frac{16}{8}$

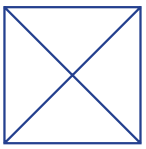
e $\frac{100}{100}$ **f** $\frac{100}{2}$ **g** $\frac{15}{1}$ **h** $\frac{20}{10}$

7 Shade $\frac{3}{4}$ of each of the following diagrams.

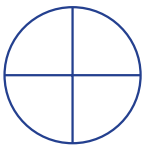
a



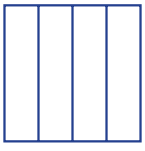
b



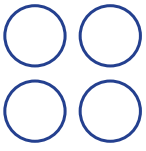
c



d



e



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8 Write the next three fractions for each of the following fraction sequences.

a $\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}, __, __, __$

b $\frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, __, __, __$

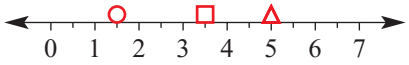
c $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, __, __, __$

d $\frac{11}{7}, \frac{10}{7}, \frac{9}{7}, \frac{8}{7}, __, __, __$

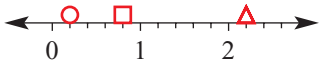
Problem-solving and Reasoning

9 What fraction matches each of the different shapes (○, □ and △) on these number lines?

a



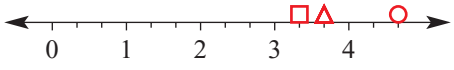
b



c



d

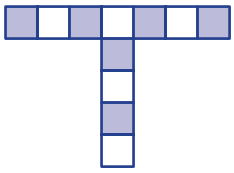


Write any mixed numerals as improper fractions as well.



10 Match each of the following diagrams to one of the fractions in the box below.

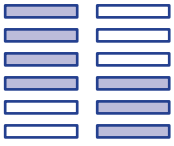
a



b



c



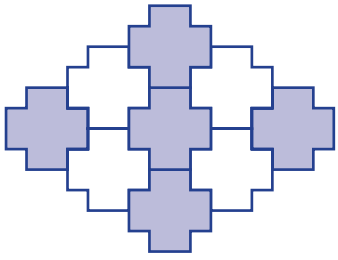
d



e



f



Fractions	$\frac{7}{12}$	$\frac{5}{9}$	$\frac{6}{11}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{6}$

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4C

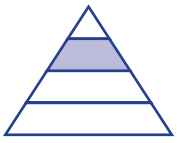
11 For each of the following, write the fraction that is describing part of the total.

- **a** After one day of a 43-kilometre hike, the students had completed 12 kilometres.
- **b** From 15 starters, 13 went on and finished the race.
- **c** Rainfall for 11 months of the year was below average.
- **d** One egg is broken in a carton that contains a dozen eggs.
- **e** Two players in the soccer team scored a goal.
- **f** The lunch stop was 144 kilometres into the 475-kilometre trip.
- **g** Seven members in the class of 20 have visited Australia Zoo.
- **h** One of the car tyres (not including the spare) is worn and needs replacing.
- **i** It rained three days this week.

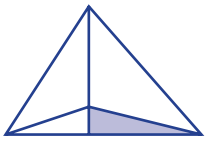


12 Which diagram has one-quarter shaded?

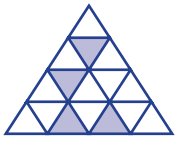
A



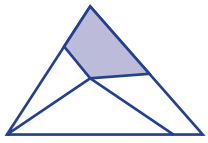
B



C



D



Enrichment: Adjusting concentration

13 a Callum pours 20 mL of water into this beaker. What fraction of 200 mL is that?

- **b** Callum adds acid to the same beaker until it holds 200 mL. How much acid did he add? What fraction of 200 mL is that?
- **c** Rose has a 200 mL beaker that is $\frac{1}{4}$ full of water. How much acid will she need to add to fill the beaker?



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4D Equivalent fractions and simplified fractions



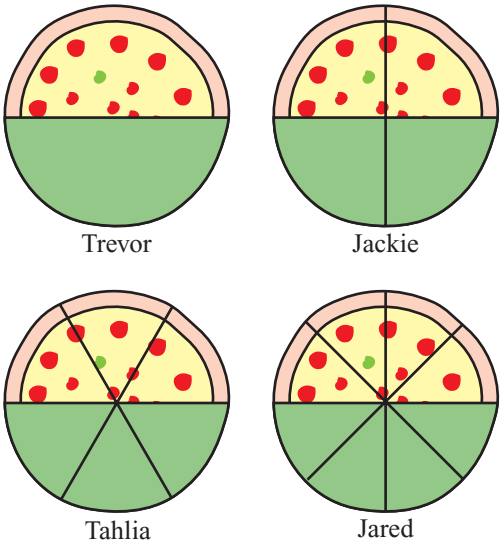
There are four quarters played out in a game of AFL football.

Fractions may look very different but still have the same value. For example, in an AFL football match, 'half-time' is the same as 'the end of the second quarter'. We can say that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Consider a group of friends eating pizzas. Each person cuts up their pizza as they like. The green shading shows the amount eaten.

By looking at the pizzas, it is clear to see that Trevor, Jackie, Tahlia and Jared have all eaten the same amount of pizza.

This means that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.



Let's start: Odd one out

- Pick the fraction that is the odd one out.
  $\frac{25}{100}, \frac{4}{16}, \frac{2}{8}, \frac{2}{5}, \frac{5}{20}$
- What could we call the other four fractions?

Key ideas

- **Equivalent fractions** are fractions that mark the same place on a number line or the same amount shaded.
  For example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.
- Equivalent fractions are produced by multiplying the numerator and denominator by the same number.
- Equivalent fractions can also be produced by dividing the numerator and denominator by the same number.
- Simplifying fractions involves writing a fraction in its 'simplest form'. To do this, the numerator and the denominator must be divided by their **highest common factor (HCF)**.

Equivalent fractions Fractions that represent the same amount

Highest common factor (HCF) The largest number that is a factor of all the given numbers

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Exercise 4D

Understanding

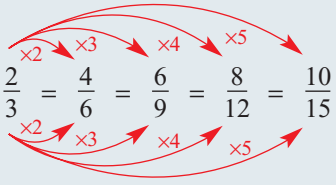
Example 12 Producing equivalent fractions

Write four equivalent fractions for $\frac{2}{3}$ by multiplying by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$ and $\frac{5}{5}$.

Solution

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$$

Explanation



1 Copy and complete the following.

a $\frac{3}{4} = \frac{\square}{8}$ (×2 above, ×2 below)

b $\frac{3}{4} = \frac{\square}{\square}$ (×3 above, ×3 below)

c $\frac{3}{4} = \frac{\square}{\square}$ (×5 above, ×5 below)

d $\frac{3}{4} = \frac{30}{40}$ (×□ above, ×□ below)

e $\frac{2}{5} = \frac{14}{35}$ (×□ above, ×□ below)

f $\frac{4}{7} = \frac{12}{21}$ (×□ above, ×□ below)

2 Write four equivalent fractions for $\frac{3}{5}$ by multiplying by $\frac{3}{3}$, $\frac{5}{5}$, $\frac{10}{10}$ and $\frac{11}{11}$.



Show the steps each time. For example: $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$

3 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\frac{1}{3} = \frac{\square}{6} = \frac{4}{\square} = \frac{\square}{30} = \frac{\square}{60} = \frac{100}{\square}$

b $\frac{2}{8} = \frac{\square}{4} = \frac{\square}{12} = \frac{6}{\square} = \frac{\square}{80} = \frac{10}{\square}$

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Example 13 Converting to simplest form

Write these fractions in simplest form.

a $\frac{18}{24}$ **b** $\frac{7}{42}$

Solution

a Method 1

$$\frac{18}{24} \overset{\div 6}{=} \frac{3}{4}$$

Explanation

Method 1: The HCF of 18 and 24 is 6.
Both the numerator and the denominator are divided by the HCF of 6.

Method 2

$$\frac{18}{24} \overset{\div 2}{=} \frac{9}{12} \overset{\div 3}{=} \frac{3}{4}$$

Method 2: Dividing by 2 and then by 3 is the same as dividing by 6.

b

$$\frac{7}{42} \overset{\div 7}{=} \frac{1}{6}$$

The HCF of 7 and 42 is 7.
The 7 is 'cancelled' from the numerator and the denominator.

4 Fill in the missing numbers to complete the following.

HCF means **H**ighest **C**ommon **F**actor.

a $\frac{10}{30} \overset{\div 10}{=} \frac{\square}{\square}$

b $\frac{4}{8} \overset{\div \square}{=} \frac{1}{4}$

c $\frac{2}{6} \overset{\div \square}{=} \frac{1}{3}$

d $\frac{9}{12} \overset{\div \square}{=} \frac{3}{4}$

e $\frac{20}{28} \overset{\div \square}{=} \frac{5}{7}$

f $\frac{9}{18} \overset{\div 9}{=} \frac{\square}{\square}$

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4D

Fluency

5 Copy and complete these equivalent fractions.

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{5}{8} = \frac{\square}{80}$ **c** $\frac{6}{11} = \frac{18}{\square}$

d $\frac{2}{7} = \frac{16}{\square}$ **e** $\frac{3}{\square} = \frac{15}{40}$ **f** $\frac{\square}{1} = \frac{14}{7}$

g $\frac{\square}{10} = \frac{24}{20}$ **h** $\frac{13}{14} = \frac{\square}{42}$ **i** $\frac{2}{7} = \frac{10}{\square}$

j $\frac{19}{20} = \frac{190}{\square}$ **k** $\frac{11}{21} = \frac{55}{\square}$ **l** $\frac{11}{\square} = \frac{44}{8}$



Always multiply the numerator (up) and denominator (down) by the same number.

$\frac{3}{4} \xrightarrow{\times 3} \frac{\square}{12}$ $\times 3$

6 By writing either = or ≠ in the box, state whether each pair of fractions is equivalent or not equivalent. The first one has been done for you.

a $\frac{1}{2} \boxed{\neq} \frac{5}{8}$ **b** $\frac{4}{8} \square \frac{2}{4}$ **c** $\frac{3}{7} \square \frac{30}{60}$

d $\frac{5}{9} \square \frac{15}{18}$ **e** $\frac{11}{15} \square \frac{33}{45}$ **f** $\frac{1}{2} \square \frac{402}{804}$

g $\frac{12}{36} \square \frac{1}{3}$ **h** $\frac{18}{24} \square \frac{21}{28}$ **i** $\frac{6}{18} \square \frac{11}{33}$



Think: $\frac{1}{2} \xrightarrow{\times 5} \frac{5}{10}$ not $\frac{5}{8}$

or: $\frac{1}{2} \xrightarrow{\times 4} \frac{4}{8}$ not $\frac{5}{8}$

7 Simplify the following fractions.

a $\frac{2}{4}$ **b** $\frac{2}{6}$ **c** $\frac{4}{6}$ **d** $\frac{5}{10}$

e $\frac{6}{8}$ **f** $\frac{6}{9}$ **g** $\frac{3}{9}$ **h** $\frac{6}{10}$



Remember: $\frac{5}{5} = 5 \div 5 = 1$

8 Write the following fractions in simplest form. Check your answers using a calculator.

a $\frac{15}{20}$ **b** $\frac{12}{18}$ **c** $\frac{10}{30}$ **d** $\frac{8}{22}$

e $\frac{14}{35}$ **f** $\frac{2}{22}$ **g** $\frac{8}{56}$ **h** $\frac{9}{27}$

i $\frac{35}{45}$ **j** $\frac{30}{50}$ **k** $\frac{12}{144}$ **l** $\frac{80}{100}$



HCF of 15 and 20 is 5.

$\frac{15}{20} \xrightarrow{\div 5} \frac{3}{4}$ $\div 5$

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Problem-solving and Reasoning

9 In each group, choose the fraction that is not in its simplest form. What should it be?

a $\frac{1}{3}, \frac{3}{8}, \frac{5}{9}, \frac{7}{14}$

b $\frac{2}{5}, \frac{12}{16}, \frac{15}{19}, \frac{13}{37}$

c $\frac{12}{19}, \frac{4}{42}, \frac{5}{24}, \frac{6}{61}$

d $\frac{7}{63}, \frac{9}{62}, \frac{11}{81}, \frac{13}{72}$

10 Which of the following fractions are equivalent to $\frac{8}{20}$?

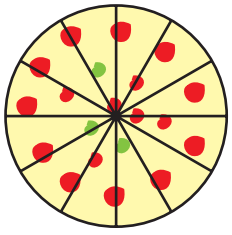
$\frac{4}{10}, \frac{1}{5}, \frac{6}{20}, \frac{8}{10}, \frac{16}{40}, \frac{2}{5}, \frac{4}{12}, \frac{12}{40}, \frac{80}{200}, \frac{1}{4}$

11 A family block of chocolate consists of 6 rows of 6 individual squares. Darcy eats 16 individual squares. What fraction of the block, in simplest terms, has Darcy eaten?





12 Jameel, Joanna and Jake are sharing a large pizza for dinner. The pizza has been cut into 12 equal pieces. Jameel would like $\frac{1}{3}$ of the pizza, Joanna would like $\frac{1}{4}$ of the pizza and Jake will eat whatever is remaining. How much does Jake eat?



Enrichment: Half-way

13 For each of the following, find the fraction that is half-way between:

a $\frac{1}{2}$ and 1

b $\frac{1}{4}$ and $\frac{3}{4}$

c $\frac{1}{5}$ and $\frac{2}{5}$

d $\frac{3}{4}$ and 1

e $\frac{1}{2}$ and $\frac{3}{4}$

f $\frac{9}{10}$ and 1

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4E Mixed numerals and improper fractions

As we have seen in this chapter, a fraction is a common way of representing part of a whole. For example, a particular car trip may require $\frac{2}{3}$ of a tank of petrol.

On many occasions, you may need whole numbers plus a part of a whole number. For example, a long car trip may require $2\frac{1}{4}$ tanks of petrol. When you have a combination of a whole number and a fraction, we call this a mixed numeral.



A long car trip may require more than one tank of petrol.

Let's start: Pizza frenzy

Tom ate $1\frac{1}{2}$ pizzas. Chandra ate $\frac{3}{2}$. Who ate the most? Discuss, showing each person's pizzas on a separate diagram.

Key ideas

Mixed numeral
A number with a whole number part and a fraction part

Improper fraction
A fraction where the numerator is greater than or equal to the denominator

- A number is said to be a **mixed numeral** when it contains a whole number plus a proper fraction.

$2\frac{3}{5}$ is a mixed numeral

whole number: 2 — proper fraction: $\frac{3}{5}$

- **Improper fractions** (fractions greater than a whole, where the numerator is greater than the denominator) can be converted to mixed numerals or whole numbers.

$\frac{15}{4} = 3\frac{3}{4}$

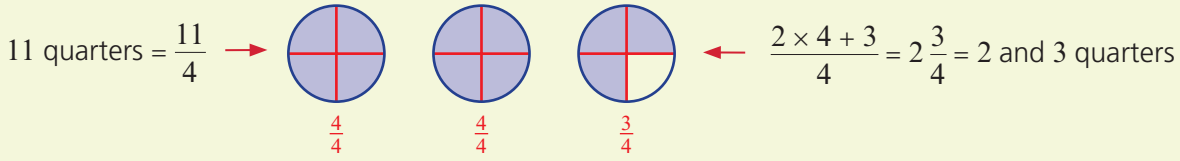
improper fraction: $\frac{15}{4}$ — mixed numeral: $3\frac{3}{4}$ — $15 \div 4 = 3$ remainder 3

$\frac{16}{4} = 4$

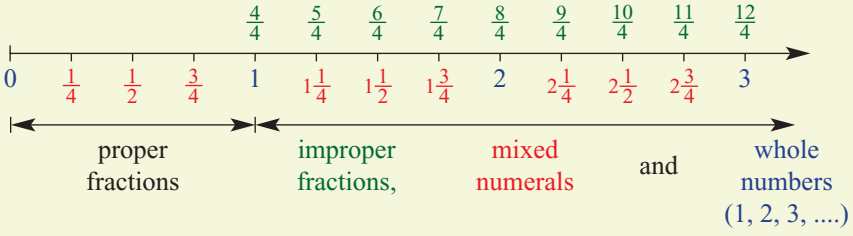
improper fraction: $\frac{16}{4}$ — whole number: 4 — $16 \div 4 = 4$

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- Mixed numerals can be converted to improper fractions, and vice versa.



- A number line helps show the different types of fractions.



Exercise 4E

Understanding

Example 14 Fractions on the number line

a Show $1\frac{3}{4}$ on a number line.

b Between which two whole numbers does it lie?

Solution

a



b $1\frac{3}{4}$ lies between 1 and 2.

Explanation

Draw a number line starting at zero and going up by quarters.

Put a dot at $1\frac{3}{4}$.

The whole number before $1\frac{3}{4}$ is 1 and the whole number after it is 2.

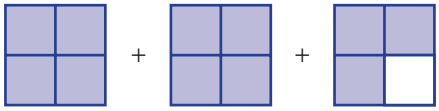
1 Between which two whole numbers do the following mixed numerals lie?

a $2\frac{1}{2}$ **b** $11\frac{1}{7}$ **c** $36\frac{8}{9}$



Draw a number line to help you decide.

2 The mixed numeral $2\frac{3}{4}$ can be represented as regions.

$2\frac{3}{4} =$  $+ \quad +$

Drilling for Gold 4E1

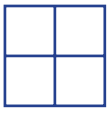
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4E

Represent the following mixed numerals using regions.

a $1\frac{1}{4}$ **b** $1\frac{3}{4}$ **c** $3\frac{2}{4}$ **d** $5\frac{2}{4}$

3 A 'window shape' consists of four panes of glass. How many panes of glass are there in the following number of 'window shapes'?



a 2 **b** 3 **c** 7 **d** 11

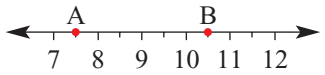
e $4\frac{1}{4}$ **f** $1\frac{3}{4}$ **g** $2\frac{2}{4}$ **h** $5\frac{4}{4}$



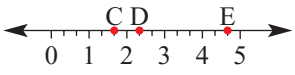
Drilling for Gold 4E2

4 What mixed numerals correspond to the letters written on each number line?

a



b



Fluency

Example 15 Converting mixed numerals to improper fractions

Convert $3\frac{1}{5}$ to an improper fraction.

Solution

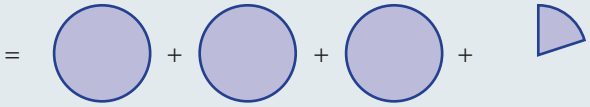
$$3\frac{1}{5} = 1 + 1 + 1 + \frac{1}{5}$$
$$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

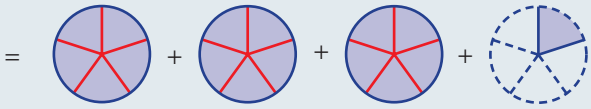
or

$$3\frac{1}{5} = \frac{15}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

Explanation

$3\frac{1}{5}$ = 3 wholes + $\frac{1}{5}$ of a whole





Short-cut method:

Step 1: Multiply the whole number part by the denominator.

Step 2: Then add the numerator: $3 \times 5 + 1 = 16$

Step 1: 3×5 Step 2: Then add 1.

That is: $3\frac{1}{5} = \frac{3 \times 5 + 1}{5}$

$$= \frac{16}{5}$$

The denominator remains 5, as we are talking about fifths.

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5 Convert these mixed numerals to improper fractions.

a $2\frac{1}{5}$ **b** $1\frac{3}{5}$ **c** $3\frac{1}{3}$ **d** $5\frac{2}{3}$ **e** $4\frac{1}{7}$

f $3\frac{3}{7}$ **g** $2\frac{1}{2}$ **h** $6\frac{1}{2}$ **i** $4\frac{2}{5}$ **j** $11\frac{1}{2}$

k $8\frac{2}{5}$ **l** $10\frac{3}{10}$ **m** $6\frac{1}{9}$ **n** $2\frac{7}{9}$ **o** $5\frac{2}{8}$

p $2\frac{5}{8}$ **q** $1\frac{11}{12}$ **r** $3\frac{5}{11}$ **s** $4\frac{5}{12}$ **t** $9\frac{7}{12}$

Example 16 Converting improper fractions to mixed numerals

Convert $\frac{11}{4}$ to a mixed numeral.

Solution

Method 1

$$\frac{11}{4} = \frac{8+3}{4}$$
$$= \frac{8}{4} + \frac{3}{4}$$
$$= 2 + \frac{3}{4}$$
$$= 2\frac{3}{4}$$

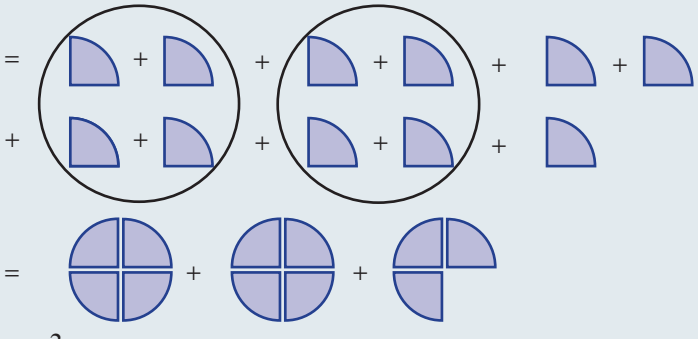
Method 2

$4\overline{)11}$ 2 remainder 3

$= 2\frac{3}{4}$

Explanation

$\frac{11}{4}$ = 11 quarters



$= 2\frac{3}{4}$

Divide the bottom (denominator) into the top (numerator). The remainder becomes the numerator.

6 Convert these improper fractions to mixed numerals.

a $\frac{7}{5}$ **b** $\frac{4}{3}$ **c** $\frac{5}{3}$ **d** $\frac{7}{4}$

e $\frac{11}{3}$ **f** $\frac{21}{5}$ **g** $\frac{16}{7}$ **h** $\frac{10}{4}$

i $\frac{12}{7}$ **j** $\frac{19}{6}$ **k** $\frac{20}{3}$ **l** $\frac{41}{4}$

m $\frac{35}{8}$ **n** $\frac{26}{5}$ **o** $\frac{48}{7}$ **p** $\frac{41}{3}$

q $\frac{37}{12}$ **r** $\frac{81}{11}$ **s** $\frac{93}{10}$ **t** $\frac{78}{7}$

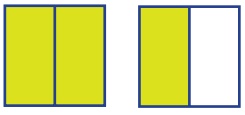
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4E

7 For each of these diagrams, write:

i the mixed numeral **ii** the improper fraction

a



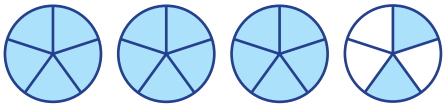
b



c



d



Problem-solving and Reasoning

8 $\frac{10}{4}$ simplifies to $\frac{5}{2}$, which equals $2\frac{1}{2}$.

Write the following fractions in two different ways.

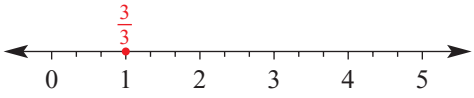
a $\frac{8}{6}$ **b** $\frac{12}{8}$ **c** $\frac{6}{4}$ **d** $\frac{20}{16}$



It's easier to simplify first.

9 Copy the number line and show the fractions on it.

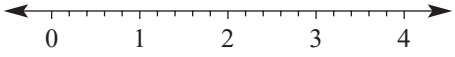
a $\frac{2}{3}, 2, \frac{5}{3}, 3\frac{1}{3}, \frac{15}{3}$





This number line is marked in thirds.

b $\frac{1}{5}, \frac{4}{5}, \frac{5}{5}, 1\frac{2}{5}, \frac{10}{5}, 3\frac{4}{5}$





This number line shows fifths.

10 Rewrite each of these patterns, using improper fractions where needed.

a $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$ **b** $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}, 2$

11 Four friends order three large pizzas for their dinner. Each pizza is cut into eight equal-sized slices. Simone has three slices, Izabella has four slices, Mark has five slices and Alex has three slices.

a Draw circles to show the three pizzas ordered.

b How many pizza slices do they eat in total?

c How much pizza do they eat in total? Give your answer as a mixed numeral.

d How many pizza slices are left uneaten?

e How much pizza is left uneaten? Give your answer as a mixed numeral.



Remember, each pizza has 8 equal-sized slices.

Enrichment: Writing fractions

12 What different fractions can you write using only the digits 1, 2 and 3? Who in your class wrote the most?





You can include improper fractions and mixed numerals. You don't need to use all three digits.

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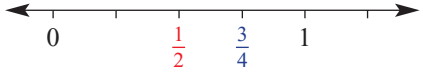
4F Ordering fractions

Just like whole numbers, fractions can be written in order.

Remember that > means *is greater than* and < means *is less than*.

A fraction is less than another fraction if it lies to the left of that fraction on a number line.

$\frac{1}{2} < \frac{3}{4}$



Let's start: The order of five



- As a warm-up activity, ask five volunteer students to arrange themselves in alphabetical order, then in height order and, finally, in birthday order.
- Each of the five students receives a large fraction card and displays it to the class.

  (For example, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{10}{2}$, $\frac{2}{3}$.)
- The rest of the class must then attempt to order the students in ascending order, according to their fraction card. It is a group decision and none of the five students should move until the class agrees on a decision.

Key ideas

- To order (or arrange) fractions we must know how to compare different fractions. There are three cases to consider.
  1. If the numerators are the same, the smallest fraction is the one with the biggest denominator, as the whole has been divided up into the most pieces.

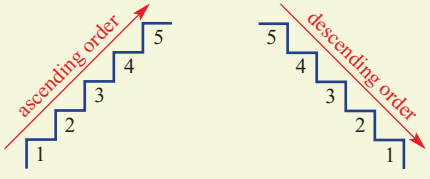
     For example: $\frac{1}{7} < \frac{1}{2}$
  2. If the denominators are the same, the smallest fraction is the one with the smallest numerator.

     For example: $\frac{3}{10} < \frac{7}{10}$
  3. Otherwise, use equivalent fractions to make fractions with the same denominator. (The **lowest common denominator (LCD)** is best.) Then compare numerators as above.
- **Ascending** order is when numbers are ordered going *up*, from smallest to largest.
- **Descending** order is when numbers are ordered going *down*, from largest to smallest.

Lowest common denominator (LCD) The smallest common multiple of the denominators of two or more fractions

Ascending Going up, from smallest to largest

Descending Going down, from largest to smallest



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Exercise 4F

Understanding

1 Circle the largest fraction in each of the following lists.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{4}{3}, \frac{2}{3}, \frac{7}{3}, \frac{5}{3}$

c $\frac{5}{11}, \frac{9}{11}, \frac{3}{11}, \frac{4}{11}$ **d** $\frac{8}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}$



When the denominators are the same, compare numerators: $\frac{6}{7}$ is more than $\frac{5}{7}$, just as 6 is more than 5.

2 State the lowest common multiple of the following sets of numbers.

a 2, 5 **b** 3, 7 **c** 5, 4 **d** 6, 5
e 3, 6 **f** 2, 10 **g** 4, 6 **h** 8, 6
i 2, 3, 5 **j** 3, 4, 6 **k** 3, 8, 4 **l** 2, 6, 5



2, 4, 6, 8, (10), ...
5, (10), 15, 20, 25, ...
The LCM of 2 and 5 is 10.

3 State the lowest common denominator of the following sets of fractions.

a $\frac{1}{3}, \frac{3}{5}$ **b** $\frac{2}{4}, \frac{3}{5}$ **c** $\frac{4}{7}, \frac{2}{3}$ **d** $\frac{2}{10}, \frac{1}{5}$

e $\frac{4}{6}, \frac{3}{8}$ **f** $\frac{5}{12}, \frac{2}{5}$ **g** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ **h** $\frac{4}{3}, \frac{3}{4}$



Find the LCM of the denominators.

4 Fill in the gaps to produce equivalent fractions.

a $\frac{2}{5} = \frac{\square}{15}$ **b** $\frac{2}{3} = \frac{\square}{12}$ **c** $\frac{1}{4} = \frac{\square}{16}$ **d** $\frac{3}{7} = \frac{\square}{14}$

e $\frac{3}{8} = \frac{\square}{40}$ **f** $\frac{5}{6} = \frac{\square}{18}$ **g** $\frac{21}{50} = \frac{\square}{100}$ **h** $\frac{3}{20} = \frac{\square}{100}$

Fluency

Example 17 Comparing fractions

Write < or > in each box to make a true mathematical statement.

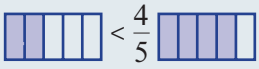
a $\frac{2}{5} \square \frac{4}{5}$ **b** $\frac{1}{3} \square \frac{1}{5}$

Solution | **Explanation**

a $\frac{2}{5} \boxed{<} \frac{4}{5}$

Denominators are the same, therefore compare numerators.

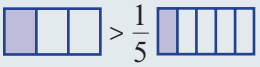
$2 < 4$ so $\frac{2}{5} < \frac{4}{5}$



b $\frac{1}{3} \boxed{>} \frac{1}{5}$

Numerators are the same.
The smaller fraction has the bigger denominator.

So $\frac{1}{3} > \frac{1}{5}$



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5 Write < or > to make true statements.

a $\frac{3}{5} \square \frac{1}{5}$ **b** $\frac{7}{9} \square \frac{2}{9}$ **c** $\frac{2}{2} \square \frac{3}{2}$ **d** $\frac{3}{8} \square \frac{7}{8}$

e $\frac{2}{3} \square \frac{1}{3}$ **f** $\frac{7}{10} \square \frac{5}{10}$ **g** $\frac{1}{6} \square \frac{5}{6}$ **h** $\frac{7}{2} \square \frac{1}{2}$

6 Write < or > to make true statements.

a $\frac{1}{4} \square \frac{1}{3}$ **b** $\frac{1}{10} \square \frac{1}{20}$ **c** $\frac{1}{7} \square \frac{1}{5}$ **d** $\frac{1}{10} \square \frac{1}{2}$

e $\frac{1}{4} \square \frac{1}{40}$ **f** $\frac{1}{6} \square \frac{1}{8}$ **g** $\frac{2}{5} \square \frac{2}{7}$ **h** $\frac{3}{50} \square \frac{3}{100}$

Example 18 Comparing fractions by using a common denominator

Which is larger, $\frac{2}{3}$ or $\frac{3}{5}$?

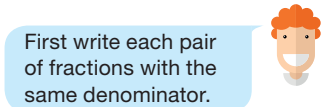
Solution	Explanation
$\frac{2}{3} = \frac{10}{15}$ $\frac{3}{5} = \frac{9}{15}$	Find a common denominator. The lowest common multiple (LCM) of 3 and 5 is 15. Produce equivalent fractions with that denominator. $\frac{2}{3} \overset{\times 5}{=} \frac{10}{15}$ $\quad \frac{3}{5} \overset{\times 3}{=} \frac{9}{15}$
$\frac{2}{3} > \frac{3}{5}$	Compare numerators: $10 > 9$ so $\frac{10}{15} > \frac{9}{15}$

Drilling for Gold 4F1 4F2

7 Decide which fraction is the largest in each pair.

a $\frac{1}{2}$ and $\frac{2}{3}$ **b** $\frac{4}{5}$ and $\frac{2}{3}$ **c** $\frac{7}{10}$ and $\frac{4}{5}$

d $\frac{3}{2}$ and $\frac{5}{4}$ **e** $\frac{3}{4}$ and $\frac{4}{9}$ **f** $\frac{1}{2}$ and $\frac{3}{5}$



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4F

Example 19 Ordering fractions

Write the fractions $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$ in ascending order.

Solution	Explanation
$\frac{45}{60}, \frac{48}{60}, \frac{40}{60}$	LCD of 3, 4 and 5 is 60. Produce equivalent fractions with denominator of 60.
$\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$	Order fractions in ascending order. (Look at the top of each fraction.)
$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$	Rewrite fractions back in original form.

8 Place the following fractions in ascending order. Hint: Find the common denominator.

a $\frac{3}{5}, \frac{8}{5}, 1\frac{2}{5}$ **b** $\frac{5}{9}, \frac{1}{3}, \frac{2}{9}$ **c** $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$

d $\frac{5}{6}, \frac{3}{5}, \frac{2}{3}$ **e** $\frac{7}{8}, \frac{1}{2}, \frac{3}{4}$ **f** $\frac{7}{10}, \frac{1}{2}, \frac{3}{5}$


Ascending order is from smallest to largest.

Problem-solving and Reasoning

9 Place the following cake fractions in decreasing order of size.

A sponge cake shared equally by four people = $\frac{1}{4}$ cake

B chocolate cake shared equally by eleven people = $\frac{1}{11}$ cake

C carrot and walnut cake shared equally by eight people = $\frac{1}{8}$ cake



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10 Serena and Dion ordered two large pizzas. Serena ate $\frac{2}{3}$ of her pizza. Dion ate $\frac{5}{8}$ of his.





a Show each person's pizzas on a separate diagram.

b Who ate the most pizza?

c Should they have ordered only one pizza?

11 Rewrite the fractions in each set with their lowest common denominator. Then write the next two fractions that would continue the pattern.

a $\frac{2}{9}, \frac{1}{3}, \frac{4}{9}, ___, ___$ **b** $\frac{1}{2}, \frac{5}{4}, 2, ___, ___$ **c** $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, ___, ___$

12 Write a fraction that lies between the following pairs of fractions.

a $\frac{1}{10}$ and $\frac{7}{10}$ **b** $\frac{1}{2}$ and $\frac{7}{10}$ **c** $\frac{1}{5}$ and $\frac{8}{10}$

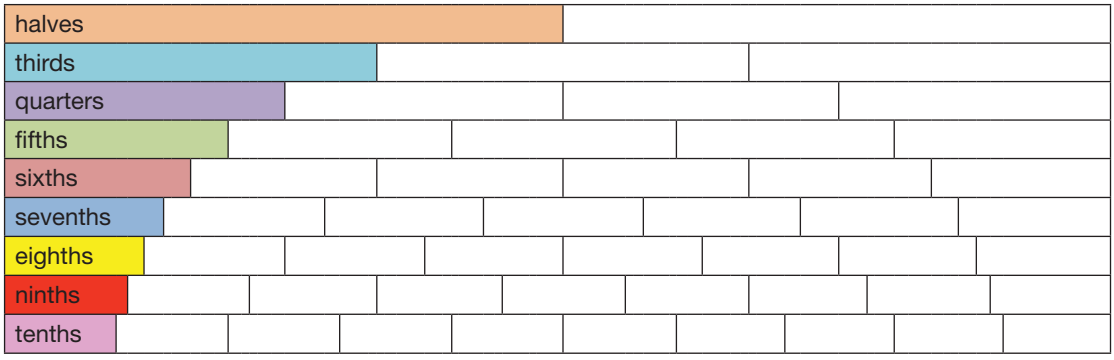
d $\frac{3}{5}, \frac{3}{4}$ **e** $\frac{1}{4}, \frac{1}{2}$ **f** $\frac{2}{7}, \frac{1}{6}$



First write each pair with the same denominator.

Enrichment: Using a fraction wall to order fractions

13 Consider the fraction wall below, in which every row has been divided into equal pieces. It may be useful to copy or print this diagram onto paper.



a Use the fraction wall to insert <, = or > between each pair of fractions.

i $\frac{1}{3} \square \frac{2}{5}$ **ii** $\frac{1}{3} \square \frac{3}{9}$ **iii** $\frac{1}{3} \square \frac{3}{10}$ **iv** $\frac{1}{4} \square \frac{2}{8}$

v $\frac{1}{4} \square \frac{2}{9}$ **vi** $\frac{1}{4} \square \frac{3}{10}$ **vii** $\frac{1}{5} \square \frac{2}{9}$ **viii** $\frac{1}{5} \square \frac{2}{10}$

ix $\frac{4}{6} \square \frac{6}{9}$ **x** $\frac{8}{9} \square \frac{9}{10}$ **xi** $\frac{2}{3} \square \frac{7}{10}$ **xii** $\frac{7}{10} \square \frac{5}{7}$

b Use the fraction wall to arrange the following fractions in ascending order.

$\frac{5}{8}, \frac{5}{6}, \frac{4}{7}, \frac{2}{3}, \frac{5}{9}, \frac{3}{4}, \frac{7}{10}, \frac{3}{5}, \frac{1}{2}$

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4G Place value in decimals and ordering decimals

Decimals are used for many measurements. For example, at the 2002 Commonwealth Games Ian Thorpe swam the 400 metres freestyle in 3 minutes 40.08 seconds, which was a new world record at the time.

It's very important to be able to use decimals, as our money works on a decimal system of dollars and cents.



If a chocolate bar costs $2, would the price still be a decimal number?

Let's start: Decimals time trial

Work with a partner. You will need six cards:

0	1	2	3	4	.

- How many different numbers can you make in 1 minute?
- What is the greatest number you can make?
- What number can you make that is the closest to 0?

Key ideas

Decimal point
Symbol that separates the whole part of a number from its fractional part

- A **decimal point** is used to separate the whole number from the decimal or fraction part.
- When dealing with decimal numbers, the place value table must be made bigger to involve tenths, hundredths, thousandths etc. We read the number 428.357 as four hundred and twenty-eight point three, five, seven.

Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousandths
4	2	8	•	3	5	7
4×100	2×10	8×1	•	$3 \times \frac{1}{10}$	$5 \times \frac{1}{100}$	$7 \times \frac{1}{1000}$
400	20	8	•	$\frac{3}{10}$	$\frac{5}{100}$	$\frac{7}{1000}$
← whole numbers			decimal point		fractions →	

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Exercise 4G

Understanding

1 Match each decimal (**a** to **e**) with the correct wording (**A** to **E**).

a	0.015	**A**	seventeen and three-tenths
b	10.835	**B**	five and thirteen-hundredths
c	5.13	**C**	zero point zero, one, six
d	0.016	**D**	ten point eight, three, five
e	17.3	**E**	fifteen-thousandths



Ones	.	Tenths	Hundredths	Thousandths
0	.	0	1	3

$0.013 = \frac{13}{1000}$ = thirteen-thousandths

2 For the number 58.237, give the value of the digit:
a 2 **b** 3 **c** 7

3 A stopwatch is stopped at 36.57 seconds.
a What is the digit displayed in the tenths column?
b What is the digit displayed in the ones column?
c What is the digit displayed in the hundredths column?
d Is this number closer to 36 or 37 seconds?

Example 20 Understanding decimal place value

What is the value (as a fraction) of the digit 8 in the following numbers?
a 12.85 **b** 6.1287

Solution **Explanation**

a The value of 8 is $\frac{8}{10}$.

The 8 is in the tenths column, so the value is eight-tenths.

Tens	Ones	.	Tenths	Hundredths
1	2	.	8	5

b The value of 8 is $\frac{8}{1000}$.

The 8 is in the thousandths column, so the value is eight-thousandths.

Ones	.	Tenths	Hundredths	Thousandths	Tens of thousandths
6	.	1	2	8	7



Drilling for Gold 4G1

4 What is the value (as a fraction) of the digit 6 in the following numbers?
a 23.612 **b** 17.46 **c** 80.016 **d** 0.693
e 1.64 **f** 8.568 13 **g** 2.3641 **h** 11.926

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4G

Fluency

Example 21 Changing to decimals

Express each of the following proper fractions and mixed numerals as decimals.

a $\frac{7}{10}$ **b** $\frac{5}{100}$ **c** $3\frac{17}{100}$

Solution	Explanation
a $\frac{7}{10} = 0.7$	$\frac{7}{10}$ means seven-tenths, so put the 7 in the tenths column.

Ones	.	Tenths
0	.	7

b $\frac{5}{100} = 0.05$	$\frac{5}{100}$ means five-hundredths, so put the 5 in the hundredths column.

Ones	.	Tenths	Hundredths
0	.	0	5

c $3\frac{17}{100} = 3.17$	$3\frac{17}{100}$ means 3 ones and seventeen-hundredths. Seventeen-hundredths is one-tenth and seven-hundredths.

Ones	.	Tenths	Hundredths
3	.	1	7

5 Express each of the following proper fractions as a decimal.

a $\frac{3}{10}$ **b** $\frac{8}{10}$ **c** $\frac{15}{100}$ **d** $\frac{23}{100}$

e $\frac{9}{10}$ **f** $\frac{12}{100}$ **g** $\frac{121}{1000}$ **h** $\frac{174}{1000}$

i $\frac{1}{10}$ **j** $\frac{11}{100}$ **k** $\frac{111}{1000}$ **l** $\frac{3}{100}$

6 Express each of the following mixed numerals as a decimal.

a $6\frac{4}{10}$ **b** $5\frac{7}{10}$ **c** $212\frac{3}{10}$ **d** $1\frac{16}{100}$

e $14\frac{83}{100}$ **f** $7\frac{51}{100}$ **g** $5\frac{7}{100}$ **h** $18\frac{612}{1000}$

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7 True (T) or false (F)?

a $0.6 = \frac{6}{10}$ **b** $0.7 = \frac{7}{100}$ **c** $0.70 = \frac{70}{100}$ **d** $0.07 = \frac{7}{100}$

e $\frac{15}{10} = 0.15$ **f** $\frac{15}{10} = 1.5$ **g** $0.6 = 0.60$ **h** $7.0 = 7$

Problem-solving and Reasoning

Example 22 Arranging decimal numbers in order

Arrange the following decimal numbers in ascending order (i.e. from smallest to largest):
3.72, 7.23, 2.73, 2.37, 7.32, 3.27

Solution	Explanation
2.37 2.73 3.27 3.72 7.23 7.32	The ones column has a higher value than the tenths column, and the tenths column has a higher value than the hundredths column. 2.73 is bigger than 2.37 because it has seven-tenths, which is bigger than three-tenths.

8 Choose the larger decimal in each pair.

a 6.1, 0.16 **b** 6.9, 9.6 **c** 0.8, 0.08
d 25.8, 28.5 **e** 0.107, 0.171 **f** 0.032, 0.203

9 Arrange each group of numbers in ascending order (i.e. from smallest to largest).

a 3.52, 3.05, 3.25, 3.55
b 30.6, 3.06, 3.6, 30.3
c 17.81, 1.718, 1.871, 11.87
d 26.92, 29.26, 29.62, 22.96, 22.69



To compare decimals, write them underneath each other. Line up the decimal points.

30.60
3.06
3.60
30.30



Gymnastics scoring involves the use of a decimal system.

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4G

10 The batting averages for five retired Australian Cricket test captains are: Adam Gilchrist 47.60, Steve Waugh 51.06, Mark Taylor 43.49, Allan Border 50.56 and Kim Hughes 37.41.



a List the five players in descending order of batting averages (i.e. from largest to smallest).

b Ricky Ponting's test batting average is 51.85. Where does this rank him in terms of the other retired Australian test captains listed above?

11 The depth of a river at 9 a.m. on six consecutive days was:

Day 1: 1.53 m Day 2: 1.58 m
Day 3: 1.49 m Day 4: 1.47 m
Day 5: 1.52 m Day 6: 1.61 m



a On which day was the river level highest?

b On which day was the river level lowest?

c On which day was the water level above 1.6 metres?

12 Which is larger?

a 0.7135 or $7 \times 1 + 1 \times \frac{1}{10} + 3 \times \frac{1}{100} + 5 \times \frac{1}{1000}$

b 1.563 or $1 \times \frac{1}{10} + 5 \times \frac{1}{100}$

Enrichment: Different decimal combinations

13 For each of the following, write as many different decimal numbers as you can. (Each digit must be used only once and all digits must be used.) How many different numbers did you get? Circle the largest number and the smallest number.

a Use the digits 0, 1 and a decimal point.

b Use the digits 0, 1, 2 and a decimal point.

c Use the digits 0, 1, 2, 3 and a decimal point.

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4H Rounding decimals

Decimal numbers sometimes contain more decimal places than we need. It is important that we are able to round decimal numbers when working with money, measuring distance or writing answers to division computations.

For example, the distance around the school oval might be 0.397 km, which rounded to 1 decimal place is 0.4 km or 400 m. The rounded figure, although not precise, is accurate enough for most applications.



Usain Bolt's 100-metre sprint world record was measured to 2 decimal places.

Let's start: Closer to $1.26 or $1.27?

Is each amount closer to $1.26 or $1.27?

$1.261	$1.269	$1.2625	$1.2675	$1.2609	$1.2699	$1.2649	$1.265

Key ideas

- **Rounding** involves approximating a decimal number using fewer digits.
- Rounding to **2** decimal places:
  In the following decimals, more than 2 decimals are given.
  A blue line has been drawn after 2 decimal places.
  The 'critical digit' is circled.
  If the critical digit is 0, 1, 2, 3 or 4, then round down.
  For example: 185.26 |(3) = 185.26 (to 2 d.p.)
  185.26 |(0)05 = 185.26 (to 2 d.p.)
  185.26 |(4)499 = 185.26 (to 2 d.p.)
  If the critical digit is 5, 6, 7, 8 or 9, then round up.
  For example: 185.26 |(5) = 185.27 (to 2 d.p.)
  185.26 |(6)05 = 185.27 (to 2 d.p.)
  185.26 |(9)499 = 185.27 (to 2 d.p.)

Rounding To make an approximation of a number with fewer digits

Ignore all digits to the right of the critical digit.



Exercise 4H

Understanding

1 Answer the following.
- **a** Is $0.763 closer to $0.76 or $0.77?
- **b** Is $10.569 closer to $10.56 or $10.57?
- **c** Is $4.036 closer to $4.03 or $4.04?
- **d** Is $4.796 closer to $4.79 or $4.80?

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4H

Example 23 Determining the critical digit

The following decimal numbers need to be rounded to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit.

a 23.5398 **b** 1.75137

Solution	Explanation
a 23.53|(9)8	Draw a line after 2 decimal places. Circle the next digit.
b 1.75|(1)37	The critical digit is always straight after the number of decimal places we want to keep.

2 The following decimals need to be rounded to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit.

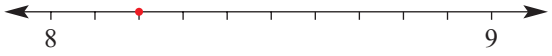
a 12.6453 **b** 4.81932 **c** 157.281 **d** 4 001 565.384 71
e 0.060 31 **f** 203.5791 **g** 66.6666 **h** 7.995123

3 State the critical digit in each of the following numbers.

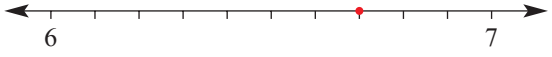
a 25.8(1)74 rounded to 1 decimal place. Critical digit is ____.
b 25.81(7)4 rounded to 2 decimal places. Critical digit is ____.
c 25.817(4) rounded to 3 decimal places. Critical digit is ____.
d 25.(8)174 rounded to the nearest whole number. Critical digit is ____.

4 For each of the following, select the closer alternative.

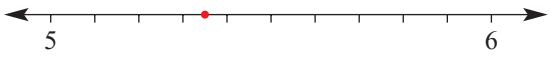
a Is 8.2 closer to 8 or 9?



b Is 6.7 closer to 6 or 7?



c Is 5.35 closer to 5 or 6?



Fluency

Example 24 Rounding decimals to 1 decimal place

Round each of the following to 1 decimal place.

a 25.682 **b** 13.5458

Ignore the digits to the right of the critical digit.



Solution	Explanation
a 25.7	25.6|(8)2 The critical digit is 8 so round up to 25.7.
b 13.5	13.5|(4)8 The critical digit is 4 so round down to 13.5.

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5 Round each of the following to 1 decimal place.

a 14.82	**b** 7.38	**c** 15.62
d 0.87	**e** 6.85	**f** 9.94
g 55.55	**h** 7.98	**i** 0.68
j 0.72	**k** 0.69	**l** 0.88


Here, the critical digit is the second decimal place.

6 Write each of the following correct to 2 decimal places.

a 3.7823	**b** 11.8627	**c** 5.9156	**d** 0.93225
e 123.456	**f** 300.0549	**g** 3.1250	**h** 9.849
i 56.2893	**j** 7.121999	**k** 29.9913	**l** 0.8971

Example 25 Rounding decimals to different decimal places

a Round 18.34728 to 3 decimal places. **b** Round 0.43917 to 2 decimal places.

Solution	Explanation
a 18.347	18.347|(2)8 The critical digit is 2, so round down.
b 0.44	0.43|(9)17 The critical digit is 9, so round up.


Ignore the digit to the right of the critical digit.

7 Round each of the following to the specified number of decimal places, given as the number in the brackets.

a 15.913 (1)	**b** 7.8923 (2)	**c** 235.62 (0)	**d** 0.5111 (0)
e 231.86 (1)	**f** 9.3951 (1)	**g** 9.3951 (2)	**h** 34.71289 (3)

8 Round each of the following to the specified number of decimal places.

a 23.983 (1)	**b** 14.8992 (2)
c 6.954 32 (0)	**d** 29.999731 (3)


To round 7.59(8) to 2 decimal places, think: 59 goes up to 60. Answer = 7.60

Skillsheet 4B

9 Round each of the following to the nearest whole number. This is the same as zero decimal places.

a 27.612	**b** 9.458	**c** 12.299	**d** 123.72
e 22.26	**f** 117.555	**g** 2.6132	**h** 10.7532

Problem-solving and Reasoning

10 Round each of the following amounts to the nearest dollar.

a $12.85	**b** $30.50	**c** $7.10	**d** $1566.80
e $120.45	**f** $9.55	**g** $1.39	**h** $36.19

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4H

11 Petrol cost 149.9 cents per litre. Yannis put 48 litres in his car. He calculated that this would cost 7195.2 cents.

- **a** What did Yannis type on his calculator to get this answer?
- **b** How much should he pay, correct to:
  - **i** the nearest cent?
  - **ii** the nearest dollar?



12 Lee rounded the decimal 74.74 □ 63 to 2 decimal places. She wrote down 74.75. What could the missing digit be?

13 List all the decimal hundredths (such as 0.39) that would round to 0.4.

Enrichment: Rounding with technology

14 Most calculators are able to round numbers correct to a specified number of places. Find out how to do this on your calculator and check your answers to questions **6** and **7**.



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4I Decimal and fraction conversions

Sometimes we use decimals to show numbers that are not whole numbers, and sometimes we use fractions. It is important to be able to change a fraction to a decimal (for example, $\frac{1}{4} = 0.25$), and change a decimal to a fraction. (We often say 'convert' instead of 'change'.)



Would you use fractions or decimals to measure long jump?

Let's start: How many do you already know?

- You probably know that the decimal 0.5 and the fraction $\frac{1}{2}$ are equivalent. List all the other decimal–fraction pairs you know.
- Could you use the decimal–fraction equivalences you know to work out some other pairs?

Key ideas

- Converting decimals to fractions:
  - When the denominator is 2, think halves. Example 1: $\frac{1}{2} = 0.5$
    Example 2: $3\frac{1}{2} = 3.5$
  - When the denominator is 4, think quarters. Example 1: $\frac{1}{4} = 0.25$
    Example 2: $\frac{3}{4} = 0.75$
  - When the denominator is 10, think tenths; e.g. $\frac{7}{10} = 0.7$
    seven-tenths; 7 in the tenths column
  - When the denominator is 100, think hundredths.
    Example 1: $\frac{3}{100} = 0.03$
    Example 2: $\frac{37}{100} = 0.37$
    the hundredths column

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– When the denominator is 5, think tenths; e.g. $\frac{3}{5} = \frac{6}{10} = 0.6$

– When the denominator is a factor of 100, think hundredths.

e.g. $\frac{17}{50} = \frac{34}{100} = 0.34$

– When all else fails, do a division by hand (or use a calculator).

e.g. $\frac{5}{8} = 8\overline{)5.^{5}0^{2}0^{4}0}$ $\begin{matrix}0.625\end{matrix}$ or $\frac{5}{8} = 5 \div 8 = 0.625$

– Sometimes, the decimal has a pattern that repeats forever. These are called **recurring decimals** or repeating decimals.

Recurring Repeating indefinitely

Example 1: $\frac{2}{3} = 0.666\ldots = 0.\dot{6}$

Example 2: $\frac{13}{11} = 1.1818\ldots = 1.\dot{1}\dot{8}$ or $1.\overline{18}$

Example 3: $\frac{5}{7} = 0.\dot{7}14285\dot{}$ or $0.\overline{714285}$

Exercise 4I

Understanding



Drilling for Gold 4I1

1 Complete each of these statements, which convert common fractions to decimals.

a $\frac{1}{2} = \frac{\square}{10} = 0.5$ **b** $\frac{1}{4} = \frac{25}{\square} = 0.25$ **c** $\frac{3}{4} = \frac{\square}{100} = 0.\square 5$ **d** $\frac{2}{\square} = \frac{4}{10} = 0.\square$

2 Complete each of these statements, which convert decimals to fractions, in simplest form.

a $0.2 = \frac{\square}{10} = \frac{1}{5}$ **b** $0.15 = \frac{\square}{100} = \frac{3}{\square}$

c $0.8 = \frac{8}{\square} = \frac{\square}{5}$ **d** $0.64 = \frac{64}{100} = \frac{\square}{25}$

3 Circle the larger number in each pair.

a $\frac{1}{10}$, 0.3 **b** $\frac{1}{2}$, 0.4 **c** 0.8, $\frac{8}{100}$

d 1.5, $\frac{15}{100}$ **e** $\frac{3}{4}$, 0.9 **f** 0.05, $\frac{1}{2}$



Convert one of the numbers so that you can compare two fractions or two decimals.

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Fluency

Example 26 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.239 **b** 10.35

Solution	Explanation
a $\frac{239}{1000}$	0.239 has 3 decimal places. So use 3 zeros (1000) in the denominator. 0.239 = 239-thousandths
b $10\frac{35}{100} = 10\frac{7}{20}$	0.35 has 2 decimal places. Use 2 zeros (100) in the denominator. 0.35 = 35-hundredths Simplify by dividing the numerator and denominator by the highest common factor of 5.

4 Convert the following decimals to fractions in their simplest form.

a 0.4	**b** 0.6	**c** 0.8	**d** 0.22
e 1.22	**f** 5.5	**g** 0.15	**h** 0.99
i 0.08	**j** 0.01	**k** 0.001	**l** 0.202
m 0.5	**n** 6.4	**o** 10.15	**p** 18.12
q 3.25	**r** 0.05	**s** 9.075	**t** 5.192

Example 27 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $5\frac{3}{5}$ **c** $\frac{7}{12}$

Solution	Explanation
a $\frac{17}{100} = 0.17$	The denominator is 100, so write 17-hundredths as a decimal.
b $5\frac{3}{5} = 5\frac{6}{10} = 5.6$	The denominator is 5, so change $\frac{3}{5}$ to $\frac{6}{10}$ and write it as a decimal.
c $\frac{7}{12} = 0.58333\ldots$ or $0.58\dot{3}$	12 is not a factor of 10, 100 or 1000. So divide the numerator (7) by the denominator (12). $12\overline{)7.0^{10}0^{4}0^{4}0^{4}0}$ $0.5833\ldots$

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41

5 Convert each of these fractions to decimals.

a $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{31}{100}$

d $\frac{79}{100}$ **e** $\frac{121}{100}$ **f** $3\frac{29}{100}$

g $\frac{123}{1000}$ **h** $\frac{3}{100}$ **i** $\frac{7}{100}$

6 Convert the following fractions to decimals by first changing the fraction to an equivalent fraction.

a $\frac{4}{5} = \frac{\square}{10}$ **b** $\frac{1}{2} = \frac{\square}{10}$ **c** $\frac{7}{20} = \frac{\square}{100}$

d $\frac{23}{50} = \frac{\square}{100}$ **e** $5\frac{19}{20} = 5\frac{\square}{100}$ **f** $3\frac{1}{4} = 3\frac{\square}{100}$

g $\frac{5}{2} = \frac{\square}{10}$ **h** $\frac{3}{8} = \frac{\square}{1000}$ **i** $\frac{7}{25} = \frac{\square}{100}$

7 Convert the following fractions to decimals by dividing the numerator by the denominator. Check your answer using a calculator.

a $\frac{1}{2}$ **b** $\frac{3}{6}$ **c** $\frac{3}{4}$

d $\frac{2}{5}$ **e** $\frac{1}{3}$ **f** $\frac{3}{8}$

g $\frac{5}{12}$ **h** $\frac{3}{7}$ **i** $\frac{2}{9}$



Use a dot to show recurring decimals: $0.333\ldots = 0.\dot{3}$

8 Copy and complete the following tables. Try to memorise these fractions and their equivalent decimal values.

a halves

Fraction	$\frac{0}{2}$	$\frac{1}{2}$	$\frac{2}{2}$
Decimal			

b thirds

Fraction	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
Decimal				

c quarters

Fraction	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
Decimal					

d fifths

Fraction	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
Decimal						

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Problem-solving and Reasoning

9 Arrange $\frac{1}{2}$, 0.75, $\frac{5}{8}$, 0.4, 0.99, $\frac{1}{4}$ from smallest to largest.

10 Copy and complete the following table.

Decimal amount	\$0.01	\$0.05	10c	20c		50c		90c	\$0.99
Fraction of \$1	$\frac{1}{100}$				$\frac{1}{4}$		$\frac{3}{4}$		

11 a Copy and complete the following fraction ↔ decimal table.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
Decimal									

You may need a calcualtor for questions **11** and **12**.

b What happens to the decimals as the denominator increases?

c Try to explain why this makes sense.

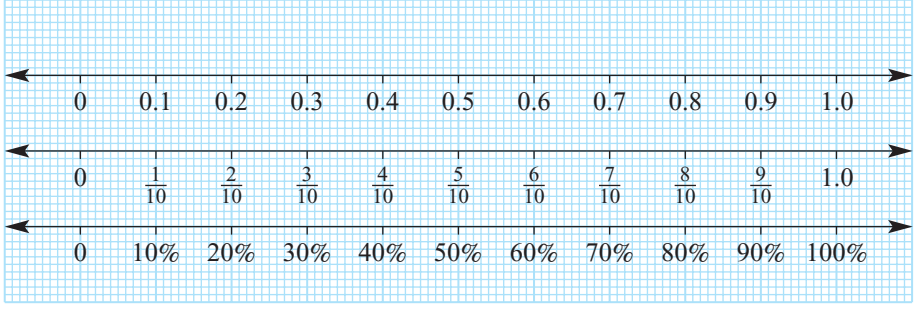
12 a Copy and complete the following decimal ↔ fraction table.

Decimal	0.1	0.2	0.25	0.4	0.5	0.6	0.75	0.8	0.9
Fraction									

b What happens to the numerator of each fraction as the decimals get larger? What happens to the denominator?

c Try to explain why this makes sense.

13 Use these number lines to help you answer the following questions.



a List in ascending order: $\frac{7}{10}$, 62%, $0.\dot{6}$, $\frac{66}{100}$, $\frac{2}{3}$, 65%, 0.67, $\frac{6}{10}$

Put an = sign between any equal numbers in your lists.

b List in descending order: 36%, 0.43, $\frac{2}{5}$, 45%, 0.4, $\frac{3}{5}$, 0.52, $\frac{6}{20}$, $0.\dot{3}$, $\frac{9}{25}$, $\frac{1}{3}$

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41

Enrichment: Looking for patterns in decimals



14 a Use a calculator (as little as possible) to convert the following fractions to decimals.
b Describe the pattern you find in each row.
c Which denominators in the table give recurring decimals?
d Which other denominators that are less than 20 give recurring decimals?


Drilling for Gold 412

halves	$\frac{1}{2}=$	$\frac{2}{2}=$	$\frac{3}{2}=$	$\frac{4}{2}=$	$\frac{5}{2}=$	$\frac{6}{2}=$
thirds	$\frac{1}{3}=$	$\frac{2}{3}=$	$\frac{3}{3}=$	$\frac{4}{3}=$	$\frac{5}{3}=$	$\frac{6}{3}=$
quarters	$\frac{1}{4}=$	$\frac{2}{4}=$	$\frac{3}{4}=$	$\frac{4}{4}=$	$\frac{5}{4}=$	$\frac{6}{4}=$
fifths	$\frac{1}{5}=$	$\frac{2}{5}=$	$\frac{3}{5}=$	$\frac{4}{5}=$	$\frac{5}{5}=$	$\frac{6}{5}=$
sixths	$\frac{1}{6}=$	$\frac{2}{6}=$	$\frac{3}{6}=$	$\frac{4}{6}=$	$\frac{5}{6}=$	$\frac{6}{6}=$
sevenths	$\frac{1}{7}=$	$\frac{2}{7}=$	$\frac{3}{7}=$	$\frac{4}{7}=$	$\frac{5}{7}=$	$\frac{6}{7}=$
eighths	$\frac{1}{8}=$	$\frac{2}{8}=$	$\frac{3}{8}=$	$\frac{4}{8}=$	$\frac{5}{8}=$	$\frac{6}{8}=$
ninths	$\frac{1}{9}=$	$\frac{2}{9}=$	$\frac{3}{9}=$	$\frac{4}{9}=$	$\frac{5}{9}=$	$\frac{6}{9}=$
tenths	$\frac{1}{10}=$	$\frac{2}{10}=$	$\frac{3}{10}=$	$\frac{4}{10}=$	$\frac{5}{10}=$	$\frac{6}{10}=$
elevenths	$\frac{1}{11}=$	$\frac{2}{11}=$	$\frac{3}{11}=$	$\frac{4}{11}=$	$\frac{5}{11}=$	$\frac{6}{11}=$
twelfths	$\frac{1}{12}=$	$\frac{2}{12}=$	$\frac{3}{12}=$	$\frac{4}{12}=$	$\frac{5}{12}=$	$\frac{6}{12}=$

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4J Connecting percentages with fractions and decimals

Percentages are related closely to fractions. A percentage is a fraction in which the denominator is 100. *Per cent* is Latin for 'out of 100'. One dollar is equivalent to 100 **cent**s and a **cent**ury is 100 years.

Percentages are used in many everyday situations. Interest rates, discounts and test results are usually described using percentages rather than fractions or decimals because it is easier to compare two different results.

Let's start: Comparing performance

Consider these netball scores achieved by four students.

- Annie scores 30 goals from 40 shots (i.e. 30 out of 40). What is her percentage?
- Bella scores 19 goals from 25 shots. What is her percentage?
- Cara scores 4 goals from 5 shots. What is her percentage?
- Dianne scores 16 goals from 20 shots. What is her percentage?

The chart below might be useful for comparing their performance.

	Annie (40 shots)	Bella	Cara	Dianne
100%	4			
90%	4			
80%	4			
70%	4			
60%	4			
50%	4			
40%	4			
30%	4			
20%	4			
10%	4			
0%				



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Key ideas

Percentage A fraction with one hundred equal parts

Per cent A figure that is a fraction of one hundred

- **Percentages** have been used for hundreds of years but the symbol we use today is fairly recent. The symbol % means **per cent**. It comes from the Latin words *per centum*, which mean 'out of 100'.

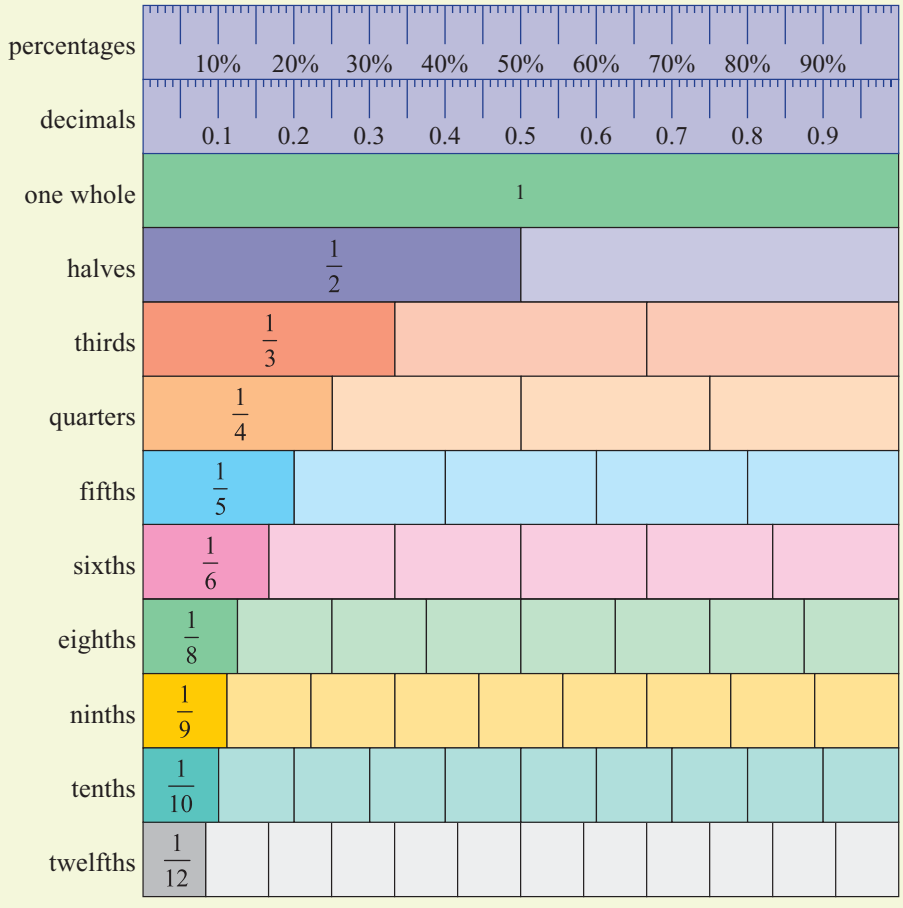
  For example: 35% means '35 out of 100' or $\frac{35}{100}$ or $35 \div 100$ or 0.35.

- Percentages are a useful way to compare fractions.

  For example: $\frac{3}{4} = \frac{75}{100} = 75\%$ and $\frac{18}{25} = \frac{72}{100} = 72\%$,

  therefore $\frac{3}{4} > \frac{18}{25}$ because $75\% > 72\%$.

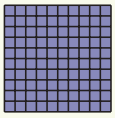
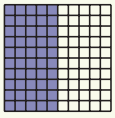
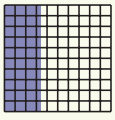
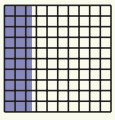
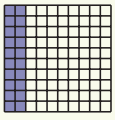
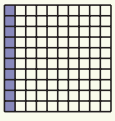
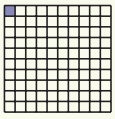
- It is important to understand the relationships and connections between fractions, decimals and percentages. The 'fraction wall' diagram below shows these very clearly.



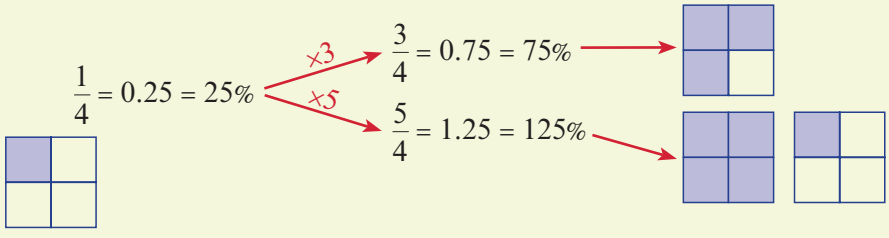
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- It is also important to memorise the most commonly used conversions. These are listed in the table below.



Words	Diagram	Fraction	Decimal	Percentage
one whole		1	1	100%
one-half		$\frac{1}{2}$	0.5	50%
one-third		$\frac{1}{3}$	0.333... or $0.\dot{3}$	$33\frac{1}{3}\%$
one-quarter		$\frac{1}{4}$	0.25	25%
one-fifth		$\frac{1}{5}$	0.2	20%
one-tenth		$\frac{1}{10}$	0.1	10%
one-hundredth		$\frac{1}{100}$	0.01	1%

- The number facts in the table can be used to do other conversions.
For example:



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Exercise 4J

Understanding



Drilling for Gold
4J2
4J3

1 The percentage equivalent to one-quarter is:
A 4% **B** 2.5% **C** 25% **D** 40%

2 The percentage equivalent to three-quarters is:
A 7.5% **B** 34% **C** 75% **D** 80%

3 The percentage equivalent to 0.1 is:
A 0.1% **B** 1% **C** 10% **D** 100%

4 Use the fraction wall (see page 168) to complete the following.

a three-quarters = $\frac{\square}{\square} = 0.__ = __\%$

b nine-tenths = $\frac{\square}{\square} = 0.__ = __\%$

c two-fifths = $\frac{\square}{\square} = 0.__ = __\%$

d four-fifths = $\frac{\square}{\square} = 0.__ = __\%$

Fluency

Example 28 Using a known number fact to make conversions

Given that $\frac{1}{5} = 20\%$, complete the following.

a $\frac{1}{5} = 20\%$, so $\frac{3}{5} = __\%$

b $\frac{1}{5} = 20\%$, so $\frac{6}{5} = __\%$

Solution	Explanation
$\frac{1}{5} = 20\%$	This should be a memorised number fact.
a $\frac{3}{5} = 20\% \times 3 = 60\%$	Multiply the number fact by 3.
b $\frac{6}{5} = 20\% \times 6 = 120\%$	Multiply the number fact by 6.

5 Complete the following.

a $\frac{1}{2} = 50\%$, so $\frac{3}{2} = __\%$

b $\frac{1}{2} = 50\%$, so $\frac{7}{2} = __\%$

c $\frac{1}{4} = 25\%$, so $\frac{3}{4} = __\%$

d $\frac{1}{4} = 25\%$, so $\frac{7}{4} = __\%$

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Example 29 Using memorised number facts

Convert the following fractions to decimals and percentages.

a seven-tenths **b** three-quarters **c** two-thirds

Solution	Explanation
a $\frac{1}{10} = \frac{10}{100} = 0.10 = 10\%$	This should be a memorised number fact.
$\therefore \frac{7}{10} = \frac{70}{100} = 0.7 = 70\%$	Multiply the number fact by 7.
b $\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$	This should be a memorised number fact.
$\therefore \frac{3}{4} = 0.75 = 75\%$	Multiply the number fact by 3.
c $\frac{1}{3} = 0.333\ldots = 33\frac{1}{3}\%$	This should be a memorised number fact.
$\therefore \frac{2}{3} = 0.666\ldots = 66\frac{2}{3}\%$	Multiply the number fact by 2.

6 Convert the following fractions to decimals and percentages. Use the fraction wall if necessary.

a three-tenths **b** three-fifths **c** five-quarters

7 **a** Use the fraction wall to write down:

i six fractions that are equivalent to 50%

ii ten fractions that are greater than 25% but less than 50%

b Of the fractions you wrote down for **ii**, which fraction is closest to 50%?

8 Use the fraction wall to complete these computations. Give your answer as a fraction in simplest form.

a $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{1}{4} + \frac{1}{4}$ **c** $\frac{1}{8} + \frac{1}{8}$

d $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$ **e** $\frac{1}{6} + \frac{1}{6}$ **f** $\frac{2}{3} + \frac{2}{3}$

9 Use the fraction wall to do these computations.

a $1 - \frac{1}{4}$ **b** $1 - \frac{1}{5}$ **c** $1 - \frac{3}{8}$

d $\frac{1}{2} - \frac{1}{4}$ **e** $\frac{1}{2} - \frac{1}{4} - \frac{1}{4}$ **f** $\frac{1}{4} - \frac{1}{8}$

10 Use the fraction wall to complete these computations.

a $1 - 0.1$ **b** $1 - 0.7$ **c** $1 - 0.25$

d $1 - 0.65$ **e** $0.25 + 0.25$ **f** $0.25 + 0.65$

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4J

Problem-solving and reasoning



11 Rachel's birthday cake is cut into ten equal-sized pieces. Rachel eats three pieces.
- **a** What percentage of the cake did Rachel eat?
- **b** What percentage of the cake remains?

12 Use the fraction wall to answer the following questions.
- **a** Which is bigger: three-quarters or two-thirds?
- **b** Which is bigger: two-thirds or three-fifths?
- **c** What is half of one-half?
- **d** What is half of one-quarter?
- **e** What fraction is exactly halfway between one-half and one-quarter?

13 Sophie's netball team wins six of their first seven games. They have three more games to play.
- **a** What is the highest percentage the team can achieve?
- **b** What is the lowest percentage the team can achieve?

14 Are the following statements true or false? Explain your answers, using the fraction wall.

a $0.5 = 0.50\%$ **b** $\frac{1}{3} = 33\%$ **c** $\frac{1}{5} = 15\%$

d $\frac{9}{10} = 90\%$ **e** $\frac{1}{8} \approx 12\%$ **f** $\frac{2}{3} > 66\%$

Enrichment: Which fractions?

15 Use the fractions in the fraction wall to solve these problems.
- **a** *Two* fractions with the *same* denominator add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
- **b** *Two* fractions with *different* denominators add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
- **c** *Three* fractions with the *same* denominator add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
- **d** *Three* fractions with *different* denominators add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
- **e** *Two* fractions with *different* denominators add up to one-half. One of them is one-tenth. What is the other one?

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4K Decimal and percentage conversions

Percentages are used for interest rates and discounts. When operating with percentages it is sometimes convenient to convert them to decimals before doing calculations.

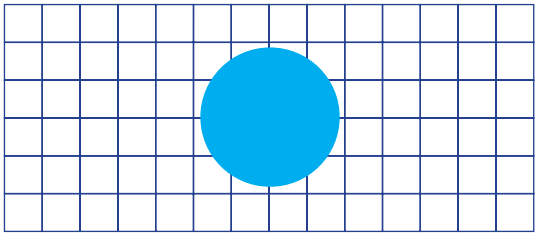
$0.375 \longleftrightarrow 37.5\%$

Let's start: The national flag of Percentia guessing competition

The mathemagical nation of Percentia has a new national flag.

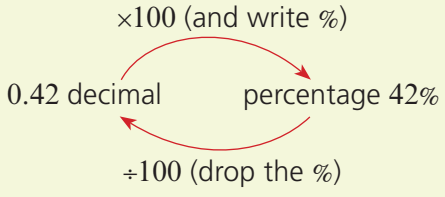
- Use a whole number percentage and a decimal to estimate the fraction of the flag covered by the blue circle.
- Compare your estimate with others in your class.
- Your teacher will be able to calculate the actual area, using a magical number called pi.

Maybe there will be a prize for the best estimator.



Key ideas

- To convert a percentage to a decimal, drop the % sign and divide by 100.
  For example: $42\% = 42 \div 100 = 0.42$



- To convert a decimal to a percentage, multiply by 100 and write a % sign.
  For example: $0.654 = 0.654 \times 100\% = 65.4\%$

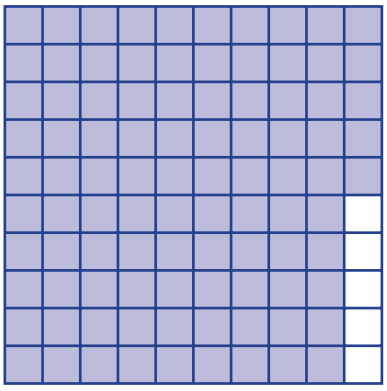
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Exercise 4K

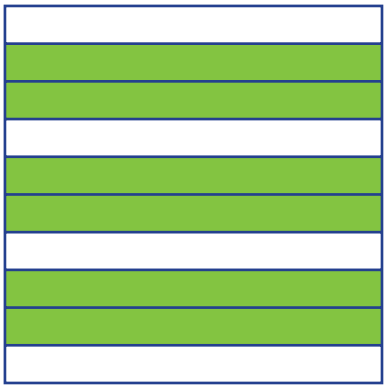
Understanding

1 What percentage of each square has been shaded?

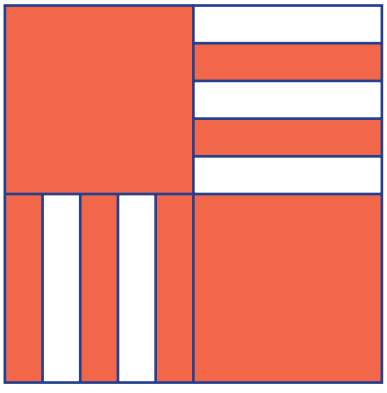
a



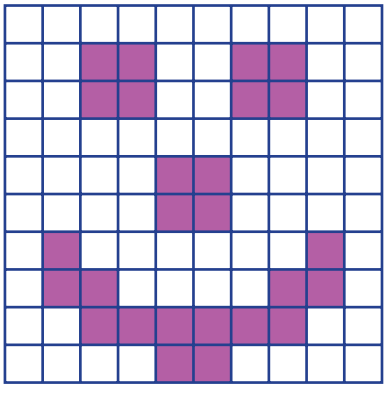
b



c



d



2 72.5% is equivalent to which of the following decimals?

A 72.5 **B** 7.25 **C** 0.725 **D** 725.0

3 0.39 is equivalent to which of the following percentages?

A 39% **B** 3.9% **C** 0.39% **D** 0.0039%

4 Coby answered half the questions correctly for a test marked out of 100.

a What score did Coby get on the test?

b What percentage did Coby get on the test?

c Assuming Coby always gets half the answers correct, find his score if the test was out of:

i 10 **ii** 200 **iii** 40 **iv** 2

d Find Coby's percentage for each of the tests in part **c**.

5 Fill in the empty boxes.

a 58% = 58 out of ☐

b 35% = ☐ out of 100

c 126% = 126 ☐ ☐ 100

d ☐% = 15 out of 100

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Fluency

Example 30 Converting percentages to decimals

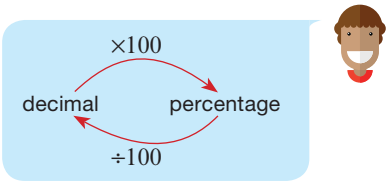
Express the following percentages as decimals.

a 30% **b** 240% **c** 12.5% **d** 0.4%

Solution	Explanation
a $30\% = 0.3$	$30 \div 100 = 30.0 = 0.30 = 0.3$
b $240\% = 2.4$	$240 \div 100 = 240. = 2.4$
c $12.5\% = 0.125$	$12.5 \div 100$ Decimal point appears to move 2 places to the left.
d $0.4\% = 0.004$	$0.4 \div 100$ Decimal point appears to move 2 places to the left.

6 Express the following percentages as decimals.

a 32%	**b** 27%	**c** 68%	**d** 54%
e 10%	**f** 12%	**g** 18%	**h** 85%
i 92%	**j** 75%	**k** 11%	**l** 60%
m 6%	**n** 9%	**o** 100%	**p** 1%
q 218%	**r** 142%	**s** 75%	**t** 199%



7 Express the following percentages as decimals.

a 22.5%	**b** 17.5%	**c** 33.33%	**d** 8.25%
e 112.35%	**f** 188.8%	**g** 150%	**h** 520%
i 0.79%	**j** 0.025%	**k** 1.04%	**l** 0.95%

Example 31 Converting decimals to percentages

Express the following decimals as percentages.

a 0.045 **b** 7.2

Solution	Explanation
a $0.045 = 4.5\%$	$0.045 \times 100 = 4.5$ Decimal point appears to move 2 places to the right.
b $7.2 = 720\%$	$7.2 \times 100 = 720.$

8 Express the following decimals as percentages.

a 0.8	**b** 0.3	**c** 0.45	**d** 0.71
e 0.416	**f** 0.375	**g** 2.5	**h** 2.314
i 0.025	**j** 0.0014	**k** 12.7	**l** 1.004

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4K

Problem-solving and Reasoning

9 At a hockey match, 65% of the crowd supports the home team. What percentage of the crowd supports the visiting team?



10 Sarah thinks that the chance of beating her brother, Tim, at tennis is 52%. What is this chance as a decimal?

11 Write the following as:

i percentages **ii** decimals

a 50 cents in the dollar
b 25 cents in the dollar
c 90 cents in the dollar
d 10 cents in the dollar
e 100 cents in the dollar

12 a Explain why Yuang could not expect to get more than 100% for his Maths test.
b What does it mean to get 50% on a test out of 40?
c If a student received a mark of 0% for a test, how many questions were answered correctly?

Enrichment: AFL ladder

13 The AFL ladder has the following column headings.

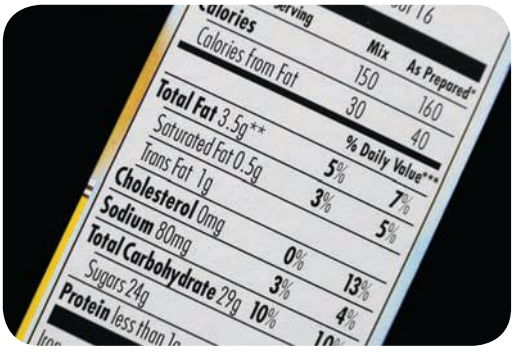
Pos	Team	P	W	L	D	F	A	%	Pts
6	Brisbane Lions	22	13	8	1	2017	1890	106.72	**54**
7	Carlton	22	13	9	0	2270	2055	110.46	**52**
8	Essendon	22	10	11	1	2080	2127	97.79	**42**
9	Hawthorn	22	9	13	0	1962	2120	92.55	**36**
10	Port Adelaide	22	9	13	0	1990	2244	88.68	**36**

a Using a calculator, can you determine how the percentage column is calculated?
b What do you think the 'F' and the 'A' column stand for?
c In their next match, Essendon scores 123 points for their team and has 76 points scored against them. What will be their new percentage?
d Find out the current For and Against points for your favourite AFL team.



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4L Fraction and percentage conversions



A fraction can be interpreted as a percentage of the total.

Percentages are used for test results, discounts and interest rates.

A percentage is another way of writing a fraction with a denominator of 100. For example, 87 per cent simply means 87 out of 100, so:

$$87\% = \frac{87}{100}$$

Let's start: Student ranking

Five students each completed a different Mathematics test.

- Matthew scored 15 out of a possible 20 marks.
- Mengna scored 36 out of a possible 50 marks.
- Miriam scored 17 out of a possible 25 marks.
- Marcus scored 7 out of a possible 10 marks.
- Melissa scored 128 out of a possible 200 marks.

Change these test results to equivalent scores out of 100, and therefore state the percentage test score for each student. Why are percentages useful in this situation?

Key ideas

- We can write percentages as fractions by changing the % sign to a denominator of 100 (meaning out of 100).
  For example: $37\% = \frac{37}{100}$
- We can use equivalent fractions to convert fractions to percentages.
  For example: $\frac{1}{4} = \frac{25}{100} = 25\%$
  – This idea works well if the denominator is 2, 4, 5, 10, 20, 25 or 50.
- You can also use the fraction button on your calculator.
  For example: $\frac{3}{8} \times 100$ gives 37.5, so $\frac{3}{8} = 37.5\%$.
- It is useful to know the following common percentages and their equivalent fractions.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{10}$
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$66\frac{2}{3}\%$	75%	10%

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Exercise 4L

Understanding

1 Change these test results to equivalent scores out of 100, and therefore state the percentage.

a 7 out of 10 = _____ out of 100 = _____%
b 24 out of 50 = _____ out of 100 = _____%
c 12 out of 20 = _____ out of 100 = _____%
d 1 out of 5 = _____ out of 100 = _____%
e 80 out of 200 = _____ out of 100 = _____%
f 630 out of 1000 = _____ out of 100 = _____%

2 Copy and complete these patterns.

a $\frac{1}{4} = 25\%$

$\frac{2}{4} = 50\%$

$\frac{3}{4} = \square$

$\frac{4}{4} = \square$

b $\frac{1}{5} = 20\%$

$\frac{2}{5} = \square$

$\frac{3}{5} = 60\%$

$\frac{4}{5} = \square$

$\frac{5}{5} = 100\%$

c $\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{2}{3} = \square$

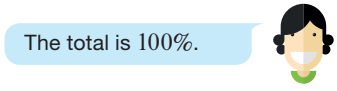
$\frac{3}{3} = \square$

d $\frac{1}{10} = 10\%$

$\frac{2}{10} = \square$

$\frac{3}{10} = \square$

$\frac{4}{10} = \square$



3 Zoe scored 100% on her fractions test. The test was out of 25. What was Zoe's mark?

4 **a** If 14% of students in Year 7 are absent due to illness, what percentage of Year 7 students are at school?

b If 80% of the Geography project has been completed, what percentage still needs to be finished?



Fluency

Example 32 Converting percentages to fractions

Express these percentages as fractions or mixed numbers in their simplest form.

a 17% **b** 36% **c** 140%

Solution	Explanation
a $17\% = \frac{17}{100}$	Change % sign to a denominator of 100.

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b $36\% = \frac{36}{100}$ — Change % sign to a denominator of 100.

$= \frac{\cancel{36}^{9}}{\cancel{100}_{25}}$ — Divide numerator and denominator by their HCF, which in this case is 4.

$= \frac{9}{25}$ — Answer is now in simplest form.

c $140\% = \frac{140}{100}$ — Change % sign to a denominator of 100.

$= \frac{\cancel{140}^{7}}{\cancel{100}_{5}}$ — Divide numerator and denominator by their HCF, which in this case is 20.

$= \frac{7}{5} = 1\frac{2}{5}$ — Convert answer to a mixed numeral.

5 Express these percentages as fractions in their simplest form. Use a calculator to check your answers.

a 11%	**b** 71%	**c** 43%	**d** 49%	**e** 25%	**f** 30%
g 15%	**h** 88%	**i** 7%	**j** 19%	**k** 21%	**l** 50%
m 70%	**n** 90%	**o** 99%	**p** 55%	**q** 125%	**r** 120%

6 Express these percentages as mixed numbers in their simplest form. Use a calculator to check your answers.

a 120%	**b** 180%	**c** 237%	**d** 401%	**e** 175%	**f** 110%
g 316%	**h** 840%	**i** 200%	**j** 205%	**k** 310%	**l** 350%



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4L

Example 33 Converting to percentages through equivalent fractions

Convert the following fractions to percentages.

a $\frac{5}{100}$ **b** $\frac{11}{25}$

Solution	Explanation
a $\frac{5}{100} = 5\%$	Denominator is already 100, therefore simply write the number as a percentage.
b $\frac{11}{25} \overset{\times 4}{=} \frac{44}{100} = 44\%$	We need the denominator to be 100. Therefore, multiply numerator and denominator by 4 to get an equivalent fraction.

7 Convert these fractions to percentages, using equivalent fractions.

a $\frac{98}{100}$ **b** $\frac{9}{100}$ **c** $\frac{79}{100}$ **d** $\frac{56}{100}$

e $\frac{8}{100}$ **f** $\frac{15}{100}$ **g** $\frac{97}{100}$ **h** $\frac{50}{100}$

i $\frac{7}{20}$ **j** $\frac{8}{25}$ **k** $\frac{43}{50}$ **l** $\frac{18}{20}$

m $\frac{56}{50}$ **n** $\frac{27}{20}$ **o** $\frac{20}{5}$ **p** $\frac{16}{10}$



First write with a denominator of 100.

Example 34 Converting to percentages using a calculator

Convert the following fractions to percentages using a calculator.

a $\frac{3}{8}$ **b** $3\frac{3}{5}$

Solution	Explanation
a $\frac{3}{8} \times 100\% = 37.5\%$	$\frac{3}{8} \times 100 = 37.5$ So $\frac{3}{8} = 37.5\%$.
b $3\frac{3}{5} \times 100\% = 360\%$	$3\frac{3}{5} \times 100 = 360$ So $3\frac{3}{5} = 360\%$.

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8 Convert these fractions to percentages using a calculator.

a $\frac{1}{8}$ **b** $\frac{1}{3}$ **c** $\frac{4}{15}$ **d** $\frac{10}{12}$

e $1\frac{3}{20}$ **f** $4\frac{1}{5}$ **g** $2\frac{36}{40}$ **h** $\frac{13}{40}$

Problem-solving and Reasoning

9 A lemon tart is cut into eight equal pieces. What percentage of the tart does each piece represent?



10 A bottle of lemonade is only 25% full.

a What fraction of the bottle has been consumed?

b What percentage of the bottle has been consumed?

c What fraction of the bottle is left?

d What percentage of the bottle is left?

11 Petrina scores 28 out of 40 on her fractions test. What is her percentage score?

12 The Sydney Kings basketball team have won 14 out of 18 games. They still have two games to play. What is the smallest and the largest percentage of games the Sydney Kings could win for the season?



Enrichment: Write your own instruction book

13 Write down the calculator steps for each conversion.

a Convert $\frac{3}{8}$ to a decimal.

b Convert 0.375 to a fraction.

c Convert 37.5% to a fraction.

d Convert $\frac{3}{8}$ to a percentage.

e Convert 0.375 to a percentage.

f Convert 37.5% to a decimal.

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4M Percentage of a quantity

Throughout life you will come across many examples where you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, salary increases and more. In this section we will focus on the mental calculation of percentages.



Retail sales usually involve the price being reduced by a percentage.

Let's start: Percentages in your head

It is a useful skill to be able to calculate percentages mentally. Calculating 10% or 1% is often a good starting point. You can then multiply or divide these values to quickly arrive at other percentage values.

With a partner, and using mental arithmetic only, calculate 10% of each of these amounts.

a \$120	**b** \$35	**c** \$160	**d** \$90
e \$300	**f** \$40	**g** \$80	**h** \$420
i \$1400	**j** \$550	**k** \$200	**l** \$60

How did you find 10% of an amount mentally?

Key ideas

- To find the percentage of a number, without using a calculator:
  1. Express the required percentage as a fraction.
  2. Change the 'of' to a multiplication sign.
  3. Perform the multiplication.
  4. Simplify the answer.
- Useful mental strategies:
  - To find 50%, divide by 2.
  - To find 10%, divide by 10.
  - To find 25%, divide by 4.
  - To find 1%, divide by 100.
- Using a calculator:
  25% of 60 = 25 ÷ 100 × 60

$$\begin{aligned} 25\% \text{ of } 60 &= \frac{1}{4} \text{ of } 60 \\ &= 60 \div 4 \\ &= 60 \div 2 \div 2 \\ &= 30 \div 2 \\ &= 15 \end{aligned}$$

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Exercise 4M

Understanding

1 Copy and complete the following.
- **a** To find 10% of \$20, you can use $20 \div \square = 2$.
- **b** To find 25% of \$20, you can use $20 \div 4 = \square$.
- **c** To find 50% of \$20, you can use $20 \div \square = \square$.

2 **a** If 10% of ★ = 3, find 20% of ★. **b** If 1% of ▲ = 7, find 5% of ▲.
c If 50% of ● = 8, find 100% of ●.

3 Use mental strategies to find:
- **a** 10% of \$500 **b** 1% of \$900 **c** 25% of 84 kilograms
- **d** 50% of 7 days **e** 75% of 84 kilograms

4 What is 100% of 8 hours?

Fluency

Example 35 Finding the percentage of a number

Find:
a 30% of 50 **b** 15% of 400 **c** 25% of 200

Solution

a 10% of 50 = 5
30% of 50 = 15

Calculator method:
30% of 50 = (30 ÷ 100) × 50
= 15

b 10% of 400 = 40, 5% of 400 = 20
15% of 400 = 60

c $25\% \text{ of } 200 = \frac{1}{4} \text{ of } 200$
$= 200 \div 4$
$= (200 \div 2) \div 2$
$= 100 \div 2$
$= 50$

Explanation

10% is easy to find mentally.
Multiply 10% by 3 to find 30%.

30% means $\frac{30}{100}$ or $30 \div 100$.

Find 10% mentally. Halve to find 5%.
Add 10% and 5% to find 15%.

$25\% = \frac{1}{4}$
One way to divide by 4 is to divide by 2 and then divide by 2 again.

Drilling for Gold 4M1

5 Find, without using a calculator:
- **a** 50% of 800 **b** 10% of 800 **c** 5% of 800
- **d** 25% of 800 **e** 1% of 800 **f** 15% of 800

6 Use mental strategies to calculate the following. Check your answers with a calculator.
- **a** 50% of 140 **b** 10% of 360 **c** 20% of 50 **d** 30% of 90
- **e** 25% of 40 **f** 25% of 28 **g** 75% of 200 **h** 80% of 250
- **i** 5% of 80 **j** 5% of 1200 **k** 5% of 880 **l** 10% of 9500
- **m** 20% of 200 **n** 20% of 400 **o** 30% of 300 **p** 30% of 700

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4M

7 Calculate the following, using a calculator to help you.

a 120% of 80	**b** 150% of 400	**c** 110% of 60	**d** 400% of 25
e 125% of 12	**f** 225% of 32	**g** 146% of 50	**h** 3000% of 20

8 Match each question with the correct answer, without using a calculator.

Questions	*Answers*
10% of \$200	\$8
20% of \$120	\$16
10% of \$80	\$20
50% of \$60	\$24
20% of \$200	\$25
5% of \$500	\$30
30% of \$310	\$40
10% of \$160	\$44
1% of \$6000	\$60
50% of \$88	\$93

Skillsheet 4C

9 Use mental strategies to find:

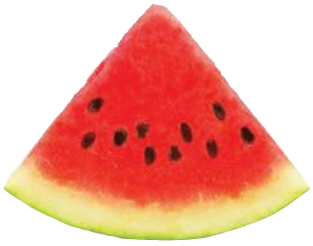
a 30% of \$120	**b** 10% of 240 millimetres
c 15% of 60 kilograms	**d** 2% of 4500 tonnes
e 20% of 40 minutes	**f** 30% of 500 centimetres
g 5% of 80 grams	**h** 25% of 20 hectares



Remember that the answers must include units.

Problem-solving and Reasoning

10 25% of teenagers say their favourite fruit is watermelon. In a survey of 48 teenagers, how many students would write watermelon as their favourite fruit?



11 Harry scored 70% on his percentages test. If the test is out of 50 marks, how many marks did Harry score?

12 In a student survey, 80% of students said they received too much homework. If 300 students were surveyed, how many students felt they get too much homework?

13 At Gladesbrook College, 10% of students walk to school, 35% of students catch public transport and the rest are driven to school. If there are 1200 students at the school, find how many students:

a walk to school

b catch public transport

c are driven to school



Enrichment: Percentage challenge

14 a Which is larger: 60% of 80 or 80% of 60?

b Tom did the following calculation: $120 \div 4 \div 2 \times 3$. What percentage of 120 did Tom find?

c **i** If 5% of an amount is \$7, what is 100% of the amount?

ii If 25% of an amount is \$3, what is $12\frac{1}{2}$% of the amount?

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4N Using fractions and percentages to compare two quantities

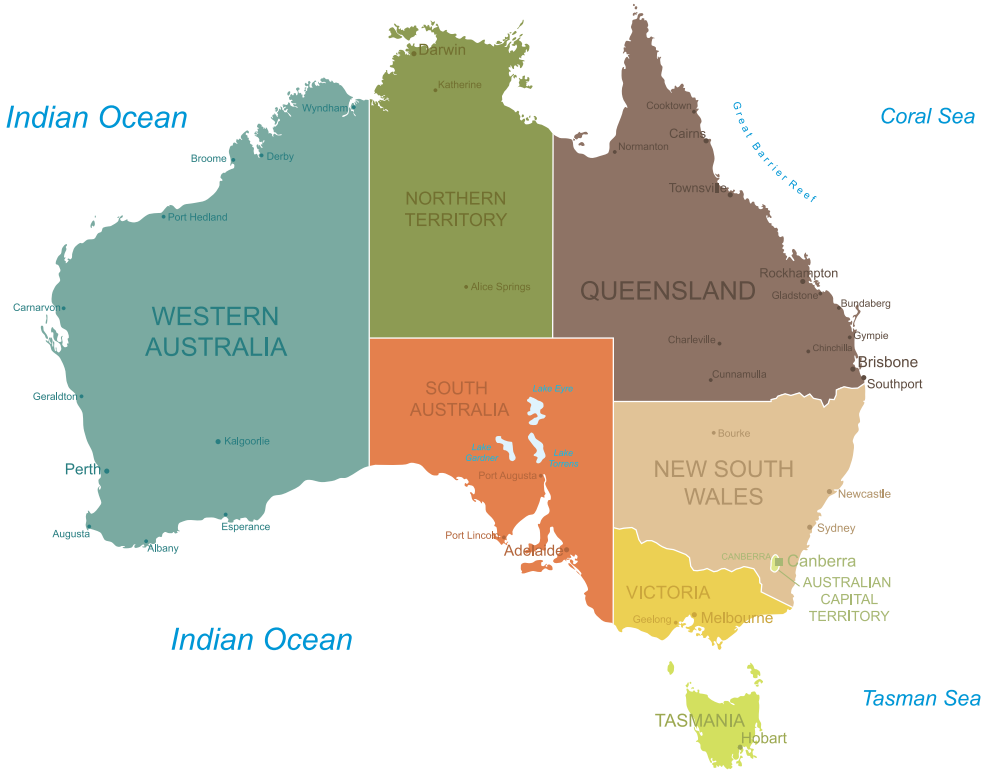
The Earth's surface is about 70% ocean. So, the proportion of land could be written as 30% (as a percentage) or $\frac{3}{10}$ (as a fraction). The ratio of land to ocean could be described as 30 parts of land to 70 parts of ocean. Alternatively, the ratio could be expressed as 3 parts of land to 7 parts of ocean.



The proportion of land to sea in this photo of the Whitsunday Islands, Queensland, could be expressed as a fraction, percentage or ratio.

Let's start: Map of Australia

All the States and Territories of Australia are different shapes and sizes.



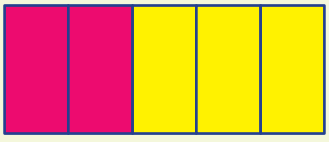
- Write a list of the eight States and Territories.
- Estimate the percentage of Australia that each one occupies. Make sure your percentages add up to 100%.
  Hint: You will need to use 2 decimal places for the ACT.

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Key ideas

Proportion
A comparison of a part to the whole or one quantity with another quantity

- We can express **proportion** as a fraction, percentage or ratio.



Red fraction $= \frac{2}{5}$

Red percentage $= \frac{2}{5} = \frac{4}{10} = \frac{40}{100} = 40\%$

Ratio of red parts to all parts $= 2:5$
Ratio of red to yellow $= 2:3$

- This can also be done using a calculator:
  $2 \div 5 \times 100 = 40$
  So $\frac{2}{5} = 40\%$.

Exercise 4N

Understanding

Example 36 Expressing fractions as percentages

Express $\frac{4}{5}$ as a percentage.

Solution	Explanation
$\frac{4}{5} = 80\%$	$\frac{4}{5} = \frac{8}{10} = \frac{80}{100}$

1 Use the diagram to complete the following.



a pink fraction $= \frac{\square}{\square}$

b green fraction $= \frac{\square}{\square}$

c pink percentage = ___%

d green percentage = ___%

e ratio of pink to green = $\square : \square$

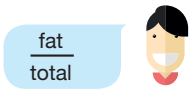
f ratio of green to pink = $\square : \square$

2 Express these fractions as percentages.

a $\frac{1}{4}$ **b** $\frac{10}{50}$ **c** $\frac{12}{25}$ **d** $\frac{99}{100}$ **e** $\frac{17}{100}$ **f** $\frac{17}{50}$ **g** $\frac{7}{20}$ **h** $\frac{9}{10}$

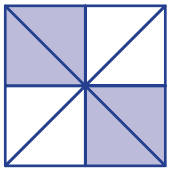
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3 A cake contains 8 grams of fat. The cake has a mass of 200 grams.



- **a** Write the fraction of the cake which is fat.
- **b** Simplify the fraction in part **a**.
- **c** Use this simplified fraction to find the percentage of fat in the cake.

4 This square shows some coloured triangles and some white triangles.



- **a** How many triangles are coloured?
- **b** How many triangles are white?
- **c** What fraction of the total is coloured?
- **d** What percentage of the total is coloured?
- **e** What fraction of the total is white?
- **f** What percentage of the total is white?
- **g** What is the ratio of coloured : white?

5 A farmer's pen has 2 black sheep and 8 white sheep.

- **a** How many sheep are there in total?
- **b** What fraction of the sheep are black?
- **c** What fraction of the sheep are white?
- **d** What percentage of the sheep are black?
- **e** What percentage of the sheep are white?
- **f** What is the ratio of black sheep to white sheep?

Fluency

Example 37 Expressing a proportion

Express 24 green ducks out of a total of 30 ducks as a fraction and then as a percentage.

Solution	Explanation
Fraction $= \frac{24}{30}$	24 out of 30 is $\frac{24}{30}$.
$= \frac{4}{5}$	Simplify the fraction.
Percentage $= 80\%$	$\frac{4}{5} = \frac{8}{10} = \frac{80}{100} = 80\%$
	Or, using a calculator, $4 \div 5 \times 100 = 80$.

6 Express each of these proportions as a fraction and then as a percentage.

- **a** 30 out of a total of 100
- **b** 3 out of a total of 5
- **c** \$10 out of a total of \$50
- **d** \$60 out of a total of \$80
- **e** 2 kg out of a total of 40 kg
- **f** 14 g out of a total of 28 g
- **g** 3 L out of a total of 12 L
- **h** 30 mL out of a total of 200 mL

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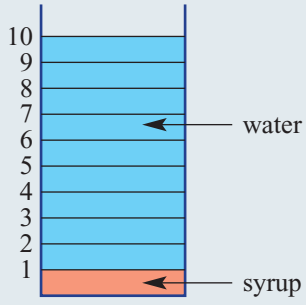
4N

Example 38 Using ratios

A glass of cordial is 1 part syrup to 9 parts water.

a Express the amount of syrup as a fraction of the total.

b Express the amount of water as a percentage of the total.



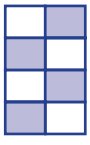
Solution	Explanation
a $\frac{1}{10}$	The total is 10 parts, including 1 part syrup.
b 90%	$\frac{9}{10} = \frac{90}{100} = 90\%$ or, using a calculator $9 \div 10 \times 100 = 90$.

7 Write each coloured area as a fraction and then as a percentage of the total area.

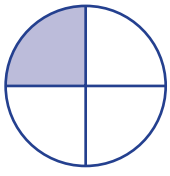
a



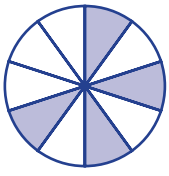
b



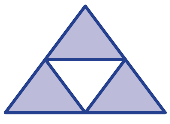
c



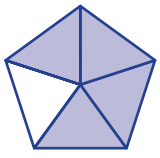
d



e



f



8 A jug of lemonade is made up of 2 parts of lemon juice to 18 parts of water.

a Express the amount of lemon juice as a fraction of the total.

b Express the amount of lemon juice as a percentage of the total.

9 A mix of concrete is made up of 1 part of cement to 4 parts of sand.

a Express the amount of cement as a fraction of the total.

b Express the amount of cement as a percentage of the total.

c Express the amount of sand as a fraction of the total.

d Express the amount of sand as a percentage of the total.

Problem-solving and Reasoning

10 Gillian pays $80 tax out of her income of $1600.

a What fraction of her income is tax?

b What percentage of her income is tax?

c What percentage of her income does she keep?

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11 During summer, the amount of water in a dam reduced from 20 megalitres to 4 megalitres.

- **a** How many megalitres of water were used?
- **b** What fraction of the water was used?
- **c** What percentage of the water was used?

12 Express the following as a fraction and percentage of the total.

- **a** 20 cents out of 500 cents
- **b** 14 days out of 35 days
- **c** 15 centimetres out of 3 metres (300 centimetres)
- **d** 15 seconds out of 5 minutes
- **e** 50 centimetres from a total of 2 metres
- **f** 1500 metres from 2 kilometres



Use the same units:
5 minutes
$= 5 \times 60$ seconds
$= 300$ seconds

13 For a recent class test, Ross scored 45 out of 50 and Maleisha scored 72 out of 100. Use percentages to show that Ross obtained the higher result.

Enrichment: Estimating test results as percentages



Drilling for Gold 4N1

14 Sometimes test results are given 'out of' strange numbers, like 87. In the table below:

- **a** Without using a calculator, estimate what each result would be when converted to a percentage. Write them in the 'My estimate' column.
- **b** Use your calculator to convert each result to a percentage. Round off to the nearest whole number. Write them in the 'Actual' column.
- **c** Calculate the difference between each estimate and actual and write them in the 'Error' column.
- **d** Find the sum of the 'Error' column. This is your score. The lower it is, the better you are at estimating.

	Test result	My estimate	Actual	Error
1	11 out of 19	%	%	
2	30 out of 48	%	%	
3	20 out of 26	%	%	
4	61 out of 75	%	%	
5	24 out of 32	%	%	
			My score is	

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Puzzles and games

1 Three cities are known as India's Golden Triangle. To find the names of these cities, complete the puzzle.
Match each of the fractions in the middle row with the equivalent fraction in the bottom row. Place the letter in the code below.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{4}{24}$	$\frac{28}{35}$	$\frac{100}{120}$	$\frac{5}{7}$	$\frac{21}{36}$	$\frac{1}{2}$	$\frac{22}{77}$	$\frac{2}{3}$	$4\frac{2}{5}$	$\frac{81}{90}$	$\frac{25}{3}$	$\frac{43}{9}$
U = $8\frac{1}{3}$	A = $\frac{5}{6}$	H = $\frac{15}{21}$	D = $\frac{4}{5}$	G = $\frac{1}{6}$	N = $4\frac{7}{9}$	I = $\frac{7}{12}$	E = $\frac{2}{7}$	P = $\frac{18}{27}$	J = $\frac{9}{10}$	R = $\frac{48}{96}$	L = $\frac{22}{5}$

2	7	9	4	5

3	1	6	3

3	12	2

10	3	5	8	11	6

2 *Forming fractions*


Drilling for Gold 4PG1

Make a set of cards that look like these shown below.

1	2	3	4	5	6	7	8	9

In the following questions, any two of these cards can be used to make a fraction. One card is used as the numerator and the other is used as the denominator.
(Hint: Use the fraction wall in **Section 4J** to help answer the questions.)

- **a** What fraction with the least value can you make?
- **b** What fraction with the greatest value can you make?
- **c** How many fractions can be simplified to give whole numbers?
- **d** What is the fraction you can make with these cards that is closest to 1 but less than 1?
- **e** What fractions can you make that are equal to 0.5?
- **f** What fractions can you make that are equal to 75%?
- **g** How many fractions can you make that are greater than 0.5 but less than 1?
- **h** Use four different cards to make two fractions that add together to give 1.
  In how many ways can this be done?
- **i** Use six different cards to make three fractions that add together to give 1.
  In how many ways can this be done?
- **j** How many fractions can you make that are greater than 1 but less than 1.5?

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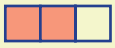
Chapter summary

Understanding fractions

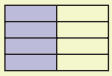
A fraction is part of a whole.

$\frac{2}{3}$ ← Number of shaded parts (numerator)

$\frac{2}{3}$ ← Number of equal parts (denominator)



Equivalent fractions



$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$

Simplifying fractions

$\div 4$

$\frac{4}{8} = \frac{1}{2}$

$\div 4$

Divide the numerator and the denominator by the HCF.

proper fraction $\frac{3}{4}$

whole number $\frac{7}{7} = 1$, $\frac{10}{5} = 2$

improper fraction $\frac{7}{5}$

mixed numeral $1\frac{2}{5}$

$1 \times 5 + 2$

mixed numeral $1\frac{2}{5}$ ⇄ improper fraction $\frac{7}{5}$

$7 \div 5 = 1$ remainder 2

Comparing fractions

- Same numerator: compare the denominators

  $\frac{1}{3} < \frac{1}{2}$

  is less than

- Same denominator: compare the numerators

  $\frac{3}{8} > \frac{1}{8}$

  is greater than

- Different denominators: change to **LCD** first (**L**owest **C**ommon **D**enominator).

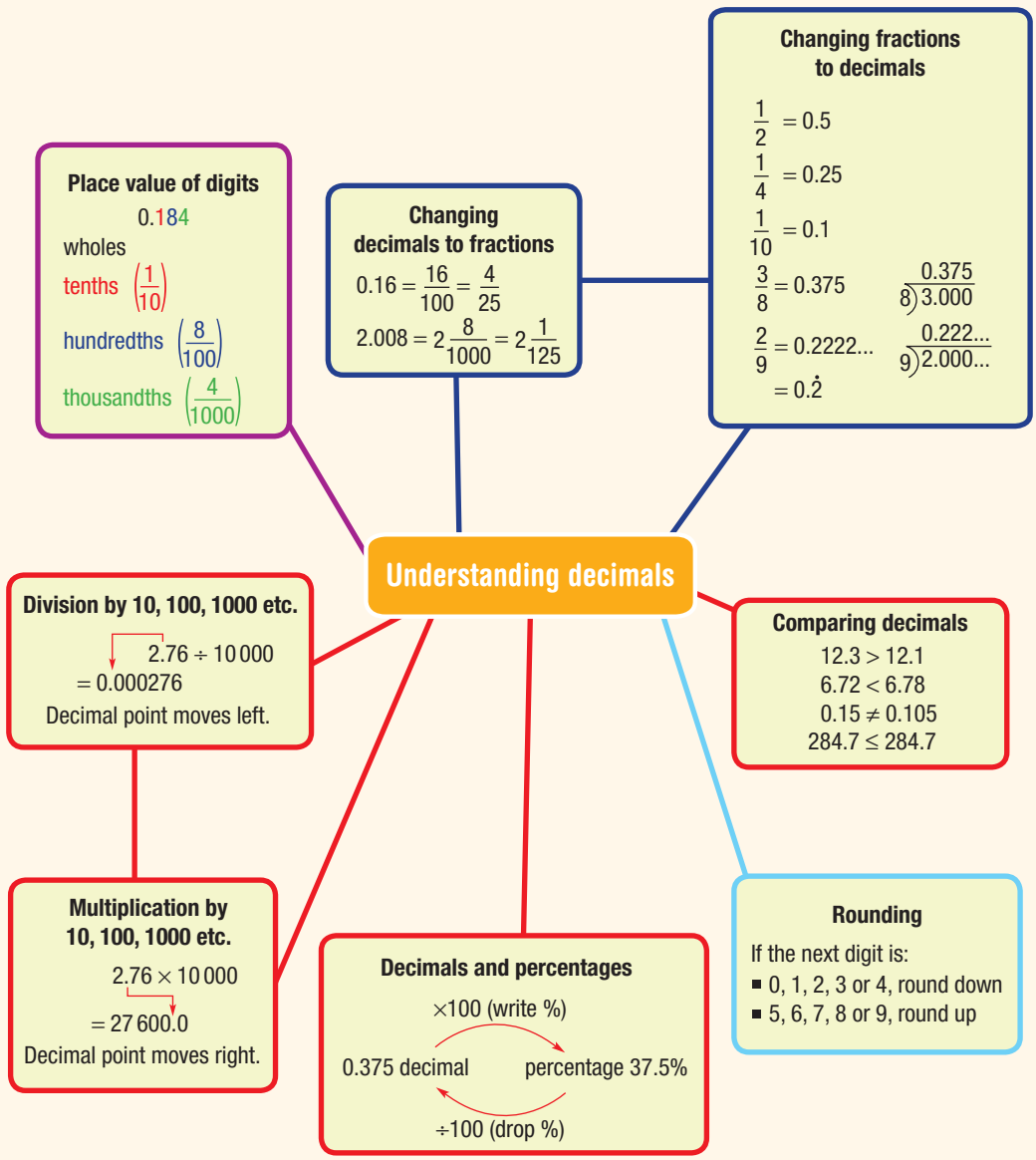
  $\frac{5}{6} = \frac{20}{24}$

  $\frac{7}{8} = \frac{21}{24}$ ← LCD

  $\therefore \frac{5}{6} < \frac{7}{8}$

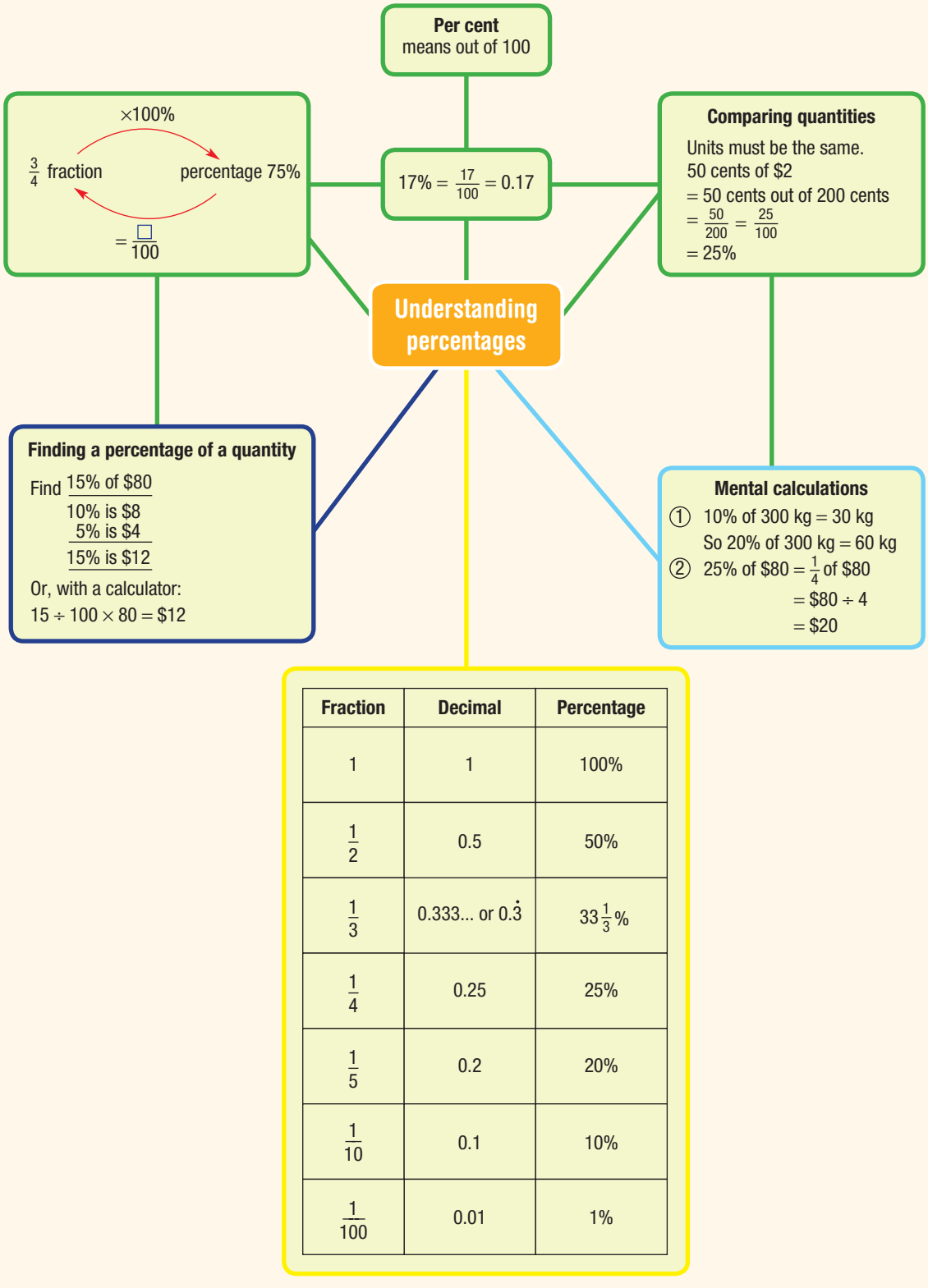
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Chapter summary



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Chapter summary



Fraction	Decimal	Percentage
1	1	100%
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	0.333... or $0.\dot{3}$	$33\frac{1}{3}$%
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{10}$	0.1	10%
$\frac{1}{100}$	0.01	1%

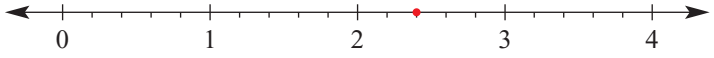
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Chapter review

T **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

Multiple-choice questions

1 Which fraction is shown on the number line?



A $\frac{2}{3}$ **B** $\frac{4}{5}$ **C** $\frac{12}{5}$ **D** $2\frac{2}{10}$ **E** $\frac{5}{12}$

2 $\frac{3}{4}$ is the same as:

A $\frac{4}{5}$ **B** $\frac{75}{100}$ **C** $\frac{7}{8}$ **D** $1\frac{1}{3}$ **E** $\frac{2}{3}$

3 $\frac{4}{5}$ is smaller than:

A $\frac{7}{10}$ **B** $\frac{3}{4}$ **C** $\frac{1}{2}$ **D** $\frac{9}{10}$ **E** $\frac{79}{100}$

4 Which is the lowest common denominator for this set of fractions? $\frac{1}{3}, \frac{1}{4}, \frac{5}{6}$

A 3 **B** 4 **C** 6 **D** 72 **E** 12

5 Maria has 15 red apples and 5 green apples. What fraction of the apples are green?

A 5 **B** $\frac{1}{3}$ **C** $\frac{2}{3}$ **D** $\frac{1}{4}$ **E** $\frac{3}{4}$



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Chapter review

6 Which of the following is true?

A $\frac{1}{2} > \frac{4}{5}$ **B** $\frac{1}{2} < \frac{3}{8}$ **C** $\frac{1}{2} > \frac{2}{5}$ **D** $\frac{1}{2} > \frac{9}{10}$ **E** $\frac{1}{2} < \frac{4}{11}$

7 Which set of fractions is ordered from smallest to largest (i.e. in ascending order)?

A $\frac{1}{2}, \frac{2}{3}, \frac{5}{12}$ **B** $\frac{1}{2}, \frac{13}{24}, \frac{1}{4}$ **C** $\frac{2}{3}, \frac{1}{2}, \frac{7}{12}$ **D** $\frac{7}{12}, \frac{1}{2}, \frac{1}{4}$ **E** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

8 0.005 is equivalent to:

A $\frac{5}{10}$ **B** $\frac{5}{100}$ **C** $\frac{5}{1000}$ **D** $\frac{5}{500}$ **E** 5

9 Which fraction is greater than 75%?

A $\frac{1}{4}$ **B** $\frac{3}{4}$ **C** $\frac{15}{24}$ **D** $\frac{4}{5}$ **E** $\frac{6}{8}$

10 What is \$5 as a percentage of \$25?

A 50% **B** 20% **C** 25% **D** 5% **E** 10%

11 $\frac{60}{14}$ can be written as:

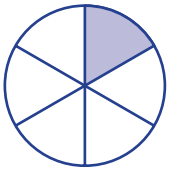
A $4\frac{2}{7}$ **B** $2\frac{4}{7}$ **C** $4\frac{2}{14}$ **D** $7\frac{4}{7}$ **E** $5\frac{1}{7}$

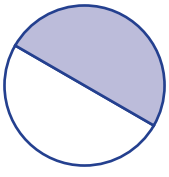
12 $\frac{3}{4}$ of a metre of material is needed for a school project. How many centimetres is this?

