

NEW CENTURY MATHS

8

NSW Stage 4

For the
**australian
curriculum**



Judy Binns
Gaspare Carrozza

Robert Yen Series Editor



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Technology sections by Sarah Hamper



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First published in 1994, *New Century Maths 8* has now been completely rewritten to reflect the spirit of the new Australian curriculum and the NSW syllabus. In this book, teachers will still find those familiar features that have made New Century Maths a leading mathematics series, such as clear worked examples, graded exercises, multiple-choice questions, Investigation, Technology, Mental skills, SkillCheck pre-chapter exercise, Power Plus extension exercise, Language of Maths, Topic overview with mind map, glossary, video tutorials and worksheets.

New features include:

- Homework sheets
- Worked solutions to selected questions
- *GeoGebra* activities in our Technology sections
- Chapter outlines showing coverage of the Working Mathematically / Proficiency strands
- *NelsonNet* student and teacher websites of print and multimedia resources
- *NelsonNetBook* web version of this book with interactive features

We always aim to write textbooks to provide flexibility for teachers to plan and teach for a variety of pathways. We thank our families and teaching colleagues for their continued support and patience, and editors Anna Pang and Alan Stewart for transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing 21st century Australian curriculum mathematics.

About the authors

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Curriculum grid: Australian curriculum

Strand and substrand	New Century Maths 7 chapter	New Century Maths 8 chapter
NUMBER AND ALGEBRA		
Number and place value	1 Integers 3 Whole numbers 5 Algebra and equations	2 Working with numbers
Real numbers	4 Fractions and percentages 7 Decimals 12 Ratios, rates and time	2 Working with numbers 6 Fractions and percentages 11 Ratios, rates and time
Money and financial mathematics	12 Ratios, rates and time	6 Fractions and percentages 11 Ratios, rates and time
Patterns and algebra	5 Algebra and equations	3 Algebra
Linear and non-linear relationships	5 Algebra and equations 9 The number plane 12 Ratios, rates and time	10 Equations 11 Ratios, rates and time 12 Graphing linear equations
MEASUREMENT AND GEOMETRY		
Using units of measurement	8 Area and volume 12 Ratios, rates and time	5 Area and volume 11 Ratios, rates and time
Shape	8 Area and volume	
Location and transformation	6 Geometry 9 The number plane	4 Geometry 8 Congruent figures
Geometric reasoning	2 Angles 6 Geometry	4 Geometry 8 Congruent figures
Pythagoras and trigonometry		1 Pythagoras' theorem
STATISTICS AND PROBABILITY		
Chance	11 Probability	9 Probability
Data representation and interpretation	10 Analysing data	7 Investigating data

Curriculum grid: NSW syllabus

Strand and substrand	New Century Maths 7 chapter	New Century Maths 8 chapter
NUMBER AND ALGEBRA		
Computation with integers	1 Integers 3 Whole numbers	2 Working with numbers
Fractions, decimals and percentages	4 Fractions and percentages 7 Decimals	2 Working with numbers 6 Fractions and percentages
Financial mathematics	12 Ratios, rates and time	6 Fractions and percentages 11 Ratios, rates and time
Proportion	12 Ratios, rates and time	11 Ratios, rates and time
Algebraic techniques 1	5 Algebra and equations	3 Algebra
Algebraic techniques 2	5 Algebra and equations	3 Algebra 12 Graphing linear equations
Indices	3 Whole numbers	2 Working with numbers 3 Algebra
Equations	5 Algebra and equations	10 Equations
Linear relationships	9 The number plane	12 Graphing linear equations
MEASUREMENT AND GEOMETRY		
Length		5 Area and volume
Area	8 Area and volume	5 Area and volume
Volume	8 Area and volume	5 Area and volume
Time	12 Ratios, rates and time	11 Ratios, rates and time
Right-angled triangles (Pythagoras)		1 Pythagoras' theorem
Angle relationships	2 Angles	4 Geometry
Properties of geometrical figures 1	6 Geometry	4 Geometry
Properties of geometrical figures 2		8 Congruent figures
STATISTICS AND PROBABILITY		
Single variable data analysis 1	10 Analysing data	7 Investigating data
Single variable data analysis 2	10 Analysing data	7 Investigating data
Probability 1	11 Probability	9 Probability
Probability 2		9 Probability

Curriculum grid: Year 8 content descriptions

Content description	New Century Maths 8 chapter
NUMBER AND ALGEBRA	
Number and place value	
NA182: Use index notation with numbers to establish the index laws with positive integral indices and the zero index	2 Working with numbers
NA183: Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies	2 Working with numbers
Real numbers	
NA152: Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line	4 Fractions and percentages
NA153: Solve problems involving addition and subtraction of fractions, including those with unrelated denominators	4 Fractions and percentages
NA154: Multiply and divide fractions and decimals using efficient written strategies and digital technologies	4 Fractions and percentages 7 Decimals
NA155: Express one quantity as a fraction of another, with and without the use of digital technologies	4 Fractions and percentages
NA156: Round decimals to a specified number of decimal places	7 Decimals
NA184: Investigate terminating and recurring decimals	2 Working with numbers
NA186: Investigate the concept of irrational numbers, including π	1 Pythagoras' theorem 5 Area and volume
NA187: Solve problems including the use of percentages, including percentage increases and decreases, with and without digital technologies	6 Fractions and percentages
NA188: Solve a range of problems involving rates and ratios, with and without digital technologies	11 Ratios, rates and time
Money and financial mathematics	
NA189: Solve problems involving profit and loss, with and without digital technologies	6 Fractions and percentages
Patterns and algebra	
NA190: Extend and apply the distributive law to the expansion of algebraic expressions	3 Algebra
NA191: Factorise algebraic expressions by identifying numerical factors	3 Algebra
NA192: Simplify algebraic expressions involving the four operations	3 Algebra
YEAR 9 NA212: Extend and apply the index laws to variables, using positive integral indices and the zero index	3 Algebra
Linear and non-linear relationships	
NA193: Plot linear relationships on the Cartesian plane with and without the use of digital technologies	12 Linear equations
NA194: Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.	10 Equations 12 Linear equations
NSW STAGE 4: Solve simple quadratic equations	10 Equations

Content description	New Century Maths 8 chapter
MEASUREMENT AND GEOMETRY	
Using units of measurement	
MG195: Choose appropriate units of measurement for area and volume and convert from one unit to another	5 Area and volume
MG196: Find perimeters and areas of parallelograms, rhombuses and kites	5 Area and volume
MG197: Investigate the relationship between the features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.	5 Area and volume
MG198: Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.	5 Area and volume
MG199: Solve problems involving duration, including using 12- and 24-hour time within a single time zone	11 Ratios, rates and time
NSW STAGE 4: Solve problems involving international time zones	11 Ratios, rates and time
Geometric reasoning	
MG200: Define congruence of plane shapes using transformations	8 Congruent figures
MG201: Develop the conditions for congruence of triangles	8 Congruent figures
MG202: Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems using reasoning.	4 Geometry 8 Congruent figures
NSW STAGE 5: Apply the result for the interior angle sum of a triangle to find, by dissection, the interior angle sum of polygons with more than three sides	4 Geometry
Pythagoras and trigonometry	
YEAR 9 MG222 / NSW STAGE 4: Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles	1 Pythagoras' theorem
STATISTICS AND PROBABILITY	
Chance	
SP204: Identify complementary events and use the sum of probabilities to solve problems	9 Probability
SP205: Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'	9 Probability
SP292: Represent such events in two-way tables and Venn diagrams and solve related problems	9 Probability
Data representation and interpretation	
SP206: Explore the practicalities and implications of obtaining representative data using a variety of investigative processes	7 Investigating data
SP207: Investigate the effect of individual data values, including outliers, on the mean and median	7 Investigating data
SP293: Explore the variation of means and proportions in representative data	7 Investigating data

Coverage of the Australian curriculum and NSW syllabus

- In addition to the curriculum grids on the previous pages, the table of contents includes codes for Australian Curriculum content descriptions. For example, **8MG199** refers to the Year 8 content description **ACM8MG199**, which stands for Australian Curriculum Mathematics, Measurement and Geometry strand, content description 199.
- This book includes some Year 7 and Year 9 content as well as additional content from the NSW syllabus only. Year 9 and NSW content are highlighted in **orange**.
- There are three content strands in Mathematics:
 - NA** = Number and Algebra
 - MG** = Measurement and Geometry
 - SP** = Statistics and Probability
- Each chapter begins with a chapter outline that includes the Working Mathematically proficiencies covered in each section:
 - U** = Understanding
 - F** = Fluency
 - PS** = Problem solving
 - R** = Reasoning
 - C** = Communicating

Further references to the Working Mathematically proficiencies can be found in the teaching program.

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NSW	12-08 Comparing linear and non-linear equations	487
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	Technology: Intersecting lines	494
8NA194	12-10 Finding the equation of a line	495
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Chapter outline		Proficiency strands				
8-01 Transformations	U	F		R	C	
8-02 Congruent figures	U	F		R	C	
8-03 Constructing triangles	U	F	PS	R	C	
8-04 Tests for congruent triangles	U	F	PS	R	C	
8-05 Proving properties of quadrilaterals	U	F	PS	R	C	
8-06 Extension: Bisecting intervals and angles	U	F		R		
8-07 Constructing parallel and perpendicular lines	U	F				

Working mathematically explained

While the three **content strands** are the 'nouns' of the mathematics curriculum, the five **Working Mathematically proficiencies** are the 'verbs': the doing and thinking processes that go hand-in-hand with the content being taught.

- Understanding** is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.
- Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.
- Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.
- Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.
- Communicating** is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

In each chapter

- **Wordbank** is a sample glossary of 6–8 terms from the chapter
- There is also a full glossary at the back of the book: in this book, all terms printed in **red** appear in the glossary
- Curriculum content descriptions (Australian and NSW) are listed at the beginning of each chapter
- **SkillCheck** and **StartUp assignment** review prerequisite skills and knowledge for the chapter

- Important facts and formulas are highlighted in a **Summary** box.

In this chapter you will:

- compare fractions using equivalence
- solve problems involving addition and subtraction of fractions, including those with unrelated denominators
- multiply and divide fractions using efficient written strategies and digital technologies
- connect fractions, decimals and percentages and carry out simple conversions
- find fractions and percentages of quantities and express one quantity as a fraction or percentage of another, with and without digital technologies
- solve problems involving the use of percentages, including percentage increases, percentage decreases and the unitary method, with and without digital technologies
- solve problems involving profit, loss, and goods and services tax (GST), with and without digital technologies

SkillCheck

Problem
StartUp assignment 6
MAT08NAWK10049

1 Copy and complete each pair of equivalent fractions.

a $\frac{1}{2} = \frac{\quad}{6}$

b $\frac{2}{3} = \frac{\quad}{12}$

c $\frac{4}{5} = \frac{16}{\quad}$

d $\frac{7}{28} = \frac{1}{\quad}$

e $\frac{3}{4} = \frac{\quad}{8}$

f $\frac{15}{20} = \frac{\quad}{4}$

g $\frac{6}{10} = \frac{24}{\quad}$

h $\frac{3}{5} = \frac{9}{\quad}$

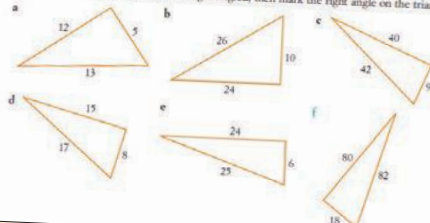
Summary

For equations with variables on both sides, perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

Exercise 1-06 Testing for right-angled triangles

1 Copy each triangle, test whether it is right-angled, then mark the right angle on the triangle.



- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications
- Worked solutions for selected exercise questions are provided.

- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.

Investigation: Scale drawings

- 1 Measure the dimensions of your bedroom and the dimensions of the items of furniture in it.
- 2 Using a scale of 1 : 100, draw a scale diagram of your bedroom on graph paper.
- 3 Draw scale diagrams of each piece of furniture on another piece of graph paper, then cut them out.
- 4 Rearrange the furniture in your bedroom on the scale diagram to see which arrangement you prefer best.

Just for the record

Dollars

Australia, the USA, Hong Kong and parts of Central America use the dollar as their unit of currency. The word 'dollar' actually originated in the town of Jachymov in Czechoslovakia, where the coins minted from its silver mine were called 'Joachimsthaler'. Coins in various German states from the 16th to the 19th centuries were called 'taler', which later became 'daler'. The early Spanish settlers introduced the 'Philippe dolers' into North America, and this later became the word 'dollar' that we know today.

Find out which countries use dollars as their unit of currency.



- **Just for the record** contains interesting facts and applications of the mathematics learnt in the chapter.

New Century Maths User's guide

- **Technology** promotes ICT in the classroom, using spreadsheets, *GeoGebra* and the Internet

Technology Substitution

1 Enter the variables and values shown into a new spreadsheet.

	A	B
1	variable	value
2	m	10
3	n	12

This means $m = 10$.

Mental skills 1 Maths without calculators

Squaring a number ending in 5 or 1

The square of a number ending in 5 always ends in 25.

For example, $35^2 = 1225$ and $105^2 = 11\,025$.

A mental calculation trick requires three easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

- **Mental skills** reinforce mental calculation strategies ('maths without calculators')

NelsonNet resources

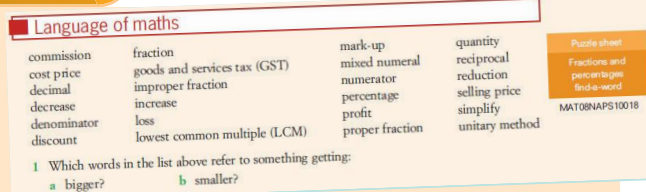
Margin icons link to print (PDF) and multimedia resources found on the NelsonNet website, www.nelsonnet.com.au. These include:

Worksheet Adding and subtracting integers	Worksheets	Video tutorial Solving equations	Video tutorials of worked examples
Puzzle sheet Solving equations	Puzzle sheets of matching activities, crosswords and find-a-word puzzles	Worked solutions Exercise 8-10	Worked solutions of selected exercise questions
Skillsheet Ordering numbers	Skillsheets of examples and exercises of prerequisite skills and knowledge	Technology worksheet Excel Graphing data	Technology worksheets: additional technology activities
Homework sheet Transformations and symmetry	Homework sheets for weekly practice and revision, including mental calculation, numeracy and literacy questions	Quiz Whole numbers	ExamView quizzes: interactive and self-marking

Each resource has a unique identifier code. For example, the video tutorial 'GST' has the code **MAT08NAVT10012**, which stands for Mathematics, Year 8, Number and Algebra strand, Video Tutorial 10012.

At the end of each chapter

- **Power plus** is an extension/challenge exercise
- **Language of maths** has a chapter word list and literacy questions
- **Topic overview** has reflection questions and an incomplete mind map
- **Chapter revision** is a review exercise with links to each exercise set of the chapter
- **Mixed revision** is a review exercise after every 3–4 chapters

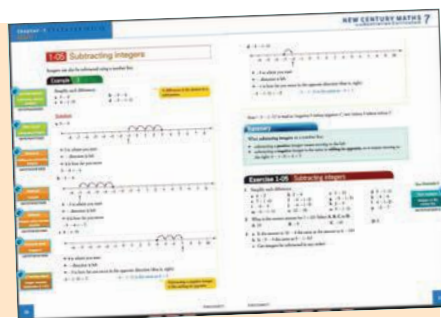


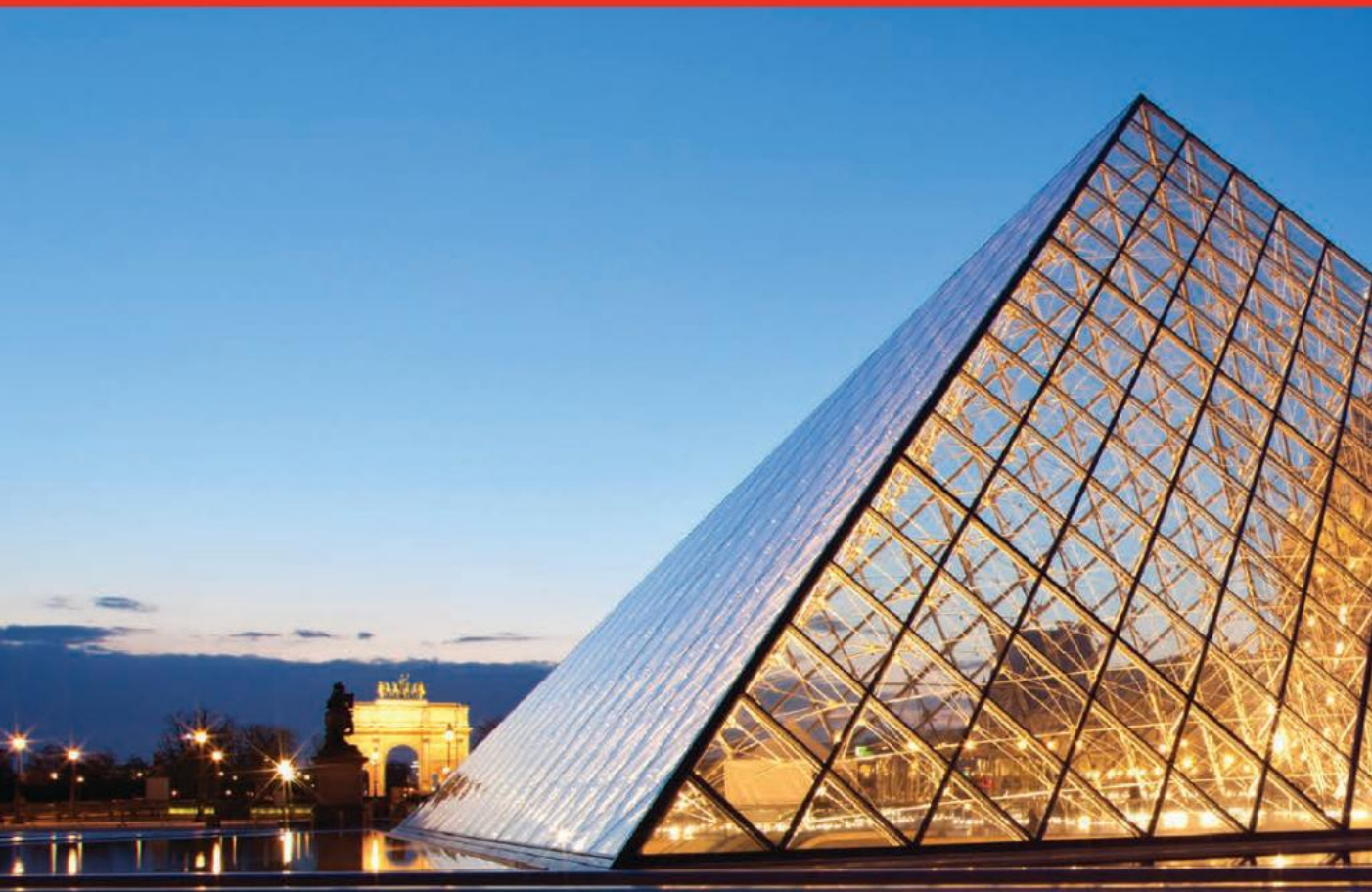
At the end of the book

- **General Revision exercise**
- **Appendix 1: Calculator skills**
- **Appendix 2: Instructional glossary and Mathematical glossary** (in this book, words printed in red also appear in the glossary)
- **Answers and index**

NelsonNetBook

- **NelsonNetBook** is the web-based version of this textbook found on NelsonNet, containing margin icon links directly to NelsonNet resources
- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students





1

Measurement and geometry

Pythagoras' theorem

Pythagoras' theorem was known to a number of ancient civilisations, including the Egyptians who used the rule to construct their pyramids as far back as 3000 BCE, but Pythagoras was the first person to write it in a mathematical way. Pythagoras' theorem expresses the relationship between the sides of a right-angled triangle. It is still used today in architecture, engineering, surveying and many other fields.



Chapter outline

	Proficiency strands			
1-01 Square roots and surds	U			
1-02 Discovering Pythagoras' theorem	U	F	R	C
1-03 Finding the hypotenuse	U	F		
1-04 Finding a shorter side	U	F		
1-05 Mixed problems		F		
1-06 Testing for right-angled triangles	U	F	R	
1-07 Pythagorean triads	U	F		
1-08 Pythagoras' theorem problems		F	PS	

This is a Year 9 topic in the Australian Curriculum but a Stage 4 (Years 7–8) topic in the NSW syllabus.

Wordbank

- converse** A rule or statement turned back-to-front; the reverse statement
- hypotenuse** The longest side of a right-angled triangle; the side opposite the right angle
- Pythagoras** An ancient Greek mathematician who discovered an important formula about the sides of a right-angled triangle
- surd** A square root (or other type of root) whose exact decimal value cannot be found
- theorem** A formal rule or a formula
- triad** A group of three related objects

In this chapter you will:

- investigate the concept of irrational numbers
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways, including using digital technologies.
- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles
- solve practical problems involving Pythagoras' theorem, approximating the answer as a decimal or giving an exact answer as a surd
- use the converse of Pythagoras' theorem to establish whether a triangle has a right angle
- identify a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third

SkillCheck

Worksheet

StartUp assignment 1

MAT08MGWK10001

1 Evaluate each expression.

a 4^2

b 12^2

c 30^2

d 2.1^2

e 10.3^2

f $3^2 + 4^2$

g $5^2 + 12^2$

h $5.4^2 - 3.2^2$

2 Evaluate each square root.

a $\sqrt{16}$

b $\sqrt{64}$

c $\sqrt{100}$

d $\sqrt{196}$

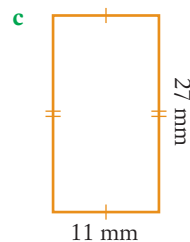
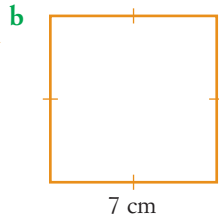
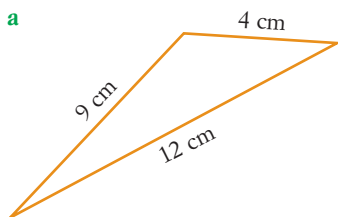
e $\sqrt{121}$

f $\sqrt{49}$

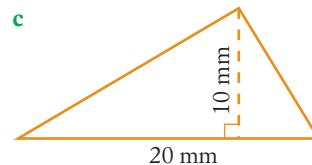
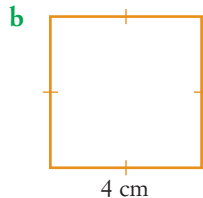
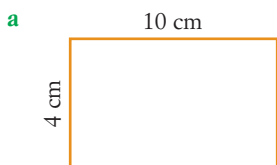
g $\sqrt{1.21}$

h $\sqrt{2.25}$

3 Find the perimeter of each shape.



4 Find the area of each shape.



5 Write each number correct to one decimal place.

a 10.33

b 4.67

c 7.654

d 0.8888

6 Evaluate each expression correct to one decimal place.

a $\sqrt{3^2 + 7^2}$

b $\sqrt{5.6^2 + 9.2^2}$

c $\sqrt{15^2 - 8^2}$

1-01 Square roots and surds

The **square root** ($\sqrt{\quad}$) of a number is the **positive** value which, if squared, will give that number. For example:

- $\sqrt{25} = 5$ because $5^2 = 25$ 'the square root of 25'
- $\sqrt{49} = 7$ because $7^2 = 49$ 'the square root of 49'

Most square roots do not give exact answers like the ones above. For example, $\sqrt{10} = 3.16227766 \dots \approx 3.2$. Such square roots are called **surds**. A surd is a square root ($\sqrt{\quad}$), cube root ($\sqrt[3]{\quad}$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$ so it is also called an **irrational** number.

Worksheet

Table of squares and square roots

MAT08MGWK10002

Worksheet

Surds

MAT08MGWK10003

Example 1

Evaluate each expression correct to two decimal places.

a $\sqrt{7^2 + 9^2}$

b $\sqrt{12^2 - 10^2}$

Solution

a $\sqrt{7^2 + 9^2} = 11.40175 \dots$
 ≈ 11.40

On a calculator, enter $\sqrt{\quad} (7 x^2 + 9 x^2) =$

OR:

$7^2 + 9^2 = 130$

On a calculator, enter $7 x^2 + 9 x^2 =$

$\sqrt{130} = 11.40175 \dots$
 ≈ 11.40

On a calculator, enter $\sqrt{\quad} \text{ANS} =$

b $\sqrt{12^2 - 10^2} = 6.63324 \dots$
 ≈ 6.63

On a calculator, enter $\sqrt{\quad} (12 x^2 - 10 x^2) =$

OR:

$12^2 - 10^2 = 140$

On a calculator, enter $12 x^2 - 10 x^2 =$

$\sqrt{140} = 6.63324 \dots$
 ≈ 6.63

On a calculator, enter $\sqrt{\quad} \text{ANS} =$

Example 2

Select the surds from this list of square roots: $\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

Solution

$$\sqrt{72} = 8.4852 \dots$$

$$\sqrt{121} = 11$$

$$\sqrt{64} = 8$$

$$\sqrt{90} = 9.4868 \dots$$

$$\sqrt{28} = 5.2915 \dots$$

so the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

Exercise 1-01 Square roots and surds

See Example 1

- 1 Evaluate each square root.

a $\sqrt{196}$

b $\sqrt{900}$

c $\sqrt{64}$

d $\sqrt{625}$

e $\sqrt{121}$

f $\sqrt{361}$

- 2 Evaluate each square root, correct to two decimal places.

a $\sqrt{12}$

b $\sqrt{45}$

c $\sqrt{1001}$

d $\sqrt{325}$

e $\sqrt{153}$

f $\sqrt{207}$

g $\sqrt{98}$

h $\sqrt{888}$

i $\sqrt{24}$

j $\sqrt{110}$

k $\sqrt{297}$

l $\sqrt{689}$

- 3 Evaluate each expression, correct to two decimal places.

a $\sqrt{12^2 - 5^2}$

b $\sqrt{5^2 + 7^2}$

c $\sqrt{6^2 + 11^2}$

d $\sqrt{17^2 - 12^2}$

e $\sqrt{8^2 - 3^2}$

f $\sqrt{4^2 + 9^2}$

g $\sqrt{(1.5)^2 + (4.2)^2}$

h $\sqrt{(12.5)^2 - (7.1)^2}$

i $\sqrt{(25.7)^2 + (18.2)^2}$

See Example 2

- 4 Which of the following is **not** a surd? Select the correct answer **A**, **B**, **C** or **D**.

A $\sqrt{10.24}$

B $\sqrt{24}$

C $\sqrt{1000}$

D $\sqrt{3}$

- 5 Select all the surds from this list of square roots.

$\sqrt{98}$

$\sqrt{9}$

$\sqrt{225}$

$\sqrt{160}$

$\sqrt{36}$

$\sqrt{52}$

$\sqrt{144}$

$\sqrt{77}$

$\sqrt{18}$

$\sqrt{196}$

$\sqrt{200}$

$\sqrt{81}$

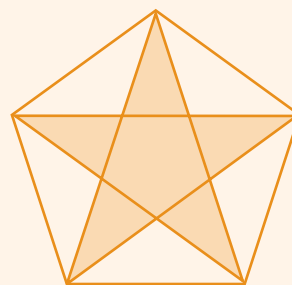
Just for the record

Pythagoras and the Pythagoreans

Pythagoras was a mathematician who lived in ancient Greece. The Pythagoreans were a group of men who followed Pythagoras.

This is the symbol of the Pythagorean secret society. Apparently they were so upset about the discovery of surds that they tried to keep it a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

Find the special name for this symbol involving a star inside a pentagon.



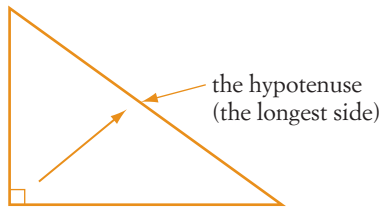
This is the symbol of the Pythagorean secret society.

1-02 Discovering Pythagoras' theorem

A **right-angled triangle** has one right angle (90°) and two smaller angles. The side opposite the right angle is the *longest* side and is called the **hypotenuse**.

Pythagoras' theorem is a rule that describes the relationship between the hypotenuse and the other two (shorter) sides.

The hypotenuse is always opposite the right angle.

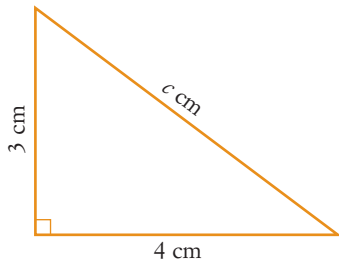


Example 3

- Construct a right-angled triangle with the two shorter sides being of length 3 cm and 4 cm.
- Measure the length of the hypotenuse.
- Evaluate $3^2 + 4^2$.
- If c is the length of the hypotenuse in centimetres, evaluate c^2 .

Solution

- a**  **b** Measuring with a ruler, the hypotenuse is 5 cm long.

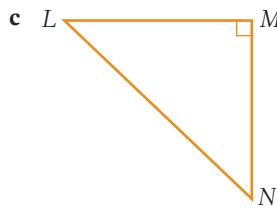
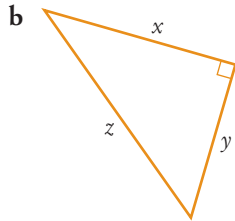
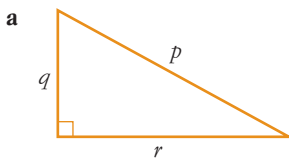


c $3^2 + 4^2 = 25$.

d $c^2 = 5^2 = 25$.

Exercise 1-02 Discovering Pythagoras' theorem

- 1 Name the hypotenuse in each triangle.



- 2 a** Draw a right-angled triangle with short sides measuring 5 cm and 12 cm.
- b** Measure the length of the hypotenuse.
- c** Evaluate $5^2 + 12^2$.

GeoGebra: Pythagoras' theorem

MAT08MGTC00004

Pythagoras' discovery

MAT08MGWK10004

A page of right-angled triangles

MAT08MGWK10005

MAT08MGWK10006

d If c is the length of the hypotenuse in centimetres, evaluate c^2 .

e Copy and complete each statement.

The square of the hypotenuse = $\underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}}$

The sum of the squares of the two shorter sides = $\underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}}$

- 3 Which equation is true for the right-angled triangle shown? Select the correct answer

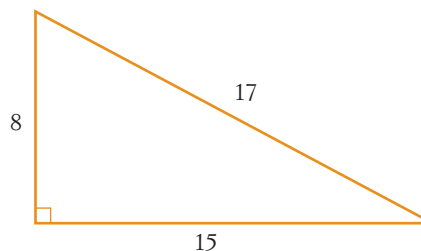
A, B, C or D.

A $8 + 15 = 17$

B $8^2 + 15^2 = 17^2$

C $8^2 + 17^2 = 15^2$

D $17^2 + 15^2 = 8^2$



- 4 a Use *GeoGebra* (or pencil, ruler and 5 mm grid paper) to draw a right-angled triangle with the shorter sides of length 6 cm and 8 cm, then measure the length of the hypotenuse. Click **View Grid** and remove **Axes** before you begin, then create the triangle using **Interval Between Two Points**. Check the lengths using **Distance or Length**.



b Copy and complete this table by:

- constructing each right-angled triangle with the measurements given for the two shorter sides a and b (using *GeoGebra* or pencil and paper)
- measuring the length, c , of the hypotenuse
- evaluating the values of a^2 , b^2 , $a^2 + b^2$, c^2

	Shorter sides (cm)		Hypotenuse (cm)				
	a	b	c	a^2	b^2	$a^2 + b^2$	c^2
a	6	8					
b	2.5	6					
c	1.5	2					
d	4	7.5					

- 5 What rule or relationship did you discover between the length of the hypotenuse and the lengths of the other two sides?

Video tutorial

Pythagoras' theorem

MAT08MGVT0001

Technology worksheet

Excel: Pythagoras' theorem

MAT08MGCT00024

Technology

Excel: Pythagoras' theorem

MAT08MGCT00009

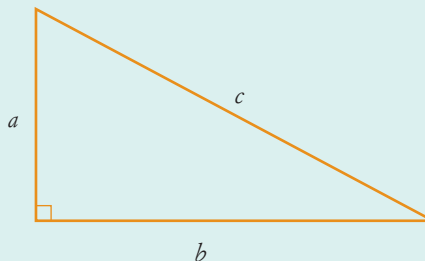
1-03 Finding the hypotenuse

Summary

Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. If c is the length of the hypotenuse, and a and b are the lengths of the other two sides, then:

$$c^2 = a^2 + b^2$$



A theorem is a formal rule or formula. Pythagoras' theorem was discovered by the ancient Greek mathematician Pythagoras. This rule is used to find the length of one side of a right-angled triangle when the other two sides are known.

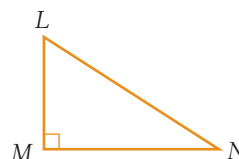
Example 4

Write Pythagoras' theorem for $\triangle LNM$.

Solution

The hypotenuse is LN , the two shorter sides are LM and MN .

So Pythagoras' theorem is $LN^2 = LM^2 + MN^2$



Example 5

Find the value of c in this triangle.

Solution

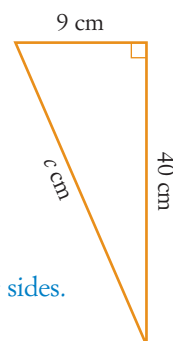
c cm is the length of the hypotenuse. Using Pythagoras' theorem:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 9^2 + 40^2 \\ &= 1681 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{1681} \\ &= 41 \end{aligned}$$

Substitute the lengths of the two shorter sides.

Use the square root to find c .



Check: A hypotenuse of 41 cm looks reasonable from the diagram: it is the longest side and a little longer than 40 cm.

Example 6

Find the value of x in this triangle:

- a as a surd
- b correct to two decimal places.

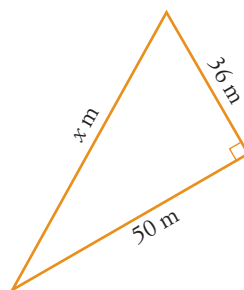
Solution

- a Write Pythagoras' theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 50^2 + 36^2 \\ &= 3796 \end{aligned}$$

$$x = \sqrt{3796}$$

Substitute the lengths of the two shorter sides.



Because the question asks for a **surd**, leave the answer in square root ($\sqrt{}$) form.

b $x = \sqrt{3796}$
 $= 61.6116\dots$
 ≈ 61.61

From part a.

Check: From the diagram, this answer looks reasonable.

Note: The surd answer is more exact than the decimal answer because it is not rounded.

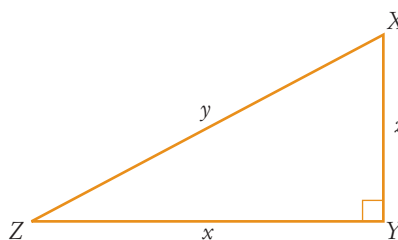
Exercise 1-03 Finding the hypotenuse

See Example 4

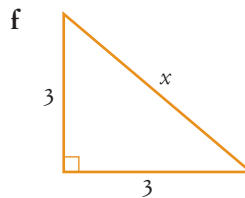
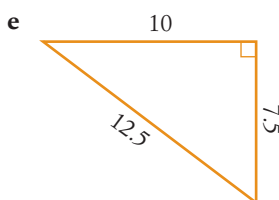
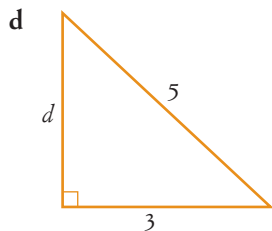
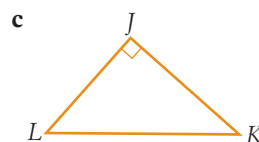
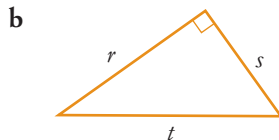
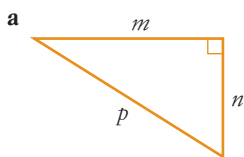
- 1 What is Pythagoras' theorem for this triangle?

Select the correct answer A, B, C or D.

- A $z^2 = x^2 + y^2$
 B $ZY^2 = ZX^2 + XY^2$
 C $XY^2 = ZY^2 + ZX^2$
 D $y^2 = x^2 + z^2$

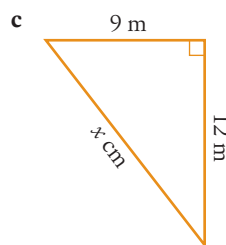
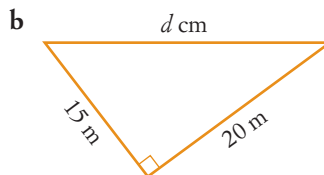
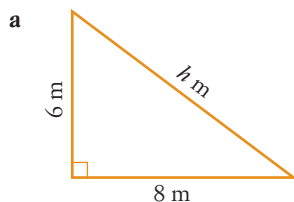


- 2 Write Pythagoras' theorem for each right-angled triangle.



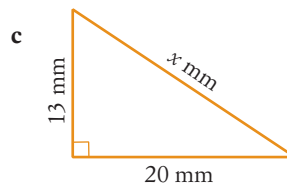
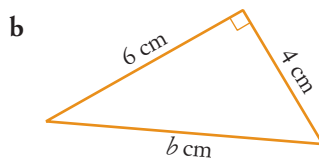
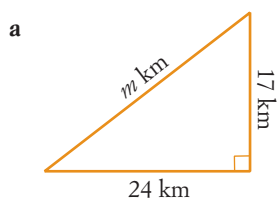
See Example 5

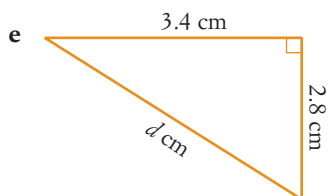
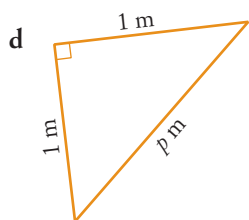
- 3 Find the value of each pronumeral.



See Example 6

- 4 Find the value of each pronumeral as a surd.





5 For each triangle in question 4, find correct to one decimal place the value of each pronumeral.

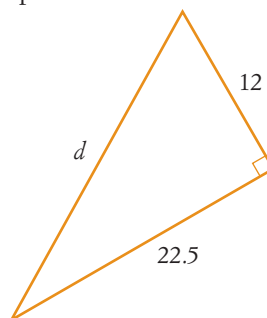
6 In this triangle, what is the value of d ? Select the correct answer **A**, **B**, **C** or **D**.

A 22.8

B 12.9

C 34.5

D 25.5

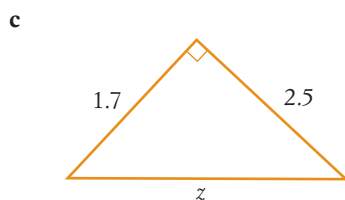
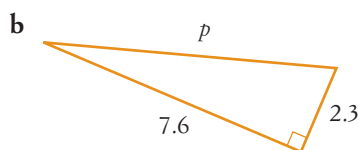
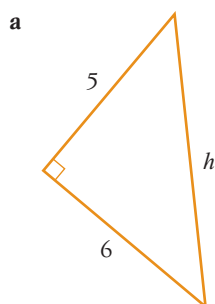


Worked solutions

Exercise 1-03

MAT08MGWS10001

7 Find the value of each pronumeral, correct to two decimal places.



8 Copy and complete each equation.

a $24^2 + 32^2 = \underline{\hspace{1cm}}^2$

b $36^2 + 15^2 = \underline{\hspace{1cm}}^2$

c $9^2 + 40^2 = \underline{\hspace{1cm}}^2$

d $21^2 + \underline{\hspace{1cm}}^2 = 35^2$

e $60^2 + \underline{\hspace{1cm}}^2 = 61^2$

f $\underline{\hspace{1cm}}^2 + 15^2 = 17^2$

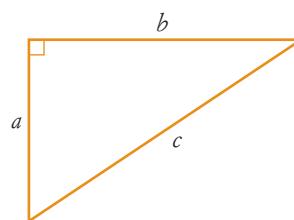
g $\underline{\hspace{1cm}}^2 + 24^2 = 51^2$

h $\underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}}^2 = 10^2$

i $\underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}}^2 = 15^2$

Technology Finding the hypotenuse

In this activity, we will create a spreadsheet to calculate the length of the hypotenuse, c , of a right-angled triangle, given the lengths of the other two sides (a and b).



- 1 Enter the headings and values shown below.

	A	B	C	D	E
1	a	b	a^2+b^2	c	
2	3	4			
3	5	12			
4	6	8			
5	7	24			
6					
7					

Remember: $a^2 + b^2$ means $a^2 + b^2$.

- 2 Enter into cell C2 the formula **=A2^2+B2^2**, and use **Fill Down** to copy this formula into cells C3 to C5.
- 3 To calculate the length of each hypotenuse in column D, we need to find the square root of the values in column C, so in cell D2, enter **=sqrt(C2)**. Then **Fill Down** to copy this formula into cells D3 to D5.
- 4 Use your spreadsheet to check your answers to questions 3 to 6 of Exercise 1-03.

1-04 Finding a shorter side

Video tutorial

Pythagoras' theorem

MAT08MGVT10001

Technology

Excel: Pythagoras' theorem

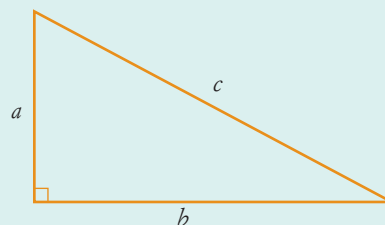
MAT08MGCT00009

Pythagoras' theorem can also be used to find the length of one of the shorter sides in a right-angled triangle if the hypotenuse and the other side are given. In this case, we *subtract* the length of the known shorter side from the length of the hypotenuse.

Summary

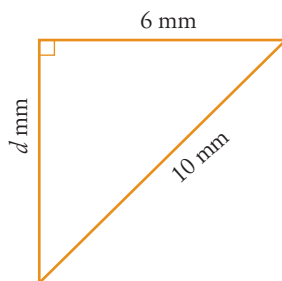
To find the length of a shorter side, a , in a right-angled triangle with hypotenuse c and other shorter side b , use Pythagoras' theorem in the form:

$$a^2 = c^2 - b^2$$



Example 7

Find the value of d in this triangle.



Solution

We want to find the length of one of the shorter sides. Use Pythagoras' theorem.

$$a^2 = c^2 - b^2$$

$$d^2 = 10^2 - 6^2$$

$$= 64$$

$$d = \sqrt{64}$$

$$= 8$$

Substitute the hypotenuse and the other side.

Take the square root to find d .

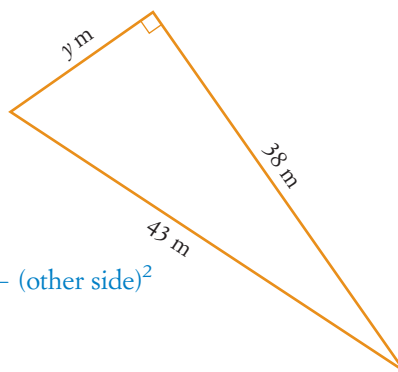
Check: From the diagram, the answer $d = 8$ looks reasonable.

Example 8

Find the value of y in this triangle:

a as a surd

b correct to two decimal places.



Solution

a $a^2 = c^2 - b^2$

$$y^2 = 43^2 - 38^2$$

$$= 405$$

$$y = \sqrt{405}$$

(Hypotenuse)² – (other side)²

Surd answer

b $y = \sqrt{405}$

$$= 20.1246\dots$$

$$\approx 20.12$$

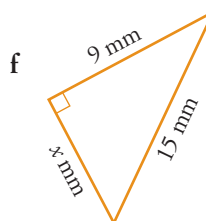
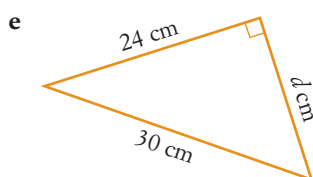
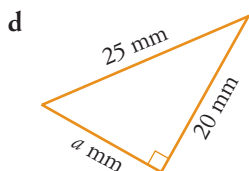
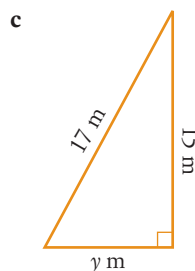
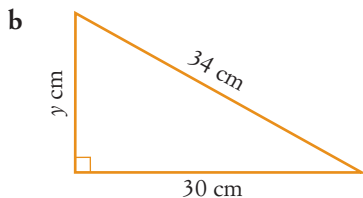
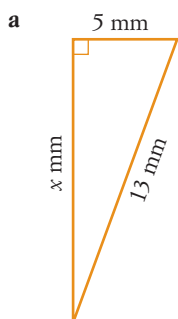
from part **a**

Check: From the diagram, this answer appears to be reasonable.

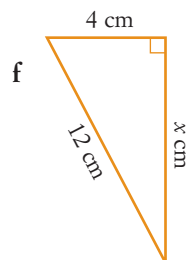
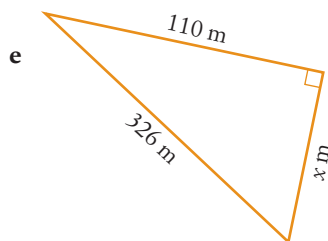
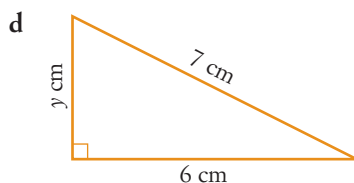
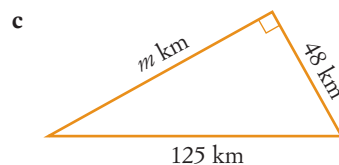
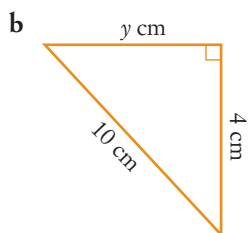
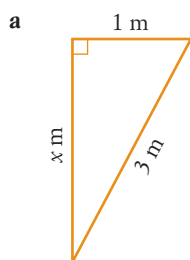
Exercise 1-04 Finding a shorter side

1 Find the value of each pronumeral.

See Example 7



See Example 8 2 Find the value of each pronumeral, giving your answer in surd form.



3 Find the value of each pronumeral in question 2, correct to one decimal place.

4 What is the value of d ? Select the correct answer

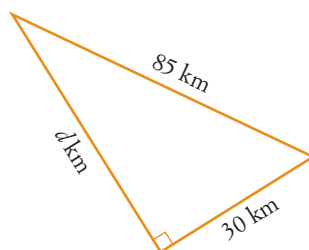
A, B, C or D.

A 74.16

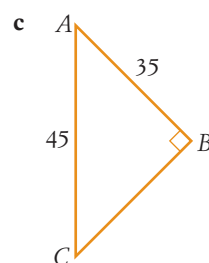
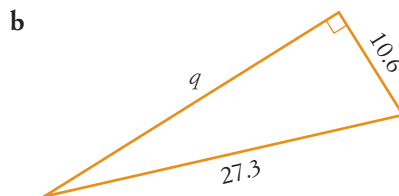
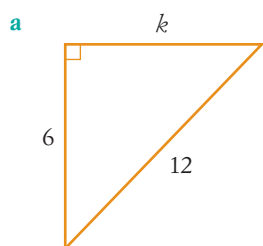
B 79.53

C 90.14

D 127.50



5 Find the value of each pronumeral, correct to two decimal places.



6 What could be the lengths of the other two sides of a right-angled triangle if:

a one of the shorter sides is $\sqrt{7}$ cm long?

b one of the shorter sides is 7 cm long?

c the hypotenuse is $\sqrt{20}$ cm long?

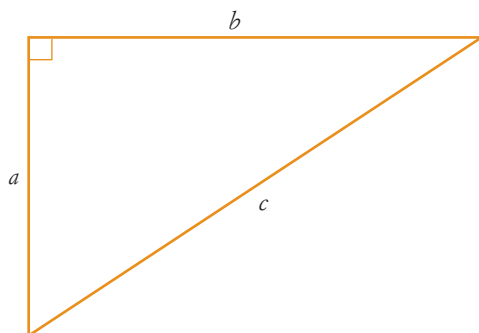
d the hypotenuse is 20 cm long?

Worked solutions

Exercise 1-04

MAT08MGWS10002

Technology Finding a shorter side



In this activity, we will create a spreadsheet to calculate the length of a shorter side, a , of a right-angled triangle, given the lengths of the hypotenuse (c) and the other side (b).

- 1 Enter the headings and values shown below.

	A	B	C	D
1	c	b	$c^2 - b^2$	a
2	25	20		
3	37	12		
4	11	7		
5	29	21		

- 2 Enter a formula into cell C2 to calculate $c^2 - b^2$, which is 225, and use **Fill Down** to copy this formula into cells C3 to C5.
- 3 To calculate the length of the shorter side (a) in column D, we need to find the square root of $c^2 - b^2$. Enter a formula into cell D2 to calculate the square root of cell C2, which is 15, and use **Fill Down** to copy this formula into cells D3 to D5.
- 4 Use your spreadsheet to check your answers to Exercise 1-04.

Mental skills 1

Maths without calculators

Squaring a number ending in 5 or 1

The square of a number ending in 5 always ends in 25.

For example, $35^2 = 1225$ and $105^2 = 11\,025$.

A mental calculation trick requires three easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

- 1 Study each example.

a To calculate 35^2 :

- deleting the 5 from 35 leaves 3
 - multiply 3 by the next consecutive number: $3 \times 4 = 12$
 - write '25' at the end: 1225
- $35^2 = 1225$

b To calculate 105^2 :

- deleting the 5 from 105 leaves 10
 - multiply 10 by the next consecutive number: $10 \times 11 = 110$
 - write 25 at the end: 11 025
- $$105^2 = 11\,025$$

2 Now calculate each square number.

a 25^2

b 55^2

c 45^2

d 85^2

e 115^2

f 7.5^2

g 95^2

h 195^2

i 1.5^2

j 65^2

k 155^2

l 245^2

The square of a number ending in 1 always ends in 1.

For example, $41^2 = 1681$ and $71^2 = 5041$.

A mental calculation trick requires three steps:

- round the number down to the nearest 10 (by subtracting 1) to make a new number
- square the new number
- to your answer, add the new number and its next consecutive number

3 Study each example.

a To calculate 41^2 :

- round 41 down to 40
 - square 40: $40^2 = 1600$
 - add 40 and 41: $1600 + 40 + 41 = 1681$
- $$41^2 = 1681$$

b To calculate 71^2 :

- round 71 down to 70
 - square 70: $70^2 = 4900$
 - $4900 + 70 + 71 = 5041$
- $$71^2 = 5041$$

4 Now calculate each square number.

a 21^2

b 101^2

c 31^2

d 91^2

e 5.1^2

f 81^2

g 61^2

h 201^2

i 1.1^2

j 4.1^2

1-05 Mixed problems

Worksheet

Finding an unknown side

MAT08MGWK10007

Skillsheet

Pythagoras' theorem

MAT08MGSS10001

To find the length of an unknown side in a right-angled triangle, follow these steps.

- 1 Decide whether the unknown side is the **hypotenuse** or one of the **shorter sides**.
- 2 To find the hypotenuse, use $c^2 = a^2 + b^2$ and *add*.
To find a shorter side, use $a^2 = c^2 - b^2$ and *subtract*.
- 3 If a **surd answer** is required, leave the answer in square root ($\sqrt{\quad}$) form. Otherwise, calculate the value correct to the required number of decimal places.

Example 9

Find the value of x as a surd.

Solution

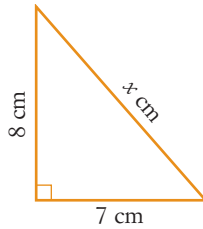
x is the hypotenuse.

$$c^2 = a^2 + b^2$$

$$x^2 = 7^2 + 8^2$$

$$= 113$$

$$x = \sqrt{113}$$



As a surd.

Puzzle sheet

Pythagoras 1

MAT09MGPS00039

Puzzle sheet

Pythagoras 2

MAT09MGPS00040

Example 10

Find the value of n correct to one decimal place.

Solution

n is a shorter side.

$$a^2 = c^2 - b^2$$

$$n^2 = 4.3^2 - 1.9^2$$

$$= 14.88$$

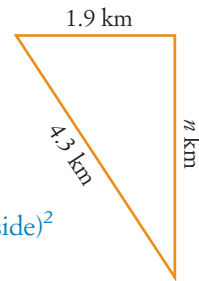
$$n = \sqrt{14.88}$$

$$= 3.85746 \dots$$

$$\approx 3.9$$

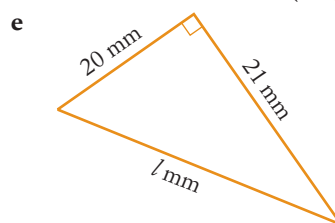
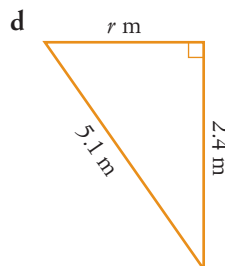
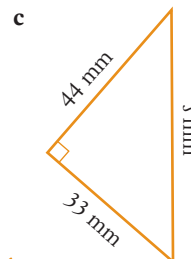
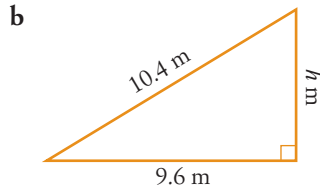
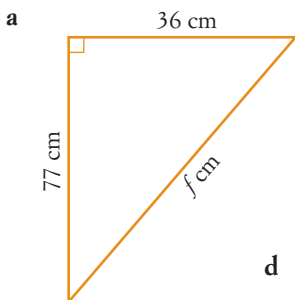
$$(\text{Hypotenuse})^2 - (\text{other side})^2$$

Find the square root.

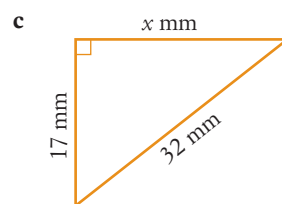
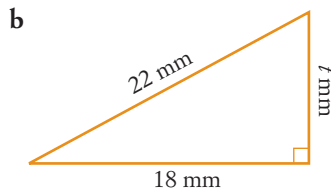
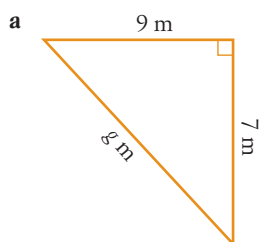


Exercise 1-05 Mixed problems

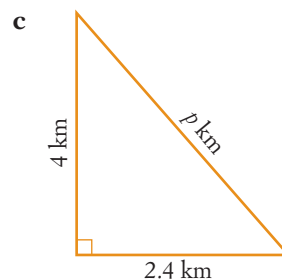
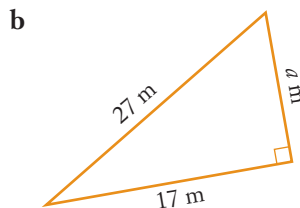
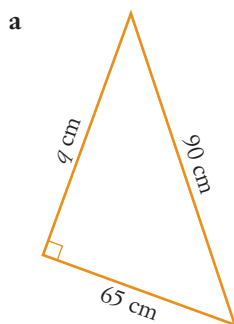
1 Find the value of each pronumeral.



See Example 9 2 Find the value of each pronumeral as a surd.

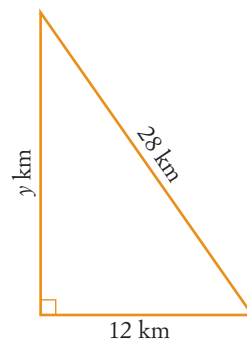


See Example 10 3 Find the value of each pronumeral correct to one decimal place.

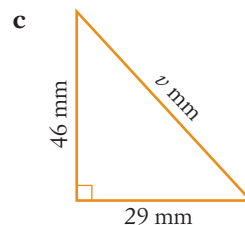
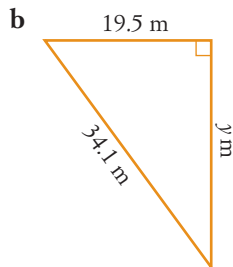
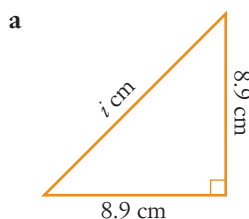


4 What is the value of y ? Select the correct answer **A**, **B**, **C** or **D**.

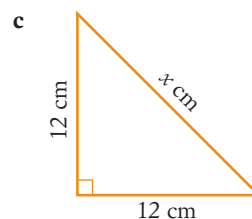
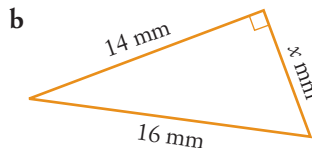
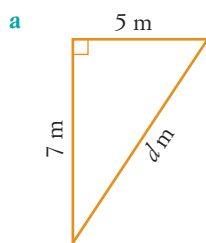
- A** 25.29
B 25.30
C 30.47
D 30.46



5 Find the value of each pronumeral correct to two decimal places.



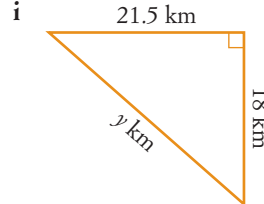
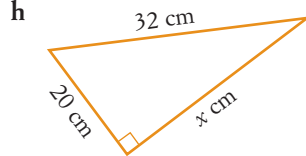
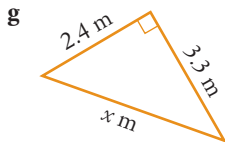
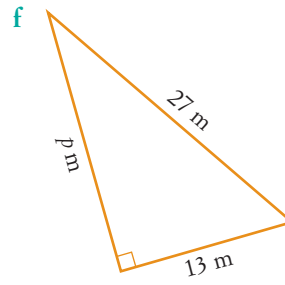
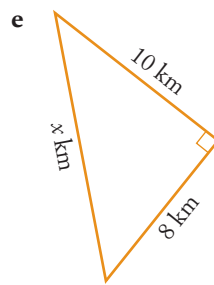
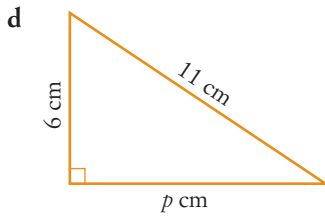
6 Find the value of each pronumeral as a surd.



Worked solutions

Exercise 1-05

MAT08MGWS10003



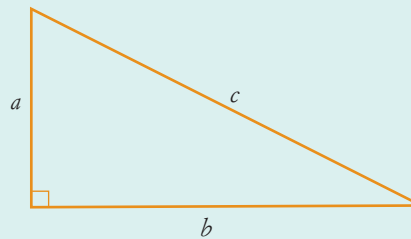
1-06 Testing for right-angled triangles

Pythagoras' theorem can be used to test whether a triangle is right-angled.

Summary

If the sides of a triangle have lengths a , b and c , where c is largest, and they follow the formula $c^2 = a^2 + b^2$, then the triangle must be right-angled.

The right angle is opposite the longest side, c .



Technology worksheet

Pythagoras' theorem
tester

MAT08MGCT10011

Video tutorial
Testing for right-angled
triangles

MAT08MGVT10002

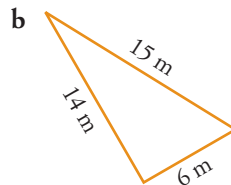
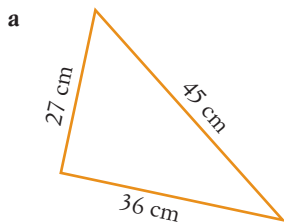
Worksheet
Pythagorean triads

MAT08MGWK00056

This is called the **converse** of Pythagoras' theorem. The word 'converse' means to 'turn around' or 'reverse'. So the converse of a theorem is the theorem written 'back-to-front'.

Example 11

Test whether each triangle is right-angled.



Solution

a $45^2 = 2025$

$$27^2 + 36^2 = 2025$$

$$\therefore 45^2 = 27^2 + 36^2$$

$\therefore$ The triangle is right-angled
(with the right angle opposite the
45 cm side).

b $15^2 = 225$

$$6^2 + 14^2 = 232$$

$$\therefore 15^2 \neq 6^2 + 14^2$$

$\therefore$ The triangle is not right-angled.

Squaring the longest side.

Squaring the two shorter sides, and adding.

The three sides follow $c^2 = a^2 + b^2$.

$\therefore$ is the symbol for 'therefore'

Squaring the longest side.

Squaring the two shorter sides, and adding.

The three sides do not follow $c^2 = a^2 + b^2$.

Exercise 1-06 Testing for right-angled triangles

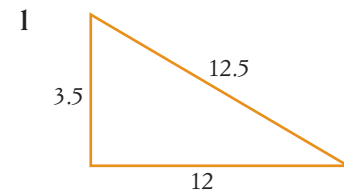
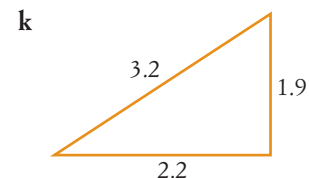
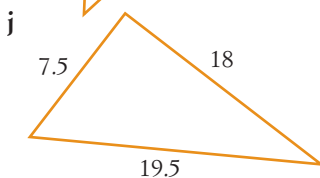
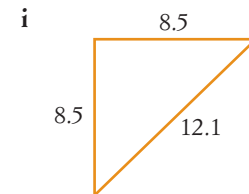
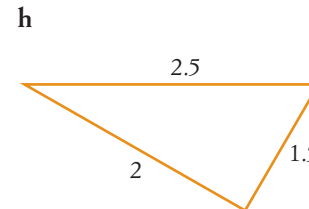
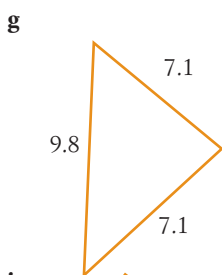
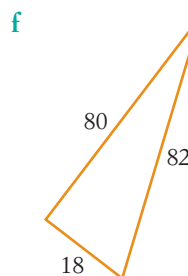
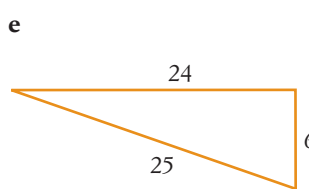
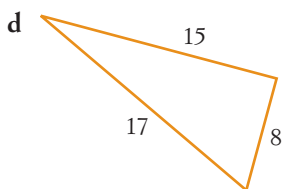
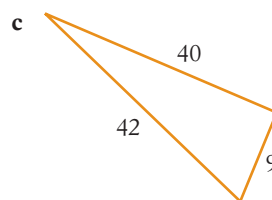
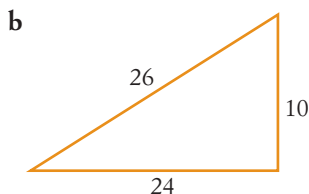
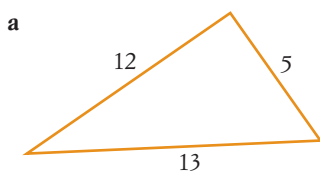
See Example 11

1 Copy each triangle, test whether it is right-angled, then mark the right angle on the triangle.

Worked solutions

Exercise 1-06

MAT08MGWS10004



1-07 Pythagorean triads

A **Pythagorean triad** or **Pythagorean triple** is any group of three numbers that follow Pythagoras' theorem, for example, (3, 4, 5) or (2.5, 6, 6.5). The word **triad** means a group of three related items ('tri-' means 3).

Worksheet

Pythagorean triads

MAT08MGWK10008

Technology worksheet

Excel: Pythagorean
triples

MAT08MGCT00025

Technology

Excel: Pythagorean
triples

MAT08MGCT00010

Summary

(a, b, c) is a **Pythagorean triad** if $c^2 = a^2 + b^2$.

Any multiple of (a, b, c) is also a Pythagorean triad.

Example 12

Test whether (5, 12, 13) is a Pythagorean triad.

Solution

$$13^2 = 169$$

Squaring the largest number.

$$5^2 + 12^2 = 169$$

Squaring the two smaller numbers, and adding.

$$\therefore 13^2 = 5^2 + 12^2$$

These three numbers follow Pythagoras' theorem.

$\therefore$ (5, 12, 13) is a Pythagorean triad.

Example 13

(3, 4, 5) is a Pythagorean triad. Create other Pythagorean triads by multiplying (3, 4, 5) by:

a 2

b 9

c $\frac{1}{2}$

Solution

a $2 \times (3, 4, 5) = (6, 8, 10)$

Checking: $10^2 = 100$

$$6^2 + 8^2 = 100$$

$$\therefore 10^2 = 6^2 + 8^2$$

$\therefore$ (6, 8, 10) is a Pythagorean triad.

b $9 \times (3, 4, 5) = (27, 36, 45)$

Checking: $45^2 = 2025$

$$27^2 + 36^2 = 2025$$

$$\therefore 45^2 = 27^2 + 36^2$$

$\therefore$ (27, 36, 45) is a Pythagorean triad.

$$\text{c } \frac{1}{2} \times (3, 4, 5) = (1.5, 2, 2.5)$$

$$\text{Checking: } 2.5^2 = 6.25$$

$$1.5^2 + 2^2 = 6.25$$

$$\therefore 2.5^2 = 1.5^2 + 2^2$$

$\therefore (1.5, 2, 2.5)$ is a Pythagorean triad.

Exercise 1-07 Pythagorean triads

See Example 12

- 1 Test whether each triad is a Pythagorean triad.

a (8, 15, 17)

b (10, 24, 26)

c (30, 40, 50)

d (5, 7, 9)

e (9, 40, 41)

f (4, 5, 9)

g (11, 60, 61)

h (7, 24, 25)

i (15, 114, 115)

- 2 Which of the following is a Pythagorean triad? Select the correct answer **A**, **B**, **C** or **D**.

A (4, 6, 8)

B (5, 10, 12)

C (6, 7, 10)

D (20, 48, 52)

- 3 Use the spreadsheet you created in **Technology: Pythagoras' theorem tester** to check your answers to questions 1 and 2.

See Example 13

- 4 For each Pythagorean triad, create another Pythagorean triad by multiplying each number in the triad by:

i a whole number

ii a fraction

iii a decimal.

a (5, 12, 13)

b (8, 15, 17)

c (30, 40, 50)

d (7, 24, 25)

Check that each answer follows Pythagoras' theorem.

- 5 Pythagoras developed a formula for finding Pythagorean triads (a, b, c) . If one number in the triad is a , the formulas for the other two numbers are $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$.

a If $a = 5$, use the formulas to find the values of b and c .

b Hence show that (a, b, c) is a Pythagorean triad.

- 6 Use the formulas to find Pythagorean triads for each value of a .

a $a = 7$

b $a = 11$

c $a = 15$

d $a = 4$

e $a = 9$

f $a = 19$

g $a = 10$

h $a = 51$

- 7 There are many other formulas for creating Pythagorean triads. Use the Internet to search for some of them.

1-08 Pythagoras' theorem problems

Worksheet

Pythagoras' theorem has many practical applications: from finding the diagonal length of a television screen to calculating the distance a ship has sailed.

Applications of
Pythagoras' theorem

MAT08MGWK10009

Example 14

The size of a TV screen is described by the length of its diagonal. If a flat screen TV is 58 cm wide and 32 cm high, what is the size of its screen? Answer to the nearest centimetre.

Solution

Let the diagonal length be d cm.

$$d^2 = 58^2 + 32^2$$

$$= 4388$$

$$d = \sqrt{4388}$$

$$= 66.24198\dots$$

$$\approx 66$$

The size of the screen is 66 cm.



d is the hypotenuse.

Worksheet

Pythagoras' theorem in 2D and 3D

MAT08MGWK10010

Worksheet

TV screens

MAT08MGWK10011

Example 15

A tower is supported by a wire that is 20 m long and attached to the ground 10 m from the base of the tower. How high does the wire reach up the tower? Answer to the nearest 0.1 m.

Solution

Let h m be how high the wire reaches up the tower.

$$h^2 = 20^2 - 10^2$$

$$= 300$$

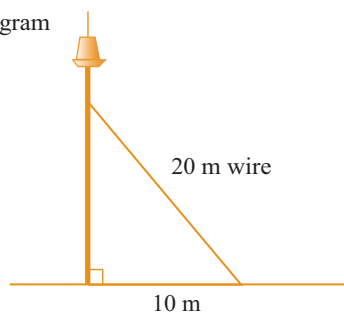
$$h = \sqrt{300}$$

$$= 17.3205$$

$$\approx 17.3$$

The wire reaches 17.3 m up the tower.

The diagram

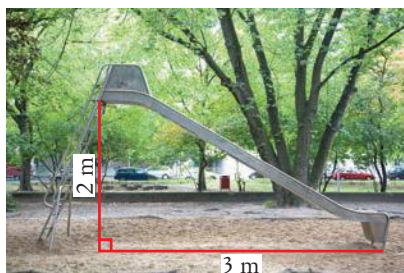


h is a shorter side.

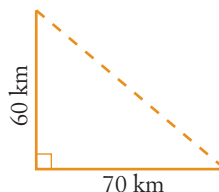
Exercise 1-08 Pythagoras' theorem problems

- Find the length of this playground slide correct to two decimal places.

See Example 14

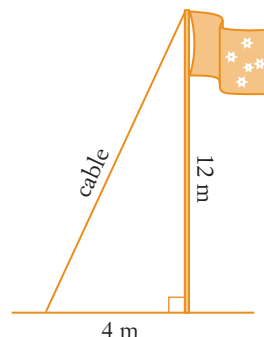


- 2 A ship sails 60 kilometres south and then 70 kilometres east. How far is it from its starting point, correct to one decimal place?



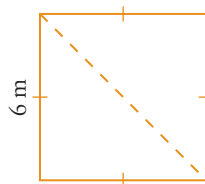
- 3 What is the length of the cable used to stabilise a flagpole that is 12 metres high, if the cable is secured to the ground 4 m from the base of the flagpole? Select the correct answer **A**, **B**, **C** or **D**.

- A** 11.3 m **B** 12.6 m
C 16 m **D** 80 m



See Example 15

- 4 Find the height of an LCD TV screen with a 51 cm diagonal if its length is 41 cm. Give your answer correct to one decimal place.
- 5 Find the length of the diagonal of a square with sides of length 6 metres. Give your answer as a surd.



- 6 A firefighter places a ladder on a window sill 4.5 m above the ground. If the foot of the ladder is 1.6 m from the wall, how long is the ladder, correct to one decimal place?

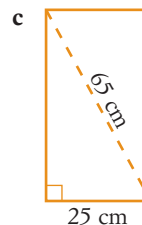
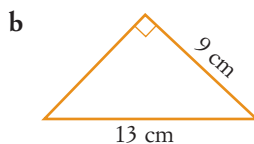


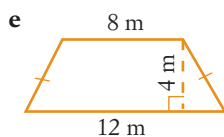
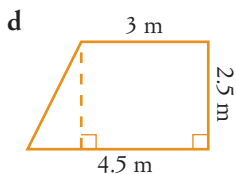
Worked solutions

Exercise 1-08

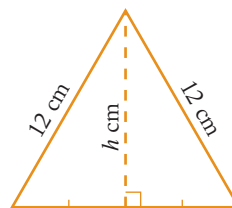
MAT08MGWS10006

- 7 By first using Pythagoras' theorem to find the length of the unknown side, find the perimeter of each shape below, correct to two decimal places, where necessary.

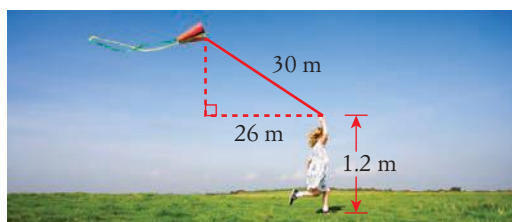




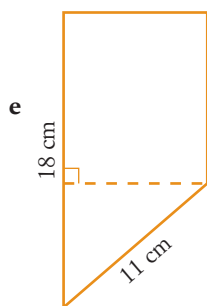
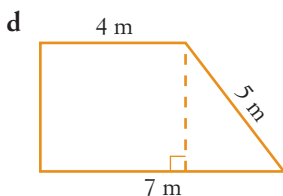
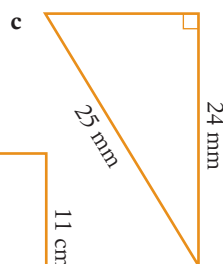
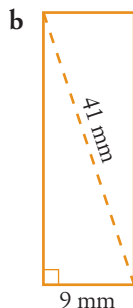
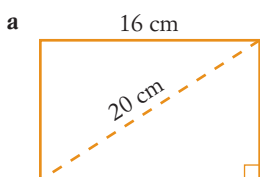
- 8** An equilateral triangle has sides of 12 cm. Find its perpendicular height, h , correct to two decimal places.



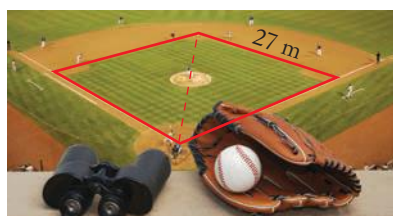
- 9** A rope is tied to the top of the 6 metre wall of a tent and tied to a peg in the ground. The peg is 2 m from the bottom of the tent. How long is the rope, correct to two decimal places?
- 10** Olga holds a kite string 1.2 m above the ground. How high is the kite above the ground, to the nearest metre?



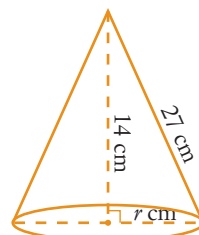
- 11** Find the area of each shape below, correct to one decimal place where necessary.



- 12** This baseball diamond is a square shape of length 27 m. What is the distance from the home plate to second base, to the nearest metre?



- 13 The slant height of a cone is 27 cm and its vertical height is 14 cm.
Find the radius of the base circle, correct to the nearest centimetre.

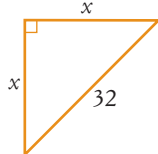


- 14 A ladder 5 m long leans against a wall, with its base 2 m from the bottom of the wall. How far does the ladder reach up the wall, correct to nearest centimetre?

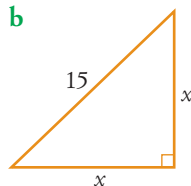
Power plus

- 1 Find the value of x in each triangle, correct to one decimal place.

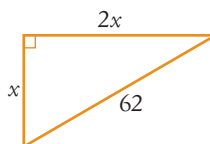
a



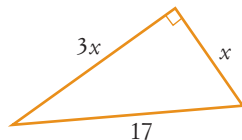
b



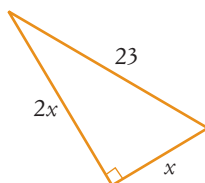
c



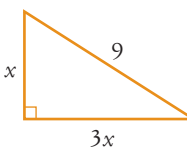
d



e



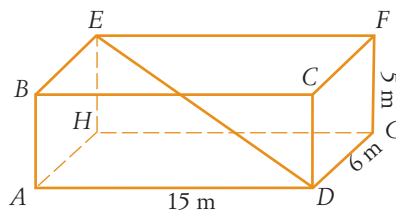
f



- 2 Find, correct to one decimal place, the length of:

a HD

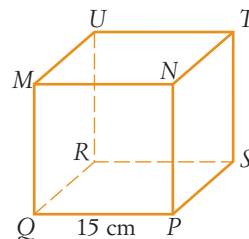
b DE



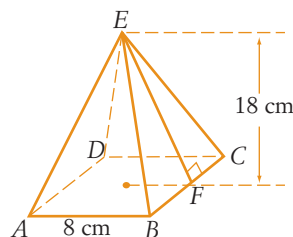
- 3 For this cube of side length 15 cm find, correct to one decimal place, the length of:

a QS , the diagonal of the base

b QT , the diagonal of the cube



- 4 For this square pyramid, find the slant height EF correct to one decimal place.



Language of maths

area	hypotenuse	right-angled	surd
converse	irrational	shorter	theorem
diagonal	perimeter	side	triad
formula	Pythagoras	square root	unknown

Puzzle sheet

Pythagoras' theorem
find-a-word

MAT08MGPS10001

- Who was **Pythagoras** and what country did he come from?
- Describe the **hypotenuse** of a right-angled triangle in two ways.
- What is another word for 'theorem'?
- For what type of triangle is **Pythagoras' theorem** used?
- What is a **surd**?
- What is the name given to a set of three numbers that follows Pythagoras' theorem?

Topic overview

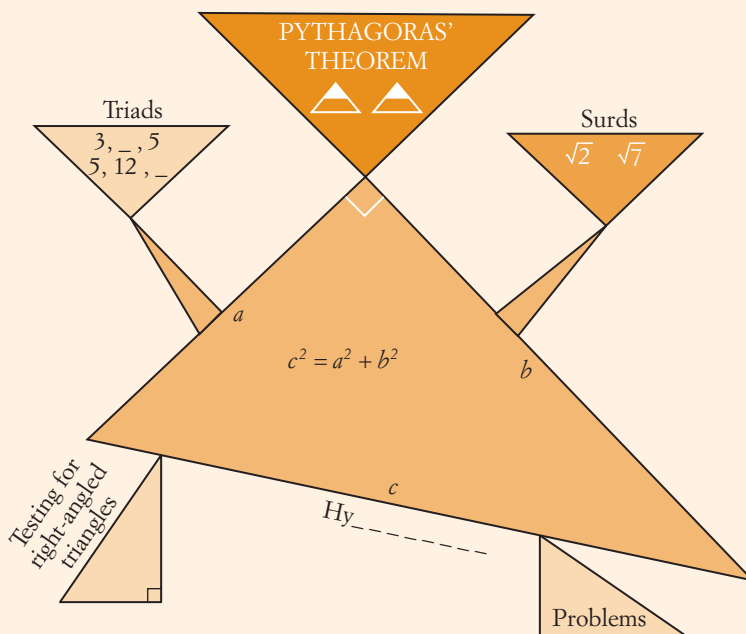
- How relevant do you think Pythagoras' theorem is to our world? Give reasons for your answer.
- Give three examples of jobs that would use Pythagoras' theorem.
- What did you find especially interesting about this topic?
- Is there any section of this topic that you found difficult? Discuss any problems with your teacher or a friend.

Worksheet

Mind map: Pythagoras' theorem

MAT08MGWK10012

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 1 revision

See Exercise 1-01

- 1 Evaluate each expression, correct to two decimal places.

a $\sqrt{203}$

b $\sqrt{15^2 + 6^2}$

c $\sqrt{7^2 - 4^2}$

d $\sqrt{3.4^2 + 5.2^2}$

See Exercise 1-01

- 2 Select all the surds from this list of square roots.

$\sqrt{25}$
 $\sqrt{169}$

$\sqrt{104}$
 $\sqrt{121}$

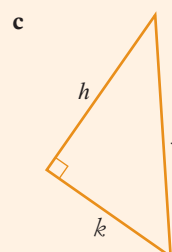
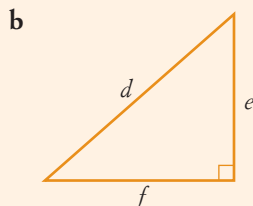
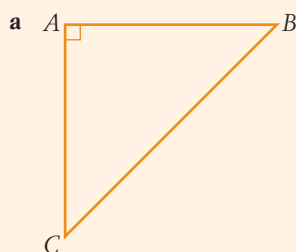
$\sqrt{96}$
 $\sqrt{45}$

$\sqrt{256}$
 $\sqrt{729}$

$\sqrt{12}$
 $\sqrt{88}$

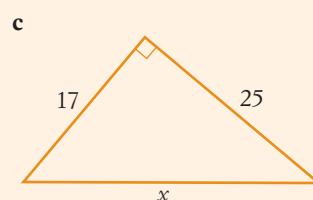
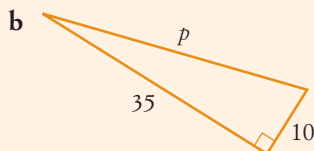
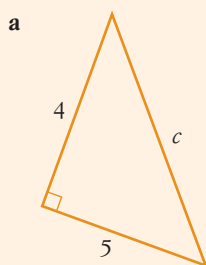
See Exercise 1-02

- 3 For each right-angled triangle, name the hypotenuse and write Pythagoras' theorem.



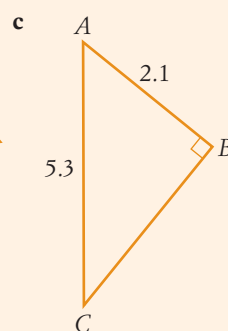
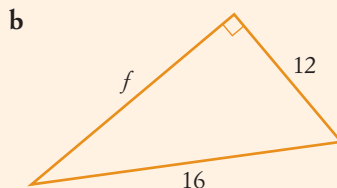
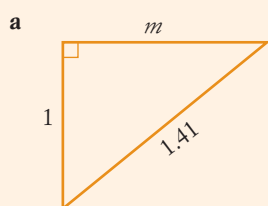
See Exercise 1-03

- 4 Find the value of each pronumeral, giving your answer as a surd.



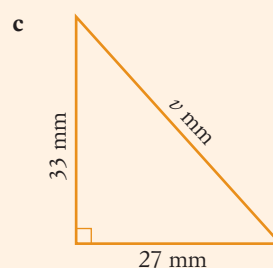
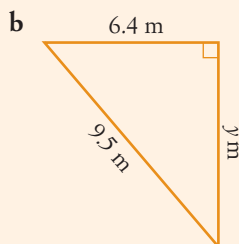
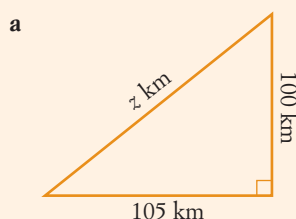
See Exercise 1-04

- 5 Find the value of each pronumeral, giving your answer correct to two decimal places.



See Exercise 1-05

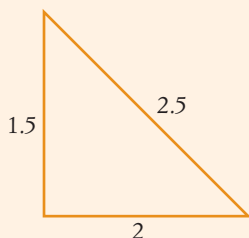
- 6 Find the value of each pronumeral, correct to one decimal place.



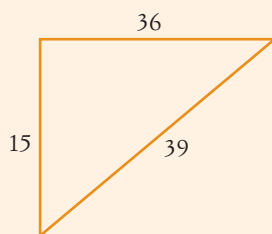
7 Test whether each triangle is right-angled.

See Exercise 1-06

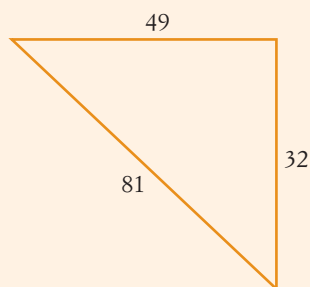
a



b



c



8 Test whether each triad is a Pythagorean triad.

See Exercise 1-07

a (15, 20, 25)

b (11, 14, 20)

c (20, 21, 29)

d (2.5, 6, 6.5)

9 Find the length of the longest pencil that can fit inside this pencil case. Answer to the nearest mm.

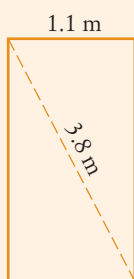
See Exercise 1-08



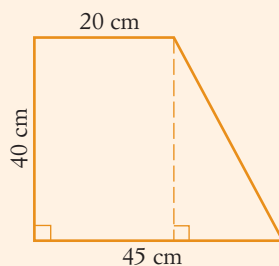
10 Find the perimeter of each shape, correct to one decimal place.

See Exercise 1-08

a



b





2

Number and algebra

Working with numbers

The use of numbers is an essential part of our lives. We need to make calculations when we shop, when we plan a party or holiday, when we do our banking and even when playing sport. We all need to be able to estimate and calculate answers effectively. In this chapter, we will revise and extend our number skills and examine the patterns involved when calculating with powers, using mental calculation, pen-and-paper methods and calculators.



Chapter outline

	Proficiency strands			
2-01 Mental calculation	U	F	R	
2-02 Adding and subtracting integers	U	F	PS	R
2-03 Multiplying integers	U	F		R
2-04 Dividing integers	U	F		R
2-05 Order of operations	U	F		
2-06 Decimals	U	F		
2-07 Multiplying and dividing decimals	U	F	PS	R
2-08 Terminating and recurring decimals	U	F		R
2-09 Powers and roots	U	F		R C
2-10 Prime factors	U	F		R C
2-11 Index laws for multiplying and dividing	U	F		R
2-12 More index laws	U	F		R

Wordbank

base (in index notation) A number raised to a power, for example, in 3^5 , 3 is the base

cube root The value which, if cubed, will give the number required, for example $\sqrt[3]{8} = 2$ because $2^3 = 8$

factor tree A diagram that lists the prime factors of a number.

index notation A way of writing powers for the repeated multiplication of a number, for example, 3^5 .

mental calculation To operate with numbers 'in your head', without using a pen or calculator

order of operations The rules for calculating an expression involving mixed operations, such as $14 - 2 \times 4 + 1$

recurring decimal A decimal that has one or more digits that repeat endlessly

terminating decimal A decimal that is not recurring, but comes to an end

In this chapter you will:

- apply the associative, commutative and distributive laws to aid mental computation
- carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies
- apply the order of operations to evaluate expressions involving directed numbers mentally, including where an operator is contained within the numerator or denominator of a fraction, for example, $\frac{15+9}{15-3}$
- investigate terminating and recurring decimals
- apply the order of operations to evaluate expressions involving indices, square and cube roots, with and without a calculator
- find square roots and cube roots of any non-square whole number using a calculator, after first estimating
- determine through numerical examples the properties of squares and square roots of products: $(ab)^2$ and $\sqrt{ab}$
- express a number as a product of its prime factors to determine whether its square root or cube root is an integer
- use index notation with numbers to establish the index laws with positive integral indices and the zero index

SkillCheck

Worksheet

StartUp assignment 2

MAT08NAWK10013

Skillsheet

Decimals

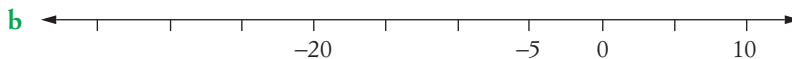
MAT08NASS10002

Skillsheet

Fractions and decimals

MAT08NASS10003

1 Copy and complete each number line.


2 Write these integers in ascending order: $-7, 6, -5, 11, -2, 1, 0, 7$.

3 Write these integers in descending order: $12, -3, 5, -1, 2, -9, 4, 8$.

4 In the decimal 2.718, name the digit in the:

a tenths place **b** thousandths place **c** hundredths place

5 Convert each decimal to a fraction.

a 0.03 **b** 0.007 **c** 0.49 **d** 0.9

6 How many decimal places has each number?

a 14.451 **b** 86.6 **c** 28.04 **d** 3.141 59

7 **a** List the factors of 15.

b List the first 5 multiples of 15.

8 State whether each number is prime or composite.

a 2 **b** 17 **c** 25 **d** 27

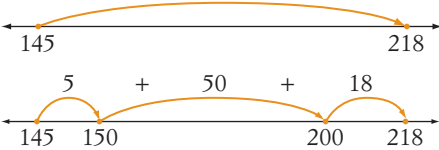
2-01 Mental calculation

The word ‘mental’ means ‘using the mind’, and mental calculation is the skill of working with numbers ‘in your head’, without using a pen or calculator. In the ‘Mental skills’ sections of *New Century Maths 7*, you learnt many mental strategies for calculating with numbers.

Example 1

Skill	Examples
Estimating by rounding numbers.	a $15 + 37 + 18 + 45 + 22$ $\approx 20 + 40 + 20 + 40 + 20$ (exact answer = 137) $= 140$
	b $18 \times 12 \approx 20 \times 11 = 220$ (exact answer = 216)
	c $504 \div 8 \approx 500 \div 10 = 50$ (exact answer = 63)

Example 2

Skill	Examples
Adding in any order using the commutative law: $a + b = b + a$ and the associative law: $(a + b) + c = a + (b + c)$	$15 + 37 + 18 + 45 + 22$ $= (15 + 45) + (18 + 22) + 37$ ← pair numbers that add to multiples of 10 $= 60 + 40 + 37$ $= 137$
Adding and subtracting 8 or 9.	a $43 + 9 = 43 + 10 - 1$ ← add 10, count back 1 $= 53 - 1 = 52$ b $97 + 8 = 97 + 10 - 2$ ← add 10, count back 2 $= 107 - 2 = 105$ c $61 - 8 = 61 - 10 + 2$ ← subtract 10, count forward 2 $= 51 + 2 = 53$
Calculating differences using a number line.	$218 - 145$ Think of this as the difference or ‘gap’ between 145 and 218 on a number line.  Add the jumps. $5 + 50 + 18 = 73$. $218 - 145 = 73$. ← This is like counting out change with money.

Example 3

Skill	Examples
Multiplying in any order using the commutative law: $a \times b = b \times a$ and the associative law: $(a \times b) \times c = a \times (b \times c)$	$7 \times 4 \times 5 = 7 \times (4 \times 5)$ $= 7 \times 20$ $= 140$
Multiplying by a multiple of 10.	$9 \times 60 = 9 \times 6 \times 10$ $= 54 \times 10$ $= 540$
Dividing by a multiple of 10.	$2700 \div 30 = 2700 \div 30$ $= 270 \div 3$ $= 90$

pair numbers that multiply to multiples of 10

Doubling and halving numbers.	a $47 \times 2 = 40 \times 2 + 7 \times 2$ $= 80 + 14$ $= 94$
	b $\frac{1}{2} \times 68 = \frac{1}{2} \times 60 + \frac{1}{2} \times 8$ $= 30 + 4$ $= 34$
	c $\frac{1}{2} \times 192 = \frac{1}{2} \times 180 + \frac{1}{2} \times 12$ $= 90 + 6$ $= 96$

Multiplying and dividing by 4 or 8.	a $29 \times 4 = 29 \times 2 \times 2$ Double 29 = 58, double 58 = 116. $29 \times 4 = 116$.	Double twice.
	b $16 \times 8 = 16 \times 2 \times 2 \times 2$ Double 16 = 32, double 32 = 64, double 64 = 128. $16 \times 8 = 128$.	Double three times.
	c $560 \div 4 = 560 \div 2 \div 2$ $\frac{1}{2} \times 560 = 280, \frac{1}{2} \times 280 = 140$. $560 \div 4 = 140$.	Halve twice.
Simplifying multiplication by factorising.	a $45 \times 16 = 9 \times 5 \times 4 \times 4$ $= 5 \times 4 \times 4 \times 9$ $= 20 \times 36$ $= 720$	
	b $12 \times 18 = 6 \times 2 \times 9 \times 2$ $= 6 \times 9 \times 2 \times 2$ $= 54 \times 4$ $= 216$	

Multiplying and dividing by 5, 15, 25, 50.	a $16 \times 15 = 16 \times \frac{1}{2} \times 30$ or $8 \times 2 \times 15$ $= 8 \times 30$ $= 240$
	b $24 \times 25 = 24 \times \frac{1}{4} \times 100$ or $6 \times 4 \times 25$ $= 6 \times 100$ $= 600$
	c $420 \div 5 = 420 \div 10 \times 2$ $= 42 \times 2$ $= 84$
	d $300 \div 25 = 300 \div 100 \times 4$ $= 3 \times 4$ $= 12$
Multiplying by 8, 9, 11, 12 using the distributive laws: $a \times (b + c)$ $= a \times b + a \times c$ $a \times (b - c)$ $= a \times b - a \times c$	a $23 \times 11 = 23 \times (10 + 1)$ $= 23 \times 10 + 23 \times 1$ $= 230 + 23$ $= 253$
	b $18 \times 12 = 18 \times (10 + 2)$ $= 18 \times 10 + 18 \times 2$ $= 180 + 36$ $= 216$
	c $17 \times 9 = 17 \times (10 - 1)$ $= 17 \times 10 - 17 \times 1$ $= 170 - 17$ $= 153$
	d $36 \times 8 = 36 \times (10 - 2)$ $= 36 \times 10 - 36 \times 2$ $= 360 - 72$ $= 288$

Short and long division

Summary

Short division is a technique for dividing by one-digit numbers (whole numbers less than 10).

Long division is a technique for dividing by numbers with two or more digits (whole numbers greater than 10).

NSW

Example 4

Evaluate each quotient.

a $520 \div 8$

b $945 \div 21$

Solution

a By short division:

$$8 \overline{)520}$$

$$8 \overline{)52} 40$$

$$8 \overline{)52} 40$$

$$520 \div 8 = 65$$

OR:

$$520 \div 8 = 520 \div 2 \div 2 \div 2$$

$$520 \div 2 = 260, \quad 260 \div 2 = 130, \quad 130 \div 2 = 65$$

$$520 \div 8 = 65$$

8 into 5 goes 0.

8 into 52 goes 6, remainder 4.

8 into 40 goes 5 (exactly).

Check by estimating: $520 \div 8 \approx 500 \div 10 = 50$
(65 is close to 50)

Halve three times.

NSW

b By long division:

$$\begin{array}{r} 45 \\ 21 \overline{)945} \\ -84 \downarrow \\ \hline 105 \\ -105 \\ \hline 0 \end{array}$$

$$945 \div 21 = 45$$

OR:

$$\begin{array}{r|l} 21 \overline{)945} & \\ -420 & 20 \text{ times} \\ \hline 525 & \\ -420 & 20 \text{ times} \\ \hline 105 & \\ -84 & 4 \text{ times} \\ \hline 21 & \\ -21 & 1 \text{ time} \\ \hline 0 & 45 \text{ times} \end{array}$$

21 into 94 is 4, remainder 10.

21 into 105 is 5.

Check by estimating: $945 \div 21 \approx 900 \div 20 = 45$

Guessing with 'easy' multiples of 21.

Exercise 2-01 Mental calculation

1 Estimate the value of each expression, then check your estimate with a calculator.

See Example 1

a $27 + 11 + 87 + 142 + 64$	b $55 + 34 - 22 - 46 + 136$	c $684 + 903$
d $35 + 81 + 110 + 22 + 7$	e $517 - 96$	f $766 - 353$
g 46×8	h 367×2	i 51×12
j $245 \div 7$	k $828 \div 3$	l $564 \div 12$

2 Evaluate each sum.

See Example 2

a $55 + 18 + 25 + 9 + 12$	b $23 + 42 + 16 + 24 + 7$	c $140 + 33 + 12 + 20 + 28$
d $37 + 11 + 29 + 5 + 13$	e $45 + 49 + 121 + 25 + 10$	f $74 + 99 + 21 + 32 + 16$

3 What is the value of $720 + 1456$? Select the correct answer **A**, **B**, **C** or **D**.

A 2176	B 2206	C 2806	D 8656

4 Evaluate each expression.

a $27 + 9$	b $45 + 9$	c $16 + 8$	d $34 + 28$
e $17 + 11$	f $22 + 21$	g $69 + 12$	h $114 + 32$
i $45 - 9$	j $27 - 9$	k $76 - 8$	l $32 - 18$
m $55 - 12$	n $183 - 11$	o $67 - 41$	p $121 - 22$

5 Evaluate each difference.

a $560 - 327$	b $1614 - 239$	c $452 - 367$

6 Write the answer to each multiplication.

a 3×8	b 4×4	c 7×5	d 6×9
e 4×9	f 8×5	g 8×8	h 7×3
i 4×6	j 6×6	k 9×3	l 7×4

7 Write the answer to each division.

a $49 \div 7$	b $20 \div 4$	c $36 \div 9$	d $27 \div 3$
e $54 \div 6$	f $28 \div 7$	g $15 \div 5$	h $72 \div 8$
i $16 \div 4$	j $45 \div 5$	k $24 \div 8$	l $40 \div 8$

8 Evaluate each product.

See Example 3

a $8 \times 2 \times 50$	b $6 \times 5 \times 3$	c $25 \times 7 \times 4$	d $4 \times 8 \times 5$
e $5 \times 2 \times 10$	f $7 \times 4 \times 25$	g $15 \times 50 \times 2$	h $3 \times 4 \times 20$
i $6 \times 6 \times 5$	j $2 \times 5 \times 32$	k $20 \times 17 \times 5$	l $10 \times 7 \times 20$

9 Evaluate each expression.

a 8×20	b 7×500	c 2×9000	d 15×20
e 3×70	f 6×300	g 9×4000	h 12×200
i $900 \div 30$	j $1200 \div 200$	k $250 \div 50$	l $320 \div 40$
m $1800 \div 90$	n $2000 \div 20$	o $42\,000 \div 6000$	p $240 \div 80$

10 How many hours are there in one week? Select the correct answer **A**, **B**, **C** or **D**.

A 84	B 151	C 168	D 240

11 Evaluate each expression.

a 85×2	b 39×2	c 57×4	d 28×4
e 16×8	f 33×8	g $\frac{1}{2} \times 648$	h $\frac{1}{2} \times 232$
i $216 \div 4$	j $488 \div 4$	k $872 \div 8$	l $208 \div 8$

Worked solutions

Exercise 2-01

MAT08NAWS10007

- 12 Evaluate each product by factorising.

a 7×40

b 25×16

c 22×6

d 13×20

e 35×6

f 21×3

g 16×12

h 18×18

- 13 Evaluate each expression.

a 22×5

b 14×5

c 28×50

d 36×15

e 44×25

f 52×50

g $800 \div 50$

h $90 \div 5$

i $120 \div 15$

j $1000 \div 25$

k $410 \div 5$

l $700 \div 50$

- 14 Nathan can type 76 words per minute. How many words can he type in 15 minutes?

- 15 Evaluate each product by multiplying by 10 first, then adding or subtracting.

a 34×9

b 51×9

c 27×8

d 72×8

e 36×11

f 41×11

g 64×12

h 22×12

See Example 4

- 16 Evaluate each quotient.

a $2640 \div 8$

b $1638 \div 9$

c $616 \div 7$

d $1584 \div 4$

e $675 \div 15$

f $756 \div 14$

g $806 \div 31$

h $729 \div 27$

- 17 There are 135 students in Year 8. If they are divided evenly into five classes, how many students will be in each class? Select the correct answer A, B, C or D.

A 21

B 25

C 27

D 29

- 18 Divide a restaurant bill of \$204 evenly among six people.

Just for the record

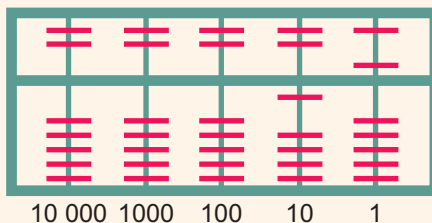
The **abacus** is often called the 'first computer'. It was invented by the Chinese in the 14th century and it is still used today to add, subtract, multiply, divide, including calculating with fractions and square roots. The word 'abacus' comes from the Greek word **abax** meaning 'calculating board'.

The abacus

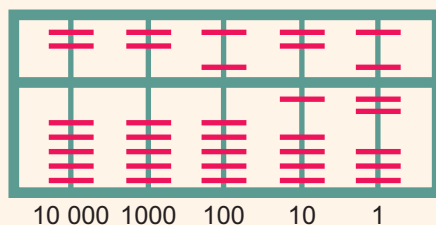


An abacus uses place value to represent numbers. Each column of beads represents a place value such as units, tens, hundreds, thousands. The beads below the centre bar represent **one** of that place value. The beads above the bar represent **five** units of that place value. Only the beads that have been moved to the bar represent numbers.

This abacus shows 15: 1 ten + 5 units.



This abacus shows 517: 5 hundreds + 1 ten + 7 units.



Draw how the three numbers 63, 492 and this year would be shown on an abacus.

2-02 Adding and subtracting integers

Integers are all the positive and negative numbers and zero. Integers can be added and subtracted on a number line. A negative number can be entered into a calculator using the **(-)** or **+/-** key.

Skillsheet

Integers

MAT08NASS10004

Skillsheet

Integers using
diagrams

MAT08NASS10005

Summary

Adding a negative number is the same as subtracting its opposite: $7 + (-4) = 7 - 4 = 3$

Subtracting a negative number is the same as adding its opposite: $7 - (-4) = 7 + 4 = 11$

Example 5

Evaluate each expression.

a $-8 + 3$

b $-8 - 3$

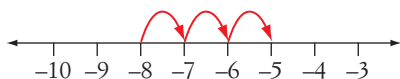
c $2 + (-6)$

d $2 - (-6)$

Solution

a $-8 + 3 = -5$

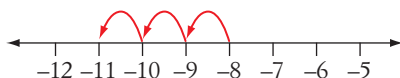
Start at -8 , go forward 3



On a calculator, enter: **(-)** 8 **+** 3 **=**.

b $-8 - 3 = -11$

Start at -8 , go back 3.



On a calculator, enter: **(-)** 8 **-** 3 **=**.

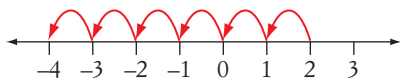
Animated example

Number lines

MAT08NAAE00002

c $2 + (-6) = 2 - 6 = -4$

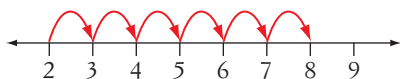
Start at 2, go back 6.



On a calculator, enter: 2 **+** **(-)** 6 **=**.

d $2 - (-6) = 2 + 6 = 8$

Start at 2, go forward 6.



On a calculator, enter: 2 **-** **(-)** 6 **=**.

Exercise 2-02 Adding and subtracting integers

See Example 5

- 1 Evaluate each sum and check your answer using a calculator.

A **sum** is the answer to an addition

a $-3 + 1$

b $-5 + 10$

c $-2 + 4$

d $-1 + 7$

e $-10 + 14$

f $-9 + 6$

g $-4 + 8$

h $-5 + 5$

i $9 + (-3)$

j $4 + (-1)$

k $6 + (-4)$

l $2 + (-2)$

m $3 + (-8)$

n $7 + (-10)$

o $5 + (-6)$

p $1 + (-9)$

- 2 Evaluate each difference and check your answer using a calculator.

A **difference** is the answer to a subtraction

a $7 - 9$

b $5 - 11$

c $3 - 10$

d $2 - 6$

e $-4 - 4$

f $-8 - 2$

g $-1 - 5$

h $-3 - 9$

i $6 - (-6)$

j $4 - (-1)$

k $7 - (-5)$

l $10 - (-8)$

m $-5 - (-4)$

n $-6 - (-9)$

o $-3 - (-3)$

p $-1 - (-2)$

- 3 At the ski resort, the temperature rose from -2°C to 7°C . What was the increase in temperature?
- 4 Rhys' bank account balance was $-\$44$, meaning he was $\$44$ in debt. He deposited $\$120$ into his account. What is his balance now? Select the correct answer **A**, **B**, **C** or **D**.
- A** $\$64$ **B** $\$74$ **C** $\$76$ **D** $\$164$
- 5 A bird dives into a river from a height of 450 cm above the water, travelling at a speed of 120 cm per second. How far below the water will it be after 4 seconds?
- 6 Evaluate each expression.
- a $12 - 3 - 11$ b $6 - 10 + 4$ c $-18 + 10 - 3$ d $-7 + 3 + 8$
- e $8 - 15 + (-6)$ f $-2 - 12 + 20$ g $3 + (-5) + 1$ h $12 - 14 - (-4)$
- i $-9 + 6 - (-1)$ j $7 - 7 + (-7)$ k $-5 + (-5) + (-5)$ l $8 - 10 + (-3)$
- 7 The temperature changed from 6°C to -8°C overnight. What was the change in temperature? Select the correct answer **A**, **B**, **C** or **D**.
- A** 2°C decrease **B** 14°C decrease **C** 14°C increase **D** 2°C increase

8 Copy and fill in the blank for each equation.

a $5 + \underline{\hspace{1cm}} = -4$

b $-2 + \underline{\hspace{1cm}} = 1$

c $-4 + \underline{\hspace{1cm}} = -9$

d $\underline{\hspace{1cm}} + (-7) = -11$

e $3 - \underline{\hspace{1cm}} = -3$

f $-6 - \underline{\hspace{1cm}} = -7$

g $-2 - \underline{\hspace{1cm}} = 5$

h $\underline{\hspace{1cm}} - 2 = -6$

i $\underline{\hspace{1cm}} - (-8) = 0$

9 If a and b are integers, test whether each equation is true or false.

a $a + b = b + a$

b $a - b = b - a$

10 The table shows the temperature of four towns at two different times of the day. Which town had the smallest change in temperature?

Town	5:00 a.m.	8:00 a.m.
Echo Bay	-6°C	-3°C
Cambridge Bay	-2°C	3°C
Dawson Creek	2°C	6°C
Fort Liard	-3°C	4°C

11 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 4$

b $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = -1$

c $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 0$

d $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = -9$

e $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 2$

f $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 4$

g $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -8$

h $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 9$

i $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -1$

12 Simplify each algebraic expression below by using integers to test them.

a $a + (-a)$

b $a - (-a)$

c $a + a$

d $a - a$

13 Investigate whether each sum or difference is always positive (+), always negative (−), or could be either positive or negative (E).

a A larger integer minus a smaller integer

b A positive integer minus a positive integer

c A positive integer plus a positive integer

d A smaller integer minus a larger integer

e A positive integer plus a negative integer

f A negative integer plus a negative integer

g A positive integer minus a negative integer

h A negative integer minus a negative integer

Investigation: Multiplying integers

1 Copy and complete each pattern.

a 9, 6, 3, 0, -3 , $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$

b 4, 2, 0, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$

c 15, 10, 5, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$

2 Copy and complete each multiplication.

a $3 \times (-7) = (-7) + (-7) + (-7) = \underline{\hspace{2cm}}$

b $2 \times (-5) = (-5) + (-5) = \underline{\hspace{2cm}}$

c $3 \times (-4) = (-4) + (-4) + (-4) = \underline{\hspace{2cm}}$

d $4 \times (-2) = (-2) + (-2) + (-2) + (-2) = \underline{\hspace{2cm}}$

e $2 \times (-6) = \underline{\hspace{2cm}}$

f $3 \times (-3) = \underline{\hspace{2cm}}$

- 3 a In pairs or as a group activity, copy the multiplication grid below onto a large piece of paper and complete the shaded section.

×	5	4	3	2	1	0	-1	-2	-3	-4	-5
5											
4											
3											
2											
1											
0											
-1											
-2											
-3											
-4											
-5											

- b Continue the pattern for the first row: 25, 20, 15, 10, 5, 0, -5, ...
 c Continue the pattern for the second row: 20, 16, 12, 8, 4, 0, -4, ...
 d Complete the next four rows.
 e Continue the pattern for each **column**.
- 4 Use your completed table to simplify each product.
 a $4 \times (-3)$ b -3×5 c $-4 \times (-2)$ d $5 \times (-1)$
 e $0 \times (-4)$ f -3×3 g $2 \times (-4)$ h $-5 \times (-5)$
- 5 How do the signs (positive or negative) of the integers in the question affect the sign of the product (answer)? Try to express the rules for multiplying integers in words.

Homework sheet

Integers

MAT08NAHS10001

Technology worksheet

Excel: Integer quiz:
Multiplication

MAT08NACT00005

Technology

Excel: Integer quiz:
Multiplication

MAT08NACT00019

2-03 Multiplying integers

Summary

When **multiplying integers**:

positive $\times$ positive = positive
 positive $\times$ negative = negative
 negative $\times$ positive = negative
 negative $\times$ negative = positive

or

If both numbers have the **same** sign, the answer is **positive**.
 If both numbers have **different** signs, the answer is **negative**.

×	+	-
+	+	-
-	-	+

Example 6

Evaluate each product.

a -3×5

b $-6 \times (-9)$

c $(-7)^2$

A **product** is the answer to a multiplication

Solution

a $-3 \times 5 = -15$

On a calculator, enter:

$(-) \times (+) = (-)$

$(-) 3 \times 5 =$

b $-6 \times (-9) = 54$

On a calculator, enter:

$(-) \times (-) = (+)$

$(-) 6 \times (-) 9 =$

c $(-7)^2 = (-7) \times (-7) = 49$

On a calculator, enter:

$(-) \times (-) = (+)$

$(-) (-) 7) x^2 =$

Exercise 2-03 Multiplying integers

1 Evaluate each product and check your answer using a calculator.

a -5×4

b $3 \times (-6)$

c $-4 \times (-8)$

d -9×5

e $10 \times (-7)$

f $-15 \times (-2)$

g $(-3)^2$

h $6 \times (-4)$

i -7×3

j $(-10)^2$

k $9 \times (-4)$

l $-8 \times (-5)$

m $4 \times (-11)$

n -20×3

o $(-6)^2$

p $-7 \times (-9)$

q -11×5

r $(-9)^2$

s $10 \times (-10)$

t $-6 \times (-7)$

2 For each product, select the correct answer A, B, C or D.

a $12 \times (-3)$

A 15

B -15

C -36

D 36

b $-3 \times 6 \times 2$

A 0

B -16

C -18

D -36

c $5 \times 3 \times (-2)$

A -15

B -30

C 15

D 30

3 Copy and fill in the blank for each equation.

a $7 \times \underline{\quad} = -14$

b $-3 \times \underline{\quad} = 12$

c $-5 \times \underline{\quad} = -25$

d $\underline{\quad} \times 3 = -21$

e $\underline{\quad} \times (-4) = 28$

f $\underline{\quad} \times 9 = -63$

4 If a and b are integers, test whether the equation $a \times b = b \times a$ is true or false.

5 What happens when you multiply an integer by its opposite?

6 Copy and complete this number grid.

$\times$	7	-5	-2	10
-4				
-9				
5				
-6				

See Example 6

Worked solutions

Exercise 2-03

MAT08NAWS10008

- 7 Evaluate each product and check your answer using a calculator.
- a $-2 \times (-2) \times 7$ b $6 \times 5 \times (-3)$ c $4 \times (-1) \times 10$
d $3 \times (-5)^2$ e $-5 \times 2 \times (-2)$ f $(-2)^3$
- 8 Investigate whether each product is always positive (+), always negative (-), or could be either positive or negative (E).
- a The product of two positive integers
b The product of two negative integers
c The product of a positive integer and a negative integer
d A positive integer squared
e A negative integer squared
- 9 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.
- a $\_\_ \times \_\_ = 20$ b $\_\_ \times \_\_ = -16$ c $\_\_ \times \_\_ = 36$
d $\_\_ \times \_\_ = -5$ e $\_\_ \times \_\_ = 9$ f $\_\_ \times \_\_ = -22$

Investigation: Dividing integers

- 1 In pairs or as a group activity, copy and complete each question.
- a $5 \times \_\_ = -20$ b $-6 \times \_\_ = 12$ c $-3 \times \_\_ = 27$
d $\_\_ \times (-4) = -16$ e $\_\_ \times 2 = -10$ f $\_\_ \times -5 = 15$
g $-20 \div 5 = \_\_$ h $27 \div (-3) = \_\_$ i $-16 \div (-4) = \_\_$
j $-14 \div 2 = \_\_$ k $-12 \div (-12) = \_\_$ l $21 \div (-3) = \_\_$
- 2 Express the rules for dividing integers in words.

Worksheet

What is the integer question?

MAT08NAWK10014

Worksheet

Integer review

MAT08NAWK10015

Puzzle sheet

Directed numbers

MAT08NAPS00006

2-04 Dividing integers

Because division is the opposite of multiplication, the rules for dividing integers are the same as those for multiplying them.

