

OXFORD

INSIGHT
MATHEMATICS

AUSTRALIAN CURRICULUM FOR NSW

STAGES 5.1/5.2

10

JOHN LEY
MICHAEL FULLER

OXFORD



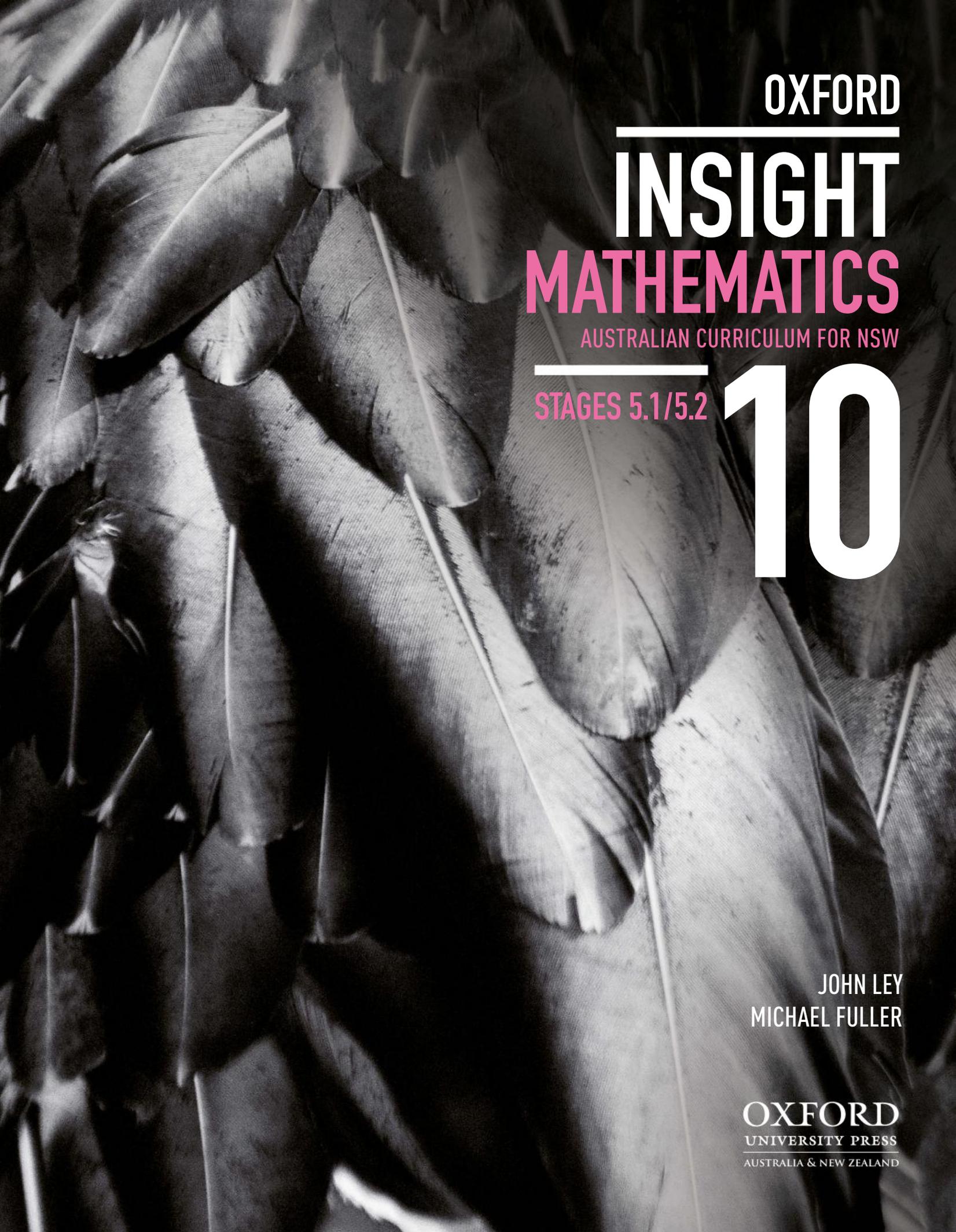
ebook

- Gives you access to all student book content and online resources
- Looks like the student book for easy reference

Click here to access online extras

oup.com.au/399klr





OXFORD

INSIGHT
MATHEMATICS

AUSTRALIAN CURRICULUM FOR NSW

STAGES 5.1/5.2

10

JOHN LEY
MICHAEL FULLER

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

OXFORD
UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trademark of Oxford University Press in the UK and in certain other countries.

Published in Australia by
Oxford University Press
253 Normanby Road, South Melbourne, Victoria 3205, Australia
© John Ley, Michael Fuller 2015

The moral rights of the author have been asserted

First published 2015

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, by licence, or under terms agreed with the appropriate reprographics rights organisation. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above.

You must not circulate this work in any other form and you must impose this same condition on any acquirer.

National Library of Australia Cataloguing-in-Publication data
Oxford Insight Mathematics 10, Stages 5.1/5.2 : Australian Curriculum for NSW / John Ley, Michael Fuller.
ISBN 978 019 552269 3 (paperback)

For secondary school students.
Mathematics—Study and teaching (Secondary).
Mathematics—New South Wales—Textbooks.
510.76

Reproduction and communication for educational purposes

The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this work, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.



For details of the CAL licence for educational institutions contact:

Copyright Agency Limited
Level 15, 233 Castlereagh Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
Email: info@copyright.com.au

Edited and proofread by Marta Veroni
Technical artwork by Rhonda Idczak and Paulene Meyer
Typeset by Idczak Enterprises
Printed by Golden Cup Printing Co. Ltd

Links to third party websites are provided by Oxford in good faith and for information only. Oxford disclaims any responsibility for the materials contained in any third party website referenced in this work.

All material identified by  is material subject to copyright under the Copyright Act 1968 (Cth) and is owned by the Australian Curriculum, Assessment and Reporting Authority 2013. This is an extract from the Australian Curriculum. **Disclaimer:** ACARA neither endorses nor verifies the accuracy of the information provided and accepts no responsibility for incomplete or inaccurate information. In particular, ACARA does not endorse or verify that:

- The content descriptions are solely for a particular year and subject;
- All the content descriptions for that year have been used; and
- The author's material aligns with the Australian Curriculum content descriptions for the relevant year and subject.

You can find the unaltered and most up to date version of this material at <http://www.australiancurriculum.edu.au>
This material is reproduced with the permission of ACARA.

CONTENTS

1 Review of Year 9 1

A Indices	2
B Collecting and analysing data	3
C Numbers of any magnitude.....	4
D Financial mathematics.....	6
E Area, surface area and volume	7
F Probability	8
G Right-angled trigonometry	10
H Similarity	11
I Linear and non-linear relationships	12
J Proportions and rates	13
K Equations and inequalities.....	14

2 Surface area and volume Measurement & Geometry 15

Diagnostic test.....	16
A Area review.....	17
B Surface areas of right cylinders.....	21
C Volumes with uniform cross-sections	23
D Volumes of composite solids.....	26
E Surface areas of composite solids.....	28
F Problems with surface area and volume	29
Language in mathematics.....	31
Check your skills	32
Review set 2A	33
Review set 2B	34
Review set 2C	35
Review set 2D	36

3 Algebra Number & Algebra 37

Diagnostic test.....	38
A Algebraic fractions	39
B The distributive law	45
C Factorising algebraic expressions	51
Language in mathematics.....	55
Check your skills	55
Review set 3A	56
Review set 3B	57
Review set 3C	57
Review set 3D	58

4 Linear relationships Number & Algebra 59

Diagnostic test.....	60
<i>Investigation 1 Distance formula</i>	<i>62</i>
A Distance, midpoint and gradient review	62
B Graphing lines	65
<i>Investigation 2 Gradients and intercepts</i>	<i>67</i>
C Straight line $y = mx + b$ form.....	68
D Equations of lines.....	72
<i>Investigation 3 Intercept form of the equation of a line</i>	<i>75</i>
E General form of a straight line equation.....	76
<i>Investigation 4 Graphs of lines</i>	<i>79</i>
F Parallel and perpendicular lines	79
G Further coordinate geometry (extension).....	82
Language in mathematics.....	84
Check your skills	84
Review set 4A	87
Review set 4B	88
Review set 4C	89
Review set 4D	90

5 Probability Statistics & Probability 91

Diagnostic test.....	92
A Review of Year 9.....	93
B Probability for multistage events.....	98
C Sampling with replacement	102
D Sampling without replacement.....	104
E The product rule in probability.....	107
F Conditional probability	115
Language in mathematics.....	117
Check your skills	118
Review set 5A	120
Review set 5B	121
Review set 5C	122
Review set 5D	123

Cumulative review 2–5 125

6 Simultaneous equations	127
Number & Algebra	
Diagnostic test.....	128
A Linear equations review.....	129
B Simultaneous equations.....	133
C Graphical solutions.....	136
<i>Investigation 1 Graphics calculator</i>	138
D Solution by substitution.....	139
E Solution by elimination.....	140
F Problem solving with two unknowns.....	143
<i>Investigation 2 Simultaneous equations solutions</i>	
	146
Language in mathematics.....	147
Check your skills.....	147
Review set 6A.....	149
Review set 6B.....	149
Review set 6C.....	150
Review set 6D.....	150

7 Geometrical proofs	151
Measurement & Geometry	
Diagnostic test.....	152
A Reasoning in geometry.....	153
B Congruent triangles.....	155
C Formal proofs of congruent triangles.....	159
D Similar triangles.....	162
<i>Investigation 1 Similar triangles</i>	162
E Polygons.....	173
<i>Investigation 2 Exterior angle sum</i>	176
Language in mathematics.....	178
Check your skills.....	178
Review set 7A.....	181
Review set 7B.....	183
Review set 7C.....	186
Review set 7D.....	188

8 Bivariate data analysis	191
Statistics & Probability	
Diagnostic test.....	192
A Variables.....	193
B Bivariate numerical data.....	194
<i>Investigation 1 Variables</i>	197
C Scatter plots.....	197
<i>Investigation 2 Investigating data</i>	204
Language in mathematics.....	205
Check your skills.....	205
Review set 8A.....	207
Review set 8B.....	208
Review set 8C.....	209
Review set 8D.....	210

9 Financial mathematics	211
Number & Algebra	
Diagnostic test.....	212
A Compound interest.....	213
<i>Investigation 1 Compound interest formula</i>	216
B Compound interest formula.....	216
C Compound interest calculations.....	219
<i>Investigation 2 Using a spreadsheet</i>	223
D Depreciation.....	224
Language in mathematics.....	227
Check your skills.....	227
Review set 9A.....	228
Review set 9B.....	229
Review set 9C.....	230
Review set 9D.....	231

Cumulative review 6–9 **232**

10 Binomial expressions and quadratics	235
Number & Algebra	
Diagnostic test.....	236
A Quadratic expressions.....	237
<i>Investigation 1 Using substitution</i>	240
B Binomial products (extension).....	240
C Quadratic trinomials.....	242
<i>Investigation 2 Square numbers</i>	244
D Simple quadratic equations.....	244
<i>Investigation 3 Special product</i>	248
E Solving quadratic equations by factorising... ..	248
Language in mathematics.....	250
Check your skills.....	250
Review set 10A.....	251
Review set 10B.....	252
Review set 10C.....	252
Review set 10D.....	252

11 Box plots	253
Statistics & Probability	
Diagnostic test.....	254
A Mean, mode, median and range.....	255
B Quartiles.....	261
C Box plots.....	266
D Comparing data sets.....	272
<i>Investigation 1 Statistical reports in the media</i>	278
Language in mathematics.....	279
Check your skills.....	279
Review set 11A.....	281
Review set 11B.....	282
Review set 11C.....	283
Review set 11D.....	284

12 Non-linear relationships	285
Number & Algebra	
Diagnostic test.....	286
A The parabola.....	287
<i>Investigation 1 Parabolas</i>	289
B Exponential graphs	290
<i>Investigation 2 Circles</i>	291
C Circles.....	292
D Miscellaneous graphs	294
Language in mathematics.....	297
Check your skills	298
Review set 12A.....	299
Review set 12B	300
Review set 12C	301
Review set 12D	302
13 Further trigonometry	303
Measurement & Geometry	
Diagnostic test.....	304
A Trigonometric ratios of acute angles.....	305
B Using trigonometry to find sides.....	309
C Using trigonometry to find angles	315
D Compass bearings.....	318
<i>Investigation 1 Complementary angles</i>	323
E Acute and obtuse angles.....	324
F Area of a triangle.....	325
G Sine rule	327
H Cosine rule	331
I Miscellaneous questions.....	334
J Practical problems (extension)	335
Language in mathematics.....	340
Check your skills	340
Review set 13A.....	342
Review set 13B	343
Review set 13C	344
Review set 13D	345
Cumulative review 10–13	347
Answers	350
Index	398

OXFORD

INSIGHT MATHEMATICS

AUSTRALIAN CURRICULUM FOR NSW

10



STUDENT BOOKS



Peerless maths content designed to support deep understanding of mathematical concepts and development of skills. Written for the Mathematics Syllabus of the Australian Curriculum in New South Wales.



- Interactive tutorials and guided examples support independent 24/7 study.
- Diagnostic tools aid student understanding.
- Teachers can manage class progress, set tests, and plan instruction to meet individual and whole-class needs.



PROFESSIONAL SUPPORT

Guided progress through the Australian Curriculum so teachers can drive student learning.

SYLLABUS GRID

Chapter	Name	Outcomes	NSW Syllabus references	AC references
1	Review of Year 9			
2	Surface area and volume	MA5.2-1WM, MA5.2-2WM, MA5.2-11MG, MA5.2-12MG	5.2 M&G Area and surface area, 5.2 M&G Volume	ACMMG242
3	Algebra	MA5.2-1WM, MA5.2-3WM, MA5.2-6NA	5.2 N&A Algebraic techniques, 5.2 N&A Equations (part)	ACMNA213, ACMNA230, ACMNA232
4	Linear relationships	MA5.1-1WM, MA5.1-3WM, MA5.1-6NA, MA5.2-1WM, MA5.2-3WM, MA5.2-9NA	5.1 N&A Linear relationships, 5.2 N&A Linear relationships	ACMNA238, ACMNA294
5	Probability	MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-17SP	5.2 S&P Probability	ACMSP225, ACMSP246, ACMSP247
CR 2–5 Cumulative review chapters 2–5				
6	Simultaneous equations	MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-8NA	5.2 N&A Equations	ACMNA237, ACMNA238
7	Geometrical proofs	MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-14MG	5.2 M&G Properties of geometrical figures	ACMMG220, ACMMG243, ACMMG244
8	Bivariate data analysis	MA5.2-1WM, MA5.2-3WM, MA5.2-16SP	5.2 S&P Bivariate data analysis	ACMSP251, ACMSP252
9	Financial mathematics	MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-4NA	5.2 N&A Financial mathematics	ACMNA229
CR 6–9 Cumulative review chapters 6–9				
10	Binomial expressions and quadratics	MA5.2-1WM, MA5.2-3WM, MA5.2-6NA	5.2 N&A Algebraic techniques	ACMNA233, ACMNA241
11	Box plots	MA5.2-1WM, MA5.2-3WM, MA5.2-15SP	5.2 S&P Single variable data analysis	ACMSP227, ACMSP248, ACMSP249, ACMSP250
12	Non-linear relationships	MA5.2-1WM, MA5.2-3WM, MA5.2-10NA	5.2 N&A Non-linear relationships	ACMNA239, ACMNA296
13	Further trigonometry	MA5.2-1WM, MA5.2-2WM, MA5.2-13MG, MA5.3-1WM, MA5.3-2WM, MA5.3-15MG	5.2 M&G Right-angled triangles (trigonometry), 5.3 M&G Trigonometry and Pythagoras' theorem	ACMMG223, ACMMG224, ACMMG245, ACMMG273, ACMMG275
CR 10–13 Cumulative review chapters 10–13				

NSW Maths 7-10 Syllabuses © Board of Studies, Teaching and Educational Standards NSW for and on behalf of the Crown in right of the state of New South Wales 2012. The Board of Studies, Teaching and Educational Standards NSW takes no responsibility for any errors in the reproduction of the BOSTES' Materials.

1

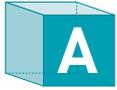


Review of Year 9

This chapter reviews the Year 9 component of the mathematics syllabus.

After completing this chapter you should be able to:

- ▶ apply index laws to simplify algebraic expressions and evaluate arithmetic expressions
- ▶ collect and analyse data and the symmetry of data distributions
- ▶ use scientific notation to write large and small numbers, convert between units and understand error in measurement
- ▶ solve financial problems involving earning and spending money
- ▶ calculate area, surface area and volume
- ▶ determine theoretical probabilities and perform probability calculations
- ▶ apply trigonometry to solve problems including angles of elevation and depression
- ▶ perform calculations involving similar figures
- ▶ determine midpoint, length and distance, and graph straight lines and non-linear graphs
- ▶ use direct and inverse proportion to solve problems
- ▶ solve linear equations and inequalities.



Indices

Exercise 1A

1 Write each in index form.

a $2 \times 2 \times 2 \times 2 \times 2$

b $5 \times 5 \times 5 \times 5 \times 5$

2 Write the base of each number.

a 9^7

b 5^{11}

3 Write each number in expanded form.

a 5^3

b 6^7

4 Use a calculator to evaluate each number.

a 2^9

b 6^5

5 State whether the following are true or false.

a $3^4 \times 2^7 = 6^{11}$

b $8^{10} \div 4^2 = 2^5$

6 Simplify each of the following, leaving the answer in index form.

a $5^{12} \times 5^{16}$

b $(4^5)^2$

c $2^8 \div 2^5$

d $7^6 \times 7$

e $5^6 \times 5^3 \div 5^9$

7 Write the meaning of:

a 2^{-6}

b $5^{\frac{1}{2}}$

c $11^{\frac{1}{3}}$

8 Evaluate:

a 5^{-2}

b $16^{\frac{1}{2}}$

c $8^{\frac{1}{3}}$

d 17^0

9 Simplify:

a $y^{10} \times y^5$

b $k^{11} \div k^6$

c $(p^7)^3$

d $\frac{t^7 \times t^7}{t^3 \times t^5}$

e $(5m^4)^2$

f $3a^5b^3 \times 2ab^7$

10 Evaluate:

a v^0

b $6v^0$

c $(6v)^0$

d $6v^0 + 1$

11 Write the meaning of:

a $x^{\frac{1}{2}}$

b $5x^{\frac{1}{2}}$

c $(5x)^{\frac{1}{2}}$

d $x^{\frac{1}{3}}$

e $5x^{\frac{1}{3}}$

12 Write the meaning of:

a z^{-3}

b $3z^{-3}$

c $(3z)^{-3}$

13 Simplify:

a $y^{-3} \times y^7$

b $e^6 \div e^{-3}$

c $(n^{-4})^4$

d $6b^{-2} \times 3b^8$

e $4k^{-5} \div 2k^{-5}$

14 State whether the following are true or false.

a $4q^0 = 4$

b $a^6 \div a^8 = a^2$

c $6m^5 \div 2m^5 = 3m$

d $5b^{-2} = \frac{5}{b^2}$

e $4q^{\frac{1}{2}} = 2\sqrt{q}$

15 Expand:

a $6(2v - 4w)$

b $a^3(2a^2 + 3a)$

c $-3(5x + 2)$

16 Expand and simplify:

a $6(m - 2) + 5(2m + 5)$

b $4(3a - b) - (2a - b)$

B Collecting and analysing data

Exercise 1B

- 1 The table below shows the monthly and annual rainfall for Sydney (Observatory Hill) from 2002 to 2011. Measurements are to the nearest millimetre.

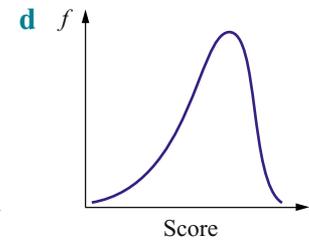
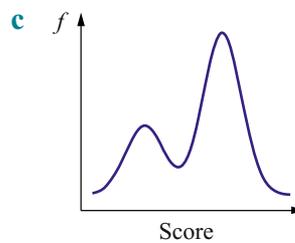
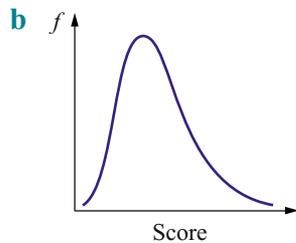
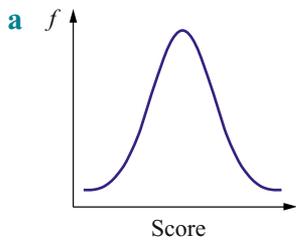
Rainfall for Sydney (mm)

Year	J	F	M	A	M	J	J	A	S	O	N	D	Annual
2002	98	348	45	68	93	28	24	20	22	6	32	75	860
2003	14	59	132	192	349	76	58	43	6	103	109	60	1200
2004	51	129	101	33	8	39	44	153	60	234	67	76	995
2005	68	125	154	33	48	79	63	2	51	43	125	25	816
2006	121	51	40	10	40	177	140	86	192	17	45	74	994
2007	45	108	65	180	10	511	67	152	41	27	170	123	1499
2008	57	258	63	147	3	127	90	44	99	67	73	54	1083
2009	25	128	61	153	126	130	53	6	16	180	13	67	956
2010	36	239	51	30	168	147	115	27	42	85	130	83	1154
2011	54	18	192	206	136	94	282	52	72	37	148	78	1369

- a In this time period, in which year was the annual rainfall:
- highest?
 - lowest?
- b How much rain fell in:
- July 2006?
 - June 2007?
 - October 2011?
- c Which month had the highest rainfall in:
- 2006?
 - 2011?
- d Which month had the lowest rainfall in:
- 2005?
 - 2008?
- e Which year had the wettest:
- February?
 - June?
 - October?
- f Which year had the driest:
- January?
 - June?
 - December?



2 State whether the shape of each distribution is symmetrical, positively skewed, negatively skewed or bimodal.



3 The numbers of goals per match scored by two soccer teams are shown in the tables below.

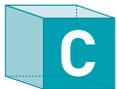
Team A

Number of goals	Number of matches
0	6
1	8
2	4
3	0
4	2

Team B

Number of goals	Number of matches
0	3
1	5
2	6
3	4
4	2

- a** Display this information in:
- i** a back-to-back histogram
 - ii** a parallel dot plot.
- b** Comment on the shape of each distribution.
- c** Compare the mean, median and range for these two distributions.
- d** Which team do you think performed better?



C Numbers of any magnitude

Exercise 1C

- 1 Write the following numbers in scientific notation.
- a** 23 000 000 000 **b** 0.000 052
- 2 Write the following as ordinary numbers.
- a** 9.8×10^7 **b** 3.7×10^{-9}
- 3 Explain why $4 \times 10^5 \neq 4^5$.
- 4 Use your calculator to evaluate the following, leaving the answer in scientific notation.
- a** $(3.4 \times 10^4) \times (4.8 \times 10^9)$ **b** $(5.6 \times 10^{10}) \div (3.5 \times 10^5)$
- c** $(4 \times 10^9)^5$ **d** $\sqrt{2.25 \times 10^{16}}$
- 5 Write the following numbers in order from smallest to largest.
 3.8×10^{15} , 4.6×10^{13} , 7.7×10^{-16} , 3.1×10^{-12}
- 6 Write the value of the digit 7 in each of the following numbers.
- a** 753.6 **b** 1407.2 **c** 76.45 **d** 7564

- 7** Round:
- a** 3470 to the nearest hundred **b** 7956 to the nearest ten
c 37.5 to the nearest whole number **d** 69 900 to the nearest thousand.
- 8** Round 7.2681 to:
- a** 1 decimal place **b** 2 decimal places **c** 3 decimal places.
- 9** Round:
- a** 4.288 correct to 2 decimal places **b** 39.97 correct to 1 decimal place.
- 10** A number was rounded to the nearest 10 and the answer was 50.
- a** What is the smallest the number could have been?
b What is the largest the number could have been? Discuss.
c Write a mathematical statement that shows the range of possible numbers.
- 11** Write the first significant figure in each of the following numbers.
- a** 160 **b** 3.201 **c** 0.006 51
- 12** Round 67.30591 to the following number of significant figures.
- a** 1 **b** 2 **c** 3 **d** 4 **e** 5
- 13** How many significant figures are there in each of the following numbers?
- a** 957 **b** 0.03 **c** 7.500 **d** 0.0035 **e** 142 000
- 14** Explain the difference between measurements of 4.65 m and 4.650 m.
- 15** How many nanowatts in 3.6 milliwatts?
- 16** Convert 43 000 metres to:
- a** kilometres **b** megametres.
- 17** The time taken for a student to complete a task in an aptitude test was measured to be 21 s.
- a** Find the limit of reading of the measuring instrument used.
b Determine the greatest possible error in the measurement.
c What are the limits of accuracy of the measurement?
d Write a mathematical statement that shows the range of values within which the true time lies.



- 18** The length and breadth of a rectangle were measured as 8 cm and 10 cm to the nearest centimetre.
- a** Calculate the perimeter of the rectangle using these measurements.
b Write down the greatest possible error in each of these measurements.
c Hence find the limits of accuracy of the length and the breadth.
d Calculate the lower and upper limits of the true perimeter.
e Find the maximum error in the answer in part **a**.
- 19** The length and breadth of a rectangle were measured to be 12.6 cm and 7.9 cm. Give a reasonable estimate of the area of this rectangle using the level of accuracy of the measurements.



Financial mathematics

Exercise 1D

- Dan earns \$423.76 per week. How much does he earn per:
a fortnight? **b** year? **c** month?
- Convert a salary of \$54 700 p.a. to the equivalent salary per:
a week **b** fortnight **c** month.
- Olivia works a 36-hour week and is paid \$25.40 per hour. How much does she earn for a week in which she works an additional 6 hours at time-and-a-half and 2 hours at double time?
- Terry earns \$710 per week. He is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Calculate his total pay for this holiday period.
- Joanne sews buttons on shirts in a clothing factory. She is paid \$0.33 per shirt. Calculate her income for a week in which she completed the following number of shirts: Monday 172, Tuesday 189, Wednesday 203, Thursday 194 and Friday 188.
- Benita sells printers. She is paid a retainer of \$180 per week plus a commission of 1.5% of sales. How much does she earn in a week in which her sales are \$33 240?
- Dennis works as a casual in a coffee shop. He gets paid \$19.40 for any hours worked from Monday to Friday, \$24.27 per hour for Saturdays and \$25.36 for Sundays. Calculate how much he earns for a week in which he works 10 hours between Monday and Friday, 4 hours on Saturday and 6 hours on Sunday.
- John's gross weekly income is \$752 per week. The deductions from his salary each week are tax \$126, superannuation \$36.78, health insurance \$41.20 and savings \$50. Calculate his take-home pay each week.
- Calculate the simple interest on \$15 000 if it is invested at 6% p.a. for:
a 3 years **b** 15 months.
- An electrical goods store offers a discount of 14% for cash purchases. Find the cash price of a toaster marked at \$67.
- List the advantages and disadvantages of using a credit card to purchase goods.
- An outdoor furniture setting costing \$1788 can be bought on terms for \$300 deposit and 24 monthly instalments of \$90.04.
a Calculate the cost of buying the furniture on terms.
b How much interest is paid?
- A washing machine costing \$1598 can be bought on the following terms: deposit \$200, the balance to be repaid over 2 years by 24 equal monthly repayments. Simple interest is charged on the balance at 15% p.a. Calculate:
a the balance owing
b the interest charged on the balance owing
c the monthly repayment.

- 14 Use the table below to calculate the monthly repayments on a loan of \$17 000 for 5 years at 10.5% p.a.

Amount per \$1000 borrowed

Loan term (months)	Annual interest rate								
	10.0%	10.5%	11.0%	11.5%	12.0%	12.5%	13.0%	13.5%	14.0%
12	87.9159	88.1486	88.3817	88.6151	88.8488	89.0829	89.3173	89.5520	89.7871
18	60.0571	60.2876	60.5185	60.7500	60.9820	61.2146	61.4476	61.6811	61.9152
24	46.1449	46.3760	46.6078	46.8403	47.0735	47.3073	47.5418	47.7770	48.0129
30	37.8114	38.0443	38.2781	38.5127	38.7481	38.9844	39.2215	39.4595	39.6984
36	32.2672	32.5204	32.7387	32.9760	33.2143	33.4536	33.6940	33.9353	34.1776
42	28.3168	28.5547	28.7939	29.0342	29.2756	29.5183	29.7621	30.0071	30.2532
48	25.3626	25.6034	25.8455	26.0890	26.3338	26.5800	26.8275	27.0763	27.3265
54	23.0724	23.3162	23.5615	23.8083	24.0566	24.3064	24.5577	24.8104	25.0647
60	21.2470	21.4939	21.7424	21.9926	22.2444	22.4979	22.7531	23.0098	23.2683

- 15 A-One Car Hire Co. charges \$70 per day all up with unlimited kilometres to rent a new Corolla. B-One Car Rentals charges \$52 per day plus 12 cents per kilometre travelled. Which company is cheaper if you are likely to travel the following distances each day?
- a 60 km b 100 km c 150 km d 500 km
- 16 Calculate the GST included in the price of a pair of shoes costing \$149, price including GST.



Area, surface area and volume

Exercise 1E

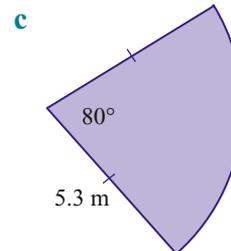
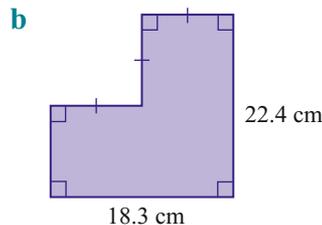
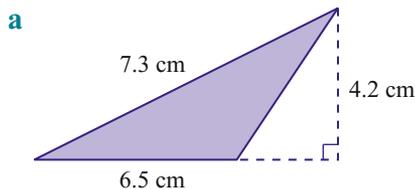
- 1 Complete the following.

a $5.13 \text{ ha} = \underline{\hspace{2cm}} \text{ m}^2$

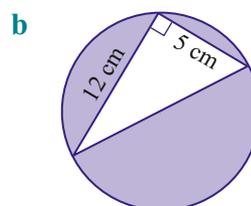
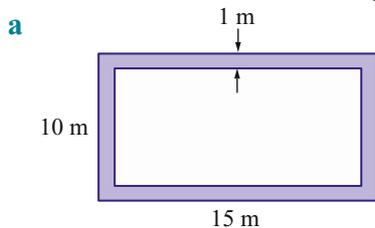
b $4 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ mm}^2$

c $2700 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$

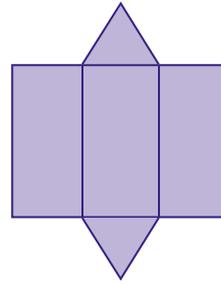
- 2 Find the area of these shapes.



- 3 Find the shaded area of each shape.

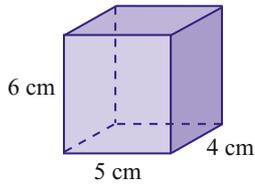


4 Name the solid whose net is shown.

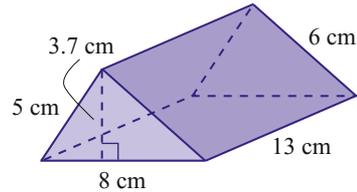


5 Calculate the surface area of each prism.

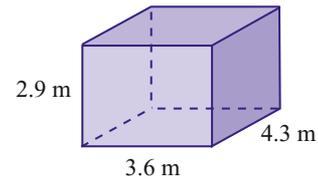
a



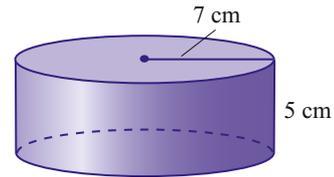
b



6 A closed metal tank, with the dimensions shown, is to be constructed. What area of metal is required for the tank?

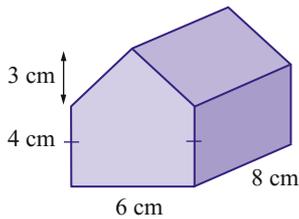


7 Find the surface area and volume of this closed cylinder.

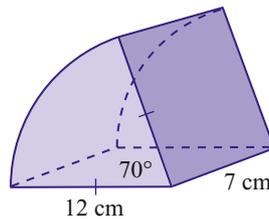


8 Find the volumes of the following solids.

a



b



F Probability

Exercise 1F

1 Four hundred car drivers were randomly selected and asked the country in which their car was manufactured. The results are shown in the table.

- Complete the table.
- Using this data, estimate the probability that another driver chosen at random will own a car manufactured in Australia.
- Estimate the probability that the driver's car has been manufactured in Japan.

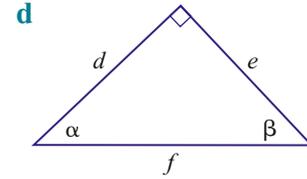
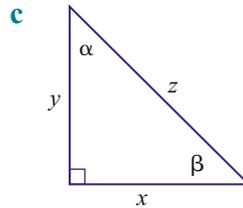
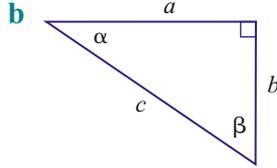
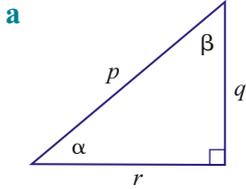
Country	Frequency	Relative frequency	Percentage
Australia	185		
Japan	93		
Korea	72		
Germany	44		
Other	6		

G

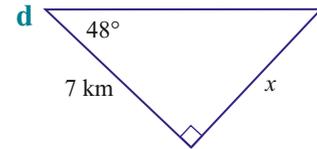
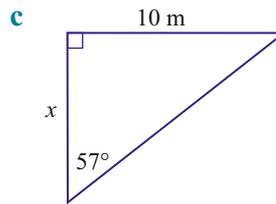
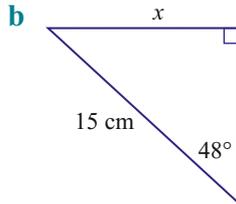
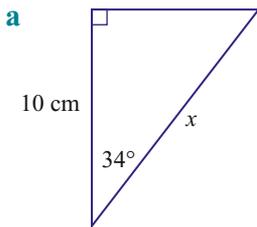
Right-angled trigonometry

Exercise 1G

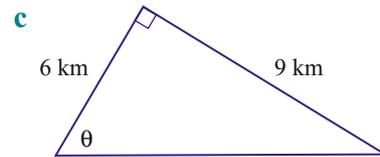
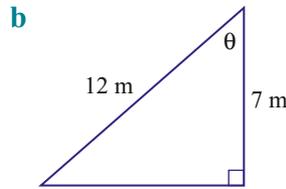
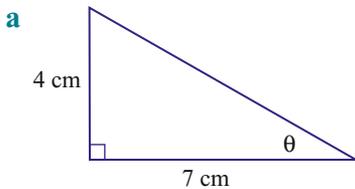
1 Write the expressions for $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\sin \beta$, $\cos \beta$ and $\tan \beta$ for each of the following triangles.



2 Find the length of the side marked x to 3 significant figures.

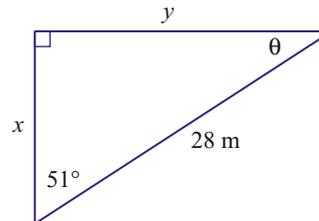


3 Find the value of θ to the nearest minute.

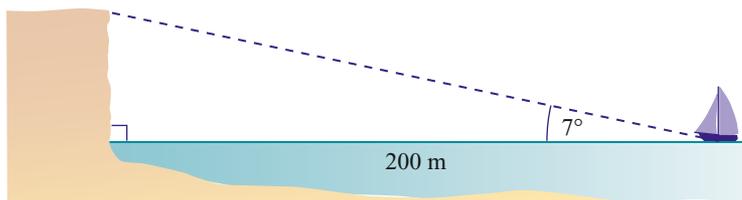


4 Solve the following problems using trigonometry.

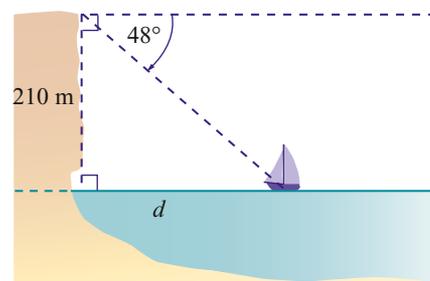
a Find all the sides and angles in the given triangle.



b The diagram shows that the angle of elevation of the top of a cliff from a boat 200 m out to sea is 7° . Calculate the height of the cliff above the boat.

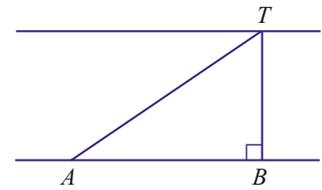


c The angle of depression from the top of a cliff 210 m above sea level to a boat is 48° . Calculate the distance of the boat from the cliff.



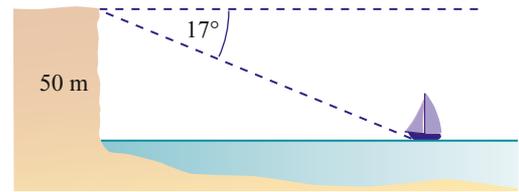
5 Solve the following problems.

a To measure the width of a river, a surveyor finds a point B directly opposite a landmark T , such as a tree, on the bank on the other side of the river. He then moves 20 m along the bank at right angles to BT to a point A . With a theodolite he measures $\angle BAT$ as $72^\circ 11'$. Calculate the width of the river to the nearest metre.



b An isosceles triangle has sides 9 cm, 9 cm and 8 cm long. Find the measure of the base angles of the triangle to the nearest minute.

c From the top of a vertical cliff 50 m high, the angle of depression to a boat straight out to sea is 17° . How far is the boat from the foot of the cliff, to the nearest metre?

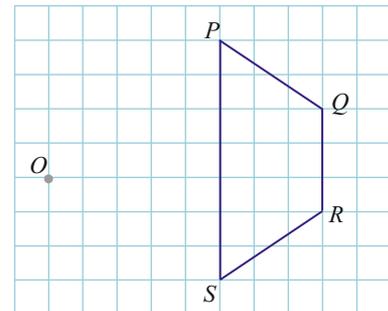


d Danielle travels 14 km west and 8 km south. Find the distance and bearing of Danielle from her starting position.

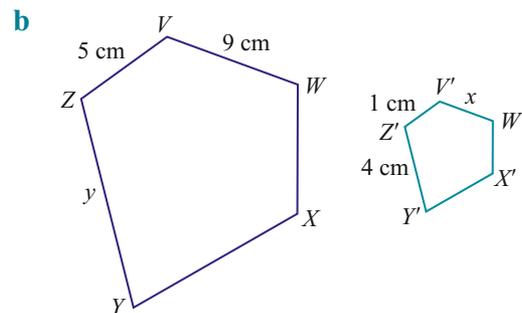
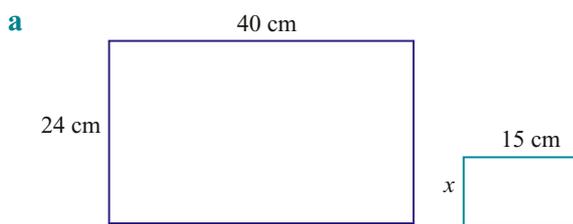
H Similarity

Exercise 1H

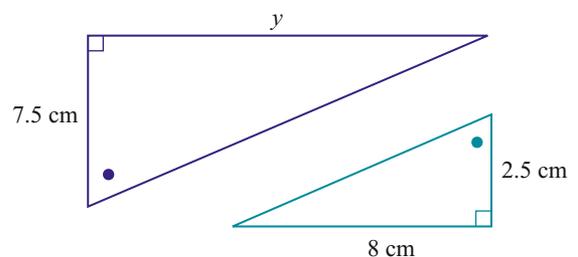
- 1 a Enlarge this figure using an enlargement factor of 2 and O as the centre of enlargement.
 b Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.



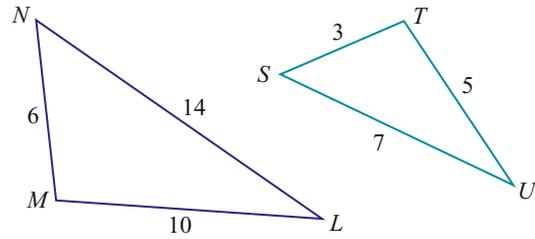
2 Each pair of figures is similar. Find the scale factor and the value of the pronumeral.



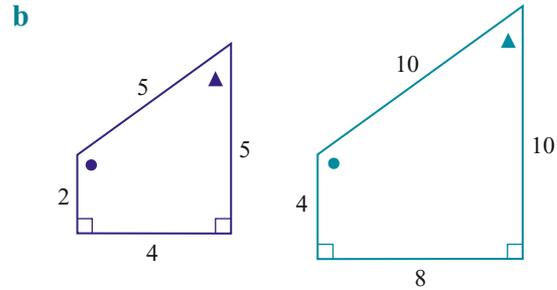
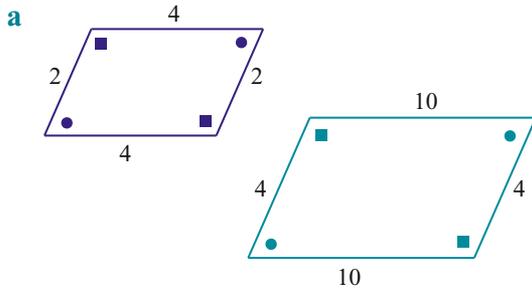
3 In this pair of similar triangles, find the value of the pronumeral.



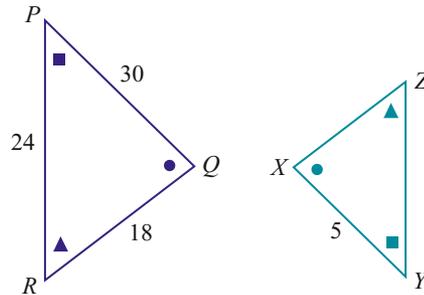
- 4 In $\triangle STU$, which side corresponds to:
- MN ?
 - NL ?
 - ML ?



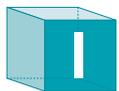
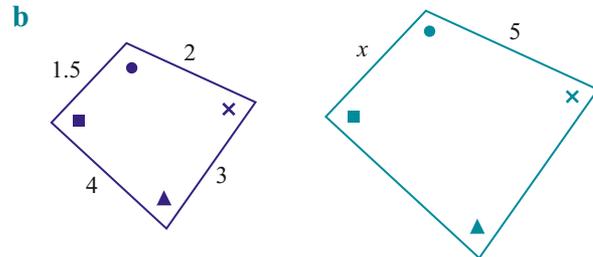
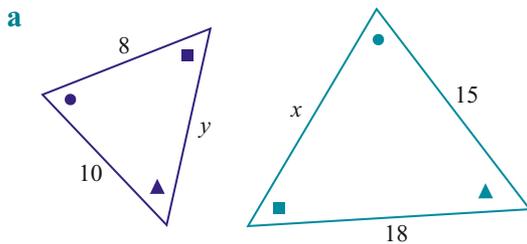
- 5 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.



- 6 a Name the matching (corresponding) sides in these similar triangles.
b Hence find the enlargement factor.



- 7 Find the enlargement factor and hence the length of the unknown sides in these pairs of similar figures.

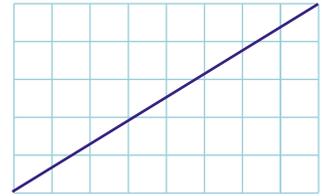


I Linear and non-linear relationships

Exercise 11

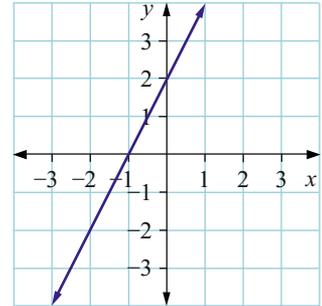
- 1 By drawing a diagram and plotting the points, find the midpoint of the line joining these points.
- $(4, 3)$ and $(12, 3)$
 - $(2, 3)$ and $(2, 9)$
 - $(4, 1)$ and $(8, 10)$
 - $(-4, 3)$ and $(6, -1)$
- 2 Using Pythagoras' theorem, find the distance between these pairs of points.
- $(4, 3)$ and $(10, 5)$
 - $(-5, 5)$ and $(2, 8)$

- 3 In this diagram, draw a right-angled triangle and find the gradient of the line.



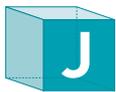
- 4 Find the gradient of the line passing through these pairs of points.
a $(-3, -2)$ and $(6, 4)$ **b** $(-3, 6)$ and $(7, 5)$

- 5 Find the gradient of the line shown on the right.



- 6 Draw a sketch of each of these lines.
a $y = 4$ **b** $y = -3$
c $y = x + 1$ **d** $y = 7 - 3x$
e $x + y = 6$ **f** $x - y = 1$

- 7 Does the point $(4, -3)$ lie on the line $y = 3x - 15$? Explain your answer.
 8 Draw a neat sketch of the relation $y = x^2$.
 9 A circle has centre $(0, 0)$ and radius $\sqrt{7}$ units. Sketch the circle and write its equation.

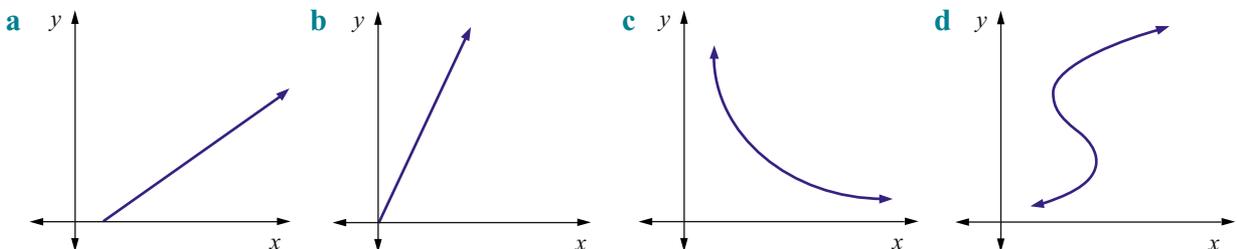


Proportions and rates

Exercise 1J

- 1 Convert:
a 4 mL/min to L/day **b** 50 km/h to m/s **c** 6.3 c/s to \$/h **d** 7 m/s to km/h
- 2 Determine whether the following quantities are in direct proportion, inverse proportion or neither.
a the radius of a circle and the area of the circle
b the time taken to lay a brick wall and the number of bricklayers (assuming they all work at the same rate)
c the thickness of a book and the number of pages (ignore the cover)

- 3 Do the following graphs show direct proportion, inverse proportion or neither?



- 4 Determine whether or not the variables in the table are in direct linear proportion.

a

x	4	9	15
y	3.2	7.2	12

b

x	5	16	18
y	12.5	40	45

5 Determine whether or not the variables in these equations are in direct linear proportion.

a $y = 1.36k$

b $P = \frac{3M}{7}$

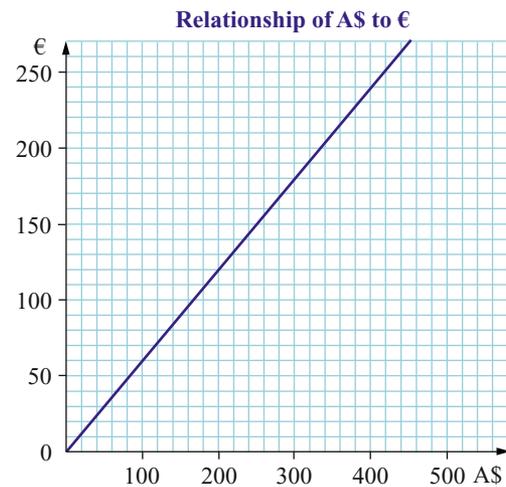
6 Use the graph converting Australian dollars (A\$) into euros (€) to convert:

a A\$400 to euros

b €120 to A\$

7 Given that the variables in the following table are in direct linear proportion, calculate the missing values.

x	5	4	
y	6		7.2



Equations and inequalities

Exercise 1K

1 Solve for x .

a $4 - 7x = 4$

b $\frac{x}{5} = -3$

c $\frac{x}{7} + 3 = -5$

d $\frac{3x - 2}{5} = 4$

2 If $y = \frac{3x - 5}{2}$ find:

a y when $x = \frac{1}{3}$

b x when $y = \frac{7}{2}$

3 If $y = 6 - 5(4 - x)$, find x when $y = 0$.

4 Solve:

a $-2(4x + 3) = 17$

b $7(3t + 1) = t - 4$

c $7(3p - 7) = 4(1 - p)$

d $x - 5(2x + 1) - 3 = 2$

e $\frac{4}{3} = \frac{x}{7}$

f $\frac{1}{4}(3x - 1) = -11$

g $\frac{3 - 4x}{3} = \frac{x + 5}{4}$

h $\frac{3x - 4}{3} - \frac{2 - 4x}{5} = 1$

i $\frac{x + 1}{5} + x = \frac{3 - x}{3} + 2$

5 Solve these problems.

a When 12 is added to twice a number, the answer is 10. Find the number.

b If a number is decreased by 3 then multiplied by 5, the result is 1 more than three times the number. Find the number.

6 The velocity of an object is given by $v^2 = u^2 + 2as$.

a Find v when $u = 10$, $a = -5$ and $s = 2$.

b Find u when $v = 20$, $a = 5$ and $s = 12$.

7 Solve:

a $x - 4 > 5$

b $-3x > 12$

c $4 - 3x \leq -7$

d $\frac{x}{3} + 3 \geq 0$

e $\frac{4 - 3x}{2} \geq 3$

f $6 - x \geq 5 - 4(x - 1)$



2

Surface area and volume

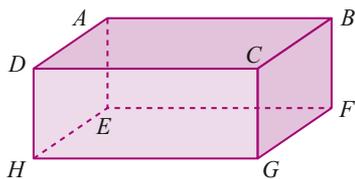
This chapter deals with calculating the surface areas and volumes of right prisms and cylinders.

After completing this chapter you should be able to:

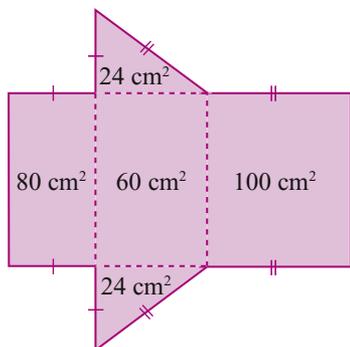
- ▶ solve problems involving the surface areas and volumes of right rectangular and triangular prisms
- ▶ calculate the surface areas and volumes of cylinders
- ▶ solve problems involving the surface areas and volumes of composite solids.

Diagnostic test

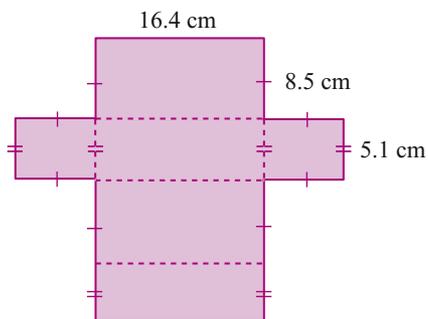
Questions 1 to 3 refer to the prism below.



- The face that corresponds to the face $ABFE$ is:
 A $DCHG$ B $BFGC$
 C $AEHD$ D $DCGH$
- The face that corresponds to the face $CBFG$ is:
 A $ABFE$ B $AEHD$
 C $DAEH$ D $DGCH$
- The face that corresponds to the face $ABCD$ is:
 A $EFHG$ B $HEFG$
 C $EFGH$ D $DCGH$
- The surface area of the net shown is:

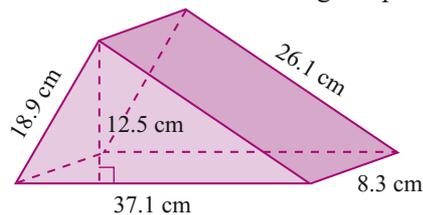


- The surface area of the net shown is:
 A 288 cm^2 B 324 cm^2
 C 232 cm^2 D 264 cm^2



- The surface area of the net shown is:
 A 643.28 cm^2 B 532.78 cm^2
 C 634.28 cm^2 D 537.28 cm^2

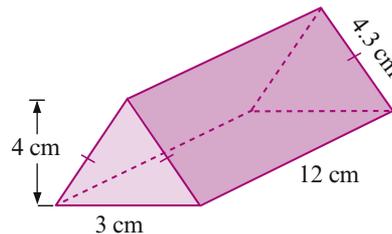
- The surface area of this triangular prism is:



- The surface area of this triangular prism is:
 A 1329.5 cm^2 B 1052.43 cm^2
 C 913.305 cm^2 D 1145.18 cm^2

- The surface area of a cube of side length 8.7 cm is:
 A 302.76 cm^2 B 658.503 cm^2
 C 454.14 cm^2 D 378.45 cm^2

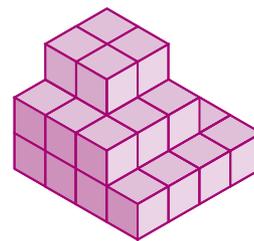
- A chocolate bar is shown below. The surface area of the wrapping to the nearest cm^2 is:



- A chocolate bar is shown below. The surface area of the wrapping to the nearest cm^2 is:
 A 150 cm^2 B 151.2 cm^2
 C 151 cm^2 D 152 cm^2

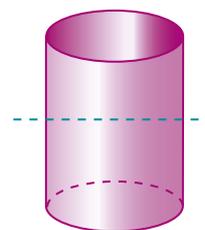
- This solid is made from 1 cm^3 cubes. The volume of the solid is:

- This solid is made from 1 cm^3 cubes. The volume of the solid is:
 A 18 cm^3
 B 29 cm^3
 C 16 cm^3
 D 30 cm^3



- The cross-section of this solid is:

- The cross-section of this solid is:
 A an oval
 B a cylinder
 C an ellipse
 D a circle

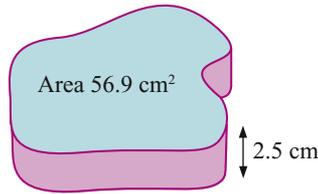


- A solid that has a circular cross-section is called a:

- A solid that has a circular cross-section is called a:
 A pyramid B cylinder
 C rectangular prism D box

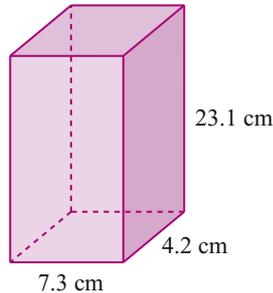
12 The volume of the solid shown is:

- A 61.4 cm³
- B 268.2 cm³
- C 307.26 cm³
- D 142.25 cm³

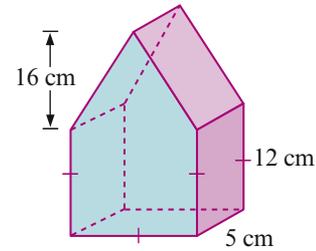


13 The volume of this solid to the nearest cm³ is:

- A 709 cm³
- B 708 cm³
- C 705 cm³
- D 710 cm³



14 The volume of this composite solid is:



- A 1680 cm³
- B 1350 cm³
- C 1200 cm³
- D 960 cm³

15 The volume of a cube of side length 8.7 cm is:

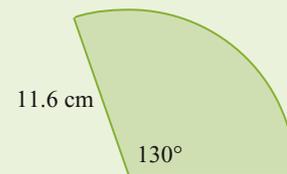
- A 302.76 cm³
- B 658.503 cm³
- C 454.14 cm³
- D 378.45 cm³

The diagnostic test questions refer to outcomes ACMMG210 and ACMMG218.

A Area review

EXAMPLE 1

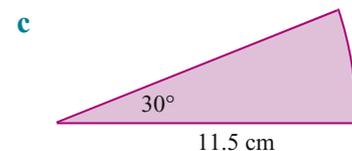
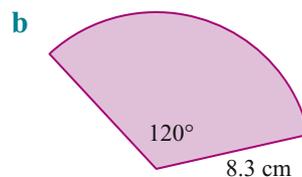
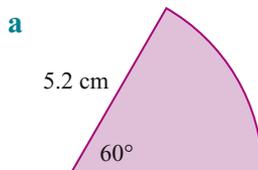
Find the area of this sector to 1 decimal place.

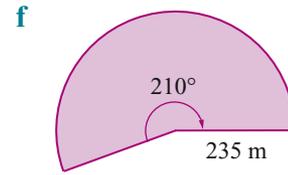
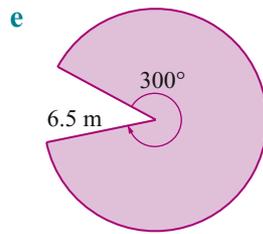
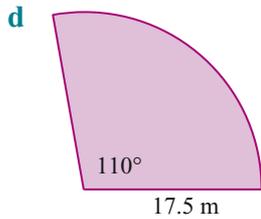


Solve	Think	Apply
$A = \frac{\theta}{360} \times \pi r^2$ $= \frac{130}{360} \times \pi \times 11.6^2$ $\approx 152.7 \text{ cm}^2$	Angle of a sector = 130°	Find the area of a sector of a circle by comparing its angle with the angle of a full circle, 360°. $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{sector angle}}{360}$ $A \text{ (of sector)} = \frac{\text{sector angle}}{360} \times \text{area of circle}$

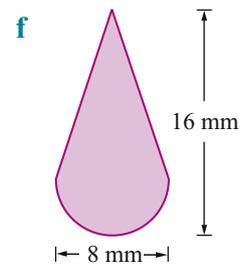
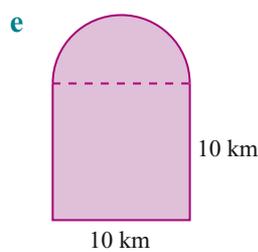
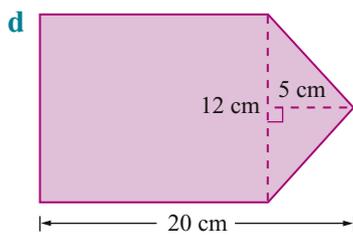
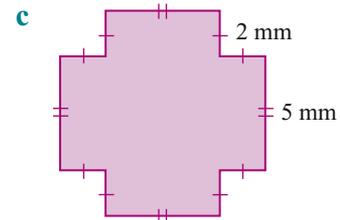
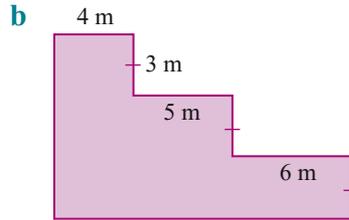
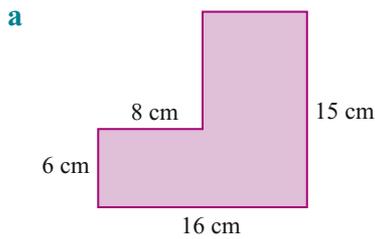
Exercise 2A

1 Calculate the area of each sector correct to 1 decimal place.



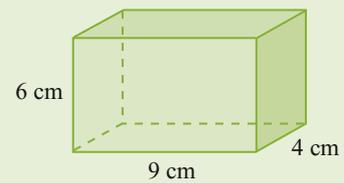


2 Find the areas of the following shapes.



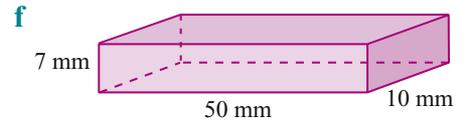
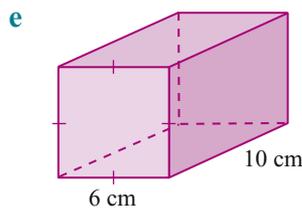
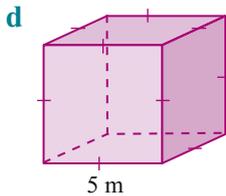
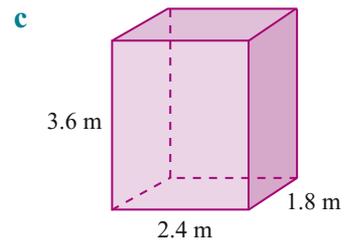
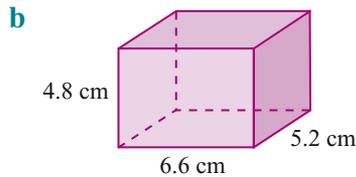
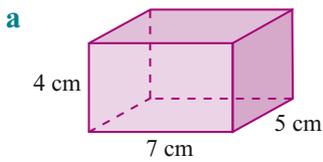
EXAMPLE 2

- a** Draw a net for this rectangular prism, showing the lengths of its edges.
b Calculate the surface area of the prism.



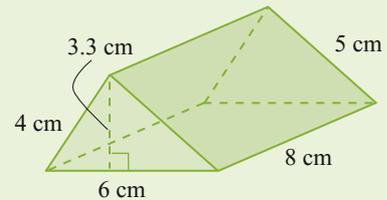
	Solve/Think	Apply
a		<p>Draw the net, identify the faces, and transfer the edge lengths from the solid to the net. Calculate the area of each face and sum these areas.</p>
b	$SA = (\text{bottom} + \text{top}) + (\text{front} + \text{back}) + (\text{left side} + \text{right side})$ $= (9 \times 4) \times 2 + (9 \times 6) \times 2 + (6 \times 4) \times 2$ $= 228 \text{ cm}^2$	

- 3** For each of the following rectangular prisms:
- Draw a net of each prism and mark its edge lengths.
 - Calculate the surface area.



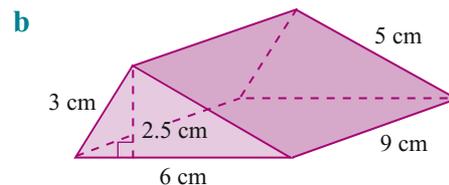
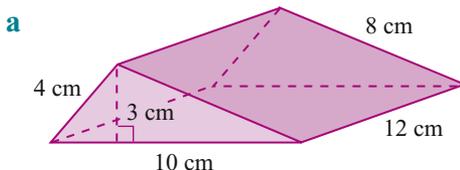
EXAMPLE 3

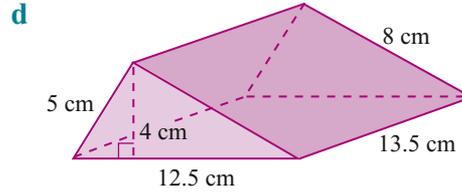
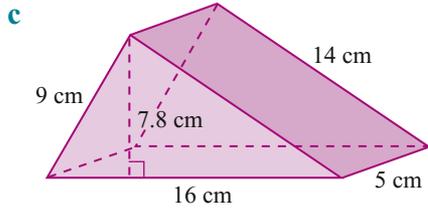
- Draw a net of this triangular prism, marking its edge lengths.
- Calculate the surface area of the prism.



	Solve/Think	Apply
a		Draw the net, identify the faces and transfer the edge lengths from the solid to the net. Calculate the area of each face and sum these areas.
b	$ \begin{aligned} SA &= \text{area of 2 triangles} + \text{area of 3 rectangles} \\ &= \left(\frac{1}{2} \times 6 \times 3.3\right) \times 2 + 8 \times 4 + 8 \times 6 + 8 \times 5 \\ &= 139.8 \text{ cm}^2 \end{aligned} $	

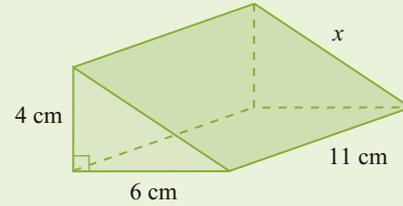
- 4** Calculate the surface area of each of the following triangular prisms.





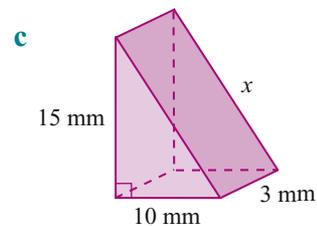
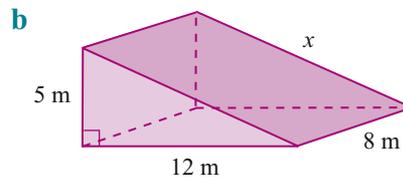
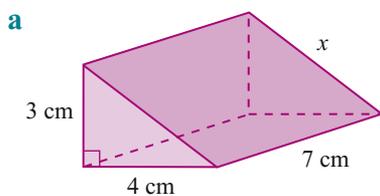
EXAMPLE 4

- a** Calculate the length of the unknown edge of this triangular prism.
- b** Draw a net of the prism.
- c** Calculate its surface area.



	Solve/Think	Apply
a	By Pythagoras' theorem: $x^2 = 4^2 + 6^2 = 52$ $\therefore x = \sqrt{52}$ $\approx 7.2 \text{ cm (1 decimal place)}$	Calculate the unknown edge using Pythagoras' theorem.
b		Draw the net and calculate the surface area as before.
c	$SA = \left(\frac{1}{2} \times 6 \times 4\right) \times 2 + 11 \times 4 + 11 \times 6 + 11 \times 7.2$ $= 213.2 \text{ cm}^2$	

- 5** For each triangular prism:
- i** Find the length of the unknown edge.
 - ii** Calculate the surface area.



B

Surface areas of right cylinders

The formula for the surface area of a cylinder can be developed by ‘cutting’ the cylinder and laying it out flat. The net then gives a formula for the surface area.

The curved part forms a rectangle of length $2\pi r$ and breadth h .

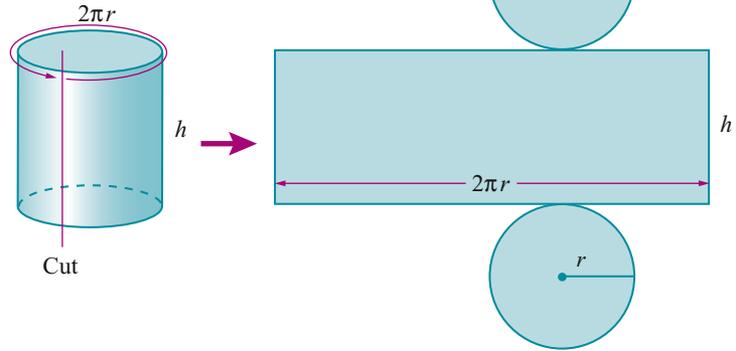
$$\begin{aligned} A &= 2(\text{area of circle}) + \text{area of rectangle} \\ &= 2 \times \pi r^2 + 2\pi r \times h \\ &= 2\pi r^2 + 2\pi rh \end{aligned}$$

The surface area of a closed cylinder is:

$$A = 2\pi r^2 + 2\pi rh$$

The surface area of a cylinder open at both ends is:

$$A = 2\pi rh$$

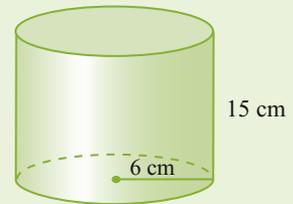


Remember: Area of a circle is πr^2 .



EXAMPLE 1

Find the surface area of this closed cylinder.

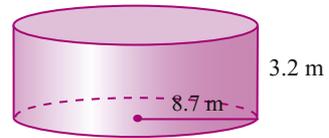


Solve	Think	Apply
$\begin{aligned} \text{Surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi \times 6^2 + 2\pi \times 6 \times 15 \\ &\approx 791.7 \text{ cm}^2 \text{ (1 decimal place)} \end{aligned}$	Radius = 6 cm Height = 15 cm	For a cylinder closed at both ends: $SA = 2\pi r^2 + 2\pi rh$

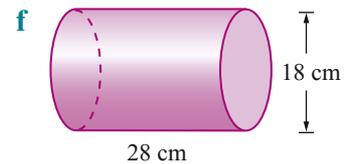
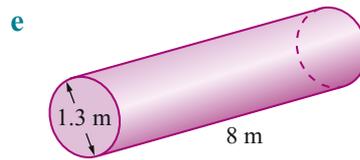
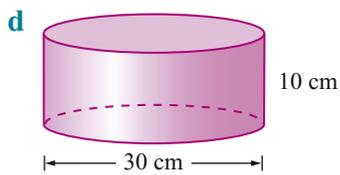
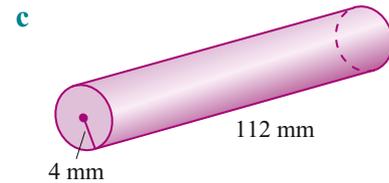
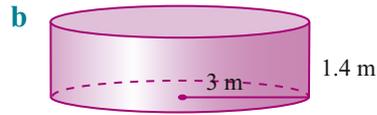
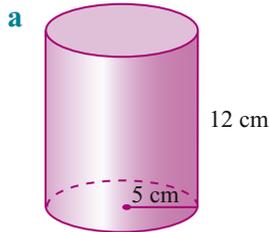
Exercise 2B

- 1 Complete to find the surface area of this closed cylinder.

$$\begin{aligned} \text{Surface area} &= 2\pi r^2 + 2\pi ______ \\ &= 2\pi \times ______^2 + 2\pi \times 8.7 \times ______ \\ &\approx ______ \text{ m}^2 \text{ (1 decimal place)} \end{aligned}$$

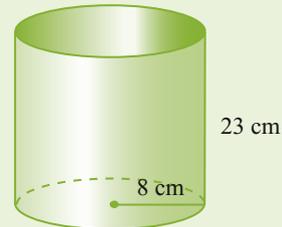


- 2 Find the surface areas of these closed cylinders to the nearest whole number.



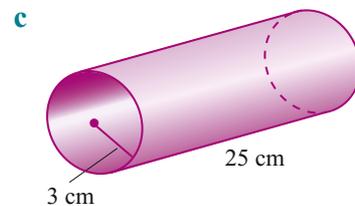
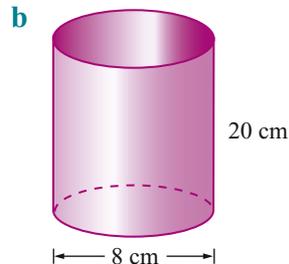
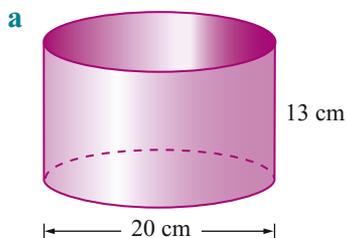
EXAMPLE 2

Find the surface area of this open cylinder.



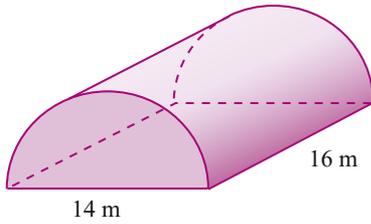
Solve/Think	Think	Apply
$\begin{aligned} \text{Surface area} &= 2 \times \pi \times 8 \times 23 \\ &= 1156.1 \text{ cm}^2 \end{aligned}$	Radius = 8 cm Height = 23 cm	For an open cylinder: $SA = 2\pi rh$

- 3 Find the surface areas of the following open cylinders.



- 4 a Determine how much paint is required to cover the outside of a cylindrical container 12 m long with diameter 10 m if each litre of paint covers 15 m^2 .
- b Which has the greater surface area: a cylinder of length 15 cm and radius 8 cm, or a cylinder of length 8 cm and radius 10 cm?

- 5 Find the surface area, correct to 1 decimal place where necessary, of:
- an open can with radius of 4 cm and height of 15 cm
 - an open-ended pipe of 10 cm radius and 5 m long
 - the closed solid shown below.



- 6 Determine the cost of painting the exterior walls and top of a cylindrical wheat silo that is 40 m high and 20 m in diameter, given that each litre of paint costs \$7.25 and covers 8 m².
- 7 Find the cost of making 125 cylindrical tennis ball containers that have diameter 7 cm and height 21 cm, given that the metal costs \$4.50 per square metre (metal base but open at the top).

EXAMPLE 3

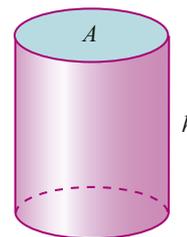
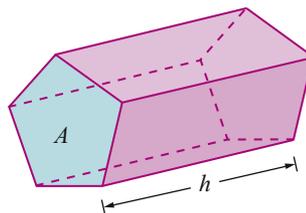
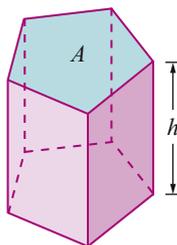
An open cylinder of radius of 8 cm has a curved surface area of 1000 cm². Find its height.

Solve	Think	Apply
$1000 = 2 \times \pi \times 8 \times h$ $= 16\pi \times h$ $h = \frac{1000}{16\pi}$ $= 19.9 \text{ cm (1 decimal place)}$	To solve $1000 = 16\pi \times h$, divide both sides by 16π .	Substitute the given information into $SA = 2\pi rh$ and solve the resulting equation.

- 8 Find the height of an open cylinder of radius 10 cm and curved surface area of 2000 cm².
- 9 Find the radius of an open cylinder of height 5 cm and curved surface area of 1500 cm².



Volumes with uniform cross-sections



The volume of a right prism (or cylinder) is given by:

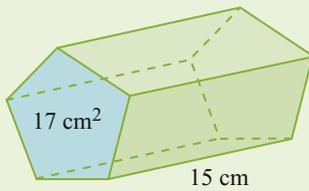
$$V = A \times h$$

where A is the area of the base (or cross-sectional area) and h is the perpendicular height.

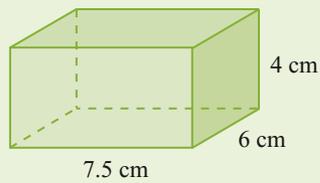
EXAMPLE 1

Find the volumes of these solids.

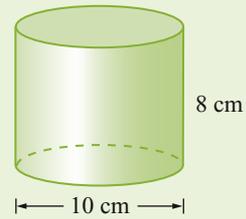
a



b



c



	Solve	Think	Apply
a	$V = 17 \times 15$ $= 255 \text{ cm}^3$	Area of base = 17 cm^2	For prisms and cylinders use $V = Ah$.
b	$V = (7.5 \times 6) \times 4$ or $(7.5 \times 4) \times 6$ or $(6 \times 4) \times 7.5$ $= 180 \text{ m}^3$	Area of base = $7.5 \times 6 \text{ cm}^2$	Choose any rectangle as the base.
c	$V = (\pi \times 5^2) \times 8$ $= 628.3 \text{ cm}^3$ (1 decimal place)	Area of base = πr^2 $= \pi \times 5^2 \text{ cm}^2$	The base is a circle.

Exercise 2C

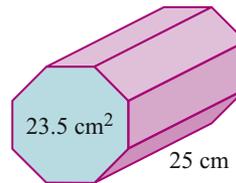
1 Complete to find the volume of this prism.

$$V = A \times h$$

where A is the area of _____ and h is the _____ height.

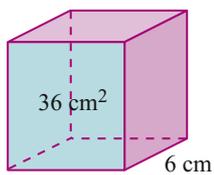
$$V = _ \times 25$$

$$= _ \text{ cm}^3$$

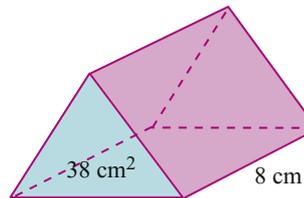


2 Calculate the volumes of these solids.

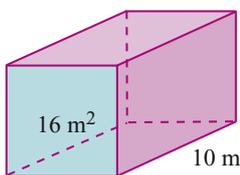
a



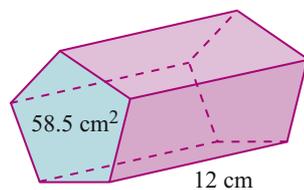
b



c

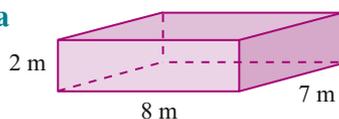


d

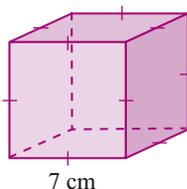


3 Calculate the area of the base and hence find the volume of each solid.

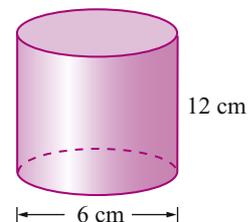
a



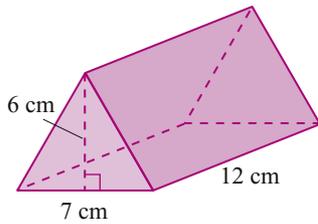
b



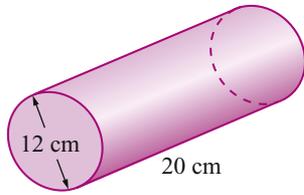
c



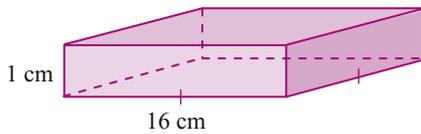
d



e



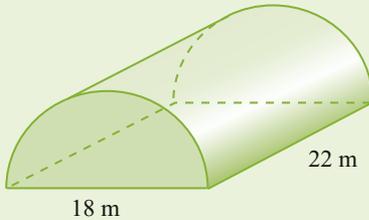
f



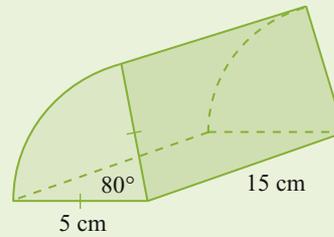
EXAMPLE 2

Calculate the volume of each solid correct to 1 decimal place.

a



b



	Solve	Think	Apply
a	$V = A \times h$ $= \frac{1}{2}\pi r^2 \times h$ $= \frac{1}{2} \times \pi \times 9^2 \times 22$ $= 2799.2 \text{ m}^3$	Diameter = 18 m so radius = 9 m The base is a semicircle so the area of the circle must be halved.	Calculate the area of the base first. Multiply by the height, which must be perpendicular to the base. The solid does not have to stand on the base.
b	$V = A \times h$ $= \frac{80}{360} \times \pi r^2 \times h$ $= \frac{80}{360} \times \pi \times 5^2 \times 15$ $= 261.8 \text{ m}^3$	The base is a sector. $A = \frac{\theta}{360} \times \pi r^2$ where $\theta = 80^\circ$. The height is 15 m.	

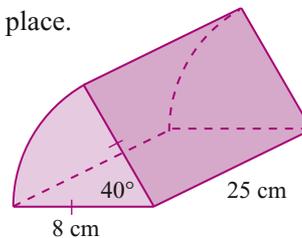
4 Complete to find the volume of this solid correct to 1 decimal place.

$$V = A \times h$$

$$= \frac{\square}{360} \times \square \times r^2 \times h$$

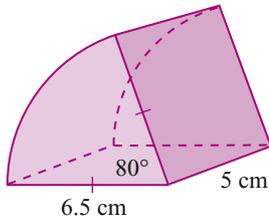
$$= \frac{\square}{360} \times \pi \times \square^2 \times \square$$

$$= \square \text{ cm}^3$$

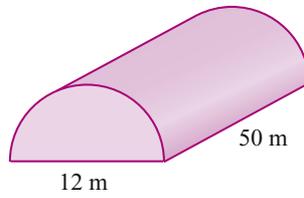


5 Calculate the volume of each solid.

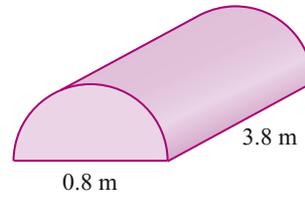
a



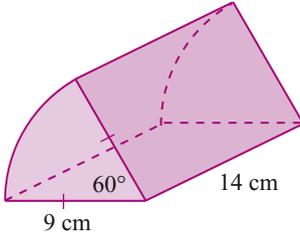
b



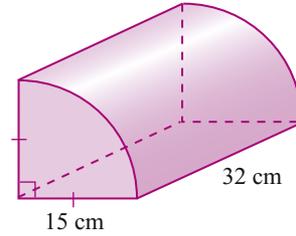
c



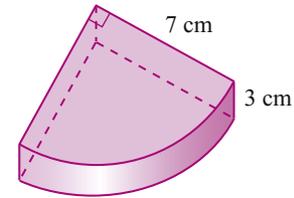
d



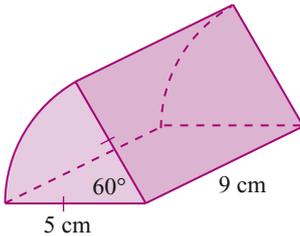
e



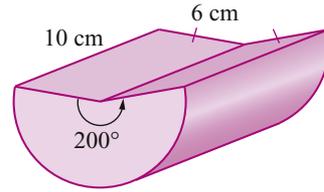
f



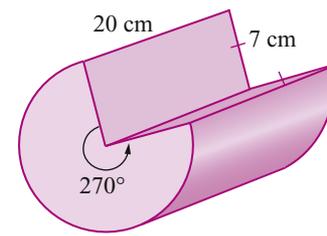
g



h



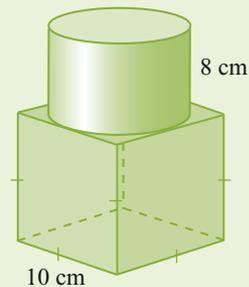
i



D Volumes of composite solids

EXAMPLE 1

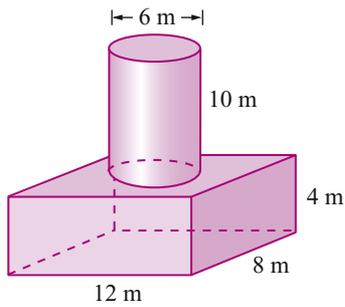
Calculate the volume of this composite solid.



Solve	Think	Apply
<p><i>Cylinder:</i> $V = \pi r^2 \times h$ $= \pi \times 5^2 \times 8$ $= 628.318\dots \text{ cm}^3$</p> <p><i>Cube:</i> $V = Ah$ $= 10 \times 10 \times 10$ $= 1000 \text{ cm}^3$</p> <p>Total volume = $628.318 + 1000$ $= 1628 \text{ cm}^3$ to nearest cm^3</p>	<p>The solid is made up of a cylinder and a cube.</p> <p><i>Cylinder:</i> Radius = $10 \div 2 = 5 \text{ cm}$ Height = 8 cm</p> <p><i>Cube:</i> $l = b = h = 10 \text{ cm}$</p>	<p>Break the composite solid into simpler solids and find the volume of each one separately.</p> <p>Combine the volumes to give the answer.</p>

Exercise 2D

- 1 Complete to find the volume of this composite solid.



The solid is a _____ and a rectangular prism.

Cylinder:

Diameter = 6 m Radius = _____

$$\begin{aligned} V &= \pi r^2 \times \text{height} \\ &= \pi \times \text{_____}^2 \times \text{_____} \\ &= \text{_____ m}^3 \end{aligned}$$

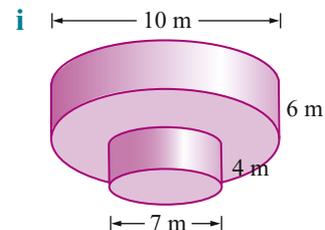
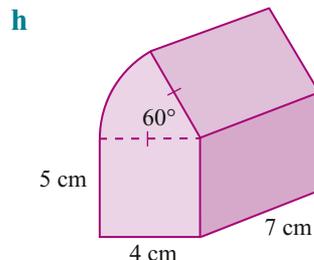
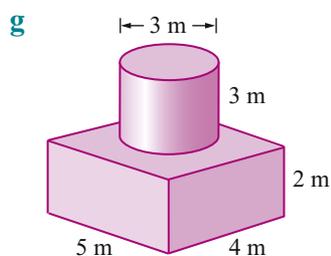
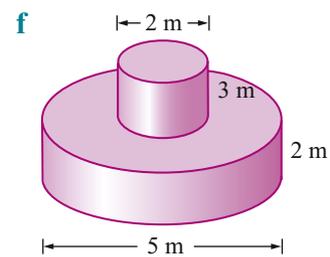
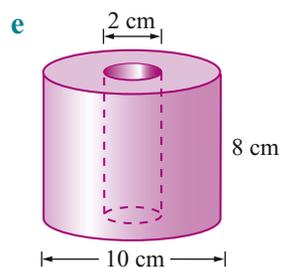
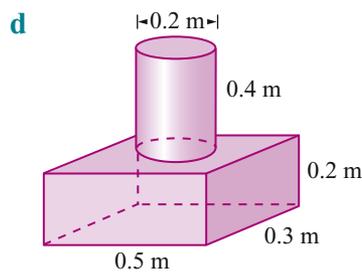
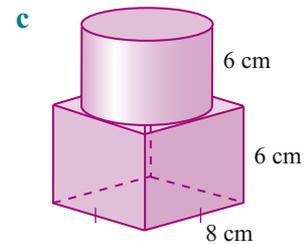
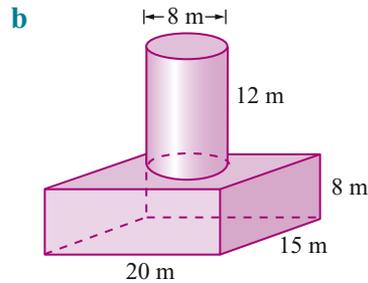
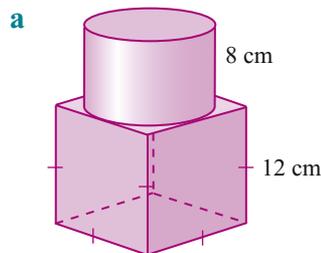
Rectangular prism:

$$\begin{aligned} V &= Ah \\ &= (12 \times \text{_____}) \times 4 \\ &= \text{_____ m}^3 \end{aligned}$$

Total volume = _____ + _____ = _____ m³ to the nearest whole number



- 2 Calculate the volume of each composite solid.

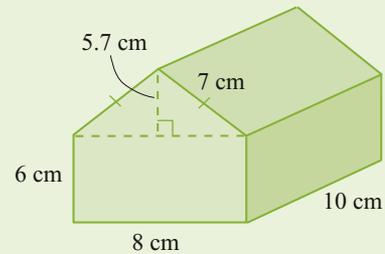


E

Surface areas of composite solids

EXAMPLE 1

Calculate the surface area of the solid shown.



Solve	Think	Apply
$\begin{aligned} \text{Area of front face} &= 8 \times 6 + \frac{1}{2} \times 8 \times 5.7 \\ &= 70.8 \text{ cm}^2 \\ \text{Total surface area} \\ &= 2 \times 70.8 + 2 \times (10 \times 6) + 2 \times (10 \times 7) + 10 \times 8 \\ &= 481.6 \text{ cm}^2 \end{aligned}$	<p>Total surface area = front + back + 4 sides + bottom</p>	<p>Find the total surface area by summing the areas of all the faces of the solid.</p>

Exercise 2E

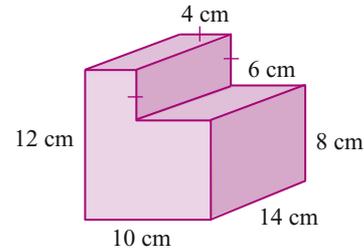
1 Complete to find the surface area of this solid.

Total surface area = front + back + side + 4 rectangles

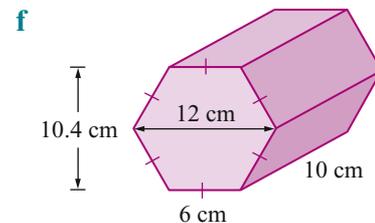
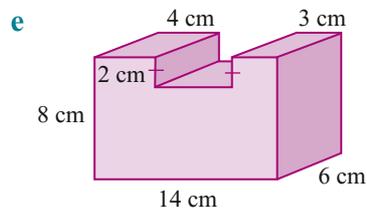
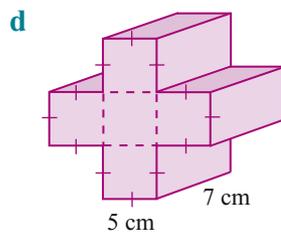
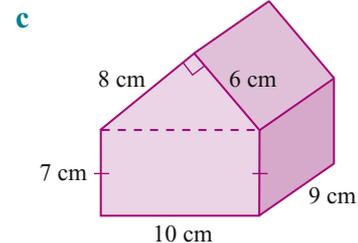
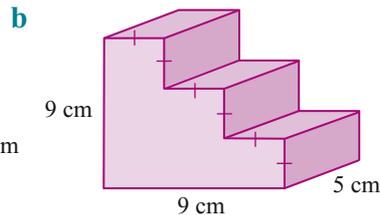
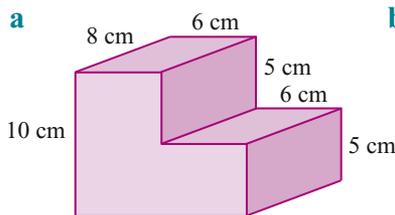
Area of front face = $__ \times 8 + 4 \times __$

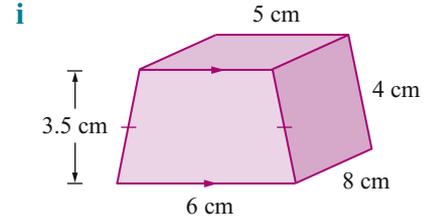
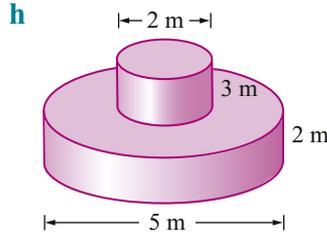
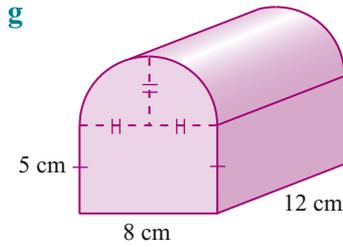
$$= 96 \text{ cm}^2$$

Total surface area = $2 \times 96 + 12 \times __ + 14 \times 8 + 14 \times __ + 14 \times __ + 14 \times 4 + 10 \times __ = __ \text{ cm}^2$



2 Calculate the surface areas of the following solids.



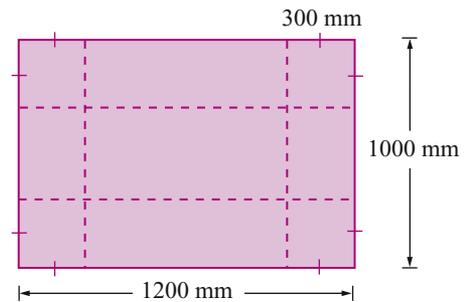


Problems with surface area and volume

Exercise 2F

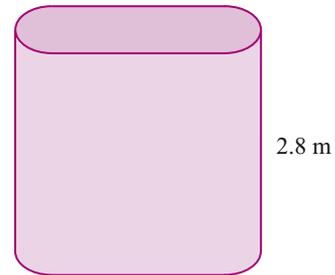
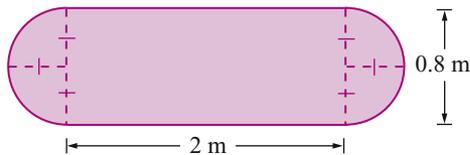
- 1** A sheet of cardboard 1200 mm by 1000 mm has squares of side-length 300 mm cut from each corner. The sides are folded up to form an open rectangular box.

- a** Calculate its internal surface area.
b What is the volume of the box?

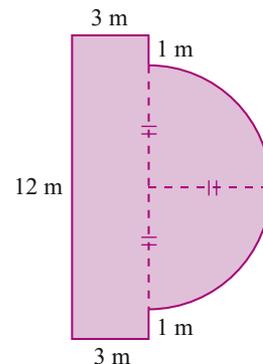


- 2** A carport and workshop are covered by a flat rectangular roof 3.6 m by 11.2 m. All the rain that falls on the roof is collected in a water tank. If 3 mm of rain falls on the roof, how much water will be collected in the tank? ($1 \text{ m}^3 = 1000 \text{ L}$)

- 3** The cross-section of this closed rainwater tank is shown beside it.

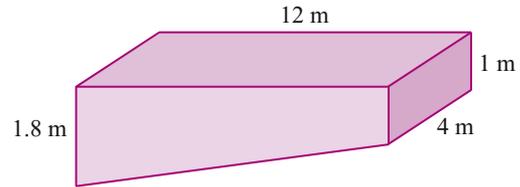


- a** Calculate the area of this cross-section.
b Hence calculate the volume of the tank.
c What is the capacity of the tank if 1 m^3 holds 1000 L?
d The tank is completely made from sheet steel that costs $\$40/\text{m}^2$. What is the cost of the steel to make this tank?
- 4** The diagram shows the design for a concrete driveway.
- a** Calculate its area.
b A concrete contractor charges $\$70/\text{m}^2$ to supply and lay concrete. How much will he charge for this job? Give the answer to the nearest dollar.
c If the concrete needs to be 100 mm deep, calculate the volume of concrete needed, in cubic metres.



- 5** The cylindrical roller for a cricket pitch is 1.5 m wide and has a radius of 0.3 m.
- Calculate the curved surface area of the roller.
 - What is the minimum number of revolutions the roller would have to make to roll the cricket pitch once if the pitch is 20 m long and 3 m wide? (Ignore any revolutions needed to turn the roller around.)

- 6** A backyard swimming pool has dimensions as shown.
- Calculate the volume of the pool.
 - How long will it take to fill the pool with water from a garden hose that can supply water at the rate of 7.5 L/min? (Use 1 m³ holds 1000 L of water.)
 - What is the cost of filling the pool if water costs \$2.75/kL?



- 7** A hollow iron pipe is 2 m long. Its external diameter is 10 cm and it is 1 cm thick. Calculate the weight of the pipe if iron weighs 8.2 g/cm³.

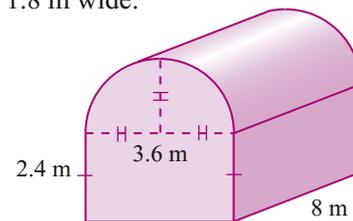
- 8** A fish tank has a rectangular base 40 cm by 20 cm. Water is poured in to a height of 24 cm.
- What is the volume of water in the tank?
 - If a further 2 litres of water is poured into the tank, by how much will the water level rise?

- 9** A pontoon with base 3 m by 3 m is floating on a lake. When a man swims out and climbs onto it the pontoon sinks 1 cm. If 1 L of water weighs 1 kg, what is the weight of the man? (*Hint*: Archimedes' principle tells us that the weight of the man is equal to the weight of water displaced.)

- 10** A 50 cm³ block of metal is made into wire of diameter 1 mm. How long will the wire be?



- 11** A greenhouse with the dimensions shown is to be covered on the top and sides only (not the front and back) with shade cloth. The shade cloth comes in 20 m rolls and is 1.8 m wide.

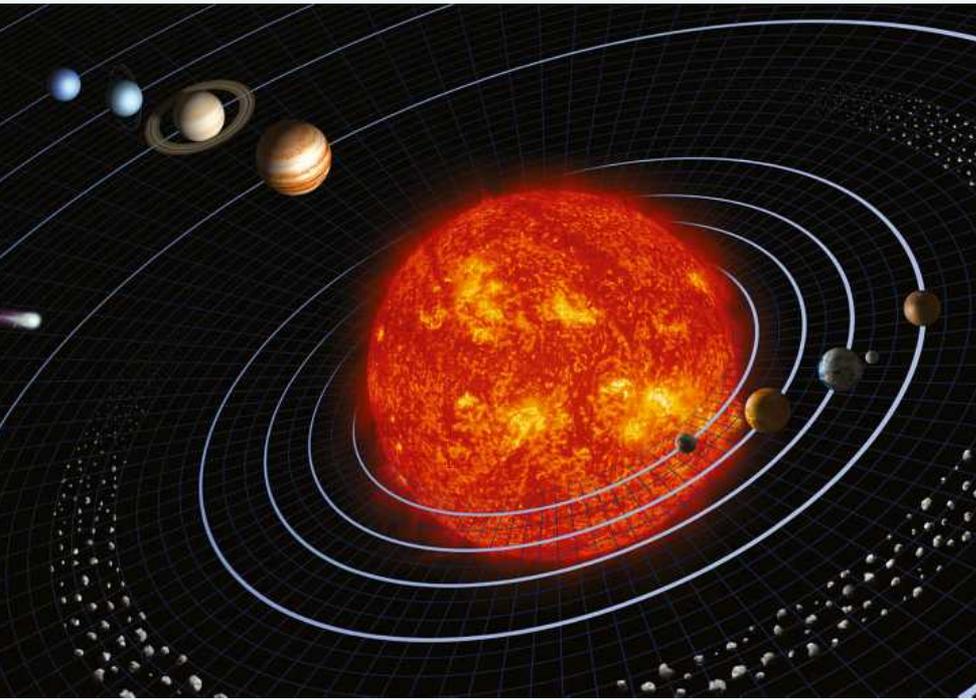


- Calculate the number of linear metres of shade cloth needed.
- How many rolls will be needed?

Language in mathematics

Johann Kepler (1571–1630)

Johann Kepler was born in the German town of Wurttemberg. As a child he was small and suffered from ill health, but he was recognised as being intelligent. He was given a scholarship to attend the University of Tubingen, where he studied first for the Lutheran ministry and then science. He studied under a master in astronomy who believed in, and taught, the Copernican theory that Earth rotated around its own axis and around the Sun. Kepler taught mathematics in Graz from 1594.



In 1600 he went to Prague and became assistant to Tycho Brahe, an important astronomer. After Brahe's death, Kepler succeeded him as astronomer and mathematician to the emperor. Kepler had access to Brahe's extensive records of observations and calculations.

Kepler believed in the Copernican theory, and became one of the founders of modern astronomy. He developed three fundamental laws of planetary motion, now known as Kepler's Laws, in 1609. These proposed, among other things, that the Sun was at the centre of our planetary system, and that the orbits of the planets were elliptical rather than circular. Sixty years later these laws helped Newton to develop his Universal Law of Gravitation.

Kepler also suggested that tides are caused by the Moon's gravitational pull on the seas. He produced tables giving the positions of the Sun, Moon and planets, which were used for about 100 years. In 1611 he proposed an improved refracting telescope, and later he suggested a reflecting telescope that was developed by Newton.

- a** How old was Kepler when he died?

b When and where did Kepler teach mathematics?

c Describe the development of Kepler's ideas concerning planetary motion.

d Research Kepler's three laws.

e For how long were Kepler's tables of positions of the Sun, Moon and planets used?

f How are tides formed?
- Rearrange these words to form a sentence.

a a circle a semicircle A half is of.

b a is of quarter quadrant A circle a.

c may way than Composite more in areas one be found.

- Use every third letter to find the sentence.

W D T R F H T G E H Y A U J R N H E G B A V F O E D F S W A A Z R D F H H J O L P
M O E B Q A U Z D S F Y O I J R B W A Q A K C G I H J T I I E O P I L L S G F H D
E A S K L A X F V B T H Q H S O E Y A P E F R H K O I P D N M U A E C S D T C G O
H N F B E T W X H A U E I O D A G I B H A J K G N H O D S N W E A D F L T Y S

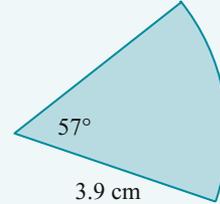
Terms

area	circle	composite	diameter	formula	prism
quadrant	quadrilateral	radius	right	sector	semicircle

Check your skills

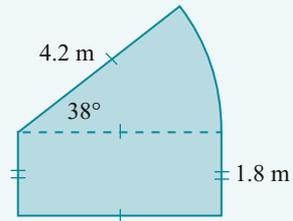
1 The area of this sector is closest to:

- A 3.87 cm^2 B 7.6 cm^2
 C 15.6 cm^2 D 222.3 cm^2



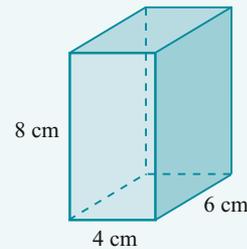
2 The area of this shape is closest to:

- A 5.85 m^2 B 9.02 m^2
 C 13.41 m^2 D 14.8 m^2



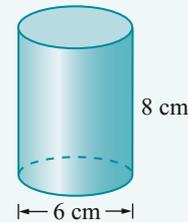
3 The surface area of this prism is:

- A 104 cm^2 B 184 cm^2
 C 192 cm^2 D 208 cm^2



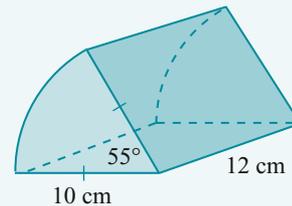
4 The surface area of this closed cylinder is:

- A 150.8 cm^2 B 207.3 cm^2
 C 226.2 cm^2 D 179.1 cm^2



5 The volume of this solid is:

- A 576 cm^3 B 144 cm^3
 C 115 cm^3 D 58 cm^3



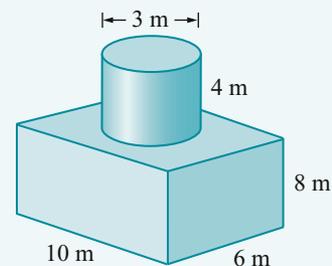
Use this diagram for questions 6 and 7.

6 The volume of this composite solid is:

- A 593.1 cm^3 B 555.4 cm^3
 C 517.7 cm^3 D 508.3 cm^3

7 The surface area of the solid is:

- A 225.7 cm^2 B 232.8 cm^2
 C 413.7 cm^2 D 420.8 cm^2



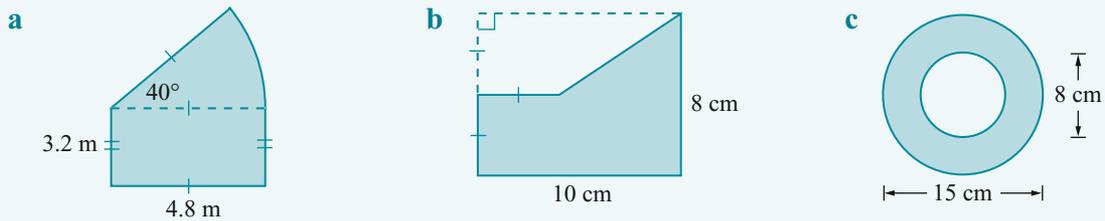
- 8** A lidded wooden box, $15\text{ cm} \times 8.5\text{ cm} \times 6\text{ cm}$, is to be lacquered inside and out with two coats of lacquer. Ignoring the thickness of the wood, the total area to be lacquered is:
- A** 537 cm^2 **B** 2148 cm^2 **C** 1074 cm^2 **D** 2685 cm^2

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4	5	6	7	8
Section	A	B	C	D	E	F

2A Review set

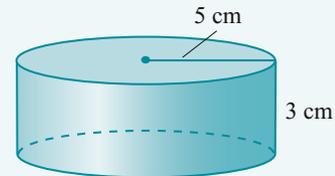
- 1** Calculate the shaded areas correct to 1 decimal place.



- 2** Calculate the surface area of each prism.



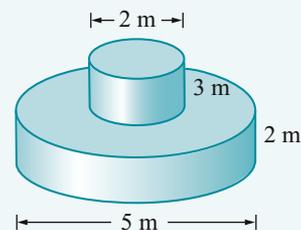
- 3** Calculate the surface area and volume of this closed cylinder.



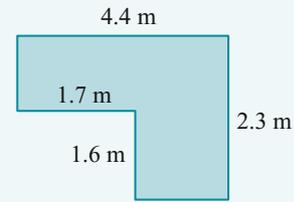
- 4** Calculate the volumes of the following solids.



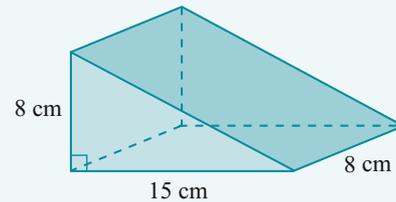
- 5** Calculate the surface area and volume of this solid.



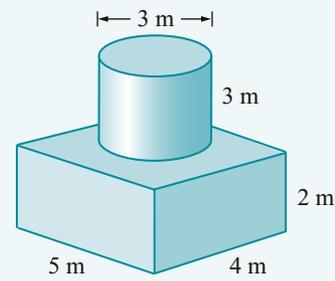
- 1 Deborah's family room is shown opposite. Calculate the cost of carpet-tiling the room if the carpet tiles costs \$119.80 per square metre.



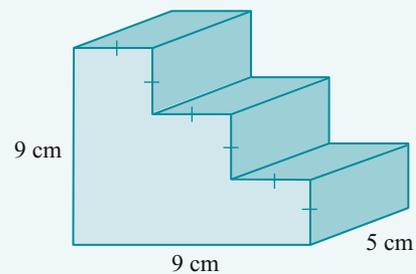
- 2 A door wedge shaped as shown is to be painted. What is the total area to be painted?



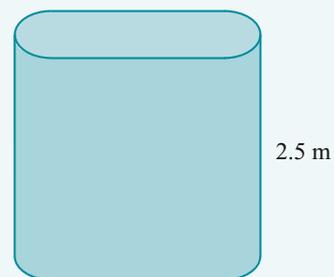
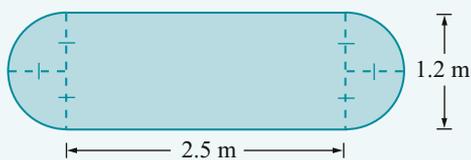
- 3 Calculate the surface area and volume of a closed cylinder with diameter 2.4 m and height 1.8 m.
- 4 Calculate the surface area of this solid.



- 5 Calculate the volume of this solid.



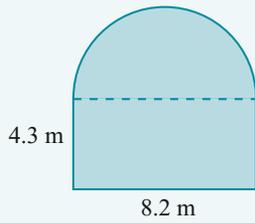
- 6 The cross-section of this rainwater tank is shown beside it.



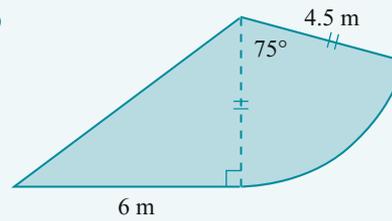
- Calculate the area of this cross-section.
- Hence calculate the volume of the tank.
- What is the capacity of the tank if 1 m^3 holds 1000 L?
- The tank was made from sheet steel that costs $\$45/\text{m}^2$. What was the cost, to the nearest dollar, of the steel used to make this tank?

- 1 Calculate the area of each shape correct to 1 decimal place.

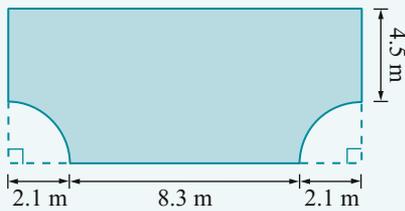
a



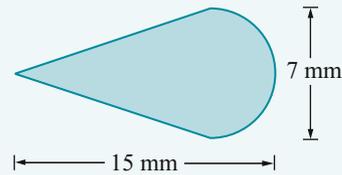
b



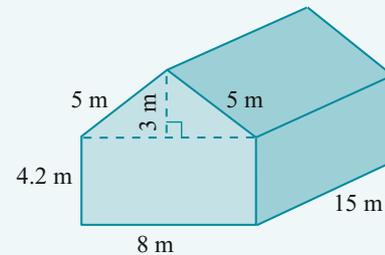
c



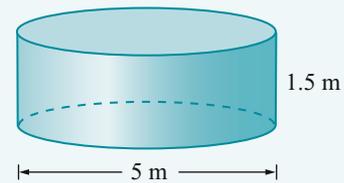
d



- 2 a The army shed shown is to be painted in camouflage colours. What area is to be camouflaged?
b Calculate the volume of the shed.

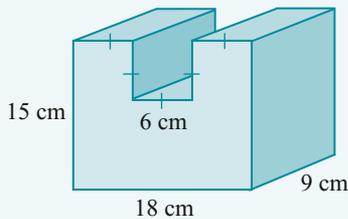


- 3 Calculate the surface area of this closed cylinder.

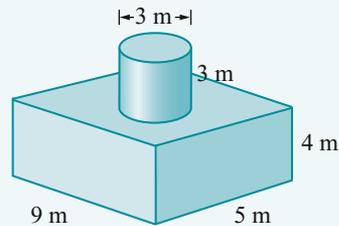


- 4 Calculate the surface area and volume of each solid.

a



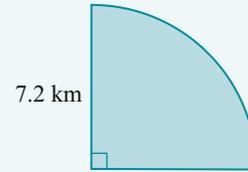
b



- 5 A hollow steel pipe is 5 m long. Its external diameter is 20 cm and it is 1.5 cm thick. Calculate the weight of the pipe to the nearest gram given that steel weighs 8.2 g/cm^3 .

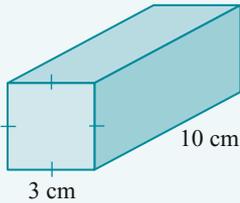


- 1 A river delta is shaped roughly like a quadrant, as shown. Calculate the population of the delta if 225 people per square kilometre live there.

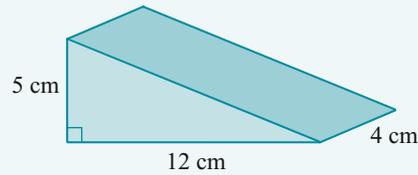


- 2 Calculate the surface area of each prism.

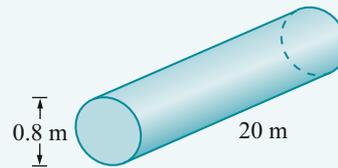
a



b

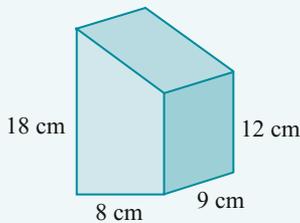


- 3 Calculate the surface area and volume of this open cylinder.

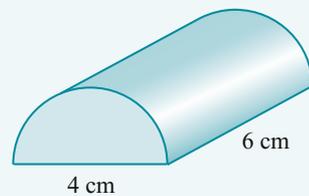


- 4 Calculate the surface area and volume of each closed solid.

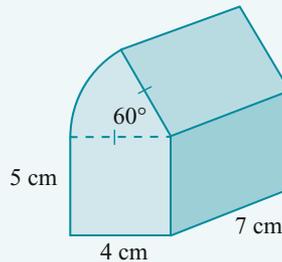
a



b

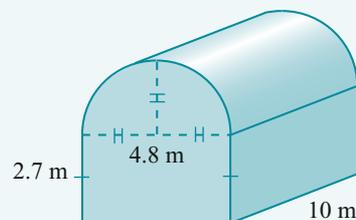


- 5 Calculate the surface area and volume of this solid.



- 6 A greenhouse with the dimensions shown is to be covered on the top and sides only (not the front and back) with shade cloth. The shade cloth comes in 15 m rolls and is 1.8 m wide.

- a Calculate the number of linear metres of shade cloth needed.
b How many rolls will be needed?





3

Algebra

This chapter deals with algebraic fractions and using the distributive law to expand and factorise algebraic expressions.

After completing this chapter you should be able to:

- ▶ perform the four operations with algebraic fractions including those with pronumerals in the denominator and those with binomial numerators
- ▶ apply the distributive law to the expansion of algebraic expressions
- ▶ and collect like terms where appropriate
- ▶ factorise algebraic expressions by taking out the highest common algebraic factor.

NSW Syllabus references: 5.2 N&A Algebraic techniques, 5.2 N&A Equations (part)

Outcomes: MA5.2-1WM, MA5.2-3WM, MA5.2-6NA

Number & algebra – ACMNA213, ACMNA230, ACMNA232



Diagnostic test

- 1 $5a - 2b + 3a - 4b =$
A $2a - 6b$ B $8a - 2b$
C $8a - 6b$ D $8a + 2b$
- 2 $7x \times 3y =$
A $73xy$ B $21xy$
C $10xy$ D x^7y^3
- 3 $\frac{12m}{8} =$
A $4m$ B $\frac{3m}{8}$ C $\frac{3m}{2}$ D $\frac{3}{2}$
- 4 $3m^4n^2 \times 4m^5n^3 =$
A $12m^9n^5$ B $12m^{20}n^6$
C $7m^9n^5$ D $7m^{20}n^6$
- 5 $\frac{a^7b^4}{a^3b} =$
A $(ab)^{24}$ B $(ab)^7$ C $\frac{a^7}{3b^4}$ D a^4b^3
- 6 $(3y^5)^3 =$
A $3y^{15}$ B $3y^8$
C $27y^{15}$ D $27y^8$
- 7 $(4p)^0 =$
A 1 B 4 C 0 D $4p$
- 8 $-5(2z - 3) =$
A $-7z + 2$ B $-7z + 15$
C $-10z + 15$ D $-10z - 15$
- 9 $6 + 4(a + 3) =$
A $4a + 18$ B $4a + 9$
C $10a + 30$ D $10a + 3$
- 10 When fully factorised $18x - 24y =$
A $2(9x - 12y)$ B $3(6x - 8y)$
C $6(3x - 24y)$ D $6(3x - 4y)$

The diagnostic test questions refer to outcomes ACMNA190, ACMNA191, ACMNA192 and ACMNA212.



A

Algebraic fractions

An algebraic fraction is one in which the numerator or denominator or both contain pronumerals.

For the numerical fraction $\frac{3}{4}$, the denominator (4) indicates the number of equal parts into which the whole has been divided and the numerator (3) indicates the number of these equal fractional parts.

In the algebraic fraction $\frac{5a}{9b}$, the denominator indicates that the whole has been divided into $9b$ equal parts and the numerator indicates that there are $5a$ of these equal parts.

For the fraction $\frac{9b}{5a}$, the denominator indicates that the whole has been divided into $5a$ equal parts and the numerator indicates that there are $9b$ of these equal parts.

The reciprocal of a fraction is the number by which the fraction must be multiplied to give a result of 1.

For example, $\frac{3}{4} \times \frac{4}{3} = 1$. Hence $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$ (and $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$).

We say that $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals of each other.

As $\frac{5a}{9b} \times \frac{9b}{5a} = 1$, $\frac{5a}{9b}$ and $\frac{9b}{5a}$ are reciprocals of each other.

EXAMPLE 1

Complete the following equivalent fractions.

a $\frac{4}{5} = \frac{\square}{35}$

b $\frac{3n}{7} = \frac{\square}{14}$

c $\frac{5x}{y} = \frac{\square}{3y}$

d $\frac{3a}{4b^2} = \frac{\square}{12b^7}$

	Solve	Think	Apply
a	$\frac{4}{5} = \frac{28}{35}$	As $5 \times 7 = 35$, $\frac{4}{5} \times \frac{7}{7} = \frac{28}{35}$	Multiply the numerator and the denominator by the same number.
b	$\frac{3n}{7} = \frac{6n}{14}$	As $7 \times 2 = 14$, $\frac{3n}{7} \times \frac{2}{2} = \frac{6n}{14}$	
c	$\frac{5x}{y} = \frac{15x}{3y}$	As $y \times 3 = 3y$, $\frac{5x}{y} \times \frac{3}{3} = \frac{15x}{3y}$	
d	$\frac{3a}{4b^2} = \frac{9ab^5}{12b^7}$	As $4b^2 \times 3b^5 = 12b^7$, $\frac{3a}{4b^2} \times \frac{3b^5}{3b^5} = \frac{9ab^5}{12b^7}$	Multiply the numerator and the denominator by the same algebraic term.

Exercise 3A

1 Complete the following equivalent fractions.

a $\frac{2y}{5} = \frac{\square}{15}$

b $\frac{4m}{3} = \frac{\square}{6}$

c $\frac{5a}{2} = \frac{\square}{12}$

d $\frac{7a}{4} = \frac{\square}{20}$

e $\frac{5k}{12} = \frac{\square}{60}$

f $\frac{5m}{2n} = \frac{\square}{6n}$

g $\frac{3t}{4w} = \frac{\square}{16w}$

h $\frac{8}{5q} = \frac{\square}{30q}$

i $\frac{6x}{5y} = \frac{\square}{5yz}$

j $\frac{3a}{4b} = \frac{\square}{4b^2}$

k $\frac{7m}{9n} = \frac{\square}{18np}$

l $\frac{2u}{3w} = \frac{\square}{15wx}$

m $\frac{5a}{4b} = \frac{\square}{4b^2}$

n $\frac{3x}{2y} = \frac{\square}{10y^3}$

o $\frac{3r}{7t} = \frac{\square}{14tz}$

p $\frac{4t}{11r} = \frac{\square}{44r^2t}$

EXAMPLE 2

Reduce each fraction to its simplest form.

a $\frac{36}{48}$

b $\frac{9t}{12}$

c $\frac{6ab}{7a}$

d $\frac{9xy}{15x}$

	Solve	Think	Apply
a	$\frac{36}{48} = \frac{3\cancel{3}6}{4\cancel{4}8} = \frac{3}{4}$	Divide the numerator and the denominator by 12, the HCF of 36 and 48.	Divide the numerator and the denominator by their highest common factor (HCF).
b	$\frac{9t}{12} = \frac{3\cancel{9}t}{4\cancel{1}2} = \frac{3t}{4}$	Divide the numerator and the denominator by 3, the HCF of 9t and 12.	
c	$\frac{6ab}{7a} = \frac{\cancel{6}ab}{7\cancel{a}} = \frac{6b}{7}$	Divide the numerator and the denominator by a, the HCF of 6ab and 7a.	
d	$\frac{9xy}{15x} = \frac{3\cancel{9}xy}{5\cancel{1}5x} = \frac{3y}{5}$	Divide the numerator and the denominator by 3x, the HCF of 9xy and 15x. This is equivalent to dividing the numerator and the denominator by 3, the HCF of 9 and 15, and x, the HCF of xy and x.	

2 Complete the following to reduce each fraction to its simplest form.

a $\frac{8t}{12} = \frac{\square}{3}$

b $\frac{10k}{15} = \frac{2k}{\square}$

c $\frac{11x}{9x} = \frac{\square}{9}$

d $\frac{4xy}{5y} = \frac{4x}{\square}$

e $\frac{12y}{9y} = \frac{\square}{3}$

f $\frac{8a}{10ab} = \frac{4}{\square}$

3 Reduce each fraction to its simplest form.

a $\frac{6x}{12}$

b $\frac{5t}{20}$

c $\frac{10y}{15}$

d $\frac{9b}{12}$

e $\frac{3p}{10p}$

f $\frac{7t}{4t}$

g $\frac{6a}{9a}$

h $\frac{18d}{24d}$

i $\frac{8a}{4ab}$

j $\frac{12pq}{9q}$

k $\frac{12xy}{16xyz}$

l $\frac{4x^3}{6x^4}$

EXAMPLE 3

Reduce each fraction to its simplest form.

a $\frac{10a^4}{8a^2}$

b $\frac{3ab^4}{9a^2b^3}$

	Solve	Think	Apply
a	$\frac{5\cancel{1}0a^4}{4\cancel{8}a^2} = \frac{5a^2}{4}$	Divide the numerator and denominator by $2a^2$, the HCF of $10a^4$ and $8a^2$. This is equivalent to dividing the numerator and denominator by 2, the HCF of 10 and 8, and a^2 , the HCF of a^4 and a^2 .	Divide the numerator and the denominator by their HCF.
b	$\frac{\cancel{3}ab^4}{3\cancel{9}a^2b^3} = \frac{b}{3a}$	Divide the numerator and denominator by $3ab^3$, the HCF of $3ab^4$ and $9a^2b^3$. This is equivalent to dividing the numerator and denominator by 3, the HCF of 3 and 9, a, the HCF of a and a^2 , and b^3 , the HCF of b^4 and b^3 .	

4 Complete the following to reduce each fraction to its simplest form.

a $\frac{7m^4}{8m^3} = \frac{\square}{8}$

b $\frac{4k^3}{5k^6} = \frac{4}{\square}$

c $\frac{5y^3}{10y^2} = \frac{\square}{2}$

d $\frac{16z^4}{12z^6} = \frac{4}{\square}$

e $\frac{7a^2b^3}{6a^3b^2} = \frac{\square}{6a}$

f $\frac{4x^2y^3}{8xy^4} = \frac{\square}{2y}$

5 Reduce each fraction to its simplest form.

a $\frac{7x^5}{6x^3}$

b $\frac{11z^4}{15z^6}$

c $\frac{4a^7}{8a^4}$

d $\frac{8x^5}{6x^2}$

e $\frac{9m^5}{6m^3}$

f $\frac{9ab^2}{15ab}$

g $\frac{5a^3b^8}{4ab^6}$

h $\frac{9x^5y^4}{5x^2y^5}$

EXAMPLE 4

Simplify the following.

a $\frac{5}{6} + \frac{3}{8}$

b $\frac{2t}{3} + \frac{4t}{5}$

c $\frac{7k}{10} - \frac{k}{5}$

d $2y - \frac{4y}{5}$

	Solve	Think	Apply
a	$\frac{5}{6} + \frac{3}{8} = \frac{20}{24} + \frac{9}{24}$ $= \frac{29}{24} = 1\frac{5}{24}$	LCD = 24 $\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$ and $\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$	Find the lowest common denominator (LCD) of the fractions. Change both fractions to equivalent fractions with this denominator. Simplify the result if possible.
b	$\frac{2t}{3} + \frac{4t}{5} = \frac{10t}{15} + \frac{12t}{15}$ $= \frac{22t}{15}$	LCD = 15 $\frac{2t}{3} \times \frac{5}{5} = \frac{10t}{15}$ and $\frac{4t}{5} \times \frac{3}{3} = \frac{12t}{15}$	
c	$\frac{7k}{10} - \frac{k}{5} = \frac{7k}{10} - \frac{2k}{10}$ $= \frac{5k}{10} = \frac{k}{2}$	LCD = 10 $\frac{k}{5} \times \frac{2}{2} = \frac{2k}{10}$	
d	$2y - \frac{4y}{5} = \frac{10y}{5} - \frac{4y}{5}$ $= \frac{6y}{5}$	LCD = 5 $\frac{2y}{1} \times \frac{5}{5} = \frac{10y}{5}$	

6 Complete the following.

a For the fractions $\frac{4m}{5}$ and $\frac{3m}{4}$ the LCD = ____.

$$\frac{4m}{5} \times \frac{\square}{\square} = \frac{\square}{20} \text{ and } \frac{3m}{4} \times \frac{\square}{\square} = \frac{\square}{20}$$

$$\text{Hence } \frac{4m}{5} + \frac{3m}{4} = \frac{\square}{20} + \frac{\square}{20}$$

$$= \frac{\square}{\square}$$

b For the fractions $\frac{19a}{20}$ and $\frac{7a}{10}$ the LCD = ____.

$$\frac{7a}{10} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$\text{Hence } \frac{19a}{20} - \frac{7a}{10} = \frac{19a}{20} - \frac{\square}{\square}$$

$$= \frac{\square}{20} = \frac{a}{4}$$

c For the fraction $\frac{4z}{9}$ and z the LCD = ____.

$$\frac{z}{1} \times \frac{\square}{\square} = \frac{\square}{9}$$

$$\text{Hence } \frac{4z}{9} + z = \frac{4z}{9} + \frac{\square}{9}$$

$$= \frac{\square}{9}$$

d For the fractions $\frac{6c}{5}$ and $\frac{2c}{7}$ the LCD = ____.

$$\frac{6c}{5} \times \frac{\square}{\square} = \frac{\square}{35} \text{ and } \frac{2c}{7} \times \frac{\square}{\square} = \frac{\square}{35}$$

$$\text{Hence } \frac{6c}{5} + \frac{2c}{7} = \frac{\square}{35} + \frac{\square}{35}$$

$$= \frac{\square}{\square}$$

7 Simplify the following.

a $\frac{5x}{11} + \frac{4x}{11}$

b $\frac{5n}{6} + \frac{2n}{3}$

c $\frac{7m}{8} - \frac{m}{3}$

d $\frac{5k}{6} + \frac{3k}{4}$

e $\frac{7b}{8} - \frac{b}{4}$

f $\frac{3a}{5} + \frac{a}{10}$

g $k + \frac{2k}{3}$

h $\frac{4z}{5} - \frac{2z}{3}$

i $4t - \frac{7t}{8}$

j $\frac{2x}{5} - \frac{3x}{10}$

k $\frac{7x}{3} - \frac{2x}{5}$

l $\frac{5t}{7} + \frac{9t}{14}$

EXAMPLE 5

Simplify the following.

a $\frac{2}{3a} + \frac{4}{5a}$

b $\frac{5k}{4m} - \frac{2k}{3m}$

	Solve	Think	Apply
a	$\frac{2}{3a} + \frac{4}{5a} = \frac{10}{15a} + \frac{12}{15a}$ $= \frac{22}{15a}$	LCD = 15a $\frac{2}{3a} \times \frac{5}{5} = \frac{10}{15a}$ and $\frac{4}{5a} \times \frac{3}{3} = \frac{12}{15a}$	Find the lowest common denominator of the fractions. Change both fractions to equivalent fractions with this denominator.
b	$\frac{5k}{4m} - \frac{2k}{3m} = \frac{15k}{12m} - \frac{8k}{12m}$ $= \frac{7k}{12m}$	LCD = 12m $\frac{5k}{4m} \times \frac{3}{3} = \frac{15k}{12m}$ and $\frac{2k}{3m} \times \frac{4}{4} = \frac{8k}{12m}$	

8 Complete to simplify the following.

a For the fractions $\frac{3}{k}$ and $\frac{5}{2k}$ the LCD = ____.

$$\frac{3}{k} \times \frac{\square}{\square} = \frac{\square}{2k}$$

$$\text{Hence } \frac{3}{k} + \frac{5}{2k} = \frac{\square}{2k} + \frac{\square}{2k}$$

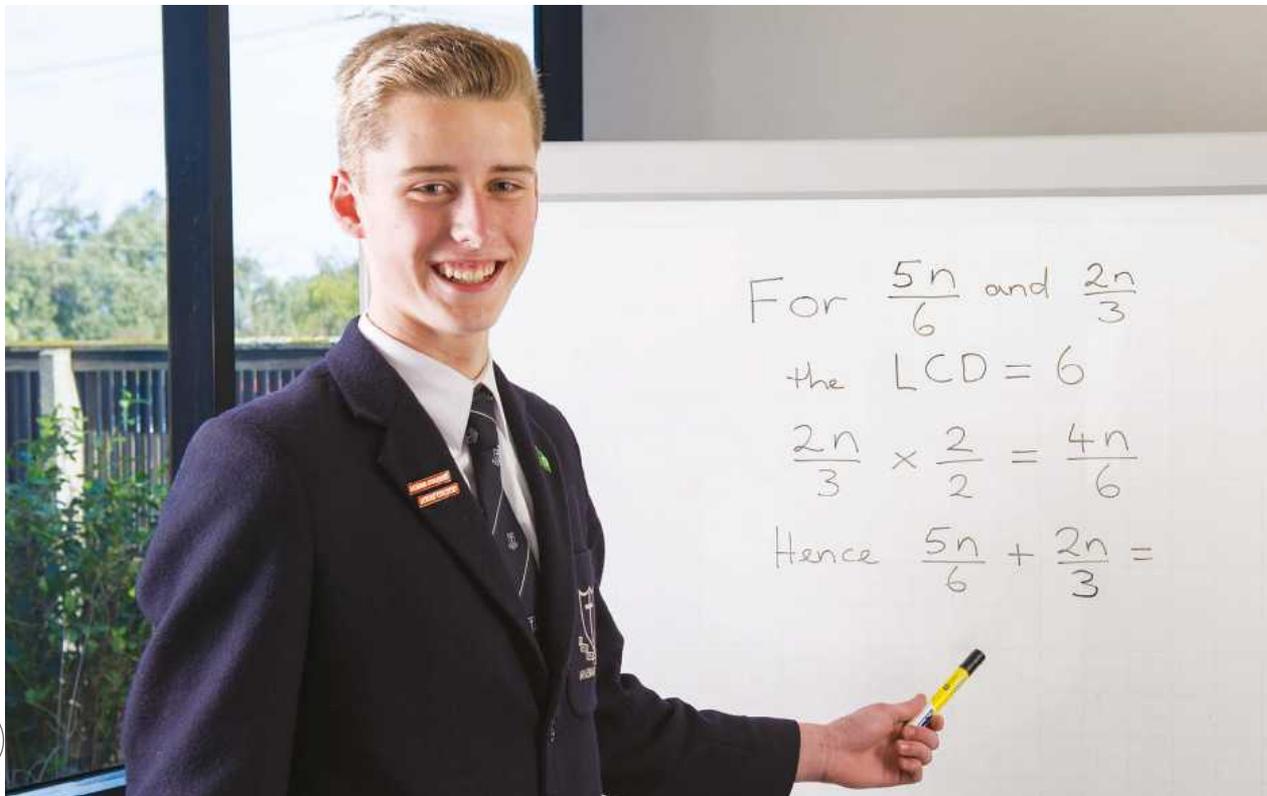
$$= \frac{\square}{2k}$$

b For the fractions $\frac{5a}{6b}$ and $\frac{3a}{7b}$ the LCD = ____.

$$\frac{5a}{6b} \times \frac{\square}{\square} = \frac{\square}{42b} \text{ and } \frac{3a}{7b} \times \frac{\square}{\square} = \frac{\square}{42b}$$

$$\text{Hence } \frac{5a}{6b} - \frac{3a}{7b} = \frac{\square}{42b} - \frac{\square}{42b}$$

$$= \frac{\square}{42b}$$



9 Simplify the following.

a $\frac{3}{x} + \frac{1}{2x}$

b $\frac{5}{3a} - \frac{3}{2a}$

c $\frac{7}{4y} + \frac{3}{5y}$

d $\frac{9}{2z} - \frac{5}{6z}$

e $\frac{2a}{3b} + \frac{4a}{b}$

f $\frac{5m}{3n} - \frac{2m}{7n}$

g $\frac{2xy}{5z} + \frac{8xy}{9z}$

h $\frac{6a}{5c} - \frac{5b}{6c}$

EXAMPLE 6

Simplify the following.

a $\frac{10}{9} \times \frac{18}{25}$

b $\frac{4m}{3} \times \frac{2n}{5}$

c $3y \times \frac{2z}{9}$

d $\frac{4b}{9} \times \frac{15}{8ab}$

	Solve	Think	Apply
a	$\frac{10}{9} \times \frac{18}{25} = \frac{\overset{2}{\cancel{10}}}{9} \times \frac{\overset{2}{\cancel{18}}}{\overset{5}{\cancel{25}}}$ $= \frac{2}{1} \times \frac{2}{5}$ $= \frac{4}{5}$	The HCF of 10 and 25 is 5 and the HCF of 9 and 18 is 9. Divide the numerator and denominator by 5 and 9. Multiply the resulting numerators and denominators.	Simplify by dividing the numerator and denominator by any common factors, then multiply the resulting numerators and denominators.
b	$\frac{4m}{3} \times \frac{2n}{5} = \frac{8mn}{15}$	$\frac{4m}{3} \times \frac{2n}{5} = \frac{4m \times 2n}{3 \times 5}$	
c	$3y \times \frac{2z}{9} = \frac{\overset{3}{\cancel{y}}}{1} \times \frac{2z}{\overset{3}{\cancel{9}}}$ $= \frac{y}{1} \times \frac{2z}{3}$ $= \frac{2yz}{3}$	The HCF of 3 and 9 is 3. Divide the numerator and denominator by 3. Multiply the resulting numerators and denominators.	
d	$\frac{4b}{9} \times \frac{15}{8ab} = \frac{\overset{4}{\cancel{b}}}{9} \times \frac{\overset{5}{\cancel{15}}}{\overset{2}{\cancel{8ab}}}$ $= \frac{1}{3} \times \frac{5}{2a}$ $= \frac{5}{6a}$	The HCF of 4 and 8 is 4, the HCF of 9 and 15 is 3, and the HCF of b and ab is b . Divide the numerators and denominators by 3, 4 and b . Multiply the resulting numerators and denominators.	

10 Complete to simplify the following.

a $\frac{3w}{4} \times \frac{7z}{5} = \frac{3w \times \square}{4 \times \square} = \frac{\square}{\square}$

b $2p \times \frac{5p}{7} = \frac{2p}{\square} \times \frac{5p}{7} = \frac{2p \times \square}{\square \times 7} = \frac{\square}{\square}$

c $\frac{5mn}{8} \times \frac{6p}{25n} = \frac{\overset{5}{\cancel{mn}}}{8} \times \frac{\overset{3}{\cancel{6p}}}{\overset{5}{\cancel{25n}}} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

11 Simplify the following.

a $\frac{3m}{5} \times \frac{10n}{7}$

b $\frac{2k}{9} \times \frac{6n}{5}$

c $\frac{4w}{3} \times \frac{9z}{8}$

d $\frac{8a}{5} \times \frac{15b}{16}$

e $\frac{3t}{5} \times \frac{10}{9u}$

f $\frac{5y}{3} \times \frac{9}{2y}$

g $\frac{7}{2z} \times \frac{3z}{14}$

h $\frac{2ab}{3} \times \frac{6}{5b}$

i $\frac{8mn}{9} \times \frac{5}{3m}$

j $\frac{3k}{15m} \times \frac{5n}{9}$

k $\frac{6pq}{5r} \times \frac{25r}{3q}$

l $\frac{12ab}{5c} \times \frac{10c^2}{3a}$

EXAMPLE 7

Simplify the following.

a $\frac{5}{8} \div \frac{3}{4}$

b $\frac{2a}{3} \div \frac{6b}{7}$

c $\frac{4}{5m} \div \frac{10}{3m}$

d $\frac{5pq}{8} \div \frac{3pr}{2q}$

	Solve	Think	Apply
a	$\frac{5}{8} \div \frac{3}{4} = \frac{5}{\cancel{2}8} \times \frac{\cancel{4}}{3}$ $= \frac{5}{2} \times \frac{1}{3}$ $= \frac{5}{6}$	To divide by $\frac{3}{4}$, multiply by its reciprocal $\frac{4}{3}$.	To divide by a fraction, multiply by its reciprocal.
b	$\frac{2a}{3} \div \frac{6b}{7} = \frac{2a}{3} \times \frac{7}{\cancel{3}6b}$ $= \frac{a}{3} \times \frac{7}{3b}$ $= \frac{7a}{9b}$	To divide by $\frac{6b}{7}$, multiply by its reciprocal $\frac{7}{6b}$.	
c	$\frac{4}{5m} \div \frac{10}{3m} = \frac{\cancel{4}}{5m} \times \frac{\cancel{3}m}{\cancel{5}10}$ $= \frac{2}{5} \times \frac{3}{5}$ $= \frac{6}{25}$	To divide by $\frac{10}{3m}$, multiply by its reciprocal $\frac{3m}{10}$.	
d	$\frac{5pq}{8} \div \frac{3pr}{2q} = \frac{5pq}{\cancel{4}8} \times \frac{\cancel{2}q}{3pr}$ $= \frac{5q}{4} \times \frac{q}{3r}$ $= \frac{5q^2}{12r}$	To divide by $\frac{3pr}{2q}$, multiply by its reciprocal $\frac{2q}{3pr}$.	

12 Complete to simplify the following.

a $\frac{3m}{5} \div \frac{2n}{9} = \frac{3m}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$

b $\frac{10p}{9} \div \frac{25}{27m} = \frac{10p}{9} \times \frac{\square}{\square} = \frac{\square}{\square}$

c $\frac{4ab}{3} \div \frac{5bc}{6} = \frac{4ab}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

13 Simplify the following.

a $\frac{2x}{3} \div \frac{8y}{5}$

b $\frac{3a}{2} \div \frac{6b}{7}$

c $\frac{5p}{3} \div \frac{10q}{9}$

d $\frac{7}{5v} \div \frac{3}{10v}$

e $\frac{16}{9w} \div \frac{8}{3w}$

f $\frac{6k}{5} \div \frac{7k}{2}$

g $\frac{4m}{3} \div \frac{2m}{5}$

h $\frac{7}{2m} \div \frac{m}{8}$

i $\frac{4xy}{3} \div \frac{2xz}{5}$

j $\frac{9p}{10km} \div \frac{6p}{5m}$

k $\frac{7mn}{p} \div \frac{m^2}{p^2}$

l $\frac{5xy}{7} \div \frac{3xy}{14}$

14 Simplify the following.

a $\frac{9a}{6ab}$

b $\frac{9x^4}{11x^2}$

c $\frac{5x^2y}{15xy^2}$

d $\frac{5k}{4} + \frac{5k}{6}$

e $3m - \frac{7m}{6}$

f $\frac{5}{2x} + \frac{4}{3x}$

g $\frac{2a}{3b} - \frac{3a}{5b}$

h $\frac{8ab}{5c} \times \frac{25c}{12a}$

i $\frac{4a}{7} \div \frac{3a}{5}$

j $\frac{3}{2x} \div \frac{9}{8x}$

k $\frac{x^6y^4}{3z} \div \frac{xy^5}{2z^2}$

l $\frac{5p^7}{3q^2} \div \frac{10p^3}{q}$

B The distributive law

The distributive law is used to expand algebraic expressions that involve grouping symbols.

$$a(b \pm c) = ab \pm ac$$

EXAMPLE 1

Expand and then simplify the following.

a $3(y + 7)$

b $5w(2w - 3z)$

c $2a^2(3a^4 + 7b)$

	Solve	Think	Apply
a	$3(y + 7) = 3y + 21$	$3(y + 7) = 3 \times y + 3 \times 7$	Use the distributive law to expand and then simplify.
b	$5w(2w - 3z) = 10w^2 - 15wz$	$5w(2w - 3z) = 5w \times 2w - 5w \times 3z$	
c	$2a^2(3a^4 + 7b) = 6a^6 + 14a^2b$	$2a^2(3a^4 + 7b) = 2a^2 \times 3a^4 + 2a^2 \times 7b$	

Exercise 3B

1 Complete the following.

a $5(y + 7) = 5 \times \underline{\quad} + 5 \times \underline{\quad}$
 $= \underline{\quad} + \underline{\quad}$

b $3a(2a - 7b) = \underline{\quad} \times 2a - \underline{\quad} \times 7b$
 $= \underline{\quad} - \underline{\quad}$

c $4p^2(2p^3 + 5q) = 4p^2 \times \underline{\quad} + 4p^2 \times \underline{\quad}$
 $= \underline{\quad} + \underline{\quad}$

2 Expand and then simplify the following.

a $4(t + 8)$

b $6(3y - 4z)$

c $5(6a + 9b)$

d $3a(2a - 4)$

e $4x(3x + 5)$

f $2m(6 - 3m)$

g $6k(3k - 4m)$

h $2b(5b - 4c)$

i $3w(4z - 5w)$

j $5m^2(2m^3 + 8n)$

k $2p^3(6p^5 - 11)$

l $4a^2(3a^7 + 2b)$

m $2x^3(4x^5 - 5y)$

n $6x^2(3x^5 - 2x^3)$

o $2a(8a - 4b + 7)$

EXAMPLE 2

Expand and then simplify the following.

a $-4(5a + 3)$

b $-2(3x - 7y)$

c $-(4m + 3n)$

d $-(6k - 5)$

	Solve	Think	Apply
a	$-4(5a + 3)$ $= -20a - 12$	$-4(5a + 3) = -4 \times 5a + (-4) \times 3$ $= -20a + (-12)$ $= -20a - 12$ <p>This can be simplified by multiplying each term in the parentheses by the term at the front. As $-4 \times 5a = -20a$ and $-4 \times (+3) = -12$, then $-4 \times (5a + 3) = -20a - 12$.</p>	<p>Use the distributive law to expand and then simplify.</p> <p>Adding $-12b$ is the same as subtracting $12b$. </p>
b	$-2(3x - 7y)$ $= -6x + 14y$	$-2(3x - 7y) = -2(3x + (-7y))$ $= -2 \times 3x + (-2) \times (-7y)$ $= -6x + 14y$ <p>This can be simplified by multiplying each term in the parentheses by the term at the front. As $-2 \times 3x = -6x$ and $-2 \times (-7y) = +14y$, then $-2(3x - 7y) = -6x + 14y$.</p>	<p>Subtracting $7y$ is the same as adding $-7y$. </p> <p>Remember: $-a = -1 \times a$. </p>
c	$-(4m + 3n)$ $= -4m - 3n$	$-(4m + 3n) = -1 \times (4m + 3n)$ $= -1 \times 4m + (-1) \times 3n$ $= -4m + (-3n)$ $= -4m - 3n$ <p>Or as $-1 \times 4m = -4m$ and $-1 \times (+3n) = -3n$, then $-1 \times (4m + 3n) = -4m - 3n$.</p>	<p>Adding $-3n$ is the same as subtracting $3n$. </p>
d	$-(6k - 5)$ $= -6k + 5$	$-(6k - 5) = -1 \times (6k - 5)$ $= -1 \times (6k + (-5))$ $= -1 \times 6k + (-1) \times (-5)$ $= -6k + 5$ <p>Or as $-1 \times 6k = -6k$ and $-1 \times (-5) = +5$, then $-1 \times (6k - 5) = -6k + 5$.</p>	

3 Complete the following.

a As $-3 \times 2a = \underline{\quad}$ and $-3 \times (+5) = \underline{\quad}$, then $-3(2a + 5) = \underline{\quad} - \underline{\quad}$.

b As $-4 \times 5t = \underline{\quad}$ and $-4 \times (-8) = \underline{\quad}$, then $-4(5t - 8) = \underline{\quad} + \underline{\quad}$.

c As $-1 \times 6m = \underline{\quad}$ and $-1 \times (+7) = \underline{\quad}$, then $-(6m + 7) = \underline{\quad} - \underline{\quad}$.

d As $-1 \times 4k = \underline{\quad}$ and $-1 \times (-2) = \underline{\quad}$, then $-(4k - 2) = \underline{\quad} + \underline{\quad}$.

4 Expand and then simplify the following.

a $-5(6t + 7)$

b $-4(3w - 7)$

c $-(7a + 9)$

d $-(5v - 2)$

e $-3(2b + 8c)$

f $-2(6x - 5y)$

g $-(4a + 9b)$

h $-(2m - 8n)$

i $-10(5p - 2q)$

j $-(3a^2 + 4b^3)$

k $-(2x^2 - 3y)$

l $-6(3x^2 - 4)$

EXAMPLE 3

Simplify the following.

a $-5a(4a - 8b)$

b $-3w^2(2w^3 + 5w^4z)$

	Solve	Think	Apply
a	$-5a(4a - 8b)$ $= -20a^2 + 40ab$	$-5a(4a - 8b) = -5a \times (4a + (-8b))$ $= -5a \times 4a + (-5a) \times (-8b)$ $= -20a^2 + 40ab$ <p>Or as $-5a \times 4a = -20a^2$ and $-5a \times (-8b) = +40ab$ then $-5a(4a - 8b) = -20a^2 + 40ab.$</p>	Use the distributive law to expand and then simplify.
b	$-3w^2(2w^3 + 5w^4z)$ $= -6w^5 - 15w^6z$	$-3w^2(2w^3 + 5w^4z) = -3w^2 \times 2w^3 + (-3w^2) \times 5w^4z$ $= -6w^5 + (-15w^6z)$ $= -6w^5 - 15w^6z$ <p>Or as $-3w^2 \times 2w^3 = -6w^5$ and $-3w^2 \times (+5w^4z) = -15w^6z$ then $-3w^2(2w^3 + 5w^4z) = -6w^5 - 15w^6z.$</p>	

5 Complete the following.

a $-5w(2w + 4z) = -10w^2 - \underline{\hspace{2cm}}$

b $-3p(5p - 2q) = -15p^2 + \underline{\hspace{2cm}}$

6 Simplify the following.

a $-2p(3q - 9r)$

b $-5x(2x + 3y)$

c $-4m(5m - 7n)$

d $-a(4a + 5b)$

e $-k(3m - 2n)$

f $-10z^4(2z^3 + 4z^5)$

g $-3m^3(4m^5 - 2m^2)$

h $-6x^4(5x^7 + 2x^5y^2)$

i $-3a(2b + 8c)$

EXAMPLE 4

Expand and then simplify the following.

a $3(a + 2) + 7$

b $3 + 2(3n - 5)$

	Solve	Think	Apply
a	$3(a + 2) + 7$ $= 3a + 6 + 7$ $= 3a + 13$	$3(a + 2) + 7$ $= 3 \times a + 3 \times 2 + 7$ <p>Multiply before adding. Collect like terms.</p> $= 3a + 13$	Expand (multiply) first and then collect like terms.
b	$3 + 2(3n - 5)$ $= 3 + 6n - 10 = -7 + 6n$ <p>or $= 6n - 7$</p>	$3 + 2(3n - 5)$ $= 3 + 2 \times 3n - 2 \times 5$ <p>Multiply before adding. Collect like terms.</p> $= -7 + 6n \text{ or } 6n - 7$	

7 Complete the following.

a $3(2b + 4) + 7 = 6b + \underline{\hspace{1cm}} + 7$
 $\quad \quad \quad = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$

b $11 + 2(3m - 4) = 11 + \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$
 $\quad \quad \quad = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$

8 Expand and then simplify the following.

a $4(a + 3) + 6$

b $2(3b - 12) + 12$

c $3(4w + 2) - 7$

d $5(2y - 3) - 2y$

e $6(3z - 1) + 4z$

f $10x + 2(4x + 3)$

g $12b + 2(3b - 5)$

h $13 + 4(y + 5)$

i $4w + 3(2w - 4)$

j $16 + 5(4e - 6)$

k $10a + 3(2a + 9)$

l $5(3c - 2) + 4c$

EXAMPLE 5

Expand and then simplify the following.

a $10 - 3(p + 2)$

b $5 - 2(4y - 3)$

	Solve	Think	Apply	
a	$10 - 3(p + 2)$ $= 10 - 3p - 6$ $= 4 - 3p$	$10 - 3(p + 2)$ $= 10 - 3 \times p + (-3) \times 2$ $= 10 - 3p + (-6)$ $= 10 - 3p - 6$ $= 10 - 6 - 3p$ $= 4 - 3p$	<p>Multiply first.</p> <p>Collect like terms.</p>	Expand the parentheses first and then collect like terms.
b	$5 - 2(4y - 3)$ $= 5 - 8y + 6$ $= 11 - 8y \text{ or } -8y + 11$	$5 - 2(4y - 3)$ $= 5 - 2 \times (4y + (-3))$ $= 5 - 2 \times 4y + (-2) \times (-3)$ $= 5 - 8y + 6$ $= 5 + 6 - 8y$ $= 11 - 8y$	<p>Multiply first.</p> <p>Collect like terms.</p>	

9 Complete the following.

a $15 - 3(b + 2) = 15 - \underline{\quad} - \underline{\quad}$
 $= \underline{\quad} + \underline{\quad}$

b $4 - 5(y - 3) = 4 - \underline{\quad} + \underline{\quad}$
 $= \underline{\quad} - \underline{\quad}$

10 Expand and then simplify the following.

a $12 - 2(a + 5)$

b $8 - 3(y - 2)$

c $9 - 4(b + 3)$

d $7v - 2(v - 6)$

e $20w - 3(2w + 5)$

f $2 - 5(3t - 4)$

g $4 - 3(5x + 2)$

h $10 - 2(3k - 1)$

i $5 - 3(3 + 4z)$

j $3w - 10(1 - 2w)$

k $12a - 3(2a + 5)$

l $15x - 4(3x - 2)$

EXAMPLE 6

Expand and then simplify $4(2m - 3) + 3(m - 2)$.

Solve	Think	Apply
$4(2m - 3) + 3(m - 2)$ $= 8m - 12 + 3m - 6$ $= 11m - 18$	$4(2m - 3) + 3(m - 2)$ $= 4 \times (2m - 3) + 3 \times (m - 2)$ $= 8m - 12 + 3m - 6$ $= 8m + 3m - 12 - 6$ $= 11m - 18$	<p>Multiply first.</p> <p>Collect like terms.</p> <p>Expand (multiply) first and then collect like terms.</p>

EXAMPLE 8

Expand and then simplify the following.

a $3m(4m - 5) + 2m(3m - 6)$

b $2p(3p + 4q) - 3p(4p - 5q)$

	Solve	Think	Apply
a	$3m(4m - 5) + 2m(3m - 6)$ $= 12m^2 - 15m + 6m^2 - 12m$ $= 18m^2 - 27m$	$3m(4m - 5) + 2m(3m - 6)$ $= 12m^2 - 15m + 6m^2 - 12m$ Multiply first. $= 12m^2 + 6m^2 - 15m - 12m$ Collect like terms. $= 18m^2 - 27m$	Expand (multiply) first and then collect like terms.
b	$2p(3p + 4q) - 3p(4p - 5q)$ $= 6p^2 + 8pq - 12p^2 + 15pq$ $= -6p^2 + 23pq \text{ or } 23pq - 6p^2$	$2p(3p + 4q) - 3p(4p - 5q)$ $= 6p^2 + 8pq - 12p^2 + 15pq$ Multiply first. $= 6p^2 - 12p^2 + 8pq + 15pq$ Collect like terms. $= -6p^2 + 23pq$	

15 Expand and then simplify the following.

a $4x(3x + 1) + 2x(5x - 2)$

b $5m(4m - 3) - 2m(9m - 2)$

c $3a^2(4a^3 - b) + 2a^2(5a^3 + 3b)$

d $2y^3(5y^2 + 3y) - 4y^2(2y^2 - y)$

e $3y(y - 5) + 4(y + 2)$

f $6a(a + 3) - 2(a - 5)$

g $2q(q - 5r) - 4(q - 5r)$

h $4z(3z + 2w) - (w - z)$

EXAMPLE 9

Simplify the following.

a $\frac{p}{3} + \frac{p+2}{4}$

b $\frac{5m-3}{6} - \frac{3m-1}{9}$

	Solve	Think	Apply
a	$\frac{p}{3} + \frac{p+2}{4}$ $= \frac{4p + 3(p+2)}{12}$ $= \frac{4p + 3p + 6}{12}$ $= \frac{7p + 6}{12}$	LCD = 12 $\frac{p}{3} + \frac{p+2}{4}$ $= \frac{4p}{12} + \frac{3(p+2)}{12}$ $= \frac{4p + 3(p+2)}{12}$	Find the lowest common denominator of the fractions. Change both fractions to equivalent fractions with this denominator and simplify.
b	$\frac{5m-3}{6} - \frac{3m-1}{9}$ $= \frac{3(5m-3) - 2(3m-1)}{18}$ $= \frac{15m-9-6m+2}{18}$ $= \frac{9m-7}{18}$	LCD = 18 $\frac{5m-3}{6} - \frac{3m-1}{9}$ $= \frac{3(5m-3)}{18} - \frac{2(3m-1)}{18}$ $= \frac{3(5m-3) - 2(3m-1)}{18}$	

16 Complete the following

$$\begin{aligned} \text{a } \frac{2m}{3} + \frac{m+4}{5} &= \frac{\square}{15} + \frac{\square(m+4)}{15} \\ &= \frac{\square + \square(m+4)}{15} \\ &= \frac{\square + \square + \square}{15} \\ &= \frac{\square + \square}{15} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{4p+3}{8} - \frac{2p-5}{6} &= \frac{\square(4p+3)}{24} - \frac{\square(2p-5)}{24} \\ &= \frac{\square(4p+3) - \square(2p-5)}{24} \\ &= \frac{\square + \square - \square + \square}{24} \\ &= \frac{\square + \square}{24} \end{aligned}$$

17 Simplify the following.

$$\text{a } \frac{m}{3} + \frac{m+5}{4}$$

$$\text{b } \frac{4p-3}{5} - \frac{2p}{7}$$

$$\text{c } \frac{k+4}{3} + \frac{k-2}{5}$$

$$\text{d } \frac{2x+5}{4} - \frac{x+3}{8}$$

$$\text{e } \frac{3y-5}{4} + \frac{2y-7}{10}$$

$$\text{f } \frac{9w}{10} - \frac{2w-5}{3}$$



Factorising algebraic expressions

When expanded

$$\begin{aligned} 5(2y+7) &= 5 \times 2y + 5 \times 7 \\ &= 10y + 35 \end{aligned}$$

Reversing the process

$$\begin{aligned} 10y + 35 &= 5 \times 2y + 5 \times 7 \\ &= 5(2y + 7) \end{aligned}$$

This second process is called factorising (or factoring).