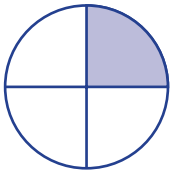
A 25 cm **B** 50 cm **C** 75 cm **D** 80 cm **E** 60 cm

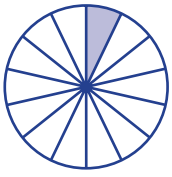
Short-answer questions

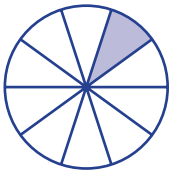
1 Write the fraction of each circle that is shaded.

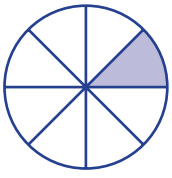
a 

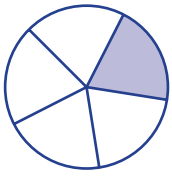
b 

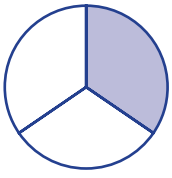
c 

d 

e 

f 

g 

h 

2 Given that $24 = 12 \times 2$ and $36 = 12 \times 3$, simplify the fraction $\frac{24}{36}$.

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Chapter review

3 Write the following fractions in simplest form.

a $\frac{18}{30}$ **b** $\frac{8}{28}$ **c** $\frac{30}{40}$

4 Convert each of the following to a mixed numeral in simplest form.

a $\frac{7}{4}$ **b** $\frac{10}{7}$ **c** $\frac{8}{6}$ **d** $\frac{15}{10}$

5 Place the correct symbol (<, = or >) in between the following pairs of fractions to make true mathematical statements.

a $\frac{2}{7} \square \frac{4}{7}$ **b** $\frac{3}{8} \square \frac{1}{8}$ **c** $\frac{2}{3} \square \frac{3}{5}$ **d** $3\frac{1}{9} \square \frac{29}{9}$

6 State the largest fraction in each list.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{3}{8}, \frac{2}{8}, \frac{5}{8}, \frac{1}{8}$

7 State the lowest common multiple for each pair of numbers.

a 2, 5 **b** 3, 7 **c** 8, 12

8 State the lowest common denominator for each set of fractions.

a $\frac{1}{2}, \frac{3}{5}$ **b** $\frac{2}{3}, \frac{3}{7}$ **c** $\frac{3}{8}, \frac{5}{12}$

9 Rearrange each set of fractions in descending order.

a $\frac{4}{4}, \frac{3}{4}, 1\frac{1}{4}, \frac{1}{4}$

b $\frac{1}{8}, \frac{1}{3}, \frac{1}{10}, \frac{1}{5}$

c $\frac{2}{3}, \frac{1}{12}, \frac{5}{6}, \frac{4}{3}$



10 Are the following statements true or false?

a $8.34 < 8.28$ **b** $4.668 > 4.67$ **c** $8.2 > 8.18$

d $3.08 \leq \frac{308}{100}$ **e** $\frac{62}{100} \geq 6.20$ **f** $\frac{7}{10} = \frac{70}{100}$

g $0.6 = 0.60$ **h** $1\% = \frac{1}{10}$ **i** $5\% = 0.05$

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11 Write \$62.876 to the nearest:

a cent **b** dollar **c** 5 cents

12 Round each of the following to 1 decimal place.

a 12.74 **b** 8.36 **c** 9.41

d 7.45 **e** 0.08 **f** 7.124

13 Round each of these to 2 decimal places.

a 12.814 **b** 423.461 **c** 15.889

d 7.2543 **e** 6.6666 **f** 3.3333…

14 Find the missing percentages and fractions.

Percentage form	10%	A	B	75%	C	150%
Fraction	D	$\frac{1}{4}$	$\frac{1}{2}$	E	$\frac{7}{25}$	F

15 **a** Find 10% of \$200. **b** Find 25% of \$840.

c Find 50% of 90 grams. **d** Find 20% of \$150.

16 Express the following as both a fraction and percentage of the total.

a 6 out of 10 **b** \$4 out of \$20

c 50 cents out of \$2 **d** 200 mL out of 2 L

Extended-response question

1 A printer produces 1200 leaflets. One-quarter of the leaflets are on green paper. Half the remaining leaflets are on blue paper. There are smudges on 10% of all the leaflets.

a How many leaflets are on green paper?

b What percentage of the leaflets are *not* on green paper?

c How many blue leaflets were printed?

d Of the blue leaflets, how many have smudges?

e How many leaflets do not have any smudges?



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Chapter 5

Probability

What you will learn

- 5A Describing probability
- 5B Theoretical probability in single-step experiments
- 5C Experimental probability in single-step experiments
- 5D Venn diagrams
- 5E Two-way tables

Strand: Statistics and Probability

Substrand: PROBABILITY

In this chapter, you will learn to:

- represent probabilities of simple and compound events.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Gambling problem or problem gambling?

Gambling activities include lotteries, online gaming, gaming machines, sports betting and table games. The people who invent and run these activities calculate the mathematical probabilities so that, in the long run, the players lose their money.

The average player of gaming machines in NSW loses $4000 every year. Australians lose about $20 billion in total every year through gambling.

It is worth thinking about Probability before becoming involved in gambling activities.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Write these fractions in simplest form.

a $\frac{10}{20}$ **b** $\frac{20}{30}$ **c** $\frac{20}{25}$ **d** $\frac{2}{8}$

2 **a** How many faces are on a standard die?
b How many sides does a coin have?
c How many letters are in the alphabet?
d How many vowels are in the alphabet?
e How many playing cards are there in a standard deck?
f How many positive numbers less then 10 are even?

3 Order these events from least likely to most likely.
A Rolling a die and it landing on the number 3.
B Flipping a coin and it landing with 'tails' showing.
C The Prime Minister of Australia being struck by lightning tomorrow.
D The internet being used in Australia in the next 20 minutes.

4 When a fair die is rolled, what is the probability of the following?
a rolling a 2 **b** rolling an even number
c rolling a number less than 5 **d** rolling a 9

5 Consider the set of numbers 4, 2, 6, 5, 9.
a How many numbers are in the set?
b How many of the numbers are even?
c What fraction of the numbers are even?

6 Copy this table into your workbook and complete.

Fraction	Decimal	Percentage
$\frac{1}{2}$		
$\frac{1}{3}$		
$\frac{1}{4}$		
$\frac{1}{5}$		
$\frac{1}{10}$		
$\frac{1}{100}$		

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5A Describing probability

Every day the Australian weather bureau estimates the chance of rain. They use measurements and observations to make predictions.



Let's start: Likely or unlikely?

Try to rank these events from least likely to most likely. Compare your answers with those of other students in the class.

1. The Sun will rise tomorrow.
2. Flipping a 'tail' on a coin.
3. Australia will win the soccer World Cup.
4. The king of spades is at the top of a shuffled deck of 52 playing cards.
5. You will live to the age of 20.



Key ideas

- There is a large amount of terminology in this topic. The following two pages include most of the key words and phrases. They can be downloaded from *Cambridge GO* and pasted into your book.

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Probability terminology



Drilling for Gold 5A1

Terminology	Example	Definition
chance experiment	rolling a fair 6-sided die	A chance experiment is an activity which may produce a variety of different results that occur randomly. The example given is a single-step experiment.
trials	rolling a die 50 times	When an experiment is performed one or more times, each occurrence is called a trial. The example given indicates 50 trials of a single-step experiment.
outcome	rolling a 5	An outcome is one of the possible results of a chance experiment.
equally likely outcomes	rolling a 5 rolling a 6	Equally likely outcomes are two or more results that have the same chance of occurring.
sample space	{1, 2, 3, 4, 5, 6}	The sample space is the set of all possible outcomes of an experiment. It is usually written inside braces, as shown in the example.
event	e.g. 1: rolling a 2 e.g. 2: rolling an even number	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.
compound event	rolling an even number (e.g. 2 or 4 or 6)	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
mutually exclusive events	rolling a 5 rolling an even number	Two or more events are mutually exclusive if they share no outcomes.
non-mutually exclusive events	rolling a 5 rolling an odd number	Events are non-mutually exclusive if they share one or more outcomes. In the example given the outcome 5 is shared.
complementary events	rolling a 2 or 3 rolling a 1, 4, 5 or 6	If *all* the outcomes in the sample space are divided into two events, they are complementary events.
complement	Rolling 2, 3, 4 or 5 is an event. Rolling a 1 or 6 is the complement.	If an experiment was performed and an event did not occur, then the complement definitely occurred.
favourable outcome(s)	In some games you must roll a 6 before you can start moving your pieces.	Outcomes are favourable when they are part of some desired event.

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theoretical probability or likelihood or chance	The probability of rolling an odd number is written as: $P(\text{odd}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$ Probabilities can be expressed as fractions, decimals and percentages.	Theoretical probability is the actual chance or likelihood that an event will occur when an experiment takes place. $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Probabilities range from 0 to 1 or 0% to 100%.
experimental probability	A die was rolled 600 times and showed a 5 on 99 occasions. The experimental probability of rolling a 5 on this die is: $P(5) \approx \frac{99}{600} = 0.165 = 16.5\%$	Sometimes it is difficult or impossible to calculate a theoretical probability, so an estimate can be found using a large number of trials. This is called the experimental probability. If the number of trials is large, then the experimental probability should be very close to the theoretical.
certain likely even chance unlikely impossible	Rolling a number below 7 Rolling a number below 6 Rolling a 1, 2 or 3 Rolling a 2 Rolling a 7	The probability is 100% or 1. The probability is 50% or 0.5 or $\frac{1}{2}$. The probability is 0% or 0.
the sum of all probabilities in an experiment	$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$ $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$	The sum of the probabilities of all the outcomes of a chance experiment is 1.
the sum of the probabilities of an event and its complement	$P(\text{rolling } 1 \text{ or } 6) = \frac{2}{6}$ $P(\text{rolling } 2, 3, 4 \text{ or } 5) = \frac{4}{6}$ $\frac{2}{6} + \frac{4}{6} = \frac{6}{6} = 1$	The sum of the probabilities of an event and its complement is 1. $P(\text{event}) + P(\text{complementary event}) = 1$

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Exercise 5A

Understanding


Drilling for Gold 5A2

1 Fill in each blank with the word *likely* or *unlikely*.

- **a** A fair coin is flipped 100 times. It is _____ that these will be 100 tails.
- **b** A page of this book is picked at random. It is _____ that the letter '**e**' will be on the page.
- **c** The television is switched on at 5 pm and left on for 3 hours. It is _____ that the News will be shown at some stage.
- **d** It is _____ that the next Australian prime minister will be 21 years old.

2 Match each of the events (**a** to **d**) with a description of how likely they are to occur (**A** to **D**).

- **a** A tossed coin landing heads up
- **b** Selecting an ace first try from a fair deck of 52 playing cards
- **c** Obtaining a number other than 6 if a fair 6-sided die is rolled
- **d** Obtaining a number greater than 8 if a fair 6-sided die is rolled

- **A** unlikely
- **B** likely
- **C** impossible
- **D** even chance

Fluency

Example 1 Describing chance

Say whether each of the following statements is true or false.

- **a** It is likely that children will go to school next year.
- **b** It is an even chance for a fair coin to display tails.
- **c** Rolling a 3 on a 6-sided die and getting heads on a coin are equally likely.
- **d** It is certain that two randomly chosen odd numbers will add up to an even number.

Solution		Explanation
a	true	Although there is a small chance that the laws might change, it is (very) likely that children will go to school next year.
b	true	There is a 50-50 or even chance of a fair coin displaying tails. It will happen, on average, half of the time.
c	false	These events are not equally likely. Flipping heads on a coin is more likely than rolling a 3 on a 6-sided die.
d	true	No matter what odd numbers are chosen, they will always add to an even number.

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3 Consider a fair 6-sided die with the numbers 1 to 6 on it. Say whether each of the following is true or false.



- **a** Rolling a 3 is unlikely.
- **b** Rolling a 5 is likely.
- **c** Rolling a 4 and rolling a 5 are equally likely events.
- **d** Rolling an even number is likely.
- **e** There is an even chance of rolling an odd number.
- **f** There is an even chance of rolling a multiple of 3.

4 Copy and complete the following, using the special words that describe chance.

- **a** If an event is guaranteed to occur, we say it is _____.
- **b** An event that is equally likely to occur or not occur has an _____ _____.
- **c** A rare event is considered _____.
- **d** An event that will never occur is called _____.

5 Match each of the events (**a** to **d**) with an equally likely event (**A** to **D**).

- **a** Rolling a 2 on a 6-sided die
- **b** Selecting a heart card from a fair deck of 52 playing cards
- **c** Flipping a coin and tails landing face up
- **d** Rolling a 1 or a 5 on a 6-sided die

- **A** Flipping a coin and heads landing face up
- **B** Rolling a 5 or a 6 on a 6-sided die
- **C** Selecting a diamond card from a fair deck of 52 playing cards
- **D** Rolling a 4 on a 6-sided die



Problem-solving and Reasoning

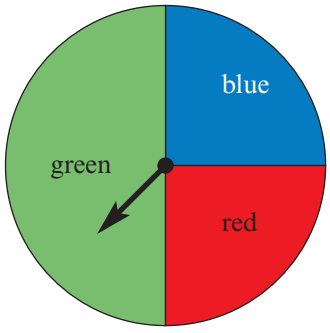
6 Give an example of:

- **a** an event that is unlikely
- **b** an event that is likely
- **c** an event that has an even chance of occurring
- **d** two events that are equally likely

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5A

7 This spinner could land with the arrow pointing to any of the three colours.



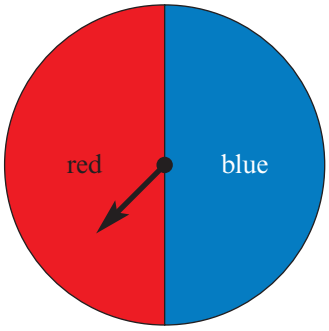
a State whether each of the following is true or false.
- **i** There is an even chance that the spinner will point to green.
- **ii** It is likely that the spinner will point to red.
- **iii** It is certain that the spinner will point to purple.
- **iv** It is equally likely that the spinner will point to red or blue.

b Use the spinner to match the events (**i** to **iv**) with the description (**A** to **D**).

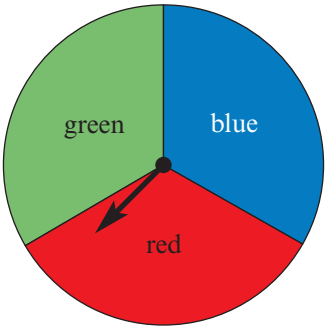
- **i** an impossible event
- **ii** a likely event
- **iii** a certain event
- **iv** two events that are equally likely

- **A** The spinner will land on blue, green or red.
- **B** The spinner will land on green.
- **C** The spinner will land on yellow.
- **D** The spinner will land on either blue or red.

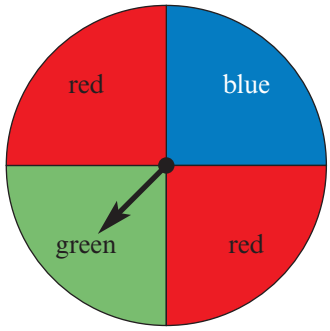
8 Three spinners are shown below. Match each spinner with the description.



spinner 1



spinner 2



spinner 3

a Has an even chance of red, but blue is unlikely.
b Blue and green are equally likely, but red is unlikely.
c Has an even chance of blue, and green is impossible.

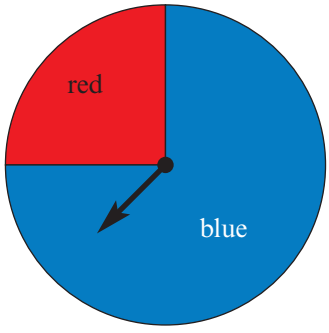
9 Draw spinners to match each of the following descriptions.
a Blue is likely, red is unlikely and green is impossible.
b Red is certain.
c Blue has an even chance, red and green are equally likely.
d Blue, red and green are all equally likely.



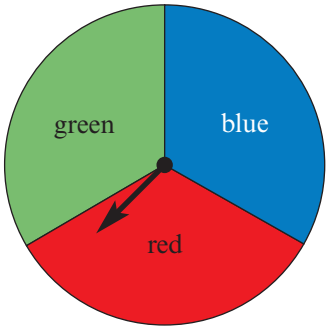
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Enrichment: Spinner fractions

10 The language of chance is a bit vague. For example, for both of the following spinners it is 'unlikely' that you will spin red, but in each case the chance of spinning red is different.



spinner 1



spinner 2

Rather than describing chance in words, we could consider the fraction of the spinner for a certain colour.

a What fraction of spinner 1 is red?
b What percentage of spinner 1 is red?
c What fraction of spinner 2 is red?
d What percentage of spinner 2 is red?
e Draw a spinner in which:
- half is red
- one-third is blue
- one-sixth is green


For part **e**, start with 6 equal pieces.



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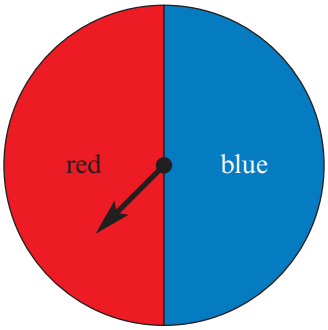
5B Theoretical probability in single-step experiments

The theoretical probability of an event is the chance that it will happen.

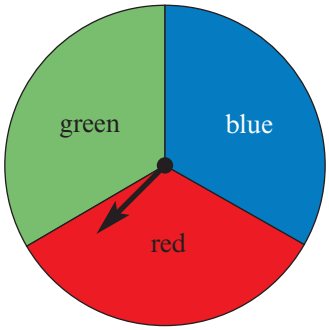
Probabilities are numbers from 0 (impossible) to 1 (certain). They can be written as fractions, decimals or percentages.

Let's start: Spinner probabilities

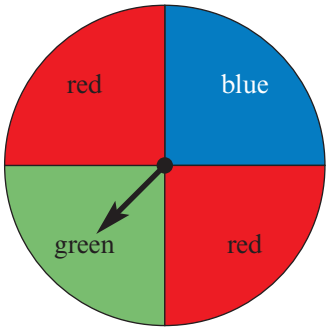
Consider these three spinners.



spinner A



spinner B



spinner C

- Describe the differences between the spinners.
- What is the probability of spinning blue for each of these spinners?
- What is the probability of spinning red for each of these spinners?

Key ideas

Theoretical probability The actual chance that an event will occur

- Many key ideas relevant to this section can be found in the list of terminology presented in Section 5A.
- Some examples of single-step experiments are:
  - tossing a coin once
  - rolling a die once
  - spinning a spinner once
  - choosing one card
- **Theoretical probability** is the chance that an event will occur when an experiment takes place.

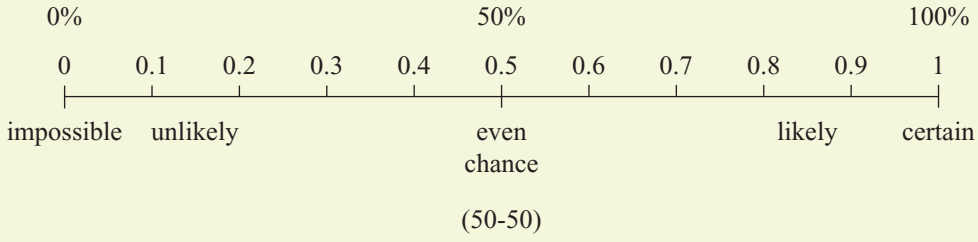
$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

For example: The chance of rolling a fair die once and getting a 2.

$P(\text{rolling a } 2) = \frac{1}{6}$

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- Probabilities can be expressed as:
  - fractions or decimals between 0 and 1
  - percentages between 0% and 100%



Exercise 5B

Understanding

1 Complete the following sentences.

a The _____ _____ is the set of possible outcomes.

b An impossible event has a probability of ____.

c If an event has a probability of 1, then it is _____.

d The higher its probability, the _____ likely the event will occur.

e Rolling a 7 on a standard die is _____.

f An event with a lower probability than another is _____ likely to occur.



Choose from:
space, zero, more, less, sample, impossible, certain.

2 A fair coin is flipped. The probability of flipping tails is $\frac{1}{2}$. Write this probability as:

a a decimal

b a percentage



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5B

Example 2 Calculating probability

A fair 6-sided die is rolled.

a List the sample space.
b Find the probability of rolling a 3, giving your answer as a fraction.
c Find the probability of rolling an even number, giving your answer as a decimal.
d Find the probability of rolling a number less than 3, giving your answer as a percentage.

Solution	Explanation
a Sample space = {1, 2, 3, 4, 5, 6}	For the sample space, we list all the possible outcomes.
b $P(3) = \frac{1}{6}$	The event can occur in one way out of six possible outcomes.
c $P(\text{even}) = \frac{1}{2} = 0.5 = 50\%$	The event can occur in three ways (i.e. 2, 4 or 6). So the probability is $\frac{3}{6} = \frac{1}{2}$ or 0.5 or 50%.
d $P(\text{less than } 3) = \frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$	The event can occur in two ways (1 or 2). So the probability is $\frac{2}{6} = \frac{1}{3}$.

3 Consider a fair 6-sided die.
a List the sample space.
b List the odd numbers on the die.
c State the probability of rolling an odd number.
d State the probability of rolling a 5.



For parts **c** and **d**, begin your answers with *P*(odd) = and *P*(5) = .

4 Match up each event (**a** to **d**) with a set of possible outcomes (**A** to **D**).

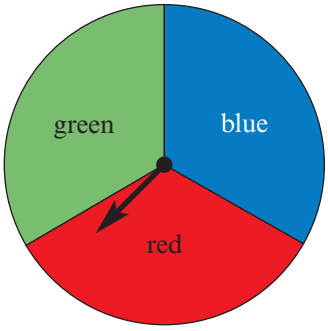
a Tossing a coin
b Rolling a die
c Selecting a suit from a fair deck of 52 playing cards
d Selecting a letter from the word POWER

A 1, 2, 3, 4, 5, 6
B P, O, W, E, R
C heads, tails
D hearts, diamonds, clubs, spades

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Fluency

5 Consider the spinner shown.





P(not red) means the probability that the spinner's arrow does not point to red.

- **a** How many outcomes are there? List them.
- **b** Find *P*(red); i.e. find the probability of the spinner pointing to red.
- **c** Find *P*(green).
- **d** Find *P*(not red).
- **e** Find *P*(yellow).

6 A spinner with the numbers 1 to 7 is spun. The numbers are evenly spaced.



- **a** List the sample space.
- **b** Find *P*(6).
- **c** Find *P*(8).
- **d** Find *P*(2 or 4).
- **e** Find *P*(even).
- **f** Find *P*(odd).
- **g** Give an example of an event having the probability of 1.

7 The letters in the word MATHS are written on 5 cards and then one is drawn from a hat.

- **a** List the sample space.
- **b** Find *P*(T), giving your answer as a decimal.
- **c** Find *P*(A), giving your answer as a decimal.
- **d** Find *P*(consonant is chosen), giving your answer as a decimal.

8 The letters in the word PROBABILITY are written on 11 cards and then one is drawn from a hat.



For part **b**, there are 2 cards out of 11 with the letter I.

- **a** Find *P*(P).
- **b** Find *P*(I).
- **c** Find *P*(letter chosen is in the word BIT).
- **d** Find *P*(not a B).
- **e** Find *P*(a vowel is chosen).
- **f** Give an example of an event with the probability of $\frac{3}{11}$.

Problem-solving and Reasoning

9 The whole numbers from 1 to 11 are written on 11 cards. If the cards are shuffled and one card is chosen at random, which of the following outcomes is most likely?

A choosing a 5
B choosing an even number
C choosing a two-digit number
D choosing an odd number

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5B

10 A bag of marbles contains 3 red marbles, 2 green marbles and 5 blue marbles. They are all equal in size and weight. A marble is chosen at random.

- **a** What is the probability that a red marble is chosen?
- **b** What is the probability that a blue marble is chosen?
- **c** What is the probability that a green marble is *not* chosen?



The probability of choosing red is not $\frac{1}{3}$ because the colours are not equally likely.

11 A box contains different coloured counters, with $P(\text{purple}) = \frac{1}{2}$, $P(\text{yellow}) = \frac{1}{4}$ and $P(\text{orange}) = \frac{1}{5}$.

- **a** Is it possible to obtain a colour other than purple, yellow or orange? If so, state the probability.
- **b** What is the minimum number of counters in the box?
- **c** If the box cannot fit more than 125 counters, what is the maximum number of counters in the box?

12 Consider this spinner, numbered 2 to 9.



- **a** List the sample space.
- **b** Find the probability that a prime number will be spun, giving your answer as a decimal. (Remember that 2 is a prime number.)
- **c** Giving your answers as decimals, state the probability of getting a prime number if each number in the spinner is:
  - **i** increased by 1
  - **ii** increased by 2
  - **iii** doubled
- **d** Design a new spinner for which $P(\text{prime}) = 1$.

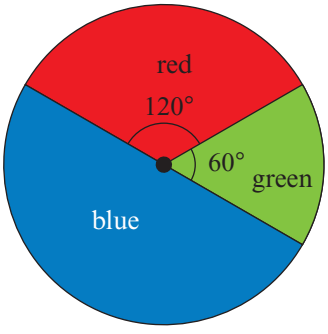


For part **c**, it will help if you draw the new spinners.

Enrichment: Designing spinners

Drilling for Gold 5B1

13 a State the probability of spinning green with this spinner.



- **b** For each of the following, design a spinner using red, green and blue sectors to obtain the desired probabilities. If it cannot be done, then explain why not.
  - **i** $P(\text{red}) = \frac{1}{2}$, $P(\text{green}) = \frac{1}{4}$, $P(\text{blue}) = \frac{1}{4}$
  - **ii** $P(\text{red}) = \frac{1}{2}$, $P(\text{green}) = \frac{1}{2}$, $P(\text{blue}) = \frac{1}{2}$
  - **iii** $P(\text{red}) = \frac{1}{4}$, $P(\text{green}) = \frac{1}{4}$, $P(\text{blue}) = \frac{1}{4}$
  - **iv** $P(\text{red}) = 0.1$, $P(\text{green}) = 0.6$, $P(\text{blue}) = 0.3$

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5C Experimental probability in single-step experiments

If the probability of an event is unknown, an experiment can help. The more trials you use in your experiment, the more likely you are to get a good estimate of the actual probability of that event.



Tossing a coin 100 times does not mean it will come up heads 50 times.

Let's start: Tossing coins

For this experiment, each class member needs a fair coin they can toss.

- Each student should toss the coin 20 times and count how many times heads occurs.
- Tally the total number of heads obtained by the class.
- How close is this total number to the number you would expect that is based on the theoretical probability of $\frac{1}{2}$? Discuss what this means.

Key ideas

- The experimental probability of an event based on a particular **experiment** is:

$$\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$$

- The expected number of occurrences = probability × number of trials

Experiment An activity which may produce a variety of different results that occur randomly

Exercise 5C

Understanding

1 Which of the following experiments would be best to see if a coin was fair?

A Flipping it once **B** Flipping it twice **C** Flipping it 20 times

2 A coin is flipped and the results are: H, H, T, H, T, T, T, H, T, T

a How many times did the coin show heads?

b How many times did the coin show tails?

c How many times was the coin flipped?

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5C

Example 3 Working with experimental probability

When using a spinner numbered 1 to 4, the following numbers come up.
1, 4, 1, 3, 3, 1, 4, 3, 2, 3.

a What is the experimental probability of getting a 3?
b What is the experimental probability of getting an even number?

Solution	Explanation
a $\frac{2}{5}$ or 0.4 or 40%	$\frac{\text{number of 3s}}{\text{number of trials}} = \frac{4}{10} = \frac{2}{5}$
b $\frac{3}{10}$	$\frac{\text{number of even results}}{\text{number of trials}} = \frac{3}{10}$

3 A 6-sided die is rolled 10 times and the following numbers come up: 2, 4, 6, 4, 5, 1, 6, 4, 4, 3. Find the experimental probability of getting:
a a 3
b a 4
c an odd number


For part **c**, count how many times a 1, 3 or 5 was rolled.

4 When a coin is tossed 100 times, the results are 53 heads and 47 tails.
a What is the experimental probability of getting a head?
b What is the experimental probability of getting a tail?
c The actual probability of getting a tail on a fair coin is $\frac{1}{2}$. Does this experiment prove that the coin is not fair?

Fluency

5 The table shows the results of spinning a spinner.

Colour	red	green	blue
Number of times	13	5	2

State the experimental probability of getting:
a red **b** green **c** blue

Example 4 Finding expected numbers

A spinner is found to land on red $\frac{1}{4}$ of the time. If it is spun 200 times, how many times would you expect it to land on red?

Solution	Explanation
50 times	Expected number = probability × number of trials = $\frac{1}{4} \times 200 = 50$

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6 A fair coin is tossed.



- **a** How many times would you expect it to show tails in 1000 trials?
- **b** How many times would you expect it to show heads in 3500 trials?
- **c** You start by tossing the coin 10 times to find the probability of the coin showing tails.
  - **i** Explain how you could get an experimental probability of 0.7.
  - **ii** If you toss the coin 100 times, are you more or less likely to get an experimental probability close to 0.5?

7 A fair 6-sided die is rolled.

- **a** How many times would you expect to get a 3 in 600 trials?
- **b** How many times would you expect to get an even number in 600 trials?
- **c** If you roll the die 600 times, is it possible that you will get an even number 400 times?
- **d** Are you more likely to obtain an experimental probability of 100% from two rolls or to obtain an experimental probability of 100% from 10 rolls?

8 The colour of the cars in a school car park is recorded.



Colour	red	silver	white	blue	purple	black
Number of cars	21	24	25	20	3	7

- **a** Based on this sample, find the probability that a randomly chosen car is:
  - **i** white
  - **ii** purple
  - **iii** silver or black
- **b** How many purple cars would you expect to see in a shopping centre car park with 2000 cars?

Problem-solving and Reasoning

9 The number of children in some families is recorded in the table shown.

Number of children	0	1	2	3	4
Number of families	5	20	32	10	3



- **a** How many families have no children?
- **b** How many families have an even number of children?
- **c** How many families participated in the survey?
- **d** Based on this experiment, what is the probability that a randomly selected family has 1 or 2 children?
- **e** Based on this experiment, what is the probability that a randomly selected family has an even number of children?
- **f** What is the total number of *children* in this survey?

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5C

10 A handful of 10 marbles of different colours is placed into a bag. A marble is selected at random, its colour recorded and then returned to the bag.

Red marble chosen	Green marble chosen	Blue marble chosen
21	32	47

a Based on the results in the table, how many marbles of each colour do you think there are? Justify your answer in a sentence.

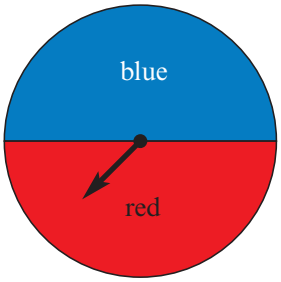
b For each of the following, state whether or not they are possible colours for the 10 marbles in the bag.

i 3 red, 3 green, 4 blue
ii 2 red, 4 green, 4 blue
iii 1 red, 3 green, 6 blue
iv 2 red, 3 green, 4 blue, 1 purple
v 2 red, 0 green, 8 blue

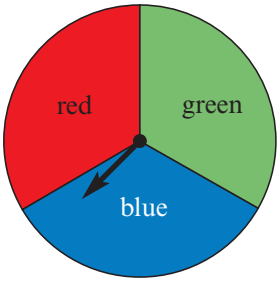
11 Match each of the experiment results (**a** to **d**) with the most likely spinner that was used (**A** to **D**).

	Red	Green	Blue
a	18	52	30
b	27	23	0
c	20	23	27
d	47	0	53

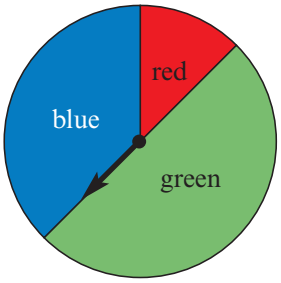
A



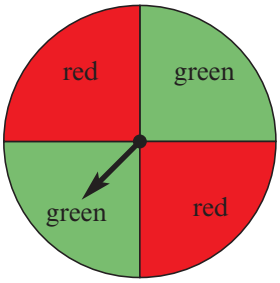
B



C



D



Enrichment: A game – What is in the hat?



Drilling for Gold 5C1

12 Your teacher will secretly place 20 blocks in a hat. There are 4 different colours. Your teacher will:

- Randomly choose a block.
- Call out the colour.
- Return the block to the hat.
- Repeat this process indefinitely.

You should record the number of times each colour is chosen by keeping a tally. Use the tally marks to estimate how many blocks of each colour there could be in the hat. Write something like 5 green, 10 red, 4 blue, 1 white and show it to your teacher. The student with the closest estimate is the winner.

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5D Venn diagrams

When two events are being considered a Venn diagram can be used to represent all possibilities. They are especially useful when survey results are being considered and converted to probabilities.

Let's start: Free dress day

On a free dress day, a Year 7 class decided to wear either pink only or green only or both pink and green. A few students came in their school uniform.

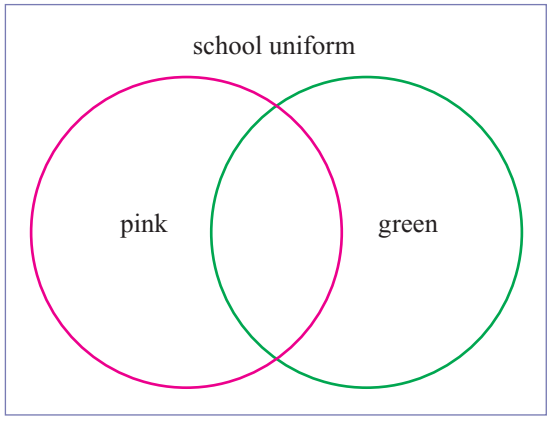


This is how the students dressed:

- 9 wore pink only
- 5 wore green only
- 8 wore both pink and green
- 4 wore school uniform

In maths class that day, the students drew a Venn diagram showing the colours that the students dressed in on free dress day.

1 Copy this Venn diagram.



2 Write the number of students in each area that matches the colours worn.

3 How many students in total wore pink?

4 How many students in total wore green?

5 How many students in total wore green or pink or both?

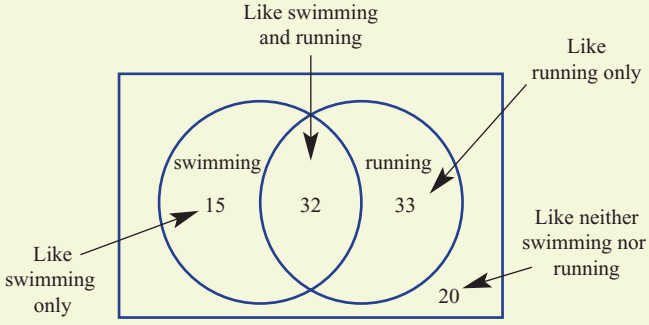
6 How many students altogether are in this class?

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Key ideas

- A **Venn diagram** is a pictorial representation using overlapping circles inside a rectangle.
  In the example below, 100 people were asked:
  – Do you like swimming? (Yes or No)
  – Do you like running? (Yes or No)
  This divides the group into four subgroups.
  Note that the four numbers add to 100.

Venn diagram
A diagram used to categorise a group into subgroups

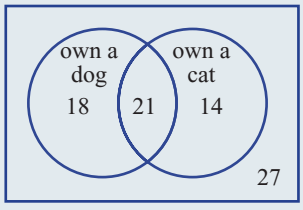


Exercise 5D

Understanding

Example 5 Interpreting a Venn diagram

This Venn diagram shows the results of a survey that asked whether a family own a dog, cat, both or neither.



a How many families surveyed own a dog?
b How many families surveyed own a cat?
c How many families surveyed own both a dog and a cat?
d How many families surveyed own a cat or a dog or both?
e How many families surveyed don't own either a dog or a cat?
f How many families participated in this survey?

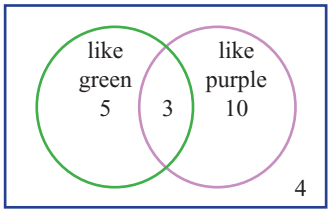
Solution	Explanation
a 39 families own a dog.	Add all the numbers in the own a dog circle; i.e. 18 + 21 = 39.
b 35 families own a cat.	Add all the numbers in the own a cat circle; i.e. 21 + 14 = 35.

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c 21 families own both.	21 is where the circles overlap. So 21 families own a cat and a dog.
d 53 families own either or both.	Add all the numbers across both circles; i.e. $18 + 21 + 14 = 53$.
e 27 families don't own either.	27 is on the outside of the circles.
f 80 families in total.	Add all the numbers in the Venn diagram; i.e. $18 + 21 + 14 + 27 = 80$.

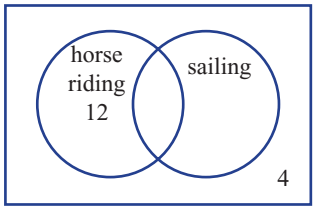
1 Students were asked if they like green and/or purple.

a How many students surveyed like green?
b How many students surveyed like purple?
c How many students surveyed like both green and purple?
d How many students surveyed like either green or purple or both?
e How many students surveyed don't like either green or purple?
f How many students participated in this survey?



2 A class of Year 7 students are asked whether they would like to try sailing or horse riding or both. Some of the results of this survey are shown in this Venn diagram.

a Copy this Venn diagram and complete it by writing these numbers in the correct parts.
i Six students would like to try both horse riding and sailing.
ii Eight students would like to try sailing but not horse riding.





The overlap of the circles shows the number of students who would like to try both horse riding and sailing.

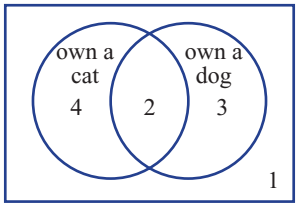
b How many students in total are in this class?

3 Look at the Venn diagram representing cat and dog ownership. State the missing number (1, 2, 3 or 4) to make the following statements true.

a The number of people surveyed who own both a cat and a dog is ____.
b The number of people surveyed who own a cat but do not own a dog is ____.
c The number of people surveyed who own neither a cat nor a dog is ____.
d The number of people surveyed who own a dog but do not own a cat is ____.



The number outside of the circles shows the number of people who don't own either a cat or a dog.



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5D

Fluency

4 A survey asked students if they liked oranges or bananas. Draw and label a Venn diagram showing the results of this survey as listed here.
15 students liked only oranges.
12 students liked both oranges and bananas.
8 students liked only bananas.
6 students prefer other fruit.



5 In a group of 30 students it is found that 10 play both cricket and soccer, 5 play only cricket and 7 play only soccer.

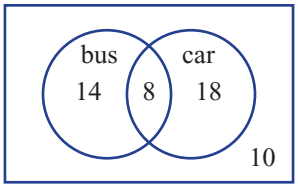
a How many students do not play cricket or soccer?
b Represent the survey findings in a Venn diagram.
c How many of the students surveyed play cricket?
d How many of the students surveyed play either cricket or soccer or both?

6 The Venn diagram below shows the results of a survey of some Year 7 students about whether they travel to school by bus or car.

a How many students in total were surveyed?
b How many students surveyed travel by bus or car but not both?
c How many students surveyed use neither?

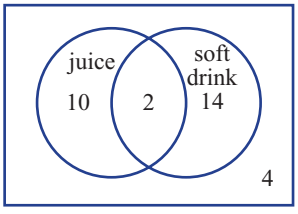


Add all the numbers in the rectangle to find the total number of students surveyed.



7 The Venn diagram shows the number of people surveyed who like juice and/or soft drinks.





a What is the total number of people surveyed who like neither juice nor soft drink?
b How many people surveyed like one but not the other?
c How many people surveyed like soft drink?



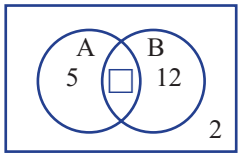
Add the numbers outside of the 'like juice' circle to find how many people surveyed do not like juice.

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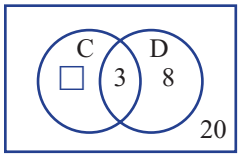
Problem-solving and Reasoning

8 Find all of the missing numbers in each of these Venn diagrams.

a Overall total = 20



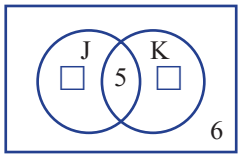
b Overall total = 40



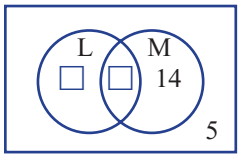


Remember that all the numbers in the Venn diagram add to the overall total.

c Total of J = 18
Total of K = 21



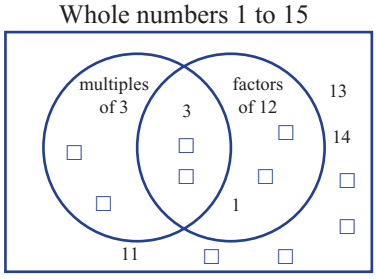
d Total of L = 26
Total of M = 21





The total in a circle is the sum of the two numbers in that circle.

9 **a** Copy and complete this Venn diagram by writing in the numbers missing from whole numbers 1 to 15.





Multiples of 3 are found when 3 is multiplied by whole numbers. So 3, 6, …

b How many numbers that are multiples of 3 are also factors of 12?

c How many factors of 12 are not multiples of 3?



Factors of 12 are numbers that divide into 12 with no remainder.

10 For this question you will select from the numbers from 1 to 30.

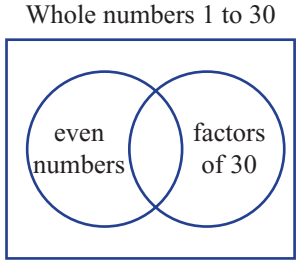
a List the even numbers.

b List the factors of 30.

c Make a large copy of this Venn diagram and copy the *values* from your lists into the correct parts of the diagram.

d How many even numbers are also factors of 30?

e How many even numbers are there in the Venn diagram?



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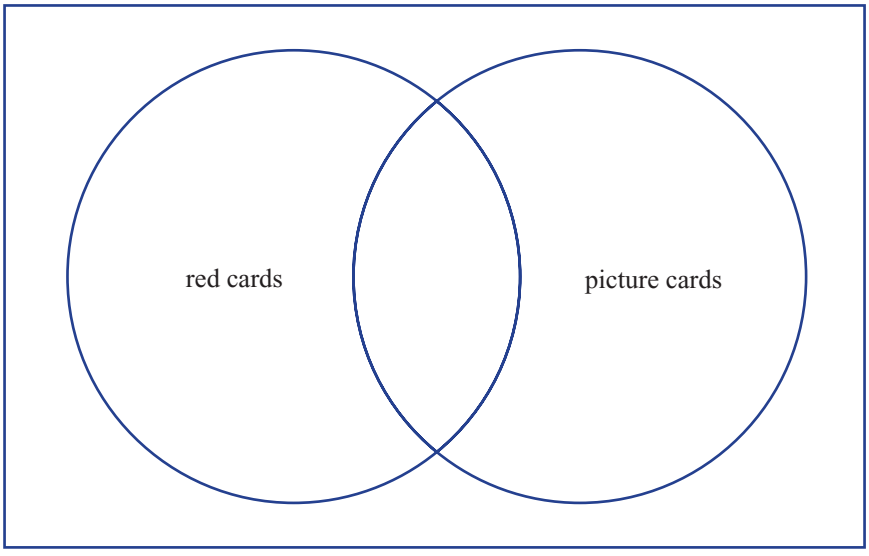
5D

Enrichment: Deck of playing cards

11 Make a large Venn diagram on a whiteboard or on the ground.



Drilling for Gold 5D1



Take a standard deck of 52 playing cards and put each card in one of the four categories.
How many cards are in each category?



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5E Two-way tables

Another way to divide a group into four categories is to use a two-way table.

Let's start: Another way to divide a deck of cards

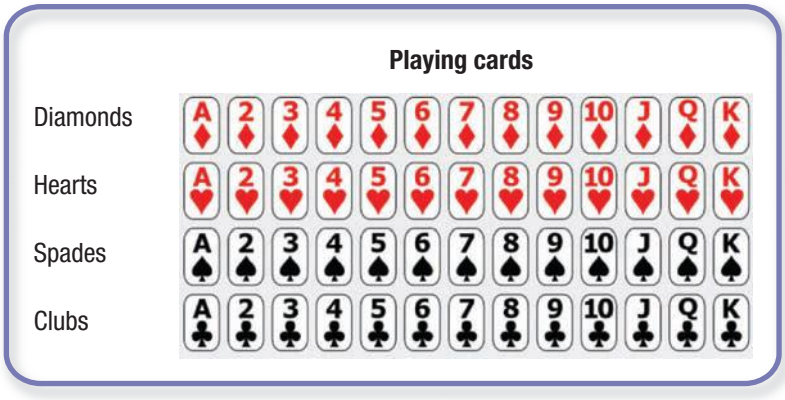
Drilling for Gold 5E1

Draw the table below on the whiteboard so that it fills the whole whiteboard.

	Picture card	Not a picture card
Red card		
Black card		

Take a standard deck of playing cards and put them in one of the four categories. (A picture card is a King, Queen or Jack.)

- How many cards are in each category?



Key ideas

A **two-way table** is similar to a Venn diagram. It divides data into four categories.

For example: A survey with two Yes/No questions.

- Do you like English?
- Do you like Maths?

Two-way table A tool for organising data into four categories

	Like Maths	Do not like Maths	Total
Like English	28	33	61
Do not like English	5	34	39
Total	33	67	100

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Exercise 5E

Understanding

Example 6

a Copy and complete this two-way table.

	Like gardening	Dislike gardening	Total
Like cooking	24	16	40
Dislike cooking	12	4	
Total			

b How many people surveyed like cooking and like gardening?
c How many people surveyed like cooking and dislike gardening?
d How many people surveyed dislike cooking and dislike gardening?
e How many people surveyed like gardening?
f How many people participated in this survey?

Solution

a

	Like gardening	Dislike gardening	Total
Like cooking	24	16	40
Dislike cooking	12	4	16
Total	36	20	56

b 24 like cooking and gardening.

c 16 like cooking but dislike gardening.

d 4 dislike both cooking and gardening.

e 36 like gardening.

f 56 people in total.

Explanation

Add the numbers in each column to get the column totals.
$24 + 12 = 36$, $16 + 4 = 20$

Add the numbers in each row to get the row totals.
$24 + 16 = 40$, $12 + 4 = 16$

The column totals add to 56.
$(36 + 20 = 56)$

The row totals also add to 56.
$(40 + 16 = 56)$

From the 'like cooking' row, move to the 'like gardening' heading, so 24.

From the 'like cooking' row, move to the 'dislike gardening' heading, so 16.

From the 'dislike cooking' row, move to the 'dislike gardening' heading, so 4.

Add the total under the 'like gardening' heading.
$24 + 12 = 36$

The total number of people is 56, which is in the bottom corner.

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1 **a** Copy and complete the two-way table by writing in the missing totals.

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	
Total		35	75

b How many people surveyed like both apples and bananas?
c How many people surveyed dislike apples and dislike bananas?
d How many people participated in the survey?

2 Copy and complete the two-way table below using the following survey results.
- 23 students like Anzac biscuits and also like lamingtons.
- 14 students like Anzac biscuits but dislike lamingtons.
- 12 students like lamingtons but dislike Anzac biscuits.
- 3 students dislike lamingtons and also dislike Anzac biscuits.

	Like lamingtons	Dislike lamingtons	Total
Like Anzac biscuits			
Dislike Anzac biscuits			
Total			





3 Here is a two-way table showing the results of a survey of teenagers about exercise. Use the numbers in this table to answer the questions below.

	Like jogging	Dislike jogging	Total
Like cycling	15	10	25
Dislike cycling	12	3	15
Total	27	13	40

a How many teenagers surveyed like jogging?
b How many teenagers surveyed like both jogging and cycling?
c How many teenagers surveyed like jogging but dislike cycling?
d How many teenagers surveyed like cycling?
e How many teenagers surveyed like cycling but dislike jogging?
f How many teenagers participated in the survey?



The number who 'like jogging' is equal to the total of the column 'like jogging'.

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5E

Fluency

Example 7 Completing a two-way table

Copy and complete this two-way table.

	Like Macs	Dislike Macs	Total
Like PCs			
Dislike PCs		3	13
Total	35	40	75

Solution

	Like Macs	Dislike Macs	Total
Like PCs	25	37	62
Dislike PCs	10	3	13
Total	35	40	75

Explanation

Start with a row or column that has only one number missing.
'Dislike PCs' row, $10 + 3 = 13$.
'Dislike Macs' column, $37 + 3 = 40$.

Now complete 'like Macs' column:
$25 + 10 = 35$

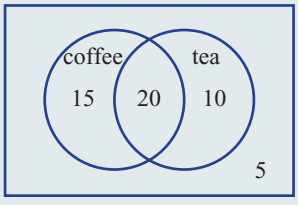
The total in the 'like PCs' row is
$25 + 37 = 62$

4 Copy and complete this two-way table.

	Like surfing	Dislike surfing	Total
Like hiking			
Dislike hiking		5	30
Total	70	15	85

Example 8 Constructing two-way tables from Venn diagrams

Consider this Venn diagram, showing the number of people surveyed who like coffee and who like tea.



a Represent the survey findings in a two-way table.
b How many people surveyed like neither tea nor coffee?
c How many people surveyed like tea?
d How many people surveyed like both coffee and tea?
e How many people surveyed like coffee or tea (or both)?

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Solution | **Explanation**

a

	Like coffee	Dislike coffee	Total
Like tea	20	10	30
Dislike tea	15	5	20
Total	35	15	50

The two-way table has the four numbers from the Venn diagram and also a 'total' column (e.g. $20 + 10 = 30$, $15 + 5 = 20$) and a 'total' row. Note that 50 in the bottom corner is both $30 + 20$ and $35 + 15$.

b 5 do not like either tea or coffee. — $50 - 20 - 15 - 10 = 5$ people who do not like either.

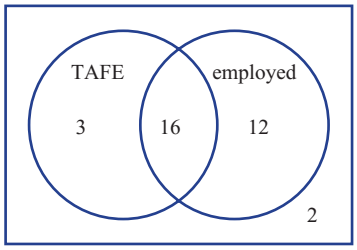
c $20 + 10 = 30$ like tea. — 10 people like tea but not coffee, but 20 people like both. In total 30 people like tea.

d 20 like both coffee and tea. — 20 out of 50 people like both coffee and tea.

e 45 like tea or coffee or both. — $15 + 20 + 10 = 45$ people like either coffee or tea or both.

5 This Venn diagram shows the survey results of a group of people who were asked two questions:

- Do you have a TAFE qualification?
- Are you currently employed?



a Copy and complete the two-way table shown below.

	Employed	Unemployed	Total
TAFE			
No TAFE			
Total			

b How many people surveyed were unemployed with no TAFE qualification?

c How many people surveyed were employed?

d How many people had a TAFE qualification and were also employed?

e How many people had a TAFE qualification or were employed (or both)?

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5E

6 The two-way table below shows the results of a poll conducted of a group of students who own mobile phones to see who pays their own bill.

	Males	Females	Total
Pay own bill	4	7	11
Do not pay own bill	8	7	15
Total	12	14	26

a How many students participated in this poll?
b How many males participated in this poll?
c How many of the students surveyed pay their own bill?



Problem-solving and Reasoning

7 Copy and complete the following two-way tables.

a

	B	Not B	Total
A	20		70
Not A			
Total		60	100

b

	B	Not B	Total
A		5	
Not A		3	7
Total	10		



'A' means 'like A'. 'Not A' means 'dislike A'.

8 A car salesman notes that among his 40 cars, there are 15 automatic cars and 10 sports cars. Only two of the sports cars are automatic.
a Create a two-way table of this situation.
b How many sports cars are not automatic?
c How many of the cars are not automatic?

9 A total of 33 students were asked whether they liked or disliked volleyball and tennis. Of these students, 12 liked both volleyball and tennis and 6 disliked volleyball but liked tennis. The total number of students who liked volleyball was 23.
a Copy and complete a two-way table.

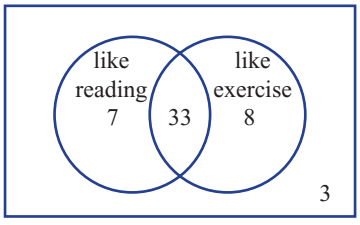
	Like volleyball	Dislike volleyball	Total
Like tennis			
Dislike tennis			
Total			

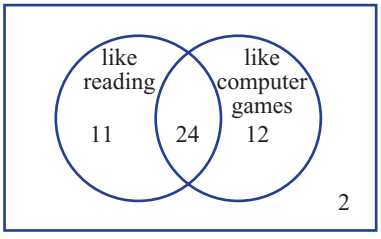
b How many students surveyed like tennis?
c What proportion of students who like tennis also like volleyball?

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Enrichment: Two-way table from two Venn diagrams

10 Two surveys showed how many students like reading compared with how many like exercise and computer games.
The results are shown in these Venn diagrams.





Copy and complete this two-way table showing this information.

	Like reading	Don't like reading	Total
Like exercise			
Don't like exercise			
Like computer games			
Don't like computer games			
Total			



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Puzzles and games

1 For each of the following, find an English word that matches the description.

a $P(\text{vowel}) = \frac{1}{2}$

b $P(\text{F}) = \frac{2}{3}$

c $P(\text{vowel}) = \frac{1}{4}$ and $P(\text{D}) = \frac{1}{4}$

d $P(1) = \frac{2}{11}$ and $P(\text{consonant}) = \frac{7}{11}$

e $P(\text{M}) = \frac{1}{7}$ and $P(\text{T}) = \frac{1}{7}$ and $P(\text{S}) = \frac{1}{7}$

f $P(\text{vowel}) = 0$ and $P(\text{T}) = \frac{1}{3}$

2 When one coin is flipped, $P(\text{heads}) = \frac{1}{2}$. For two coins, the probability that both are heads is $\frac{1}{4}$. For three coins, $P(\text{all heads}) = \frac{1}{8}$. How many coins are flipped if the probability of getting all heads is $\frac{1}{64}$?

3 There are four possibilities of arranging two children: BB, BG, GB, GG.

a Write down the eight different ways of arranging 3 children.

b How many different ways can you arrange 4 children?

4 Two girls, Andrea and Imogen, and a boy, Marcus, are going to sit side-by-side to watch a movie. If they sit down randomly, what is the chance that the two girls sit together?

5 Five people are surveyed. They are asked if they prefer bottled water, tap water or mineral water. Two possible results are shown. How many more are there?

Bottled	Tap	Mineral
3	1	1
2	3	0

6 A bag of 24 marbles contains blue, green and red marbles. If you pick one marble at random (without looking), there is an even chance that it will be blue. Given that there are twice as many green marbles as red marbles, how many of each colour are there?



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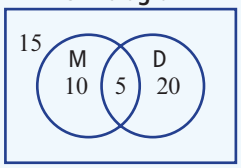
Probability

Presenting data from a survey

50 people were asked:

- Are you male? (Yes/No)
- Do you drive a car? (Yes/No)

Venn diagram

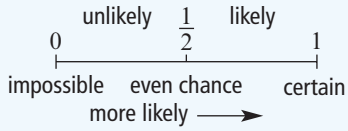


Two-way table

	Drive	Don't drive
Male	5	10
Not male	20	15

Theoretical probability

How likely an event is



Experimental probability

Use an experiment or survey or simulation to estimate probability.

e.g. Spinner lands on blue 47 times out of 120, so

experimental probability $= \frac{47}{120}$

Expected number is

P(event) × number of trials

e.g. Flip coin 100 times, expected number of heads

$= \frac{1}{2} \times 100 = 50$

e.g. Roll die 36 times, expected number of 5s

$= \frac{1}{6} \times 36 = 6$

Probabilities can be given as fractions, decimals or percentages.

e.g. 25%, $\frac{1}{4}$, 0.25

e.g. 70%, $\frac{7}{10}$, 0.7

More trials make the experimental probability more reliable or accurate.

Experiment: Toss a coin

Sample space: {H, T}

$P(\text{H}) = \frac{1}{2}$

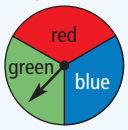
$P(\text{T}) = \frac{1}{2}$

Experiment: Spin the spinner

Sample space: {red, green, blue}

$P(\text{spin red}) = \frac{1}{3}$

$P(\text{don't spin blue}) = \frac{2}{3}$



Experiment: Roll a fair die

Sample space: {1, 2, 3, 4, 5, 6}

$P(\text{roll a } 5) = \frac{1}{6}$

$P(\text{roll odd number}) = \frac{3}{6} = \frac{1}{2}$

Experiment: Select a playing card and note its suit.

Sample space: {♠, ♦, ♣, ♥}

$P(\diamondsuit) = \frac{1}{4}$

$P(\heartsuit \text{ or } \clubsuit) = \frac{2}{4} = \frac{1}{2}$



$P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 The results of a survey are shown below. Each student learns only one instrument.

Instrument	piano	violin	drums	guitar
Number of students	10	2	5	3

Based on the survey, the experimental probability that a randomly selected student is learning the guitar is:

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 3 **D** $\frac{3}{5}$ **E** $\frac{3}{20}$



2 Which of the following events has a probability of 50%?
- **A** Rolling a number greater than 4 on a fair 6-sided die
- **B** Choosing a vowel from the word CAT
- **C** Tossing a fair coin and getting heads
- **D** Choosing the letter T from the word TOE
- **E** Spinning an odd number on a spinner numbered 1 to 7

3 Each letter of the word APPLE is written separately on five cards. One card is then chosen at random. *P*(letter P) is:

A 0 **B** 0.2 **C** 0.4 **D** 0.5 **E** 1

4 A fair 6-sided die is rolled 600 times. The expected number of times that the number rolled is either a 1 or a 2 is:

A 100 **B** 200 **C** 300 **D** 400 **E** 600

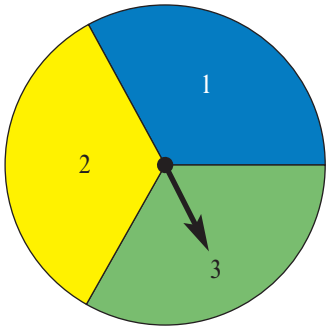
5 The letters of the word STATISTICS are placed on 10 different cards and placed into a hat. If a card is drawn at random, the probability that it will show a vowel is:

A 0.2 **B** 0.3 **C** 0.4 **D** 0.5 **E** 0.7

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Chapter review

6 The spinner shown at right is spun. The probability of spinning an odd number is:



A 25% **B** $33\frac{1}{3}\%$ **C** 50%

D $66\frac{2}{3}\%$ **E** 100%

7 A coin is tossed. The probability of obtaining tails is:

A $\frac{2}{3}$ **B** 4 **C** $\frac{1}{2}$

D $\frac{3}{8}$ **E** $\frac{1}{8}$

8 An experiment is conducted in which three dice are rolled and the results are added. In 12 of the 100 trials, the sum of the faces is 11. Based on this, the experimental probability of having three faces add to 11 is:

A $\frac{11}{100}$ **B** $\frac{12}{111}$ **C** $\frac{3}{25}$ **D** 12 **E** $\frac{1}{2}$

9 Rachel has a fair coin. She has tossed 'heads' five times in a row. Rachel tosses the coin one more time. What is the probability of tossing 'tails'?

A 0 **B** 1 **C** $\frac{1}{2}$

D less than $\frac{1}{2}$ **E** more than $\frac{1}{2}$

10 When a fair die is rolled, what is the probability that the number is even but not less than 3?

A 0 **B** $\frac{1}{6}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$ **E** $\frac{2}{3}$

Short-answer questions

1 For each of the following descriptions, choose the probability from the set $0, \frac{1}{8}, \frac{3}{4}, 1, \frac{19}{20}$ that matches best.

a certain **b** highly unlikely **c** highly likely

d likely **e** impossible

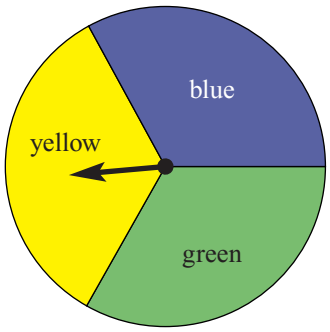
2 List the sample space for each of the following experiments.

a A fair 6-sided die is rolled.

b A fair coin is tossed.

c A letter is chosen from the word DESIGN.

d Spinning the spinner shown opposite.



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Chapter review

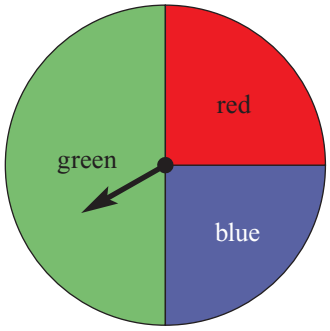
3 Vin spins a spinner with nine equal sectors, which are numbered 1 to 9.
- **a** How many outcomes are there?
- **b** Find the probability of spinning:
  - **i** an odd number
  - **ii** a multiple of 3
  - **iii** a number greater than 10
  - **iv** a prime number less than 6
  - **v** a factor of 8



4 One card is chosen at random from a standard deck of 52 playing cards. Find the probability of drawing:
- **a** a red king
- **b** a king or queen
- **c** a jack of diamonds
- **d** a picture card (i.e. king, queen or jack)

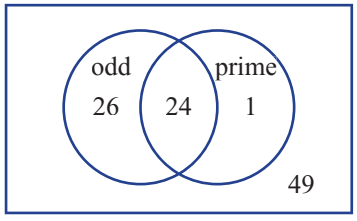
5 A coin is tossed 100 times, with the outcome 42 heads and 58 tails.
- **a** What is the experimental probability of getting heads? Give your answer as a percentage.
- **b** What is the actual probability of getting heads if the coin is fair? Give your answer as a percentage.

6 Consider the spinner shown.
- **a** State the probability that the spinner lands in the green section.
- **b** State the probability that the spinner lands in the blue section.
- **c** Grace spins the spinner 100 times. What is the expected number of times it would land in the red section?
- **d** She spins the spinner 500 times. What is the expected number of times it would land in the green section?



Extended-response questions

1 The Venn diagram shows how many numbers between 1 and 100 are odd and how many are prime.
Consider the numbers 1 to 100.
- **a** How many are odd?
- **b** How many prime numbers are there?
- **c** How many numbers are both odd and prime?



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Chapter review

2 The two-way table below shows the results of a survey on car ownership and public transport usage.

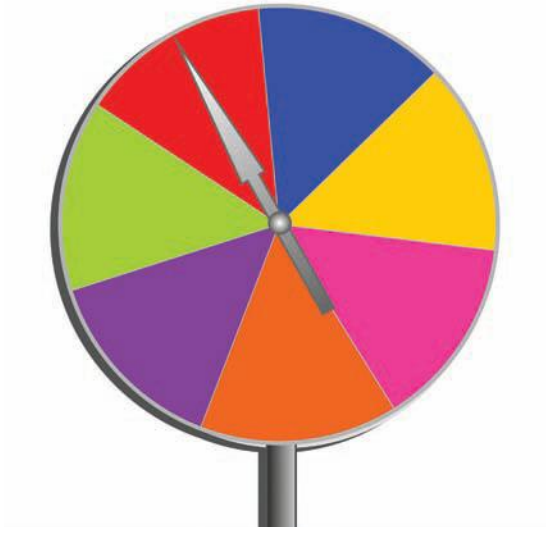
	Uses public transport	Does not use public transport	Total
Owns a car	20	80	
Does not own a car	65	35	
Total			

a Copy and complete the table.
b How many people participated in the survey?
c How many people surveyed own a car?
d How many people surveyed use public transport even though they own a car?

3 A spinner is made using the numbers 1, 3, 5 and 10 in four sectors. The spinner is spun 80 times and the results obtained are shown in the table.

Number on spinner	Frequency
1	30
3	18
5	11
10	21
	80

a Which sector on the spinner occupies the largest area? Explain.
b Two sectors of the spinner have the same area. Which two numbers do you think have equal areas and why?
c What is the experimental probability of obtaining a 1 on the next spin?
d Draw an example of what you think the spinner might look like, in terms of the area covered by each of the four numbers.



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Chapter 6

Computation with decimals and fractions

What you will learn

6A Adding and subtracting decimals
6B Adding fractions
6C Subtracting fractions
6D Multiplying fractions
6E Multiplying and dividing decimals by 10, 100, 1000 etc.
6F Multiplying by a decimal
6G Dividing fractions
6H Dividing decimals

Strand: Number and Algebra

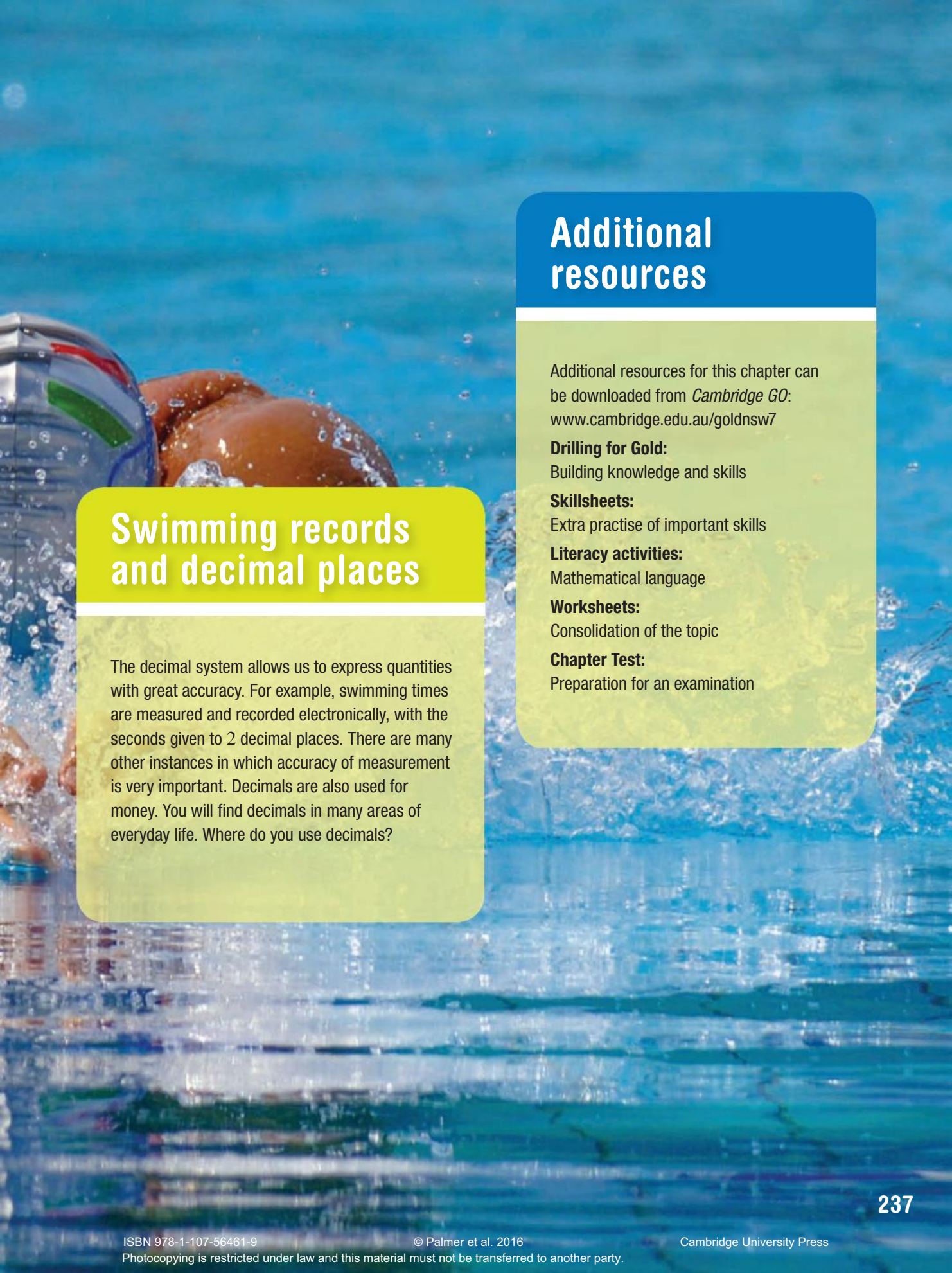
Substrand: FRACTIONS, DECIMALS AND PERCENTAGES

In this chapter, you will learn to:

- operate with fractions, decimals and percentages.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Swimming records and decimal places

The decimal system allows us to express quantities with great accuracy. For example, swimming times are measured and recorded electronically, with the seconds given to 2 decimal places. There are many other instances in which accuracy of measurement is very important. Decimals are also used for money. You will find decimals in many areas of everyday life. Where do you use decimals?

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Complete the following.

a $\frac{1}{10} = 0.___$ **b** $\frac{3}{10} = 0.___$ **c** $\frac{17}{10} = 1.___$

d $\frac{1}{100} = 0.0___$ **e** $\frac{1}{1000} = 0._____$ **f** $\frac{47}{10} = ___.___$

2 Write the decimal for:

a one-half **b** one-quarter **c** three-quarters

3 Write the following cents as dollars.

a 70 cents **b** 85 cents **c** 100 cents

d 5 cents **e** 105 cents **f** 3 cents

4 Find how many cents are in:

a half a dollar **b** one-quarter of \$1

c three-quarters of \$1 **d** half of \$5

5 Find the cost of:

a two labels at 45 cents each

b 10 pears at \$1.05 each

c $1\frac{1}{2}$ boxes of mangoes at \$15 a box

d three pens at 27 cents a pen

6 Tom paid \$20 for 200 photos to be printed. What was the cost of each print?

7 \$124 is shared between eight people. If each share is the same amount, how much does each person receive?

8 Complete:

a \$8.50 × 10 = ____ **b** \$6 – \$5.90 = ____ **c** \$10 – \$7.30 = ____

d \$70 ÷ 100 = ____ **e** \$6.90 + \$4.30 = ____ **f** \$20 – \$19.76 = ____

9 **a** Take \$5 away from \$12.

b Take \$2.10 away from \$5.

c Add \$1.70 and \$2.25.

d Add \$12.50 to \$17.25.

e Take \$2.50 away from \$10.

10 Calculate how much change from \$100 Calvin receives when he spends:

a \$12.50 **b** \$7.40 **c** \$79.10

11 Find the total of these amounts: \$7, \$5.50, \$4.90, \$12 and \$9.15.

12 Complete these computations.

a
$$\begin{array}{r} 329 \\ +194 \\ \hline \end{array}$$

b
$$\begin{array}{r} 1024 \\ -\ 185 \\ \hline \end{array}$$

c
$$\begin{array}{r} 104 \\ \times\ 3 \\ \hline \end{array}$$

d $5\overline{)6185}$

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6A Adding and subtracting decimals

If you can add and subtract whole numbers, then you can add and subtract decimals. They follow the same process. Just remember to keep the decimal points lined up when adding in columns.

Let's start: How fast can you calculate?



- Add $10 + 7 + 13 + 15 + 5$.
  Now add $1 + 0.7 + 1.3 + 1.5 + 0.5$.
- What about using money?
  Add $\$1 + \$0.70 + \$1.30 + \$1.50 + \$0.50$.
- Which calculation was the fastest?
- Now try $10 - 7.85$ and \$10 less \$7.85.

Calculating with money often seems easier!

Key ideas

- When adding or subtracting decimals, the decimal points and each of the decimal places must be lined up.

$$\begin{array}{r} 1.56 \\ +2.70 \\ \hline 4.26 \end{array}$$

Writing an extra zero will help.

Line up the decimal points and the digits.

Exercise 6A

Understanding

1 The numbers 7.12, 8.5 and 13.032 are to be added. Which of the following is the best way to write these numbers ready for addition?

A $\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$ **B** $\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$ **C** $\begin{array}{r} 7.120 \\ 8.500 \\ +13.032 \\ \hline \end{array}$ **D** $\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$

2 Which of the following is the correct way to present the subtraction $77.81 - 6.3$?

A $\begin{array}{r} 77.81 \\ -\ 6.3 \\ \hline 84.11 \end{array}$ **B** $\begin{array}{r} 77.81 \\ -\ 6.30 \\ \hline 71.51 \end{array}$ **C** $\begin{array}{r} 77.81 \\ -6.3 \\ \hline 14.81 \end{array}$ **D** $\begin{array}{r} 77.81 \\ -\ 6.3 \\ \hline 77.18 \end{array}$

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6A

3 Copy each sum and put the decimal point in the correct place in the answer.

a
$$\begin{array}{r} 5.16 \\ +3.41 \\ \hline 8\ 57 \end{array}$$

b
$$\begin{array}{r} 3.047 \\ 0.522 \\ +1.610 \\ \hline 5\ 179 \end{array}$$

c
$$\begin{array}{r} 15.9 \\ +\ 0.056 \\ \hline 15\ 956 \end{array}$$

4 Show how you would write your working for:

a $0.571 + 1.209 + 3.528$ **b** $1.97 - 0.43$ **c** $12.4 - 8.35$

Fluency

Example 1 Adding decimals

Find:

a $8.31 + 5.93$ **b** $64.8 + 3.012 + 5.94$

Solution	Explanation
a $\begin{array}{r} {}^{1}8.31 \\ +\ 5.93 \\ \hline 14.24 \end{array}$	Write: $\begin{array}{r} 8.31 \\ +5.93 \\ \hline . \end{array}$ Then add.
b $\begin{array}{r} {}^{1}6{}^{1}4.800 \\ 3.012 \\ +\ 5.940 \\ \hline 7\ 3.752 \end{array}$	Put decimal points under one another. Fill missing decimal places with zeros. Add, using the procedure for whole numbers.

5 Find each of the following.

a $\begin{array}{r} 1.2 \\ +5.6 \\ \hline \end{array}$ **b** $\begin{array}{r} 12.61 \\ +\ 2.35 \\ \hline \end{array}$ **c** $\begin{array}{r} 2.83 \\ +1.04 \\ \hline \end{array}$ **d** $\begin{array}{r} 7.90 \\ +1.09 \\ \hline \end{array}$

e $\begin{array}{r} 13.25 \\ +14.72 \\ \hline \end{array}$ **f** $\begin{array}{r} 7.23 \\ 16.31 \\ +\ 2.40 \\ \hline \end{array}$ **g** $\begin{array}{r} 210.0 \\ 22.3 \\ +\ 15.1 \\ \hline \end{array}$ **h** $\begin{array}{r} 47.81 \\ 6.98 \\ +\ 3.52 \\ \hline \end{array}$

6 Find each of the following.

a $1.5 + 1.1$ **b** $5.6 + 0.2$
c $0.6 + 0.3$ **d** $0.9 + 0.09$
e $6.42 + 2.05$ **f** $9.072 + 6.435$
g $12.45 + 3.61$ **h** $5.37 + 13.81 + 2.15$



Rewrite the sums as shown in Example 1, lining up the decimal points.

7 Find the value of the following.

a $\$3.50 + \$3.50$ **b** $\$2.80 + \$2.80$ **c** $\$9.99 + \$4.50$
d $\$3.25 + \$4.99$ **e** $\$99 + \$46$ **f** $\$2.50 + \$2.50 + \$2.50$

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Example 2 Subtracting decimals

Find:

a 5.83 – 3.12 **b** 146.35 – 79.5 **c** 24.9 – 3.52

Solution	Explanation
a 5.83 –3.12 2.71	Write: 5.83 –3.12 .  Then subtract.
b 13 15 1 1~~4~~6.35 – 79.50 66.85	Put decimal points under one another. Fill missing decimal places with zeros. Subtract, using the procedure for whole numbers.
c 8 1 24.90 – 3.52 21.38	Put decimal points under one another. Fill missing decimal places with zeros. Subtract, using the procedure for whole numbers.

8 Evaluate the following.

a $5 – $2.50 **b** $5 – $1.25 **c** $10 – $1.40
d $10 – $7.50 **e** $20 – $4.20 **f** $50 – $32.50


Drilling for Gold 6A1

9 Find:

a 0.99 – 0.20 **b** 0.756 – 0.240 **c** 1.2 – 0.8 **d** 0.99 – 0.26

e 17.2 – 5.1 **f** 128.63 – 14.50 **g** 23.94 – 17.61 **h** 158.32 – 87.53


Skillsheet 6A

10 Find:

a 5 – 4.5 **b** 5.6 – 2.4
c 23.7 – 2.5 **d** 0.98 – 0.5
e 14.8 – 2.5 **f** 234.6 – 103.2
g 25.9 – 3.67 **h** 31.657 – 18.2
i 412.1 – 368.83 **j** 5312.271 – 364.93


Rewrite the subtractions as shown in Example 2.

11 Find the total of 0.808, 4.376 and 11.005.

Problem-solving and Reasoning

12 Stuart wants to raise $100 for a charity. He already has three donations of $30.20, $10.50 and $5.00.

a How much has Stuart raised already?
b How much does Stuart still need to raise?

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6A

13 Daily rainfalls for four days over Easter were 12.5 mm, 3.3 mm, 0.6 mm and 33 mm. What was the total rainfall over the four-day Easter holiday?



14 Bryce's normal body temperature is 36.9°C. During a fever, Bryce's temperature rose to 40.2°C. What is the difference in temperature?



15 Sophie has $75.49 in her bank account at the start of the month. During the month she deposits $37.50 and withdraws $57.90. How much money does she have at the end of the month?

Enrichment: Decimals magic square

16 Complete these magic squares.

a

0.6		0.2
0.1		
0.8		

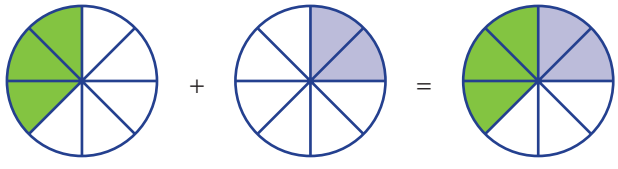
b

1.6	0.5		0.4
0.3		0.6	1.5
	1.1		
1.3		1.2	

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6B Adding fractions

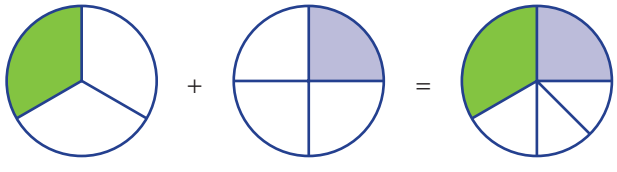
Fractions with the same denominator can be easily added together.



$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

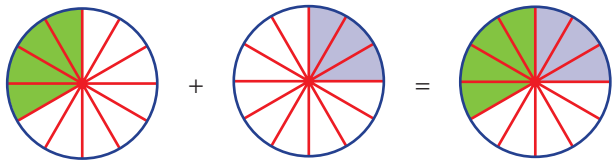
three-eighths + two-eighths = five-eighths

Fractions with different denominators cannot be added together so easily.



$\frac{1}{3} + \frac{1}{4} = ?$

But with a common denominator it is possible.



$\frac{1}{3} + \frac{1}{4} = ?$

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

four-twelfths + three-twelfths = seven-twelfths

Let's start: 'Like' addition

As a class, discuss which of the following pairs of numbers can be simply added together without having to change them in some way.

- **a** 6 goals, 2 goals
- **b** 11 goals, 5 behinds
- **c** 56 runs, 3 wickets
- **d** 6 hours, 5 minutes
- **e** 21 seconds, 15 seconds
- **f** 47 minutes, 13 seconds
- **g** 15 cm, 3 m
- **h** 2.2 km, 4.1 km
- **i** 5 kg, 1680 g
- **j** $\frac{2}{7}, \frac{3}{7}$
- **k** $\frac{1}{4}, \frac{1}{2}$
- **l** $2\frac{5}{12}, 1\frac{1}{3}$

You can see that, when adding, the units need to be the same. With fractions, the 'units' are the denominators.

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Key ideas

Denominator The number in a fraction below the vinculum

Numerator The number in a fraction above the vinculum

Equivalent Having the same value

- To add fractions, they must have the same **denominator**.
  - If the denominator is the same, keep it and add the **numerators**.
    For example: $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$
  - If the denominators are different, change one or both fractions so that the denominators become equal. Remember to use **equivalent** fractions and the lowest common denominator (LCD).
    For example: $\frac{3}{5}+\frac{1}{10}=\frac{6}{10}+\frac{1}{10}$
    $=\frac{7}{10}$
  - Simplify all answers.
    For example: $\frac{4}{10}=\frac{2}{5}$ and $\frac{4}{3}=1\frac{1}{3}$

Exercise 6B

Understanding

1 Copy the following sentences into your workbook and fill in the gaps.

a To add two fractions together, they must have the same _____.

b When adding fractions together, if they have the same _____, you simply add the _____.

c When adding two or more fractions where the _____ are different, you must find the _____ _____ _____.

d After carrying out the addition of fractions, you should always _____ your answer to see if it can be _____.

Choose from: *denominator, denominators, check, multiple, numerators, lowest, simplified, common.*



2 Use words to complete these additions.

a one-third + one-third = _____-thirds

b one-quarter + two-_____ = three-quarters

c three-tenths + _____-tenths = seven-tenths

d one-fifth + three-fifths = four-_____

e _____-half + _____-half = two-halves (i.e. one whole)

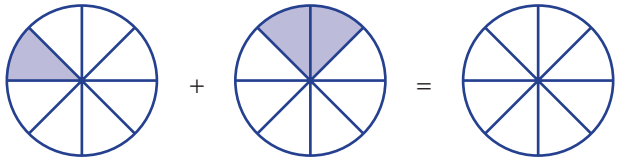
3 Copy and complete the following.

a 

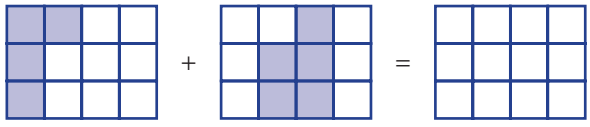
b 

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c



d



4 True (T) or false (F)?

a $\frac{1}{6}+\frac{3}{6}=\frac{4}{6}$ **b** $\frac{2}{5}+\frac{4}{5}=\frac{6}{10}$ **c** $\frac{1}{11}+\frac{3}{11}=\frac{4}{11}$ **d** $\frac{3}{5}+\frac{4}{5}=1\frac{2}{5}$

e $\frac{2}{7}+\frac{2}{7}=\frac{2}{7}$ **f** $\frac{7}{12}+\frac{4}{12}=\frac{11}{12}$ **g** $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **h** $\frac{1}{100}+\frac{2}{100}=\frac{3}{200}$

Fluency

Example 3 Adding 'like' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{3}{5}$ **b** $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}$

Solution	Explanation
a $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$	The denominators are the same ('like'), so simply add the numerators. one-fifth + three-fifths = four-fifths
b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}=\frac{14}{11}$ $=1\frac{3}{11}$	Denominators are the same, so add the numerators. We are adding elevenths. Simplify the answer by converting to a mixed numeral.

5 Copy and complete:

a $\frac{1}{8}+\frac{4}{8}=\frac{\square}{8}$ **b** $\frac{2}{7}+\frac{3}{7}=\frac{\square}{7}$ **c** $\frac{1}{5}+\frac{3}{5}=\frac{\square}{5}$ **d** $\frac{3}{11}+\frac{6}{11}=\frac{\square}{11}$

e $\frac{5}{8}+\frac{2}{8}=\frac{\square}{8}$ **f** $\frac{1}{12}+\frac{6}{12}=\frac{\square}{12}$ **g** $\frac{3}{15}+\frac{4}{15}=\frac{\square}{15}$ **h** $\frac{3}{9}+\frac{2}{9}=\frac{\square}{9}$

i $\frac{3}{5}+\frac{1}{5}=\frac{\square}{5}$ **j** $\frac{2}{7}+\frac{4}{7}=\frac{\square}{7}$ **k** $\frac{1}{10}+\frac{6}{10}=\frac{\square}{10}$ **l** $\frac{77}{100}+\frac{4}{100}=\frac{\square}{100}$

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6B

6 Add these fractions and write your answers using mixed numerals.

a $\frac{6}{7}+\frac{3}{7}$ **b** $\frac{7}{10}+\frac{6}{10}$ **c** $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$

d $\frac{12}{19}+\frac{3}{19}+\frac{8}{19}$ **e** $\frac{7}{10}+\frac{4}{10}$ **f** $\frac{4}{5}+\frac{3}{5}$

g $\frac{6}{7}+\frac{4}{7}$ **h** $\frac{8}{11}+\frac{6}{11}$ **i** $\frac{99}{100}+\frac{2}{100}$



Example 4 Adding 'unlike' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{1}{2}$ **b** $\frac{3}{4}+\frac{5}{6}$

Solution **Explanation**

a $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}$
$=\frac{7}{10}$

LCD of 5 and 2 is 10.
Write equivalent fractions with the LCD.

$\frac{1}{5}=\frac{2}{10}$ ($\times 2$ above and below) $\quad \frac{1}{2}=\frac{5}{10}$ ($\times 5$ above and below)

Denominators are the same, so add numerators.



b $\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}$
$=\frac{19}{12}$
$=1\frac{7}{12}$

LCD of 4 and 6 is 12.
Write equivalent fractions with the LCD.

$\frac{3}{4}=\frac{9}{12}$ ($\times 3$ above and below) $\quad \frac{5}{6}=\frac{10}{12}$ ($\times 2$ above and below)

Denominators are the same, so add numerators.
Simplify answer to a mixed numeral.

7 Copy and complete the following additions.

a $\frac{3}{10}+\frac{2}{5}$ (LCD = 10)

$=\frac{3}{10}+\frac{\square}{10}$

$=\frac{\square}{10}$

b $\frac{1}{2}+\frac{3}{8}$ (LCD = 8)

$=\frac{\square}{8}+\frac{3}{8}$

$=\frac{\square}{8}$

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c $\frac{1}{4}+\frac{2}{3}$ (LCD = 12)

$=\frac{\square}{12}+\frac{8}{12}$

$=\frac{\square}{12}$

d $\frac{2}{8}+\frac{1}{6}$ (LCD = 24)

$=\frac{\square}{24}+\frac{\square}{24}$

$=\frac{\square}{24}$

$=\frac{5}{12}$



Drilling for Gold 6B1

8 Add the following fractions.

a $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{3}+\frac{3}{5}$ **c** $\frac{1}{2}+\frac{1}{6}$ **d** $\frac{1}{4}+\frac{1}{3}$

e $\frac{2}{5}+\frac{1}{4}$ **f** $\frac{1}{5}+\frac{3}{4}$ **g** $\frac{2}{7}+\frac{1}{3}$ **h** $\frac{3}{8}+\frac{1}{5}$

i $\frac{3}{5}+\frac{5}{6}$ **j** $\frac{4}{7}+\frac{3}{4}$ **k** $\frac{8}{11}+\frac{2}{3}$ **l** $\frac{2}{3}+\frac{3}{4}$

Use similar steps to Question 7.



Example 5 Adding mixed numerals

Simplify:

a $3\frac{2}{3}+4\frac{2}{3}$ **b** $2\frac{5}{6}+3\frac{3}{4}$

Solution | **Explanation**

a $3+4+\frac{2}{3}+\frac{2}{3}=7+\frac{4}{3}$

$=7+1\frac{1}{3}$

$=8\frac{1}{3}$

Add the whole number parts together.
Add the fraction parts together.
Noting that $\frac{4}{3}=1\frac{1}{3}$, simplify the answer.

b $2+3+\frac{5}{6}+\frac{3}{4}$

$=5+\frac{10}{12}+\frac{9}{12}$

$=5+\frac{19}{12}$

$=5+1\frac{7}{12}$

$=6\frac{7}{12}$

Add the whole number parts together.
LCD of 6 and 4 is 12.
Write equivalent fractions with LCD.
Add the fraction parts together.
Noting that $\frac{19}{12}=1\frac{7}{12}$, simplify the answer.
Another method involves converting to improper fractions at the start.
For example, $2\frac{5}{6}+3\frac{3}{4}=\frac{17}{6}+\frac{15}{4}$.

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6B

9 Simplify:

a $1\frac{1}{5}+2\frac{3}{5}$ **b** $3\frac{2}{7}+4\frac{1}{7}$ **c** $11\frac{1}{4}+1\frac{2}{4}$ **d** $1\frac{3}{9}+4\frac{2}{9}$

e $5\frac{2}{3}+4\frac{2}{3}$ **f** $8\frac{3}{6}+12\frac{4}{6}$ **g** $9\frac{7}{11}+9\frac{7}{11}$ **h** $4\frac{3}{5}+7\frac{4}{5}$

10 Simplify:

a $2\frac{2}{3}+1\frac{3}{4}$ **b** $5\frac{2}{5}+1\frac{5}{6}$ **c** $3\frac{1}{2}+8\frac{2}{3}$ **d** $5\frac{4}{7}+7\frac{3}{4}$

e $8\frac{1}{2}+6\frac{3}{5}$ **f** $12\frac{2}{3}+6\frac{4}{9}$ **g** $17\frac{8}{11}+7\frac{3}{4}$ **h** $9\frac{7}{12}+5\frac{5}{8}$

Problem-solving and Reasoning

11 Dad gives you $\frac{2}{5}$ of the money for a movie ticket and Mum gives you $\frac{1}{5}$.

a What fraction of the ticket do your parents pay for?

b What fraction is left for you to pay?

c If the ticket costs $\$10$, how much do you pay?



12 Julie owns $\frac{1}{3}$ of a company and Sean owns $\frac{1}{4}$.

a What fraction of the company do they own together?

b What fraction of the company is left?

13 Mark spends $\frac{1}{3}$ of the day at school and $\frac{3}{8}$ of the day asleep.

a What fraction of the day has been used?

b What fraction of the day is left for fun?

c How many hours of fun does he get?



Enrichment: Raise it to the max, lower it to the min

14 a Using the numbers 1, 2, 3, 4, 5 and 6 only once, arrange them in the boxes below to, first, produce the maximum possible answer, and then the minimum possible answer. Work out the maximum and minimum possible answers.

$$\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}$$

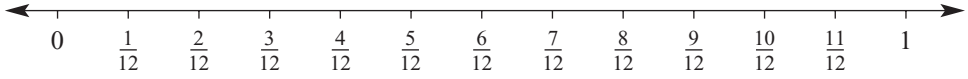
b Repeat the process for four fractions using the digits 1 to 8 only once each. Again, state the maximum and minimum possible answers.

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6C Subtracting fractions

The rule for subtracting fractions is very similar to adding fractions. Before you can subtract, the fractions must have the same denominator.

Let's start: Subtraction on a number line



Use the number line to find:

- $\frac{9}{12} - \frac{2}{12}$
- $\frac{11}{12} - \frac{1}{2}$
- $1 - \frac{9}{12}$

Key ideas

- Fractions with the same denominator can be easily subtracted.
- When subtracting mixed numerals, you may need to trade a whole. For example:

$7\frac{1}{8} - 2\frac{3}{8}$ $\frac{1}{8}$ is not big enough to have $\frac{3}{8}$ subtracted from it.

$6\frac{9}{8} - 2\frac{3}{8}$ Therefore, we choose to trade a whole from the 7.

(Note: $7\frac{1}{8}$ is equivalent to $6\frac{9}{8}$.)

- An alternative method involves first converting both mixed numerals to improper fractions.

For example: $7\frac{1}{8} - 2\frac{3}{8} = \frac{57}{8} - \frac{19}{8}$

Exercise 6C

Understanding

1 Copy the following sentences and fill in the blanks.

a To subtract one fraction from another, you must have a common _____.

b One easy method of producing a common denominator is to simply _____ the two denominators.

c The problem with finding a common denominator that is not the lowest common denominator is that you have to deal with larger numbers. You also need to _____ your answer at the final step.

d The LCD of 6 and 12 is _____.

e The LCD of 3 and 5 is _____.

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6C

2 Copy and complete the following subtractions.

a three-tenths – two-tenths = _____-tenth

b three-quarters – _____-quarters = one-quarter

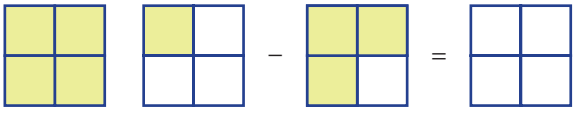
c four-_____ – three-fifths = one-fifth

d one whole – two-fifths = _____-fifths

e five-eighths – two-_____ = three-eighths

3 Copy and complete the following.

a 

b 

c 

4 Copy and complete these computations.

a $\frac{3}{7} - \frac{2}{7} = \frac{\square}{7}$

b $\frac{8}{13} - \frac{5}{13} = \frac{\square}{13}$

c $\frac{1}{3} - \frac{1}{4}$

$= \frac{\square}{12} - \frac{\square}{12}$

$= \frac{\square}{12}$

d $\frac{4}{5} - \frac{2}{3}$

$= \frac{\square}{15} - \frac{\square}{15}$

$= \frac{\square}{15}$

Fluency

Example 6 Subtracting 'like' and 'unlike' fractions

Simplify:

a $\frac{7}{9} - \frac{2}{9}$

b $\frac{5}{6} - \frac{1}{4}$

Solution	Explanation
a $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$	Denominators are the same, therefore we are ready to subtract the second numerator from the first.
b $\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12}$	The LCD of 6 and 4 is 12. Write equivalent fractions with the LCD.
$= \frac{7}{12}$	$\frac{5}{6} \overset{\times 2}{=} \frac{10}{12}$ (×2 above and below) $\quad \frac{1}{4} \overset{\times 3}{=} \frac{3}{12}$ (×3 above and below)
	We have the same denominators now, so subtract second numerator from the first. The denominator stays the same (i.e. twelfths).

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5 Simplify:

a $\frac{5}{7} - \frac{3}{7}$ **b** $\frac{4}{11} - \frac{1}{11}$ **c** $\frac{12}{18} - \frac{5}{18}$ **d** $\frac{2}{3} - \frac{1}{3}$

e $\frac{3}{5} - \frac{3}{5}$ **f** $\frac{6}{9} - \frac{2}{9}$ **g** $\frac{5}{19} - \frac{2}{19}$ **h** $\frac{17}{23} - \frac{9}{23}$

i $\frac{84}{100} - \frac{53}{100}$ **j** $\frac{41}{50} - \frac{17}{50}$ **k** $\frac{23}{25} - \frac{7}{25}$ **l** $\frac{7}{10} - \frac{3}{10}$

6 Simplify the following. Check your answers using a calculator.

a $\frac{2}{3} - \frac{1}{4}$ **b** $\frac{3}{5} - \frac{1}{2}$ **c** $\frac{3}{5} - \frac{3}{6}$ **d** $\frac{4}{7} - \frac{1}{4}$

e $\frac{1}{2} - \frac{1}{3}$ **f** $\frac{3}{4} - \frac{1}{9}$ **g** $\frac{8}{11} - \frac{1}{3}$ **h** $\frac{4}{5} - \frac{2}{3}$

i $\frac{3}{4} - \frac{5}{8}$ **j** $\frac{11}{20} - \frac{2}{5}$ **k** $\frac{5}{12} - \frac{7}{18}$ **l** $\frac{7}{9} - \frac{2}{3}$





Example 7 Subtracting mixed numerals

Simplify:

a $5\frac{2}{3} - 3\frac{1}{4}$ **b** $3\frac{1}{5} - 1\frac{3}{4}$

Solution	Explanation
a $5\frac{2}{3} - 3\frac{1}{4}$	Find the LCD of the fraction. The LCD of $\frac{2}{3}$ and $\frac{1}{4}$ is 12.
$= 5\frac{8}{12} - 3\frac{3}{12}$	Make the denominators the same. $\frac{2}{3} = \frac{8}{12}$ ($\times 4$ above, $\times 4$ below) $\quad \frac{1}{4} = \frac{3}{12}$ ($\times 3$ above, $\times 3$ below)
$= (5 - 3) + \left(\frac{8}{12} - \frac{3}{12}\right)$	Subtract the whole numbers.
$= 2 + \left(\frac{8}{12} - \frac{3}{12}\right)$	
$= 2\frac{5}{12}$	Subtract the fractions.

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6C

Solution	Explanation
b Method 1: Trading 1 from the whole number to the fraction	
$3\frac{1}{5} - 1\frac{3}{4}$	Find the LCD of the fractions. LCD of $\frac{1}{5}$ and $\frac{3}{4}$ is 20.
$= 3\frac{4}{20} - 1\frac{15}{20}$	Make the denominators the same. $\frac{1}{5} \overset{\times 4}{=} \frac{4}{20}$ $\quad \frac{3}{4} \overset{\times 5}{=} \frac{15}{20}$
$= 2\frac{24}{20} - 1\frac{15}{20}$	Trade a whole (1) from 3 so that the numerators can be subtracted. $3\frac{4}{20} = 2 + 1\frac{4}{20} = 2\frac{24}{20}$
$= 1\frac{9}{20}$	Subtract whole numbers and then subtract the fractions.
Method 2: Converting to improper fractions	
$3\frac{1}{5} - 1\frac{3}{4}$	
$= 3\frac{4}{20} - 1\frac{15}{20}$	Find the LCD and make denominators the same.
$= \frac{64}{20} - \frac{35}{20}$	Change each mixed numeral to an improper fraction.
$= \frac{29}{20}$	Subtract the fractions.
$= 1\frac{9}{20}$	Simplify.

7 Simplify:

a $3\frac{4}{5} - 2\frac{1}{5}$ **b** $23\frac{5}{7} - 15\frac{2}{7}$ **c** $8\frac{11}{14} - 7\frac{9}{14}$

d $3\frac{5}{9} - \frac{3}{9}$ **e** $6\frac{2}{3} - 4\frac{1}{4}$ **f** $5\frac{3}{7} - 2\frac{1}{4}$

For some of these, you will first need to make denominators the same.



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8 Simplify the following. Check your answers using a calculator.

a $5\frac{1}{3} - 2\frac{2}{3}$ **b** $8\frac{2}{5} - 3\frac{4}{5}$ **c** $13\frac{1}{2} - 8\frac{5}{6}$

d $12\frac{2}{9} - 7\frac{1}{3}$ **e** $8\frac{5}{12} - 3\frac{3}{4}$ **f** $1\frac{3}{5} - \frac{7}{9}$

Skillsheet 6B

You will need to 'trade a whole' for some of these.

Problem-solving and Reasoning

9 A family block of chocolate is made up of 60 small squares of chocolate. Marcia eats 10 blocks, Jon eats 9 blocks and Holly eats 5 blocks. What fraction of the block of chocolate is left?

10 Three friends split a restaurant bill. One pays $\frac{1}{2}$ of the bill and one pays $\frac{1}{3}$ of the bill. What fraction of the bill must the third friend pay?

11 A full container of flour weighs $\frac{4}{5}$ kg. The empty container weighs $\frac{1}{20}$ kg. How much does the flour weigh?



12 Copy and complete these two subtractions, using improper fractions.

a $2\frac{1}{4} - 1\frac{2}{3}$

$= \frac{9}{\square} - \frac{\square}{3}$

$= \frac{\square}{12} - \frac{\square}{12}$

$= \frac{\square}{12}$

b $2\frac{3}{5} - 1\frac{1}{2}$

$= \frac{\square}{5} - \frac{\square}{2}$

$= \frac{\square}{10} - \frac{\square}{10}$

$= \frac{\square}{10}$

$= \square\frac{\square}{\square}$

Enrichment: Missing numbers

13 Using the fraction wall in Section 4J and/or a calculator, arrange four different numbers in the four boxes to make the equation true.

$$\frac{\square}{\square} - \frac{\square}{\square} = \frac{1}{2}$$

Use a calculator to help.

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6D Multiplying fractions

When we multiply whole numbers together, we end up with a number larger than (or equal to) the numbers we started with.

$2 \times 3 = 6$ $5 \times 7 = 35$ $15 \times 1 = 15$

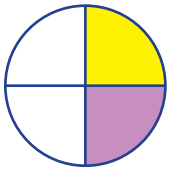
But when we multiply fractions together, things can be different. Consider half of half an apple, for example.



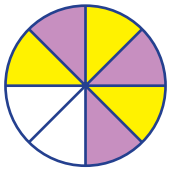
$\frac{1}{2}$ an apple $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$

Let's start: Parts of a pizza

- How does this diagram show half of half a pizza?



- What is half of three-quarters of a pizza? (Hint: Chop each quarter in half.)



Use diagrams to investigate other fraction multiplications.
What shortcut (or rule) can you find to help multiply fractions together?

Key ideas

- Fractions do *not* need to have the same denominator to be multiplied together.
- To multiply fractions, multiply the numerators together and multiply the denominators together.
  - For example: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$
- If possible, *simplify*, *divide* or *cancel* fractions before multiplying. (Remember, you can only cancel tops with bottoms.)
  - For example: $\frac{\cancel{6}^{2}}{5} \times \frac{2}{\cancel{9}_{3}} = \frac{4}{15}$
- Mixed numerals must be changed to improper fractions before multiplying.
- Final answers should be written in simplest form.

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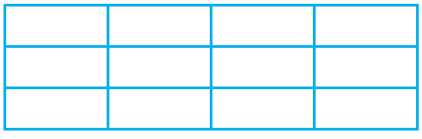
Exercise 6D

Understanding

1 Copy these sentences into your workbook and fill in the blanks.

- **a** A proper fraction has a value that is between _____ and _____.
- **b** An improper fraction is always greater than _____.
- **c** A mixed numeral consists of two parts, a _____ _____ part and a _____ _____ part.

2 Copy the grid shown here.



- **a** On your diagram, use blue to shade $\frac{1}{3}$ of the grid.
- **b** Now use red to shade $\frac{1}{4}$ of the shaded blue.
- **c** You have now shaded $\frac{1}{4}$ of $\frac{1}{3}$. What fraction is this of the original grid?

Example 8 Finding a simple fraction of a quantity

Find $\frac{2}{3}$ of 15 bananas.

Solution

$\frac{2}{3}$ of 15 bananas

$= \left(\frac{1}{3} \text{ of } 15\right) \times 2$

$= 15 \div 3 \times 2$

$= 10$ bananas

Explanation

Divide 15 bananas into 3 equal groups.
Therefore, 5 in each group.
Take 2 of the groups.



Answer is 10 bananas.
Short-cut: $\frac{2}{3}$ of $15 = 15 \div 3 \times 2$



Drilling for Gold 6D1

3 Use drawings to show the answer to these problems.

- **a** $\frac{1}{3}$ of 12 lollies
- **b** $\frac{1}{5}$ of 10 pencils
- **c** $\frac{2}{3}$ of 18 donuts
- **d** $\frac{3}{4}$ of 16 boxes
- **e** $\frac{3}{8}$ of 32 dots
- **f** $\frac{3}{7}$ of 21 triangles
- **g** $\frac{1}{10}$ of \$20
- **h** $\frac{3}{5}$ of 10 kg
- **i** $\frac{4}{7}$ of 21 triangles
- **j** $\frac{4}{5}$ of 15 stars



The denominator tells you how many equal groups to make. The numerator tells you how many of those groups to circle.

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6D

4 **a** If $\frac{1}{3}$ of ● = 4, what is $\frac{2}{3}$ of ●?

b If $\frac{1}{10}$ of ▲ = 7, what is $\frac{3}{10}$ of ▲?

c If $\frac{1}{2}$ of ★ = 4, what is $\frac{3}{2}$ of ★?

d If $\frac{1}{5}$ of 10 = 2, what is $\frac{4}{5}$ of 10?

e If $\frac{1}{100}$ of 200 = 2, what is $\frac{21}{100}$ of 200?

f If $\frac{1}{10}$ of P = 3, what is $\frac{10}{10}$ of P?

Fluency

5 Find:

a $\frac{2}{3}$ of 18 **b** $\frac{1}{5}$ of 45 **c** $\frac{2}{3}$ of 24 **d** $\frac{3}{5}$ of 25

e $\frac{2}{7}$ of 42 **f** $\frac{1}{4}$ of 16 **g** $\frac{4}{5}$ of 100 **h** $\frac{3}{7}$ of 77

i $\frac{1}{3}$ of 6 **j** $\frac{1}{3}$ of 60 **k** $\frac{2}{3}$ of 60 **l** $\frac{10}{9}$ of 18

m $\frac{2}{5}$ of 100 **n** $\frac{3}{4}$ of 16 **o** $\frac{5}{9}$ of 27 **p** $\frac{3}{4}$ of 20



$\frac{2}{3}$ of 18
$= 18 \div 3 \times 2$



Drilling for Gold 6D2

Example 9 Multiplying proper fractions

Find:

a $\frac{2}{3} \times \frac{1}{5}$ **b** $\frac{3}{4} \times \frac{8}{9}$

Solution	Explanation
a $\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5}$	Multiply the numerators together. Multiply the denominators together.
$= \frac{2}{15}$	The answer is in simplest form.
b $\frac{3}{4} \times \frac{8}{9} = \frac{{}^{1}\cancel{3} \times {}^{2}\cancel{8}}{{}_{1}\cancel{4} \times \cancel{9}_{3}}$	Cancel first. Then multiply numerators together and denominators together.
$= \frac{2}{3}$	

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6 Evaluate:

a $\frac{3}{4} \times \frac{1}{5}$ **b** $\frac{2}{7} \times \frac{1}{3}$ **c** $\frac{2}{3} \times \frac{5}{7}$ **d** $\frac{4}{9} \times \frac{2}{5}$

e $\frac{2}{3} \times \frac{3}{5}$ **f** $\frac{4}{7} \times \frac{1}{4}$ **g** $\frac{3}{4} \times \frac{1}{3}$ **h** $\frac{5}{9} \times \frac{9}{11}$

i $\frac{3}{6} \times \frac{5}{11}$ **j** $\frac{2}{3} \times \frac{4}{8}$ **k** $\frac{8}{11} \times \frac{3}{4}$ **l** $\frac{2}{5} \times \frac{10}{11}$

m $\frac{2}{7}$ of $\frac{3}{5}$ **n** $\frac{3}{4}$ of $\frac{2}{5}$ **o** $\frac{5}{10}$ of $\frac{4}{7}$ **p** $\frac{6}{9}$ of $\frac{3}{12}$



Where possible, cancel first so that you multiply smaller numbers. Remember that 'of' means multiply.

7 Find:

a $\frac{5}{2} \times \frac{7}{3}$ **b** $\frac{6}{5} \times \frac{11}{7}$ **c** $\frac{6}{4} \times \frac{11}{5}$ **d** $\frac{9}{6} \times \frac{13}{4}$

e $\frac{8}{5} \times \frac{10}{3}$ **f** $\frac{21}{4} \times \frac{8}{6}$ **g** $\frac{10}{7} \times \frac{21}{5}$ **h** $\frac{14}{9} \times \frac{15}{7}$

i $\frac{8}{5} \times \frac{1}{2}$ **j** $\frac{4}{3} \times \frac{1}{5}$ **k** $\frac{3}{2} \times \frac{1}{4}$ **l** $\frac{3}{4} \times \frac{5}{3}$



Multiply improper fractions just as if they were proper fractions, then simplify your answer.

Example 10 Multiplying mixed numerals

Find:

a $2\frac{1}{3} \times 1\frac{2}{5}$ **b** $6\frac{1}{4} \times 2\frac{2}{5}$

Solution	Explanation
a $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5}$	Convert mixed numerals to improper fractions.
$= \frac{49}{15}$	Multiply numerators together. Multiply denominators together.
$= 3\frac{4}{15}$	Write the answer in simplest form.
b $6\frac{1}{4} \times 2\frac{2}{5} = \frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{5}}}$	Convert to improper fractions. Simplify fractions by cancelling.
$= \frac{15}{1}$	Multiply numerators and denominators together.
$= 15$	Write the answer in simplest form.

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6D

Skillsheet 6C

8 Find:

a $1\frac{3}{5} \times 2\frac{1}{3}$ **b** $1\frac{1}{7} \times 1\frac{2}{9}$ **c** $3\frac{1}{4} \times 2\frac{2}{5}$ **d** $4\frac{2}{3} \times 5\frac{1}{7}$

e $\frac{6}{5} \times \frac{8}{3}$ **f** $\frac{1}{2} \times \frac{3}{8}$ **g** $\frac{3}{4}$ of $5\frac{1}{3}$ **h** $7\frac{1}{2} \times 4\frac{2}{5}$

i $\frac{3}{7} \times \frac{2}{3}$ **j** $1\frac{1}{2} \times 2\frac{1}{4}$ **k** $\frac{8}{9} \times \frac{6}{20}$ **l** $\frac{15}{4} \times \frac{8}{5}$



First convert mixed numerals to improper fractions.

Problem-solving and Reasoning



9 At one school, $\frac{2}{5}$ of the Year 7 students are boys.

a What fraction of the Year 7 students are girls?

b If there are 120 Year 7 students, how many boys and girls are there?

10 Julia was injured during the netball season. She was able to play only $\frac{2}{3}$ of the matches. The season consisted of 21 matches. How many games did Julia miss as a result of injury?

11 a Blake spends $\frac{3}{4}$ of an hour on his homework. How many minutes is this?

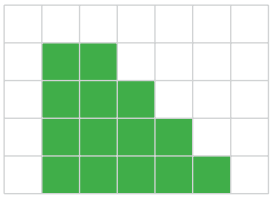
b Perform this calculation: $60 \div 4 \times 3$. What do you notice?

c Now find the number of minutes in $\frac{2}{3}$ of an hour in a similar way.

12 The diagram shows a plan of Joel's garden. The shaded section is grass. The rest is paved.

a What fraction of the garden is grass?

b If half the grass is removed and replaced with pavers, what fraction of the garden will remain grass?



Enrichment: Missing numbers

13 Find a way to arrange four different numbers in the four boxes to make the equation true.

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{1}{2}$$

Can you find another way?

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6E Multiplying and dividing decimals by 10, 100, 1000 etc.

In this section, we will be multiplying decimals by numbers such as 10, 100, 1000 etc.

Let's start: Does the decimal point really 'move'?

Consider the number 2.58.
Working from left to right:

- The digit 2 is in the units column.
- The decimal point sits between the units and the tenths, as it always does.
- The digit 5 is in the tenths column.
- The digit 8 is in the hundredths column.

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
		2	•	5	8	

Now, imagine that you buy 10 items for \$2.58 each. The cost is \$25.80.

- Did the digits change?
- Did the decimal point move? If so, which way and how many places?
- Or was it that the digits moved and the decimal point stayed still?
- If so, which way did the digits move? By how many places?

Key ideas

- Every number contains a decimal point but it is usually not shown in integers.
  For example: 345 is 345.0 and 2500 is 2500.0.
- Extra zeros can be added in the columns to the right of the decimal point without changing the value of the decimal.
  For example: $12.5 = 12.50 = 12.500 = 12.5000$ etc.
- When a decimal is multiplied by 10, **the decimal point stays still** and the digits all move 1 place to the left.
  However, it is easier to visualise the decimal point moving 1 place to the right.
  For example: $23\overset{\downarrow}{.}758 \times 10 = 237.58$

Operation	Visualisation	Example
Multiplying a decimal by 1000	Decimal point moves 3 places to the right.	$23.758 \times 1000 = 23758$
Multiplying a decimal by 100	Decimal point moves 2 places to the right.	$23.758 \times 100 = 2375.8$
Multiplying a decimal by 10	Decimal point moves 1 place to the right.	$23.758 \times 10 = 237.58$

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Operation	Visualisation	Example
Dividing a decimal by 10	Decimal point moves 1 place to the left.	$23.758 \div 10 = 2.3758$
Dividing a decimal by 100	Decimal point moves 2 places to the left.	$23.758 \div 100 = 0.23758$
Dividing a decimal by 1000	Decimal point moves 3 places to the left.	$23.758 \div 1000 = 0.023758$

Exercise 6E

Understanding

1 How many places has the decimal point appeared to move in each of these multiplications?

a $278.71 \times 10 = 2787.1$
b $15.389 \times 100 = 1538.9$
c $15.98513 \times 10\,000 = 159851.3$
d $48.9 \div 100 = 0.489$
e $10.076 \div 10 = 1.0076$

2 Fill in the correct number of zeros in the multiplier to make the following product statements correct. The first one has been done for you.

a $56.321 \times 1[0\,0] = 5632.1$
b $27.9234 \times 1[\quad] = 27\,923.4$
c $0.03572 \times 1[\quad] = 3.572$
d $3200 \times 1[\quad] = 320\,000\,000$

3 Fill in the correct number of zeros in the divisor to make the following division statements correct. The first one has been done for you.

a $2345.1 \div 1[0\,0\,0] = 2.3451$
b $7238.4 \div 1[\quad] = 72.384$
c $0.00367 \div 1[\quad] = 0.000367$
d $890 \div 1[\quad] = 0.0089$

4 How many places, and in what direction, would the decimal point appear to move if the following operations occur?

a $\times 100$
b $\div 10$
c $\times 1\,000\,000$
d $\div 1$
e $\div 1000$
f $\times 1000$
g $\times 10$
h $\div 10\,000\,000$

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Fluency

Example 11 Multiplying by 10, 100, 1000 etc.