Summary

When **dividing integers**:

positive $\div$ positive = positive
positive $\div$ negative = negative
negative $\div$ positive = negative
negative $\div$ negative = positive

or

If both numbers have the **same** sign, the answer is **positive**.

If both numbers have **different** signs, the answer is **negative**.

$\div$	+	-
+	+	-
-	-	+

Example 7

Simplify each quotient.

a $16 \div (-2)$ b $-20 \div (-10)$ c $\frac{-27}{9}$

A **quotient** is the answer to a division

Solution

a $16 \div (-2) = -8$

On a calculator, enter:

b $-20 \div (-10) = 2$

On a calculator, enter:

c $\frac{-27}{9} = -27 \div 9 = -3$

On a calculator, enter:

$(+) \div (-) = (-)$

16 $\div$ $(-)$ 2 $=$

$(-) \div (-) = (+)$

$(-)$ 20 $\div$ $(-)$ 10 $=$

$(-) \div (+) = (-)$

$(-)$ 27 $\div$ 9 $=$

Exercise 2-04 Dividing integers

1 Evaluate each quotient and check your answer using a calculator.

a $36 \div (-4)$	b $-15 \div (-3)$	c $-14 \div 2$	d $60 \div (-10)$
e $-25 \div (-5)$	f $-28 \div 4$	g $45 \div (-9)$	h $-32 \div 16$
i $-36 \div (-6)$	j $56 \div (-7)$	k $-24 \div 3$	l $-42 \div (-6)$
m $40 \div (-8)$	n $-20 \div (-1)$	o $-81 \div 9$	p $24 \div (-4)$

2 For each quotient, select the correct answer A, B, C or D.

a $-45 \div (-5)$	A 9	B -9	C -50	D -40
b $\frac{42}{-6}$	A 36	B -7	C 7	D -48

3 Evaluate each quotient.

a $\frac{-18}{2}$	b $\frac{16}{-4}$	c $\frac{-30}{-5}$	d $\frac{21}{-3}$
e $\frac{-32}{-8}$	f $\frac{-100}{25}$	g $\frac{-81}{-9}$	h $\frac{56}{-8}$

4 If a and b are integers, test whether the equation $a \div b = b \div a$ is true or false.

5 What happens when you divide an integer:

a by itself b by its opposite?

6 Evaluate each expression and check your answer using a calculator.

a $-28 \div 2 \div 7$	b $45 \div (-3) \div (-5)$	c $36 \div (-9) \div 2$
d $-90 \div 3 \div (-1)$	e $100 \div 2 \div (-10)$	f $-24 \div (-6) \div (-4)$
g $8 \times (-5) \div 2$	h $-4 \times 7 \div 14$	i $35 \div (-5) \times 10$

See Example 7

Worked solutions

Exercise 2-04

MAT08WS10009

7 Copy and fill in the blank for each equation.

a $-12 \div \underline{\hspace{1cm}} = 2$

b $15 \div \underline{\hspace{1cm}} = -3$

c $-9 \div \underline{\hspace{1cm}} = -1$

d $\underline{\hspace{1cm}} \div (-4) = -8$

e $\underline{\hspace{1cm}} \div 12 = -2$

f $\underline{\hspace{1cm}} \div (-10) = -10$

8 Simplify each algebraic expression by using integers to test them.

a $a \times a$

b $(-a) \times (-a)$

c $a \div a$

d $(-a) \div a$

9 Investigate whether each quotient is always positive (+), always negative (-), or could be either positive or negative (E).

a The quotient of two positive integers

b The quotient of two negative integers

c The quotient of a positive integer and a negative integer

10 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.

a $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 9$

b $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -5$

c $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 3$

d $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -4$

e $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 10$

f $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = -7$

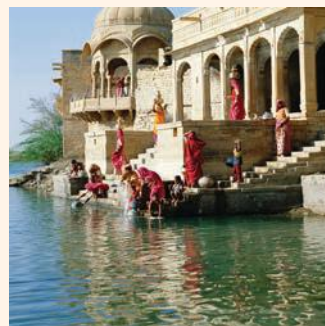
Just for the record

Brahmagupta

Brahmagupta was a famous Indian mathematician who lived from 598 to 670 CE. He wrote important works on mathematics and astronomy, and discovered the number zero. Brahmagupta's writings contained two remarkable ideas that were ahead of his time.

- He defined zero as the result of subtracting a number from itself.
- He developed calculation rules for positive and negative integers, which he called 'fortunes' and 'debts'.

Why do you think he called integers 'fortunes' and 'debts'?
Use the Internet to find out what some of these rules were.



Mental skills 2A

Maths without calculators

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

$0 \text{ dp} + 1 \text{ dp} = 1 \text{ dp}$ (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ ($2.4 \approx 3$)

b $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.3$

$1 \text{ dp} + 1 \text{ dp} = 2 \text{ dp}$

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ ($0.3 \approx 0.25$)

c $7 \times 3 = 21$, so $0.07 \times 0.3 = 0.021$

$\uparrow \quad \uparrow \quad \uparrow$
 2 dp + 1 dp = 3 dp

By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ ($0.021 \approx 0.02$)

2 Now evaluate each product.

a 0.7×5

b 12×0.2

c 0.4×0.3

d $(0.6)^2$

e 8×0.1

f 0.03×0.9

g 4×0.05

h 1.1×8

i 0.3×0.8

j 0.2×0.06

k 9×0.2

l 0.07×0.4

3 Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

a $1.5 \times 2.3 = 3.45$

$\uparrow \quad \uparrow \quad \uparrow$
 1 dp + 1 dp = 2 dp (Estimate: $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

$\uparrow \quad \uparrow \quad \uparrow$
 0 dp + 2 dp = 2 dp
 (Estimate: $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

$\uparrow \quad \uparrow \quad \uparrow$
 2 dp + 0 dp = 2 dp
 (Estimate: $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a 3.9×17

b 39×170

c 39×0.17

d 0.39×1.7

e 3.9×1.7

f 390×1.7

g 3.9×0.17

h 3.9×170

i 3900×1.7

j 39×1.7

k 39×0.017

l 0.39×0.17

2-05 Order of operations

When evaluating a mixed expression such as $18 \div (-2 + 1) \times 2$, there is a specific order in which the different operations are performed.

Summary

Order of operations

- 1 Simplify any expression inside grouping symbols (brackets): start with the innermost brackets first.
- 2 Simplify any multiplication ($\times$) and division ($\div$), from left to right.
- 3 Simplify any addition ($+$) and subtraction ($-$), from left to right.

Video tutorial

Order of operations

MAT08NAVT10002

Puzzle sheet

Order of operations
puzzle

MAT08NAPS10002

Skillsheet

Order of operations

MAT08NASS10006

Beware of cheap calculators that do not follow the 'order of operations' rules!

Example 8

Evaluate each expression.

a $5 \times 2 + 3 \times (-9)$

b $2 \times [(25 - 4) \div 3]$

Solution

a $5 \times 2 + 3 \times (-9) = 10 + (-27)$
 $= -17$

Multiply first: $5 \times 2 = 10$, and $3 \times (-9) = -27$

On a calculator, enter: 5 $\times$ 2

+ 3 $\times$ (-) 9 =.

b $2 \times [(25 - 4) \div 3]$

Innermost brackets first: $25 - 4 = 21$

$= 2 \times [21 \div 3]$

Next brackets: $21 \div 3 = 7$

$= 2 \times 7$

$= 14$

On a calculator, enter: 2 $\times$

((25 - 4)

$\div$ 3) =.

Example 9

Simplify each fraction.

a $\frac{8 + 16}{40 \div 10}$

b $\frac{10 - 30}{6^2}$

Solution

To simplify each fraction, evaluate the numerator and denominator separately, then divide the numerator by the denominator.

a $\frac{8 + 16}{40 \div 10} = \frac{24}{4}$
 $= 6$

On a calculator, enter:

(8 + 16) $\div$ (40 $\div$ 10) =.

OR if using MATH Mode, press $\frac{\square}{\square}$ and enter $8 + 16$ and $40 \div 10$ separately into the blank spaces.

b $\frac{10 - 30}{6^2} = \frac{-20}{36}$
 $= -\frac{5}{9}$

Simplifying the fraction.

Exercise 2-05 Order of operations

1 Evaluate each expression and check your answer on a calculator.

a $15 \div (7 - 4)$	b $[14 + (-9)] \times 6$	c $20 - (5 - 7)$
d $8 \times (-3 + 5 - 6)$	e $-2 \times (10 - 9) + 28$	f $(16 + 8) \div 2 - 4$
g $18 \div (5 - 2) \times (-2)$	h $[4 - (-7)] \times 3 - 10$	i $-26 \div (14 + 12)$
j $(7 - 10) \times 10 \div 6$	k $5 \times [(22 - 10) \div (-3)] + 1$	l $30 + [7 \times (2 - 6)] - 2$

See Example 8

2 Evaluate each expression and check your answer on a calculator.

a $8 + 5 \times (-2)$	b $7 - 2 \times 3$	c $6 \times 5 - (-1)$
d $12 + (-6) \div 3$	e $3 \times 6 + 2 \times 5$	f $9 - 11 + 15 \div 3$
g $-5 \times 10 + 16 \div 2$	h $-3 \times 6 - (-2) \times 5$	i $42 \div 6 + 9$
j $17 + 8 - 3 \times 2$	k $4 \times 3 - (-7) + 5$	l $16 - 3 \times (-4) \div 2$

Worked solutions

Exercise 2-05

MAT08NAWS10010

3 Simplify each fraction.

a $\frac{5 \times 2}{16 + 4}$	b $\frac{19 + 5}{18 - 6}$	c $\frac{9 \times 3}{40 - 10}$	d $\frac{58 + 8}{-7 - 4}$
e $\frac{4^2}{16 \div (-2)}$	f $\frac{5 \times 3 - 1}{16 + 10 \times 4}$	g $\frac{28 - 5 \times 3}{(56 - 30) \div 2}$	h $\frac{25 \div 5 + 13}{\sqrt{36}}$

See Example 9

4 Which expression is equal to 4? Select the correct answer A, B, C or D.

- A $20 \div 4 + 6 \times 2$ B $20 \div (4 + 6) \times 2$ C $20 \div [(4 + 6) \times 2]$ D $(20 \div 4 + 6) \times 2$

5 Copy and complete for each equation.

a $-8 \times 3 - \underline{\hspace{1cm}} = -32$	b $\underline{\hspace{1cm}} \times (5 - 9) = -20$	c $(8 - \underline{\hspace{1cm}}) \times 7 = 35$
d $9 \times \underline{\hspace{1cm}} + (-2) = -20$	e $24 \div (-3) \times \underline{\hspace{1cm}} = 8$	f $\underline{\hspace{1cm}} + 5 \times 3 = -5$
g $(-2 - \underline{\hspace{1cm}}) \times (-4) = -16$	h $(-3 - 9) \div \underline{\hspace{1cm}} = -2$	i $\underline{\hspace{1cm}} \div (-4) + 6 = 1$

6 Copy each equation and insert grouping symbols in the correct places to make the equation true.

a $8 - 3 + 7 = -2$	b $40 - 10 \times 5 = 150$	c $27 \div 9 \div 3 = 9$
d $8 + 4 - 3 \times 2 = 10$	e $8 + 4 - 3 \times 2 = 18$	f $6 + 4 \times (-1) - 1 = -11$
g $13 + 3 \div 4 - 6 = -8$	h $100 \div 10 + 10 + 5 = 10$	i $100 \div 10 + 10 + 5 = 4$

7 Use all the integers 5, -2 and 10 and grouping symbols to complete each equation.

a $\underline{\hspace{2cm}} = 30$	b $\underline{\hspace{2cm}} = -1$	c $\underline{\hspace{2cm}} = -60$

Technology Order of operations

- Enter these 7 values in column A of a new spreadsheet: 36, 7, 8.5, 18, 50, 2, -3.
- In column B, write an appropriate formula to evaluate each expression in the given cell. Try to predict the answer before you enter each formula. The first one has been done for you.

B1: $36 + 7 - 2$

In cell B1, enter **=A1+A2-A6**

B2: $(7 + 8.5) \div 2$

B3: $7 + 8.5 \div 2$

B4: $50 - 2 \times (-3)$

B5: $18 \times 7 + 36 \times 8.5$

B6: $18 \times (7 + 36) \times 8.5$

B7: $\frac{50}{2 + 18}$

B8: $50 + 8.5 - 2 - (-3)$

B9: $7 \times 2 + 50 \div 2$

B10: $\frac{50 \times (2 + 18 + 8.5)}{-3}$

Skillsheet

Spreadsheets

MAT08NASS10007

Technology worksheet

Review of spreadsheets

MAT08NACT10012

2-06 Decimals

Summary

When ordering decimals:

- list them in a column with the decimal points in line so that the place values can be compared
- fill any gaps at the end with zeros
- ‘ascending order’ means going up, from smallest to largest
- ‘descending order’ means going down, from largest to smallest.

Example 10

Write these decimals in ascending order: 3.5, 3.751, 3.15, 3.157.

Solution

3.500		List the numbers in a column with the decimal points aligned.
3.751		Fill any gaps at the end with zeros.
3.150		Order the decimals from smallest to largest by comparing place values.
3.157		Smallest is 3.150, next is 3.157, next is 3.500, largest is 3.751.

In ascending order, the decimals are 3.15, 3.157, 3.5, 3.751.

Summary

To **round a decimal**, cut it at the required decimal place and look at the digit in the next place:

- if that digit is less than 5 (that is, 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is, 5, 6, 7, 8 or 9), **round up**.

Example 11

- a Round 5.261 to the nearest tenth.
b Write 14.8239 correct to two decimal places.

Solution

- a 'nearest tenth' = to one decimal place

5.2|61

cut the next digit is 6 (greater than 5), so round **up** to 5.3

$$5.261 \approx 5.3$$

- b 14.82|39

cut the next digit is 3 (less than 5), so round **down** to 14.82

$$14.8239 \approx 14.82$$

Summary

When **adding and subtracting decimals**, keep decimal points below one another.

Example 12

Evaluate each expression.

a $2.7 + 44.3 + 13.25$

b $23.8 - 5.65$

Solution

a

$$\begin{array}{r} 2.70 \\ 44.30 \\ +13.25 \\ \hline 60.25 \end{array}$$

Line up decimal points in the same column.
Fill any gap with zeros.

Check by estimating: $2.7 + 44.3 + 13.25 \approx 3 + 44 + 10 = 57$ (60.25 is close to 57.)

b

$$\begin{array}{r} 23.80 \\ -5.65 \\ \hline 18.15 \end{array}$$

Line up decimal points in the same column.
Fill any gap with zeros.

Check by estimating: $23.8 - 5.65 \approx 24 - 6 = 18$ (18.15 is close to 18.)

Exercise 2-06 Decimals

See Example 10

- Which of the following decimals is the largest? Select the correct answer **A**, **B**, **C** or **D**.
A 6.408 **B** 6.48 **C** 6.048 **D** 6.4
- Write each set of decimals in ascending order.
a 4.35, 4.3, 4.05, 4.035 **b** 17.2103, 17.12, 12.173, 17.231
c 14.8, 14.08, 14.801, 14.81 **d** 0.3, 0.295, 0.032, 0.2
- Write each set of decimals in descending order.
a 1.612, 1.621, 1.61, 1.16 **b** 3.05, 3.053, 3.035, 3.305
- How many decimal places has each decimal?
a 3.824 **b** 1.09 **c** 51.6 **d** 22
e 12.573 **f** 450.304 **g** 788.5 **h** 0.45

See Example 11

- Write each decimal correct to one decimal place.
a 3.851 **b** 4.0736 **c** 0.3333 **d** 7.34
e 15.0801 **f** 2.976 **g** 2.048 **h** 16.1919
- Round each decimal to the nearest hundredth.
a 68.9109 **b** 107.0594 **c** 3.5963 **d** 4.7077
e 3.198 **f** 32.999 **g** 19.7291 **h** 11.254
- Write each decimal correct to three decimal places.
a 9.7043 **b** 13.167 54 **c** 0.08281 **d** 53.094 23
- Round \$460.39512 to the nearest:
a cent **b** dollar **c** tenth **d** 5 cents **e** thousandth

See Example 12

- Evaluate each expression, then check your answer with an estimate.
a $28.51 + 136.4$ **b** $7.2 + 18.16$ **c** $34.7 + 29.4 + 8.5$
d $127.81 - 36.2$ **e** $46.5 - 30.8$ **f** $100.87 - 23.6$
g $20.03 - 1.59$ **h** $12.56 - 9.88$ **i** $71.5 - 4.82$
j $65.21 - 13.6$ **k** $9 - 4.36$ **l** $3.671 - 1.28$
- What is the value of $0.52 + 14.025$? Select the correct answer **A**, **B**, **C** or **D**.
A 65.025 **B** 19.225 **C** 14.545 **D** 14.077
- Yesterday, the temperature dropped from 24.1°C to 16.5°C .
 What was the difference between the two temperatures?

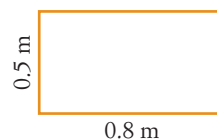
Worked solutions

Exercise 2-06

MAT08NAWS10011



- 12 Monique built a wooden frame with dimensions 0.8 m by 0.5 m.
How much wood will be left from a 3.4 m length of timber?



Worked solutions

Exercise 2-06

MAT08NAWS10011

- 13 Copy and complete each equation with the correct decimals.
- a $2.8 + \underline{\hspace{1cm}} = 5.84$ b $\underline{\hspace{1cm}} - 7.6 = 5.14$ c $18.75 - \underline{\hspace{1cm}} = 13.3$
d $\underline{\hspace{1cm}} + 0.83 = 1.719$ e $\underline{\hspace{1cm}} - 1.07 = 3.256$ f $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 4.89$
- 14 Tahir bought the following items: an exercise book for \$2.70, two pens for \$1.60 each, a drink for \$1.50 and a packet of chips for \$2.65.
- a How much did Tahir spend in total?
b If he paid with a \$20 note, how much change did he receive?

Investigation: Rounding up or down?

Sometimes, the decision on whether to round an answer up or down depends on the situation. In groups of two to four, discuss each of the following situations and decide whether it is more appropriate to round up or to round down. You must give a reason for your choice.

- 1 Seven friends have dinner at a restaurant and the total bill is \$156. They decide to share the bill evenly: $\$156 \div 7 = \$22.285\ 71\dots$
How much should each friend pay, to the nearest dollar: \$22 or \$23?

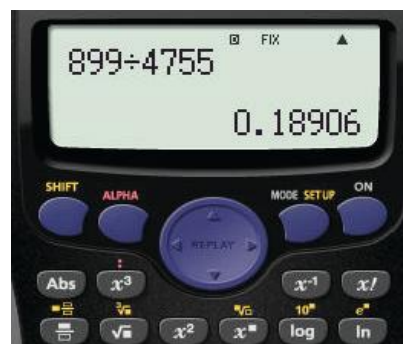


- 2 You have a budget of \$45 for buying drinks for a party. One can of drink costs \$0.94 at the supermarket: $\$45 \div \$0.94 = 47.872\ 34\dots$
How many whole cans of drink can you buy: 47 or 48?
- 3 You need to find the average number of people living in each home in your street. You survey 48 homes and count a total of 166 people: $166 \div 48 = 3.458\ 33\dots$
What is the average whole number of people living in each house: 3 or 4?
- 4 You need to paint the walls of a house with a total surface area of 334 m^2 . One tin of paint covers 64 m^2 : $334 \div 64 = 5.218\ 75$
How many whole tins of paint do you need for the job: 5 or 6?
- 5 Jodie has \$2098 in her bank account and the bank pays her 4.71% in interest:
 $4.71\% \times \$2098 = \$98.8158\dots$
How much interest will the bank pay Jodie: \$98.81 or \$98.82?

Technology The FIX mode on a calculator

Most scientific calculators have a FIX mode that rounds a displayed answer to a given number of decimal places. However, the exact answer is still stored in the calculator's memory to make sure that any further calculations involving this answer are accurate.

Find out how to use the FIX mode on your calculator. The FIX mode has been used to set this calculator to display answers to five decimal places.



Worksheet

Decimal cards

MAT08NAWK10016

Homework sheet

Decimals 1

MAT08NAHS10002

Technology worksheet

Movie night

MAT08NACT10001

Worksheet

Decimal number grids

MAT08NAWK00033

Puzzle sheet

Operations with
decimals

MAT08NAPS00017

2-07 Multiplying and dividing decimals

Summary

When **multiplying decimals**, the number of decimal places in the answer equals the *total* number of decimal places in the question.

Example 13

Evaluate each product.

a 0.8×0.3

b 7.2×5

c 5.31×1.3

Solution

a $8 \times 3 = 24$

Count the number of decimal places in the question.

$$0.8 \times 0.3 = 0.24$$

b $72 \times 5 = 360$

7.2 has one decimal place, 5 has no decimal places.

$$\begin{aligned} 7.2 \times 5 &= 36.0 \\ &= 36 \end{aligned}$$

Multiply without decimal points first.

0.8 has one decimal place, 0.3 has one decimal place.

Write the answer with $1 + 1 = 2$ decimal places.

Check by estimating: $0.8 \times 0.3 \approx 1 \times 0.3 = 0.3$
($0.24 \approx 0.3$)

Multiply without decimal points first.

Write the answer with $1 + 0 = 1$ decimal place.

Check by estimating: $7.2 \times 5 \approx 7 \times 5 = 35$
($36 \approx 35$)

c 531

$\times 13$

1593

5310

6903

$$5.31 \times 1.3 = 6.903$$

Multiply without decimal points first.

5.31 has 2 decimal places, and 1.3 has 1 decimal place.

Write the answer with $2 + 1 = 3$ decimal places.

Check by estimating: $5.31 \times 1.3 \approx 5 \times 1 = 5$

(6.903 $\approx$ 5)

Summary

When **dividing a decimal by a whole number**:

- rewrite the question in 'short division' form
- make the decimal point in the answer line up with the decimal point in the question
- add zeros to the end of the decimal being divided, if needed.

When **dividing a decimal by another decimal**:

- make the second decimal a whole number by moving the decimal point the required number of places to the right
- move the point in the first decimal the same number of places to the right
- divide the new first number by the whole number.

This works because we are multiplying both decimals by the same power of 10 before dividing.

Example 14

Evaluate each quotient.

a $6.28 \div 5$

b $12.4 \div 0.04$

c $4.98 \div 1.2$ (requires long division)

Solution

a
$$\begin{array}{r} 1.256 \\ 5 \overline{)6.12830} \end{array}$$

Write $6.28 \div 5$ as a short division.

- 5 into 6 goes 1, remainder 1
- Make the decimal point in the answer line up with the decimal point in the question
- 5 into 12 goes 2, remainder 2.
- 5 into 28 goes 5, remainder 3.
- Add a zero to 6.28 so that you can complete the division.
- 5 into 30 goes 6 exactly.

$$6.28 \div 5 = 1.256.$$

Check by estimating: $6.28 \div 5 \approx 6 \div 5 \approx 1$
(1.256 $\approx$ 1)

$$\begin{aligned} \text{b } 12.4 \div 0.04 &= 1240 \div 004 \\ &= 1240 \div 4 \\ &= 310 \end{aligned}$$

Move both decimal points two places to the right so that 0.04 becomes a whole number.

We added a zero to the end of 12.4 so that we can move the point two places to the right.

Check by estimating: $1240 \div 4 \approx 1200 \div 4 \approx 300$
($310 \approx 300$)

NSW

$$\text{c } 4.98 \div 1.2 = 49.8 \div 12$$

$$\begin{array}{r} 4.15 \\ 12 \overline{)49.80} \\ \underline{-48} \\ 18 \\ \underline{-12} \\ 60 \\ \underline{-60} \\ 0 \end{array}$$

$$4.98 \div 1.2 = 4.15$$

Move both decimal points one place to the right so that 1.2 becomes a whole number.

Write $49.8 \div 12$ as a long division:

12 into 49 is 4, remainder 1

12 into 18 is 1, remainder 6

12 into 60 is 5 exactly

Check by estimating:

$$4.98 \div 1.2 \approx 5 \div 1 = 5 \quad (4.15 \approx 5)$$

Exercise 2-07 Multiplying and dividing decimals

See Example 13

- 1 Evaluate each product, then check your answer with an estimate.

a 6.5×4

b 0.45×9

c 2.3×5

d 7×3.9

e 0.7×0.3

f 0.8×0.11

g 0.05×0.4

h 4.2×0.3

i 6.3×4.5

j 28.7×3.1

k 1.3×0.62

l 0.72×0.51

- 2 Phillip was building a fence. He needed 9.4 metres of wire at \$5.80 per metre. Which of the following is the cost of the wire? Select the correct answer A, B, C or D.

A \$15.20

B \$54.52

C \$152.00

D \$545.20

- 3 Thao's mobile phone plan charges \$20 per month plus \$0.18 for each phone call. How much will Thao need to pay if she made 112 calls in one month?

Worked solutions

Exercise 2-07

MAT08NAWS10012

See Example 14

- 4 Evaluate each quotient, then check your answer with an estimate.

a $24.8 \div 4$

b $114.1 \div 7$

c $7.83 \div 3$

d $0.695 \div 5$

e $63.3 \div 0.2$

f $4.173 \div 0.06$

g $18.5 \div 0.05$

h $23.22 \div 0.4$

i $14.5 \div 25$

j $6.24 \div 1.2$

k $238 \div 1.4$

l $0.252 \div 0.21$

- 5 What is the value of $12.92 \div 0.04$? Select the correct answer A, B, C or D.

A 32.3

B 323

C 3.23

D 3230

- 6 Brad uses 7.24 metres of wood to make a garden bed fence.



- a How much wood does he need to make 6 garden bed fences?
b If one of the garden beds is square-shaped, how long is each side?
- 7 Evaluate each expression.
a $0.3 \times 0.7 + 5.7$ b $2.8 \div (7.4 - 7.2)$ c $3.62 + 4.08 \div 2$
- 8 Jordan's car holds 45 litres of petrol. If the price of petrol is 149.9 cents per litre, how much will Jordan need to pay to fill the tank? Give your answer to the nearest 5 cents.
- 9 Copy and complete each equation with the correct decimals.
a $0.3 \times \underline{\hspace{1cm}} = 2.4$ b $\underline{\hspace{1cm}} \times 0.6 = 1.2$ c $\underline{\hspace{1cm}} \times 0.7 = 0.028$
d $3.1 \times \underline{\hspace{1cm}} = 2.79$ e $83.4 \div \underline{\hspace{1cm}} = 6$ f $\underline{\hspace{1cm}} \div 8.1 = 3.2$
- 10 A car travels 145.8 kilometres on 9 litres of petrol. How many kilometres could it travel on one litre of petrol?
- 11 Anja earned \$169.20 for working a 12-hour shift. How much was earned each hour?
- 12 Henry has a faulty calculator that does not show the decimal point. For each calculation in the table shown, write the correct answer.

	Calculation	Answer with missing decimal point
a	3.42×12	4104
b	4.145×0.2	829
c	37.3×8.8	32824
d	0.03×157.64	47292
e	8.3902×0.3	251706

2-08 Terminating and recurring decimals

When converting a fraction to a decimal, or dividing two numbers, the decimal answer can be **terminating** or **recurring**.

Terminating decimals, such as 0.625, have a definite number of decimal places, while **recurring decimals**, such as $0.272727\dots$, written as $0.\dot{2}7$ or $0.2\overline{7}$, have one or more digits that repeat endlessly. We use dots or lines to mark the repeating section. For example, $0.259\ 259\ 259\dots = 0.\dot{2}5\dot{9}$ or $0.2\overline{59}$.

'Terminate' means 'to stop' while 'recurring' means 'repeating'.

Example 15

Convert each fraction to a recurring decimal.

a $\frac{5}{6}$

b $\frac{2}{11}$

Solution

a $\frac{5}{6}$ means $5 \div 6$

$$\begin{array}{r} 0.8333 \dots \\ 6 \overline{)5.0000 \dots} \end{array}$$

Add zeros to complete the division.

$$\frac{5}{6} = 0.8333 \dots = 0.8\dot{3} \text{ or } 0.8\overline{3}$$

A recurring decimal.

b $\frac{2}{11}$ means $2 \div 11$

$$\begin{array}{r} 0.18181 \dots \\ 11 \overline{)2.00000 \dots} \end{array}$$

$$\frac{2}{11} = 0.181818 \dots = 0.1\dot{8} \text{ or } 0.1\overline{8}$$

A recurring decimal.

Example 16

Evaluate each quotient.

a $2.8 \div 9$

b $0.7 \div 0.12$

Solution

a $2.8 \div 9$

$$\begin{array}{r} 0.3111 \dots \\ 9 \overline{)2.8000 \dots} \end{array}$$

$$2.8 \div 9 = 0.3111 \dots = 0.3\dot{1} \text{ or } 0.3\overline{1}$$

A recurring decimal.

b $0.7 \div 0.12 = 70 \div 12$

$$\begin{array}{r} 5.8333 \dots \\ 12 \overline{)70.0000 \dots} \end{array}$$

$$0.7 \div 0.12 = 5.8333 \dots = 5.8\dot{3} \text{ or } 5.8\overline{3}$$

A recurring decimal.

Exercise 2-08 Terminating and recurring decimals**See Example 15**

- 1**
- Convert each fraction to a terminating decimal.

a $\frac{2}{5}$

b $\frac{3}{8}$

c $\frac{3}{4}$

d $\frac{2}{2}$

e $\frac{2}{4}$

f $\frac{6}{8}$

g $\frac{3}{5}$

h $\frac{2}{8}$

i $\frac{5}{8}$

j $\frac{7}{8}$

k $\frac{4}{8}$

l $\frac{5}{5}$

- 2**
- Explain why some of the fractions in question
- 1**
- have the same decimal value.

Homework sheet

Decimals 2

MAT08NAHS10003

3 Rewrite each recurring decimal using dot notation.

- a 0.6666... b 4.2727... c 0.2828... d 3.8333...
e 9.607607... f 0.15191519... g 0.05252... h 12.23444...

4 Write each recurring decimal showing the repeated pattern.

- a 0.8583 b 0.165 c 0.27 d 0.461538

5 Convert each fraction to a recurring decimal.

- a $\frac{1}{9}$ b $\frac{1}{6}$ c $\frac{5}{6}$ d $\frac{1}{7}$ e $\frac{2}{3}$
f $\frac{2}{7}$ g $\frac{2}{9}$ h $\frac{3}{7}$ i $\frac{4}{7}$ j $\frac{4}{9}$
k $\frac{4}{6}$ l $\frac{5}{9}$ m $\frac{6}{7}$ n $\frac{7}{9}$ o $\frac{5}{7}$

See Example 15

6 Copy and complete the table and note the pattern.

Fraction	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$
Decimal						

7 Evaluate each expression as a recurring decimal.

- a $11 \div 3$ b $15.4 \div 9$ c $58.43 \div 0.11$ d $1.96 \div 0.6$

See Example 16

Mental skills 2B Maths without calculators

Dividing decimals

To divide one decimal by another, first move the decimal points in **both** decimals the same number of places to the right so that the second decimal is a **whole number**.

1 Study each example.

a $0.24 \div 0.06 = 24 \div 6 = 4$

b $0.45 \div 0.5 = 4.5 \div 5 = 0.9$

c $0.006 \div 0.3 = 0.06 \div 3 = 0.02$

d $27 \div 0.9 = 270 \div 9 = 30$

e $1.6 \div 0.4 = 16 \div 4 = 4$

f $5.6 \div 0.07 = 560 \div 7 = 80$

2 Now evaluate each quotient.

a $0.25 \div 0.5$

b $63 \div 0.7$

c $3.2 \div 0.4$

d $0.18 \div 0.2$

e $2.7 \div 0.03$

f $0.042 \div 0.06$

g $4 \div 0.5$

h $1.2 \div 0.04$

i $0.072 \div 0.9$

j $0.35 \div 0.1$

k $0.28 \div 0.07$

l $0.033 \div 0.11$

3 Study each example.

Given that $112 \div 14 = 8$, evaluate each expression.

a $112 \div 1.4 = 112.0 \div 1.4$
 $= 1120 \div 14$
 $= 112 \times 10 \div 14$
 $= 112 \div 14 \times 10$
 $= 8 \times 10$
 $= 80$

Estimate: $112 \div 1.4 \approx 112 \div 1 = 112$

b $0.112 \div 0.14 = 0.112 \div 0.14$
 $= 11.2 \div 14$
 $= 112 \div 10 \div 14$
 $= 112 \div 14 \div 10$
 $= 8 \div 10$
 $= 0.8$

Estimate: $0.112 \div 0.14 \approx 0.1 \div 0.1 = 1$

$$\begin{aligned}
 \text{c } 1120 \div 1.4 &= 11200 \div 14 \\
 &= 112 \times 100 \div 14 \\
 &= 112 \div 14 \times 100 \\
 &= 8 \times 100 \\
 &= 800
 \end{aligned}$$

Estimate: $1120 \div 1.4 \approx 1120 \div 1 = 1120$

$$\begin{aligned}
 \text{d } 1.12 \div 14 &= 112 \div 100 \div 14 \\
 &= 112 \div 14 \div 100 \\
 &= 8 \div 100 \\
 &= 0.08
 \end{aligned}$$

Estimate: $1.12 \div 14 \approx 1.12 \div 10 = 0.112$

4 Now given that $368 \div 23 = 16$, evaluate each quotient.

a $36.8 \div 2.3$

b $368 \div 2.3$

c $3.68 \div 2.3$

d $0.368 \div 0.23$

e $36.8 \div 23$

f $3.68 \div 0.23$

g $36.8 \div 0.23$

h $0.368 \div 2.3$

i $0.368 \div 23$

j $3.68 \div 0.023$

k $3.68 \div 23$

l $0.368 \div 230$

Worksheet

Powers and roots

MAT08NAWK10017

Skillsheet

Index notation

MAT08NASS10008

Skillsheet

Square roots and cube roots

MAT08NASS10009

2-09 Powers and roots

Powers and the use of index notation allow us to write repeated multiplication in a shorter way.
 $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$ '4 to the power of 6'

In 4^6 , the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.

4^6
 ↑ base
 ← power

Summary

On a calculator:

- the square of a number can be found using the x^2 key
- the cube of a number can be found using the x^3 key
- any power of a number can be found using the y^x , x^y or $\wedge$ key.

Example 17

Evaluate each power.

a 11^3

b $(-8)^4$

Solution

a $11^3 = 1331$

On a calculator, enter: 11 x^3 =

b $(-8)^4 = 4096$

On a calculator, enter: ((-) 8) y^x 4 =

Summary

- The **square root** ($\sqrt{\quad}$) of a number is the **positive** value which, if squared, will give that number.
- The **cube root** ($\sqrt[3]{\quad}$) of a given number is the value which, if cubed, will give that number.

For example:

$$\begin{array}{lll} \sqrt{36} = 6 & \text{because } 6^2 = 36 & \text{'the square root of 36'} \\ \sqrt[3]{125} = 5 & \text{because } 5^3 = 125 & \text{'the cube root of 125'} \end{array}$$

Most roots do not give exact answers like the ones above, and are called **surd**s. For example, $\sqrt{7} = 2.645751311... \approx 2.6$. A surd is a square root ($\sqrt{\quad}$), cube root ($\sqrt[3]{\quad}$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$ so it is also called an **irrational** number.

Example 18

Evaluate each cube root correct to two decimal places.

a $\sqrt[3]{729}$

b $\sqrt[3]{100}$

Solution

a $\sqrt[3]{729} = 9$

On a calculator, enter: $\sqrt[3]{\quad}$ 729 =

b $\sqrt[3]{100} = 4.6415 \dots \approx 4.64$

On a calculator, enter: $\sqrt[3]{\quad}$ 100 =

Example 19

Estimate the value of $\sqrt{50}$ correct to one decimal place.

Solution

There is no *exact* answer for the square root of 50, because there isn't a number which, if squared, equals 50 exactly. However, we can find a decimal whose square is close to 50.

Noting that:

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

we can tell that $\sqrt{50}$ must lie somewhere between 7 and 8. Also, it is much closer to 7 because 50 is just over 49.

So we can estimate that $\sqrt{50} \approx 7.1$.

(In fact, $7.1^2 = 50.41$)

Exercise 2-09 Powers and roots

1 Write each expression using index notation.

a $7 \times 7 \times 7$

b $4 \times 4 \times 4 \times 4 \times 4$

c $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

d $-8 \times (-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8)$

See Example 17

2 Evaluate each power.

a $(-8)^2$

b 2^3

c $(-6)^3$

d 3^4

e 7^1

f $(-2)^5$

g 10^3

h 4^4

i $(-11)^6$

j $3^9 - 9^3$

k 4×3^5

l $7^4 + 2 \times 8^2$

m $16^4 \div 2^{10} - 5^2$

n $(-4)^6 - 18 \div 3^2$

3 Fill in each blank to make the equation true.

a $2^{\square} = 8$

b $3^{\square} = 27$

c $(-10)^{\square} = 100$

d $4^{\square} = 4096$

e $5^{\square} = 125$

f $(-3)^{\square} = -243$

4 What is the value of $(1.3)^2$? Select the correct answer **A**, **B**, **C** or **D**.

A 1.9

B 1.69

C 2.6

D 1.09

5 Which is smaller: 100^3 or 3^{100} ?

6 Which is larger: 10^2 or 2^{10} ?

7 a Evaluate $(2 \times 3)^2$ without using a calculator.

b Evaluate $2^2 \times 3^2$ without using a calculator.

c Does $(2 \times 3)^2 = 2^2 \times 3^2$? Explain your answer.

8 a Evaluate $(4 \times 5)^2$.

b Evaluate $4^2 \times 5^2$.

c Does $(4 \times 5)^2 = 4^2 \times 5^2$?

9 Use the pattern you found in questions 7 and 8 to complete each equation.

a $(3 \times 8)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

b $(a \times b)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

10 Copy and complete each pattern.

a $18^2 = (6 \times 3)^2 = 6^2 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

b $22^2 = (2 \times 11)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

c $16^2 = (2 \times \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

d $15^2 = (\underline{\hspace{1cm}} \times 5)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

e $30^2 = (10 \times \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

f $28^2 = (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

11 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2				16			49					
Number cubed, x^3								512		1000		

12 Evaluate each root.

a $\sqrt{784}$	b $\sqrt{256}$	c $\sqrt{289}$	d $\sqrt{1089}$
e $\sqrt[3]{8}$	f $\sqrt[3]{343}$	g $\sqrt[3]{2197}$	h $\sqrt[3]{216}$

See Example 18

13 Evaluate each root correct to one decimal place.

a $\sqrt{37}$	b $\sqrt[3]{900}$	c $\sqrt{502}$	d $\sqrt[3]{6.5}$
e $\sqrt[3]{-495}$	f $\sqrt{2000}$	g $\sqrt{1.1}$	h $\sqrt[3]{1103}$

Use the table from question 11 to answer questions 14 to 16.

14 Between which two consecutive whole numbers does $\sqrt{80}$ lie? Select the correct answer A, B, C or D.

See Example 19

A 40 and 41	B 9 and 10	C 79 and 81	D 8 and 9

15 Between which two consecutive whole numbers does $\sqrt[3]{130}$ lie? Select the correct answer A, B, C or D.

A 4 and 5	B 10 and 11	C 11 and 12	D 5 and 6

16 Between which two consecutive whole numbers does $\sqrt[3]{31}$ lie?

17 Estimate, correct to one decimal place, the value of each square root, then check your estimate using a calculator.

a $\sqrt{33}$	b $\sqrt{105}$	c $\sqrt{68}$

18 Select all the surds from this list of roots.

$\sqrt{6}$ $\sqrt{2025}$ $\sqrt{62}$ $\sqrt{961}$ $\sqrt{14}$ $\sqrt[3]{181}$ $\sqrt[3]{49}$ $\sqrt[3]{1000}$ $\sqrt[3]{674}$ $\sqrt[3]{1331}$

19 a Evaluate $\sqrt{4 \times 25}$.

b Evaluate $\sqrt{4} \times \sqrt{25}$.

c What do you notice about your answers to parts a and b?

20 a Evaluate $\sqrt{27 \times 3}$.

b Evaluate $\sqrt{27} \times \sqrt{3}$.

c What do you notice about your answers to parts a and b?

21 Use the pattern you found in questions 19 and 20 to complete each equation.

a $\sqrt{16 \times 9} = \text{---} \times \text{---}$	b $\sqrt{a \times b} = \text{---} \times \text{---}$

22 Copy and complete each pattern.

a $\sqrt{64} = \sqrt{4} \times \sqrt{16} = \text{---} \times \text{---} = \text{---}$

b $\sqrt{441} = \sqrt{9} \times \sqrt{\text{---}} = \text{---} \times \text{---} = \text{---}$

c $\sqrt{144} = \sqrt{36} \times \sqrt{\text{---}} = \text{---} \times \text{---} = \text{---}$

d $\sqrt{2025} = \sqrt{81} \times \sqrt{\text{---}} = \text{---} \times \text{---} = \text{---}$

Technology Powers and roots

- 1 Start a new spreadsheet and enter the label **Powers** in cell A1.
- 2 In cell A2, enter **=2^3** to evaluate 2^3 .
- 3 In cell A3, enter **=3^5** to evaluate 3^5 .
- 4 In cell A4, enter **=4^2** to evaluate 4^2 .

- 5 Write an appropriate formula to evaluate each expression in the given cell.
A5: 6.1^2 A6: 4.25^2 A7: $5^2 \times 3^2$ A8: $(5 \times 3)^2$
- 6 Does your answer in cell A7 equal your answer in cell A8?
- 7 Write an appropriate formula to evaluate each expression in the given cell.
A9: 2.9^3 A10: 15^4 A11: 8.3^5 A12: $(-1)^3 + (-2)^4$ A13: $-1^3 + (-2^4)$
- 8 Does your answer in cell A12 equal your answer in cell A13?
- 9 In cell B1, enter **Square roots**.
- 10 In cell B2, enter **=sqrt(49)** to evaluate $\sqrt{49}$ (=sqrt stands for 'square root')
- 11 Write an appropriate formula to evaluate each square root in the given cell.
B3: $\sqrt{81}$ B4: $\sqrt{625}$ B5: $\sqrt{10\,000}$
B6: $\sqrt{71\,289}$ B7: $\sqrt{0.064}$ B8: $\sqrt{2.89}$
B9: $\sqrt{16.9}$ (Use **Format Cells** to round to 2 decimal places)
B10: $\sqrt{\frac{81}{25}}$

Worksheet

Perfect and amicable numbers

MAT08NAWK10018

Puzzle sheet

Crossnumber puzzles

MAT08NAPS10003

Puzzle sheet

Crossnumber challenges

MAT08NAPS10004

Skillsheet

Factors and divisibility

MAT08NASS10010

Skillsheet

Prime factors by repeated division

MAT08NASS10011

Homework sheet

Prime factors

MAT08NAHS10006

2-10 Prime factors

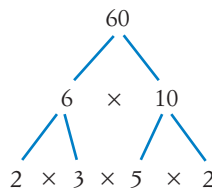
Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

Example 20

Write 60 as a product of its prime factors.

Solution

Draw a factor tree for 60.

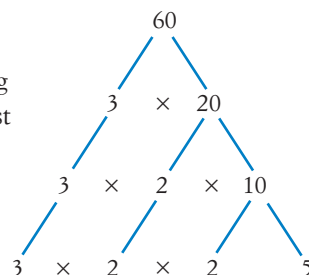


Stop when all of the factors are prime.

So 60 as a product of its prime factors is:

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 60:



$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

Using prime factors to find the highest common factor

Summary

The **highest common factor** (HCF) or **greatest common divisor** (GCD) of two (or more) numbers is the largest number that is a factor of **both (or all)** of these numbers.

To find the HCF of two numbers using their prime factors:

- 1 circle common prime factors
- 2 multiply them together.

Divisor is just another name for factor.

Example 21

Find the HCF of 36 and 45:

- a by listing factors
- b using factor trees.

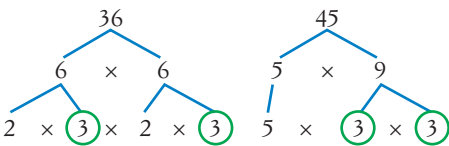
Solution

- a Factors of $36 = 1, 2, 3, 4, 6, 9, 12, 18, 36$.

Factors of $45 = 1, 3, 5, 9, 15, 30, 45$.

The common factors are 1, 3, 9.

The highest common factor is 9.

- b 
HCF = $3 \times 3 = 9$.

- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Multiply 3 and 3 to calculate the HCF

Using prime factors to find the lowest common multiple

Summary

The **lowest common multiple** (LCM) of two (or more) numbers is the smallest number that is a multiple of **both (or all)** of these numbers.

To find the LCM of two numbers using their prime factors:

- 1 circle common prime factors
- 2 cross out **one** of each pair of common factors
- 3 multiply the remaining factors.

also called **least** common multiple

Example 22

Find the LCM of 36 and 45:

a by listing multiples

b using factor trees.

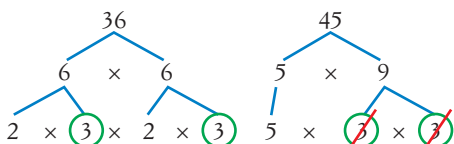
Solution

a Multiples of 36 = 36, 72, 108, 144, **180**, ...

Multiples of 45 = 45, 90, 135, **180**, 225, ...

The lowest common multiple is **180**.

b



$$\text{LCM} = (2 \times 3 \times 2 \times 3) \times 5 = 36 \times 5 = 180$$

- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Cross out the two 3s in the second factor tree
- Multiply the remaining factors to calculate the LCM

Finding square and cube roots using prime factors

Example 23

Use a factor tree to find the value of:

a $\sqrt{196}$

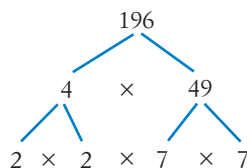
b $\sqrt[3]{216}$

Solution

a Draw a factor tree for 196.

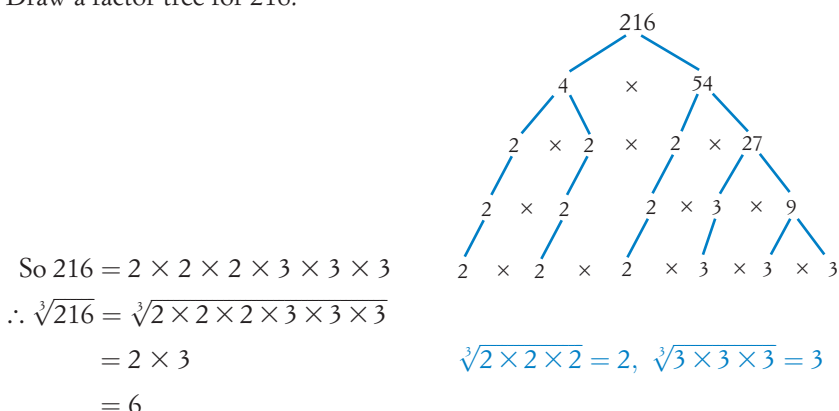
$$\text{So } 196 = 2 \times 2 \times 7 \times 7$$

$$\begin{aligned} \therefore \sqrt{196} &= \sqrt{2 \times 2 \times 7 \times 7} \\ &= 2 \times 7 \\ &= 14 \end{aligned}$$



$$\sqrt{2 \times 2} = 2, \sqrt{7 \times 7} = 7$$

b Draw a factor tree for 216.



Exercise 2-10 Prime factors

- Use a factor tree to write each number as a product of its prime factors, in index notation.

a 27	b 108	c 588	d 84
e 600	f 51	g 125	h 432
- What is 42 expressed as a product of its prime factors? Select the correct answer **A**, **B**, **C** or **D**.
A 2×21 **B** $3 \times 3 \times 7$ **C** $2 \times 3 \times 7$ **D** 3×14
- Draw two different factor trees for 80 and show that both give the same prime factors.
- Find the highest common factor of each pair of numbers.

a 8, 28	b 14, 35	c 20, 5	d 45, 60
e 24, 90	f 72, 24	g 16, 48	h 30, 18
- Use factor trees to find the highest common factor of each pair of numbers.

a 45, 60	b 208, 78	c 33, 176
- Find the lowest common multiple of each pair of numbers.

a 9, 8	b 4, 6	c 10, 22	d 7, 4
e 6, 10	f 12, 8	g 9, 45	h 11, 10
- Use factor trees to find the lowest common multiple of each pair of numbers.

a 90, 15	b 28, 36	c 16, 20
- Use a factor tree to evaluate each root.

a $\sqrt{2025}$	b $\sqrt{441}$	c $\sqrt{256}$
e $\sqrt[3]{4096}$	f $\sqrt[3]{729}$	g $\sqrt[3]{5832}$

See Example 20

See Example 21

Worked solutions

Exercise 2-10

MAT08NAWS10013

See Example 22

See Example 23

Investigation: Index laws for multiplying and dividing

Multiplying terms with the same base

The terms 3^6 and 3^2 have the same base, 3.

$$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \text{ and } 3^2 = 3 \times 3.$$

What happens when we multiply them?

$$3^6 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^8 = 6561.$$

1 Copy and complete each pattern.

a $2^4 \times 2^3 = (2 \times \_ \times \_ \times \_) \times (2 \times \_ \times \_) = 2^{\_} = \_$

b $7^2 \times 7^2 = (7 \times \_) \times (\_ \times \_) = 7^{\_} = \_$

c $10^2 \times 10^3 = (\_ \times \_) \times (\_ \times \_ \times \_) = 10^{\_} = \_$

d $4^3 \times 4^5 = (\_) \times (\_) = 4^{\_} = \_$

e $5^6 \times 5 = (\_) \times \_ = 5^{\_} = \_$

2 Copy and complete each pattern.

a $3^3 \times 3^7 = 3^{\_}$

b $2^5 \times 2^3 = 2^{\_}$

c $9^6 \times 9^4 = 9^{\_}$

d $6^3 \times 6^3 = 6^{\_}$

e $8^8 \times 8 = 8^{\_}$

3 Copy and complete each statement.

a $a^4 \times a^5 = \_$

b $a^m \times a^n = \_$

c When multiplying terms with the same base, $\_$ the powers.

Dividing terms with the same base

What happens when we divide 3^6 by 3^2 ?

$$3^6 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3^1 \times 3^1} = 3 \times 3 \times 3 \times 3 = 3^4 = 81.$$

1 Copy and complete each pattern.

a $2^5 \div 2^3 = \frac{2 \times 2 \times 2^1 \times 2^1 \times 2^1}{2^1 \times 2^1 \times 2^1} = \_ \times \_ = 2^{\_} = \_$

b $7^8 \div 7^4 = \frac{7 \times 7 \times 7 \times \_ \times \_ \times \_ \times \_}{7 \times 7 \times 7 \times \_} = \_ \times \_ \times \_ \times \_ = 7^{\_} = \_$

c $4^6 \div 4^3 = \frac{\_ \times \_ \times \_ \times \_ \times \_ \times \_}{\_ \times \_ \times \_} = \_ \times \_ \times \_ = 4^{\_} = \_$

d $3^6 \div 3 = \_ = \_ = 3^{\_} = \_$

e $10^7 \div 10^2 = \_ = \_ = 10^{\_} = \_$

2 Copy and complete each pattern.

a $3^5 \div 3^2 = 3^{\_}$

b $10^9 \div 10^5 = 10^{\_}$

c $8^6 \div 8^4 = 8^{\_}$

d $6^7 \div 6 = 6^{\_}$

e $2^6 \div 2^5 = 2^{\_}$

3 Copy and complete each statement.

a $a^5 \div a^3 = \_$

b $a^m \div a^n = \_$

c When dividing terms with the same base, $\_$ the powers.

2-11 Index laws for multiplying and dividing

Homework sheet

Mental calculation and powers

MAT08NAHS10004

Maths clip

Powers, indices and exponents

MAT08NAMC00002

Summary

When **multiplying terms with the same base**, *add* the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, *subtract* the powers.

$$a^m \div a^n = a^{m-n}$$

These are called **index laws** because **index** is another name for 'power'.

Example 24

Use index notation to simplify each product.

a $5^3 \times 5^4$

b $6^4 \times 6^5$

c $2 \times 2^3 \times 2^4$

Solution

a $5^3 \times 5^4 = 5^{3+4} = 5^7$

Adding the powers.

b $6^4 \times 6^5 = 6^{4+5} = 6^9$

Note: Do **not** multiply the bases.

c $2 \times 2^3 \times 2^4 = 2^{1+3+4} = 2^8$

Note: $2 = 2^1$

Example 25

Use index notation to simplify each quotient.

a $5^8 \div 5^3$

b $\frac{7^5}{7^2}$

c $6^{11} \div 6^{10}$

Solution

a $5^8 \div 5^3 = 5^{8-3} = 5^5$

Subtracting the powers.

b $7^5 \div 7^2 = 7^{5-2} = 7^3$

Note: Do **not** divide the bases.

c $6^{11} \div 6^{10} = 6^{11-10} = 6^1$

Exercise 2-11 Index laws for multiplying and dividing

1 Use index notation to simplify each product.

a $6^3 \times 6^2$

b $4^3 \times 4^4$

c $2^5 \times 2^6$

d 8×8^9

e $11^4 \times 11^4$

f $7^3 \times 7^7$

g 5×5^6

h $6^6 \times 6^{10}$

i $2^3 \times 2^2 \times 2^4$

j $5^3 \times 5^4 \times 5^2$

k $2^4 \times 2^4 \times 2^6$

l $3^2 \times 3^9 \times 3^3 \times 3^2$

2 Which term is equal to $2^3 \times 2^3 \times 2$? Select the correct answer **A**, **B**, **C** or **D**.

A 2^6

B 2^9

C 8^7

D 2^7

See Example 24

- 3 Simplify each product, then evaluate it as a whole number.

a $5^2 \times 5^4$

b $7^6 \times 7^2$

c $3^4 \times 3^9$

d $4^3 \times 4^3$

f $9^2 \times 9^5$

g $6^5 \times 6^4$

e $8^4 \times 8^6$

f 7×7^3

See Example 25

- 4 Use index notation to simplify each quotient.

a $2^6 \div 2^2$

b $3^9 \div 3^7$

c $7^8 \div 7^2$

d $\frac{2^9}{2^4}$

e $5^{11} \div 5^9$

f $10^{11} \div 10^7$

g $11^{12} \div 11^5$

h $6^{18} \div 6^{11}$

i $2^5 \div 2^4$

j $\frac{16^{13}}{16^7}$

k $35^{12} \div 35$

l $\frac{20^7}{20^6}$

- 5 Which term is equal to
- $5^8 \div 5^2$
- ? Select the correct answer A, B, C or D.

A 1^6

B 5^4

C 1^5

D 5^6

- 6 Simplify each quotient, then evaluate it as a whole number.

a $2^6 \div 2^2$

b $5^9 \div 5^7$

c $4^5 \div 4^2$

d $10^9 \div 10^4$

e $\frac{8^{11}}{8^8}$

f $6^{10} \div 6^2$

g $11^4 \div 11^3$

h $\frac{3^7}{3}$

- 7 Copy and complete this table of powers.

Number, n	n^2	n^3	n^4	n^5	n^6
2	4	8			
3		27		243	
4	16		256		
5		125			15 625

- 8 Use your answers from the table in question 7 to evaluate each expression.

a $2^3 \times 2^2$

b $3^2 \times 3^2$

c $5^2 \times 5$

d $5^3 \times 5^2$

e $4^4 \times 4^2$

f $2^2 \times 3^2 \times 3^3 \times 2$

g $4^5 \div 4^2$

h $5^6 \div 5$

i $\frac{3^6}{3^4}$

j $2^5 \div 2^4$

k $\frac{4^4}{4^3}$

l $3^3 \div 3^3$

- 9 Copy and complete each equation.

a $5^2 \times \underline{\quad} = 5^6$

b $\underline{\quad} \div 3^4 = 3^3$

c $2^7 \div \underline{\quad} = 2^4$

- 10 Simplify each algebraic expression.

a $a^3 \times a^7$

b $x^2 \times x^4$

c $b^4 \times b$

d $y^3 \times y^2$

e $d^6 \div d^2$

f $x^5 \div x^3$

g $\frac{a^8}{a^7}$

h $\frac{r^5}{r}$

Investigation: More index laws

Raising a term with a power to another power

What happens when we raise 2^3 to a power of 4? How do we simplify $(2^3)^4$ using index notation?

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3} = 2^{12}$$

- 1 Copy and complete each pattern.

a $(3^2)^5 = \underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = 3^{\underline{\quad}} = 3^{\underline{\quad}}$

b $(7^4)^2 = \underline{\quad} \times \underline{\quad} = 7^{\underline{\quad}} = 7^{\underline{\quad}}$

c $(16^3)^3 = \underline{\quad} \times \underline{\quad} \times \underline{\quad} = 16^{\underline{\quad}} = 16^{\underline{\quad}}$

d $(2^6)^4 = \_ \times \_ \times \_ \times \_ = 2\_\_ = 2\_\_$

e $(5^2)^3 = \_ \times \_ \times \_ = 5\_\_ = 5\_\_$

2 Copy and complete each statement.

a $(a^5)^2 = \_\_\_\_\_\_$

b $(a^m)^n = \_\_\_\_\_\_$

c When a term with a power is raised to another power, $\_\_\_\_\_\_$ the powers.

The power of zero

We know that $3^2 = 3 \times 3$ and $3^1 = 3$, but what is the value of 3^0 (3 to the power of 0)?

1 a Copy and complete this pattern.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^3 = 3 \times \_ \times \_ = \_\_\_$

$3^2 = \_\_\_\_\_\_ = \_\_\_$

$3^1 = \_\_\_\_\_\_ = \_\_\_$

b As the powers of 3 decrease by 1, what happens to its calculated value?

c According to the pattern, what should be the value of 3^0 ?

2 a Use index notation to simplify $2^5 \div 2^5$.

b But $2^5 \div 2^5 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$. What is the answer to this?

c What is any number divided by itself equal to?

d What is the value of 2^0 ?

3 a Use index notation to simplify $5^2 \times 5^0$. What do you notice about the answer?

b If a number multiplied by 5^0 equals itself, what must be the value of 5^0 ?

c Use index notation to simplify $10^6 \div 10^0$. What do you notice about the answer?

d If a number divided by 10^0 equals itself, what must be the value of 10^0 ?

4 Copy and complete each statement.

a $4^0 = \_\_\_\_\_\_$

b $a^0 = \_\_\_\_\_\_$

c When any number is raised to a power of zero, the answer is always $\_\_\_\_\_\_$.

2-12 More index laws

Summary

When a **term with a power** is raised to **another power**, **multiply** the powers.

$$(a^m)^n = a^{m \times n}$$

Any number raised to the **power of zero** is equal to **1**.

$$a^0 = 1$$

Worksheet

What is the power question?

MAT08NAWK10019

Homework sheet

Powers and divisibility

MAT08NAHS10005

Homework sheet

Powers and prime factors

MAT08NAHS10007

Video tutorial

Index laws

MAT08NAVT10003

Example 26

Use index notation to simplify each expression.

a $(6^2)^5$

b $(7^3)^3$

Solution

a $(6^2)^5 = 6^{2 \times 5} = 6^{10}$

Multiply the powers.

b $(7^3)^3 = 7^{3 \times 3} = 7^9$

Example 27

Evaluate each expression.

a 6^0

b $(3 \times 4)^0$

c 3×4^0

Solution

a $6^0 = 1$

Any number raised to the power of 0 equals 1.

b $(3 \times 4)^0 = 12^0 = 1$

c $3 \times 4^0 = 3 \times 1 = 3$

Exercise 2-12 More index laws**See Example 26****1** Use index notation to simplify each expression.

a $(2^9)^2$

b $(5^2)^3$

c $(7^4)^5$

d $(8^2)^7$

e $(3^8)^3$

f $(9^6)^4$

g $(6^7)^2$

h $(11^5)^5$

i $(8^{10})^3$

j $(4^9)^5$

k $[(-2)^4]^7$

l $[(-7)^3]^6$

2 Which term is equal to $(3^4)^2$? Select the correct answer **A**, **B**, **C** or **D**.

A 9^6

B 6^4

C 3^6

D 3^8

3 Simplify each expression, then evaluate it as a whole number.

a $(5^2)^4$

b $(2^5)^3$

c $(3^9)^2$

d $(6^3)^4$

e $(7^2)^3$

f $(4^6)^2$

g $(8^0)^5$

h $(4^3)^3$

See Example 27**4** Evaluate each expression.

a 9^0

b 3^0

c 10^0

d 16^0

e $(-2)^0$

f 6^0

g $(7 - 3)^0$

h $7 - 3^0$

i $(-5)^0$

j 7×3^0

k $(7 \times 3)^0$

l $2 + 3^0$

m $(6 + 6)^0$

n $6^0 + 6^0$

o $6 \div 6^0$

p $10 - 10^0$

5 Evaluate $6^2 \div 2^0$. Select the correct answer **A**, **B**, **C** or **D**.

A 3

B 36

C 18

D 6

Worked solutions

Exercise 2-12

MAT08NAWS10015

- 6 Copy and complete this table of powers of 2.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
		4			32						1024	

- 7 Use your answers from question 6 to evaluate each expression.

a $(2^5)^3$ **b** $(2^6)^2$ **c** $(2^2)^2$ **d** $(2^3)^3$
e $(2^0)^4$ **f** $(2^2)^4$ **g** $(2^{10})^0$ **h** $(2^7)^1$

- 8 Copy and complete each equation.

a $(6^2)^- = 6^8$ **b** $(5^-)^3 = 5^9$ **c** $(10^-)^- = 10^{18}$

- 9 Simplify each algebraic expression.

a $(a^4)^2$ **b** $(x^6)^6$ **c** $(d^2)^5$ **d** $(n^3)^6$
e p^0 **f** $4 \times y^0$ **g** $(4 \times y)^0$ **h** $a^0 - b^0$

Power plus

- 1 Even if we don't know the exact value of a surd, we can use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ to simplify a surd if one of its factors is a square number. Study each example.

- $\sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$
- $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$
- $\sqrt{36} = \sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$
- $\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$

Simplify each surd.

a $\sqrt{81}$ **b** $\sqrt{8}$ **c** $\sqrt{27}$ **d** $\sqrt{50}$
e $\sqrt{72}$ **f** $\sqrt{45}$ **g** $\sqrt{180}$ **h** $\sqrt{128}$
i $\sqrt{28}$ **j** $\sqrt{200}$ **k** $\sqrt{48}$ **l** $\sqrt{245}$

- 2 **a** Use index notation to simplify $2^4 \div 2^5$.

b But $2^4 \div 2^5 = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$. What is the answer to this?

c What is the value of 2^{-1} ?
d What is the value of 2^{-2} ?

- 3 **a** Does $(-2)^5 = -2^5$?

b Does $(-3)^4 = -3^4$?

c For what values of n does $(-1)^n = -1$?

d For what value of x is x^0 not defined?

- 4 Scientific notation is a special way of writing very large or very small numbers using powers of 10. For example:

- 25 600 can be written as $2.56 \times 10 \times 10 \times 10 \times 10 = 2.56 \times 10^4$ or
 $\boxed{2.56} \boxed{04}$ on a calculator because $2.56 \times 10^4 = 2.56 \times 10 \times 10 \times 10 \times 10$
- 0.001678 can be written as $1.678 \div 10 \div 10 \div 10 = 1.678 \times 10^{-3}$ or
 $\boxed{1.678} \boxed{-03}$ on a calculator because $1.678 \times 10^{-3} = 1.678 \div 10 \div 10 \div 10$

Puzzle sheet

Crossnumber
challenges

MAT08NAPS10004

Note that the first number is a decimal between 1 and 10 and the second number is a power of 10.

Convert each number written in scientific notation to an ordinary number.

a 2.4×10^4

b 4.55×10^5

c 9.33×10^{-2}

d 6.38×10^{-2}

e 8.7×10^6

f 5.82×10^{-3}

g 1.26×10^{-1}

h 2.69×10^3

i 3.14×10^8

Can you see a quick way of writing the answer each time?

- 5** Write each number in scientific notation.

a 12 000

b 345 000 000

c 0.007

d 4000

e 0.0005

f 0.00041

g 1 920 000

h 0.000 361

i 0.000 000 063 7

- 6** **a** What is the largest number that can be displayed on your calculator?
b What is the smallest?

Language of maths

base	factor tree	long division	quotient
consecutive	highest common factor	lowest common multiple	recurring decimal
cube root	grouping symbol	mental calculation	round
decimal places	index laws	order of operations	square root
estimate	index notation	prime factors	surd
evaluate	integer	product	terminating decimal

Puzzle sheet

Numbers crossword

MAT08NAPS10005

- If you round a decimal to the nearest hundredth, how many decimal places is this?
- Write a numerical expression that requires the use of:
 - 'order of operations' rules
 - long division
- Write 'the cube root of -64 ' using mathematical symbols, then evaluate it.
- In 8^5 , what are the names given to the '8' and the '5'?
- What is another name for:
 - grouping symbols?
 - index?
- What diagram is used to write a number as a product of its prime factors?

Topic overview

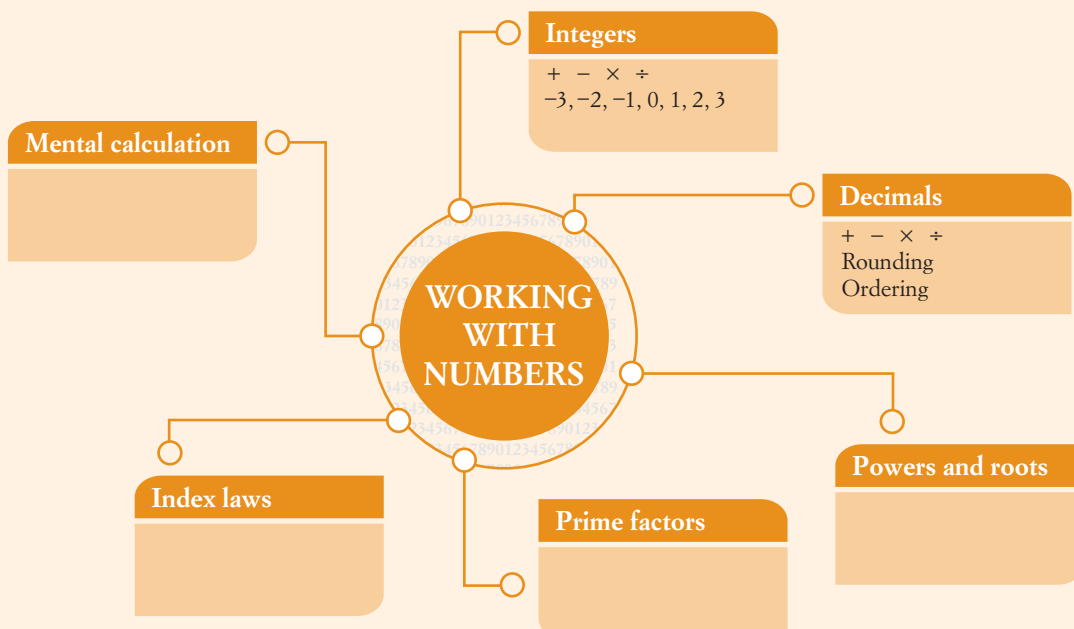
- What parts of this chapter do you remember from last year?
- Are there any parts of this chapter that you do not understand?
- Discuss any problems with your teacher or a friend.

Worksheet

Mind map: Working with numbers

MAT08NAWK10020

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 2 revision

See Exercise 2-01

1 Use mental calculation strategies to evaluate each expression.

a 7×200

b 14×15

c $8000 \div 50$

d $\frac{1}{2} \times 262$

e $68 + 19$

f 35×11

g 65×4

h 8×60

i $26 + 61 + 156 + 19 + 14$

j $8 \times 25 \times 4$

k 23×12

l $168 \div 4$

m $2700 \div 300$

n 40×70

o 37×9

p 18×5

q $384 \div 8$

r $970 - 245$

s $4 \times 8 \times 5$

See Exercise 2-01

2 Evaluate each quotient.

a $812 \div 7$

b $846 \div 9$

c $396 \div 18$

d $2139 \div 23$

See Exercise 2-02

3 Evaluate each expression.

a $-4 + 6$

b $5 - (-2)$

c $-3 - 8$

d $-2 - (-2)$

See Exercise 2-03

4 Evaluate each product.

a -2×7

b $-3 \times (-8)$

c $(-7)^2$

d $16 \times (-5)$

See Exercise 2-04

5 Evaluate each quotient.

a $36 \div (-4)$

b $-28 \div 7$

c $24 \div (-2)$

d $-15 \div (-5)$

See Exercise 2-05

6 Evaluate each expression.

a $2 + (13 - 8) \times 11$

b $14 - 5 \times (-3)$

c $15 - 18 \div 3 \times 2$

d $80 - [(7 - 10) \times (-6)]$

e $\frac{8-3}{100 \div 4}$

f $\frac{15 \times (-6 + 8)}{24 \div 2^2}$

See Exercise 2-06

7 Write these decimals in descending order: 0.417, 0.47, 0.147, 0.471.

See Exercise 2-06

8 Write each number correct to the number of decimal places shown in the brackets.

a 0.473 [1]

b 13.1051 [2]

c 98.4273 [3]

See Exercise 2-07

9 Evaluate each expression.

a $37.4 - 6.9$

b $13.3 + 0.82 + 5.6$

c 2.6×4

d 5.5×0.6

e 0.71×1.3

f $(2.5)^2$

g $9.57 \div 4$

h $8.12 \div 0.7$

See Exercise 2-08

10 Convert each fraction to a decimal.

a $\frac{2}{9}$

b $\frac{7}{8}$

c $\frac{1}{6}$

See Exercise 2-09

11 Evaluate each expression.

a 7^3

b 11^5

c $(-5)^4$

d $\sqrt{400}$

e $\sqrt[3]{27}$

f $\sqrt[3]{-125}$

See Exercise 2-09

12 Without using a calculator, estimate $\sqrt{31}$ to one decimal place.

See Exercise 2-09

13 Copy and complete:

a $20^2 = (\_\_ \times 10)^2$

b $\sqrt{196} = \sqrt{\_\_} \times \sqrt{49}$

$= \_\_^2 \times 10^2$

$= \_\_ \times \_\_$

$= \_\_ \times \_\_$

$= \_\_$

$= \_\_$

- 14** **a** Find the highest common factor of 36 and 42. See Exercise 2-10
b Find the lowest common multiple of 20 and 14.
- 15** Use factor trees to write 132 and 88 as products of their prime factors, then use them to find the highest common factor of 132 and 88. See Exercise 2-10
- 16** Use factor trees to write 12 and 45 as products of their prime factors, then use them to find the lowest common multiple of 12 and 45. See Exercise 2-10
- 17** Use a factor tree to write 784 as a product of its prime factors, then use it to evaluate $\sqrt{784}$. See Exercise 2-10
- 18** Simplify each expression using index notation. See Exercise 2-11
- | | | | |
|---------------------------|---------------------------|-----------------------------|---------------------------------------|
| a $4^2 \times 4^5$ | b $6^4 \times 6^7$ | c $12^4 \times 12^4$ | d $9^3 \times 9$ |
| e $2^7 \div 2^3$ | f $5^9 \div 5^6$ | g $3^8 \div 3^7$ | h $6^{11} \div 6^4 \times 6^5$ |
- 19** Simplify each expression using index notation. See Exercise 2-12
- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| a $(3^4)^2$ | b $(7^2)^5$ | c $(6^3)^3$ | d $(3^5)^3$ |
|--------------------|--------------------|--------------------|--------------------|
- 20** Evaluate each expression. See Exercise 2-12
- | | | | |
|-----------------|----------------------------|-------------------------|----------------------|
| a 3^0 | b 6^0 | c 4×4^0 | d $9 - 9^0$ |
| e 10^0 | f $(12 \times 3)^0$ | g $(5 - 3)^0$ | h $5^0 - 3^0$ |

Mixed revision 1

See Exercise 2-09

1 Evaluate each expression.

a 7^3 **b** $(-35)^2$ **c** 4^4
d $\sqrt{400}$ **e** $\sqrt[3]{2744}$ **f** $\sqrt{2.25}$

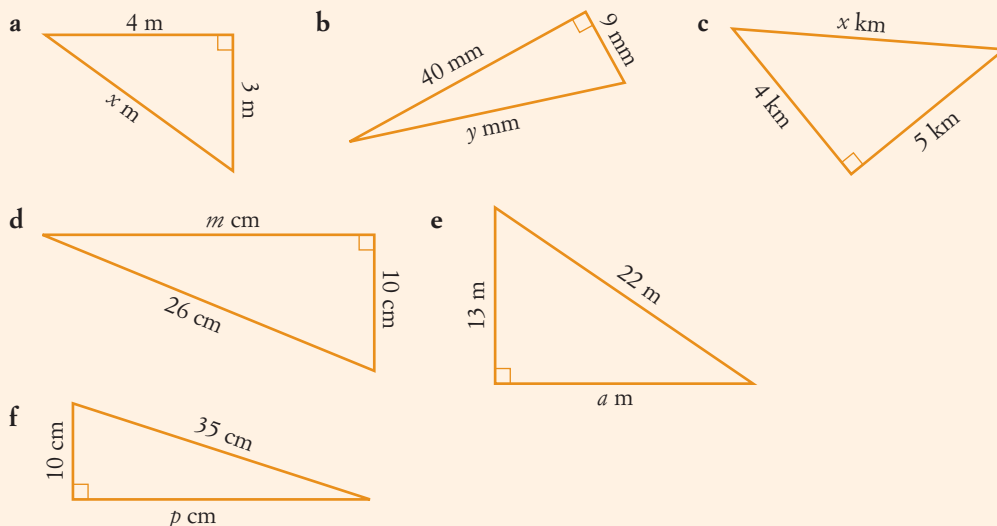
See Exercise 2-01

2 Evaluate each expression without using a calculator.

a 28×5 **b** 15×9 **c** $8 \times 3 \times 5$
d 16×30 **e** $720 \div 4$ **f** $300 \div 20$

See Exercise 1-05

3 Find the value of each pronumeral, correct to one decimal place where necessary.



See Exercises 2-02,
2-03, 2-04

4 Evaluate each expression.

a $-3 + 11$ **b** $6 \times (-5)$ **c** $-11 - 9$ **d** $3 - (-2)$
e $-7 \times (-4)$ **f** $\frac{42}{-7}$ **g** $7 - 9$ **h** $-4 - (-6)$
i $-6 - 10$ **j** -4×7 **k** $-54 \div (-9)$ **l** $75 \div (-5)$

See Exercise 2-05

5 Evaluate each expression.

a $4 + 7 \times 2$ **b** $16 - 28 \div (-4)$ **c** $(-32 + 11) \div 7$
d $32 \div (-8) - 7 \times 1$ **e** $(3 + 7 \times 5) \div 2$ **f** $(17 - 6) \times (3 - 12)$
g $\frac{12 + 7}{4}$ **h** $\frac{8.8}{6.5 - 5.4}$ **i** $\frac{64 - 12}{15 + 11}$

See Exercise 3-03

6 If $a = 3$ and $b = 5$, then evaluate each expression.

a $2b + a$ **b** a^2 **c** $5a - b$ **d** $ab - b$

See Exercise 3-05

7 Simplify each expression.

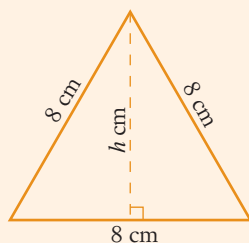
a $3k + 2j - k + 2j$ **b** $5p - p$ **c** $4p + 3q + 7p + 2q$ **d** $10x + 7y - 2x - y$

See Exercise 2-06

8 Evaluate each expression.

a $1.35 + 0.2 - 5.78$ **b** $35.3 - 12.91$

- 9 Evaluate each expression. See Exercise 2-07
a 0.8×9 **b** 1.5×0.3 **c** $2.1 \div 5$ **d** $5.52 \div 0.3$
- 10 Write each statement as an algebraic expression. See Exercise 3-02
a 5 more than k **b** twenty times k
c half of k **d** 3 lots of k increased by 10
e the difference between k and 12 **f** k multiplied by itself
- 11 Simplify each expression. See Exercises 3-06, 3-07
a $5 \times 2p$ **b** $12m \div 2m$ **c** $-4x \times 3xy$
d $3a \times 2b \times 4$ **e** $\frac{14xy}{7xy}$ **f** $\frac{5a \times 4a}{10a}$
- 12 Use a factor tree to write each number as a product of its prime factors, in index notation. See Exercise 2-10
a 180 **b** 875
- 13 Use factor trees to find the highest common factor of each pair of numbers. See Exercise 2-10
a 28 and 72 **b** 36 and 90
- 14 Use index notation to simplify each expression. See Exercises 2-11, 2-12
a $2^4 \times 2^3$ **b** $7^6 \times 7^5$ **c** $5^8 \div 5^3$ **d** $8^5 \div 8^4$
e $(3^4)^2$ **f** $(11^3)^5$ **g** 7^0 **h** $(8-5)^0$
- 15 Simplify each expression. See Exercise 3-08
a $5y^2 \times 3y^4$ **b** $20y^7 \div 4y^2$ **c** $\frac{35m^3n^7}{5m^3n^2}$
d $(m^4)^5$ **e** $(3k^2)^2$ **f** $(5m^3n^2)^3$
- 16 Find the height, h cm, of an equilateral triangle with sides 8 cm long. Write your answer: See Exercise 1-08
a as a surd **b** correct to two decimal places.



- 17 Use factor trees to find the lowest common multiple of each pair of numbers. See Exercise 2-10
a 12 and 18 **b** 18 and 27
- 18 Expand and simplify each expression. See Exercise 3-09
a $4(x+2)$ **b** $6(m-4)$ **c** $-7(2k+1)$
d $-(3p+q)$ **e** $2(m+2)+4m$ **f** $8+4(p-2)$
- 19 Factorise each expression. See Exercises 3-11, 3-19
a $2m+4$ **b** d^2-d **c** $6p+12q$
d $-12x-24$ **e** $2ab+3a$ **f** $-2x^2+6x$



3

Number and algebra

Algebra

Today, algebra is an important day-to-day tool that can be applied to all types of real-world situations. It is used to describe the relationships between variables. For example, algebra can be used to figure out how much weight a bridge can carry or predict how many people will be living in your city in twenty years.



Chapter outline

	Proficiency strands				
3-01 Variables	U	F		R	C
3-02 From words to algebraic expressions	U	F	PS	R	C
3-03 Substitution	U	F	PS		
3-04 Collecting variables	U	F		R	C
3-05 Adding and subtracting terms	U	F		R	C
3-06 Multiplying terms	U	F		R	C
3-07 Dividing terms	U	F		R	C
3-08 Extension: The index laws	U	F		R	C
3-09 Expanding expressions	U	F		R	
3-10 Factorising algebraic terms	U	F		R	
3-11 Factorising expressions	U	F		R	
3-12 Factorising with negative terms	U	F		R	

Wordbank

algebraic expressions An expression that describes a quantity by using variables and numerals, such as $3x + 4y - 8$

algebraic term A part of an algebraic expression, such as $3x$ in $4x^2 + 3x - 11$

evaluate To find the value of an expression

expand To rewrite an expression such as $4(2k + 4)$ without brackets

factorise The opposite of expand; to rewrite an expression, such as $8k + 16$, with brackets

like terms Terms that have exactly the same variables, for example, $5p$ and $2p$

substitute To replace a variable with a number

variable (or **pronumeral**) A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

In this chapter you will:

- introduce the concept of variables as a way of representing numbers using letters
- extend and apply the laws and properties of arithmetic to algebraic terms and expressions
- move fluently between algebraic and word representations as descriptions of the same situation
- create algebraic expressions and evaluate them by substituting a given value for each variable
- simplify algebraic expressions involving the four operations
- extend and apply the distributive law to the expansion of algebraic expressions
- factorise a single algebraic term, for example, $6ab = 3 \times 2 \times a \times b$
- factorise algebraic expressions by identifying numerical and algebraic factors

SkillCheck

Worksheet

StartUp assignment 3

MAT08NAWK10021

1 Evaluate each expression.

a $3 - 4$

b $-5 + 9$

c $-8 - 2$

d $3 - (-2)$

e -3×6

f $-12 \div (-4)$

g $-12 \div 4 - 10$

h $-4 \times (-8)$

i $(-6)^2$

j 2^3

k $(-1)^3$

l 2×3^2

2 Simplify each algebraic expression.

a $7 \times p$

b $a \times b$

c $k \times k$

d $n \times n \times n$

e $2 \times p \times m$

f $p \times k \times d$

g $y \times 6$

h $t \times 3 \times q$

i $6d - 7d$

j $q + q + q$

k $a + a + a + a$

l $1x$

3 Choose which operation (+, −, × or ÷) is related to each word.

a sum**b** product**c** difference**d** quotient**e** decrease**f** increase**g** more than**h** less than4 If $x = 8$ and $y = 2$, evaluate each expression.

a $x + y$

b $3x$

c $x - y$

d $5y$

e xy

f $9 - y$

g $x + 17$

h $2x + 4$

i $3y - 2$

j $x \div y$

k $5x \div y$

l $4x + 5y$

5 Find:

a the sum of 8 and 5**b** double 9**c** 6 increased by 10**d** the difference between 14 and 11**e** the product of 3 and 7**f** how many times 5 divides into 40

Worksheet

What does the symbol mean?

MAT08NAWK10022

3-01 Variables

In algebra, a **variable** or **pronumeral** is a symbol, usually a letter of the alphabet, that stands for a number. It is called a variable because its value can vary (change).

When writing algebraic expressions, we use the following abbreviations.

- $3 \times k = 3k$ for multiplication, we leave out the '×' symbol
- $m \div 4 = \frac{m}{4}$ for division, we can write in fraction form
- $r \times r = r^2$ for powers
- $1x = x$ for multiplying by 1
- $c \times a \times 6 = 6ac$ write the number first, then the variables in alphabetical order

Example 1

Simplify each expression.

a $p + p + p + p$

d $m \times n \times 6$

g $y \times y \times y$

b $d \times 5$

e $4v + 2v$

h $5a - 4a$

c $r \div 7$

f $5 \times k \times 2 \times h$

i $b \times b \times 15$

Solution

a $p + p + p + p = 4p$

4 lots of p .

b $d \times 5 = 5d$

Write the number first.

c $r \div 7 = \frac{r}{7}$

d $m \times n \times 6 = 6mn$

Number first, then variables in alphabetical order.

e $4v + 2v = 6v$

$(v + v + v + v) + (v + v) = 6$ lots of v

f $5 \times k \times 2 \times h = 5 \times 2 \times k \times h$
 $= 10hk$

$(a + a + a + a + a) - (a + a + a + a)$
 $=$ one a left

g $y \times y \times y = y^3$

Just a : no need to write the '1'.

h $5a - 4a$

$= 1a$

$= a$

The '2' (squared) belongs to the b only.

i $b \times b \times 15 = 15b^2$

Example 2

Use order of operations to simplify each expression.

a $2 \times m - 4$

b $15 - k \div 3$

c $13 \div (d \times d) + 4$

Solution

a $2 \times m - 4 = 2m - 4$

 $\times$ first, then $-$

b $15 - k \div 3 = 15 - (k \div 3)$

 $\div$ first, then $-$

$$= 15 - \frac{k}{3}$$

c $13 \div (d \times d) + 4 = 13 \div d^2 + 4$

 $()$ first, then $\div$, then $+$

$$= \frac{13}{d^2} + 4$$

Example 3

Write each expression in **expanded form**.

a $5bc$

b $-4kr^2$

c $x^2 - \frac{t}{11}$

Note that expanding is the opposite of simplifying.

Solution

a $5bc = 5 \times b \times c$

b $-4kr^2 = -4 \times k \times r \times r$

c $x^2 - \frac{t}{11} = x \times x - t \div 11$

Exercise 3-01 Variables

See Example 1 1 Simplify each expression.

a $w \times 5$

b $m \times m$

c $3 \times c \times b \times a$

d $k + k + k + k + k + k$

e $m \div 8$

f $f + f + f$

g $12m + 4m$

h $7 \times w \times 3 \times x$

i $11p - 2p$

j $4 \times t \times t$

k $6d - 5d$

l $b \times c \times d \times 16$

m $26 \div n$

n $h \times 9 \times h$

o $9x + 11x$

p $a + a - a$

q $12q - 4q$

r $2 \times c \times 6 \times n$

s $3 \times r \times d \times (-2) \times d$

t $a + a + a + c + c$

u $a \times a \times a \times x \times x$

See Example 2 2 Use order of operations to simplify each expression.

a $13 \div (a \times 2)$

b $9 + 4 \times m$

c $(e - 6) \div 5$

d $4 \times t - 8$

e $y \times y + z \times z$

f $k \div 9 - n$

g $p \div (9 + n)$

h $22 - e \div 2$

i $11 + u \times u \times 11$

- 3 Which expression is equal to $m \times m \times 5 + 2$? Select the correct answer **A**, **B**, **C** or **D**.
A $5m^2 + 2$ **B** $5m^3 + 2$ **C** $5m + 2$ **D** $m^2 + 7$
- 4 Explain the meaning of:
a pq **b** $\frac{3y}{x}$ **c** $1x = x$
- 5 Which expression is **not** equal to $4p$? Select the correct answer **A**, **B**, **C** or **D**.
A $5p - p$ **B** p^4 **C** $p + p + p + p$ **D** $p \times 4$
- 6 Write each expression in expanded form.
a $8st$ **b** $-2y^2$ **c** $\frac{8}{f}$ **d** $r^2 + t^2$
e $4b^2d$ **f** $5xy - 2a$ **g** $\frac{x-2}{3}$ **h** q^3
i $16 - 2d^2$ **j** $\frac{-4d}{3}$ **k** h^2i^2 **l** $4j - \frac{9}{d}$
- 7 What is $7e^3$ in expanded form? Select the correct answer **A**, **B**, **C** or **D**.
A $7e \times 7e \times 7e$ **B** $7e + 7e + 7e$ **C** $7 \times e + e + e$ **D** $7 \times e \times e \times e$

Worked solutions

Exercise 3-01

MAT08NAWS10016

See Example 3

Investigation: The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic.
 For example, if we add zero to any number, the answer is still that number.
 If we let N stand for any number: $N + 0 = N$

In groups of two to four students, answer the following questions.

- 1 Describe in words what each number rule below means.
a $N \times 1 = N$ **b** $N \times 0 = 0$
c $a - b \neq b - a$ **d** $a + b + c = b + a + c$
e $N - 0 = N$ **f** $ab = ba$
- 2 Write each of the following rules algebraically using variables.
a Any number divided by 1 equals itself.
b Multiplying a number by 8 is the same as doubling it three times.
c Any three numbers can be multiplied together in any order.
d Any number added to itself is the same as multiplying that number by 2.
e Any number subtracted from itself equals 0.
f Any number multiplied by its reciprocal equals 1.
- 3 Is each of the following equations true or false? Test your answer by substituting a number for the variable and checking.
a $a \div b = b \div a$ **b** $N \div N = 1$
c $4a - a = 4$ **d** a is a factor of a
e If N is even, then $N + 3$ is odd **f** $\frac{1}{2}N = N - 2$
g $a + (-a) = 2a$ **h** $0 \div N = 0$
i $N \div 0 = 0$ **j** $N \div 1 = N$
- 4 If k is an odd number, what is an expression for:
a the previous odd number? **b** the next even number?

Puzzle sheet

What's the expression?

MAT08NAPS10006

Skillsheet

Algebraic expressions

MAT08NASS10012

Worksheet

Writing algebraic
expressions

MAT08NAWK00002

3-02 From words to algebraic expressions

Example 4

If x is a variable representing any number, write an algebraic expression for:

- a four times the number
- b the number divided by six
- c the previous consecutive number
- d one-third of the number

consecutive means 'following in order': for example, 7, 8, 9 are consecutive numbers

Solution

a $4 \times x = 4x$

b $x \div 6$ or $\frac{x}{6}$

c $x - 1$

d $\frac{1}{3}x = \frac{x}{3}$

The number before x is $x - 1$
(for example, the number before 4 is $4 - 1 = 3$).

Example 5

Think of a number, y , multiply it by three and subtract seven. Write an algebraic expression for the answer.

Solution

First, we multiply the number y by 3 to get $y \times 3 = 3y$.

Then, we subtract 7 from our result to get the answer, $3y - 7$.

Example 6

Describe the algebraic expression $\frac{a+b}{2}$ in words.

Solution

$\frac{a+b}{2}$ is the sum of a and b , divided by 2, or the average of a and b .

Exercise 3-02 From words to algebraic expressions

- 1 If n represents any number, write an algebraic expression for:
- | | |
|--|---|
| a five times the number | b the number plus six |
| c the number divided by 10 | d the next consecutive number |
| e half of the number | f 20 less than the number |
| g the number multiplied by itself | h 20 minus the number |
| i the previous consecutive number | j double the number |
| k 6 divided by the number | l the difference between the number and 8 |
| m the product of the number and (-4) | n the square root of the number |
- 2 If a , b and c represent any three numbers, write an algebraic expression for:
- | | |
|--|---|
| a the sum of a , b and c | b the product of a and b |
| c the difference between b and c , where b is greater than c | e three times b , plus a |
| d the product of a and c , plus b | g how much a is greater than b |
| f c divided by b | i c less than b |
| h five more than $(b \text{ times } c)$ | k the square root of the sum of a and c |
| j a times b divided by c | |
| l the product of a and b , decreased by seven times c | |
- 3 A cricketer scored a total of x runs in 6 innings. What was her average score? Select the correct answer **A**, **B**, **C**, or **D**.
- | | |
|----------------|---------------------|
| A $6x$ | B $6 \div x$ |
| C x^6 | D $x \div 6$ |



- 4 Write an algebraic expression for the answer to each instruction.
- Think of a number (y), multiply it by 3 and add 8
 - Think of a number (p), divide it by 3 and then subtract 6
 - Think of a number (M), multiply it by 4 and then add 27
 - Think of a number (k), multiply it by 5 and then divide it by 2
 - Think of a number (x), divide it by 6 and add 13
 - Think of a number (B), halve it and then subtract 2
 - Think of a number (R), divide it by 9 and then add 10
 - Think of a number (n), multiply it by 7 and then multiply it by 3

- 5 Describe each algebraic expression in words.

a $x + y - z$	b $5K + L$	c $xy - 3$	d $\frac{W}{5}$
e $3x - 1$	f $\frac{y-x}{10}$	g $2N + 21$	h $7 - e$

See Example 4

See Example 5

See Example 6

Worked solutions

Exercise 3-02

MAT08NAWS10017

- 6 Write an expression for:
- a the number of girls in a class if there are b boys and a total of t students
 - b the number of bottles of drink needed for a party if each bottle can be shared by 5 guests and there are n guests
 - c the amount of change in dollars that Sandra receives if she paid for 3 hamburgers costing $\$p$ each with $\$y$ in cash
 - d the perimeter of an isosceles triangle with equal sides of length r and an uneven side of length 9
 - e the cost of one egg if a carton of a dozen eggs costs $\$a$
 - f the perimeter of a square of length x
 - g the number of rows of chairs needed to place 160 chairs in the hall if each row contains r chairs
 - h the total cost of attending the pool for x adults and y children, where admission is $\$5$ per adult and $\$3$ per child



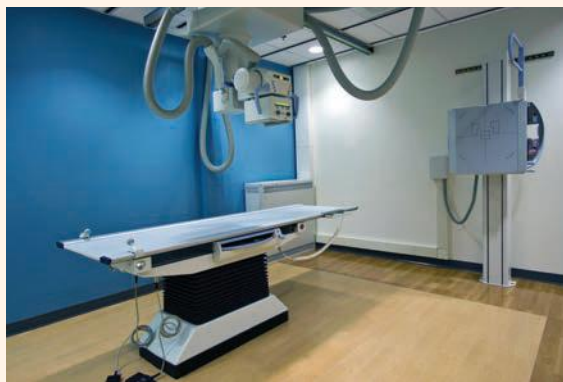
Just for the record

In algebra the letter 'x' is used quite frequently as a variable. During the 9th century, the Arabic word *al-jabr* ('algebra') described the process of moving variables from one side of an equation to the other to find the value of an unknown. The word for 'thing' or 'object' (the unknown number) in Arabic was *shei*, translated into Greek as *xei*, and shortened to *x*. *Xenos* is also the Greek word for unknown, stranger or foreigner, which could explain why early European mathematicians used the letter *x* in algebra.

One further explanation is that ancient scholars could only write using feathers dipped in ink. The two easiest letters to write were *x* and *y*. Mathematicians needed variables they could write quickly without mixing them up. As a result, *x* and *y* are still the two most commonly used variables in algebra.

Find examples of X-words that involve something unknown, such as X-ray, Brand X or X factor.

The X factor



3-03 Substitution

Puzzle sheet

Formula 1 game

MAT08NAPS10007

Homework sheet

Algebraic notation and substitution

MAT08NAHS10016

The word **substitution** means replacing one thing with another. In algebra, substitution means to replace a variable with a value to evaluate an algebraic expression.

Example 7

If $p = 3$ and $q = 14$, evaluate each expression.

a $2p + q$

b $\frac{pq + 2}{4}$

c $5q - p^2$

Solution

a $2p + q = 2 \times 3 + 14$
 $= 20$

b $\frac{pq + 2}{4} = \frac{3 \times 14 + 2}{4}$
 $= \frac{44}{4}$
 $= 11$

c $5q - p^2 = 5 \times 14 - 3^2$
 $= 61$

Substituting into formulas

A **formula** is a general rule that is written as an algebraic equation and shows the relationship between variables. Solving mathematical problems often involves substituting values into formulas.

Example 8

The formula for the perimeter of a rectangle is $P = 2l + 2w$, where P is the perimeter, l is the length and w is the width. Use this formula to calculate the perimeter of a rectangle with length 12 m and width 9.5 m.

Solution

When length $l = 12$ and width $w = 9.5$,

$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 12 + 2 \times 9.5 \\ &= 43 \text{ m} \end{aligned}$$

Exercise 3-03 Substitution

1 If $x = 5$ and $y = 6$, what is the value of $2x - y$? Select the correct answer **A**, **B**, **C** or **D**.

A 19

B -2

C 4

D 16

2 If $a = 3$, $b = 5$ and $c = 2$, evaluate each expression.

a $a + b$

b $4a$

c c^2

d $a + b + c$

e abc

f $3b - a$

g $c + a - b$

h $4a - 2c$

i $10 - b$

j $ab + c^2$

k $c - b$

l $\frac{2a}{c}$

See Example 7

- 3 If $d = 10$, $e = -3$ and $f = 5$, evaluate each expression.

a def

b f^2

c e^3

d $d \div f$

e $de \div 2$

f $d^2 - f^2$

g $7e$

h $3e + 2d - f$

i $f + d - e$

j $\frac{d}{2} - 3f$

k $6ef$

l $\frac{e+f}{4}$

- 4 If $p = -3$, what is the value of $2p^2$? Select the correct answer A, B, C or D.

A 36

B 18

C -18

D -36

Worked solutions

Exercise 3-03

MAT08NAWS10018

- 5 If $y = -1$ and $z = 4$, evaluate each expression.

a $3 - 4z$

b $-3y^2$

c $5(z - 2)$

d $y(z + 1)$

e $\frac{y}{z}$

f $\frac{z}{4}$

g $\frac{y+z}{9}$

h $\frac{10}{3+y}$

- 6 If $n = 6$, which of the following expressions is equal to 22? Select the correct answer A, B, C or D.

A $\frac{n+4}{2}$

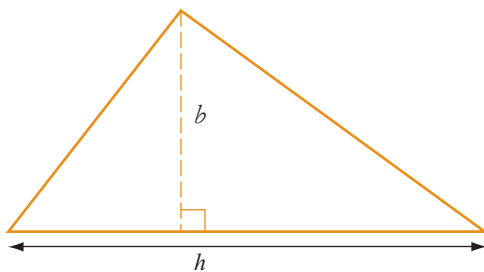
B $\frac{2n+4}{2}$

C $\frac{n^2}{2} + 4$

D $\frac{n^2+4}{2}$

See Example 8

- 7 The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area of the triangle, b is the length of the base and h is the height.



Find the area of a triangle with base length 12 m and height 7 m.

- 8 A plumber charges according to the formula $C = 42h + 65$, where C is the charge in dollars and h is the number of hours worked. How much does the plumber charge for working for $3\frac{1}{2}$ hours?



- 9 The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$, where F is the temperature in Fahrenheit and C is the temperature in Celsius. Use the formula to convert 26°C to Fahrenheit.
- 10 The number of hours of sleep recommended for children has the formula $H = 17 - \frac{A}{2}$, where H is the number of hours and A is the age of the child. Find the number of hours of sleep recommended for a 6-year-old.

- 11 The formula for the cost of a party is $C = 45n + 500$, where C is the cost in dollars and n is the number of guests. Use this formula to calculate the cost of a party for 120 guests.

Technology Substitution

- 1 Enter the variables and values shown into a new spreadsheet.

	A	B
1	variable	value
2	m	10
3	n	12
4	p	4
5	q	0
6	r	2.7
7	s	-8
8	t	25
9	u	-5

This means $m = 10$.

- 2 In cell D1, enter the formula **=B4-B9** to evaluate $p - u$. You should get 9 because $4 - (-5) = 9$.
- 3 In cell D2, enter the formula **=B2+B3** to evaluate $m + n$. You should get 22 because $10 + 12 = 22$.
- 4 For each cell below, enter a formula to evaluate the given expression.

D3: pr

D4: $\frac{t}{u}$

D5: $n + t - p$

D6: $n - \frac{m}{u}$

D7: $\frac{n - m}{u}$

D8: $\frac{s}{p} \times r$

D9: p^3

D10: $(s + n) \times (p + r)$

D11: $t + pq + s$

D12: $\sqrt{t}$

D13: $mnpq$

D14: the average of r , s , t and u

D15: $\frac{6m - n}{p + s}$

D16: $\sqrt{n + p}$

D17: ts^2

D18: $n^3 - r^2$

3-04 Collecting variables

Let the variable x stand for an unknown number. Let x be represented by a cup that holds the unknown number of balls.



Then the total number of balls represented in the diagram below can be written using the algebraic expression $3 \times x + 5 = 3x + 5$.



Note that even though we don't know how many balls are in each cup, it is the same number of balls, x , and we can still describe the total number of balls algebraically.

Let y stand for an unknown number that is different from x . Let y be represented by an envelope that contains this number of balls.



Then the total number of balls represented in the diagram below can be written using the algebraic expression $4x + 3y + 1$.



Exercise 3-04 Collecting variables

- 1 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls.



- 2 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls and each envelope holds y balls.



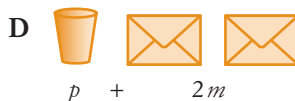
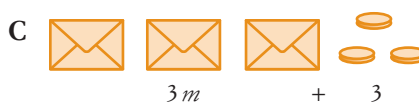
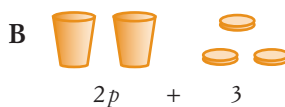
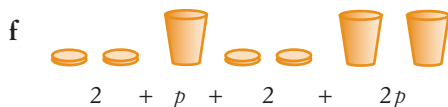
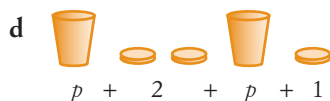
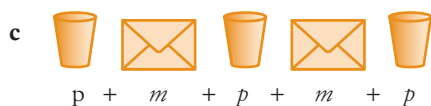
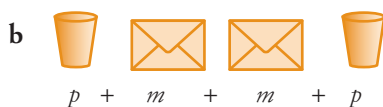
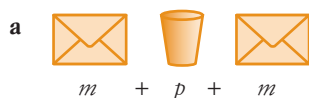
3 Draw cups and balls to represent each algebraic expression, if each cup holds x balls.

a $x + 3$	b $x + x + 1$
c $4 + 2x$	d $2 + x + 2$
e $x + 5 + 2x$	f $1 + 2x + 3 + x$

4 If N stands for the number of paperclips in each envelope, which diagram represents $3N + 2$ paperclips? Select the correct answer A, B, C or D.



5 If m stands for the number of coins in each envelope and p stands for the number of coins in each cup, match each expression in the left-hand column with the correct expression in the right-hand column.



6 Match each expression in the left column with the correct expression on the right.

a   - 
 $m + p - m$

A 
 $2m$

b   - 
 $m + 4 - 2$

B  
 $2p + 2$

c   - 
 $2m + p - p$

C 
 $2p$

d   - 
 $2p + 6 - 4$

D 
 p

e   - 
 $2m + 2p - 2m$

E  
 $2p + 2m$

f   - 
 $2p + 3m - m$

F  
 $m + 2$

7 Simplify each expression.

a   
 $p + m + p$

b   - 
 $m + 2p - p$

c    
 $2p + 2 + p + 3$

d    +   - 
 $p + m + 2 + p + m - 1$

e   -  - 
 $3p + 2m - m - p$

8 Simplify $9a + 4b - 5a + 2b$. Select the correct answer A, B, C or D.

A $14a + 6b$

B $10ab$

C $4a + 6b$

D $14a + 11b$

9 Simplify each expression.

a $3d + 2d + 4a$

b $6h + 4r + 5r - h$

c $e + e - e$

d $2k + 3 + 2k - 6$

e $3x + 4z - x + 2z$

f $2s - p - s + 3p$

g $6u + 4 - 2u + 8$

h $7n - 6 + 5 - 3n$

i $20 + 6i - 3i - 20$

Worked solutions

Exercise 3-04

MAT08NAWS10019

Mental skills 3A Maths without calculators

Simplifying multiplication by factorising

Sometimes, one big multiplication may be made into many simpler little multiplications if we break one or both terms into two 'easier' factors. Then we use the property that numbers can be multiplied in any order.

1 Study each example.

$$\begin{aligned} \text{a } 24 \times 8 &= 6 \times 4 \times 2 \times 4 \\ &= 4 \times 4 \times 6 \times 2 \\ &= 16 \times 6 \times 2 \\ &= 96 \times 2 \\ &= 192 \end{aligned}$$

Look for pairs of numbers that you can multiply easily and rearrange

$$\begin{aligned} \text{b } 15 \times 6 &= 3 \times 5 \times 6 \\ &= 3 \times 30 \\ &= 90 \end{aligned}$$

$$\begin{aligned} \text{c } 14 \times 36 &= 7 \times 2 \times 6 \times 6 \\ &= 6 \times 7 \times 6 \times 2 \\ &= 42 \times 6 \times 2 \\ &= 252 \times 2 \\ &= 504 \end{aligned}$$

$$\begin{aligned} \text{d } 30 \times 22 &= 10 \times 3 \times 22 \\ &= 10 \times 66 \\ &= 660 \end{aligned}$$

Note: For each question, there are many different possible ways of arriving at the correct result.

2 Now evaluate each product by factorising first.

$$\text{a } 30 \times 24$$

$$\text{b } 18 \times 27$$

$$\text{c } 25 \times 33$$

$$\text{d } 16 \times 8$$

$$\text{e } 28 \times 20$$

$$\text{f } 12 \times 18$$

$$\text{g } 21 \times 9$$

$$\text{h } 36 \times 12$$

$$\text{i } 16 \times 35$$

$$\text{j } 22 \times 28$$

$$\text{k } 9 \times 15$$

$$\text{l } 27 \times 25$$

3-05 Adding and subtracting terms

What is the fastest way to mentally calculate the sum $3 + 3 + 5 + 3 + 5 + 5 + 3 + 3 + 5$? Do you think of collecting all the 3s together and all the 5s together like this?

$$\begin{aligned} 3 + 3 + 5 + 3 + 5 + 5 + 3 + 3 + 5 &= (3 + 3 + 3 + 3 + 3) + (5 + 5 + 5 + 5) \\ &= (5 \times 3) + (4 \times 5) \\ &= 15 + 20 \\ &= 35 \end{aligned}$$

We can do the same thing algebraically. Suppose an unknown number p is represented by a cup



and another unknown number m is represented by an envelope



Consider the following two sums.

a

$$p + m + p + m = 2p + 2m$$

b

$$m + p + m + p + m = 3m + 2p$$

Note that we can add all the p terms together, and we can add all the m terms together, but we cannot add the ps and ms together because p and m represent different numbers. We cannot simplify the answers $2p + 2m$ and $3m + 2p$ because p and m are different.

These are examples of collecting **like terms**, terms that contain exactly the same variable(s), such as $4h$, $5h$ and h . Here are some more examples.

$$\begin{array}{lll} 3x \text{ and } -5x & 4mw \text{ and } 2mw & 12m \text{ and } m \\ -6d \text{ and } 10d & 5ab \text{ and } 2ba & xyz \text{ and } 2yzx \end{array}$$

Note: $ab = ba$

Here are some examples of **unlike terms**, terms that have different variables.

$$3x \text{ and } 5m \qquad 4mw \text{ and } 2mn \qquad -6d \text{ and } 10d^2$$

We can add or subtract like terms because the variables in them represent the same number. We can only add or subtract things that are the same. For example:

$$3m + 2p - 2m = m + 2p$$

$$(3 \text{ lots of } m) + (2 \text{ lots of } p) - (2 \text{ lots of } m) = (3 \text{ lots of } m - 2 \text{ lots of } m) + (2 \text{ lots of } p)$$

$$= (1 \text{ lot of } m) + (2 \text{ lots of } p)$$

This makes sense because if $m = 5$ and $p = 8$, then

$$\begin{aligned} 3m + 2p - 2m &= (3 \times 5) + (2 \times 8) - (2 \times 5) \\ &= (3 \times 5) - (2 \times 5) + (2 \times 8) \quad \text{grouping like terms} \\ &= (1 \times 5) + (2 \times 8) \quad 3 \text{ lots of } 5 - 2 \text{ lots of } 5 = 1 \text{ lot of } 5 \end{aligned}$$

Summary

When **adding and subtracting terms**, only **like terms** can be collected.
Like terms have exactly the same variables.

Example 9

Select the like terms from each list below.

a $5x, -x, x^2, xy, \frac{1}{2}x, -8x, \frac{4x}{3}$ b $3ab, 5, a, ba, -2b, 4ab^2, -ab$

Solution

a $5x, -x, xy, \frac{1}{2}x, -8x, \frac{4x}{3}$ are like terms because they each have a single x in them.

b $3ab, ba$ and $-ab$ are like terms because each contains ab .

Example 10

Simplify each expression by collecting like terms.

- a $7d + 3e + 3d + e$ b $6m - 8 + 3m + 6$
c $6xy + 7x^2 - 3yx - 12x^2$ d $14 - 4t - 9t + 2$

Solution

$$\text{a } (7d) + (3e) + (3d) + (e) = (7d + 3d) + (3e + e) \\ = 10d + 4e$$

Grouping like terms.

$$\text{b } (6m) - 8 + (3m) + 6 = (6m + 3m) - 8 + 6 \\ = 9m - 2$$

The + or - sign goes with the term after it.

$$\text{c } 6xy + 7x^2 - 3yx - 12x^2 = 6xy - 3xy + 7x^2 - 12x^2 \\ = 3xy - 5x^2$$

$$3yx = 3xy$$

$$\text{d } 14 - 4t - 9t + 2 = 14 + 2 - 4t - 9t \\ = 16 - 13t$$

Video tutorial

Adding and subtracting like terms

MAT08NAVT10004

Animated example

Collecting like terms

MAT08NAEE00012

Exercise 3-05 Adding and subtracting terms

- 1 Select the like terms from each list below.

- a $4p, 2y, 2p, 5z, p$ b $m^2, m, n, 3m^2, mn$ c $5ac, 4a, 7ca, 3, 5$
d $vw, 5v, 9v, w, v^2, 2wv$ e $p, 6pq, 2qp, 3pq, 7q$ f $2d, 5, 3d^3, 7d^2, 9, 3d, 2$
g $4mn, 3m, 4, 2nm, mn, 2n$ h $x^2y, 2x, 3y, 4x^2y$ i $7, 2a, 4b, 5ba, 9, ab, 2$

- 2 What are the like terms in $3ab, 2a^2, 2b^2a, 2p^2, 2ba$? Select the correct answer A, B, C or D.

- A $2a^2, 2p^2$ B $2a^2, 2b^2a, 2p^2, 2ba$ C $3ab, 2ba$ D $2b^2a, 2ba$

- 3 Simplify each expression by collecting like terms.

- a $4k + 7k$ b $5mn + 2nm$ c $xy + xy$
d $3abc + 4abc + 2bac$ e $3x^2 + 2x^2 + x^2$ f $ef^2 + 4ef^2$
g $8d - 3d$ h $12mk - 7mk$ i $12xy - 7yx - xy$
j $8de - 12de$ k $6k^2 - 3k^2$ l $4w^2 - w^2$

- 4 Which expression is $2m + 4p - 5m - 2p$ simplified? Select the correct answer A, B, C or D.

- A $-mp$ B $7m + 6p$ C $-3m + 2p$ D $-3p$

- 5 Simplify each expression.

- a $5k - 2j + 6j$ b $10ab + 2 - ab$ c $12m^2 - 5m^2 + 2m$
d $12s^2 - 7st - 2s^2$ e $4mn + 7mn - mn$ f $14k + 3 - 5k$
g $y^2 + 2y^2 - y^2$ h $9d - 4e + 3d$ i $7 - 2x - 3$
j $15gb - 8gb + gb$ k $4 - 2q + 5$ l $5a + 6c - 4c$

- 6 Simplify each expression.

- a $4x - y - x - 2y$ b $7ab + 2bc - 3ab + bc$
c $4k^2 + 3 + 5k^2 + 8$ d $4bc - 8a + 2bc - a$

Worked solutions

Exercise 3-05

MAT08NAWS10020

e $x^2 - 6x - x + 7$

g $3 + 4y + 8y + 11$

i $7g + 8b - b - g$

k $20a + 11m + 32a - 11m$

m $5 - 2y - 5 - 4y$

f $p^2 + q^2 + 3p^2 - q^2$

h $11r - 3s - 4r + 6s$

j $6x^2 - 4x - 9x + 6$

l $3q - 7 - 8q + 12q$

n $s^2 + 4s^2 + 12 - 5s^2$

- 7 Which expression is
- $a^5 + a^5$
- simplified? Select the correct answer A, B, C or D.

A a^{10}

B a^{25}

C $2a^5$

D $2a^{10}$

- 8 Write an algebraic expression involving a sum that simplifies to:

a $2x + 6$

b $6m + 12p$

c $x^2 - x$

- 9 Write an algebraic expression involving a difference that simplifies to:

a $2x + 6$

b $6m + 12p$

c $x^2 - x$

3-06 Multiplying terms

What is the fastest way to mentally calculate the product $6 \times 4 \times 5 \times 3$?

Do you think of rearranging the numbers to pair convenient factors like this?

$$\begin{aligned} 6 \times 4 \times 5 \times 3 &= (4 \times 5) \times (6 \times 3) \\ &= 20 \times 18 \\ &= 360 \end{aligned}$$

We can do the same thing algebraically. As numbers can be multiplied in any order, we can multiply like *and* unlike terms.