To factorise, or factor, an algebraic expression means to write it as the product of its factors. Reversing the distributive law gives:

$$ab \pm ac = a(b \pm c)$$

EXAMPLE 1

Factorise these expressions.

a $10z + 16$

b $15m - 20n$

	Solve	Think	Apply
a	$10z + 16 = 2(5z + 8)$	The HCF of $10z$ and $16 = 2$. $10z = 2 \times 5z$ and $16 = 2 \times 8$ Hence $10z + 16 = 2 \times 5z + 2 \times 8$ $= 2 \times (5z + 8)$ $= 2(5z + 8)$	Factorise each term using the highest common factor as one factor and apply the distributive law.
b	$15m - 20n = 5(3m - 4n)$	The HCF of $15m$ and $20n = 5$. $15m = 5 \times 3m$ and $20n = 5 \times 4n$ Hence $15m - 20n = 5 \times 3m - 5 \times 4n$ $= 5 \times (3m - 4n)$ $= 5(3m - 4n)$	

Exercise 3C

- 1 Complete the following to factorise $18a + 12b$.

The HCF of $18a$ and $12b = \underline{\hspace{2cm}}$.

$18a = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ and $12 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

Hence $18a + 12b = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

- 2 Factorise these expressions.

a $14s + 10$

b $9y - 6$

c $8k + 12b$

d $15w - 25z$

e $12 + 9k$

f $24p - 18q$

g $6x + 3y + 9z$

h $24a - 12b - 18c$

EXAMPLE 2

Factorise the following by finding the highest negative common factor.

a $-4m + 6$

b $-8k - 12m$

	Solve	Think	Apply
a	$-4m + 6 = -2(2m - 3)$	The highest negative common factor of $-4m$ and $6 = -2$. $-4m = -2 \times 2m$ and $6 = -2 \times (-3)$ Hence $-4m + 6 = -2 \times 2m + (-2) \times (-3)$ $= -2 \times (2m + (-3))$ $= -2(2m - 3)$	Factorise each term using the highest negative common factor as one factor and apply the distributive law.
b	$-8k - 12m = -4(2k + 3m)$	The highest negative common factor of $-8k$ and $-12m = -4$. $-8k = -4 \times 2k$ and $-12m = -4 \times 3m$ Hence $-8k - 12m = -8k + (-12m)$ $= -4 \times 2k + (-4) \times 3m$ $= -4(2k + 3m)$	

- 3 Complete the following to factorise each expression by finding the highest negative common factor.

a $-5y + 10$

The highest negative common factor of $-5y$ and $10 = -\underline{\hspace{2cm}}$.

$-5y = -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ and $10 = -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

Hence $-5y + 10 = -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= -\underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

b $-12p - 8q$

The highest negative common factor of $-12p$ and $-8q = \underline{\hspace{2cm}}$.

$-12p = -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ and $-8q = -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

Hence $-12p - 8q = -12p + -8q$
 $= -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + -\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= -\underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

- 4 Factorise the following by finding the highest negative common factor.

a $-6t + 8$

b $-5k - 10$

c $-8p + 20q$

d $-12x - 15y$

e $-16 + 8p$

f $-18 - 24w$

EXAMPLE 3

Factorise the following.

a $pq + pr$

b $6mn - 4m$

c $12k^2 + 8k$

	Solve	Think	Apply
a	$pq + pr = p(q + r)$	The HCF of pq and $pr = p$. $pq = p \times q$ and $pr = p \times r$ Hence $pq + pr = p \times q + p \times r$ $= p(q + r)$	Factorise each term using the highest common factor as one factor and apply the distributive law.
b	$6mn - 4m = 2m(3n - 2)$	The HCF of 6 and 4 = 2 and the HCF of mn and $m = m$. Hence the HCF of $6mn$ and $4m = 2m$. $6mn = 2m \times 3n$ and $4m = 2m \times 2$ Hence $6mn - 4m = 2m \times 3n - 2m \times 2$ $= 2m(3n - 2)$	
c	$12k^2 + 8k = 4k(3k + 2)$	The HCF of 12 and 8 = 4 and the HCF of k^2 and $k = k$. Hence the HCF of $12k^2$ and $8k = 4k$. $12k^2 = 4k \times 3k$ and $8k = 4k \times 2$ Hence $12k^2 + 8k = 4k \times 3k + 4k \times 2$ $= 4k(3k + 2)$	

5 Complete the following to factorise each expression.

a $15pq + 18p$

The HCF of 15 and 18 = ___ and the HCF of pq and $p =$ ___. Hence the HCF of $15pq$ and $18p =$ ___.

$15pq =$ ___ \times ___ and $18p =$ ___ \times ___

Hence $15pq + 18p =$ ___ \times ___ $+$ ___ \times ___

$=$ ___($\text{---} + \text{---}$)

b $6y^2 - 30y$

The HCF of 6 and 30 = ___ and the HCF of y^2 and $y =$ ___. Hence HCF of $6y^2$ and $30y =$ ___.

$6y^2 =$ ___ \times ___ and $30y =$ ___ \times ___

Hence $6y^2 - 30y =$ ___ \times ___ $-$ ___ \times ___

$=$ ___($\text{---} - \text{---}$)

c $y^2 + 7y =$ ___($y + 7$)

d $m^2 - 3m = m(\text{---} - \text{---})$

e $3mn + 4m =$ ___($3n + 4$)

f $9p - 5pq = p(\text{---} - \text{---})$

g $x^2 + 5xy =$ ___($x + 5y$)

h $2bc - b^2 = b(\text{---} - \text{---})$

6 Factorise the following.

a $8pq - 12pr$

b $p^2 + 3p$

c $9ab + 6b$

d $x^2 - 7x$

e $12mn - 16n$

f $2k^2 + 4k$

g $6km - 8m^2$

h $10z^2 - 5z$

i $15ab + 3a^2$

j $24t^2 - 18tw$

k $7pq + 5pr + 11ps$

l $4ab + 6bc - 10bd$

7 a State whether the following statements are true or false.

i $12ab + 16a = 2(6ab + 8a)$

ii $12ab + 16a = 4(3ab + 4a)$

iii $12ab + 16a = 4a(3b + 4)$

b An algebraic expression is said to be fully factorised when the highest common factor is used. Which of the statements above shows that the expression has been fully factorised? (*Note:* An expression that has been factorised by using a common factor that is not the HCF is said to be partially factorised.)

8 State whether the following expressions are partially or fully factorised.

a $16xy + 20x = 4x(4y + 5)$

b $18mn - 12n = 3n(6m - 4)$

c $3a^2 + 6a = a(3a + 6)$

d $12pq + 9p^2 = 3p(4q + 3p)$

EXAMPLE 4

Factorise the following.

a $7a^3 + 4a^5$

b $15p^2q^3 - 12pq^4$

	Solve	Think	Apply
a	$7a^3 + 4a^5 = a^3(7 + 4a^2)$	The HCF of 7 and 4 = 1 and the HCF of a^3 and $a^5 = a^3$. Hence the HCF of $7a^3$ and $4a^5 = a^3$. $7a^3 = a^3 \times 7$ and $4a^5 = a^3 \times 4a^2$ Hence $7a^3 + 4a^5 = a^3 \times 7 + a^3 \times 4a^2$ $= a^3(7 + 4a^2)$	Factorise each term using the highest common factor as one factor and apply the distributive law.
b	$15p^2q^3 - 12pq^4 = 3pq^3(5p - 4q)$	The HCF of 15 and 12 = 3, the HCF of p^2 and $p = p$ and the HCF of q^3 and $q^4 = q^3$. Hence the HCF of $15p^2q^3$ and $12pq^4 = 3pq^3$. $15p^2q^3 = 3pq^3 \times 5p$ and $12pq^4 = 3pq^3 \times 4q$ Hence $15p^2q^3 - 12pq^4 = 3pq^3(5p - 4q)$	

9 Complete to factorise the following.

a $5z^4 + 3z^2$

The HCF of 5 and 3 = ___ and the HCF of z^4 and $z^2 =$ ___.

Hence the HCF of $5z^4$ and $3z^2 =$ ___.

$5z^4 =$ ___ \times ___ and $3z^2 =$ ___ \times ___

Hence $5z^4 + 3z^2 =$ ___ \times ___ $+$ ___ \times ___
 $=$ ___($+$ ___)

b $9a^3b^2 - 6a^2b^4$

The HCF of 9 and 6 = ___, the HCF of a^3 and $a^2 =$ ___ and the HCF of b^2 and $b^4 =$ ___.

Hence the HCF of $9a^3b^2$ and $6a^2b^4 =$ ___.

$9a^3b^2 =$ ___ \times ___ and $6a^2b^4 =$ ___ \times ___

Hence $9a^3b^2 - 6a^2b^4 =$ ___ \times ___ $-$ ___ \times ___
 $=$ ___($-$ ___)

c $14w^5 + 8w^3 = 2w^3(\text{___} + \text{___})$

d $20k^6 - 15k^2 = \text{___}(4k^4 - 3)$

e $7x^3y^8 + 5x^4y^5 = x^3y^5(\text{___} + \text{___})$

f $9a^5b^6 - 3a^3b^8 = \text{___}(3a^2 - b^2)$

10 Factorise the following.

a $6t^4 + 7t^2$

b $8k^{10} - 12k^5$

c $16n^7 + 24n^9$

d $9a^3b^5 + 8a^5b^2$

e $18m^4n^2 - 9m^3n^5$

f $30x^5y^7 - 20x^3y^7$

g $21xy - 3x + 9x^2$

h $16m^2 + 8mn - 12m$

6 $4(3k - 2) + 5(2k + 9) =$

A $22k + 53$

B $22k + 37$

C $2k + 53$

D $2k + 37$

7 $\frac{4q + 1}{5} - \frac{q + 3}{4} =$

A $3q - 2$

B $\frac{11q + 4}{20}$

C $\frac{11q - 11}{20}$

D $\frac{11q + 19}{20}$

8 When factorised fully $-12w - 8 =$

A $-4(3w + 2)$

B $-4(3w - 2)$

C $-4(3w - 8)$

D $-4(3w + 8)$

9 When factorised fully $4x^2 - 6x =$

A $2x^2(2x^2 - 3)$

B $2(2x^2 - 3x)$

C $x(4x - 6)$

D $2x(2x - 3)$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–4	5–7	8, 9
Section	A	B	C

3A Review set

1 Complete to make equivalent fractions.

a $\frac{5k}{3} = \frac{\square}{15}$

b $\frac{4x}{5y} = \frac{\square}{30y}$

c $\frac{2a}{3b} = \frac{\square}{12b^2}$

2 Reduce these fractions to the simplest form.

a $\frac{16k}{48}$

b $\frac{6a}{4ab}$

c $\frac{8x^3y^2}{10xy^3}$

3 Simplify:

a $\frac{4t}{3} - \frac{5t}{8}$

b $w + \frac{2w}{3}$

c $\frac{3}{x} + \frac{1}{2x}$

4 Simplify:

a $\frac{4d}{5} \times \frac{3e}{7}$

b $\frac{5b}{9} \times \frac{27}{10ab}$

c $\frac{2w}{3} \div \frac{8w}{9}$

d $\frac{4mn}{3p} \div \frac{8m}{pq}$

5 Expand and simplify the following where possible.

a $4x(3x + 2)$

b $2p^4(3p^2 - 7q)$

c $-6(3x^2 - 2y)$

d $5(2v - 3) + 7$

e $12 - 3(7 - 2x)$

f $3(2p + 5q) + 4(3p - 2q)$

6 Simplify:

a $\frac{x + 2}{5} - \frac{x + 3}{6}$

b $\frac{3a}{2} + \frac{5a - 1}{7}$

7 Factorise:

a $3xy + 12y^2$

b $-8m - 10$

c $15x - 9xy - 6x^2$

d $8m^3n^2 - 6m^2n^3$

3B Review set

1 Complete the following to make equivalent fractions.

a $\frac{7t}{4} = \frac{\square}{20}$

b $\frac{5x}{2y} = \frac{\square}{10y}$

c $\frac{4m}{3n} = \frac{\square}{12n^2}$

2 Reduce these fractions to the simplest form.

a $\frac{15p}{18}$

b $\frac{4mn}{6mnp}$

c $\frac{10k^2m^2}{15km^4}$

3 Simplify:

a $\frac{4p}{5} + \frac{3p}{4}$

b $\frac{12k}{5} - 2k$

c $\frac{1}{3x} + \frac{4}{5x}$

4 Simplify:

a $\frac{3w}{4} \times \frac{5w}{2}$

b $\frac{5m}{7n} \times \frac{14n}{25}$

c $\frac{4z}{5} \div \frac{8z}{15}$

d $\frac{8a^3b^2}{9c} \div \frac{2ab}{21}$

5 Expand and simplify where possible.

a $5k(2k - 7m)$

b $3d^2(4d^2 - 7e)$

c $-4m(3m - 7)$

d $6(4z + 3) - 15$

e $7a - 3a(2a - 5)$

f $5(3x - 2y) - 2(x + 5y)$

6 Simplify:

a $\frac{2x - 1}{3} - \frac{4x - 3}{2}$

b $\frac{4k}{3} + \frac{6k - 4}{5}$

7 Factorise:

a $-8k - 12$

b $7a^2 + 3ab$

c $12ab - 9 + 6b^2$

d $12x^4y^3 - 16x^2y$

3C Review set

1 Complete the following to make equivalent fractions.

a $\frac{7m}{4} = \frac{\square}{16}$

b $\frac{2k}{m} = \frac{\square}{9mn}$

c $\frac{4w}{3z} = \frac{\square}{3z^2}$

2 Reduce these fractions to the simplest form.

a $\frac{8k}{15k}$

b $\frac{6p}{26pq}$

c $\frac{30a^4b^3}{20ab^2}$

3 Simplify:

a $\frac{6k}{7} - \frac{3k}{5}$

b $3t + \frac{2t}{3}$

c $\frac{2}{5w} + \frac{3}{4w}$

4 Simplify:

a $\frac{7a}{2b} \times \frac{3c}{5d}$

b $\frac{6p}{5q} \times \frac{15q}{4}$

c $\frac{3ab}{4} \div \frac{7a}{8}$

d $\frac{4p^2}{3q^2} \div \frac{2p}{q}$

5 Expand and simplify where possible.

a $3s(2s - 1)$

c $-6a(2a + 5)$

e $3a(2a + 4b - 5c)$

b $7q^2(4q - 5p)$

d $19 - 2(5e - 6)$

f $5z(z - 2) + 3z(1 + z)$

6 Simplify:

a $\frac{7z}{8} + \frac{4z - 2}{3}$

b $\frac{5x - 3}{2} - \frac{2x + 1}{4}$

7 Factorise:

a $-24d - 16e$

c $9a^3b^4 - 12ab^3$

b $6pq + 16p^2$

d $2x^3y - 6x^2y + 4xy$

3D Review set

1 Complete to make equivalent fractions.

a $\frac{3h}{5} = \frac{\square}{20}$

b $\frac{2ab}{3} = \frac{\square}{18}$

c $\frac{4x}{3y} = \frac{\square}{6xy}$

2 Reduce these fractions to simplest form.

a $\frac{24}{36p}$

b $\frac{18xy}{20x}$

c $\frac{10m^4n^5}{5mn^3}$

3 Simplify:

a $\frac{2k}{7} + \frac{3k}{4}$

b $\frac{2p}{3} - \frac{5p}{6}$

c $\frac{8}{3w} + \frac{6}{5w}$

4 Simplify:

a $\frac{2}{3a} \times \frac{5}{7b}$

b $\frac{7w}{4z} \times \frac{8}{21w}$

c $\frac{4e}{3} \div \frac{5e}{6}$

d $\frac{3a^2b}{4c} \div \frac{9ab}{2c}$

5 Expand and simplify where possible.

a $7d(d - 3e)$

c $-2p^2(3p^2 + q)$

e $4a(3 + 2a) - a(3a + 2)$

b $10k(4k + 9m)$

d $7 + 3(6z - 2)$

f $3x^2(7 - 2x + 4x^2)$

6 Simplify:

a $\frac{4w - 1}{6} + \frac{2w + 5}{12}$

b $\frac{2x + 3y}{6} - \frac{x - 2y}{5}$

7 Factorise:

a $-15q + 12$

c $28x + 14x^2 - 7x^3$

b $12abc + 6bc$

d $a^3b^3 - a^2b^2$



4

Linear relationships

This chapter deals with distance, midpoint and gradient formulas and the use and application of various forms of the equation of the straight line.

After completing this chapter you should be able to:

- ▶ use the distance, midpoint and gradient formulas
- ▶ apply the formulas to worded coordinate geometrical problems
- ▶ graph straight lines in $y = mx + b$ form
- ▶ find the equation of a straight line
- ▶ rearrange equations of straight lines into various forms
- ▶ sketch straight lines given their intercepts
- ▶ demonstrate that two lines are perpendicular if the product of their gradients is -1
- ▶ find the equation of a line parallel or perpendicular to a given line.

Diagnostic test

1 The midpoint of the join of (4, 3) and (10, 3) is:

- A (14, 6) B (6, 3)
C (7, 3) D (6, 6)

2 The midpoint of the join of (-3, -5) and (-3, 11) is:

- A (-6, 6) B (-3, 6)
C (-6, 3) D (-3, 3)

3 The midpoint of the join of (1, 4) and (8, 0) is:

- A (9, 4) B (7, 4)
C $(4\frac{1}{2}, 2)$ D $(3\frac{1}{2}, 2)$

4 The midpoint of the join of (-5, 1) and (7, -5) is:

- A (1, -2) B (2, -4)
C (6, 6) D (6, 3)

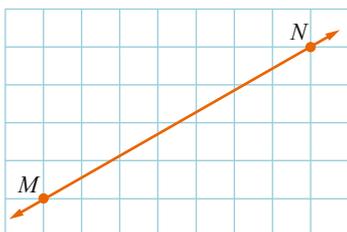
5 The distance between points (7, 1) and (2, 9) is:

- A $\sqrt{89}$ B $\sqrt{181}$
C $\sqrt{39}$ D $\sqrt{39}$

6 The distance between points (-5, 7) and (4, -5) is:

- A $\sqrt{63}$ B 15
C 5 D $\sqrt{21}$

7 The slope of MN is:

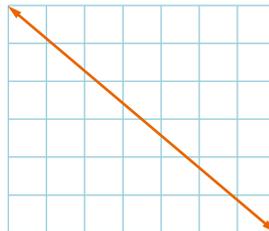


- A +3 B -3
C $\frac{4}{7}$ D $\frac{7}{4}$

8 The gradient of the join of $A(-5, -1)$ and $B(3, 5)$ is:

- A $+\frac{3}{4}$ B $-\frac{3}{4}$
C $+\frac{4}{3}$ D $-\frac{4}{3}$

9 The gradient of this line is:

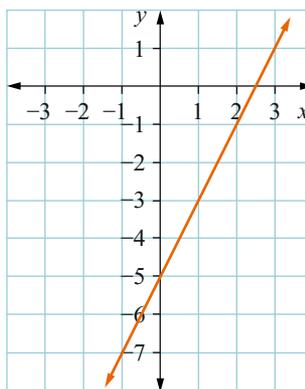


- A $\frac{6}{7}$ B $\frac{7}{6}$
C $-\frac{6}{7}$ D $-\frac{7}{6}$

10 The gradient of the join of $A(-5, 9)$ and $B(7, 5)$ is:

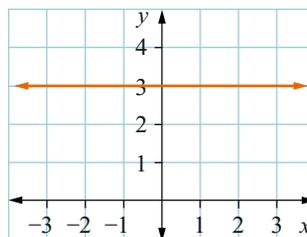
- A $+\frac{1}{3}$ B $-\frac{1}{3}$
C +3 D -3

11 The gradient of this line is:



- A +2 B -2
C $+\frac{1}{2}$ D $-\frac{1}{2}$

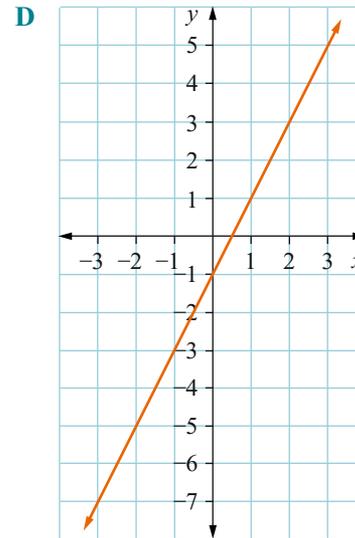
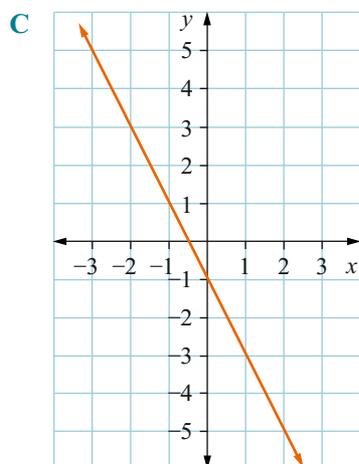
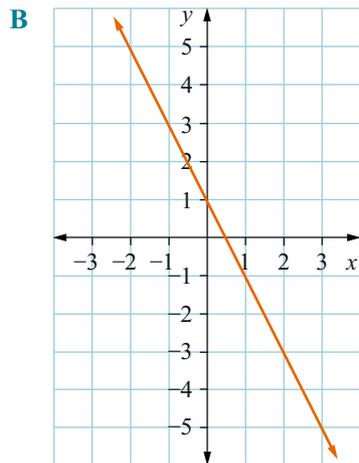
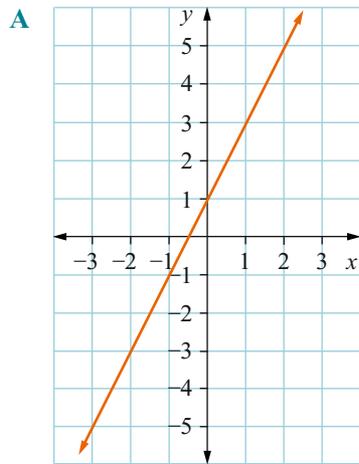
12 The equation of this line is:



- A $y = 3$ B $x = 3$
C $y = 3x$ D $x = 3y$

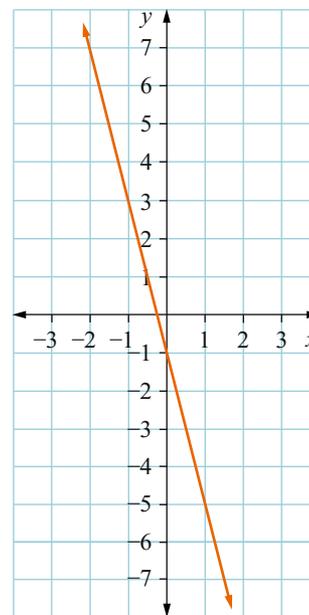
- 13** Complete this table of values to determine that the graph of $y = -2x + 1$ is:

x	-2	0	2
y			



- 14** By completing this table of values for each equation, determine that the equation of this graph is:

x	-2	0	2
y			



- A** $y = 3x - 1$ **B** $y = -3x + 1$
C $y = 4x - 1$ **D** $y = -4x - 1$

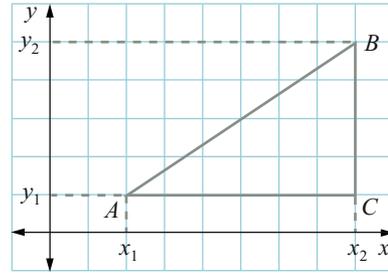
- 15** The line containing the point $(2, -2)$ is:
A $y = 3x - 8$ **B** $y = -3x - 3$
C $y = 4x + 14$ **D** $y = -4x - 11$

The diagnostic test questions refer to outcomes ACMNA214, ACMNA215, ACMNA239, ACMNA294 and ACMNA296. 

Investigation 1 Distance formula

The purpose of this investigation is to develop a formula to find the distance between the two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

- Use the graph to find an expression involving y_2 and y_1 for the length BC .
- Use the graph to find an expression involving x_2 and x_1 for the length AC .
- Use Pythagoras' rule to find an expression for d , the length AB .
- Confirm that the distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



- Check the formula by showing that the distance between $(-1, 4)$ and $(5, 2)$ is $\sqrt{40}$ units. Use $(-1, 4)$ for (x_1, y_1) and $(5, 2)$ for (x_2, y_2) .
- Show that the choice for (x_1, y_1) and (x_2, y_2) does not affect the outcome by using $(5, 2)$ for (x_1, y_1) and $(-1, 4)$ for (x_2, y_2) to show the distance is $\sqrt{40}$ units.
- Use the distance formula to find the distance between the following pairs of points.
 - $(4, 5)$ and $(-6, 3)$
 - $(-5, -3)$ and $(2, -7)$
 - $(0, 0)$ and $(14, 4)$
 - $(-11, 13)$ and $(2, -5)$
- Lianna uses the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to find the distance between two points.
 - Show that this formula works for the points $(-5, -3)$ and $(2, -7)$.
 - Expand the expression $(x_1 - x_2)^2 + (y_1 - y_2)^2$.
 - Expand the expression $(x_2 - x_1)^2 + (y_2 - y_1)^2$.
 - Explain why both formulas work.
- Complete the following. In the distance formula:
 - d stands for _____
 - x_1 is _____
 - x_2 is _____
 - y_1 is _____
 - y_2 is _____



Distance, midpoint and gradient review

EXAMPLE 1

Use the distance formula to find the distance between the points $T(2, 6)$ and $U(5, 1)$.

Solve	Think	Apply
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 2)^2 + (1 - 6)^2}$ $= \sqrt{3^2 + (-5)^2}$ $= \sqrt{9 + 25}$ $= \sqrt{34} \text{ units}$	Use $x_1 = 2, y_1 = 6$ and $x_2 = 5, y_2 = 1$ and substitute.	Choose a point to be (x_1, y_1) and the other (x_2, y_2) . It does not matter which is chosen as (x_1, y_1) .

Exercise 4A

- 1 Complete the following to find the distance between $A(-3, 5)$ and $B(6, -4)$.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (\underline{\quad} - \underline{\quad})^2} & x_1 &= -3, y_1 = \underline{\quad} \\
 &= \sqrt{(6 - (-3))^2 + (\underline{\quad} - \underline{\quad})^2} & x_2 &= 6, y_2 = \underline{\quad} \\
 &= \sqrt{9^2 + (\underline{\quad})^2} \\
 &= \sqrt{\underline{\quad}} \text{ units}
 \end{aligned}$$

- 2 Use the distance formula to find the distance between these pairs of points.

- a** $A(-2, 3)$ and $B(5, 4)$
 b $A(0, 4)$ and $B(3, 6)$
 c $A(0, 3)$ and $B(6, 4)$
d $C(1, 1)$ and $D(6, -2)$
 e $P(1, -2)$ and $Q(-4, 3)$
 f $W(2, 4)$ and $X(-3, -4)$

EXAMPLE 2

Use the midpoint formula to find the midpoint of the line segment joining $A(-4, 3)$ and $B(8, -2)$.

Solve	Think	Apply
$ \begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-4 + 8}{2}, \frac{3 + (-2)}{2} \right) \\ &= \left(2, \frac{1}{2} \right) \\ \text{The midpoint is } &\left(2, \frac{1}{2} \right). \end{aligned} $	<p>Let A be (x_1, y_1), so $x_1 = -4$, $y_1 = 3$, and B be (x_2, y_2), so $x_2 = 8, y_2 = -2$.</p>	<p>The x-coordinate of the midpoint of an interval is the average of the x-values of the endpoints. Similarly the y-coordinate is the average of the y-values.</p>

- 3 Complete to find the midpoint of the join of $A(5, -3)$ and $B(-9, 5)$.

$$\begin{aligned}
 \text{Midpoint} &= \left(\frac{x_1 + \square}{2}, \frac{y_1 + \square}{2} \right) & x_1 &= 5, y_1 = \underline{\quad} \\
 &= \left(\frac{\square + (-9)}{2}, \frac{-3 + \square}{2} \right) & x_2 &= \underline{\quad}, y_2 = \underline{\quad} \\
 &= (\underline{\quad}, \underline{\quad})
 \end{aligned}$$

- 4 Use the midpoint formula to find the midpoint of the join of:

- a** $(2, 3)$ and $(6, 9)$
 b $(-2, 6)$ and $(8, -1)$
 c $(-5\frac{1}{2}, 4\frac{1}{2})$ and $(7, -1)$
 d $(-5, -6)$ and $(2, 4)$

EXAMPLE 3

The midpoint of $A(2, 5)$ and $B(x_2, y_2)$ is $(4, -1)$. Find the coordinates of B .

Solve	Think	Apply
$ \begin{aligned} x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\ 4 &= \frac{2 + x_2}{2} & -1 &= \frac{5 + y_2}{2} \\ 8 &= 2 + x_2 & -2 &= 5 + y_2 \\ x_2 &= 6 & y_2 &= -7 \\ B \text{ has coordinates } &(6, -7). \end{aligned} $	<p>Let the midpoint be (x, y), then $x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$.</p> <p>Substitute the values: $(x, y) = (4, -1)$, $(x_1, y_1) = (2, 5)$</p>	<p>When finding one of the endpoints, set the endpoints as (x_2, y_2) and substitute the values into the formula.</p>

- 5** The midpoint of $A(3, 7)$ and $B(x_2, y_2)$ is $(-7, 3)$. Complete to find the coordinates of B .

Let the midpoint be (x, y) , then $x = \frac{x_1 + \square}{2}$ and $y = \frac{y_1 + y_2}{\square}$.

Substitute the values: $(x, y) = (-7, \underline{\quad})$ and $(x_1, y_1) = (\underline{\quad}, 7)$

$$-7 = \frac{\square + x_2}{2}$$

$$3 = \frac{7 + \square}{2}$$

$$-14 = 3 + x_2$$

$$\underline{\quad} = 7 + y_2$$

$$x_2 = \underline{\quad}$$

$$y_2 = \underline{\quad}$$

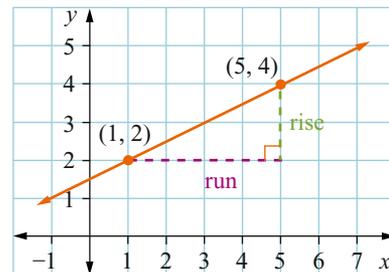
B has coordinates $(\underline{\quad}, -1)$.

- 6** The midpoint of $A(3, -2)$ and $B(x_2, y_2)$ is $(5, -4)$. Find the coordinates of B .

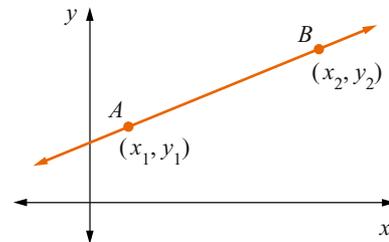
- 7** The midpoint of XY is $(-3, 5)$. X has coordinates $(2, 7)$. Find the coordinates of Y .

- 8 a i** Find values for the vertical rise and horizontal run as shown in the triangle.

- ii** Calculate the gradient using $\text{gradient} = \frac{\text{rise}}{\text{run}}$.



- b i** Copy the diagram shown on the right.



- ii** Draw in the triangle as shown.

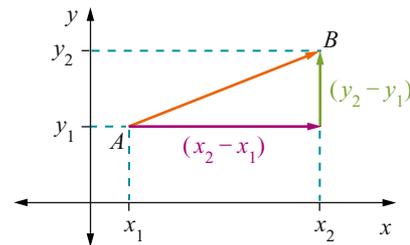
- iii** If A is (x_1, y_1) and B is (x_2, y_2) then from the diagram:

$$\text{rise} = y_2 - y_1 \text{ and run} = x_2 - x_1$$

The vertical rise from A to B is $y_2 - y_1$
(the difference between the y -coordinates)

The horizontal run from A to B is $x_2 - x_1$
(the difference between the x -coordinates).

The symbol for gradient is m . Complete: $m = \frac{y_2 - \square}{\square - \square}$



EXAMPLE 4

Use the gradient formula to find the slope of the line passing through $A(4, 7)$ and $B(7, 3)$.

Solve	Think	Apply
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3 - 7}{7 - 4}$ $= -\frac{4}{3}$	$A(4, 7)$ is (x_1, y_1) $B(7, 3)$ is (x_2, y_2)	Choose one point as (x_1, y_1) and the other as (x_2, y_2) .

- 9 Complete to use the gradient formula to find the slope of the line passing through (4, -2) and (10, 6).

$$m = \frac{y_2 - y_1}{x_2 - \square}$$

$$= \frac{6 - \square}{10 - \square} = \frac{4}{3}$$

$$A = (4, -2) = (x_1, \underline{\quad})$$

$$B = (10, 6) = (\underline{\quad}, y_2)$$

- 10 Use the gradient formula to find the slope of the line passing through A and B .

a $A(2, 3)$ and $B(5, 8)$

b $A(4, -1)$ and $B(2, 7)$

c $A(-3, 2)$ and $B(4, 2)$

d $A(-2, 5)$ and $B(0, 6)$

e $A(-1, -2)$ and $B(-3, -7)$

f $A(3, 1)$ and $B(3, 5)$

g $A(4, 0)$ and $B(6, 0)$

h $A(-1, -2)$ and $B(3, -1)$

i $A(4, 3)$ and $B(-4, -1)$

- 11 Angus wants to use the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$ to find gradient. He says that if the expression $\frac{y_2 - y_1}{x_2 - x_1}$ is multiplied on the top and bottom by -1 , the formulas are equal.

a Test both formulas on the points $A(2, 3)$ and $B(-7, 5)$.

b Explain why Angus is correct.

B Graphing lines

Tables of values can be used to plot points that give the graphs of equations of straight lines. Recall from Year 9 that graphs with positive gradients go *uphill*, and graphs with negative gradients go *downhill*.

EXAMPLE 1

Draw the graph of the lines with these equations.

a $y = x + 3$

b $y = -\frac{1}{2}x$

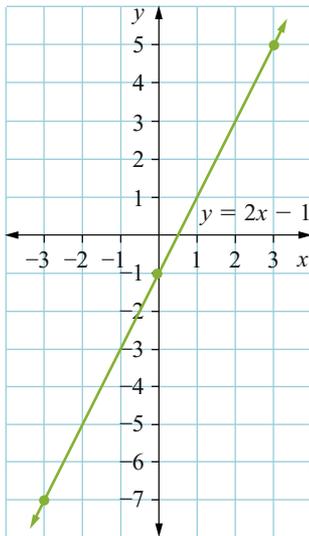
c $y = 2x - 1$

Using a table of values will help. !

	Solve	Think	Apply												
a		$y = x + 3$ <table border="1"> <tr> <td>x</td> <td>-3</td> <td>-2</td> <td>0</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>0</td> <td>1</td> <td>3</td> <td>5</td> <td>6</td> </tr> </table> $x = -3$ $y = -3 + 3 = 0$ $x = -2$ $y = -2 + 3 = 1$	x	-3	-2	0	2	3	y	0	1	3	5	6	<p>When graphing these equations, the line is extended past the plotted points with an arrow on each end to show that it continues in both directions. Write the equation on the line.</p> <p>It is only necessary to use two points, but it is advisable to use at least three points.</p>
x	-3	-2	0	2	3										
y	0	1	3	5	6										
b		$y = -\frac{1}{2}x$ <table border="1"> <tr> <td>x</td> <td>-4</td> <td>-2</td> <td>0</td> <td>2</td> <td>4</td> </tr> <tr> <td>y</td> <td>2</td> <td>1</td> <td>0</td> <td>-1</td> <td>-2</td> </tr> </table> $x = -4$ $y = -\frac{1}{2}(-4) = 2$ $x = -2$ $y = -\frac{1}{2}(-2) = 1$	x	-4	-2	0	2	4	y	2	1	0	-1	-2	
x	-4	-2	0	2	4										
y	2	1	0	-1	-2										

EXAMPLE 1 CONTINUED

c



Solve

Think/Apply

$$y = 2x - 1$$

x	-3	-2	0	2	3
y	-7	-5	-1	3	5

$$x = -3$$

$$y = 2(-3) - 1 = -7$$

$$x = -2$$

$$y = 2(-2) - 1 = -5$$

Exercise 4B

1 Complete each table of values below using the rule provided. Plot each set of ordered pairs on separate axes and draw the straight line through the points.

a $y = x + 1$

x	-2	-1	0	1	2
y					

b $y = x - 1$

x	-2	-1	0	1	2
y					

c $y = -x - 2$

x	-2	-1	0	1	2
y					

d $y = 3x - 1$

x	-2	-1	0	1	2
y					

e $y = 2x + 2$

x	-2	-1	0	1	2
y					

f $y = -\frac{1}{2}x + 1$

x	-2	-1	0	1	2
y					

g $y = -2x + 8$

x	-2	-1	0	1	2
y					

h $y = 1 - 3x$

x	-2	-1	0	1	2
y					

2 Consider the straight lines in question 1.

- a** Write a list of the equations with a positive gradient.
- b** Write a list of the equations with a negative gradient.
- c** What is the difference between these groups of equations?
- d** Without drawing the graph, state whether each of these equations has a positive or a negative gradient.
 - i** $y = 2x - 1$
 - ii** $y = -3x + 4$
 - iii** $y = 5 - 7x$
 - iv** $y = 3 + 2x$
 - v** $y = 7x - 1$
 - vi** $y = -5x + 2$

- 3** When making up a table of values from rules, I unfortunately mixed them up. Can you sort out which graph belongs to which table of values?

A $y = 2x$

B $y = -x$

C $y = \frac{1}{2}x$

D $y = x + 2$

E $y = 2 - x$

a

x	-4	-2	0	2	4
y	-2	-1	0	1	2

b

x	0	1	2	3	4
y	0	2	4	6	8

c

x	-4	-2	0	1	3
y	-2	0	2	3	5

d

x	-2	0	1	2	3
y	4	2	1	0	-1

e

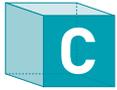
x	-3	-2	-1	0	1
y	3	2	1	0	-1

Investigation 2 Gradients and intercepts

A graphics calculator could be used in this investigation.

- 1 a** On the same number plane draw the graphs of:
 $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, $y = 2x + 3$
 - b** What do you notice about the gradients of these lines?
 - c** Write the coordinates of the y -intercept of each line. What do you notice about the y -intercept and the equation of the line?
- 2 a** On the same number plane draw the graphs of:
 $y = 2x + 1$, $y = 3x + 1$, $y = 5x + 1$,
 $y = x + 1$
 - b** What do you notice about the gradients of these lines? Compare each gradient with the coefficient of x .
 - c** What do you notice about the y -intercept of each line? How is this shown in the equation?
- 3 a** Sketch the line $y = 3x - 2$.
 - b** Change the gradient to 4 and sketch the new line.
 - c** Is the new line parallel to $y = 3x - 2$? Explain.
- 4 a** Sketch the line $y = 3x - 1$.
 - b** What equation will give a line parallel to $y = 3x - 1$ but with a y -intercept of 2?
 - c** Sketch the line.





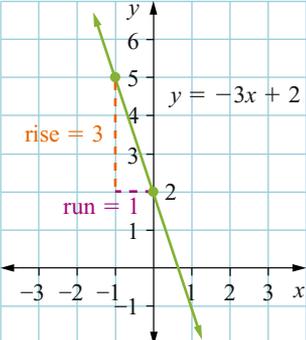
Straight line $y = mx + b$ form

From Investigation 2 it can be seen that:

- if a straight line has gradient m and y -intercept b , it has equation $y = mx + b$
- if two straight lines have the same gradient, they are parallel.

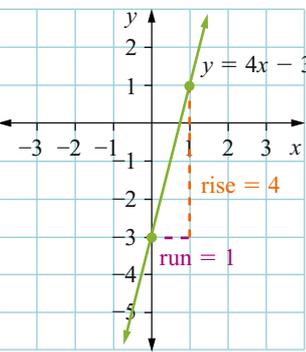
EXAMPLE 1

- a** Find the equation of the line with gradient -3 and y -intercept 2 .
b Sketch the line.

	Solve	Think	Apply
a	$y = -3x + 2$	As $m = -3$ and $b = 2$, the equation is $y = -3x + 2$.	Substitute the m and b values into the equation $y = mx + b$.
b		Plot the point $(0, 2)$. As the gradient is -3 , the line slopes downhill with a rise of 3 and a run of -1 .	Using a rise of -3 and a run of 1 would give the same graph.

EXAMPLE 2

- a** Find the equation of the line with gradient 4 and y -intercept -3 .
b Sketch the line.

	Solve	Think	Apply
a	$y = 4x - 3$	As $m = 4$ and $b = -3$, the equation is $y = 4x - 3$.	Substitute the m and b values into the equation $y = mx + b$.
b		Plot the point $(0, -3)$. As the gradient is $+4$, the line slopes uphill with a rise of 4 and a run of 1 .	To check the graph, substitute an x -value into the equation; that point should lie on the line.

Exercise 4C

- 1 a Complete to find the equation of the line with gradient -3 and y -intercept 5 .

Gradient = $m =$ ___

y -intercept = $b =$ ___

The equation is $y = mx + b$.

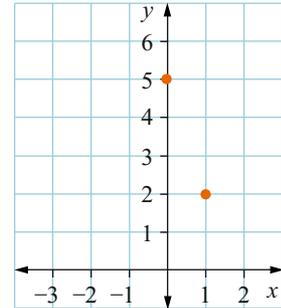
$\therefore y =$ ___ $x + 5$

- b Complete the diagram on the right to sketch the line.

Plot the point (___, 5). As the gradient is ___, the line slopes ____.

with a rise of ___ and a run of ___.

Draw a line through the points.



- 2 Find the equation of the line with:

a gradient 2 and y -intercept 7

c gradient -3 and y -intercept -1

e gradient -2 and y -intercept 6

g gradient 3 and y -intercept 0

b gradient 4 and y -intercept -6

d gradient -2 and y -intercept 2

f gradient 1 and y -intercept 3

h gradient -1 and y -intercept 2 .

- 3 Sketch each of the lines in question 2.

EXAMPLE 3

- a Find the equation of the line with gradient $-\frac{2}{3}$ and y -intercept 1 .

- b Sketch the line.

	Solve	Think	Apply
a	$y = -\frac{2}{3}x + 1$	As $m = -\frac{2}{3}$ and $b = 1$, the equation is $y = -\frac{2}{3}x + 1$.	Substitute the m and b values into the equation $y = mx + b$.
b		Plot the point $(0, 1)$. As the gradient is $-\frac{2}{3}$, the line slopes downhill with a rise of -2 and a run of 3 .	Using a rise of 2 and a run of -3 would give the same graph.

- 4 Find the equation of the line with:

a gradient $-\frac{3}{4}$ and y -intercept 1

c gradient $\frac{2}{3}$ and y -intercept -1

e gradient $-\frac{1}{3}$ and y -intercept 4

g gradient $\frac{2}{5}$ and y -intercept 0

b gradient $\frac{2}{3}$ and y -intercept 2

d gradient $\frac{4}{5}$ and y -intercept -2

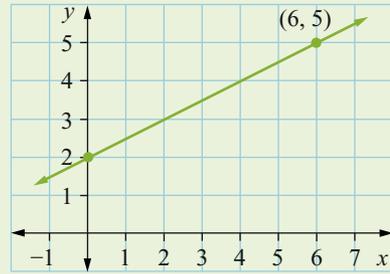
f gradient $-\frac{1}{4}$ and y -intercept -1

h gradient $-\frac{2}{5}$ and y -intercept 4 .

- 5 Sketch each of the lines in question 4.

EXAMPLE 4

Find the gradient of the given line.

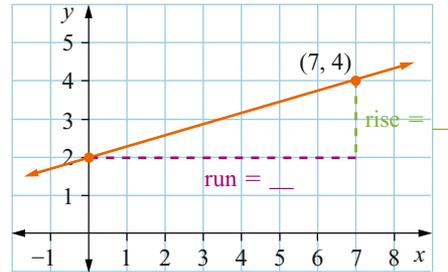


Solve	Think	Apply
$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$	<p>Draw a right-angled triangle, labelling the rise and run.</p>	<p>Any two points can be chosen to find the gradient. The gradient formula can be used.</p>

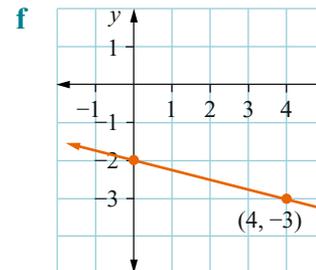
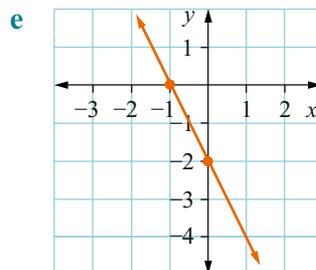
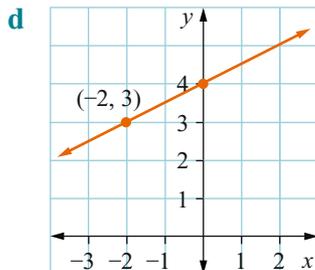
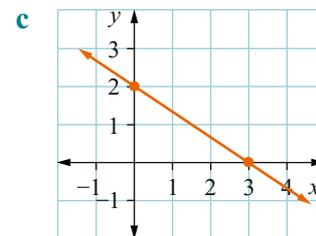
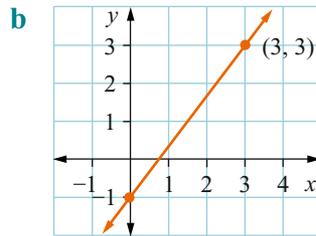
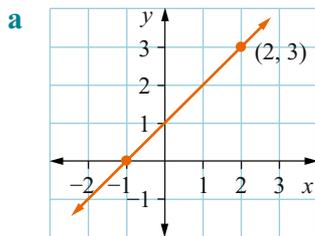
- 6 Complete the diagram to find the gradient of the line.
Write the length of the sides of the triangle on the diagram.

$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\square} \\ &= \frac{2}{\square} \\ &= \underline{\quad} \end{aligned}$$

Gradient is uphill so it is $\underline{\quad}$.



- 7 Find the gradient of the following lines.



8 Draw the graph of the lines with the following equations.

a $y = \frac{1}{2}x + 2$

b $y = 2x + 1$

c $y = -x + 3$

d $y = -3x + 2$

e $y = -\frac{1}{2}x$

f $y = -2x - 2$

g $y = \frac{3}{2}x$

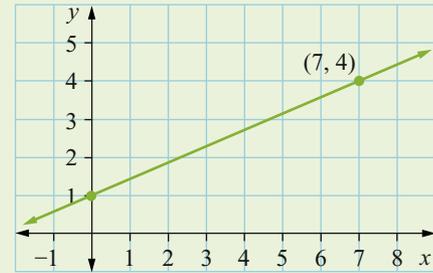
h $y = \frac{2}{3}x + 2$

i $y = -\frac{3}{4}x - 1$

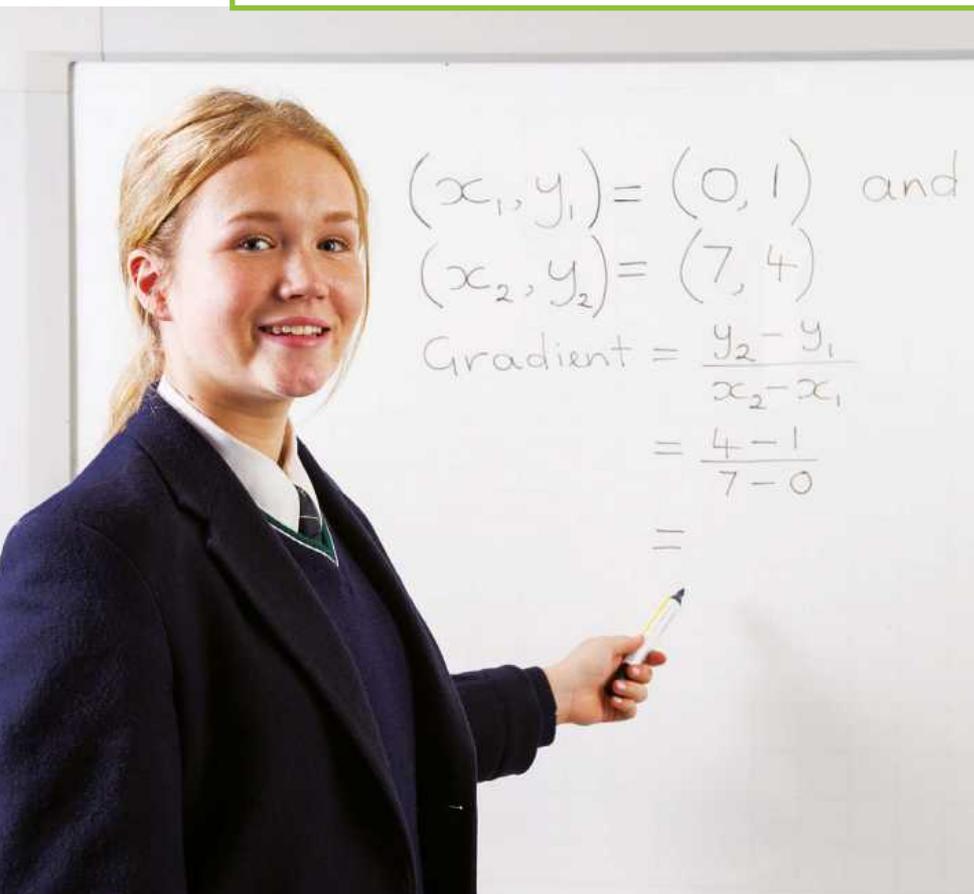
9 Write the gradient and y -intercept of each of the lines in question 8.

EXAMPLE 5

Find the equation of this straight line.

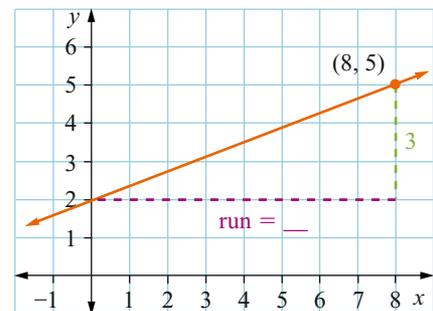


Solve	Think	Apply
$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{3}{7}$ <p>y-intercept is 1. The equation is $y = \frac{3}{7}x + 1$.</p>	<p>Draw a right-angled triangle, labelling the rise and run.</p>	<p>The gradient formula and the points (0, 1) and (7, 4) could be used.</p>



$$\begin{aligned} (x_1, y_1) &= (0, 1) \text{ and} \\ (x_2, y_2) &= (7, 4) \\ \text{Gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 1}{7 - 0} \\ &= \end{aligned}$$

10 Complete to find the equation of the line shown below.



Gradient = $\frac{\square}{\text{run}}$

$= \frac{3}{\square}$

y -intercept is \square .

The equation is $y = \frac{3}{\square}x + \square$.

- 11** Find the equation of the lines in question 7.
- 12** The equations of three lines are $y = 3x + 1$, $y = 2x + 1$, $y = 3x - 2$.
- Without sketching the lines, which two lines are parallel? Why?
 - Sketch all three lines on the same number plane. What do you notice?
 - Which of the above three lines would have something in common with these lines?
 - $y = 2x + 4$
 - $y = 5x - 2$
- Explain your answers.

EXAMPLE 6

Rewrite the equation $4x - 3y - 12 = 0$ in the form $y = mx + b$. Hence find the gradient and y -intercept.

Solve	Think	Apply
$4x - 3y - 12 = 0$ $4x - 12 = 3y$ $3y = 4x - 12$ $y = \frac{4}{3}x - 4$ Gradient is $m = \frac{4}{3}$ and y -intercept is -4 .	Add $3y$ to both sides. Swap sides. Divide both sides by 3 .	Use equation-solving techniques to rearrange the terms. $4x - 3y - 12 = 0$ is called the general form of an equation of a straight line and will be examined later in this chapter.

- 13** Complete to rewrite the equation $4x - 5y - 20 = 0$ in the form $y = mx + b$ and hence find the gradient and y -intercept.
- $$4x - 5y - 20 = 0$$
- $$4x - 5y + \underline{\quad} - 20 = 0 + 5y \quad \text{Add } 5y \text{ to both sides.}$$
- $$4x - 20 = 5y$$
- $$5y = \underline{\quad} \quad \text{Swap sides.}$$
- $$y = \frac{\square}{\square}x - 4 \quad \text{Divide both sides by } 5.$$
- Gradient is $m = \underline{\quad}$.
 y -intercept is $\underline{\quad}$.
- 14** Rewrite the following equations in $y = mx + b$ form and hence find the gradient and y -intercept.
- | | | |
|----------------------------|-----------------------------|----------------------------|
| a $x + 2y - 4 = 0$ | b $3x + 2y - 24 = 0$ | c $2x - y + 4 = 0$ |
| d $4x - 2y - 6 = 0$ | e $5x + 2y + 10 = 0$ | f $3x + 2y - 8 = 0$ |
| g $4x - y - 6 = 0$ | h $3x - 2y + 17 = 0$ | i $8x - 2y - 7 = 0$ |

D Equations of lines

The equation of a line can be determined if we know:

- the gradient of the line, and
- the coordinates of a point on the line.

If a straight line has gradient m and passes through the point with coordinates (x_1, y_1) , then its equation is:

$$y - y_1 = m(x - x_1) \quad \text{and} \quad \frac{y - y_1}{x - x_1} = m$$

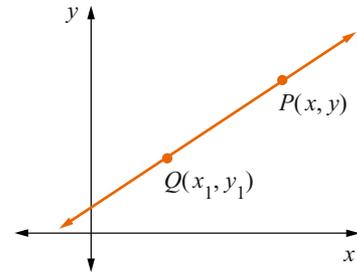
Proof

Suppose $P(x, y)$ is any point on the line with gradient m .

Equating the slopes gives:

$$\frac{y - y_1}{x - x_1} = m$$

or $y - y_1 = m(x - x_1)$



EXAMPLE 1

Find the equation of the line that has a gradient of 4 and passes through $(-1, 2)$.

Solve	Think	Apply
$y - 2 = 4(x - (-1))$ $= 4(x + 1)$ $= 4x + 4$ <p>The equation is $y = 4x + 4$.</p>	<p>The equation of the line is</p> $y - y_1 = m(x - x_1)$ <p>where $(x_1, y_1) = (-1, 2)$ and $m = 4$.</p>	<p>The point–gradient formula is a rearrangement of the gradient formula.</p>

Exercise 4D

- 1 Complete to find the equation of the line that has a gradient of 5 and passes through $(-3, 4)$.

Gradient $m = \underline{\hspace{2cm}}$

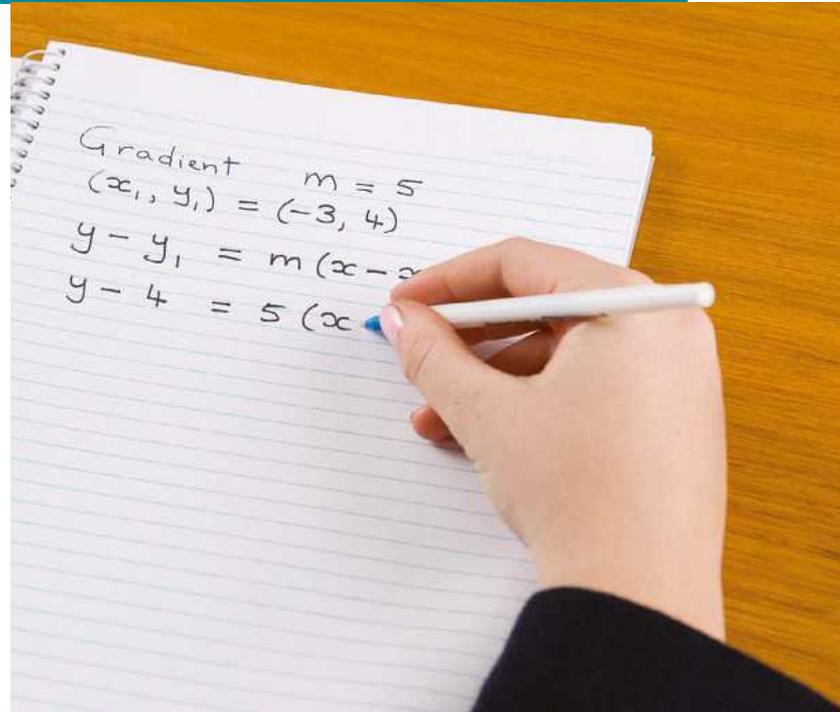
$(x_1, y_1) = (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

$$y - y_1 = m(x - x_1)$$

$$y - \underline{\hspace{1cm}} = 5(x - \underline{\hspace{1cm}})$$

$$y - 4 = 5x + \underline{\hspace{1cm}} \quad \text{Add 4 to both sides.}$$

$$y = 5x + \underline{\hspace{1cm}}$$



- 2 Find the equation of the line that has:

a a gradient of 3 and passes through $(2, -3)$

c a gradient of $-\frac{1}{2}$ and passes through $(7, 4)$

e a gradient of $\frac{1}{3}$ and passes through $(3, 7)$

g a gradient of $-\frac{1}{2}$ and passes through $(3, 4)$

i a gradient of 4 and passes through $(-\frac{1}{2}, \frac{1}{2})$

b a gradient of -2 and passes through $(-4, 2)$

d a gradient of 5 and passes through $(2, -5)$

f a gradient of $-\frac{1}{2}$ and passes through $(-5, -2)$

h a gradient of 6 and passes through $(0, 0)$

j a gradient of 0 and passes through $(5, 4)$.

EXAMPLE 2

Use $y = mx + b$ to find the equation of the line that has a gradient of 3 and passes through $(-3, 5)$.

Solve	Think	Apply
$y = mx + b$ $5 = 3(-3) + b$ $= -9 + b$ $14 = b$ The equation is $y = 3x + 14$.	$y = 3x + b$ as $m = 3$. We substitute $x = -3$ and $y = 5$ as $(-3, 5)$ lies on the line.	The equation can be found using $y = mx + b$. It can also be found using $y - y_1 = m(x - x_1)$.

3 Find the equation of the line that has:

- | | |
|--|---|
| a a gradient of 2 and passing through $(3, -4)$ | b a gradient of 1 and passing through $(5, -2)$ |
| c a gradient of -3 and passing through $(8, -2)$ | d a gradient of -2 and passing through $(-4, -4)$ |
| e a gradient of 5 and passing through $(3, 0)$ | f a gradient of $\frac{1}{2}$ and passing through $(6, 8)$ |
| g a gradient of $-\frac{1}{3}$ and passing through $(0, 5)$ | h a gradient of 8 and passing through $(0, 0)$ |
| i a gradient of $\frac{1}{3}$ and passing through $(9, 9)$ | j a gradient of 0 and passing through $(6, 2)$. |

EXAMPLE 3

Find the equation of the line that passes through the points $A(-1, 5)$ and $B(2, -3)$.

Solve	Think	Apply
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-3 - 5}{2 - (-1)} = -\frac{8}{3}$ $y - 5 = -\frac{8}{3}(x - (-1))$ $= -\frac{8}{3}(x + 1)$ $y = -\frac{8}{3}x - \frac{8}{3} + 5$ The equation is $y = -\frac{8}{3}x + \frac{7}{3}$ or $y = -\frac{8}{3}x + 2\frac{1}{3}$.	First find the gradient of AB using $(-1, 5)$ for (x_1, y_1) and $(2, -3)$ for (x_2, y_2) . Then find the equation of the line using $y - y_1 = m(x - x_1)$. Add 5 to both sides.	$B(2, -3)$ could have been used as (x_1, y_1) to give the same equation.

4 Complete to find the equation of the line that passes through the points $A(3, -2)$ and $B(7, 5)$.

Find the gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $(x_1, y_1) = (3, \underline{\quad})$
 $\quad\quad\quad = \frac{5 - \underline{\quad}}{\underline{\quad} - 3} = \frac{\underline{\quad}}{\underline{\quad}}$ $(x_2, \underline{\quad}) = (\underline{\quad}, 5)$

Use $y - y_1 = m(x - x_1)$ to find the equation.

$$y - \underline{\quad} = \frac{7}{4}(x - \underline{\quad})$$

$$y + 2 = \frac{7}{4}x - \frac{\underline{\quad}}{\underline{\quad}}$$

$$y = \frac{7x}{4} - \frac{\underline{\quad}}{\underline{\quad}} - \frac{\underline{\quad}}{\underline{\quad}}$$

$$= \frac{7}{4}x - \frac{\underline{\quad}}{\underline{\quad}}$$

Subtract 2 from both sides.

- 5 Find the equation of the line passing through the following pairs of points.
- a $A(2, 3)$ and $B(4, 7)$
 - b $A(0, 2)$ and $B(-2, 4)$
 - c $A(-1, -3)$ and $B(5, -5)$
 - d $A(6, 3)$ and $B(4, 1)$
 - e $A(5, -2)$ and $B(2, -5)$
 - f $P(0, 0)$ and $Q(3, 5)$
 - g $P(-3, 5)$ and $Q(1, -2)$
 - h $L(-3, -2)$ and $M(0, 4)$
 - i $X(2, 2)$ and $Y(-3, -1)$
 - j $X(0, 6)$ and $Y(-4, 0)$

EXAMPLE 4

Find the equation of the line that cuts the y -axis at 3 and the x -axis at -2 .

Solve	Think	Apply
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0 - 3}{-2 - 0}$ $= \frac{-3}{-2} = \frac{3}{2}$ $m = \frac{3}{2}, b = 3$ $\therefore y = \frac{3}{2}x + 3$ <p>The equation is $y = \frac{3}{2}x + 3$.</p>	<p>The y-axis is cut when $x = 0$, thus $(0, 3)$ is one point.</p> <p>The x-axis is cut when $y = 0$, thus $(-2, 0)$ is the other point.</p> <p>Use $y = mx + b$ with $m = \frac{3}{2}$ and $b = 3$.</p>	<p>Once the gradient is found then $y = mx + b$ can be used to find the equation.</p> <p>The equation can also be found using $y - y_1 = m(x - x_1)$.</p>

- 6 Find the equation of the line:
- a cutting the y -axis at -4 and the x -axis at 2
 - b cutting the y -axis at 7 and the x -axis at -2
 - c cutting the y -axis at -5 and the x -axis at -3
 - d with y -intercept -3 and x -intercept 2.

Investigation 3 Intercept form of the equation of a line

- 1 The x -intercept of a line is $(a, 0)$ and the y -intercept is $(0, b)$.
 - a Calculate the gradient of the line through these two points.
 - b Find the equation of the line using the gradient and the point $(a, 0)$.
 - c Repeat using the gradient and the point $(0, b)$.
 - d Show that the point $(a, 0)$ lies on the line $\frac{x}{a} + \frac{y}{b} = 1$.
 - e Show that the point $(0, b)$ lies on the line $\frac{x}{a} + \frac{y}{b} = 1$.
 - f Without calculating, write the equation of the line with x -intercept $(2, 0)$ and y -intercept $(0, 3)$.
 - g Use the points from part f and the equations from parts b and c to show that the equation of the line with x -intercept $x = a$ and y -intercept $y = b$ is $\frac{x}{a} + \frac{y}{b} = 1$.
- 2 Repeat Exercise 4D question 6. Check that your solutions are correct by rearranging the equation.



General form of a straight line equation

The general form of the equation of a straight line is:

$$Ax + By + C = 0 \quad \text{where } A, B \text{ and } C \text{ are all integers and } A \geq 0.$$

EXAMPLE 1

Write each equation in general form.

a $y = 3x - 1$

b $y = -\frac{2}{3}x + 4$

	Solve	Think	Apply
a	$y = 3x - 1$ $0 = 3x - y - 1$ $3x - y - 1 = 0$	Subtract y from both sides. Swap sides.	Rearrange using the usual equation-solving techniques. The coefficient of x must be positive and all values integers.
b	$y = -\frac{2}{3}x + 4$ $0 = -\frac{2}{3}x - y + 4$ $= -2x - 3y + 12$ $-2x - 3y + 12 = 0$ $2x + 3y - 12 = 0$	Subtract y from both sides. Multiply both sides by 3. Swap sides. Multiply by -1 so that $A \geq 0$.	

Exercise 4E

1 Complete to write $y = 3x - 7$ in general form.

$$y = 3x - 7$$

$$0 = 3x \text{ --- } - 7 \quad \text{Subtract } y \text{ from both sides.}$$

$$3x \text{ --- } - 7 = 0 \quad \text{Swap sides.}$$

2 Write the following equations in general form.

a $y = 2x + 1$

b $y = 5x - 2$

c $y = 2x + 5$

d $y = -2x - 5$

e $y = -3x + 4$

f $y = \frac{1}{2}x + 2$

g $y = -\frac{1}{2}x - 5$

h $y = -\frac{2}{3}x - 3$

i $y = -\frac{3}{4}x - \frac{2}{3}$

j $\frac{2}{3}y = \frac{1}{4}x + 1$

k $y = \frac{1}{2}x - \frac{3}{4}$

l $\frac{1}{5}y = x + \frac{1}{2}$

EXAMPLE 2

Rewrite the equation $4x - 3y - 12 = 0$ in the form $y = mx + b$ and hence find the gradient and y -intercept.

Solve	Think	Apply
$4x - 3y - 12 = 0$ $4x - 12 = 3y$ $3y = 4x - 12$ $y = \frac{4}{3}x - 4$ Gradient is $m = \frac{4}{3}$ and y -intercept is -4 .	Add $3y$ to both sides. Swap sides. Divide both sides by 3.	The variable y needs to be the subject with a coefficient of 1.

- 3** Complete to write $2x - 5y + 6 = 0$ in $y = mx + b$ form and hence find the gradient and y -intercept.

$$2x - 5y + 6 = 0$$

$$2x - 5y \quad + 6 = 5y$$

$$5y = 2x \quad +$$

$$\quad = \frac{2}{5}x + \quad$$

$$\text{Gradient} = m = \quad$$

$$y\text{-intercept} = b = \quad$$

Add 5y to both sides.

Swap sides.

Divide both sides by 5.

- 4** Rewrite each equation in $y = mx + b$ form and hence find the gradient and y -intercept.

a $x + 2y - 4 = 0$

b $3x + 2y - 24 = 0$

c $2x - y + 4 = 0$

d $4x - 2y - 6 = 0$

e $5x + 2y + 10 = 0$

f $3x + 2y - 8 = 0$

g $4x - y - 6 = 0$

h $3x - 2y + 17 = 0$

i $8x - 2y - 7 = 0$

- 5** Find b for the points below, which lie on the line with the given equation.

a $(2, b)$ $x + 2y = -4$

b $(-1, b)$ $3x - 4y = 6$

c $(b, 4)$ $5x + 2y = 1$

d $(b, -3)$ $4x - y = 8$

EXAMPLE 3

Find the x - and y -intercepts of the line with equation $4x - 3y - 12 = 0$.

Solve	Think	Apply
$4x - 3(0) - 12 = 0$ $4x - 12 = 0$ $4x = 12$ $x = 3$ The x -intercept is 3.	The x -intercept is when $y = 0$.	Substitute 0 for y and then solve the resulting linear equation.
$4(0) - 3y - 12 = 0$ $-3y - 12 = 0$ $-3y = 12$ $y = -4$ The y -intercept is -4 .	The y -intercept is when $x = 0$.	Substitute 0 for x and then solve the resulting linear equation.

- 6** Complete to find the x - and y -intercepts of the line with equation $5x - 7y + 35 = 0$.

The x -intercept is when $y = \quad$.

$$5x - 7(\quad) + 35 = 0$$

$$5x + 35 = 0$$

$$5x = \quad$$

$$x = \quad$$

The x -intercept is \quad .

The y -intercept is when $x = \quad$.

$$5(0) - 7y + 35 = 0$$

$$-7y \quad = 0$$

$$-7y = \quad$$

$$y = \quad$$

The y -intercept is \quad .

Subtract 35 from both sides.

Divide both sides by \quad .

Subtract 35 from both sides.

Divide both sides by \quad .

7 Find the x - and y -intercepts of the following lines.

a $x + 2y - 8 = 0$

c $2x - y + 6 = 0$

e $5x + 2y + 20 = 0$

g $4x - y - 5 = 0$

i $9x - 2y - 5 = 0$

b $3x + 2y - 12 = 0$

d $3x - 2y - 36 = 0$

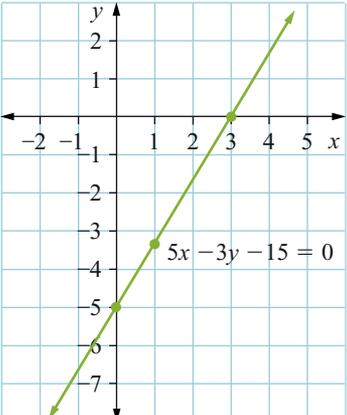
f $3x + 2y - 5 = 0$

h $3x - 2y + 15 = 0$

j $5x + 20y - 8 = 0$

EXAMPLE 4

Draw the graph of the line with equation $5x - 3y - 15 = 0$.

Solve	Think	Apply
<p>The y-intercept is -5. The x-intercept is 3.</p> 	<p>Find the x- and y-intercepts.</p> <p>When $x = 0$</p> $5(0) - 3y - 15 = 0$ $-3y - 15 = 0$ $-3y = 15$ $y = -5$ <p>When $y = 0$</p> $5x - 3(0) - 15 = 0$ $5x - 15 = 0$ $5x = 15$ $x = 3$	<p>Use a third point as a check. Try $x = 1$.</p> $5(1) - 3y - 15 = 0$ $5 - 3y - 15 = 0$ $-3y - 10 = 0$ $-3y = 10$ $y = -\frac{10}{3}$ <p>$(1, -\frac{10}{3})$ is on the line.</p>

8 Complete to draw the graph of the line with equation $6x - 5y + 30 = 0$.

Find the x - and ___ intercepts.

When $x = 0$

$$6(\underline{\quad}) - 5y + 30 = 0$$

$$\underline{\quad} = -30$$

$$y = \underline{\quad}$$

Subtract 30 from both sides.

When $y = 0$

$$6x - 5(\underline{\quad}) + 30 = 0$$

$$6x + \underline{\quad} = 0$$

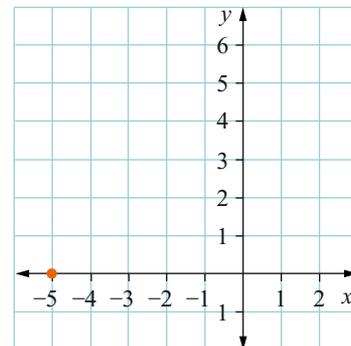
$$\underline{\quad} = -30$$

$$x = \underline{\quad}$$

Subtract ___ from both sides.

Plot x - and y -intercepts: $(-5, 0)$ and $(\underline{\quad}, 6)$.

Draw the line.



9 Find the x - and y -intercepts and then draw the graph of the line for each equation.

a $x + 2y - 8 = 0$

c $2x - 3y - 4 = 0$

e $x + y - 5 = 0$

g $3x - 4y - 12 = 0$

i $x - 2y = 0$

b $3x - y - 6 = 0$

d $4x + 3y - 8 = 0$

f $x - y + 5 = 0$

h $5x + 2y + 10 = 0$

j $2x - 5y - 5 = 0$

Investigation 4 Graphs of lines

- 1 a** On the same number plane draw the straight lines with these equations.
 $y = 2x, y = 2x + 1, y = 2x - 3, y = 2x - 1$
- b** What do you notice about these lines?
- 2 a** On the same number plane draw the straight lines with these equations.
 $y = -3x + 1, 3x + y - 2 = 0, 6x + 2y + 3 = 0$
- b** What do you notice about these three lines?
- c** Rewrite the second and third equations in $y = mx + b$ form and find their gradients.
- 3** Complete: Straight lines are _____ if their _____ are equal.
- 4 a** On the same number plane draw the straight lines with equations $y = 2x + 1$ and $y = -\frac{1}{2}x + 2$.
- b** What do you notice about these lines?
- 5 a** On the same number plane draw the straight lines with equations $y = 3x - 1$ and $x + 3y - 6 = 0$.
- b** What do you notice about these lines?
- c** Rewrite $x + 3y - 6 = 0$ in $y = mx + b$ form and find the gradient.
- 6** Complete: Straight lines are _____ if the product of their gradients is _____.

Use a graphics calculator. 

F Parallel and perpendicular lines

From Investigation 4 it can be seen that if two straight lines have gradients m_1 and m_2 then:

- the lines are parallel if $m_1 = m_2$
- the lines are perpendicular if $m_1 \times m_2 = -1$.

EXAMPLE 1

- a** Find the equation of the line parallel to the line $y = -5x - 7$ and passing through the point $(2, 3)$.
- b** Find the equation of the line parallel to the line $3x - 6y + 8 = 0$ and passing through the point $(-1, -2)$.

	Solve	Think	Apply
a	$y = -5x + b$ $3 = -5(2) + b$ $= -10 + b$ $b = 13$ The equation is $y = -5x + 13$.	$m = -5$, as the lines are parallel. Substitute $(2, 3)$.	Parallel lines have equal gradients. Find the gradient by inspection or calculation then use $y = mx + b$ or $y - y_1 = m(x - x_1)$.
b	Use $y - y_1 = m(x - x_1)$ with $m = \frac{1}{2}$ and $(-1, -2)$ for (x_1, y_1) . $y - (-2) = \frac{1}{2}(x - (-1))$ $y + 2 = \frac{1}{2}(x + 1)$ $y = \frac{1}{2}x + \frac{1}{2} - 2$ The equation is $y = \frac{1}{2}x - 1\frac{1}{2}$.	Rearrange $3x - 6y + 8 = 0$ into $y = mx + b$ form to find the gradient. $3x + 8 = 6y$ $\frac{3x}{6} + \frac{8}{6} = y$ $y = \frac{1}{2}x + \frac{4}{3}$ $m = \frac{1}{2}$	

Exercise 4F

1 Which of the following pairs of lines are parallel?

A $y = 3x + 1$

$y = 3x - 5$

D $y = 4x - 3$

$y = 4 - 3x$

G $4x - 3y + 5 = 0$

$3x + 4y + 2 = 0$

B $y = 2x - 1$

$y = 2x$

E $y = 2x - 5$

$2x - y + 4 = 0$

H $2x + 3y - 2 = 0$

$2x + 3y - 5 = 0$

C $y = 5x + 3$

$y = 3x + 5$

F $y = -x - 5$

$x - 2y + 3 = 0$

I $3x - 2y - 5 = 0$

$y = \frac{2}{3}x + 3$

2 Complete to find the equation of the line parallel to $y = 3x - 5$ and passing through the point $(-4, 3)$.

As the lines are parallel the gradient is _____. So the equation is $y = 3x + b$.

Substitute $(-4, 3)$ into the equation.

$$3 = 3(\underline{\quad}) + b$$

$$3 = \underline{\quad} + b$$

Add _____ to both sides.

$$\underline{\quad} = b$$

The equation is $y = \underline{\quad}x + 15$.

3 Find the equation of the line:

a parallel to the line $y = 2x - 5$ and passing through the point $(1, 4)$

b parallel to the line $y = -7x - 2$ and passing through the point $(-5, -2)$

c parallel to the line $y = -\frac{1}{2}x + 5$ and passing through the origin

d parallel to the line $5x - 7y + 3 = 0$ and passing through the point $(2, -3)$

e passing through $(-1, -3)$ and parallel to the line passing through the points $(1, 5)$ and $(3, 6)$

f with y -intercept -2 and parallel to the line segment joining $(-7, 5)$ and the origin.

EXAMPLE 2

a Is the line $y = 3x - 5$ perpendicular to the line $2x + 6y + 9 = 0$?

b What is the gradient of the line perpendicular to the line $5x - 2y + 4 = 0$?

c Find the equation of the line passing through $(6, -3)$ and perpendicular to the line $y = -\frac{2}{3}x + 4$.

	Solve	Think	Apply
a	$2x + 6y + 9 = 0$ $6y = -2x - 9$ $y = \frac{-2x}{6} - \frac{9}{6}$ $y = -\frac{1}{3}x - \frac{3}{2}$ <p>$2x + 6y + 9 = 0$ has gradient $m_2 = -\frac{1}{3}$.</p> <p>$y = 3x - 5$ has gradient $m_1 = 3$.</p> <p>The lines are perpendicular.</p>	<p>Rearrange $2x + 6y + 9 = 0$ into $y = mx + b$ form.</p> <p>Divide both sides by 6.</p> $m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$	<p>Lines with gradients m_1 and m_2 are perpendicular if $m_1 \times m_2 = -1$.</p>
b	$5x - 2y + 4 = 0$ $5x + 4 = 2y$ $\frac{5x}{2} + \frac{4}{2} = y$ $y = \frac{5}{2}x + 2 \text{ so } m = \frac{5}{2}$ <p>The perpendicular line has gradient $-\frac{2}{5}$.</p>	<p>Rearrange $5x - 2y + 4 = 0$ into $y = mx + b$ form.</p> <p>Divide both sides by 2.</p> <p>$5x - 2y + 4 = 0$ has gradient $m_1 = \frac{5}{2}$.</p> $m_1 \times m_2 = \frac{5}{2} \times -\frac{2}{5} = -1$	

EXAMPLE 2 CONTINUED

	Solve	Think/Apply
c	$y = \frac{3}{2}x + b$ $-3 = \frac{3}{2}(6) + b$ $= 9 + b$ $\therefore b = -12$ The equation of the line is $y = \frac{3}{2}x - 12$.	$y = -\frac{2}{3}x + 4$ has gradient $m_1 = -\frac{2}{3}$. The gradient of the perpendicular line $m_2 = \frac{3}{2}$. $m_1 \times m_2 = -\frac{2}{3} \times \frac{3}{2} = -1$ Substitute (6, -3) into the perpendicular line $y = \frac{3}{2}x + b$.

- 4** Complete to determine if the line $y = -2x + 5$ is perpendicular to $x - 2y - 4 = 0$.

$y = -2x + 5$ has gradient $m_1 = \underline{\hspace{2cm}}$.

Rearrange $x - 2y - 4 = 0$

$x - 2y + 2y - 4 = \underline{\hspace{2cm}}$

$x - \underline{\hspace{2cm}} = 2y$

Add 2y to both sides.

$\frac{1}{2}x - \underline{\hspace{2cm}} = y$

Divide both sides by 2.

Gradient $m_2 = \underline{\hspace{2cm}}$

$m_1 \times m_2 = \underline{\hspace{2cm}} \times \frac{1}{2}$

$= \underline{\hspace{2cm}}$

The lines are perpendicular.

- 5** Which of the following pairs of lines are perpendicular?

A $y = 2x - 5$

$y = 2x$

B $y = -3x + 5$

$y = \frac{1}{3}x - 2$

C $y = 4x + 7$

$y = -4x + 3$

D $y = \frac{2}{5}x - 1$

$y = -\frac{5}{2}x - 1$

E $3x - 4y + 5 = 0$

$4x + 3y - 2 = 0$

F $7x + 5y + 3 = 0$

$5y - 7x = 0$

- 6** Find the gradients of the lines perpendicular to the given lines.

a $y = -\frac{3}{5}x + 2$

b $y = 7x - 2$

c $y = \frac{3}{2}x - 5$

d $y = -\frac{4}{5}x - 7$

e $3x - 2y + 1 = 0$

f $5x - 7y + 7 = 0$

- 7** Find the equation of the line:

a perpendicular to $y = 5x - 2$ and passing through $(-3, 2)$

b perpendicular to $y = -\frac{1}{4}x + 7$ and passing through $(0, 5)$

c passing through the origin and perpendicular to $y = \frac{3}{4}x + \frac{1}{3}$

d passing through $(-2, 5)$ and perpendicular to $3x - 4y + 12 = 0$

e with y-intercept 7 and perpendicular to $5x + 2y - 7 = 0$

f passing through $(-2, -5)$ and perpendicular to the line segment joining $(2, 3)$ and $(5, -3)$

g passing through the origin and perpendicular to the line segment joining $(3, 0)$ and $(0, -5)$.

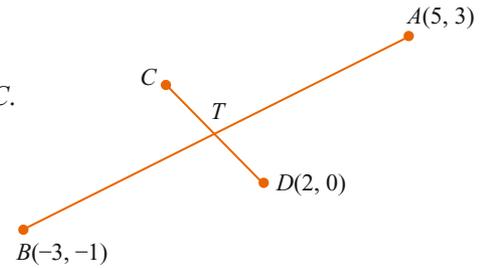
- 8 a** Find the equation of all lines that are perpendicular to $y = \frac{1}{2}x - 3$.

b Find the equation of all lines that are parallel to $y = 3x - 2$.

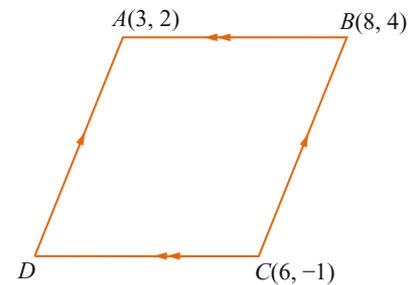
c Write the equation of all lines parallel to $3x - 5y + 6 = 0$.

d Write the equation of all lines perpendicular to $5x - 3y + 7 = 0$.

- 2** Classify $\triangle LMN$ with vertices $L(-2, -1)$, $M(0, 3)$ and $N(4, 1)$ as scalene, isosceles or equilateral. Give a reason for your answer.
- 3** Triangle ABC has $A(-1, 4)$, $B(2, -1)$ and $C(5, 2)$ as vertices. Find the length of the line segment from A to the midpoint of BC .
- 4 a** By finding the equation of line AB and substituting the point C show that points $A(2, 1)$, $B(-2, -11)$ and $C(4, 7)$ all lie on a straight line.
b Show that the points $A(-3, 13)$, $B(3, 1)$ and $C(4, -1)$ all lie on the same straight line.
- 5** Line segments AB and CD bisect each other at T .
a Using A and B , find the coordinates of the midpoint T .
b Use the midpoint T and the point D to find the coordinates of C .



- 6** $ABCD$ is a parallelogram. Use the fact that the diagonals of the parallelogram bisect each other to find the coordinates of D .



- 7** The triangle XYZ has vertices $X(1, 2)$, $Y(2, 5)$ and $Z(4, 1)$.
a Find the length of each of the sides XY , YZ and XZ .
b Use Pythagoras' rule to decide if $\triangle XYZ$ is right angled. Give a reason for your answer.
c Find the gradient of the sides XY , YZ and XZ .
d Two lines are perpendicular, that is they meet at right angles, if the product of their gradients is -1 . Multiply each pair of gradients together to find the pair that gives -1 .
e Use the gradients to decide if $\triangle XYZ$ is right angled. Give a reason for your answer.
- 8** Is the triangle with vertices $D(-2, -1)$, $E(1, -1)$ and $F(-2, 3)$ right angled? Give a reason for your answer.
- 9** The triangle PQR has vertices $P(-2, 5)$, $Q(3, -1)$ and $R(-4, -7)$.
a Find the coordinates of S , the midpoint of PQ .
b Find the coordinates of T , the midpoint of PR .
c Show that the length of QR is twice the length of ST .
- 10** The quadrilateral $PQRS$ has vertices $P(2, 4)$, $Q(5, 1)$, $R(-1, -2)$ and $S(-4, 1)$.
a Prove that $PQRS$ is a parallelogram by showing that:
i the opposite sides are equal
ii the diagonals bisect each other because they have the same midpoint.
b i Find the gradient of each side.
ii Is $PQRS$ a rectangle; that is, are the sides perpendicular?
c Is $PQRS$ a rhombus; that is, are all sides equal?

Language in mathematics

1 Rearrange these words to form a sentence.

- a run over Gradient rise is
- b y -intercept b is symbol for the
- c is the m gradient symbol for
- d ordinates Midpoint average of is the the
- e theorem distance Pythagoras' The to is formula related
- f $y = mx + b$ straight line gradient–intercept form is the a of

2 Complete these glossary terms by adding the vowels.

- a q t n
- b f rm l
- c gr d nt
- d p r ll l
- e v rt c l r s
- f h r z nt l r n

3 Use every third letter to complete a sentence.

D F T E D W V G O H U L I O I K J N H G E F D S A S A E R R T G E B N P M
 I E K J R G T P Q A E Z S N W E D R F I D C C S H U K O L P L A D U R I
 O I D E F A S T F V H B K E N J P M H R K I O L O D P O U T U C X Y T T
 T O R R F E E T D D H F F E X F I V H R N P G O I R Y T A E D D S V I H
 U E K A N E D T A S S C F I V G S F D N E S E A E G A E A A E T D R I F
 G V H N E M J O T G N A X E H U A J K N U P D Z C T V B H M K E L P Y I
 O A U Y R U I E I J P H G A I E R E T A R E L E T L R T E A S L C V I B H
 F N G T D S H F G E H N I M J R I K G U Y R T R A E D D E D I S W E A D
 N A S T C V S B N A N N R M M E S S E R R Q A A U S S A Q W L

Terms

average	distance formula	equation	formula	general form
gradient	gradient–intercept form	graph	horizontal run	intercept
linear relationship	midpoint	parallel	perpendicular	point
Pythagoras' theorem	rearrange	sketch	straight line	tables of values
vertical rise	x -intercept	y -intercept		

Check your skills

1 The distance between the points (5, 3) and (1, 7) is:

- A $\sqrt{16}$
- B $\sqrt{32}$
- C $\sqrt{136}$
- D $\sqrt{8}$

2 The distance between points (−5, 8) and (6, −2) is:

- A $\sqrt{11}$
- B $\sqrt{21}$
- C $\sqrt{37}$
- D $\sqrt{221}$

3 The midpoint of the join of (5, 5) and (9, 5) is:

- A (14, 5)
- B (7, 5)
- C (4, 10)
- D (4, 0)

4 The midpoint of the join of (2, −4) and (2, 8) is:

- A (2, 2)
- B (2, 4)
- C (0, 12)
- D (4, 4)

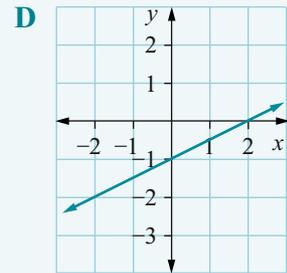
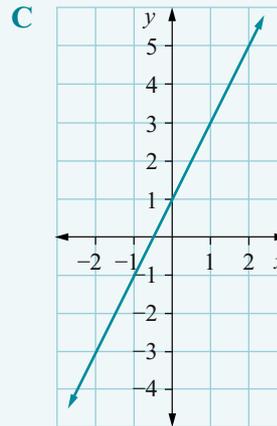
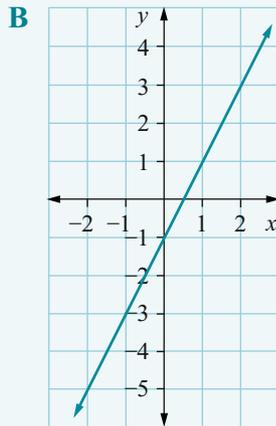
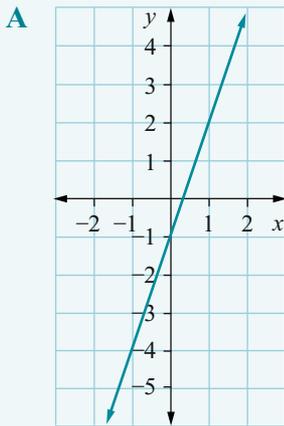
5 The midpoint of the join of (−1, 3) and (7, 1) is:

- A (3, 2)
- B (8, 2)
- C (8, 4)
- D (4, 2)

- 6 The midpoint of the join of $(-6, 3)$ and $(5, -9)$ is:
A $(-1, -6)$ **B** $(-\frac{1}{2}, -3)$ **C** $(\frac{1}{2}, 3)$ **D** $(-5\frac{1}{2}, 6)$
- 7 The midpoint of PQ is $(4, -3)$. If P has coordinates $(7, 2)$ then the coordinates of Q are:
A $(5\frac{1}{2}, -\frac{1}{2})$ **B** $(-10, 7)$ **C** $(1, -8)$ **D** $(1\frac{1}{2}, -\frac{1}{2})$
- 8 The gradient of the line passing through the points $(4, 3)$ and $(-5, 7)$ is:
A $\frac{9}{4}$ **B** -10 **C** $-\frac{1}{10}$ **D** $-\frac{4}{9}$
- 9 The formula for gradient is:
A $m = \frac{x_2 - x_1}{y_2 - y_1}$ **B** $m = \frac{y_2 + y_1}{x_2 + x_1}$ **C** $m = \frac{y_2 - y_1}{x_2 - x_1}$ **D** $m = \frac{x_2 + x_1}{y_2 + y_1}$
- 10 The length of the line segment from $(1, 3)$ to the midpoint of $(-5, 3)$ and $(3, -1)$ is:
A $\sqrt{8}$ **B** $\sqrt{36}$ **C** $\sqrt{20}$ **D** $\sqrt{80}$

- 11 By using this table, the graph of $y = 2x - 1$ is:

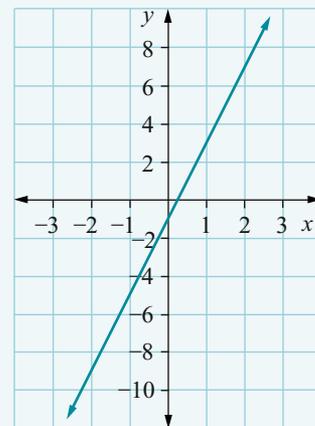
x	-2	0	2
y			



- 12 By completing this table of values for each equation, the equation of this graph is:

x	-2	0	2
y			

- A** $y = 3x - 1$ **B** $y = -3x + 1$
C $y = 4x - 1$ **D** $y = -4x - 1$



- 13 The equation of the line corresponding to the values in the table is:

x	-4	-3	-2	-1	0	1	2	3	4
y	7	5	3	1	-1	-3	-5	-7	-9

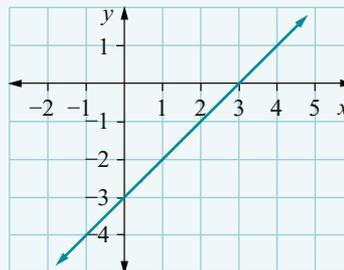
- A** $y = 2x - 1$ **B** $y = -2x - 1$ **C** $y = x + 3$ **D** $y = -x + 3$

14 The equation of the line with gradient -3 and y -intercept 5 is:

- A** $y = -3x + 5$ **B** $y = 5x - 3$ **C** $x = -3y + 5$ **D** $x = 5y - 3$

15 The equation of the line shown at right is:

- A** $y = -x - 3$ **B** $y = -x + 3$
C $y = x + 3$ **D** $y = x - 3$



16 The equation $4x - 5y + 20 = 0$ in gradient–intercept form is:

- A** $5y = 4x + 20$ **B** $y = \frac{4x}{5} + 4$ **C** $y = -\frac{4x}{5} - 20$ **D** $x - \frac{5y}{4} + 5 = 0$

17 The equation of the line passing through $(-2, -6)$ with gradient 2 is:

- A** $y = 2x + 3$ **B** $y = 2x - 7$ **C** $y = 2x - 2$ **D** $y = 2x + 7$

18 The equation of the line passing through $A(-4, 5)$ and $B(2, -13)$ is:

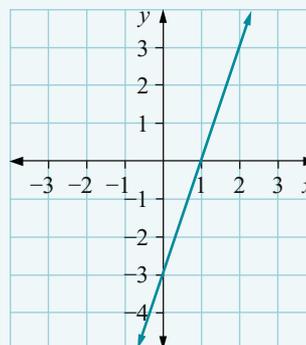
- A** $y = -x - 1$ **B** $y = x + 5$ **C** $y = -3x - 7$ **D** $y = -x + 1$

19 The equation of the line cutting the y -axis at -4 and the x -axis at $+5$ is:

- A** $y = \frac{4}{5}x - 4$ **B** $y = -\frac{4}{5}x - 4$ **C** $y = \frac{5x}{4} - 4$ **D** $y = -\frac{5x}{4} - 4$

20 The equation of the line shown on the right is:

- A** $y = 3x - 3$ **B** $y = -3x - 3$
C $y = -x + 3$ **D** $y = 2x - 3$



21 The point $(a, 1)$ lies on the line $y = 3x - 2$, so the value of a is:

- A** 0 **B** 1 **C** 2 **D** 3

22 In general form $y = \frac{1}{2}x - \frac{4}{3}$ is:

- A** $6y - 3x + 8 = 0$ **B** $3x - 6y - 8 = 0$
C $3x + 2y - 4 = 0$ **D** $\frac{1}{2}x - y - \frac{4}{3} = 0$

23 When written in gradient–intercept form, the equation $3x + 2y + 8 = 0$ is:

- A** $2y = 3x + 4$ **B** $3x - 2y = -8$
C $y = -\frac{3x}{2} + 8$ **D** $y = -\frac{3x}{2} - 4$

24 The x - and y -intercepts of $4x + 3y - 12 = 0$ are:

- A** 4 and -3 **B** 3 and 4 **C** -4 and 3 **D** -3 and 4

25 A line parallel to $y = -3x + 1$ is:

- A** $y = \frac{1}{3}x + 1$ **B** $y = -\frac{1}{3}x + 1$ **C** $y = -3x + 3$ **D** $y = -x + 3$

26 A line perpendicular to $3x + 5y + 7 = 0$ is:

- A** $3x + 5y - 2 = 0$ **B** $3x - 5y - 7 = 0$ **C** $3x + 5y + 7 = 0$ **D** $y = \frac{5x}{3} + 2$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–10	11, 12	13–16	17–21	22–24	25, 26
Section	A	B	C	D	E	F

4A Review set

1 For the points $A(3, -1)$ and $B(-5, 0)$, find:
a the distance AB **b** the midpoint of AB **c** the gradient of AB .

2 The midpoint of $A(4, 2)$ and $B(x, y)$ is $(-2, 7)$. Find the coordinates of B .

3 Find the fourth vertex of the parallelogram $ABCD$ for $A(-7, 11)$, $B(6, 5)$ and $C(3, 8)$.

4 a Complete the following table of values for $y = 2x + 3$.

x	-3	-2	-1	0	1	2	3
y							

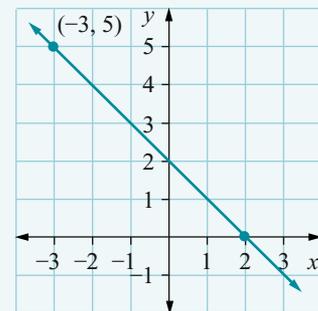
- b** Plot the resulting ordered pairs on a set of coordinate axes and draw the straight line through the points.
c Does the point $(-10, -17)$ lie on the line?

5 a Write the equation of the line with y -intercept -2 and gradient -3 .
b Sketch the line.

6 a Write the equation of the line with gradient $\frac{2}{3}$ and y -intercept -1 .
b Sketch the line.

7 Consider the straight line shown on the right.

- a** Calculate the gradient.
b Find the y -intercept.
c Write the equation of the line.



8 a Write the equation $3x - 2y + 12 = 0$ in the form $y = mx + b$.
b Sketch the line.

9 For the points $A(3, -1)$ and $B(-5, 0)$, find:
a the gradient of AB
b the equation of the line through A and B
c the x - and y -intercepts of the line AB
d the equation of the line perpendicular to AB passing through A
e the equation of the line parallel to AB passing through the point $(3, 0)$.

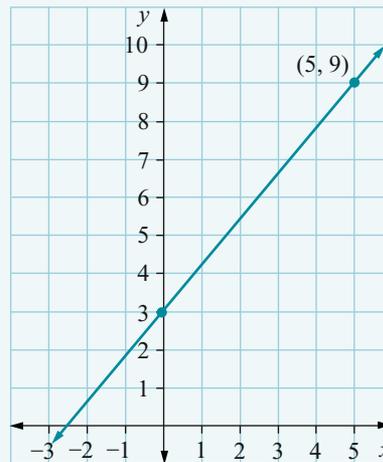
- 10** **a** Write $3y = 4x - 7$ in general form.
b Write the equation of the line passing through $(-4, 7)$ and $(2, 6)$ in general form.
c Sketch the line $3x - 4y + 12 = 0$.
- 11** **a** Find k if $2x + ky = 5$ is perpendicular to $x - 3y = 11$.
b $A(-1, 3)$, $B(2, 4)$ and $C(t, -1)$ are collinear. Find t .

4B Review set

- 1** For the points $A(0, 0)$ and $B(-7, 4)$, find:
a the distance AB **b** the midpoint of AB **c** the gradient of AB .
- 2** The triangle ABC has vertices $A(1, 1)$, $B(5, 3)$ and $C(3, -1)$. Find the length of each side and classify the triangle as scalene, isosceles or equilateral.
- 3** **a** Complete this table of values for $y = -5x + 7$.

x	-3	-2	-1	0	1	2	3
y							

- b** Sketch $y = -5x + 7$.
- 4** **a** Write the equation of the line with y -intercept 3 and gradient -2 .
b Sketch the line.
- 5** Find the equation of this straight line shown on the right.



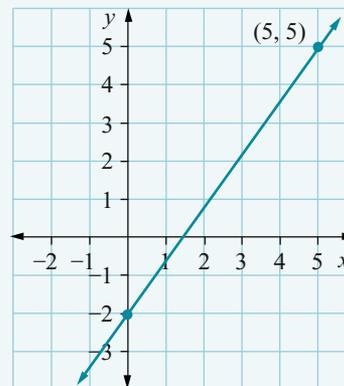
- 6** Find the equation of the line passing through $M(-4, 2)$ and $N(3, 7)$.
- 7** Find the equation of the line passing through $(-4, 5)$ and perpendicular to $3x - 2y + 7 = 0$.
- 8** Write $y = -\frac{2}{3}x - 4$ in general form.
- 9** Find the equation of the perpendicular bisector of the join of $(-2, 3)$ and $(4, 5)$.

4C Review set

- 1 For the points $P(-6, 3)$ and $Q(-1, -5)$, find:
- the distance PQ
 - the midpoint of PQ
 - the gradient of PQ .
- 2 The midpoint of $L(x, y)$ and $M(0, -1)$ is $(3, -4)$. Find the coordinates of L .
- 3 Find the coordinates of D for parallelogram $ABCD$ if $A(-2, 3)$, $B(1, 7)$ and $C(5, 1)$ are vertices.
- 4 a Complete the following table of values for $y = -3x + 2$.

x	-3	-2	-1	0	1	2	3
y							

- Plot the resulting ordered pairs on a set of coordinate axes and draw the straight line through the points.
 - Does the point $(-5, 17)$ lie on the line?
- 5 a Write the equation of the line with gradient 4 and y -intercept -2 .
- Write the line.
- 6 a Write the equation of the line with gradient $-\frac{3}{4}$ and y -intercept -2 .
- Sketch the line.
- 7 Consider the straight line on the right.
- Calculate the gradient.
 - Find the y -intercept.
 - Write the equation of the line.



- Write the equation $5x - 4y + 20 = 0$ in the form $y = mx + b$.
 - Sketch the line.
- 9 For the points $P(-6, 3)$ and $Q(-1, -5)$, find:
- the gradient of PQ
 - the equation of the line through P and Q
 - the x - and y -intercepts of the line PQ
 - the equation of the line perpendicular to PQ and passing through Q
 - the equation of the line parallel to PQ and passing through the origin.
- 10 a Write $4x - 7y + 8 = 0$ in gradient–intercept form.
- Find the equation of the line passing through $(4, -2)$ and the origin in general form.
 - Sketch the line $6x - 7y + 9 = 0$.

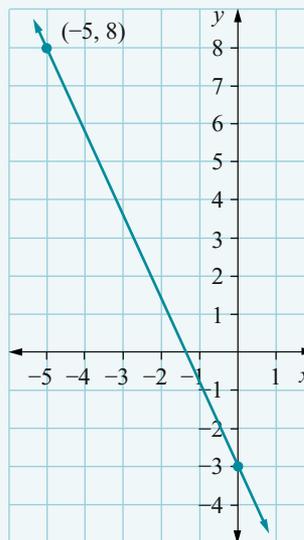
4D

Review set

- 1 For the points $A(-3, 5)$ and $B(2, 4)$, find:
- a the distance AB b the midpoint of AB c the gradient of AB .
- 2 The triangle LMN has vertices $L(2, 3)$, $M(3, 6)$ and $N(5, 2)$.
- a Calculate the length of each side.
b Determine whether $\triangle LMN$ is right-angled. Give a reason for your answer.
- 3 a Complete the table of values for $y = -\frac{1}{2}x + 3$.

x	-4	-2	0	2	4
y					

- b Sketch $y = -\frac{1}{2}x + 3$.
c What is the gradient and y -intercept of $y = -\frac{1}{2}x + 3$?
- 4 a Write the equation of the line with gradient 2 and y -intercept -8 .
b Sketch the line.
- 5 Write the equation of the straight line shown on the right.



- 6 a Find the equation of the line with gradient 3 and passing through $(-2, 4)$.
b Find the equation of the line parallel to $3x - 5y = 8$ and passing through the origin.
- 7 Write $y = -\frac{3x}{4} + 2$ in general form.
- 8 Does the point $(1, 4)$ lie on the line $3x - 5y + 2 = 0$? Explain your answer.



5 Probability

This chapter deals with multistage events.

After completing this chapter you should be able to:

- ▶ list outcomes, describe the results and calculate the probabilities of multistage events, with and without replacement
- ▶ distinguish informally between dependent and independent events
- ▶ recognise that for independent events $P(A \text{ and } B) = P(A) \times P(B)$
- ▶ record outcomes of three-step chance experiments, with and without replacement, using organised lists, tables and tree diagrams
- ▶ calculate the probabilities of events in which a condition restricts the sample space
- ▶ critically evaluate conditional statements used in descriptions of chance situations.

NSW Syllabus references: 5.2 S&P Probability

Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-17SP

STATISTICS & PROBABILITY – ACMSP225, ACMSP246, ACMSP247

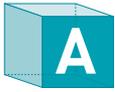


Diagnostic test

- 1 A normal six-sided die is thrown once. The number of outcomes in the sample space is:
A 1 B 3 C 5 D 6
- 2 A ticket is selected at random from a hat containing 1 blue, 2 red and 1 green ticket. The sample space is:
A {blue, red, green} B {blue}
C {red} D {not red}
- 3 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of randomly selecting a purple ball is:
A 1 B 3 C $\frac{1}{3}$ D $\frac{2}{3}$
- 4 The probability of an event is $\frac{2}{5}$. As a percentage this is:
A 20% B 40%
C 50% D none of these
- 5 A hat contains 3 blue and 5 black tickets. If one ticket is chosen at random from the hat, the probability that it is blue is:
A $\frac{1}{2}$ B $\frac{3}{5}$ C 3 D $\frac{3}{8}$
- 6 A letter is chosen randomly from the word PROBABILITY. The probability that the letter chosen is B is:
A $\frac{1}{11}$ B $\frac{2}{11}$ C 1 D $\frac{2}{9}$
- 7 A term describing a probability of about 80% is:
A certain B high probability
C evens D low probability
- 8 A coin is tossed once. An impossible event would be getting:
A heads B tails
C heads or tails D a six
- 9 When the probabilities of all outcomes of an event are added together their total is:
A 1 B 0
C between 1 and 0 D impossible
- 10 An eight-sided die with the numbers 1 to 8 is thrown once. The probability that the number thrown is *not* a 5 is:
A $\frac{1}{8}$ B $\frac{5}{8}$ C $\frac{7}{8}$ D $\frac{8}{8}$
- 11 The probability that Teesha wins a game is $\frac{2}{7}$. Thus the probability that she loses is:
A $\frac{2}{7}$ B $\frac{7}{2}$
C $\frac{5}{7}$ D none of these
- 12 The probability of an event is $\frac{3}{4}$. The probability of the complementary event is:
A $\frac{3}{4}$ B $\frac{2}{4}$ C $\frac{1}{4}$ D 0

The diagnostic test questions refer to outcome ACMSP226.





Review of Year 9

In Year 9 the following facts were discovered.

$$\text{Relative frequency} = \frac{\text{frequency of an event}}{\text{total number of trials}}$$

Note: Number of trials = number of times the experiment is repeated.

The relative frequency of an event can be used to predict the probability of an event occurring. When used in this way, **relative frequency** is often referred to as **experimental probability**. It is based on the assumption that the number of times that an event has occurred in the past is an indication of how often it will occur in the future.

As the number of trials increases, the relative frequency gets closer to the theoretical probability. (It is said that the probability estimates become more stable as the number of trials increases.) The **theoretical probability** can be thought of as being the likelihood of an event occurring under ideal conditions. Hence the probability of an event can be estimated from experimental data by using the relative frequency of the event, as long as the number of trials is large enough.

Venn diagrams and two-way tables are useful methods for displaying information.

EXAMPLE 1

A cylindrical can is tossed 200 times and the number of times it landed on its side and on an end is shown in the table.

- a Write down the relative frequency of each outcome.
- b Estimate the probability that in a future toss of the can it will land on its:
 - i end
 - ii side.

Outcome	Frequency
End	51
Side	149

	Solve	Think	Apply
a	End: $\frac{51}{200}$ or 25.5% Side: $\frac{149}{200}$ or 74.5%	Relative frequency = $\frac{\text{frequency of event}}{\text{total number of trials}}$	The probability of an event can be estimated from experimental data by using the relative frequency of the event.
b i	25.5%	Probability of 'land on end' \approx relative frequency of 'land on end'	
ii	74.5%	Probability of 'land on side' \approx relative frequency of 'land on side'	

Exercise 5A

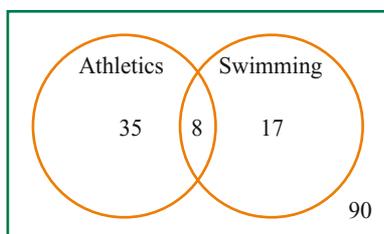
- 1 A drawing pin is dropped 300 times and the number of times it landed 'point up' and 'point down' is shown in the table.
 - a Write down the relative frequency of each outcome.
 - b Estimate the probability that in a future drop of the drawing pin it will land:
 - i point up
 - ii point down.

Outcome	Frequency
Point up	93
Point down	207

EXAMPLE 2 CONTINUED

	Solve	Think
e	$P(\text{plays both tennis and soccer})$ $= \frac{2}{30} = \frac{1}{15}$	Number of students who play both tennis and soccer = number in the intersection of the circles = 2
f	$P(\text{plays tennis or soccer or both})$ $= \frac{24}{30} = \frac{4}{5}$	Number of students who play tennis or soccer or both = number inside the circles = $8 + 2 + 14$ = 24
g	$P(\text{plays tennis or soccer but not both})$ $= \frac{22}{30} = \frac{11}{15}$	Number of students who play tennis or soccer but not both = number inside the two circles, but excluding the intersection = $8 + 14 = 22$

- 5** The Venn diagram represents the number of students in Year 10 who were chosen for the school athletics and swimming teams.

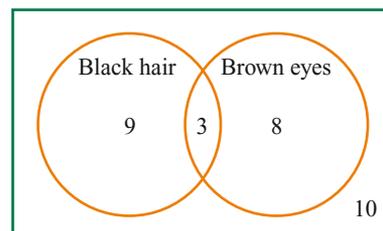


What is the probability that a Year 10 student chosen at random:

- a** was chosen for the athletics team?
- b** was not chosen for the athletics team?
- c** was chosen for the athletics team but not the swimming team?
- d** was chosen for the swimming team?
- e** was not chosen for the swimming team?
- f** was chosen for the swimming team but not the athletics team?
- g** was chosen for neither team?
- h** was chosen for both teams?
- i** was chosen for either the athletics team or the swimming team or both?
- j** was chosen for either the athletics team or the swimming team but not both?



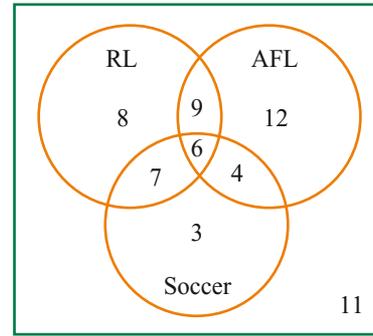
- 6** The Venn diagram represents the number of students in a class who have black hair and brown eyes. What is the probability that a student chosen at random from this class:



- a** has black hair?
- b** does not have black hair?
- c** has black hair but not brown eyes?
- d** has brown eyes?
- e** does not have brown eyes?
- g** does not have black hair or brown eyes?
- i** has black hair or brown eyes or both?
- f** has brown eyes but not black hair?
- h** has both black hair and brown eyes?
- j** has black hair or brown eyes but not both?

7 The Venn diagram represents the results of a survey of which sports people liked to watch. Calculate the probability that a person chosen at random from this group:

- a** does not watch rugby league
- b** watches rugby league but not soccer or AFL
- c** watches soccer and AFL but not rugby league
- d** does not watch any of these sports
- e** watches all three sports
- f** watches rugby league or AFL or both
- g** watches AFL or soccer but not both?



EXAMPLE 3

The two-way table shows the data collected from a survey of Year 10 students.

	Left-handed	Right-handed	
Male	3	9	12
Female	7	11	18
	10	20	30

What is the probability that a student chosen at random from this class is:

- a** female?
- b** left-handed?
- c** not left-handed?
- d** male and right-handed?
- e** male or right-handed or both?
- f** male or right-handed but not both?
- g** neither male nor right-handed?
- h** right-handed but not male?

	Solve	Think	Apply
a	$P(\text{female}) = \frac{18}{30} = \frac{3}{5}$	Number of females = 18	Find the number in the appropriate row and column and calculate the probability.
b	$P(\text{left-handed}) = \frac{10}{30} = \frac{1}{3}$	Number of left-handed students = 10	
c	$P(\text{not left-handed}) = \frac{20}{30} = \frac{2}{3}$	Number not left-handed students = number right-handed students = 20	
d	$P(\text{male and right-handed}) = \frac{9}{30} = \frac{3}{10}$	Number who are male and right-handed = 9	
e	$P(\text{male or right-handed or both}) = \frac{23}{30}$	Number who are male or right-handed or both = $3 + 9 + 11 = 23$	
f	$P(\text{male or right-handed but not both}) = \frac{14}{30} = \frac{7}{15}$	Number who are male or right-handed but not both = $3 + 11 = 14$	
g	$P(\text{neither male nor right-handed}) = \frac{7}{30}$	Number who are neither male nor right-handed = number who are female and left-handed = 7	
h	$P(\text{right-handed but not male}) = \frac{11}{30}$	Number who are right-handed but not male = number of right-handed females = 11	

- 8 The information in the table was collected from a group of athletes.

	Heavy	Light	
Tall	9	7	16
Short	5	9	14
	14	16	30

Calculate the probability that an athlete chosen at random from this group is:

- | | | |
|-------------------------------------|---------------------------------|--------------------------------------|
| a tall | b short | c heavy |
| d light | e short and light | f short and heavy |
| g tall and light | h tall and heavy | i tall or light or both |
| j tall or light but not both | k short or heavy or both | l short or heavy but not both |
| m neither short nor heavy | n neither tall nor light | o heavy but not short |
| p light but not tall. | | |

- 9 The data in the table was collected from a survey of people attending a fitness class.

	Male	Female	
Overweight	18	9	27
Not overweight	30	43	73
	48	52	100

Calculate the probability that a person chosen at random from this group is:

- a** female and overweight
- b** male but not overweight
- c** male or overweight or both
- d** male or overweight but not both
- e** male and overweight
- f** female but not overweight
- g** female or overweight or both
- h** female or overweight but not both
- i** neither overweight nor female.



- 10 Students were surveyed about whether they played soccer or netball. The results are shown in the table.

	Soccer	Not soccer	
Netball	12	5	17
Not netball	4	7	11
	16	12	28

Calculate the probability that a student chosen at random from this group plays:

- | | | |
|-------------------------------------|---------------------------------|------------------------------------|
| a soccer | b soccer and netball | c soccer or netball or both |
| d neither soccer nor netball | e soccer but not netball | f netball but not soccer. |

EXAMPLE 2 CONTINUED

	Solve	Think	Apply
b i	$P(\text{HR}) = \frac{3}{14}$	Total number of equally likely outcomes = 14 Number of favourable outcomes = 3	Use: Probability = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
ii	$P(\text{TB}) = \frac{2}{14} = \frac{1}{7}$	Number of favourable outcomes = 2	

Exercise 5B

1 A bag contains 3 red discs and 1 green disc. A coin is tossed and a disc is chosen at random from the bag.

a Complete the following to list the sample space.

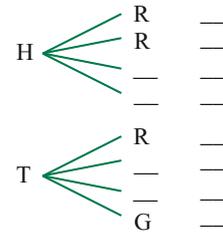
R for red disc

G for ___ disc

H for ___

___ for tails

Sample space = HR, H___, ___R, ___G, TR, T___, ___R, ___G



b Complete to find the probability of getting tails and a red disc.

$$\begin{aligned} \text{Total number of outcomes} &= ___ + 4 \\ &= ___ \end{aligned}$$

$$P(\text{TR}) = \frac{3}{\square}$$

2 a A coin is tossed and a die is thrown. List the sample space.

b Find the probability of getting:

i a tail and a 4

ii a tail and an odd number

iii a head and a 6

iv a head and an even number.

3 a A jar contains 2 red discs and 1 blue disc. A coin is tossed and a disc is selected at random from the jar. List the sample space.

b Find the probability that the result is:

i a head and a red disc

ii a head and a blue disc

iii a tail and a red disc.

4 a A spinner has the numbers 1 to 4, each with an equal chance of occurring. A coin is tossed and the spinner is spun. List all the equally likely outcomes of this experiment.

b Find the probability of obtaining:

i a head and a 2

ii a tail and a 3

iii a head and an even number.

5 a A bag contains 2 green discs and 1 black disc. A die is rolled and a disc is chosen at random from the bag. List the sample space.

b What is the probability of getting:

i a 3 and a green disc?

ii a 5 and a black disc?

iii an odd number and a green disc?

iv an even number and a black disc?

- 6 a** Marie and Peter are planning a family with 2 children. Assume the chance of having a boy or a girl are equally likely. List all the possible outcomes.
- b** Find the probability of having:
- i** 2 boys
 - ii** a boy and a girl, in that order
 - iii** a boy and a girl, in any order
 - iv** at least 1 girl.

EXAMPLE 3

- a** Two dice are thrown. List the sample space.
- b** What is the probability of obtaining:
- i** a 3 and a 5?
 - ii** a double?
 - iii** at least one 6?

	Solve	Think	Apply
a		Show the sample space using a lattice diagram. Each point on the lattice represents the numbers on each die.	Use an organised method to combine the outcomes of each simple event.
b i	$P(3 \text{ and } 5) = \frac{2}{36} = \frac{1}{18}$	Total number of equally likely possible outcomes = 36 Number of favourable outcomes = 2	Use: Probability $= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
ii	$P(\text{a double}) = \frac{6}{36} = \frac{1}{6}$	Number of favourable outcomes = 6	
iii	$P(\text{at least one } 6) = \frac{11}{36}$	Number of favourable outcomes = 11	

- 7** Two dice are thrown.
- a** Complete the lattice diagram.
- b** Complete to calculate the probability of obtaining:

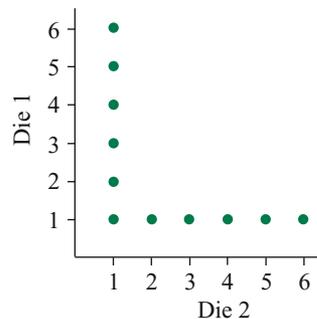
i a 5 and a 1
Total outcomes = ____
Number of favourable outcomes = ____

$$P(5 \text{ and } 1) = \frac{2}{\square} = \frac{\square}{\square}$$

ii a double
____ = ____

Number of favourable outcomes = 6

$$P(\text{double}) = \frac{\square}{36} = \frac{\square}{\square}$$



- 8** Two dice are thrown. Find the probability of throwing:
- a** a 4 and a 6
 - b** a 3 and a 2
 - c** double 6
 - d** at least one 2
 - e** a 5 and an even number.

- 13 a** A family has 3 children. Assuming the chance of having a boy or a girl is equally likely, list the sample space.
- b** Calculate the probability of having:
- 3 boys
 - 2 girls and a boy, in that order
 - 2 girls and a boy, in any order
 - no girls
 - at least 1 girl.
- 14 a** A spinner has the numbers 1 to 4, each with an equal chance of occurring. A bag contains a blue, a red and a green disc. A coin is tossed, the spinner is spun and a disc is selected from the bag. List all the equally likely outcomes of this experiment.
- b** Calculate the probability of obtaining:
- a head, a 4 and a blue disc
 - a tail, an even number and a red disc
 - a head, a 1 and either a blue or a green disc
 - a head, a number < 4 and a green disc.



C Sampling with replacement

EXAMPLE 1

- a** A bag contains one red, one white and one blue ball. A ball is selected at random, its colour noted and the ball replaced. A second ball is then selected at random. List the sample space.
- b** Calculate the probability of selecting:
- 2 white balls
 - a red ball and a blue ball, in that order
 - balls of different colours.

	Solve	Think	Apply												
a	<table border="0"> <tr> <td>1st ball</td> <td>2nd ball</td> <td>Sample space</td> </tr> <tr> <td>R</td> <td> <ul style="list-style-type: none"> R W B </td> <td> <ul style="list-style-type: none"> RR RW RB </td> </tr> <tr> <td>W</td> <td> <ul style="list-style-type: none"> R W B </td> <td> <ul style="list-style-type: none"> WR WW WB </td> </tr> <tr> <td>B</td> <td> <ul style="list-style-type: none"> R W B </td> <td> <ul style="list-style-type: none"> BR BW BB </td> </tr> </table>	1st ball	2nd ball	Sample space	R	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> RR RW RB 	W	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> WR WW WB 	B	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> BR BW BB 	If the first ball is replaced there are 3 possible outcomes for the second selection. Hence there are 9 possible, equally likely outcomes for the combined events.	If the first ball is replaced, then the number of outcomes for the second-stage event is the same as for the first-stage event.
1st ball	2nd ball	Sample space													
R	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> RR RW RB 													
W	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> WR WW WB 													
B	<ul style="list-style-type: none"> R W B 	<ul style="list-style-type: none"> BR BW BB 													
b i	$P(WW) = \frac{1}{9}$	Number of favourable outcomes = 1													
ii	$P(RB) = \frac{1}{9}$	Number of favourable outcomes = 1													
iii	$P(\text{different colours}) = \frac{6}{9} = \frac{2}{3}$	Number of favourable outcomes = 6													

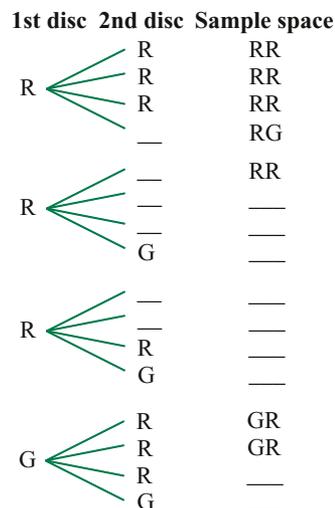
EXAMPLE 2

- a** A bag contains 3 red and 2 blue discs. A disc is selected at random from the bag and its colour noted. The first disc is replaced and a second disc selected. Find the sample space.
- b** Calculate the probability of getting:
- i** 2 red discs
 - ii** in order, a red and a blue disc
 - iii** different coloured discs.

	Solve			Think			Apply	
a	<p>1st disc</p> <p>2nd disc</p> <p>Sample space</p> <p>RR RR RR RB RB</p>	<p>1st disc</p> <p>2nd disc</p> <p>Sample space</p> <p>BR BR BR BB BB</p>					<p>If the first disc is replaced, there are 5 possible outcomes for the second selection. Hence there are 25 possible, equally likely outcomes for the combined events.</p>	<p>If the first disc is replaced, the number of outcomes for the second-stage event is the same as for the first-stage event.</p>
b i	$P(RR) = \frac{9}{25}$			Number of favourable outcomes = 9				
ii	$P(RB) = \frac{6}{25}$			Number of favourable outcomes = 6				
iii	$P(\text{different colours}) = P(RB \text{ or } BR) = \frac{12}{25}$			Number of favourable outcomes = 12				

Exercise 5C

- 1 a** A bag contains 3 red discs and a green disc. A disc is selected at random, its colour noted and the disc replaced. A second disc is then selected at random. Complete the tree diagram to list the sample space.



- b** Complete to calculate the probability that the discs selected are different colours.

Total number of possible outcomes = ___

Number of outcomes with different colours

$$= 1 + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$P(RG \text{ or } GR) = \frac{6}{\square} = \frac{\square}{\square}$$

- 2** A bag contains 2 red balls and 2 black balls. A ball is chosen at random from the bag and replaced before a second ball is selected.

- a** Draw a tree diagram to find the sample space.

- b** Determine the probability of getting:

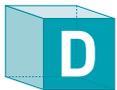
i 2 red balls

iii balls of different colours

ii a red and a black ball, in that order

iv balls of the same colour.

- 3** A bag contains 3 blue counters, 2 red counters and 1 white counter. A counter is chosen at random and replaced before a second counter is selected.
- a** Draw a tree diagram to find the sample space.
- b** Calculate the probability of getting:
- i** 2 blue counters
 - ii** a blue and a red counter, in that order
 - iii** a red and a white counter, in any order
 - iv** counters of different colours
 - v** counters of the same colour.
- 4** The digits 1, 2, 3 and 4 are written on separate cards and shuffled. A card is selected at random, the number is noted and the card replaced. A second card is selected to form a two-digit number.
- a** List the sample space.
- b** Calculate the probability that the number formed:
- i** is 23
 - ii** is even
 - iii** is greater than 21
 - iv** contains a 1
 - v** is in the forties.
- 5** Two letters are chosen from the word SUM. The first letter is replaced before the second letter is selected.
- a** List the sample space.
- b** Find the probability that:
- i** the two letters are the same
 - ii** exactly one of the letters is 'S'
 - iii** the letters are 'U' and 'M', in any order
 - iv** at least one 'M' is chosen
 - v** at most one 'U' is chosen.



D Sampling without replacement

EXAMPLE 1

- a** A bag contains one red, one white and one blue ball. A ball is selected at random, its colour noted and the ball is not replaced. A second ball is then selected at random. List the sample space.
- b** If the first ball is not replaced, find the probability of selecting:
- i** 2 white balls
 - ii** a red ball and a blue ball, in that order
 - iii** balls of different colours.

	Solve	Think	Apply												
a	<table style="display: inline-table; border: none;"> <tr> <td style="text-align: right; padding-right: 10px;">1st ball</td> <td style="text-align: center;">2nd ball</td> <td style="text-align: left; padding-left: 10px;">Sample space</td> </tr> <tr> <td style="text-align: right;">R</td> <td style="text-align: center;"> $\begin{array}{l} \diagup \text{ W} \\ \diagdown \text{ B} \end{array}$ </td> <td style="text-align: left;">RW RB</td> </tr> <tr> <td style="text-align: right;">W</td> <td style="text-align: center;"> $\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ B} \end{array}$ </td> <td style="text-align: left;">WR WB</td> </tr> <tr> <td style="text-align: right;">B</td> <td style="text-align: center;"> $\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ W} \end{array}$ </td> <td style="text-align: left;">BR BW</td> </tr> </table>	1st ball	2nd ball	Sample space	R	$\begin{array}{l} \diagup \text{ W} \\ \diagdown \text{ B} \end{array}$	RW RB	W	$\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ B} \end{array}$	WR WB	B	$\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ W} \end{array}$	BR BW	If the first ball is not replaced there are only 2 possible outcomes for the second selection. Hence there are 6 possible, equally likely outcomes for the combined events.	If the first ball is not replaced, then the number of outcomes for the second-stage event is one less than for the first-stage event.
1st ball	2nd ball	Sample space													
R	$\begin{array}{l} \diagup \text{ W} \\ \diagdown \text{ B} \end{array}$	RW RB													
W	$\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ B} \end{array}$	WR WB													
B	$\begin{array}{l} \diagup \text{ R} \\ \diagdown \text{ W} \end{array}$	BR BW													
b i	$P(WW) = 0$	Number of favourable outcomes = 0													
ii	$P(RB) = \frac{1}{6}$	Number of favourable outcomes = 1													
iii	$P(\text{different colours}) = \frac{6}{6} = 1$	Number of favourable outcomes = 6													

- 3** A bag contains 3 blue counters, 2 red counters and 1 white counter. Two counters are chosen at random one after the other from the bag. The first counter is *not* replaced before the second counter is selected.
- a** Draw a tree diagram to find the sample space.
- b** Calculate the probability of getting:
- | | |
|--|---|
| i 2 blue counters | ii a blue and red counter, in that order |
| iii a red and a white counter, in any order | iv counters of different colours |
| v counters of the same colour. | |
- 4** The digits 1, 2, 3 and 4 are written on separate cards and shuffled. Two cards are selected at random one after the other to form a two-digit number. The first card is *not* replaced before the second card is selected.
- a** List the sample space.
- b** Calculate the probability that the number formed:
- | | | |
|------------------------|------------------------------|-------------------------------|
| i is 23 | ii is even | iii is greater than 21 |
| iv contains a 1 | iv is in the forties. | |
- 5** Two letters are chosen one after the other from the letters of the word SUM. The first letter is *not* replaced before the second letter is selected.
- a** List the sample space.
- b** Calculate the probability that:
- | | |
|--|---|
| i the two letters are the same | ii exactly one of the letters is 'S' |
| iii the letters are 'U' and 'M', in any order | iv at least one 'M' is chosen |
| v at most one 'U' is chosen. | |
- 6**
- a** A doubles pair is selected from 3 right-handed and 2 left-handed tennis players. List the sample space.
- b** Calculate the probability that the pair consists of:
- | | |
|---|---------------------------------|
| i 2 right-handed players | ii 2 left-handed players |
| iii one left-handed and one right-handed player. | |
- 7**
- a** The president and secretary of the school council are to be chosen at random from Charo, Sue, Jack and Bill. Draw a tree diagram to list all the possibilities.
- b** Calculate the probability that:
- | | |
|---|---|
| i Sue is the president and Bill is the secretary | ii Charo is the president and either Jack or Bill is the secretary |
| iii Bill is not the president | iv Jack is the president or the secretary. |





The product rule in probability

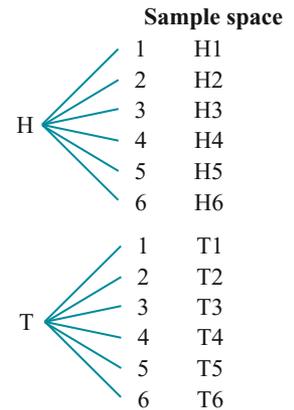
Consider an experiment in which a coin is tossed and a die is thrown. The sample space can be found using a tree diagram as shown. The sample space consists of 12 equally likely outcomes and, for example, $P(\text{a head and a } 5) = \frac{1}{12}$.

Now consider the simple events 'tossing a head' and 'rolling a 5'.

When a coin is tossed $P(H) = \frac{1}{2}$ and when a die is rolled $P(5) = \frac{1}{6}$.

Note: The result of the first event (tossing a coin) can in no way affect the result of the second event (rolling a die). These are known as **independent events**.

$$\begin{aligned}
 P(\text{a head and a } 5) &= \frac{1}{12} \\
 &= \frac{1}{2} \times \frac{1}{6} \\
 &= P(H) \times P(5)
 \end{aligned}$$



In general, if A and B are independent events:

$$P(A \text{ and } B) = P(A) \times P(B)$$

This is known as the **product rule of probability**. This rule can be extended to more than two independent events.

If $A, B, C \dots$ are independent events then:

$$P(A \text{ and } B \text{ and } C \text{ and } \dots) = P(A) \times P(B) \times P(C) \times \dots$$

An example of **dependent events** would be selecting two aces, without replacement, from a normal pack of cards. For the first selection there are 4 aces to choose from in the 52 cards, but if the first card selected is an ace and it is not replaced then there are only 3 aces left to choose from in the remaining 51 cards. In this case the result of the first selection affects the result of the second.

EXAMPLE 1

A coin is tossed and a die is rolled. Calculate the probability of getting:

- a a tail and a 6
- b a head and an odd number
- c a tail and a number greater than 4.

	Solve	Think	Apply
a	$ \begin{aligned} P(\text{T and } 6) \\ &= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \end{aligned} $	$ \begin{aligned} P(\text{T}) &= \frac{1}{2} \text{ and } P(6) = \frac{1}{6} \\ P(\text{T and } 6) &= P(\text{T}) \times P(6) \end{aligned} $	The events are independent so $P(A \text{ and } B) = P(A) \times P(B)$.
b	$ \begin{aligned} P(\text{H and odd number}) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned} $	$ \begin{aligned} P(\text{H}) &= \frac{1}{2} \\ P(\text{odd number}) &= \frac{3}{6} = \frac{1}{2} \\ P(\text{H and odd number}) \\ &= P(\text{H}) \times P(\text{odd number}) \end{aligned} $	
c	$ P(\text{T and number } >4) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} $	$ \begin{aligned} P(\text{T}) &= \frac{1}{2} \\ P(\text{number } >4) &= \frac{2}{6} = \frac{1}{3} \\ P(\text{T and number } >4) \\ &= P(\text{T}) \times P(\text{number } >4) \end{aligned} $	

Exercise 5E

1 A jar contains 3 black discs and 2 gold discs. A coin is tossed and a disc is selected at random from the jar. Complete the following to calculate the probability that the result is:

a a tail and a gold disc

$$P(\text{tail}) = \frac{1}{\square} \quad P(\text{gold disc}) = \frac{\square}{5}$$

$$P(\text{tail and gold disc}) = \frac{1}{2} \times \frac{\square}{5} = \frac{\square}{\square} = \frac{\square}{\square}$$

b a head and a black disc

$$P(\text{head}) = \frac{\square}{2} \quad P(\text{black disc}) = \frac{3}{\square}$$

$$P(\text{head and black disc}) = \frac{\square}{2} \times \frac{3}{\square} = \frac{\square}{\square}$$

2 A jar contains 5 red discs and 4 blue discs. A coin is tossed and a disc is selected at random from the jar. Calculate the probability that the result is:

a a head and a red disc

b a head and a blue disc

c a tail and a red disc.

3 A bag contains 4 green discs and 3 black discs. A die is rolled and a disc is chosen at random from the bag. Calculate the probability of getting:

a a 3 and a green disc

b a 5 and a black disc

c an odd number and a green disc

d an even number and a black disc.

4 A bag contains 10 cards numbered 0 to 9 and a box contains 26 cards with each of the letters A, B, C, ..., Z occurring once. A card is chosen at random from the bag and the box. Calculate the probability of getting:

a a 4 and a W

b an even number and a K

c a 9 and a vowel

d a 5 and a letter of the word 'MATHS'

e a number greater than 5 and a Z.

5 A dice is rolled twice. What is the probability of getting these numbers, in order?

a a 5 and then a 3

b double 6

c a number >4 and then an even number

6 A coin is tossed, a die is rolled and a card is selected from a normal pack. Calculate the probability of getting:

a a head, a 1 and a spade

b a tail, an even number and a king

c a head, a number <3 and a red ace

d a tail, a number >2 and a picture card.

7 A die is rolled three times. Calculate the probability of getting these numbers, in order:

a a 6, a 5 and a 4

b a 2, an odd number and a 4

c three 6s

d a 1, an even number and a number >4

e an odd number, an even number and a number <3 .

EXAMPLE 2

A bag contains 3 white and 4 red balls. Another bag contains 2 white and 3 red balls. A ball is chosen from each bag. Calculate the probability of getting:

a 2 white balls

b 2 red balls

c a white and a red ball, in that order

d a white and a red ball, in any order

e at least 1 white ball.

	Solve	Think	Apply
a	$P(WW) = \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$	$P(W \text{ from 1st bag}) = \frac{3}{7}, P(W \text{ from 2nd bag}) = \frac{2}{5}$ $P(2 \text{ white balls})$ $= P(W \text{ from 1st bag}) \times P(W \text{ from 2nd bag})$	These events are independent so use the probability product rule.

EXAMPLE 3 CONTINUED

	Solve	Think	Apply
b	$P(RR) = \frac{4}{7} \times \frac{3}{5}$ $= \frac{12}{35}$	The probability of 2 red balls is found by multiplying the probabilities along the branches leading to the result red and red. This is equivalent to $P(RR) = P(R) \times P(R)$.	<i>Note:</i> A tree diagram lists all the possible, equally likely outcomes at each stage, but for a probability tree similar outcomes are grouped and only the <i>results</i> at each stage are listed.
c	$P(WR) = \frac{3}{7} \times \frac{3}{5}$ $= \frac{9}{35}$	The probability of a white and then a red ball is found by multiplying the probabilities along the branches leading to the result white and red, in that order. This is equivalent to $P(WR) = P(W) \times P(R)$.	
d	$P(WR \text{ or } RW)$ $= P(WR) + P(RW)$ $= \frac{3}{7} \times \frac{3}{5} + \frac{4}{7} \times \frac{2}{5} = \frac{17}{35}$	The probability of a white and red ball, in any order, is found by multiplying the probabilities along the branches leading to the results white and then red and red and then white and adding.	
e	$P(WR \text{ or } RW \text{ or } WW)$ $= P(WR) + P(RW) + P(WW)$ $= \frac{3}{7} \times \frac{3}{5} + \frac{4}{7} \times \frac{2}{5} + \frac{3}{7} \times \frac{2}{5}$ $= \frac{23}{35}$	$P(\text{at least one white ball})$ $= P(\text{exactly one white ball or 2 white balls})$ $= P(WR \text{ or } RW \text{ or } WW)$. Multiply the probabilities along the appropriate branches and add the results.	

8 A jar contains 4 black discs and 2 green discs. Another jar contains 3 black discs and 4 green discs. A disc is chosen from each jar. Complete the following to calculate the probability of getting:

a a black disc from the first jar and a green disc from the second jar

$$P(\text{black disc}) = \frac{4}{\square} \quad P(\text{green disc}) = \frac{\square}{7}$$

$$P(\text{black disc then green disc}) = \frac{4}{\square} \times \frac{\square}{7} = \frac{\square}{\square}$$

b a black disc and a green disc in any order.

Part **a** had $P(\text{black disc then green disc}) = \frac{\square}{\square}$

$$P(\text{green disc then black disc}) = P(G) \times P(\square)$$

$$= \frac{2}{6} \times \frac{\square}{7} = \frac{\square}{\square}$$

$$P(\text{green disc and black disc in any order}) = \frac{16}{42} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

9 A jar contains 7 red discs and 3 blue discs. Another jar contains 4 red discs and 5 blue discs. A disc is chosen from each jar.

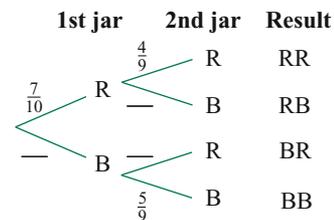
a Complete the probability tree diagram.

b Complete to calculate the probability that the discs chosen are the same colours.

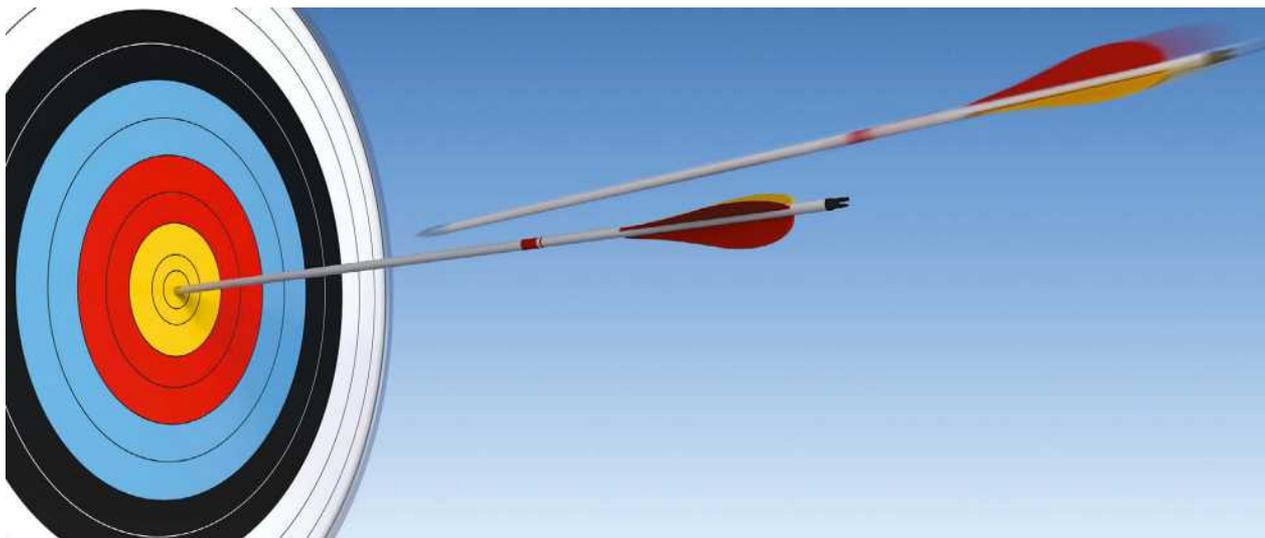
$$P(RR) = \frac{7}{10} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$P(BB) = \frac{\square}{\square} \times \frac{5}{9} = \frac{\square}{\square}$$

$$P(\text{same colour}) = \frac{28}{90} + \frac{\square}{\square} = \frac{\square}{\square}$$



- 10** A bag contains 5 white and 3 red balls. Another bag contains 3 white and 6 red balls. A ball is chosen from each bag.
- Draw a probability tree showing the possible results for each selection and the combined results.
 - Calculate the probability of getting:
 - 2 white balls
 - 2 red balls
 - a white and a red ball, in that order
 - a white and a red ball, in any order
 - at least one white ball.
- 11** Class 10A has 14 boys and 19 girls. Class 10B has 16 boys and 12 girls. A student is selected at random from each class.
- Draw a probability tree showing the possible results for each class and the combined results.
 - Calculate the probability of selecting:
 - 2 boys
 - 2 girls
 - a boy and a girl, in any order.
- 12** A student enters the 100 m and 400 m events at the school swimming carnival. His chance of winning the 100 m event is 0.8 and his chance of winning the 400 m event is 0.6.
- Draw a probability tree showing the possible results for each event and the combined results.
 - Calculate the probability that he will:
 - win both events
 - not win either event
 - win the 100 m but not win the 400 m
 - win at least one event.
- 13** The probability that a throat infection will be cured by a particular drug is 0.9. Two people with a throat infection are treated with this drug.
- Draw a probability tree showing the possible results for each person and the combined results.
 - Calculate the probability that:
 - both people will be cured
 - neither person will be cured
 - only one will be cured
 - at least one will be cured.
- 14** Matt and Adele are archers. The probability that Matt will hit the bullseye of the target is $\frac{4}{5}$ and the probability that Adele will hit the bullseye is $\frac{6}{7}$.
- Draw a probability tree showing the results for each person and the combined results.
 - They both shoot at the target. Calculate the probability that:
 - both hit the bullseye
 - neither hits the bullseye
 - only one of them hits the bullseye
 - at least one of them hits the bullseye.



EXAMPLE 4

There are 15 novels and 8 maths books on a bookshelf. Two books are selected at random from the shelf.

- a** The first book is replaced before the second book is selected.
b The first book is *not* replaced before the second book is selected.

Calculate the probability of choosing:

- i** a novel and then a maths book **ii** 2 novels **iii** at least 1 maths book.

	Solve	Think	Apply													
a	<table border="1"> <thead> <tr> <th>1st book</th> <th>2nd book</th> <th>Result</th> </tr> </thead> <tbody> <tr> <td rowspan="2">$\frac{15}{23}$ N</td> <td>$\frac{15}{23}$ N</td> <td>NN</td> </tr> <tr> <td>$\frac{8}{23}$ M</td> <td>NM</td> </tr> <tr> <td rowspan="2">$\frac{8}{23}$ M</td> <td>$\frac{15}{23}$ N</td> <td>MN</td> </tr> <tr> <td>$\frac{8}{23}$ M</td> <td>MM</td> </tr> </tbody> </table>	1st book	2nd book	Result	$\frac{15}{23}$ N	$\frac{15}{23}$ N	NN	$\frac{8}{23}$ M	NM	$\frac{8}{23}$ M	$\frac{15}{23}$ N	MN	$\frac{8}{23}$ M	MM	<p>If the first book is replaced there are 23 possible outcomes at each stage.</p> <p>At the first stage, $P(\text{novel}) = \frac{15}{23}$ and $P(\text{maths book}) = \frac{8}{23}$.</p> <p>At the second stage, $P(\text{novel}) = \frac{15}{23}$ and $P(\text{maths book}) = \frac{8}{23}$.</p>	<p>Draw a probability tree and multiply along the branches leading to the desired event.</p>
1st book	2nd book	Result														
$\frac{15}{23}$ N	$\frac{15}{23}$ N	NN														
	$\frac{8}{23}$ M	NM														
$\frac{8}{23}$ M	$\frac{15}{23}$ N	MN														
	$\frac{8}{23}$ M	MM														
i	$P(\text{a novel then a maths book}) = \frac{15}{23} \times \frac{8}{23} = \frac{120}{529}$	Multiply along the branches leading to a novel and then a maths book.														
ii	$P(2 \text{ novels}) = \frac{15}{23} \times \frac{15}{23} = \frac{225}{529}$	Multiply along the branches leading to a novel and then a novel.														
iii	$\begin{aligned} P(\text{at least 1 maths book}) &= P(\text{MN}) + P(\text{NM}) + P(\text{MM}) \\ &= \frac{8}{23} \times \frac{15}{23} + \frac{15}{23} \times \frac{8}{23} + \frac{8}{23} \times \frac{8}{23} \\ &= \frac{304}{529} \end{aligned}$	$P(\text{at least 1 maths book}) = P(\text{exactly 1 maths book or 2 maths books}).$ Multiply the probabilities along the appropriate branches and add the results.														
b	<table border="1"> <thead> <tr> <th>1st book</th> <th>2nd book</th> <th>Result</th> </tr> </thead> <tbody> <tr> <td rowspan="2">$\frac{15}{23}$ N</td> <td>$\frac{14}{22}$ N</td> <td>NN</td> </tr> <tr> <td>$\frac{8}{22}$ M</td> <td>NM</td> </tr> <tr> <td rowspan="2">$\frac{8}{23}$ M</td> <td>$\frac{15}{22}$ N</td> <td>MN</td> </tr> <tr> <td>$\frac{7}{22}$ M</td> <td>MM</td> </tr> </tbody> </table>	1st book	2nd book	Result	$\frac{15}{23}$ N	$\frac{14}{22}$ N	NN	$\frac{8}{22}$ M	NM	$\frac{8}{23}$ M	$\frac{15}{22}$ N	MN	$\frac{7}{22}$ M	MM	<p>If the first book is not replaced there are 23 possible outcomes at the first stage and 22 at the second stage.</p> <p>At the first stage: $P(\text{novel}) = \frac{15}{23}$ and $P(\text{maths book}) = \frac{8}{23}$.</p> <p>At the second stage: if the first book is a novel, $P(\text{novel}) = \frac{14}{22}$ and $P(\text{maths book}) = \frac{8}{22}$, or if the first book is a maths book, $P(\text{novel}) = \frac{15}{22}$ and $P(\text{maths book}) = \frac{7}{22}$.</p>	<p>Draw a probability tree and multiply along the branches leading to the desired event.</p>
1st book	2nd book	Result														
$\frac{15}{23}$ N	$\frac{14}{22}$ N	NN														
	$\frac{8}{22}$ M	NM														
$\frac{8}{23}$ M	$\frac{15}{22}$ N	MN														
	$\frac{7}{22}$ M	MM														
i	$P(\text{a novel then a maths book}) = \frac{15}{23} \times \frac{8}{22} = \frac{60}{253}$	Multiply along the branches leading to a novel and then a maths book.														
ii	$P(2 \text{ novels}) = \frac{15}{23} \times \frac{14}{22} = \frac{105}{253}$	Multiply along the branches leading to a novel and then a novel.														

EXAMPLE 4 CONTINUED

	Solve	Think/Apply
iii	$P(\text{at least 1 maths book})$ $= P(NM) + P(MN) + P(MM)$ $= \frac{15}{23} \times \frac{8}{22} + \frac{8}{23} \times \frac{15}{22} + \frac{8}{23} \times \frac{7}{22}$ $= \frac{148}{253}$	$P(\text{at least 1 maths book})$ $= P(\text{exactly 1 maths book or 2 maths books})$ Multiply the probabilities along the appropriate branches and add the results.

Note: We can also use the result for complementary events in Example 4 section iii for parts a and b:

- a $P(\text{at least 1 maths book}) = 1 - P(\text{no maths books})$
 $= 1 - P(2 \text{ novels})$
 $= 1 - \frac{15}{23} \times \frac{15}{23}$
 $= 1 - \frac{225}{529} = \frac{304}{529}$
- b $P(\text{at least 1 maths book}) = 1 - P(\text{no maths books})$
 $= 1 - \frac{15}{23} \times \frac{14}{22}$
 $= 1 - \frac{105}{253} = \frac{148}{253}$

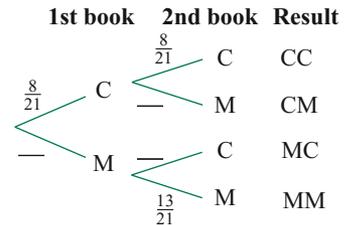


- 15 There are 8 cookbooks and 13 maths books on a bookshelf. Two books are selected at random from the bookshelf.

- a The first book is replaced before the second book is selected. Complete to find the probability that the books are the same type.

$$P(CC) + P(MM) = \frac{8}{21} \times \frac{8}{21} + \frac{\square}{\square} \times \frac{\square}{\square}$$

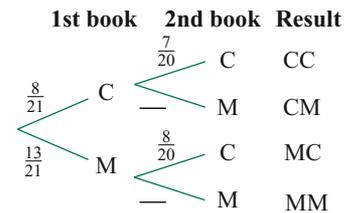
$$= \underline{\hspace{2cm}}$$



- b The first book is *not* replaced before the second book is selected. Complete to find the probability that the books are the same type.

$$P(CC) + P(MM) = \frac{\square}{\square} \times \frac{\square}{\square} + \frac{13}{21} \times \frac{\square}{\square}$$

$$= \underline{\hspace{2cm}}$$



- 16 There are 12 novels and 9 science books on a bookshelf. Two books are selected at random from the bookshelf.

- a The first book is replaced before the second book is selected.
 b The first book is *not* replaced before the second book is selected.
 Find the probability of choosing:

- i a novel and then a science book ii 2 novels iii at least one science book.

- 17 A bag contains 8 red discs and 11 black discs. Two discs are selected at random.

- a The first disc is replaced before the second disc is selected.
 b The first disc is *not* replaced before the second disc is selected.
 Find the probability of selecting:

- i 2 red discs ii 2 black discs
 iii discs of different colours iv at most 1 red disc.

- 21 One hundred tickets are sold in a raffle that has 2 prizes. If you buy 4 tickets, what is the probability of:
 a winning both prizes? b winning second prize only? c winning a prize?
- 22 Twenty tickets are sold in a raffle that has 2 prizes. If you buy 3 tickets, what is the probability of:
 a winning both prizes? b winning second prize only? c winning a prize?
- 23 Twenty tickets are sold in a raffle that has 3 prizes. If you buy 3 tickets, what is the probability of:
 a winning all three prizes? b winning second prize only? c winning a prize?



Conditional probability

The probability that an event occurs, given that another event has already occurred, is called **conditional probability**. In this section you will investigate and evaluate conditional statements in probability.

Exercise 5F

- 1 A family has 4 girls. The probability that the fifth child is a girl is $\frac{1}{2}$. Explain whether or not this statement is true.
- 2 At the toss, the captain of the cricket team has called 'heads' five times in a row and lost. The vice-captain says to call 'heads' again because it is overdue. Is this a valid statement? Explain your answer.

EXAMPLE 1

If a die is thrown and the result is an odd number, what is the probability that it is a 5?

Solve	Think	Apply
$P(5) = \frac{1}{3}$	The effect of the condition is to reduce the sample space. There are only three odd numbers possible, so $S = \{1, 3, 5\}$ and there is one favourable outcome among these.	Find the reduced sample space and identify the favourable outcomes.

- 3 A die is thrown and the result is an even number. Complete to find the probability that the number is 6.
 The number of even outcomes = ____
 $P(6) = \frac{\square}{3}$
- 4 If a die is thrown and the result is an even number, what is the probability that it is a 2?
- 5 A die is thrown and it is known that the result is a number less than 5. What is the probability that it is a 3?
- 6 A card is selected at random from a normal pack. Calculate the probability that it is:
 a an ace, given that it is a heart b a king, given that it is black
 c a spade, given that it is a jack d red, given that it is a queen.
- 7 Two dice are rolled. If the sum of the two numbers shown is at least 10, what is the probability that the two numbers are the same?
- 8 Two dice are rolled. If the sum of the two numbers is 7, what is the probability that the numbers are 1 and 6?

- 9 A drawer contains 6 black and 4 grey socks. Two socks are selected at random, one after the other without replacement. If the first sock is grey, what is the probability that the two socks are matching colours?
- 10 The table shows data collected from 100 male Year 10 students.

	Wear glasses	Do not wear glasses	
Left-handed	14	9	23
Right-handed	35	42	77
	49	51	100

If one of these students is selected at random, what is the probability that:

- a he wears glasses given that he is left-handed?
- b he is left-handed given that he does not wear glasses?
- 11 A bag contains 3 red balls, 2 black balls and 1 green ball. Two balls are chosen at random from the bag one after the other.
- a The first ball is replaced before the second ball is selected.
- b The first ball is *not* replaced before the second ball is selected.

If the first ball chosen is red, what is the probability of getting:

- i 2 red balls? ii a red and a black ball? iii balls of different colours?
- 12 A bag contains 3 black balls, 3 white balls and 1 yellow ball. Two balls are chosen at random from the bag one after the other.
- a The first ball is replaced before the second ball is selected.
- b The first ball is *not* replaced before the second ball is selected.

If the first ball chosen is black, what is the probability of getting:

- i 2 black balls? ii a black and a white ball?
- iii a black and a yellow ball? iv balls of different colours?
- 13 Two coins are tossed.
- a List the sample space.
- b If one coin is heads, what is the probability that the other coin is tails?

- 14 A bag contains the ace of hearts, the ace of diamonds, the ace of spades and the ace of clubs. Two cards are selected at random from the bag. If one of the cards is the ace of hearts, find the probability that the other is the ace of diamonds:
- a with replacement
- b without replacement.

- 15 Two dice are rolled. Find the probability that the outcomes on the dice are different given that both numbers are odd.



When your number's up – on average

How long will you live? This is a tantalising question throughout our lives. For those planning finances for retirement, it assumes special significance as they attempt to allocate financial resources to last for the remainder of their days. For the individual there is no way of determining a precise answer, but for the male and female populations there are satisfactory ways of establishing accurate life expectancy.

The results are of fundamental importance for organisations such as insurance companies, when they calculate payments for commonly used retirement products. In addition, these results can be used by individuals to determine accurately their chances of living to or beyond any specified age.

The means for considering life expectancy in the future are essentially based on the assumption that, in a population, patterns for deaths of women and men established in the immediate past are unlikely to change substantially in the immediate future. So past records of death in mortality tables can be used to predict future patterns with a high degree of accuracy.

Examples of figures extracted from a particular mortality table for males are given below. It gives the probability, in percentage terms, of a male who has attained a certain age living to another designated age.