Evaluate:

a 36.532×100 **b** $4.31 \times 10\,000$

Solution	Explanation
a $36.532 \times 100 = 3653.2$	100 has two zeros, so the decimal point appears to move 2 places to the right. $36.\overset{\curvearrowright}{5}\overset{\curvearrowright}{3}2$
b $4.31 \times 10\,000 = 43\,100$	The decimal point appears to move 4 places to the right and additional zeros are needed. $4.\overset{\curvearrowright}{3}\overset{\curvearrowright}{1}\overset{\curvearrowright}{0}\overset{\curvearrowright}{0}$

5 Calculate:

a 4.87×10 **b** 35.283×10 **c** 422.27×10
d 14.304×100 **e** 5.69923×1000 **f** 1.25963×100
g 12.7×1000 **h** 154.23×1000 **i** $0.34 \times 10\,000$
j 213.2×10 **k** $867.1 \times 100\,000$ **l** $0.00516 \times 100\,000\,000$

Example 12 Dividing by 10, 100, 1000 etc.

Evaluate:

a $268.15 \div 10$ **b** $7.82 \div 1000$

Solution	Explanation
a $268.15 \div 10 = 26.815$	10 has one zero, so the decimal point appears to move 1 place to the left. $26\overset{\curvearrowleft}{8}.15$
b $7.82 \div 1000 = 0.00782$	The decimal point appears to move 3 places to the left and additional zeros are needed. $.\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{7}82$

6 Calculate:

a $42.7 \div 10$ **b** $353.1 \div 10$
c $24.422 \div 10$ **d** $5689.3 \div 100$
e $12\,135.18 \div 1000$ **f** $93\,261.1 \div 10\,000$
g $2.9 \div 100$ **h** $13.62 \div 10\,000$
i $0.54 \div 1000$ **j** $36.7 \div 100$
k $0.02 \div 10\,000$ **l** $1000.04 \div 100\,000$



7 Calculate:

a 22.913×100 **b** 0.03167×1000 **c** $4.9 \div 10$
d $22.2 \div 100$ **e** $6348.9 \times 10\,000$ **f** $1.0032 \div 1000$

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6E

Example 13 Working with the 'invisible' decimal point

Evaluate:

a $567 \times 10\,000$ **b** $23 \div 1000$

Solution	Explanation
a $567 \times 10\,000 = 5\,670\,000$	As no decimal point is shown in the question, it must be at the very end of the number. Four additional zeros must be put in because we are multiply by $10\,000$. 5670000
b $23 \div 1000 = 0.023$	Decimal point appears to move 3 places to the left because we are dividing by 1000. 0.023

8 Calculate:

a 156×100 **b** 43×1000 **c** 2251×10 **d** 16×1000
e 2134×100 **f** 2134×1000 **g** 7×1000 **h** $99 \times 100\,000$
i $34 \times 10\,000$ **j** $156 \div 10$ **k** $156 \div 100$ **l** $156 \div 1000$
m $87 \div 10$ **n** $87 \div 100$ **o** $87 \div 1000$ **p** $16 \div 1000$
q $7 \div 1000$ **r** $34 \div 10\,000$

Problem-solving and Reasoning

9 A service station charges $1.37 per litre of petrol. How much will it cost Tanisha to fill her car with 100 litres of petrol?

10 Wendy is on a mobile phone plan that charges her 3 cents per text message. On average, Wendy sends 10 text messages per day. What will it cost Wendy for 100 days of sending text messages at this rate? Give your answer in cents and then convert it to dollars.



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11 Darren wishes to purchase 10 000 shares at \$2.12 per share.

a What is the cost of the shares?

b There is an additional \$200 fee. How much will it cost Darren to buy the shares?

12 Choose 10, 100 or 1000 to complete the following.

a $5.67 \times 10 \div \square = 5.67$

b $18.5 \div 100 \times \square = 1.85$

c $900 \div \square \times 1 = 9$

d $56 \div \square \div \square = 0.56$

e $3.4 \times \square \div 10 = 340$

Enrichment: Scoring system for diving

13 At the 2012 London Olympics, a new scoring system for diving was introduced. There are seven judges, who each give a score. The two highest and two lowest scores are disregarded. The remaining three are added. This is multiplied by the degree of difficulty.

What is the score for this dive?

Degree of difficulty is 3.2.

7.5 7.5 8.5 7.5 8.0 7.5 7.5



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6F Multiplying by a decimal

Many real-life applications involve the multiplication of decimal numbers. Examples include finding the area of a block of land that is 34.5 metres long and 5.2 metres wide, or pricing a 4.5 hour job at a rate of $21.75 per hour.

You can use whole-number methods to multiply decimals. However, you must place the decimal point in the correct position in your final answer.



Let's start: Multiplication musings

We can use what we know about multiplying fractions to multiply decimals.

For example: $0.4 \times 1.6 = \frac{4}{10} \times \frac{16}{10}$

$= \frac{64}{100}$

$= 0.64$

Now try these.

- 0.7×0.3
- 0.07×0.03
- 0.007×0.03

What do you notice about the decimal places in the question and the decimal places in the final answer?

Key ideas

- When multiplying decimals, we use the following rule.
  The total number of decimal places in the answer is the same as the total number of decimal places in the question.
  e.g. $6.25 \times 4.5 = 28.125$ (3 decimal places) $2.41 \times 6 = 14.46$ (2 decimal places) $0.0002 \times 5 = 0.0010 = 0.001$ (4 decimal places)

- To multiply decimals:
  - Count the total number of decimal places in the question.
    e.g. 5.34×1.2 ← 3 decimal places in the question
  - Now multiply, ignoring the decimal points.

$$\begin{array}{r} 534 \\ \times\ 12 \\ \hline 1068 \\ 5340 \\ \hline 6408 \end{array}$$

Decimal points ignored here.



  - Put in the decimal point into your answer.
    $5.34 \times 1.2 = 6.408$ ← 3 decimal places in the answer

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Exercise 6F

Understanding

1 State the number of decimal places for each of the following.

a 5.9	**b** 1.805	**c** 5.12	**d** 0.0072
e 4.9	**f** 0.49	**g** 4.87	**h** 5.29643

2 Work out the total number of decimal places in the each of the following products.

a 4×6.3	**b** 3.52×76	**c** 42×5.123
d 8.71×11.2	**e** 5.283×6.02	**f** 2.7×10.3
g 4.87×3241.21	**h** 0.003×3	**i** $0.001\,03 \times 0.0045$

3 Put the decimal point into each of the following answers so that the statement is true.

a $6.4 \times 3 = 192$	**b** $6.4 \times 0.3 = 192$	**c** $0.64 \times 0.3 = 192$
d $15.2 \times 0.1 = 152$	**e** $97.3 \times 0.2 = 1946$	**f** $0.18 \times 0.42 = 00756$

4 Copy and complete the rule for multiplying decimal numbers. (See the Key ideas in this section.) The total number of decimal places _____ must equal the number of _____ in the answer.

Fluency



Drilling for Gold 6F1

Example 14 Multiplying decimals

Calculate:

a 0.56×3 **b** 4.13×0.3

Solution	Explanation
a $\begin{array}{r} 56 \\ \times\ 3 \\ \hline 168 \end{array}$ $0.56 \times 3 = 1.68$	Multiply, ignoring the decimal point. There are 2 decimal places in the question, so there will be 2 decimal places in the answer.
b $\begin{array}{r} 413 \\ \times\ 3 \\ \hline 1239 \end{array}$ $4.13 \times 0.3 = 1.239$	Ignore both decimal points. Multiply. Total of 3 decimal places in the question, so there must be 3 decimal places in the answer.

5 Calculate:

a 1.2×4	**b** 8.4×2	**c** 75×0.1	**d** 5.8×5
e 9.8×2	**f** 9.8×0.2	**g** 0.6×4	**h** 0.6×0.4
i 0.8×0.7	**j** 0.9×0.3	**k** 7.4×0.1	**l** 0.9×0.9



We usually write 5.60 as 5.6.

6 Find:

a 5.64×0.2	**b** 18.09×0.3	**c** 5.08×0.7
d 18.5×0.04	**e** 7.8×0.3	**f** 11.6×0.7



If there are 3 decimal places in the question, there are 3 decimal places in the answer.

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6F

7 Calculate:

a 14×7.2 **b** 3×72.82 **c** 1.293×12
d 3.4×6.8 **e** 5.4×2.3 **f** 0.34×16
g 43.21×7.2 **h** 0.023×0.042



Skillsheet 6D

8 Calculate and then round your answer to the nearest dollar.

a $5 \times \$6.30$ **b** $3 \times \$7.55$
c $4 \times \$18.70$ **d** $\$5.64 \times 0.5$
e $\$10.48 \times 0.2$ **f** $\$7.86 \times 1.5$

Problem-solving and Reasoning

9 What is the cost of 37.25 litres of petrol at \$1.55 per litre? Give your answer to the nearest cent.

10 Anita requires 4.21 m of material for each dress she is making. She is planning to make a total of seven dresses. How much material does she need?

11 The net weight of a can of spaghetti is 0.445 kg. Find the net weight of eight cans of spaghetti.

12 **a** If $68 \times 57 = 3876$, what is the answer to 6.8×5.7? Why?
b If $23 \times 32 = 736$, what is the answer to 2.3×32? Why?
c If $250 \times 300 = 75000$, what is the answer to 2.5×0.3? Why?
d What is 7×6? What is the answer to 0.7×0.6? Why?
e What about 0.07×0.6?



Remember to count the decimal places.
6.8×5.7 has two places.

Enrichment: Using your calculator

13 Joseph buys the follow items at the supermarket. He gives the cashier \$80.

4 chocolate bars @\$1.85 each
3 loaves of bread @\$3.19 each
newspaper @\$1.40
2×2 litres of milk @\$3.70 each
washing powder @\$8.95
toothpaste @\$4.95
2 kg sausages @\$5.99 per kg
tomato sauce @\$3.20
2 packets of Tim Tams @\$3.55 each
5×1.25 litres of soft drink @\$0.99 each



a How much change does Joseph receive?
b How could he be given this change if he receives at least one note?

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6G Dividing fractions

Remember that division is the opposite of multiplication. Thinking of division as 'how many' helps us to understand dividing fractions.

For example, to find $\frac{1}{2} \div \frac{1}{4}$, think: How many quarters are in a half?

Consider a strip of paper that is divided into four equal sections. Half the strip is shaded.



There are two quarters in our half.

Therefore, $\frac{1}{2} \div \frac{1}{4} = 2$, which means there are two quarters in $\frac{1}{2}$.

Let's start: Using division patterns

Use patterns to help you find the missing numbers.

$20 \div 4 = 5$
$20 \div 2 = 10$
$20 \div 1 = 20$
$20 \div \frac{1}{2} = \square$
$20 \div \frac{1}{4} = \square$

Can you see an easy way to find the following?

- $30 \div \frac{1}{2}$
- $12 \div \frac{1}{4}$
- $20 \div \frac{1}{8}$
- $10 \div \frac{1}{3}$

Key ideas

- Dividing by $\frac{1}{2}$ is equivalent to multiplying by 2.

  For example: $20 \div \frac{1}{2} = 40$ $\quad$ $20 \times 2 = 40$

- 2 is the **reciprocal** of $\frac{1}{2}$ (and vice versa).
  - The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
  - The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$.
- To divide by a fraction, you can multiply by its reciprocal.

  For example: $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$ $\quad$ $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1}$
- Always find the reciprocal of the second fraction.
- When dividing, mixed numbers must be changed to improper fractions.

Reciprocal A fraction formed by swapping the numerator and denominator

Finding a reciprocal is called inverting, flipping or turning the fraction upside down.





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Exercise 6G

Understanding

1 Copy these sentences, filling in the blanks as you go.

a $10 \div \frac{1}{2}$ is the same as 10 ___ 2.

b $24 \div \frac{1}{4}$ is the same as $24 \times$ ___.

c To find half of a number you can _____ by 2.

d The _____ of $\frac{3}{5}$ is $\frac{5}{3}$.

e To invert a fraction you _____ it upside down.

f When dividing fractions we change any mixed numbers into _____ fractions.

2 Which of the following is the correct first step for finding $\frac{3}{5} \div \frac{4}{7}$?

A $\frac{3}{5} \times \frac{7}{4}$ **B** $\frac{5}{3} \times \frac{4}{7}$ **C** $\frac{5}{3} \times \frac{7}{4}$

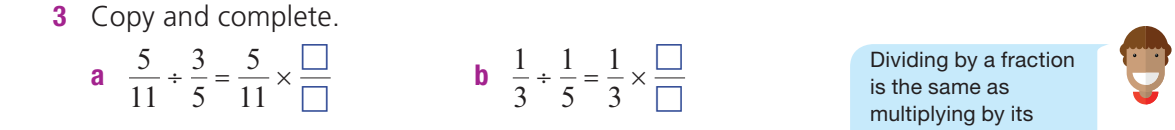
3 Copy and complete.

a $\frac{5}{11} \div \frac{3}{5} = \frac{5}{11} \times \frac{\square}{\square}$ **b** $\frac{1}{3} \div \frac{1}{5} = \frac{1}{3} \times \frac{\square}{\square}$

c $\frac{7}{10} \div \frac{12}{17} = \frac{7}{10} \times \frac{\square}{\square}$ **d** $\frac{8}{3} \div 3 = \frac{8}{3} \times \frac{\square}{\square}$

e $\frac{7}{10} \div \frac{1}{2} = \frac{7}{10} \times \frac{\square}{\square}$ **f** $1\frac{1}{2} \div \frac{1}{4} = \frac{3}{2} \times \frac{\square}{\square}$

g $\frac{3}{5} \div \frac{1}{10} = \frac{3}{5} \times \frac{\square}{\square}$ **h** $1\frac{1}{2} \div 2\frac{1}{3} = \frac{\square}{\square} \times \frac{3}{7}$





4 Make each sentence correct by inserting the word *more* or *less* in the gap.

a $10 \div 2$ gives an answer that is _____ than 10.

b $10 \div \frac{1}{2}$ gives an answer that is _____ than 10.

c $\frac{3}{4} \div \frac{2}{3}$ gives an answer that is _____ than $\frac{3}{4}$.

d $\frac{3}{4} \times \frac{3}{2}$ gives an answer that is _____ than $\frac{3}{4}$.

e $\frac{5}{7} \div \frac{8}{5}$ gives an answer that is _____ than $\frac{5}{7}$.

f $\frac{5}{7} \times \frac{5}{8}$ gives an answer that is _____ than $\frac{5}{7}$.

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Fluency

Example 15 Finding reciprocals

State the reciprocal of the following.

a $\frac{2}{3}$ **b** 5 **c** $1\frac{3}{7}$

Solution	Explanation
a Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.	The numerator and denominator are swapped.
b Reciprocal of 5 is $\frac{1}{5}$.	Think of 5 as $\frac{5}{1}$ and then swap the numerator and denominator.
c Reciprocal of $1\frac{3}{7}$ is $\frac{7}{10}$.	Convert $1\frac{3}{7}$ to the improper fraction $\frac{10}{7}$, and then swap the numerator and denominator.

5 What is the reciprocal of each of the following?

a $\frac{5}{7}$ **b** $\frac{3}{5}$ **c** $\frac{2}{9}$ **d** $\frac{1}{8}$

e $\frac{1}{3}$ **f** $\frac{1}{10}$ **g** $\frac{3}{10}$ **h** $\frac{5}{4}$

i $\frac{12}{1}$ **j** $\frac{101}{1}$ **k** $\frac{1}{9}$ **l** 1

6 First change each of the following to an improper fraction, then find its reciprocal.

a $1\frac{1}{2}$ **b** $1\frac{1}{5}$ **c** $2\frac{1}{2}$ **d** $1\frac{2}{5}$

e $2\frac{3}{4}$ **f** $2\frac{1}{3}$ **g** $4\frac{3}{5}$ **h** $1\frac{5}{6}$



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6G

Example 16 Dividing a fraction by a whole number

Find:

a $\frac{5}{8} \div 3$ **b** $2\frac{3}{11} \div 5$

Solution	Explanation
a $\frac{5}{8} \div \frac{3}{1} = \frac{5}{8} \times \frac{1}{3}$	Change the ÷ sign to a × sign and find the reciprocal.
$= \frac{5}{24}$	Multiply the numerators and denominators.
b $2\frac{3}{11} \div \frac{5}{1} = \frac{25}{11} \div \frac{5}{1}$	Convert the mixed numeral to an improper fraction. Write 5 as an improper fraction.
$= \frac{{}^{5}\cancel{25}}{11} \times \frac{1}{\cancel{5}^{1}}$	Change the ÷ sign to a × sign and find the reciprocal. Simplify by cancelling.
$= \frac{5}{11}$	Multiply numerators and denominators.

7 Find:

a $\frac{3}{4} \div 2$ **b** $\frac{5}{11} \div 3$ **c** $\frac{8}{5} \div 4$ **d** $\frac{15}{7} \div 3$

e $2\frac{1}{4} \div 3$ **f** $5\frac{1}{3} \div 4$ **g** $12\frac{4}{5} \div 8$ **h** $1\frac{13}{14} \div 9$



Remember, the reciprocal of 2 is $\frac{1}{2}$.

Example 17 Dividing a whole number by a fraction

Find:

a $6 \div \frac{1}{3}$ **b** $24 \div \frac{3}{4}$

Solution	Explanation
a $6 \div \frac{1}{3} = \frac{6}{1} \times \frac{3}{1}$	Instead of $\div \frac{1}{3}$, change to $\times \frac{3}{1}$.
$= \frac{18}{1}$	Simplify.
$= 18$	
b $24 \div \frac{3}{4} = \frac{{}^{8}\cancel{24}}{1} \times \frac{4}{\cancel{3}^{1}}$	Instead of $\div \frac{3}{4}$, change to $\times \frac{4}{3}$.
$= 32$	Cancel and simplify.

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8 Find:

a $5 \div \frac{1}{4}$ **b** $7 \div \frac{1}{3}$ **c** $10 \div \frac{1}{10}$ **d** $24 \div \frac{1}{5}$

e $12 \div \frac{2}{5}$ **f** $15 \div \frac{3}{8}$ **g** $14 \div \frac{7}{2}$ **h** $10 \div \frac{3}{2}$

Example 18 Dividing a fraction by a fraction

Find:

a $\frac{3}{5} \div \frac{3}{8}$ **b** $2\frac{2}{5} \div 1\frac{3}{5}$

Solution	Explanation
a $\frac{3}{5} \div \frac{3}{8} = \frac{3}{5} \times \frac{8}{3}$	Change the ÷ to × and find the reciprocal.
$= \frac{8}{5}$	Cancel and simplify.
$= 1\frac{3}{5}$	
b $2\frac{2}{5} \div 1\frac{3}{5} = \frac{12}{5} \div \frac{8}{5}$	Convert mixed numerals to improper fractions.
$= \frac{{}^{3}\cancel{12}}{\cancel{5}_{1}} \times \frac{\cancel{5}^{1}}{\cancel{8}^{2}}$	Change the ÷ sign to a × sign and find the reciprocal.
$= \frac{3}{2}$	Cancel, multiply and simplify.
$= 1\frac{1}{2}$	

Skillsheet 6C

9 Find:

a $\frac{2}{7} \div \frac{2}{5}$ **b** $\frac{1}{5} \div \frac{1}{4}$ **c** $\frac{3}{7} \div \frac{6}{11}$

d $\frac{2}{3} \div \frac{8}{9}$ **e** $2\frac{1}{4} \div 1\frac{1}{3}$ **f** $4\frac{1}{5} \div 3\frac{3}{10}$

g $12\frac{1}{2} \div 3\frac{3}{4}$ **h** $9\frac{3}{7} \div 12\frac{4}{7}$ **i** $1\frac{1}{4} \div \frac{5}{6}$



First convert any mixed numerals to improper fractions.

Problem-solving and Reasoning



10 If $2\frac{1}{4}$ leftover pizzas are to be shared between three friends, what fraction of pizza will each friend receive? Use a calculator to check your answer.

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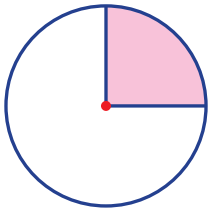
6G

11 A car travels 180 kilometres in $1\frac{1}{2}$ hours. Calculate this car's speed. Your answer will be in km/h. Use a calculator to check your answer.

Speed = distance ÷ time



12 Ceanna colours $\frac{1}{4}$ of her circle pink.

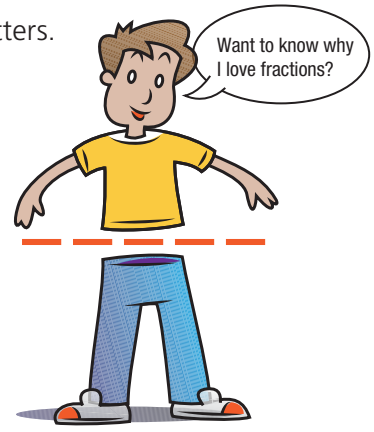


She divides the rest of the circle into three equal sectors and colours them blue, purple and green. What fraction of the circle is purple?

Enrichment: A puzzling question!

13 *Why do I love fractions?*

To find out, work out the value of each of these 15 letters.



$A = \frac{2}{3} + 1\frac{1}{3}$	$O = \frac{2}{7} + \frac{2}{7}$	$E = \frac{1}{4} + \frac{1}{5}$
$N = 2 - \frac{3}{4}$	$M = \frac{1}{10} + \frac{1}{2}$	$C = \frac{1}{4} - \frac{1}{5}$
$F = \frac{2}{3}$ of 6	$R = \frac{1}{2} - \frac{3}{10}$	$S = \frac{1}{9}$ of 27
$T = 2\frac{1}{2} + 1\frac{1}{4}$	$I = \frac{5}{6} - \frac{1}{3}$	$L = \frac{7}{12} - \frac{1}{4}$
$K = 1\frac{1}{2} \times 4$	$W = \frac{5}{6} \times \frac{6}{5}$	$H = 2 \div \frac{1}{4}$

Now use those values to decode the answer.

4	$\frac{1}{5}$	2	$\frac{1}{20}$	$3\frac{3}{4}$	$\frac{1}{2}$	$\frac{4}{7}$	$1\frac{1}{4}$	3

$\frac{3}{5}$	2	6	$\frac{9}{20}$

$\frac{3}{5}$	$\frac{9}{20}$

4	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{3}$

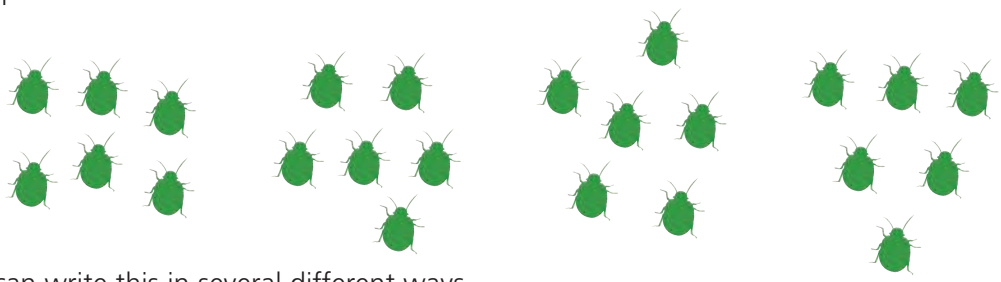
1	8	$\frac{4}{7}$	$\frac{1}{3}$	$\frac{9}{20}$

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6H Dividing decimals

For multiplication, we treat decimals like whole numbers until the very end of the computation. For division, however, we try to change the question so that we divide by a whole number instead of a decimal.

First, let's review the terminology for division. Suppose we divide 24 by 4 to make groups of 6.



We can write this in several different ways.

$24 \div 4 = 6$ or $\frac{24}{4} = 6$ or $4\overline{)24}$ with quotient 6

- 24 is the dividend (i.e. the amount you have or the number being divided).
- 4 is the divisor (i.e. the number doing the dividing).
- 6 is the quotient (or the answer).

Dividend ÷ Divisor = Quotient

Let's start: Division decisions

Use a calculator to find:

- $100 \div 2$ and $10 \div 0.2$
- $60 \div 3$ and $6 \div 0.3$
- $1.56 \div 0.02$ and $156 \div 2$

What do you notice about each pair?

Can you think of an easy way to divide 21.464 by 0.02 without using a calculator?

Key ideas

- Division of decimal numbers by whole numbers:
  - Work out as you would any other division question.
  - The decimal point in the **quotient** (i.e. the answer) goes directly above the decimal point in the **dividend**.

  For example: $60.524 \div 4$

$$4\overline{)6^{2}0.5^{1}24} \quad 15.131$$

Quotient Another word for answer

Dividend The amount you have or the number being divided

Divisor The number doing the dividing

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- Division of decimal numbers by other decimals:
  - Change the **divisor** (i.e. the number after the ÷ sign) into a whole number.
  - Whatever change you make to the divisor, you also make to the dividend.
  - Then carry out the division by a whole number.
  - For example: $24.56 \div 0.2$

$$\overset{\times 10}{\curvearrowright}\quad\overset{\times 10}{\curvearrowright}$$
$$24.56 \div 0.2 = 245.6 \div 2 \qquad 0.2 \times 10 = 2 \text{ (which is a whole number)}$$
$$= 122.8 \qquad \text{Therefore, multiply } 24.56 \text{ by } 10.$$

Exercise 6H

Understanding

1 For $36.52 \div 0.4 = 91.3$, which of the following uses the correct words?

A 36.52 is the divisor, 0.4 is the dividend and 91.3 is the quotient.
B 36.52 is the dividend, 0.4 is the divisor and 91.3 is the quotient.
C 36.52 is the quotient, 0.4 is the dividend and 91.3 is the divisor.
D 36.52 is the divisor, 0.4 is the quotient and 91.3 is the dividend.

2 Copy and complete these products, converting decimals to whole numbers.

a $0.3 \times \square = 3$ **b** $0.03 \times \square = 3$
c $0.003 \times \square = 3$ **d** $1.2 \times \square = 12$
e $0.07 \times \square = 7$ **f** $0.05 \times \square = 5$

3 Rewrite each question so that you are dividing by a whole number. The first one has been done for you.

a $1.2 \div 0.3 = 12 \div 3$ **b** $1.8 \div 0.2$ **c** $15.2 \div 0.1$
d $56.42 \div 0.02$ **e** $3.8 \div 0.1$ **f** $3.8 \div 0.01$
g $4.846 \div 0.02$ **h** $0.07 \div 0.2$ **i** $7.2 \div 0.9$

4 Copy and complete:

a $3.2456 \div 0.3 = \square \div 3$ **b** $120.432 \div 0.12 = \square \div 12$
c $0.00345 \div 0.0001 = \square \div 1$ **d** $1234.12 \div 0.004 = \square \div \square$

Fluency

Example 19 Dividing decimals by whole numbers

Calculate:

a $42.837 \div 3$ **b** $0.0234 \div 4$

Solution	Explanation
a 14.279 $3\overline{)4^{1}2.8^{2}3^{2}7}$	Simply divide by the whole number. Remember that the decimal point in the answer goes directly above the decimal point in the dividend.

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b 0.00585

$$4\overline{)0.02^{2}3^{3}4^{2}0} \quad 0.00\,5\,8\,5$$

Remember to write a zero in the answer every time the divisor 'doesn't go'. Make sure the decimal points line up.

Write a zero at the end of the dividend so that you can complete the division.

5 Calculate:

a $8.4 \div 2$	**b** $30.5 \div 5$	**c** $64.02 \div 3$
d $2.822 \div 4$	**e** $4.713 \div 3$	**f** $2.156 \div 7$
g $38.786 \div 2$	**h** $1491.6 \div 4$	**i** $0.0144 \div 6$
j $234.21 \div 2$	**k** $3.417 \div 5$	**l** $0.01025 \div 4$
m $1.56 \div 4$	**n** $1.85 \div 5$	**o** $1.225 \div 7$
p $53.7 \div 6$	**q** $28.08 \div 3$	**r** $210.2 \div 2$

Example 20 Dividing decimals by decimals

Calculate:

a $62.316 \div 0.03$ **b** $0.03152 \div 0.002$

Solution	Explanation
a $62.316 \div 0.03$ $= 6231.6 \div 3$ $= 2077.2$ $3\overline{)62^{2}3^{2}1.6}$ $\quad 20\,7\,7.2$	We want to divide by a whole number, so we need to make the 0.03 into 3. $62.316 \div 0.03$ Carry out the division for $6231.6 \div 3$.
b $0.03152 \div 0.002$ $= 31.52 \div 2$ $= 15.76$ $2\overline{)3^{1}1.^{1}5^{1}2}$ $\quad 1\,5\,.7\,6$	Change 0.002 into 2. $0.03152 \div 0.002$ Carry out the division for $31.52 \div 2$. Remember to line up the decimal points.

6 Calculate:

a $6.14 \div 0.2$	**b** $23.25 \div 0.3$	**c** $2.144 \div 0.08$
d $5.1 \div 0.6$	**e** $0.3996 \div 0.009$	**f** $45.171 \div 0.07$
g $0.0032 \div 0.04$	**h** $0.04034 \div 0.8$	**i** $10.78 \div 0.011$
j $4.003 \div 0.005$	**k** $0.948 \div 1.2$	**l** $432.2 \div 0.0002$

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6H

Skillsheet 6D

7 Find:

a $1500 \div 200$ **b** $1500 \div 20$ **c** $1500 \div 2$
d $1500 \div 0.2$ **e** $1500 \div 0.02$ **f** $1500 \div 0.002$



$1500 \div 200$
$= 15 \div 2$

8 Calculate the following, rounding your answers to 2 decimal places.

a $35.5\,\text{kg} \div 3$ **b** $\$213.25 \div 7$ **c** $182.6\,\text{m} \div 0.6$
d $287\,\text{g} \div 1.2$ **e** $482.523\,\text{L} \div 0.5$ **f** $\$5235.50 \div 9$

Problem-solving and Reasoning

9 Timber can be purchased in lengths of 2.4 m. How many pieces of timber measuring 0.3 m long can be cut from one 2.4 m length?

10 a A water bottle can hold 600 mL of water. How many water bottles can be filled from a large drink container that can hold 16 L?

b How many 400 mL bottles are needed for 16 L?

11 Rose paid \$12.72 to fill her ride-on lawnmower with 8 L of fuel. What was the price per litre of the fuel she purchased?



12 Find how many 20-cent pieces are needed to make:

a \$0.80 **b** \$1.20 **c** \$5.40
d \$7.80 **e** \$15.20 **f** \$1000

Enrichment: Calculations without calculators

13 Given that $5 \times 7 = 35$, find the value of:

a $35 \div 7$ **b** $35 \div 0.7$ **c** $35 \div 0.07$ **d** $35 \div 0.007$

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1 Find the missing numbers in the following sums.

a
$$\begin{array}{r} 3.\square \\ +\,4.6 \\ \hline \square.3 \end{array}$$

b
$$\begin{array}{r} 8.\square 9 \\ +\,\square.75 \\ \hline \square 4.4\square \end{array}$$

c
$$\begin{array}{r} 1.\square 1 \\ +\,\square\square.11 \\ \hline 11.1\square \end{array}$$

d
$$\begin{array}{r} \square.3\square 6 \\ 2.\square 43 \\ +\,1.89\square \\ \hline \square 1.395 \end{array}$$

2 Find the missing digits in these division questions.

a $3\overline{)2.67}$ becomes $3\overline{)2.\square\square 7}$

b $3\overline{)1.62}$ becomes $3\overline{)1.\square 2}$

c $\square\overline{)10.7\square}$ becomes $\square\overline{)2.\square 5}$

d $\square\overline{)15.\square29}$ becomes $\square\overline{)2.14\square}$

3 *What room can nobody enter?*

To find out, solve each problem. Then use your answers to unlock the code below.

A Find the total of \$7.06, \$24.95, \$1.05 and \$3.50.

O Calculate \$69.97 – \$15.98.

M Find \$100 – 4 × \$16.05.

S Tracy bought five books at \$24.95 each. How much change did she receive from \$150?

H Nensi spent \$216 on nine CDs. How much did each CD cost?

U Find the cost of seven cans of soft drink if a dozen cans cost \$16.20.

M Find the cost of six slices of cake at \$1.25 per slice.

O Find the total cost of four books at \$4.95 each, three pens at 95 cents each and one eraser at \$1.05.

R My-Anh spent \$23.15 on items for school and another \$3.95 on chocolate. How much change did she get from \$30?

\$36.56

\$7.50	\$9.45	\$25.25	\$24	\$2.90	\$53.99	\$23.70	\$35.80

4 *Fraction dice game*

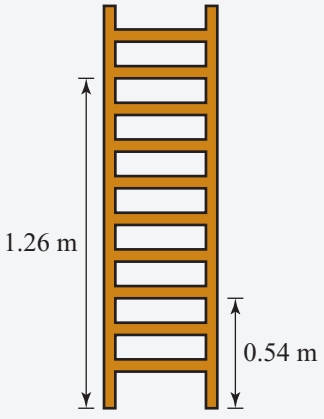
Two different-coloured dice are required. Choose one die for the numerator and one die for the denominator.

For example: $\frac{\text{red die result}}{\text{blue die result}} = \frac{5}{3}$ or $\frac{\text{red die result}}{\text{blue die result}} = \frac{4}{1}$.

Players take turns to throw both dice and record their fraction results.

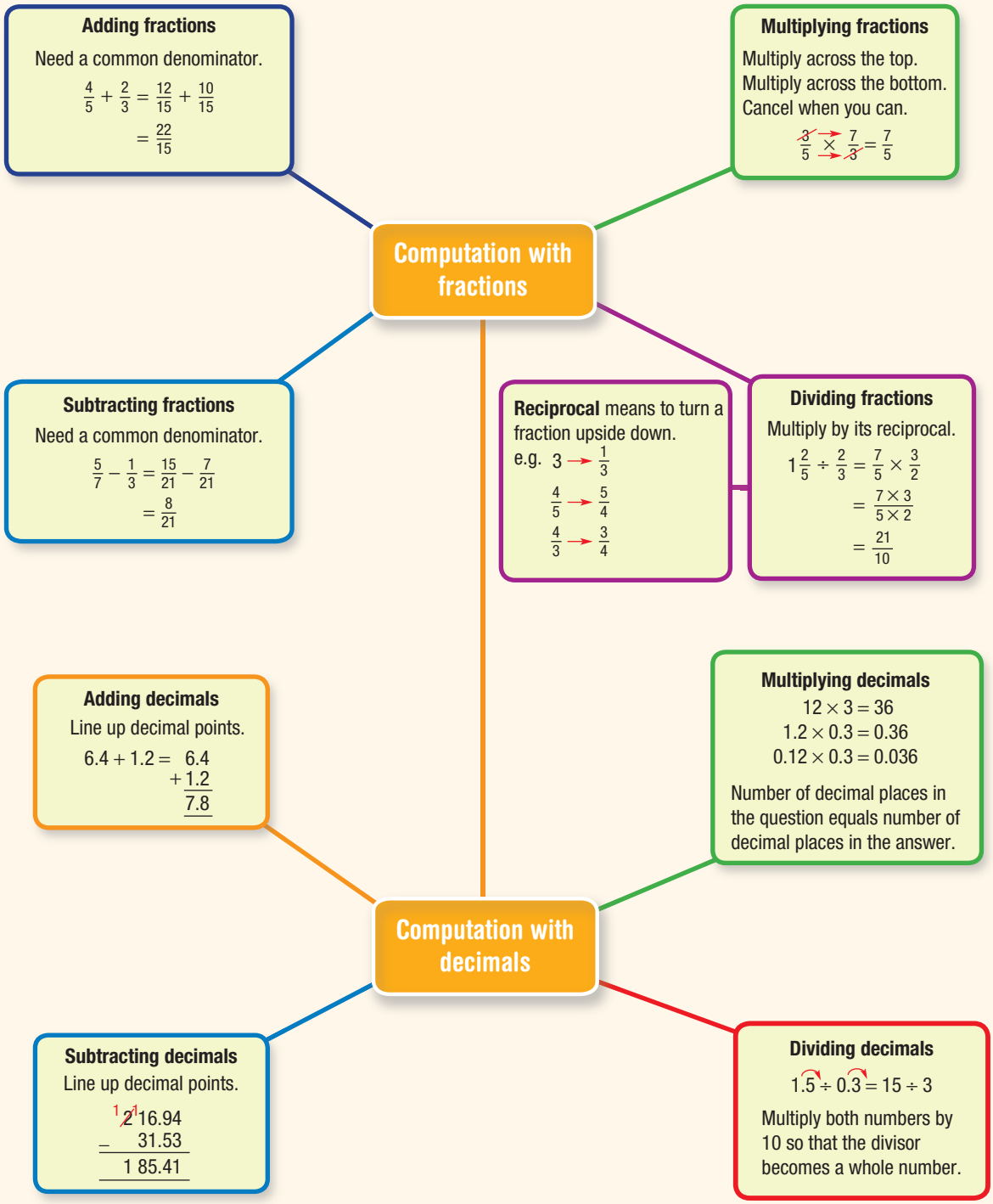
After an equal number of turns, each player then adds all their results together and the winner is the player with the largest number.

5 The rungs in this ladder are evenly spaced. The height above the ground is shown for two of the rungs. Find the height above the ground of each of the other rungs.



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Chapter summary



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Chapter review

T **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

Multiple-choice questions

1 $\frac{3}{4} + \frac{3}{4}$ is equal to:
A $\frac{6}{8}$ **B** $\frac{9}{16}$ **C** $\frac{9}{4}$ **D** $1\frac{1}{2}$ **E** $\frac{3}{4}$

2 0.2×6 is equal to:
A 0.12 **B** 1.2 **C** 12 **D** 120 **E** 0.8

3 0.2×0.3 is equal to:
A 6 **B** 0.6 **C** 0.06 **D** 0.006 **E** 60

4 $0.36 \div 1000$ is equal to:
A 3.6 **B** 360 **C** 0.036 **D** 0.0036 **E** 0.000 36

5 6.2×0.2 is equal to:
A 1.24 **B** 12.4 **C** 0.124 **D** 124 **E** 0.0124

6 What is the answer to 0.08×0.6?
A 0.48 **B** 4.8 **C** 0.0048 **D** 0.048 **E** 48

7 $\frac{1}{2} \div \frac{1}{4}$ is equal to:
A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** $\frac{1}{8}$ **D** 2 **E** 4

8 Which computation has an incorrect answer?
A $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ **B** $\frac{3}{4} + \frac{5}{12} = \frac{8}{16}$ **C** $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$ **D** $1\frac{3}{4} - \frac{1}{2} = 1\frac{1}{4}$ **E** $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

9 Bread rolls at a bakery are 60 cents each or six for $3.50. The cost of eight bread rolls is:
A $3.60 **B** $4.80 **C** $4.70 **D** $4.10 **E** $21

10 Aiden spends $7.25 on a sandwich, $2.55 on a drink and $2.70 on a doughnut. His change from $20 is:
A $12.75 **B** $18 **C** $12.50 **D** $7.50 **E** $2



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Chapter review

Short-answer questions

1 Find the value of:

a $0.5 + 0.5$ **b** $0.4 + 0.8$ **c** $0.25 + 0.75$

d $0.5 + 0.75$ **e** $1.5 + 1.5$ **f** $2.5 + 0.25$

2 Find the value of:

a $1 - 0.5$ **b** $1 - 0.2$ **c** $1 - 0.25$

d $2 - 1.1$ **e** $2.8 - 1.2$ **f** $3.5 - 2.5$

3 Find the value of:

a \$10 – \$7.50 **b** \$5 – \$3.25 **c** \$20 – \$12.55

4 Determine the simplest answer for each of the following.

a $\frac{3}{8} + \frac{1}{8}$ **b** $\frac{1}{3} + \frac{1}{2}$ **c** $\frac{3}{8} + \frac{5}{6}$

d $2\frac{7}{15} + 3\frac{3}{10}$ **e** $\frac{7}{8} - \frac{3}{8}$ **f** $5\frac{1}{4} - 2\frac{3}{4}$

5 Find:

a $\frac{1}{3} \times 21$ **b** $\frac{4}{5}$ of 100 **c** $\frac{3}{4}$ of 16

d $\frac{8}{10} \times \frac{25}{4}$ **e** $\frac{2}{3}$ of $\frac{1}{4}$ **f** $3\frac{1}{8} \times 2\frac{2}{5}$

6 Determine the reciprocal of each of the following.

a $\frac{3}{4}$ **b** $\frac{7}{12}$ **c** $2\frac{3}{4}$ **d** 8

7 Perform these divisions.

a $\frac{6}{10} \div 3$ **b** $64 \div 3\frac{1}{5}$ **c** $\frac{2}{3} \div \frac{1}{6}$ **d** $1\frac{1}{2} \div \frac{3}{4}$

8 Evaluate:

a $1.2 + 0.4$ **b** $0.36 + 1.2$ **c** $19.4 + 0.194$

d $7.6 + 1.2 + 0.8$ **e** $20 + 1.9$ **f** $6.4 - 3.1$

g $47.06 - 1.12$ **h** $200 - 156.5$ **i** \$15 – \$7.24

j \$1.60 + \$5.40 **k** \$19.46 + \$10.34 – \$5

9 True (T) or false (F)?

a $1.37 \times 100 = 137$ **b** $9.4 \div 10 = 940$ **c** $8.7 \div 10 = 0.87$

d $18 \div 10 = 1.8$ **e** $5\% = 0.5$ **f** $\frac{1}{4} = 1.4$

g $12 \div 0.4 = 120 \div 4$ **h** $10\% \times 6 = 0.1 \times 6$ **i** $9.81 \times 10 \div 10 = 9.81$

j $\frac{7}{100} = 70\%$ **k** $25\% = 2.5$ **l** $56.1 \div 10 = 5.61$

10 Calculate the following.

a 2.4×8 **b** 9×7.11 **c** 2.3×8.4

d $3.8 \div 4$ **e** $12.16 \div 8$ **f** $3 \div 0.5$

g $4 \div 0.25$ **h** $1.2 \div 0.4$ **i** $3.42 \div 0.1$

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Extended-response questions

1 Find the answer to these practical situations.

a Jessica is paid $125.70 for 10 hours of work and Jaczinda is paid $79.86 for 6 hours of work. Who receives the higher rate of pay per hour, and by how much?

b Petrol is sold for 124.9 cents per litre. Jacob buys 10 L of petrol for his car. Find the total price he pays, to the nearest 5 cents.

c The cost of a ticket at the movies is $14.50 for an adult and $11.20 for a child. Find the cost for 1 adult and 2 children.

d A pie costs $2.70 at the school canteen. Zara bought 2 pies and a drink. The total was $8.60.

i How much change from $10 did Zara receive?

ii How much was her drink?

e Jason buys one coffee every workday, from different shops. Last week the prices were: $2.70, $3.30, $3.50, $3.50, $4.50

i Find the total cost for the week.

ii Find the average cost by dividing the total cost by the number of coffees.

iii How much should Jason budget for the 200 days he works in a year?



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Chapter 1: Computation with positive integers

Multiple-choice questions

1 Five thousand, two hundred and six is:
A 526 **B** 50 206 **C** 5260 **D** 5026 **E** 5206

2 The place value of 8 in 4837 is:
A 8 thousand **B** 8 hundred **C** zero **D** 8 tens **E** 8 ones

3 The remainder when 650 is divided by 4 is:
A 0 **B** 4 **C** 1 **D** 2 **E** 3

4 $18 - 3 \times 4$ simplifies to:
A 12 **B** 81 **C** 6 **D** 60 **E** 30

5 $(7 - 2 + 65) \div 10$ is equal to:
A 7 **B** 10 **C** 11 **D** 70 **E** 32

Short-answer questions

1 Write the missing number.
a 0, 2, ___, 6, 8, 10
b 13, 9, 5, ___
c 101, 202, ___, 404

2 Write the number for:
a seven thousand, three hundred and twenty-four
b twelve thousand and ninety-two

3 Calculate:
a $3712 + 1204 + 46$ **b** $1438 - 619$ **c** 49×3
d 380×20 **e** $525 \div 5$ **f** $411 \div 3$

4 True or false?
a $15 < 6 \times 2$ **b** $9 \times 6 > 45$ **c** $23 = 40 \div 2 + 3$

5 How much more than 9×11 is 11×11?

6 Calculate:
a $4 + 2 \times 3$ **b** $10 \div 5 + 3$ **c** $20 - 15 \div 5$
d $7 \times 6 - 4 \times 3$ **e** $8 \times 8 - 16 \div 2$ **f** $12 \times (6 - 2)$
g $9 \times (2 \times 4) - 10$ **h** $24 \div 6 \times 4$ **i** $56 - (7 - 5) \times 7$

7 Are the following true or false?
a $4 \times 25 \times 0 = 1000$ **b** $0 \div 10 = 0$ **c** $8 \div 0 = 0$
d $8 \times 7 = 7 \times 8$ **e** $20 \div 4 = 20 \div 2 \div 2$ **f** $8 + 5 + 4 = 8 + 9$

8 Put in brackets to make each of the following true.
a $2 + 3 \times 4 = 20$ **b** $10 - 2 \div 8 = 1$ **c** $4 \times 6 - 2 \div 8 = 2$

9 Round each number to the nearest ten.
a 12 **b** 35 **c** 137

10 Round each number to the nearest hundred.
a 129 **b** 87 **c** 1451

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Extended-response questions

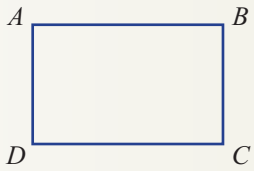
1 Tom works as a labourer, earning $25 an hour on weekdays and $60 an hour on weekends.

- **a** During a particular week, Tom works 7 a.m. to 2 p.m. Monday to Thursday. How many hours does he work that week?
- **b** How much does Tom earn for this work?
- **c** If Tom works 5 hours on Saturday in the same week, what is his total income for the week?
- **d** How many more hours on a Friday must Tom work to earn the same amount as working 5 hours on a Saturday? (Hint: Work out how much he earns on Saturday.)

Chapter 2: Angle relationships

Multiple-choice questions

1 $ABCD$ is a rectangle. Which statement is correct?

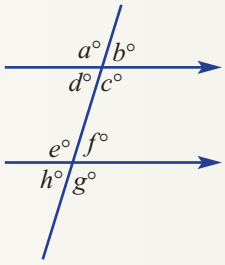


- **A** AB is perpendicular to BC.
- **B** $AB = BC$
- **C** $AD \parallel AB$
- **D** $\angle BAD$ is acute.
- **E** AD is perpendicular to BC.

2 An angle of 181° is classified as:

A acute **B** reflex **C** straight **D** obtuse **E** sharp

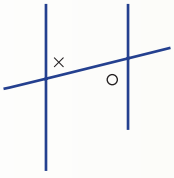
3 Which two angles represent alternate angles?



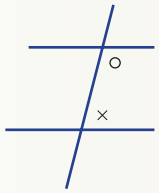
- **A** a° and e°
- **B** d° and f°
- **C** a° and f°
- **D** g° and b°
- **E** c° and f°

4 Which of the following shows a pair of cointerior angles?

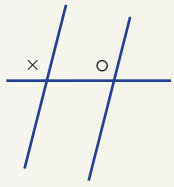
A



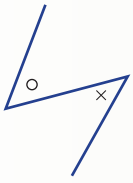
B



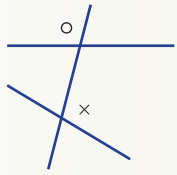
C



D



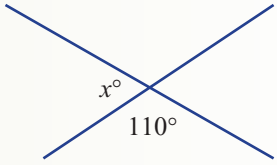
E



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5 The value of x is:

A 140 **B** 110 **C** 220
D 70 **E** 65



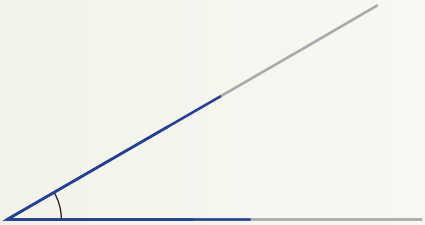
Short-answer questions

1 Give the name of the type of angle which is:

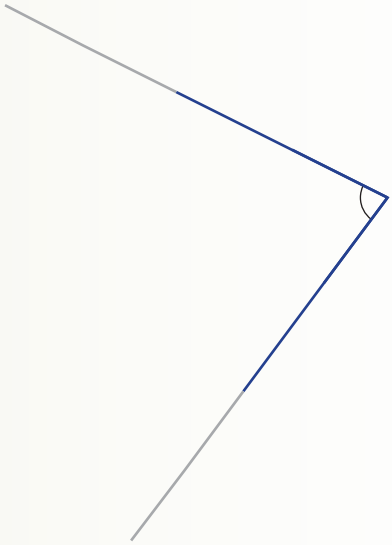
a between 0° and 90° **b** exactly 90° **c** between 90° and 180°
d exactly 180° **e** between 180° and 360° **f** exactly 360°

2 Measure these angles using a protractor.

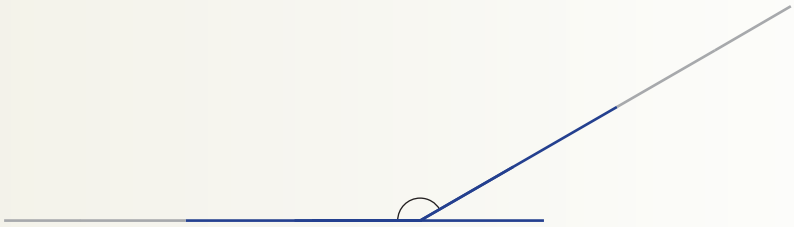
a



b



c

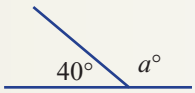


3 What is the complement of 65°?

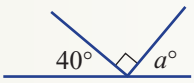
4 What is the supplement of 102°?

5 Find the value of a in each of the following angles.

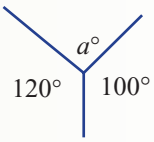
a



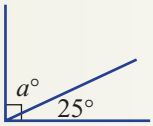
b



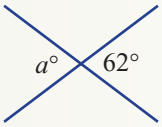
c



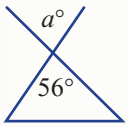
d



e

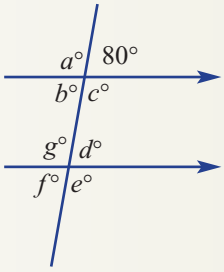


f

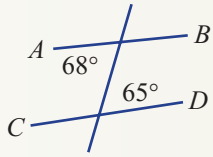


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6 Find the value of each pronumeral in the diagram.

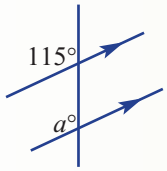


7 Explain why AB is *not* parallel to CD.

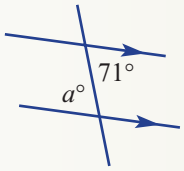


8 Find the value of a.

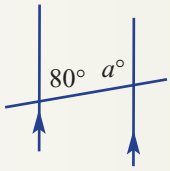
a



b



c



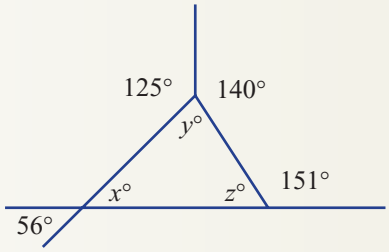
Extended-response questions

1 Consider the diagram shown.

a Find the value of:

i x **ii** y **iii** z

b What is the value of $x + y + z$?



Chapter 3: Computation with positive and negative integers

Multiple-choice questions

1 Which statement is incorrect?

A $-2 > -4$ **B** $0 < 5$ **C** $0 < -10$ **D** $-9 < -8$ **E** $-5 < 3$

2 $5 - 7$ is equal to:

A -2 **B** 2 **C** 12 **D** -12 **E** 0

3 $4 + (-2)$ is equal to:

A -6 **B** 2 **C** 6 **D** -2 **E** -4

4 $9 - (-3)$ is equal to:

A 13 **B** -12 **C** -6 **D** 6 **E** 12

5 The origin is the point at the centre of the Cartesian plane with coordinates:

A $(0, 1)$ **B** $(1, 1)$ **C** $(1, 0)$ **D** $(0, 0)$ **E** $(10, 10)$

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Short-answer questions

1 For each of the following, *insert* > (is greater than) or < (is less than).

a $-3 \square 3$ **b** $-4 \square -5$ **c** $-21 \square -4$

2 Find the value of:

a $4 - 5$ **b** $1 - 6$ **c** $0 - 10$

d $-4 - 2$ **e** $-7 - 11$ **f** $-37 - 40$

3 Find the value of:

a $2 + (-1)$ **b** $11 + (-7)$ **c** $5 + (-7)$

d $-1 + (-4)$ **e** $-10 + (-2)$ **f** $-31 + (-26)$

4 Find the value of:

a $3 - (-2)$ **b** $6 - (-1)$ **c** $11 - (-13)$

d $-4 - (-3)$ **e** $-13 - (-7)$ **f** $-11 - (-13)$

5 Copy and complete:

a $\square + 9 = -6$ **b** $4 - \square = 7$ **c** $-2 + \square = -10$

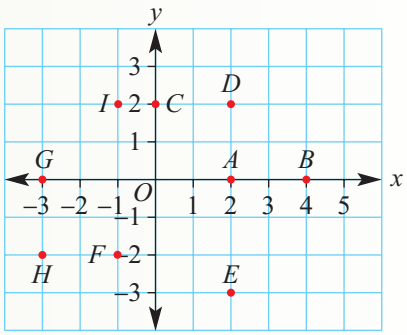
6 Evaluate the following.

a $-6 + (-4)$ **b** $6 - (-4)$ **c** $2 \times (-4 - 6)$

d $2 \times 6 - (-4)$ **e** $-4 + 6 \times 6$ **f** $4 \times 6 - (-4) + (-4)$

Extended-response questions

1 Consider this Cartesian plane.



a Give the coordinates of each point.

b Name any point(s) with a y-coordinate of zero. Where does each point lie?

c Find the distance between points:

i A and B **ii** D and E

d What shape is formed by joining the points $IDAG$?

e Decode: (2, 2), (2, –3), (0, 2), (–1, 2), (2, 2), (2, –3) using the letters on the Cartesian plane.

Chapter 4: Understanding fractions, decimals and percentages

Multiple-choice questions

1 Which of the following is equivalent to $\frac{12}{7}$?

A $\frac{24}{7}$ **B** $1\frac{5}{7}$ **C** $1\frac{5}{12}$ **D** $\frac{112}{17}$ **E** $\frac{7}{12}$

2 $80 + \frac{6}{10} + \frac{7}{100}$ is the same as:

A 8067 **B** 867 **C** 80.67 **D** 80.067 **E** 80.607

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3 $\frac{350}{450}$ in simplest form is:

A $\frac{35}{45}$ **B** $\frac{4}{5}$ **C** $\frac{3}{4}$ **D** $\frac{3.5}{4.5}$ **E** $\frac{7}{9}$

4 What fraction of \$2 is 40 cents?

A $\frac{1}{20}$ **B** $\frac{20}{1}$ **C** $\frac{5}{1}$ **D** $\frac{1}{5}$ **E** $\frac{1}{40}$

5 $\frac{3}{8}$ is the same as:

A 0.375 **B** 3.8 **C** 0.38 **D** 2.6 **E** 38%

Short-answer questions

1 Arrange $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{3}{10}$ in *ascending* order.

2 Express $5\frac{2}{3}$ as an improper fraction.

3 Are the following true or false?

a $0.5 = 50\%$ **b** $0.15 = \frac{5}{20}$ **c** $38\% = 0.19$

d $126\% = 1.26$ **e** $\frac{4}{5} = 0.08$ **f** $1\frac{3}{4} = 1.75$

4 Write 15% as a simple fraction.

5 Find 25% of \$480.

6 Find 20% of \$400.

7 Are the following true or false?

a To find 25% of an amount, divide the amount by 4.
b 10% of an amount = amount ÷ 10
c 20% of 50 = 2 × 10% of 50
d 1% of an amount = amount ÷ 100

8 Which is larger, $\frac{2}{3}$ or 60%?

Extended-response questions

1 Caleb's cold and flu prescription states: 'Take two pills three times a day with food.' The bottle contains 54 pills.

a How many pills does Caleb take each day?
b What fraction of the bottle remains after Day 1?
c How many days will it take for the pills to run out?
d If Caleb takes his first dose Friday night before going to bed, on what day will he take his last dose?

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Chapter 5: Probability

Multiple-choice questions

The following information is relevant for questions 1 and 2.
A survey asked 60 participants what is their favourite colour. The results are shown below.

Favourite colour	blue	pink	green	purple	black
No. of people	12	20	6	12	10

1 The number of people surveyed who listed purple as their favourite colour is:
A 12 **B** 20 **C** 6 **D** 60 **E** 10

2 Based on this survey, the experimental probability that a randomly selected person's favourite colour is black is:
A 1 **B** $\frac{1}{2}$ **C** $\frac{1}{6}$ **D** 10% **E** $\frac{1}{3}$

3 Each letter from the word MATHEMATICS is written on cards and shuffled. One card is chosen at random. The probability of choosing a vowel is:
A $\frac{3}{10}$ **B** $\frac{4}{10}$ **C** $\frac{3}{11}$ **D** $\frac{4}{11}$ **E** $\frac{5}{11}$

4 In the previous question, what is the probability of choosing the complement (i.e. choosing a consonant)?
A $\frac{7}{10}$ **B** $\frac{6}{10}$ **C** $\frac{8}{11}$ **D** $\frac{7}{11}$ **E** $\frac{6}{11}$

5 Which of the following could be used to describe an event that has a probability of 0.9?
A unlikely **B** likely **C** even chance **D** certain **E** impossible

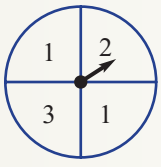
Short-answer questions

1 An 8-sided die has the numbers 1, 2, 3, 4, 5, 6, 7, 8 on its faces. It is rolled once. Find the following probabilities.
a $P(4)$ **b** P(odd number) **c** P(even and above 5)
d P(prime number) **e** P(not a prime number)

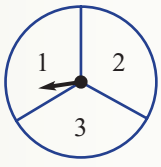
2 Consider the spinners A to D.



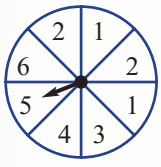
spinner A



spinner B



spinner C



spinner D

a Which spinner has the lowest probability of landing on the number 1 in a single spin?
b Which spinner has a 50% probability of landing on the number 1 in a single spin?
c What is the probability that spinner C will land on the number 2?

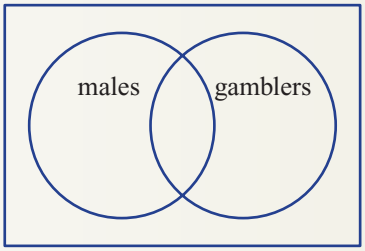
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3 A standard die is rolled.
- **a** What is the sample space?
- **b** What is the probability of rolling a 2?
- **c** What is the probability of rolling an odd number?

4 A group of people were asked two questions:
– Do you spend money on gambling activities?
– Are you male or female?
- **a** Copy the two-way table and fill in all the empty cells.

	Gambles	Never gambles	Total
Male			
Female	20	65	
Total	100	100	

- **b** How many males were surveyed?
- **c** How many people were surveyed?
- **d** Use the data in the table to complete the Venn diagram shown opposite.



5 In the game of SCRABBLE®, there are 100 tiles.

A 9		H 2		O 8		V 2	
B 2		I 9		P 2		W 2	
C 2		J 1		Q 1		X 1	
D 4		K 1		R 6		Y 2	
E 12		L 4		S 4		Z 1	
F 2		M 2		T 6			2
G 3		N 6		U 4			

If one tile is chosen at random, give the probability that it is:
- **a** blank
- **b** a vowel
- **c** not a vowel
- **d** worth 10 points
- **e** a letter that is able to be drawn with one stroke of a pen, without retracing or lifting the pen

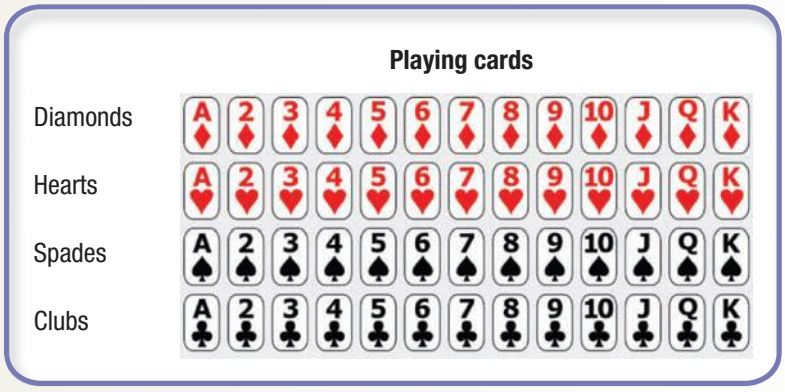
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Extended-response question

1 A standard deck of playing cards contains 13 cards of each suit: hearts, diamonds, clubs and spades. Each suit has an ace, king, queen, jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. One card is drawn at random from the deck.

Find the following probabilities.

a P(heart) **b** P(club) **c** P(diamond or spade)
d P(ace of hearts) **e** P(2 or 3) **f** P(king)
g P(ace or heart) **h** P(queen or club) **i** P(not a king)



Chapter 6: Computation with decimals and fractions

Multiple-choice questions

1 Which of the following is equivalent to $\frac{12}{7}$?

A $\frac{24}{7}$ **B** $1\frac{5}{7}$ **C** $1\frac{5}{12}$ **D** $\frac{112}{17}$ **E** $\frac{7}{12}$

2 Select the incorrect statement.

A $0.707 > 0.7$ **B** $0.770 = \frac{77}{100}$ **C** $0.07 \times 0.7 = 0.49$
D $0.7 \times \frac{1}{10} = 0.07$ **E** $0.7 \times 10 = 7$

3 The best estimate for 23.4×0.96 is:

A 234 **B** 230 **C** 0.234 **D** 23 **E** 20

4 $2\frac{1}{2} \div \frac{3}{4}$ is the same as:

A $\frac{5}{2} \times \frac{4}{3}$ **B** $\frac{5}{2} \times \frac{3}{4}$ **C** $\frac{2}{5} \div \frac{3}{4}$ **D** $\frac{2}{5} \times \frac{4}{3}$ **E** $\frac{3}{2} \times \frac{3}{4}$

5 $6.8 \div 0.4$ is the same as:

A $68 \div 4$ **B** $680 \div 4$ **C** 1.7 **D** $\frac{4}{68}$ **E** $7 \div 0.05$

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Short-answer questions

1 Find the value of:

a $\frac{1}{4}+\frac{1}{4}$ **b** $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ **c** $\frac{1}{4}+\frac{1}{2}$

2 Find the value of:

a $1-0.9$ **b** $4-2.9$ **c** 5×0.2

3 Find the value of:

a $\$3.50\times3$ **b** $\$20-\$8.25$ **c** $\$50-\$26.50$

4 Evaluate:

a $15-10.93$ **b** $19.7+240.6+9.03$ **c** $20-0.99$

d 0.6×0.4 **e** 0.3×0.3 **f** $\frac{12}{0.2}$

5 Find:

a $1.24-0.407$ **b** $1.2+1.8-0.6$ **c** 0.6×0.07

6 If $369\times123=45\,387$, write down the value of:

a 3.69×1.23 **b** 0.369×123 **c** 0.369×0.123

7 Find:

a 36.49×1000 **b** $1.8\div100$ **c** 19.43×200

8 Find each of the following.

a $\frac{2}{3}+\frac{1}{4}$ **b** $4-1\frac{1}{3}$ **c** $2\frac{1}{2}+3\frac{3}{4}$

d $\frac{2}{5}\times\frac{1}{2}$ **e** $\frac{2}{3}\div\frac{1}{6}$ **f** $1\frac{1}{5}\times\frac{5}{12}$

Extended-response question

1 The cost of petrol is 116.5 cents per litre.

a Find the cost of 5 L of petrol, correct to the nearest cent.

b Mahir pays cash for 5 L of petrol for his motorcycle. What is the amount that he pays, correct to the nearest 5 cents?

c How much change from $10 does Mahir receive?

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Chapter 7

Time

What you will learn

7A Units of time
7B Working with time
7C Using time zones

Strand: Measurement and Geometry

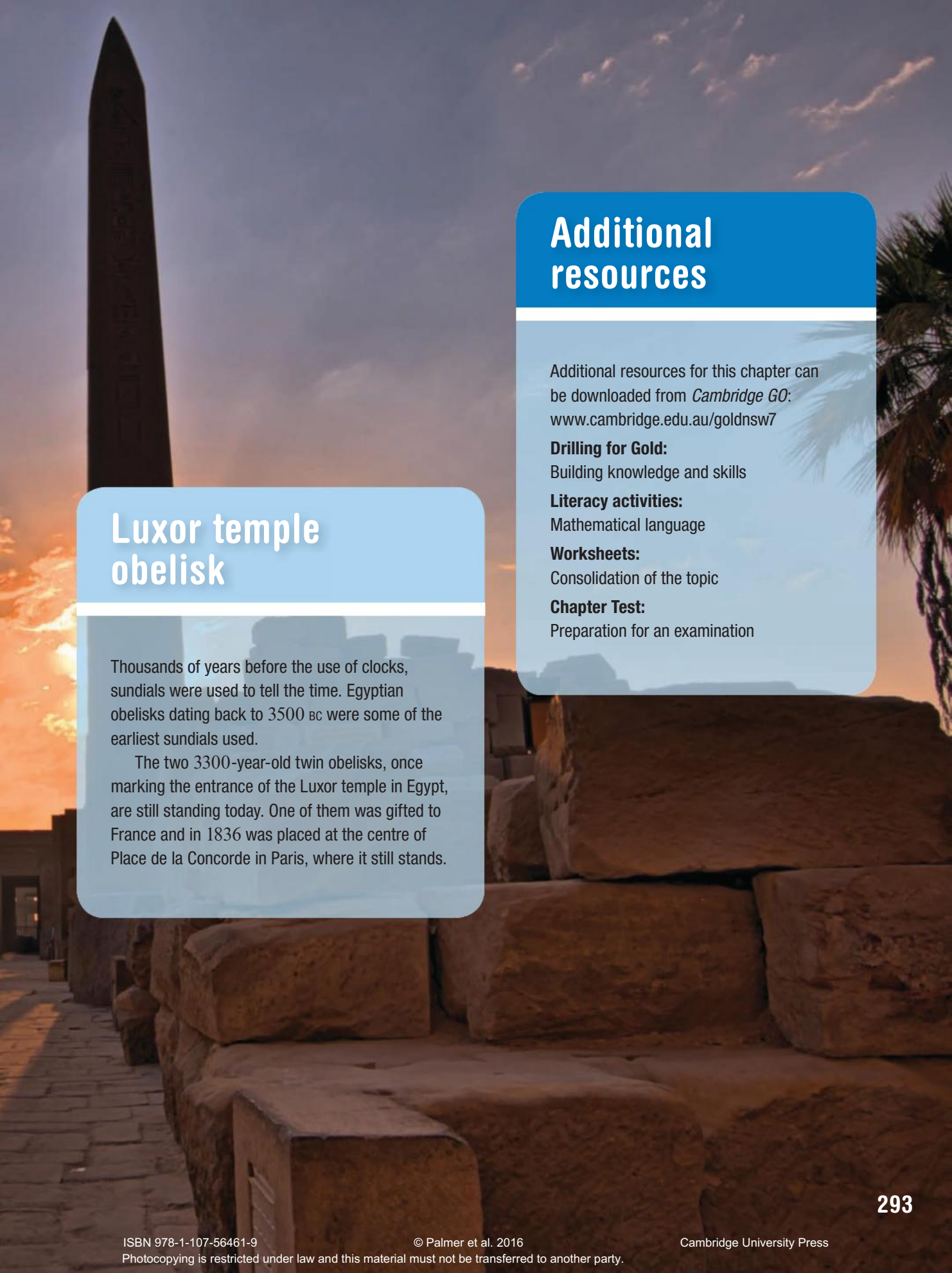
Substrand: TIME

In this chapter, you will learn to:

- perform calculations of time that involve mixed units
- interpret time zones.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Luxor temple obelisk

Thousands of years before the use of clocks, sundials were used to tell the time. Egyptian obelisks dating back to 3500 BC were some of the earliest sundials used.

The two 3300-year-old twin obelisks, once marking the entrance of the Luxor temple in Egypt, are still standing today. One of them was gifted to France and in 1836 was placed at the centre of Place de la Concorde in Paris, where it still stands.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 How many:

a hours in one day?
b seconds in one minute?
c minutes in one hour?
d days in one week?
e months in one year?
f days in one year (not a leap year)?

2 What day is it:

a 3 days after Tuesday?
b 6 days before Sunday?
c 3 weeks after Wednesday?
d 10 minutes after 11:55 p.m. Saturday?

3 Give the time, using a.m. or p.m., that matches these descriptions.

a 2 hours after 3 p.m.
b 1 hour before 2:45 a.m.
c 6 hours before 10:37 a.m.
d 4 hours after 4:49 p.m.
e $1\frac{1}{2}$ hours after 2:30 p.m.
f $3\frac{1}{2}$ hours before 7:15 p.m.
g 2 hours before 12:36 p.m.
h 5 hours after 9:14 a.m.

4 Convert the following to the units shown in brackets.

a 60 seconds (minutes)
b 120 minutes (hours)
c 49 days (weeks)
d 6 hours (minutes)

5 Melissa watched two movies on the weekend. One lasted 1 hour 36 minutes and the other lasted 2 hours 19 minutes.

a What was the total time Melissa spent watching movies, in hours and minutes?
b What was the total time in minutes?



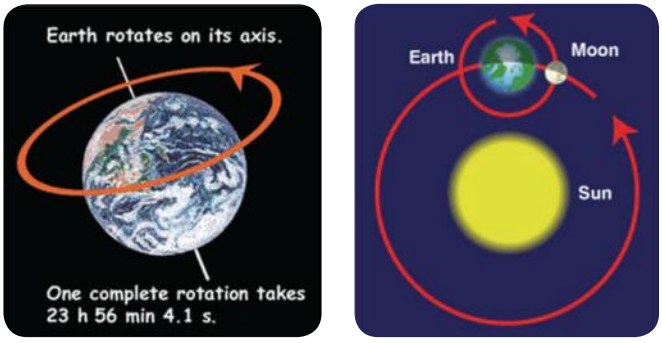
6 Write the following times as they would be displayed on a digital clock; e.g. 8:15.

a 3 o'clock
b half past 2
c noon
d ten to 9
e five to 12
f quarter to 6

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7A Units of time

Time is measured using several different units. There are seconds, minutes, hours, days, weeks, months, years etc. A year, which is approximately $365\frac{1}{4}$ days, is the length of time it takes for the Earth to orbit (i.e. rotate around) the Sun. A month is how long it takes for the Moon to orbit around the Earth. At the same time, the Earth is rotating on its axis. A full rotation of the Earth takes one day (approximately 24 hours).



It takes a year for the Earth to orbit the Sun.

The calendar we use today is called the Gregorian calendar

Number	1	2	3	4	5	6	7	8	9	10	11	12
Month	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Days	31	28/29	31	30	31	30	31	31	30	31	30	31

The following rhyme can help us remember how many days there are in each month.
Thirty days has September, April, June and November
All the rest have thirty-one, except for February alone
Which has twenty-eight days clear and twenty-nine in each leap year.

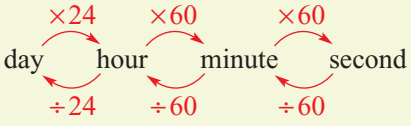
Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- When is the next leap year?
- Why do we have a leap year?
- Which months have 31 days?
- What do BC (or BCE) and AD (or CE) mean on time scales?

Key ideas

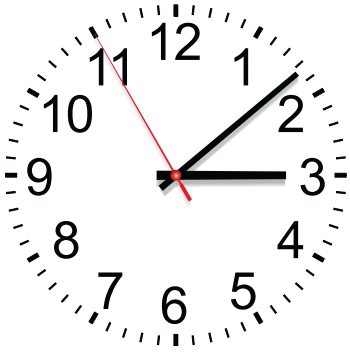
- Units of time include:
  - 1 **minute** (min) = 60 **seconds** (s)
  - 1 **hour** (h) = 60 **minutes** (min)
  - 1 **day** = 24 **hours** (h)
  - 1 **week** = 7 **days**
  - 1 **year** = 12 **months**
  - 1 **decade** = 10 **years**
  - 1 **century** = 100 **years**
  - 1 **millenium** = 1000 **years**



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- a.m. (ante meridiem) means before midday.
  p.m. (post meridiem) means after midday.
- **24-hour time** shows the number of hours and minutes after midnight.
  – 0330 is 3:30 a.m.
  – 1530 is 3:30 p.m.
- DMS conversion: Most calculators have a DMS (Degrees, Minutes and Seconds) button that converts time in fraction or decimal form to hours, minutes and seconds.
  – For example: 2.26 hours → 2°15′36″, meaning 2 hours, 15 minutes and 36 seconds.

Drilling for Gold 7A1 7A2



An analogue clock or watch has moving hands that rotate to tell time.



A digital clock or watch shows time using numbers that change as time passes.

Exercise 7A

Understanding

1 Which months of the year contain:
a 28 or 29 days? **b** 30 days? **c** 31 days?

2 From options **A** to **F**, match up the time units with the most appropriate description.

a single heartbeat	**A** 1 hour
b 40 hours of work	**B** 1 minute
c duration of Maths class	**C** 1 day
d time spent in Year 7	**D** 1 week
e 200-metre run	**E** 1 year
f flight from Australia to England	**F** 1 second

3 State whether you would multiply by 60 or divide by 60 when converting:
a 3 hours to minutes **b** 240 seconds to minutes
c 120 minutes to hours **d** 6 minutes to seconds



Refer back to the conversion diagram in Key ideas.



4 Find the number of:
a seconds in 2 minutes **b** minutes in 180 seconds
c hours in 120 minutes **d** minutes in 4 hours
e hours in 3 days **f** days in 48 hours
g weeks in 35 days **h** days in 40 weeks



Drilling for Gold 7A3

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Fluency

Example 1 Converting units of time

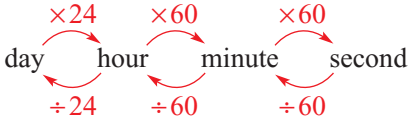
Convert these times to the units shown in brackets.

a 3 days (minutes) **b** 30 months (years)

Solution	Explanation
a $3\text{ days} = 3 \times 24 \times 60\text{ min}$ $= 4320\text{ min}$	1 day = 24 hours 1 hour = 60 minutes
b $30\text{ months} = 30 \div 12\text{ years}$ $= 2\frac{1}{2}\text{ years}$	There are 12 months in 1 year.

5 Convert these times to the units shown in brackets.

a 3 h (min) **b** 10.5 min (s)
c 240 s (min) **d** 90 min (h)
e 6 days (h) **f** 72 h (days)
g 1 week (h) **h** 1 day (min)
i 14 400 s (h) **j** 20 160 min (weeks)
k 2 weeks (min) **l** 24 h (s)
m 3.5 h (min) **n** 0.25 min (s)
o 36 h (days) **p** 270 min (h)
q 75 s (min) **r** 7200 s (h)



Example 2 Using 24-hour time

a Convert 4:30 p.m. to 24-hour time.
b Convert 1945 hours to 12-hour time.

Solution	Explanation
a $4.30\text{ p.m.} = 1200 + 0430$ $= 1630\text{ hours}$	Since the time is p.m., add 12 hours to 0430 hours.
b $1945\text{ hours} = 7{:}45\text{ p.m.}$	Since the time is after 1200 hours, subtract 12 hours.

6 Convert the following from 12-hour time to 24-hour time.

a 11:30 a.m. **b** 1:30 a.m. **c** 12 noon
d 1:30 p.m. **e** 11:30 p.m. **f** 11:59 p.m.



3 a.m. = 0300 h
noon = 1200 h
3 p.m. = 1500 h
8 p.m. = 2000 h

7 Convert the following from 24-hour time to 12-hour time.

a 0015 h **b** 0930 h **c** 1147 h
d 1430 h **e** 1945 h **f** 2330 h

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7A

Drilling for Gold 7A4

Example 3 Converting to hours, minutes and seconds

Convert the following.

a Convert (mentally) $4\frac{1}{3}$ hours to hours and minutes.

b Use a calculator to convert 6.42 hours to hours, minutes and seconds.

Solution	Explanation
a $4\frac{1}{3} = 4$ hours, 20 minutes	$\frac{1}{3}$ of an hour is 20 minutes because $\frac{1}{3}$ of $60 = 20$.
b $4.42 \rightarrow 4°25'12'' = 4$ hours, 25 minutes and 12 seconds	Use the DMS button on your calculator. Ensure your calculator is in Degree mode.



$\frac{1}{4}$ hour = 15 min
$\frac{1}{2}$ hour = 30 min
$\frac{3}{4}$ hour = 45 min

8 Convert the following to hours and minutes.

a $2\frac{1}{2}$ hours **b** $4\frac{1}{4}$ hours **c** $1\frac{1}{3}$ hours
d 6.5 hours **e** 3.75 hours **f** 9.25 hours

9 Use the DMS button on your calculator to convert these time durations to decimals.

a 3 hours 12 minutes **b** 3 hours 50 min **c** 3 hours 45 min

10 Use the DMS button on your calculator to convert the following to hours, minutes and seconds. For example, 6.21 hours = 6 h 12 min 36 s.

a 7.12 hours **b** 2.28 hours **c** 3.05 hours **d** 8.93 hours

Problem-solving & reasoning

11 Marion reads the following times on an airport display panel. What will be on the display 6 hours later?

a 0630 **b** 1425 **c** 1927



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12 When there are 365 days in a year, how many weeks are there in a year? Round your answer to 2 decimal places.

13 For how many days have you been alive?

14 Tobias says: "There are 4 weeks in every month, so 6 months is 24 weeks."
- **a** Explain why he is incorrect.
- **b** Give a more accurate answer.

15 **a** To convert from hours to seconds, what single number do you multiply by?
- **b** To convert from days to minutes, what single number do you multiply by?
- **c** To convert from seconds to hours, what single number do you divide by?
- **d** To convert from minutes to days, what single number do you divide by?


Refer back to the conversion diagram in Key ideas.

Enrichment: Very small units of time

16 The stopwatch on a mobile phone uses hundredths of a second.

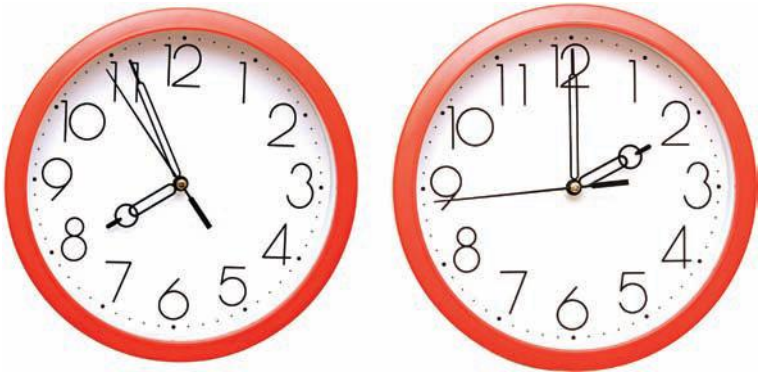


Use the internet to research the following.
- **a** How quickly can you start and then stop the stopwatch on a phone?
- **b** What is a millisecond and how many milliseconds are there are in 1 second?
- **c** How many nanoseconds are there are in 1 second?
- **d** What is a zeptosecond?
- **e** What is a yoctosecond?

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7B Working with time

In this section, addition and subtraction will be used to perform computations of time duration and time of day.



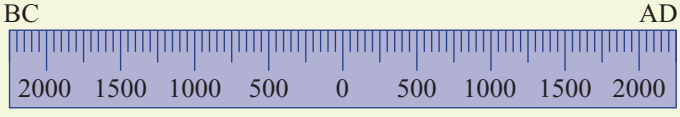
Let's start: What is your mental strategy?

Answer the following questions using a mental strategy. Compare your strategy with those used by other students.

- A movie begins at 1:45 p.m. and finishes at 4:10 p.m. What is the duration of the movie?
- A train leaves the city station at 8:40 a.m. and arrives in town $2\frac{1}{2}$ hours later. At what time does the train arrive?
- The construction of the Great Pyramid of Giza began in 2560 BC. How old is the pyramid now?

Key ideas

- Mental strategies can be used to do computations involving time.
  - The total time to build two models, which took 45 minutes and 55 minutes each, is $55 + 5 + 40 = 1$ hour 40 minutes.
  - The time duration of a taxi ride beginning at 2:50 p.m. and ending at 3:35 p.m. is $35 + 10 = 45$ minutes or $60 - 15 = 45$ minutes.
- The BC and AD timeline is similar to a number line in that it extends in both directions, with 0 representing the birth of Jesus Christ. The time duration from 500 BC to 2000 AD is therefore $500 + 2000 = 2500$ years.



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Exercise 7B

Understanding

1 State whether each of the following is true or false.
- **a** There are 60 seconds in 1 hour.
- **b** 12 noon is between morning and afternoon.
- **c** There are 35 minutes between 9:35 a.m. and 10:10 a.m.
- **d** There are 17 minutes between 2:43 p.m. and 3:10 p.m.
- **e** The total of 39 minutes and 21 minutes is 1 hour.

2 Find what is the time duration from:
- **a** 12 noon to 6:30 p.m.
- **b** 12 midnight to 10:45 a.m.
- **c** 12 midnight to 4:20 p.m.
- **d** 11 a.m. to 3:30 p.m.


You may find it easier to work in 24-hour time.

3 Add these time durations.
- **a** 1 h 30 min and 2 h 30 min
- **b** 4 h 30 min and 1 h 30 min
- **c** 2 h 15 min and 1 h 15 min
- **d** 6 h 15 min and 2 h 30 min
- **e** 3 h 45 min and 1 h 30 min
- **f** 4 h 45 min and 2 h 45 min

Fluency

Example 4 Calculating time duration and time difference


Drilling for Gold 7B1

- **a** Calculate the time duration from 4:35 p.m. to 9:10 p.m.
- **b** Calculate the time difference between 5 h 20 min and 9 h 5 min.

Solution	Explanation
a Time duration = 4 h + 25 min + 10 min = 4 h 35 min	There are 4 hours from 5 p.m. to 9 p.m., another 25 minutes before 5 p.m., and then 10 minutes after 9 p.m. Calculator method: 21°10′ – 16°35′ Note the use of 24-hour time.
b Time difference = 3 h 45 min	Mental method: 5 h 20 min to 6 h = 0 h 40 min 6 h to 9 h = 3 h 9 h to 9 h 5 min = 0 h 5 min 3 h 45 min Calculator method: 9°5′ – 5°20′ = 3 h 45 min

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7B



Convert to 24-hour time.

4 Mentally calculate these time intervals. Check your answers using a calculator.

a 2:40 a.m. to 4:45 a.m.
b 4:20 p.m. to 6:30 p.m.
c 1:50 p.m. to 5:55 p.m.
d 12:07 p.m. to 2:18 p.m.
e 6:40 a.m. to 8:30 a.m.
f 1:30 a.m. to 5:10 a.m.
g 10:35 p.m. to 11:22 p.m.
h 3:25 a.m. to 6:19 a.m.
i 6:18 a.m. to 9:04 a.m.
j 7:51 p.m. to 11:37 p.m.

5 Find what is the time:

a 4 hours after 2:30 p.m.
b 10 hours before 7 p.m.
c $3\frac{1}{2}$ hours before 10 p.m.
d $7\frac{1}{2}$ hours after 9 a.m.
e $6\frac{1}{4}$ hours after 11:15 a.m.
f $1\frac{3}{4}$ hours before 1:25 p.m.

6 Calculate the time difference between the following.
(You may wish to use the DMS or ° ′ ″ button on your calculator.)

a 2 h 40 min and 3 h 45 min
b 2 h 40 min and 3 h 20 min
c 2 h 40 min and 5 h 55 min
d 2 h 40 min and 7 h 10 min

7 For each of the following, add the time durations to find the total time. Give your answers in hours, minutes and seconds. Use a calculator where appropriate.

a $2\frac{1}{4}$ hours and $3\frac{1}{2}$ hours
b $5\frac{1}{3}$ hours and $2\frac{2}{3}$ hours
c 6.2 hours and 2.9 hours
d 0.3 hours and 4.2 hours
e 2 h 40 min 10 s and 1 h 10 min 18 s
f 10 h 50 min 18 s and 2 h 30 min 12 s

Problem-solving & reasoning

8 A ferry takes Selina from Cabarita to Circular Quay in 23 minutes and 28 seconds. The return trip takes 19 minutes and 13 seconds. What is Selina's total travel time? Check your answer using a calculator.



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Example 5 Using timetables

Use this train timetable for Bathurst to Penrith to answer the following.

Station	a.m.	p.m.
Bathurst	7:11	2:41
Lithgow	8:15	3:45
Bell	8:32	4:02
Mount Victoria	8:42	4:13
Katoomba	8:57	4:29
Springwood	9:29	5:01
Penrith	9:54	5:30

a How long does it take to travel from:
- **i** Bathurst to Lithgow in the morning?
- **ii** Lithgow to Penrith in the morning?
- **iii** Bathurst to Penrith in the afternoon?

b Luke travels from Lithgow to Bell in the morning and then from Bell to Katoomba in the afternoon. What is Luke's total travel time?

Solution	Explanation
a i 1 h 4 min	8:15 is 1 hour plus 4 minutes after 7:11.
ii 1 h + 45 min – 6 min = 1 h 39 min	1 hour and 45 minutes takes 8:15 to 10:00, so subtract 6 minutes to get 9:54.
iii 3 h – 11 min = 2 h 49 min	3 hours after 2:41 is 5:41, so subtract 11 minutes.
b 17 min + 27 min = 44 min	8:15 to 8:32 is 17 minutes, and 4:02 to 4:29 is 27 minutes. This gives a total of 44 minutes.

9 Use this train timetable to answer the following questions.

Station	a.m.	p.m.
Fairfield	7:32	2:43
Granville	7:44	2:56
Auburn	7:48	2:59
Ashfield	8:01	3:12
Redfern	8:11	3:23



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7B

a How long does it take to travel from:
- **i** Fairfield to Auburn in the morning?
- **ii** Granville to Redfern in the morning?
- **iii** Auburn to Redfern in the afternoon?
- **iv** Fairfield to Redfern in the afternoon?

b Does it take longer to travel from Fairfield to Redfern in the morning or afternoon?

c Jeremiah travels from Fairfield to Auburn in the morning and then from Auburn to Redfern in the afternoon. What is Jeremiah's total travel time?

10 It is believed that Newgrange (in Ireland) was built in 3200 BC. How old is it?



11 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

12 Adrian arrived at school at 8:09 a.m. and left at 3:37 p.m. How many hours and minutes was Adrian at school?

13 Janelle spent 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva. What was Janelle's total travel time?

Enrichment: Time challenges

14 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day

b per hour

c per minute

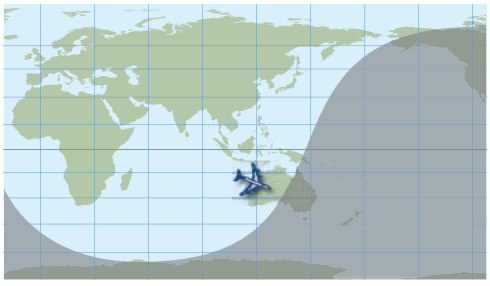
d per second (in cents)

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7C Using time zones

When the Sun sets at 6 p.m. in Sydney, it is still broad daylight in Perth and it is 'yesterday' in Alaska. To overcome these differences, the world has been divided into time zones.

In Australia, we use three main time zones: the Western, Central and Eastern Standard Time zones.



The shadow moves from right to left as the Earth rotates.

Let's start: Time zone discussion

In groups or as a class, discuss what you know about Australian and international time zones. You may wish to include:

- time zones and time differences within Australia
- the current time in other cities
- daylight saving
- the timing of telecasts of sporting events around the world
- jet lag.

Key ideas

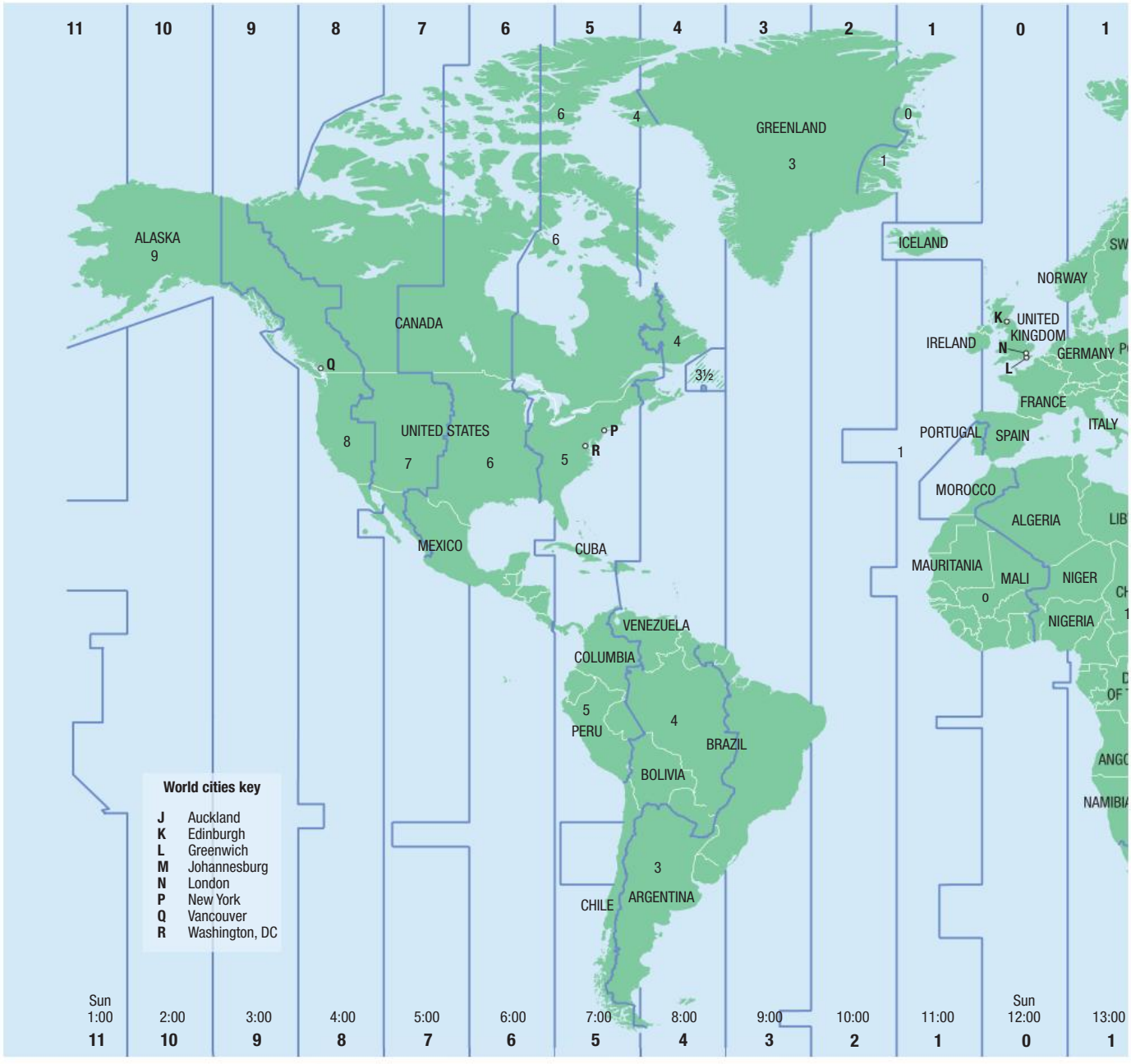
- The map on the following pages shows the time zones all around the world.
- Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.

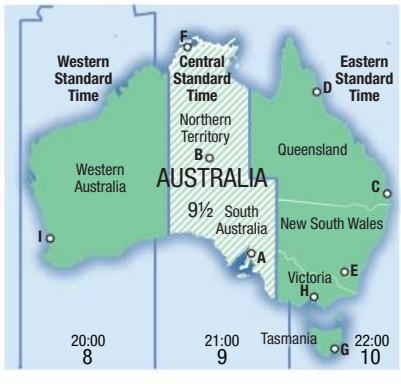
Perth (WA) (4 p.m.) $\xrightarrow{\text{add 2 hours}}$ Sydney (NSW) (6 p.m.)

Perth (WA) (4 p.m.) $\xleftarrow{\text{subtract 2 hours}}$ Sydney (NSW) (6 p.m.)

- Daylight saving involves moving clocks forward by one hour, to provide people with more daylight in the evening and less daylight in the morning.
- The International Date Line separates one calendar day from the next. So crossing the date line from west to east subtracts one day.

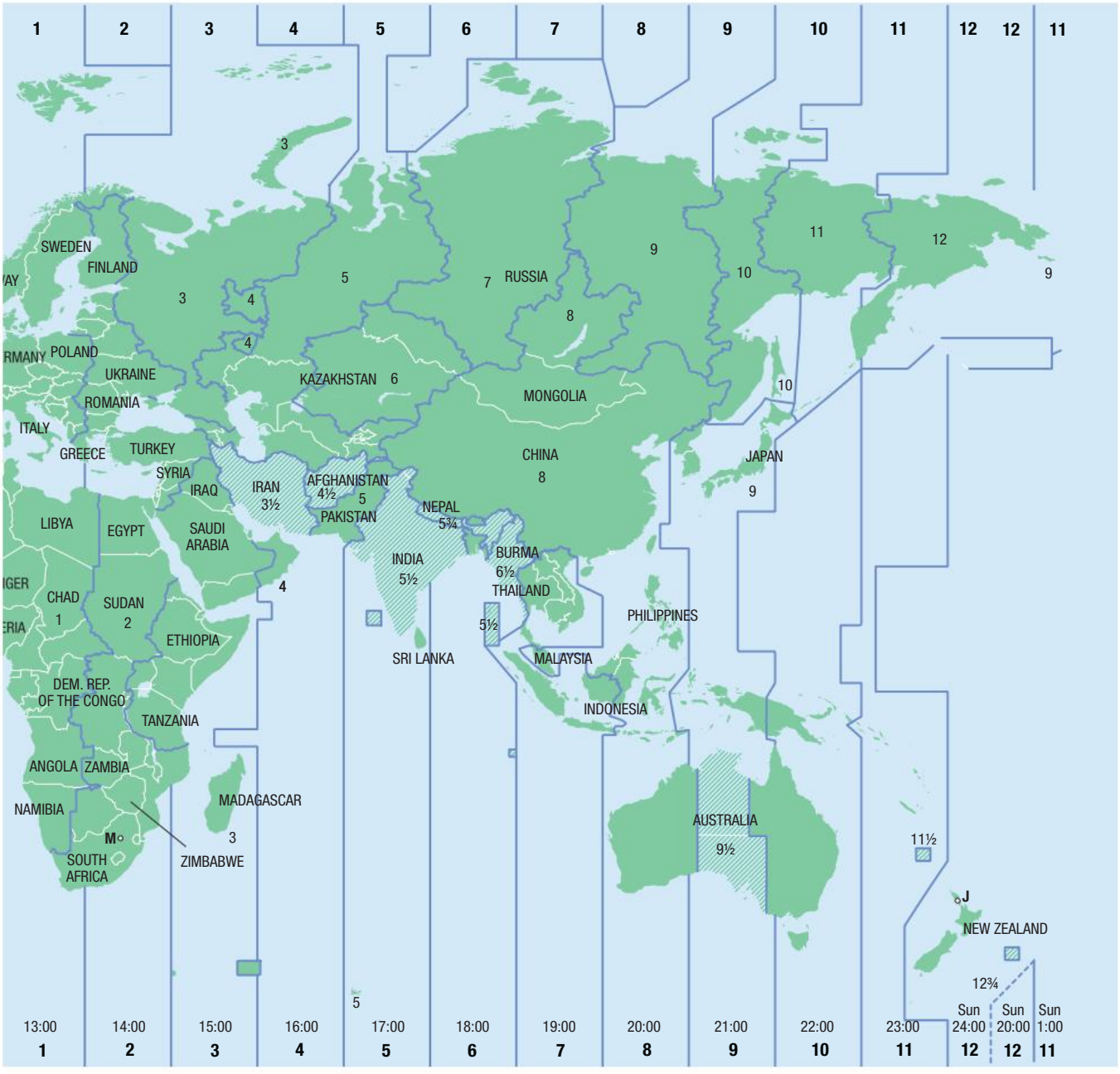
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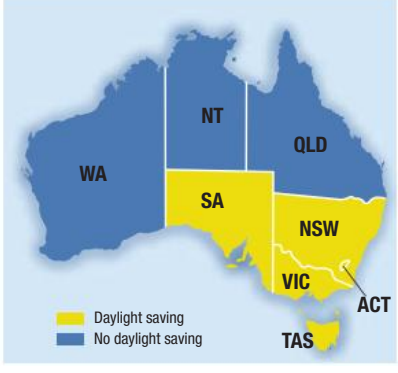




Australian cities key
A Adelaide
B Alice Springs
C Brisbane
D Cairns
E Canberra, ACT
F Darwin
G Hobart
H Melbourne
I Perth

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Exercise 7C

Understanding

1 Which Australian States and Territories use daylight saving time?

2 **a** How many hours in front of Coordinated Universal Time (UTC) are these countries and Australian States?

i Victoria, Australia **ii** South Australia **iii** Western Australia
iv Thailand **v** China **vi** Egypt

b How many hours behind Coordinated Universal Time (UTC) are the following countries?

i Iceland **ii** east Brazil
iii Columbia **iv** Peru

3 When it is 10 a.m. Monday in New Zealand, what day of the week is it in the USA?

Fluency

Example 6 Working with Australian time zones

Use the Australian standard time zones map (on the previous pages) to help with these questions.

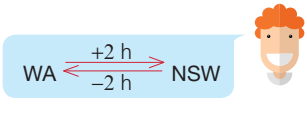
When it is 8:30 a.m. in New South Wales, what time is it in each of the following?

a Queensland **b** Northern Territory **c** Western Australia

Solution	Explanation
a 8:30 a.m.	Using standard time, NSW and Qld are in the same time zone.
b 8:00 a.m.	NT is UTC + $9\frac{1}{2}$ hours, whereas NSW is UTC + 10 hours. So NT is $\frac{1}{2}$ hour behind.
c 6:30 a.m.	WA is UTC + 8 hours and so is 2 hours behind NSW.

4 Use the Australian standard time zones map (on the previous pages) to help answer the following. When it is 10 a.m. in New South Wales, what time is it in these States and Territories?

a Victoria **b** South Australia
c Tasmania **d** Northern Territory
e Western Australia **f** The Australian Capital Territory (ACT)



5 Use the Australian standard time zones map (on the previous pages) to help answer the following. When it is 4:30 p.m. in Western Australia, what time is it in the following States?

a South Australia **b** New South Wales **c** Tasmania

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Example 7 Using time zones


Drilling for Gold 7C1

Coordinated Universal Time (UTC) is based on the time in Greenwich, United Kingdom. Use the world time zones map (on the previous pages) to answer the following.

a When it is 2 p.m. UTC, find the time in these places.
i France **ii** China **iii** Queensland **iv** Alaska

b When it is 9:35 a.m. in New South Wales, Australia, find the time in these places.
i Alice Springs **ii** Perth
iii London **iv** central Greenland

Solution	Explanation
a i 2 p.m. + 1 hour = 3 p.m.	Use the time zones map to see that France is to the east of Greenwich and is in a zone that is 1 hour ahead.
ii 2 p.m. + 8 hours = 10 p.m.	From the time zones map, China is 8 hours ahead of Greenwich.
iii 2 p.m. + 10 hours = 12 midnight	Queensland uses Eastern Standard Time, which is 10 hours ahead of Greenwich.
iv 2 p.m. – 9 hours = 5 a.m.	Alaska is to the west of Greenwich, in a time zone that is 9 hours behind.
b i 9:35 a.m. $-\frac{1}{2}$ hour = 9:05 a.m.	Alice Springs uses Central Standard Time, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 9:35 a.m. – 2 hours = 7:35 a.m.	Perth uses Western Standard Time, which is 2 hours behind Eastern Standard Time.
iii 9:35 a.m. – 10 hours = 11:35 p.m. (the day before)	UTC (time in Greenwich, United Kingdom) is 10 hours behind EST.
iv 9:35 a.m. – 13 hours = 8:35 p.m. (the day before)	Central Greenland is 3 hours behind UTC in Greenwich, so is 13 hours behind EST.

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7C

6 Use the time zones map (on the previous pages) to find the time in the following places, when it is 10 a.m. UTC.

a Spain **b** Turkey **c** Tasmania **d** Darwin
e Argentina **f** Peru **g** Alaska **h** Portugal

7 Use the time zones map (on the previous pages) to find the time in these places, when it is 3:30 p.m. in New South Wales.

a United Kingdom **b** Libya
c Sweden **d** Perth
e Japan **f** central Greenland
g Alice Springs **h** New Zealand
i China



Problem-solving and Reasoning

8 What is the time difference between these places?

a United Kingdom and Kazakhstan **b** South Australia and New Zealand
c Queensland and Egypt **d** Peru and Angola (in Africa)

9 Rick in NSW wants to watch a soccer match playing at 2 p.m. in England. What time will it be in NSW?

10 In London a rowing race is scheduled to begin at 11:35 a.m. What time will this be in Western Australia?

11 A 2-hour football match starts at 2:30 p.m. Eastern Standard Time (EST) in Newcastle, NSW. What time will it be in the United Kingdom when the match finishes?

12 It is 3 p.m. on 29 March in Perth. What is the time and date in these places?

a Italy **b** Alaska **c** Chile



13 Use the daylight saving time zones map to help answer the following. During daylight saving time, when it is 9:30 a.m. in Sydney, what time is it in the following States?

a Queensland **b** Victoria
c South Australia **d** Western Australia

14 During daylight saving time, Alice drives from New South Wales to Queensland. How will she need to adjust her watch when she crosses the border?

15 Elsa departs on an 11-hour flight from South Africa to Perth at 6:30 a.m. on 25 October. Give the time and date of her arrival in Perth.

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Enrichment: Time zone anomalies

16 There are a number of interesting anomalies associated with time zones. You may wish to use the internet to help explore these topics.

- **a** Usually, States and Territories to the east are ahead of those in the west. During daylight saving time, however, this is not true for all States in Australia. Can you find these States and explain why?
- **b** Broken Hill is in New South Wales but does not use the New South Wales time zone. Explore.
- **c** Does Lord Howe Island (part of New South Wales) use the same time as New South Wales all year round? Discuss.



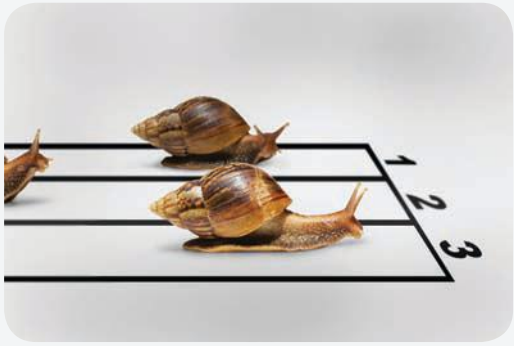
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Puzzles and games

1 Teaghan takes 7 hours to fly from Dubbo, New South Wales to Esperance, Western Australia. She departs at 7 a.m. What is the time in Esperance when she arrives? (Use Western Standard Time.)

2 When it is a Tuesday on 25 October in a particular year, what day will it be on 25 October in the following year, if it is not a leap year?

3 The average time for five snails to complete a race is 2 min 30 s. Four of the snails' race times are 2 min 20 s, 3 min, 2 min 10 s and 1 min 50 s. What is the fifth snail's race time?



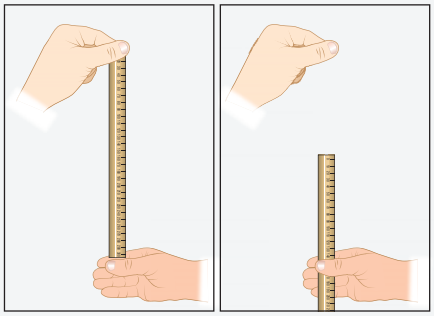
Investigating your reaction times

You will need:

- 30 cm ruler

What to do:

- One person holds the ruler vertically 30 cm from the catcher's open hand.
- The other person catches the falling ruler when it is released.
- Record the distance (in centimetres) that the ruler fell.



For example:

Trial	1	2	3	4	5	6	7	8	9	10	Mean
Maree right hand	12.5 cm	11 cm	13.5 cm								
Maree left hand											

- Repeat the experiment 10 times with the right hand and then 10 times with the left hand.
- Use a calculator to find the mean by adding all the measurements of the right hand together and dividing by 10. Find the mean of the left hand.
  - **a** With which hand did you respond fastest?
  - **b** Do boys react more quickly than girls?
  - **c** Did you get faster/slower with each turn or stay the same?
  - **d** Try this at home with your family or caregivers to see if young people react more quickly than older people.

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Chapter summary

Conversion

× 24 × 60 × 60
day hour minute second
÷ 24 ÷ 60 ÷ 60

24-hour time

9:26 a.m. is 0926
5:03 p.m. is 1703

Time zones

UTC (Coordinated Universal Time) is the time in Greenwich, England (United Kingdom).
- Countries to the east are ahead
- Countries to the west are behind

Time

DMS conversion

1.75 hours ↔
1 hour 45 min

Australian time zones

State/ Territory	Standard	Daylight saving
WA	+ 8	+ 8
NT	+ $9\frac{1}{2}$	+ $9\frac{1}{2}$
SA	+ $9\frac{1}{2}$	+ $10\frac{1}{2}$
Tas.	+ 10	+ 11
Vic.	+ 10	+ 11
NSW	+ 10	+ 11
ACT	+ 10	+ 11
Qld	+ 10	+ 10

Mental strategies

Finding duration
4:35 to 6:44 = 2 h + 9 min
6:19 to 8:07 = 2 h – 12 min
= 1 h 48 min

Addition

2 h 26 min 12 s + 3 h 24 min + 56 s
= 5 h + 50 min + 68 s
= 5 h 51 min 8 s

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Chapter review

Additional consolidation and review material, including literacy activities worksheets and a chapter test, can be downloaded from *Cambridge GO.*

Multiple-choice questions

1 The number of minutes in 3 hours is:
A 180 **B** 60 **C** 90 **D** 300 **E** 200

2 How many years are there in 48 months?
A 2 **B** 2.5 **C** 2 **D** 3.5 **E** 4

3 When written using 24-hour time, 4:26 p.m. is:
A 0626 **B** 1226 **C** 0426 **D** 1426 **E** 1626

4 Converting $2\frac{3}{4}$ hours to hours and minutes gives:
A 2 h 75 min **B** 2 h 34 min **C** 2 h 23 min
D 2 h 45 min **E** 2 h 30 min

5 Converting 2.64 hours to hours, minutes and seconds gives:
A 2 h 40 min 12 s **B** 2 h 38 min 24 s **C** 3 h 4 min 0 s
D 2 h 30 min 10 s **E** 2 h 60 min 4 s

6 The time taken to make and assemble two chairs is 3 hours 40 minutes and 15 seconds and 2 hours 38 minutes and 51 seconds. Hence, the total build time is:
A 5 h 58 min 6 s **B** 6 h 20 min 6 s **C** 6 h 19 min 6 s
D 6 h 19 min 66 s **E** 6 h 18 min 6 s

7 The time difference between 3:36 a.m. and 4:27 a.m. is:
A 51 min **B** 49 min **C** 41 min **D** 39 min **E** 61 min

8 How many hours is Western Australia behind New South Wales during winter?
A 5 **B** 4 **C** 3 **D** 2 **E** 1.5

9 If it is noon during daylight saving time in South Australia, what time is it in Queensland?
A 2 p.m. **B** 2:30 p.m. **C** 1 p.m.
D 12:30 p.m. **E** 11:30 a.m.

10 When it is 4 a.m. UTC, the time in Sydney is:
A 1:30 p.m. **B** 1 p.m. **C** 2 p.m. **D** 3 p.m. **E** 3 a.m.

Short-answer questions

1 Convert these times to the units shown in brackets.
a $1\frac{1}{2}$ h (min) **b** 120 s (min) **c** 48 h (days)
d 3 weeks (days) **e** 1 day (min) **f** 1800 s (h)

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Chapter review

2 Re-write these times, using the system shown in brackets.

a 4 a.m. (24-hour time)
b 3:30 p.m. (24-hour time)
c 7:19 p.m. (24-hour time)
d 0635 (a.m./p.m.)
e 1251 (a.m./p.m.)
f 2328 (a.m./p.m.)

3 Re-write these times, using hours and minutes.

a $3\frac{1}{2}$ hours **b** $4\frac{1}{3}$ hours **c** 6.25 hours **d** 1.75 hours

4 Use the DMS button on your calculator to convert the following to hours, minutes and seconds.

a 3.6 hours **b** 6.92 hours **c** 11.44 hours

5 Michael is catching a train leaving at 1330 in London and arriving at 1503 in York. What will be Michael's travel time?



6 Calculate the time difference. Give your answer in hours, minutes and seconds.

a 7:43 a.m. to 1:36 p.m.
b 2 h 30 min 10 s to 6 h 36 min 5 s
c 5 h 52 min 6 s to 7 h 51 min 7 s
d 0931 to 1309

7 Use this train timetable for Telarah to Newcastle to answer the following questions.

Station	a.m.	p.m.
Telarah	7:30	2:52
Metford	7:42	3:04
Sandgate	7:55	3:16
Hamilton	8:10	3:30
Newcastle	8:16	3:36

a How long does it take to travel from:
i Telarah to Sandgate in the morning?
ii Metford to Newcastle in the morning?
iii Sandgate to Newcastle in the afternoon?
iv Telarah to Newcastle in the afternoon?

b Does it take longer to travel from Telarah to Newcastle in the morning or afternoon?

c Ashdi travels from Telarah to Sandgate in the morning, then from Sandgate to Newcastle in the afternoon. What is Ashdi's total travel time?

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Chapter review

8 Use the Australian time zones map to help answer these questions.

a During winter it is 7:45 a.m. in South Australia. What time is it in:

i New South Wales? **ii** Western Australia?

b During Australian daylight saving time it is 4:36 p.m. in New South Wales. What time is it in:

i Western Australia? **ii** Queensland?

9 An AFL match telecast begins at 2:10 p.m. Eastern Standard Time. At what time will someone in the Northern Territory need to switch on the television if they want to watch the game?



10 How many days are there in these months?

a April **b** May **c** June

d July **e** August **f** September

Extended-response questions

1 Use the world time zones map to answer these questions.

a When it is 11 a.m. UTC, state the time in:

i Sydney **ii** Ethiopia **iii** Pakistan

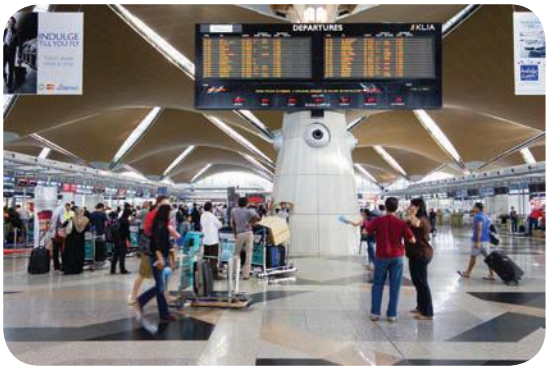
b When it is 3:30 p.m. in New South Wales, state the time in:

i Zimbabwe **ii** China **iii** Bolivia

c When it is 6 a.m. Tuesday in New South Wales, state the day of the week in:

i India **ii** Canada **iii** Hawaii

d Chris flies from Sydney, leaving at 8 a.m., and travels for 7 hours, arriving in Kuala Lumpur, Malaysia. What is the time in Kuala Lumpur when he arrives?



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Chapter review

2 It takes Scott 8 minutes to walk from his house in Cabarita to the ferry wharf. He likes to be at the wharf 5 minutes before the ferry arrives. The timetable below shows the ferries he could catch to the city.

Cabarita	6:12 a.m.	6:43 a.m.	6:48 a.m.	7:13 a.m.	7:18 a.m.	7:33 a.m.	7:43 a.m.	7:48 a.m.
Darling Harbour	6:40 a.m.	–	7:23 a.m.	–	7:53 a.m.	8:08 a.m.	–	8:23 a.m.
Circular Quay	–	7:14 a.m.	–	7:44 a.m.	–	–	8:14 a.m.	–

On Monday, Scott has a meeting in Martin Place at 8 a.m. He is going to catch a ferry from Cabarita to either Darling Harbour or Circular Quay. If he goes to Darling Harbour, it will take him 15 minutes to walk to Martin Place. If he goes to Circular Quay, the walk will take 20 minutes.

a How long does it take the 6:12 a.m. ferry to reach Darling Harbour?

b Copy and complete the following times, assuming Scott catches the 7:18 a.m. ferry.

Leave home at	
Arrive at ferry wharf	
Ferry leaves at	7:18 a.m.
Ferry arrives at	
Walk to Martin Place by	

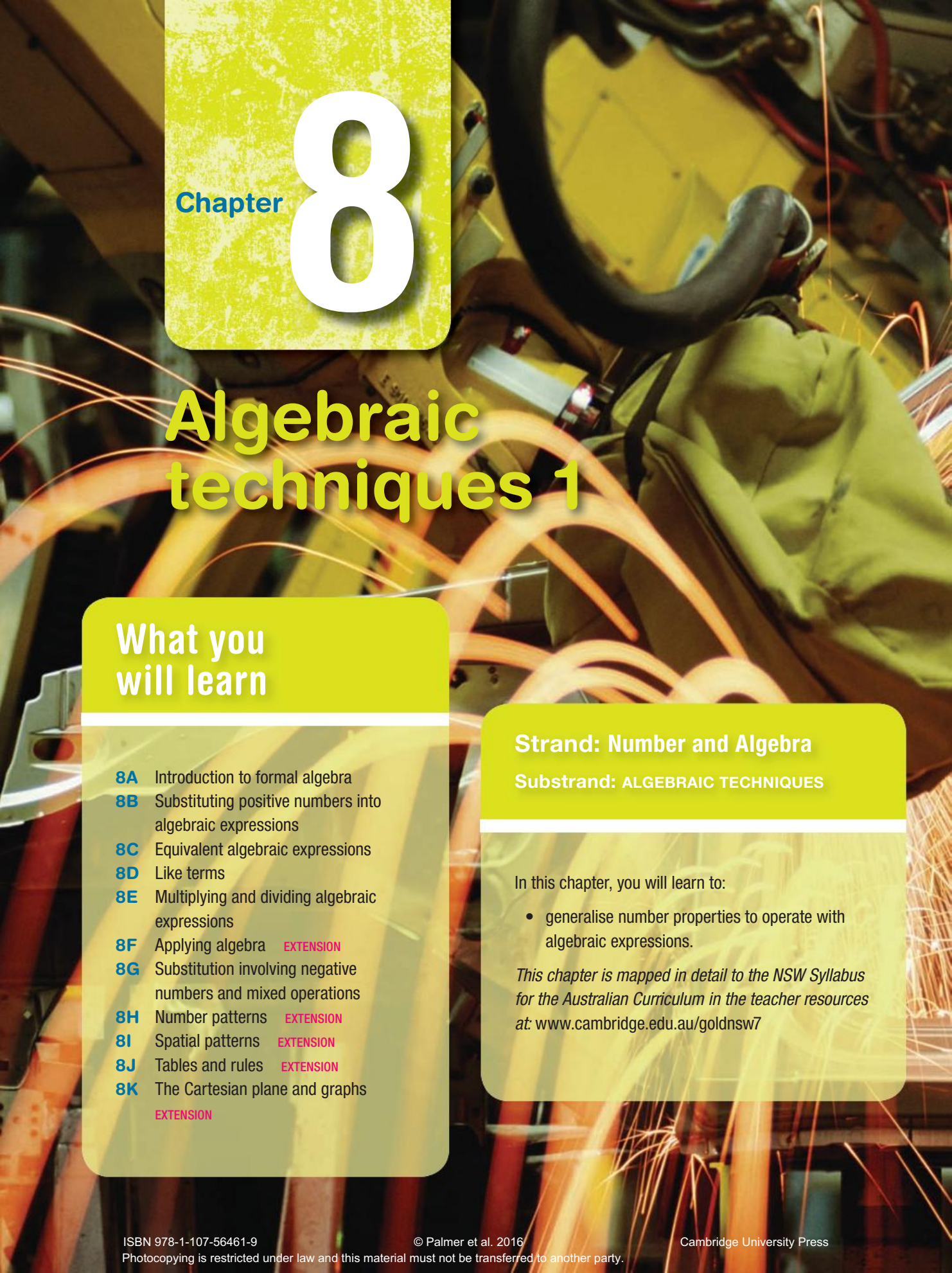
c **i** What is the latest ferry Scott can catch and still make it to the meeting on time?

ii Copy and complete the following details for that trip.