Example 11

Simplify each product.

a $7b \times 3a$

b $8x \times (-2y)$

c $5d \times 6d \times 4$

d $9m^2 \times 4p$

e $4a \times 3b \times (-2a)$

Solution

$$\begin{aligned} \text{a } 7b \times 3a &= 7 \times b \times 3 \times a \\ &= 7 \times 3 \times a \times b \\ &= 21ab \end{aligned}$$

Group numbers and variables separately.

$$\begin{aligned} \text{b } 8x \times (-2y) &= 8 \times (-2) \times x \times y \\ &= -16xy \end{aligned}$$

$$\begin{aligned} \text{c } 5d \times 6d \times 4 &= 5 \times 6 \times 4 \times d \times d \\ &= 120d^2 \end{aligned}$$

$$\begin{aligned} \text{d } 9m^2 \times 4p &= 9 \times 4 \times m^2 \times p \\ &= 36m^2p \end{aligned}$$

$$\begin{aligned} \text{e } 4a \times 3b \times (-2a) &= 4 \times 3 \times (-2) \times a \times a \times b \\ &= -24a^2b \end{aligned}$$

Summary

When **multiplying algebraic terms**, multiply the numbers first, then multiply the variables. Write all variables in alphabetical order.

Example 12

Use index notation to simplify each expression.

a $k \times k \times k \times k$

b $j \times j \times j \times j \times j \times j$

Solution

a $k \times k \times k \times k = k^4$

b $j \times j \times j \times j \times j \times j = j^6$

Exercise 3-06 Multiplying terms

- 1 Which expression is $2p \times 5pq$ simplified? Select the correct answer **A**, **B**, **C** or **D**.

A $10pq$

B $7p^2q$

C $7pq$

D $10p^2q$

See Example 11

- 2 Simplify each expression.

a $5 \times 3y$

b $8 \times 2m$

c $4k \times m$

d $4t \times 5$

e $2 \times f \times 6$

f $2x \times 8y$

g $10p \times 3m$

h $3b \times 6d$

i $3d \times 4d$

j $5 \times 4m \times 6n$

k $2y \times 3y \times 8$

l $3r \times 3 \times 3r$

m $2 \times 4a \times 9b$

n $x \times xy \times y$

o $2j \times 3k \times 7$

p $5rst \times 2rs$

q $3ac \times 6cd$

r $hjk \times hj \times jk$

s $4q^2 \times 6p^2$

t $2m \times 3n \times mn$

- 3 Simplify each expression.

a $-4b \times 2d$

b $6m \times (-4m)$

c $-2 \times 5k \times 9l$

d $20 \times (-3p)$

e $9k \times (-7)$

f $-15b \times (-5b)$

g $-3m \times mn \times 4n$

h $6abc \times (-2ab)$

i $5r \times (-2r) \times (-4)$

j $9d \times (-4e) \times 6d$

k $-8x \times 9x \times (-2)$

l $5m \times 4 \times (-2m)$

Worked solutions

Exercise 3-06

MAT08NAWS10021

- 4 Which expression simplifies to $14a^2n^2$? Select the correct answer **A**, **B**, **C** or **D**.

A $2a \times an \times 7$

B $2a \times 7n \times an$

C $7a^2 \times 2n$

D $7a^2n^2 \times 2a^2n^2$

- 5 Simplify each expression.

a $y \times y \times y \times y$

b $a \times a \times a$

c $2 \times c \times c$

d $t \times t \times t \times t \times t$

e $q \times q \times q \times 7$

f $k \times k \times k \times l \times l$

g $m \times m \times m \times n \times n$

h $-5 \times p \times p \times p \times p \times k \times k \times k$

i $-3 \times y \times y \times y$

j $d \times d \times (-1) \times e \times e \times f$

k $q \times q \times r \times r \times r \times r \times 4$

l $3p + 2 \times 4p$

m $10m \times 4n - 5mn$

n $7a^2 - 2a \times 3a$

o $6r \times 2y - y \times 10r$

p $5p - 4p \times 8$

- 6 Write an algebraic expression involving a product that simplifies to:

a $18de$

b $-20a^2y$

c $5pr^2$

See Example 12

3-07 Dividing terms

Summary

When **dividing algebraic terms**, divide the numbers first, then divide the variables. Write all variables in alphabetical order.

Worksheet

Simplifying algebraic expressions

MAT08NAWK10024

Homework sheet

Simplifying algebraic expressions

MAT08NAHS10017

Example 13

Simplify each quotient.

a $20p \div 5$

b $\frac{24y}{8y}$

c $\frac{-2m}{4mn}$

d $12a^2b \div 3a$

Solution

$$\begin{aligned} \text{a } 20p \div 5 &= \left(\frac{20}{5}\right)p \\ &= 4p \end{aligned}$$

Group numbers and variables separately.

$$\begin{aligned} \text{b } \frac{24y}{8y} &= \left(\frac{24}{8}\right) \frac{\cancel{y}}{\cancel{y}} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{-2m}{4mn} &= \left(\frac{-2}{4}\right) \frac{\cancel{m}}{\cancel{m}n} \\ &= \left(\frac{-1}{2}\right) \frac{1}{n} \\ &= -\frac{1}{2n} \end{aligned}$$

$$\begin{aligned} \text{d } 12a^2b \div 3a &= \left(\frac{12}{3}\right) \frac{\cancel{a}^2b}{\cancel{a}} \\ &= 4ab \end{aligned}$$

Exercise 3-07 Dividing terms

See Example 13 1 Which expression is $\frac{55m^2n}{5m}$ simplified? Select the correct answer A, B, C or D.

A $11mn$

B $11m^2n$

C $50mn$

D $11n$

2 Simplify each quotient.

a $12m \div 3$

b $36e \div 4$

c $21p \div (-3)$

d $\frac{81x}{9}$

e $\frac{60w}{15}$

f $\frac{8m}{m}$

g $-25k \div 5k$

h $30y \div 40y$

i $28pq \div 14q$

j $\frac{63a}{-9a}$

k $\frac{4pq}{20q}$

l $\frac{-56mn}{8m^2}$

3 Simplify each quotient.

a $14m^2 \div 7$	b $6e^2 \div (-2e)$	c $-5p^2 \div p$	d $\frac{14x^2}{2x}$
e $\frac{6wm}{-3m}$	f $\frac{8m^2n}{2mn}$	g $32k^2y \div (-4k)$	h $9jk^2 \div 12jk^2$
i $\frac{15d^2e^2}{3de}$	j $\frac{3a^2b^2c}{3ab^2c}$	k $\frac{-24acb}{3bc}$	l $\frac{-10m^2n}{5mn}$

4 Which expression is $12x \div (-3) \times 2x$ simplified? Select the correct answer A, B, C or D.

A $-2x$ B -2 C $-8x$ D $-8x^2$

5 Simplify each expression.

a $-18pq \div 6pq$	b $48p \div (-8) \div 2p$	c $y \times 3y \times (-5y)$
d $9d \times 4e \div 6d$	e $8x \times 9x \div 12$	f $48m^2 \div 12m \times 2n$
g $\frac{15c}{5c}$	h $\frac{a \times 3ab}{ab}$	i $\frac{2ab}{10bc}$
j $\frac{3d \times 4e}{6e}$	k $\frac{15mn^2p}{3m^2n^2}$	l $\frac{-4x^2y \times 3y}{2xy \times (-6xy)}$
m $6x + 4x \div 2$	n $14r \div r + 9r$	o $24p - \frac{18p^2}{3p}$

6 Write an algebraic expression involving a quotient that simplifies to:

a $7m$ b $-3ab$ c $5pr^2$

Worked solutions

Exercise 3-07

MAT08NAWS10022

Investigation: Equivalent expressions

1 State whether each statement below is true or false, then check by substituting a number for the variable and testing whether the left-hand side equals the right-hand side.

a $6 + 4m + 4 = 4m + 10$	b $5x + 3x = 8x^2$	c $4k - 7k = -3k$
d $7g + 8h - h - g = 7h + 6g$	e $3 + 7n + 13 + 2n = 25n$	f $3p \times 7q = 21pq$
g $21m \div 3m = 7m$	h $5y - 4y = y$	i $\frac{-16a}{4} = -4a$

3-08 Extension: The index laws

Puzzle sheet

Indices puzzle

MAT08NAPS10011

In Chapter 2, Working with numbers, we discovered properties about powers, called **index laws**. Study these examples.

- $2^3 \times 2^4 = 2^{3+4} = 2^7$
- $7^5 \times 7 = 7^{5+1} = 7^6$
- $4^6 \div 4^2 = 4^{6-2} = 4^4$
- $\frac{10^3}{10^2} = 10^{3-2} = 10^1$

Summary

When **multiplying powers** with the same base, **add** the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing powers** with the same base, **subtract** the powers.

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

Study these examples.

- $(3^4)^2 = 3^{4 \times 2} = 3^8$
- $(8^5)^3 = 8^{5 \times 3} = 8^{15}$
- $5^0 = 1$
- $8^0 = 1$

Summary

When **raising a term with a power** to another power, **multiply** the powers.

$$(a^m)^n = a^{m \times n}$$

Any number raised to the **power of zero** equals **1**.

$$a^0 = 1$$

Now we can apply these index laws algebraically to variables.

Example 14

Simplify each expression.

a $p^6 \times p^2$

b $3m^2n \times 7m^3n^5$

c $a^{12} \div a^4$

d $\frac{42y^6}{7y^2}$

e $(d^4)^5$

f $(4m^2)^3$

g $2k^0$

h $(2k)^0$

Solution

a $p^6 \times p^2 = p^{6+2}$
 $= p^8$

adding the powers

b $3m^2n \times 7m^3n^5 = 3 \times 7 \times m^{2+3} \times n^{1+5}$
 $= 21m^5n^6$

c $a^{12} \div a^4 = a^{12-4}$
 $= a^8$

subtracting the powers

d $\frac{42y^6}{7y^2} = \frac{42}{7} \times y^{6-2}$
 $= 6y^4$

e $(d^4)^5 = d^{4 \times 5} = d^{20}$

f $(4m^2)^3 = 4^3 \times (m^2)^3$
 $= 64m^{2 \times 3}$
 $= 64m^6$

g $2k^0 = 2 \times k^0$
 $= 2 \times 1$
 $= 2$

h $(2k)^0 = 1$

multiplying the powers

because $(4m^2)^3 = 4m^2 \times 4m^2 \times 4m^2$

the zero power belongs to k only

the zero power belongs to $(2k)$

Exercise 3-08 The index laws

- 1 Which expression is $a^2b \times a^4b^3$ simplified? Select the correct answer A, B, C or D.

A a^8b^3 B a^8b^4 C a^6b^3 D a^6b^4

- 2 Simplify each product.

a $m^5 \times m^2$ b $k^8 \times k$ c $y^4 \times y^6$ d $x^7 \times x$
e $n^4 \times n^4$ f $q^2 \times q^8$ g $b^4 \times b^{10} \times b^2$ h $s^3 \times s^5 \times s^7$
i $6k^3 \times 7k^3$ j $5y^3 \times 4y$ k $d^4 \times 3d^5$ l $11e^3 \times 5e$
m $h^4n^2 \times h^6n^5$ n $10c^3d^4 \times (-3c^3d)$ o $f^5g^4 \times f^3g^7$ p $-5p^3q \times 2p^4q$

- 3 Which expression is $\frac{36d^6e^4}{12d^3e^4}$ simplified? Select the correct answer A, B, C or D.

A $3d^3$ B $3d^3e$ C $3d^2e$ D $3d^2$

- 4 Simplify each quotient.

a $y^6 \div y^5$ b $p^4 \div p$ c $\frac{q^5}{q^2}$ d $n^8 \div n^3$
e $\frac{k^{11}}{k^9}$ f $x^7 \div x^5$ g $r^8 \div r^8$ h $m^{12} \div m^5 \div m^2$
i $q^5 \times q^3 \div q^7$ j $45g^6 \div 5g^2$ k $18p^7 \div 6p^3$ l $a^{12} \div a^6 \div a$
m $66x^4y^2 \div 6x^3y$ n $\frac{x^4 \times x^2}{x^3}$ o $\frac{70k^5l^6}{10k^2l}$ p $\frac{15e^3f}{21ef^2}$

- 5 Which expression is $(a^4b^2)^3$ simplified? Select the correct answer A, B, C or D.

A a^7b^5 B $a^{12}b^6$ C $a^{12}b^5$ D a^7b^6

- 6 Simplify each expression.

a $(y^8)^2$ b $(x^2)^5$ c $(y^3)^3$ d $(m^2)^9$
e $(k^7)^3$ f $(t^{20})^2$ g $(3m)^2$ h $(2m^4)^3$
i $(xy)^3$ j $(p^3q)^4$ k $(3h^4)^4$ l $(k^3z)^5$
m $(2kj)^4$ n $(4c^3d)^2$ o $(5x^2y^8)^3$ p $(2m^4p^2)^6$

- 7 Simplify each expression.

a y^0 b $(xy)^0$ c $(2r - r)^0$ d $5u^0$
e $\left(\frac{x}{2}\right)^0$ f $(2k^7)^0$ g $m^8 \div m^8$ h $h^2 \div (h^0)^3$
i $(b^3)^2 \div 2b^0$ j $(3x)^0 \times (4x)^2$ k $(d^0)^5 \times (d^8)^0$ l $26x^4y^3 \div 2x^3y^3$
m $4m^0 + 3m - 3k^0$ n $\frac{27b^3d}{9bd}$ o $(8h)^2 \times 2h^0 \div 4h^2$ p $\frac{6ef^4}{12ef^4}$

See Example 14

Worked solutions

Exercise 3-08

MAT08NAWS10023

Mental skills 3B

Maths without calculators

Multiplying by 9, 11, 99 or 101

We can use expanding when multiplying by a number near 10 or near 100.

1 Study each example.

$$\begin{aligned}\mathbf{a} \quad 25 \times 11 &= 25 \times (10 + 1) \\ &= 25 \times 10 + 25 \times 1 \\ &= 250 + 25 \\ &= 275\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 14 \times 9 &= 14 \times (10 - 1) \\ &= 14 \times 10 - 14 \times 1 \\ &= 140 - 14 \\ &= 126\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 32 \times 12 &= 32 \times (10 + 2) \\ &= 32 \times 10 + 32 \times 2 \\ &= 320 + 64 \\ &= 384\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad 7 \times 99 &= 7 \times (100 - 1) \\ &= 7 \times 100 - 7 \times 1 \\ &= 700 - 7 \\ &= 693\end{aligned}$$

$$\begin{aligned}\mathbf{e} \quad 27 \times 101 &= 27 \times (100 + 1) \\ &= 27 \times 100 + 27 \times 1 \\ &= 2700 + 27 \\ &= 2727\end{aligned}$$

$$\begin{aligned}\mathbf{f} \quad 18 \times 8 &= 18 \times (10 - 2) \\ &= 18 \times 10 - 18 \times 2 \\ &= 180 - 36 \\ &= 144\end{aligned}$$

2 Now evaluate each product.

$$\mathbf{a} \quad 16 \times 11$$

$$\mathbf{b} \quad 33 \times 11$$

$$\mathbf{c} \quad 29 \times 9$$

$$\mathbf{d} \quad 45 \times 9$$

$$\mathbf{e} \quad 62 \times 11$$

$$\mathbf{f} \quad 7 \times 101$$

$$\mathbf{g} \quad 18 \times 101$$

$$\mathbf{h} \quad 36 \times 99$$

$$\mathbf{i} \quad 19 \times 8$$

$$\mathbf{j} \quad 45 \times 12$$

$$\mathbf{k} \quad 21 \times 102$$

$$\mathbf{l} \quad 6 \times 98$$

3-09 Expanding expressions

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of two other numbers. Look at these two examples.

$$\begin{aligned}\mathbf{a} \quad 23 \times 12 &= 23 \times (10 + 2) \\ &= (10 + 2) + (10 + 2) + (10 + 2) + \dots && 23 \text{ times} \\ &= 10 + 10 + 10 + \dots + 2 + 2 + 2 + \dots && 23 \text{ times each} \\ &= (23 \times 10) + (23 \times 2) \\ &= 230 + 46 \\ &= 276\end{aligned}$$

$$\text{so } 23 \times (10 + 2) = 23 \times 10 + 23 \times 2$$

Puzzle sheet

Expandominoes

MAT08NAPS00035

Skillsheet

Algebra using diagrams

MAT08NASS10013

Video tutorial

Expanding and simplifying expressions

MAT08NAVT10005

Worksheet

Expanding brackets

MAT08NAWK00057

$$\begin{aligned}
 \text{b } 35 \times 9 &= 35 \times (10 - 1) \\
 &= (10 - 1) + (10 - 1) + (10 - 1) + \dots && 35 \text{ times} \\
 &= 10 + 10 + 10 + \dots - 1 - 1 - 1 - \dots && 35 \text{ times each} \\
 &= (35 \times 10) - (35 \times 1) \\
 &= 350 - 35 \\
 &= 315
 \end{aligned}$$

$$\text{so } 35 \times (10 - 1) = 35 \times 10 - 35 \times 1$$

Puzzle sheet

Expanding

MAT08NAPS00034

Puzzle sheet

Expand the brackets

MAT08NAPS00003

Puzzle sheet

Expand negative
brackets

MAT08NAPS00004

Summary

The distributive law of arithmetic

If a , b and c stand for numbers, then:

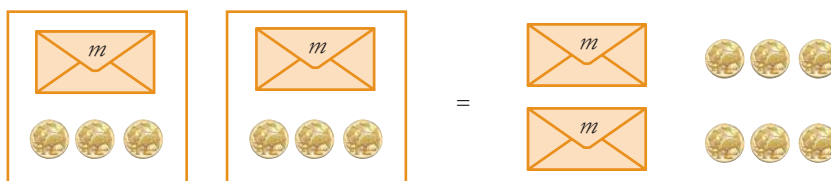
$$a(b + c) = ab + ac$$

$a(b + c)$ means $a \times (b + c)$

$$a(b - c) = ab - ac$$

We can also apply the distributive law algebraically to variables. Suppose an unknown number m is represented by an envelope  that contains m coins.

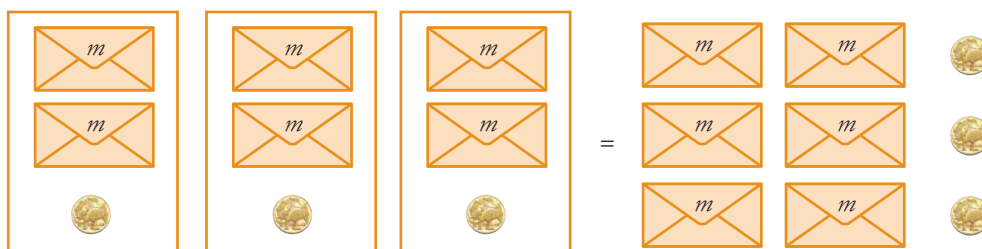
Then $2(m + 3)$ or '2 lots of $(m + 3)$ ', can be represented by this diagram.



It can be seen that $2(m + 3) = 2 \times m + 2 \times 3 = 2m + 6$.

The number outside the brackets is *multiplied* by each term inside.

Here is another example for $3(2m + 1)$.



It can be seen that $3(2m + 1) = 3 \times 2m + 3 \times 1 = 6m + 3$.

Summary

Removing the brackets in an algebraic expression by multiplying each term inside by the term outside is called **expanding** the expression.

Example 15

Expand each expression.

a $7(p + 4)$

d $-2(a + 3)$

b $3(x - 5)$

e $-(4d - 2)$

c $5(2m - 7)$

f $w(9w - 1)$

Solution

$$\begin{aligned} \text{a } 7(p + 4) &= 7 \times p + 7 \times 4 \\ &= 7p + 28 \end{aligned}$$

7 is multiplied by each term inside the brackets separately.

$$\begin{aligned} \text{b } 3(x - 5) &= 3 \times x - 3 \times 5 \\ &= 3x - 15 \end{aligned}$$

$$\begin{aligned} \text{c } 5(2m - 7) &= 5 \times 2m - 5 \times 7 \\ &= 10m - 35 \end{aligned}$$

$$\begin{aligned} \text{d } -2(a + 3) &= -2 \times a + (-2) \times 3 \\ &= -2a + (-6) \\ &= -2a - 6 \end{aligned}$$

-2 is multiplied by each term inside the brackets separately.

$$\begin{aligned} \text{e } -(4d - 2) &= -1(4d - 2) \\ &= -1 \times 4d - 1 \times (-2) \\ &= -4d + 2 \end{aligned}$$

-(4d - 2) is the same as -1 × (4d - 2).

$$\begin{aligned} \text{f } w(9w - 1) &= w \times 9w - w \times 1 \\ &= 9w^2 - w \end{aligned}$$

Example 16

Expand and simplify each expression.

a $2(x + 5) + 6(x - 4)$

b $9k - 4(k - 3)$

c $3p - (p + 6)$

Solution

$$\begin{aligned} \text{a } 2(x + 5) + 6(x - 4) &= 2 \times x + 2 \times 5 + 6 \times x + 6 \times (-4) \\ &= 2x + 10 + 6x - 24 \\ &= 8x - 14 \end{aligned}$$

Expanding.

Collecting like terms.

$$\begin{aligned} \text{b } 9k - 4(k - 3) &= 9k - 4 \times k - 4 \times (-3) \\ &= 9k - 4k + 12 \\ &= 5k + 12 \end{aligned}$$

$$\begin{aligned} \text{c } 3p - (p + 6) &= 3p - 1(p + 6) \\ &= 3p - 1 \times p - 1 \times 6 \\ &= 3p - p - 6 \\ &= 2p - 6 \end{aligned}$$

Exercise 3-09 Expanding expressions

See Example 15

1 Expand each expression.

a $5(m + 3)$	b $8(m - 2)$	c $2(j + 10)$	d $3(a + 2)$
e $4(h - 7)$	f $6(x + y)$	g $9(4 - b)$	h $11(y - 6)$
i $4(2k + 5)$	j $3(5m - 2)$	k $6(3x + 4)$	l $10(2a + 5b)$

2 Expand each expression.

a $-3(n + 4)$	b $-4(v - 5)$	c $-7(d + 3)$	d $-10(k - 2)$
e $-2(d + e)$	f $-5(5 - z)$	g $-6(h + 1)$	h $-(m - 1)$
i $-9(5 + 2x)$	j $-(2f + 3)$	k $-3(u - v)$	d $-8(4 - k)$

3 Which expression is $-3(a - 2b)$ simplified? Select the correct answer A, B, C, or D.

- A $-3a - 2b$ B $-3a + 6b$ C $-3a - 6b$ D $-3a + 5b$

4 Expand each expression.

a $x(y + 6)$	b $p(q - 2)$	c $a(a + 4)$	d $b(3b - 1)$
e $d(d + 3e)$	f $2(4m + 5n)$	g $5k(3k - 2)$	h $7j(3j + ik)$
i $x(4x + y)$	j $-2(4p - 3q)$	k $k(k - 12)$	l $-m(4 - m)$
m $4n(2n + 5m)$	n $2x(1 - 7x)$	o $-f(2f + 4)$	p $-(5j - 7k)$

5 Which expression is $3y(y - 4)$ simplified? Select the correct answer A, B, C, or D.

- A $3y + 12$ B $3y^2 - 7y$ C $3y^2 - 12y$ D $3y + 7$

6 Explain the difference between $2a + 1$ and $2(a + 1)$.

7 Scott says that, to find the perimeter of a rectangle, you double its length, double its width, and add the answers together. Thomas says that you add the length and width first, then double the answer. Who is correct? Can you write each of their answers algebraically?

8 Which expression is $12 - 8(x + 2)$ simplified? Select the correct answer A, B, C, or D.

- A $4(x + 2)$ B $4x + 8$ C $-8x + 14$ D $-8x - 4$

See Example 16

9 Expand and simplify each expression.

a $6(a + 2) + 14$	b $4k + 2(k + 3)$	c $3(q - 3m) + 5m$
d $8 + 6(p - 5)$	e $18 - (2y + 14)$	f $31 - 7(t + 2)$
g $6(2h - 1) + 11$	h $4x - 4(3x - 2)$	i $2(9n - 2) + 8$

10 Check that $8(2b - 3) = 16b - 24$ by substituting any value for b and showing that the answers to both sides are equal.

11 Expand and simplify each expression.

a $3(a + 4) + 2(a + 1)$	b $2(x + 5) + 6(x - 1)$
c $7(4 + k) + 5(2k - 2)$	d $5(2t + 1) + 3(4t + 2)$
e $3(n + 5) - 2(n + 4)$	f $10(d + 2) - 6(d + 3)$
g $8(4y + 1) - 5(2c - 3)$	h $3(p - 4) - 5(p - 1)$
i $m(m + 1) + 3(m + 1)$	j $4(d + e) + 3(d - e)$
k $2f(3f + 4) - 4(2f - 1)$	l $5e(1 - 7e) - e(3e + 4)$

12 Copy and complete each blank.

a $2(\underline{\hspace{1cm}}) = 2x + 10$	b $4(\underline{\hspace{1cm}}) = 4r - 12$	c $3(\underline{\hspace{1cm}}) = 24k - 9$
d $5(\underline{\hspace{1cm}}) = 10a + 35$	e $-2(\underline{\hspace{1cm}}) = -2y - 18$	f $-3(\underline{\hspace{1cm}}) = -6t + 21$

Worked solutions

Exercise 3-09

MAT08NAWS10024

3-10 Factorising algebraic terms

Skillsheet

Algebra using diagrams

MAT08NASS10013

Skillsheet

Factorising using
diagrams

MAT08NASS10014

The **factors** of a number are those numbers that divide into it evenly. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.

Algebraic terms also have factors. The **factors** of an algebraic term are those numbers or terms that divide into it evenly. For example, $10xy = 2 \times 5 \times x \times y$, so *some* of the factors of $10xy$ are 2, 5, x , y , 10, $2y$ and $10x$. Often there are too many to list them all easily.

Example 17

List four factors of $40a^2b$.

Solution

There are many ways of multiplying terms to get $40a^2b$, such as:

$$8 \times 5 \times a \times a \times b$$

$$4 \times 10 \times a^2 \times b$$

$$40 \times a^2 \times b$$

So four factors of $40a^2b$ are: 4, 5, a and a^2b . (Note: It is impractical to list them all.)

The highest common factor of algebraic terms

The **highest common factor (HCF)** (or **greatest common divisor (GCD)**) of two numbers is the largest number that divides into both numbers evenly. For example, the HCF of 18 and 30 is 6.

The **highest common factor (HCF)** of two algebraic terms is the largest term that divides into both terms evenly. For example, the HCF of $18ab$ and $30b$ is $6b$.

Summary

To find the **highest common factor (HCF)** of two or more algebraic terms:

- find the HCF of the numbers
- find the HCF of the variables
- multiply them together to make the highest common algebraic factor.

Example 18

Find the greatest common divisor of each pair of terms.

a $16x$ and $40xy$

b $12r$ and 28

Solution

- a** First, find the HCF of the numbers.

The factors of 16 are 1, 2, 4, 8, 16.

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

So the HCF of 16 and 40 is 8.

Then find the HCF of the variables:

The HCF of x and xy is x .

$\therefore$ The HCF of $16x$ and $40xy$ is $8 \times x = 8x$.

b First, find the HCF of the numbers:

The factors of 12 are 1, 2, 3, 4, 6, 12.

The factors of 28 are 1, 2, 4, 7, 14, 28.

So the HCF of 12 and 28 is 4.

Then find the HCF of the variables:

The second term (28) has no variable so the HCF of the variables is 1.

$\therefore$ The HCF of $12r$ and 28 is $4 \times 1 = 4$.

Exercise 3-10 Factorising algebraic terms

1 List the factors of each number.

a 15

b 8

c 17

d 25

e 55

f 42

2 Find the highest common factor of each pair of numbers.

a 9 and 6

b 6 and 14

c 18 and 24

d 16 and 12

e 33 and 22

f 15 and 25

g 21 and 9

h 45 and 30

3 List four factors of each term.

a $5m$

b $7pq$

c $10x$

d $18d$

e $4k^2$

f $21bc$

g $32x^2y$

h $15ab$

4 Which term is **not** a factor of $36m^2n$? Select the correct answer **A**, **B**, **C** or **D**.

A $4m$

B $9n$

C $8mn$

D $12m^2n$

5 Find the highest common factor of each pair of terms.

a $3xy$ and $12x$

b $4j$ and $24j$

c $6x$ and $4ax$

d $21kj$ and $14k$

e 5 and $10p$

f $16m$ and $12m^2$

g $48cd$ and $32bc$

h de and de^2

i 9 and $27x^2$

j $3m$ and $6n$

k $22m$ and $4mn$

l $16y$ and $4y$

m a^2x and ay

n $64j$ and 32

o $15w^2$ and $20wy$

p $22x^2y$ and $36xy^2$

q cd and ad

r $36t$ and $12ty^2$

6 Which term is the highest common factor of $8x^2$ and $12xy$? Select the correct answer **A**, **B**, **C** or **D**.

A 4

B $2x$

C $4x$

D $24x^2y$

See Example 17

Worked solutions

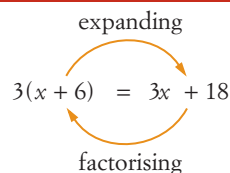
Exercise 3-10

MAT08NAWS10025

3-11 Factorising expressions

Expanding an algebraic expression means removing its brackets by multiplying terms.

Factorising is the opposite of expanding. To factorise an algebraic expression, find the HCF of all terms and insert brackets.



Skillsheet

Algebra using diagrams

MAT08NASS10013

Skillsheet

Factorising using diagrams

MAT08NASS10014

Puzzle sheet

Factorising

MAT08NAPS00036

Summary

To factorise an algebraic expression:

- find the HCF of all terms and write it outside the brackets
- divide each term by the HCF and write them inside the brackets
- $ab + ac = a(b + c)$
- $ab - ac = a(b - c)$

To check that the answer is correct, expand it.

Example 19

Factorise each expression.

a $2x + 6$

b $7 - 7y$

c $3ab - 5b$

d $r^2 + 7rs$

Solution

a $2x + 6 = 2 \times x + 2 \times 3$

$$= 2(x + 3)$$

HCF of $2x$ and 6 is 2 .Think: $2 \times \underline{\quad} = 2x$, $2 \times \underline{\quad} = 6$ Check by expanding: $2(x + 3) = 2x + 6$

b $7 - 7y = 7 \times 1 - 7 \times y$

$$= 7(1 - y)$$

HCF of 7 and $7y$ is 7 .Think: $7 \times \underline{\quad} = 7$, $7 \times \underline{\quad} = 7y$ Check by expanding: $7(1 - y) = 7 - 7y$

c $3ab - 5b = b \times 3a - b \times 5$

$$= b(3a - 5)$$

HCF of $3ab$ and $5b$ is b .Think: $b \times \underline{\quad} = 3ab$, $b \times \underline{\quad} = 5b$ Check by expanding: $b(3a - 5) = 3ab - 5b$

d $r^2 + 7rs = r \times r + r \times 7s$

$$= r(r + 7s)$$

HCF of r^2 and $7rs$ is r .Think: $r \times \underline{\quad} = r^2$, $r \times \underline{\quad} = 7rs$ Check by expanding: $r(r + 7s) = r^2 + 7rs$

Exercise 3-11 Factorising expressions

1 Find the highest common factor of the terms in each expression.

a $10a + 2b$

b $4 - 12x$

c $ax + xy$

d $y^2 - y$

2 Copy and complete each factorisation.

a $2x + 4 = 2(\underline{\quad})$

b $4p - 16 = 4(\underline{\quad})$

c $12k - 15 = 3(\underline{\quad})$

d $5j - 20k = 5(\underline{\quad})$

e $3x^2 + 3y^2 = 3(\underline{\quad})$

f $5x - 15 = \underline{\quad}(x - 3)$

g $12x - 6 = \underline{\quad}(2x - 1)$

h $2a + 8b = \underline{\quad}(a + 4b)$

- i $4pq + 10 = \underline{\hspace{2cm}}(2pq + 5)$ j $5 + 15x = 5(\underline{\hspace{2cm}})$
 k $xy + xz = x(\underline{\hspace{2cm}})$ l $ab - ac = a(\underline{\hspace{2cm}})$
 m $pq - p = p(\underline{\hspace{2cm}})$ n $cd + bc = c(\underline{\hspace{2cm}})$
 o $a^2 + 2a = a(\underline{\hspace{2cm}})$ p $5hk - 8h = \underline{\hspace{2cm}}(5k - 8)$
 q $24mn + 11n = \underline{\hspace{2cm}}(24m + 11)$ r $xy + y^2 = \underline{\hspace{2cm}}(x + y)$
 s $3k^2 + 4jk = \underline{\hspace{2cm}}(3k + 4j)$ t $ef - 2f^2 = \underline{\hspace{2cm}}(e - 2f)$

3 Factorise each expression.

- a $4x + 8$ b $3m + 6$ c $5a - 10$ d $15m - 10a$
 e $8x - 4y$ f $12m - 16w$ g $hk - h$ h $2xy - xm$
 i $12p + 5pr$ j $x^2 + 5x$ k $4m - m^2$ l $y^2 - 11y$

4 Which expression is $16k^2 + 18k$ factorised completely? Select the correct answer A, B, C or D.

- A $2(8k^2 + 9k)$ B $k(16k + 18)$ C $2k(8k + 9)$ D $4k(4k + 9)$

5 Copy and complete each factorisation.

- a $2am + 2an = 2a(\underline{\hspace{2cm}})$ b $6xy - 3x = 3x(\underline{\hspace{2cm}})$ c $4ap + 2a = 2a(\underline{\hspace{2cm}})$
 d $5mn - 5mp = 5m(\underline{\hspace{2cm}})$ e $3xy - 12x = 3x(\underline{\hspace{2cm}})$ f $2pq + 2pr = 2p(\underline{\hspace{2cm}})$
 g $14x - 2xy = 2x(\underline{\hspace{2cm}})$ h $20kj + 5j = 5j(\underline{\hspace{2cm}})$ i $9xy - 12yz = 3y(\underline{\hspace{2cm}})$

6 Factorise each expression.

- a $7ab + 14a$ b $2x - 6xy$ c $3m - 9mw$ d $10xy - 25xw$
 e $8am - 12mp$ f $9xy + 6my$ g $3x + 9xy$ h $ab + abc$
 i $xyz + xya$ j $18m - 24mn$ k $2ac + 3abc$ l $pqr + mpq$
 m $rs + 7r$ n $4s^2 + 16$ o $p^2 - 2p$ p $6k^3 - 2k$
 q $3j + j^2$ r $7n - n^2$ s $7e + 21e^2$ t $6q - 12q^2$
 u $a^4 + a^2$ v $x^2y + xy^2$ w $3m^2n + 9m$ x $12a^2b^2c^2 + 3abc$

Worked solutions

Exercise 3-11

MAT08NAWS10026

3-12 Factorising with negative terms

Summary

When factorising an algebraic expression whose first term is *negative*, include the negative sign in the factor.

$$-ab - ac = -a(b + c)$$

$$-ab + ac = -a(b - c)$$

Example 20

Factorise each expression.

- a $-5x - 10$ b $-9n + 21$ c $-gh - h$ d $-mn + n^2$

Puzzle sheet

Expandominoes

MAT08NAPS00035

Puzzle sheet

Factorising puzzle

MAT08NAPS10010

Worksheet

Algebra review

MAT08NAWK10025

Skillsheet

Algebra using diagrams

MAT08NASS10013

Skillsheet

Factorising using diagrams

MAT08NASS10014

Video tutorial

Factorising expressions

MAT08NAVT10006

Puzzle sheet

Factorising with negative terms

MAT08NAPS00037

Homework sheet

Expanding and factorising expressions

MAT08NAHS10018

Homework sheet

Algebra review

MAT08NAHS10019

Quiz

Algebraic expressions

MAT08NAQZ00009

Solution

$$\mathbf{a} \quad -5x - 10 = -5 \times x + (-5) \times 2$$

$$= -5(x + 2)$$

$$\mathbf{b} \quad -9n + 21 = -3 \times 3n + (-3) \times (-7)$$

$$= -3(3n + (-7))$$

$$= -3(3n - 7)$$

$$\mathbf{c} \quad -gh - h = -h \times g + (-h) \times 1$$

$$= -h(g + 1)$$

$$\mathbf{d} \quad -mn + n^2 = -n \times m + (-n) \times (-n)$$

$$= -n(m + (-n))$$

$$= -n(m - n)$$

HCF of $-5x$ and -10 is -5 .Think: $-5 \times \underline{\quad} = -5x$, $-5 \times \underline{\quad} = -10$ Check by expanding: $-5(x + 2) = -5x - 10$ HCF of $-9n$ and 21 is -3 .Think: $-3 \times \underline{\quad} = -9n$, $-3 \times \underline{\quad} = 21$ Check by expanding: $-3(3n - 7) = -9n + 21$ HCF of $-gh$ and $-h$ is $-h$.Think: $-h \times \underline{\quad} = -gh$, $-h \times \underline{\quad} = -h$ Check by expanding: $-h(g + 1) = -gh - h$ HCF of $-mn$ and n^2 is $-n$.Think: $-n \times \underline{\quad} = -mn$, $-n \times \underline{\quad} = n^2$ Check by expanding: $-n(m - n) = -mn + n^2$

Exercise 3-12 Factorising with negative terms

Worksheet

Algebraic expressions review

MAT08NAWK00059

- 1 Copy and complete each factorisation.

$$\mathbf{a} \quad -6k - 6 = -6(\underline{\quad})$$

$$\mathbf{b} \quad -mn - mp = -m(\underline{\quad})$$

$$\mathbf{c} \quad -5r + 10 = -5(\underline{\quad})$$

$$\mathbf{d} \quad -3k + 27 = -3(\underline{\quad})$$

$$\mathbf{e} \quad -ac + ad = \underline{\quad}(c - d)$$

$$\mathbf{f} \quad -y^2 - 7y = \underline{\quad}(y + 7)$$

$$\mathbf{g} \quad -12y - 30 = \underline{\quad}(2y + 5)$$

$$\mathbf{h} \quad -4f + f^2 = \underline{\quad}(4 - f)$$

See Example 20

- 2 Factorise each expression.

$$\mathbf{a} \quad -4x + 12$$

$$\mathbf{b} \quad -3m - 9$$

$$\mathbf{c} \quad -10x + 20$$

$$\mathbf{d} \quad -15x + 10m$$

$$\mathbf{e} \quad -6x - 8m$$

$$\mathbf{f} \quad -5x - 30$$

$$\mathbf{g} \quad -a - ab$$

$$\mathbf{h} \quad -4xyz + 8abc$$

$$\mathbf{i} \quad -20 + 4x$$

$$\mathbf{j} \quad -dx - dy$$

$$\mathbf{k} \quad -2m + 5mp$$

$$\mathbf{l} \quad -24x - 16y$$

- 3 Which expression is
- $-14wy - 28wz$
- fully factorised? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

$$\mathbf{A} \quad -2(7wy + 14wz)$$

$$\mathbf{B} \quad -14w(y + 2z)$$

$$\mathbf{C} \quad 14w(-y - 2z)$$

$$\mathbf{D} \quad -2w(7y + 14z)$$

Worked solutions

Exercise 3-12

MAT08NAWS10027

- 4 Factorise each expression.

$$\mathbf{a} \quad -16xy - 24xk$$

$$\mathbf{b} \quad -20x + 12xy$$

$$\mathbf{c} \quad -8mw + 12m$$

$$\mathbf{d} \quad 5e - 15ef$$

$$\mathbf{e} \quad -ab - abc$$

$$\mathbf{f} \quad xya + xyw$$

$$\mathbf{g} \quad -12c - 18cw$$

$$\mathbf{h} \quad -2rt + 7rst$$

$$\mathbf{i} \quad 20fh + 5h$$

$$\mathbf{j} \quad -19bw - 38w$$

$$\mathbf{k} \quad -abc + abcd$$

$$\mathbf{l} \quad -abc - abcd$$

- 5
- a**
- Substitute
- $w = 2$
- into the expressions
- $-w^2 + 7w$
- and
- $-w(w - 7)$
- .

$$\mathbf{b} \quad \text{Is } -w^2 + 7w = -w(w - 7) \text{ for any value of } w?$$

- 6 Which expression is
- $-ab + a^2$
- fully factorised? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

$$\mathbf{A} \quad -a(b - a)$$

$$\mathbf{B} \quad a(-b - a)$$

$$\mathbf{C} \quad -a(b + a)$$

$$\mathbf{D} \quad -a(b + a^2)$$

Power plus

1 Simplify each expression.

a $-7p - 8q + 3p - 5q + 4q$

c $3k - 2j - 5 - 4k + 6j + 9$

e $4x^2 + 6x - x^2 - 12x$

g $-d^2 + 7 + 6e^2 - 3d^2 - 2e^2 + 9$

b $-m - 3n - 5m - 7mn$

d $a^2 + 2b^2 + 3b^2 - a^2$

f $xy + yz + xz + 4xy + 3xz + yz$

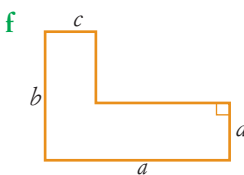
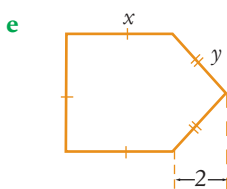
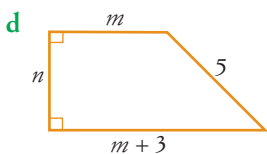
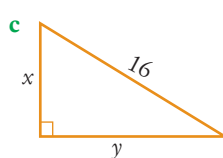
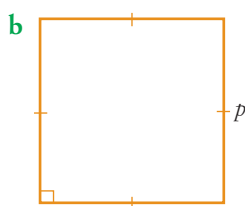
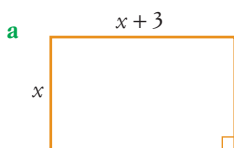
h $5f^2 + 6f + 2 + 8f^2 - 1$

2 For each of these shapes, write an algebraic expression for:

i its perimeter

ii its area.

(Give your answer in simplest form.)



3 Expand and simplify each expression.

a $x(x - 3) + 5(x - 3)$

c $-3(y - 4) + 7(y - 3)$

e $p(p - 3q + 2r) + 2p(4r - 3p - 2q)$

g $12 - 5(4f - 6g - 3)$

i $(x + 3)(x + 5)$

b $m(a + 4) + m(a - 7)$

d $4(2a + b + 3c) - 2(a + 3b - c)$

f $7k^2 + 3(k^2 - 3k - 1) - (6k^2 - 4k + 2)$

h $5(2x^2 - 5x + 3) - 4(3x^2 - 8x - 25)$

j $(p - 1)(p + 10)$

4 Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

a $p + q$

b pq

c $r^2 + t^2$

d $q^2 - 6p + rt$

e $\frac{pq}{rt}$

f $6 - 2r + p$

g $2p + 3q + 4r - 5t$

h $\frac{p+4}{2}$

i $\frac{q+r}{p+t}$

j $\frac{r+7}{5} - \frac{q-t}{5}$

k $\frac{p+3}{10} + \frac{r-2}{2}$

l $q + pr - qt$

5 Factorise each expression.

a $9pq + 5p - 2p^2$

b $44k - 33l - 11m$

c $2x - 7x^2 + 3xy$

d $x(y + 2) + 3(y + 2)$

e $a(b - 3) - 7(b - 3)$

f $4(k + 1) - j(k + 1)$

6 Simplify each expression.

a $\frac{m}{2} + \frac{m}{2}$

b $\frac{7n}{10} - \frac{3n}{10}$

c $\frac{2k}{3} + \frac{k}{3}$

d $\frac{3d}{5} - \frac{d}{5}$

e $\frac{5f}{8} + \frac{7f}{8}$

f $\frac{3c}{10} + \frac{4c}{10}$

g $\frac{4}{x} + \frac{2}{x}$

h $\frac{7}{a} + \frac{5}{b}$

Chapter 3 review

Language of maths

Puzzle sheet

Algebra find-a-word

MAT08NAPS10012

base	expand	highest common factor (HCF)	pronumeral
brackets	expanded form	index laws	substitute
collecting	expression	indices	term
consecutive	factor	like terms	unknown
distributive law	factorise	power	unlike terms
evaluate	formula		variable

- 1 Why does the word 'variable' have that name? Look up its meaning in the dictionary.
- 2 What are **unlike terms**?
- 3 How do you **expand** an algebraic expression?
- 4 What is the name given to the largest algebraic term that divides evenly into two or more given terms?
- 5 What is the opposite of **expanding** an algebraic expression?
- 6 Look up some non-mathematical meanings of the word 'factor'. Use 'factor' in a sentence to show one of these meanings.

Topic overview

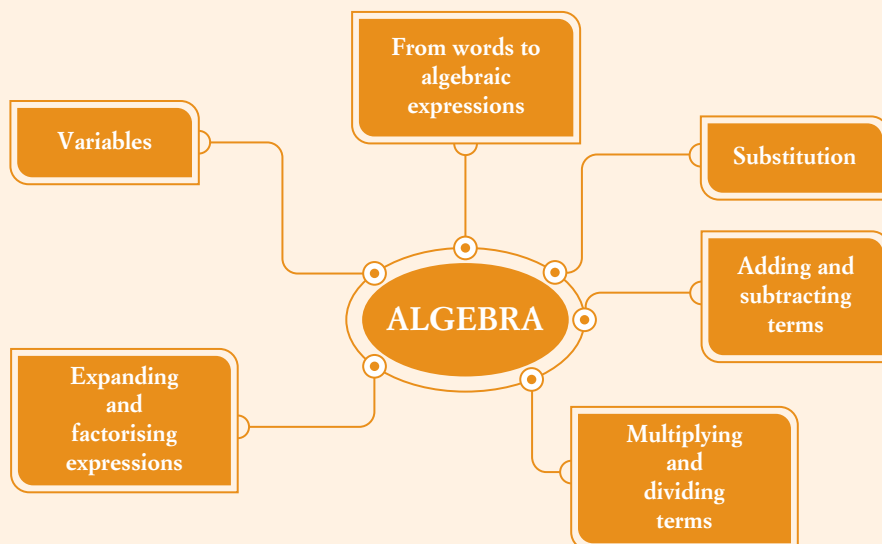
Worksheet

Mind map: Algebra

MAT08NAWK10026

- What parts of algebra did you remember from Year 7? What new rules have you learnt?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give two examples of jobs where algebra is needed.

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



1 Simplify each expression.

See Exercise 3-01

a $p \times 6$	b $r \times (-1)$	c $4 \times d \times g$
d $t + t + t + t + t$	e $14m + 6m$	f $11k - 6k$
g $6 \times y \times y$	h $16 \div a$	i $5 \times c \times 4 \times d$

2 Write each expression in expanded form.

See Exercise 3-01

a cd	b $3y^2$	c $\frac{w}{6}$	d $c^2 + y$

3 Write each statement as an algebraic expression.

See Exercise 3-02

a 4 more than y	b six times b minus 3	c p divided by q

4 If $x = 5$ and $y = -2$, evaluate each expression.

See Exercise 3-03

a $x + y$	b xy	c $3x - 2y$	d $6y^2$
e $6x \div y$	f $x^2 + y^2$		

5 Simplify each expression.

See Exercise 3-05

a $5k + 7k$	b $29d - 12d$	c $4m + 18m - 5m$
d $13m - 3m + 8m$	e $4ac + 4ac - 5ca$	f $11cd + 7cd - 9cd$
g $7x + 3y + 3x + 7y$	h $13r + 4s + 11s + 2r$	i $33h + 6g - 5h + g$

6 Simplify each expression.

See Exercise 3-06

a $5 \times 2x$	b $2b \times 7d$	c $11m \times 6$	d $3p \times 8q$
e $4 \times 2w \times 4w$	f $15j \times 5$	g $3m \times 6m \times 2$	h $4p \times 3p \times 5p$

7 Simplify each expression.

See Exercise 3-07

a $50x \div 2$	b $28b \div 7d$	c $66m \div 6$	d $\frac{32rs}{4}$
e $\frac{15p}{3p}$	f $14d^2 \div 2$	g $14e^2 \div 2e$	h $13b^2 \div b$
i $\frac{18x^2}{9x}$	j $\frac{44m^2n^2}{11m}$	k $\frac{16e^2g}{4eg}$	l $\frac{35a^2e^2}{7ae}$

8 Expand and simplify each expression.

See Exercise 3-09

a $6(n + 2)$	b $-5(2b + 7)$	c $j(9j - 3)$
d $10 + 2(n + 4)$	e $22v - 3(v + 5)$	f $5(y + 2) + 4(y - 7)$

9 Find the highest common factor of each pair of terms.

See Exercise 3-10

a $6a$ and 12	b $3p^2$ and $18p$	c $7t$ and 14

10 Factorise each expression.

See Exercise 3-11

a $3m + 9$	b $5h - 35$	c $6a + 9ab$	d $10h - 12hm$
e $xy + xyz$	f $cd - de$	g $a^2 - 6a$	h $8kj + 24j^2$

11 Factorise each expression.

See Exercise 3-12

a $-2 - 4q$	b $-4f - 16$	c $-12 + 15d$	d $-ab + b$
e $-16b - 24bd$	f $-4k^2 - k$	g $-x^2 + 7x$	h $-h^2i - hi^2$

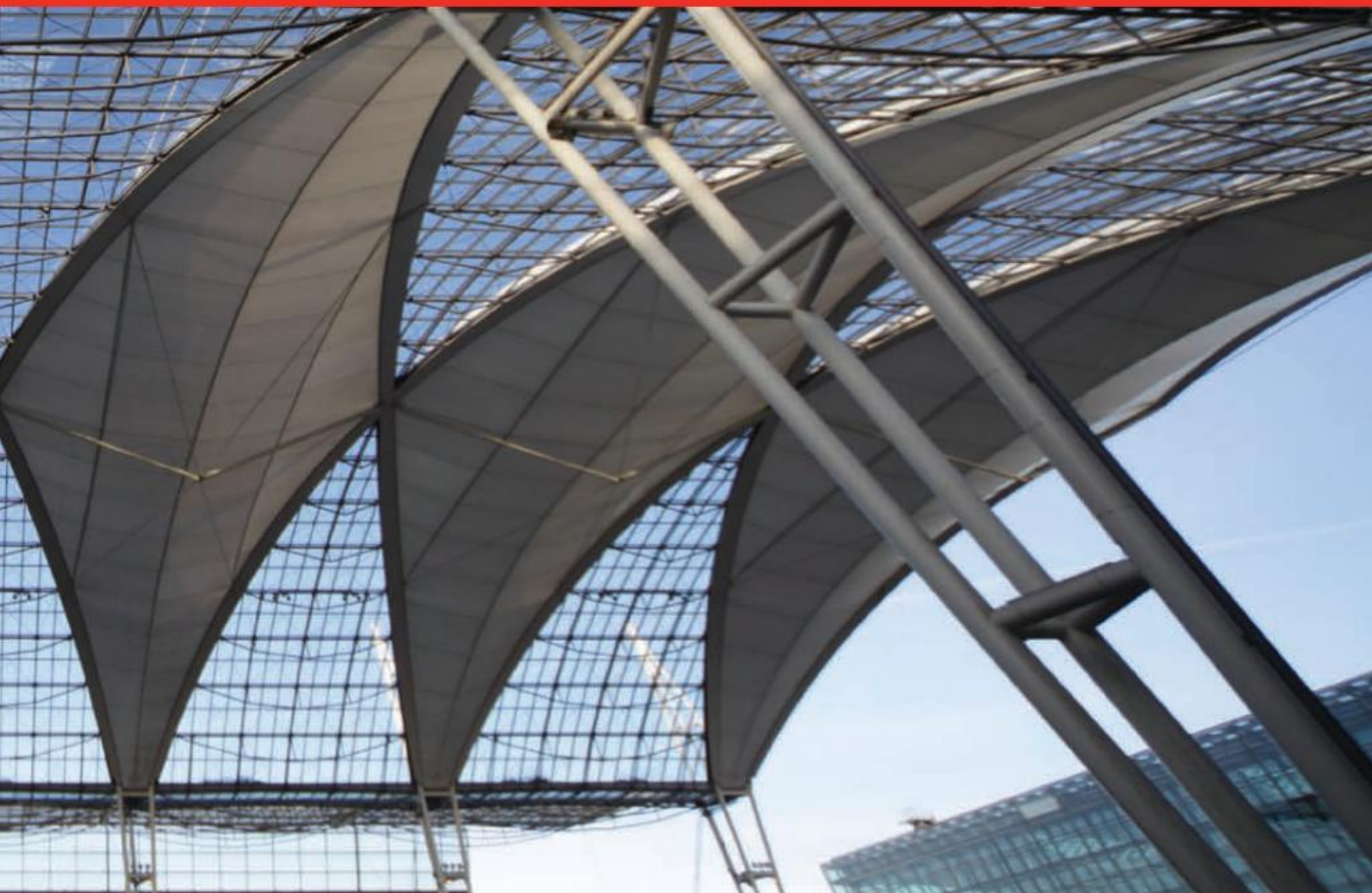


4

Measurement and geometry

Geometry

Geometry is everywhere. Angles, parallel lines, triangles and quadrilaterals can be found all around us, in our homes, on transport, in construction, art and nature. This scene from Munich airport in Germany shows the importance of angles, lines and shapes in architecture and design.



Chapter outline

	Proficiency strands				
4-01 Angle geometry	U	F	PS	R	C
4-02 Angles on parallel lines	U	F	PS	R	C
4-03 Line and rotational symmetry	U	F			C
4-04 Classifying triangles	U	F		R	C
4-05 Classifying quadrilaterals	U	F		R	C
4-06 Properties of quadrilaterals	U	F		R	C
4-07 Angle sums of triangles and quadrilaterals	U	F	PS	R	
4-08 Extension: Angle sum of a polygon	U	F	PS	R	

Wordbank

angle sum The total of the sizes of the angles in a shape, such as a triangle

bisect To cut in half

convex quadrilateral A quadrilateral whose vertices all point outwards.

diagonal An interval joining two non-adjacent vertices of a shape

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

scalene triangle A triangle with no equal sides

supplementary angles Two angles that add to 180°

In this chapter you will:

- investigate angles on a straight line, angles at a point and vertically opposite angles and use results to find unknown angles
- identify corresponding, alternate and co-interior angles when two parallel lines are crossed by a transversal, and the relationships between them
- investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning
- identify line and rotational symmetries
- classify triangles according to their side and angle properties
- distinguish between convex and non-convex quadrilaterals
- describe squares, rectangles, rhombuses, parallelograms, kites and trapeziums
- apply the angle sum of a triangle and quadrilateral and that any exterior angle of a triangle equals the sum of the two interior opposite angles

SkillCheck

Worksheet

StartUp assignment 4

MAT08MGWK10027

- Draw two different examples of each type of angle.

a acute angle	b right angle	c obtuse angle	d reflex angle
- Classify each angle size below as being acute, obtuse, right, reflex, straight or a revolution.

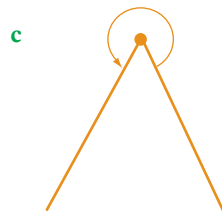
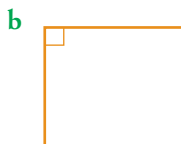
a 25°	b 100°	c 300°	d 128°
e 90°	f 360°	g 286°	h 180°

Skillsheet

Types of angles

MAT08MGSS10015

- Name each type of angle(s) marked.



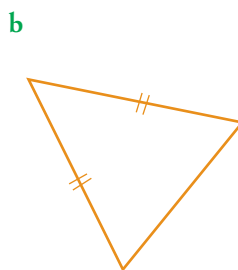
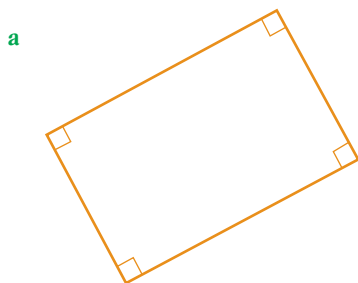
- Find the value of x in each equation.

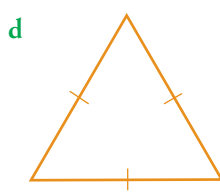
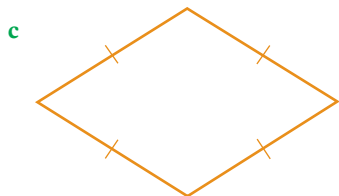
a $x + 30 = 90$

b $x + 57 = 180$

c $x + 121 + 77 = 360$

- Copy each shape, name it and draw all axes of symmetry.





- 6 Write the order of rotational symmetry of each shape in question 5.
- 7 Which quadrilateral has all four sides equal and all four angles equal? Select the correct answer **A**, **B**, **C** or **D**.
- A** parallelogram **B** rhombus **C** rectangle **D** square
- 8 Draw a scalene triangle.
- 9 **a** Draw a parallelogram and its diagonals.
b Are the lengths of the diagonals you drew in part **a** equal?

4-01 Angle geometry

Classifying angles

Right angle	Straight angle	Revolution
90° (quarter-turn) 	180° (half-turn) 	360° (complete turn)
Acute angle	Obtuse angle	Reflex angle
Less than 90° 	Between 90° and 180° 	Between 180° and 360°

Skillsheet

Starting GeoGebra

MAT08MGSS10019

Skillsheet

Starting Geometer's Sketchpad

MAT08MGSS10020

Puzzle sheet

Angles: A dog day

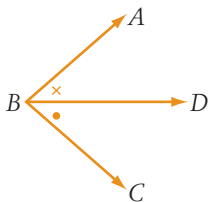
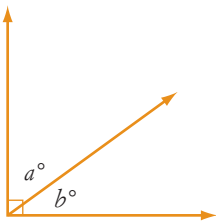
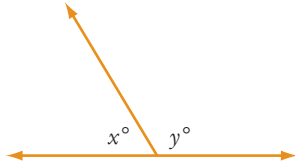
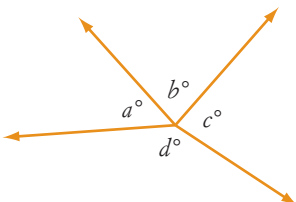
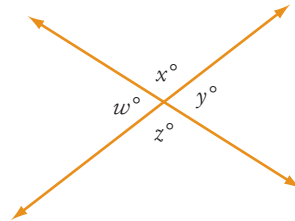
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Worksheet

Straight angles, right angles and revolutions

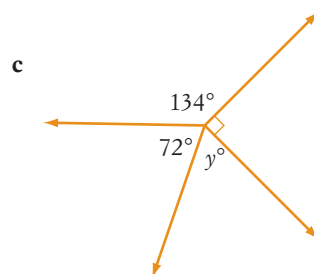
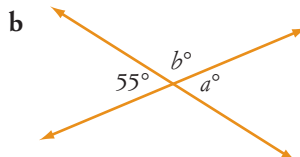
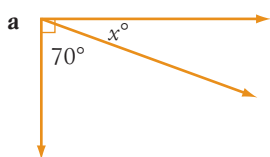
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Angle facts

Adjacent angles	Complementary angles	Angles in a right angle
Angles next to each other, sharing a common arm. $\angle ABD$ and $\angle DBC$ are adjacent.	Angles that have a sum of 90° , for example, 35° and 55° .	Are complementary. $a + b = 90$
	Supplementary angles Angles that have a sum of 180° , for example, 140° and 40° .	
Angles on a straight line	Angles at a point	Vertically opposite angles
Are supplementary. $x + y = 180$	(In a revolution) Add up to 360° . $a + b + c + d = 360$	Are equal. $w = y$ and $x = z$
		

Example 1

Find the value of each pronumeral, giving reasons.



Solution

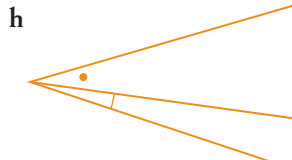
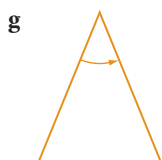
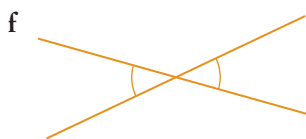
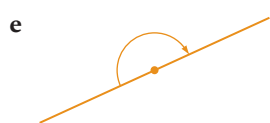
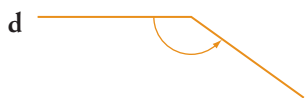
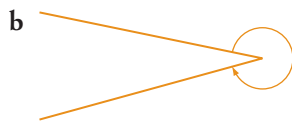
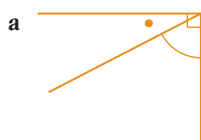
a $x + 70 = 90$ (Angles in a right angle.)
 $x = 90 - 70$
 $= 20$

The reason is written inside brackets.

- b** $a = 55$ (Vertically opposite angles.)
 $b + 55 = 180$ (Angles on a straight line.)
 $b = 180 - 55$
 $= 125$
- c** $y + 72 + 134 + 90 = 360$ (Angles at a point.)
 $y + 296 = 360$
 $y = 360 - 296$
 $= 64$

Exercise 4-01 Angle geometry

- 1 Classify each type of angle(s).



- 2 For each angle size, find the complementary angle.

- a** 16° **b** 33° **c** 71° **d** 2°

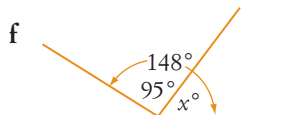
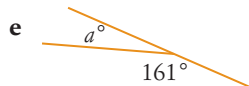
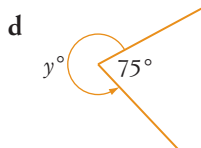
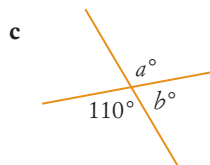
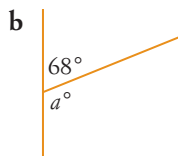
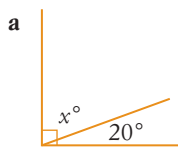
- 3 For each angle size, find the supplementary angle.

- a** 25° **b** 149° **c** 107° **d** 48°

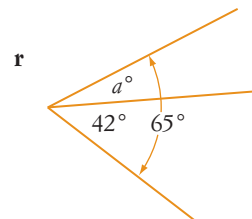
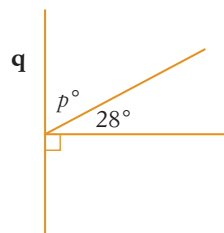
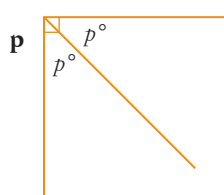
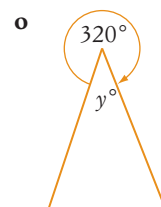
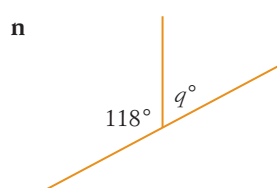
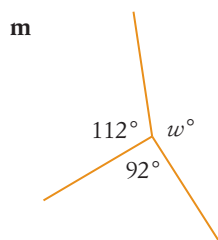
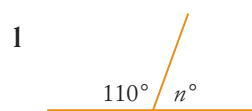
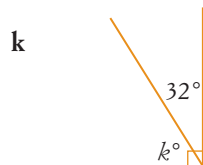
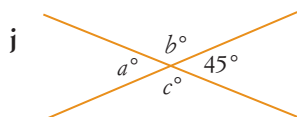
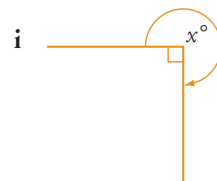
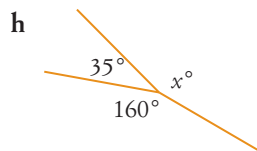
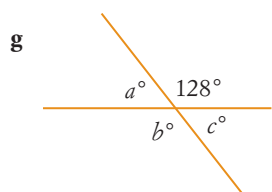
- 4 What is the sum of the angles at a point? Select the correct answer **A**, **B**, **C** or **D**.

- A** 90° **B** 180° **C** 270° **D** 360°

- 5 Find the value of each pronumeral, giving reasons.



See Example 1

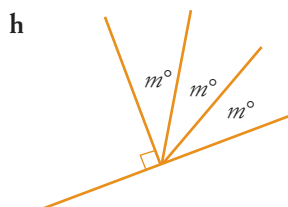
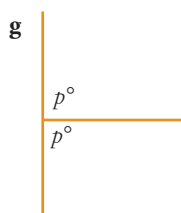
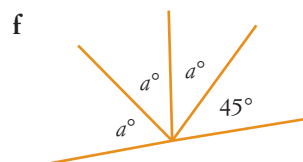
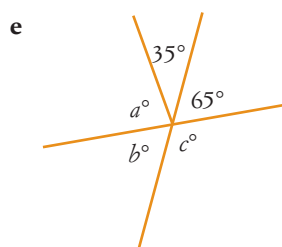
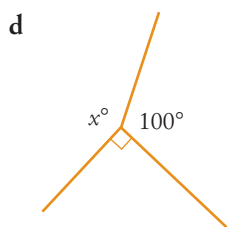
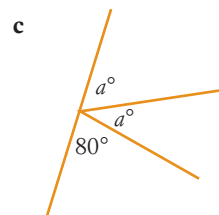
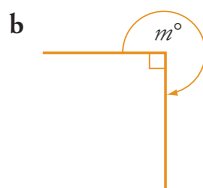
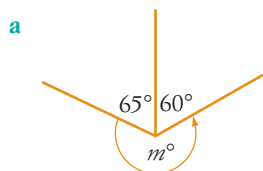


Worked solutions

Exercise 4-01

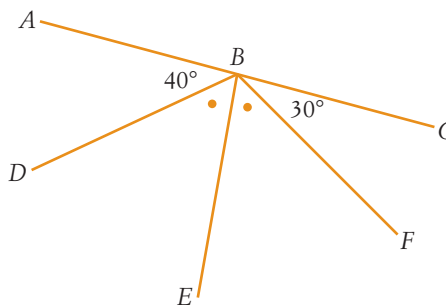
MAT08MGWS10028

6 Find the value of each pronumeral, giving reasons.



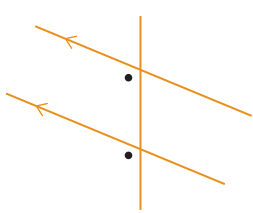
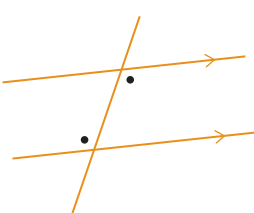
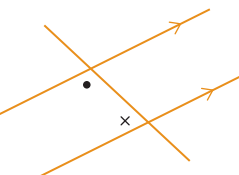
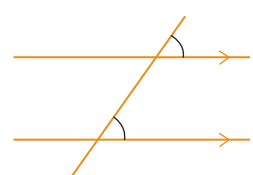
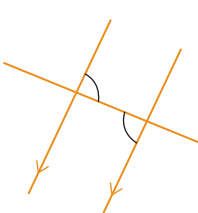
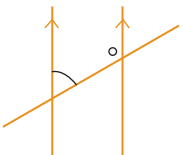
- 7 ABC is a straight line and EB bisects $\angle DBF$.
Which of the following is the size of $\angle EBC$?
Select the correct answer A, B, C or D.

A 95° B 110°
C 85° D 140°



4-02 Angles on parallel lines

When parallel lines are crossed by another line (called a transversal), special pairs of angles are formed.

Corresponding angles	Alternate angles	Co-interior angles
Corresponding angles on parallel lines are equal. 	Alternate angles on parallel lines are equal. 	Co-interior angles on parallel lines are supplementary (add to 180°). 
		

- Corresponding angles are in 'matching' positions on the *same* side of the transversal: 'corresponding' means 'matching'
- Alternate angles are between the parallel lines on *opposite* sides of the transversal: 'alternate' means 'going back and forth in turns'
- Co-interior angles are between the parallel lines on the *same* side of the transversal: 'co-interior' means 'together inside'

Worksheet

Finding the unknown angle 1

MAT08MGWK10028

Skillsheet

Angles and parallel lines

MAT08MGSS10016

Homework sheet

Angle geometry

MAT08MGHS10030

Worksheet

Angles in parallel lines

MAT08MGWK00039

Puzzle sheet

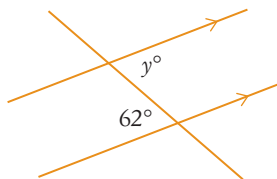
Angles in parallel lines

MAT08MGPS00021

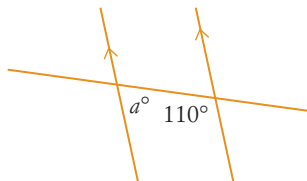
Example 2

Find the value of each pronumeral, giving reasons.

a



b



Solution

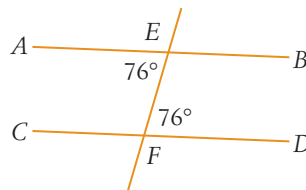
a $y = 62$ (Alternate angles on parallel lines)

b $a + 110 = 180$ (Co-interior angles on parallel lines)

$$\begin{aligned} a &= 180 - 110 \\ &= 70 \end{aligned}$$

Example 3

Prove that the lines AB and CD are parallel.



Solution

$\angle AEF$ and $\angle EFD$ are alternate angles.

$$\angle AEF = \angle EFD = 76^\circ$$

$\therefore AB \parallel CD$

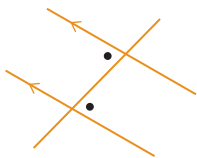
‘ $\therefore AB \parallel CD$ ’ means ‘Therefore line AB is parallel to line CD ’

(Alternate angles are equal)

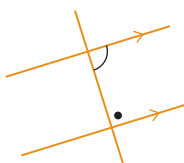
Exercise 4-02 Angles on parallel lines

1 Is each marked pair of angles corresponding, alternate or co-interior?

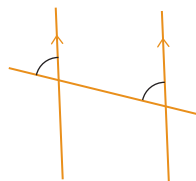
a



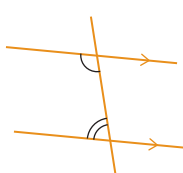
b



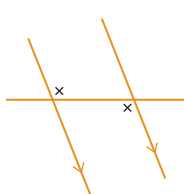
c



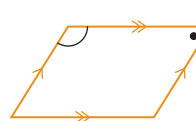
d



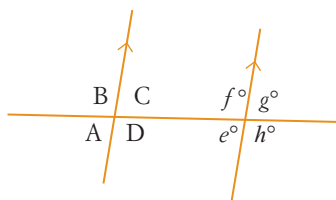
e



f

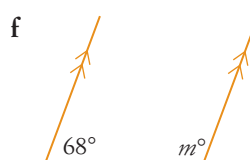
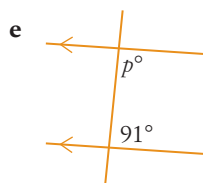
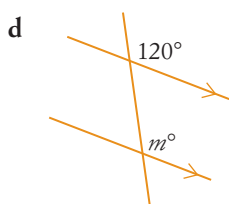
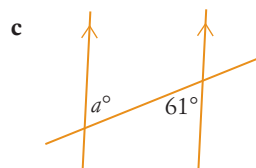
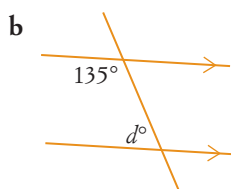
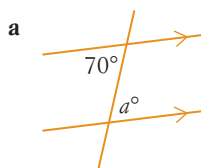


- 2 Which angle is corresponding to the angle marked g° ?
Select A, B, C or D.

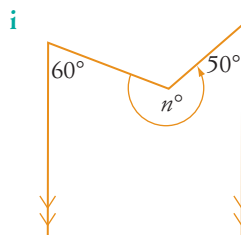
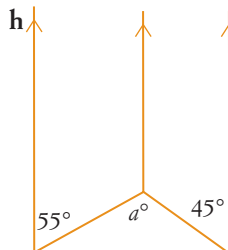
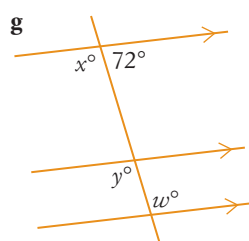
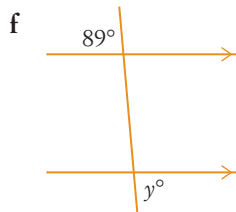
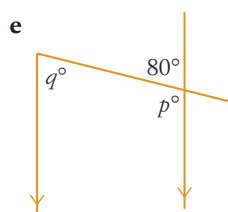
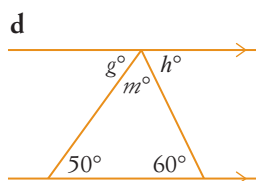
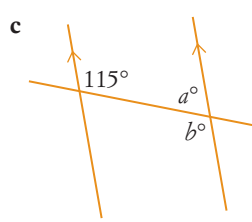
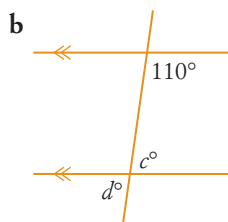
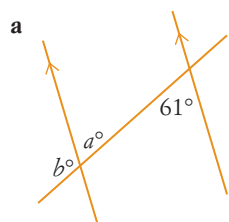


- 3 For the diagram in question 2, name an angle that is:
a co-interior to D
b alternate to the angle marked e°
c equal to C
d corresponding to B
e supplementary to the angle marked e° .
- 4 Find the value of the pronumeral in each diagram, giving reasons for your answers.

See Example 2



- 5 Find the value of each pronumeral, giving reasons.

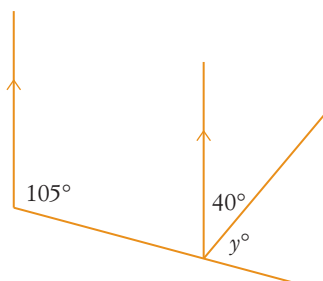


Worked solutions

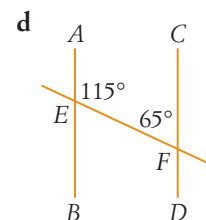
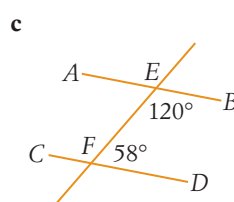
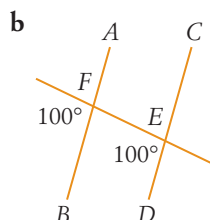
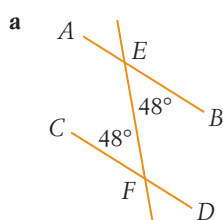
Exercise 4-02

MAT08MGWS10029

- 6 What is the value of y in this diagram?
Select the correct answer **A**, **B**, **C** or **D**.
- A** 85 **B** 40
C 35 **D** 65



See Example 3 7 For each diagram, decide whether AB is parallel to CD . If it is, then prove it.



Worksheet

Symmetry

MAT08MGWK10029

Skillsheet

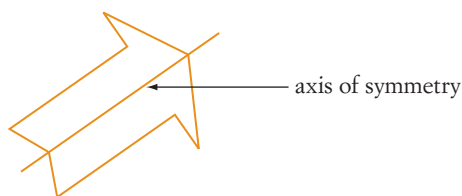
Line and rotational
symmetry

MAT08MGSS10017

4-03 Line and rotational symmetry

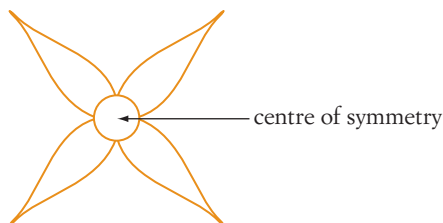
A shape is symmetrical if it looks the same after it has undergone a change of position or a movement. The two types of symmetry are **line symmetry** and **rotational symmetry**.

When a shape with **line symmetry** is folded along a line, called an **axis of symmetry**, the two halves of the shape fit exactly on top of each other. One half is the reflection or mirror-image of the other half.



This shape has one axis of symmetry.

When a shape with **rotational symmetry** is rotated (spun) about a point, called the **centre of symmetry**, it fits exactly on itself at least once before one full revolution (360°). The number of times the shape fits on itself in one revolution is called its **order of rotational symmetry**.



This shape has rotational symmetry of order 4.

Exercise 4-03 Line and rotational symmetry

1 Copy each shape and mark its axes of symmetry.

a



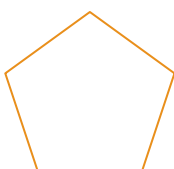
b



c



d



e



f

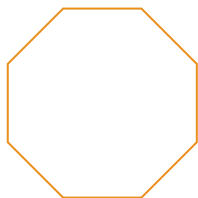


2 Count the number of axes of symmetry of each shape.

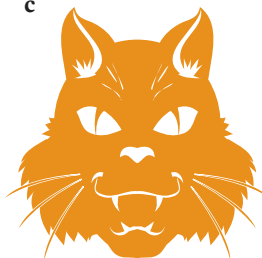
a



b



c



d



3 Decide whether each shape has rotational symmetry. If it does, state the order of rotational symmetry.

a



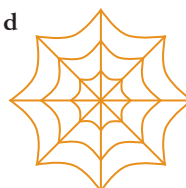
b



c



d



4 a Copy the capital letters below that have line symmetry and draw their axes of symmetry.

D H I K M N O R S V X Z

b Copy the capital letters above that have rotational symmetry and mark their centre of symmetry.

5 Draw a shape or pattern that has:

a one axis of symmetry

b two axes of symmetry

c four axes of symmetry

6 Draw a shape or pattern that has rotational symmetry:

a of order 2

b of order 4

- 7 For each shape:
- find how many axes of symmetry it has
 - state whether it has rotational symmetry and if it does, state the order

a



b



c



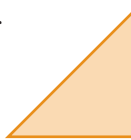
d



e



f



- 8 What shape has:
- an infinite number of axes of symmetry?
 - an infinite order of rotational symmetry?

Worksheet

Properties of triangles

MAT08MGWK10030

Worksheet

Triangle geometry

MAT08MGWK10031

Skillsheet

Naming shapes

MAT08MGSS10018

Homework sheet

Symmetry and triangles

MAT08MGHS10031

Puzzle sheet

Classifying triangles

MAT08MGPS00003

4-04 Classifying triangles

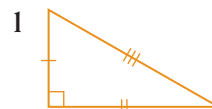
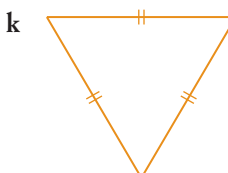
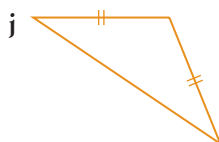
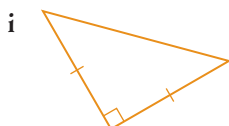
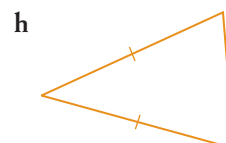
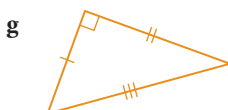
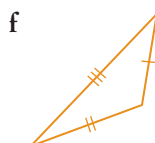
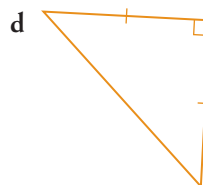
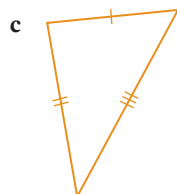
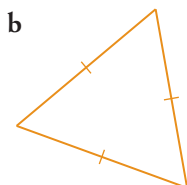
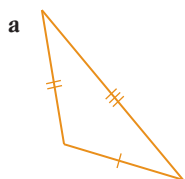
Triangles can be classified in two ways: by their sides or by their angles.

Classifying by sides		
Equilateral triangle	Isosceles triangle	Scalene triangle
Three equal sides (Also three equal angles, each 60°)	Two equal sides (Also two equal angles, opposite the equal sides)	No equal sides (Also no equal angles)

Classifying by angles		
Acute-angled triangle	Obtuse-angled triangle	Right-angled triangle
Three acute angles (less than 90°)	One obtuse angle (between 90° and 180°)	One right angle (90°)

Exercise 4-04 Classifying triangles

1 Classify each triangle according to its sides and angles.



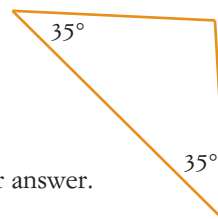
2 Sketch a triangle that is:

- a right-angled and isosceles
c scalene and obtuse-angled
e right-angled and scalene

- b equilateral
d acute-angled and scalene
f acute-angled and isosceles

3 Which of the following describes this triangle when we classify it by sides and angles? Select the correct answer **A**, **B**, **C** or **D**.

- A** isosceles and obtuse-angled **B** isosceles and acute-angled
C scalene and obtuse-angled **D** scalene and acute-angled

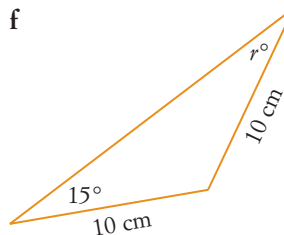
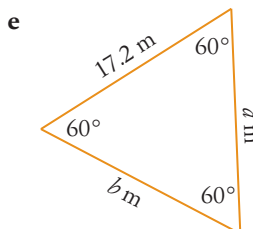
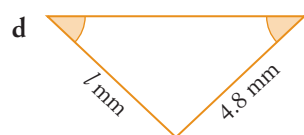
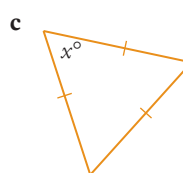
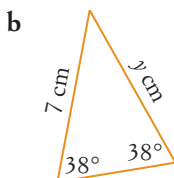
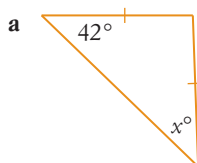


4 Is it possible to draw an obtuse-angled equilateral triangle? Justify your answer.

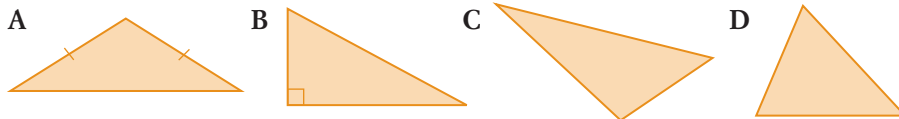
5 Which triangles in question 1 have:

- a line symmetry? b rotational symmetry?

6 Find the value of each pronumeral, giving a reason.



- 7 Is it possible to draw a triangle with two obtuse angles? Why?
- 8 Which triangle is both obtuse-angled and scalene? Select A, B, C or D.



- 9 Copy and complete this table.

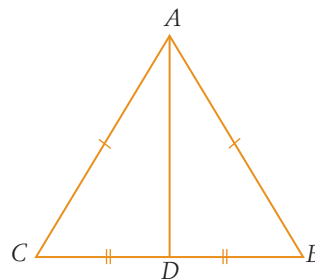
Triangle	Number of axes of symmetry	Order of rotational symmetry
Equilateral triangle		
Isosceles triangle		
Scalene triangle		

Investigation: The perpendicular bisector in an isosceles triangle

$\triangle ABC$ is an isosceles triangle with $AC = AB$.

It has one axis of symmetry, AD .

- Why is $CD = DB$?
- Why is $\angle ADC = \angle ADB$?
- What is the size of $\angle ADC$ and $\angle ADB$?
- AD bisects side CB . What does 'bisect' mean?
- $AD \perp CB$. What does ' $\perp$ ' mean?
- 'In an isosceles triangle, the axis of symmetry is the perpendicular bisector of the uneven side.' Explain what this means in your own words.



Mental skills 4A

Maths without calculators

Converting fractions and decimals to percentages

To convert a fraction or decimal into a percentage, multiply it by 100%.

- 1 Study each example.

a $\frac{2}{5} = \frac{2}{5} \times 100\% = \frac{2}{\cancel{5}_1} \times \cancel{100}^{20}\% = 2 \times 20\% = 40\%$

b $\frac{24}{40} = \frac{24}{40} \times 100\% = \frac{24^3}{\cancel{40}_5} \times 100\% = \frac{3}{\cancel{5}_1} \times \cancel{100}^{20}\% = 3 \times 20\% = 60\%$

2 Now convert each fraction to a percentage.

a $\frac{7}{10}$

b $\frac{33}{50}$

c $\frac{27}{60}$

d $\frac{22}{25}$

e $\frac{24}{32}$

f $\frac{30}{40}$

g $\frac{60}{75}$

h $\frac{4}{5}$

i $\frac{11}{20}$

j $\frac{28}{80}$

k $\frac{15}{50}$

l $\frac{16}{20}$

m $\frac{54}{60}$

n $\frac{18}{40}$

o $\frac{13}{25}$

3 Study each example.

a $0.41 = 0.41 \times 100\%$
 $= 0.41$
 $= 41\%$

b $0.08 = 0.08 \times 100\%$
 $= 0.08$
 $= 8\%$

c $0.9 = 0.9 \times 100\%$
 $= 0.90$
 $= 90\%$

d $0.375 = 0.375 \times 100\%$
 $= 0.375$
 $= 37.5\%$

4 Now convert each decimal to a percentage.

a 0.25

b 0.68

c 0.17

d 0.6

e 0.1

f 0.333

g 0.59

h 0.702

i 0.84

j 0.7

k 0.428

l 0.055

m 0.91

n 0.7825

o 0.314

Just for the record

It's all Greek or Latin to me!

Many of our words in geometry come from Greek or Latin. Latin was the language of the ancient Roman Empire.

Word	Origin	Meaning
Equilateral	Latin: <i>aequus latus</i>	Equal sides
Equiangular	Latin: <i>aequus angulus</i>	Equal corners
Isosceles	Greek: <i>isos skelos</i>	Equal legs
Scalene	Greek: <i>skalenos</i>	Uneven leg
Acute	Latin: <i>acutus</i>	Sharp
Obtuse	Latin: <i>obtusus</i>	Dull or blunt
Reflex	Latin: <i>reflexus</i>	Bent back
Triangle	Latin: <i>tri angulus</i>	Three corners
Rectangle	Latin: <i>rectus angulus</i>	Right corners
Quadrilateral	Latin: <i>quadri latus</i>	Four sides
Polygon	Greek: <i>poly gonon</i>	Many angles
Diagonal	Greek: <i>dia gonios</i>	From angle to angle
Trapezium/Trapezoid	Latin/Greek: <i>trapeza</i>	Small table

Explain what this sentence means, and illustrate with a diagram: 'A rhombus is equilateral but not equiangular'.

Worksheet

Properties of
quadrilaterals

MAT08MGWK10032

4-05 Classifying quadrilaterals

A quadrilateral is any shape with four straight sides. A quadrilateral may be convex or non-convex.

Worksheet

Classifying
quadrilaterals

MAT08MGWK10033

Worksheet

Always, sometimes,
never true

MAT08MGWK10034

Puzzle sheet

What shape am I?

MAT08MGPS10013

Video tutorial

Classifying
quadrilaterals

MAT08MGVT00002

Puzzle sheet

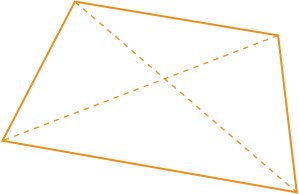
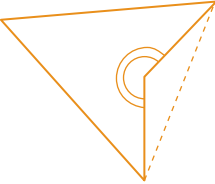
Singing in the car

MAT08MGPS00004

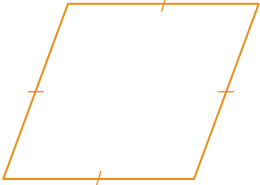
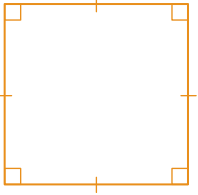
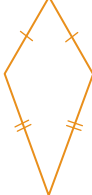
Worksheet

Classifying triangles
and quadrilaterals

MAT08MGWK00010

Convex quadrilateral	Non-convex quadrilateral
	
• All vertices (corners) point outwards. • All diagonals lie within the shape. • All angles are less than 180°.	• One vertex points inwards. • One diagonal lies outside the shape. • One angle is more than 180° (reflex angle).

There are six special types of quadrilaterals.

Trapezium	Parallelogram	Rectangle
		
One pair of parallel sides	Two pairs of parallel sides	Four right angles
Rhombus	Square	Kite
		
Four equal sides	Four equal sides and four right angles	Two pairs of equal adjacent sides
		← 'Adjacent' means 'next to each other'.

Exercise 4-05 Classifying quadrilaterals

1 Is each quadrilateral convex or non-convex?

a

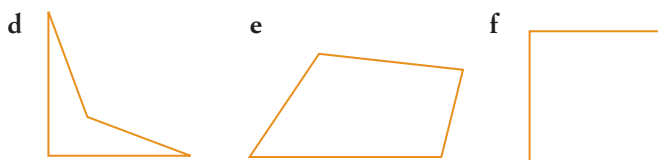


b

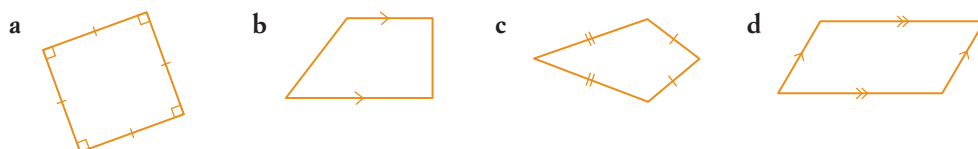


c





2 Name each quadrilateral.



3 Name all the quadrilaterals that have:

- a four right angles b exactly one pair of parallel sides
c four equal sides d opposite sides equal
e opposite sides parallel f two pairs of equal adjacent sides

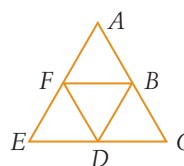
4 Which quadrilateral is also called a 'diamond'?

5 Copy and complete this table.

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Rectangle		
Parallelogram		
Trapezium		
Rhombus		
Square		
Kite		

6 In the diagram on the right, what type of quadrilateral is:

- a $ACDF$? b $FBCD$?



7 A parallelogram is any quadrilateral with both pairs of opposite sides parallel. Which of the following is **not** a special type of parallelogram? Select the correct answer **A**, **B**, **C** or **D**.

- A** square **B** kite **C** rectangle **D** rhombus

8 Sketch each of these quadrilaterals, showing its main features.

- a rectangle b trapezium c rhombus d kite

9 Which of the following statements are always true? (Explain your answers)

- a A rhombus is a square.
b A square is a rhombus.
c A rectangle is a parallelogram.
d A parallelogram is a quadrilateral with its opposite sides parallel and equal.
e The diagonals of a parallelogram meet at right angles.
f A square is a rectangle.
g A rectangle is a square.

Technology Properties of quadrilaterals

Technology

GeoGebra: Making quadrilaterals

MAT08MGTC00005

Technology

GeoGebra: Quadrilateral sides and angles

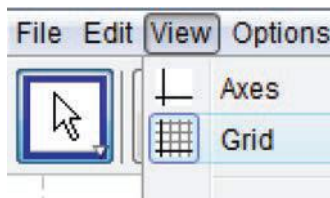
MAT08MGTC00002

Technology

GeoGebra: Quadrilateral diagonals

MAT08MGTC00003

This activity will use GeoGebra to construct quadrilaterals. For each quadrilateral, set up your drawing page by clicking **View**. Make sure that **Grid** is ticked and **Axes** is not ticked.

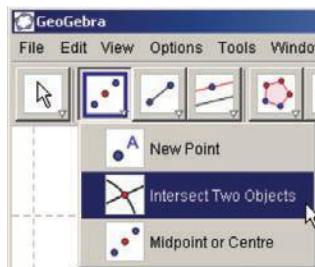


Square

- 1 To draw a square, click on **Regular Polygon**. Use the grid to draw an interval of length 7.5 cm. Select 4 points. Make sure that the labels are showing.
- 2 Use **interval between two points** to construct the two diagonals for the square. Select **distance or length** to measure the length of each diagonal in the square. What do you notice?
- 3 Now measure the **Angle** of each vertex of the square. What do you notice?
- 4 Draw another square with side length 9 cm. Repeat steps 2 and 3. List two properties of the square.

Rectangle

- 1 To draw a rectangle, click on **Polygon**. Use the grid to draw a rectangle with sides 6 cm by 3 cm.
- 2 Use **distance or length** to measure each side length of the rectangle.
- 3 Now measure the **Angle** of each vertex to check your accuracy. Correct any inaccurate sides using **Move**. What symbol is shown on each vertex when the angle is shown as exactly 90°?
- 4 Draw another rectangle with length 5 cm and width 8.4 cm. Use the instructions given in step 2 above to measure the side lengths and angles and also to correct any inaccurate sides and/or angles.
- 5 Use **interval between two points** to construct the two diagonals for each of your rectangles. Select **distance or length** to measure the length of each diagonal in every rectangle. What do you notice?
- 6 Complete this property: The _____ in a rectangle are _____.
- 7 In one rectangle, select  **Intersect two objects** and click on the two diagonals.

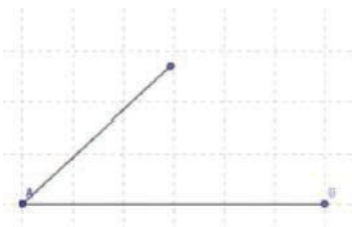


Use **distance or length** to measure the distance from the intersection point to the vertex for each diagonal. Repeat for the second rectangle. What do you notice?

- 8 Complete this property: The _____ of a rectangle _____ each other.

Parallelogram

- 1 Select  **Interval with given length from point**. Make the interval 6 cm long. Click **Show label**.
- 2 Now select **Interval with given length from point** and click point A. Make the interval 4 cm. Use **Move** and drag the new interval as shown below. Label the new point, C.

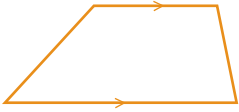


- 3 Click  **Parallel line** and select line AB and point C. Now select AC and point B.
- 4 Use **Intersect two objects** to create the missing vertex of the parallelogram. Label the vertex.
- 5 Use **distance or length** to show the length of each side of the parallelogram. What do you notice?
- 6 Complete the following: The _____ sides of a parallelogram are _____.
- 7 Now use **Angle** to find the size of $\angle CAB$ and $\angle CDB$. Repeat for $\angle ACD$ and $\angle ABD$. What do you notice?
- 8 Complete the following: The _____ angles of a parallelogram are _____.
- 9 Now draw the diagonals of the parallelogram. Use **distance or length** to show the length of each diagonal. What do you notice?
- 10 Complete the following: The diagonals of a parallelogram are _____.
- 11 Do the diagonals of a parallelogram bisect each other? Repeat step 7 from 'Rectangle' to help you.
- 12 Complete the following: The diagonals of a parallelogram _____ bisect each other.
- 13 Drag any vertices of the parallelogram that you can. Is it possible to draw other parallelograms with the same dimensions, 6 cm by 4 cm? What do you notice?
- 14 Use your GeoGebra skills to **accurately** construct other quadrilaterals such as a rhombus, kite or trapezium.

4-06 Properties of quadrilaterals

Exercise 4-06 Properties of quadrilaterals

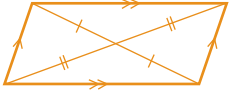
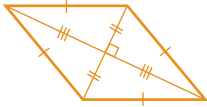
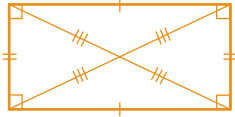
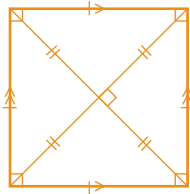
- 1 Copy the table below or use the link to print one out. Accurately draw each of the six special quadrilaterals below on a sheet of paper, then cut them out. Use your shapes to help you complete the table of quadrilateral properties.

a Trapezium 	• One pair of opposite sides are _____

Worksheet

Properties of
quadrilaterals

MAT08MGWK10032

b Kite 	- Two pairs of adjacent sides are _____ - One pair of opposite _____ are equal. - Diagonals intersect at _____
c Parallelogram 	_____ sides are equal. - Opposite _____ are parallel. - Opposite angles are _____ - Diagonals _____ each other.
d Rhombus 	- All _____ are equal. _____ sides are _____ _____ angles are _____ - Diagonals bisect each other at _____ angles. - Diagonals _____ the angles of the rhombus.
e Rectangle 	- Opposite sides are _____ - Opposite sides are also _____ - All angles are _____ - Diagonals are _____ - Diagonals _____ each other.
f Square 	- All sides are _____ - All angles are _____ - Opposite _____ are parallel. - Diagonals are _____ - Diagonals bisect each other at _____

2 Which quadrilaterals have each property?

a Opposite sides are equal

c Opposite angles are equal

e Diagonals bisect each other

g Adjacent sides are of different lengths

i All angles are equal

b Diagonals cross at right angles

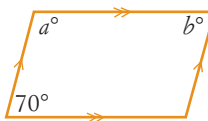
d One pair of opposite sides are parallel

f Opposite sides are parallel

h Two equal diagonals

j All sides are equal

3 Copy and fill in the blanks to find the values of a and b .

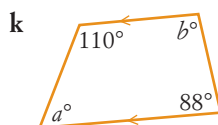
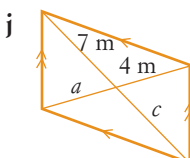
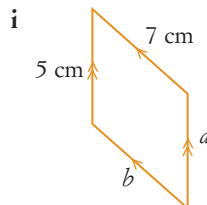
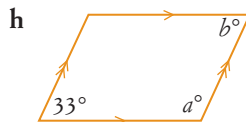
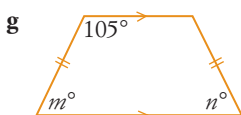
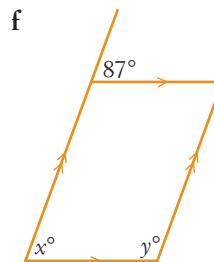
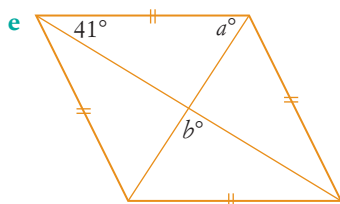
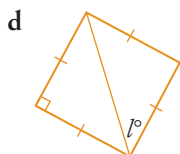
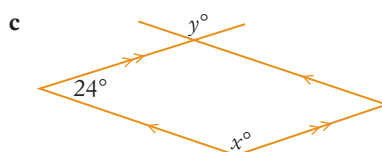
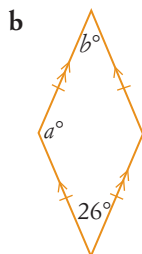
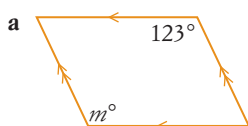


$$a + 70 = 180 \text{ (_____ angles on _____ lines)}$$

$$a = \underline{\hspace{2cm}}$$

$$b = \underline{\hspace{2cm}} \text{ (opposite _____ of a parallelogram)}$$

4 Find the value of each pronumeral.



5 I am a quadrilateral with opposite sides equal and parallel.

My diagonals are equal and I have two axes of symmetry.

Which quadrilateral am I? Select the correct answer **A**, **B**, **C** or **D**.

A parallelogram **B** rectangle **C** square **D** rhombus

6 I am a quadrilateral with opposite sides equal.

My diagonals bisect each other and meet at right angles.

Which quadrilateral am I? Select **A**, **B**, **C** or **D**.

A parallelogram **B** trapezium **C** rectangle **D** rhombus

7 A rectangle is a quadrilateral with four right angles.

Which one of the following is a special type of rectangle? Select **A**, **B**, **C** or **D**.

A square **B** kite **C** parallelogram **D** rhombus

8 A rhombus is a quadrilateral with all sides equal.

Which one of the following is a special type of rhombus? Select **A**, **B**, **C** or **D**.

A square **B** kite **C** parallelogram **D** rectangle

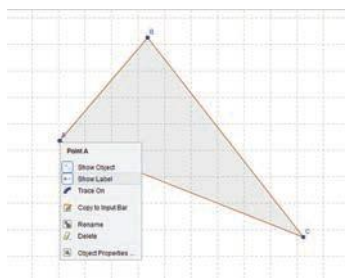
Technology Exterior angle of a triangle

In this activity we will use GeoGebra to discover an important property about the interior and exterior angles of any triangle. To set up your drawing page, click **View** and make sure that **Grid** is selected and **Axes** is not selected.

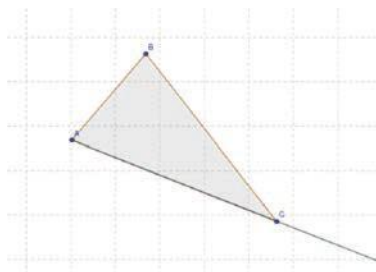


- 1 Draw a triangle using the **polygon** tool.

Right-click on each vertex and select **Show label** if no labels are showing.



- 2 Select  **Ray through two points** from the third icon menu and draw a ray from A through C.



- 3 Select  **New Point** from the second icon menu. Insert the new point on ray AC outside interval AC. **Show label D**.
- 4 Use  **Angle** from the eighth icon menu to find the size of $\angle ABC$, $\angle BAC$ and $\angle BCD$.
- 5 Calculate $\angle ABC + \angle BAC$.
- 6 Compare your answer from question 5 with the size of $\angle BCD$. What do you notice?
- 7 Copy and complete: The _____ angle of a triangle is _____ to the sum of the _____ opposite _____ angles.

4-07

Angle sums of triangles and quadrilaterals

Worksheet

Find the unknown angle 2

MAT08MGWK10035

Video tutorial

Angle sums of triangles and quadrilaterals

MAT08MGVT10007

Video tutorial

Angle in polygons

MAT08MGVT00006

Homework sheet

Quadrilaterals and angle sums

MAT08MGHS10032

Quiz

Shapes and angles

MAT08MGQZ00006

Puzzle sheet

Mixed angles

MAT08MGPS00023

Puzzle sheet

Angles in triangles

MAT08MGPS00022

Technology

GeoGebra: Exterior angle of a triangle

MAT08MGTC00007

Worksheet

Angles in triangles

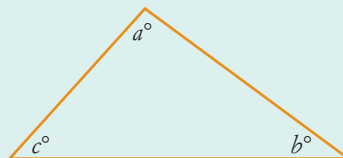
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Angle sum of a triangle

Summary

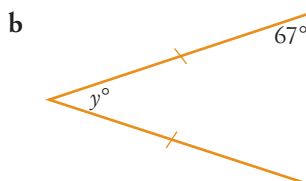
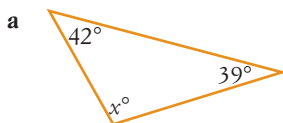
The **angle sum of a triangle** is 180°

$$a + b + c = 180$$



Example 4

Find the value of each pronumeral, giving reasons.



Solution

a $x + 42 + 39 = 180$ (Angle sum of a triangle)

$$x = 180 - 42 - 39$$

$$= 99$$

b $y + 67 + 67 = 180$ (Angle sum of an isosceles triangle)

$$y = 180 - 134$$

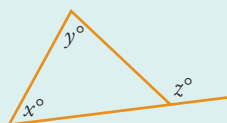
$$= 46$$

The exterior angle of a triangle

Summary

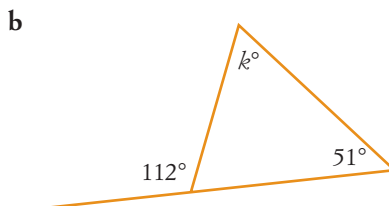
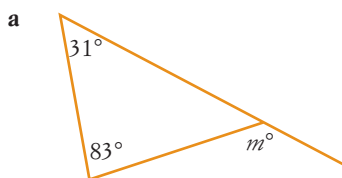
The **exterior angle of a triangle** is equal to the sum of the two interior opposite angles.

$$z = x + y$$



Example 5

Find the value of each pronumeral, giving reasons.



Solution

a $m = 83 + 31$ (Exterior angle of a triangle)
 $= 114$

b $k + 51 = 112$ (Exterior angle of a triangle)
 $k = 112 - 51$
 $= 61$

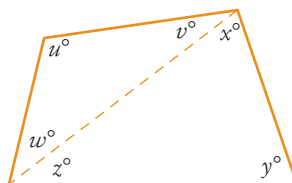
Angle sum of a quadrilateral

Any quadrilateral can be divided into two triangles along one of its diagonals. Because the angles in each triangle add to 180° , the angles in both triangles add to $2 \times 180^\circ = 360^\circ$.

$$u^\circ + v^\circ + w^\circ = 180^\circ \text{ and } x^\circ + y^\circ + z^\circ = 180^\circ$$

$$\therefore \text{Angle sum of quadrilateral} = 180^\circ + 180^\circ$$

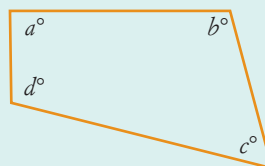
$$= 360^\circ$$



Summary

The **angle sum of a quadrilateral** is 360° .

$$a + b + c + d = 360$$

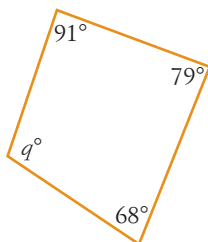


This property is true for both convex and non-convex quadrilaterals.

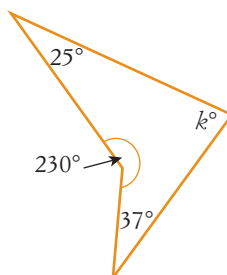
Example 6

Find the value of each pronumeral, giving reasons.

a



b



Solution

a $q + 91 + 79 + 68 = 360$ (Angle sum of a quadrilateral)

$$q + 238 = 360$$

$$q = 360 - 238$$

$$= 122$$

b $k + 25 + 37 + 230 = 360$ (Angle sum of a quadrilateral)

$$k + 292 = 360$$

$$k = 360 - 292$$

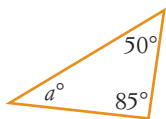
$$= 68$$

Exercise 4-07 Angle sums of triangles and quadrilaterals

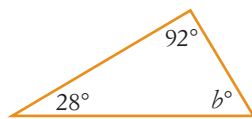
1 Find the value of each pronumeral.

See Example 4

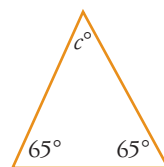
a



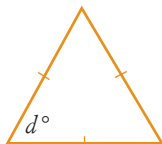
b



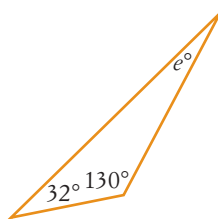
c



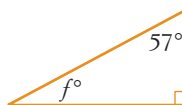
d



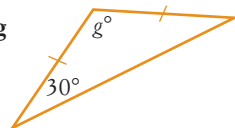
e



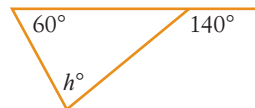
f



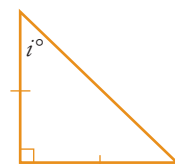
g



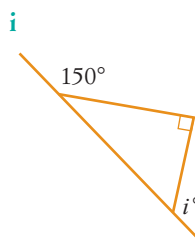
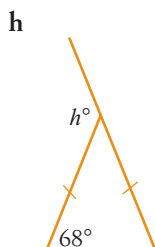
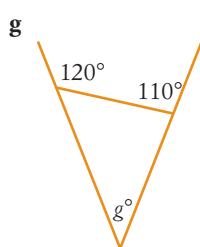
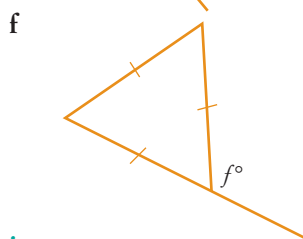
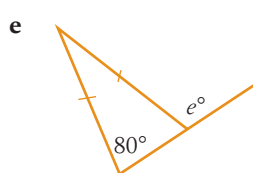
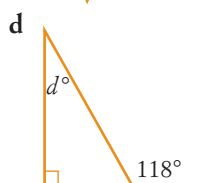
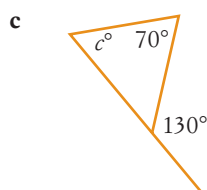
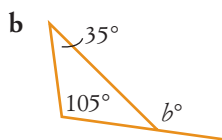
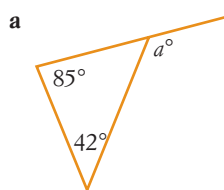
h



i



See Example 5 2 Find the value of each pronumeral.

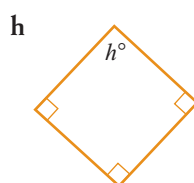
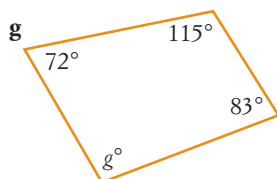
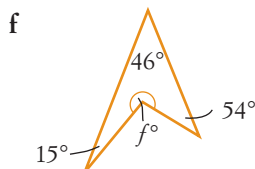
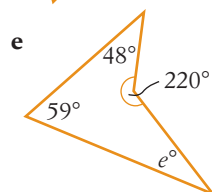
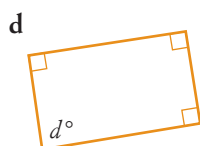
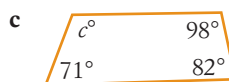
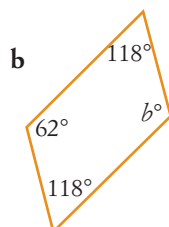
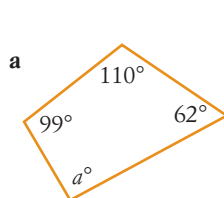


Worked solution

Exercise 4-07

MAT08MGWS10031

See Example 6 3 Find the value of each pronumeral.



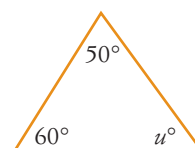
4 Which of the following is the value of u ? Select the correct answer A, B, C or D.

A 50

B 60

C 70

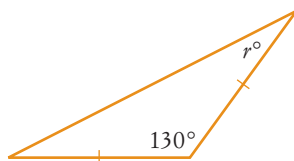
D 80



- 5 Which of the following is the value of r ?

Select A, B, C or D.

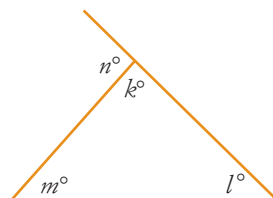
- A 25 B 35
C 50 D 100



- 6 Which equation is correct for the triangle on the right?

Select A, B or C.

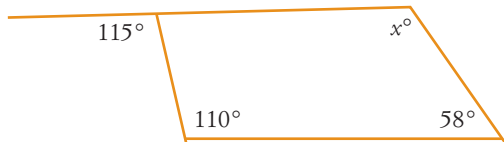
- A $n = l + m$ B $n = k + m$ C $n = l + k$



- 7 Which of the following is the value of x ?

Select A, B, C or D.

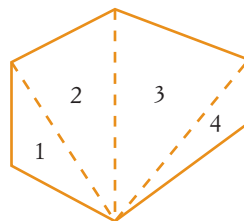
- A 115 B 65
C 127 D 122



Investigation: Angle sum of a polygon

We know that the angle sum of a triangle is 180° and that the angle sum of a quadrilateral is 360° , but how do we find the angle sum of other convex polygons?

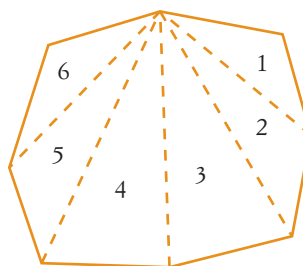
A hexagon can be divided into four triangles by drawing the diagonals from one vertex.



$$\begin{aligned}\text{The sum of the angles in a hexagon} &= (\text{angle sum of a triangle}) \times 4 \\ &= 180^\circ \times 4 \\ &= 720^\circ\end{aligned}$$

An octagon can be divided into six triangles by drawing the diagonals from one vertex.

$$\text{The total of the angles in an octagon} = 180^\circ \times 6 = 1080^\circ.$$



- 1 How is the number of sides related to the number of triangles formed in a shape?