For example, a man who is currently 65 years of age has an 80% chance of reaching 70, a 57% chance of reaching 75, a 33% chance of reaching 80 and so on across the line in the table. Alternatively, from a random sample of 100 males aged 65 alive today, 80 could be expected to reach 70 years of age, 57 to reach 75 years, 33 to reach 80 years and so on. For organisations such as insurance companies that deal with large groups of retirees, mortality tables provide invaluable information for calculating such products as life annuities. By dealing with sufficiently large groups of people, insurance companies can remove some of the uncertainty associated with planning finances for retirement.

% chance of reaching a designated age

		Designated age (males)									
		55	60	65	70	75	80	85	90	95	100
Current age	55	100	92	80	64	45	14	12	4	1	0
	60		100	87	69	49	29	13	4	1	0
	65			100	80	57	33	15	4	1	0
	70				100	71	42	19	6	1	0
	75					100	59	26	8	1	0
	80						100	44	13	3	0
	85							100	30	6	1
	90								100	19	2
	95									100	12
	100										100



The following questions relate to this article.

- If a man is currently 55 years old, what is the chance that he will reach the age of:
 - 65 years?
 - 75 years?
 - 85 years?
 - 95 years?
- The table tells us that a 55-year-old man has a 0% chance of reaching 100. But a 0% chance means an impossibility. Discuss.
- If 1000 60-year-old males were selected at random, how many of them would you expect to reach the age of:
 - 65 years?
 - 70 years?
 - 75 years?
 - 80 years?
- Friends Bill and Joe are currently 55 and 60 years old respectively. What is the probability that they will both be alive in 20 years time?
- Five male classmates now 55 years old meet at a reunion and agree to meet again in 10 years time. What is the probability that they will all reach 65 years of age?
- Find out what is meant by an ‘annuity’ and a ‘life annuity’.

Word maze

The maze contains the following words, not necessarily in the order given. Movement in any direction, except diagonally, is possible. Work your way through the maze and find these words:

coins chance dice event experiment
 expectation likelihood outcome probability
 sample space spinner tree diagram

	N	T	E	X	P	E	R	E	S	out →
	E	N	E	M	I	R	E	X	N	I
	V	T	L	I	K	E	N	P	C	O
	E	E	C	H	I	L	N	E	N	O
in →	D	I	O	S	P	I	C	T	I	
	E	L	D	O	M	A	R	G	A	T
	S	P	S	E	O	U	T	A	I	D
	P	M	A	C	H	C	C	O	M	E
	A	P	R	N	A	Y	T	I	E	E
	C	E	O	B	A	B	I	L	T	R

Terms

- | | | | |
|-------------------------|--------------------------|---------------------------|------------------------------|
| chance | complementary event | conditional probability | dependent event |
| experiment | experimental probability | independent event | lattice diagram |
| multistage event | outcome | probability tree | product rule of probability |
| relative frequency | sample space | sampling with replacement | sampling without replacement |
| theoretical probability | tree diagram | two-way table | Venn diagram |

Check your skills

- A 6-sided die is rolled 50 times and the results are shown in the table.

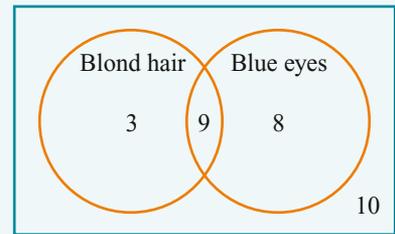
Result	1	2	3	4	5	6
Frequency	7	9	12	8	10	4

The relative frequency of the result 4 is:

- A** 8 **B** $\frac{8}{50}$ **C** 6 **D** $\frac{6}{21}$

- 2 The Venn diagram represents the number of students in a class who have blond hair and blue eyes. The probability that a student chosen at random from this class has both blond hair and blue eyes is:

- A $\frac{2}{3}$ B $\frac{11}{30}$
 C $\frac{2}{5}$ D $\frac{3}{10}$



- 3 The table shows the number of males and females in a class who were born in Australia and overseas. The probability of choosing a male who was born overseas from this class is:

	Born in Australia	Born overseas	
Male	9	5	14
Female	11	2	13
	20	7	27

- A $\frac{5}{27}$ B $\frac{5}{7}$
 C $\frac{5}{14}$ D $\frac{1}{3}$

- 4 A bag contains 2 red balls, 3 blue balls and 1 white ball. A spinner has the numbers 1 to 5 on its 5 equal sectors. The spinner is spun, a ball is chosen from the bag and a coin is tossed. The number of equally likely outcomes in the sample space is:

- A 30 B 40 C 50 D 60

- 5 A jar contains 2 black, 2 white and 5 red counters. Two counters are selected from the jar *with* replacement. The number of equally likely outcomes in the sample space is:

- A 18 B 45 C 72 D 81

- 6 A jar contains 2 black, 2 white and 5 red counters. Two counters are selected from the jar *without* replacement. The number of equally likely outcomes in the sample space is:

- A 18 B 45 C 72 D 81

- 7 A six-sided die is rolled and a coin is tossed. The probability of getting a head and a number less than 5 is:

- A $\frac{1}{3}$ B $\frac{5}{12}$ C $\frac{2}{3}$ D $\frac{5}{6}$

- 8 A bag contains 4 red, 3 blue and 2 green discs. Two discs are selected at random one after the other. The first disc is not replaced before a second disc is selected. The probability of a blue and green disc, in any order is:

- A $\frac{2}{27}$ B $\frac{4}{27}$ C $\frac{1}{12}$ D $\frac{1}{6}$

- 9 A card is selected from a normal playing pack, its suit noted and the card replaced in the pack. Second and third cards are selected in the same way. The probability of getting 3 cards of the same suit is:

- A $\frac{3}{64}$ B $\frac{1}{64}$ C $\frac{1}{16}$ D $\frac{1}{256}$

- 10 Twenty tickets are sold in a raffle which has 2 prizes. If you buy 5 tickets, the probability of winning at least one prize is:

- A $\frac{17}{38}$ B $\frac{1}{19}$ C $\frac{15}{76}$ D $\frac{21}{38}$

- 11 Two coins are tossed. Given that one of them is 'heads', the probability that the other is tails is:

- A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{1}{4}$ D $\frac{1}{3}$

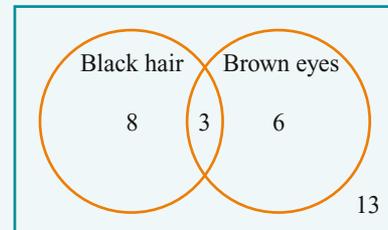
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4	5	6	7–10	11
Section	A	B	C	D	E	F

- 1 One hundred batteries were selected at random from a production line and tested. The results are shown in the table.
- What is the relative frequency of the result 20–29 hours?
 - Estimate the probability that a battery chosen at random from this manufacturer will last less than 10 hours.

Battery life (h)	Frequency
<10	3
10–19	21
20–29	60
30–49	15
40–49	1

- 2 The Venn diagram represents the number of students in a class who have black hair and brown eyes. What is the probability that a student chosen at random from this class will have:
- black hair and brown eyes?
 - black hair or brown eyes but not both?
 - black hair or brown eyes or both?
 - neither black hair nor brown eyes?
 - black hair but not brown eyes?



- 3 The table shows the number of students in a class who study history and art.

	Study art	Do not study art	
Study history	7	11	18
Do not study history	6	5	11
	13	16	29

Find the probability that a student chosen at random from this class will study:

- history or art or both
 - history or art but not both
 - art and history
 - neither history nor art
 - art but not history.
- 4 A bag contains 3 red balls, 2 blue balls and 1 white ball. A spinner has the numbers 1 to 5 on its 5 equal sectors. The spinner is spun and a ball is chosen at random from the bag. Find the probability of getting:
- a 5 and a blue ball
 - an odd number and a red ball
 - an even number and a white ball.
- 5 A coin is tossed three times. What is the probability of getting:
- 3 tails?
 - 2 heads and a tail, in that order?
 - 2 heads and a tail, in any order?
 - at least one tail?
- 6 A box contains 7 red balls, 4 white balls and 2 black balls. Two balls are selected at random.
- The first ball is replaced before the second ball is selected.
 - The first ball is *not* replaced before the second ball is selected.
- Find the probability of selecting:
- 2 red balls
 - 2 white balls
 - iii a black and a white ball, in any order
 - iv a red and a white ball, in any order
 - v at least 1 white ball.

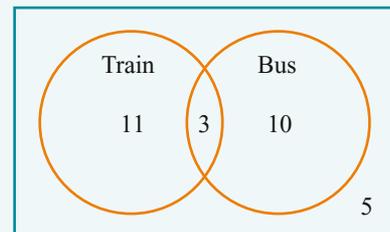
- 6** Two-digit numbers are formed by selecting from the digits 1, 2, 3, 4:
a with replacement **b** without replacement.
 Find the probability that the number formed is:
i 31 **ii** odd **iii** greater than 23
iv contains a 4 **v** in the twenties.
- 7** Twenty tickets are sold in a raffle that has 3 prizes. If you buy 4 tickets, what is the probability of winning:
a all 3 prizes? **b** second prize only? **c** at least one prize?
- 8** A bag contains 4 red balls, 3 black balls and 1 green ball. Two balls are chosen at random from the bag one after the other.
a The first ball is replaced before the second ball is selected.
b The first ball is *not* replaced before the second ball is selected.
 If the first ball chosen is red, what is the probability of getting:
i 2 red balls? **ii** a red and a black ball? **iii** balls of different colours?

5C Review set

- 1** A card was selected from a normal playing pack and its value noted. (An ace was counted as 1 and the picture cards as 10.) This was repeated 150 times. The results are shown in the table.
a What is the relative frequency of the result <6 ?
b Estimate the probability that the value of the card chosen is ≥ 6 .

Value	Frequency
<6	49
≥ 6	101

- 2** The Venn diagram represents the methods used by students in a class to travel to school. What is the probability that a student travels by:
a both train and bus? **b** train or bus but not both?
c train or bus or both? **d** neither train nor bus?
e train but not bus?



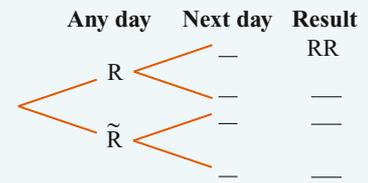
- 3** The table shows the results of a Year 10 class in the term tests.

	Passed Mathematics	Did not pass Mathematics	
Passed English	24	3	27
Did not pass English	2	1	3
	26	4	30

Find the probability that a student chosen at random:

- a** passed Mathematics **b** passed Mathematics and English
c passed Mathematics or English or both **d** passed Mathematics or English but not both
e failed both subjects **f** passed Mathematics but not English
g passed English but not Mathematics.
- 4** Class 10A has 15 boys and 13 girls. Class 10B has 14 boys and 15 girls. A student is selected at random from each class. Find the probability of selecting:
a 2 boys **b** 2 girls **c** a boy and a girl (in any order).

- 5** During summer in Brisbane the probability that it will rain on any day is 0.3. The probability of rain the day after a rainy day is 0.8, and the probability of rain after a non-rainy day is 0.2.



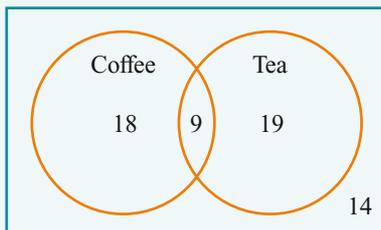
- a** Complete this probability tree where R is the event it rains and \tilde{R} is the event it does not rain.
- b** Find the probability that on two consecutive days:
- i** it rains on both days
 - ii** it rains on the first day but not the second
 - iii** it rains on the second day but not on the first
 - iv** it rains on only one of the two days
 - v** it does not rain on either day
 - vi** it rains on at least one of the days.
- 6** There are 7 novels and 5 science books on a bookshelf. Two books are selected at random.
- a** The first book is replaced before the second book is selected.
- b** The first book is *not* replaced before the second book is selected.
- Find the probability of choosing:
- i** a novel then a science book
 - ii** 2 novels
 - iii** at least one science book.
- 7** Twenty tickets are sold in a raffle that has 3 prizes. If you buy 5 tickets, what is the probability of:
- a** winning all three prizes? **b** winning second prize only? **c** winning a prize?
- 8** Three coins are tossed. If it is known that two of the coins show heads, what is the probability that the other shows tails?

5D Review set

- 1** A drawing pin and a coin are tossed 100 times. The results are shown in the table.
- a** What is the relative frequency of the event pin up and tails?
- b** Estimate the probability that in another toss of the pin and the coin the result will be pin down and heads?
- c** How could we obtain a better estimate for the probability in part **b**?

Event	Frequency
Pin up and H	13
Pin up and T	17
Pin down and H	32
Pin down and T	38

- 2** The Venn diagram represents data collected from a group of people about whether they drank tea or coffee.



What is the probability that a person chosen at random from this group drinks:

- a** coffee?
- b** coffee but not tea?
- c** both tea and coffee?
- d** coffee or tea or both?
- e** coffee or tea but not both?
- f** neither coffee nor tea?



- 3 The table shows the results of a survey of the newspaper read by people while travelling to work on the train.

	Read the <i>Herald</i>	Do not read the <i>Herald</i>	
Read the <i>Telegraph</i>	2	19	21
Do not read the <i>Telegraph</i>	15	4	19
	17	23	40

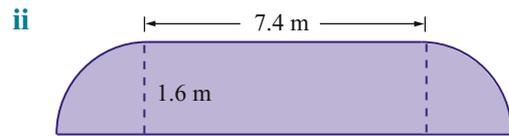
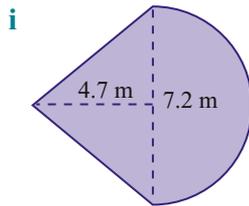
Find the probability that a person chosen at random from this group reads:

- a the *Herald*
- b the *Herald* but not the *Telegraph*
- c the *Telegraph* but not the *Herald*
- d both the *Herald* and the *Telegraph*
- e the *Herald* or the *Telegraph* or both
- f the *Herald* or the *Telegraph* but not both
- g neither the *Herald* nor the *Telegraph*.

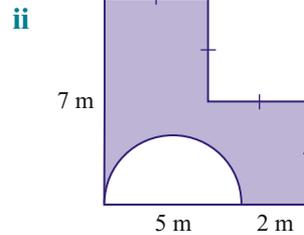
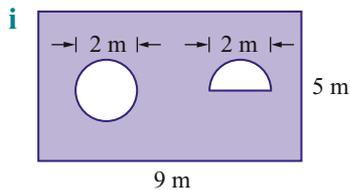


- 4 a Urn A contains 6 white and 3 red balls. Urn B contains 5 white and 2 red balls. An urn is chosen at random and a ball selected from that urn. Draw a probability tree for this situation.
- b Find the probability that the ball is:
- i red
 - ii white.
- 5 Nora has a 70% chance of solving a problem. Jill has a 60% chance of solving the same problem. They both attempt the problem independently of each other. Find the probability that:
- a they both solve the problem
 - b neither of them solves the problem
 - c the problem is solved.
- 6 Two cards are selected at random from a normal playing pack.
- a The first card is replaced before the second card is selected.
 - b The first card is *not* replaced before the second card is selected.
- Find the probability of selecting:
- i 2 hearts
 - ii 2 cards of the same suit
 - iii a diamond and a spade, in that order
 - iv a diamond and a spade, in any order.
- 7 One hundred tickets are sold in a raffle that has 2 prizes. If you buy 10 tickets, what is the probability of:
- a winning both prizes
 - b winning second prize only
 - c winning a prize.
- 8 Two coins are tossed. Find the probability that they both fall heads given that at least one is a head.

1 a Find the areas of these shapes.



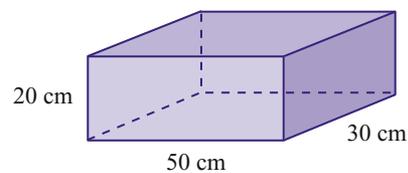
b Find the shaded areas of these figures.



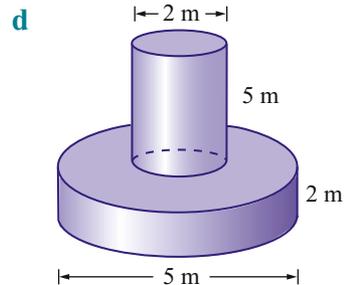
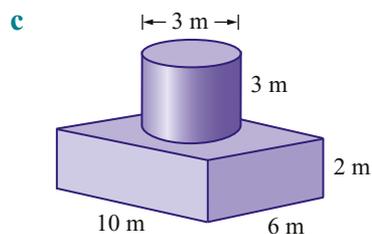
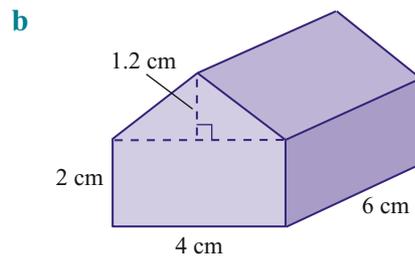
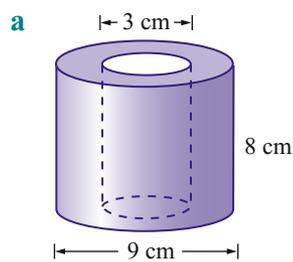
2 a For this rectangular prism, find the:

i surface area **ii** volume.

b Find the curved surface area of a paint roller of radius 2 cm and length 30 cm.



3 Find the volumes of the following figures to the nearest whole cm^3 .



4 a Complete to make equivalent fractions.

i $\frac{7m}{3} = \frac{\square}{12}$

ii $\frac{5k}{3m} = \frac{\square}{15mn}$

iii $\frac{2ab}{5} = \frac{\square}{20}$

b Simplify:

i $\frac{9k}{15k}$

ii $\frac{4mn}{6mnp}$

iii $\frac{4k}{3} - \frac{7k}{5}$

iv $\frac{3}{4w} + \frac{4}{5w}$

v $\frac{4z}{5} \times \frac{8}{15y}$

vi $\frac{4p^2}{3q^3} \div \frac{2p}{5q^2}$

vii $15 - 3(2e - 7)$

viii $\frac{4x}{3} + \frac{2x - 1}{5}$

ix $\frac{3x - 1}{2} - \frac{5 - 2x}{5}$

c Factorise:

i $15t - 12$

ii $-16d + 20e$

iii $15a^3b^4 - 25a^2b^2$

5 a For the points $L(-3, 5)$ and $M(1, -3)$ find the following.

- i** the distance LM **ii** the midpoint of LM **iii** the gradient of LM

b i Complete the table of values for $y = 2x - 3$.

x	-3	-2	-1	0	1	2	3
y							

ii Plot the ordered pairs on a set of coordinate axes. Draw the straight line graph through the points.

c i Find the equation of the line with gradient 3 and y -intercept -4 .

ii Sketch the line.

d Consider the given straight line.

i Find the gradient.

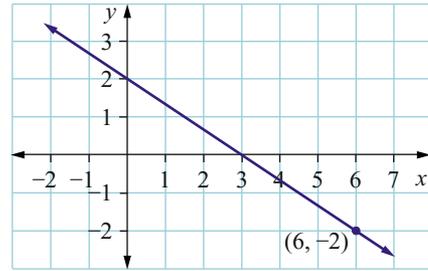
ii Find the y -intercept.

iii Write the equation of the line.

e i Write the line with equation $3x + 2y - 6 = 0$

in the form $y = mx + b$.

ii Sketch the line.



6 a A ball is selected at random from a bag containing 7 red, 5 blue and 2 orange balls. Determine the probability of selecting:

- i** a red ball **ii** a blue ball **iii** a blue ball or an orange ball
iv a non-red ball **v** a non-black ball **vi** a green ball.

b Two dice are thrown. Find the probability of obtaining:

- i** a 3 and a 5 **ii** a 2 and a 3 **iii** a double 2 **iv** any double.

c There are five right-handed tennis players and three left-handed tennis players in a team. One player is selected and plays a match. After the match a player is selected from the eight players.

i Draw a tree diagram showing this information.

ii Find the probability of selecting:

- a right-hander and a right-hander
- a left-hander and a left-hander
- a right-hander first and a left-hander second
- the same player twice.

d On my bookshelf there are 9 novels and 7 science books. I select a book at random, note its type and do not return it. I then select another book in the same way. Find the probability of choosing:

- i** a novel then a science book **ii** two novels **iii** at least one science book.

e This table represents data collected from 100 Year 10 students.

	Short	Tall	
Glasses	33	27	60
No glasses	30	10	40
	63	37	100

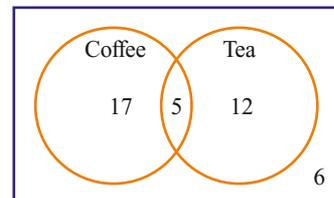
i Find the probability of selecting a tall student who wears glasses.

ii Find the probability of selecting a short student who does not wear glasses.

iii Describe a student who has a $\frac{1}{10}$ chance of being selected.

f The Venn diagram shows the number of people surveyed who drank tea or coffee. What is the probability that a person chosen at random from this group drinks:

- i** coffee? **ii** coffee but not tea?
iii both tea and coffee? **iv** coffee or tea but not both?



g Two dice are rolled and the sum of the two numbers shown is greater than 9. What is the probability that the numbers are the same?



6

Simultaneous equations

This chapter deals with solving simultaneous equations.

After completing this chapter you should be able to:

- ▶ solve simultaneous equations using trial and error
- ▶ solve simultaneous equations using graphical methods
- ▶ solve simultaneous equations algebraically using substitution and elimination methods
- ▶ solve simple word problems resulting in simultaneous equations.

NSW Syllabus references: 5.2 N&A Equations

Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-8NA

NUMBER & ALGEBRA – ACMNA237, ACMNA238



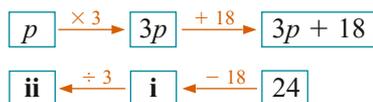
Diagnostic test

- 1 By inspection, the value of the pronumeral in $x + 9 = 10$ is:
A 19 **B** -1 **C** 1 **D** -19

- 2 The option that shows how the expression $2x - 3$ is built is:

A $x \xrightarrow{\times 2} 2x \xrightarrow{-3} \square$
B $x \xrightarrow{-3} x - 3 \xrightarrow{\times 2} \square$
C $-3 \xrightarrow{\times x} -3x \xrightarrow{\times 2} \square$
D $2 \xrightarrow{\times x} 2x \xrightarrow{+3} \square$

- 3 The equation $3p + 18 = 24$ has been solved using backtracking techniques.



The values of parts **i** and **ii** using this technique are:

- A** **i** 42 **ii** 14
B **i** 3 **ii** 1
C **i** 42 **ii** 12
D **i** 6 **ii** 2
- 4 The value of T in the equation $3T = -9$ is:
A -3 **B** 0 **C** $\frac{1}{3}$ **D** 3

- 5 The value of G in the equation $\frac{5G-2}{7} = 3$ is:
A $3\frac{4}{5}$ **B** $4\frac{3}{5}$ **C** $4\frac{4}{5}$ **D** $3\frac{3}{5}$

- 6 The value of m in the equation $3m - 9 = 5m + 5$ is:
A 2 **B** -7 **C** -2 **D** 7

- 7 When $x = 7$, $y = -3$ and $z = 0$, the value of the expression $z(4x - 2y)$ is:
A 34 **B** 6 **C** 22 **D** 0

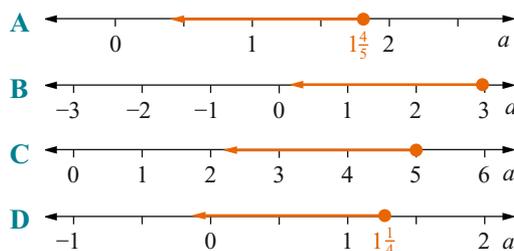
- 8 When $x = 0$, $y = 3$ and $z = -3$, the value of the expression $z(4x - 2y)$ is:
A 18 **B** 58 **C** 127 **D** 156

- 9 Ten is added to four times a certain number and the result is 18.

An equation using x as a certain number is:

- A** $40 + x = 18$ **B** $4x + 10 = 18$
C $4(x + 10) = 18$ **D** $40x + 10 = 18$

- 10 The graph that satisfies the inequality $a - 6 \leq -3$ is:



The diagnostic test questions refer to outcomes ACMNA234, ACMNA235, ACMNA236 and ACMNA240.



A

Linear equations review

Linear equations are equations of the form (or can be simplified to the form) $ax + b = 0$, where a and b are constants and x is the unknown (or variable).

EXAMPLE 1

Solve these linear equations.

a $7x - 9 = -5$

b $17 = 8 - 4x$

	Solve	Think	Apply
a	$7x - 9 = -5$ $7x - 9 + 9 = -5 + 9$ $7x = 4$ $\frac{7x}{7} = \frac{4}{7}$ $x = \frac{4}{7}$	<p>Add 9 to both sides.</p> <p>Divide both sides by 7.</p>	<p>Add or subtract numbers to both sides until the pronumeral is on one side and a number is on the other side.</p> <p>Multiply or divide to solve.</p>
b	$17 = 8 - 4x$ $17 - 8 = 8 - 4x - 8$ $9 = -4x$ $\frac{9}{-4} = \frac{-4x}{-4}$ $\frac{-9}{4} = x$ $x = -\frac{9}{4} = -2\frac{1}{4}$	<p>Subtract 8 from both sides.</p> <p>Divide both sides by -4.</p> <p>Write the solution with x on the left-hand side.</p>	

Exercise 6A

1 Solve for x in the following equations.

a $x + 3 = 10$

b $3x = -9$

c $3x + 6 = 0$

d $3x - 4 = -6$

e $5x + 8 = 2$

f $8x - 6 = 10$

g $6 + 7x = -2$

h $6x - 7 = -1$

i $6 - x = -5$

j $3 - 2x = 7$

k $3 - 7x = -2$

l $11 = 3 - 2x$

m $8 = 3 - 2x$

n $-15 = 3 - 6x$



2 Check the given solution by substitution and state whether or not it is correct.

a $2x + 8 = 15$ ($x = 7$)

b $7 + 5x = 9$ ($x = 2$)

c $-15 = 6 - 7x$ ($x = 3$)

d $\frac{x}{5} - 3 = 6$ ($x = \frac{9}{5}$)

EXAMPLE 2

Given that $y = 5x - 3$, find x when $y = -18$.

Solve	Think	Apply
$y = 5x - 3$ $-18 = 5x - 3$ $-18 + 3 = 5x - 3 + 3$ $-15 = 5x$ $\frac{-15}{5} = \frac{5x}{5}$ $-3 = x$ $x = -3$	<p>Substitute $y = -18$. Add 3 to both sides.</p> <p>Divide both sides by 5.</p>	<p>Often when substituting and solving an equation, the pronumeral is on the right-hand side. Solve as normal and then write the pronumeral on the left-hand side.</p>

3 a Given that $y = 3x - 5$, find x when $y = 5$.

b Given that $y = 4x + 2$, find x when $y = 11$.

c Given that $y = 7 - 5x$, find x when $y = 0$.

d Given that $y = 4 - 3x$, find x when $y = -3$.

e Given that $y = 5 - 7x$, find x when $y = -5$.

f Given that $y = 3x - 5$, find x when $y = 8$.

EXAMPLE 3

Solve the following equations.

a $5x + 2 = 3x - 5$

b $15 - 2x = 11 + x$

	Solve	Think	Apply
a	$5x + 2 = 3x - 5$ $5x + 2 - 3x = 3x - 5 - 3x$ $2x + 2 = -5$ $2x + 2 - 2 = -5 - 2$ $2x = -7$ $\frac{2x}{2} = \frac{-7}{2}$ $x = -\frac{7}{2} = -3\frac{1}{2}$	<p>Subtract $3x$ from both sides.</p> <p>Subtract 2 from both sides.</p> <p>Divide both sides by 2.</p>	<p>Eliminate the pronumeral from one side of the equation by adding or subtracting one of the pronumeral terms. Solve the resulting equation in the same way as in the previous example.</p>
b	$15 - 2x = 11 + x$ $15 - 2x + 2x = 11 + x + 2x$ $15 = 11 + 3x$ $15 - 11 = 11 + 3x - 11$ $4 = 3x$ $\frac{4}{3} = \frac{3x}{3}$ $\frac{4}{3} = x$ $x = 1\frac{1}{3}$	<p>Add $2x$ to both sides.</p> <p>Subtract 11 from both sides.</p> <p>Divide both sides by 3.</p> <p>Swap the pronumeral to the left-hand side.</p>	

4 Solve the following equations with integer solutions.

a $5x + 2 = 2x + 14$

b $3x + 7 = 11 - x$

c $5 + x = 8 - 2x$

d $3x - 4 = 5x - 2$

e $3 - x = x + 7$

f $4 - 2x = 3 - x$

g $2x - 3 = x + 6$

h $5x - 9 = 1 + 6x$

i $3x - 5 = 7 - x$

5 Solve the following equations.

a $8x + 7 = 4x - 2$

b $7x + 3 = 2x + 7$

c $5 + 2x = 11 - x$

d $x - 3 = 5x + 7$

e $3 + x = 17 + 4x$

f $15 - 3x = 2 - x$

g $2x + 5 = 9 - 2x$

h $3x - 5 = 5x + 9$

i $5 - 7x = 3x + 2$

j $5a + 3 = a - 1$

k $4 - 3s = 2s + 17$

l $9x - 4 = 3 + 4x$

m $11a - 7 = 5a + 12$

n $3y - 5 = -14 - 2y$

o $7p = 15 - 3p$

EXAMPLE 4

By substituting, check the solutions to the following equations.

a $2x - 5 = 10 - 3x$ ($x = 3$)

b $5x + 2 = 2x - 7$ ($x = 2$)

	Solve	Think	Apply
a	Does $2x - 5 = 10 - 3x$ when $x = 3$? LHS: $2 \times 3 - 5 = 1$ RHS: $10 - 3 \times 3 = 1$ LHS = RHS $\therefore x = 3$ is the solution.	Substitute 3 for x on both sides of the equation. Left-hand side = 1 Right-hand side = 1 $x = 3$ is a solution.	Substitute the value of x and evaluate both sides of the equation. Both sides must give the same value for that value of x to be a solution.
b	Does $5x + 2 = 2x - 7$ when $x = 2$? LHS: $5 \times 2 + 2 = 12$ RHS: $2 \times 2 - 7 = -3$ $12 \neq -3$ $\therefore x = 2$ is not the solution.	Substitute 2 for x on both sides of the equation. Left-hand side = 12 Right-hand side = -3 This is not a solution.	The actual value of the sides is not relevant.

6 By substituting, check the solutions to the following equations.

a $3x + 9 = 4 + 2x$ ($x = 1$)

b $9a + 2 = 7a - 4$ ($a = -3$)

c $7a - 5 = 3 - a$ ($a = 2$)

d $15 - 2x = 6 + x$ ($x = 3$)

e $2x - 3 = 7 - 4x$ ($x = \frac{5}{3}$)

f $5x - 7 = 3 + x$ ($x = 3\frac{1}{2}$)

EXAMPLE 5

Solve $5(x + 1) - 2x = 7$.

Solve	Think	Apply
$5(x + 1) - 2x = 7$ $5x + 5 - 2x = 7$ $3x + 5 = 7$ $3x + 5 - 5 = 7 - 5$ $3x = 2$ $x = \frac{2}{3}$	Expand the brackets. Collect the like terms. Subtract 5 from both sides. Divide both sides by 3.	The number in front of the brackets is multiplied by each term within the brackets.

7 Solve for x in each equation.

a $2(x + 1) - 1 = 8$

c $3(x + 2) - 7 = 11$

e $4(2x - 1) + 7 = 0$

g $3 - 2(x + 1) = -4$

b $5(1 - 3x) = -4$

d $2(x + 1) + 3(x - 1) = 6$

f $11 - 2(x - 1) = 7$

h $7 - (2 - x) = 2x$

EXAMPLE 6

Given that $y = 3 - 5(x + 4)$, find x when $y = -32$.

Solve	Think	Apply
$y = 3 - 5(x + 4)$ $-32 = 3 - 5(x + 4)$ $= 3 - 5x - 20$ $-32 = -17 - 5x$ $-32 + 17 = -17 - 5x + 17$ $-15 = -5x$ $\frac{-15}{-5} = \frac{-5x}{-5}$ $3 = x$ $x = 3$	<p>Substitute $y = -32$.</p> <p>Expand.</p> <p>Collect like terms.</p> <p>Add 17 to both sides.</p> <p>Divide both sides by -5.</p>	<p>Substitute the value, simplify both sides if possible, then solve the equation. The pronumeral is often on the right-hand side of the equation.</p> <p>The most common error is to multiply the second term in the brackets incorrectly.</p> <p>Be vigilant with the signs.</p>

8 a Given that $y = 7 - 3(x + 2)$, find x when $y = -5$.

b Given that $y = 5 - 4(x - 3)$, find x when $y = 37$.

c Given that $y = 4 - 5(2x - 5)$, find x when $y = 12$

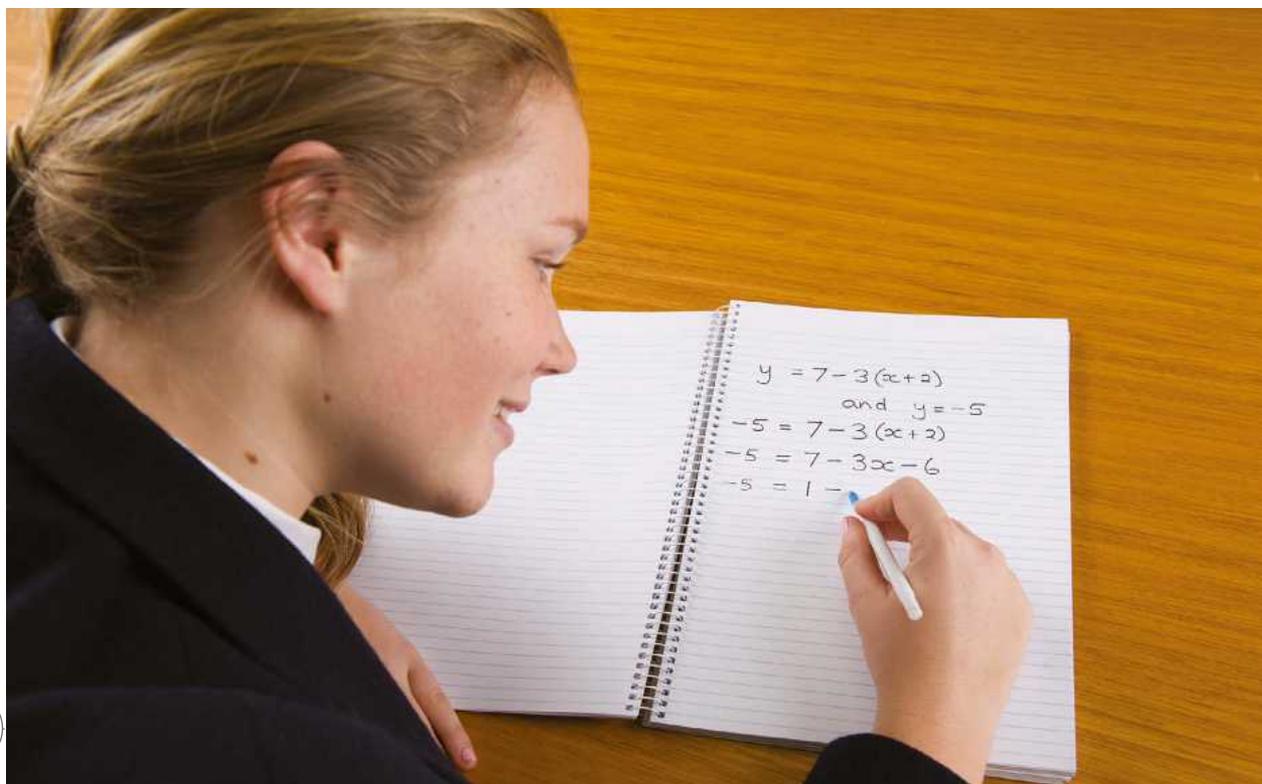
d Given that $y = 14 - 3(2x - 8)$, find x when $y = 0$.

e Given that $y = 3x - 2(5x + 1)$, find x when $y = -16$.

f Given that $y = 4x - 3(5 - 2x)$, find x when $y = 8$.

g Given that $y = 3(2x - 1) - 4(x + 2)$, find x when $y = -3$.

h Given that $y = 4(1 - 3x) - 2(1 - x)$, find x when $y = 2$.



B

Simultaneous equations

Consider the following problem. Sarah bought five pieces of fruit each of which was either an apple or a banana. The apples cost 20 cents each and the bananas cost 25 cents each. The total cost of the fruit was \$1.05. How many apples and bananas did Sarah buy?

One method of solving this problem is to introduce two unknowns and set up two equations that need to be solved simultaneously. We will solve the problem using this approach.

Suppose that there are x apples and y bananas.

Then $x + y = 5$ (as there are 5 pieces of fruit).

The apples cost 20 cents each, so the total cost for x apples is $20x$.

Similarly, the cost for y bananas is $25y$.

As the total cost is \$1.05 or 105 cents, $20x + 25y = 105$.

For the equation $x + y = 5$, the possible values for x or y are 0, 1, 2, 3, 4 or 5, as shown in the table.

x	y	$x + y$	$20x + 25y$
0	5	5	125
1	4	5	120
2	3	5	115
3	2	5	110
4	1	5	105
5	0	5	115

This possibility satisfies both equations.

Hence, the solution is 4 apples and 1 banana.

Trial and error

EXAMPLE 1

Complete a table to find the simultaneous solution of $x + y = 6$ and $3x + 2y = 16$, given that x and y are integers.

Solve				Think	Apply
x	y	$x + y$	$3x + 2y$	List the possible values for x . As $x + y = 6$, fill in the values of y . Next calculate the values of $3x + 2y$. Keep going until the value 16 is found in the $3x + 2y$ column.	The x - and y -values must satisfy both equations. There is only one pair of values that will satisfy both equations.
0	6	6	12		
1	5	6	13		
2	4	6	14		
3	3	6	15		
4	2	6	16		
5	1	6	17		
6	0	6	18		
The solution is $x = 4$ and $y = 2$.					

Exercise 6B

- 1 Complete this table to find the simultaneous solution of $x + y = 5$ and $5x + 3y = 19$. Remember to look for 19 in the $5x + 3y$ column. Complete the y column first.

x	y	$x + y$	$5x + 3y$
0		5	
1		5	
2		5	
3	2	5	
4		5	
5		5	

- 2 Complete this table to find the simultaneous solution of $x + y = 7$ and $3x - 2y = 11$.

x	y	$x + y$	$3x - 2y$
0		7	
1		7	
2			
3			
4			
5			
6			
7			

EXAMPLE 2

By completing a table, find the integers x and y that satisfy the equations $y = x + 3$ and $5x - 2y = 6$ simultaneously.

Solve			Think	Apply
x	$y = x + 3$	$5x - 2y$	In this example y is 3 more than x . So start with $x = 0$ and $y = 3$. Fill in the values for $5x - 2y$. Look for 6 in the last column.	The x - and y -values must satisfy both equations. There is only one pair of values that will satisfy both equations.
0	$y = 0 + 3 = 3$	$5(0) - 2(3) = -6$		
1	$y = 1 + 3 = 4$	$5(1) - 2(4) = -3$		
2	$y = 2 + 3 = 5$	$5(2) - 2(5) = 0$		
3	$y = 3 + 3 = 6$	$5(3) - 2(6) = 3$		
4	$y = 4 + 3 = 7$	$5(4) - 2(7) = 6$		
5	$y = 5 + 3 = 8$	$5(5) - 2(8) = 9$		
The solution is $x = 4$ and $y = 7$.				

- 3** Complete this table to find the integers x and y that satisfy the equations $y = 5 - x$ and $3x - 2y = 10$ simultaneously.
The solution is $x = \underline{\quad}$ and $y = \underline{\quad}$.

x	$y = 5 - x$	$3x - 2y$
0	$y = 5 - 0 = 5$	$3(0) - 2(5) = \underline{\quad}$
1	$y = 5 - 1 = \underline{\quad}$	$3(1) - 2(\underline{\quad}) = \underline{\quad}$
2		
3		
4		
5		$3(5) - 2(0) = 15$

- 4** By completing a table as in Examples 1 and 2, find integers x and y that satisfy these equations simultaneously.
- a** $x + y = 4$
 $3x + 5y = 14$
- b** $x + y = 11$
 $4x + 3y = 40$
- c** $y = x + 2$
 $9x - 4y = 7$
- d** $y = 6 + x$
 $8x - 3y = -3$

EXAMPLE 3

Check if the possible solution $x = 5$ and $y = 3$ is correct for the pair of simultaneous equations $x - y = 2$ and $3x + 4y = 17$.

Solve	Think	Apply
$x - y = 2$ $5 - 3 = 2$ This is correct. $3x + 4y = 17$ $3(5) + 4(3) = 27$ This is not correct. $\therefore x = 5$ and $y = 3$ is not a solution.	Substitute $x = 5$ and $y = 3$ into each equation.	Substitute the values for x and y into each equation. The solution must satisfy both equations.

- 5** Complete to check if the possible solution $x = 7$ and $y = 3$ is correct for this pair of simultaneous equations.
 $x - y = 4$ and $3x - 8y = -3$
- $x - y = 4$
Test: $7 - \underline{\quad} = 4$ This is $\underline{\quad}$.
 $3x - 8y = -3$
Test: $3(7) - 8(\underline{\quad}) = \underline{\quad}$ This is $\underline{\quad}$.
 $\therefore x = 7$ and $y = 3$ is $\underline{\quad}$.
- 6** A possible solution is given for each pair of simultaneous equations. Check if the solution is correct.
- a** $x - y = 3$
 $2x - y = 11$
Solution: $x = 5, y = 2$
- b** $x + y = 9$
 $2x - y = 6$
Solution: $x = 5, y = 4$
- c** $a + b = 2$
 $a - b = 8$
Solution: $a = 5, b = -3$
- d** $2p + q = 7$
 $3p + 2q = 10$
Solution: $p = 4, q = -1$
- 7** Find by trial and error the simultaneous solution of the following equation pairs, given that the solutions are integers.
- a** $x + y = 4$
 $2x - y = 5$
- b** $x + y = 6$
 $2x + y = 10$
- c** $a - b = 1$
 $2a + 3b = 2$
- d** $p - q = 3$
 $5p + 2q = 29$

C

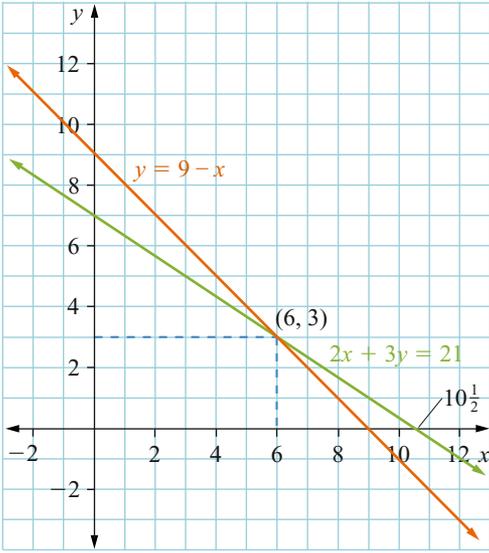
Graphical solutions

Equations such as $x + y = 9$ and $2x + 3y = 21$ have linear (straight-line) graphs. If the two straight-line graphs are drawn, their point of intersection is the simultaneous solution.

In this section linear graphs are used to solve simultaneous equations.

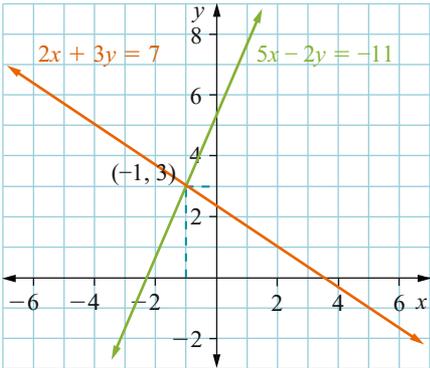
EXAMPLE 1

Graph $y = 9 - x$ and $2x + 3y = 21$ to find their point of intersection.

Solve	Think	Apply																
<p>Draw a table of values for each equation.</p> <p>$y = 9 - x$</p> <table border="1"> <tr> <td>x</td> <td>0</td> <td>4</td> <td>9</td> </tr> <tr> <td>y</td> <td>9</td> <td>5</td> <td>0</td> </tr> </table> <p>$2x + 3y = 21$</p> <table border="1"> <tr> <td>x</td> <td>0</td> <td>3</td> <td>9</td> </tr> <tr> <td>y</td> <td>7</td> <td>5</td> <td>1</td> </tr> </table> <p>Graph both lines and find the point of intersection.</p>  <p>The solution is $x = 6$ and $y = 3$.</p>	x	0	4	9	y	9	5	0	x	0	3	9	y	7	5	1	<p>When $x = 0$, $y = 9 - 0 = 9$ When $x = 4$, $y = 9 - 4 = 5$ When $x = 9$, $y = 9 - 9 = 0$</p> <p>When $x = 0$, $2(0) + 3y = 21$ $y = 7$ When $x = 3$, $2(3) + 3y = 21$ $6 + 3y = 21$ $3y = 15$ $y = 5$ When $x = 9$, $2(9) + 3y = 21$ $18 + 3y = 21$ $3y = 3$ $y = 1$</p> <p>We can graph the values in each of the tables. The point of intersection is $(6, 3)$. The solution to the simultaneous equations $y = 9 - x$ and $2x + 3y = 21$ is $x = 6$ and $y = 3$.</p>	<p>Choose any x-value and find the corresponding y-value by substituting and solving the equation.</p> <p>Choose at least three points. Using the x- and y-intercepts by substituting $y = 0$ and $x = 0$ is an efficient method of drawing the line. This is demonstrated in the next example.</p>
x	0	4	9															
y	9	5	0															
x	0	3	9															
y	7	5	1															

EXAMPLE 2

Solve these equations simultaneously by drawing the graphs of $2x + 3y = 7$ and $5x - 2y = -11$.

Solve	Think	Apply
$2x + 3y = 7$ $2(0) + 3y = 7$ $3y = 7$ $y = 2\frac{1}{3}$ $2x + 3y = 7$ $2x + 3(0) = 7$ $2x = 7$ $x = 3\frac{1}{2}$ $5x - 2y = -11$ $5(0) - 2y = -11$ $-2y = -11$ $y = 5\frac{1}{2}$ $5x - 2(0) = -11$ $5x = -11$ $x = -2\frac{1}{5}$	<p>Substitute $x = 0$ to find the y-intercept.</p> <p>Substitute $y = 0$ to find the x-intercept.</p> <p>Substitute $x = 0$ to find the y-intercept.</p> <p>Substitute $y = 0$ to find the x-intercept.</p>	<p>The solution to simultaneous equations can have positive or negative values.</p> <p>The solutions do not have to be integers but they are in this section to make calculations easier.</p>
<p>Graph both lines and find the point of intersection.</p> 	<p>Graph these points and draw the lines.</p> <p>The solution is $x = -1, y = 3$.</p> <p>Check by substituting:</p> $2x + 3y = 2(-1) + 3(3)$ $= 7$ $5x - 2y = 5(-1) - 2(3)$ $= -11$ <p>The solution is correct.</p>	
<p>The solution is $x = -1, y = 3$.</p>		

Exercise 6C

- 1 a Use these tables of values to graph $2x - 3y = -7$ and $4x + y = 7$.

$$2x - 3y = -7$$

x	0	2	4
y	$2\frac{1}{3}$	$3\frac{2}{3}$	5

$$4x + y = 7$$

x	0	2	4
y	7	-1	-9

- b Find the point of intersection of the lines $2x - 3y = -7$ and $4x + y = 7$.
- c Check that your solution from part b is correct.

- 2 a** Graph $3x - 2y = 8$ and $2x + 5y = 18$ using these tables of values.

$$3x - 2y = 8$$

x	0	3	6
y	-4	$\frac{1}{2}$	5

$$2x + 5y = 18$$

x	0	4	9
y	$3\frac{3}{5}$	2	0

- b** Find the point of intersection of these two graphs.
c Check that the point from part **b** is the solution to both equations.

- 3** Use graphical methods to solve these equations simultaneously.

a $y = x - 3$

$y = 1 - x$

d $3x + y = 5$

$2x - 3y = 18$

g $3x + y = -5$

$2x + 3y = -8$

b $x - y = 1$

$y = 2x$

e $y = 3x - 8$

$3x + y = -2$

h $x - 3y = -9$

$2x - 3y = -12$

c $4x + 3y = 10$

$x - 2y = -3$

f $2x + y = 4$

$2x - y = 8$

i $x - 2y = 4$

$2x - 4y = 8$

Investigation 1 Graphics calculator

- 1** Use the graphics calculator to sketch the graphs of $y = 2x + 3$ and $y = 8 - 3x$. Find their point of intersection. These instructions are for a Casio fx-9860G AU series.

Step 1: Select GRAPH from the MAIN MENU.

Step 2: Press **F3** for graph type and choose $Y=$ by pressing **F1**.

Step 3: To enter the equation $y = 2x + 3$, press 2 **X,θ,T** **+** 3 (the **X,θ,T** key is used to enter x).

Press **EXE** to enter the equation.

To make the calculator sketch the graph, press **F6**, the DRAW command.

Press **EXIT** to return.

Step 4: To enter the equation $y = 8 - 3x$, press 8 **-** 3 **X,θ,T** (to enter x).

Press **EXE** to enter the equation.

To make the calculator sketch the graph, press **F6**, the DRAW command.

Step 5: Both graphs now appear on the screen.

The point of intersection is in the first quadrant. Use the up arrow if necessary to view.

Step 6: To find the coordinates of the point of intersection use the trace function.

Press TRACE **F1** and a cursor will appear at the left of the graph entered second: $y = 8 - 3x$. Use the left and right arrows to move the cursor to the point of intersection.

The correct solution is $x = 1$ and $y = 5$.



The coordinates are only approximate due to the accuracy of the calculator. **!**

- 2 Use the graphics calculator to sketch the graphs in Exercise 6C.
The equations must have y as their subject. If not, then they must be rearranged.

For example: $2x + 3y = 7$

becomes $2x + 3y - 2x = 7 - 2x$ Subtract $2x$ from both sides.

$$3y = 7 - 2x$$

$$\frac{3y}{3} = \frac{7 - 2x}{3} \quad \text{Divide both sides by 3.}$$

$$y = \frac{7 - 2x}{3}$$

D Solution by substitution

Simultaneous equations can be solved algebraically. The first method is by **substitution**. One variable is substituted into the other equation so that the expression is reduced to an equation with only one variable. The method of solution by substitution is often used when at least one of the given equations has either x or y as the subject.

As in Example 1 below, we can use substitution to solve $y = 2x + 3$ and $3x - 4y = 8$, as the first equation has y as the subject. This method involves replacing y in the second equation with the value of y in the first equation (in this example by $2x + 3$). The result is an equation containing only one unknown (x in this case).

EXAMPLE 1

Solve these equations simultaneously using substitution.

a $y = 2x + 3$ and $3x - 4y = 8$

b $y = 3x - 5$ and $y = 10 - 2x$

	Solve	Think	Apply
a	$y = 2x + 3$ ① $3x - 4y = 8$ ② $3x - 4(2x + 3) = 8$ $3x - 8x - 12 = 8$ $-5x - 12 = 8$ $-5x = 20$ $x = -4$ $y = 2(-4) + 3$ $= -8 + 3 = -5$ The solution is $x = -4$ and $y = -5$.	Substitute $2x + 3$ for y in equation ②. Expand the bracket. Simplify and solve for x . Substitute $x = -4$ into equation ① to obtain the y -value.	This method is most appropriate when one or both of the equations have either x or y as the subject. It is good practice to substitute the solution into both equations to check it is correct. This is the only method for solving simultaneous equations when one equation is not linear.
b	$y = 3x - 5$ ① $y = 10 - 2x$ ② $3x - 5 = 10 - 2x$ $5x - 5 = 10$ $5x = 15$ $x = 3$ $y = 3(3) - 5$ $= 9 - 5 = 4$ The solution is $x = 3$ and $y = 4$.	As both equations have y as the subject, put them equal to each other. Solve the resulting equation for x . Substitute $x = 3$ into equation ① to obtain the y -value.	

Exercise 6D

- 1 Complete to solve the simultaneous equations $y = 2x - 7$ and $3x - 2y = 11$ using substitution.

$$y = 2x - 7 \quad \textcircled{1}$$

$$3x - 2y = 11 \quad \textcircled{2}$$

Substitute _____ for y in equation $\textcircled{2}$.

$$3x - 2(\text{_____}) = 11$$

Expand the brackets.

$$3x - 4x \text{ _____} = 11$$

$$\text{_____} + 14 = 11$$

$$\text{_____} + 14 - 14 = 11 \text{ _____}$$

Subtract 14 from both sides.

$$-x = \text{_____}$$

$$x = \text{_____}$$

Substitute $x = \text{_____}$ into equation $\textcircled{1}$ to obtain the y -value.

$$y = 2(\text{_____}) - 7$$

$$= \text{_____}$$

The solution is $x = \text{_____}$ and $y = \text{_____}$.

- 2 Solve the following pairs of simultaneous equations using the substitution method.

a $x = 1 + 2y$

$$2x + y = 17$$

d $x = 2y - 6$

$$2x + y = 8$$

g $x = 1 - 2y$

$$y + 3x = 13$$

b $y = x - 4$

$$y = 2x - 3$$

e $3x - 2y = 5$

$$y = 3 - 4x$$

h $x = 3y + 12$

$$3x - 2y = 8$$

c $y + 4x = 6$

$$y = 2x + 3$$

f $y = 2x - 8$

$$y = 8x + 4$$

i $y = -2x + 3$

$$y = x + 6$$

- 3 Solve these equations by substitution.

a $y = 1 - 2x$

$$y = 3 - 4x$$

d $y = 4 - x$

$$3y = 5x$$

b $x = 2y + 2$

$$2x + y = 9$$

e $y = 5 - 2x$

$$2x + 3y = 13$$

c $y = 2x + 5$

$$5x - y = 1$$

f $x = 3y + 14$

$$x = 2y - 1$$



Solution by elimination

A second method of solving simultaneous equations algebraically is by **elimination**. Solution by elimination is used for simultaneous equations such as $3x + 2y = -2$ and $5x - y = 27$. We make the coefficients of either x or y equal and then add (or subtract) the resulting equations.

There is always a choice as to whether we eliminate x or y . However, our choice is usually determined by the ease of eliminating one of the variables.

If we make the coefficients of one of the variables equal in size but opposite in sign, we can add these terms to eliminate them.

EXAMPLE 1

Solve $3x + 2y = -2$ and $5x - y = 27$ using the elimination method.

Solve	Think	Apply
$3x + 2y = -2$ ① $5x - y = 27$ ② $3x + 2y = -2$ $10x - 2y = 54$ <hr/> $13x = 52$ $x = 4$ $3(4) + 2y = -2$ $12 + 2y = -2$ $2y = -14$ $y = -7$ The solution is $x = 4$ and $y = -7$.	Label the equations ① and ②. Multiply both sides of equation ② by 2. Add the equations and solve for x . Substitute $x = 4$ into equation ①. <i>Check by substituting:</i> In ①: $3x + 2y = 3(4) + 2(-7)$ $= 12 - 14 = -2$ In ②: $5x - y = 5(4) - (-7)$ $= 20 + 7 = 27$	In this case only one equation had to be changed to be able to eliminate a variable. Often both equations must be multiplied by different numbers, as in the next example.

EXAMPLE 2

Solve $5x + 3y = 12$ and $7x + 2y = 19$ using the elimination method.

Solve	Think	Apply
$5x + 3y = 12$ ① $7x + 2y = 19$ ② $10x + 6y = 24$ $-21x - 6y = -57$ <hr/> $-11x = -33$ $x = 3$ $5(3) + 3y = 12$ $15 + 3y = 12$ $3y = -3$ $y = -1$ Or eliminate the x variable. $35x + 21y = 84$ $-35x - 10y = -95$ <hr/> $11y = -11$ $y = -1$ The solution is $x = 3$ and $y = -1$.	To eliminate the y variable: <ul style="list-style-type: none"> • Multiply equation ① by 2. • Multiply equation ② by -3. • Add to eliminate y. Solve for x . Substitute $x = 3$ into ①. To eliminate the x variable: <ul style="list-style-type: none"> • Multiply equation ① by 7. • Multiply equation ② by -5. • Add to eliminate x. $y = -1$ and $x = 3$ by substitution.	In this case both equations must be multiplied by different numbers to be able to eliminate a variable.

Exercise 6E

1 Write the equation that results when:

a $2x - 3y = 5$ is multiplied by 2

c $2x + 5y = 1$ is multiplied by 4

e $5x - y = 2$ is multiplied by 5

b $x + 3y = 7$ is multiplied by -3

d $3x - 2y = 8$ is multiplied by -2

f $-2x + 5y = -1$ is multiplied by -1

2 What equation results when the following pairs of equations are added?

a $3x + 2y = 6$

$x - 2y = 10$

d $3x - y = 4$

$-3x + 4y = 2$

b $3x - y = 8$

$x + y = 8$

e $5x - y = 6$

$-5x + 3y = -8$

c $x + y = 5$

$x - y = 7$

f $-8x + 2y = 11$

$8x - 3y = -7$

3 Complete to solve $3x + 2y = 4$ and $2x + y = 1$ using the elimination method.

$3x + 2y = 4$

①

$2x + y = 1$

②

To eliminate y multiply equation ② by ____.

$$\begin{array}{r} 3x + 2y = 4 \\ -4x - 2y = -2 \\ \hline \quad = 2 \\ x = -2 \end{array}$$

Add to eliminate y .

Substitute $x = -2$ into equation ①.

$$\begin{aligned} 3(\underline{\quad}) + 2y &= 4 \\ \underline{\quad} + \underline{\quad} + 2y &= 4 + \underline{\quad} \\ 2y &= \underline{\quad} \\ y &= \underline{\quad} \end{aligned}$$

Add ____ to both sides.

The solution is $x = -2$ and $y = \underline{\quad}$.

4 Solve the following equations simultaneously using the elimination method.

a $3x + y = 13$

$x - y = 3$

c $x + 3y = 13$

$-x + y = 7$

e $2x + 3y = 11$

$7x - y = 50$

g $4x + y = 19$

$3x + 4y = -2$

i $6x + 5y = -2$

$3x - y = 13$

k $3x + 2y = 7$

$8x + 7y = 12$

m $2x + 7y = -51$

$3x - 2y = 11$

o $2x - 7y = 14$

$5x - 7y = 34$

b $2x - y = 8$

$3x + y = 7$

d $5x + 2y = -19$

$3x - 4y = -1$

f $2x + y = 1$

$x + 3y = -12$

h $7x + 2y = -5$

$3x - 5y = -49$

j $4x - 3y = 12$

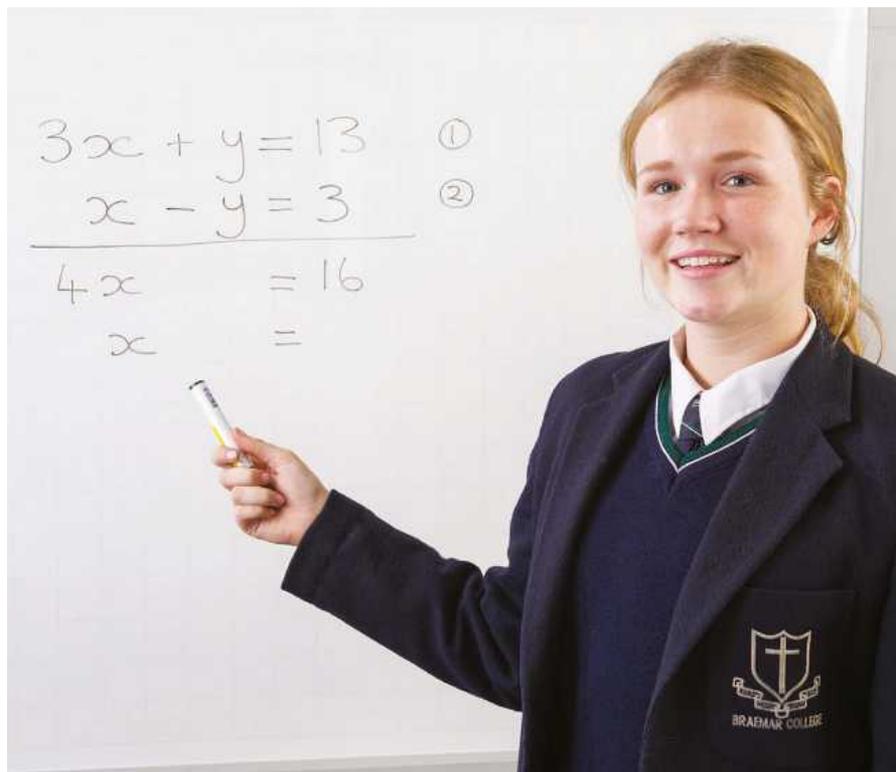
$-x + 5y = -3$

l $3x + 7y = 47$

$7x + 3y = 43$

n $3x + y = -10$

$2x - y = 5$



- 2** Find two equations in x and y and solve algebraically.
- The difference between two numbers is 84 and their sum is 278. What are the numbers?
 - One number exceeds another by 11. The sum of the two numbers is 5. What are the numbers?
 - The larger of two numbers is four times the smaller number. The sum of the two numbers is 85. What are the numbers?
 - Find two integers such that three times the smaller is 33 more than twice the larger, and twice the smaller plus five times the larger equals 250.
- 3** Find two equations in x and y , draw their graphs and solve graphically.
- Find two numbers with a sum of 12 and a difference of 2.
 - Find two numbers with a sum of 20 and a difference of 4.
 - The sum of two numbers is 10. When one number is added to twice the other number the result is 16. Find the numbers.
 - The sum of two numbers is 14. When one number is added to twice the other number the result is 25. Find the numbers.
 - Find two numbers with a sum of 12 and with half their difference equal to 2.

● EXAMPLE 2

Five apples and 4 bananas cost a total of \$2.05, and 6 apples and 5 bananas cost of total of \$2.50. Find the cost of an apple and the cost of a banana.

Solve	Think	Apply
$5x + 4y = 205$ ① $6x + 5y = 250$ ② $25x + 20y = 1025$ $-24x - 20y = -1000$ $x = 25$ $5(25) + 4y = 205$ $125 + 4y = 205$ $4y = 80$ $y = 20$ An apple costs 25 cents and a banana costs 20 cents.	Write the equations with the totals in cents. Multiply equation ① by 5. Multiply equation ② by -4 . Add to eliminate y . Substitute $x = 25$ into equation ①.	Ensure that the units are consistent in the two equations.



- 4 Two hammers and one screwdriver cost a total of \$59, and one hammer and three screwdrivers cost a total of \$42. Complete to form simultaneous equations to find the price of each tool. Let the number of hammers be h and the number of screwdrivers be d .

Then $2h + \underline{\hspace{1cm}} = 59$ ①

and $h + \underline{\hspace{1cm}} = 42$ ②

Multiply equation ② by 2 to eliminate $\underline{\hspace{1cm}}$.

$$2h + d = 59 \quad \text{①}$$

$$2h + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{②}$$

$$\hline -5d = \underline{\hspace{1cm}}$$

$$d = 5$$

Substitute $\underline{\hspace{1cm}}$ into equation ②.

$$h + \underline{\hspace{1cm}} = 42$$

$$h = \underline{\hspace{1cm}}$$

The hammer costs $\underline{\hspace{1cm}}$ and the screwdriver costs $\underline{\hspace{1cm}}$.



- 5 Form simultaneous equations and solve using any method.
- Theatre tickets for four adults and three children cost \$74. Theatre tickets for two adults and five children cost \$58. Calculate the price of an adult's ticket and a child's ticket.
 - Three blankets and a sheet cost \$190 altogether, and two sheets and a blanket cost a total of \$100. Calculate the cost of one blanket and one sheet.
 - Seven apples and eight oranges cost \$4.06 altogether. At the same shop three apples and one orange cost a total of \$1.06. Calculate the cost of an apple and the cost of an orange.
 - A purse contains \$3.75 in 5-cent and 20-cent coins. If there are 33 coins altogether, how many of each type of coin are in the purse?
 - I collect only 50-cent and \$1 coins. My collection consists of 43 coins and their total value is \$35. How many of each coin type do I have?

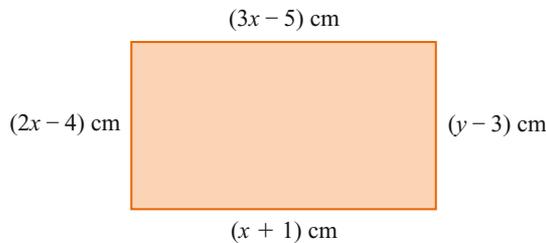
Extension

- 6 A particular brand of milk is sold in either 1 L or 2 L cartons. If a delicatessen owner orders 120 L of milk and receives 97 cartons, how many cartons of each size did they receive?
- 7 A yard holds rabbits and pheasants only. There are 35 heads and 98 feet in the yard. How many rabbits and pheasants are in the yard?



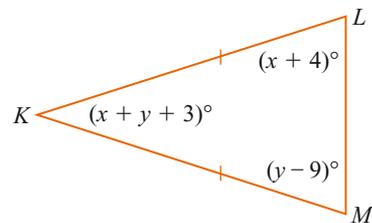
- 8** A baker's basket contains bread rolls and sandwich loaves. The loaves weigh 750 g and the rolls 150 g. If the total number of loaves and rolls is 16 and their total weight is 6 kg, how many rolls and how many loaves are in the basket?

- 9** Find x and y in this rectangle.



- 10** An equilateral triangle has sides of length $(3x - y)$ cm, $(x + 5)$ cm and $(y + 3)$ cm. Find the length of each side.

- 11** KLM is an isosceles triangle. Find x and y and hence find the measure of the angle at K .



- 12** Twelve years ago Jane was five times as old as Anne. In 3 years time Anne will be half Jane's age. How old is each girl at the moment?
- 13** Nine years ago a mother was three times as old as her son. In eight years from now the sum of their ages will be 78. How old are they today?
- 14** The weekly wages of Sam and Ben are in the ratio of 2 : 1. Their living costs are in the ratio of 9 : 4. If each saves \$100 per week, find their weekly incomes.

Investigation 2 Simultaneous equations solutions

- 1 a** Graph the equations $y = 2x + 1$ and $y = 2x + 3$.
b What is the solution when $y = 2x + 1$ and $y = 2x + 3$ are solved simultaneously? Why?
c Solve the equations simultaneously using the elimination method. What happens?
d Write a pair of simultaneous equations that have no solution.
- 2 a** Graph the equations $x + 2y = 5$ and $2x + 4y = 10$. What do you notice?
b How many solutions are there when $x + 2y = 5$ and $2x + 4y = 10$ are solved simultaneously?
c Solve $x + 2y = 5$ and $2x + 4y = 10$ simultaneously using the elimination method. What happens?
d Write a pair of simultaneous equations that have an infinite number of solutions.
- 3** Write an equation that when solved simultaneously with $y = 3x + 2$ gives:
a 0 solutions **b** 1 solution **c** an infinite number of solutions.
- 4** Write three pairs of linear equations that when solved simultaneously have:
a 0 solutions **b** 1 solution **c** an infinite number of solutions.
 Have your equations checked.

Language in mathematics

Albert Einstein (1879–1955)

Albert Einstein was born in Ulm, Germany. His family moved to Munich where Einstein started his education at a school that instilled a very rigid discipline.

Young Albert did not take well to strict regimentation and showed very little academic enthusiasm and ability. His family moved to Milan, Italy, leaving him behind in Munich to continue his schooling. Einstein left school with poor grades in a number of subjects. It was said that the reason he could not cope was because of the stifling education system, but in fact he could not bear to be away from the rest of his family.

A few years later he recommenced studying in Switzerland and finally spent four years at the Polytechnic Academy in Zurich concentrating on Physics. His first position after graduating was as a mathematics teacher, and he later worked as an examiner in the patents office.

In later years he proved to be one of the greatest thinkers of all time. He offered explanations for many observed physical phenomena that were previously unexplained, and predicted many useful results through mathematics that ultimately led to new discoveries. Einstein published three great papers that changed theoretical physics.

Einstein deduced the equation $E = mc^2$ where E represents energy, m mass and c the velocity of light, in 1905 in a paper on the special theory of relativity. This equation indicated the enormous energy content of small masses, and the validity of his deduction was demonstrated in 1945 by the enormous energy in atomic explosions.

In 1933 Einstein renounced his German citizenship and moved to Princeton in the USA. He spent the last 20 years of his life working there, serving as an inspiration to other scientists around him, and campaigning for the international control of atomic power.

- a** How old would Albert Einstein be today?

b Why did Einstein have difficulty at school?

c How can so much energy be released from something as small as an atom?

d Einstein believed that imagination is more important than knowledge. What do you think he might have meant by this?
- Insert vowels to complete these terms.

a s _ l _ t _ _ n **b** s _ m _ lt _ n _ _ _ s **c** _ q _ _ t _ _ n

d v _ r _ _ bl _ **a** s _ bst _ t _ t _ _ n **f** s _ lv _
- Write a paragraph describing the methods of solving simultaneous equations.

Terms

consecutive solution	elimination solve	equation substitution	linear unknown	pronumeral value	simultaneous variable
-------------------------	----------------------	--------------------------	-------------------	---------------------	--------------------------

Check your skills

- The solution to $3x - 5 = -6$ is:

A $x = -\frac{1}{3}$ **B** $x = \frac{1}{3}$ **C** $x = -\frac{11}{3}$ **D** $x = \frac{11}{3}$
- Given $y = 7 - 2x$ and $y = -8$, x equals:

A $7\frac{1}{2}$ **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$ **D** $-7\frac{1}{2}$

3 The solution to $8 - 5x = 2x + 3$ is:

A $x = \frac{7}{5}$

B $x = \frac{5}{7}$

C $x = \frac{5}{3}$

D $x = -\frac{5}{3}$

4 $x = -2$ is not a solution of:

A $3x + 5 = 3 + 2x$

B $7 - 2x = 3x + 17$

C $4x + 1 = 7 - 5x$

D $6x + 4 = 2x - 4$

5 The solution to the simultaneous equations $y = x - 3$ and $5x - 2y = 27$ is:

A (5, 2)

B (6, 3)

C (6, 5)

D (7, 4)

6 The pair of simultaneous equations that has a solution of $x = -5$ and $y = 3$ is:

A $y = x + 8$

B $y = x + 8$

C $x + y = 2$

D $y - x = 8$

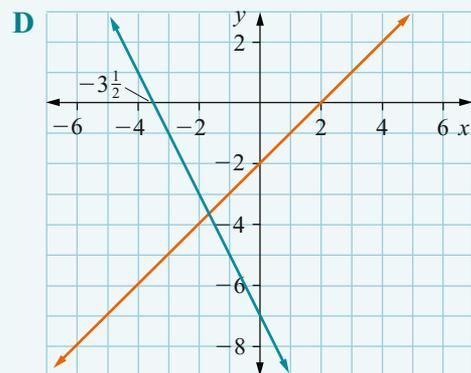
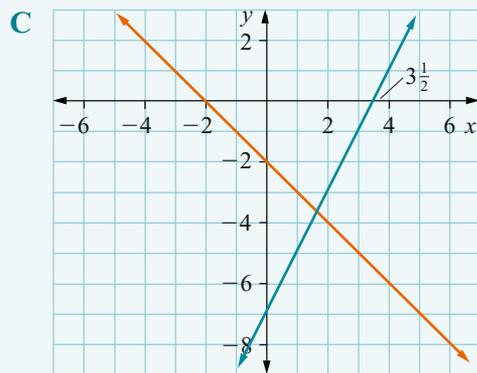
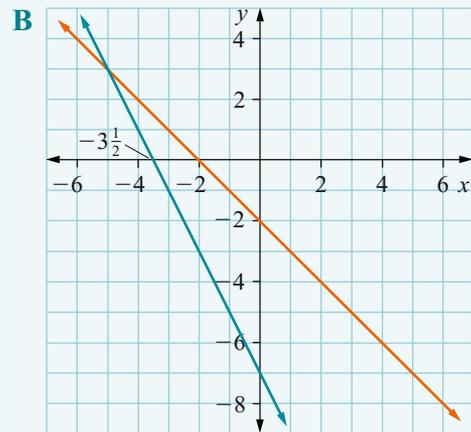
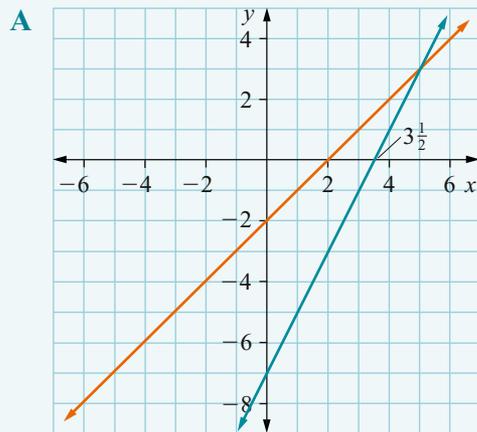
$2x - 3y = 16$

$3x - 2y = -21$

$5x + y = 28$

$4x - 3y = 26$

7 The diagram showing the graphical solution of $y = x - 2$ and $2x - y = 7$ is:



8 When solved by substituting $y = x + 3$ into $3x - 5y = -11$, the x -value is:

A -3

B -2

C -1

D 0

9 When solving $5x - 3y = 8$ and $2x + y = 4$ by elimination, the first line after eliminating y would be:

A $7x - 2y = 12$

B $7x = 12$

C $x = 4$

D $11x = 20$

10 Two numbers that have a sum of 20 and a difference of 4 are:

A 8 and 12

B 10 and 10

C 16 and 12

D 16 and 4

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1-4	5, 6	7	8	9	10
Section	A	B	C	D	E	F

6A Review set

- 1 Solve these equations for x .
- a** $2x + 5 = -3$ **b** $4x = 2x + 11$ **c** $\frac{x}{3} = \frac{1}{6}$
d $3(2x + 1) = 4$ **e** $2(3 - x) = 3(x + 5)$ **f** $4 - 3(2 - x) = 7$
- 2 Find y when $x = -3$, given that $y = \frac{4 - 2x}{5}$.
- 3 **a** Find y when $x = 1$ and $2x - 3y = -7$.
b What equation results when $8x - 3y = -1$ is multiplied by 3?
c What equation results when $3x - 5y = -1$ and $7x + 5y = 21$ are added?
- 4 On the same set of axes, draw accurate graphs of the lines with equations $y = 2x - 3$ and $y = 2 - 3x$.
a Use your graph to solve $y = 2x - 3$ and $y = 2 - 3x$ simultaneously.
b Check your answer to part **a** by using the substitution method.
- 5 **a** Solve $y = 2x + 1$ and $y = -x - 5$ simultaneously.
b Are the lines with equations $y = 2x + 1$, $y = -x - 5$ and $y = -3x - 9$ concurrent? (Three or more lines are concurrent if they all meet at a common point of intersection.)
- 6 Solve the following problems.
a John is two years older than Paula. Three times John's age plus four times Paula's age is 55 years. How old are John and Paula?
b In my purse I have 21 coins, all of which are either 5-cent or 20-cent coins. Their total value is \$2.25. How many 20-cent coins do I have?

6B Review set

- 1 Solve these equations for x .
- a** $3x - 1 = 2$ **b** $6x = 11 - 3x$ **c** $4 - 3x = 2x + 1$
d $2(4x + 1) = 3$ **e** $3(5 - 2x) = 5(x - 3)$ **f** $5 - 2(5 - 2x) = 3$
- 2 Find t when $a = -2$, given that $t = \frac{5a - 4}{3}$.
- 3 **a** Find y when $x = 2$ and $3x - 5y = 7$.
b What equation results when $7x - 2y = 8$ is multiplied by 4?
c What equation results when $2x - 7y = 3$ and $5x + 7y = -7$ are added?
- 4 On the same set of axes, draw accurate graphs of the lines with equations $y = 3x - 2$ and $y = 8 - 2x$.
a Use your graph to solve $y = 3x - 2$ and $y = 8 - 2x$ simultaneously.
b Check your answer to part **a** by using the substitution method.
- 5 Use the elimination method to solve $3x - 5y = 21$ and $5x - 4y = 22$ simultaneously.
- 6 Solve these problems.
a The sum of two numbers is 10. When one number is added to twice the other, the result is 16. Find the numbers.
b If I have 10-cent and 20-cent coins only with a total value of \$6.90 and I have 36 coins altogether, how many 10-cent coins do I have?
- 7 Explain why the pair of simultaneous equations $x + 2y = 6$ and $x + 2y = 8$ has no solution.

6C

Review set

1 Solve these equations.

a $4x + 5 = 12$

b $5 - 3x = -7$

c $3x - 2 = x + 6$

d $-3(2a + 5) = 15$

e $2(3a - 4) = a + 9$

f $3(2p - 3) = 4(p + 1)$

2 Consider $y = \frac{2x - 3}{5}$.

a Find y when $x = \frac{1}{2}$.

b Find x when $y = 3$.

3 a What equation results when $3x - 7y = 5$ is multiplied by -4 ?

b At what point does the line $y = 3x - 4$ meet the y -axis?

c Find y when $x = -5$, given that $2x - y = 8$.

d What equation results when $3x - 4y = 6$ and $2x + 4y = 14$ are added?

4 a On the same set of axes graph the lines with equations $y = x + 5$ and $y = -4x - 5$.

b Hence solve the equations $y = x + 5$ and $y = -4x - 5$ simultaneously.

5 a Solve $x = 5 - y$ and $3x - 7y = -1$ simultaneously using the substitution method.

b Solve $3x - 2y = 12$ and $4x + 3y = -1$ simultaneously using the elimination method.

6 a Two chocolate frogs and one lollipop cost a total of 78 cents, and three chocolate frogs and two lollipops cost \$1.32. Find the cost if I buy one chocolate frog and one lollipop.

b Four cricket balls and three tennis balls weigh a total of 1060 g. Two cricket balls and five tennis balls weigh 740 g. Find the weight of each type of ball.

6D

Review set

1 Solve these equations.

a $3x - 2 = 5$

b $4 - 7x = 3$

c $4x - 7 = 3x + 5$

d $-2(4x + 3) = 15$

e $4(3t + 1) = t - 2$

f $4(3p - 7) = 5(1 - p)$

2 Given that $y = \frac{3x - 5}{3}$, find:

a y when $x = \frac{1}{3}$

b x when $y = \frac{7}{3}$

3 a What equation results when $2x - 5y = 11$ is multiplied by 3?

b At what point does the line $y = 3x - 5$ cut the y -axis?

c Find y when $x = -3$, given $3x - 2y = 11$.

d What equation results when $7x - 3y = 8$ and $5x + 3y = 7$ are added?

4 a Graph the lines with equations $y = x - 3$ and $y = 2x - 8$ on the same set of axes.

b Solve $y = x - 3$ and $y = 2x - 8$ simultaneously.

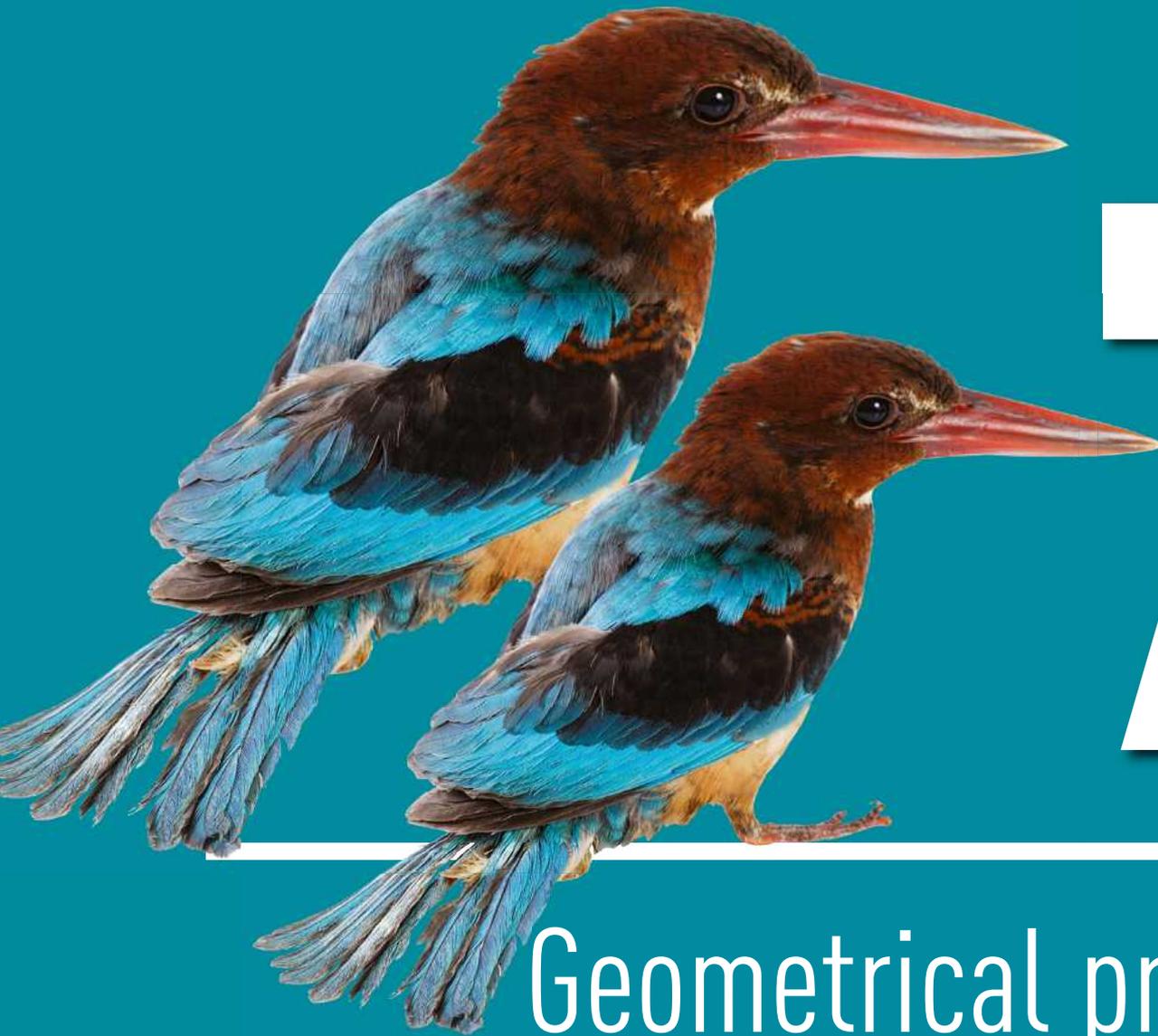
c Solve simultaneously using the substitution method to check your answer.

5 Solve $5x - 3y = 29$ and $2x + y = 5$ simultaneously by the elimination method.

6 Solve these problems.

a I wish to blend brand A coffee at \$7 per kg with brand B coffee at \$11 per kg. If the total weight is 10 kg, and the total cost is \$86, how many kilograms of each coffee do I use?

b In 5 years time Pam will be twice as old as Sam was two years ago. Pam is 8 years older than Sam. How old is Sam?



7

Geometrical proofs

This chapter deals with the use of logical reasoning, including congruence and similarity, to complete numerical and theoretical exercises involving geometrical figures.

At the end of this chapter you should be able to:

- ▶ formulate proofs involving congruent triangles and angle properties
- ▶ explain similarity and develop the minimum conditions for triangles to be similar
- ▶ apply logical reasoning, including the use of congruence and similarity, to proofs and numeral exercises involving plane shapes
- ▶ find the sum of the interior and exterior angles of polygons
- ▶ find the size of interior and exterior angles of regular polygons.

NSW Syllabus references: 5.2 M&G Properties of geometrical figures

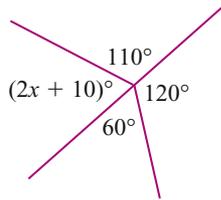
Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-14MG

Measurement & geometry – ACMMG220, ACMMG243, ACMMG244



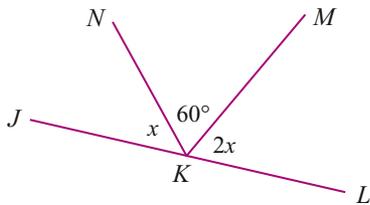
Diagnostic test

- 1 The value of x is:



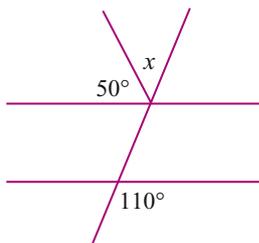
- A 25 B 30 C 50 D 55

- 2 $\angle MKL$ equals:



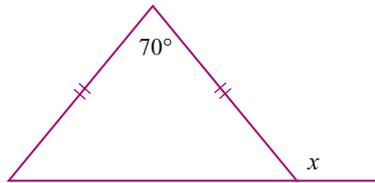
- A 40° B 80° C 90° D 120°

- 3 The value of x is:



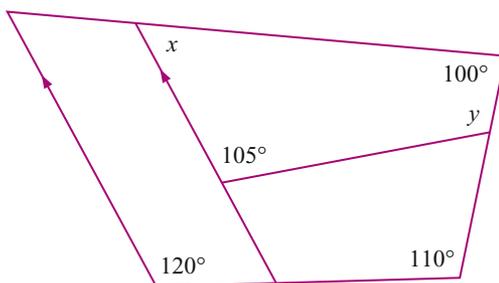
- A 60° B 70° C 80° D 110°

- 4 The value of x is:



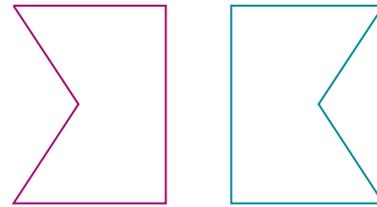
- A 55° B 70° C 125° D 140°

- 5 The values of x and y in this diagram are:



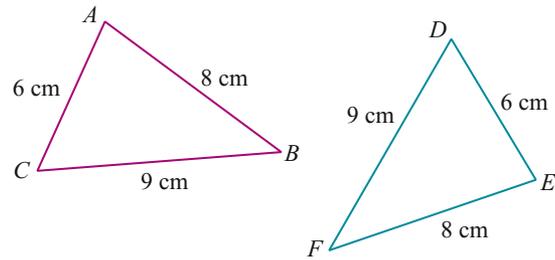
- A $x = 45^\circ, y = 110^\circ$ B $x = 45^\circ, y = 125^\circ$
 C $x = 30^\circ, y = 110^\circ$ D $x = 30^\circ, y = 125^\circ$

- 6 The transformation that could not have been used to produce this pair of congruent figures is:



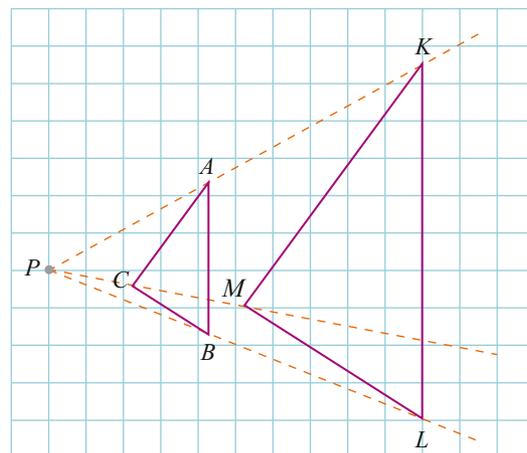
- A reflection
 B reflection and translation
 C rotation
 D translation

- 7 The triangles shown below are congruent. The correct statement of congruency is:



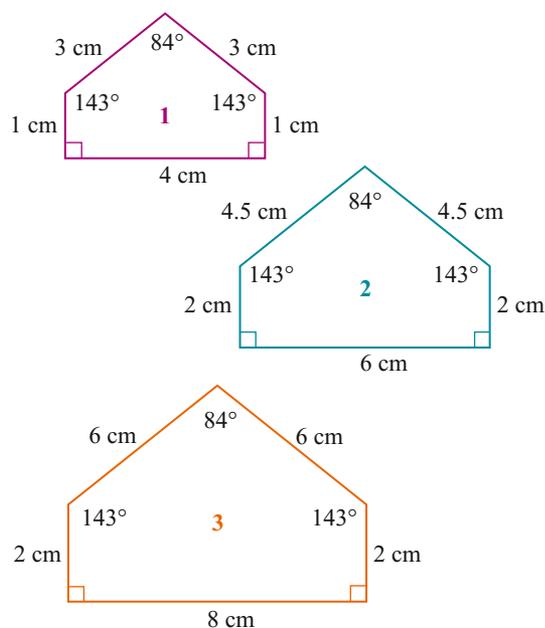
- A $\triangle ABC \equiv \triangle DEF$
 B $\triangle ABC \equiv \triangle EFD$
 C $\triangle ABC \equiv \triangle FDE$
 D $\triangle CBA \equiv \triangle DEF$

- 8 $\triangle ABC$ has been enlarged to $\triangle KLM$ using P as the centre of enlargement. $PA = 24$ mm and $AK = 32$ mm. The enlargement factor is:



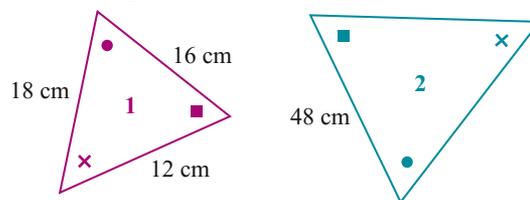
- A $\frac{4}{3}$ B $\frac{3}{4}$ C $\frac{7}{3}$ D $\frac{3}{7}$

9 The figures that are similar are:



- A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3

10 These triangles are similar.



The enlargement factor from triangle 1 to triangle 2 is:

- A $2\frac{2}{3}$ B 3 C 4 D 4.8

11 Which of the following statements is not true?

- A Congruent figures are identical in all respects except for position.
B Congruent figures have the same shape and the same size.
C Similar figures have the same size but different shape.
D Similar figures have the same shape but not necessarily the same size.

The diagnostic test questions refer to outcomes ACMMG141, AMMG163, AMMG164, AMMG166, AMMG200 and AMMG220.

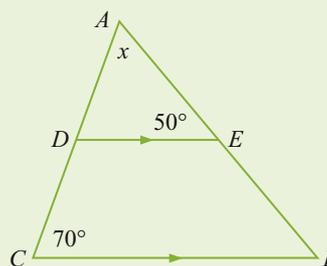
A

Reasoning in geometry

In this section geometrical facts, properties and relationships are applied to proofs and numerical exercises involving diagrams and plane shapes, with appropriate reasons.

EXAMPLE 1

Find x , giving reasons.



Solve/Think

$$\begin{aligned}\angle ADE &= \angle ACB = 70^\circ && \text{(Corresponding angles and } DE \parallel CB.) \\ x + 70^\circ + 50^\circ &= 180^\circ && \text{(Angle sum of a triangle is } 180^\circ.) \\ x + 120^\circ &= 180^\circ \\ x &= 60^\circ\end{aligned}$$

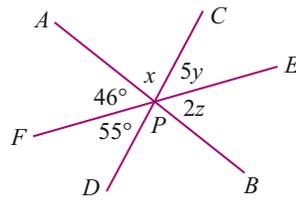
Apply

The diagram contains triangles and parallel lines. Look for appropriate properties of these to use.

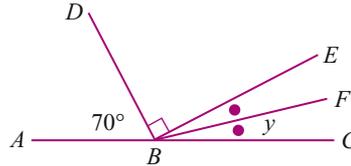
Exercise 7A

1 Complete the following to find the values of the pronumerals in the following diagrams, giving reasons.

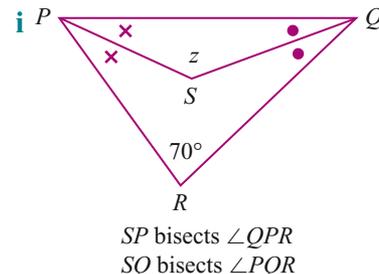
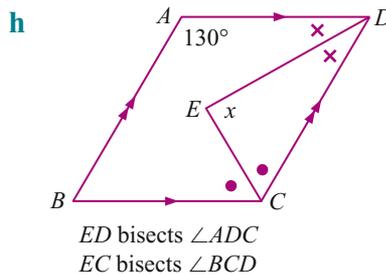
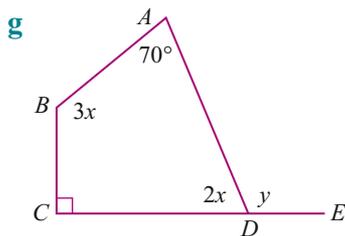
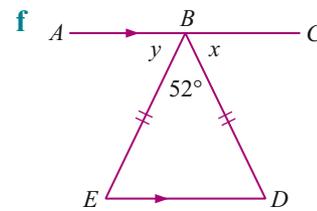
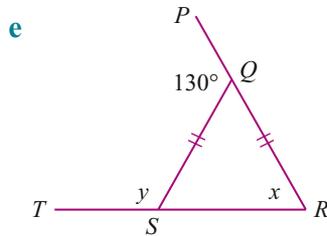
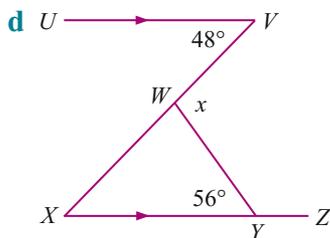
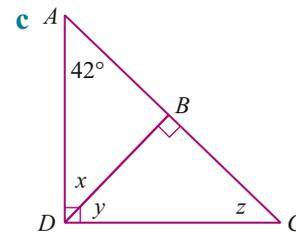
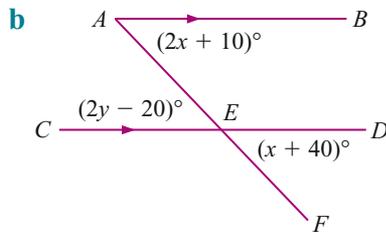
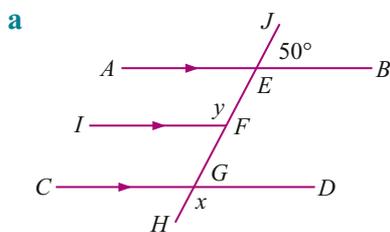
- a $5y = 55^\circ$ ()
 $y = \underline{\hspace{2cm}}$
 $2z = \underline{\hspace{2cm}}$ ()
 $z = \underline{\hspace{2cm}}$
 $x + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 180^\circ$ ()
 $x = \underline{\hspace{2cm}}$



- b BF bisects $\angle EBC$.
 $2y + \underline{\hspace{2cm}} + 90 + 70 = \underline{\hspace{2cm}}$ ()
 $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $y = \underline{\hspace{2cm}}$

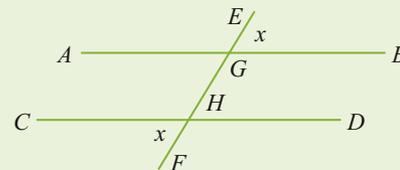


2 Find the values of the pronumerals in the following diagrams, giving reasons.



EXAMPLE 2

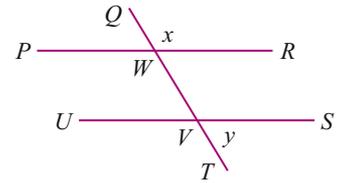
Given that $\angle EGB = \angle CHF$, prove that $AB \parallel CD$.



Solve/Think	Apply
$\angle AGH = \angle EGB = x$ (vertically opposite angles) Hence $\angle CHF = \angle AGH = x$ $\therefore AB \parallel CD$ (corresponding angles on lines AB and CD)	Use the properties of intersecting lines and parallel lines.

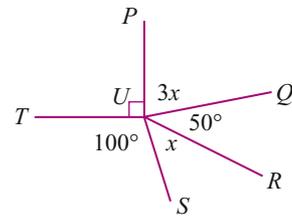
- 3 a** Given $x + y = 180^\circ$, complete the following to prove that $PR \perp US$.

$$\begin{aligned} \angle PWV &= \angle QWR = \underline{\hspace{2cm}} && (\underline{\hspace{2cm}}) \\ \angle UVW &= \angle \underline{\hspace{2cm}} = \underline{\hspace{2cm}} && (\underline{\hspace{2cm}}) \\ \angle PWV + \angle UVW &= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} && (\text{given}) \\ \therefore PR &\perp US && (\underline{\hspace{2cm}}) \end{aligned}$$

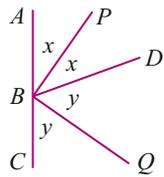


- b** Complete the following to prove that TUQ is a straight line.

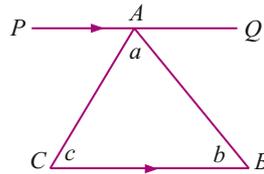
$$\begin{aligned} 3x + 50 + x + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} && (\underline{\hspace{2cm}}) \\ \underline{\hspace{2cm}} + \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ x &= \underline{\hspace{2cm}} \\ \angle PUQ &= 3 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \\ \therefore TUQ &\text{ is a straight line.} && (\underline{\hspace{2cm}}) \end{aligned}$$



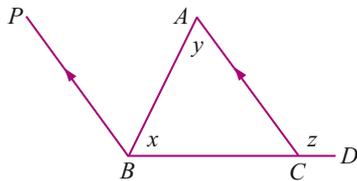
- 4 a** ABC is a straight line. PB bisects $\angle ABD$. QB bisects $\angle DBC$. Prove that $\angle PBQ = 90^\circ$.



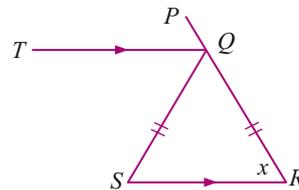
- b** $PQ \perp CB$. Prove $a + b + c = 180^\circ$; that is, the sum of the interior angles of a triangle is 180° .



- c** $BP \perp CA$. Prove that $z = x + y$.



- d** $QR = QS$ and $TQ \perp SR$. Prove that TQ bisects $\angle PQS$.



B Congruent triangles

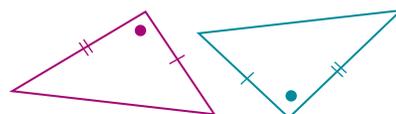
Congruent triangles have the same size and shape; that is, they have three pairs of equal matching sides and three pairs of equal matching angles. They are formed by performing one or more of the transformations translation, rotation or reflection. It is not necessary to show that all matching sides and angles are equal to prove congruency. The four sets of minimum conditions used are known as the congruency tests for triangles.

Tests for congruent triangles

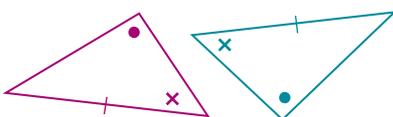
- 1** If three sides of one triangle are equal to three sides of another triangle, then the two triangles are congruent. (SSS)



- 2** If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent. (SAS)



- 3** If two angles and one side of a triangle are equal to two angles and the matching side of another triangle, then the triangles are congruent. (AAS)

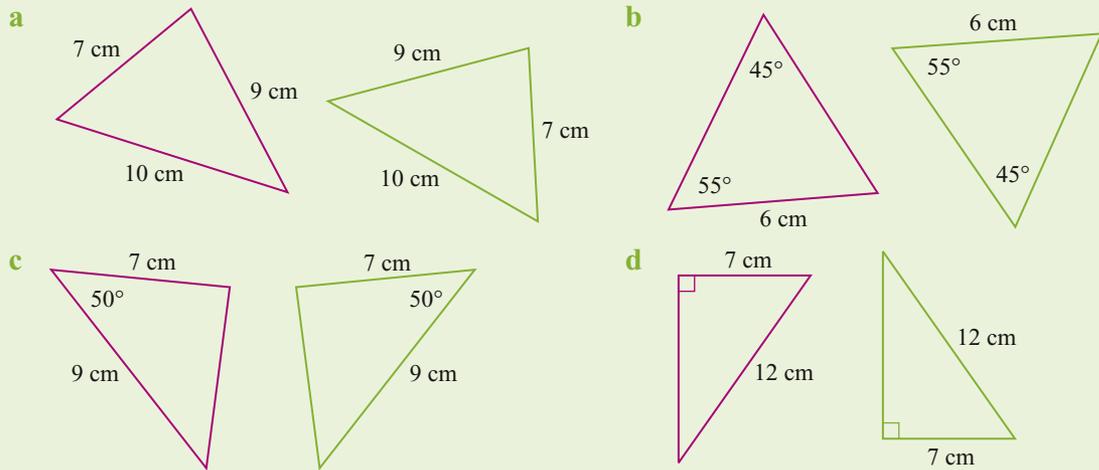


- 4** If the hypotenuse and a second side of a right-angled triangle are equal to the hypotenuse and a second side of another, then the triangles are congruent. (RHS)



EXAMPLE 1

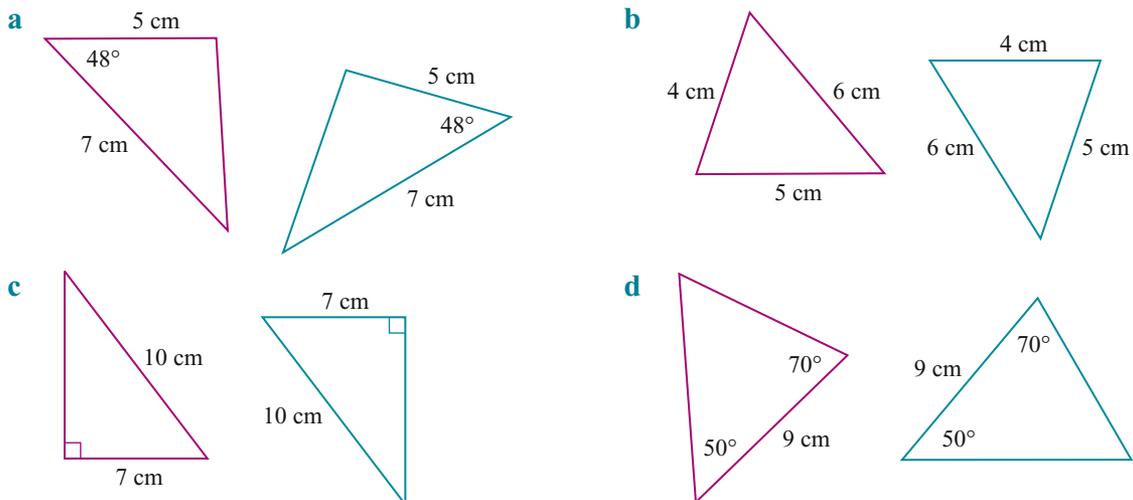
State the congruency test used to show that the following pairs of triangles are congruent.

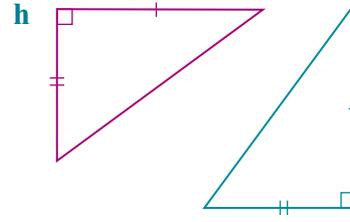
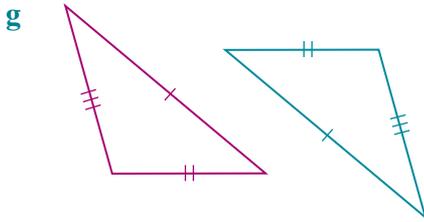
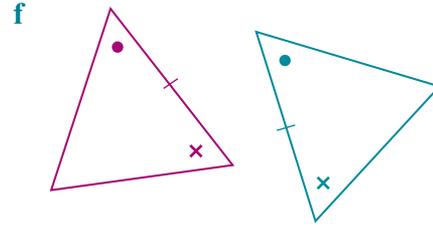
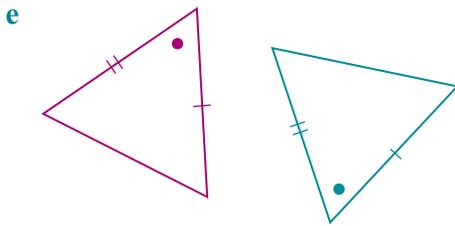


	Solve	Think	Apply
a	SSS	There are three pairs of equal sides.	Look for pairs of equal matching sides and angles that will satisfy one of the congruency tests.
b	AAS	There are two pairs of equal angles and the pair of equal sides are matching sides.	
c	SAS	There are two pairs of equal sides and the angles included by these sides are equal.	
d	RHS	The hypotenuse and a second side of the first right-angled triangle are equal to the hypotenuse and a second side of the second right-angled triangle.	

Exercise 7B

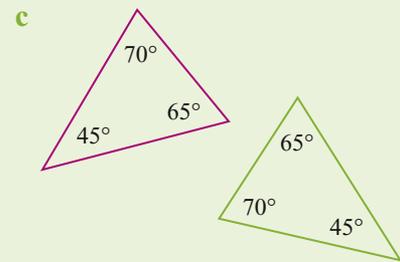
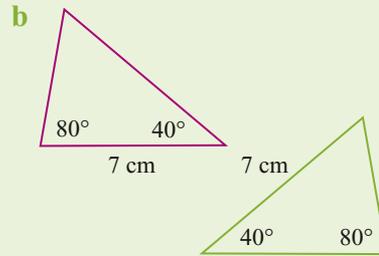
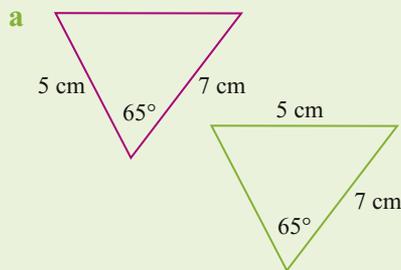
- 1 State the congruency test used to show that the following pairs of triangles are congruent. (Diagrams are not drawn to scale.)





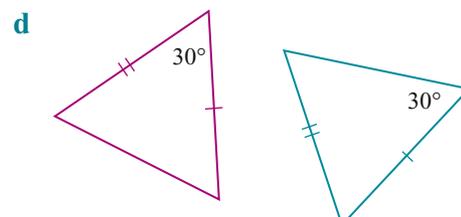
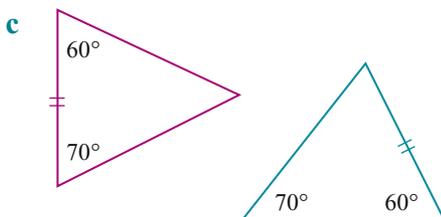
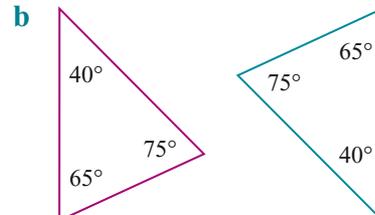
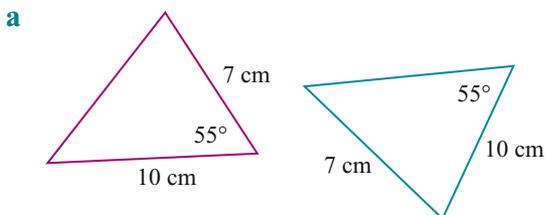
EXAMPLE 2

State why the following pairs of triangles are not congruent.

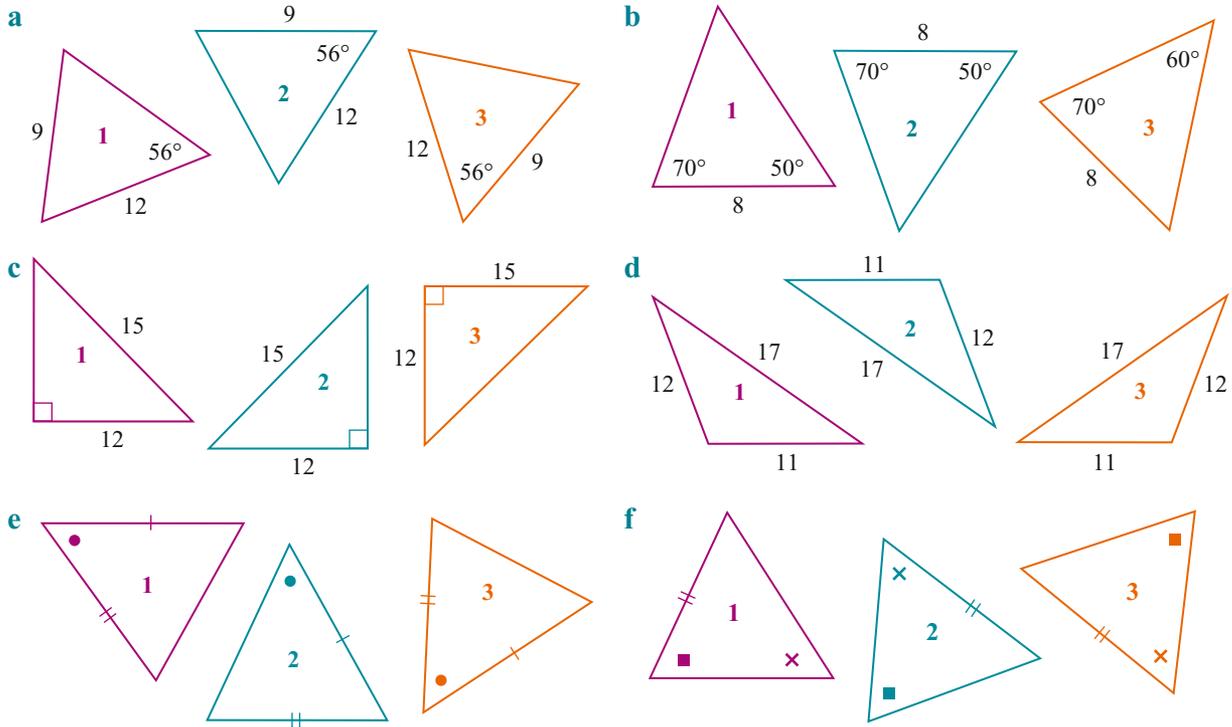


	Solve/Think	Apply
a	The 65° angle in the second triangle is not the included angle.	For the SAS test to apply, both the angles must be included by the pairs of equal sides.
b	The equal sides are not matching sides in the triangles.	For the AAS test to apply, the pair of equal sides must be matching (or corresponding) sides.
c	Three pairs of equal angles do not necessarily make the triangles congruent. These triangles have the same shape but not necessarily the same size.	If two triangles have three pairs of equal angles, then they have the same shape but not necessarily the same size.

2 State why the following pairs of triangles are not congruent.

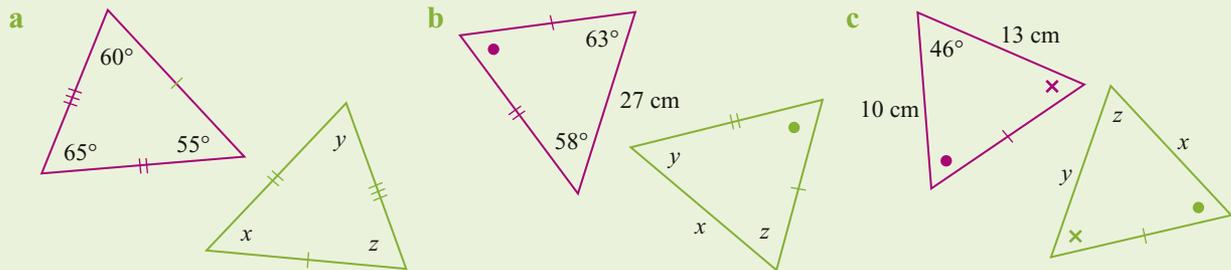


3 Which triangles are congruent? Give a reason. (Diagrams are not to scale and all lengths are in centimetres.)



EXAMPLE 3

State why the following pairs of triangles are congruent and find the values of the pronumerals.



	Solve	Think	Apply
a	SSS: $x = 55^\circ, y = 65^\circ, z = 60^\circ$	There are three pairs of equal sides. Matching angles are opposite matching sides; hence, x and 55° , y and 65° , and z and 60° are pairs of matching angles.	Use one of the congruency tests to show that the triangles are congruent. The remaining pairs of matching angles and sides are then equal.
b	SAS: $x = 27 \text{ cm}, y = 58^\circ, z = 63^\circ$	There are two pairs of equal sides and the angles included by these sides are equal. x and 27 cm are a pair of matching sides, and y and 58° , and z and 63° are pairs of matching angles.	
c	AAS: $x = 10 \text{ cm}, y = 13 \text{ cm}, z = 46^\circ$	There are two pairs of equal angles and the pair of equal sides are matching sides.	

4 State why the following pairs of triangles are congruent and find the values of the pronumerals.

a

b

c

d

e

f

g

h

C

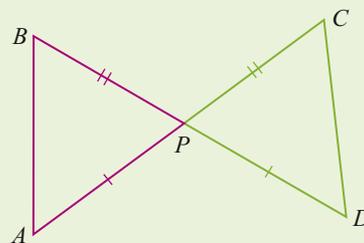
Formal proofs of congruent triangles

EXAMPLE 1

AC and BD are straight lines that intersect at P such that $AP = PD$ and $BP = PC$.

Prove that:

- $\triangle ABP \equiv \triangle DCP$
- $AB = DC$
- $\angle BAP = \angle CDP$



\equiv is the symbol for congruency. It is read as 'is congruent to'.

	Solve	Think	Apply
a	$AP = DP$ (given) $BP = CP$ (given) $\angle APB = \angle DPC$ (Vertically opposite angles are equal.) Hence $\triangle ABP \equiv \triangle DCP$ (SAS)	Data is the information given about the triangles, either on the diagram or listed. There are two pairs of equal sides and the angles included by these sides are equal.	Look for pairs of equal matching sides and angles that will satisfy one of the congruency tests.
b	$AB = DC$ (Matching sides in congruent \triangle s are equal.)		
c	$\angle BAP = \angle CDP$ (Matching angles in congruent \triangle s are equal.)		

Exercise 7C

1 Complete the following to prove that:

a $\triangle ACD \equiv \triangle ACB$

In $\triangle ACD$ and $\triangle ACB$:

$AD = \underline{\hspace{2cm}}$ (given)

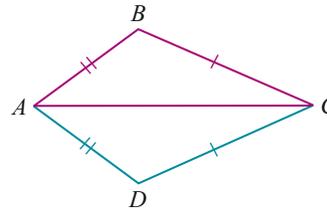
$DC = \underline{\hspace{2cm}}$ ()

AC is .

$\therefore \triangle ACD \equiv \triangle ACB$ ()

b $\angle DAC = \angle BAC$ ()

c $\angle ADC = \angle ABC$ ()



2 Complete the following to prove that:

a $\triangle ABP \equiv \triangle DCP$

In $\triangle ABP$ and $\triangle DCP$:

$\angle BAP = \underline{\hspace{2cm}}$ (given)

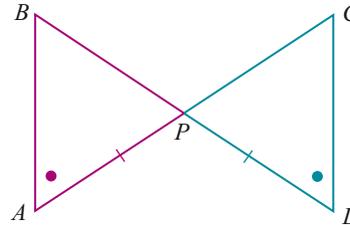
$\angle APB = \underline{\hspace{2cm}}$ ()

$AP = \underline{\hspace{2cm}}$ (given)

$\therefore \triangle ABP \equiv \triangle DCP$ ()

b $BP = CP$ ()

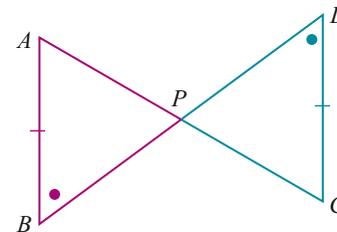
c $AB = \underline{\hspace{2cm}}$ ()



3 Prove the following.

a $\triangle ABP \equiv \triangle CDP$

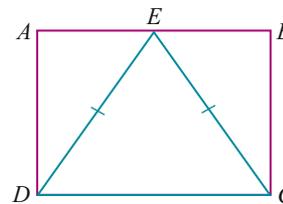
b $AP = CP$



4 $ABCD$ is a rectangle and $DE = CE$. Prove the following.

a $\triangle AED \equiv \triangle BEC$

b E is the midpoint of AB (that is $AE = BE$).

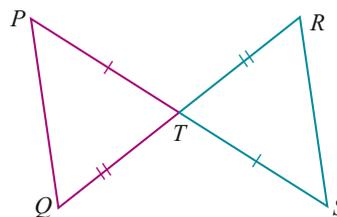


5 Prove the following.

a $\triangle PTQ \equiv \triangle STR$

b $PQ = SR$

c $\angle PQT = \angle SRT$



6 PQS is an isosceles triangle with $PS = PQ$ and $PR \perp SQ$.

Prove the following.

a $\triangle PRS \equiv \triangle PRQ$

b $\angle PSR = \angle PQR$

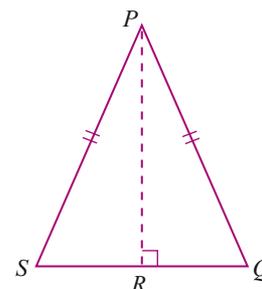
c $SR = QR$

d $\angle SPR = \angle QPR$

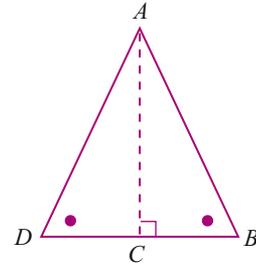
e Complete the following two properties of isosceles triangles which have been proven above.

i The angles opposite the equal sides of an isosceles triangle .

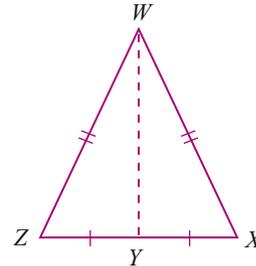
ii A line drawn from the vertex of an isosceles triangle and perpendicular to its base bisects the and the of the triangle.



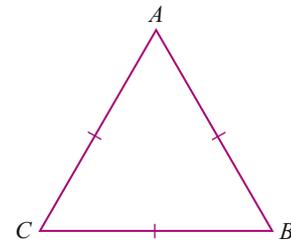
- 7** $\angle ADC = \angle ABC$ and $AC \perp DB$. Prove the following.
- $\triangle ACD \equiv \triangle ACB$
 - $\triangle ABD$ is an isosceles triangle.
 - Complete the following: If two angles of a triangle are equal, then the sides opposite these angles are ____; that is, it is an ____ triangle.



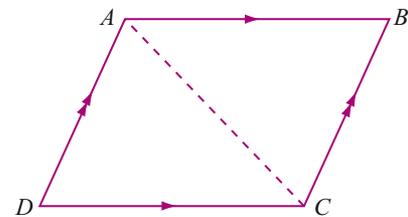
- 8** $WZ = WX$ and Y is the midpoint of ZX ($ZY = YX$). Prove the following.
- $\triangle WZY \equiv \triangle WXY$
 - $\angle WYZ = \angle WYX$
 - $\angle WYZ = 90^\circ$ and $\angle WYX = 90^\circ$
 - $\angle ZWY = \angle XWY$
 - Complete: A line drawn from the vertex of an isosceles triangle to the midpoint of its base is ____ to the base and ____ the angle at the vertex.



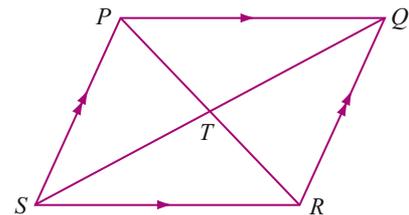
- 9** An equilateral triangle is one with three sides of equal length.
- Are all equilateral triangles isosceles triangles?
 - $\triangle ABC$ is an equilateral triangle. Using the properties of isosceles triangles, complete the following.
 $AC = AB$ so $\angle C = \angle$ ____.
 $AB = BC$ so $\angle C = \angle$ ____.
Hence $\angle C = \angle$ ____ = \angle ____; that is, the three angles of an equilateral triangle are ____.
 - Prove that each angle of an equilateral triangle is 60° .



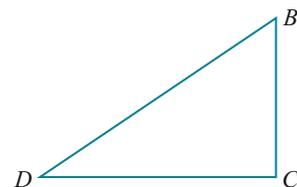
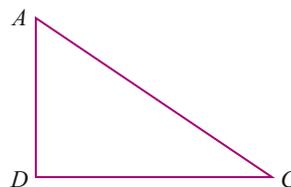
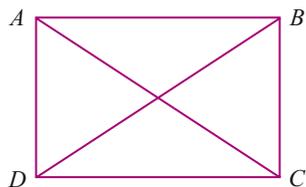
- 10** $ABCD$ is a parallelogram. ($AB \parallel DC$ and $AD \parallel BC$)
Prove the following.
- $\triangle ABC \equiv \triangle CDA$
 - $AB = DC$
 - $AD = BC$
 - $\angle B = \angle D$
 - Complete:
 - The opposite sides of a parallelogram ____.
 - The opposite angles of a parallelogram ____.



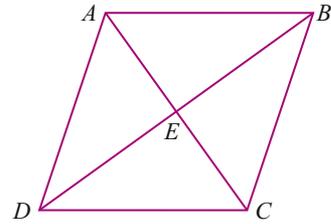
- 11** $PQRS$ is a parallelogram. The diagonals intersect at T .
Using the result of question 10, prove the following.
- $\triangle PQT \equiv \triangle RST$
 - $PT = RT$
 - $QT = ST$
 - Complete: The diagonals of a parallelogram ____.



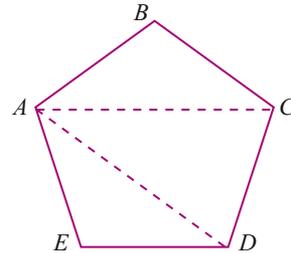
- 12** The diagram shows the rectangle $ABCD$ with $\triangle ADC$ and $\triangle BCD$ drawn separately. Prove the following.
- $\triangle ADC \equiv \triangle BCD$
 - $AC = BD$
 - Complete: The diagonals of a rectangle are ____.



- 13** $ABCD$ is a rhombus (a parallelogram with all sides equal).
The diagonals intersect at E . Prove the following.
- Using the result from question 11 part d, $\triangle ADE \equiv \triangle ABE$.
 - $\angle AED = \angle AEB = 90^\circ$
 - $\angle DAE = \angle BAE$
 - Complete:
 - The diagonals of a rhombus intersect at _____.
 - The diagonals of a rhombus _____ the angles of the rhombus.



- 14** $ABCDE$ is a regular pentagon (that is all sides and angles equal).
- Prove that $AC = AD$.
 - Hence prove that all the diagonals are equal in length.



D Similar triangles

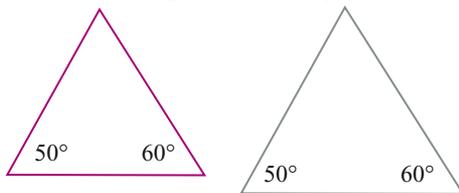
Two triangles are congruent if they have exactly the same shape and size.
If the shape is identical, but not the size, two triangles are similar.

'In proportion' means
'in the same ratio'. !

If two triangles are similar, then they have three pairs of angles that are equal and three pairs of matching sides that are in proportion. Conversely, if two triangles have three pairs of angles that are equal and three pairs of matching sides that are in proportion, then the triangles are similar.

Investigation 1 Similar triangles

- 1 a** Using a protractor, draw two equiangular triangles (two triangles with two pairs of equal angles).
Note that if two pairs of angles are equal, the third pair must also be equal. An example is shown.



- Measure the lengths of the sides of the triangles to the nearest millimetre.
- Complete the statements:
 - $\frac{\text{longest side of the large triangle}}{\text{longest side of the small triangle}} = \underline{\hspace{2cm}}$
 - $\frac{\text{shortest side of the large triangle}}{\text{shortest side of the small triangle}} = \underline{\hspace{2cm}}$
 - $\frac{\text{other side of the large triangle}}{\text{other side of the small triangle}} = \underline{\hspace{2cm}}$
- If two triangles are equiangular, are the lengths of their matching sides in proportion?
- If two triangles are equiangular, are they similar?

The matching sides in similar triangles
are opposite pairs of equal angles. !



- 2** a Using a ruler and a pair of compasses, draw a triangle with sides 3.5 cm, 4.5 cm and 5.5 cm.
 b Draw another triangle with sides 7 cm, 9 cm and 11 cm.
 c Measure the angles of both triangles to the nearest degree.
 d What is the ratio of the lengths of the matching sides?
 e What is the enlargement factor?
 f If two triangles have their sides in the same ratio, are they equiangular?
 g If two triangles have their sides in the same ratio, are they similar?
- 3** a Draw a triangle with sides 3 cm and 4 cm, making the angle between these sides 50° .
 b Draw another triangle with sides 6 cm and 8 cm and the included angle 50° .
 c Measure the angles of both triangles to the nearest degree. Are these triangles equiangular?
 d Measure the length of the third side of each triangle. Are the lengths of the three pairs of matching sides in proportion?
 e Are these triangles similar?
 f Draw another triangle with sides 6 cm and 8 cm, and an angle that is not the included angle 50° .
 g By measuring the other angles and side of this triangle, determine whether or not it is similar to the first two triangles.
 h If two triangles have two pairs of sides in proportion and a pair of angles equal, are they similar?
- 4** a Draw a right-angled triangle with hypotenuse 6 cm and one other side 4 cm.
 b Draw another right-angled triangle with hypotenuse 9 cm and one other side 6 cm.
 c Measure the angles of both triangles. Are these triangles equiangular?
 d Measure the length of the third side of each triangle. Are the lengths of the three pairs of matching sides in proportion?
 e Are these triangles similar?

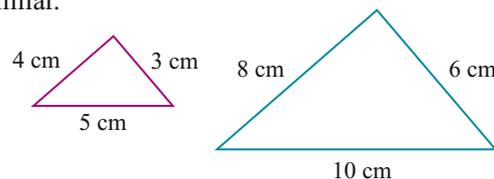
From Investigation 1 it can be seen that it is not necessary to show that there are three pairs of equal angles and three pairs of matching sides in proportion for two triangles to be similar. There are four sets of minimum conditions that can be used. These four tests for similar triangles are listed below.

Tests for similar triangles

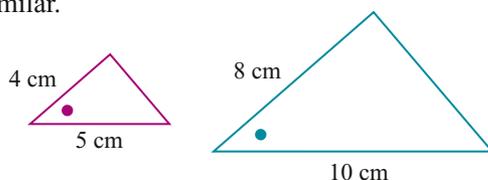
- 1** If two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.



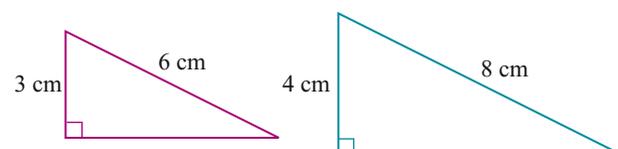
- 2** If the three sides of one triangle are proportional to three sides of another triangle, then the triangles are similar.



- 3** If two sides of one triangle are proportional to two sides of another triangle and the included angles are equal, then the triangles are similar.

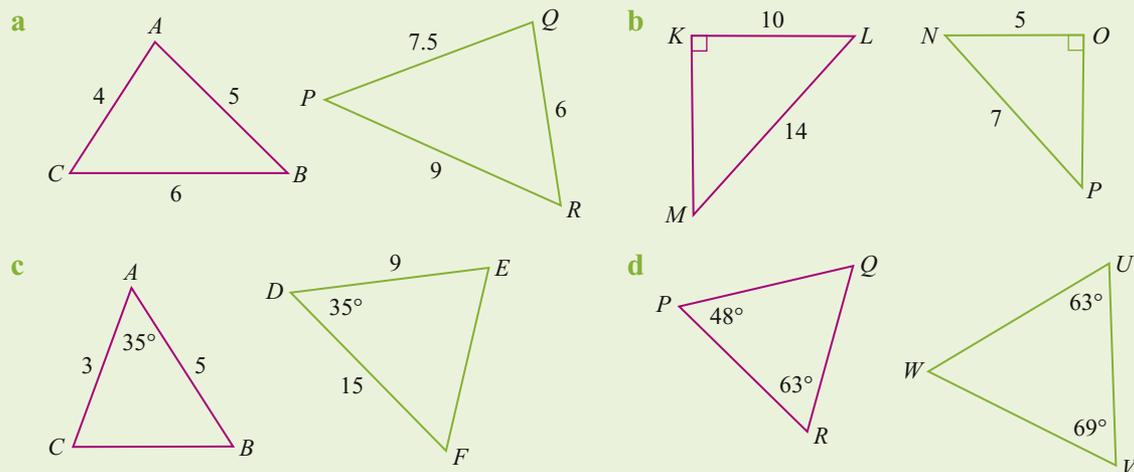


- 4** If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the triangles are similar.



EXAMPLE 1

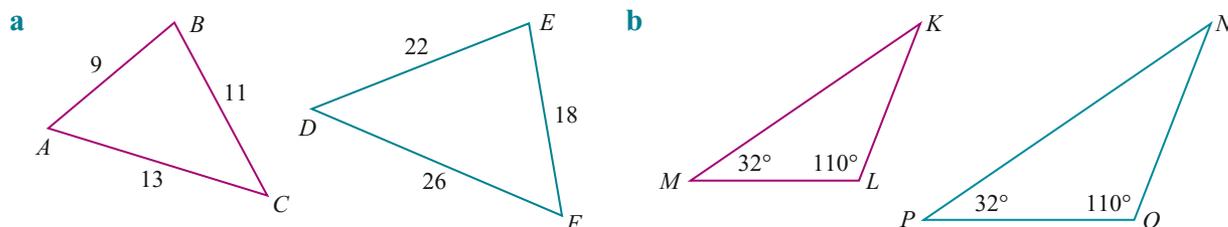
State the test used to show that the following pairs of triangles are similar. (Diagrams are not to scale and all lengths are in centimetres.)

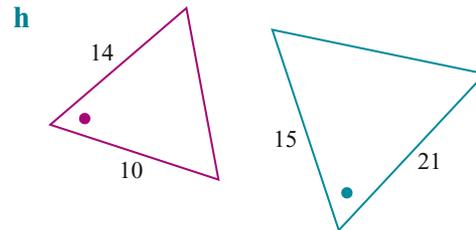
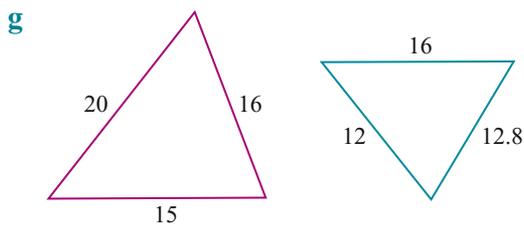
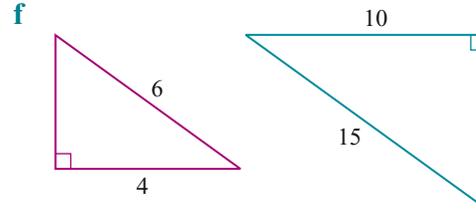
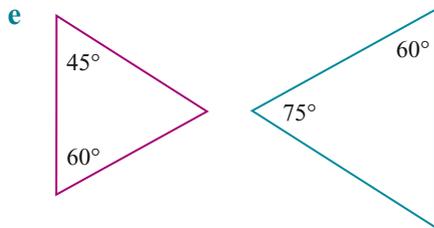
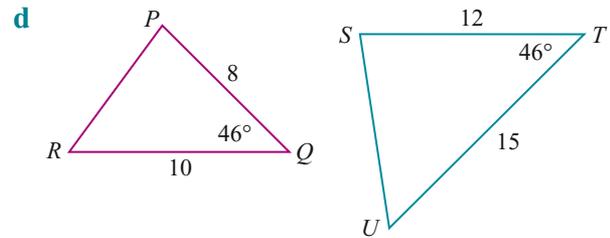
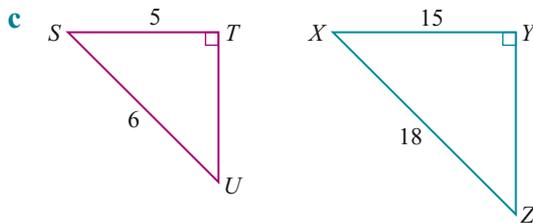


	Solve	Think	Apply
a	Test 2: There are three pairs of sides in proportion.	$\frac{PR}{BC} = \frac{QR}{AC} = \frac{PQ}{BA} = 1.5$ Start with the ratio of the longest sides in the two triangles, then the shortest sides, then the other pair of sides.	Look for pairs of equal angles and pairs of matching sides in proportion that will satisfy one of the tests for similarity.
b	Test 4: The hypotenuse and a second side of right-angled $\triangle ONP$ are proportional to the hypotenuse and a second side of right-angled $\triangle KLM$.	$\frac{ON}{KL} = \frac{NP}{LM} = 0.5$ $\angle O = \angle K = 90^\circ$	
c	Test 3: Two sides of $\triangle FDE$ are proportional to two sides of $\triangle BAC$ and the included angles are equal.	$\frac{DF}{AB} = \frac{DE}{AC} = 3$ $\angle FDE = \text{included } \angle BAC$.	
d	Test 1: Two angles of $\triangle PQR$ are equal to two angles of $\triangle UVW$.	$\angle U = \angle R$ $\angle W = 48^\circ$ (Angle sum of $\triangle UVW = 180^\circ$.) Hence $\angle W = \angle P$	

Exercise 7D

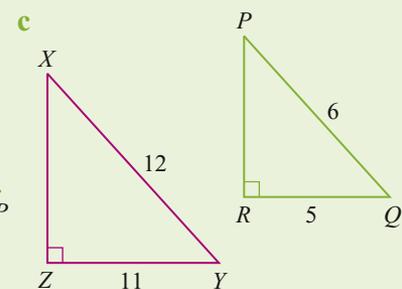
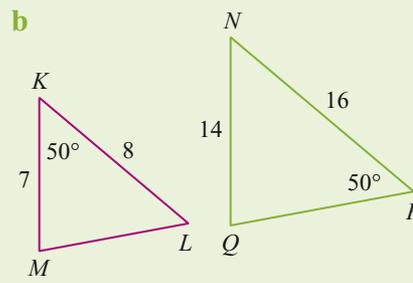
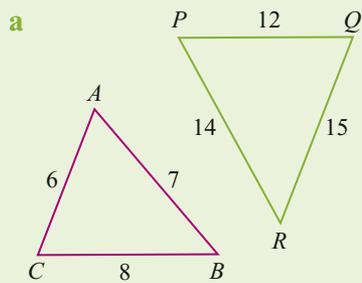
1 State the test used to show that the following pairs of triangles are similar. (All lengths are in centimetres.)





EXAMPLE 2

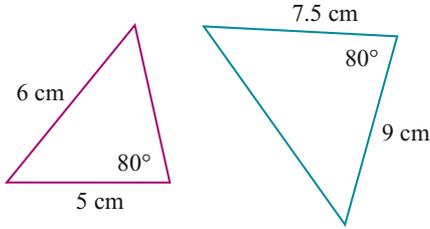
State why the following pairs of triangles are not similar. (Diagrams are not to scale and all lengths are in centimetres.)



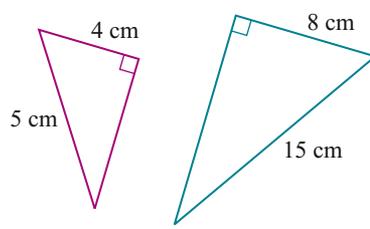
	Solve	Think	Apply
a	Only two pairs of sides are in proportion.	$\frac{PR}{AB} = \frac{PQ}{AC} = 2$ but $\frac{QR}{CB} = \frac{15}{8} \neq 2$	Check that all conditions of the test are satisfied.
b	$\angle P$ is not included by the sides in proportion.	$\frac{NP}{KL} = \frac{NQ}{KM} = 2$ but $\angle P$ is not included by the sides NP and NQ .	
c	The proportion of the second sides of the triangles is not the same as the proportion of the hypotenuses of the triangles.	$\angle R = \angle Z = 90^\circ$ $\frac{PQ}{XY} = \frac{6}{12} = 0.5$ but $\frac{RQ}{ZY} = \frac{5}{11} \neq 0.5$ $\therefore \frac{PQ}{XY} \neq \frac{RQ}{ZY}$	

2 State why the following pairs of triangles are not similar.

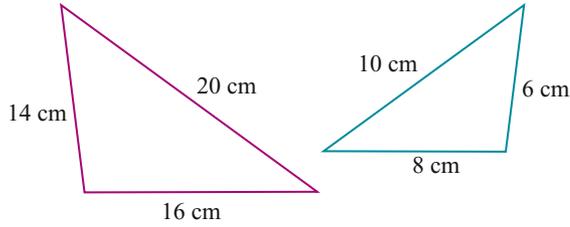
a



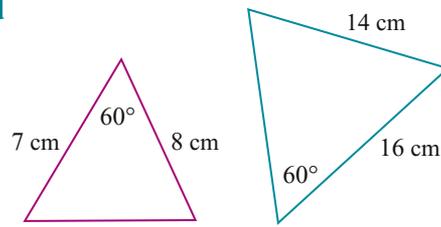
b



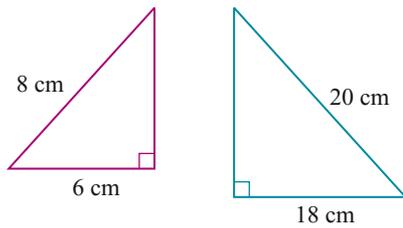
c



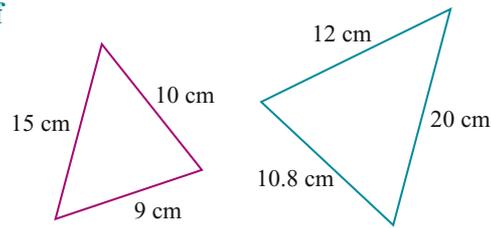
d



e



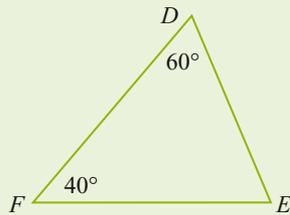
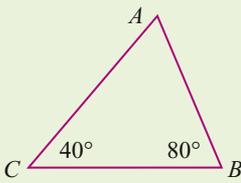
f



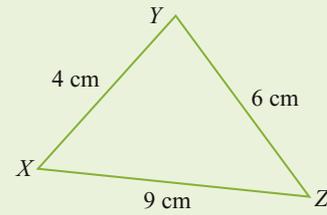
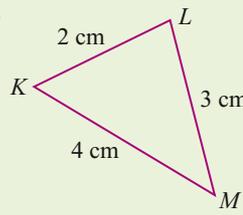
EXAMPLE 3

Determine whether the following pairs of triangles are similar.

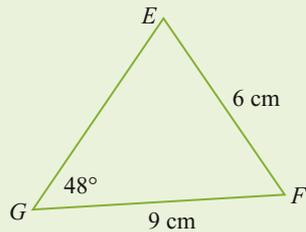
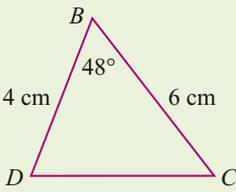
a



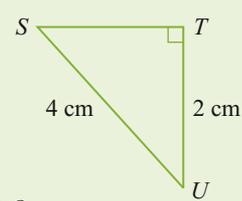
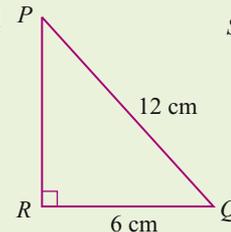
b



c



d



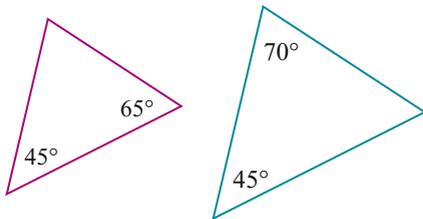
	Solve	Think	Apply
a	$\angle A = 60^\circ$ or $\angle E = 80^\circ$: Hence $\triangle ABC \sim \triangle DEF$. The triangles are similar.	$\angle A = 60^\circ$ (Angle sum of $\triangle ABC = 180^\circ$.) or $\angle E = 80^\circ$ (Angle sum of $\triangle DEF = 180^\circ$.) Two pairs of equal angles.	Look for pairs of equal angles and pairs of matching sides in proportion that will satisfy one of the four tests for similar triangles. .. is the symbol for similarity. It is read as 'is similar to'.
b	$\frac{XY}{KL} = \frac{4}{2} = 2$ $\frac{YZ}{LM} = \frac{6}{3} = 2$ but $\frac{XZ}{KM} = \frac{9}{4} \neq 2$ The triangles are not similar.	It is not possible to find three pairs of matching sides that are in proportion.	

EXAMPLE 3 CONTINUED

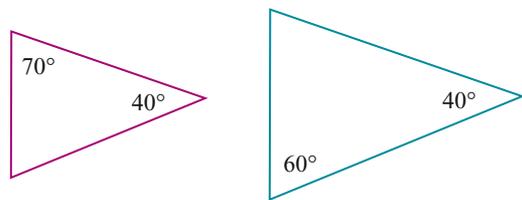
	Solve	Think	Apply
c	$\frac{BD}{EF} = \frac{BC}{GF} = \frac{2}{3}$ $\angle DBC = \angle EGF$ but $\angle EGF$ is not the angle included by EF and GF . The triangles are not similar.	Two pairs of sides are in proportion but the equal angles are not both the included angles.	Look for pairs of equal angles and pairs of matching sides in proportion that will satisfy one of the four tests for similar triangles.
d	$\angle PRQ = \angle STU (= 90^\circ)$ $\frac{PQ}{SU} = \frac{RQ}{TU} = \frac{3}{1}$ Hence $\triangle PQR \sim \triangle STU$.	The triangles are right-angled. The hypotenuse and a second side of the right-angled triangle are proportional to those of the other right-angled triangle.	

3 Determine whether the following pairs of triangles are similar. Give reasons. (Diagrams are not drawn to scale.)

a

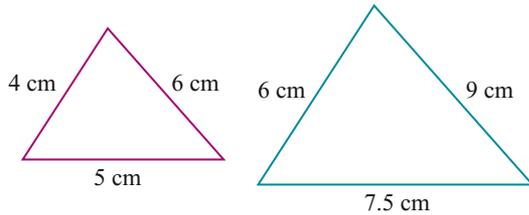


b

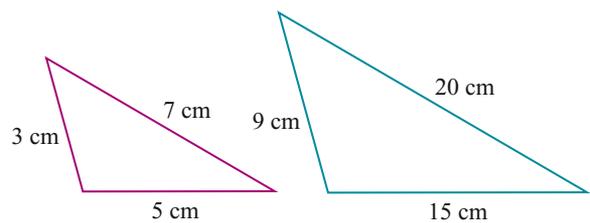


For parts a and b find the remaining angle in each triangle. !

c

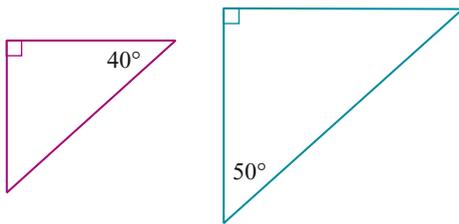


d

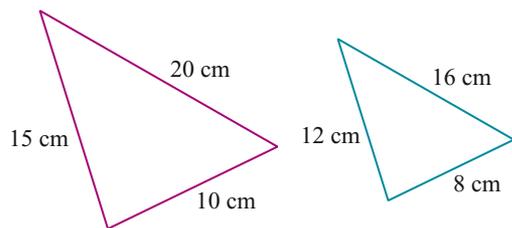


For part c find the ratios of the lengths of the longest, shortest and third pair of sides. !

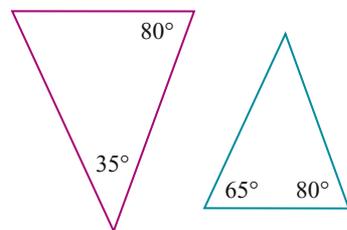
e



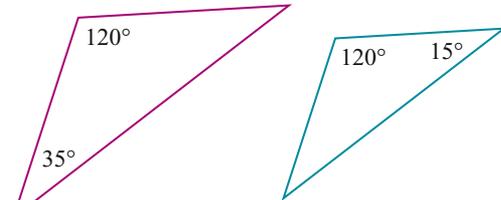
f



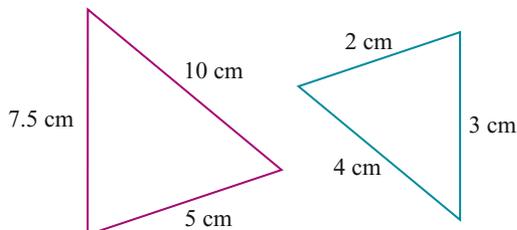
g



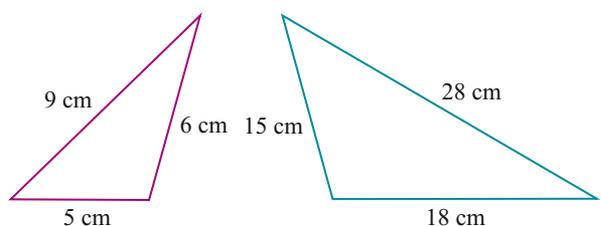
h

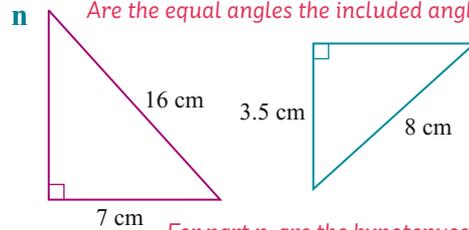
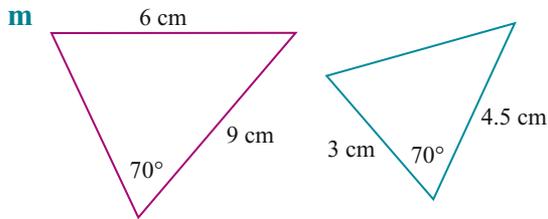
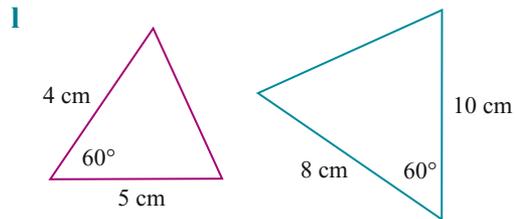
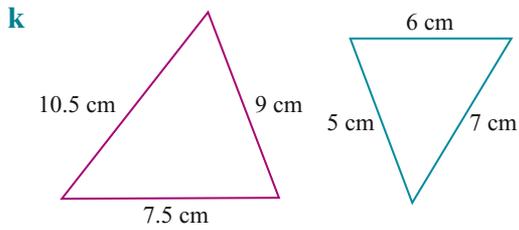


i



j



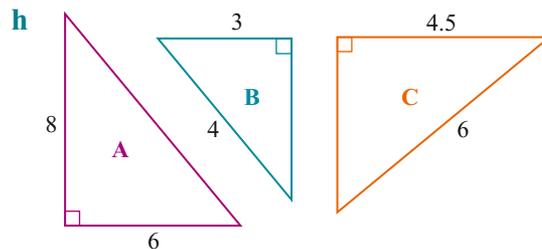
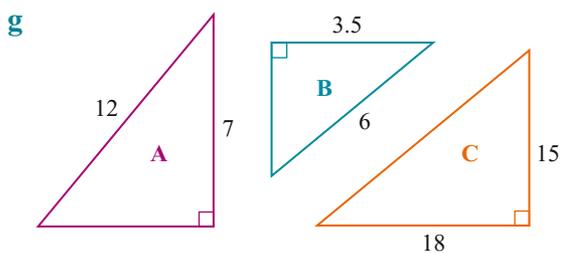
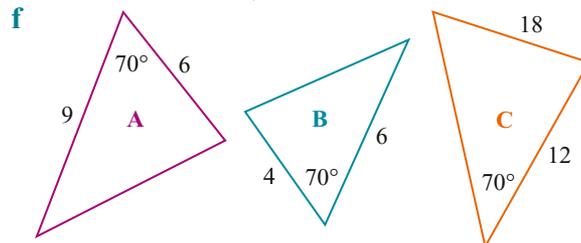
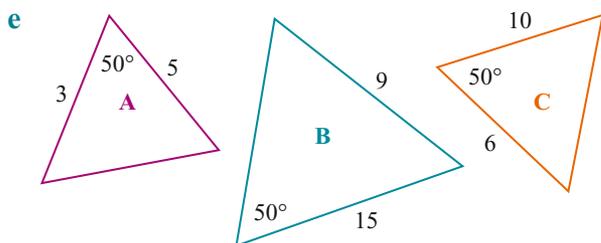
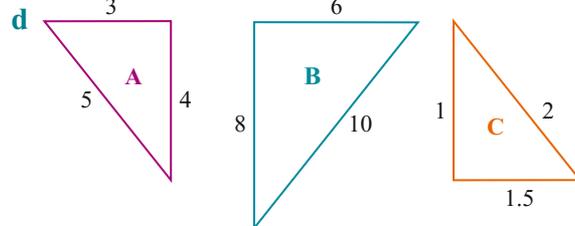
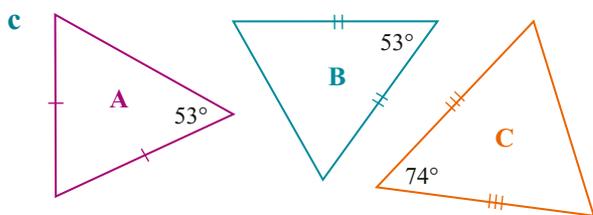
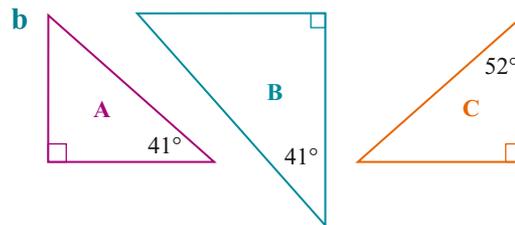
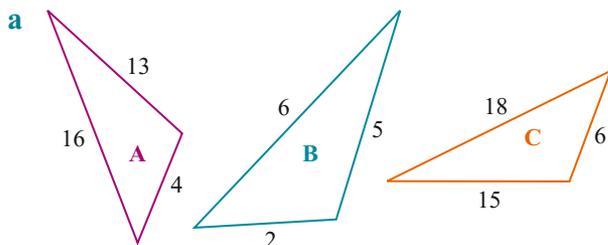


For part l, are the sides in proportion? Are the equal angles the included angles?

For part n, are the hypotenuses in proportion to the other pair of given sides?

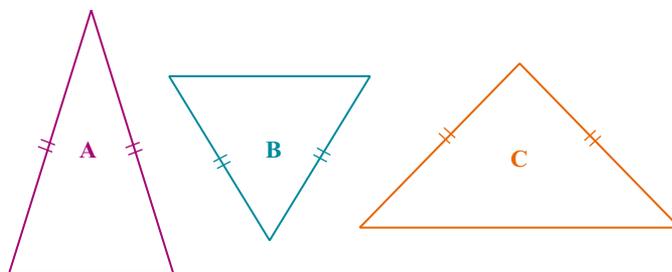


4 Which triangles are similar? Give reasons. (All measurements are in centimetres.)



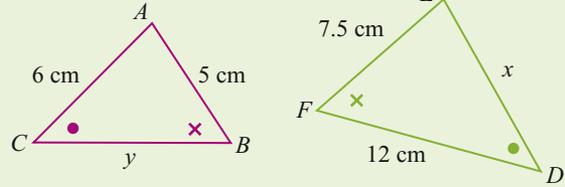
5 Here are three isosceles triangles.

- a** Measure all angles.
- b** Are any of these triangles equiangular?
- c** Comment on the statement 'All isosceles triangles are similar'.
- d** Draw two similar isosceles triangles.



EXAMPLE 4

Find the lengths of the unknown sides in this pair of similar triangles.