Leave home at	
Arrive at ferry wharf	
Ferry leaves at	
Ferry arrives at	
Walk to Martin Place by	



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Chapter 8

Algebraic techniques 1

What you will learn

8A Introduction to formal algebra
8B Substituting positive numbers into algebraic expressions
8C Equivalent algebraic expressions
8D Like terms
8E Multiplying and dividing algebraic expressions
8F Applying algebra EXTENSION
8G Substitution involving negative numbers and mixed operations
8H Number patterns EXTENSION
8I Spatial patterns EXTENSION
8J Tables and rules EXTENSION
8K The Cartesian plane and graphs EXTENSION

Strand: Number and Algebra

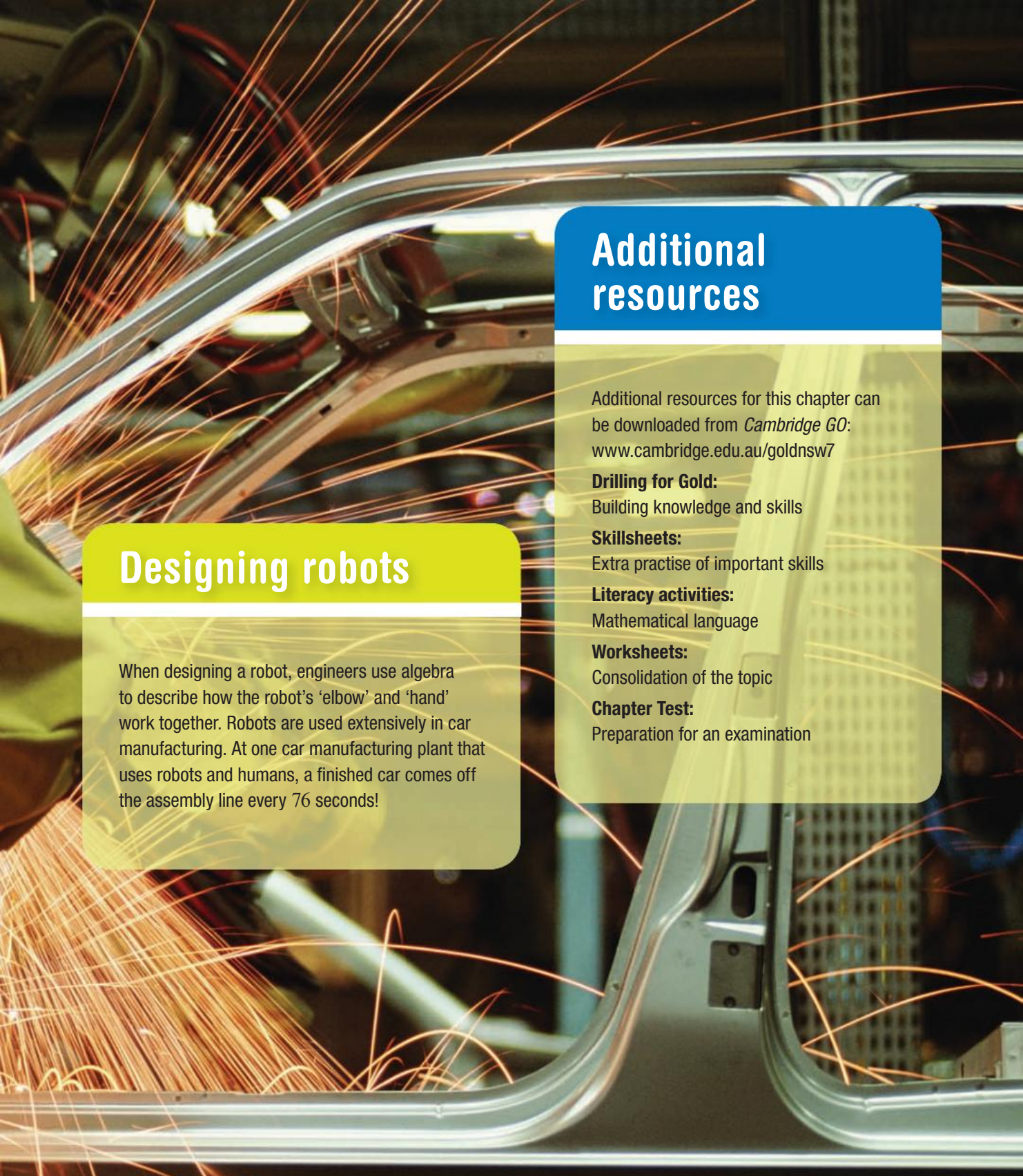
Substrand: ALGEBRAIC TECHNIQUES

In this chapter, you will learn to:

- generalise number properties to operate with algebraic expressions.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Designing robots

When designing a robot, engineers use algebra to describe how the robot's 'elbow' and 'hand' work together. Robots are used extensively in car manufacturing. At one car manufacturing plant that uses robots and humans, a finished car comes off the assembly line every 76 seconds!

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Find the value of:

a $3 + 6$ **b** $12 - 5$

c 12×4 **d** $20 \div 5$

2 Find the result when:

a 4 and 9 are added **b** 3 is multiplied by 7

c 12 is divided by 3 **d** 10 is halved

3 If $\square = 7$, write the value of each of the following.

a $\square + 4$ **b** $\square - 2$

c $12 - \square$ **d** $3 \times \square$

4 Write the value of $\square \times 4$ if:

a $\square = 2$ **b** $\square = 9$

c $\square = 10$ **d** $\square = 2.5$

5 If $\blacktriangle = 10$, write the value of:

a $\blacktriangle + 7$ **b** $\blacktriangle - 2$

c $\blacktriangle \div 2$ **d** $\blacktriangle + \blacktriangle$

6 Find the value of each of the following.

a $4 \times 3 + 8$ **b** $4 \times (3 + 8)$

c $4 \times 3 + 2 \times 5$ **d** $4 \times (3 + 2) \times 5$

7 Find the value of each of the following.

a $50 - (3 \times 7 + 9)$ **b** $24 \div 2 - 6$

c $24 \div 6 - 2$ **d** $24 \div (6 - 2)$

8 If $\square = 5$, write the value of each of the following.

a $\square - 4$ **b** $\square \times 2 - 1$

c $\square \div 5 + 2$ **d** $\square \times 7 + 10$

9 Find the value of each of the following.

a 5 more than 12 **b** The sum of 8 and 6

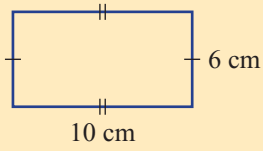
c 4 added to 7 **d** 12 less than 20

e 15 subtracted from 22 **f** double 9

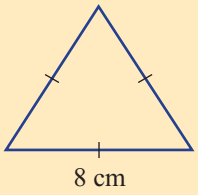
g The product of 7 and 5 **h** 4 is tripled, then 5 is added

10 Find the perimeter of each of these shapes.

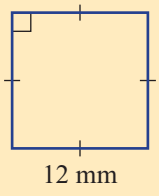
a



b



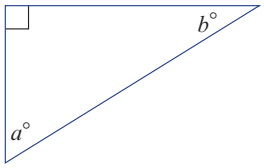
c



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8A Introduction to formal algebra

Letters can be used to stand for unknown numbers. In Chapter 2, letters were used to represent the number of degrees in angles. For example, in the diagram opposite the values of a and b could be 60 and 30 or 70 and 20.



Let's start: Pick any number and try this!

Choose a number between 1 and 10.
Multiply it by 2.
Increase the result by 8.
Divide the result by 2.
Subtract the number you chose.

- What did you get?
- Ask your friends what they got.
- Try it with a number bigger than 10.

Algebra can be used to explain why tricks like this work for *all* numbers.



Key ideas

- A list of all the terminology used in this chapter is provided on the following page.
- Algebra is used to describe the rules and conventions of numbers and arithmetic.
- When doing algebra, we use the equals symbol (i.e. =) to indicate that two or more things have exactly the same value. This is called an **identity** because they are identical.

  For example: $3 + 5$ gives 8, which is the same as $5 + 3$.
  So we can write $3 + 5 = 5 + 3$.

  '$=$' 'is identical to' or 'has the same value as'

- In the following table, the letters a, b and c could represent any numbers.

Drilling for Gold 8A1

	Number fact	Algebra fact	Notes
1	$5 + 3 = 3 + 5$	$a + 3 = 3 + a$	Numbers can be added in any order.
2	$5 + 3 = 3 + 5$	$a + b = b + a$	
3	$4 + 5 + 1 = 1 + 4 + 5$	$b + c + a = a + b + c$	
4	$-3 + 5 = 5 - 3$	$-3 + a = a - 3$	The negative sign 'belongs' to the number 3.
5	$5 \times 3 = 3 \times 5$ Numbers can be multiplied in any order.	$a \times 3 = 3 \times a$	The multiplication symbol is usually 'invisible'. $a \times 3 = 3 \times a = 3a$

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	Number fact	Algebra fact	Notes
6	$5 \times 3 = 3 \times 5$	$a \times b = b \times a$	This can be written as: $a \times b = ab$
7	$5 + 5 + 5 = 3 \times 5$ meaning '3 lots of 5'.	$a + a + a = 3 \times a$	This can be written as: $a + a + a = 3a$
8	$2 \div 8 = \frac{2}{8}$	$a \div 8 = \frac{a}{8}$	Division and fractions are related to each other. The first number in the division is the numerator in the fraction.
9	$2 \div 8 = \frac{2}{8}$	$a \div b = \frac{a}{b}$	



Drilling for Gold 8A2

Terminology	Example	Definition
pronumeral	a or b or c	A letter of an alphabet or a symbol used to represent one or more numerical values
variable	a or b or c	A pronumeral that represents more than one numerical value
expression or algebraic expression	$3a + 5$	A statement containing numbers and pronumerals that are connected by mathematical operations but containing no equals sign
term	The expression $3a + 5$ contains two terms.	One of the components of an expression
like terms	$3a$ and $5a$ are like terms. $3a$ and $5a^2$ are *not* like terms.	Two or more terms that contain the same pronumerals
constant or constant term	In the expression $3a + 5$, the number 5 is called the constant or the constant term.	The part of an expression without any pronumerals
coefficient	In the expression $3a + b + 5$: • The coefficient of a is 3. • The coefficient of b is 1.	A numeral placed before a pronumeral to indicate that the pronumeral is multiplied by that factor
equivalent expressions	$3a + 5$ and $5 + 3a$	Expressions that will always have the same numerical value as each other when the pronumerals are replaced with any number
simplify	$3a + 5a$ simplifies to $8a$. $3a + 5$ can't be simplified.	To find the simplest possible equivalent expression

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Terminology	Example	Definition
identity	$3a + 5 = 5 + 3a$ or $3a + 5 \equiv 5 + 3a$	A statement that indicates that two expressions will have the same numerical value when the pronumerals are replaced with numbers. The symbol $\equiv$ is sometimes used in identities.
substitute	When $a = 3$, $a + 5$ becomes $3 + 5$.	To replace pronumerals with numerical values
substitution	When $a = 3$, the value of $a + 5$ is 8.	A process in which pronumerals are replaced with numbers
evaluate	Evaluate $a + 5$ when $a = 3$.	To calculate the numerical value of an expression in which all the pronumerals have been given a value

Exercise 8A

Understanding



Drilling for Gold 8A3

1 The simplest way to write $7 \times a$ is:

A $a7$ **B** $7a$ **C** a^7 **D** $\frac{7}{a}$



In algebra, $5 \times b$ is written $5b$.

2 Another way to write $5 \div m$ is:

A $\frac{m}{5}$ **B** $m5$ **C** $\frac{5}{m}$ **D** $5m$

Example 1 Listing terms

List the individual terms in the expression $3a + b + 13$.

Solution	Explanation
There are three terms: $3a$, b and 13.	Each part of the expression is a term.

3 List the terms in each of the following.

a $2x + 7y$ **b** $3a + 2c + e$ **c** $5q + 3r + 2s$ **d** $7d + 5f + 17$

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8A

Example 2 Stating coefficients

What is the coefficient of each pronumeral in the expression $3a + 2b + 13c$?

Solution	Explanation
The coefficient of a is 3. The coefficient of b is 2. The coefficient of c is 13.	The coefficient is the number in front of a pronumeral.

4 **a** What is the coefficient of x in $3x + 5y$?
b What is the coefficient of b in $7a + 13b + 5c$?
c What is the coefficient of k in $10 + 4k$?

Fluency

5 For each of the following expressions, state:
i the number of terms
ii the coefficient of n



Terms can be added or subtracted to form an expression.

a $17n + 24$
b $31 + 27a + 15n$
c $15nw + 21n + 15$
d $15a - 32b + 2n + 4xy$
e $n + 51$
f $5bn - 12 + \frac{d}{5} + 12n$

Example 3 Writing expressions from word descriptions

Write an expression for the following.
a 5 more than k
b 3 less than m
c the sum of a and b
d double the value of x
e the product of c and d

Solution	Explanation
a $k + 5$ or $5 + k$	5 must be added to k to get 5 more than k.
b $m - 3$	3 is subtracted from m.
c $a + b$ or $b + a$	a and b are added to obtain their sum.
d $2 \times x$ or just $2x$	x is multiplied by 2. The multiplication sign is optional.
e $c \times d$ or just cd or dc	c and d are multiplied to obtain their product.



Drilling for Gold 8A4, 8A5

6 Write an expression for the following.
a 3 more than x
b the sum of k and 5
c 2 is added to b
d 3 less than g
e 4 is subtracted from H
f 6 is subtracted from M

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7 Write an expression for the following without using $\times$ or $\div$.

a double the value of u
b 4 lots of y
c 3 is multiplied by x
d the product of k and 10
e y is divided by 8
f half of z
g a is tripled, then 4 is added
h p is doubled, then 12 is added


'Product' tells you to use multiplication. The product of 3 and 10 is $3 \times 10 = 30$.

Problem-solving and Reasoning

8 In a room there are k people, and then 5 people leave. How many people are now in the room?

A $k + 5$ **B** 5 **C** $k - 5$ **D** $5k$

9 Nicholas buys 10 lolly bags from a supermarket.

a If there are 7 lollies in each bag, how many lollies does he buy in total?
b If there are n lollies in each bag, how many lollies does he buy in total?


For part **b**, write an expression involving n.



10 Mikayla is paid $\$x$ per hour at her job. Write an expression for each of the following.

a How much does Mikayla earn if she works 8 hours?
b If Mikayla gets a pay rise of \$3 per hour, what is her new hourly wage?
c If Mikayla works for 8 hours at the increased hourly rate, how much does she earn?

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8A

11 Read 'Let's start' again (see page 321).
Copy and complete this table.

Words	Algebraic expression
Choose a number	n
a Multiply it by 2.	
b Increase the result by 8.	
c Divide the result by 2.	
d Subtract the number you chose.	
e What did you get?	

12 If b is an even number greater than 3, are these statements true or false?

a $b + 1$ must be even.
b $b + 2$ could be odd.
c $5 + b$ could be greater than 10.
d $5b$ must be greater than b.
e $2b$ must be greater than 10.
f $\frac{b}{2}$ is a whole number.

Enrichment: Restaurant algebra

13 A group of people go out to a restaurant, and the total amount they must pay (in dollars) is A. They split the bill equally. Write expressions to answer the following questions.

a If there are 4 people in the group, how much do they each pay?
b If there are n people in the group, how much do they each pay?
c One of the n people has a voucher that reduces the total bill by \$20.
  i How much does each person pay now?
  ii If the bill is \$200 and $n = 6$, how much does each person end up paying?



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8B Substituting positive numbers into algebraic expressions

To evaluate an expression, replace the pronumerals (such as x and y) with numbers to get a final result. For example, when $a = 11$ we can evaluate $4 + a$ to get 15. This is called substitution.



Let's start: Sum to 10

The pronumerals x and y could stand for any number.

- When $x = 3$ and $y = 7$, what is $x + y$?
- When $x + y = 10$, what other numbers could x and y be? Try to list as many pairs as possible.
- Can you make $x \times y$ equal 10?

Key ideas

- To **evaluate** an expression or to **substitute** values, replace each pronumeral in an expression with a number to obtain a final value.
  For example, if $x = 3$ and $y = 8$, evaluating $x + 2y$ gives $3 + 2 \times 8 = 19$.
- A term like $4a$ means $4 \times a$. When substituting a number we must include the multiplication sign, since the number 42 is very different from the product 4×2.

Evaluate To calculate the numerical value of an expression in which all the pronumerals have been given a value

Substitute To replace pronumerals with numerical values

Exercise 8B

Understanding

1 When ★ = 10, find the value of:
a ★ + 5 **b** ★ − 2 **c** ★ × 2 **d** ★ ÷ 2

2 When □ = 4, find the value of:
a □ + 2 **b** 7 × □ **c** □ − 3 **d** □ ÷ 2

3 When ▲ = 3, find the value of:
a ▲ + 7 **b** ▲ × 5 **c** 8 − ▲ **d** 12 ÷ ▲

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8B

4 Use the correct order of operations to evaluate the following.

a $4 + 2 \times 5$ **b** $7 - 3 \times 2$

c $3 \times 6 - 2 \times 4$ **d** $(7 - 3) \times 2$

Remember: First ( ), then × and ÷, then + and −.



5 Find the value of the following.

a When a is 4, what is $a + 3$?

b When b is 5, what is $b + 12$?

c When x is 10, what is $12 \times x$?

d When r is 7, what is $r - 2$?

$a + 3 = 4 + 3 = ?$



Fluency

Example 4 Substituting into an expression

Given that $t = 5$, evaluate:

a $t + 7$ **b** $8t$ **c** $10t + 4 - t$

Solution	Explanation
a $t + 7 = 5 + 7$ $= 12$	Replace t with 5 and then evaluate the expression, which now contains no pronumerals.
b $8t = 8 \times t$ $= 8 \times 5$ $= 40$	Insert × between 8 and t, then substitute in 5. If the multiplication sign is not included, we might get a completely wrong answer of 85.
c $10t + 4 - t = 10 \times 5 + 4 - 5$ $= 50 + 4 - 5$ $= 49$	Replace t with 5 before evaluating. Note that the multiplication (10×5) is calculated *before* the addition and subtraction.

6 Given that $a = 6$, evaluate:

a $a + 2$ **b** $7 \times a$ **c** $a - 3$ **d** $a \div 2$

7 When $b = 5$, find the value of:

a $4b$ **b** $10b$ **c** $7b$ **d** $20b$

$4b$ means $4 \times b$.



8 Calculate the value of $12 + b$ when:

a $b = 5$ **b** $b = 8$ **c** $b = 60$ **d** $b = 0$



Drilling for Gold 8B1

9 Substitute $x = 5$ and then evaluate each of the following.

a $x + 3$ **b** $x \times 2$ **c** $14 - x$

d $2x + 4$ **e** $3x + 2$ **f** $13 - 2x$

g $2(x + 2)$ **h** $20 \div x + 3$ **i** $4x - 2$

Substitute means replace x with 5.



10 When $y = 3$, state the value of:

a $5y + 2 - y$ **b** $3 + 4y + 2y$ **c** $y \times 7 + 5 - y$

d $y \times (1 + y)$ **e** $y + 2y + 3y$ **f** $(10 - y) \times y$

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Example 5 Substituting multiple pronumerals

Substitute $x = 4$ and $y = 7$ to evaluate $5x + y + 8$.

Solution	Explanation
$5x + y + 8 = 5 \times x + y + 8$ $= 5 \times 4 + 7 + 8$ $= 20 + 7 + 8$ $= 35$	Insert the implied multiplication sign between 5 and x before substituting the values for x and y.

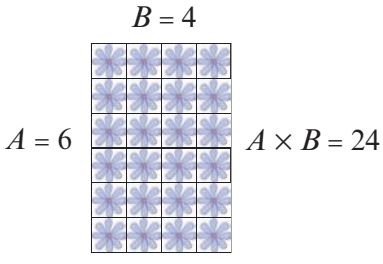
11 Substitute $a = 2$ and $b = 3$ and then evaluate:

a $2a + 4$ **b** $2b - 2$ **c** $a + b$
d $3a + b$ **e** $5a - 2b$ **f** ab
g $a + 2b$ **h** $b - a$ **i** $5a + 2 + b$

Problem-solving and Reasoning

12 A number is substituted for b in the expression $7 + b$ and gives the result 12. What is the value of b?

13 Assume A and B are two numbers, where $AB = 24$. What values could A and B equal if they are whole numbers? Try to list as many as possible.



14 Rachel adds 4 to her age, then she doubles the result. This gives her the number 80. How old is Rachel?

Enrichment: Missing values

15 Copy and complete the table.

x	5	9	12			
$x + 6$	11			7		
$4x$	20				24	28

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8C Equivalent algebraic expressions

Sometimes two expressions will evaluate to give the same result, no matter what numbers the pronumerals stand for.

For example, $B + B$ and $2 \times B$ will always give the same result.

$B = 3$ → $B + B = 6$, $2 \times B = 6$

$B = 11$ → $B + B = 22$, $2 \times B = 22$

If this is true for every value of B, then $B + B$ and $2 \times B$ are equivalent expressions.

Let's start: Odd one out

Here are four expressions:

$2 \times B + 6$ $\quad 6 + B + B$ $\quad (B + 3) \times 2$ $\quad B + 6$

One of them is not equivalent to the others.

- Copy and complete the table to help you find the odd one out. (The first row has already been done for you.)

	$2 + B + 6$	$6 + B + B$	$(B + 3) + 2$	$B + 6$
$B = 0$	$2 \times 0 + 6 = 6$	$6 + 0 + 0 = 6$	$(0 + 3) \times 2 = 6$	$0 + 6 = 6$
$B = 1$				
$B = 2$				

- Can you draw pictures to show why the 'odd one out' in the table is not equivalent to the other three expressions? (For example, if B is the number of marbles in a bag, you could draw 2 bags and 6 extra marbles to show $2 \times B + 6$.)

Key ideas

Equivalent expressions Expressions that will have the same numerical value as each other when the pronumerals are replaced with any number

- **Equivalent expressions** are *always* equal in value, no matter what numbers are substituted for the pronumerals.

  For example: $3a + 6$ is equivalent to $6 + 3a$ and to $3a + 3 + 3$.



= a + a + a + 6
= 3a + 3 + 3
= 2a + 6 + a
= . . .

Let $a = 1$	Let $a = 1$	Let $a = 1$
$3a + 6$	$6 + 3a$	$3a + 3 + 3$
$= 3 \times 1 + 6$	$= 6 + 3 \times 1$	$= 3 \times 1 + 3 + 3$
$= 3 + 6$	$= 6 + 3$	$= 3 + 3 + 3$
$= 9$	$= 9$	$= 9$

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Exercise 8C

Understanding



Remember that $4x = 4 \times x$.

1 **a** When $x = 3$, what does $x + 6$ equal?
b When $x = 3$, what does $4x$ equal?
c When $x = 3$, are $x + 6$ and $4x$ equal to each other?

2 **a** When $a = 5$, evaluate $a + 4$.
b When $a = 5$, evaluate $2 + a + 2$.
c When $a = 5$, do $a + 4$ and $2 + a + 2$ equal each other?

3 Copy and complete:
Two expressions that are always equal are called _____ expressions.

4 True or false? Explain your answer with a sentence.
'No matter what number you choose for $\square$, the values of $\square + 6$ and $6 + \square$ are equal.'

Example 6 Using tables to decide equivalence

Fill in a table to help you decide if $3a + 6$ and $(a + 2) \times 3$ are equivalent.
Use $a = 0$, $a = 1$, $a = 2$, $a = 3$.

Solution

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$3a + 6$	6	9	12	15
$(a + 2) \times 3$	6	9	12	15

They appear to be equivalent.

Explanation

$3a + 6$ and $(a + 2) \times 3$ are equal for all chosen values of a, so they appear to be equivalent. Drawing pictures cinfirms this:

$\boxed{a}\boxed{a}\boxed{a}\!::$ $\quad$ $\boxed{a}:\boxed{a}:\boxed{a}:$

$3a + 6$ $\quad$ $(a + 2) \times 3$

5 **a** Copy and complete the following table.

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$2a + 2$				
$(a + 1) + 2$				

b Fill in the gap: $2a + 2$ and $(a + 1) \times 2$ appear to be _____ expressions.
c Draw pictures to show they *are* equivalent.

Fluency

6 **a** Copy and complete the following table.

	$B = 0$	$B = 1$	$B = 2$	$B = 3$
$5B + 3$				
$6B + 3$				

b Are $5B + 3$ and $6B + 3$ equivalent expressions?



Equivalent expressions are equal for *all* values.

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8C

7 Copy and complete this table.

	$6x + 5$	$4x + 5 + 2x$
$x = 1$		
$x = 2$		
$x = 3$		
$x = 4$		

8 For each of the following pairs, decide if they are equivalent (E) or not equivalent (N).

a $k + 6$ and $k \times 4$

b $k \times 3$ and $2 \times k + k$

c $k + 2$ and $1 + k + 1$

d $k + 10$ and $k \times 10$

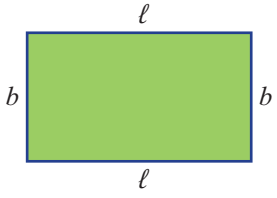


Try making k stand for different numbers (e.g. $k = 0$, $k = 1$, $k = 2$ etc.) in a table.

Problem-solving and Reasoning

9 Give an example of an expression that is equivalent to $4y$.

10 The perimeter of this rectangle is given by $b + \ell + b + \ell$. Write an equivalent expression for the perimeter.



11 The expressions $a + b$ and $b + a$ are equivalent and only contain two terms. How many expressions are equivalent to $a + b + c$ and contain only three terms?

12 Prove that no two of these expressions are equivalent: $4 + x$, $4x$, $x - 4$, $x \div 4$.



Find a value of x that gives four different values.

Enrichment: Sometimes the same, always the same or never the same?

13 On the following game board, each box has a partner box. Write all the matches. (For example, A1 and C2 match because $3a + 2a$ is equivalent to $5a$.)

	Column A	Column B	Column C	Column D
Row 1	$3a + 2a$	$6a$	$4a + 2$	$7a$
Row 2	$5 - a - 2a$	10	$5a$	$2 \times (a + 5) - 2a$
Row 3	$(1 + 2a) \times 2$	$2a + 5a$	$2a \times 3$	$5 - 3a$

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14 Although the following pairs of expressions are *not* equivalent for all values of x, you can find a value of x for which they *are* equal. What is that value? The first one has been done for you.

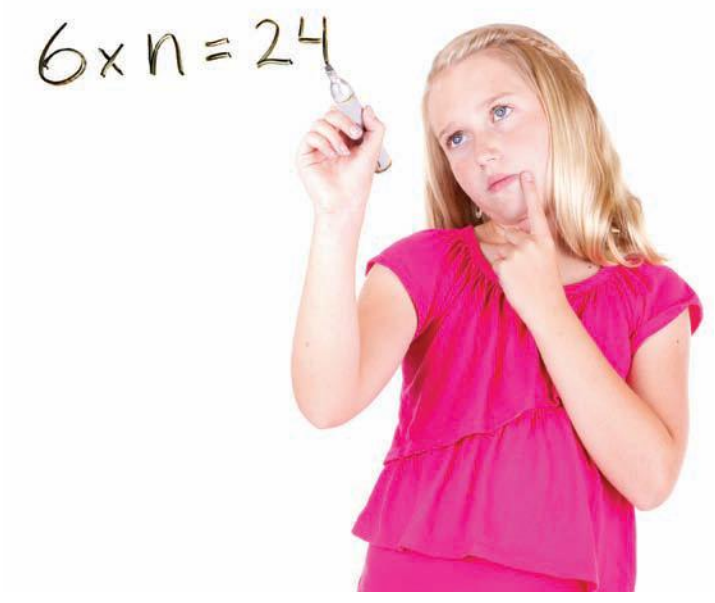
a $2x$ and $x + 5$ are equivalent when $x = 5$.

b	$2x$	$x + 3$	**c**	$3x$	$x + 2$
d	$5x$	$x + 8$	**e**	x	$2x$
f	x	$3x$	**g**	$4x$	$5x$
h	$4x$	$3x + 4$	**i**	$x + x$	$x - x$
j	$x \times x$	$x \div x$	**k**	x	$x \times x$
l	$x + x$	$x \times x$	**m**	$x - 1$	$x \div 2$

15 Every question contains a pair of expressions. For each pair, choose one of the following.

A They will *never* have the same value.
B They will *always* have the same value.
C They have *one* value of x that makes them equal.
D They have *two* values of x that make them equal.

a	x	$7x$	**b**	$x + 1$	x
c	$x - 5$	$5 - x$	**d**	$x + x + x$	$3x$
e	$x \times 2$	$x \times x$	**f**	$x + 1$	$x - 1$
g	$x \times x$	$x - 1$	**h**	$x \times x$	$x \times x \times x$



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8D Like terms

In this section we will be combining like terms to make expressions simpler.

Let's start: Adding numbers, adding pronumerals

	Numbers				Pronumerals				
	3	3	3	3	x	x	x	x	
	3	3	3	3	y	y	y		
	5	5	5	5	z	z	z	z	z
Total	$8 \times \square + 4 \times \square$				What goes here?				

Key ideas

Like terms Two or more terms that contain the same pronumerals

Simplify To find the simplest possible equivalent expression

- **Like terms** have exactly the same pronumerals, although not necessarily in the same order.

Like	Not like
$3x$ and $5x$	$3x$ and $5y$
$-12a$ and $7a$	$11d$ and $4c$
$5ab$ and $6ba$	$-8ab$ and $5a$
$4x^2$ and $3x^2$	x^2 and x

- We **simplify** expressions by combining like terms.
  For example: $3x + 5x$ simplifies to $8x$.
  $12a - 7a$ simplifies to $5a$.

Exercise 8D

Understanding

1 True or false?
a $x + y = xy$ **b** $x + x = 2x$ **c** $3x - x = 2x$ **d** $5x - 4x = 1$

2 **a** When $a = 7$, what is $2a$?
b When $a = 7$, what is $3a$?
c When $a = 7$, what is $2a + 3a$?
d When $a = 7$, what is $5a$?

$2a = 2 \times a$
$= 2 \times 7$
$= 14$



3 Evaluate the following, using $x = 5$.
a $10x$ **b** $4x$ **c** $10x - 4x$ **d** $6x$

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4 Copy and complete the following sentences.

a $3x$ and $5x$ are _____ terms.

b $4x$ and $3y$ are not _____ _____.

c $4xy$ and $4yx$ are like _____.

d $12a$ and $5ab$ are not like _____ because they have different pronumerals.

Fluency

Example 7 Identifying like terms

Which of the following pairs are like terms?

a $3x$ and $2x$ **b** $3a$ and $3b$ **c** $2ab$ and $5ba$

d $2a$ and $4ab$ **e** $7ab$ and $9aba$

Solution	Explanation
a $3x$ and $2x$ are like terms.	The pronumerals are the same.
b $3a$ and $3b$ are not like terms.	The pronumerals are different.
c $2ab$ and $5ba$ are like terms.	The pronumerals are the same, even though they are written in a different order (one a and one b).
d $2a$ and $4ab$ are not like terms.	The pronumerals are not exactly the same (the first term contains only a and the second term has a and b).
e $7ab$ and $9aba$ are not like terms.	The pronumerals are not exactly the same (the first term contains one a and one b, but the second term contains two a terms and one b).



Drilling for Gold 8D1

5 Classify the following pairs as like terms (L) or not like terms (N).

a $7a$ and $4b$ **b** $3a$ and $10a$ **c** $18x$ and $32x$

d $4a$ and $4b$ **e** 7 and $10b$ **f** x and $4x$

g $5x$ and 5 **h** $12ab$ and $4ab$ **i** $7cd$ and $12cd$

j $3abc$ and $12abc$ **k** $3ab$ and $2ba$ **l** $4cd$ and $3dce$

Example 8 Adding or subtracting like terms

Simplify:

a $10x + 4x$ **b** $7a - 2a$ **c** $6x - x$

d $6x - 5x$ **e** $6x - 6x$

Solution	Explanation
a $10x + 4x = 14x$	$10x$ and $4x$ are like terms, so they are combined $(10 + 4 = 14)$.

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8D

Solution	Explanation
b $7a - 2a = 5a$	$7a$ and $2a$ are like terms, so they can be subtracted. $7a$ take away $2a$ leaves $5a$.
c $6x - x = 5x$	$6x - x = 6x - 1x$ $= 5x$
d $6x - 5x = x$	$6x - 5x = 1x$, which is just x.
e $6x - 6x = 0$	$6x - 6x = 0x$, which is just 0.

Drilling for Gold 8D2 8D3

6 Simplify the following by collecting like terms.

a $3x + 2x$ **b** $4a + 2a$ **c** $6q + 10q$ **d** $b + 2b$
e $6cd + 3cd$ **f** $2qr + 4qr$ **g** $8ab + ab$ **h** $9cf + 2cf$

Remember that b is the same as $1b$ or $1 \times b$.

7 Simplify the following by collecting like terms.

a $7x - 3x$ **b** $8a - 5a$ **c** $12q - 2q$ **d** $7b - b$
e $10cd - 2cd$ **f** $8qr - 6qr$ **g** $8ab - ab$ **h** $10cf - 7cf$

Example 9 Simplifying by using like terms

Simplify the following by collecting like terms.

a $7b + 2 + 3b$ **b** $12d - 4d + d$ **c** $5 + 12a + 4b - 2 - 3a$

Solution	Explanation
a $7b + 2 + 3b = 10b + 2$	$7b$ and $3b$ are like terms, so they are combined. They cannot be combined with 2 because it contains no pronumerals.
b $12d - 4d + d = 9d$	All the terms here are like terms. Remember that d means $1d$ when combining them.
c $5 + 12a + 4b - 2 - 3a$ $= 12a - 3a + 4b + 5 - 2$ $= 9a + 4b + 3$	$12a$ and $3a$ are like terms. We subtract $3a$ because it has a minus sign in front of it. We can also combine the 5 and the 2 as they are like terms.

8 Simplify the following by collecting like terms.

a $2a + a + 4b + b$ **b** $5a + 2a + b + 8b$ **c** $3x - 2x + 2y + 4y$
d $4a + 2 + 3a$ **e** $7 + 2b + 5b$ **f** $3k - 2 + 3k$

Skillsheet 8A

9 **a** $7f + 12 - 2f$ **b** $4a - 4 + 5b + b$
c $3x + 7x + 3y - 4x + y$ **d** $10a + 3 + 4b - 2a$
e $4 + 10g - 3g$ **f** $10x + 4x + 31y - y$
g $10 + 7y - 3x + 5x + 2y$ **h** $11a + 4 - 3a + 9$
i $3b + 4b + c + 5b - c$ **j** $8 + 3d - 5 + 2d$

For $7f + 12 - 2f$, the sign in front of $2f$ tells us to subtract that term from $7f$.

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Problem-solving and Reasoning

10 Ravi and Marissa each work for n hours per week. Ravi earns \$10 per hour and Marissa earns \$12 per hour.

- **a** Write an expression for the amount Ravi earns in one week.
- **b** Write an expression for the amount Marissa earns in one week.
- **c** Write a simplified expression for the total amount Ravi and Marissa earn in one week.

11 The length of the line segment shown could be expressed as $a + a + 3 + a + 1$.



- **a** Write the length as a simplified expression.
- **b** What is the length of the segment if a is equal to 5?

12 a Show, using a table of values, that $3x + 2x$ is not equivalent to $6x$.

- **b** Prove that $3x + 2y$ is not equivalent to $5x$.
- **c** Prove that $3x + 2y$ is not equivalent to $5xy$.



For part **b**, try choosing x and y to be different from each other.

Enrichment: Algebraic marbles

13 Let x represent the number of marbles in a bag. Xavier bought 4 bags and Cameron bought 7 bags.

- **a** Write simplified expressions for:
  - **i** the number of marbles Xavier has
  - **ii** the number of marbles Cameron has
  - **iii** the total number of marbles that Xavier and Cameron have
  - **iv** the number of *extra* marbles that Cameron has compared to Xavier
- **b** if x is 12, how many marbles do they each have?



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8E Multiplying and dividing algebraic expressions

Remember that $4a$ means $4 \times a$, and $6xy = 6 \times x \times y$.
Also, a fraction like $\frac{b}{3}$ means $b \div 3$.



Let's start: Putting in the multiplication symbol

- Rewrite the following expression by putting in all the multiplication signs.
  $7abc + 2de + 3f$
- How can the following expression be written without multiplication signs?
  Put the pronumerals in alphabetic order to find a hidden message.
  $s \times i \times m \times c \times h \times p + v \times l \times u + s \times p \times c \times i \times h$

Key ideas

- $a \times b$ is written as ab and $a \times a$ is written as a^2.
  For example: $a \times 2 = 2a$
  $a \times 1 = 1a = a$
  $a \times 0 = 0a = 0$
- $a \div b$ is written as $\frac{a}{b}$.
  For example: $a \div 2 = \frac{a}{2}$
  $a \div 1 = \frac{a}{1} = a$
  $1 \div a = \frac{1}{a}$
- In a product, numbers should be written first and pronumerals are usually written in alphabetical order.
  For example: $a \times 2 \times b$ is written $2ab$.
- When dividing, any common factor in the numerator and denominator can be cancelled. Common factors can be numbers or pronumerals.
  For example: $\frac{{}^{2}\cancel{4}\ a\ \cancel{b}^{1}}{{}_{1}\cancel{2}\ \cancel{b}_{1}\ c} = \frac{2a}{c}$

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Exercise 8E

Understanding

1 True (T) or false (F)?

a $4 \times n$ can be written as $4n$.
b $n \times 3$ can be written as $3n$.
c $4 \times b$ can be written as $b + 4$.
d $a \times b$ can be written as ab.
e $a \times 5$ can be written as $50a$.
f $a \times a$ can be written as $2a$.

2 Simplify these fractions.

a $\frac{10}{20}$ **b** $\frac{6}{18}$ **c** $\frac{12}{20}$ **d** $\frac{15}{20}$

3 **a** Simplify the fraction $\frac{12}{18}$.

b Simplify the fraction $\frac{2000}{3000}$.

c Simplify $\frac{2a}{3a}$.



a $\frac{12}{18} = \frac{2 \times 6}{3 \times 6}$
b $\frac{2000}{3000} = \frac{2 \times 1000}{3 \times 1000}$
c $\frac{2a}{3a} = \frac{2 \times a}{3 \times a}$



Drilling for Gold 8E1

4 Match these expressions (**a** to **f**) with the conventional way to write them (**A** to **F**).

a $2 \times u$	**A** $3u$
b $u \times u$	**B** $\frac{5}{u}$
c $5 \div u$	**C** $2u$
d $u \times 3$	**D** $\frac{u}{5}$
e $u \div 5$	**E** $7u$
f $u \times 7$	**F** u^2

Fluency

Example 10 Simplifying expressions with multiplication

a Write $4 \times a \times b \times c$ without multiplication signs.
b Simplify $4a \times 2b \times 3c$, giving your final answer without multiplication signs.

Solution	Explanation
a $4 \times a \times b \times c = 4abc$	When pronumerals are written next to each other they are being multiplied.
b $4a \times 2b \times 3c = 4 \times a \times 2 \times b \times 3 \times c$ $= 4 \times 2 \times 3 \times a \times b \times c$ $= 24abc$	First insert the missing multiplication signs. Rearrange to bring the numbers to the front. $4 \times 2 \times 3 = 24$ and $a \times b \times c = abc$.

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8E

5 Write each of these expressions without multiplication signs.

a $2 \times x$ **b** $5 \times p$ **c** $7 \times r$ **d** $11 \times s$
e $10 \times a \times b$ **f** $5 \times c \times d$ **g** $x \times 4$ **h** $x \times x$

6 Write each of these expressions without any multiplication signs.

a $5 \times 2 \times a \times b$ **b** $2 \times 8 \times x \times y$
c $2 \times b \times 5$ **d** $x \times 7 \times z \times 4$
e $2 \times a \times 3 \times b \times 6 \times c$ **f** $8 \times d \times 2 \times e \times 3 \times f$
g $7 \times 3 \times a \times 2 \times b$ **h** $a \times 2 \times b \times 7 \times 3 \times c$
i $7 \times a \times 12 \times b \times c$ **j** $a \times 3 \times a$



For part **c**, reorder: $2 \times b \times 5 = 2 \times 5 \times b$

7 Simplify these expressions.

a $3a \times 12$ **b** $7d \times 9$ **c** $2 \times 4e$
d $3 \times 5a$ **e** $4a \times 3b$ **f** $7e \times 9g$
g $8a \times bc$ **h** $4d \times 7af$ **i** $a \times 3b \times 4c$
j $2a \times 4b \times c$ **k** $4d \times 3e \times 5fg$ **l** $2cb \times 3a \times 4d$



Multiply the numbers and write the pronumerals in alphabetical order.

Example 11 Simplifying expressions with division

a Write $x \div 3$ as a fraction.

b Simplify the expression $\frac{8ab}{12b}$.

Solution	Explanation
a $x \div 3 = \frac{x}{3}$	Divisions can be written as fractions.
b $\frac{8ab}{12b} = \frac{8 \times a \times b}{12 \times b}$	Insert multiplication signs to help spot common factors.
$= \frac{2 \times \cancel{4} \times a \times \cancel{b}}{3 \times \cancel{4} \times \cancel{b}}$	8 and 12 have a common factor of 4.
$= \frac{2a}{3}$	Cancel out the common factors of 4 and b.

8 Write each expression without a division sign.

a $x \div 5$ **b** $z \div 2$ **c** $a \div 12$ **d** $b \div 5$
e $2 \div x$ **f** $5 \div d$ **g** $x \div y$ **h** half of a

Skillsheet 8B

9 Simplify the following expressions by dividing by any common factors.

a $\frac{2x}{5x}$ **b** $\frac{5a}{5a}$ **c** $\frac{9ab}{4b}$ **d** $\frac{2ab}{5a}$
e $\frac{2x}{4}$ **f** $\frac{9x}{12}$ **g** $\frac{10a}{15a}$ **h** $\frac{30y}{40y}$
i $\frac{4a}{2}$ **j** $\frac{21x}{7x}$ **k** $\frac{4xy}{2x}$ **l** $\frac{9x}{3xy}$

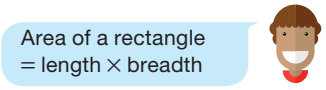


Remember that $\frac{a}{1} = a$ and $\frac{a}{a} = 1$.

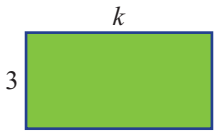
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Problem-solving and Reasoning

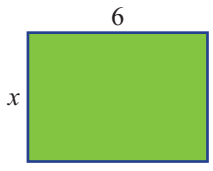
10 Write a simplified expression for the area of each rectangle.



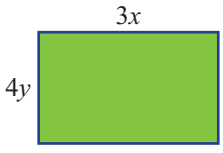
a



b



c



11 Five friends go to a restaurant. They split the bill evenly, so each pays the same amount.

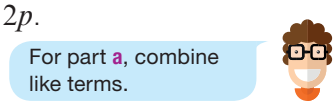
a If the total cost is $\$100$, how much do they each pay?

b If the total cost is $\$C$, how much do they each pay? Write an expression.

12 The expression $3 \times 2p$ is the same as the expression $2p + 2p + 2p$.

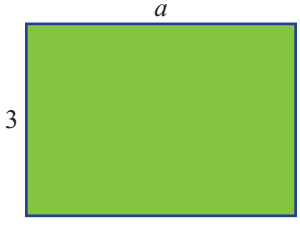
a What is a simpler expression for $2p + 2p + 2p$?

b $3 \times 2p$ is shorthand for $3 \times 2 \times p$.
How does this relate to your answer in part **a**?



Enrichment: Doubling rectangles

13 The area of this rectangular paddock is $3a$.



Both the length and the breadth of the paddock are now doubled.

a Draw the new paddock, showing its dimensions.

b Write a simplified expression for the area of the new paddock.

c Divide the area of the new paddock by the area of the old paddock. What do you notice?

d What happens to the area of the original paddock when you triple both the length and the breadth?

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8F Applying algebra

EXTENSION

An algebraic expression can be used to describe problems relating to many different everyday situations, including costs, speed and sporting results.

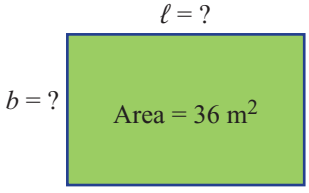


In many sports, results and details can be expressed using algebra.

Let's start: Garden bed area

The garden shown at right has an area of 36 m^2, but the length and breadth are unknown. Area = length × breadth.

- What are some possible values that ℓ and b could equal?
- Try to find the dimensions of the garden that make the fencing around the outside as small as possible.



Key ideas

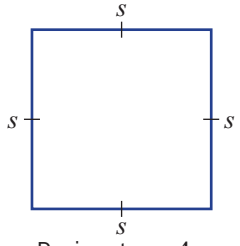
- Many different situations can be modelled with algebraic expressions. For example, an algebraic expression for perimeter is $2\ell + 2b$.
- To apply an expression, the pronumerals should be defined clearly. Then, known values should be substituted for the pronumerals.
  For example: If $\ell = 5$ and $b = 10$, then
  Perimeter $= 2 \times 5 + 2 \times 10$
  $= 30$

Exercise 8F

Understanding

1 **a** If $w = 3$, find the value of $4w$.
b If $c = 2$, find the value of $c + 8$.
c Evaluate $3d$, given that $d = 10$.
d Evaluate $7 + 4f$ when $f = 5$.

2 The perimeter of this square is given by $4s$.
a If $s = 5$, find the value of $4s$.
b Find a square's perimeter when $s = 7$.
c What is a square's perimeter if its side length is 10?
d If a square has a side length of 25, what is its perimeter?

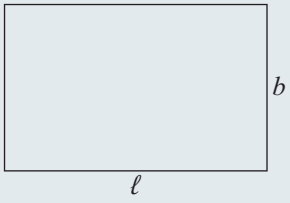


Perimeter $= 4s$

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Example 12 Applying an expression

The perimeter of a rectangle is given by the expression $2\ell + 2b$, where b is the breadth and ℓ is the length. Find the perimeter of a rectangle when $b = 4$ and $\ell = 7$.



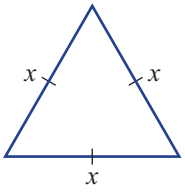
Solution	Explanation
Perimeter is given by $2\ell + 2b = 2(4) + 2(7)$ $= 8 + 14$ $= 22$	To apply the rule, we substitute $\ell = 4$ and $b = 7$ into the expression. Evaluate using the order of operations.

3 The area of a rectangle is given by the expression $\ell \times b$, where ℓ is its length and b is its breadth.

a Find the area when $\ell = 5$ and $b = 7$.

b Find the area when $\ell = 2$ and $b = 10$.

4 The following diagram shows an equilateral triangle.



a The perimeter is $x + x + x$. Simplify this expression.

b Use your answer to part **a** to find the perimeter when $x = 12$.



For part **a**, combine like terms.

Fluency

5 Exercise books cost $3 each.

a How much does it cost for 12 exercise books?

b How much does it cost for 7 exercise books?

c Write an expression for the cost of n exercise books.

6 Mustafa says that he will work for 2 more hours than Kristina.

a If Kristina works for 4 hours, how long will Mustafa work?

b If Kristina works for 9 hours, how long will Mustafa work?

c If Kristina works for t hours, write an expression for the number of hours Mustafa works.



For part **c**, give an expression for 2 more than t.

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8F

Example 13 Constructing expressions from problem descriptions

a Write an expression for the total cost, in dollars, of 10 bottles, if each bottle costs $\$x$.

b A plumber charges a \$30 call-out fee plus \$60 per hour.

i Complete the table.

Hours	1	2	3	4	5
Total cost (\$)					

ii Write an expression for the total cost.

Solution

a $10x$

b **i**

Hours	1	2	3	4	5
Total cost (\$)	90	150	210	270	330

ii $30 + 60n$

Explanation

Each of the 10 bottles costs $\$x$, so the total cost is $10 \times x = 10x$.

For a 1-hour job:
$\$30 + \$60 = \$90$
For a 2-hour job:
$\$30 + \$60 \times 2 = \$150$

For each hour, the plumber charges \$60, so must pay $60 \times n = 60n$. The \$30 call-out fee is added to the total bill.

7 If pencils cost $\$x$ each, write an expression for the cost of:

a 10 pencils

b 3 packets of pencils, if each packet contains 5 pencils

c k pencils

8 If pens cost \$2 each, write an expression for the cost of n pens.

9 A carpenter charges a \$40 call-out fee and then \$80 per hour. This means the total cost for x hours of work is $40 + 80x$.

a How much would it cost for a 2-hour job (i.e. $x = 2$)?

b How much would it cost for a job that takes 8 hours?

c The call-out fee is increased to \$50. What is the new expression for the total cost of x hours?



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Problem-solving and Reasoning

10 A plumber charges a $\$50$ call-out fee and $\$100$ per hour.

a Copy and complete the table below.

Hours	1	2	3	4	5
Total cost ($)					

b Find the total cost if the plumber works for t hours. Give an expression.

c Substitute $t = 30$ into your expression to find how much it will cost for the plumber to work 30 hours.

11 To hire a tennis court, you must pay a $\$5$ booking fee plus $\$10$ per hour.

a What is the cost of booking a court for 2 hours?

b What is the cost of booking a court for x hours? Write an expression.

c A tennis coach hires a court for 7 hours. Substitute $x = 7$ into your expression to find the total cost.

12 In AFL football a goal is worth 6 points and a behind is worth 1 point. This means the total score for a team is $6g + b$, if g goals and b behinds are scored.

a What is the score for a team that has scored 5 goals and 3 behinds?

b What are the values of g and b for a team that has scored 8 goals and 5 behinds?

c If a team has a score of 20, this could be because $g = 2$ and $b = 8$. What are the other possible values of g and b?



Enrichment: Mobile phone mayhem

13 Rochelle and Emma are on different mobile phone plans, as shown below.

Rochelle's plan	Emma's plan
20-cent connection fee 60 cents per minute	80-cent connection fee 40 cents per minute



a Write an expression for the cost of making a t minute call using Rochelle's phone.

b Write an expression for the cost of making a t minute call using Emma's phone.

c Whose phone plan would be cheaper for a 7-minute call?

d What length of call would cost exactly the same for both phones?

e Investigate current mobile phone plans and describe how they compare to Rochelle's and Emma's plans.

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8G Substitution involving negative numbers and mixed operations

Algebraic expressions are used to describe many real-world situations. For example, the speed of an accelerating rocket could be shown by $100 + 20t$ metres per second, where t is the time in seconds.

To work out the speed after 5 seconds, we would substitute $t = 5$ into $100 + 20t$ to give $100 + 20 \times 5 = 200$ seconds.

In this section, we will look at the process of substitution using both positive and negative integers.



Let's start: Order matters

Two students substitute the values $a = -2$, $b = 5$ and $c = -7$ into the expression $ac - bc$. Some of the different answers received are 21, -49, -21 and 49.

- Which answer is correct and what errors were made in the calculation of the other three incorrect answers?

Key ideas

- Substitute into an expression by replacing pronumerals with numbers.

When $a = -2$ and $b = 5$, then:

$b - a = 5 - (-2)$
$= 5 + 2$
$= 7$

$2b + a = 2 \times 5 + (-2)$
$= 10 - 2$
$= 8$

- Use brackets around negative numbers to avoid confusion with other symbols.
- When working with more than one operation and with positive and/or negative numbers:
  - Deal with brackets first. ( )
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

$5 \times 2 - (3 - (-2)) \div 5 = 10 - 5 \div 5$
$= 10 - 1$
$= 9$

2nd: 10 (from 5×2)
1st: 5 (from $3 - (-2)$)
3rd: 1 (from $5 \div 5$)
last: 9 (from $10 - 1$)

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Exercise 8G

Understanding

1 Find the value of each expression when $a = 3$.

a $a + 3$ **b** $2 - a$ **c** $2 \times a$ **d** $4 \times a - 10$
e $-2 + a$ **f** $-3 - a$ **g** $6 \div a$ **h** $-4 - a$

2 Which of the following shows the correct substitution of $a = -2$ into the expression $a - 5$?

A $2 - 5$ **B** $-2 + 5$ **C** $-2 - 5$ **D** $2 + 5$

3 Which of the following shows the correct substitution of $x = -3$ into the expression $2 - x$?

A $-2 - (-3)$ **B** $2 - (-3)$ **C** $-2 + 3$ **D** $-3 + 2$

Fluency

Example 14 Substituting integers

Evaluate the following expressions using $a = 3$ and $b = -5$.

a $2 + 4a$ **b** $a - b$ **c** $2a + b$

Solution	Explanation
a $2 + 4a = 2 + 4 \times 3$ $= 2 + 12$ $= 14$	Replace a with 3 and evaluate the multiplication before doing the addition.
b $a - b = 3 - (-5)$ $= 3 + 5$ $= 8$	Replace a with 3 and b with -5. To subtract -5, add 5.
c $2a + b = 2 \times 3 + (-5)$ $= 6 - 5$ $= 1$	Replace a with 3 and b with -5, and evaluate the multiplication before the addition. To add -5, subtract 5.

4 Evaluate the following expressions using $a = 3$ and $b = -2$.

a $2a + 1$ **b** $4a - 3$ **c** $a - 6$
d $3a - 20$ **e** $2 - a$ **f** $4 - 3a$
g $-1 - a$ **h** $-2 - 2a$ **i** $b - 4$
j $b + 8$ **k** $b - a$ **l** $b + 2a$
m $b + 7a$ **n** $a - b$ **o** $2a - b$
p $3a + b$ **q** $9 \div a - b$ **r** $b - 12 \div a$



Substitute $a = 3$ and/or $b = -2$, then work out the answer.

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8G

Example 15 Substituting with brackets

Evaluate the following, using $x = -2$ and $y = 7$.

a $3 \times (10 - y)$ **b** $(y - x) \div 3$

Solution	Explanation
a $3 \times (10 - y) = 3 \times (10 - 7)$ $= 3 \times 3$ $= 9$	Substitute $y = 7$. Deal with brackets before other operations.
b $(y - x) \div 3 = (7 - (-2)) \div 3$ $= 9 \div 3$ $= 3$	Substitute $x = -2$ and $y = 7$. Evaluate the brackets, using $7 - (-2) = 7 + 2$.

5 Evaluate the following expressions, using $x = 5$ and $y = -3$.

a $2 \times (7 - x)$ **b** $5 \times (8 - x)$ **c** $(2 + x) \times 4$
d $(-2 + x) \div 1$ **e** $3 \times (1 - y)$ **f** $4 \times (7 - y)$
g $(2 + y) \times 6$ **h** $(4 + y) \div 2$ **i** $(x + y) \times 4$
j $(x + y) \div 1$ **k** $5 \times (x - y)$ **l** $10 \div (x - y)$



Remember to deal with brackets first, before doing other operations.



Skillsheet 8C

6 Evaluate the following if $x = -4$ and $y = 3$.

a $x - y - 6$ **b** $2y + x + 1$ **c** $3y - x + 7$ **d** $2 - x - y$
e $5y - x$ **f** $2 - x + 3y$ **g** $-4 + x - 2y$ **h** $-6 + x + 7y$

Problem-solving and Reasoning

7 The volume of water running into a tank is given by the expression $10 + 5t$ litres, where t is the time in minutes. Find the volume of water after:

a 2 minutes ($t = 2$) **b** 5 minutes ($t = 5$) **c** 20 minutes ($t = 20$)



8 Find, by trial and error, the value of a to make each statement true.

a $a + 6$ equals 10 **b** $a - 7$ equals -2
c $2a + 1$ equals 7 **d** $2 - a$ equals 6
e $6 - a$ equals 13 **f** $-1 - a$ equals 0
g $3 + a$ equals -2 **h** $-4 + a$ equals -3
i $-10 - a$ equals -4

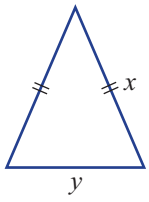


Guess a value for a and substitute to see if you are correct. If not, make a better guess.

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9 Albert substitutes $c = -10$ into $10 - c$ and gets 0. Is he correct? If not, what is the correct answer?

10 The formula for the perimeter, P, of this isosceles triangle is $P = 2x + y$.



a Use the formula to find P when:

i $x = 2$ and $y = 1$ **ii** $x = 4$ and $y = -2$

b What problems are there with part **a ii** above?

11 Write two different expressions involving x that give an answer of -10 when $x = -5$.

Enrichment: It's cold in here!

12 Karen had a new freezer delivered. When she switched it on, the temperature in the freezer was $20°$C. The temperature will fall until it is $-20°$C, then it will stay constant. The temperature falls by $4°$C every hour.

a Copy and complete this table.

Hours	0	1	2	3	4	5
Temperature (°C)	20					

b How long will it take for the temperature to fall to $-20°$C?

c Which of the following expressions would describe the temperature in the fridge while it is falling?

A $20 + 4n$ **B** $20 - 4n$ **C** $4n + 20$ **D** $4n - 20$ **E** $4 - 20n$

d When the temperature reached $-20°$C, Karen opened the freezer and left the door slightly open. The temperature rose by $3°$C every hour. Write an expression for this.



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8H Number patterns

EXTENSION

In the following sections, algebraic skills will be applied to number patterns. Scientists use algebra and number patterns to study the world and make predictions.





Many careers involve the use of mathematical patterns and rules.

Let's start: How many grains of rice?

A chess board has 64 squares.
If you put:

- 5 grains on the first square
- 10 grains on the second square
- 15 grains on the third square
- 20 grains on the fourth square

- How many grains will you need to place on the 64th square?
- How many grains will you need to place in total?

Key ideas

Sequence A set of numbers that follow a pattern

Term One of the numbers in a number pattern

- A list of numbers that follow a rule is called a **number pattern** or a **sequence**. For example: 4, 7, 10, 13, …
- Each separate number in the sequence is called a **term**.
- In the example above, the 4th term is 13. The rule 'start with 3 and add 2 to each term' gives 3, 5, 7, 9, 11, …
- To find the pattern rule for a sequence, ask:
  - Are the terms increasing or decreasing by a fixed amount?

$$25, \overset{+3}{\rightarrow} 28, \overset{+3}{\rightarrow} 31, \overset{+3}{\rightarrow} 34, \ldots \qquad 50, \overset{-5}{\rightarrow} 45, \overset{-5}{\rightarrow} 40, \overset{-5}{\rightarrow} 35, \ldots$$

  - Are the terms being multiplied or divided by the same amount?

$$4, \overset{\times 3}{\rightarrow} 12, \overset{\times 3}{\rightarrow} 36, \overset{\times 3}{\rightarrow} 118, \ldots \qquad 64, \overset{\div 2}{\rightarrow} 32, \overset{\div 2}{\rightarrow} 16, \overset{\div 2}{\rightarrow} 8, \ldots$$

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Exercise 8H

Understanding



Look back at the Key ideas.

1 Write the missing words.

a When a list of numbers follows a pattern that can be described by a rule, it is called a _____.

b Each separate number in a sequence is called a _____.

2 List the next three terms in each of these patterns.

a $8, \overset{+4}{\rightarrow}\ 12, \overset{+4}{\rightarrow}\ 16, \overset{+4}{\rightarrow}\ ___, ___, ___$

b $100, \overset{-5}{\rightarrow}\ 95, \overset{-5}{\rightarrow}\ 90, \overset{-5}{\rightarrow}\ ___, ___, ___$

c $64, \overset{\div 2}{\rightarrow}\ 32, \overset{\div 2}{\rightarrow}\ 16, \overset{\div 2}{\rightarrow}\ ___, ___, ___$

d $5, \overset{\times 2}{\rightarrow}\ 10, \overset{\times 2}{\rightarrow}\ 20, \overset{\times 2}{\rightarrow}\ ___, ___, ___$

3 List the first five terms of the sequence for each of these pattern rules.

a Start with 8 and keep adding 3.

b Start with 32 and keep subtracting 1.

c Start with 52 and keep subtracting 4.

d Start with 123 and keep adding 7.



4 List the first five terms of the following number patterns.

a Start with 3 and keep multiplying by 2.

b Start with 5 and keep multiplying by 4.

c Start with 240 and keep dividing by 2.

d Start with 625 and keep dividing by 5.

Fluency

Example 16 Finding patterns that change by a fixed amount

Find the next three terms for these number patterns.

a 6, 18, 30, 42, ___, ___, ___ **b** 99, 92, 85, 78, ___, ___, ___

Solution	Explanation
a 54, 66, 78	The same number is being added to each term. Keep adding 12 to find the next three terms. $6, \overset{+12}{\rightarrow}\ 18, \overset{+12}{\rightarrow}\ 30, \overset{+12}{\rightarrow}\ 42, \overset{+12}{\rightarrow}\ 54, \overset{+12}{\rightarrow}\ 66, \overset{+12}{\rightarrow}\ 78$
b 71, 64, 57	Keep subtracting 7 to find the next three terms. $99, \overset{-7}{\rightarrow}\ 92, \overset{-7}{\rightarrow}\ 85, \overset{-7}{\rightarrow}\ 78, \overset{-7}{\rightarrow}\ 71, \overset{-7}{\rightarrow}\ 64, \overset{-7}{\rightarrow}\ 57$

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8H

5 Find the next three terms for these sequences.

a 3, 8, 13, 18, ___, ___, ___
b 4, 14, 24, 34, ___, ___, ___
c 26, 23, 20, 17, ___, ___, ___
d 106, 108, 110, 112, ___, ___, ___
e 63, 54, 45, 36, ___, ___, ___
f 9, 8, 7, 6, ___, ___, ___
g 101, 202, 303, 404, ___, ___, ___
h 75, 69, 63, 57, ___, ___, ___


Look for a fixed number that is added or subtracted.

Example 17 Finding patterns that involve multiplication or division

Find the next three terms for the following number patterns.

a 2, 6, 18, 54, ___, ___, ___
b 256, 128, 64, 32, ___, ___, ___

Solution	Explanation
a 162, 486, 1458	Each term is being multiplied by the same number. Keep multiplying by 3 to find the next three terms. ×3 ×3 ×3 ×3 ×3 ×3 2, 6, 18, 54, 162, 486, 1458
b 16, 8, 4	Keep dividing by 2 to find the next three terms. ÷2 ÷2 ÷2 ÷2 ÷2 ÷2 256, 128, 64, 32, 16, 8, 4

6 Find the next three terms for the following number patterns.

a 2, 4, 8, 16, ___, ___, ___
b 5, 10, 20, 40, ___, ___, ___
c 96, 48, 24, ___, ___, ___
d 1215, 405, 135, ___, ___, ___
e 11, 22, 44, 88, ___, ___, ___
f 7, 70, 700, 7000, ___, ___, ___
g 256, 128, 64, 32, ___, ___, ___
h 1216, 608, 304, 152, ___, ___, ___


Is each term being multiplied or divided by the same number?

7 Find the missing numbers in each of the following number patterns.

a 62, 56, ___, 44, 38, ___, ___
b 15, ___, 35, ___, ___, 65, 75
c 4, 8, 16, ___, ___, 128, ___
d 3, 6, ___, 12, ___, 18, ___
e 88, 77, 66, ___, ___, ___, 22
f 2997, 999, ___, ___, 37


These patterns can involve + or − or × or ÷.

Example 18 Finding pattern rules

For each of these sequences, write the pattern rule in words.

a 2, 10, 50, 250, …
b 6, 10, 14, 18, …
c 32, 16, 8, 4, …
d 32, 28, 24, 20, …

Solution	Explanation
a Start with 2 and multiply each term by 5.	×5 ×5 ×5 2, 10, 50, 250, …

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Solution	Explanation
b Start with 6 and add 4 to each term.	+4 +4 +4 6, 10, 14, 18, ...
c Start with 32 and divide each term by 2.	÷2 ÷2 ÷2 32, 16, 8, 4, ...
d Start with 32 and subtract 4 from each term.	−4 −4 −4 32, 28, 24, 20, ...

8 Use words to write the pattern rule for each sequence.

a 19, 17, 15, 11, …
b 48, 24, 12, 6, …
c 50, 56, 62, 68, …
d 1, 3, 9, 27, …
e 625, 125, 25, 5, …
f 75, 72, 69, 66, …

Problem-solving and Reasoning

9 Write the next three terms in each of the following sequences.

a 3, 5, 8, 12, ___, ___, ___
b 1, 2, 4, 7, 11, ___, ___, ___
c 10, 8, 11, 9, 12, ___, ___, ___
d 25, 35, 30, 40, 35, ___, ___, ___


Look for a pattern in the increases or decreases.

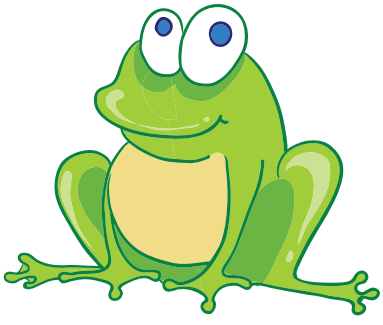
10 A frog has fallen to the bottom of a well that is 6 metres deep.

- On the first day the frog climbs 3 metres up the wall of the well.
- On the second day it slides back 2 metres.
- On the third day it climbs up 3 metres.
- On the fourth day it slides back 2 metres.

The frog continues following this pattern until it reaches the top of the well and hops away.

a Write a sequence of numbers to show the frog's height above the bottom of the well at the end of each day.
b How many days does it take the frog to get out of the well?


Draw a diagram of the well. Use arrows to show the movement of the frog.



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8H

11 Only some of the following number lists are sequences. For each list that *is* a sequence, write the pattern rule.

a 4, 12, 36, 108, 324, …
b 19, 17, 15, 13, 11, …
c 212, 223, 234, 245, 256, …
d 8, 10, 13, 17, 22, …
e 64, 32, 16, 8, 4, …
f 5, 15, 5, 15, 5, …
g 2, 3, 5, 7, 11, …
h 75, 72, 69, 66, 63, …



12 Copy and complete each of the following. Give the special name for each type of numbers.

a 1, 4, 9, 16, 25, 36, ___, ___, ___
b 1, 1, 2, 3, 5, 8, 13, ___, ___, ___
c 1, 8, 27, 64, 125, ___, ___, ___
d 2, 3, 5, 7, 11, 13, 17, ___, ___, ___
e 4, 6, 8, 9, 10, 12, 14, 15, ___, ___, ___
f 121, 131, 141, 151, ___, ___, ___

Choose from: *composite numbers, cube numbers, even numbers, Fibonacci numbers, negative numbers, odd numbers, palindromes, prime numbers, square numbers, triangular numbers.*

You may use the internet to search for the meaning of these words.

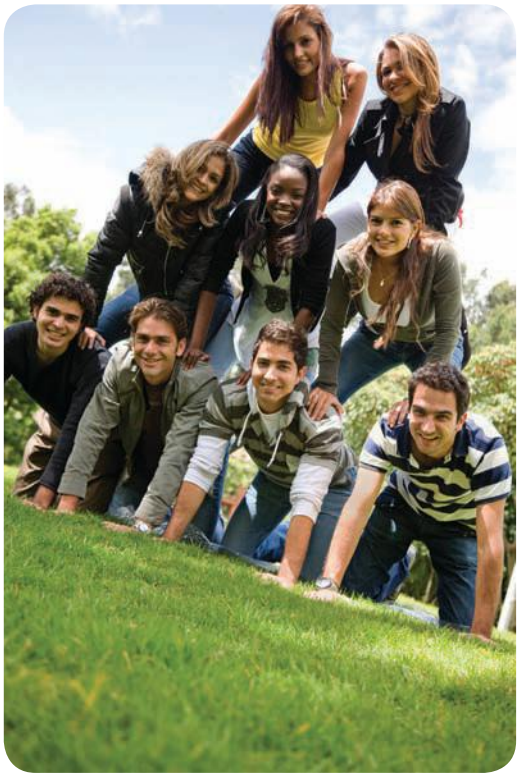
Enrichment: Human pyramids



Draw a diagram.

13 When making a human pyramid, each new row has one less person than the row below. The pyramid is complete when there is a row of only one person on the top.

Write down a number pattern for a human pyramid with 10 students on the bottom row. How many people are needed to make this pyramid?



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8I Spatial patterns

EXTENSION

Repeated geometric shapes form interesting spatial patterns. Architects often use spatial patterns in the design of buildings. Artists also use repeated geometric shapes in designs to be printed on curtains, tiles and wallpaper.



A pattern rule can be used to find the number of pickets for a given number of fence posts.

Let's start: Stick patterns

Copy these shapes, using matchsticks or toothpicks. Then build the next three shapes in the pattern.



How many sticks would you need to make the shape with:

- 10 triangles?
- 100 triangles?

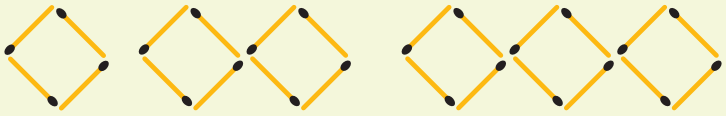
If you know the number of triangles, how could you find the number of sticks? Discuss this with a partner and then write your answer.

Use similar steps to explore the number of squares in the following pattern.



Key ideas

- A spatial pattern is a sequence of geometrical shapes. For example:



  - The number of squares in each term makes a number sequence.
  - The number of sticks in each term makes another number sequence.

- A table of values shows the number of shapes and the number of sticks.

Number of squares	1	2	3	4	5
Number of sticks	4	8	12	16	20

- A pattern rule tells how many sticks are needed for a certain number of shapes.
  For example: Number of sticks $= 4 \times$ number of shapes

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Exercise 8I

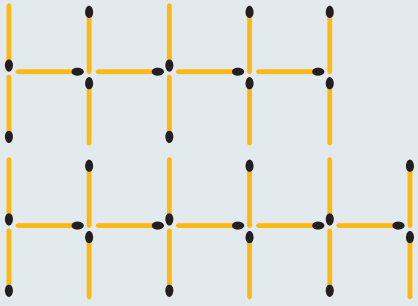
Understanding

Example 19 Drawing spatial patterns

Draw the next two shapes in this spatial pattern.

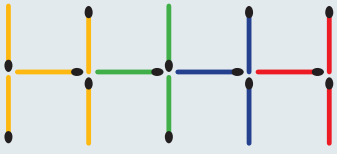


Solution

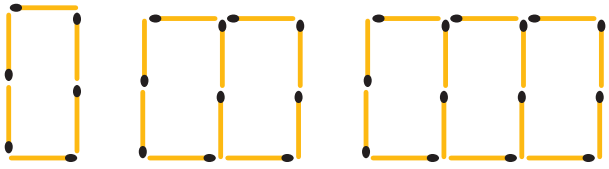


Explanation

Look for the pattern.
5 sticks are used to start with.
Then 3 sticks are added each time.



1 Anwar used matchsticks to begin a pattern of rectangles.



shape 1 shape 2 shape 3

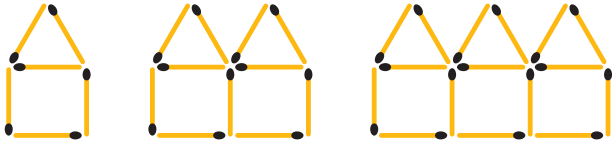
Write the missing words or numbers for each of these.

a Each shape shows some _____ made with matchsticks.

b Shape 1 has ___ rectangle, shape 2 has ___ rectangles and shape 3 has ___ rectangles.

c Shape 1 has ___ sticks, shape 2 has ___ sticks and shape 3 has ___ sticks.

2 Jayne used matchsticks to make a spatial pattern of houses.





How many extra matches are needed to change 1 house into 2 houses?

Copy and complete this table.

Number of houses	1	2	
Number of sticks			

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3 Draw the next two terms for each of these spatial patterns.

a

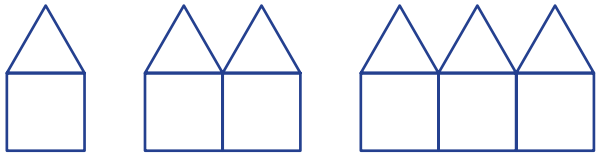


b





c



d



Fluency

Example 20 Finding a general rule for a spatial pattern

a Draw the next two shapes in this spatial pattern.



b Complete the table.

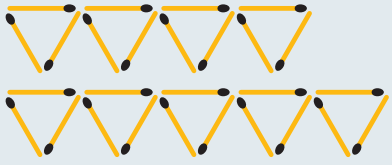
Number of triangles	1	2	3	4	5
Number of sticks required	3				

c Complete this pattern rule: Number of sticks = ☐ × number of triangles

d How many sticks would you need for 20 triangles?

Solution

a



b

No. of triangles	1	2	3	4	5
No. of sticks	3	6	9	12	15

c Number of sticks = 3 × number of triangles

d Number of sticks = 3 × 20 triangles = 60 sticks

Explanation

Follow the pattern by adding one triangle each time.

An extra 3 sticks are required to make each new triangle.

3 sticks are required per triangle.

20 triangles × 3 sticks each

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8I

4 **a** Draw the next two shapes in this spatial pattern.





For part **c**, check that your pattern rule works for all values in the table.

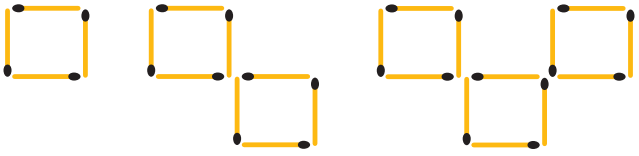
b Copy and complete this table.

Number of crosses	1	2	3	4	5
Number of sticks					

c Copy and complete this pattern rule: Number of sticks = ☐ × number of crosses

d How many sticks would you need for 10 crosses?

5 **a** Draw the next two shapes for this spatial pattern.



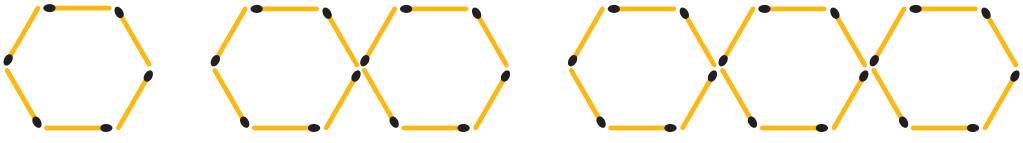
b Copy and complete this table.

Number of squares	1	2	3	4	5
Number of sticks					

c Copy and complete the pattern rule:
Number of sticks = ☐ × number of squares

d How many sticks would you need for 12 squares?

6 **a** Draw the next two shapes for this spatial pattern.



b Copy and complete this table.

Number of hexagons	1	2	3	4	5
Number of sticks					



How many extra matches would you need to add another hexagon?

c Copy and complete the pattern rule:
Number of sticks = ☐ × number of hexagons

d How many sticks would you need for 20 hexagons?



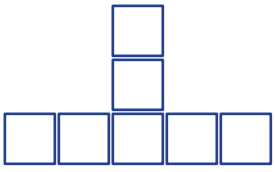
Skillsheet 8D

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Problem-solving and Reasoning

7 List the shapes (**A** to **D**) in the correct order to make a spatial pattern. (Start with the smallest shape.) Then draw the next shape in the sequence.

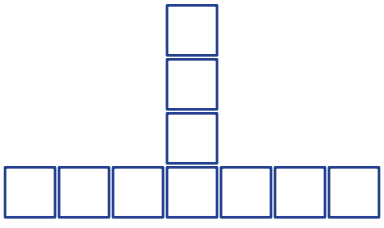
A



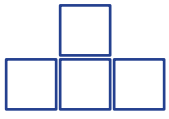
B



C



D



Example 21 Finding more challenging rules

a Draw the next two shapes for this spatial pattern.



b Copy and complete this table.

No. of squares	0	1	2	3	4
No. of sticks	1	$1 + \square \times 1 = \square$	$1 + \square \times 2 = \square$	$1 + \square \times 3 = \square$	$1 + \square \times 4 = \square$

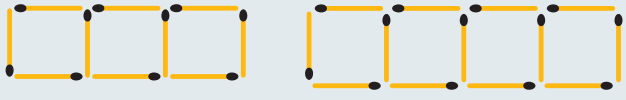
c Copy and complete the rule for the pattern:
Number of sticks = $1 + \square \times$ number of squares

d How many sticks are needed to make 30 squares this way?

e How many squares could be made from 25 sticks?

Solution

a



b

No. of squares	0	1	2	3	4
No. of sticks	1	$1 + 3 \times 1 = 4$	$1 + 3 \times 2 = 7$	$1 + 3 \times 3 = 10$	$1 + 3 \times 4 = 13$

Explanation

Add 3 sticks at a time to complete each new square.

Count the squares. Complete the calculations, then count sticks in the diagrams to check.

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Solution	Explanation
c Number of sticks = $1 + 3 \times$ number of squares	The number of sticks is 1 more than 3 times the number of squares.
d 91 sticks	$1 + 3 \times 30 = 91$
e 8 squares	$25 - 1 = 24$, $24 \div 3 = 8$

8 a Draw the next two shapes for this spatial pattern.





How many extra matches are needed to form a triangle?

b Copy and complete this table.

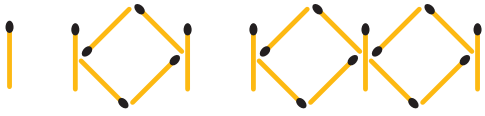
Number of triangles	0	1	2	3	4
Number of sticks	1	$1 + \square = \square$	$1 + \square \times 2 = \square$	$1 + \square \times 3 = \square$	$1 + \square \times 4 = \square$

c Copy and complete the rule for this pattern:
Number of sticks = $1 + \square \times$ number of triangles

d How many sticks are needed to make 12 triangles this way?

e How many triangles could be made from 81 sticks?

9 a Draw the next two shapes in this spatial pattern.





Copy the last shape and add more sticks to make the next shape.

b Copy and complete this table.

Number of shapes	0	1	2	3	4
Number of sticks	1	$1 + \square = \square$	$1 + \square \times 2 = \square$	$1 + \square \times 3 = \square$	$1 + \square \times 4 = \square$

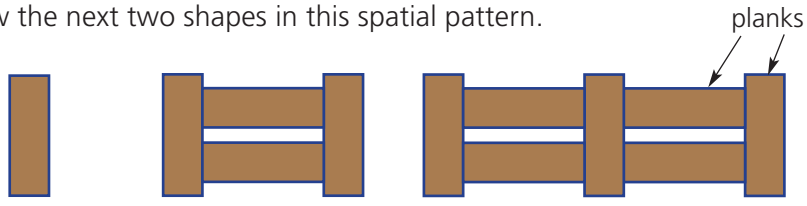
c Copy and complete the rule for this pattern:
Number of sticks = $1 + \square \times$ number of shapes

d How many sticks are needed to make 20 shapes this way?

e How many shapes could be made from 86 sticks?

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10 a Draw the next two shapes in this spatial pattern.



b Copy and complete this table.

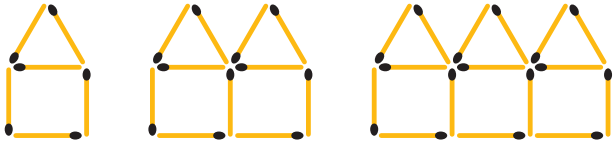
Number of fence sections	0	1	2	3	4
Number of planks	1	$1 + \square = \square$	$1 + \square \times 2 = \square$	$1 + \square \times 3 = \square$	$1 + \square \times 4 = \square$

c Copy and complete the pattern rule:
Number of planks = $1 + \square \times$ number of fence sections

d How many planks would you need to make 9 fence sections?

e How many fence sections can be made from 43 planks?

11 Which rule correctly describes this spatial pattern?



A Number of sticks = 5 × number of houses + 1

B Number of sticks = 6 × number of houses + 1

C Number of sticks = 6 × number of houses

D Number of sticks = 5 × number of houses

Enrichment: Design your own spatial pattern

12 Design a spatial pattern to fit the following number patterns.

a 4, 7, 10, 13, …

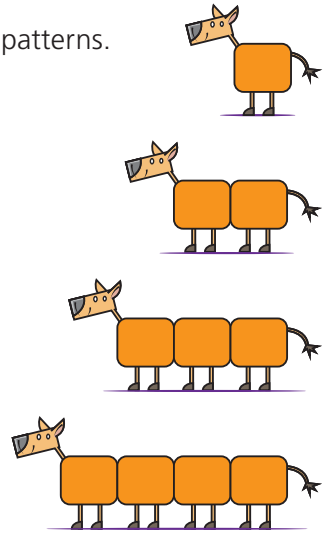
b 4, 8, 12, 16, …

c 3, 5, 7, 9, …

d 3, 6, 9, 12, …

e 5, 8, 11, 14, …

f 6, 11, 16, 21, …



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8J Tables and rules

EXTENSION

In the previous section, we investigated rules for spatial patterns. Rules are also useful for many everyday situations.

It can be helpful to think of a rule as a 'machine'. You feed in one number (the input), and another number (the output) comes out. For example, Mary was 3 years old when her brother Tim was born. If you know Tim's age (the input), what is a rule for finding Mary's age (the output)?

Showing some values in a table makes it is easy to 'see' the rule.

Tim's age (*input*)	0	1	7	3
Mary's age (*output*)	3	4	10	6

+3

Rule: Mary's age = Tim's age + 3
Output = *input* + 3

Let's start: What's the story?

Each of the following stories tells how an input and output are related.

	Story	*Input*	*Output*
1	Connor is 5 years younger than his brother Declan.	Connor's age	Declan's age
2	Liam earns $5 for every car he washes.	Number of cars washed	Amount ($) earned
3	Jayce and 4 friends share some lollies equally.	Total number of lollies	Number of lollies each person gets

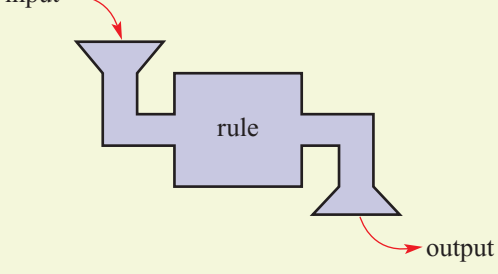
- Which story matches the following table of input and output values? How did you decide?

Input	2	1	7	3	6	?
Output	10	5	35	15	?	50

- How would you find the missing values?
- Pick one of the other stories and make up your own table of values.

Key ideas

- A rule shows the relationship between two amounts that can vary. It is used to calculate the **output** (answer) from the **input** (starting number).



Output The result from applying a rule to the input

Input A number that is changed according to some rule

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- A **table of values** shows inputs and outputs.
  To make a table of values:
  – Choose some input values.
  – Use the rule to calculate the output values.
- To find the rule from a table of values, try different operations (+, –, ×, ÷) until you find a rule that works for *all* the values.
  For example:

Input	5	8	12
Output	15	24	36

To get the outputs, has the same number been:
– added to each input? (no)
– subtracted from each input? (no)
– multiplied by each input? (yes)
The rule is: *Output* = 3 × *input*

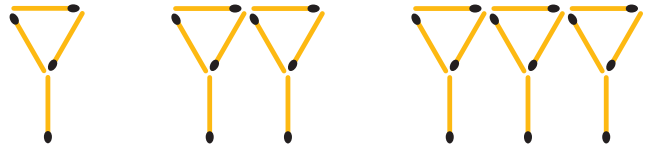
Table of values Pairs of numbers related by a rule, often used for plotting points on a Cartesian plane

Exercise 8J

Understanding

1 Here is a spatial pattern. Each 'flower' is made with 4 sticks.

1 flower 2 flowers 3 flowers



a Copy and complete this rule:
Output (sticks) = 4 × *input* (_____)

b Copy and complete this table of values.

Number of flowers (*input*)	1	2	3	4	5
Number of sticks (*output*)					

×4

2 Ebony is 3 years older than José.

a Copy and complete this rule:
Output (_____) = *input* (José's age) + 3

b Copy and complete this table of values.

José's age (*input*)	1	3	7	12	15
Ebony's age (*output*)					

+3

3 Jake earns $8 an hour.

a Copy and complete this rule.
Output (amount earned) = 8 × _____ (hours worked)

b Copy and complete this table of values.

Hours worked (*input*)	1	2	3	4	5
Amount earned (*output*)					

×8

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8J

Example 22 Substituting an input value to find the output value

Use each rule to find the output when the input = 12.

a *Output* = *input* × 2
b *Output* = *input* − 2
c *Output* = *input* + 2
d *Output* = *input* ÷ 2

Solution	Explanation
a *Output* = 12 × 2 = 24	Replace *input* by 12, then multiply by 2.
b *Output* = 12 − 2 = 10	Replace *input* by 12, then subtract 2.
c *Output* = 12 + 2 = 14	Replace *input* by 12, then add 2.
d *Output* = 12 ÷ 2 = 6	Replace *input* by 12, then divide by 2.

4 Use each rule to find the output when the input = 8.

a *Output* = *input* × 2
b *Output* = *input* − 2
c *Output* = *input* + 2
d *Output* = *input* ÷ 2

Fluency

Example 23 Completing a table of values

Complete each table for the given rule.

a *Output* = *input* − 2

Input	3	5	7	12	20
Output					

b *Output* = (3 × *input*) + 1

Input	4	2	9	12	0
Output					

Solution

a *Output* = *input* − 2

Input	3	5	7	12	20
Output	1	3	5	10	18

b *Output* = (3 × *input*) + 1

Input	4	2	9	12	0
Output	13	7	28	37	1

Explanation

Put each *input* value in turn into the rule.
e.g. When *input* is 3:
Output = 3 − 2 = 1

Put each *input* value in turn into the rule.
e.g. When *input* is 4:
Output = (3 × 4) + 1 = 13

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5 Copy and complete each table for the given rule.



a *Output* = *input* + 3

Input	4	5	6	7	10
Output					

b *Output* = *input* × 2

Input	5	1	3	21	0
Output					

c *Output* = *input* − 8

Input	11	18	9	44	100
Output					

d *Output* = *input* ÷ 5

Input	5	15	55	0	100
Output					

6 Copy and complete each table for the given rule.



a *Output* = (10 × *input*) − 3

Input	1	2	3	4	5
Output					

b *Output* = (*input* ÷ 2) + 4

Input	6	8	10	12	14
Output					

c *Output* = (3 × *input*) + 1

Input	5	12	2	9	0
Output					

d *Output* = (2 × *input*) − 4

Input	3	10	11	7	50
Output					

Example 24 Finding a rule from a table of values

Find the rule for each of these tables of values.

a

Input	3	4	5	6	7
Output	12	13	14	15	16

b

Input	1	2	3	4	5
Output	7	14	21	28	35

Solution	Explanation
a *Output* = *input* + 9	Each *output* value is 9 more than the *input* value.
b *Output* = *input* × 7 or *Output* = 7 × *input*	By inspection, it can be observed that each *output* value is 7 times bigger than the *input* value.



7 State the rule for each of these tables of values.



a

Input	4	5	6	7	8
Output	5	6	7	8	9

b

Input	1	2	3	4	5
Output	4	8	12	16	20

c

Input	10	8	3	1	14
Output	21	19	14	12	25

d

Input	6	18	30	24	66
Output	1	3	5	4	11

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8J

Problem-solving and Reasoning

8 Copy and complete the missing values in the table and state the rule.

Input	4	10	13	24			5	11	2
Output			39		42	9	15		6

9 Match each rule (**A** to **D**) with the correct table of values (**a** to **d**).
Rule **A**: *Output* = *input* − 5
Rule **B**: *Output* = *input* + 1
Rule **C**: *Output* = 4 × *input*
Rule **D**: *Output* = 5 + *input*



The rule must be true for each input number in the table.

a

Input	20	14	6
Output	15	9	1

b

Input	8	10	12
Output	13	15	17

c

Input	4	5	6
Output	5	6	7

d

Input	4	3	2
Output	16	12	8

10 When Zac was born, his grandfather placed $100 in a special account to save for his education. He adds $50 to the account every time Zac has a birthday.

a Copy and complete this table for Zac's birthday account.

Zac's age in years (*input*)	0	1	2	3
Amount ($) in account (*output*)				

b Copy and complete this rule for Zac's birthday account:
Output = ☐ × *input* + ☐

c How much will be in the account when Zac turns 18?

11 Cindy has saved $64 so far for the school ski trip. She has just started working at the local vet, helping to clean out pens and feed the animals. Cindy gets paid $8 an hour and saves all her wages towards the ski trip.

a Copy and complete this table for Cindy's savings.

Hours worked (*input*)	0	2	5	10
Cindy's total savings (*output*)				

b Copy and complete this rule for Cindy's savings:
Output = ☐ × *input* + ☐

c Cindy wants to save $200 to pay for the ski trip. How many hours will she need to work?



For part **c**, first work out how much more Cindy needs to earn.

12 Complete these two different rules so that they each give an output of 7 when the input is 3.

a *Output* = ☐ × *input* + ☐

b *Output* = ☐ × *input* − ☐

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Enrichment: Finding harder rules

13 a The following rules all involve two operations. Find the rule for each of these tables of values.

i *Output* = ☐ × *input* – ☐

Input	4	5	6	7	8
Output	5	7	9	11	13

ii *Output* = ☐ × *input* + ☐

Input	1	2	3	4	5
Output	5	9	13	17	21

iii *Output* = ☐ × *input* – ☐

Input	10	8	3	1	14
Output	49	39	14	4	69

iv *Output* = *input* ÷ ☐ + ☐

Input	6	18	30	24	66
Output	3	5	7	6	13

v *Output* = ☐ × *input* + ☐

Input	4	5	6	7	8
Output	43	53	63	73	83

vi *Output* = ☐ × *input* – ☐

Input	1	2	3	4	5
Output	0	4	8	12	16

b Use an Excel spreadsheet to make each of the tables in part **a**. Each time, enter a formula so that the computer calculates all the output values.

Here is an example of the formulas for part **a i** above. Try extending your tables using other input numbers.

	A	B
1	13 (i)	
2	input	output
3	4	=2*A3-3
4	5	=2*A4-3
5	6	=2*A5-3
6	7	=2*A6-3
7	8	=2*A7-3
8		



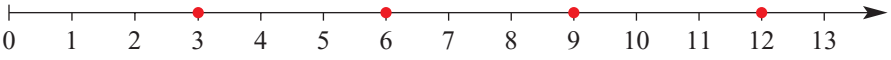
When entering Excel formulas:
- Always start with =.
- Use * for ×.
- Instead of typing 'A3' just click on cell A3.
- To copy the formula into other cells, fill down by dragging the + cursor.

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8K The Cartesian plane and graphs

EXTENSION

In previous sections we looked at number sequences and spatial patterns. We used rules and tables of values to describe them. Another way of showing a pattern is by plotting points. For example, we could use a number line to show the simple pattern 3, 6, 9, 12, …

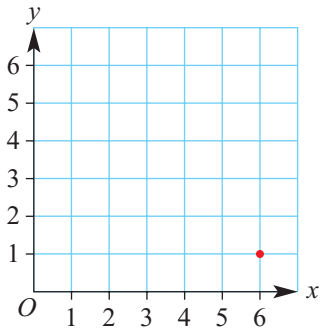


However, when we work with two sets of values (i.e. inputs and outputs) we need two dimensions. Instead of a number line, we use a Cartesian plane.

Let's start: I'm thinking of two numbers

I am thinking of two positive numbers.
They add together to give 7.
They could be $x = 6$, $y = 1$, which is graphed as $(6, 1)$ on the Cartesian plane opposite.

- What else could they be?
  Find another 5 points and plot them.
- What do you notice about the position of those points?



Key ideas

Number plane A diagram on which two numbers can be used to locate any point

Cartesian plane Another name for the number plane

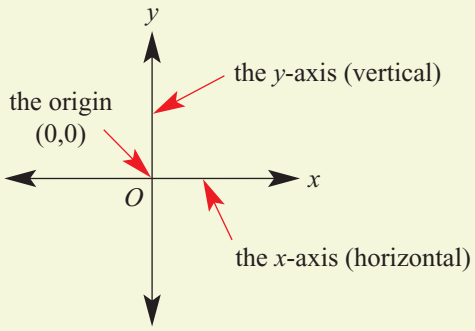
x-axis Horizontal axis of the number plane

y-axis Vertical axis of the number plane

Origin The point on the number plane with coordinates $(0, 0)$

Coordinates Numbers or letters used to give a location or position, often an ordered pair written in the form (x, y)

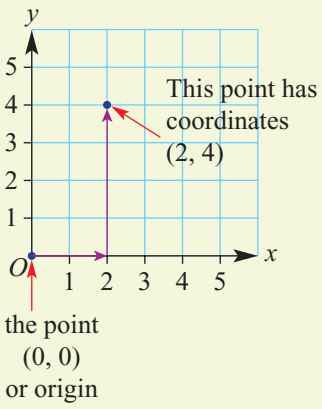
- A **number plane** is a grid for plotting points.
  It is also called the **Cartesian plane**.
  Important features of a number plane are:
  - the **x-axis** and **y-axis**: these are horizontal and vertical number lines
  - the **origin**: the point where the x-axis and y-axis meet



- Points are located by a grid reference system of **coordinates**.
  - The point (x, y) means:
    - Start from the origin.
    - Go x units across to the left or right.
    - Go y units up or down.
  - For $(2, 4)$ the x-coordinate is 2 and the y-coordinate is 4. To plot this point, start at the origin and go 2 units right, then 4 units up.

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- For a rule describing a pattern with *input* and *output*:
  - The input is the x-value.
  - The output is the y-value.

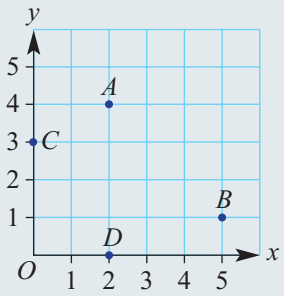


Exercise 8K

Understanding

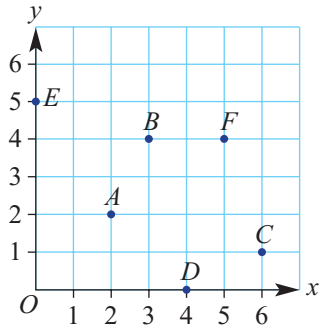
Example 25 Finding coordinates of plotted points

Match each point on the grid with its correct coordinates.
(0, 3) (2, 4) (5, 1) (2, 0)

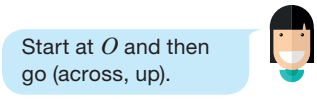


Solution	Explanation
A (2, 4)	To locate A, start at O, go across 2 and up 4.
B (5, 1)	To locate B, start at O, go across 5 and up 1.
C (0, 3)	To locate C, start at O, don't move across at all, then up 3.
D (2, 0)	To locate D, start at O, go across 2 and don't go up at all.

1 Match each point on the grid with its correct coordinates.



(4, 0)
(0, 5)
(6, 1)
(3, 4)
(2, 2)
(5, 4)

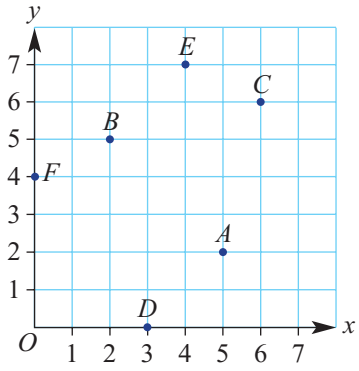


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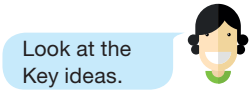
8K

2 Copy and complete the coordinates for each point on this number plane.

a $A(5, \square)$
b $B(\square, 5)$
c $C(\square, \square)$
d $D(3, \square)$
e $E(\square, \square)$
f $F(\square, 4)$



3 Copy and complete the following sentences.

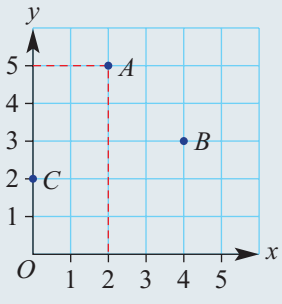


a The horizontal axis is known as the _____.
b The _____ is the vertical axis.
c The point at which the axes intersect is called the _____.
d The first number in the bracket is always the number on the _____ axis.
e The second number is always the _____.
f The letter _____ comes before _____ in the dictionary, and the _____ -coordinate comes before the _____ -coordinate on the number plane.

Example 26 Plotting points on a number plane

Draw a number plane and plot these points on it.
$A(2, 5)$ $B(4, 3)$ $C(0, 2)$

Solution



Explanation

Draw a number plane with both axes labelled from 1 to 5.
The numbers go on the grid lines, not in the spaces.
Label the horizontal axis x and the vertical axis y. Label the origin, O.
(2, 5) means go across 2 from O (along the x-axis) and then go up 5 units. Plot the point and label it A.
$B(4, 3)$ means (4 across, 3 up).
$C(0, 2)$ means (0 across, 2 up), so C is on the y-axis.

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4 On grid paper, draw a number plane, with the numbers 1 to 6 marked on each axis. Don't forget to label the origin, O. Plot and label these points:
$A(3, 5)$ $B(4, 2)$ $C(0, 3)$ $D(1, 0)$ $E(6, 6)$

Fluency

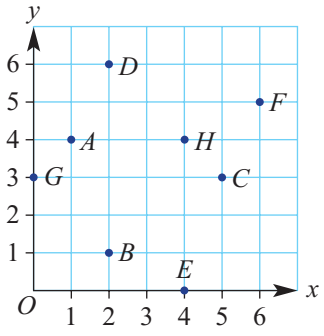
5 Use grid paper to draw a number plane with the x- and y-axis numbered from 1 to 10.
Plot each group of points and join them in order, using a ruler.

a	(4, 10) (2, 8) (5, 9)	Name this type of angle.
b	(2, 6) (4, 6) (4, 3)	Name this type of angle.
c	(6, 8) (8, 8) (8, 5) (6, 5) (6, 8)	Name this shape.
d	(5, 1) (6, 4) (7, 1) (5, 1)	Name this type of triangle.
e	(0, 0) (0, 3) (3, 0) (0, 0)	Name this type of triangle.
f	(7, 3) (9, 3) (10, 6)	Name this type of angle.

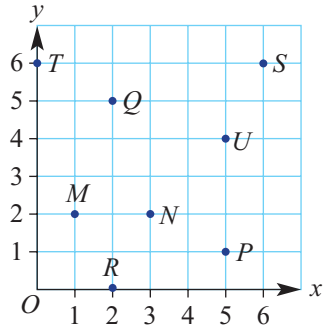
6 Draw a number plane from 1 to 8 on both axes. Plot the following points on the grid and join them in the order they are given.
(2, 7), (6, 7), (5, 5), (7, 5), (6, 2), (5, 2), (4, 1), (3, 2), (2, 2), (1, 5), (3, 5), (2, 7)

7 Write down the coordinates of each of these labelled points.

a



b




The coordinates are (x, y) or (across, up).



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Example 27 Drawing a graph

For the rule *output* = *input* + 1:

a Copy and complete the table of values.

b List in the table the coordinates of each point.

c Plot each pair of points on the number plane.

Input (x)	*Output* (y)
0	1
1	
2	
3	

Solution

a

Input (x)	*Output* (y)
0	1
1	2
2	3
3	4

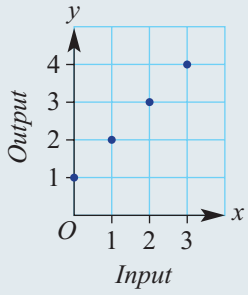
Explanation

Use the rule to find each *output* value for each *input* value. The rule is: *Output* = *input* + 1, so add 1 to each *input* value.

b (0, 1), (1, 2), (2, 3) and (3, 4)

The coordinates of each point are (*input*, *output*).

c



Plot each (x, y) pair as a point: (x units across from O, y units up).

8 For the given rule *output* = *input* + 2:

a Copy and complete the given table of values.

Input (x)	*Output* (y)
0	2
1	
2	
3	

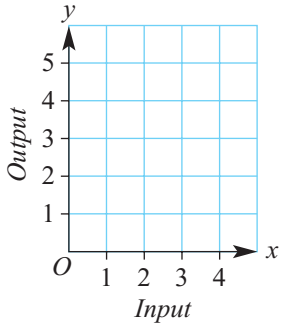
b List the coordinates of each point.



Coordinates are (*input*, *output*) = (x, y).

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c Plot each point on a number plane, like the one below.



Skillsheet 8D

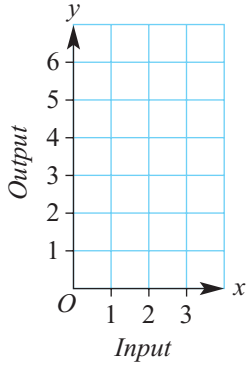
9 For the given rule *output* = *input* × 2:

a Copy and complete the given table of values.

Input (x)	*Output* (y)
0	
1	
2	
3	

b List the coordinates of each point.

c Plot each point on a number plane, like the one below.

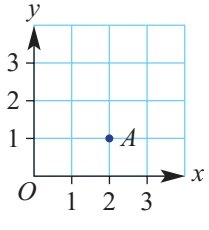


Problem-solving and Reasoning

10 Draw a number plane from 1 to 5 on both axes. Place a cross on five points with coordinates that have the same x value and y value.

11 a Which point is directly above point A?

A (2, 0)
B (0, 2)
C (2, 3)
D (3, 2)
E (1, 2)

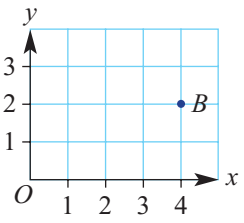


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11 b Which point is 2 units to the left of point B?

A (0, 2)
B (1, 2)
C (2, 2)
D (4, 0)
E (4, 4)



12 a Plot the following points on a number plane and join the points in the order given, to draw the basic shape of a house.
(1, 5), (0, 5), (5, 10), (10, 5), (1, 5), (1, 0), (9, 0), (9, 5)



Draw a number plane with 1 to 10 on both axes.

b Draw a door and list the coordinates of the four corners of the door.

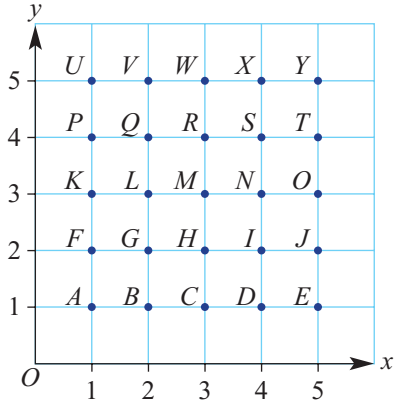
c Draw a window and list the coordinates of the four corners of the window.

d Draw a chimney and list the coordinates of the four points needed to draw the chimney.



Enrichment: Secret messages

13 A grid system can be used to make secret messages. Lars decides to arrange the letters of the alphabet on a number plane in the following manner.



a Decode Lars' following message:
(3, 2), (5, 1), (2, 3), (1, 4)

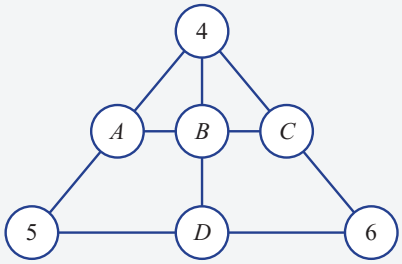
b Code the word 'secret'.

c To increase the difficulty of the code, Lars does not include brackets or commas and he uses the origin to indicate the end of a word.
What do the following numbers mean?
13515500154341513400145354001423114354.

d Code the phrase: 'BE HERE AT SEVEN'.

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1 Each line of three numbers in the triangle adds to 12.
Find the value of A, B, C and D.



2 Find the values of the pronumerals in the following tables.

a

			Sum
	a	b	c
	d	24	32
Sum	12	e	48

b

			Product
	a	b	18
	2	c	d
Product	12	e	180

3 Copy and complete the following table, in which x and y are always whole numbers.

x	2				
y	7	6		12	
$3x$		6	9		
$x + 2y$			9		7
xy				0	5

4 In a mini-sudoku, the digits 1 to 4 occupy each square. No row, column or 2×2 block has the same digit twice. Find the value of each of the pronumerals in the following mini-sudoku.

a	3	2	c
c	d	e	f
2	g	$d+1$	h
i	1	j	k

5 In a magic square the sum of each row, column and diagonal is the same. Find the value of the pronumerals to make the following into magic squares. Confirm your answer by writing out the magic square as a grid of numbers.

a

5	12	A
B	C	6
D	E	F

(Magic sum is 24.)

b

A	B	C
$A-1$	$A+1$	$B-C$
$B-1$	$C-1$	$A+C$

(Magic sum is 15.)

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Chapter summary

Algebraic techniques

Creating expressions

6 more than k: $k + 6$
Product of 4 and x: $4x$
10 less than b: $b - 10$
Half of q: $\frac{q}{2}$
The sum of a and b is tripled: $3(a + b)$

Mathematical convention

$3a$ means $3 \times a$.
$\frac{a}{10}$ means $a \div 10$.
$1a$ is just a.
$a \div 2$ is $\frac{1}{2}a$ or $\frac{a}{2}$.
$\frac{a}{1}$ is a.
$0a$ is 0.
$x \div x$ is 1.
$\frac{x}{x}$ is 1.

Algebraic expressions

Combinations of numbers, pronumerals and operations
e.g. $2xy + 3yz$, $\frac{12}{x} - 3$

Pronumerals

Letters used to represent numbers

Terms

A component of an expression
e.g. $4x$, $10y$, $\frac{a}{3}$, 12
In $4x + 5$:
- There are two terms.
- The constant term is 5.
- The coefficient of x is 4.

Equivalent expressions

Always equal when pronumerals are substituted
e.g. $2x + 3$ and $3 + 2x$ are equivalent.
$4(3x)$ and $12x$ are equivalent.

To **simplify** an expression, find a simpler expression that is equivalent.

Substitution

Replacing pronumerals with values
e.g. $5x + 2y$ when $x = 10$ and $y = 3$ becomes $5(10) + 2(3) = 50 + 6 = 56$

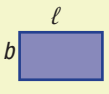
Combining like terms

Gives a way to simplify
e.g. $4a + 2 + 3a = 7a + 2$
$3b + 5c + 2b - c = 5b + 4c$
$12xy + 3x - 5yx = 7xy + 3x$

Like terms have exactly the same pronumerals.
$5a$ and $3a$ ✓
$2ab$ and $12ba$ ✓
$7ab$ and $2a$ ×
$2a$ and a^2 ×

Applications

Expressions are used widely
- Area is $\ell \times b$
- Perimeter is $2\ell + 2b$



- Cost is $50 + 90x$

call-out fee: 50
hourly rate: $90x$

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 In the expression $3x + 2y + 7z$ the coefficient of y is:
A 3 **B** 2 **C** 4 **D** 7 **E** 16

2 If $b = 7$, then $b + 5$ is equal to:
A 5 **B** 7 **C** 57 **D** 12 **E** 75

3 If $t = 5$ and $u = 7$, then $2t + u$ is equal to:
A 17 **B** 32 **C** 24 **D** 257 **E** 70

4 When two expressions are always equal (e.g. $2k$ and $k + k$), they are called:
A pronumerals **B** equivalent **C** coefficient
D variables **E** constant terms


Drilling for Gold 8R1 8R2

5 Which of the following pairs does *not* consist of two like terms?
A $3x$ and $5x$ **B** $3y$ and $12y$ **C** $3ab$ and $2ab$
D $3d$ and $5c$ **E** $3xy$ and xy

6 How many terms are there in the expression $3a + 4b + 5c + 6d$?
A 1 **B** 2 **C** 3 **D** 4 **E** 6

7 A fully simplified expression equivalent to $2a + 4 + 3b + 5a$ is:
A 4 **B** $5a + 5b + 4$ **C** $10ab + 4$
D $7a + 3b + 4$ **E** $11ab$

8 The simplified form of $4x \times 3yz$ is:
A $43xyz$ **B** $12xy$ **C** $12xyz$ **D** $12yz$ **E** $4x3yz$

9 The simplified form of $\frac{21ab}{3ac}$ is:
A $\frac{7b}{c}$ **B** $\frac{7ab}{ac}$ **C** $\frac{21b}{3c}$ **D** 7 **E** $\frac{b}{7c}$

10 A number is doubled and then 5 is added. The result is tripled. The number is represented by k. An expression for this description is:
A $3(2k + 5)$ **B** $6(k + 5)$ **C** $2k + 5$ **D** $2k + 15$ **E** $30k$

Short-answer questions

1 **a** How many terms are in the expression $5a + 3b + 7c + 12$?
b What is the constant term in the expression above?

2 Write an expression for each of the following.
a 7 is added to u **b** k is tripled **c** 10 is subtracted from h

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Chapter review

3 Find the value of $x + 4$ when:
a x is 2 **b** x is 100 **c** x is 12 **d** x is 17

4 When $k = 8$, evaluate:
a $k + 5$ **b** $3k$ **c** $k - 6$ **d** $2k + 1$

5 When $u = 12$, find the value of:
a $u + 3$ **b** $2u$ **c** $\frac{24}{u}$ **d** $\frac{u}{3}$

6 When $p = 3$ and $q = 5$, find the value of:
a pq **b** $p + q$ **c** $2(q - p)$ **d** $4p + 3q$

7 Copy and complete this table.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$4x$				
$3x + x$				

8 Classify the following pairs of expressions as equivalent (E) or not equivalent (N).
a $5x$ and $2x + 3x$ **b** $4x + 7 + 2x$ and $13x$
c $3c - c$ and $2c$ **d** $2 + 3b$ and $3 + 2b$

9 Classify the following pairs as like terms (L) or not like terms (N).
a $2x$ and $5x$ **b** $7ab$ and $2a$ **c** $3p$ and p
d $4ab$ and $4aba$ **e** $8t$ and $2t$ **f** $3p$ and 3

10 Simplify the following by collecting like terms.
a $2x + 3 + 5x$ **b** $12p - 3p + 2p$ **c** $12b + 4a + 2b + 3a + 4$
d $12mn + 3m + 2n + 5mn$ **e** $1 + 2c + 4h - 3o + 5c$ **f** $7u + 3v + 2uv - 3u$

11 Simplify the following expressions involving products.
a $3a \times 4b$ **b** $2xy \times 3z$ **c** $12f \times g \times 3h$ **d** $8k \times 2 \times 4lm$

12 Simplify the following expressions involving quotients.
a $\frac{3u}{2u}$ **b** $\frac{12y}{20y}$ **c** $\frac{2ab}{6b}$ **d** $\frac{12xy}{9yz}$

13 If a tin of paint weighs 9 kg, write an expression for the weight of t tins of paint.

14 If there are g girls and b boys in a class, write an expression for the total number of children.

15 Analena owns x fiction books and twice as many non-fiction books. Write an expression for the total number of books that Analena owns.

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Extended-response questions

1 To hire a tennis court, Cat must pay $\$20$ per hour. The total cost for n hours is $20n$.

- **a** How much would it cost in total to hire the court for 3 hours?
- **b** If $n = 3$, state the value of $20n$.
- **c** If Cat wants to hire the court at night she must pay $\$30$ per hour to pay for the lights. How much would 3 hours at night cost?
- **d** Write an expression for the total cost of hiring the court for n hours at night.
- **e** On one occasion, Cat hired the court for 2 hours during the day and then another 2 hours during the night. What was the total cost?



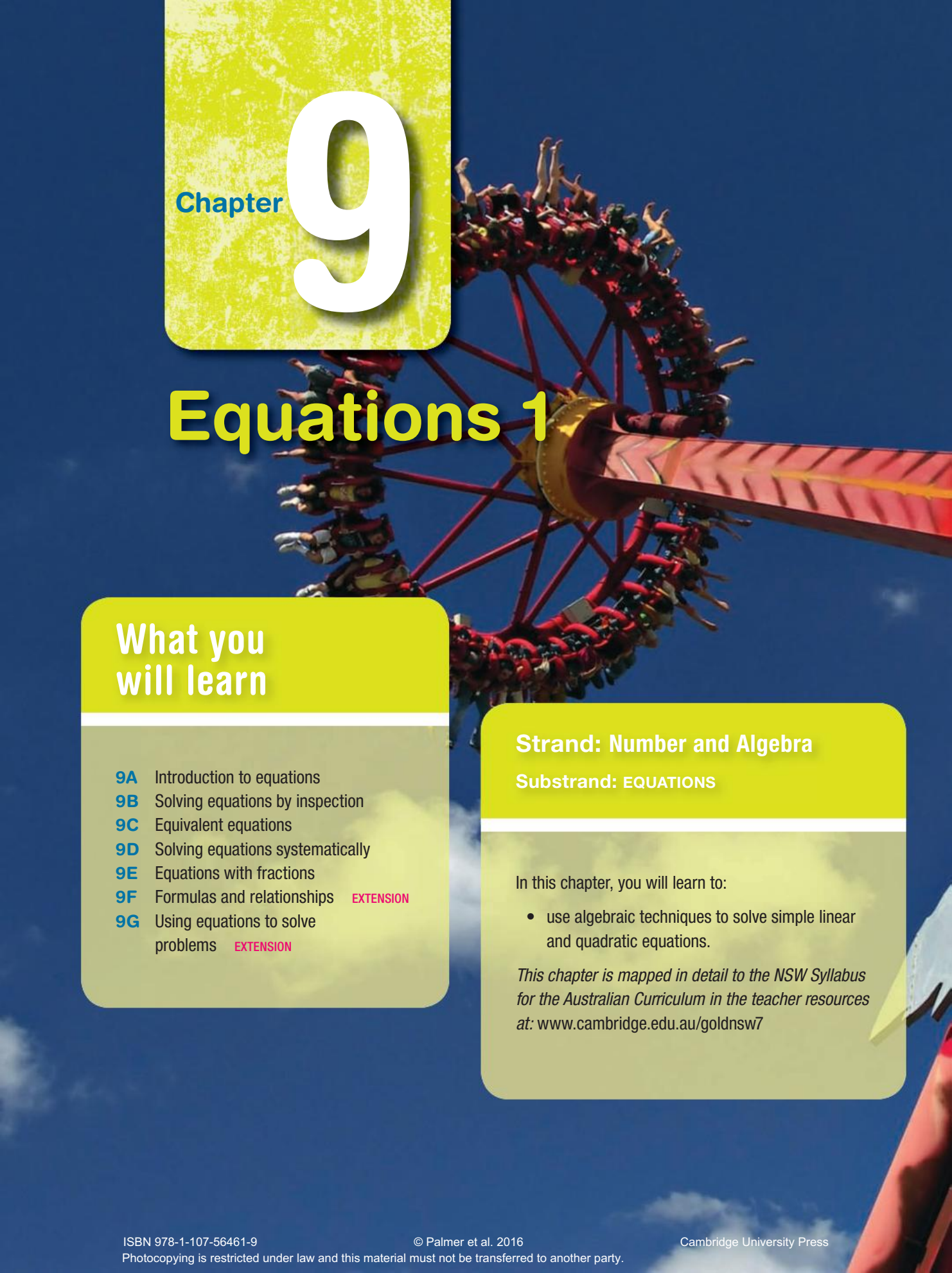
2 A taxi driver charges $\$2$ to pick up passengers and then $\$1.50$ per kilometre travelled.