- 2 Copy and complete the following sentences.

The angle sum of a polygon with n sides is

$$A = 180 \times (\text{number of sides} - \underline{\hspace{2cm}})^\circ$$

$$\text{or: } A = 180 \times (n - \underline{\hspace{2cm}})^\circ$$

- 3 Find the angle sum of each polygon.

a 11-sided polygon

b 20-sided polygon

c 14-sided polygon

Worksheet

Angle sum of a polygon

MAT08MGWK10036

Technology worksheet

Angle sum of a polygon

MAT08MGCT10002

4-08 Extension: Angle sum of a polygon

A **convex polygon** has all vertices pointing outwards.

A **regular polygon** has all sides the same length and all angles the same size.

Example 7

These shapes are all hexagons (six-sided).

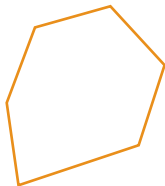
a Which hexagons are convex?

b Which hexagon is regular?

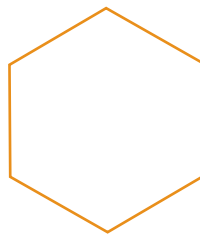
i



ii



iii



Solution

a Hexagons **ii** and **iii** are convex because all of their vertices point outwards.

b Hexagon **iii** is regular, because all its sides are equal and all its angles are equal.

Summary

The **angle sum of a polygon** with n sides is given by the formula $A = 180(n - 2)^\circ$.

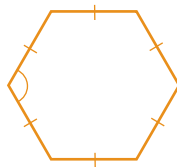
This property applies to both convex and non-convex polygons.

Example 8

Find the size of one angle in a **regular** hexagon.

Solution

A hexagon has six sides ($n = 6$).



$$\begin{aligned}\text{Angle sum of a hexagon} &= 180(6 - 2)^\circ \\ &= 180 \times 4^\circ \\ &= 720^\circ\end{aligned}$$

For a regular hexagon:

$$\begin{aligned}\text{one angle} &= 720^\circ \div 6 \\ &= 120^\circ\end{aligned}$$

$\therefore$ Each angle in a regular hexagon is 120° .

Worksheet

Angles in regular polygons

MAT08MGWK00041

Exercise 4-08 Angle sum of a polygon

- 1 Copy these polygons and, beneath each drawing, write their correct name (chosen from the list). Say if each polygon is **regular** or **irregular**, **convex** or **non-convex**.

See Example 7

hexagon
decagon
quadrilateral

nonagon
octagon
pentagon

heptagon
triangle

a



b



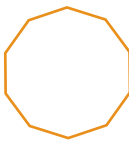
c



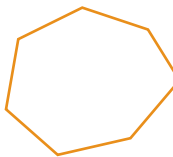
d



e



f



g



h



- 2 Copy and complete this table.

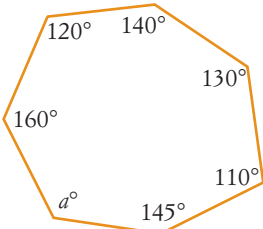
Polygon	Number of sides	Sum of angles inside polygon
hexagon		
heptagon		
octagon		
nonagon		
decagon		

- 3 Find the sum of the interior angles in:

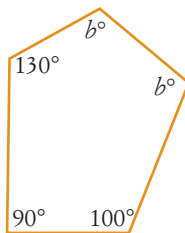
a a 15-agon b a 20-agon c a 25-agon d a 100-agon

- 4 Find the value of each pronumeral.

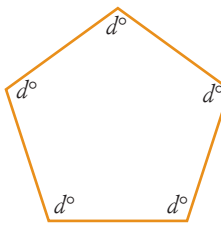
a

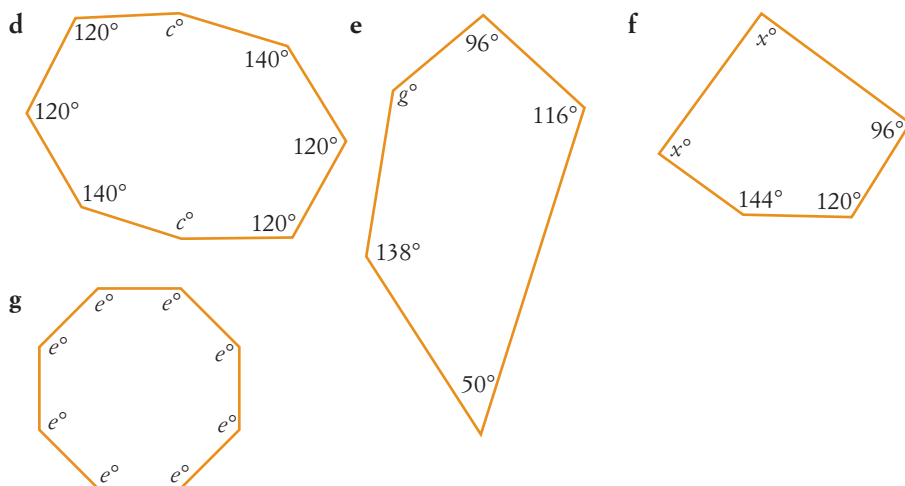


b



c





See Example 8 5 Find the size of one interior angle in each **regular** polygon.

- a** square **b** equilateral triangle **c** regular hexagon
d regular octagon **e** regular decagon **f** regular pentagon
g regular dodecagon

Worked solution

6 Find the number of sides of the polygon that has an angle sum of:

- a** 2160° **b** 5760° **c** 4320° **d** 9180° **e** $22\,140^\circ$

Exercise 4-08

MAT08MGWS10032

Mental skills 4B

Maths without calculators

Converting decimals and percentages to fractions

1 Consider each of the following examples.

$$\text{a } 0.35 = \frac{35}{100} = \frac{7}{20}$$

(two decimal places, two zeros in the denominator)

$$\text{b } 0.8 = \frac{8}{10} = \frac{4}{5}$$

(one decimal place, one zero in the denominator)

$$\text{c } 0.64 = \frac{64}{100} = \frac{16}{25}$$

$$\text{d } 0.22 = \frac{22}{100} = \frac{11}{50}$$

2 Now convert each decimal to a fraction.

- a** 0.75 **b** 0.28 **c** 0.3 **d** 0.14 **e** 0.06
f 0.85 **g** 0.32 **h** 0.49 **i** 0.56 **j** 0.9
k 0.72 **l** 0.65 **m** 0.2 **n** 0.24 **o** 0.53

3 Consider each of the following examples.

$$\text{a } 26\% = \frac{26}{100} = \frac{13}{50}$$

$$\text{b } 40\% = \frac{40}{100} = \frac{2}{5}$$

$$\text{c } 8\% = \frac{8}{100} = \frac{2}{25}$$

$$\text{d } 95\% = \frac{95}{100} = \frac{19}{20}$$

4 Now convert each percentage to a fraction.

a 76%

b 10%

c 80%

d 45%

e 88%

f 56%

g 75%

h 31%

i 68%

j 5%

k 60%

l 54%

m 6%

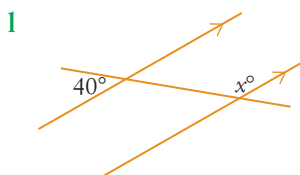
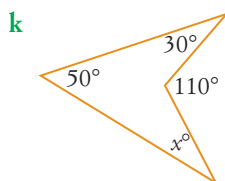
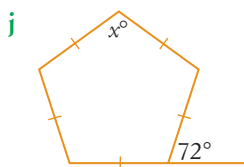
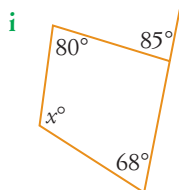
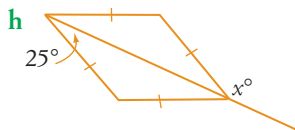
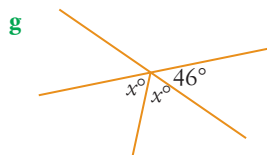
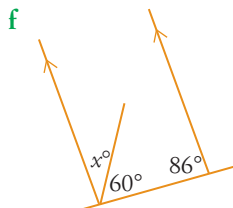
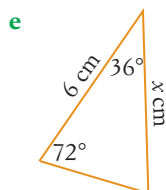
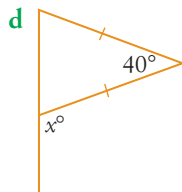
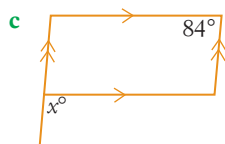
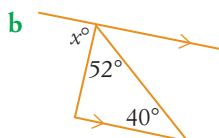
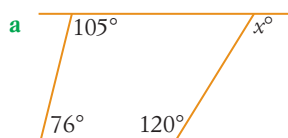
n 49%

o 82%

Power plus

As questions become more complex, it may not be possible to find the answer in one step. It may be necessary to find another angle first.

1 Find the value of x in each diagram. (Give reasons for all steps.)



Chapter 4 review

Language of maths

Puzzle sheet

Geometry crossword

MAT08MGPS10014

acute

alternate

angle sum

axis/axes

co-interior

complementary

convex

corresponding

diagonal

equilateral

exterior angle

interior angle

isosceles

kite

obtuse

parallelogram

quadrilateral

reflex

rhombus

rotational symmetry

scalene

supplementary

trapezium

vertically opposite

- 1 What are **supplementary angles**?
- 2 Name the types of angles associated with parallel lines cut by a transversal.
- 3 What type of triangle has one angle that is greater than 90° ?
- 4 What is the **angle sum** of a quadrilateral?
- 5 Illustrate the difference between an **interior angle** and an **exterior angle** of a triangle.
- 6 What does **equilateral** mean? What is the common name for an 'equilateral parallelogram'?

Topic overview

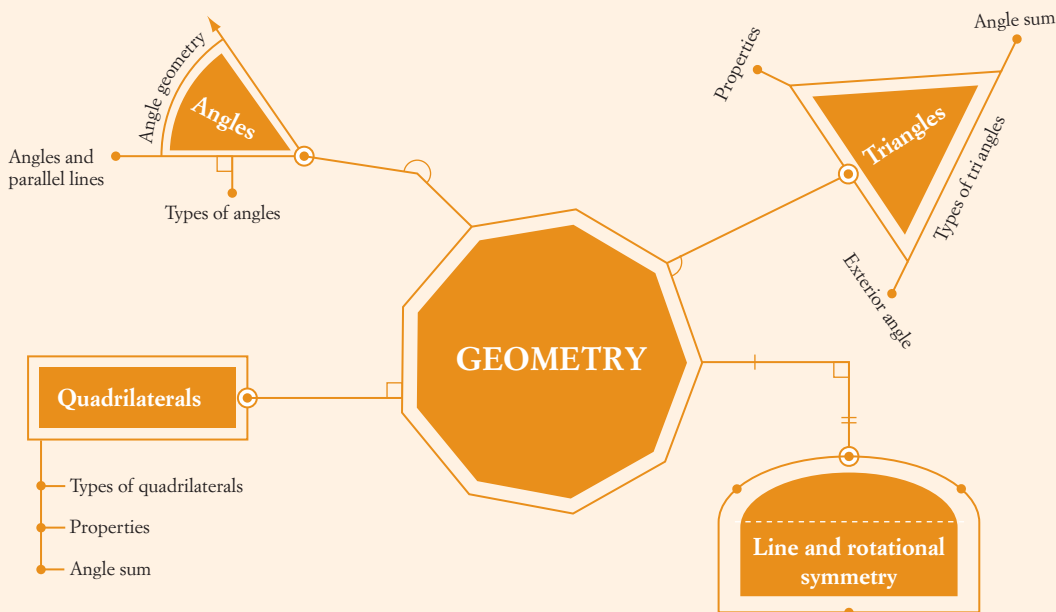
Worksheet

Mind map: Geometry

MAT08MGWK10037

- Do you think this chapter is useful? Why? What did you learn in this chapter?
- How confident do you feel with geometry?
- List anything in this chapter that you did not understand. Show your teacher.

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



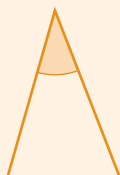
1 Name each type of angle.

See Exercise 4-01

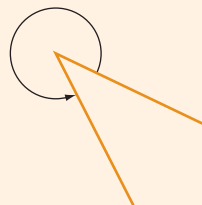
a



b



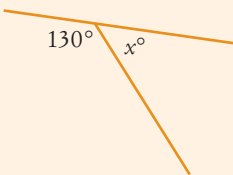
c



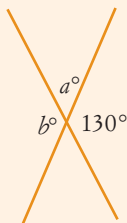
2 Find the value of each pronumeral, giving reasons.

See Exercise 4-01

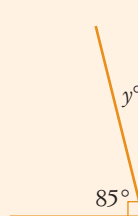
a



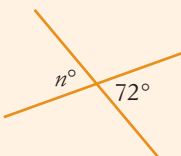
b



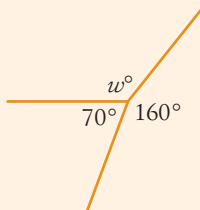
c



d



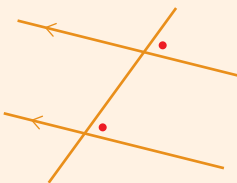
e



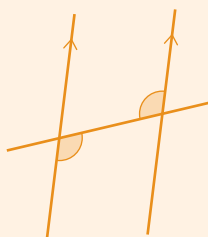
3 Name each type of angle pair.

See Exercise 4-02

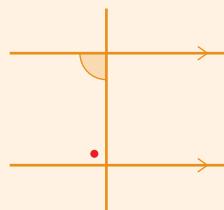
a



b



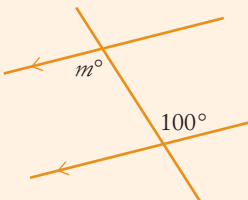
c



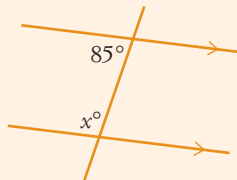
4 Find the value of each pronumeral, giving reasons.

See Exercise 4-02

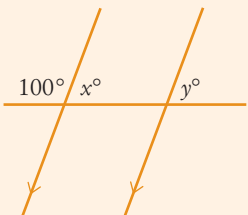
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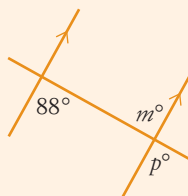
b



c



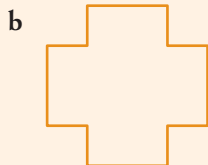
d



Chapter 4 revision

See Exercise 4-03

- 5 Copy each shape and mark in its axes of symmetry.

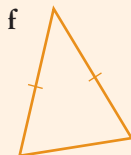
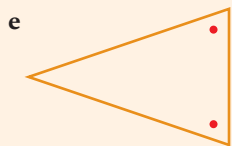
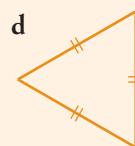
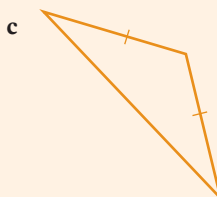
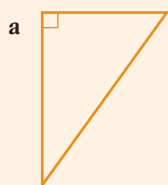


See Exercise 4-03

- 6 For each shape in question 5, decide whether or not it has rotational symmetry. If it does, state the order of rotational symmetry.

See Exercise 4-04

- 7 Classify each of these triangles by its sides and angles.



See Exercise 4-05

- 8 Draw a neat diagram of each of the following quadrilaterals.

a a parallelogram

b a kite

c a trapezium

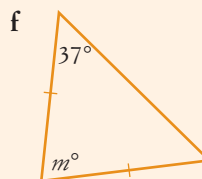
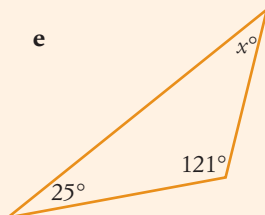
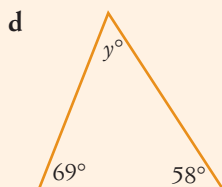
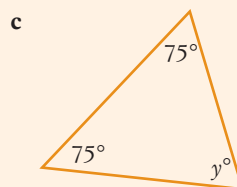
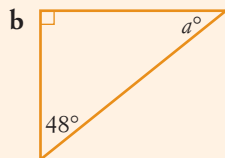
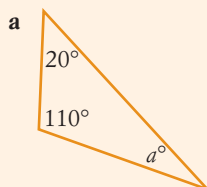
See Exercise 4-06

- 9 Name all the quadrilaterals that have the following properties.

- a** all sides equal in length
- b** no parallel sides
- c** all angles right angles
- d** two pairs of opposite angles equal
- e** diagonals bisect each other

See Exercise 4-07

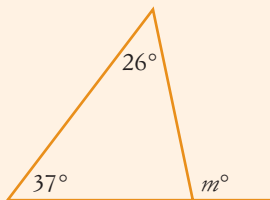
- 10 Find the value of each pronumeral.



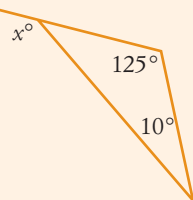
11 Find the value of each pronumeral.

See Exercise 4-07

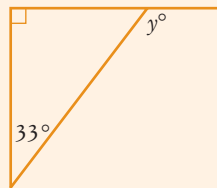
a



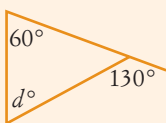
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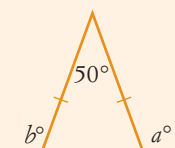
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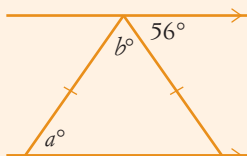
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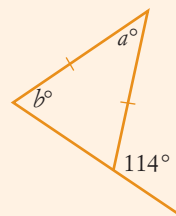
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f



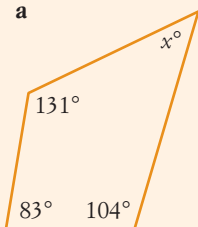
g



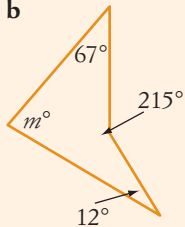
12 Find the value of each pronumeral.

See Exercise 4-07

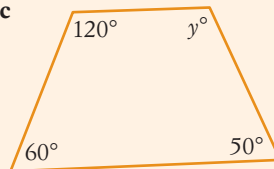
a



b



c





5

Measurement and geometry

Area and volume

In 1978, Prime Minister Malcolm Fraser announced a competition to design a new Parliament House to be built for Australia's bicentenary in 1988. The winning design was submitted by architects Mitchell, Giurgola and Thorp. The building was constructed in seven years and was opened on 9 May 1988 by Queen Elizabeth II. The Parliament House site covers 32 hectares of Capital Hill while the building itself is 300 metres wide, 300 metres long with a floor area of 250 000 m².



Chapter outline

	Proficiency strands				
5-01 Perimeter	U	F	PS	R	
5-02 Metric units for area	U	F	PS	R	C
5-03 Areas of rectangles, triangles and parallelograms	U		PS	R	
5-04 Areas of composite shapes	U	F	PS		
5-05 Area of a trapezium	U		PS	R	
5-06 Areas of kites and rhombuses	U		PS	R	
5-07 Parts of a circle	U				C
5-08 Circumference of a circle	U	F	PS	R	
5-09 Area of a circle	U	F	PS	R	
5-10 Metric units for volume	U	F	PS	R	C
5-11 Volume of a prism	U	F	PS	R	C
5-12 Volume of a cylinder	U	F	PS	R	
5-13 Volume and capacity	U	F	PS		

Wordbank

capacity The amount of fluid (liquid or gas) in a container

circumference The perimeter of a circle; a circle's outer boundary

cross-section A 'slice' of a solid, taken across the solid rather than along it

cubic metre The volume of a cube that measures 1 m by 1 m by 1 m

perpendicular height The height of a shape taken at right angles to its base

pi (π) A special irrational number, approximately 3.1416, used in calculating circular measurements

quadrant A quarter of a circle, a sector containing a right angle

radius The distance from the centre of a circle to the circle's edge

In this chapter you will:

- find perimeters and areas of parallelograms, trapeziums, rhombuses and kites
- choose appropriate units of measurement for area and convert from one unit to another
- establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving
- investigate the line symmetries and the rotational symmetry of circles and of diagrams involving circles, such as a sector and a circle with a marked chord or tangent
- investigate the relationship between features of circles such as circumference, area, radius and diameter
- use formulas to solve problems involving circumference and area
- find the perimeter and area of quadrants, semicircles, sectors and composite figures
- solve problems involving circles and parts of circles, giving an exact answer in terms of π and an approximate answer
- choose appropriate units of measurement for volume and convert from one unit to another
- develop the formulas for volumes of rectangular and triangular prisms and prisms in general
- connect volume and capacity and their units of measurement
- solve problems involving volume and capacity of right prisms and cylinders

SkillCheck

Worksheet

StartUp assignment 5

MAT08MGWK10038

1 Copy and complete each statement.

a 20 cm = ____ mm

b 350 cm = ____ m

c 2500 m = ____ km

d 46 mm = ____ cm

e 4 km = ____ m

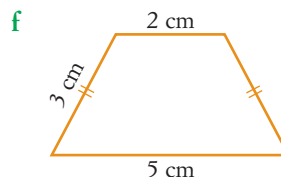
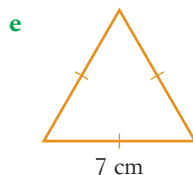
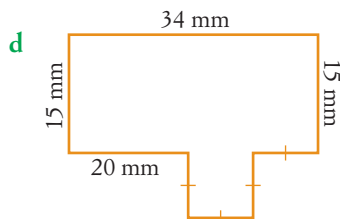
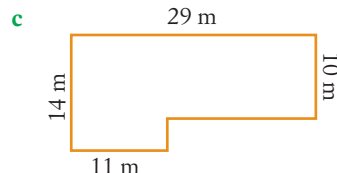
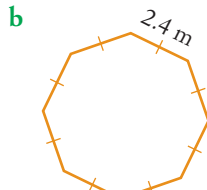
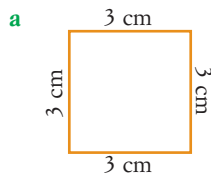
f 5200 m = ____ km

g 600 m = ____ km

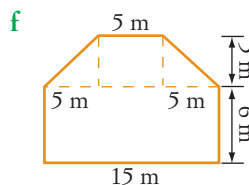
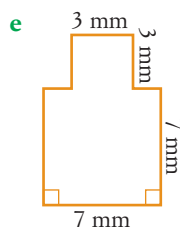
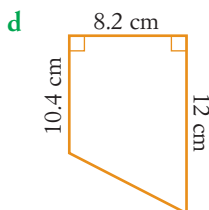
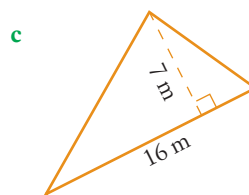
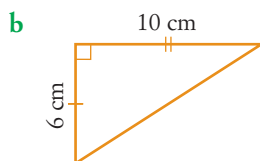
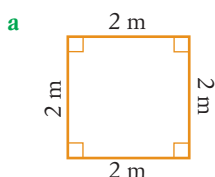
h 8200 mm = ____ m

i 0.4 m = ____ mm

2 Find the perimeter of each shape.



3 Find the area of each shape.



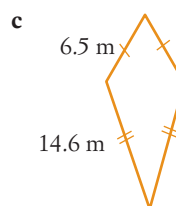
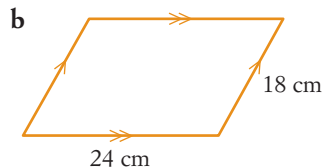
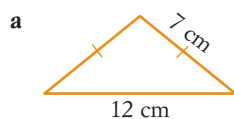
5-01 Perimeter

Summary

The **perimeter** of a shape is the distance around the shape.
It is the sum of the lengths of the sides of the shape.

Example 1

Find the perimeter of each shape.



Solution

a Perimeter = $7 + 7 + 12$
= 26 cm

Adding the three sides of the isosceles triangle.

b Perimeter = $24 + 18 + 24 + 18$
= 84 cm

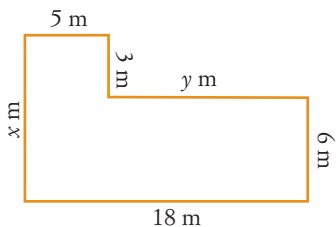
In a parallelogram, opposite sides are equal.

c Perimeter = $6.5 + 6.5 + 14.6 + 14.6$
= 42.2 m

In a kite, adjacent sides are equal.

Example 2

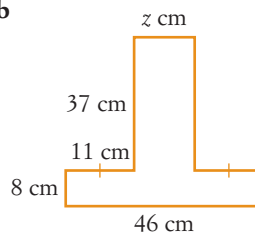
Find the perimeter of each composite shape.

a**Solution****a** Find the unknown sides first.

$$x = 3 + 6 = 9$$

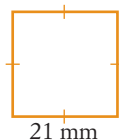
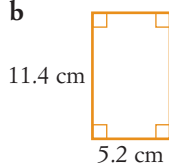
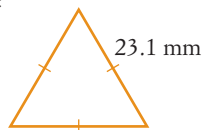
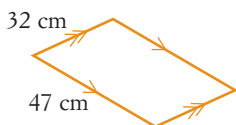
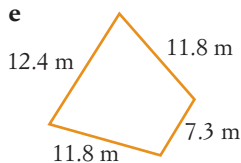
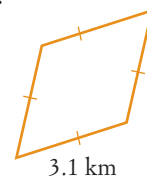
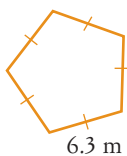
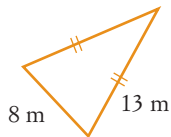
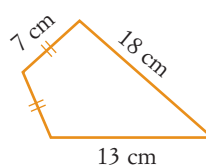
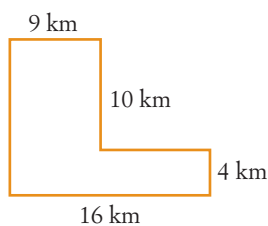
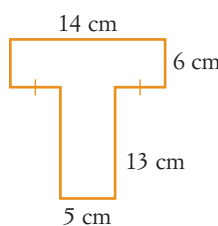
$$y = 18 - 5 = 13$$

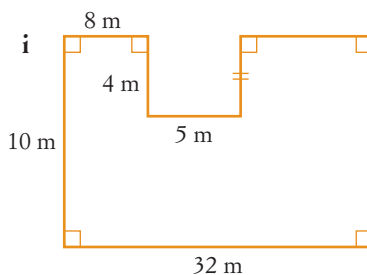
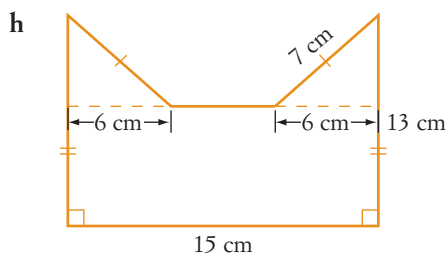
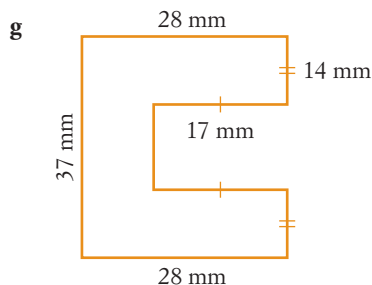
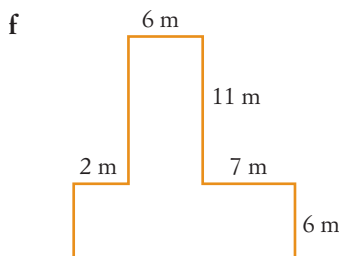
$$\begin{aligned}\text{Perimeter} &= 9 + 5 + 3 + 13 + 6 + 18 \\ &= 54 \text{ m}\end{aligned}$$

b**b** Find the unknown sides first.

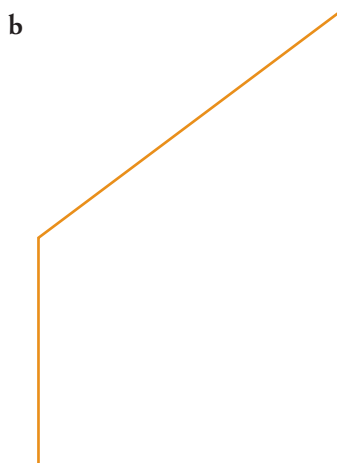
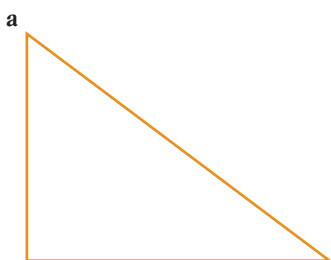
$$z = 46 - 11 - 11 = 24$$

$$\begin{aligned}\text{Perimeter} &= 8 + 11 + 37 + 24 + 37 \\ &\quad + 11 + 8 + 46 \\ &= 182 \text{ cm}\end{aligned}$$

Exercise 5-01 Perimeter**See Example 1****1** Find the perimeter of each shape.**a****b****c****d****e****f****See Example 2****2** Find the perimeter of each shape.**a****b****c****d****e**



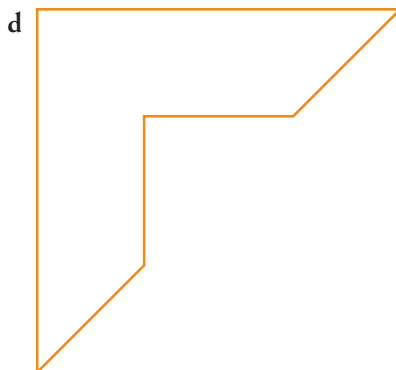
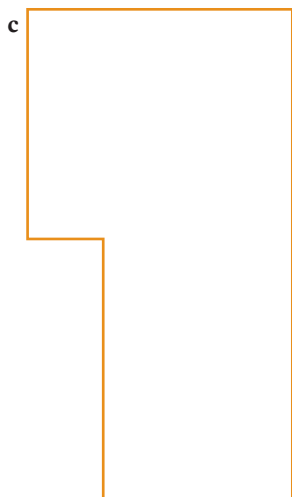
- 3 Find the perimeter of:
- a square of side length 3 cm
 - a parallelogram with adjacent sides of length 7 m and 10 m
 - an equilateral triangle of side length 5 cm
 - an isosceles triangle with equal sides of 9 m and a side of 4 m
- 4 A square has a perimeter of 24 cm. What is the length of one side? Select the correct answer
A, B, C or D.
- A 6 cm B 8 cm C 12 cm D 96 cm
- 5 A rectangle has a perimeter of 40 m. If its length is 13 m, find its width.
- 6 A rhombus has a perimeter of 18 m. Find the length of one side.
- 7 A kite has a perimeter of 26 cm. If one of its longer sides is 9 cm, find the length of a shorter side.
- 8 Measure the sides of each shape in millimetres and find its perimeter.



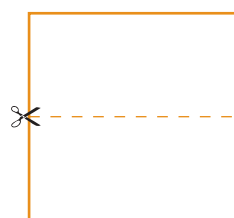
Worked solutions

Exercise 5-01

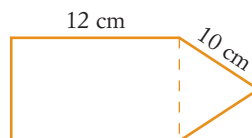
MAT08MGWS10033



- 9 A square is cut in half as shown. The two rectangles have a combined perimeter of 24 cm. Find the perimeter of the original square.



- 10 This composite shape is made up of a rectangle and an equilateral triangle. Find its perimeter. Select the correct answer **A**, **B**, **C** or **D**.



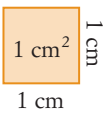
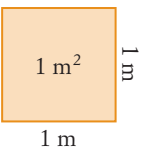
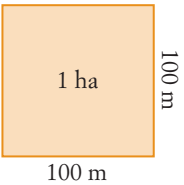
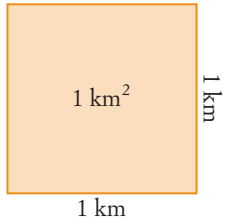
A 22 cm **B** 44 cm **C** 54 cm **D** 64 cm

- 11 Which of the following lengths is the closest to the perimeter of your classroom? Select the correct answer **A**, **B**, **C** or **D**.

A 250 m **B** 2.5 km **C** 250 cm **D** 25 m

5-02 Metric units for area

The **area** of a shape is the amount of surface covered by the shape. Area is measured in **square units**.

square centimetre (cm ²)	square metre (m ²)
$1 \text{ cm} = 10 \text{ mm}$ $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$ $= 10 \text{ mm} \times 10 \text{ mm}$ $= 100 \text{ mm}^2$ 	$1 \text{ m} = 100 \text{ cm}$ $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$ $= 100 \text{ cm} \times 100 \text{ cm}$ $= 10\,000 \text{ cm}^2$ 
hectare (ha)	square kilometre (km ²)
$1 \text{ ha} = 100 \text{ m} \times 100 \text{ m}$ $= 10\,000 \text{ m}^2$ 	$1 \text{ km} = 1000 \text{ m}$ $1 \text{ km}^2 = 1 \text{ km} \times 1 \text{ km}$ $= 1000 \text{ m} \times 1000 \text{ m}$ $= 1\,000\,000 \text{ m}^2$ $1 \text{ km}^2 = 100 \text{ ha}$ 

A **square centimetre** is about the area of a fingernail.

A **square metre** is approximately the area of the floor of a large shower recess.

A **hectare** is about the area of two rugby league fields, side-by-side.

A **square kilometre** is about the area of a theme park, like Dreamworld on the Gold Coast.

Note that, while $1 \text{ cm} = 10 \text{ mm}$, $1 \text{ cm}^2 = 100 \text{ mm}^2$ (double the number of 0s), and while $1 \text{ m} = 100 \text{ cm}$, $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ (double the number of 0s).

Summary

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

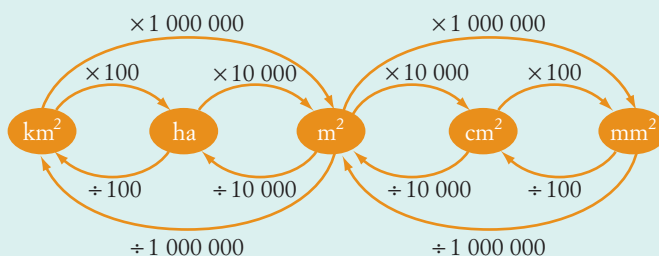
$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$

$$1 \text{ m}^2 = 1\,000\,000 \text{ mm}^2$$

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

$$1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$$

$$1 \text{ km}^2 = 100 \text{ ha}$$



Example 3

Convert:

a 800 mm^2 to cm^2

b 4.6 m^2 to cm^2

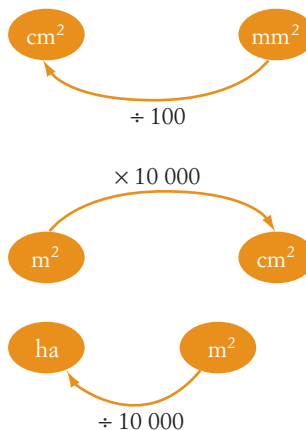
c $25\,000\,000 \text{ m}^2$ to ha

Solution

a $800 \text{ mm}^2 = 800 \div 100 \text{ cm}^2$
 $= 8 \text{ cm}^2$

b $4.6 \text{ m}^2 = 4.6 \times 10\,000 \text{ cm}^2$
 $= 46\,000 \text{ cm}^2$

c $25\,000\,000 \text{ m}^2 = 25\,000\,000 \div 10\,000 \text{ ha}$
 $= 25 \text{ ha}$



Exercise 5-02 Metric units for area

- 1 What units of area would you use when measuring the area of:
- a your home? b your pencil case? c a football field?
d the state of NSW? e your body? f a desk top?
- 2 How many square centimetres in 3.57 m^2 ? Select the correct answer A, B, C, or D.
A 35 700 B 357 C 0.357 D 0.0357

See Example 3

- 3 Copy and complete each statement.

a $100 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$	b $750 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
c $4500 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$	d $34\,000 \text{ m}^2 = \underline{\hspace{2cm}} \text{ km}^2$
e $60\,000 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ m}^2$	f $432\,000 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$
g $950 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ m}^2$	h $7600 \text{ m}^2 = \underline{\hspace{2cm}} \text{ ha}$
i $4.8 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$	j $30 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
k $850 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	l $6.9 \text{ km}^2 = \underline{\hspace{2cm}} \text{ m}^2$
m $54 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	n $700 \text{ ha} = \underline{\hspace{2cm}} \text{ m}^2$
o $0.48 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$	p $0.45 \text{ km}^2 = \underline{\hspace{2cm}} \text{ m}^2$

- 4 How many square metres in one square kilometre? Select the correct answer A, B, C, or D.
A 1000 B 10 000 C 100 000 D 1 000 000

- 5 A large cattle station in South Australia has an area of $30\,028 \text{ km}^2$. What is this in hectares?
- 6 The Cradle Mountain–St Clair National Park in Tasmania has an area of $134\,805 \text{ ha}$. What is this in km^2 ?
- 7 The centre of Sydney is approximately a rectangle bounded by George Street, Circular Quay, Macquarie and College Streets and Liverpool Street. This rectangle is about 1.75 km by 0.5 km . What is its area in:

- a km^2 ? b hectares?

- 8 Copy and complete each statement.

a $7.600 \text{ m}^2 = \underline{\hspace{2cm}} \text{ mm}$	b $0.7 \text{ ha} = \underline{\hspace{2cm}} \text{ cm}^2$
c $8 \text{ m}^2 = \underline{\hspace{2cm}} \text{ mm}^2$	d $850 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
e $6.90 \text{ ha}^2 = \underline{\hspace{2cm}} \text{ m}^2$	f $15.2 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$
g $8500 \text{ m}^2 = \underline{\hspace{2cm}} \text{ ha}$	h $49\,000\,000 \text{ m}^2 = \underline{\hspace{2cm}} \text{ km}^2$

- 9 The area of Australia is $7\,682\,300 \text{ km}^2$. Western Australia has an area of $2\,526\,000 \text{ km}^2$. Calculate to one decimal place the area of Western Australia as a percentage of the area of Australia.
- 10 Sydney Airport has an area of 881 hectares. Sydney Harbour has an area of about 5500 hectares.
- a What is the area of Sydney Airport in square kilometres?
- b How many 'Sydney Airports' would fit onto Sydney Harbour? Answer correct to one decimal place.

Worked solutions

Exercise 5-02

MAT08MGWS10034

5-03

Areas of rectangles, triangles and parallelograms

Video tutorial

Area formulas for triangles and quadrilaterals

MAT08MGVT00010

Skillsheet

What is area?

MAT08MGSS10021

Homework sheet

Area 1

MAT08MGHS10026

Worksheet

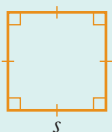
Rectangle and triangle areas

MAT08MGWK00065

Summary

Area formulas

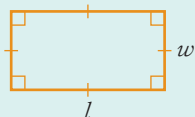
Square



$$A = (\text{side})^2$$

$$A = s^2$$

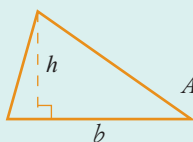
Rectangle



$$A = \text{length} \times \text{width}$$

$$A = lw$$

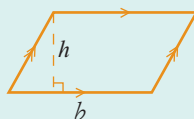
Triangle



$$A = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$A = \frac{1}{2}bh$$

Parallelogram

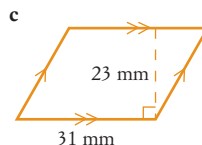
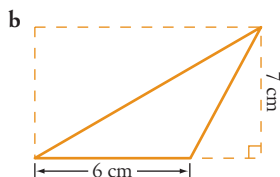
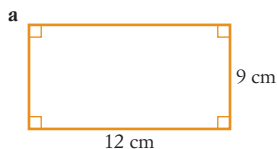


$$A = \text{base} \times \text{perpendicular height}$$

$$A = bh$$

Example 4

Find the areas of the following shapes.



Solution

a $A = lw$

$$A = 12 \times 9$$

$$= 108 \text{ cm}^2$$

Area of a rectangle = length $\times$ width

b $A = \frac{1}{2}bh$

$$= \frac{1}{2} \times 6 \times 7$$

$$= 21 \text{ mm}^2$$

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

c $A = bh$

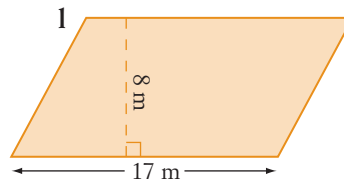
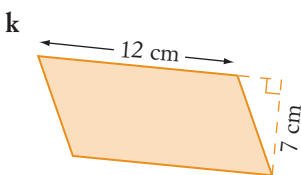
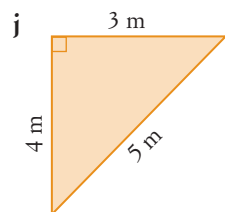
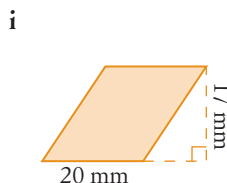
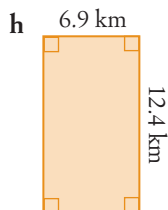
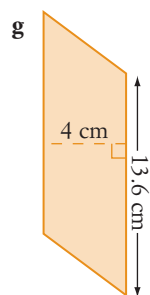
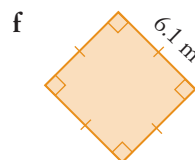
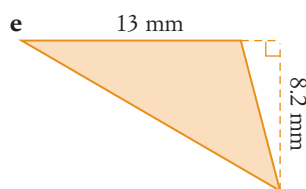
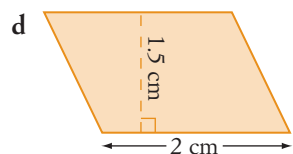
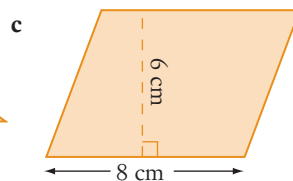
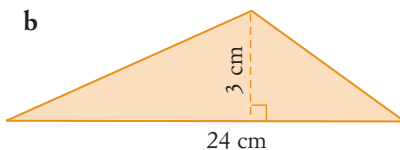
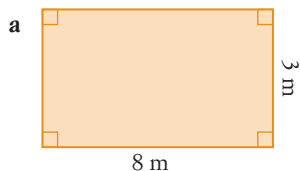
$$= 31 \times 23$$

$$= 713 \text{ m}^2$$

Area of a parallelogram = base $\times$ height

Exercise 5-03 Areas of rectangles, triangles and parallelograms

See Example 4 1 Find the area of each of these shapes.

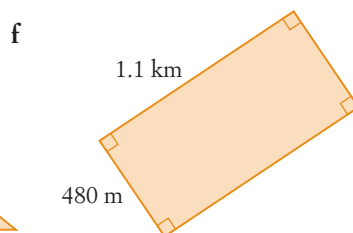
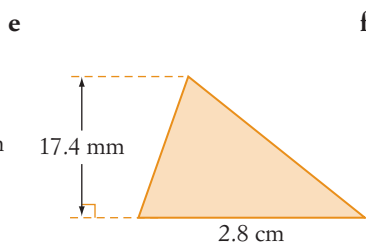
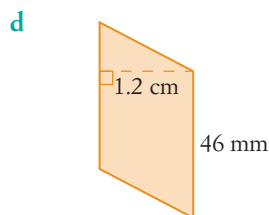
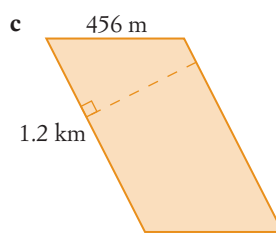
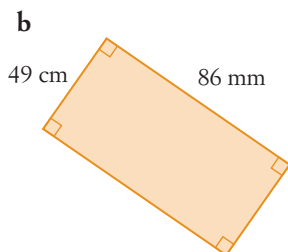
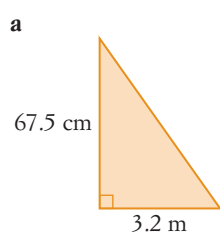


Worked solutions

Exercise 5-03

MAT08MGWS10035

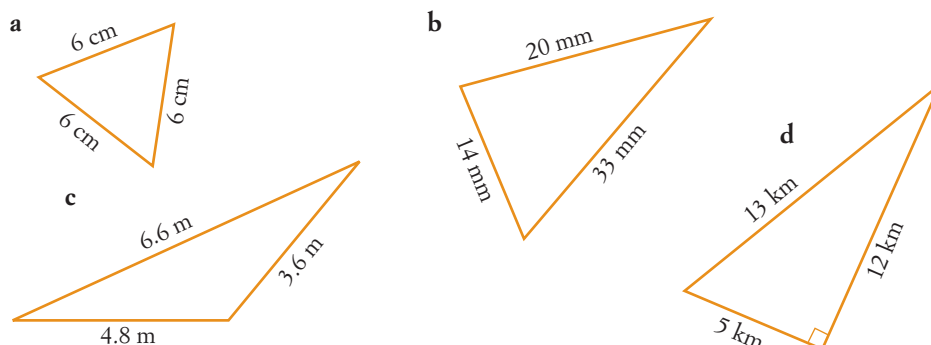
2 Find the area of these shapes. Make sure that you change all lengths to the same units first.



- 3 One hectare of land is subdivided into blocks. If the average block measures 20 metres by 25 metres, how many blocks would there be? Select the correct answer **A**, **B**, **C** or **D**.
A 50 **B** 25 **C** 40 **D** 20
- 4 A square has an area of 25 cm^2 . Find the length of one side.
- 5 A rectangle has an area of 27 m^2 . If its length is 9 m, find its width.
- 6 A triangle has an area of 35 cm^2 . If its base length is 10 cm, what is its perpendicular height?
- 7 A rectangle has an area of 72 mm^2 . What could its length and width be?
- 8 The area of a square is 900 m^2 . If the area of the square quadrupled, what would be the length of the new square? Give your answer to one decimal place.
- 9 The area of a triangle can also be found using **Heron's formula** if you know the lengths of all three sides a , b and c :

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \text{half the perimeter} = \frac{a+b+c}{2}$$

Find the area of each triangle using Heron's formula, correct to one decimal place.



Worked solutions

Exercise 5-03

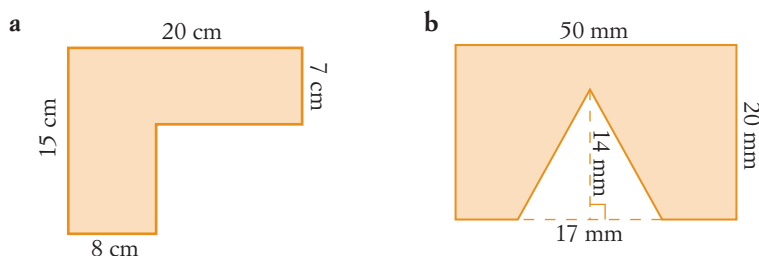
MAT08MGWS10035

5-04 Areas of composite shapes

Shapes made by combining other simpler shapes are called composite shapes. To find the areas of composite shapes, suitable dividing lines must be made.

Example 5

Find the area of each composite shape



Worksheet

Composite areas

MAT08MGWK10039

Worksheet

Odd areas

MAT08MGWK10040

Puzzle sheet

Areas 1

MAT08MGPS00043

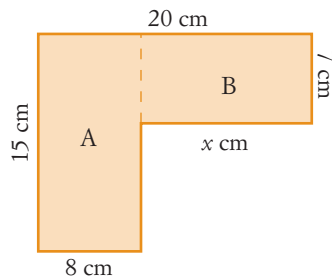
Solution

a Method 1

Divide the shape into two rectangles, A and B,
and find any unknown lengths.

$$x = 20 - 8 = 12$$

$$\begin{aligned}\text{Area of shape} &= \text{area of A} + \text{area of B} \\ &= 8 \times 15 + 12 \times 7 \\ &= 204 \text{ cm}^2\end{aligned}$$

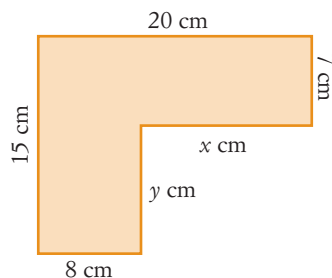
**Method 2**

The area can also be found by subtracting areas.

$$x = 20 - 8 = 12$$

$$y = 15 - 7 = 8$$

$$\begin{aligned}\text{Area} &= \text{large rectangle} - \text{small rectangle} \\ &= 20 \times 15 - 12 \times 8 \\ &= 204 \text{ cm}^2\end{aligned}$$

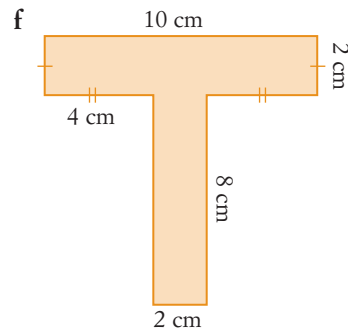
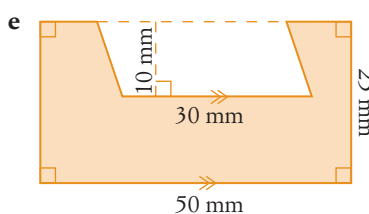
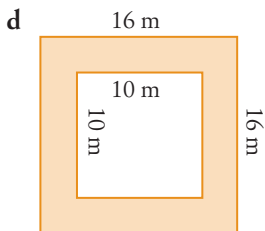
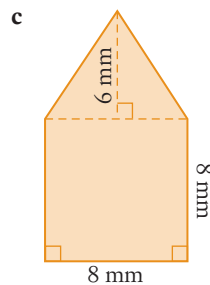
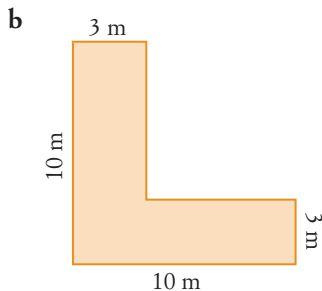
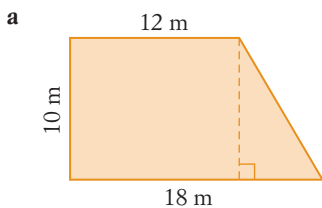
**b Area = area of rectangle – area of triangle**

$$\begin{aligned}&= 50 \times 20 - \frac{1}{2} \times 17 \times 14 \\ &= 1000 - 119 \\ &= 881 \text{ mm}^2\end{aligned}$$

Exercise 5-04 Areas of composite shapes

See Example 5

1 Find the area of each composite shape.



Mental skills 5A

Maths without calculators

Converting fractions and percentages to decimals

To convert a fraction into a decimal, change the denominator to a power of 10. This may require simplifying the fraction first.

- 1 Study each example.

a $\frac{31}{50} = \frac{31 \times 2}{50 \times 2} = \frac{62}{100} = 0.62$

b $\frac{18}{40} = \frac{18 \div 2}{40 \div 2} = \frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 0.45$

- 2 Now convert each fraction into a decimal.

a $\frac{7}{10}$

b $\frac{15}{25}$

c $\frac{14}{200}$

d $\frac{9}{20}$

e $\frac{49}{50}$

f $\frac{12}{30}$

To convert a percentage into a decimal, divide by 100 by moving the decimal point two places to the left.

- 3 Study each example.

a $18\% = 0.18$

b $70\% = 0.70 = 0.7$

c $11.4\% = 0.114$

d $2.9\% = 0.029$

- 4 Now convert each percentage to a decimal.

a 61%

b 4%

c 90%

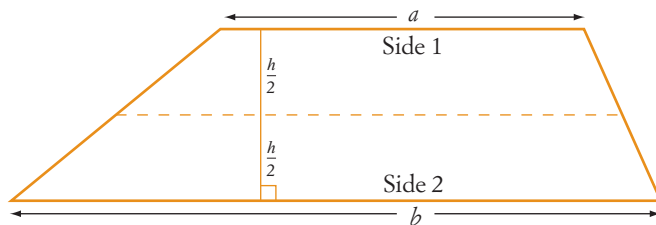
d 38%

e 27.1%

f 0.7%

Investigation: Area of a trapezium

- 1 Draw two copies of this trapezium onto a sheet of paper. Draw a dotted line, parallel to the sides labelled a and b , halfway down the height of each trapezium and label the sides as shown.



- 2 Cut out one trapezium and then cut along the dotted line to make two smaller trapeziums. Join the pieces to make a long parallelogram.
- 3 By measuring the height and base of the parallelogram to the nearest millimetre, find its area.
- 4 By comparing the measurements of the parallelogram to the measurements of the original trapezium, suggest a general formula for finding the area of any trapezium. Check your answer with your teacher.
- 5 Cut out the original trapezium and paste it and the parallelogram into your book.

5-05 Area of a trapezium

Video tutorial

Area formulas for
triangles and
quadrilaterals

MAT08MGVT00010

Technology

GeoGebra: Area of a
trapezium

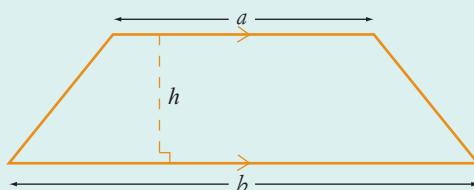
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Summary

Area of a trapezium

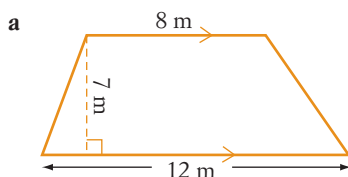
$A = \frac{1}{2} \times \text{sum of lengths of parallel sides} \times \text{perpendicular height}$

$$A = \frac{1}{2}(a + b)h$$



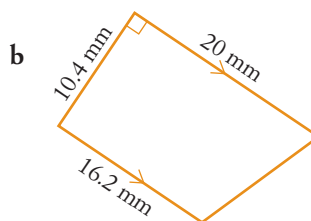
Example 6

Find the area of each trapezium.



Solution

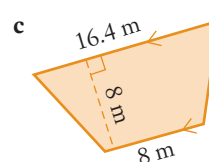
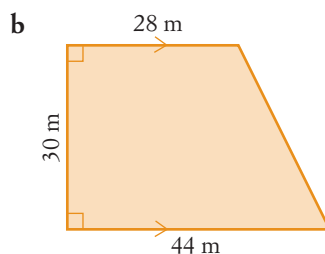
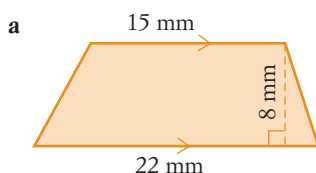
$$\begin{aligned} \mathbf{a} \quad A &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2}(8 + 12) \times 7 \\ &= 70 \text{ m}^2 \end{aligned}$$



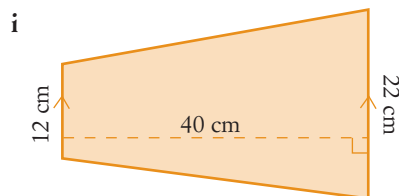
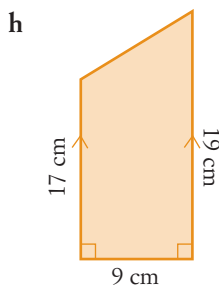
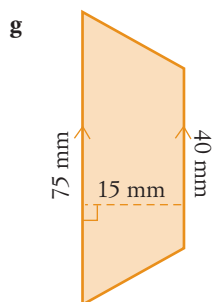
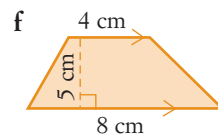
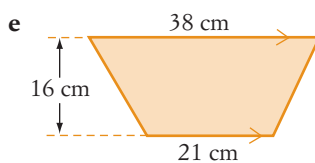
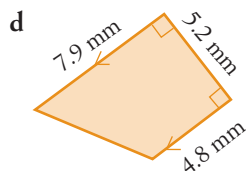
$$\begin{aligned} \mathbf{b} \quad A &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2}(20 + 16.2) \times 10.4 \\ &= 188.24 \text{ mm}^2 \end{aligned}$$

Exercise 5-05 Area of a trapezium

1 Find the area of each trapezium.



See Example 6

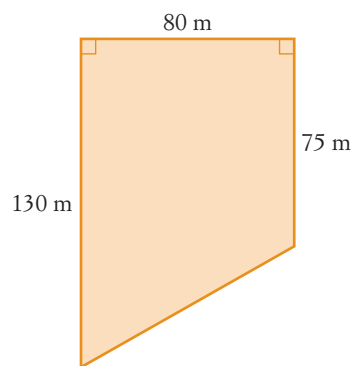


Worked solutions

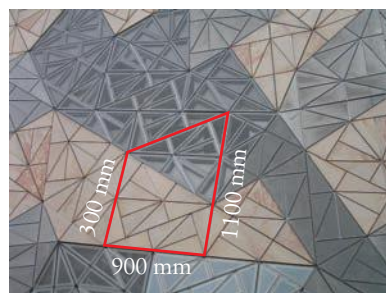
Exercise 5-05

MAT08MGWS10037

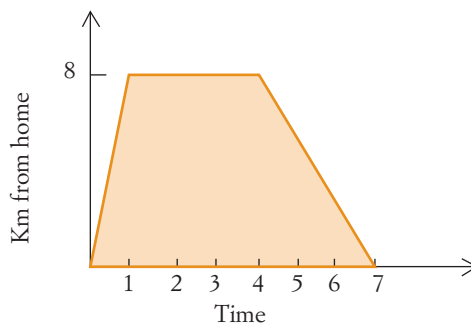
- 2 **a** Find the area of the block of land shown here.
b Council regulations state that $\frac{1}{5}$ of the land must be reserved for gardens. How much land is available for building?



- 3 Find the area of the trapezoidal shape in the photo of the exterior of Federation Square, Melbourne.



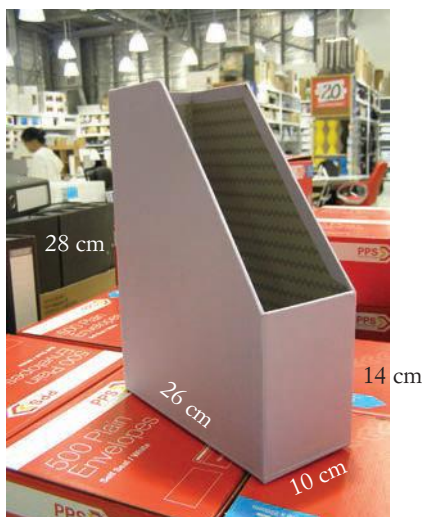
- 4 The travel graph shows a trip made by Sabrina.



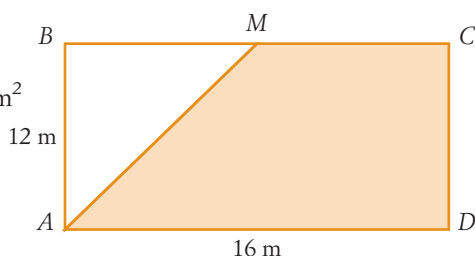
What is the shaded area in the graph? Select **A**, **B**, **C** or **D**.

- A** 44 units² **B** 48 units² **C** 42 units² **D** 40 units²

- 5 A magazine holder has trapezoidal sides as shown.
- Find the area of one of the trapezoidal sides.
 - Find the total area of cardboard needed to build this holder.

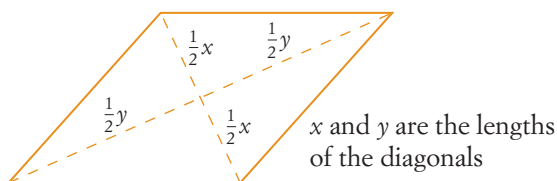
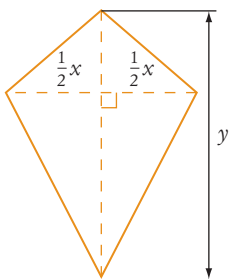


- 6 In the diagram, M is halfway between B and C . Calculate the shaded area. Select **A**, **B**, **C** or **D**.
- A** 96 m^2 **B** 132 m^2 **C** 224 m^2 **D** 144 m^2



Investigation: Area of a kite and a rhombus

- 1 Draw two copies of the kite and rhombus shown below on a sheet of paper. Draw the diagonals of length x and y and label them as shown. Notice that they cross at right angles. For the kite, one diagonal (x) is bisected by the other (y), while for the rhombus, *both* diagonals bisect each other.



- Cut out one of your kites and rhombuses and cut along their diagonals to make four triangles each. Join each set of triangles to make a rectangle.
- By comparing the measurements of the kite to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any kite. Check your answer with your teacher.
- What is the definition of a kite?
 - Is a rhombus a special type of kite?

- 5 By comparing the measurements of the rhombus to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any rhombus. Check your answer with your teacher.
- 6 Cut out the original kite and rhombus and paste all shapes into your book.

Homework sheet

Area 2

MAT08MGHS10027

Video tutorial

Area formulas for
triangles and
quadrilaterals

MAT08MGVT00010

Worksheet

Quadrilateral and
triangle areas

MAT08MGWK00066

Puzzle sheet

Areas of quadrilaterals
and triangles

MAT08MGPS00044

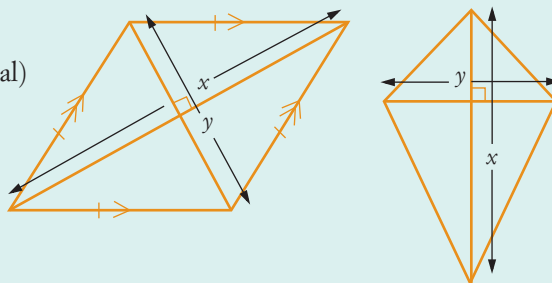
5-06 Areas of kites and rhombuses

Summary

Area of a kite or a rhombus

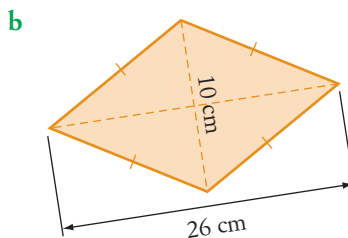
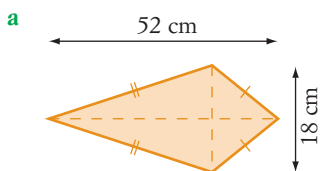
$$A = \frac{1}{2} \times (\text{diagonal}) \times (\text{other diagonal})$$

$$A = \frac{1}{2}xy$$



Example 7

Find the area of each shape.



Solution

a

$$\begin{aligned} A &= \frac{1}{2}xy \\ &= \frac{1}{2} \times 18 \times 52 \\ &= 468 \text{ cm}^2 \end{aligned}$$

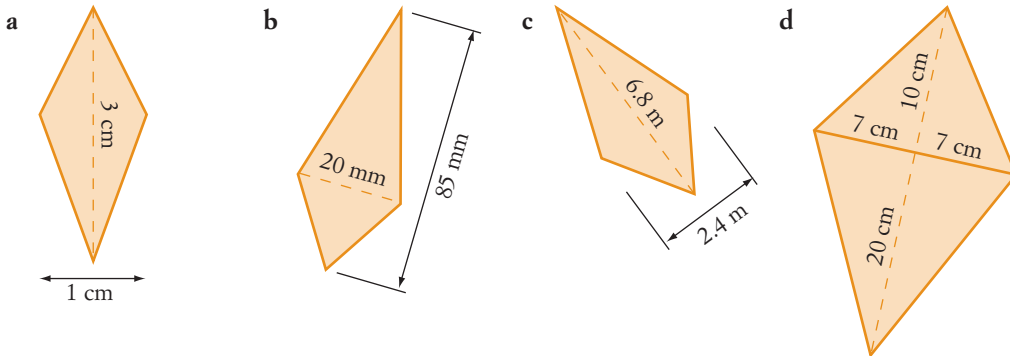
b

$$\begin{aligned} A &= \frac{1}{2}xy \\ &= \frac{1}{2} \times 10 \times 26 \\ &= 130 \text{ cm}^2 \end{aligned}$$

Exercise 5-06 Areas of kites and rhombuses

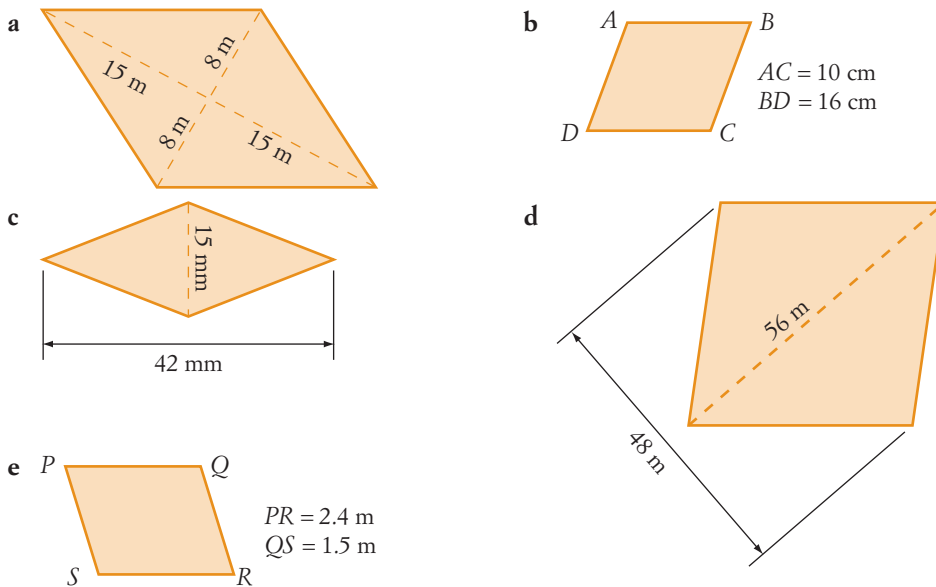
1 Find the area of each kite.

See Example 7



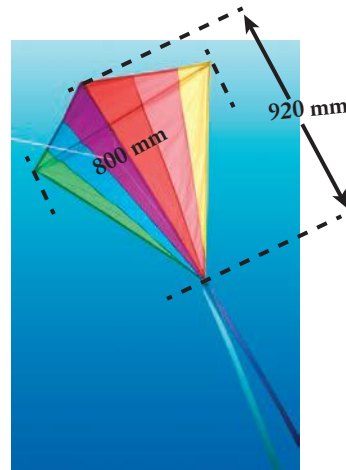
2 Find the area of each rhombus.

See Example 7

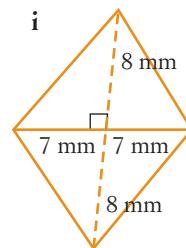
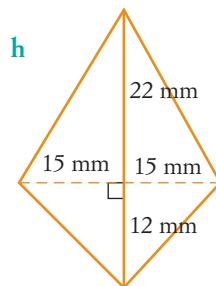
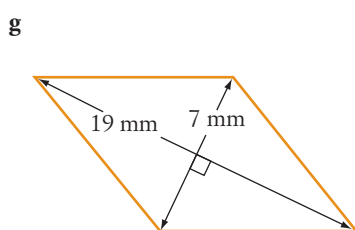
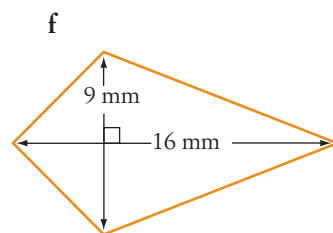
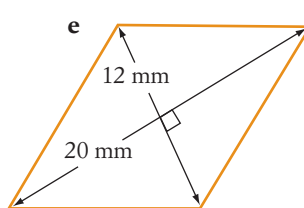
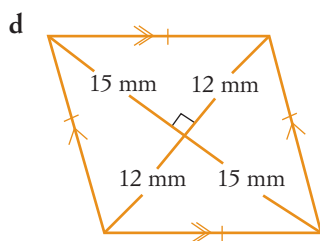
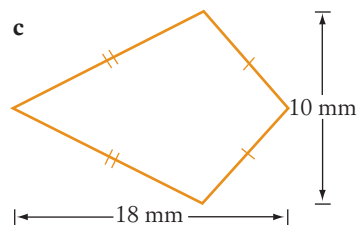
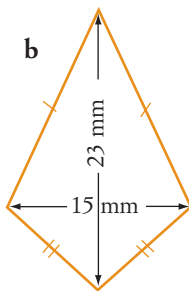
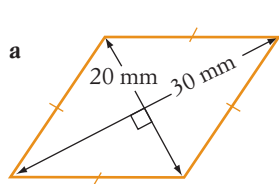


3 What area of material in cm^2 is needed to build this kite?
Select **A**, **B**, **C** or **D**.

- A** 36.8 cm^2 **B** 3680 cm^2
C 73.6 cm^2 **D** 7360 cm^2



4 Find the area of each shape.



Worked solutions

Exercise 5-06

MAT08MGWS10038

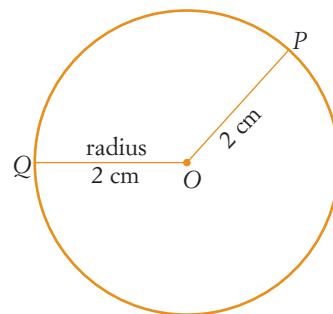
Worksheet

Parts of a circle

MAT08MGWK10041

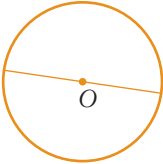
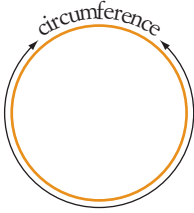
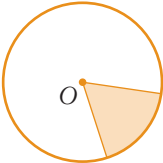
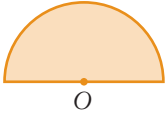
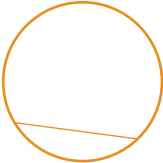
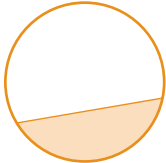
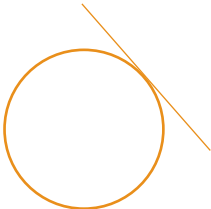
5-07 Parts of a circle

The circle is a completely round shape. Every point on a circle is the same distance from its centre. The circle shown has centre O , and points P and Q are exactly 2 cm from O . This distance, from the centre to the edge of the circle, is called the **radius** of the circle (*plural: radii*).



Circle words

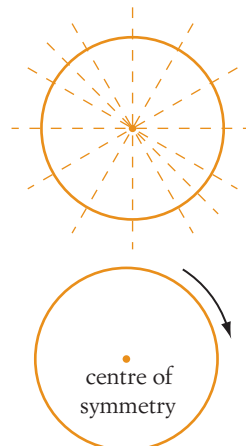
There are special words that describe the parts of a circle or the related lines.

Diameter	Circumference	Arc
An interval running from one side of a circle to the other side and through the circle's centre 	The perimeter of a circle 	Part of the circumference 
Sector	Quadrant	Semicircle
A 'pizza-slice' of a circle cut along two radii 	A sector that is a quarter of a circle, with angle 90° 	Half a circle 
Chord	Segment	Tangent
An interval joining any two points on a circle 	A region cut off by a chord 	A line outside a circle that touches it at exactly one point 

Circle symmetry

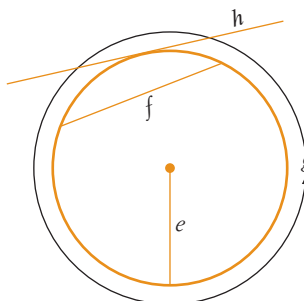
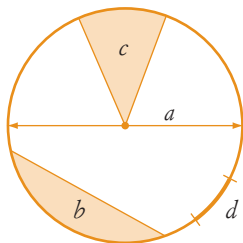
A circle can be folded in half in an infinite number of ways, along any diameter, so it has an *infinite* number of axes of symmetry.

A circle can also be rotated through any angle size and still map onto itself, so it has an *infinite* order of rotational symmetry. The centre of symmetry is the centre of the circle.



Exercise 5-07 Parts of a circle

- Use a ruler and compasses to construct a circle of:
 - radius 4 cm
 - radius 2.5 cm
 - radius 56 mm
- Name the parts of the circle marked by letters in these diagrams.



- Match each of the following words to the correct description below.

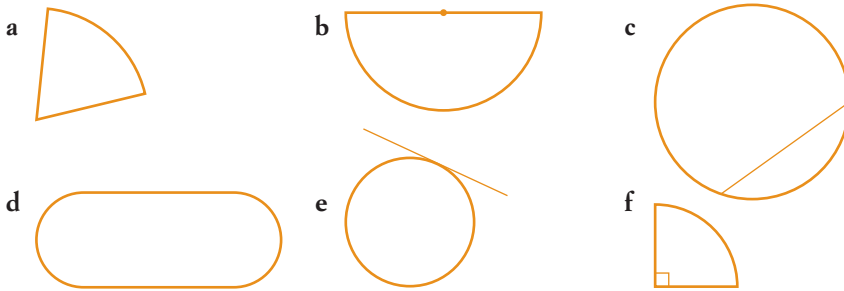
circumference	radius	sector
tangent	arc	quadrant
segment	diameter	chord

 - the distance from the centre of a circle to its side
 - a quarter of a circle
 - a line that touches the outside of a circle once
 - the distance from one side of a circle to the other side, through the circle's centre
 - a line segment from one side of the circle to the other side, not through the centre
 - part of the circumference of a circle
 - the area inside a circle formed by two radii and an arc
 - the distance around a circle
 - the area inside a circle formed by a chord and an arc
- An ellipse is a 'flattened' circle. Which of the following is the number of axes of symmetry of an ellipse? Select the correct answer **A**, **B**, **C** or **D**.

A 1	B 2	C 4	D an infinite number
- Which of the following is the order of rotational symmetry of an ellipse? Select **A**, **B**, **C** or **D**.

A 1	B 2	C 4	D infinite number
- Sketch a diagram of each term.

a diameter	b circumference	c chord
d sector	e tangent	f segment
g quadrant	h arc	i radius
- Copy each shape below. Then state:
 - the number of axes of symmetry in the shape and draw them
 - the order of rotational symmetry of the shape.

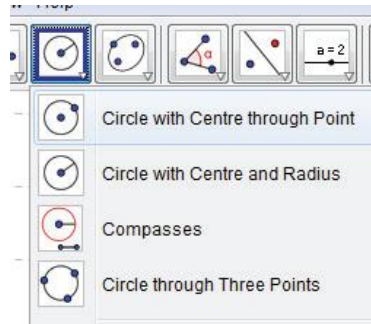


- 8 Using a radius of 4 cm, draw:
- a semicircle
 - a quadrant
 - a sector whose angle at the centre of the circle is 60°
 - an arc that is longer than half the circumference of the circle
 - a circle with a chord of length 2.5 cm
 - a circle with a tangent of length 4 cm.

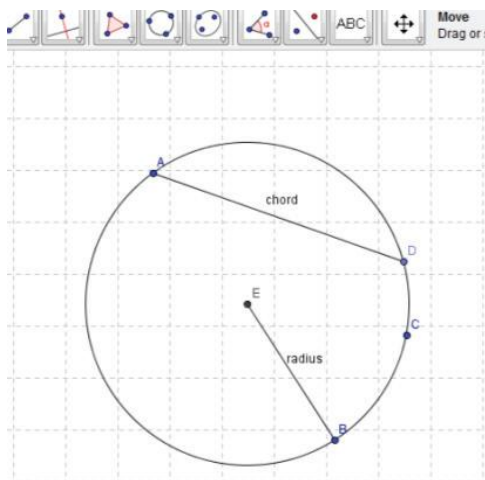
Technology Circle parts

In this activity you will use GeoGebra to construct a circle and display all the circle parts.

- Click **View Grid** and click off the **Axes**.
To construct a large circle, use **Circle with Centre through Point** or **Circle with Centre and Radius** (suggest radius of 4 units) or **Circle through Three Points**.



- Click **Interval between Two Points** and draw a radius and a chord in your circle.
- Click **Insert Text** to label the chord and the radius (as shown).



- 4 Construct each of the circle parts below on your page and label each one.

centre diameter circumference sector arc
segment semicircle tangent quadrant

- 5 What are **concentric circles**? Find out and construct a diagram showing three or more concentric circles. Remember to clearly label the centre of the circle.

Investigation: Measuring the circumference of a circle

Worksheet

Discovering pi

MAT08MGWK10042

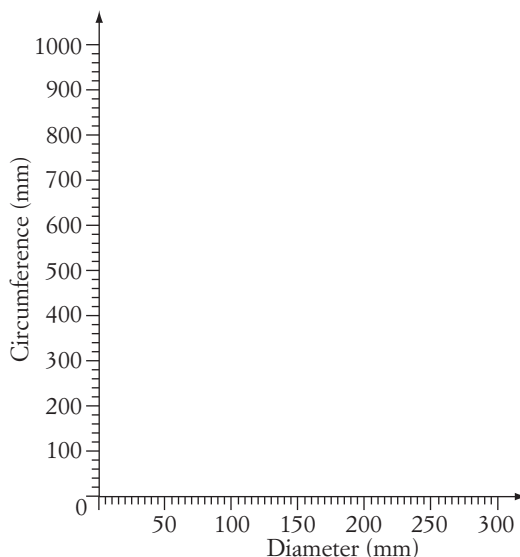
This is an indoor measuring activity to be done in groups of three.

You will need: A measuring tape, or some string and a ruler; graph paper; six round objects such as cans, round cake tins, pipes, money, drums, bottles, and so on.

- 1 Copy the following results table into your notebook.

Object	Diameter, d (mm)	Circumference, C (mm)	The ratio $\frac{C}{d}$
a			
b			
c			
d			
e			
f			

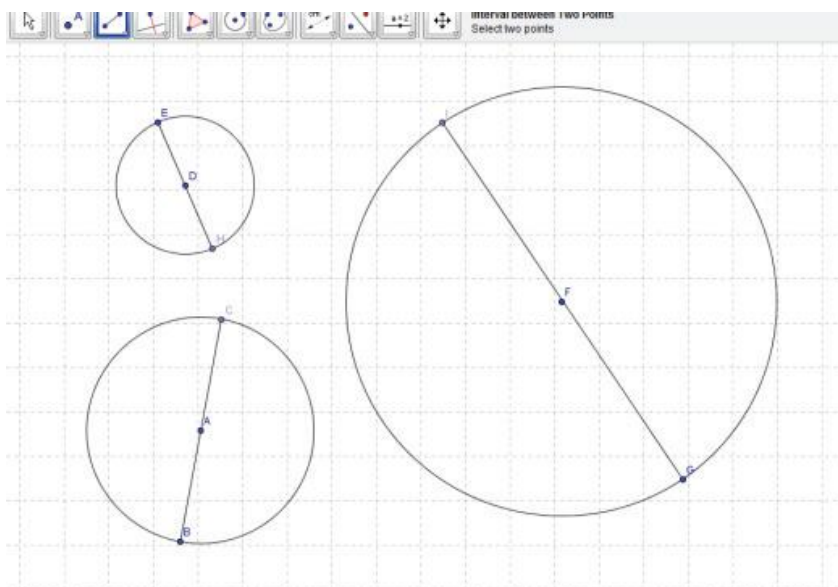
- 2 Measure (in millimetres) to find the diameter and the circumference of six different objects. Record your results in the table.
- 3 Calculate the ratio $\frac{C}{d}$ for each object. Round your answers to two decimal places.
- 4 Draw a graph similar to the one below, showing the diameter and the circumference for each object. What do you notice about the graph?



- 5 It seems that there is a formula for finding the circumference of a circle. Copy and complete: $C = d \times$ _____

Technology The circumference and diameter of a circle

- 1 Use GeoGebra to construct three different-sized circles. Use **Interval between Two Points** to construct the diameter of each circle.



- 2 Use **Distance or Length** to calculate each circle's circumference and diameter.
- 3 For each circle, use your calculator to find $\frac{\text{length of circumference}}{\text{length of diameter}}$
- 4 What do you notice? Discuss this with another student.

5-08 Circumference of a circle

The **circumference** of a circle can be found by multiplying its **diameter** by a special number called **pi** (pronounced 'pie'), represented by the Greek letter π . For any circle,

$$\frac{\text{circumference}}{\text{diameter}} = \pi = 3.14 \dots$$

Pi is often estimated as 3.14, but a more accurate value can be found on your calculator when you press the π key (you may need to press **SHIFT** first). As a decimal, the digits of π continue endlessly without any repeats or patterns:

$$\pi = 3.141\,592\,653\,589\,793 \dots$$

so like $\sqrt{2}$ it is called an **irrational number**.

This number was named 'pi' by the Swiss mathematician Euler in 1737.

The formula for the circumference of a circle is $C = \pi \times \text{diameter} = \pi d$.

Because the diameter of a circle is double its radius, the circumference, C , of a circle with radius r is $C = \pi \times 2 \times \text{radius} = 2\pi r$.

Worksheet

A page of circles

MAT08MGWK10043

Worksheet

Circumference and
area

MAT08MGWK10044

Puzzle sheet

Circle crossword

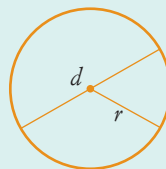
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Summary

The **circumference (perimeter) of a circle** is:

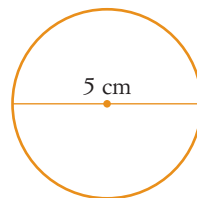
$$C = \pi \times \text{diameter} \quad \text{or} \quad C = 2 \times \pi \times \text{radius}$$

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$



Example 8

- a Estimate the circumference of a circle with a diameter of 5 cm.
- b Calculate the circumference of the circle:
- correct to two decimal places
 - in terms of π



Solution

a $C = \pi d$

$$= \pi \times 5$$

$$\approx 3 \times 5$$

$$= 15 \text{ cm}$$

π is approximately 3.

b i $C = \pi d$

$$= \pi \times 5$$

$$= 15.707\ 963\ \dots$$

$$\approx 15.71 \text{ cm}$$

On your calculator enter $\pi \times 5 =$.

ii $C = \pi d$

$$= \pi \times 5$$

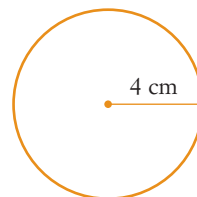
$$= 5\pi \text{ cm}$$

Like writing an answer as a surd in Pythagoras' theorem problems, writing an answer in terms of π is more exact as there is no rounding involved.

Example 9

Calculate the circumference of the circle:

- a correct to one decimal place
- b in terms of π



Solution

a $C = 2\pi r$

$$= 2 \times \pi \times 4$$

$$= 25.13274\ \dots$$

$$\approx 25.1 \text{ cm}$$

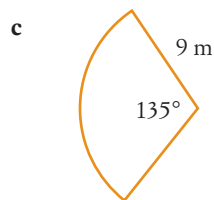
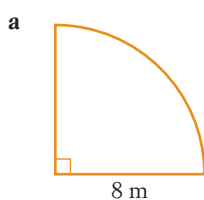
b $C = 2\pi r$

$$= 2 \times \pi \times 4$$

$$= 8\pi \text{ cm}$$

Example 10

Find, correct to two decimal places, the perimeter of:

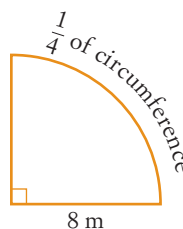


Solution

a Perimeter of quadrant

$$\begin{aligned} &= \frac{1}{4} \times \text{circumference} + \text{radius} + \text{radius} \\ &= \frac{1}{4} \times 2 \times \pi \times 8 + 8 + 8 \\ &= 28.566\,370 \dots \\ &\approx 28.57 \text{ cm} \end{aligned}$$

$$C = 2\pi r$$

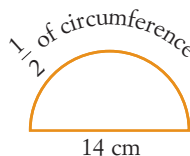


From the diagram, a circumference of 28.57 cm looks reasonable

b Perimeter of semicircle

$$\begin{aligned} &= \frac{1}{2} \times \text{circumference} + \text{diameter} \\ &= \frac{1}{2} \times \pi \times 14 + 14 \\ &= 35.991\,148 \dots \\ &\approx 35.99 \text{ cm} \end{aligned}$$

$$C = \pi d$$



c Perimeter of sector

$$\begin{aligned} &= \frac{135}{360} \times \text{circumference} + \text{radius} + \text{radius} \\ &= \frac{135}{360} \times \pi \times 9 \times 2 + 9 + 9 \\ &= 39.205\,75 \dots \\ &\approx 39.21 \text{ m} \end{aligned}$$

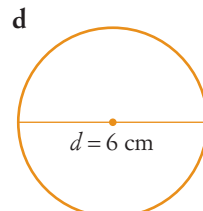
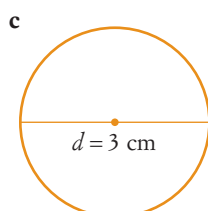
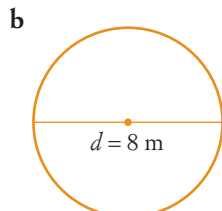
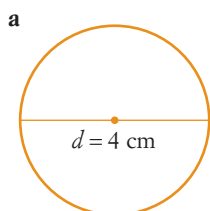
$$C = 2\pi r$$

There are 360° in a circle, but a sector is a fraction of a circle

Exercise 5-08 Circumference of a circle

1 Estimate the circumference of each circle.

See Example 8



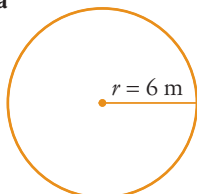
2 Calculate, correct to two decimal places, the circumference of each circle in question 1.

3 Calculate the circumference of each circle in question 1 in terms of π .

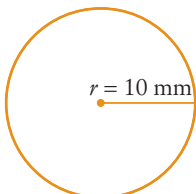
See Example 9

4 Estimate the circumference of each circle.

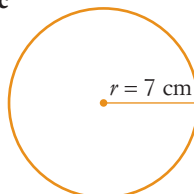
a



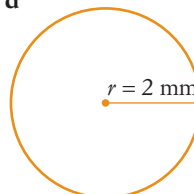
b



c



d



5 Calculate, correct to one decimal place, the circumference of each circle in question 4.

6 Calculate the circumference of each circle in question 4 in terms of π .

7 A child's inflatable swimming pool has a diameter of 1.4 m. Find its circumference.

8 Tina's bicycle has wheels with a diameter of 60 cm.

a How far does the bicycle move when a wheel turns around once?

b If Tina cycles 900 m to school, how many complete turns does the bicycle wheel make?

9 The Earth has a radius of 6370 km. Find the distance around the Equator.

10 A 20-cent coin has a radius of 16 mm. Calculate its circumference.

11 This tin of tomatoes has a diameter of 75 mm. If the label wraps around the tin completely, how long is the label? Answer correct to the nearest millimetre.



12 Calculate, correct to 2 decimal places, the circumference of a circle with:

a a diameter of 2 cm

b a diameter of 7 cm

c a radius of 3 cm

d a radius of 5 cm

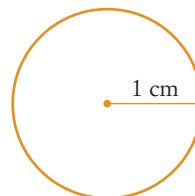
13 Which of the following intervals best shows the circumference of this circle? Select A, B, C or D.

A _____

B _____

C _____

D _____



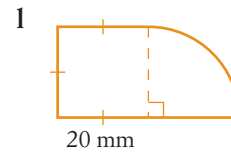
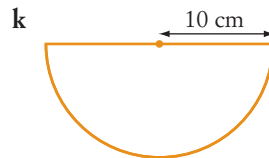
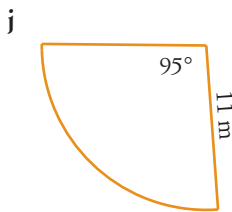
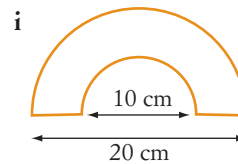
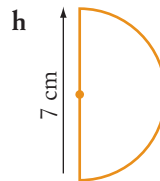
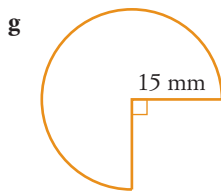
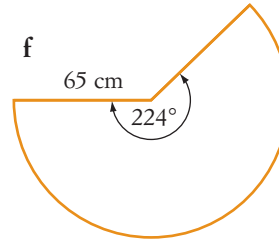
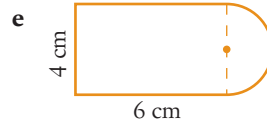
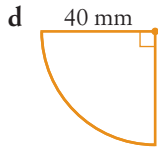
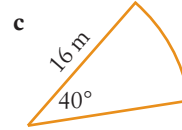
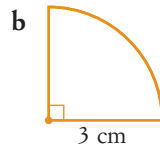
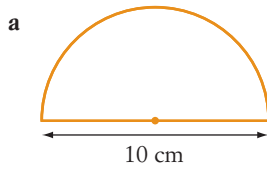
Worked solutions

Exercise 5-08

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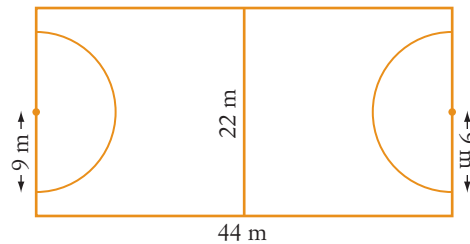
14 Calculate the perimeter of each shape, correct to one decimal place.

See Example 10



15 Tape has been placed on all the lines of this indoor hockey pitch. How much tape was used? Express the answer:

- a correct to one decimal place
- b in terms of π .



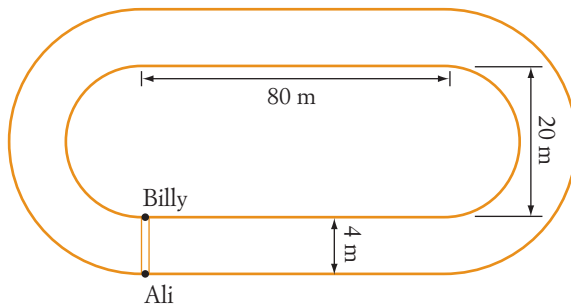
Worked solutions

Exercise 5-08

MAT08MGWS10039

16 A circle has a circumference of 50.265 cm. Find its diameter, correct to the nearest centimetre.

17 Ali and Billy raced each other around this athletic track. Ali ran along the outside perimeter while Billy ran along the inside perimeter. After one lap of the track, who ran the longer distance, and by how much? Answer correct to the nearest metre.



Mental skills 5B

Maths without calculators

Comparing fractions, decimals and percentages

To compare or order fractions, express them with a common denominator first. To compare or order decimals, express them with the same number of decimal places first.

1 Study each example.

a Which fraction is smaller: $\frac{4}{10}$ or $\frac{3}{8}$?

Using a common denominator of 80 (8×10)	Using a common denominator of 40 (the LCM of 10 and 8)
$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$	$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$
$\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$	$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$
By comparing numerators: $\frac{30}{80} < \frac{32}{80}$	By comparing numerators: $\frac{15}{40} < \frac{16}{40}$
$\therefore \frac{3}{8}$ is smaller.	$\therefore \frac{3}{8}$ is smaller.

b Arrange these decimals in ascending order: 0.407, 0.47, 0.047, 0.4.

Express all decimals with three decimal places by inserting zeros at the end where necessary.

0.407 0.470 0.047 0.400

By comparing digits in the same decimal place, the decimals from smallest to largest are: 0.047 0.400 0.407 0.470

That is, 0.047, 0.4, 0.407, 0.47

2 Now find the smaller number in each pair.

a $\frac{2}{3}$ and $\frac{3}{5}$

b $\frac{1}{4}$ and $\frac{1}{3}$

c 0.15 and 0.105

d 3.826 and 3.68

e $\frac{3}{7}$ and $\frac{4}{10}$

f $\frac{5}{6}$ and $\frac{7}{8}$

g 2.87 and 2.817

h 0.5301 and 0.503

i $\frac{1}{5}$ and $\frac{2}{9}$

3 Arrange each set of numbers in ascending order.

a 0.81, 0.082, 0.821

b 3.5, 3.51, 3.55, 3.513

c $\frac{2}{3}$, $\frac{1}{6}$, $\frac{2}{5}$

d 0.007, 0.07, 0.7, 0.707

e $\frac{3}{8}$, $\frac{2}{10}$, $\frac{1}{2}$

f $\frac{1}{4}$, $\frac{4}{10}$, $\frac{3}{5}$

- 4 Study this example.

Arrange these numbers in descending order: 68% , $\frac{13}{20}$, $0.\dot{6}$, $\frac{3}{5}$.

To order fractions, decimals and percentages, express them all as percentages first.

As percentages: $68\% = 68\%$, $\frac{13}{20} = 65\%$, $0.\dot{6} = 66.66\ldots\%$, $\frac{3}{5} = 60\%$.

Largest to smallest, this is 68% , 66.66% , 65% , 60% , that is, 68% , $0.\dot{6}$, $\frac{13}{20}$, $\frac{3}{5}$.

- 5 Now arrange each set of numbers in descending order.

a 0.25 , $\frac{1}{6}$, 16% , $\frac{1}{5}$

b 27% , $\frac{1}{3}$, 0.4 , 0.28

c 0.05 , 50% , 6% , $\frac{1}{8}$

d $\frac{3}{4}$, 0.639 , 55% , $\frac{2}{5}$

e 69% , 0.609 , $\frac{2}{3}$, 0.6

f $\frac{2}{9}$, 0.105 , 17% , 22.5%

Just for the record

Never-ending pi!

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279\ 502\ 884\ 197\ 169\ 399\ 375\ 105\ 820\ldots$

Ancient civilisations knew about the value of π , estimating it as 3. Over time, the calculations have improved due to better formulas and technology. Since the first computer, the ENIAC, was invented in 1949, much progress has been made. In 2010, two Japanese and American system engineers took 90 days to calculate π to five trillion decimal places.

Year	Person/Country	Number of decimal places
1855	Shanks, England	527
1949	ENIAC computer, USA	2037
1973	CDC 7600 computer, France	1 000 000
1988	Kanada, Hitachi S-820 computer, Japan	200 000 000
1989	Chudnovsky brothers, USA	1 000 000 000
1999	Kanada and Takahashi, Hitachi SR8000 computer, Japan	206 158 430 000
2002	Kanada, Hitachi SR8000 computer, Japan	1 241 100 000 000
2010	Shigeru Kondo, Alexander Yee, Japan and USA	5 trillion
???	???	???

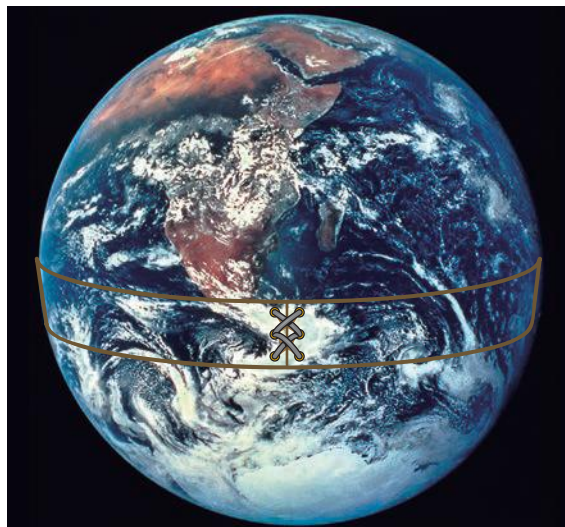
In 2006, Japanese counsellor Akira Haraguchi memorised π to 100 000 decimal places. It took him over 16 hours to recite it. He had to stop after three hours as he lost his place, and had to start from the beginning.

On average, how many digits would Haraguchi have recited per minute?

Investigation: Belt around the Equator

Imagine that we wrapped a giant belt tightly around the Earth, along the Equator. This belt would touch the Earth at all points on its circumference, assuming the Earth was a perfect sphere or ball (with no mountains or valleys).

Now suppose we added one extra metre to the length of the belt. Then it would become loose and not touch the Earth any more. There would be a small gap between the Earth and the belt. How wide is this gap?

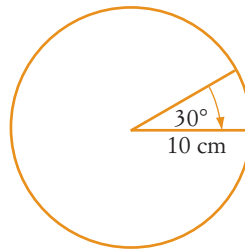
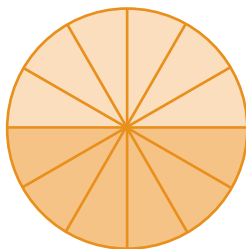


- 1 In a group of two to four people, guess whether you could:
 - a slip your hand between the belt and the ground
 - b crawl under the belt
 - c sit under the belt
 - d stand under the belt.
- 2 If the diameter of the Earth is 12 755 metres, calculate to two decimal places, the length of the tight belt.
- 3 Add 1 metre to the length of the tight belt.
- 4 Calculate the diameter of the longer, looser belt.
- 5 So what is the length of the gap between the Earth and the loose belt?
- 6 Check whether your guesses in question 1 were correct.

Investigation: Area by cutting out sectors

You will need: compasses, a ruler, a protractor and a pair of scissors.

- 1 Draw a circle of radius 10 cm. Use your protractor to divide the circle into 12 sectors, each with angle size 30° .

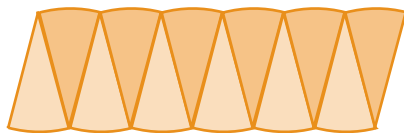


Worksheet

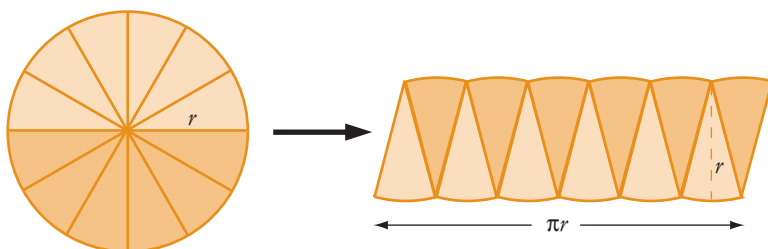
Approximating the
area of a circle

MAT08MGWK00067

- 2 Cut out all the sectors and arrange them into a shape like a parallelogram.



- 3 Use a ruler to measure the *base* and the *height* of your 'parallelogram'.
4 Now calculate the area of the 'parallelogram'.
5 Which formula gives the better approximation for the area of this circle: $A = 3 \times r^2$, or $A = 4 \times r^2$?
6 We will now find the actual formula for the area of a circle. A circle of radius r is cut up into many sectors and rearranged into a 'parallelogram'.



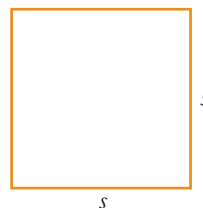
What is the formula for the circumference of this circle?

- 7 Explain why the length of the 'parallelogram' is πr .
8 What is the formula for the area of this 'parallelogram'?
9 Explain why the area of the circle is πr^2 .

5-09 The area of a circle

The area of a square is calculated by squaring its length. $A = s \times s = s^2$

In a similar way, the area of a circle is calculated by squaring its radius and multiplying by π .



Worksheet

A page of circles

MAT08MGWK10043

Worksheet

Circumference and area

MAT08MGWK10044

Worksheet

Area ID

MAT08MGWK10045

Homework sheet

Circumference and area of a circle

MAT08MGHS10028

Worksheet

Area and perimeter investigations

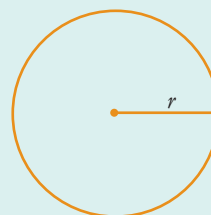
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Summary

The **area of a circle** is:

$$A = \pi \times (\text{radius})^2$$

$$A = \pi r^2$$



Puzzle sheet

Circle areas

MAT08MGPS00045

Technology worksheet

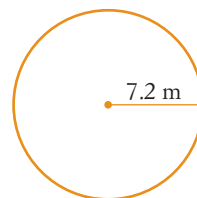
Excel: Area calculator

MAT08MGCT00012

Example 11

Calculate the area of this circle:

- a** correct to 2 decimal places
b in terms of π



Solution

$$\begin{aligned}\mathbf{a} \quad A &= \pi r^2 \\ &= \pi \times 7.2^2 \\ &= 162.8601 \dots \\ &\approx 162.86 \text{ m}^2\end{aligned}$$

On a calculator enter: $\pi \times 7.2^2 =$

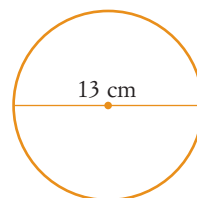
$$\begin{aligned}\mathbf{b} \quad A &= \pi r^2 \\ &= \pi \times 7.2^2 \\ &= 51.84\pi \text{ m}^2\end{aligned}$$

Area is measured in square units, such as m^2 .

Example 12

Find the area of this circle:

- a** correct to one decimal place
b in terms of π



Solution

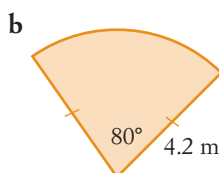
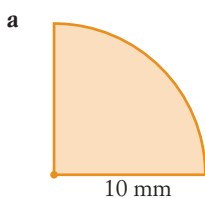
$$\begin{aligned}\mathbf{a} \quad r &= \frac{1}{2} \times 13 = 6.5 \text{ cm} \\ A &= \pi r^2 \\ &= \pi \times 6.5^2 \\ &= 132.7322 \dots \\ &\approx 132.7 \text{ cm}^2\end{aligned}$$

The radius is half of the diameter

$$\begin{aligned}\mathbf{b} \quad \text{From a,} \\ A &= \pi \times 6.5^2 \\ &= 42.25\pi \text{ cm}^2\end{aligned}$$

Example 13

Find, correct to one decimal place, the area of each shape.



Solution

$$\begin{aligned}\text{a Area of quadrant} &= \frac{1}{4} \times \text{area of circle} \\ &= \frac{1}{4} \times \pi r^2 \\ &= \frac{1}{4} \times \pi \times 10^2 \\ &= 78.5398 \dots \\ &\approx 78.5 \text{ mm}^2 \text{ (correct to two decimal places)}\end{aligned}$$

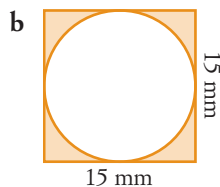
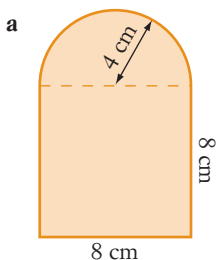
Area is measured in square units, such as mm^2

$$\begin{aligned}\text{b Area of sector} &= \frac{80}{360} \times \text{area of circle} \\ &= \frac{80}{360} \times \pi \times 4.2^2 \\ &= 12.315\,043 \dots \\ &\approx 12.3 \text{ m}^2\end{aligned}$$

There are 360° in a circle, but a sector is a fraction of a circle

Example 14

Find, correct to two decimal places, the area of each shaded region.



Solution

$$\begin{aligned}\text{a Area} &= \text{area of square} + \text{area of semicircle} \\ &= s^2 + \frac{1}{2} \times \pi r^2 \\ &= 8^2 + \frac{1}{2} \times \pi \times 4^2 \\ &= 89.132\,741 \dots \\ &\approx 89.13 \text{ cm}^2\end{aligned}$$

(correct to two decimal places)

$$\begin{aligned}\text{b Shaded area} &= \text{area of square} - \text{area of circle} \\ &= s^2 - \pi r^2 \\ &= 15^2 - \pi \times 7.5^2 \\ &= 48.285\,413 \dots \\ &\approx 48.29 \text{ mm}^2\end{aligned}$$

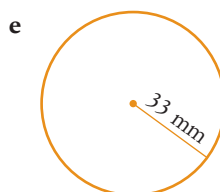
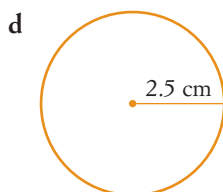
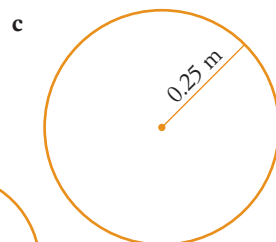
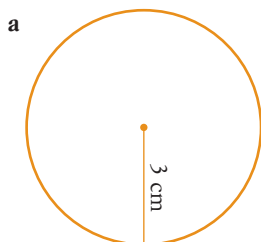
Radius: $r = \frac{1}{2} \times 15 = 7.5 \text{ mm}$

(correct to two decimal places)

Exercise 5-09 The area of a circle

See Example 11

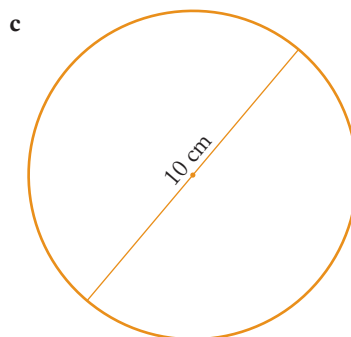
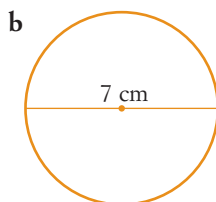
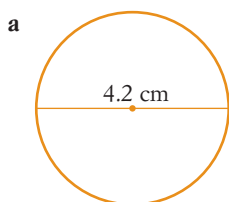
- 1 Calculate, correct to two decimal places, the area of each circle.



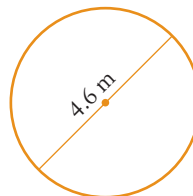
- 2 Calculate the area of each circle in question 1 in terms of π .

See Example 12

- 3 Calculate, correct to one decimal place, the area of each circle.



- 4 Calculate the area of each circle in question 3 in terms of π .
- 5 A sprinkler on the school playing field sprays water in a circular pattern of radius 13.1 m. Calculate the area being sprayed.
- 6 Kevlar is a very strong light plastic. What area of it is needed to make a solid bicycle wheel of diameter 685.5 mm? Give your answer in square centimetres, correct to two decimal places.
- 7 A dinner plate has a radius of 14 cm. Calculate its area in terms of π .
- 8 What is the area of this circle, correct to one decimal place? Select the correct answer **A**, **B**, **C** or **D**.
A 14.5 m^2 **B** 66.5 m^2 **C** 7.2 m^2 **D** 16.6 m^2



Worked solutions

Exercise 5-09

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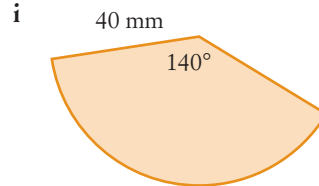
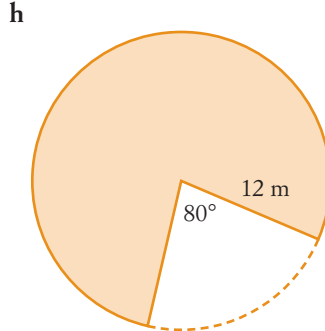
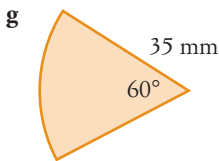
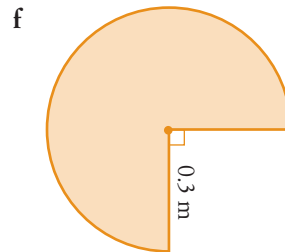
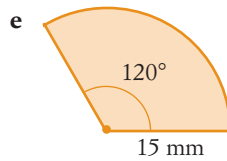
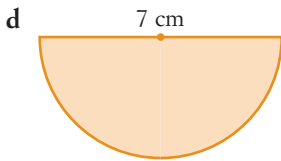
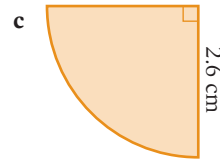
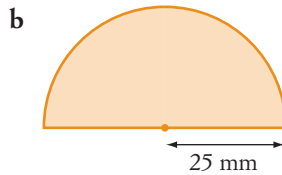
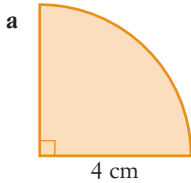
- 9 The circular floor of a fishpond is to be covered in plastic. Find, correct to one decimal place, the area of plastic needed if the diameter of the pond is 2.8 m.
- 10 Find the approximate radius of a circular tree trunk whose cross-sectional area is 454.40 cm^2 . Give your answer correct to two decimal places.

- 11 The area of a DVD is approximately 113 cm^2 . Which of the following is closest to its diameter? Select **A**, **B**, **C** or **D**.

A 12 cm **B** 13 cm **C** 12.5 cm **D** 13.1 cm

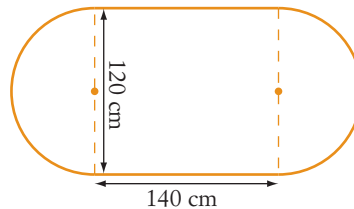
See Example 13

- 12 Find, correct to one decimal place, the area of each shape.



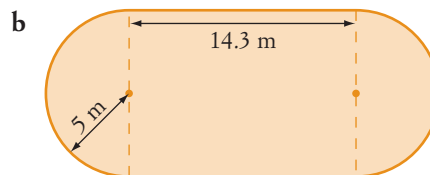
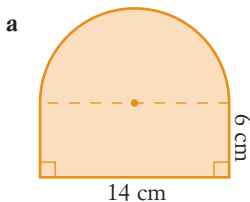
- 13 What is the area of this shape? Select the closest answer **A**, **B**, **C** or **D**.

A $28\,110 \text{ cm}^2$ **B** $22\,455 \text{ cm}^2$
C $62\,039 \text{ cm}^2$ **D** $11\,570 \text{ cm}^2$



See Example 14

- 14 Find the area of each shaded region, correct to two decimal places. *Hint: Where appropriate, combine two semicircles to make a whole circle.*

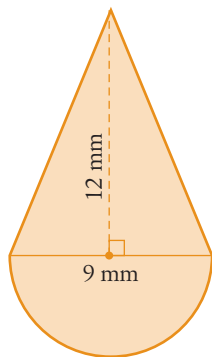


Worked solutions

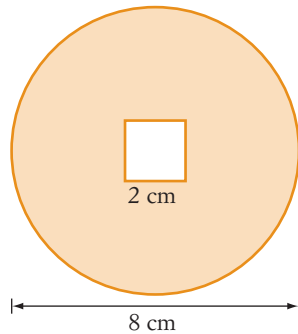
Exercise 5-09

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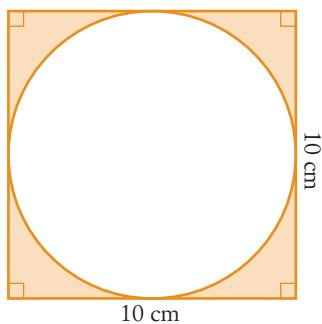
c



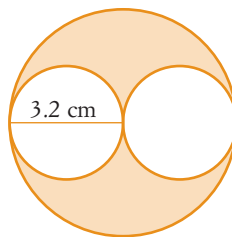
d



e

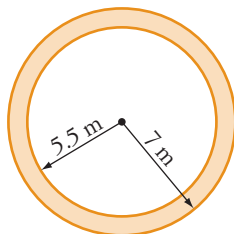


f

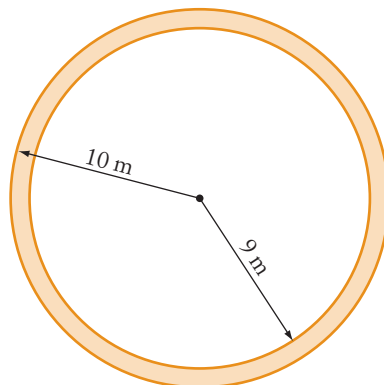


- 15 Each ring-shape or donut-shape shown below is called an **annulus**, made from two circles that are concentric (with the same centre) but of different size.

a

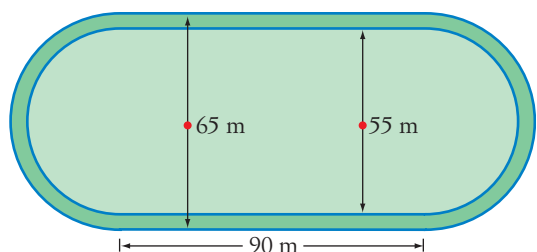


b



For each annulus, find, correct to one decimal place, the area of:

- the larger circle
 - the smaller circle
 - the (shaded) annulus
- 16 The dimensions of a running track are shown. The ends are circular.