Solve	Think	Apply
<p><i>Method 1</i></p> <p>To find x:</p> $k = \frac{7.5}{5} = 1.5$ $x = 1.5 \times 6 = 9$ <p>To find y:</p> $k = \frac{5}{7.5} = \frac{2}{3}$ $y = \frac{2}{3} \times 12 = 8$ $\therefore x = 9 \text{ cm and } y = 8 \text{ cm}$ <p><i>Method 2</i></p> $\frac{x}{6} = \frac{7.5}{5}$ $x = 6 \times \frac{7.5}{5} = 9$ $\frac{y}{12} = \frac{5}{7.5}$ $y = 12 \times \frac{5}{7.5} = 8$ $\therefore x = 9 \text{ cm and } y = 8 \text{ cm}$	<p><i>Method 1</i></p> <p>To find x: EF and AB are a pair of matching sides.</p> $k = \frac{EF}{AB} = \frac{7.5}{5} = 1.5$ <p>The length of each side in the second triangle is 1.5 times the length of its matching side in the first triangle. Hence $DE = k \times CA$.</p> <p>To find y: AB and EF are a pair of matching sides.</p> $k = \frac{AB}{EF} = \frac{5}{7.5} = \frac{2}{3}$ <p>The length of each side in the first triangle is $\frac{2}{3}$ times the length of its matching side in the second triangle. Hence $CB = k \times DF$.</p> <p><i>Method 2</i></p> <p>AB and EF, BC and DF, CA and DE are pairs of corresponding sides in the two triangles. Take the ratio of each pair in the same order.</p>	<p><i>Method 1</i></p> <p>Find the scale factor, k, using a known pair of matching sides. Apply this scale factor to the side corresponding to the unknown side.</p> <p><i>Method 2</i></p> <p>Identify the pairs of corresponding sides and equate their ratios in the appropriate order.</p>

- 6 Complete the following to find the lengths of the unknown sides in the similar triangles shown.

Method 1

To find x : $k = \frac{16.25}{\square} = \underline{\hspace{2cm}}$

$x = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

To find y : $k = \frac{\square}{16.25} = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

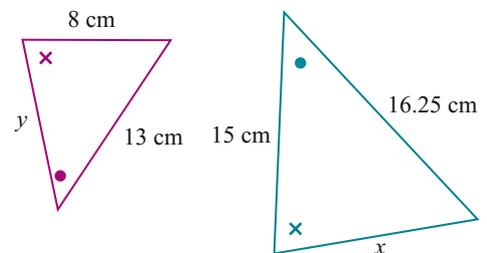
Method 2

$$\frac{x}{\square} = \frac{16.25}{\square}$$

$$x = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

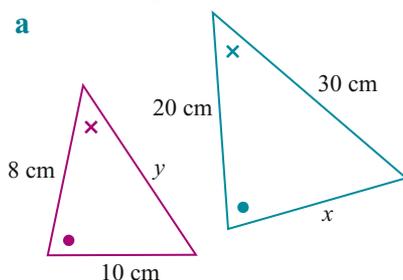
$$\frac{y}{\square} = \frac{\square}{16.25}$$

$$y = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

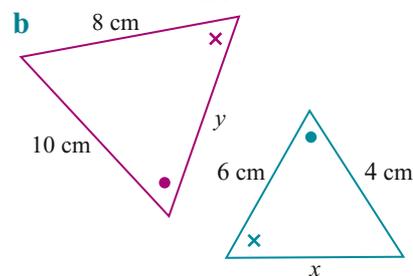


- 7 Find the lengths of the unknown sides in the following pairs of similar triangles.

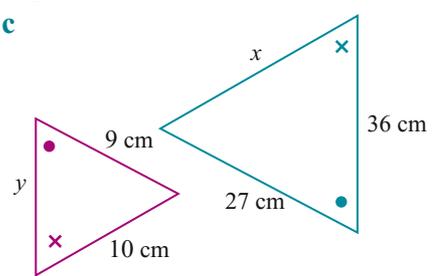
a

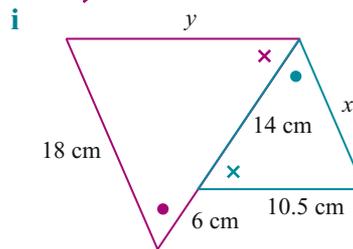
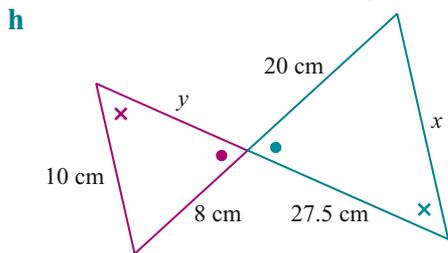
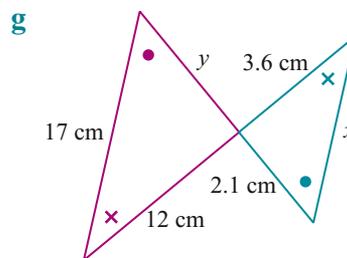
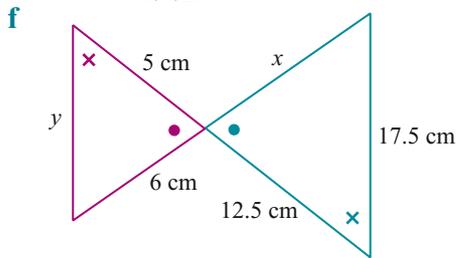
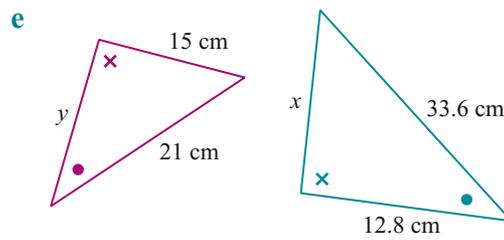
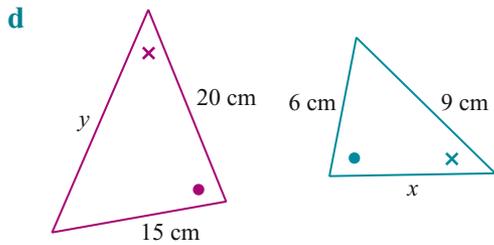


b



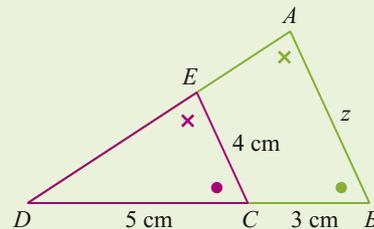
c





EXAMPLE 5

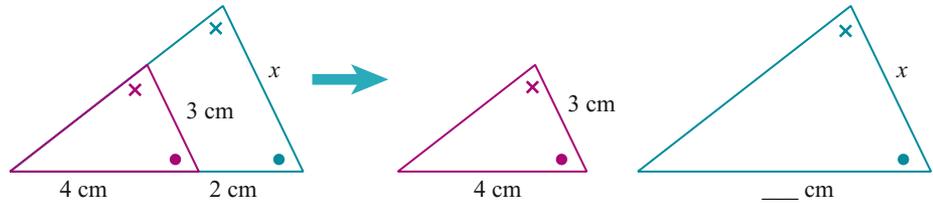
Find the length of the unknown side in the following pair of similar triangles.



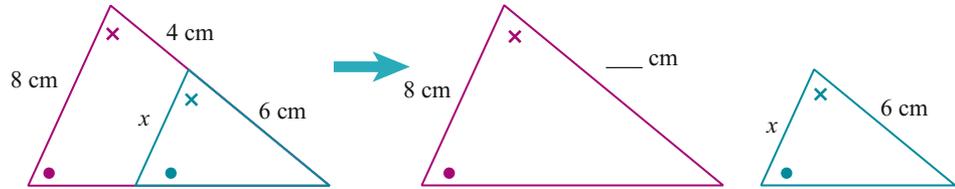
Solve	Think	Apply
<p><i>Method 1</i></p> $k = \frac{8}{5}$ $z = \frac{8}{5} \times 4$ $= 6.4 \text{ cm}$ <p><i>Method 2</i></p> $\frac{z}{4} = \frac{8}{5}$ $z = 4 \times \frac{8}{5}$ $= 6.4 \text{ cm}$	<p>Draw $\triangle ABD$ and $\triangle ECD$ separately.</p> <p><i>Method 1</i> DB and DC are a pair of corresponding sides. Hence $k = \frac{DB}{DC} = \frac{8}{5}$. AB and EC are corresponding sides. Hence $AB = k \times EC$.</p> <p><i>Method 2</i> AB and EC, DB and DC are pairs of corresponding sides. Hence $\frac{AB}{EC} = \frac{DB}{DC}$.</p>	<p>Draw the similar triangles separately.</p> <p><i>Method 1</i> Find the scale factor using a known pair of matching sides. Apply this factor to the side corresponding to the unknown side.</p> <p><i>Method 2</i> Identify the pairs of matching sides and equate their ratios in the appropriate order.</p>

8 Complete the following to find the lengths of the unknown sides in the following pairs of similar triangles.

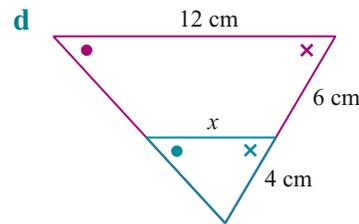
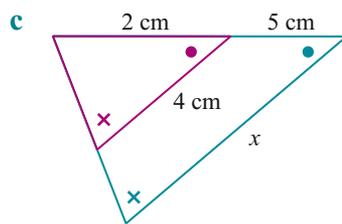
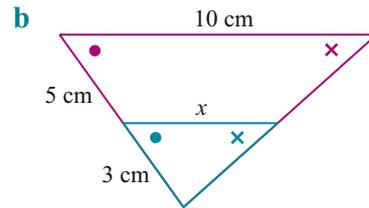
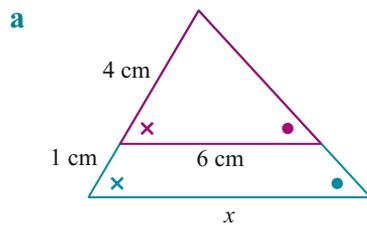
a $\frac{x}{\square} = \frac{\square}{4}$
 $x = \underline{\hspace{2cm}}$



b $\frac{x}{\square} = \frac{6}{\square}$
 $x = \underline{\hspace{2cm}}$



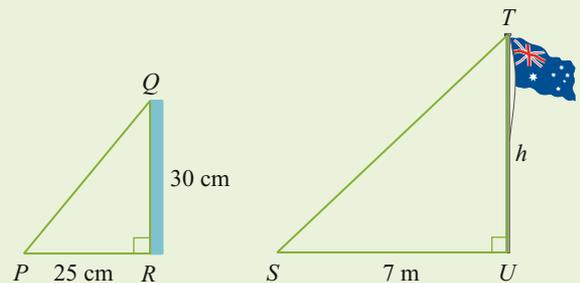
9 Find the length of the unknown side in the following pairs of similar triangles.



EXAMPLE 6

When a 30 cm ruler stands vertically on the ground it casts a shadow 25 cm long. At the same time a flagpole casts a shadow of length 7 m.

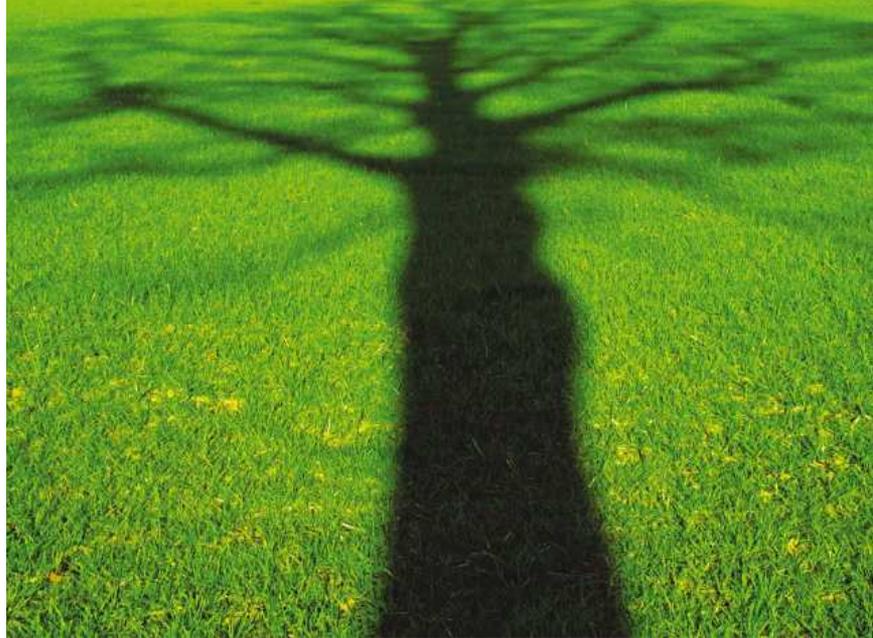
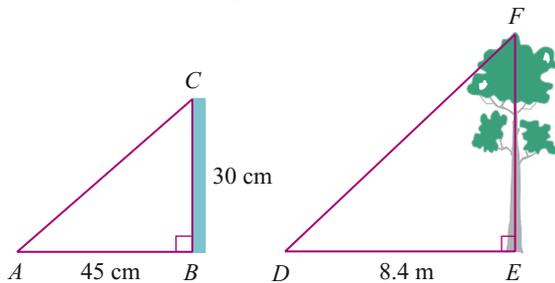
- a Name a pair of similar triangles, giving reasons for your answer.
 b What is the height of the flagpole?



	Solve	Think	Apply
a	$\angle U = \angle R$ and $\angle S = \angle P$ $\triangle STU \sim \triangle PQR$ because two angles of one triangle are equal to two angles of the other triangle.	$\angle U = \angle R = 90^\circ$ as the ruler and the flagpole are vertical to the ground. $\angle TSU = \angle QPR$ as the sun's rays make the same angle with the ground.	Look for pairs of equal angles and pairs of matching sides in proportion that will satisfy one of the four tests for similar triangles.
b	$\frac{h}{30} = \frac{700}{25}$ $h = 30 \times \frac{700}{25}$ $= 840 \text{ cm or } 8.4 \text{ m}$	TU and QR , SU and PR are pairs of matching sides in similar triangles. Hence $\frac{TU}{QR} = \frac{SU}{PR}$. Change measurements to the same units.	Identify the pairs of corresponding sides and equate their ratios in the appropriate order.

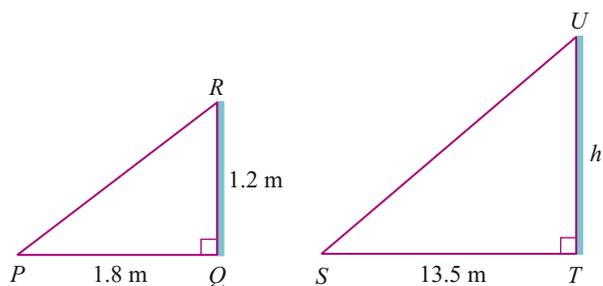
- 10** On a sunny day the shadow cast by a tree is 8.4 m long. At the same time the shadow cast by a 30 cm ruler is 45 cm long.

- a** Name a pair of similar triangles, giving reasons for your answer.
b What is the height of the tree?



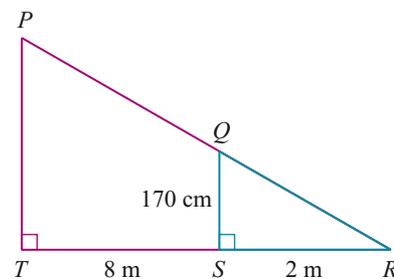
- 11** A fence surrounding a football field is 1.2 m high. At a certain time of the day the shadow of the fence was 1.8 m long and the shadow of the goalposts was 13.5 m long.

- a** Name a pair of similar triangles, giving reasons for your answer.
b What is the height of the goalposts?



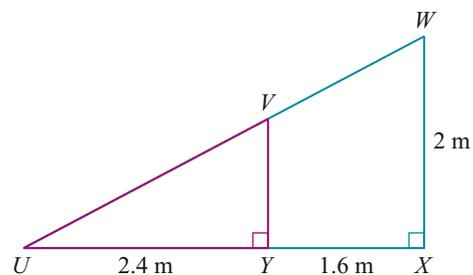
- 12** When a man of height 170 cm stands 8 m from the base of a street light his shadow is 2 m long.

- a** Name a pair of similar triangles, giving reasons.
b What is the height of the light above the ground?



- 13** A ramp for a BMX track is to be built with two vertical supports VY and WX . The dimensions are shown in the diagram.

- a** Name a pair of similar triangles, giving reasons.
b Calculate the length of the inner support (VY).



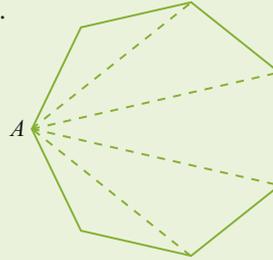
E

Polygons

A polygon is a plane shape with straight sides. Polygons are named according to the number of sides. The simplest polygon is a triangle, which has an interior angle sum of 180° . A quadrilateral has four sides and an angle sum of 360° .

EXAMPLE 1

Find the sum of the interior angles of a heptagon.



A heptagon and a septagon are different terms meaning a seven-sided figure. 

Solve	Think	Apply
Choose one vertex and draw all the diagonals from it. $\text{Sum of interior angles} = 5 \times 180^\circ = 900^\circ$	A heptagon is a polygon with 7 sides. The diagonals divide the heptagon into five triangles. The sum of the interior angles of the heptagon is equal to the sum of the interior angles of the five triangles.	Choose any vertex and draw all the diagonals from it. The sum of the interior angles of the polygon is equal to the sum of the interior angles of the triangles formed.

Exercise 7E

- Draw any pentagon (5-sided polygon) and label one of its vertices A .
 - Draw all the diagonals from A .
 - Complete: There are ____ triangles in the pentagon, hence interior angle sum = $___ \times 180^\circ = ___$.
- Repeat question 2 with different polygons, drawing diagonals from one vertex only. Complete the table below. Example 1 shows this for the heptagon.

Polygon	Number of sides	Number of triangles	Angle sum of polygon
Quadrilateral	4	2	$2 \times 180^\circ = 360^\circ$
Pentagon	5		
Hexagon			
Heptagon	7	5	$5 \times 180^\circ = 900^\circ$
Octagon			
Nonagon			
Decagon			
n -gon			

3 Complete the following statement:

The sum of the measure of the interior angles of any n -sided polygon is $___ \times 180^\circ$.

EXAMPLE 2

Find the size of each interior angle of a regular dodecagon.

A dodecagon has 12 sides. !

Solve	Think	Apply
$\begin{aligned} \text{Sum of angles} &= (12 - 2) \times 180^\circ \\ &= 1800^\circ \\ \text{Each angle} &= 1800^\circ \div 12 \\ &= 150^\circ \end{aligned}$	<p>Sum of angles of dodecagon $= (12 - 2) \times 180^\circ$.</p> <p>A regular polygon has all its sides and interior angles equal.</p>	<p>Find the sum of the interior angles and then divide by the number of equal interior angles.</p>

4 Complete the following statement to find the size of each angle in a regular pentagon.

Sum of interior angles = $___$

Each interior angle = $___ \div ___ = ___$

5 Find the size of each interior angle in these regular polygons.

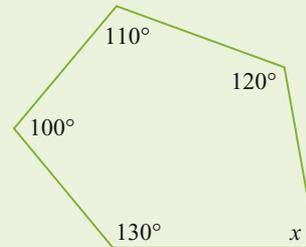
- a hexagon b heptagon c octagon d nonagon e decagon

6 A regular polygon has 24 sides.

- a Find the sum of the interior angles. b Find the size of each interior angle.

EXAMPLE 3

Find x , giving reasons for your answers.



Solve	Think	Apply
$\begin{aligned} \text{Angle sum of polygon} &= 3 \times 180^\circ = 540^\circ \\ x + 130^\circ + 100^\circ + 110^\circ + 120^\circ &= 540^\circ \\ x + 460^\circ &= 540^\circ \\ x &= 80^\circ \end{aligned}$	<p>Angle sum of an n-gon is $(n - 2) \times 180^\circ$.</p> <p>This is a pentagon so the angle sum = $3 \times 180^\circ = 540^\circ$.</p>	<p>Find the sum of the angles of the polygon. Form an equation from which the value of the pronumeral can be found.</p>

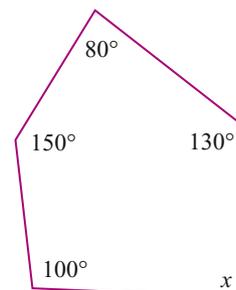
7 Complete the following to find the value of x in the pentagon.

Angle sum of polygon = $___$

$x + 130^\circ + ___ + ___ + ___ = ___$

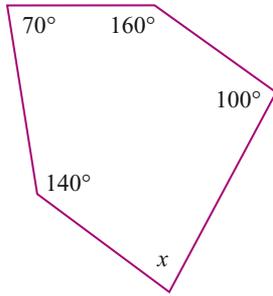
$x + ___ = ___$

$x = ___$

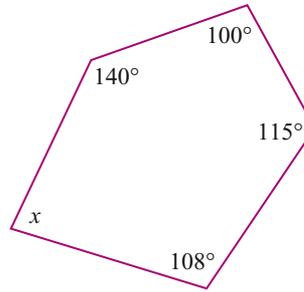


8 Find the value of x in these pentagons. Give a reason for your answer.

a



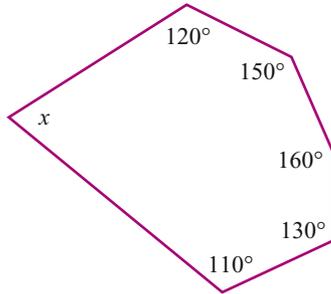
b



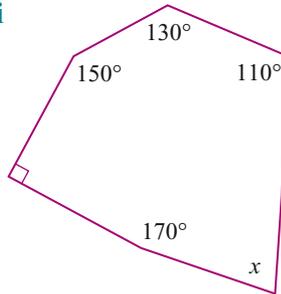
9 a Find the angle sum of a hexagon.

b Find the value of x in these hexagons.

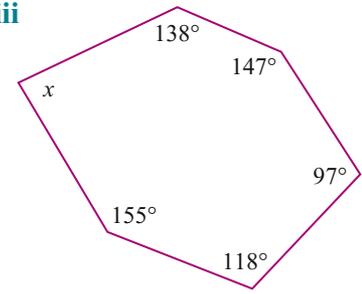
i



ii

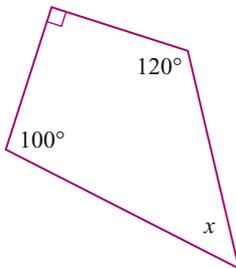


iii

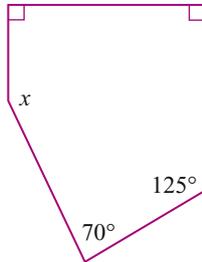


10 Find x , giving a brief reason.

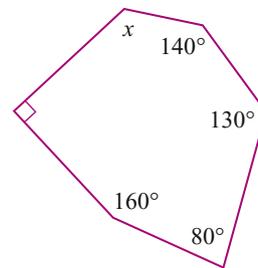
a



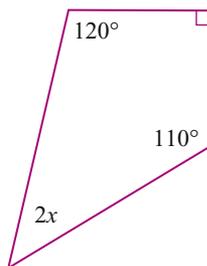
b



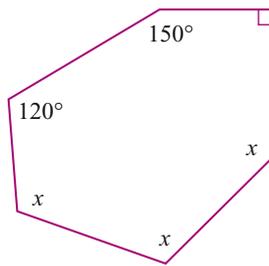
c



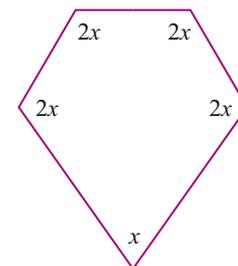
d



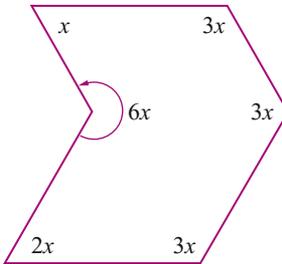
e



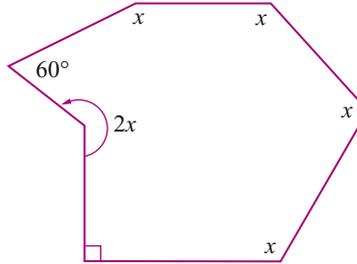
f



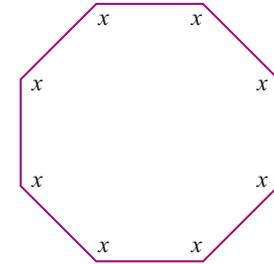
g



h



i



11 A pentagon has three right angles and two other equal angles. Find the size of each of these two equal angles.

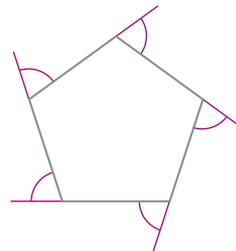
12 A hexagon has two right angles and all other angles equal. Find the size of each of the equal angles.

Investigation 2 Exterior angle sum

The exterior angles of a polygon are formed by extending each side in the same order, as shown in the diagram. Each exterior angle is the angle between the extended side and the side that is adjacent to it.

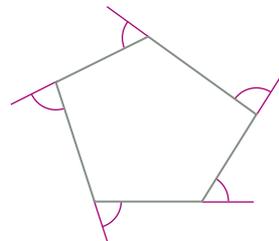
1 The five exterior angles of a pentagon are shown. As this is a regular pentagon all exterior angles are equal.

- Measure the exterior angles.
- Find the sum of the five exterior angles.



2 A non-regular pentagon is shown opposite.

- Measure the five exterior angles.
- Find the sum of the exterior angles.



3 a Draw a hexagon.

- Measure the exterior angles.
- Find the sum of the exterior angles.

4 a Draw an octagon.

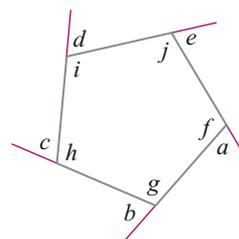
- Measure the exterior angles.
- Find the sum of the exterior angles.

5 Complete the following.

The sum of the exterior angles of any polygon is ____.

6 Complete the following.

- $(a + f) = \underline{\hspace{2cm}}$
 - $(b + g) = \underline{\hspace{2cm}}$
- The sum of the exterior and interior angles is $(a + f) + (b + g) + (c + h) + (d + i) + (e + j) = 5 \times \underline{\hspace{2cm}}$.
- The sum of the interior angles is $f + g + h + i + j = 3 \times \underline{\hspace{2cm}}$.
- The sum of the exterior angles is ____.



EXAMPLE 4

- Find the size of each exterior angle of a regular decagon.
- Hence, find the size of each interior angle.
- Find the angle sum of a decagon.

	Solve	Think	Apply
a	Exterior angle $= \frac{360^\circ}{10}$ $= 36^\circ$	From Investigation 2, the sum of the exterior angles of any polygon is 360° . A regular decagon has 10 equal exterior angles.	Exterior angle $= \frac{360^\circ}{n}$ where n is the number of equal exterior angles (number of equal sides).
b	Each interior angle $= 180^\circ - 36^\circ = 144^\circ$	The sum of each interior angle and its exterior angle is 180° .	Interior angle $= 180^\circ - \text{exterior angle}$.
c	Angle sum $= 144^\circ \times 10$ $= 1440^\circ$	A regular decagon has 10 equal interior angles.	Multiply the size of each interior angle by the number of angles.

- 13** Complete the following. For a regular octagon, sum of exterior angles = ____.
 Size of each exterior angle = ____ \div ____ = ____
 Hence, size of each interior angle = $180^\circ -$ ____ = ____.
 Angle sum of octagon = ____ \times ____ = ____
- 14 a** Find the size of each exterior angle of a regular 20-sided polygon.
b Find the size of each interior angle.
c Hence, find the angle sum of a 20-sided polygon.

EXAMPLE 5

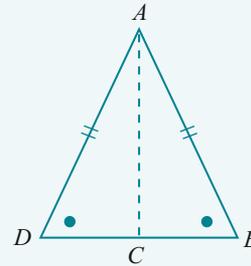
- a** The measure of each exterior angle of a regular polygon is 12° . How many sides does this polygon have?
b An irregular polygon has one exterior angle of 80° and all others 7° . How many sides does it have?

	Solve	Think	Apply
a	$12^\circ = \frac{360^\circ}{n}$ $n = \frac{360^\circ}{12^\circ} = 30$ The polygon has 30 sides.	Each exterior angle = $\frac{360^\circ}{n}$ where n is the number of sides. Substitute exterior angle = 12° .	Use $n = \frac{360^\circ}{\text{angle size}}$
b	$80^\circ + (n - 1) \times 7^\circ = 360^\circ$ $80^\circ + 7^\circ(n - 1) = 360^\circ$ $7^\circ(n - 1) = 280^\circ$ $n - 1 = 40$ $n = 41$ The polygon has 41 sides.	Let n equal the number of sides. If one exterior angle is 80° , then the other $(n - 1)$ exterior angles are each 7° .	The sum of the exterior angles of any polygon is 360° .

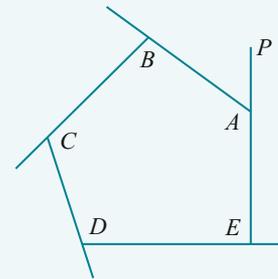
- 15 a** Each exterior angle of a regular polygon is 15° . Complete to find the number of sides of the polygon.
 Exterior angle = $\frac{360^\circ}{n}$
 ____ = $\frac{360^\circ}{n}$
 $n = \frac{360^\circ}{\square} =$ ____ \therefore The polygon has ____ sides.
- b** An irregular polygon has one exterior angle of 60° and the rest are 10° . Complete to find the number of sides of the polygon.
 $60^\circ + (\text{____}) \times 10^\circ = 360^\circ$
 $10(\text{____}) =$ ____
 ____ = ____
 $n =$ ____ \therefore The polygon has ____ sides.
- 16** Find the number of sides in regular polygons with these exterior angles.
a 10° **b** 18° **c** 24° **d** 90°
- 17** An irregular polygon has one exterior angle 100° and all others 13° . How many sides does it have?
- 18** Find the number of sides in an irregular polygon with:
a one exterior angle of 120° and all others 12° **b** one exterior angle of 45° and all others 15° .
- 19** An irregular polygon has two exterior angles twice the size of the others. Find the number of sides if:
a the other exterior angles are 15° **b** the other exterior angles are 20° .

Language in mathematics

- 1 Explain why all congruent triangles are similar but not all similar triangles are congruent.
- 2 Define and list all the properties of these triangles.
 - a isosceles triangles
 - b equilateral triangles
- 3 What is the meaning of the term 'equiangular'? Are all equiangular figures similar?
- 4 If two angles of a triangle are equal to two angles of another, explain why the remaining third pair of angles must be equal.
- 5 a Explain why $\triangle ABC$ is not congruent to $\triangle ADC$.
 b What extra piece of information would make them congruent?



- 6 Define the exterior angle of a polygon.
- 7 Write two equivalent formulas for the sum of the interior angles of a polygon. Explain the meaning of any pronumerals used.
- 8 $ABCDE$ is a diagram of a pentagonal-shaped field. The sides have been produced as shown. Cherie stands at A facing P , turns and walks to B . She then turns and walks to C , turns and walks to D , turns and walks to E and then turns and walks back to A . To complete this circuit, through what angle has Cherie walked?



Terms

bisect	congruent	congruency	enlargement	equilateral	exterior
included	interior	isosceles	matching	parallelogram	polygon
proportional	similar	similarity	vertices	vertex	

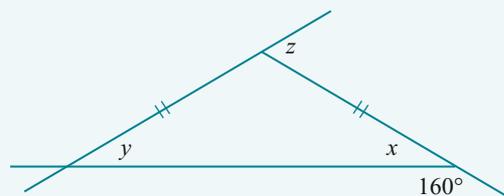
Check your skills

- 1 The following is the setting out used to find the value of the pronumeral z in the diagram.

$$\begin{aligned}
 x &= 20^\circ && (\underline{\quad}) \\
 y &= 20^\circ && (\underline{\quad}) \\
 \therefore z &= 40^\circ && (\underline{\quad})
 \end{aligned}$$

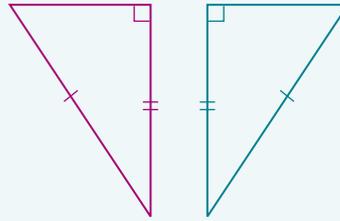
The reason that is not used in the above proof is:

- A The exterior angle of a triangle equals the sum of the interior opposite angles.
- B A straight angle is 180° .
- C The angles opposite the equal sides in an isosceles triangle are equal.
- D Vertically opposite angles are equal.



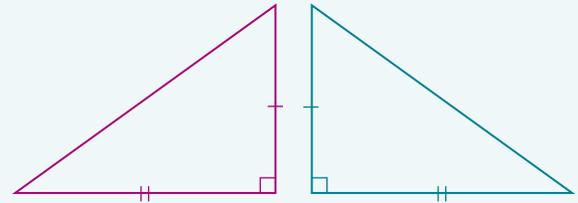
- 2 The congruency test used to show that these triangles are congruent is:

A SSS B SAS
C AAS D RHS



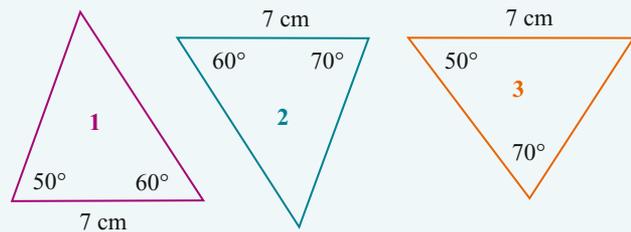
- 3 The congruency test used to show that these triangles are congruent is:

A SSS B SAS
C AAS D RHS



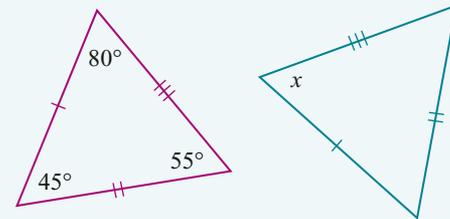
- 4 The triangles that are congruent are:

A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3



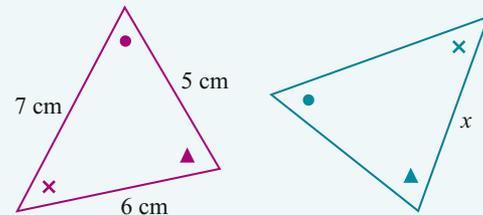
- 5 The value of the pronumeral in this pair of congruent triangles is:

A 45° B 55°
C 80° D 35°



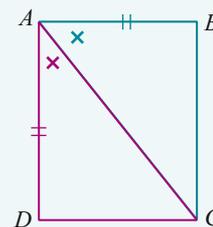
- 6 The value of the pronumeral in this pair of congruent triangles is:

A 5 cm B 6 cm
C 7 cm D 18 cm



- 7 In $\triangle ABC$ and $\triangle ADC$:

$AB = AD$ (____)
 $AC = AC$ (common)
 $\angle BAC = \angle DAC$ (given)
 $\therefore \triangle ABC \equiv \triangle ADC$ (____)

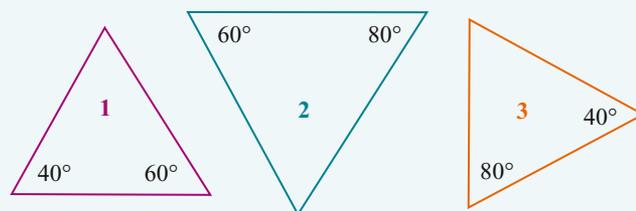


The missing statements in the above proof are:

A given, AC , SAS B isosceles triangle, AC , SAS
C given, AB , SSS D isosceles triangle, AB , SAS

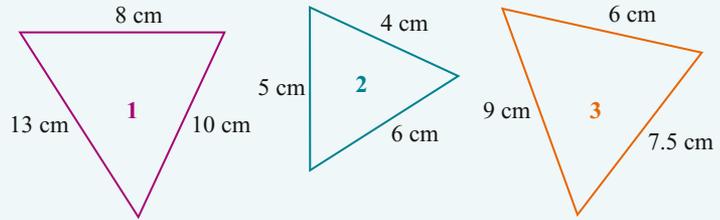
- 8 The similar triangles are:

A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3



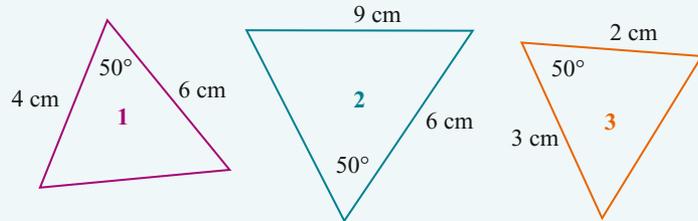
9 The similar triangles are:

- A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3



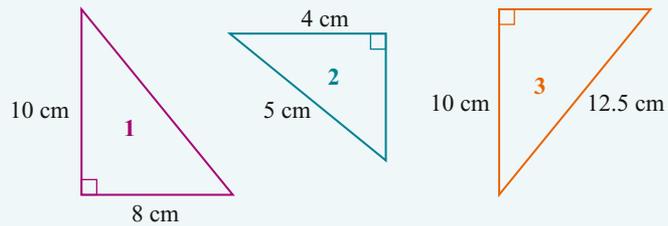
10 The similar triangles are:

- A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3



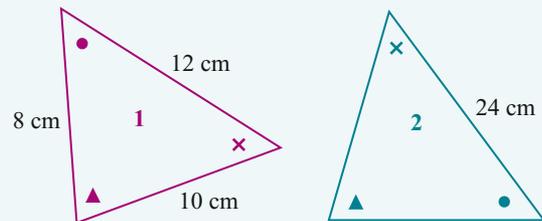
11 The similar triangles are:

- A 1 and 2 B 2 and 3
C 1 and 3 D 1, 2 and 3



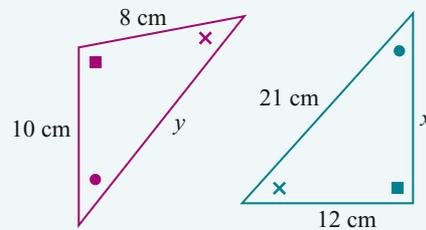
12 These triangles are similar. The enlargement factor (from 1 to 2) is:

- A 2 B 3
C 2.4 D 14



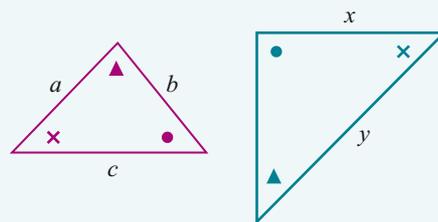
13 The values of the pronumerals in these similar triangles are:

- A $x = 15, y = 31.5$ B $x = 15, y = 14$
C $x = 6\frac{2}{3}, y = 31.5$ D $x = 6\frac{2}{3}, y = 14$



14 The statement about these triangles that is correct is:

- A $\frac{x}{a} = \frac{y}{b}$ B $\frac{x}{b} = \frac{y}{a}$
C $\frac{x}{c} = \frac{y}{a}$ D $\frac{x}{c} = \frac{y}{b}$



15 The angle sum of an eight-sided polygon is:

- A 540° B 720° C 900° D 1080°

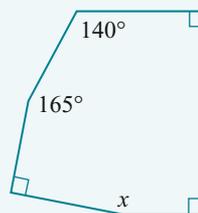
16 A ten-sided polygon is called a:

- A pentagon B nonagon C decagon D hexagon

17 A regular polygon with 24-sides has interior angle size of:

- A 3960° B 165° C 4320° D 180°

- 18 The value of x in this figure is:
 A 145° B 235°
 C 325° D 35°



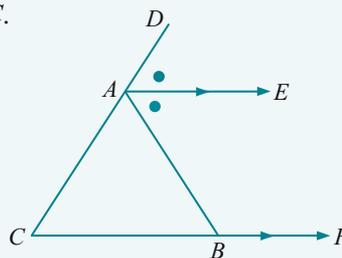
- 19 A heptagon has three right angles and the other angles are all equal. The size of the equal angles is:
 A 90° B 112.5° C 157.5° D 247.5°
- 20 The exterior angle of a 30-sided regular polygon is:
 A 12° B 168° C 360° D 5040°
- 21 An irregular polygon has one exterior angle of 100° and all others 13° . The number of sides of this polygon is:
 A 20 B 21 C 30 D 113

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

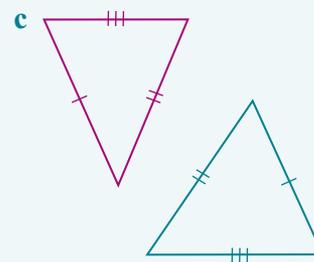
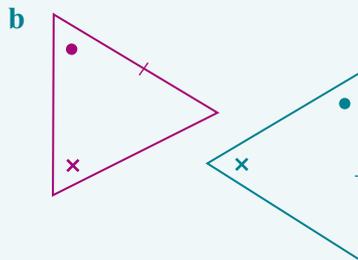
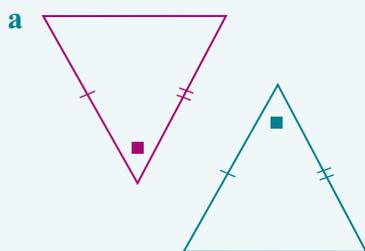
Question	1	2–6	7	8–14	15–21
Section	A	B	C	D	E

7A Review set

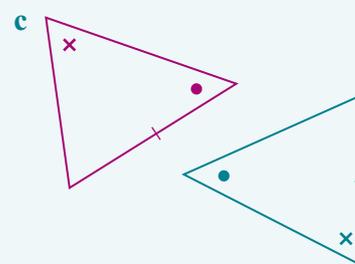
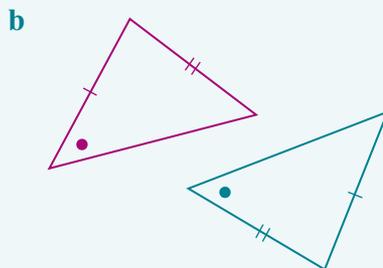
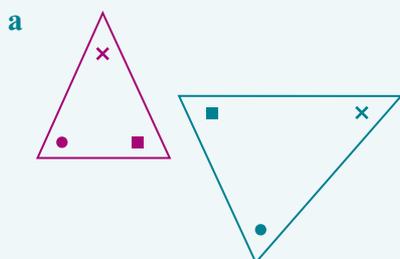
- 1 $AE \perp CF$. AE bisects $\angle DAB$. Prove, giving reasons, that $AB = AC$.



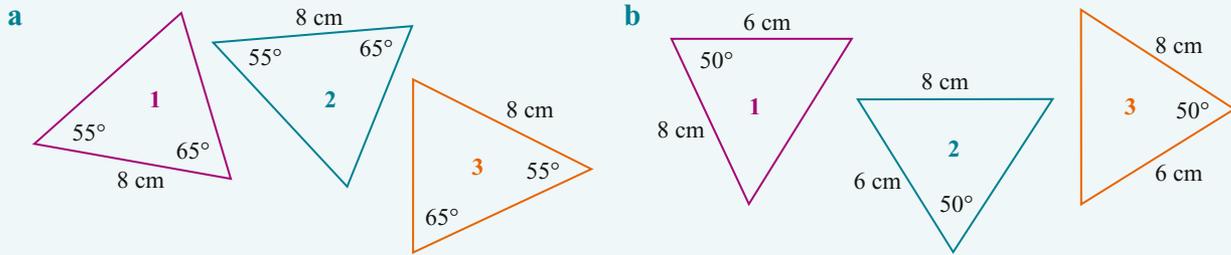
- 2 State the congruency test used to show that the following triangles are congruent.



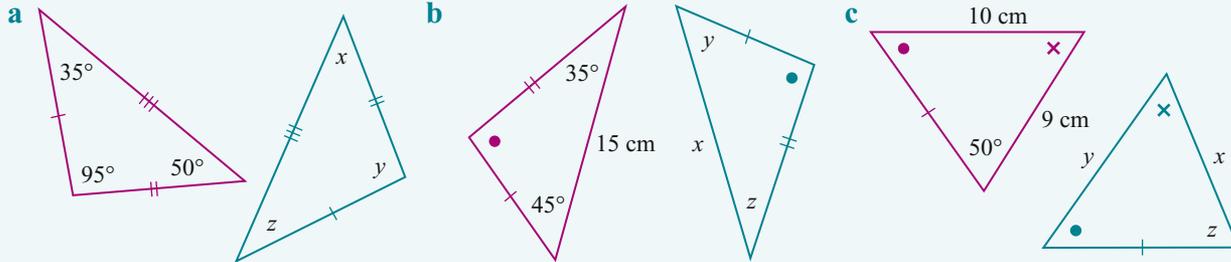
- 3 State why the following pairs of figures are not congruent.



4 Which triangles are congruent? Give a reason for your answer.



5 State why the following pairs of triangles are congruent and hence find the values of the pronumerals.

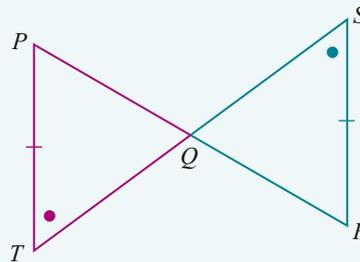


6 Complete the proofs that $\triangle PQT \equiv \triangle RQS$ and $PQ = RQ$.

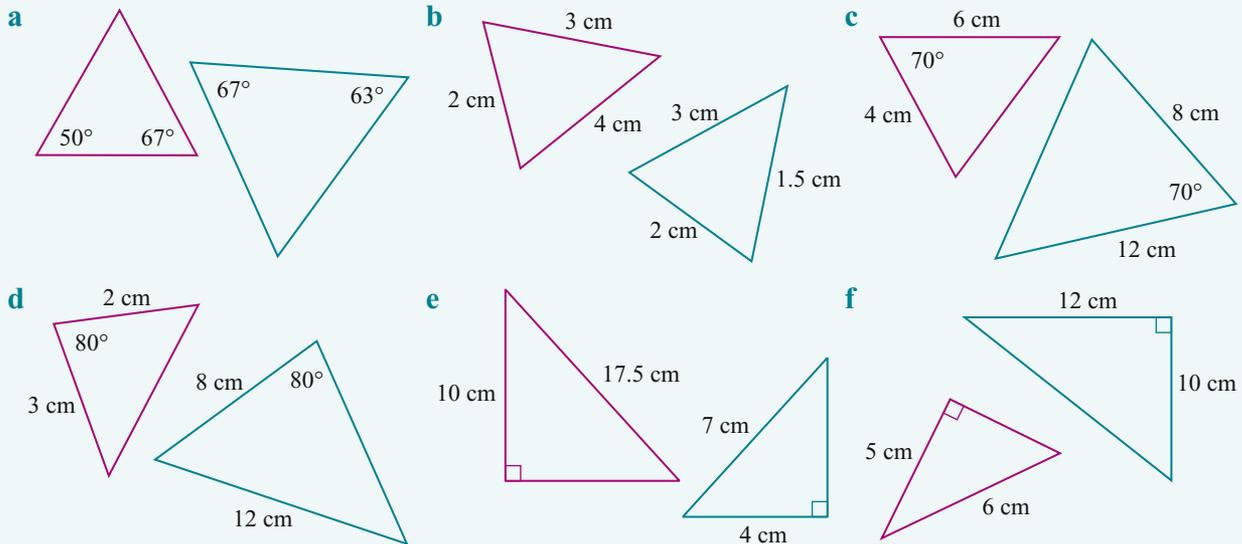
a In $\triangle PQT$ and $\triangle RQS$:

- $PT = RS$ (given)
 $\angle PTQ = \angle RSQ$ ()
 $\angle PQT = \angle RQS$ ()
 $\therefore \triangle PQT \equiv \triangle RQS$ ()

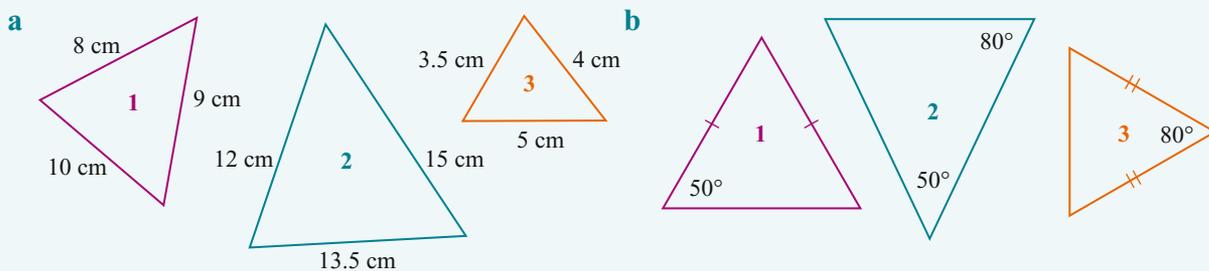
b $\therefore PQ = RQ$ ()

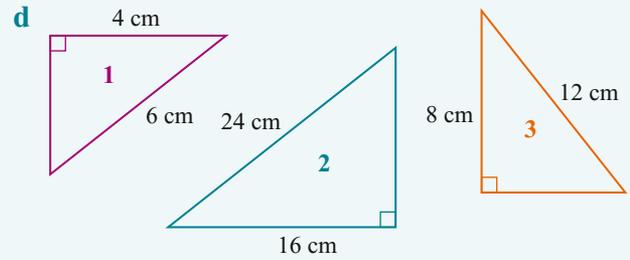
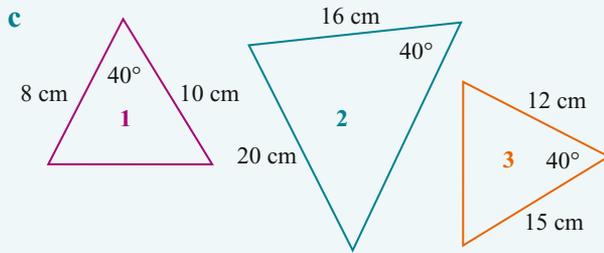


7 State with reasons whether or not the following pairs of triangles are similar.

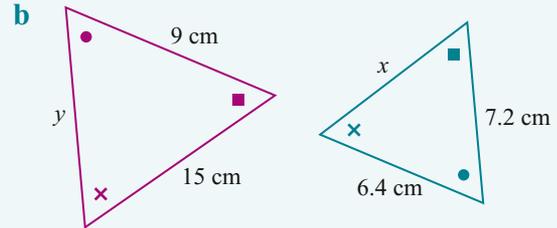
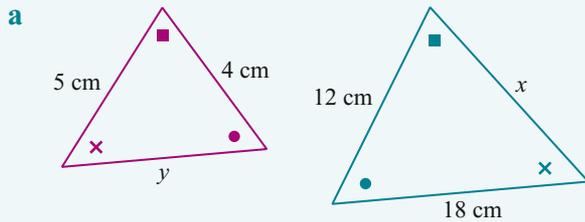


8 Which triangles are similar? State the test used.

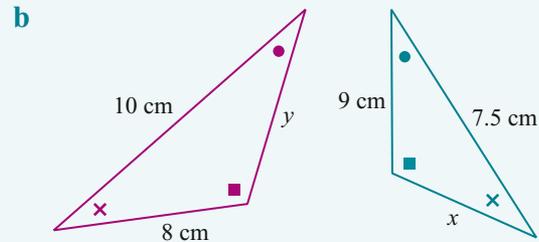
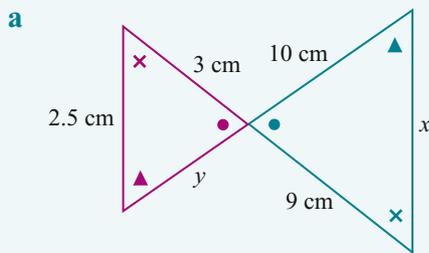




9 Find the enlargement factor and hence the length of the unknown sides in these pairs of similar triangles.

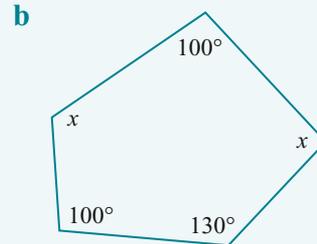
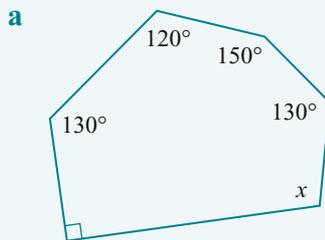


10 Use the ratios of matching sides to find the length of the unknown sides in these pairs of similar triangles.



- 11**
- Draw a hexagon.
 - Draw all the diagonals from one vertex.
 - Find the angle sum of a hexagon.

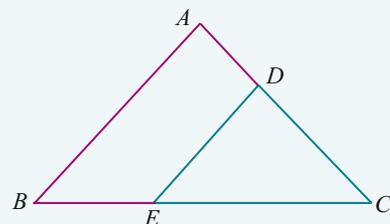
12 Find x , giving a reason for your answer.



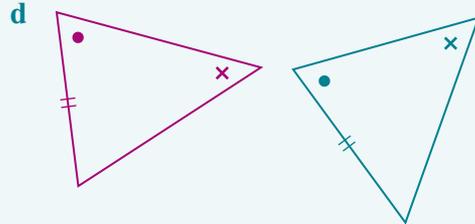
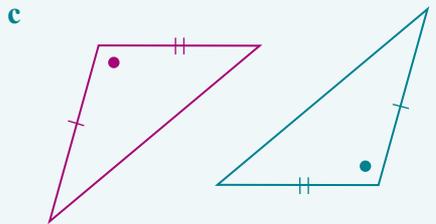
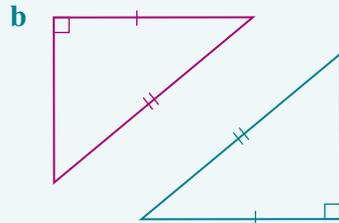
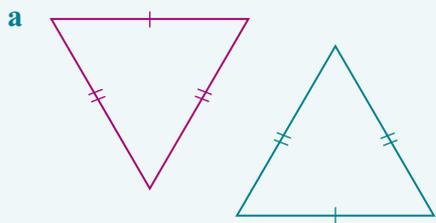
- 13** Consider a regular octagon.
- What is the size of each interior angle?
 - What is the sum of the exterior angles?
- 14** A regular polygon has exterior angles 10° .
- How many sides does it have?
 - What is the sum of the interior angles?

7B Review set

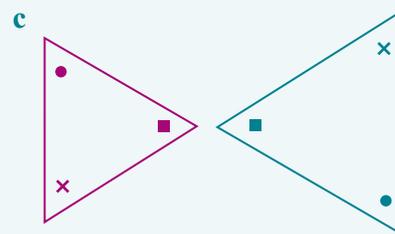
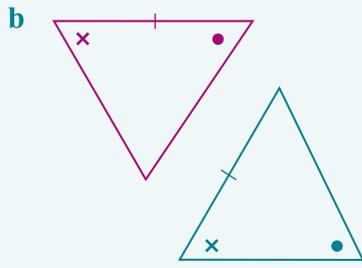
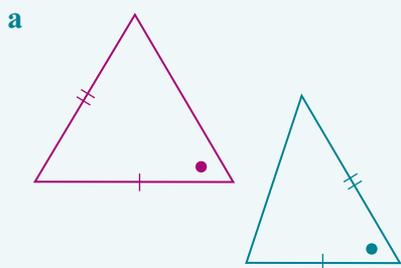
1 $AB = AC$ and $DC = DE$. Prove with reasons that $AB \parallel DE$.



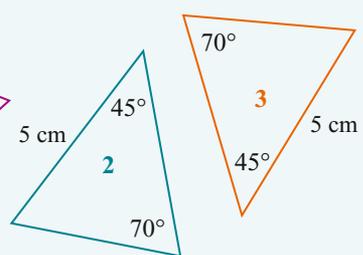
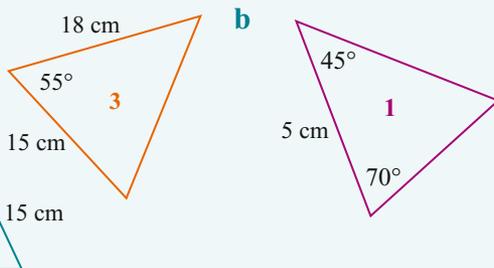
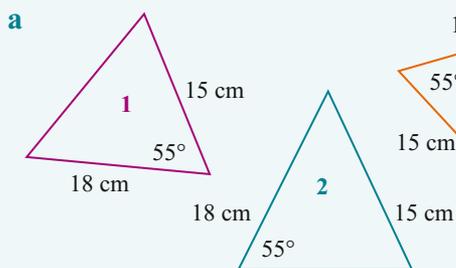
2 State the congruency test used to show that the following triangles are congruent.



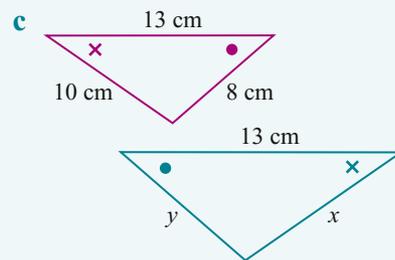
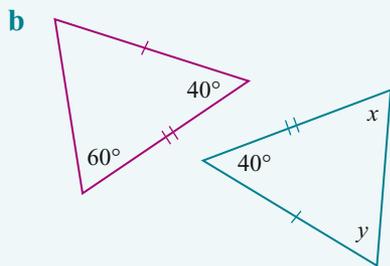
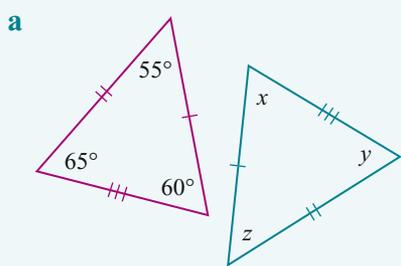
3 State why the following pairs of figures are not congruent.



4 Which triangles are congruent? Give a reason for your answer.



5 State why the following pairs of triangles are congruent. Hence find the values of the pronumerals.

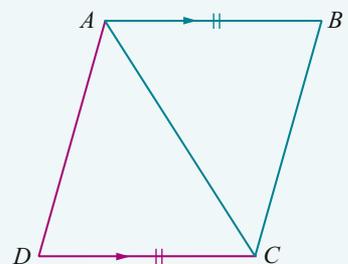


6 Complete the proofs that $\triangle ABC \equiv \triangle CDA$ and $AD = CB$.

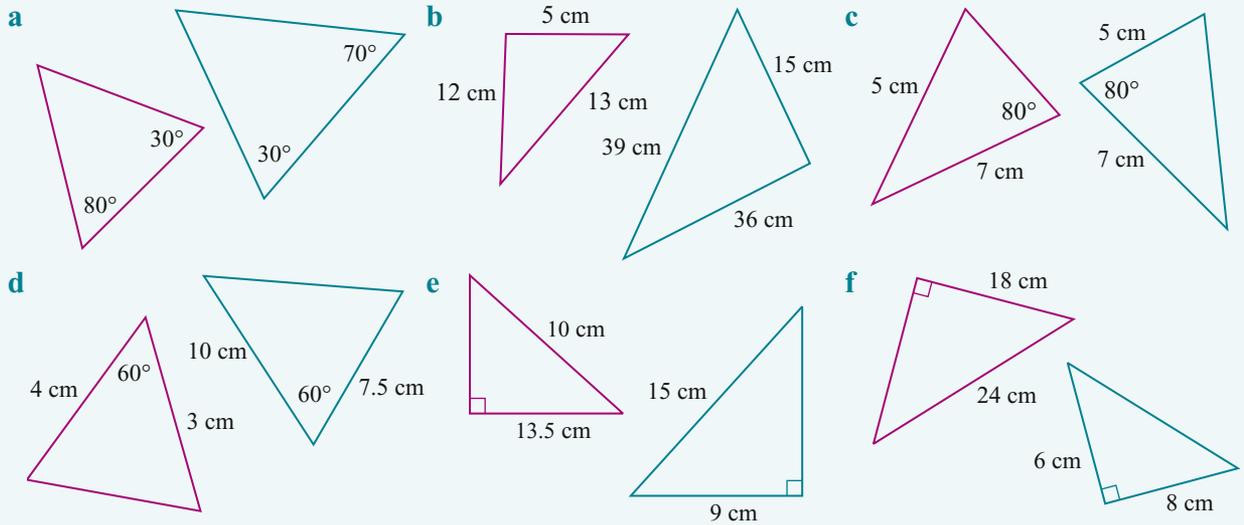
a In $\triangle ABC$ and $\triangle CDA$:

- $AB = \underline{\hspace{2cm}}$ (given)
- $AC = \underline{\hspace{2cm}}$ (common)
- $\angle BAC = \angle DCA$ (___)
- $\therefore \triangle ABC \equiv \triangle CDA$ (___)

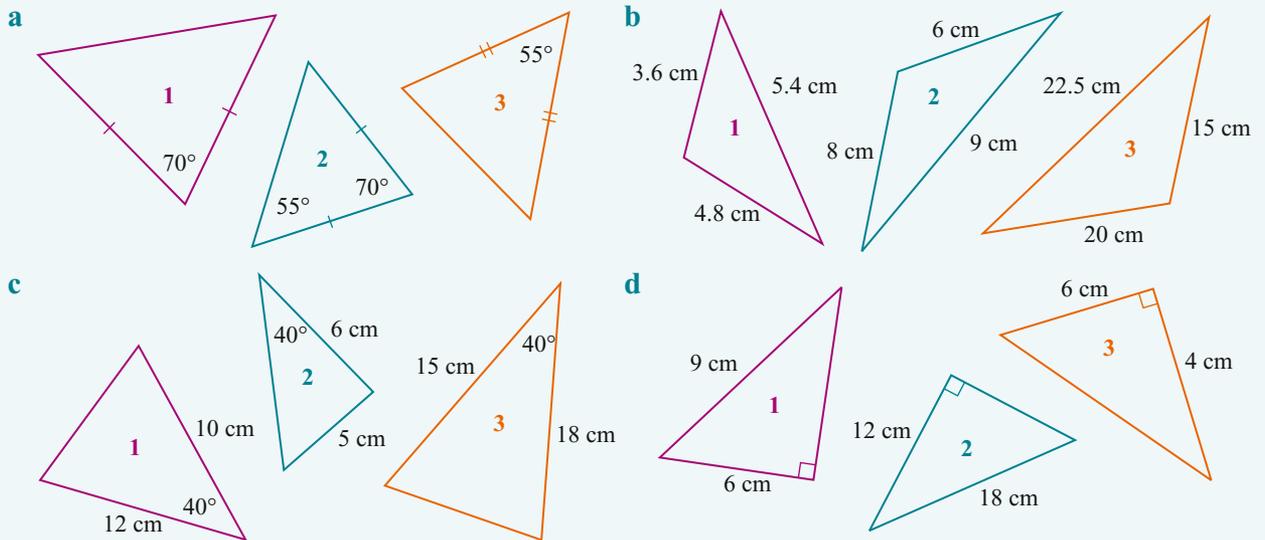
b $\therefore AD = \underline{\hspace{2cm}}$ (matching sides of congruent triangles)



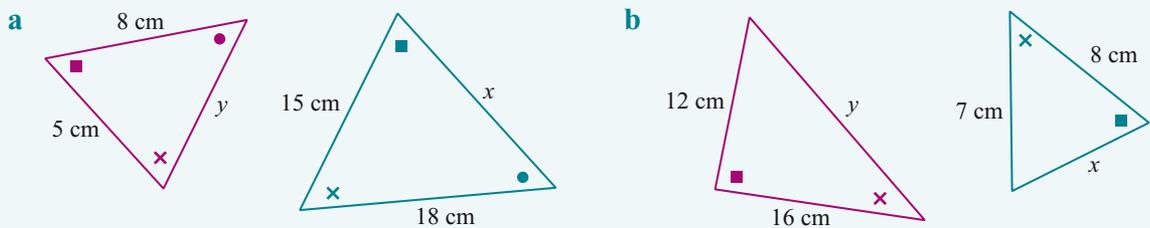
7 State with reasons whether or not the following pairs of triangles are similar.



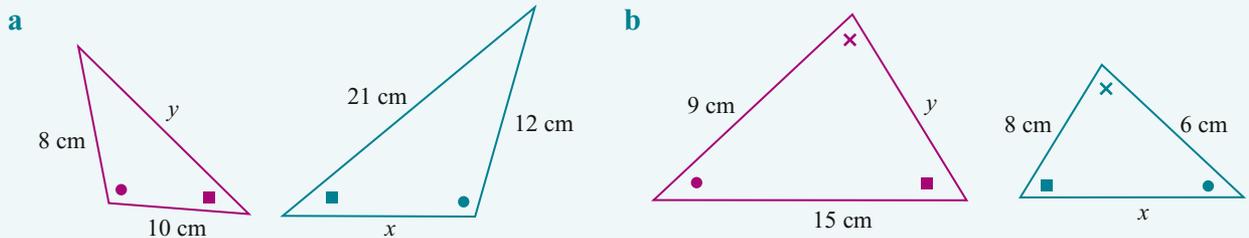
8 Which triangles are similar? State the test used.



9 Find the enlargement factor and hence the length of the unknown sides in these pairs of similar triangles.

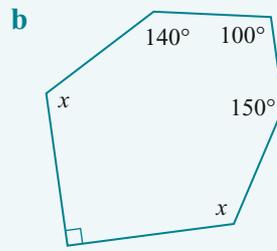
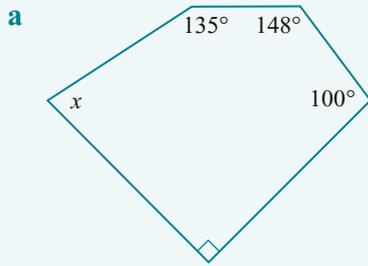


10 Use proportion to find the length of the unknown sides in these pairs of similar triangles.



- 11 a** Find the angle sum of a dodecagon (12 sides).
b Find the size of each interior angle of a regular dodecagon.

12 Find x , giving a reason for your answer.



13 A nine-sided figure has three right angles and all other angles equal. What is the measure of each of these equal angles?

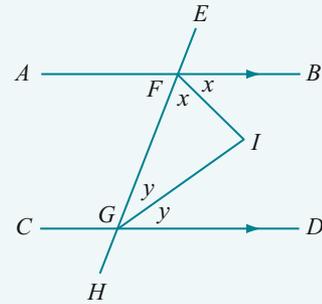
14 a Find the size of each exterior angle of a 25-sided regular polygon.

b Find the size of each interior angle.

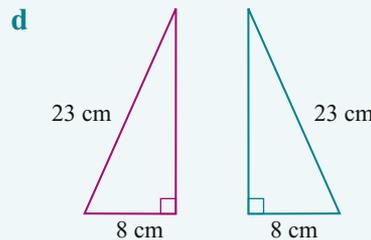
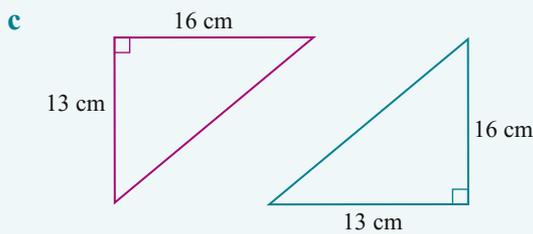
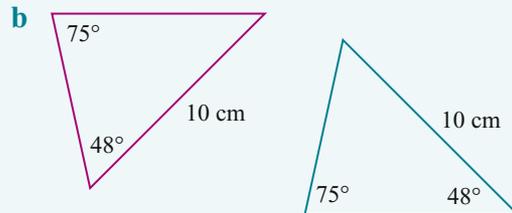
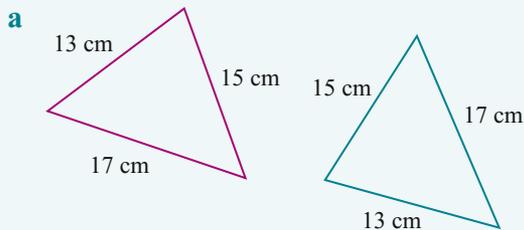
c Hence, find the angle sum of a 25-sided polygon.

7C Review set

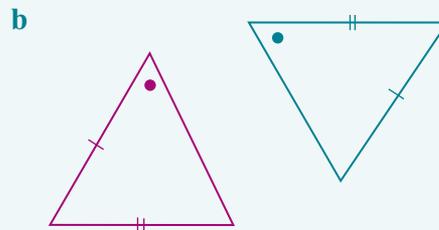
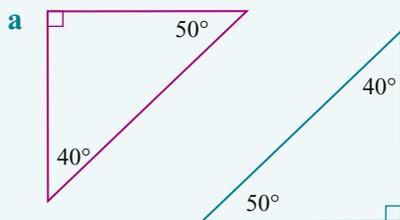
1 IF bisects $\angle BFG$ and IG bisects $\angle FGD$. Prove that $\angle FIG = 90^\circ$.



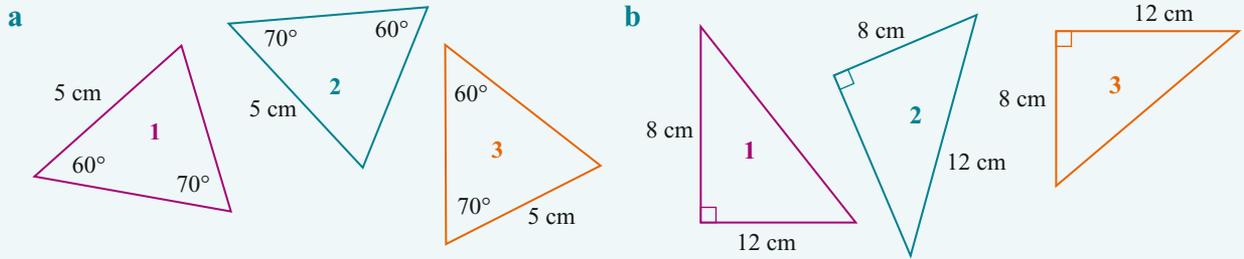
2 State the congruency test used to show that the following pairs of triangles are congruent.



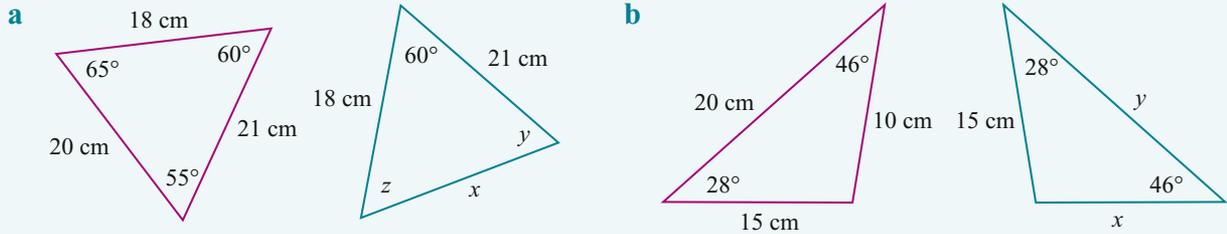
3 State why the following pairs of triangles are not congruent.



4 Which triangles are congruent. Give a reason for your answer.

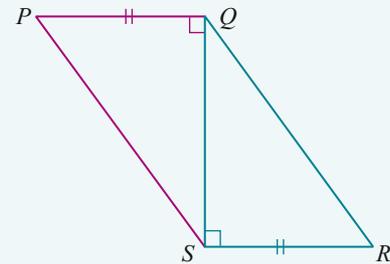


5 State why the following pairs of triangles are congruent and find the values of the pronumerals.

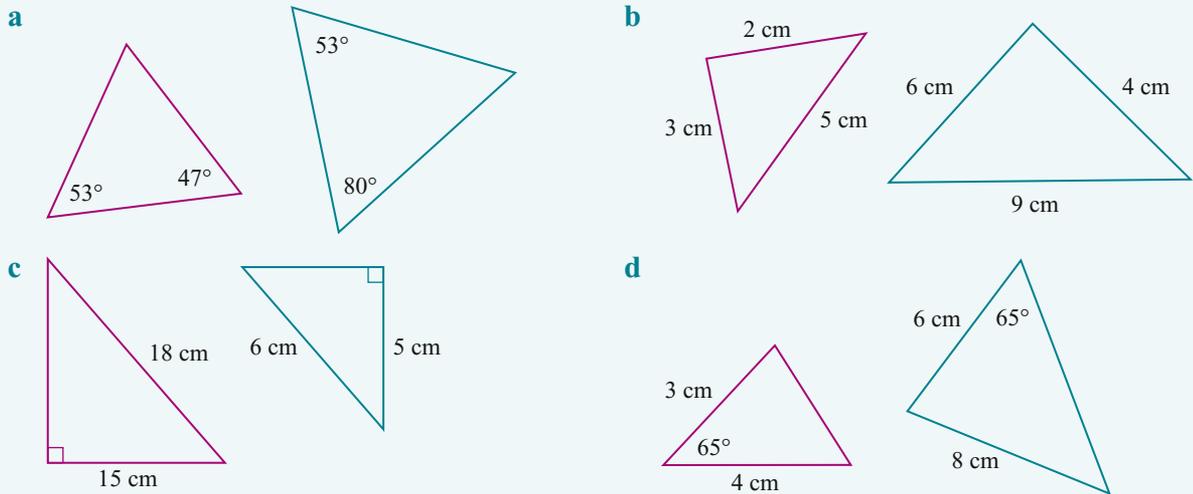


6 $PQ = RS$. $\angle PQS = \angle QSR = 90^\circ$. Prove the following.

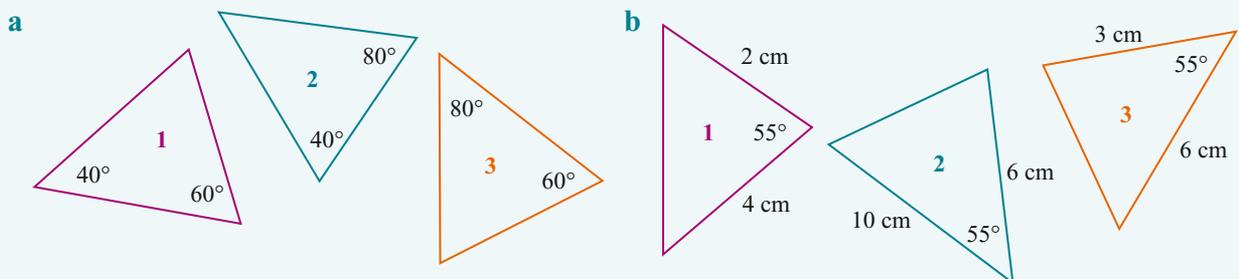
- $\triangle PQS \equiv \triangle RSQ$
- $PS = RQ$
- $PS \perp RQ$

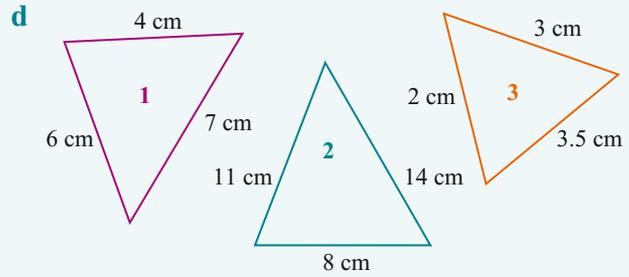
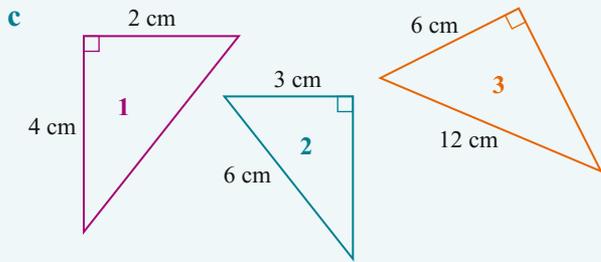


7 State with reasons whether or not the following triangles are similar.

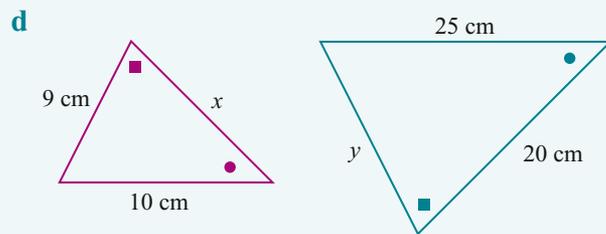
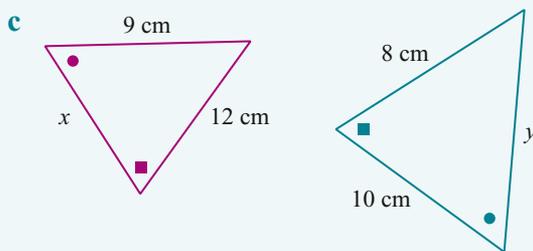
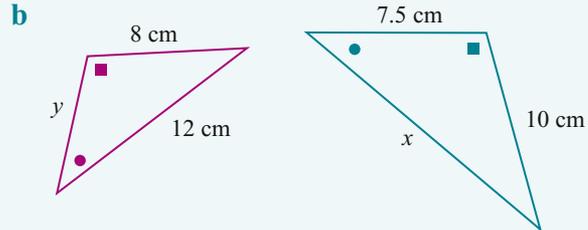
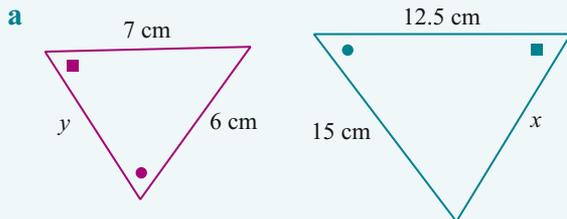


8 Which triangles are similar? State the test used.





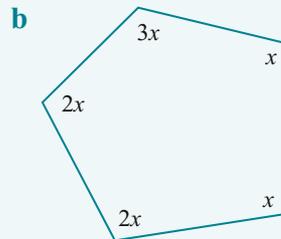
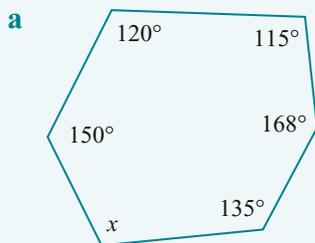
9 Find the lengths of the unknown sides in the following pairs of similar triangles.



10 A regular polygon has 36 sides.

- a** Find the sum of the interior angles.
- b** Find the size of each interior angle.

11 Find x , giving a reason for your answer.

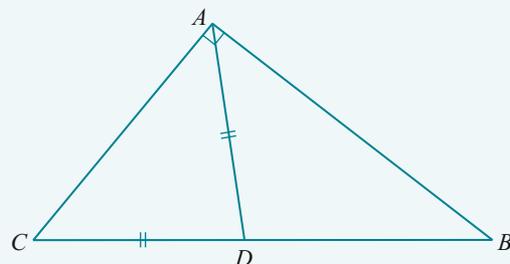


12 A regular pentagon has two right angles and all other angles equal. Find the size of the equal angles.

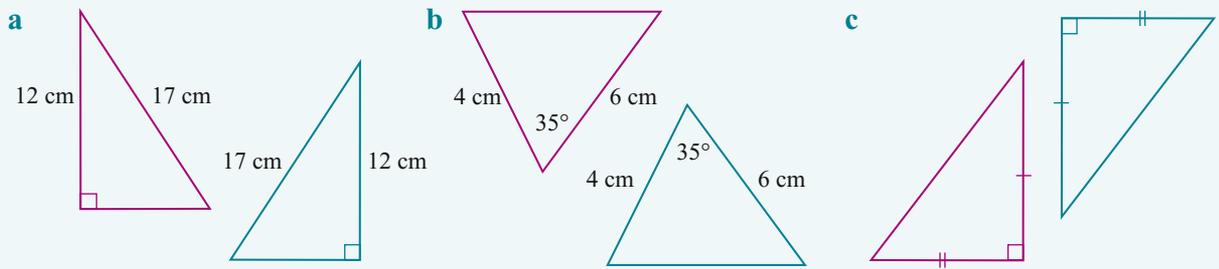
13 Find the size of the exterior angle in a regular 30-sided figure.

7D Review set

- 1** $\triangle ABC$ is right-angled at A . $DA = DC$. Prove that $DA = DB$.
Hint: Let $\angle ACD = x$.



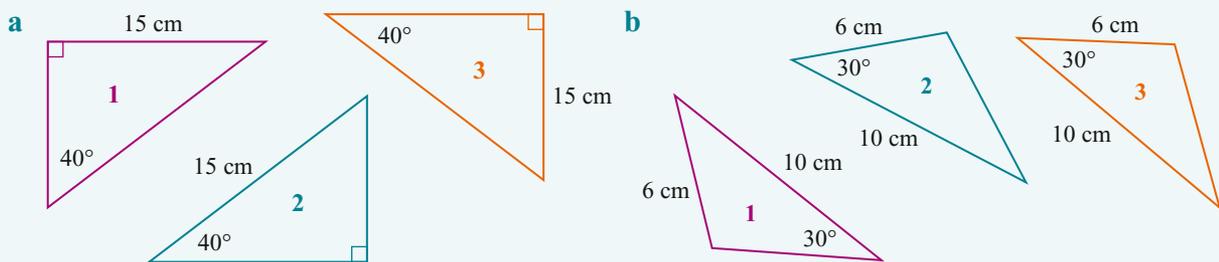
2 State the congruency test used to show that the following pairs of triangles are congruent.



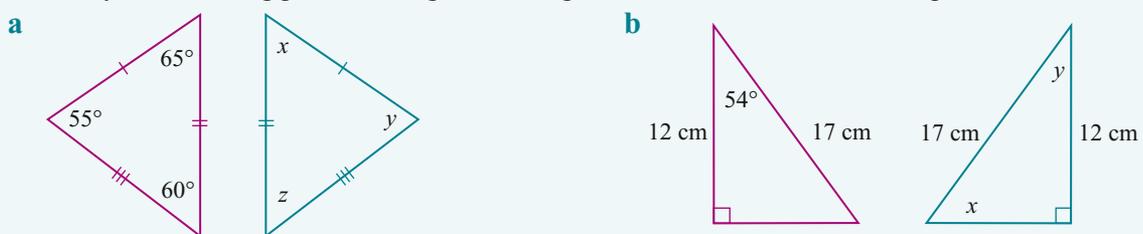
3 State why the following pairs of triangles are not congruent.



4 Which triangles are congruent? Give a reason for your answer.

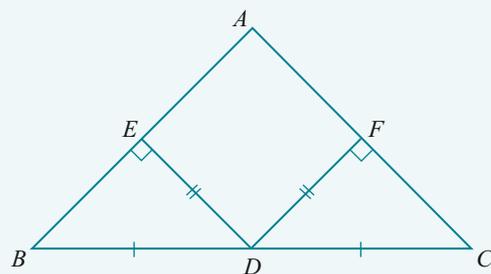


5 State why the following pairs of triangles are congruent and find the values of the pronumerals.

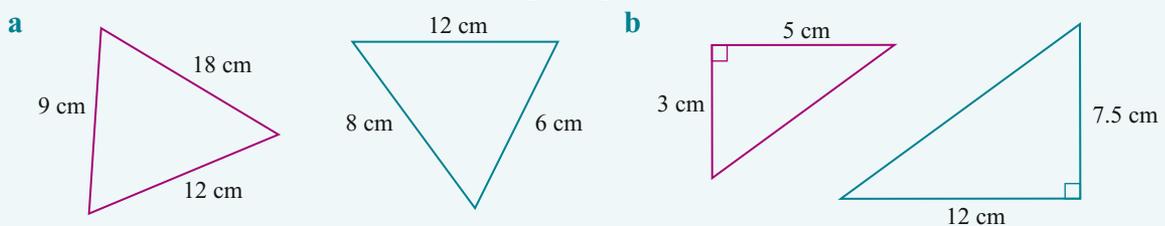


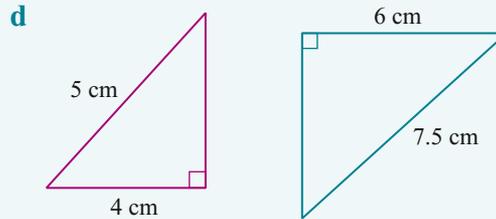
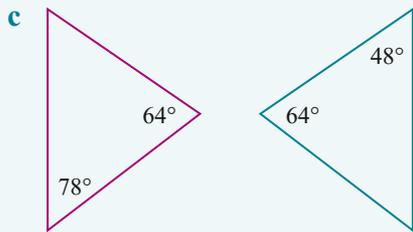
6 Prove the following.

- a** $\triangle BED \equiv \triangle CFD$
b $\triangle ABC$ is isosceles.

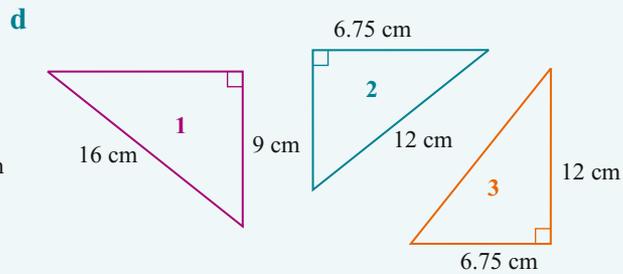
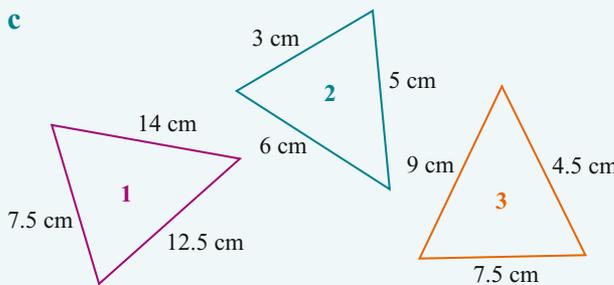
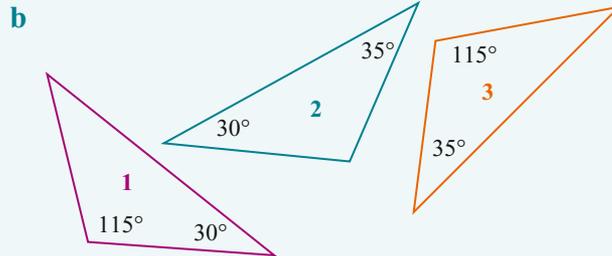
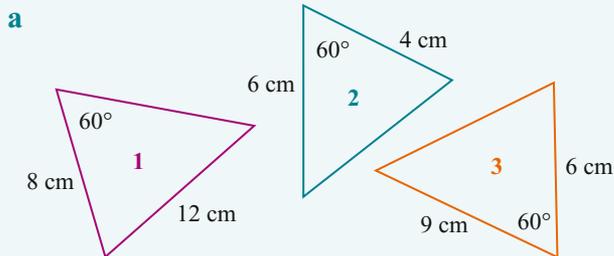


7 State with reasons whether or not the following triangles are similar.

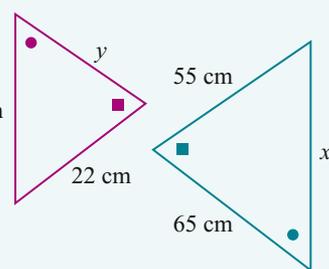
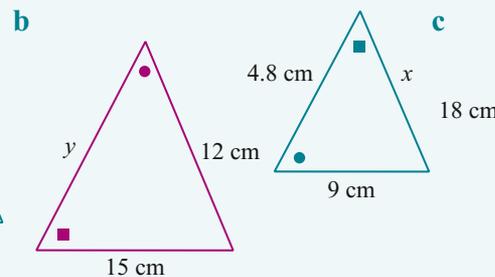
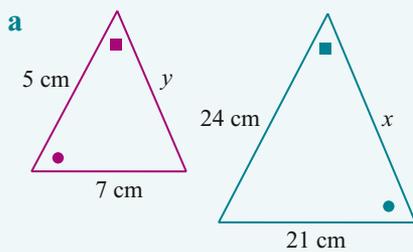




8 Which triangles are similar? State the test used.

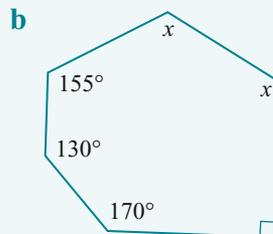
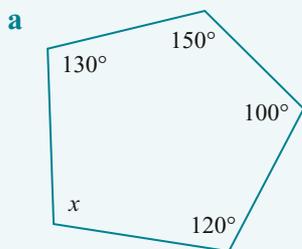


9 Find the lengths of the unknown sides in the following pairs of similar triangles.



- 10**
- Draw a pentagon.
 - Draw all diagonals from one vertex.
 - What is the angle sum of a pentagon?

11 Find x , giving a reason for your answer.



12 A hexagon has three right angles and all other angles equal. Find the size of the equal angles.

13 Find the size of the exterior angle in a regular octagon.



8

Bivariate data analysis

This chapter deals with the analysis of bivariate data.

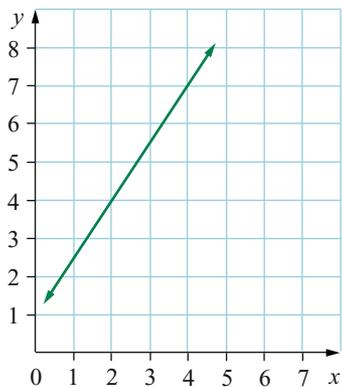
After completing this chapter you should be able to:

- ▶ recognise the difference between dependent and independent variables
- ▶ distinguish bivariate data from single variable data
- ▶ describe bivariate data where the independent variable is time
- ▶ construct a scatter plot with and without technology
- ▶ describe informally the strength and direction of the relationship between two variables
- ▶ make predictions from a scatter plot
- ▶ find the equation of a line of best fit.

Diagnostic test

- 1 For the straight line shown below, when $y = 4$, $x =$

A 5 B $5\frac{1}{2}$ C 7 D 2



- 2 The point that does not lie on the line

$$y = \frac{1}{2}x - 3 \text{ is:}$$

A (4, -1) B (-4, -5)
C (3, $-1\frac{1}{2}$) D (-3, $1\frac{1}{2}$)

- 3 If the point $(-2, 3)$ lies on the line $y = -x + c$ then the value of c is:

A 5 B -5 C 1 D -1

- 4 The gradient of the line joining the points $P(2, -1)$ and $Q(-3, 14)$ is:

A $\frac{1}{3}$ B $-\frac{1}{3}$ C 3 D -3

- 5 The equation of the line joining the points P and Q in question 4 is:

A $y = \frac{1}{3}x - 1\frac{2}{3}$ B $y = -\frac{1}{3}x - \frac{1}{3}$
C $y = 3x - 7$ D $y = -3x + 5$

- 6 The equation of the line passing through $(-2, 3)$ with gradient 2 is:

A $y = 2x + 3$ B $y = 2x - 7$
C $y = 2x - 2$ D $y = 2x + 7$

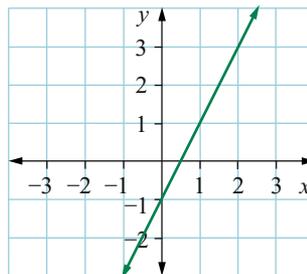
- 7 The equation of the line passing through $K(-3, 2)$ and $L(4, -5)$ is:

A $y = -x - 1$ B $y = x + 5$
C $y = 3x - 7$ D $y = -x + 1$

- 8 The equation of the line cutting the y -axis at 4 and the x -axis at -5 is:

A $y = \frac{4}{5}x + 4$ B $y = -\frac{4}{5}x + 4$
C $y = \frac{5}{4}x + 4$ D $y = -\frac{5}{4}x + 4$

- 9 The equation of this line is:



A $y = \frac{2}{3}x - 1$ B $y = \frac{3}{2}x - 1$
C $y = -x + 3$ D $y = 2x - 1$

- 10 The value of a if $(a, -2)$ lies on the line $y = 3x - 2$ is:

A 0 B 1 C 2 D 3

- 11 In general form $y = \frac{1}{2}x - \frac{2}{3}$ is:

A $6y - 3x + 4 = 0$ B $3x - 6y - 4 = 0$
C $3x + 2y - 2 = 0$ D $\frac{1}{2}x - y - \frac{2}{3} = 0$

- 12 When written in gradient-intercept form the equation $3x - 2y + 8 = 0$ is:

A $2y = 3x + 4$ B $3x - 2y = -8$
C $y = \frac{3}{2}x + 4$ D $y = \frac{3}{2}x + 8$

- 13 The x - and y -intercepts of $3x - 4y - 12 = 0$ are:

A 4 and -3 B 3 and -4
C -4 and 3 D -3 and 4

The diagnostic test questions refer to outcomes ACMNA215, ACMNA238 and ACMNA294.



A

Variables

Variables are quantities that vary or change in value. In practical situations the value of one variable usually depends on the value of another. For example, the number of people waiting at a bus stop depends on the time of day. The first variable, number of people waiting at the bus stop, is called the **dependent variable** and the second, time of day, the **independent variable**.

When graphing the relationship between two variables, the independent variable is placed on the horizontal axis and the dependent variable on the vertical axis.

EXAMPLE 1

Consider the statements below.

- i State the variables being compared.
 - ii Decide which variable is the dependent variable to be graphed on the vertical axis.
- a The height of a burning candle decreases with time.
- b The bigger the hall, the more people can attend the concert.

	Solve	Think	Apply
a i	The height of the candle and the time	The height of the candle depends on the length of time it has been burning. The length of time the candle has been burning does not depend on the height of the candle.	State the two variables being compared and determine which variable is dependent on the value of the other.
ii	The height of the candle		
b i	The size of hall and the number of people	The number of people who can attend the concert depends on the size of the hall. The size of the hall does not depend on the number of people attending a concert.	
ii	The number of people		

Exercise 8A

- 1 Consider the statements below.
- i State the variables being compared.
 - ii Decide which variable is the dependent variable to be graphed on the vertical axis.
- a The weight of a baby increases with age.
- b The value of a diamond increases with its weight.
- c The share of a lottery prize decreases as the number of winners increases.
- d The more goods sold, the greater the commission.
- e The more hours worked, the greater the income.
- f For a fixed distance, the higher the average speed the shorter the time.
- g The more income earned, the more tax paid.
- h The greater the speed, the greater the distance it takes to stop.



B

Bivariate numerical data

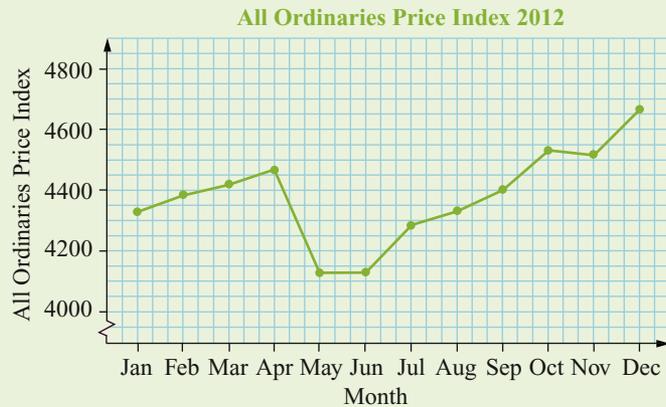
Single variable data analysis examines one variable only; for example, the length of an arm. The data can be analysed using measures of central tendency (mean, mode, median) and measures of spread (for example range).

Bivariate data analysis compares two variables; for example, the length of an arm and height. The data can be graphed using one variable as the dependent variable and the other as the independent variable. One of the most frequent uses of these graphs is a comparison of a quantity over time, where time is the independent variable. These are sometimes referred to as time series.

EXAMPLE 1

The All Ordinaries Price Index is a measure of the collective value of share prices on the Australian Stock Exchange. This graph shows the end of month values of the index for 2012.

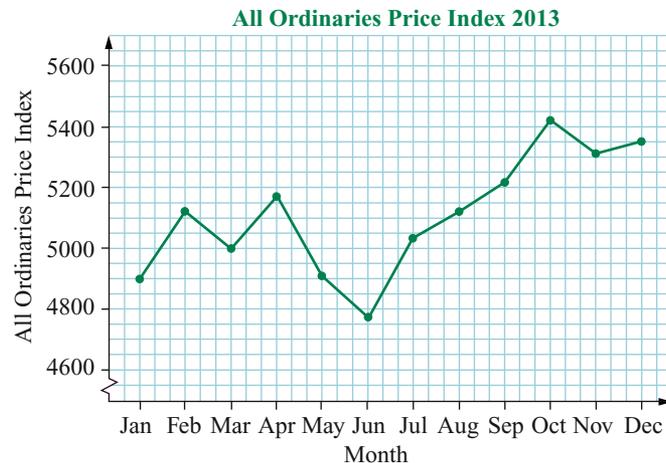
- a** In which month(s) was the index:
- highest?
 - lowest?
- b** Describe the trend for this year.



	Solve	Think	Apply
a i	December	Find the highest and lowest points of the graph.	Determine the highest and lowest points on the graph.
ii	May and June		
b	The index increased steadily until April, dropped for May and June, then gradually increased until it reached its highest value in December.	The trend was upwards until April, then downwards until June, then steadily upwards to the December value.	Look for downwards, horizontal and upwards trends in the graph.

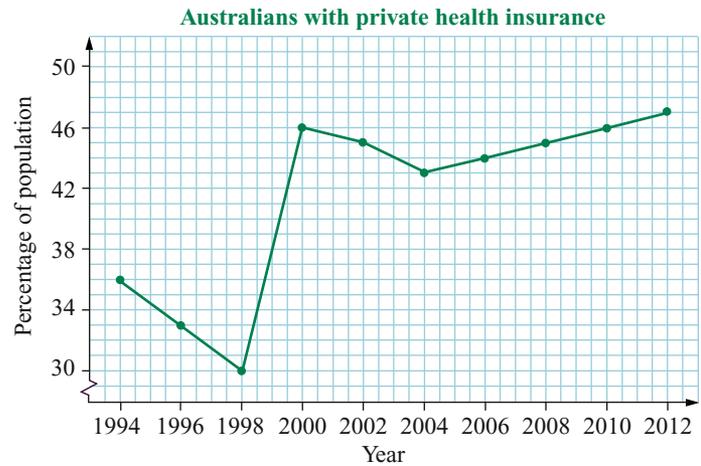
Exercise 8B

- 1** This graph shows the monthly values of the All Ordinaries Price Index for 2013.
- a** In which month(s) was the index:
- highest?
 - lowest?
- b** Describe the trend for this year.
- c** Compare the value of the index in 2012 and 2013.



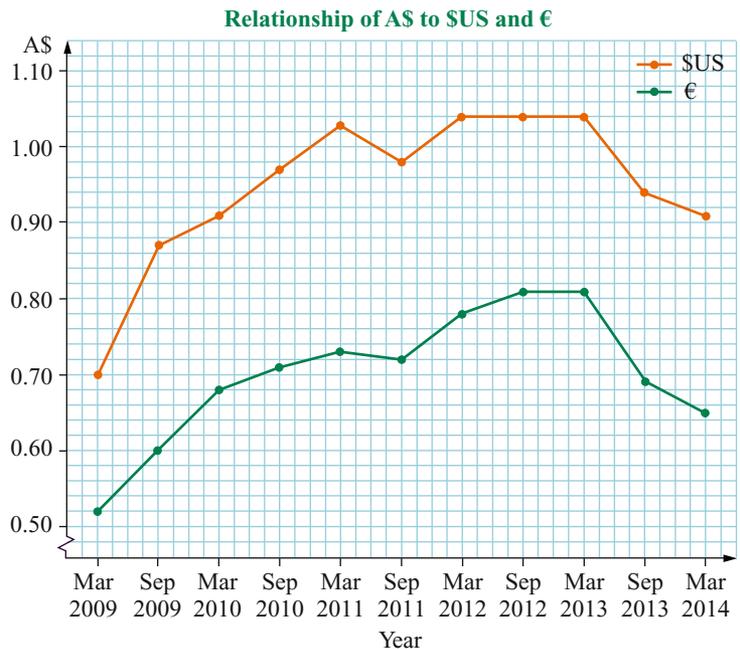
- 2** The graph shows the percentage of Australians who had private health insurance cover in the years 1994 to 2012.

- a** What percentage of the population had private cover in:
i 1996? **ii** 2006?
b Comment on the trends.
c During this time period, the government introduced a 30% rebate on private health insurance to encourage more people to take out private cover. From the graph, state the year in which this might have happened.



- 3** The graph shows the six-monthly exchange rates for the Australian dollar (A\$) against the US dollar (\$US) and the euro (€) from March 2009 to 2014.

- a** How many US dollars was the Australian dollar worth in:
i September 2009?
ii March 2014?
b How many euros was the Australian dollar worth in:
i March 2010?
ii September 2012?
c Which currency exchange rate varied more over this time period?
d When did the A\$ reach its maximum value compared with the \$US? What was this value?
e When did the A\$ reach its maximum value compared with the euro? What was this value?
f What was the biggest fall in the exchange rate with the \$US in this time period? When did this occur?
g Did the biggest fall in the exchange rate with the euro also occur at this time?
h Give a brief description of the change in the exchange rates over this time period.



- 4 a** The table shows the price from 2008 to 2013 paid by milk processing companies in New South Wales to dairy farmers for the milk they produced. Draw a graph using time as the independent variable. Comment on any trends in the price paid over this period.

Year	2008	2009	2010	2011	2012	2013
Price (c/L)	48.6	52.4	48.7	48.3	47.4	46.4

- b** This table shows the average price paid for milk across Australia for the same period. Draw a graph of these prices on the same axes as in part **a**. Compare the price paid in New South Wales with the average price paid in Australia.

Year	2008	2009	2010	2011	2012	2013
Price (c/L)	49.6	42.4	37.3	43.2	42.0	40.2

5 This table shows the mean monthly maximum temperatures ($^{\circ}\text{C}$) in Sydney and Melbourne for 2012.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	26	26	25	24	21	18	18	20	23	23	24	26
Melbourne	27	27	24	22	17	14	15	15	19	21	23	26

- On the same axes draw graphs showing the maximum temperatures in each city. Use time as the independent variable.
- Which city had the colder winter?
- In which city did the biggest fall occur from one month to the next?
 - What was the size of this fall?
- In which city did the biggest increase occur from one month to the next?
 - What was the size of this increase?
- Comment on the shape of these graphs, giving possible reasons.

6 This table shows the mean monthly rainfall (mm) in Sydney and Melbourne for 2012.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	139	111	270	187	37	244	56	19	24	29	52	45
Melbourne	29	60	59	51	83	72	59	53	39	29	37	30

- On the same axes draw graphs showing the monthly rainfall in each city. Use time as the independent variable.
- Which city was the wetter in 2012?
- In Sydney, which month(s) was/were the:
 - driest?
 - wettest?
- In Melbourne, which month(s) was/were the:
 - driest?
 - wettest?
- Which city had the wetter winter?
- Comment on the shape of the graphs. Are there any trends?

7 The graph shows the number of short-term visitor arrivals to Australia for the years 2003 to 2013. The majority of these arrivals are tourists.

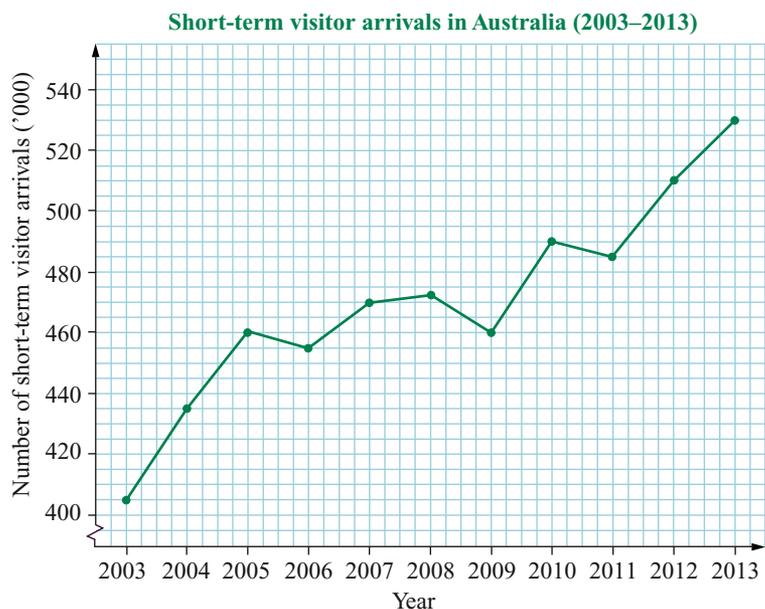
- How many visitors arrived in:
 - 2005?
 - 2012?

- When was the largest increase in the number of visitors from one year to the next?
 - What was this increase?

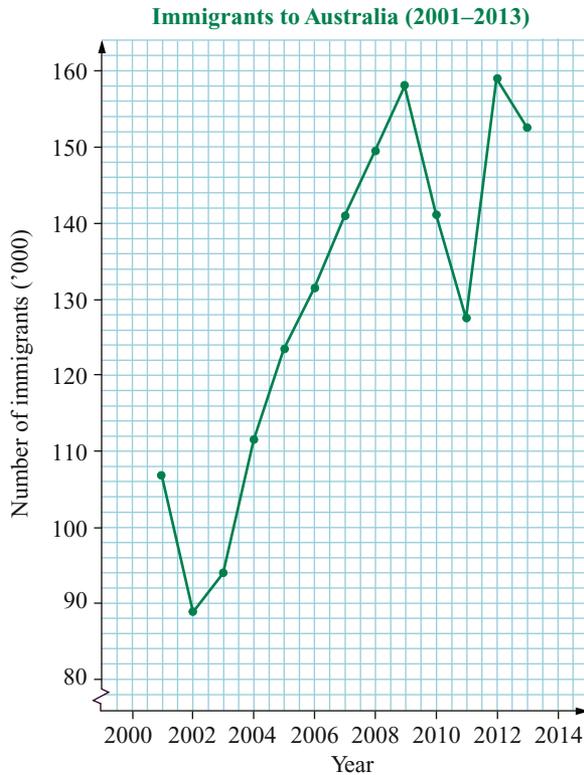
- Were there any decreases in number of visitors from one year to the next?
 - When did this occur?

- The global financial crisis (GFC) had a significant effect on the number of visitors to Australia. From the graph, in which year do you think the GFC occurred?

- Comment on any trends evident in the graph.



- 8 a** The graph below shows the number of immigrants to Australia for 2001 to 2013. How many arrived in:
- i** 2001? **ii** 2011?
 - b** In which year was the number of immigrants:
 - i** highest? **ii** lowest?
 - c** Between which years was there the largest increase in the number of immigrants?
 - d** Between which years was there the largest decrease in the number of immigrants?
 - e** Comment on the shape of the graph and any trends that are evident.



Investigation 1 Variables

Investigate a topic of interest for which graphs of one variable against another can be plotted. Represent the dependent numerical variable against the independent variable, in an appropriate graphical form. For example, plot retail sales, the number of building approvals or job advertisements in your state, city or town against time. Describe the changes in the dependent variable over time and suggest reasons for these changes with reference to relevant national or international events.

C Scatter plots

The aim of many statistical investigations is to determine whether there is a relationship, or association, between the two variables being investigated. For example, medical researchers might be interested in the relationship, if any, between the dose of a drug and the number of patients cured. A business might be interested in the relationship, if any, between the amount of money spent on advertising and an increase in sales.

In this section we will investigate a way of illustrating bivariate data so that a relationship, if it exists, can be identified. A simple method to illustrate numerical data relating two variables is to plot the data as ordered pairs. The resulting diagram is known as a scatter plot (or scattergram).

EXAMPLE 1

The heights and weights of 10 students were measured and the results listed in the table.

Student	1	2	3	4	5	6	7	8	9	10
Height (cm)	179	165	160	179	152	168	168	165	166	166
Weight (kg)	60	55	58	67	48	64	61	52	65	55

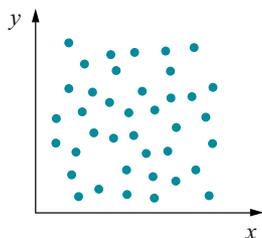
- Illustrate this data on a scatter plot.
- Draw a trend line if it exists.
- Determine whether there is a possible relationship between the variables.

	Solve	Think	Apply
a	<p>Height versus weight</p>	<p>Points are plotted as ordered pairs with height as the independent variable and weight as the dependent variable. In this case a line can be drawn that roughly passes through this set of points.</p>	<p>You can use a spreadsheet to plot the data.</p> <p><i>Step 1:</i> Put the data in a table. <i>Step 2:</i> Highlight the table. <i>Step 3:</i> Click on Chart from the menu bar. <i>Step 4:</i> Choose x-y Scatter from Chart type.</p>
b			
c	<p>There is a trend that as height increases so does weight, but the scatter of the points from the trend line indicates only a weak link between the variables.</p>	<p>There does not appear to be a mathematical connection between the variables that would allow the weight of a student to be predicted accurately from their height.</p>	<p>This line is called a 'line of best fit'. There are different methods of finding the line of best fit, but in this section we will approximate it by drawing a general trend line that has approximately half the points above it and half below it.</p>

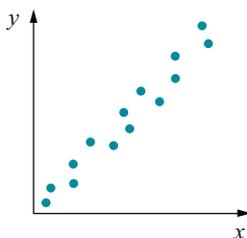
Analysing scatter plots

If the points on a scatter plot are scattered at random, as shown in graph A below, there is no relationship between the variables.

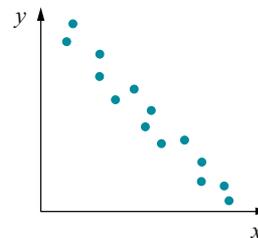
If the points are scattered along a straight line as shown in graphs B and C below, there is a linear relationship between the variables.



Graph A



Graph B



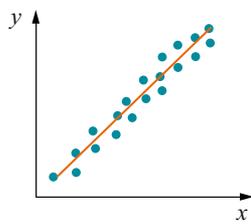
Graph C

If there is an upwards trend, that is the trend line slopes up from left to right as in graph B, there is a positive relationship between the variables. For a positive relationship, large values of one variable, x , are associated with large values of another variable, y , and small values of the variable x are associated with small values of the variable y .

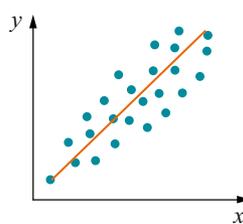
If there is a downwards trend, that is the trend line slopes down from left to right as in graph C, there is a negative relationship between the variables. For a negative relationship, large values of variable x are associated with small values of the variable y , and small values of variable x are associated with large values of variable y .

Further, the association can be classified as strong or weak.

- If the points are closely spread about the trend line, as in graph D, the association is strong or high.
- If there is a trend, but the points are widely scattered about the trend line, as in graph E, the association is weak or low.



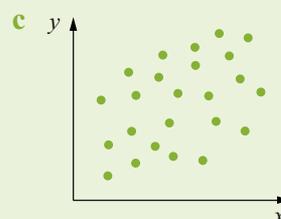
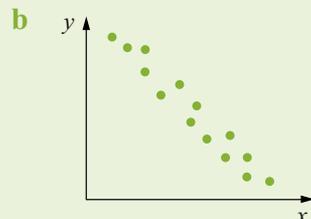
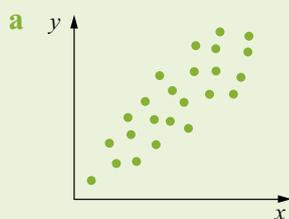
Graph D

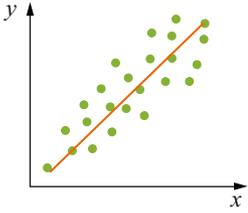
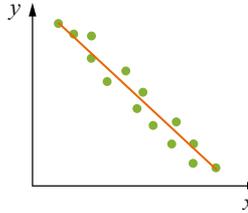
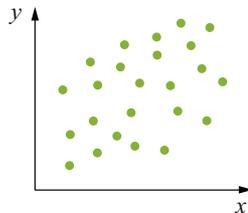


Graph E

EXAMPLE 2

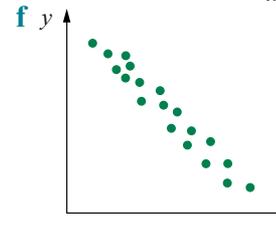
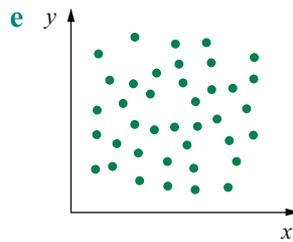
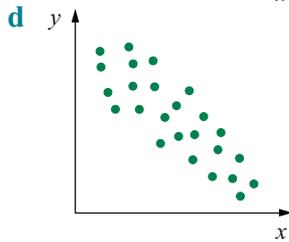
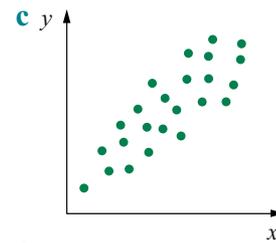
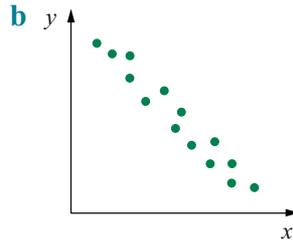
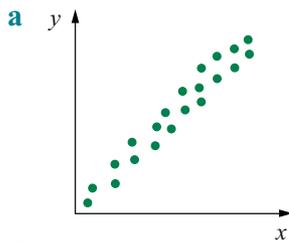
Classify these scatter plots for the strength and direction of the relationship between the variables.



	Solve	Think	Apply
a	Weak positive 	There is an upwards linear trend, but the points are quite scattered about the trend line.	Look for upwards or downwards trends to determine the direction of the relationship. Consider the amount of the scatter of points from the trend line to determine the strength of the relationship. Randomly scattered points indicate no relationship between the variables. <i>The strength of the relationship between two variables is called correlation.</i> !
b	Strong negative 	There is a downwards linear trend with some spread, but with the points quite close to the trend line.	
c	No association 	The points are randomly scattered on the number plane. There is no upwards or downwards trend.	

Exercise 8C

1 For the scatter plots drawn below, classify the strength and direction of the relationship between the variables.



2 For each set of bivariate data given in the tables below:

i draw a scatter plot

ii determine the strength and direction of the relationship between the variables.

a

x	0.5	1	1.5	2	2.5	3	3.5	4
y	1.3	1.7	2.1	2.6	2.5	3	3.5	3.8

b

x	1	2	3	4	5	6	7	8
y	5	2.5	3	3.5	2	1	3	1

3 Draw a scatter plot for two variables that have:

a a strong positive relationship

b a weak negative relationship

c no relationship

d a weak positive association

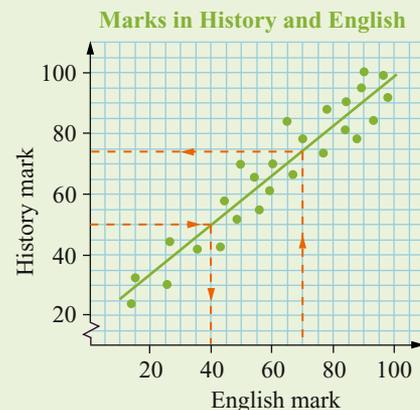
e a strong negative association.

EXAMPLE 3

This scatter plot shows the English marks and History marks for a group of students. A line of best fit has been drawn for these points. Use the line to predict:

a the History marks for a student who scores 70 in English

b the English mark for a student who scores 50 in History.



	Solve	Think	Apply
a	74	Draw a vertical line from 70 on the English axis to meet the line of best fit. From this point, draw a horizontal line to meet the History axis. Read off the mark, about 74, which is the predicted History mark for a student who scores 70 in English.	Locate the point on the line with the given x -coordinate (English mark) and read off the y -coordinate (History mark) of the point.

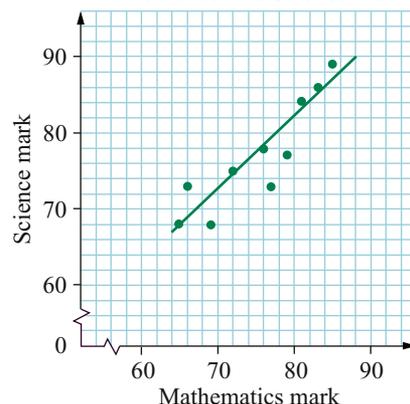
EXAMPLE 3 CONTINUED

	Solve	Think	Apply
b	40	Draw a horizontal line from 50 on the History axis to meet the line of best fit. From this point, draw a vertical line to meet the English axis. Read off the mark, 40, which is the predicted English mark for a student who scores 50 in History.	Find the appropriate x -coordinate (English mark) using the line of best fit.

4 This scatter plot shows the marks in Mathematics and Science for a group of students. A line of best fit has been drawn for these points. Use the line of best fit to predict:

- a** the Science mark of a student who scored the following in Mathematics
- i** 65 **ii** 72 **iii** 84
- b** the Mathematics mark of a student who scored the following in Science
- i** 70 **ii** 76 **iii** 89

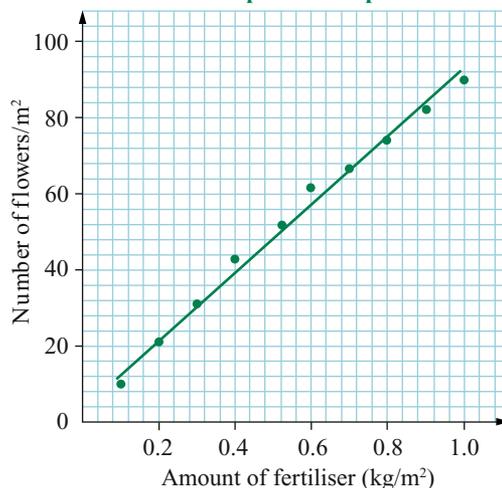
Mathematics and Science marks



5 The scatter plot shows the amount of fertiliser applied (in kg/m^2) and the number of flowers produced per square metre in an experimental garden. A line of best fit has been drawn for these points. Use the line of best fit to predict:

- a** the number of flowers produced by applying these amounts of fertiliser
- i** $0.32 \text{ kg}/\text{m}^2$ **ii** $0.63 \text{ kg}/\text{m}^2$
- b** the amount of fertiliser needed to produce these numbers of flowers/ m^2
- i** $25/\text{m}^2$ **ii** $70/\text{m}^2$

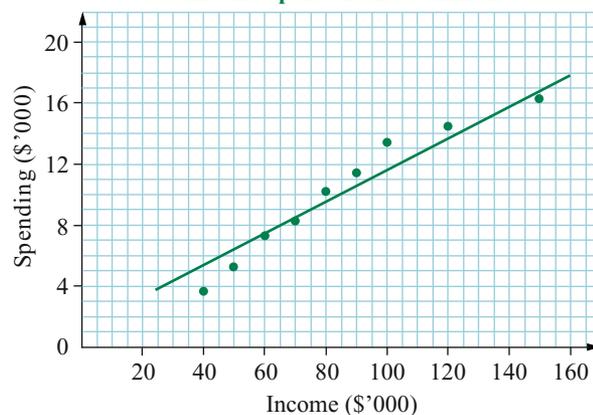
Fertiliser per flowers produced



6 The scatter plot shows the results of a survey of the annual income of a group of people who work in the city and the amount they spent on entertainment. A line of best fit has been drawn for these points. Use the line of best fit to predict:

- a** the amount spent on entertainment by a person who has an annual income of
- i** \$55 000 **ii** \$75 000
- iii** \$110 000
- b** the annual income of a person who spends the following amount on entertainment
- i** \$5000 **ii** \$9500 **iii** \$14 500

Income spent on entertainment



- 8 The data in the table below shows the age and value of a sample of a particular brand of car.

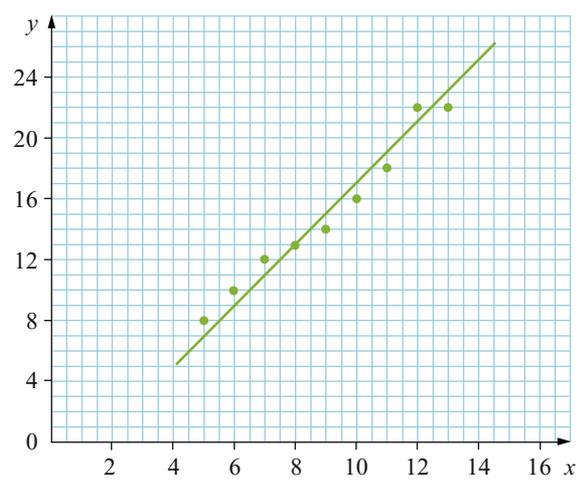
Age (years)	1	1	2	2	3	3	3	4	4	5	5	5	6	6
Value ('000)	27	26	25	25.5	22.5	23	22	18	17.5	14	15	16	13	11.5

- a Draw a scatter plot for the data given in the table.
 b Determine the strength and direction of the relationship between the variables.
 c Draw a line of best fit on the scatter plot.
 d Use the line of best fit to predict:
 i the value of a car of this brand after $3\frac{1}{2}$ years ii the age of a car with a value of \$15 000.

EXAMPLE 5

x	5	6	7	8	9	10	11	12	13
y	8	10	12	13	14	16	18	22	22

- a Draw a scatter plot for the data given in the table above.
 b Draw a line of best fit on the scatter plot.
 c Find the equation of the line of best fit.
 d Use the equation to predict the value of:
 i y when $x = 15$ ii x when $y = 20$.

	Solve/Think	Apply
a b		Plot as points on a number plane.
c	<p>The points (5, 7) and (13, 23) lie on this line.</p> $\text{Gradient of line} = \frac{23 - 7}{13 - 5} = 2$ <p>Equation is of the form $y = 2x + c$.</p> <p>As the point (5, 7) lies on the line:</p> $7 = 2 \times 5 + c$ $7 = 10 + c$ $c = -3$ <p>The equation of the line of best fit is $y = 2x - 3$.</p>	Choose any two suitable points on the line of best fit. Use these points to express the equation in the form $y = mx + c$.
d	<p>i When $x = 15$, $y = 27$.</p> <p>ii When $y = 20$, $x = 11.5$.</p>	Substitute the given values into the equation and solve to find the unknown.

9

x	1	2	3	4	5	6	7	8
y	14	14	10	10	6	5	4	1

- a Draw a scatter plot for the data given in the table above.
- b Draw a line of best fit on the scatter plot.
- c Find the equation of the line of best fit.
- d Use the equation to predict the value of:
 - i y when $x = 10$
 - ii x when $y = 12$.

10 The data in the table shows a comparison of the fitness level of a group of 16-year-old females, measured on a scale of 1 to 20, and the number of hours spent each week playing sport.

Time (h)	1	1	2	2	3	4	5	8	10	15	18	20
Fitness level	3	2	4	3	5	5	6	9	10	15	17	19

- a Draw a scatter plot for the data given in the table above.
- b Determine the strength and direction of the relationship between the variables.
- c Draw a line of best fit on the scatter plot.
- d Find the equation of the line of best fit.
- e Use the equation to predict:
 - i the fitness level of a student who plays sport for 12 hours per week
 - ii the hours of sport a student would need to play each week to attain a fitness level of 16.

Investigation 2 Investigating data

- 1 Investigate a matter of interest that involves two numerical variables such as height versus arm span, hand span, shoe size or head circumference.
 - a Draw a scatter plot for the data (with or without the use of technology).
 - b Determine the strength and direction of the relationship, if any, between the variables.
 - c Draw a line of best fit on the scatter plot.
 - d Find the equation of the line of best fit.



Language in mathematics

- 1 Explain the following, giving examples:
 - a the difference between bivariate data and single variable data
 - b the meaning of the terms 'dependent variable' and 'independent variable'.
- 2 Why are line graphs the most appropriate method of representing data collected over time?
- 3
 - a What is meant by a 'line of best fit'?
 - b For what purpose is a line of best fit used?

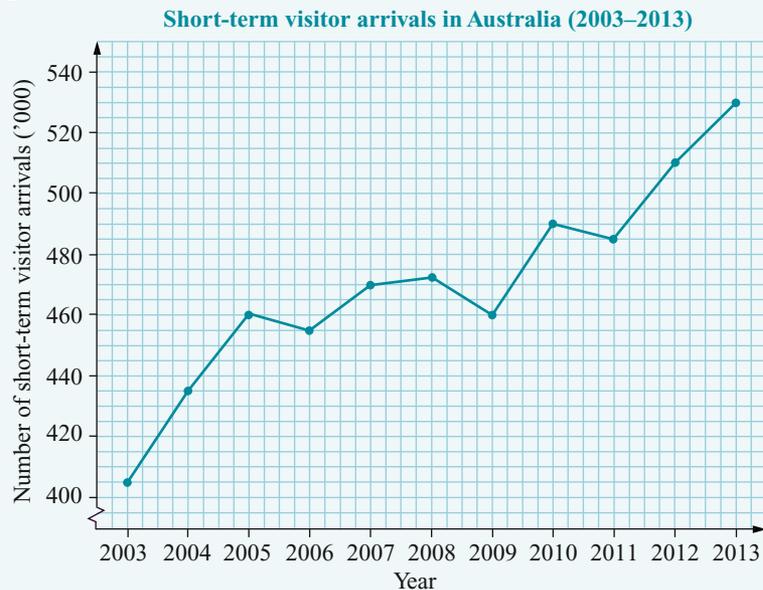
Terms

association	bivariate	correlation	dependent	independent	line of best fit
relationship	scatter plot	time series	trend	trend line	variable

Check your skills

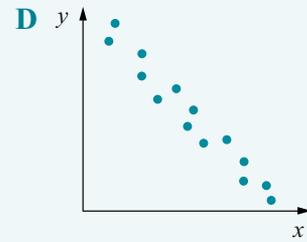
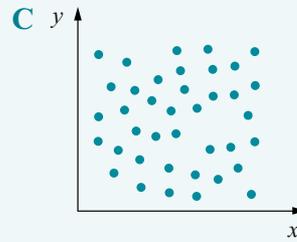
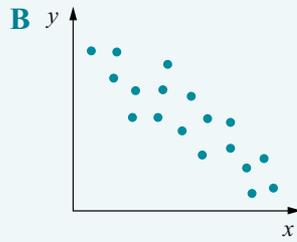
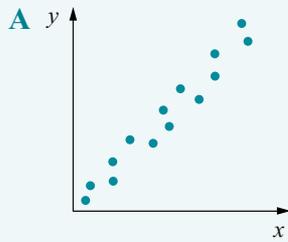
- 1 Which statement is correct for 'the number of people waiting at the station during the day'?
 - A The number of people waiting at the station is the independent variable.
 - B The time of day is the independent variable.
 - C The time of day would be placed on the vertical axis.
 - D The number of people waiting at the station would be placed on the horizontal axis.

The graph below shows the number of short-term visitor arrivals to Australia for the years 2003 to 2013. Use the graph to answer questions 2 and 3.



- 2 Which statement is true for the number of visitors?
 - A The largest increase in the number of visitors occurred between 2008 and 2009.
 - B The number of visitors increased between years on three occasions.
 - C The largest increase in visitors was from 2011 to 2012.
 - D The number of visitors decreased between years on three occasions.
- 3 The number of short-term visitor arrivals was approximately the same in the years:
 - A 2005 and 2006
 - B 2007 and 2008
 - C 2009 and 2010
 - D 2011 and 2012

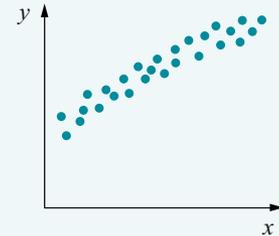
4 The scatter plot below that shows a weak negative relationship between the variables is:



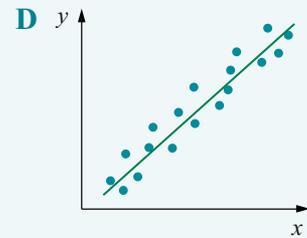
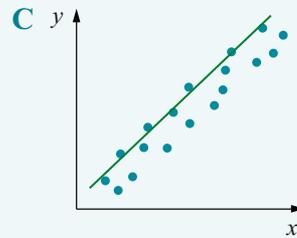
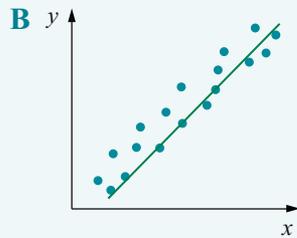
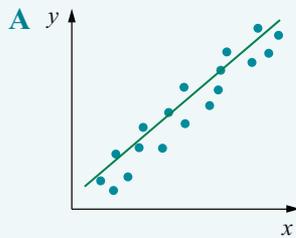
5 This scatter plot shows that the relationship between the variables is:

- A** strong positive
C no relationship

- B** weak positive
D strong negative

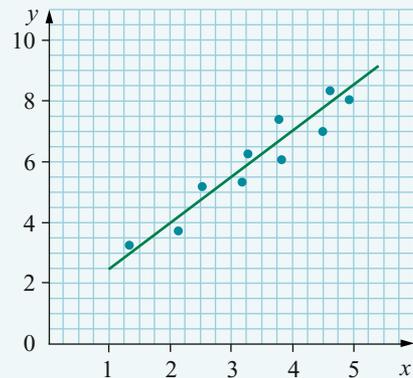


6 The diagram that illustrates a line of best fit for the data graphed in the scatter plot is:



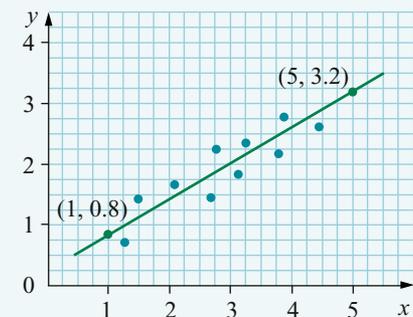
7 Using the line of best fit in the diagram, the value of x when $y = 3.5$ is:

- A** 1.3 **B** 1.7
C 6.3 **D** 7.3



8 The equation of the line of best fit in the diagram using the information given is:

- A** $y = 0.6x + 0.2$
B $y = 0.6x + 0.52$
C $y = \frac{5}{3}x - \frac{13}{15}$
D $y = \frac{5}{3}x - \frac{1}{3}$



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

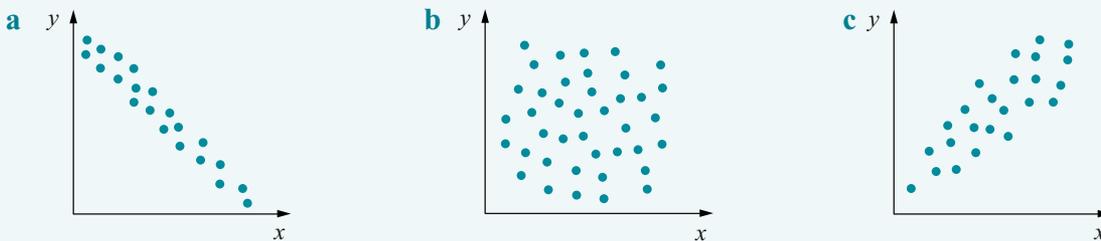
Question	1	2, 3	4-8
Section	A	B	C

8A Review set

- 1 Consider the statements below.
 - i State the variables being compared.
 - ii Decide which variable is the dependent variable to be graphed on the vertical axis.
 - a The fewer people there are, the bigger the slice of cake.
 - b The older a car is, the lower its value.
- 2 The graph shows the exchange rates of the Australian dollar (A\$) against the British pound (GBP) from March 2010 to December 2013.



- a How many British pounds was the Australian dollar worth in:
 - i September 2010?
 - ii March 2011?
 - iii June 2012?
 - b When was the (A\$) worth more than 0.65 GBP?
 - c
 - i When did the (A\$) reach its maximum value compared with the GBP in this time period?
 - ii What was this value?
 - d What was the biggest fall in the exchange rate with the GBP in this time period?
 - e When did this occur?
 - f Comment on any trends in the change in the exchange rates over this time period.
- 3 Classify the strength and direction of the relationship between the variables shown on the scatter plots below.



4

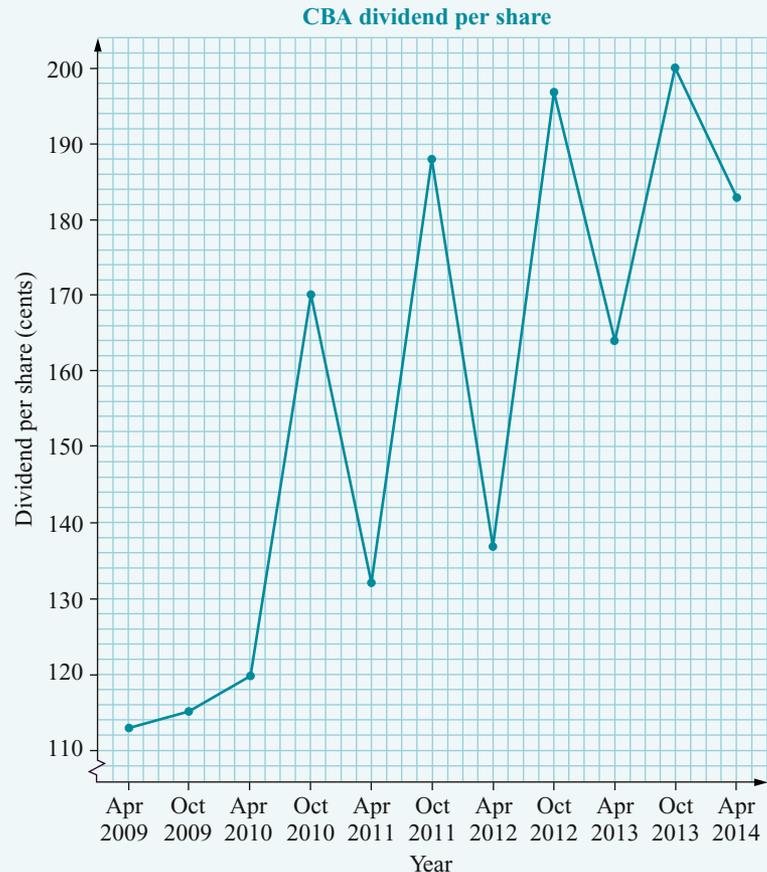
x	10	15	20	25	30	35	40	45	50
y	11	17	22	26	33	38	42	46	52

- a Draw a scatter plot for the data given in the table above.
- b Determine the strength and direction of the relationship between the variables.
- c Draw a line of best fit on the scatter plot.
- d Find the equation of the line of best fit.
- e Use the equation to predict the value of:
 - i y when $x = 32$
 - ii x when $y = 24$

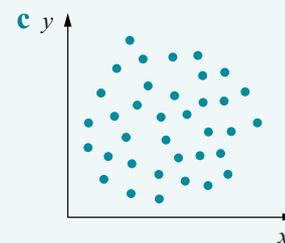
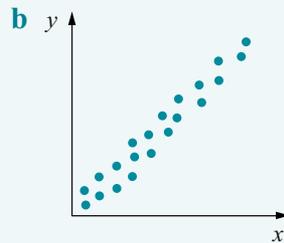
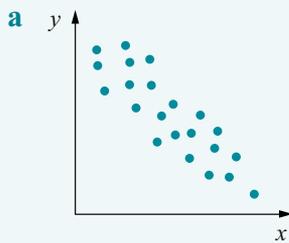
- 1 Consider the statements below.
- State the variables being compared.
 - Decide which variable is the dependent variable to be graphed on the vertical axis.
- The greater the speed of the car, the greater the distance it takes to stop.
 - The more alcohol consumed, the lower the ability to drive.

- 2 The graph shows the six-monthly dividends per share paid by the Commonwealth Bank (CBA) from April 2009 to April 2014.

- What was the dividend paid in:
 - April 2010?
 - October 2011?
 - April 2013?
- What was the maximum dividend paid in this time interval?
 - When did this occur?
- In which six-monthly period did the dividend paid have the:
 - largest increase?
 - largest decrease?
- Give a brief description of the change in the dividends paid over this time period.



- 3 Classify the strength and direction of the relationship between the variables shown on the scatter plots below.



4

x	5	6	7	8	9	10	11	12	13	14
y	35	33	28	27	22	22	15	13	10	7

- Draw a scatter plot for the data given in the table above.
- Determine the strength and direction of the relationship between the variables.
- Draw a line of best fit on the scatter plot.
- Find the equation of the line of best fit.
- Use the equation to predict the value of:
 - y when $x = 15$
 - x when $y = 20$

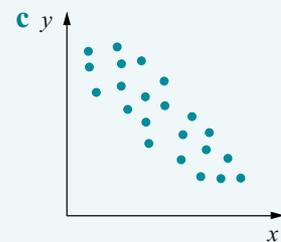
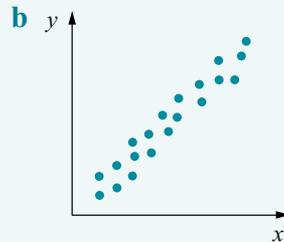
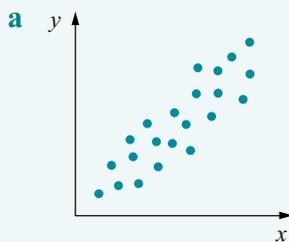
- 1 Consider the statements below.
- State the variables being compared.
 - Decide which variable is the dependent variable to be graphed on the vertical axis.
- a The more police cars on the road, the fewer accidents occur.
- b As the radius of a sphere increases, its volume also increases.

- 2 The graph shows the number of people working in the mining industry in Australia for the years 1992 to 2012.

- a Approximately how many people were employed in:
- 1994?
 - 2001?
 - 2009?
- b i What was the lowest number of people employed in this time period?
ii When did this occur?
- c Estimate when the 'mining boom' began in Australia.
- d Comment on any trends in the change of employment over this time period.



- 3 Classify the strength and direction of the relationship between the variables shown on the scatter plots below.



4

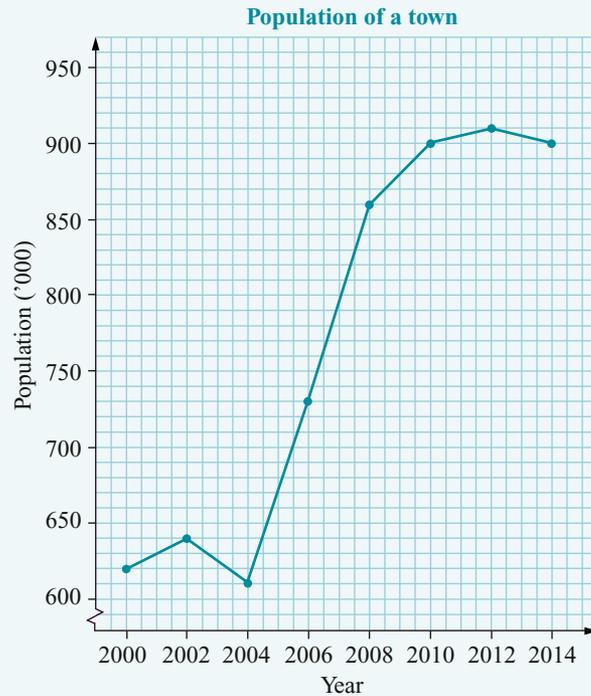
x	10	12	16	22	26	29	33	37	42	49
y	2.6	2.7	3	3.4	3.5	3.9	4.0	4.1	4.5	4.8

- Draw a scatter plot for the data given in the table above.
- Determine the strength and direction of the relationship between the variables.
- Draw a line of best fit on the scatter plot.
- Find the equation of the line of best fit.
- Use the equation to predict the value of:
 - y when $x = 30$
 - x when $y = 3.2$

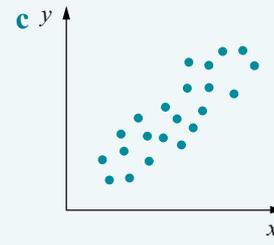
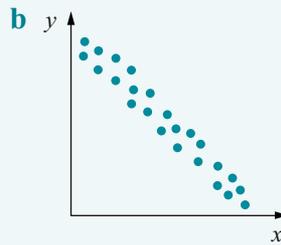
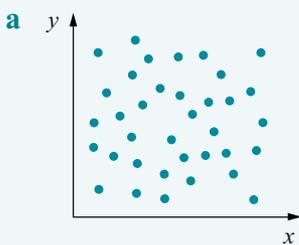
- 1 Consider the statements below.
- State the variables being compared.
 - Decide which variable is the dependent variable to be graphed on the vertical axis.
- a The greater the speed, the shorter the travel time.
- b The more rainy days, the greater the number of umbrellas sold.

- 2 The graph shows the number of people living in a town for the years 2000 to 2012.

- a What was the population of the town in:
- 2002?
 - 2005?
 - 2012?
- b
- What was the lowest population in this period of time?
 - When did this occur?
- c A new mine opened near this town in the time period shown. When does the graph indicate that this occurred?
- d Give a brief description of the change in population over this time period.



- 3 Classify the strength and direction of the relationship between the variables shown on the scatter plots below.



4

x	10	20	30	40	50	60	70	80
y	2.1	2.0	1.8	1.7	1.5	1.3	1.0	0.9

- Draw a scatter plot for the data given in the table above.
- Determine the strength and direction of the relationship between the variables.
- Draw a line of best fit on the scatter plot.
- Find the equation of the line of best fit.
- Use the equation to predict the value of:
 - y when $x = 25$
 - x when $y = 1.6$



9

Financial mathematics

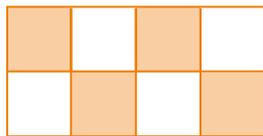
This chapter deals with solving problems involving compound interest and depreciation.

After completing this chapter you should be able to:

- ▶ calculate compound interest by repeated multiplication and using a table
- ▶ use the compound interest formula
- ▶ calculate depreciation using the compound interest formula.

Diagnostic test

- 1 The percentage of this shape that is shaded is:



- A $\frac{1}{2}$ B 50% C 0.5 D $\frac{4}{8}\%$
- 2 41% expressed as a fraction is:
A $4\frac{1}{10}$ B 0.41 C 0.041 D $\frac{41}{100}$
- 3 125% expressed as a simplified fraction is:
A $1\frac{1}{4}$ B $\frac{125}{100}$ C $1\frac{25}{100}$ D $\frac{25}{20}$
- 4 $8\frac{1}{5}\%$ expressed as a decimal is:
A 0.0082 B 0.082
C 0.82 D 8.2
- 5 $\frac{4}{5}$ expressed as a percentage is:
A 20% B 40%
C 60% D 80%
- 6 3.56 expressed as a percentage is:
A 356% B 35.6%
C 3.56% D 0.356%
- 7 \$270 expressed as a percentage of \$300 is:
A 90% B 111.1%
C 11.1% D 10%
- 8 330 mL expressed as a percentage of 2L is:
A 16500% B 1650%
C 165% D 16.5%
- 9 12% of 90 km is:
A 10.8 km B 12.2 km
C 14.6 km D 18.3 km
- 10 $12\frac{1}{2}\%$ of 400 m is:
A 0.05 m B 0.5 m C 5 m D 50 m
- 11 The simple interest on \$2490 at 4.5% p.a. for 5 years is:
A \$112.05 B \$2602.05
C \$560.25 D \$3050.25
- 12 Michelle invested \$3000 for 4 years and earned \$780 in interest. The annual rate of interest was:
A 26% B 6.5% C 1.04% D 4%

Use the values $x = 4$ and $y = 5$ for questions 13 to 15.

- 13 The value of $3xy$ is:
A 345 B 17 C 12 D 60
- 14 The value of $3x - 2y$ is:
A 2 B 9 C 50 D 24
- 15 The value of $2x^2$ is:
A 64 B 32 C 576 D 16
- 16 If $y = 6$ the value of $\frac{5y}{2}$ is:
A 28 B 15 C 35 D 53
- 17 If $k = 5$ the value of $3(k + 2)$ is:
A 21 B 17 C 37 D 11
- 18 Given $k = 5$, the value of $\frac{k + 10}{5}$ is:
A 11 B 3 C 7 D 10

The diagnostic test questions refer to outcomes ACMNA157 and ACMNA176.





Compound interest

If the interest earned each year on an investment is reinvested, the size of the principal on which interest is calculated increases each year. Hence the amount of interest earned also increases. This is called **compound interest**.

EXAMPLE 1

\$6000 is invested for 3 years at 5% p.a. interest, compounded yearly.

- a Calculate the value of the investment after 3 years.
- b What is the total amount of interest earned?

'Compounded yearly' means that the interest is reinvested each year. !

	Solve	Think/Apply																				
a	After 3 years, balance $= \$6615 + \330.75 $= \$6945.75$	For the 1st year, the principal is \$6000. 1st year interest $= 5\%$ of \$6000 $= \frac{5}{100} \times \$6000 = \300 For the 2nd year, the principal is $\$6000 + \$300 = \$6300$. 2nd year interest $= 5\%$ of \$6300 $= \frac{5}{100} \times \$6300 = \315 For the 3rd year, the principal is $\$6300 + \$315 = \$6615$. 3rd year interest $= 5\%$ of \$6615 $= \frac{5}{100} \times \$6615 = \330.75 Hence, the balance after 3 years $= \$6615 + \$330.75 = \$6945.75$. We can set this out in a table. <table border="1" style="margin: 10px auto;"> <thead> <tr> <th>Year</th> <th>Principal</th> <th>Interest (at 5%)</th> <th>Balance at end of year</th> </tr> </thead> <tbody> <tr> <td>1st</td> <td>\$6000</td> <td>\$300</td> <td>\$6300</td> </tr> <tr> <td>2nd</td> <td>\$6300</td> <td>\$315</td> <td>\$6615</td> </tr> <tr> <td>3rd</td> <td>\$6615</td> <td>\$330.75</td> <td>a \$6945.75</td> </tr> <tr> <td></td> <td>Total</td> <td>b \$945.75</td> <td></td> </tr> </tbody> </table>	Year	Principal	Interest (at 5%)	Balance at end of year	1st	\$6000	\$300	\$6300	2nd	\$6300	\$315	\$6615	3rd	\$6615	\$330.75	a \$6945.75		Total	b \$945.75	
Year	Principal	Interest (at 5%)	Balance at end of year																			
1st	\$6000	\$300	\$6300																			
2nd	\$6300	\$315	\$6615																			
3rd	\$6615	\$330.75	a \$6945.75																			
	Total	b \$945.75																				
b	Total interest earned $= \$6945.75 - \6000 (or $\$300 + \$315 + \$330.75$) $= \$945.75$	The total interest earned is the value of the investment at the end of the term minus the initial principal (amount invested), or the sum of all the interest payments.																				

Exercise 9A

- 1 \$5000 is invested for 3 years at 4% p.a. interest, compounded yearly. Complete this table to find the value of the investment after 3 years and the total amount of interest earned.

Year	Principal	Interest (at 4%) p.a.	Balance at end of year
1st	\$5000	\$200	\$__
2nd	\$5200	\$__	\$__
3rd	\$__	\$__	\$__
	Total	\$__	

- 2** \$9000 is invested for 3 years at 2% p.a. interest, compounded yearly.
- Calculate the value of the investment after 3 years.
 - What is the total amount of interest earned.
- 3** \$8400 is invested for 3 years at 6% p.a. interest, compounded yearly.
- Calculate the value of the investment after 3 years.
 - What is the total amount of interest earned?

● EXAMPLE 2

This example uses the constant multiplier method for the data in Example 1. \$6000 is invested for 3 years at 5% p.a. interest, compounded yearly.

The number 1.05 is called a constant multiplier. 

- Calculate the value of the investment after 3 years.
- What is the total amount of interest earned?

	Solve	Think	Apply
a	The value of the investment after 3 years is \$6945.75.	Balance at the end of the 1st year = principal + 5% of principal = 100% of \$6000 + 5% of \$6000 or 105% of \$6000 = $\frac{105}{100} \times \$6000 = 1.05 \times \$6000 = \$6300$ Balance at the end of the 2nd year = principal + 5% of principal = 100% of \$6300 + 5% of \$6300 or 105% of \$6300 = $1.05 \times \$6300 = \6615 Balance at the end of the 3rd year = principal + 5% of principal = 100% of \$6615 + 5% of \$6615 or 105% of \$6615 = $1.05 \times \$6615 = \6945.75	The balance at the end of each year can be found by repeated multiplication by a constant multiplier. In this case $\frac{105}{100} = 1.05$.
b	Total interest earned = \$6945.75 – \$6000 = \$945.75	Total interest earned is the principal at the end of the term minus the initial principal, or the sum of all the interest payments.	

● EXAMPLE 3

- Find the amount to which \$8600 grows in 3 years at 4% p.a. interest, compounded yearly.
- Hence calculate the total amount of interest earned over the 3 years.

	Solve	Think	Apply
a	The value of the investment after 3 years is \$9673.83.	Constant multiplier = $100\% + 4\% = \frac{104}{100} = 1.04$ Balance at end of 1st year = $\$8600 \times 1.04 = \8944 Balance at end of 2nd year = $\$8944 \times 1.04 = \9301.76 Balance at end of 3rd year = $\$9301.76 \times 1.04 = \9673.83	The principal is multiplied by as many of the constant multipliers as there are years.
b	Interest earned = \$9673.83 – \$8600 = \$1073.83	Total interest earned is the principal at the end of the term minus the initial principal.	

4 Complete the following using the constant multiplier method.

a Find the amount to which \$5000 grows in 3 years at 4% p.a. interest, compounded yearly.

$$\begin{aligned}\text{The constant multiplier} &= 100\% + ______ \\ &= \frac{\square}{100} \\ &= 1.04\end{aligned}$$

$$\text{Balance at end 1st year} = ___ \times 1.04 = \$5200$$

$$\text{Balance at end 2nd year} = \$5200 \times 1.04 = ______$$

$$\text{Balance at end 3rd year} = ___ \times ___ = \$5624.32$$

b What is the total amount of interest earned?

$$\begin{aligned}\text{Interest} &= \$5624.32 - ______ \\ &= ______\end{aligned}$$

5 a Calculate the amount to which \$12 000 grows in 3 years at 5% p.a. interest, compounded yearly.

b What is the total amount of interest earned?

6 Complete the table below to find:

a the amount to which \$25 000 grows in 3 years at 3% p.a. interest, compounded yearly

b the total amount of interest earned.

Year	Principal	Balance at end of year
1st	\$25 000	$\$25\,000 \times 1.03 = \$25\,750$
2nd	\$25 750	$\$25\,750 \times 1.03 = ______$
3rd		

7 Use a table similar the one in question 6 to find:

a the amount to which \$16 000 grows in 2 years at 8% p.a. interest, compounded yearly

b the total amount of interest earned.

8 Use a table to find:

a the amount to which \$5800 grows in 3 years at 4.5% p.a. interest, compounded yearly

b the total amount of interest earned.

9 \$14 200 is invested for 2 years at 2.9% p.a. interest, compounded yearly.

a Calculate the value of the investment after 2 years.

b What is the total amount of interest earned?

10 \$16 000 is invested for 3 years at 7.2% p.a. interest, compounded annually.

a Calculate the value of the investment after 3 years.

b What is the total amount of interest earned?

11 a Calculate the amount to which \$11 000 will grow in 3 years if it is invested at 8% p.a.:

i simple interest

ii compound interest.

b Which method of calculating interest produces the greater amount and by how much?

12 a Calculate the amount to which \$30 000 will grow in 4 years if it is invested at 5.6% p.a.:

i simple interest

ii compound interest.

b Which method of calculating interest produces the greater amount and by how much?

Investigation 1 Compound interest formula

Complete the following to calculate the amount to which \$5000 will grow if invested for 10 years at 7% p.a. interest, compounding yearly.

$$\text{Balance at end of 1st year} = \$5000 \times 1.07$$

$$\text{Balance at end of 2nd year} = (\$5000 \times 1.07) \times 1.07 = \$5000 \times 1.07^2$$

$$\text{Balance at end of 3rd year} = (\$5000 \times 1.07^2) \times 1.07 = \$5000 \times \underline{\quad}$$

$$\text{Balance at end of 4th year} = (\$5000 \times 1.07^3) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 5th year} = (\$5000 \times 1.07^4) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 6th year} = (\$5000 \times 1.07^5) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 7th year} = (\$5000 \times 1.07^6) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 8th year} = (\$5000 \times 1.07^7) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 9th year} = (\$5000 \times 1.07^8) \times 1.07 = \underline{\quad}$$

$$\text{Balance at end of 10th year} = \$5000 \times 1.07^{\square}$$

In general, the amount \$A to which \$P will grow if invested for n years at r% p.a. compound interest is given by:

$$A = P(1 + R)^n \text{ where } R = \frac{r}{100}$$

This is known as the **compound interest formula**.