- **a** State the total cost if the trip length is:
  - **i** 10 kilometres
  - **ii** 20 kilometres
  - **iii** 100 kilometres
- **b** Write an expression for the total cost of travelling a distance of k kilometres.
- **c** Use your expression to find the total cost of travelling 40 kilometres.
- **d** Another taxi driver charges $\$6$ to pick up passengers and then $\$1.20$ per kilometre. Write an expression for the total cost of travelling k kilometres in this taxi.

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Chapter 9

Equations 1



What you will learn

9A Introduction to equations
9B Solving equations by inspection
9C Equivalent equations
9D Solving equations systematically
9E Equations with fractions
9F Formulas and relationships EXTENSION
9G Using equations to solve problems EXTENSION

Strand: Number and Algebra

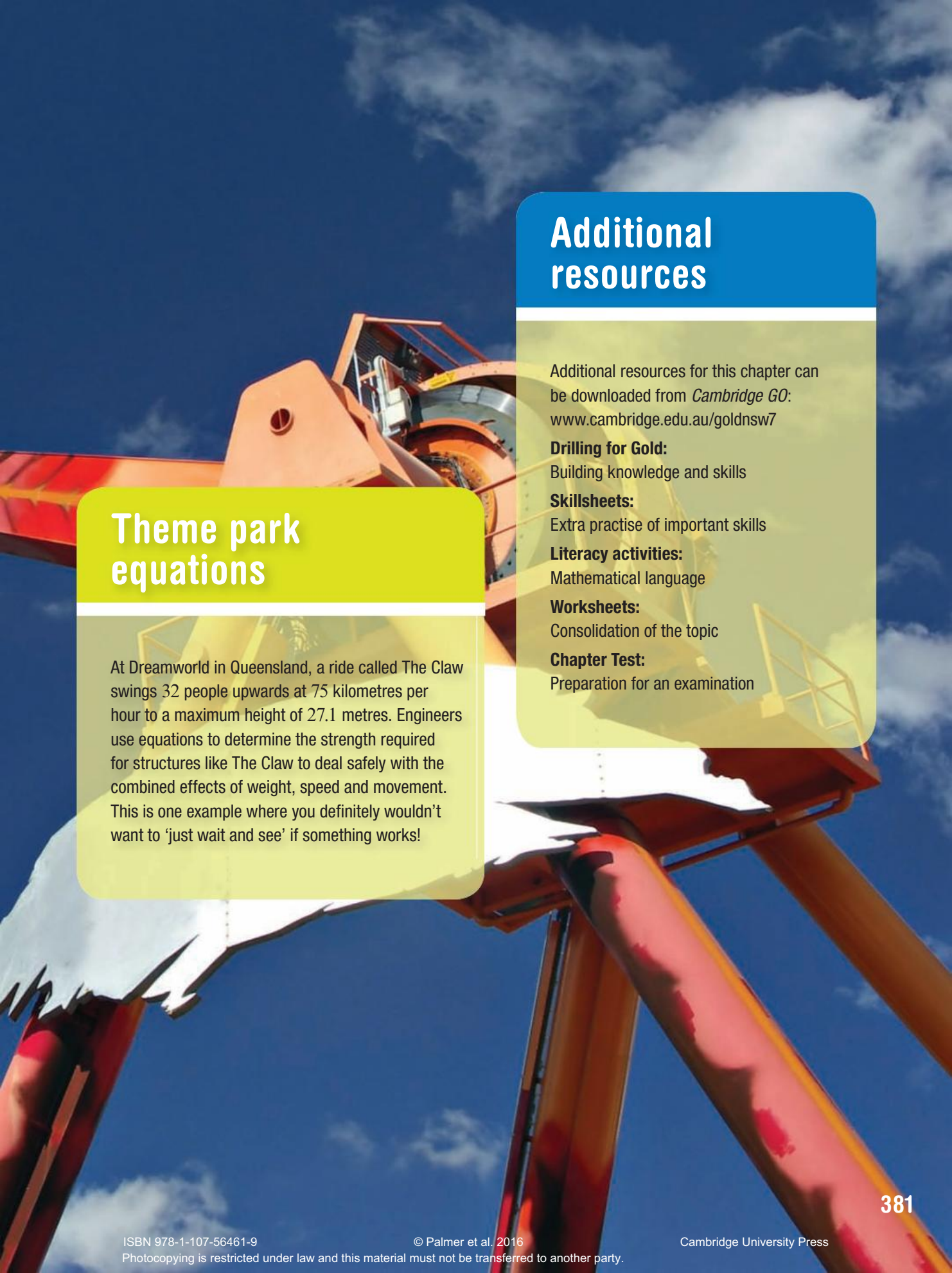
Substrand: EQUATIONS

In this chapter, you will learn to:

- use algebraic techniques to solve simple linear and quadratic equations.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Theme park equations

At Dreamworld in Queensland, a ride called The Claw swings 32 people upwards at 75 kilometres per hour to a maximum height of 27.1 metres. Engineers use equations to determine the strength required for structures like The Claw to deal safely with the combined effects of weight, speed and movement. This is one example where you definitely wouldn't want to 'just wait and see' if something works!

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Fill in each missing number.

a $7 + 5 = \square$ **b** $3 + 12 = \square$ **c** $2 \times 8 = \square$ **d** $19 - 12 = \square$

2 Fill in the missing numbers.

a $\square + 3 = 10$ **b** $50 - \square = 30$ **c** $\square + 3 = 19$ **d** $100 \div \square = 20$

3 If $\square = 5$, state whether each of these equations is true or false.

a $\square - 2 = 5$ **b** $\square \times 3 = 15$ **c** $20 \div 4 = \square$ **d** $7 \times \square = 42$

4 When $a = 3$, find the value of:

a $a + 4$ **b** $8 - a$ **c** $a \times 5$ **d** $a + 21$

5 When $n = 6$, state the value of:

a $n \div 2$ **b** $n \times 4 + 3$ **c** $8 - n$ **d** $12 \div n + 4$

6 The expression $n + 3$ can be described as 'the sum of n and 3'. Write expressions for:

a the sum of k and 5 **b** double p **c** 7 lots of y

7 Copy and complete the tables.

a

n	1	2	3	4	5
$5 \times n$	5				

b

n	2	4	6	8	10
$n - 2$					

c

n	1	2	5	8	9
$2n$					

8 For each of the following operations, state the opposite operation.

a $\times$ **b** $+$ **c** $\div$ **d** $-$

9 Find each of the following.

a the sum of 15 and 12
b the product of 8 and 5
c triple 6
d double 8, then add 10
e 12 more than 5
f 20 divided by 5, then add 10
g add 10 to 20, then divide that answer by 5

10 a If Mia is 12 years old and Oliver is 15 years old, what will their ages be in 5 years' time?

b Ethan is paid \$7 per hour for mowing lawns. How much would he earn in 4 hours?

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9A Introduction to equations

Consider the equation $\frac{f}{5} + 2 = 8$.

To the left of the equals sign is the expression $\frac{f}{5} + 2$. To the right is the expression 8.

In this example, there is only one value of f that will make this equation true. The value of f is 30.



Let's start: Equations – True or false?

Rearrange the following five symbols to make four different statements.

$$5,\ 2,\ 3,\ +,\ =$$

- Which of them are true? Which are false?
- How many true statements could you make?

Key ideas

Drilling for Gold 9A1

- An expression contains no equals sign (e.g. $2x + 3$).
- An **equation** is a mathematical statement that two expressions are equal.
- Equations have a left-hand side (LHS) and a right-hand side (RHS), with an equals sign in between.

$$\underbrace{2x + 3}_{\text{LHS}} = \underbrace{4y - 2}_{\text{RHS}}$$

equals sign

- Equations can be true (e.g. $2 + 3 = 5$) or false (e.g. $5 + 7 = 21$).
- The value(s) that make a solution true are called solutions.

  For example: The solution of $3x = 12$ is $x = 4$.

Equation
A statement in which it may be possible to find value(s) for the pronumerals that will make the statement true. The values are called solutions

Exercise 9A

Understanding

1 **a** Is $7 + 4 = 11$ true or false?
b Is $3 \times 2 = 6$ true or false?
c Is $2 = 6 - 1$ true or false?

2 Classify each of these statements as true or false.
a $2 + 3 = 5$ **b** $3 + 2 = 6$ **c** $6 = 5 - 1$ **d** $6 = 2 + 4$

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9A

3 Put a number in the box to make a true statement.

a $7 + 2 = \square$ **b** $\square = 5 \times 3$ **c** $10 = 8 + \square$ **d** $20 = 2 \times \square$

4 If k is 5, what is the value of $4 + k$?

Example 1 Identifying equations

Which of the following are equations?

a $3 + 5 = 8$ **b** $7 + 7 = 18$ **c** $2 + 12$ **d** $4 = 12 - x$ **e** $3 + u$

Solution	Explanation
a $3 + 5 = 8$ is an equation.	There are two expressions (i.e. $3 + 5$ and 8) separated by an equals sign.
b $7 + 7 = 18$ is an equation.	There are two expressions separated by an equals sign. Although this equation is false, it is still an equation.
c $2 + 12$ is not an equation.	This is just a single expression. There is no equals sign.
d $4 = 12 - x$ is an equation.	There are two expressions separated by an equals sign.
e $3 + u$ is not an equation.	There is no equals sign, so this is not an equation.

5 Classify each of the following as an equation (E) or not an equation (N).

a $7 + x = 9$ **b** $2 + 2$ **c** $2 \times 5 = t$
d $10 = 5 + x$ **e** $2 = 2$ **f** $7 \times u$
g $10 \div 4 = 3p$ **h** $3 = e + 2$ **i** $x + 5$

Fluency

Example 2 Classifying statements by comparing sides

State whether the following statements are true or false.

a $10 + 15 = 30 - 5$ **b** $2 + 5 + 6 = 10 - 3$

Solution	Explanation
a true	LHS (left-hand side) is 25. RHS is 25. LHS = RHS, so statement is true.
b false	LHS = 13 and RHS = 7. They are different so the statement is false.

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6 Write true or false for each of the following statements.

a $3 \times 2 = 5 + 1$ **b** $100 - 90 = 2 \times 5$ **c** $30 \times 2 = 32 + 5$

d $12 - 4 = 4$ **e** $2(3 - 1) = 4 + 2$ **f** $5 - (2 + 1) = 7 - 4$

g $2 = 17 - 14 - 1$ **h** $10 + 2 = 12 - 4$ **i** $2 \times 3 = 1 + 2 + 3$

7 If $x = 3$, state whether each of these equations is true or false.

a $5 + x = 7$ **b** $x + 1 = 4$

c $10 = 13 + x$ **d** $6 = 2x$



8 Consider the equation $4 + 3x = 2x + 9$.

a When $x = 5$, state the value of the left-hand side (LHS).

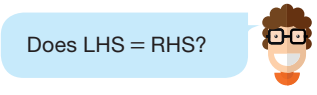
b When $x = 5$, state the value of the right-hand side (RHS).

c Is the equation $4 + 3x = 2x + 9$ true or false when $x = 5$?

9 If $b = 4$, are the following statements true or false?

a $5b + 2 = 22$ **b** $10 \times (b - 3) = b + b + 2$

c $12 - 3b = 5 - b$ **d** $b \times (b + 1) = 20$



Problem-solving and Reasoning



Example 3 Writing equations from a description

Write equations for each of the following.

a The sum of x and 5 is 22.

b A deck of cards costs $\$x$. The cost of 7 decks is $91.

c Priya's age is currently j. In 5 years' time her age will equal 17.

Solution	Explanation
a $x + 5 = 22$	The sum of x and 5 is written $x + 5$.
b $7x = 91$	$7x$ means $7 \times x$ and this number must equal the total cost.
c $j + 5 = 17$	In 5 years' time Priya's age will be 5 more than her current age, so $j + 5$ must be 17.

10 Write equations for each of the following.

a The sum of 3 and x is equal to 10.

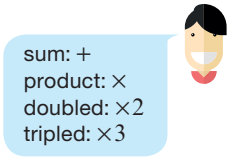
b When k is multiplied by 5, the result is 1005.

c The sum of a and b is 22.

d When d is doubled, the result is 78.

e The product of 8 and x is 56.

f When p is tripled, the result is 21.



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9A

11 Write an equation for each of the following, using the letter n to represent the unknown number.



- **a** A number is doubled and the result is 240.
- **b** A number is increased by 2 and the result is 240.
- **c** Half of a number is 240.
- **d** The number 240 is two less than a number.
- **e** The sum of double a number and triple the number is 240.
- **f** A number is multiplied by 3 and the result is increased by 2 to give 240.
- **g** A number is increased by 2 and the result is multiplied by 3 to give 240.
- **h** An even number and the following even number are added to give 240.
- **i** The sum of three consecutive whole numbers is 240.
- **j** A number is tripled and then halved to give 240.
- **k** Half of one-third of a number is 240.
- **l** The sum of two more than a number and half of the number is 240.
- **m** The sum of half of a number and one-third of a number is 240.
- **n** The difference between a number and half of the number is 240.
- **o** Half of a number is 240 more than one-third of the number.
- **p** The square of a number is 240.
- **q** The sum of a number and the square of a number is 240.

12 Write true equations for each of these problems. You do not need to solve them.

- **a** Chairs cost $\$c$ at a store. The cost of 6 chairs is \$546.
- **b** A plumber charges $\$k$ per hour. The cost of 7 hours' work is \$567.
- **c** Pens cost $\$a$ each and pencils cost $\$b$. Twelve pens and three pencils cost \$28 in total.
- **d** Amy is f years old. In 10 years' time her age will be 27.



13 a Find a value of m that would make this equation true: $10 = m + 7$.

- **b** Find a possible value of k that would make this equation true: $k \times (8 - k) = 12$.



Enrichment: Equation permutations

14 For each of the following, rearrange the symbols to make a true equation.

- **a** 6, 2, 3, ×, =
- **b** 1, 4, 5, –, =
- **c** 2, 2, 7, 10, –, ÷, =
- **d** 2, 4, 5, 10, –, ÷, =

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9B Solving equations by inspection

Solving an equation involves finding the value of the pronumeral that makes it true. Some equations can be solved by trying different numbers until one works or using known number facts. Others are not so easy to solve. Later in the chapter you will learn a systematic method for solving equations.

$x + 5 = 12$ $x = 7$ Easily solved!	$\frac{2x}{3} - 7 = 11$ $x = ???$ You need a plan B!
Section 9B	Section 9C to 9G

Let's start: Finding the missing value

- What number goes in the box?
  $16 - \square = 9$ $\quad 27 = 15 + 3 \times \square$ $\quad 2 \times \square + 4 = 17$ $\quad 2 \times \square \div 3 - 7 = 11$
- Which equations were easy?

Key ideas

- In the equation $x + 5 = 12$, the pronumeral x is called an **unknown**.
- In the equation $x + 5 = 12$, there is only one value of x that makes the equation true.
- Some equations can be solved without using a formal method.
- It is quite easy to **solve** the equation $x + 5 = 12$ if you know that $7 + 5 = 12$. This is called solving by inspection.
- The **solution** is $x = 7$ because $7 + 5 = 12$.

Unknown The pronumeral in an equation

Solve To find the value(s) of the unknown(s) so that the equation is a true statement

Solution The value of an unknown that makes an equation true

Exercise 9B

Understanding

1 State whether each of the following equations is true or false.

a $10 + 7 = 19$ **b** $5 = 12 - 7$

c $8 \times 2 = 3 + 11$ **d** $2 + 5 \times 3 = 17$

Use the order of operations:
$2 + \underbrace{5 \times 3}_{15} = 2 + 15$

2 If the missing number is 5, classify each of the following equations as true or false.

a $\square + 3 = 8$ **b** $10 \times \square + 2 = 46$ **c** $10 - \square = 5$ **d** $8 - \square = 3$

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9B

3 For the equation $\square + 7 = 13$:
- **a** Find the value of the LHS (left-hand side) when $\square = 5$.
- **b** Find the value of the LHS when $\square = 10$.
- **c** Find the value of the LHS when $\square = 6$.
- **d** What value of $\square$ would make the LHS equal to 13?

Example 4 Finding the solution by inspection

For each of these equations, find the value of the missing number that would make it true.

a $\square \times 7 = 35$ **b** $20 - \square = 14$

Solution	Explanation
a 5	Think: What number multiplied by 7 equals 35? $\boxed{5} \times 7 = 35$ is a true equation.
b 6	Think: 20 minus what number equals 14? $20 - \boxed{6} = 14$ is a true equation.

4 Find the value of the missing numbers.

a $4 + \square = 7$ **b** $2 \times \square = 12$ **c** $\square \times 4 = 80$ **d** $\square + 12 = 31$

5 What value should go in the box to make a true equation?

a $12 = 3 + \square$ **b** $5 = 8 - \square$ **c** $42 = \square \times 7$ **d** $8 = \square \div 3$

Fluency



Drilling for Gold 9B1

Example 5 Solving equations by inspection

Solve each of the following equations by inspection.

a $c + 12 = 30$ **b** $5 \times b = 20$ **c** $2x + 13 = 21$

Solution	Explanation
a $c + 12 = 30$ $c = 18$	$\boxed{18} + 12 = 30$ is a true equation.
b $5 \times b = 20$ $b = 4$	$5 \times \boxed{4} = 20$ is a true equation.
c $2x + 13 = 21$ $x = 4$	$2x$ means $2 \times x$. Trying a few values: $x = 10$ makes LHS $= 20 + 13 = 33$, which is too large. $x = 3$ makes LHS $= 6 + 13 = 19$, which is too small. $x = 4$ makes LHS $= 21$.

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6 Solve the following equations by inspection.

a $8 \times y = 64$ **b** $l \times 3 = 18$ **c** $4 - d = 2$

d $l + 2 = 14$ **e** $a - 2 = 4$ **f** $s + 7 = 19$

g $x \div 8 = 1$ **h** $12 = e + 4$ **i** $13 = 21 - s$

7 Solve the following equations by inspection. (See part **c** of Example 5.)

a $2p - 1 = 5$ **b** $3p + 2 = 14$ **c** $4q - 4 = 8$

d $4v + 4 = 24$ **e** $2b - 1 = 1$ **f** $5u + 1 = 21$

g $5g + 5 = 20$ **h** $3d - 5 = 13$ **i** $8 = 3m - 4$

8 Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

a $x = 6 - x$ **b** $7 + x = 2 \times x$

c $10 - x = x + 2$ **d** $15 - 2x = x$



An equation is true if its LHS and RHS are equal.

Problem-solving and Reasoning

9 Find the value of the number in each of these examples.

a A number is doubled and the result is 22.

b 3 less than a number is 9.

c Half of a number is 8.

d 7 more than a number is 40.

10 Justine is paid \$10 an hour for x hours. During a particular week, she earns \$180.

a Write an equation involving x to describe this situation.

b Solve the equation by inspection to find x.

11 Karim's weight is w kg and his brother is twice as heavy, weighing 70 kg.

a Write an equation involving w to describe this situation.

b Solve the equation by inspection to find w.



Karim's brother weighs $2w$ kg.

12 Yanni's current age is y years old. Twelve years from now he will be three times older.

a Write an equation involving y to describe this situation.

b Solve the equation by inspection to find y.

Enrichment: Multiple variables

13 Equations with two unknowns can sometimes be solved by inspection. For each of the following equations find, by inspection, one pair of values for x and y that make them true.

a $x + y = 8$ **b** $x - y = 2$

c $x \times y = 6$ **d** $x + y = x \times y$



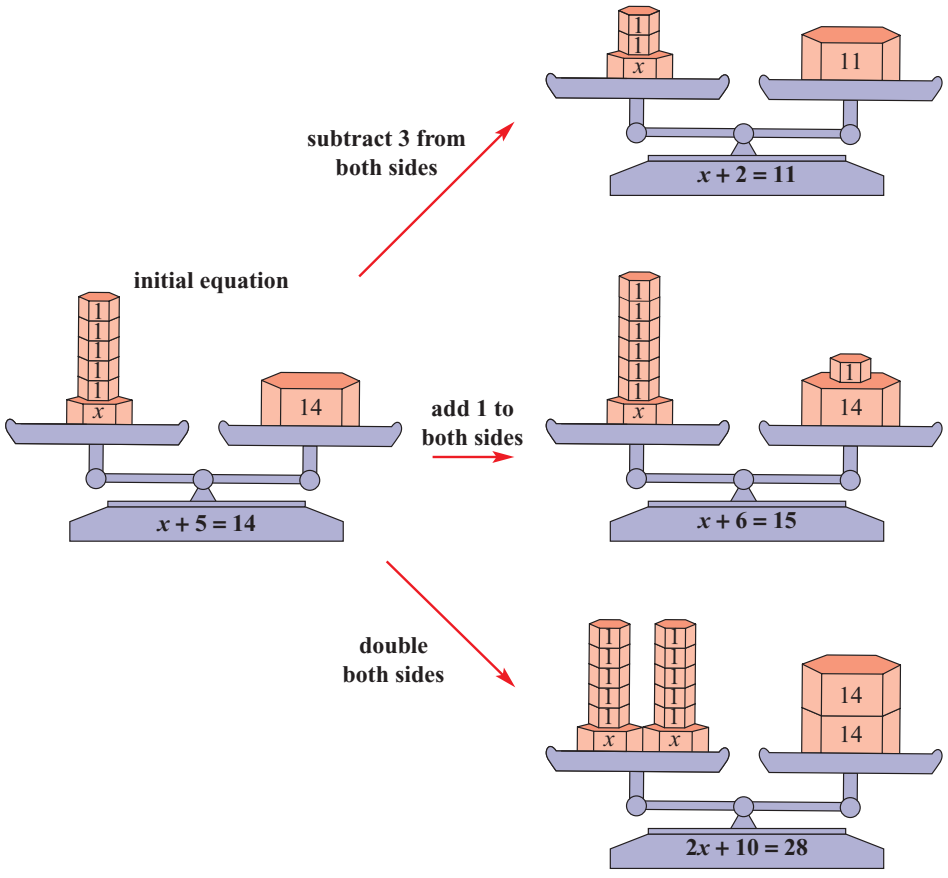
For $x + y = 8$, you could try:
If $x = 1$ and $y = 1$, $1 + 1 = 8$ (false)
If $x = 1$ and $y = 2$, $1 + 2 = 8$ (false)
If $x = 7$ and $y = 3$, $7 + 3 = 8$ (false)
Keep going until your equation is true.

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9C Equivalent equations

An equation is like a balanced scale. The two sides have equal mass.

Sometimes, two equations express the same thing. For example, the equations $x + 5 = 14$, $x + 6 = 15$ and $x + 7 = 16$ are all made true by the same value of x (i.e. $x = 9$).

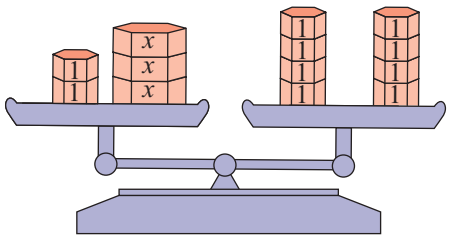


A true equation stays true if we 'do the same thing to both sides', such as adding a number or multiplying by a number. If we do the same thing to both sides then we will have an equivalent equation.

Let's start: Equations as scales

The scales in the diagram show $2 + 3x = 8$.

- What would the scales look like if two '1 kg' blocks were removed from both sides?
- What would the scales look like if the two '1 kg' blocks were removed only from the left-hand side? (Try to show whether they would be level.)
- Use scales to illustrate why $4x + 3 = 4$ and $4x = 1$ are equivalent equations.



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Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - Adding a number to both sides
  - Subtracting a number from both sides
  - Multiplying both sides by a number other than zero
  - Dividing both sides by a number other than zero
  - Swapping the left-hand side with the right-hand side.

Example 6 Applying an operation to LHS and RHS

For each equation, apply the given operation to both sides and then simplify.

a $2 + x = 5$ [add 4 to both sides] **b** $7x = 10$ [multiply both sides by 2]

c $30 = 20b$ [divide both sides by 10] **d** $7q - 4 = 10$ [add 4 to both sides]

Solution	Explanation
a $2 + x = 5$ → (+4) → $6 + x = 9$	4 is added to both sides.
b $7x = 10$ → (×2) → $14x = 20$	Both sides are multiplied by 2.
c $30 = 20b$ → (÷10) → $3 = 2b$	Both sides are divided by 10.
d $7q - 4 = 10$ → (+4) → $7q = 14$	4 is added to both sides.

Exercise 9C

Understanding

1 Add 10 to both sides of each of these equations.

a $10d + 5 = 20$ **b** $7e = 31$ **c** $2a = 12$ **d** $x = 12$

2 Match up each of these equations (**a** to **e**) with its equivalent equation (**A** to **E**), where 3 has been added to both sides.

a $10 + x = 14$

b $x + 1 = 13$

c $12 = x + 5$

d $x + 10 = 11x$

e $12x = 120$

A $12x + 3 = 123$

B $x + 13 = 11x + 3$

C $13 + x = 17$

D $x + 4 = 16$

E $15 = x + 8$

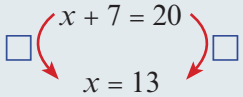
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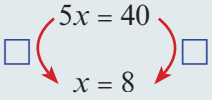
9C

Fluency

Example 7 Making equivalent equations

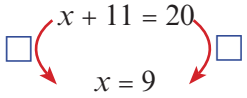
What is missing from the boxes?

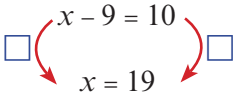
a 

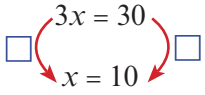
b 

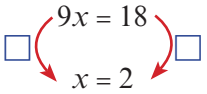
Solution	Explanation
a –7	7 has been subtracted from both sides.
b ÷5	Both sides have been divided by 5.

3 What is missing from the boxes?

a 

b 

c 

d 

4 Consider the equation $5x = 20$. Write down the equation you get when you:

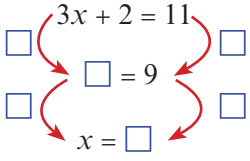
a add 3 to both sides
b subtract 3 from both sides
c multiply both sides by 3
d divide both sides by 5

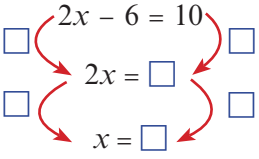
5 For each equation, show the result of applying the listed operations to both sides. (Note: [+1] means 'add 1 to both sides'.)

a $5 + x = 10$ [+1]
b $3x = 7$ [×2]
c $12 = 8q$ [÷4]
d $9 + a = 13$ [–3]
e $7 + b = 10$ [+5]
f $5 = 3b + 7$ [–5]
g $2 = 5 + a$ [+2]
h $12x - 3 = 3$ [+5]
i $7p - 2 = 10$ [+2]

Problem-solving and Reasoning

6 Copy and complete:

a 

b 

7 Which equations have the solution $x = 5$?

a $2x = 10$
b $2x = 25$
c $x \div 5 = 1$
d $3x - 1 = 16$
e $3x + 1 = 16$
f $6 = 1 + x$

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8 For each of the following equations, write an equivalent equation that you can get in one operation. Your equation should be simpler (i.e. smaller) than the original.

a $2q + 7 = 9$ **b** $10x + 3 = 10$ **c** $x \div 12 = 5$ **d** $6 + 2x = 11$



9 Sometimes two equations that look quite different can be equivalent. Show that $3x + 2 = 14$ and $10x + 1 = 41$ are equivalent by copying and completing the following.

$3x + 2 = 14$

-2 -2

$3x = 12$

$\div 3$ $\div 3$

$___ = ___$

$\times 10$ $\times 10$

$___ = ___$

$+1$ $+1$

$10x + 1 = 41$

Enrichment: Mutiplying both sides by zero is bad!

10 As stated in the rules for equivalence, which are listed in Key ideas, multiplying both sides by zero is not permitted.

a Write the result of multiplying both sides of the following statements by zero.

i $3 + x = 5$ **ii** $2 + 2 = 4$ **iii** $2 + 2 = 5$

b Explain in a sentence why multiplying by zero does not give a useful equivalent equation.

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9D Solving equations systematically

Solving equations by inspection can be very difficult when the solution is not a small whole number.

In this section, you will learn how to solve equations using a systematic process. A similar reversal of procedures occurs with equivalent equations.

Here are three equivalent equations.

$$x = 3$$
$$\times 2 \downarrow \quad \times 2 \downarrow$$
$$2x = 6$$
$$+4 \downarrow \quad +4 \downarrow$$
$$2x + 4 = 10$$

$$\frac{2x+1}{3} - \frac{x}{5} = 11$$

Some equations are not easily solved by inspection.

We can undo the operations around x by doing the opposite operation in the reverse order.

$$2x + 4 = 10$$
$$-4 \downarrow \quad -4 \downarrow$$
$$2x = 6$$
$$\div 2 \downarrow \quad \div 2 \downarrow$$
$$x = 3$$

Because these equations are equivalent, this means that the solution to $2x + 4 = 10$ is $x = 3$.

Let's start: Attempting solutions

Georgia, Khartik and Lucas try to solve the equation $4x + 8 = 40$.

They present their attempted solutions below.

Georgia

$$4x + 8 = 40$$
$$\div 4 \downarrow \quad \div 4 \downarrow$$
$$x + 8 = 10$$
$$-8 \downarrow \quad -8 \downarrow$$
$$x = 2$$

Khartik

$$4x + 8 = 40$$
$$-8 \downarrow \quad +8 \downarrow$$
$$4x = 48$$
$$\div 4 \downarrow \quad \div 4 \downarrow$$
$$x = 12$$

Lucas

$$4x + 8 = 40$$
$$-8 \downarrow \quad -8 \downarrow$$
$$4x = 32$$
$$\div 4 \downarrow \quad \div 4 \downarrow$$
$$x = 8$$

- Which of the students has the correct solution? Justify your choice by substituting each student's final answer into the original equation.
- For each of the two students with the incorrect answer, explain the mistake they have made in their attempt to make equivalent equations.
- What operations would you do to both sides if the original equation was $7x - 10 = 11$?

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Key ideas

- Sometimes it is very difficult to solve an equation by inspection, so a systematic approach is required.
- To solve an equation, find a simpler equivalent equation. Repeat this process until the solution is found. For example:

$$5x + 2 = 17 \quad (-2, -2)$$
$$5x = 15 \quad (\div 5, \div 5)$$
$$x = 3$$

- A solution can be checked by **substituting** the value to see if the equation is true (i.e. LHS = RHS).

$$\text{LHS} = 5x + 2 \qquad \text{RHS} = 17 \checkmark$$
$$= 5 \times 3 + 2$$
$$= 17 \checkmark$$

Substituting
Replacing pronumerals with numerical values

Exercise 9D

Understanding

1 State whether each of the following equations is true or false.

a $x + 4 = 7$, when $x = 3$.
b $b - 2 = 7$, when $b = 5$.
c $f \times 4 = 20$, when $f = 3$.
d $g + 5 = 3g$, when $g = 2$.

2 Consider the equation $7x = 42$.

a Copy and complete the following.

$$7x = 42 \quad (\div 7, \div 7)$$
$$x = __$$

b What is the solution to the equation $7x = 42$?

3 Copy and complete the following, showing which operation was used. Remember that the same operation must be used for both sides!

a $5 + a = 30$ ☐ ☐ $a = 25$

b $10b = 72$ ☐ ☐ $b = 7.2$

c $12 = \frac{c}{4}$ ☐ ☐ $48 = c$

d $8 = c - 12$ ☐ ☐ $20 = c$

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9D

Fluency

Example 8 Solving one-step equations

Solve each of the following equations systematically.

a $5x = 30$ **b** $17 = y - 21$ **c** $10 = \frac{q}{3}$

Solution	Explanation
a $5x = 30$ $\div 5$ ↙ ↘ $\div 5$ $x = 6$ The solution is $x = 6$.	The opposite of $\times 5$ is $\div 5$. By dividing both sides by 5, we get an equivalent equation. Recall that $5x \div 5$ simplifies to x.
b $17 = y - 21$ $+21$ ↙ ↘ $+21$ $38 = y$ The solution is $y = 38$.	The opposite of -21 is $+21$. Write the pronumeral on the LHS.
c $10 = \frac{q}{3}$ $\times 3$ ↙ ↘ $\times 3$ $30 = q$ The solution is $q = 30$.	Multiplying both sides by 3 gives an equivalent equation that is simpler. Note that $\frac{q}{3} \times 3 = q$. Write the pronumeral on the LHS.

4 Solve the following equations systematically.

a $6m = 54$	**b** $g - 9 = 2$	**c** $s \times 9 = 81$	**d** $i - 9 = 1$
e $7 + t = 9$	**f** $8 + q = 11$	**g** $4y = 48$	**h** $7 + s = 19$
i $24 = j \times 6$	**j** $12 = \ell + 8$	**k** $1 = v \div 2$	**l** $19 = 7 + y$
m $k \div 5 = 1$	**n** $2 = y - 7$	**o** $8z = 56$	**p** $13 = 3 + t$
q $b \times 10 = 120$	**r** $p - 2 = 9$	**s** $5 + a = 13$	**t** $n - 2 = 1$

5 Copy and complete the following to solve the given equations systematically.

a
$7a + 3 = 38$
□ ↙ ↘ □
$7a = 35$
□ ↙ ↘ □
$__ = __$

b
$4b - 10 = 14$
$+10$ ↙ ↘ $+10$
$__ = __$
□ ↙ ↘ □
$__ = __$

c
$2(q + 6) = 20$
$\div 2$ ↙ ↘ $\div 2$
$q + 6 = __$
□ ↙ ↘ □
$__ = __$

d
$5 = \frac{x}{10} + 3$
-3 ↙ ↘ -3
$__ = \frac{x}{10}$
□ ↙ ↘ □
$__ = __$

6 For each equation, state the first operation you would apply to both sides to solve it.

a $2x + 3 = 9$	**b** $4x - 7 = 33$
c $5a + 15 = 50$	**d** $22 = 2b - 34$

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Example 9 Solving two-step equations

Solve each of the following equations systematically and check the solution.

a $7 + 4a = 23$ **b** $3d - 2 = 16$ **c** $13 = 4 + 3e$

Solution	Explanation
a $7 + 4a = 23$ -7 / -7 $4a = 16$ $\div 4$ / $\div 4$ $a = 4$	At each step, try to make the equation simpler by applying an operation to both sides. The opposite of $+7$ is -7 and the opposite of $\times 4$ is $\div 4$.
Check: LHS $= 7 + 4a$ RHS $= 23$ ✓ $= 7 + 4 \times 4$ $= 7 + 16$ $= 23$ ✓	Check that the solution is correct by substituting $a = 4$ back into the equation to show LHS = RHS.
b $3d - 2 = 16$ $+2$ / $+2$ $3d = 18$ $\div 3$ / $\div 3$ $d = 6$	At each step, try to make the equation simpler by applying an operation to both sides. The opposite of -2 is $+2$ and the opposite of $\times 3$ is $\div 3$.
Check: LHS $= 3d - 2$ RHS $= 16$ ✓ $= 3 \times 6 - 2$ $= 18 - 2$ $= 16$ ✓	Check that the solution is correct by substituting $d = 6$ back into the equation to show LHS = RHS.
c $13 = 4 + 3e$ -4 / -4 $9 = 3e$ $\div 3$ / $\div 3$ $3 = e$	At each step, try to make the equation simpler by applying an operation to both sides. The opposite of -4 is $+4$ and the opposite of $\div 3$ is $\times 3$.
The solution is $e = 3$.	Swap LHS & RHS. Write $e = 3$, not $3 = e$.
Check: LHS $= 13$ ✓ RHS $= 4 + 3e$ $= 4 + 3 \times 3$ $= 4 + 9$ $= 13$ ✓	Check that the solution is correct by substituting $e = 3$ back into the equation to show LHS = RHS.

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9D

7 For each of the following equations:
- **i** Solve the equation systematically, showing your steps.
- **ii** Check your solution by substituting the value into the LHS and RHS.

a $6f - 2 = 64$ **b** $4k + 9 = 29$ **c** $5x - 4 = 41$
d $3a - 24 = 3$ **e** $5k - 9 = 31$ **f** $3a + 6 = 36$
g $2n - 8 = 14$ **h** $4n + 6 = 10$ **i** $1 = 2g - 7$
j $30 = 3q - 3$ **k** $3z - 4 = 26$ **l** $17 = 9 + 8p$
m $10d + 7 = 47$ **n** $38 = 6t - 10$ **o** $9u + 2 = 47$
p $7 = 10c - 3$ **q** $10 + 8q = 98$ **r** $80 = 4y + 32$
s $4q + 32 = 40$ **t** $7 + 6u = 67$

8 Solve the following equations, giving your solutions as fractions.

a $4x + 5 = 8$ **b** $3 + 5k = 27$ **c** $22 = 6w + 14$
d $10 = 6 + 3x$ **e** $6 = 8x + 1$ **f** $3x + 6 = 7$

9 Solve the following equations systematically.
(Note: The solutions for these equations are negative numbers.)

a $4r + 30 = 2$ **b** $2x + 12 = 6$ **c** $20 + t = 4$
d $y + 40 = 16$ **e** $-3x = 15$ **f** $4 = 2k + 22$
g $2x = -12$ **h** $5x + 20 = 0$ **i** $0 = 2x + 3$

Problem-solving and Reasoning

10 For each of the following, write an equation and solve it systematically.
- **a** The sum of x and 5 is 12.
- **b** The product of 2 and y is 10.
- **c** When b is doubled and then 6 is added, the result is 44.
- **d** 7 is subtracted from k. This result is tripled, giving 18.

11 Danny gets paid \$12 per hour, plus a bonus of \$50 for each week. In one week he earned \$410.
- **a** Write an equation to describe this, using n for the number of hours worked.
- **b** Solve the equation systematically and state the number of hours worked.

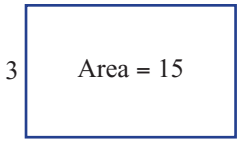


12 Jenny buys 12 pencils and 5 pens for the new school year. The pencils cost \$1.00 each.
- **a** If pens cost \$$x$ each, write an expression for the total cost, in dollars.
- **b** The total cost was \$14.50. Write an equation to describe this.
- **c** Solve the equation systematically, to find the total cost of each pen.
- **d** Check your solution by substituting your value of x into $12 + 5x$.

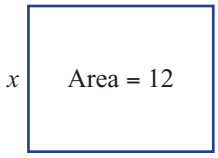
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13 Write equations and solve them systematically to find the value of the pronumeral in each of the following diagrams.

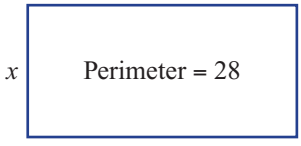
a



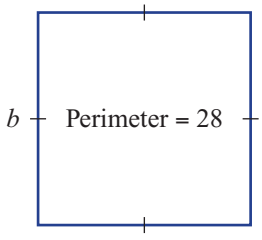
b



c

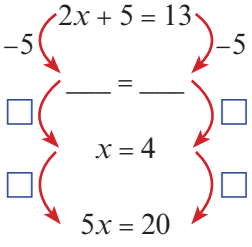


d



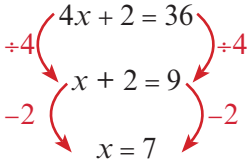
14 Write five different equations that give a solution of $x = 2$.

15 Show that $2x + 5 = 13$ and $5x = 20$ are equivalent by filling in the missing steps.

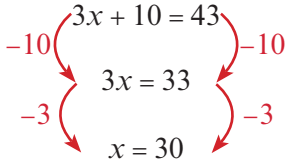


16 Nicola has attempted to solve four equations. Describe the error she has made in each case.

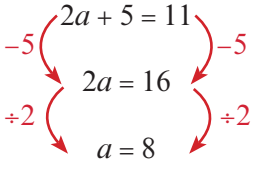
a



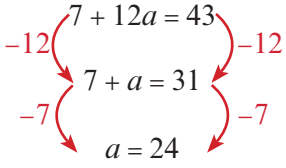
b



c



d



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9D

Enrichment: Equations with pronumerals on both sides

17 If an equation has a pronumeral on both sides, you can subtract it from one side and then use the same method as before. For example:

$5x + 4 = 3x + 10$

$-3x$ $-3x$

$2x + 4 = 10$

-4 -4

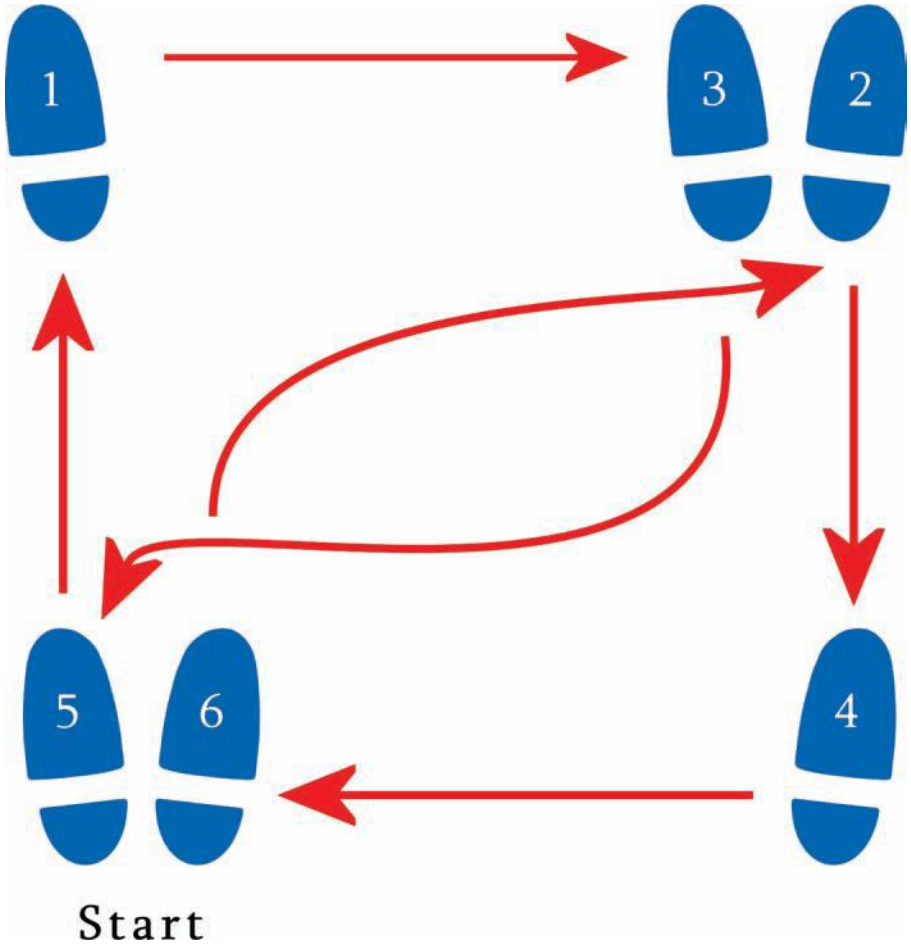
$2x = 6$

$\div 2$ $\div 2$

$x = 3$

Solve the following equations using this method.

a $5x + 2 = 3x + 10$
b $8x - 1 = 4x + 3$
c $5 + 12\ell = 20 + 7\ell$
d $2 + 5t = 4t + 3$
e $12s + 4 = 9 + 11s$
f $9b - 10 = 8b + 9$
g $5j + 4 = 10 + 2j$
h $3 + 5d = 6 + 2d$



Just like dance steps, a strict order must be followed when solving equations systematically.

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9E Equations with fractions

Equations that involve fractions can be difficult to solve by inspection.

The expression $\frac{x}{5}$ can also be written as $x \div 5$.

Equations such as $\frac{x}{5} = 8$ can be solved systematically or by inspection.

$\frac{x}{5} = 8$

$\times 5$ ↓ ↓ $\times 5$

$x = 40$

Let's start: Who is correct?

Steve and Lauren were asked to solve the following problem.

A number is divided by 3 and then increased by 5.
The result is 15.
What is the number?

They both wrote the equation $\frac{x}{3} + 5 = 15$ and then attempted to solve it systematically.

Steve did this:

$\frac{x}{3} + 5 = 15$

$\times 3$ ↓ ↓ $\times 3$

$x + 5 = 45$

-5 ↓ ↓ -5

$x = 40$

Lauren did this:

$\frac{x}{3} + 5 = 15$

-5 ↓ ↓ -5

$\frac{x}{3} = 10$

$\times 3$ ↓ ↓ $\times 3$

$x = 30$

- Substitute the solutions into the equation to decide which person is correct.
- Now solve the following problem using an equation.

  A number is increased by 5.
  The result is divided by 3 to give 15.
  What is the number?

Key ideas

- Recall that $\frac{a}{b}$ means $a \div b$, so $\frac{x}{5}$ means $x \div 5$ and 'half of x' means $\frac{x}{2}$.
- To solve an equation that has a fraction on one side, multiply *both* sides by the denominator.

Example 1: $x \div 5 = 4$

$\frac{x}{5} = 4$

$\times 5$ ↓ ↓ $\times 5$

$x = 20$

Example 2: $(x - 5) \div 2 = 6$

$\frac{x - 5}{2} = 6$

$\times 2$ ↓ ↓ $\times 2$

$x - 5 = 12$

$+5$ ↓ ↓ $+5$

$x = 17$

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- Do not multiply by the denominator until the fraction is on its own on one side.

$\frac{x}{3} + 5 = 8$ ×3 ×3 → $x + 5 = 24$ ✗ Do *not* do this!

$\frac{x}{3} + 5 = 8$ −5 −5 → $\frac{x}{3} = 3$ ×3 ×3 → $x = 9$ ✓ Do this.

- Check your solution using substitution.

For example: LHS $= \frac{x}{3} + 5$ RHS $= 8$ ✓

$= \frac{9}{3} + 5$

$= 3 + 5$

$= 8$ ✓

- Sometimes it is wise to swap the LHS and RHS.

For example, $12 = \frac{x}{3} + 1$ becomes $\frac{x}{3} + 1 = 12$, which is easier to solve.

Exercise 9E

Understanding

1 Check whether the following statements are true or false.

a $\frac{15}{5} = 3$ **b** $\frac{7+2}{3} = 5$ **c** $\frac{8-6}{2} = 1$ **d** $3 + \frac{10}{2} = 9$

$\frac{15}{5}$ means $15 \div 5$.



2 If $x = 24$, classify the following as true or false.

a $\frac{x}{8} = 3$ **b** $\frac{x}{4} = 9$ **c** $\frac{x}{3} = 10$ **d** $\frac{x}{6} = 4$

3 The opposite of adding 5 is subtracting 5. State the opposite of:

a multiplying by 4 **b** subtracting 2 **c** dividing by 7 **d** adding 11

Example 10 Solving equations with fractions

Solve $\frac{a}{7} = 3$.

Solution

$\frac{a}{7} = 3$ ×7 ×7 → $a = 21$

Explanation

Multiplying both sides by 7 removes the denominator of 7.

Check your solution.

LHS $= 21 \div 7 = 3$ ✓ RHS $= 3$ ✓

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4 Copy and complete the following.

a

$\frac{b}{4} = 11$

$\times 4$ ↓ $\times 4$

$b = __$

b

$\frac{d}{5} = 3$

$\times 5$ ↓ $\times 5$

$d = __$

c

$\square$ $\frac{z}{10} = 2$ $\square$

↓

$z = __$

5 For each of the following equations, choose the appropriate first step (**A** to **D**) needed to solve it.

a $\frac{x}{3} = 10$

b $\frac{x}{3} + 2 = 5$

c $\frac{x - 3}{2} = 1$

d $\frac{x}{2} - 3 = 5$

A Multiply both sides by 2.

B Multiply both sides by 3.

C Subtract 2 from both sides.

D Add 3 to both sides.

Fluency

6 Solve:

a $\frac{b}{4} = 20$ **b** $\frac{d}{50} = 4$ **c** $6.5 = \frac{z}{10}$

7 Solve the following equations.

Skillsheet 9A

a $\frac{m}{6} = 2$ **b** $\frac{c}{9} = 2$ **c** $s \div 8 = 2$ **d** $r \div 5 = 2$

e $\frac{u}{2} = 6$ **f** $\frac{y}{5} = 10$ **g** $1 = \frac{x}{2}$ **h** $3 = \frac{a}{4}$

Example 11 Solving two-step equations with fractions

Solve each of the following.

a $\frac{d + 5}{3} = 12$ **b** $10 = \frac{m}{5} + 2$

Solution

a

$\frac{d + 5}{3} = 12$

$\times 3$ ↓ $\times 3$

$d + 5 = 36$

-5 ↓ -5

$d = 31$

Explanation

Multiply both sides by 3 to remove the denominator of 3.
Then solve $d + 5 = 36$ in the usual way.
Check your solution.

$\text{LHS} = \frac{31 + 5}{3}$ $\quad$ RHS = 12 ✓

$= \frac{36}{3}$

$= 12$ ✓

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9E

Solution	Explanation
b $10 = \frac{m}{5} + 2$	Swap the LHS and RHS.
$\frac{m}{5} + 2 = 10$	Subtract 2 to get the fraction on its own on the LHS.
-2 ↓ $\quad -2$ ↓	
$\frac{m}{5} = 8$	Then multiply both sides by the denominator (5).
$\times 5$ ↓ $\quad \times 5$ ↓	
$m = 40$	Check your solution.
	LHS $= \frac{m}{5} + 2$ $\qquad$ RHS $= 10$ ✓
	$= \frac{40}{5} + 2$
	$= 8 + 2 = 10$ ✓

8 Solve the following equations.

a $\frac{d+15}{12} = 2$ **b** $\frac{y+5}{11} = 1$ **c** $\frac{j+8}{11} = 1$ **d** $\frac{b-2}{2} = 1$

e $\frac{w+5}{11} = 1$ **f** $\frac{s+2}{5} = 1$ **g** $1 = \frac{v-4}{7}$ **h** $1 = \frac{f-2}{7}$

Skillsheet 9B

9 Solve the following equations.

a $\frac{m}{2} + 7 = 12$ **b** $\frac{q}{3} - 2 = 1$ **c** $\frac{k}{4} - 3 = 7$ **d** $4 + \frac{x}{5} = 10$

e $\frac{a}{2} + 3 = 5$ **f** $\frac{x}{5} - 1 = 10$ **g** $8 = \frac{y}{2} + 5$ **h** $1 = \frac{a}{7} - 2$

Drilling for Gold 9E1

10 Solve the following equations.

a $\frac{x+5}{2} = 7$ **b** $\frac{x}{2} + 5 = 7$ **c** $\frac{p}{3} - 5 = 2$ **d** $\frac{p-5}{3} = 2$

e $\frac{q}{2} = 10$ **f** $\frac{r-3}{2} = 12$ **g** $12 = \frac{r}{3} + 5$ **h** $10 = \frac{x}{5} - 2$



Don't multiply until the fraction is on its own.

Problem-solving and Reasoning

11 In each of the following cases, write an equation and solve it to find the number.

a A number, t, is halved and the result is 9.
b A number, x, is divided by 10 and the result is 8.
c 4 is subtracted from q and this is halved, giving a result of 3.
d 3 is added to x and the result is divided by 4, giving a result of 2.
e A number, y, is divided by 4 and then 3 is added, giving a result of 5.

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12 A group of five people go out for dinner and split the bill evenly. They each pay $22.



a If b represents the total cost of the bill, in dollars, write an equation to describe this situation.

b Solve this equation algebraically.

c What is the total cost of the bill?

13 a Explain the difference between $\frac{x+3}{5}$ and $\frac{x}{5}+3$.

b What is the first operation you would apply to both sides to solve:

i $\frac{x+3}{5}=7$? **ii** $\frac{x}{5}+3=7$?



For part **c**, try a few values of x and look for a pattern.

c Are there any values of x for which $\frac{x+3}{5}$ and $\frac{x}{5}+3$ are equal to each other?

Enrichment: Three-step equations with fractions

14 Sometimes an equation involving fractions takes more than two steps to solve. For example:

$$\frac{2x-3}{5}=3$$

$\times 5$ $\times 5$

$$2x-3=15$$

$+3$ $+3$

$$2x=18$$

$\div 2$ $\div 2$

$$x=9$$

Solve the following equations, showing all three steps.

a $\frac{3a-4}{2}=1$ **b** $\frac{5k-3}{4}=3$ **c** $7=\frac{2q+11}{3}$

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9F Formulas and relationships EXTENSION

The area of a rectangle is found by multiplying the length by the breadth. This can be written as $A = \ell \times b$. Another example is the equation for speed, which is $s = \frac{d}{t}$. Equations such as these are called formulas.



Let's start: Different formulas

- List all the formulas you can think of.
- What is this formula: $A = \frac{h}{2}(a + b)$?
- Using the internet or a library, find the longest and/or most complicated formula you can, and try to explain what it does.

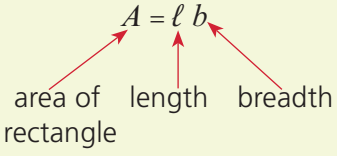
Key ideas

Formula, Equation, Rule A statement in which it may be possible to find values for the pronumerals that will make the statement true. The values are called solutions.

Variable A pronumeral that represents more than one value

- A **formula** is a **rule** or **equation** that shows the relationship between two or more **variables**.

  For example:



- In the example above there are three variables, whereby $A = 6$, $\ell = 2$ and $b = 3$ is one solution for this formula because $6 = 2 \times 3$.
- $A = \ell b$ has three variables: A, ℓ and b. If two variables are known, then the formula can be used to find the unknown variable.

Exercise 9F

Understanding

1 List the variables in each of these equations.

a $F = 3g$ **b** $x = 2y - 1$ **c** $A = B + C - 2$ **d** $g = 3.2d + 5$

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2 **a** Substitute $g = 5$ into the expression $3g$ and find the value of $3g$.
b If $t = 3$, state the value of $t + 6$.
c What is $5x + 2$ when x is 10?



3 Substitute and find the value of each expression.
a $x = 3$ into the expression $5x$
b $x = 7$ into the expression $4(x + 2)$
c $y = 3$ into the expression $20 - 4y$
d $y = 10$ into the expression $\frac{y + 4}{7}$

4 Find the value of $G + H$ when:
a $G = 2$ and $H = 3$
b $G = 7$ and $H = 1$
c $G = 2$ and $H = 12$
d $G = 10$ and $H = 100$

Fluency

Example 12 Applying a formula

Consider the formula $k = 3b + 2$. Find the value of k when:
a $b = 5$ **b** $b = 10$

Solution	Explanation
a $k = 3b + 2$ $= 3 \times 5 + 2$ $= 17$	Copy the formula. Substitute $b = 5$ into the equation, then evaluate.
b $k = 3b + 2$ $= 3 \times 10 + 2$ $= 32$	Copy the formula. Substitute $b = 10$ into the equation, then evaluate.

5 Consider the formula $d = t + 7$. Find the value of d when:
a $t = 2$ **b** $t = 10$ **c** $t = 6$

6 Consider the rule $n = 2m + 1$. Find n when:
a $m = 3$ **b** $m = 4$ **c** $m = 100$



7 Consider the formula $F = ma$. Find F when:
a $m = 3$ and $a = 7$ **b** $m = 9$ and $a = 2$ **c** $m = 1$ and $a = 30$

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9F

Example 13 Substituting a formula and then solving

Consider the formula $k = 3b + 2$. Find the value of b when $k = 23$.

Solution

Solve $23 = 3b + 2$.

$23 = 3b + 2$
-2 -2
$21 = 3b$
$\div 3$ $\div 3$
$7 = b$

Therefore, $b = 7$.

Explanation

Substitute $k = 23$ into the equation. Now solve the equation to find the value of b.

Check your solution.
LHS = 23 ✓ RHS = $3 \times 7 + 2 = 23$ ✓

8 You are given the formula $m = 4x$.
- **a** Solve the equation $12 = 4x$.
- **b** Find the value of x when $m = 20$.

9 You are given the formula $A = 3b + 4$.
- **a** If $b = 2$, what is A ?
- **b** If $A = 34$, what is b?
- **c** Find b when $A = 25$.



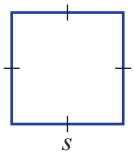
For parts **b** and **c**, set up an equation and then solve it.

Skillsheet 9C

10 Consider the formula $y = 5 + 3x$. Find:
- **a** y if $x = 6$
- **b** x if $y = 17$
- **c** x if $y = 26$

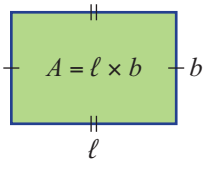
Problem-solving and Reasoning

11 The perimeter, P, of a square is given by the formula $P = 4s$, where s is the side length.



- **a** Find the perimeter when:
  - **i** $s = 7$
  - **ii** $s = 10$
- **b** If the perimeter is 48, solve the equation $P = 4s$ to find the side length.

12 The formula for the area of a rectangle is $A = \ell \times b$, where ℓ is the rectangle's length and b is the rectangle's breadth.



- **a** Find the value of A when $\ell = 7$ and $b = 5$.
- **b** Set up and solve an equation to find the length of a rectangle when $A = 20$ and $b = 4$.
- **c** A rectangle is drawn for which $A = 25$ and $\ell = 5$.
  - **i** Set up and solve an equation to find b.
  - **ii** What type of rectangle is this?

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Enrichment: Converting temperatures



13 To convert between temperatures in degrees Celsius (°C) and degrees Fahrenheit (°F), the rule is $F = 1.8C + 32$.

For parts **c** and **d**, round to the nearest whole degree.

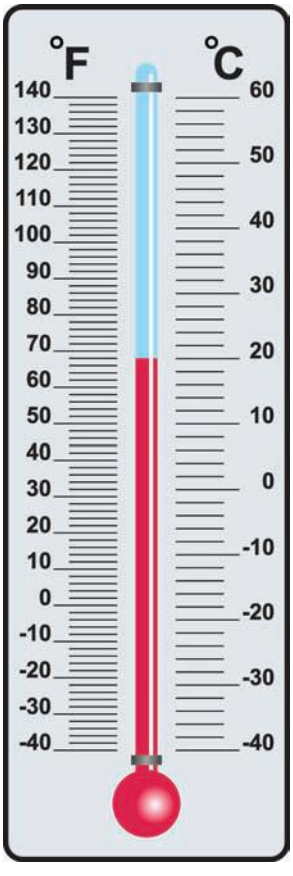


a Find F when $C = 20$.

b Find C when $F = 50$.

c What is the temperature in degrees Fahrenheit when it is 23°C outside?

d In Florida last week, the temperature hit 97°F. What is this in degrees Celsius? Solve an appropriate equation and round your answer to the nearest degree.



e Marieko claims that the temperature in her city varies between 68°F and 95°F. What is the difference, in degrees Celsius, between these two temperatures?

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9G Using equations to solve problems

EXTENSION

Equations can be used to solve problems that may arise in the real world.



Let's start: Stationery shopping

Sylvia bought 10 pencils and 2 pens for \$25.
She knows that the pens cost \$3.50 each.
How much did each pencil cost?

- Describe how you got your answer.
- How much will Karl pay for 6 pencils and 5 pens?

Key ideas

- To solve a problem, follow these steps.

Use a pronumeral to stand in for the unknown.	e.g. Let p = the cost of a pencil.
↓	
Write an equation to describe the problem.	e.g. $10p + 2 \times 3.5 = 25$.
↓	
Solve the equation.	This can be done by inspection or systematically.
↓	
Make sure that you answer the original question and the solution seems reasonable and realistic.	Don't forget to include the correct units (e.g. dollars, years, cm).

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Exercise 9G

Understanding

1 Match each of the descriptions (**a** to **e**) with the correct algebraic expression (**A** to **E**).

a the sum of x and 3
b twice the value of x
c 2 less than x
d half of x
e the product of 5 and x

A $2x$
B $\frac{x}{2}$
C $5x$
D $x + 3$
E $x - 2$

2 Solve the following equations.

a $5x = 30$ **b** $7a + 2 = 16$ **c** $2k - 3 = 15$

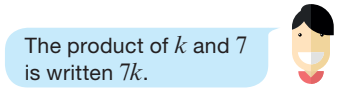
3 Alysha notices that buying 4 pens costs \$12. Which of the following is the cost of one pen?

A \$4 **B** \$12 **C** \$1 **D** \$3 **E** \$16

Fluency

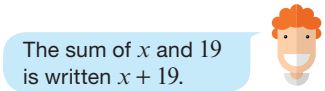
4 The product of k and 7 is 42.

a Write an equation to describe this fact.
b Solve the equation to find the value of k.



5 The sum of x and 19 is 103.

a Write an equation to describe this fact.
b Solve the equation to find the value of x.



Example 14 Solving a problem using equations

The sum of Kate's age now, and her age next year, is 19. How old is Kate?

Solution	Explanation
Let k = Kate's current age. $k + (k + 1) = 19$	Use a pronumeral to stand in for the unknown. Write an equation to describe the situation. Note that $k + 1$ is Kate's age next year.
$2k + 1 = 19$ -1 -1 $2k = 18$ $\div 2$ $\div 2$ $k = 9$	Simplify the LHS and then solve the equation.

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9G

Solution	Explanation
Check: LHS $= 2k + 1$ RHS $= 19$ ✓ $= 2 \times 9 + 1$ $= 19$ ✓	Check the solution by substituting the value back into the equation.
Kate is currently 9 years old.	Answer the original question in a sentence.


Drilling for Gold 9G1

6 Millie buys 12 pens for a total cost of $18.
- **a** Use a pronumeral for the cost of one pen.
- **b** Write an equation to describe the problem.
- **c** Solve the equation.
- **d** What is the cost of one pen?

7 Launz buys four new tyres for his car. He also buys a smaller spare tyre for his trailer, which costs $160. The total cost is $1400.
- **a** Use a pronumeral for the cost of a car tyre.
- **b** Write an equation to describe the problem.
- **c** Solve the equation.
- **d** What is the cost of a car tyre?

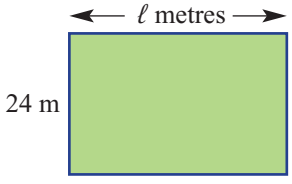


8 Jonas is paid $17 per hour and gets paid a bonus of $65 each week. One particular week he earned $643.
- **a** Use a pronumeral for the number of hours Jonas worked.
- **b** Write an equation to describe the problem.
- **c** Solve the equation.
- **d** How many hours did Jonas work in that week?

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Problem-solving and Reasoning

9 This rectangular paddock has an area of 720 m^2.

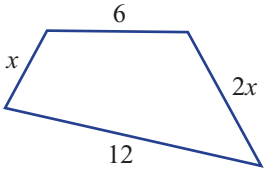



The perimeter is the total distance around the paddock.

a Write an equation to describe the problem, using ℓ for the paddock's length.
b Solve the equation.
c How long is the paddock?
d What is the paddock's perimeter?

10 A number is doubled, then 3 is added. This gives a final result of 31. Set up and solve an equation to find the original number, showing all the steps clearly.

11 a The perimeter of this shape is 30. Find the value of x.
b Is it possible for the perimeter to equal 15? Why or why not?



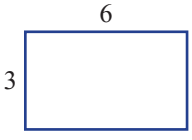
12 Marco and Sara's combined age is 30. Given that Sara is 2 years older than Marco, write an equation and find Marco's age.


If Marco's age is m, then Sara's age is $m + 2$.



Enrichment: Curious rectangles

13 The formulas for the area and perimeter of rectangles are $A = \ell \times b$ and $P = 2\ell + 2b$.
a Calculate the area and the perimeter of this rectangle (where $\ell = 6$ and $b = 3$). What do you notice?
b Find some other solution to the equation $\ell b = 2\ell + 2b$ by inspection. (There are many solutions.)
c Are there any solutions when the rectangle is a square (i.e. $\ell = b$)? Justify your answer.



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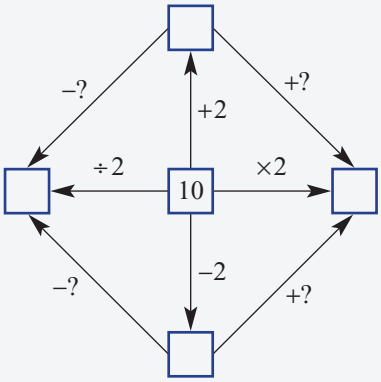
Puzzles and games

1 Find the unknown number in the following puzzles.

- **a** A number is added to half of itself and the result is 39.
- **b** A number is doubled, then tripled, then quadrupled. The result is 696.
- **c** One-quarter of a number is subtracted from 100 and the result is 8.
- **d** Half of a number is added to 47, and the result is the same as the original number doubled.
- **e** A number is increased by 4, the result is doubled and then 4 is added again to give an answer of 84.

2 Find the values of x and y that will make *both* these equations true.
$x + y = 20$ and $x \times y = 91$

3 Some numbers in this two-dimensional flowchart are missing. Find the sum of the four numbers that are shown by question marks.



4 *What did the student expect when she solved the puzzle?*
Find the answer by solving the following equations. If the solution is $x = 1$, the letter is A. If the solution is $x = 2$, the letter is B and so on.

$3x + 2 = 5$	$16 = 2x + 10$	$5x - 10 = 65$	$\frac{x}{3} + 1 = 7$	$2 = \frac{x + 2}{8}$	$2x + 10 = 60$	$\frac{x}{2} + 3 = 8$	$3x + 2 = 47$	$20 - x = 9$	$4x - 3 = 17$



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5 In a farmer's paddock there are sheep and ducks. Farmer Jess says to her grandson, 'There are 41 animals in this paddock.' Grandson James says to his grandma, 'There are 134 animal legs in this paddock.'
How many sheep and how many ducks are in the paddock?

6 Michelle looks at how many matchsticks are required to make a pattern of triangles.



a Copy and complete the table.

Number of triangles, n	1	2	3	4	5
Number of matches, M					

b Find a formula relating M and n.
c How many matchsticks would Michelle need to make 100 triangles?

7 Bryce and Jordon have ages that are 5 years apart. When their ages are added, the result is 21. How old is each boy?

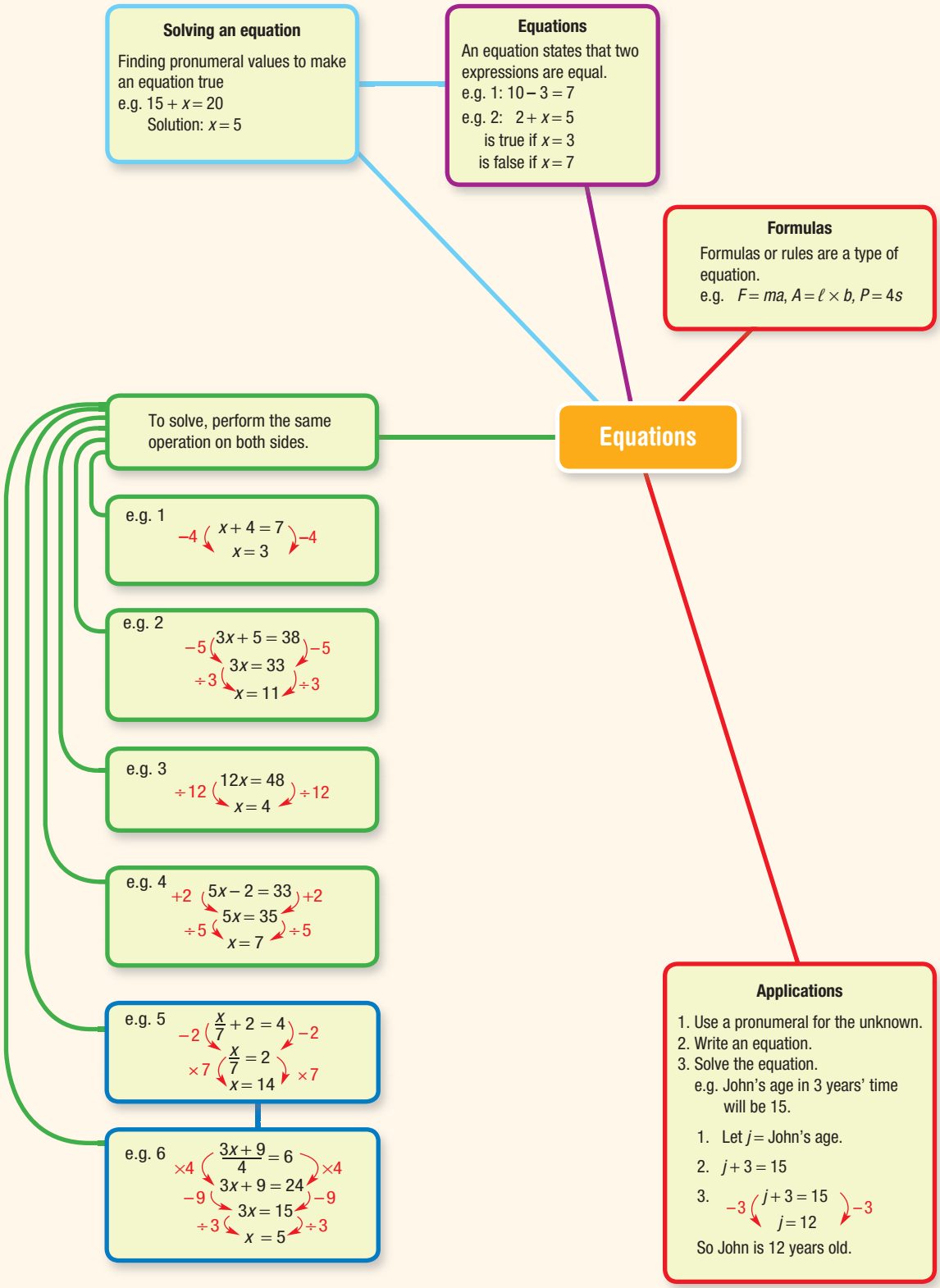
8 Simon says, "When I was born my father was 36."
a How old will Simon be when his age is half that of his father's?
b How old will Simon be when his age is one-quarter of his father's age?

9 Half of one-third of a number is 20. What is the number?



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Chapter summary



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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 Which one of the following statements is *false*?
A $3 + 4 = 7$ **B** $10 - 2 = 8$ **C** $5 + 5 = 2 + 8$
D $7 - 4 = 4$ **E** $5 + 11 = 17 - 1$

2 If $x = 3$, which one of the following equations is true?
A $4x = 21$ **B** $2x + 4 = 12$ **C** $9 - x = 6$
D $2 = x + 1$ **E** $x - 3 = 4$

3 When x is tripled and then 11 is added, the result is 53. This can be written as:
A $3x + 11 = 53$ **B** $3(x + 11) = 53$ **C** $\frac{x}{3} + 11 = 53$
D $\frac{x + 11}{3} = 53$ **E** $3x - 11 = 53$

4 Which of the following values of x makes the equation $3x = 24$ true?
A 2 **B** 4 **C** 6 **D** 8 **E** 10

5 The solution of the equation $2a + 4 = 10$ is:
A $a = 3$ **B** $a = 7$ **C** $a = 12$ **D** $a = 28$ **E** $a = 1$

6 To solve $3a + 5 = 17$, the first step to apply to both sides is:
A add 5 **B** divide by 3 **C** subtract 17
D divide by 5 **E** subtract 5

7 The solution to $2t - 4 = 6$ is:
A $t = 1$ **B** $t = 3$ **C** $t = 5$ **D** $t = 7$ **E** $t = 9$

8 The solution of $\frac{x}{7} = 10$ is:
A $x = 35$ **B** $x = 70$ **C** $x = 20$ **D** $x = 30$ **E** $x = 5$

9 The solution to the equation $10 = 3p - 5$ is:
A $p = 5$ **B** $p = 20$ **C** $p = 15$ **D** $p = 7$ **E** $p = 1$

10 A formula relating F, m and a is $F = ma$. When $F = 60$ and $a = 6$, m equals:
A 18 **B** 4 **C** 30 **D** 2 **E** 10

Short-answer questions

1 Classify each of the following statements as true or false.
a $4 + 2 = 10 - 2$ **b** $2(3 + 5) = 4(1 + 3)$ **c** $5w + 1 = 11$, if $w = 2$
d $2x + 5 = 12$, if $x = 4$ **e** $y = 3y - 2$, if $y = 1$ **f** $4 = z + 2$, if $z = 3$

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Chapter review

2 Write an equation for each of the following situations. You do not need to solve the equations.

a The sum of 2 and u is 22.
b The product of k and 5 is 41.
c When z is tripled the result is 36.
d The sum of a and 12 is 15.

3 Solve the following equations by inspection.

a $x + 1 = 4$ **b** $x + 8 = 14$ **c** $10 = 11 - y$
d $y - 7 = 2$ **e** $5a = 10$ **f** $2 = a \div 5$

4 Solve:

a $4x = 20$ **b** $7q = 42$ **c** $a + 4 = 25$ **d** $12 = k - 6$
e $2k + 3 = 23$ **f** $5w - 7 = 33$ **g** $3b + 5 = 20$ **h** $8 = 10r - 12$

5 Solve:

a $5x = 15$ **b** $r + 25 = 70$ **c** $5 = x - 4$
d $13 = 2r + 5$ **e** $10 = 4q + 2$ **f** $66 = 8u + 2$

6 Solve the following equations.

a $\frac{u}{4} = 6$ **b** $\frac{p}{2} = 8$ **c** $3 = \frac{x + 1}{3}$
d $\frac{y}{2} + 10 = 30$ **e** $4 = \frac{y + 20}{7}$ **f** $\frac{x}{3} + 4 = 24$

7 Consider the equation $4(x + 3) = 2x + 14$.

a Is $x = 2$ a solution? (Check the LHS and RHS.)
b Show that $x = 1$ is a solution.

8 The formula for the area of a rectangle is $A = \ell \times b$, where ℓ is the length and b is the breadth.

a Find the area when $b = 10$ and $\ell = 45$.
b If $A = 24$ and $\ell = 3$, find the value of b.
c Give an example of a rectangle that has an area of 40.

9 Consider the rule $F = 3a + 2b$. Find:

a F when $a = 10$ and $b = 3$
b b when $F = 27$ and $a = 5$
c a when $F = 25$ and $b = 8$

10 For each of the following problems, write an equation and solve it.

a A number is multiplied by 4 and the result is 20. What is the number?
b The sum of 5 and a number is 30. What is the number?
c Juanita's mother is twice as old as Juanita. The sum of their ages is 60. How old is Juanita?
d A square has a perimeter of 20. What is its side length?

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Chapter review

Extended-response questions

1 Mya is paid $\$12$ per hour.

- **a** How much is she paid if she works for 10 hours?
- **b** Which of the following formulas correctly relates her total wage W (in dollars) and the number n of hours worked?

  A $W = 12$ **B** $W = 12n$ **C** $W = 12 + n$ **D** $W = n - 12$
- **c** If $n = 5$, what is the value of her total wage, W?
- **d** If her wage one day is $\$84$, how many hours did she work?
- **e** Write an equation (but do not solve it) for the following problem. During a particular week, Mya worked n hours and earned $\$252$.

2 Leo's mobile phone plan charges a 15-cents connection fee and then 2 cents per second for every call. The total cost is given by $C = 15 + 2t$, where C is the cost in cents and t is the time in seconds.

- **a** How much does a 30-second call cost?
- **b** How much would a 1-minute call cost?
- **c** Solve the equation $15 + 2t = 39$.
- **d** If a call cost 39 cents, how long did it last?
- **e** If a call cost $\$1.15$, how long did it last?
- **f** On a particular day, Leo makes two calls. The first call lasted 20 seconds and the second lasted twice as long. What was the total cost of these two calls?


For part **b**, $t = 60$.



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Chapter 10

Measurement and computation of length, perimeter and area

What you will learn

10A Using and converting units of length
10B Perimeter of rectilinear figures
10C Pi and circumference of circles
10D Arc length and perimeter of sectors and composite figures
10E Units of area and area of rectangles
10F Area of triangles
10G Area of parallelograms
10H Mass and temperature

Strand: Measurement and Geometry

Substrand: LENGTH AND AREA

In this chapter, you will learn to:

- calculate the perimeters of plane shapes and the circumference of circles
- use formulas to calculate the areas of quadrilaterals
- convert units of area.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Measurement everywhere

The Eiffel Tower in France is painted with 50 tonnes of paint every 7 years. The Great Wall of China is more than 6000 km long. The world's smallest country is Vatican City in Rome, with an area of 0.44 km^2. The maximum temperature on Mars is about 20°C. The distance between the Earth and the Moon is about $380\,000$ km. Imagine trying to describe facts about the world around us without using any form of measurement!

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Consider these units of measurement: centimetre, kilogram and metre.
- **a** Which unit measures mass?
- **b** Which two units measure distance?

2 Measure the length of **a**, **b** and **c**, in millimetres.
- **a** _____
- **b** _____
- **c** ____

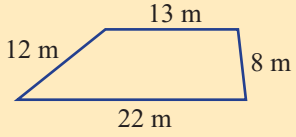
3 Arrange these units from smallest to largest.
- **a** centimetre (cm), kilometre (km), metre (m), millimetre (mm)
- **b** gram (g), kilogram (kg), milligram (mg), tonne (t)

4 Find how many:
- **a** millimetres are in a centimetre
- **b** centimetres are in a metre
- **c** grams are in a kilogram
- **d** metres are in a kilometre

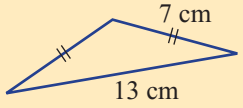
5 Calculate:
- **a** 2×1000
- **b** $200 \div 100$
- **c** $56\,000 \div 1000$
- **d** 2.5×1000

6 Find the total distance around each shape.

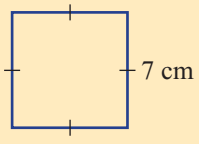
a



b

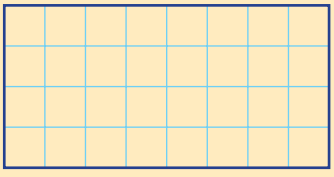


c

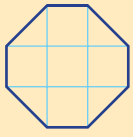


7 How many unit squares make up the area of these shapes?

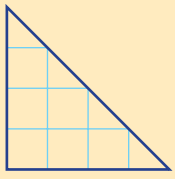
a



b



c



8 Give the most appropriate unit (e.g. metres) for measuring each of the following.
- **a** the distance between two towns
- **b** your weight
- **c** the length of a school lesson
- **d** the edges of this page

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10A Using and converting units of length

When builders are estimating the cost of a job, they use:
- metres for the length of timber and guttering
- millimetres for the length of nails and screws
- square metres for the area of floors, walls and ceilings
- kilometres for the distance they need to drive to the job each day.

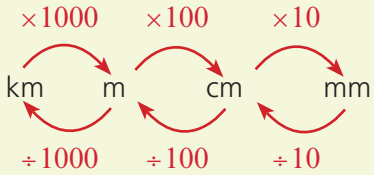


Let's start: How good is your estimate?

Without using a ruler, estimate the length of the long edge of this page.
- Now use a ruler to measure the length, in centimetres.
- Convert your answer to millimetres and to metres.
- How many pages would be needed to reach 1 kilometre? Explain how you calculated your answer.

Key ideas

- The **metre** (m) is the basic metric unit of length in the **metric system**.
  - 1 centimetre (cm) = 10 millimetres (mm)
  - 1 metre (m) = 100 centimetres (cm)
  - 1 kilometre (km) = 1000 metres (m)



- Conversion
  - When converting to a smaller unit, multiply by 10 or 100 or 1000. The decimal point appears to move to the right. For example:

$2.3\text{ m} = (2.3 \times 100)\text{ cm} = 230\text{ cm}$ $\quad 28\text{ cm} = (28 \times 10)\text{ mm} = 280\text{ mm}$

  - When converting to a larger unit, divide by a power of 10 (i.e. 10, 100, 1000). The decimal point appears to move to the left. For example:

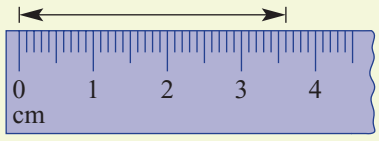
$47\text{ m} = (47 \div 10)\text{ cm} = 4.7\text{ cm}$ $\quad 4600\text{ m} = (4600 \div 1000)\text{ km} = 4.6\text{ km}$

Metre The standard metric unit for length, equal to 100 centimetres

Metric system A measurement system using the base-ten number system

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- When reading scales, be sure about what units are showing on the scale. This scale shows 36 mm or 3.6 cm.
- Measure from 0, not the edge of the ruler.



Exercise 10A

Understanding



Drilling for Gold 10A1

1 List four metric units of length.



One of these units is the metre.

2 Write the missing number or word in these sentences.
- **a** To convert from metres to centimetres, multiply by _____.
- **b** To convert from metres to kilometres, divide by _____.
- **c** When converting from centimetres to metres, you _____ by 100.
- **d** When converting from kilometres to metres, you _____ by 1000.

3 **a** When multiplying by 100 (e.g. 3.21×100), does the decimal point appear to move left or right?
- **b** When dividing by 100 (e.g. $32.1 \div 100$), does the decimal point appear to move left or right?

Fluency

Example 1 Choosing metric lengths

Which metric unit would be the most appropriate for measuring these lengths?
a width of a large room **b** thickness of glass in a window

Solution	Explanation
a metres (m)	Using mm or cm would give a very large number, and using km would give a number that is very small.
b millimetres (mm)	The thickness of glass is likely to be around 5 mm.

4 Which metric unit would be the most appropriate for measuring the following?
- **a** distance between two towns
- **b** width of a small drill bit
- **c** height of a flag pole
- **d** length of a garden hose
- **e** width of a small desk
- **f** distance across a city

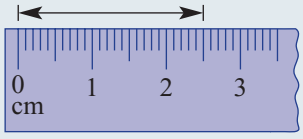


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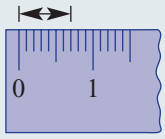
Example 2 Reading length scales

Read the scales on these rulers to measure the marked length.

a



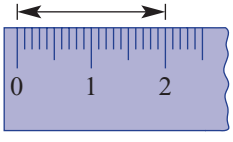
b



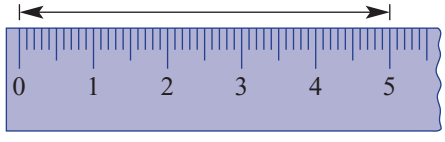
Solution	Explanation
a 25 mm or 2.5 cm	Every centimetre is 10 mm.
b 7 mm or 0.7 cm	Every centimetre is 10 mm.

5 These rulers show centimetres with millimetre divisions. Read the scale to measure the marked length, in centimetres and millimetres (e.g. 5.7 cm, 57 mm).

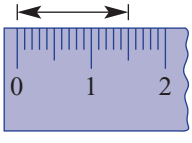
a



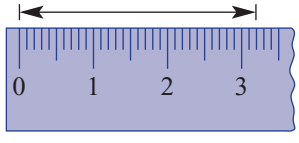
b



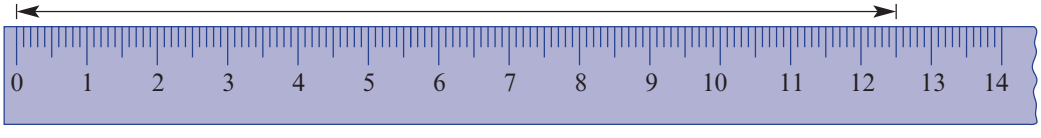
c



d



e



Example 3 Converting to smaller units of length

Convert to the units given in brackets.

a 3 m (cm) **b** 2.8 km (m)

Solution	Explanation
a $3\text{ m} = (3 \times 100)\text{ cm}$ $= 300\text{ cm}$	$1\text{ m} = 100\text{ cm}$ Multiply, as you are converting to a smaller unit.
b $2.8\text{ km} = (2.8 \times 100)\text{ m}$ $= 2800\text{ m}$	There are 1000 m in 1 km, so multiply by 1000.

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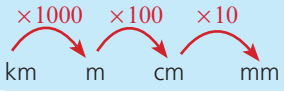
10A

6 Convert these measurements to the units shown in brackets.

a 5 cm (mm)
b 2 m (cm)
c 3.5 km (m)
d 26.1 m (cm)
e 2.2 km (m)
f 5.3 cm (mm)
g 6.2 m (cm)
h 20 cm (mm)
i 6.84 m (cm)
j 0.02 km (m)
k 38 m (cm)
l 6.7 m (cm)



When converting *larger* units to smaller units, *multiply* by 10, 100 or 1000.



Example 4 Converting to larger units of length

Convert to the units given in the brackets.
a 39 mm (cm) **b** 580 m (km)

Solution	Explanation
a 39 mm = (39 ÷ 10) cm = 3.9 cm	There are 10 mm in 1 cm so divide by 10.
b 580 m = (580 ÷ 1000) km = 0.58 km	There are 1000 m in 1 km so divide by 1000.

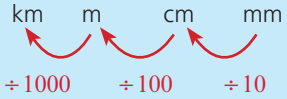

Drilling for Gold 10A2

7 Convert these measurements to the units shown in the brackets.

a 40 mm (cm)
b 500 cm (m)
c 4200 m (km)
d 472 mm (cm)
e 360 cm (m)
f 32 mm (cm)
g 50 000 m (km)
h 27 000 m (km)
i 362 mm (cm)
j 0.4 mm (cm)
k 9261 mm (cm)
l 4230 m (km)



When converting *smaller* units to larger units, *divide* by 10, 100 or 1000.



Problem-solving and Reasoning

8 Choose which metric unit (i.e. mm, cm, m or km) would be the most suitable for measuring the length indicated in these photos.



Choose the unit for the real-life length, not the length of the arrow in the photo!

a

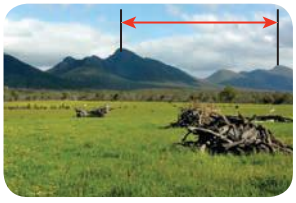


b

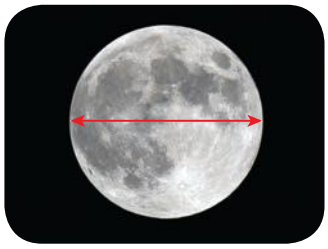


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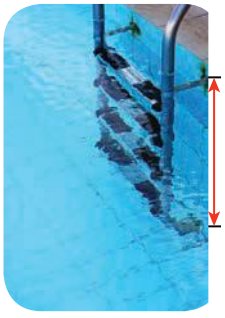
c



d



e



f



9 Add these lengths and give the result in the units shown in brackets.

a 2 cm and 5 mm (cm) **b** 8 cm and 2 mm (mm)
c 2 m and 50 cm (m) **d** 7 m and 30 cm (cm)
e 6 km and 200 m (m) **f** 25 km and 732 m (km)



First convert one of the measurements so that both lengths have the unit shown in brackets.

10 Use subtraction to find the difference between the measurements, and give your answer with the units shown in brackets.

a 9 km, 500 m (km) **b** 3.5 m, 40 cm (cm) **c** 0.2 m, 10 mm (cm)

11 Arrange these measurements from smallest to largest.

a 38 cm, 540 mm, 0.5 m **b** 0.02 km, 25 m, 160 cm, 2100 mm
c 0.003 km, 20 cm, 3.1 m, 142 mm **d** 0.001 km, 0.1 m, 1000 cm, 10 mm

12 Joe widens a 1.2 m doorway by 50 mm. What is the new width of the doorway, in centimetres?



Convert all the measurements to the same units.

13 Steel chain costs $8.20 per metre. How much does it cost to buy chain of the following lengths?

a 1.5 m **b** 80 cm **c** 500 mm

14 Mount Everest is moving with the Indo-Australian plate at a rate of about 10 cm per year. How many years will it take to move 1 km?



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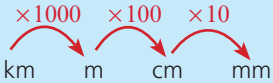
10A

15 Convert to the units shown in the brackets.

a 3 m (mm)
b 6 km (cm)
c 2.4 m (mm)
d 0.04 km (cm)
e 47 000 cm (km)
f 913 000 mm (m)
g 216 000 mm (km)
h 0.5 mm (m)
i 0.002 km (m)



When converting *larger* units to smaller units, *multiply* by 10, 100 or 1000.





Drilling for Gold
10A3
10A4

Enrichment: Estimating length

16 Here are the lengths 1 mm and 1 cm.

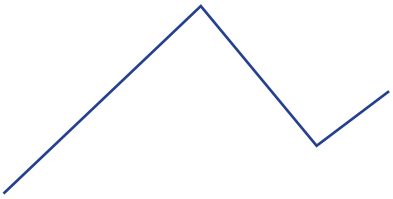
1 mm 1 cm

Use these diagrams as a guide to estimate the length of the following lines.

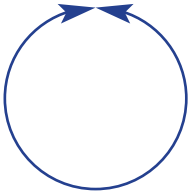
a _
b __
c _____
d _____
e _____

17 Estimate the length of each line or curve, in centimetres.

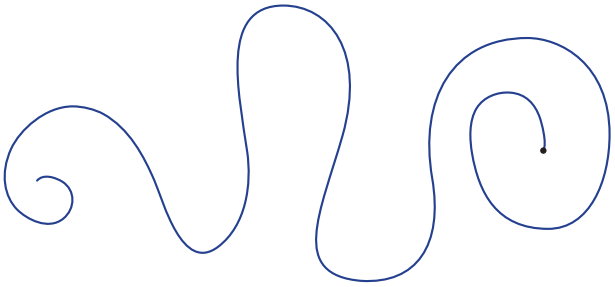
a



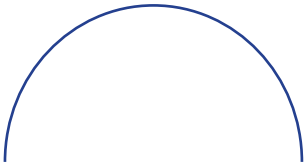
b



c



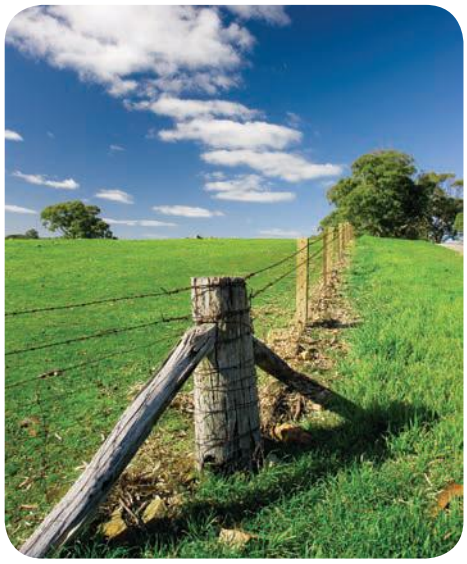
d



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10B Perimeter of rectilinear figures

The distance around the outside of a two-dimensional shape is called the perimeter. The word *perimeter* comes from the Greek words *peri*, meaning 'around', and *metron*, meaning 'measure'. Perimeter is used when calculating the length of fencing around a block of land or the length of timber required to frame a picture.

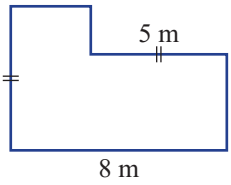


This fence marks the perimeter of a paddock.

Let's start: The L-shaped perimeter

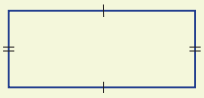
This shape has only two given measurements. All angles are 90°.

- For each of the other sides, state whether it is possible to find the length. Give reasons for each answer.
- Is it possible to find the perimeter of the entire shape? Explain your answer.

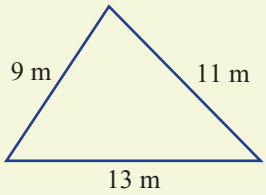


Key ideas

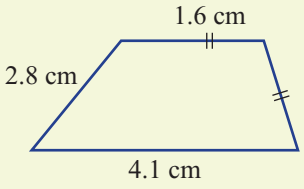
- **Perimeter**, often denoted as P, is the total distance (length) around the outside of a two-dimensional shape.
- Perimeter is calculated by adding together the length of all sides.
- Sides with the same markings are of equal length.



Perimeter The total distance (length) around the outside of a figure



$P = 9 + 11 + 13$
$= 33$ m



$P = 1.6 + 1.6 + 2.8 + 4.1$
$= 10.1$ cm

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Exercise 10B

Understanding

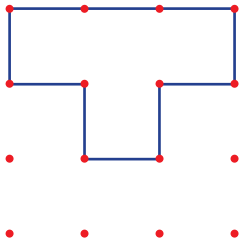
1 Choose a suitable word to complete each sentence.

a The distance around the outside of a shape is called the _____.

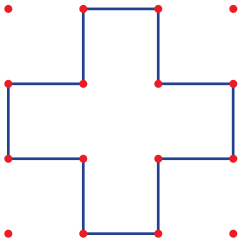
b Sides with the same markings are of _____ length.

2 These shapes are drawn on 1 cm grids. Give the perimeter of each.

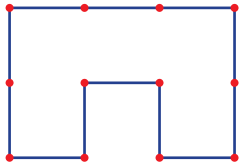
a



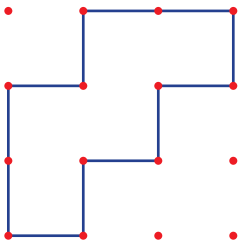
b



c



d





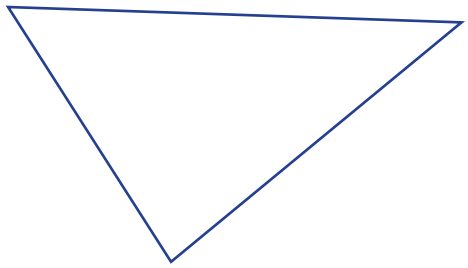
The distance between each pair of red dots is 1 cm.



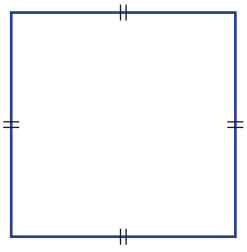
Drilling for Gold 10B1

3 Use a ruler to measure the lengths of the sides of these shapes, in millimetres, and then find the perimeter.

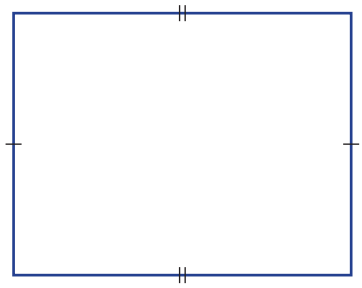
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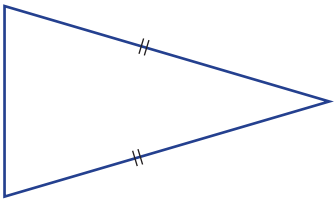
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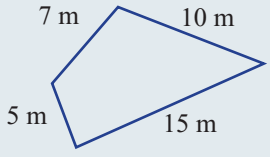
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Fluency

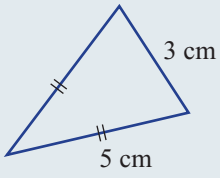
Example 5 Finding the perimeter

Find the perimeter of each of these shapes.

a



b



Solution

a $P = 5 + 7 + 10 + 15$
$= 37$ m

b $P = 2 \times 5 + 3$
$= 13$ cm

Explanation

Add all the side lengths to find the perimeter.

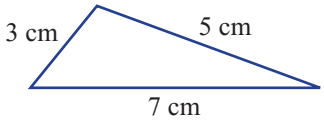
There are two equal lengths of 5 cm and one length of 3 cm.

4 Find the perimeter of these shapes. (Diagrams are not drawn to scale.)

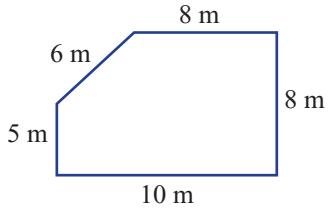


Sides with the same markings are of equal length.
Add all the sides to find the perimeter.

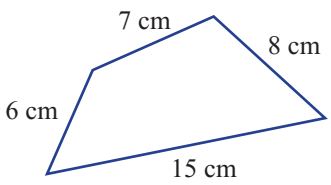
a



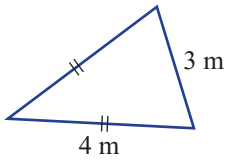
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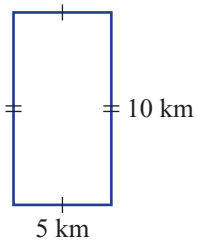
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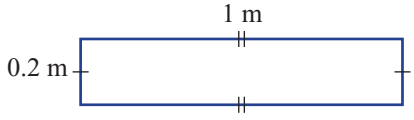
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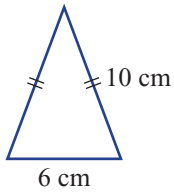


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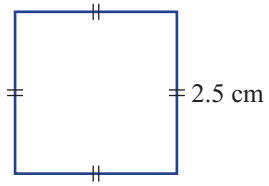


Skillsheet 10A

g



h

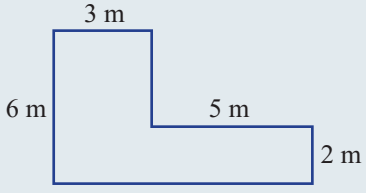


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10B

Example 6 Finding perimeter when sides are unknown

Find the perimeter of this L-shape.

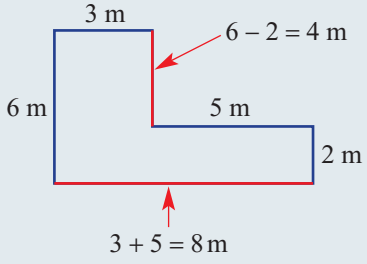


Solution

$P = 6 + 3 + 4 + 5 + 2 + 8$
$= 28$ m

Explanation

First find the missing side lengths.



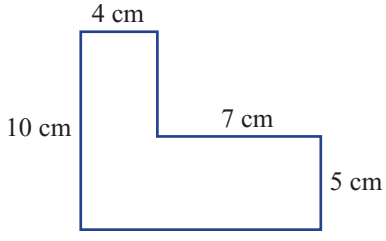
Then add all the side lengths.

5 Find the perimeter of each L-shape. All corner angles are 90°.

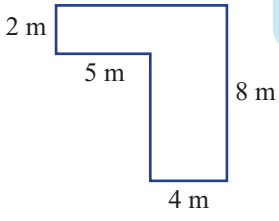


First find the length of the missing sides.

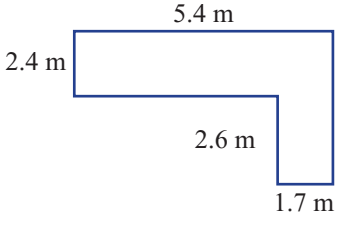
a



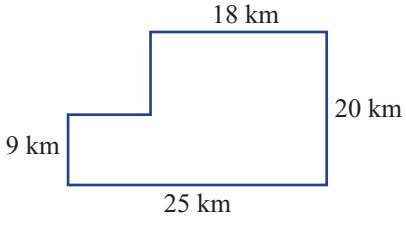
b



c



d



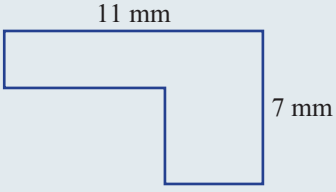
Problem-solving and Reasoning

6 **a** A square has a side length of 2.1 cm. Find its perimeter.
b A rectangle has a length of 4.8 m and a breadth of 2.2 m. Find its perimeter.
c An equilateral triangle has all sides the same length. If each side is 15.5 mm, find its perimeter.

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Example 7 Using a rectangle when sides are unknown

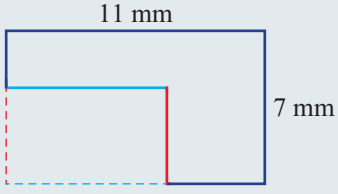
Use a rectangle to help find the perimeter of this L-shape.



Solution

$P = 2 \times 7 + 2 \times 11$
$= 14 + 22$
$= 36$ mm

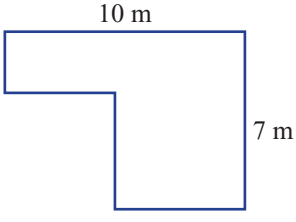
Explanation



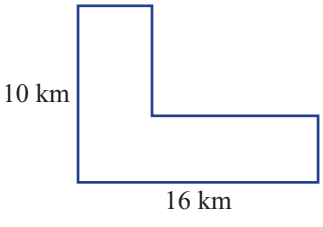
The perimeter of the L-shape is the same as the perimeter of the rectangle.

7 Use a rectangle to help you find the perimeter of each of these shapes.

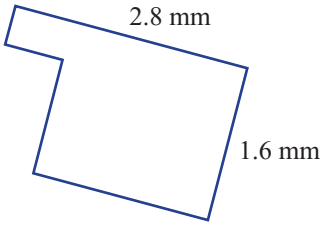
a



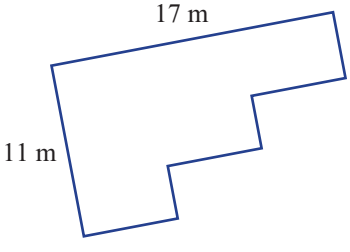
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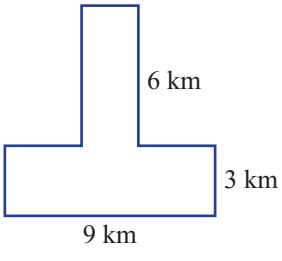
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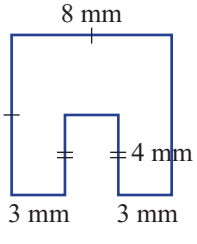
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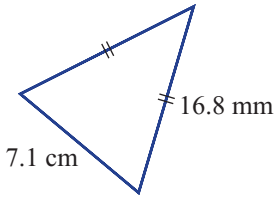
8 A horse paddock is to be fenced on all sides. It is rectangular in shape, with a length of 242 m and a breadth of 186 m. If fencing costs $25 per metre, find the cost of fencing required. Use a calculater to help you.

9 Find the perimeter of each of these shapes. Give your answers in centimetres.

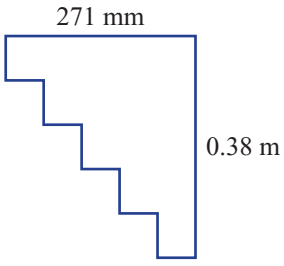


First convert all side lengths to centimetres before adding them.

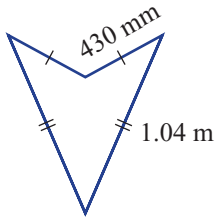
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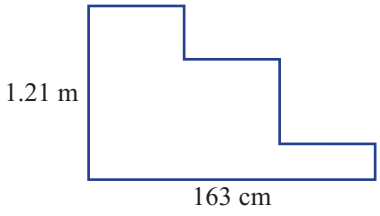
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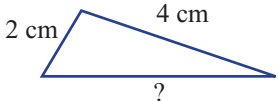


d



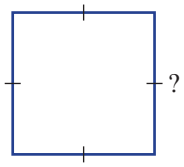
10 The perimeter of each shape is given. Find the missing length of each.

a



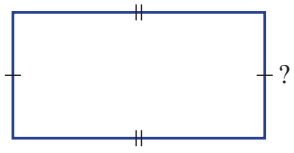
$P = 11$ cm

b



$P = 20$ m

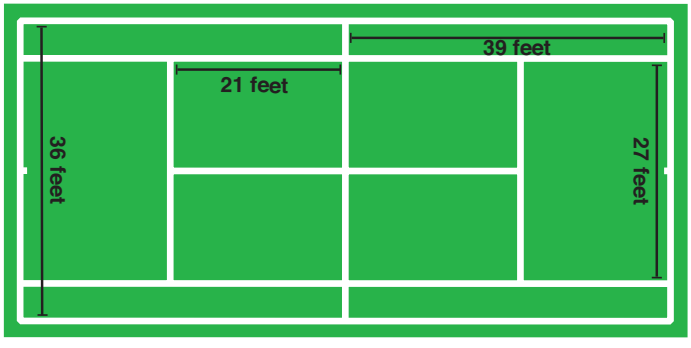
c



$P = 38$ km

Enrichment: Tennis court markings

11 A grass tennis court has white chalk lines. All the measurements are shown in the diagram and given in feet.



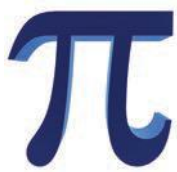
a Find the total number of feet of chalk required to mark all the lines of the tennis court.

b Use the internet to investigate how many centimetres are in 1 foot.

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10C Pi and circumference of circles

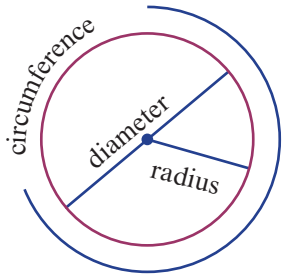
In this section you will meet a special number called pi. On your calculator you will find a π button, which is the symbol used for pi. When pressed, it displays 3.1415926... Pi is a very useful number for calculating curved distances, areas and volumes.



The symbol for pi

Let's start: 1 circumference = ? diameters

Step 1: Wrap a piece of string around a can.
Step 2: Cut the string so that the length of the string is the circumference.
Step 3: Stretch the string across the diameter and mark it.
Step 4: Repeat step 3 as many times as you can.

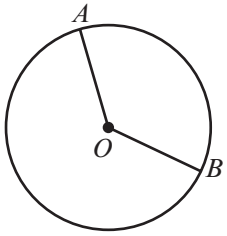


- How many diameters are in the circumference?
- Try this experiment with a wider cylinder.
- Try it again with a narrower cylinder.
- Using a trundle wheel, try it with the circle on a soccer field or netball court.
- What do you notice?

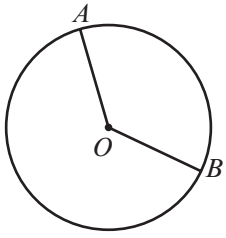
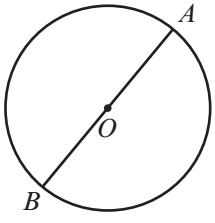
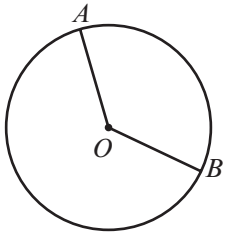




Key ideas

Terminology	Example	Definition
circle		A plane shape that is perfectly round. All points on the edge of a circle are the same distance from a point called the centre (O).

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Terminology	Example	Definition
circumference (C)		1. The edge of a circle. Points A and B are on the circumference. 2. The perimeter of a circle.
diameter (d)		1. A line segment (or interval) that passes through the centre of a circle with endpoints on the circumference. AB is a diameter. 2. The length of the line segment AB.
radius (r) (plural radii or radiuses)		1. A line segment (or interval) with one endpoint at the centre and the other on the circumference. OA and OB are radii. 2. The length of the line segment OA, which is half the diameter.

Irrational number A decimal that continues forever without ever forming a repeating pattern. A number that can't be written as a fraction $\frac{a}{b}$, where a and b are integers

Formula A statement in which it may be possible to find values for the pronumerals that will make the statement true

- In *every* circle,
  (circumference) ÷ (diameter) ≒ 3.14159
  i.e. $C \div d \fallingdotseq 3.14159$
  This number is called pi (π), which is an **irrational number**.


Drilling for Gold 10C1

$$C \div d = \pi$$

or $$\frac{C}{d} = \pi$$

Multiplying both LHS and RHS by d gives:

$$C = \pi \times d$$

This is the **formula** for the circumference of a circle. It can also be written as:

$$C = 2\pi r$$

because the diameter is twice the radius (r).

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Exercise 10C

Understanding

1 Evaluate the following using a calculator and round to 2 decimal places.

a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

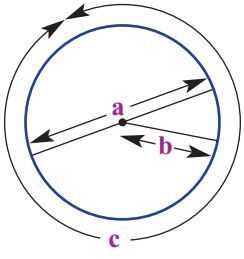
2 Write down the value of π, correct to:

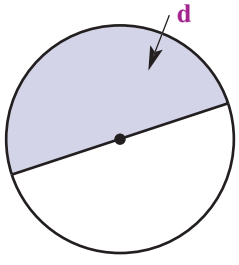
a 1 decimal place
b 2 decimal places
c 3 decimal places

Use the π button on your calculator.



3 Name the features of the circles shown.





4 A circle was measured to have circumference (C) 81.7 m and diameter (d) 26.0 m, correct to 1 decimal place. Calculate $C \div d$. What do you notice?

Fluency

5 Answer true or false to the following questions.

a The distance from the centre of a circle to its circumference is called the diameter.

b $\pi = \frac{C}{d}$ **c** $C = \pi r$

d The radius is twice the diameter. **e** The diameter is twice the radius.

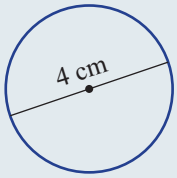
Example 8 Finding the circumference with a calculator

Find the circumference of these circles, correct to 2 decimal places. Use a calculator for the value of pi.

a



b



Solution	Explanation
a $C = 2\pi r$ $= 2 \times \pi \times 3.5$ $= 7\pi$ $= 21.99$ m (to 2 d.p.)	Since r is given, you can use $C = 2\pi r$. Alternatively, use $C = \pi d$ with $d = 7$. 7π is called 'the exact value'.
b $C = \pi d$ $= \pi \times 4$ $= 4\pi$ $= 12.57$ cm (to 2 d.p.)	Substitute $d = 4$ into the rule $C = \pi d$ or use $C = 2\pi r$ with $r = 2$. 4π is called 'the exact value'.

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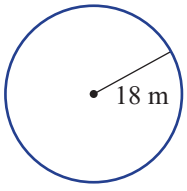
10C

6 Find the circumference of these circles, correct to 2 decimal places. Use a calculator for the value of pi.

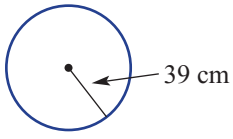
a



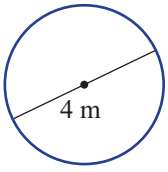
b



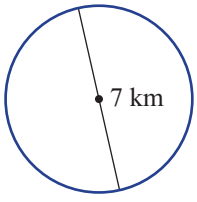
c



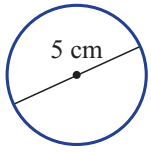
d



e



f



$C = 2 \times \pi \times r$
or
$C = \pi \times d$



Problem-solving and Reasoning

7 A water tank has a diameter of 3.5 m. Find its circumference, correct to 1 decimal place.



8 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

9 Which of these circles has the largest circumference?

A a circle with radius 5 cm
B a circle with diameter 11 cm
C a circle with diameter 9.69 cm
D a circle with radius 5.49 cm

10 The table shows some approximate circle measurements. Which students have incorrect measurements?

	r	C
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	13.8 m

11 What is the difference in the circumference of a circle with radius 2 cm and a circle with diameter 3 cm? Use 3 as an approximation to pi.

12 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

Enrichment: History of pi

13 Pi (π) has been used throughout the world for almost 4000 years. Use the internet to research:

a the different values that have been used for pi by different civilisations
b interesting facts about pi
c Pi Day

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10D Arc length and perimeter of sectors and composite figures

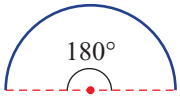
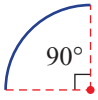
In this section we will use the formula $C = \pi d$ to find the perimeter of fractions of circles, which are called sectors.



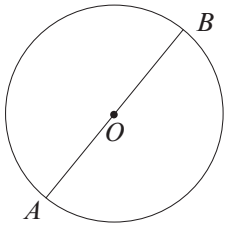
Pizzas are usually cut into sectors.

Let's start

Copy this table into your workbook and complete it. The first two rows have been completed for you.

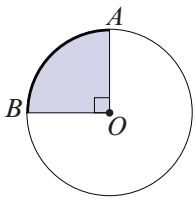
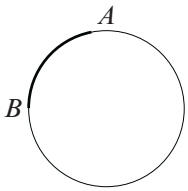
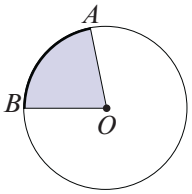
Angle	Fraction of circle	Arc length	Diagram
180°	$\frac{180}{360} = \frac{1}{2}$	$\ell = \frac{1}{2} \times \pi d$	
90°	$\frac{90}{360} = ___$	$\ell = ___ \times ___$	
60°			
45°			
30°			
θ			

Key ideas

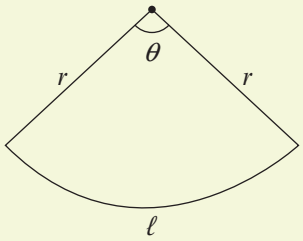
Terminology	Example	Definition
semicircle		Half a circle. In the diagram, the diameter AB creates two semicircles.

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Drilling for Gold 10D1

Terminology	Example	Definition
quadrant		A sector that is exactly one-quarter of a circle. The diagram shows quadrant AOB.
arc		Part of the circumference of a circle. In the diagram there are two arcs called AB. The shorter one is called the minor arc and the longer one is called the major arc.
sector		A region inside a circle bounded by two radii and an arc. The diagram shows sector AOB.

- In the diagram:
  r = radius of circle
  θ = number of degrees in angle at centre of circle
  ℓ = length of arc



  Formula for length of arc:

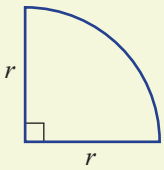
$$\ell = \frac{\theta}{360} \times 2\pi r \quad \text{or} \quad \ell = \frac{\theta}{360} \times \pi d$$

- The sector also has two straight edges.
  Formula for perimeter of sector:

$$P = \frac{\theta}{360} \times 2\pi r + 2r \quad \text{or} \quad P = \frac{\theta}{360} \times \pi d + d$$

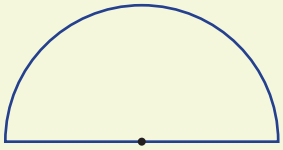
- Common circle portions

quadrant



$$P = \frac{1}{4} \times 2\pi r + 2r$$

semicircle



$$P = \frac{1}{2} \times 2\pi r + 2r$$

- A **composite figure** is made up of more than one basic shape.

composite figure
A combination of shapes joined together

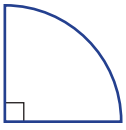
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Exercise 10D

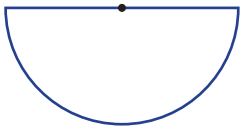
Understanding

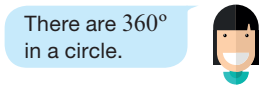
1 What fraction of a circle is shown in these diagrams? Name each shape.

a



b



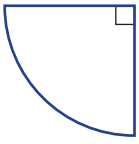


2 What fraction of a circle is shown in these sectors? Simplify your fraction.

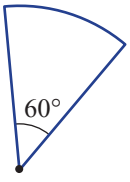
a



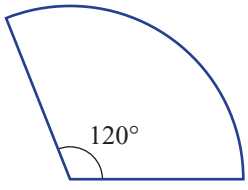
b



c



d



e

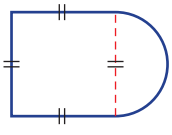


f

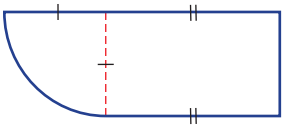


3 Name the two basic shapes that make up these composite figures.

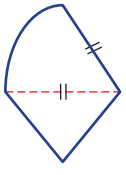
a



b



c

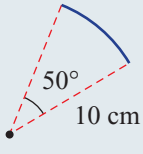


Fluency

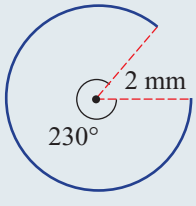
Example 9 Finding an arc length

Find the length of these arcs for the given angles, correct to 2 decimal places.

a



b



Solution

a $\ell = \frac{50}{360} \times 2\pi \times 10$

$= 8.73$ cm (to 2 d.p.)