- Find the combined area of the (dark green) track and the (light green) grassed central area.
- Find the area of the grassed area.
- Find the area of the track.
- It is decided to cover the track with synthetic grass that costs \$24.25 per square metre. How much will the track surface cost?

5-10 Metric units for volume

Skillsheet

What is volume?

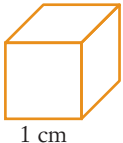
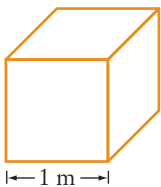
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Worksheet

Volume

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The **volume** of a solid is the amount of space it occupies. In the metric system, volume is measured in **cubic units**.

cubic centimetre (cm^3)	cubic metre (m^3)
$1 \text{ cm} = 10 \text{ mm}$ $1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$ $= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$ $= 1000 \text{ mm}^3$	$1 \text{ m} = 100 \text{ cm}$ $1 \text{ m}^3 = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$ $= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$ $= 1\,000\,000 \text{ cm}^3$
	

A **cubic centimetre** is about the volume of a person's tooth or a medical pill.

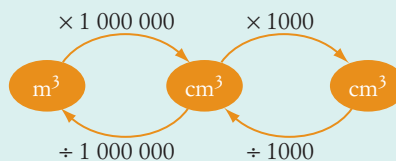
A **cubic metre** is about the volume of two washing machines.

Note that, while $1 \text{ cm} = 10 \text{ mm}$, $1 \text{ cm}^3 = 1\,000 \text{ mm}^3$ (triple the number of 0s), and while $1 \text{ m} = 100 \text{ cm}$, $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$ (triple the number of 0s).

Summary

$$1 \text{ cm}^3 = 1000 \text{ mm}^3$$

$$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$



Example 15

Convert:

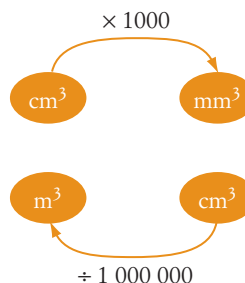
a 36 cm^3 to mm^3

b $84\,000\,000 \text{ cm}^3$ to m^3

Solution

a $36 \text{ cm}^3 = 36 \times 1000 \text{ mm}^3$
 $= 36\,000 \text{ mm}^3$

b $84\,000\,000 \text{ cm}^3 = 84\,000\,000 \div 1\,000\,000 \text{ m}^3$
 $= 84 \text{ m}^3$



Exercise 5-10 Metric units for volume

- 1 What unit of volume would you use when measuring the volume of:
- a a bedroom? b a backpack? c a mobile phone?
 d a concert hall? e your body? f an aeroplane?
- 2 A swimming pool has a volume of 38 m^3 . How many cubic centimetres is this pool? Select the correct answer **A, B, C, or D**.
- A 380 000 B 3 800 000 C 38 000 000 D 380 000 000
- 3 Copy and complete each statement.
- a $5000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$ b $1.6 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
 c $2 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$ d $4 \text{ m}^3 = \underline{\hspace{2cm}} \text{ mm}^3$
 e $6000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$ f $8.2 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
 g $0.007 \text{ m}^3 = \underline{\hspace{2cm}} \text{ mm}^3$ h $9\,600\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
 i $4000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$ j $160\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
 k $250 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$ l $12 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$
 m $0.18 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$ n $200\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
- 4 The volume of a chest of drawers is $272\,000 \text{ cm}^3$. Convert this to cubic metres.
- 5 What is the approximate volume of a brick? Select the correct answer **A, B, C, or D**.
- A 1000 cm^2 B 20 cm^2 C 1600 cm^3 D 2100 cm^3
- 6 How many cubic millimetres in 2.3 m^3 ? Select the correct answer **A, B, C, or D**.
- A 2 300 000 000 B 2300 C 23 000 D 0.0023
- 7 Match the correct volume (**A to G**) with each of the items (**a to g**) listed.
- a a bottle of liquid paper A 200 m^3
 b a box of tissues B 3890 m^3
 c a glass of water C 1250 cm^3
 d a bottle of lemonade D 5000 cm^3
 e a classroom E $20\,000 \text{ mm}^3$
 f a school hall F 250 cm^3
 g a cereal package G 2200 cm^3



- 8 The volume of a lunchbox is 2460 cm^3 . Convert this to cubic millimetres.

See Example 15

Worked solutions

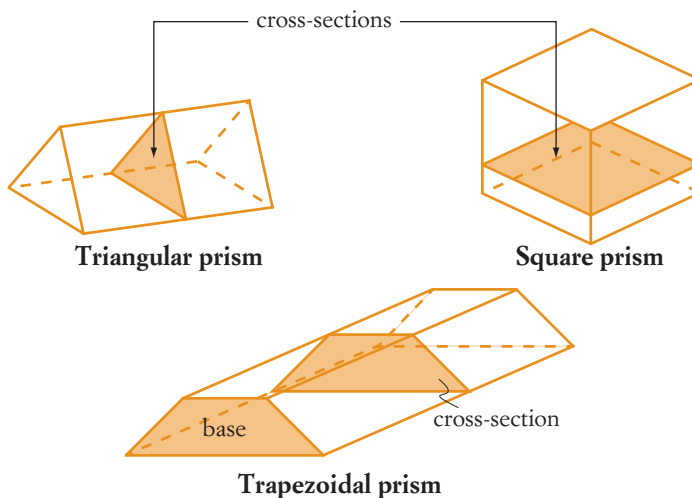
Exercise 5-10

MAT08MGWS10041

5-11 Volume of a prism

A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it.

If a solid has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides, not rounded), then the solid is called a **prism**. The following solids are all prisms.

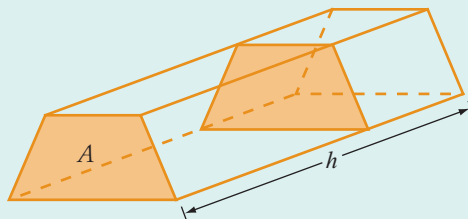


Prisms take their names from their cross-section. For example, the prism shown above is called a **trapezoidal prism** because its cross-sections are all trapeziums.

Because a prism is made up of identical cross-section 'slices' along its length, its volume can be calculated by multiplying the **area of the cross-section** by its **perpendicular height** (the length or depth of the prism).

Summary

Volume of a prism = area of cross-section $\times$ perpendicular height
 $= Ah$



Skillsheet

Solid shapes

MAT08MGSS10022

Skillsheet

What is volume?

MAT08MGSS10023

Worksheet

Volume

MAT08MGWK00070

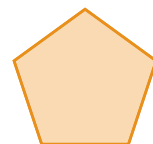
Worksheet

Measuring shapes
review

MAT08MGWK00071

Example 16

- What is the name of this polygon?
- Draw a prism with this polygon as a cross-section.
- What shape are the side faces?
- What is the name of the prism?



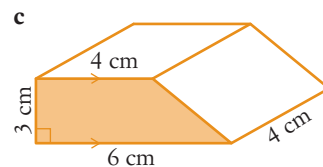
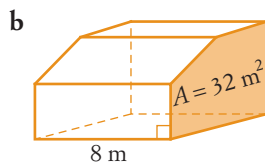
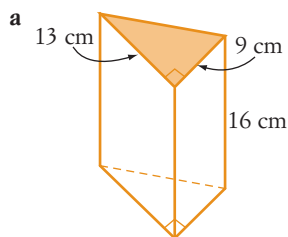
Solution

- Pentagon
-

- The side faces are all rectangles.
- This is a pentagonal prism.

Example 17

Find the volume of each prism.



Solution

$$\begin{aligned} \text{a } A &= \frac{1}{2} \times 9 \times 13 \\ &= 58.5 \text{ cm}^2 \\ V &= 58.5 \times 16 \\ &= 963 \text{ cm}^3 \end{aligned}$$

Area of shaded triangular cross-section.

$V = Ah$, where height $h = 16$ cm.

$$\begin{aligned} \text{b } A &= 32 \text{ m}^2 \\ V &= 32 \times 8 \\ &= 256 \text{ m}^3 \end{aligned}$$

Area of shaded cross-section

$V = Ah$, where height $h = 8$.

$$\begin{aligned} \text{c } A &= \frac{1}{2} \times (4 + 6) \times 3 \\ &= 15 \text{ cm}^2 \\ V &= 15 \times 4 \\ &= 60 \text{ cm}^3 \end{aligned}$$

Cross-section is a trapezium.

$V = Ah$, where height $h = 4$.

Video tutorial

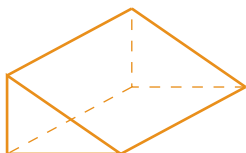
Volume of a prism

MAT08MGVT10009

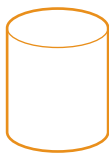
Exercise 5-11 Volume of a prism

1 State whether each solid is a prism or not.

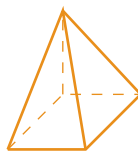
a



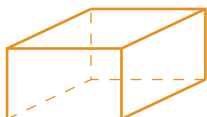
b



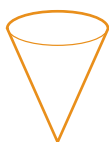
c



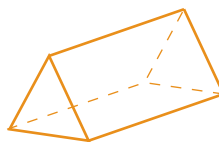
d



e



f



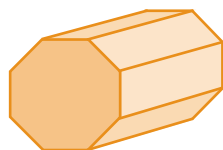
2 For each prism:

i draw its cross-section

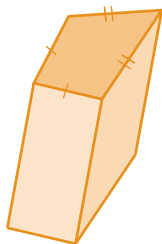
ii write the name of the prism

See Example 16

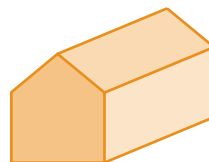
a



b



c



3 Draw each prism named below and shade its base.

a square prism

b isosceles triangular prism

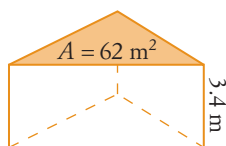
c trapezoidal prism

d hexagonal prism

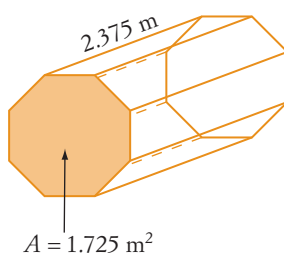
4 Find the volume of each prism.

See Example 17

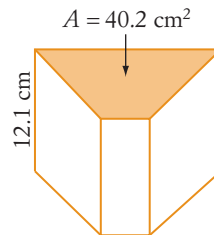
a



b

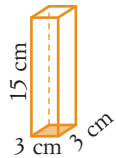


c

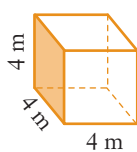


5 Find the volume of each rectangular prism using the formula $V = lwh$.

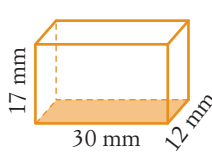
a



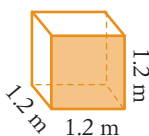
b



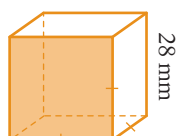
c



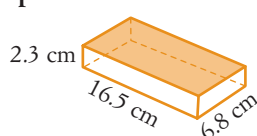
d



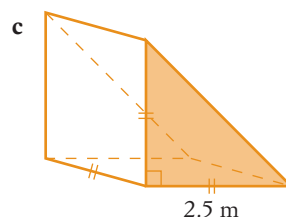
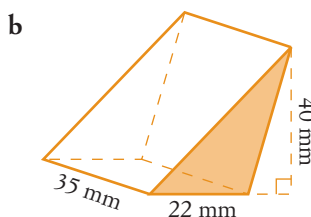
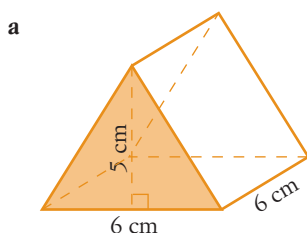
e



f



- 6 Find the volume of each triangular prism.

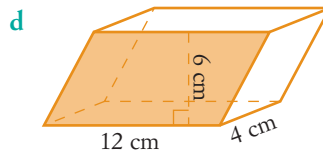
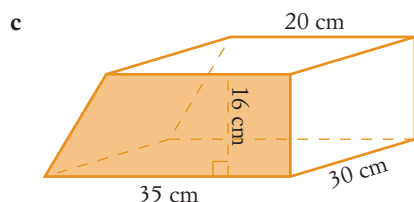
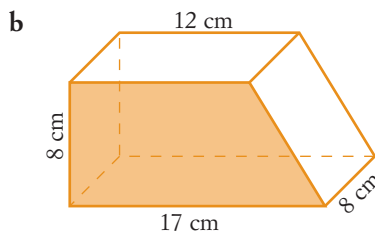
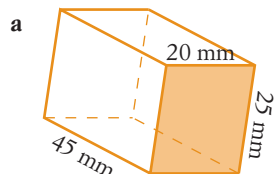


Worked solutions

Exercise 5-11

MAT08MGWS10042

- 7 Find the volume of each prism.

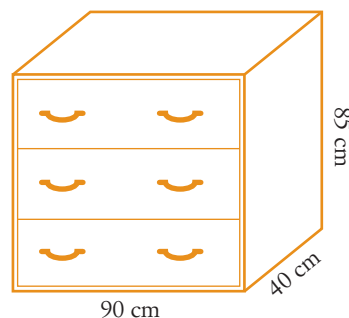


- 8 Find the volume of this chest of drawers.

Give the answer in m^3 :

- a** by calculating the volume in cm^3 , then converting to m^3
b by converting each length to metres first, then calculating the volume.

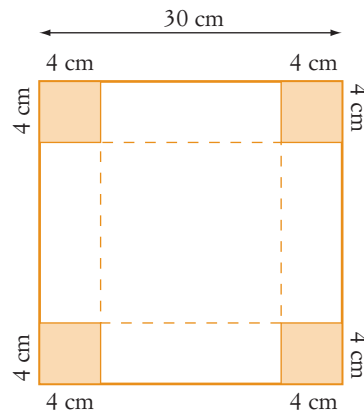
Which method is easier?



- 9 A square sheet of metal has a side length of 30 cm.

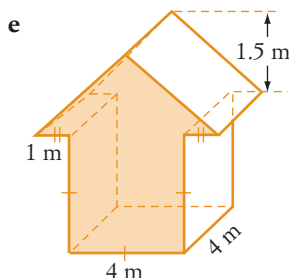
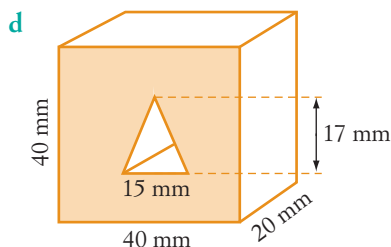
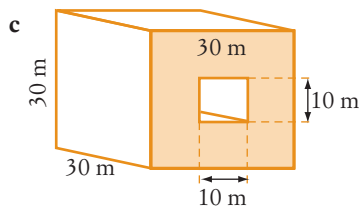
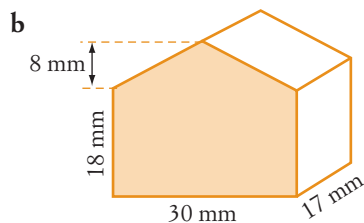
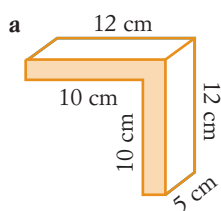
Four identical squares of length 4 cm are cut away from the corners, as shown. Which of the following is the volume of the solid formed when the remaining shape is folded along the dotted lines? Select **A**, **B**, **C** or **D**.

- A** 1936 cm^2
B 2704 cm^2
C 64 cm^2
D 3600 cm^2



- 10 A triangular prism has base length 6 m, height 10 m and volume 150 m^3 . What is its length?
 11 A cube has a volume of 343 m^3 . What is its side length?

12 Calculate the volume of each prism.



Worked solutions

Exercise 5-11

MAT08MGWS10042

13 A rectangular garden is 12 metres long and 4 metres wide. It is filled with soil to a depth of 15 centimetres. Calculate the volume of the soil. Select the correct answer **A**, **B**, **C** or **D**.

A 0.72 m^3 **B** 7.2 m^3 **C** 72 m^3 **D** 720 m^3

14 A triangular prism has a volume of 36 cm^3 . What could its length, perpendicular height and length be?

5-12 Volume of a cylinder

A cylinder is like a 'circular prism' because its cross-sections are all identical circles. Because of this, we can also apply the prism formula $V = Ah$ to calculate the volume of a cylinder.

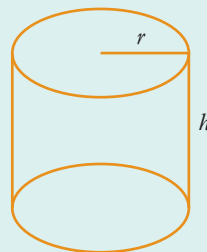
But for a circle, $A = \pi r^2$, so:

$$\text{Volume} = Ah = \pi r^2 \times h = \pi r^2 h$$

Summary

$$V = \pi \times (\text{radius})^2 \times \text{perpendicular height}$$

$$V = \pi r^2 h$$



Example 18

Find the volume of this cylinder, correct to one decimal place.

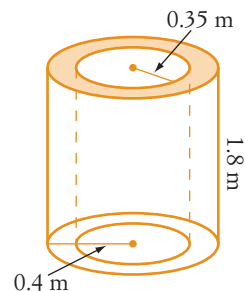
Solution

$$\begin{aligned} V &= \pi \times 5^2 \times 12.5 \\ &= 981.7477 \dots \\ &\approx 981.7 \text{ cm}^3 \end{aligned}$$

$$V = \pi r^2 h$$

**Example 19**

Calculate the volume of concrete needed to make this pipe, correct to two decimal places.



Solution

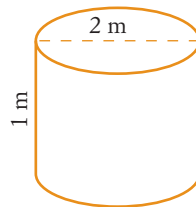
$$\begin{aligned} \text{Volume of concrete} &= \text{volume of big cylinder} - \text{volume of small cylinder (hole)} \\ &= \pi \times 0.4^2 \times 1.8 - \pi \times 0.35^2 \times 1.8 \\ &= 0.212057 \dots \\ &\approx 0.21 \text{ m}^3 \end{aligned}$$

Exercise 5-12 Volume of a cylinder

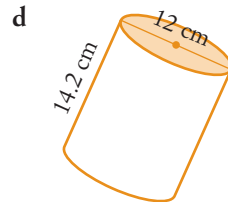
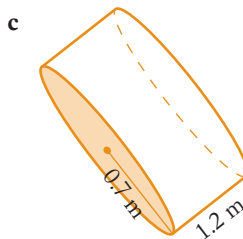
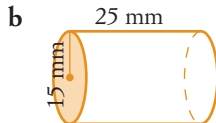
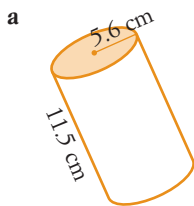
See Example 18

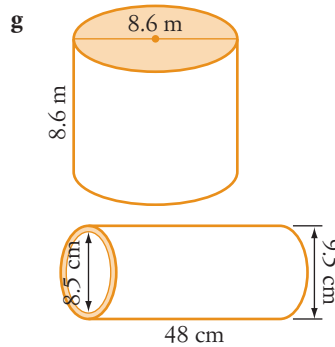
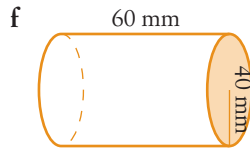
- 1 What is the volume of this cylinder? Select the closest answer **A**, **B**, **C** or **D**.

A 2 m^3 **B** 12 m^3 **C** 3 m^3 **D** 6 m^3

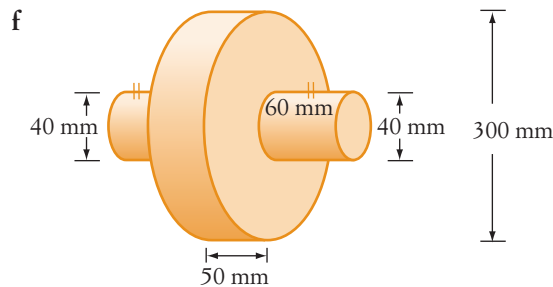
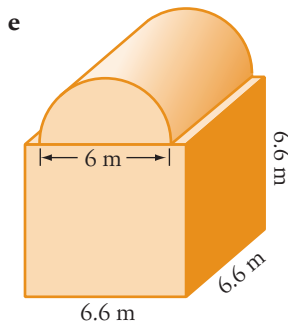
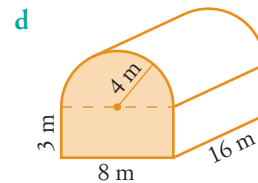
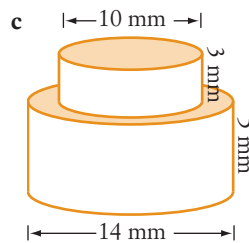
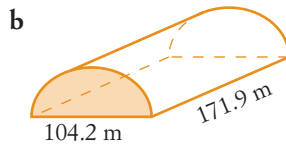
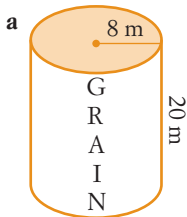


- 2 Find, correct to one decimal place, the volume of each cylinder.





- 3 This metal pipe has an inner diameter of 8.5 cm and an outer diameter of 9.5 cm.
- What is the inner radius?
 - What is the outer radius?
 - Calculate, to two decimal places, the volume of metal needed to make the pipe.
- 4 A manufacturer is experimenting with a new size of can. Which of these options provides the larger volume?
- A can of radius 3.95 cm and height 11.8 cm
 - A can of diameter 8.1 cm and height 15 cm
- 5 A cylinder has a radius of 5 cm and a volume of 628.32 cm^3 . Find its height correct to the nearest centimetre.
- 6 Find the volume of each solid, correct to two decimal places.



- 7 A cylinder has a volume of 552.92 m^3 and a height of 11 m. Find its radius correct to the nearest metre.

See Example 19

Worked solutions

Exercise 5-12

MAT08MGWS10043

5-13 Volume and capacity

Capacity is the amount of fluid (liquid or gas) in a container, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

Worksheet

Volume and capacity

MAT08MGWK10047

Homework sheet

Volume and capacity

MAT08MGHS10029

- A large drop of water is about 1 mL
- A teaspoon holds about 5 mL
- A tall standard carton of milk holds 1 L
- A small rainwater tank holds about 1 kL



A can of drink holds 375 mL.



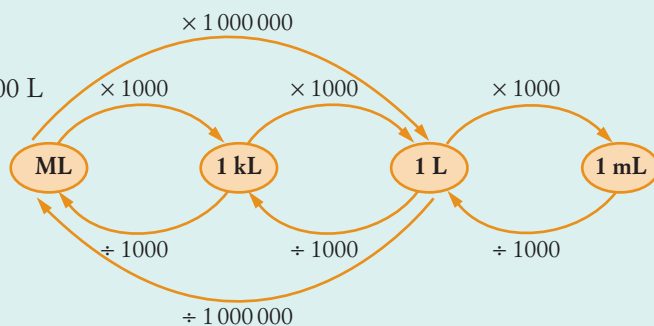
A can of paint holds 4 L.

Summary

$$1 \text{ L} = 1000 \text{ mL}$$

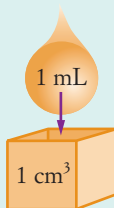
$$1 \text{ kL} = 1000 \text{ L}$$

$$1 \text{ ML} = 1000 \text{ kL} = 1\,000\,000 \text{ L}$$

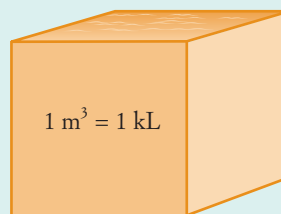


$$1 \text{ cm}^3 \text{ contains } 1 \text{ mL}$$

$$1 \text{ m}^3 \text{ contains } 1000 \text{ L or } 1 \text{ kL}$$



$$\times 1\,000\,000 =$$



Example 20

Convert:

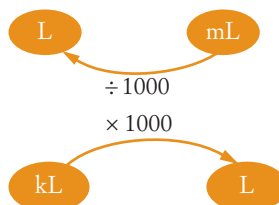
a 3400 mL to L

b 2.9 m³ to L

Solution

$$\begin{aligned} \text{a } 3400 \text{ mL} &= 3400 \div 1000 \text{ L} \\ &= 3.4 \text{ L} \end{aligned}$$

$$\begin{aligned} \text{b } 2.9 \text{ m}^3 &= 2.9 \text{ kL} \\ &= 2.9 \times 1000 \text{ L} \\ &= 2900 \text{ L} \end{aligned}$$

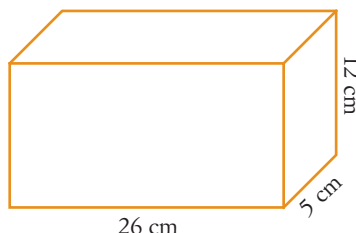


Example 21

Find the capacity of this container in litres.

Solution

$$\begin{aligned} V &= 32 \times 8 \times 15 & V &= lwh \\ &= 3840 \text{ cm}^3 \\ \text{Capacity} &= 3840 \text{ mL} & 1 \text{ cm}^3 &= 1 \text{ mL} \\ &= 3840 \div 1000 \text{ L} \\ &= 3.84 \text{ L} \\ \text{Capacity} &= 3840 \text{ mL or } 3.84 \text{ L} \end{aligned}$$



Quiz

Measuring shapes
and time

MAT08MGQZ00010

Exercise 5-13 Volume and capacity

- State what unit of capacity you would use when measuring:
 - a glass of water
 - a dam
 - a car's petrol tank
 - a bottle of medicine
 - an office water cooler
 - an Olympic swimming pool
- Copy and complete the following.

a 5000 mL = _____ L	b 3.4 kL = _____ L
c 1.6 L = _____ mL	d 4000 cm ³ = _____ L
e 2980 kL = _____ ML	f 7.1 ML = _____ L
g 875 L = _____ kL	h 8.2 m ³ = _____ L
i 0.8 ML = _____ kL	j 1850 mL = _____ L
k 5.4 kL = _____ L	l 900 000 L = _____ m ³
m 6 kL = _____ L	n 3500 mm ³ = _____ mL
o 1.2 m ³ = _____ mL	
- What is the capacity of a regular bottle of cough medicine? Select the closest answer **A**, **B**, **C** or **D**.

A 200 mL	B 500 mL	C 1500 mL	D 2000 mL
- The internal dimensions of a refrigerator are 150 cm (height), 60 cm (width) and 40 cm (depth). What is the capacity of the refrigerator? Select the closest answer **A**, **B**, **C** or **D**.

A 350 L	B 400 L	C 250 L	D 300L
- A rectangular fish tank is 90 cm long and 30 cm wide, and is filled to a depth of 45 cm. Find the volume of water in:

a cm ³	b litres
- A can of drink has a radius of 3 cm and a height of 15 cm. Find its volume to the nearest millilitre.
- A swimming pool is the shape of a rectangular prism 50 m × 18 m × 2 m. Find its volume in litres.

See Example 20

See Example 21

Worked solutions

Exercise 5-13

MAT08MGWS10044

Homework sheet

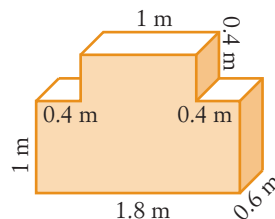
Volume and capacity

MAT08MGHS10029

- 8 A tank in the shape of a rectangular prism has dimensions as shown on the right.
- Find the volume of the tank in cm^3 .
 - How many litres of petrol would fit in the tank?



- 9 Samantha is inviting 18 friends to a party. She calculates that each person will drink 1300 mL of soft drink.
- How many litres of soft drink must she buy?
 - If Samantha intends to buy large 2.5 L bottles of drink, how many bottles must she buy?
- 10 A gardener orders a cubic metre of topsoil. She wants to spread it in her garden to a depth of 10 cm. If her garden is rectangular, what might be its dimensions?
- 11 How many litres of rainwater were collected by a tank when 40 mm of rain fell on a roof 12 m long and 3.6 m wide?
- 12 A plastic road block barrier is shown on the right. It is filled with water to weigh it down.
- Find the volume of the barrier.
 - How many litres of water would it take to fill the barrier?



- 13 The tank on a fuel tanker is in the shape of a cylinder 6 metres long, with a diameter of 2 metres.



- Find, correct to three decimal places, the volume of the tank in cubic metres.
 - How many litres of fuel can this tank hold?
- 14 A tap leaks 10 mL of water every 50 seconds. How many litres of water will be lost in:
- 1 hour?
 - 1 day?
- 15 A cylindrical can holds 2 L of pesticide. What could its radius and height be?

Worked solutions

Exercise 5-13

MAT08MGWS10044

Just for the record

Warragamba Dam

Warragamba Dam is one of the largest domestic water supply dams in the world and the largest in Australia. It provides 80 per cent of the water supply for four million people in the Sydney region. The dam wall was built between 1948 and 1960, across a section of the Nepean-Hawkesbury river system. The wall is 142 metres high. The lake formed by the dam holds 2057 gegalitres of water, four times as much as Sydney Harbour. The lake is called Lake Burragorang, which is an Aboriginal name meaning ‘tribe which lives in a valley where there is plenty of game’.



How many litres are there in 2057 gegalitres?

Investigation: Drinking the swimming pool dry

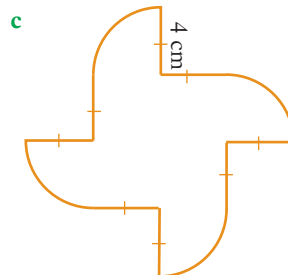
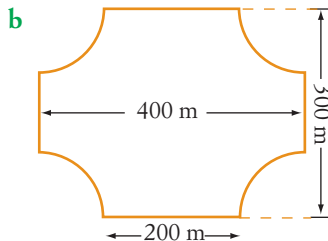
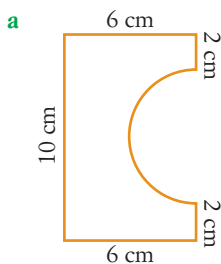
Charlotte’s mum told her that it was healthy to drink six full glasses of water each day. Assume that Charlotte uses a cylindrical glass with a height of 9 cm and a diameter of 7 cm.

- 1 Calculate the volume of water Charlotte drinks each day, in millilitres, correct to four decimal places.
- 2 How much water will Charlotte drink in a year? Answer in litres, correct to four decimal places.
- 3 The family’s swimming pool is in the shape of a cylinder with a diameter of 8.3 metres, and is filled to a constant depth of 1.2 metres. What is the amount of water in the pool? Answer in litres, correct to four decimal places.
- 4 How long would it take Charlotte to drink the equivalent of the swimming pool? Select the closest answer **A**, **B**, **C** or **D**.
A 3 months **B** 6 months **C** 1 year **D** 3 years



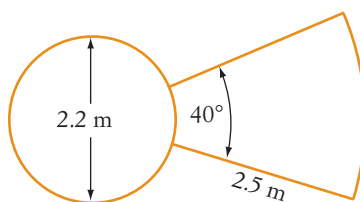
Power plus

- 1 Find, correct to 2 decimal places, the perimeter of each shape.



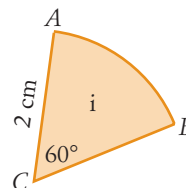
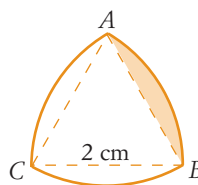
- 2 This diagram shows how the field is marked out for the shot-put at an athletics event. Calculate, correct to one decimal place:

- a** the area of the field
b the total length of the lines used.

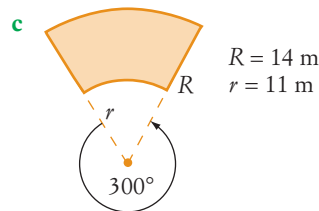
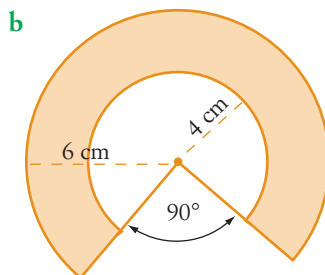
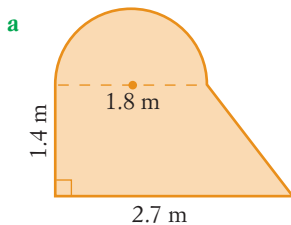


- 3 Mathsland has introduced a new 30-cent coin, as shown on the right in the first diagram.

$\angle ABC$ is an equilateral triangle of length 2 cm, and arc AB is shown in the second diagram centred at C .



- a** Find, correct to 3 decimal places, the area of the sector in the second diagram, marked **i**.
b Use Pythagoras' theorem to prove that the perpendicular height of $\angle ABC$ is $\sqrt{3}$ cm.
c Calculate the area of $\angle ABC$ correct to 3 decimal places.
d Calculate the area of the shaded segment in the first diagram.
e Hence calculate the area of the 30-cent coin.
- 4 Find, correct to 2 decimal places, the area of each shape.



$R = 14$ m
 $r = 11$ m

Language of maths

arc	cubic metre	pi (π)	segment
base	cylinder	prism	semicircle
capacity	diameter	quadrant	square metre
chord	hectare	radius/radii	tangent
circumference	perimeter	rectangular prism	triangular prism
cross-section	perpendicular height	sector	volume

Puzzle sheet

Area and volume
find-a-word

MAT08MGPS10016

- Why are there two formulas for the circumference of a circle?
- What is meant by the **perpendicular height** of a shape?
- What is the correct name for a 'pizza slice' of a circle bounded by two radii and an arc?
- In the formula $A = \frac{1}{2}xy$ for the area of a rhombus, what do x and y stand for?
- What name is given to a 'slice' of a solid taken across the solid, rather than along it?
- What shape could be called a 'circular prism'?

Topic overview

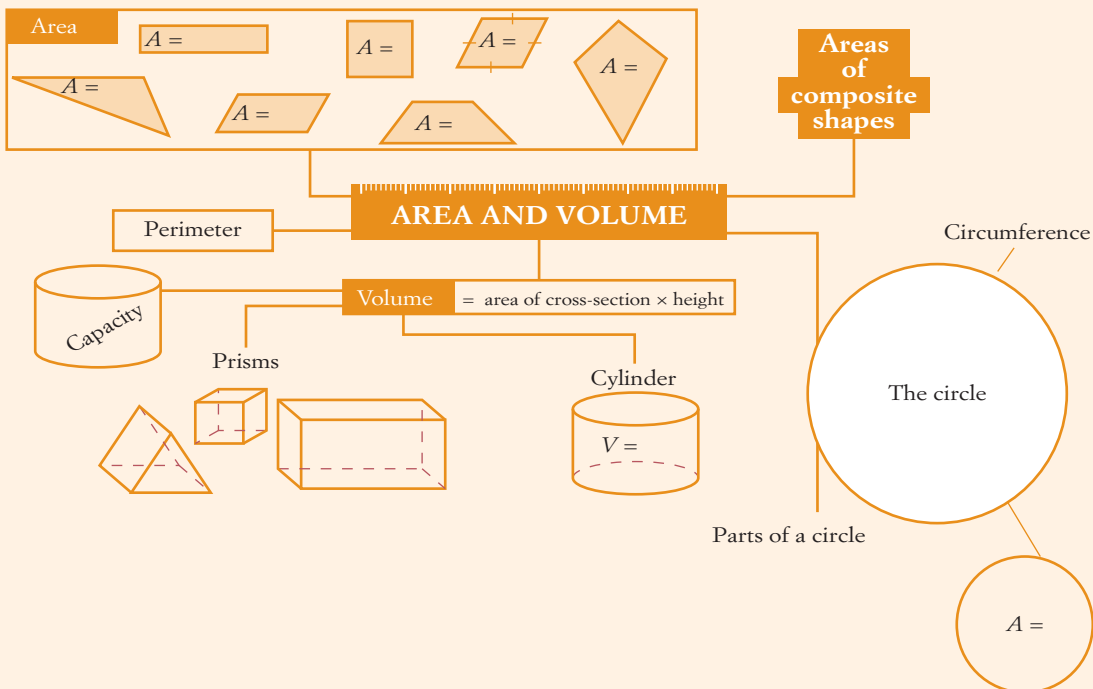
- What new things did you learn in this chapter?
- How often do you measure items? Give examples of the types of objects measured in your home.
- List four occupations that rely on being able to measure accurately.
- Did you have any problems with any section of this topic? If you did, discuss them with your teacher or a friend.

Worksheet

Mind map: Area and
volume

MAT08MGWK10048

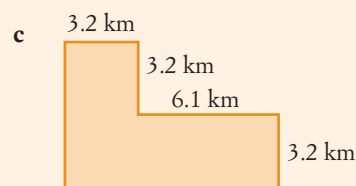
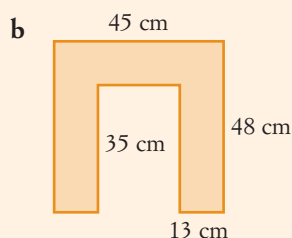
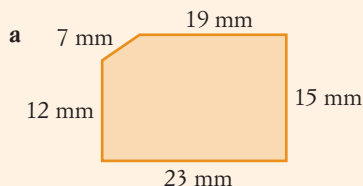
Copy and complete this overview, filling in the correct formulas. Add colour or other things to help you remember the overview. Check your overview with your teacher.



Chapter 5 revision

See Exercise 5-01

1 Find the perimeter of each shape.



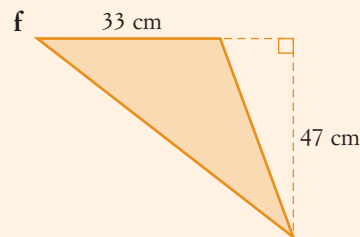
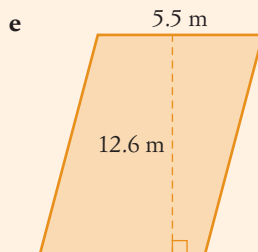
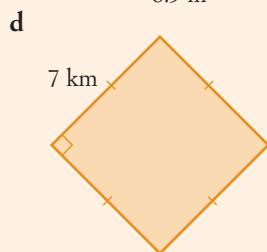
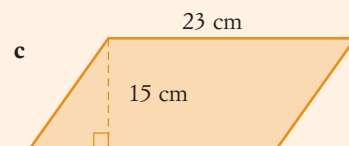
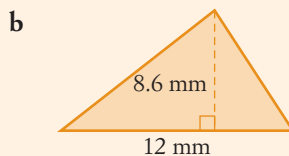
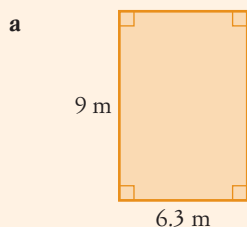
See Exercise 5-02

2 Copy and complete each statement.

- a** $200 \text{ mm}^2 = \underline{\hspace{1cm}} \text{ cm}^2$
 b $7 \text{ m}^2 = \underline{\hspace{1cm}} \text{ cm}^2$
 c $8000 \text{ cm}^2 = \underline{\hspace{1cm}} \text{ m}^2$
d $30 \text{ km}^2 = \underline{\hspace{1cm}} \text{ m}^2$
 e $70\,000 \text{ cm}^2 = \underline{\hspace{1cm}} \text{ m}^2$
 f $770 \text{ mm}^2 = \underline{\hspace{1cm}} \text{ cm}^2$
g $700 \text{ mm}^2 = \underline{\hspace{1cm}} \text{ m}^2$
 h $7.2 \text{ ha} = \underline{\hspace{1cm}} \text{ m}^2$
 i $5.8 \text{ mm}^2 = \underline{\hspace{1cm}} \text{ cm}^2$

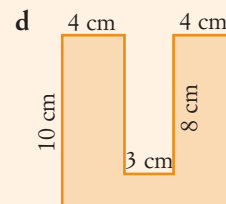
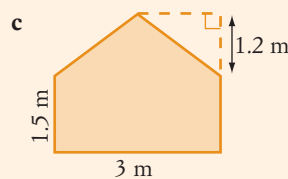
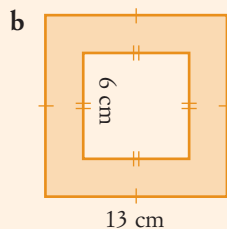
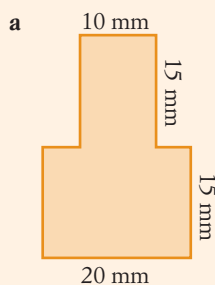
See Exercise 5-03

3 Find the area of each of the following shapes.



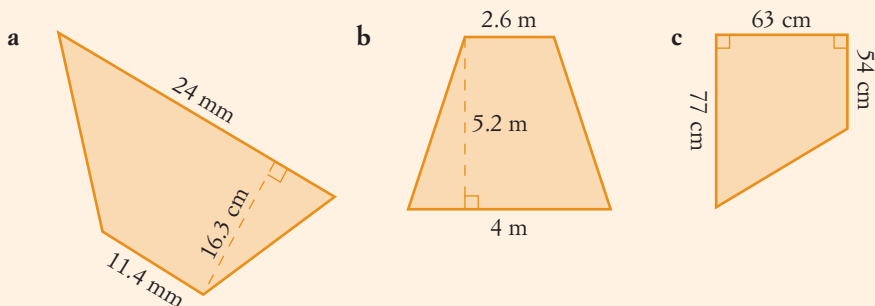
See Exercise 5-04

4 Find the area of each shape.



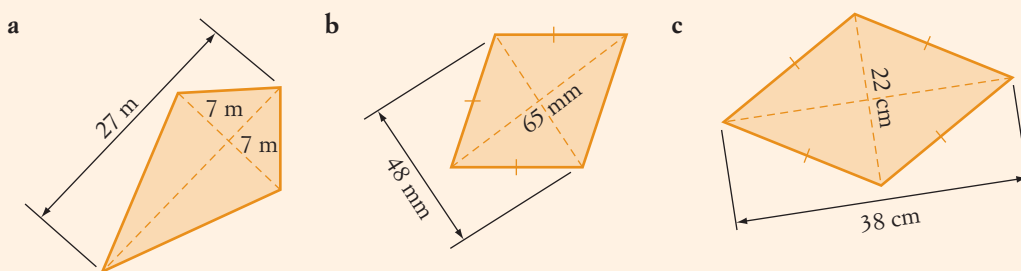
5 Find the area of each shape.

See Exercise 5-05



6 Find the area of each shape.

See Exercise 5-06



7 Name each part of the circle labelled.

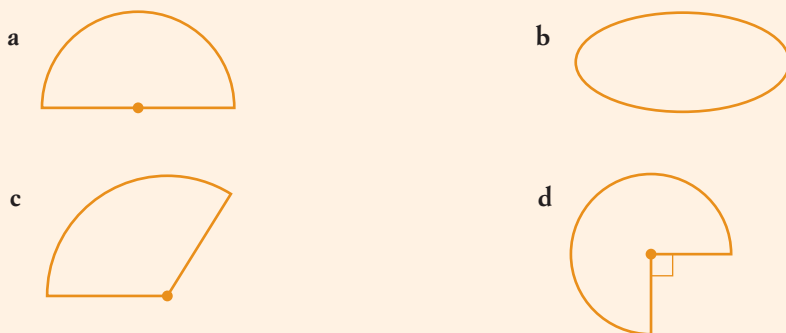
See Exercise 5-07



8 For each shape, state:

See Exercise 5-07

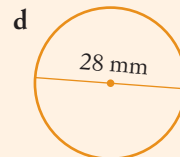
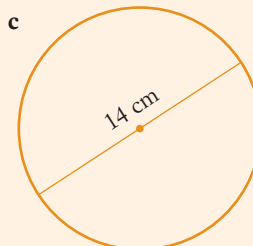
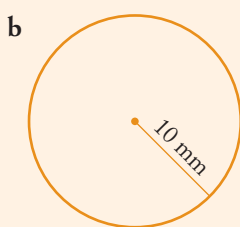
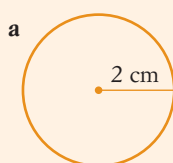
- i the number of axes of symmetry
- ii the order of rotational symmetry



Chapter 5 revision

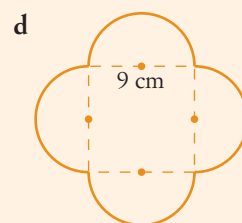
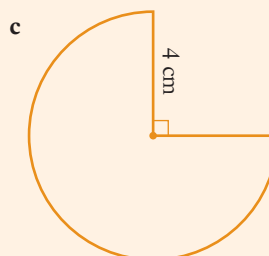
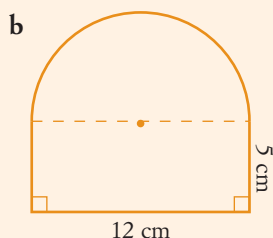
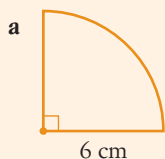
See Exercise 5-08

- 9 Calculate, correct to one decimal place, the circumference of each circle below.



See Exercise 5-08

- 10 Calculate the perimeter of each shape, correct to two decimal places.



See Exercise 5-09

- 11 Calculate the area of each circle in question 9:

- i in terms of π
ii correct to one decimal place

See Exercise 5-09

- 12 Calculate, correct to two decimal places, the area of each shape in question 10.

See Exercise 5-10

- 13 Copy and complete each statement.

a $8.2 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$

c $2\,000\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$

e $8 \text{ m}^3 = \underline{\hspace{2cm}} \text{ mm}^3$

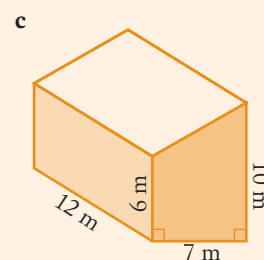
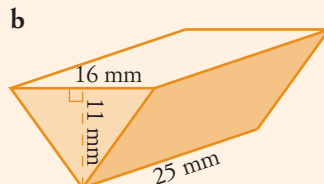
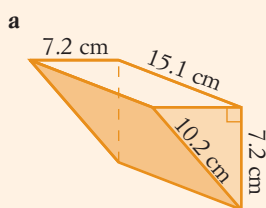
b $3.4 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$

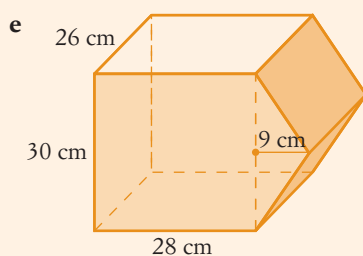
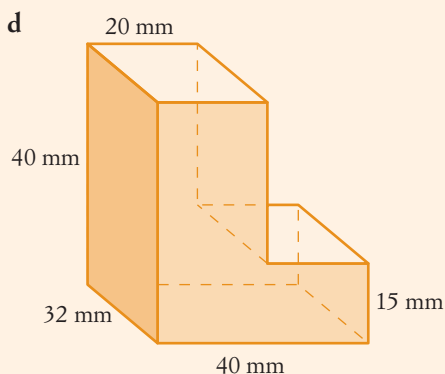
d $1 \text{ km}^3 = \underline{\hspace{2cm}} \text{ m}^3$

f $45\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$

See Exercise 5-11

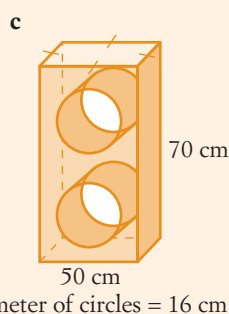
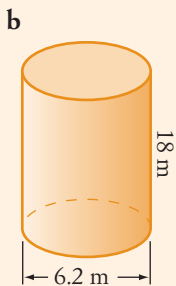
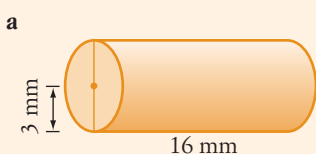
- 14 Find the volume of each prism.





15 Find the volume of each solid correct to two decimal places.

See Exercise 5-12



16 Copy and complete each statement.

See Exercise 5-12

a 3 L = _____ mL

b 2500 mL = _____ L

c 6.5 kL = _____ L

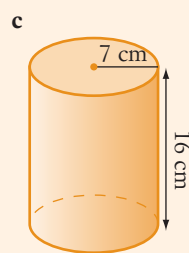
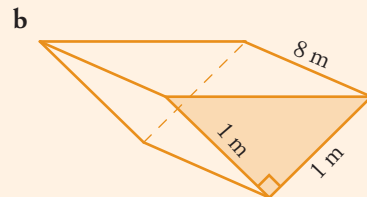
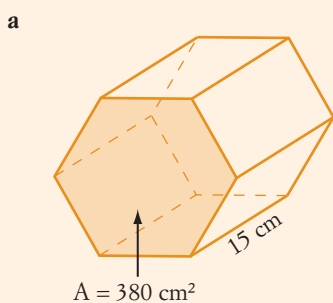
d 7.2 mm³ = _____ mL

e 120 mL = _____ L

f 35 cm³ = _____ L

17 Find the capacity of each shape in litres.

See Exercise 5-13



Mixed revision 2

See Exercise 6-01

- 1 Simplify each fraction.

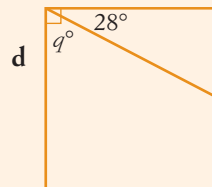
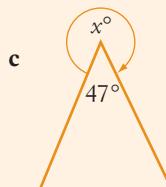
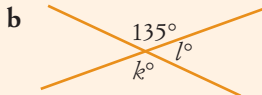
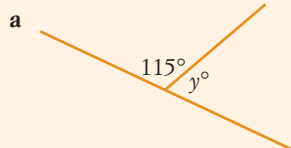
a $\frac{12}{24}$

b $\frac{7}{21}$

c $\frac{15}{25}$

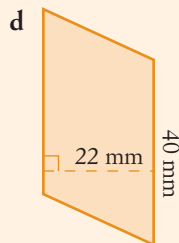
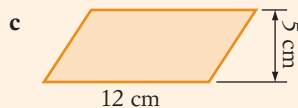
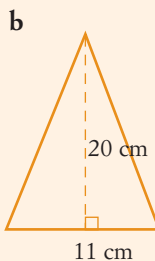
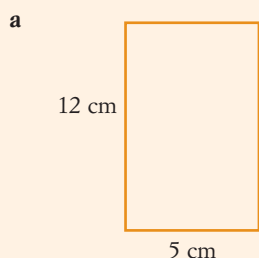
See Exercise 4-01

- 2 Find the value of each pronumeral.



See Exercise 5-03

- 3 Find the area of each shape.



See Exercise 5-07

- 4 Draw each part of a circle.

a chord

b segment

c arc

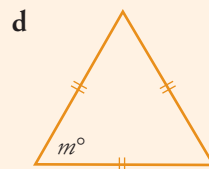
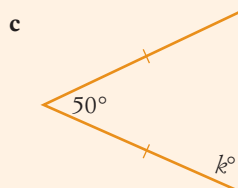
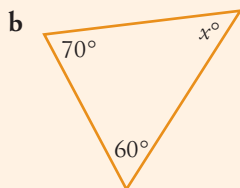
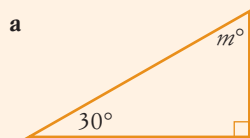
d quadrant

See Exercise 4-04

- 5 Classify each triangle:

i by sides

ii by angles.



See Exercise 4-07

- 6 Find the value of each pronumeral in question 5.

See Exercise 6-01

- 7 Convert each mixed numeral into an improper fraction.

a $2\frac{1}{2}$

b $1\frac{3}{4}$

c $4\frac{2}{5}$

8 Convert each percentage into:

See Exercise 6-04

i a simple fraction

ii a decimal.

a 25%

b 40%

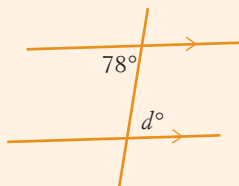
c 85%

d $12\frac{1}{2}\%$

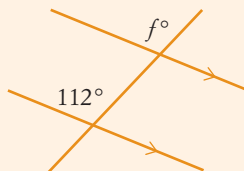
9 Find the value of each pronumeral.

See Exercise 4-02

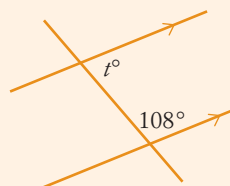
a



b



c



10 Convert each number into a percentage.

See Exercise 6-04

a 0.34

b $\frac{3}{4}$

c 0.215

d $\frac{5}{8}$

11 Name each quadrilateral whose definition is given below.

See Exercise 4-05

a The quadrilateral with opposite sides parallel.

b The quadrilateral with four right angles.

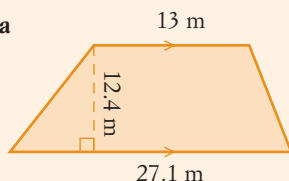
c The quadrilateral with four equal sides and four equal angles.

d The quadrilateral with two pairs of equal adjacent sides.

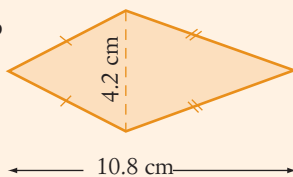
12 Find the area of each shape.

See Exercise 5-05, 5-06

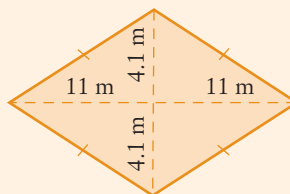
a



b



c



13 Convert each improper fraction into a mixed numeral.

See Exercise 6-01

a $\frac{11}{2}$

b $\frac{19}{3}$

c $\frac{14}{4}$

14 Evaluate each expression.

See Exercises 6-02, 6-03

a $\frac{2}{5} + \frac{4}{5}$

b $\frac{1}{3} + \frac{2}{5}$

c $\frac{3}{5} - \frac{1}{4}$

d $1\frac{3}{5} + 2\frac{1}{2}$

e $2 - \frac{2}{3}$

f $\frac{1}{2} \times \frac{3}{5}$

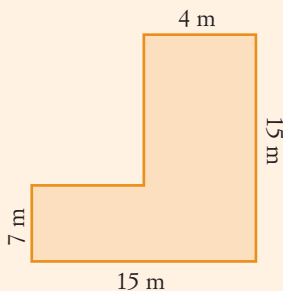
g $\frac{3}{4} \times \frac{4}{5}$

h $\frac{4}{5} \div \frac{1}{2}$

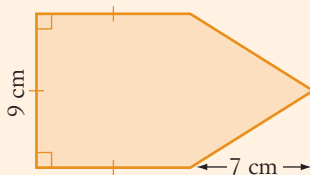
15 Find the area of each shape.

See Exercise 5-04

a



b



Mixed revision 2

See Exercise 5-08

- 16 Find, correct to two decimal places, the circumference of a circle with:

a radius 5 cm

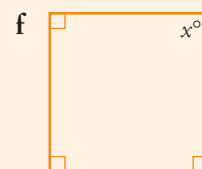
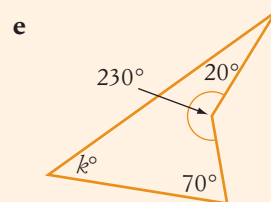
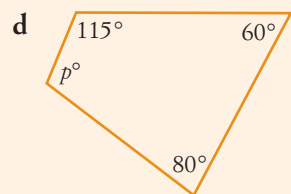
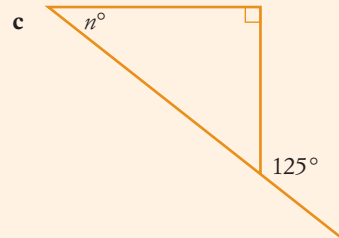
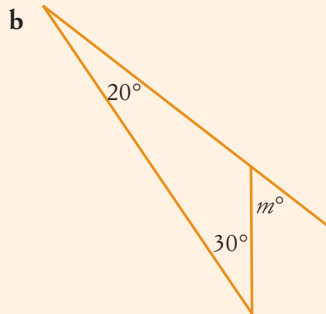
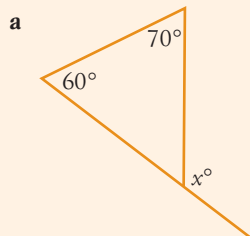
b diameter 7 cm

See Exercise 5-09

- 17 Find, correct to two decimal places, the area of each circle described in question 16.

See Exercise 4-07

- 18 Find the value of each pronumeral.



See Exercise 6-07

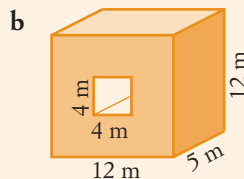
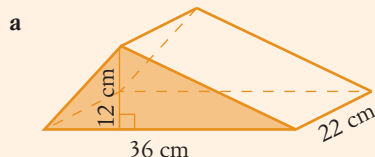
- 19 School canteen prices are to increase by 15%. Find the new price for a \$2 sandwich.

See Exercise 5-13

- 20 What is the capacity in litres of a box measuring 48 cm by 33 cm by 17 cm?

See Exercise 5-11

- 21 Find the volume of each prism.



See Exercise 5-02

- 22 Convert:

a 3.6 m^2 to cm^2

b 875 mm^2 to cm^2

See Exercise 6-04

- 23 Arrange these numbers in ascending order: 63% , $\frac{3}{5}$, 0.67 , $\frac{6}{7}$.

See Exercise 4-06

- 24 True or false?

a The sides of a rhombus are equal.

b The diagonals of a rectangle bisect each other at right angles.

c The opposite sides of a parallelogram are equal.

d The opposite sides of a trapezium are parallel.

See Exercise 6-05

- 25 Find:

a 2% of \$15

b $\frac{2}{5}$ of 1320 kg

c $\frac{3}{4}$ of \$6.40

d 25% of 1 hour (in minutes)

26 What percentage is:

See Exercise 6-06

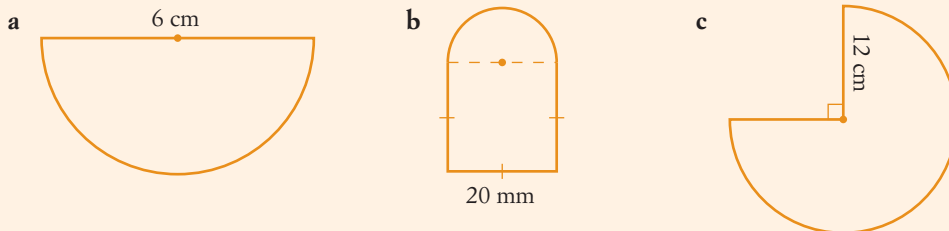
- a 40 cm of 1 m? b 350 mL of 1 litre? c 45 minutes of $1\frac{1}{2}$ hours?

27 The population of Banksia decreased by 3.5%, from 3400 people. Find the new population.

See Exercise 6-07

28 Find, correct to two decimal places, the perimeter of each shape.

See Exercise 5-08



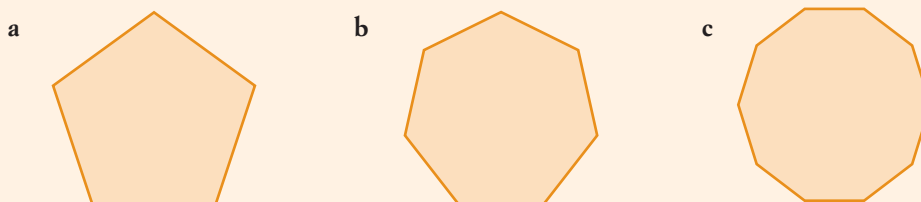
29 Find, correct to two decimal places, the area of each shape in question 28.

See Exercise 5-09

30 For each regular polygon, find:

See Exercise 4-08

- i its angle sum ii the size of one angle.



31 Mark saves 15% of his weekly income. If he saves \$102, what is his weekly income?

See Exercise 6-09

32 A computer has a cost price of \$1100 and a selling price of \$1599.

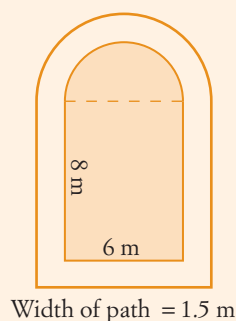
See Exercise 6-10

- a Calculate the profit made.
b Calculate, correct to one decimal place, the percentage profit on the cost price.

33 Mr Moneybags wants to put a new garden with a path around it in the grounds of his country estate. The plan for this garden is shown on the right. Find, correct to one decimal place:

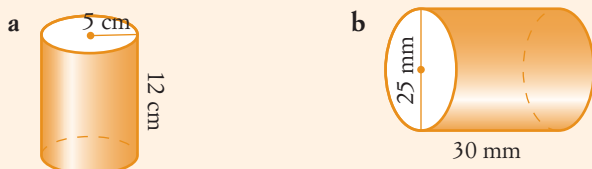
See Exercise 5-09

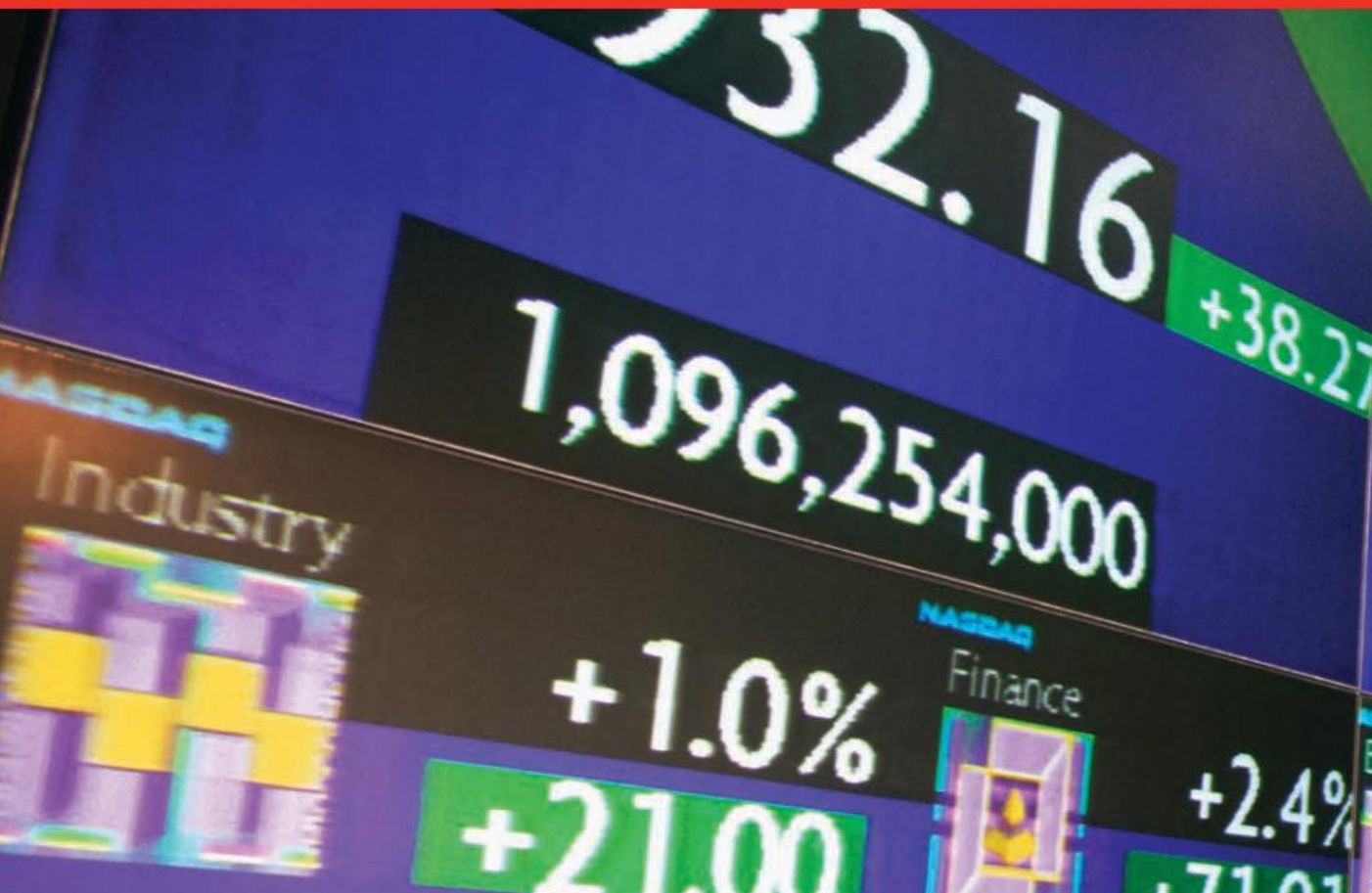
- a the area of the garden
b the area of the path.



34 Find, correct to one decimal place, the volume of each cylinder.

See Exercise 5-12





Number and algebra

6

Fractions and percentages

A percentage is a special type of fraction whose denominator is 100, for example, $95\% = \frac{95}{100}$. The symbol % is an abbreviation of '100' and 'per cent', which comes from the Latin *per centum*, meaning 'by the hundred'. Percentages are all around us: with interest earned on savings, discounts on purchases, profits for businesses, opinion polls, sports statistics, school reports, ... Percentages are a useful way of making comparisons. An understanding of percentages is vital.



Chapter outline

	Proficiency strands				
6-01 Fractions	U	F			
6-02 Adding and subtracting fractions	U	F	PS		
6-03 Multiplying and dividing fractions	U	F	PS		
6-04 Percentages, fractions and decimals	U	F		C	
6-05 Fraction and percentage of a quantity	U	F		C	
6-06 Expressing amounts as fractions and percentages	U	F		C	
6-07 Percentage increase and decrease	U	F	PS	C	
6-08 Percentages without calculators	U	F	PS	R	
6-09 The unitary method	U	F	PS	R	C
6-10 Profit, loss and GST	U	F	PS	R	C
6-11 Percentage problems	U	F	PS		C

Wordbank

cost price The price an item costs the retailer

discount The saving made between the original price of an item and the reduced price

improper fraction A fraction such as $\frac{7}{5}$, in which the numerator is larger than or equal to the denominator.

loss The amount lost when selling an item at a lower price

GST Goods and services tax, a 10% tax added to the original price of an item or service

profit The amount made when selling an item at a higher price

selling price The price at which an item is sold by the retailer

unitary method A method for finding the whole amount when a percentage of that amount is known, by first finding the size of 1%

In this chapter you will:

- compare fractions using equivalence
- solve problems involving addition and subtraction of fractions, including those with unrelated denominators
- multiply and divide fractions using efficient written strategies and digital technologies
- connect fractions, decimals and percentages and carry out simple conversions
- find fractions and percentages of quantities and express one quantity as a fraction or percentage of another, with and without digital technologies
- solve problems involving the use of percentages, including percentage increases, percentage decreases and the unitary method, with and without digital technologies
- solve problems involving profit, loss, and goods and services tax (GST), with and without digital technologies

SkillCheck

Worksheet

StartUp assignment 6

MAT08NAWK10049

- 1 Copy and complete each pair of equivalent fractions.

a $\frac{1}{2} = \frac{\quad}{6}$

b $\frac{2}{3} = \frac{\quad}{12}$

c $\frac{4}{5} = \frac{16}{\quad}$

d $\frac{7}{28} = \frac{1}{\quad}$

e $\frac{3}{4} = \frac{\quad}{8}$

f $\frac{15}{20} = \frac{\quad}{4}$

g $\frac{6}{10} = \frac{24}{\quad}$

h $\frac{3}{5} = \frac{9}{\quad}$

- 2 Write each percentage as a fraction.

a 17%

b 3%

c 51%

d 63%

e 89%

- 3 Evaluate each difference.

a $100\% - 30\%$

b $100\% - 95\%$

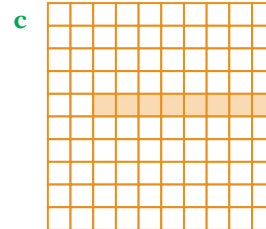
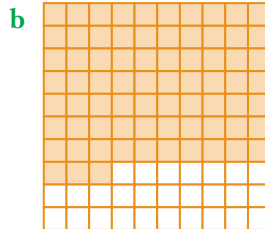
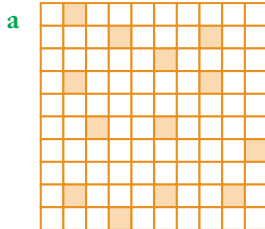
c $100\% - 39\%$

d $100\% - 71\%$

e $100\% - 12\%$

f $100\% - 7\%$

- 4 Write the percentage that is shaded in each diagram.



- 5 Write the percentage that is unshaded in each diagram in question 4.

- 6 On a plane, 10% of the seats are first-class and 25% are business class. The rest are economy class. What percentage of the seating is economy class?

- 7 Write all of the factors of 100.

- 8 Evaluate each expression.

a 50% of \$80

b 50% of 12 kg

c 25% of \$20

d 25% of 40 kg

e 10% of \$30

f 10% of 20 kg

6-01 Fractions

Scientific calculators have a fraction key for entering fractions:  or $\text{a}^{\text{b/c}}$

Some calculators have two ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into two blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). LINE mode is used in the following examples.

Example 1

Convert $\frac{27}{4}$ into a mixed numeral.

Solution

$$\frac{27}{4} = 27 \div 4$$

$$= 6 \text{ remainder } 3$$

$$= 6\frac{3}{4}$$

To find the number of 'wholes' in 27 quarters, divide 27 by 4.

Write the remainder in the numerator of a fraction.

Or on a calculator, enter: 27 $\text{a}^{\text{b/c}}$ 4 =.

On a calculator, a mixed numeral can be converted to an improper fraction using the improper fraction key: d/c or $\frac{\square}{\square}$ (this may require the **SHIFT** or **2ndF** key).

Example 2

Convert $4\frac{2}{5}$ to an improper fraction.

Solution

$$4\frac{2}{5} = \frac{5 \times 4 + 2}{5} = \frac{22}{5}$$

$$\text{This works because } 4\frac{2}{5} = 4 + \frac{2}{5} = \frac{5 \times 4}{5} + \frac{2}{5}$$

Or on a calculator, enter: 4 $\text{a}^{\text{b/c}}$ 2 $\text{a}^{\text{b/c}}$ 5 = d/c .

Example 3

Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?

Solution

Method 1

Convert both fractions so that they share a common denominator of $10 \times 8 = 80$.

$$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80} \text{ and } \frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$$

$$\text{By comparing numerators, } \frac{32}{80} > \frac{30}{80}$$

$$\therefore \frac{4}{10} \text{ is larger.}$$

Note that we multiply the numerator and denominator of each fraction by the denominator of the **other** fraction

Worksheet

Fractions wall

MAT08NAWK10050

Skillsheet

Fractions

MAT08NASS10024

Skillsheet

Improper fractions and mixed numerals

MAT08NASS10025

Animated example

Rational numbers

MAT08NAAE00007

Maths clip

Working out fractions

MAT08NAMC00003

Skillsheet

Equivalent fractions

MAT08NASS10026

Method 2

Another suitable denominator is the lowest common denominator (LCD) of both denominators.

The LCD of 10 and 8 is 40.

Convert both fractions.

$$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40} \text{ and } \frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

By comparing numerators, $\frac{16}{40} > \frac{15}{40}$.

$\therefore \frac{4}{10}$ is larger.

Example 4

Simplify each fraction.

a $\frac{10}{15}$

b $\frac{36}{60}$

Solution

a $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

Or on a calculator, enter: 10 $\boxed{a^{b/c}}$ 15 $\boxed{=}$.

b $\frac{36}{60} = \frac{36 \div 6}{60 \div 6} = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

or $\frac{36}{60} = \frac{36 \div 12}{60 \div 12} = \frac{3}{5}$

Or on a calculator, enter: 36 $\boxed{a^{b/c}}$ 60 $\boxed{=}$.

We can simplify in one step if we divide by 12, the HCF of 36 and 60.

Exercise 6-01 Fractions**See Example 1**

- 1 Convert each improper fraction into a mixed numeral or whole number.

a $\frac{3}{2}$

b $\frac{11}{3}$

c $\frac{9}{4}$

d $\frac{12}{5}$

e $\frac{20}{4}$

f $\frac{47}{11}$

See Example 2

- 2 Convert each mixed numeral into an improper fraction.

a $3\frac{1}{2}$

b $4\frac{1}{3}$

c $5\frac{3}{4}$

d $7\frac{2}{3}$

e $3\frac{4}{5}$

f $7\frac{2}{9}$

g $10\frac{1}{7}$

h $9\frac{3}{10}$

- 3 Write an improper fraction that can be converted into a whole number.

- 4 Which is larger: a proper fraction or a mixed numeral?

See Example 3

- 5 For each pair of fractions, find the larger fraction.

a $\frac{1}{2}, \frac{1}{3}$

b $\frac{2}{3}, \frac{3}{4}$

c $\frac{3}{8}, \frac{1}{3}$

d $2\frac{3}{5}, 2\frac{7}{10}$

e $\frac{11}{15}, \frac{3}{5}$

f $\frac{5}{12}, \frac{2}{5}$

Worked solutions**Exercise 6-01**

MAT08NAWS10045

6 Simplify each fraction.

a $\frac{5}{10}$

b $\frac{4}{12}$

c $\frac{12}{26}$

d $\frac{18}{24}$

e $\frac{15}{25}$

f $\frac{16}{28}$

g $\frac{32}{48}$

h $\frac{60}{100}$

i $\frac{44}{77}$

j $\frac{20}{35}$

k $\frac{21}{35}$

l $\frac{72}{80}$

7 Which one of these fractions is **not** equivalent to $\frac{10}{25}$? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{5}$

B $\frac{2}{5}$

C $\frac{20}{50}$

D $\frac{40}{100}$

8 Convert each improper fraction into a simplified mixed numeral.

a $\frac{9}{6}$

b $\frac{18}{6}$

c $\frac{45}{10}$

d $\frac{36}{20}$

See Example 4

Worked solutions

Exercise 6-01

MAT08NAWS10045

6-02 Adding and subtracting fractions

Puzzle sheet

Adding and subtracting fractions

MAT08NAPS00018

Worksheet

Fraction arithmagons

MAT08NAWK00035

Summary

- To **add or subtract fractions**, convert them (if needed) so that they have the same denominator, then simply add or subtract just the numerators.
- To **add or subtract mixed numerals**, add or subtract the whole numbers and fractions separately.

Example 5

Evaluate each expression.

a $\frac{1}{3} + \frac{5}{6}$

b $\frac{5}{7} - \frac{2}{3}$

c $3 - \frac{3}{4}$

d $1\frac{2}{3} + 4\frac{1}{5}$

Solution

a Common denominator = $3 \times 6 = 18$

$$\frac{1}{3} = \frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

$$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

$$\begin{aligned} \frac{1}{3} + \frac{5}{6} &= \frac{6}{18} + \frac{15}{18} \\ &= \frac{21}{18} \\ &= \frac{7}{6} \\ &= 1\frac{1}{6} \end{aligned}$$

Writing equivalent fractions using the common denominator.

Note that we multiply the numerator and denominator of each fraction by the denominator of the **other** fraction

OR Lowest common denominator of 3 and 6 = 6:

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\frac{5}{6} = \frac{5}{6}$$

$$\begin{aligned} \frac{1}{3} + \frac{5}{6} &= \frac{2}{6} + \frac{5}{6} \\ &= \frac{7}{6} \\ &= 1\frac{1}{6} \end{aligned}$$

Writing equivalent fractions using the lowest common denominator.

OR on a calculator, enter: 1 $\frac{a}{b/c}$ 3 + 5 $\frac{a}{b/c}$ 6 = .

b Common denominator = $7 \times 3 = 21$:

$$\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21}$$

$$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

$$\begin{aligned} \frac{5}{7} - \frac{2}{3} &= \frac{15}{21} - \frac{14}{21} \\ &= \frac{1}{21} \end{aligned}$$

Using a common denominator.

OR on a calculator, enter: 5 $\frac{a}{b/c}$ 7 - 2 $\frac{a}{b/c}$ 3 = .

$$\begin{aligned} \text{c } 3 - \frac{3}{4} &= 2 + 1 - \frac{3}{4} \\ &= 2 + \frac{4}{4} - \frac{3}{4} \\ &= 2\frac{1}{4} \end{aligned}$$

OR on a calculator, enter: 3 - 3 $\frac{a}{b/c}$ 4 = .

$$\begin{aligned} \text{d } 4\frac{1}{5} - 1\frac{2}{3} &= 4 - 1 + \frac{1}{5} - \frac{2}{3} \\ &= 3 + \frac{3}{15} - \frac{10}{15} \\ &= 3 - \frac{7}{15} \\ &= 2 + \frac{15}{15} - \frac{7}{15} \\ &= 2\frac{8}{15} \end{aligned}$$

OR on a calculator, enter: 4 $\frac{a}{b/c}$ 1 $\frac{a}{b/c}$ 5 - 1 $\frac{a}{b/c}$ 2 $\frac{a}{b/c}$ 3 = .

Exercise 6-02 Adding and subtracting fractions

1 Evaluate each expression.

a $\frac{1}{5} + \frac{3}{5}$

b $\frac{3}{4} - \frac{1}{4}$

c $\frac{3}{8} + \frac{2}{8}$

d $\frac{7}{8} + \frac{4}{8}$

2 Evaluate each expression.

a $\frac{2}{5} + \frac{3}{10}$

b $\frac{2}{3} - \frac{3}{7}$

c $\frac{3}{5} + \frac{1}{4}$

d $\frac{1}{2} - \frac{1}{4}$

e $\frac{1}{6} + \frac{3}{8}$

f $\frac{7}{10} - \frac{2}{3}$

g $\frac{4}{9} + \frac{5}{6}$

h $\frac{7}{8} - \frac{5}{12}$

i $\frac{5}{8} + \frac{1}{2}$

j $1 - \frac{6}{7}$

k $4 - \frac{4}{9}$

l $3 - \frac{2}{5}$

3 Joe bought a length of timber to build fences for his garden beds. He used $\frac{1}{5}$ of the wood for the first garden and $\frac{3}{8}$ of the wood for the second garden. What fraction of the wood remains for the third garden?

4 What fraction goes in the blank? $\frac{7}{8} - \text{---} = \frac{1}{4}$. Select the correct answer A, B, C or D.

A $\frac{6}{4}$

B $\frac{8}{12}$

C $\frac{3}{8}$

D $\frac{5}{8}$

5 Copy and complete each blank with the correct fraction.

a $\frac{1}{4} + \text{---} = \frac{2}{3}$

b $\text{---} - \frac{2}{5} = \frac{1}{6}$

6 A bottle is three-quarters full. One-third of the liquid is then poured out. What fraction remains in the bottle?

7 Danielle used half a sheet of adhesive plastic to cover her books, while Christina used $\frac{2}{5}$ of the same sheet. What fraction of the original sheet remains? Select the correct answer A, B, C or D.

A $\frac{3}{5}$

B $\frac{4}{5}$

C $\frac{1}{10}$

D $\frac{9}{10}$

8 What is the value of $2\frac{7}{8} + 1\frac{3}{8}$? Select the correct answer A, B, C or D.

A $4\frac{1}{4}$

B $3\frac{4}{5}$

C $3\frac{1}{2}$

D $13\frac{1}{4}$

9 Evaluate each expression.

a $1\frac{1}{2} + \frac{2}{5}$

b $2\frac{1}{4} + 1\frac{3}{8}$

c $7\frac{3}{8} + 9\frac{1}{3}$

d $3\frac{4}{5} + 1\frac{2}{9}$

e $6\frac{2}{5} - 4\frac{7}{8}$

f $4\frac{3}{4} - 2\frac{4}{5}$

g $2\frac{1}{3} - 1\frac{1}{2}$

h $4\frac{3}{5} - 2\frac{9}{10}$

10 In a magic square, each row, column and diagonal must add to the same total. Complete each magic square with appropriate fractions.

a

4	$1\frac{1}{2}$	
$\frac{1}{2}$		$4\frac{1}{2}$
3		

b

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

See Example 5

Worked solutions

Exercise 6-02

MAT08NAWS10046

Homework sheet

Fractions

MAT08NAHS10008

Worksheet

Multiplying and dividing
fractions number grids

MAT08NAWK00036

Worksheet

Fractions and decimals
review

MAT08NAWK00037

6-03 Multiplying and dividing fractions

Summary

- To **multiply fractions**, multiply the numerators and denominators separately, then simplify if possible (sometimes, it is easier to simplify the fractions first).
- To **divide by a fraction**, $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.
- To **multiply or divide mixed numerals**, convert them to improper fractions first.

Example 6

Evaluate each expression.

a $\frac{3}{5} \times \frac{2}{7}$

b $\frac{2}{3} \times \frac{3}{8}$

c $\frac{4}{5} \div \frac{2}{3}$

d $1\frac{1}{2} \times 2\frac{2}{3}$

Solution

$$\begin{aligned} \text{a } \frac{3}{5} \times \frac{2}{7} &= \frac{3 \times 2}{5 \times 7} \\ &= \frac{6}{35} \end{aligned}$$

OR on a calculator, enter: 3 $\boxed{a\frac{b}{c}}$ 5 $\boxed{\times}$ 2 $\boxed{a\frac{b}{c}}$ 7 $\boxed{=}$

$$\begin{aligned} \text{b } \frac{2}{3} \times \frac{3}{8} &= \frac{2 \times 3}{3 \times 8} \\ &= \frac{6}{24} \\ &= \frac{1}{4} \end{aligned}$$

OR by simplifying first before multiplying:

$$\begin{aligned} \frac{2}{3} \times \frac{3}{8} &= \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{4}{\cancel{8}}} \\ &= \frac{1}{4} \end{aligned}$$

Because $\frac{2}{8} = \frac{1}{4}$, $\frac{3}{3} = \frac{1}{1}$

OR on a calculator, enter: 2 $\boxed{a\frac{b}{c}}$ 3 $\boxed{\times}$ 3 $\boxed{a\frac{b}{c}}$ 8 $\boxed{=}$

$$\begin{aligned} \text{c } \frac{4}{5} \div \frac{2}{3} &= \frac{4}{5} \times \frac{3}{2} \\ &= \frac{12}{10} \\ &= \frac{6}{5} \\ &= 1\frac{1}{5} \end{aligned}$$

OR by simplifying first before multiplying:

$$\begin{aligned} \frac{4}{5} \div \frac{2}{3} &= \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{2}}} = \frac{6}{5} = 1\frac{1}{5} \end{aligned}$$

OR on a calculator, enter: 4 $\boxed{a^{b/c}}$ 5 $\boxed{\div}$ 2 $\boxed{a^{b/c}}$ 3 $\boxed{=}$

$$\begin{aligned} \text{d } 1\frac{1}{2} \times 2\frac{2}{3} &= \frac{3}{2} \times \frac{8}{3} \\ &= \frac{\cancel{3}^1}{2} \times \frac{8}{\cancel{3}_1} \\ &= \frac{4}{1} \\ &= 4 \end{aligned}$$

OR on a calculator, enter: 1 $\boxed{a^{b/c}}$ 1 $\boxed{a^{b/c}}$ 2 $\boxed{\times}$ 2 $\boxed{a^{b/c}}$ 2 $\boxed{a^{b/c}}$ 3 $\boxed{=}$

Exercise 6-03 Multiplying and dividing fractions

1 Evaluate each product.

a $\frac{1}{2} \times \frac{2}{3}$

b $\frac{2}{3} \times \frac{5}{7}$

c $\left(\frac{1}{8}\right)^2$

d $\frac{9}{10} \times \frac{5}{6}$

e $\frac{3}{5} \times \frac{3}{4}$

f $1\frac{1}{2} \times 3$

g $3\frac{1}{4} \times 3\frac{4}{5}$

h $1\frac{1}{3} \times 2\frac{1}{3}$

2 What is the value of $\frac{7}{10} \div \frac{1}{5}$? Select the correct answer A, B, C or D.

A $\frac{7}{50}$

B $1\frac{1}{5}$

C $1\frac{2}{5}$

D $3\frac{1}{2}$

3 a What number's reciprocal is itself?

b What number does not have a reciprocal?

c What is the product of a number and its reciprocal?

d What is the reciprocal of the reciprocal of $\frac{1}{3}$?

4 Evaluate each quotient.

a $\frac{1}{7} \div \frac{2}{3}$

b $\frac{2}{5} \div \frac{3}{4}$

c $\frac{5}{9} \div \frac{5}{6}$

d $\frac{9}{10} \div \frac{3}{5}$

e $4 \div \frac{1}{4}$

f $6 \div \frac{2}{3}$

f $\frac{5}{8} \div 5$

g $\frac{2}{5} \div 6$

i $\frac{7}{8} \div \frac{1}{4}$

j $10 \div 3\frac{1}{7}$

k $3\frac{1}{2} \div 1\frac{1}{4}$

l $3\frac{3}{4} \div 1\frac{1}{10}$

5 From a box of chocolates, Lindy takes $\frac{3}{8}$ of the chocolates and shares them equally among her four children. What fraction of the box of lollies does each child receive?

6 Copy and fill in each blank.

a $\frac{3}{4} \times \text{---} = \frac{21}{32}$

b $\text{---} \times \frac{1}{5} = \frac{2}{15}$

c $\text{---} \div \frac{2}{3} = \frac{1}{7}$

d $\frac{4}{15} \div \text{---} = \frac{8}{21}$

7 a When a number is divided by a proper fraction, does the number increase or decrease? Give reasons for your answer.

b When a number is multiplied by an improper fraction, does the number increase or decrease? Give reasons for your answer.

See Example 6

Worked solutions

Exercise 6-03

MAT08NAWS10047

Worksheet

Fractions wall

MAT08NAWK10050

Worksheet

Decimals wall

MAT08NAWK10051

Worksheet

Percentages wall

MAT08NAWK10052

Worksheet

Fractions, decimals
and percentages

MAT08NAWK10053

Skillsheet

Multiplying and dividing
by 10, 100, 1000

MAT08NASS10028

6-04 Percentages, fractions and decimals

A **percentage** is a fraction whose denominator is 100.

Example 7

Convert each percentage into a simplified fraction.

a 55%

b 130%

c $37\frac{1}{2}\%$

Solution

$$\begin{aligned} \text{a } 55\% &= \frac{55}{100} \\ &= \frac{11}{20} \end{aligned}$$

$$\begin{aligned} \text{b } 130\% &= \frac{130}{100} \\ &= \frac{13}{10} \\ &= 1\frac{3}{10} \end{aligned}$$

$$\begin{aligned} \text{c } 37\frac{1}{2}\% &= \frac{37\frac{1}{2}}{100} \\ &= \frac{37\frac{1}{2} \times 2}{100 \times 2} \\ &= \frac{75}{200} \\ &= \frac{3}{8} \end{aligned}$$

Example 8

Convert each percentage into a decimal.

a 8%

b 43.6%

c $18\frac{1}{2}\%$

Solution

$$\begin{aligned} \text{a } 8\% &= \frac{8}{100} \\ &= 8 \div 100 \\ &= 0.08 \end{aligned}$$

$$\begin{aligned} \text{b } 43.6\% &= \frac{43.6}{100} \\ &= 43.6 \div 100 \\ &= 0.436 \end{aligned}$$

$$\begin{aligned} \text{c } 18\frac{1}{2}\% &= \frac{18\frac{1}{2}}{100} \\ &= 18.5 \div 100 \\ &= 0.185 \end{aligned}$$

← To convert a percentage into a decimal mentally, move the decimal point 2 places to the left

Summary

To convert a fraction or a decimal into a percentage, multiply it by 100%

Example 9

Convert each fraction into a percentage.

a $\frac{11}{20}$

b $\frac{7}{8}$

Solution

a $\frac{11}{20} = \frac{11}{20} \times 100\%$
 $= 55\%$

b $\frac{7}{8} = \frac{7}{8} \times 100\%$
 $= 87\frac{1}{2}\%$

Example 10

Convert each decimal into a percentage.

a 0.65

b 0.267

Solution

a $0.65 = 0.65 \times 100\%$
 $= 65\%$

b $0.267 = 0.267 \times 100\%$
 $= 26.7\%$

← To convert a decimal into a percentage mentally just move the decimal point 2 places to the right

Summary

To order fractions, percentages and decimals, convert each into a percentage first.

Example 11

Arrange in ascending order: 0.667, 66%, $\frac{7}{11}$.

Solution

Convert all numbers into percentages.

$0.667 \times 100\% = 66.7\%$

$66\% = 66\%$

$\frac{7}{11} \times 100\% = 63\frac{7}{11}\%$

Arrange the percentages from smallest to largest: $63\frac{7}{11}\%$, 66% and 66.7%

So, in ascending order, the numbers are $\frac{7}{11}$, 66% and 0.667.

Exercise 6-04 Percentages, fractions and decimals

See Example 7

- 1 Convert each percentage into a simplified fraction.

a 60%	b 75%	c 31%	d 8%
e 30%	f 85%	g 99%	h 3%
i 160%	j 135%	k 25%	l 250%

See Example 8

- 2 Which decimal is equal to $62\frac{1}{2}\%$? Select the correct answer A, B, C or D.

A 62.12	B 62.5	C 0.625	D 0.0625

Worked solutions

Exercise 6-04

MAT08NAWS10048

- 3 Convert each percentage to a decimal.

a 18%	b 82%	c 2%	d 50%
e 120%	f 51.1%	g 79%	h $12\frac{1}{2}\%$
i 16.3%	j 4%	k 18.7%	l $5\frac{1}{4}\%$

See Example 9

- 4 Convert each fraction into a percentage.

a $\frac{17}{100}$	b $\frac{7}{10}$	c $\frac{13}{50}$	d $\frac{11}{20}$
e $\frac{5}{8}$	f $\frac{24}{25}$	g $\frac{2}{3}$	h $\frac{5}{4}$
i $1\frac{2}{5}$	j $\frac{27}{40}$	k $\frac{1}{16}$	l $\frac{4}{9}$

See Example 10

- 5 Convert each decimal into a percentage.

a 0.38	b 0.55	c 0.96	d 0.625
e 0.08	f 0.054	g 0.6	h 0.003
i 1.9	j 0.405	k 1.26	l 0.114

- 6 Copy and complete this table.

	Fraction	Decimal	Percentage
a		0.65	
b		0.6	
c			20%
d			84%
e	$\frac{1}{2}$		
f	$\frac{1}{8}$		
g			36%
h	$\frac{5}{8}$		
i		0.73	
j	$\frac{1}{3}$		
k			$66\frac{2}{3}\%$

See Example 11

- 7 For each pair of numbers, determine which is larger.

a 75% and 0.73	b $\frac{4}{25}$ and 14%	c $\frac{2}{3}$ and 0.64
d 18% and $\frac{1}{7}$	e 0.22 and $\frac{1}{4}$	f 55% and $\frac{6}{11}$
g 0.93 and $\frac{19}{20}$	h 60% and $\frac{13}{25}$	i $\frac{1}{6}$ and 0.08

8 Arrange each set of numbers in ascending order.

a $\frac{4}{5}$, 78%, 0.75, $\frac{9}{11}$

b 22%, $\frac{1}{4}$, 0.29, $\frac{7}{20}$

c 0.62, $\frac{3}{5}$, 57%, 0.605

9 Which list of numbers is arranged in descending order? Select the correct answer A, B, C or D.

A $\frac{4}{11}$, $\frac{2}{5}$, 0.41, 43%

B $\frac{2}{5}$, 0.41, $\frac{4}{11}$, 43%

C 43%, 0.41, $\frac{2}{5}$, $\frac{4}{11}$

D 0.41, $\frac{4}{11}$, 43%, $\frac{2}{5}$

10 Arrange each set of numbers in descending order.

a $\frac{9}{20}$, 0.47, $\frac{2}{5}$, 43%

b 0.08, 86%, $\frac{21}{25}$, 0.88

c 0.905, $\frac{19}{20}$, 91%, $\frac{9}{10}$

Worked solutions

Exercise 6-04

MAT08NAWS10048

Mental skills 6A Maths without calculators

Finding 10%, 20% and 5%

To find 10% or $\frac{1}{10}$ of a number, simply divide the number by 10 by moving the decimal point one place to the left.

1 Study each example.

a $10\% \times 150 = 150 \div 10 = 15$

b $10\% \times \$1256.80 = \$1256.80 \div 10 = \$125.68$

c $10\% \times 4917 = 4917 \div 10 = 491.7$

d $10\% \times \$48.55 = \$48.55 \div 10 = \$4.855$

2 Now find 10% of each amount.

a 190

b \$75

c 875

d \$202

e \$37.60

f 400

g \$9.25

h 896

i \$2700

j \$3.80

k \$1527.60

l \$72.50

m 3154

n \$10.70

o 426

p \$24 317.60

20% is 10% doubled so to find 20% of a number, first find 10% then double it.

3 Study each example.

a $20\% \times 700$

$10\% \times 700 = 70$

$\therefore 20\% \times 700 = 70 \times 2 = 140$

c $20\% \times 325$

$10\% \times 325 = 32.5$

$\therefore 20\% \times 325 = 32.5 \times 2 = 65$

b $20\% \times \$876$

$10\% \times \$876 = \$87.60$

$\therefore 20\% \times \$876 = \$87.60 \times 2 = \$175.20$

d $20\% \times \$38.50$

$10\% \times \$38.50 = \$3.85$

$\therefore 20\% \times \$38.50 = \$3.85 \times 2 = \$7.70$

4 Now find 20% of each amount.

a 50

b 620

c \$2450

d \$8.60

e 72

f \$12 700

g 390

h \$5.80

i \$45

j \$84

k \$4600

l 320

5% is half of 10%, so to find 5% of a number first find 10% then divide it by 2.

5 Study each example.

a $5\% \times 180$

$$10\% \times 180 = 18$$

$$\therefore 5\% \times 180 = 18 \div 2 = 9$$

c $5\% \times 120$

$$10\% \times 120 = 12$$

$$\therefore 5\% \times 120 = 12 \div 2 = 6$$

b $5\% \times \$76$

$$10\% \times \$76 = \$7.60$$

$$\therefore 5\% \times \$76 = \$7.60 \div 2 = \$3.80$$

d $5\% \times \$142.20$

$$10\% \times \$142.20 = \$14.22$$

$$\therefore 5\% \times \$142.20 = \$14.22 \div 2 = \$7.11$$

6 Now find 5% of each amount.

a 2000

b \$12

c 50

d \$27

e \$36.80

f \$84

g 800

h 130

i \$9.60

j \$138

k \$72

l 840

Puzzle sheet

Percentage
crossnumber

MAT08NAPS00019

6-05 Fraction and percentage of a quantity

Example 12

Find:

a $\frac{1}{5} \times 45$

b $\frac{7}{8} \times \$32$

c $\frac{1}{3}$ of one year

d $\frac{4}{5}$ of 2 kg (in grams)

Solution

a $\frac{1}{5} \times 45 = 45 \div 5$
 $= 9$

b $\frac{7}{8} \times \$32 = \left(\frac{1}{8} \times \$32\right) \times 7$
 $= \$4 \times 7$
 $= \$28$

c $\frac{1}{3}$ of 1 year $= \frac{1}{3} \times 12$ months
 $= 4$ months

Convert 1 year to 12 months.

d $\frac{4}{5}$ of 2 kg $= \frac{4}{5} \times 2000$ g
 $= \left(\frac{1}{5} \times 2000 \text{ g}\right) \times 4$
 $= 400 \text{ g} \times 4$
 $= 1600 \text{ g}$

Convert 2 kg to 2000 g.

Summary

To find a **percentage of a quantity**, calculate:

$$\frac{\text{percentage}}{100} \times \text{quantity} \quad \text{or} \quad \text{percentage} \div 100 \times \text{quantity}$$

Puzzle sheet

Percentages

MAT08NAPS00039

Example 13

Find:

- a** 8% of \$400 **b** 12.5% of 1 hour (in minutes) **c** 20% of 3 m (in centimetres).

Solution

$$\begin{aligned} \text{a } 8\% \text{ of } \$400 &= \frac{8}{100} \times \$400 && \text{or } 8 \div 100 \times \$400 \text{ or } 0.08 \times \$400 \\ &= \$32 \end{aligned}$$

$$\begin{aligned} \text{b } 12.5\% \text{ of } 1 \text{ h} &= 12.5\% \times 60 \text{ min} && \text{Convert 1 h to 60 min} \\ &= \frac{12.5}{100} \times 60 \text{ min} && \text{or } 12.5 \div 100 \times 60 \text{ or } 0.125 \times 60 \\ &= 7.5 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{c } 20\% \text{ of } 3 \text{ m} &= 20\% \times 300 \text{ cm} && \text{Convert 3 m to 300 cm} \\ &= \frac{20}{100} \times 300 \text{ cm} && \text{or } 20 \div 100 \times 300 \text{ or } 0.2 \times 300 \\ &= 60 \text{ cm} \end{aligned}$$

Exercise 6-05 Fraction or percentage of a quantity

1 Find:

a $\frac{3}{5} \times 40$	b $\frac{1}{4} \times 28$	c $\frac{1}{6} \times 24$
d $\frac{2}{3} \times 15$	e $\frac{7}{10} \times 60$	f $\frac{5}{8} \times 16$
g $\frac{3}{4}$ of 1 km (in metres)	h $\frac{1}{3}$ of 1 day (in hours)	i $\frac{2}{5}$ of 1 L (in mL)
j $\frac{1}{8}$ of 1 t (in kgs)	k $\frac{5}{6}$ of 1 year (in months)	l $\frac{7}{12}$ of 1 hour (in minutes)

2 What is 35% of \$75? Select the correct answer **A**, **B**, **C** or **D**.

- A** \$40 **B** \$110 **C** \$26.25 **D** \$48.75

3 Find:

a 11% of \$500	b 2% of 250 kg	c 13% of 150 L
d 21% of 600 cm	e 15% of \$450	f 5% of 5700 g
g 42% of 1128 m	h 112% of 256 km	i 19.4% of 785 mL
j 7.1% of \$220	k 23.6% of \$380	l 11.3% of 403 kg
m 110% of 95	n 150% of 302	o 130% of \$2010
p 105% of 120 m	q 170% of 350 g	r 115% of \$400

See Example 12

See Example 13

- 4 Taffy the cat had a mass of 2.7 kg when she was found. If her mass increased by $\frac{1}{4}$, how much did she gain?



- 5 A **discount** is a saving made between the original price of an item and the cheaper price.
- How much do you *save* if you get a 25% discount on a \$420 games system?
 - What would be the discount price of the games system?
- 6 An Airbus can seat 351 passengers. If the plane flew with $\frac{2}{3}$ of the seats occupied, how many passengers were on the flight?
- 7 Calculate each of the following. (Convert to a smaller unit first.)
- | | | |
|--------------------------------|--------------------------------|-----------------------------|
| a 12% of 8 m | b 35% of 1 tonne | c 20% of 12 hours |
| d $2\frac{1}{2}\%$ of 3 litres | e 72% of 150 kg | f $7\frac{1}{2}\%$ of 12 km |
| g 35% of 10 days | h $12\frac{1}{2}\%$ of 4 hours | i 67.5% of 40 g |
- 8 12% of the 525 swimmers at the swimming carnival wore black caps. How many swimmers wore black caps?
- 9 Haroula earns \$17.20 an hour in her part-time job. If she is given a 5% pay increase, by how much does her hourly rate increase?
- 10 If 42% of the 650 road deaths occurred on country roads, how many road deaths happened in the country?
- 11 In a city of 3 million people, 1% of the population are doctors. How many doctors are there? Select **A**, **B**, **C** or **D**.
- | | | | |
|------------|---------------|--------------|-----------------|
| A 3 | B 3000 | C 300 | D 30 000 |
|------------|---------------|--------------|-----------------|
- 12 One-twentieth of the cars produced in a factory were found to have steering defects. If 340 cars were produced, how many had steering defects?
- 13 How many children are there in a crowd of 40 530 if $\frac{3}{5}$ are children?
- 14 If 70% of the seats at a rock concert must be sold to make a profit, how many must be sold in a 2000-seat theatre?
- 15 Copy and complete.
- | | |
|---|---|
| a $20\% \times \underline{\hspace{1cm}} = \$18$ | b $15\% \times \underline{\hspace{1cm}} = \$75$ |
|---|---|

Worked solutions

Exercise 6-05

MAT08NAWS10049

Technology Discounts

In this activity, you will calculate the discount and sale price of items in a department store given their original price.

- 1 Enter the following data into a spreadsheet. Make the headings in row 1 **bold**.

	A	B	C	D	E
1	Item	Original Price	% Discount	Discount	Sale Price
2	Backpack	\$39.95	10%	=B2*C2	
3	DVD	\$15.00	5%		
4	Beach towel	\$20	30%		
5	Book	\$29.95	15%		
6	Swimwear	\$60	20%		
7	MP3 player	\$145	10%		

- 2 Cell D2 shows the formula for calculating the discount on the backpack. Right-click on D2 and **Fill Down** to copy the formula down to cell D7. This will calculate the discount that applies to each item. To add \$ signs to the values, with D2 to D7 still selected, right-click and select **Format Cells** and **Currency** with **two decimal places**.
- 3 In cell E2, enter the formula =B2-D2 to calculate the sale price of the backpack. Use **Fill Down** to calculate the sale prices of the remaining items. To add \$ signs to the values, with E2 to E7 still selected, right-click and select **Format Cells** and **Currency** with **two decimal places**.

6-06

Expressing amounts as fractions and percentages

Summary

To write an amount as a fraction of a whole amount:

- write the amount in the numerator of the fraction
- write the whole amount (total) in the denominator

$$\text{Fraction} = \frac{\text{amount}}{\text{whole amount}}$$

To write an amount as a percentage of a whole amount:

- write the amount in the numerator of a fraction
- write the whole amount (total) in the denominator
- multiply by 100%

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

Worksheet

Percentages

MAT08NAWK00061

Technology worksheet

Online discounts

MAT08NACT10013

Technology worksheet

Coffee shop sales

MAT08NACT10003

Quiz

Fractions and decimals

MAT08NAQZ00005

Example 14

There were 250 people at the school fete, and 160 of them were children.

- a** What fraction of the people at the fete were children?
b What percentage of the people at the fete were children?

Solution

$$\begin{aligned}\text{a Fraction} &= \frac{\text{Number of children}}{\text{Total number of people}} \\ &= \frac{160}{250} \\ &= \frac{16}{25}\end{aligned}$$

$$\begin{aligned}\text{b Percentage} &= \frac{160}{250} \times 100\% \\ &= 64\%\end{aligned}$$

Example 15

Express 36 minutes as:

- a** a fraction of an hour
b a percentage of an hour

Solution

Quantities need to be expressed in the same units, so change 1 hour to 60 minutes.

$$\text{a Fraction} = \frac{36}{60} = \frac{3}{5}$$

$$\begin{aligned}\text{b Percentage} &= \frac{36}{60} \times 100\% \\ &= 60\%\end{aligned}$$

Exercise 6-06 Expressing amounts as fractions and percentages

See Example 14

- 1 Convert each test mark into a simplified fraction.

a 50 out of 100	b 38 out of 50	c 87 out of 100
d 8 out of 12	e 5 out of 20	f 45 out of 120
- 2 Convert each test mark in question 1 into a percentage.

- 3 A hockey team scored 8 goals. If one player scored 5 of them, what is this as a percentage of the team score?
- 4 In a class of 25 students, 6 ride bikes, 10 walk to school and the rest catch the bus.
 - a What fraction walk to school?
 - b What percentage catch the bus?
- 5 Wakeel answered 21 questions correctly out of 24 in his driving test. If the pass mark is 95%, did he pass?
- 6 The World Cup cricket final attracted 86 000 people to the game, but 94 500 tickets were sold before the match.
 - a What fraction of the sold tickets were used?
 - b What percentage (correct to the nearest whole number) of the sold tickets were not used?
- 7 Tania earns \$1340 a week. She pays \$428.80 in tax and saves \$180 a week.
 - a What percentage of Tania's earnings is paid in tax?
 - b What fraction of her earnings does Tania save?
 - c What percentage (correct to one decimal place) of her earnings does Tania save?
- 8 Louise sells a house for \$458 000. If the real estate agent is paid a commission of \$22 900, what percentage of the sale price does the agent receive?
- 9 The Great Gals are having a sale on microwave ovens. The sale price of each oven is listed below, along with the discount.



Worked solutions

Exercise 6-06

MAT08NAWS10050

i



\$149
Save \$20

ii



\$179
Save \$26

iii



\$219
Save \$30

- a Calculate the original price of each microwave oven.
 - b Calculate, to one decimal place, the percentage discount on each oven.
 - c Which oven has the greatest percentage discount?
- 10 Joshua bought a pair of roller skates for \$180 and sold them at a profit of \$45. What is the profit as a percentage? Select the correct answer **A**, **B**, **C** or **D**.

A 25%	B 4%	C 45%	D 20%
 - 11 Express each measurement as a simplified fraction.

a 5 min of 1 h	b 250 mL of 1 L	c 700 m of 1 km	d 230 kg of 1 t
e 75c of \$6	f 40 min of 4h	g 300 g of 2 kg	h \$3.80 of \$14
i 12 h of 4 days	j 75 mm of 20 cm	k 800 m of 1.5 km	l 400 mL of 3.5 L
 - 12 Express each measurement in question 11 as a percentage.
 - 13 What is 40 minutes as a percentage of 2 hours? Select the correct answer **A**, **B**, **C** or **D**.

A 20%	B 5%	C $33\frac{1}{3}\%$	D 0.33%

See Example 15

- 14 A football team is scoring well in its matches if its points percentage is over 100, according to the formula:

$$\text{Points percentage} = \frac{\text{Points for}}{\text{Points against}} \times 100$$

where 'points for' are the total number of points the team has scored and 'points against' are the total number of points the other teams have scored when playing against that team.

- a Calculate, correct to two decimal places, the points percentage for each team listed in the table.

Team	Points for	Points against	Team	Points for	Points against
Broncos	391	313	Raiders	405	466
Bulldogs	410	366	Roosters	296	479
Cowboys	383	452	Sea Eagles	434	235
Dragons	321	395	Sharks	328	279
Eels	407	352	Storm	478	207
Knights	332	382	Tigers	498	642
Panthers	357	469	Titans	319	399
Rabbitohs	275	283	Warriors	688	454

- b What are the top four teams based on these results?

Just for the record

Dollars

Australia, the USA, Hong Kong and parts of Central America use the dollar as their unit of currency. The word 'dollar' actually originated in the town of Jachymov in Czechoslovakia, where the coins minted from its silver mine were called 'Joachimsthaler'. Coins in various German states from the 16th to the 19th centuries were called 'taler', which later became 'daler'. The early Spanish settlers introduced the 'Philippe dolers' into North America, and this later became the word 'dollar' that we know today.

Find out which countries use dollars as their unit of currency.



Investigation: Percentages in sport



Percentages are a good way of comparing sporting performances. Commentators often give statistics in the form of percentages so that we can compare the success of teams or players.

- 1 In netball, goal shooters rarely have the same number of shooting chances. To work out their success rates, we must use percentages to compare the number of shooting chances with the number of successful shots.

The following statistics were recorded in a match between Australia and New Zealand.

	Australia		New Zealand	
	Shooting chances	Goals	Shooting chances	Goals
1st quarter	15	11	9	9
2nd quarter	25	17	14	13
3rd quarter	18	14	23	20
4th quarter	27	20	18	17

- a Calculate as a percentage to one decimal place:
 - i each team's success rate for each quarter
 - ii the total success rate for each team.
 - b Comment on the relationship between the success rates and the final scores. What can you say about each team?
- 2 At any stage during the season in the National Basketball League (NBL), the teams will have played different numbers of games. The highest position on the ladder is awarded to the team with the highest **wins percentage**, which is calculated as follows:

$$\text{Wins percentage} = \frac{\text{Number of games won}}{\text{Number of games played}} \times 100\%$$

At one stage of a season the teams had won and lost the number of games shown below.

Team	Won	Lost
Adelaide	6	6
Cairns	8	7
Gold Coast	7	4
Melbourne	6	9
New Zealand	9	4
Perth	3	11
Sydney	11	3
Townsville	8	4
Wollongong	2	12



Calculate the wins percentage for each team and construct the 'NBL Ladder' showing the teams in the correct order.

Worksheet

Percentage shortcuts

MAT08NAWK10054

Worksheet

Discounts and special offers

MAT08NAWK10055

Video tutorial

Percentage increase and decrease

MAT08NAVT10010

Homework sheet

Percentages 1

MAT08NAHS10009

Puzzle sheet

Discounts

MAT08NAPS00040

Worksheet

Discounts

MAT08NAWK00062

6-07 Percentage increase and decrease

- Percentage increase** means to increase (make bigger) a quantity by a percentage
- Percentage decrease** means to decrease (make smaller) a quantity by a percentage

Example 16

Increase \$200 by 7%.

Solution

Method 1

$$\begin{aligned} 7\% \text{ of } \$200 &= \frac{7}{100} \times \$200 \\ &= \$14 \end{aligned}$$

$$\$200 + \$14 = \$214$$

Method 2

$$(100\% + 7\%) \text{ of } \$200 = 107\% \text{ of } \$200$$

$$\begin{aligned} &= \frac{107}{100} \times \$200 \\ &= \$214 \end{aligned}$$

$$\text{or } 7 \div 100 \times \$200$$

$$\text{or } 0.07 \times \$200$$

Increasing by 7% is the same as calculating 107%

$$\text{or } 107 \div 100 \times \$200$$

$$\text{or } 1.07 \times \$200$$

Example 17

Decrease \$150 by 12%.

Solution

Method 1

$$\begin{aligned} 12\% \text{ of } \$150 &= \frac{12}{100} \times \$150 \\ &= \$18 \\ \$150 - \$18 &= \$132 \end{aligned}$$

$$\text{or } 12 \div 100 \times \$150 \text{ or } 0.12 \times \$150$$

Method 2

$$\begin{aligned} (100\% - 12\%) \text{ of } \$150 &= 88\% \text{ of } \$150 \\ &= \frac{88}{100} \times \$150 \\ &= \$132 \end{aligned}$$

Decreasing by 12% is the same as calculating 88%

$$\begin{aligned} \text{or } 88 \div 100 \times \$150 \\ \text{or } 0.88 \times \$150 \end{aligned}$$

Example 18

The price of a watch increases by 15%. If its original price was \$35, find its new price.

Solution

Method 1

$$\begin{aligned} 15\% \text{ of } \$35 &= \$5.25 \\ \$35 + \$5.25 &= \$40.25 \end{aligned}$$

Method 2

$$\begin{aligned} (100\% + 15\%) \text{ of } \$35 &= 115\% \text{ of } \$35 \\ &= \$40.25 \end{aligned}$$

Example 19

Find the price of a computer game system, originally priced at \$420, after a 9% discount.