B Compound interest formula

From Investigation 1 the compound interest formula was found to be:

$$A = P(1 + R)^n$$

where \$P is the initial amount invested (principal)

$R = \frac{r}{100}$ and r is the percentage interest rate per compounding period

n is the number of compounding periods

\$A is the final amount (includes principal and interest).

EXAMPLE 1

Use the compound interest formula to calculate:

- the amount to which \$10 000 grows if invested for 8 years at 4.5% p.a. compound interest
- the amount of interest earned over this period.

	Solve	Think	Apply
a	$A = \$10\,000 \left(1 + \frac{4.5}{100}\right)^8$ $= \$10\,000(1.045)^8$ $= \$14\,221.01$	$A = P(1 + R)^n$ $P = 10\,000$ $r = 4.5$ $n = 8$	Substitute the values into the compound interest formula $A = P(1 + R)^n$. The amount is always the original amount plus interest.
b	$\text{Interest} = \$14\,221.01 - \$10\,000$ $= \$4221.01$	Subtract the original amount invested.	

Exercise 9B

- Complete the following using the compound interest formula.
 - Find the amount to which \$6000 grows if it is invested for 5 years at 3% p.a. compound interest.

$$A = \square \left(1 + \frac{\square}{100}\right)^5 = \square (1.03)^\square = \square$$
 - The total amount of interest earned over this period = $\square - \$6000 = \square$
- Use the compound interest formula to calculate:
 - the amount to which \$18 000 grows if it is invested for 7 years at 6% p.a. compound interest
 - the amount of interest earned over this period.
- If I invest \$25 000 at 6.5% p.a. compound interest, how much will I have in 10 years time?
 - Calculate the amount of interest earned over this period.
- If I invest \$5000 at 4.7% p.a. compound interest, how much will I have in 8 years time?
 - Calculate the amount of interest earned over this period.

EXAMPLE 2

Use the compound interest formula to calculate the amount to which \$10 000 grows if it is invested for 5 years at 9% p.a. interest, compounded:

- a** monthly **b** quarterly **c** six-monthly.

	Solve	Think	Apply
a	$A = \$10\,000 \left(1 + \frac{0.75}{100}\right)^{60}$ $= \$10\,000(1.0075)^{60}$ $= \$15\,656.81$	In this case the time period is monthly. Hence $r = \frac{9}{12}\% = 0.75\%$ per month and $n = 5 \times 12 = 60$ months.	The compound interest formula $A = P(1 + R)^n$ where $R = \frac{r}{100}$ can be adapted for this question by using r as the interest rate per time period and n as the number of time periods. <i>Note:</i> The more often the interest is reinvested, the greater the final value of the investment.
b	$A = \$10\,000 \left(1 + \frac{2.25}{100}\right)^{20}$ $= \$10\,000(1.0225)^{20}$ $= \$15\,605.09$	The time period is quarterly. Hence $r = \frac{9}{4}\% = 2.25\%$ per quarter and $n = 5 \times 4 = 20$ quarters.	
c	$A = \$10\,000 \left(1 + \frac{4.5}{100}\right)^{10}$ $= \$10\,000(1.045)^{10}$ $= \$15\,529.69$	The time period is six-monthly. Hence $r = \frac{9}{2}\% = 4.5\%$ per 6 months and $n = 5 \times 2 = 10$ six-monthly periods.	

- Complete the following using the compound interest formula to calculate the amount to which \$10 000 grows if it is invested for 3 years at 12% p.a. interest, compounded:
 - monthly: $r = \frac{\square}{12}\% = \square\%$ per month and $n = 3 \times \square = \square$ months

$$A = 10\,000 \left(1 + \frac{\square}{100}\right)^\square = 10\,000(\square)^{\square} = \square$$
 - quarterly: $r = \frac{12}{\square}\% = \square\%$ per quarter and $n = \square \times 4 = \square$ quarters

$$A = 10\,000 \left(1 + \frac{\square}{100}\right)^{\square} = 10\,000(\square)^{\square} = \square$$
 - six-monthly: $r = \frac{\square}{2}\% = \square\%$ per 6 months and $n = \square \times 2 = \square$ six-month periods

$$A = 10\,000 \left(1 + \frac{\square}{100}\right)^\square = 10\,000(\square)^{\square} = \square$$

- 10** Calculate the interest charged on these credit card cash advances.
- a** \$600 for 19 days when the annual interest rate is 16%
 - b** \$350 for 22 days when the annual interest rate is 16%
 - c** \$200 for 25 days when the annual interest rate is 19%
 - d** \$500 for 15 days when the annual interest rate is 21%
 - e** \$400 for 13 days when the annual interest rate is 14%



Compound interest calculations

At times the final amount of an investment is given and calculations are used to find the principal, the time period or the interest rate.

EXAMPLE 1

Calculate how much money (principal) must be invested at 4% p.a. interest compounding annually to have \$10 000 at the end of 3 years.

Solve	Think	Apply
$10\,000 = P(1 + 0.04)^3$ $= P(1.04)^3$ $\frac{10\,000}{(1.04)^3} = P$ $P = \$8889.96$	$A = P(1 + R)^n$ <p>Substitute the values into the formula. As the interest compounds annually, $R = 0.04$ and $n = 3$.</p>	<p>Carefully check the compounding period. Substitute the answer into the formula to check.</p>

Exercise 9C

- 1** Complete the following to find the amount that must be invested at 6% p.a. interest compounding annually to have \$4000 at the end of 5 years.

$$\begin{aligned} \square &= P\left(1 + \frac{\square}{100}\right)^\square \\ &= P(1.06)^\square \\ \frac{4000}{\square} &= P \\ P &= \square \end{aligned}$$

- 2** Calculate the amount that must be invested at 7% p.a. interest compounding annually to have \$6000 at the end of 4 years.
- 3** Calculate the amount that must be invested at 2.5% p.a. interest compounding annually to have \$1000 at the end of 7 years.
- 4** Calculate the amount that must be invested at 3.6% p.a. interest compounding annually to have \$800 at the end of 6 years.
- 5** Calculate the amount that must be invested at 1.8% p.a. interest compounding annually to have \$400 at the end of 10 years.

EXAMPLE 2

Calculate how much money must be invested at 6% p.a. interest compounding quarterly to have \$5000 after 2 years.

Solve	Think	Apply
$5000 = P(1 + 0.015)^8$ $= P(1.015)^8$ $\frac{5000}{(1.015)^8} = P$ $P = \$4438.56$	$A = P(1 + R)^n$ <p>The investment compounds quarterly. Divide 0.06 by 4 to obtain R. Multiply 2 by 4 to obtain n.</p>	<p>Calculate the values of R and n. In general, R is divided by the number of compounding periods per annum and n is multiplied.</p>

- 6 Complete the following to find the amount of money that must be invested at 8% p.a. interest compounding quarterly to have \$3000 at the end of 3 years.

$$R = \underline{\quad} \div 4 = \underline{\quad} \text{ and } n = 3 \times \underline{\quad} = \underline{\quad}$$

$$A = P(1 + R)^n$$

$$\underline{\quad} = P(1 + 0.02)^{\square}$$

$$\underline{\quad} = P(1.\underline{\quad})^{\square}$$

$$P = \frac{3000}{\square} = \underline{\quad}$$

- 7 Calculate the amount that must be invested at 3.2% p.a. interest compounding quarterly to have \$7000 at the end of 8 years.
- 8 Calculate the amount that must be invested at 6% p.a. interest compounding monthly to have \$500 at the end of 6 years.
- 9 Calculate the amount that must be invested at 4.8% p.a. interest compounding monthly to have \$1200 at the end of 5 years.
- 10 Calculate the amount that must be invested at 2.4% p.a. interest compounding monthly to have \$10 000 at the end of 8 years.
- 11 Calculate the amount that must be invested at 5.1% p.a. interest compounding monthly to have \$450 at the end of 3 years.

EXAMPLE 3

What annual compound interest rate is required to increase \$2000 to \$2500 over 5 years?

Solve	Think	Apply
$2500 = 2000(1 + R)^5$ $\frac{2500}{2000} = (1 + R)^5$ $1.25 = (1 + R)^5$ $\sqrt[5]{1.25} = (1 + R)$ $1.0456\dots = 1 + R$ $\therefore R = 0.0456\dots \approx 0.046$ <p>The interest rate required is 4.6% p.a.</p>	$A = P(1 + R)^n$ <p>Substitute the values. As the compounding period is annual, $n = 5$.</p> $R = \frac{r}{100} \text{ so } r = R \times 100$	<p>Substitute the values and solve the resulting equation. The value of R is for that compounding time period.</p>



- 12** Complete to find the annual compound interest rate required to increase \$1000 to \$1200 over 4 years.

$$A = \underline{\quad}, P = \underline{\quad} \text{ and } \underline{\quad} = 4$$

$$A = P(1 + R)^n$$

$$1200 = \underline{\quad} (1 + R)^\square$$

$$\frac{1200}{\underline{\quad}} = (1 + R)^\square$$

$$\underline{\quad} = (1 + R)^\square$$

$$\sqrt[4]{1.2} = 1 + R$$

$$R = \underline{\quad} - 1 \approx 0.0466$$

The interest rate required is $\underline{\quad}\%$ p.a.

- 13** What annual compound interest rate is required to increase \$500 to \$800 over 6 years?
14 What annual compound interest rate is required to increase \$3500 to \$4000 over 4 years?
15 What annual compound interest rate is required to increase \$200 to \$300 over 8 years?
16 What annual compound interest rate is required to increase \$450 to \$500 over 3 years?

● EXAMPLE 4

What interest rate compounding monthly is required to increase \$500 to \$700 over 3 years?

Solve	Think	Apply
$700 = 500(1 + R)^{36}$ $\frac{700}{500} = (1 + R)^{36}$ $1.4 = (1 + R)^{36}$ $\sqrt[36]{1.4} = 1 + R$ $1.00939 = 1 + R$ $R = 0.00939$ $r = 0.00939 \times 12 \times 100$ $= 11.27\% \text{ p.a.}$ <p>The interest rate required is 11.27% p.a.</p>	$A = P(1 + R)^n$ $A = 700, P = 500$ <p>3 years is 36 months so $n = 36$.</p> <p>Subtract 1 from both sides. This is R for 1 month. Multiply R by 12 to make the rate annual. Multiply by 100 to make it a percentage.</p>	<p>Substitute the values and calculate. The number of time periods depends on the frequency of compounding the interest. The value of R is in terms of the compounding period.</p>

- 17** Complete to find the interest rate compounding monthly required to increase \$900 to \$1200 over 6 years.

$$A = \underline{\quad}, \underline{\quad} = 900 \text{ and } n = \underline{\quad} \times 12 = \underline{\quad}$$

$$A = P(1 + R)^n$$

$$1200 = \underline{\quad}(1 + R)^\square$$

$$\frac{1200}{\underline{\quad}} = (1 + R)^\square$$

$$\underline{\quad} = (1 + R)^{72}$$

$$\sqrt[72]{1.333} = \underline{\quad}$$

$$R = \underline{\quad} - 1 = \underline{\quad}$$

$$r = \underline{\quad} \times 12 \times \underline{\quad} = \underline{\quad}$$

- 18** What interest rate compounding monthly is required to increase \$8000 to \$10 000 over 5 years?
19 What interest rate compounding monthly is required to increase \$100 to \$125 over 4 years?

- 20** What interest rate compounding quarterly is required to increase \$400 to \$450 over 3 years?
- 21** What interest rate compounding quarterly is required to increase \$900 to \$1200 over 6 years?
- 22** What interest rate compounding half yearly is required to increase \$1000 to \$1300 over 8 years?

● EXAMPLE 5

The amount of \$7300 is obtained when \$6000 is invested at 5% interest, compounding annually. Use guess, check and refine to calculate the value of n .

Solve	Think	Apply
$A = P(1 + R)^n$ $7300 = 6000(1.05)^n$ $6000(1.05)^3 = \$6945.75$ $6000(1.05)^6 = \$8040.57$ $6000(1.05)^4 = \$7293.04$ <p>The time is approximately 4 years.</p>	<p>Substitute the values into the compound interest formula.</p> <p>Try $n = 3$ too small</p> <p>Try $n = 6$ too big</p> <p>Try $n = 4$ very close</p>	<p>Try different values for n.</p> <p>If the value is less, then use a bigger value of n.</p>

- 23** The amount of \$2020 is obtained when \$1500 is invested at 5.1% p.a. interest, compounding annually. Use guess, check and refine to calculate the value of n .

$$A = P(1 + R)^n$$

$$2020 = ___(1 + 0.051)^n = 1500(1.051)^n$$

Try $n = 4$: $1500(1.051)^4 = ___ \quad$ too $___$

Try $n = 8$: $1500(1.051)^8 = ___ \quad$ too $___$

Try $n = 6$: $1500(1.051)^6 = ___ \quad$ very close

The time is approximately 6 years.

- 24** The amount of \$2700 is obtained when \$2000 is invested at 3.4% p.a. interest, compounding annually. Use guess, check and refine to calculate the value of n .
- 25** The amount of \$800 is obtained when \$600 is invested at 4.2% p.a. interest, compounding annually. Use guess, check and refine to calculate the value of n .

Extension

● EXAMPLE 6

How long would it take to obtain \$1000 if \$600 is invested at 4.8% p.a. interest, compounding monthly? Use the guess, check and refine method. Give the answer to the nearest month.

Solve	Think	Apply
$A = P(1 + R)^n$ $1000 = 600(1 + 0.004)^n$ $600(1.004)^{100} = \$894$ $600(1.004)^{150} = \$1091.96$ $600(1.004)^{130} = \$1008.17$ $600(1.004)^{128} = \$1000.15$ <p>The time is 10 years 8 months.</p>	<p>Substitute using</p> $R = 0.048 \div 12 = 0.004$ <p>Try $n = 100$ too small</p> <p>Try $n = 150$ too big</p> <p>Try $n = 130$ very close</p> <p>Try $n = 128$ correct</p> <p>Thus $n = 128$ months or 10 years 8 months.</p>	<p>The value of n will be approximate and may take many attempts to obtain the correct value.</p>

- 26** The amount of \$600 is obtained when \$550 is invested at 3.2% p.a. interest, compounding quarterly. Use guess, check and refine to calculate the value of n .
- 27** The amount of \$1400 is obtained when \$1300 is invested at 5% p.a. interest, compounding quarterly. Use guess, check and refine to calculate the value of n .
- 28** The amount of \$820 is obtained when \$750 is invested at 3.6% p.a. interest, compounding monthly. Use guess, check and refine to calculate the value of n .
- 29** The amount of \$5100 is obtained when \$5000 is invested at 2.4% p.a. interest, compounding monthly. Use guess, check and refine to calculate the value of n .

Investigation 2 Using a spreadsheet

The following spreadsheets can be used for compound interest calculations. Enter the formulas as shown to perform the calculations. Use the spreadsheets to confirm the answers to the questions in Exercise 9C.

- 1** This spreadsheet can be used to calculate the final amount of an investment when the principal, interest rate and time period are known.

	A	B	C	D
1	Compound interest calculator			
2	Principal	Interest rate as a percentage	Number of years	Number of compounding periods per year
3	8889.96	6	3	1
4				
5	Final amount			
6	$=A3*(1+B3/(100*D3))^(C3*D3)$			
7				
8				
9				

- 2** This spreadsheet can be used to calculate the interest rate required to achieve a final amount of an investment when the principal and time period are known.

	A	B	C	D
1	Compound interest calculator			
2	Principal	Final amount	Number of years	Number of compounding periods per year
3	2000	2500	5	1
4				
5	Interest rate	Annual rate as a percentage		
6	$= (B3/A3)^{(1/C3)} - 1$			
7		$= D3 * A6 * 100$		
8				
9				

- 3 This spreadsheet can be used to calculate the time period required to achieve a final amount of an investment when the principal and time period are known. Enter the formulas for this spreadsheet and then fill down until the amount required is shown. The number in column A shows the number of time periods.

	A	B	C	D
1	Compound interest calculator			
2	Principal	Final amount	Interest rate as a percentage	Number of compounding periods per year
3	5000	5100	2.4	1
4				
5	Time period	Amount		
6	1	=A\$3*(1+\$C\$3/(\$D\$3*100))^A6		Fill down A7 and B7 until the amount is located.
7	=A6+1	=A\$3*(1+\$C\$3/(\$D\$3*100))^A7		
8				

D Depreciation

Many items such as cars, machinery and computers lose value over time because of wear and tear from usage, becoming out of date, and so on. This loss is called **depreciation**.

EXAMPLE 1

- a A farm tractor depreciates each year by 20% of its value at the start of the year. Calculate the value after 3 years of a new tractor that costs \$35 000.
- b Find the amount the tractor has been depreciated over the 3 years.



	Solve	Think	Apply
a	Value after 1 year = 80% of \$35 000 = \$28 000 Value after 2 years = 80% of \$28 000 = \$22 400 Value after 3 years = 80% of \$22 400 = \$17 920	As the tractor loses 20% of its value each year, its value at the end of the year will be $100\% - 20\% = 80\%$ of its value at the start of the year.	Depreciation is similar to compound interest except that the 'interest' is subtracted, thus reducing the value.
b	Amount of depreciation = \$35 000 - \$17 920 = \$17 080	Subtract the depreciated value from the original cost.	

Exercise 9D

- 1** A new car that costs \$30 000 is depreciated by 20% of its value at the start of each year.
- a** Complete the following to find the car's value after 3 years.
 Rate of depreciation = $100\% - \underline{\hspace{1cm}} = 80\%$
 Value after 1 year = 80% of $\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
 Value after 2 years = 80% of $\underline{\hspace{1cm}} = 19\,200$
 Value after 3 years = 80% of $19\,200 = \underline{\hspace{1cm}}$
- b** What is the amount of depreciation over the 3 years?
 Amount of depreciation = $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- 2 a** Calculate the value after 2 years of a new car that costs \$25 000 and depreciates by 18% of its value at the start of each year.
- b** Find the amount of depreciation over the 2 years.
- 3 a** Calculate the value after 3 years of a new computer that costs \$4000 and depreciates by 25% of its value at the start of each year.
- b** Find the amount of depreciation over the 3 years.

EXAMPLE 2

- a** A new car that costs \$35 000 depreciates by 22% of its value at the start of each year. Calculate the value after 3 years.
- b** Find the amount of depreciation over the 3 years

	Solve	Think/Apply
a	$V = P(1 - R)^n$ $= \$35\,000 \left(1 - \frac{22}{100}\right)^3$ $= \$16\,609 \text{ to the nearest dollar}$	The compound interest formula can be adapted for these calculations as follows. $V = P(1 - R)^n$ where V is the depreciated value, P is original value, R is the rate of depreciation per year and n is the number of years.
b	Amount of depreciation $= \$35\,000 - \$16\,609 = \$18\,391$	Subtract the depreciated value from the original amount.

- 4** Use the depreciation (compound interest) formula to complete the following.
- a** A new forklift that costs \$48 000 depreciates by 25% of its value at the start of each year. Calculate the forklift's value after 3 years.
 $P = \underline{\hspace{1cm}}, R = 25\%$ and $n = \underline{\hspace{1cm}}$

$$V = P(1 - R)^n$$

$$= \underline{\hspace{1cm}} \left(1 - \frac{25}{100}\right)^{\underline{\hspace{1cm}}} = \underline{\hspace{1cm}}$$
- b** What is the amount of depreciation over the 3 years?
 Amount of $\underline{\hspace{1cm}} = 48\,000 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- 5 a** Calculate the value after 5 years of a photocopier that costs \$21 000 and depreciates by 18% of its value at the start of each year.
- b** What is the amount of depreciation over the 5 years?

- 6 a** Calculate the value after 2 years of a computer that costs \$2500 and depreciates by 16% of its value at the start of each year.
- b** What is the amount of depreciation over the 2 years?
- 7 a** Calculate the present value of office furniture purchased 6 years ago for \$11 900 if the rate of depreciation is 15% p.a.
- b** How much has the furniture depreciated in value?
- 8 a** What is the current value of a security system installed 5 years ago for \$17 600 if the rate of depreciation is 28% p.a.?
- b** How much has the security system depreciated in value?

● EXAMPLE 3

A new grand piano costs \$32 000 and depreciates by 15% of its value at the start of each year. Use the guess and refine method to find how long it will take for the piano to depreciate in value to \$16 000.

Solve	Think	Apply
$V = P(1 - R)^n$ $= \$32\,000\left(1 - \frac{15}{100}\right)^n$ $= \$32\,000(0.85)^n$	Use the depreciation formula to find the value of the piano after n years. Substitute the values.	The value of n will be approximate and may take many attempts to obtain the correct or a close value.
$V = \$32\,000(0.85)^3$ $= \$19\,652$	Try $n = 3$ too high	<i>Use the guess, check and refine method.</i> 
$V = \$32\,000(0.85)^4$ $= \$16\,704$	Try $n = 4$ close	
$V = \$32\,000(0.85)^5$ $= \$14\,199$	Try $n = 5$ too low	
The piano is worth \$16 000 after approximately 4 years.		

- 9** The price of a new boat is \$28 000. It depreciates by 20% of its value at the start of each year. Complete the following using the guess and refine method to find how long it will take for the boat to depreciate in value to \$11 000.

$$V = \underline{\quad}, P = \$28\,000 \text{ and } R = \underline{\quad}$$

$$V = P(1 - R)^n$$

$$11\,000 = \underline{\quad}(1 - 0.2)^n$$

$$11\,000 = 28\,000(0.8)^n$$

$$\text{Try } n = 8: 28\,000(0.8)^8 = \underline{\quad} \quad \text{too } \underline{\quad}$$

$$\text{Try } n = 4: 28\,000(0.8)^4 = \underline{\quad} \quad \text{close}$$

The boat is worth \$11 000 after approximately 4 years.

- 10** The price of a new motor bike is \$36 000. It depreciates by 22% of its value at the start of each year. Use the guess and refine method to find how long it will take for the motor bike to be worth half its original price.



Language in mathematics

- Using examples, explain the difference between simple interest and compound interest.
- Complete the following words by replacing the vowels.

a pr__nc__p__l	b m__n__m__m	c __nv__stm__nt
d m__lt__pl__r	e r__p__ym__nt	
- Three of the following words have been spelt incorrectly. Write these words with their correct spelling.

a quarterly	b constant	c depreciation
d intrest	e reducible	f purchase

Terms

annually balance compound interest constant multiplier depreciation equivalent
 interest investment principal quarterly reducible repayment

Check your skills

- The calculations for \$5000 invested for 3 years at 4% p.a. interest compounded yearly are set out in the table.

Year	Principal	Interest (at 4%) p.a.	Balance at end of year
1st	\$5000	\$200	x
2nd	\$5200	y	\$5408
3rd	\$5408	\$216.32	\$5624.32

The amounts missing from the table above are:

- | | |
|---------------------------|---------------------------|
| A $x = \$5000, y = \200 | B $x = \$5000, y = \208 |
| C $x = \$5200, y = \208 | D $x = \$5200, y = \200 |
- The calculations for the amount to which \$24 000 grows in 3 years at 6.5% p.a. interest compounded yearly are set out in the table.

Year	Principal	Balance at end of year
1st	\$24 000	$\$24\,000 \times x = \$25\,560$
2nd	\$25 560	$\$25\,560 \times 1.065 = \$27\,221.40$
3rd	\$27 221.40	$= \$y$

The amounts missing from the table above are:

- | | |
|-------------------------------|----------------------------|
| A $x = 0.065, y = 28\,990.79$ | B $x = 0.065, y = 1769.39$ |
| C $x = 1.065, y = 28\,990.79$ | D $x = 1.065, y = 1769.39$ |
- \$12 000 is invested for 3 years at 4.5% p.a. interest, compounded yearly. The amount of interest earned is:

A \$13 693.99	B \$1693.99	C \$1620	D \$13 620
---------------	-------------	----------	------------
 - The amount to which an investment of \$9000 grows if it is invested for 7 years at 3% p.a. interest compounded yearly is:

A $\$9000(1 + 0.07)^3$	B $\$9000(1 + 0.03) \times 7$
C $\$(9000 + (1 + 0.03)^7)$	D $\$9000(1 + 0.03)^7$

- 5** The amount of interest earned when \$15 000 is invested for 12 years at 6% p.a. interest compounded yearly is (to the nearest dollar):
A \$30 183 **B** \$29 607 **C** \$10 800 **D** \$15 183
- 6** The amount to which an investment of \$10 000 grows if invested for 4 years at 3% p.a. interest compounded monthly is:
A \$10 100.38 **B** \$11 273.28 **C** \$32 714.90 **D** \$41 322.52
- 7** The interest charged on a credit card cash advance of \$500 for 18 days if the annual interest rate is 24% and compound interest is charged daily is (to the nearest cent):
A \$120 **B** \$0.33 **C** \$5.92 **D** \$5.95
- 8** If \$6000 is invested at 4% p.a. interest for 7 years, the investment will earn the greatest amount when the interest is compounded:
A yearly **B** six-monthly **C** quarterly **D** monthly
- 9** The amount of money that must be invested at 8% p.a. interest compounding quarterly to have \$5000 at the end of 3 years is:
A \$1986 **B** \$3942 **C** \$3969 **D** \$54 712
- 10** A new car that costs \$28 000 depreciates by 22% of its value at the start of each year. Its value after 5 years is:
A $\{28\,000(1 + 0.22) \times 5\}$ **B** $\{28\,000(1 - 0.22) \times 5\}$
C $\{28\,000(1 + 0.22)^5\}$ **D** $\{28\,000(1 - 0.22)^5\}$
- 11** The amount of depreciation over 3 years of a computer that costs \$18 000 and depreciates by 28% of its value at the start of the year is (to the nearest dollar):
A \$15 120 **B** \$11 282 **C** \$6718 **D** \$2880

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3–8	9	10, 11
Section	A	B	C	D

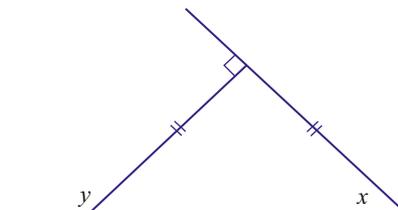
9A Review set

- 1** \$8000 is invested for 3 years at 7% p.a. interest, compounded (reinvested) yearly.
a Calculate the value of the investment after 3 years.
b What is the total amount of interest earned?
- 2** Complete the table below to calculate:
a the amount to which \$30 000 grows in 3 years at 4% p.a. interest, compounded yearly
b the total amount of interest earned.

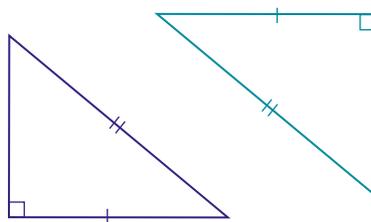
Year	Principal	Balance at end of year
1st	\$30 000	$\$30\,000 \times 1.04 = \$31\,200$
2nd	\$31 200	$\$31\,200 \times 1.04 = \$______$
3rd	$\$______$	$\$______$

- 1 a** On the same set of axes, draw accurate graphs of $y = 4x - 3$ and $y = 11 - 3x$.
- i** Use your graph to solve $y = 4x - 3$ and $y = 11 - 3x$ simultaneously.
 - ii** Check your answer by solving algebraically.
- b** Solve $y = 5x - 2$ and $y = 3x + 6$ simultaneously by substitution.
- c** Solve $2x - 3y = 8$ and $5x + 2y = -3$ simultaneously by elimination.
- d** Three pies and five sausage rolls cost \$10.15 while two pies and three sausage rolls cost \$6.45. Find the cost of one pie and one sausage roll.

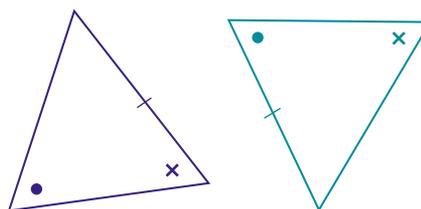
- 2 a** Find the value of x and y .



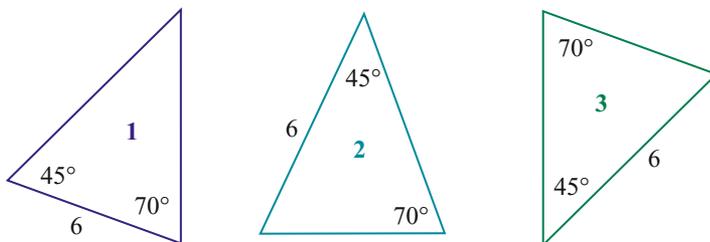
- b** State the congruency test used to show that these triangles are congruent.



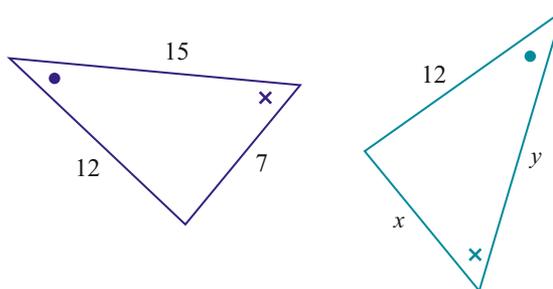
- c** State why the following pairs of figures are not congruent.



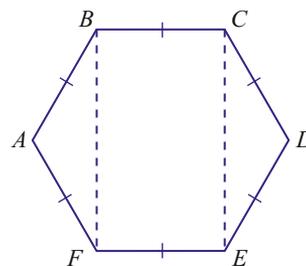
- d** Which triangles are congruent? Give a reason for your answer.



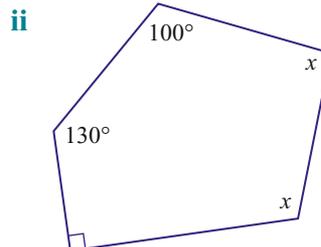
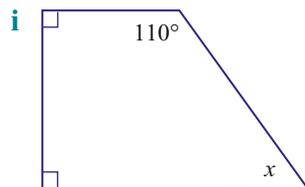
- e** State the congruency test showing that these triangles are congruent, and hence find the values of the pronumerals.



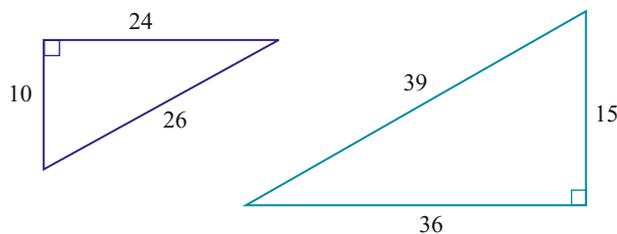
- f** $ABCDEF$ is a regular hexagon; that is, all sides are equal and all interior angles are equal. Prove that $BF = CE$.



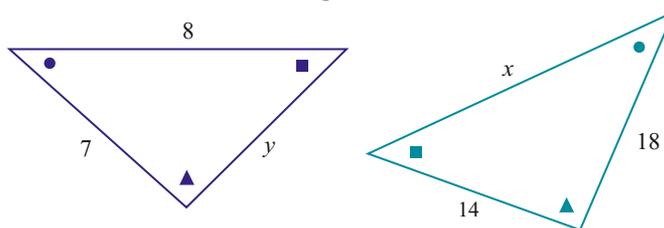
- g** Find x , giving a reason for your answer.



- h** Determine whether or not the following triangles are similar. If they are similar, state the test used.



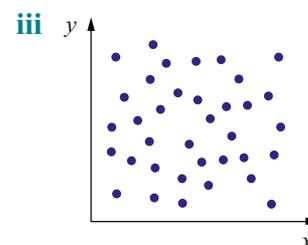
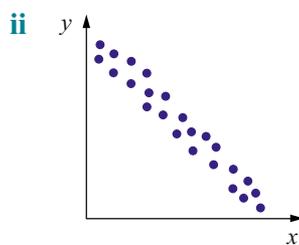
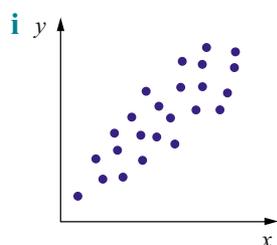
- i** Find the enlargement factor and hence the length of the unknown sides in the similar triangles shown.



- 3 a** Consider this statement: ‘The more sunny days, the more sunscreen sold’.

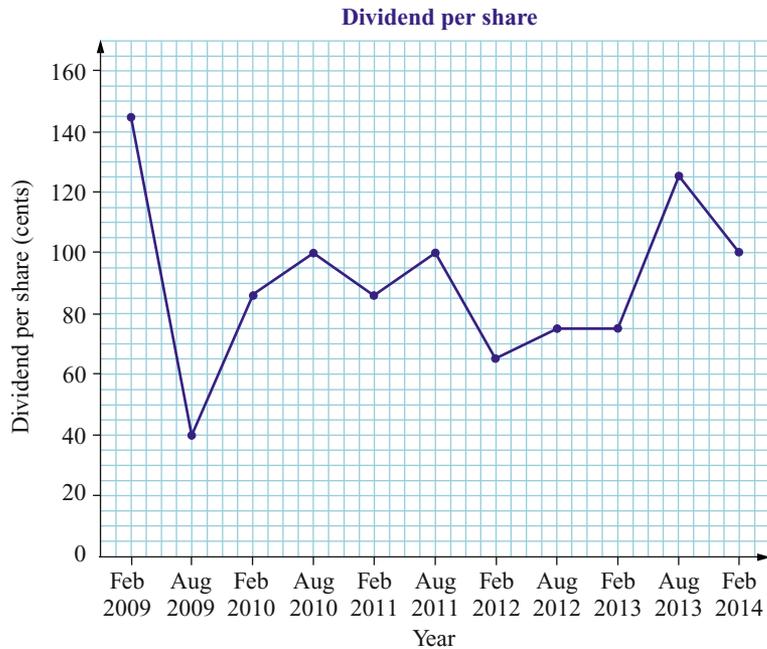
- i** State the variables being compared.
ii Decide which is the dependent variable to be graphed on the vertical axis.

- b** For the scatter plots classify the strength and direction of the relationship between the variables.



4 The graph shows the six-monthly dividends for a share.

- a What was the dividend paid in
- February 2010?
 - August 2011?
 - February 2013?
- b i What was the maximum dividend paid in this time interval?
- When did this occur?
- c In which six-monthly period was there the biggest:
- increase in the dividend paid?
 - decrease in the dividend paid?
- d Give a brief description of the change in the dividends paid over this time period?



5

x	15	16	17	18	19	20	21	22	23	24
y	42	40	35	34	29	29	22	20	17	14

- a Draw a scatter plot for the data given in the table.
- b Determine the strength and direction of the relationship between the variables.
- c Draw a line of best fit on the scatter plot.
- d Find the equation of the line of best fit.
- e Use the equation to find the value of:
- y when $x = 26$
 - x when $y = 27$

- 6 a \$3600 is invested for 3 years at 3.9% p.a. interest, compounding yearly.
- Calculate the value of the investment after 3 years.
 - What is the total amount of interest earned?
- b i Calculate the amount to which \$20 000 grows in 3 years at 6% p.a. interest, compounded yearly.
- What is the total amount of interest earned?
- c Calculate the amount to which \$15 000 will grow in 5 years if it is invested at 4.5% p.a.:
- simple interest
 - compound interest.
 - Which method of calculating interest produces the greater amount, and by how much?
- d Calculate the amount to which \$15 000 will grow in 5 years if it is invested at 8% p.a. and interest is compounded:
- monthly
 - quarterly
 - six-monthly.
- e Calculate the compound interest charged on a credit card cash advance of \$500 for 17 days if the annual interest rate is 28% and the interest is charged daily.
- f i Calculate the value after 4 years of a new car that costs \$38 000 and depreciates by 19% of its value at the start of each year.
- What is the amount of depreciation over the 3 years?
- g The purchase price of a new car is \$25 700. It depreciates by 18% of its value at the start of each year. Use the guess and refine method to find how long it will take for the car to depreciate in value to \$12 000.



10

Binomial expressions and quadratics

This chapter deals with expanding binomial products, factorising quadratic expressions and solving simple quadratic equations.

After completing this chapter you should be able to:

- ▶ expand quadratic expressions
- ▶ factorise quadratic trinomial expressions
- ▶ solve simple quadratic equations
- ▶ solve quadratic equations by factorising.

Diagnostic test

- 1 The diagram shows 2 cups and 3 marbles.



If there are y marbles in each cup, an expression for the total number of marbles is:

- A $2 \times y + 3$ B $3 \times y + 2$
 C $2 \times y + 3 \times y$ D $2(3 + y)$
- 2 The value of $4 \times p - 2$ when $p = 5$ is:
 A 43 B 18 C 12 D 5
- 3 The expression that does not simplify to $36m$ is:
 A $36 \times m$ B $3 \times 6 \times m$
 C $6 \times 6 \times m$ D $4 \times 9 \times m$
- 4 When $x = 3$ and $y = 8$, the value of $4xy$ is:
 A 438 B 20 C 152 D 96
- 5 The expressions that is not equivalent to $\frac{2r}{3p}$ is:
 A $2r \div 3p$ B $2 \times r \div 3p$
 C $2 \times r \div (3 \times p)$ D $3p \div 2r$
- 6 When $t = 4$, the value of $\frac{3t + 8}{4}$ is:
 A 5 B $10\frac{1}{2}$ C 9 D $\frac{11}{4}$
- 7 The missing numbers in the pattern 3, 7, __, 15, __, 23 are:
 A 11 and 27 B 10 and 18
 C 10 and 19 D 11 and 19
- 8 $5a + 7a + 3 =$
 A $12a^2 + 3$ B $12a + 3$
 C 15a D $15a^2$
- 9 $-8g \times 4m =$
 A -32 B $-32gm$
 C $4gm$ D $-84gm$
- 10 $6ab \div 18b =$
 A $\frac{a}{3}$ B 3a C $3ab^2$ D $\frac{3}{a}$
- 11 $\frac{6k}{11} + \frac{4k}{11} =$
 A $\frac{10k}{22}$ B $\frac{5k}{11}$ C $\frac{2k}{11}$ D $\frac{10k}{11}$
- 12 $\frac{2x}{3} + \frac{x}{5} =$
 A $\frac{3x}{8}$ B $\frac{2x^2}{15}$ C $\frac{13x}{15}$ D $\frac{2x^2}{8}$
- 13 When expanded $a(x - y) =$
 A $x - ay$ B $ax - ay$
 C $ax - y$ D $ax - a^2y$
- 14 When expanded $6(2m - 5) =$
 A $8m - 11$ B $12m - 5$
 C $12m - 30$ D $2m - 30$
- 15 When factorised $15x - 20 =$
 A $5(3x - 20)$ B $5(3x - 4)$
 C $x(15 - 20)$ D $-5x$
- 16 When $m = 5$ and $n = -3$, the value of the expression $3m - 2n$ is:
 A 9 B 2 C -19 D 21
- 17 By inspection, the value of the pronumeral if $x + 9 = 10$ is:
 A 19 B -1 C 1 D -19
- 18 The value of T in the equation $3T = -9$ is:
 A -3 B 0 C $\frac{1}{3}$ D 3
- 19 The value of G in the equation $\frac{5G - 2}{7} = 3$ is:
 A $3\frac{4}{5}$ B $4\frac{3}{5}$ C $4\frac{4}{5}$ D $3\frac{3}{5}$
- 20 The value of m in the equation $3m - 9 = 5m + 5$ is:
 A 2 B -7 C -2 D 7
- 21 When $x = 7$, $y = -3$ and $z = 0$, the value of the expression $z(4x - 2y)$ is:
 A 34 B 6 C 22 D 0
- 22 Given $s = p^3 + \sqrt{m^2 - n}$, the value of s when $p = 5$, $m = 4$ and $n = 12$ is:
 A 18 B 58 C 127 D 156
- 23 An equation using x as the number for: 'Four times a certain number is added to ten and the result is eighteen' is:
 A $40 + x = 18$ B $4x + 10 = 18$
 C $4(x + 10) = 18$ D $40x + 10 = 18$

The diagnostic test questions refer to outcomes ACMNA190, ACMNA191, ACMNA192, ACMNA234, ACMNA235 and ACMNA240.





Quadratic expressions

Quadratic expressions are of the form $ax^2 + bx + c$ where x is a variable and a, b and c are constants, with $a \neq 0$.

EXAMPLE 1

A rectangle has length 4 cm more than its breadth. Write an expression for the area.

Solve	Think	Apply
$A = (x + 4) \times x$ $= x(x + 4)$ $= x^2 + 4x \text{ cm}^2$	<div style="text-align: center;">$x + 4$</div>  <p>Let the breadth be x cm. Then the length is $(x + 4)$ cm. Area = length \times breadth</p>	Define the first variable and then obtain an expression for the second variable.

Exercise 10A

- 1 A rectangle has length 6 cm more than its breadth. Complete to write an expression for the area.

Let breadth be x cm.

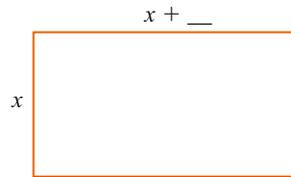
Then length is _____ cm.

Area = length \times breadth

$$A = (x + __) \times x$$

$$= x(__)$$

$$= x^2 + __ \text{ cm}^2$$



- 2 A rectangle has length 5 cm more than its breadth. Write an expression for the area.
- 3 A rectangle has length 3 cm more than its breadth. Write an expression for the area.
- 4 A triangle has perpendicular height 3 cm more than its base length. Write an expression for the area.
- 5 What is an expression for the area of the rectangle with sides $(x + 3)$ and $(x + 1)$?
- 6 What is an expression for the area of the triangle with base $(x + 7)$ and height $(x + 2)$?



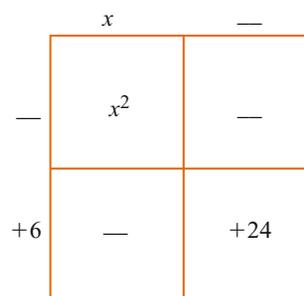
EXAMPLE 2

Expand and simplify the expression for the area of a rectangle with sides $(x + 3)$ and $(x + 2)$.

Solve	Think	Apply
$\begin{aligned} \text{Area} &= (x + 3)(x + 2) \\ &= x(x + 2) + 3(x + 2) \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$		Use a diagram to find the four terms.

- 7** Complete to expand and simplify the expression for the area of a rectangle with sides $(x + 4)$ and $(x + 6)$.

$$\begin{aligned} \text{Area} &= (x + 4)(\underline{\quad}) \\ A &= x(\underline{\quad}) + 4(\underline{\quad}) \\ &= x^2 + \underline{\quad} + 4x + \underline{\quad} \\ &= x^2 + \underline{\quad} + 24 \end{aligned}$$



- 8** Expand and simplify the expression for the area of a rectangle with these sides.

a $(x + 5)$ and $(x + 1)$

b $(x + 7)$ and $(x + 2)$

c $(x + 3)$ and $(x + 8)$

EXAMPLE 3

Expand and simplify the following.

a $(x + 3)(x - 5)$

b $(x - 2)(x - 1)$

c $(x - 5)(x + 7)$

	Solve	Think/Apply
a	$\begin{aligned} (x + 3)(x - 5) &= x(x - 5) + 3(x - 5) \\ &= x^2 - 5x + 3x - 15 \\ &= x^2 - 2x - 15 \end{aligned}$	Split the first bracket then multiply each term by the second bracket. Simplify your answer.
b	$\begin{aligned} (x - 2)(x - 1) &= x(x - 1) - 2(x - 1) \\ &= x^2 - x - 2x + 2 \\ &= x^2 - 3x + 2 \end{aligned}$	
c	$\begin{aligned} (x - 5)(x + 7) &= x(x + 7) - 5(x + 7) \\ &= x^2 + 7x - 5x - 35 \\ &= x^2 + 2x - 35 \end{aligned}$	

- 9** Complete to expand and simplify the following.

a $(x + 3)(x + 7) = x(\underline{\quad}) + 3(\underline{\quad})$
 $= x^2 + \underline{\quad} + 3x + \underline{\quad}$
 $= x^2 + \underline{\quad} + 21$

b $(x - 5)(x - 2) = x(x - 2) \underline{\quad} (x - 2)$
 $= x^2 - 2x \underline{\quad} + \underline{\quad}$
 $= x^2 - 7x + \underline{\quad}$

10 Expand and simplify the following.

a $(x + 4)(x + 2)$

b $(x - 3)(x + 2)$

c $(x + 6)(x - 2)$

d $(x + 4)(x - 3)$

e $(x - 5)(x + 3)$

f $(x + 9)(x - 5)$

g $(x - 10)(x + 3)$

h $(x - 4)(x - 7)$

i $(x + 7)(x - 1)$

j $(x - 3)(x - 2)$

k $(x - 6)(x - 9)$

l $(x - 12)(x - 3)$

EXAMPLE 4

If $x = 3$ and $y = -2$, find the value of:

a $3x^2 - 2x + 5$

b $(x + 2)(y + 3)$

c $(3x - 2)(x + 5)$

	Solve	Think/Apply
a	$3x^2 - 2x + 5 = 3(3)^2 - 2(3) + 5$ $= 26$	Substitute the values then evaluate the equation.
b	$(x + 2)(y + 3) = (3 + 2)(-2 + 3)$ $= (5)(1)$ $= 5$	
c	$(3x - 2)(x + 5) = (3(3) - 2)(3 + 5)$ $= (7)(8)$ $= 56$	

11 If $x = 4$ and $y = -3$, find the value of:

a $2x^2 - 3x + 1$

b $4x^2 + 2x - 1$

c $y^2 - 3y + 5$

d $4y^2 - 7y - 2$

e $5x^2 - 7x + 1$

f $(x + 3)(y - 2)$

g $(x - 5)(x + 2)$

h $(y - 3)(y + 5)$

i $(3x - 7)(2x + 1)$

j $(4x + 1)(3y - 2)$

k $(5x - 2)(3x + 1)$

l $(8y - 2)(y + 1)$

EXAMPLE 5

Expand and simplify the following.

a $(2x - 5)(x + 3)$

b $(5x - 7)(2x + 5)$

c $(3x + 2)(2x - 1)$

	Solve	Think/Apply
a	$(2x - 5)(x + 3) = 2x(x + 3) - 5(x + 3)$ $= 2x^2 + 6x - 5x - 15$ $= 2x^2 + x - 15$	Split the first bracket and multiply the second bracket by each item. Collect like terms.
b	$(5x - 7)(2x + 5) = 5x(2x + 5) - 7(2x + 5)$ $= 10x^2 + 25x - 14x - 35$ $= 10x^2 + 11x - 35$	
c	$(3x + 2)(2x - 1) = 3x(2x - 1) + 2(2x - 1)$ $= 6x^2 - 3x + 4x - 2$ $= 6x^2 + x - 2$	

12 Expand and simplify the following.

a $(2x + 3)(x - 1)$

d $(3x + 2)(3x - 5)$

g $(4x + 1)(3x - 1)$

j $(7x - 2)(7x + 2)$

b $(2x - 5)(x - 8)$

e $(2x + 9)(3x - 2)$

h $(2x - 5)(3x - 2)$

k $(5x - 3)(2x - 5)$

c $(3x - 5)(x - 5)$

f $(4x - 5)(4x + 5)$

i $(2x + 3)(4x - 5)$

l $(4x + 1)(3x - 5)$

13 Expand and simplify the following.

a $3x + (x - 5)(x + 2)$

d $(x + 5)(x + 2) - 5x$

g $4 - (x - 3)(x + 2)$

b $6x + (2x - 1)(3x + 4)$

e $(3x - 7)(x - 2) + 5x$

h $7x - (2x + 1)(x - 5)$

c $(x + 2)(x - 7) - 2x$

f $(4x - 2)(x + 3) - 4x^2 + 2$

i $6x - (x - 2)(2x - 3)$

Investigation 1 Using substitution

Rachel and Diana have different answers to the same question.

Rachel has $(3x - 2)(4x + 7) = 12x^2 + 13x - 14$.

Diana has $(3x - 2)(4x + 7) = 12x^2 + 25x - 14$.

Answer the following questions to determine who is correct.

1 Substitute $x = 1$ and evaluate these three expressions.

a $(3x - 2)(4x + 7)$

b $12x^2 + 13x - 14$

c $12x^2 + 25x - 14$

2 Who is correct, Rachel or Diana?

3 Robert substitutes $x = 0$ into the three expressions in question **1** and decides that both Rachel and Diana are correct.

a Evaluate each expression when $x = 0$.

b Explain the flaw in Robert's reasoning.

4 How can you ensure your substitution will work?

B Binomial products (extension)

A **binomial product** is the product of expressions that each have two terms, such as $(x + 1)(x + 3)$. The following are two methods of expanding special types of binomial products. It is still correct to expand using the method from Section A. Simply write $(x + a)^2$ as $(x + a)(x + a)$ first.

Perfect squares expansion

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

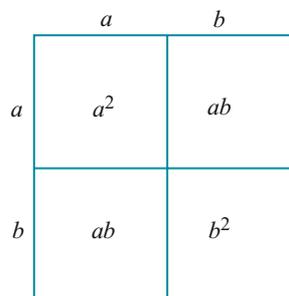
So $(a + b)^2 = a^2 + 2ab + b^2$

and similarly $(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares expansion

$$\begin{aligned}(a + b)(a - b) &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - b^2\end{aligned}$$

So $(a + b)(a - b) = a^2 - b^2$



EXAMPLE 1

Expand and simplify the following.

a $(x + 5)^2$

b $(x - 3)^2$

c $(4x - 5)^2$

	Solve	Think/Apply
a	$(x + 5)^2 = x^2 + 2 \times x \times 5 + 5^2$ $= x^2 + 10x + 25$	Use the perfect square expansion. Make sure the middle term is calculated correctly. The last term is always positive.
b	$(x - 3)^2 = x^2 - 2 \times x \times 3 + 3^2$ $= x^2 - 6x + 9$	
c	$(4x - 5)^2 = (4x)^2 - 2 \times 4x \times 5 + 5^2$ $= 16x^2 - 40x + 25$	

Exercise 10B

1 Complete to expand and simplify the following.

a $(x + 3)^2 = x^2 + 2 \times \underline{\quad} \times \underline{\quad} + 3^2$
 $= x^2 + \underline{\quad} + 9$

b $(2x - 5)^2 = (2x)^2 - 2 \times \underline{\quad} \times \underline{\quad} + (-5)^2$
 $= \underline{\quad} - 20x + \underline{\quad}$

2 Expand the following perfect squares.

a $(x + 2)^2$

b $(x + 6)^2$

c $(y + 10)^2$

d $(3x + 1)^2$

e $(2x + 3)^2$

f $(4a + 5)^2$

g $(5x + 4)^2$

h $(3y + 2)^2$

i $(4 + 3x)^2$

j $(1 + 2x)^2$

k $(5 + 2x)^2$

l $(4 + 5x)^2$

3 Expand the following perfect squares.

a $(x - 2)^2$

b $(x - 6)^2$

c $(y - 9)^2$

d $(3x - 1)^2$

e $(2x - 3)^2$

f $(5a - 4)^2$

g $(3x - 4)^2$

h $(4y - 1)^2$

i $(5 - 2x)^2$

j $(1 - 3x)^2$

k $(5 - 3x)^2$

l $(4 - 2x)^2$

4 Expand these perfect squares.

a $(x + 5)^2$

b $(x - 5)^2$

c $(2x - 7)^2$

d $(2x + 7)^2$

e $(3x - 5)^2$

f $(4x + 3)^2$

g $(5 + 2x)^2$

h $(3 - 7x)^2$

EXAMPLE 2

Expand the following using the difference of two squares.

a $(x + 5)(x - 5)$

b $(2x - 3)(2x + 3)$

	Solve	Think/Apply
a	$(x + 5)(x - 5) = x^2 - 5^2$ $= x^2 - 25$	Use the difference of two squares expansion. There is no middle term and the second term is always negative.
b	$(2x - 3)(2x + 3) = (2x)^2 - (3)^2$ $= 4x^2 - 9$	

5 Complete to expand and simplify.

a $(x + 3)(x - 3) = x^2 - (\quad)^2$
 $= \quad^2 - \quad$

b $(3x - 1)(3x + 1) = (\quad)^2 - (1)^2$
 $= \quad - 1$

6 Expand and simplify using the difference of two squares.

a $(x + 5)(x - 5)$

b $(x + 4)(x - 4)$

c $(x + 6)(x - 6)$

d $(x - 10)(x + 10)$

e $(x + 1)(x - 1)$

f $(2x - 5)(2x + 5)$

g $(3x - 2)(3x + 2)$

h $(5x + 1)(5x - 1)$

i $(7x + 8)(7x - 8)$

7 Expand and simplify using one of the rules.

a $(x - 10)^2$

b $(x + 7)^2$

c $(x + 8)(x - 8)$

d $(x + 11)(x - 11)$

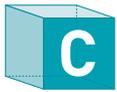
e $(x + 4)(x + 4)$

f $(x - 7)(x + 7)$

g $(x - 7)^2$

h $(x - 12)(x + 12)$

i $(x + 12)(x + 12)$



Quadratic trinomials

First expand $(x + a)(x + b)$.

$$\begin{aligned} (x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \\ &= x^2 + (a + b)x + ab \end{aligned}$$

So, the factorisation of $x^2 + (a + b)x + ab = (x + a)(x + b)$.

The sum of the numbers a and b is the coefficient of the term x and the product of the numbers a and b is the constant term.

The sign of the coefficient of x is the same as the sign of the larger of a or b .

EXAMPLE 1

Factorise the following.

a $x^2 + 5x + 6$

b $x^2 + 7x + 10$

c $x^2 + 7x + 12$

	Solve	Think	Apply
a	$x^2 + 5x + 6 = (x + 3)(x + 2)$	Two numbers that add to give 5 and have a product of 6 are 3 and 2.	The constant term is the product of a and b and the sign indicates adding or subtracting.
b	$x^2 + 7x + 10 = (x + 5)(x + 2)$	Two numbers that add to give 7 and have a product of 10 are 5 and 2.	
c	$x^2 + 7x + 12 = (x + 4)(x + 3)$	Two numbers that add to give 7 and have a product of 12 are 4 and 3.	

Exercise 10C

1 Complete to factorise $x^2 + 12x + 11$.

Find two numbers that add to give ___ and whose ___ is 11. These numbers are 11 and ___.

$\therefore x^2 + 12x + 11 = (x + 11)(x + \quad)$

2 Fully factorise the following.

a $x^2 + 8x + 7$

b $x^2 + 8x + 12$

c $x^2 + 13x + 12$

d $x^2 + 10x + 9$

e $x^2 + 10x + 24$

f $x^2 + 13x + 30$

g $x^2 + 11x + 30$

h $x^2 + 12x + 20$

i $x^2 + 9x + 20$

j $x^2 + 9x + 18$

k $x^2 + 19x + 18$

l $x^2 + 13x + 42$

EXAMPLE 2

Factorise the following.

a $x^2 - 4x + 3$

b $x^2 - 8x + 12$

	Solve	Think	Apply
a	$x^2 - 4x + 3 = (x - 3)(x - 1)$	Two numbers whose sum is -4 and product is 3 are -3 and -1 .	A positive constant term has both factors with the same sign.
b	$x^2 - 8x + 12 = (x - 6)(x - 2)$	Two numbers whose sum is -8 and product is 12 are -6 and -2 .	

3 Complete to factorise $x^2 - 7x + 12$.

As the constant term is positive, both factors have the same ____.

Find two numbers that add to give ____ and whose ____ is 12 .

These numbers are -4 and ____.

$\therefore x^2 - 7x + 12 = (x - 4)(\underline{\hspace{1cm}})$

4 Fully factorise the following.

a $x^2 - 6x + 5$

b $x^2 - 8x + 7$

c $x^2 - 12x + 11$

d $x^2 - 6x + 8$

e $x^2 - 9x + 8$

f $x^2 - 7x + 10$

g $x^2 - 11x + 10$

h $x^2 - 8x + 15$

i $x^2 - 16x + 15$

j $x^2 - 9x + 14$

k $x^2 - 15x + 14$

l $x^2 - 11x + 24$

EXAMPLE 3

Factorise the following.

a $x^2 - 3x - 10$

b $x^2 + x - 6$

	Solve	Think	Apply
a	$x^2 - 3x - 10 = (x - 5)(x + 2)$	Two numbers whose sum is -3 and product is -10 are -5 and 2 .	A negative constant term has factors with different signs.
b	$x^2 + x - 6 = (x + 3)(x - 2)$	Two numbers whose sum is 1 and product is -6 are 3 and -2 .	

5 Complete to factorise $x^2 - 2x - 15$.

As the constant term is negative, the factors have ____ signs.

Find ____ numbers that add to give ____ and whose ____ is -15 .

The numbers are -5 and ____.

$\therefore x^2 - 2x - 15 = (x - 5)(\underline{\hspace{1cm}})$.

6 Fully factorise the following.

a $x^2 + 7x - 8$

b $x^2 + 3x - 10$

c $x^2 + x - 2$

d $x^2 + x - 42$

e $x^2 + 4x - 12$

f $x^2 - 11x - 12$

g $x^2 - 5x - 24$

h $x^2 + 5x - 24$

i $x^2 + 4x - 21$

j $x^2 - 20x - 21$

k $x^2 + 17x - 60$

l $x^2 + 3x - 54$

7 Fully factorise the following.

a $x^2 + 19x + 18$

d $x^2 + 15x + 54$

g $x^2 - 16x + 64$

j $x^2 + 2x - 35$

b $x^2 - 7x - 18$

e $x^2 + 53x - 54$

h $x^2 + 12x - 64$

k $x^2 + 7x - 30$

c $x^2 + 17x - 18$

f $x^2 - 25x - 54$

i $x^2 - 30x - 64$

l $x^2 - 15x + 50$

Investigation 2 Square numbers

1 Find 3×3 and -3×-3 . Compare the answers.

2 Find:

a 4×4 and -4×-4

c -6×-6 and 6×6

b 5×5 and -5×-5

d -10×-10 and 10×10

3 What do you notice about the answers to each part in question 2?

4 The solution to the equation $x^2 = 49$ is found by finding a number that when multiplied by itself gives 49. What are the two answers?

5 The solution to $x^2 = 64$ is $x = 8$ or -8 . Explain why there are two answers.

6 Are there always two answers to $x^2 = c$ where c is a number? Explain, considering c as positive or negative.



D Simple quadratic equations

From Investigation 2 it can be seen that there are two solutions to the equation $x^2 = c$.

This equation is called a **quadratic equation** because the variable x has a power of 2.

EXAMPLE 1

Solve the following.

a $x^2 = 25$

b $x^2 = 169$

	Solve	Think/Apply
a	$x^2 = 25$ $x = \pm\sqrt{25}$ $= \pm 5$	Find the square root of both sides. Use the \pm symbol to indicate that there is a positive and a negative answer. \pm is the symbol for 'plus or minus'.
b	$x^2 = 169$ $x = \pm\sqrt{169}$ $= \pm 13$	

Exercise 10D

1 Solve the following.

a $x^2 = 9$

b $x^2 = 16$

c $x^2 = 64$

d $x^2 = 144$

e $x^2 = 49$

f $x^2 = 121$

g $x^2 = 36$

h $x^2 = 81$

i $x^2 = 100$

j $x^2 = 4$

k $x^2 = 225$

l $x^2 = 289$

EXAMPLE 2

Solve the following.

a $x^2 = 10$

b $x^2 = 43$

	Solve	Think/Apply
a	$x^2 = 10$ $x = \pm\sqrt{10}$ $= \pm 3.16$ (2 decimal places)	Find the square root of both sides. There is always a positive and a negative answer so we use \pm .
b	$x^2 = 43$ $x = \pm\sqrt{43}$ $= \pm 6.56$ (2 decimal places)	

2 Solve the following, giving the answers to 2 decimal places.

a $x^2 = 12$

b $x^2 = 51$

c $x^2 = 19$

d $x^2 = 47$

e $x^2 = 83$

f $x^2 = 28$

g $x^2 = 68$

h $x^2 = 91$

i $x^2 = 193$

j $x^2 = 200$

k $x^2 = 39$

l $x^2 = 17$

EXAMPLE 3

Solve the following.

a $5x^2 = 80$

b $3x^2 = 75$

	Solve	Think	Apply
a	$5x^2 = 80$ $x^2 = 16$ $x = \pm\sqrt{16}$ $= \pm 4$	Divide both sides by 5. Find the square root of both sides.	You can only find the square root of both sides when x^2 is a number. Quadratic equations have two answers. 
b	$3x^2 = 75$ $x^2 = 25$ $x = \pm\sqrt{25}$ $= \pm 5$	Divide both sides by 3. Find the square root of both sides.	

3 Complete to solve the following.

$$5x^2 = 45$$

$$\frac{5x^2}{5} = \frac{45}{\square}$$

$$x^2 = \underline{\hspace{2cm}}$$

$$x = \pm\sqrt{\square}$$

$$= \pm 3$$

4 Solve the following.

a $2x^2 = 18$

c $8x^2 = 72$

e $7x^2 = 175$

g $7x^2 = 252$

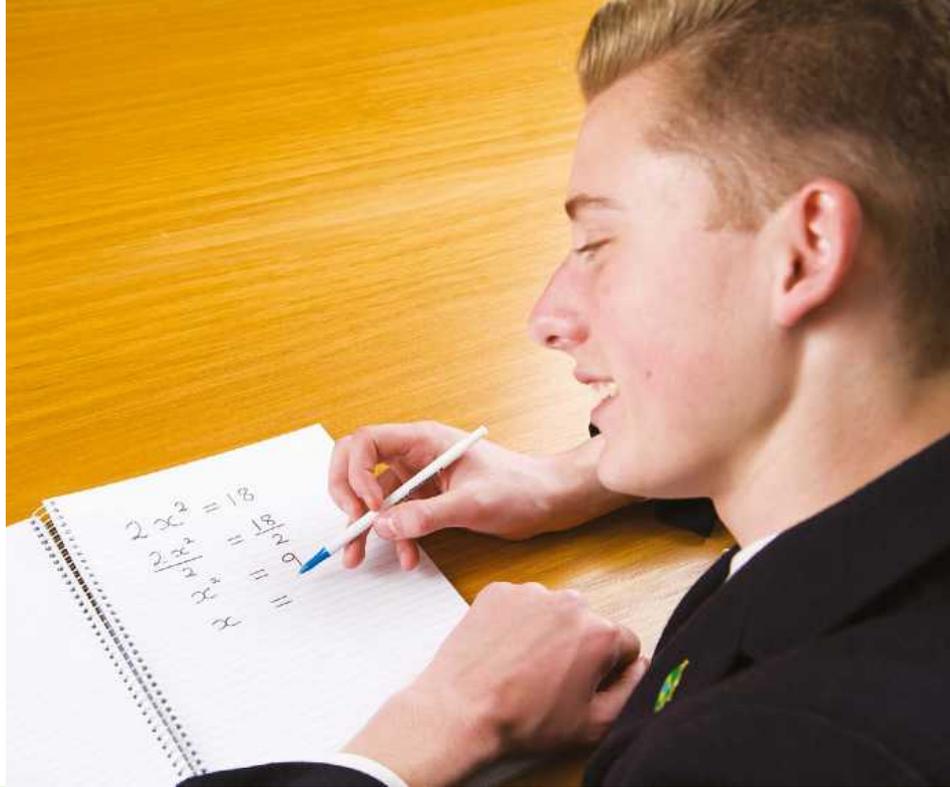
i $4x^2 = 676$

b $5x^2 = 180$

d $3x^2 = 48$

f $10x^2 = 160$

h $6x^2 = 294$



EXAMPLE 4

Solve the following.

a $9x^2 = 25$

b $81x^2 = 49$

	Solve	Think	Apply
a	$9x^2 = 25$ $x^2 = \frac{25}{9}$ $x = \pm\sqrt{\frac{25}{9}}$ $= \pm\frac{5}{3}$	<p>Divide both sides by 9.</p> <p>Find the square root of the numerator and the denominator.</p>	<p>You can only find the square root of both sides when x^2 is a number.</p>
b	$81x^2 = 49$ $x^2 = \frac{49}{81}$ $x = \pm\sqrt{\frac{49}{81}}$ $= \pm\frac{7}{9}$	<p>Divide both sides by 9.</p> <p>Find the square root of the numerator and the denominator.</p>	

5 Complete to solve the following.

$$16x^2 = 49$$

$$\frac{16x^2}{16} = \frac{49}{\square}$$

$$x^2 = \frac{49}{\square}$$

$$x = \pm\sqrt{\frac{49}{\square}} = \pm\frac{7}{\square}$$

6 Solve the following.

a $4x^2 = 49$

d $81x^2 = 16$

g $144x^2 = 49$

b $100x^2 = 81$

e $49x^2 = 144$

h $81x^2 = 100$

c $25x^2 = 16$

f $121x^2 = 64$

i $36x^2 = 169$

EXAMPLE 5

Solve the following.

a $3x^2 = 21$

b $5x^2 = 12$

	Solve	Think	Apply
a	$3x^2 = 21$ $x^2 = 7$ $x = \pm\sqrt{7}$ $= \pm 2.65$ (2 decimal places)	Divide both sides by 3. Find the square root of both sides.	If the number is not a square number use a calculator for an approximation.
b	$5x^2 = 12$ $x^2 = \frac{12}{5}$ $x = \pm\sqrt{\frac{12}{5}}$ $= \pm 1.55$ (2 decimal places)	Divide both sides by 5. Find the square root of both sides.	

7 Complete to solve the following correct to 2 decimal places.

$$11x^2 = 37$$

$$\frac{11x^2}{\square} = \frac{37}{11}$$

$$x^2 = \frac{37}{11}$$

$$x = \pm\sqrt{\frac{\square}{\square}} = \pm \text{---}$$

8 Solve, giving answers to 2 decimal places if necessary.

a $7x^2 = 56$

b $4x^2 = 12$

c $11x^2 = 66$

d $13x^2 = 47$

e $7x^2 = 18$

f $5x^2 = 23$

Extension

EXAMPLE 6

Solve $4x^2 - 5 = 20$.

Solve	Think	Apply
$4x^2 - 5 = 20$ $4x^2 = 25$ $x^2 = \frac{25}{4}$ $x = \pm\sqrt{\frac{25}{4}}$ $= \pm\frac{5}{2}$	Add 5 to both sides. Divide both sides by 4. Find the square root of both sides.	The last step is to find the square root.

9 Solve, leaving the answer as a fraction if necessary.

a $9x^2 - 8 = 56$

b $4x^2 + 3 = 52$

c $81x^2 + 7 = 107$

d $49x^2 - 20 = 5$

Investigation 3 Special product

- Consider the equation $xy = 12$.
 - If $x = 3$ what is the value of y ?
 - If $x = 6$ what is the value of y ?
 - If $x = -2$ what is the value of y ?
 - If $x = \frac{1}{2}$ what is the value of y ?
 - How many solutions are there to the equation $xy = 12$?
 - How many solutions are there to the equation $xy = 15$?
- Consider the equation $xy = 0$.
 - If $x = 3$ what is the value of y ?
 - If $x = 5$ what is the value of y ?
 - If $x = -4$ what is the value of y ?
 - If $y = 6$ what is the value of x ?
 - If $y = -2$ what is the value of x ?
 - What do all the pairs of solutions to $xy = 0$ have in common?



E Solving quadratic equations by factorising

From Investigation 3 it can be seen that in the case of $xy = 0$ then either $x = 0$ or $y = 0$ or $x = y = 0$.

EXAMPLE 1

Solve the following quadratic equations by first factorising.

a $3x^2 + 6x = 0$

b $4x^2 - 8x = 0$

	Solve	Think	Apply
a	$3x^2 + 6x = 0$ $3x(x + 2) = 0$ $\therefore 3x = 0$ or $(x + 2) = 0$ $x = 0$ or $x = -2$	Factorise by taking out the common factor of $3x$.	When factorised the equation is of the form $xy = 0$. Therefore put each factor equal to zero.
b	$4x^2 - 8x = 0$ $4x(x - 2) = 0$ $\therefore 4x = 0$ or $(x - 2) = 0$ $x = 0$ or $x = 2$	Factorise by taking out the common factor of $4x$.	<i>Note:</i> This cannot be done if the product does not equal 0.

Exercise 10E

- 1** Complete to solve the following.

$$9x^2 + 36x = 0$$

$$9x(x + \underline{\quad}) = 0$$

$$9x = 0 \text{ or } x + \underline{\quad} = 0$$

$$x = 0 \text{ or } \underline{\quad}$$

- 2** Solve the following by factorising.

a $4x^2 + 12x = 0$

b $7x^2 - 21x = 0$

c $x^2 - 5x = 0$

EXAMPLE 2

Solve the following quadratic equations by factorising.

a $x^2 - 8x + 12 = 0$

b $x^2 + 2x - 15 = 0$

	Solve	Think	Apply
a	$x^2 - 8x + 12 = 0$ $(x - 6)(x - 2) = 0$ $x - 6 = 0$ or $x - 2 = 0$ $x = 6$ or 2	Factorise using the factors of 12 that add to give 8 (6 and 2).	When we solve quadratic equations, if $a \times b = 0$ then $a = 0$ or $b = 0$ or $a = b = 0$.
b	$x^2 + 2x - 15 = 0$ $(x + 5)(x - 3) = 0$ $x + 5 = 0$ or $x - 3 = 0$ $x = -5$ or 3	Factorise using the factors of 15 that add to give 2 (5 and -3).	

3 a $x^2 + 7x + 10 = 0$ Find the factors of ___ that add to give ___ (5 and ___).

$$(x + 5)(_) = 0$$

$$x + 5 = 0 \text{ or } _ = 0$$

$$x = -5 \text{ or } _$$

b $x^2 + 5x - 14 = 0$ Find the factors of ___ that add to give ___ (7 and $-_$).

$$(x + 7)(_) = 0$$

$$x + 7 = 0 \text{ or } _ = 0$$

$$x = -7 \text{ or } _$$

4 Solve the following quadratic equations by factorising.

a $x^2 - 3x - 4 = 0$

b $x^2 + 5x + 4 = 0$

c $x^2 + 2x - 24 = 0$

d $x^2 + 7x + 10 = 0$

e $x^2 - 5x + 4 = 0$

f $x^2 + 10x + 25 = 0$

g $x^2 + 7x - 18 = 0$

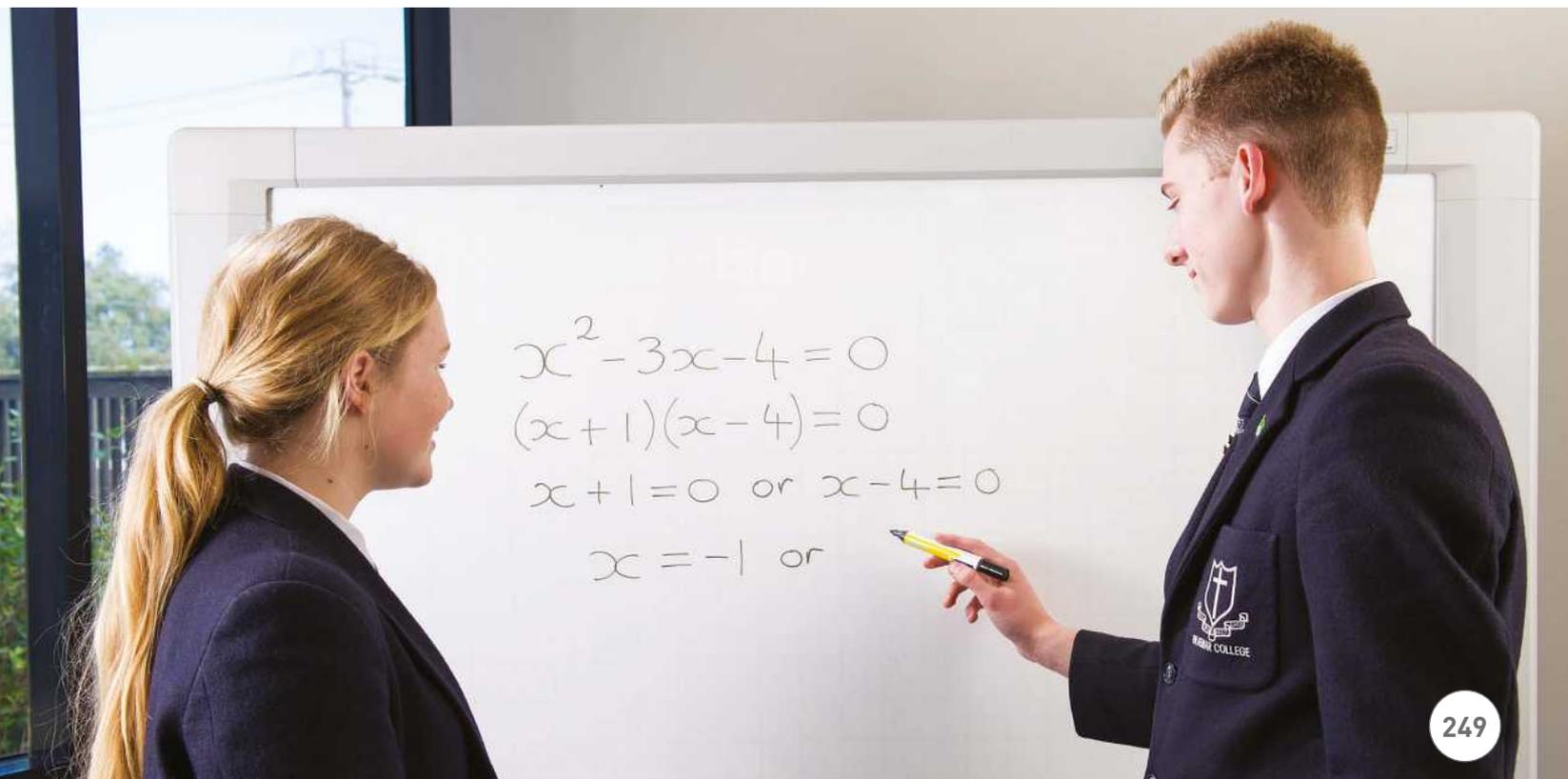
h $x^2 + 11x - 60 = 0$

i $x^2 - 11x + 24 = 0$

j $x^2 - 12x + 36 = 0$

k $x^2 + 14x - 15 = 0$

l $x^2 + 8x + 12 = 0$



Language in mathematics

Sir William Rowan Hamilton (1805–1865)

William Rowan Hamilton was born in Dublin in 1805, the son of a solicitor. His ability was evident at an early age, and by 13 years of age he had managed to learn 13 different languages. This mastery of languages helped him to become one of the few great mathematicians with a facility for involved mental calculations. In 1823 Hamilton entered Trinity College in Dublin and was appointed to the post of Andrew's Professor of Astronomy and Royal Astronomer to Ireland while still an undergraduate. In 1827 he moved to Dunsink Observatory just outside Dublin. Hamilton continued his work in physics, astronomy and algebra, and in 1835 was knighted for his contribution to science. In the same year he discovered quaternions—a very important step in the development of modern algebra. He continued to work on quaternion theory and spent the last 20 years of his life as a recluse.

The result of his research *The Elements of Quaternions* was published after his death in 1866. He was honoured by many foreign academics for his contributions in each of his fields of interest.

- How old was Hamilton when he became Professor of Astronomy and Royal Astronomer to Ireland?
 - For what was he knighted? When?
 - How old was Hamilton when he moved to Dublin?
 - What were Hamilton's main interests?
 - What is a recluse?
- Rearrange these words to form sentences.
 - opposite Expanding is factorising of the
 - highest Always factor factorise common possible using the
 - brackets brackets the the by in outside Multiply term all terms the the

- Use every third letter to find a sentence.

H K T Q T O F G C Y U H F D E S A C E E K R R A T L F A C A B T C I E T Q W O O
P R D F I Z T S N H A Z Y S K N O J T L P B U A C T B E I R S O A A N I O Y P R
O D T U G T C H Y A E R N A X E V T I N H T M O H Q Q E A E R F E E G E X T T P
H J A A A N C V D B G Y E D O Y H U K H R A R A T I N O I S B K W C D E X S R W
E O R G R H J S K M U N B B G F S D Q T E H I O L T F X U F G T U U E E U A N J
N A D U T H M U J B W Q E A S R C T I B U N N I T O P O F D T G H H T E E A E Q
S H U N Y E F G S J K T E R I T H O A S N F G A Y U N I O D L L T A S H C F E V
Y A N U N M I S O L W W C E E V R T N T Y J O I E S A S E D F E G H I J K F L O
T T R H Q A E Z X Y C V A B N R M K E H J E G F Q S A U Q W A E R L

- Give an algebraic example of five glossary terms.

Terms

algebra	binomial product	common factor	expand	expression
factorise	quadratic equation	solve	substitution	trinomial

Check your skills

- If $(x - \square)^2 = x^2 - \Delta x + 16$ then:

A $\square = 4, \Delta = 8$

B $\square = 16, \Delta = 8$

C $\square = 256, \Delta = 128$

D $\square = 16, \Delta = 4$

- 2** When factorised, $3x^2 - 6x =$
A $-3x$ **B** $-3x^3$ **C** $3x(x - 2)$ **D** $3(x^2 - 2x)$
- 3** When factorised, $x^2 - y^2 =$
A $(x - y)(x + y)$ **B** $(x - y)^2$ **C** $(x + y)^2$ **D** $2(x - y)$
- 4** When factorised, $4p^2 - 25q^2 =$
A $(2p - 5q)^2$ **B** $(2p - 5q)(2p + 5q)$ **C** $(4p - 25q)(4p + 25q)$ **D** $(4p - 25q)^2$
- 5** When factorised, $x^2 - 10x + 25 =$
A $(x + 5)(x - 5)$ **B** $(x + 5)^2$ **C** $(x - 10)^2$ **D** $(x - 5)^2$
- 6** When factorised, $x^2 - 5x - 24 =$
A $(x + 8)(x - 3)$ **B** $(x - 12)(x + 2)$ **C** $(x - 8)(x + 3)$ **D** $(x + 12)(x - 2)$
- 7** When factorised $x^2 + x - 12 =$
A $(x - 4)(x + 3)$ **B** $(x + 4)(x - 3)$ **C** $(x - 6)(x + 2)$ **D** $(x + 6)(x - 2)$
- 8** A solution to $3x^2 = 12$ is:
A $x = +\sqrt{12}$ **B** $x = -\sqrt{12}$ **C** $x = 4$ **D** $x = -2$
- 9** The solutions of $5x^2 = 9$ are:
A $x = \pm\frac{3}{\sqrt{5}}$ **B** $x = \pm\sqrt{\frac{3}{5}}$ **C** $x = \pm\sqrt{9}$ **D** $x = \pm\sqrt{45}$
- 10** The solutions of $3x^2 - 9x = 0$ are:
A 0 and 3 **B** 0 and -3 **C** 0 and 9 **D** -3 and 9
- 11** The solutions of $x^2 - 7x - 18 = 0$ are:
A -9 and 2 **B** 9 and -2 **C** 6 and -3 **D** -6 and 3

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3–5	6, 7	8, 9	10, 11
Section	A	B	C	D	E

10A Review set

- 1** Expand and simplify if possible.
a $(x + 5)(x - 3)$ **b** $(2x - 7)(3x - 8)$ **c** $(x - 8)^2$
d $(3x - 4)^2$ **e** $(x - 2)(x + 2)$ **f** $(3x - 5)(3x + 5)$
- 2** If $x = 3$, evaluate $3x^2 - 5x + 8$.
- 3** Factorise:
a $x^2 + 7x - 8$ **b** $x^2 + x - 6$ **c** $x^2 - 7x + 10$
- 4** Solve:
a $x^2 = 12$ **b** $5x^2 = 49$
- 5** Factorise and solve:
a $x^2 - 8x + 7 = 0$ **b** $x^2 + 11x - 12 = 0$ **c** $x^2 - 4x - 5 = 0$

10B Review set

1 Expand and simplify if possible.

a $(x + 2)(x - 11)$

d $(8x - 5)^2$

b $(3x - 8)(4x - 3)$

e $(x - 3)(x + 3)$

c $(x + 4)^2$

f $(5x - 3)(5x + 3)$

2 If $x = -2$, evaluate $2x^2 - 9x + 5$.

3 Factorise:

a $x^2 - 3x + 2$

b $x^2 + x - 20$

c $x^2 - 2x - 24$

4 Solve the following.

a $x^2 = 16$

b $5x^2 = 2$

5 Factorise and solve:

a $6x^2 - 12x = 0$

b $x^2 - 7x - 8 = 0$

c $x^2 + 3x - 10 = 0$

10C Review set

1 Expand and simplify if possible.

a $(x - 3)(x + 4)$

d $(4p - 5)^2$

b $(2x - 5)(3x - 7)$

e $(x - 3)(x + 3)$

c $(x - 4)^2$

f $(4x - 3)(4x + 3)$

2 Use $x = 3$ to show that $6x^2 - x - 2 = (3x - 2)(2x + 1)$.

3 Factorise:

a $x^2 - x - 12$

b $x^2 + 9x + 8$

c $x^2 + x - 30$

4 Why doesn't the equation $3x^2 = -5$ have a solution?

5 Factorise and solve:

a $x^2 + x - 20 = 0$

b $x^2 - 4x - 5 = 0$

c $x^2 - 9x + 8 = 0$

10D Review set

1 Expand and simplify, if possible.

a $(x - 2)(x + 11)$

d $(6z - 5)^2$

b $(3x - 7)(4x + 2)$

e $(x - 9)(x + 9)$

c $(y + 3)^2$

f $(6x - 5)(6x + 5)$

2 Use $x = -1$ to show that $10x^2 - x - 3 = (5x - 3)(2x + 1)$.

3 Factorise:

a $x^2 + 4x - 21$

b $x^2 + 2x - 15$

c $x^2 + 6x + 9$

4 Solve:

a $x^2 = 25$

b $7x^2 = 9$

5 Factorise and solve:

a $x^2 + 8x + 15 = 0$

b $x^2 - 8x - 20 = 0$

c $x^2 - 7x + 12 = 0$

11



Box plots

This chapter deals with the construction and interpretation of box plots.

At the end of this chapter you should be able to:

- ▶ find the upper and lower extremes, the median, and the upper and lower quartiles for sets of numerical data
- ▶ calculate the range and interquartile range
- ▶ compare the relative merits of range and interquartile range as measures of spread
- ▶ construct a box plot using the median, upper and lower quartiles, and upper and lower extremes of a set of data
- ▶ compare two or more sets of data using parallel box plots
- ▶ determine quartiles from data displayed in histograms and dot plots and use these to draw box plots
- ▶ identify skewed and symmetrical sets of data displayed in histograms and dot plots
- ▶ evaluate survey data reported in the digital media and elsewhere.

Diagnostic test

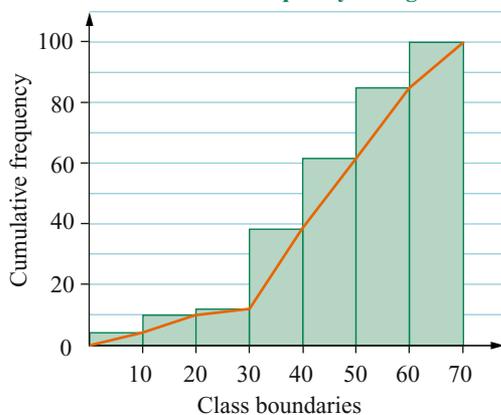
Use the table below to answer questions 1 to 4.

Score	5	6	7	8	9	10
Frequency	4	7	12	17	11	5

- The mean of the data in the table is closest to:
A 7 B 7.7 C 8 D 7.5
- The median of the data in the table is closest to:
A 17 B 7.7 C 8 D 7.5
- The mode of the data in the table is closest to:
A 17 B 7.7 C 8 D 7.5
- The cumulative frequency for the score of 8 is:
A 40 B 17 C 8 D 23
- The ogive is the:
A frequency polygon
B frequency histogram
C cumulative frequency polygon
D cumulative frequency histogram

Use this cumulative frequency histogram and polygon to answer questions 6 and 7.

Cumulative frequency histogram



- The cumulative frequency for the 40–49 class is:
A 61 B 44.5 C 26
D It is impossible to determine without the exact scores.
- An estimate for the median is closest to:
A 40–49 class B 50 C 45
D It is impossible to determine without the exact scores.

- Which frequency distribution table represents the following scores?

12, 30, 38, 49, 13, 28, 33, 17, 21, 31, 23, 32, 25, 26, 39, 36, 42, 46, 36, 50, 48, 32, 45, 57, 43, 51, 49, 53, 42, 33

x	f
10–19	3
20–29	6
30–39	9
40–49	8
50–59	4

x	f
10–19	3
20–29	5
30–39	10
40–49	8
50–59	4

x	f
10–19	4
20–29	5
30–39	9
40–49	9
50–59	3

x	f
10–19	4
20–29	4
30–39	10
40–49	7
50–59	5

- The class centre for the 32–38 class is:
A 32 B 38 C 70 D 35

Use this table to answer questions 10 to 12.

Class	Class centre	Frequency	fx
10–16		3	
17–23		15	
24–30		8	
31–37		12	
38–44		5	

- The class centre for the 38–44 class is:
A 84 B 41 C 5 D 10–44
- The mean for the data is closest to:
A 27 B 10 C 8.6
D It is impossible to determine without the exact scores.
- The modal class is:
A 24–30 B 17–23
C 24–30 D 31–37

The diagnostic test questions refer to outcomes ACMSP170 and ACMSP171.



Mean, mode, median and range

This section reviews the three measures of central tendency, mean, mode and median, and the measure of spread or range.

Mean

The **mean** is the statistical term most thought of when the word ‘average’ is used. The mean of a set of scores is calculated by adding all the scores and dividing this sum by the number of scores. \bar{x} is the symbol used to represent the mean.

For example, for the scores 3, 7, 8, 9 and 9:

$$\bar{x} = \frac{3 + 7 + 8 + 9 + 9}{5} = \frac{36}{5} = 7.2$$

Mode

The **mode** is the score that occurs most often; that is, it is the score with the highest frequency. It is the most commonly occurring score. For example, for the scores 3, 7, 8, 9 and 9, the mode is 9 (as it occurs more frequently than any other score).

A set of scores may be **bimodal**; that is, have two modes. For example, 2, 3, 3, 3, 4, 4, 4, 5, 6, 8 is bimodal as it has two modes; namely, 3 and 4.

Median

The **median** of a set of scores is the middle score (or the average of the two middle scores) after the scores have been arranged in ascending order (that is from smallest to largest).

- For an *odd number* of scores, there is one middle score.

If there are n scores in ascending order, the median is the value of the score in the $\left(\frac{n+1}{2}\right)$ th position.

For example, for the scores 7, 9, 3, 9, 8, the ascending order is 3, 7, 8, 9, 9.

$$n = 5, \text{ so } \frac{n+1}{2} = 3$$

The median is the 3rd score; that is, the median is 8.

There are two numbers below the median and two numbers above the median. !

- For an *even number* of scores, there are two middle scores so the median is not always one of the scores.

For example, consider the scores 4, 7, 9, 6, 5, 9, 3, 7.

The ascending order is 3 4 5 6 7 7 9 9
1st 2nd 3rd 4th 5th 6th 7th 8th

As $n = 8$, the median is the number midway between the 4th and 5th scores.

$$\text{The median} = \frac{6+7}{2} \text{ (the average of the 4th and 5th scores)} = 6.5.$$

There are four scores below the median and four scores above the median. !

Range

The range of a set of data is a measure of its spread. It is found by subtracting the lowest score from the highest score.

$$\text{Range} = \text{highest score} - \text{lowest score}$$

Consider these two data sets:

Set A: 3, 5, 7, 9, 11

Set B: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Both sets have a mean of 7. However, set B is obviously more spread out and the range highlights this fact.

For set A: range = 11 - 3 = 8

For set B: range = 13 - 1 = 12

EXAMPLE 1

For the scores 12, 13, 14, 14, 14, 15, 16, find the:

- a** mean **b** mode **c** median **d** range.

	Solve	Think	Apply
a	Mean = 14	$\text{Mean} = \frac{12 + 13 + 14 + 14 + 14 + 15 + 16}{7}$ $= \frac{98}{7}$ $\bar{x} = 14$	Use the definitions for mean, mode, median and range to find these measures.
b	Mode = 14	As 14 occurs more frequently than any other score, the mode = 14.	
c	Median = 14	To find the median, arrange the scores in ascending order. Find the middle score by crossing out equal numbers of scores from each end. 12 13 14 (14) 14 15 16 Median = 14	
d	Range = 4	Range = highest score – lowest score $= 16 - 12$ $= 4$	

EXAMPLE 2

For the scores 110, 106, 114, 109, 114, 107, find the:

- a** mean **b** mode **c** median **d** range.

	Solve	Think	Apply
a	Mean = 110	$\text{Mean} = \frac{110 + 106 + 114 + 109 + 114 + 107}{6}$ $= \frac{660}{6}$ $\bar{x} = 110$	Use the definitions for mean, mode, median and range to find these measures.
b	Mode = 114	As 114 occurs more frequently than any other score, the mode = 114.	
c	Median = 109.5	To find the median, first arrange the scores in ascending order. Cross out scores from each end until there are two numbers left. The median is the average of them. 106 107 (109) (110) 114 114 $\text{Median} = \frac{109 + 110}{2}$ $= 109.5$	
d	Range = 8	Range = highest score – lowest score $= 114 - 106$ $= 8$	

Exercise 11A

- 1** For the score 4, 5, 6, 6, 7, 9, 9, 9, 10, complete to find the following.
- a** Mean = $\frac{4 + 5 + \square + \square + 7 + \square + \square + \square + 10}{\square} = \underline{\hspace{1cm}}$ (1 decimal place)
- b** Mode: the mode is the score that occurs $\underline{\hspace{1cm}}$ often. As 9 occurs $\underline{\hspace{1cm}}$ times, the mode = $\underline{\hspace{1cm}}$.
- c** Median: cross out 4, 5, 6, 6, 7, 9, 9, 9, 10 from either end.
~~4~~ ~~5~~ ~~6~~ ~~6~~ ~~7~~ ~~9~~ ~~9~~ ~~9~~ ~~10~~
- The middle score is $\underline{\hspace{1cm}}$. Median = $\underline{\hspace{1cm}}$
- d** Range = highest $\underline{\hspace{1cm}}$ - $\underline{\hspace{1cm}}$ score = $10 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- 2** Find the mean of the following sets of data. Answer to 1 decimal place if necessary.
- | | |
|---|---|
| a 1, 3, 4, 5, 8, 8, 9 | b 1, 2, 2, 3, 4, 5, 6, 8, 9 |
| c 10, 12, 13, 15, 16 | d 20, 20, 20, 23, 25, 27 |
| e 51, 52, 54, 55, 57, 57, 58, 59 | f 0, 0, 1, 3, 3, 3, 5, 6, 6, 7, 8, 9 |
| g 105, 101, 104, 101, 101, 102 | h 3, 4, 9, 5, 1, 8, 3, 2, 0 |
- 3** Find the mode, if there is one, of the scores in question 1.
- 4** Find the median of the scores in question 1.
- 5** Find the range of the scores in question 1.
- 6** The number of lollies in 10 packets were 15, 18, 17, 15, 16, 14, 15, 18, 16, 19. Find the:
- | | |
|---|---|
| a range | b mean number of lollies per packet |
| c modal number of lollies per packet | d median number of lollies per packet. |
- 7 a i** Find the mean of the first eight counting numbers: 1, 2, 3, 4, 5, 6, 7, 8.
- ii** Subtract 3 from each of the first eight counting numbers and then find the mean of these numbers.
- iii** How has the mean changed?
- b i** Add 20 to each of the first eight counting numbers and then find the mean of these numbers.
- ii** How has the mean changed?
- c i** Find the median of the first eight counting numbers.
- ii** Add 3 to each of the first eight counting numbers and then find the median of these numbers.
- iii** How has the median changed?
- d i** Find the range of the first eight counting numbers.
- ii** Add 3 to each of the first eight counting numbers and find the range.
- iii** What has happened to the range?
- 8** A shoe store had a special on women's running shoes and sold the following sizes: 6, 10, 4, 7, 8, 7, 6, 5, 7, 8, 7, 5, 6, 4, 3, 7
- a** Find the:
- | | |
|-----------------|-------------------|
| i range | ii mean |
| iii mode | iv median. |
- b** Which of the mean, mode and median would be of most use to the shop owner?

A set of scores with two modes is called bimodal. 

The average female shoe size is 7.  This refers to the mode not the mean.



- 9 The Lighthouse Lamp Company has a total of 30 employees whose annual salaries are listed below.

1 general manager	\$260 000
1 marketing manager	\$100 000
1 accountant	\$120 000
1 engineer	\$100 000
1 warehouse manager	\$100 000
15 production workers	\$50 000 each
10 tradespeople	\$60 000 each

- a What is the total annual wages bill for this company?
- b Calculate the mean wage for the employees.
- c How many employees earn:
- less than this mean wage?
 - more than this mean wage?
- d What is the median wage?
- e What is the modal wage?
- f In a wage determination case for the employees of this company, which measure of central tendency would you use to support your argument if you were the representative for:
- the general manager?
 - the production workers?
- g Which measure of central tendency is the most appropriate to represent the wages of the employees of this company? Give reasons for your answer.



- 10 a Find the mean and median of these scores: 15, 16, 16, 17, 19, 20, 430.
- b Find the mean and median leaving out the score 430.
- c The score 430 is called an **outlier** score because it is an extremely large score compared with all the other scores. An outlier can also be a very small score when compared with the other scores. Which measure, mean or median, is most affected by the outlier?
- 11 A batsman's scores for six innings are 55, 73, 96, 88, 34, 64.
- Find the mean and median.
 - In his next innings he scores 0 runs. Find the new mean and median.
 - In cricket a score of 0 is called a 'duck'. For this batsman's scores, what statistical name would you give the score of 0?
 - Is the mean or median most affected by the score of 0?

EXAMPLE 3

The mean of five scores is 12.2. What is the sum of the scores?

Solve/Think	Apply
Let S = sum of scores. $\frac{S}{5} = 12.2$ $S = 12.2 \times 5 = 61$ The sum of the scores is 61.	Substitute the given information into the formula: $\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$ Solve the resulting equation.

- 12** The mean of 8 scores is 11.5. Complete to find the sum of scores. Let S = sum of ____.

$$\frac{S}{\square} = 11.5$$

$$S = 11.5 \times \underline{\quad} = \underline{\quad}$$

The sum of the scores is ____.

- 13 a** The mean of eight scores is 7.5. What is the sum of the scores?
b The mean of nine scores is 11.6. What is the sum of the scores?
c While on an outback trip, Bill drove, on average, 262 km per day for a period of 12 days. How far did Bill drive in total while on the trip?
d The mean monthly sales of a clothing store is \$15 467. Calculate the total sales of the store for the year.

EXAMPLE 4

Find x if 10, 7, 3, 6 and x have a mean of 8.

Solve/Think	Apply
$\frac{10 + 7 + 3 + 6 + x}{5} = 8$ $\frac{26 + x}{5} = 8$ $26 + x = 40$ $x = 14$	Write an algebraic expression for the sum of the scores and then substitute the given information into the formula: $\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$ Solve the resulting equation.

- 14** Complete to find x if 13, 10, 6, 9 and x have a mean of 11.

$$\frac{13 + 10 + \square + 9 + \square}{\square} = 11$$

$$\frac{\square + x}{\square} = 11$$

$$\underline{\quad} + x = 55$$

$$x = 55 - \underline{\quad} = \underline{\quad}$$

- 15 a** Find x if 8, 11, 5, 7 and x have a mean of 8.
b Find x if 3, 15, 7, 9, 11 and x have a mean of 10.
c Find x if 5, 9, 11, 12, 13, 14, 17 and x have a mean of 12.
d Find a , given that 3, 0, a , a , 4, a , 6, a and 3 have a mean of 4.
e Over the complete assessment period, Jenny averaged 35 out of a possible 40 marks for her eight tests. However, when checking her files, she could only find seven of the tests. For these she scored 29, 36, 32, 38, 35, 34 and 39. Can you determine how many marks out of 40 she scored for the eighth test?

EXAMPLE 5

A cricketer played 12 innings and had a mean of 38.5 runs. He then scored 12 and 71 runs in the next two innings. Find the cricketer's new mean number of runs.

Solve/Think	Apply
<p>Let S = sum of scores</p> $\frac{S}{12} = 38.5$ $S = 462$ <p>New mean = $\frac{462 + 12 + 71}{14}$</p> $= \frac{545}{14} \approx 38.9$	<p>Find the sum of the scores for the first 12 innings. Use this to determine the sum of the cricketer's 14 scores. Calculate the mean of these 14 scores.</p> <p style="color: red; text-align: right;">There are $12 + 2 = 14$ scores in total. </p>

- 16** An AFL player played 8 matches and has a mean of 4.5 goals per game. Complete to find his new mean if he scores 6 goals and 8 goals in the next two matches.

Let S = sum of scores

$$\frac{S}{8} = \underline{\quad}$$

$$S = 8 \times \underline{\quad} = \underline{\quad}$$

New $\underline{\quad} = \frac{36 + 6 + \square}{8 + \square}$

$$= \frac{\square}{10}$$

$$= \underline{\quad}$$



- 17 a** A netballer played 14 matches and had a mean of 16.5 goals per game. In the next two matches she threw 21 goals and 24 goals. Calculate her new mean.
- b** A cricketer played 11 matches and had a mean of 23 runs per game. In the next two games she scored 41 and 35 runs. Calculate her new mean.
- c** A tennis player averaged 8 aces per match in his first six matches. In the next three matches he served 6, 11 and 13 aces. Calculate his new average.
- 18** A sample of 12 measurements has a mean of 16.5, and a sample of 15 measurements has a mean of 18.6. What is the mean of all 27 measurements?
- 19** Fifteen of 31 measurements are below 10 cm and 12 measurements are above 11 cm. Find the median if the other four measurements are 10.1 cm, 10.4 cm, 10.7 cm and 10.9 cm.
- 20** The mean and the median of a set of nine measurements are both 12. If seven of the measurements are 7, 9, 11, 13, 14, 17 and 19, find the other two measurements.

For a large number of scores:

- 25% of the scores $< Q_1$ and 75% of the scores $> Q_1$
- 50% of the scores $< Q_2$ and 50% of the scores $> Q_2$
- 75% of the scores $< Q_3$ and 25% of the scores $> Q_3$.

Hence the interquartile range is a measure of the spread of the middle 50% of the data. It is often a better measure of dispersion (the spread of the scores) than the range because it is not affected by outliers in the data.

EXAMPLE 1

Find the lower, middle and upper quartiles of these scores and then find the interquartile range.

21 22 22 23 25 26 27

Solve/Think	Apply
<p>There are seven scores so the median is the fourth score $\left(\frac{7+1}{2} = 4\right)$.</p> <p style="text-align: center;"> 21 22 22 23 25 26 27 ↑ Q_2 </p> <p>The median is 23. The data is divided into two parts each with three scores. Cross out in each half to find the quartiles.</p> <p style="text-align: center;"> 21 22 22 <u>23</u> 25 26 27 ↑ ↑ ↑ Q_1 Q_2 Q_3 </p> <p>Lower quartile = 22 Upper quartile = 26 Interquartile range = $Q_3 - Q_1$ = $26 - 22 = 4$</p>	<p>Divide the scores into two parts with equal numbers of scores in each by finding the median. The median is the middle quartile.</p> <p>The lower quartile is the middle score of the lower group.</p> <p>The upper quartile is the middle score of the upper group.</p>

Exercise 11B

1 For these scores complete the following to find the lower, middle and upper quartiles and then the interquartile range.

17 23 23 27 29 30 34

a Median: cross out from either end to find the median.

~~17~~ 23 23 27 29 30 ~~34~~

Median = ____

b Quartiles: cross out in each half to find the quartiles.

~~17~~ 23 23 27 29 30 34

Lower quartile = 23

Upper quartile = ____

c Interquartile range = ____ quartile - ____ quartile

= ____ - 23 = ____

2 For the scores 19, 20, 22, 27, 28, 30, 31, find:

a the median

b the lower quartile

c the upper quartile

d the interquartile range.

6 Find the interquartile range for the following scores.

a 15, 16, 16, 20, 22, 23, 25

c 30, 32, 35, 35, 35, 37, 38

e 50, 50, 52, 55, 55, 57, 57, 58, 60, 60

g 23, 23, 23, 24, 25, 26, 28, 28, 29, 32

i 11, 12, 14, 18, 18, 20, 22, 25, 25, 26, 30

b 11, 13, 13, 14, 14, 15, 18

d 2, 3, 3, 4, 5, 5, 6, 7

f 15, 15, 16, 17, 17, 18, 20, 21, 21, 22

h 33, 35, 38, 42, 43, 44, 52, 53, 55, 58, 61, 64, 66, 67

j 46, 50, 50, 53, 54, 54, 58, 58, 58, 60, 62, 62, 66, 66

EXAMPLE 3

Find the interquartile range for the following scores.

a 30, 32, 32, 33, 35, 40, 41, 42, 45

b 9, 5, 7, 11, 10, 4, 14, 7

	Solve/Think	Apply
a	<p>Find the median by crossing off from each end.</p> <p style="text-align: center;">30 32 32 33 35 40 41 42 45</p> <p>Median = 35</p> <p>The middle score in the set 30, 32, 32, 33 is:</p> $Q_1 = \frac{32 + 32}{2} = 32$ <p>The middle score in the set 40, 41, 42, 45 is:</p> $Q_3 = \frac{41 + 42}{2} = 41.5$ <p>Interquartile range = $41.5 - 32$ = 9.5</p>	<p>Find the upper and lower quartiles and hence the interquartile range.</p>
b	<p>Arrange in ascending order to find the median.</p> <p style="text-align: center;">4 5 7 7 9 10 11 14</p> <p>Median = $\frac{7 + 9}{2} = 8$</p> <p>The middle score in the set 4, 5, 7, 7 is:</p> $Q_1 = \frac{5 + 7}{2} = 6$ <p>The middle score in the set 9, 10, 11, 14 is:</p> $Q_3 = \frac{10 + 11}{2} = 10.5$ <p>Interquartile range = $10.5 - 6$ = 4.5</p>	

7 Complete to find the interquartile range of these scores.

1 1 2 2 2 3 4 5 6

Find the ____ by crossing off from each end.

~~1~~ 1 2 2 2 3 4 5 ~~6~~

Median = ____

The ____ score in the set 1, 1, 2, 2 is:

$$Q_1 = \frac{\square + \square}{2} = \underline{\hspace{2cm}}$$

The middle ____ in the set 3, 4, 5, 6 is:

$$Q_3 = \frac{\square + 5}{2} = \underline{\hspace{2cm}}$$

Interquartile ____ = $4.5 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

- 8 Find the interquartile range for the following scores.
- a 42, 45, 45, 48, 53, 61, 64, 68, 71 b 170, 170, 170, 185, 188, 189, 194, 196, 203
- c 2, 3, 5, 8, 8, 8, 9, 9, 9, 11, 15, 17, 18 d 15, 17, 20, 23, 28, 35, 42, 44
- e 95, 102, 95, 89, 92, 103, 90, 98 f 5, 2, 3, 9, 7, 11, 1, 5, 7, 13

EXAMPLE 4

Find the interquartile range from this stem-and-leaf plot.

Stem	Leaf
4	2 4 6
5	0 3 7 7 9 9
6	1 3 4 5 9
7	0 2 4 7
8	1 5 5 6 7

Solve/Think

There are 23 scores so the median is the 12th score.
 Alternatively cross off the numbers to find the median.
 Median = 64
 There are 11 scores above the median and 11 scores below the median.
 The lower quartile is the 6th score.
 $Q_1 = 57$
 The upper quartile is the 12th + 6 = 18th score.
 $Q_3 = 77$
 Interquartile range = $77 - 57$
 $= 20$

Apply

Find the upper and lower quartiles and hence the interquartile range.

- 9 Complete the following to find the interquartile range from this stem-and-leaf plot.

Stem	Leaf
7	1 1 2 2 3 3 7 8 9
8	2 3 3 6 8 8

There are ____ scores.

Cross off from each end to find the ____.

____ = 78

Cross off again in the lower group to find ____ quartile.

$Q_1 =$ ____

Cross off again in the upper group to find ____ quartile.

____ = 83

Interquartile ____ = $83 -$ ____ = ____

Stem	Leaf
7	1 1 2 2 3 3 7 8 9
8	2 3 3 6 8 8

- 10 Find the interquartile range from these stem-and-leaf plots.

Remember to put in order first. ! . . .

a

Stem	Leaf
10	4 5 6 8 9
11	2 2 2 3 5 6 8 8 9
12	0 0 2 2 5 6 7 7
13	1 1 1 6

b

Stem	Leaf
85	5 8 4 5 3 4 2
86	1 2 1 1 1 2 3 4 7 6
87	2 3 4 8 4 5 1 3 6 9 9
88	5 3 6 8 7 4 3 0 3

- 11 a Consider the scores 8, 8, 9, 10, 12, 14, 148. Determine:
- i the range ii the interquartile range.
- b Explain why the interquartile range is a better measure of spread than the range for this set of data.

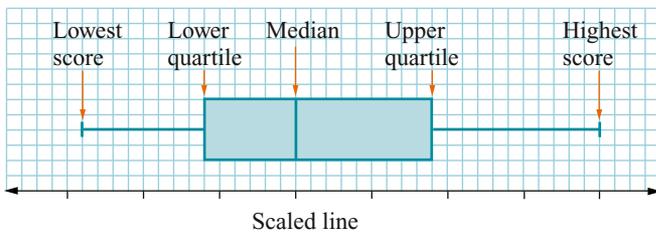
C

Box plots

A box plot uses five especially selected numbers to display information about numerical scores in a graphical form. The numbers used are the extremes (the highest and lowest scores), the median (the middle score) and the upper and lower quartiles. These five numbers make up the five-number summary.

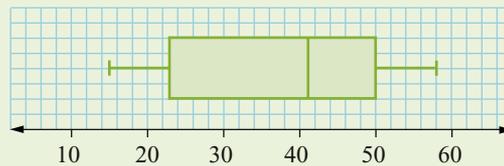
A box plot is used to show the range and middle half of ranked data. Ranked data is numerical data such as numbers. The middle half of the data is represented by the box. The highest and lowest scores are joined to the box by straight lines. The regions above the upper quartile and below the lower quartile each contain 25% of the data.

The five-number summary is shown in the diagram.



EXAMPLE 1

From this box plot, find the following.



- | | | | |
|----------|-------------------------|--------------------------|--------------------------------|
| a | i highest score | ii lowest score | iii range of the scores |
| b | median | | |
| c | i upper quartile | ii lower quartile | iii interquartile range |

	Solve/Think	Apply
a	i Highest score = 58	Read the values of the quartiles and extremes from the box plot. Calculate the range and interquartile range from these values. <i>Note:</i> From these results we can say that: <ul style="list-style-type: none"> • The bottom 25% of the scores take values from 15 up to, but less than, 23. • The top 25% of the scores take values from, but not including, 50 up to 58. • The middle 50% of the scores lie between 23 and 50. The median is closer to the upper quartile than to the lower quartile, so the top half of the scores are clustered closer to the median than the bottom half.
	ii Lowest score = 15	
	iii Range = $58 - 15 = 43$	
b	Median = 41	
c	i $Q_3 = 50$	
	ii $Q_1 = 23$	
	iii Interquartile range = $50 - 23 = 27$	

Exercise 11C

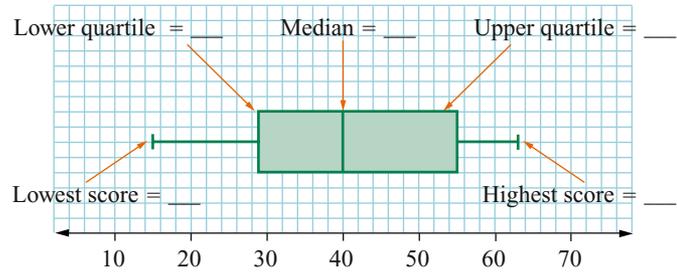
1 Complete the values shown on the box plot and the following.

Range = $___ - 15$

= $___$

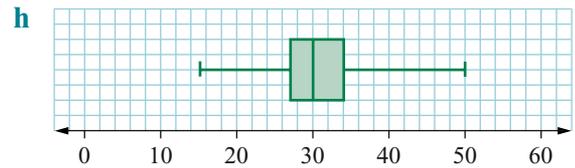
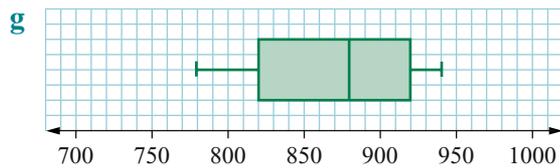
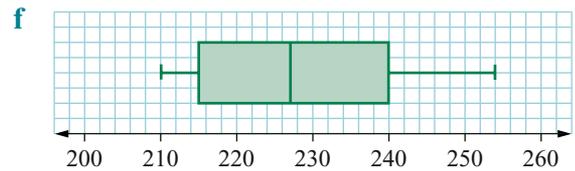
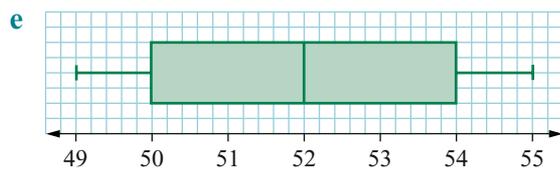
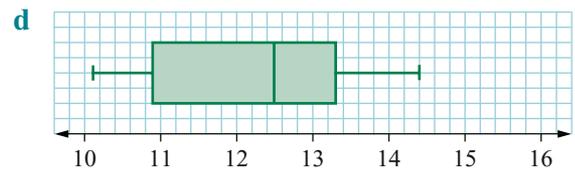
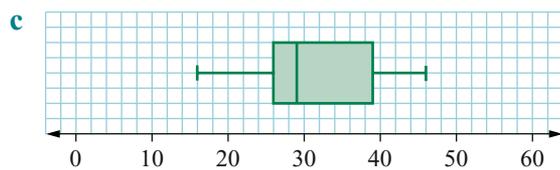
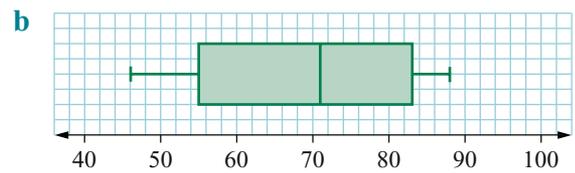
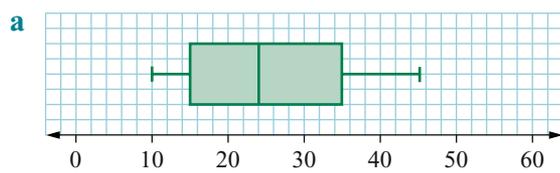
Interquartile range = $55 - ___$

= $___$



2 From the box plots shown below, find the following.

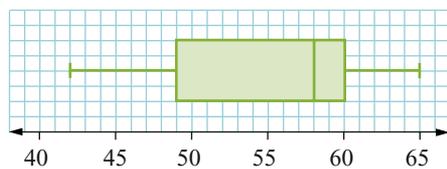
- i** highest score
- ii** lowest score
- iii** range of the scores
- iv** median
- v** upper quartile
- vi** lower quartile
- vii** interquartile range



EXAMPLE 2

Draw a box plot for ranked data with highest score 65, lowest score 42, median 58, upper quartile 60 and lower quartile 49.

Solve/Think



Apply

Use the three quartiles to form the box. Plot the extreme values and join them to the box.

3 Draw box plots for ranked data with the following values.

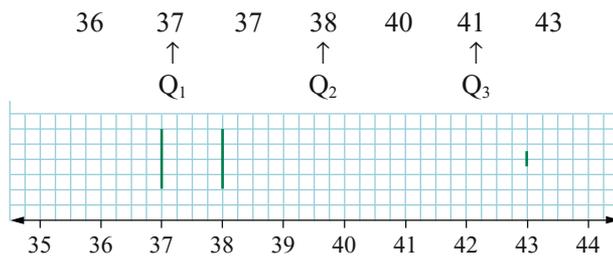
	Highest score	Lowest score	Median	Upper quartile	Lower quartile
a	40	15	28	32	23
b	153	130	141	148	139
c	28	6	10	18	7
d	83	71	78	80	73
e	9	1	5	7	3

EXAMPLE 3

Draw a box plot for the scores 21, 22, 22, 23, 25, 26, 27.

Solve	Think	Apply
	<p>Median is 23. Upper quartile is 26. Lower quartile is 22. Highest score is 27. Lowest score is 21.</p>	<p>Determine the values of the quartiles and extremes. Draw the box plot.</p>

4 Complete the following to draw a box plot for these scores.



Median = ____
 Upper quartile = ____
 Lower quartile = ____
 ____ score = ____
 ____ score = 36

5 Draw box plots for the following scores.

a 34, 35, 36, 36, 37, 38, 39, 39, 39, 40

b 4, 5, 8, 8, 10, 12, 12, 14, 15, 19

c 21, 21, 23, 24, 24, 24, 26, 28, 30

d 89, 90, 92, 95, 95, 98, 102, 103

e 18, 20, 22, 23, 25, 29, 30, 30, 30, 31

f 1, 3, 4, 4, 5, 5, 5, 7, 11, 15



EXAMPLE 4

Construct a box plot for the data in this stem-and-leaf plot.

Stem	Leaf
4	2 4 6
5	0 3 7 7 9 9
6	1 3 4 5 9
7	0 2 4 7
8	1 5 5 6 7

Solve	Think	Apply
	<p>There are 23 leaves and hence 23 scores. Median = 12th score = 64 Lower quartile = 6th score $Q_1 = 57$ Upper quartile = 18th score $Q_3 = 77$ Highest score = 87 Lowest score = 42</p>	<p>Determine the values of the quartiles and extremes. Draw the box plot.</p>

- 6 Complete the following to draw a box plot for the data in this stem-and-leaf plot.

There are 17 leaves and hence ___ scores.

Median = ___ score = ___

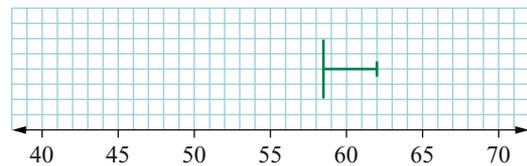
Lower ___ = $\frac{\square + \square}{2} = \underline{\hspace{2cm}}$

___ quartile = $\frac{58 + \square}{2} = \underline{\hspace{2cm}}$

Lowest score = ___

___ score = 62

Stem	Leaf
4	1 1 2 5 8
5	2 2 2 3 4 6 8 8 9
6	0 0 2



- 7 Draw box plots for the data in the following stem-and-leaf plots.

a

Stem	Leaf
2	1 1 3 5 6 8 8
3	2 2 3 3 3 4 5 6 8
4	0 0 3 4 4 4 5 5 7 7 8
5	5 5 6 8

b

Stem	Leaf
10	8 8 9
11	1 1 2 3 3 3 4 4 5 8 8 9
12	0 2 2 3 4 4 4 5 5 8 8 8 9
13	0 0 0 1 1

c

Stem	Leaf
7	1 1 2 3 3 3 7 8 9
8	2 2 2 3 5 6 8 8 9
9	5 5 5 5 6 6 6 9 9

d

Stem	Leaf
5	1 1 2 3 3 4 5 6 6 7 7 8
6	0 1 2 3 4 4 5 5 8 8
7	2 3 4 5 5 5 6 7 8 8

e

Stem	Leaf
33	7 7 7 7
34	3 4 4 5 5 5 5 6 9
35	0 0 3 7 7 8 8 9
36	0 1 2 3 3 4 7 7 9
37	1 1 1 2

f

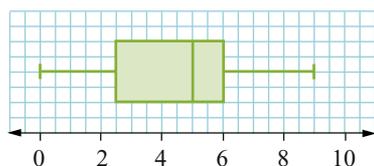
Stem	Leaf
21	0 3 3 3 4 5 6 7 8
22	1 1 1 1 2 2 3 4 6 7
23	2 3 4 4 5 5 8
24	1 2 3 3 4 4 5 6 8 9 9

EXAMPLE 5

Draw a box plot for the data in this frequency distribution table.

Score	Frequency	Cumulative frequency
0	1	1
1	3	4
2	4	8
3	3	11
4	3	14
5	7	21
6	4	25
7	5	30
8	1	31
9	1	32

Solve



Think

As there are 32 scores, the median is the average of the 16th and 17th scores.

$$\begin{aligned} \text{Median} &= \frac{16\text{th score} + 17\text{th score}}{2} \\ &= \frac{5 + 5}{2} \\ &= 5 \end{aligned}$$

There are 16 scores above and 16 scores below the median.

The 16th score is found by looking for the first score in the cumulative frequency column that is equal to or greater than 16. Go across from this value to find the score required, in this case 5. All other scores are found in this way.

The lower quartile is the average of the 8th and 9th scores.

$$\begin{aligned} Q_1 &= \frac{8\text{th score} + 9\text{th score}}{2} \\ &= \frac{2 + 3}{2} \\ &= 2.5 \end{aligned}$$

The upper quartile is the average of the 24th and 25th scores.

$$\begin{aligned} Q_3 &= \frac{24\text{th score} + 25\text{th score}}{2} \\ &= \frac{6 + 6}{2} \\ &= 6 \end{aligned}$$

Highest score = 9

Lowest score = 0

Apply

If it is not given, complete a cumulative frequency column to determine the values of the quartiles and extremes.

Draw the box plot.

- 8 Complete the following to draw a box plot for the data in this frequency distribution table.

As there are ___ scores the median is the 12th score.

Median = ___

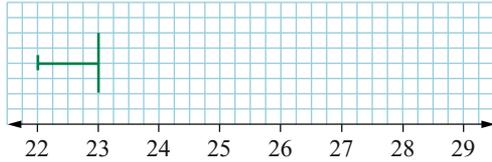
There are 11 scores ___ the median and ___ scores below the median.

Lower quartile is the ___th score = 23

Upper ___ is the 18th score = ___

Highest score = ___

Lowest score = ___



Score	Frequency	Cumulative frequency
22	5	
23	6	
24	5	
25	3	
26	1	
27	2	
28	1	

- 9 Copy these tables and add a cumulative frequency column. Calculate the necessary information and draw a box plot for each data set.

a

Score	Frequency
12	20
13	18
14	15
15	15
16	17
17	15

b

Score	Frequency
53	15
54	30
55	13
56	3
57	9
58	30

c

Score	Frequency
110	5
111	22
112	26
113	25
114	17
115	5

d

Score	Frequency
32	6
33	8
34	9
35	13
36	9
37	3
38	2

e

Score	Frequency
47	4
48	7
49	12
50	21
51	10
52	6

f

Score	Frequency
0	1
1	2
2	3
3	4
4	2
5	5
6	4

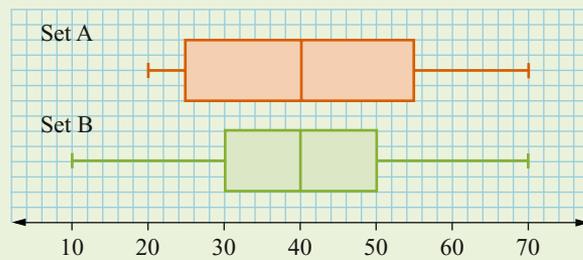
D

Comparing data sets

EXAMPLE 1

Two data sets are shown in these parallel box plots.

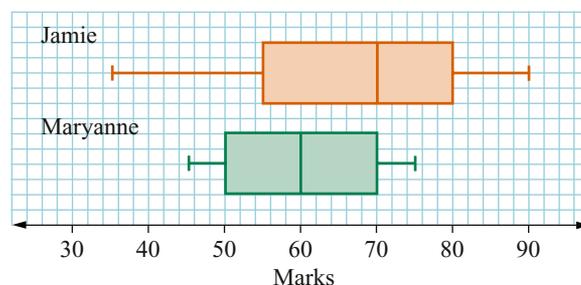
- Describe any similarities in the data sets.
- Compare the range of set A with that of set B.
- For which data set is the middle 50% clustered more closely to the median?
- In which data set is the top 50% of scores more closely clustered to the median?



	Solve	Think	Apply
a	The greatest score and the median are the same for both data sets.	Greatest score of both is 70. Median of both is 40.	Use the known proportion of values between the quartiles as well as the extreme values to analyse the data.
b	The range of set A is less than the range of set B.	Range of set A = $70 - 20 = 50$ Range of set B = $70 - 10 = 60$ Set B has the greater spread of scores.	
c	Set B	IQR of set A = $55 - 25 = 30$ IQR of set B = $50 - 30 = 20$ The spread of scores about the median is less for set B than set A.	
d	Set B	As the top 50% of scores are spread over the same interval, the scores between Q_2 and Q_3 will show any clustering. Q_3 is closer to the median for set B than for set A. The 25% of scores between Q_2 and Q_3 (and hence the top 50%) for set B are closer to their median.	

Exercise 11D

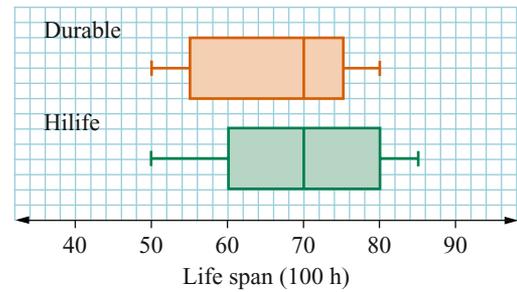
- A class has eight assessment tasks over a year. The parallel box plots show a summary of the marks for the assessments for two students, Jamie and Maryanne.



- Who scored the highest mark?
 - Who scored the lowest mark?
- What was the range of marks for each student?
 - Who had the greater spread of marks?
- What was the interquartile range of marks for each student?
 - Whose marks were the more consistent?
- Who had more marks over 70?
- Assuming each assessment task had the same weighting, who do you think finished the year with the higher overall assessment? Give reasons for your answer.

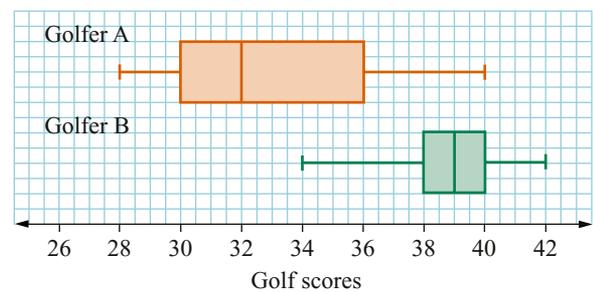
2 These parallel box plots show the life span of two brands of light globes ($\times 100$ hours).

- Describe any similarities in the data.
- Which brand had the globe with the
 - greatest life span?
 - shortest life span?
- What was the range of life spans for each brand?
 - Which brand had the greater spread of life spans?
- What was the interquartile range for each brand?
 - What does this indicate about the middle 50% of life spans for each brand?
- Which brand lasts longer? Give reasons for your answer.

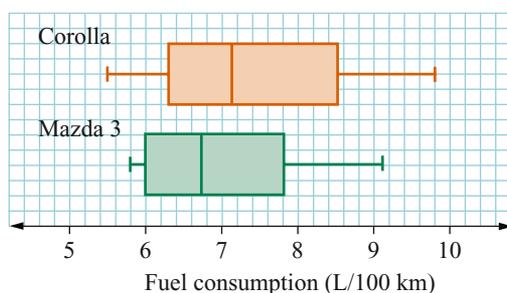


3 The scores of two golfers over 10 rounds are displayed in these parallel box plots.

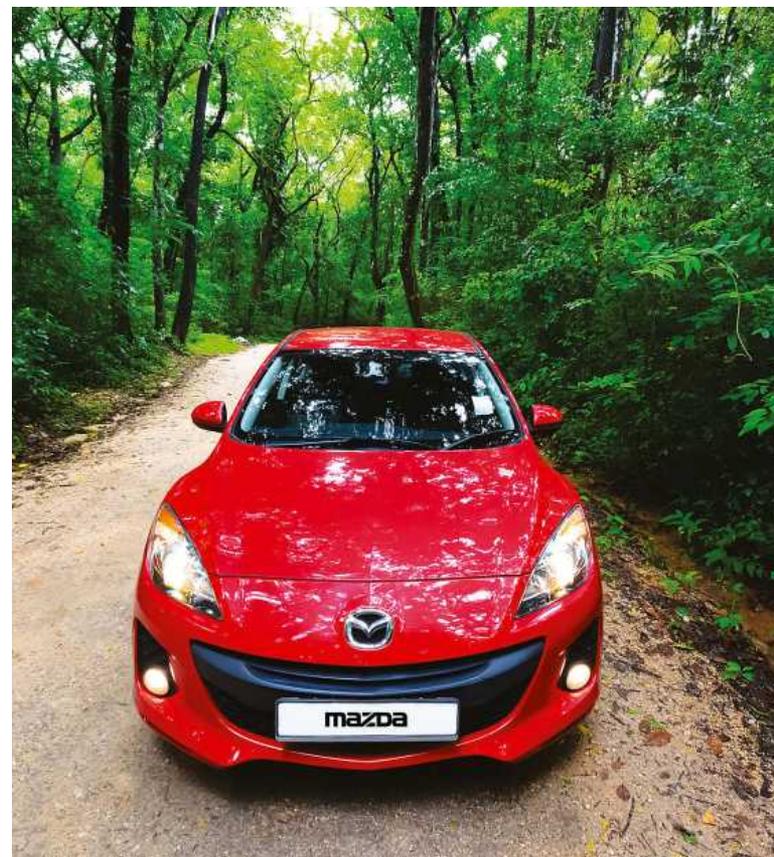
- Which golfer scored:
 - the highest score?
 - the lowest score?
- Which golfer had the greatest range of results?
- What was the interquartile range for each golfer?
 - Which golfer was more consistent?
- Which golfer had the higher median score?



4 A new Toyota Corolla and Mazda 3 were each taken for ten test drives over the same routes. These parallel box plots show a summary of the fuel consumption, in L/100 km, of each vehicle.

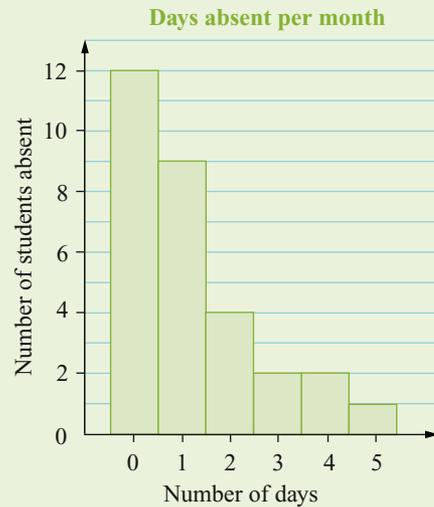


- Which car recorded the:
 - highest fuel consumption?
 - lowest fuel consumption?
- Which car had the greater range of results?
- Which car demonstrated the more consistent fuel consumption over all routes? Give a reason.
- Which car used less than 7 L/100 km more often?
- Which car had the better overall fuel consumption. Give reasons.



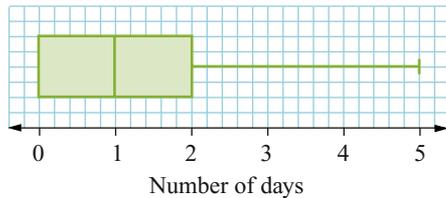
EXAMPLE 2

The histogram shows the number of days in a month on which students in a Year 10 class were absent. Draw a box plot for this data.



Solve

Number of days	Number of students absent	Cumulative frequency
0	12	12
1	9	21
2	4	25
3	2	27
4	2	29
5	1	30



Think

The frequency of each score can be found from the histogram and put in a frequency distribution table. Add a cumulative frequency column.

From the cumulative frequency column:

$$Q_2 = \frac{15\text{th} + 16\text{th scores}}{2} = 1$$

$$Q_1 = 8\text{th score} = 0$$

$$Q_3 = 23\text{rd score} = 2$$

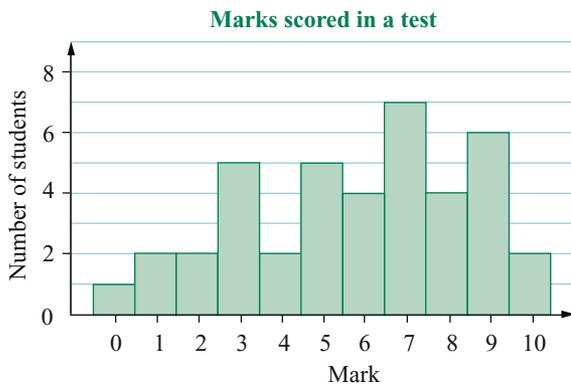
$$\text{Lowest score} = 0$$

$$\text{Highest score} = 5$$

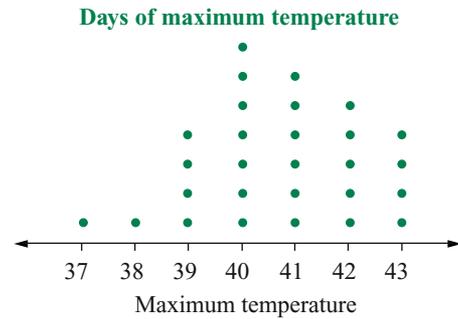
Apply

Put the information shown in the histogram into a frequency distribution table and add a cumulative frequency column. Use the cumulative frequency column to find the quartiles and add the extreme scores to make a five-number summary for the data. Draw the box plot.

- 5 The histogram shows the marks scored by a class in a test. Draw a box plot for this data.

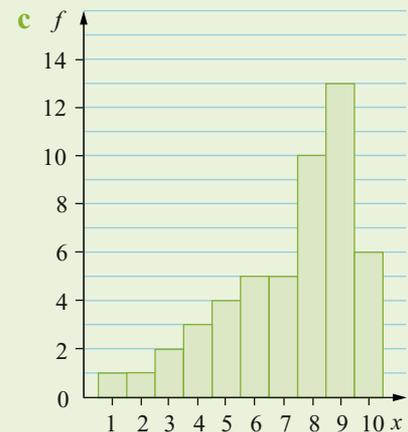
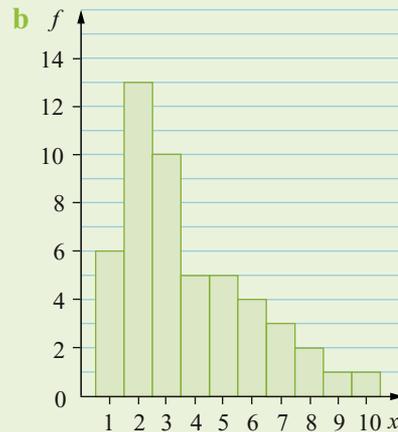
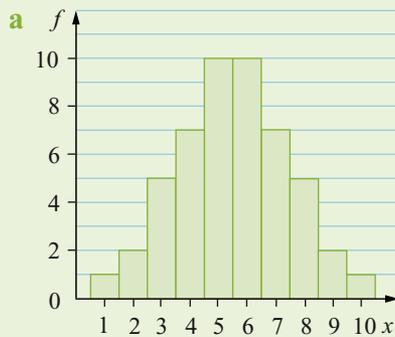


- 6 The dot plot shows the maximum daily temperatures for February. Draw a box plot for this information.



EXAMPLE 3

- i Draw a box plot for the data shown in each of the histograms below.
 ii Describe and compare the features of each histogram and its corresponding box plot.

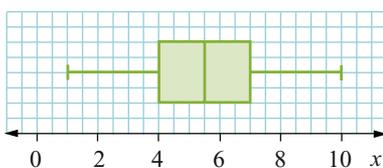


a i

From the histogram:

x	f	cf
1	1	1
2	2	3
3	5	8
4	7	15
5	10	25
6	10	35
7	7	42
8	5	47
9	2	49
10	1	50

So the box plot is:



Solve

Think

Apply

Using the information from the histogram, first compile a cumulative frequency table. Determine the position of the box and the relative spread between the quartiles and extremes in the box plot.

For this data:

$$\begin{aligned} \text{Median} &= \frac{25\text{th} + 26\text{th scores}}{2} \\ &= \frac{5 + 6}{2} \\ &= 5.5 \end{aligned}$$

Upper quartile is 38th score = 7

Lower quartile is 13th score = 4

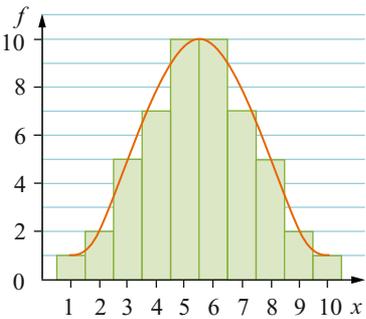
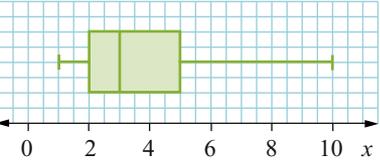
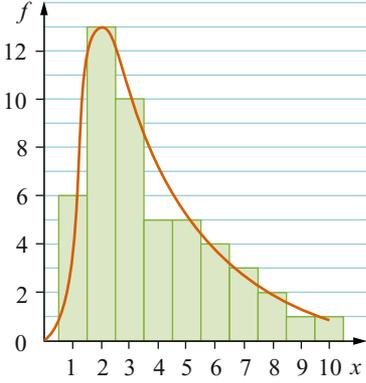
Highest score = 10

Lowest score = 1

Q_1 and Q_3 are equally spaced from the median. The lowest and highest scores are equally spaced from Q_1 and Q_3 respectively.

Put the information shown in the histogram into a frequency distribution table and add a cumulative frequency column. Use the cumulative frequency column to find the quartiles and add the extreme scores to make a five-number summary for the data. Draw the box plot.

EXAMPLE 3 CONTINUED

	Solve	Think	Apply																																	
ii	 <p>The histogram shows that the data is symmetrical. The corresponding box plot is also symmetrical.</p>																																			
b i	<p>From the histogram:</p> <table border="1" data-bbox="294 685 657 1166"> <thead> <tr> <th>x</th> <th>f</th> <th>cf</th> </tr> </thead> <tbody> <tr><td>1</td><td>6</td><td>6</td></tr> <tr><td>2</td><td>13</td><td>19</td></tr> <tr><td>3</td><td>10</td><td>29</td></tr> <tr><td>4</td><td>5</td><td>34</td></tr> <tr><td>5</td><td>5</td><td>39</td></tr> <tr><td>6</td><td>4</td><td>43</td></tr> <tr><td>7</td><td>3</td><td>46</td></tr> <tr><td>8</td><td>2</td><td>48</td></tr> <tr><td>9</td><td>1</td><td>49</td></tr> <tr><td>10</td><td>1</td><td>50</td></tr> </tbody> </table> <p>So the box plot is:</p> 	x	f	cf	1	6	6	2	13	19	3	10	29	4	5	34	5	5	39	6	4	43	7	3	46	8	2	48	9	1	49	10	1	50	<p>Using the information from the histogram, first compile a cumulative frequency table.</p> <p>For this data: Median = 3 Upper quartile = 5 Lower quartile = 2 Highest score = 10 Lowest score = 1</p>	<p>Use the cumulative frequency column to find the quartiles and add the extreme scores to make a five-number summary for the data. Draw the box plot.</p>
x	f	cf																																		
1	6	6																																		
2	13	19																																		
3	10	29																																		
4	5	34																																		
5	5	39																																		
6	4	43																																		
7	3	46																																		
8	2	48																																		
9	1	49																																		
10	1	50																																		
ii	 <p>The histogram shows that the data is positively skewed. This is equivalent to saying that most of the data is on the left of the data set. The box plot shows the 'box' is to the far left of the data set.</p>	<p>The tail of the frequency curve is on the right. The lower 50% of the data is compressed between the scores 1 and 3. The upper 50% is widely spread between 3 and 10. Q_1 is closer than Q_3 to the median, showing the relative compression of this 25% of scores.</p>																																		

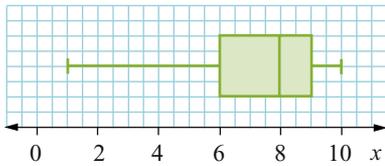
EXAMPLE 3 CONTINUED

c i

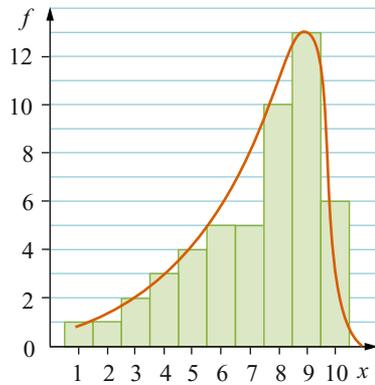
From the histogram:

x	f	xf
1	1	1
2	1	2
3	2	4
4	3	7
5	4	11
6	5	16
7	5	21
8	10	31
9	13	44
10	6	50

So the box plot is:



ii



The histogram shows that the data is negatively skewed. The corresponding box plot shows the 'box' is to the far right of the data set.

Solve

Think

Apply

Using the information from the histogram, first compile a cumulative frequency table.

For this data:

Median = 8

Upper quartile = 9

Lower quartile = 6

Highest score = 10

Lowest score = 1

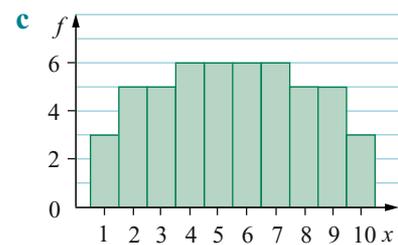
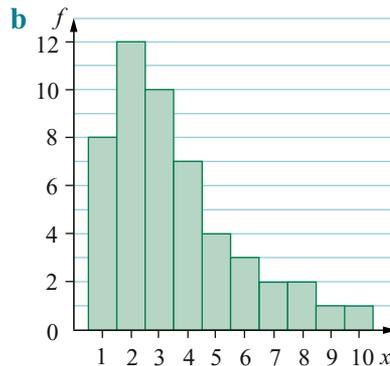
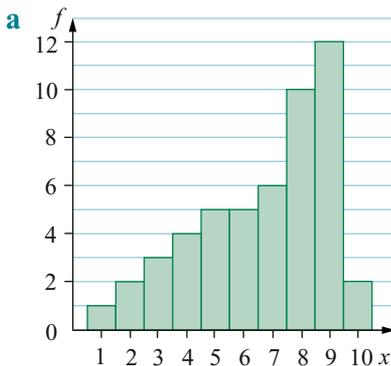
Use the cumulative frequency column to find the quartiles and add the extreme scores to make a five-number summary for the data. Draw the box plot.

The tail of the frequency curve is on the left as most of the data is on the right of the data set. The upper 50% of the data is compressed between the scores 8 and 10. The lower 50% is widely spread between 1 and 8. Also Q_3 is closer than Q_1 to the median, showing the relative compression of this 25% of scores.

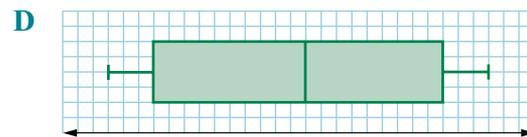
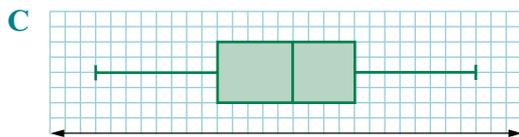
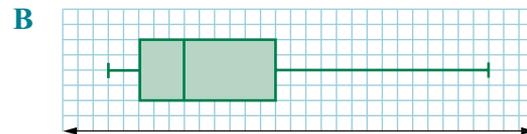
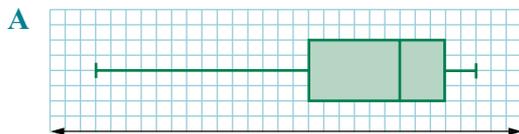
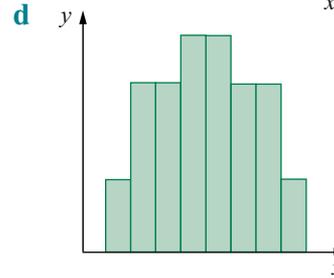
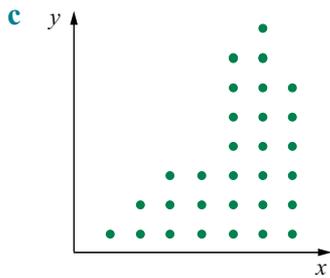
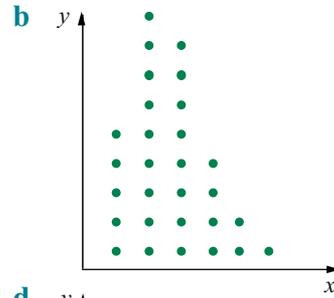
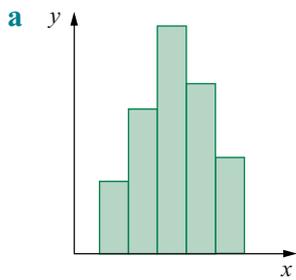
Determine the position of the box and the relative spread between the quartiles and extremes in the box plot.

7 i Draw a box plot for the data shown in each of the histograms below.

ii Describe and compare the features of each histogram and its corresponding box plot.



8 Match each histogram or dot plot with its corresponding box plot.



Investigation 1 Statistical reports in the media

1 Investigate survey data reported in the digital media and elsewhere to critically evaluate the reliability and validity of the source of the data and its usefulness. Describe bias that may exist due to the way in which the data was obtained. These are questions to consider:

- Who instigated and/or funded the research?
- Is the sample being used representative of the population?
- Is the sample big enough?
- Do the questions contain bias?
- Is the research recent?

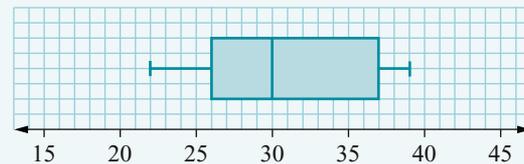


- 4 The mode of the scores 8, 11, 11, 12, 14, 15, 15, 15, 16, 17, 20 is:
 A 15 B 14 C 12 D 11
- 5 The mean of 11, 15, 16, 19, 21 and x is 17. The value of x is:
 A 20 B 21 C 16.4 D 10

Use the scores 15, 16, 17, 18, 18, 18, 20, 21, 21, 25 to answer questions 6 to 8.

- 6 The lower quartile is:
 A 25 B 21 C 17 D 15
- 7 The upper quartile is:
 A 21 B 18 C 17 D 10
- 8 The interquartile range is:
 A 10 B 18 C 5 D 4

Use the information in this box plot to answer questions 9 to 11.



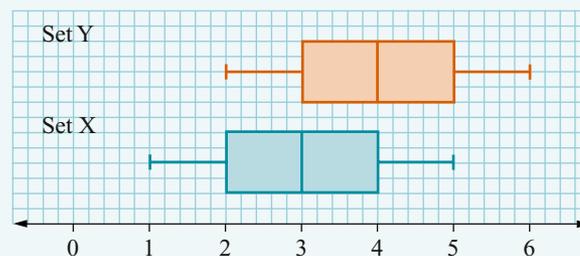
- 9 The range is:
 A 39 B 17
 C 11 D 30
- 10 The median is:
 A 39 B 17 C 11 D 30
- 11 The interquartile range is:
 A 39 B 17 C 11 D 30

Use the data in this stem-and-leaf plot to answer questions 12 to 14.

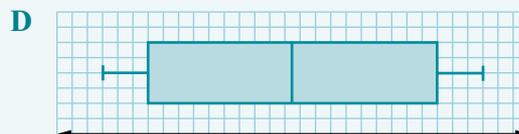
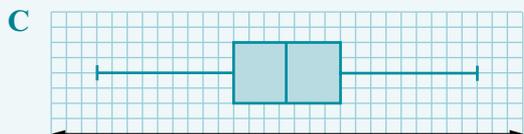
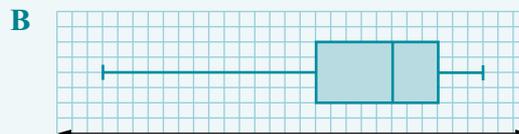
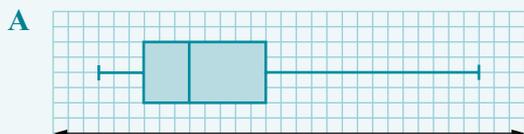
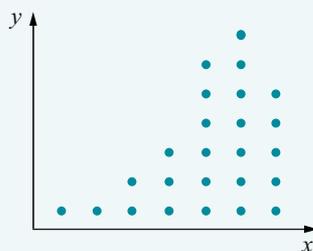
Stem	Leaf
4	8 8 9
5	1 3 5 5 8
6	1 1 2 4 6 7 7 8 9 9
7	1 2 2 2 3 5 5 6 6 7 7 7 9
8	0 3 6 6 6 6 7 8 8 9 9
9	1 2 3 6 7 8 8 8

- 12 The median is:
 A 86 B 75.5
 C 65 D 50
- 13 The interquartile range is:
 A 50 B 86
 C 21 D 11
- 14 The lowest and highest scores are:
 A 48 and 98 B 65 and 86 C 0 and 9 D 48 and 75

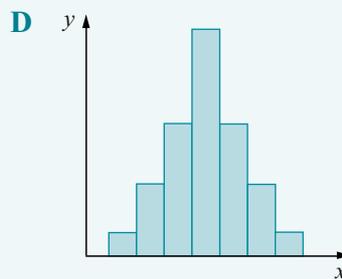
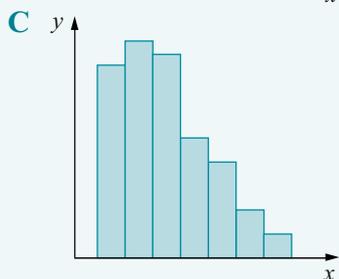
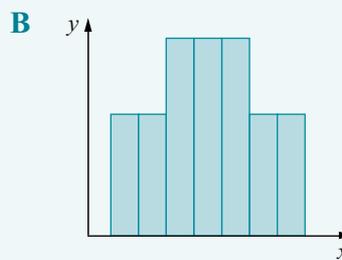
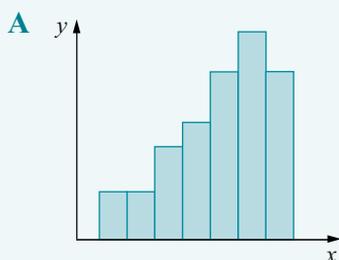
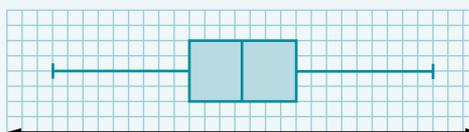
- 15 For the parallel box plots shown on the right, which statement is not true?
 A The range is the same for both data sets.
 B The interquartile range is the same for both data sets.
 C The median of set X is greater than the median of set Y.
 D Both data sets are symmetrical.



16 The box plot that best matches the given dot plot is:



17 The histogram that best matches this box plot is:



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–5	6–8	9–14	15–17
Section	A	B	C	D

11A Review set

1 For the scores 6, 7, 7, 9, 10, 11, 14, find:

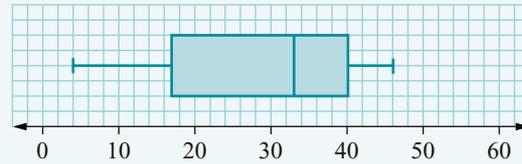
- a the mean
c the range

- b the median
d the interquartile range.

2 Find x when the mean of 7, 12, 18, 16 and x is 15.

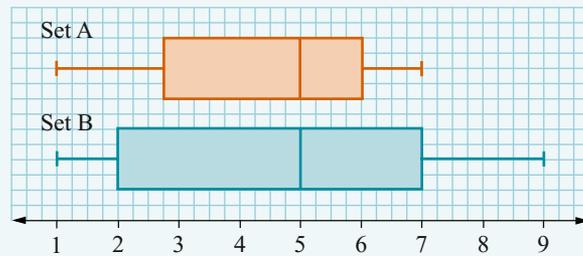
3 From the box plot shown, find:

- a the highest score
- b the lowest score
- c the range
- d the median
- e the upper quartile
- f the lower quartile
- g the interquartile range.



4 The diagram shows parallel box plots for the data in sets A and B.

- a What are the similarities between these sets of data?
- b Which data set has the greater range?
- c Which data set has the greater spread of the middle 50% of its scores?
- d Compare the spread of the lower 50% of scores in each data set.
- e If the box plots represent the marks of two classes on a test, which class do you think was more consistent?



11B Review set

1 For the scores 2, 4, 6, 9, 9, 10, find:

- a the mean
- b the median
- c the range
- d the interquartile range.

2 a The mean of six scores is 14. What is the sum of the scores?

b If 11, 15, 12, 11, 8 and x have a mean of 13, find x .

3 a Find the range of the scores in these frequency distribution tables.

b What is the interquartile range?

i

Score	Frequency
9	6
10	5
11	9
12	11
13	3
14	6

ii

Score	Frequency
25	6
26	10
27	10
28	13
29	6
30	2

4 Draw box plots for the following data sets.

a 3, 4, 7, 7, 9, 11, 11, 13, 14, 18

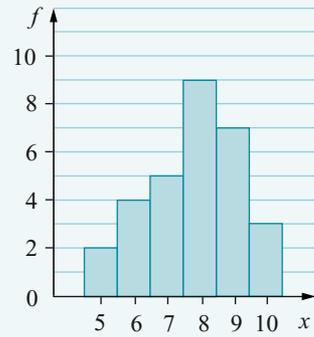
c

Stem	Leaf
6	3 3 3 3
7	3 4 4 5 5 5 6 6 8
8	0 0 2 5 5 9 9 9
9	0 1 3 3 3 4 7 7 9
10	3 3 3 4

b

Score	Frequency
15	18
16	16
17	13
18	13
19	15
20	13

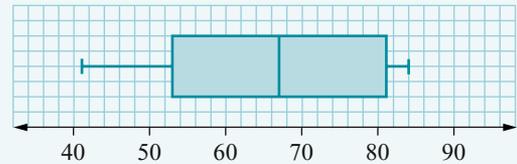
- 5 a** Draw a box plot for the data shown in the histogram.
b Describe how the features of the histogram are shown in the corresponding box plot.



11C Review set

- 1** For the scores 11, 11, 12, 13, 15, 15, 15, 16, 19, 20, 21, 21, find:
a the mean
b the median
c the range
d the interquartile range.
- 2** Find x when the mean of 17, 22, 38, 36 and x is 30.

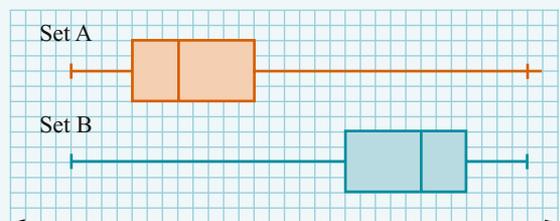
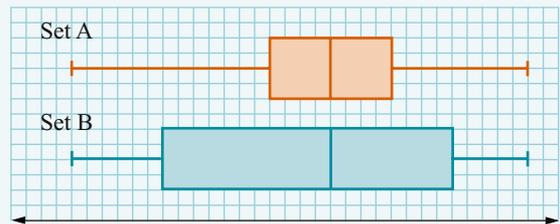
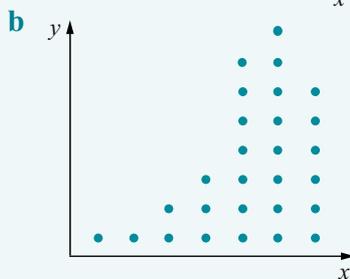
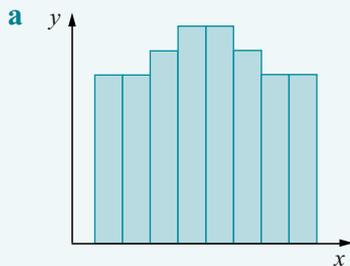
- 3** From the box plot, find:
a the highest score
b the lowest score
c the range
d the median
e the upper quartile
f the lower quartile
g the interquartile range.