Explanation

The fraction of the full circumference is $\frac{50}{360}$ and the full circumference is $2\pi r$, where $r = 10$.

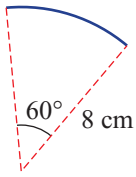
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10D

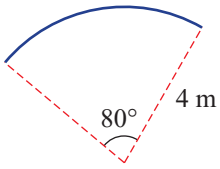
Solution	Explanation
b $\ell = \frac{230}{360} \times 2\pi \times 2$ $= 8.03$ mm (to 2 d.p.)	The fraction of the full circumference is $\frac{230}{360}$ and the full circumference is $2\pi r$.

4 Find the length of the following arcs for the given angles, correct to 2 decimal places.

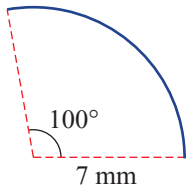
a



b



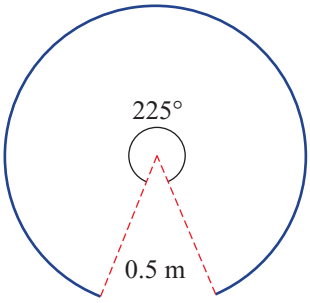
c



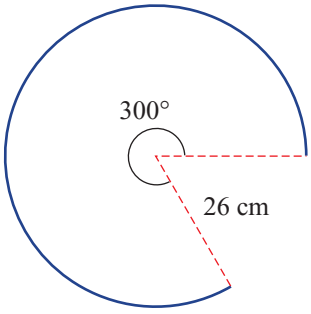
Use $\frac{\theta}{360} \times 2\pi r$ or $\frac{\theta}{360} \times \pi d$.



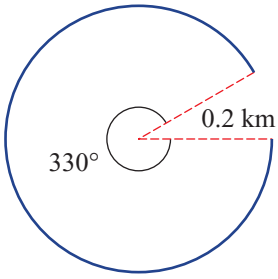
d



e



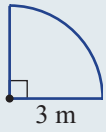
f



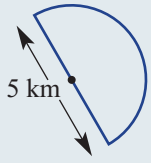
Example 10 Finding the perimeter of a sector

Find the perimeter of these sectors, correct to 1 decimal place.

a



b

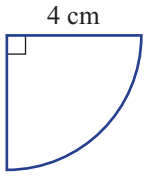


Solution	Explanation
a $P = \frac{1}{4} \times 2\pi \times 3 + 2 \times 3$ $= 10.7$ m (to 1 d.p.)	The arc length is one-quarter of the circumference and included are two radii, each of 3 m.
b $P = \frac{1}{2} \times \pi \times 5 + 5$ $= 12.9$ km (to 1 d.p.)	A semicircle's perimeter consists of half the circumference of a circle plus a full diameter.

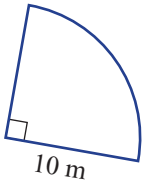
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5 Find the perimeter of these sectors, correct to 1 decimal place.

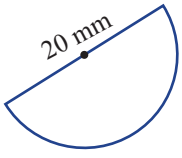
a



b



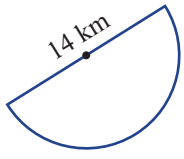
c



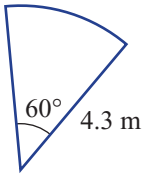


Don't forget to include the straight sides.

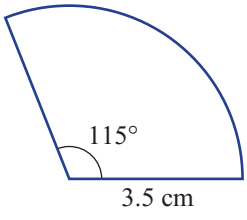
d



e



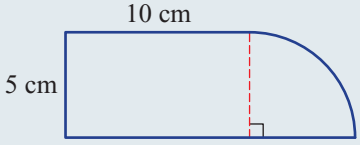
f



Problem-solving and Reasoning

Example 11 Finding the perimeter of a composite figure

Find the perimeter of the following composite figure, correct to 1 decimal place.



Solution

$P = 10 + 5 + 10 + 5 + \frac{1}{4} \times 2\pi \times 5$

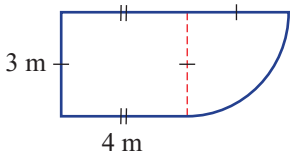
$= 37.9$ cm (to 1 d.p.)

Explanation

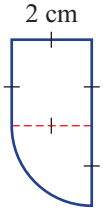
There are two straight sides of 10 cm and 5 cm shown in the diagram. The radius of the circle is 5 cm, so the straight edge at the base of the diagram is 15 cm long. The arc is a quarter circle.

6 Find the perimeter of these composite figures, correct to 1 decimal place.

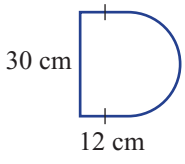
a



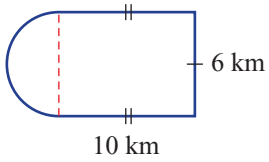
b



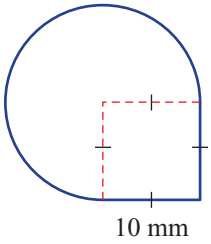
c



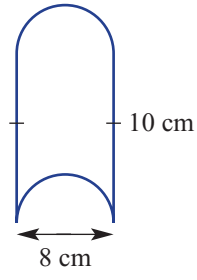
d



e



f



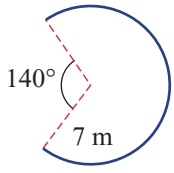
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10D

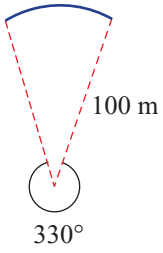
7 A window consists of a rectangular part of length 2 m and breadth 1 m, with a semicircular top having a diameter of 1 m. Find its perimeter, correct to the nearest cm. Use a calculator to help you.

8 For these sectors, find only the length of the arc, correct to 2 decimal places.

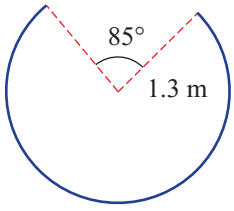
a



b

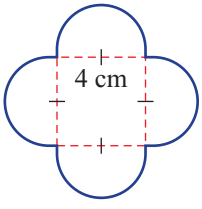


c

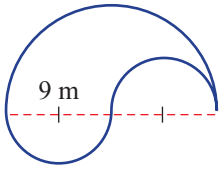


9 Calculate the perimeter of these shapes, correct to 2 decimal places.

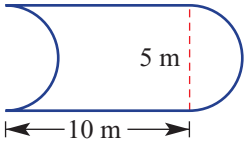
a



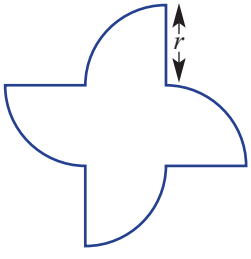
b



c



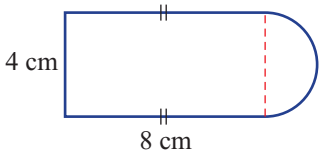
10 Give reasons why the circumference of this composite figure can be found by simply using the rule $P = 2\pi r + 4r$.



Enrichment: Exact values and perimeters

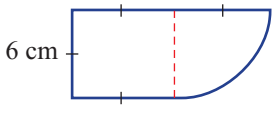
11 The working to find the exact perimeter of this composite figure is given by:

$$P = 2 \times 8 + 4 + \frac{1}{2}\pi \times 4$$
$$= (20 + 80\pi) \text{ cm}$$

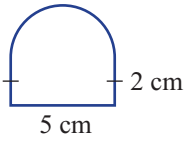


Find the exact perimeter of the following composite figures.

a



b

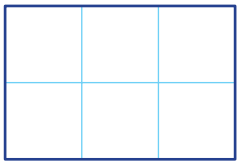


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10E Units of area and area of rectangles

The number of square units inside the boundary of a closed shape gives the measurement called area.

The number of square centimetres inside this rectangle is 6, so the area is written as 6 sq. cm or 6 cm^2.



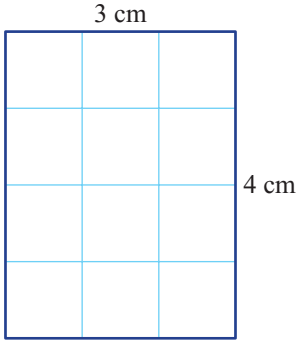


Other shapes may be measured in larger or smaller units. For example, the area of a coin might be 200 mm^2 and the area of a park might be 8000 m^2.

Let's start: The 12 cm^2 rectangle

Consider an area of 12 square centimetres (12 cm^2).

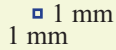
- Draw examples of rectangles that have this area, showing the dimensions. One example is shown here. You might find it helpful to draw on grid paper.
- How many different rectangles (with whole number dimensions) are possible?
- How many different rectangles are possible if there is no restriction on the type of numbers used for length and breadth?

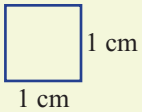


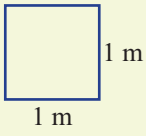
Key ideas


Drilling for Gold 10A1

- The metric units of **area** include:
  - 1 square millimetre (1 mm^2)
  - 1 square centimetre (1 cm^2)
  - 1 square metre (1 m^2)





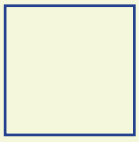


(Not drawn to scale.)

Area The amount of surface that a two-dimensional shape covers

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– 1 square kilometre (1 km^2)



(Not drawn to scale.)

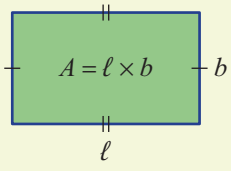
– 1 hectare (1 ha) (10 000 m^2)



(Not drawn to scale.)

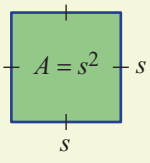
Dimension
A measurement belonging to a shape

- The word **dimensions** is used for the lengths of the sides of rectangles. Other words used include: length (ℓ), breadth (b), base (b), height (h) and width (w).
- On a grid, you can count squares to find the area of a rectangle, or multiply the number of rows by the number of columns.
- The area of a rectangle is the product of length (ℓ) and breadth (b).



$$A = \ell b$$

- The area of a square is the square of the side length (s).



$$A = s^2$$

Exercise 10E

Understanding

1 Arrange the following from smallest to largest.
1 cm^2, 1 m^2, 1 ha, 1 km^2, 1 mm^2

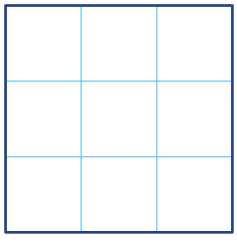


1 ha is not as large as 1 km^2.

2 Match the object (**a** to **e**) with the most appropriate unit (**A** to **E**).

a the area of a small farm	**A** square mm
b the area of Australia	**B** square cm
c the area of the floor in a room	**C** square m
d the area of this page	**D** hectares
e the area of a freckle	**E** square km

3 For this square drawn on a 1 cm grid, find:
- **a** the number of single 1 cm squares
- **b** the side length (s)
- **c** the square of the side length (s^2)





For part **b**, give the side length in cm.

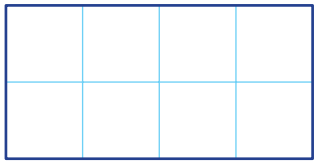
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4 For this rectangle drawn on a 1 cm grid, find:

a the number of single 1 cm squares

b the length (ℓ) and the breadth (b)

c length (ℓ) $\times$ breadth (b)



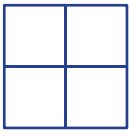
Fluency



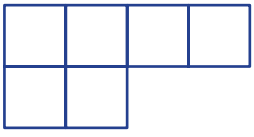
Drilling for Gold 10E1

5 Count the number of squares to find each area, in square units.

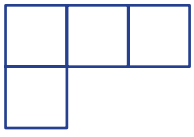
a



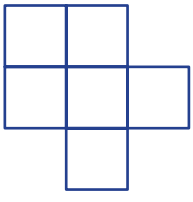
b



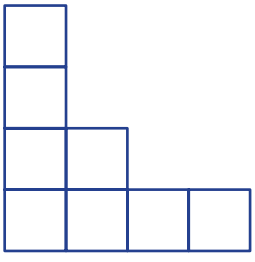
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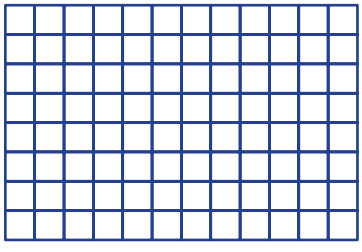
d



e

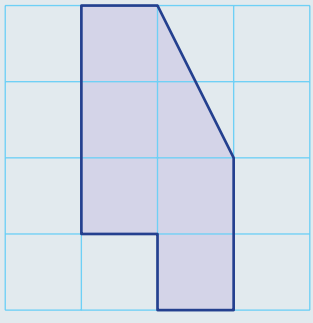


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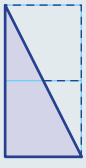


Example 12 Counting areas

Count the number of squares to find the area of the shape drawn on this 1 cm grid.



Solution	Explanation
6 cm^2	There are 5 full squares and half of 2 squares in the triangle, giving 5 plus 1 more.



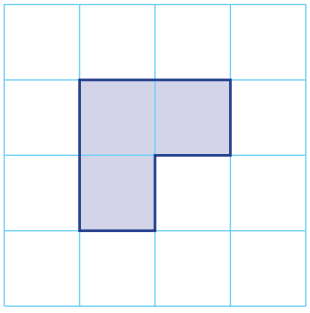
$\frac{1}{2}$ of 2 = 1

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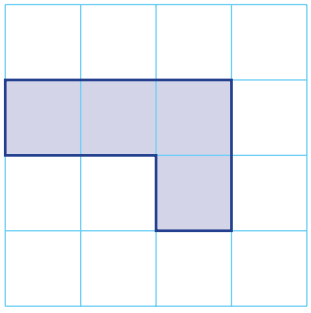
10E

6 Count the number of squares to find the area of these shapes on centimetre grids.

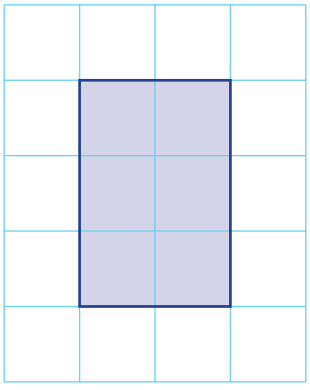
a



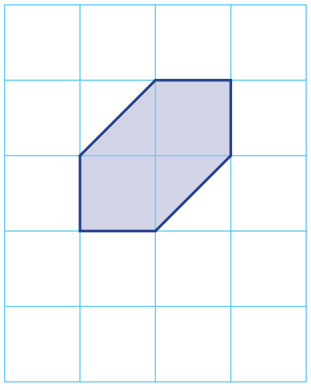
b



c



d

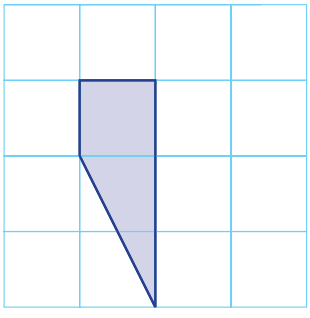




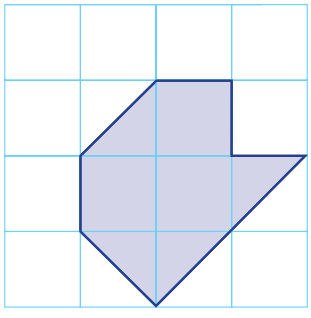
is $\frac{1}{2}$ cm^2.



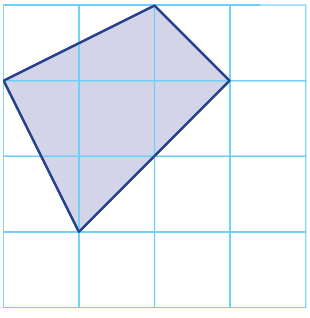
e



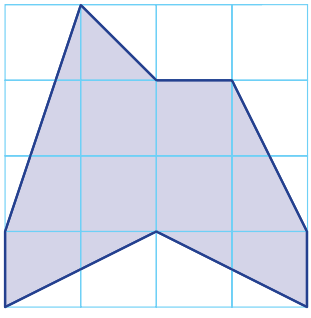
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g

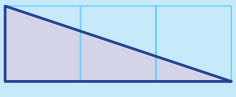


h





is $\frac{1}{2}$ of $2 = 1$ cm



is $\frac{1}{2}$ of $3 = 1.5$ cm^2

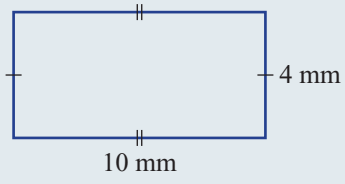


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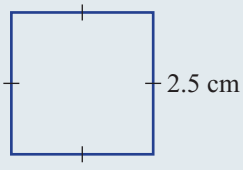
Example 13 Areas of rectangles and squares

Find the area of this rectangle and square.

a



b



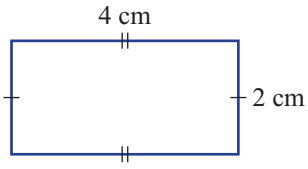
Solution	Explanation
a $A = \ell \times b$ $= 10 \times 4$ $= 40 \text{ mm}^2$	Multiply the length and breadth to find the area of a rectangle.
b $A = s^2$ $= 2.5^2$ $= 6.25 \text{ cm}^2$	The side length is s. $(2.5)^2 = 2.5 \times 2.5$

7 Find the area of these rectangles and squares. Diagrams are not drawn to scale.

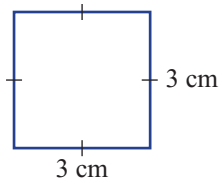
$A = \ell b$



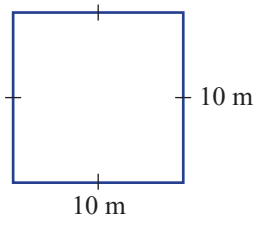
a



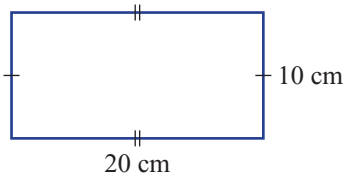
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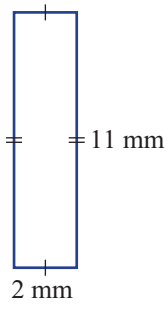
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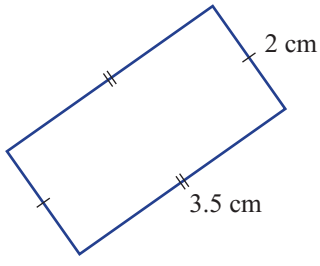
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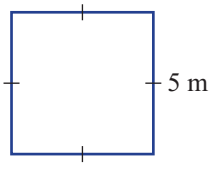
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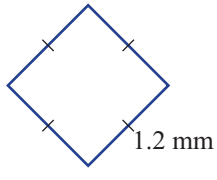
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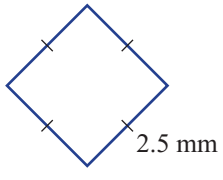
g



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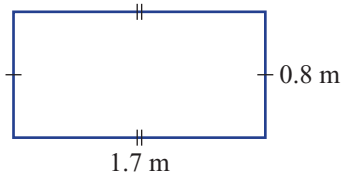


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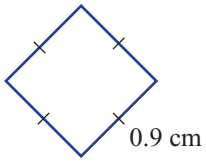
10E

Skillsheet 10B

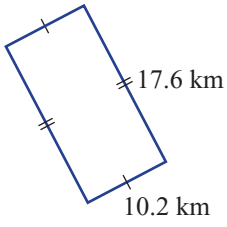
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k



l



Problem-solving and Reasoning

8 A rectangular soccer field is to be laid with new grass. The field is 100 m long and 50 m wide. Find the area of grass to be laid.

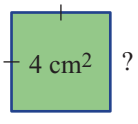
9 Glass is to be cut for a square window of side length 50 cm. Find the area of glass required for the window.

10 Find the side length of each of these squares.

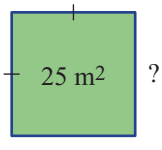
Use trial and error if you are unsure.



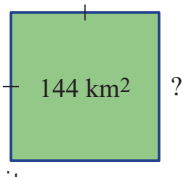
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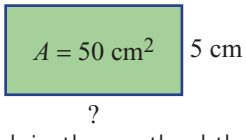
11 a A square has a perimeter of 20 cm. Find its area.
b A square has an area of 9 cm². Find its perimeter.
c A square's area and perimeter are the same number. How many units is the side length?

First try to work out the side length of each square.

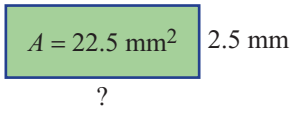


12 a Find the missing length for each of these rectangles.

i

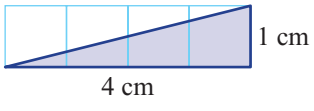


ii



b Explain the method that you used for finding the missing lengths of the rectangles above.

13 Explain why the area shaded here is exactly 2 cm².



Enrichment: Renovation work

Drilling for Gold 10E2

14 Two hundred square tiles, each measuring 10 cm by 10 cm, are used to tile an open floor area. Find the area of flooring that is tiled.

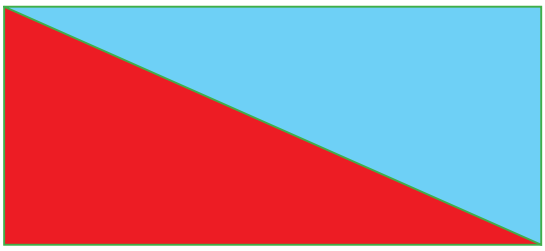
15 The carpet chosen for a room costs $70 per square metre. The room is rectangular and is 6 m long by 5 m wide. What is the cost of carpeting the room?

16 Tanisha wants to paint a house wall that is 11 m long and 3 m high. Two coats of paint are needed. The paint suitable to do the job can be purchased only in whole numbers of litres and covers an area of 15 m² per litre. How many litres of paint will Tanisha need to purchase?

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10F Area of triangles

In the diagram, each right-angled triangle occupies half of the rectangle.

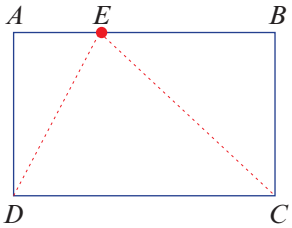


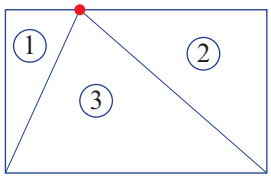
Half of the rectangle is red and half is blue.

Let's start: Is every triangle 'half a rectangle'?

Start with a sheet of A4 paper, $ABCD$, in landscape orientation (i.e. sideways).
Now follow these steps.

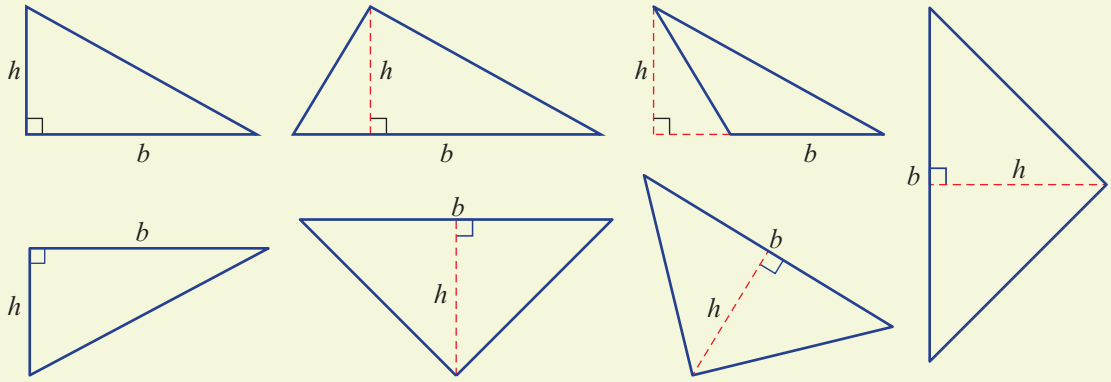
1. Choose a random point on AB. Call it E.
2. Using a ruler and pencil, join E to D and E to C.
3. Label the three triangles ①, ② and ③.
4. Cut the sheet into three triangles.
   - Is it possible to cover ③ with ① and ② without any gaps or overlaps?
   - What does this tell you about the area of ③ compared with the area of ① and ②?
   - What does this tell you about the area of ③ compared with the area of the rectangle?





Key ideas

- In rectangles, the dimensions are often called length (ℓ) and breadth (b).
- In triangles, the dimensions are usually called **base** (b) and **height** (h).
- The base can be *any* side of the triangle.
- The height is **perpendicular** to the base.



- The area of a rectangle is $A = \ell \times b$.
- A triangle is half of a rectangle, so the area is $\frac{1}{2} \times$ base $\times$ height.

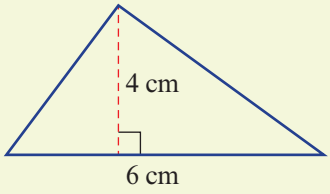
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- The formula for the area of a triangle is:

$$A = \frac{1}{2}bh$$

A = area
b = length of the base
h = perpendicular height

Note: This can be also written as $A = \frac{bh}{2}$ or $A = b \times h \div 2$.



base One of three sides of a triangle

height The perpendicular distance from the base of a triangle to the opposite vertex

perpendicular Meeting at right angles

$A = \frac{1}{2} \times 6 \times 4$ or $A = \frac{1}{2} \times 6 \times 4$ or $A = \frac{1}{2} \times 6 \times 4$

$= \frac{1}{2} \times 24$ $= 3 \times 4$ $= 6 \times 2$

$= 12$ cm^2 $= 12$ cm^2 $= 12$ cm^2



Mental strategy: It is okay to 'halve' the base *or* the height *or* the product $b \times h$.

Exercise 10F

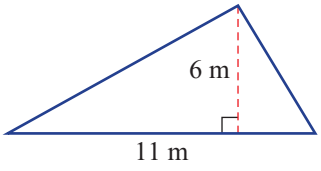
Understanding

1 For each of these triangles, what length would be used as the base?

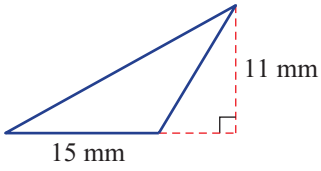


The base is one of the side lengths of the triangle.

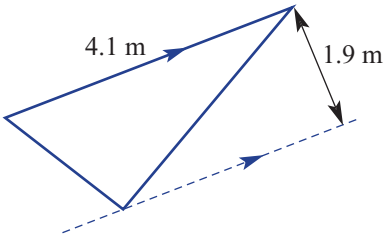
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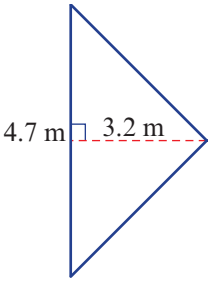
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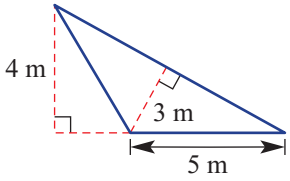
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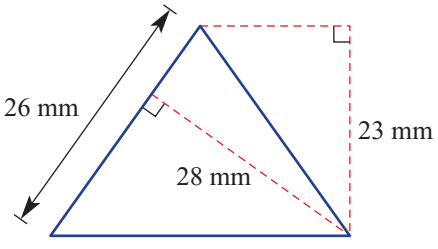
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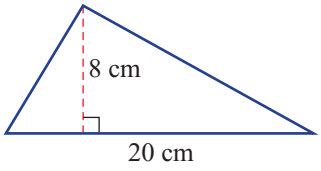
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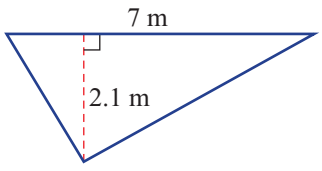
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2 For each of these triangles, what length would be used as the height?

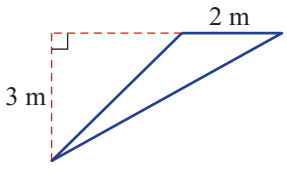
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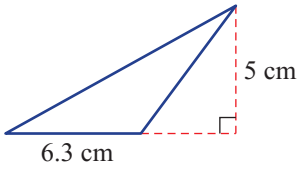
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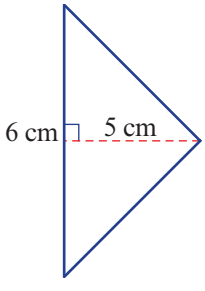
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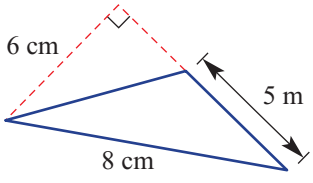
d



e



f



3 Using the formula $A = \frac{1}{2}bh$, find the value of A when:

a $b = 5$ and $h = 4$ **b** $b = 7$ and $h = 16$

c $b = 2.5$ and $h = 10$ **d** $b = 10$ and $h = 12$

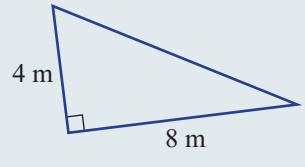


In part **a**, $\frac{1}{2} \times 5 \times 4 = 5 \times 2 = ?$

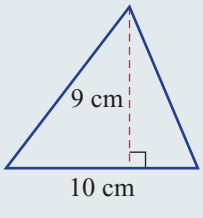
Example 14 Computation of areas of triangles

Find the area of each given triangle.

a



b



Solution	Explanation
a $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 8 \times 4$ $= 16 \text{ m}^2$	Use 8 m for the base and 4 m for the height (or the other way around). Mental strategy: $\frac{1}{2} \times 8 \times 4$ can be 4×4 or 8×2 or $\frac{1}{2}$ of 32.
b $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 9$ $= 45 \text{ cm}^2$	Use the formula and substitute the values for base length and height. Mental strategy: $\frac{1}{2} \times 10 \times 9$ can be 5×9 or $\frac{1}{2}$ of 90.

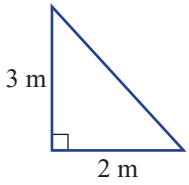
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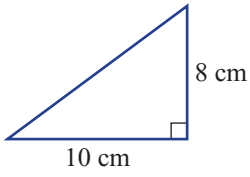
Fluency

4 Find the area of each triangle given. Remember to write units with your answer; e.g. 5 m^2.

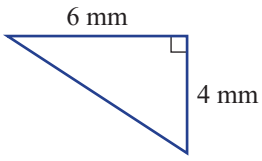
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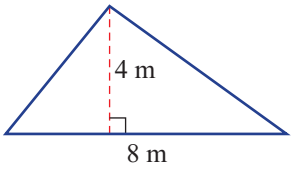
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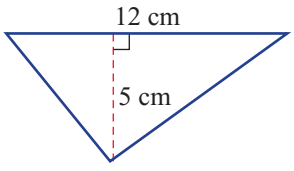
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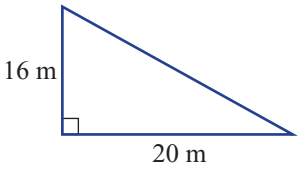
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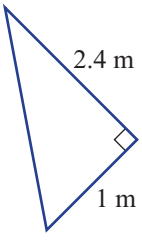
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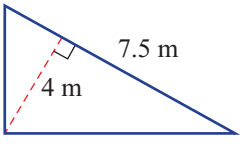
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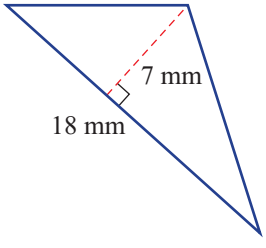
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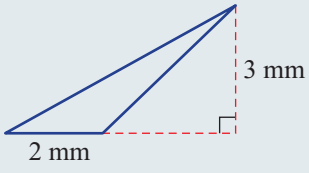
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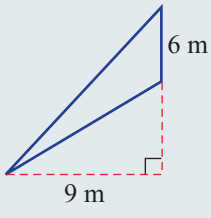
Example 15 Area of obtuse-angled triangles

A triangle is obtuse-angled if it contains exactly one angle greater than 90° but less than 180°.
Find the area of each of these obtuse-angled triangles.

a



b

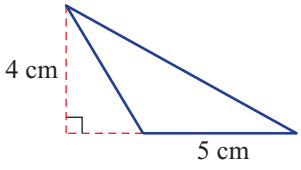


Solution	Explanation
a $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 2 \times 3$ $= 3$ mm^2	The base length is 2 mm, so use $b = 2$. The height is 3 mm, so use $h = 3$.
b $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 9$ $= 27$ m^2	The length measure of 9 m is marked at 90° to the side marked 6 m. So 6 m is the base and 9 m is the height.

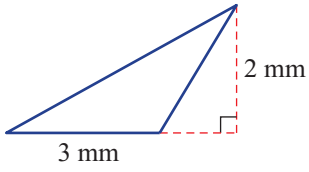
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5 Find the area of each obtuse-angled triangle.

a



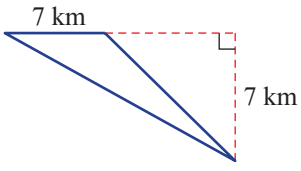
b



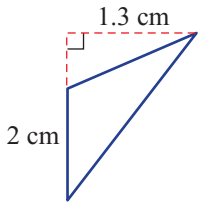
Remember to write units with your answer; e.g. 5 m^2.



c

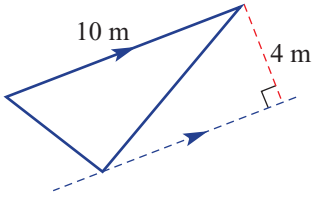


d

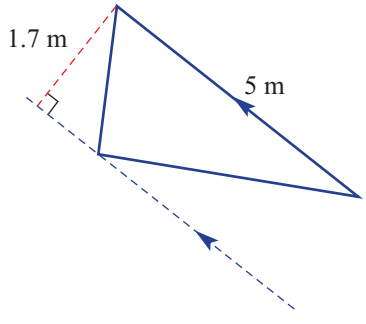


Skillsheet 10B

e

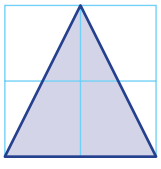


f

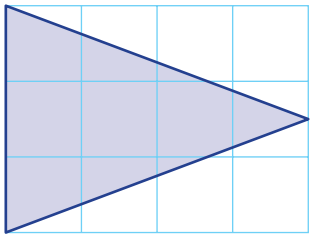


6 Find the area of these triangles, which have been drawn on 1 cm grids. Give your answer in cm^2.

a

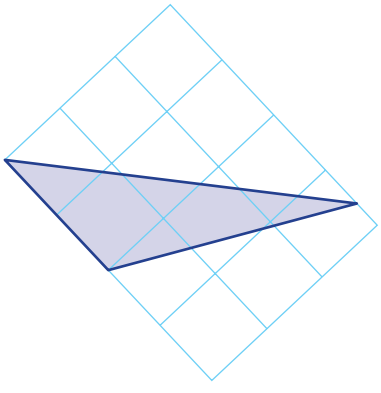


b

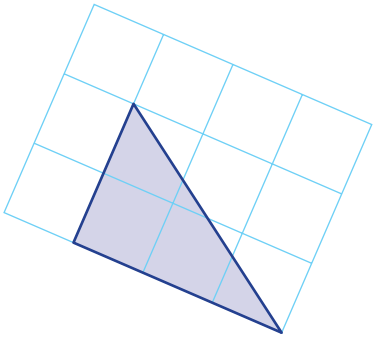


Choose b and h carefully!

c



d

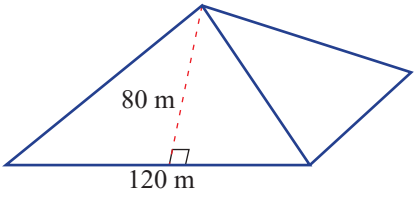


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10F

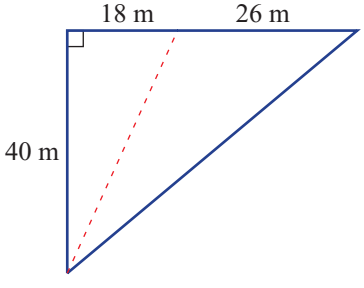
Problem-solving and Reasoning

7 A square pyramid has a base length of 120 m and a triangular face of height 80 m. Find the area of one triangular face of the pyramid.





8 A farmer uses fencing to divide a triangular piece of land into two smaller triangles, as shown. What is the difference in the two areas?

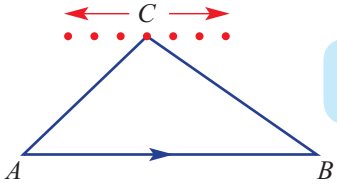


9 A rectangular block of land measuring 40 m long by 24 m wide is cut in half along a diagonal. Find the area of each triangular block of land.

10 A yacht must have two of its sails replaced as they have been damaged by a recent storm. One sail has a base length of 2.5 m and a height of 8 m. The bigger sail has a base length of 4 m and a height of 16 m. If the cost of sail material is $150 per square metre, find the total cost of replacing the yacht's damaged sails.



11 If the vertex C for this triangle moves left or right (to one of the red dots) will the area of the triangle change? Justify your answer.

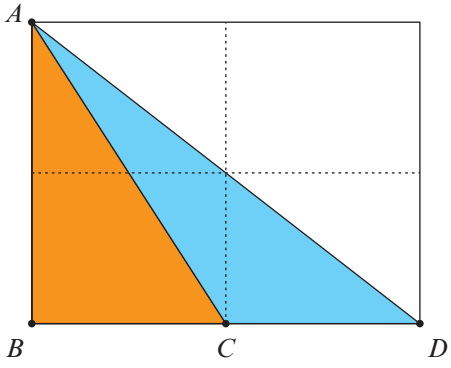


Try building this in a dynamic geometry computer program.



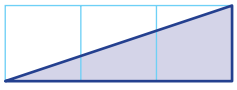
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12 Explain why triangle ABC and triangle ACD have the same area.



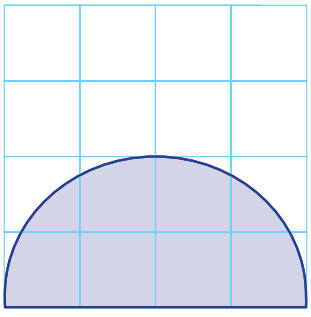
Enrichment: Estimating curved areas

13 This diagram shows a shaded region that is $\frac{1}{2}$ of 3 cm^2 = 1.5 cm^2.

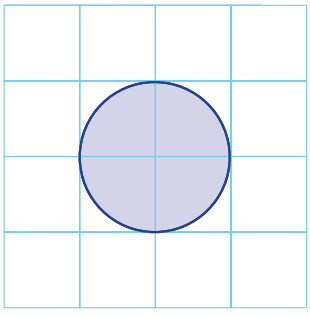


Using triangles like the one shown here, and by counting whole squares also, estimate the areas of these shapes below. Give your answer correct to the nearest square centimetre.

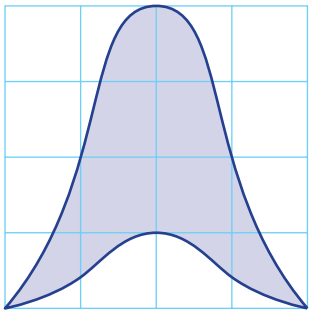
a



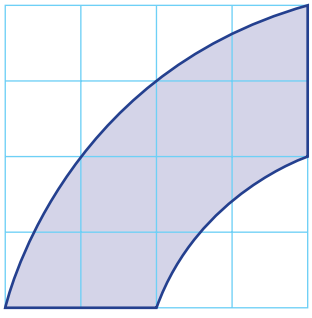
b



c



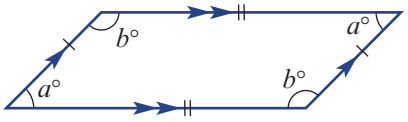
d



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10G Area of parallelograms

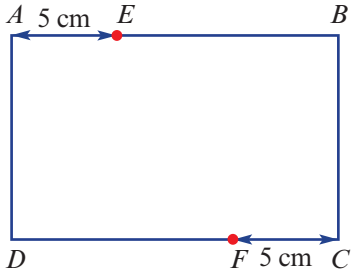
A parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are the same length and opposite angles are equal.



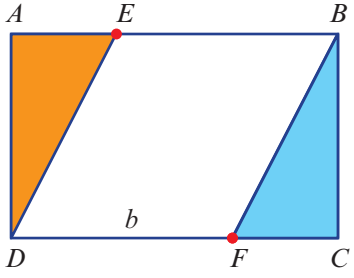
Let's start: 'Discover' the formula for area of parallelograms

Steps:

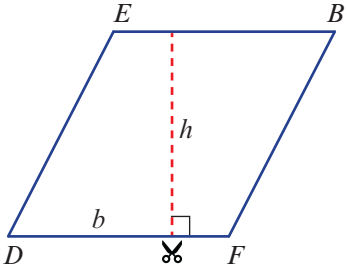
1. Start with an A4 sheet of paper, $ABCD$.
2. Mark points E and F, 5 cm from A and C, as shown.



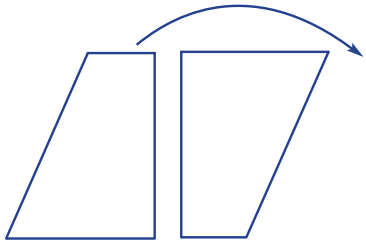
3. Join D to E and B to F, as shown.
4. Cut off ΔAED and ΔBCF. Throw them away to leave parallelogram $EBFD$. Label the base b.



5. Draw a perpendicular height, label it h, and cut the parallelogram into two pieces.



6. Swap the position of the left piece and the right piece to make a familiar shape.



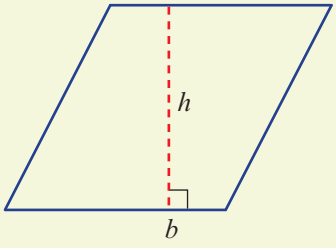
- What shape do you now have?
- What is the area of that shape?
- What is the area of a parallelogram?

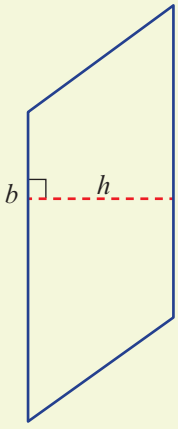
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Key ideas

Parallelogram
A quadrilateral with both pairs of opposite sides parallel

- For parallelograms, the dimensions are called base (b) and perpendicular height (h).
- The base can be *any* side of the **parallelogram**.
- The height is perpendicular to the base.





- The area of a parallelogram is base × perpendicular height.
- The formula for the area of a parallelogram is:

$A = b \times h$

A = area
b = length of the base
h = perpendicular height

Exercise 10G

Understanding

1 Which of the following is the correct formula for the area of a parallelogram?

A $A = \frac{1}{2}bh$ **B** $A = \pi r^2$ **C** $A = bh$ **D** $A = b \div h$

2 Copy and complete the following, using the given values of b and h.

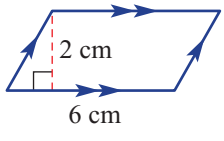
a $b = 5, h = 7$
$A = bh$
$= ___ \times ___$
$= 35$

b $b = 20, h = 3$
$A = ___$
$= 20 \times ___$
$= ___$

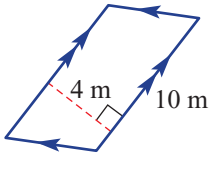
c $b = 8, h = 2.5$
$A = ___$
$= ___ \times ___$
$= ___$

3 For each of these parallelograms, state the base and the height that might be used to find the area.

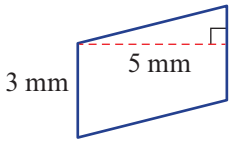
a



b

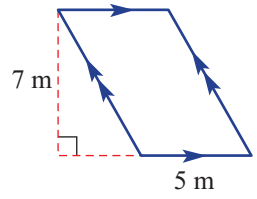


c

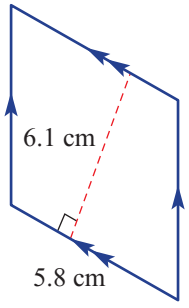


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d



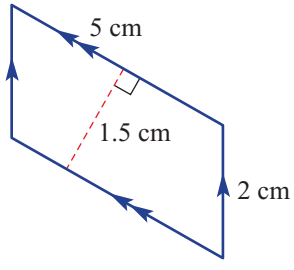
e



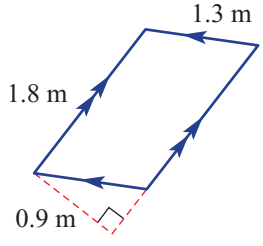


For parts **f** to **h**, choose the base (side) that is at 90° to the perpendicular height.

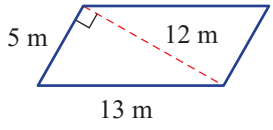
f



g



h

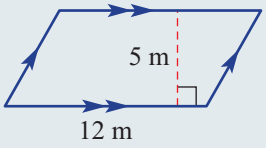


Fluency

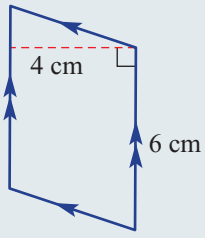
Example 16 Computation of area of parallelograms

Find the area of these parallelograms.

a



b



Solution	Explanation
a $A = bh$ $= 12 \times 5$ $= 60\ \text{m}^2$	Choose the given side as the base (12 m) and note the perpendicular height is 5 m.
b $A = bh$ $= 6 \times 4$ $= 24\ \text{cm}^2$	Use the base length 6 cm and the perpendicular height 4 cm.

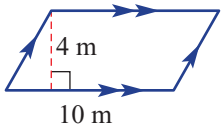


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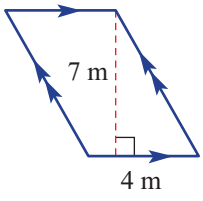
10G

4 Find the area of these parallelograms.

a

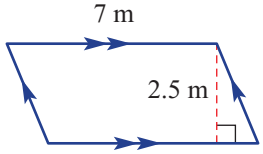


b

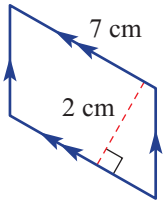


b = base
h = height
$A = b \times h$

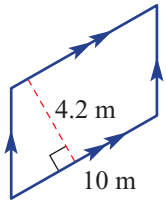
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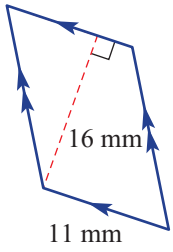
d



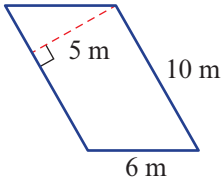
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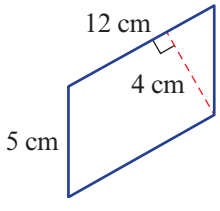
f



g



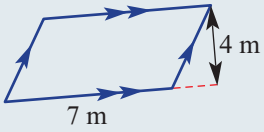
h



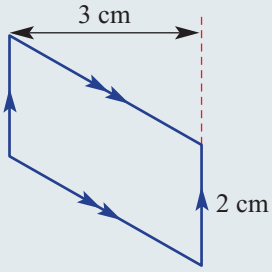
Example 17 Finding area with the height shown outside the shape

Find the area of these parallelograms.

a



b



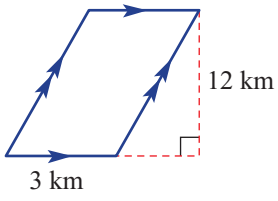
Solution	Explanation
a $A = bh$ $= 7 \times 4$ $= 28$ m^2	Choose the base of 7 m and the height 4 m, which is at 90° to the base.
b $A = bh$ $= 2 \times 3$ $= 6$ cm^2	Use the given side as the base (2 cm), noting that the height is 3 cm.

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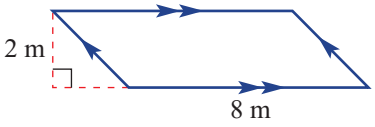
Skillsheet 10B

5 Find the area of these parallelograms.

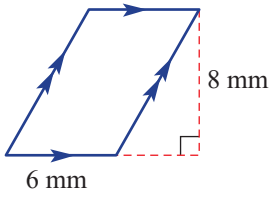
a



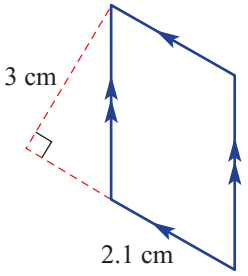
b



c



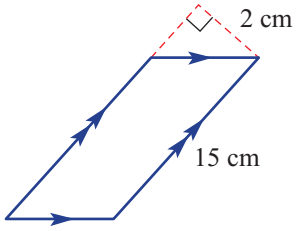
d



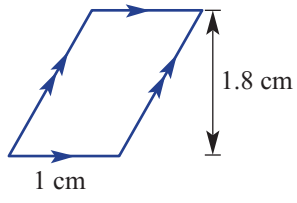


b = base
h = height
$A = b \times h$

e

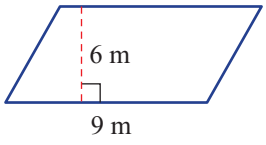


f



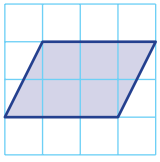
Problem-solving and Reasoning

6 The floor of an office space is in the shape of a parallelogram. The longest sides are 9 m and the distance between them is 6 m. Find the area of the office floor.

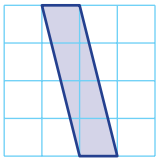


7 These parallelograms are on 1 cm grids (not drawn to scale). Find each area.

a



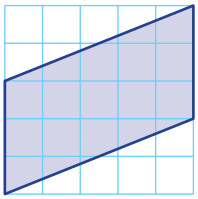
b



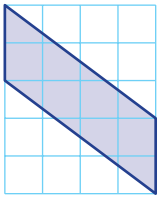


First identify the base and the height.

c



d



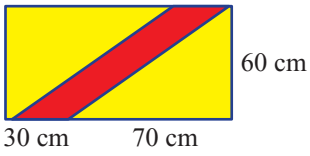
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10G

8 A proposed rectangular flag for a new country is yellow with a red stripe in the shape of a parallelogram, as shown. Find:

a the area of the red stripe

b the yellow area



9 Find the height of a parallelogram when its:

a area = 10 m^2 and base = 5 m

b area = 28 cm^2 and base = 4 cm



Use $A = bh$ and substitute the given information.

10 Find the base of a parallelogram when its:

a area = 40 cm^2 and height = 4 cm

b area = 150 m^2 and height = 30 m

11 A large wall in the shape of a parallelogram is to be painted with a special red paint, which costs \$20 per litre. Each litre of paint covers 5 m^2. The wall has a base length of 30 m and a height of 10 m. Find the cost of painting the wall.

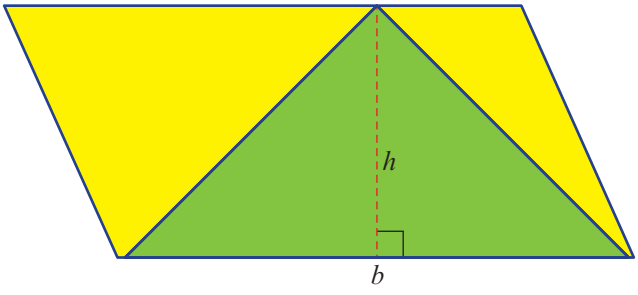


First work out the area of the wall. Then find the number of litres of paint needed.

12 A parallelogram includes a green triangular area, as shown. What fraction of the total area is the green area? Give reasons for your answer.



Compare the area formulas for a triangle and a parallelogram.





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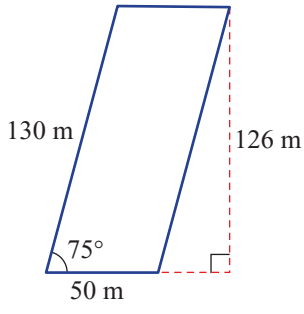
Enrichment: Glass façade

13 The Puerta de Europa (Gate of Europe) towers are twin office buildings in Madrid, Spain. They look like normal rectangular glass-covered skyscrapers but they lean towards each other at an angle of $15°$ to the vertical.



For each building:

- The perpendicular height of the building is 126 metres.
- The base is a square with 50 m sides.
- The two vertical walls (i.e. the front and the back) are parallelograms, as shown in the diagram.
- The two slanting walls are rectangles.
- All four walls are covered with glass.



Answer the following for *one* of the towers.

a Find the area of one of the sloping (rectangular) walls.

b Find the area of one of the sides that is vertical (a parallelogram).

c Calculate the total area of all four sides of the tower.

d If glass costs $\$180$ per square metre, find the cost of covering the tower with glass. (Assume that glass covers the entire surface, ignoring the beams.)

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10H Mass and temperature

Mass relates to how heavy an object is. We use the metric units milligrams, grams, kilograms and tonnes to measure mass. The mass of a large elephant is about 4000 kg or 4 tonnes, whereas the mass of an ant is about 2 milligrams.

Temperature tells us how hot or cold something is. Anders Celsius (1701–1744), a Swedish scientist, developed a scale for temperature.







Let's start: Matching a mass or temperature

Work with a partner or group.

Name an object, place or situation to match each of these temperatures.

- 0°C
- 20°C
- 50°C
- 100°C
- 250°C
- –10°C

Name an object whose mass would be measured using:

- tonnes
- kilograms
- grams
- milligrams

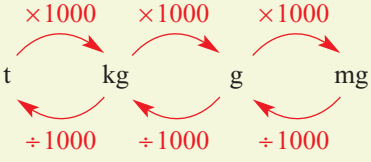
Key ideas

Mass The measure of the amount of matter in an object

Kilogram Unit of metric measurement for mass

Celsius Scale used for measuring temperature, where water freezes at 0° and boils at 100°

- The basic unit for **mass** is the **kilogram** (kg).
  - 1 litre of water has a mass of 1 kilogram.
- Metric units for mass include:
  - 1 gram (g) = 1000 milligrams (mg)
  - 1 kilogram (kg) = 1000 grams (g)
  - 1 tonne (t) = 1000 kilograms (kg)



- The common unit for temperature is degrees **Celsius** (°C).
  - 0°C is the freezing point of water.
  - 100°C is the boiling point of water.



Drilling for Gold 10A1

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Exercise 10H

Understanding


Drilling for Gold 10H1

1 Write the missing word or number in these sentences.

a There are ____ grams in 1 kilogram.
b There are 1000 _____ in 1 gram.
c There are 1000 kilograms in 1 _____.
d Water boils at ____°C.
e Water freezes at ____°C.

2 Choose the mass (**A** to **F**) that best matches the given object (**a** to **f**).

a human hair
b 10-cent coin
c bottle
d large book
e large bag of sand
f truck

A 300 g
B 40 kg
C 100 mg
D 1.5 kg
E 13 t
F 5 g


mg is milligrams
g is grams
kg is kilograms
t is tonnes

3 Choose the temperature (**A** to **D**) that best matches the description (**a** to **d**).

a temperature of coffee
b temperature of tap water
c temperature of oven
d temperature in Antarctica

A 15°C
B 50°C
C –20°C
D 250°C

Fluency

Example 18 Reading scales

Read this temperature scale.



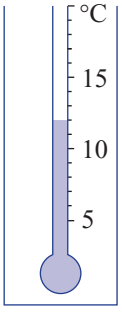
Solution	Explanation
3°C	The markings each represent 1°C and the temperature is 3 markings above zero.

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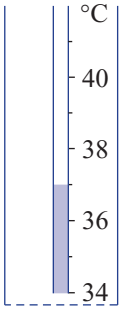
10H

4 Read these temperature scales.

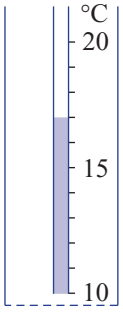
a



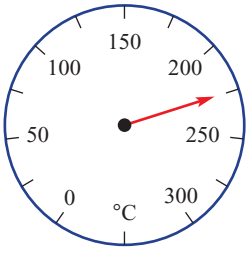
b



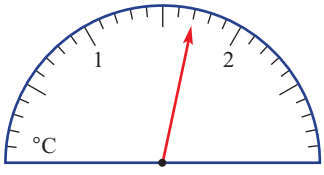
c



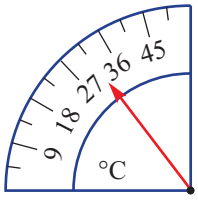
d



e

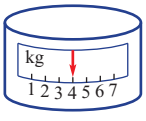


f

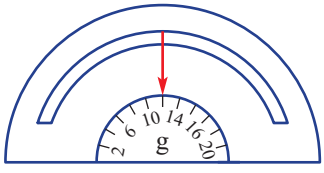


5 Read these mass scales.

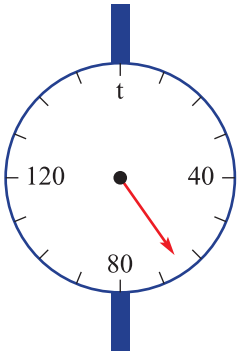
a



b



c



Example 19 Converting units of mass

Convert to the units shown in brackets.

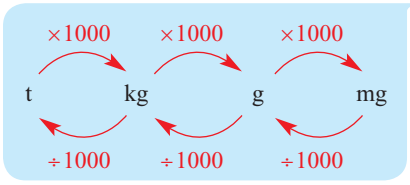
a 2.4 kg (g) **b** 170 000 kg (t)

Solution	Explanation
a 2.4 kg = (2.4 × 1000) g = 2400 g	1 kg = 1000 g Multiply because you are changing to a smaller unit.
b 170 000 kg = (170 000 ÷ 1000) t = 170 t	1 t = 1000 kg Divide because you are changing to a larger unit.

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6 Convert to the units shown in the brackets.

a 2 kg (g)	**b** 7 kg (g)	**c** 6.2 kg (g)
d 5.8 kg (g)	**e** 6000 g (kg)	**f** 8900 g (kg)
g 900 g (kg)	**h** 450 g (kg)	**i** 5 t (kg)
j 0.6 t (kg)	**k** 2400 kg (t)	**l** 4320 kg (t)
m 3 g (mg)	**n** 4.2 g (mg)	**o** 7500 mg (g)



7 Convert to the units shown in brackets.

a 4620 mg (g)	**b** 21 600 kg (t)	**c** 0.47 t (kg)
d 312 g (kg)	**e** 27 mg (g)	**f** $\frac{3}{4}$ t (kg)
g $\frac{1}{8}$ kg (g)	**h** 10.5 g (kg)	**i** 210 000 kg (t)
j 0.47 t (kg)	**k** 592 000 mg (g)	**l** 0.08 kg (g)

Problem-solving and Reasoning

8 The temperature of water in a cup of tea is initially 95°C. After half an hour the temperature is 62°C. What is the drop in temperature?



9 An oven is initially at a room temperature of 25°C. The oven dial is turned to 172°C. What is the expected increase in temperature?

10 A small truck delivers 0.06 t of stone for a garden. Write the mass of stones using these units.

a kg **b** g **c** mg

11 A box contains 20 blocks of cheese, each weighing 150 g. Find the total mass of cheese in the following units.

a g **b** kg

12 Add all the mass measurements and give the result in kg.

a 3 kg, 4000 g, 0.001 t

b 2.7 kg, 430 g, 930 000 mg, 0.0041 t

13 Arrange these mass measurements from smallest to largest.

a 2.5 kg, 370 g, 0.1 t, 400 mg **b** 0.000 32 t, 0.41 kg, 710 g, 290 000 mg

14 A 10 kg bag of flour is used at a rate of 200 g per day. How many days will the bag of flour last?

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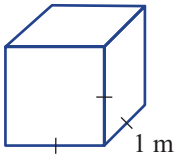
10H

Enrichment: Weighing water

15 Every litre of water has a mass of one kilogram. What is the mass of these volumes of water?

a 1 mL **b** 1 kL **c** 1 ML

16 Suppose that the container below was filled with water. Calculate the mass of water in the container, in kg.



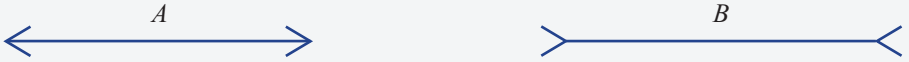


The heaviest organism to ever live on Earth is the blue whale.

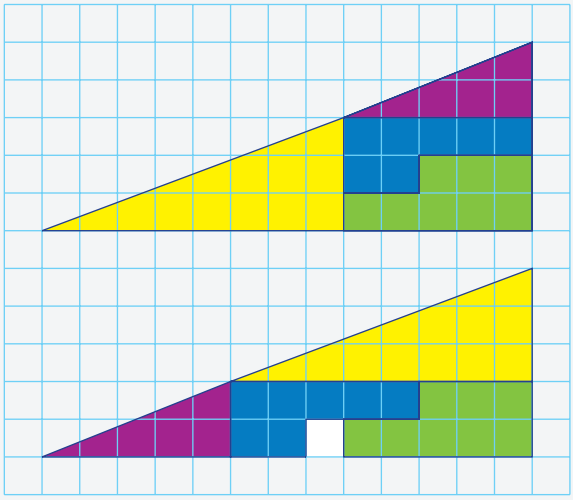
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Puzzles and games

1 Without measuring, state which line looks longer: A or B? Then measure to check your answer.

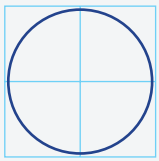


2 Do these two shapes have the same area? Explain the 'hole' in the second shape.

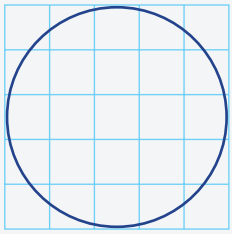


3 Count squares to estimate the area of these circles.

a

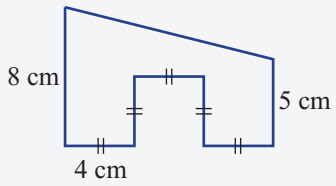


b

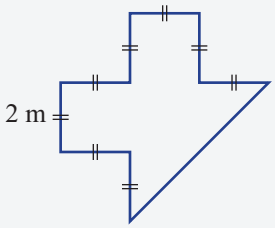


4 Find the areas of these composite shapes.

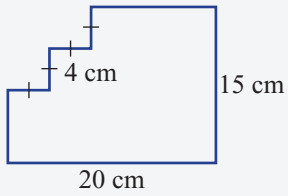
a



b

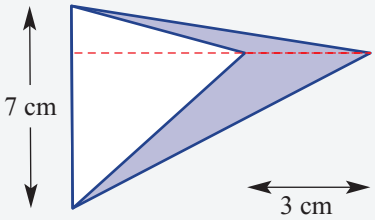


c



5 You have two sticks of length 3 m and 5 m. Neither stick is marked with a scale. How could you use the sticks to mark a length of 1 m?

6 Find the area of the shaded region.



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Chapter summary

Measurement and computation of length, perimeter and area

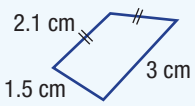
Length

Metric units for length

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Perimeter

The sum of all the sides



$P = 2 \times 2.1 + 1.5 + 3$
$= 8.7$ cm

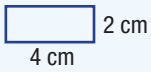
Area

The amount of surface covered by a shape

Metric units for area

mm^2
cm^2
m^2
ha
km^2
1 ha = 10 000 m^2

Rectangle



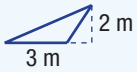
$A = \ell b$
$= 4 \times 2$
$= 8$ cm^2

Square



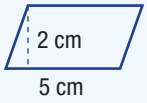
$A = s^2$
$= 9^2$
$= 81$ m^2

Triangle



$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 3 \times 2$
$= 3$ m^2

Parallelogram



$A = bh$
$= 5 \times 2$
$= 10$ cm^2



Drilling for Gold 10R1, 10R2

Mass

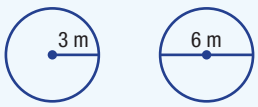
Often called weight

1 g = 1000 mg
1 kg = 1000 g
1 t = 1000 kg

Temperature

°C (Celsius)
0° = freezing point of water
100° = boiling point of water

Circumference

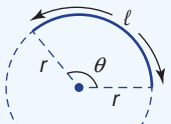


$C = 2\pi r$
$C = 2 \times \pi \times 3$
$C = (6\pi)$ m

$C = \pi d$
$C = \pi \times 6$
$C = (6\pi)$ m

Circular arc

is a portion of the circumference of a circle

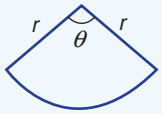


Arc length

Fraction = $\frac{\theta}{360}$
$\ell = \frac{\theta}{360} \times 2\pi r$
$\ell = \frac{\theta}{360} \times \pi d$

Perimeter of a sector

$P = \frac{\theta}{360} \times 2\pi r + 2r$



$\frac{\theta}{360} \times 2\pi r$

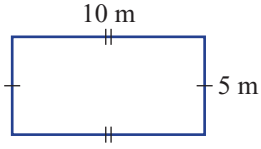
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Chapter review

T **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

Multiple-choice questions

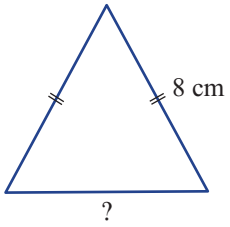
1 The perimeter of this rectangle is:



A 25 cm **B** 40 m **C** 15 m
D 30 m **E** 30 m^2

2 A square has side length 3 cm. Its area is:
A 9 cm **B** 12 cm^2 **C** 6 cm
D 6 cm^2 **E** 9 cm^2

3 The triangle has a perimeter of 20 cm. What is the missing base length?



A 6 cm **B** 8 cm **C** 4 cm
D 16 cm **E** 12 cm

4 The area of a rectangle with length 2 m and width 5 m is:
A 10 m^2 **B** 5 m^2 **C** 5 m **D** 5 m^3 **E** 10 m

5 A triangle has base length 3.2 cm and height 4 cm. What is its area?
A 25.6 cm^2 **B** 12.8 cm **C** 12.8 cm^2 **D** 6 cm **E** 6.4 cm^2

6 If the diameter of a circle is 6 cm, the radius is:
A 12 cm **B** 3 cm **C** (6π) cm **D** (12π) cm **E** (3π) cm

7 If the diameter of a circle is 6 cm, the circumference is:
A 12 cm **B** 3 cm **C** (6π) cm **D** (12π) cm **E** (3π) cm

8 If the diameter of a semicircle is 6 cm, the perimeter is:
A 12 cm **B** $(3\pi + 6)$ cm **C** $(6\pi + 6)$ cm
D $(6\pi + 12)$ cm **E** $(3\pi + 12)$ cm

9 Gravel is being loaded onto a truck at a rate of 20 kg per second. How many minutes will it take to load all of the 9 tonnes of gravel?
A 0.75 min **B** 45 min **C** 7.3 min
D 450 min **E** 7.5 min

10 The base length of a parallelogram is 10 cm and its height is 4 cm. The parallelogram's area is:
A 20 cm^2 **B** 40 cm^2 **C** 30 cm^2
D 4 cm^2 **E** 40 m^2

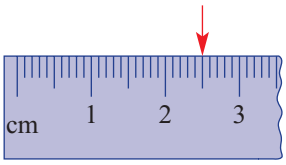
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Chapter review

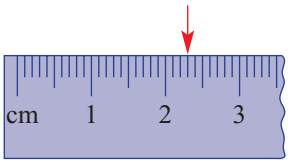
Short-answer questions

1 Read these scales.

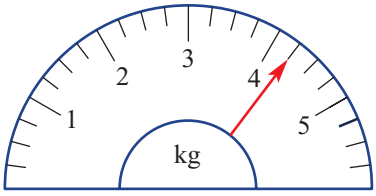
a



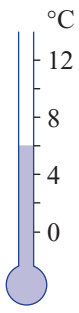
b



c



d



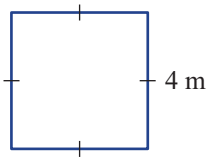


2 Convert to the units shown in brackets.

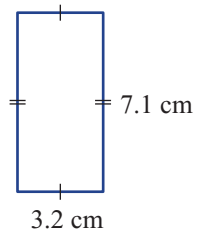
a 5 cm (mm)
b 200 cm (m)
c 3.7 km (m)
d 3600 cm (m)
e 7.1 kg (g)
f 24 000 mg (g)
g 22 000 kg (t)
h 2.5 t (kg)
i 60 mm (cm)

3 Find the perimeter of these shapes. All square corners are 90°.

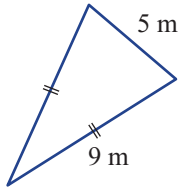
a



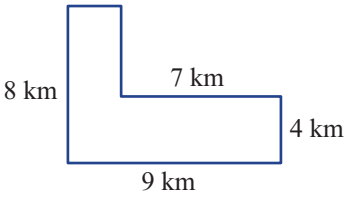
b



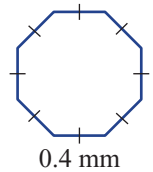
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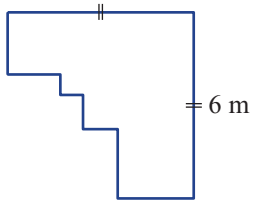
d



e



f

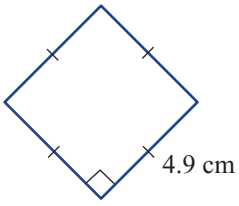


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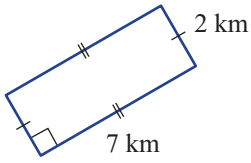
Chapter review

4 Find the area of each of the following shapes.

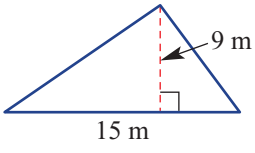
a



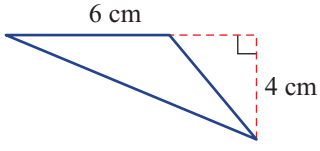
b



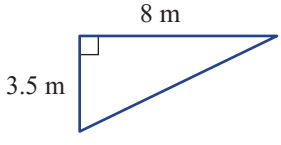
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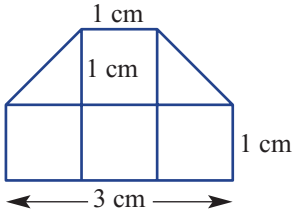
d



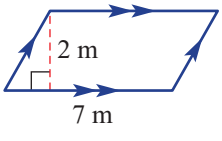
e



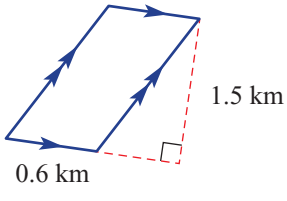
f



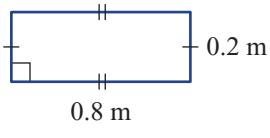
g



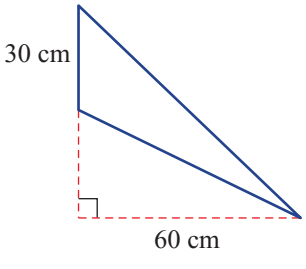
h



i

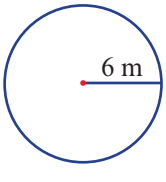


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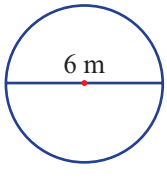


5 Calculate the circumference, in terms of π.

a



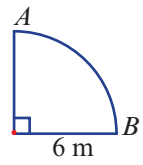
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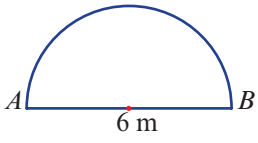
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6 Calculate the arc length AB, in terms of π.

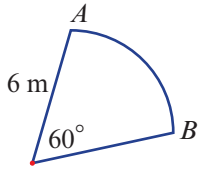
a



b



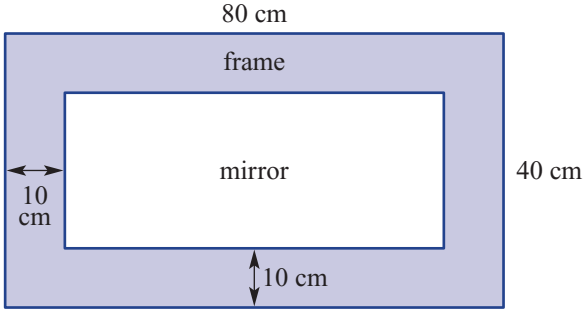
c



7 Calculate the perimeter of the shapes in Question **6**. Give your answers in terms of π.

Extended-response question

1 A mirror is surrounded by a 10 cm wide frame so that the total dimensions are 80 cm by 40 cm, as shown.



a Find the outside perimeter of the frame.

b Find the total area of the framed mirror.

c Find the length and breadth of the mirror alone.

d Find the perimeter of the mirror alone.

e Find the area of the mirror alone.

f Find the area of the frame.

g Find the cost of the frame if it is \$5 per 50 cm^2.

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Chapter 11

Introducing indices

What you will learn

11A Divisibility tests
11B Prime numbers
11C Using indices
11D Prime decomposition
11E Squares, square roots, cubes and cube roots

Strand: Number and Algebra

Substrand: INDICES

In this chapter, you will learn to:

- operate with positive-integer and zero indices of numerical bases.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw7

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Number patterns in architecture

The Louvre Palace in Paris is the one of the world's largest museums and is visited by over 8 million people a year. Visitors enter the museum through a giant glass pyramid. Each triangular side of the pyramid has 17 rows of glass panels, and each panel is in the shape of a rhombus. There is one glass panel in the top row, two in the second row, three in the third row and so on. So we can see that the architect has used the spatial patterns of rhombus shapes, and also number patterns, to design this huge glass pyramid. Many patterns such as this involve the use of indices.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw7

Drilling for Gold:
Building knowledge and skills

Literacy activities:
Mathematical language

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Find:
a 10×10 **b** $10 \times 10 \times 10$ **c** $10 \times 10 \times 10 \times 10 \times 10 \times 10$

2 Find:
a $20 \div 1$ **b** $20 \div 2$ **c** $20 \div 3$ **d** $20 \div 4$
e all the numbers that divide exactly into 20 with no remainder

3 Find the remainder when 30 is divided by:
a 2 **b** 3 **c** 4 **d** 5

4 True or false?
a 2 is a factor of 25 **b** 3 is a factor of 25
c 4 is a factor of 25 **d** 5 is a factor of 25

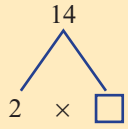
5 Write four different products of two numbers that are each equal to 24.

6 Replace $\square$ with $<$ (is less than) or $>$ (is greater than).
a 3 is smaller than 7, so we write $3 \square 7$.
b 12 is bigger than 5, so we write $12 \square 5$.

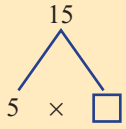
7 **a** $2^2 = 2 \times 2 =$ ____ **b** $3^2 = 3 \times 3 =$ ____
c $4^2 =$ ____ $\times$ ____ $=$ ____ **d** $5^2 =$ ____ $\times$ ____ $=$ ____
e $6^2 =$ ____ $\times$ ____ $=$ ____ **f** $7^2 =$ ____ $\times$ ____ $=$ ____

8 Copy and complete the following.
a $8 = 4 \times$ __ **b** $15 = 3 \times$ __ **c** $12 \times$ __ $= 48$
d $4 \times$ __ $= 16 = 8 \times$ ___ **e** $12 \times 3 =$ __ $= 4 \times$ __

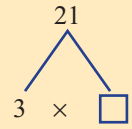
9 Copy and complete these factor trees.
a



b



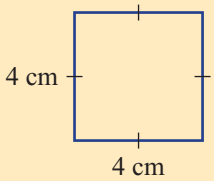
c



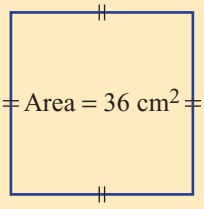
10 **a** Find $33 \div 3$. **b** Is 33 divisible by 3?
c Find $32 \div 3$. **d** Is 32 divisible by 3?

11 Match each description (**a** to **d**) with the correct sequence (**A** to **D**).
a factors of 10 **A** 4, 8, 12, 16,…
b multiples of 4 **B** 1, 2, 5, 10.
c factors of 16 **C** 5, 10, 15, 20,…
d multiples of 5 **D** 1, 2, 4, 8, 16.

12 Find:
a the area of this square



b the side length of this square



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11A Divisibility tests

It is sometimes useful to know whether a number is exactly divisible by another number. For example, 20 is divisible by 2 because $20 \div 2 = 10$ with no remainder. However, 20 is not divisible by 3 because $20 \div 3 = 6$ with remainder 2.

There are simple divisibility tests for 2, 3, 4, 5, 6, 8, 9 and 10, but not 7.

Let's start: Exploring remainders

Work with a partner to sort these divisions into two groups:

- There is NO remainder.
- There IS a remainder.

$20 \div 4$ $60 \div 5$ $28 \div 9$ $20 \div 7$ $48 \div 10$ $55 \div 11$ $72 \div 7$ $93 \div 3$ $56 \div 5$ $26 \div 8$ $88 \div 4$

Key Ideas

Divisibility test
A way to determine whether a whole number is divisible by another whole number, without actually doing the division

- A number is divisible by another number when there is no remainder after the division. For example, 84 is divisible by 4 because $84 \div 21 = 4$ exactly, with no remainder. This means:
  - 4 is a factor of 84.
  - 84 is a multiple of 4.
- **Divisibility tests**
  All positive integers are divisible by 1.

2
Numbers that end in 0, 2, 4, 6 or 8 are even and can be divided by 2.
32, 14, 206, 68, 30

3
The sum of the digits must be divisible by 3.
63, 27, 12, 6, 48

4
The number formed from the last two digits must be divisible by 4.
12, 312, 16, 216

5
The last digit must be 0 or 5.
30, 50, 75, 85, 125

6
The number must be divisible by 2 and 3.
48, 24, 24, 6, 72

7
There is no test for divisibility by 7.
49, 56, 63, 70

8
The number formed from the last three digits must be divisible by 8.
64, 40, 120, 320, 4320

9
The sum of the digits must be divisible by 9.
54, 216, 621, 882

10
The last digit must be 0.
90, 200, 650, 2130

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Exercise 11A

Understanding

1 **a** Write down the first 10 positive integers that are divisible by 2.
b Write down the first 10 positive integers that are divisible by 3.
c In your answers for parts **a** and **b**, circle all the numbers that are divisible by 6.
d Copy and complete:
If a number is divisible by 2 and ____, then it is also divisible by ___.

2 **a** We say that 24 is _____ by 3 because $24 \div 3 = 8$ with no _____.
b Even numbers are all divisible by ___.
c Even numbers end in ___, ___, ___, ___ or ___.
d 432 is _____ by 3 because the sum of the digits is ___ + ___ + ___ = ___ and ___ is divisible by 3.
e 432 is a number divisible by both 2 and 3, so it is also divisible by ___.


Look back at the Key ideas.

3 Who am I? Match each clue (**a** to **c**) with the correct description (**A** to **C**).

a The sum of my digits is divisible by 3.	**A** divisible by 2
b I am an even number.	**B** divisible by 3
c I am even and the sum of my digits is divisible by 3.	**C** divisible by 6

4 Write the missing words or numbers.
a A number that ends in 0 is divisible by both ___ and ___.
b A number that ends in 5 is divisible by ___.
c The last two digits of 316 form the number ___, and ___ is divisible by 4, so 316 is also divisible by ___.
d The last three digits of 5328 form the number ___, and ___ is divisible by 8, so 5328 is also divisible by ___.


Drilling for Gold 11A1

5 Write the missing words or numbers.
a In 2583, the sum of the digits is ___ + ___ + ___ + ___ = ___.
b 2583 is _____ by 3 because the sum of its digits ___ is divisible by ___.
c 2583 is also _____ by 9 because the sum of its digits ___ is divisible by ___.
d 2583 is not divisible by 6 because it is an _____ number.

Example 1 Using divisibility rules for 2, 3, 6 and 8

Which of the numbers 148, 63, 462, 6387 and 7168 are divisible by:
a 2 **b** 3 **c** 6 **d** 8?

Solution	Explanation
a 148, 462, 7168	Only even numbers are divisible by 2. The numbers 63 and 6387 are odd.
b 63, 462, 6387	A number is divisible by 3 when the sum of its digits is divisible by 3. For 148: $1 + 4 + 8 = 13$ but $13 \div 3 = 4$ with rem. 1 ✘

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	For 63: $6 + 3 = 9$ and $9 \div 3 = 3$ ✓ In a similar way, test 462 ✓ 6387 ✓ and 7168 ✘
c 462	A number is divisible by 6 when even and divisible by 3. $4 + 6 + 2 = 12$ and $12 \div 3 = 4$ ✓
d 7168	Divisible by 8 when the number formed from the last 3 digits is divisible by 8. $168 \div 8 = 21$ ✓

6 **a** Which of these numbers are divisible by 2?
3, 6, 13, 14, 8, 17, 21, 54, 22, 34, 33, 50, 18, 35, 46

b Which of these numbers are divisible by 3?
12, 14, 18, 20, 22, 30, 27, 23, 54, 50, 36, 42, 13, 24, 43

c Which of these numbers are divisible by 6?
12, 24, 28, 38, 63, 60, 87, 225, 54, 252, 36, 92, 66, 84, 143

d Which of these numbers are divisible by 8?
35, 168, 7168, 40, 5032, 9338, 248, 7831, 6400, 9568

Divisibility tests:
2: Last digit even or 0
3: Sum of digits ÷ 3
6: Divisible by 2 and 3
8: Last 3 digits ÷ 8



Example 2 Using divisibility tests for 5, 10, 4 and 9

Which of the numbers 540, 918, 8775 and 3924 are divisible by:
a 10 **b** 5 **c** 4 **d** 9?

Solution	Explanation
a 540	Divisible by 10 when number ends in 0.
b 540, 8775	Divisible by 5 when number ends in 0 or 5.
c 540, 7924	Divisible by 4 when last two digits divisible by 4. For 540: $40 \div 4 = 10$ ✓ For 918: $18 \div 4 = 4$ rem. 2 ✘ In a similar way, test 75 ✘ and 24 ✓
d 540, 918, 3924, 8775	Divisible by 9 when sum of digits divisible by 9. For 540: $5 + 4 + 0 = 9$ and $9 \div 9 = 1$ ✓ For 918: $9 + 1 + 8 = 18$ and $18 \div 9 = 2$ ✓ In a similar way, test 8775 ✓ and 3924 ✓

7 **a** Which of these numbers are divisible by 5?
35, 52, 125, 13, 15, 100, 113, 112, 32, 515, 408, 730, 105

b Which of these numbers are divisible by 10?
20, 64, 800, 98, 290, 610, 85, 265, 590, 52, 39, 90, 160

Divisibility tests:
5: Last digit 5 or 0
10: Last digit 0
4: Last two digits ÷ 4
9: Sum of digits ÷ 9



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11A

c Which of these numbers are divisible by 4?
16, 32, 220, 10, 12, 28, 213, 432, 72, 316, 424, 1836, 135

d Which of these numbers are divisible by 9?
27, 432, 456, 88, 99, 387, 63, 55, 720, 85, 253, 2799

Fluency

Example 3 Applying divisibility tests

Are the following statements true or false?

a 54 327 is divisible by 3

b 765 146 is divisible by 8

Solution	Explanation
a Digit sum = 21 True: 54 327 is divisible by 3.	5 + 4 + 3 + 2 + 7 = 21 21 is divisible by 3.
b $8\overline{)14^{6}6}$ 18 rem. 2 False: 765 146 is not divisible by 8.	Check whether the last three digits are divisible by 8.

8 Are the following statements true or false?

Apply the divisibility tests.



a 23 562 is divisible by 3
b 39 245 678 is divisible by 4
c 1 295 676 is divisible by 9
d 213 456 is divisible by 8
e 3 193 457 is divisible by 6
f 2 000 340 is divisible by 10
g 51 345 678 is divisible by 5
h 215 364 is divisible by 6
i 9543 is divisible by 6
j 25 756 is divisible by 2
k 56 789 is divisible by 9
l 324 534 565 is divisible by 5
m 2 345 176 is divisible by 8
n 329 541 is divisible by 10
o 225 329 is divisible by 3
p 356 781 276 is divisible by 9
q 164 567 is divisible by 8
r 2 002 002 002 is divisible by 4



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Example 4 Testing divisibility

Carry out divisibility tests on the given number and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412								

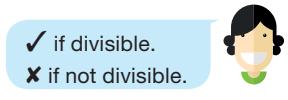
Solution

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412	✓	✓	✓	✘	✓	✘	✘	✘

Explanation

48 569 412 is an even number and therefore is divisible by 2.
48 569 412 has a digit sum of 39 and therefore is divisible by 3, but not by 9.
48 569 412 is divisible by 2 and 3, therefore it is divisible by 6.
The last two digits are 12, which is divisible by 4.
The last three digits are 412, which is not divisible by 8.
The last digit is a 2 and therefore is not divisible by 5 or 10.

9 Copy the table. Carry out the divisibility tests on the given numbers, filling in the table with ticks or crosses.

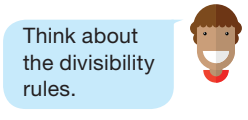


Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567								
28 080								
189 000								
1 308 150								
1 062 347								

Problem-solving and Reasoning

10 Give a reason why:

a 8631 is not divisible by 2
b 31 313 is not divisible by 3
c 426 is not divisible by 4
d 5044 is not divisible by 5
e 87 548 is not divisible by 6
f 214 125 is not divisible by 8
g 3 333 333 is not divisible by 9
h 56 405 is not divisible by 10



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11A

11 Give the remainder when:
- **a** 326 is divided by 3
- **b** 21 154 is divided into groups of four
- **c** 72 is divided into six groups
- **d** 45 675 is shared into five groups



12 The game of 'clusters' involves a group getting into smaller-sized groups. Players get into groups as quickly as possible once the cluster size has been called out. If a year level consists of 88 students, which cluster sizes would mean that no students are left out of a group? Give all possible answers.



13 Ivan's age is a two-digit number. It is divisible by 2, 3, 6 and 9. How old is Ivan if you know that he is older than 20 but younger than 50?

Enrichment: A very large number

14
- **a** Is the number 968 362 396 392 139 963 359 divisible by 3?
- **b** Many of the digits in the number above can actually be ignored when calculating the digit sum. Which numbers can be ignored and why?
- **c** To determine if the number above is divisible by 3, only five of the 21 digits actually need to be added together. Find this 'reduced' digit sum.
- **d** Make a list of large numbers. Include some numbers that are divisible by 3 and other numbers that are not.
- **e** Swap lists with a classmate. See how quickly you can find each other's numbers that are divisible by 3.



The world population at 8:47 a.m. on 5 February 2015 was 7 292 758 595. Search the internet for a world population clock and watch it grow.

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11B Prime numbers

The number 20 has six factors: 1, 2, 4, 5, 10 and 20.
The number 19 has only two factors: 1 and 19.
The number 19 is a prime number and 20 is a composite number.

There are some interesting prime numbers with remarkable patterns in their digits, such as 12 345 678 901 234 567 891. You can also get palindromic primes, such as 111 191 111 and 123 494 321. Below is a palindromic prime number. It also reads the same upside down or when viewed in a mirror.



Computers have allowed mathematicians to find larger and larger prime numbers.

I888081808881

Let's start: Find the prime numbers

Elise is trying to decide which of the following are prime numbers.

$$138 \quad 27 \quad 31 \quad 91 \quad 98 \quad 57 \quad \frac{3}{7} \quad 101$$
$$71 \quad 99 \quad -19 \quad 83 \quad 85 \quad 60 \quad 202$$

- Which numbers could she eliminate straight away? Why?
- How could Elise use divisibility tests to eliminate some of the other numbers?
- Which of the numbers do you think are prime?

Key ideas

- A **prime number** is a positive **integer** that has only two factors; i.e. itself and 1.
  - The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.
- A number that has more than two factors is called a **composite number**.
- The number 1 has only one factor ($1 \times 1 = 1$). It is neither prime nor composite.
- The number 2 is prime. It is the only even prime number.

Prime number
An integer greater than 1 that has only two factors: itself and 1

Integer A whole number

Composite number
An integer greater than 1 with three or more factors

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Exercise 11B

Understanding

1 Copy and complete the following.
- **a** A _____ number has only two factors: __ and _____.
- **b** A number that has more than 2 factors is a _____ number.

2 **a** The factors of 12 are 1, 2, 3, 4, 6 and 12. Is 12 a prime number?
- **b** The factors of 13 are 1 and 13. Is 13 a prime number?

3 **a** List the first 10 prime numbers.
- **b** List the first 10 composite numbers.



4 **a** What is the first prime number greater than 10?
- **b** What is the first composite number greater than 10?
- **c** What is the first prime number greater than 20?

Fluency

Example 5 Is a number prime or composite?

Which of these numbers are prime and which are composite: 22, 35, 17, 11, 9, 5.

Solution	Explanation
Prime: 5, 11, 17	5, 11, 17 have only two factors (1 and itself).
Composite: 9, 22, 35	$9 = 3 \times 3$, $22 = 2 \times 11$, $35 = 5 \times 7$

5 Is each of the following a prime (P) or composite (C) number?

a 14	**b** 23	**c** 70	**d** 37
e 51	**f** 27	**g** 29	**h** 3
i 8	**j** 49	**k** 99	**l** 59
m 2	**n** 31	**o** 39	**p** 89



Example 6 Finding prime factors

Find the prime numbers that are factors of 30.

Solution	Explanation
Factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30 Prime factors are 2, 3 and 5.	Find all the factor pairs: 1×30, 2×15, 3×10, 5×6 Determine which factors are prime. 1 is not a prime number. $2 = 2 \times 1$, $3 = 3 \times 1$, $5 = 5 \times 1$

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6 Find the prime numbers that are factors of:

a 42 **b** 39 **c** 60
d 25 **e** 28 **f** 36



7 List the composite numbers between:

a 30 and 40 **b** 50 and 60 **c** 80 and 90

8 List the prime numbers between:

a 20 and 30 **b** 40 and 50 **c** 70 and 80

Problem-solving and Reasoning

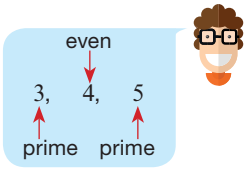
9 The following are not prime numbers, yet they are the product (×) of two primes. Find the two primes that multiply to give:

a 15 **b** 21 **c** 35
d 55 **e** 143 **f** 133

10 Which one of the following numbers has factors that are all prime numbers, except itself and 1?
12, 14, 16, 18, 20



11 Twin primes are pairs of primes that are separated from each other by only one even number. For example, 3 and 5 are twin primes. Find three more pairs of twin primes.



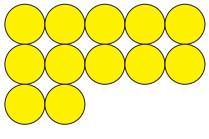
12 Answer these questions for each of the arrays (**a** to **h**) below.

i Is it possible to rearrange the counters into a rectangle with two or more equal rows?
ii If it is possible, state the dimensions of the new rectangle.
iii State the number of counters and whether this a prime or composite number.

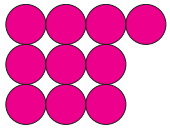
a



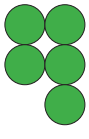
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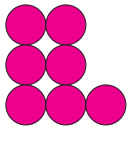
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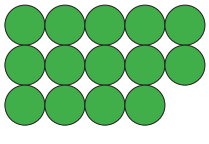
d



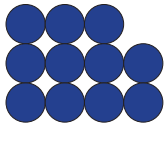
e



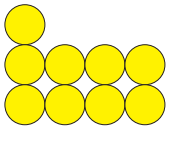
f



g



h



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11B

Enrichment: Prime car race

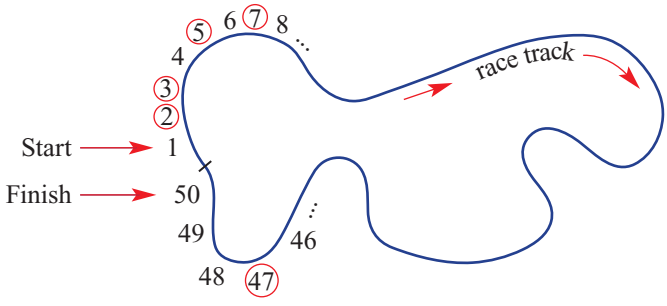
13 Play this game with a classmate. You will need a large sheet of paper, a die and two 'race cars' (counters or erasers).



- On the sheet of paper, draw a curvy line for your race track.
- Write the numbers 1 to 50 along your track and circle all the prime numbers. For example:



Drilling for Gold 11B1



- Start with both cars on 1. Take turns to roll the die and drive your car that number of places along the track.
- If your car lands on a prime number, it needs a pit stop (flat tyre, low fuel etc.) and you miss a turn.
- The car that wins reaches the finish line first. (You must roll the correct number to land exactly on 50.)

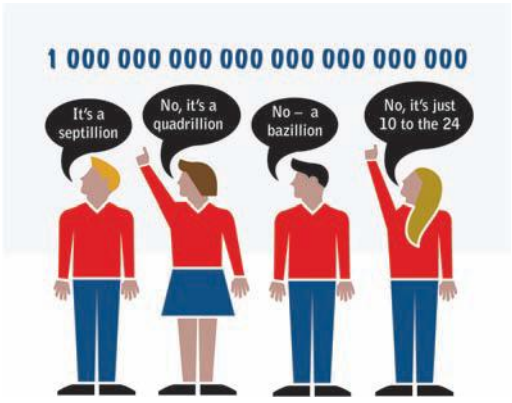
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11C Using indices

When a number is multiplied by itself we often write that product in index form. For example:

$1000 = 10 \times 10 \times 10$ (factor form)
$= 10^3$ (index form)

For 10^3 we say '10 to the power of 3' or '10 cubed'.



Let's start: An easier way

What is an easier way of writing:

- $10 \times 10 \times 10 \times 10$, other than 10 000?
- $10 \times 10 \times 10 \times 10 \times 10 \times 10$, other than 1 000 000?
- $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, other than 1024?

Key ideas

- **Powers** are used to help write expressions involving repeated multiplication in index form.
  expanded form $\longrightarrow$ $8 \times 8 \times 8 \times 8 \times 8 = 8^5$ $\longleftarrow$ index form
- The expression 8^5 is written in **index form**. It is read as '8 to the power of 5'.
  The 8 is called the **base**.
  The 5 is called the **index** or **exponent**.
  The 5 indicates that 8 appears five times in the product.
- In the statement $4^3 = 8^2$, the 3 and 2 are called **indices**. This is the plural of 'index'.
- 5×5 and 5^2 are read as '5 to the power of 2' or '5 squared'.
  $5 \times 5 \times 5$ and 5^3 are read as '5 to the power of 3' or '5 cubed'.
  $5 \times 5 \times 5 \times 5$ and 5^4 are read as '5 to the power of 4'.
- 5^b reads as '5 to the power of b'. In expanded form it would look like:

$$\underbrace{5 \times 5 \times 5 \times 5 \times 5 \ldots \times 5}_{\text{The number 5 is written } b \text{ times.}}$$

- Powers take priority in the order of operations.
  For example: $3 + 2 \times 4^2 = 3 + 2 \times 16$
  $= 3 + 32$
  $= 35$

Base The number that is being raised to a power or index

Index, Exponent, Power The number of times the base appears in the product

Index form A method of writing numbers that are multiplied by themselves

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- **Note:** $2^3 \neq 2 \times 3$, therefore $2^3 \neq 6$. This is a common mistake that must be avoided. **Instead:** $2^3 = 2 \times 2 \times 2 = 8$.
- The powers of 2 are 2, 4, 8, 16, 32, ...
- The powers of 3 are 3, 9, 27, ...
- The powers of 10 are 10, 100, 1000, ...

Exercise 11C

Understanding

1 Write the missing words.

a The product $2 \times 2 \times 2$ is called the _____ form of 8.

b 2^3 is called the _____ form of 8.

c 2^3 reads: 'two to the _____ of 3'.

d In 2^3, the special name for the 2 is the _____ number.

e In 2^3, the special name for the 3 is the _____ or _____.

2 Copy and complete each product of repeated factors.

a $3^2 = 3 \times \square$

b $2^4 = 2 \times \square \times \square \times \square$

c $5^3 = 5 \times \square \times \square$

d $8^5 = \square \times \square \times \square \times \square \times \square$

Example 7 Relating factor form and index form

Copy and complete this table.

Factor form	Index form	Base	Index or exponent
$3 \times 3 \times 3 \times 3$	3^4		
$5 \times 5 \times 5 \times 5 \times 5 \times 5$			
		2	5

Solution

Factor form	Index form	Base	Index or exponent
$3 \times 3 \times 3 \times 3$	3^4	3	4
$5 \times 5 \times 5 \times 5 \times 5 \times 5$	5^6	5	6
$2 \times 2 \times 2 \times 2 \times 2$	2^5	2	5

Explanation

The factor 3 appears 4 times, so 4 is the index or exponent.

The factor 5 appears 6 times.

The base of 2 is the factor and it appears 5 times.

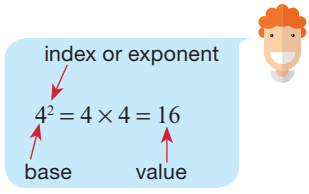
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3 Copy and complete this table.

Factor form	Index form	Base	Index
	7^3		
$5 \times 5 \times 5$			
		2	6
	6^4		

4 Copy and complete the table.

Index form	Base number	Index	Value
2^3	2	3	8
5^2			
10^4			
2^7			
1^{12}			
12^1			
0^5			



Fluency

Example 8 Converting to index form

Simplify these products by writing them in index form.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **b** $3 \times 3 \times 2 \times 3 \times 2 \times 3$

Solution	Explanation
a $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$	The factor 5 appears six times in the product.
b $3 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^2 \times 3^4$	2 appears twice in the product. 3 appears four times in the product.

5 Simplify these products by writing in index form.

a $3 \times 3 \times 3$
b $2 \times 2 \times 2 \times 2 \times 2$
c $15 \times 15 \times 15 \times 15$
d $10 \times 10 \times 10 \times 10$
e 6×6
f $20 \times 20 \times 20$
g $1 \times 1 \times 1 \times 1 \times 1 \times 1$
h $4 \times 4 \times 4$
i 100×100
j $3 \times 3 \times 5 \times 5$
k $2 \times 2 \times 7 \times 7 \times 7$
l $9 \times 9 \times 12 \times 12$
m $8 \times 8 \times 5 \times 5 \times 5$
n $6 \times 3 \times 6 \times 3 \times 6 \times 3$
o $13 \times 7 \times 13 \times 7 \times 7 \times 7$
p $4 \times 13 \times 4 \times 4 \times 7$
q $10 \times 9 \times 10 \times 9 \times 9$
r $2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 2$



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11C

Example 9 Expanding expressions in index form

Write each of the following in factor form and find the value.

a 2^4 **b** $2^3 \times 5^2$

Solution	Explanation
a $2^4 = 2 \times 2 \times 2 \times 2$ $= 16$	The factor 2 appears four times. Calculate the value.
b $2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5$ $= 8 \times 25$ $= 200$	The factor 2 appears three times. The factor 5 appears twice. Calculate the value.



6 Write in factor form. (Do not find the value.)

a 2^4 **b** 17^2 **c** 9^3 **d** 3^7
e 14^4 **f** 8^8 **g** 10^5 **h** 54^3
i $3^5 \times 2^3$ **j** $4^3 \times 3^4$ **k** $7^2 \times 5^3$ **l** $4^6 \times 9^3$
m 5×7^4 **n** $2^2 \times 3^3 \times 4^1$ **o** $11^5 \times 9^2$ **p** $20^3 \times 30^2$

7 Copy and complete the following.

a $3 \times 2 =$ ___ but $3^2 =$ ___
b $2 \times 4 =$ ___ but $2^4 =$ ___
c $5 \times 2 =$ ___ but $5^2 =$ ___
d $6 \times 2 =$ ___ but $6^2 =$ ___

8 Write in factor form and find the value.

a 2^5 **b** 8^2 **c** 10^3 **d** $3^2 \times 2^3$
e 10^4 **f** $2^3 \times 5^3$ **g** $1^6 \times 2^6$ **h** $11^2 \times 1^8$

Problem-solving and Reasoning

Example 10 Evaluating expressions with index form

Evaluate:

a $7^2 - 6^2$ **b** $2 \times 3^3 + 10^2 + 1^7$

Solution	Explanation
a $7^2 - 6^2 = 7 \times 7 - 6 \times 6$ $= 49 - 36$ $= 13$	Write powers in factor form. Do multiplication before subtraction.
b $2 \times 3^3 + 10^2 + 1^7$ $= 2 \times 27 + 100 + 1$ $= 54 + 100 + 1$ $= 155$	$2^3 = 27$, $10^2 = 100$, $1^7 = 1$ Do multiplication first, then addition.

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9 Evaluate:

a $3^2 + 4^2$ **b** $2 \times 5^2 - 7^2$ **c** $8^2 - 2 \times 3^3$

d $(9 - 5)^3$ **e** $2^4 \times 2^3$ **f** $2^7 - 1 \times 2 \times 3 \times 4 \times 5$

g $1^4 + 2^3 + 3^2 + 4^1$ **h** $10^3 - 10^2$ **i** $(1^{27} + 1^{23}) \times 2^2$

10 Find the index number for each of the following.

a $16 = 2^{\square}$ **b** $16 = 4^{\square}$ **c** $64 = 4^{\square}$ **d** $64 = 2^{\square}$

e $27 = 3^{\square}$ **f** $100 = 10^{\square}$ **g** $49 = 7^{\square}$ **h** $625 = 5^{\square}$

11 Write one of the symbols <, = or > in the box to make the following statements true.

a $2^6 \square 2^9$ **b** $8^3 \square 8^2$

c $2^4 \square 4^2$ **d** $3^2 \square 4^2$

e $6^4 \square 5^3$ **f** $12^2 \square 3^4$

g $11^2 \square 2^7$ **h** $1^8 \square 2^3$

< means is less than. > means is greater than. The signs point to the smaller number.

12 Five friends receive the same text message at the same time. Each of the five friends then forwards it to five other friends and each of these people also sends it to five other friends. How many people does the text message reach?

Enrichment: The power of email

Assume that everyone uses email and everyone can read!

13 A chain email is sent to 10 people. Five minutes later, each of them sends it to 10 other people. That is, after 10 minutes, 110 people (10 + 100) will have received the email.

a How many people will have received the email after:

i 15 minutes? **ii** 30 minutes?

b If the email always goes to a new person, how long would it take until everyone in Australia has received the message? (Australia has approximately 23 million people.)

c How long would it take until everyone in the world has received the message? (The world population is approximately 7 billion people.)



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11D Prime decomposition

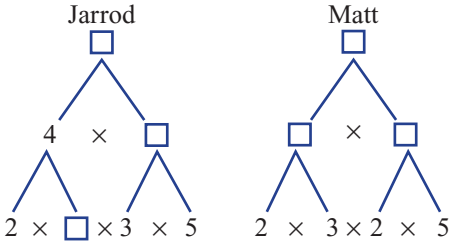
Every composite number can be written as the product of prime factors. Factor trees help us to work out the prime factors. When there are repeated factors, we write that product in index form.

$4 = 2 \times 2$
$6 = 2 \times 3$
$8 = 2 \times 2 \times 2$
$10 = 2 \times 5$
$12 = 2 \times 2 \times 3$

The first five composite numbers, expressed as products of prime factors

Let's start: Factor trees

For homework, Jarrod and Matt each drew a factor tree. Then their little brother rubbed out some of the numbers.





- Can you find the missing numbers? Copy and complete each factor tree.
- What was the boys' homework question?
- How is Jarrod's factor tree different from Matt's?
- What is the same about both factor trees?
- Can you draw a different factor tree that answers the homework question correctly?

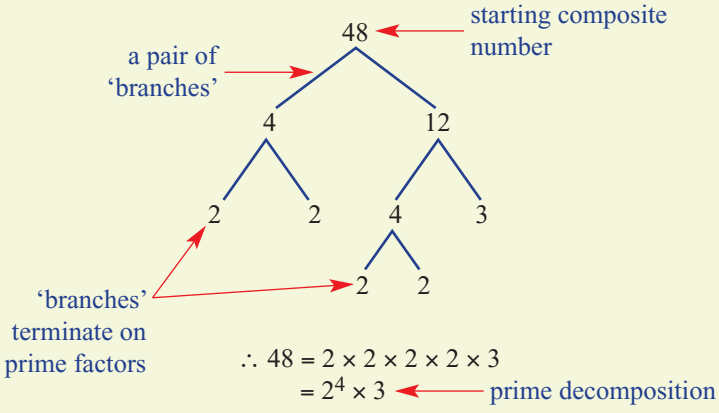
Key ideas

Factor tree
An illustrated decomposition of a number into a product of prime factors

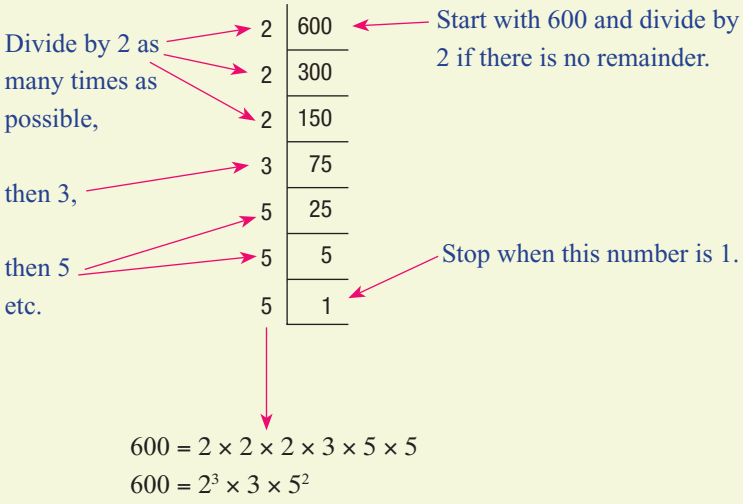
- Every **composite number** can be expressed as a product of its **prime factors**.
- A **factor tree** can be used to show the prime factors of a composite number.
- Each 'branch' of a factor tree eventually terminates in a prime factor.

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- Powers are often used to efficiently represent composite numbers in prime factor form.
  For example:



- It does not matter with which pair of factors you start a factor tree. The final set of prime factors will always be the same.
- It is conventional to write the prime factors in ascending (i.e. increasing) order. For example: $600 = 2^3 \times 3 \times 5^2$
- Here is another way to decompose a number, using division by prime numbers.



$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$
$600 = 2^3 \times 3 \times 5^2$

Sometimes it may be necessary to divide by 2, 3, 5, 7, 11, 13 or any prime number.

- Some calculators can decompose a number. The Casio fx-82AU PLUS 11 and fx-100AU PLUS have the word FACT above the ° ′ ″ button. Press 600 = SHIFT FACT.

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Exercise 11D

Understanding

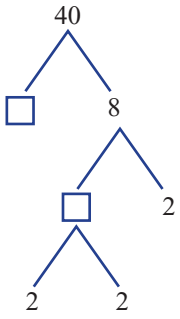
1 Sort the following list of numbers into two groups: composite numbers and prime numbers.
2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15, 23, 27


A prime number only has two factors: 1 and itself.

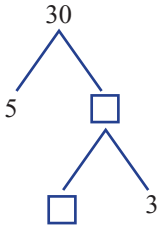

Drilling for Gold 11D1

2 Copy and complete the following factor trees.

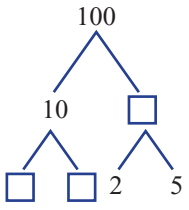
a



b



c



3 Copy and complete these repeated divisions with prime numbers.

a

2	12
2	□
□	3
	1

b

□	30
□	15
□	5
	1

c

□	66
3	33
□	□
	□

4 Rewrite these with the prime factors in ascending order. Then write using powers.

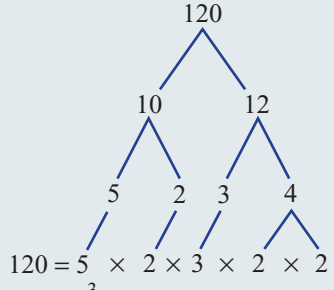
a $2 \times 3 \times 2$
b $3 \times 2 \times 3$
c $5 \times 3 \times 3 \times 5$
d $2 \times 3 \times 4 \times 3 \times 2$
e $2 \times 3 \times 3 \times 2 \times 2$
f $5 \times 3 \times 3 \times 3 \times 3 \times 5$
g $7 \times 2 \times 3 \times 7 \times 2$
h $3 \times 3 \times 2 \times 11 \times 11 \times 2$


Ascending means smallest to largest.

Fluency

Example 11 Using a factor tree to find prime factors

Draw a factor tree for the number 120. Then write 120 as the product of prime factors. (Use index form.)

Solution	Explanation
	Start with 120. Find any pair of factors, such as 10×12 or 20×6.
	$120 = 10 \times 12$
	$10 = 5 \times 2$, $12 = 3 \times 4$
$120 = 5 \times 2 \times 3 \times 2 \times 2$ $120 = 2^3 \times 3 \times 5$	All factors are now primes. Use index form for repeated factors. The power 3 shows that the factor of 2 appears three times.

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5 For each number, draw a factor tree. Then write the number as a product of prime factors, using powers.

a 14	**b** 16	**c** 18	**d** 20
e 24	**f** 36	**g** 44	**h** 56
i 64	**j** 72	**k** 75	**l** 80



Write prime factors in ascending order. Use powers for repeated factors.

Example 12 Using repeated division to find prime factors

Use repeated division with prime numbers to find the prime factors of 126. Then write 126 as a product of prime factors with powers.

Solution		Explanation	
2 | 126		$126 \div 2 = 63$	126 is even, so divide by 2: $126 \div 2 = 63$.
3 | 63		$63 \div 3 = 21$	63 is not divisible by 2 but it is divisible
3 | 21		$21 \div 3 = 7$	by 3: $63 \div 3 = 21$ and $21 \div 3 = 7$.
7 | 7		$7 \div 7 = 1$	7 is a prime number.
1			
$126 = 2 \times 3 \times 3 \times 7$		Write prime factors in ascending order.	
$= 2 \times 3^2 \times 7$		$3 \times 3 = 3^2$	

6 Use repeated division with prime numbers to find the prime factors of 96. Then write 96 as a product of prime factors with powers.



Start by dividing 96 by 2, as many times as you can.

7 Use repeated division with prime numbers to help you write each of these numbers as a product of prime factors with powers.

a 32	**b** 40	**c** 81	**d** 144
e 120	**f** 500	**g** 1800	**h** 1250

Problem-solving and Reasoning

8 Match the correct composite number (**a** to **d**) to its set of prime factors (**A** to **D**).

a 120	**A** $2 \times 3 \times 5^2$
b 150	**B** $2^2 \times 3^2 \times 5$
c 144	**C** $2^4 \times 3^2$
d 180	**D** $2 \times 3 \times 2 \times 5 \times 2$



Look for easy ways to multiply.

$$\overbrace{2 \times 3}^{6} \times \overbrace{5 \times 5}^{25}$$

$$6 \times 25$$

$$\overbrace{6}^{10} \times \overbrace{25}^{15}$$

9 Find the smallest number that has the first five prime numbers as prime factors.

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11D

10 Draw four different factor trees that each show the prime factors of 24. (For two trees to be different, they must show different combinations of factors, not just the same factors in a different order.)



11 Only one of the following is the correct set of prime factors for 424.

A $2^2 \times 3^2 \times 5$ **B** $2 \times 3^2 \times 5^2$ **C** 53×8 **D** $2^3 \times 53$

i Explain why **A** and **B** are wrong.

ii Why is option **C** wrong?

iii Show that option **D** is the correct answer.



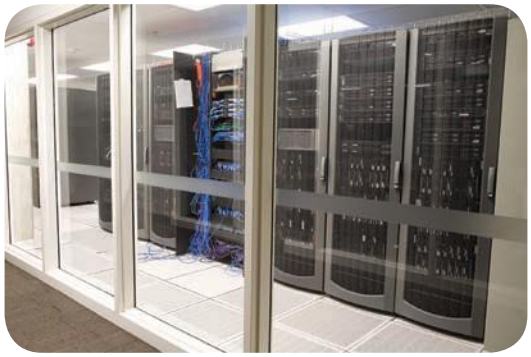
Remember the rules for divisibility.

Enrichment: Four different prime factors



12 Only 17 composite numbers smaller than 1000 have four prime factors.
For example: $546 = 2 \times 3 \times 7 \times 13$
By considering the prime factor possibilities, see how many of the other 16 composite numbers you can find. Express each of them in prime factor form.

13 Use the internet to find the largest-known prime number.

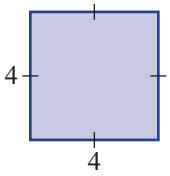


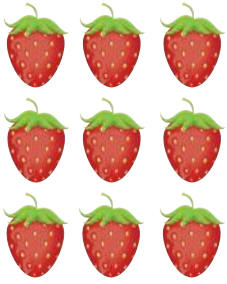
Supercomputers like this have been used to search for prime numbers with millions of digits.

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11E Squares, square roots, cubes and cube roots

We can picture a square number as the area of a square or the number of dots in a square array. For example:





Area $= 4 \times 4$
$= 16$
16 is a square number.

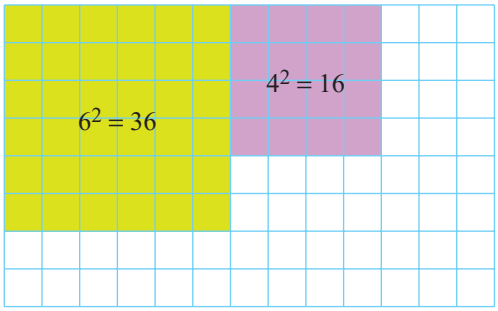
Number of strawberries = 3 rows of 3
$= 3 \times 3$
$= 3^2$
$= 9$
9 is a square number.

Finding a square root of a number is the opposite of squaring a number.
We use the symbol $\sqrt{\ }$ to show the square root of a number.
The pictures above show that $\sqrt{16} = 4$ and $\sqrt{9} = 3$.

Let's start: Shading squares

You will need a sheet of 1 cm grid paper.

- Shade as many different-sized squares as you can fit onto the page.
- Write each area in index form.
  For example, you might begin with:



Key ideas

- If you multiply a whole number by itself, the result is a **square number**.
  For example: $5^2 = 5 \times 5 = 25$, so 25 is a square number.
  - The first 12 square numbers are:

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
1	4	9	16	25	36	49	64	81	100	121	144

Square number The result of multiplying a number by itself

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Square root (of a given number) The number that, when multiplied by itself, gives the number

Cube (of a number) The third power of a number

Cube root (of a number) The number that must be cubed to give that number

- The **square root** of a given number multiplied by itself results in the given number.
  - The symbol for square root is $\sqrt{\ }$.
  - Finding a square root of a number is the opposite of squaring a number.
    For example: $4^2 = 16$, so $\sqrt{16} = 4$.
    We read this as: '4 squared equals 16, so the square root of 16 equals 4.'