Solution

Method 1

$$\begin{aligned} 9\% \text{ of } \$420 &= \$37.80 \\ \$420 - \$37.80 &= \$382.20 \end{aligned}$$

Method 2

$$\begin{aligned} (100\% - 9\%) \text{ of } \$420 &= 91\% \text{ of } \$420 \\ &= \$382.20 \end{aligned}$$

Exercise 6-07 Percentage increase and decrease

See Example 16

1 Increase:

- a \$150 by 5%
d \$2500 by 6%

- b 400 by 20%
e 95 kg by 60%

- c 60 km by 22%
f 10 L by 33%

See Example 17

2 Decrease:

- a \$440 by 60%
d \$325 by 25%

- b 120 by 15%
e 2000 L by 38%

- c 110 kg by 8%
f \$1570 by 3%

See Example 18

3 Wooden chairs cost \$172 to make. Calculate the selling price if the chairs are marked up by 35% when sold.

4 Julie buys a bike for \$2700 and sells it a year later, making a 15% profit.

- a How much profit did Julie make?
b What was the selling price of the bike?

5 Geeva's weekly pay of \$980 increased by 4.5%. What is his new pay?

6 Yumi's height of 168 cm increased by 2%. What is her new height?

7 A department store has a mark-up of 200% on clothing. If the store buys a vest for \$12, what will be its selling price after the mark-up? Select the correct answer **A**, **B**, **C** or **D**.

- A** \$32 **B** \$14.40 **C** \$24 **D** \$36

See Example 19

8 A car dealer offers a 15% discount on all new car purchases. What would you pay for a car marked at \$21 990?

9 Calculate the sale price of a pair of sports shoes that cost \$165 after a discount of 12%.

10 After speed cameras were installed, the road toll of 840 deaths decreased by 5%. What is the new road toll?



11 Aaron bought a house for \$464 000 and sold it 10 years later, making a 150% profit. Calculate the selling price of the house.

12 What is the new price of a pair of jeans worth \$75 if they are discounted by 25%? Select the correct answer **A**, **B**, **C** or **D**.

- A** \$50 **B** \$56.25 **C** \$18.75 **D** \$55

13 A netball sells for \$45, but you receive 11% discount if you buy two. What is the discounted price of two netballs?

14 The owner of a store buys a DVD recorder for \$470. She adds a mark-up of \$70 but, at sale time, offers a 30% discount.

- a Find the selling price after the mark-up.
b Find the discounted price at sale time.
c How much loss did the store owner incur on the DVD recorder?
d Calculate the percentage loss correct to one decimal place.

15 Find the number which if increased by 18% gives 767.

16 Winter coats priced at \$830 were reduced by 15% at the end-of-season sale. On the last weekend of the sale, they were reduced by a further 40% off the discounted price. What was the final price of a coat?



17 Find the number which if decreased by 40% gives 306.

18 A digital tablet with an original price of \$680 has 10% GST added to it. It is then sold at an end-of-year sale for '10% off'. Is the sale price of the tablet more than, less than, or equal to its original price? Justify your answer by calculation.



Worked solutions

Exercise 6-07

MAT08NAWS10051

6-08 Percentages without calculators

If we know the equivalent fractions for common percentages, then we can find percentages mentally.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{10}$	$\frac{1}{8}$
Percentage	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	20%	40%	60%	10%	$12\frac{1}{2}\%$

Skillsheet

Mental percentages

MAT08NASS10029

Worksheet

Percentages without calculators

MAT08NAWK10057

Example 20

Calculate each percentage mentally.

a $33\frac{1}{3}\%$ of \$1800

b $12\frac{1}{2}\%$ of 72 kg

Solution

$$\begin{aligned}\text{a } 33\frac{1}{3}\% \text{ of } \$1800 &= \frac{1}{3} \text{ of } \$1800 \\ &= \$600\end{aligned}$$

$$\begin{aligned}\text{b } 12\frac{1}{2}\% \text{ of } 72 \text{ kg} &= \frac{1}{8} \text{ of } 72 \text{ kg} \\ &= 9 \text{ kg}\end{aligned}$$

Example 21

a Increase \$470 by 10%.

b Decrease \$75 by 20%.

Solution

$$\begin{aligned}\text{a } 10\% \text{ of } \$470 &= \frac{1}{10} \times \$470 \\ &= \$47 \\ \$470 + \$47 &= \$517\end{aligned}$$

$$\begin{aligned}\text{b } 20\% \text{ of } \$75 &= \frac{1}{5} \times \$75 && (\text{or } 10\% \times \$75 \times 2) \\ &= \$15 \\ \$75 - \$15 &= \$60\end{aligned}$$

Note: More examples and exercises on increasing and decreasing an amount by a percentage mentally can be found in **Mental skills 6B: Percentage increase and decrease** following **Exercise 6-08**.

Exercise 6-08 Percentages without calculators

Complete this exercise without using a calculator.

See Example 20

1 Find 25% of each amount.

a 32 kg

b 180 mm

c 24 L

d \$1000

2 Find each percentage.

a 50% of 36 m

b 75% of 12 g

c $12\frac{1}{2}\%$ of 24 h

d 20% of 5 L

e $66\frac{2}{3}\%$ of \$30

f 60% of 20 km

g 25% of 20 mL

h $33\frac{1}{3}\%$ of 18 days

i 75% of 100 kg

3 Find 10% of each amount.

a 60 tonnes

b 40 hours

c 300 sheep

d \$90

Worked solutions

Exercise 6-08

MAT08NAWS10052

- 4 Find each percentage.
- a 10% of 70 b 50% of \$80 c 40% of 120 minutes
d 30% of \$12 e 60% of 20 cm f 20% of 400 mL
- 5 What is 5% of 80 days? Select the correct answer A, B, C or D.
A 8 days B 4 days C 40 days D 80 days
- 6 Find $12\frac{1}{2}\%$ of each amount.
- a \$72 b 40 kg c 160 kL
- 7 Increase:
- a \$80 by 25% b 140 by 10% c \$150 by $33\frac{1}{3}\%$
d 28 by 50% e \$240 by $12\frac{1}{2}\%$ f 45 by 20%
- 8 Decrease:
- a \$310 by 10% b 120 by 20% c \$80 by $12\frac{1}{2}\%$
d 60 by 5% e \$900 by $66\frac{2}{3}\%$ f 2000 by 75%

See Example 21

Worked solutions

Exercise 6-08

MAT08NAWS10052

Mental skills 6B Maths without calculators

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{2}{3}\%$	75%
Fraction	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

- 1 Consider each example.

- a Increase 80 by 20%

$$\begin{aligned}
 20\% \text{ of } 80 &= \frac{1}{5} \times 80 & \text{or} & & 10\% \text{ of } 80 &= 8 \\
 &= 80 \div 5 & & & 20\% \text{ of } 80 &= 8 \times 2 \\
 &= 16 & & & &= 16 \\
 80 + 16 &= 96
 \end{aligned}$$

- b Increase \$36 by $66\frac{2}{3}\%$

$$\begin{aligned}
 66\frac{2}{3}\% \text{ of } 36 &= \frac{2}{3} \times 36 \\
 &= \$36 \div 3 \times 2 \\
 &= \$12 \times 2 \\
 &= \$24 \\
 \$36 + \$24 &= \$60
 \end{aligned}$$

- 2 Now increase:

- a \$280 by 10% b 45 by $33\frac{1}{3}\%$ c 25 by 20% d \$400 by $12\frac{1}{2}\%$
e 64 by 50% f \$72 by 25% g \$55 by 10% h 90 by $66\frac{2}{3}\%$
i 120 by 75% j \$80 by 5% k \$250 by 20% l 70 by 40%

3 Consider each example.

a Decrease 225 by $33\frac{1}{3}\%$

$$\begin{aligned} 33\frac{1}{3}\% \text{ of } 225 &= \frac{1}{3} \times 225 \\ &= 225 \div 3 \\ &= 75 \\ 225 - 75 &= 150 \end{aligned}$$

b Decrease \$70 by 15%

$$\begin{aligned} 10\% \text{ of } \$70 &= \frac{1}{10} \times \$70 = \$7 \\ \therefore 5\% \text{ of } \$70 &= \frac{1}{2} \times \$7 = \$3.50 \\ \therefore 15\% \text{ of } \$70 &= (10\% \times \$70) + (5\% \times \$70) \\ &= \$7 + \$3.50 \\ &= \$10.50 \\ \$70 - \$10.50 &= \$59.50 \end{aligned}$$

4 Now decrease:

a \$480 by 25%

b 60 by $33\frac{1}{3}\%$

c 110 by 20%

d \$25 by 10%

e 900 by 50%

f \$72 by $12\frac{1}{2}\%$

g \$320 by 75%

h 150 by $66\frac{2}{3}\%$

i \$63 by 20%

j \$100 by $12\frac{1}{2}\%$

k 250 by 10%

l \$150 by 30%

Worksheet

The unitary method

MAT08NAWK10056

Video tutorial

The unitary method

MAT08NAVT10011

6-09 The unitary method

If we know the percentage of an amount, but not the actual amount, we can use the **unitary method** to find the whole amount. The word **unit** means ‘one,’ and with the unitary method we find 1% first.

Example 22

If 15% of an amount is \$75, what is the amount?

Solution

$$15\% \text{ of amount} = \$75$$

$$\begin{aligned} 1\% \text{ of amount} &= \$75 \div 15 \\ &= \$5 \end{aligned}$$

$$\begin{aligned} 100\% \text{ of amount} &= \$5 \times 100 \\ &= \$500 \end{aligned}$$

The amount is \$500.

Find 1% first by dividing by 15.

Then find the whole amount by multiplying by 100.

Check: $15\% \times \$500 = \$75$

Summary

When using the **unitary method** to find an amount given a **percentage** of the amount:

- first find 1% of the amount by dividing by the known percentage
- then multiply by 100 to find the whole amount (100%).

Example 23

Farmer Brown lost 30% of her sheep in a flood. If she had 560 sheep left, how many sheep did she have before the flood?



Solution

Because 30% of the total were lost, $100\% - 30\% = 70\%$ of the total were left.

So: 70% of the total = 560 sheep

$$\begin{aligned} 1\% \text{ of the total} &= 560 \div 70 \\ &= 8 \text{ sheep} \end{aligned}$$

$$\begin{aligned} 100\% \text{ of the total} &= 8 \times 100 \\ &= 800 \text{ sheep} \end{aligned}$$

Farmer Brown had 800 sheep before the flood.

Check: $70\% \times 800 = 560$

Exercise 6-09 The unitary method

- What is the whole amount if:

a 10% of it is \$70?	b 25% of it is \$140?	c 17% of it is \$782?
d 8.5% of it is \$161.50?	e 16% of it is 64 cm?	f 20% of it is 56 kg?
g 42% of it is 1.26 m?	h 30% of it is 45 minutes?	i 70% of it is \$280?
j 110% of it is \$396?	k 45% of it is 180 kg?	l 120% of it is 132 minutes?
- The deposit of 15% on a kitchen stove is \$180. What is the price of the stove?
- There were 51 870 men in the crowd at the football match. If this was 65% of the crowd, what was the total attendance?

See Example 22

Worked solutions

Exercise 6-09

MAT08NAWS10053

- 4 Last year Joanna paid \$23 009.28 in income tax. If this represents 28% of her income, calculate her income.
- 5 Joe sold his car for \$12 200, making a loss of 20% on the original price. What was the original price?
- 6 A cake decorator makes 18% profit on cakes she decorates. If she sells a cake for \$330.40, what was the original cost of the cake? Select the closest answer **A**, **B**, **C** or **D**.
- A** \$184 **B** \$238 **C** \$271 **D** \$280



- 7 A town's population increased by 5%, or 1200 people. What was the old population?
- 8 The interest earned on an investment is \$168. If the interest rate is 5.6%, how much was invested?
- 9 Shania buys a dress that has been discounted by 25%. If the sale price is \$225, what was the original price of the dress?
- 10 Matthew buys a game system for \$348 at a discount of 20%. What was the original price of the game system?

Worksheet

Profit and loss

MAT08NAWK10058

6-10 Profit, loss and GST

Profit and loss

Retailers (store owners) buy goods and sell them to people.

- **Cost price** is how much they buy the item for
- **Selling price** is how much they sell them for
- If they sell the item for **more** than what they paid for it, they make a **profit**
- If they sell the item for **less** than what they paid for it, they make a **loss**

The **percentage profit** or **percentage loss** is usually calculated as a percentage of the cost price.

Maths clip

Business maths

MAT08NAMC00006

Example 24

The Book Bin buys books for \$18 each and sells them for \$22.50. For each book, calculate:

- a the profit
- b the percentage profit.

Solution

a Cost price = \$18

Selling price = \$22.50

$$\begin{aligned}\text{Profit} &= \$22.50 - \$18 \\ &= \$4.50\end{aligned}$$

$$\text{selling price} - \text{cost price}$$

b Percentage profit = $\frac{\$4.50}{\$18} \times 100\%$
= 25%

$$\frac{\text{profit}}{\text{cost price}} \times 100\%$$

Example 25

Renae buys a car for \$17 500 and sells it 2 years later for \$15 000. Find the percentage loss correct to one decimal place.

Solution

Cost price = \$17 500

Selling price = \$15 000

$$\begin{aligned}\text{Loss} &= \$17\,500 - \$15\,000 \\ &= \$2500\end{aligned}$$

$$\text{cost price} - \text{selling price}$$

$$\begin{aligned}\text{Percentage loss} &= \frac{\$2500}{\$17\,500} \times 100\% \\ &= 14.2857 \dots \% \\ &\approx 14.3\%\end{aligned}$$

$$\frac{\text{loss}}{\text{cost price}} \times 100\%$$

Example 26

Cooper buys a mountain bike for \$850 and sells it a year later at 13% profit. Find the selling price.



Solution

$$\begin{aligned}\text{Profit} &= 13\% \times \$850 \\ &= \$110.50\end{aligned}$$

$$\begin{aligned}\text{Selling price} &= \$850 + \$110.50 \\ &= \$960.50\end{aligned}$$

$$\begin{aligned}\text{OR Selling price} &= (100\% + 13\%) \times \$850 && \text{Increasing \$850 by 13\%.} \\ &= 113\% \times \$850 \\ &= \$960.50\end{aligned}$$

Video tutorial

GST

MAT08NAVT10012

GST

Goods and Services Tax (GST) is a tax paid to the government on most goods (items) and services that we purchase. In Australia, GST is charged at 10% of the original price and is generally included in the marked price of the good or service.

Example 27

The selling price of a TV is \$722 + 10% GST. Calculate:

- a the GST payable
- b the selling price

Solution

$$\begin{aligned}\text{a GST} &= 10\% \text{ of } \$722 \\ &= \$72.20\end{aligned}$$

$$\begin{aligned}\text{b Selling price} &= \$722 + \$72.20 \\ &= \$794.20\end{aligned}$$

$$\begin{aligned}\text{OR Selling price} &= 110\% \times \$722 && \text{Increasing \$722 by 10\%.} \\ &= \$794.20\end{aligned}$$

Example 28

The selling price of a lounge suite is \$2695 with GST included. How much of this price is the GST?

Solution

$$\begin{aligned}\text{Selling price} + \text{GST} &= 100\% + 10\% = 110\% \\ 110\% \text{ of the selling price} &= \$2695 \\ 10\% \text{ GST} &= \$2695 \div 11 \\ &= \$245\end{aligned}$$

Exercise 6-10 Profit, loss and GST

- 1 In each situation, state:
 - i the cost price
 - ii whether a profit or loss was made.
 - a A car was bought for \$18 700 and sold for \$15 000.
 - b A house was bought for \$543 000 and sold for \$674 000.
 - c A computer was sold for \$350 when it originally cost \$799.
 - d An antique desk sold for \$8000 when it was purchased for \$6800.
- 2 For each situation, find:
 - i the profit
 - ii the percentage profit, correct to one decimal place where necessary.
 - a cost price \$85 selling price \$102
 - b cost price \$415 selling price \$600
 - c cost price \$2.75 selling price \$4.20
- 3 For each situation, find:
 - i the loss
 - ii the percentage loss, correct to one decimal place where necessary.
 - a cost price \$19 selling price \$14
 - b cost price \$2700 selling price \$1450
 - c cost price \$79 selling price \$58
- 4 A 3D TV costing \$2500 is sold for \$3500. Find:
 - a the profit
 - b the percentage profit.
- 5 A car costing \$17 000 is sold for \$15 200. Find:
 - a the loss
 - b the percentage loss correct to one decimal place
- 6 Calculate the selling price for each situation.
 - a cost price \$200 profit 17%
 - b cost price \$42 profit 70%
 - c cost price \$720 loss 35%
 - d cost price \$1400 loss 9.5%
 - e cost price \$2.50 profit 150%
 - f cost price \$27 loss 40%
- 7 A block of land is purchased by Erin and Mark for \$133 000. Six years later, they sell it for \$164 000. Calculate their percentage profit correct to two decimal places.
- 8 Rose buys a bed for \$2700. She later sells it for \$1500. Calculate her percentage loss, correct to one decimal place.
- 9 For each item, calculate the GST payable and the final price.
 - a a car priced at \$20 900
 - b a home entertainment unit priced at \$1810
 - c an ice cream priced at \$3.00
 - d a theatre ticket priced at \$145
 - e plumber's fees of \$180
 - f a DVD priced at \$27

See Example 24

See Example 25

Worked solutions

Exercise 6-10

MAT08NAWS10054

See Example 26

See Example 27

- See Example 28** 10 Given that the final price of each item has GST included, calculate the GST and the original price correct to the nearest cent.
- | | |
|----------------------------|---------------------------------|
| a accountant's fees: \$792 | b piano lessons: \$198 per term |
| c refrigerator: \$924 | d box of chocolates: \$18.70 |
| e batteries: \$34.65 | f diary: \$16.70 |

Investigation: Uses of percentages

- 1 Prepare a one-page percentage collage for class display. Look through newspapers, magazines and brochures for examples to cut out. Find photographs or draw situations where percentages are used.
- 2 Select two examples of situations where percentages are used. Write a set of ten questions for each example, using the rules about percentages you have learnt. Calculate the answers.
- 3 Swap the questions you wrote in question 2 with others in the class. Each person should answer at least three sets of questions. Ask the student who prepared the questions to mark your answers. If there are any disagreements, check with your teacher.

Worksheet

Percentage problems

MAT08NAWK10059

Homework sheet

Percentages 2

MAT08NAHS10010

Homework sheet

Fractions and percentages revision

MAT08NAHS10011

Quiz

Personal business calculations

MAT08NAQZ00011

6-11 Percentage problems

Exercise 6-11 Percentage problems

- 1 In a box of 180 oranges, 15% are not ripe. How many oranges are not ripe?



- 2 A test result is 65 out of 80. What percentage is this?
- 3 Southside High's hockey team wins 12 games out of 15. What percentage is this?
- 4 A shirt costing \$55 is reduced by 25%. What is the new selling price?
- 5 In some mixed paint, 25% of the mixture is blue, 15% is yellow and the remainder is white. What percentage is white?

Worked solution

Exercise 6-11

MAT08NAWS10055

- 6 Westvale High's basketball team scored 1065 points in a season. Claire shot 22% of the total points. How many points did Claire score?
- 7 At Upper Darling High, 60% of students are girls. How many boys are there in this school of 870 students?
- 8 What percentage of pure gold is 9 carat gold, if pure gold is 24 carat?
- 9 When a floor was being tiled, 830 green tiles were used out of a total of 2075 tiles. What percentage of the floor is made of green tiles?
- 10 In a kindergarten class of 24 students, there were 3 absent students. Find the percentage of students absent from the class.
- 11 A real estate agent earns $2\frac{1}{2}\%$ commission on a sale of \$345 000. How much does the agent earn?
- 12 A worker's pay rate of \$13.65/hour is increased to \$15.20/hour. Find the increase as a percentage of the original pay rate.
- 13 Find how much rent is paid if it is 30% of a weekly income of \$575.
- 14 Anton receives a 7% pay rise. If his old salary was \$67 000 per year, what is his new salary? Select the correct answer **A, B, C** or **D**.
A \$67 007 **B** \$67 469 **C** \$71 690 **D** \$113 900

Worked solution

Exercise 6-11

MAT08NAWS10055

Technology Weekly budget

In this activity, we will use a spreadsheet to calculate a weekly budget for Abdul, a full-time student with a part-time job in a café.

- 1 Enter Abdul's weekly expenses into a spreadsheet. Highlight cells B2 to B8, right-click on **Format Cells** and **Currency** with **two decimal places**.

	A	B	C
1	Expenses	Weekly Budget	% of total
2	Rent	157.40	
3	Food	80.00	
4	Fares	34.50	
5	Entertainment	37.00	
6	Mobile phone	11.75	
7	Savings	25.00	
8	Total Expenses		

- 2 In cell B8, write a formula to calculate the sum of Abdul's weekly expenses.
- 3 In cell C2, write a formula to calculate the percentage of total expenses spent on rent. Right-click on **Format Cells** and **Number** with **one decimal place**.
- 4 Select cell C2 and **Fill Down** to cell C8 to copy the formula and calculate the percentage of total expenses belonging to each item.
- 5 Highlight cells A1 to B7. Click **Insert, Pie Chart** to create a pie chart (sector graph) and give your pie chart an appropriate title.
- 6 Point your mouse (do **not** click) in each sector of the pie chart to read off the percentage breakdown (in brackets) of Abdul's expenses.

Power plus

- 1 A retailer pays \$55 for a chair, then marks it up 60% for the selling price, then adds 10% of this price for GST. Calculate the final selling price.
- 2 Some items increase in value over time, such as jewellery, antiques and real estate. This is called **appreciation**. A gold chain costs \$1500 and appreciates by 8.5% per year. Find:
 - a the value of the chain after one year
 - b the value of the chain after two years.
- 3 The Blewes family buys a home for \$480 000, and it appreciates at 4% per year. Calculate the home's value:
 - a after one year
 - b after two years
 - c after five years
- 4 a A number is increased by 25%, then the result is decreased by 25%. Is the final answer more than or less than the original number?
 b If a number is increased by 25%, then by what percentage must the result be decreased so that the answer is equal to the original number?
- 5 Most items lose value over time. This is called **depreciation**. A Ferrari car costs \$220 000 and depreciates at 15% per year. Calculate how much the Ferrari is worth:



- a after 1 year
 - b after 2 years
 - c after 5 years.
- 6 a A number is decreased by $33\frac{1}{3}\%$, then the result is increased by $33\frac{1}{3}\%$. Is the final answer more than or less than the original number?
 b If a number is decreased by $33\frac{1}{3}\%$, then by what percentage must the result be increased so that the answer is equal to the original number?

Language of maths

commission	fraction	mark-up	quantity
cost price	goods and services tax (GST)	mixed numeral	reciprocal
decimal	improper fraction	numerator	reduction
decrease	increase	percentage	selling price
denominator	loss	profit	simplify
discount	lowest common multiple (LCM)	proper fraction	unitary method

Puzzle sheet

Fractions and percentages
find-a-word

MAT08NAPS10018

- Which words in the list above refer to something getting:
a bigger? **b** smaller?
- What is the product of a number and its reciprocal?
- Name one good and one service that would have a **GST**.
- The 'cent' in percentage means 100. Find other 'cent' words that are related to 100.
- What do we mean when we say that someone gives '110%' effort or support to something?
- Copy and complete: "A **loss** is made by a retailer if the **cost price** is _____ than the **selling price**."

Topic overview

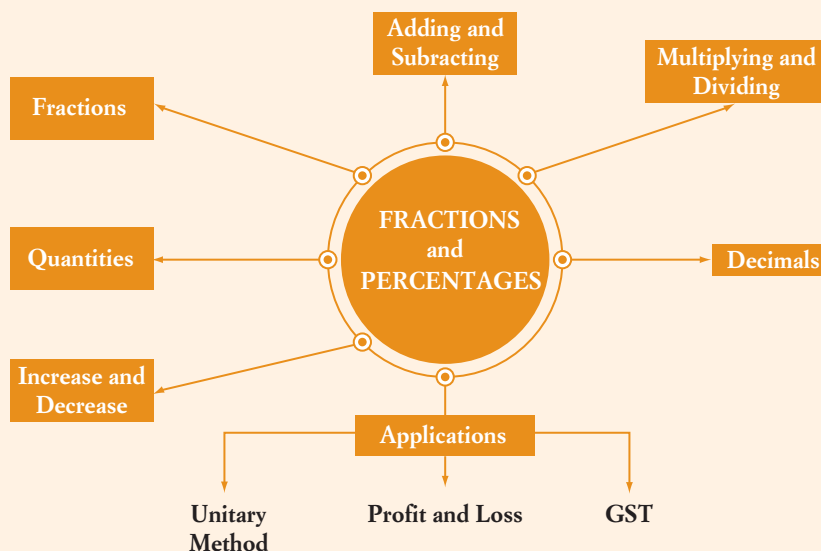
- List any rules you remember from your work on fractions and percentages.
- Name at least three jobs where percentages would be used.
- Is there anything you still do not understand about fractions or percentages? See your teacher.
- Give examples of where you may use fractions or percentages in the future.

Worksheet

Mind map: Fractions and percentages

MAT08NAWK10060

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 6 revision

See Exercise 6-01

1 Simplify each fraction.

a $\frac{6}{8}$

b $\frac{28}{48}$

c $\frac{35}{45}$

See Exercise 6-01

2 Convert each improper fraction into a mixed numeral.

a $\frac{15}{4}$

b $\frac{22}{5}$

c $\frac{26}{6}$

See Exercise 6-01

3 Convert each mixed numeral to an improper fraction.

a $4\frac{1}{2}$

b $3\frac{2}{3}$

c $6\frac{4}{5}$

See Exercise 6-01

4 Arrange these fractions in descending order: $\frac{5}{6}$, $\frac{7}{12}$, $\frac{1}{2}$, $\frac{3}{4}$

See Exercise 6-02

5 Evaluate each expression.

a $\frac{1}{3} + \frac{2}{5}$

b $\frac{4}{7} - \frac{1}{2}$

c $\frac{3}{4} + \frac{2}{3}$

d $2\frac{1}{5} + 4\frac{1}{2}$

e $7 - \frac{5}{6}$

f $5\frac{3}{8} - 2\frac{1}{2}$

See Exercise 6-03

6 Evaluate each expression.

a $\frac{7}{8} \times \frac{4}{9}$

b $\frac{4}{5} \times \frac{2}{3}$

c $\frac{6}{7} \times \frac{5}{8}$

d $2\frac{1}{4} \times \frac{3}{10}$

e $\frac{1}{6} \div \frac{5}{12}$

f $5\frac{3}{8} \div 1\frac{1}{2}$

See Exercise 6-04

7 Convert each percentage into a simplified fraction.

a 74%

b 29%

c 65%

d 157%

See Exercise 6-04

8 Convert each percentage into a decimal.

a 12%

b 16.2%

c 2%

d 5.4%

See Exercise 6-04

9 Convert each fraction into a percentage.

a $\frac{7}{8}$

b $\frac{12}{50}$

c $\frac{2}{9}$

d $\frac{33}{40}$

See Exercise 6-04

10 Convert each decimal into a percentage.

a 0.56

b 0.613

c 0.7

d 0.048

See Exercise 6-04

11 Arrange these numbers in ascending order: $\frac{3}{4}$, 0.725, 77%, 0.7

See Exercise 6-05

12 Evaluate each expression.

a 15% of 40

b $\frac{1}{4}$ of 64

c 27.5% of \$880

d $\frac{2}{3}$ of 1 year (in months)

e 120% of 90 L

f 35% of 6 m (in cm)

See Exercise 6-06

13 Gianni scored 68 out of 80 in a maths test. Write his score as:

a a simplified fraction

b a percentage

See Exercise 6-06

14 Westvale Christian College has 375 boys and 405 girls. What percentage (correct to one decimal place) of the students are boys?

See Exercise 6-07

15 A refrigerator with a marked-up price of \$545 is discounted by 40%. Calculate its sale price.

See Exercise 6-07

16 Judy is to receive a salary increase of 3.5%. If she currently earns \$65 000, what will her new salary be?

- 17 Calculate each expression in your head and write down the answer. See Exercise 6-09
a 20% of 55 tonnes **b** $12\frac{1}{2}\%$ of 96 cm **c** 75% of 12 minutes
- 18 A deposit of 18% paid on a car is \$26 480. What is the cost of the car? See Exercise 6-10
- 19 James bought a guitar for \$750, then sold it for \$600. Calculate his percentage loss on the cost price. See Exercise 6-10
- 20 The price of a mobile phone is \$95.70 including GST. Calculate: See Exercise 6-11
a the amount of GST included in this price
b the price of the phone before GST was added
- 21 Ishween sold \$850 worth of books last week. If she receives 5% commission on all sales, calculate her commission. See Exercise 6-11



7

Statistics and probability

Investigating data

The Australian Bureau of Statistics is the official organisation in charge of collecting data for government departments. The data collected covers many areas, from the height of Australian men, the amount of petrol sold across the country to the number of internet users and providers. This information is used by many people to make decisions that affect governments and businesses.



Chapter outline

	Proficiency strands				
7-01 Organising and displaying data	U	F	PS	R	C
7-02 Types of data	U	F		R	C
7-03 The mean and mode	U	F	PS	R	
7-04 The median and range	U	F	PS	R	
7-05 Analysing frequency tables	U	F	PS	R	
7-06 Dot plots and stem-and-leaf plots	U	F	PS	R	C
7-07 Frequency histograms and polygons	U	F	PS	R	C
7-08 Sampling	U	F			C
7-09 Designing survey questions	U	F	PS		C
7-10 Comparing samples and populations	U	F	PS	R	C
7-11 Analysing data	U	F	PS	R	C

Wordbank

bias Something unwanted that causes a sample to not truly represent the population

categorical data Non-numerical data that can be classified into categories (for example, people's hair colour, or their favourite football team)

frequency How often a score appears in a data set

frequency histogram A special column graph that shows the frequencies of numerical data

frequency polygon A special line graph that shows the frequencies of numerical data

measure of location An average, middle or typical value of a data set; a general name for the mean, median or mode

median The middle score of a data set when the scores are arranged in order, or the average of the two middle scores

numerical data Data that can be measured and counted using values (for example people's height, or the number of CDs they own)

In this chapter you will:

Maths clip

Statistics

MAT08SPMC00001

- interpret and construct divided bar graphs, sector graphs and line graphs with and without ICT
- construct and compare a range of data displays including stem-and-leaf plots and dot plots
- use a tally to organise data into a frequency distribution table
- recognise data as numerical (either discrete or continuous) or categorical
- calculate mean, median, mode and range for sets of data, and interpret these statistics in the context of data
- describe and interpret data displays using median, mean and range
- draw and interpret frequency histograms and polygons
- investigate techniques for collecting data, including census, sampling and observation
- identify and investigate issues involving numerical data collected from primary and secondary sources
- explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes
- construct appropriate survey questions and a related recording sheet to collect both numerical and categorical data about an issue of interest
- explore the variation of means and proportions in random samples drawn from the same population
- investigate the effect of individual data values, including outliers, on the mean and median

SkillCheck

Worksheet

StartUp assignment 7

MAT08SPWK10061

- 1 A group of school students visiting Japan have the following ages.

14	13	12	15	13	14	12
14	15	14	14	13	13	

Show this information on a dot plot.

- 2 The masses (in kilograms) of some hospital patients are:

68	59	63	68	54	48	49
64	47	48	59	68	30	

- a Show this data on an ordered stem-and-leaf plot.
b What is the outlier?

- 3 This back-to-back stem-and-leaf plot shows the points scored for and against the Koala Rugby team in each match, at one stage in the season.

- a What is the highest number of points scored by the Koalas in one game?
b How many matches have the Koalas played so far?
c How many points have been scored against the Koalas?

For	Against
4	0 6
8 4	1 4 6 6
2 2 0 0	2 2 9 9
	3 0 1
8 1	4 7
3	5

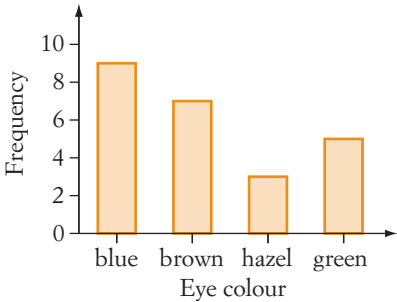
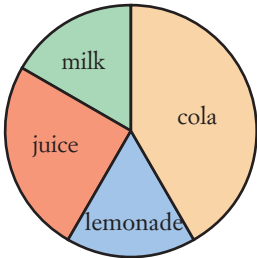
- 4 A group of people was surveyed about the number of phone calls they made on a Saturday night. The results are shown below.

6	4	0	3	1	2	2	3
5	4	3	3	4	1	0	6
6	3	4	3	1	5	4	3

- Draw a column graph for this data.
- How many people were surveyed?
- What was the most common number of phone calls?
- What was the least common number of phone calls?
- What was the highest number of phone calls?

7-01 Organising and displaying data

Data means a collection of facts or raw information. Data can be organised and displayed in a variety of ways so that we can **analyse** (study) them in more detail. Three examples are shown in the table below.

Graph	Description
Column graph 	- Mostly used for data that are in categories - Uses columns to show the quantity or frequency of each category - Also called a 'bar chart' - The horizontal axis (going across) shows the categories - The vertical axis (going up) shows the frequency
Sector graph Preferred drink 	- Shows parts of a whole in circle form - Uses sectors of a circle to show sizes of parts - Also called a 'pie chart' or 'circle graph'

Skillsheet

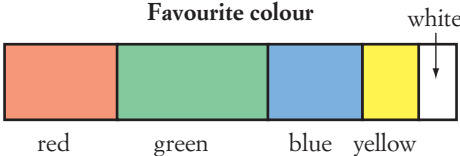
Displaying data

MAT08SPSS10031

Technology worksheet

Olympics medal tally

MAT08SPCT10004

Graph	Description
Divided bar graph Favourite colour  red green blue yellow	- Shows parts of a whole in rectangular form - Uses sections of a rectangle to show sizes of parts

Example 1

A sample of 80 people was surveyed on their favourite TV sport, and the results are shown below.

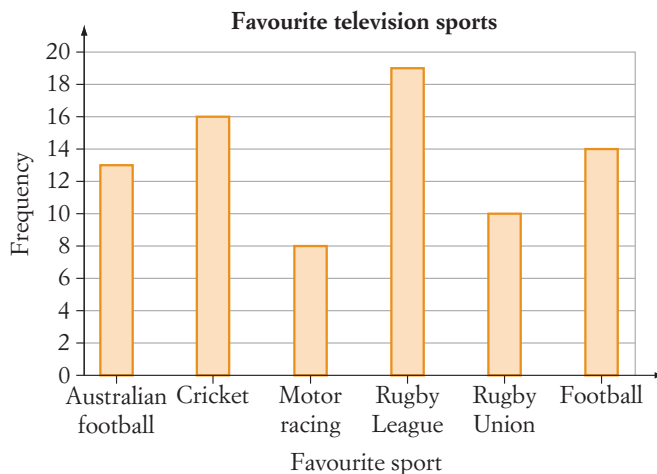
TV sport	Australian football	Cricket	Motor racing	Rugby league	Rugby union	Football
Frequency	13	16	8	19	10	14

Display this data on a:

- a column graph b sector graph c divided bar graph.

Solution

- a The highest frequency is 19 (for rugby league), so use a scale of 1 unit = 2 on the 'Frequency' axis. Label both axes, draw a column for each category, then label the graph.



- b** Draw a large circle, then calculate the angle sizes for each sector using fractions of 360° (because there are 360° in a revolution).
Total number of people in sample = 80.



Sector angles

Australian football: $\frac{13}{80} \times 360^\circ = 58.5^\circ$

Motor racing: $\frac{8}{80} \times 360^\circ = 36^\circ$

Rugby Union: $\frac{10}{80} \times 360^\circ = 45^\circ$

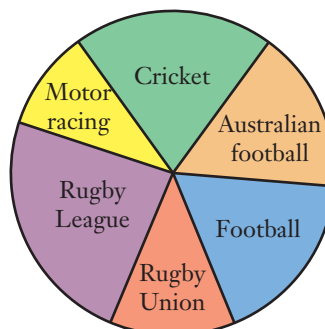
Cricket: $\frac{16}{80} \times 360^\circ = 72^\circ$

Rugby League: $\frac{19}{80} \times 360^\circ = 85.5^\circ$

Football: $\frac{14}{80} \times 360^\circ = 63^\circ$

Construct the sector graph with the calculated angles, then label the sectors and the graph.

Favourite television sports



- c** Draw a rectangle whose length will represent 80, the total frequency (the whole), for example, 12 cm. Then calculate the length of each section using fractions of 12 cm.

Section sizes

Australian football: $\frac{13}{80} \times 12 \text{ cm} = 1.95 \text{ cm}$

Motor racing: $\frac{8}{80} \times 12 \text{ cm} = 1.2 \text{ cm}$

Rugby Union: $\frac{10}{80} \times 12 \text{ cm} = 1.5 \text{ cm}$

Cricket: $\frac{16}{80} \times 12 \text{ cm} = 2.4 \text{ cm}$

Rugby League: $\frac{19}{80} \times 12 \text{ cm} = 2.85 \text{ cm}$

Football: $\frac{14}{80} \times 12 \text{ cm} = 2.1 \text{ cm}$

Construct the divided bar graph with the calculated section lengths, then label the sections and the graph.

Favourite television sports



Frequency tables

Worksheet

Frequency tables

MAT08SPWK00002

A **frequency table** (or **frequency distribution table**) can be used to efficiently organise and count a large set of numerical data scores. The **frequency** of a score is the number of times that score appears in the set of data.

Example 2

Twenty people were surveyed about the number of cars in their household. The results were:

2 2 1 2 0 3 2 1 1 4
1 1 1 2 2 0 3 2 1 2

Complete a frequency table for the data.

Solution

Draw a table with columns for Score, Tally and Frequency, as shown below. The lowest score is 0 and the highest is 4, so list the numbers 0 to 4 in the Score column.

For each score in the data set, draw a tally mark in the tally column. Tally means 'to count', and we are counting how often each score occurs.

The first score is a 2, so mark a 'I' in the 'Tally' column beside 2. The second score is another 2, so mark another 'I' there. Continue until you have counted all of the scores.

Score	Tally	Frequency
0	II	
1	IIII	
2	IIII	
3	II	
4	I	
Total		

IIII means a tally of 5.

For each score, count up the tally marks and write the answer in the 'Frequency' column. Then write the total in the final row and check that it is equal to the total number of scores.

Score	Tally	Frequency
0	II	2
1	IIII	7
2	IIII	8
3	II	2
4	I	1
Total		20

Exercise 7-01 Organising and displaying data

- 1 A class of 30 PE students was surveyed about the day of the week that each student trained for their chosen sport. The results are shown in the table below.

See Example 1

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of students	3	13	4	7	3

Display this data in a sector graph. Label the graph.

- 2 The colours of the jelly beans in a jar were recorded and counted.



Colour	Red	Black	Purple	Pink	Orange	White
Frequency	11	5	2	4	6	8

Display this data as a column graph, using an appropriate scale. Label the graph and both axes.

- 3 A sample of men were surveyed on the number of hours they slept the previous night. The results are shown in this table:

Number of hours	6	7	8	9	10	11
Frequency	3	9	11	6	5	2

Display this data as a divided bar graph, 9 cm long. Label the graph.

Worked solutions

Exercise 7-01

MAT08SPWS10056

- 4 This table shows the number of kilolitres of water used by a household in different areas over a year.

Area	Garden	Shower	Toilet	Kitchen
Water usage (kL)	90	520	140	50

If this information is shown in a divided bar graph of length 10 cm, which of the following is the length of the section representing 'Shower'? Select the correct answer **A**, **B**, **C** or **D**.

- A** 1.5 cm **B** 1.9 cm **C** 5.2 cm **D** 6.5 cm

See Example 2

- 5 The results of a spelling quiz (out of 10) for a class of students are as shown.

7	7	8	9	7	5	4	8	9
9	8	10	8	7	5	5	4	1
8	8	9	7	7	4	8	3	1

- Complete a frequency table for this data.
 - What was the highest score?
 - How many students scored 4 marks in the quiz?
 - What was the most frequent score?
 - Draw a column graph of this data.
- 6 Thirty customers were asked to name their preferred car colour. Their answers were coded as follows: B = black, S = silver, W = white, U = blue, R = red.

B	B	W	R	W	R	B	S	U
W	R	B	S	B	U	W	B	W
S	B	B	R	R	U	W	B	B

- Complete a frequency table for this data.
 - Draw a divided bar graph for this data.
 - Which is the most popular colour for this group of customers?
- 7 A sample of women were asked how many siblings (brothers and sisters) they had. The results were as follows:

4	0	2	3	2	4	3	5	4	3
0	0	1	2	3	2	2	1	4	3
1	1	1	5	2	5	0	1	2	1
0	1	3	2	3	4	1	1	3	3

- Complete a frequency table for this data.
- What was the highest number of siblings?
- What was the mode (the most frequent score)?
- Illustrate this data on a sector graph.

Technology Graphing life expectancy

Life expectancy describes the age to which a person is expected to live, based on statistical data covering different factors.

- 1 Enter the data below into a new spreadsheet. It compares the life expectancy at birth for males and females in 10 countries.



	A	B	C	D
1	Life Expectancy at Birth			
2				
3	Country	Male (years)	Female (years)	Total (years)
4	Australia	77.75	83.63	80.62
5	Bangladesh	62.81	62.86	62.84
6	Brazil	68.3	76.38	72.24
7	Canada	76.98	83.86	80.34
8	Chad	46.17	48.27	47.2
9	China	71.13	74.82	72.88
10	India	66.28	71.17	68.59
11	Spain	76.46	83.32	79.78
12	Sweden	78.39	83	80.63
13	USA	75.15	80.97	78

Source: www.cia.gov

- 2 Highlight all four columns, including headings. Click **Insert** and **2-D Column Graph**. Give the graph an appropriate title and label the axes.
- 3 Which country has:
 - a the lowest total life expectancy?
 - b the highest total life expectancy?
 - c the lowest male life expectancy?
 - d the lowest female life expectancy?
 - e the highest male life expectancy?
 - f the highest female life expectancy?
- 4 Suggest reasons why some countries would have higher life expectancies than others.
- 5 In column E, calculate the difference between the male and female life expectancies for each country.
- 6 Generally, do males or females live longer?
- 7 Which country has the greatest difference between its male and female life expectancies?
- 8 Which country has the least difference?

7-02 Types of data

Statistical data may be in the form of **categories** (such as jelly bean colour or favourite sport), or in the form of **numbers** (such as quiz scores or number of cars owned). There are two types of data.

Categorical data are usually words and can be grouped into categories, such as a car's colour or the suburb where you live.

Numerical data (or **quantitative data**) are numbers describing things that can be counted or measured, like the number of soccer goals scored or a person's height.

Example 3

Classify each of the following types of data as categorical or numerical.

a makes of cars	b people's salaries
c weights of athletes	d favourite radio stations

Solution

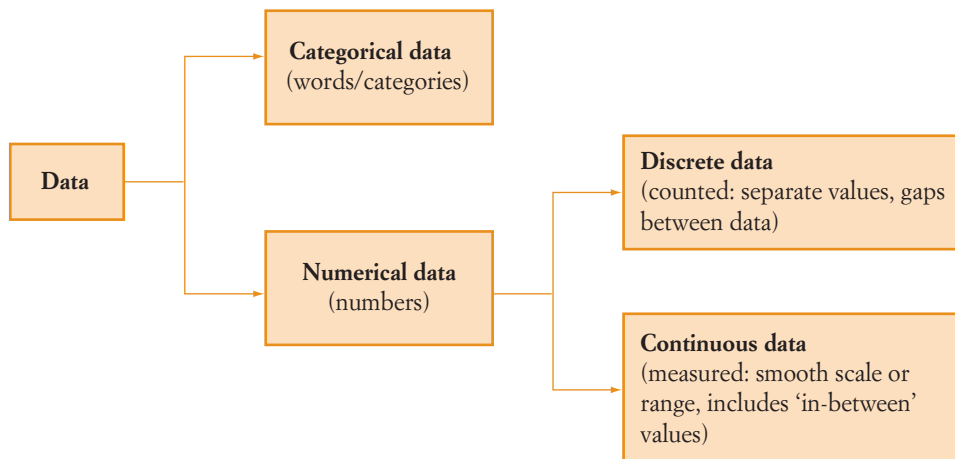
a Makes of cars is categorical data.	Involves words or categories.
b People's salaries is numerical data.	Involves numbers.
c Weights of athletes is numerical data.	Involves numbers.
d Favourite radio stations is categorical data.	Involves words or categories.

Types of numerical data

Numerical data can be further divided into **discrete data** and **continuous data**.

Discrete data are counted or measured and can only take on separate, distinct values, with 'gaps' or 'jumps'. 'In-between' values are not allowed. For example, the number of children in a family is discrete because it could be 0, 1, 2, etc., but never 2.3 or $1\frac{2}{5}$.

Continuous data are measured on a 'smooth' scale without any gaps, and can take on a full range of values. 'In-between' values are allowed. For example, a town's temperature is continuous because it could be 28°C, 29°C, 29.5°C, 29.8°C, etc.



Example 4



Classify each of the following types of numerical data as discrete or continuous:

- a** number of rooms in a home
- b** loudness of aeroplanes
- c** people's shoe sizes
- d** weight of whales

Solution

a Number of rooms is discrete data.	Takes on separate values.
b Loudness of aeroplanes is continuous data.	Takes on a smooth scale of values.
c People's shoe sizes is discrete data.	Takes on separate values.
d Weight of whales is continuous data.	Takes on a smooth scale of values.

Exercise 7-02 Types of data

- 1** Classify each type of data as categorical (C) or numerical (N).
 - a** masses of Year 8 students
 - b** brands of computers
 - c** prices of gelato ice cream
 - d** numbers of TV sets owned by family members
 - e** amounts of rainfall in Adelaide
 - f** causes of tooth decay
 - g** sets won in a tennis match
 - h** speed of trucks

See Example 3

- See Example 4**
- 2 Classify each type of numerical data as discrete (D) or continuous (C).
 - a number of homes with solar power
 - b patients' body temperatures
 - c sizes of the crowds at rock concerts
 - d star ratings of motels
 - e times taken for athletes to run 100 metres
 - f school populations
 - g amount of cough medicine taken by children
 - h number of road accidents in Grafton last weekend
 - 3 Which of the following is **not** discrete data? Select the correct answer **A, B, C** or **D**.
 - A The number of shoes you own.
 - B The number of blue-eyed students in your school.
 - C The number of options in a multiple choice test.
 - D The average temperature for the month.
 - 4 Classify each type of data as categorical (C), numerical discrete (ND) or numerical continuous (NC).
 - a people's hair colours
 - b customer ratings of a restaurant's level of service (poor, satisfactory, good, very good)
 - c number of points players scored in a netball match
 - d film classification (G, PG, M, MA, R)
 - e the age you turn this year
 - f heights of bridges
 - g number of staff employed
 - h countries where cars were manufactured

Worked solutions

Exercise 7-02

MAT08SPWS10057



7-03 The mean and mode

When we analyse data, we summarise the information and try to find patterns and trends to draw some conclusions from them. We can use the **mean**, **mode**, **median** and **range** to make comparisons between sets of data.

The mean, mode and median are known as **measures of location** (averages): they are typical or central values that represent all of the data in the set.

The range is a **measure of spread**: it describes how spread out the scores are.

The mean

The **mean** is found by adding all the scores and dividing by the number of scores.

Summary

The symbol for the mean is $\bar{x}$.

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

- The mean is a single value that represents all of the scores.
- *All* of the scores are used in the calculation of the mean.
- The mean is *not* necessarily one of the scores.

The mode

The **mode** is the most common or frequent score (or scores). A set of data may have more than one mode, or no mode at all.

Summary

The **mode** is the score (or scores) that occurs most often:
the score with highest frequency.

← **Think:** mode = 'most often'

- The mode can be one or more values.
- The mode does not depend on all of the scores in the data set.
- The mode is one (or more) of the scores.

Example 5

The number of goals scored by Julia's soccer team in each match over a season were:

3 2 1 4 0 2 3 1 3 1 1 0

a Find the mean.

b Find the mode.

Solution

$$\begin{aligned} \text{a Mean } \bar{x} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{3 + 2 + 1 + 4 + 0 + 2 + 3 + 1 + 3 + 1 + 1 + 0}{12} \\ &= \frac{21}{12} \\ &= 1.75 \end{aligned}$$

Note that the value of the mean is at the centre of the scores.

b The mode is 1.

It is the most common score (occurs 4 times).

The mean from a calculator's statistics mode

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the scores from Example 5 using your calculator's statistics mode.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 3 = 2 = , etc. to enter in column AC to leave table	3 M+ 2 M+ , etc.
Calculate the mean. ($\bar{x} = 1.75$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 12$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio graphics	Texas instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting and pressing CLEAR STAT EDIT

Operation	Casio graphics	Texas instruments graphics
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes	With cursor on L1 CLEAR ENTER
Enter data.	3 EXE 2 EXE , etc. to enter in List 1 column	3 ENTER 2 ENTER , etc. to enter in List 1 column
Calculate the mean. ($\bar{x} = 1.75$) Calculate the sum of scores. ($\Sigma x = 21$) Check the number of scores. ($n = 12$)	F6 CALC SET to make these settings (if different): 1Var XList: List1 1Var Freq: 1 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats ENTER to calculate many statistics (scroll down for more)

Exercise 7-03 The mean and mode

- 1 The residents in a street were surveyed about the number of children living in each household. The results were:

See Example 5

2 2 1 2 0 3 2 1 1 4 1 0

- Find the mean, correct to one decimal place.
 - Find the mode.
- 2 For each set of data, find:
- the mean (rounded to two decimal places, if needed)
 - the mode(s).
- 6 8 7 6
 - 7 9 5 2 2 6 2
 - 11 7 7 6 1 2 4 1
 - 96 53 96 45 64 76 54 54 96
 - 47.31 47.20 47.24 47.21 47.28 47.26
- 3 Use your calculator's statistics mode to calculate the mean of each data set in Question 2.
- 4 Find the mode of each set of categorical data.
- Holden, Ford, Nissan, Holden, Hyundai, Ford, Hyundai, Nissan, Hyundai, Toyota, Toyota
 - heart, spade, spade, diamond, spade, club, heart, heart, diamond, spade
 - Chinese, Korean, Japanese, Korean, Chinese, Japanese, Japanese, Korean, Korean
- 5 Why is it not possible to find the **mean** of each data set in Question 4?
- 6 In a diving competition, the judges awarded the following scores.
- 6.2 7.0 8.2 8.8 7.0 9.1 7.0 8.1 8.0
- Find the mean of these scores.
 - Find the mode of these scores.
 - Which measure of location represents this set of scores better? Give a reason for your answer.

Worked solutions

Exercise 7-03

MAT08SPWS10058

- 7 What is the difference between the mean and the mode of the following set of scores?

20 60 20 50 40

Select the correct answer **A, B, C** or **D**.

A 20 **B** 12 **C** 18 **D** 2

- 8 The ages of the members of the Carrozza family are:

1 3 11 25 26 30 31 52 58

The ages of the members of the Binns family are:

5 8 16 21 25 29 40 63 93

- Calculate the mean age (to one decimal place) of each family.
 - What is the main difference between these two sets of ages?
 - What effect does this difference have on the means?
- 9 Two Year 8 classes scored the following marks for their PE assignments.
- 8Y:** 5 5 5 5 6 6 6 7 7 7 8
- 8Z:** 3 3 4 5 5 6 6 7 9 9 10

- Calculate the mean mark (to one decimal place) for each class.
- Describe the differences between the way each class has its marks spread out.
- Class 8X scored the following marks, but Lisa was away.

8X: 4 5 5 6 6 6 7 7 7 10

What mark would Lisa have to score so that her class has the same mean as the other two classes?

Worked solutions

Exercise 7-03

MAT08SPWS10058

- 10 The mean points score per game by a basketball team over a season of 30 games was 98. What was the total number of points scored over the season?



- 11 The takings at the school canteen were:
- Monday \$1600 Tuesday \$2400 Wednesday \$1500 Thursday \$1750.
- Find the average takings for these four days.
 - How much should the canteen earn on Friday to make the average daily takings \$2000?
- 12 **a** Find 4 scores that have a mean of 10 and a mode of 11.
- b** Find 7 scores that have a mean of 13 and a mode of 5.

7-04 The median and range

Worksheet

Mean, median, mode

MAT08SPWK10064

Skillsheet

Statistical measures

MAT08SPSS10032

Homework sheet

Statistics 1

MAT08SPHS10038

The median

The **median** is the middle score or the average of the two middle scores.

Summary

When the scores are ordered, the **median** is:

- the middle score if there is an odd number of scores
- the average of the two middle scores if there is an even number of scores.

Think: 'Median' sounds like 'medium', which is halfway between small and large.

A 'median' strip is the strip of grass or concrete down the *middle* of a highway.

- The median is a single value that represents all of the scores.
- The median does not depend on all of the scores in the data set, so it is not affected by outliers (extremely high or low scores).
- The median is often one of the scores.

The range

The **range** is a measure of the spread of a set of data, and is simply the difference between the highest and lowest scores.

Summary

Range = highest score – lowest score

Puzzle sheet

Ranges and averages

MAT08SPPS00001

Puzzle sheet

Data puzzles

MAT08SPPS00002

Example 6

The ages of the members of the Carrozza family are:

3 1 25 58 26 52 30 31 11

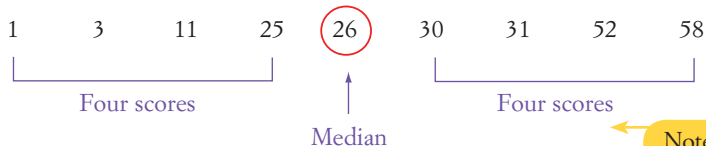
Find:

a the median

b the range

Solution

- a First, rewrite the scores in order.



Note that the value of the median is at the centre of the scores.

There are 9 scores, an **odd** number, so there is **one** middle score.

$\frac{1}{2} \times 9 = 4\frac{1}{2}$, so round up to 5 and count to the 5th score (circled above) to find the median.

$$\text{Median} = 26$$

- b Range = $58 - 1$
= 57

Highest score – lowest score

Example 7

The lap times (in seconds) of six cyclists were:

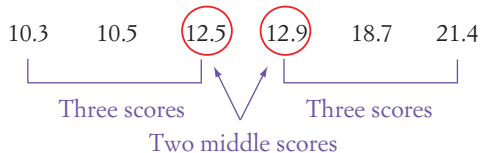
12.5 21.4 10.5 18.7 12.9 10.3

Find:

- a the median b the range

Solution

- a Rewrite the scores in order:



There are 6 scores, an **even** number, so there are **two** middle scores.

$\frac{1}{2} \times 6 = 3$, so count to the 3rd and 4th scores (circled above) and find their average.

$$\begin{aligned} \text{Median} &= \frac{12.5 + 12.9}{2} \\ &= 12.7 \end{aligned}$$

12.7 is halfway between 12.5 and 12.9

- b Range = $21.4 - 10.3$
= 11.1



Highest score – lowest score

Exercise 7-04 The median and range

- 1 What is the median of this set of data?

10 3 5 9 4 3 10

Select the correct answer **A, B, C** or **D**.

A 6 **B** 7 **C** 5 **D** 4

See Example 6

- 2 For each set of data, find:

i the median ii the range.

a 11 9 7 4

b 46 50 48 47 50 40

c 12 13 11 14 10

d 84 50 0 28 14 99 51 66 51

e 57 57 57 57 57 57 57

f 64.2 99 99.2 71.2 93.5 61.8

- 3 Manish likes to gamble on horse races. His winnings or losses (in dollars) each day are as follows (losses are shown as negative).

8	21	-16	-25	21	-25	16
11	-12	12	-25	17	-25	14

- a** Calculate the mean correct to the nearest cent.
b Find the median and the mode.
c As Manish wins more often than he loses, should he continue to gamble on horse races? Give a reason for your answer.

- 4 Eleven houses were sold in Jacaranda Avenue. The selling prices are listed below.

\$580 000	\$485 000	\$660 000	\$758 000	\$644 000	\$1 670 000
\$815 000	\$686 000	\$666 500	\$720 800	\$805 000	

- a** Find the mode.
b Find the median.
c Find the mean.
d Which measure of location best describes this set of house prices? Give a reason for your answer.



Worked solutions

Exercise 7-04

MAT08SPWS10059

- 5 Two tenpin bowlers recorded the following scores over 10 games.

<i>Aldo:</i>	175	175	160	165	180	145	150	177	177	177
<i>Peter:</i>	173	173	174	175	175	176	176	177	179	179

- a Calculate the mean for each bowler.
 b Calculate the range for each bowler.
 c A third player, Rob, recorded the following scores over nine games.

Rob: 134 175 175 144 176 150 187 166 180

What score would Rob need to achieve in his tenth game to have the same mean score as Aldo?

- d Who would you choose, and why, if you were asked to present an award for:
 i the more talented bowler?
 ii the more consistent bowler?



- 6 The favourite party food for a group of three-year-old children was recorded.

popcorn	fruit	chocolate	fruit	popcorn
chocolate	chocolate	fruit	fruit	fruit

Which is the only statistical measure that can be found for this data? Select the correct answer **A**, **B**, **C** or **D**.

A mean **B** median **C** mode **D** range

- 7 Jim is the coach of the Panthers athletics team. He needs to pick his best runner for the 200 m race. He has recorded the times (in seconds) for the last few races of Daniel and Cristian.

Daniel	29.4	31	27.5	32.5	28	
Cristian	28.5	27	30.5	28.5	31.5	28.5

- a Calculate the mean for each runner.
 b Calculate the range for each runner.
 c Who do you think Jim should choose as the runner for the next event? Give a reason for your answer.

- 8 A group of 8 friends counted the number of letters in their surnames. The results were:

4 6 7 5 4 6 9 □

where □ represents a missing score.

Find a possible value of □ if the median is 6 and the range is 5.

- 9 a Find 8 scores that have a median of 8 and a range of 8.
 b Find 7 scores from 3 to 9 inclusive that have a mode of 4 and a median of 6.

Worked solutions

Exercise 7-04

MAT08SPWS10059

Technology Mean, mode, median and range

Using a spreadsheet

- 1 Enter the data below into a new spreadsheet, listing the daily maximum temperatures in Camden for one week in January 2011.

	A	B	C	D	E
1	Day	Temperature (deg C)			
2	Saturday	31.5		Mean	
3	Sunday	32.2		Mode	
4	Monday	33		Median	
5	Tuesday	36		Maximum	
6	Wednesday	36.8		Minimum	
7	Thursday	35.9		Range	
8	Friday	26.7			

- 2 Enter each formula into the given cell to calculate each statistic.

Cell E2: =average(B2:B8)

Cell E3: =mode(B2:B8)

Cell E4: =median(B2:B8)

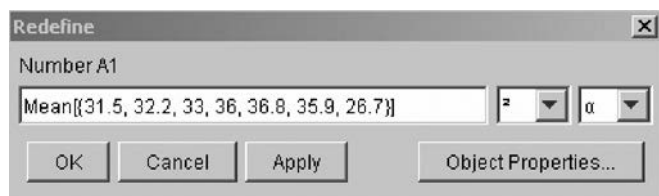
Cell E5: =max(B2:B8)

Cell E6: =min(B2:B8)

Cell E7: =E5-E6

Using GeoGebra

- 1 Click **View** and **Spreadsheet view**.
- 2 To calculate the **mean**, in cell A1 enter **me** and you should see the complete word **Mean[]**.
- 3 Click between [] and enter the data set, with commas, then click **Enter**.
Mean[31.5, 32.2, 33, 36, 36.8, 35.9, 26.7]
- 4 Double-click on cell A1. This shows the data entered.



- 5 To calculate the **mode(s)**, in cell A2 enter the data set, then click **Enter**.
Mode[31.5, 32.2, 33, 36, 36.8, 35.9, 26.7].
- 6 To calculate the **median**, in cell A3 enter the data set, then click **Enter**.
Median[31.5, 32.2, 33, 36, 36.8, 35.9, 26.7].

Technology worksheet

Students' marks

MAT08SPCT10005

Technology worksheet

Finding the mean,
median, mode and
range

MAT08SPCT00001

Mental skills 7

Maths without calculators

Reading linear scales

Understanding and reading the scale on a measuring instrument, on a number line or on the axis of a graph is an important mathematical skill.

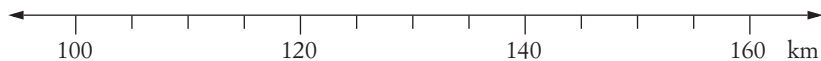
Skillsheet

Reading linear scales

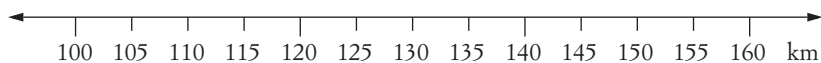
MAT08SPSS10030

1 Study each example.

a Complete the missing values on this scale.



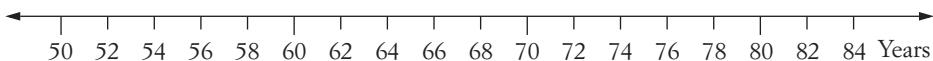
- First, choose two values on the scale, say 100 and 120.
- Count the number of intervals ('spaces') between the two values. There are four intervals between 100 and 120.
- To find the size of each interval, divide the difference between the two values by the number of intervals.
- Difference = $120 - 100 = 20$ km
- Number of intervals = 4
- Size of an interval = $20 \div 4 = 5$ km
- Use the calculated size of an interval to complete the missing values.



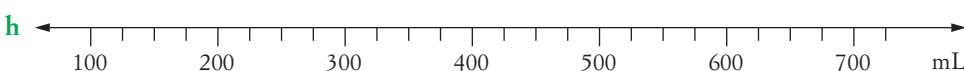
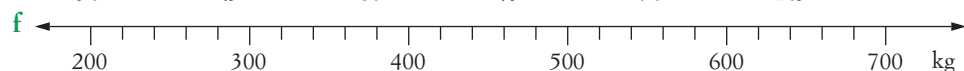
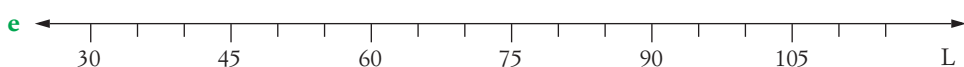
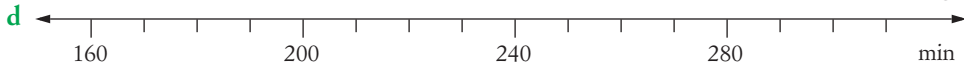
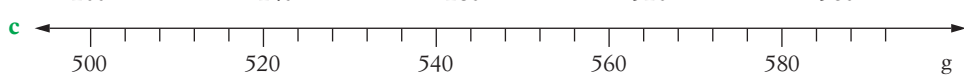
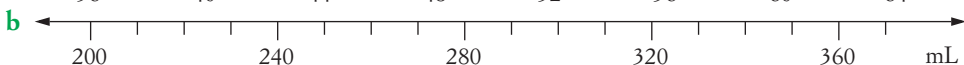
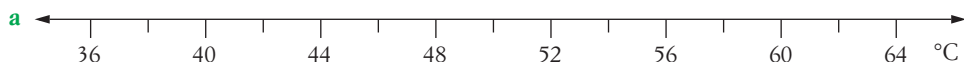
b Complete the values on this scale.



- Choose 50 and 60 on the scale.
- Number of intervals (between 50 and 60) = 5
- Difference (between 50 and 60) = $60 - 50 = 10$ years
- Size of an interval = $10 \div 5 = 2$ years.



2 Now copy and complete the following scales.



Investigation: Gym friends

Seven friends joined a gym to lose weight. The table below shows the ages and the weight loss for each person.

	Sex	Age	Weight loss
Brenda	F	36	4 kg
Peter	M	34	28 kg
Mark	M	28	21 kg
Fiona	F	33	2 kg
Manjeet	F	30	6 kg
Mia	F	34	3 kg
Natasa	F	29	6 kg

In the following questions, round all mean answers to two decimal places.

- What is the mean age?
- Find the mean age of the males.
- What is the median age of the group? Who has this age?
- What is the median age of the males? Who has this age?
- If the men were not counted, would the mean age or the median age be affected more?
- Predict what would happen to the mean age if Imraan (aged 21) joined the group. Test your prediction.
- What was the mean age two years ago? How does this compare to the group's mean age now?
- Compare the mean weight loss with the median weight loss of the whole group. Who has the median weight loss?
- Compare the mean weight loss and the median weight loss of the females only. Who has the median weight loss now?
- When Diego joined the group, the mean weight loss of the group became 20 kg. How much weight did Diego lose?



Investigation: Finding the mean of a large set of data

How do we find the mean of a large number of scores? Is there a simpler way than adding all the scores, then dividing the total by the number of scores?

A group of students was surveyed about the number of hours they watched television on Saturday. The results are shown below.

6 4 0 3 1 2 2 3 5 4 3 3
4 1 0 6 6 3 4 3 1 5 4 3

We can calculate the mean of this set of data using three different methods.

Method 1: Adding all the scores and dividing by the number of scores

- 1 Find the sum of all of the scores in the data set.
- 2 How many scores are there in the data set?
- 3 Hence show that the mean is approximately 3.17 hours.

Method 2: Arranging the scores in order first to simplify the addition

- 1 Sort all of the scores in ascending order and group the equal scores:

0 0 1 1 1 2 2 3 3 3 3 3 3 4 4 4 4 5 5 6 6 6

- 2 To find the sum of all the scores, notice that we can multiply each score by its frequency and add those amounts together.

Evaluate $(0 \times 2) + (1 \times 3) + (2 \times 2) + (3 \times 7) + (4 \times 5) + (5 \times 2) + (6 \times 3)$.

- 3 Hence show that the mean is approximately 3.17 hours.

Method 3: Using a frequency table with an extra column

This is a more organised version of Method 2.

- 1 Copy the frequency table on the right and complete the frequency column for the data.
- 2 What is the total number of scores?
- 3 To help calculate the sum of all of the scores, in the 'Frequency $\times$ Score' column, multiply each score by its frequency.
- 4 What is the sum of all the scores?
- 5 Hence show that the mean is approximately 3.17 hours.

Score	Frequency	Frequency $\times$ Score
0	2	0
1	3	3
2	2	4
3		
4		
5		
6		
Total		

Worksheet

The mean and fx tables

MAT08SPWK00003

Video tutorial

Analysing frequency tables

MAT08SPVT10013

7-05 Analysing frequency tables**Example 8**

The number of minutes each train is late at a railway station in one afternoon is shown below:

2 4 1 0 0 3 2 3
3 2 0 3 2 1 3

Copy and complete this frequency table for the data and use it to find:

- a the range
- b the mode
- c the median
- d the mean, correct to one decimal place.

Score, x	Frequency, f	fx
0		
1		
2		
3		
4		
Total		

Solution

fx means $f \times x$, so in the fx column, multiply each score (x) by its frequency (f):

$$3 \times 0 = 0$$

$$2 \times 1 = 2$$

$$4 \times 2 = 8$$

Score, x	Frequency, f	fx
0	3	0
1	2	2
2	4	8
3	5	15
4	1	4
Total	15	29

a Range = $4 - 0$
= 4

b Mode = 3

c There are 15 scores (an odd number), so the median (middle) is the **8th** score. According to the frequency table, the scores in order are:

0, 0, 0 1, 1 2, 2, **2**, 2 3, 3, 3, 3, 3 4

The first 3 scores are 0, the next 2 scores are 1, the next 4 scores are 2, and so on.

$\therefore$ The 8th score (circled) is 2.

Median = 2.

d Mean $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$.

The fx column multiplies each score by its frequency so its total, 29, is the sum of all the scores. If we write all the scores, we can see that the fx column groups and adds the scores.

$$\underbrace{0, 0, 0}_{\text{Sum } 0} \quad \underbrace{1, 1}_2 \quad \underbrace{2, 2, 2, 2}_8 \quad \underbrace{3, 3, 3, 3, 3}_{15} \quad \underbrace{4}_4$$

The f column shows the frequency of each score so its total, 15, is the number of scores.

$$\begin{aligned} \text{Mean} &= \frac{\text{sum of } fx}{\text{sum of } f} \\ &= \frac{29}{15} \\ &= 1.9333 \dots \\ &= 1.9 \end{aligned}$$

Highest score – lowest score

The score with the highest frequency (5)

Half of 15 is $7\frac{1}{2}$, so round up to 8 and count the 8th score

Summary

For data in a frequency table:

- the **mean** is $\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f}$
- the **mode** is the score(s) with the highest frequency, f
- the **range** is the difference between the last and first scores in the score (x) column

Example 9

The ages of children at a play centre are shown in the frequency table. Copy and complete the table and use it to find:

Score, x	Frequency, f	fx
5	3	
6	4	
7	5	
8	2	
9	2	
Total		

- a the mode b the mean
c the median d the range.



Solution

Score, x	Frequency, f	fx
5	3	15
6	4	24
7	5	35
8	2	16
9	2	18
Total	16	108

- a Mode = 7 Has the highest frequency, 5.

b Mean: $\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f}$

$$= \frac{108}{16}$$

$$= 6.75$$

- c There are 16 scores (an even number), so there are two middle scores.

5 5 5 6 6 6 6 7 7 7 8 8 9 9

↑ ↑
The two middle scores

Half of 16 is 8, so count the 8th and 9th scores (circled) and find the average of them.

$$\begin{aligned}\text{Median} &= \frac{7+7}{2} \\ &= 7\end{aligned}$$

d Range = $9 - 5$
= 4

Note that the mean and median are close to each other and at the centre of the scores.

The mean of data in a frequency table using a calculator's statistics mode

Data from a frequency table can be entered into a scientific or graphics calculator using the statistics mode. Follow the instructions in the table below to calculate the mean of the children's ages from Example 9 using your calculator's statistics mode.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 5 = 6 = , etc. to enter in x column 3 = 4 = , etc. to enter in FREQ column AC to leave table	5 2ndF STO 3 M+ 6 2ndF STO 4 M+ , etc.
Calculate the mean. ($\bar{x} = 6.75$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 16$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio graphics	Texas instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting and pressing CLEAR STAT EDIT

Operation	Casio graphics	Texas instruments graphics
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2	With cursor on L1 CLEAR ENTER Repeat for L2
Enter data.	5 EXE 6 EXE , etc. to enter in List 1 column 3 EXE 4 EXE , etc. to enter in List 2 column	5 ENTER 6 ENTER , etc. to enter in List 1 column 3 ENTER 4 ENTER , etc. to enter in List 2 column
Calculate the mean. ($\bar{x} = 6.75$) Calculate the number of scores. ($\Sigma x = 108$) Check the number of scores. ($n = 16$)	F6 CALC SET to make these settings (if different): 1Var XList: List1 1Var Freq: List 2 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats and type 'L1, L2' by pressing 2nd 1 · 2nd 2 ENTER to calculate many statistics (scroll down for more)

Exercise 7-05 Analysing frequency tables

In this exercise, round all means to one decimal place where necessary.

See Examples 8, 9

- 1 This table shows how many times a sample of children visited a doctor over 3 months. Copy and complete the table, and use it to find:

- the number of people surveyed
- the median
- the mean
- the mode
- the range.

Number of visits, x	Frequency, f	fx
0	5	
1	6	
2	14	
3	3	
4	5	
8	1	
Total		

- 2 A sample of hospital patients were surveyed on the number of hours they slept the previous night. The results are shown below.

6 7 9 8 10 11 9 10 8 9 7 6
11 9 8 8 8 10 10 9 7 8 9 10

- How many people were surveyed?
- What was the highest number of hours people slept?
- Use a frequency table with an fx column to find the mean.
- Find the range.
- Find the median.

- 3 The judge's scores for students' speeches (out of 10) for an English class are as shown.

7	7	8	9	7	5	4	8	9
9	8	10	8	7	5	5	4	1
8	8	9	7	7	4	8	3	1

- Complete a frequency table for this data.
 - What is the mode?
 - What is the mean?
 - What is the median?
 - What is the range?
- 4 A class of students was asked how many children there were in their families.
The results were:

2	4	5	1	2	3	1	3	2	4	2	4
2	4	2	3	1	5	6	2	2	1	3	3

Complete a frequency table for this data, including an $\sum fx$ column, and use it to find:

- the range
 - the mode
 - the median
 - the mean
- 5 This frequency table shows the heights in metres of a sample of students.

Height (m)	Frequency
1.42	4
1.44	5
1.46	8
1.48	11
1.5	6
1.52	3

Find:

- the number of students in the sample
 - the mean using your calculator's statistics mode
 - the range
 - the median
 - the mode
- 6 Carmelina recorded the number of DVDs bought by the people entering her shop during one hour. The results are shown in the table.

Number of DVDs	Frequency
0	7
1	5
2	13
3	4
4	1
5	2
6	1

What is the median number of DVDs bought?
Select the correct answer **A**, **B**, **C** or **D**.

- A** 2 **B** 3 **C** 13 **D** 4

Worked solutions

Exercise 7-05

MAT08SPWS10060

- 7 Thirty people were asked how often they went to a cinema in the previous month. The results were:

4 5 3 1 3 7 1 1 4 0
 0 4 3 2 4 2 4 7 6 6
 1 3 5 5 1 3 6 7 0 3



Construct a frequency table and use it to find:

- the range
 - the mode
 - the median
 - the mean.
- 8 A pigeon breeders' club has 25 members. This frequency table shows the number of pigeons each member has.

Score, x	Frequency, f
8	3
10	4
14	3
15	6
17	2
18	4
20	3

- What is the range?
- What is the median?
- Use your calculator's statistics mode to find the mean.



Technology Daily temperatures

- 1 Enter the following data into a spreadsheet: a frequency table showing the maximum daily temperatures in Coffs Harbour over July 2011 (source: www.bom.gov.au).

	A	B	C
1	Temperatures (x)	Frequency (f)	fx
2	15	1	
3	16	3	
4	17	3	
5	18	7	
6	19	7	
7	20	2	
8	21	7	
9	22	0	
10	23	1	
11	Total:		
12	Mean:		
13	Range:		

- 2 How many temperatures are in this data set? In cell B11, enter an appropriate formula for the sum of the Frequency column.
- 3 To calculate the mean, first complete the fx column. In cell C2, enter the formula for multiplying the values in the A and B columns, then **Fill Down** to copy the formula into cells C3 to C10.
- 4 In cell C11, enter an appropriate formula for the sum of the fx column.
- 5 In cell B12, calculate the mean by entering the formula for dividing the sum of the fx column by the sum of the f column.
- 6 In cell B13, calculate the range by entering the formula for subtracting the lowest temperature from the highest temperature.

7-06 Dot plots and stem-and-leaf plots

Dot plots

A **dot plot** is a graph that uses dots to show the frequency of each score. A dot plot shows:

- any gaps in the data
- any **clusters**, where scores are grouped or bunched together
- any **outliers**: extremely high or low scores that are very different from the rest of the scores
- how the scores are spread out.

An **outlier** 'lies outside' the other scores in the data set.
Outlier is pronounced 'out-ly-er'

Worksheet

Stem-and-leaf plots

MAT08SPWK10066

Technology worksheet

Dot plots Fx -Stat

MAT08SPCT10006

Worksheet

What data?

MAT08SPWK10067

Example 10

Homework sheet

Statistics 2

MAT08SPHS10039

Technology

GeoGebra: Statistics

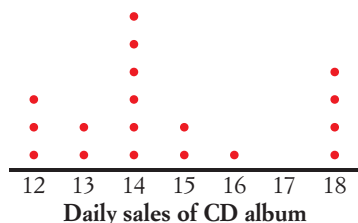
MAT08SPTC00001

Quiz

Statistics

MAT08SPQZ00001

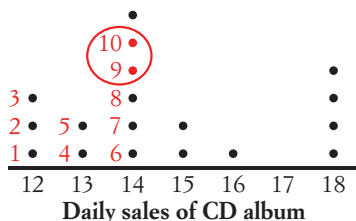
A music store recorded the daily sales figure of a particular CD album while it was popular, and displayed the results in a dot plot.



- On how many days were exactly 13 copies sold?
- What is the range?
- Find the mode.
- Find the median.
- Calculate the mean, correct to one decimal place.

Solution

- 13 copies were sold on 2 days 2 dots above 13
- Range = $18 - 12$ Highest score – lowest score
 $= 6$
- Mode = 14 The score with the most dots.
- There are 18 scores (18 dots). This is an even number, so there are two middle scores (the 9th and 10th scores). By counting the dots, or by crossing out pairs of dots at each end, we can see that the middle scores (circled) are 14 and 14.



$$\begin{aligned}\text{Median} &= \frac{14 + 14}{2} \\ &= 14\end{aligned}$$

- Mean: $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$ (or $\frac{\text{sum of } fx}{\text{sum of } f}$)

Number of scores = 18 18 dots

$$\begin{aligned}\bar{x} &= \frac{3 \times 12 + 2 \times 13 + 6 \times 14 + 2 \times 15 + 16 + 4 \times 18}{18} \\ &= 14.6666\dots \\ &= 14.7\end{aligned}$$

Note that the mean and the median are close to each other and at the centre of the scores.

Note: This answer can also be found using a frequency table and/or your calculator's statistics mode.

Stem-and-leaf plots

A **stem-and-leaf plot** (or stemplot) is like a sideways column graph but one that lists the actual scores on the horizontal columns. An **ordered** stem-and-leaf plot shows:

- all the scores, listed from smallest to largest
- any modes (the most common scores)
- any clusters or outliers
- how the scores are spread out.

Example 11

The heights (in centimetres) of a sample group of Year 8 boys and girls were measured.

Girls: 148 155 153 137 155 137 146 165 157 168
Boys: 140 152 143 139 166 144 136 151 130 138

- Show this data on a back-to-back stem-and-leaf plot.
- Find the median for each set of data.
- Find the range for each set of data.
- Comment on the differences between the data for girls and boys.

Solution

- Heights of Year 8 students in centimetres

Girls				Boys		
7	7	13		0	6	8 9
8	6	14		0	3	4
7	5	15		1	2	
8	5	16		6		

- Median: There are 10 scores in each set.
The median is the average of the 5th and 6th scores (circled in red on the plot).

$$\text{Median:} \quad \text{Girls} = \frac{153 + 155}{2} = 154 \quad \text{Boys} = \frac{140 + 143}{2} = 141.5$$

- Range: Girls = $168 - 137 = 31$ Boys = $166 - 130 = 36$
- The girls in the sample are generally taller (higher median) but the boys' heights are more spread out (higher range).

Video tutorial

Back-to-back
stem-and-leaf plots

MAT08SPVT10014

Exercise 7-06 Dot plots and stem-and-leaf plots

In this exercise, round all mean answers to one decimal place where necessary.

See Example 10

1 For each dot plot, find:

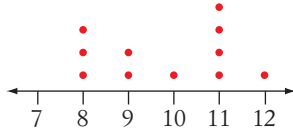
i the range

ii the mode

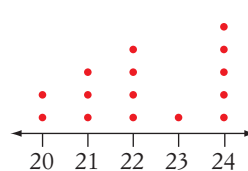
iii the median

iv the mean.

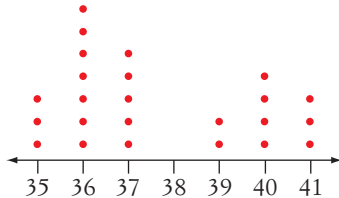
a



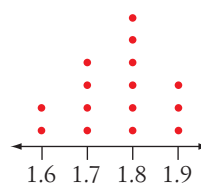
b



c



d



2 The number of goals a football player kicked for each match last season were:

2	7	3	2	4	0	3	2	4	3	5
3	1	1	2	4	2	5	3	0	1	0

a Draw a dot plot to represent this data.

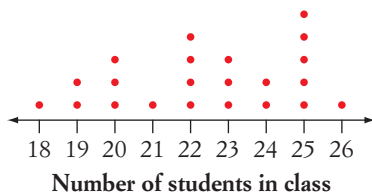
b How many matches were there last season?

c Which score is an outlier?

d Calculate the median.

e Calculate the mean.

3 This dot plot shows the number of students in each class at Northvale Primary School.



a How many students are in the smallest class?

b Find the range.

c Find the mode.

d Calculate the mean.

e What percentage (to the nearest whole number) of classes have over 20 students?

f Which of the following values is the median? Select the correct answer **A**, **B**, **C** or **D**.

A 11

B 22

C 22.5

D 23

- 4 The number of songs Pamela downloaded each day was:

1, 2, 0, 2, 3, 3, 3, 1, 1, 1, 6, 2, 3, 0

- Draw a dot plot for this data.
- How many days are represented by this dot plot?
- What is the outlier?
- Find the median.
- Calculate the mean.
- If this pattern continues in the following weeks, how many songs would she expect to download on average in a week?

- 5 The quiz marks out of 10 for two Year 8 classes are shown below.

8 Hamper:	3	2	0	1	5	8	6	7	6	3	
	5	4	5	6	7	9	2	5	7		
8 Yen:	7	6	3	7	8	1	9	4	6	7	2
	7	2	8	10	9	9	5	7	8	9	10

- Draw a dot plot of the marks for each class.
- What is the mode of the marks for class 8 Yen?
- What is the median of the marks for class 8 Hamper?
- What is the range of the marks for class 8 Yen?
- Draw a frequency table of the data for class 8 Hamper and use it to calculate the mean.
- Which do you think is the 'better' class? Give a reason for your answer.

- 6 For each stem-and-leaf plot below, find:

- the range
- the mode
- the median
- the mean.

a

Stem	Leaf
1	0 2 3
2	1 4 4 5 6
3	3 3 3 7
4	1 2 3 5 9

b

Stem	Leaf
7	3 4 5 7 7
8	1 2 2 3 8 9
9	0 4 4 4 4 6 7 9

c

Stem	Leaf
10	0 1 1 2
11	7 8 8 8 9
12	3 6 6 7
13	1 1 1 1 1 8
14	0

d

Stem	Leaf
0	5 5 6
1	4 4 7 7 7 7 7
2	0 3 8 8
3	
4	9 9 9

- 7 A survey shows that the number of students served at the school canteen daily over a three-week period is:

105	76	97	108	114	106	124	101
112	98	95	105	117	101	92	

- Show this information on a stem-and-leaf plot.
- What is the outlier?
- Find the mode.
- What was the highest score?
- On what stems were the scores clustered?
- On what percentage (to one decimal place) of days were more than 100 students served?

See Example 11

- 8 The heights in centimetres of the students in a Science class are:

156	154	158	167	164	163
155	166	159	158	137	163
171	160	164	169	157	161
160	165	145	175	167	167

- Make a stem-and-leaf plot for this data.
 - On what stem are the data clustered?
 - How many students were measured?
 - How many students were shorter than 150 cm?
 - Calculate the range.
 - Find the median.
- 9 The times taken, in minutes, to drive between two towns along two different roads are shown on the back-to-back stem-and-leaf plot below.

Fuller's Ridge Rd							Great Southern Drive					
						2	6	8				
9	8	8	5	1	1	3	0	2	4	7	8	
	3	3	2	0	0	4	0	1	2	6	9	9
						5	0	1	3	7		
					2	6	2					
				1	0	7	1					

- How many trips were made along the Great Southern Drive?
- How many trips along Fuller's Ridge Road took over an hour?
- Which road is the faster one to take to drive between the two towns? Explain your answer.
- Which statement is true for the data sets above? Select the correct answer **A**, **B**, **C** or **D**.



- The range is the same for both data sets
- There is a difference of 5 minutes in the median travel times
- The mean travel time for Fuller's Ridge Road is 36 minutes
- A travel time of 71 minutes along the Great Southern Drive is an outlier

- 10 This back-to-back stem-and-leaf plot shows the number of penalties that two rugby teams received for each match last season.

Wee Waa Reds								Narrabri Lions			
		9	8	4	4	2	0	7	9	9	
8	6	5	5	2	2	2	1	2	3	3	6 8
				3	1	1	2	5	6	8	9
						0	3	0	0	1	3

- How many games did these teams play in one season?
- How many times did the Narrabri Lions receive fewer than 20 penalties in a match?
- Find the highest number of penalties received by the Wee Waa Reds in a game.
- Find the mode for the Narrabri Lions.
- Which value was an outlier for the Wee Waa Reds?
- Which team generally had a higher number of penalties?
- Which team had a greater range in their number of penalties? Give a reason for your answer.

Investigation: Measures of location

- Find six scores where the mean equals the median.
- Find five scores where the median equals the mode.
- Find four scores where the mean, the median and mode all differ.
- Find five scores where the mean, median and mode are the same.
- Seven judges score a gymnast out of a possible 10. Their scores are:

6 7 8 6 8 9 6

The scoring system requires that the lowest and highest scores are rejected and that the mean of the remaining five gives the final score. What is the final score?

- In four games Duong scores 23, 11, 17 and 26 points. What score must Duong achieve in his next game so that he can average 20 on the five games? (Show the working or reasoning that you use.)
- Members of a group of 11 film students rate a movie out of a possible 10. Their ratings are:

7 9 9 4 6 4 8 9 7 8 5

The group cannot agree on a group rating for the movie. In one sentence, comment on each of these suggestions.

- Kim thinks that the rating should be 9 (the mode) because that was the rating given the most often.
- Vlado thinks that the rating should be 7 (the median) because it is not swayed by extremely high or extremely low ratings.
- Thanh thinks that the rating should be 6.9 (the mean) because it considers the ratings of all the students.

What overall rating would you give the film based on these opinions?

- Find three numbers for which the median is 15, the mean is 13 and the range is 12.
- Find five numbers for which the median is 3, the mean is 4, and the mode is 7.

Worksheet

Statistical match-up

MAT08SPWK00004

Technology worksheet

Frequency histograms

MAT08SPCT10007

Worksheet

Analysing data

MAT08SPWK00006

7-07 Frequency histograms and polygons

Numerical data can be graphed on a **frequency histogram**, a special type of column graph, or a **frequency polygon**, a special type of line graph.

Example 12

Twenty households were surveyed about how many phones, including mobile phones, they owned.

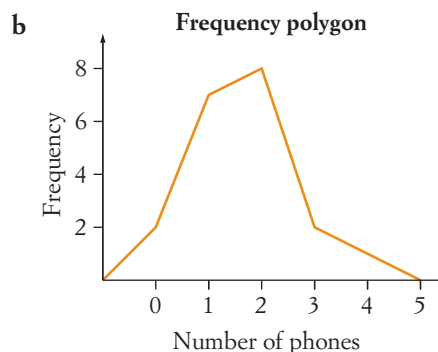
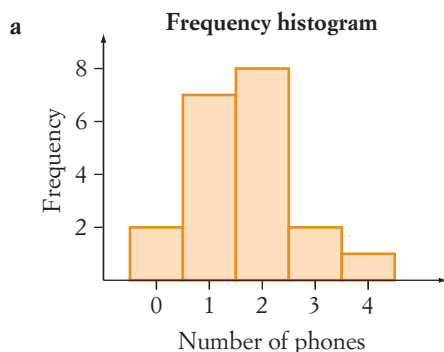
The results are shown in the frequency table on the right.

Graph this data as:

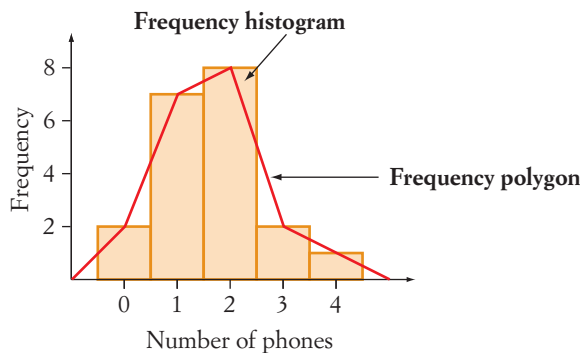
- a a frequency histogram
- b a frequency polygon.

Number of phones	Frequency
0	2
1	7
2	8
3	2
4	1
Total	20

Solution



Note: A frequency polygon can be constructed from a frequency histogram by joining the midpoints of the tops of the columns.



Summary

Frequency histogram

- column graph that looks like a row of office buildings
- frequency is shown on the vertical axis
- the scores are shown on the horizontal axis
- the columns stand together without gaps between them
- the columns are centred on the scores
- there is a gap of half a column width on the left (between the frequency axis and the first column)

Frequency polygon

- line graph that looks like a mountain
- frequency is shown on the vertical axis
- the scores are shown on the horizontal axis
- begins and ends on the horizontal axis
- called a 'polygon' because the graph and the horizontal axis together make a shape with straight sides

Exercise 7-07 Frequency histograms and polygons

- 1 The students in a class were surveyed about how many pets they owned, and the results are shown in this frequency table.

See Example 12



- Graph this data as a frequency polygon.
- How many students were surveyed?
- How many students had no pets?
- What percentage of students had 2 pets?
- Find the median.
- Calculate the mean correct to one decimal place.

Number of pets	Frequency
0	7
1	11
2	9
3	3

- 2 Hadieya counted the number of phone calls she made each day for 30 days. The results are as shown.

5	3	6	2	4	3	3	5	4	7
5	1	6	3	2	3	5	6	1	6
2	4	3	1	5	4	2	7	3	2

- Complete a frequency table for this data.
- Draw a combined frequency histogram and polygon of the data.
- On how many days did Hadieya make three calls?
- How many phone calls were made over the 30 days?
- On how many days did Hadieya make fewer than three calls?
- What was the mode?
- What was the mean (correct to one decimal place)?

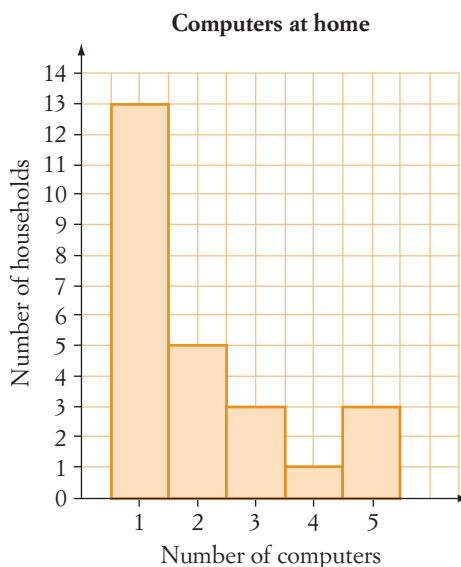
Worked solutions

Exercise 7-07

MAT08SPWS10062

- 3 This histogram shows the number of computers found in a sample of households.

- How many households were surveyed?
- Where are most of the scores clustered?
- Find the mode.
- Find the range.
- Complete a frequency table, including an fx column, for the data and use it to calculate the mean.
- Find the median.

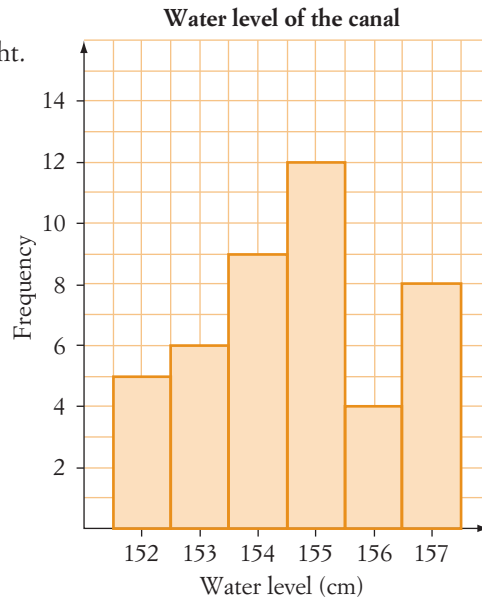


- 4 The numbers of goals scored by a soccer team in each match last season were:

2	0	1	2	3	4	3	1	2	0
2	1	4	3	2	5	1	2	0	3

- Complete a frequency table for this data.
- Draw a frequency histogram and frequency polygon of the data on the same axes.
- How many matches were played by the team?
- What was the highest number of goals scored?
- What was the mode?
- What was the mean?
- In how many matches were more than two goals scored?
- In what percentage of matches were no goals scored?

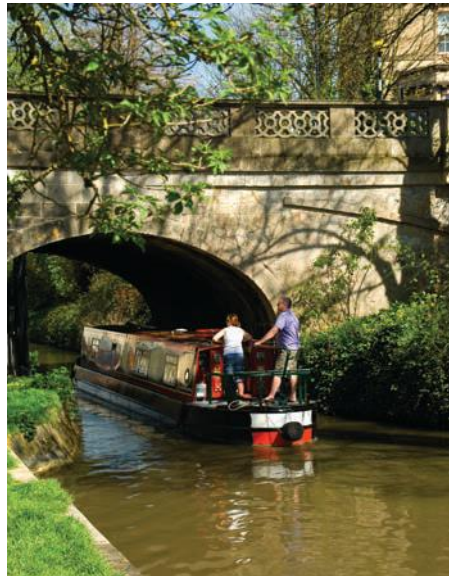
- 5 The water level of a canal was recorded over 44 days and graphed on the histogram on the right.



- a Use the histogram to copy and complete the frequency table below.

Water level (cm)	Frequency
152	
153	
154	
155	
156	
157	
Total	

- b On how many days was the water level 156 cm high?
- c What is the range?
- d What is the mode?
- e What is the mean, correct to two decimal places?
- f On how many days was the water level lower than 154 cm?
- g What is the median?
- h There needs to be at least 155 cm of water in the canal for a boat to travel on it safely. On how many days would the boat be able to use the canal?



7-08 Sampling

Sample vs census

In statistics, the **population** means all of the items being studied, whether they be people, cars, businesses or houses. Statistical data is collected in a variety of ways, such as surveys, Internet polls, interviews and questionnaires. To collect information, statisticians usually survey a large representative group called a **sample**. For example, to investigate opinions about school uniform, we can survey a selection of students, teachers and parents. The bigger the sample, the more representative and accurate it will be.

To collect information about a whole population, **all** people or items must be surveyed. This is called taking a **census**. For example, to plan the locations of childcare centres, the government may need to count the number of babies in different areas.

Sample	Census
Surveys a selected representative group of people or items.	Surveys all people or items (in the population).
Only gives approximate information about the population. Helps us make predictions about the population.	Gives exact information about the population.
Simple and inexpensive to survey.	Complex and expensive to survey.
Can be done quickly.	Takes a lot of time to collect and process the information.

In Australia, a national census is conducted every 5 years to count the Australian population and obtain important information about every person. It takes place in a year ending in 1 or 6, such as 2016 and 2021.

Example 13

Should a sample or a census be used to investigate each of the following?

- The most popular car colour.
- Whether energy-saving light globes really use less electricity.
- The number of retired people living on the Gold Coast.
- People's views on which nation will win the rugby World Cup.

Solution

- Sample, because it is easier and faster than asking all car-owners in Australia.
- Sample, because it is easier and faster than testing every light globe.
- Census, because the exact number is required.
- Sample, because it is very difficult to ask all people

Random sampling and bias

Remember that in probability, a **random experiment** is a situation where every possible outcome has the same chance of occurring. When taking a sample, it is important that each person or item in the population has an equal chance of being chosen to be surveyed. This is called **random sampling**. To find a sample of students to survey about school uniform, we could place students' names in a box, mix them up, and then choose 100 names without looking. Alternatively, we could use a computer or calculator to generate random numbers to select students.

A sample that is not random is called a **biased sample**, and is not truly representative of the population. For example, if you survey the first 100 students to arrive at school today, or the first 100 students on the school roll, then not all students in the school have an equal chance of being selected. Your sample would not represent students who arrive at school later or whose names begin with letters at the end of the alphabet.

Example 14

A random sample of students is to be selected for their views on a new school uniform. Will each of the following sampling methods provide a sample that is representative or biased?

If biased, explain why.

- a Randomly selecting one student from each class.
- b Selecting twenty students from your mathematics class.
- c Selecting the students from the school's netball teams.
- d Randomly selecting students from the school roll by rolling a die and counting down the roll.

Solution

- a Representative. Every student in the school has an equal chance.
- b Biased, because your mathematics class does not represent the other Year groups (or even the other Year 8 classes) at your school.
- c Biased, because the netball teams do not represent the whole school and will have mainly female students.
- d Representative. Every student in the school has an equal chance.

Exercise 7-08 Sampling

- 1 Is a sample or a census more appropriate for each situation?
 - a Testing coffee for taste.
 - b Finding the number of migrants from Japan.
 - c Obtaining general views on road safety.
 - d Finding out people's opinions on whether Australia should become a republic.
 - e Testing a flu vaccine for effectiveness.
 - f Counting the number of Indonesian-speaking children in Australian schools.
 - g Finding the population of Gundagai.
 - h Finding the number of people in Australia over 40, to plan for health care.

See Example 13

- 2 For each situation, state whether a sample or a census would be more appropriate and how the data could be collected.
 - a A car dealer wants to find the level of customer satisfaction with his business.
 - b To find the most popular TV program on Friday nights.
 - c To find the busiest airport in Australia.
 - d A principal wishes to find the least liked subject in the school.
 - e The number of people who support whaling.
 - f The average height of students in your school.
 - g The amount of recyclable garbage each household produces in a week.
 - h The number of times the students in your class visit a restaurant in a month.

See Example 14

- 3 For each survey, describe the total population, give a suitable way of choosing a representative sample, write any problems that may be encountered and explain how bias could be minimised.
 - a The attitudes of people to smoking in public.
 - b Whether girls are better than boys at mathematics.
 - c The choice of music at a school disco.
 - d Which party people will vote for at the next state election.
- 4 In each survey, state whether the sample is random. If not, explain why you think it is biased.
 - a To find participants for a survey of students' study habits, a teacher wrote a message in the daily notices asking for volunteers.
 - b To ensure that items being made in a factory were of good quality, two items coming off the assembly line were tested at a randomly selected time each hour.
 - c In a poll to find voters' preferences in the next election, every 100th person listed in the telephone book was called. If there was no answer, the next person listed in the telephone book was called, and so on.
 - d In a survey about school uniform, the school council posted letters to all parents of students at the school asking for their opinion on the issue.
 - e In a survey about the best song of the decade, listeners of a rock radio station were asked to ring and nominate a song.
- 5 The student council wants to survey students on their attitude to mobile phone usage at school. Fifteen students from each Year group were selected to take part in the survey.
 - a Do you believe the results of this survey would be reliable? Explain your answer.
 - b What factors should have been considered in selecting the sample?
 - c How would you have selected the students to be surveyed?

Technology Random numbers

Random numbers can be used to select people or items for a sample. A calculator or spreadsheet can be set up to generate a random number from 1 to 100, for example, and that number can be matched to the name of a person or item from a list. To generate such a number on:

- a **scientific calculator**, type $\text{RAN\#} \times 100 + 1 =$ and press $=$ repeatedly for more random numbers
 - a **graphics calculator**, enter $\text{int}(\text{ran\#} \times 100 + 1)$ from the probability functions and press EXE , then press EXE repeatedly for more random numbers
 - a **spreadsheet**, type $=\text{INT}(\text{RAND}() * 100 + 1)$ into a cell and press the **ENTER** key, then the **F9** key repeatedly for more random numbers.
- 1 Use your calculator or spreadsheet to generate 12 random numbers from 1 to 100.
 - 2 How many students are there in your class?
 - 3 Use your calculator or spreadsheet to generate 12 random numbers from 1 to the number of students in your class.
 - 4 Use a class roll to number the students in your class from 1 onwards. Suppose you need to choose a random sample of 9 students from your class for a survey on homework. Use your calculator or spreadsheet to generate 9 random numbers to select the 9 students for your survey.
 - 5 Choose four students from each Year 8 maths class to answer some questions. Devise several different methods to ensure that your selection is random. Prepare a short class talk about your methods.

Investigation: Statistics in the media

We live in a world of 24-hour news: from TV, newspapers, the Internet, Twitter, blogs, etc. To detect bias in news, we need to consider where the news item comes from and what types of samples the statistics are based on. Is the information supplied by a reporter, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, large sample, unrepresentative sample, or a phone or Internet poll of volunteers? Each may have a particular bias that may influence how the story is reported.

Find examples of news items or surveys reported in a newspaper, magazine or the Internet and investigate the following questions.

- a Where did it come from?
- b Who wrote the story?
- c Does it show any bias?
- d Can it be supported by other news providers?
- e What type of sampling was used? Was it representative?
- f How many people were questioned?
- g When was the survey conducted?

Just for the record

The Australian Census

The national Australian Census is conducted every 5 years, on the second Tuesday in August. The Census in 2011 marked 100 years of Australian censuses, the first being held in April 1911. Many changes had occurred over 100 years:

1911 Census	2011 Census
7300 Census collectors	40 000 Census collectors
Collectors mainly walked, but some travelled by horseback or bicycle.	Collectors walked, drove and some even used helicopters to reach remote areas.
Collectors covered the costs of feeding their horse.	Collectors receive a fuel allowance.
The Census form asked whether you were a subject of the British Empire.	The Census form asked for your cultural heritage, language spoken at home, country of birth, ancestry and Aboriginal or Torres Strait Islander status.

Since 2006, Australians have been able to complete the Census form online and send it electronically.

Source: Australian Bureau of Statistics

Investigate the significant changes that have been made to the Australian Census in the last ten years.



7-09 Designing survey questions

Worksheet

Canteen survey

MAT08SPWK10070

Worksheet

Student survey form

MAT08SPWK10071

When designing a survey, it is important that the questions asked are clear, fair and not biased. A good survey question:

- uses plain English
- is not vague and does not have a 'double meaning'
- is not too open-ended, and does not allow too many different answers
- asks for only one piece of information
- does not try to influence a particular answer unfairly (this is also called **bias**)
- protects a person's privacy and does not ask for personal information, such as phone numbers
- provides choices, such as check boxes or a scale of 1 to 5 to select from.

Example 15

Explain what is wrong with each survey question and suggest an alternative.

- 'How many phones are there in your home?'
- 'What is your e-mail address?'
- 'How long did you have to wait before someone answered your call?'

Solution

- This question is not clear. Does it include mobile phones? Do two or more phones using the same line count as one phone? A better question would be: 'How many fixed-line phone numbers does your home have?'
- This question is too personal. This should be an optional question, perhaps adding 'May we contact you for further information if required?'
- This question has too many possible answers. It would be better to provide options:
☐ less than 3 minutes ☐ 3 to 5 minutes ☐ over 5 minutes

Exercise 7-09 Designing survey questions

- Explain what is wrong with each survey question and suggest an alternative.
 - 'Are you happy?'
 - 'How did you enjoy your meal at our restaurant?'
 - 'What is your religion?'
 - 'Soldiers have died for our Australian flag, so do you think it should be changed?'
 - 'How much do you earn?'
 - 'How often do you eat takeaway food?'
 - 'What brand of computer do you own?'

A IBM B Hitachi C Macintosh

See Example 15

- 2 Some Year 8 students designed this questionnaire to investigate the breakfast choices of students. They were particularly interested in whether there is a difference between what the boys and girls have for breakfast.

BREAKFAST QUESTIONNAIRE					
Please complete each blank or tick the box or boxes that apply to you.					
Age : _____		Male ☐		Female ☐	
1 What do you eat for breakfast?					
Porridge	☐	Rice bubbles	☐	Eggs	☐
Toast	☐	Corn flakes	☐	Muesli	☐
2 What do you drink?					
Orange juice	☐	Milk	☐	Tea	☐
Coffee	☐	Apple juice	☐		
3 Do you think you have a healthy breakfast?			Yes ☐	No ☐	
Give reasons for your answer:					
.....					
.....					

In a small group, discuss the following.

- a Do you think there are enough questions on the questionnaire? If you think more questions need to be added, write them down.
 - b Do you think the wording of the questions can be improved? If yes, replace the questions.
 - c Do the questions take into account cultural differences? Is it possible to improve the questions to minimise this problem?
- 3 As a group activity, design a survey about:
- the types of cars owned by the families of students in the class, or
 - the most popular types of food sold in the school canteen.

Determine the suitable wording of each question, and provide check box choices where appropriate.

7-10 Comparing samples and populations

Statisticians do not study just one sample. They generally take a number of samples from a population and calculate the statistics of each sample. The results of each sample are meant to estimate the statistics of the whole population. Also, the larger the sample sizes, the better the estimates of the population.



Example 16

The ages of 100 people working at a theme park are:

18	19	28	27	20	20	24	40	24	19
30	35	26	24	22	19	27	27	23	29
28	40	21	17	20	22	23	21	24	23
22	26	36	25	16	24	36	25	19	25
34	39	45	20	35	21	33	27	19	33
18	27	34	30	25	34	37	29	25	27
24	24	16	18	25	21	26	31	26	25
18	19	26	22	25	22	45	38	34	43
57	24	48	18	20	30	28	22	26	35
29	21	21	28	16	22	35	24	18	23

- Randomly select five samples of 12 ages from this population of employees and for each sample, calculate the mean to one decimal place, the median, the number of employees aged under 20 and the number aged between 30–39.
- From your sample statistics, estimate the population mean, median, the total number of employees aged under 20 and the total number aged between 30–39.

Solution

- a Sample 1: 40, 19, 17, 26, 25, 27, 34, 18, 19, 43, 22, 23
 Sample 2: 19, 19, 25, 36, 23, 24, 16, 18, 17, 21, 24, 18
 Sample 3: 24, 22, 43, 25, 16, 34, 35, 25, 24, 22, 20, 40
 Sample 4: 35, 24, 20, 35, 48, 26, 28, 18, 17, 25, 22, 22
 Sample 5: 27, 22, 19, 25, 25, 25, 35, 34, 35, 39, 34, 24

Each sample of 12 ages is selected randomly from the above population

	Mean	Median	No. aged under 20	No. aged 30–39
Sample 1	26.1	24	4	1
Sample 2	21.7	20	6	1
Sample 3	27.5	24.5	1	2
Sample 4	26.7	24.5	2	3
Sample 5	28.7	26	1	5

The values are similar for different samples but not close, because each sample is small and does not represent the whole population accurately.

- b We can estimate the population statistics by taking averages of the samples.

$$\text{Estimate of mean} = \frac{26.1 + 21.7 + 27.5 + 26.7 + 28.7}{5} = 26.14$$

$$\text{Estimate of median} = \frac{24 + 20 + 24.5 + 24.5 + 26}{5} = 23.8$$

$$\text{Average number of employees aged under 20 per sample} = \frac{4 + 6 + 1 + 2 + 1}{5} = 2.8$$

$$\begin{aligned} \therefore \text{Estimated number of employees aged under 20 in population} &= \frac{2.8}{12} \times 100 \\ &= 23.333 \dots \\ &\approx 23 \end{aligned}$$

$$\begin{aligned} \text{Average number of employees aged between 30–39 per sample} &= \frac{1 + 1 + 2 + 3 + 5}{5} \\ &= 2.4 \end{aligned}$$

$$\begin{aligned} \therefore \text{Estimated number of employees aged between 30–39 in population} &= \frac{2.4}{12} \times 100 \\ &= 20 \end{aligned}$$

Exercise 7-10 Comparing samples and populations

See Example 16

- 1 a Copy this table.

	Mean	Median	No. aged under 20	No. aged 30–39
Sample 1				
Sample 2				
Sample 3				
Sample 4				
Sample 5				

- b As in Example 16, randomly select five samples of 12 ages from the population of 100 theme park employees and for each sample complete the table above by calculating the mean to one decimal place, the median, the number of employees aged under 20 and the number aged between 30–39.
- c Are the values similar or different across the samples? Why or why not?
- d From your sample statistics, estimate the population mean, median, the total number of employees aged under 20 and the total number aged between 30–39.
- e Add a row of the table for 'Population' and calculate the population mean, median, the total number of employees aged under 20 and the total number aged between 30–39.
- f How do the statistics of each sample compare to the population statistics?
- g How do your estimated statistics from part d compare to the population statistics?

- 2 a Copy this table.

Sample size	Mean	Median	% aged under 20	% aged 30–39
12				
24				
36				
48				
60				
Population				

- b Select another random sample of 12 ages from the population given in Example 16 and calculate and record in the table correct to one decimal place (where appropriate) the mean, the median, the *percentage* of employees aged under 20 and the *percentage* aged between 30–39.
- c Repeat for random samples of 24 ages, 36 ages, 48 ages and 60 ages, recording your results in the table.
- d Are the values similar or different across the samples? Why?
- e Copy your results from Question 1e for the population of 100 ages into the last row of the table.
- f How do your sample statistics compare to the population statistics?
- g Do larger samples represent the population better? Are their statistics closer to the population statistics?

Worksheet

Analysing data

MAT08SPWK00006

Homework sheet

Statistics 3

MAT08SPHS10040

Technology worksheet

Population polygons

MAT08SPCT10008

7-11 Analysing data

The table below describes the four statistical measures and shows when it is more appropriate to use each one.

Statistical measure	Features	When is it appropriate to use?
Mean $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$ For a frequency table: $\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f}$	- For numerical data - Depends on all the scores in the data set - Affected by outliers - Can be a decimal that is not one of the actual scores	When the data set does not have many outliers.
Mode Most common score(s)	- For numerical and categorical data - There may be more than one mode, or no mode at all - Not affected by outliers	When the most common score or category is needed.
Median Middle score, or average of the two middle scores, when scores are arranged in order	- For numerical data. - Can be one of the scores - Not affected by outliers	When the data set has many outliers.
Range Highest score – lowest score	- For numerical data - Depends on the highest and lowest scores only	When a measure of spread is needed.

Example 17

A group of 8 children was surveyed about the amount of pocket money (in dollars) they received each week. The results were:

20 32 32 40 18 32 18 48

a Find:

i the mean ii the mode iii the median iv the range

b If the last student's pocket money increased from \$48 to \$75, describe how this would affect:

i the mean ii the mode iii the median iv the range

c Which measure of location best represents the new set of data?

Solution

a i Mean:

$$\begin{aligned}\bar{x} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{20 + 32 + 32 + 40 + 18 + 32 + 18 + 48}{8} \\ &= \frac{240}{8} \\ &= \$30\end{aligned}$$

ii Mode = \$32

iii Rewriting the eight scores in order:

18 18 20 **32** **32** 32 40 48

There are two middle scores, 32 and 32 (circled above).

$$\begin{aligned}\text{Median} &= \frac{32 + 32}{2} \\ &= \$32\end{aligned}$$

iv Range = $48 - 18$
= \$30

b If the eighth student's pocket money increased to \$75, this score would be a high outlier compared to the others.

i The mean should increase.

$$\begin{aligned}\text{Mean: } \bar{x} &= \frac{267}{8} \\ &= \$33.375\end{aligned}$$

ii The mode will not change because \$32 is still the most common score.

iii The median will not change because the order of the scores has not been affected.

18 18 20 **32** **32** 32 40 75

The two middle scores are still 32 and 32 (circled above).

iv The range will increase because the highest score is now even higher.

$$\begin{aligned}\text{Range} &= 75 - 18 \\ &= \$57\end{aligned}$$

c The median is the best measure of location because it is not affected by the outlier of \$75.

Note:

- The mode (\$32) is a good measure but it only represents three of the scores.
- The mean (\$33.375) is least appropriate because it has been affected by the outlier and is now higher than most of the scores.

Exercise 7-11 Analysing data

In this exercise, round all mean answers to one decimal place where necessary.