- 4 a** Find the range and interquartile range for the scores in this frequency distribution table.
b Draw a histogram for the scores in the table.
c Comment on the shape of the distribution.

Score	Frequency
16	4
17	6
18	8
19	15
20	23
21	14

- 5** Select the data set in the box plot that best matches the given histogram or dot plot.



1 For the scores 65, 61, 64, 61, 61, 62, find:

- a the mean
- b the median
- c the range
- d the interquartile range.

2 a If 9, 6, 2, 5 and x have a mean of 7, find x .

b The mean of eight scores is 5.25. What is the sum of the scores?

3 a For this stem-and-leaf plot, find:

- i the mean
- ii the range
- iii the median
- iv the interquartile range.

b Is this distribution symmetrical or skewed? Explain.

Stem	Leaf
4	1 2 2 3
5	2 4 6 9 9
6	1 3 4 5 5 6 7 7 8
7	0 0 2 3 3 3 8
8	5 6 7 9
9	0 1 2

4 Draw box plots for the following data sets.

a 1, 1, 3, 4, 5, 5, 5, 6, 7, 10, 11

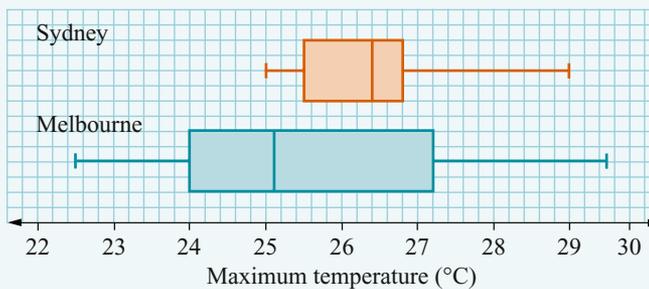
b

Score	Frequency
21	23
22	28
23	15
24	31
25	12

c

Stem	Leaf
18	1 2 3 3 5 6 7 8
19	1 1 1 1 2 2 3 6 6 7 9
20	1 3 4 5 6 6 7 8
21	0 0 0 1 3 4 5 9

5 The box plot shows the mean daily maximum temperatures in Sydney and Melbourne for the month of January. Compare and describe the features of the weather illustrated by these displays.



12



Non-linear relationships

This chapter deals with the graphs of simple non-linear relationships.

After completing this chapter you should be able to:

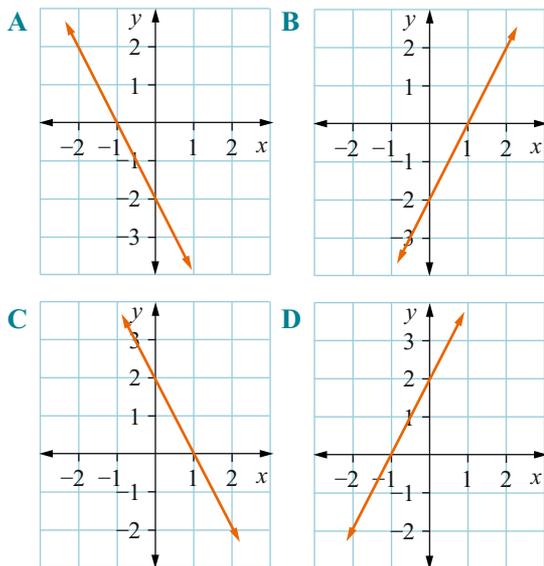
- ▶ graph parabolic relationships of the form $y = ax^2 + c$
- ▶ determine the x -coordinate of a point on a parabola given its y -coordinate
- ▶ sketch, compare and describe the features of simple exponential curves
- ▶ recognise and describe equations that represent circles with centre the origin and radius r
- ▶ sketch circles of the form $x^2 + y^2 = r^2$
- ▶ identify graphs and equations of straight lines, parabolas, circles and exponentials
- ▶ match the graphs of straight lines, parabolas, circles and exponentials to the appropriate equations
- ▶ sort and classify different types of graphs.

Diagnostic test

- 1 The equation of the line with gradient $\frac{2}{3}$ and y -intercept -5 is:

A $y = -5x + \frac{2}{3}$ B $y = 5x - \frac{2}{3}$
 C $y = \frac{2}{3}x + 5$ D $y = \frac{2}{3}x - 5$

- 2 The graph of $y = 2x - 2$ is:

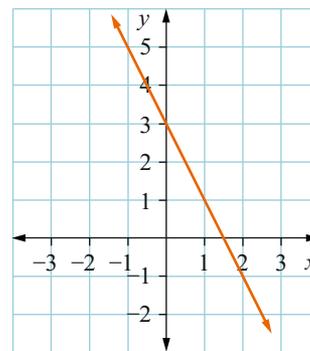


- 3 The graph of the straight line $x - 2y - 8 = 0$ has:

A gradient = -2 and y -intercept = -8
 B gradient = 2 and y -intercept = 8
 C gradient = $\frac{1}{2}$ and y -intercept = -4
 D gradient = $-\frac{1}{2}$ and y -intercept = 4

- 4 The equation of the line graph is:

A $y = 2x - 3$
 B $y = -2x + 3$
 C $y = \frac{1}{3}x - 3$
 D $y = -\frac{1}{3}x - 3$



5

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

The equation of the line that passes through the points in the table is:

A $y = -4x$ B $y = -4x - 1$
 C $y = x - 2$ D $y = 3x - 2$

- 6 The y -coordinate of the point on the curve $y = 2x^2 - 3x + 1$ with x -coordinate -2 is:

A 15 B 6 C 0 D -4

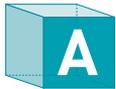
- 7 The value of $(-2)^{-3}$ is:

A 6 B -8 C $\frac{1}{8}$ D $-\frac{1}{8}$

- 8 The distance of the point $(8, 15)$ from the origin is:

A 23 units B 17 units
 C 15 units D 7 units

The diagnostic test questions refer to outcome ACMNA193, ACMNA214, ACMNA215 and ACMNA294.



The parabola

This exercise explores some of the properties of the parabola. A parabola is a graph of the form $y = ax^2 + bx + c$ and is either a  or  shaped curve. The turning point is called the vertex of the parabola.

Graphics calculators may be used to help draw the graphs.



Exercise 12A

- 1 a** Complete this table of values for the parabola $y = x^2$.
b Graph $y = x^2$.

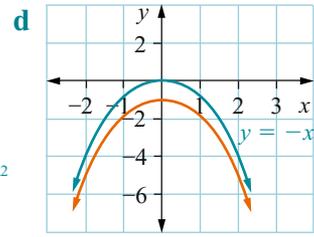
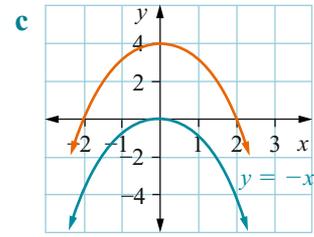
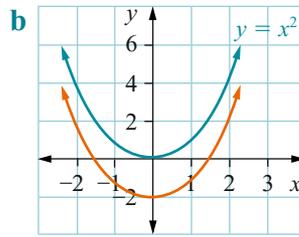
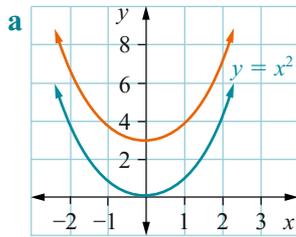
x	-4	-3	-2	-1	0	1	2	3	4
y									

- c** Write the coordinates of the vertex of the parabola.
- 2 a** On the same number plane, graph:
i $y = x^2, y = 2x^2, y = \frac{1}{2}x^2$ **ii** $y = x^2, y = 3x^2, y = 4x^2, y = \frac{1}{4}x^2$
b Comment on the effect of the coefficient of x^2 on the graph of $y = ax^2$.
- 3** Graph $y = -x^2$ by first completing the table.

x	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
y											

- 4 a** On the same number plane, graph:
i $y = -x^2, y = -2x^2, y = -3x^2$ **ii** $y = -x^2, y = -\frac{1}{2}x^2, y = -\frac{1}{3}x^2$
b Comment on the effect of the coefficient of x^2 on the graph of $y = -ax^2$.
- 5 a** On the same number plane, graph $y = x^2, y = x^2 + 1, y = x^2 + 3$.
b Write the coordinates of the vertex of each parabola.
c Comment on the effect of the constant term on the graph of $y = x^2 + c$.
- 6 a** On the same number plane, graph $y = x^2, y = x^2 - 3, y = x^2 - 5$.
b Write the coordinates of the vertex of each parabola.
c Comment on the effect of the constant term on the graph of the parabola.
- 7 a** On the same number plane, graph $y = -x^2, y = -x^2 + 2, y = -x^2 - 2$.
b Write the coordinates of the vertex of each parabola.
c Comment on the effect of the constant term on the graph of the parabola.
- 8 a** On the same number plane, graph $y = 2x^2, y = 2x^2 + 2, y = 2x^2 - 2$.
b Write the coordinates of the vertex of each parabola.
c Comment on the effect of the constant term on the graph of the parabola.
- 9** Find the equation of the graph with the same shape as $y = x^2$ but with vertex:
a $(0, -1)$ **b** $(0, 1)$ **c** $(0, -3)$ **d** $(0, -5)$ **e** $(0, 4\frac{1}{2})$

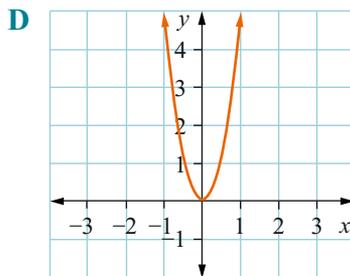
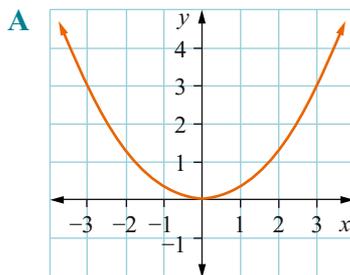
- 10** The sketch shows the new position of the graph of $y = \pm x^2$ after it has been translated up or down. Write the equation of each new parabola.



- 11** Match the following equations with the parabolas drawn below.

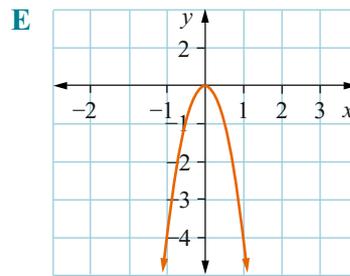
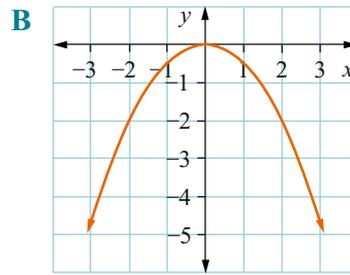
a $y = 5x^2$

d $y = -\frac{1}{2}x^2$



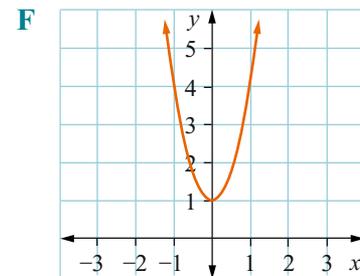
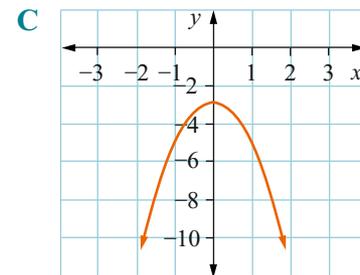
b $y = \frac{1}{3}x^2$

e $y = 3x^2 + 1$



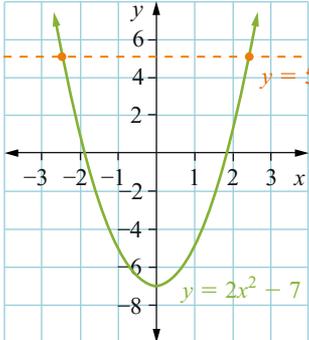
c $y = -4x^2$

f $y = -2x^2 - 3$



EXAMPLE 1

Find the x -coordinate(s) of the point(s) on the parabola $y = 2x^2 - 7$ whose y -coordinate is 5.

Solve	Think	Apply
$5 = 2x^2 - 7$ $12 = 2x^2$ $x^2 = 6$ $x = \pm\sqrt{6}$	<p>Substitute $y = 5$ into $y = 2x^2 - 7$. For $x^2 = 6$ there are two solutions, $\sqrt{6}$ and $-\sqrt{6}$, written $\pm\sqrt{6}$.</p> <p>The sketch of the graph shows that there are two points on the parabola whose y-coordinate is 5.</p> 	<p>Substitute the given y-coordinate into the equation of the parabola, solve the resulting equation.</p>



- 12** Complete to find the x -coordinate(s) of the point(s) on the parabola $y = 3x^2 - 1$ whose y -coordinate is 17.

$$\begin{aligned}
 y &= 3x^2 - 1 \\
 17 &= \underline{\quad} - 1 \\
 17 + \underline{\quad} &= 3x^2 - 1 + \underline{\quad} \\
 \underline{\quad} &= 3x^2 \\
 x^2 &= 6 \\
 x &= \pm \underline{\quad}
 \end{aligned}$$

- 13** Find the x -coordinate(s) of the point(s) on the parabola:

- | | |
|---|---|
| a $y = x^2 - 7$ whose y -coordinate is 3 | b $y = 2x^2 - 3$ whose y -coordinate is 11 |
| c $y = 3x^2 + 1$ whose y -coordinate is 4 | d $y = 4x^2 - 5$ whose y -coordinate is 4 |
| e $y = 2x^2 + 7$ whose y -coordinate is 57 | f $y = 7x^2 - 9$ whose y -coordinate is 5. |

Investigation 1 Parabolas

The parabola with equation of the form $y = (x - b)^2 + c$ can be drawn by starting with $y = x^2$.

- 1 a** To sketch $y = (x - 3)^2 + 1$ first sketch $y = x^2$.
- Translate the graph so that the vertex is at $(3, 0)$.
 - Translate the graph up 1 unit.
- b** Where is the vertex now?
- 2 a** To sketch $y = (x + 3)^2 - 1$ first sketch $y = x^2$.
- Translate the graph so that the vertex is at $(-3, 0)$.
 - Translate the graph down 1 unit.
- b** Where is the vertex now?
- 3** Sketch the following parabolas starting with $y = x^2$.
- | | | |
|------------------------------|------------------------------|------------------------------|
| a $y = (x - 2)^2 + 3$ | b $y = (x + 2)^2 + 2$ | c $y = (x - 3)^2 + 2$ |
| d $y = (x + 5)^2 - 3$ | e $y = (x + 1)^2 - 3$ | f $y = (x - 2)^2 - 1$ |
- g** vertex $(-3, 5)$ and passing through $(2, 0)$
- 4** Sketch the following parabolas starting with $y = -x^2$.
- | | |
|-------------------------------|-------------------------------|
| a $y = -(x + 1)^2 + 1$ | b $y = -(x + 2)^2 - 2$ |
| c $y = -(x - 2)^2 + 3$ | d $y = -(x - 4)^2 + 3$ |
- 5** Complete the following for the graph $y = (x - b)^2 + c$. Use up, down, left or right.
- | | |
|--|--|
| a If $b > 0$, the graph of $y = x^2$ is moved _____. | b If $b < 0$, the graph of $y = x^2$ is moved _____. |
| c If $c > 0$, the graph of $y = x^2$ is moved _____. | d If $c < 0$, the graph of $y = x^2$ is moved _____. |
- 6** A parabola has vertex at $(1, 10)$ and cuts the x -axis at $(4, 0)$. Find its equation.
- Start with $y = a(x - b)^2 + c$.* **!** . . .
- 7** Repeat question 6 for the following parabolas.
- | | |
|--|--|
| a vertex $(2, 5)$ and passing through $(3, 0)$ | b vertex $(1, 4)$ and passing through $(2, 0)$ |
| c vertex $(5, 7)$ and passing through $(-1, 0)$ | d vertex $(-3, 5)$ and passing through $(2, 0)$ |

B

Exponential graphs

Exponential graphs have the variable x in the power of the equation, which makes them different from all other graphs in this chapter.

The general form of an exponential graph is $y = ka^x$ where a is a constant and, in this course, $k = \pm 1$.

Exercise 12B

- 1 a Complete this table of values for $y = 2^x$.

x	-3	-2	-1	0	1	2	3	4
y								

- b Graph the values from the table.
 c Are there any negative values for y ? Explain.
 d What happens to the value of y as x gets:
 i larger (consider $x = 5, 10, 20, \dots$)?
 ii smaller (consider $x = -5, -10, -20, \dots$)?
 e An asymptote is a straight line that a curve approaches (gets closer to) but does not cross. This curve has an asymptote. What is its equation?

- 2 a Using the x -values in question 1, complete a similar table of values for $y = 3^x$.

- b Graph $y = 3^x$.
 c Compare $y = 3^x$ and $y = 2^x$. Discuss common features.
 d Use your answer to part c to predict the shape of the graph of $y = 5^x$.

- 3 a Complete this table of values for $y = 2^{-x}$.

x	-3	-2	-1	0	1	2	3	4
y								

- b Use the table of values to graph $y = 2^{-x}$.
 c Describe the features of the graph, comparing with $y = 2^x$.
 d What happens to the value of y as x gets:
 i larger (consider $x = 5, 10, 20, \dots$)?
 ii smaller (consider $x = -5, -10, -20, \dots$)?
 e Describe the asymptote. How is it different from that for $y = 2^x$?

- 4 a Sketch $y = -2^x$, noting that it is the negative of 2^x ; that is, the sign of each y -value is the opposite of that for $y = 2^x$.

- b Describe the features of the graph of $y = -2^x$.

- 5 a Sketch $y = -2^{-x}$.

- b Compare this graph with $y = 2^{-x}$, noting similarities and differences.

- 6 Without completing a table of values, sketch each set of graphs on the same number plane.

a $y = 3^x$ and $y = 5^x$

b $y = 3^x$ and $y = 3^{-x}$

c $y = 3^x$ and $y = -3^x$

d $y = 5^x, y = 5^{-x}, y = -5^{-x}$ and $y = -5^x$

7 Sketch each set of graphs on the same number plane.

a $y = 2^x, y = 2^x + 2, y = 2^x - 2$

b $y = -3^x, y = -3^x + 2, y = -3^x - 2$

c $y = 2^{-x}, y = 2^{-x} + 3, y = 2^{-x} - 3$

d $y = -2^{-x}, y = -2^{-x} + 2, y = -2^{-x} - 3$

8 a What is the value of y when $x = 0$ for:

i $y = a^x$?

ii $y = a^{-x}$?

What common feature of the graphs of $y = a^x$ does this illustrate?

b What is the value of y when $x = 0$ for:

i $y = -a^x$?

ii $y = -a^{-x}$?

What common feature of the graphs of $y = -a^x$ does this illustrate?

EXAMPLE 1

Find the x -coordinate of the point on the graph of $y = 6^x$ whose y -coordinate is 216.

Solve	Think	Apply
$6^x = 216$ Try $x = 1$ $y = 6^1 = 6$ $x = 2$ $y = 6^2 = 36$ $x = 3$ $y = 6^3 = 216$ If the y -coordinate = 216, the x -coordinate = 3.	Substitute $y = 216$ into the equation $y = 6^x$.	Substitute the value of the y -coordinate into the equation of the curve and solve the resulting equation by trial and error.

9 Complete the following to find the x -coordinate of the point on the graph of $y = 7^x$ whose y -coordinate is 2401.

Try $x = 3$ $y = 7^3 = \underline{\hspace{2cm}}$ Too small

$x = 24$ $y = 7^{24} = \underline{\hspace{2cm}}$

If the y -coordinate = 2401, the x -coordinate = $\underline{\hspace{2cm}}$

10 Find, by trial and error, the x -coordinate of the given points on the following curves.

a $y = 3^x$ whose y -coordinate = 81

b $y = 2^x$ whose y -coordinate = 32

c $y = 5^x$ whose y -coordinate = 125

d $y = 10^x$ whose y -coordinate = 1 000 000

Investigation 2 Circles

1 The point $(5, 12)$ lies on the circumference of a circle with centre $O(0, 0)$.

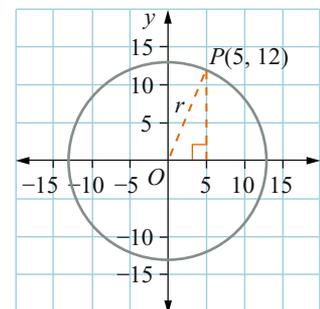
a Using the right-angled triangle shown in the diagram, find the radius, r , of the circle.

b Show that $Q(-5, 12)$ and $R(0, -13)$ also lie on the circumference of this circle; that is, show that $OQ = OR = OP$.

c Write down the coordinates of three other points that lie on the circumference of this circle.

d If $T(x, y)$ lies on the circumference, show that $x^2 + y^2 = 169$.

e Verify that the equation of the circle centre $O(0, 0)$ and radius r units is $x^2 + y^2 = r^2$.



C

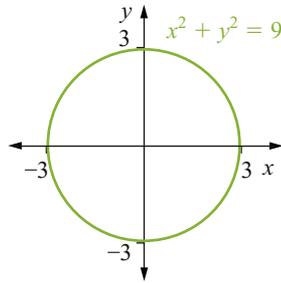
Circles

From Investigation 2 we can see that the equation of the circle with centre the origin O and radius r units is $x^2 + y^2 = r^2$.

EXAMPLE 1

Find the radius of the circle with equation $x^2 + y^2 = 9$. Sketch the graph.

Solve/Think	Apply
$r^2 = 9$ $r = \sqrt{9}$ $= 3$ (radius > 0)	When the equation of a circle is in the form $x^2 + y^2 = r^2$, the radius is r .



Exercise 12C

1 Find the radius and draw a sketch of the following circles.

a $x^2 + y^2 = 64$

b $x^2 + y^2 = 81$

c $x^2 + y^2 = 36$

d $x^2 + y^2 = 144$

e $x^2 + y^2 = 100$

f $x^2 + y^2 = 20$

g $x^2 + y^2 = \frac{9}{4}$

h $9x^2 + 9y^2 = 1$

i $16x^2 + 16y^2 = 9$

2 Write the equation of the circle with centre $(0, 0)$ and radius:

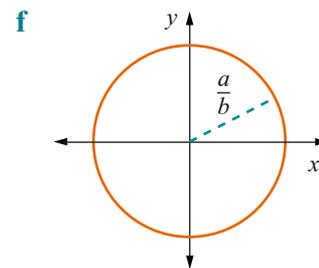
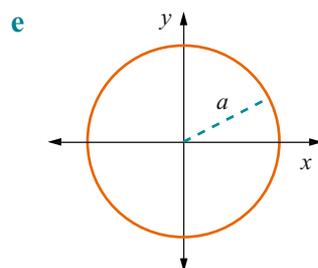
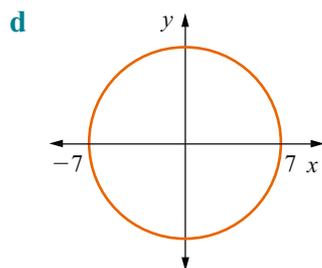
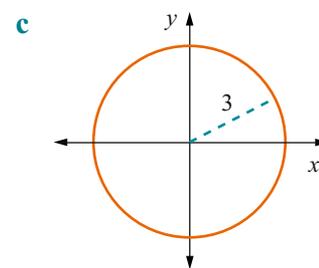
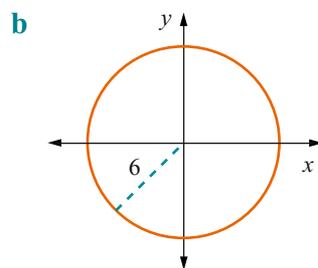
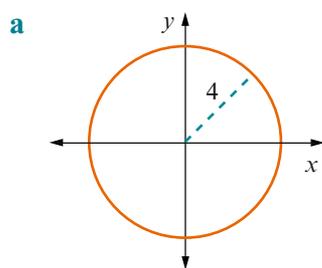
a 3 units

b 15 units

c 1.5 units

d $\sqrt{7}$ units

3 Write the equation of the following circles.



4 Show that the equation of a circle with centre $(0, 0)$ and radius $\frac{\sqrt{5}}{2}$ is $4x^2 + 4y^2 = 5$.

EXAMPLE 2

Find the equation of the circle with its centre at the origin and $(-5, 6)$ on its circumference.

Solve/Think	Apply
<p>Let the equation of the circle be $x^2 + y^2 = r^2$.</p> <p>As the point $(-5, 6)$ lies on the circle $(-5)^2 + 6^2 = r^2$</p> $61 = r^2$ $r = \sqrt{61}$ <p>Hence the equation of the circle is $x^2 + y^2 = 61$.</p>	<p>Let the equation of the circle be $x^2 + y^2 = r^2$.</p> <p>Substitute the coordinates of the given point into this equation to find r^2.</p>

- 5** Complete to find the equation of the circle with its centre at the origin and point $(3, -1)$ on its circumference.

The general form of a _____ with centre $(0, \text{---})$ and radius r units is:

$$x^2 + \text{---}^2 = \text{---}^2$$

$$(3)^2 + (\text{---})^2 = r^2$$

$$\text{---} = r^2$$

$$r = \sqrt{10}$$

Hence the equation of the circle is $x^2 + \text{---} = 10$.

- 6** Find the equation of the circles, centre the origin, with the following points on their circumference.

a $(2, 5)$

b $(-6, -3)$

c $(7, -5)$

d $(0, 8)$

e $(-1, -1)$

- 7** Which of the following equations represent circles with centre the origin?

A $x^2 + y^2 = 1$

B $x^2 = 3 - y^2$

C $x^2 - y^2 = 25$

D $\frac{x^2 + y^2}{3} = 3$

E $\frac{x^2}{2} + \frac{y^2}{3} = 1$

F $\frac{x^2}{5} - \frac{y^2}{2} = 1$

G $\frac{x^2}{5} + \frac{y^2}{5} = 1$

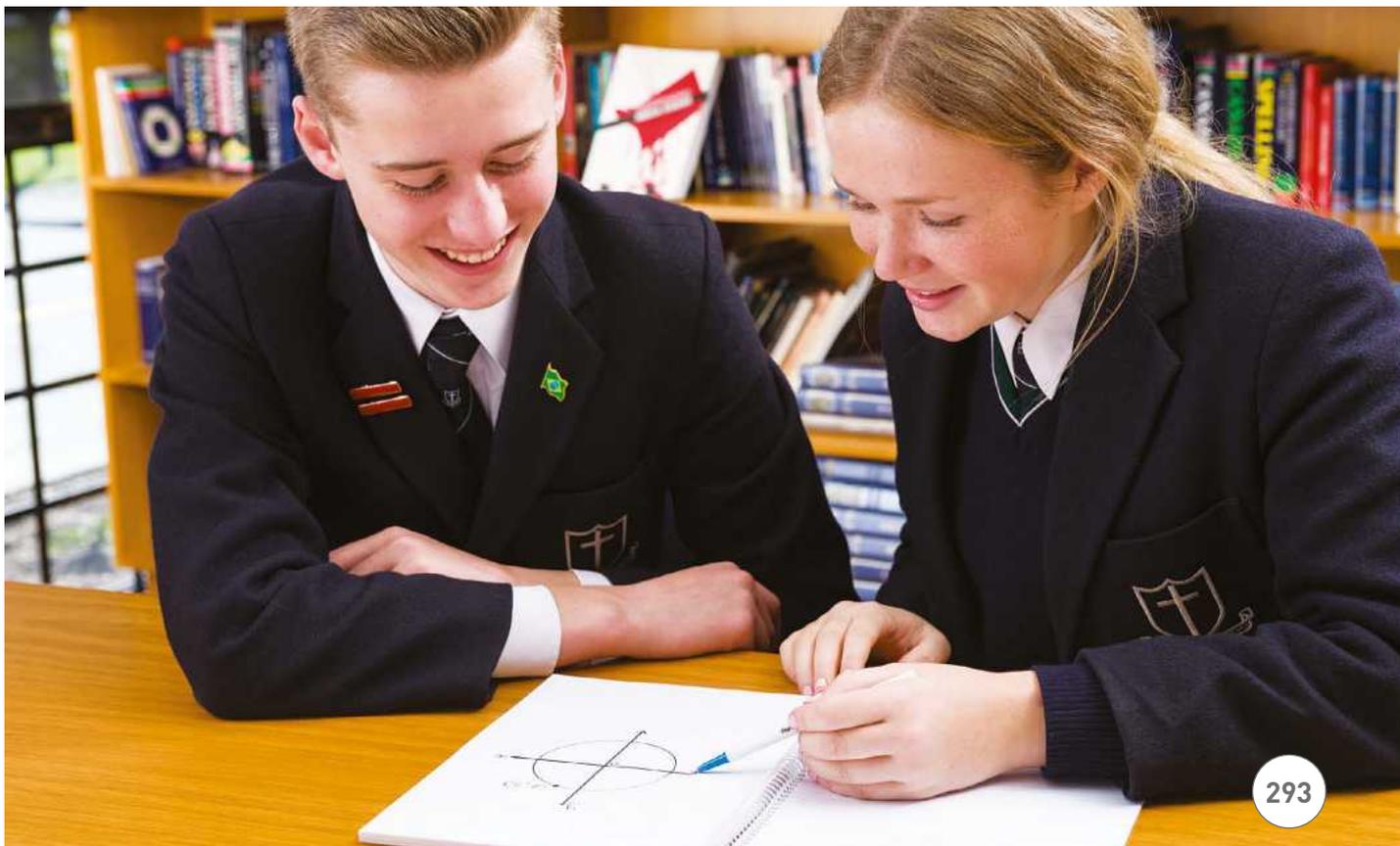
H $(x - 3)^2 + y^2 = 25$

I $x = \pm\sqrt{9 - y^2}$

J $y = \pm\sqrt{1 - x^2}$

K $y = \sqrt{16 - x^2}$

L $y = -\sqrt{4 - x^2}$



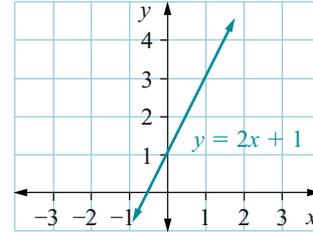
D

Miscellaneous graphs

This section explores the connection between algebraic and graphical representations of straight lines, parabolas, exponential curves and circles. A summary of each type follows.

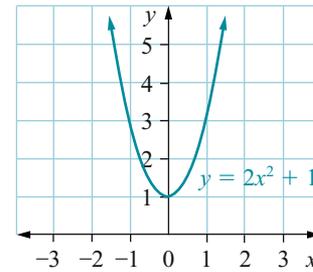
Straight lines

These are the graphs of equations of the form $y = mx + b$. They are straight lines with gradient m and y -intercept of b . $y = 2x + 1$ has gradient = 2 and y -intercept = 1.



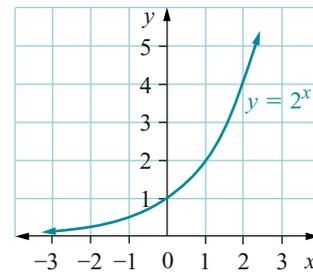
Parabolas

These are graphs of equations of the form $y = ax^2 + bx + c$ where c is the y -intercept.



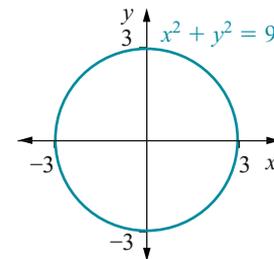
Exponential curves

These are graphs of equations of the form $y = a^x$ where a is a constant. They all pass through the point $(0, 1)$.



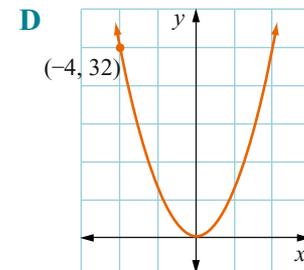
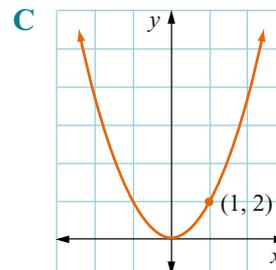
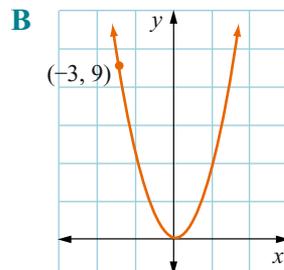
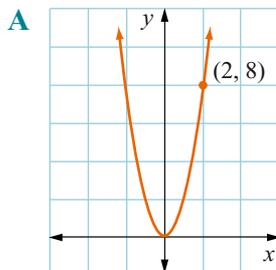
Circles

These are graphs of equations of the form $x^2 + y^2 = r^2$. The centre of the circle is $(0, 0)$ and the radius is r .

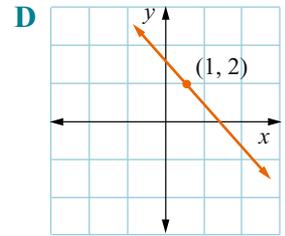
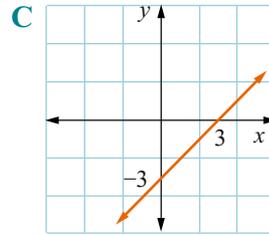
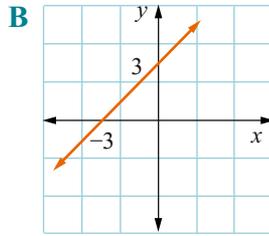
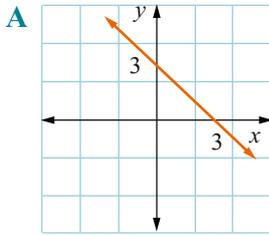


Exercise 12D

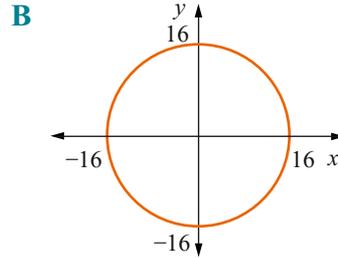
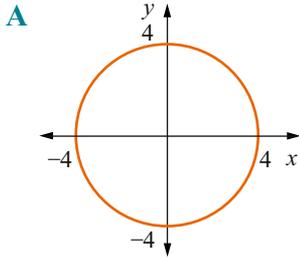
1 Test the point on each graph to decide which of these parabolas have equation $y = 2x^2$.



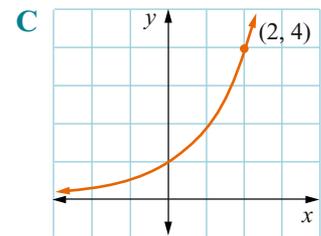
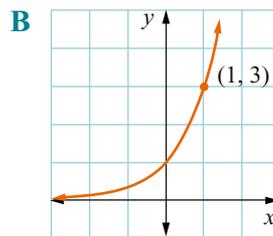
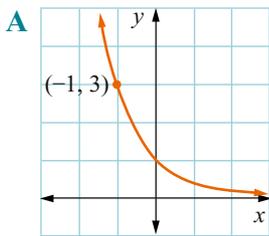
2 Test a point on each graph to decide which of these straight lines have equation $y = 3 - x$.



3 Test a point on each graph to decide which of these circles has equation $x^2 + y^2 = 16$.

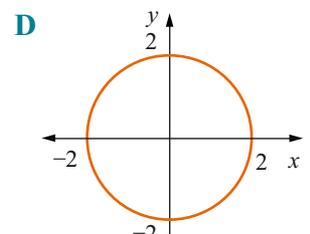
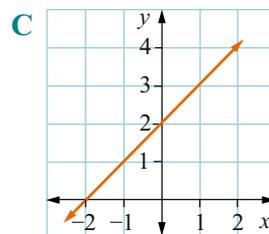
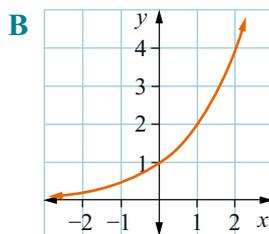
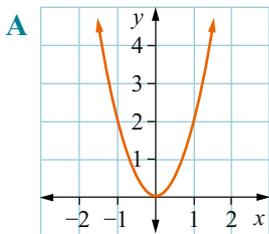


4 Test a point on each graph to decide which of these exponential graphs is $y = 3^x$.

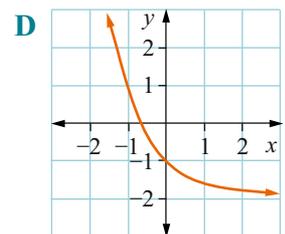
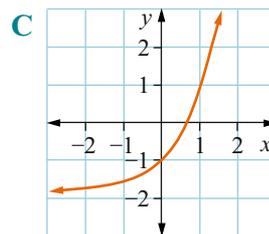
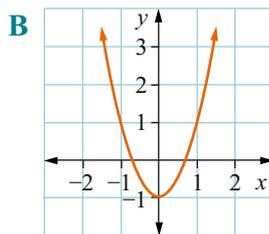
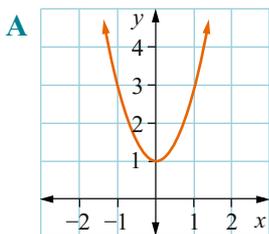


5 Determine which of the following could be the graph of:

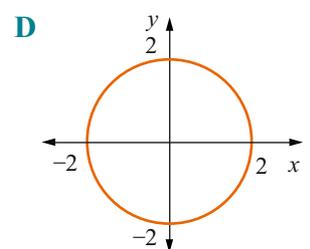
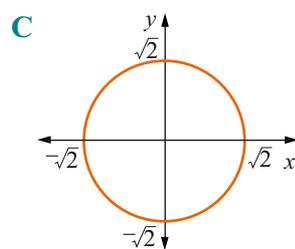
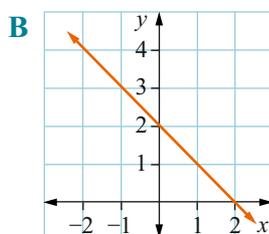
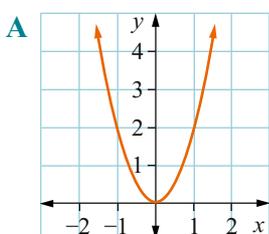
a $y = 2^x$



b $y = 2x^2 - 1$



c $x^2 + y^2 = 2$



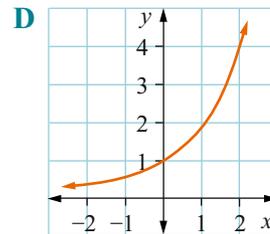
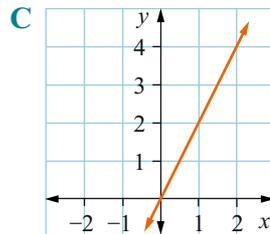
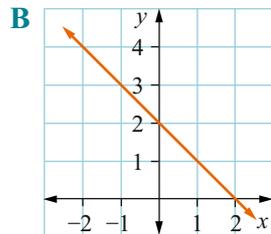
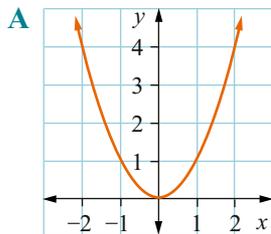
6 Match the following equations with the graphs below.

a $y = 2 - x$

b $y = 2x$

c $y = x^2$

d $y = 2^x$



7 Match the following equations with the graphs below.

a $y = 2x^2$

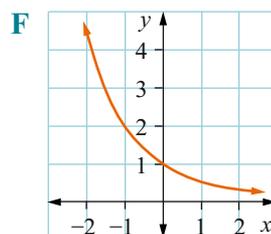
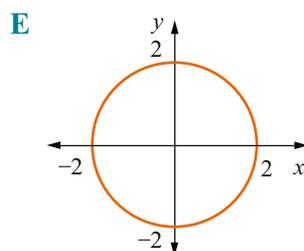
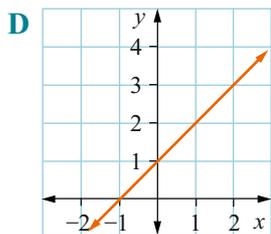
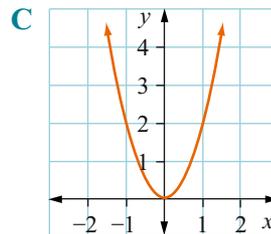
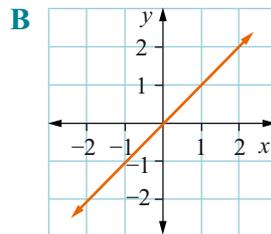
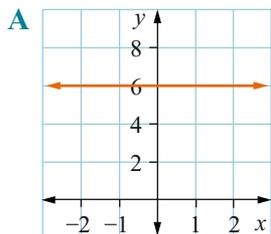
b $y = 6$

c $x^2 + y^2 = 4$

d $y = x + 1$

e $y = x$

f $y = 2^{-x}$



8 Match the following equations with the graphs below.

a $y = 3^x$

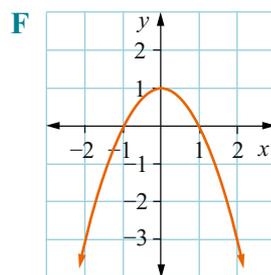
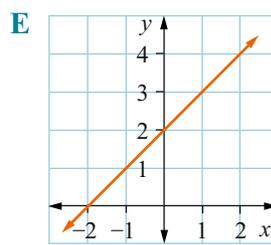
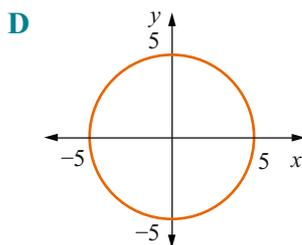
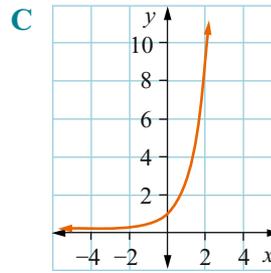
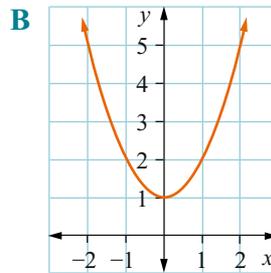
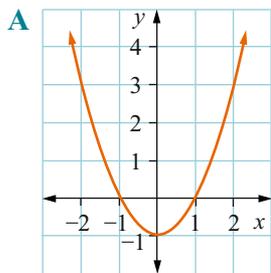
b $x^2 + y^2 = 25$

c $y = x^2 + 1$

d $y = x + 2$

e $y = x^2 - 1$

f $y = 1 - x^2$



Language in mathematics

Benoit Mandelbrot (1924–2010)

Benoit Mandelbrot was largely responsible for the present interest in fractal geometry. He showed how fractals can occur in many different places in both mathematics and nature.

Mandelbrot was born in Poland in 1924 into a family with a very academic tradition. As a young boy, he was introduced to mathematics by his two uncles. The family emigrated to France in 1936. Mandelbrot attended school in Paris until the start of World War II, when his family moved to the country. The war, the constant threat of poverty and the need to survive often kept him away from school. Mandelbrot now attributes much of his success to this unconventional education.

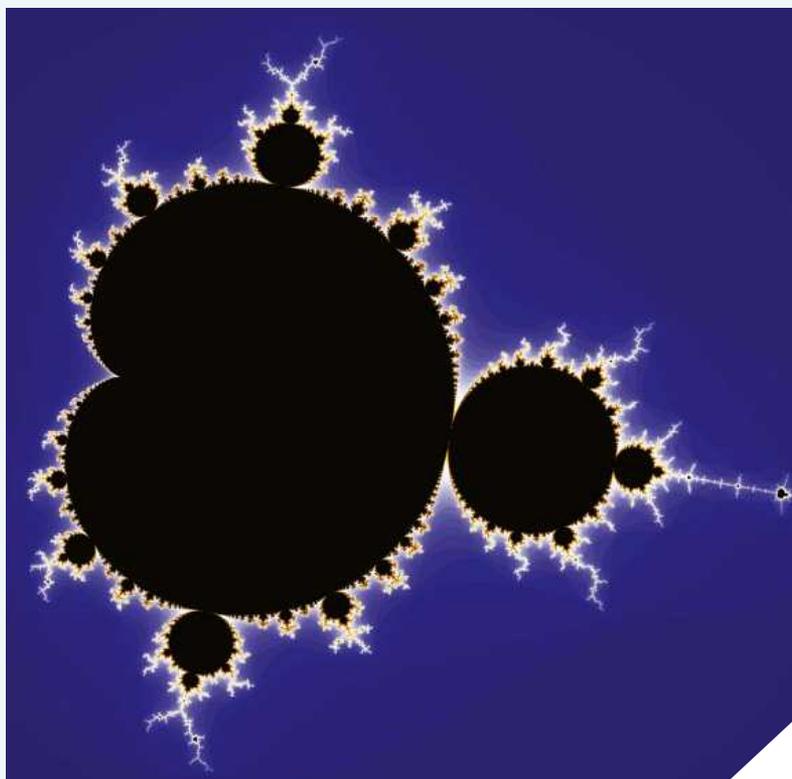
After completing his studies in France, Mandelbrot went to the United States where he studied further and later worked for IBM and Harvard University. With the aid of computer graphics, he was able to discover some of the most beautiful fractals known today. To do this he had to develop not only new mathematical ideas, but also some of the first computer programs to print graphics.

The Mandelbrot set is a connected set of points that are generated through an iteration on quadratic polynomials of the form $y = x^2 + c$. (Iteration means to repeat a process over and over again.) This gives a sequence, or list, of values. This sequence of values must remain within a certain distance of the origin forever to be in the Mandelbrot set, otherwise the pattern does not continue.

The same pattern is repeated over and over, with the size decreasing after each iteration. In this picture the Mandelbrot set is shown in black and as you zoom in on the picture the shapes and patterns are repeated indefinitely.

Fractals occur in physics in the description of the behaviour of turbulence, as the foundations of what is now known as chaotic systems, in economics in the behaviour of the stock exchange, and in the growth of mammalian cells. In the garden, the difference between the flower heads of broccoli and cauliflower can be exactly characterised in fractal theory.

Mandelbrot received numerous honours and prizes in recognition of his remarkable achievements.



1 Answer these questions based on the notes on Mandelbrot.

- How old was Mandelbrot when the family moved to France?
- To what does he attribute his success?
- What is it about a sequence that make it a Mandelbrot set?
- In your own words, define an iteration and give an example.
- Investigate some fractals other than the Mandelbrot set.

2 Rearrange these words to form a sentence.

- is a graph A with parabola x^2
- moves constant up or term a graph down parabola The the of

3 Use every third letter to complete a sentence.

S D T H T H W U E R B G M O R A D A N O P P Q H R G S T H O U O F S C E T R X F
 U P O W O A C N V U E P O N Q E T D U I T E A F Y L K G S A S A D R N T T D U X
 P G S A I O R E R A A S B F G O J K L O I A Y T S R E A W Q R Q A E Z S B C D O
 R F T T H H J U C I K U O L R P O V I S E A Z S S X W D C I C F T G B H H N T J
 M H K L E L P E O K X I J P U H O Y G N T F E R D N E S T W Q I W E A T U L O P
 A L G S D E A T U N F S U A G M H B B D E E Y U R R G T S E O F T T G U H K O E
 D T P H D O J H W S D E G S R H T O H S F K U X T C A D T N I U D Y G T R D H W
 A E C F P B H A I Y R S A A C A B G F O J Y L K G A N G A U E S A G X G A T H R
 O A R T U J H Y H E T G P F H O H F W J F E J R R I R O A A F V F T H M W K P O

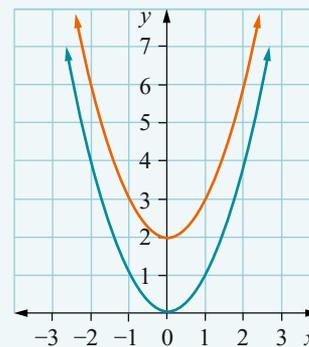
Terms

asymptote	circle	circumference	coefficient	constant term	equation
exponential	graph	linear	number plane	parabola	parabolic
radius	quadratic relationship		square	straight line	symmetrical
variable	vertex				

Check your skills

1 The graphs drawn have equations:

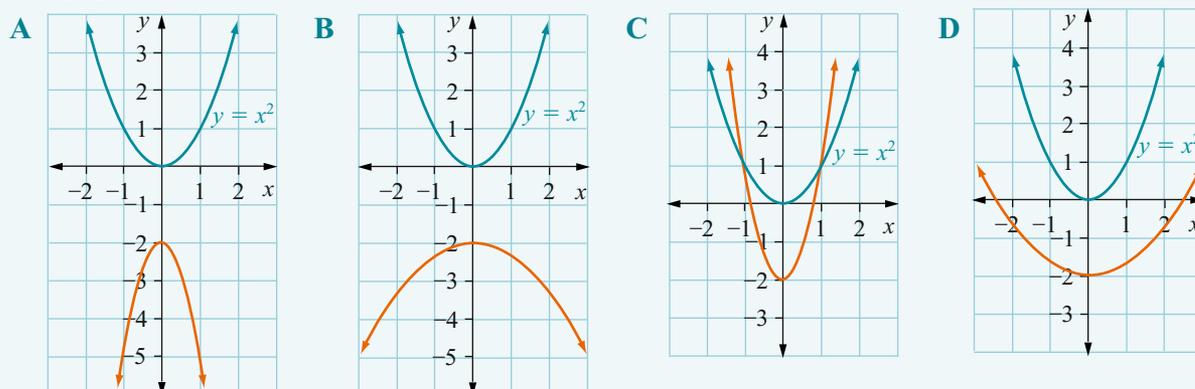
- A** $y = x^2$ and $y = x^2 + 2$
B $y = x^2$ and $y = -x^2$
C $y = x^2$ and $y = 3x^2$
D $y = -x^2$ and $y = -x^2 + 2$



2 The graph with the same shape as $y = x^2$ and vertex $(0, 5)$ has equation:

- A** $y = x^2 + 5$ **B** $y = -x^2 + 5$ **C** $y = -x^2 - 5$ **D** $y = x^2 - 5$

3 The graph of $y = 3x^2 - 2$ could be:



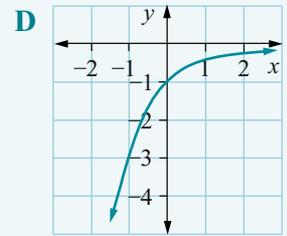
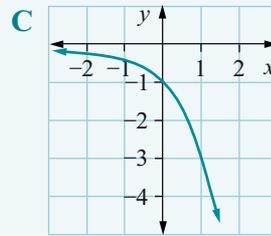
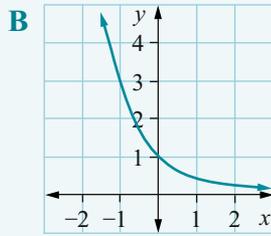
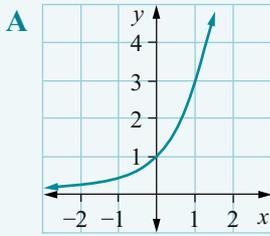
4 The x -coordinates of the points on $y = 2x^2 - 5$ with y -coordinate $y = 7$ are:

- A** $\pm\sqrt{12}$ **B** $\pm\sqrt{6}$ **C** $\pm\sqrt{5}$ **D** $\pm\sqrt{2}$

5 The y -intercept of the parabola $y = x^2 - 3x + 7$ is:

- A** -7 **B** 7 **C** -3 **D** 1

6 The graph of $y = -3^x$ is:



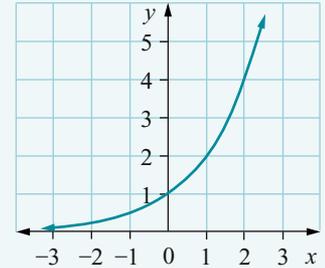
7 The equation of the exponential curve shown is:

A $y = 2^x$

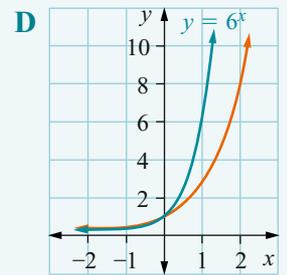
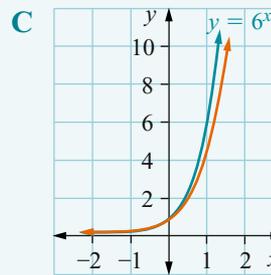
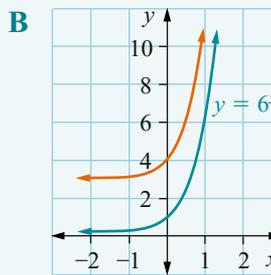
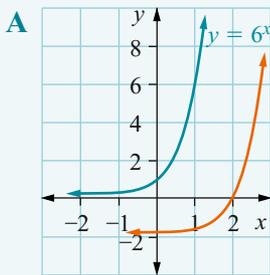
B $y = 2^{-x}$

C $y = 5^x$

D $y = 5^{-x}$



8 Which of the following could be the graph of $y = 4^x$?



9 The x -coordinate of the point on the graph of $y = 3^x$ with y -coordinate = 243 is:

A 2

B 3

C 4

D 5

10 The radius of the circle $x^2 + y^2 = 16$ is:

A 16

B 8

C 4

D 2

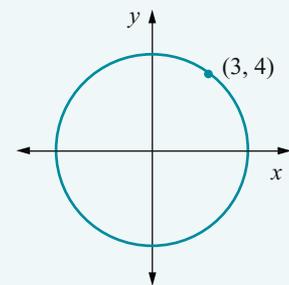
11 The equation of the circle shown is:

A $x^2 + y^2 = 9$

B $x^2 + y^2 = 16$

C $x^2 + y^2 = 25$

D $x^2 + y^2 = 49$



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–5	6–9	10–11
Section	A	B	C

12A Review set

1 On the same number plane, sketch graphs of:

a $y = x^2$

b $y = 3x^2$

c $y = \frac{1}{2}x^2$

d $y = x^2 + 3$

2 Find the x -coordinate(s) of the point(s) on the following parabolas.

a $y = 3x^2 - 1$ with y -coordinate = 5

b $y = 2x^2 + 3$ with y -coordinate = 21

3 On the same number plane, sketch graphs of:

a $y = 3^x$

b $y = 3^{-x}$

c $y = -3^x$

d $y = -3^{-x}$

e $y = 3^x + 1$

4 Find the x -coordinate of the point on the graph of $y = 2^x$ that has y -coordinate = 16.

5 Write the equation of the circle with centre at the origin and a radius of:

a 5 units

b $\sqrt{5}$ units

6 Find the equation of the circle with centre the origin and passing through the point (5, 7).

7 Match each of the following equations with the graphs below.

a $y = 3 - x$

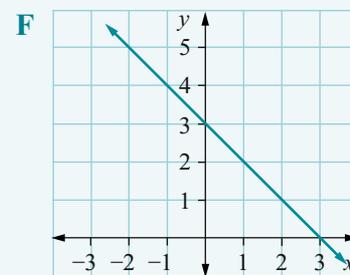
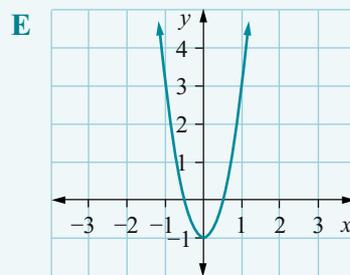
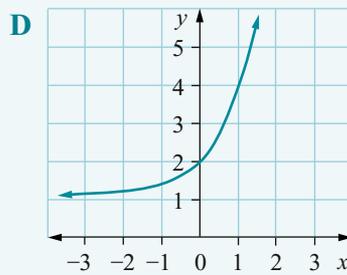
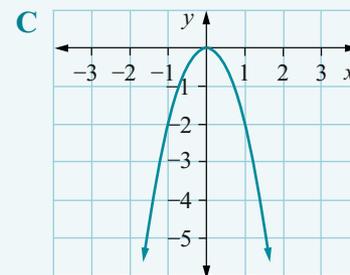
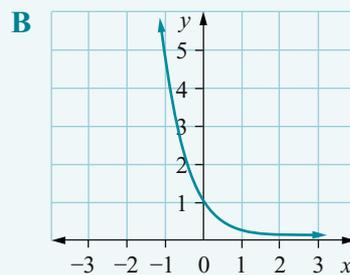
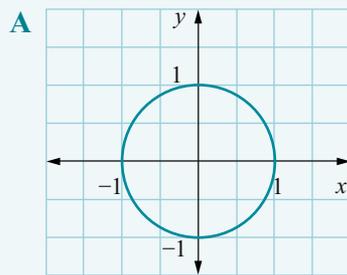
b $y = -2x^2$

c $y = 4x^2 - 1$

d $y = 5^{-x}$

e $y = 3^x + 1$

f $x^2 + y^2 = 1$



12B Review set

1 On the same number plane sketch graphs of:

a $y = -x^2$

b $y = -2x^2$

c $y = -\frac{1}{2}x^2$

d $y = -x^2 - 3$

2 Find the x -coordinate(s) of the point(s) on the following parabolas.

a $y = 4x^2 + 1$ with y -coordinate = 37

b $y = 2x^2 - 3$ with y -coordinate = 17

3 On the same number plane, sketch graphs of:

a $y = 2^x$

b $y = 2^{-x}$

c $y = -2^x$

d $y = -2^{-x}$

e $y = 2^x - 1$

4 Find the x -coordinate of the point on the graph of $y = 3^x$ which has y -coordinate = 81.

5 Write the equation of the circle with centre at the origin and a radius of:

a 4 units

b $\sqrt{3}$ units

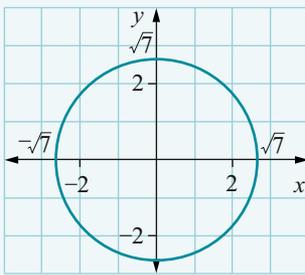
6 Find the equation of the circle with centre the origin and passing through the point (5, 12).

7 Match each of the following equations with the graphs below.

a $y = 2x - 4$

d $y = -5^{-x}$

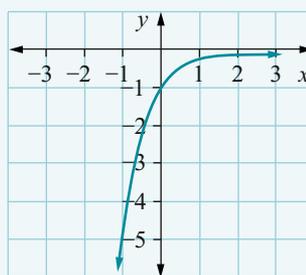
A



b $y = -3x^2$

e $y = 5^x - 1$

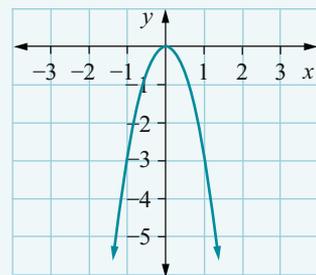
B



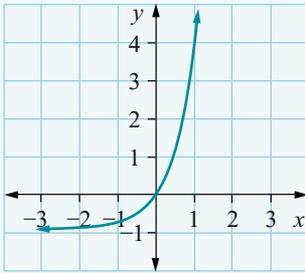
c $y = 2x^2 + 5$

f $x^2 + y^2 = 7$

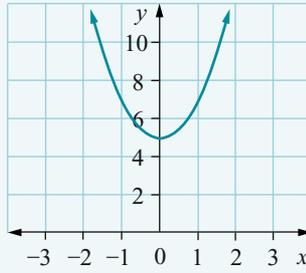
C



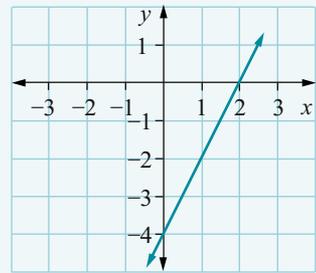
D



E



F



12C Review set

1 On the same number plane sketch graphs of:

a $y = x^2$

b $y = -3x^2$

c $y = -\frac{2}{3}x^2$

d $y = 2x^2 + 2$

2 Find the x -coordinate(s) of the point(s) on the following parabolas.

a $y = x^2 + 11$ with y -coordinate = 47

b $y = 5x^2 + 17$ with y -coordinate = 42

3 On the same number plane, sketch graphs of:

a $y = 4^x$

b $y = 4^{-x}$

c $y = -4^x$

d $y = -4^x + 3$

e $y = -4^x - 2$

4 Find the x -coordinate of the point on the graph of $y = 10^x$ that has y -coordinate = 100 000.

5 Write the equation of the circle with centre at the origin and a radius of:

a 10 units

b $\sqrt{10}$ units

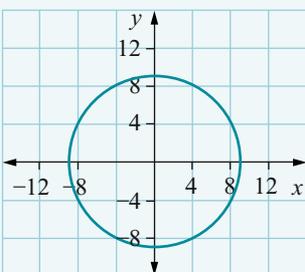
6 Find the equation of the circle with centre the origin and passing through the point (16, 30).

7 Match each of the following equations with the graphs shown.

a $x + y + 1 = 0$

d $y = -4^{-x}$

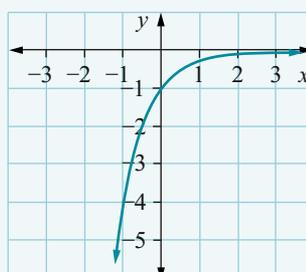
A



b $y = -\frac{1}{3}x^2$

e $y = 4^x + 3$

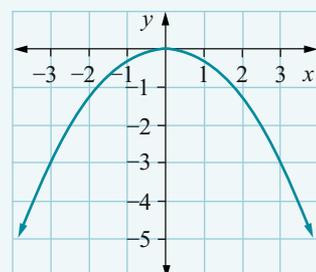
B

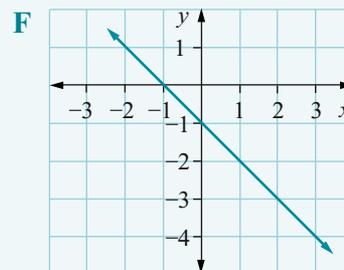
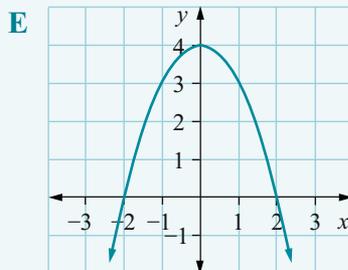
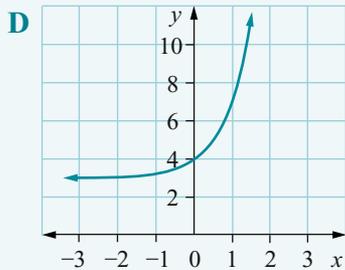


c $y = -x^2 + 4$

f $x^2 + y^2 = 81$

C





12D Review set

1 On the same number plane sketch graphs of:

a $y = -x^2$

b $y = -3x^2$

c $y = -\frac{3}{2}x^2$

d $y = -x^2 + 2$

2 Find the x -coordinate(s) of the point(s) on the following parabolas.

a $y = 2x^2 - 13$ with y -coordinate = 19

b $y = 3x^2 + 9$ with y -coordinate = 21

3 On the same number plane, sketch graphs of:

a $y = 3^x + 2$

b $y = 3^x - 2$

c $y = -3^x + 2$

d $y = -3^{-x} + 2$

e $y = -3^x - 2$

4 Find the x -coordinate of the point on the graph of $y = 2^x$ which has y -coordinate = 128.

5 Write the equation of the circle with centre at the origin and a radius of:

a 7 units

b $\sqrt{\frac{3}{2}}$ units

6 Find the equation of the circle with centre the origin and passing through the point $(-4, 8)$.

7 Match each of the following equations with the graphs below.

a $x + 2y - 4 = 0$

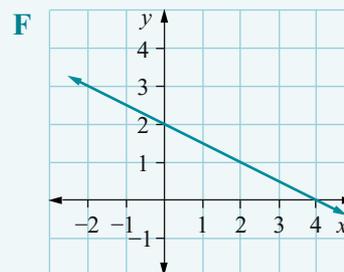
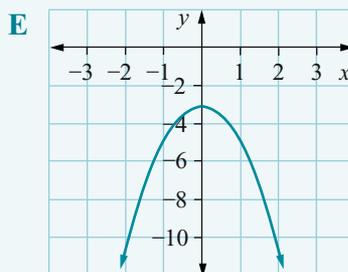
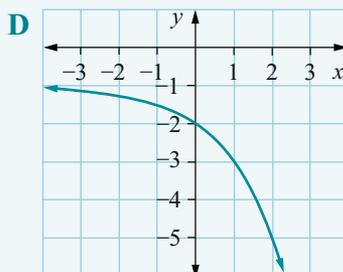
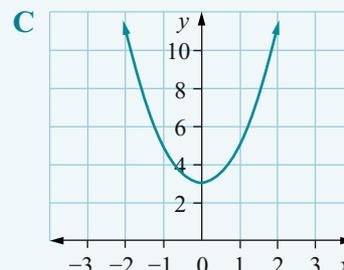
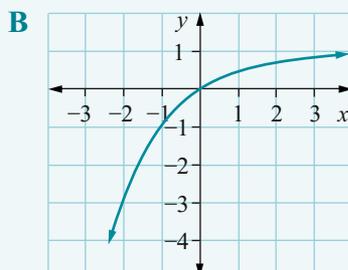
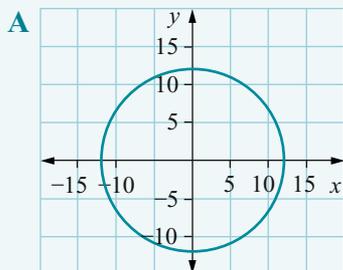
b $y = 2x^2 + 3$

c $y = -2x^2 - 3$

d $y = -2^{-x} + 1$

e $y = -2^x - 1$

f $x^2 + y^2 = 144$





13

Further trigonometry

● This chapter deals with trigonometric relationships and problem solving.

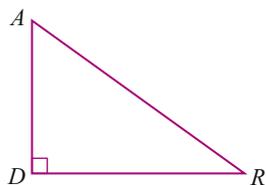
After completing this chapter you should be able to:

- ▶ define complementary angles
- ▶ use trigonometric ratios and obtuse angles
- ▶ solve problems using the sine rule and the cosine rule
- ▶ find the area of non-right-angled triangles
- ▶ draw diagrams and solve problems involving trigonometry.

Diagnostic test

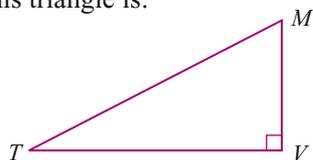
- 1 The side opposite $\angle A$ in this triangle is:

A AD
 B AR
 C DR
 D the hypotenuse



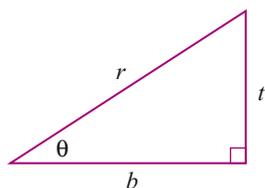
- 2 The hypotenuse of this triangle is:

A MT
 B MV
 C TV
 D m



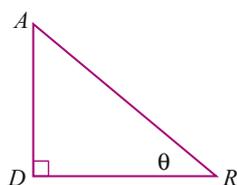
- 3 The expression for $\sin \theta$ in this triangle is:

A $\frac{r}{t}$ B $\frac{b}{t}$
 C $\frac{t}{r}$ D $\frac{t}{b}$



- 4 The expression for $\tan \theta$ in this triangle is:

A $\frac{AD}{DR}$ B $\frac{DR}{AD}$
 C $\frac{AR}{AD}$ D $\frac{DR}{AR}$



- 5 The value of $\cos 53^\circ$ is closest to:

A 53 B 0.6018
 C 0.8192 D -0.9848

- 6 The value of θ in the expression $\tan \theta = 3.466$ is closest to:

A 74° B 0.0606
 C 19° D 3

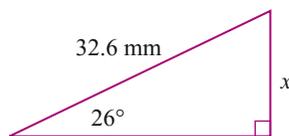
- 7 The value of $\angle A$ in the expression

$\cos \theta = \frac{11.5}{25}$ is closest to:

A 0.9799 B 0.999 96
 C 62° D 63°

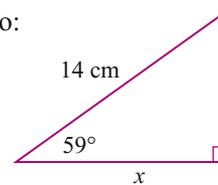
- 8 The value of x is closest to:

A 14.3 mm
 B 74.4 mm
 C 29.3 mm
 D 15.9 mm



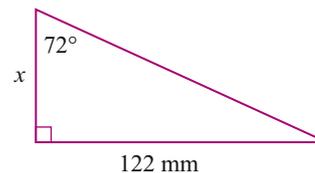
- 9 The value of x is closest to:

A 27.2 cm
 B 12 cm
 C 23.3 cm
 D 7.2 cm



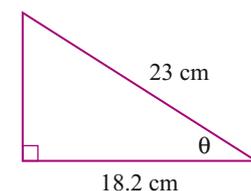
- 10 The value of x is closest to:

A 375 mm
 B 40 mm
 C 128 mm
 D 116 mm



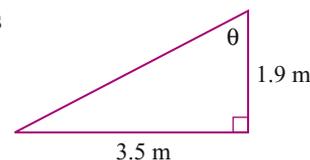
- 11 The value of θ in this triangle is closest to:

A 52° B 38°
 C 1° D 51°



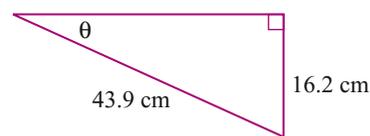
- 12 The value of θ in this triangle is closest to:

A 62° B 28°
 C 33° D 57°



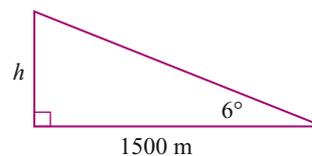
- 13 The value of θ in this triangle is closest to:

A 68°
 B 70°
 C 32°
 D 22°



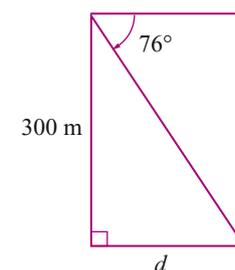
- 14 The angle of elevation of the top of a cliff from a boat 1500 m out to sea is 6° . The height of the cliff above the boat is closest to:

A 150 m
 B 14 272 m
 C 160 m
 D 1492 m



- 15 The angle of depression of the top of a cliff 300 m above sea-level to a boat is 76° . The distance of the boat from the cliff is closest to:

A 1203 m B 75 m
 C 73 m D 1240 m



The diagnostic test questions refer to outcomes ACMMG222, ACMMG223, ACMMG224 and ACMMG245.



A

Trigonometric ratios of acute angles

In Year 9 trigonometry, angles were rounded to the nearest degree.

For greater accuracy, we can measure angles not only in **degrees** ($^{\circ}$) but also in parts of degrees, as decimals or using the units **minutes** ($'$) and **seconds** ($''$).

$$1 \text{ degree} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

In this chapter we will be measuring angles accurate to the nearest minute. For example, angle 53 degrees 18 minutes is written $53^{\circ}18'$.

EXAMPLE 1

Find the following correct to 4 decimal places.

a $\cos 84.3^{\circ}$

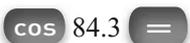
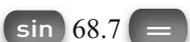
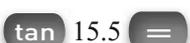
b $\sin 68.7^{\circ}$

c $\tan 15.5^{\circ}$

d $\cos 78^{\circ}15'$

e $\sin 11^{\circ}12'$

f $\tan 17^{\circ}58'$

	Solve	Think	Apply
a	$\cos 84.3^{\circ} \approx 0.0993$		Make sure your calculator is in degree mode. Some calculators have a  key instead of a  key.
b	$\sin 68.7^{\circ} \approx 0.9317$		
c	$\tan 15.5^{\circ} \approx 0.2773$		
d	$\cos 78^{\circ}15' \approx 0.2036$		On some calculators the second  is not required.
e	$\sin 11^{\circ}12' \approx 0.1942$		
f	$\tan 17^{\circ}58' \approx 0.3243$		



Exercise 13A

1 Find the following correct to 4 decimal places.

- | | | |
|------------------------------|------------------------------|------------------------------|
| a $\sin 36.8^\circ$ | b $\cos 14.23^\circ$ | c $\tan 8.11^\circ$ |
| d $\cos 65.25^\circ$ | e $\cos 89^\circ 21'$ | f $\tan 18^\circ 23'$ |
| g $\tan 68^\circ 23'$ | h $\sin 45^\circ 21'$ | i $\cos 57^\circ 51'$ |
| j $\cos 33^\circ 21'$ | k $\tan 21^\circ 33'$ | l $\sin 11^\circ 11'$ |

EXAMPLE 2

Evaluate the following correct to 4 decimal places.

- a** $12 \cos 15.6^\circ$ **b** $5 \sin 11^\circ 15'$ **c** $\frac{3 \tan 11^\circ 51'}{\cos 23^\circ 15'}$

	Solve	Think	Apply
a	$12 \cos 15.6^\circ \approx 11.5580$	12 \times cos 15.6 $=$	Remember that the fraction line is a grouping symbol. Insert brackets if you are not sure of the order of operations.
b	$5 \sin 11^\circ 15' \approx 0.9755$	5 \times sin 11 DMS 15 DMS $=$	
c	$\frac{3 \tan 11^\circ 51'}{\cos 23^\circ 15'} \approx 0.6851$	3 \times tan (11 DMS 51 DMS) \div cos (23 DMS 15) DMS $=$	

2 Evaluate the following correct to 4 decimal places.

- | | | |
|--|---|--|
| a $8 \cos 23.1^\circ$ | b $5 \tan 16.4^\circ$ | c $15 \sin 48.18^\circ$ |
| d $23 \sin 75^\circ 12'$ | e $8.3 \tan 58^\circ 51'$ | f $12.3 \cos 27^\circ 48'$ |
| g $\frac{9 \sin 11^\circ 51'}{\sin 31^\circ}$ | h $\frac{8 \tan 16^\circ 16'}{\sin 15^\circ}$ | i $\frac{12.3 \cos 48^\circ}{\sin 16^\circ 15'}$ |
| j $\frac{8.7 \tan 75^\circ 14'}{13.2}$ | k $\frac{4.2 \cos 18.3^\circ}{6.8}$ | l $\frac{3 \sin 83^\circ 12'}{16.5}$ |
| m $\frac{4 \sin 18^\circ \cos 18^\circ}{3}$ | n $\frac{11 \tan 16^\circ \cos 14^\circ}{\sin 12^\circ}$ | o $\frac{8.3 \cos 11^\circ 15'}{\sin 11^\circ 15'}$ |

Using trigonometric ratios to find angles

You can work backwards on a calculator to find an angle from one of the trigonometric ratios, by using one of the key combinations **SHIFT** **tan** or **SHIFT** **sin** or **SHIFT** **cos**. These may appear on your calculator display as \tan^{-1} or \sin^{-1} or \cos^{-1} .

For example, if $\sin \theta = 0.4369$

then $\theta = \sin^{-1} 0.4369$

where $\sin^{-1} 0.4369$ means 'the angle whose sine is 0.4369'.

Similarly, \cos^{-1} means 'the angle whose cosine is' and \tan^{-1} means 'the angle whose tangent is'.

EXAMPLE 3

Find θ to the nearest:

i degree

ii minute.

a $\sin \theta = 0.6314$

b $\tan \theta = 3.6$

c $\cos \theta = 0.8$

	Solve	Think	Apply
a	i $\sin \theta = 0.6314$ $\theta = 39.153\dots$ $\approx 39^\circ$		<p>Make sure your calculator is in degree mode.</p> <p>Ensure that  is pressed before the trigonometric ratios so that the answer is an angle. The half-way point for rounding is 30 seconds. Below 30 seconds round down; 30 seconds or more, round up.</p> <p>Note that some calculators require  before  to convert to minutes and seconds.</p>
	ii $\theta = 39^\circ 9' 12.55\dots''$ $\approx 39^\circ 9'$ to the nearest minute as $12.55\dots < 30$	 As the seconds are less than 30, round the minutes down.	
b	i $\tan \theta = 3.6$ $\theta = 74.475\dots$ $\approx 74^\circ$		
	ii $\theta = 74^\circ 28' 33.20\dots''$ $\approx 74^\circ 29'$ to the nearest minute as $33.2 > 30$	 As the seconds are greater than or equal to 30, round the minutes up.	
c	i $\cos \theta = 0.8$ $\theta = 36.869\dots$ $\approx 37^\circ$		
	ii $\theta = 36^\circ 52' 11.63\dots''$ $\approx 36^\circ 52'$ to the nearest minute as $11.63 < 30$		

3 Write these calculator displays as angles to the nearest minute.

a $43^\circ 27' 14.2''$

b $62^\circ 15' 58.13''$

c $14^\circ 3' 0''$

d $81^\circ 53' 30''$

e $21^\circ 59' 48.72''$

f $10^\circ 1' 28.42''$

g $35^\circ 28' 18.3''$

h $72^\circ 51' 38.5''$

i $27^\circ 53' 58.1''$

j $39^\circ 35' 11.3''$

k $68^\circ 54' 41.2''$

l $0^\circ 3' 34.2''$

4 Find the value of θ to the nearest:

i degree

ii minute.

a $\sin \theta = 0.3625$

b $\cos \theta = 0.1445$

c $\tan \theta = 2.1351$

d $\cos \theta = 0.6731$

e $\tan \theta = 4.1371$

f $\sin \theta = 0.1113$

g $\tan \theta = 0.0371$

h $\sin \theta = 0.5512$

i $\cos \theta = 0.0314$

j $\sin \theta = 0.0027$

k $\tan \theta = 23.7215$

l $\cos \theta = 0.9811$

m $\cos \theta = 0.6614$

n $\sin \theta = 0.6262$

o $\tan \theta = 0.2222$

EXAMPLE 4

Find θ to the nearest:

i degree

ii minute.

a $\sin \theta = \frac{5}{9}$

b $\cos \theta = \frac{6}{13}$

c $\tan \theta = \frac{18}{7}$

	Solve	Think	Apply
a i	$\sin \theta = \frac{5}{9}$ $\theta = 33.74\dots$ $\approx 34^\circ$		<p>Make sure that the calculator is in degree mode.</p> <p>Press SHIFT first to obtain an angle.</p> <p>Put the fraction in brackets before pressing =.</p> <p>Round accordingly.</p> <p>Note that some calculators require SHIFT DMS to convert to minutes and seconds.</p>
	ii $\theta = 33^\circ 44' 56.35\dots''$ $\approx 33^\circ 45'$	DMS As the seconds are greater than 30, round the minutes up.	
b i	$\cos \theta = \frac{6}{13}$ $\theta = 62.51\dots$ $\approx 63^\circ$		
	ii $\theta = 62^\circ 30' 48.86\dots''$ $\approx 62^\circ 31'$	DMS As the seconds are greater than 30, round the minutes up.	
c i	$\tan \theta = \frac{18}{7}$ $\theta = 68.74\dots$ $\approx 69^\circ$		
	ii $\theta = 68^\circ 44' 58.18\dots''$ $\approx 68^\circ 45'$	DMS As the seconds are greater than 30, round the minutes up.	

5 Find the value of θ to the nearest:

i degree

ii minute.

a $\tan \theta = \frac{14}{3}$

b $\cos \theta = \frac{3}{11}$

c $\sin \theta = \frac{11}{18}$

d $\sin \theta = \frac{4}{29}$

e $\tan \theta = \frac{6}{7}$

f $\cos \theta = \frac{14}{17}$

g $\sin \theta = \frac{0.013}{0.214}$

h $\cos \theta = \frac{6.2}{15}$

i $\tan \theta = \frac{11.27}{15}$

j $\cos \theta = \frac{1}{3}$

k $\sin \theta = \frac{3}{4}$

l $\tan \theta = \frac{4}{3}$

6 Find angle A to the nearest minute given that:

a $\cos A = 0.7$

b $\sin A = 0.642$

c $\tan A = 3.265$

d $\sin A = 0.435$

e $\tan A = 38.188$

f $\cos A = 0.434$

g $\tan A = 0.733$

h $\sin A = 0.349$

i $\cos A = 0.144$

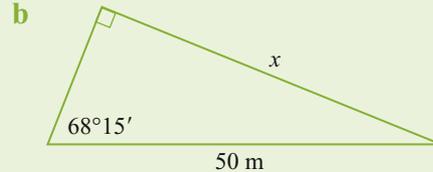
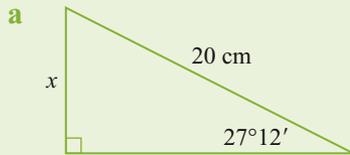
B

Using trigonometry to find sides

This section revises Year 9 material and extends to include angles in degrees and minutes.

EXAMPLE 1

Use the sine ratio to find the value of x correct to 1 decimal place.



	Solve	Think	Apply
a	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 27^\circ 12' = \frac{x}{20}$ $\therefore x = 20 \sin 27^\circ 12'$ $\approx 9.1 \text{ cm}$		<p>x is opposite the given angle in both triangles. The sine ratio is used when the opposite side and hypotenuse are the sides given.</p> <p>When finding the opposite side, multiply the hypotenuse by the sine of the angle.</p>
b	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 68^\circ 15' = \frac{x}{50}$ $\therefore x = 50 \sin 68^\circ 15'$ $\approx 46.4 \text{ m}$		

Exercise 13B

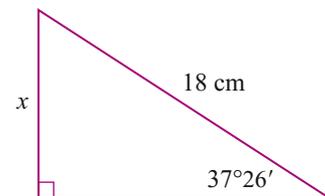
- 1 Using the sine ratio, complete the following to find the value of x correct to 1 decimal place.

$$\sin \theta = \frac{\square}{\text{hypotenuse}}$$

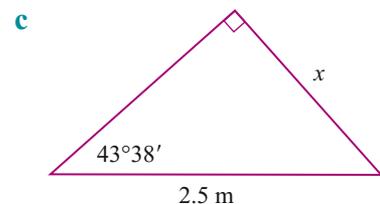
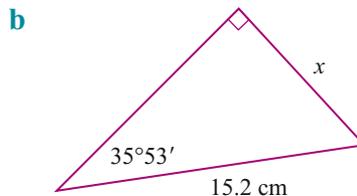
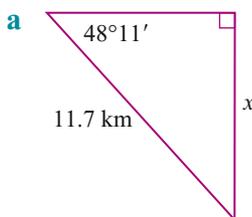
$$\sin 37^\circ 26' = \frac{\square}{18}$$

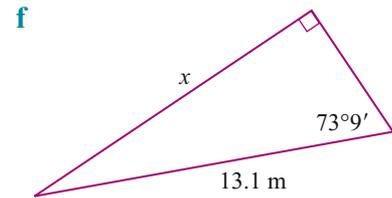
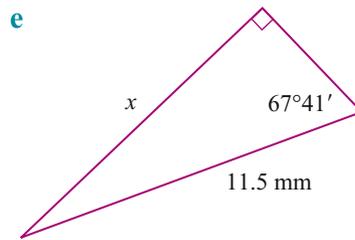
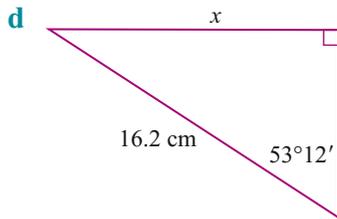
$$x = 18 \times \underline{\hspace{1cm}}$$

$$\approx \underline{\hspace{1cm}}$$



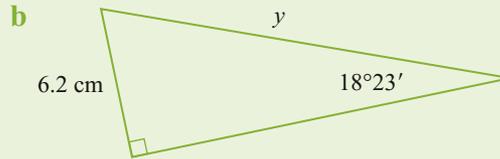
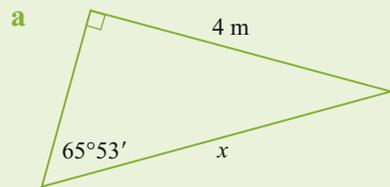
- 2 Use the sine ratio to find the value of x correct to 1 decimal place.





EXAMPLE 2

Use the sine ratio to find the length of the hypotenuse correct to 1 decimal place.



	Solve	Think	Apply
a	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 65^\circ 53' = \frac{4}{x}$ $x \sin 65^\circ 53' = 4$ $x = \frac{4}{\sin 65^\circ 53'}$ $= 4.382\dots$ $\approx 4.4 \text{ m}$	<p>x is the hypotenuse.</p> <p>4 \div sin 65 DMS 53 DMS =</p>	<p>When finding the hypotenuse, divide the opposite side by the sine of the angle.</p> <p>Enter degrees and minutes using the DMS key.</p>
b	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 18^\circ 23' = \frac{6.2}{y}$ $y \sin 18^\circ 23' = 6.2$ $y = \frac{6.2}{\sin 18^\circ 23'}$ $= 19.659\dots$ $\approx 19.7 \text{ cm}$	<p>y is the hypotenuse.</p> <p>6.2 \div sin 18 DMS 23 DMS =</p>	

- 3** Using the sine ratio, complete the following to find the length of the hypotenuse correct to 1 decimal place.

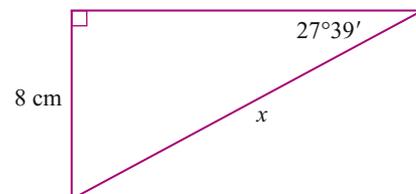
$$\sin \theta = \frac{\text{opposite}}{\square}$$

$$\sin \square = \frac{\square}{x}$$

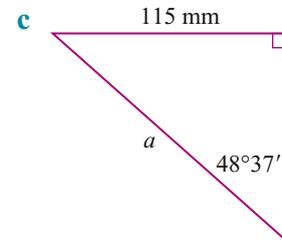
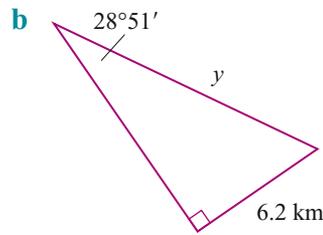
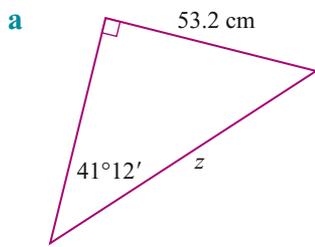
$$x \sin \square = 8$$

$$x = \frac{8}{\square}$$

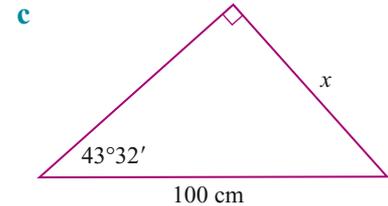
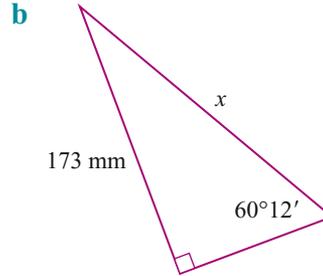
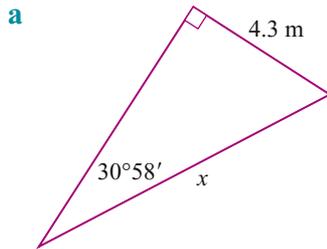
$$\approx \square \text{ cm}$$



4 Use the sine ratio to find the length of the hypotenuse correct to 1 decimal place.

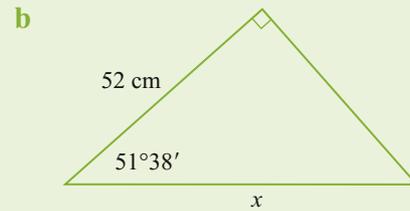
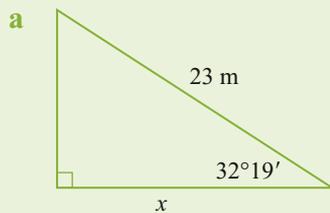


5 Find the unknown sides correct to 1 decimal place.



EXAMPLE 3

Use the cosine ratio to find the value of x correct to 1 decimal place.



	Solve	Think	Apply
a	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos 32^\circ 19' = \frac{x}{23}$ $\therefore x = 23 \cos 32^\circ 19'$ $= 19.437\dots$ $\approx 19.4 \text{ m}$	<p>x is adjacent to the given angle.</p> <p>23 \times \cos 32 DMS 19 DMS $=$</p>	<p>The cosine ratio is used when the adjacent side and hypotenuse are the sides given. As with sine, multiply when finding the adjacent side and divide by the cosine of the angle when finding the hypotenuse.</p>
b	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos 51^\circ 38' = \frac{52}{x}$ $\therefore x = \frac{52}{\cos 51^\circ 38'}$ $= 83.777\dots$ $\approx 83.8 \text{ cm}$	<p>x is the hypotenuse.</p> <p>52 \div \cos 51 DMS 38 DMS $=$</p>	

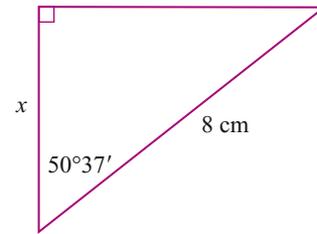
- 6 Using the cosine ratio, complete the following to find the value of x correct to 1 decimal place.

$$\cos \theta = \frac{\square}{\text{hypotenuse}}$$

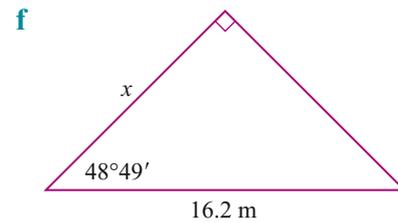
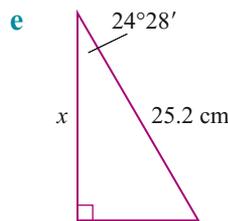
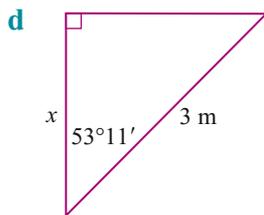
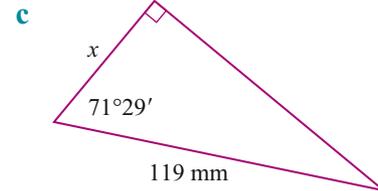
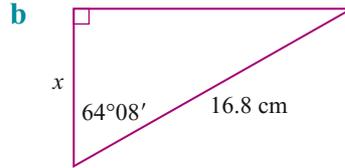
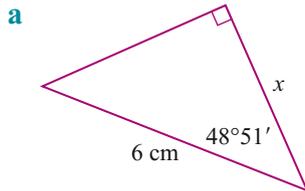
$$\cos \square = \frac{x}{\square}$$

$$\therefore x = 8 \cos \square$$

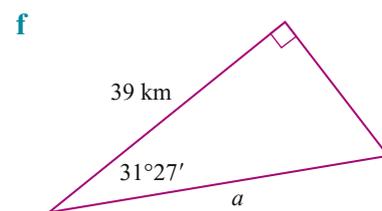
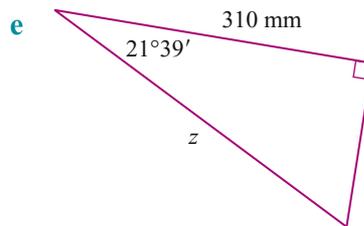
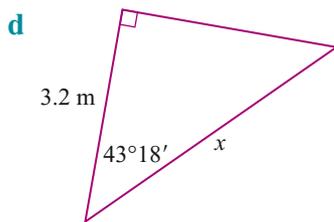
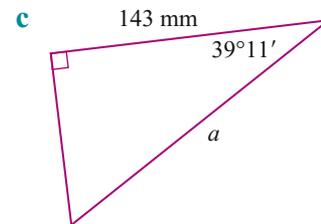
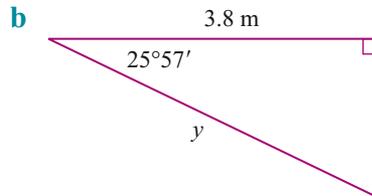
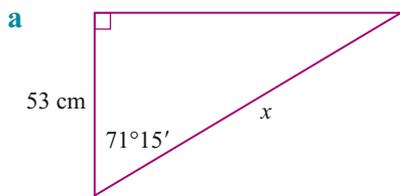
$$\approx \square \text{ m}$$



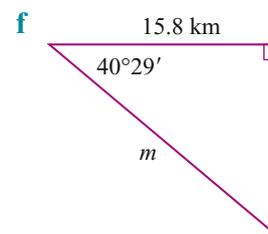
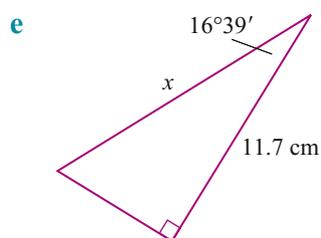
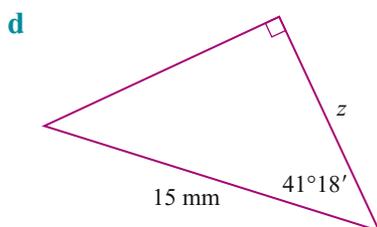
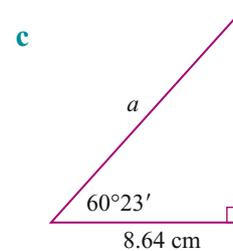
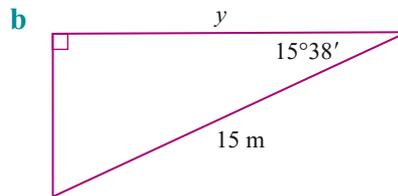
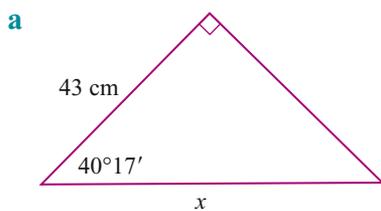
- 7 Use the cosine ratio to find the value of x correct to 1 decimal place.



- 8 Use the cosine ratio to find the length of the hypotenuse correct to 1 decimal place.

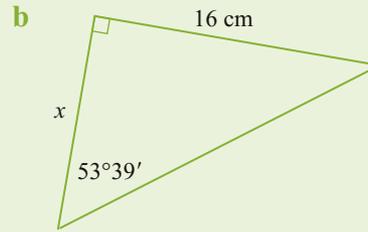
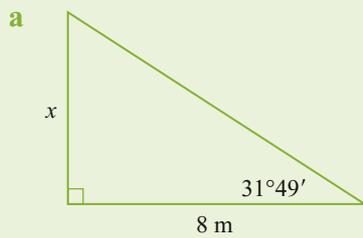


- 9 Find the unknown sides correct to 1 decimal place.



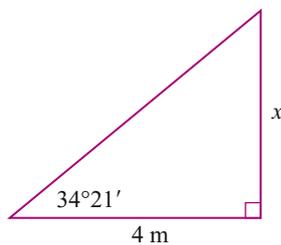
EXAMPLE 4

Use the tangent ratio to find the value of x correct to 1 decimal place.



	Solve	Think	Apply
a	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan 31^\circ 49' = \frac{x}{8}$ $\therefore x = 8 \tan 31^\circ 49'$ $= 4.963\dots$ $\approx 5.0 \text{ m}$	x is opposite the given angle. $8 \times \tan 31 \text{ DMS } 49 =$	The tangent ratio is used when the hypotenuse is not given. Identify the opposite and adjacent sides. When finding the opposite side, multiply the side and the tangent of the angle. When finding the adjacent side, divide by the tangent of the angle.
b	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan 53^\circ 39' = \frac{16}{x}$ $\therefore x = \frac{16}{\tan 53^\circ 39'}$ $= 11.774\dots$ $\approx 11.8 \text{ cm}$	x is adjacent to the given angle. $16 \div \tan 53 \text{ DMS } 39 \text{ DMS } =$	

10 Using the tangent ratio, complete the following to find the value of x correct to 1 decimal place.

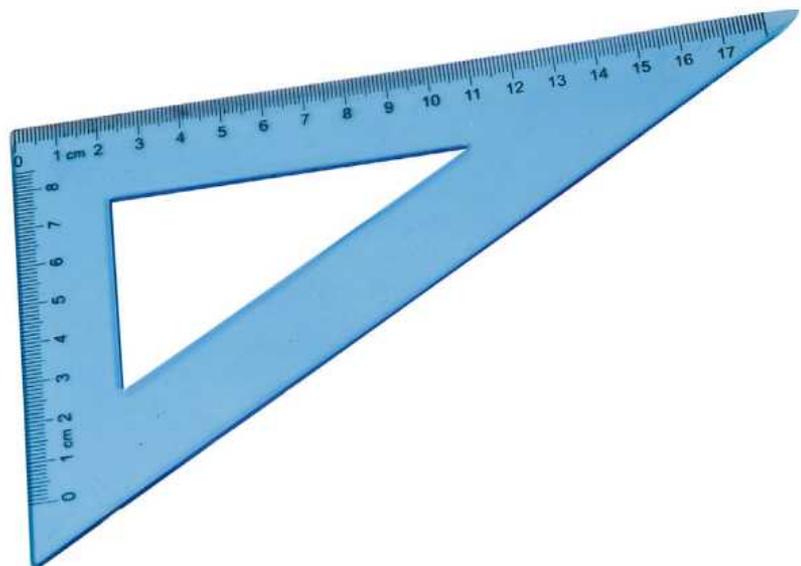


$$\tan \theta = \frac{\square}{\text{adjacent}}$$

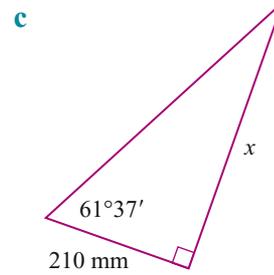
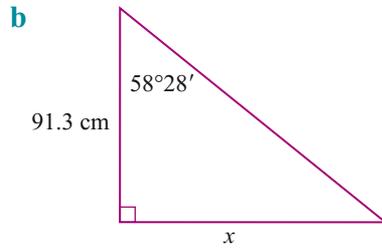
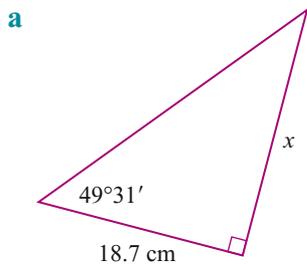
$$\tan \square = \frac{x}{\square}$$

$$x = 4 \tan \square$$

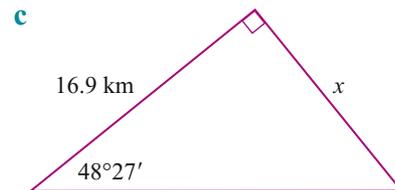
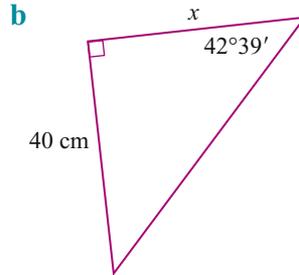
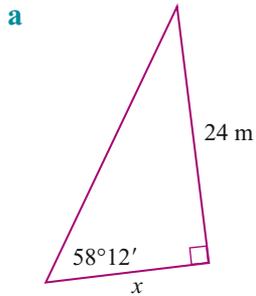
$$\approx \square \text{ m}$$



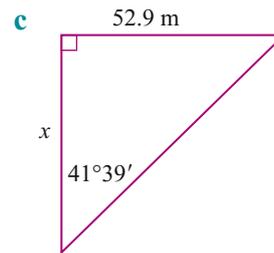
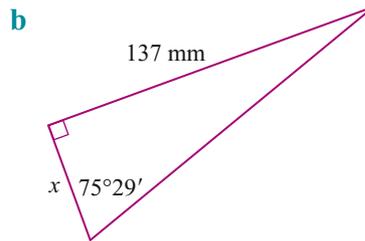
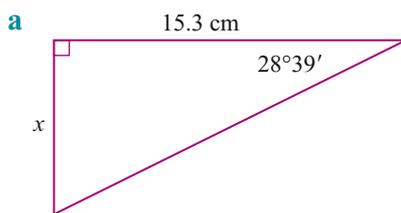
11 Use the tangent ratio to find the value of x correct to 1 decimal place.



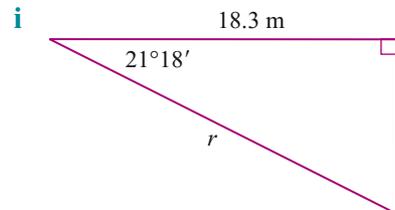
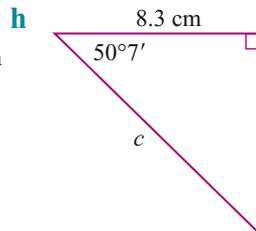
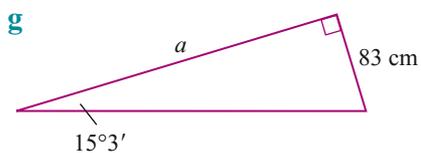
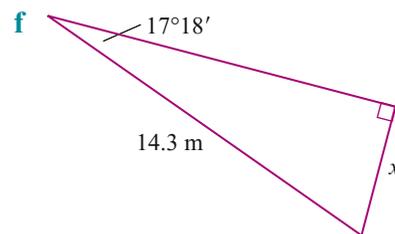
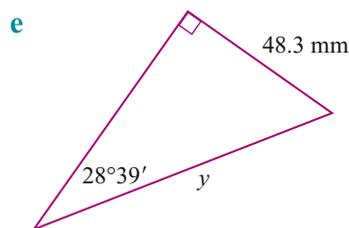
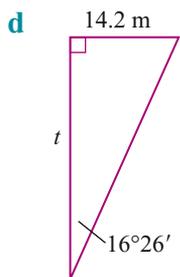
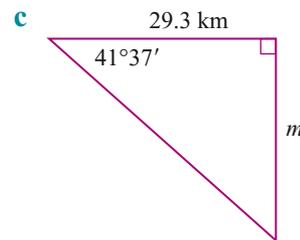
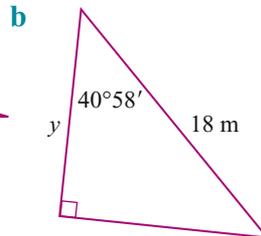
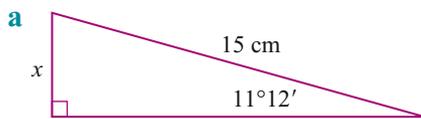
12 Use the tangent ratio to find the value of x correct to 1 decimal place.



13 Use the tangent ratio to find the value of x correct to 1 decimal place.



14 Use the sine, cosine or tangent ratios to find each unknown side correct to 1 decimal place.

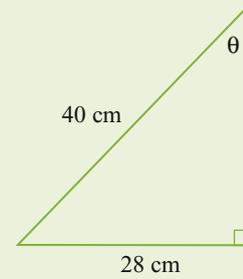


C

Using trigonometry to find angles

EXAMPLE 1

Use the sine ratio to find the value of θ to the nearest minute.



Solve	Think	Apply
$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $= \frac{28}{40}$ $\therefore \theta = 44^{\circ}25'37.21\dots''$ $\approx 44^{\circ}26'$	<p>The side opposite θ and the hypotenuse are given.</p>	<p>Press SHIFT before sin to obtain an angle.</p>

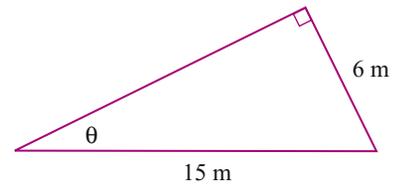
Exercise 13C

- 1 Using the sine ratio, find the value of θ to the nearest minute.

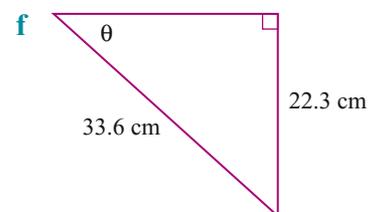
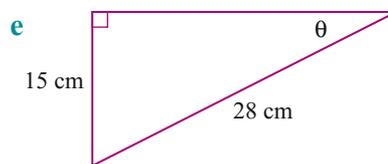
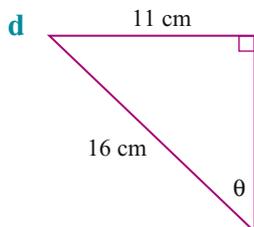
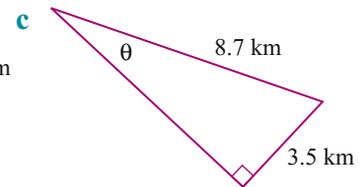
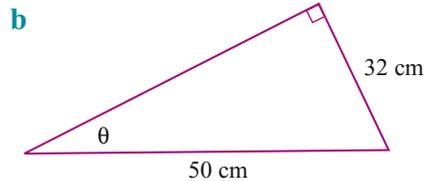
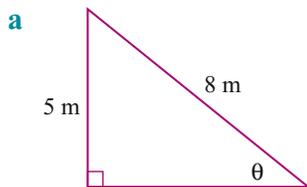
$$\sin \theta = \frac{\square}{\text{hypotenuse}}$$

$$= \frac{\square}{15}$$

$$\therefore \theta \approx \underline{\quad}$$

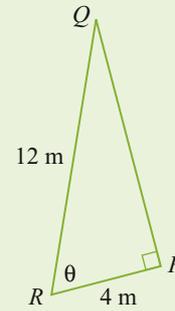


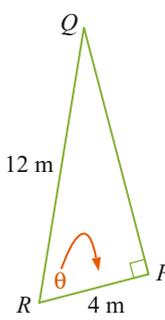
- 2 Use the sine ratio to find the value of θ to the nearest minute.



EXAMPLE 2

Use the cosine ratio to find the value of θ to the nearest minute.



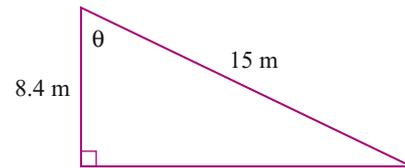
Solve	Think	Apply
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $= \frac{4}{12}$ $\therefore \theta = 70^{\circ}31'43.60\dots''$ $\approx 70^{\circ}32'$	<p>The side adjacent to θ and the hypotenuse are given.</p> 	<p>Press SHIFT before COS to obtain an angle.</p>
		

- 3 Using the cosine ratio, find the value of θ to the nearest minute.

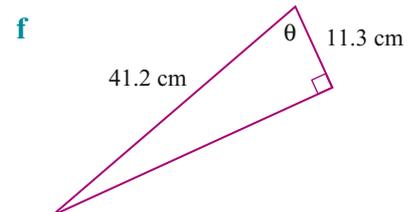
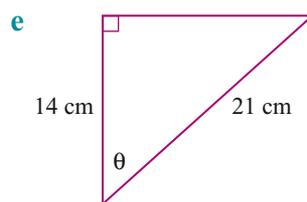
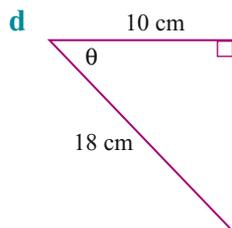
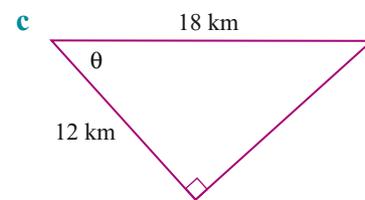
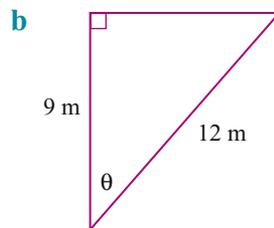
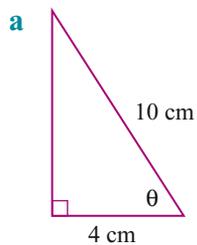
$$\cos \theta = \frac{\text{adjacent}}{\square}$$

$$= \frac{8.4}{\square}$$

$$\therefore \theta \approx \underline{\quad}$$

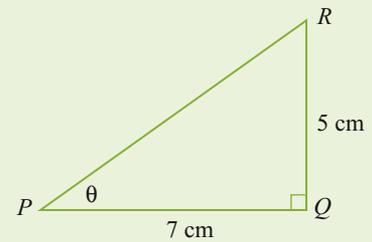


- 4 Using the cosine ratio, find the value of θ to the nearest minute.



EXAMPLE 3

Use the tangent ratio to find the value of θ to the nearest minute.



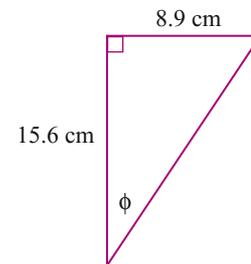
Solve	Think	Apply
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $= \frac{5}{7}$ $\therefore \theta = 35^{\circ}32'15.64\dots''$ $\approx 35^{\circ}32'$	<p>The sides opposite and adjacent to θ are given.</p> 	<p>Press SHIFT before tan to obtain an angle.</p>

- 5 Using the tangent ratio, find the value of ϕ to the nearest minute.

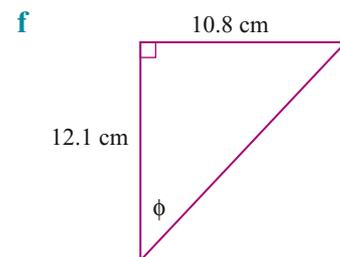
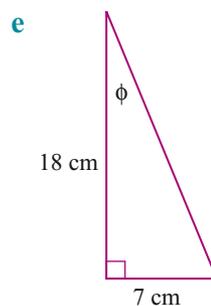
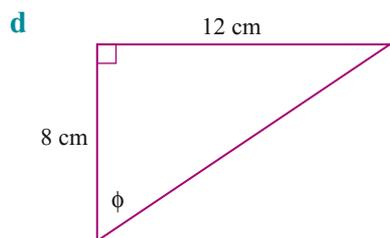
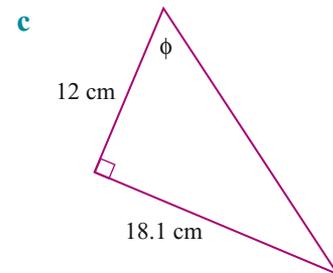
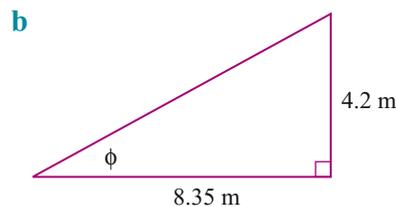
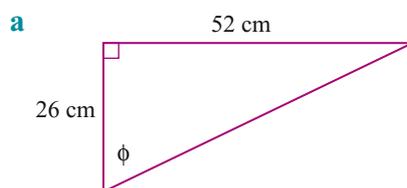
$$\tan \phi = \frac{\square}{\square}$$

$$= \frac{8.9}{\square}$$

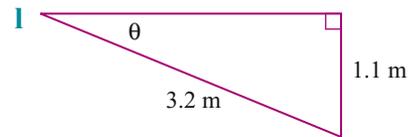
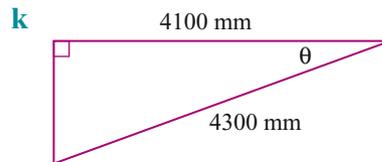
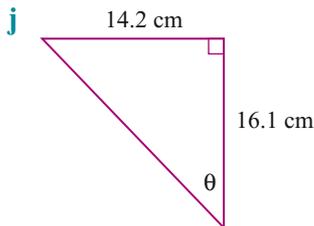
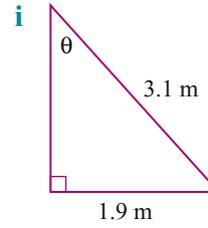
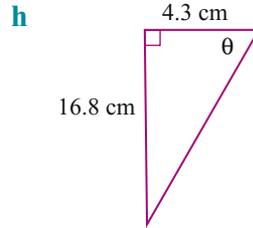
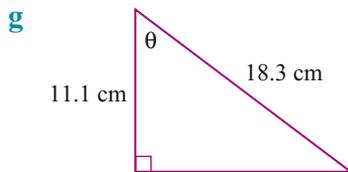
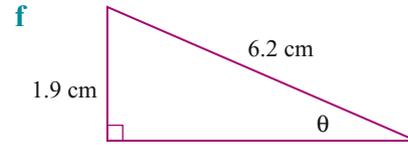
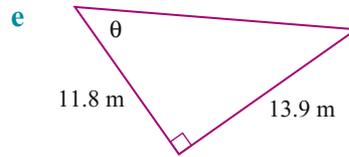
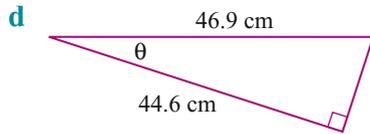
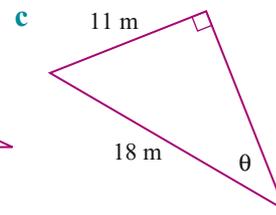
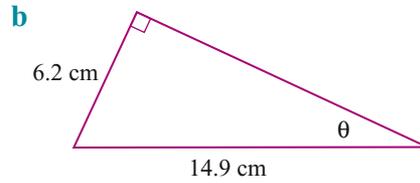
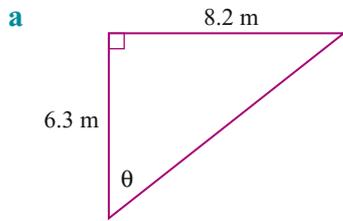
$$\therefore \phi \approx \underline{\quad}$$



- 6 Using the tangent ratio, find the value of ϕ to the nearest minute.



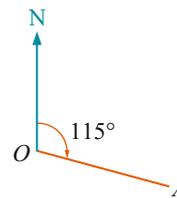
7 Use the trigonometric ratios to find each unknown angle to the nearest minute.



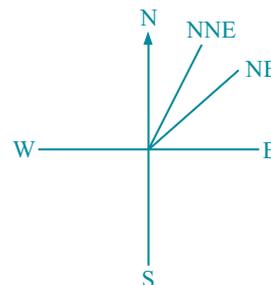
D Compass bearings

Plotting a course for a ship or an aircraft requires accurate directions. These directions are usually given in the form of bearings. The agreed convention is that the direction of travel is measured by a clockwise rotation from the true north direction. The bearing of A from O is the measure of the angle between the line OA and the line through O in the true north direction. The angles are always written using three digits.

This angle is the bearing of A from O . It is written as 115°T .



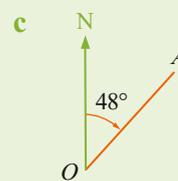
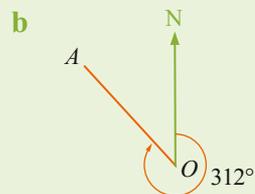
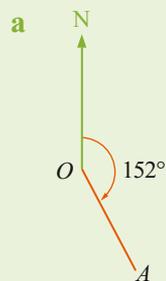
Bearings are sometimes given using the compass rose. In this case the bearings are given with respect to north, south, east and west. For example, NNE is shown.





EXAMPLE 1

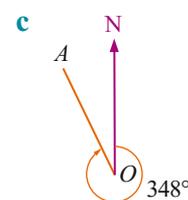
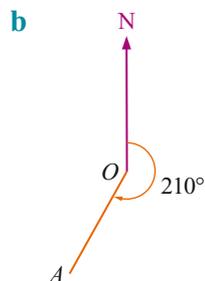
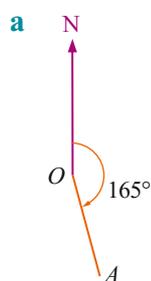
Write the bearing of A from O as shown in each diagram.

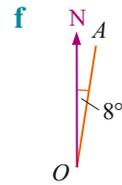
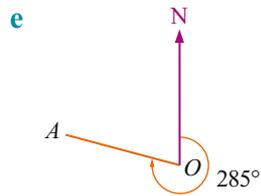
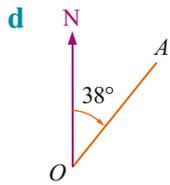


	Solve	Think	Apply
a	The bearing is 152°T .	Clockwise 152° .	There must be three digits in the bearing. The bearing is the clockwise turning from north.
b	The bearing is 312°T .	Clockwise 312° .	
c	The bearing is 048°T .	Clockwise 048° .	

Exercise 13D

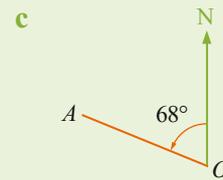
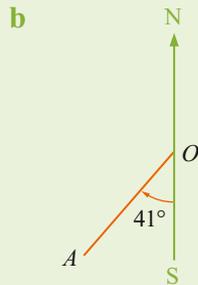
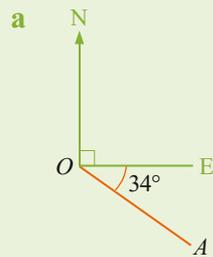
1 Write the bearings of A from O for each of the following.

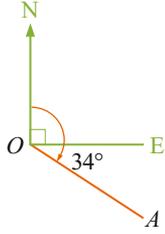
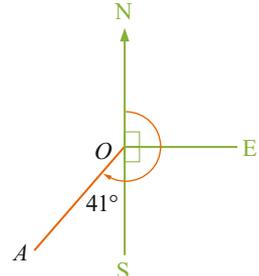
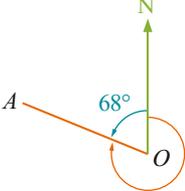




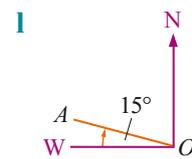
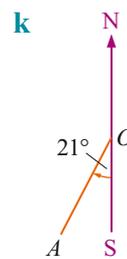
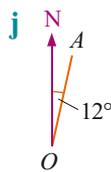
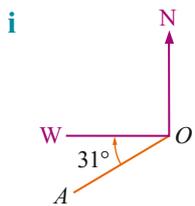
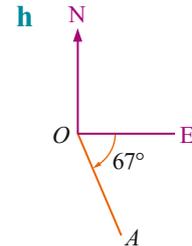
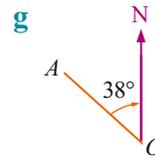
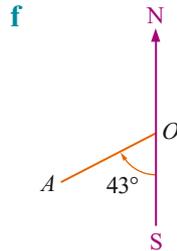
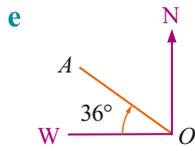
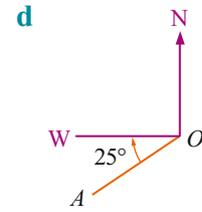
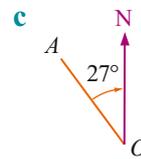
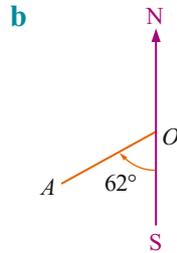
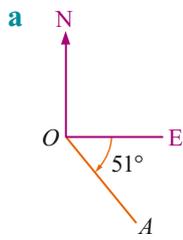
EXAMPLE 2

Write the bearing of A from O .



	Solve	Think	Apply
a	Bearing is $90^\circ + 34^\circ = 124^\circ\text{T}$.	The angle NOE is 90° . 	The angle from north in a clockwise direction must be found for the bearing. Add or subtract as required. Bearings will never be greater than 360° .
b	Bearing is $180^\circ + 41^\circ = 221^\circ\text{T}$.	The angle NOS is 180° . 	
c	The bearing is $360^\circ - 68^\circ = 292^\circ\text{T}$.	68° is anticlockwise, so subtract from 360° . 	

2 Write the bearing of A from O shown below.



EXAMPLE 3

Draw a diagram to represent the position of A from O for each of the following compass bearings.

a 110°T

b 048°T

c 328°T

	Solve	Think	Apply
a		Clockwise 110° from north.	Always turn in a clockwise direction from north.
b		Clockwise 48° from north.	
c		Clockwise 328° from north.	

3 Draw a diagram to represent the position of A from O for each of these compass bearings.

a 128°T

b 022°T

c 312°T

d 231°T

e 005°T

f 285°T

g 185°T

h 300°T

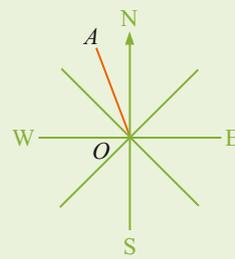
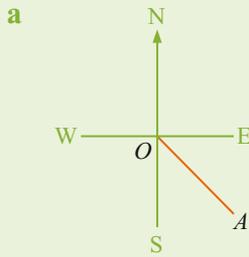
i 073°T

j 355°T

k 133°T

l 099°T

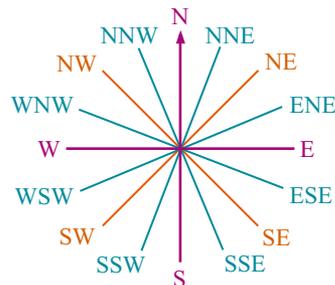
EXAMPLE 4



- i** Write the compass bearing shown in each diagram above.
- ii** Find $\angle NOA$.
- iii** Write as a true bearing.

	Solve	Think	Apply
a i	The bearing is SE.	OA is in the middle of south and east.	Each of the main compass points is 90° . The bearing divides the angle into two angles of 45° .
ii	$\angle NOA = 90^\circ + 45^\circ$ $= 135^\circ$	East is 90° from north.	
iii	$135^\circ T$	The angle from north.	
b i	The bearing is NNW.	AO is between NW and N.	The angle between these dividers is 22.5° .
ii	$\angle NOA = 90^\circ + 90^\circ + 90^\circ$ $+ 45^\circ + 22.5^\circ$ $= 337.5^\circ$	A is close to north, so the bearing is close to 360° .	
iii	$337.5^\circ T$	The angle from north.	

4 Here is a compass rose.



Always put the north or south part of the bearing first. **!**

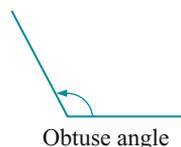
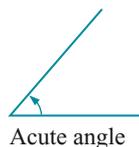
- a** Find the angle between:
 - i** N and E
 - ii** S and SW
 - iii** W and NW
 - iv** E and ESE
 - v** SW and WSW
 - vi** W and NNW
- b** Write each of these compass bearings as true bearings.
 - i** NNE
 - ii** ENE
 - iii** SE
 - iv** SSE
 - v** SSW
 - vi** WSW
 - vii** WNW
 - viii** NW



E

Acute and obtuse angles

Trigonometry can be extended to non-right-angled triangles. Angles between 0° and 180° will be examined; that is, we will look at both acute angles (less than 90°) and obtuse angles (between 90° and 180°).



EXAMPLE 1

Find the value of the following correct to 4 decimal places.

a $\sin 65^\circ$

b $\cos 173^\circ 15'$

c $\tan 116.6^\circ$

	Solve	Think/Apply
a	$\sin 65^\circ = 0.9063$	Check the correct use of your calculator to evaluate trigonometric functions. Usually either  or  is used for degrees and minutes.
b	$\cos 173^\circ 15' = -0.9931$	
c	$\tan 116.6^\circ = -1.9970$	

Exercise 13E

1 Use a calculator to find these values correct to 4 decimal places.

a $\sin 80^\circ$

b $\cos 121^\circ$

c $\tan 101^\circ$

d $\cos 11^\circ 15'$

e $\tan 171.3^\circ$

f $\cos 144.3^\circ$

g $\cos 91^\circ 12'$

h $\sin 115^\circ 18'$

i $\cos 135.9^\circ$

j $\tan 111^\circ 11'$

k $\sin 59^\circ 15'$

l $\tan 175^\circ 8'$

2 a Complete the following table of values for $y = \sin \theta$. Give the values correct to 2 decimal places.

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
y										

θ	100°	110°	120°	130°	140°	150°	160°	170°	180°
y									

b Plot the points from part **a** to draw the graph of $y = \sin \theta$ for θ between 0° and 180° .

c Look at the values of $\sin 20^\circ$ and $\sin 160^\circ$. What do you notice?

d Look at the values of $\sin 30^\circ$ and $\sin 150^\circ$. What do you notice?

e Complete: $\sin \theta = \sin (180 - \underline{\quad})$.

3 a Complete a table of values similar to the table in question **2** above for $y = \cos \theta$. Give the values correct to 2 decimal places.

b Plot the points from part **a** to draw the graph of $y = \cos \theta$ for θ between 0° and 180° .

c Look at the values of $\cos 20^\circ$ and $\cos 160^\circ$. What do you notice?

d Look at the values of $\cos 30^\circ$ and $\cos 150^\circ$. What do you notice?

e Complete: $\cos \theta = -\cos (180 - \underline{\quad})$.

- 4 Use the rules $\sin \theta = \sin (180 - \theta)$, $\cos \theta = -\cos (180 - \theta)$ and $\tan \theta = -\tan (180 - \theta)$, and the table in question 1 of Investigation 1 to evaluate:
- | | | |
|--------------------|--------------------|--------------------|
| a $\sin 140^\circ$ | b $\cos 130^\circ$ | c $\tan 120^\circ$ |
| d $\tan 170^\circ$ | e $\cos 150^\circ$ | f $\sin 120^\circ$ |
| g $\sin 160^\circ$ | h $\tan 140^\circ$ | i $\cos 110^\circ$ |

EXAMPLE 2

Find the value of θ to the nearest minute.

- | | |
|--|--|
| a $\cos \theta = 0.4312$ | b $\cos \theta = -0.5318$ |
| c $\sin \theta = 0.2713$ ($0^\circ < \theta < 90^\circ$) | d $\sin \theta = 0.2713$ ($90^\circ < \theta < 180^\circ$) |

	Solve	Think	Apply
a	$\cos \theta = 0.4312$ $\theta = 64^\circ 27'$	The value of $\cos \theta$ is positive, so the angle is acute.	The three trigonometric ratios sine, cosine and tan are positive for acute angles. Tan and cosine are negative if the angle is obtuse. But sine is positive for both acute and obtuse angles, and further information is required when determining the angle.
b	$\cos \theta = -0.5318$ $\theta = 122^\circ 8'$	The value of $\cos \theta$ is negative, so the angle is obtuse.	
c	$\sin \theta = 0.2713$ ($0^\circ < \theta < 90^\circ$) $\theta = 15^\circ 44'$	The angle is restricted to acute.	
d	$\sin \theta = 0.2713$ $\therefore \theta = 15^\circ 44'$ But $90^\circ < \theta < 180^\circ$, so it is obtuse. $\therefore \theta = 180^\circ - 15^\circ 44'$ $= 164^\circ 16'$	The angle is restricted to obtuse, so the acute angle given must be subtracted from 180° .	

- 5 Find the value of θ . All angles are obtuse.
- | | | |
|---------------------------|---------------------------|--------------------------|
| a $\cos \theta = -0.9312$ | b $\cos \theta = -0.4718$ | c $\sin \theta = 0.4113$ |
| d $\sin \theta = 0.2771$ | e $\sin \theta = 0.6643$ | f $\sin \theta = 0.8118$ |
- 6 Using a graphics calculator or other technology, investigate trigonometric graphs, noting any points of interest.

F Area of a triangle

The formula for the area of a triangle of base b and perpendicular height h is $\text{Area} = \frac{1}{2}bh$.

Examine the area of $\triangle ABC$ with base AC and perpendicular height h .

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times CA \times BX \\ &= \frac{1}{2}bh \end{aligned} \quad \textcircled{1}$$

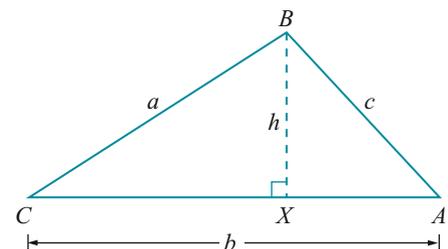
In $\triangle BXC$:

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{a}$$

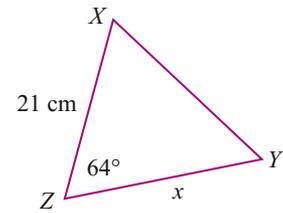
$$\therefore h = a \sin C \quad \textcircled{2}$$

Combining $\textcircled{1}$ and $\textcircled{2}$ gives:

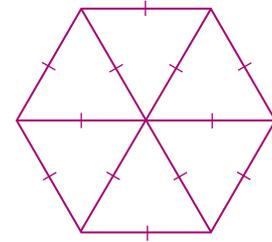
$$\text{Area} = \frac{1}{2}b(a \sin C) = \frac{1}{2}ab \sin C$$



- 3 Triangle XYZ has an area of 230 cm^2 . Find the value of x correct to 1 decimal place.



- 4 Find the area of the given hexagon with sides 5 cm.



Sine rule

The sine rule is one method used to find sides and angles in non-right-angled triangles. The sine rule states that in any $\triangle ABC$ with sides and angles as shown:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Proof of sine rule

In $\triangle ABC$ the area may be found in three ways:

$$\text{Area} = \frac{1}{2}bc \sin A \quad \text{Area} = \frac{1}{2}ac \sin B \quad \text{Area} = \frac{1}{2}ab \sin C$$

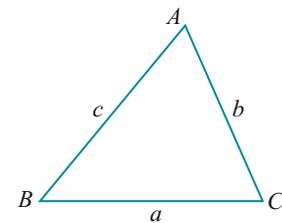
Equating these gives: $\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$

Dividing each part by $\frac{1}{2}abc$ gives:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine rule is used to find:

- a side, given two angles and one side, or
- an angle, given two sides and a non-included angle.



EXAMPLE 1

Find the length of AC correct to 1 decimal place.



Solve	Think	Apply
$\frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{b}{\sin 71^\circ} = \frac{14}{\sin 33^\circ}$ $b = 14 \times \frac{\sin 71^\circ}{\sin 33^\circ}$ $\approx 24.3 \text{ cm}$	<p>Let $AC = b \text{ cm}$.</p> <p>Use the sine rule.</p> <p>Substitute the values for c, $\angle B$ and $\angle C$.</p> <p>Multiply both sides by $\sin 71^\circ$.</p>	<p>The angles must be opposite sides.</p> <p>Rename the vertices if required.</p>



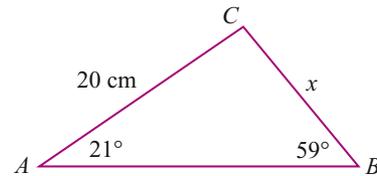
Exercise 13G

- 1 Complete to find the value of x correct to 1 decimal place.

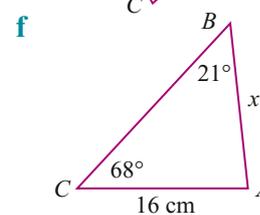
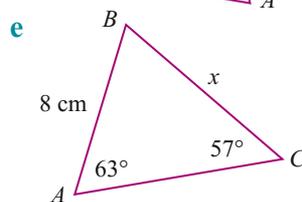
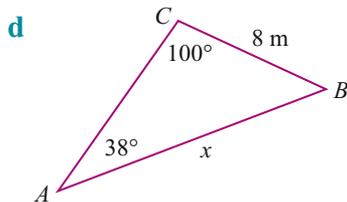
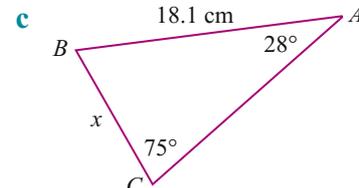
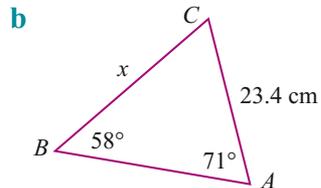
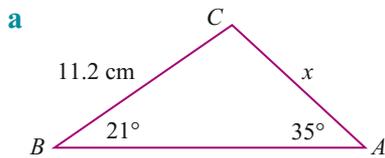
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 21^\circ} = \frac{\square}{\sin \square}$$

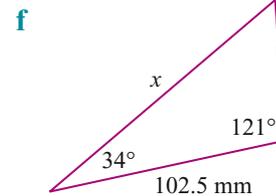
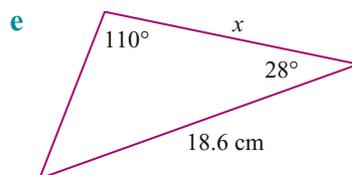
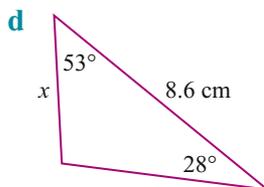
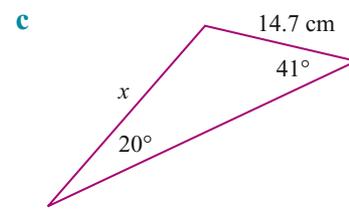
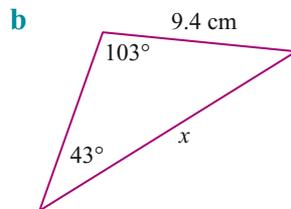
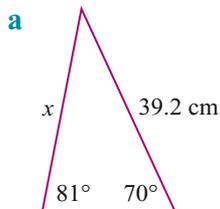
$$x = \frac{\square}{\square} \times \square \approx 8.4 \text{ cm}$$



- 2 Find the value of x correct to 1 decimal place.

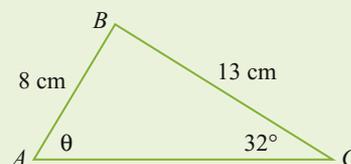


- 3 Find the value of x correct to 1 decimal place.



EXAMPLE 2

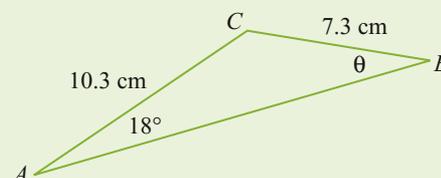
Find the value(s) of θ in this triangle.



Solve	Think	Apply
$\frac{\sin A}{a} = \frac{\sin C}{c}$ $\frac{\sin \theta}{13} = \frac{\sin 32^\circ}{8}$ $\sin \theta = \frac{13 \sin 32^\circ}{8}$ $\theta = 59^\circ 27'$ <p>Or $\theta = 180^\circ - 59^\circ 27'$ $= 120^\circ 33'$ $\therefore \theta = 59^\circ 27'$ or $120^\circ 33'$</p>	<p>Use the sine rule.</p> <p>Substitute the values for a, c and $\angle C$.</p> <p>Multiply both sides by 13.</p>	<p>The length of AC is not given, so both the acute and the obtuse values for θ must be calculated. The calculator will give the acute angle.</p> <p>The obtuse angle is found by subtracting from 180°.</p> <p>$\sin \theta = \sin (180^\circ - \theta)$</p>

EXAMPLE 3

Find the value of θ in this triangle.



Solve	Think	Apply
$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin 18^\circ}{7.3} = \frac{\sin \theta}{10.3}$ $\frac{10.3 \sin 18^\circ}{7.3} = \sin \theta$ $\theta = 25^\circ 51'$ <p>Or $\theta = 180^\circ - 25^\circ 51'$ $= 154^\circ 9'$ $\therefore \theta = 25^\circ 51'$ or $154^\circ 9'$</p>	<p>Use the sine rule.</p> <p>Substitute the values for a, b and $\angle A$.</p> <p>Multiply both sides by 10.3.</p> <p>$\sin \theta = \sin (180^\circ - \theta)$</p>	<p>The length of AB is not given, so both the acute and the obtuse values for θ must be calculated.</p> <p>Had $\angle C$ been given as obtuse, then only the acute angle for θ would be a solution. A triangle can have only one obtuse angle.</p>

- 4 Complete to find the two values for θ to the nearest minute.

$$\frac{\sin A}{a} = \frac{\sin B}{\square}$$

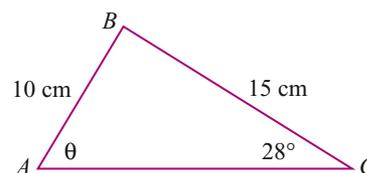
$$\frac{\sin \theta}{15} = \frac{\sin \square}{\square}$$

$$\sin \theta = \frac{15 \sin \square}{\square}$$

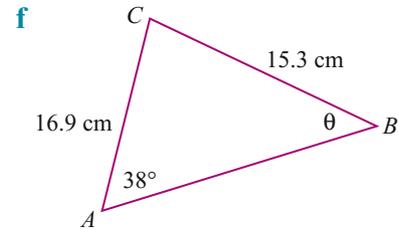
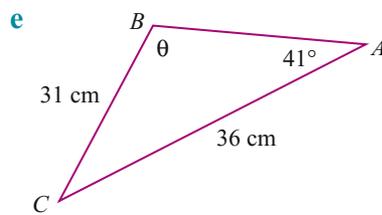
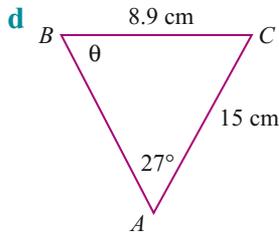
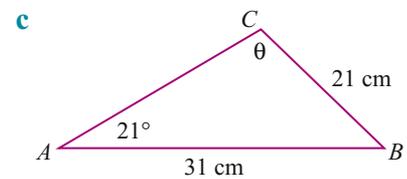
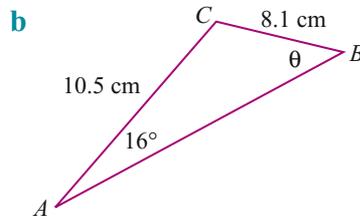
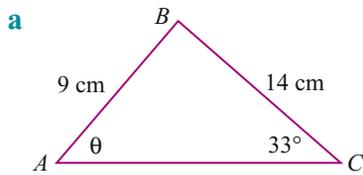
$$\theta = \underline{\hspace{2cm}}$$

Or $\theta = 180^\circ - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$\therefore \theta = \underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$ as $\angle B$ is not given and length AC is not given.

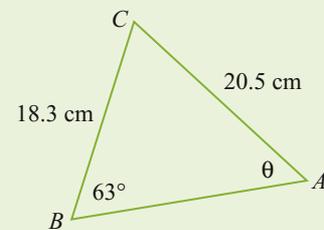


5 Find the two values for θ to the nearest minute.



EXAMPLE 4

Find the value(s) of θ in the triangle.



Solve	Think	Apply
$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{18.3} = \frac{\sin 63^\circ}{20.5}$ $\sin \theta = \frac{18.3 \sin 63^\circ}{20.5}$ $\theta = 52^\circ 42'$ <p>Or $\theta = 180^\circ - 52^\circ 42'$ $= 127^\circ 18'$</p> <p>$\therefore \theta = 52^\circ 42'$ is the only solution.</p>	<p>Use the sine rule.</p> <p>Substitute the values for a, b and $\angle B$.</p> <p>Multiply both sides by 18.3.</p> <p>But, $127^\circ 18' + 63^\circ = 190^\circ 18'$, which is $> 180^\circ$. So an obtuse angle is impossible in this triangle.</p>	<p>The angle sum of a triangle is 180°. In this case $\angle A$ cannot be obtuse as the angle sum of just two angles exceeds 180°. Usually only triangles with small values for the known angle can have acute and obtuse values for the unknown angle.</p>

6 Complete to find the value(s) of θ . Decide if there are two values or only one based on information in the diagram.

$$\frac{\sin C}{c} = \frac{\sin \square}{a}$$

$$\frac{\sin \theta}{18.6} = \frac{\sin \square}{\square}$$

$$\sin \theta = \frac{18.6 \sin \square}{\square}$$

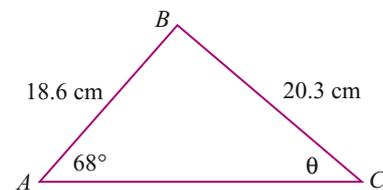
$$\theta = 58^\circ 10'$$

Or $\theta = 180^\circ - 58^\circ 10'$

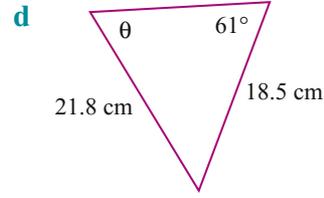
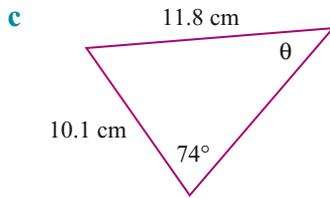
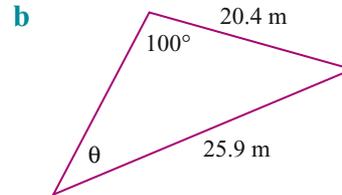
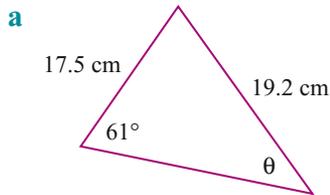
$$= \underline{\hspace{2cm}}$$

But $121^\circ 50' + 68^\circ = \underline{\hspace{2cm}}$, which is greater than $\underline{\hspace{2cm}}$.

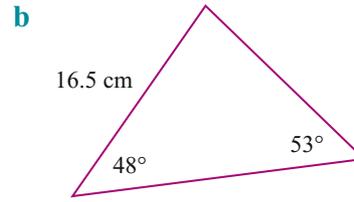
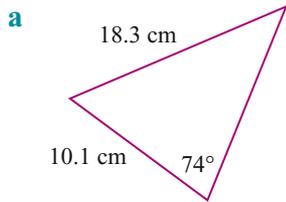
$\therefore \theta = \underline{\hspace{2cm}}$ is the only answer.



7 Find the value(s) of θ . Decide if there are two values or only one based on information in the diagram.



8 Find the area of the following triangles. (*Hint: An angle or side may need to be calculated.*)



9 **a** Find the value of $\sin 90^\circ$.

b What happens to the sine rule if one angle is 90° ? Explain.

H Cosine rule

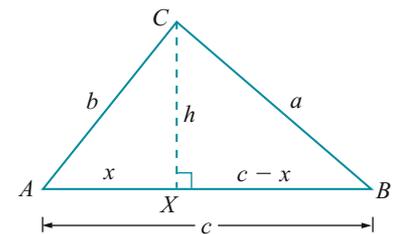
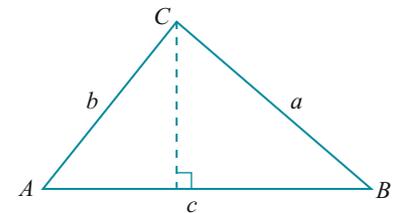
The **cosine rule** is another method used to find sides and angles in non-right-angled triangles.

In any $\triangle ABC$ with sides and angles as shown:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Proof of cosine rule

Using Pythagoras' rule in $\triangle ACX$:

$$b^2 = h^2 + x^2$$

$$h^2 = b^2 - x^2 \quad \text{①}$$

Using Pythagoras' rule in $\triangle BCX$:

$$a^2 = h^2 + (c - x)^2$$

$$h^2 = a^2 - (c - x)^2 \quad \text{②}$$

As ① and ② both equal h^2 they equal each other:

$$a^2 - (c - x)^2 = b^2 - x^2$$

$$a^2 - c^2 + 2cx - x^2 = b^2 - x^2$$

$$a^2 = b^2 + c^2 - 2cx$$

But, in $\triangle ACX$: $\cos A = \frac{x}{b}$

$\therefore x = b \cos A$

Thus $a^2 = b^2 + c^2 - 2cx$

becomes $a^2 = b^2 + c^2 - 2bc \cos A$

The cosine rule is used to find:

- the third side, given two sides and the included angle, or
- an angle, given three sides.

Finding an angle

Rearranging $a^2 = b^2 + c^2 - 2bc \cos A$

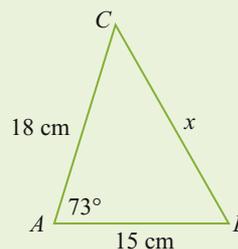
gives $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

which is a more convenient form for finding angles.

Likewise $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ and $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

EXAMPLE 1

Find x correct to 1 decimal place.



Solve	Think	Apply
$a^2 = b^2 + c^2 - 2bc \cos A$ $x^2 = 18^2 + 15^2 - 2 \times 18 \times 15 \times \cos 73^\circ$ $x = \sqrt{18^2 + 15^2 - 2 \times 18 \times 15 \times \cos 73^\circ}$ $\approx 19.8 \text{ cm}$	<p>Use the cosine rule. Substitute the values for b, c and $\angle A$. Complete the calculation in one application.</p>	<p>The unknown side must be opposite the angle in order to use the cosine rule. The most common error is to omit finding the square root at the end of the calculation.</p>

Exercise 13H

- 1 Using the cosine rule, complete the following to find the value of x correct to 1 decimal place.

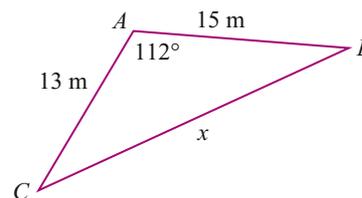
$$a^2 = b^2 + c^2 - \underline{\hspace{2cm}}$$

$$x^2 = 13^2 + \underline{\hspace{1cm}}^2 - 2 \times 13 \times \underline{\hspace{1cm}} \times \cos \underline{\hspace{1cm}}$$

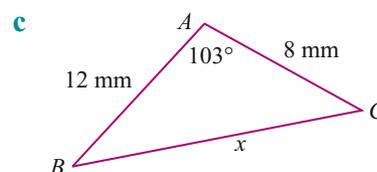
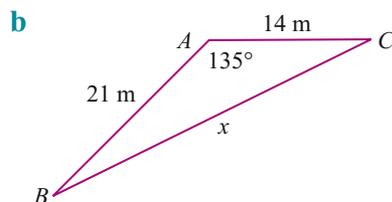
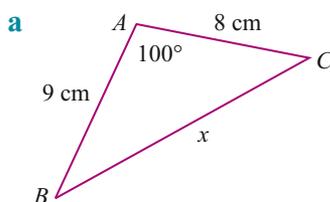
$$= \underline{\hspace{2cm}}$$

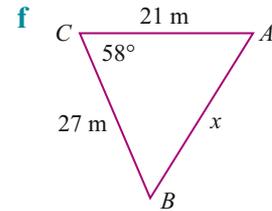
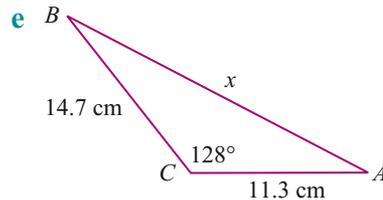
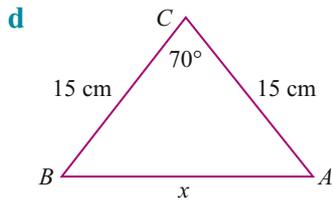
$$x = \sqrt{540.096\dots}$$

$$\approx \underline{\hspace{1cm}} \text{ cm}$$



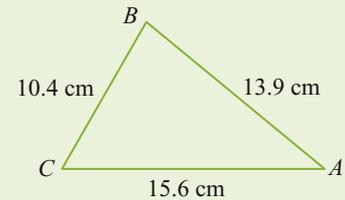
- 2 Use the cosine rule to find the value of x correct to 1 decimal place.





EXAMPLE 2

Use the cosine rule to find the smallest angle in $\triangle ABC$.



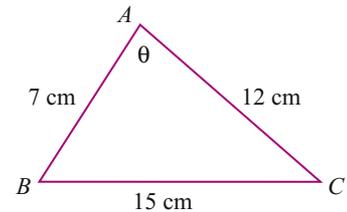
Solve	Think	Apply
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{15.6^2 + 13.9^2 - 10.4^2}{2 \times 15.6 \times 13.9}$ $\theta \approx 40^\circ 47'$	<p>The smallest angle is opposite the shortest side.</p> <p>Substitute the values for a, b and c.</p> <p>Use inverse cos to obtain the angle.</p>	<p>As $\cos \theta = -\cos (180^\circ - \theta)$, there is only one solution for angles found using the cosine rule.</p>

- 3** Complete the following to find the value of θ in this triangle. Give your answer to the nearest minute.

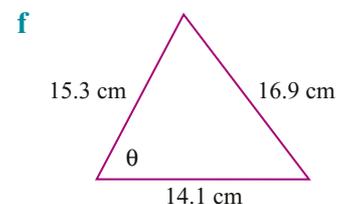
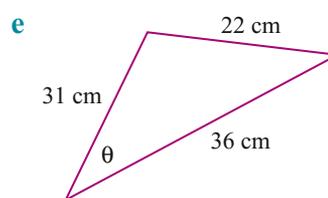
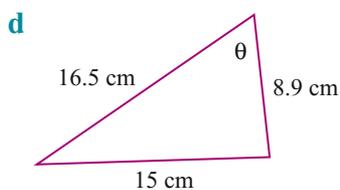
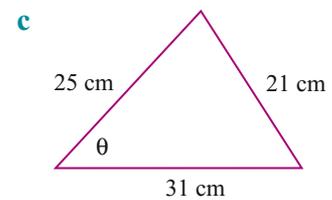
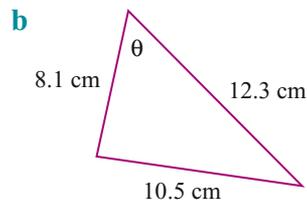
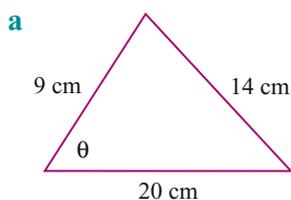
$$\cos A = \frac{b^2 + \square^2 - \square}{2bc}$$

$$\cos \theta = \frac{\square^2 + \square^2 - 15^2}{2 \times 7 \times 12}$$

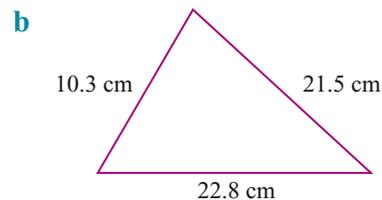
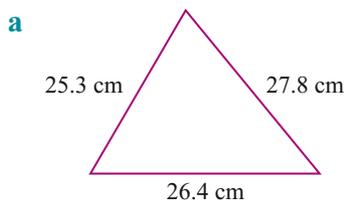
$$\theta \approx \underline{\quad}$$



- 4** Find the value of θ in each triangle. Give your answer to the nearest minute.

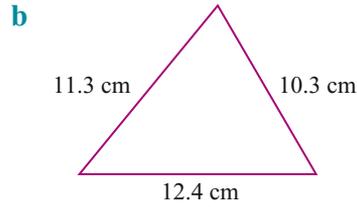
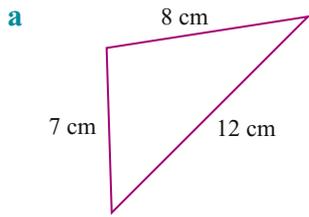


- 5** Find all three angles in the following triangles.

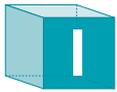


- 6 a** Find the smallest angle in a triangle with sides 8 cm, 9 cm and 10 cm.
b Find the largest angle in a triangle with sides 18.3 cm, 29.8 cm and 24.4 cm.

7 Find the area of each triangle.



- 8 a** Find the value of $\cos 90^\circ$.
b What happens to the cosine rule when the angle is 90° ? Explain your answer.



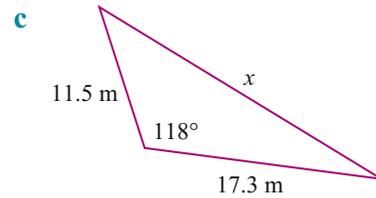
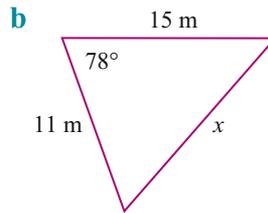
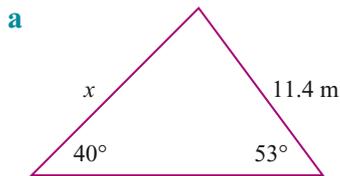
Miscellaneous questions

Sometimes it is difficult to decide which rule to use. In general, if a question mentions:

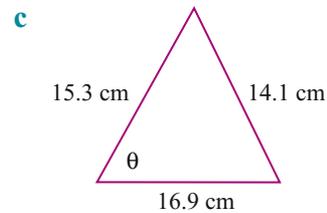
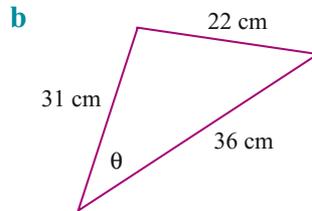
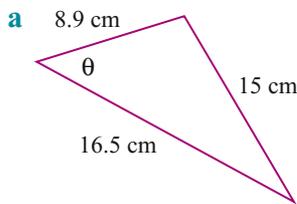
- three sides and one angle, one of which is to be found, use the cosine rule
- two sides and two angles, one of which is to be found, use the sine rule.