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$
1	2	3	4	5	6	7	8	9	10	11	12

- When a number is raised to a power of 3, it produces a cube.
  For example, the number '2 cubed' is written:
  $2^3 = 2 \times 2 \times 2 = 8$
- Cube root is the reverse:
  For example: $1^3 = 1$ so $\sqrt[3]{1} = 1$
  $2^3 = 8$ so $\sqrt[3]{8} = 2$
  $3^3 = 27$ so $\sqrt[3]{27} = 3$
  $4^3 = 64$ so $\sqrt[3]{64} = 4$
- Order of operations
  - Squares and **cubes** are powers, so evaluate first.
    For example: $2 \times 3^2 + 4 = 2 \times 9 + 4$
  - Square roots and **cube roots** are like brackets.
    For example: $\sqrt{16 + 9} = \sqrt{25} = 5$
- Calculators have buttons for square, square root, cube and cube root.

Exercise 11E

Understanding

1 Copy and complete this table of square numbers.

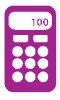
1^2			4^2	5^2		7^2		9^2	
	4	9			36		64		100

Look back at the Key ideas.



2 Copy and complete this table of square roots.

$\sqrt{1}$	$\sqrt{4}$		$\sqrt{16}$		$\sqrt{36}$			$\sqrt{81}$	
		3		5		7	8		10



3 Copy and complete this table of cubes.

1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3

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4 Use your calculator to find:

a $\sqrt[3]{3375}$ **b** $\sqrt[3]{8000}$ **c** $\square^3 = 64$

d $\sqrt{10\,000}$ **e** $\sqrt{1\,000\,000}$ **f** $\square^3 = 15\,625$

Example 13 Finding squares and square roots

Drilling for Gold 11E1

Write the value of each of these.

a 4^2 **b** 4 squared **c** $(4)^2$ **d** 4 to the power 2

e square root of 36 **f** the side length of a square with area 64 cm^2

Solution	Explanation
a $4^2 = 16$	$4^2 = 4 \times 4$
b 4 squared = 16	4 squared is the same as 4^2.
c $(4)^2 = 16$	$(4)^2 = 4^2$
d 4 to the power 2 = 16	4 to the power 2 is the same as 4^2.
e square root of 36 = 6	$\sqrt{36} = 6$ because $6 \times 6 = 36$.
f side length = 8 cm	side length = $\sqrt{64}$ and $8 \times 8 = 64$.

5 Evaluate:

a 6^2 **b** 5 squared **c** $(11)^2$

d 10 to the power of 2 **e** 7^2 **f** 12×12

g $13 \times 13 \times 13$ **h** 11 cubed **i** 12 to the power of 3

6 Evaluate:

a $\sqrt{25}$ **b** the square root of 16 **c** $\sqrt{100}$

d the side length of a square that has an area of 49 cm^2

e the cube root of 64 **f** the square root of 64

Fluency

Example 14 Evaluating squares and square roots

Find the value of:

a 6^2 **b** 200^2 **c** $\sqrt{64}$ **d** $\sqrt{1600}$

Solution	Explanation
a $6^2 = 36$	$6^2 = 6 \times 6$
b $200^2 = 40\,000$	$200 \times 200 = 40\,000$
c $\sqrt{64} = 8$	$\sqrt{64} = 8$ because $8 \times 8 = 64$.
d $\sqrt{1600} = 40$	$\sqrt{1600} = 40$ because $40 \times 40 = 1600$.

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11E

7 Find the value of:

a 8^2 **b** 7^2 **c** 1^2 **d** 12^2
e 3^2 **f** 15^2 **g** 5^2 **h** 0^2
i 11^2 **j** 100^2 **k** 30^2 **l** 40^2

$8^2 = 8 \times 8 = 64$



8 Find the value of:

a $\sqrt{25}$ **b** $\sqrt{9}$ **c** $\sqrt{1}$ **d** $\sqrt{121}$
e $\sqrt{0}$ **f** $\sqrt{81}$ **g** $\sqrt{49}$ **h** $\sqrt{16}$
i $\sqrt{4}$ **j** $\sqrt{144}$ **k** $\sqrt{400}$ **l** $\sqrt{169}$
m $\sqrt{2500}$ **n** $\sqrt{6400}$ **o** $\sqrt{8100}$ **p** $\sqrt{729}$

For $\sqrt{25}$, think: $\square \times \square = 25$



Example 15 Evaluating expressions involving squares and square roots

Evaluate:

a $\sqrt{64} + \sqrt{36}$ **b** $\sqrt{8^2 + 6^2}$ **c** $3^2 - \sqrt{9} + 1^2$

Solution	Explanation
a $\sqrt{64} + \sqrt{36} = 8 + 6$ $= 14$	Find square roots first. Then add.
b $\sqrt{8^2 + 6^2} = \sqrt{64 + 36}$ $= \sqrt{100}$ $= 10$	The square root sign is like a bracket: $\sqrt{(8^2 + 6^2)} = \sqrt{(8 \times 8 + 6 \times 6)}$ Multiply and add to give: $\sqrt{64 + 36} = \sqrt{100}$.
c $3^2 - \sqrt{9} + 1^2 = 9 - 3 + 1$ $= 7$	Evaluate squares and square roots first. $3^2 = 3 \times 3$, $\sqrt{9} = 3$, $1^2 = 1 \times 1$

9 Evaluate:

a $\sqrt{9} + \sqrt{16}$ **b** $\sqrt{3^2 + 4^2}$ **c** $\sqrt{9} \times \sqrt{16}$
d $3^2 + 5^2 - \sqrt{16}$ **e** 4×4^2 **f** $8^2 - 0^2 + 1^2$
g $1^2 \times 2^2 \times 3^2$ **h** $\sqrt{5^2 - 3^2}$ **i** $\sqrt{81} - 3^2$
j $6^2 \div 2^2 \times 3^2$ **k** $\sqrt{9} \times \sqrt{64} \div \sqrt{36}$ **l** $\sqrt{12^2 + 5^2}$

Remember the order of operations.



Problem-solving and Reasoning

10 This arrangement of dots shows that 9 is a square number.



a Show, using dots, that 6 is *not* a square number.
b Show, using dots, that 16 is a square number.

11 List all the square numbers between 50 and 101.

12 List all the square numbers between 101 and 200. (Hint: There are only four.)

13 **a** Show that $3^2 + 4^2 = 5^2$.
b Does $5^2 + 6^2 = 7^2$?
c Does $6^2 + 8^2 = 10^2$?
d Find some other true sums of square numbers like the one in part **a**.

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Enrichment: Number dot patterns


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14 For each of the following:

i Copy the dot pattern and draw the next three terms.

ii How many dots are added each time?

iii What patterns did you notice?

a odd numbers



b even numbers



c square numbers



d triangular numbers



15 a Copy and complete the table.

Square numbers	1	4	9							
Triangular numbers	1	3	6							

Now let's show that every square number greater than 1 can be written as the sum of two triangular numbers. For example: $4 = 1 + 3$.

The first five triangular numbers are 1, 3, 6, 10 and 15.

The expression below can be used to find *any* triangular number.

$$\frac{n^2 + n}{2}$$

For example, to find the 20th triangular number, substitute $n = 20$ into the formula.

$$\frac{20^2 + 20}{2} = \frac{400 + 20}{2}$$
$$= \frac{420}{2}$$
$$= 210$$

b Use the formula to find the 50th triangular number.

c Use the formula to find the 100th triangular number.

d What is the smallest 4-digit triangular number?

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Puzzles and games

1 To play *Prime Drop* with a partner, you will need a number chart (with circles around the prime numbers and square numbers coloured in) and a die.

Start →

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

← Finish

- Each player must roll a 1 to start.
- Take turns to roll the die and move that number of places on the chart.
- If you land on a prime number, move back to the previous prime number (or to 1).
- If you land on a square number, have another turn.
- To finish, you must land on 50. (If the number you roll is too big, move forward to 50 and then move backwards.)

2 *Why did the elephant go, 'Baa, baa'?*
To find out, work out the following powers and square roots. Then decode the answer below.

A 2^2 **E** 8^3 **G** $\sqrt{121}$ **H** 3^2 **I** $\sqrt{144}$
K 4^4 **L** $\sqrt{256}$ **N** 6^2 **O** 6^3 **P** $\sqrt{196}$
R 9^2 **S** 7^2 **T** 1^3 **U** $\sqrt{225}$ **W** 5^3

12	1		125	4	49		49	14	512	4	256	12	36	11							
4	36	216	1	9	512	81		16	4	36	11	15	4	11	512						

3 *What number am I?*

a Read the following clues to work out each mystery number.

i I have three digits.
I am divisible by 5.
I am odd.
The product of my digits is 15.
The sum of my digits is less than 10.
I am less than 12×12.

ii I have three digits.
The sum of my digits is 12.
My digits are all even.
My digits are all different.
I am divisible by 4.
The sum of my units and tens digits equals my hundreds digit.

iii I have three digits.
I am odd and divisible by 5 and 9.
The product of my digits is 180.
The sum of my digits is less than 20.
I am greater than 30^2.

b Make up two of your own mystery number puzzles and submit your clues to your teacher.

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Chapter summary

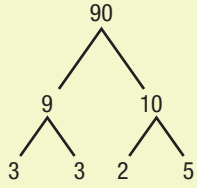
Introducing indices

Prime numbers
5: factors 1 and 5
17: factors 1 and 17
A prime number has only two factors: 1 and itself.

0 and 1 are neither prime nor composite.

Composite numbers
10: factors 1, 2, 5, 10
62: factors 1, 2, 31, 62
Composite numbers have three or more factors.

Factor trees



$90 = 2 \times 3 \times 3 \times 5$
$90 = 2 \times 3^2 \times 5$

Repeated division by primes

2	90
5	45
3	9
3	3
	1

$90 = 2 \times 3 \times 3 \times 5$
$= 2 \times 3^2 \times 5$

$90 = 2 \times 3^2 \times 5$ is a prime decomposition (i.e. a product of prime factors).

Powers

$243 = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{factor form}} = \underbrace{3^5}_{\text{index form}}$

base

index or exponent

Squares and square roots
$4^2 = 16$ so $\sqrt{16} = 4$
$5^2 = 25$ so $\sqrt{25} = 5$
$10^2 = 100$ so $\sqrt{100} = 10$

Cubes and cube roots
'2 cubed' $= 2^3 = 2 \times 2 \times 2$
$2^3 = 8$ so $\sqrt[3]{8} = 2$
$5^3 = 125$ so $\sqrt[3]{125} = 5$

Division

$\frac{8}{2} = 4$ no remainder
So 8 is divisible by 2 and 2 is a factor of 8.
$\frac{9}{2} = 4$ remainder 1
So 9 is not divisible by 2.

Divisibility tests
2: last digit even or 0
3: sum digits ÷ 3
4: last two digits ÷ 4
5: last digit 0 or 5
6: ÷ by 2 and 3
8: last 3 digits ÷ 8
9: sum digits ÷ 9
10: last digit 0

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Chapter review

Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.

Multiple-choice questions

1 What is the smallest prime number?
A 0 **B** 1 **C** 2 **D** 3 **E** 4

2 What is the smallest composite number?
A 0 **B** 1 **C** 2 **D** 3 **E** 4

3 Which of the following numbers is a prime number?
A 21 **B** 77 **C** 11 **D** 22 **E** 1

4 Which of the following groups of numbers include one prime and two composite numbers?
A 2, 10, 7 **B** 54, 7, 11 **C** 9, 32, 44 **D** 5, 17, 23 **E** 18, 3, 12

5 $7 \times 7 \times 7 \times 7 \times 7$ can be simplified to:
A 5^7 **B** 7^5 **C** 7×5 **D** 75 **E** 77 777

6 Evaluate $\sqrt{3^2 + 4^2}$.
A 7 **B** 5 **C** 14 **D** 25 **E** 6

7 Evaluate $3^2 + 2^3$.
A 12 **B** 15 **C** 16 **D** 17 **E** 3125

8 The prime factor form of 48 is:
A $2^4 \times 3$ **B** $2^2 \times 3^2$ **C** 2×3^3 **D** 3×4^2 **E** $2^3 \times 6$

9 Evaluate $4^3 - 3 \times (2^4 - 3^2)$.
A 427 **B** 18 **C** 43 **D** 320 **E** 68

10 Which number is *not* divisible by 3?
A 25 697 403 **B** 31 975 **C** 7 297 008
D 28 650 180 **E** 38 629 634 073

Short-answer questions

1 Write the missing words or numbers.
a A number that ends in 0 is divisible by both ___ and ___.
b 264 is _____ by 3 because the sum of the digits is ___ + ___ + ___ = ___ and __ is divisible by 3.
c 576 is _____ by 9 because the sum of the digits is ___+ ___ + ___ = ___ and __ is _____ by 9.
d 344 is _____ by 4 because the last two digits are ___ and ___ is _____ by 4.

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Chapter review

2 206, 48, 56, 621, 320, 85, 63, 14, 312
From the list of numbers above, write the numbers that are divisible by:
a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

3 Answer these questions.
a Is the number 1 a prime number?
b Is the number 1 a composite number?
c 5 has only two factors; i.e. 1 and 5. Is 5 a prime number?
d List the factors of 10. Is 10 a prime or composite number?
e Sort these numbers into prime or composite numbers: 2, 3, 8, 7, 11, 15, 20.
f Write the factors of 20 in two groups: prime and composite numbers.
g List the prime numbers between 15 and 25.

4 Answer these questions.
a For $16 = 4^2$, what number is the base? What number is the index or exponent?
b Write the product $5 \times 5 \times 7 \times 7 \times 7$ in index form.
c Write $2^3 \times 3^2$ in factor form and find the number value.
d Evaluate $3^2 + 5^2 - 3 \times 2$. (Show your steps.)
e Find the missing powers: $125 = 5^{\square}$; $32 = 2^{\square}$; $100\,000 = 10^{\square}$.

5 For each number, draw a factor tree. Then write the number with powers of prime factors.
a 50 **b** 16 **c** 54

6 Copy and complete each of these tables.

a

Index form	4^2		7^2	
Value		36		81

b

Square root form	$\sqrt{16}$		$\sqrt{100}$	
Value		5		12

7 Find the value of:
a 1^{99} **b** $\sqrt{3^2} + \sqrt{4^2}$ **c** $\sqrt{3^2 + 4^2}$
d $\sqrt{3^2} \times \sqrt{4^2}$ **e** $4^2 - \sqrt{25} + \sqrt{7^2}$ **f** $10^3 \div \sqrt{3^2 + 4^2}$
g 8 to the power of 2
h 11 squared
i the square root of 81
j the side length of a square that has area = 25 cm^2
k the side length of a square that has area = 400 cm^2

8 Copy and complete:
a $\square^3 = 1$, so $\sqrt[3]{1} = \square$ **b** $2^3 = \square$, so $\sqrt[3]{\square} = \square$
c $5^3 = \square$, so $\sqrt[3]{\square} = \square$ **d** $\square^3 = 1000$, so $\sqrt[3]{1000} = \square$

9 Find the value of:
a $2^2 + 3^2$ **b** $2^2 \times 3^2$ **c** $2^2 - 3^2$
d $(2 + 3)^2$ **e** $(2 + 3)^3$ **f** $2^3 + 3^3$

10 The four boxes contain four different prime numbers.
$\square \times \square \times \square \times \square = x$
What is the smallest value of x?

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Chapter review



Drilling for Gold 11R1

Extended-response questions

1 The number 12 is the smallest number that is divisible by 2, 3, and 4.
Find the smallest number that is divisible by:

a 3, 4 and 5 **b** 4, 5 and 6 **c** 5, 6 and 7 **d** 8, 9 and 10

2 Copy this grid into your book or download it from *Cambridge GO*.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

a Cross out 1 because it is neither prime nor composite.
b Highlight 2 because it is prime. Then cross out all the other multiples of 2.
c Highlight 3 because it is prime. Then cross out all the other multiples of 3, using a different coloured pen.
d Highlight 5 because it is prime. Then cross out all the other multiples of 5, using a different coloured pen.
e Highlight 7 because it is prime. Then cross out all the other multiples of 7, using a different coloured pen.
f Circle all the remaining numbers.
g Write down all the numbers that have been circled. This is the set of prime numbers less than 100. (Hint: There are 25 of them.)
h Which numbers got 'crossed out' three times?

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Chapter 7: Time

Multiple-choice questions

1 An ancient building is dated back to 2500 BC. How old was it in 2014?
A 4514 years **B** 486 years **C** 10 000 years
D 2500 years **E** 2014 years

2 Anastasia walks to school in 25 minutes and 54 seconds, then home in 37 minutes and 17 seconds. What is Anastasia's total walking time?
A 62 min 11 s **B** 1 h 2 min 11 s **C** 53 min 11 s
D 17 min 37 s **E** 1 h 3 min 11 s

3 How many hours behind New South Wales is South Australia?
A $\frac{1}{2}$ h **B** 1 h **C** $1\frac{1}{2}$ h **D** 2 h **E** $2\frac{1}{2}$ h

4 6.1 hours is the same as:
A 6 h 1 min **B** 6 h 10 min **C** 6 h 12 min
D 6 h 6 min **E** 6 h 10 min 10 s

5 When it is 3 a.m. in New York, what time is it in Sydney, using Australian standard time?
A 3 a.m. **B** 12 noon **C** 5 p.m. **D** 3 p.m. **E** 6 p.m.

Short-answer questions

1 Write the following, using the units shown in brackets.
a $2\frac{1}{2}$ days (h) **b** 180 min (h) **c** 3.5 min (min and s)
d $6\frac{1}{2}$ h (h and min) **e** 9:45 p.m. (24-h time) **f** 1326 (a.m./p.m.)
g 5.75 h (h and min) **h** 6.32 h (h, min, s)

2 Calculate these time intervals.
a 2:25 a.m. to 3:10 a.m. **b** 6:18 p.m. to 8:09 p.m.
c 6 h 40 min 10 s to 7 h 51 min 11 s **d** 2 h 18 min 50 s to 4 h 10 min 40 s

3 **a** When it is 9 a.m. UTC, what time is it in the following places?
i New South Wales **ii** Western Australia
iii Iraq **iv** central Greenland
v Alaska **vi** New Zealand
b When it is 4:20 p.m. in New South Wales during daylight saving time, what time is it in the following places?
i Western Australia **ii** South Australia
iii Queensland **iv** Tasmania

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4 Use the given train timetable for Richmond to Chatswood to answer the following questions.

Station	a.m.	p.m.
Richmond	6:37	2:56
Seven Hills	7:21	3:40
Parramatta	7:32	3:52
Central	8:07	4:25
Chatswood	8:35	4:53

a How long does it take to travel from:
- **i** Richmond to Parramatta in the morning?
- **ii** Seven Hills to Chatswood in the morning?
- **iii** Parramatta to Chatswood in the afternoon?
- **iv** Richmond to Chatswood in the afternoon?

b Does it take longer to travel from Richmond to Chatswood in the morning or afternoon?

c Domenic travels from Richmond to Parramatta in the morning, then from Parramatta to Chatswood in the afternoon. What is Domenic's total travel time?

5 How many hours and minutes are there between 2:30 p.m. Monday and 11:45 a.m. Tuesday?

6 A busy airport operates 18 hours a day. An aircraft lands every 9 minutes. How many aircraft arrive at the airport each day?

7 The local time in Sydney and Melbourne is 2 hours behind Auckland and 10 hours ahead of London. When it is 5 p.m. in Auckland, what time is it in London?

Extended-response question

1 Anchen plans to travel from Sydney to Cairns, to Alice Springs, to Perth and then return home.

a It is currently 4:30 p.m. at home (Australian standard time). What is the current time in these places?
- **i** Cairns
- **ii** Alice Springs
- **iii** Perth

b Anchen leaves Sydney at 0635 hours and arrives in Cairns at 0914 hours. What is the duration of the flight?

c After staying a week in Cairns, Anchen leaves on a 2 hour and 30 minute flight to Alice Springs. If he leaves at 1:30 p.m., what will be the time in Alice Springs when he arrives?

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Chapter 8: Algebraic techniques 1

Multiple-choice questions

1 When $x = 3$, $5 + x$ equals:
A 2 **B** 8 **C** 3 **D** 53 **E** 15

2 The sum of x and y is written as:
A $x + y$ **B** $2x$ **C** xy **D** $x - y$ **E** $x \div y$

3 When $a = 7$, $5a + 2$ is equal to:
A 7 **B** 59 **C** 37 **D** 33 **E** 52

4 $4a + 3b + c + 5b + c$ is equivalent to:
A $32ab$ **B** $4a + 8b + 2c$ **C** $8a + 4b$ **D** $64abc$ **E** $4a + 8b$

5 $2a \times 3b \times 4c$ simplifies to:
A $6ab + c$ **B** $7abc$ **C** 24 **D** $24abc$ **E** $24 + abc$

Short-answer questions

1 Consider the expression $5x + 7y + 3x$.
a How many terms are in this expression?
b What is the coefficient of y?
c Simplify this expression by combining the like terms.

2 Write an algebraic expression for each of the following.
a the sum of x and 3
b the product of a and 12
c the sum of double x and triple y
d w divided by 6
e double x taken from y

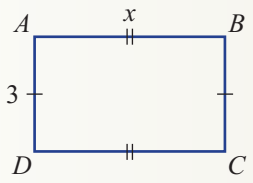
3 For the following pairs, state if they are like (L) or not like (N) terms.
a $2a$ and $3a$ **b** $7k$ and $7m$ **c** $4xy$ and $2xy$ **d** $3ab$ and $5b$

4 If $m = 6$, find the value of each of the following.
a $m + 7$ **b** $2m - 1$ **c** $6m + 3$
d $2(m - 3)$ **e** $\frac{m}{2}$ **f** $5m - 6$

5 Evaluate the expression $6x + 3y$ when $x = 5$ and $y = 2$.

6 Simplify each of the following.
a $6a + 4a$ **b** $7x - 3x$ **c** $9a + 2a + a$
d $m + m - m$ **e** $6 + 2a + 3a$ **f** $x + y + 3x + y$

7 **a** Write an expression for the perimeter of rectangle $ABCD$.
b Write an expression for the area of rectangle $ABCD$.

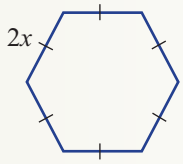


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8 Find the missing term.
a $3a \times ____ = 18abc$ **b** $9b - ____ = 4b$ **c** $2p + 2p + 2p = 6____$

9 The cost of one pencil is $\$p$. Write an expression for the cost of:
a 10 pencils **b** 25 pencils

10 Write the simplest expression for the perimeter of this figure.



Extended-response question

1 A bottle of soft drink costs $3 and a pie costs $2.
a Find the cost of:
i 2 bottles of soft drink and 5 pies
ii x bottles of soft drink
iii x bottles of soft drink and 5 pies
b Write an expression for the cost of buying x bottles of soft drink and y pies.
c Substitute $x = 2$ and $y = 5$ into your expression from part **b**.

Chapter 9: Equations 1

Multiple-choice questions

1 The solution to the equation $x - 3 = 7$ is:
A $x = 4$ **B** $x = 10$ **C** $x = 9$ **D** $x = 11$ **E** $x = 3$

2 The solution to the equation $2x + 6 = 12$ is:
A $x = 4.5$ **B** $x = 2$ **C** $x = 7$ **D** $x = 6$ **E** $x = 3$

3 $m = 4$ is a solution to:
A $3m + 12 = 0$ **B** $\frac{m}{4} = 16$ **C** $2 = 10 - 2m$
D $m + 4 = 0$ **E** $3m - 6 = 2$

4 The solution to $2p - 3 = 7$ is:
A $p = 4$ **B** $p = 5$ **C** $p = 2$ **D** $p = 10$ **E** $p = 3$

5 Ying thinks of a number. If he multiplies the number by 5 and then subtracts 4, the result is 35. What equation represents this information?
A $y + 9 = 35$ **B** $5y - 4 = 35$ **C** $5y + 4 = 35$
D $5(y + 4) = 35$ **E** $y + 20 = 35$

Short-answer questions

1 Classify the following as true or false.
a $2 + 17 = 10 + 8$ **b** $5 = 10 - 5$ **c** $x + 2 = 10$ when $x = 8$
d $a - 3 = 7$ when $a = 4$ **e** $12 = 20 - k$ when $k = 6$ **f** $y - 3 = 3$ when $y = 6$

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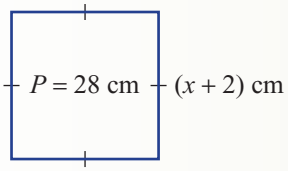
2 Solve:
a $x + 9 = 12$ **b** $\frac{x}{9} = 12$ **c** $x - 9 = 12$ **d** $9x = 12$

3 Solve:
a $3x + 3 = 9$ **b** $2y - 7 = 3$ **c** $\frac{x}{2} + 6 = 12$ **d** $\frac{m - 1}{3} = 2$

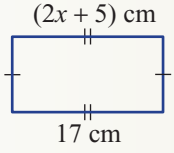
4 Consider the formula $P = 4w$.
a Find the value of P when:
i $w = 2$ **ii** $w = 5$ **iii** $w = 12$
b Solve the equation $24 = 4w$.
c Find the value of w if $P = 24$.
d Find the value of w if $P = 44$.

5 Use your knowledge of geometry and shapes to find the value of x in each of the following.

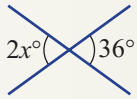
a



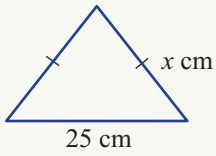
b



c



6 The perimeter of this triangle is 85 cm. Write an equation and then solve it to find the value of x.



Extended-response question

1 The cost of hiring a hall for an event is \$200 plus \$40 per hour.
a What is the cost of hiring the hall for 3 hours?
b What is the cost of hiring the hall for 5 hours?
c Write a formula for the cost, $\$C$, of hiring the hall for n hours.
d If the cost of hiring the hall totals \$460, for how many hours was it hired?

Chapter 10: Measurement and computation of length, perimeter and area

Multiple-choice questions

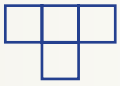
1 17 mm is the same as:
A 0.17 m **B** 0.17 cm **C** 1.7 cm **D** 170 cm **E** 1.7 m

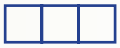
2 2.6 m is the same as:
A 260 mm **B** 26 cm **C** 260 km
D 2600 mm **E** 26 000 mm

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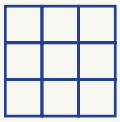
3 Which of the following shapes has the largest perimeter?

A 

B 

C 

D 

E 

4 The perimeter of this square is 12 cm. Its area is:

A 4 cm^2 **B** 144 cm^2 **C** 9 cm^2

D 16 cm^2 **E** 8 cm^2



5 Which is the correct formula for the circumference of a circle?

A $C = 2\pi r$ **B** $C = \pi r^2$ **C** $C = \pi r$ **D** $C = 2\pi d$ **E** $C = \pi d^2$

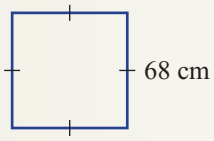
Short-answer questions

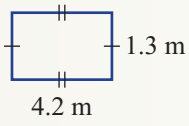
1 Complete these conversions.

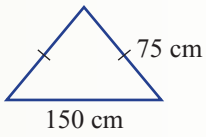
a 5 m = ____ cm **b** 6 km = _____ m

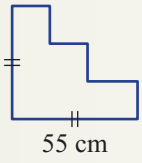
c 180 mm = _____ cm **d** 180 cm = _____ m

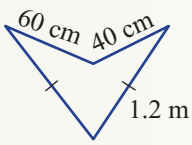
2 Find the perimeter of each of the following.

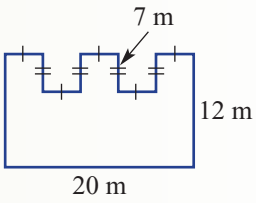
a 

b 

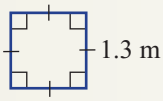
c 

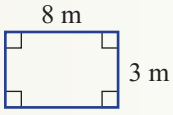
d 

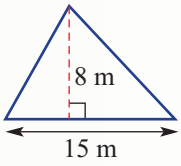
e 

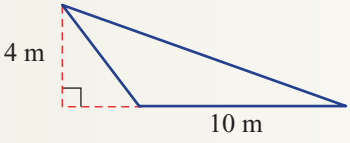
f 

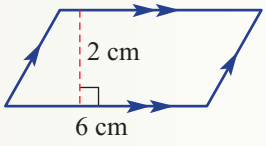
3 Find the area of each of the following.

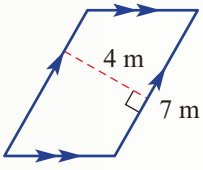
a 

b 

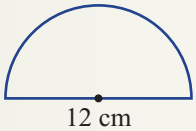
c 

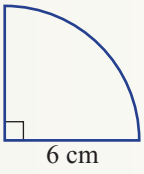
d 

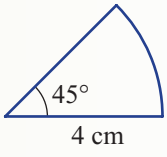
e 

f 

4 Find the exact perimeter of the following sectors.

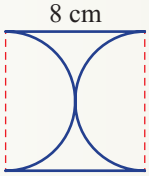
a 

b 

c 

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5 Find the exact perimeter of this composite figure, in which two semicircles have been removed from a square.



6 Convert to the units shown in the brackets.

a 3 t (kg) **b** 2 kg (g) **c** 6500 g (kg) **d** 500 kg (t)
e 5000 mg (g) **f** 24 g (mg) **g** 0.05 kg (g) **h** 20 mg (g)

7 Copy and complete:

a Water freezes at ____°C.
b Water boils at ____°C.

Extended-response question

1 Robert has been given 36 m of fencing with which to build the largest rectangular enclosure that he can, using whole number side lengths.

a Draw three possible enclosures and calculate the area of each one.
b What are the dimensions of the rectangle that gives the largest possible area?
c If Robert chooses the dimensions in part **b** and puts a post on each corner, and then posts every metre along the boundary, how many posts will he need?
d If each post costs $25, what will be the total cost of the posts?

Chapter 11: Introducing indices

Multiple-choice questions

1 The first prime number after 14 is:

A 15 **B** 21 **C** 16 **D** 19 **E** 17

2 The first composite number after 14 is:

A 15 **B** 21 **C** 16 **D** 19 **E** 17

3 $2 \times 2 \times 2 \times 3$ is the same as:

A 6×3 **B** $2^3 \times 3$ **C** 8^3 **D** 6^3 **E** 4^3

4 Evaluating $3^2 - \sqrt{25} + 3$ gives:

A 8 **B** 5 **C** 4 **D** 17 **E** 7

5 The number 48 in prime factor form is:

A $2^4 \times 5$ **B** $2 \times 3 \times 5$ **C** $2^3 \times 3^2$ **D** $2^4 \times 3$ **E** $2^3 \times 3$

Short-answer questions

1 Write down all the numbers between 10 and 20 that are divisible by:

a 3 **b** 7 **c** 9

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2 Write down all the numbers less than 20 that have exactly three factors. (Hint: They are square numbers.)

3 Write down all the prime numbers between 20 and 40. (Hint: There are four of them.)

4 Find the value of:

a $2 \times 2 \times 2$ **b** $2 \times 2 \times 3$ **c** $2 \times 3 \times 3$

d $3 \times 3 \times 3$ **e** $2 \times 2 \times 5$ **f** $2 \times 3 \times 5$

5 Find the value of:

a 11^2 **b** $6^2 \times 2^2$ **c** $33 - 2^3$

6 Find:

a 7 to the power of 2 **b** 12 squared **c** the square root of 81

7 Copy and complete each of these tables.

a

Index form	3^2		6^2	
Value		25		64

b

Square root form	$\sqrt{9}$		$\sqrt{36}$	
Value		5		8

8 Which of the numbers 1080, 536, 135, 930 and 316 are divisible by the following?

a 2 **b** 3 **c** 4 **d** 5 **e** 10

9 Is the number 60 divisible by the following?

a 1 **b** 2 **c** 3 **d** 4 **e** 5

f 6 **g** 7 **h** 8 **i** 9 **j** 10

10 Use Question **9** to write 60 as a product of prime factors:

$60 = \square \times \square \times \square \times \square$

so

$60 = \square^2 \times \square \times \square$

Extended-response question

1 The number 6 is called a perfect number. Here is why:

$1 \times 6 = 6$ and $2 \times 3 = 6$

Factors of 6 are 1, 2, 3 and 6.

$1 + 2 + 3 + 6 = 12$

Half of 12 is 6, which is the number we started with.

a Show that 10 is *not* a perfect number.

b Find the next perfect number after 6. (Hint: It is less than 40.)

c The third perfect number is 496. Use the internet to find all the factors of 496. Prove that it is a perfect number.

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Answers

Chapter 1

Pre-test

1 **a** 11 **b** 137 **c** 104 **d** 10 102
2 **a** C **b** A **c** D **d** B
3 **a** 57 **b** 116 **c** 2044 **d** 11 002
4 **a** 13 **b** 37 **c** 89 **d** 8000
e 26 **f** 28
5 **a** 42, 49, 56, 63 **b** 54, 63, 72, 81
c 66, 77, 88, 99
6 **a** 14 **b** 23 **c** 119 **d** 150
e 210 **f** 1110 **g** 500 **h** 900
i 9 **j** 32 **k** 79 **l** 79
m 112 **n** 398 **o** 699 **p** 901
7 **a** 30 **b** 63 **c** 144 **d** 88
e 56 **f** 130 **g** 1100 **h** 8000
i 5 **j** 2 **k** 11 **l** 4
m 10 **n** 9 **o** 3 **p** 6
8 **a** 37, 58, 59, 62, 73 **b** 103, 130, 301, 310
c 24 319, 24 913, 24 931, 29 143
9 **a** 0 **b** 1 **c** 1 **d** 2
10 **a** 1 **b** 1 **c** 3 **d** 1
e 1 **f** 3 **g** 7 **h** 4

Exercise 1A

1 **a** 0 **b** 5 **c** 2 **d** 7
2 **a** 46 **b** 263 **c** 7421 **d** 36 015
3 **a** one hundred and fifty
b one thousand five hundred
c one thousand and fifty
d ten thousand five hundred
e fifteen thousand
f one hundred and fifty thousand
4 **a** $\neq$ **b** $<$ **c** $\geq$
d $=$ **e** $>$ **f** $\leq$
5 **a** 7 **b** 70 **c** 70
d 700 **e** 700 **f** 7000
6 **a** 3 **b** 30 **c** 30 **d** 300
e 30 **f** 3000 **g** 3 **h** 30 000
7 **a** 20 **b** 2000 **c** 200 **d** 200 000
8 **a** true **b** false **c** true **d** true
e false **f** true **g** false **h** true
i true **j** false **k** false **l** true
m false **n** true
9 **a** 44, 45, 54, 55
b 29, 92, 279, 729, 927
c 4, 23, 136, 951
d 345, 354, 435, 453, 534, 543
e 12 345, 31 254, 34 512, 54 321
f 1001, 1010, 1100, 10 001, 10 100
10 **a** 872 **b** 13 469
11 **a** 6 **b** 6 **c** 24
12 **a** **i** 7000 **ii** 46 000
iii 712 000 **iv** 5 000 000
v 44 000 000 **vi** 6 000 000 000
vii 437 000 000 000 **viii** 15 000 000 000 000
b 1 with 100 zeros

Exercise 1B

1 **a** addition **b** subtraction **c** subtraction
d addition **e** addition **f** addition
g subtraction **h** addition **i** subtraction
2 **a** 7 **b** 10 **c** 69
d 4 **e** 12 **f** 20
3 **a** **i** 8 **ii** 27 **iii** 132
b **i** 6 **ii** 16 **iii** 8
4 **a** 18 **b** 19 **c** 32
d 140 **e** 21 **f** 9
5 **a** 34 **b** 46 **c** 59
d 64 **e** 97 **f** 579
6 **a** 11 **b** 36 **c** 51
d 112 **e** 233 **f** 132
7 **a** 24 **b** 75 **c** 95 **d** 133
8 **a** 24 **b** 26 **c** 108
d 222 **e** 317 **f** 5017
9 **a** 20 **b** 30 **c** 20
d 27 **e** 20 **f** 30
10 $79 **11** 38 hours **12** 107 runs
13 **a** 12 **b** 27 **c** 107
d 133 **e** 14 **f** 90
g 109 **h** 0 **i** 3
14 107 cards
15 **a** true **b** true **c** true
d false **e** true **f** false
g false **h** true **i** true
16 **a**

6	1	8
7	5	3
2	9	4

b

10	15	8
9	11	13
14	7	12

c

15	20	13
14	16	18
19	12	17

d

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Exercise 1C

1 **a** 13 **b** 13 **c** 20 **d** 22
e 17 **f** 101 **g** 144 **h** 110
i 1005 **j** 143 **k** 201 **l** 1105

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2 **a** 7 **b** 2 **c** 5 **d** 6
e 8 **f** 27 **g** 67 **h** 84
i 15 **j** 92 **k** 29 **l** 979

3 **a** 8 **b** 1 **c** 1 **d** 6
e 2 **f** 1 **g** 8 **h** 3

4 **a** 87 **b** 99 **c** 41 **d** 86
e 81 **f** 51 **g** 92 **h** 81

5 **a** 222 **b** 322 **c** 521 **d** 920
e 226 **f** 745 **g** 1923 **h** 5080

6 **a** 77 **b** 192 **c** 418 **d** 4208

7 **a** 28 **b** 19 **c** 19 **d** 58
e 58 **f** 29 **g** 26 **h** 39

8 **a** 149 **b** 273 **c** 656 **d** 888
e 36 **f** 112 **g** 79 **h** 72

9 1854 sheep

10 576 kilometres

11 **a**
$$\begin{array}{r} 3\,\boxed{8} \\ +5\;3 \\ \hline \boxed{9}\;1 \end{array}$$
b
$$\begin{array}{r} 1\,1\,4 \\ +\;\;7\,7 \\ \hline \boxed{1}9\,1 \end{array}$$
c
$$\begin{array}{r} 6\,\boxed{2} \\ -2\;8 \\ \hline \boxed{3}\;4 \end{array}$$
d
$$\begin{array}{r} 2\,\boxed{6}\,5 \\ -\boxed{1}\,8\,\boxed{4} \\ \hline 8\;1 \end{array}$$

12 **a** Answers may vary.
b Different combinations in the middle column can be used to create the sum.

13 452 and 526.

14 Answers may vary.

Exercise 1D

1 **a** 10, 12, 14 **b** 15, 18, 21 **c** 35, 42, 49
d 20, 24, 28 **e** 44, 55, 66 **f** 36, 45, 54

2 **a** 4 **b** 2 **c** 11
d 2 **e** 4 **f** 7

3 **a** 22 **b** 27 **c** 32 **d** 56 **e** 28
f 60 **g** 44 **h** 77 **i** 108 **j** 72
k 21 **l** 54 **m** 30 **n** 110 **o** 144
p 40 **q** 49 **r** 63 **s** 132 **t** 72
u 55 **v** 22 **w** 24 **x** 96 **y** 36

4 **a** 105 **b** 124 **c** 93
d 132 **e** 115 **f** 217
g 198 **h** 252 **i** 336

5 **a** 57 **b** 38 **c** 58
d 116 **e** 90 **f** 126
g 117 **h** 196 **i** 234

6 **a** 70 **b** 90 **c** 110
d 180 **e** 96 **f** 54
g 96 **h** 72 **i** 72

7 **a** 25 **b** 9 **c** 81

8 **a** 66 **b** 129 **c** 432 **d** 165
e 148 **f** 459 **g** 336 **h** 472
i 258 **j** 2849 **k** 2630 **l** 31 581

9 **a** 3 **b** 0 **c** 5 **d** 2

10 1680 metres

11 $264

12 116 cards

13 **a** true **b** true **c** false
d true **e** true **f** true
g false **h** true **i** false

14 **a**
$$\begin{array}{r} 3\,9 \\ \times\;\;3 \\ \hline 2\,\boxed{7}3 \end{array}$$
b
$$\begin{array}{r} 2\,5 \\ \times\;\boxed{5} \\ \hline 1\,2\,5 \end{array}$$
c
$$\begin{array}{r} 7\,9 \\ \times\;\boxed{3} \\ \hline 2\,3\,7 \end{array}$$
d
$$\begin{array}{r} 1\,3\,2 \\ \times\;\;\;8 \\ \hline 1\,0\,\boxed{5}6 \end{array}$$
e
$$\begin{array}{r} 2\,\boxed{7} \\ \times\;\;7 \\ \hline \boxed{1}8\,9 \end{array}$$
f
$$\begin{array}{r} \boxed{3}\boxed{9} \\ \times\;\;\;9 \\ \hline 3\;51 \end{array}$$

15 Three ways: (0, 1), (1, 5), (2, 9). You cannot carry a number to the hundreds column.

Exercise 1E

1 **a** 10 **b** 100 **c** 1000 **d** 10

2 **a** 81 **b** 78 **c** 460 **d** 726

3 **a** 120 **b** 120 **c** 250 **d** 720
e 180 **f** 180 **g** 350 **h** 540
i 350 **j** 200 **k** 1000 **l** 3000

4 **a** 400 **b** 290 **c** 1830
d 4600 **e** 37 000 **f** 1920
g 301 000 **h** 248 000 **i** 50 000
j 63 000 **k** 14 410 **l** 29 100 000

5 **a** 240 **b** 540 **c** 520
d 630 **e** 340 **f** 1440
g 6440 **h** 22 500 **i** 41 400
j 460 000 **k** 63 400 **l** 9 387 000

6 **a** 252 **b** 286 **c** 434 **d** 645
e 407 **f** 1368 **g** 1232 **h** 1254
i 2116 **j** 2268 **k** 2116 **l** 5184

7 $420

8 $2176

9 81

10 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6

11 $6020

12 86 400 seconds

13 **a** 84 000 **b** 3185

14 100, 121, 144, 169, 196, 225, 256, 289

Exercise 1F

1 **a** 0 **b** 0 **c** 0 **d** 0
e 4 **f** 0 **g** 3 **h** 0

2 **a** 4, 6 **b** 19, 27

3 **a** 1 **b** 2 **c** 2 **d** 5

4 **a** 4 **b** 3 **c** 6 **d** 5
e 7 **f** 9 **g** 8 **h** 11
i 10 **j** 4 **k** 11 **l** 12
m 8 **n** 12 **o** 11 **p** 12

5 **a** 31 **b** 21 **c** 21 **d** 21
e 21 **f** 11 **g** 32 **h** 16

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Exercise 1F cont.

6 **a** 29 **b** 19 **c** 24 **d** 9
e 39 **f** 14 **g** 29 **h** 9
7 **a** 22 **b** 31 **c** 17 **d** 7
8 **a** 26 **b** 1094 **c** 0 **d** 0
9 **a** $23\frac{2}{3}$ **b** $13\frac{1}{7}$ **c** $69\frac{1}{2}$ **d** $41\frac{1}{6}$
e $43\frac{2}{5}$ **f** $126\frac{2}{4}$ **g** $264\frac{2}{3}$ **h** $90\frac{4}{9}$
i $543\frac{1}{4}$ **j** $20\,333\frac{2}{3}$ **k** $818\frac{3}{5}$ **l** 10 001
10 **a** 1 **b** 1 **c** 5 **d** 5
11 13 packs
12 124 packs
13 67 posts
14 15 taxis
15 $204
16 58
17 **a** $33\frac{8}{11}$ **b** $54\frac{8}{17}$ **c** $31\frac{1}{13}$
d $108\frac{1}{15}$ **e** $91\frac{16}{23}$ **f** $123\frac{26}{56}$
18

1	6	20	56
40	28	2	3
14	5	24	4
12	8	7	10

Exercise 1G

1 **a** up **b** down **c** up
d up **e** down **f** down
2 **a** up **b** up **c** down **d** down
e down **f** down **g** up **h** up
i down **j** up **k** up **l** down
3 **a** down **b** down **c** up **d** up
e up **f** down **g** down **h** up
i up **j** down **k** up **l** up
4 **a** 60 **b** 30 **c** 120 **d** 190
e 200 **f** 900 **g** 100 **h** 600
i 2000 **j** 1000 **k** 8000 **l** 10 000
5 **a** 20 **b** 30 **c** 100 **d** 900
e 200 **f** 700 **g** 100 **h** 1000
i 6000 **j** 90 000 **k** 10 000 **l** 10
6 **a** 120 **b** 160 **c** 100 **d** 12
e 10 **f** 3000 **g** 1200 **h** 3
i 40 **j** 2000 **k** 4000 **l** 100
7 $1200
8 ≈ 2100 scoops
9 ≈ 1200 sheep
10 ≈ 8 people
11 **a** larger **b** smaller **c** smaller **d** larger
12 **a** 200 **b** 100 000 **c** 800
d 3 000 000
13 Compare your answer with a friend's.

Exercise 1H

1 **a** multiplication **b** brackets
c brackets **d** multiplication
2 **a** addition **b** division
c multiplication **d** multiplication
e division **f** addition
g division **h** multiplication
i division **j** subtraction
k multiplication **l** division
3 **a** 7 **b** 19 **c** 7 **d** 20
e 3 **f** 2 **g** 3 **h** 24
i 1 **j** 23 **k** 21 **l** 0
m 18 **n** 32 **o** 2 **p** 22
q 22 **r** 38 **s** 153 **t** 28
u 200
4 **a** 10 **b** 3 **c** 2 **d** 9
e 8 **f** 0 **g** 22 **h** 2
i 9 **j** 18 **k** 3 **l** 10
m 121 **n** 20 **o** 1 **p** 16
5 **a** true **b** false **c** false **d** true
6 **a** 27 **b** 10 **c** 8 **d** 77
e 30 **f** 21 **g** 192
7 75 books
8 45 TVs
9 **a** $(4 + 2) \times 3 = 18$ **b** $9 \div (12 - 9) = 3$
c $2 \times (3 + 4) - 5 = 9$ **d** $(3 + 2) \times (7 - 3) = 20$
e $(10 - 7) \div (21 - 18) = 1$ **f** $(4 + 10) \div (21 \div 3) = 2$
10 **a** no **b** yes **c** no
d yes **e** yes **f** no
11 **a** 58 **b** 18 **c** 13
d 28 **e** 22
12 **a** $[20 - (31 - 19)] \times 2 = 16$
b $50 \div (2 \times 5) - 4 = 1$
c $(25 - 19) \times (3 + 7) \div 12 + 1 = 6$

Puzzles and games

1 **a**

15	20	13
14	16	18
19	12	17

b

11	16	9
10	12	14
15	8	13

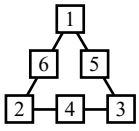
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2 **a** $(5 + 2) \times 3 = 21$

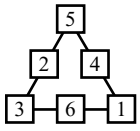
b $(16 - 8) \div (10 - 6) = 2$

c $4 + (2 \times 7 - 1) \times 3 = 43$

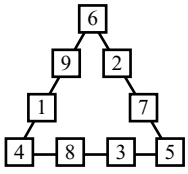
3 **a** **i**



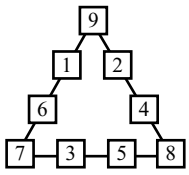
ii



b **i**



ii



4 **a**

4	3	2	1
2	1	4	3
3	4	1	2
1	2	3	4

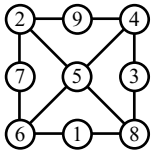
b

4	1	3	2
2	3	4	1
3	2	1	4
1	4	2	3

c

1	3	2	4
4	2	3	1
2	4	1	3
3	1	4	2

5



6 Answers may vary; e.g.

a $\begin{array}{r} \boxed{2}1\boxed{7} \\ \times \quad 7 \\ \hline \boxed{1}5\,1\boxed{9} \end{array}$ **b** $\begin{array}{r} 2\,9\,\boxed{5} \\ \times \quad 3 \\ \hline 8\,\boxed{8}\boxed{5} \end{array}$

Multiple-choice questions

1 B	**2** C	**3** E	**4** A	**5** B
6 A	**7** D	**8** C	**9** B	**10** A

Short-answer questions

1 **a** 137, 173, 317, 371, 713, 731

b 199, 999, 1000, 1001, 1010, 1090, 1900

2 **a** 50 **b** 5000 **c** 50 000

3 **a** 459 **b** 363 **c** 95 **d** 217

4 **a** 128 **b** 355 **c** 191 **d** 739

5 **a** 95 **b** 132 **c** 220

d 41 **e** 33 **f** 24

g 29 000 **h** 10 800 **i** 14 678

6 **a** 156 **b** 840 **c** 1413 **d** 351

e 45 **f** $101\frac{3}{9}$ **g** $46\frac{5}{7}$ **h** $7540\frac{2}{4}$

7 **a** $\begin{array}{r} 2\,\boxed{2}3 \\ +\,7\,3\boxed{8} \\ \hline 9\,6\,1 \end{array}$ **b** $\begin{array}{r} \boxed{7}2\boxed{9} \\ -\,4\,\boxed{7}3 \\ \hline 2\,5\,6 \end{array}$

c $\begin{array}{r} \boxed{4}1 \\ \times \quad 7 \\ \hline 2\,8\,\boxed{7} \end{array}$ **d** $4\,\boxed{5}\,8\,\overline{)\,9\,1\,6}$

8 **a** 70 **b** 3300 **c** 1000

9 **a** 800 **b** 400 **c** 5000 **d** 10

10 **a** 24 **b** 4 **c** 1

d 92 **e** 14 **f** 3

g 20 **h** 0 **i** 13

Extended-response questions

1 **a** 90 loads **b** $27 500

c $4000

2 **a** 313 sweets

b 78

c jelly beans, 5

Chapter 2

Pre-test

1 **a** C **b** D **c** E

d B **e** A **f** F

2 **a** 90° **b** 180° **c** 360°

d 270° **e** 45° **f** 315°

3 **a** 60° **b** 140° **c** 125° **d** 80°

Exercise 2A

1 **a** F **b** A **c** B

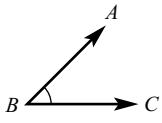
d E **e** D **f** C

2 **a** $\bullet P$

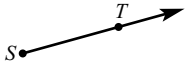
b



c



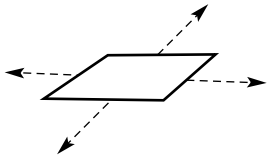
d



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Exercise 2A cont.

e



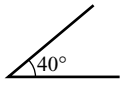
f



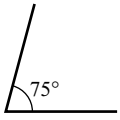
3 **a** ray **b** line **c** segment
4 **a** point T **b** line CD **c** angle BAC
d plane **e** ray PQ **f** segment ST
5 **a** $\angle AOB$ **b** $\angle XYZ$ **c** $\angle BOC$
d $\angle BAC$ **e** $\angle BEA$ **f** $\angle AOC$
6 **a** collinear **b** concurrent
7 **a** yes **b** no
8 **a** no **b** yes
9 **a** 3 **b** 6
c 8 **d** 14
10 $\angle ADB$, $\angle BDA$, $\angle FDB$, $\angle BDF$, $\angle FDE$, $\angle EDF$, $\angle ADE$, $\angle EDA$
11 All segments should intersect at the same point; i.e. are concurrent.

Exercise 2B

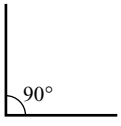
1 **a** acute **b** obtuse **c** right
d straight **e** revolution **f** acute
g reflex **h** reflex **i** obtuse
2 Answers may vary.
3 **a** 50° **b** 145°
c 160° **d** 55°
e 90° **f** 250°
4 **a** 40° **b** 55° **c** 90°
d 125° **e** 165° **f** 180°
g 230° **h** 270° **i** 325°
5 **a** acute **b** acute **c** right
d obtuse **e** obtuse **f** straight
g reflex **h** reflex **i** reflex
6 **a**



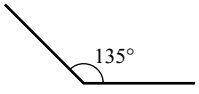
b



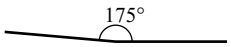
c



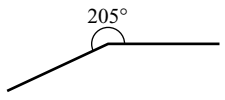
d



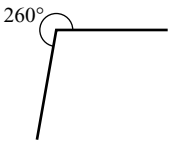
e



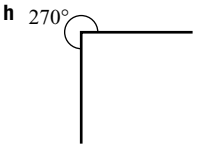
f



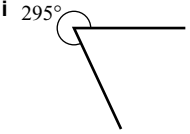
g



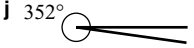
h



i



j



7 **a** 55° **b** 29° **c** 35° **d** 130°
8 **a** 2 **b** 3 **c** 4
9 **a** 180° **b** 360° **c** 90° **d** 270°
e 30° **f** 120° **g** 330° **h** 6°
10 Use the revolution to get 360° − 60° = 300°.
11 Answers will vary.
12 **a** 180° **b** 90° **c** 120° **d** 30°
13 **a** 115° **b** 127.5° **c** 85° **d** 77.5°

Exercise 2C

1 **a, b** a and b should add to 90°.
c complementary
2 **a, b** a and b should add to 180°.
c supplementary
3 **a** a and c should be equal; b and c should be equal.
b a, b, c and d should add to 360°.
c vertically opposite angles
4 **a** $\angle ABD$ **b** $\angle CBF$ **c** $\angle CBT$
5 **a** $\angle UOV$ **b** $\angle MON$ **c** $\angle BOA$
6 **a** $\angle MFS$ or $\angle SFM$ **b** $\angle TFS$ or $\angle SFT$
c $\angle BFS$ or $\angle SFB$ **d** $\angle SFX$ or $\angle XFS$
7 **a** 60 **b** 15 **c** 69 **d** 135
e 70 **f** 141 **g** 115 **h** 37
i 50 **j** 240 **k** 270 **l** 140
8 **a** N **b** N **c** S **d** N
e C **f** C **g** C **h** S
9 **a** 80° **b** 50° **c** 20° **d** 85°
10 **a** 170° **b** 140° **c** 110° **d** 50°
11 **a** $\angle BOC$ **b** $\angle AOD$ and $\angle BOC$ **c** $\angle COD$
12 **a** 30 **b** 75 **c** 60
d 135 **e** 45 **f** 130
13 **a** No, should add to 90°. **b** Yes, they add to 180°.
c Yes, they add to 360°. **d** Yes, they are equal.
e No, they should be equal.
f No, they should add to 360°.
14 24°
15 **a** 30 **b** 60 **c** 60
d 45 **e** 180 **f** 36

Exercise 2D

1 **a** 4 **b** no
2 **a** 2 **b** yes
3 **a** equal **b** supplementary
c equal **d** equal
4 **a** $\angle DEH$ **b** $\angle BEF$ **c** $\angle DEB$ **d** $\angle CBG$
5 **a** $\angle FEG$ **b** $\angle DEB$ **c** $\angle GEB$ **d** $\angle ABC$
6 **a** 130, corresponding **b** 70, corresponding
c 110, alternate **d** 120, alternate

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e 130, vertically opposite **f** 67, vertically opposite
g 65, cointerior **h** 118, cointerior
i 100, corresponding **j** 117, vertically opposite
k 116, cointerior **l** 116, alternate
m 110, cointerior **n** 48, cointerior
o 117, corresponding

7 **a** $a = 70$, $b = 70$, $c = 110$
b $a = 120$, $b = 120$, $c = 60$
c $a = 98$, $b = 82$, $c = 82$, $d = 82$
d $a = 90$, $b = 90$, $c = 90$
e $a = 95$, $b = 85$, $c = 95$
f $a = 61$, $b = 119$

8 **a** No, corresponding angles should be equal.
b Yes, alternate angles are equal.
c Yes, cointerior angles are supplementary.
d Yes, corresponding angles are equal.
e No, alternate angles should be equal.
f No, cointerior angles should be supplementary.

9 **a** 35 **b** 41 **c** 110
d 30 **e** 60 **f** 141

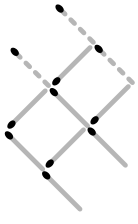
10 **a** 65 **b** 100 **c** 62
d 67 **e** 42 **f** 57
g 100 **h** 130 **i** 59

11 **a** $a = 120$, $b = 120$
b 60
c opposite angles are equal

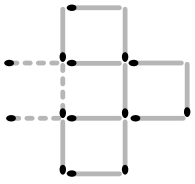
12 **a** 60 **b** 115 **c** 123

Puzzles and games

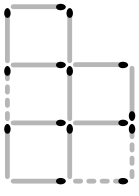
1



2 **a**



b



3 720°

4 **a** 135° **b** 77.5° **c** 143.5°.

5 **a** All angles are 90° or 45° or 135°.
b i, ii There are four different convex hexagons. Going around the hexagon, the angles are:
(1) 90°, 90°, 135°, 135°, 135°, 135°
Angle sum = 720°
(2) 90°, 135°, 135°, 90°, 135°, 135°
Angle sum = 720°
(3) 90°, 135°, 135°, 90°, 135°, 135°
Angle sum = 720°
(4) 90°, 135°, 135°, 90°, 135°, 135°
Angle sum = 720°
There are many non-convex hexagons.
c The angle sum of a hexagon is 720°.

Multiple-choice questions

1 C **2** B **3** D **4** E **5** D
6 E **7** C **8** A **9** E **10** B

Short-answer questions

1 **a** segment AB **b** $\angle AOB$ **c** point O
d plane **e** ray AB **f** line AB

2 **a** acute, 35° **b** reflex, 210°
c obtuse, 115° **d** reflex, 305°

3 Check by measuring with a protractor. A right angle is 90°.

4 **a** 20 **b** 230 **c** 35
d 41 **e** 15° **f** 38

5 **a** $a°$ and $b°$ **b** $a°$ and $d°$
c $a°$ and $c°$ **d** $b°$ and $c°$
e $c°$ and $d°$ or $b°$ and $d°$

6 **a** 61 **b** 128 **c** 59

7 **a** Yes, corresponding angles are equal.
b No, alternate angles should be equal.
c No, cointerior angles should be supplementary.

8 **a** 100 **b** 95 **c** 30

Extended-response questions

1 **a** **i** 30° **ii** 180° **iii** 270°
b **i** 30° **ii** 210° **iii** 15°
c **i** 180° **ii** 90° **iii** 60°

2 **a** 12 pieces **b** 30
c **i** 15 **ii** 22.5
iii 20 **iv** 24

Chapter 3

Pre-test

1 **a** 11 **b** 5 **c** 7 **d** 22
e 0 **f** 6 **g** 1 **h** 12
i 1 **j** 8 **k** 9 **l** 13

2 **a** < **b** < **c** < **d** >
e < **f** < **g** > **h** >

3 **a** 5°C **b** 2°C **c** 7°C

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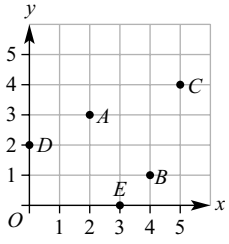
Pre-test cont.

4 **a** 30 **b** 77 **c** 72 **d** 39

5 **a** 5 **b** 11 **c** 9 **d** 7

6 $A(1, 1)$, $B(3, 2)$, $C(2, 3)$

7



8 6°

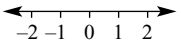
Exercise 3A

1 **a** 1 **b** 0 **c** –4
d –1 **e** –18 **f** –36

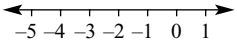
2 **a** –2, 2 **b** 0, 2 **c** –7, –5 **d** –5, –3, 0

3 **a** greater **b** less **c** greater
d less **e** greater **f** less

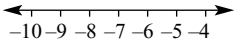
4 **a**



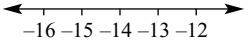
b



c



d



5 **a** right **b** left **c** left
d right **e** left **f** right

6 **a** –2 **b** –6 **c** 3 **d** 7

7 **a** < **b** > **c** > **d** <
e > **f** < **g** < **h** >
i < **j** > **k** < **l** >
m < **n** > **o** > **p** <

8 **a** 4°C **b** –1°C **c** –7°C **d** –25°C

9 **a** false **b** false **c** false **d** true
e false **f** false **g** true **h** false

10 **a** –2, –1, 0, 1, 2, 3, 4
b –7, –6, –5, –4, –3, –2, –1, 0
c –2, –1, 0, 1
d –4, –3, –2, –1, 0
e –3, –2, –1, 0, 1, 2, 3
f –9, –8, –7, –6, –5, –4

11 **a** –10, –6, –3, –1, 0, 2, 4
b –304, –142, –2, 0, 1, 71, 126

12 **a** 0, –1, –2 **b** –2, 0, 2
c –5, –10, –15 **d** –44, –46, –48
e –79, –75, –71 **f** –101, –201, –301

13 **a** 4°C **b** 0°C **c** –2°C **d** –10°C

14 **a** –2 **b** 1 **c** –1 **d** –7

Exercise 3B

1 **a** 1 **b** –2 **c** –9
d –1 **e** –4 **f** –8

2 **a** right **b** right **c** left **d** left

3 **a** D **b** A **c** B **d** C

4 **a** 1 **b** 3 **c** 2 **d** 1
e –5 **f** –3 **g** 1 **h** 4
i –1 **j** –3 **k** –2 **l** –2
m –4 **n** –8 **o** –1 **p** 2

5 **a** –2 **b** –1 **c** –8 **d** –19
e –3 **f** –7 **g** –15 **h** –13
i –4 **j** –10 **k** –15 **l** –7
m –41 **n** –12 **o** –22 **p** –47

6 **a** 2 **b** –10 **c** –2 **d** 2
e 4 **f** –20 **g** –4 **h** 4

7 **a** 5 **b** –9 **c** 1 **d** –13
e 1 **f** –8 **g** –7 **h** –13
i 6 **j** –4 **k** –12 **l** 2

8 **a** 5 **b** 9 **c** 5 **d** 2
e 5 **f** 7 **g** 3 **h** 10
i 5 **j** 16 **k** –4 **l** –5
m –6 **n** –13 **o** –30 **p** –113

9 **a** \$145 **b** \$55 **c** \$5250

10 **a** 3°C **b** –3°C **c** –46°C

11 69°C

12 **a** 59 m **b** 56 m

13 Other combinations may be possible.
a –, + **b** +, –, –
c +, +, –, + **d** –, +, +, +, –

14 Answers may vary.
a 2 + 3 = 5 **b** not possible
c –2 + 3 = 1 **d** –4 + 2 = –2
e 7 – 5 = 2 **f** 2 – 4 = –2
g not possible **h** –3 – 4 = –7

Exercise 3C

1 **a** To add (–5), subtract 5.
b To add (–8), subtract 8.
c To add (–6), subtract 6.
d To add (–1), subtract 1.

2 **a** 3 **b** 3 **c** 2 **d** 2

3 **a** C **b** A **c** D **d** B

4 **a** 1 **b** 5 **c** 6 **d** 2
e –3 **f** –5 **g** –2 **h** –4
i –3 **j** –22 **k** –35 **l** –80
m –10 **n** –29 **o** –50 **p** –112
q –109 **r** –113

5 **a** 5 **b** 7 **c** –12 **d** –7
e –15 **f** –42

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6 **a** 4 **b** 9 **c** 0
d –8 **e** –7 **f** –10
g –1 **h** –17 **i** –33
7 **a** –3 **b** –10 **c** –4
d 4 **e** –1 **f** 4
g 5 **h** 5 **i** –1
j –2 **k** –2 **l** –14
8 $190 of debt
9 **a** **i** $8000
ii –$6000
b $2000
10 **a** no **b** no **c** yes
11 **a**

–2	0	5
8	1	–6
–3	2	4

b

–13	–11	–6
–3	–10	–17
–14	–9	–7

Exercise 3D

1 **a** 3 **b** 6
c 4 **d** 11
e 15 **f** 312
2 **a** 3, 5 **b** 6, –2 **c** –3, 7
d –11, –7 **e** –4, –2 **f** –1, –6
g –7, –11 **h** –12, 27
3 **a** D **b** A **c** B **d** C
4 **a** 5 **b** 11 **c** 50
d 6 **e** –4 **f** –3
g –5 **h** 10 **i** 2
j 1 **k** 0 **l** 40
m 28 **n** 34 **o** –22
5 **a** 8 **b** 11 **c** 3
d 4 **e** –5 **f** –5
6 **a** –2 **b** 1 **c** –2
d –6 **e** –7 **f** –14
g 6 **h** –3 **i** 1
7 **a** 0 **b** –5 **c** 8
d 12 **e** –9 **f** 5
g –6 **h** –91 **i** –15
j 6 **k** 17 **l** 11
8 **a** –1 **b** –5 **c** –4
d 4 **e** 2 **f** –24
g –6 **h** –5 **i** 2
j –6 **k** –9 **l** –6
9 –12 m
10 –$35 000
11 –$30
12 Answers will vary.
13 **a** +, + **b** +, – **c** –, –
d +, – **e** –, + **f** +, –

Exercise 3E

1 **a**

□	△	□ × △
3	5	15
2	5	10
1	5	5
0	5	0
–1	5	–5
–2	5	–10
–3	5	–15

b

□	△	□ × △
3	–5	–15
2	–5	–10
1	–5	–5
0	–5	0
–1	–5	5
–2	–5	10
–3	–5	15

2 **a** 15, 3 **b** –15, –3 **c** –15, –3 **d** 15, –3
3 **a** + **b** – **c** – **d** +
e + **f** – **g** + **h** –
4 **a** + **b** – **c** –
d – **e** – **f** +
5 **a** –20 **b** –54 **c** –40 **d** –99
e 6 **f** –42 **g** –72 **h** 99
i –40 **j** –64 **k** 35 **l** –32
m 60 **n** –44 **o** 9 **p** –60
6 **a** –5 **b** –2 **c** –4 **d** –30
e –2 **f** –3 **g** –3 **h** 3
i –2 **j** 8 **k** –9 **l** 5
m 11 **n** 1 **o** –8 **p** 8
7 **a** –20 **b** 21 **c** –12
d –23 **e** 50 **f** –9
g –15 **h** 39 **i** –18
8 **a** 25 **b** 36 **c** 49
d 64 **e** 81
9 **a** –3 **b** –5 **c** 7 **d** 6
e –3 **f** –72 **g** –252 **h** –30
10 negative
11 **a** ×, ÷ **b** ×, ÷ **c** ÷, × **d** ÷, ÷
12 –8 and 3
13 **a** $(-2 + 1) \times 3 = -3$
b $-10 \div (3 - (-2)) = -2$
c $-8 \div (-1 + 5) = -2$
d $(-1 - 4) \times (2 + (-3)) = 5$
e $(-4 + -2) \div (10 + (-7)) = -2$
f $20 + ((2 - 8) \times (-3)) = 38$

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Exercise 3E cont.

g $(1 - (-7) \times 3) \times 2 = 44$
h $(4 + -5 \div 5) \times (-2) = -6$

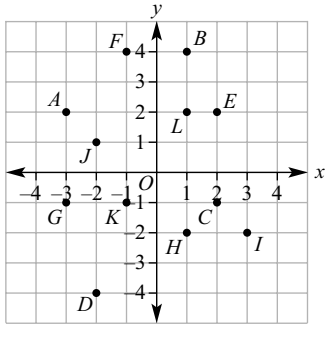
Exercise 3F

1 **a** right, up **b** right down
c left, up **d** left, down

2 **a** D **b** B
c A **d** C
e E **f** H
g F **h** G

3 $A(2, 1)$, $B(3, -2)$, $C(-1, -4)$, $D(-2, 2)$, $E(4, 3)$, $F(2, -3)$, $G(-3, -1)$, $H(-4, 4)$

4 **a, b**



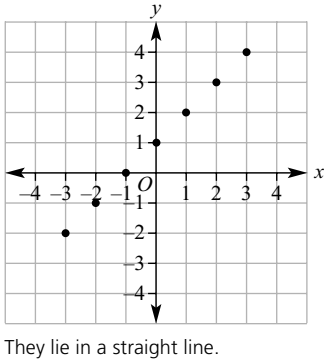
5 $A(3, 0)$, $B(0, -2)$, $C(-1, 0)$, $D(0, 4)$, $E(0, 2)$, $F(1, 0)$, $G(0, -4)$, $H(-3, 0)$

6 **a** 9 **b** 18 **c** 15 **d** 6
e 10 **f** 2 **g** 1

7 **a** triangle **b** rectangle **c** trapezium **d** kite

8 28 km

9 **a**



b They lie in a straight line.

10 $y = 2$

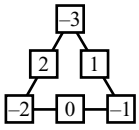
11 [☐☐] IS THE ORIGIN

Puzzles and games

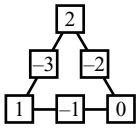
1 a house

2 **a** $-3 \boxtimes (4 \boxplus -2) = -6$
b $-2 \boxtimes 5 \boxtimes -1 \boxplus 11 = 21$ or
$-2 \boxtimes 5 \boxplus -1 \boxplus 11 = 21$
c $(1 \boxminus 30 \boxdiv -6) \boxdiv -2 = -3$ or
$1 \boxtimes 30 \boxdiv -6 \boxminus -2 = -3$

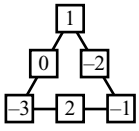
3 **a**



b



c



4 **a** −81, 243, −729 **b** 4, −2, 1
c −10, −15, −21 **d** −8, −13, −21

5 **a** 11 and −3 **b** 21 and −10

Multiple–choice questions

1 C **2** E **3** B **4** D **5** C
6 A **7** D **8** C **9** C **10** C

Short–answer questions

1 **a** −3, 0 **b** 1, −2 **c** −6, −2 **d** −1, −11

2 **a** < **b** < **c** > **d** <

3 **a** −5 **b** −2 **c** −15
d 1 **e** −1 **f** 2
g −7 **h** −25 **i** −11
j −7 **k** −47 **l** −131

4 **a** 1 **b** 3 **c** −3
d −7 **e** −2 **f** −5
g 12 **h** −18 **i** 7
j −17 **k** 3 **l** 3

5 **a** −6 **b** 5 **c** −11
d 5 **e** −6 **f** −6
g 5 **h** −29 **i** −6

6 **a** −1 **b** −9 **c** −1
d 2 **e** −21 **f** −2
g −87 **h** 30

7 **a** 4 **b** −5 **c** −3
d 7 **e** 30 **f** 10
g 1 **h** 40

8 $A(3, 0)$, $B(2, 3)$, $C(-1, 2)$, $D(-4, -2)$, $E(0, -3)$, $F(4, -4)$

Extended–response questions

1 **a** 16°C **b** −31°C **c** 8°C
d 19°C **e** 27°C

2 rocket

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Chapter 4

Pre-test

1 C

2 **a**  **b**  **c**  **d** 

3 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$
d $\frac{1}{10}$ **e** $\frac{3}{4}$

4 C

5 B

6 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{4}{5}$

7 **a** $2\frac{3}{4}$ **b** $1\frac{1}{2}$ **c** $9\frac{1}{2}$ **d** $5\frac{1}{4}$

8 one-quarter of a block

9 **a** $2, 2\frac{1}{2}, 3$ **b** $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ **c** $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}$ **d** $\frac{4}{6}, \frac{5}{6}, \frac{6}{6}$

10 **a** $3 \times \frac{1}{2}$ **b** $4 \times \frac{3}{4}$
c $\frac{5}{6} \times 1 = \frac{5}{6}$ **d** $\frac{6}{8} \div 1 = \frac{6}{8}$

11 **a** \$7.50 **b** \$40 **c** 75c **d** \$2

12 **a** true **b** true **c** true **d** false

Exercise 4A

1 **b** $12 \times 2 = 24$ or $2 \times 12 = 24$
c $24 \times 1 = 24$ or $1 \times 24 = 24$
$3 \times 8 = 24$ or $8 \times 3 = 24$
$4 \times 6 = 24$ or $6 \times 4 = 24$

2 **a** $1 \times 12 = 12$, $2 \times 6 = 12$, $3 \times 4 = 12$; 1, 2, 3, 4, 6, 12
b $1 \times 5 = 5$; 1, 5
c $1 \times 30 = 30$, $2 \times 15 = 30$, $3 \times 10 = 30$, $5 \times 6 = 30$
1, 2, 3, 5, 6, 10, 15, 30

3 **a** 5, 10, 15, 20, 25, 30
b 10, 20, 30, 40, 50, 60
c 7, 14, 21, 28, 35, 42

4 **a** 2, 4, 6, 8, 10, 12, 14, 16 **b** 2
c 2 **d** 1, 3, 5, 7, 9, 11, 13, 15

5 **a** factors **b** 2, 3, 4 **c** 12, 24, 30
d multiples **e** multiples

6 **a** 1, 2, 5, 10 **b** 1, 2, 3, 4, 6, 8, 12, 24
c 1, 17 **d** 1, 2, 3, 4, 6, 9, 12, 18, 36
e 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
f 1, 2, 3, 6, 7, 14, 21, 42
g 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
h 1, 2, 3, 4, 6, 12 **i** 1, 2, 4, 7, 14, 28

7 **a** 5, 10, 15, 20, 25, 30 **b** 8, 16, 24, 32, 40, 48
c 12, 24, 36, 48, 60, 72 **d** 7, 14, 21, 28, 35, 42
e 20, 40, 60, 80, 100, 120
f 75, 150, 225, 300, 375, 450
g 15, 30, 45, 60, 75, 90
h 100, 200, 300, 400, 500, 600
i 37, 74, 111, 148, 185, 222

8 **a** 3, 18 **b** 5 **c** 1, 2, 5, 10, 25, 50
d 1, 2, 4, 5, 10, 20, 25, 50, 100

9 **a** 22; 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
b 24; 5, 10, 15, 20, 25, 30
c 21; 11, 22, 33, 44, 55, 66, 77, 88, 99, 110
d 49; 12, 24, 36, 48, 60, 72, 84

10 **a** 4 **b** 6 **c** 16 **d** 12

11 **a** **i** 4, 8, 12, 16, 20, 24
ii 5, 10, 15, 20, 25, 30
b Zane 5 laps, Matt 4 laps **c** 20 minutes

12 **a** 13 **b** 2 **c** 11 **d** 1

13 **a** 12 pens (1 m × 1 m), 6 pens (2 m × 1 m), 4 pens (3 m × 1 m) or 3 pens (1 m × 4 m)
b **i** 32 pens (1 m × 1 m), 8 pens (2 m × 2 m) or 2 pens (4 m × 4 m)
ii 54 pens (1 m × 1 m) or 6 pens (3 m × 3 m)
c **i** 16 pens (1 m × 2 m), 8 pens (1 m × 4 m) or 4 pens (2 m × 4 m)
ii 27 pens (1 m × 2 m), 18 pens (1 m × 3 m) or 9 pens (2 m × 3 m)
d 25 pens (2 m × 3 m), 15 pens (2 m × 5 m), 10 pens (3 m × 5 m), 6 pens (5 m × 5 m), 5 pens (5 m × 6 m or 3 m × 10 m or 2 m × 15 m), 3 pens (2 m × 25 m or 5 m × 10 m), or 2 pens (3 m × 25 m or 5 m × 15 m)

Exercise 4B

1 **a** highest common factor
b factors
c common
d lowest common multiple
e multiples, common

2 **a** 1, 2, 4 **b** 4

3 Factors of 18 are 1, 2, 3, 6, 9 and 18.
Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.
HCF = 6

4 **a** 24, 48 **b** 24

5 Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81 and 90.
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105 and 120.
LCM = 45

6 **a** 1 **b** 1 **c** 2 **d** 3
e 4 **f** 15 **g** 10 **h** 3

7 **a** 20 **b** 21 **c** 30 **d** 40
e 12 **f** 10 **g** 30 **h** 60
i 42

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Exercise 4B cont.

8 **a** 7 black ants, 11 red ants **b** 77 mm

9 **a** 3 teaspoons, 2 spoons **b** 42 cm **c** LCM

10 **a** 100 cm, 200 cm, 300 cm **b** LCM = 100

11 **a** 6 roses per bunch **b** 6 = HCF

c 6 red, 7 pink, 5 cream bunches

12 **a** 8, 16 **b** 24, 32

13 **b** after 12 minutes

c Lilli 8 laps, Aliyah 6 laps, Ciara 4 laps

d 15 mins more for Aliyah, 45 mins more for Ciara

Exercise 4C

1 **a** 9 **b** 7

2 proper: b, e, f, g

improper: a, c, h, i, l, o, p

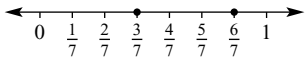
whole numbers: d, j, k, m, n

3

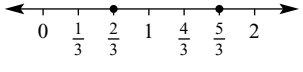
4	1	$\frac{1}{4}$	one-quarter
8	7	$\frac{7}{8}$	seven-eighths
3	2	$\frac{2}{3}$	two-thirds
12	5	$\frac{5}{12}$	five-twelfths
8	8	$\frac{8}{8}$	one whole

4 **a** $\frac{1}{2}, \frac{3}{2} = 1\frac{1}{2}, \frac{5}{2} = 2\frac{1}{2}$ **b** $\frac{1}{3}, \frac{2}{3}, \frac{4}{3} = 1\frac{1}{3}, \frac{7}{3} = 2\frac{1}{3}$

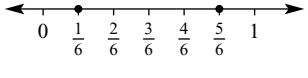
5 **a**



b



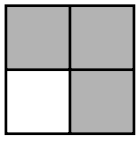
c



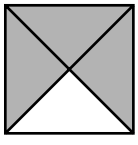
6 **a** 1 **b** 1 **c** 1 **d** 2

e 1 **f** 50 **g** 15 **h** 2

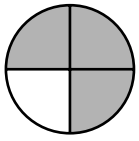
7 **a**



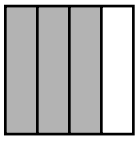
b



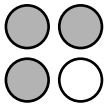
c



d



e



8 **a** $\frac{7}{5}, \frac{8}{5}, \frac{9}{5}$ **b** $\frac{9}{8}, \frac{10}{8}, \frac{11}{8}$

c $\frac{5}{3}, \frac{6}{3}, \frac{7}{3}$ **d** $\frac{7}{7}, \frac{6}{7}, \frac{5}{7}$

9 **a** $\bigcirc = 1\frac{1}{2}, \frac{3}{2}; \square = 3\frac{1}{2}, \frac{7}{2}; \triangle = 5, \frac{10}{2}$

b $\bigcirc = \frac{1}{5}; \square = \frac{4}{5}; \triangle = 2\frac{1}{5}, \frac{11}{5}$

c $\triangle = \frac{3}{7}; \square = 1\frac{4}{7}, \frac{11}{7}; \bigcirc = 2\frac{2}{7}, \frac{16}{7}$

d $\square = 3\frac{1}{3}, \frac{10}{3}; \triangle = 3\frac{2}{3}, \frac{11}{3}; \bigcirc = 4\frac{2}{3}, \frac{14}{3}$

10 **a** $\frac{6}{11}$ **b** $\frac{4}{8} = \frac{1}{2}$ **c** $\frac{7}{12}$

d $\frac{5}{6}$ **e** $\frac{3}{12} = \frac{1}{4}$ **f** $\frac{5}{9}$

11 **a** $\frac{12}{43}$ **b** $\frac{13}{15}$ **c** $\frac{11}{12}$

d $\frac{1}{12}$ **e** $\frac{2}{11}$ **f** $\frac{144}{475}$

g $\frac{7}{20}$ **h** $\frac{1}{4}$ **i** $\frac{3}{7}$

12 C

13 **a** $\frac{1}{10}$ **b** 180 mL, $\frac{9}{10}$ **c** 150 mL

Exercise 4D

1 **a** 6 **b** $\frac{9}{12}$ **c** $\frac{15}{20}$

d 10 **e** 7 **f** 3

2 $\frac{9}{15}, \frac{15}{25}, \frac{30}{50}, \frac{33}{55}$

3 **a** 2, 12, 10, 20, 300 **b** 1, 3, 24, 20, 40

4 **a** $\frac{1}{3}$ **b** 4 **c** 2

d 3 **e** 4 **f** $\frac{1}{2}$

5 **a** 9 **b** 50 **c** 33 **d** 56

e 8 **f** 2 **g** 12 **h** 39

i 35 **j** 200 **k** 105 **l** 2

6 **a** ≠ **b** = **c** ≠ **d** ≠

e = **f** = **g** = **h** =

i =

7 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{1}{2}$

e $\frac{3}{4}$ **f** $\frac{2}{3}$ **g** $\frac{1}{3}$ **h** $\frac{3}{5}$

8 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$ **d** $\frac{4}{11}$

e $\frac{2}{5}$ **f** $\frac{1}{11}$ **g** $\frac{1}{7}$ **h** $\frac{1}{3}$

i $\frac{7}{9}$ **j** $\frac{3}{5}$ **k** $\frac{1}{12}$ **l** $\frac{4}{5}$

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9 **a** $\frac{7}{14} = \frac{1}{2}$ **b** $\frac{12}{16} = \frac{3}{4}$ **c** $\frac{4}{42} = \frac{2}{21}$ **d** $\frac{7}{63} = \frac{1}{9}$

10 $\frac{4}{10}, \frac{16}{40}, \frac{2}{5}, \frac{80}{200}$

11 $\frac{4}{9}$

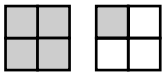
12 Jameel 4, Joanna 3, Jack 5

13 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$
d $\frac{7}{8}$ **e** $\frac{5}{8}$ **f** $\frac{19}{20}$

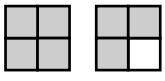
Exercise 4E

1 **a** 2 and 3 **b** 11 and 12 **c** 36 and 37

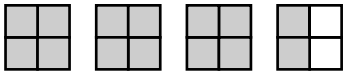
2 **a**



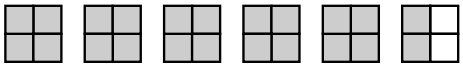
b



c



d



3 **a** 8 **b** 12 **c** 28 **d** 44
e 17 **f** 7 **g** 10 **h** 24

4 **a** $A\ 7\frac{1}{2}$, $B\ 10\frac{1}{2}$

b $C\ 1\frac{2}{3}$, $D\ 2\frac{1}{3}$, $E\ 4\frac{2}{3}$

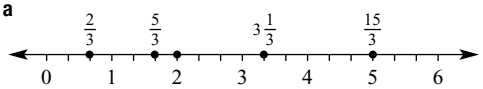
5 **a** $\frac{11}{5}$ **b** $\frac{8}{5}$ **c** $\frac{10}{3}$ **d** $\frac{17}{3}$
e $\frac{29}{7}$ **f** $\frac{24}{7}$ **g** $\frac{5}{2}$ **h** $\frac{13}{2}$
i $\frac{22}{5}$ **j** $\frac{23}{2}$ **k** $\frac{42}{5}$ **l** $\frac{103}{10}$
m $\frac{55}{9}$ **n** $\frac{25}{9}$ **o** $\frac{42}{8}$ **p** $\frac{21}{8}$
q $\frac{23}{12}$ **r** $\frac{38}{11}$ **s** $\frac{53}{12}$ **t** $\frac{115}{12}$

6 **a** $1\frac{2}{5}$ **b** $1\frac{1}{3}$ **c** $1\frac{2}{3}$ **d** $1\frac{3}{4}$
e $3\frac{2}{3}$ **f** $4\frac{1}{5}$ **g** $2\frac{2}{7}$ **h** $2\frac{1}{2}$
i $1\frac{5}{7}$ **j** $3\frac{1}{6}$ **k** $6\frac{2}{3}$ **l** $10\frac{1}{4}$
m $4\frac{3}{8}$ **n** $5\frac{1}{5}$ **o** $6\frac{6}{7}$ **p** $13\frac{2}{3}$
q $3\frac{1}{12}$ **r** $7\frac{4}{11}$ **s** $9\frac{3}{10}$ **t** $11\frac{1}{7}$

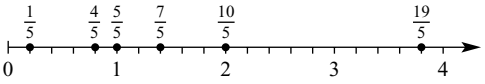
7 **a** **i** $1\frac{1}{2}$ **ii** $\frac{3}{2}$ **b** **i** $2\frac{1}{2}$ **ii** $\frac{5}{2}$
c **i** $2\frac{1}{3}$ **ii** $\frac{7}{3}$ **d** **i** $3\frac{2}{5}$ **ii** $\frac{17}{5}$

8 **a** $\frac{4}{3}, 1\frac{1}{3}$ **b** $\frac{3}{2}, 1\frac{1}{2}$ **c** $\frac{3}{2}, 1\frac{1}{2}$ **d** $\frac{5}{4}, 1\frac{1}{4}$

9 **a**

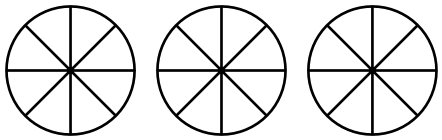


b



10 **a** $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}$ **b** $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$

11 **a**



b 15 **c** $1\frac{7}{8}$ **d** 9 **e** $1\frac{1}{8}$

12 $\frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{1}, \frac{2}{3}, \frac{3}{2}, \frac{1}{23}, \frac{1}{32}, \frac{2}{13}, \frac{2}{31}, \frac{3}{12}, \frac{3}{21}, \frac{12}{3}, \frac{21}{3}, \frac{13}{2}, \frac{31}{2}, \frac{23}{1}, \frac{32}{1}, 1\frac{2}{3}, 2\frac{1}{3}, 3\frac{1}{2}$

Exercise 4F

1 **a** $\frac{5}{7}$ **b** $\frac{7}{3}$ **c** $\frac{9}{11}$ **d** $\frac{8}{5}$

2 **a** 10 **b** 21 **c** 20 **d** 30
e 6 **f** 10 **g** 12 **h** 24
i 30 **j** 12 **k** 24 **l** 30

3 **a** 15 **b** 20 **c** 21 **d** 10
e 24 **f** 60 **g** 12 **h** 12

4 **a** 6 **b** 8 **c** 4 **d** 6
e 15 **f** 15 **g** 42 **h** 15

5 **a** > **b** > **c** < **d** <
e > **f** > **g** < **h** >

6 **a** < **b** > **c** < **d** <
e > **f** > **g** > **h** >

7 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$
c $\frac{4}{5}$ **d** $\frac{3}{2}$
e $\frac{3}{4}$ **f** $\frac{3}{5}$

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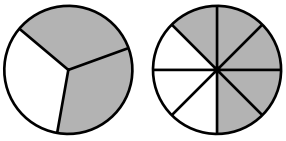
Exercise 4F cont.

8 **a** $\frac{3}{5}, 1\frac{2}{5}, \frac{8}{5}$ **b** $\frac{2}{9}, \frac{1}{3}, \frac{5}{9}$ **c** $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$

d $\frac{3}{5}, \frac{2}{3}, \frac{5}{6}$ **e** $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$ **f** $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$

9 $\frac{1}{4}, \frac{1}{8}, \frac{1}{11}$

10 **a**



b Serena **c** No, together they ate $1\frac{7}{24}$ pizzas.

11 **a** $\frac{5}{9}, \frac{6}{9}$ **b** $\frac{11}{4}, \frac{14}{4}$ **c** $\frac{2}{5}, \frac{1}{2}$

12 Answers will vary. Sample answers:

a $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{2}$

d $\frac{13}{20}$ **e** $\frac{1}{3}$ **f** $\frac{5}{21}$

13 **a** **i** < **ii** = **iii** > **iv** =

v > **vi** < **vii** < **viii** =

ix = **x** < **xi** < **xii** <

b $\frac{1}{2}, \frac{5}{9}, \frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3}, \frac{7}{10}, \frac{3}{4}, \frac{5}{6}$

Exercise 4G

1 **a** E **b** D **c** B

d C **e** A

2 **a** $\frac{2}{10}$ **b** $\frac{3}{100}$ **c** $\frac{7}{1000}$

3 **a** 5 **b** 6 **c** 7 **d** 37

4 **a** $\frac{6}{10}$ **b** $\frac{6}{100}$ **c** $\frac{6}{1000}$ **d** $\frac{6}{10}$

e $\frac{6}{10}$ **f** $\frac{6}{100}$ **g** $\frac{6}{100}$ **h** $\frac{6}{1000}$

5 **a** 0.3 **b** 0.8 **c** 0.15 **d** 0.23

e 0.9 **f** 0.12 **g** 0.121 **h** 0.174

i 0.1 **j** 0.11 **k** 0.111 **l** 0.03

6 **a** 6.4 **b** 5.7 **c** 212.3 **d** 1.16

e 14.83 **f** 7.51 **g** 5.07 **h** 18.612

7 **a** T **b** F **c** T **d** T

e F **f** T **g** T **h** T

8 **a** 6.1 **b** 9.6 **c** 0.8

d 28.5 **e** 0.171 **f** 0.203

9 **a** 3.05, 3.25, 3.52, 3.55 **b** 3.06, 3.6, 30.3, 30.6

c 1.718, 1.871, 11.87, 17.81

d 22.69, 22.96, 26.92, 29.26, 29.62

10 **a** Waugh, Border, Gilchrist, Taylor, Hughes

b first

11 **a** Day 6 **b** Day 4 **c** Day 6

12 **a** $7 \times 1 + 1 \times \frac{1}{10} + \frac{3}{100} + 5 \times \frac{1}{1000}$ **b** 1.563

13 **a** (0.1), (1.0) (2 ways)

b (0.12), 0.21, 1.02, 1.20, 2.01, 2.10, 10.2, 12.0, 20.1, (21.0) (10 ways)

c 0.123, 0.132, 0.213, 0.231, 0.312, 0.321, 1.023, 1.032, 1.203, 1.230, 1.302, 1.320, 2.013, 2.031, 2.103, 2.130, 2.301, 2.310, 3.012, 3.021, 3.102, 3.120, 3.201, 3.210, 10.23, 10.32, 12.03, 12.30, 13.02, 13.20, 20.13, 20.31, 21.03, 21.30, 23.01, 23.10, 30.12, 30.21, 31.02, 31.20, 32.01, 32.10, 102.3, 103.2, 120.3, 123.0, 130.2, 132.0, 201.3, 203.1, 210.3, 213.0, 230.1, 231.0, 301.2, 302.1, 310.2, 312.0, 320.1, 321.0 (60 ways)

Exercise 4H

1 **a** \$0.76 **b** \$10.57 **c** \$4.04 **d** \$4.80

2 **a** 5 **b** 9 **c** 1 **d** 4

e 0 **f** 9 **g** 6 **h** 5

3 **a** 1 **b** 7 **c** 4 **d** 8

4 **a** 8 **b** 7 **c** 5

5 **a** 14.8 **b** 7.4 **c** 15.6 **d** 0.9

e 6.9 **f** 9.9 **g** 55.6 **h** 8.0

i 0.7 **j** 0.7 **k** 0.7 **l** 0.9

6 **a** 3.78 **b** 11.86 **c** 5.92 **d** 0.93

e 123.46 **f** 300.05 **g** 3.13 **h** 9.85

i 56.29 **j** 7.12 **k** 29.99 **l** 0.90

7 **a** 15.9 **b** 7.89 **c** 236 **d** 1

e 231.9 **f** 9.4 **g** 9.40 **h** 34.713

8 **a** 24.0 **b** 14.90 **c** 7 **d** 30.000

9 **a** 28 **b** 9 **c** 12 **d** 124

e 22 **f** 118 **g** 3 **h** 11

10 **a** \$13 **b** \$31 **c** \$7 **d** \$1567

e \$120 **f** \$10 **g** \$1 **h** \$36

11 **a** 149.9×48

b **i** 7195 cents **ii** \$72

12 5, 6, 7, 8 or 9

13 0.35, 0.36, 0.37, 0.38, 0.39, 0.40, 0.41, 0.42, 0.43, 0.44

14 It depends on your calculator.

Exercise 4I

1 **a** 5 **b** 100 **c** 75, 7 **d** 5, 4

2 **a** 2 **b** 15, 20 **c** 10, 4 **d** 16

3 **a** 0.3 **b** $\frac{1}{2}$ **c** 0.8 **d** 1.5

e 0.9 **f** $\frac{1}{2}$

4 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{4}{5}$ **d** $\frac{11}{50}$

e $1\frac{11}{50}$ **f** $5\frac{1}{2}$ **g** $\frac{3}{20}$ **h** $\frac{99}{100}$

i $\frac{2}{25}$ **j** $\frac{1}{100}$ **k** $\frac{1}{1000}$ **l** $\frac{101}{500}$

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m $\frac{1}{2}$ **n** $6\frac{2}{5}$ **o** $10\frac{3}{20}$ **p** $18\frac{3}{25}$

q $3\frac{1}{4}$ **r** $\frac{1}{20}$ **s** $9\frac{3}{40}$ **t** $5\frac{24}{125}$

5 **a** 0.7 **b** 0.9 **c** 0.31

d 0.79 **e** 1.21 **f** 3.29

g 0.123 **h** 0.03 **i** 0.07

6 **a** $\frac{8}{10} = 0.8$ **b** $\frac{5}{10} = 0.5$

c $\frac{35}{100} = 0.35$ **d** $\frac{46}{100} = 0.46$

e $5\frac{95}{100} = 5.95$ **f** $3\frac{25}{100} = 3.25$

g $\frac{25}{10} = 2.5$ **h** $\frac{375}{1000} = 0.375$

i $\frac{28}{100} = 0.28$

7 **a** 0.5 **b** 0.5 **c** 0.75

d 0.4 **e** $0.\dot{3}$ **f** 0.375

g $0.41\dot{6}$ **h** $0.\overline{428571}$ **i** $0.\dot{2}$

8 **a** 0, 0.5, 1 **b** 0, $0.\dot{3}$, $0.\dot{6}$, 1

c 0, 0.25, 0.5, 0.75, 1

d 0, 0.2, 0.4, 0.6, 0.8, 1.0

9 $\frac{1}{4}$, 0.4, $\frac{1}{2}$, $\frac{5}{8}$, 0.75, 0.99

10

$0.01	$0.05	10c	20c	25c	50c	75c	90c	$0.99
$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{9}{10}$	$\frac{99}{100}$

11 **a** 0.5, $0.\dot{3}$, 0.25, 0.2, $0.1\dot{6}$, $0.\overline{142857}$, 0.125, $0.\dot{1}$, 0.1

b They get smaller.

c Check with your teacher.

12 **a** $\frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{4}{5}, \frac{9}{10}$

b They get larger.

c Check with your teacher.

13 **a** $\frac{6}{10}$, 62%, 65%, $\frac{66}{100}$, $0.\dot{6} = \frac{2}{3}$, 0.67, $\frac{7}{10}$

b $\frac{3}{5}$, 0.52, 45%, 0.43, $\frac{2}{5} = 0.4$ $\frac{9}{25} = 36\%$, $0.\dot{3} = \frac{1}{3}, \frac{6}{20}$

14 **a** See table below.

b Answers will vary.

c 3, 6, 7, 9, 11, 12

d 13, 14, 15, 17, 18, 19

halves	$\frac{1}{2} = 0.5$	$\frac{2}{2} = 1.0$	$\frac{3}{2} = 1.5$	$\frac{4}{2} = 2.0$	$\frac{5}{2} = 2.5$	$\frac{6}{2} = 3.0$
thirds	$\frac{1}{3} = 0.\dot{3}$	$\frac{2}{3} = 0.\dot{6}$	$\frac{3}{3} = 1.0$	$\frac{4}{3} = 1.\dot{3}$	$\frac{5}{3} = 1.\dot{6}$	$\frac{6}{3} = 2.0$
quarters	$\frac{1}{4} = 0.25$	$\frac{2}{4} = 0.5$	$\frac{3}{4} = 0.75$	$\frac{4}{4} = 1.0$	$\frac{5}{4} = 1.25$	$\frac{6}{4} = 1.5$
fifths	$\frac{1}{5} = 0.2$	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$	$\frac{4}{5} = 0.8$	$\frac{5}{5} = 1.0$	$\frac{6}{5} = 1.2$
sixths	$\frac{1}{6} = 1.\dot{6}$	$\frac{2}{6} = 0.\dot{3}$	$\frac{3}{6} = 0.5$	$\frac{4}{6} = 0.\dot{6}$	$\frac{5}{6} = 0.8\dot{3}$	$\frac{6}{6} = 1.0$
sevenths	$\frac{1}{7} = 0.14$	$\frac{2}{7} = 0.29$	$\frac{3}{7} = 0.43$	$\frac{4}{7} = 0.57$	$\frac{5}{7} = 0.71$	$\frac{6}{7} = 0.86$
eighths	$\frac{1}{8} = 0.125$	$\frac{2}{8} = 0.25$	$\frac{3}{8} = 0.375$	$\frac{4}{8} = 0.5$	$\frac{5}{8} = 0.625$	$\frac{6}{8} = 0.75$
ninths	$\frac{1}{9} = 0.\dot{1}$	$\frac{2}{9} = 0.\dot{2}$	$\frac{3}{9} = 0.\dot{3}$	$\frac{4}{9} = 0.\dot{4}$	$\frac{5}{9} = 0.\dot{5}$	$\frac{6}{9} = 0.\dot{6}$
tenths	$\frac{1}{10} = 0.1$	$\frac{2}{10} = 0.2$	$\frac{3}{10} = 0.3$	$\frac{4}{10} = 0.4$	$\frac{5}{10} = 0.5$	$\frac{6}{10} = 0.6$
elevenths	$\frac{1}{11} = 0.\overline{09}$	$\frac{2}{11} = 0.0\overline{18}$	$\frac{3}{11} = 0.\overline{27}$	$\frac{4}{11} = 0.\overline{36}$	$\frac{5}{11} = 0.\overline{45}$	$\frac{6}{11} = 0.\overline{54}$
twelfths	$\frac{1}{12} = 0.08\dot{3}$	$\frac{2}{12} = 0.1\dot{6}$	$\frac{3}{12} = 0.25$	$\frac{4}{12} = 0.\dot{3}$	$\frac{5}{12} = 0.41\dot{6}$	$\frac{6}{12} = 0.5$

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Exercise 4J

1 C **2** C **3** C

4 **a** $\frac{3}{4} = 0.75 = 75\,\%$ **b** $\frac{9}{10} = 0.9 = 90\,\%$

c $\frac{2}{5} = 0.4 = 40\,\%$ **d** $\frac{4}{5} = 0.8 = 80\,\%$

5 **a** 150 **b** 350 **c** 75 **d** 175

6 **a** $\frac{3}{10} = 0.3 = 30\,\%$ **b** $\frac{3}{5} = 0.6 = 60\,\%$

c $\frac{5}{4} = 1.25 = 125\,\%$

7 **a** **i** $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ **ii** $\frac{1}{3}, \frac{2}{5}, \frac{2}{6}, \frac{3}{8}, \frac{3}{9}, \frac{4}{9}, \frac{3}{10}, \frac{4}{10}, \frac{4}{12}, \frac{5}{12}$

b $\frac{4}{9}$

8 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

d 1 **e** $\frac{1}{3}$ **f** $\frac{4}{3} = 1\frac{1}{3}$

9 **a** $\frac{3}{4}$ **b** $\frac{4}{5}$ **c** $\frac{5}{8}$

d $\frac{1}{4}$ **e** 0 **f** $\frac{1}{8}$

10 **a** 0.9 **b** 0.3 **c** 0.75

d 0.35 **e** 0.5 **f** 0.9

11 **a** 30% **b** 70%

12 **a** three-quarters **b** two-thirds

c one-quarter **d** one-eighth

e three-eighths

13 **a** 90% **b** 60%

14 **a** This is true because 0.5 is one-half and so is 50%.

b This is false because one-third is $\frac{33}{99}$, not $\frac{33}{100}$.

c This is false because $\frac{1}{5} = \frac{2}{10} = \frac{20}{100} = 20\%$, not 15%.

d This is true because $\frac{9}{10} = \frac{90}{100} = 90\%$.

e This is true because $\frac{1}{2} = 50\%$ and $\frac{1}{4} = 25\%$, so $\frac{1}{8} = 12.5\%$, which is close to 12%.

f This is true because $\frac{1}{3} = 33\frac{1}{3}\%$ so $\frac{2}{3} = 66\frac{2}{3}\%$, which is greater than 66%.

15 **a** $\frac{1}{4} + \frac{1}{4}, \frac{2}{6} + \frac{1}{6}, \frac{2}{8} + \frac{2}{8}, \frac{1}{8} + \frac{3}{8}, \frac{1}{10} + \frac{4}{10}, \frac{2}{10} + \frac{3}{10}$, $\frac{1}{12} + \frac{5}{12}, \frac{2}{12} + \frac{4}{12}, \frac{3}{12} + \frac{3}{12}$

b $\frac{1}{3} + \frac{1}{6}, \frac{1}{4} + \frac{2}{8}, \frac{1}{5} + \frac{3}{10}, \frac{1}{3} + \frac{2}{12}, \frac{3}{9} + \frac{1}{6}, \frac{3}{12} + \frac{2}{8}$, $\frac{2}{6} + \frac{2}{12}, \frac{1}{6} + \frac{4}{12}, \frac{3}{9} + \frac{2}{12}$ etc.

c $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}, \frac{1}{8} + \frac{1}{8} + \frac{2}{8}, \frac{1}{10} + \frac{1}{10} + \frac{3}{10}, \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$, $\frac{2}{12} + \frac{2}{12} + \frac{2}{12}, \frac{1}{12} + \frac{2}{12} + \frac{3}{12}, \frac{1}{12} + \frac{1}{12} + \frac{4}{12}$

d $\frac{1}{4} + \frac{1}{6} + \frac{1}{12}$

e $\frac{2}{5}$

Exercise 4K

1 **a** 95% **b** 60% **c** 75% **d** 26%

2 C

3 A

4 **a** 50 **b** 50%

c **i** 5 **ii** 100 **iii** 20 **iv** 1

d **i** 50% **ii** 50% **iii** 50% **iv** 50%

5 **a** 100 **b** 35 **c** out of **d** 15

6 **a** 0.32 **b** 0.27 **c** 0.68 **d** 0.54

e 0.1 **f** 0.12 **g** 0.18 **h** 0.85

i 0.92 **j** 0.75 **k** 0.11 **l** 0.6

m 0.06 **n** 0.09 **o** 1 **p** 0.01

q 2.18 **r** 1.42 **s** 0.75 **t** 1.99

7 **a** 0.225 **b** 0.175 **c** 0.3333 **d** 0.0825

e 1.1235 **f** 1.888 **g** 1.50 **h** 5.20

i 0.0079 **j** 0.000 25 **k** 0.0104 **l** 0.0095

8 **a** 80% **b** 30% **c** 45% **d** 71%

e 41.6% **f** 37.5% **g** 250% **h** 231.4%

i 2.5% **j** 0.14% **k** 1270% **l** 100.4%

9 35%

10 0.52

11 **a** **i** 50% **ii** 0.5

b **i** 25% **ii** 0.25

c **i** 90% **ii** 0.9

d **i** 10% **ii** 0.1

e **i** 100% **ii** 1

12 **a** 100% is all questions correct.

b 20 out of 40, half the answers correct.

c No questions answered correctly.

13 **a** F ÷ A × 100

b F: points scored for the team; A: points scored against the team

c 100%

Exercise 4L

1 **a** 70, 70 **b** 48, 48 **c** 60, 60

d 20, 20 **e** 40, 40 **f** 63, 63

2 **a** $\frac{1}{4} = 25\%, \frac{2}{4} = 50\%, \frac{3}{4} = 75\%, \frac{4}{4} = 100\%$

b $\frac{1}{5} = 20\%, \frac{2}{5} = 40\%, \frac{3}{5} = 60\%, \frac{4}{5} = 80\%, \frac{5}{5} = 100\%$

c $\frac{1}{3} = 33\frac{1}{3}\%, \frac{2}{3} = 66\frac{2}{3}\%, \frac{3}{3} = 100\%$

d $\frac{1}{10} = 10\%, \frac{2}{10} = 20\%, \frac{3}{10} = 30\%, \frac{4}{10} = 40\%$

3 Zoe scored full marks; i.e. $\frac{25}{25}$.

4 **a** 86% **b** 20%

5 **a** $\frac{11}{100}$ **b** $\frac{71}{100}$ **c** $\frac{43}{100}$ **d** $\frac{49}{100}$

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e $\frac{1}{4}$ **f** $\frac{3}{10}$ **g** $\frac{3}{20}$ **h** $\frac{22}{25}$

i $\frac{7}{100}$ **j** $\frac{19}{100}$ **k** $\frac{21}{100}$ **l** $\frac{1}{2}$

m $\frac{7}{10}$ **n** $\frac{9}{10}$ **o** $\frac{99}{100}$ **p** $\frac{11}{20}$

q 1.25 **r** 1.2

6 **a** $1\frac{1}{5}$ **b** $1\frac{4}{5}$ **c** $2\frac{37}{100}$ **d** $4\frac{1}{100}$

e $1\frac{3}{4}$ **f** $1\frac{1}{10}$ **g** $3\frac{4}{25}$ **h** $8\frac{2}{5}$

i 2 **j** $2\frac{1}{20}$ **k** $3\frac{1}{10}$ **l** $3\frac{1}{2}$

7 **a** 98% **b** 9% **c** 79% **d** 56%

e 8% **f** 15% **g** 97% **h** 50%

i 35% **j** 32% **k** 86% **l** 90%

m 112% **n** 135% **o** 400% **p** 160%

8 **a** $12\frac{1}{2}\%$ **b** $33\frac{1}{3}\%$ **c** $26\frac{2}{3}\%$ **d** $83\frac{1}{3}\%$

e 115% **f** 420% **g** 290% **h** $32\frac{1}{2}\%$

9 $12\frac{1}{2}\%$

10 **a** $\frac{3}{4}$ **b** 75% **c** $\frac{1}{4}$ **d** 25%

11 70%

12 70%, 80%

13 Answers will vary.

Exercise 4M

1 **a** $\boxed{10}$ **b** $\boxed{5}$ **c** $20 \div 2 = 10$

2 **a** 6 **b** 35 **c** 16

3 **a** $\$500 \div 10 = \$50$ **b** $\$900 \div 100 = \$9$

c $84 \text{ kg} \div 4 = 21 \text{ kg}$

d 7 days ÷ 2 = 3 days, 12 hours

e $84 \text{ kg} \div 4 \times 3 = 63 \text{ kg}$

4 8 hours

5 **a** 400 **b** 80 **c** 40

d 200 **e** 8 **f** 120

6 **a** 70 **b** 36 **c** 10 **d** 27

e 10 **f** 7 **g** 150 **h** 200

i 4 **j** 60 **k** 44 **l** 950

m 40 **n** 80 **o** 90 **p** 210

7 **a** 96 **b** 600 **c** 66 **d** 100

e 15 **f** 72 **g** 73 **h** 600

8 10% of \$200 = \$20 5% of \$500 = \$25

20% of \$120 = \$24 30% of \$310 = \$93

10% of \$80 = \$8 10% of \$160 = \$16

50% of \$60 = \$30 1% of \$6000 = \$60

20% of \$200 = \$40 50% of \$88 = \$44

9 **a** \$36 **b** 24 mm **c** 9 kg

d 90 tonnes **e** 8 min **f** 150 cm

g 4 g **h** 5 hectares

10 12 students

11 35 marks

12 240 students

13 **a** 120 **b** 420 **c** 660

14 **a** They are the same. **b** $37\frac{1}{2}\%$

c **i** \$140 **ii** \$1.50

Exercise 4N

1 **a** $\frac{3}{10}$ **b** $\frac{7}{10}$ **c** 30% **d** 70%

e 3 : 7 **f** 7 : 3

2 **a** 25% **b** 20% **c** 48% **d** 99%

e 17% **f** 34% **g** 35% **h** 90%

3 **a** $\frac{8}{200}$ **b** $\frac{1}{25}$ **c** 4%

4 **a** 4 **b** 4 **c** $\frac{1}{2}$ **d** 50%

e $\frac{1}{2}$ **f** 50% **g** 4 : 4 or 1 : 1

5 **a** 10 **b** $\frac{1}{5}$ **c** $\frac{4}{5}$ **d** 20%

e 80% **f** 2 : 8 or 1 : 4

6 **a** $\frac{3}{10}$, 30% **b** $\frac{3}{5}$, 60% **c** $\frac{1}{5}$, 20% **d** $\frac{3}{4}$, 75%

e $\frac{1}{20}$, 5% **f** $\frac{1}{2}$, 50% **g** $\frac{1}{4}$, 25% **h** $\frac{3}{20}$, 15%

7 **a** $\frac{3}{5}$, 60% **b** $\frac{1}{2}$, 50% **c** $\frac{1}{4}$, 25%

d $\frac{2}{5}$, 40% **e** $\frac{3}{4}$, 75% **f** $\frac{4}{5}$, 80%

8 **a** $\frac{1}{10}$ **b** 10%

9 **a** $\frac{1}{5}$ **b** 20% **c** $\frac{4}{5}$ **d** 80%

10 **a** $\frac{80}{1600} = \frac{1}{20}$ **b** 5% **c** 95%

11 **a** 16 megalitres **b** $\frac{16}{20} = \frac{4}{5}$ **c** 80%

12 **a** $\frac{1}{25}$, 4% **b** $\frac{2}{5}$, 40% **c** $\frac{1}{20}$, 5%

d $\frac{1}{20}$, 5% **e** $\frac{1}{4}$, 25% **f** $\frac{3}{4}$, 75%

13 $\frac{45}{50} = 90\%$ $\frac{72}{100} = 72\%$ Ross scored the higher result.

14 **a** Answers will vary.

b 58%, 63%, 77%, 81%, 75%

c Answers will vary.

d Answers will vary.

Puzzles and games

1 DELHI AGRA AND JAIPUR

2 **a** $\frac{1}{9}$ **b** $\frac{9}{1}$ **c** 14 **d** $\frac{8}{9}$

e $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ **f** $\frac{3}{4}, \frac{6}{8}$ **g** 16 **h** eight ways

i three ways **j** 9

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Multiple-choice questions

1 C **2** B **3** D **4** E
5 D **6** C **7** E **8** C
9 D **10** B **11** A **12** C

Short-answer questions

1 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{14}$
e $\frac{1}{10}$ **f** $\frac{1}{8}$ **g** $\frac{1}{5}$ **h** $\frac{1}{3}$

2 $\frac{2}{3}$

3 **a** $\frac{3}{5}$ **b** $\frac{2}{7}$ **c** $\frac{3}{4}$

4 **a** $1\frac{3}{4}$ **b** $1\frac{3}{7}$ **c** $1\frac{1}{3}$ **d** $1\frac{1}{2}$

5 **a** $\frac{2}{7} \leqslant \frac{4}{7}$ **b** $\frac{3}{8} \geqslant \frac{1}{8}$
c $\frac{2}{3} \geqslant \frac{3}{5}$ **d** $3\frac{1}{9} \leqslant \frac{29}{9}$

6 **a** $\frac{5}{7}$ **b** $\frac{5}{8}$

7 **a** 10 **b** 21 **c** 24

8 **a** 10 **b** 21 **c** 24

9 **a** $1\frac{1}{4}, \frac{4}{4}, \frac{3}{4}, \frac{1}{4}$ **b** $\frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}$ **c** $\frac{4}{3}, \frac{5}{6}, \frac{2}{3}, \frac{1}{12}$

10 **a** false **b** false **c** true
d true **e** false **f** true
g true **h** false **i** true

11 **a** \$62.88 **b** \$63 **c** \$62.90

12 **a** 12.7 **b** 8.4 **c** 9.4
d 7.5 **e** 0.1 **f** 7.1

13 **a** 12.81 **b** 423.46 **c** 15.89
d 7.25 **e** 6.67 **f** 3.33

14 A = 25%, B = 50%, C = 28%, D = $\frac{1}{10}$, E = $\frac{3}{4}$, F = $1\frac{1}{2}$

15 **a** \$20 **b** \$210 **c** 45 g **d** \$30

16 **a** $\frac{3}{5}$, 60% **b** $\frac{1}{5}$, 20% **c** $\frac{1}{4}$, 25% **d** $\frac{1}{10}$, 10%

Extended-response question

1 **a** 300 **b** 75% **c** 450
d 45 **e** 1080

Chapter 5

Pre-test

1 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $\frac{1}{4}$

2 **a** 6 **b** 2 **c** 26 **d** 5
e 52 **f** 4

3 C, A, B, D

4 **a** $\frac{1}{6}$ **b** $\frac{3}{6}$ or $\frac{1}{2}$ **c** $\frac{4}{6}$ or $\frac{2}{3}$ **d** 0

5 **a** 5 **b** 3 **c** $\frac{3}{5}$

6

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{10}$	0.1	10%
$\frac{1}{100}$	0.01	1%

Exercise 5A

1 **a** unlikely **b** likely **c** likely **d** unlikely

2 **a** D **b** A **c** B **d** C

3 **a** true **b** false **c** true
d false **e** true **f** false

4 **a** certain **b** even chance **c** unlikely
d impossible

5 **a** D **b** C **c** A **d** B

6 Answers will vary.

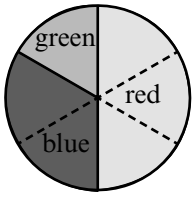
7 **a** **i** true **ii** false **iii** false **iv** true
b **i** C **ii** B **iii** A **iv** D

8 **a** spinner 3 **b** spinner 2 **c** spinner 1

9 Answers will vary.

10 **a** $\frac{1}{4}$ **b** 25% **c** $\frac{1}{3}$ **d** $33\frac{1}{3}\%$

e Hint: Start with 6 equal pieces.



Exercise 5B

1 **a** sample space **b** 0
c certain **d** more
e impossible **f** less

2 **a** 0.5 **b** 50%

3 **a** {1, 2, 3, 4, 5, 6} **b** {1, 3, 5}
c $P(\text{odd}) = \frac{1}{2}$ **d** $P(5) = \frac{1}{6}$

4 **a** C **b** A **c** D **d** B

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5 **a** 3: red, green, blue **b** $\frac{1}{3}$ **c** $\frac{1}{3}$
d $\frac{2}{3}$ **e** 0

6 **a** {1, 2, 3, 4, 5, 6, 7} **b** $\frac{1}{7}$ **c** 0
d $\frac{2}{7}$ **e** $\frac{3}{7}$ **f** $\frac{4}{7}$
g Number chosen is less than 8; other solutions possible.

7 **a** {M, A, T, H, S} **b** 0.2
c 0.2 **d** 0.8

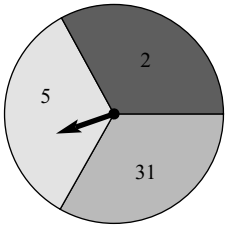
8 **a** $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{5}{11}$
d $\frac{9}{11}$ **e** $\frac{4}{11}$
f Choosing a letter in the word TRY; other solutions possible.