See Example 17

- 1** The distances (in kilometres) that a salesperson travelled each day for 10 days were:

49 77 63 56 49 40 56 40 74 49

a Find:

- i** the range **ii** the mode **iii** the median **iv** the mean.

b Which measure of location best represents the data?

- 2** The ages of the mathematics teachers at Westvale Christian College are:

49 32 37 32 25 41 39

a Find:

- i** the range **ii** the mode **iii** the median **iv** the mean

b The 39-year-old teacher was replaced by a 22-year-old teacher. Without recalculating, describe how this will affect:

- i** the range **ii** the mode **iii** the median **iv** the mean

- 3** In a swimming competition, each swimmer was awarded one of the grades A, B, C, D or E. One squad obtained the results below.

Grade	A	B	C	D	E
Number of swimmers	4	9	8	6	3

a How many swimmers were in the squad?

b What is the mode?

c Why isn't it possible to find the mean or median of these grades?

- 4** The annual incomes ($\times \$1000$) of 12 people are:

84 57 121 75 81 134 66 484 210 93 125 90

a Find (in dollars):

- i** the range **ii** the median **iii** the mean

b Why isn't the mean or mode an appropriate measure of location for the incomes?

c Remove the outlier from the set of data and recalculate the mean and median. Are they closer together now?

Worked solutions

Exercise 7-11

MAT08SPWS10063

- 5** The heights (in centimetres) of a team of basketball players are:

204 196 181 185 190 198 193

a Find:

- i** the mean **ii** the median **iii** the mode **iv** the range

b The 190 cm player is replaced by a 181 cm player. Without recalculating, describe how this will affect:

- i** the mean **ii** the median **iii** the mode **iv** the range

-

- 9 Two judges rate eight contestants in a talent quest.

Dani	8	5	6	8	5	9	2	6
Kyle	7	8	5	4	9	5	5	6

- Draw a dot plot for each judge's marks.
- Calculate the range and median for each judge.
- Describe differences in the way that each judge allocates marks.

Power plus

- A small class obtained these results in an exam:
66 68 74 76 82 79
 - Find the mean of these marks.
 - The teacher realised there was an error in the marking and added three more marks to each score. Find the mean of the new scores.
 - What effect did the extra three marks have on the mean?
 - What effect does adding or subtracting the same number for all of the data have on the mean?
- Margaret sat five exams. Her average mark was 74%. What mark should Margaret obtain in the sixth exam if she wishes her average mark for the six exams to be 77%?
- If the mean of a set of scores is 20, must 20 be one of the scores? Give an example to illustrate your answer.
- Five items of data were collected, but the figures were lost. The mean of the data was known to be 8 and the median was 9.
 - What was the total of the data?
 - What could the data have been?
 - If you are now told that the range is 7, what could the set of data have been?
- Four sisters work at the same bank. One earns \$500 per week, another earns \$800 per week. The mean weekly wage of the four sisters is \$2000. Is this possible? How?

Language of maths

analyse	discrete	measure of location	population
bias	dot plot	measure of spread	random
categorical	frequency histogram	median	range
census	frequency polygon	mode	sample
continuous	frequency table	numerical	stem-and-leaf plot
cluster	mean	outlier	survey

Puzzle sheet

Data crossword

MAT08SPPS10019

- 1 Name the **measures of location**.
- 2 What word means how often a score is listed in a set of data?
- 3 What type of data is described by words rather than numbers?
- 4 Is a **frequency polygon** a column graph or a line graph?
- 5 What is a survey of the whole population called?
- 6 What word describes a sample that misrepresents the population or a survey question that tries to lead you to answer in a particular way?

Topic overview

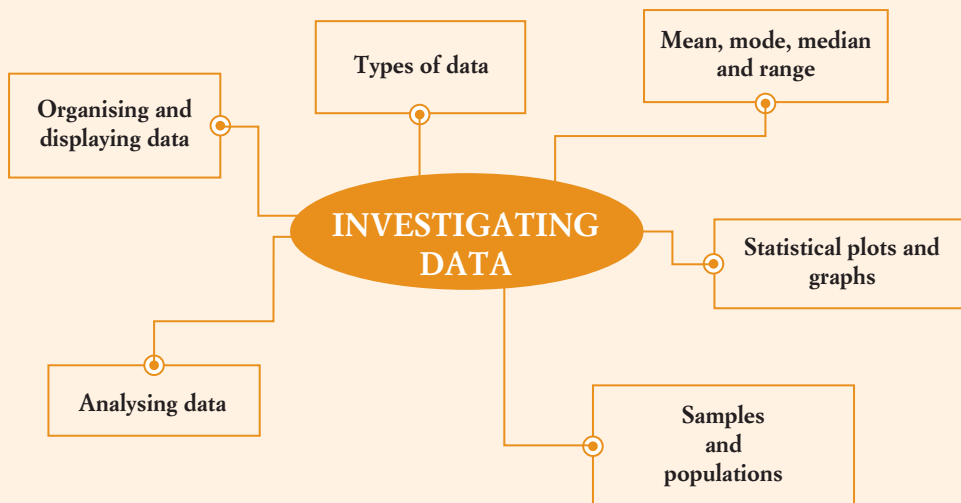
- What did you find easy in this chapter? What don't you understand? Talk to your teacher.
- Give some examples of situations where statistics and measures of location are used. Explain, where possible, which measure of location is being used.

Worksheet

Mind map:
Investigating data

MAT08SPWK10072

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 7 revision

In this exercise, round all mean values to two decimal places where necessary.

See Exercise 7-01

- 1 Twenty people were asked how many computers they owned in their households. The results were as follows:

3 1 2 3 4 3 2 1 4 3
3 2 4 6 3 5 4 2 3 4

- Complete a frequency table for this data.
- What was the most frequent score?
- How many people owned four computers?
- What percentage of people owned one computer?
- Draw a sector graph of this data.

See Exercise 7-02

- 2 Classify each type of data below as categorical (C), numerical discrete (ND) or numerical continuous (NC).

- life of a battery
- type of milk sold each day
- number of rooms in homes
- volume of water in pools
- person's gender
- weight of babies

See Exercise 7-03

- 3 For each set of data, find:

- the mean
 - the mode
- a 6 12 11 12 10 6 6 10 6 b 8 4 1 1 4 1 3 6 c 20 21 23 24 25

See Exercise 7-04

- 4 For each set of data in Question 3, find:

- the median
- the range

See Exercise 7-05

- 5 The number of goals scored in each soccer match by the Southwest Strikers is shown in this frequency table.

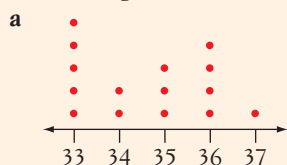
Score, x	Frequency, f	fx
0	3	
1	7	
2	6	
3	4	
4	2	
5	1	
6	1	
Total		

- Copy and complete the frequency table.
- How many matches did the Strikers play?
- Find the range.
- Find the mode.
- Find the median.
- Find the mean (rounded to two decimal places).

See Exercise 7-06

- 6 For each plot below, find:

- the range
- the median
- the mode
- the mean



b

Stem	Leaf
12	3 5 6
13	1 1
14	0 8 8 8 8
15	7 8

7 The daily number of people visiting a house that was for sale was:

See Exercise 7-07

1	3	5	6	5	4	5	4	3	5
3	5	6	6	5	4	3	3	5	4
1	2	2	5	4	3	6	6	2	5

- Arrange this information into a frequency table.
- Draw a frequency histogram and frequency polygon.
- On how many days did the house receive more than 4 visitors?
- Find the mode.
- Find the median.

8 Determine whether a sample or a census would be more appropriate for each survey below.

See Exercise 7-08

- Electing a Year 8 representative for student council.
- Investigating audience reaction to a new film.
- Testing the safety of a new car.
- Finding how many people live in Australia.

9 A school of 1200 students wants to survey a sample of students to determine how often they use mobile phones.

See Exercise 7-08

- State a suitable way of choosing a representative sample of 50 students.
- Write one problem that may be encountered in conducting the survey.
- Explain how bias could be minimised.

10 Write four questions which could be used in the survey described in Question 9.

See Exercise 7-09

11 Explain what is wrong with each survey question and suggest an alternative.

See Exercise 7-09

- 'How big is your house?'
- 'Is it time that Australia grew up and became a republic?'

12 Twenty-one Year 8 students were asked to find out the ages of their mothers. The results were:

See Exercise 7-11

30	35	35	35	36	36	36	36	36	37	37
37	37	38	39	39	41	43	43	45	45	

- Find the median.
- Find the mode.
- Find the mean.
- Which measure of location is most appropriate for representing this data?
- Predict how many 39-year-old mothers there would be in a Year 8 group of 120 students.
- Suppose one of the ages is incorrect: one of the 39s should actually be 40. Without recalculating, describe how this will affect:
 - the mean
 - the median
 - the mode
 - the range

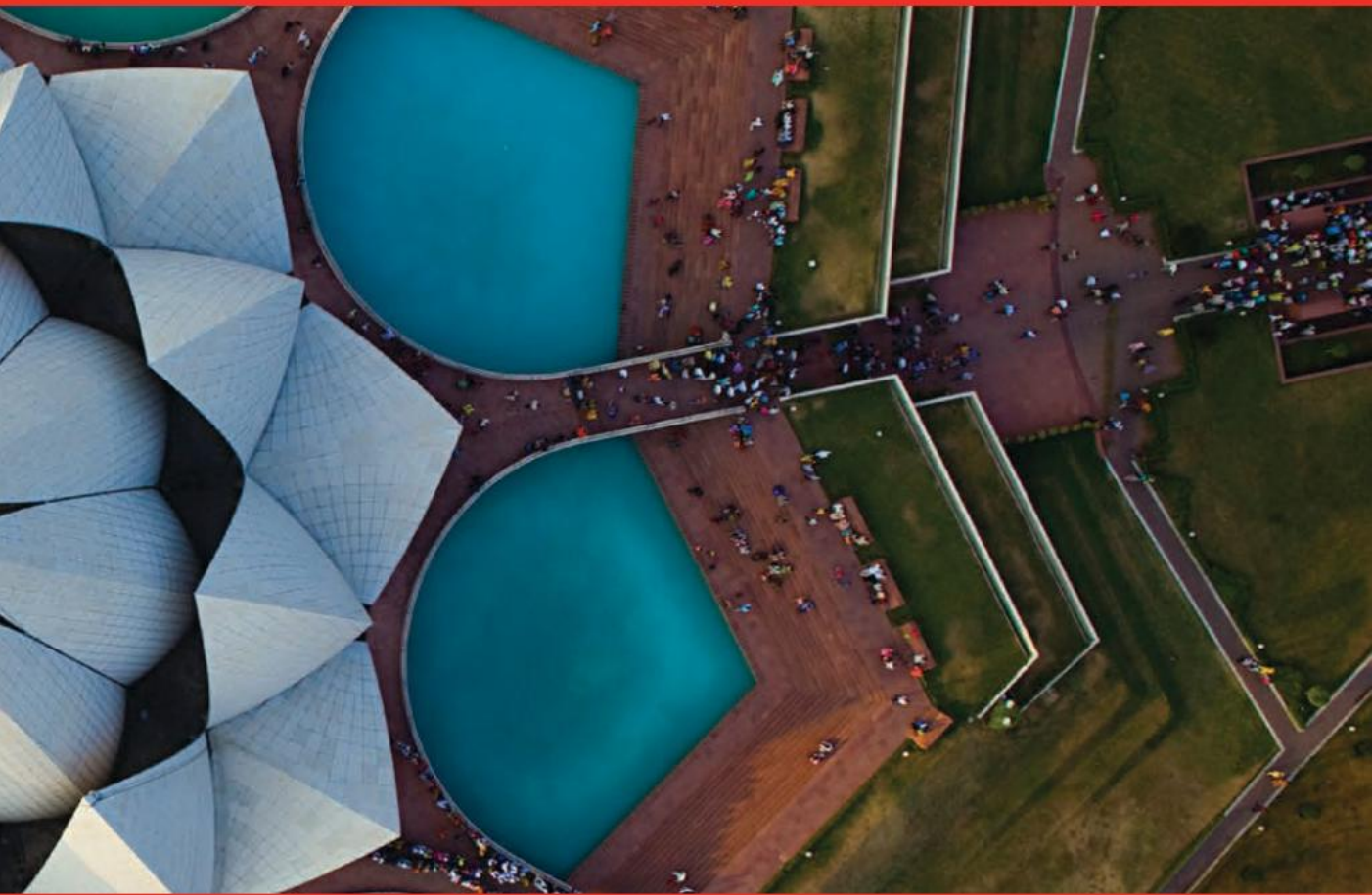


8

Measurement and geometry

Congruent figures

Congruent figures are used in areas such as design, photography, building and art. They are also used to prove geometric properties and to solve practical problems.



Chapter outline

	Proficiency strands				
8-01 Transformations	U	F		R	C
8-02 Congruent figures	U	F		R	C
8-03 Constructing triangles	U	F	PS	R	
8-04 Tests for congruent triangles	U	F	PS	R	C
8-05 Proving properties of quadrilaterals	U	F	PS	R	C
8-06 Extension: Bisecting intervals and angles	U	F		R	
8-07 Constructing parallel and perpendicular lines	U	F			

Wordbank

congruence test One of four tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical; exactly the same (symbol: $\equiv$)

image A transformed shape after it has been translated, reflected or rotated

included angle The angle between two given sides of a shape

RHS The congruence test 'right angle-hypotenuse-side'

SAS The congruence test 'side-angle-side', where the included angle is between the two sides

superimpose To place one figure on top of another congruent figure so that sides and angles match

transformations Translations (slides), reflections (flips) or rotations (spins or turns)

In this chapter you will:

- describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates
- define congruence of plane shapes using transformations
- name the vertices in matching order when using the symbol $\equiv$ in a congruence statement
- construct triangles using the conditions for congruence
- develop the conditions for congruence of triangles (the SSS, SAS, AAS and RHS rules)
- establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning
- use transformations of congruent triangles to verify some of the properties of special quadrilaterals, including properties of the diagonals
- construct parallel and perpendicular lines using their properties, a pair of compasses and a ruler, and dynamic geometry software

SkillCheck

Worksheet

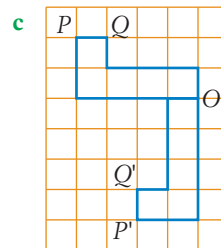
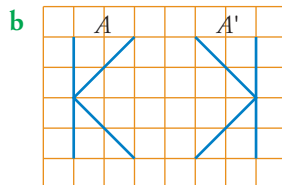
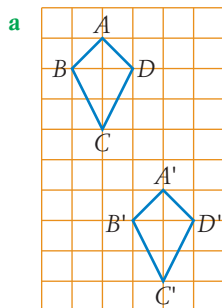
StartUp assignment 8

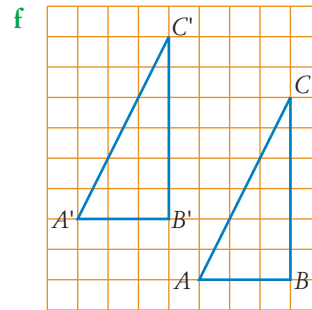
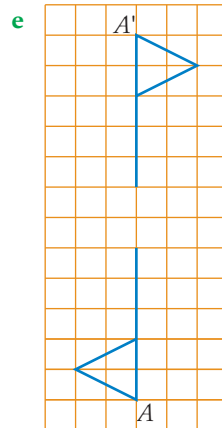
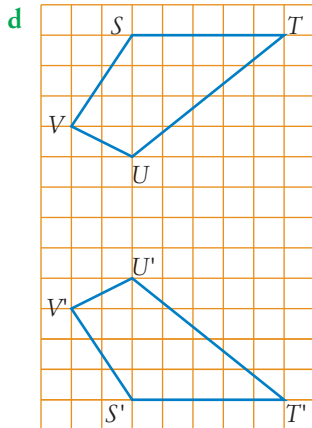
MAT08MGWK10072

- Draw each of the special quadrilaterals below and mark all axes of symmetry.

a rectangle	b square	c parallelogram
d rhombus	e trapezium	f kite
- Name all quadrilaterals that have:

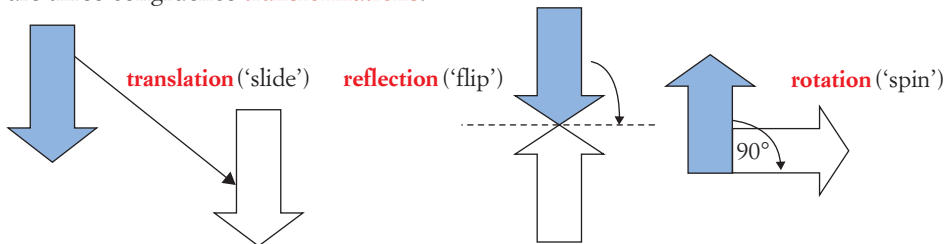
a all angles 90°	b opposite sides parallel
c opposite sides equal	d one pair of parallel sides
e four equal sides	f two pairs of adjacent sides equal
- State the transformation (translation, rotation or reflection) used on each original figure to form the image.





8-01 Transformations

There are three congruence **transformations**.



'Congruence' means identical and 'transformation' means change. Even though a shape has changed position (transformed), it still has the same shape and size (congruence).

- The original shape is called the **original**
- The transformed shape is called the **image**

Composite transformations

Combinations of translations, reflections and rotations can be applied one after the other. These are called **composite transformations**.

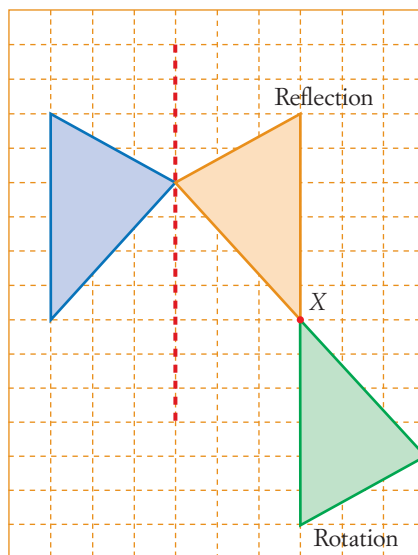
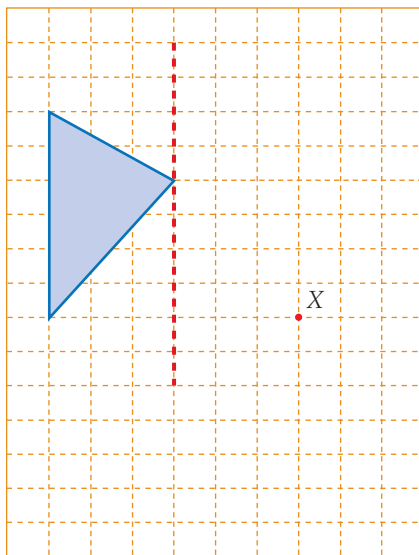
Worksheet

Transformations

MAT08MGWK10073

Example 1

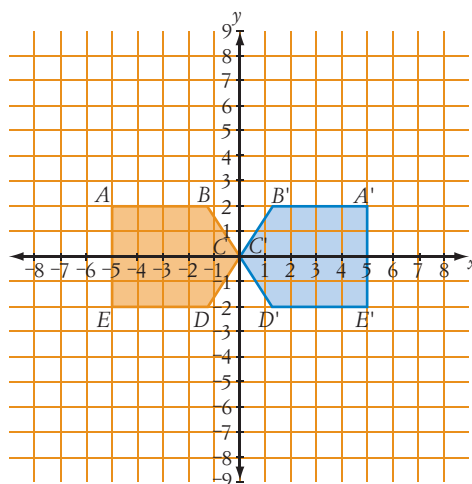
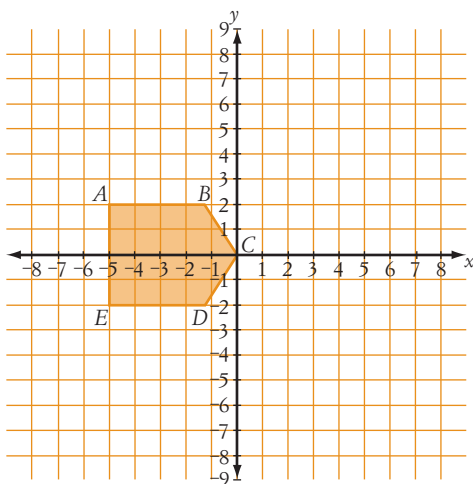
Reflect the triangle below across the dotted line and then rotate the image 180° about X .

Solution**Transformations on a number plane**

When a point or vertex P of an original shape is transformed, the corresponding point or vertex on the image is labelled P' , pronounced 'P-dash' or 'P-prime'.

Example 2

- a Reflect the pentagon $ABCDE$ across the y -axis to create the image $A'B'C'D'E'$.

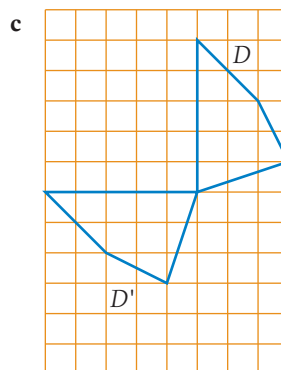
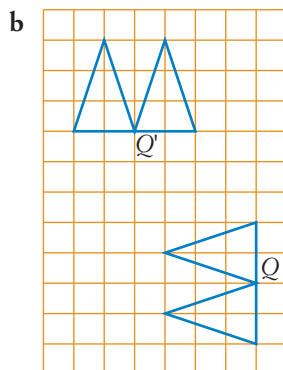
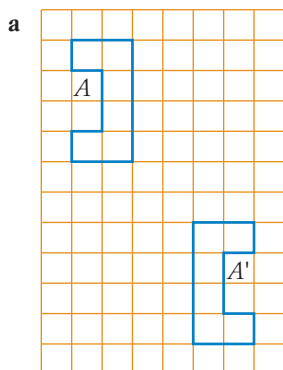
Solution**a**

- b Compare the coordinates of each vertex of $ABCDE$ to its matching vertex in $A'B'C'D'E'$.
- b $A(-5, 2) \rightarrow A'(5, 2)$
 $B(-1, 2) \rightarrow B'(1, 2)$
 $C(0, 0) \rightarrow C'(0, 0)$
 $D(-1, -2) \rightarrow D'(1, -2)$
 $E(-5, -2) \rightarrow E'(5, -2)$

When reflected in the y -axis, the x -coordinate of each vertex changes sign while the y -coordinate stays the same.

Exercise 8-01 Transformations

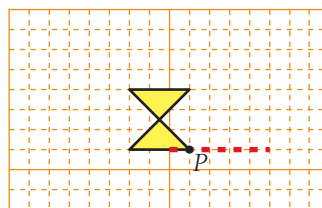
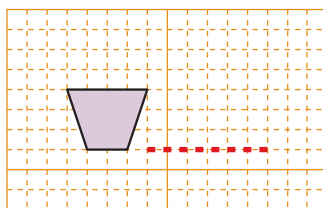
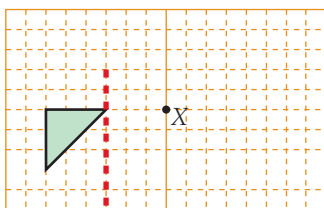
- 1 For each diagram below, state the combination of two transformations used on the original figure to form the image.



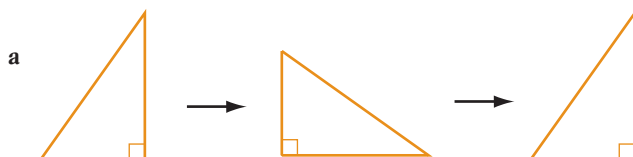
- 2 Copy each shape onto grid paper and draw the image after performing the stated composite transformations.

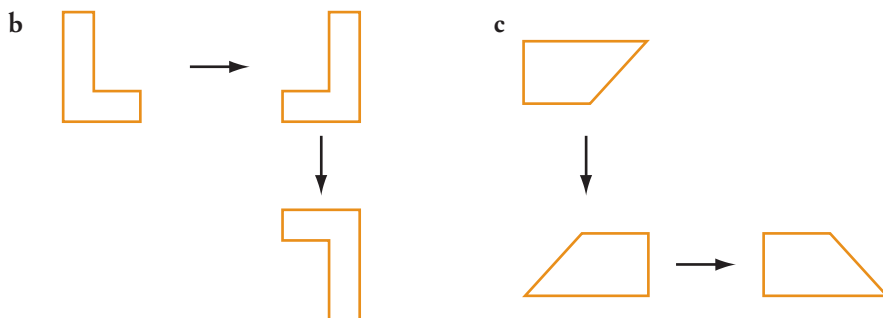
See Example 1

- a Reflect across the line, rotate 90° clockwise about X .
- b Translate 5 units right, then reflect across the line.
- c Rotate 90° clockwise about P , then reflect across the line, then translate 3 units left.

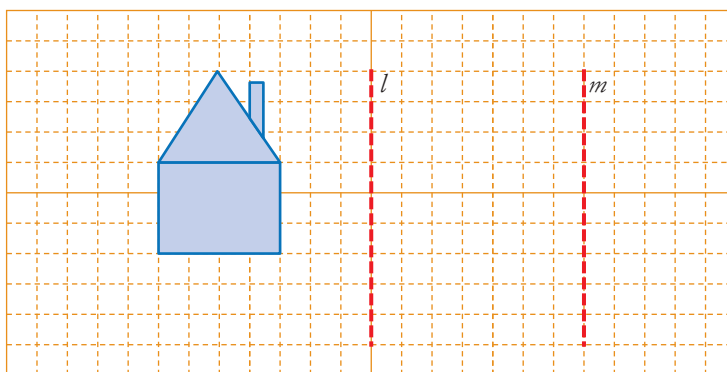


- 3 Each diagram below has been transformed twice.
- Name the two transformations that have been performed.
 - Name one transformation that would give the same result as the two transformations.



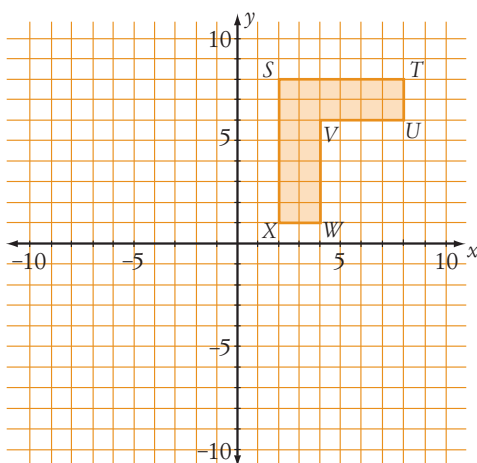


- 4 a Reflect the house shape across line l and then across line m .



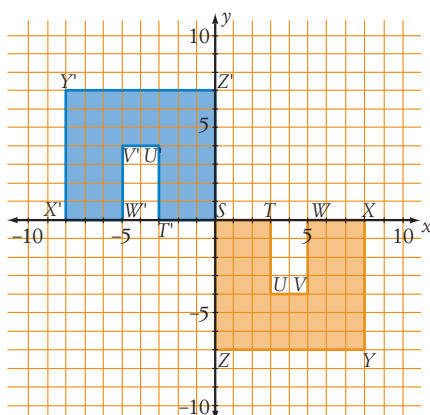
- b Which single transformation would give the same result as the two reflections in part a?

See Example 2 5 a Copy and reflect the L-shape across the x -axis to create the image $S'T'U'V'W'X'$.



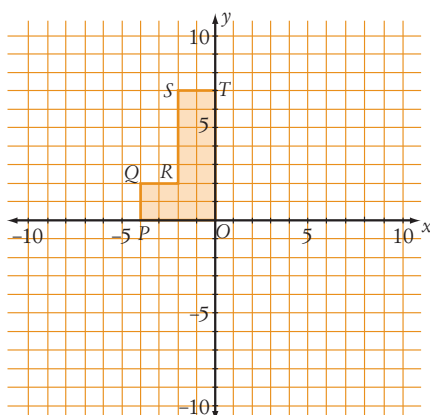
- b Compare the coordinates of each vertex in the original figure to its matching vertex in the image figure.

- 6 a Accurately describe the transformation that has been performed on $STUVWXYZ$.



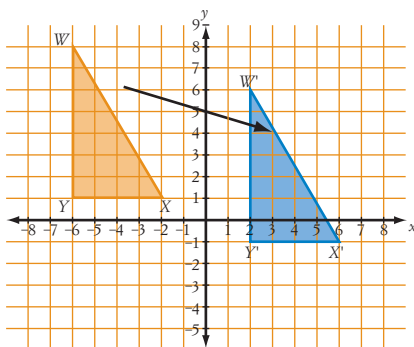
- b Compare the coordinates of each vertex in the original shape to its matching vertex in the image.

- 7 a Copy and rotate $OPQRST$ about O through an angle of 180° .



- b Write the coordinates of the new position of Q and compare them to its original coordinates.

- 8 a Accurately describe the transformation that has been performed on $\triangle WXY$.



- b Compare the coordinates of the vertices of $\triangle WXY$ to those of the image $\triangle W'X'Y'$.

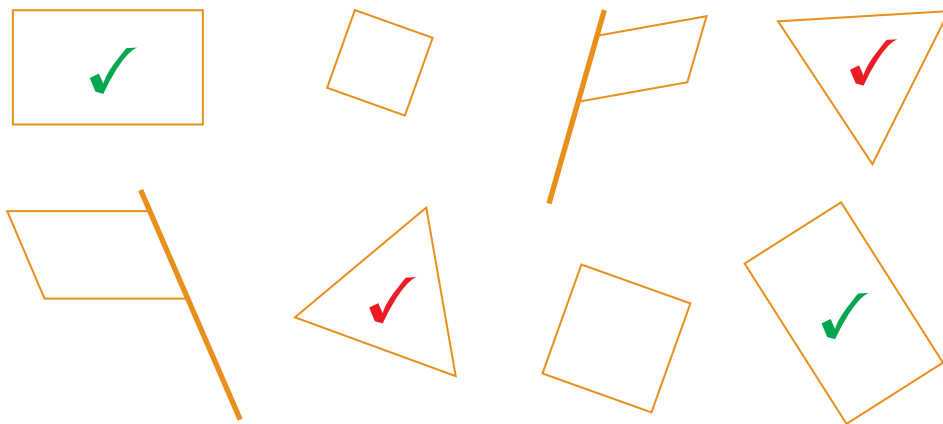
Worksheet

A page of congruent
triangles

MAT08MGWK10074

8-02 Congruent figures

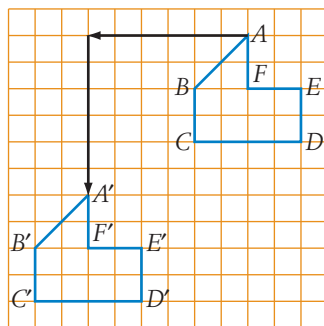
Two shapes are **congruent** if they have exactly the same shape and size. Congruent means 'identical'.



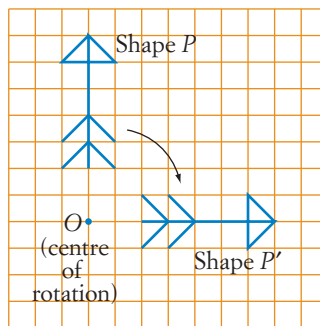
- The two rectangles are congruent (marked by green ticks)
- The two triangles are congruent (marked by red ticks)
- The two squares are not congruent, because they are not the same size
- The two flags are not congruent, because they are not the same size

One way of testing whether two shapes are congruent is to **superimpose** one shape onto the other, that is, to move it to a position on top of the other so that the sides and angles match. Congruent figures may be identified by superimposing them through a combination of translations, rotations and reflections.

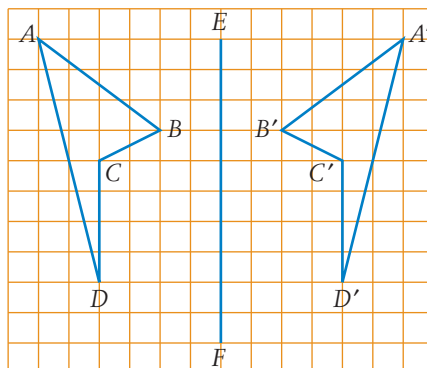
These two shapes are congruent because $ABCDEF$ can be superimposed onto $A'B'C'D'E'F'$ by translation.



These two figures are congruent because arrow P can be superimposed onto arrow P' by rotation.

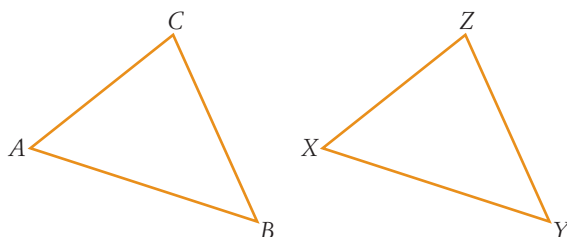


These two quadrilaterals are congruent because $ABCD$ can be superimposed onto $A'B'C'D'$ by reflection.



The congruence symbol $\equiv$

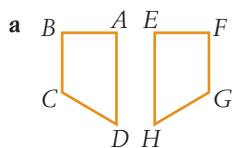
The symbol for 'is congruent to' is a special equals sign, written as ' $\equiv$ ' (which also means 'is identical to'). The two triangles below are congruent, so we can write $\triangle ABC \equiv \triangle XYZ$, which is read 'triangle ABC is congruent to triangle XYZ '.



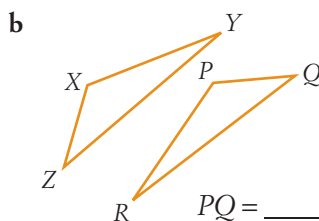
When using this notation, we must make sure that the vertices (angles) of the congruent figures are written in matching order: $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

Example 3

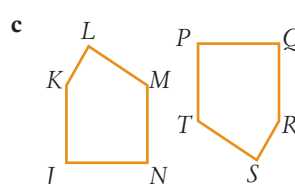
For each pair of congruent figures, identify the required matching side and angle, then write the statement relating the figures using the congruency symbol.



$\angle B = \underline{\hspace{2cm}}$
 $CD = \underline{\hspace{2cm}}$



$PQ = \underline{\hspace{2cm}}$
 $\angle R = \underline{\hspace{2cm}}$



$\angle J = \underline{\hspace{2cm}}$
 $TS = \underline{\hspace{2cm}}$

Solution

a $\angle B = \angle F$ **b** $PQ = XZ$ **c** $\angle J = \angle Q$
 $CD = GH$ $\angle R = \angle Y$ $TS = ML$
 $ABCD \equiv EFGH$ $\triangle XYZ \equiv \triangle PRQ$ $JKLMN \equiv QRSTP$

Note that the vertices of congruent figures are named in matching order: equal angles are paired

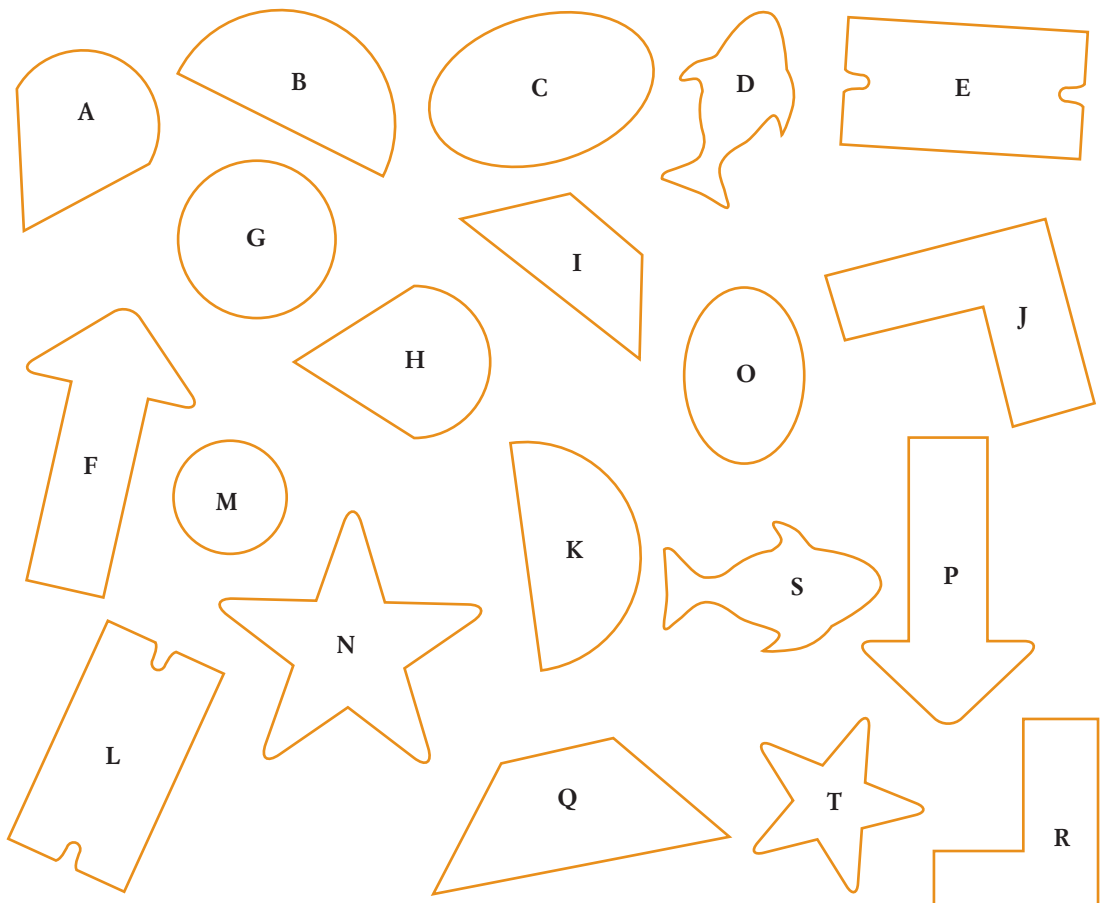
Summary

For **congruent figures**:

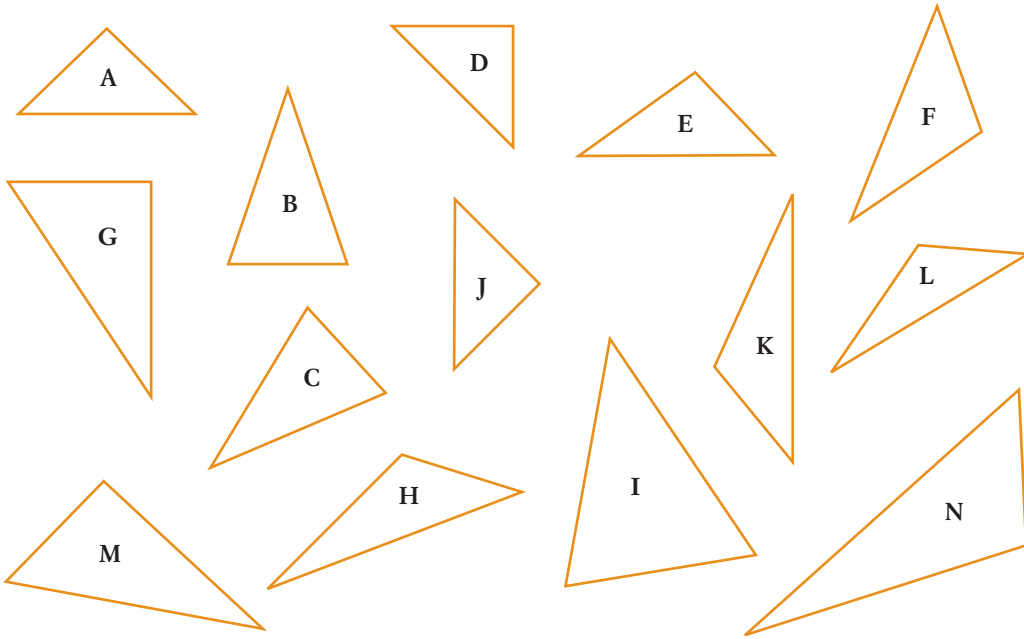
- matching sides are equal
- matching angles are equal
- the symbol for 'is congruent to' is $\equiv$
- vertices are named in matching order.

Exercise 8-02 Congruent figures

- 1 By measuring or examining, find the four matching pairs of congruent shapes from the diagrams below.

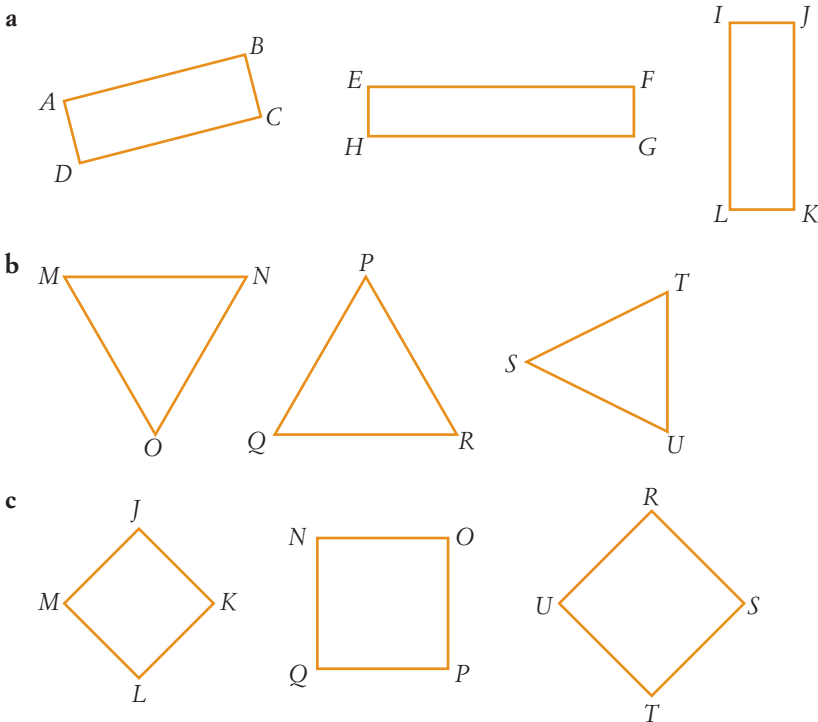


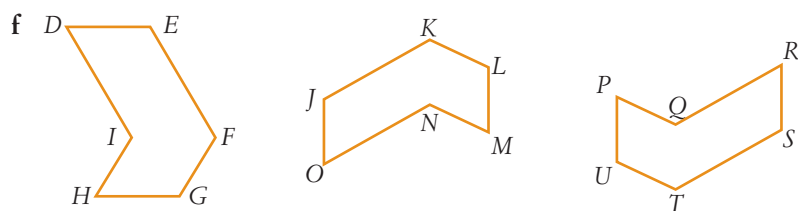
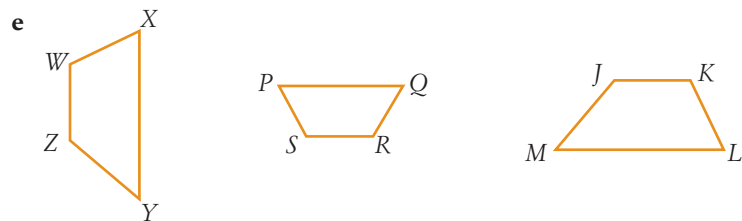
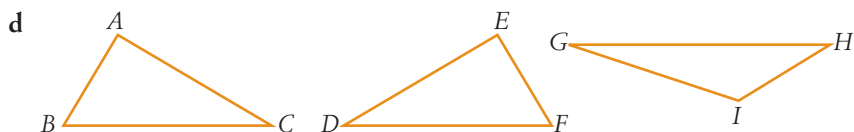
- 2 Of the triangles below:
- a which two triangles are congruent to triangle A?
 - b which triangle is congruent to triangle B?
 - c which triangle is congruent to triangle K?



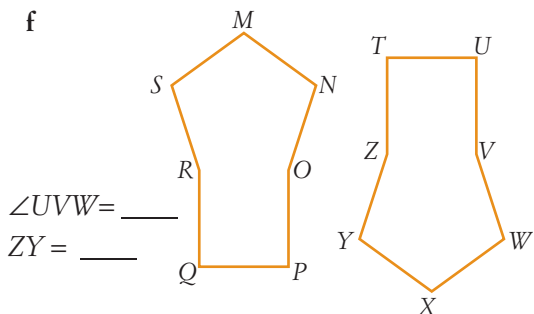
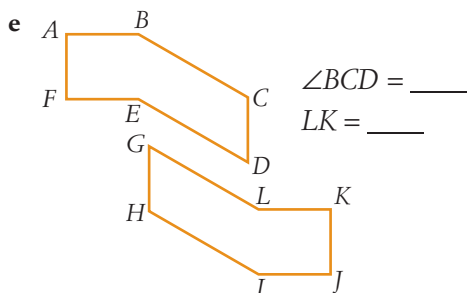
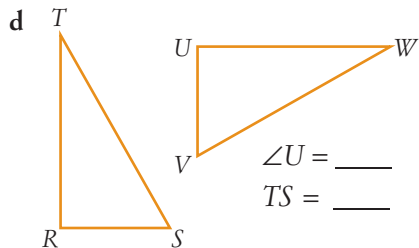
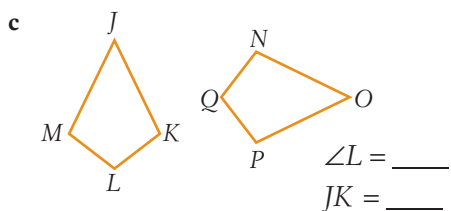
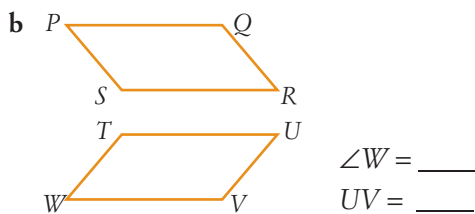
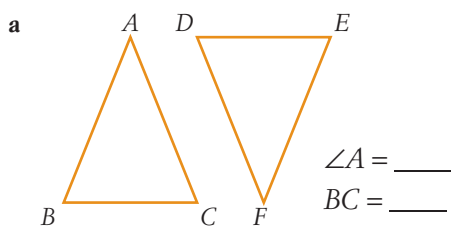
- 3 For each set of shapes, identify the two congruent figures, using the correct notation and matching order of vertices.

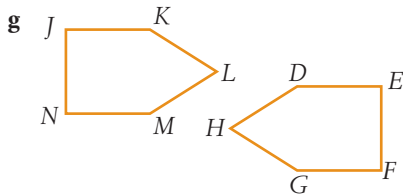
See Example 3





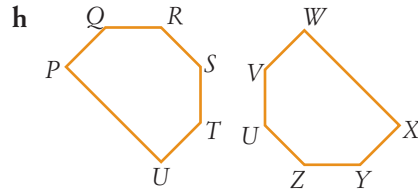
4 Complete the matching angle and side in each pair of congruent figures.





$$\angle NML = \underline{\hspace{2cm}}$$

$$EF = \underline{\hspace{2cm}}$$



$$\angle QPU = \underline{\hspace{2cm}}$$

$$RS = \underline{\hspace{2cm}}$$

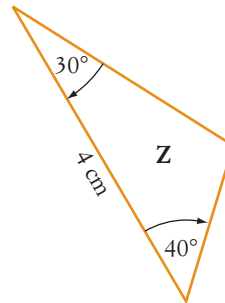
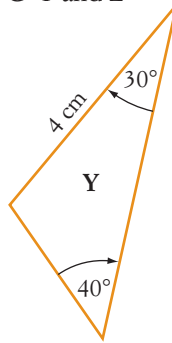
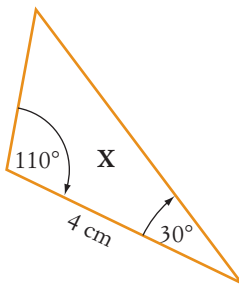
5 Which of the triangles below are congruent? Select the correct answer **A**, **B**, **C** or **D**.

A X and Y

B X and Z

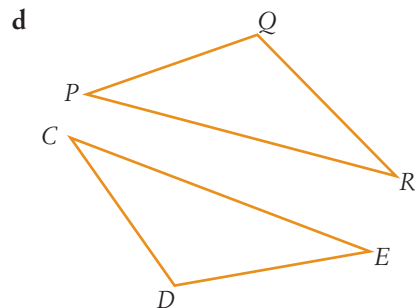
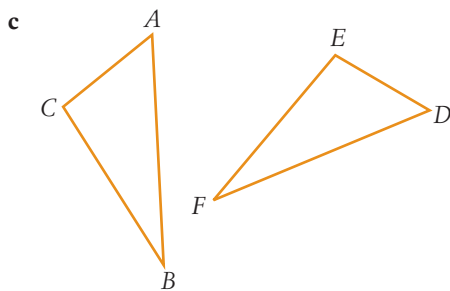
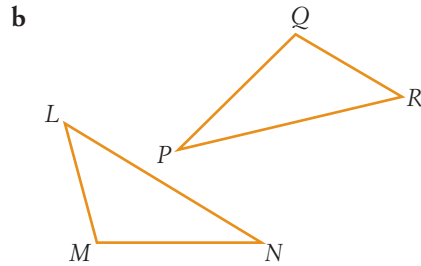
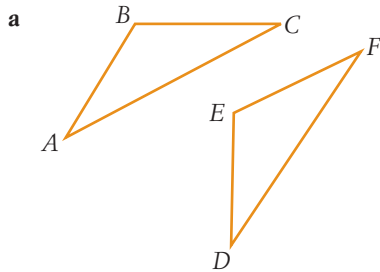
C Y and Z

D X, Y and Z



6 For each pair of congruent triangles, list:

- the three pairs of matching sides
- the three pairs of matching angles.

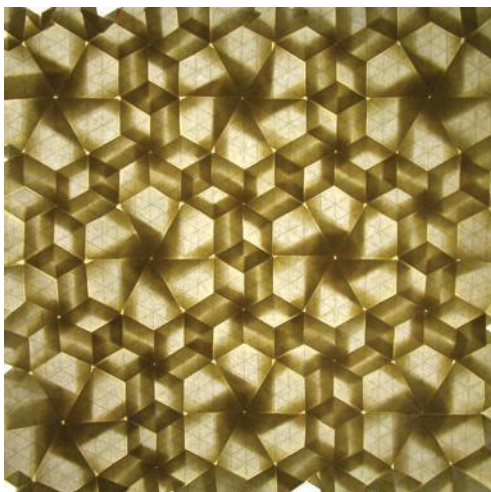


7 Draw a rectangle and its diagonals. You now have four triangles. Shade congruent triangles using the same colours.

8 Repeat question 7 using a parallelogram.

Investigation: Congruence in design

Congruent figures appear in a variety of art and design work. Some examples are shown below.



- 1 Find five examples of congruent figures in the real world. Look at logos and design; artwork, including the work of Escher; tessellations including paving and walls; and decoration in other cultures, including Aboriginal and Islamic design. Put together a presentation of your examples, showing clearly the congruent shapes that have been used.
- 2 Produce your own design or piece of art based on congruent shapes.

8-03 Constructing triangles

Worksheet

Drawing different triangles

MAT08MGWK10075

Puzzle sheet

Triangle constructions group clues

MAT08MGPS10020

To construct (draw accurately) a triangle, we need to know the length of its sides and the size of its angles. We can construct triangles using geometrical instruments (ruler, protractor and compasses) or dynamic geometry software such as *GeoGebra*.

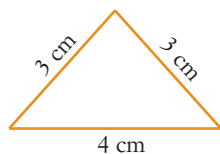
The following examples will show you how to construct triangles. *Hint:* Draw a rough sketch before beginning the construction.

Example 4

Construct an isosceles triangle with sides 3 cm, 3 cm and 4 cm.

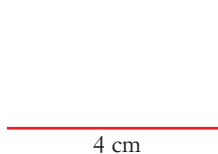
Solution

Step 1:



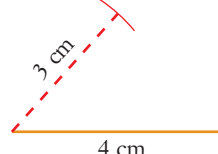
Do a rough sketch.

Step 2:



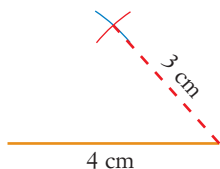
Use a ruler.

Step 3:



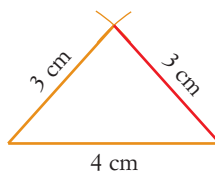
Use a ruler and compasses.

Step 4:



Use a ruler and compasses.

Step 5:



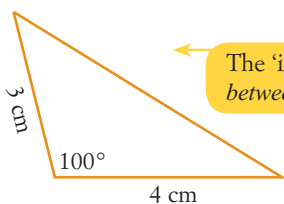
Use a ruler to complete the triangle.

Example 5

Construct a triangle with two sides of length 3 cm and 4 cm and an **included angle** of 100° .

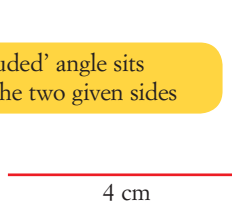
Solution

Step 1:



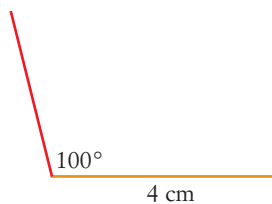
Do a rough sketch.

Step 2:



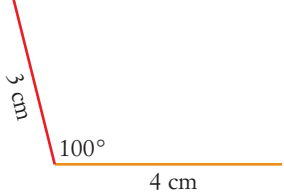
Use a ruler to draw the 4 cm side.

Step 3:



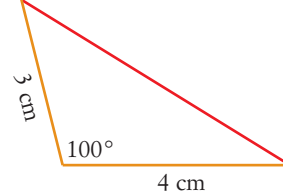
Use a protractor to measure 100° .

Step 4:



Use a ruler to draw the 3 cm side.

Step 5:



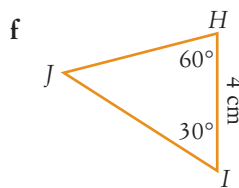
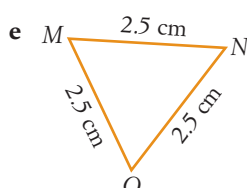
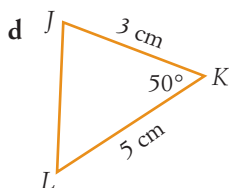
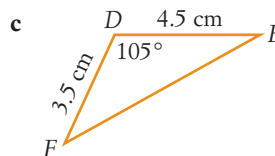
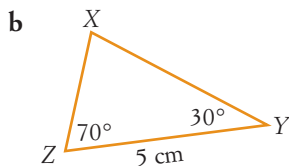
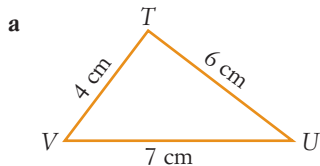
Complete the triangle.

The 'included' angle sits *between* the two given sides

Exercise 8-03 Constructing triangles

1 Construct each triangle accurately.

See Examples 4, 5



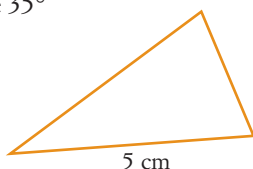
Worksheet
Congruent or different
triangles?
MAT08MGWK10076

- 2 a Which triangle in question 1 is equilateral?
b What is the size of each angle?
- 3 a Which triangle in question 1 is right-angled?
b Measure the length of its shortest side.
- 4 How many measurements about the sides and angles of a triangle are needed to construct a congruent triangle? As a group activity for 3–4 students, for each triangle described below, try to construct different triangles (if possible) with the same measurements. If you can't, and all the triangles drawn are **congruent**, then the measurements that were given are enough. But if one or more of the triangles drawn are different, then the measurements that were given are not enough.

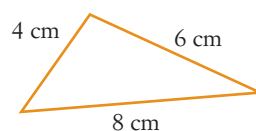
For each triangle described:

- i each group member constructs the triangle with the given measurements
ii determine whether your triangle is 'congruent to' or 'different from' the triangles made by the others in your group (you may need to cut them out first to superimpose them)
iii label the shape 'congruent' or 'different' and paste it into your book.

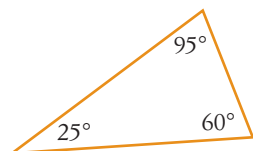
- a one side and one angle: 5 cm and any angle 35°



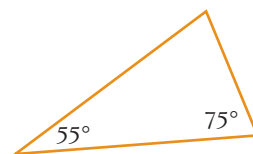
- b 3 sides: 4 cm, 6 cm and 8 cm



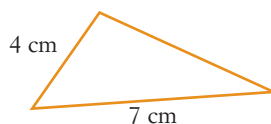
- c 3 angles: 25° , 95° and 60°



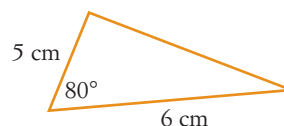
- d 2 angles and one side: 55° , 75° and any side 8 cm



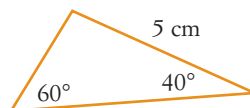
- e 2 sides and one angle: 4 cm, 7 cm and any angle 40°



- f 2 sides and included angle: 5 cm, 6 cm and included angle 80°



- g 2 angles and particular side: 60° , 40° and the side opposite the 60° angle 5 cm



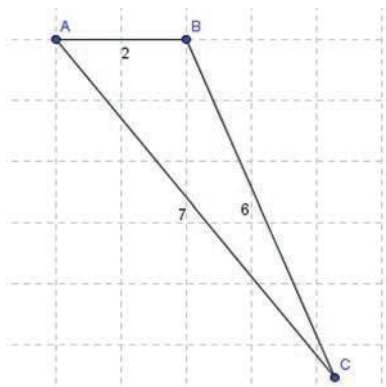
- 5 Consider the triangles you constructed in question 4. Which set of measurements produced congruent triangles? Suggest a reason why.

Technology Tests for congruent triangles

We will use *GeoGebra* to test whether two triangles are congruent. It is best to do this activity in small groups. Select **Grid** on, and **Axes** and **Algebra View** off. Also select **Options Rounding** and **0 Decimal Places**.

Given three sides: SSS

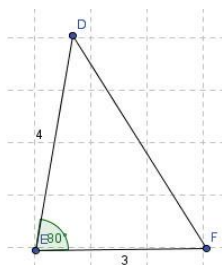
- 1 Construct this triangle using **Interval between Two Points**. Use **Distance** to adjust the side lengths to be 2 cm, 6 cm and 7 cm.



- 2 Can you construct a **different** triangle with the same three side lengths? What do you notice about the second triangle? Is it congruent (exactly the same shape and size) to the first triangle?

Given two sides and the included angle: SAS

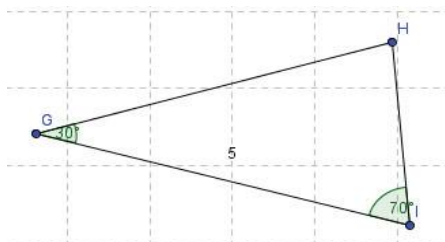
- 1 Construct this triangle using **Interval between Two Points** and **Distance** to draw sides of length 3 cm and 4 cm and **Angle** to adjust the angle between them to be exactly 80° .



- 2 Can you construct a different triangle with the same two sides and an included angle of 80° ? Or is it congruent to the first triangle?

Given two angles and a side: AAS

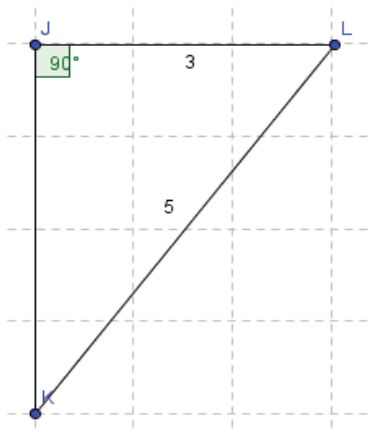
- 1 Construct this triangle with a side length of 5 units between two angles of size 30° and 70° .



- 2 Can you construct a different triangle with the same two angles and a matching side of 5 cm? Or is it congruent to the first triangle?

Given a right angle, hypotenuse and side: RHS

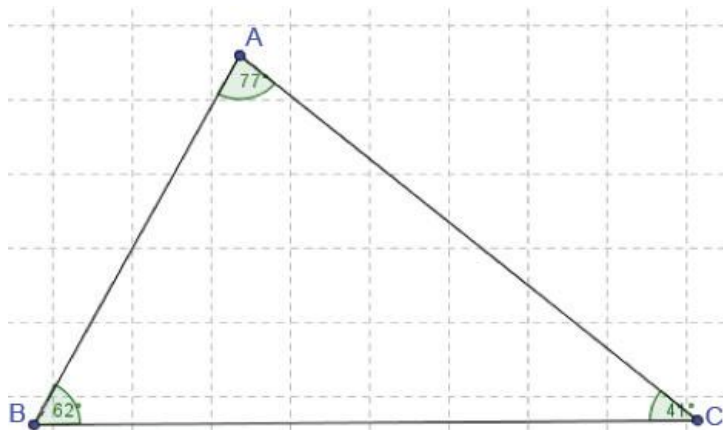
- 1 Construct this right-angled triangle with one of the shorter sides being 3 cm and the hypotenuse 5 cm.



- 2 Can you construct a different triangle with the same two sides and right angle? Or is it congruent to the first triangle?

Given three angles: AAA

- 1 Construct this triangle with angles 77° , 62° and 41° .



- 2 Can you construct a different triangle with the three angles? Or is it congruent to the first triangle?
- 3 Are all triangles with angles 77° , 62° and 41° congruent?

Investigation: Tests for congruent triangles

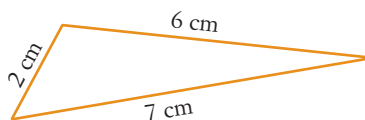
You will need: paper, scissors and geometrical instruments.

How much information about a triangle is necessary for us to be able to draw a congruent triangle? This group investigation will look at the conditions required to show that two triangles are congruent, based on your answers to questions 5 and 6 from the previous exercise. It is best to complete this investigation in small groups, with every person constructing their own triangle.

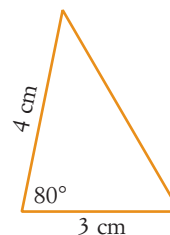
For each triangle described below:

- every person in the group constructs the triangle accurately
- cut the triangle out and compare it with others in the group
- decide whether all of the triangles in your group are congruent.

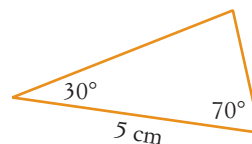
1 Given three sides: SSS



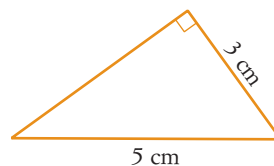
2 Given two sides and the included angle: SAS



3 Given two angles and a side: AAS



4 Given a right angle, hypotenuse and side: RHS



From this investigation, you can see that you don't need to know all the measurements (sides and angles) of two triangles to determine whether they are congruent. Only **three** measurements are needed, which gives us a method of testing for congruent triangles (described in the next section).

Puzzle sheet

Tests for congruent triangles

MAT08MGPS10021

Homework sheet

Congruent figures 1

MAT08MGHS10033

8-04 Tests for congruent triangles

Summary

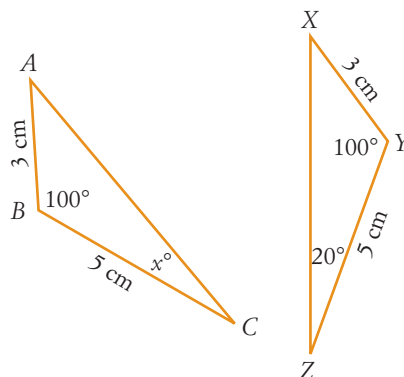
There are four tests for proving congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

Two triangles are congruent if:

- the three sides of one triangle are equal to the three sides of the other triangle (SSS rule)
- two sides and the **included** angle of one triangle are equal to two sides and the **included** angle of the other triangle (SAS rule)
- two angles and one side of one triangle are equal to two angles and the matching side of the other triangle (AAS rule)
- they are right-angled and the hypotenuse and another side of one triangle are equal to the hypotenuse and another side of the other triangle (RHS rule).

Example 6

- Which test shows that these two triangles are congruent?
- Use the correct notation to write a congruency statement relating these two triangles.
- Find x .



Solution

a $AB = XY = 3 \text{ cm}$

S

$\angle B = \angle Y = 100^\circ$

A

$BC = YZ = 5 \text{ cm}$

S

The congruence test is SAS.

b $\triangle ABC \equiv \triangle XYZ$

Matching order of vertices.

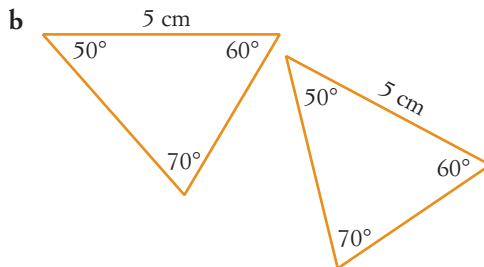
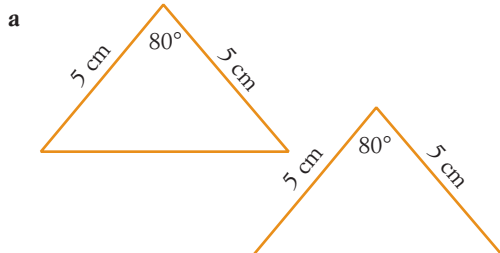
c $\angle C$ matches $\angle Z$.

$\therefore x = 20$

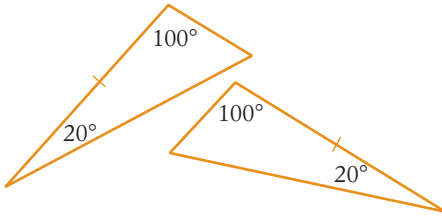
Exercise 8-04 Tests for congruent triangles

See Example 6

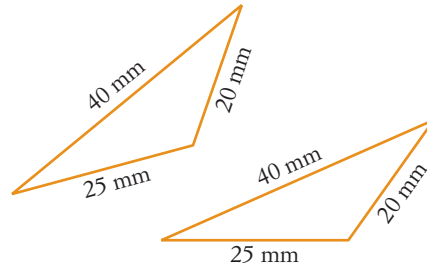
- 1 For each pair of triangles, state which rule (SSS, SAS, AAS or RHS) shows that they are congruent.



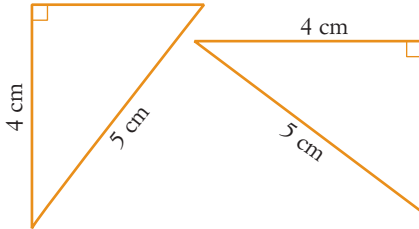
c



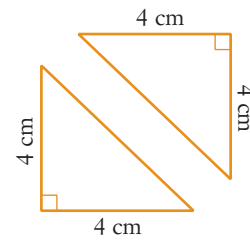
d



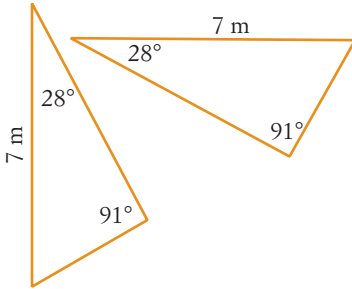
e



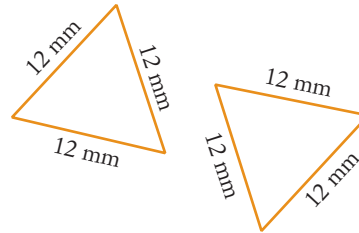
f



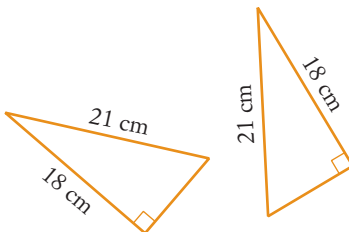
g



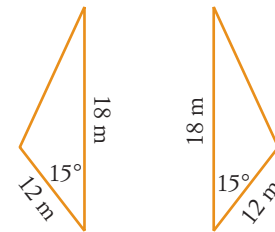
h



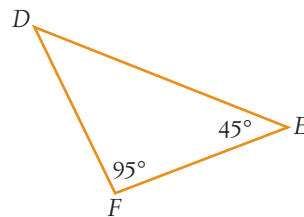
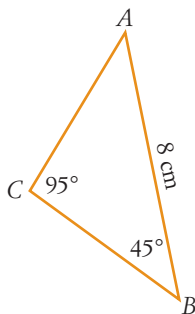
i



j



- 2 a Which of the following is the additional information needed to prove that these two triangles are congruent? Select the correct answer **A**, **B**, **C** or **D**.



A $\angle EDF = 40^\circ$

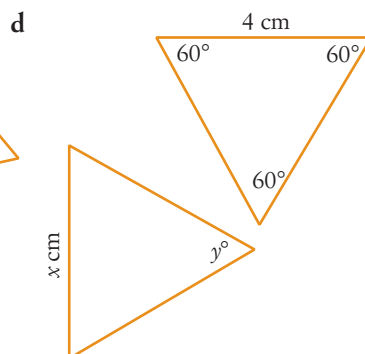
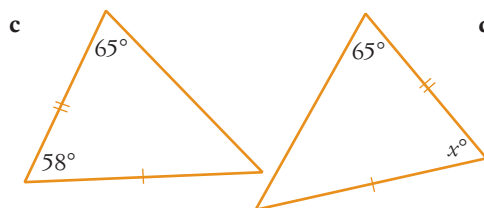
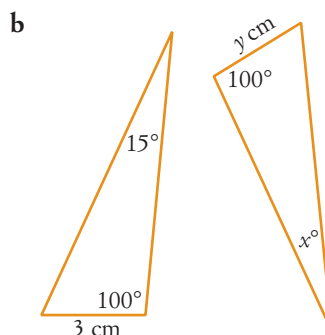
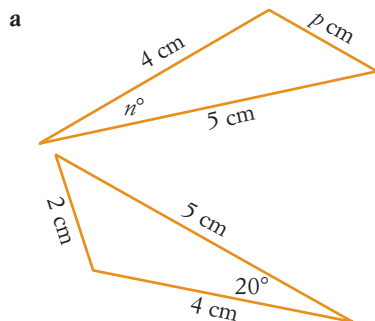
B $DE = 8\text{ cm}$

C $DF = 8\text{ cm}$

D $EF = 8\text{ cm}$

- b Use the correct notation to write a congruency statement relating these two triangles.

- 3 Find the value of each pronumeral if each pair of triangles are congruent.

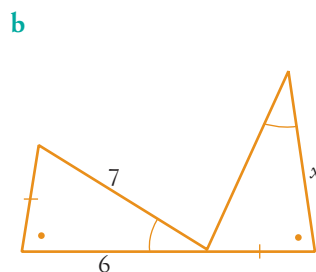
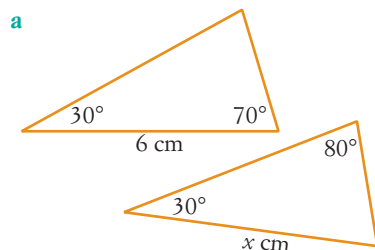


Worked solutions

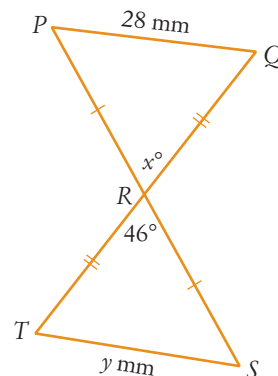
Exercise 8-04

MAT08MGWS10064

- 4 For each pair of triangles, state which test can be used to prove that they are congruent and then find the value of the pronumeral.



- 5
- What geometrical rule shows that $x = 46$ in the diagram?
 - Which test can be used to prove that the two triangles are congruent?
 - Use the correct notation to write a congruency statement relating these two triangles.
 - Find y .
 - Which angle in $\triangle RST$ is equal in size to $\angle P$?
 - Hence why are sides PQ and TS parallel?



Islamic art

This image displays a complex, multi-layered mandala pattern, characteristic of traditional rug or tapestry designs. The central motif is a vibrant sunburst or starburst in shades of orange and red, surrounded by concentric rings of geometric shapes, including triangles, squares, and circles. The colors transition through a spectrum of reds, oranges, yellows, greens, and blues, creating a rich, textured appearance. The overall design is highly symmetrical and intricate, with a dense arrangement of small, repeating geometric elements that form a larger, harmonious whole.

Maths without calculators

1 Study each example.

- $$11:45 \text{ a.m.} + 7 \text{ hours} = 6:45 \text{ p.m.}$$

Count: '11:45, 12:45, 1:45, 2:45, 3:45, 4:45, 5:45, 6:45'

$$\begin{aligned} 6:45 \text{ p.m.} + 40 \text{ minutes} &= 6:45 \text{ p.m.} + 15 \text{ minutes} + 25 \text{ minutes} \\ &= 7:00 \text{ p.m.} + 25 \text{ minutes} \\ &= 7:25 \text{ p.m.} \end{aligned}$$

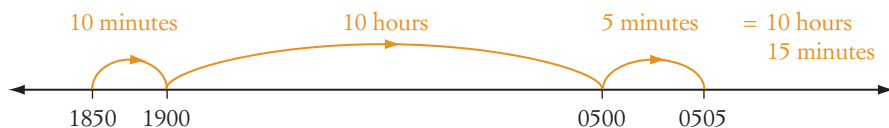
[illegible]

- $$1850 + 10 \text{ hours} = 0450 \text{ (next day).}$$

Count: '1850, 1950, 2050, 2150, 2250, 2350, 0050, 0150, 0250, 0350, 0450'

$$\begin{aligned} 0450 + 15 \text{ min} &= 0450 \text{ hours} + 10 \text{ min} + 5 \text{ min} \\ &= 0500 + 5 \text{ min} \\ &= 0505 \text{ (next day)} \end{aligned}$$

OR:



2 Now find the time of day:

a 3 hours 20 minutes after 9:05 a.m.	b 5 hours 40 minutes after 7:30 p.m.
c 4 hours 35 minutes after 6:15 p.m.	d 11 hours 10 minutes after 11:45 a.m.
e 2 hours 45 minutes after 0325 hours	f 7 hours 5 minutes after 1705 hours
g 8 hours 30 minutes after 12:40 a.m.	h 4 hours 55 minutes after 10:20 p.m.
i 6 hours 25 minutes after 0435 hours	j 2 hours 15 minutes after 2050 hours
k 9 hours 50 minutes after 2:30 p.m.	l 3 hours 10 minutes after 8:25 a.m.

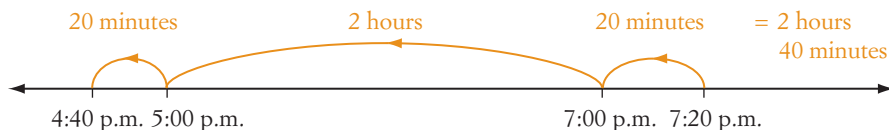
3 Study each example.

a What is the time 2 hours and 40 minutes before 7:20 p.m.?

7:20 p.m. – 2 hours = 5:20 p.m. Count back: '7:20, 6:20, 5:20'

$$\begin{aligned}
 5:20 \text{ p.m.} - 40 \text{ minutes} &= 5:20 \text{ p.m.} - 20 \text{ minutes} - 20 \text{ minutes} \\
 &= 5:00 \text{ p.m.} - 20 \text{ minutes} \\
 &= 4:40 \text{ p.m.}
 \end{aligned}$$

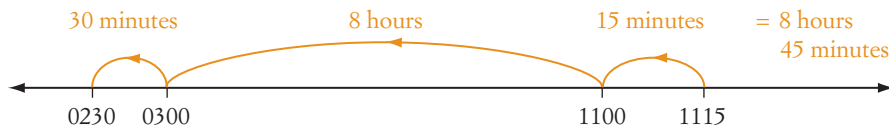
OR:

**b** What is the time 8 hours and 45 minutes before 1115?

1115 – 8 hours = 0315

Count back: '1115, 1015, 0915, 0815, 0715, 0615, 0515, 0415, 0315' (or $11 - 8 = 3$).
$$\begin{aligned}
 0315 - 45 \text{ min} &= 0315 - 15 \text{ min} - 30 \text{ min} \\
 &= 0300 - 30 \text{ min} \\
 &= 0230
 \end{aligned}$$

OR:



4 Now find the time of day:

a 1 hour 15 minutes before 7:20 p.m.	b 4 hours 40 minutes before 11:20 a.m.
c 3 hours 20 minutes before 3:30 p.m.	d 5 hours 35 minutes before 8:25 a.m.
e 2 hours 10 minutes before 1455 hours	f 3 hours 45 minutes before 0740 hours
g 5 hours 25 minutes before 4:15 a.m.	h 9 hours 30 minutes before 9:45 p.m.
i 4 hours 20 minutes before 2005 hours	j 2 hours 15 minutes before 0615 hours

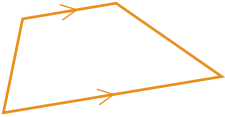
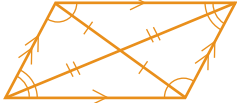
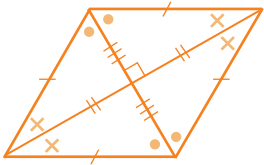
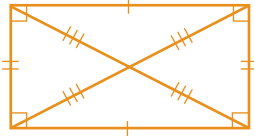
8-05 Proving properties of quadrilaterals

Worksheet

Proving properties
of quadrilaterals

MAT08MGWK10077

In Chapter 4, Geometry, we examined many properties of the special quadrilaterals, as shown in the table below.

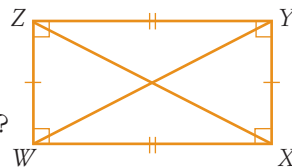
Quadrilateral	Properties
Trapezium 	- One pair of parallel sides - No axes of symmetry
Kite 	- Two pairs of equal adjacent sides - One pair of opposite angles equal - One axis of symmetry - Diagonals intersect at right angles
Parallelogram 	- Opposite sides are parallel and equal - Opposite angles are equal - No axes of symmetry - Diagonals bisect each other
Rhombus 	- Four equal sides - Two axes of symmetry - A special type of parallelogram - Diagonals bisect each other at right angles - Diagonals bisect the angles of the rhombus
Rectangle 	- All four angles measure 90° - Two axes of symmetry - A special type of parallelogram - Diagonals are equal and bisect each other
Square 	- Four equal sides, four angles of 90° - Four axes of symmetry - A special type of rhombus and rectangle - Diagonals are equal and bisect each other at right angles - Diagonals bisect the angles of the square

We can now use congruent triangles to prove some of these properties.

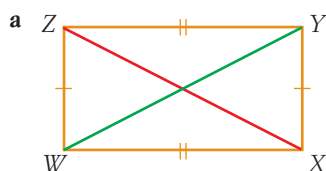
Example 7

$ZYXW$ is a rectangle so its opposite sides are equal and all angles are 90° .

- $\triangle ZXW$ and $\triangle YWX$ are both right-angled triangles that have WX as their base. Which test proves that they are congruent?
- Which side is equal to ZX ?
- What does this prove about the diagonals of a rectangle?



Solution



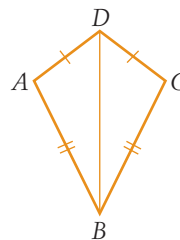
$ZW = YX$, $\angle ZWX = \angle YXW = 90^\circ$, WX is shared. Therefore, SAS proves that they are congruent.

- The side YW is equal to ZX .
- The diagonals of a rectangle are equal in length.

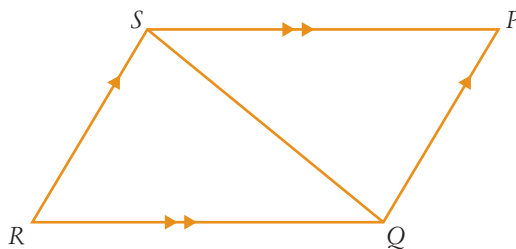
Exercise 8-05 Proving properties of quadrilaterals

See Example 7

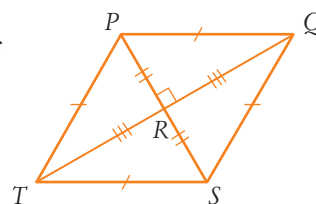
- $ABCD$ is a kite so it has two pairs of equal adjacent sides. The diagonal DB is its axis of symmetry that divides it into two congruent triangles.
 - Which test proves that the two triangles are congruent?
 - Which angle is equal to $\angle A$?
 - Which angle is equal to $\angle ADB$? Copy the diagram and mark both angles with a dot.
 - Draw the other diagonal AC , intersecting DB at point X .
 - This creates four triangles. Looking at your diagram, which congruence test proves that $\triangle DAX \equiv \triangle DCX$?
 - Which side is equal to AX ? Mark both sides with three dashes.
 - Which angle is equal to $\angle DXA$?
 - What is the size of $\angle DXA$? Mark this on your diagram.
 - What does this prove about the diagonals of a kite?



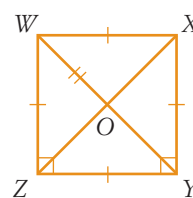
- $SPQR$ is a parallelogram so its opposite sides are parallel.
 - Why is $\angle PSQ = \angle RQS$? What type of angles are they? Copy the diagram and mark both angles with a dot.
 - Which angle is equal to $\angle PQS$? Mark both angles with an arc.



- c The diagonal SQ divides the parallelogram into two congruent triangles. Which test proves that they are congruent?
- d Which side is equal to SP ? Mark both sides with a dash.
- e Which side is equal to PQ ? Mark both sides with two dashes.
- f What does this prove about the opposite sides of a parallelogram?
- g Which angle is equal to $\angle P$? Mark both angles with a cross.
- h What does this prove about the opposite angles of a parallelogram?
- 3 Draw the other diagonal PR of the parallelogram from question 2, crossing SQ at X .
- a Which angle is equal to $\angle SPX$? Mark both angles with little circles.
- b Which test proves that the top and bottom triangles joined at X are congruent?
- c Which side is equal to SX ? Mark both sides with 3 dashes.
- d Which side is equal to PX ? Mark both sides with 4 dashes.
- e What does this prove about the diagonals of a parallelogram?
- 4 $PQST$ is a rhombus so all sides are equal. A rhombus is a special type of parallelogram so its diagonals should also bisect each other as shown.
- a The two diagonals divide the rhombus into four congruent triangles. Which test proves that the four triangles are congruent?
- b What is the size of the four angles around point R ? Why? Copy the diagram and mark those angles with the correct symbol.
- c What does this prove about the diagonals of a rhombus?
- d The bigger triangle, $\triangle PQT$, is **isosceles** so mark its two equal angles with a dot.
- e Mark the other two angles that are equal to the angles marked with a dot.
- f Mark the remaining angles in the congruent triangles with a cross since they are equal.
- g What does this prove about the diagonals of a rhombus?



- 5 $WXYZ$ is a square so all sides are equal and all angles are 90° .
- a Which congruence test proves that $\triangle WYZ \equiv \triangle XZY$?
- b Which side is equal to WY ?
- c What does this prove about the diagonals of a square?
- d A square is a special type of parallelogram so its diagonals bisect each other. Copy the diagram and mark all sides that are equal to WO with double dashes.
- e The two diagonals divide the square into four congruent triangles. Which test proves that they are congruent?
- f What is the size of the four angles around point O ? Mark those angles with the correct symbol.
- g What does this prove about the diagonals of a square?
- h As the four congruent triangles are also **isosceles** triangles, mark all equal angles with a dot. What is the size of these angles?
- i What does this prove about the diagonals of a square?



Worked solutions

Exercise 8-05

MAT08MGWS10065

Worksheet

Bisecting intervals
and angles

MAT08MGWK10078

8-06

Extension: Bisecting intervals
and angles

An **interval** is a section of a line that has a definite length.

We can apply the properties of geometric figures and use geometrical instruments to bisect intervals and angles.

Example 8

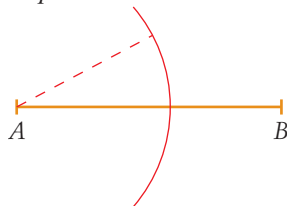
Bisect the interval AB .



Solution

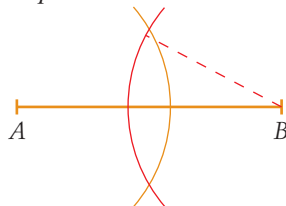
The following method involves constructing an 'invisible rhombus' whose diagonal is the interval AB . We use the property that the diagonals of a rhombus bisect each other.

Step 1:



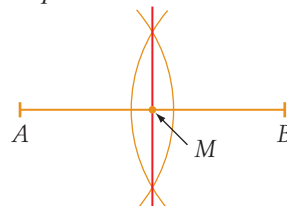
Open your compasses out to more than half the length of AB and draw a large arc from A .

Step 2:



Draw another arc the same distance from B .

Step 3:

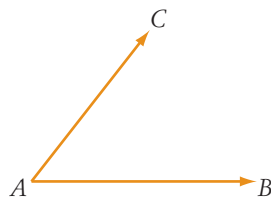


Use a ruler and mark the midpoint M . Check with a ruler that M is halfway.

Use a ruler to check that the interval has been bisected.

Example 9

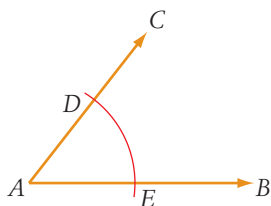
Bisect $\angle CAB$.



Solution

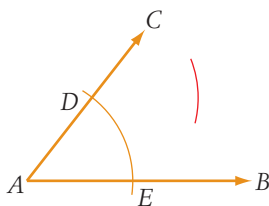
The following method involves constructing an 'invisible rhombus' with the arms of the angle being two sides of the rhombus, and a diagonal bisecting $\angle A$.

Step 1:



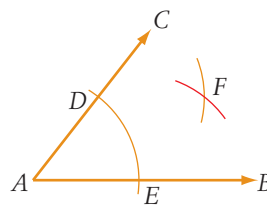
Use compasses to draw a large arc from A .

Step 2:



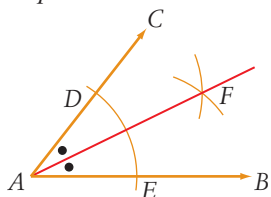
Draw another arc the same distance from D .

Step 3:



Draw another arc the same distance from E .

Step 4:

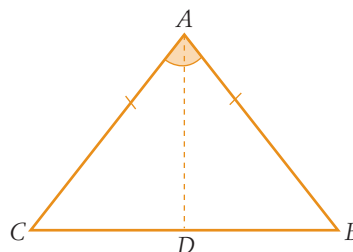


Use a ruler to join F to A to cut the angle in half.

Use a protractor to check that the angle has been bisected.

Exercise 8-06 Bisecting intervals and angles

- Draw three different intervals and bisect them.
- Draw two acute and two obtuse angles and bisect them.
- Draw a reflex angle and bisect it.
- Construct an angle of 90° by bisecting a straight angle.
 - Bisect the 90° angle to form a 45° angle.
- Draw an interval of length 10 cm and use compasses to divide it into four equal subintervals. Check by measuring with a ruler.
- Construct an isosceles triangle and label it ABC as shown in the diagram.
 - Bisect $\angle A$ and extend the line to cut BC at D .
 - Which congruency test proves that $\triangle ACD \equiv \triangle ABD$?
 - Hence which angle is equal to $\angle C$? Check by measuring with a protractor.
 - Measure the size of $\angle ADB$. What do you notice?
 - Measure the lengths of CD and DB . What do you notice?
- Draw $\angle PQR$ of any size and use a pair of compasses to bisect $\angle Q$.
 - Check with a protractor that $\angle Q$ has been bisected.



See Example 8

See Example 9

Technology worksheet

Bisecting angles

MAT08MGCT10009

Worksheet

A page of intervals

MAT08MGWK10079

Homework sheet

Congruent figures 2

MAT08MGHS10034

Homework sheet

Congruent figures
revision

MAT08MGHS10035

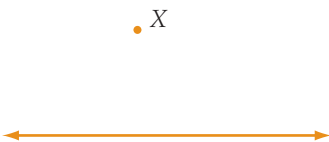
8-07

Constructing parallel and perpendicular lines

We can apply the properties of geometric figures and use geometrical instruments to construct parallel and perpendicular lines.

Example 10

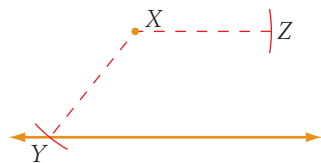
Use compasses to draw a line through X that is parallel to the given line.



Solution

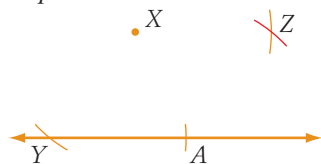
The following method involves constructing an 'invisible rhombus' to create parallel lines.

Step 1:



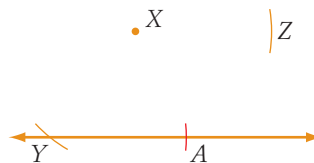
Use compasses from X to mark two large arcs at Y and Z .

Step 3:



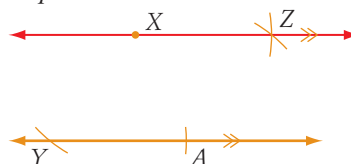
Use compasses from A to mark an arc with the same radius to cross the arc at Z .

Step 2:



Use compasses from Y to mark an arc with the same radius at A on the line.

Step 4:



Join XZ to construct a line parallel to AY .

Example 11

Use compasses to draw a perpendicular line through point B .



Solution

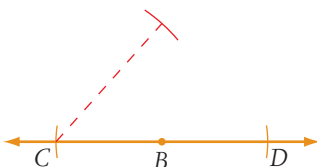
The following method involves constructing an 'invisible isosceles triangle' on the line, with its axis of symmetry making a 90° angle at B . Also, note that drawing a perpendicular at B is the same as bisecting a straight angle with vertex B .

Step 1:



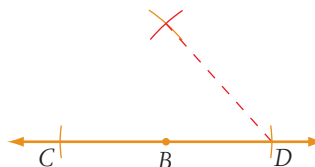
Use compasses to draw two arcs from B .

Step 2:



Open compasses wider to draw an arc from C .

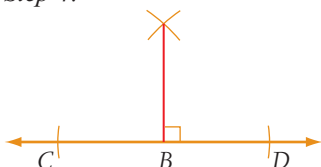
Step 3:



Use compasses to draw an arc with the same distance from D .

Use a protractor or set square to check that the line is perpendicular (at 90°) to CD .

Step 4:



Join B to where the two arcs cross.

Example 12

Use compasses to draw a perpendicular line through point P .

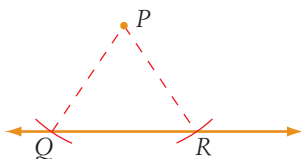
P



Solution

The following method involves constructing an 'invisible rhombus' with the line being one of its diagonals and P being one of its vertices. We are using the property that the diagonals of a rhombus cross at right angles.

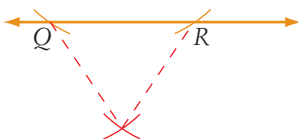
Step 1:



Use compasses from P to mark two arcs with the same radius on the line.

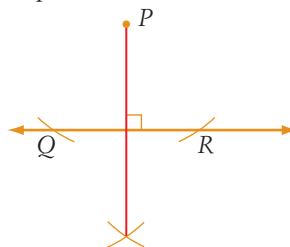
Step 2:

P



Use compasses from Q and R to mark two intersecting arcs with the same radius below the line.

Step 3:



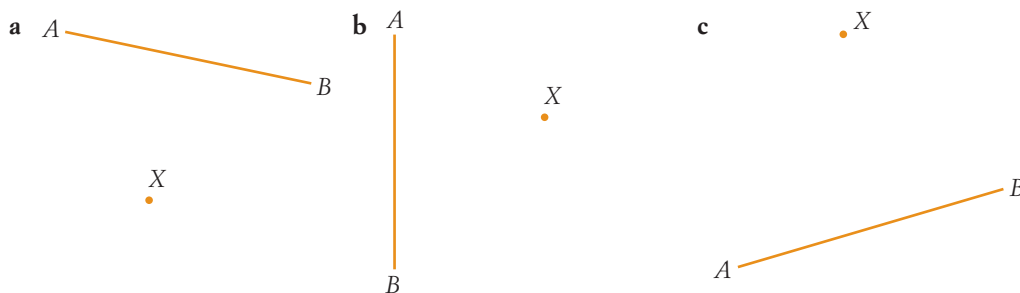
Join P to where the two arcs cross.

Use a protractor or set square to check that the line is perpendicular to QR .

Exercise 8-07 Constructing parallel and perpendicular lines

See Example 10

- 1 Copy each diagram and construct a line parallel to AB through X .



- 2 a Draw two intervals that are parallel and of different lengths.
b Join their ends to make a quadrilateral.
c What type of quadrilateral have you constructed?
- 3 a Draw two intervals that are parallel and of the same length.
b Join their ends to make a quadrilateral.
c What type of quadrilateral have you constructed?

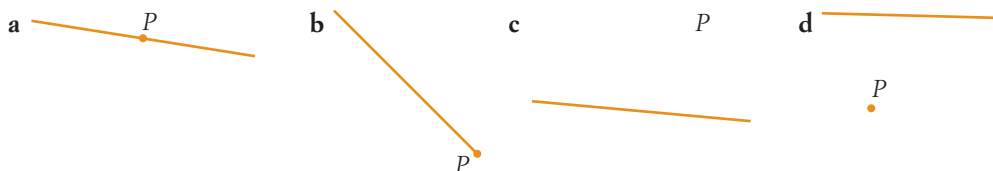
See Example 11

- 4 Draw a line and mark a point, X , on it. Construct a perpendicular line that passes through X using:

a compasses b a protractor c a set square

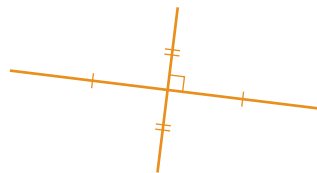
See Example 12

- 5 Draw a line and mark a point, X , above or below it. Construct a perpendicular line that passes through X .
- 6 Copy each diagram and construct a perpendicular line through P .



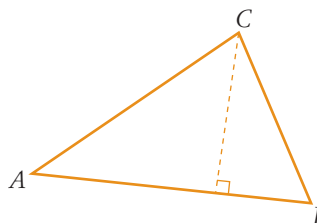
- 7 a Draw a point, M , on a line and use compasses to construct a perpendicular through M .
b When we construct the perpendicular through M , we are really constructing an ‘invisible’ isosceles triangle. Draw the ‘invisible’ triangle on your diagram from part a.
c The perpendicular is an axis of symmetry that divides the isosceles triangle into two congruent triangles. Mark on your diagram all pairs of matching equal sides and state which congruence test proves that the two triangles are equal.
d Why do you think that the line through M is a perpendicular?
- 8 a Draw a point, P , above a line AB and use compasses to construct a perpendicular through P .
b Can you ‘see’ the ‘invisible’ rhombus? Draw it.
c Which property of the rhombus proves that the line through P is perpendicular to AB ?

- 9 a Draw an interval and mark its midpoint.
b Draw a different-sized interval that is perpendicular to the first interval so that they have the same midpoint (as shown in the diagram).
c Join the ends of the interval to make a quadrilateral.
d What type of quadrilateral have you constructed?



- 10 a Draw an interval and mark its midpoint.
b Draw a same-sized interval that is perpendicular to the first interval so that they have the same midpoint.
c Join the ends of the interval to make a quadrilateral.
d What type of quadrilateral have you constructed?

- 11 a Draw a large triangle, ABC .
b From each vertex, construct a line that is perpendicular to the opposite side of the triangle. (These lines are called the **altitudes** of the triangle.)
c What do you notice about the three altitudes of a triangle?



Mental skills 8B Maths without calculators

Time differences

- 1 Study each example.

- a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

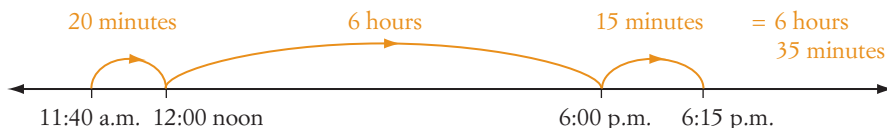
Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

5 hours + 20 min + 15 min = 6 hours 35 min

OR:



- b What is the time difference between 1645 and 2320?

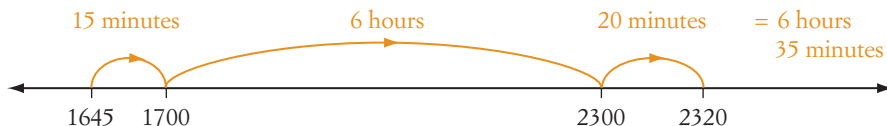
From 1645 to 2245 = 6 hours ($22 - 16 = 6$)

From 2245 to 2300 = 15 min

From 2300 to 2320 = 20 min

6 hours + 15 minutes + 20 minutes = 6 hours 35 minutes

OR:



2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.	b 6:20 p.m. and 12:00 midnight
c 4:45 p.m. and 8:10 p.m.	d 2:35 a.m. and 10:50 a.m.
e 1:05 p.m. and 12:30 a.m.	f 9:35 a.m. and 11:15 a.m.
g 0425 and 0935	h 1440 and 2025
i 7:55 a.m. and 3:50 p.m.	j 2:40 p.m. and 10:20 p.m.

Power plus

Worksheet

Congruent triangle
proofs

MAT08MGWK10080

1 Copy and complete each formal proof of congruent triangles.

a $WXYZ$ is a kite.

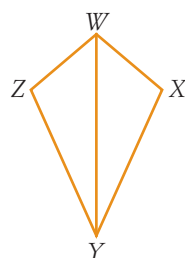
In $\triangle WYZ$ and $\triangle WYX$:

$WZ = \underline{\hspace{2cm}}$ (equal sides of a kite)

$ZY = \underline{\hspace{2cm}}$ ()

Side $\underline{\hspace{2cm}}$ is common to both triangles.

$\triangle WYZ \equiv \triangle WYX$ (SSS)



b ABC is an isosceles triangle, where $AB = AC$.

D is the midpoint of BC .

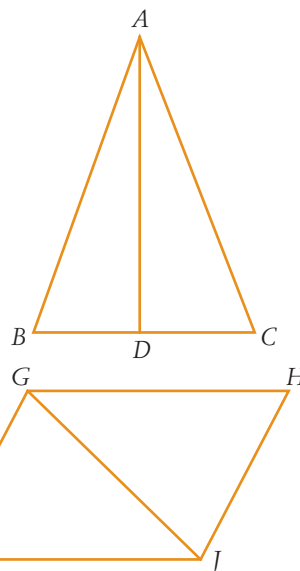
In $\triangle ABD$ and $\triangle ACD$:

$AB = \underline{\hspace{2cm}}$ ()

$BD = \underline{\hspace{2cm}}$ (D is)

AD is $\underline{\hspace{2cm}}$ to both triangles.

$\triangle ABD \equiv \triangle ACD$ ()



c $GHJK$ is a parallelogram.

In $\triangle GHJ$ and $\triangle JKG$:

$GH = \underline{\hspace{2cm}}$ ()

$HJ = \underline{\hspace{2cm}}$ ()

$\angle GHJ = \angle \underline{\hspace{2cm}}$ ()

$\triangle \underline{\hspace{2cm}} \equiv \triangle \underline{\hspace{2cm}}$ ()

2 a What additional property makes a parallelogram a rectangle?

b What makes a kite a rhombus?

c What makes a rectangle a square?

3 Name all quadrilaterals whose:

a opposite angles are equal

c diagonals are equal

e opposite sides are parallel

b diagonals intersect at 90°

d angles are all 90°

f diagonals bisect each other.

Language of maths

AAS	diagonal	parallel	SSS
bisect	hypotenuse	perpendicular	superimpose
compasses	image	reflection	transformation
congruence test	included angle	RHS	translation
congruent ($\equiv$)	matching	rotation	vertex/vertices
construct	original	SAS	

Puzzle sheet

Congruent triangles
find-a-word

MAT08MGPS10022

- What are the three **congruence transformations**?
- What word means:
 - to cut in half?
 - to draw accurately?
 - to place one shape on top of another shape so that it fits?
- What does the phrase 'vertices must be named in matching order' mean?
- What does **RHS** stand for:
 - in congruent triangles?
 - in solving equations?
- What is an **included angle**?

Topic overview

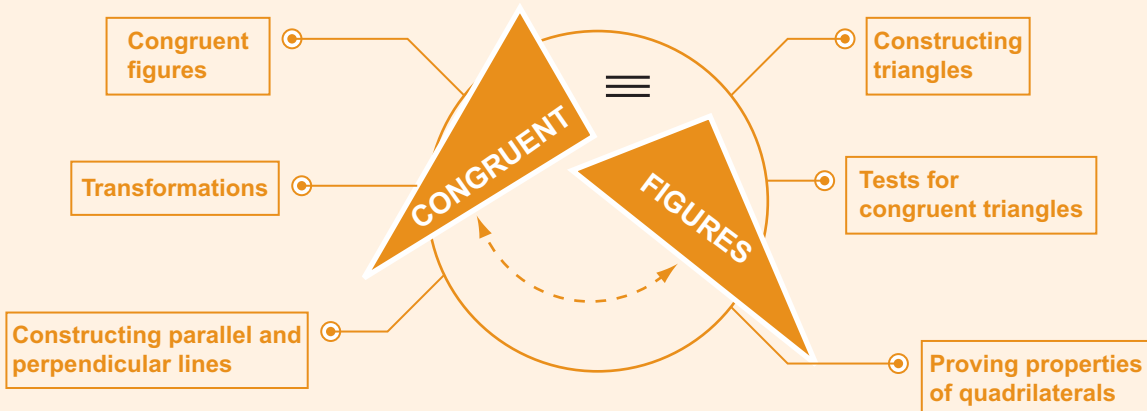
- How useful do you think this chapter is to you?
- Which sections did you find the most interesting? Which did you find the most difficult?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet

Mind map: Congruent
figures

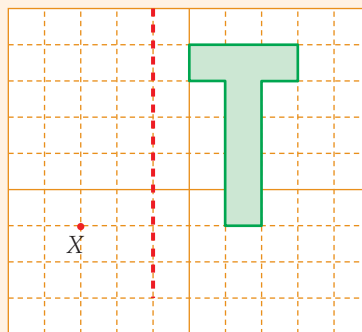
MAT08MGWK10081



Chapter 8 revision

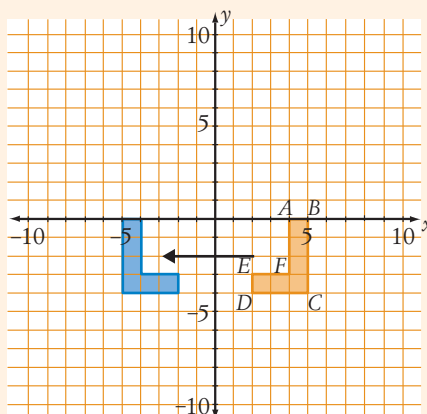
See Exercise 8-01

- 1 Copy this diagram, reflect the shape across the line, then rotate it 270° clockwise about the point X .



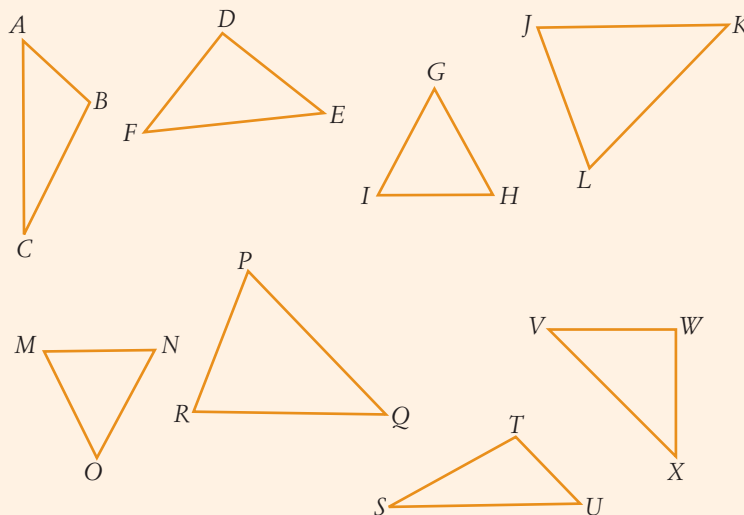
See Exercise 8-01

- 2 a Accurately describe the transformation that has been performed on the orange L-shape $ABCDEF$.
b Write the coordinates of the new position of C and compare them to its original coordinates.



See Exercise 8-02

- 3 From the diagrams, match four pairs of congruent triangles and for each pair:
a name one pair of matching sides
b name one pair of matching angles

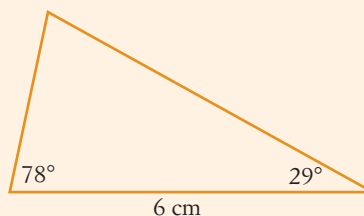


- c Write the congruence statement using the $\equiv$ notation and match the order of the vertices.

- 4 This triangle has a base of length 6 cm that is bounded by angles of size 78° and 29° .

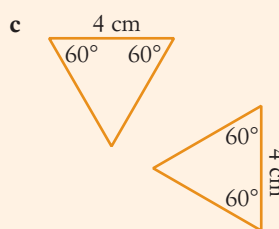
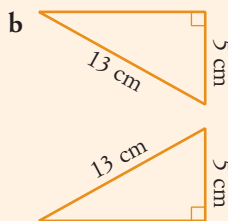
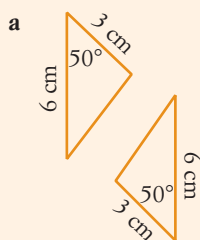
See Exercise 8-03

- a Construct a triangle that is congruent to this triangle.
b Is it possible to construct another triangle with the same measurements that is not congruent?



- 5 Which congruence test (SSS, SAS, AAS or RHS) proves that each pair of triangles is congruent?

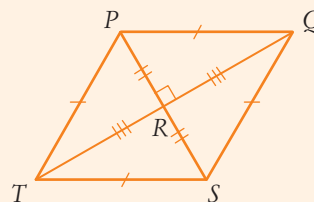
See Exercise 8-04



- 6 The diagonals of this rhombus divide the rhombus into four congruent triangles.

See Exercise 8-05

- a Which test proves that the four triangles are congruent?
b Hence explain why the diagonals cross at right angles.
c Name the three angles that are equal to $\angle PQR$.
d True or false? The diagonals of a rhombus:



- i are equal ii bisect each other iii bisect the angles of the rhombus

- 7 Copy each diagram below and construct a line parallel to BC going through P .

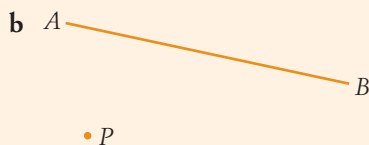
See Exercise 8-07



• P

- 8 Copy each diagram below and construct a line perpendicular to AB going through P .

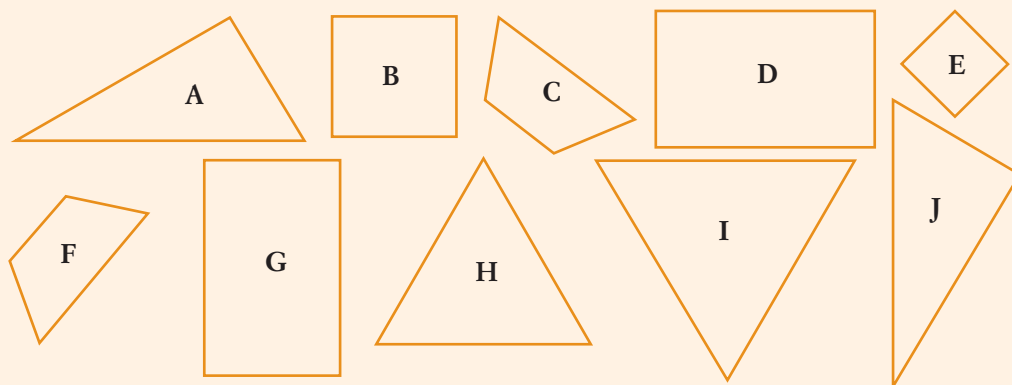
See Exercise 8-07



Mixed revision 3

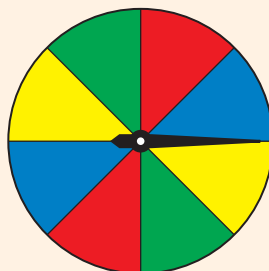
See Exercise 8-02

- 1 Identify the three pairs of congruent shapes from the shapes below.



See Exercise 9-01

- 2 If this spinner is spun, find the probability, as a percentage, that the arrow lands on:
 a green b a traffic light colour c a colour that is not red



See Exercise 7-01

- 3 Mrs Smart gave her class a quick quiz. The results are shown below.

Mark	Tally	Frequency
6		
7		
8		5
9		
10		3

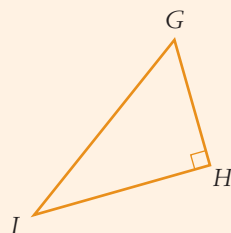
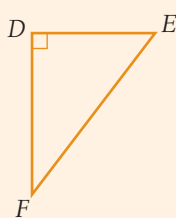
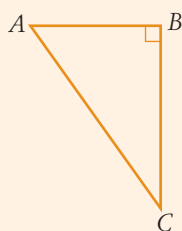
- a Copy and complete the table.
 b How many students are there in Mrs Smart's class?

See Exercise 7-07

- 4 For the quiz marks in question 3, construct a frequency histogram and polygon.

See Exercise 8-03

- 5 a Name the two congruent triangles below, using ' $\triangle \_\_\_ \equiv \triangle \_\_\_$ ' notation.
 b For the congruent triangles, name one pair of matching sides and one pair of matching angles.



- 6 A die was rolled 40 times and the results are shown here.

See Exercise 7-01

5 1 5 2 3 2 6 5 2 5 4 2 5 2 6 2 6 4 3 1
5 5 3 6 4 2 6 5 6 5 5 2 3 3 3 3 3 1 2 1

- a** Draw a frequency table for these results.
b Draw a divided bar graph to represent this information.

- 7 Is each type of data categorical or numerical?

See Exercise 7-02

- a** numbers of cars owned **b** favourite drinks
c types of hairstyle

- 8 Is each type of data discrete or continuous?

See Exercise 7-02

- a** lap times of swimmers **b** distances between towns
c points scored by players on a basketball team

- 9 A die is rolled. Find the probability that the number that comes up is:

See Exercise 9-01

- a** even **b** at least 2 **c** is a composite number

- 10 Construct a dot plot for these Mathematics test marks.

See Exercise 7-06

22 16 10 8 12 11 15 13 12 13 11 16
11 12 10 10 12 12 15 11 13 15 17 13

- a** Find the median. **b** Find the mode.
c What is the outlier? **d** Where are the marks clustered?