Exercise 131

1 Find x correct to 1 decimal place.



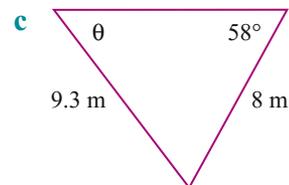
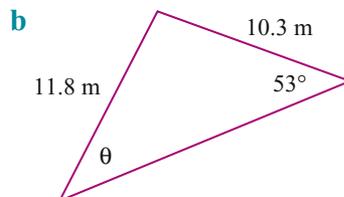
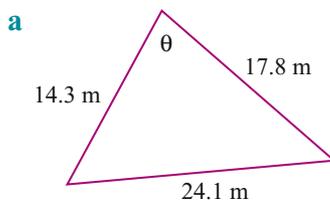
2 Find the value of θ to the nearest minute.

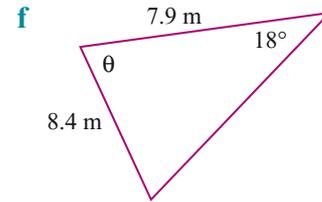
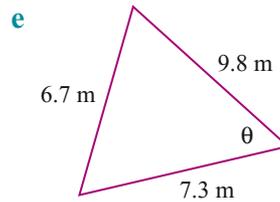
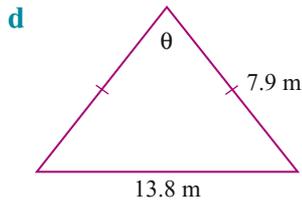


3 Find the area of each of the triangles in question 1.

4 Find the area of each of the triangles in question 2.

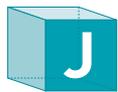
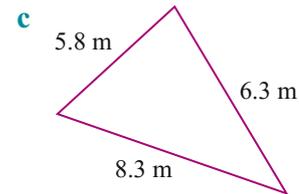
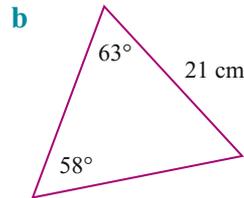
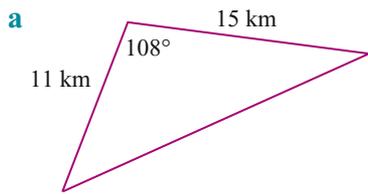
5 Find the value of θ to the nearest minute.





6 Find the area of each triangle in question 5.

7 Find all unknown sides and angles in these triangles. Give angles to the nearest degree.



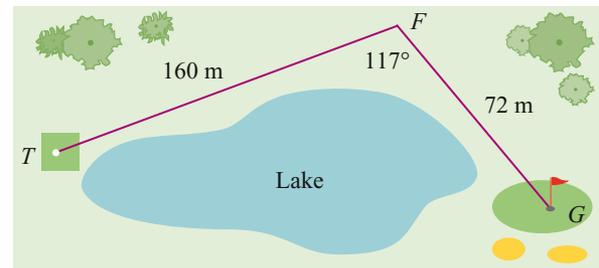
Practical problems (extension)

When solving worded problems it is important to accurately place all the information on a diagram. The solution may involve a combination of trigonometric techniques. The answer should be stated in the context of the question.

Exercise 13J



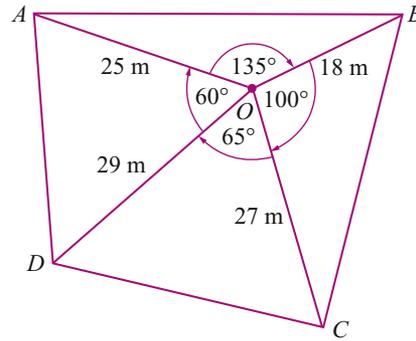
- 1** A golfer hits the ball 160 m from T to F . He then hits it from F to G , a distance of 72 m. If $\angle TFG$ is 117° , find the direct distance from T to G .



- 2** Two trees at A and B are 15 km apart. A third tree C is located so that $\angle CAB = 43^\circ$ and $\angle CBA = 64^\circ$.
- a** Find the distance of C from A .
- b** Find the distance of C from B .
- 3** The minute and hour hands of a clock are 15 cm and 9 cm long respectively. Find the distance between the points of the hands at 2 pm.

- 4** A hiker walks 8 km due west from A to B and then 3 km in a direction $215^\circ T$ to C .
- a** Find the distance AC .
- b** Find the bearing of A from C .
- 5** A soccer goal is 8 m wide. A forward shoots for goal when he is 18 m from one post and 23 m from the other. Within what angle must the shot be made to score a goal?

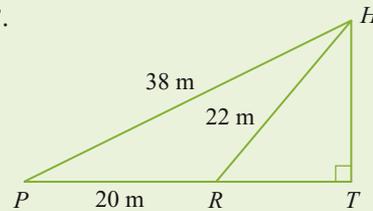
- 6 A radial survey was completed from a point O as shown in the diagram.
- Find the area of the paddock $ABCD$.
 - Find the perimeter of $ABCD$.



EXAMPLE 1

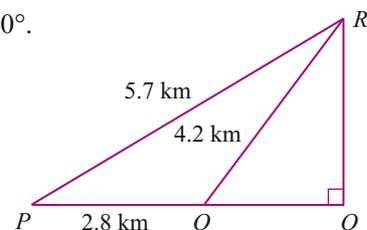
In the diagram $PR = 20$ m, $RH = 22$ m, $PH = 38$ m and $\angle RTH = 90^\circ$.

- Use the cosine rule to find $\angle PRH$, and hence $\angle HRT$.
- Find HT to the nearest whole number.
- If TH is north, write the bearing of H from R .

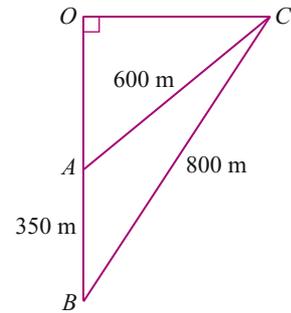


	Solve	Think	Apply
a	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{20^2 + 22^2 - 38^2}{2 \times 20 \times 22}$ $A = 129^\circ 31'$ $\angle PRH = 129^\circ 31'$ $\angle HRT = 180^\circ - 129^\circ 31' = 50^\circ 29'$	Relabel the triangle for $\angle R$. A straight line is 180° .	A clear diagram with all the information is essential. Sometimes drawing separate triangles helps to find sides and angles. The sine and cosine rules are only required for non-right-angled triangles. For right-angled triangles use the sin, cos or tan ratios.
b	$\sin 50^\circ 29' = \frac{HT}{22}$ $HT = 22 \sin 50^\circ 29'$ $= 16.97\dots$ $\therefore HT \approx 17 \text{ m}$	Use $\triangle HRT$ with $\angle HRT = 50^\circ 29'$ from part a and $HR = 22$ m.	
c	Bearing of H from $R = 040^\circ T$.	Draw a north line starting from R . Find the angle from north to H : $\angle NRH$. 	

- 7 In the diagram $PQ = 2.8$ km, $QR = 4.2$ km, $PR = 5.7$ km and $\angle QOR = 90^\circ$.
- Use the cosine rule to find $\angle PQR$ and hence find $\angle RQO$.
 - Find OR correct to 1 decimal place.
 - If OR is north, write the bearing of R from Q .



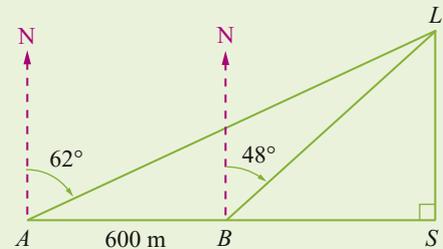
- 8 In the diagram $AB = 350$ m, $AC = 600$ m, $BC = 800$ m and $\angle COA = 90^\circ$.
- Find $\angle BAC$ and $\angle OAC$.
 - Find OA and OC to the nearest 10 m.
 - If BO is north, find the bearing of C from A , and C from B .



EXAMPLE 2

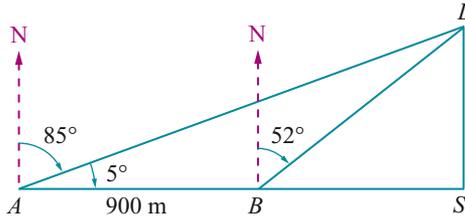
A boat is travelling due east. The bearing of a lighthouse L is 062°T . After the boat has travelled 600 m the bearing of the lighthouse is 048°T . The diagram shows this information.

- Find $\angle LAB$, $\angle LBS$ and $\angle ABL$.
- Use the sine rule to find the distance from the boat to the lighthouse.
- Find the distance the boat now needs to travel to be due south of the lighthouse.



	Solve	Think	Apply
a	$\begin{aligned}\angle LAB &= 90^\circ - 62^\circ \\ &= 28^\circ \\ \angle LBS &= 90^\circ - 48^\circ \\ &= 42^\circ \\ \angle ABL &= 90^\circ + 48^\circ \\ &= 138^\circ\end{aligned}$	<p>The angle between east and north is 90°. $\angle ABL$ is obtuse. The diagram shows the information.</p>	<p>It is important to have an accurate diagram showing all the information from the question and the diagram if supplied. Look for right angles, especially when using compass directions. Sometimes drawing separate triangles can help.</p>
b	$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{x}{\sin 28^\circ} &= \frac{600}{\sin 14^\circ} \\ x &= \frac{600}{\sin 14^\circ} \times \sin 28^\circ \\ &= 1164.35\dots \\ &\approx 1164\end{aligned}$ <p>The distance from the boat to the lighthouse is 1164 m.</p>	<p>Use $\angle ABL$ and the sine rule. $\angle ALB$ needs to be found. $\angle ALB = 180^\circ - 138^\circ - 28^\circ = 14^\circ$</p> <p>Multiply both sides by $\sin 28^\circ$.</p> <p>Answer in a sentence.</p>	<p>If a triangle contains a right angle, use direct trigonometric ratios not the sine or cosine rules.</p>
c	$\begin{aligned}\cos 42^\circ &= \frac{y}{1164} \\ y &= 1164 \times \cos 42^\circ \\ &= 865\end{aligned}$ <p>The boat must travel about 865 m to be due south of the lighthouse.</p>	<p>Use $\triangle BLS$ which is right-angled. $\angle LBS = 42^\circ$ and side $BL = 1164$ from part b.</p>	

- 9 A boat is travelling due east. The bearing of a lighthouse is 085°T . After the boat has travelled 900 m the bearing of the lighthouse is 052°T . The diagram shows this information.

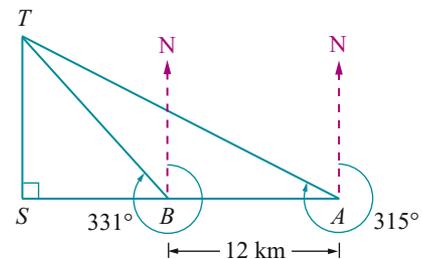


- Find $\angle ABL$.
- Use the sine rule to find the distance from the boat to the lighthouse.
- Find the distance the boat needs to travel to be due south of the lighthouse.



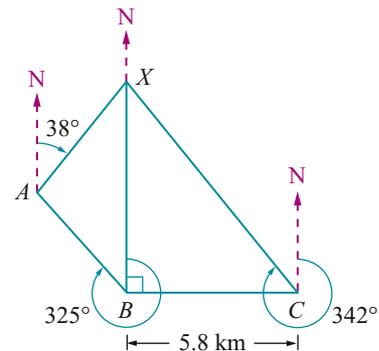
- 10 An aeroplane is travelling due west. The bearing of a control tower is 315°T . After the aeroplane has travelled 12 km the bearing of the tower is 331°T . The diagram shows this information.

- Find $\angle ABT$.
- Use the sine rule to find the distance from the aeroplane to the tower.
- Find the distance the aeroplane needs to travel to be due south of the tower.



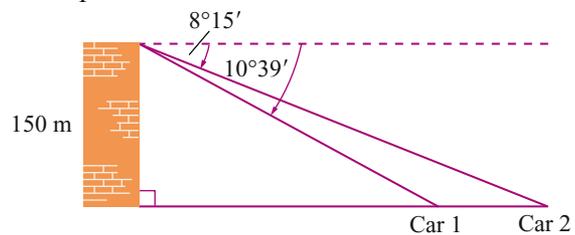
- 11 Roads intersect as shown in the diagram. Roads BX and BC are perpendicular. The bearing of road BA is 325°T . The bearing of road CX is 342°T . The distance BC is 5.8 km. The bearing of road AX is 38°T .

- Find $\angle BCX$. Hence find the distance BX .
- Find $\angle AXB$, $\angle XBA$ and $\angle BAX$.
 - Hence find the distance from B to X via A .



- 12 Tamryn notices that the angle of elevation to the top of a building is 22° . She walks 120 m towards the building and the angle of elevation is now 28° . How tall is the building?

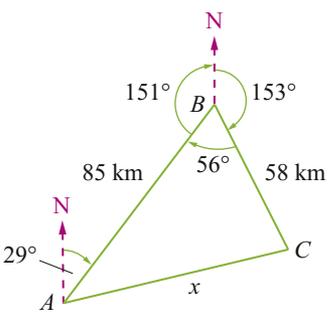
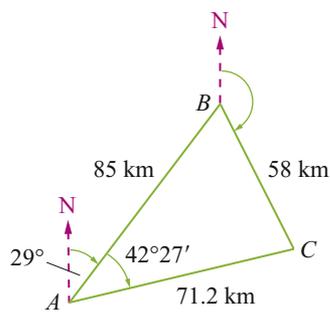
- 13 From the top of a building the angles of depression to two cars are $8^\circ 15'$ and $10^\circ 39'$ respectively. If the building is 150 m high, how far apart are the cars?



EXAMPLE 3

A rally car is driven 85 km along a road from A to B in a direction 029°T . It then travels 58 km along a road to C on a bearing of 153°T .

- a** Find the distance of the car from its starting point.
b Find the bearing of the car from its starting point.

	Solve	Think	Apply
a	 $b^2 = a^2 + c^2 - 2ac \cos B$ $x^2 = 58^2 + 85^2 - 2 \times 58 \times 85 \times \cos 56^\circ$ $x = 71.241\dots$ $= 71.2 \text{ (1 decimal place)}$ <p>Distance from the starting point is 71.2 km.</p>	<p>Draw a diagram. $\angle NAB = 29^\circ$ therefore $\angle NBA = 180^\circ - 29^\circ = 151^\circ$, as the two north lines are parallel.</p> <p>Using 153° and 151°, $\angle B$ is calculated to be 56°, as angles at a point equal 360°.</p> <p>Use the cosine rule in $\triangle ABC$ to calculate AC.</p>	<p>All instances of a north line for each bearing create parallel lines and transversals forming co-interior angles. As co-interior angles are supplementary, other angles can be found.</p> <p>In most cases angles or sides calculated in one part are used in the next part. Drawing separate triangles may help.</p>
b	 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $= \frac{71.2^2 + 85^2 - 58^2}{2 \times 71.2 \times 85}$ $\therefore A \approx 42^\circ 27'$ <p>Bearing C from $A = 29^\circ + 42^\circ 27'$ $= 71^\circ 27'\text{T}$</p>	<p>Draw a diagram. First find $\angle BAC$.</p> <p>Use the cosine rule in $\triangle ABC$.</p> <p>Draw a north line on the diagram at A. The bearing of C from A is the angle between AN and AC $= \angle NAB + \angle BAC$.</p>	

- 14** A car drives 128 km from A to B on a bearing of 053°T . The car then travels 93 km along a road C in a direction 138°T .
- a** Find the distance of the car from the starting point.
b Find the bearing of A from C .
- 15** A plane flies 320 km in a direction 283°T , then it turns and flies 525 km in a direction 192°T .
- a** Find the distance of the plane from its starting point.
b Find the bearing of the plane from its starting point.

Language in mathematics

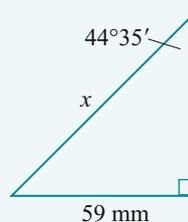
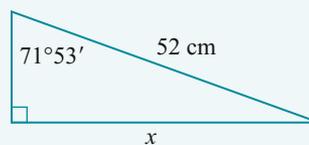
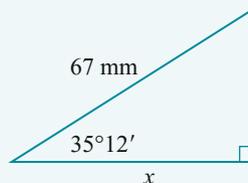
- 1 Use the words from the list below to complete these sentences.
 angle, angle, cosine, cosine, negative, non-right-angled, obtuse, ratio, right-angled, rule, side, side, sine, sine, two
- a In the study of trigonometry, ____ triangles are considered first, enabling the three ratios ____, cosine and tangent to be defined.
- b For obtuse angles, the ____ ratio is still positive but the cosine ____ is ____, making ____ angles easier to find using the ____ rule than the sine ____.
- c In ____ triangles, the sine rule is used when ____ angles and one side are given to find another ____ or when two sides and one non-included angle are given to find another ____.
- d The ____ rule is used to find a third ____ when two sides and the included angle are given or to find an ____ when three sides are given.

Terms

acute angle	adjacent side	angle	bearing	compass bearing
cosine rule	degrees	gradient	hypotenuse	minutes
obtuse angle	opposite side	ratio	right angle	sine rule
tangent	trigonometric ratios	trigonometry	true bearing	

Check your skills

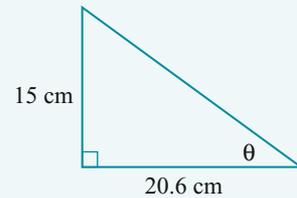
- 1 The value of $\cos 28^\circ 3'$ is closest to:
 A 29 B 0.8825 C 0.1392 D 0.1736
- 2 The value of θ in the expression $\tan \theta = 4.29$ is closest to:
 A 76° B $76^\circ 52'$ C $76^\circ 53'$ D 0.075°
- 3 The value of angle A in the expression $\sin A = \frac{0.56}{1.8}$ is closest to:
 A 0.311° B 0.0098 C $71^\circ 2'$ D $18^\circ 8'$
- 4 The value of x in this triangle is closest to:
 A 82 mm B 55 mm
 C 39 mm D 47 mm



- 5 The value of x in this triangle is closest to:
 A 151 cm B 160 cm
 C 49 cm D 55 cm
- 6 The value of x in this triangle is closest to:
 A 82 mm B 60 mm
 C 83 mm D 84 mm

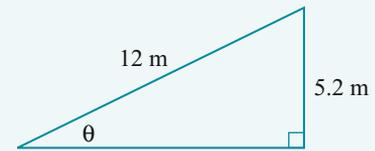
7 The value of θ in the triangle is closest to:

- A 36° B 55°
 C 46° D 43°



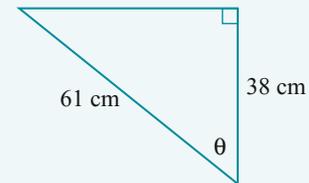
8 The value of θ in the triangle is closest to:

- A 23° B 66°
 C 26° D 64°



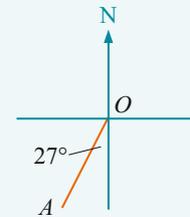
9 The value of θ in the triangle is closest to:

- A 38° B 51°
 C 47° D 58°



10 The bearing of A from O shown in the diagram is:

- A 027°T B 153°T
 C 207°T D 063°T



11 The compass midway bearing between south and south-east is:

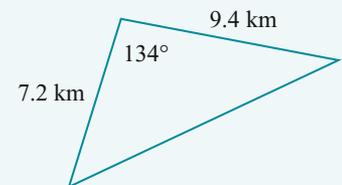
- A SSE B SES C ESS D WSE

12 A value of θ when $\sin \theta = 0.4713$ is closest to:

- A $61^\circ 53'$ B $151^\circ 53'$ C $29^\circ 16'$ D none of these

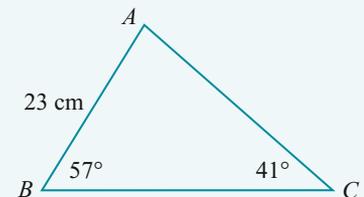
13 The area of this triangle is closest to:

- A 24.3 km^2 B 48.7 km^2
 C 2.8 km^2 D 9069 km^2



14 The length of AC is closest to:

- A 32.0 cm B 16.5 cm
 C 31.2 cm D 29.4 cm



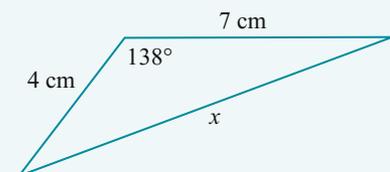
15 The value of θ in this triangle is closest to:

- A $157^\circ 20'$ B $22^\circ 40'$
 C $23^\circ 47'$ D $2^\circ 31'$

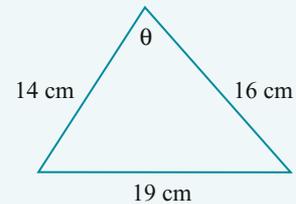


16 The value of x is closest to:

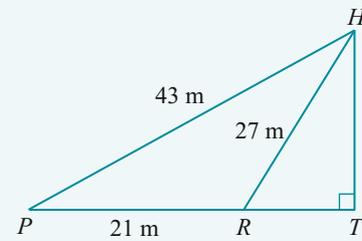
- A 106.6 cm B 10.3 cm
 C 6.7 cm D 44.7 cm



- 17 The value of θ is closest to:
A $78^\circ 16'$ **B** $11^\circ 43'$
C $11^\circ 29'$ **D** $78^\circ 17'$



Use this description and diagram to answer questions 18 to 20.
 In the diagram $PR = 21$ m, $RH = 27$ m, $PH = 43$ m and $\angle RTH = 90^\circ$.



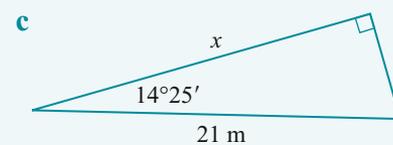
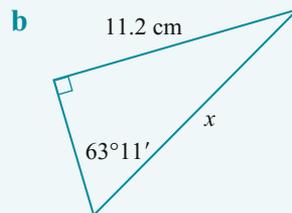
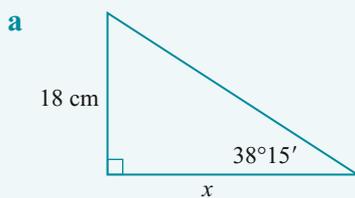
- 18 When the cosine rule is used to find $\angle PRH$, the size of $\angle HRT$ is closest to:
A $126^\circ 47'$ **B** $53^\circ 13'$
C $23^\circ 2'$ **D** $30^\circ 13'$
- 19 The length of HT is closest to:
A 27.5 m **B** 29.4 m **C** 21.6 m **D** 13.2 m
- 20 If TH is north, the bearing of H from R to the nearest degree is:
A $053^\circ T$ **B** $037^\circ T$ **C** $233^\circ T$ **D** $127^\circ T$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

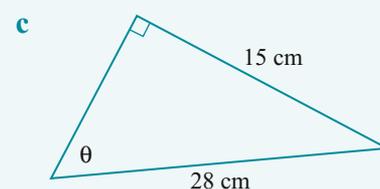
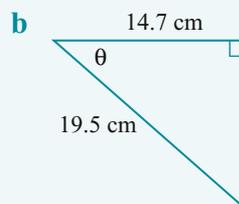
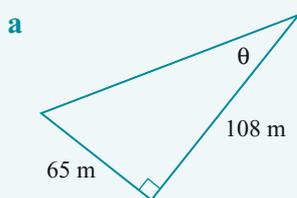
Question	1–3	4–6	7–9	10, 11	12	13	14, 15	16, 17	18–20
Section	A	B	C	D	E	F	G	H	J

13A Review set

- 1 Find the length of the side marked x correct to 3 significant figures.

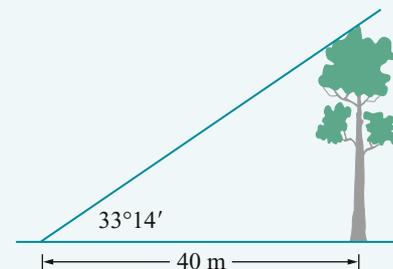


- 2 Find θ to the nearest minute.



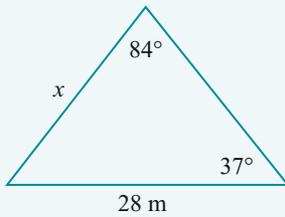
- 3 Solve the following problems involving trigonometry.

- a** The shadow of a tree is 40 m in length and the angle of elevation from the end of the shadow to the top of the tree is $33^\circ 14'$. Find the height of the tree to the nearest $\frac{1}{10}$ of a metre.
- b** An aeroplane takes off at a constant angle of $20^\circ 30'$. When it has flown 1000 m, what is its altitude to the nearest metre?

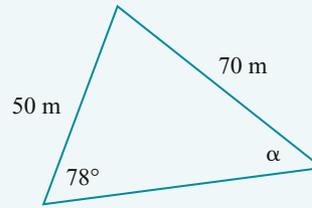


4 Use the sine rule to find:

a x to the nearest metre

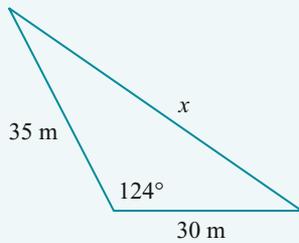


b α to the nearest degree.

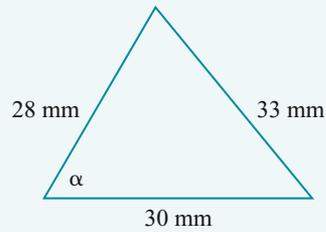


5 Use the cosine rule to find:

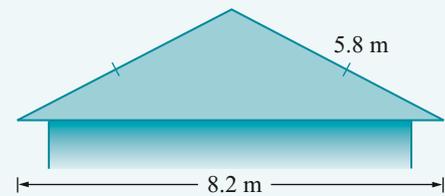
a x to the nearest metre



b α to the nearest degree.

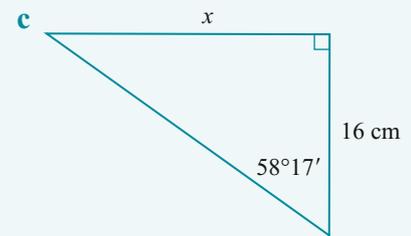
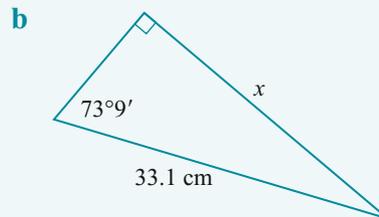
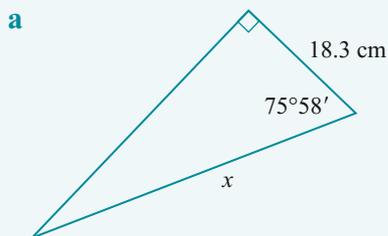


6 A roof truss has equal lengths of 5.8 m and is 8.2 m across, as shown in the diagram. Find the angle between the two equal beams.

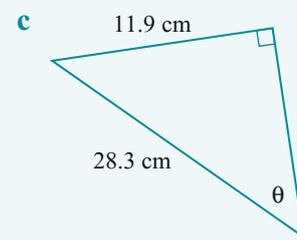
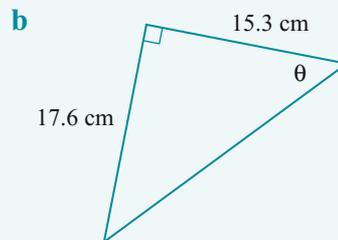
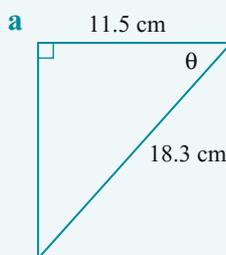


13B Review set

1 Find the length of the side marked x correct to 1 decimal place.



2 Find θ to the nearest minute.



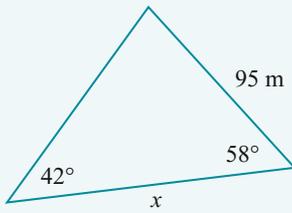
3 Solve these problems using trigonometry.

a A circle of radius 6 cm has diameter AB . If AC is a chord 5 cm long, find the measure of $\angle BAC$ to the nearest degree. ($\angle ACB = 90^\circ$)

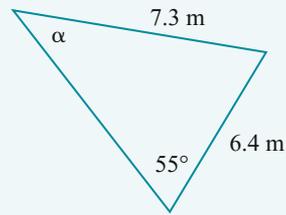
b Town A is 155 km south of town B . Town C is 218 km west of town B . Find the bearing and distance of town C from town A .

4 Use the sine rule to find:

a x to the nearest metre

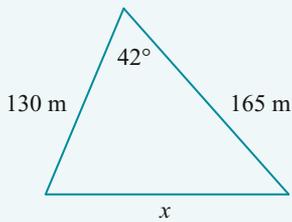


b α to the nearest degree.

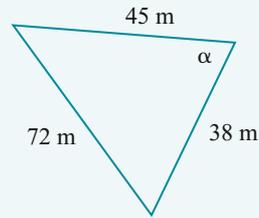


5 Use the cosine rule to find:

a x to the nearest metre



b α to the nearest degree.

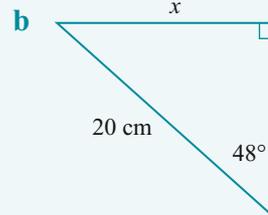
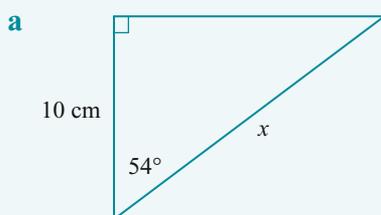


6 Two forest fire stations are 25 km apart. One is due north of the other and they report a fire at 143°T and 58°T respectively. Find the distance from each station to the fire.

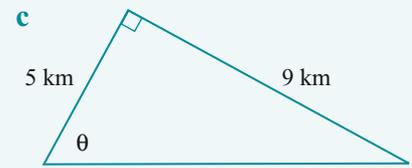
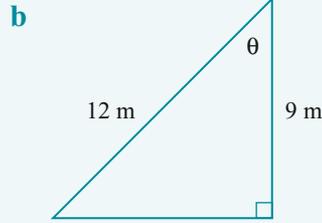
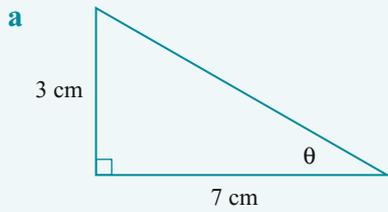


13C Review set

1 Find the length of the side marked x correct to 3 significant figures.



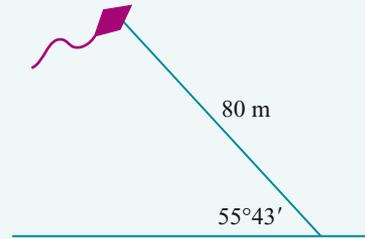
2 Find θ to the nearest minute.



3 Solve these problems using trigonometry.

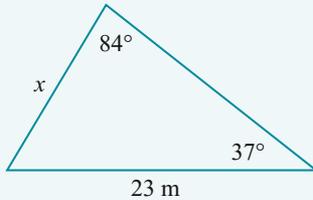
a A kite string is pinned to the ground. The string is 80 m long and makes an angle of $55^\circ 43'$ with the ground.

b Find the size of all angles of a triangle with sides 3 cm, 4 cm and 5 cm.

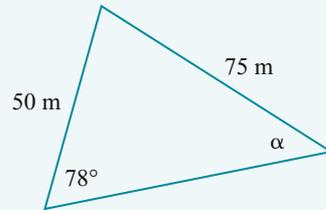


4 Use the sine rule to find:

a x to the nearest metre

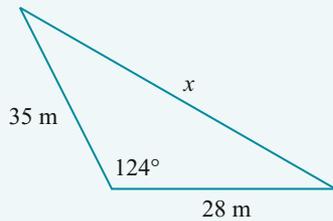


b α to the nearest degree.

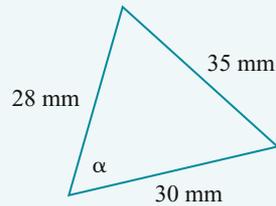


5 Use the cosine rule to find:

a x to the nearest metre



b α to the nearest degree.

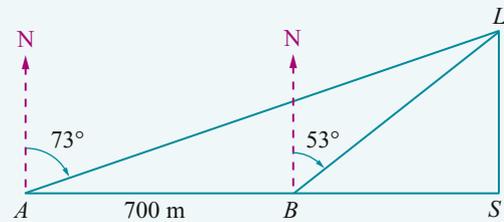


6 A boat is travelling due east. The bearing of a lighthouse is $073^\circ T$. The boat travels 700 m and the bearing of the lighthouse is now $053^\circ T$.

a Calculate $\angle LAB$, $\angle LBS$ and $\angle ABL$.

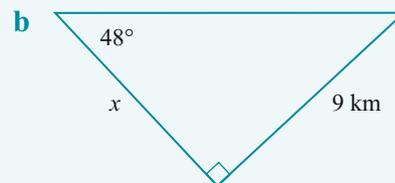
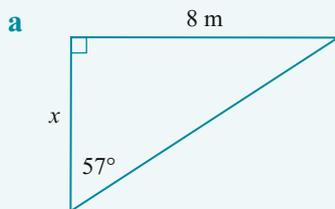
b Use the sine rule to find the distance from the boat to the lighthouse.

c Find the distance the boat needs to travel, on the same course, to be due south of the lighthouse.

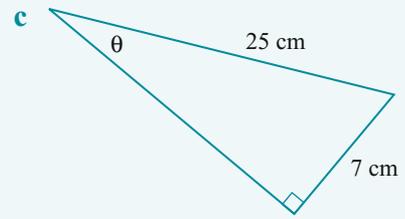
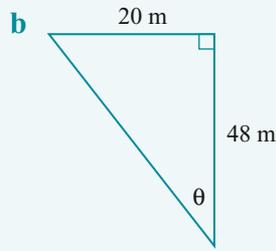
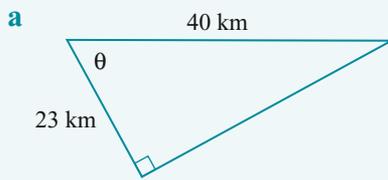


13D Review set

1 Find the length of the side marked x correct to 1 decimal place.



2 Find θ to the nearest minute.



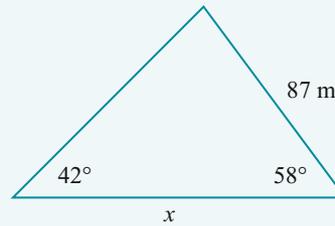
3 Solve these problems using trigonometry.

a A ladder is 5 m long and makes an angle of 75° with the ground. How far up the wall does it reach (to the nearest 10 cm)?

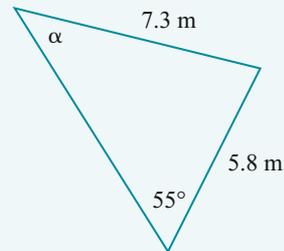
b The bearing of a boat B from a port P is 147°T . If the boat is 83 km south of the port, find the distance of the port from the boat.

4 Use the sine rule to find:

a x to the nearest metre

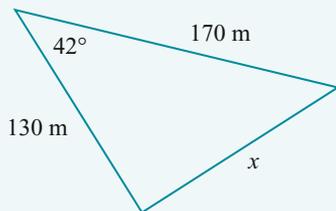


b α to the nearest degree.

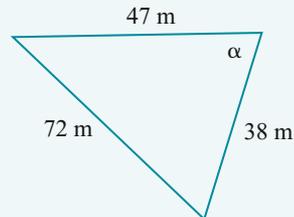


5 Use the cosine rule to find:

a x to the nearest metre



b α to the nearest degree.



6 From a point P , a yacht sails 10 km in a direction 139°T to a point Q . It turns and sails 7 km to R on a bearing of 038°T .

a Find the distance PR .

b Find the bearing of P from R .

1 a Expand and simplify:

i $(x - 3)(x + 5)$

iv $(x - 3)^2$

ii $(2x - 7)(x + 3)$

v $(7x - 2)^2$

iii $(x + 3)(x + 3)$

vi $(5x + 2)(5x - 2)$

b Factorise:

i $x^2 + 8x + 16$

ii $x^2 - 2x - 15$

iii $x^2 + 7x + 10$

c Solve:

i $x^2 = 36$

ii $3x^2 = 5$

d Factorise and solve:

i $x^2 + 7x + 12 = 0$

ii $x^2 + 4x - 12 = 0$

iii $x^2 - 6x + 8 = 0$

iv $x^2 + 8x - 9 = 0$

v $x^2 - 10x + 9 = 0$

vi $x^2 + 6x - 27 = 0$

2 For the scores 12, 17, 11, 12, 15, 16, 16, 16, 17, find the:

a mean (to 2 decimal places)

b mode

c median

d range

e upper quartile

f lower quartile

g interquartile range.

3 a 8, 3, 12, 7 and x have a mean of 9. Find x .

b The mean of eight scores is 27. What is the sum of scores?

c The mean of eight scores is 27. If a score of 18 is added, find the new mean.

4 Here are scores in a frequency distribution table.

a Find the:

i mean

ii mode

iii median

iv range

v upper quartile

vi lower quartile

vii interquartile range.

b Draw a box plot for the data.

c Draw a histogram for the data.

Score	Frequency
4	8
5	3
6	0
7	2
8	9
9	12
10	6

5 Here are scores in a frequency distribution table.

a Draw a box plot for the data.

b Is the distribution symmetrical? Explain your answer.

Score	Frequency
9	1
10	4
11	11
12	6
13	12
14	1

6 a For this stem-and-leaf plot, find the:

i mean (to 1 decimal place)

ii median

iii mode

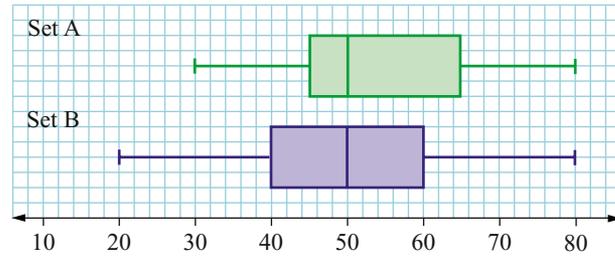
iv range.

b Is this distribution normal or skewed? Explain.

c Draw a box-and-whisker plot for this information.

Stem	Class B
11	0 8 9
12	1 2 2 2 2 3 4 5 6
13	2 3 4 4 4 5 5 5 6 7 8 9
14	0 1 2 2 5 8 8 9
15	3 5 7 7
16	1 9

- 7** Two box plots are shown.
- Describe any similarities in the data.
 - Compare the range of set A and set B.
 - For which set of scores is the middle 50% clustered more closely to the median?
 - In which data set is the top 50% of scores more closely clustered to the median?



- 8** Consider this table of values.

x	0	1	2	3	4	5	6
y	-2	-1	2	7	14	23	34

- Plot the points and sketch the graph.
- What kind of graph is this?

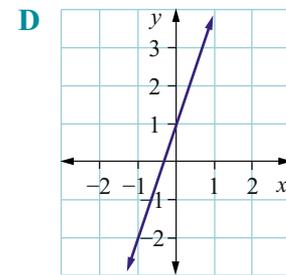
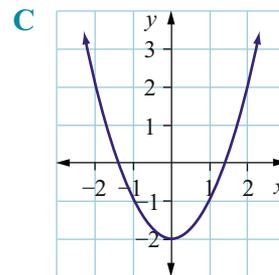
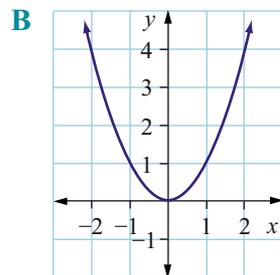
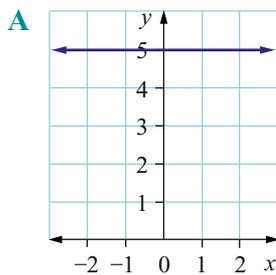
- 9** Match the following equations with their graphs.

a $y = x^2$

b $y = 3x + 1$

c $y = 5$

d $y = x^2 - 2$

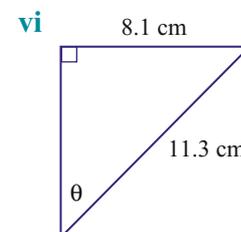
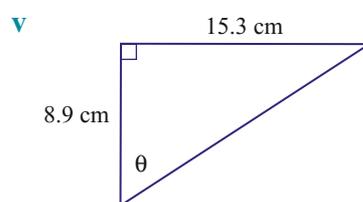
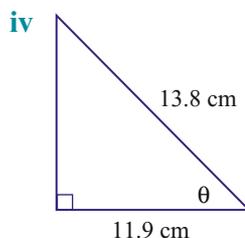
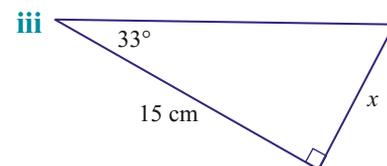
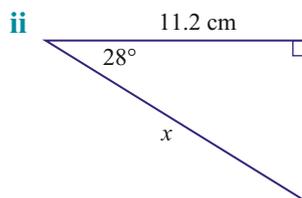
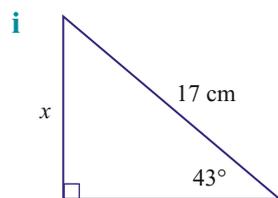


- 10 a** Consider the parabola $y = x^2 - 5x + 4$.

- Find the y -intercept.
- Find the x -intercepts.
- Sketch the graph.

- Find the length of the radius and sketch the circle $x^2 + y^2 = 81$.
 - What is the equation of a circle with centre the origin and radius $\frac{3}{4}$?
- On the same number plane sketch the graphs of $y = 3^x$ and $y = 5^x$.

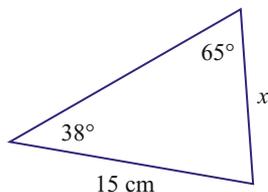
- 11 a** Find the value of the unknown side or angle. Give lengths to 1 decimal place and angles to the nearest minute.



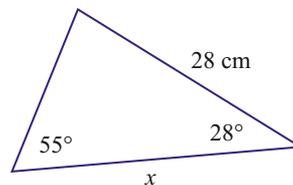
- b** **i** The bearing of town A from town B is 243°T . The distance from A to B is 85 km. How far is B east of A ?
ii Maree walks 3 km east and 8 km south. Find the distance and bearing of Maree from her starting position.
iii An isosceles triangle has sides 8 cm, 8 cm and 5 cm long. Find the measure of the base angles of the triangle to the nearest minute.

c Use the sine rule to find x to the nearest centimetre.

i

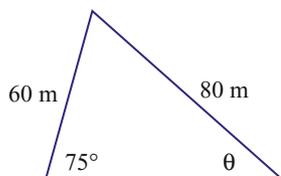


ii

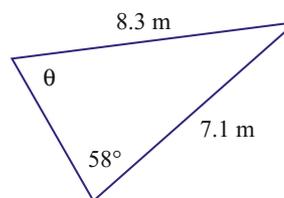


d Use the sine rule to find θ to the nearest minute.

i

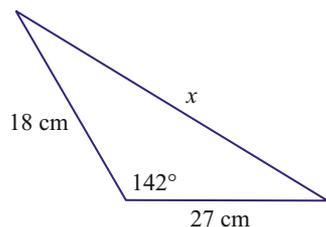


ii

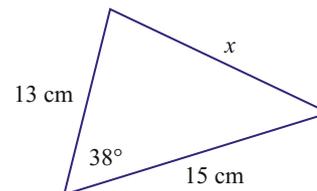


e Use the cosine rule to find x to the nearest centimetre.

i

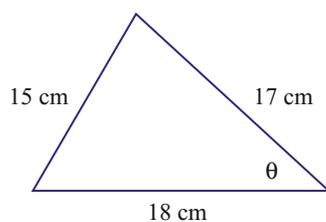


ii

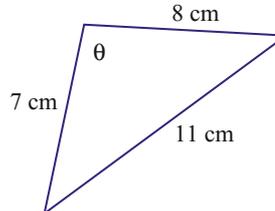


f Use the cosine rule to find θ to the nearest minute.

i



ii



- 12 a** Genevieve and Lauren walk along roads that are at an angle of 130° to each other. Genevieve travels 3.2 km to C and Lauren walks 2.8 km to B . Find the distance BC . They start at the same point.
b A car is driven 85 km from town A to town B on a bearing of 075°T . It turns to a bearing of 157°T and travels a further 118 km to town C .
i Find the distance of the car from the starting point.
ii Find the bearing of town A from town C .
c Two trees X and Y are 200 m apart. They lie on an east–west line. The bearing from a lookout to X is 057°T and Y is 311°T . Find the distance from each tree to the lookout.
d The bearing of town T from town Z is 314°T . The distance from T to Z is 850 km. How far is Z east of T ?
e Danielle travels 14 km west and 9 km south. Find the distance and bearing of Danielle from her starting position.

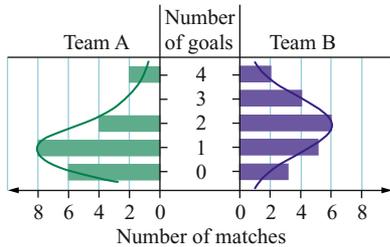
CHAPTER 1 REVIEW OF YEAR 9

Exercise 1A

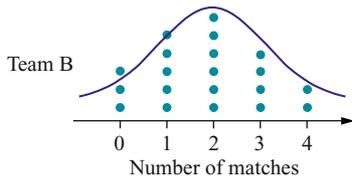
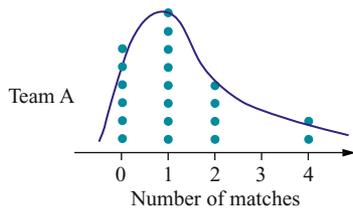
- 1 a 2^5 b 5^5
 2 a base = 9 b 5
 3 a $5 \times 5 \times 5$ b $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
 4 a 512 b 7776
 5 a False b False
 6 a 5^{28} b 4^{10} c 2^3 d 7^7 e $5^0 = 1$
 7 a $\frac{1}{2^6}$ b $\sqrt{5}$ c $\sqrt[3]{11}$
 8 a $\frac{1}{25}$ b 4 c 2 d 1
 9 a y^{15} b k^5 c p^{21}
 d t^6 e $25m^8$ f $6a^6b^{10}$
 10 a 1 b 6 c 1 d 7
 11 a \sqrt{x} b $5\sqrt{x}$ c $\sqrt{5x}$ d $\sqrt[3]{x}$ e $5\sqrt[3]{x}$
 12 a $\frac{1}{z^3}$ b $\frac{3}{z^3}$ c $\frac{1}{27z^3}$
 13 a y^4 b e^9 c $n^{-16} = \frac{1}{n^{16}}$
 d $18b^6$ e 2
 14 a True b False c False
 d True e False
 15 a $12v - 24w$ b $2a^5 + 3a^4$ c $-15x - 6$
 16 a $16m + 13$ b $10a - 3b$

Exercise 1B

- 1 a i 2007 ii 2005
 b i 140 mm ii 511 mm iii 37 mm
 c i September ii July
 d i August ii May
 e i 2002 ii 2007 iii 2004
 f i 2003 ii 2002 iii 2005
 2 a Symmetrical b Positively skewed
 c Bimodal d Negatively skewed
 3 a i



ii



b Team A: positively skewed; team B: symmetrical

- c Team A: mean = 1.2, median = 1, range = 4
 Team B: mean = 1.85, median = 2, range = 4
 Team B has a larger mean and median, indicating that it scored more goals than team A. Ranges are equal.
 d Team B performed better with more goals and higher mean number of goals per match.

Exercise 1C

- 1 a 2.3×10^{10} b 5.2×10^{-5}
 2 a 98 000 000 b 0.000 000 003 7
 3 $4 \times 10^5 = 400\ 000$ and
 $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$
 4 a 1.632×10^{14} b 1.6×10^5
 c 1.024×10^{48} d 1.5×10^8
 5 7.7×10^{-16} , 3.1×10^{-12} , 4.6×10^{13} , 3.8×10^{15}
 6 a 700 b 7 c 70 d 7000
 7 a 3500 b 7960 c 38 d 70 000
 8 a 7.3 b 7.27 c 7.268
 9 a 4.29 b 40.0
 10 a 45 b < 55 c $45 \leq x < 55$
 11 a 1 b 3 c 6
 12 a 70 b 67 c 67.3 d 67.31 e 67.306
 13 a 3 b 1 c 4 d 2 e 3, 4, 5 or 6
 14 4.65 m accurate to 1 cm. 4.650 m accurate to 0.1 cm.
 15 3.6×10^6 nanowatts
 16 a 43 km b 0.043 Mm
 17 a 1 s b ± 0.5 s
 c 20.5 to 21.5 s
 d $20.5 \text{ s} \leq \text{measurement} < 21.5 \text{ s}$
 18 a 36 cm b ± 0.5 cm
 c 7.5 to 8.5 cm and 9.5 to 10.5 cm
 d 34 cm to 38 cm e ± 2 cm
 19 Area is 99.54 cm^2 , lower limit of area is 98.5175 cm^2 , upper limit of area is 100.5675 cm^2 , estimate = 99.5 cm^2

Exercise 1D

- 1 a \$847.52 b \$22 035.52 c \$1836.29
 2 a \$1051.92 b \$2103.85 c \$4558.33
 3 \$1244.60 4 \$3337
 5 \$312.18 6 \$678.60
 7 \$443.24 8 \$498.02
 9 a \$2700 b \$1125
 10 \$57.62
 11 Advantages: instant purchase, purchase goods without having the cash.
 Disadvantages: interest high, impulse buying.
 12 a \$2460.96 b \$672.96
 13 a \$1398 b \$419.40 c \$75.73
 14 \$365.40
 15 a B b B c Equal d A
 16 \$13.55

Exercise 1E

- 1 a 51 300 b 400 c 0.27
 2 a 13.7 cm^2 b 326.2 cm^2 c 19.6 m^2
 3 a 46 m^2 b 103 cm^2
 4 Triangular prism
 5 a 148 cm^2 b 276.6 cm^2
 6 76.78 m^2
 7 SA = 527.8 cm^2 , $V = 769.7 \text{ cm}^3$
 8 a 264 cm^3 b 615.8 cm^3

Exercise 1F

1 a	Country	Frequency	Relative frequency	Percentage (%)
	Australia	185	$\frac{185}{400}$	46.25
	Japan	93	$\frac{93}{400}$	23.25
	Korea	72	$\frac{72}{400}$	18
	Germany	44	$\frac{44}{400}$	11
	Other	6	$\frac{6}{400}$	1.5

- b** 46.25% **c** 23.25%
2 a $\frac{1}{6}$ **b** $\frac{2}{6} = \frac{1}{3}$ **c** $\frac{2}{6} = \frac{1}{3}$
3 True in theory.
4 $P(6) = \frac{1}{6}$, so would expect one 6 in six throws.
5 a $\frac{5}{20} = \frac{1}{4}$ **b** $\frac{8}{20} = \frac{2}{5}$ **c** $\frac{7}{20}$ **d** $\frac{3}{5}$
6 a $\frac{4}{30} = \frac{2}{15}$ **b** $\frac{17}{30}$ **c** $\frac{21}{30} = \frac{7}{10}$
d $\frac{9}{30} = \frac{3}{10}$ **e** $\frac{10}{30} = \frac{1}{3}$
7 a $\frac{26}{33}$ **b** $\frac{18}{33} = \frac{6}{11}$ **c** $\frac{8}{33}$
d $\frac{7}{33}$ **e** $\frac{6}{33} = \frac{2}{11}$
8 a $\frac{1}{15}$ **b** $\frac{4}{15}$ **c** $\frac{4}{45}$
9 a $\frac{1}{8}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{7}{8}$
10 a i $\frac{1}{4}$ **ii** $\frac{25}{256}$ **iii** $\frac{30}{256} = \frac{15}{128}$
iv $\frac{5}{16}$ **v** $\frac{135}{256}$
b i $\frac{7}{30}$ **ii** $\frac{1}{12}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{3}$ **v** $\frac{13}{24}$

Exercise 1G

- 1 a** $\sin \alpha = \frac{q}{p}$, $\cos \alpha = \frac{r}{p}$, $\tan \alpha = \frac{q}{r}$
 $\sin \beta = \frac{r}{p}$, $\cos \beta = \frac{q}{p}$, $\tan \beta = \frac{r}{q}$
b $\sin \alpha = \frac{b}{c}$, $\cos \alpha = \frac{a}{c}$, $\tan \alpha = \frac{b}{a}$
 $\sin \beta = \frac{a}{c}$, $\cos \beta = \frac{b}{c}$, $\tan \beta = \frac{a}{b}$
c $\sin \alpha = \frac{x}{z}$, $\cos \alpha = \frac{y}{z}$, $\tan \alpha = \frac{x}{y}$
 $\sin \beta = \frac{y}{z}$, $\cos \beta = \frac{x}{z}$, $\tan \beta = \frac{y}{x}$
d $\sin \alpha = \frac{e}{f}$, $\cos \alpha = \frac{d}{f}$, $\tan \alpha = \frac{e}{d}$
 $\sin \beta = \frac{d}{f}$, $\cos \beta = \frac{e}{f}$, $\tan \beta = \frac{d}{e}$
2 a 12.1 cm **b** 11.1 cm **c** 6.49 m **d** 7.77 km
3 a 29°45' **b** 54°19' **c** 56°19'
4 a $\theta = 39^\circ$, $x = 17.6$ m, $y = 21.8$ m
b The height is 24.6 m.
c The boat is 189 m from the cliff.
5 a 62 m **b** 63°37' **c** 164 m
d 16.1 km, 240°T

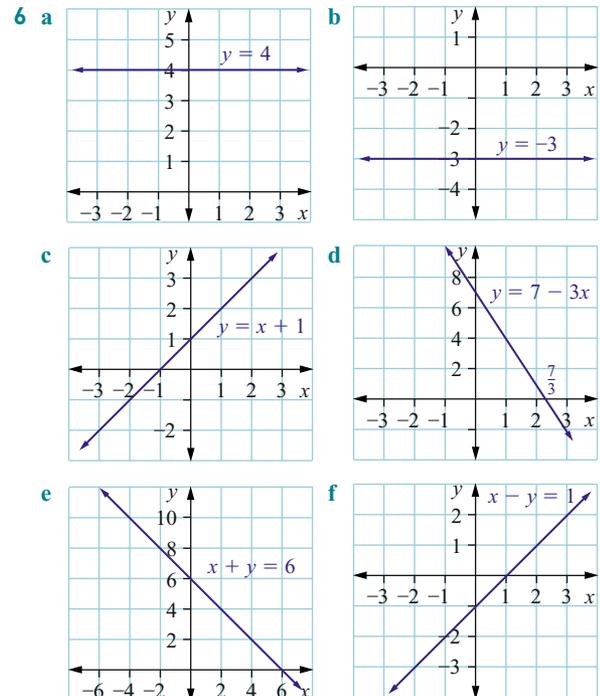
Exercise 1H

- 1 b** PQ and $P'Q'$, QR and $Q'R'$, RS and $R'S'$, PS and $P'S'$
2 a Scale factor = $\frac{3}{8}$, $x = 9$ cm
b Scale factor = $\frac{1}{5}$, $x = \frac{9}{5}$ cm, $y = 20$ cm

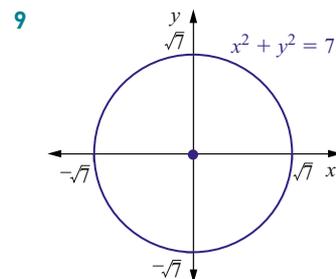
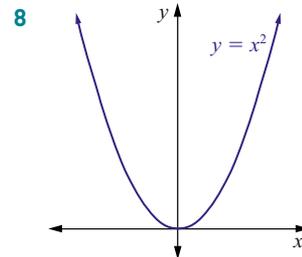
- 3** $y = 24$ cm
4 a TS **b** SU **c** TU
5 a Not similar **b** Similar; scale factor = 2
6 a PQ and YX , QR and XZ , PR and YZ
b Enlargement factor = $\frac{1}{6}$
7 a Enlargement factor = $\frac{3}{2}$, $x = 12$, $y = 12$
b Enlargement factor = $\frac{5}{2}$, $x = 3.75$

Exercise 1I

- 1 a** (8, 3) **b** (2, 6) **c** (6, 5 $\frac{1}{2}$) **d** (1, 1)
2 a $\sqrt{40} \approx 6.3$ **b** $\sqrt{58} \approx 7.6$
3 $\frac{7}{11}$
4 a $\frac{2}{3}$ **b** $-\frac{1}{10}$
5 2



7 Yes since $-3 = 3(4) - 15$



Exercise 1J

- 1 a 5.76 L/day b 13.9 m/s
 c \$226.80/h d 25.2 km/h
- 2 a Neither b Inverse c Direct
- 3 a Neither b Direct c Inverse d Neither
- 4 a Yes b Yes
- 5 a Yes b Yes
- 6 a €240 b \$200
- 7 $y = 4.8, x = 6$

Exercise 1K

- 1 a $x = 0$ b $x = -15$
 c $x = -56$ d $x = \frac{22}{3}$
- 2 a $y = -2$ b $x = 4$
- 3 $x = \frac{14}{5}$
- 4 a $x = -\frac{23}{8}$ b $t = -\frac{11}{20}$ c $p = \frac{53}{25}$
 d $x = -\frac{10}{9}$ e $x = \frac{28}{3}$ f $x = -\frac{43}{3}$
 g $x = -\frac{3}{19}$ h $x = \frac{41}{27}$ i $x = \frac{42}{23}$
- 5 a -1 b 8
- 6 a $v = \sqrt{80} \approx 8.9$ b $u = \sqrt{280} \approx 16.7$
- 7 a $x > 9$ b $x < -4$ c $x \geq \frac{11}{3}$
 d $x \geq -9$ e $x \leq -\frac{2}{3}$ f $x \geq 1$

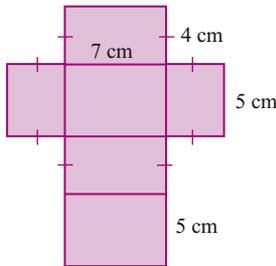
CHAPTER 2 SURFACE AREA AND VOLUME

Diagnostic test

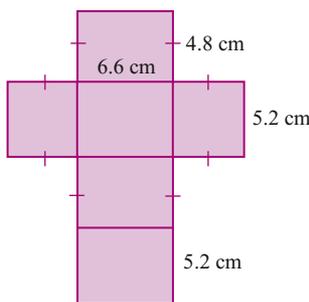
- 1 D 2 C 3 C 4 A 5 B
 6 D 7 C 8 C 9 B 10 D
 11 B 12 D 13 B 14 C 15 B

Exercise 2A

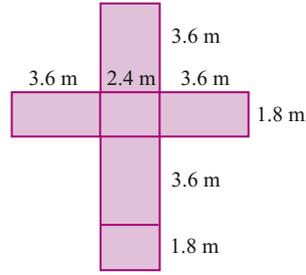
- 1 a 14.2 cm² b 72.1 cm² c 34.6 cm²
 d 294.0 m² e 110.6 m² f 101 205.1 m²
- 2 a 168 cm² b 84 m² c 65 mm²
 d 210 cm² e 139.3 km² f 73.1 mm²
- 3 a i ii 166 cm²



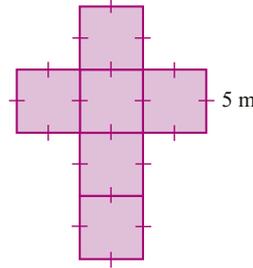
- b i ii 181.92 cm²



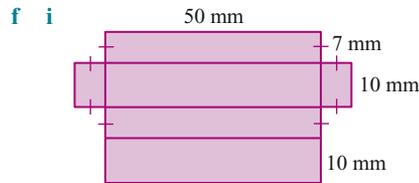
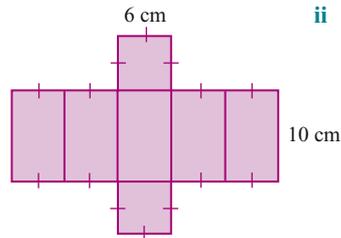
- c i ii 38.88 m²



- d i ii 150 m²



- e i ii 312 cm²



- ii 1840 mm²

- 4 a 294 cm² b 141 cm²
 c 319.8 cm² b 394.25 cm²
- 5 a i $x = 5$ cm ii 96 cm²
 b i $x = 13$ m ii 300 cm²
 c i $x \approx 18.0$ mm ii 279 mm²

Exercise 2B

- 1 Surface area = $2\pi r^2 + 2\pi rh$
 $= 2\pi \times 8.7^2 + 2\pi \times 8.7 \times 3.2$
 ≈ 650.5 m² (1 decimal place)
- 2 a 534 cm² b 83 m² c 2915 mm²
 d 2356 cm² e 35 m² f 2092 cm²
- 3 a 816.8 cm² b 502.7 cm² c 471.2 cm²
- 4 a 35.6 L b length = 15 cm, radius = 8 cm
- 5 a 427.3 cm² b 31 416 cm² (or 3.1416 m²)
 c 729.8 cm²
- 6 354 L needed, \$2566.50 7 \$28.14
 8 31.8 cm 9 47.7 cm

Exercise 2C

- 1 $V = A \times h$ where A is the area of base and h is the perpendicular height.
 $V = 23.5 \times 25 = 587.5$ cm³
- 2 a 216 cm³ b 304 cm³
 c 160 m³ d 702 cm³

- 3 a 112 m^3 b 343 cm^3 c 339.3 cm^3
 d 252 cm^3 e 2262 cm^3 f 256 cm^3
- 4 $V = A \times h$
 $= \frac{\theta}{360} \times \pi \times r^2 \times h$
 $= \frac{40}{360} \times \pi \times 8^2 \times 25$
 $= 558.5 \text{ cm}^3$
- 5 a 147 cm^3 b 2827 m^3 c 0.96 m^3
 d 594 cm^3 e 5655 cm^3 f 115 cm^3
 g 118 cm^3 h 628 cm^3 i 2309 cm^3

Exercise 2D

- 1 The solid is a cylinder and a rectangular prism.
 Cylinder: $V = \pi r^2 \times h$ Diameter = 6 m
 $= \pi \times 3^2 \times 10$ Radius = 3 m
 $= 282.74 \text{ m}^3$
 Rectangular prism: $V = Ah$
 $= (12 \times 8) \times 4 = 384 \text{ m}^3$
 Total volume = $282.74 + 384$
 $= 667 \text{ m}^3$ to the nearest whole number
- 2 a 2633 cm^3 b 3003 m^3 c 686 cm^3
 d 0.043 m^3 e 603.2 cm^3 f 48.7 m^3
 g 61.2 m^3 h 198.6 cm^3 i 625 m^3

Exercise 2E

- 1 Total surface area = front + back + side + 4 rectangles
 Area of front face = $10 \times 8 + 4 \times 4$
 $= 96 \text{ cm}^2$
 Total surface area = $2 \times 96 + 12 \times 14 + 14 \times 8$
 $+ 14 \times 6 + 14 \times 4 + 14 \times 4$
 $+ 10 \times 14$
 $= 808 \text{ cm}^2$
- 2 a 532 cm^2 b 288 cm^2 c 530 cm^2
 d 670 cm^2 e 484 cm^2 f 547.7 cm^2
 g 497.1 cm^2 h 89.5 m^2 i 190.5 cm^2

Exercise 2F

- 1 a 0.84 m^2 b 0.072 m^3
 2 121 L
 3 a 2.103 m^2 b 5.887 m^3
 c 5887 L d $\$897.70$
 4 a 75.27 m^2 b $\$5269$ c 7.53 m^3
 5 a 2.827 m^2 b 21.22 revolutions
 6 a 67.2 m^3 b 6 days 5 h 20 min
 c $\$184.80$
 7 46.4 kg
 8 a $19\,200 \text{ cm}^3$ b 2.5 cm
 9 90 kg 10 63.66 m
 11 a 52.25 linear m b 3 rolls

Language in mathematics

- 2 a A semicircle is a half of a circle.
 b A quadrant is a quarter of a circle.
 c Composite areas may be found in more than one way.
- 3 The area of a rhombus or a kite is half the product of the diagonals.

Check your skills

- 1 B 2 C 3 D 4 B 5 A
 6 D 7 C 8 B

Review set 2A

- 1 a 23.4 m^2 b 52 cm^2 c 126.4 cm^2
 2 a 214 cm^2 b 314.6 cm^2
 3 $251.3 \text{ cm}^2, 235.6 \text{ cm}^3$
 4 a 231 cm^3 b 366.5 cm^3
 5 $89.5 \text{ m}^2, 48.7 \text{ m}^3$

Review set 2B

- 1 $\$886.52$
 2 440 cm^2 3 $22.6 \text{ m}^2, 8.1 \text{ m}^3$
 4 104.3 m^2 5 270 cm^3
 6 a 4.13 m^2 b 10.3 m^3
 c $10\,300 \text{ L}$ d $\$1358$

Review set 2C

- 1 a 61.7 m^2 b 26.8 m^2
 c 75.6 m^2 d 59.5 mm^2
 2 a 367.2 m^2 b 684 m^3
 3 62.8 m^2
 4 a $1170 \text{ cm}^2, 2106 \text{ cm}^3$ b $230.3 \text{ cm}^2, 201.2 \text{ cm}^3$
 5 $357\,435 \text{ g}$ or 357.435 kg

Review set 2D

- 1 9161 people
 2 a 138 cm^2 b 180 cm^2
 3 $51.3 \text{ m}^2, 10.1 \text{ m}^3$
 4 a $672 \text{ cm}^2, 1080 \text{ cm}^3$ b $74.3 \text{ cm}^2, 37.7 \text{ cm}^3$
 5 $212.1 \text{ cm}^2, 198.6 \text{ cm}^3$
 6 a ≈ 78 linear m b 6 rolls

CHAPTER 3 ALGEBRA

Diagnostic test

- 1 C 2 B 3 C 4 A 5 D
 6 C 7 A 8 C 9 A 10 D

Exercise 3A

- 1 a $\frac{6y}{15}$ b $\frac{8m}{6}$ c $\frac{30a}{12}$ d $\frac{35a}{20}$
 e $\frac{25k}{60}$ f $\frac{15m}{6n}$ g $\frac{12t}{16w}$ h $\frac{48}{30q}$
 i $\frac{6xz}{5yz}$ j $\frac{3ab}{4b^2}$ k $\frac{14mp}{18np}$ l $\frac{10wx}{15wx}$
 m $\frac{5ab}{4b^2}$ n $\frac{15xy^2}{10y^3}$ o $\frac{6rz}{14tz}$ p $\frac{16r^2t}{44r^2t}$
- 2 a $\frac{2t}{3}$ b $\frac{2k}{3}$ c $\frac{11}{9}$ d $\frac{4x}{5}$ e $\frac{4}{3}$ f $\frac{4}{5b}$
 3 a $\frac{x}{2}$ b $\frac{t}{4}$ c $\frac{2y}{3}$ d $\frac{3b}{4}$ e $\frac{3}{10}$ f $\frac{7}{4}$
 g $\frac{2}{3}$ h $\frac{3}{4}$ i $\frac{2}{b}$ j $\frac{4p}{3}$ k $\frac{3}{4z}$ l $\frac{2}{3x}$
 4 a $\frac{7m}{8}$ b $\frac{4}{5k^3}$ c $\frac{y}{2}$ d $\frac{4}{3z^2}$ e $\frac{7b}{6a}$ f $\frac{x}{2y}$
 5 a $\frac{7x^2}{6}$ b $\frac{11}{15z^2}$ c $\frac{a^3}{2}$ d $\frac{4x^3}{3}$
 e $\frac{3m^2}{2}$ f $\frac{3b}{5}$ g $\frac{5a^2b^2}{4}$ h $\frac{9x^3}{5y}$
- 6 a For the fractions $\frac{4m}{5}$ and $\frac{3m}{4}$ the LCD = 20.
 $\frac{4m}{5} \times \frac{4}{4} = \frac{16m}{20}$ and $\frac{3m}{4} \times \frac{5}{5} = \frac{15m}{20}$
 Hence $\frac{4m}{5} + \frac{3m}{4} = \frac{16m}{20} + \frac{15m}{20} = \frac{31m}{20}$

- b** For the fractions $\frac{19a}{20}$ and $\frac{7a}{10}$ the LCD = 20.
 $\frac{7a}{10} \times \frac{2}{2} = \frac{14a}{20}$
Hence $\frac{19a}{20} - \frac{7a}{10} = \frac{19a}{20} - \frac{14a}{20} = \frac{5a}{20} = \frac{a}{4}$
- c** For the fractions $\frac{4z}{9}$ and z the LCD = 9.
 $\frac{z}{1} \times \frac{9}{9} = \frac{9z}{9}$
Hence $\frac{4z}{9} + z = \frac{4z}{9} + \frac{9z}{9} = \frac{13z}{9}$
- d** For the fractions $\frac{6c}{5}$ and $\frac{2c}{7}$ the LCD = 35.
 $\frac{6c}{5} \times \frac{7}{7} = \frac{42c}{35}$ and $\frac{2c}{7} \times \frac{5}{5} = \frac{10c}{35}$
Hence $\frac{6c}{5} + \frac{2c}{7} = \frac{42c}{35} + \frac{10c}{35} = \frac{52c}{35}$
- 7 a** $\frac{9x}{11}$ **b** $\frac{3n}{2}$ **c** $\frac{13m}{24}$ **d** $\frac{19k}{12}$
e $\frac{5b}{8}$ **f** $\frac{7a}{10}$ **g** $\frac{5k}{3}$ **h** $\frac{2z}{15}$
i $\frac{25t}{8}$ **j** $\frac{x}{10}$ **k** $\frac{29x}{15}$ **l** $\frac{19t}{14}$
- 8 a** For the fractions $\frac{3}{k}$ and $\frac{5}{2k}$ the LCD = 2k.
 $\frac{3}{k} \times \frac{2}{2} = \frac{6}{2k}$
Hence $\frac{3}{k} + \frac{5}{2k} = \frac{6}{2k} + \frac{5}{2k} = \frac{11}{2k}$
- b** For the fractions $\frac{5a}{6b}$ and $\frac{3a}{7b}$ the LCD = 42b.
 $\frac{5a}{6b} \times \frac{7}{7} = \frac{35a}{42b}$ and $\frac{3a}{7b} \times \frac{6}{6} = \frac{18a}{42b}$
Hence $\frac{5a}{6b} - \frac{3a}{7b} = \frac{35a}{42b} - \frac{18a}{42b} = \frac{17a}{42b}$
- 9 a** $\frac{7}{2x}$ **b** $\frac{1}{6a}$ **c** $\frac{47}{20y}$ **d** $\frac{11}{3z}$
e $\frac{14a}{3b}$ **f** $\frac{29m}{21n}$ **g** $\frac{58xy}{45z}$ **h** $\frac{36a - 25b}{30c}$
- 10 a** $\frac{3w}{4} \times \frac{7z}{5} = \frac{3w \times 7z}{4 \times 5} = \frac{21wz}{20}$
b $2p \times \frac{5p}{7} = \frac{2p}{1} \times \frac{5p}{7} = \frac{2p \times 5p}{1 \times 7} = \frac{10p^2}{7}$
c $\frac{5mn}{8} \times \frac{6p}{25n} = \frac{5mn \times 6p}{8 \times 25n} = \frac{m \times 3p}{4 \times 5} = \frac{3mp}{20}$
- 11 a** $\frac{6mn}{7}$ **b** $\frac{4kn}{15}$ **c** $\frac{3wz}{2}$ **d** $\frac{3ab}{2}$
e $\frac{2t}{3u}$ **f** $\frac{15}{2} = 7\frac{1}{2}$ **g** $\frac{3}{4}$ **h** $\frac{4a}{5}$
i $\frac{40n}{27}$ **j** $\frac{kn}{9m}$ **k** 10p **l** 8bc
- 12 a** $\frac{3m}{5} \div \frac{2n}{9} = \frac{3m}{5} \times \frac{9}{2n} = \frac{27m}{10n}$
b $\frac{10p}{9} \div \frac{25}{27m} = \frac{10p}{9} \times \frac{27m}{25} = \frac{6pm}{5}$
c $\frac{4ab}{3} \div \frac{5bc}{6} = \frac{4ab}{3} \times \frac{6}{5bc} = \frac{8a}{5c}$
- 13 a** $\frac{5x}{12y}$ **b** $\frac{7a}{4b}$ **c** $\frac{3p}{2q}$ **d** $\frac{14}{3} = 4\frac{2}{3}$
e $\frac{2}{3}$ **f** $\frac{12}{35}$ **g** $\frac{10}{3} = 3\frac{1}{3}$ **h** $\frac{28}{m^2}$
i $\frac{10y}{3z}$ **j** $\frac{3}{4k}$ **k** $\frac{7np}{m}$ **l** $\frac{10}{3} = 3\frac{1}{3}$
- 14 a** $\frac{3}{2b}$ **b** $\frac{9x^2}{11}$ **c** $\frac{x}{3y}$ **d** $\frac{25k}{12}$
e $\frac{11m}{6}$ **f** $\frac{23}{6x}$ **g** $\frac{a}{15b}$ **h** $\frac{10b}{3}$
i $\frac{20}{21}$ **j** $\frac{4}{3} = 1\frac{1}{3}$ **k** $\frac{2x^5z}{3y}$ **l** $\frac{p^4}{6q}$

Exercise 3B

- 1 a** $5(y + 7) = 5 \times y + 5 \times 7$
 $= 5y + 35$
b $3a(2a - 7b) = 3a \times 2a - 3a \times 7b$
 $= 6a^2 - 21ab$
c $4p^2(2p^3 + 5q) = 4p^2 \times 2p^3 + 4p^2 \times 5q$
 $= 8p^5 + 20p^2q$
- 2 a** $4t + 32$ **b** $18y - 24z$
c $30a + 45b$ **d** $6a^2 - 12a$
e $12x^2 + 20x$ **f** $12m - 6m^2$
g $18k^2 - 24km$ **h** $10b^2 - 8bc$
i $12wz - 15w^2$ **j** $10m^5 + 40m^2n$
k $12p^8 - 22p^3$ **l** $12a^9 + 8a^2b$
m $8x^8 - 10x^3y$ **n** $18x^7 - 12x^5$
o $16a^2 - 8ab + 14a$
- 3 a** As $-3 \times 2a = -6a$ and $-3 \times (+5) = -15$,
then $-3(2a + 5) = -6a - 15$.
b As $-4 \times 5t = -20t$ and $-4 \times (-8) = 32$,
then $-4(5t - 8) = -20t + 32$.
c As $-1 \times 6m = -6m$ and $-1 \times (+7) = -7$,
then $-(6m + 7) = -6m - 7$.
d As $-1 \times 4k = -4k$ and $-1 \times (-2) = 2$,
then $-(4k - 2) = -4k + 2$.
- 4 a** $-30t - 35$ **b** $-12w + 28$
c $-7a - 9$ **d** $-5v + 2$
e $-6b - 24c$ **f** $-12x + 10y$
g $-4a - 9b$ **h** $-2m + 8n$
i $-50p + 20q$ **j** $-3a^2 - 4b^3$
k $-2x^2 + 3y$ **l** $-18x^2 + 24$
- 5 a** $-5w(2w + 4z) = -10w^2 - 20wz$
b $-3p(5p - 2q) = -15p^2 + 6pq$
- 6 a** $-6pq + 18pr$ **b** $-10x^2 - 15xy$
c $-20m^2 + 28mn$ **d** $-4a^2 - 5ab$
e $-3km + 2kn$ **f** $-20z^7 - 40z^9$
g $-12m^8 + 6m^5$ **h** $-30x^{11} - 12x^9y^2$
i $-6ab - 24ac$
- 7 a** $3(2b + 4) + 7 = 6b + 12 + 7 = 6b + 19$
b $11 + 2(3m - 4) = 11 + 6m - 8 = 6m + 3$
- 8 a** $4a + 18$ **b** $6b - 12$
c $12w - 1$ **d** $8y - 15$
e $22z - 6$ **f** $18x + 6$
g $18b - 10$ **h** $4y + 33$
i $10w - 12$ **j** $20e - 14$
k $16a + 27$ **l** $19c - 10$
- 9 a** $15 - 3(b + 2) = 15 - 3b - 6 = -3b + 9$
b $4 - 5(y - 3) = 4 - 5y + 15 = 19 - 5y$
- 10 a** $2 - 2a$ **b** $14 - 3y$
c $-3 - 4b$ **d** $5v + 12$
e $14w - 15$ **f** $22 - 15t$
g $-2 - 15x$ **h** $12 - 6k$
i $-4 - 12z$ **j** $23w - 10$
k $6a - 15$ **l** $3x + 8$
- 11 a** $4(b + 5) + 3(b - 2) = 4b + 20 + 3b - 6$
 $= 7b + 14$
b $2(y - 6) + 4(y + 1) = 2y - 12 + 4y + 4$
 $= 6y - 8$
- 12 a** $13k + 9$ **b** $15m + 11$
c $14p + 6$ **d** $13a - 6$
e $25x - 11$ **f** $14y + 1$
g $18v - 17$ **h** $22x + 2y$
i $23a - 9b$ **j** $38a - 6a^2$
- 13 a** $4(2a - 3) - 2(3a + 1) = 8a - 12 - 6a - 2 = 2a - 14$
b $3(4x + 3) - 4(x - 2) = 12x + 9 - 4x + 8 = 8x + 17$

- 14 a $4k + 9$ b $2w + 26$
 c $9t - 10$ d $13z - 8$
 e $-2a + 14$ f $-d - 18$
 g $10x + 15$ h $4a - 12$
- 15 a $22x^2$ b $2m^2 - 11m$
 c $22a^5 + 3a^2b$ d $10y^5 - 2y^4 + 4y^3$
 e $3y^2 - 11y + 8$ f $6a^2 + 16a + 10$
 g $2q^2 - 10qr - 4q + 20r$ h $12z^2 + 8wz - w + z$

16 a $\frac{2m}{3} + \frac{m+4}{5} = \frac{10m}{15} + \frac{3(m+4)}{15}$
 $= \frac{10m + 3(m+4)}{15}$
 $= \frac{10m + 3m + 12}{15}$
 $= \frac{13m + 12}{15}$

b $\frac{4p+3}{8} - \frac{2p-5}{6} = \frac{3(4p+3)}{24} - \frac{4(2p-5)}{24}$
 $= \frac{3(4p+3) - 4(2p-5)}{24}$
 $= \frac{12p + 9 - 8p + 20}{24}$
 $= \frac{4p + 29}{24}$

- 17 a $\frac{7m+15}{12}$ b $\frac{18p-21}{35}$ c $\frac{8k+14}{15}$
 d $\frac{3x+7}{8}$ e $\frac{19y-39}{20}$ f $\frac{7w+50}{30}$

Exercise 3C

- 1 The HCF of $18a$ and $12b = 6$.
 $18a = 6 \times 3a$ and $12b = 6 \times 2b$
 Hence $18a + 12b = 6 \times 3a + 6 \times 2b$
 $= 6(3a + 2b)$
- 2 a $2(7s + 5)$ b $3(3y - 2)$
 c $4(2k + 3b)$ d $5(3w - 5z)$
 e $3(4 + 3k)$ f $6(4p - 3q)$
 g $3(2x + y + 3z)$ h $6(4a - 2b - 3c)$
- 3 a The highest negative common factor of $-5y$ and $10 = -5$.
 $-5y = -5 \times y$ and $10 = -5 \times (-2)$
 Hence $-5y + 10 = -5 \times y + (-5) \times (-2)$
 $= -5(y - 2)$
- b The highest negative common factor of $-12p$ and $-8q = -4$.
 $-12p = -4 \times 3p$ and $-8q = -4 \times 2q$
 Hence $-12p - 8q = -12p + (-8q)$
 $= -4 \times 3p + (-4) \times 2q$
 $= -4(3p + 2q)$
- 4 a $-2(3t - 4)$ b $-5(k + 2)$
 c $-4(2p - 5q)$ d $-3(4x + 5y)$
 e $-8(2 - p)$ f $-6(3 + 4w)$
- 5 a The HCF of 15 and $18 = 3$ and the HCF of pq and $p = p$. Hence the HCF of $15pq$ and $18p = 3p$.
 $15pq = 3p \times 5q$ and $18p = 3p \times 6$
 Hence $15pq + 18p = 3p \times 5q + 3p \times 6$
 $= 3p(5q + 6)$
- b The HCF of 6 and $30 = 6$ and the HCF of y^2 and $y = y$. Hence HCF of $6y^2$ and $30y = 6y$.
 $6y^2 = 6y \times y$ and $30y = 6y \times 5$
 Hence $6y^2 - 30y = 6y \times y - 6y \times 5$
 $= 6y(y - 5)$

- c $y^2 + 7y = y(y + 7)$ d $m^2 - 3m = m(m - 3)$
 e $3mn + 4m = m(3n + 4)$ f $9p - 5pq = p(9 - 5q)$
 g $x^2 + 5xy = x(x + 5y)$ h $2bc - b^2 = b(2c - b)$
- 6 a $4p(2q - 3r)$ b $p(p + 3)$
 c $3b(3a + 2)$ d $x(x - 7)$
 e $4n(3m - 4)$ f $2k(k + 2)$
 g $2m(3k - 4m)$ h $5z(2z - 1)$
 i $3a(5b + a)$ j $6t(4t - 3w)$
 k $p(7q + 5r + 11s)$ l $2b(2a + 3c - 5d)$
- 7 a i True ii True iii True

b Part iii

- 8 a Fully b Partially
 c Partially d Fully
- 9 a The HCF of 5 and $3 = 1$ and the HCF of z^4 and $z^2 = z^2$.
 Hence the HCF of $5z^4$ and $3z^2 = z^2$.
 $5z^4 = z^2 \times 5z^2$ and $3z^2 = z^2 \times 3$
 Hence $5z^4 + 3z^2 = z^2 \times 5z^2 + z^2 \times 3$
 $= z^2(5z^2 + 3)$
- b The HCF of 9 and $6 = 3$, the HCF of a^3 and $a^2 = a^2$ and the HCF of b^2 and $b^4 = b^2$.
 Hence the HCF of $9a^3b^2$ and $6a^2b^4 = 3a^2b^2$.
 $9a^3b^2 = 3a^2b^2 \times 3a$ and $6a^2b^4 = 3a^2b^2 \times 2b^2$
 Hence $9a^3b^2 - 6a^2b^4 = 3a^2b^2 \times 3a - 3a^2b^2 \times 2b^2$
 $= 3a^2b^2(3a - 2b^2)$
- c $14w^5 + 8w^3 = 2w^3(7w^2 + 4)$
 d $20k^6 - 15k^2 = 5k^2(4k^4 - 3)$
 e $7x^3y^8 + 5x^4y^5 = x^3y^5(7y^3 + 5x)$
 f $9a^3b^6 - 3a^3b^8 = 3a^3b^6(3a^2 - b^2)$
- 10 a $t^2(6t^2 + 7)$ b $4k^5(2k^5 - 3)$
 c $8n^7(2 + 3n^2)$ d $a^3b^2(9b^3 + 8a^2)$
 e $9m^3n^2(2m - n^3)$ f $10x^3y^7(3x^2 - 2)$
 g $3x(7y - 1 + 3x)$ h $4m(4m + 2n - 3)$

Language in mathematics

- 1 reduce, substitute, apply, equivalent
- 2 a factorise b reciprocal
 c equivalent d algebraic
- 3 a The reciprocal is the number by which a fraction must be multiplied to equal 1. A fraction multiplied by its reciprocal equals 1.
 For example, $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$.
- b The highest common factor of two terms is the largest number that divides into the two terms. For example, the HCF of 8 and 12 is 4 .
- c The lowest common denominator is the smallest number into which the denominators of the fractions added or subtracted divide exactly. For example, the LCD of $\frac{1}{2}$ and $\frac{1}{3}$ is 6 or $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.
- d If the HCF is factorised out of the expression, then the expression is fully factorised. If a common factor, not the HCF, is used the factorisation is only partial.
- 4 a B b D c A d E e C
- 5 They are reciprocals.

Check your skills

- 1 B 2 D 3 B 4 B 5 A
 6 B 7 C 8 A 9 D

Review set 3A

- 1 a $\frac{25k}{15}$ b $\frac{24x}{30y}$ c $\frac{8ab}{12b^2}$

- 2 a $\frac{k}{3}$ b $\frac{3}{2b}$ c $\frac{4x^2}{5y}$
 3 a $\frac{17t}{24}$ b $\frac{5w}{3}$ c $\frac{7}{2x}$
 4 a $\frac{12de}{35}$ b $\frac{3}{2a}$ c $\frac{3}{4}$ d $\frac{nq}{6}$
 5 a $12x^2 + 8x$ b $6p^6 - 14p^4q$
 c $-18x^2 + 12y$ d $10v - 8$
 e $-9 + 6x$ or $6x - 9$ f $18p + 7q$
 6 a $\frac{x-3}{30}$ b $\frac{31a-2}{14}$
 7 a $3y(x+4y)$ b $-2(4m+5)$
 c $3x(5-3y-2x)$ d $2m^2n^2(4m-3n)$

Review set 3B

- 1 a $\frac{35t}{20}$ b $\frac{25x}{10y}$ c $\frac{16mn}{12n^2}$
 2 a $\frac{5p}{6}$ b $\frac{2}{3p}$ c $\frac{2k}{3m^2}$
 3 a $\frac{31p}{20}$ b $\frac{2k}{5}$ c $\frac{17}{15x}$
 4 a $\frac{15w^2}{8}$ b $\frac{2m}{5}$ c $\frac{3}{2} = 1\frac{1}{2}$ d $\frac{28a^2b}{3c}$
 5 a $10k^2 - 35km$ b $12d^4 - 21d^2e$
 c $-12m^2 + 28m$ d $24z + 3$
 e $22a - 6a^2$ f $13x - 20y$
 6 a $\frac{-8x+7}{6}$ b $\frac{38k-12}{15}$
 7 a $-4(2k+3)$ b $a(7a+3b)$
 c $3(4ab-3+2b^2)$ d $4x^2y(3x^2y^2-4)$

Review set 3C

- 1 a $\frac{28m}{16}$ b $\frac{18kn}{9mn}$ c $\frac{4wz}{3z^2}$
 2 a $\frac{8}{15}$ b $\frac{3}{13q}$ c $\frac{3a^3b}{2}$
 3 a $\frac{9k}{35}$ b $\frac{11t}{3}$ c $\frac{23}{20w}$
 4 a $\frac{21ac}{10bd}$ b $\frac{9p}{2}$ c $\frac{6b}{7}$ d $\frac{2p}{3q}$
 5 a $6s^2 - 3s$ b $28q^3 - 35pq^2$
 c $-12a^2 - 30a$ d $31 - 10e$
 e $6a^2 + 12ab - 15ac$ f $8z^2 - 7z$
 6 a $\frac{53z-16}{24}$ b $\frac{8x-7}{4}$
 7 a $-8(3d+2e)$ b $2p(3q+8p)$
 c $3ab^3(3a^2b-4)$ d $2xy(x^2-3x+2)$

Review set 3D

- 1 a $\frac{12h}{20}$ b $\frac{12ab}{18}$ c $\frac{8x^2}{6xy}$
 2 a $\frac{2}{3p}$ b $\frac{9y}{10}$ c $2m^3n^2$
 3 a $\frac{29k}{28}$ b $-\frac{p}{6}$ c $\frac{58}{15w}$
 4 a $\frac{10}{21ab}$ b $\frac{2}{3z}$ c $\frac{8}{5} = 1\frac{3}{5}$ d $\frac{a}{6}$
 5 a $7d^2 - 21de$ b $40k^2 + 90km$
 c $-6p^4 - 2p^2q$ d $1 + 18z$
 e $10a + 5a^2$ f $21x^2 - 6x^3 + 12x^4$
 6 a $\frac{10w+3}{12}$ b $\frac{4x+27y}{30}$
 7 a $-3(5q-4)$ b $6bc(2a+1)$
 c $7x(4+2x-x^2)$ d $a^2b^2(ab-1)$

CHAPTER 4 LINEAR RELATIONSHIPS

Diagnostic test

- 1 C 2 D 3 C 4 A 5 A
 6 B 7 C 8 A 9 C 10 B
 11 A 12 A 13 B 14 D 15 A

Exercise 4A

1 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $x_1 = -3, y_1 = 5$
 $= \sqrt{(6 - (-3))^2 + (-4 - 5)^2}$ $x_2 = 6, y_2 = -4$
 $= \sqrt{9^2 + (-9)^2}$
 $= \sqrt{162}$ units

- 2 a $\sqrt{50}$ b $\sqrt{13}$ c $\sqrt{37}$
 d $\sqrt{34}$ e $\sqrt{50}$ f $\sqrt{89}$

3 Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $x_1 = 5, y_1 = -3$
 $= \left(\frac{5 + (-9)}{2}, \frac{-3 + 5}{2} \right)$ $x_2 = -9, y_2 = 5$
 $= (-2, 1)$

- 4 a (4, 6) b $(3, 2\frac{1}{2})$ c $(\frac{3}{4}, 1\frac{3}{4})$ d $(-\frac{3}{2}, -1)$

5 Let the midpoint be (x, y), then

$x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$.

Substitute the values $(x, y) = (-7, 3)$ and $(x_1, y_1) = (3, 7)$.

$-7 = \frac{3 + x_2}{2}$ $3 = \frac{7 + y_2}{2}$

$-14 = 3 + x_2$ $6 = 7 + y_2$
 $x_2 = -17$ $y_2 = -1$

B has coordinates (-17, -1).

- 6 (7, -6) 7 (-8, 3)

- 8 a i Rise = 2, run = 4 ii $m = \frac{1}{2}$

b iii $m = \frac{y_2 - y_1}{x_2 - x_1}$

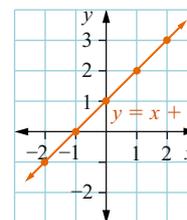
9 $m = \frac{y_2 - y_1}{x_2 - x_1}$ $A = (4, -2) = (x_1, y_1)$
 $= \frac{6 - (-2)}{10 - 4} = \frac{4}{3}$ $B = (10, 6) = (x_2, y_2)$

- 10 a $\frac{5}{3}$ b -4 c 0 d $\frac{1}{2}$
 e $\frac{5}{2}$ f Undefined g 0 h $\frac{1}{4}$
 i $\frac{1}{2}$

Exercise 4B

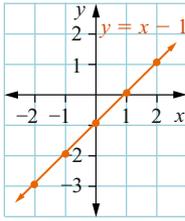
- 1 a $y = x + 1$

x	-2	-1	0	1	2
y	-1	0	1	2	3



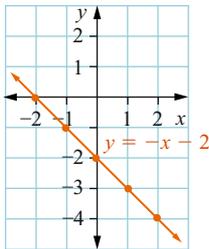
b $y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



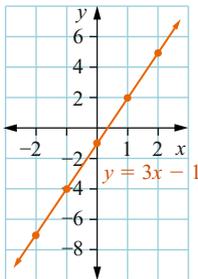
c $y = -x - 2$

x	-2	-1	0	1	2
y	0	-1	-2	-3	-4



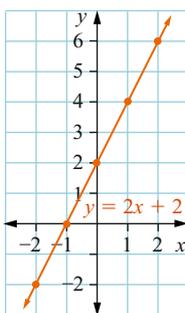
d $y = 3x - 1$

x	-2	-1	0	1	2
y	-7	-4	-1	2	5



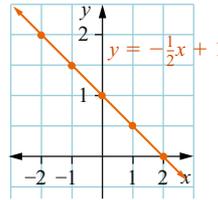
e $y = 2x + 2$

x	-2	-1	0	1	2
y	-2	0	2	4	6



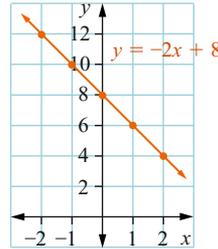
f $y = -\frac{1}{2}x + 1$

x	-2	-1	0	1	2
y	2	$1\frac{1}{2}$	1	$\frac{1}{2}$	0



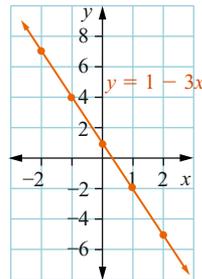
g $y = -2x + 8$

x	-2	-1	0	1	2
y	12	10	8	6	4



h $y = 1 - 3x$

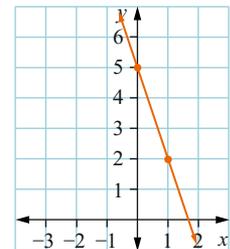
x	-2	-1	0	1	2
y	7	4	1	-2	-5



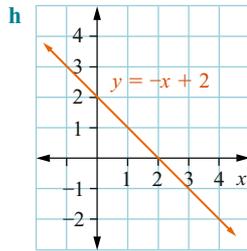
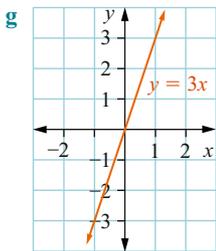
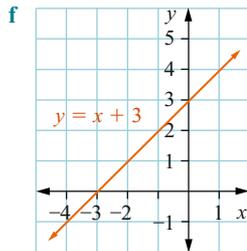
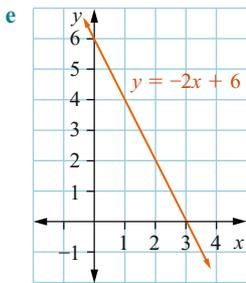
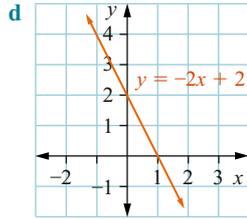
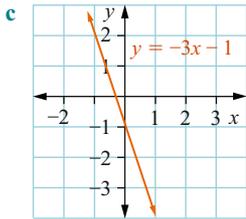
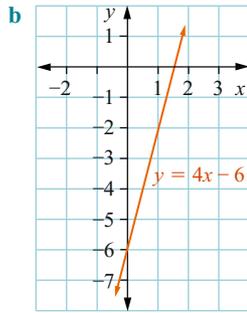
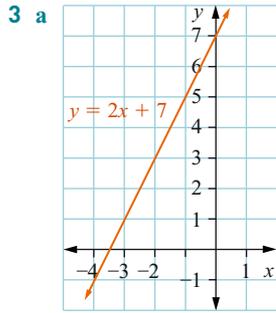
- 2 a** $y = x + 1, y = x - 1, y = 3x - 1, y = 2x + 2$
b $y = -x - 2, y = -\frac{1}{2}x + 1, y = -2x + 8, y = 1 - 3x$
c If the coefficient of x is positive then the gradient is positive, and if the coefficient is negative so too is the gradient.
d **i** Positive **ii** Negative **iii** Negative
iv Positive **v** Positive **vi** Negative
3 a C **b** A **c** D **d** E **e** B

Exercise 4C

- 1 a** Gradient = $m = -3$
 y -intercept = $b = 5$
 The equation is $y = mx + b$.
 $\therefore y = -3x + 5$



- b** Plot the point (0, 5). As the gradient is negative, the line slopes downhill with a rise of -3 and a run of 1.
2 a $y = 2x + 7$ **b** $y = 4x - 6$
c $y = -3x - 1$ **d** $y = -2x + 2$
e $y = -2x + 6$ **f** $y = x + 3$
g $y = 3x$ **h** $y = -x + 2$



4 a $y = -\frac{3}{4}x + 1$

c $y = \frac{2}{3}x - 1$

e $y = -\frac{1}{3}x + 4$

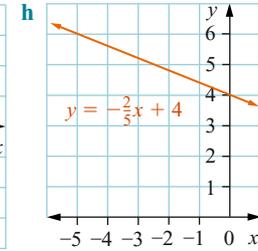
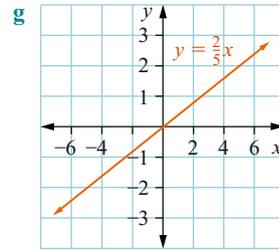
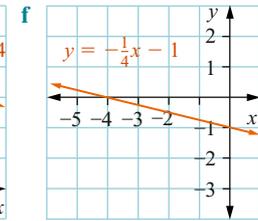
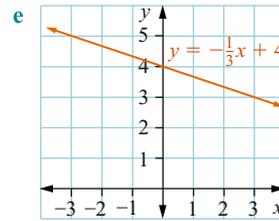
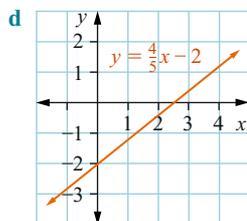
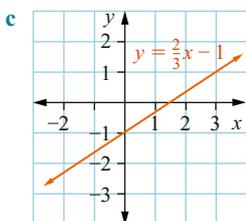
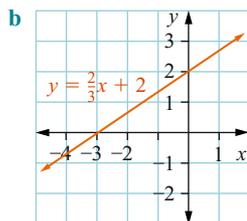
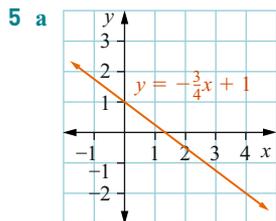
g $y = \frac{2}{5}x$

b $y = \frac{2}{3}x + 2$

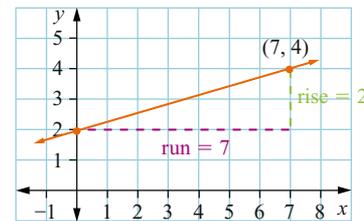
d $y = \frac{4}{5}x - 2$

f $y = -\frac{1}{4}x - 1$

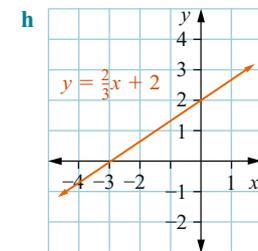
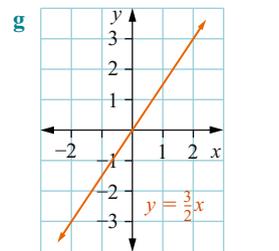
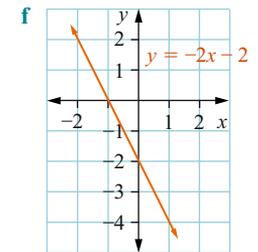
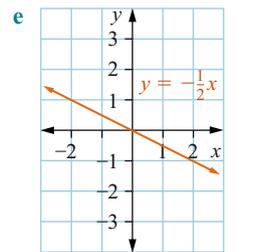
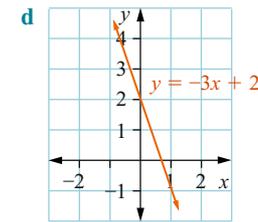
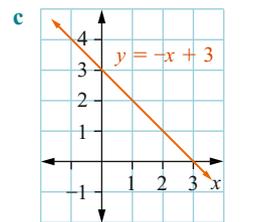
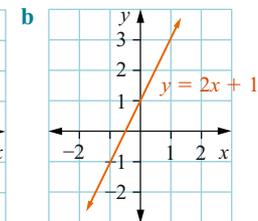
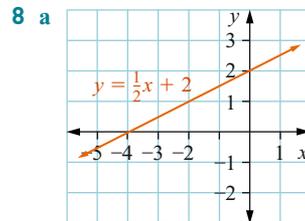
h $y = -\frac{2}{5}x + 4$

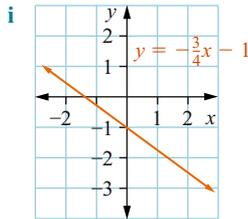


6 Gradient = $\frac{\text{rise}}{\text{run}} = \frac{2}{7}$
Gradient is uphill so it is positive.



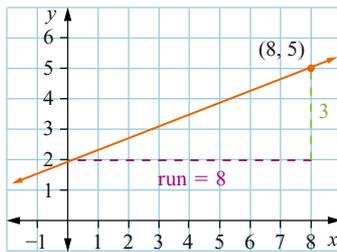
7 a 1 **b** $\frac{4}{3}$ **c** $-\frac{2}{3}$ **d** $\frac{1}{2}$ **e** -2 **f** $-\frac{1}{4}$



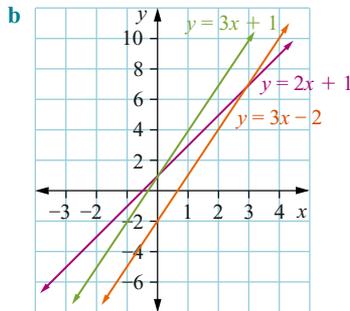


- 9 a** $m = \frac{1}{2}, b = 2$ **b** $m = 2, b = 1$
c $m = -1, b = 3$ **d** $m = -3, b = 2$
e $m = -\frac{1}{2}, b = 0$ **f** $m = -2, b = -2$
g $m = \frac{3}{2}, b = 0$ **h** $m = \frac{2}{3}, b = 2$
i $m = -\frac{3}{4}, b = -1$

- 10** Gradient = $\frac{\text{rise}}{\text{run}} = \frac{3}{8}$
 y -intercept is 2.
 The equation is $y = \frac{3}{8}x + 2$.



- 11 a** $y = x + 1$ **b** $y = \frac{4}{3}x - 1$
c $y = -\frac{2}{3}x + 2$ **d** $y = \frac{1}{2}x + 4$
e $y = -2x - 2$ **f** $y = -\frac{1}{4}x - 2$
- 12 a** $y = 3x + 1, y = 3x - 2$; they have the same gradient



$y = 3x + 1$ and $y = 2x + 1$ have the same y -intercept

- c i** $y = 2x + 1$, parallel as it has the same gradient
ii $y = 3x - 2$, as it has the same y -intercept
- 13** $4x - 5y - 20 = 0$
 $4x - 5y + 5y - 20 = 0 + 5y$
 $4x - 20 = 5y$
 $5y = 4x - 20$
 $y = \frac{4}{5}x - 4$
 Gradient is $m = \frac{4}{5}$.
 y -intercept is -4 .
- 14 a** $y = -\frac{1}{2}x + 2, m = -\frac{1}{2}, b = 2$
b $y = -\frac{3}{2}x + 12, m = -\frac{3}{2}, b = 12$
c $y = 2x + 4, m = 2, b = 4$
d $y = 2x - 3, m = 2, b = -3$
e $y = -\frac{5}{2}x - 5, m = -\frac{5}{2}, b = -5$
f $y = -\frac{3}{2}x + 4, m = -\frac{3}{2}, b = 4$

- g** $y = 4x - 6, m = 4, b = -6$
h $y = \frac{3}{2}x + 8\frac{1}{2}, m = \frac{3}{2}, b = 8\frac{1}{2}$
i $y = 4x - 3\frac{1}{2}, m = 4, b = -3\frac{1}{2}$

Exercise 4D

- 1** Gradient $m = 5$
 $(x_1, y_1) = (-3, 4)$
 $y - y_1 = m(x - x_1)$
 $y - 4 = 5(x - (-3))$
 $y - 4 = 5x + 15$
 $y = 5x + 19$
- 2 a** $y = 3x - 9$ **b** $y = -2x - 6$
c $y = -\frac{1}{2}x + 7\frac{1}{2}$ **d** $y = 5x - 15$
e $y = \frac{1}{3}x + 6$ **f** $y = -\frac{1}{2}x - 4\frac{1}{2}$
g $y = -\frac{3}{2}x + 8\frac{1}{2}$ **h** $y = 6x$
i $y = 4x + 3\frac{1}{2}$ **j** $y = 4$
- 3 a** $y = 2x - 10$ **b** $y = x - 7$
c $y = -3x + 22$ **d** $y = -2x - 12$
e $y = 5x - 15$ **f** $y = \frac{1}{2}x + 5$
g $y = -\frac{1}{3}x + 5$ **h** $y = 8x$
i $y = \frac{1}{3}x + 6$ **j** $y = 2$
- 4** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{7 - 3} = \frac{7}{4}$
 Use $y - y_1 = m(x - x_1)$
 $y - (-2) = \frac{7}{4}(x - 3)$
 $y + 2 = \frac{7}{4}x - \frac{21}{4}$
 $y = \frac{7}{4}x - \frac{21}{4} - \frac{8}{4}$
 $y = \frac{7}{4}x - \frac{29}{4}$
- 5 a** $y = 2x - 1$ **b** $y = -x + 2$
c $y = -\frac{1}{3}x - 3\frac{1}{3}$ **d** $y = x - 3$
e $y = x - 7$ **f** $y = \frac{5}{3}x$
g $y = -\frac{7}{4}x - \frac{1}{4}$ **h** $y = 2x + 4$
i $y = \frac{3}{5}x + \frac{4}{5}$ **j** $y = \frac{3}{2}x + 6$
- 6 a** $y = 2x - 4$ **b** $y = \frac{7}{2}x + 7$
c $y = -\frac{5}{3}x - 5$ **d** $y = \frac{3}{2}x - 3$

Exercise 4E

- 1** $y = 3x - 7$
 $0 = 3x - y - 7$
 $3x - y - 7 = 0$
- 2 a** $2x - y + 1 = 0$ **b** $5x - y - 2 = 0$
c $2x - y + 5 = 0$ **d** $2x + y + 5 = 0$
e $3x + y - 4 = 0$ **f** $x - 2y + 4 = 0$
g $x + 2y + 10 = 0$ **h** $2x + 3y + 9 = 0$
i $9x + 12y + 8 = 0$ **j** $3x - 8y + 12 = 0$
k $2x - 4y - 3 = 0$ **l** $10x - 2y + 5 = 0$
- 3** $2x - 5y + 6 = 0$
 $2x - 5y + 5y + 6 = 5y$
 $5y = 2x + 6$
 $y = \frac{2}{5}x + \frac{6}{5}$
 Gradient = $m = \frac{2}{5}$
 y -intercept = $b = \frac{6}{5}$

4 a $y = -\frac{1}{2}x + 2$, $m = -\frac{1}{2}$, $b = 2$

b $y = -\frac{3}{2}x + 12$, $m = -\frac{3}{2}$, $b = 12$

c $y = 2x + 4$, $m = 2$, $b = 4$

d $y = 2x - 3$, $m = 2$, $b = -3$

e $y = -\frac{5}{2}x - 5$, $m = -\frac{5}{2}$, $b = -5$

f $y = -\frac{3}{2}x + 4$, $m = -\frac{3}{2}$, $b = 4$

g $y = 4x - 6$, $m = 4$, $b = -6$

h $y = \frac{3}{2}x + 8\frac{1}{2}$, $m = \frac{3}{2}$, $b = 8\frac{1}{2}$

i $y = 4x - 3\frac{1}{2}$, $m = 4$, $b = -3\frac{1}{2}$

5 a -3 b $-\frac{9}{4}$ c $-\frac{7}{5}$ d $\frac{5}{4}$

6 x -intercept is when $y = 0$.

$$5x - 7(0) + 35 = 0$$

$$5x + 35 = 0$$

$$5x = -35$$

$$x = -7 \quad \text{Divide both sides by 5.}$$

The x -intercept is -7 .

y -intercept is when $x = 0$.

$$5(0) - 7y + 35 = 0$$

$$-7y + 35 = 0$$

$$-7y = -35$$

$$y = 5 \quad \text{Divide both sides by 7.}$$

The y -intercept is 5 .

7 a $8, 4$ b $4, 6$ c $-3, 6$

d $12, -18$ e $-4, -10$ f $\frac{5}{3}, \frac{5}{2}$

g $\frac{5}{4}, -5$ h $-5, 7\frac{1}{2}$ i $\frac{5}{9}, -2\frac{1}{2}$

j $\frac{8}{5}, \frac{2}{5}$

8 Find the x - and y -intercepts.

When $x = 0$

$$6(0) - 5y + 30 = 0$$

$$-5y = -30 \quad \text{Subtract 30 from both sides.}$$

$$y = 6$$

When $y = 0$

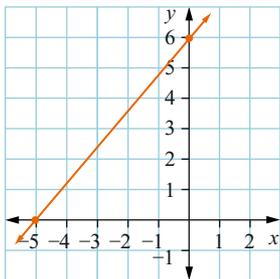
$$6x - 5(0) + 30 = 0$$

$$6x + 30 = 0 \quad \text{Subtract 30 from both sides.}$$

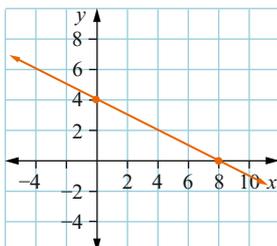
$$6x = -30$$

$$x = -5$$

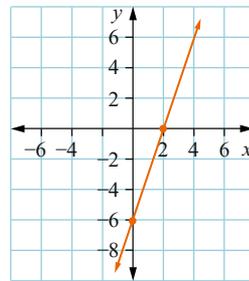
Plot x - and y -intercepts: $(-5, 0)$ and $(0, 6)$.



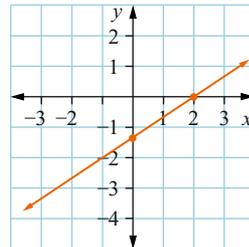
9 a $x + 2y - 8 = 0$



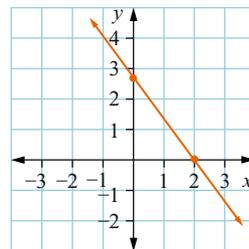
b $3x - y - 6 = 0$



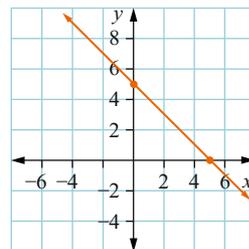
c $2x - 3y - 4 = 0$



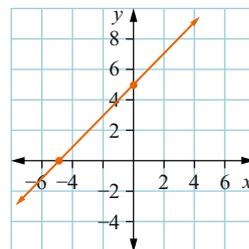
d $4x + 3y - 8 = 0$



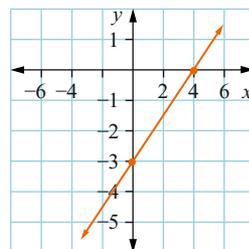
e $x + y = 5$



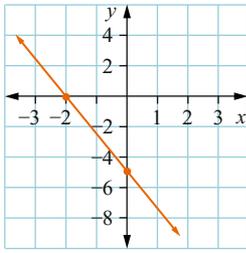
f $x - y + 5 = 0$



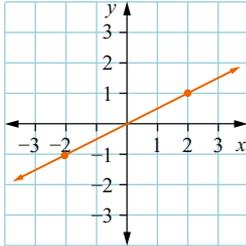
g $3x - 4y - 12 = 0$



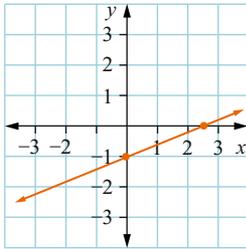
h $5x + 2y + 10 = 0$



i $x - 2y = 0$



j $2x - 5y - 5 = 0$



Exercise 4F

- A, B, E, H
- As the lines are parallel the gradient is 3.
So the equation is $y = 3x + b$.
Substitute $(-4, 3)$ into the equation.
 $3 = 3(-4) + b$
 $3 = -12 + b$ Add 12 to both sides.
 $15 = b$
The equation is $y = 3x + 15$.
- $y = 2x + 2$
 - $y = -7x - 37$
 - $y = -\frac{1}{2}x$
 - $5x - 7y - 31 = 0$
 - $y = \frac{1}{2}x - 2\frac{1}{2}$
 - $y = -\frac{5}{7}x - 2$
- $y = -2x + 5$ has gradient $m_1 = -2$.
Rearrange $x - 2y - 4 = 0$
 $x - 2y + 2y - 4 = 2y$
 $x - 4 = 2y$
 $\frac{1}{2}x - 2 = y$
Gradient $m_2 = \frac{1}{2}$
 $m_1 \times m_2 = -2 \times \frac{1}{2} = -1$
- B, D, E
- $\frac{5}{3}$
 - $-\frac{1}{7}$
 - $-\frac{2}{3}$
 - $\frac{5}{4}$
 - $-\frac{2}{3}$
 - $-\frac{7}{5}$
- $y = -\frac{1}{5}x + \frac{7}{5}$
 - $y = 4x + 5$
 - $y = -\frac{4}{3}x$
 - $4x + 3y - 7 = 0$
 - $2x - 5y + 35 = 0$
 - $y = \frac{1}{2}x - 4$
 - $y = -\frac{3}{5}x$

- $y = -2x + c$
 - $y = 3x + c$
 - $3x - 5y + c = 0$
 - $3x + 5y + c = 0$
- $4x + 3y + c = 0$
 - Coefficients of x and y are interchanged and the sign of y is changed.
- $2x - 7y + c = 0$
- Same coefficients of x and y .
- $7x - 5y + c = 0$
 - $5x + 7y + c = 0$

Exercise 4G

- $d_{PQ} = \sqrt{5}$, $d_{QR} = \sqrt{18}$, $d_{PR} = \sqrt{17}$
 - Scalene as all sides are different lengths.
- $d_{LM} = \sqrt{20}$, $d_{MN} = \sqrt{20}$, $d_{LN} = \sqrt{40}$, $\triangle LMN$ is isosceles.
- 5.7
- $y = 3x - 5$
 - $y = -2x + 7$
- $(1, 1)$
 - $(0, 2)$
- $(1, -3)$
- $d_{XY} = \sqrt{10}$, $d_{YZ} = \sqrt{20}$, $d_{XZ} = \sqrt{10}$
 - $\triangle XYZ$ is right angled since $XY^2 + XZ^2 = YZ^2$
 - $m_{XY} = 3$, $m_{YZ} = -2$, $m_{XZ} = -\frac{1}{3}$
 - $m_{XY} \times m_{XZ} = -1$
 - Yes, as $m_{XY} \times m_{XZ} = -1$
- Yes, DE and DF are perpendicular.
- $S = (\frac{1}{2}, 2)$
 - $T = (-3, -1)$
- $d_{QR} = \sqrt{85}$
 - $d_{ST} = \sqrt{\frac{85}{4}} = \frac{1}{2}\sqrt{85}$, \therefore true
- $d_{PQ} = d_{RS} = \sqrt{18}$, $d_{QR} = d_{PS} = \sqrt{45}$
 - Midpoint $PR = (\frac{1}{2}, 1)$
Midpoint $QS = (\frac{1}{2}, 1)$, \therefore diagonals bisect
 - $m_{PQ} = -1$, $m_{QR} = \frac{1}{2}$, $m_{RS} = -1$, $m_{PS} = \frac{1}{2}$
 - Sides are parallel and non-perpendicular so it is not a rectangle.
 - No

Language in mathematics

- Gradient is rise over run.
 - b is the symbol for y -intercept.
 - m is the symbol for gradient.
 - Midpoint is the average of the ordinates.
 - The distance formula is related to Pythagoras' theorem.
 - $y = mx + b$ is the gradient-intercept form of a straight line.
- equation
 - formula
 - gradient
 - parallel
 - vertical rise
 - horizontal run
- Two lines are perpendicular if the product of their gradients is negative one and they are parallel if their gradients are equal.

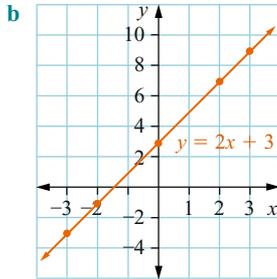
Check your skills

- | | | | | |
|------|------|------|------|------|
| 1 B | 2 D | 3 B | 4 A | 5 A |
| 6 B | 7 C | 8 D | 9 C | 10 A |
| 11 B | 12 C | 13 B | 14 A | 15 D |
| 16 B | 17 C | 18 C | 19 A | 20 A |
| 21 B | 22 B | 23 D | 24 B | 25 C |
| 26 D | | | | |

Review set 4A

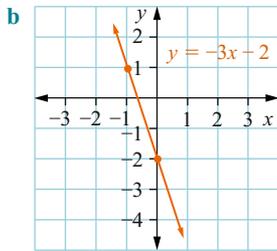
- 1 a $\sqrt{65}$ b $(-1, -\frac{1}{2})$ c $-\frac{1}{8}$
 2 $(-8, 12)$ 3 $(-10, 14)$
 4 a $y = 2x + 3$

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

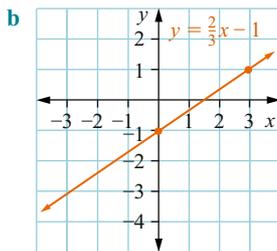


c Yes

- 5 a $y = -3x - 2$

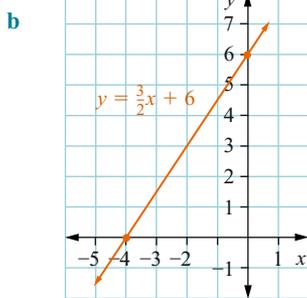


- 6 a $y = \frac{2x}{3} - 1$



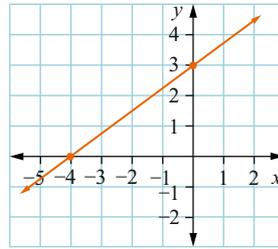
- 7 a $m = -1$ b 2
 c $y = -x + 2$

- 8 a $y = \frac{3x}{2} + 6$



- 9 a $-\frac{1}{8}$ b $y = -\frac{1}{8}x - \frac{5}{8}$
 c x-intercept = -5 , y-intercept = $-\frac{5}{8}$
 d $y = 8x - 25$ e $y = -\frac{1}{8}x + \frac{3}{8}$

- 10 a $4x - 3y - 7 = 0$ b $x + 6y - 38 = 0$
 c $3x - 4y + 12 = 0$

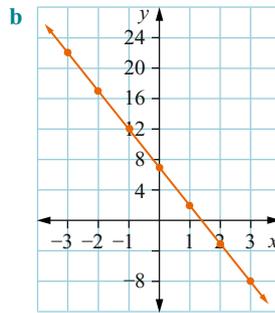


- 11 a $\frac{2}{3}$ b $t = -13$

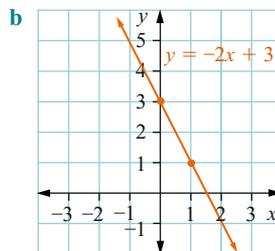
Review set 4B

- 1 a $\sqrt{65}$ b $(-3\frac{1}{2}, 2)$ c $-\frac{4}{7}$
 2 $d_{AB} = \sqrt{20}$, $d_{BC} = \sqrt{20}$, $d_{AC} = \sqrt{8}$
 Isosceles since $d_{AB} = d_{BC}$
 3 a $y = -5x + 7$

x	-3	-2	-1	0	1	2	3
y	22	17	12	7	2	-3	-8



- 4 a $y = -2x + 3$

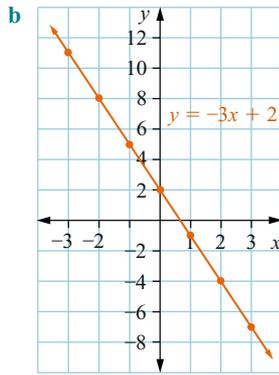


- 5 $y = \frac{6x}{5} + 3$
 6 $5x - 7y + 34 = 0$
 7 $3x + 2y + 2 = 0$
 8 $2x + 3y + 12 = 0$
 9 $y = -3x + 7$

Review set 4C

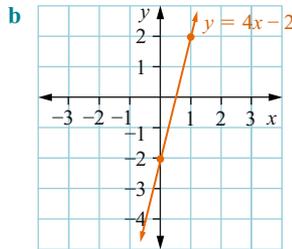
- 1 a $\sqrt{89}$ b $(-3\frac{1}{2}, -1)$ c $-\frac{8}{5}$
 2 $(6, -7)$
 3 $(2, -3)$
 4 a $y = -3x + 2$

x	-3	-2	-1	0	1	2	3
y	11	8	5	2	-1	-4	-7

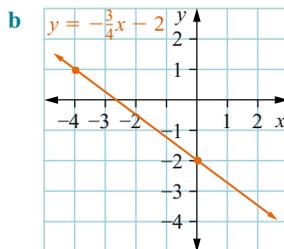


c Yes

5 a $y = 4x - 2$



6 a $y = -\frac{3}{4}x - 2$

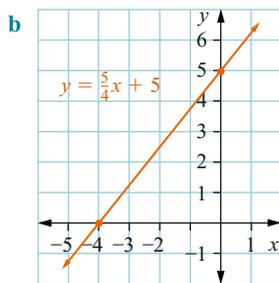


7 a $m = \frac{7}{5}$

b -2

c $y = \frac{7}{5}x - 2$

8 a $y = \frac{5}{4}x + 5$



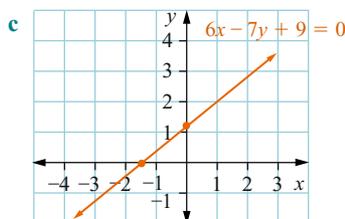
9 a $-\frac{8}{5}$

b $y = -\frac{8}{5}x - 6\frac{3}{5}; 8x + 5y + 33 = 0$

c x -intercept $-4\frac{1}{8}$, y -intercept $-6\frac{3}{5}$

d $5x - 8y - 35 = 0$ **e** $y = -\frac{8}{5}x$

10 a $y = \frac{4}{7}x + \frac{8}{7}$ **b** $x + 2y = 0$



Review set 4D

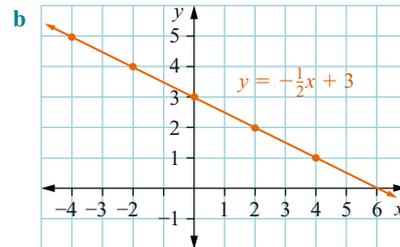
1 a $\sqrt{26}$ **b** $(-\frac{1}{2}, \frac{9}{2})$ **c** $-\frac{1}{5}$

2 a $d_{LM} = \sqrt{10}$, $d_{MN} = \sqrt{20}$, $d_{LN} = \sqrt{10}$

b $(\sqrt{10})^2 + (\sqrt{10})^2 = (\sqrt{20})^2$, \therefore right-angled

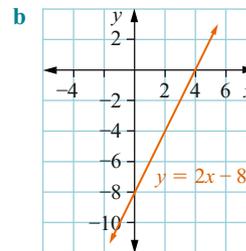
3 a $y = -\frac{1}{2}x + 3$

x	-4	-2	0	2	4
y	5	4	3	2	1



c Gradient $= -\frac{1}{2}$, y -intercept $= 3$

4 a $y = 2x - 8$



5 $y = -\frac{11}{5}x - 3$

6 a $y = 3x + 10$

b $3x - 5y = 0$

7 $3x + 4y - 8 = 0$

8 No, as $3(1) - 5(4) + 2 \neq 0$

CHAPTER 5 PROBABILITY

Diagnostic test

1 D **2** A **3** C **4** B **5** D

6 B **7** B **8** D **9** A **10** C

11 C **12** C

Exercise 5A

1 a up 31%, down 69%

b i $\approx 31\%$

ii $\approx 69\%$

2 a 29% **b** 56.5%

c 14.5% **d** 85.5%

3 a $\approx 51\%$

b $\approx 49\%$

4 a 23% **b** 17.8%

c 38.2% **d** 56%

5 a $\frac{43}{150}$ **b** $\frac{107}{150}$

c $\frac{35}{150} = \frac{7}{30}$ **d** $\frac{25}{150} = \frac{1}{6}$

e $\frac{125}{150} = \frac{5}{6}$

f $\frac{17}{150}$

g $\frac{90}{150} = \frac{3}{5}$

h $\frac{8}{150} = \frac{4}{75}$

i $\frac{60}{150} = \frac{2}{5}$

j $\frac{52}{150} = \frac{26}{75}$

6 a $\frac{12}{30} = \frac{2}{5}$

b $\frac{18}{30} = \frac{3}{5}$

c $\frac{9}{30} = \frac{3}{10}$

d $\frac{11}{30}$

e $\frac{19}{30}$

f $\frac{8}{30} = \frac{4}{15}$

g $\frac{10}{30} = \frac{1}{3}$

h $\frac{3}{30} = \frac{1}{10}$

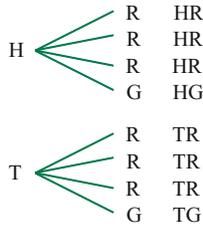
i $\frac{20}{30} = \frac{2}{3}$

j $\frac{17}{30}$

- 7 a $\frac{30}{60} = \frac{1}{2}$ b $\frac{8}{60} = \frac{2}{15}$ c $\frac{4}{60} = \frac{1}{15}$ d $\frac{11}{60}$
 e $\frac{6}{60} = \frac{1}{10}$ f $\frac{46}{60} = \frac{23}{30}$ g $\frac{31}{60}$
 8 a $\frac{16}{30} = \frac{8}{15}$ b $\frac{14}{30} = \frac{7}{15}$ c $\frac{14}{30} = \frac{7}{15}$ d $\frac{16}{30} = \frac{8}{15}$
 e $\frac{9}{30} = \frac{3}{10}$ f $\frac{5}{30} = \frac{1}{6}$ g $\frac{7}{30}$ h $\frac{9}{30} = \frac{3}{10}$
 i $\frac{25}{30} = \frac{5}{6}$ j $\frac{18}{30} = \frac{3}{5}$ k $\frac{23}{30}$ l $\frac{18}{30} = \frac{3}{5}$
 m $\frac{7}{30}$ n $\frac{5}{30} = \frac{1}{6}$ o $\frac{9}{30} = \frac{3}{10}$ p $\frac{9}{30} = \frac{3}{10}$
 9 a $\frac{9}{100}$ b $\frac{30}{100} = \frac{3}{10}$ c $\frac{57}{100}$ d $\frac{39}{100}$
 e $\frac{18}{100} = \frac{9}{50}$ f $\frac{43}{100}$ g $\frac{70}{100} = \frac{7}{10}$ h $\frac{61}{100}$
 i $\frac{30}{100} = \frac{3}{10}$
 10 a $\frac{16}{28} = \frac{4}{7}$ b $\frac{12}{28} = \frac{3}{7}$ c $\frac{21}{28} = \frac{3}{4}$ d $\frac{7}{28} = \frac{1}{4}$
 e $\frac{4}{28} = \frac{1}{7}$ f $\frac{5}{28}$

Exercise 5B

- 1 a R for red disc
 G for green disc
 H for heads
 T for tails
 Sample space
 = {HR, HR, HR, HG,
 TR, TR, TR, TG}
- b Total number of outcomes
 = 4 + 4 = 8
 $P(\text{TR}) = \frac{3}{8}$



- 2 a H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6
 b i $\frac{1}{12}$ ii $\frac{1}{4}$ iii $\frac{1}{12}$ iv $\frac{1}{4}$

3 a

H	R	R	B
T	R	R	B

- b i $\frac{1}{3}$ ii $\frac{1}{6}$ iii $\frac{1}{3}$

4 a

H	1	2	3	4
T	1	2	3	4

- b i $\frac{1}{8}$ ii $\frac{1}{8}$ iii $\frac{1}{4}$

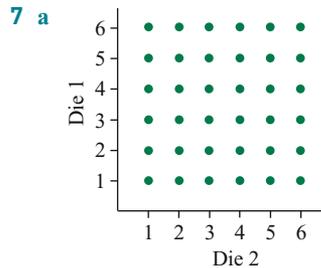
5 a

G	1	2	3	4	5	6
G	1	2	3	4	5	6
B	1	2	3	4	5	6

- b i $\frac{1}{9}$ ii $\frac{1}{18}$ iii $\frac{1}{3}$ iv $\frac{1}{6}$

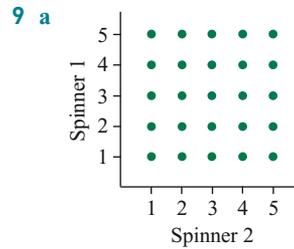
- 6 a BB, BG, GB, GG

- b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{2}$ iv $\frac{3}{4}$



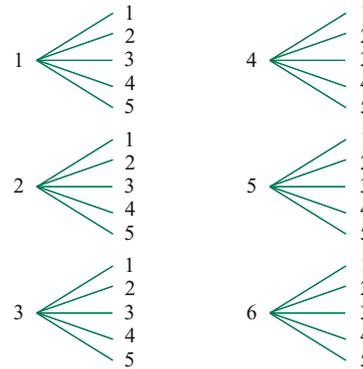
- b i Total outcomes = 36
 Number of favourable outcomes = 2
 $P(5 \text{ and } 1) = \frac{2}{36} = \frac{1}{18}$
 ii Total outcomes = 36
 Number of favourable outcomes = 6
 $P(\text{double}) = \frac{6}{36} = \frac{1}{6}$

- 8 a $\frac{1}{18}$ b $\frac{1}{18}$ c $\frac{1}{36}$ d $\frac{11}{36}$ e $\frac{1}{6}$



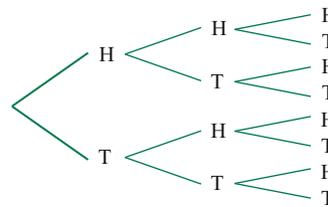
- b i $\frac{2}{25}$ ii $\frac{3}{25}$ iii $\frac{1}{5}$ iv $\frac{9}{25}$

- 10 a
- | | | | |
|------|---------|------|---------|
| Disc | Spinner | Disc | Spinner |
|------|---------|------|---------|



- b i $\frac{1}{30}$ ii $\frac{1}{6}$ iii $\frac{2}{15}$ iv $\frac{1}{3}$

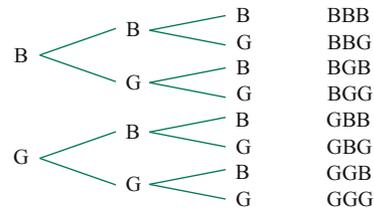
- 11 a 1st coin 2nd coin 3rd coin



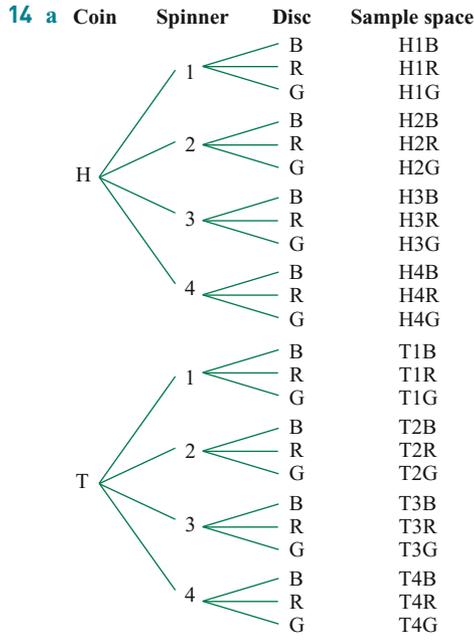
- b There are 3 branchings of the tree.
 They are THT, TTH and HTT.
 The probability is $\frac{3}{8}$.

- 12 a $\frac{1}{8}$ b $\frac{1}{8}$ c $\frac{1}{8}$ d $\frac{7}{8}$ e $\frac{1}{2}$

- 13 a 1st child 2nd child 3rd child Sample space

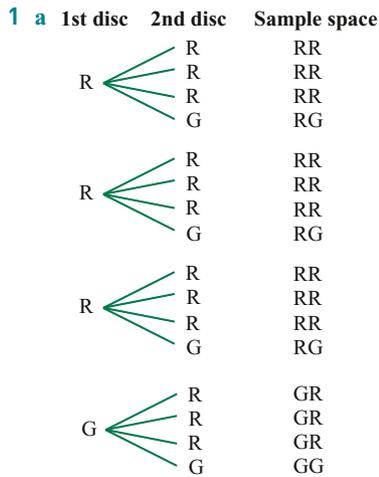


- b i $\frac{1}{8}$ ii $\frac{1}{8}$ iii $\frac{3}{8}$ iv $\frac{1}{8}$ v $\frac{7}{8}$

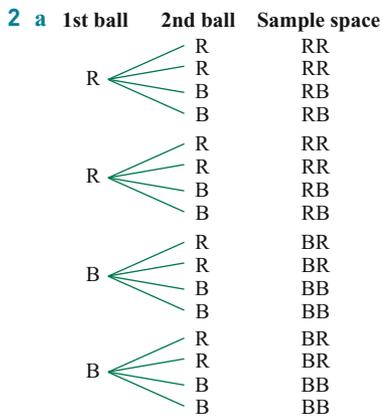


- b** i $\frac{1}{24}$ ii $\frac{1}{12}$ iii $\frac{1}{12}$ iv $\frac{1}{8}$

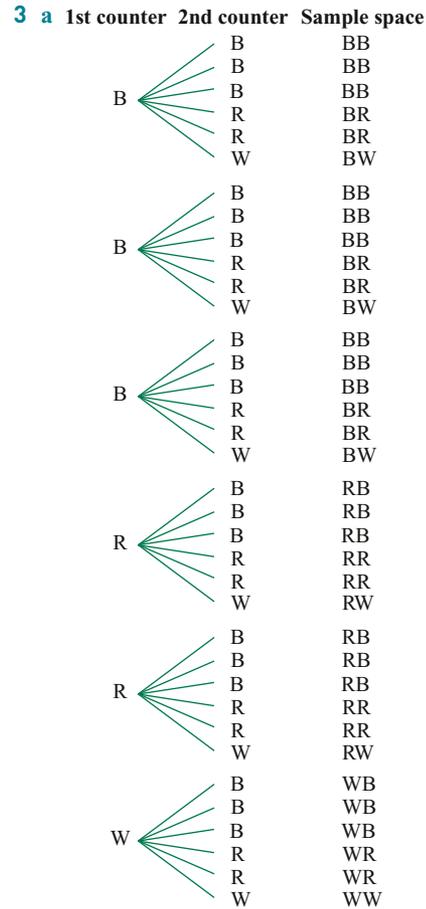
Exercise 5C



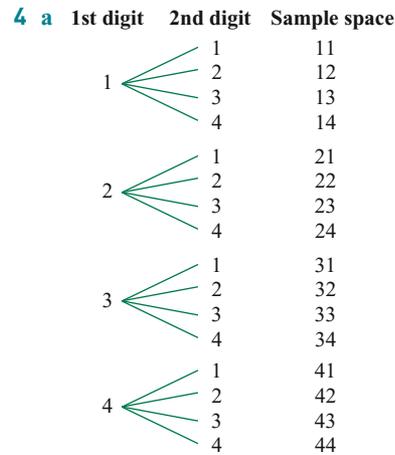
- b** Total number of possible outcomes = 16
 Number of outcomes with different colours
 = 1 + 1 + 1 + 3 = 6
 $P(\text{RG or GR}) = \frac{6}{16} = \frac{3}{8}$



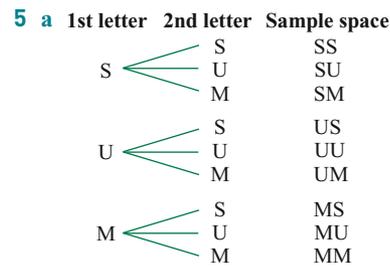
- b** i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{2}$ iv $\frac{1}{2}$



- b** i $\frac{1}{4}$ ii $\frac{1}{6}$ iii $\frac{1}{9}$ iv $\frac{11}{18}$ v $\frac{7}{18}$

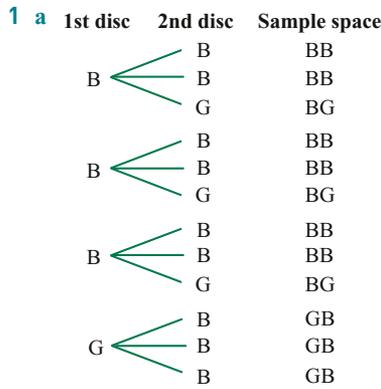


- b** i $\frac{1}{16}$ ii $\frac{1}{2}$ iii $\frac{11}{16}$ iv $\frac{7}{16}$ v $\frac{1}{4}$

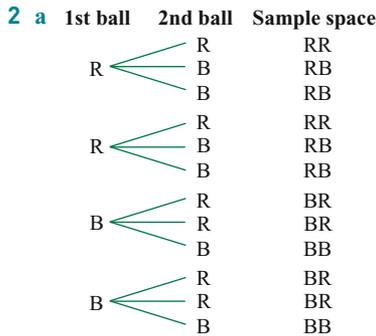


- b** i $\frac{1}{3}$ ii $\frac{4}{9}$ iii $\frac{2}{9}$ iv $\frac{5}{9}$ v $\frac{8}{9}$

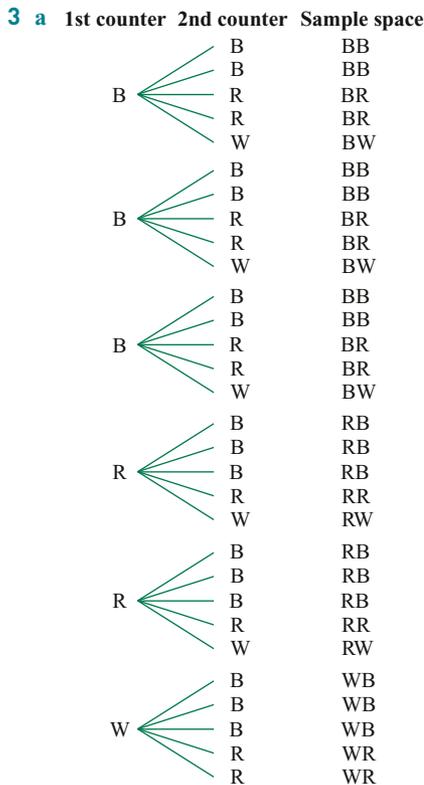
Exercise 5D



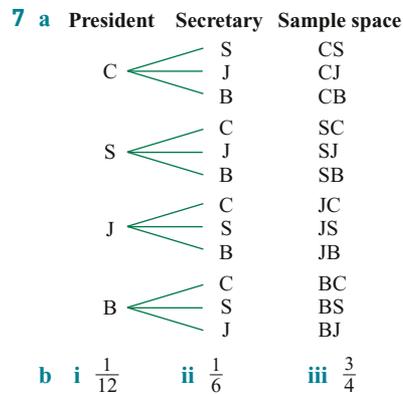
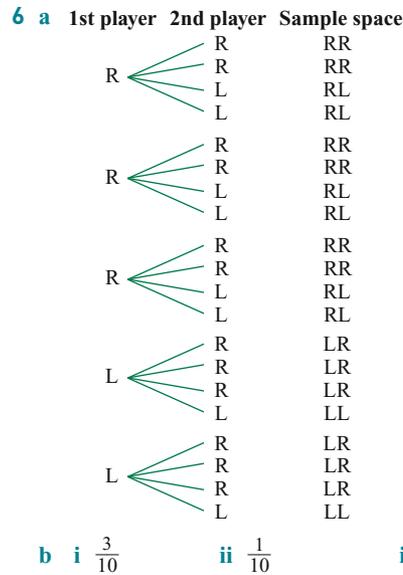
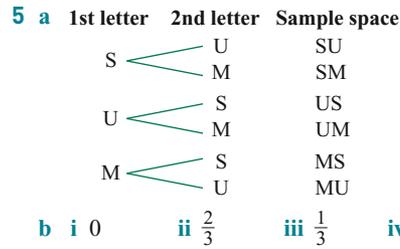
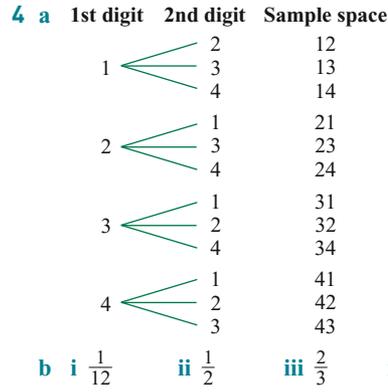
b Total number of outcomes = $3 + 3 + 3 + 3 = 12$
 Number of outcomes with different colours
 = $1 + 1 + 1 + 3 = 6$
 $P(\text{RG or GR}) = \frac{6}{12} = \frac{1}{2}$



b i $\frac{1}{6}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{1}{3}$



b i $\frac{1}{5}$ ii $\frac{1}{5}$ iii $\frac{2}{15}$ iv $\frac{11}{15}$ v $\frac{4}{15}$



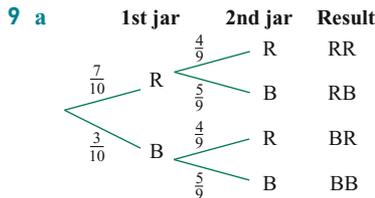
Exercise 5E

1 a $P(\text{tail}) = \frac{1}{2}$
 $P(\text{gold disc}) = \frac{2}{5}$
 $P(\text{tail and gold disc}) = \frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$

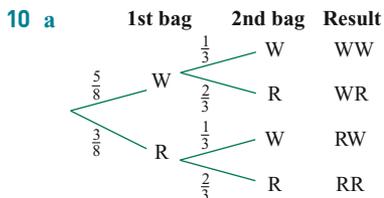
b $P(\text{head}) = \frac{1}{2}$
 $P(\text{black disc}) = \frac{3}{5}$
 $P(\text{head and black disc}) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

- 2 a** $\frac{5}{18}$ **b** $\frac{2}{9}$ **c** $\frac{5}{18}$
3 a $\frac{2}{21}$ **b** $\frac{1}{14}$ **c** $\frac{2}{7}$ **d** $\frac{3}{14}$
4 a $\frac{1}{260}$ **b** $\frac{1}{52}$ **c** $\frac{1}{52}$ **d** $\frac{1}{52}$ **e** $\frac{1}{65}$
5 a $\frac{1}{36}$ **b** $\frac{1}{36}$ **c** $\frac{1}{6}$
6 a $\frac{1}{48}$ **b** $\frac{1}{52}$ **c** $\frac{1}{156}$ **d** $\frac{1}{13}$
7 a $\frac{1}{216}$ **b** $\frac{1}{72}$ **c** $\frac{1}{216}$ **d** $\frac{1}{36}$ **e** $\frac{1}{12}$

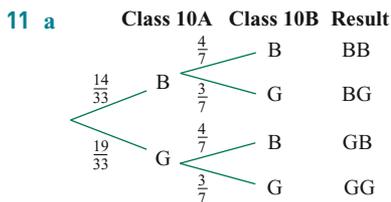
8 a $P(\text{black disc}) = \frac{4}{6} = \frac{2}{3}$
 $P(\text{green disc}) = \frac{4}{7}$
 $P(\text{black disc and green disc}) = \frac{4}{6} \times \frac{4}{7} = \frac{16}{42}$
b Part **a** had $P(\text{black disc then green disc}) = \frac{16}{42}$
 $P(\text{green disc then black disc}) = P(G) \times P(B)$
 $= \frac{2}{6} \times \frac{3}{7} = \frac{6}{42}$
 $P(\text{green disc and black disc in any order})$
 $= \frac{16}{42} + \frac{6}{42} = \frac{22}{42} = \frac{11}{21}$



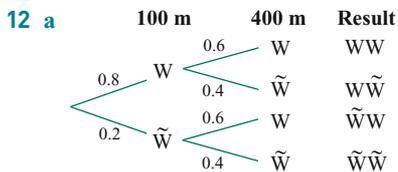
b $P(RR) = \frac{7}{10} \times \frac{4}{9} = \frac{28}{90}$
 $P(BB) = \frac{3}{10} \times \frac{5}{9} = \frac{15}{90}$
 $P(\text{same colour}) = \frac{28}{90} + \frac{15}{90} = \frac{43}{90}$



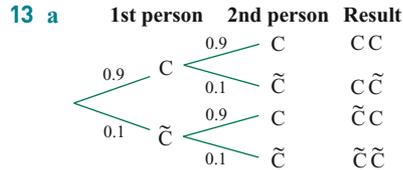
b i $\frac{5}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{5}{12}$ **iv** $\frac{13}{24}$ **v** $\frac{3}{4}$



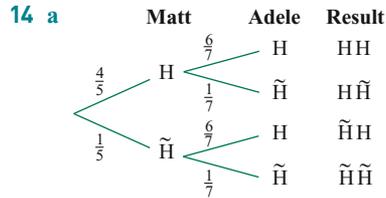
b i $\frac{8}{33}$ **ii** $\frac{19}{77}$ **iii** $\frac{118}{231}$



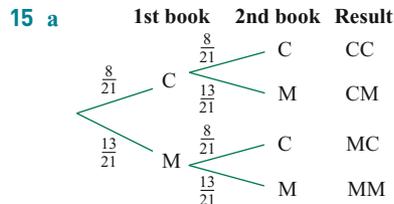
b i 0.48 **ii** 0.08 **iii** 0.32 **iv** 0.92



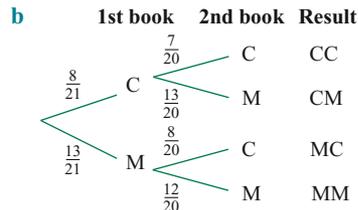
b i 0.81 **ii** 0.01 **iii** 0.18 **iv** 0.99



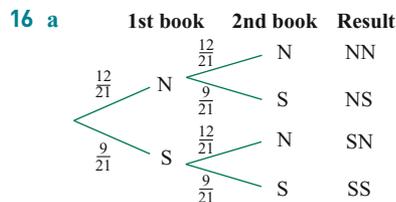
b i $\frac{24}{35}$ **ii** $\frac{1}{35}$ **iii** $\frac{2}{7}$ **iv** $\frac{34}{35}$



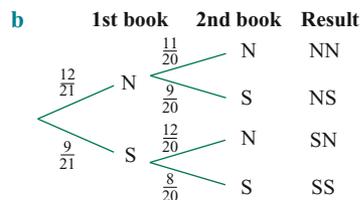
$P(CC) + P(MM) = \frac{8}{21} \times \frac{8}{21} + \frac{13}{21} \times \frac{13}{21} = \frac{233}{441}$



$P(CC) + P(MM) = \frac{8}{21} \times \frac{7}{20} + \frac{13}{21} \times \frac{12}{20} = \frac{212}{420} = \frac{53}{105}$



i $\frac{12}{49}$ **ii** $\frac{16}{49}$ **iii** $\frac{33}{49}$



i $\frac{9}{35}$ **ii** $\frac{11}{35}$ **iii** $\frac{24}{35}$

17 a i $\frac{64}{361}$ **ii** $\frac{121}{361}$ **iii** $\frac{176}{361}$ **iv** $\frac{297}{361}$

b i $\frac{28}{171}$ **ii** $\frac{55}{171}$ **iii** $\frac{88}{171}$ **iv** $\frac{107}{171}$

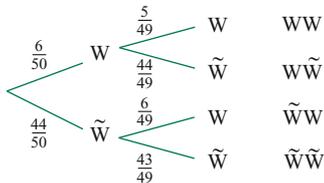
18 a i $\frac{1}{4}$ **ii** $\frac{25}{144}$ **iii** $\frac{5}{72}$ **iv** $\frac{5}{12}$ **v** $\frac{95}{144}$

b i $\frac{5}{22}$ **ii** $\frac{5}{33}$ **iii** $\frac{5}{66}$ **iv** $\frac{5}{11}$ **v** $\frac{15}{22}$

19 a i $\frac{64}{343}$ **ii** $\frac{27}{343}$ **iii** $\frac{144}{343}$ **iv** $\frac{135}{343}$

b i $\frac{4}{35}$ **ii** $\frac{1}{35}$ **iii** $\frac{18}{35}$ **iv** $\frac{13}{35}$

20 1st prize 2nd prize Result



a $P(WW) = \frac{6}{50} \times \frac{5}{49} = \frac{30}{2450} = \frac{3}{245}$
 b $P(\tilde{W}\tilde{W}) = \frac{44}{50} \times \frac{6}{49} = \frac{264}{2450} = \frac{132}{1225}$
 c $P(\text{win a prize}) = 1 - P(\tilde{W}\tilde{W})$
 $= 1 - \left(\frac{44}{50} \times \frac{43}{49}\right) = \frac{558}{2450} = \frac{279}{1225}$

- 21 a $\frac{1}{825}$ b $\frac{32}{825}$ c $\frac{13}{165}$
 22 a $\frac{3}{190}$ b $\frac{51}{380}$ c $\frac{27}{95}$
 23 a $\frac{1}{1140}$ b $\frac{34}{285}$ c $\frac{23}{57}$

Exercise 5F

- 1 True. The sex of the 5th child is independent of the previous 4 births.
 2 No. There is still an even chance of heads or tails on the 6th toss because all the tosses are independent.
 3 The number of even outcomes = 3

$P(6) = \frac{1}{3}$

- 4 $\frac{1}{3}$ 5 $\frac{1}{4}$
 6 a $\frac{1}{13}$ b $\frac{1}{13}$ c $\frac{1}{4}$ d $\frac{1}{2}$
 7 $\frac{1}{3}$ 8 $\frac{1}{3}$ 9 $\frac{1}{3}$
 10 a $\frac{14}{23}$ b $\frac{9}{51} = \frac{3}{17}$ c $\frac{1}{2}$
 11 a i $\frac{1}{2}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$
 b i $\frac{2}{5}$ ii $\frac{2}{5}$ iii $\frac{3}{5}$
 12 a i $\frac{3}{7}$ ii $\frac{3}{7}$ iii $\frac{1}{7}$ iv $\frac{4}{7}$
 b i $\frac{1}{3}$ ii $\frac{1}{2}$ iii $\frac{1}{6}$ iv $\frac{2}{3}$
 13 a HH, HT, TH, TT b $\frac{2}{3}$
 14 a $\frac{1}{4}$ b $\frac{1}{3}$
 15 $\frac{2}{3}$

Language in mathematics

- 1 a 80% b 45% c 12% d 1%
 2 It is less than 0.5%, so the number has been rounded to the nearest whole number.
 3 a 870 b 690 c 490 d 290
 4 $\frac{261}{2000} \approx 13\%$ 5 $0.33 \approx 33\%$

Check your skills

- 1 B 2 D 3 A 4 D 5 D
 6 C 7 A 8 D 9 B 10 A
 11 B

Review set 5A

- 1 a $\frac{60}{100} = \frac{3}{5}$ b $\frac{3}{100}$
 2 a $\frac{1}{10}$ b $\frac{7}{15}$ c $\frac{17}{30}$ d $\frac{13}{30}$ e $\frac{4}{15}$

- 3 a $\frac{24}{29}$ b $\frac{17}{29}$ c $\frac{7}{29}$ d $\frac{5}{29}$ e $\frac{6}{29}$
 4 a $\frac{1}{15}$ b $\frac{3}{10}$ c $\frac{1}{15}$
 5 a $\frac{1}{8}$ b $\frac{1}{8}$ c $\frac{3}{8}$ d $\frac{7}{8}$
 6 a i $\frac{49}{169}$ ii $\frac{16}{169}$ iii $\frac{16}{169}$ iv $\frac{56}{169}$ v $\frac{88}{169}$
 b i $\frac{7}{26}$ ii $\frac{1}{13}$ iii $\frac{4}{39}$ iv $\frac{14}{39}$ v $\frac{7}{13}$
 7 a $\frac{3}{1225}$ b $\frac{141}{2450}$ c $\frac{1222}{1225}$
 8 $\frac{1}{5}$

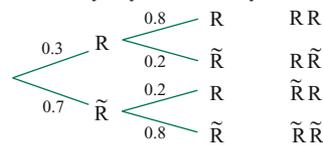
Review set 5B

- 1 a Group 1: $\frac{6}{10} = \frac{3}{5}$, Group 2: $\frac{17}{30}$, Group 3: $\frac{55}{100} = \frac{11}{20}$
 b Group 3
 2 a $\frac{3}{25}$ b $\frac{14}{25}$ c $\frac{17}{25}$ d $\frac{8}{25}$ e $\frac{1}{5}$
 3 a 0.27 b 0.17 c 0.42 d 0.69
 e 0.59 f 0.31 g 0.1
 4 a 0.36 b 0.16 c 0.48 d 0.84
 5 a 0.64 b 0.32 c 0.96
 6 a i $\frac{1}{16}$ ii $\frac{1}{2}$ iii $\frac{9}{16}$ iv $\frac{7}{16}$ v $\frac{1}{4}$
 b i $\frac{1}{12}$ ii $\frac{1}{2}$ iii $\frac{7}{12}$ iv $\frac{1}{2}$ v $\frac{1}{4}$
 7 a $\frac{1}{285}$ b $\frac{8}{57}$ c $\frac{29}{57}$
 8 a i $\frac{1}{2}$ ii $\frac{3}{8}$ iii $\frac{1}{2}$
 b i $\frac{3}{7}$ ii $\frac{3}{7}$ iii $\frac{4}{7}$

Review set 5C

- 1 a $\frac{49}{150}$ b $\frac{101}{150}$
 2 a $\frac{3}{29}$ b $\frac{21}{29}$ c $\frac{24}{29}$ d $\frac{5}{29}$ e $\frac{11}{29}$
 3 a $\frac{13}{15}$ b $\frac{4}{5}$ c $\frac{29}{30}$ d $\frac{1}{6}$
 e $\frac{1}{30}$ f $\frac{1}{15}$ g $\frac{1}{10}$
 4 a $\frac{15}{58}$ b $\frac{195}{812}$ c $\frac{407}{812}$

5 a Any day Next day Result



- b i 0.24 ii 0.06 iii 0.14
 iv 0.2 v 0.56 vi 0.44
 6 a i $\frac{35}{144}$ ii $\frac{49}{144}$ iii $\frac{95}{144}$
 b i $\frac{35}{132}$ ii $\frac{7}{22}$ iii $\frac{15}{22}$
 7 a $\frac{1}{114}$ b $\frac{35}{228}$ c $\frac{137}{228}$
 8 $\frac{3}{4}$

Review set 5D

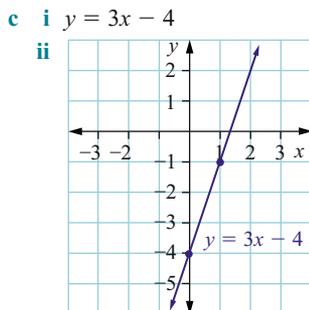
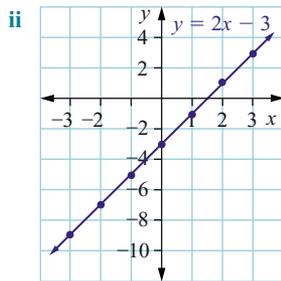
- 1 a $\frac{17}{100}$ b $\frac{32}{100} = \frac{8}{25}$
 c Perform more trials of the experiment
 2 a $\frac{9}{20}$ b $\frac{3}{10}$ c $\frac{3}{20}$
 d $\frac{23}{30}$ e $\frac{37}{60}$ f $\frac{7}{30}$

- 3 a $\frac{17}{40}$ b $\frac{3}{8}$ c $\frac{19}{40}$ d $\frac{2}{40} = \frac{1}{20}$
 e $\frac{36}{40} = \frac{9}{10}$ f $\frac{17}{20}$ g $\frac{1}{10}$
- 4 a
- | | | | | |
|--------------------------------|-----|---------------|------|--------|
| | Urn | | Ball | Result |
| $\frac{1}{2}$
$\frac{1}{2}$ | A | $\frac{2}{3}$ | W | AW |
| | | $\frac{1}{3}$ | R | AR |
| | B | $\frac{5}{7}$ | W | BW |
| | | $\frac{2}{7}$ | R | BR |
- b i $\frac{13}{42}$ ii $\frac{29}{42}$
- 5 a 0.42 = 42% b 0.12 = 12% c 0.88 = 88%
- 6 a i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{1}{16}$ iv $\frac{1}{8}$
 b i $\frac{3}{51} = \frac{1}{17}$ ii $\frac{12}{51} = \frac{4}{17}$
 iii $\frac{169}{2652} = \frac{13}{204}$ iv $\frac{169}{1326} = \frac{13}{102}$
- 7 a $\frac{1}{110}$ b $\frac{1}{11}$ c $\frac{21}{110}$
- 8 $\frac{1}{3}$

CUMULATIVE REVIEW: 2-5

- 1 a i 37.3 m² ii 15.9 m²
 b i 40.3 m² ii 26.9 m²
- 2 a i 6200 cm² ii 30 000 cm³
 b 377 cm²
- 3 a 452 cm³ b 62.4 cm³
 c 141 m³ d 55 m³
- 4 a i 28m ii 25kn iii 8ab
 b i $\frac{3}{5}$ ii $\frac{2}{3p}$ iii $-\frac{k}{15}$
 iv $\frac{31}{20w}$ v $\frac{32z}{75y}$ vi $\frac{10p}{3q}$
 vii 36 - 6e viii $\frac{26x - 3}{15}$ ix $\frac{19x - 15}{10}$
- c i 3(5t - 4) ii -4(4d - 5e) or 4(-4d + 5e)
 iii 5a²b²(3ab² - 5)
- 5 a i $\sqrt{80}$ ii (-1, 1) iii -2

b i	x	-3	-2	-1	0	1	2	3
	y	-9	-7	-5	-3	-1	1	3



- d i $-\frac{2}{3}$ ii 2 iii $y = -\frac{2}{3}x + 2$
 e i $y = -\frac{3x}{2} + 3$
- ii
-

- 6 a i $\frac{7}{14} = \frac{1}{2}$ ii $\frac{5}{14}$ iii $\frac{1}{2}$
 iv $\frac{7}{14} = \frac{1}{2}$ v 1 vi 0
- b i $\frac{2}{36} = \frac{1}{18}$ ii $\frac{1}{18}$ iii $\frac{1}{36}$ iv $\frac{1}{6}$
- c i
-
- ii $\frac{25}{64}, \frac{9}{64}, \frac{15}{64}, \frac{1}{8}$
- d i $\frac{21}{80}$ ii $\frac{3}{10}$ iii $\frac{7}{10}$
- e i $\frac{27}{100}$ ii $\frac{3}{10}$
- iii A tall person who does not wear glasses
- f i $\frac{22}{40} = \frac{11}{20}$ ii $\frac{17}{40}$
 iii $\frac{5}{40} = \frac{1}{8}$ iv $\frac{29}{40}$
- g $\frac{1}{3}$

CHAPTER 6 SIMULTANEOUS EQUATIONS

Diagnostic test

- 1 C 2 A 3 D 4 A 5 B
 6 B 7 D 8 A 9 B 10 B

Exercise 6A

- 1 a $x = 7$ b $x = -3$ c $x = -2$
 d $x = -\frac{2}{3}$ e $x = -\frac{6}{5}$ f $x = 2$
 g $x = -\frac{8}{7}$ h $x = 1$ i $x = 11$
 j $x = -2$ k $x = \frac{5}{7}$ l $x = -4$
 m $x = -\frac{5}{2}$ n $x = 3$
- 2 a No b No c Yes d No
- 3 a $x = \frac{10}{3}$ b $x = \frac{9}{4}$ c $x = \frac{7}{5}$
 d $x = \frac{7}{3}$ e $x = \frac{10}{7}$ f $x = \frac{13}{3}$
- 4 a $x = 4$ b $x = 1$ c $x = 1$
 d $x = -1$ e $x = -2$ f $x = 1$
 g $x = 9$ h $x = -10$ i $x = 3$
- 5 a $x = -\frac{9}{4}$ b $x = \frac{4}{5}$ c $x = 2$
 d $x = -\frac{5}{2}$ e $x = -\frac{14}{3}$ f $x = \frac{13}{2}$
 g $x = 1$ h $x = -7$ i $x = \frac{3}{10}$
 j $a = -1$ k $s = -\frac{13}{5}$ l $x = \frac{7}{5}$
 m $x = \frac{19}{6}$ n $y = -\frac{9}{5}$ o $p = \frac{3}{2}$

- 6 a No b Yes c No
 d Yes e Yes f No
- 7 a $x = \frac{7}{2}$ b $x = \frac{3}{5}$ c $x = 4$
 d $x = \frac{7}{5}$ e $x = -\frac{3}{8}$ f $x = 3$
 g $x = \frac{5}{2}$ h $x = 5$
- 8 a $x = 2$ b $x = -5$ c $x = \frac{17}{10}$
 d $x = \frac{19}{3}$ e $x = 2$ f $x = \frac{23}{10}$
 g $x = 4$ h $x = 0$

Exercise 6B

1

x	y	x + y	5x + 3y
0	5	5	15
1	4	5	17
2	3	5	19
3	2	5	21
4	1	5	23
5	0	5	25

The solution is $x = 2$ and $y = 3$.