9 D

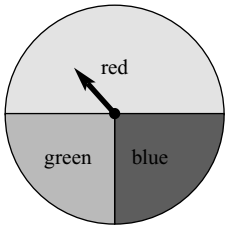
10 **a** 30% **b** 50% **c** 80%

11 **a** yes, $\frac{1}{20}$ **b** 20 **c** 120

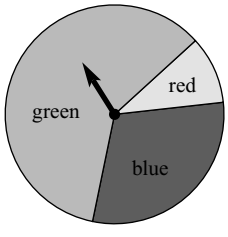
12 **a** {2, 3, 4, 5, 6, 7, 8, 9} **b** 0.5
c **i** 0.375 **ii** 0.375 **iii** 0
d Possible spinner shown:



13 **a** $\frac{1}{6}$
b **i**



ii Cannot be done because adds to more than 1.
iii Cannot be done because adds to less than 1.
iv



Exercise 5C

1 C

2 **a** 4 **b** 6 **c** 10

3 **a** $\frac{1}{10}$ **b** $\frac{2}{5}$ **c** $\frac{3}{10}$

4 **a** $\frac{53}{100}$ **b** $\frac{47}{100}$ **c** no

5 **a** $\frac{13}{20}$ **b** $\frac{1}{4}$ **c** $\frac{1}{10}$

6 **a** 500 **b** 1750 **c** **i** 7 tails **ii** more

7 **a** 100 **b** 300
c yes (but this is very unlikely) **d** from 2 rolls

8 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{100}$ **iii** $\frac{31}{100}$ **b** 60

9 **a** 5 **b** 35 **c** 70
d $\frac{26}{35}$ **e** $\frac{1}{2}$ **f** 126

10 **a** 2 red, 3 green and 5 blue
b **i** yes **ii** yes **iii** yes **iv** yes **v** no

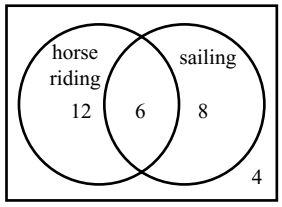
11 **a** C **b** D **c** B **d** A

12 Answers will vary.

Exercise 5D

1 **a** 8 **b** 13 **c** 3
d 18 **e** 4 **f** 22

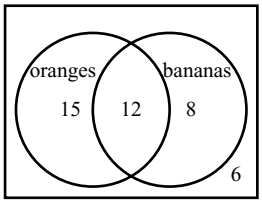
2 **a**



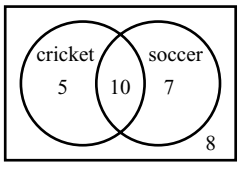
b Total = 12 + 6 + 8 + 4 = 30 students

3 **a** 2 **b** 4 **c** 1 **d** 3

4



5 **a** 8 **b**



c 15 **d** 22

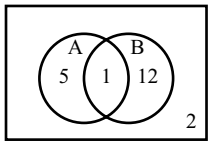
6 **a** 14 + 8 + 18 + 10 = 50 **b** 32 **c** 10

7 **a** 4 **b** 24 **c** 16

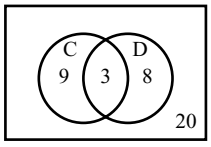
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Exercise 5D cont.

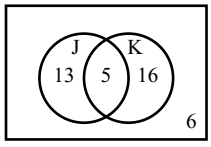
8 a



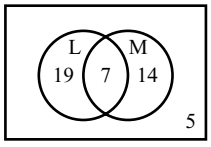
b



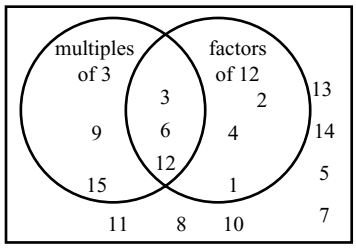
c



d



9 a Whole numbers 1 to 15

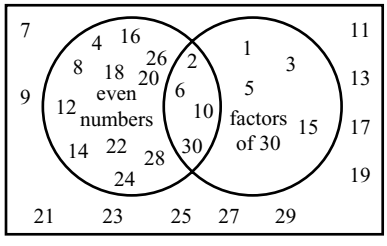


b 3 **c** 3

10 a 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

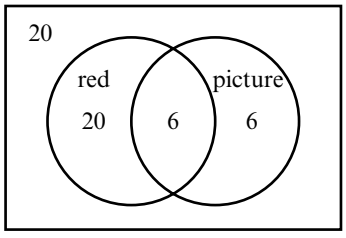
b 1, 2, 3, 5, 6, 10, 15, 30

c Whole numbers 1 to 30



d 4 **e** 15

11



Exercise 5E

1 a

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	30
Total	40	35	75

b 30 **c** 20 **d** 75

2

	Like lamingtons	Dislike lamingtons	Total
Like Anzacs	23	14	37
Dislike Anzacs	12	3	15
Total	35	17	52

3 a 27 **b** 15 **c** 12

d 25 **e** 10 **f** 40

4

	Like surfing	Dislike surfing	Total
Like hiking	45	10	55
Dislike hiking	25	5	30
Total	70	15	85

5 a

	Employed	Unemployed	Total
TAFE	16	3	19
No TAFE	12	2	14
Total	28	5	33

b 2 **c** 28 **d** 16 **e** 31

6 a 26 **b** 12 **c** 11

7 a

	B	Not B	Total
A	20	50	70
Not A	20	10	30
Total	40	60	100

b

	B	Not B	Total
A	6	5	11
Not A	4	3	7
Total	10	8	18

8 a

	Sports	Not Sports	Total
Automatic	2	13	15
Not Automatic	8	17	25
Total	10	30	40

b 8 **c** 25

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9 a

	Like volleyball	Dislike volleyball	Total
Like tennis	12	6	18
Dislike tennis	11	4	15
Total	23	10	33

b 18 **c** $\frac{12}{18} = \frac{2}{3}$

10

	Like reading	Don't like reading	Total
Like exercise	33	8	41
Don't like exercise	7	3	10
Like computer games	24	12	36
Don't like computer games	11	2	13
Total	75	25	100

Puzzles and games

1 Answers will vary.
2 6 coins
3 a BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG
b 16
4 $\frac{2}{3}$
5 19 more ways (21 in total)
6 12 blue, 8 green, 4 red

Multiple-choice questions

1 E **2** C **3** C **4** B **5** B
6 D **7** C **8** C **9** C **10** C

Short-answer questions

1 a 1 **b** $\frac{1}{8}$ **c** $\frac{19}{20}$
d $\frac{3}{4}$ **e** 0

2 a {1, 2, 3, 4, 5, 6} **b** {heads, tails}
c {D, E, S, I, G, N} **d** {blue, yellow, green}

3 a 9
b i $\frac{5}{9}$ **ii** $\frac{1}{3}$ **iii** 0 **iv** $\frac{1}{3}$ **v** $\frac{4}{9}$

4 a $\frac{1}{26}$ **b** $\frac{2}{13}$ **c** $\frac{1}{52}$ **d** $\frac{3}{13}$

5 a 42% **b** 50%

6 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** 25 **d** 250

Extended-response questions

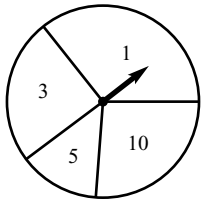
1 a 50 **b** 25 **c** 24

2 a

	Uses public transport	Does not use public transport	Total
Owns a car	20	80	100
Does not own a car	65	35	100
Total	85	115	200

b 200 **c** 100 **d** 20

3 a Sector 1 because it has the most occurrences.
b 3 and 10, as they have the closest outcomes.
c $\frac{3}{8}$
d



Chapter 6

Pre-test

1 a 0.1 **b** 0.3 **c** 1.7
d 0.01 **e** 0.001 **f** 4.7
2 a 0.5 **b** 0.25 **c** 0.75
3 a \$0.70 **b** \$0.85 **c** \$1.00
d \$0.05 **e** \$1.05 **f** \$0.03
4 a 50 **b** 25 **c** 75 **d** 250
5 a \$0.90 **b** \$10.50 **c** \$22.50 **d** \$0.81
6 10c
7 \$15.50
8 a \$85 **b** \$0.10 **c** \$2.70
d \$0.70 **e** \$11.20 **f** \$0.24
9 a \$7 **b** \$2.90 **c** \$3.95
d \$29.75 **e** \$7.50
10 a \$87.50 **b** \$92.60 **c** \$20.90
11 \$38.55
12 a 523 **b** 839 **c** 312 **d** 1237

Exercise 6A

1 C
2 B
3 a 8.57 **b** 5.179 **c** 15.956
4 a 0.571 + 1.209 + 3.528 **b** 1.97 − 0.43 **c** 12.40 − 8.35
5 a 6.8 **b** 14.96 **c** 3.87 **d** 8.99
e 27.97 **f** 25.94 **g** 247.4 **h** 58.31

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Exercise 6A cont.

6 **a** 2.6 **b** 5.8 **c** 0.9 **d** 0.99
e 8.47 **f** 15.507 **g** 16.06 **h** 21.33

7 **a** \$7 **b** \$5.60 **c** \$14.49
d \$8.24 **e** \$145 **f** \$7.50

8 **a** \$2.50 **b** \$3.75 **c** \$8.60
d \$2.50 **e** \$15.80 **f** \$17.50

9 **a** 0.79 **b** 0.516 **c** 0.4 **d** 0.73
e 12.1 **f** 114.13 **g** 6.33 **h** 70.79

10 **a** 0.5 **b** 3.2 **c** 21.2 **d** 0.48
e 12.3 **f** 131.4 **g** 22.23 **h** 13.457
i 43.27 **j** 4947.341

11 16.189

12 **a** \$45.70 **b** \$54.30

13 49.4 mm

14 3.3°C

15 \$55.09

16 **a**

0.6	0.7	0.2
0.1	0.5	0.9
0.8	0.3	0.4

Magic number = 1.5

b

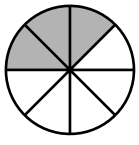
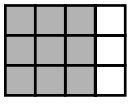
1.6	0.5	0.9	0.4
0.3	1.0	0.6	1.5
0.2	1.1	0.7	1.4
1.3	0.8	1.2	0.1

Magic number = 3.4

Exercise 6B

1 **a** denominator
b denominator, numerators
c denominators, lowest common denominator
d check, simplified

2 **a** two- **b** -quarters **c** four-
d -fifths **e** one-, one-

3 **a**  **b** 
c  **d** 

4 **a** T **b** F **c** T **d** T
e F **f** T **g** T **h** F

5 **a** $\frac{5}{8}$ **b** $\frac{5}{7}$ **c** $\frac{4}{5}$ **d** $\frac{9}{11}$
e $\frac{7}{8}$ **f** $\frac{7}{12}$ **g** $\frac{7}{15}$ **h** $\frac{5}{9}$
i $\frac{4}{5}$ **j** $\frac{6}{7}$ **k** $\frac{7}{10}$ **l** $\frac{81}{100}$

6 **a** $1\frac{2}{7}$ **b** $1\frac{3}{10}$ **c** $1\frac{4}{5}$ **d** $1\frac{4}{19}$
e $1\frac{1}{10}$ **f** $1\frac{2}{5}$ **g** $1\frac{3}{7}$ **h** $1\frac{4}{11}$
i $1\frac{1}{100}$

7 **a** $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **b** $\frac{4}{8}+\frac{3}{8}=\frac{7}{8}$
c $\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$ **d** $\frac{6}{24}+\frac{4}{24}=\frac{10}{24}$

8 **a** $\frac{3}{4}$ **b** $\frac{14}{15}$ **c** $\frac{2}{3}$ **d** $\frac{7}{12}$
e $\frac{13}{20}$ **f** $\frac{19}{20}$ **g** $\frac{13}{21}$ **h** $\frac{23}{40}$
i $1\frac{13}{30}$ **j** $1\frac{9}{28}$ **k** $1\frac{13}{33}$ **l** $1\frac{5}{12}$

9 **a** $3\frac{4}{5}$ **b** $7\frac{3}{7}$ **c** $12\frac{3}{4}$ **d** $5\frac{5}{9}$
e $10\frac{1}{3}$ **f** $21\frac{1}{6}$ **g** $19\frac{3}{11}$ **h** $12\frac{2}{5}$

10 **a** $4\frac{5}{12}$ **b** $7\frac{7}{30}$ **c** $12\frac{1}{6}$ **d** $13\frac{9}{28}$
e $15\frac{1}{10}$ **f** $19\frac{1}{9}$ **g** $25\frac{21}{44}$ **h** $15\frac{5}{24}$

11 **a** $\frac{3}{5}$ **b** $\frac{2}{5}$ **c** \$4

12 **a** $\frac{7}{12}$ **b** $\frac{5}{12}$

13 **a** $\frac{17}{24}$ **b** $\frac{7}{24}$ **c** 7 hours

14 **a** $1\frac{3}{20}, 9\frac{5}{6}$ **b** $1\frac{97}{210}, 14\frac{3}{4}$

Exercise 6C

1 **a** denominators **b** multiply **c** simplify
d twelve **e** fifteen

2 **a** one- **b** two- **c** -fifths
d three- **e** -eighths

3 **a**  **b**  **c** 

4 **a** 1 **b** 3 **c** 4, 3, 1 **d** 12, 10, 2

5 **a** $\frac{2}{7}$ **b** $\frac{3}{11}$ **c** $\frac{7}{18}$ **d** $\frac{1}{3}$
e 0 **f** $\frac{4}{9}$ **g** $\frac{3}{19}$ **h** $\frac{8}{23}$
i $\frac{31}{100}$ **j** $\frac{12}{25}$ **k** $\frac{16}{25}$ **l** $\frac{2}{5}$

6 **a** $\frac{5}{12}$ **b** $\frac{1}{10}$ **c** $\frac{1}{10}$ **d** $\frac{9}{28}$
e $\frac{1}{6}$ **f** $\frac{23}{36}$ **g** $\frac{13}{33}$ **h** $\frac{2}{15}$
i $\frac{1}{8}$ **j** $\frac{3}{20}$ **k** $\frac{1}{36}$ **l** $\frac{1}{9}$

7 **a** $1\frac{3}{5}$ **b** $8\frac{3}{7}$ **c** $1\frac{1}{7}$ **d** $3\frac{2}{9}$
e $2\frac{5}{12}$ **f** $3\frac{5}{28}$

8 **a** $2\frac{2}{3}$ **b** $4\frac{3}{5}$ **c** $4\frac{2}{3}$ **d** $4\frac{8}{9}$
e $4\frac{2}{3}$ **f** $\frac{37}{45}$

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9 $\frac{3}{5}$

10 $\frac{1}{6}$

11 $\frac{3}{4}$ kg, 750 g

12 a $\frac{9}{4} - \frac{5}{3} = \frac{27}{12} - \frac{20}{12} = \frac{7}{12}$

b $\frac{13}{5} - \frac{3}{2} = \frac{26}{10} - \frac{15}{10} = \frac{11}{10} = 1\frac{1}{10}$

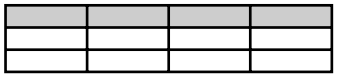
13 Answers may vary.

Exercise 6D

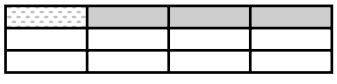
1 a 0, 1 **b** 1

c whole number; proper fraction

2 a

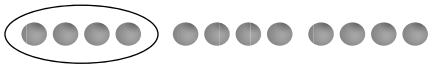


b

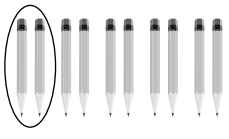


c $\frac{1}{12}$

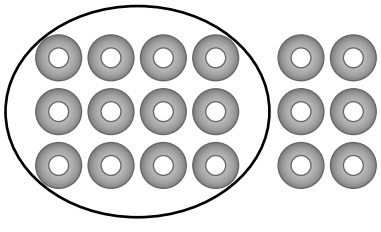
3 a



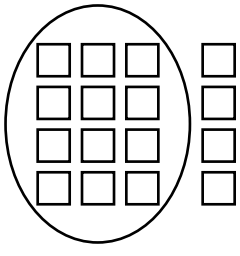
b



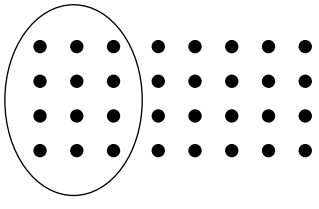
c



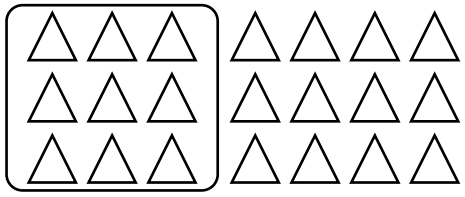
d



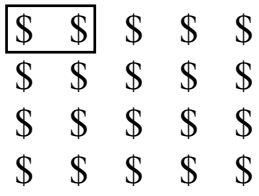
e



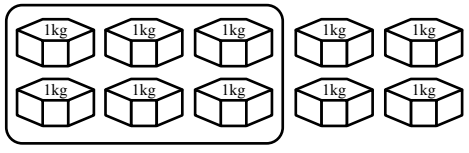
f



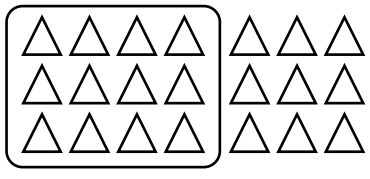
g



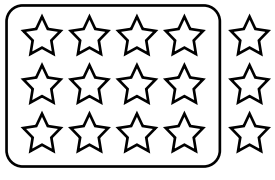
h



i



j



4 a 8 **b** 21 **c** 12
d 8 **e** 42 **f** 30

5 a 12 **b** 9 **c** 16 **d** 15
e 12 **f** 4 **g** 80 **h** 33
i 2 **j** 20 **k** 40 **l** 20
m 40 **n** 12 **o** 15 **p** 15

6 a $\frac{3}{20}$ **b** $\frac{2}{21}$ **c** $\frac{10}{21}$ **d** $\frac{8}{45}$
e $\frac{2}{5}$ **f** $\frac{1}{7}$ **g** $\frac{1}{4}$ **h** $\frac{5}{11}$
i $\frac{5}{22}$ **j** $\frac{1}{3}$ **k** $\frac{6}{11}$ **l** $\frac{4}{11}$
m $\frac{6}{35}$ **n** $\frac{3}{10}$ **o** $\frac{2}{7}$ **p** $\frac{1}{6}$

7 a $5\frac{5}{6}$ **b** $1\frac{31}{35}$ **c** $3\frac{3}{10}$
d $4\frac{7}{8}$ **e** $5\frac{1}{3}$ **f** 7
g 6 **h** $3\frac{1}{3}$ **i** $\frac{4}{5}$
j $\frac{4}{15}$ **k** $\frac{3}{8}$ **l** $\frac{5}{4} = 1\frac{1}{4}$

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Exercise 6D cont.

8 **a** $3\frac{11}{15}$ **b** $1\frac{25}{63}$ **c** $7\frac{4}{5}$ **d** 24
e $3\frac{1}{5}$ **f** $\frac{3}{16}$ **g** 4 **h** 33
i $\frac{2}{7}$ **j** $3\frac{3}{8}$ **k** $\frac{4}{15}$ **l** 6

9 **a** $\frac{3}{5}$ **b** 48 boys, 72 girls

10 7 games

11 **a** 45 minutes **b** 45
c $60 \div 3 \times 2 = 40$

12 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$

13 Answers may vary.

Exercise 6E

1 **a** 1 **b** 2 **c** 4
d 2 **e** 1

2 **a** 00 **b** 000 **c** 00 **d** 00000

3 **a** 000 **b** 00 **c** 0 **d** 00000

4 **a** right 2 places **b** left 1 place
c right 6 places **d** no change
e left 3 places **f** right 3 places
g right 1 place **h** left 7 places

5 **a** 48.7 **b** 352.83 **c** 4222.7 **d** 1430.4
e 5699.23 **f** 125.963 **g** 12 700 **h** 154 230
i 3400 **j** 2132 **k** 86 710 000 **l** 516 000

6 **a** 4.27 **b** 35.31 **c** 2.4422 **d** 56.893
e 12.13518 **f** 9.32611 **g** 0.029
h 0.001362 **i** 0.00054 **j** 0.367
k 0.000002 **l** 0.0100004

7 **a** 2291.3 **b** 31.67 **c** 0.49
d 0.222 **e** 63 489 000 **f** 0.0010032

8 **a** 15 600 **b** 43 000 **c** 22 510 **d** 16 000
e 213 400 **f** 2 134 000 **g** 7000
h 9 900 000 **i** 34 0000 **j** 15.6 **k** 1.56
l 0.156 **m** 8.7 **n** 0.87 **o** 0.087
p 0.016 **q** 0.007 **r** 0.0034

9 \$137

10 3000 cents, \$30

11 **a** \$21 200 **b** \$21 400

12 **a** 10 **b** 10 **c** 100 **d** 10, 10 **e** 1000

13 72

Exercise 6F

1 **a** 1 **b** 3 **c** 2 **d** 4
e 1 **f** 2 **g** 2 **h** 5

2 **a** 1 **b** 2 **c** 3
d 3 **e** 5 **f** 2
g 4 **h** 3 **i** 9

3 **a** 19.2 **b** 1.92 **c** 0.192
d 1.52 **e** 19.46 **f** 0.0756

4 in the question; decimal places

5 **a** 4.8 **b** 16.8 **c** 7.5 **d** 29
e 19.6 **f** 1.96 **g** 2.4 **h** 0.24
i 0.56 **j** 0.27 **k** 0.74 **l** 0.81

6 **a** 1.128 **b** 5.427 **c** 3.556 **d** 0.74
e 2.34 **f** 8.12

7 **a** 100.8 **b** 218.46 **c** 15.516 **d** 23.12
e 12.42 **f** 5.44 **g** 311.112
h 0.000966

8 **a** \$31.50, \$32 **b** \$22.65, \$23
c \$74.80, \$75 **d** \$2.82, \$3
e \$2.10, \$2 **f** \$11.79, \$12

9 \$57.74

10 29.47 m

11 3.56 kg

12 **a** 38.76 **b** 73.6 **c** 0.75 **d** 42, 0.42
e 0.042

13 **a** \$13.10
b \$10 note, \$2, \$1 coins and a 10c piece (Other answers are possible.)

Exercise 6G

1 **a** × **b** 4 **c** divide
d reciprocal **e** flip **f** improper

2 A

3 **a** $\frac{5}{11} \times \frac{5}{3}$ **b** $\frac{1}{3} \times \frac{5}{1}$ **c** $\frac{7}{10} \times \frac{17}{12}$ **d** $\frac{8}{3} \times \frac{1}{3}$
e $\frac{7}{10} \times \frac{2}{1}$ **f** $\frac{3}{2} \times \frac{4}{1}$ **g** $\frac{3}{5} \times \frac{10}{1}$ **h** $\frac{3}{2} \times \frac{3}{7}$

4 **a** less **b** more **c** more
d more **e** less **f** less

5 **a** $\frac{7}{5}$ **b** $\frac{5}{3}$ **c** $\frac{9}{2}$ **d** $\frac{8}{1}$
e $\frac{3}{1}$ **f** $\frac{10}{1}$ **g** $\frac{10}{3}$ **h** $\frac{4}{5}$
i $\frac{1}{12}$ **j** $\frac{1}{101}$ **k** 9 **l** 1

6 **a** $\frac{2}{3}$ **b** $\frac{5}{6}$ **c** $\frac{2}{5}$ **d** $\frac{5}{7}$
e $\frac{4}{11}$ **f** $\frac{3}{7}$ **g** $\frac{5}{23}$ **h** $\frac{6}{11}$

7 **a** $\frac{3}{8}$ **b** $\frac{5}{33}$ **c** $\frac{2}{5}$ **d** $\frac{5}{7}$
e $\frac{3}{4}$ **f** $1\frac{1}{3}$ **g** $1\frac{3}{5}$ **h** $\frac{3}{14}$

8 **a** 20 **b** 21 **c** 100 **d** 120
e 30 **f** 40 **g** 4 **h** $6\frac{2}{3}$

9 **a** $\frac{5}{7}$ **b** $\frac{4}{5}$ **c** $\frac{11}{14}$
d $\frac{3}{4}$ **e** $1\frac{11}{16}$ **f** $1\frac{3}{11}$
g $3\frac{1}{3}$ **h** $\frac{3}{4}$ **i** $1\frac{1}{2}$

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10 $\frac{3}{4}$

11 120 km/h

12 $\frac{1}{4}$

13 FRACTIONS MAKE ME FEEL WHOLE

Exercise 6H

1 B

2 **a** 10 **b** 100 **c** 1000 **d** 10
e 100 **f** 100

3 **a** $12 \div 3$ **b** $18 \div 2$ **c** $152 \div 1$
d $5642 \div 2$ **e** $38 \div 1$ **f** $380 \div 1$
g $484.6 \div 2$ **h** $0.7 \div 2$ **i** $72 \div 9$

4 **a** 32.456, 3 **b** 12 043.2, 12
c 34.5, 1 **d** 1 234 120, 4

5 **a** 4.2 **b** 6.1 **c** 21.34
d 0.7055 **e** 1.571 **f** 0.308
g 19.393 **h** 372.9 **i** 0.0024
j 117.105 **k** 0.6834 **l** 0.0025625
m 0.39 **n** 0.37 **o** 0.175
p 8.95 **q** 9.36 **r** 105.1

6 **a** 30.7 **b** 77.5 **c** 26.8
d 8.5 **e** 44.4 **f** 645.3
g 0.08 **h** 0.050425 **i** 980
j 800.6 **k** 0.79 **l** 2 161 000

7 **a** 7.5 **b** 75 **c** 750
d 7500 **e** 75 000 **f** 750 000

8 **a** 11.83 kg **b** \$30.46 **c** 304.33 m
d 239.17 g **e** 965.05 L **f** \$581.72

9 8

10 **a** 26.67, so 26 can be filled **b** 40

11 \$1.59 per L

12 **a** 4 **b** 6 **c** 27
d 39 **e** 76 **f** 5000

13 **a** 5 **b** 50 **c** 500 **d** 5000

Puzzles and games

1 **a** 7, 8 **b** 6, 5, 1, 4 **c** 0, 1, 0, 2 **d** 7, 5, 1, 6, 1

2 **a** 8, 9 **b** 9 **c** 5, 5, 1 **d** 7, 0, 7

3 A MUSHROOM

4 Answers will vary.

5 0.3 m, 0.42 m, 0.66 m, 0.78 m, 0.90 m, 1.02 m, 1.14 m, 1.38 m

Multiple-choice questions

1 D **2** B **3** C **4** E **5** A
6 D **7** D **8** B **9** C **10** D

Short-answer questions

1 **a** 1 **b** 1.2 **c** 1
d 1.25 **e** 3 **f** 2.75

2 **a** 0.5 **b** 0.8 **c** 0.75
d 0.9 **e** 1.6 **f** 1

3 **a** \$2.50 **b** \$1.75 **c** \$7.45

4 **a** $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $1\frac{5}{24}$
d $5\frac{23}{30}$ **e** $\frac{1}{2}$ **f** $2\frac{1}{2}$

5 **a** 7 **b** 80 **c** 12
d 5 **e** $\frac{1}{6}$ **f** $7\frac{1}{2}$

6 **a** $\frac{4}{3}$ **b** $\frac{12}{7}$ **c** $\frac{4}{11}$ **d** $\frac{1}{8}$

7 **a** $\frac{1}{5}$ **b** 20 **c** 4 **d** 2

8 **a** 1.6 **b** 1.56 **c** 19.594 **d** 9.6
e 21.9 **f** 3.3 **g** 45.94 **h** 43.5
i \$7.76 **j** \$7 **k** \$24.80

9 **a** T **b** F **c** T **d** T
e F **f** F **g** T **h** T
i T **j** F **k** F **l** T

10 **a** 19.2 **b** 63.99 **c** 19.32
d 0.95 **e** 1.52 **f** 6
g 16 **h** 3 **i** 34.2

Extended-response question

1 **a** Jessica \$12.57; Jaczinda \$13.31; hence, Jaczinda earns higher pay rate by 74c per hour.
b \$12.49, \$12.50 to the nearest 5 cents
c \$36.90
d **i** \$1.40
ii \$3.20
e **i** \$17.50
ii \$3.50
iii \$700

Semester review 1

Computation with positive integers

Multiple-choice questions

1 E
2 B
3 D
4 C
5 A

Short-answer questions

1 **a** 4 **b** 1 **c** 303

2 **a** 7324 **b** 12 092

3 **a** 4962 **b** 819 **c** 147
d 7600 **e** 105 **f** 137

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4 **a** false **b** true **c** true
5 22
6 **a** 10 **b** 5 **c** 17
d 30 **e** 56 **f** 48
g 62 **h** 16 **i** 42
7 **a** false **b** true **c** false
d true **e** true **f** true
8 **a** $(2 + 3) \times 4 = 10$ **b** $(10 - 2) \div 8 = 1$
c $4 \times (6 - 2) \div 8 = 2$
9 **a** 10 **b** 40 **c** 140
10 **a** 100 **b** 100 **c** 1500

Extended-response question

1 **a** 28 **b** \$700 **c** \$1000 **d** 12 h

Angle relationships

Multiple-choice questions

1 A
2 B
3 B
4 B
5 D

Short-answer questions

1 **a** acute **b** right **c** obtuse
d straight **e** reflex **f** revolution
2 **a** 30° **b** 80° **c** 150°
3 25°
4 78°
5 **a** $a = 140$ **b** $a = 50$ **c** $a = 140$
d $a = 65$ **e** $a = 62$ **f** $a = 56$
6 $a = 100$, $b = 80$, $c = 100$, $d = 80$, $e = 100$, $f = 80$, $g = 100$
7 Because the alternate angles are not equal.
8 **a** 115 **b** 71 **c** 100

Extended-response question

1 **a** **i** $x = 56$
ii $y = 95$
iii $z = 29$
b $x + y + z = 180$

Computation with positive and negative integers

Multiple-choice questions

1 C
2 A
3 B
4 E
5 D

Short-answer questions

1 **a** < **b** > **c** <
2 **a** −1 **b** −5 **c** −10
d −6 **e** −18 **f** −77
3 **a** 1 **b** 4 **c** −2
d −5 **e** −12 **f** −57
4 **a** 5 **b** 7 **c** 24
d −1 **e** −6 **f** 2
5 **a** −15 **b** −3 **c** −8
6 **a** −10 **b** 10 **c** −20
d 16 **e** 32 **f** 24

Extended-response question

1 **a** $A(2, 0)$, $B(4, 0)$, $C(0, 2)$, $D(2, 2)$, $E(2, -3)$, $F(-1, -2)$, $G(-3, 0)$, $H(-3, -2)$, $I(-1, 2)$
b A, B, O and G; all lie on the x-axis.
c **i** 2 units **ii** 5 units
d trapezium **e** DECIDE

Understanding fractions, decimals and percentages

Multiple-choice questions

1 B **2** C **3** E
4 D **5** A

Short-answer questions

1 $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$
2 $\frac{17}{3}$
3 **a** true **b** false **c** false
d true **e** false **f** true
4 $\frac{3}{20}$
5 \$120
6 \$80
7 **a** true **b** true **c** true **d** true
8 $\frac{2}{3}$

Extended-response question

1 **a** 6 **b** $\frac{8}{9}$ **c** 9
d second dose on Sunday week

Probability

Multiple-choice questions

1 A
2 C
3 D
4 D
5 B

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Short-answer questions

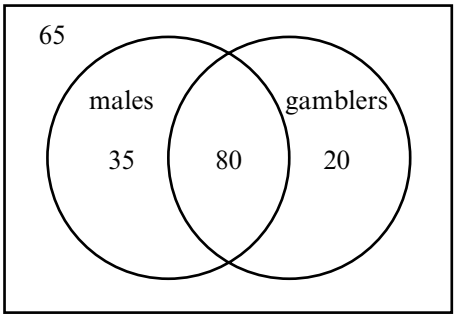
1 **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$
d $\frac{1}{2}$ **e** $\frac{1}{2}$

2 **a** A **b** B **c** $\frac{1}{3}$

3 **a** {1, 2, 3, 4, 5, 6} **b** $\frac{1}{6}$ **c** $\frac{1}{2}$

4 **a** 80, 35, 115, 85, 200 **b** 115 **c** 200

d



5 **a** 2% **b** 42% **c** 58%
d 2% **e** 62%

Extended-response question

1 **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$
d $\frac{1}{52}$ **e** $\frac{2}{13}$ **f** $\frac{1}{13}$
g $\frac{4}{13}$ **h** $\frac{4}{13}$ **i** $\frac{48}{52} = \frac{12}{13}$

Computation with decimals and fractions

Multiple-choice questions

1 B **2** C **3** D
4 A **5** A

Short-answer questions

1 **a** $\frac{1}{2}$ **b** $1\frac{1}{2}$ **c** $\frac{3}{4}$
2 **a** 0.1 **b** 1.1 **c** 1
3 **a** \$10.50 **b** \$11.75 **c** \$23.50
4 **a** 4.07 **b** 269.33 **c** 19.01
d 0.24 **e** 0.09 **f** 60
5 **a** 0.833 **b** 2.4 **c** 0.042
6 **a** 4.5387 **b** 45.387 **c** 0.045387
7 **a** 36 490 **b** 0.018 **c** 3886
8 **a** $\frac{11}{12}$ **b** $2\frac{2}{3}$ **c** $6\frac{1}{4}$
d $\frac{1}{5}$ **e** 4 **f** $\frac{1}{2}$

Extended-response question

1 **a** \$5.83 **b** \$5.85 **c** \$4.15

Chapter 7

Pre-test

1 **a** 24 **b** 60 **c** 60
d 7 **e** 12 **f** 365
2 **a** Friday **b** Monday **c** Wednesday **d** Sunday
3 **a** 5 p.m. **b** 1:45 a.m. **c** 4:37 a.m. **d** 8:49 p.m.
e 4 p.m. **f** 3:45 p.m. **g** 10:36 a.m. **h** 2:14 p.m.
4 **a** 1 min **b** 2 h **c** 7 weeks **d** 360 min
5 **a** 3 h 55 min **b** 235 min
6 **a** 3:00 **b** 2:30 **c** 12:00
d 8:50 **e** 11:55 **f** 5:45

Exercise 7A

1 **a** February
b April, June, September, November
c January, March, May, July, August, October, December

2 **a** F **b** D **c** A
d E **e** B **f** C
3 **a** multiply **b** divide **c** divide **d** multiply
4 **a** 120 s **b** 3 min **c** 2 h **d** 240 min
e 72 h **f** 2 days **g** 5 weeks **h** 280 days
5 **a** 180 min **b** 630 s **c** 4 min
d 1.5 h **e** 144 h **f** 3 days
g 168 h **h** 1440 min **i** 4 h
j 2 weeks **k** 20 160 min **l** 86 400 s
m 210 min **n** 15 s **o** 1.5 days
p 4.5 h **q** 1.25 min **r** 2 h
6 **a** 1130 h **b** 1330 h **c** 1200 h
d 1330 h **e** 2330 h **f** 2359 h
7 **a** 12:15 a.m. **b** 9:30 a.m. **c** 11:47 a.m.
d 2:30 p.m. **e** 7:45 p.m. **f** 11:30 p.m.
8 **a** 2 h 30 min **b** 4 h 15 min **c** 1 h 20 min
d 6 h 30 min **e** 3 h 45 min **f** 9 h 15 min
9 **a** 3.2 h **b** $3.8\dot{3}$ h **c** 3.75 h
10 **a** 7 h 7 min 12 s **b** 2 h 16 min 48 s
c 3 h 3 min **d** 8 h 55 min 48 s
11 **a** 1230 **b** 2025 **c** 0127
12 52.14 weeks
13 Answers will vary.
14 **a** 4 weeks is 28 days. Most months have 30 or 31 days.
b 6 months is half of a year, which is approximately 26 weeks.
15 **a** 3600 **b** 1440 **c** 3600 **d** 1440
16 Answers will vary.

Exercise 7B

1 **a** false **b** true **c** true
d false **e** true
2 **a** 6 h 30 min **b** 10 h 45 min
c 16 h 20 min **d** 4 h 30 min

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Exercise 7B cont.

3 **a** 4 h **b** 6 h **c** 3 h 30 min
d 8 h 45 min **e** 5 h 15 min **f** 7 h 30 min
4 **a** 2 h 5 min **b** 2 h 10 min
c 4 h 5 min **d** 2 h 11 min
e 1 h 50 min **f** 3 h 40 min
g 47 min **h** 2 h 54 min
i 2 h 46 min **j** 3 h 46 min
5 **a** 6:30 p.m. **b** 9 a.m. **c** 6:30 p.m.
d 4:30 p.m. **e** 5:30 p.m. **f** 11:40 a.m.
6 **a** 1 h 5 min **b** 40 min
c 3 h 15 min **d** 4 h 30 min
7 **a** 5 h 45 min **b** 8 h
c 9 h 6 min **d** 4 h 30 min
e 3 h 50 min 28 s **f** 13 h 20 min 30 s
8 42 min 41 s
9 **a** **i** 16 min **ii** 27 min **iii** 24 min **iv** 40 min
b afternoon **c** 40 min
10 Add 3200 to the current year.
11 17 min 28 s
12 7 h 28 min
13 23 h 15 min
14 **a** $900 **b** $90 **c** $1.50 **d** 2.5c

Exercise 7C

1 SA, Vic., Tas., NSW, ACT
2 **a** **i** 10 **ii** 9.5 **iii** 8 **iv** 7
v 8 **vi** 2
b **i** 0 **ii** 3 **iii** 5 **iv** 5
3 Sunday
4 **a** 10 a.m. **b** 9:30 a.m. **c** 10 a.m.
d 9:30 a.m. **e** 8 a.m. **f** 10 a.m.
5 **a** 6 p.m. **b** 6:30 p.m. **c** 6:30 p.m.
6 **a** 11 a.m. **b** 12 noon **c** 8 p.m. **d** 7:30 p.m.
e 7 a.m. **f** 5 a.m. **g** 1 a.m. **h** 10 a.m.
7 **a** 5:30 a.m. **b** 6:30 a.m. **c** 6:30 a.m.
d 1:30 p.m. **e** 2:30 p.m. **f** 2:30 a.m.
g 3 p.m. **h** 5:30 p.m. **i** 1:30 p.m.
8 **a** 6 h **b** 2.5 h **c** 8 h **d** 6 h
9 midnight
10 7:35 p.m.
11 6:30 a.m.
12 **a** 8 a.m. 29 March
b 10 p.m. 28 March
c 3 a.m. 29 March
13 **a** 8:30 a.m. **b** 9:30 a.m.
c 9 a.m. **d** 6:30 a.m.
14 turn back 1 hour
15 11:30 p.m. 25 October
16 **a** SA is ahead of Qld in daylight saving time. Qld does not use daylight saving time.
b Broken Hill uses Central Standard Time, as per SA.
c Lord Howe Island is UTC + 10:30 or 11:00 during daylight saving.

Puzzles and challenges

1 12 noon **2** Wednesday **3** 3 min 10 s

Multiple-choice questions

1 A **2** E **3** E **4** D **5** B
6 C **7** A **8** D **9** E **10** C

Short-answer questions

1 **a** 90 min **b** 2 min **c** 2 days
d 21 days **e** 1440 min **f** 0.5 h
2 **a** 0400 **b** 1530 **c** 1919
d 6:35 a.m. **e** 12:51 p.m. **f** 11:28 p.m.
3 **a** 3 h 30 min **b** 4 h 20 min
c 6 h 15 min **d** 1 h 45 min
4 **a** 3 h 36′ **b** 6 h 55′ 12″ **c** 11 h 26′ 24″
5 1 h 33 min
6 **a** 5 h 53 min **b** 4 h 5 min 55 s
c 1 h 59 min 1 s **d** 3 h 38 min
7 **a** **i** 25 min **ii** 34 min **iii** 20 min **iv** 44 min
b morning **c** 45 min
8 **a** **i** 8:15 a.m. **ii** 6:15 a.m.
b **i** 1:36 p.m. **ii** 3:36 p.m.
9 1:40 p.m.
10 **a** 30 **b** 31 **c** 30
d 31 **e** 31 **f** 30

Extended-response questions

1 **a** **i** 9 p.m. **ii** 2 p.m. **iii** 4 p.m.
b **i** 7:30 a.m. **ii** 1:30 p.m. **iii** 1:30 a.m.
c **i** Tuesday **ii** Monday **iii** Monday
d 1 p.m.
2 **a** 28 min
b 7:05 a.m., 7:13 a.m., 7:18 a.m., 7:53 a.m., 8:08 a.m.
c **i** 6:48 a.m.
ii 6:35 a.m., 6:43 a.m., 6:48 a.m., 7:23 a.m., 7:38 a.m.

Chapter 8

Pre-test

1 **a** 9 **b** 7 **c** 48 **d** 4
2 **a** 13 **b** 21 **c** 4 **d** 5
3 **a** 11 **b** 5 **c** 5 **d** 21
4 **a** 8 **b** 36 **c** 40 **d** 10
5 **a** 17 **b** 8 **c** 5 **d** 20
6 **a** 20 **b** 44 **c** 22 **d** 100
7 **a** 20 **b** 6 **c** 2 **d** 6
8 **a** 1 **b** 9 **c** 3 **d** 45
9 **a** 17 **b** 14 **c** 11 **d** 8
e 7 **f** 18 **g** 35 **h** 17
10 **a** 32 cm **b** 24 cm **c** 48 mm

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Exercise 8A

1 B **2** C

3 **a** $2x$, $7y$ **b** $3a$, $3c$, e **c** $5q$, $3r$, $2s$ **d** $7d$, $5f$, 17

4 **a** 3 **b** 13 **c** 4

5 **a** **i** 2 **ii** 17
b **i** 3 **ii** 15
c **i** 3 **ii** 21
d **i** 4 **ii** 2
e **i** 2 **ii** 1
f **i** 4 **ii** 12

6 **a** $x + 3$ **b** $k + 5$ **c** $b + 2$
d $g - 3$ **e** $H - 4$ **f** $M - 6$

7 **a** $2u$ **b** $4y$ **c** $3x$ **d** $10k$
e $\frac{y}{8}$ **f** $\frac{z}{2}$ **g** $3a + 4$ **h** $2p + 12$

8 C

9 **a** 70 **b** $10n$

10 **a** $8x$ **b** $x + 3$ **c** $8(x + 3)$

11 **a** $2n$ **b** $2n + 8$ **c** $n + 4$
d n **e** 4

12 **a** false **b** false **c** true
d true **e** false **f** true

13 **a** $\frac{\$A}{4}$ **b** $\frac{\$A}{n}$ **c** **i** $\frac{\$A - 20}{n}$ **ii** \$30

Exercise 8B

1 **a** 15 **b** 8 **c** 20 **d** 5

2 **a** 6 **b** 28 **c** 1 **d** 2

3 **a** 10 **b** 15 **c** 5 **d** 4

4 **a** 14 **b** 1 **c** 10 **d** 8

5 **a** 7 **b** 17 **c** 120 **d** 5

6 **a** 8 **b** 42 **c** 3 **d** 3

7 **a** 20 **b** 50 **c** 35 **d** 100

8 **a** 17 **b** 20 **c** 72 **d** 12

9 **a** 8 **b** 10 **c** 9
d 14 **e** 17 **f** 3
g 14 **h** 7 **i** 18

10 **a** 14 **b** 21 **c** 23
d 12 **e** 18 **f** 21

11 **a** 8 **b** 4 **c** 5
d 9 **e** 4 **f** 6
g 8 **h** 1 **i** 15

12 5

13 1 and 24, 2 and 12, 3 and 8, 4 and 6

14 36

15

x	5	9	12	1	6	7
$x + 6$	11	15	18	7	12	13
$4x$	20	36	48	4	24	28

Exercise 8C

1 **a** 9 **b** 12 **c** no

2 **a** 9 **b** 9 **c** yes

3 equivalent

4 True. When adding numbers, order does not matter.

5 **a**

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$2a + 2$	2	4	6	8
$(a + 1) \times 2$	2	4	6	8

b equivalent **c** a a : and a • a •

6 **a**

	$B = 0$	$B = 1$	$B = 2$	$B = 3$
$5B + 3$	3	8	13	18
$6B + 3$	3	9	15	21

b no

7

	$6x + 5$	$4x + 5 + 2x$
$x = 1$	11	11
$x = 2$	17	17
$x = 3$	23	23
$x = 4$	29	29

8 **a** N **b** E **c** E **d** N

9 $y + y + y + y$; other answers are possible.

10 $2(w + 1)$; other answers are possible.

11 6

12 If $x = 8$, all four expressions have different values.

13 A1 and C2, A2 and D3, A3 and C1, B1 and C3, B2 and D2, B3 and D1

14 **a** $x = 5$ **b** $x = 3$ **c** $x = 1$
d $x = 2$ **e** $x = 0$ **f** $x = 0$
g $x = 0$ **h** $x = 4$ **i** $x = 0$
j $x = 1$ **k** $x = 0$, $x = 1$ **l** $x = 0$, $x = 2$
m $x = 2$

15 **a** C **b** A **c** C **d** B
e D **f** A **g** A **h** D

Exercise 8D

1 **a** false **b** true **c** true **d** false

2 **a** 14 **b** 21 **c** 35 **d** 35

3 **a** 50 **b** 20 **c** 30 **d** 30

4 **a** like **b** like terms **c** terms **d** terms

5 **a** N **b** L **c** L **d** N
e N **f** L **g** N **h** L
i L **j** L **k** L **l** N

6 **a** $5x$ **b** $6a$ **c** $16q$ **d** $3b$
e $9cd$ **f** $6qr$ **g** $9ab$ **h** $11cf$

7 **a** $4x$ **b** $3a$ **c** $10q$ **d** $6b$
e $8cd$ **f** $2qr$ **g** $7ab$ **h** $3cf$

8 **a** $3a + 5b$ **b** $7a + 9b$ **c** $x + 6y$
d $7a + 2$ **e** $7 + 7b$ **f** $6k - 2$

9 **a** $5f + 12$ **b** $4a + 6b - 4$ **c** $6x + 4y$
d $8a + 4b + 3$ **e** $7g + 4$ **f** $14x + 30y$
g $2x + 9y + 10$ **h** $8a + 13$ **i** $12b$
j $5d + 3$

10 **a** $10n$ **b** $12n$ **c** $22n$

11 **a** $3a + 4$ **b** 19

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Exercise 8D cont.

12 a

	$3x + 2x$	6
$x = 1$	5	6
$x = 2$	10	12
$x = 3$	15	18

b For example, if $x = 2$ and $y = 4$, then $3x + 2y = 14$ but $5x = 10$.

c For example, if $x = 5$ and $y = 10$, then $3x + 2y = 35$ but $5xy = 250$.

13 a i $4x$ **ii** $7x$ **iii** $11x$ **iv** $3x$

b Xavier has 48. Cameron has 84.

Exercise 8E

1 a true **b** true **c** false
d true **e** false **f** false

2 a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{3}{5}$ **d** $\frac{3}{4}$

3 a $\frac{2}{3}$ **b** $\frac{2}{3}$ **c** $\frac{2}{3}$

4 a C **b** F **c** B
d A **e** D **f** E

5 a $2x$ **b** $5p$ **c** $7r$ **d** $11s$
e $10ab$ **f** $5cd$ **g** $4x$ **h** x^2

6 a $10ab$ **b** $16xy$ **c** $10b$ **d** $28xz$
e $36abc$ **f** $48def$ **g** $42ab$ **h** $42abc$
i $84abc$ **j** $3a^2$

7 a $36a$ **b** $63d$ **c** $8e$
d $15a$ **e** $12ab$ **f** $63eg$
g $8abc$ **h** $28adf$ **i** $12abc$
j $8abc$ **k** $60defg$ **l** $24abcd$

8 a $\frac{x}{5}$ **b** $\frac{z}{2}$ **c** $\frac{a}{12}$ **d** $\frac{b}{5}$
e $\frac{2}{x}$ **f** $\frac{5}{d}$ **g** $\frac{x}{y}$ **h** $\frac{a}{2}$

9 a $\frac{2}{5}$ **b** 1 **c** $\frac{9a}{4}$ **d** $\frac{2b}{5}$
e $\frac{x}{2}$ **f** $\frac{3x}{4}$ **g** $\frac{2}{3}$ **h** $\frac{3}{4}$
i $2a$ **j** 3 **k** $2y$ **l** $\frac{3}{y}$

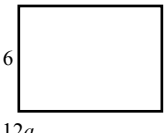
10 a $3k$ **b** $6x$ **c** $12xy$

11 a \$20 **b** $\frac{\$C}{5}$

12 a $6p$

b $3 \times 2p$ also simplifies to $6p$, so they are equivalent.

13 a



b $12a$

c $\frac{12a}{3a}$ simplifies to 4. It has four times the area.

d The area is multiplied by 9.

Exercise 8F

1 a 12 **b** 10 **c** 30 **d** 27

2 a 20 **b** 28 **c** 40 **d** 100

3 a 35 **b** 20

4 a $3x$ **b** 36

5 a \$36 **b** \$21 **c** $3n$

6 a 6 **b** 11 **c** $t + 2$

7 a $10x$ **b** $15x$ **c** kx

8 $2n$

9 a \$200 **b** \$680 **c** $50 + 80x$

10 a

Hours	1	2	3	4	5
Total cost (\$)	150	250	350	450	550

b $100t + 50$ **c** \$3050

11 a \$25 **b** $10x + 5$ **c** \$75

12 a 33 **b** $g = 8$ and $b = 5$

c $g = 3$ and $b = 2$, $g = 1$ and $b = 14$, $g = 0$ and $b = 20$

13 a $0.2 + 0.6t$ **b** $0.8 + 0.4t$ **c** Emma's **d** 3 min

e Answers will vary.

Exercise 8G

1 a 6 **b** −1 **c** 6 **d** 2
e 1 **f** −6 **g** 2 **h** −7

2 C

3 B

4 a 7 **b** 9 **c** −3 **d** −11
e −1 **f** −5 **g** −4 **h** −8
i −6 **j** 6 **k** −5 **l** 4
m 19 **n** 5 **o** 8 **p** 7
q 5 **r** −6

5 a 4 **b** 15 **c** 28 **d** 3
e 12 **f** 40 **g** −6 **h** $\frac{1}{2}$
i 8 **j** 2 **k** 40 **l** $1\frac{1}{4}$

6 a −13 **b** 3 **c** 20 **d** 3
e 19 **f** 15 **g** −14 **h** 11

7 a 20 L **b** 35 L **c** 110 L

8 a 4 **b** 5 **c** 3
d −4 **e** −7 **f** −1
g −5 **h** 1 **i** −6

9 no, 20

10 a i 5 **ii** 6

b A negative length is not possible.

11 Answer may vary.

12 a

Hours	0	1	2	3	4	5	6	7	8	9	10
Temp.	20	16	12	8	4	0	−4	−8	−12	−16	−20

b 10 hours **c** B **d** $-20 + 3n$

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Exercise 8H

1 **a** sequence **b** term

2 **a** 20, 24, 28 **b** 85, 80, 75
c 8, 4, 2 **d** 40, 80, 160

3 **a** 8, 11, 14, 17, 20 **b** 32, 31, 30, 29, 28
c 52, 48, 44, 40, 36 **d** 123, 130, 137, 144, 151

4 **a** 3, 6, 12, 24, 48 **b** 5, 20, 80, 320, 1280
c 240, 120, 60, 30, 15 **d** 625, 125, 25, 5, 1

5 **a** 23, 28, 33 **b** 44, 54, 64
c 14, 11, 8 **d** 114, 116, 118
e 27, 18, 9 **f** 5, 4, 3
g 505, 606, 707 **h** 51, 45, 39

6 **a** 32, 64, 128 **b** 80, 160, 320
c 12, 6, 3 **d** 45, 15, 5
e 176, 352, 704 **f** 70000, 700000, 7000000
g 16, 8, 4 **h** 76, 38, 19

7 **a** 50, 32, 26 **b** 25, 45, 55 **c** 32, 64, 256
d 9, 15, 21 **e** 55, 44, 33 **f** 333, 111

8 **a** Start with 19 and subtract 2 from each term.
b Start with 48 and divide each term by 2.
c Start with 50 and add 6 to each term.
d Start with 1 and multiply each term by 3.
e Start with 625 and divide every term by 5.
f Start with 75 and subtract 3 from each term.

9 **a** 17, 23, 30 **b** 16, 22, 29
c 10, 13, 11 **d** 45, 40, 50

10 **a** 3, 1, 4, 2, 5, 3, 6 **b** 7 days

11 **a** $\times 3$ **b** -2 **c** $+11$
d not a sequence **e** $\div 2$
f not a sequence **g** not a sequence
h -3

12 **a** 49, 64, 81; square numbers
b 21, 44, 65; Fibonacci numbers
c 216, 343, 512; cube numbers (i.e. 6^3, 7^3, 8^3)
d 19, 23, 29; prime numbers
e 16, 18, 20; composite numbers
f 161, 171, 181; palindromes

13 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (total = 55)

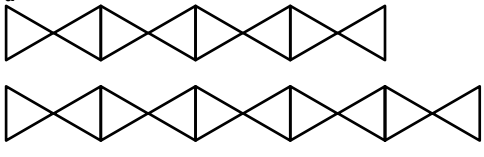
Exercise 8I

1 **a** rectangles **b** 1, 2, 3 **c** 6, 10, 14

2

Number of houses	1	2	3
Number of matchsticks	6	11	16

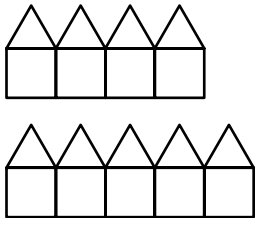
3 **a**



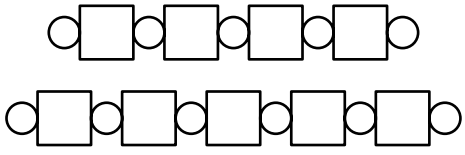
b



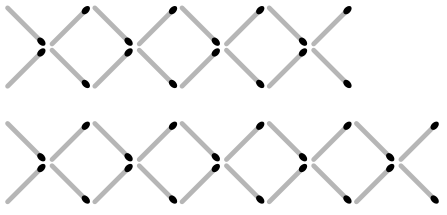
c



d



4 **a**



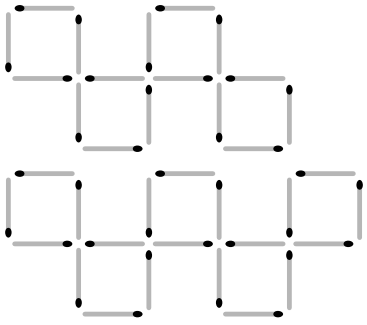
b

Number of crosses	1	2	3	4	5
Number of sticks	4	8	12	16	20

c Number of sticks = 4 × number of crosses

d 40 sticks

5 **a**



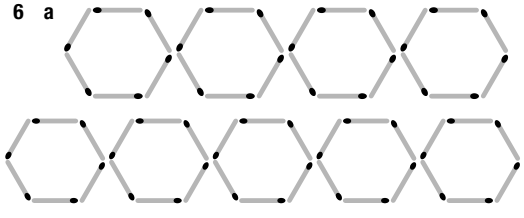
b

Number of squares	1	2	3	4	5
Number of sticks	4	8	12	16	20

c Number of sticks = 4 × number of squares

d 48 sticks

6 **a**



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Exercise 8I cont.

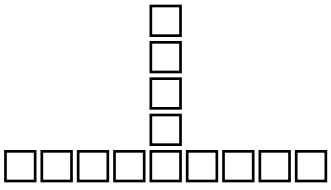
b

Number of hexagons	1	2	3	4	5
Number of sticks	6	12	18	24	30

c Number of sticks = 6 × number of hexagons

d 120 sticks

7 B, D, A, C



8 a



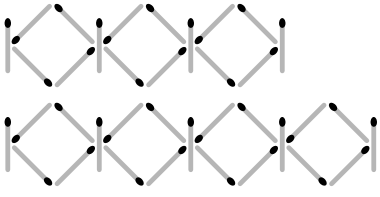
b

Number of triangles	0	1	2	3	4
Number of sticks	1	1 + 2 = 3	1 + 2 × 2 = 5	1 + 2 × 3 = 7	1 + 2 × 4 = 9

c Number of sticks = 1 + 2 × number of triangles

d 25 sticks **e** 40 triangles

9 a



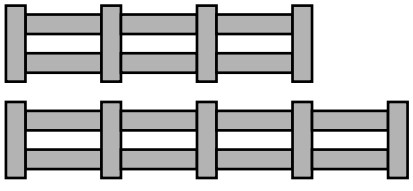
b

Number of shapes	0	1	2	3	4
Number of sticks	1	1 + 5 = 6	1 + 5 × 2 = 11	1 + 5 × 3 = 16	1 + 5 × 4 = 21

c Number of sticks = 1 + 5 × number of shapes

d 101 sticks **e** 17 shapes

10 a



b

Number of fence sections	0	1	2	3	4
Number of planks	1	1 + 3 = 4	1 + 3 × 2 = 7	1 + 3 × 3 = 10	1 + 3 × 4 = 13

c Number of planks = 1 + 3 × number of fence sections

d 28 planks **e** 14 fence sections

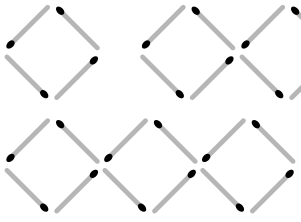
11 A

12 Answers may vary.

a



b



c



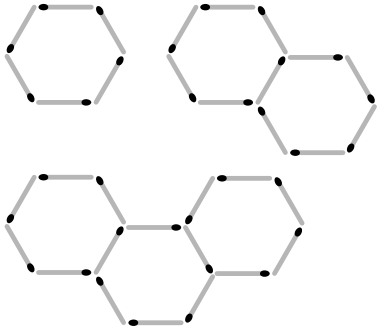
d



e



f



Exercise 8J

1 a flowers

b

Number of flowers (*input*)	1	2	3	4	5
Number of sticks (*output*)	4	8	12	16	20

2 a Ebony's age

b

José's age (*input*)	1	3	7	12	15
Ebony's age (*output*)	4	7	10	15	18

3 a input

b

Hours worked (*input*)	1	2	3	4	5
Amount earned (*output*)	$8	$16	$24	$32	$40

4 a 16 **b** 6 **c** 10 **d** 4

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5 a

Input	4	5	6	7	10
Output	7	8	9	10	13

b

Input	5	1	3	21	0
Output	10	2	6	42	0

c

Input	11	18	9	44	100
Output	3	10	1	36	92

d

Input	5	15	55	0	100
Output	1	3	11	0	20

6 a

Input	1	2	3	4	5
Output	7	17	27	37	47

b

Input	6	8	10	12	14
Output	7	8	9	10	11

c

Input	5	12	2	9	0
Output	16	37	7	28	1

d

Input	3	10	11	7	50
Output	2	16	18	10	96

7 a *output* = *input* + 1 **b** *output* = 4 × *input*
c *output* = *input* + 11 **d** *output* = *input* ÷ 6

8

Input	4	10	13	24	14	3	5	11	2
Output	12	30	39	72	42	9	15	33	6

output = 3 × *input*

9 a A **b** D **c** B **d** C

10 a

Zac's age in years (*input*)	0	1	2	3
Amount ($) in account (*output*)	$100	$150	$200	$250

b *output* = 50 × *input* + 100 **c** $1000

11 a

Hours worked (*input*)	0	2	5	10
Cindy's total saving (*output*)	64	80	104	144

b *output* = 8 × *input* + 64 **c** 17 hours

12 a *output* = 2 × *input* + 1 **b** *output* = 3 × *input* − 2
There are other correct answers.

13 a i *output* = 2 × *input* − 3
ii *output* = 4 × *input* + 1
iii *output* = 5 × *input* − 1
iv *output* = *input* ÷ 6 + 2
v *output* = 10 × *input* + 3
vi *output* = 4 × *input* − 4

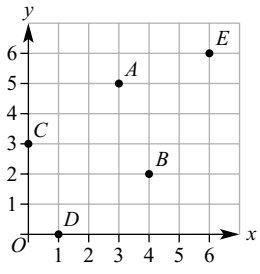
Exercise 8K

1 *A*(2, 2), *B*(3, 4), *C*(6, 1), *D*(4, 0), *E*(0, 5), *F*(5, 4)

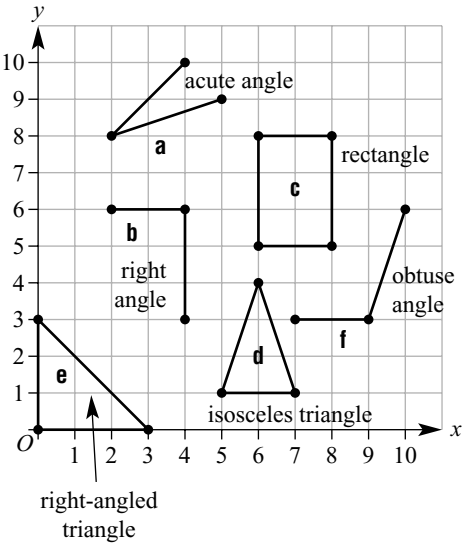
2 a *A*(5, 2) **b** *B*(2, 5)
c *C*(6, 6) **d** *D*(3, 0)
e *E*(4, 7) **f** *F*(0, 4)

3 a *x*-axis **b** *y*-axis **c** origin
d *x*-coordinate **e** *y*-coordinate **f** *x*, *y*, *x*, *y*

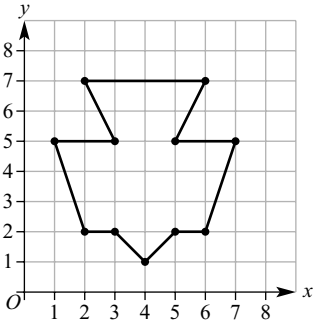
4



5



6



7 a *A*(1, 4), *B*(2, 1), *C*(5, 3), *D*(2, 6), *E*(4, 0), *F*(6, 5), *G*(0, 3), *H*(4, 4)
b *M*(1, 2), *N*(3, 2), *P*(5, 1), *Q*(2, 5), *R*(2, 0), *S*(6, 6), *T*(0, 6), *U*(5, 4)

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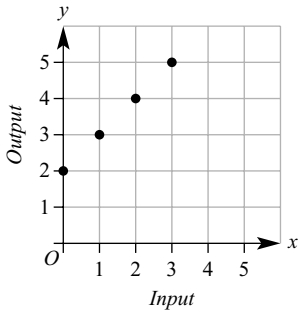
Exercise 8K cont.

8 a

Input (*x*)	*Output* (*y*)
0	2
1	3
2	4
3	5

b (0, 2), (1, 3), (2, 4), (3, 5)

c

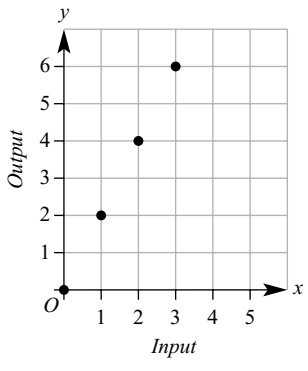


9 a

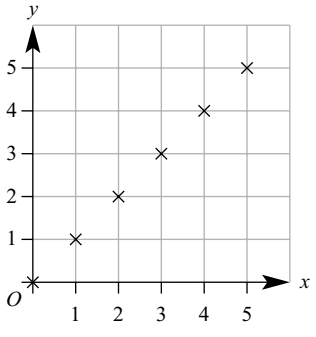
Input (*x*)	*Output* (*y*)
0	0
1	2
2	4
3	6

b (0, 0), (1, 2), (2, 4), (3, 6)

c

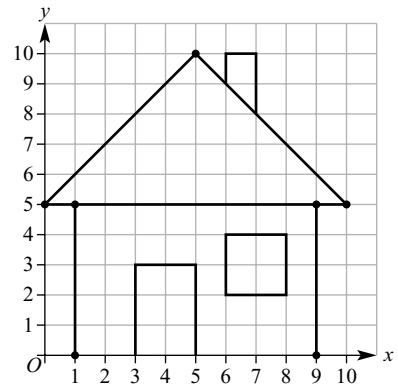


10



11 a C **b** C

12 a Includes sample door, window and chimney (for **b**–**d**). Other answers are possible for these.



b (3, 0), (3, 3), (5, 3), (5, 0)

c (6, 2), (6, 4), (8, 4), (8, 2)

d (6, 9), (6, 10), (7, 10), (7, 8)

13 a HELP

b (4, 4), (5, 1), (3, 1), (3, 4), (5, 1), (5, 4)

c KEY UNDER POT PLANT

d 21510032513451001154004451255143

Puzzles and games

1 $A = 3$, $B = 7$, $C = 2$, $D = 1$

2 a $a = 4$, $b = 12$, $c = 16$, $d = 8$, $e = 36$

b $a = 6$, $b = 3$, $c = 5$, $d = 10$, $e = 15$

3

x	2	2	3	0	5
y	7	6	3	12	1
$3x$	6	6	9	0	15
$x + 2x$	16	14	9	24	7
xy	14	12	9	0	5

4 $a = 1$, $c = 4$, $d = 2$, $e = 1$, $f = 3$, $g = 4$, $h = 1$, $i = 3$, $j = 4$, $k = 2$.

5 a $A = 7$, $B = 10$, $C = 8$, $D = 9$, $E = 4$, $F = 11$

b $A = 4$, $B = 9$, $C = 2$

Multiple-choice questions

1	B	**2**	D	**3**	A	**4**	B	**5**	D
6	D	**7**	D	**8**	C	**9**	A	**10**	A

Short-answer questions

1	**a** 4	**b** 12		
2	**a** $u + 7$	**b** $3k$	**c** $h - 10$	
3	**a** 6	**b** 104	**c** 16	**d** 21
4	**a** 13	**b** 24	**c** 2	**d** 17
5	**a** 15	**b** 24	**c** 2	**d** 4
6	**a** 15	**b** 8	**c** 4	**d** 27

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7

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$4x$	0	4	8	12
$3x + x$	0	4	8	12

8 **a** E **b** N **c** E **d** N

9 **a** L **b** N **c** L
d N **e** L **f** N

10 **a** $7x + 3$ **b** $11p$ **c** $7a + 14b + 4$
d $3m + 17mn + 2n$ **e** $1 + 7c + 4h - 3o$
f $4u + 3v + 2uv$

11 **a** $12ab$ **b** $6xyz$ **c** $36fgh$ **d** $64klm$

12 **a** $\frac{3}{2}$ **b** $\frac{3}{5}$ **c** $\frac{a}{3}$ **d** $\frac{4x}{3z}$

13 $9t$

14 $g + b$

15 $3x$

Extended-response questions

1 **a** \$60 **b** \$60 **c** \$90
d $30n$ **e** \$100

2 **a** **i** \$17 **ii** \$32 **iii** \$152
b $2 + 1.5k$ **c** \$62 **d** $6 + 1.2k$

Chapter 9

Pre-test

1 **a** 12 **b** 15 **c** 16 **d** 7

2 **a** 7 **b** 20 **c** 16 **d** 5

3 **a** false **b** true **c** true **d** false

4 **a** 7 **b** 5 **c** 15 **d** 24

5 **a** 3 **b** 27 **c** 2 **d** 6

6 **a** $k + 5$ **b** $2p$ **c** $7y$

7 **a**

n	1	2	3	4	5
$5 \times n$	5	10	15	20	25

b

n	2	4	6	8	10
$n - 2$	0	2	4	6	8

c

n	1	2	5	8	9
$2n$	2	4	10	16	18

8 **a** ÷ **b** − **c** × **d** +

9 **a** 27 **b** 40 **c** 18 **d** 26
e 17 **f** 14 **g** 6

10 **a** Mia is 17 and Oliver is 20.
b \$28

Exercise 9A

1 **a** true **b** true **c** false

2 **a** true **b** false **c** false **d** true

3 **a** 9 **b** 15 **c** 2 **d** 10

4 9

5 **a** E **b** N **c** E
d E **e** E **f** N
g E **h** E **i** N

6 **a** true **b** true **c** false
d false **e** false **f** false
g true **h** false **i** true

7 **a** false **b** true **c** false **d** true

8 **a** 19 **b** 19 **c** true

9 **a** true **b** true **c** false **d** true

10 **a** $3 + x = 10$ **b** $5k = 1005$
c $a + b = 22$ **d** $2d = 78$
e $8x = 56$ **f** $3p = 21$

11 **a** $2n = 240$ **b** $n + 2 = 240$
c $\frac{n}{2} = 240$ **d** $240 = n - 2$
e $2n + 3n = 240$ **f** $3n + 2 = 240$
g $(n + 2) \times 3 = 240$ **h** $n + n + 2 = 240$
i $n + n + 1 + n + 2 = 240$
j $n \times 3 \div 2 = 240$ or $\frac{3n}{2} = 240$
k $n \div 2 \div 3 = 240$ or $n \times \frac{1}{2} \times \frac{1}{3} = 240$
l $n + 2 + \frac{n}{2} = 240$ **m** $\frac{n}{2} + \frac{n}{3} = 240$
n $n - \frac{n}{2} = 240$ **o** $\frac{n}{2} = \frac{n}{3} + 240$
p $n^2 = 240$ **q** $n + n^2 = 240$

12 **a** $6c = 546$ **b** $7k = 567$
c $12a + 3b = 28$ **d** $f + 10 = 27$

13 **a** $m = 3$ **b** $k = 2$ or $k = 6$

14 **a** $6 = 2 \times 3$; other solutions are possible.
b $5 - 4 = 1$; other solutions are possible.
c $10 \div 2 = 7 - 2$; other solutions are possible.
d $4 - 2 = 10 \div 5$; other solutions are possible.

Exercise 9B

1 **a** false **b** true **c** false **d** true

2 **a** true **b** false **c** true **d** true

3 **a** 12 **b** 17 **c** 13 **d** 6

4 **a** 3 **b** 6 **c** 20 **d** 19

5 **a** 9 **b** 3 **c** 6 **d** 24

6 **a** $y = 8$ **b** $l = 6$ **c** $d = 2$
d $l = 12$ **e** $a = 6$ **f** $s = 12$
g $x = 8$ **h** $e = 8$ **i** $s = 8$

7 **a** $p = 3$ **b** $p = 4$ **c** $q = 3$
d $v = 5$ **e** $b = 1$ **f** $u = 4$
g $g = 3$ **h** $d = 6$ **i** $m = 4$

8 **a** $x = 3$ **b** $x = 7$
c $x = 4$ **d** $x = 5$

9 **a** 11 **b** 12 **c** 16 **d** 33

10 **a** $10x = 180$ **b** $x = 18$

11 **a** $2w = 70$ **b** $w = 35$

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Exercise 9B cont.

12 **a** $y + 12 = 3y$ **b** $y = 6$

13 **a** $x = 2$ and $y = 6$; other solutions are possible.
b $x = 12$ and $y = 10$; other solutions are possible.
c $x = 12$ and $y = 0.5$; other solutions are possible.
d $x = 2$ and $y = 2$; other solutions are possible.

Exercise 9C

1 **a** $10d + 15 = 30$ **b** $7e + 10 = 41$
c $2a + 10 = 22$ **d** $x + 10 = 22$

2 **a** C **b** D **c** E **d** B **e** A

3 **a** -11 **b** $+9$ **c** $\div 3$ **d** $\div 9$

4 **a** $5x + 3 = 23$ **b** $5x - 3 = 17$
c $15x = 60$ **d** $x = 4$

5 **a** $6 + x = 11$ **b** $6x = 14$ **c** $3 = 2q$
d $6 + a = 10$ **e** $12 + b = 15$ **f** $0 = 3b + 2$
g $4 = 7 + a$ **h** $12x + 2 = 8$ **i** $7p = 12$

6 **a**
$3x + 2 = 11$
-2 / -2
$3x = 9$
$\div 3$ / $\div 3$
$x = 3$

b
$2x - 6 = 10$
$+6$ / $+6$
$2x = 16$
$\div 2$ / $\div 2$
$x = 8$

7 a, c, e, f

8 **a** $2q = 2$ **b** $10x = 7$ **c** $x = 60$ **d** $2x = 5$

9
$3x + 2 = 14$
-2 / -2
$3x = 12$
$\div 3$ / $\div 3$
$x = 4$
$\times 10$ / $\times 10$
$10x = 40$
$+1$ / $+1$
$10x + 1 = 41$

10 **a** **i** $0 = 0$ **ii** $0 = 0$ **iii** $0 = 0$
b Regardless of original equation, will always result in $0 = 0$.

Exercise 9D

1 **a** true **b** false **c** false **d** false

2 **a** 6 **b** $x = 6$

3 **a** -5 **b** $\div 10$ **c** $\times 4$ **d** $+12$

4 **a** $m = 9$ **b** $g = 11$ **c** $s = 9$ **d** $i = 10$
e $t = 2$ **f** $q = 3$ **g** $y = 12$ **h** $s = 12$
i $j = 4$ **j** $l = 4$ **k** $v = 2$ **l** $y = 12$
m $k = 5$ **n** $y = 9$ **o** $z = 7$ **p** $t = 10$
q $b = 12$ **r** $p = 11$ **s** $a = 8$ **t** $n = 3$

5 **a**
$7a + 3 = 38$
-3 / -3
$7a = 35$
$\div 7$ / $\div 7$
$a = 5$

b
$4b - 10 = 14$
$+10$ / $+10$
$4b = 24$
$\div 4$ / $\div 4$
$b = 6$

c
$2(q + 6) = 20$
$\div 2$
$q + 6 = 10$
-6 / -6
$q = 4$

d
$5 = \frac{x}{10} + 3$
-3 / -3
$2 = \frac{x}{10}$
$\times 10$ / $\times 10$
$20 = x$

6 **a** subtract 3 **b** add 7 **c** subtract 15 **d** add 34

7 **a** $f = 11$ **b** $k = 5$ **c** $x = 9$ **d** $a = 9$
e $k = 8$ **f** $a = 10$ **g** $n = 11$ **h** $n = 1$
i $g = 4$ **j** $q = 11$ **k** $z = 10$ **l** $p = 1$
m $d = 4$ **n** $t = 8$ **o** $u = 5$ **p** $c = 1$
q $q = 11$ **r** $y = 12$ **s** $q = 2$ **t** $u = 10$

8 **a** $x = \frac{3}{4}$ **b** $k = \frac{24}{5}$ **c** $w = \frac{4}{3}$
d $x = \frac{4}{3}$ **e** $x = \frac{5}{8}$ **f** $x = \frac{1}{3}$

9 **a** $r = -7$ **b** $x = -3$ **c** $t = -16$ **d** $y = -24$
e $x = -5$ **f** $k = -9$ **g** $x = -6$ **h** $x = -4$
i $x = -\frac{3}{2}$

10 **a** $x + 5 = 12 \rightarrow x = 7$ **b** $2y = 10 \rightarrow y = 5$
c $2b + 6 = 44 \rightarrow b = 19$ **d** $3(k - 7) = 18 \rightarrow k = 13$

11 **a** $12n + 50 = 410$ **b** $n = 30$h

12 **a** $12 + 5x$ **b** $12 + 5x = 14.5$
c $x = 0.5$, so pens cost 50 cents.
d $12 + 5\ (0.5) = 14.5$

13 **a** $3b = 15 \rightarrow b = 5$ **b** $4x = 12 \rightarrow x = 3$
c $2(10 + x) = 28 \rightarrow x = 4$ **d** $4b = 28 \rightarrow b = 7$

14 Examples include: $x + 1 = 3$, $7x = 14$, $21 - x = 19$, $\frac{4}{x} = x$, $\frac{x}{2} = 1$.

15
$2x + 5 = 13$
-5 / -5
$2x = 8$
$\div 2$ / $\div 2$
$x = 4$
$\times 5$ / $\times 5$
$5x = 20$

16 **a** First step, $4x + 2$ is not completely divided by 4.
b Second step, LHS divided by 3, RHS has 3 subtracted.
c First step, RHS has 5 added not subtracted.
d First step, LHS has $11a$ subtracted, not 12.

17 **a** $x = 4$ **b** $x = 1$ **c** $l = 3$ **d** $t = 1$
e $s = 5$ **f** $b = 19$ **g** $j = 2$ **h** $d = 1$

Exercise 9E

1 **a** true **b** false **c** true **d** false

2 **a** true **b** false **c** false **d** true

3 **a** dividing by 4 **b** adding 2
c multiplying by 7 **d** subtracting 11

4 **a**
$\frac{b}{4} = 11$
$\times 4$ / $\times 4$
$b = 44$

b
$\frac{d}{5} = 3$
$\times 5$ / $\times 5$
$d = 15$

c
$\frac{z}{10} = 2$
$\times 10$ / $\times 10$
$z = 20$

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5 **a** B **b** C **c** A **d** D

6 **a** $b = 80$ **b** $d = 200$ **c** $z = 65$

7 **a** $m = 12$ **b** $c = 18$
c $s = 16$ **d** $r = 10$
e $u = 12$ **f** $y = 50$
g $x = 2$ **h** $a = 12$

8 **a** $d = 9$ **b** $y = 6$
c $j = 3$ **d** $b = 4$
e $w = 6$ **f** $s = 3$
g $v = 11$ **h** $f = 9$

9 **a** $m = 10$ **b** $q = 9$
c $k = 40$ **d** $x = 30$
e $a = 4$ **f** $x = 55$
g $y = 6$ **h** $a = 21$

10 **a** $x = 9$ **b** $x = 4$
c $p = 21$ **d** $p = 11$
e $q = 20$ **f** $r = 27$
g $r = 21$ **h** $x = 60$

11 **a** $\frac{t}{2} = 9 \rightarrow t = 18$ **b** $\frac{x}{10} = 8 \rightarrow x = 80$
c $\frac{q-4}{2} = 3 \rightarrow q = 10$ **d** $\frac{x+3}{4} = 2 \rightarrow x = 5$
e $\frac{y}{4} + 3 = 5 \rightarrow y = 8$

12 **a** $\frac{b}{5} = 22$ **b** $b = 110$ **c** \$110

13 **a** The different order in which 3 is added and the result is divided by 5.
b **i** multiply by 5 **ii** subtract 3
c no

14 **a** $a = 2$ **b** $k = 3$ **c** $q = 5$

Exercise 9F

1 **a** F, g **b** x, y **c** A, B, C **d** g, d

2 **a** 15 **b** 9 **c** 52

3 **a** 15 **b** 36 **c** 8 **d** 2

4 **a** 5 **b** 8 **c** 14 **d** 110

5 **a** 9 **b** 17 **c** 13

6 **a** $m = 7$ **b** $m = 9$ **c** $m = 201$

7 **a** 21 **b** 18 **c** 30

8 **a** $x = 3$ **b** $x = 5$

9 **a** $A = 10$ **b** $b = 10$ **c** $b = 7$

10 **a** $y = 23$ **b** $x = 4$ **c** $x = 7$

11 **a** **i** $P = 28$ **ii** $P = 40$ **b** $s = 12$

12 **a** $A = 35$ **b** Length is 5.
c **i** $b = 5$ **ii** square

13 **a** $F = 68$ **b** $C = 10$ **c** 73°F
d 36°C **e** 15°C

Exercise 9G

1 **a** D **b** A **c** E **d** B **e** C

2 **a** $x = 6$ **b** $a = 2$ **c** $k = 9$

3 D

4 **a** $7k = 42$ **b** $k = 6$

5 **a** $x + 19 = 103$ **b** $x = 84$

6 **a** Let p = cost of one pen. **b** $12p = 18$
c $p = 1.5$ **d** \$1.50

7 **a** Let t = cost of a car tyre. **b** $4t + 160 = 1400$
c $t = 310$ **d** \$310

8 **a** Let h = number of hours worked.
b $17h + 65 = 643$
c $h = 34$ **d** 34 hours

9 **a** $24\ell = 720$ **b** $\ell = 30$ **c** 30 m **d** 108 m

10 $2x + 3 = 31 \rightarrow x = 14$

11 **a** $x = 4$ **b** No. It must be greater than 18.

12 14 years old

13 **a** Both equal 18.
b Examples: $l = 4$ and $b = 4$, $l = 12$ and $b = 2.4$
c Yes, if the width equals 4.

Puzzles and games

1 **a** 26 **b** 29 **c** 368
d $31\frac{1}{3}$ **e** 36

2 7 and 13

3 30

4 A CORNY JOKE

5 26 sheep, 15 ducks

6 **a**

n	1	2	3	4	5
M	3	5	7	9	11

b $M = 2n + 1$ **c** 201

7 8 and 13

8 **a** 36 **b** 12

9 120

Multiple-choice questions

1 D **2** C **3** A **4** D **5** A
6 E **7** C **8** B **9** A **10** E

Short-answer questions

1 **a** false **b** true **c** true
d false **e** true **f** false

2 **a** $2 + u = 22$ **b** $5k = 41$
c $3z = 36$ **d** $a + 12 = 15$

3 **a** $x = 3$ **b** $x = 6$ **c** $y = 1$
d $y = 9$ **e** $a = 2$ **f** $a = 10$

4 **a** $x = 5$ **b** $q = 6$ **c** $a = 21$
d $k = 18$ **e** $k = 10$ **f** $w = 8$
g $b = 5$ **h** $r = 2$

5 **a** $x = 3$ **b** $r = 45$ **c** $x = 9$
d $r = 4$ **e** $q = 2$ **f** $u = 8$

6 **a** $u = 24$ **b** $p = 16$ **c** $x = 8$
d $y = 40$ **e** $y = 8$ **f** $x = 60$

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7 **a** no **b** LHS = 16, RHS = 16 ∴ solution
8 **a** $A = 450$ **b** $l = 8$
c $b = 5$, $l = 8$; other solutions are possible.
9 **a** 36 **b** 6 **c** 3
10 **a** 5 **b** 25 **c** 20 **d** 5

Extended-response questions

1 **a** $120 **b** B **c** $60
d 7 hours **e** $252 = 12n$
2 **a** 75 cents **b** $1.35
c $t = 12$ **d** 12 seconds
e 50 seconds **f** $1.50

Chapter 10

Pre-test

1 **a** kilogram **b** metre and centimetre
2 **a** 40 mm **b** 60 mm **c** 8 mm
3 **a** mm, cm, m, km **b** mg, g, kg, t
4 **a** 10 **b** 100 **c** 1000 **d** 1000
5 **a** 2000 **b** 2 **c** 56 **d** 2500
6 **a** 55 cm **b** 27 cm **c** 28 cm
7 **a** 32 **b** 7 **c** 8
8 **a** kilometres **b** kilograms
c minutes **d** centimetres

Exercise 10A

1 millimetre, centimetre, metre, kilometre
2 **a** 100 **b** 1000 **c** divide **d** multiply
3 **a** right **b** left
4 **a** kilometres **b** millimetres **c** metres
d metres **e** centimetres **f** kilometres
5 **a** 2 cm, 20 mm **b** 5 cm, 50 mm
c 1.5 cm, 15 mm **d** 3.2 cm, 32 mm
e 12.5 cm, 125 mm
6 **a** 50 mm **b** 200 cm
c 3500 m **d** 2610 cm
e 2200 m **f** 53 mm
g 620 cm **h** 200 mm
i 684 cm **j** 20 m
k 3800 cm **l** 670 cm
7 **a** 4 cm **b** 5 m **c** 4.2 km **d** 47.2 cm
e 3.6 m **f** 3.2 cm **g** 50 km **h** 27 km
i 36.2 cm **j** 0.04 cm **k** 926.1 cm **l** 4.23 km
8 **a** metres **b** millimetres
c kilometres **d** kilometres
e metres **f** centimetres
9 **a** 2.5 cm **b** 82 mm **c** 2.5 m **d** 730 cm
e 6200 m **f** 25.732 km
10 **a** 8.5 km **b** 310 cm **c** 19 cm
11 **a** 38 cm, 0.5 m, 540 mm
b 160 cm, 2100 mm, 0.02 km, 25 m
c 142 mm, 20 cm, 0.003 km, 3.1 m
d 10 mm, 0.1 m, 0.001 km, 1000 cm
12 125 cm
13 **a** $12.30 **b** $6.56 **c** $4.10
14 10 000 years
15 **a** 3000 mm **b** 600 000 cm **c** 2400 mm
d 4000 cm **e** 0.47 km **f** 913 m
g 0.216 km **h** 0.0005 m **i** 2 m
16 **a** 2 mm **b** 5 mm **c** 2 cm
d 4 cm **e** 8 cm
17 **a** 9 cm **b** 10 cm **c** 25 cm **d** 6 cm

Exercise 10B

1 **a** perimeter **b** equal
2 **a** 10 cm **b** 12 cm **c** 12 cm **d** 12 cm
3 **a** 150 mm **b** 120 mm **c** 160 mm **d** 115 mm
4 **a** 15 cm **b** 37 m **c** 36 cm **d** 11 m
e 30 km **f** 2.4 m **g** 26 cm **h** 10 cm
5 **a** 42 cm **b** 34 m **c** 20.8 m **d** 90 km
6 **a** 8.4 cm **b** 14 m **c** 46.5 mm
7 **a** 34 m **b** 52 km **c** 8.8 mm
d 56 m **e** 36 km **f** 40 mm
8 $21400
9 **a** 10.46 cm **b** 130.2 cm **c** 294 cm **d** 568 cm
10 **a** 5 cm **b** 5 m **c** 7 km
11 **a** 516 ft **b** 30.48 cm

Exercise 10C

1 **a** 15.71 **b** 40.84 **c** 18.85 **d** 232.48
2 **a** 3.1 **b** 3.14 **c** 3.142
3 **a** diameter **b** radius **c** circumference
d semicircle
4 Answer is close to pi.
5 **a** false **b** true **c** false
d false **e** true
6 **a** 12.57 mm **b** 113.10 m **c** 245.04 cm
d 12.57 m **e** 21.99 km **f** 15.71 cm
7 11.0 m
8 12 566 m
9 B
10 Svenya and Andre
11 3 cm
12 $d = 2r$, so $2\pi r$ is the same as πd.

Exercise 10D

1 **a** $\frac{1}{4}$, quadrant **b** $\frac{1}{2}$, semicircle
2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$
d $\frac{1}{3}$ **e** $\frac{3}{4}$ **f** $\frac{5}{8}$

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3 **a** square, semicircle **b** quadrant, rectangle
c sector, triangle
4 **a** 8.38 cm **b** 5.59 m **c** 12.22 mm
d 1.96 m **e** 136.14 cm **f** 1.15 km
5 **a** 14.3 cm **b** 35.7 m **c** 51.4 mm
d 36.0 km **e** 13.1 m **f** 14.0 cm
6 **a** 18.7 m **b** 11.1 cm **c** 101.1 cm
d 35.4 km **e** 67.1 mm **f** 45.1 cm
7 657 cm
8 **a** 26.88 m **b** 52.36 m **c** 6.24 m
9 **a** 25.13 cm **b** 56.55 m **c** 35.71 m
10 The four arcs make one full circle ($2\pi r$) and the four radii make $4r$.
11 **a** $(24 + 3\pi)$ cm **b** $(9 + 2.5\pi)$ cm

Exercise 10E

1 1 mm^2, 1 cm^2, 1 m^2, 1 ha, 1 km^2
2 **a** D **b** E **c** C **d** B **e** A
3 **a** 9 **b** 3 cm and 3 cm **c** 9
4 **a** 8 **b** 4 cm and 2 cm **c** 8
5 **a** 4 square units **b** 6 square units
c 4 square units **d** 6 square units
e 8 square units **f** 96 square units
6 **a** 3 cm^2 **b** 4 cm^2
c 6 cm^2 **d** 3 cm^2
e 2 cm^2 **f** 5 cm^2
g 4.5 cm^2 **h** 9 cm^2
7 **a** 8 cm^2 **b** 9 cm^2 **c** 100 m^2
d 200 cm^2 **e** 22 mm^2 **f** 7 cm^2
g 25 m^2 **h** 1.44 mm^2 **i** 6.25 mm^2
j 1.36 m^2 **k** 0.81 cm^2 **l** 179.52 km^2
8 5000 m^2
9 2500 cm^2
10 **a** 2 cm **b** 5 m **c** 12 km
11 **a** 25 cm^2 **b** 12 cm **c** 4 units
12 **a** **i** 10 cm **ii** 9 mm
b Divide the area by the given length.
13 half of a rectangle with area 4 cm^2
14 20 000 cm^2 = 2 m^2
15 \$2100
16 5 L

Exercise 10F

1 **a** 11 m **b** 15 mm **c** 4.1 m
d 4.7 m **e** 5 m **f** 26 mm
2 **a** 8 cm **b** 2.1 cm **c** 3 m
d 5 cm **e** 5 cm **f** 6 cm
3 **a** 10 **b** 56 **c** 12.5 **d** 60
4 **a** 3 m^2 **b** 40 cm^2 **c** 12 mm^2
d 16 m^2 **e** 30 cm^2 **f** 160 m^2
g 1.2 m^2 **h** 15 m^2 **i** 63 mm^2
5 **a** 10 cm^2 **b** 3 mm^2 **c** 24.5 km^2
d 1.3 cm^2 **e** 20 m^2 **f** 4.25 m^2
6 **a** 2 cm^2 **b** 6 cm^2 **c** 3 cm^2 **d** 3 cm^2
7 4800 m^2
8 160 m^2
9 480 m^2
10 \$6300
11 No, the base and height are always the same.
12 Both triangles have $b = 1$ cm and $h = 2$ cm.
13 **a** 6 cm^2 **b** 3 cm^2 **c** 7 cm^2 **d** 8 cm^2

Exercise 10G

1 C
2 **a** $A = bh$
$= 5 \times 7$
$= 35$
b $A = bh$
$= 20 \times 3$
$= 60$
c $A = bh$
$= 8 \times 2.5$
$= 20$
3 **a** $b = 6$ cm, $h = 2$ cm **b** $b = 10$ m, $h = 4$ m
c $b = 3$ mm, $h = 5$ mm **d** $b = 5$ m, $h = 7$ m
e $b = 5.8$ cm, $h = 6.1$ cm **f** $b = 5$ cm, $h = 1.5$ cm
g $b = 1.8$ m, $h = 0.9$ m **h** $b = 5$ m, $h = 12$ m
4 **a** 40 m^2 **b** 28 m^2
c 17.5 m^2 **d** 14 cm^2
e 42 m^2 **f** 176 mm^2
g 50 m^2 **h** 48 cm^2
5 **a** 36 km^2 **b** 16 m^2 **c** 48 mm^2
d 6.3 cm^2 **e** 30 cm^2 **f** 1.8 cm^2
6 54 m^2
7 **a** 6 cm^2 **b** 4 cm^2 **c** 15 cm^2 **d** 8 cm^2
8 **a** 1800 cm^2 **b** 4200 cm^2
9 **a** 2 m **b** 7 cm
10 **a** 10 cm **b** 5 m
11 \$1200
12 half; Area (parallelogram) = bh and
Area (triangle) = $\frac{1}{2}bh$
13 **a** 6500 m^2 **b** 6300 m^2
c 25 600 m^2 **d** \$4 608 000

Exercise 10H

1 **a** 1000 **b** milligrams **c** tonne
d 100°C **e** 0°C
2 **a** C **b** F **c** A
d D **e** B **f** E
3 **a** B **b** A **c** D **d** C
4 **a** 12°C **b** 37°C **c** 17°C
d 225°C **e** 1.7°C **f** 31.5°C

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Exercise 10H cont.

5 **a** 4 kg **b** 12 g **c** 65 t
6 **a** 2000 g **b** 7000 g **c** 6200 g **d** 5800 g
e 6 kg **f** 8.9 kg **g** 0.9 kg **h** 0.45 kg
i 5000 kg **j** 600 kg **k** 2.4 t **l** 4.32 t
m 3000 mg **n** 4200 mg **o** 7.5 g
7 **a** 4.620 g **b** 21.6 t **c** 470 kg **d** 0.312 kg
e 0.027 g **f** 750 kg **g** 125 g **h** 0.0105 kg
i 210 t **j** 470 kg **k** 592 g **l** 80 g
8 33°C
9 147°C
10 **a** 60 kg **b** 60 000 g **c** 60 000 000 mg
11 **a** 3000 g **b** 3 kg
12 **a** 8 kg **b** 8.16 kg
13 **a** 400 mg, 370 g, 2.5 kg, 0.1 t
b 290 000 mg, 0.000 32 t, 0.41 kg, 710 g
14 50 days
15 **a** 1 g **b** 1 t **c** 1000 t
16 1000 kg

Puzzles and games

1 B
2 No. Although both shapes look like triangles, this is an optical illusion. The 'hypotenuse' (longest side) on each shape is not quite straight.
3 **a** 3 cm^2 **b** 20 cm^2
4 **a** 62 cm^2 **b** 16 m^2 **c** 252 cm^2
5 Mark a length of 5 m, then use the 3 m stick to reduce this to 2 m. Place the 3 m stick on the 2 m length to show a remainder of 1 m.
6 10.5 cm^2

Multiple-choice questions

1 D **2** E **3** C **4** A **5** E
6 B **7** C **8** B **9** E **10** B

Short-answer questions

1 **a** 2.5 cm **b** 2.3 cm **c** 4.25 kg **d** 6°C
2 **a** 50 mm **b** 2 m **c** 3700 m
d 36 m **e** 7100 g **f** 24 g
g 22 t **h** 2500 kg **i** 6 cm
3 **a** 16 m **b** 20.6 cm **c** 23 m
d 34 km **e** 3.2 mm **f** 24 m
4 **a** 24.01 cm^2 **b** 14 km^2
c 67.5 m^2 **d** 12 cm^2
e 14 m^2 **f** 5 cm^2
g 14 m^2 **h** 0.9 km^2
i 0.16 m^2 **j** 900 cm^2
5 **a** (12π) m **b** (6π) m
6 **a** (3π) m **b** (3π) m **c** (2π) m
7 **a** $(3\pi + 12)$ m
b $(3\pi + 6)$ m
c $(2\pi + 12)$ m

Extended-response question

1 **a** 240 cm **b** 3200 cm^2 **c** 20 cm, 60 cm
d 160 cm **e** 1200 cm^2 **f** 2000 cm^2 **g** \$200

Chapter 11

Pre-test

1 **a** 100 **b** 1000 **c** 1 000 000
2 **a** 20 **b** 10 **c** $6\frac{2}{3}$ **d** 5
e 20, 10, 5, 4, 2, 1
3 **a** 0 **b** 0 **c** 2 **d** 0
4 **a** false **b** false **c** false **d** true
5 $24 \times 1 = 24$, $12 \times 2 = 24$, $6 \times 4 = 24$, $3 \times 8 = 24$
6 **a** $3 < 7$ **b** $12 > 5$
7 **a** 4 **b** 9 **c** 16 **d** 25
e 36 **f** 49
8 **a** $8 = 4 \times 2$ **b** $15 = 3 \times 5$ **c** $12 \times 4 = 48$
d $4 \times 4 = 16 = 8 \times 2$ **e** $12 \times 3 = 36 = 4 \times 9$
9 **a** 14 → 2×7 **b** 15 → 5×3 **c** 21 → 3×7
10 **a** 11 **b** yes **c** 10 remainder 2
d no
11 **a** B **b** A **c** D **d** C
12 **a** $A = 16$ cm^2 **b** side = 6 cm

Exercise 11A

1 **a** 2, 4, (6), 8, 10, (12), 14, 16, (18), 20
b 3, (6), 9, (12), 15, (18), 21, (24), 27, (30)
c see above
d 3, 6
2 **a** divisible, remainder **b** 2
c 0, 2, 4, 6 or 8
d divisible, 4 + 3 + 2 = 9, 9 **e** 6
3 **a** B **b** A **c** C
4 **a** 5, 10 **b** 5
c 16, 16, 4 **d** 328, 328, 8
5 **a** 2 + 5 + 8 + 3 = 18 **b** divisible, 18, 3
c divisible, 18, 9 **d** odd
6 **a** 6, 14, 8, 54, 22, 34, 50, 18, 46
b 12, 18, 30, 27, 54, 36, 42, 24
c 12, 24, 60, 54, 252, 36, 66, 84
d 168, 7168, 40, 5032, 248, 6400, 9568
7 **a** 35, 125, 15, 100, 515, 730, 105
b 20, 800, 290, 610, 590, 90, 160
c 16, 32, 220, 12, 28, 432, 72, 316, 424, 1836
d 27, 432, 99, 387, 63, 720, 2799
8 **a** true **b** false **c** true
d true **e** false **f** true
g false **h** true **i** false
j true **k** false **l** true
m true **n** false **o** false
p true **q** false **r** false

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9

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567	×	✓	×	×	×	×	✓	×
28 080	✓	✓	✓	✓	✓	✓	✓	✓
189 000	✓	✓	✓	✓	✓	✓	✓	✓
1 308 150	✓	✓	×	✓	✓	×	✓	✓
1 062 347	×	×	×	×	×	×	×	×

10 **a** not even
b digits do not sum to a multiple of 3
c 26 is not divisible by 4
d last digit is not 0 or 5
e not divisible by 3 (sum of digits is not divisible by 3)
f 125 is not divisible by 8 and it is not even
g sum of digits is not divisible by 9
h last digit is not 0

11 **a** 2 **b** 2 **c** 0 **d** 0

12 2, 4, 8, 11, 22, 44

13 36

14 **a** yes
b Multiples of 3; adding a multiple of 3 does not change the result of the divisibility test for 3.
c 18

Exercise 11B

1 **a** prime, 1, itself **b** composite

2 **a** no **b** yes

3 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
b 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

4 **a** 11 **b** 12 **c** 23

5 **a** C **b** P **c** C **d** P
e C **f** C **g** P **h** P
i C **j** C **k** C **l** P
m P **n** P **o** C **p** P

6 **a** 2, 3, 7 **b** 3, 13 **c** 2, 3, 5
d 5 **e** 2, 7 **f** 2, 3

7 **a** 32, 33, 34, 35, 36, 38, 39
b 51, 52, 54, 55, 56, 57, 58
c 81, 82, 84, 85, 86, 87, 88

8 **a** 23, 29 **b** 41, 43, 47 **c** 71, 73, 79

9 **a** 3, 5 **b** 3, 7 **c** 5, 7
d 5, 11 **e** 11, 13 **f** 7, 19

10 14

11 5 and 7, 11 and 13, 17 and 19, as well as other pairs

12 **a** no, 3, prime
b yes, 6×2 or 4×3, 12, composite
c yes, 5×2, 10, composite
d no, 5, prime
e no, 7, prime
f yes, 7×2, 14, composite
g no, 11, prime
h yes, 3×3, 9, composite

Exercise 11C

1 **a** expanded **b** index **c** power
d base **e** index, exponent

2 **a** $3^2 = 3 \times 3$
b $2^4 = 2 \times 2 \times 2 \times 2$
c $5^3 = 5 \times 5 \times 5$
d $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

3

Factor form	Index form	Base	Index
$7 \times 7 \times 7$	7^3	7	3
$5 \times 5 \times 5$	5^3	5	3
$2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6	2	6
$6 \times 6 \times 6 \times 6$	6^4	6	4

4

Index form	Base number	Index	Value
2^3	2	3	8
5^2	5	2	25
10^4	10	4	10 000
2^7	2	7	128
1^{12}	1	12	1
12^1	12	1	12
0^5	0	5	0

5 **a** 3^3 **b** 2^5 **c** 15^4
d 10^4 **e** 6^2 **f** 20^3
g 1^6 **h** 4^3 **i** 100^2
j $3^2 \times 5^2$ **k** $2^2 \times 7^3$ **l** $9^2 \times 12^2$
m $5^3 \times 8^2$ **n** $3^3 \times 6^3$ **o** $7^4 \times 13^2$
p $4^3 \times 7^1 \times 13^1$ **q** $9^3 \times 10^2$ **r** $2^3 \times 3^2 \times 5^2$

6 **a** $2 \times 2 \times 2 \times 2$
b 17×17
c $9 \times 9 \times 9$
d $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
e $14 \times 14 \times 14 \times 14$
f $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
g $10 \times 10 \times 10 \times 10 \times 10$
h $54 \times 54 \times 54$
i $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$
j $4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3$

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Exercise 11C cont.

k $7 \times 7 \times 5 \times 5 \times 5$
l $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 9 \times 9 \times 9$
m $5 \times 7 \times 7 \times 7 \times 7$
n $2 \times 2 \times 3 \times 3 \times 3 \times 4$
o $11 \times 11 \times 11 \times 11 \times 11 \times 9 \times 9$
p $20 \times 20 \times 20 \times 30 \times 30$

7 **a** 6, 9 **b** 8, 16 **c** 10, 25 **d** 12, 36

8 **a** 32 **b** 64 **c** 1000 **d** 72
e 10 000 **f** 1000 **g** 64 **h** 121

9 **a** 25 **b** 1 **c** 10
d 64 **e** 128 **f** 8
g 22 **h** 900 **i** 8

10 **a** 4 **b** 2 **c** 3 **d** 6
e 3 **f** 2 **g** 2 **h** 4

11 **a** < **b** > **c** = **d** <
e > **f** > **g** < **h** <

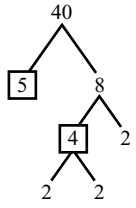
12 125

13 **a** **i** 1110 people **ii** 1 111 110 people
b 40 min
c 50 min

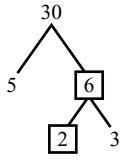
Exercise 11D

1 composite: 4, 8, 9, 12, 15, 27
prime: 2, 3, 5, 7, 11, 13, 23

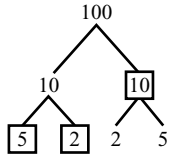
2 **a**



b



c



3 **a**

2	12
2	6
3	3
	1

b

2	30
3	15
5	5
	1

c

2	66
3	33
11	11
	1

4 **a** $2^2 \times 3$ **b** 2×3^2
c $3^2 \times 5^2$ **d** $2^2 \times 3^2 \times 4$
e $2^3 \times 3^2$ **f** $3^4 \times 5^2$
g $2^2 \times 3 \times 7^2$ **h** $2^2 \times 3^2 \times 11^2$

5 **a** 2×7 **b** 2^4 **c** 2×3^2
d $2^2 \times 5$ **e** $2^3 \times 3$ **f** $2^2 \times 3^2$
g $2^2 \times 11$ **h** $2^3 \times 7$ **i** 2^6
j $2^3 \times 3^2$ **k** 3×5^2 **l** $2^4 \times 5$

6

2	96
2	48
2	24
2	12
2	6
3	3
	1

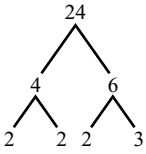
$96 = 2^5 \times 3$

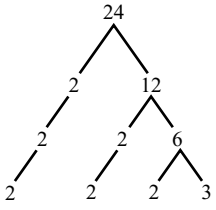
7 **a** $32 = 2^5$ **b** $40 = 2^3 \times 5$
c $81 = 3^4$ **d** $144 = 2^4 \times 3^2$
e $120 = 2^3 \times 3 \times 5$ **f** $500 = 2^2 \times 5^3$
g $1800 = 2^3 \times 3^2 \times 5^2$ **h** $1250 = 2 \times 5^4$

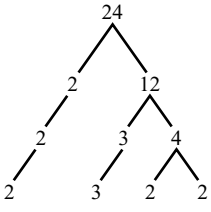
8 **a** D **b** A **c** C **d** B

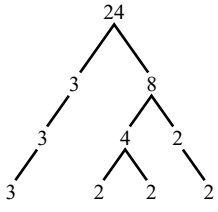
9 2310

10

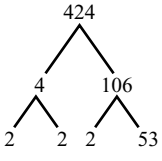








11 **i** 424 cannot have a factor of 5.
ii 8 is not a prime number.
iii



$424 = 2^3 \times 53$

12 $2 \times 3 \times 5 \times 7 = 210$
$2 \times 3 \times 5 \times 11 = 330$
$2 \times 3 \times 5 \times 13 = 390$
$2 \times 3 \times 5 \times 17 = 510$
$2 \times 3 \times 5 \times 19 = 570$
$2 \times 3 \times 5 \times 23 = 690$
$2 \times 3 \times 5 \times 27 = 810$
$2 \times 3 \times 5 \times 29 = 870$
$2 \times 3 \times 5 \times 31 = 930$
$2 \times 3 \times 7 \times 11 = 462$
$2 \times 3 \times 7 \times 13 = 546$
$2 \times 3 \times 7 \times 17 = 714$
$2 \times 3 \times 7 \times 19 = 798$
$2 \times 3 \times 7 \times 23 = 966$
$2 \times 5 \times 7 \times 11 = 770$
$2 \times 5 \times 7 \times 13 = 910$
$2 \times 3 \times 11 \times 13 = 858$

Exercise 11E

1 See Key ideas for table.

2 See Key ideas for table.

3 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

4 **a** 15 **b** 20
c 4 **d** 100
e 1000 **f** 25

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5 **a** 36 **b** 25 **c** 121
d 100 **e** 49 **f** 144
g 2197 **h** 1331 **i** 1728

6 **a** 5 **b** 4 **c** 10
d 7 cm **e** 4 **f** 8

7 **a** 64 **b** 49 **c** 1 **d** 144
e 9 **f** 225 **g** 25 **h** 0
i 121 **j** 10 000 **k** 900 **l** 1600

8 **a** 5 **b** 3 **c** 1 **d** 11
e 0 **f** 9 **g** 7 **h** 4
i 2 **j** 12 **k** 20 **l** 13
m 50 **n** 80 **o** 90 **p** 27

9 **a** 7 **b** 5 **c** 12 **d** 30
e 64 **f** 65 **g** 36 **h** 4
i 0 **j** 81 **k** 4 **l** 13

10 **a** A square is not possible. **b** Draw 4×4 square.

11 64, 81, 100

12 121, 144, 169, 196

13 **a** $9 + 16 = 25 = 5^2$
b No: $25 + 36 = 61 \neq 49$
c Yes: $36 + 64 = 100 = 10^2$
d Many answers are possible, e.g. $5^2 + 12^2 = 13^2$, $9^2 + 12^2 = 15^2$

14 **a** **ii** 2 more dots
iii always one dot by itself
b **ii** 2 more dots
iii same number of dots in each row
c **ii** 3, 5, 7, 9, 11 dots
iii dots form a square, alternate odd and even numbers
d **ii** 2, 3, 4, 5, 6 dots
iii $1 + 2 = 3$, $1 + 2 + 3 = 6$, $1 + 2 + 3 + 4 = 10$

15 **a**

Square numbers	16	25	36	49	64	81	100
Triangular numbers	10	15	21	28	36	45	55

b 1275 **c** 5050 **d** 1035

Puzzles and games

2 IT WAS SPEAKING ANOTHER LANGUAGE

3 **a** **i** 135 **ii** 624 **iii** 945
b Answers will vary.

Multiple-choice questions

1 C **2** E **3** C **4** E **5** B
6 B **7** D **8** A **9** C **10** B

Short-answer questions

1 **a** 5, 10
b divisible, $2 + 6 + 4 = 12$, 12
c divisible, $5 + 7 + 6 = 18$, 18, divisible
d divisible, 44, 44, divisible

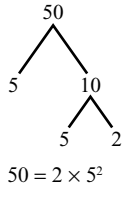
2 **a** 14, 48, 56, 206, 312, 320
b 48, 63, 312, 621
c 48, 56, 320, 312
d 85, 320
e 48, 312
f 48, 56, 320, 312
g 63, 621
h 320

3 **a** no
b no
c yes
d 1, 2, 5, 10, composite number
e prime: 2, 3, 7, 11; composite: 8, 15, 20
f prime: 2, 5; composite: 4, 10, 20
g 17, 19, 23

4 **a** base = 4, index or exponent = 2
b $5^2 \times 7^3$
c $2 \times 2 \times 2 \times 3 \times 3 = 72$
d 28
e 3, 5, 5

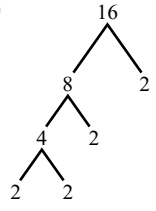
5 Answers may vary.

a



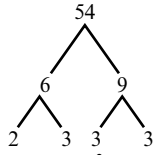
$50 = 2 \times 5^2$

b



$16 = 2^4$

c



$54 = 2 \times 3^3$

6 **a**

Index form	4^2	6^2	7^2	9^2
Value	16	36	49	81

b

Square root form	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{100}$	$\sqrt{144}$
Value	4	5	10	12

7 **a** 1 **b** 7 **c** 5 **d** 12
e 18 **f** 200 **g** 64 **h** 121
i 9 **j** 5 cm **k** 20 cm

8 **a** $1^3 = 1$, so $\sqrt[3]{1} = 1$.
b $2^3 = 8$, so $\sqrt[3]{8} = 2$.
c $5^3 = 125$, so $\sqrt[3]{125} = 5$.
d $10^3 = 1000$, so $\sqrt[3]{1000} = 10$.

9 **a** 13 **b** 36 **c** -5
d 25 **e** 125 **f** 35

10 210

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Extended-response questions

1 **a** 60 **b** 60 **c** 210 **d** 360

2 **g** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

h 30, 42, 60, 70, 84, 90

Semester review 2

Time

Multiple-choice questions

1 A **2** E **3** A

4 D **5** E

Short-answer questions

1 **a** 60 h **b** 3 h **c** 3 min 30 s

d 6 h 30 min **e** 2145 **f** 1:26 p.m.

g 5 h 45 min **h** 6 h 19 min 12 s

2 **a** 45 min **b** 1 h 51 min

c 1 h 11 min 1 s **d** 1 h 51 min 50 s

3 **a** **i** 7 p.m. **ii** 5 p.m.

iii noon **iv** 6 a.m.

v midnight **vi** 9 p.m.

b **i** 1:20 p.m. **ii** 3:50 p.m.

iii 3:20 p.m. **iv** 4:20 p.m.

4 **a** **i** 55 min **ii** 1 h 14 min

iii 1 h 1 min **iv** 1 h 57 min

b morning **c** 1 h 56 min

5 21 h 15 min

6 120

7 5 a.m.

Extended-response question

1 **a** **i** 4:30 p.m **ii** 4 p.m. **iii** 2:30 p.m.

b 2 h 39 min **c** 3:30 p.m.

Algebraic techniques 1

Multiple-choice questions

1 B **2** A **3** C **4** B **5** D

Short-answer questions

1 **a** 3 **b** 7 **c** $8x + 7y$

2 **a** $x + 3$ **b** $12a$ **c** $2x + 3y$

d $\frac{w}{6}$ **e** $y - 2x$

3 **a** L **b** N **c** L **d** N

4 **a** 13 **b** 11 **c** 39

d 6 **e** 3 **f** 24

5 36

6 **a** $10a$ **b** $4x$ **c** $12a$

d m **e** $6 + 5a$ **f** $4x + 2y$

7 **a** $6 + 2x$ **b** $3x$

8 **a** $6bc$ **b** $5b$ **c** p

9 **a** $10p$ **b** $25p$

10 $12x$

Extended-response question

1 **a** **i** \$16 **ii** $3x$ **iii** $3x + 10$

b $3x + 2y$ **c** 16

Equations 1

Multiple-choice questions

1 B **2** E **3** C **4** B **5** B

Short-answer questions

1 **a** false **b** true **c** true

d false **e** false **f** true

2 **a** $x = 3$ **b** $x = 108$ **c** $x = 21$ **d** $x = \frac{4}{3}$

3 **a** $x = 2$ **b** $y = 5$ **c** $x = 12$ **d** $m = 7$

4 **a** **i** 8 **ii** 20 **iii** 48

b $w = 6$ **c** $w = 6$ **d** $w = 11$

5 **a** $x = 5$ **b** $x = 6$ **c** $x = 18$

6 $2x + 25 = 85 \rightarrow x = 30$

Extended-response question

1 **a** \$320 **b** \$400

c $C = 200 + 40n$ **d** $6\frac{1}{2}$ hours

Measurement and computation of length, perimeter and area

Multiple-choice questions

1 C **2** D **3** E **4** C **5** A

Short-answer questions

1 **a** 500 **b** 6000 **c** 18 **d** 1.8

2 **a** 272 cm **b** 11 m **c** 300 cm

d 220 cm **e** 3.4 m **f** 92 m

3 **a** 1.69 m^2 **b** 24 m^2 **c** 60 m^2

d 20 m^2 **e** 12 cm^2 **f** 28 m^2

4 **a** $(6\pi + 12)$ cm **b** $(3\pi + 12)$ cm **c** $(\pi + 8)$ cm

5 $(8\pi + 16)$ cm

6 **a** 3000 kg **b** 2000 g **c** 6.5 kg **d** 0.5 t

e 5 g **f** 24 000 mg **g** 50 g **h** 0.02 g

7 **a** 0 **b** 100

Extended-response question

1 **a** Many answers are possible; e.g. 8 m × 10 m, 4 m × 14 m, 15 m × 3 m

b 9 m × 9 m (area = 81 m^2) **c** 36 posts

d \$900

Introducing indices

Multiple-choice questions

1 E **2** A **3** B **4** E **5** D

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Short-answer questions

1 **a** 12, 15, 18 **b** 14 **c** 18

2 4, 9

3 23, 29, 31, 37

4 **a** 8 **b** 12 **c** 18
d 27 **e** 20 **f** 30

5 **a** 121 **b** 144 **c** 25

6 **a** 49 **b** 144 **c** 9

7 **a**

Index form	3^2	5^2	6^2	8^2
Value	9	25	36	64

b

Square root form	$\sqrt{9}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{64}$
Value	3	5	6	8

8 **a** 1080, 536, 930, 316 **b** 1080, 135, 930
c 1080, 536, 316 **d** 1080, 135, 930
e 1080, 930

9 **a** yes **b** yes **c** yes **d** yes
e yes **f** yes **g** no **h** no
i no **j** yes

10 $2^2 \times 3 \times 5$

Extended-response question

1 **a** $1 + 2 + 5 + 10 = 18$
$18 \div 2 = 9$, not 10.

b It is 28 because
$1 + 2 + 4 + 7 + 14 + 28 = 56$
$56 \div 2 = 28$

c $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496 = 992$
$992 \div 2 = 496$

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A

acute angle 52
AD 300
adjacent 60
algebraic expression 322
algorithm 13
alternate angles 67
approximation 30
arc 440
area 445
ascending 120, 147
associative law 17

B

base 451, 489
BC 300

C

Cartesian plane 106, 368
Celsius 465
certain 203
chance 203
chance experiment 202
circle 435
circumference 436
coefficient 322
cointerior angles 67
collinear 47
commutative law 17
compensating 9
complement 202
complementary 60
complementary events 202
composite figure 440
composite number 485
compound event 202
concurrent 47
constant 322
constant term 322
Coordinated Universal Time (UTC) 305
coordinates 368
corresponding angles 67
cube 500
cube root 500

D

decimal point 152
denominator 131, 244
descending 147
diameter 436
dimension 446
distributive law 18
dividend 25, 273
divisibility tests 479
divisor 25, 274

E

equally likely outcomes 202
equation 383, 406
equivalent 244, 391
equivalent expressions 322, 330
equivalent fractions 137
estimate 30
evaluate 323, 327
even chance 203
event 202
expanded notation 5
experiment 213
experimental probability 203
exponent 489
expression 322

F

factor 120
factor tree 494
favourable outcome 202
formula 406, 436
fraction 131

G

Greenwich Mean Time (GMT) 305

H

height 451
highest common factor (HCF) 126, 137

I

identity 321, 323
impossible 203
improper fraction 131, 142
index 489
index form 489
input 362
integer 485
integers 85
interval 48
irrational number 436

K

kilogram 465

L

like terms 322, 334
likelihood 203
likely 203
line 47
lowest common denominator (LCD) 147
lowest common multiple (LCM) 126

M

magnitude 85
mass 465
metre 423
metric system 423
mixed numeral 142
multiple 120
mutually exclusive events 202

N

negative number 85
non-mutually exclusive events 202
number line 85
number plane 106, 368
numerator 131, 244

O

obtuse angle 52
order of operations 34
origin 106, 368
outcome 202
output 362

P

parallel lines 66
parallelogram 459
partitioning 9
per cent 168
percentage 168
perimeter 429
perpendicular 61, 451
place value 5
plane 47
point 47
power 489

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prime factor 494
prime number 485
product 17
pronumeral 322
proper fraction 131
proportion 186
protractor 53

Q

quadrant 440
quotient 25, 273

R

radius 436
ray 48
reciprocal 267
recurring decimals 162
reflex angle 52
remainder 25
revolution 52, 61
right angle 52
rounding 30, 157
rule 406

S

sample space 202
sector 440
segment 48
semicircle 439
sequence 350
simplify 322, 334
solution 387
solve 387
square number 499
square root 500
straight angle 52
substitute 323, 327
substituting 395
substitution 323
supplementary 60

T

table of values 363
term 322, 350
the sum of all probabilities in an experiment 203
the sum of the probabilities of an event and its complement 203
theoretical probability 203, 208
transversal 66
trial 202
two-way table 223

U

unknown 387
unlikely 203

V

variable 322, 406
Venn diagram 218
vertex 48
vertically opposite 61
vinculum 131

X

x-axis 106, 368

Y

y-axis 106, 368

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