2

x	y	x + y	3x - 2y
0	7	7	-14
1	6	7	-9
2	5	7	-4
3	4	7	1
4	3	7	6
5	2	7	11
6	1	7	16
7	0	7	21

The solution is $x = 5$ and $y = 2$.

3

x	y = 5 - x	3x - 2y
0	$y = 5 - 0 = 5$	$3(0) - 2(5) = -10$
1	$y = 5 - 1 = 4$	$3(1) - 2(4) = -5$
2	$y = 5 - 2 = 3$	$3(2) - 2(3) = 0$
3	$y = 5 - 3 = 2$	$3(3) - 2(2) = 5$
4	$y = 5 - 4 = 1$	$3(4) - 2(1) = 10$
5	$y = 5 - 5 = 0$	$3(5) - 2(0) = 15$

The solution is $x = 4$ and $y = 1$.

- 4 a $x = 3, y = 1$ b $x = 7, y = 4$
 c $x = 3, y = 5$ d $x = 3, y = 9$
- 5 Test: $7 - 3 = 4$ This is correct.
 $3x - 8y = -3$
 Test: $3(7) - 8(3) = -3$ This is correct.
 $\therefore x = 7$ and $y = 3$ is a solution.
- 6 a No b Yes
 c Yes d Yes
- 7 a $x = 3, y = 1$ b $x = 4, y = 2$
 c $a = 1, b = 0$ d $p = 5, q = 2$

Exercise 6C

- 1 b $x = 1, y = 3$
 2 b $x = 4, y = 2$
 3 a $x = 2, y = -1$ b $x = -1, y = -2$
 c $x = 1, y = 2$ d $x = 3, y = -4$
 e $x = 1, y = -5$ f $x = 3, y = -2$
 g $x = -1, y = -2$ h $x = -3, y = 2$
 i Infinitely many solutions

Exercise 6D

- 1 Substitute $2x - 7$ for y in equation (2).
 $3x - 2(2x - 7) = 11$
 $3x - 4x + 14 = 11$
 $-x + 14 = 11$
 $-x + 14 - 14 = 11 - 14$
 $-x = -3$
 $x = 3$
 Substitute $x = 3$ into equation (1) to obtain the y -value.
 $y = 2(3) - 7 = -1$
 The solution is $x = 3$ and $y = -1$.
- 2 a $x = 7, y = 3$ b $x = -1, y = -5$
 c $x = \frac{1}{2}, y = 4$ d $x = 2, y = 4$
 e $x = 1, y = -1$ f $x = -2, y = -12$
 g $x = 5, y = -2$ h $x = 0, y = -4$
 i $x = -1, y = 5$
- 3 a $x = 1, y = -1$ b $x = 4, y = 1$
 c $x = 2, y = 9$ d $x = 1\frac{1}{2}, y = 2\frac{1}{2}$
 e $x = \frac{1}{2}, y = 4$ f $x = -31, y = -15$

Exercise 6E

- 1 a $4x - 6y = 10$ b $-3x - 9y = -21$
 c $8x + 20y = 4$ d $-6x + 4y = -16$
 e $25x - 5y = 10$ f $2x - 5y = 1$
- 2 a $4x = 16$ b $4x = 16$
 c $2x = 12$ d $3y = 6$
 e $2y = -2$ f $-y = 4$
- 3 To eliminate y multiply equation (2) by -2 .
 $3x + 2y = 4$
 $-4x - 2y = -2$
 $-x = 2$
 $x = -2$
 Substitute $x = -2$ into equation (1).
 $3(-2) + 2y = 4$
 $-6 + 6 + 2y = 4 + 6$
 $2y = 10$
 $y = 5$
 The solution is $x = -2$ and $y = 5$.
- 4 a $x = 4, y = 1$ b $x = 3, y = -2$
 c $x = -2, y = 5$ d $x = -3, y = -2$
 e $x = 7, y = -1$ f $x = 3, y = -5$
 g $x = 6, y = -5$ h $x = -3, y = 8$
 i $x = 3, y = -4$ j $x = 3, y = 0$
 k $x = 5, y = -4$ l $x = 4, y = 5$
 m $x = -1, y = -7$ n $x = -1, y = -7$
 o $x = \frac{20}{3}, y = -\frac{2}{21}$

Exercise 6F

1 Then $x + y = 201$ (1)
and $x - y = 37$ (2)
 $2x = 238$
 $x = 119$

Substitute $x = 119$ into equation (1).

$119 + y = 201$
 $y = 201 - 119$
 $y = 82$

The numbers are 119 and 82.

2 a 181, 97 b 8, -3
c 68, 17 d 35, 36
3 a 7, 5 b 12, 8 c 4, 6
d 11, 3 e 8, 4

4 Then $2h + d = 59$ (1)
and $h + 3d = 42$ (2)

Multiply equation (2) by 2 to eliminate h .

$2h + d = 59$ (1)
 $2h + 6d = 84$ (2)
 $-5d = -25$
 $d = 5$

Substitute $d = 5$ into equation (2).

$h + 15 = 42$
 $h = 27$

The hammer costs \$27 and the screwdriver costs \$5.

- 5 a Adult \$14, child \$6
b Sheet \$22, blanket \$56
c Apples 26 cents, oranges 28 cents
d 19×5 -cent coins, 14×20 -cent coins
e 16×50 -cent coins, $27 \times \$1$ coins
6 23×2 L cartons, 74×1 L cartons
7 14 rabbits, 21 pheasants
8 6 loaves, 10 rolls
9 $x = 3$ cm, $y = 5$ cm
10 12 cm
11 $x = 39^\circ$, $y = 52^\circ$, $\angle K = 94^\circ$
12 Anne, 17 years; Jane, 37 years
13 Mother, 42 years; son, 20 years
14 Sam \$1000, Ben \$500

Language in mathematics

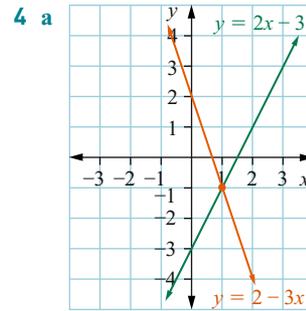
- 2 a solution b simultaneous
c equation d variable
e substitution f solve

Check your skills

- 1 A 2 A 3 B 4 C 5 D
6 B 7 A 8 B 9 D 10 A

Review set 6A

- 1 a $x = -4$ b $x = \frac{11}{2}$
c $x = \frac{1}{2}$ d $x = \frac{1}{6}$
e $x = -\frac{9}{5}$ f $x = 3$
2 $y = 2$
3 a $y = 3$ b $24x - 9y = -3$
c $10x = 20$



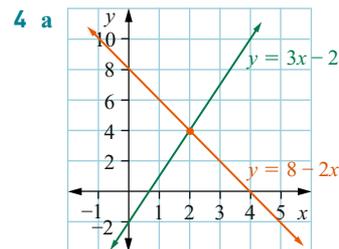
b $x = 1, y = -1$

- 5 a $x = -2, y = -3$ b Yes

- 6 a Paula is 7, John is 9.
b 8×20 -cent coins (13×5 -cent coins)

Review set 6B

- 1 a $x = 1$ b $x = \frac{11}{9}$ c $x = \frac{3}{5}$
d $x = \frac{1}{8}$ e $x = \frac{30}{11}$ f $x = 2$
2 $t = -\frac{14}{3}$
3 a $y = -\frac{1}{5}$ b $28x - 8y = 32$
c $7x = -4$

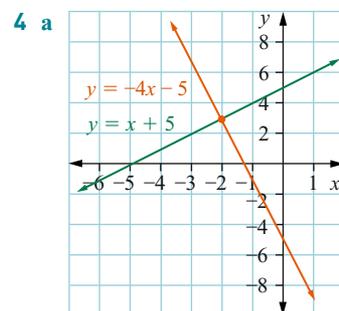


b $x = 2, y = 4$

- 5 $x = 2, y = -3$
6 a 4 and 6
b 3×10 -cent coins (33×20 -cent coins)
7 They are parallel, therefore they never intersect.

Review set 6C

- 1 a $x = \frac{7}{4}$ b $x = 4$
c $x = 4$ d $a = -5$
e $a = \frac{17}{5}$ f $p = \frac{13}{2}$
2 a $y = -\frac{2}{5}$ b $x = 9$
3 a $-12x + 28y = -20$ b $(0, -4)$
c $y = -18$ d $5x = 20$

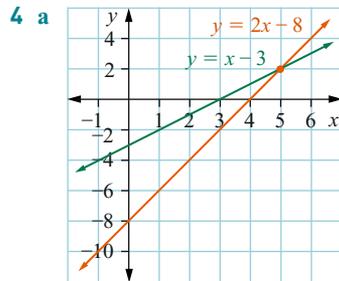


b $x = -2, y = 3$

- 5 a $x = \frac{17}{5}, y = \frac{8}{5}$ b $x = 2, y = -3$
 6 a 54 cents; frog = 24 cents, lollipop = 30 cents
 b Tennis ball, 60 g; cricket ball, 220 g

Review set 6D

- 1 a $x = \frac{7}{3}$ b $x = \frac{1}{7}$
 c $x = 12$ d $x = -\frac{21}{8}$
 e $t = -\frac{6}{11}$ f $p = \frac{33}{17}$
 2 a $y = -\frac{4}{3}$ b $x = 4$
 3 a $6x - 15y = 33$ b $(0, -5)$
 c $y = -10$ d $12x = 15$



- b $x = 5, y = 2$ c $x = 5, y = 2$
 5 $x = 4, y = -3$
 6 a 6 kg brand A, 4 kg brand B
 b 17

CHAPTER 7 GEOMETRICAL PROOFS

Diagnostic test

- 1 B 2 B 3 A 4 C 5 D
 6 C 7 B 8 C 9 C 10 B
 11 C

Exercise 7A

- 1 a $5y = 55^\circ$ (Vertically opposite angles are equal.)
 $y = 11^\circ$
 $2z = 46^\circ$ (Vertically opposite angles are equal.)
 $z = 23^\circ$
 $x + 46^\circ + 55^\circ = 180^\circ$ (DPC is a straight line.)
 $x = 79^\circ$
 b $2y + 90^\circ + 70^\circ = 180^\circ$ (ABC is a straight line.)
 $2y = 20^\circ$
 $y = 10^\circ$
 2 a $\angle EGD = \angle JEB = 50^\circ$ (Corresponding angles and $AB \cdot CD$.)
 $\angle EGD + \angle DGH = 180^\circ$ ($\angle EGH$ is a straight angle.)
 $50^\circ + x = 180^\circ$
 $x = 130^\circ$
 $\angle FGC = \angle HGD = 130^\circ$ (Vertically opposite angles are equal.)
 $\angle EFI = \angle FGC$ (Corresponding angles and $IF \cdot CD$.)
 $y = 130^\circ$
 b $\angle BAE = \angle DEF$ (Corresponding angles and $AB \cdot CD$.)
 $2x + 10^\circ = x + 40$
 $\therefore x = 30^\circ$
 $\angle AEC = \angle DEF$ (Vertically opposite angles are equal.)
 $2y - 20^\circ = 70^\circ$
 $y = 45^\circ$

- c $\angle ABD = 90^\circ$ ($\angle ABC$ is a straight angle.)
 $x + 90^\circ + 42^\circ = 180^\circ$ (Angle sum of a \triangle is 180° .)
 $x = 48^\circ$
 $y = 42^\circ$ ($\angle ADC$ is 90° .)
 $z = 48^\circ$ (Angle sum of \triangle is 180° .)
 d $\angle WXY = \angle UVW = 48^\circ$
 (Alternate angles and $UV \cdot XZ$.)
 $x = 48^\circ + 56^\circ = 104^\circ$ (Exterior angle of a \triangle equals the sum of interior opposite angles.)
 e $\angle QSR = x$ (Angles opposite equal sides of a \triangle are equal.)
 $130 = x + x$ (Exterior angles of a \triangle equal the sum of interior opposite angles.)
 $x = 65^\circ$
 $y = 115^\circ$ ($\angle TSR = 180^\circ$)
 f $\angle BDE = \angle CBD = x$ (Alternate angles and $AC \cdot ED$.)
 $\angle DEB = \angle BDE = x$ (Angles opposite equal sides of a \triangle are equal.)
 $x + x + 52^\circ = 180^\circ$ (Angle sum of a \triangle is 180° .)
 $x = 64^\circ$
 $\angle ABE = \angle BED = y$ (Alternate angles and $AC \cdot ED$.)
 $y = 64^\circ$
 g $3x + 70^\circ + 2x + 90^\circ = 360^\circ$ (Angle sum of a quadrilateral is 360° .)
 $5x + 160^\circ = 360^\circ$
 $x = 40^\circ$
 $\angle ADE + \angle ADC = 180^\circ$ ($\angle CDE$ is a straight angle.)
 $y + 80^\circ = 180^\circ$
 $y = 100^\circ$
 h $\angle ADC = 50^\circ$ (Co-interior angles and $AB \cdot CD$.)
 $\angle EDC = 25^\circ$ (ED bisects $\angle ADC$.)
 $\angle BCD = 130^\circ$ (Co-interior angles and $AD \cdot BC$.)
 $\angle DCE = 65^\circ$ (EC bisects $\angle BCD$.)
 $x + 25^\circ + 65^\circ = 180^\circ$ (Angle sum of a \triangle is 180° .)
 $x = 90^\circ$
 i Let $\angle SPQ = x$ and let $\angle SQP = y$.
 $2x + 2y + 70^\circ = 180^\circ$ (Angle sum of $\triangle PQR$ is 180° .)
 $2(x + y) = 110^\circ$
 $x + y = 55^\circ$
 $x + y + z = 180^\circ$ (Angle sum of $\triangle PQS$ is 180° .)
 $55^\circ + z = 180^\circ$
 $z = 125^\circ$
 3 a $\angle PWV = \angle QWR = x$ (Vertically opposite angles are equal.)
 $\angle UVW = \angle SVT = y$ (Vertically opposite angles are equal.)
 $\angle PWV + \angle UVW = x + y = 180^\circ$ (given)
 $\therefore PR \cdot US$ ($\angle PWV$ and $\angle UVW$ are co-interior and supplementary.)
 b $3x + 50^\circ + x + 100^\circ + 90^\circ = 360^\circ$
 (Angle sum at a point is 360° .)
 $4x + 240^\circ = 360^\circ$
 $x = 30^\circ$
 $\angle PUQ = 3 \times 30^\circ = 90^\circ$
 $\therefore TUQ$ is a straight line. ($\angle TUP + \angle PUQ = 180^\circ$)
 4 a $2x + 2y = 180^\circ$ (ABC is a straight line.)
 $2(x + y) = 180^\circ$
 $x + y = 90^\circ$
 $\angle PBQ = \angle PBD + \angle DBQ = x + y = 90^\circ$

- b** $\angle QAB = \angle CBA = b$ (Alternate angles and $PQ \parallel CB$)
 $\angle PAC = \angle BCA = c$ (Alternate angles and $PQ \parallel CB$)
 $\angle PAC + \angle CAB + \angle BAQ = 180^\circ$
(PAQ is a straight line.)
 $c + a + b = 180^\circ$
- c** $\angle PBA = y$ (Alternate angles and $PB \parallel AC$)
 $\angle ACD = \angle PBC$ (Corresponding angles and $PB \parallel AC$)
 $z = x + y$
- d** $\angle PQT = x$ (Corresponding angles and $TQ \parallel SR$)
 $\angle RSQ = x$ (Angles opposite equal sides of a Δ are equal.)
 $\angle TQS = \angle RSQ = x$ (Alternate angles and $TQ \parallel SR$)
 $\angle PQT = \angle RSQ = x$
 TQ bisects $\angle PQS$.

Exercise 7B

- 1 a** SAS **b** SSS **c** RHS **d** AAS
e SAS **f** AAS **g** SSS **h** SAS
- 2 a** The 55° angle in the second triangle is not the included angle.
b Three pairs of equal angles do not necessarily make the triangles congruent.
c The equal sides are not matching sides.
d The 30° angle in the second triangle is not the included angle.
- 3 a** 2 and 3 (SAS) **b** 1, 2 and 3 (AAS)
c 1 and 2 (RHS) **d** 1, 2 and 3 (SSS)
e 1 and 3 (SAS) **f** 2 and 3 (AAS)
- 4 a** SSS, $x = 60^\circ, y = 55^\circ, z = 65^\circ$
b SAS, $x = 30^\circ, y = 40^\circ$
c AAS, $x = 8 \text{ cm}, y = 7 \text{ cm}$
d RHS, $x = 6 \text{ cm}, y = 70^\circ, z = 20^\circ$
e SAS, $x = 15 \text{ cm}, y = 55^\circ, z = 58^\circ$
f SSS, $x = 55^\circ, y = 77^\circ, z = 48^\circ$
g AAS, $x = 10 \text{ cm}, y = 12 \text{ cm}, z = 50^\circ$
h RHS, $x = 40^\circ, y = 50^\circ, z = 6 \text{ cm}$

Exercise 7C

- 1 a** In $\triangle ACD$ and $\triangle ACB$:
 $AD = AB$ (given)
 $DC = BC$ (given)
 AC is common.
 $\therefore \triangle ACD \equiv \triangle ACB$ (SSS)
- b** $\angle DAC = \angle BAC$ (Matching angles in congruent Δ s are equal.)
- c** $\angle ADC = \angle ABC$ (Matching angles in congruent Δ s are equal.)
- 2 a** In $\triangle ABP$ and $\triangle DCP$:
 $\angle BAP = \angle CDP$ (given)
 $\angle APB = \angle DPC$ (Vertically opposite angles are equal.)
 $AP = DP$ (given)
 $\therefore \triangle ABP \equiv \triangle DCP$ (AAS)
- b** $BP = CP$ (Matching sides in congruent Δ s are equal.)
- c** $AB = DC$ (Matching sides in congruent Δ s are equal.)
- 3 a** In $\triangle ABP$ and $\triangle CDP$:
 $\angle ABP = \angle CDP$ (given)
 $\angle APB = \angle CPD$ (Vertically opposite angles are equal.)
 $AB = CD$ (given)
 $\therefore \triangle ABP \equiv \triangle CDP$ (AAS)

- b** $AP = CP$ (Matching sides in congruent Δ s are equal.)
- 4 a** In $\triangle AED$ and $\triangle BEC$:
 $ED = EC$ (given)
 $AD = BC$ (Opposite sides of a rectangle are equal.)
 $\angle DAE = \angle EBC$ ($= 90^\circ$, $ABCD$ is a rectangle.)
 $\therefore \triangle AED \equiv \triangle BEC$ (RHS)
- b** $AE = BE$ (Matching sides in congruent Δ s are equal.)
 $\therefore E$ is the midpoint of AB .
- 5 a** In $\triangle PTQ$ and $\triangle STR$:
 $PT = ST$ (given)
 $QT = RT$ (given)
 $\angle PTQ = \angle STR$ (Vertically opposite angles are equal.)
 $\therefore \triangle PTQ \equiv \triangle STR$ (SAS)
- b** $PQ = SR$ (Matching sides in congruent Δ s are equal.)
- c** $\angle PQT = \angle SRT$ (Matching angles in congruent Δ s are equal.)
- 6 a** In $\triangle PRS$ and $\triangle PRQ$:
 $PS = PQ$ (given)
 $\angle PRS = \angle PRQ$ ($= 90^\circ$, $PR \perp SQ$)
 PR is common.
 $\therefore \triangle PRS \equiv \triangle PRQ$ (RHS)
- b** $\angle PSR = \angle PQR$ (Matching angles in congruent Δ s are equal.)
- c** $SR = QR$ (Matching sides in congruent Δ s are equal.)
- d** $\angle SPR = \angle QPR$ (Matching angles in congruent Δ s are equal.)
- e i** The angles opposite the equal sides of an isosceles triangle are equal.
ii A line drawn from the vertex of an isosceles triangle and perpendicular to its base bisects the base and the vertex angle of the triangle.
- 7 a** In $\triangle ACD$ and $\triangle ACB$:
 $\angle ADC = \angle ABC$ (given)
 $\angle ACD = \angle ACB$ ($= 90^\circ$, $AC \perp DB$)
 AC is common.
 $\therefore \triangle ACD \equiv \triangle ACB$ (AAS)
- b** $AD = AB$ (Matching sides in congruent Δ s are equal.)
 $\therefore ABC$ is an isosceles triangle.
- c** If two angles of a triangle are equal, then the sides opposite these angles are equal; that is, it is an isosceles triangle.
- 8 a** In $\triangle WZY$ and $\triangle WXY$:
 $WZ = WX$ (given)
 $ZY = XY$ (given)
 $WY = WY$ (common)
 $\therefore \triangle WZY \equiv \triangle WXY$ (SSS)
- b** $\angle WYZ = \angle WYX$ (Matching angles in congruent Δ s are equal.)
- c** $\angle WYZ + \angle WYX = 180^\circ$ ($\angle ZYX$ a straight angle.)
 $\therefore \angle WYZ = \angle WYX = 90^\circ$
- d** $\angle ZWY = \angle XWY$ (Matching sides in congruent Δ s are equal.)
- e** A line drawn from the vertex of an isosceles triangle to the midpoint of its base is perpendicular to the base and bisects the angle at the vertex.

- 9 a** Yes
b $AC = AB$ so $\angle C = \angle B$
 $AB = AC$ so $\angle C = \angle A$
Hence $\angle C = \angle A = \angle B$; that is, the three angles of an equilateral triangle are equal.
c Let $\angle A = \angle B = \angle C = x$ then
 $x + x + x = 180^\circ$
 $3x = 180^\circ$
 $x = 60^\circ$
- 10 a** In $\triangle ABC$ and $\triangle CDA$:
 $\angle BAC = \angle DCA$ (Alternate angles and $AB \parallel DC$.)
 $\angle BCA = \angle DAC$ (Alternate angles and $AD \parallel BC$.)
 $AC = CA$ (common)
 $\therefore \triangle ABC \equiv \triangle CDA$ (AAS)
b $AB = DC$ (Matching sides in congruent \triangle s are equal.)
c $AD = BC$ (Matching sides in congruent \triangle s are equal.)
d $\angle B = \angle D$ (Matching angles in congruent \triangle s are equal.)
e i The opposite sides of a parallelogram are equal.
ii The opposite angles of a parallelogram are equal.
- 11 a** In $\triangle PQT$ and $\triangle RST$:
 $\angle QPT = \angle SRT$ (Alternate angles and $PQ \parallel SR$.)
 $\angle PQT = \angle RST$ (Alternate angles and $PQ \parallel SR$.)
 $PQ = RS$ (proven in question 10)
 $\therefore \triangle PQT \equiv \triangle RST$ (AAS)
b $PT = RT$ (Matching sides in congruent \triangle s are equal.)
c $QT = ST$ (Matching sides in congruent \triangle s are equal.)
d The diagonals of a parallelogram bisect each other.
- 12 a** In $\triangle ADC$ and $\triangle BCD$:
 $AD = BC$ (Opposite sides of a rectangle are equal.)
 DC is common.
 $\angle ADC = \angle BCD$ ($= 90^\circ$, $ABCD$ is a rectangle.)
 $\therefore \triangle ADC \equiv \triangle BCD$ (SAS)
b $AC = BD$ (Matching sides in congruent \triangle s are equal.)
c The diagonals of a rectangle are equal.
- 13 a** In $\triangle ADE$ and $\triangle ABE$:
 $AD = AB$ (All sides of a rhombus are equal.)
 $DE = BE$ (Diagonals of a parallelogram bisect.)
 AE is common.
 $\triangle ADE \equiv \triangle ABE$ (SSS)
b $\angle AED = \angle AEB$ (Matching angles in congruent \triangle s are equal.)
 $\angle AED + \angle AEB = 180^\circ$ ($\angle DEB$ a straight angle.)
 $\therefore \angle AED = \angle AEB = 90^\circ$
c $\angle DAE = \angle BAE$ (Matching angles in congruent \triangle s are equal.)
d i The diagonals of a rhombus intersect at right angles.
ii The diagonals of a rhombus bisect the angles of the rhombus.
- 14 a** In $\triangle ABC$ and $\triangle AED$:
 $AB = AE$ (given)
 $BC = ED$ (given)
 $\angle ABC = \angle AED$ (given)
 $\therefore \triangle ABC \equiv \triangle AED$ (SAS)
 $\therefore AC = AD$ (Matching angles in congruent \triangle s are equal.)

- b** Similarly, $\triangle ABC \equiv \triangle CDE \therefore AC = CE$
 $\triangle AED \equiv \triangle BCD \therefore AD = BD$
 $\triangle ABE \equiv \triangle DCE \therefore BE = CE$
 $AC = AD = CE = BD = BE$

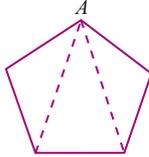
Exercise 7D

- 1 a** Test 2 (Three pairs of sides are in proportion.)
b Test 1 (Three pairs of equal angles.)
c Test 4 (The hypotenuse and a second side in each right-angled triangle are in proportion.)
d Test 3 (Two pairs of sides are in proportion and included angles are equal.)
e Test 1 (Three pairs of equal angles.)
f Test 4 (The hypotenuse and a second side in each right-angled triangle are in proportion.)
g Test 2 (Three pairs of sides are in proportion.)
h Test 3 (Two pairs of sides are in proportion and included angles are equal.)
- 2 a** The 80° angle in the first triangle is not included by the sides in proportion.
b The second sides are not in the same proportion as the hypotenuses.
c Only two pairs of sides are in proportion.
d The 60° angle in the second triangle is not the included angle.
e The second sides are not in the same proportion as the hypotenuses.
f Only two pairs of sides are in proportion.
- 3 a** Yes: Test 1 (Three pairs of equal angles.)
b No: Not all pairs of angles are equal.
c Yes: Test 2 (Three pairs of sides are in proportion.)
d No: Only two pairs of sides are in proportion.
e Yes: Test 1 (Three pairs of equal angles.)
f Yes: Test 2 (Three pairs of sides are in proportion.)
g Yes: Test 1 (Three pairs of equal angles.)
h No: Not all pairs of angles are equal.
i Yes: Test 2 (Three pairs of sides are in proportion.)
j No: Only two pairs of sides are in proportion.
k Yes: Test 2 (Three pairs of sides are in proportion.)
l Yes: Test 3 (Two pairs of sides are in proportion and included angles are equal.)
m No: The first 70° angle is not the included angle.
n Yes: Test 4 (The hypotenuse and a second side in each right-angled triangle are in proportion.)
- 4 a** B, C (Test 2) **b** A, B (Test 1)
c A, C (Test 1 or Test 3) **d** A, B (Test 2)
e A, C (Test 3) **f** A, B (Test 3)
g A, B (Test 4) **h** B, C (Test 4)
5 b No **c** Not true
- 6 Method 1**
To find x :
 $k = \frac{16.25}{13} = 1.25$
 $x = 1.25 \times 8 = 10$
To find y :
 $k = \frac{13}{16.25} = \frac{4}{5}$
 $y = \frac{4}{5} \times 15 = 12$
- Method 2**
 $\frac{x}{8} = \frac{16.25}{13}$
 $x = 8 \times \frac{16.25}{13} = 10$
 $\frac{y}{15} = \frac{13}{16.25}$
 $y = 15 \times \frac{13}{16.25} = 12$
- 7 a** $x = 25$ cm, $y = 12$ cm **b** $x = 3.2$ cm, $y = 15$ cm
c $x = 30$ cm, $y = 12$ cm **d** $x = 8$ cm, $y = 22.5$ cm
e $x = 24$ cm, $y = 8$ cm **f** $x = 15$ cm, $y = 7$ cm
g $x = 5.1$ cm, $y = 7$ cm **h** $x = 25$ cm, $y = 11$ cm
i $x = 12.6$ cm, $y = 15$ cm

- 8 a $\frac{x}{3} = \frac{6}{4} \therefore x = 4.5$ cm
 b $\frac{x}{8} = \frac{6}{10} \therefore x = 4.8$ cm
- 9 a $x = 7.5$ cm b $x = 3.75$ cm
 c $x = 14$ cm d $x = 4.8$ cm
- 10 a $\triangle FED \sim \triangle CBA$ ($\angle E = \angle B$, $\angle D = \angle A$)
 b $h = 5.6$ m
- 11 a $\triangle UTS \sim \triangle RQP$ ($\angle T = \angle Q$, $\angle S = \angle P$)
 b $h = 9$ m
- 12 a $\triangle PRT \sim \triangle QRS$ ($\angle T = \angle S$, $\angle R = \angle R$)
 b $h = 8.5$ m
- 13 a $\triangle VYU \sim \triangle WXU$ ($\angle VYU = \angle WXU$, $\angle U$ is common)
 b $VY = 1.2$ m

Exercise 7E

1 a, b



c There are 3 triangles in the pentagon, hence interior angle sum = $3 \times 180^\circ = 540^\circ$.

Polygon	Number of sides	Number of triangles	Angle sum of polygon
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Nonagon	9	7	1260°
Decagon	10	8	1440°
n -gon	n	$n - 2$	$(n - 2) \times 180^\circ$

- 3 The sum of the measure of the interior angles in any n -sided polygon is $(n - 2) \times 180^\circ$.
- 4 Sum of interior angles = 540°
 Each interior angle = $540^\circ \div 5 = 108^\circ$
- 5 a 120° b $128\frac{4}{7}^\circ$ c 135°
 d 140° e 144°
- 6 a 3960° b 165°
- 7 Angle sum of polygon = $(5 - 2) \times 180^\circ = 540^\circ$
 $x + 130^\circ + 80^\circ + 150^\circ + 100^\circ = 540^\circ$
 $x + 460^\circ = 540^\circ$
 $x = 80^\circ$
- 8 a 70° (Angle sum of a pentagon is 540° .)
 b 77° (Angle sum of a pentagon is 540° .)
- 9 a 720°
 b i 50° ii 70° iii 65°
- 10 a $x = 50^\circ$ (Angle sum of a quadrilateral is 360° .)
 b $x = 165^\circ$ (Angle sum of a pentagon is 540° .)
 c $x = 120^\circ$ (Angle sum of a hexagon is 720° .)
 d $x = 20^\circ$ (Angle sum of a quadrilateral is 360° .)
 e $x = 120^\circ$ (Angle sum of a hexagon is 720° .)
 f $x = 60^\circ$ (Angle sum of a pentagon is 540° .)
 g $x = 40^\circ$ (Angle sum of a hexagon is 720° .)
 h $x = 125^\circ$ (Angle sum of a heptagon is 900° .)
 i $x = 135^\circ$ (Angle sum of an octagon is 1080° .)
- 11 135° 12 135°

- 13 For a regular octagon, sum of exterior angles = 360° .
 Size of each exterior angle = $360^\circ \div 8 = 45^\circ$
 Hence, size of each interior angle = $180^\circ - 45^\circ = 135^\circ$
 Angle sum of octagon = $8 \times 135^\circ = 1080^\circ$
- 14 a 18° b 162° c 3240°
- 15 a Exterior angle = $\frac{360^\circ}{n}$
 $15^\circ = \frac{360^\circ}{n}$
 $n = \frac{360^\circ}{15^\circ} = 24$
 The polygon has 24 sides.
- b $60^\circ + (n - 1) \times 10^\circ = 360^\circ$
 $10(n - 1) = 300^\circ$
 $n - 1 = 30^\circ$
 $n = 31$
 The polygon has 31 sides.
- 16 a 36 sides b 20 sides
 c 15 sides d 4 sides
- 17 21 sides
- 18 a 21 sides b 22 sides
 19 a 22 sides b 16 sides

Language in mathematics

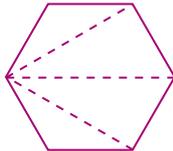
- 1 Congruent triangles have the same shape (and size) so they are similar. Similar triangles have the same shape but not necessarily the same size, hence they are not necessarily congruent.
- 2 a An isosceles triangle has two equal sides.
Properties:
 Angles opposite equal sides are equal.
 A line from the vertex drawn perpendicular to its base bisects the base and the angle at the vertex.
 A line drawn from the vertex to the midpoint of its base is perpendicular to the base and bisects the angle at the vertex.
- b An equilateral triangle has 3 equal sides.
Properties:
 All the properties of isosceles triangles. All angles equal 60° .
- 3 Two plane figures are equiangular if all the angles of one are equal to all the angles of the other. Yes, they have the same shape.
- 4 The remaining angle in each triangle is 180° minus the sum of the other two angles (which are equal in each triangle).
- 5 a $AB = AD$, $AC = AC$, $\angle B = \angle D$ but angles B and D are not included by the equal sides.
 b i If $CB = CD$ the triangles would be congruent (SAS or SSS).
 ii If $\angle BAC = \angle DAC$, the triangles would be congruent (AAS).
 iii If $\angle ACB = \angle ACD$, the triangles would be congruent (AAS).
- 6 The exterior angles of polygons are formed by extending the sides in the same order. Each exterior angle is the angle between the extended side and the side that is adjacent to it.
- 7 $S = (n - 2) \times 180^\circ$ or $S = n \times 180^\circ - 360^\circ$ where S is the sum of the angles and n is the number of sides of the polygon.
- 8 360°

Check your skills

- 1 D 2 D 3 B 4 C 5 C
 6 B 7 A 8 D 9 B 10 C
 11 B 12 A 13 B 14 C 15 D
 16 C 17 B 18 A 19 C 20 A
 21 B

Review set 7A

- 1 $\angle ABC = \angle EAB$ (Alternate angles and $AE \cdot CF$)
 $\angle ACB = \angle DAE$ (Corresponding angles and $AE \cdot CF$)
 $\therefore \angle ABC = \angle ACB$ ($\angle EAB = \angle DAE$, given)
 $\therefore AB = AC$ (Sides opposite equal angles in a Δ are equal.)
- 2 a SAS b AAS c SSS
- 3 a Three pairs of equal angles do not make two triangles congruent.
 b The equal angles are not the included angles.
 c The equal sides are not matching sides.
- 4 a 1, 2 (AAS) b 1, 3 (SAS)
- 5 a SSS, $x = 50^\circ, y = 95^\circ, z = 35^\circ$
 b SAS, $x = 15 \text{ cm}, y = 45^\circ, z = 35^\circ$
 c AAS, $x = 9 \text{ cm}, y = 10 \text{ cm}, z = 50^\circ$
- 6 a In $\triangle PQT$ and $\triangle QRS$:
 $PT = RS$ (given)
 $\angle PTQ = \angle RSQ$ (given)
 $\angle PQT = \angle QRS$ (Vertically opposite angles are equal.)
 $\therefore \triangle PQT \equiv \triangle QRS$ (AAS)
 b $PQ = RQ$ (Matching sides in congruent Δ s are equal.)
- 7 a Yes: Test 1 (Three pairs of equal angles.)
 b No: Only two pairs of sides in proportion.
 c Yes: Test 3 (Two pairs of sides are in proportion and included angles are equal.)
 d No: The 80° angle in the second triangle is not included by the sides in proportion.
 e Yes: The hypotenuses and a second side in each right-angled triangle are in proportion.
 f No: The sides in proportion do not include the hypotenuses of each triangle.
- 8 a 1, 2 (Test 2) b 1, 2, 3 (Test 1 or Test 3)
 c 1, 3 (Test 3) d 1, 2, 3 (Test 4)
- 9 a $k = 3, x = 15 \text{ cm}, y = 6 \text{ cm}$
 b $k = 0.8, x = 12 \text{ cm}, y = 8 \text{ cm}$
- 10 a $x = 7.5 \text{ cm}, y = 3.3 \text{ cm}$ b $x = 6 \text{ cm}, y = 12 \text{ cm}$
- 11 a, b c $4 \times 180^\circ = 720^\circ$



- 12 a $x = 100^\circ$ (Angle sum of a hexagon is 720°)
 b $x = 105^\circ$ (Angle sum of a pentagon is 540°)
- 13 a 135° b 360°
- 14 a 36 b 6120°

Review set 7B

- 1 $\angle DEC = \angle DCE$ ($DC = DE$)
 $\angle ABC = \angle ACB$ ($AB = AC$)
 $\angle DEC = \angle ABC$ ($\angle DCE = \angle ACB$)
 (Corresponding angles on sides AB and DE .)
 $\therefore AB \parallel DE$

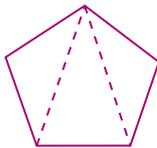
- 2 a SSS b RHS c SAS d AAS
- 3 a The angle of first triangle is not the included angle.
 b The equal sides are not matching sides.
 c Three pair of equal angles do not make triangles congruent.
- 4 a 1, 3 (SAS) b 2, 3 (AAS)
- 5 a SSS, $x = 60^\circ, y = 65^\circ, z = 55^\circ$
 b SAS, $x = 60^\circ, y = 80^\circ$
 c AAS, $x = 10 \text{ cm}, y = 8 \text{ cm}$
- 6 a $AB = CD$ (given)
 $AC = CA$ (common)
 $\angle BAC = \angle DCA$ (Alternate angles and $AB \parallel DC$.)
 $\therefore \triangle ABC \equiv \triangle CDA$ (SAS)
 b $AD = BC$
- 7 a Yes: Test 1 (Three pairs of equal angles.)
 b Yes: Test 2 (Three pairs of sides are in proportion.)
 c No: The angle of the first triangle is not the included angle.
 d Yes: Test 3 (Two pairs of sides are in proportion and included angles are equal.)
 e No: The second sides are not in the same proportion as the hypotenuses.
 f No: The sides in the second triangle do not include the hypotenuse.
- 8 a 1, 2 (Test 1) b 1, 2, 3 (Test 2)
 c 1, 3 (Test 3) d 1, 2 (Test 4)
- 9 a $k = 3, x = 24 \text{ cm}, y = 6 \text{ cm}$
 b $k = \frac{1}{2}, x = 6 \text{ cm}, y = 14 \text{ cm}$
- 10 a $x = 15 \text{ cm}, y = 14 \text{ cm}$ b $x = 10 \text{ cm}, y = 12 \text{ cm}$
- 11 a 1800° b 150°
- 12 a $x = 67^\circ$ (Angle sum of a pentagon is 540°)
 b $x = 120^\circ$ (Angle sum of a hexagon is 720°)
- 13 165°
- 14 a 14.4° b 165.6° c 4140°

Review set 7C

- 1 $\angle BFG + \angle FGD = 180^\circ$
 (Co-interior angles and $AB \parallel CD$)
 $2x + 2y = 180^\circ$
 $x + y = 90^\circ$
 $\angle FIG = 180^\circ - (x + y)$ (Angle sum of a Δ is 180° .)
 $\angle FIG = 180^\circ - 90^\circ = 90^\circ$
- 2 a SSS b AAS c SAS d RHS
- 3 a Three pairs of equal angles do not make triangles congruent.
 b The equal angles are not included angles.
- 4 a 2, 3 (AAS) b 1, 3 (SAS)
- 5 a SAS, $x = 20 \text{ cm}, y = 55^\circ, z = 65^\circ$
 b AAS, $x = 10 \text{ cm}, y = 20 \text{ cm}$
- 6 a In $\triangle PQS$ and $\triangle RSQ$:
 $PQ = RS$ (given)
 QS is common.
 $\angle PQS = \angle RSQ$ (given)
 $\therefore \triangle PQS \equiv \triangle RSQ$ (SAS)
 b $PS = RQ$ (Matching sides in congruent Δ s are equal.)
 c $\angle PSQ = \angle RQS$ (Matching angles in congruent Δ s are equal.)
 $\therefore PS \parallel RQ$ (Alternate angles are equal.)
- 7 a Yes: Test 1 (Three pairs of equal angles.)
 b No: Only two pairs of sides are in proportion.

- c Yes: Test 4 (The hypotenuse and a second side in each right-angled triangle are in proportion.)
 d No: The 65° angle in the second triangle is not the included angle.
- 8 a 1, 2, 3 (Test 1) b 1, 3 (Test 3)
 c 2, 3 (Test 4) d 1, 3 (Test 2)
- 9 a $x = 17.5$ cm, $y = 5$ cm b $x = 15$ cm, $y = 6$ cm
 c $x = 15$ cm, $y = 6$ cm d $x = 8$ cm, $y = 22.5$ cm
- 10 a 6120° b 170°
- 11 a $x = 32^\circ$ (Angle sum of a hexagon is 720° .)
 b $x = 60^\circ$ (Angle sum of a pentagon is 540° .)
- 12 120° 13 12°

Review set 7D

- 1 In $\triangle ADC$:
 $\angle CAD = \angle ACD = x$ ($DC = DA$)
 $\therefore \angle DAB = 90^\circ - x$ ($\angle CAB = 90^\circ$)
 In $\triangle ABC$:
 $\angle C + \angle A + \angle B = 180^\circ$ (Angle sum of a \triangle is 180° .)
 $x + 90^\circ + \angle B = 180^\circ$
 $x + \angle B = 90^\circ$
 $\angle B = 90^\circ - x$
 $\therefore \angle B = \angle DAB$
 $DA = DB$ (Equal sides opposite equal angles in $\triangle ADB$.)
- 2 a RHS b SAS c SAS
- 3 a The angle in the second triangle is not the included angle.
 b The equal sides are not matching sides.
- 4 a 1, 3 (AAS) b 2, 3 (SAS)
- 5 a SSS, $x = 65^\circ$, $y = 55^\circ$, $z = 60^\circ$
 b RHS, $x = 36^\circ$, $y = 54^\circ$
- 6 a In $\triangle BED$ and $\triangle CFD$:
 $DE = DF$ (given)
 $DB = DC$ (given)
 $\angle BED = \angle CFD$ (given)
 $\therefore \triangle BED \equiv \triangle CFD$ (RHS)
 b $\angle B = \angle C$ (Matching angles in congruent \triangle s are equal.)
 $\therefore AB = AC$
 $\triangle ABC$ is an isosceles triangle.
- 7 a Yes: Test 2 (Three pairs of sides are in proportion.)
 b No: The sides that include the equal angles are not in proportion.
 c No: There is only one pair of equal angles.
 d Yes: Test 4 (The hypotenuse and a second side in each right-angled triangle are in proportion.)
- 8 a 2, 3 (Test 3) b 1, 2, 3 (Test 1)
 c 2, 3 (Test 2) d 1, 2 (Test 4)
- 9 a $x = 15$ cm, $y = 8$ cm b $x = 11.25$ cm, $y = 6.4$ cm
 c $x = 45$ cm, $y = 26$ cm
- 10 a, b
- 
- c $3 \times 180^\circ = 540^\circ$
- 11 a $x = 40^\circ$ (Angle sum of a pentagon is 540° .)
 b $x = 87.5^\circ$ (Angle sum of a hexagon is 720° .)
- 12 150° 13 45°

CHAPTER 8 BIVARIATE DATA ANALYSIS

Diagnostic test

- | | | | | |
|------|------|------|-----|------|
| 1 D | 2 D | 3 C | 4 D | 5 D |
| 6 D | 7 A | 8 B | 9 D | 10 A |
| 11 B | 12 C | 13 A | | |

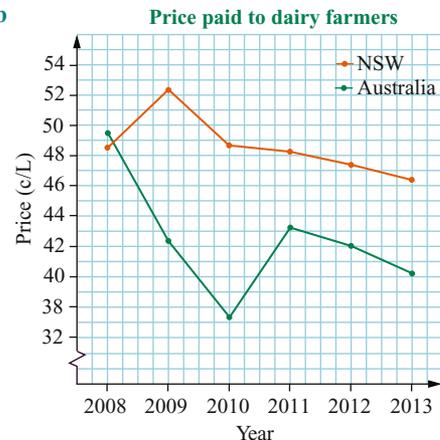
Exercise 8A

- | | | | |
|-------|--|---------------------|------------|
| 1 a i | Weight, age | ii | Weight |
| b i | Value, weight | ii | Value |
| c i | Lottery prize share, number of winners | | |
| | ii | Lottery prize share | |
| d i | Goods sold, commission | ii | Commission |
| e i | Hours worked, income | ii | Income |
| f i | Average speed, time | ii | Time |
| g i | Income earned, tax paid | ii | Tax paid |
| h i | Speed, distance | ii | Distance |

Exercise 8B

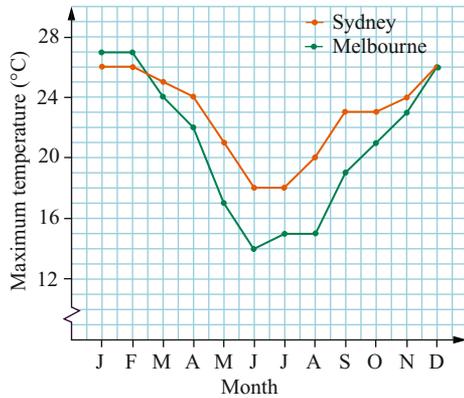
- 1 a i October ii June
 b The index increased from Jan to Feb, decreased from Feb to Mar, increased from Mar to Apr, decreased significantly from Apr to Jun then increased considerably from Jun to Oct, decreasing again in Nov before slightly increasing in Dec.
 c For both years there was an upwards trend in the early months of the year, a dip in May–June, followed by another upwards trend.
- 2 a i 33% ii 44%
 b Steady decrease from 1994 to 1998, then a significant increase from 1998 to the year 2000, where there was a slight decrease for the next 4 years followed by a steady increase to 2012.
 c 1998 or 1999
- 3 a i \$0.87 ii \$0.91
 b i \$0.68 ii \$0.81
 c \$US
 d March 2012 to March 2013; \$1.04
 e September 2012 to March 2013; \$0.81
 f \$0.10, March–September 2013
 g Yes
 h The exchange rate for both the \$US and the € increased steadily from March 2009 to March 2011. Both decreased in September 2011, then increased to reach maximum values before decreasing steadily from March 2013 to March 2014.

4 a, b



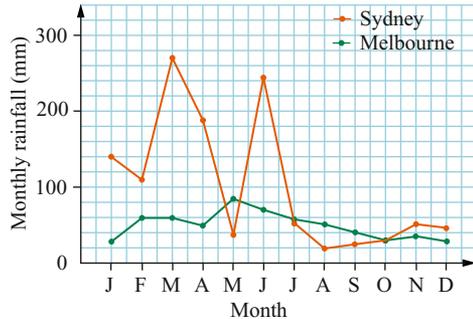
- a** After an increase in 2009, the price paid to dairy farmers in NSW has been decreasing.
b Overall, the average paid across Australia is significantly less than that paid in NSW; however, the difference in prices was less and more consistent from 2011 to 2013.

5 a Mean monthly maximum temperatures



- b** Melbourne
c **i** Melbourne **ii** 5° (April to May)
d **i** Melbourne **ii** 4° (Sept to Oct)
e Both graphs represent a trough. The temperatures at the start and end of the year are high due to the summer weather, and decrease through the colder winter months then increase again in the warmer months. Melbourne had a bigger trough as it had a colder winter.

6 a Monthly rainfall in 2012

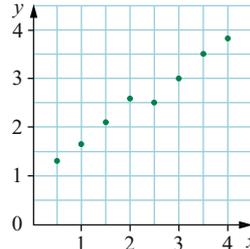


- b** Sydney
c **i** August **ii** March
d **i** January and October **ii** May
e Sydney
f As we would expect, in Melbourne the rainfall increases over the middle of the year (winter) and decreases over the warmer (summer) months. In Sydney larger than expected rainfalls in March, April and June resulted in two large peaks.
7 a **i** 460 000 **ii** 510 000
b **i** 2003–04 and 2009–10 **ii** 30 000
c **i** Yes
ii 2005–06, 2008–09, 2010–11
iii 2008–09
d Overall, the graph is increasing steadily over time.

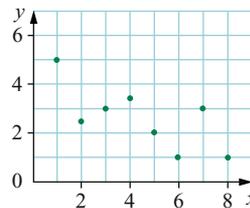
- 8 a** **i** 107 000 **ii** 127 500
b **i** 2012 **ii** 2002
c 2011–12 **d** 2001–02 and 2009–10
e Overall, the graph is steadily increasing over time with the exception of significant decreases in 2002, 2010, 2011 and a smaller decrease in 2013.

Exercise 8C

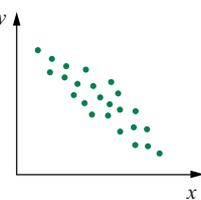
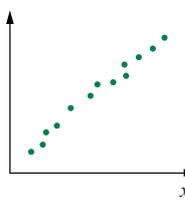
- 1 a** Strong positive **b** Strong negative
c Weak positive **d** Weak negative
e No association **f** Strong negative
2 a **i** **ii** Strong positive



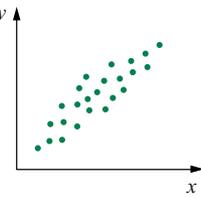
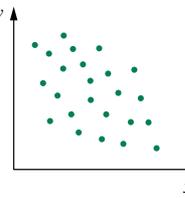
- b** **i** **ii** Weak negative



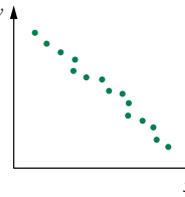
- 3 a** **b**



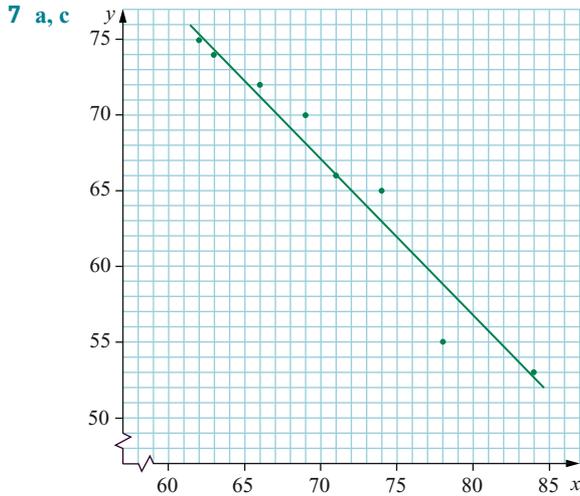
- c** **d**



- e**



- 4 a** **i** 68 **ii** 75 **iii** 86
b **i** 67 **ii** 74 **iii** 87
5 a **i** 32/m² **ii** 60/m²
b **i** 0.24 kg/m² **ii** 0.74 kg/m²
6 a **i** \$7000 **ii** \$9000 **iii** \$12 500
b **i** \$35 000 **ii** \$80 000 **iii** \$130 000



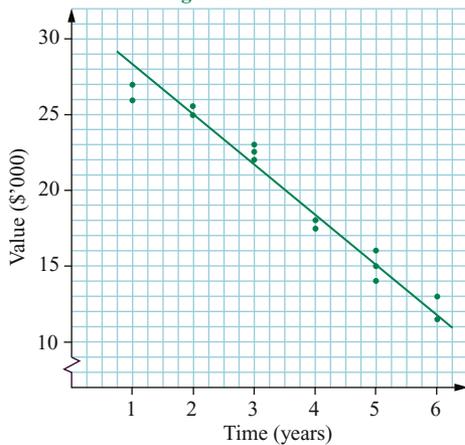
b Strong negative

d i 67

ii 77

8 a, c

Age and value of a car

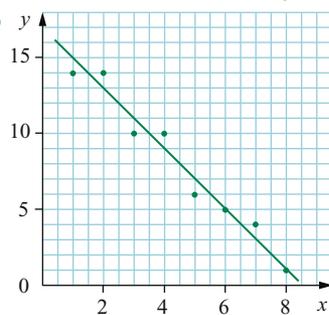


b Strong negative

d i \$20 000

ii 5 years

9 a, b

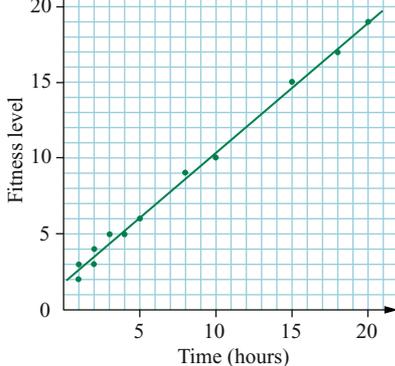


c $y = -2x + 17$

d i -3

ii 2.5

10 a, c



b Strong positive

d Fitness = $0.85 \times \text{time} + 1.8$

e i 12

ii 16.7 h

Language in mathematics

1 a Single variable data analysis examines one variable only; for example, length of foot. Bivariate data compares two variables; for example, length of foot and height.

b The value of the dependent variable will depend on the value of the independent variable. The dependent variable is placed on the y-axis.

2 Line graphs allow a comparison of quantities over time.

3 a The line of best fit is drawn on a scatter plot to give an indication of the relationship between variables.

b The line of best fit enables calculations to be made relating one variable to the other.

Check your skills

1 B

2 D

3 B

4 B

5 A

6 D

7 B

8 A

Review set 8A

1 a i Number of people, size of slice of cake

ii Size of slice of cake

b i Age of car, value of car

ii Value of car

2 a i £0.61 **ii** £0.64 **iii** £0.65

b Dec 2010, June 2011, Dec 2011, Mar 2013

c i Mar 2013

ii £0.69

d £0.08

e March–June 2013

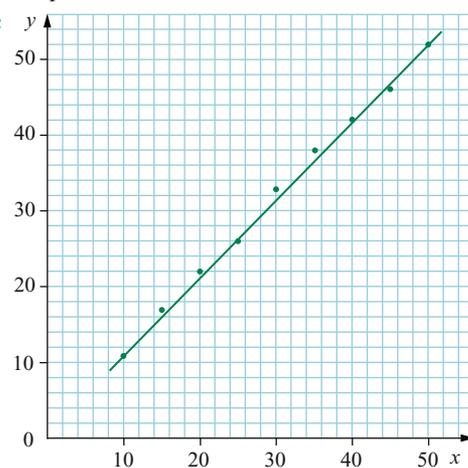
f The exchange rate increased from June to December 2010. It was relatively steady until December 2012, peaked in March 2013 and has steadily declined since then.

3 a Strong negative

b No association

c Weak positive

4 a, c



b Strong positive

d $y = x + 1.6$

e i 33.6

ii 22.4

Review set 8B

1 a i Speed, distance **ii** Distance

b i Alcohol consumed, driving ability

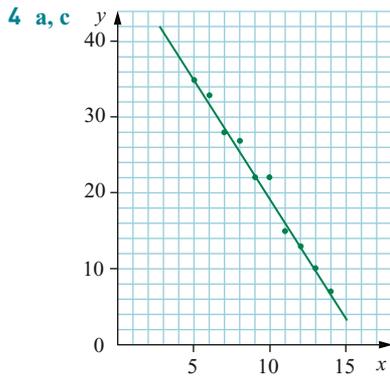
ii Driving ability

2 a i \$1.20 **ii** \$1.88 **iii** \$1.64

b i \$2.00

ii Oct 2013

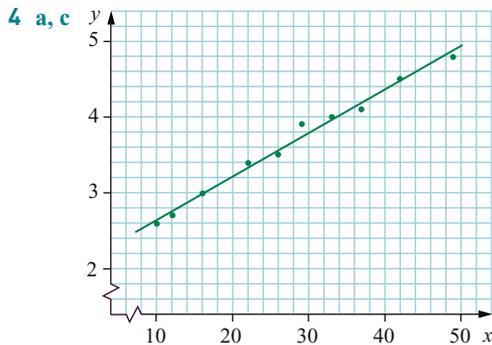
- c** **i** Apr–Oct 2012 **ii** Oct 2011–Apr 2012
d Every 6 months the dividends increase then decrease; over time the amounts are increasing.
- 3 a** Weak negative **b** Strong positive
c No association



- b** Strong negative
d $y = -3.2x + 51.4$
e **i** 3.4 **ii** 9.8

Review set 8C

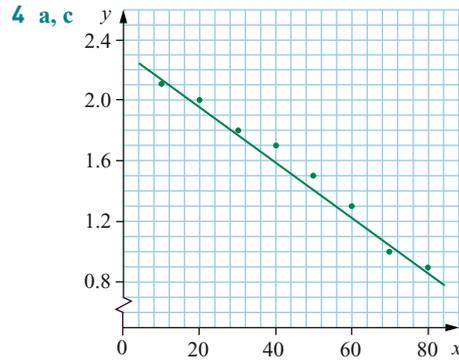
- 1 a** **i** Number of police cars, number of accidents
ii Number of accidents
- b** **i** Radius, volume **ii** Volume
- 2 a** **i** 100 000 **ii** 80 000
iii 160 000
- b** **i** 75 000 **ii** Year 2000
c 2010
- d** There was a steady decrease in employment from 1992 to 2000. Employment then started increasing, and continued to increase for the next 12 years.
- 3 a** Weak positive **b** Strong positive
c Weak negative



- b** Strong positive **d** $y = 0.06x + 2$
e **i** 3.8 **ii** 20

Review set 8D

- 1 a** **i** Speed, time **ii** Time
b **i** Number of rainy days, number of umbrellas sold
ii Number of umbrellas sold
- 2 a** **i** 640 000 **ii** 670 000 **iii** 910 000
b **i** 610 000 **ii** 2004
c 2005–06
- d** After a slight decrease in population in 2004, the population significantly increased until 2008 when the increase in the population slowed down.
- 3 a** No association **b** Strong negative
c Weak positive



- b** Strong negative **d** $y = -0.018x + 2.3$
e **i** 1.85 **ii** 39

CHAPTER 9 FINANCIAL MATHEMATICS

Diagnostic test

- 1** B **2** D **3** A **4** B **5** D
6 A **7** A **8** D **9** A **10** D
11 C **12** B **13** D **14** A **15** B
16 B **17** A **18** B

Exercise 9A

1

Year	Principal	Interest	Balance
1st	\$5000	\$200	\$5200
2nd	\$5200	\$208	\$5408
3rd	\$5408	\$216.32	\$5624.32
Total		\$624.32	

The investment is \$5624.32 and interest earned is \$624.32.

2

Year	Principal	Interest	Balance
1st	\$9000	\$180	\$9180
2nd	\$9180	\$183.60	\$9363.60
3rd	\$9363.60	\$187.27	\$9550.87
Total		\$550.87	

- a** \$9550.87 **b** \$550.87

3

Year	Principal	Interest	Balance
1st	\$8400	\$504	\$8904
2nd	\$8904	\$534.24	\$9438.24
3rd	\$9438.24	\$566.29	\$10 004.53
Total		\$1604.53	

- a** \$10 004.53 **b** \$1604.53

- 4 a** The constant multiplier = $100\% + 4\%$
 $= \frac{104}{100} = 1.04$
 Balance at end 1st year = $\$5000 \times 1.04 = \5200
 Balance at end 2nd year = $\$5200 \times 1.04 = \5408
 Balance at end 3rd year = $\$5408 \times 1.04 = \5624.32

- b** Interest = $\$5624.32 - \$5000 = \$624.32$

- 5 a** \$13 891.50 **b** \$1891.50

Year	Principal	Balance
2nd	\$25 750	\$26 522.50
3rd	\$26 522.50	\$27 318.18

- a \$27 318.18 b \$2318.18

Year	Principal	Balance
2nd	\$16 000	\$17 280
3rd	\$17 280	\$18 662.40

- a \$18 662.40 b \$2662.40

Year	Principal	Balance
1st	\$5800	\$6061
2nd	\$6061	\$6333.75
3rd	\$6333.75	\$6618.76

- a \$6618.76 b \$818.76
 9 a \$15 035.54 b \$835.54
 10 a \$19 710.80 b \$3710.80
 11 a i \$13 640 ii \$13 856.83
 b Compound interest by \$216.83
 12 a i \$36 720 ii \$37 305.85
 b Compound interest by \$585.85

Exercise 9B

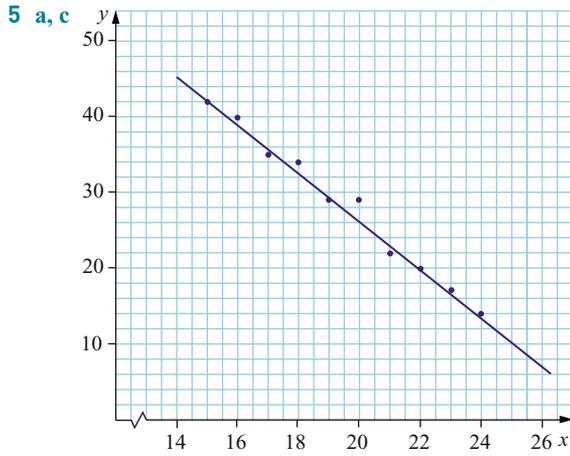
- 1 a $A = 6000\left(1 + \frac{3}{100}\right)^5$
 $= 6000(1.03)^5$
 $= \$6955.64$
 b Interest = \$6955.64 - \$6000
 $= \$955.64$
 2 a \$27 065.34 b \$9065.34
 3 a \$46 928.44 b \$21 928.44
 4 a \$7220.10 b \$2220.10
 5 a $r = \frac{12}{12}\% = 1\%$ per month
 $n = 3 \times 12 = 36$ months
 $A = 10\,000\left(1 + \frac{1}{100}\right)^{36}$
 $= 10\,000(1.01)^{36}$
 $= \$14\,307.69$
 b $r = \frac{12}{4}\% = 3\%$ per quarter
 $n = 3 \times 4$
 $= 12$ quarters
 $A = 10\,000\left(1 + \frac{3}{100}\right)^{12}$
 $= 10\,000(1.03)^{12} = \$14\,257.61$
 c $r = \frac{12}{2}\% = 6\%$ per 6 months
 $n = 3 \times 2 = 6$ six-month periods
 $A = 10\,000\left(1 + \frac{6}{100}\right)^6$
 $= 10\,000(1.06)^6 = \$14\,185.19$
 6 a \$6352.45 b \$6344.93
 c \$6333.85
 7 a i \$23 232.34 ii \$23 223.68 iii \$23 210.82
 b i \$3232.34 ii \$3223.68
 iii \$3210.82; monthly
 8 D

- 9 Daily interest rate = $\frac{23}{365}\%$
 $A = 500\left(1 + \left(\frac{23}{365} \div 100\right)\right)^{11}$
 $= \$503.48$
 Interest charged = \$3.48

- 10 a \$5.02 b \$3.39 c \$2.62
 d \$4.33 e \$2.00

Exercise 9C

- 1 $4000 = P\left(1 + \frac{6}{100}\right)^5$
 $= P(1.06)^5$
 $\frac{4000}{(1.06)^5} = P$
 $P = \$2989.03$
 2 \$4577.37 3 \$841.27
 4 \$647.04 5 \$334.64
 6 $R = 8\% \div 4 = 2\%$
 $n = 3 \times 4 = 12$
 $A = P(1 + R)^n$
 $3000 = P(1 + 0.02)^{12}$
 $= P(1.02)^{12}$
 $P = \frac{3000}{(1.02)^{12}}$
 $= \$2365.48$
 7 \$5424.52 8 \$349.15
 9 \$944.41 10 \$8254.65
 11 \$386.28
 12 $A = 1200, P = 1000$ and $n = 4$
 $A = P(1 + R)^n$
 $1200 = 1000(1 + R)^4$
 $\frac{1200}{1000} = (1 + R)^4$
 $1.2 = (1 + R)^4$
 $\sqrt[4]{1.2} = 1 + R$
 $R = 1.0466 - 1 \approx 0.0466$
 The interest rate required is 4.7%.
 13 8.1% 14 3.4%
 15 5.2% 16 3.6%
 17 $A = 1200, P = 900$ and $n = 6 \times 12 = 72$
 $A = P(1 + R)^n$
 $1200 = 900(1 + R)^{72}$
 $\frac{1200}{900} = (1 + R)^{72}$
 $1.333 = (1 + R)^{72}$
 $\sqrt[72]{1.333} = 1 + R$
 $R = 1.004 - 1 = 0.004$
 $r = 0.004 \times 12 \times 100 = 4.8\%$
 18 4.5% 19 5.6%
 20 3.9% 21 4.8%
 22 3.3%
 23 $A = P(1 + R)^n$
 $2020 = 1500(1 + 0.051)^n = 1500(1.051)^n$
 Try $n = 4$: $1500(1.051)^4 = \$1830.22$ too small
 Try $n = 8$: $1500(1.051)^8 = \$2233.12$ too big
 Try $n = 6$: $1500(1.051)^6 = \$2021.66$ very close
 The time is approximately 6 years.
 24 9 years 25 7 years
 26 11 quarters 27 6 quarters
 28 30 months 29 10 months



- b** Strong negative
e i 6.8
6 a i \$4037.84
b i \$23 820.32
c i \$18 375
iii Compound by \$317.73
d i \$22 347.69
iii \$22 203.66
e Interest is \$6.56
f i \$16 357.75
g About 4 years
- d** $y = -3.2x + 90$
ii 19.7
ii \$437.84
ii \$3820.32
ii \$18 692.73
ii \$22 289.21
ii \$21 642.25

CHAPTER 10 BINOMIAL EXPRESSIONS AND QUADRATICS

Diagnostic test

- 1 A 2 B 3 B 4 D 5 D
 6 A 7 D 8 B 9 B 10 A
 11 D 12 C 13 B 14 C 15 B
 16 D 17 C 18 A 19 B 20 B
 21 D 22 C 23 B

Exercise 10A

- 1 Let breadth be x cm.

Then length is $x + 6$ cm.

Area = length \times breadth

$$A = (x + 6) \times x$$

$$= x(x + 6)$$

$$= x^2 + 6x \text{ cm}^2$$

2 $x^2 + 5x$

4 $\frac{1}{2}(x^2 + 3x)$

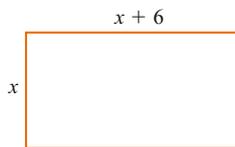
6 $\frac{1}{2}(x + 7)(x + 2)$

7 Area = $(x + 4)(x + 6)$

$$A = x(x + 6) + 4(x + 6)$$

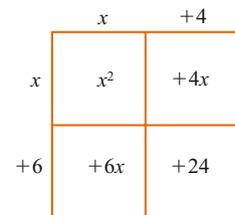
$$= x^2 + 6x + 4x + 24$$

$$= x^2 + 10x + 24$$



3 $x^2 + 3x$

5 $(x + 3)(x + 1)$



8 a $x^2 + 6x + 5$

c $x^2 + 11x + 24$

b $x^2 + 9x + 14$

9 a $(x + 3)(x + 7) = x(x + 7) + 3(x + 7)$

$$= x^2 + 7x + 3x + 21$$

$$= x^2 + 10x + 21$$

b $(x - 5)(x - 2) = x(x - 2) - 5(x - 2)$

$$= x^2 - 2x - 5x + 10$$

$$= x^2 - 7x + 10$$

10 a $x^2 + 6x + 8$

c $x^2 + 4x - 12$

e $x^2 - 2x - 15$

g $x^2 - 7x - 30$

i $x^2 + 6x - 7$

k $x^2 - 15x + 54$

11 a 21 b 71

e 53 f -35

i 45 j -187

12 a $2x^2 + x - 3$

c $3x^2 - 20x + 25$

e $6x^2 + 23x - 18$

g $12x^2 - x - 1$

i $8x^2 + 2x - 15$

k $10x^2 - 31x + 15$

13 a $x^2 - 10$

c $x^2 - 7x - 14$

e $3x^2 - 8x + 14$

g $-x^2 + x + 10$

i $-2x^2 + 13x - 6$

b $x^2 - x - 6$

d $x^2 + x - 12$

f $x^2 + 4x - 45$

h $x^2 - 11x + 28$

j $x^2 - 5x + 6$

l $x^2 - 15x + 36$

c 23 d 55

g -6 h -12

k 234 l 52

b $2x^2 - 21x + 40$

d $9x^2 - 9x - 10$

f $16x^2 - 25$

h $6x^2 - 19x + 10$

j $49x^2 - 4$

l $12x^2 - 17x - 5$

b $6x^2 + 11x - 4$

d $x^2 + 2x + 10$

f $10x - 4$

h $-2x^2 + 16x + 5$

Exercise 10B

1 a $(x + 3)^2 = x^2 + 2 \times x \times 3 + 3^2$

$$= x^2 + 6x + 9$$

b $(2x - 5)^2 = (2x)^2 - 2 \times 2x \times (-5) + (-5)^2$

$$= 4x^2 - 20x + 25$$

2 a $x^2 + 4x + 4$

c $y^2 + 20y + 100$

e $4x^2 + 12x + 9$

g $25x^2 + 40x + 16$

i $16 + 24x + 9x^2$

k $25 + 20x + 4x^2$

3 a $x^2 - 4x + 4$

c $y^2 - 18y + 81$

e $4x^2 - 12x + 9$

g $9x^2 - 24x + 16$

i $25 - 20x + 4x^2$

k $25 - 30x + 9x^2$

4 a $x^2 + 10x + 25$

c $4x^2 - 28x + 49$

e $9x^2 - 30x + 25$

g $25 + 20x + 4x^2$

5 a $(x + 3)(x - 3) = x^2 - 3^2 = x^2 - 9$

b $(3x - 1)(3x + 1) = (3x)^2 - (1)^2 = 9x^2 - 1$

6 a $x^2 - 25$

c $x^2 - 36$

e $x^2 - 1$

g $9x^2 - 4$

i $49x^2 - 64$

7 a $x^2 - 20x + 100$

c $x^2 - 64$

e $x^2 + 8x + 16$

g $x^2 - 14x + 49$

i $x^2 + 24x + 144$

b $x^2 + 12x + 36$

d $9x^2 + 6x + 1$

f $16a^2 + 40a + 25$

h $9y^2 + 12y + 4$

j $1 + 4x + 4x^2$

l $16 + 40x + 25x^2$

b $x^2 - 12x + 36$

d $9x^2 - 6x + 1$

f $25a^2 - 40a + 16$

h $16y^2 - 8y + 1$

j $1 - 6x + 9x^2$

l $16 - 16x + 4x^2$

b $x^2 - 10x + 25$

d $4x^2 + 28x + 49$

f $16x^2 + 24x + 9$

h $9 - 42x + 49x^2$

b $x^2 - 16$

d $x^2 - 100$

f $4x^2 - 25$

h $25x^2 - 1$

b $x^2 + 14x + 49$

d $x^2 - 121$

f $x^2 - 49$

h $x^2 - 144$

Exercise 10C

- 1 Find two numbers that add to give 12 and whose product is 11. These numbers are 11 and 1.

$$\therefore x^2 + 12x + 11 = (x + 11)(x + 1)$$

- 2 **a** $(x + 7)(x + 1)$ **b** $(x + 6)(x + 2)$
c $(x + 12)(x + 1)$ **d** $(x + 9)(x + 1)$
e $(x + 6)(x + 4)$ **f** $(x + 10)(x + 3)$
g $(x + 6)(x + 5)$ **h** $(x + 10)(x + 2)$
i $(x + 5)(x + 4)$ **j** $(x + 6)(x + 3)$
k $(x + 18)(x + 1)$ **l** $(x + 7)(x + 6)$

- 3 As the constant term is positive, both factors have the same sign. Find two numbers that add to give -7 and whose product is 12. These numbers are -4 and -3 .

$$\therefore x^2 - 7x + 12 = (x - 4)(x - 3)$$

- 4 **a** $(x - 5)(x - 1)$ **b** $(x - 7)(x - 1)$
c $(x - 11)(x - 1)$ **d** $(x - 4)(x - 2)$
e $(x - 8)(x - 1)$ **f** $(x - 5)(x - 2)$
g $(x - 10)(x - 1)$ **h** $(x - 5)(x - 3)$
i $(x - 15)(x - 1)$ **j** $(x - 7)(x - 2)$
k $(x - 14)(x - 1)$ **l** $(x - 8)(x - 3)$

- 5 As the constant term is negative, the factors have opposite signs. Find two numbers that add to give -2 and whose product is -15 . The numbers are -5 and $+3$.

$$\therefore x^2 - 2x - 15 = (x - 5)(x + 3)$$

- 6 **a** $(x + 8)(x - 1)$ **b** $(x + 5)(x - 2)$
c $(x + 2)(x - 1)$ **d** $(x + 7)(x - 6)$
e $(x + 6)(x - 2)$ **f** $(x - 12)(x + 1)$
g $(x - 8)(x + 3)$ **h** $(x + 8)(x - 3)$
i $(x + 7)(x - 3)$ **j** $(x - 21)(x + 1)$
k $(x + 20)(x - 3)$ **l** $(x + 9)(x - 6)$
- 7 **a** $(x + 18)(x + 1)$ **b** $(x - 9)(x + 2)$
c $(x + 18)(x - 1)$ **d** $(x + 9)(x + 6)$
e $(x + 54)(x - 1)$ **f** $(x - 27)(x + 2)$
g $(x - 8)(x - 8) = (x - 8)^2$
h $(x + 16)(x - 4)$ **i** $(x - 32)(x + 2)$
j $(x - 5)(x + 7)$ **k** $(x - 3)(x + 10)$
l $(x - 5)(x - 10)$

Exercise 10D

- 1 **a** $x = \pm 3$ **b** $x = \pm 4$ **c** $x = \pm 8$
d $x = \pm 12$ **e** $x = \pm 7$ **f** $x = \pm 11$
g $x = \pm 6$ **h** $x = \pm 9$ **i** $x = \pm 10$
j $x = \pm 2$ **k** $x = \pm 15$ **l** $x = \pm 17$
- 2 **a** ± 3.46 **b** ± 7.14 **c** ± 4.36
d ± 6.86 **e** ± 9.11 **f** ± 5.29
g ± 8.25 **h** ± 9.54 **i** ± 13.89
j ± 14.14 **k** ± 6.24 **l** ± 4.12

3 $\frac{5x^2}{5} = \frac{45}{5}$
 $x^2 = 9$
 $x = \pm\sqrt{9} = \pm 3$

- 4 **a** $x = \pm 3$ **b** $x = \pm 6$ **c** $x = \pm 3$
d $x = \pm 4$ **e** $x = \pm 5$ **f** $x = \pm 4$
g $x = \pm 6$ **h** $x = \pm 7$ **h** $x = \pm 13$

5 $\frac{16x^2}{16} = \frac{49}{16}$
 $x^2 = \frac{49}{16}$
 $x = \pm\sqrt{\frac{49}{16}} = \pm\frac{7}{4}$

- 6 **a** $x = \pm\frac{7}{2}$ **b** $x = \pm\frac{9}{10}$ **c** $x = \pm\frac{4}{5}$
d $x = \pm\frac{4}{9}$ **e** $x = \pm\frac{12}{7}$ **f** $x = \pm\frac{8}{11}$
g $x = \pm\frac{7}{12}$ **h** $x = \pm\frac{10}{9}$ **i** $x = \pm\frac{13}{6}$

7 $\frac{11x^2}{11} = \frac{37}{11}$
 $x^2 = \frac{37}{11}$
 $x = \pm\sqrt{\frac{37}{11}} = \pm 1.83$

- 8 **a** $x = \pm 2.83$ **b** $x = \pm 1.73$ **c** $x = \pm 2.45$
d $x = \pm 1.90$ **e** $x = \pm 1.60$ **f** $x = \pm 2.14$
- 9 **a** $x = \pm\frac{8}{3}$ **b** $x = \pm\frac{7}{2}$
c $x = \pm\frac{10}{9}$ **d** $x = \pm\frac{5}{7}$

Exercise 10E

- 1 $9x(x + 4) = 0$
 $9x = 0$ or $x + 4 = 0$
 $x = 0$ or -4
- 2 **a** $x = 0$ or $x = -3$ **b** $x = 0$ or $x = 3$
c $x = 0$ or $x = 5$
- 3 **a** Find the factors of 10 that add to give 7 (5 and 2).
 $(x + 5)(x + 2) = 0$
 $x + 5 = 0$ or $x + 2 = 0$
 $x = -5$ or -2
- b** Find the factors of -14 that add to give 5 (7 and -2).
 $(x + 7)(x - 2) = 0$
 $x + 7 = 0$ or $x - 2 = 0$
 $x = -7$ or 2
- 4 **a** $x = 4$ or -1 **b** $x = -1$ or -4
c $x = -6$ or 4 **d** $x = -2$ or -5
e $x = 1$ or 4 **f** $x = -5$ or -5
g $x = -9$ or 2 **h** $x = -15$ or 4
i $x = 3$ or 8 **j** $x = 6$
k $x = -15$ or 1 **l** $x = -2$ or -6

Language in mathematics

- 1 **a** 18
b For his contribution to science, 1835
c 22
d Physics, astronomy, algebra
e Someone who shuns other people
- 2 **a** Expanding is the opposite of factorising.
b Always factorise using the highest possible common factor.
c Multiply all the terms in the brackets by the term outside the brackets.
- 3 To check a factor is a solution you can either expand your answer or substitute a number into the question and the answer to see if they are equal.

Check your skills

- 1 A 2 C 3 A 4 B 5 D
6 C 7 B 8 D 9 A 10 A
11 B

Review set 10A

- 1 **a** $x^2 + 2x - 15$ **b** $6x^2 - 37x + 56$
c $x^2 - 16x + 64$ **d** $9x^2 - 24x + 16$
e $x^2 - 4$ **f** $9x^2 - 25$

- 2 20
 3 a $(x + 8)(x - 1)$ b $(x + 3)(x - 2)$
 c $(x - 5)(x - 2)$
 4 a $x = \pm\sqrt{12}$ b $x = \pm\frac{7}{\sqrt{5}}$
 5 a $x = 1$ or 7 b $x = 1$ or -12
 c $x = -1$ or 5

Review set 10B

- 1 a $x^2 - 9x - 22$ b $12x^2 - 41x + 24$
 c $x^2 + 8x + 16$ d $64x^2 - 80x + 25$
 e $x^2 - 9$ f $25x^2 - 9$
 2 31
 3 a $(x - 2)(x - 1)$ b $(x + 5)(x - 4)$
 c $(x - 6)(x + 4)$
 4 a $x = \pm 4$ b $x = \pm\sqrt{\frac{2}{5}}$
 5 a $x = 0$ or 2 b $x = -1$ or 8 c $x = 2$ or -5

Review set 10C

- 1 a $x^2 + x - 12$ b $6x^2 - 29x + 35$
 c $x^2 - 8x + 16$ d $16p^2 - 40p + 25$
 e $x^2 - 9$ f $16x^2 - 9$
 2 Both expressions equal 49.
 3 a $(x - 4)(x + 3)$ b $(x + 8)(x + 1)$
 c $(x + 6)(x - 5)$
 4 Cannot take the square root of a negative number.
 5 a $x = 4$ or -5 b $x = 5$ or -1
 c $x = 1$ or 8

Review set 10D

- 1 a $x^2 + 9x - 22$ b $12x^2 - 22x - 14$
 c $y^2 + 6y + 9$ d $36z^2 - 60z + 25$
 e $x^2 - 81$ f $36x^2 - 25$
 2 Both expressions equal 8.
 3 a $(x + 7)(x - 3)$ b $(x + 5)(x - 3)$
 c $(x + 3)(x + 3) = (x + 3)^2$
 4 a $x = \pm 5$ b $x = \pm\frac{3}{\sqrt{7}}$
 5 a $x = -3$ or -5 b $x = -2$ or 10 c $x = 3$ or 4

CHAPTER 11 BOX PLOTS

Diagnostic test

- 1 B 2 C 3 C 4 A 5 C
 6 A 7 C 8 B 9 D 10 B
 11 A 12 B

Exercise 11A

- 1 a Mean = $\frac{4 + 5 + 6 + 6 + 7 + 9 + 9 + 9 + 10}{9} = 7.2$
 b Mode: the mode is the score that occurs most often.
 As 9 occurs 3 times, the mode = 9.
 c Median: cross out 4, 5, 6, 6, 7, 9, 9, 9, 10 from either end.
 The middle score is 7. Median = 7
 d Range = highest score - lowest score
 = $10 - 4 = 6$
 2 a 5.4 b 4.4 c 13.2 d 22.5
 e 55.4 f 4.3 g 102.3 h 3.9
 3 a 8 b 2 c none d 20
 e 57 f 3 g 101 h 3

- 4 a 5 b 4 c 13 d 21.5
 e 56 f 4 g 101.5 h 3
 5 a 8 b 8 c 6 d 7
 e 8 f 9 g 4 h 9
 6 a 5 b 16.3 c 15 d 16
 7 a i 4.5 ii 1.5 iii Reduced by 3
 b i 24.5 ii Increased by 20
 c i 4.5 ii 7.5 iii Increased by 3
 d i 7 ii 7 iii Unchanged
 8 a i 7 ii 6.3 iii 7 iv 6.5
 b The mode as it is the most popular so they need more of that size.
 9 a \$2 030 000 b \$67 667
 c i 25 ii 5
 d \$55 000 e \$50 000
 f i Mean ii Mode
 g Median: half the employees earn more, half earn less.
 10 a Mean = 76.1, median = 17
 b Mean = 17.2, median = 16.5
 c Mean
 11 a Mean = 68.3, median = 68.5
 b Mean = 58.6, median = 64
 c Outlier d Mean
 12 Let S = sum of scores.
 $\frac{S}{8} = 11.5$
 $S = 11.5 \times 8 = 92$
 The sum of the scores is 92.
 13 a 60 b 104.4
 c 3144 km d \$185 604
 14 $\frac{13 + 10 + 6 + 9 + x}{5} = 11$
 $\frac{38 + x}{5} = 11$
 $38 + x = 55$
 $x = 55 - 38 = 17$
 15 a 9 b 15 c 15 d 5 e 37
 16 Let S = sum of scores
 $\frac{S}{8} = 4.5$
 $S = 8 \times 4.5 = 36$
 New mean = $\frac{36 + 6 + 8}{8 + 2}$
 = $\frac{50}{10} = 5$
 17 a 17.25 b 25.3 c 8.7
 18 17.7
 19 10.1 cm 20 12 and 6

Exercise 11B

- 1 a Median: cross out from either end to find the median.
~~17~~ 23 23 ~~27~~ 29 30 34
 Median = 27
 b Quartiles: cross out in each half to find the quartiles.
~~17~~ 23 23 ~~27~~ 29 30 34
 Lower quartile = 23
 Upper quartile = 30
 c Interquartile range = upper quartile - lower quartile
 = $30 - 23 = 7$
 2 a 27 b 20 c 30 d 10
 3 a i 7 ii 3 iii 10 iv 7
 b i 18 ii 15 iii 20 iv 5
 c i 345 ii 295 iii 485 iv 190
 d i 2.8 ii 1.5 iii 4.2 iv 2.7

- 4 There are 10 scores so the median (middle quartile) is the average of 5th and 6th scores.

Cross out from each end.

~~8~~ ~~8~~ ~~12~~ ~~12~~ 13 | 14 ~~14~~ ~~17~~ ~~19~~ ~~20~~

$$\text{Median} = \frac{13 + 14}{2} = 13.5$$

There are now 5 scores in each half.

The lower quartile is the middle of these 5 scores:

~~8~~ ~~8~~ 12 | ~~12~~ ~~13~~

Lower quartile = 12

The upper quartile is the middle of these 5 scores:

~~14~~ ~~14~~ 17 | ~~19~~ ~~20~~

Upper quartile = 17

$$\begin{aligned} \text{Interquartile range} &= \text{upper quartile} - \text{lower quartile} \\ &= 17 - 12 = 5 \end{aligned}$$

- 5 a i 11 ii 9 iii 11 iv 2
 b i 21 ii 17 iii 22 iv 5
 c i 20.5 ii 15.5 iii 25 iv 9.5
 d i 26.5 ii 17 iii 36.5 iv 19.5
 6 a 7 b 2 c 5 d 2.5
 e 6 f 5 g 5 h 19
 i 11 j 9

- 7 Find the median by crossing off from each end.

~~1~~ ~~1~~ ~~2~~ ~~2~~ ② | 3 ~~4~~ ~~5~~ ~~6~~

Median = 2

The middle score in the set 1, 1, 2, 2 is:

$$Q_1 = \frac{1 + 2}{2} = 1.5$$

The middle score in the set 3, 4, 5, 6 is:

$$Q_3 = \frac{4 + 5}{2} = 4.5$$

$$\text{Interquartile range} = 4.5 - 1.5 = 3$$

- 8 a 21 b 25 c $6\frac{1}{2}$
 d 20 e 9 f 6

- 9 There are 15 scores.

Cross off from each end to find the median.

Stem	Leaf
7	1 1 ② 2 3 3 7 ⑧ 9
8	2 3 ③ 6 8 8

Median = 78

Cross off again in the lower group to find lower quartile.

$Q_1 = 72$

Cross off again in the upper group to find upper quartile.

$Q_3 = 83$

$$\text{Interquartile range} = 83 - 72 = 11$$

- 10 a 14 b 18.5

- 11 a i Range = 140 ii IQR = 6

b The outlier of 148 affects the range but not IQR.

Exercise 11C

- 1 Median = 40

Upper quartile = 55

Lower quartile = 29

Highest score = 63

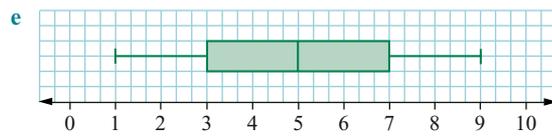
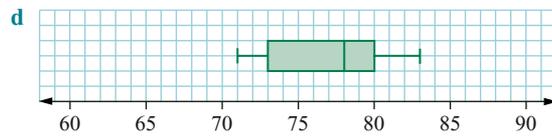
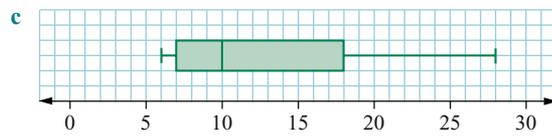
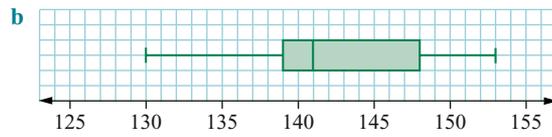
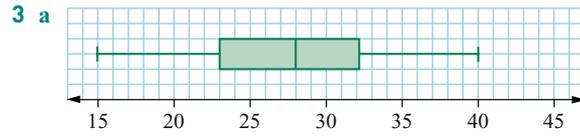
Lowest score = 15

$$\text{Range} = 63 - 15 = 48$$

$$\text{Interquartile range} = 55 - 29 = 26$$

- 2 a i 45 ii 10 iii 35 iv 24
 v 35 vi 15 vii 20
 b i 88 ii 46 iii 42 iv 71
 v 83 vi 55 vii 28

- c i 46 ii 16 iii 30 iv 29
 v 39 vi 26 vii 13
 d i 14.4 ii 10.1 iii 4.3 iv 12.5
 v 13.3 vi 10.9 vii 2.4
 e i 55 ii 49 iii 6 iv 52
 v 54 vi 50 vii 4
 f i 254 ii 210 iii 44 iv 227
 v 240 vi 215 vii 25
 g i 940 ii 780 iii 160 iv 880
 v 920 vi 820 vii 100
 h i 50 ii 15 iii 35 iv 30
 v 34 vi 27 vii 7



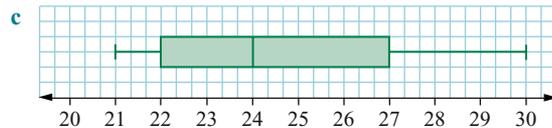
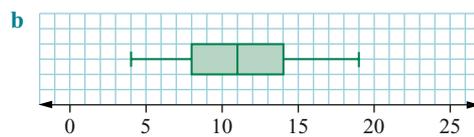
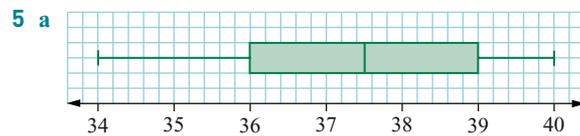
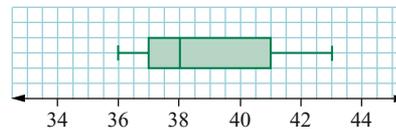
- 4 Median = 38

Upper quartile = 41

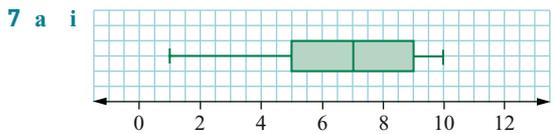
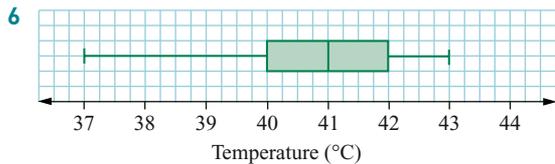
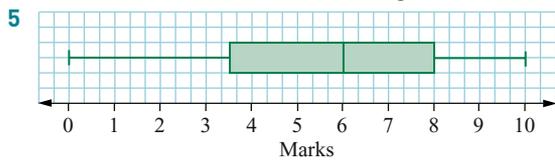
Lower quartile = 37

Highest score = 43

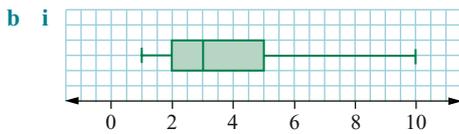
Lowest score = 36



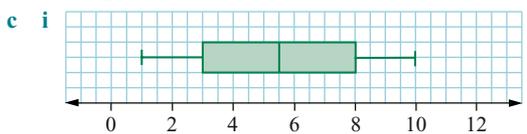
- 2 a Same minimum scores and same median value
 b i Hilife
 ii Durable, Hilife
 c i Durable = 3000, Hilife = 3500
 ii Hilife
 d i Durable = 2000, Hilife = 2000
 ii They have the same spread: 50% of the life spans are 2000 hours
 e Hilife: Higher values at the 25% and 75% mark
- 3 a i Golfer B ii Golfer A
 b Golfer A
 c i Golfer A: 6; Golfer B; 2
 ii Golfer B
 d Golfer B
- 4 a i Corolla ii Corolla
 b Corolla
 c Mazda 3: smaller range and smaller IQR
 d Mazda 3
 e Mazda 3: lower 50% of scores compared to Corolla



ii The histogram is negatively skewed and so the upper 50% of the values of the boxplot are compressed between 7 and 10.



ii The histogram is positively skewed and so the lower 50% of the values in the boxplot are compressed between 1 and 3.



ii The histogram is symmetrical and bell-shaped, and so the boxplot is also symmetrical.

- 8 a C b B c A d D

Language in mathematics

- 1 a mean b quartile
 c skewed distribution d upper quartile
 e box plots
- 2 a The range measures the spread of all the data set; the interquartile range measures the spread of the middle 50% of data.
 b Better to use when there are outliers.

- 3 a The range is unaffected by outliers.
 b The range is a measure of spread.
 c The median is the middle score when the scores are arranged in order.
- 4 The mean and median are measures of central tendency while the range and interquartile range are measures of spread. The interquartile range is the only measure of spread that is not affected by outliers.

Check your skills

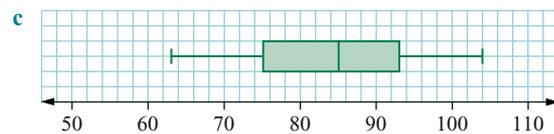
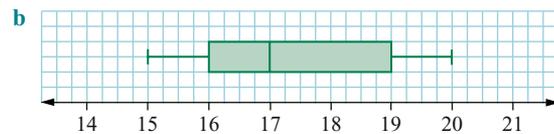
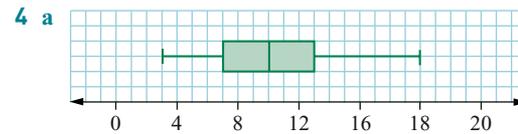
- 1 B 2 C 3 A 4 A 5 A
 6 C 7 A 8 D 9 B 10 D
 11 C 12 B 13 C 14 A 15 C
 16 B 17 D

Review set 11A

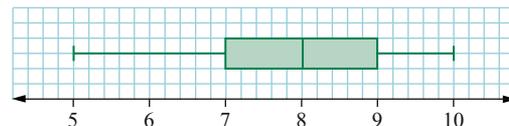
- 1 a 9.14 b 9 c 8 d 4
 2 22
 3 a 46 b 4 c 42 d 33
 e 40 f 17 g 23
- 4 a Same minimum value, same median value
 b Set B
 c Set B
 d The lower 50% of the scores in both sets have the same spread.
 e Class A, smaller range, smaller IQR: more consistent

Review set 11B

- 1 a 6.7 b 7.5 c 8 d 5
 2 a 84 b 21
 3 a i 5 ii 5
 b i 2 ii 2



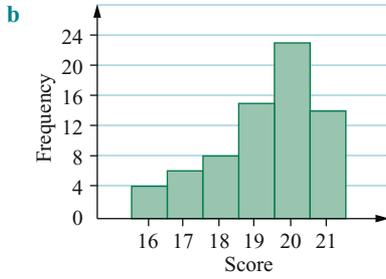
- 5 The histogram has a slight negative skew and so the median is placed slightly to the right and the middle 50% of scores are closer to the highest score than the lowest score.



Review set 11C

- 1 a 15.75 b 15 c 10 d 7
 2 37
 3 a 84 b 41 c 43 d 67
 e 81 f 53 g 28

4 a 5, 2



c Negatively skewed

5 a Set A

b Set B

Review set 11D

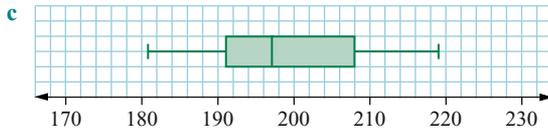
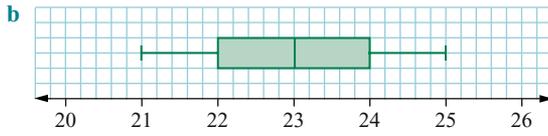
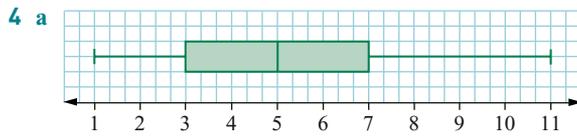
1 a 62.3 b 61.5 c 4 d 3

2 a 13 b 42

3 a i 67.6 ii Range = 51

iii Median = 67 iv IQR = 16.5

b Close to symmetrical, very slight positive skew



5 Sydney has a smaller range and IQR indicating that the temperatures over January did not vary as much as they did in Melbourne. Melbourne had more lower temperatures in January.

CHAPTER 12 NON-LINEAR RELATIONSHIPS

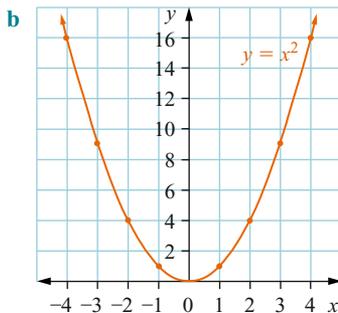
Diagnostic test

1 D 2 B 3 C 4 B 5 D

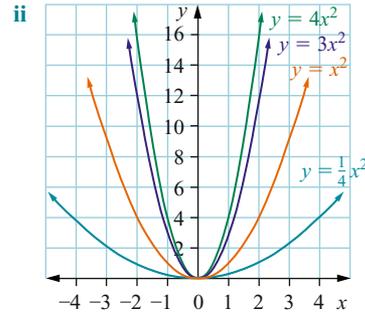
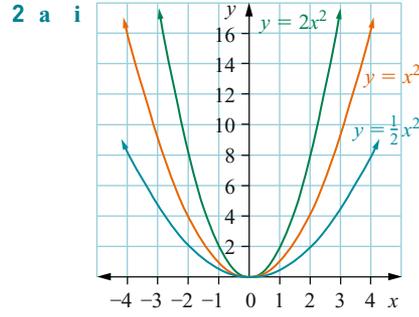
6 A 7 D 8 B

Exercise 12A

1 a	x	-4	-3	-2	-1	0	1	2	3	4
	y	16	9	4	1	0	1	4	9	16

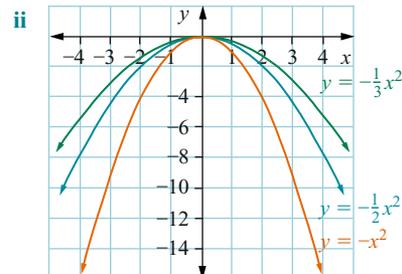
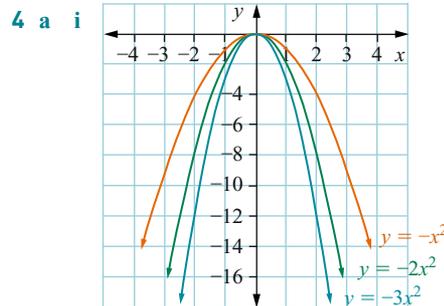
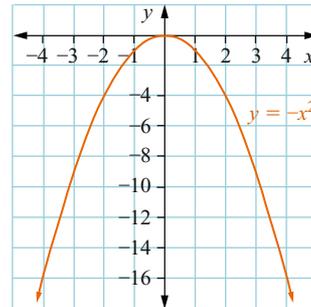


c (0, 0)

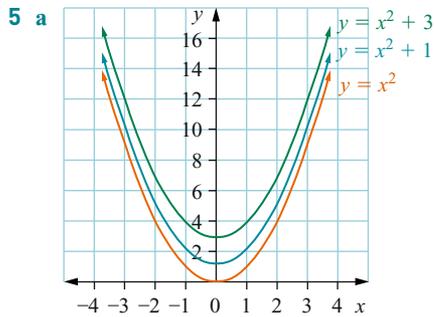


b The coefficient a affects the 'width' of parabola $y = x^2$: as a increases, the width decreases; that is, the graph becomes steeper.

3	x	-4	-3	-2	-1	-1/2	0	1/2	1	2	3	4
	y	-16	-9	-4	-1	-1/4	0	-1/4	-1	-4	-9	-16

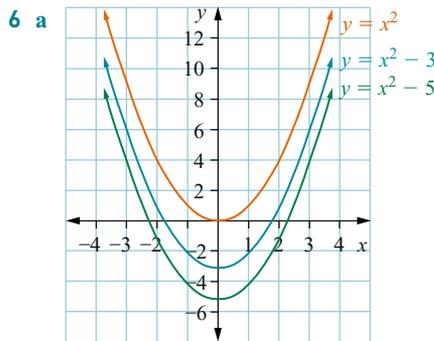


b The coefficient a affects the 'width' of the parabola; negative a inverts the parabola.



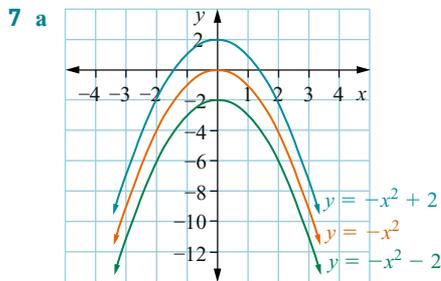
b $(0, 0), (0, 1), (0, 3)$

c The constant c affects the position of the vertex of the parabola. Positive values of c translate the parabola up c units.



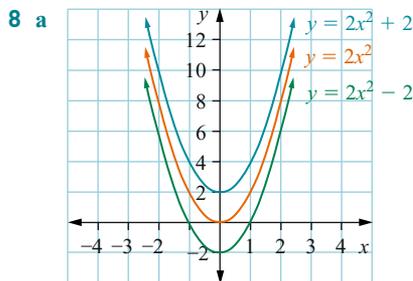
b $(0, 0), (0, -3), (0, -5)$

c The constant c affects the position of the vertex of the parabola. Negative values of c translate the parabola down c units.



b $(0, 0), (0, 2), (0, -2)$

c The constant (c) affects the position of the vertex of the parabola. Positive values of c translate the parabola up. Negative values of c translate the parabola down.



b $(0, 0), (0, 2), (0, -2)$

c The constant c affects the position of the vertex of the parabola. Positive values of c translate the parabola up c units. Negative values of c translate the parabola down.

9 a $y = x^2 - 1$ b $y = x^2 + 1$

c $y = x^2 - 3$ d $y = x^2 - 5$

e $y = x^2 + 4\frac{1}{2}$

10 a $y = x^2 + 3$ b $y = x^2 - 2$

c $y = -x^2 + 4$ d $y = -x^2 - 1$

11 a D b A c E

d B e F f C

12 $y = 3x^2 - 1$

$17 = 3x^2 - 1$

$17 + 1 = 3x^2 - 1 + 1$

$18 = 3x^2$

$x^2 = 6$

$x = \pm\sqrt{6}$

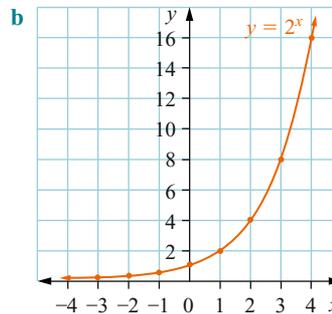
13 a $\pm\sqrt{10}$ b $\pm\sqrt{7}$ c ± 1

d $\pm 1\frac{1}{2}$ e ± 5 f $\pm\sqrt{2}$

Exercise 12B

1 a $y = 2^x$

x	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



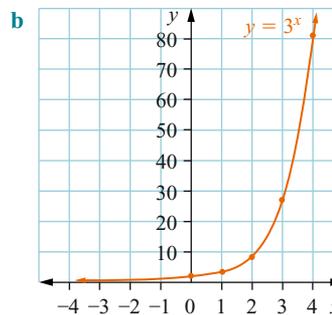
c No, graph remains above the x -axis. Negative values of x reduce the y -value but it remains positive.

d i y increases ii y decreases

e $y = 0$

2 a $y = 3^x$

x	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81

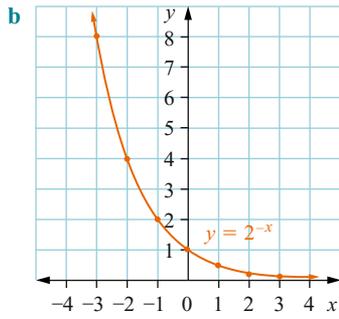


c $y = 3^x$ is steeper and both have asymptote $y = 0$.

d $y = 5^x$ is steeper than $y = 3^x$.

3 a $y = 2^{-x}$

x	-3	-2	-1	0	1	2	3	4
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

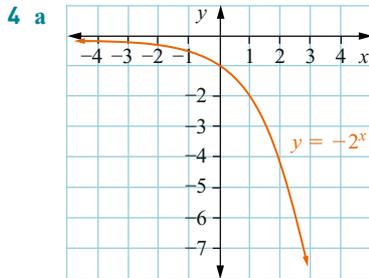


c It is a reflection of $y = 2^x$ in the y -axis.

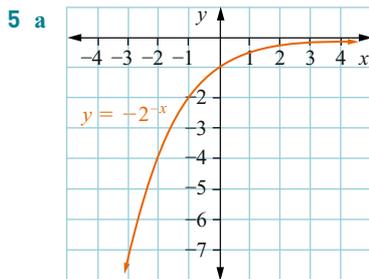
d i y decreases ii y increases

e $y = 0$ but $y \rightarrow 0$ as $x \rightarrow \infty$.

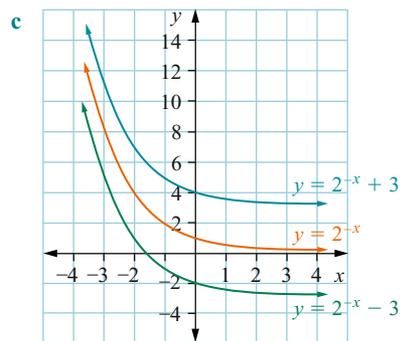
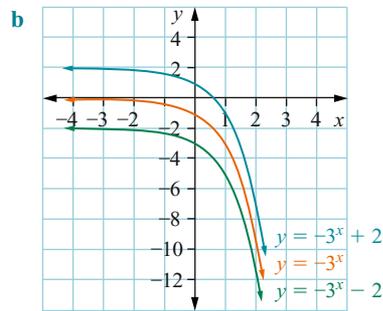
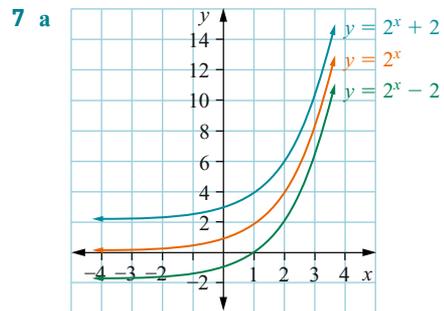
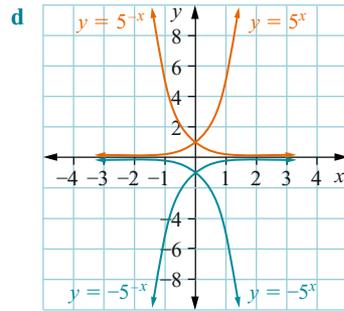
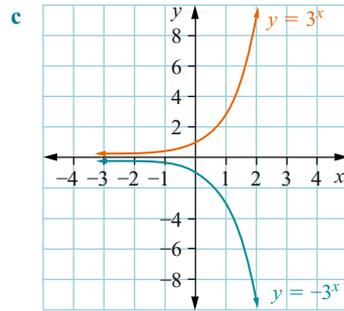
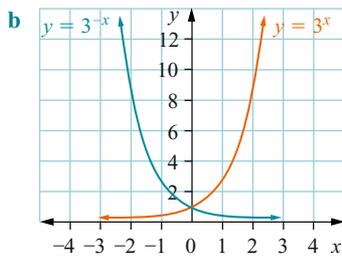
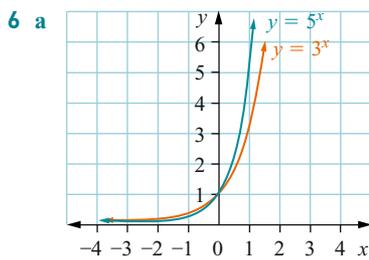
For $y = 2^x$, $y \rightarrow 0$ as $x \rightarrow -\infty$.

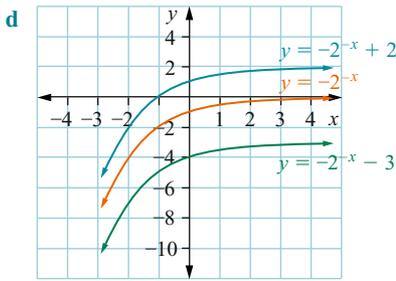


b It is a reflection of $y = 2^x$ in the x -axis.



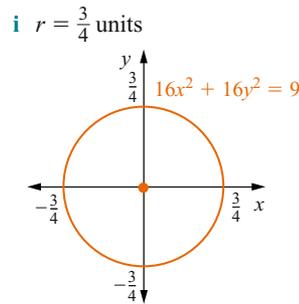
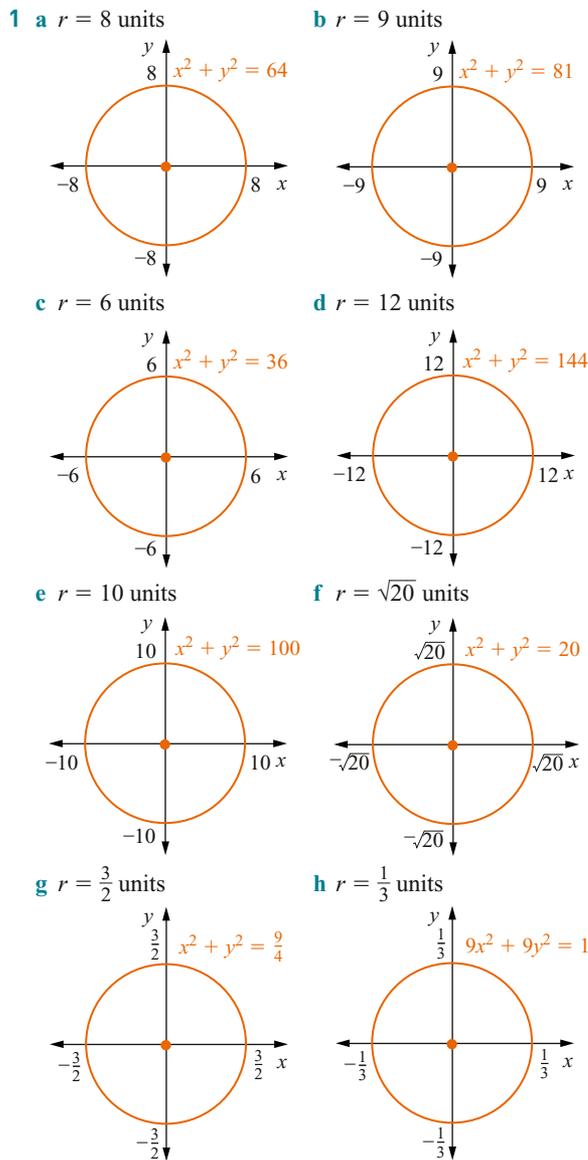
b It is a reflection of $y = 2^{-x}$ in y -axis and x -axis.





- 8 a** **i** 1 **ii** 1
 All graphs of this type must go through the point (0, 1).
b **i** -1 **ii** -1
 All graphs of this type must go through the point (0, -1).
9 Try $x = 3$ $y = 7^3 = 343$ Too small
 $x = 4$ $y = 7^4 = 2401$ correct
 If the y -coordinate = 2401, the x -coordinate = 4.
10 a $x = 4$ **b** $x = 5$ **c** $x = 3$ **d** $x = 6$

Exercise 12C



- 2 a** $x^2 + y^2 = 9$ **b** $x^2 + y^2 = 15^2 = 225$
c $x^2 + y^2 = (1.5)^2 = 2.25$ **d** $x^2 + y^2 = 7$
3 a $x^2 + y^2 = 4^2 = 16$ **b** $x^2 + y^2 = 6^2 = 36$
c $x^2 + y^2 = 3^2 = 9$ **d** $x^2 + y^2 = 7^2 = 49$
e $x^2 + y^2 = a^2$ **f** $x^2 + y^2 = \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$
4 $x^2 + y^2 = \left(\frac{\sqrt{5}}{2}\right)^2$, $x^2 + y^2 = \frac{5}{4}$, $4x^2 + 4y^2 = 5$
5 The general form of a circle with centre (0, 0) and radius r units is:

$$x^2 + y^2 = r^2$$

$$(3)^2 + (-1)^2 = r^2$$

$$10 = r^2$$

$$r = \sqrt{10}$$

Hence the equation of the circle is $x^2 + y^2 = 10$.

6 a $x^2 + y^2 = 29$ **b** $x^2 + y^2 = 45$
c $x^2 + y^2 = 74$ **d** $x^2 + y^2 = 64$
e $x^2 + y^2 = 2$
7 A, B, D, G, I, J

Exercise 12D

- 1** A, C and D **2** A and D
3 A **4** B
5 a B **b** B **c** C
6 a B **b** C **c** A **d** D
7 a C **b** A **c** E
d D **e** B **f** F
8 a C **b** D **c** B
d E **e** A **f** F

Language in mathematics

- 1 a** 12
b To his unconventional education with its interruptions
c The sequence must remain within a certain distance of the origin forever.
2 a A parabola is a graph with x^2 .
b The constant term of a parabola moves the graph up or down.
3 The graphs of exponentials and parabolas are both curves with the exponential as a number to the power of x and the parabola as x to the power of two.

Check your skills

- 1** A **2** A **3** C **4** B **5** B
6 C **7** A **8** C **9** D **10** C
11 C

CHAPTER 13 FURTHER TRIGONOMETRY

Diagnostic test

- 1 C 2 A 3 C 4 A 5 B
 6 A 7 D 8 A 9 D 10 B
 11 B 12 A 13 D 14 C 15 B

Exercise 13A

- 1 a 0.5990 b 0.9693 c 0.1425
 d 0.4187 e 0.0113 f 0.3323
 g 2.5236 h 0.7114 i 0.5321
 j 0.8353 k 0.3949 l 0.1939
- 2 a 7.3586 b 1.4716 c 11.1786
 d 22.2369 e 13.7320 f 10.8803
 g 3.5884 h 9.0191 i 29.4119
 j 2.5004 k 0.5864 l 0.1805
 m 0.3919 n 14.7202 o 41.7269
- 3 a $43^{\circ}27'$ b $62^{\circ}16'$ c $14^{\circ}3'$
 d $81^{\circ}54'$ e $22^{\circ}0'$ f $10^{\circ}1'$
 g $35^{\circ}28'$ h $72^{\circ}52'$ i $27^{\circ}54'$
 j $39^{\circ}35'$ k $68^{\circ}55'$ l $0^{\circ}4'$
- 4 a i 21° ii $21^{\circ}15'$
 b i 82° ii $81^{\circ}42'$
 c i 65° ii $64^{\circ}54'$
 d i 48° ii $47^{\circ}42'$
 e i 76° ii $76^{\circ}25'$
 f i 6° ii $6^{\circ}23'$
 g i 2° ii $2^{\circ}7'$
 h i 33° ii $33^{\circ}27'$
 i i 88° ii $88^{\circ}12'$
 j i 0° ii $0^{\circ}9'$
 k i 88° ii $87^{\circ}35'$
 l i 11° ii $11^{\circ}9'$
 m i 49° ii $48^{\circ}36'$
 n i 39° ii $38^{\circ}46'$
 o i 13° ii $12^{\circ}32'$
- 5 a i 78° ii $77^{\circ}54'$
 b i 74° ii $74^{\circ}10'$
 c i 38° ii $37^{\circ}40'$
 d i 8° ii $7^{\circ}56'$
 e i 41° ii $40^{\circ}36'$
 f i 35° ii $34^{\circ}34'$
 g i 3° ii $3^{\circ}29'$
 h i 66° ii $65^{\circ}35'$
 i i 37° ii $36^{\circ}55'$
 j i 71° ii $70^{\circ}32'$
 k i 49° ii $48^{\circ}35'$
 l i 53° ii $53^{\circ}8'$
- 6 a $45^{\circ}34'$ b $39^{\circ}56'$ c $72^{\circ}58'$
 d $25^{\circ}47'$ e $88^{\circ}30'$ f $64^{\circ}17'$
 g $36^{\circ}14'$ h $20^{\circ}26'$ i $81^{\circ}43'$

Exercise 13B

- 1 $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\sin 37^{\circ}26' = \frac{x}{18}$
 $x = 18 \times \sin 37^{\circ}26' \approx 10.9 \text{ cm}$
- 2 a 8.7 km b 8.9 cm c 1.7 m
 d 13.0 cm e 10.6 mm f 12.5 m

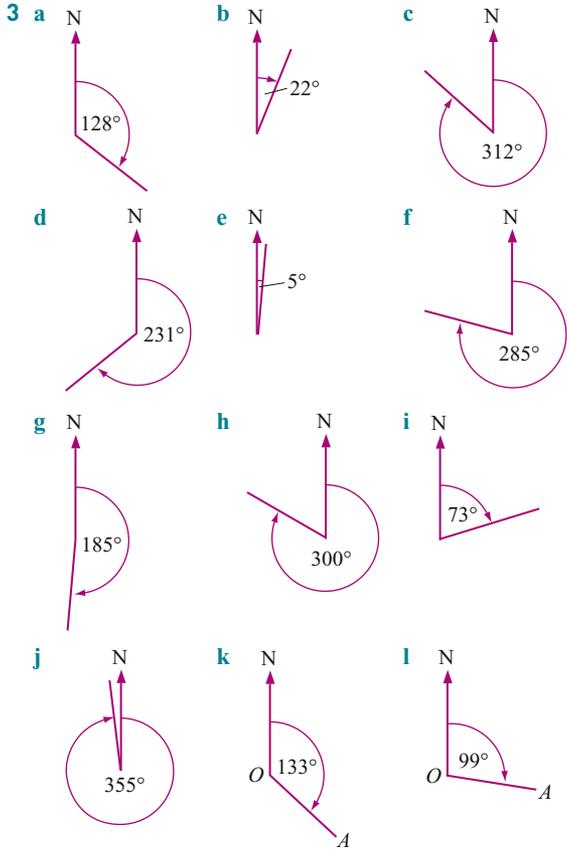
- 3 $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\sin 27^{\circ}39' = \frac{8}{x}$
 $x \sin 27^{\circ}39' = 8$
 $x = \frac{8}{\sin 27^{\circ}39'} \approx 17.2 \text{ cm}$
- 4 a 80.8 cm b 12.8 km c 153.3 mm
 5 a 8.4 m b 199.4 mm c 68.9 cm
- 6 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\cos 50^{\circ}37' = \frac{x}{8}$
 $\therefore x = 8 \cos 50^{\circ}37' \approx 5.1 \text{ m}$
- 7 a 3.9 cm b 7.3 cm c 37.8 mm
 d 1.8 m e 22.9 cm f 10.7 m
- 8 a 164.9 cm b 4.2 m c 184.5 mm
 d 4.4 m e 333.5 mm f 45.7 km
 9 a 56.4 cm b 14.4 m c 17.5 cm
 d 11.3 mm e 12.2 cm f 20.8 mm
- 10 $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
 $\tan 34^{\circ}21' = \frac{x}{4}$
 $x = 4 \tan 34^{\circ}21' \approx 2.7 \text{ m}$
- 11 a 21.9 cm b 148.8 cm c 388.7 mm
 12 a 14.9 m b 43.4 cm c 15.0 km
 13 a 8.4 cm b 35.5 mm c 59.5 m
 14 a 2.9 cm b 13.6 m c 26.0 km
 d 48.1 m e 100.7 mm f 4.3 m
 g 308.7 cm h 12.9 cm i 19.6 m

Exercise 13C

- 1 $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
 $= \frac{6}{15}$
 $\therefore \theta \approx 23^{\circ}35'$
- 2 a $38^{\circ}41'$ b $39^{\circ}48'$ c $23^{\circ}43'$
 d $43^{\circ}26'$ e $32^{\circ}24'$ f $41^{\circ}35'$
- 3 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $= \frac{8.4}{15}$
 $\therefore \theta \approx 55^{\circ}57'$
- 4 a $66^{\circ}25'$ b $41^{\circ}25'$ c $48^{\circ}11'$
 d $56^{\circ}15'$ e $48^{\circ}11'$ f $74^{\circ}5'$
- 5 $\tan \phi = \frac{\text{opposite}}{\text{adjacent}}$
 $= \frac{8.9}{15.6}$
 $\therefore \phi \approx 29^{\circ}42'$
- 6 a $63^{\circ}26'$ b $26^{\circ}42'$ c $56^{\circ}27'$
 d $56^{\circ}19'$ e $21^{\circ}15'$ f $41^{\circ}45'$
 7 a $52^{\circ}28'$ b $24^{\circ}35'$ c $37^{\circ}40'$
 d $18^{\circ}1'$ e $49^{\circ}40'$ f $17^{\circ}51'$
 g $52^{\circ}40'$ h $75^{\circ}39'$ i $37^{\circ}48'$
 j $41^{\circ}25'$ k $17^{\circ}33'$ l $20^{\circ}6'$

Exercise 13D

- 1 a 165°T b 210°T c 348°T
 d 038°T e 285°T f 008°T
 2 a 141°T b 242°T c 333°T
 d 245°T e 306°T f 223°T
 g 322°T h 157°T i 239°T
 j 012°T k 201°T l 285°T



- 4 a i 90° ii 45° iii 45°
 iv 22½° v 22½° vi 67½°
 b i 022.5°T ii 067.5°T
 iii 135°T iv 157.5°T
 v 202.5°T vi 247.5°T
 vii 292.5°T viii 315°T

Exercise 13E

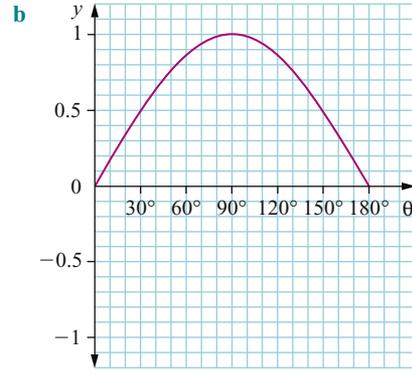
- 1 a 0.9848 b -0.5150
 c -5.1446 d 0.9808
 e -0.1530 f -0.8121
 g -0.0209 h 0.9041
 i -0.7181 j -2.5804
 k 0.8594 l -0.0851

2 a

θ	0°	10°	20°	30°	40°	50°	60°
y	0	0.17	0.34	0.5	0.64	0.77	0.87

θ	70°	80°	90°	100°	110°	120°	130°
y	0.94	0.98	1	0.98	0.94	0.87	0.77

θ	140°	150°	160°	170°	180°
y	0.64	0.5	0.34	0.17	0



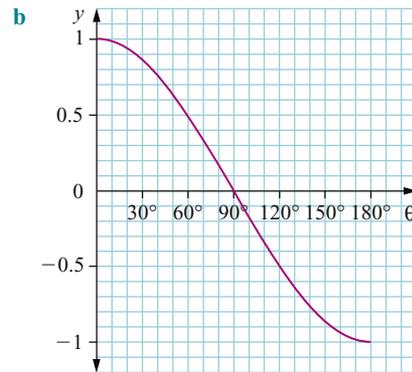
- c They are equal. d They are equal.
 e $\sin \theta = \sin (180^\circ - \theta)$

3 a

θ	0°	10°	20°	30°	40°	50°	60°
y	1	0.98	0.94	0.87	0.77	0.64	0.5

θ	70°	80°	90°	100°	110°	120°	130°
y	0.34	0.17	0	-0.17	-0.34	-0.5	-0.64

θ	140°	150°	160°	170°	180°
y	-0.77	-0.87	-0.94	-0.98	-1



- c They are equal in magnitude but opposite in sign.
 d They are equal in magnitude but opposite in sign.
 e $\cos \theta = -\cos (180^\circ - \theta)$
 4 a 0.643 b -0.643 c -1.732
 d -0.176 e -0.866 f 0.866
 g 0.342 h -0.839 i -0.342
 5 a 158°37' b 118°9' c 155°43'
 d 163°55' e 138°22' f 125°44'

Exercise 13F

- 1 Area = $\frac{1}{2}ab \sin C$
 $= \frac{1}{2} \times 7 \times 9 \times \sin 130^\circ$
 $\approx 24.1 \text{ cm}^2$
 2 a 35.45 cm² b 103.94 m² c 46.77 mm²
 d 105.72 cm² e 65.45 cm² f 240.42 cm²
 3 24.4 cm 4 64.95 cm²

Exercise 13G

- 1 $\frac{a}{\sin A} = \frac{b}{\sin B}$
 $\frac{x}{\sin 21^\circ} = \frac{20}{\sin 59^\circ}$
 $x = \frac{20}{\sin 59^\circ} \times \sin 21^\circ \approx 8.4 \text{ cm}$

- 2 a 7.0 cm b 26.1 cm c 8.8 cm
 d 12.8 cm e 8.5 cm f 41.4 cm
- 3 a 37.3 cm b 13.4 cm c 28.2 cm
 d 4.1 cm e 13.2 cm f 207.9 cm

$$4 \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{15} = \frac{\sin 28^\circ}{10}$$

$$\sin \theta = \frac{15 \sin 28^\circ}{10}$$

$$\theta = 44^\circ 46'$$

Or $\theta = 180^\circ - 44^\circ 46' = 135^\circ 14'$
 $\therefore \theta = 44^\circ 46'$ or $135^\circ 14'$

- 5 a $57^\circ 55'$, $122^\circ 5'$ b $20^\circ 56'$, $159^\circ 4'$
 c $31^\circ 56'$, $148^\circ 4'$ d $49^\circ 55'$, $130^\circ 5'$
 e $49^\circ 38'$, $130^\circ 22'$ f $33^\circ 52'$, $146^\circ 8'$

$$6 \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin \theta}{18.6} = \frac{\sin 68^\circ}{20.3}$$

$$\sin \theta = \frac{18.6 \sin 68^\circ}{20.3}$$

$$\theta = 58^\circ 10'$$

Or $\theta = 180^\circ - 58^\circ 10'$
 $= 121^\circ 50'$

But $121^\circ 50' + 68^\circ = 189^\circ 50'$, which is greater than 180° .
 $\therefore \theta = 58^\circ 10'$ is only answer.

- 7 a $52^\circ 52'$ b $50^\circ 52'$
 c $55^\circ 22'$ d $47^\circ 55'$

- 8 a 88.8 cm^2 b 124.3 cm^2

- 9 a 1
 b It becomes $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

Exercise 13H

- 1 $a^2 = b^2 + c^2 - 2bc \cos A$
 $x^2 = 13^2 + 15^2 - 2 \times 13 \times 15 \times \cos 112^\circ$
 $= 540.096$
 $x = \sqrt{540.096 \dots}$
 $\approx 23.2 \text{ m}$
- 2 a 13.0 cm b 32.4 m c 15.8 mm
 d 17.2 cm e 23.4 cm f 23.9 m

$$3 \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \theta = \frac{12^2 + 7^2 - 15^2}{2 \times 12 \times 7}$$

$$\theta \approx 100^\circ 59'$$

- 4 a $37^\circ 40'$ b $57^\circ 38'$
 c $42^\circ 23'$ d $64^\circ 30'$
 e $37^\circ 24'$ f $70^\circ 2'$
- 5 a $65^\circ 1'$, $55^\circ 35'$, $59^\circ 24'$ b $69^\circ 36'$, $26^\circ 41'$, $83^\circ 43'$
- 6 a $49^\circ 28'$ b $87^\circ 17'$
- 7 a 26.9 cm^2 b 54.6 cm^2
- 8 a $\cos 90^\circ = 0$
 b It becomes $c^2 = a^2 + b^2$, Pythagoras' rule.

Exercise 13I

- 1 a 14.2 m b 16.7 m c 24.9 m
 2 a $64^\circ 30'$ b $37^\circ 24'$ c $51^\circ 39'$
 3 a 80.8 m^2 b 80.7 m^2 c 87.8 m^2
 4 a 66.3 cm^2 b 339.0 cm^2 c 101.4 cm^2
 5 a $96^\circ 43'$ b $44^\circ 12'$ c $46^\circ 51'$
 d $121^\circ 43'$ e $43^\circ 7'$ f $145^\circ 6'$

- 6 a 126.4 m^2 b 60.3 m^2 c 36.0 m^2
 d 26.5 m^2 e 24.4 m^2 f 19.0 m^2
- 7 a Side = 21.2 km; angles = 30° ; 42°
 b Angle = 59° ; sides = 22.1 cm; 21.2 cm
 c Angles = 44° ; 49° ; 87°

Exercise 13J

- 1 203 m
 2 a 14.1 km b 10.7 km
 3 13 cm
 4 a 10 km b 076°T
 5 $17^\circ 39'$
 6 a 1067 m^2 b 132 m
 7 a $\angle PQR = 107^\circ$, $\angle RQO = 73^\circ$
 b 4.0 km c 017°T
 8 a $\angle BAC = 112^\circ$, $\angle OAC = 68^\circ$
 b $OA = 220 \text{ m}$, $OC = 560 \text{ m}$
 c 068°T , 044°T
 9 a 142° b 144 m c 113 m
 10 a 119° b 30.78 km c 14.92 km
 11 a 72° , 17.9 m
 b i 38° , 35° , 107° ii 22.2 km
 12 202 m
 13 237 m
 14 a 164.6 km b 267°T
 15 a 610 km b 224°T

Language in mathematics

- 1 a right-angled, sine
 b sine, ratio, negative, obtuse, cosine, rule
 c non-right-angled, two, side, angle
 d cosine, side, angle

Check your skills

- 1 B 2 C 3 D 4 B 5 C
 6 D 7 A 8 C 9 B 10 C
 11 A 12 B 13 A 14 D 15 B
 16 B 17 D 18 B 19 C 20 B

Review set 13A

- 1 a 22.8 cm b 12.5 cm c 20.3 m
 2 a $31^\circ 3'$ b $41^\circ 5'$ c $32^\circ 24'$
 3 a 26.2 m b 350 m
 4 a 17 m b 44°
 5 a 57 m b 69°
 6 90°

Review set 13B

- 1 a 75.5 cm b 31.7 cm c 25.9 cm
 2 a $51^\circ 4'$ b 49° c $24^\circ 52'$
 3 a 65° b $305^\circ 25' \text{T}$, 267 km
 4 a 140 m b 46°
 5 a 111 m b 120°
 6 21 km, 15 km

Review set 13C

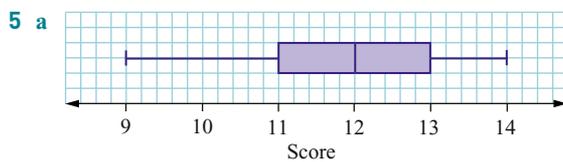
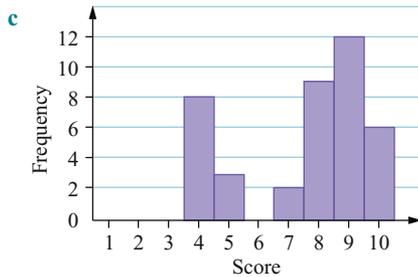
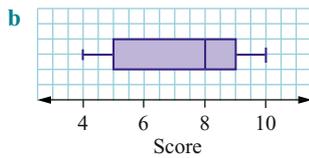
- 1 a 17.0 cm b 14.9 cm
 2 a $23^\circ 12'$ b $41^\circ 25'$ c $60^\circ 57'$
 3 a 66.1 m b 90° , $36^\circ 52'$, $53^\circ 8'$
 4 a 14 m b 41°
 5 a 56 m b 74°
 6 a 17° , 37° , 143° b 598 m c 478 m

Review set 13D

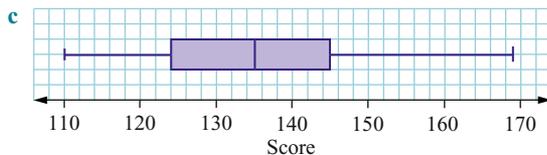
- 1 a 5.2 m b 8.1 km
 2 a 54°54' b 22°37' c 16°16'
 3 a 4.8 m b 99 km
 4 a 128 m b 41°
 5 a 114 m b 115°
 6 a 11 km b 281°T

CUMULATIVE REVIEW: 10-13

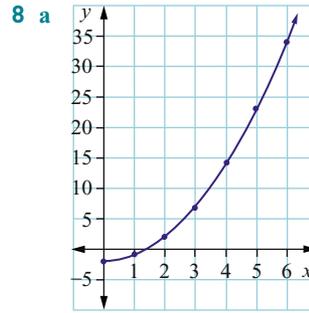
- 1 a i $x^2 + 2x - 15$ ii $2x^2 - x - 21$
 iii $x^2 + 6x + 9$ iv $x^2 - 6x + 9$
 v $49x^2 - 28x + 4$ vi $25x^2 - 4$
 b i $(x + 4)^2$ ii $(x - 5)(x + 3)$
 iii $(x + 5)(x + 2)$
 c i $x = \pm 6$ ii $x = \pm\sqrt{3}$
 d i $x = -3$ or -4 ii $x = -6$ or 2
 iii $x = 2$ or 4 iv $x = -9$ or 1
 v $x = 1$ or 9 vi $x = -9$ or 3
 2 a 14.67 b 16 c 16 d 6
 e 16.5 f 12 g 4.5
 3 a $x = 15$ b 216 c 26
 4 a i 7.525 ii 9 iii 8 iv 6
 v 9 vi 5 vii 4



- b Yes, symmetrical (very slight negative skew).
 6 a i 136.4 ii 135
 iii 122 iv 59
 b Positively skewed

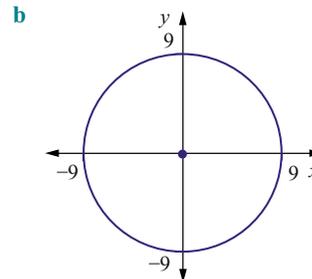
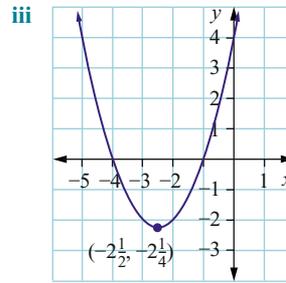


- 7 a The greatest score, the median and the interquartile range are the same in both data sets.
 b Range for set A < range for set B
 c They are the same. d Set B

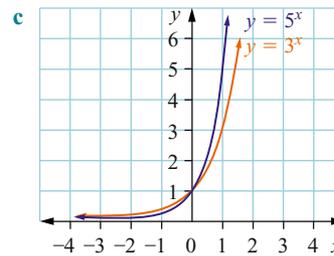


b It is the right-hand half of a parabola.

- 9 a B b D c A d C
 10 a i $y = 4$ ii $x = -4$ and -1



- i $r = 9$ units
 ii $x^2 + y^2 = \frac{9}{16}$



- 11 a i 11.6 cm ii 12.7 cm
 iii 9.7 cm iv 30°25'
 v 59°49' vi 45°48'
 b i 75.7 km ii 8.54 km, 159°27'T
 iii 71°47'
 c i 10 cm ii 34 cm
 d i 46°25' ii 46°30'
 e i 43 cm ii 9 cm
 f i 50°39' ii 94°6'
 12 a 5.44 km
 b i 154.7 km ii 304°T
 c 113 m and 136 m d 611 km
 e 16.6 km, 237°T

INDEX

- AAS (congruent triangle test) 155
- acute angles
 finding using trigonometric ratios 306–8
 non-right-angled triangles 324–5
 trigonometric ratios 305–6
- algebra 3, 38
- algebraic expressions
 expanding and simplifying 45–51, 236
 factorising 51–4, 236
- algebraic fractions 39–45
- angle geometry 152
- area
 circle 21
 sector 17–18
 triangle 325–7
- association (scatter plots) 200
- $Ax + By + C = 0$ (general form of a straight line equation) 76–8
- bimodal scores 255
- binomial products 240
 difference of two squares expansion 240, 241–2
 expanding 238–40
 perfect squares expansion 240, 241, 242
- bivariate numerical data 194–7
- box plots 266–71, 274–8
 parallel 272–3
- circle
 area 21
 graphs 291–3, 294
 compass bearings 318–22
 compass rose 318, 322
- composite solids
 surface areas 28–9
 volumes 26–8
- compound interest 213–15
 calculations 219–24
- compound interest formula 216–19
- conditional probability 115–16
- congruent figures 152, 153
- congruent triangles 155
 formal proofs 159–62
 tests for 155–9
- coordinate geometry extension 82–3
- correlation 199
- cosine ratio 311–12, 316, 318
- cosine rule 331–4
- cumulative frequency histogram 254
- cylinders, surface area 21–3
- data, collecting and analysing 3–4
- data sets, comparing 272–8
- degrees 305
- denominator 39
- dependent events 107, 112–14
- dependent variable 193
- depreciation 224–6
- difference of two squares expansion 240, 241–2
- direct proportion 13–14
- distance formula 62–3
- distributive law 45–51
- Einstein, Albert 147
- elimination method, simultaneous equations 140–2
- equation of the line 68–9, 71–5, 192, 286
 graphs 65–7, 69, 71, 78, 286, 294
- equation of a straight line, general form 76–8
- equations 14, 128, 236
 linear 129–32
 quadratic 244–7
 simultaneous 133–9
- equivalent fractions 39
- expanding
 algebraic expressions 45–51
 binomial products 238–42
- experimental probability 93–4
- exponential curves 290–1, 294
- exterior angle sum of a polygon 176–7
- factorising
 algebraic expressions 51–4, 236
 quadratic trinomials 242–4
 to solve quadratic equations 248–9
- financial mathematics 6–7, 212
 compound interest 213–24
 depreciation 224–6
- fractions
 equivalent 39
 simplifying 40–5
- frequency curves 76, 277
- frequency distribution tables 254, 270–1, 274–5

general form of a straight line equation	76–8	numbers of any magnitude	4–5
geometrical reasoning	153–5	numerator	39
gradient formula	64–5	obtuse angles, non-right-angled triangles	324–5
gradient of the line	70	outliers	258
gradients		parabolas	287–9, 294
and parallel lines	68, 79–80	parallel box plots	272–3
and perpendicular lines	80–2	parallel lines, and gradients	68, 79–80
graphical solutions, simultaneous equations	136–9	perfect squares expansion	240, 241, 242
graphs	294–6	perpendicular lines, and gradients	80–2
circles	292–3, 294	polygons	173
exponential curves	290–1, 294	exterior angle sum	176–7
parabolas	287–9, 294	interior angle sum	173–5
straight lines	65–7, 69, 71, 78, 286, 294	positive relationship (scatter plots)	200
Hamilton, Sir William Rowan	251	prisms, surface area	18–20
highest common factor (HCF)	40	probability	8–9, 92, 93–7
histograms	274–8	conditional	115–16
independent events	107–14	dependent events	107, 112–14
independent variable	193	experimental	93–4
indices	2–3	independent events	107–14
indirect proportion	13–14	multistage events	98–102
inequations	14	product rule	107–15
interior angle sum of a polygon	173–5	sampling with replacement	102–4
interquartile range	261–6	sampling without replacement	104–6
Kepler, Johann	31	theoretical	93
lattice diagrams	100	probability trees	109–10, 112–14
life expectancy	117–18	product rule of probability	107–15
linear equations, review	129–32	proportion	13–14
linear relationships	12–13, 60–83	quadratic equations	244–7
lower quartile	261–5	solving by factorising	248–9
lowest common denominator (LCD)	41	quadratic expressions	237–40
Mandelbrot, Benoit	297	quadratic trinomials	242–3
mean	254, 255–61	quartiles	261–6
measures of central tendency	254, 255–60	range	255–7
choosing the appropriate measure	261	rates	13
measures of spread	255–7	rectangular prism, surface area	18–19
median	254, 255–64	relative frequency	93–4
midpoint formula	63–4	RHS (congruent triangle test)	155
minutes	305	right-angled trigonometry	10–11, 304–18
mode	254, 255–61	sampling with replacement	102–4
multistage events	98–102	sampling without replacement	104–6
negative relationship (scatter plots)	200	SAS (congruent triangle test)	155
non-linear relationships	13	scale factor	11, 169–70
circles	291–3, 294	scatter plots	197–204
exponential graphs	290–1, 294	analysing	198–9
parabola	287–9, 294	seconds	305
non-right-angled triangles	324–5, 334–5	sector, area	17–18
area of a triangle	325–7	similar figures	153
cosine rule	331–4	similar triangles	162–72
practical problems	335–9	tests for	163–7
sine rule	326–30	similarity	11–12

simplifying			
algebraic expressions	45–51, 236		
fractions	40–5		
simultaneous equations	133–5		
graphical solutions	136–9		
problem solving with two unknowns	143–6		
solution by elimination	140–2		
solution by substitution	139–40		
sine ratio	309–11, 315, 318		
sine rule	327–31		
SSS (congruent triangle test)	155		
stem-and-leaf plots	265, 269		
straight line			
equation of the line	68–9, 71–5		
general form of a straight line equation	76–8		
graphs	65–7, 69, 71, 78, 286, 294		
$y = mx + b$ form	68–72		
strong association (scatter plots)	200		
substitution method, simultaneous equations	139–40		
surface area	8, 16		
composite solids	28–9		
cylinders	21–3		
prisms	18–20		
problems	29–30		
tangent ratio	313–14, 317–18		
theoretical probability	93		
transformations	152		
tree diagrams	98–9, 101, 102–3, 104–5, 107, 110		
trend lines (scatter plots)	198, 199		
triangle, area	325–7		
triangular prism, surface area	19–20		
trigonometric ratios			
of acute angles	305–6, 324–5		
to find angles	306–8, 315–18		
to find sides	309–14		
trigonometry			
non-right-angled triangles	324–39		
right-angled	10–11, 304–18		
two-way tables	96–7		
upper quartile	261–5		
variables	193		
bivariate data	194–7		
dependent	193		
independent	193		
relationships between (scatter plots)	199–200		
Venn diagrams	94–6		
vertex	187		
volume(s)	8, 16–17		
composite solids	26–8		
problems	30		
with uniform cross-sections	23–6		
weak association (scatter plots)	200		
$y = mx + b$ form of a straight line	68–72		

Acknowledgements

The author and the publisher wish to thank the following copyright holders for reproduction of their material.

Corbis/Image Source, front cover; Corbis/Bettmann, back cover and p. vi. **Chapter 1:** Shutterstock/Paul Reeves Photography, p. 1; iStockphoto/George Clerk, p. 3; Shutterstock/Angela Waye, p. 5. **Chapter 2:** Shutterstock/withGod, p. 15; Shutterstock/Edw,p. 23; Shutterstock/Arina P Habich, p. 25; Shutterstock/Axel Bueckert, p. 27; Shutterstock/Pan Xunbin, p. 30; NASA, p. 31; Shutterstock/MC_Noppadol ,p. 35; **Chapter 3:** Shutterstock/Eric Isselee, p. 37; Lindsay Edwards, pp. 42, 49. **Chapter 4:** Shutterstock/Eric Isselee, p. 59; Lindsay Edwards, pp. 67, 71, 73. **Chapter 5:** Shutterstock/Kokhanchikov, p. 91; Shutterstock/Susan Leggett, p. 95; Shutterstock/wavebreakmedia, p. 97; Shutterstock/LuckyImages, p. 102; Shutterstock/Chen WS, p. 106; Shutterstock/Olivier Le Moal, p. 111; iStockphoto/David_Ahn, p. 113; Brent Parker Jones, p. 116; iStockphoto/davidf, p. 117; Shutterstock/Kzenon, p. 123; Shutterstock/bikeriderlondon, p. 124. **Chapter 6:** Shutterstock/Stephen Mcsweeney, p. 127; Lindsay Edwards, pp. 129, 132, 138, 142; Shutterstock/Mikhail Zahranichny, p. 144; Shutterstock/Tyler Olson, p. 145 (screwdrivers); Shutterstock/panbazil, p. 145 (rabbit); Shutterstock/Tsekhmister, p. 145 (pheasant); Shutterstock/Borislav Bajkic, p. 146; **Chapter 7:** Shutterstock/fuyu liu, p. 151; Lindsay Edwards, p. 162; iStockphoto/urbancow, p. 172 (tree); Shutterstock/Eric Gevaert, p. 172 (people). **Chapter 8:** Shutterstock/

weter 777, p. 191; Shutterstock/everything possible, p. 193; Shutterstock/RyFlip, p. 197; Shutterstock/Zurijeta, p. 204. **Chapter 9:** Shutterstock/withGod, p. 211; Shutterstock/Valua Vitaly, p. 218; Shutterstock/alexskopje, p. 221; Shutterstock/stefan11, p. 224; Shutterstock/ChameleonsEye, p. 226. **Chapter 10:** Shutterstock/Phant, p. 235; Shutterstock/marekuliasz, p. 237; Lindsay Edwards, pp. 246, 249. **Chapter 11:** Shutterstock/Eric Isselee, p. 253; iStockphoto/carlosalvarez, p. 257; Shutterstock, p. 258 (lamps); Shutterstock/Mitch Gunn, p. 258 (cricket); Shutterstock/Neale Cousland, p. 260; Shutterstock/Aaron Amat, p. 266; Lindsay Edwards, p. 268; Shutterstock/Somchai Som, p. 273 (bulb); Shutterstock/SurangaSL, p. 273 (car); Shutterstock/wavebreakmedia, p. 274; Shutterstock/jannoon028, p. 278. **Chapter 12:** Shutterstock/smishonja, p. 285; Shutterstock/Taras Vyshnya, p. 289; Lindsay Edwards, p. 293; Shutterstock/Steve Buckley, p. 297. **Chapter 13:** Shutterstock/Anan Kaewkhammul, p. 303; Lindsay Edwards, p. 305; Shutterstock/oksana2010, p. 313; Shutterstock/Sergei Drozd, p. 319; Shutterstock/rangizzz, p. 322; Lindsay Edwards, p. 328; Shutterstock/Lichtmeister, p. 335; Shutterstock/Samot,p. 338 (lighthouse); Shutterstock/hxdbzxy,p. 328 (building); Shutterstock/Peter J. Wilson, p. 344; Shutterstock/Johnny Habell, p. 346.

Every effort has been made to trace the original source of copyright material contained in this book. The publisher will be pleased to hear from copyright holders to rectify any errors or omissions.



OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

ISBN 978-0-19-552269-3



visit us at: oup.com.au or
contact customer service: cs.au@oup.com