- 11 Construct a triangle with:

See Exercise 8-03

- a** sides of 7 cm, 5 cm and 9 cm
b sides of 4 cm and 5 cm and an included angle of 65°

- 12 For the scores below, find:

See Exercise 7-03

90 89 95 92 90 92 97 93 90 97

- a** the mean **b** the mode

- 13 For the scores in question 12, find:

See Exercise 7-04

- a** the median **b** the range

- 14 A pack of cards contains 10 blue, 6 purple and 4 pink cards. One card is drawn from the pack at random. Find the probability that this card is not purple.

See Exercise 9-02

- 15 The ages of people taking out a policy with an insurance company are shown below.

See Exercise 7-05

34 32 28 30 33 28 30 29 33 28 34 34
28 30 32 32 34 31 30 31 28 30 28 30

- a** Construct a frequency table of the ages, including an 'fx' column.
b What fraction of the people are under 30 years of age?
c For the above information, find:
i the mean (to one decimal place) **ii** the median
iii the mode **iv** the range.

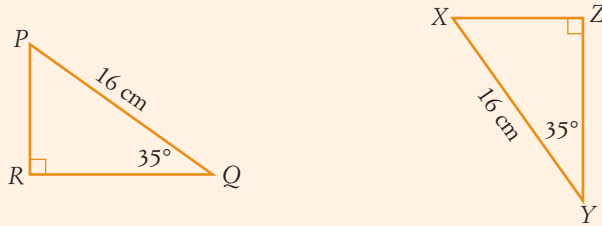
Mixed revision 3

See Exercise 9-03

- 16** In a group of 50 Year 9 students, 34 study History, 25 study Japanese and 11 study both.
- Represent this information in a Venn diagram.
 - How many students study Japanese but not History?
 - What is the probability that a student randomly chosen from this group studies neither History nor Japanese?

See Exercise 8-04

- 17**
- Name the 4 tests for congruent triangles.
 - Which test can be used to prove that these triangles are congruent?



See Exercise 9-04

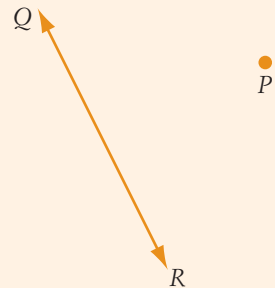
- 18** Year 8 students were surveyed on whether they participated in weekend sport.
- Copy and complete this two-way table.

	Plays weekend sport	Does not play weekend sport	
Male	49		
Female		45	84
		68	

- How many students are in Year 8?
- How many Year 8 students play weekend sport?
- If a student is chosen at random from Year 8, what is the probability that:
 - she is female?
 - the student is male and plays sport but not both?

See Exercise 8-07

- 19** Copy this diagram and use compasses and ruler to construct a line through P
- perpendicular to QR
 - parallel to QR .



- 20 The marks scored by ten students in Maths and Science exams are shown below.

See Exercise 7-11

Maths	35	58	87	43	52	96	23	35	48	58
Science	43	69	73	62	54	88	24	40	40	60

- a Show these marks on a back-to-back stem-and-leaf plot.
 b Find the median and the range for each exam.
 c Based on these results, which exam do you think was more difficult? Justify your answer.
- 21 The nationalities of players in a junior tennis tournament are shown below.

See Exercise 9-05

Australian	USA	UK	Asia	Europe
3	9	4	20	28

Find the probability that a player selected at random from this group:

- a is Australian b comes from Asia
 c is not American d is from Europe or America.



9

Statistics and probability

Probability

Millions of Australians play lotto or buy lottery tickets in the hope of winning lots of money, but the chance of winning any of these games is very very small. Lotto is a national game of chance in which 6 balls are randomly selected from a barrel of balls numbered 1 to 45. To win, you must predict the correct 6 numbers to share in a first prize of at least one million dollars. However, the chance of doing this is 1 in 8 145 060, which means that even if you played 100 Lotto games per week, it would take you, on average, 1561 years to win it.



Chapter outline

	Proficiency strands				
9-01 Probability	U	F	PS		C
9-02 Complementary events	U	F	PS	R	C
9-03 Venn diagrams	U	F	PS	R	C
9-04 Two-way tables	U	F	PS	R	C
9-05 Probability problems	U	F	PS	R	
9-06 Experimental probability	U	F	PS	R	

Wordbank

at least Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, ..., that is, '2 or more'

complementary event The 'opposite' event (for example the complementary event to selecting an Ace from a deck of cards is *not* selecting an Ace)

expected frequency The expected number of times an event will occur over repeated trials

mutually exclusive events Events or categories that have no items in common

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-way table A table that shows the number of items belonging to overlapping categories

Venn diagram A diagram that uses circles (usually overlapping) to group items into categories

In this chapter you will:

Maths clip

Using probability

MAT08SPMC00005

- assign probabilities to the outcomes of events and determine probabilities for events
- identify complementary events and use the sum of probabilities to solve problems
- describe events using language of 'at least,' exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'
- represent events in two-way tables and Venn diagrams and solve related problems
- recognise the difference between mutually and non-mutually exclusive events
- convert representations of the relationship between attributes in Venn diagrams to two-way tables
- solve probability problems involving single-step experiments such as card, dice and other games
- compare observed frequencies across experiments with expected frequencies

SkillCheck

Worksheet

StartUp assignment 9

MAT08SPWK10082

Skillsheet

The language of chance

MAT08SPSS10033

- Convert each number into a decimal.
a $\frac{17}{20}$ **b** $\frac{3}{8}$ **c** 2% **d** 30%
- List all the possible outcomes for each situation.
a tossing a coin **b** rolling a die
c the gender of a baby **d** the colour of traffic lights
e the result of a football game **f** the result of a driving test
- Convert each number into a percentage.
a 0.7 **b** 0.56 **c** $\frac{2}{3}$ **d** $\frac{6}{25}$
- Evaluate each expression.
a $1 - \frac{1}{3}$ **b** $1 - \frac{2}{5}$ **c** $1 - \frac{3}{8}$
- Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer **A**, **B**, **C**, or **D**.
A certain **B** definite **C** even chance **D** probable
- Convert each number into a simplified fraction.
a 0.28 **b** 0.02 **c** 64% **d** 80%
- Is the chance of each event greater than or less than $\frac{1}{2}$?
a You being home by 4 p.m. this afternoon
b You winning Lotto one day
c You listening to the radio today
d You learning to drive next year
e You going interstate this year
f You sending a text message on your mobile phone today

9-01 Probability

In probability (chance) situations, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

An **event** consists of one or more outcomes. For example, the event that a baby is born in a summer month consists of the outcomes {November, December, January}. An event has the symbol E , and we can calculate its probability as a fraction.

Summary

$P(E)$ means 'the probability of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

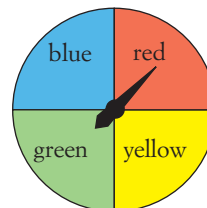
or $P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating

The **probability** of an event can be written as a fraction, decimal or percentage. For example, the probability of tossing tails on a coin can be written as $\frac{1}{2}$, 0.5 or 50%.

Example 1

- Write the sample space for this spinner.
- Is each outcome equally likely?
- Find the probability that the spinner lands on red.
- Find the probability that the spinner lands on a 'traffic light' colour.



Solution

- The sample space is {red, yellow, green, blue}.
- Each coloured region is equal in size ($\frac{1}{4}$ of the circle), so each outcome is equally likely.
- $P(\text{red}) = \frac{1}{4}$
- $P(\text{traffic light colour}) = P(\text{red or yellow or green})$
 $= \frac{3}{4}$

One chance in 4.

3 favourable outcomes out of 4.

Puzzle sheet

Greedy pig game

MAT08SPPS10023

Skillsheet

Sample space

MAT08SPSS10034

Worksheet

Basic probability

MAT08SPWK00050

Puzzle sheet

Basic probability

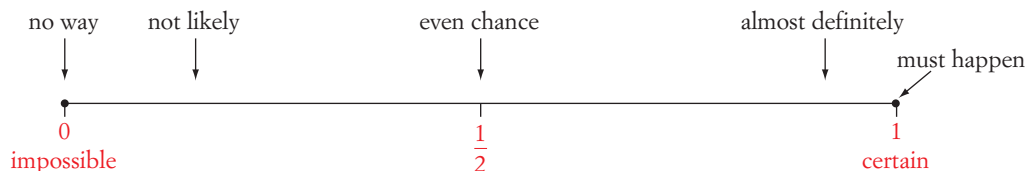
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The language of probability

Probability term	In Example 1
An experiment is a situation involving chance that leads to results called outcomes.	Spinning a spinner
A trial is one go or run of the experiment.	One spin of the spinner
An outcome is the result of an experiment.	The arrow landing on red
The sample space is the set of all possible outcomes.	{red, yellow, green, blue}
An event is one or more outcomes of an experiment.	The arrow landing on a 'traffic light' colour: red, yellow or green
In a random experiment, every possible outcome has the same chance of occurring.	All spins on this spinner are random because every colour has the same chance

The range of probability

Because probability is a proper fraction, its value must range from 0 to 1, or as a percentage, from 0% to 100%.



Example 2

A die is rolled. Find the probability that the number rolled is:

- a divisible by 3
- b a factor of 6, as a decimal
- c less than 7
- d at least 4, as a percentage

Solution

There are 6 possible outcomes when a die is rolled: {1, 2, 3, 4, 5, 6}

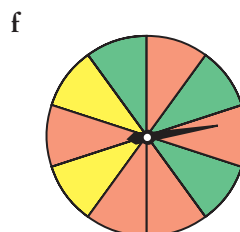
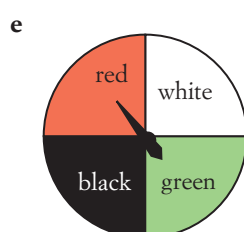
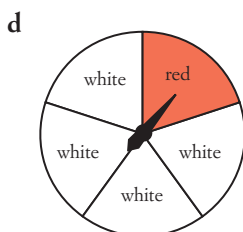
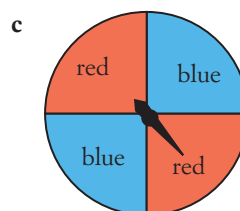
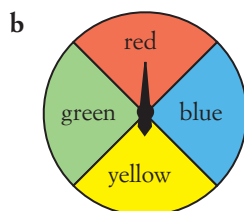
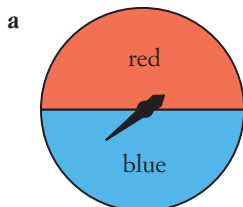
- a $P(\text{divisible by } 3) = \frac{2}{6} = \frac{1}{3}$ 2 numbers {3 and 6}.
- b $P(\text{a factor of } 6) = \frac{4}{6} = \frac{2}{3} = 0.\dot{6}$ 4 numbers {1, 2, 3, 6}.
- c $P(\text{less than } 7) = \frac{6}{6} = 1$ All outcomes are less than 7: a certain event.
- d $P(\text{at least } 4) = \frac{3}{6} = \frac{1}{2} = 50\%$ 3 numbers {4, 5, 6}.

'at least 4' means the smallest number is 4; that is, 4 or more

Exercise 9-01 Probability

See Example 1

- 1 For each spinner:
 - i write down the sample space
 - ii for one spin, calculate the probability that the spinner lands on red.



- 2 For each experiment, count the number of possible outcomes and state whether each outcome is equally likely.
 - a tossing a coin
 - b the result of a rugby match when Australia plays New Zealand
 - c the first letter of a person's name
 - d the gender of a baby
 - e the last digit of a car number plate
 - f the result of a driving test
- 3 List the outcomes in each event.
 - a rolling an odd number on a die
 - b selecting a vowel from the letters of the alphabet
 - c having a house number greater than 4 but less than 10
 - d having a birthday in a month beginning with A
 - e living in a state of Australia
 - f being in a high school grade
- 4 A money box contains four \$2 coins, three \$1 coins, two 50c coins, six 20c coins and five 10c coins. It is shaken and one coin falls out at random. Calculate each probability below.

- a $P(50c \text{ coin})$ - c $P(10c \text{ or } 20c \text{ coin}), \text{ as a percentage}$ - e $P(\text{gold coin}), \text{ as a decimal}$	- b $P(\\$1 \text{ coin}), \text{ as a decimal}$ - d $P(\text{not a } 10c \text{ coin})$ - f $P(\text{a coin under } \\$1), \text{ as a percentage}$

See Example 2

Exercise 9-01

MAT08SPWS10066

-
- A glass jar is shown, partially filled with 10 small, round balls. The balls are colored: 4 yellow, 3 red, 2 green, and 1 blue. The jar is light blue and has a white rim.

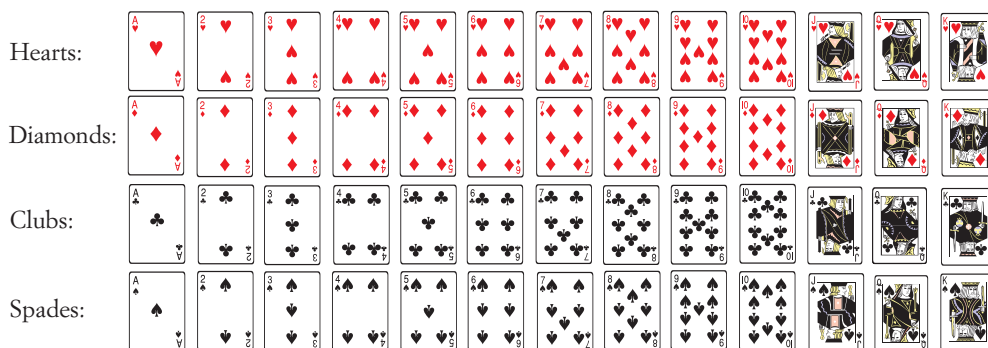
- ara
- 
- A fan of 13 playing cards, specifically the Ace through King of Diamonds, displayed in a curved arrangement. The cards are white with red and black symbols. The Ace of Diamonds is on the left, followed by the 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King of Diamonds on the right. The cards are slightly overlapping, showing the top of each card.

Outcome	5	6	7	8
Probability as a fraction	$\frac{1}{4}$			
Probability as a decimal	0.25			
Probability as a percentage	25%			

- H must happen

MAT08NASS10033

- 12 The 52 cards in a standard deck of playing cards are shown below, divided evenly into four suits: hearts, diamonds, clubs, spades.



The cards are shuffled and one is taken out at random.

- How many outcomes are in the sample space?
 - Is each outcome equally likely?
 - How many Aces are in the deck?
 - How many hearts cards are there?
 - How many red 7s are there?
 - Find the probability of selecting:
 - a red card
 - a clubs card
 - the King of spades
 - a Queen
 - a card with an even number
 - a black picture card
- 13 The weather forecaster says that the probability of a rainy day in April is 30%.
- Is this a high probability or a low probability?
 - About how many rainy days are expected in April?
- 14 Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:
- your name?
 - a girl's name?
 - someone aged 14?
 - someone with brown hair?
- 15 An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting:
- a lemonade is 50%?
 - an orange drink is 0.25?
 - a lime drink is $\frac{1}{12}$?
 - a cola can is $\frac{2}{5}$?

Investigation: Complementary events

In everyday life, we use the word 'complementary' to describe things that go together and 'complete the picture' when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that 'complementary' angles add to 90° .

In probability, **complementary events** are events that together make up all the possible outcomes.

Event	Complementary event
Tossing a tail on a coin	Tossing a head on a coin
Rolling a 6 on a die	Rolling any of the other numbers, from 1 to 5, on a die
Raining	Not raining
Being born on a Monday	Being born on a day other than Monday

- Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.
 - What is $P(\text{Tues})$, the probability of being born on a Tuesday?
 - What is $P(\text{not Tues})$, the probability of being born on a day other than Tuesday?
 - What do you notice about $P(\text{Tues}) + P(\text{not Tues})$?
- A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.
 - Find $P(\text{orange})$.
 - Find $P(\text{not orange})$.
 - What do you notice about $P(\text{orange}) + P(\text{not orange})$?
- A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.
 - Find $P(\text{boy})$.
 - Find $P(\text{girl})$.
 - What do you notice about $P(\text{boy}) + P(\text{girl})$?
- Copy and complete the following sentence.
The probability of an event _____ the probability of its complementary event must always equal _____.

Just for the record



Ancient dice



Objects similar to dice were used by people in prehistoric times to cast magic spells or predict the future. These were made of the anklebones of sheep, buffalo, or other animals. Dice have also been found in Egyptian tombs dating from 2000 BCE and Chinese excavations dating from 600 BCE. The ancient Greeks and Romans made dice similar to ours out of bone, ivory, stone and metal. Dice games were first studied by the Italian mathematician Girolamo Cardano in the 16th century, and so the study of probability was born. There are two kinds of modern dice. **Perfect dice** have sharp edges and corners and are used mostly in gambling casinos, while round-cornered dice are generally used to play social and board games.

Investigate the rules for placing the numbers on the faces of a die.

9-02 Complementary events

Puzzle sheet

Complementary events

MAT08SPPS00030

In any situation, the probabilities of all possible outcomes must add to 1.

Complementary events are events that together make up all the possible outcomes, such as when tossing a coin, a head and a tail. The **complement** of an event E , are all those outcomes that are *not* E , or that are the 'opposite' of E .

Because an event and its complement covers all possible outcomes, the sum of their probabilities must equal 1.

Summary

$$P(E) + P(\text{not } E) = 1$$

$$\text{or } P(\text{not } E) = 1 - P(E)$$

$$\text{or } P(\text{complementary event}) = 1 - P(\text{event})$$

$$\text{or } P(\text{event not occurring}) = 1 - P(\text{event occurring})$$

If the probability is written in percentage form, then $P(\text{not } E) = 100\% - P(E)$

Example 3

A pack of 20 cards contains 10 red, 6 yellow and 4 green cards. One card is drawn from the pack at random. Find the probability that this card is:

a yellow

b not yellow

c not green.

Solution

$$\mathbf{a} \quad P(\text{yellow}) = \frac{6}{20} = \frac{3}{10}$$

$$\mathbf{b} \quad P(\text{not yellow}) = 1 - P(\text{yellow})$$

$$= 1 - \frac{3}{10}$$

$$= \frac{7}{10}$$

← The complement of 'yellow' is 'not yellow', which is 'red or green'

Note that $P(\text{yellow}) + P(\text{not yellow}) = \frac{3}{10} + \frac{7}{10} = 1$, which covers all possible outcomes.

$$\mathbf{c} \quad P(\text{green}) = \frac{4}{20} = \frac{1}{5}$$

$$P(\text{not green}) = 1 - P(\text{green})$$

$$= 1 - \frac{1}{5}$$

$$= \frac{4}{5}$$

← The complement of 'green' is 'not green', which is 'red or white'

Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{5} + \frac{4}{5} = 1$, which covers all possible outcomes.

Video tutorial

Complementary events

MAT08SPVT10015

Exercise 9-02 Complementary events

- 1 Write the complement of each event below.
 - a Tossing tails on a coin
 - b Cloudy day tomorrow
 - c Selecting a white chocolate from a box containing white and brown chocolates
 - d Rolling a 6 on a die
 - e Winning a game of hockey
 - f Selecting a hearts card from a deck of cards
 - g Being born in winter
 - h Being under 15 years old
- 2 A die is rolled. What is the probability that the result is:
 - a a 3? b not a 3? c not even?
 - d not prime? e at least 2? f at most 4?
- 3 On the shelf there are 6 books: 3 Mathematics, 2 History and 1 Sport. If one book is chosen at random, what is the probability of selecting:
 - a a History book?
 - b a book that is not History?
 - c a book that is not Sport?
 - d a book that is not Science?
- 4 A car park has 450 cars, 100 motor bikes and 10 buses. One of them is selected at random.
 - a What is the probability that it is a car?
 - b What is the probability that it is not a car? Select the correct answer **A, B, C** or **D**.

A $\frac{45}{56}$
B $\frac{110}{450}$
C $\frac{10}{560}$
D $\frac{11}{56}$
- 5 What is the probability that your maths teacher was born in a month:
 - a beginning with the letter J?
 - b that does not begin with the letter J?
- 6 What is the decimal probability that a mobile phone number selected at random does not end in 0 or 1?
- 7 Tahnee buys 5 tickets in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Tahnee not winning the prize?
- 8 A jar contains 40 red, 25 blue, 50 black and 15 white jelly beans. One jelly bean is selected at random. What is the probability that it is:
 - a white? b not white? c not yellow?
 - d not blue or black? e not red? f not black or white?
- 9 Write the probability of the event that is **complementary** to each of the following events.
 - a The probability of choosing a Jack from a pack of cards is $\frac{1}{13}$.
 - b The chance of shooting a basketball hoop is 61%.
 - c The probability of winning a prize is 0.15.

See Example 3

Worked solutions

Exercise 9-02

MAT08SPWS10067

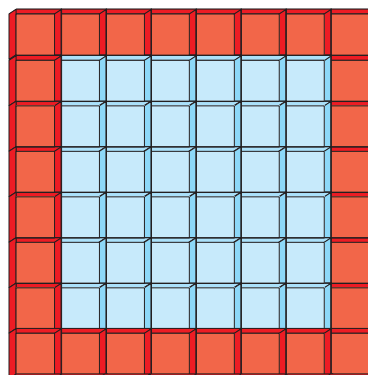
- 10 In a bag of toy cars there are only three colours: red, blue and white. If you take out a car at random, the chance of it being red is 0.4, and the chance of it being white is 0.25.
- What is the chance of selecting 'red or white'?
 - What is the chance of the car you select being blue?
 - If the bag holds 40 cars, how many of each colour would you expect to find?
 - What is the chance of the car you select being pink?
- 11 Four students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.
- Find the probability that Emily was not chosen.
 - Find the probability that the captain is a boy.
 - What is the chance that the captain is not a boy?
 - What is the chance that the captain is the teacher?
- 12 The probability that it will rain this weekend is 85%. What is the probability that it won't rain?
- 13 Which of the following is the complementary event to 'winning a race'? Select the correct answer **A**, **B**, **C** or **D**.
- | | |
|-------------------------------|---------------------------------|
| A coming last | B coming second or third |
| C not winning the race | D coming second |

Worked solutions

Exercise 9-02

MAT08SPWS10067

- 14 A game involves throwing a coin into a grid of squares. For a player to win, the coin must land in a red square. If the coin lands outside the grid or between squares, the throw is not counted and the player has another try.



For each throw, what is the probability of:

- not winning?
 - winning?
- 15 The letters of the word PROBABILITY are written on separate cards and one is selected randomly. What is the probability that a letter drawn out is:
- not P?
 - not a vowel?
 - not I?
 - not A or B?
- 16 In a family there are Mum and Dad, two daughters and a son. Each day, they take it in turns to wash the dishes after dinner. If you visit the family, what is the chance that the person washing up is:
- male?
 - not a parent?
 - not Mum?
 - not male?

Worksheet

Venn diagrams

MAT08SPWK10083

Puzzle sheet

Venn diagrams group clues

MAT08SPPS10024

Homework sheet

Probability 1

MAT08SPHS10036

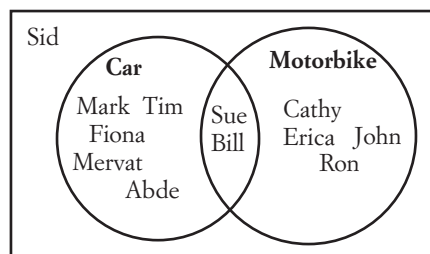
9-03 Venn diagrams

A **Venn diagram** is a diagram that uses circles (usually overlapping) to group items into categories. A rectangle represents the whole group while the circles represent categories. Items common to two or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

Example 4

A group of people were surveyed on the type of vehicle they drove, and the results are shown on the Venn diagram below.

- How many people were surveyed?
- How many people drive a car?
- How many people drive a car or ride a motorbike?
- Who drives a car and rides a motorbike?
- Who only rides a motorbike?
- Why is Sid outside of the circles?



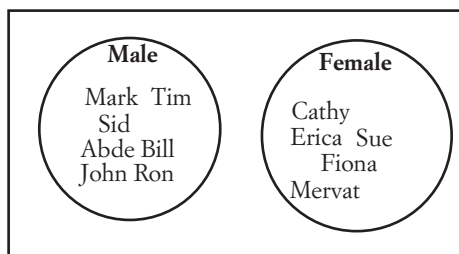
Solution

- 12 people were surveyed.
- 7 people drive a car.
- 11 people drive a car or ride a motorbike.
- Sue and Bill can drive a car and ride a bike.
- Cathy, Erica, John and Ron can only ride a bike.
- Sid does not drive a car or ride a bike.

The people within the Car circle.
All the people within the circles.
The people in the intersection of the circles.

Mutually exclusive vs overlapping events

Sometimes, two categories must be represented on a Venn diagram as two separate circles because it is not possible for them to overlap. Here is an example:



In this case, it is not possible to be male *and* female. This is an example of **mutually exclusive events**. Mutual means 'shared feature' and exclusive means 'belonging to one group only' (such as an exclusive party or an exclusive news story). However, most Venn diagrams (as in Examples 4 and 5) describe **overlapping events**, or **non-mutually exclusive events**.

'And' vs 'or'

For two categories or events A and B, the phrase '**A and B**' means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a motorbike' in Example 4 means to do both things.

If A and B are **overlapping**, the phrase '**A or B**' means to have A or B or both. For example, 'to drive a car' **or** 'to ride a motorbike' in Example 4 means to drive a car only, or to ride a motorbike only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.

If A and B are **mutually exclusive**, the phrase '**A or B**' means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.

Example 5

30 students in a class were surveyed on how they relaxed after school. Here are the results:

- 18 play on their computer
- 15 play sport
- 6 play sports and play on their computer
 - a Show this information on a Venn diagram.
 - b If one student is selected at random from this class, find the probability that the student:
 - i plays sport but not on their computer
 - ii plays sport or on their computer but not both

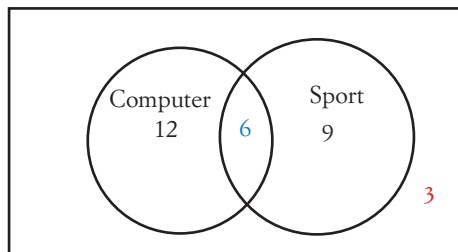


Solution

- a When drawing Venn diagrams always begin with the intersection.
 - 6 students belong to both groups
 - 18 play on the computer but 6 have already been counted, so $18 - 6 = 12$ play on the computer only
 - 15 play sport but 6 have already been counted, so $15 - 6 = 9$ play sport only
 - $12 + 6 + 9 = 27$ but there are 30 students in the class, which means that 3 students do not belong to either group

b i $P(\text{plays sport but not computer}) = \frac{9}{30} = \frac{3}{10}$

ii $P(\text{plays sport or computer but not both}) = \frac{12 + 9}{30} = \frac{21}{30} = \frac{7}{10}$



Video tutorial

Venn diagrams

MAT08SPVT10016

Animated example

Venn diagrams 1

MAT08SPAE00014

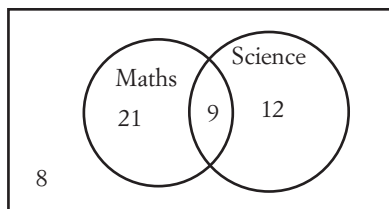
Animated example

Venn diagrams 2

MAT08SPAE00015

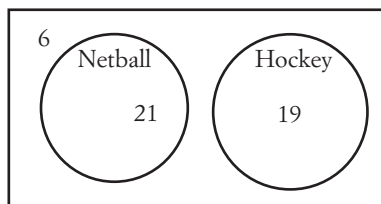
Exercise 9-03 Venn diagrams

- See Example 4** 1 A group of students were asked whether they liked Maths or Science at school. The Venn diagram shows the results of this survey.



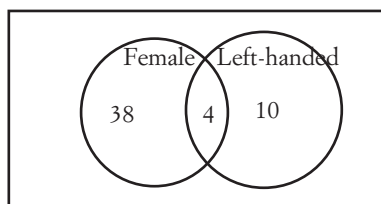
How many students:

a were surveyed?	b liked Maths but not Science?
c liked Maths or Science?	d liked Maths or Science but not both?
e did not like either subject?	f liked Maths and Science?
- 2 The Venn diagram below shows the sports played by a number of female students.



- Are netball and hockey mutually exclusive or not?
- How many students play neither netball nor hockey?
- How many students play netball or hockey?
- Find the total number of students surveyed.
- How many students play netball and hockey?

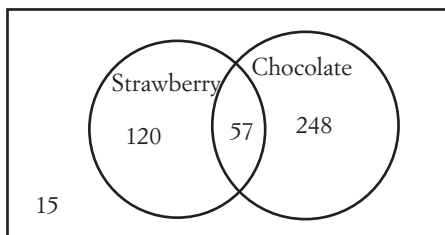
- See Example 5** 3 This Venn diagram compares the categories Female and Left-handed.



- Is Female and Left-handed mutually exclusive or not?
- Find the total number of people represented in the diagram.
- How many left-handed females are there?
- What is the decimal probability (correct to 3 decimal places) that a person randomly chosen from this group is:

i female?	ii female but not left-handed?
iii female or left-handed?	iv a left-handed male?
- Describe the type of person that would be represented outside the two circles on the Venn diagram.

- 4 A survey was carried out by an ice-cream shop to decide whether chocolate or strawberry was the more popular flavour.



- How many people were surveyed?
 - How many people liked strawberry or chocolate but not both?
 - How many people liked neither strawberry nor chocolate?
 - If a person is randomly chosen from the survey, what is the probability that the person likes both chocolate and strawberry? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{57}{440}$
B $\frac{368}{440}$
C $\frac{425}{440}$
D $\frac{57}{425}$
- 5 The Tourism Council surveyed 130 people to find whether they preferred New South Wales or Queensland as a holiday destination.
- | | |
|--------------------|----|
| NSW | 44 |
| Queensland | 66 |
| NSW and Queensland | 20 |
- Construct a Venn diagram to represent the results.
 - How many people prefer NSW or Queensland?
 - How many people prefer NSW or Queensland but not both?
 - What is the probability that a person prefers NSW exclusively?
 - What is the probability that a person does not prefer NSW or Queensland?
- 6 a Draw a Venn diagram representing the eye colours of this group of students.
- | | |
|-----------------------------|----|
| Blue eyes | 32 |
| Brown eyes | 38 |
| Neither blue nor brown eyes | 10 |
- Are these groups mutually exclusive or not? Why?
 - How many students are in the group?
 - How many students have blue or brown eyes?
 - How many students have blue and brown eyes?
 - What is the percentage probability that a student chosen at random from this group does not have brown eyes?

Worked solutions

Exercise 9-03

MAT08SPWS10068

- 7 A shopkeeper surveyed the first 30 customers to see what they bought.
- 20 bought milk 13 bought bread 1 bought neither milk nor bread
- a Show this information on a Venn diagram.
- b How many customers bought milk and bread?
- c How many customers bought milk only?
- d What is the probability that a customer randomly chosen:
- i bought bread only? ii did not buy milk?
- iii bought milk or bread? iii bought milk or bread but not both?

Worksheet

Two-way tables

MAT08SPWK10084

Worksheet

Two-way probability tables

MAT08SPWK00051

Puzzle sheet

Two-way probability tables

MAT08SPPS00029

9-04 Two-way tables

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot be represented by Venn diagrams easily.

Example 6

Fifty students were surveyed on whether they liked dogs or cats more as pets. The results were sorted into a two-way table.

- a How many students do not like dogs or cats?
- b How many students like cats?
- c How many boys like cats?
- d How many students:
- i like cats or are boys?
- ii like cats or are boys but not both?

	Preferred pet	
	Dog	Cat
Boys	12	4
Girls	11	15

- e What is the probability that a student selected at random from the survey likes dogs?

Solution

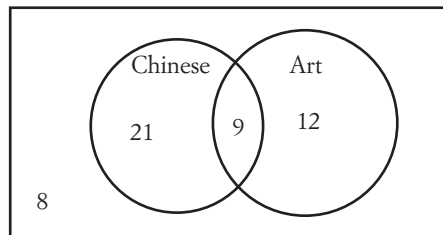
- a $12 + 4 + 11 + 15 = 42$ Adding the values in the table.
Students who do not like dogs or cats = $50 - 42$ 50 students in survey.
 $= 8$
- b Students who like cats = $4 + 15$
 $= 19$
- c Boys who like cats = 4
- d i Students who like cats or who are boys = $12 + 4 + 15$
 $= 31$
ii Students who like cats or who are boys but not both = $12 + 15$
 $= 27$

e Students who like dogs = $12 + 11$
= 23

Probability of selecting a student who likes dogs = $\frac{23}{50}$

Example 7

A group of 50 Year 9 students were grouped in a Venn diagram according to whether they took Chinese or Art as an elective subject.



Represent this information on the two-way table below.

	Chinese	Not Chinese	
Art			
Not Art			

Solution

From the Venn diagram:

- 9 students take Chinese and Art
- 21 students take Chinese but not Art
- 12 students take Art but not Chinese
- 8 students do not take Chinese or Art

	Chinese	Not Chinese	
Art	9	12	21
Not Art	21	8	29
	30	20	50

Exercise 9-04 Two-way tables

See Example 6

- 1 A primary school class was surveyed on whether its students could swim. The results are shown below.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

- How many students are in the class?
 - How many students are boys or cannot swim?
 - How many students are boys and cannot swim?
 - What is the probability that a student randomly selected from this class is a girl?
 - What is the probability that a student selected at random is:
 - a non-swimmer?
 - a girl who can swim?
- 2 The players of a soccer club were divided into groups according to their age and weight.

	Heavy	Light
Junior	64	96
Senior	144	32

- How many players does the club have?
 - How many players are juniors or light?
 - How many players are juniors or light but not both?
 - What is the probability that a player selected at random is:
 - a senior?
 - a junior and heavy?
 - is a senior or light?
- 3 This incomplete table describes the audience watching a movie at a cinema.

	Under 18	Over 18	
Female		142	198
Male	45		
			344

- Copy and complete the table.
- How many males were in the audience?
- How many under 18 females were there?
- If a person is selected at random from the audience, what is the probability that the person:
 - is male and over 18?
 - is male or over 18?
 - is male or over 18 but not both?
 - is over 18?

- 4 A group of children were asked whether they liked carrots. The table shows the results.

	Likes carrots	Dislikes carrots	
Boys	75		200
Girls		40	
	145		

- a Copy and complete the table.
b How many children dislike carrots?
c What is the probability that a child randomly selected is:
i a girl?
ii a boy or dislikes carrots?
iii a boy and dislikes carrots?
d What is the chance that a child randomly selected is a girl and likes carrots? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{29}{62}$

B $\frac{11}{31}$

C 1

D $\frac{7}{31}$

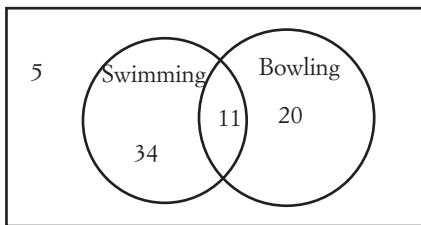


Worked solutions

Exercise 9-04

MAT08SPWS10069

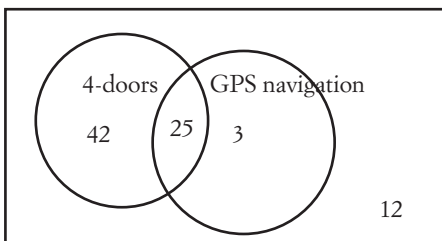
- 5 The Venn diagram below shows the number of students who participate in swimming or bowling regularly. Copy and complete the two-way table for this data.



	Swimming	Not swimming	
Bowling			
Not bowling			

See Example 7

- 6 The Venn diagram compares two features of the cars on display at Rusty Motors.



- a Copy and complete the two-way table.

	GPS navigation	No GPS navigation	
4-doors			
Not 4-doors			

- b A car is chosen at random from the car yard. What is the probability that it has:
i GPS navigation?
ii 4 doors?
iii 4 doors and GPS navigation?
iv no GPS navigation?

- 7 A group of university students were tested to see if they needed glasses. The table shows the results.

	Needs glasses	Does not need glasses	
Male	22		82
Female		85	98
		145	

- a Copy and complete the table.
- b If a student is chosen randomly from this group, what is the probability that the student:
- i is female?
 - ii needs glasses?
 - iii is female and needs glasses?
 - iv is male or does not need glasses?
 - v is female or needs glasses?
 - vi is male or does not need glasses but not both?

Technology Rolling a die

In this activity, a graphics calculator is used to simulate the rolling of a die.

Note: This activity uses a Casio graphics calculator.

- 1 Copy the table shown below, for the possible outcomes.

Outcome	Number of times rolled
1	
2	
3	
4	
5	
6	

- 2 Using the **RUN** mode, enter the following formula: **Int(Ran# × 6 + 1)** as shown below.

OPTN NUM Int EXIT (PROB Ran# × 6 + 1)
 (F6)(F4)(F2) (F3)(F4)

- 3 Repeat the simulation 20 times and record the results in your table.
- 4 Are certain numbers more likely to be rolled than others? (For example is a 2 more likely to be rolled than a 5?) Do your results reflect this?
- 5 Compare your results with the simulated results of other members in your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

Mental skills 9

Maths without calculators

Multiplying or dividing by a multiple of 10

1 Study each example.

a $4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800$

b $5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300$

c $12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480$

d $3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96$ (by estimation, $3 \times 30 = 90 \approx 96$)

e $4.5 \times 50 = 4.5 \times 5 \times 10 = 22.5 \times 10 = 225$ (by estimation, $5 \times 50 = 250 \approx 225$)

f $9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880$ (by estimation, $9 \times 200 = 1800 \approx 1880$)

2 Now evaluate each product.

a 8×2000

b 3×70

c 11×900

d 2×300

e 4×4000

f 5×80

g 7×70

h 1.3×40

i 2.5×600

j 6.8×200

k 3.9×50

l 4.4×4000

3 Study each example.

a $8000 \div 400 = 80\cancel{00} \div 10\cancel{0} \div 4 = 80 \div 4 = 20$

b $200 \div 50 = 20\cancel{0} \div 10 \div 5 = 20 \div 5 = 4$

c $6000 \div 20 = 600\cancel{0} \div 10 \div 2 = 600 \div 2 = 300$

d $282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4$

e $3520 \div 40 = 352\cancel{0} \div 10 \div 4 = 352 \div 4 = 88$

f $8940 \div 200 = 8940 \div 10\cancel{0} \div 2 = 89.4 \div 2 = 44.7$

4 Now evaluate each quotient.

a $560 \div 70$

b $2500 \div 50$

c $3200 \div 400$

d $440 \div 20$

e $160 \div 40$

f $1500 \div 30$

g $450 \div 50$

h $744 \div 80$

i $2550 \div 300$

j $846 \div 200$

k $576 \div 60$

l $2040 \div 50$

9-05 Probability problems

The following exercise involves all the probability ideas that have been covered so far.

Exercise 9-05 Probability problems

1 There were 28 students at the SRC conference: 17 were born in Australia, 3 in Italy, 4 in Vietnam, 1 in Japan and 3 in Sweden. One student was chosen at random. Find the probability that the student was:

a born in Australia

b born in Europe

c not born in Japan.

Puzzle sheet

Matching probabilities

MAT08SPPS10025

Worksheet

Probability review

MAT08SPWK10085

Quiz

Probability

MAT08SPQZ00008

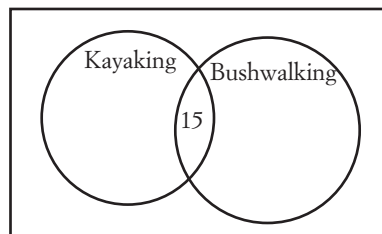
- 2 The letters of the word SUCCESS are written on cards. A card is selected at random and the letter noted.
 - a List the sample space.
 - b Is each letter equally likely to be selected? Explain.
 - c Find the chance that the letter chosen:
 - i is an S
 - ii is a vowel
 - iii is not a C
 - iv is also a letter of the word FAIL.
- 3 Alex selects one sock at random from a bag containing two black, two blue and two red socks.
 - a List the sample space.
 - b Find the probability that Alex selects:
 - i a blue sock
 - ii a black sock
 - iii a pink sock
 - c What is the complementary event to choosing a red sock? What is the probability of this event?
- 4 A spinner is evenly divided into 5 sections numbered 1, 2, 3, 4 and 5. For one spin, find the probability, as a percentage, that it lands on:
 - i 2
 - ii an even number
 - iii a number less than 5
 - iv a number at most 5
- 5 Sophia bought four tickets in a lottery in which there were 100 000 tickets. What is the probability that she won first prize? Select the correct answer A, B, C or D.

A 0.000 0025 B 0.000 25 C 0.04 D 0.000 04
- 6 One ball is selected at random from a barrel of balls numbered 1 to 100. Find the probability that the number shown on the ball is:
 - a 12
 - b greater than 40
 - c even
 - d at most 85
- 7 Stathis flipped a coin 7 times and a tail showed each time. What is the chance of a tail showing on the next toss?
- 8 A computer selected a random number from 1 to 15 inclusive. Find the probability that the number is:
 - a odd
 - b not prime
 - c a multiple of 3 or 5
 - d a factor of 12
- 9 In a tennis tournament there are 60 players. Of these, 34 are from NSW, 20 are from Victoria and 6 are from Queensland. Find the probability that a player selected at random from the tournament:
 - a is from NSW
 - b is from Queensland
 - c is not from Victoria
 - d is from Queensland or Victoria.
- 10 40 students at a school camp can select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. There weren't any students who did not choose an activity.
 - a Copy and complete the Venn diagram representing these students.
 - b What is the probability that a student, chosen at random:
 - i chose both activities?
 - ii did not choose bushwalking?

Worked solutions

Exercise 9-05

MAT08SPWS10070



- 11 The probability that a carton of eggs contains any broken eggs is 0.1. Find the probability that a carton contains no broken eggs.
- 12 A sample of students from a sports high school were surveyed on whether they participated in hockey or judo. The results are shown in this two-way table.

	Judo	Not Judo	
Hockey	128		360
Not hockey	100	40	

- a Copy and complete the table.
- b What is the probability that a student chosen randomly:
- i does judo?
 - ii doesn't do hockey?
 - iii does judo but not hockey?
 - iv does hockey and judo?
 - v does hockey or judo?
 - vi does hockey or judo but not both?

Worked solutions

Exercise 9-05

MAT08SPWS10070

9-06 Experimental probability

A probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called the **theoretical probability** (or **calculated probability**).

We can also determine the probability of an event based on the results of an experiment or trial that has been repeated many times, such as the safety testing of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** (or **relative frequency**).

Worksheet

Coins probability

MAT08SPWK10086

Worksheet

Dice probability

MAT08SPWK10087

Worksheet

Spinning chance

MAT08SPWK10088

Homework sheet

Probability 2

MAT08SPHS10037

Summary

Experimental probability has these formulas:

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

$$\text{Expected frequency} = \text{theoretical probability} \times \text{number of trials.}$$

Example 8

Video tutorial

Experimental probability

MAT08SPVT10017

Declan rolled a die 60 times and recorded the results in a table.

Outcome	1	2	3	4	5	6
Frequency	10	11	13	7	8	11

- What is the theoretical probability of rolling a 5 or 6 on a die?
- For 60 rolls, what is the expected number of times of getting a 5 or 6? How does this compare with the actual number of times?
- What is the experimental probability of rolling a 5 or 6?

Solution

a $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

b Expected number of 5s or 6s $= \frac{1}{3} \times 60$ probability $\times$ number of trials
 $= 20$

From the table, the observed number of 5s and 6s
 $= 8 + 11 = 19$, which is close to the expected number, 20.

c Experimental $P(5 \text{ or } 6) = \frac{8 + 11}{60} = \frac{19}{60}$

Exercise 9-06 Experimental probability

See Example 8

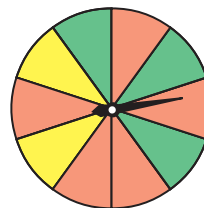
- Erica rolled a biased die 60 times and the number 2 came up 15 times.
 - What is the experimental probability of rolling a 2?
 - If the same die was rolled 500 times, what is the expected frequency of rolling a 2?
- Copy this table.

Outcome	Tally	Frequency
Head		
Tail		
	Total	

- Flip a coin 50 times and record the results in the table.
- Find, as a decimal, the experimental probability of flipping:
 - a head
 - a tail
- Find, as a decimal, the theoretical probability of flipping:
 - a head
 - a tail
- How do the experimental probabilities compare with the theoretical probabilities?

- 3 Josie spun this spinner 80 times and found the following results:

Outcome	Red	Green	Yellow
Frequency	44	25	11



- What is the theoretical probability of spinning red?
 - For 80 spins, what is the expected number of times of spinning red? How does this compare with the actual number of times?
 - What is the experimental probability of spinning red?
 - What is the theoretical probability of spinning yellow?
 - Find the expected frequency of spinning yellow over 80 spins and compare this with the observed frequency.
 - What is the experimental probability of spinning yellow?
- 4 The matches in a sample of matchboxes were counted. The number of matches in each box are recorded below.

Number of matches	48	49	50	51	52	53
Number of matchboxes	3	6	20	16	4	1

- How many matchboxes were tested?
 - What is the experimental probability of finding 51 matches in a box?
 - What is the experimental probability of finding fewer than 50 matches in a box?
 - In 1000 matchboxes, how many matchboxes would you expect to contain:
 - exactly 50 matches?
 - at least 50 matches?
- 5 a Copy this table.

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		
Total		

- Roll a die 72 times and record the results in the table.
- Find the theoretical probability of rolling:
 - 4
 - a number greater than 2
 - an even number
- When rolling a die 72 times, what is the expected frequency of rolling:
 - 4?
 - a number greater than 2?
 - an even number?
- How do your expected frequencies compare with the actual frequencies from the table?
- Find the experimental probability of rolling:
 - 4
 - a number greater than 2
 - an even number
- If a die was rolled 1000 times, how many times should 4 come up:
 - based on the theoretical probability?
 - based on the experimental probability?

- 6 Tamara tossed a coin many times and got 135 heads and 115 tails. Calculate, as a percentage, the experimental probability of tails with this coin.
- 7 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

- a Is each outcome equally likely?
- b Do you think this die is biased (unfair)? Give a reason for your answer.
- c Write, as a percentage, the experimental probability of rolling a 1 on this die.
- d If this die was rolled 100 times, how many times would you expect 3 to come up?

- 8 A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

- a Find, as a decimal, the experimental probability of rolling a sum:
- i of 11
 - ii of 5 or 6
 - iii that is odd
 - iv 6 or more
 - v at most 6
 - vi that is a composite number
- b Which sum(s):
- i was most likely?
 - ii had a probability of $\frac{3}{25}$?
 - iii was least likely?
 - iv had a probability of 16%?
 - v was second-most likely?
 - vi had a probability of $\frac{1}{10}$?

Worked solutions

Exercise 9-06

MAT08SPWS10071

Technology worksheet

Excel: Simulating a spinner

MAT08SPCT00010

Technology

Excel: Spinner simulator

MAT08SPCT00024

Technology worksheet

Random number generator

MAT08SPCT00011

Technology Tossing a coin

In this activity, a spreadsheet is used to simulate the tossing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails). We will use the command **RAND** to generate these random numbers.

- In cell A1 enter the label 'Tossing a coin'.
- In cell A2, enter the formula **=INT(RAND()*2+1)**. Press Enter and either 1 or 2 should appear randomly in the cell.
- Select cell A2 and **Fill Down** to cell A11 to generate random numbers (1 or 2), to simulate the tossing of a coin 10 times.
- Your results will probably be different from those of other students in the class. Compare.

- 5 Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 ‘tosses’).

Trial	Number of heads	Number of tails
1–10		
11–20		
21–30		
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		
Total		

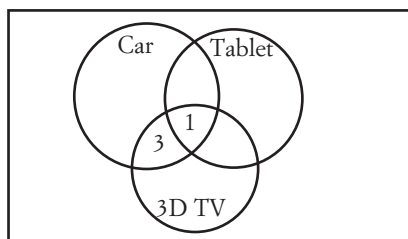
- 6 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a ‘double arrow’) and clicking.
- 7 Enter the results for Trials 11–20 in your table.
- 8 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.
- 9 a For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?
- b Compare this with the actual frequency.
- 10 Calculate, as a decimal, the theoretical and experimental probabilities of tossing heads.
- 11 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.
- 12 Combine the results of students in your class to calculate the experimental probability of tossing heads, as a decimal. How does this compare with the theoretical probability?

Power plus

- 1 A market research survey of 50 people shows that 42 people own a car, 20 own a digital tablet and 8 own a 3D TV. Of these:

- 4 own a car and a 3D TV
- 3 own a 3D TV and a tablet
- 14 own a car and a tablet
- 1 person has a car, a tablet and a 3D TV

Copy and complete the following Venn diagram to represent the survey results.



- 2 Two dice are rolled and the numbers are **multiplied** together to arrive at a score.

- a Copy and complete this table to show all possible products.

		First die						
		×	1	2	3	4	5	6
Second die	1	1	2	3				
	2	2	4					
	3	3	6					
	4			12				
	5							
	6							

- b How many different products are possible?
- c Why isn't each product equally likely?
- d Which product is most likely?
- e Which product is least likely?
- f What is the probability of a product of:
- i 6? ii 20? iii at least 20?
- g Which product has a probability of $\frac{1}{12}$?
- 3 Three friends decide to play a game with two dice. Danielle wins if the sum of the numbers is 3 or 5, and Vanessa wins if the total is 6 or 8. Any other total means that Karla wins. Is the game fair? Explain your answer.
- 4 A committee of 2 people is to be selected from two boys (Paul and Sumeet) and two girls (Tash and Nadine).
- a List all the committees you can form.
- b If you were to choose a committee at random, what is the probability that it would include Tash?

Language of maths

at least	experimental probability	observed frequency	sample space
certain	improbable	outcome	theoretical probability
complementary	impossible	overlapping	trial
event	inclusive	probable	two-way table
exclusive	likely	random	unlikely
expected frequency	mutually exclusive	relative frequency	Venn diagram

Puzzle sheet

Probability crossword

MAT08SPPS10026

- 1 What is the meaning of **sample space**?
- 2 What is the **complementary event** to winning a soccer match?
- 3 What does it mean when a person is selected '**at random**' for a survey?
- 4 Draw a Venn diagram with categories 'Year 7 students' and 'Year 8 students'. Are these categories **mutually exclusive** or not?
- 5 What term means the number of times an event should occur over repeated trials?
- 6 Explain what 'is right-handed or drives a car' means exactly, given that they are **overlapping** events.

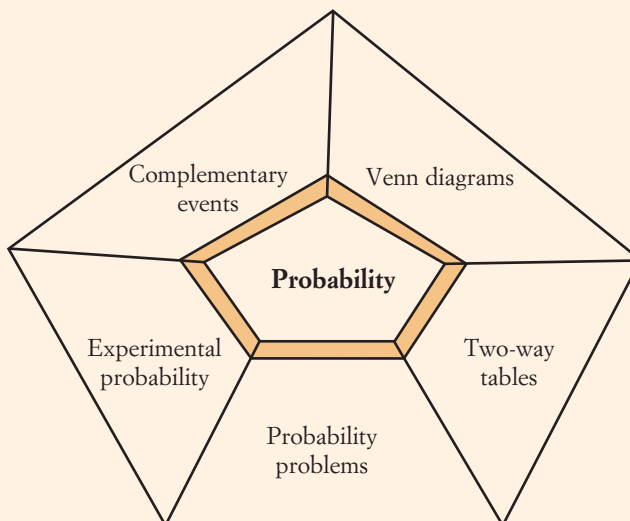
Topic overview

- Write in your own words what you have learnt in this chapter.
- Write which parts of the chapter were new to you.
- Copy and complete:

The things I understand about probability are...

The things I am still not confident in doing in this chapter are...

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet

Mind map: Probability

MAT08SPWK10089

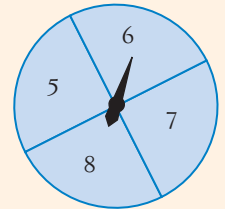
Chapter 9 revision

See Exercise 9-01

- 1 A die is rolled. Find the probability that the number that comes up is:
a 1 **b** more than 2 **c** odd **d** composite

See Exercise 9-01

- 2 **a** Write the sample space for this spinner.
b Find, as a percentage, the probability that the number spun is:
i 5 **ii** at least 5 **iii** 5 or less **iv** less than 5



See Exercise 9-02

- 3 Write the complement of each event and its probability.
a Choosing an Ace from a standard deck of cards
b Rolling a factor of 6 on a die
c Buying the winning ticket out of 1000 tickets sold

See Exercise 9-02

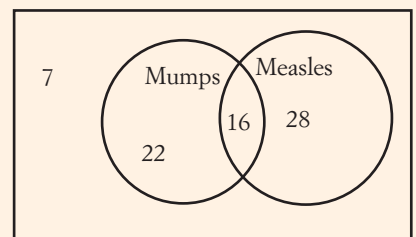
- 4 A golfer has a probability of 74% of sinking a putt. What is the probability that he will miss a putt?

See Exercise 9-02

- 5 A truck carries 240 boxes of lemonade, 305 boxes of soap, 335 boxes of paper and 120 boxes of pens. The driver chooses one box at random. What is the probability that it is:
a a box of lemonade? **b** a box of pens or soap?
c a box of paper? **d** not a box of paper?

See Exercise 9-03

- 6 The medical history of a group of children is shown in the following Venn diagram.
a How many children are in the group?
b How many children have had the mumps and the measles?
c How many children have had the mumps or the measles but not both?
d What is the probability that one child selected at random from this group has had:
i the measles? **ii** the mumps or the measles?
iii neither the mumps nor the measles?



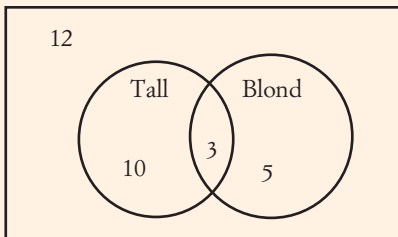
See Exercise 9-04

- 7 A group of people were surveyed on whether they watched cricket or soccer on TV.
 The results were sorted into a two-way table.
a How many people were surveyed?
b How many people watch soccer?
c How many people watch soccer or cricket?
d What is the probability that a person selected at random from the survey:
i watches neither soccer nor cricket?
ii watches soccer or cricket but not both?
iii does not watch cricket?
iv does not watch soccer?

TV sport		
	Cricket	Not cricket
Soccer	28	15
Not soccer	20	7

- 8 Represent the data in the Venn diagram on the two-way table.

See Exercise 9-04



	Tall	Not tall	
Blond			
Not blond			

- 9 An eight-sided die has 2 red, 3 white, 1 blue and 2 yellow faces. If the die is rolled, find the decimal probability that the face that comes up is:
- a blue b yellow c not red d white or yellow
- 10 A die was rolled 80 times and the numbers 1 or 6 came up 25 times.
- a What is the experimental probability of rolling 1 or 6, as a percentage?
- b What is the theoretical probability of rolling 1 or 6, as a percentage?
- c Calculate the expected frequency of rolling 1 or 6 over 80 trials.

See Exercise 9-05

See Exercise 9-06

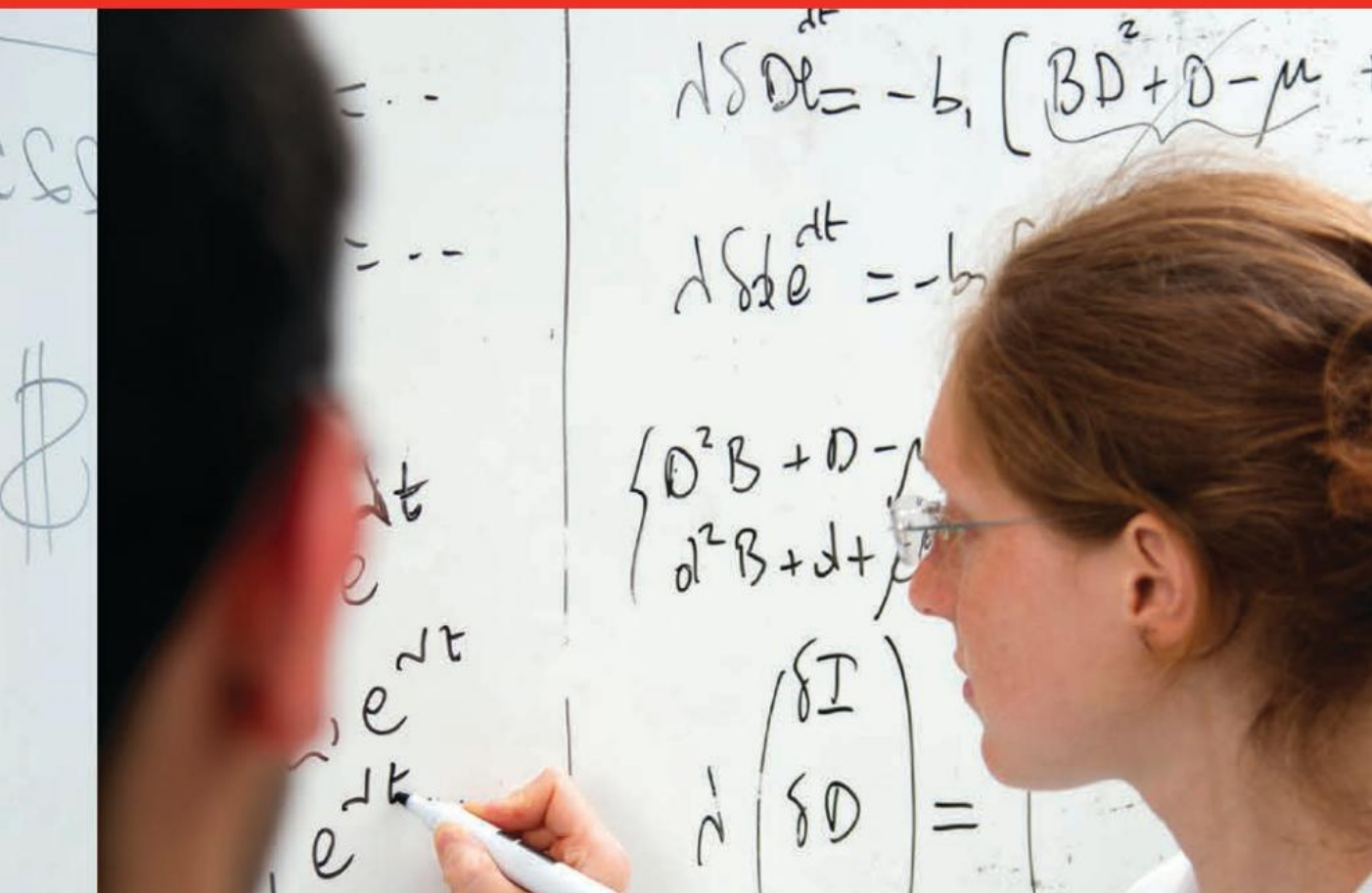


10

Number and algebra

Equations

One of the most common ways to solve complex practical problems is to use equations. By relating the various aspects of a problem using variables, we can often find the best solution, or set of solutions. Equations are used in numerous professions and trades, from accountancy to zoology. In this chapter, we will learn the techniques involved in solving equations.



Chapter outline

	Proficiency strands				
10-01 One-step equations	U	F		R	
10-02 Two-step equations	U	F		R	
10-03 Equations with variables on both sides	U	F		R	
10-04 Equations with brackets	U	F		R	
10-05 Simple quadratic equations $x^2 = c$	U	F		R	C
10-06 Equation problems	U	F	PS	R	C

Wordbank

backtracking method A method of solving equations by 'undoing', or performing inverse (opposite) operations in reverse order

balancing method A method of solving equations by using the same operations on both sides of the equation

equation A mathematical statement that two quantities are equal, involving algebraic expressions and an equals sign (=)

inverse operation An opposite used in solving an equation, for example, the inverse operation of multiplying is adding

solve (an equation) To find the value of an unknown variable in an equation

solution The answer to an equation or problem, the correct value(s) of the variable that makes an equation true

substitute To replace a variable with a number

variable A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

In this chapter you will:

Video tutorial

Solving equations

MAT08NAVT00012

- solve linear equations using algebraic techniques
- verify solutions by substitution
- solve real-life problems by using pronumerals to represent unknowns
- solve simple quadratic equations of the form $x^2 = c$

SkillCheck

Worksheet

StartUp assignment 10

MAT08NAWK10090

- 1 Find the number represented by $\square$ each time.

a $\square + 3 = 10$

b $4 \times \square = 28$

c $12 - \square = 4$

d $\square \div 3 = 9$

e $9 + \square = 15$

f $\square - 7 = 16$

g $\square \times 6 = 30$

h $24 \div \square = 8$

i $10 + \square = 20$

j $\square - 5 = 0$

k $\frac{1}{2} \times \square = 4$

l $\square \div 5 = 4$

- 2 Evaluate each expression.

a $5 - 8$

b $-8 + 3$

c $3 \times (-4)$

d $-6 \div 3$

e $-2 + 5$

f $-2 - 9$

g $-4 \times (-9)$

h $-12 \div (-4)$

i $-3 - 7$

j $80 \div (-10)$

k $6 - 15$

l -8×9

- 3 If $d = 7$, evaluate:

a $2d + 3$

b $3d - 4$

c $\frac{d+1}{2}$

d $18 - 2d$

- 4 If $x = -3$, evaluate:

a $5x + 9$

b $4 - x$

c $\frac{x-3}{-1}$

d $2x - 8$

- 5 Simplify each expression.

a $3n - 2n$

b $8x + 3x$

c $5r - r$

d $6t + 2t$

- 6 Expand each expression.

a $2(m + 3)$

b $3(x - 2)$

c $4(k + 7)$

d $5(d - 1)$

e $6(a - 3)$

f $4(b + 8)$

g $5(2q - 6)$

h $9(5j + 1)$

- 7 What is the opposite operation to:

a adding?

b dividing?

c multiplying?

d subtracting?

Worksheet

Equations 1

MAT08NAWK10091

Puzzle sheet

Equations dominoes

MAT08NAPS10027

10-01 One-step equations

An **equation** is a statement involving a variable (such as x), numbers and an equals ($=$) sign, for example, $2x - 5 = 11$. In Year 7 we learnt how to **solve** equations, that is, to find the value of the variable that makes the equation true. This value is called the **solution** to the equation.

There are two algebraic methods for solving equations.

The **balancing method** involves representing both sides of an equation as balance scales, and solving the equation by performing the same operation on both sides to keep it 'balanced'.

The **backtracking** method involves using a flow chart to show what operations are performed on the variable (say, x) to create the equation, then using a reverse flow chart to undo each operation by performing the inverse (opposite) operation in reverse order.

Puzzle sheet

One-step equations

MAT08NAPS00046

Worksheet

One-step equations

MAT08NAWK00073

Summary

To solve an equation, aim to have the variable (such as x) on one side of the equation and a number on the other side, in the form:

$$x = \text{---}$$

Check your solution by substituting it back into the equation.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

Example 1

Solve each equation.

a $m + 4 = 12$

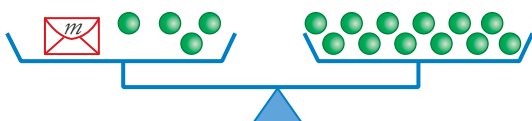
b $3y = 18$

c $k - 3 = 5$

d $\frac{t}{8} = 6$

Solution

a Method 1: The balancing method



$$m + 4 = 12$$

$$m + 4 - 4 = 12 - 4$$

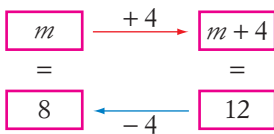
Subtracting 4 from both sides.

Check: $8 + 4 = 12$.



$$m = 8$$

Method 2: The backtracking method



$$m + 4 = 12$$

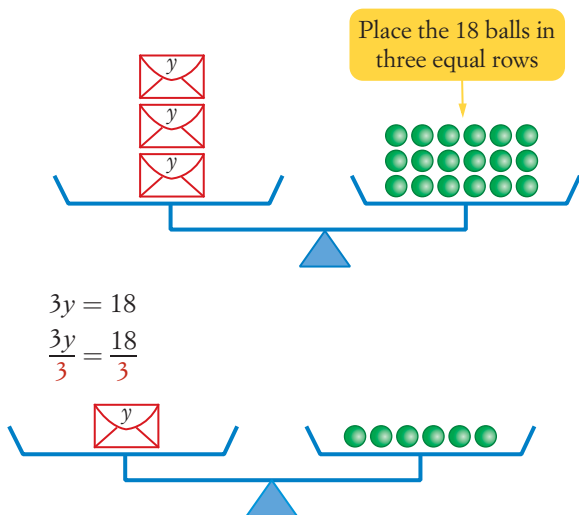
$$m = 12 - 4$$

$$m = 8$$

Undo '+ 4' by subtracting 4.

Check: $8 + 4 = 12$.

b Method 1: The balancing method



$$3y = 18$$

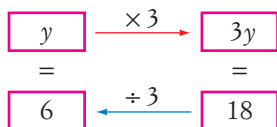
$$\frac{3y}{3} = \frac{18}{3}$$

Dividing both sides by 3.

$$\text{Check: } 3 \times 6 = 18$$

$$y = 6$$

Method 2: The backtracking method



$$3y = 18$$

$$y = \frac{18}{3}$$

$$y = 6$$

Undo ' $\times 3$ ' in $3y$ by dividing by 3.

$$\text{Check: } 3 \times 6 = 18$$

These are all **one-step** equations because they require only one step or operation to solve.

c Method 1: The balancing method

$$k - 3 = 5$$

$$k - 3 + 3 = 5 + 3$$

$$k = 8$$

Adding 3 to both sides.

$$\text{Check: } 8 - 3 = 5$$

Method 2: The backtracking method

$$k - 3 = 5$$

$$k = 5 + 3$$

$$k = 8$$

Undo ' $- 3$ ' by adding 3.

$$\text{Check: } 8 - 3 = 5$$

d Method 1: The balancing method

$$\frac{t}{8} = 6$$

$$\frac{t}{8} \times 8 = 6 \times 8$$

$$t = 48$$

Multiplying both sides by 8.

$$\text{Check: } \frac{48}{8} = 6$$

Method 2: The backtracking method

$$\frac{t}{8} = 6$$

$$t = 6 \times 8$$

$$t = 48$$

Undo ' $\div 8$ ' by multiplying by 8.

$$\text{Check: } \frac{48}{8} = 6$$

Exercise 10-01 One-step equations

1 Solve each equation, showing the working.

a $x + 5 = 12$

b $y + 13 = 36$

c $m + 9 = 21$

d $k + 27 = 54$

e $k + 6 = 2$

f $s + 3 = 1$

g $n + 9 = -4$

h $p + 17 = -3$

i $m + 8 = 0$

j $g + 4.5 = 20$

k $a + 1\frac{3}{4} = 12$

l $k + 2.7 = -5$

2 Solve each equation, showing the working.

a $t - 11 = 40$

b $w - 7 = 4$

c $a - 5 = 2$

d $j - 23 = 51$

e $q - 12 = 17$

f $f - 42 = 68$

g $x - 9 = 24$

h $g - 10 = 1$

i $j - 3 = -5$

j $w - 9 = -2$

k $m - 12 = -27$

l $d - 1 = -1$

m $y - 17 = 3.9$

n $n - 2.1 = 5.9$

o $b - \frac{4}{5} = \frac{1}{4}$

p $s - 2\frac{1}{3} = 4\frac{1}{2}$

3 Solve each equation, showing the working.

a $6x = 24$

b $4g = 16$

c $10y = 90$

d $5b = 45$

e $12d = 108$

f $9c = 81$

g $23p = 115$

h $2m = 26$

i $5x = -25$

j $7q = -42$

k $-3c = 54$

l $-15x = -120$

m $4p = 37$

n $8r = 18$

o $3q = -10$

p $9w = -12$

4 Solve each equation, showing the working.

a $\frac{s}{3} = 5$

b $\frac{m}{7} = 6$

c $\frac{d}{10} = 12$

d $\frac{w}{2} = 29$

e $\frac{a}{3} = -4$

r $\frac{f}{-2} = 9$

g $\frac{r}{5} = -3$

h $\frac{m}{-7} = 1$

i $\frac{x}{-8} = -6$

j $\frac{t}{2} = \frac{3}{4}$

k $\frac{n}{-4} = -4$

l $\frac{x}{3} = -\frac{1}{6}$

5 Which of the following is the solution to $3x = -12$? Select the correct answer **A**, **B**, **C** or **D**.

A $x = -9$

B $x = -36$

C $x = -4$

D $x = -15$

6 Write an equation whose solution is:

a $m = 4$

b $x = 0$

c $r = 7$

d $d = -5$

See Example 1

Worked solutions

Exercise 10-01

MAT08NAWS10072

Skillsheet

Solving equations by
balancing

MAT08NASS10035

Skillsheet

Solving equations by
backtracking

MAT08NASS10036

Skillsheet

Solving equations
using diagrams

MAT08NASS10037

10-02 Two-step equations

Two-step equations require two steps or operations to solve.

Example 2

Homework sheet

Equations 1

MAT08NAHS10023

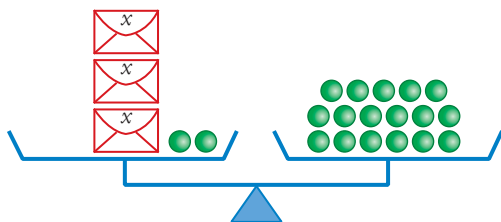
Solve each equation.

a $3x + 2 = 17$

b $\frac{k}{5} + 4 = 7$

Solution

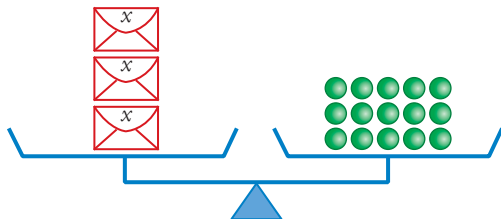
a **Method 1: The balancing method**



$$3x + 2 = 17$$

$$3x + 2 - 2 = 17 - 2$$

Step 1: Subtracting 2 from both sides.



$$3x = 15$$

$$\frac{3x}{3} = \frac{15}{3}$$

Step 2: Dividing both sides by 3.

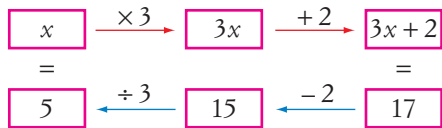
Check: $3 \times 5 + 2 = 17$



$$x = 5$$

Method 2: The backtracking method

$$3x + 2 = 17$$



$$3x = 17 - 2$$

$$3x = 15$$

$$x = \frac{15}{3}$$

$$x = 5$$

Step 1: Undo '+ 2' by subtracting 2.

Step 2: Undo '× 3' in 3x by dividing by 3.

Check: $3 \times 5 + 2 = 17$

b **Method 1: The balancing method**

$$\frac{k}{5} + 4 = 7$$

$$\frac{k}{5} + 4 - 4 = 7 - 4$$

$$\frac{k}{5} = 3$$

Step 1: Subtracting 4 from both sides.

Worksheet

Backtracking

MAT08NAWK10092

$$\frac{k}{5} \times 5 = 3 \times 5$$

$$k = 15$$

Method 2: The backtracking method

$$\frac{k}{5} + 4 = 7$$

$$\frac{k}{5} = 7 - 4$$

$$\frac{k}{5} = 3$$

$$k = 3 \times 5$$

$$k = 15$$

Step 2: Multiplying both sides by 5.

$$\text{Check: } \frac{15}{5} + 4 = 3 + 4 = 7$$

Step 1: Undo '+ 4' by subtracting 4.

Step 2: Undo '÷ 5' by multiplying by 5.

$$\text{Check: } \frac{15}{5} + 4 = 3 + 4 = 7$$

Example 3

Solve each equation.

a $\frac{2h}{3} = 4$

b $\frac{x+1}{2} = 7$

Solution

a Method 1: The balancing method

$$\frac{2h}{3} = 4$$

$$\frac{2h}{3} \times 3 = 4 \times 3$$

$$2h = 12$$

$$\frac{2h}{2} = \frac{12}{2}$$

$$h = 6$$

Method 2: The backtracking method

$$\frac{2h}{3} = 4$$

$$2h = 4 \times 3$$

$$2h = 12$$

$$h = \frac{12}{2}$$

$$h = 6$$

b Method 1: The balancing method

$$\frac{x+1}{2} = 7$$

$$\frac{x+1}{2} \times 2 = 7 \times 2$$

$$x+1 = 14$$

$$x+1 - 1 = 14 - 1$$

$$x = 13$$

Step 1: Multiplying both sides by 3.

Step 2: Dividing both sides by 2.

$$\text{Check: } \frac{2 \times 6}{3} = 4$$

Step 1: Undo '÷ 3' by multiplying by 3.

Step 2: Undo '× 2' in 2h by dividing by 2.

$$\text{Check: } \frac{2 \times 6}{3} = 4$$

Step 1: Multiplying both sides by 2.

Step 2: Subtracting 1 from both sides.

$$\text{Check: } \frac{13+1}{2} = 7$$

Method 2: The backtracking method

$$\frac{x+1}{2} = 7$$

$$x+1 = 7 \times 2$$

$$x+1 = 14$$

$$x = 14 - 1$$

$$x = 13$$

Step 1: Undo ' $\div 2$ ' by multiplying by 2.

Step 2: Undo '+ 1' by subtracting 1.

Check: $\frac{13+1}{2} = 7$

Exercise 10-02 Two-step equations

See Example 2

- 1 Solve each equation, showing all steps. Remember to check your answers.

a $2m + 7 = 19$

b $3x - 5 = 13$

c $5k + 12 = 52$

d $6w - 17 = 19$

e $4h + 21 = 9$

f $8d - 5 = -27$

g $-2x + 18 = 4$

h $-3a - 9 = -6$

i $14c + 2 = 37$

j $12p - 10 = 86$

k $9y + 11 = 101$

l $2r - 15 = 16$

- 2 Solve each equation.

a $\frac{m}{2} + 4 = 13$

b $\frac{c}{5} + 9 = 12$

c $\frac{d}{7} + 2 = 8$

d $\frac{k}{3} + 15 = 6$

e $\frac{h}{2} + 7 = 17$

f $\frac{x}{4} + 2 = -4$

g $\frac{n}{5} - 9 = 4$

h $\frac{t}{3} - 6 = -2$

i $\frac{a}{8} - 2 = -6$

j $\frac{x}{6} - 16 = -13$

k $\frac{v}{15} - 2 = 4$

l $\frac{b}{3} - 4 = 3$

- 3 The following is Liam's incorrect solution for $7x + 5 = 13$.

$$7x + 5 = 13$$

$$7x = 13 - 5$$

Line 1

$$7x = 8$$

Line 2

$$x = \frac{8}{7}$$

Line 3

$$x = 1\frac{1}{8}$$

Line 4

In which of the following was the error made? Select **A**, **B**, **C** or **D**.

A Line 1

B Line 2

C Line 3

D Line 4

See Example 3

- 4 Solve each equation.

a $\frac{3x}{2} = 9$

b $\frac{15x}{9} = 15$

c $\frac{2x}{3} = 8$

d $\frac{3N}{5} = 3$

e $\frac{6N}{7} = 18$

f $\frac{5B}{2} = -10$

g $\frac{3x}{4} = -3$

h $\frac{2x}{5} = -4$

i $\frac{5m}{2} = 11$

j $\frac{-x}{3} = 8$

k $\frac{-4x}{5} = 1$

l $\frac{-2x}{3} = -10$

- 5 Which of the following is the solution to $\frac{k-12}{9} = 6$? Select the correct answer **A**, **B**, **C** or **D**.

A $k = 27$

B $k = 42$

C $k = 66$

D $k = 162$

6 Solve each equation.

a $\frac{x+1}{3} = 2$

b $\frac{x-3}{2} = 3$

c $\frac{N+2}{5} = 1$

d $\frac{N-3}{4} = 5$

e $\frac{N+8}{5} = 6$

f $\frac{x+1}{2} = -2$

g $\frac{k-5}{2} = -5$

h $\frac{m+2}{3} = 11$

7 Write 4 equations, one of each type shown in questions 1, 2, 4 and 5, that have the solution $p = 2$.

Worked solutions

Exercise 10-02

MAT08NAWS10073

10-03

Equations with variables on both sides

Technology worksheet

Excel: Solving linear equations

MAT08NACT00017

Technology

Excel: Linear equation solver

MAT08NACT00027

For equations with variables on both sides, such as $3x + 4 = 2x + 7$, we can only use the **balancing method**, not the backtracking method.

Summary

For equations with variables on both sides, perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

Video tutorial

Equations with variables on both sides

MAT08NAVT10018

Video tutorial

Solving equations

MAT08NAVT00012

Animated example

Formally solving equations

MAT08NAAE00020

Worksheet

Equations with unknowns on both sides

MAT08NAWK00076

Puzzle sheet

Equations with unknowns on both sides

MAT08NAPS00048

Example 4

Solve each equation.

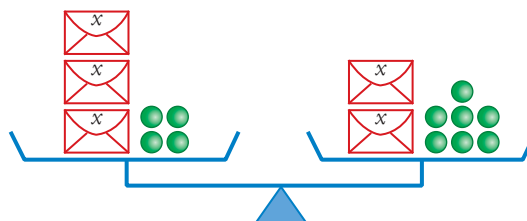
a $3x + 4 = 2x + 7$

b $5n - 3 = 2n - 15$

c $2d - 8 = -5d - 71$

Solution

a $3x + 4 = 2x + 7$

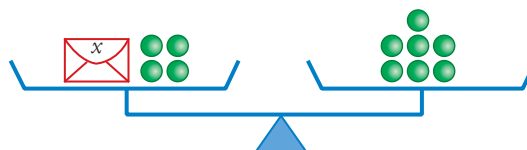


$$3x + 4 - 2x = 2x + 7 - 2x$$

$$x + 4 = 7$$

Subtracting $2x$ from both sides to remove it from the RHS.

(RHS = 'right-hand side')



$$x + 4 - 4 = 7 - 4$$

$$x = 3$$

Check:

$$\text{LHS} = 3 \times 3 + 4 = 13$$

$$\text{RHS} = 2 \times 3 + 7 = 13$$

$$\text{LHS} = \text{RHS.}$$

b $5n - 3 = 2n - 15$

$$5n - 3 - 2n = 2n - 15 - 2n$$

$$3n - 3 = -15$$

$$3n - 3 + 3 = -15 + 3$$

$$3n = -12$$

$$\frac{3n}{3} = \frac{-12}{3}$$

$$n = -4$$

Check:

$$\text{LHS} = 5 \times (-4) - 3 = -23$$

$$\text{RHS} = 2 \times (-4) - 15 = -23$$

$$\text{LHS} = \text{RHS}$$

c $2d - 8 = -5d - 71$

$$2d - 8 + 5d = -5d - 71 + 5d$$

$$7d - 8 = -71$$

$$7d - 8 + 8 = -71 + 8$$

$$7d = -63$$

$$\frac{7d}{7} = \frac{-63}{7}$$

$$d = -9$$

Check:

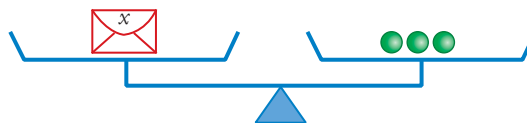
$$\text{LHS} = 2 \times (-9) - 8 = -26$$

$$\text{RHS} = -5 \times (-9) - 71 = -26$$

$$\text{LHS} = \text{RHS.}$$

Subtracting 4 from both sides to remove it from the LHS.

(LHS = 'left-hand side')



Subtracting $2n$ from both sides to remove it from the RHS.

Now this is a two-step equation.

Adding 3 to both sides to remove it from the LHS.

Now this is a one-step equation.

Dividing both sides by 3.

Adding $5d$ to both sides to remove it from the RHS.

Now this is a two-step equation.

Add 8 to both sides to remove it from the LHS.

Now this is a one-step equation.

Dividing both sides by 7.

Exercise 10-03 Equations with variables on both sides

- 1 Solve each equation, showing all steps. Remember to check your answers.
- a $3a + 6 = a + 18$ b $2k + 4 = k + 8$ c $3x + 6 = 2x + 9$
d $6p + 4 = p + 19$ e $6n + 5 = 2n + 17$ f $5q + 6 = 4q + 12$
g $5y + 14 = 3y + 14$ h $4a + 7 = 2a + 21$ i $2r + 9 = r + 15$
- 2 What is the solution to the equation $4x - 5 = 2x + 7$? Select the correct answer **A**, **B**, **C** or **D**.
A $x = 1$ **B** $x = 2$ **C** $x = 4$ **D** $x = 6$
- 3 Solve each equation.
- a $5a - 3 = 2a + 6$ b $6x - 2 = 3x + 6$ c $3d - 6 = d - 4$
d $8p - 15 = 3p - 10$ e $3m + 5 = -m + 26$ f $4x + 6 = -6x + 56$
g $4x + 3 = -2x + 7$ h $2r + 18 = -5r + 11$ i $3y - 12 = -y + 6$
- 4 Here is Bree's solution for $3x - 4 = -2x + 8$.
- | | |
|--------------------|--------|
| $3x - 4 = -2x + 8$ | |
| $3x + 2x - 4 = 8$ | Line 1 |
| $5x - 4 = 8$ | Line 2 |
| $5x = 8 - 4$ | Line 3 |
| $5x = 4$ | Line 4 |
| $x = \frac{5}{4}$ | Line 5 |
- In which lines did Bree make mistakes? Select the correct answer **A**, **B**, **C** or **D**.
A Lines 1 and 3 **B** Lines 2 and 5 **C** Lines 1 and 4 **D** Lines 3 and 5
- 5 Solve each equation.
- a $3d + 4 = d$ b $10k = 12 - 8k$ c $5 - p = p + 9$
d $7 + x = 6x + 22$ e $6k - 11 = k$ f $3m + 20 = 7m - 2$
g $4t = 12 - 4t$ h $8j - 17 = 10j$ i $6 - 3q = 8 - q$
- 6 Write an equation with x on both sides that has the solution $x = 7$.

See Example 4

Worked solutions

Exercise 10-03

MAT08NAWS10074

Worked solutions

Exercise 10-03

MAT08NAWS10074

10-04 Equations with brackets

Worksheet

Equations 2

MAT08NAWK10093

Video tutorial

Equations with brackets

MAT08NAVT10019

Worksheet

Writing equations

MAT08NAWK10094

Puzzle sheet

Equations match

MAT08NAPS10028

Summary

For equations with brackets (grouping symbols), expand the expressions and then solve as usual.

Example 5

Solve each equation.

a $3(x + 5) = 9$

b $2(3 - y) = 4y + 4$

c $5(r - 3) = 2(r + 6)$

Worksheet

Equations 3 (Extension)

MAT08NAWK10095

Homework sheet

Equations 2

MAT08NAHS10024

Video tutorial

Solving equations

MAT08NAVT00012

Solution

$$\begin{aligned} \text{a} \quad & 3(x + 5) = 9 \\ & 3x + 15 = 9 \end{aligned}$$

$$\begin{aligned} 3x + 15 - 15 &= 9 - 15 \\ 3x &= -6 \end{aligned}$$

$$\begin{aligned} \frac{3x}{3} &= \frac{-6}{3} \\ x &= -2 \end{aligned}$$

$$\begin{aligned} \text{b} \quad & 2(3 - y) = 4y + 4 \\ & 6 - 2y = 4y + 4 \\ 6 - 2y - 4y &= 4y + 4 - 4y \\ \\ & 6 - 6y = 4 \\ 6 - 6y - 6 &= 4 - 6 \end{aligned}$$

$$\begin{aligned} -6y &= -2 \\ \frac{-6y}{-6} &= \frac{-2}{-6} \\ y &= \frac{1}{3} \end{aligned}$$

Check:

$$\text{LHS} = 2 \times \left(3 - \frac{1}{3}\right) = 5\frac{1}{3}$$

$$\text{RHS} = 4 \times \frac{1}{3} + 4 = 5\frac{1}{3}$$

LHS = RHS

$$\begin{aligned} \text{c} \quad & 5(r - 3) = 2(r + 6) \\ & 5r - 15 = 2r + 12 \\ 5r - 15 - 2r &= 2r + 12 - 2r \\ \\ & 3r - 15 = 12 \\ 3r - 15 + 15 &= 12 + 15 \end{aligned}$$

$$\begin{aligned} 3r &= 27 \\ \frac{3r}{3} &= \frac{27}{3} \\ r &= 9 \end{aligned}$$

Check:

$$\text{LHS} = 5 \times (9 - 3) = 5 \times 6 = 30$$

$$\text{RHS} = 2 \times (9 + 6) = 2 \times 15 = 30$$

LHS = RHS

Expanding the expression to make it a two-step equation.

Can you think of a way to solve this equation without expanding?

Subtracting 15 from both sides.

Dividing both sides by 3.

Check: $3(-2 + 5) = 3 \times 3 = 9$

Expanding the brackets.

Subtracting $4y$ from both sides to remove it from the RHS.

Now this is a two-step equation.

Subtracting 6 from both sides to remove it from the LHS.

Dividing both sides by (-6) .

Expanding both sides.

Subtract $2r$ from both sides to remove it from the RHS.

Now this is a two-step equation.

Add 15 to both sides to remove it from the LHS.

Dividing both sides by 3.

Exercise 10-04 Equations with brackets

1 Solve each equation, showing all steps. Remember to check your answers.

a $2(x + 3) = 8$	b $3(m + 2) = 18$	c $5(k + 1) = 35$	d $4(p + 4) = 32$
e $10(j + 2) = 50$	f $7(d + 4) = 63$	g $3(x - 4) = 15$	h $3(x - 4) = 12$
i $5(x - 1) = 10$	j $8(x - 5) = 24$	k $4(k - 5) = 28$	l $2(q - 7) = 26$

2 Solve each equation.

a $2(6 - x) = 6$	b $3(4 - p) = 24$	c $2(1 - q) = 8$	d $3(16 - h) = 15$
e $2(4 + r) = -8$	f $3(e - 16) = -15$	g $5(z - 3) = -10$	h $-7(y + 2) = 35$
i $9(6 - a) = 0$	j $5(d + 3) = -20$	k $-2(3 - k) = 12$	l $4(6 - c) = -16$

3 What is the solution to $2(12 - 3x) = 30$? Select the correct answer **A**, **B**, **C** or **D**.

A $x = -2$	B $x = -1$	C $x = 1$	D $x = 2$

4 Solve each equation.

a $6(x + 1) = 3x + 3$	b $2(r + 2) = r + 5$	c $3(p + 3) = 2p + 2$
d $2(x + 1) = x$	e $5(p - 2) = 3p$	f $4(e - 2) = 2e + 4$
g $5(y + 2) = 3y + 12$	h $3(a - 5) = 5 - 2a$	i $3(2w + 1) = 3w - 15$

5 Here is Robert's solution for $7(h - 4) = 3h + 20$.

$7(h - 4) = 3h + 20$	Line 1
$7h - 4 = 3h + 20$	Line 2
$7h - 4 - 3h = 3h + 20 - 3h$	Line 3
$4h - 4 = 20$	Line 4
$4h - 4 + 4 = 20 + 4$	Line 5
$4h = 24$	Line 6
$\frac{4h}{4} = \frac{24}{4}$	Line 7
$h = 6$	

In which lines did Robert make a mistake? Select the correct answer **A**, **B**, **C** or **D**.

A Line 1	B Line 3	C Line 5	D Line 7

6 Solve each equation.

a $5(x - 1) = 4(x + 2)$	b $6(x + 2) = 4(x + 6)$	c $4(k + 3) = 3(k - 2)$
d $7(y + 2) = 4(y + 5)$	e $3(v + 2) = 2(v + 5)$	f $4(x - 2) = 2(x + 7)$
g $3(p + 1) = 5(p - 1)$	h $3(2s + 1) = 5(s + 2)$	i $5(d + 4) = 2(2d - 1)$

7 Write an equation involving brackets whose solution is:

a $c = 5$	b $k = -1$	c $n = 2$	d $q = -4$

See Example 5

Worked solutions

Exercise 10-04

MAT08NAWS10075

Mental skills 10 Maths without calculators

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a To multiply by 5, halve the number, then multiply by 10.

$$\begin{aligned}
 18 \times 5 &= 18 \times \frac{1}{2} \times 10 && (\text{or } 9 \times 2 \times 10) \\
 &= 9 \times 10 \\
 &= 90
 \end{aligned}$$

- b To multiply by 50**, halve the number, then multiply by 100.

$$\begin{aligned} 26 \times 50 &= 26 \times \frac{1}{2} \times 100 && (\text{or } 13 \times 2 \times 100) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

- c To multiply by 25**, quarter the number, then multiply by 100.

$$\begin{aligned} 44 \times 25 &= 44 \times \frac{1}{4} \times 100 && (\text{or } 11 \times 4 \times 25) \\ &= 11 \times 100 \\ &= 1100 \end{aligned}$$

- d To multiply by 15**, halve the number, then multiply by 30.

$$\begin{aligned} 8 \times 15 &= 8 \times \frac{1}{2} \times 30 && (\text{or } 4 \times 2 \times 15) \\ &= 4 \times 30 \\ &= 120 \end{aligned}$$

- e To divide by 5**, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$$\begin{aligned} 140 \div 5 &= 140 \div 10 \times 2 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

- f To divide by 50**, divide by 100 and double the answer. This is because there are two 50s in every 100.

$$\begin{aligned} 400 \div 50 &= 400 \div 100 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

- g To divide by 25**, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$$\begin{aligned} 600 \div 25 &= 600 \div 100 \times 4 \\ &= 6 \times 4 \\ &= 24 \end{aligned}$$

- h To divide by 15**, divide by 30 and double the answer. This is because there are two 15s in every 30.

$$\begin{aligned} 240 \div 15 &= 240 \div 30 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

- 2** Now evaluate each expression.

a 32×5	b 14×5	c 48×5	d 18×50
e 52×50	f 36×25	g 28×5	h 12×25
i 12×15	j 22×35	k $90 \div 5$	l $170 \div 5$
m $230 \div 5$	n $1300 \div 50$	o $900 \div 50$	p $300 \div 25$
q $1000 \div 25$	r $360 \div 45$	s $210 \div 15$	t $360 \div 15$

Investigation: Solving $x^2 = c$

- 1 What is the inverse operation of 'squaring'?
- 2 The equation $x^2 = 9$ has two solutions. What are the two numbers, x , which when squared gives 9?
- 3 What are the solutions for each equation? What is the pattern found in the answers?
a $x^2 = 25$ **b** $x^2 = 100$ **c** $x^2 = 1$
- 4 Study this example:

$$x^2 = 49$$

$$x = \pm\sqrt{49}$$

$$= \pm 7$$

$$\text{which means } x = \sqrt{49} \text{ or } -\sqrt{49}$$

$$\text{which means } x = 7 \text{ or } -7$$

Check: When $x = 7$, $x^2 = 7^2 = 49$

When $x = -7$, $x^2 = (-7)^2 = 49$

Use the same method to solve each equation and check your answers:

a $x^2 = 81$

b $x^2 = 64$

10-05 Simple quadratic equations $x^2 = c$

NSW

An equation involving a pronumeral squared, such as $x^2 = 25$ or $3x^2 - 4 = 7$, is called a **quadratic equation**. In this section, we will solve simple quadratic equations of the type $x^2 = c$, where c is a number.

Example 6

Solve each quadratic equation.

a $x^2 = 36$

b $x^2 = 121$

c $x^2 = 40$, writing the solution correct to one decimal place

d $x^2 = 83$, writing the solution as a surd

Solution

a $x^2 = 36$

$$x = \pm\sqrt{36}$$

$$= \pm 6$$

Finding the square root of both sides.

The equation has two solutions, $x = 6$ or $x = -6$.

b $x^2 = 121$

$$x = \pm\sqrt{121}$$

$$= \pm 11$$

Finding the square root of both sides.

The equation has solutions $x = 11$ or $x = -11$.

c $x^2 = 40$

$$x = \pm\sqrt{40}$$

$$= \pm 6.2345\dots$$

$$\approx \pm 6.3$$

The equation has solutions $x \approx 6.3$ or $x \approx -6.3$.

d $x^2 = 83$

$$x = \pm\sqrt{83}$$

Leaving the answer as a surd.

$$x = \sqrt{83} \text{ or } -\sqrt{83}$$

Summary

The simple quadratic equation $x^2 = c$ (where c is a positive number) has two solutions, $x = \pm\sqrt{c}$ (which means $x = \sqrt{c}$ or $x = -\sqrt{c}$).

Exercise 10-05 Simple quadratic equations $x^2 = c$

See Example 6

- 1 Solve each quadratic equation.

a $x^2 = 81$

b $x^2 = 144$

c $x^2 = 1$

d $x^2 = 169$

e $m^2 = 5041$

f $u^2 = 1849$

- 2 Solve each quadratic equation, writing the solution correct to two decimal places.

a $x^2 = 13$

b $x^2 = 54$

c $x^2 = 88$

d $t^2 = 129$

e $h^2 = 946$

f $z^2 = 527$

- 3 Solve each quadratic equation, writing the solution as a surd.

a $x^2 = 41$

b $x^2 = 30$

c $x^2 = 48$

d $a^2 = 126$

e $p^2 = 75$

f $b^2 = 509$

- 4 Many simple quadratic equations have two solutions (one positive, one negative) but there is one simple quadratic equation that has only **one** solution. What is this equation and what is its solution?

- 5 The equation $x^2 = -100$ has **no solutions**. Explain why.

- 6 Write two more simple quadratic equations that have no solutions.

- 7 Copy and complete:

a $x^2 = c$ has two solutions if c is _____.

b $x^2 = c$ has no solutions if c is _____.

c $x^2 = c$ has one solution if c is _____.

10-06 Equation problems

Worksheet

Equations problems

MAT08NAWK00075

Worksheet

Working with formulas

MAT08NAWK00077

Homework sheet

Equations 2

MAT08NAHS10024

Homework sheet

Equations revision

MAT08NAHS10025

Animated example

Writing equations

MAT08NAAE00019

Video tutorial

Solving equations

MAT08NAVT00012

Worksheet

Solving equations
review

MAT08NAWK00078

Quiz

Solving equations

MAT08NAQZ00012

Summary

To solve word problems that require an equation to be solved:

- choose your pronumeral
- translate the words into an equation
- solve the equation
- write a sentence that answers the problem.

Example 7

- a** When a number is doubled and 11 is subtracted, the result is 23. Find the number.
b Five times a number is the same as six more than three times the number. What is the number?

Solution

- a** Let the number be x .

Translating the words into an equation:

$$x \times 2 - 11 = 23$$

$$2x - 11 = 23$$

Solving the equation:

$$2x - 11 + 11 = 23 + 11$$

$$2x = 34$$

$$\frac{2x}{2} = \frac{34}{2}$$

$$x = 17$$

Adding 11 to both sides.

Dividing both sides by 2.

Check: $2 \times 17 - 11 = 23$

The number is 17.

- b** Let n represent the number.

$$5n = 3n + 6$$

$$5n - 3n = 3n + 6 - 3n$$

$$2n = 6$$

$$\frac{2n}{2} = \frac{6}{2}$$

$$n = 3$$

Subtracting $3n$ from both sides.

Dividing both sides by 2.

Check:

$$\text{LHS} = 5 \times 3 = 15$$

$$\text{RHS} = 3 \times 3 + 6 = 15$$

$$\text{LHS} = \text{RHS}$$

The number is 3.

Example 8

A repairman charges for fixing washing machines using the formula:

$$C = 32h + 45$$

where C is the charge in dollars and h is the number of hours the job takes.

Find:

- a the charge for a job that takes 3 hours
- b the number of hours worked if the charge is \$205.



Solution

- a Substitute $h = 3$ into the formula.

$$\begin{aligned} C &= 32h + 45 \\ &= 32 \times 3 + 45 \\ &= 141 \end{aligned}$$

The charge is \$141.

- b Substitute $C = 205$ into the formula and solve the equation.

$$\begin{aligned} 205 &= 32h + 45 \\ 32h + 45 &= 205 \end{aligned}$$

Swap the sides of the equation so that ' h ' is on the LHS.

$$\begin{aligned} 32h + 45 - 45 &= 205 - 45 \\ 32h &= 160 \\ \frac{32h}{32} &= \frac{160}{32} \\ h &= 5 \end{aligned}$$

The number of hours worked is 5.

Exercise 10-06 Equation problems

- 1 Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information.
 - a Four family tickets for a theme park cost \$352. How much does each ticket cost? (Let t represent the price of a family ticket.)
 - b Twenty cans of drink cost \$18.00. How much does each can cost? (Let c represent the cost of one can.)
 - c A number is tripled and the result is 99. What is the number? (Let x represent the unknown number.)
 - d A number has 7 added to it and the result is 23. Find the number. (Let n represent the unknown number.)



See Example 7

- 2 Select the correct equation **A**, **B**, **C** or **D** for each problem. Then use the equation to solve the problem.
 - a If 23 is subtracted from a number, the answer is 79. What is the number?
A $79 + N = 23$ **B** $23N = 79$ **C** $23 - N = 79$ **D** $N - 23 = 79$
 - b The sum of a number and 18 is 67. What is the number?
A $N + 67 = 18$ **B** $N + 18 = 67$ **C** $18N = 67$ **D** $N - 18 = 67$
 - c When a number is multiplied by 23, the answer is 1288. What is the number?
A $1288N = 23$ **B** $N + 23 = 1288$ **C** $1288 - N = 23$ **D** $23N = 1288$
 - d When a number is divided by 14, the answer is 42. What is the number?
A $N - 14 = 42$ **B** $\frac{N}{14} = 42$ **C** $14N = 42$ **D** $14 + N = 42$
 - e Marika buys a car that costs \$13 499. She pays a deposit of \$4525. How much does she owe?
A $4525N = 13\ 499$ **B** $N - 4525 = 13\ 499$
C $N + 4525 = 13\ 499$ **D** $N = 13\ 499 + 4525$
- 3 For each problem, write an equation and find the unknown number.
 - a When an unknown number is divided by 5, the result is 15.
 - b When 12 is subtracted from a number, the answer is 37.
 - c When a number is subtracted from 52, the answer is 25.
 - d An unknown number is multiplied by 3, then 8 is subtracted. The result is 40.
 - e An unknown number is halved, then 7 is added. The result is 13.
 - f The sum of a number and 4 is doubled. The result is 44.
 - g The sum of a number and 6 is divided by 3 and the result is 7.
 - h The product of a number and 5 is decreased by 2. The result is 28.
- 4 Translate each problem into an equation, then solve the equation to solve the problem.
 - a Year 8 is holding a disco to raise money. Each ticket bought by students raises \$5 but the costs of running the disco total \$130. How many tickets must be sold to make a profit of \$300? (Let n stand for the number of tickets sold.)

Worked solutions

Exercise 10-06

MAT08NAWS10076

- b In eight years Kelly's age will be twice what it is now. How old is Kelly now? (Let n stand for Kelly's age now.)
- c Eight sheep have the same mass as three sheep and one cow. If the cow's mass is 500 kg, what is the mass of one sheep? (Let s stand for the mass of one sheep.)



- d If you multiply Liam's favourite number by 3 and add 1, you get the same answer as if you multiplied the number by 5 and took away 11. What is the number? (Let x represent the number.)
- e Mr Yen says, 'If you add 15 to my age and multiply by 7, the answer is 294.' How old is Mr Yen? (Let a stand for his age.)
- f The area of a rhombus is calculated by multiplying its diagonals and dividing by 2. If a rhombus has an area of 88 cm^2 and one diagonal is 11 cm, what is the length of the other diagonal? (Let d represent the length of the diagonal.)

See Example 8

- 5 A temperature in degrees Celsius ($^{\circ}\text{C}$) can be converted to degrees Fahrenheit ($^{\circ}\text{F}$) using the formula $F = \frac{9C}{5} + 32$.

a Convert 25°C to $^{\circ}\text{F}$.

b Convert 108°F to $^{\circ}\text{C}$.

- 6 The charge, $\$C$, for hiring a hall for an event is $C = 150 + 2N$, where N stands for the number of people at the event. Find:

a the charge when 225 people are at the event

b the number of people at the event when the charge is $\$394$.

- 7 The cost, $\$y$, of a classified ad in the local newspaper is $y = 0.8w + 3.5$, where w is the number of words in the ad. Find:

a the cost of a 13-word ad

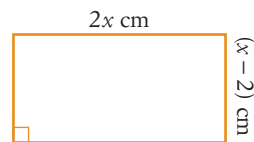
b the number of words in an ad costing $\$25.90$

- 8 The perimeter of this rectangle is 47 cm. Find:

a the value of x

b the length of the rectangle

c the width of the rectangle.

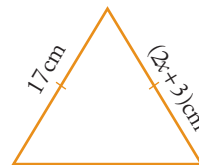


- 9 The profit, $\$P$, made by a DVD store is given by $P = 5x - 900$, where x represents the number of DVDs sold. Find:

a the profit made when 195 DVDs are sold

b the number of DVDs sold if the profit is $\$435$.

- 10 Find the value of x in this isosceles triangle.



Worked solutions

Exercise 10-06

MAT08NAWS10076

Power plus

- 1 Find the value of x in each of the following.

a $2(x + 1) + 2(x - 1) = 12$

c $4(2x - 1) - 5(x - 2) = 6$

e $\frac{x+7}{4} = \frac{6(x-1)}{3}$

g $\frac{2x}{3} - \frac{x}{6} = 10$

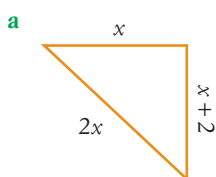
b $2(x + 4) - 3(x - 1) = 9$

d $2 - (3x + 5) = 4(x + 1)$

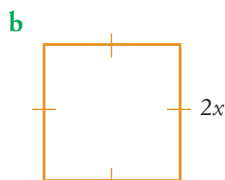
f $\frac{2(x+1)}{3} = \frac{5(x-2)}{2}$

h $\frac{3x}{4} + \frac{9x}{10} = 44$

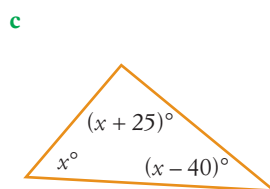
- 2 Write an equation and solve it to find the unknown values in each of the following.



Perimeter = 30 cm



Area = 100 cm²



- 3 Each of the following equations has two solutions. Find them.

a $x^2 = 25$

b $x^2 - 7 = 9$

c $2x^2 = 72$

d $3x^2 + 8 = 20$

e $(x + 4)^2 = 9$

f $x^2 + 2x = 15$

- 4 Diophantus was a famous mathematician who was the first to abbreviate his mathematical thoughts using symbols. He is known as the Greek father of algebra and when he died, one of his admirers wrote the following riddle about his life:

Diophantus' youth lasted $\frac{1}{6}$ of his life. He grew a beard after $\frac{1}{12}$ more. After $\frac{1}{7}$ more of his life, Diophantus married; 5 years later he had a son. The son lived exactly $\frac{1}{2}$ as long as his father, and Diophantus died just 4 years after his son. All this adds to the years Diophantus lived.

Write this riddle as an equation and solve it to find how long Diophantus lived.

(Hint: Let x years equal his life.)

Chapter 10 review

Language of maths

Puzzle sheet
Equations crossword
MAT08NAPS10029

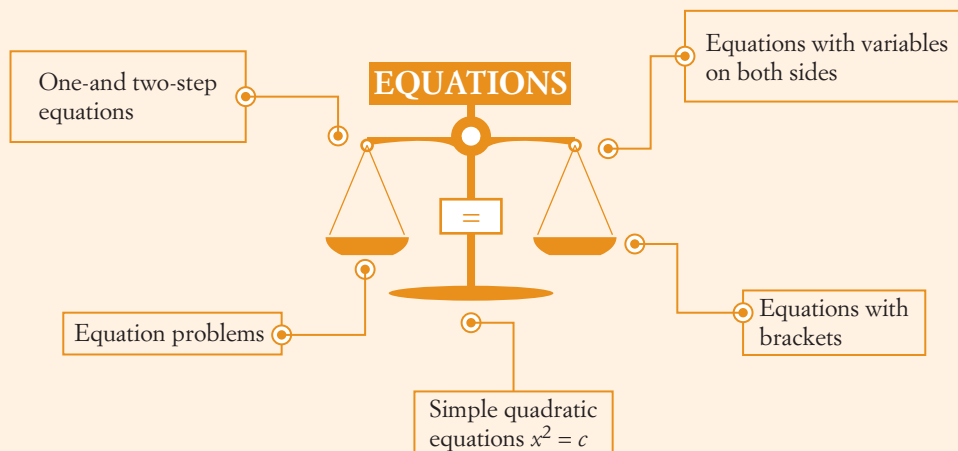
backtracking	expand	pronumeral	substitution
balancing	formula	quadratic equation	two-step equation
brackets	inverse operation	RHS (right-hand side)	undoing
check	LHS (left-hand side)	solution	unknown
equation	one-step equation	solve	variable

- 1 What are the two algebraic methods for solving equations?
- 2 Which method involves 'undoing' operations?
- 3 What does the word 'solution' mean?
- 4 Which word in the list means 'opposite'?
- 5 Why is the **variable** in an equation sometimes called an **unknown**?
- 6 What word means to rewrite an algebraic expression by removing the brackets?
- 7 What does **RHS** stand for:
 - a in congruent triangles?
 - b in solving equations?

Topic overview

Worksheet
Mind map: Equations
MAT08NAWK10098

- Which parts of this topic did you find easy? What did you already know?
- Give examples of some problems that might be solved using equations.
- Are there any parts of this topic that you still don't understand? Talk to your teacher about them.
- In what sort of careers would people use equations?
- Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



1 Solve each of the following equations.

See Exercise 10-01

a $k + 6 = 13$

b $x - 3 = 8$

c $a + 3 = 17$

d $a - 12 = 21$

e $3x = 12$

f $10f = 120$

g $\frac{m}{4} = 7$

h $\frac{x}{3} = 8$

i $w + 9 = 3$

j $k - 5 = -7$

k $-2m = -16$

l $\frac{x}{3} = -12$

2 Solve each equation.

See Exercise 10-02

a $4p + 3 = 23$

b $3m + 17 = 8$

c $2x - 12 = 18$

d $\frac{h}{7} + 5 = 16$

e $\frac{n}{2} - 8 = -12$

f $\frac{a}{5} - 8 = 4$

g $\frac{2y}{5} = 4$

h $\frac{x+2}{3} = 4$

i $\frac{n-5}{2} = 10$

j $\frac{5x}{2} = -15$

k $\frac{k+9}{7} = 5$

l $\frac{d-1}{5} = -6$

3 Solve each equation.

See Exercise 10-03

a $3x + 4 = x + 6$

b $5u - 3 = 2u + 6$

c $12h - 8 = 8h + 4$

d $3v - 4 = 7v + 8$

e $2x + 9 = 7x - 4$

f $9 - 5t = 3t - 15$

4 Solve each equation.

See Exercise 10-04

a $2(n + 3) = 18$

b $3(x + 1) = 15$

c $10(x - 3) = -10$

d $5(x - 2) = 3x + 4$

e $4(y + 1) = y + 18$

f $-2(d - 2) = 2d - 20$

g $3(r - 1) = 2(r + 9)$

h $7(a + 5) = 3(a + 9)$

i $2(2n - 4) = 2(5 - n)$

5 Solve each equation:

See Exercise 10-05

a $x^2 = 64$

b $x^2 = 45$, correct to one decimal place

c $x^2 = 27$, as a surd

6 **a** If a number is doubled and has 4 added to it, the answer is 14. What is the number?

See Exercise 10-06

b If a number has 4 added to it and is doubled, the answer is 14. What is the number?

7 **a** Kay is paid \$45 for each jumper she knits. If n is the number of jumpers Kay knits to earn a total of \$270, which equation can be used to find the value of n ? Select the correct answer **A**, **B**, **C** or **D**.

See Exercise 10-06

A $\frac{n}{45} = 270$

B $45n = 270$

C $n + 45 = 270$

D $270 - n = 45$

b Solve the equation to find the value of n .

8 Seven times a number is the same as nine more than four times the same number. Use an equation to find the number.

See Exercise 10-06

9 The sum (S) of the interior angles (in degrees) of a polygon is given by $S = 180n - 360$, where n is the number of sides. Find:

See Exercise 10-06

a the sum of the interior angles when a polygon has 9 sides

b the number of sides in a polygon whose angle sum is 1080° .



11

**Number and algebra,
Measurement and geometry**

Ratios, rates and time

Ratios and rates are used in many everyday situations to compare things. Ratios are used when baking cakes and cooking meals, mixing fertiliser, reading maps and house plans. Rates are used when calculating the price of phone calls, measuring household electricity usage and calculating the speed of a car.



Chapter outline

	Proficiency strands			
11-01 Equivalent ratios	U			C
11-02 Simplifying ratios	U	F		
11-03 Ratio problems	U	F	PS	C
11-04 Scale maps and plans	U	F	PS	C
11-05 Dividing a quantity in a given ratio	U	F		
11-06 Rates	U			C
11-07 Best buys	U	F	PS	R C
11-08 Rate problems	U	F	PS	C
11-09 Speed	U	F	PS	C
11-10 Travel graphs	U	F	PS	R C
11-11 Sketching informal graphs	U	F	PS	R C
11-12 Time differences	U	F	PS	C
11-13 International time zones	U	F	PS	C

Wordbank

best buy When comparing different brands or sizes during shopping, this is the item with the lowest unit cost and is the best value for money

per (symbol '/') A word used in rates to mean 'for each'

scaled length A length on a map or plan that represents an actual length, usually much smaller but in proportion to it

speed A rate that compares distance travelled with time taken

time zone A region of the world where all places experience the same time of day

travel graph A line graph that describes a journey and shows distance travelled over time

unit cost The price of one item or unit, such as 1 mL or 1 g

unitary method A method of finding a quantity by finding one part first

In this chapter you will:

Maths clip

Why do we have ratios?

MAT08NAMC00004

- recognise and solve problems involving simple ratios
- divide a quantity in a given ratio
- solve a range of problems involving rates and ratios, with and without digital technologies
- convert given information into rates
- investigate and calculate 'best buys'
- use travel graphs to investigate and compare the distance travelled to and from school
- interpret features of travel graphs such as the slope of lines and the meaning of horizontal lines
- investigate, interpret and analyse graphs from authentic data
- sketch informal graphs to model familiar events, for example, noise level during the lesson
- solve problems involving duration, including using 12- and 24-hour time within a single time zone
- **solve problems involving international time zones**

SkillCheck

Worksheet

StartUp assignment 11

MAT08NAWK10099

1 Copy and complete.

a $2 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

b $3 \text{ h} = \underline{\hspace{1cm}} \text{ min}$

c $3000 \text{ kg} = \underline{\hspace{1cm}} \text{ t}$

d $2.5 \text{ L} = \underline{\hspace{1cm}} \text{ mL}$

e $380 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

f $180 \text{ mg} = \underline{\hspace{1cm}} \text{ g}$

g $3 \text{ min} = \underline{\hspace{1cm}} \text{ s}$

h $8.5 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

i $480 \text{ min} = \underline{\hspace{1cm}} \text{ h}$

j $7500 \text{ mL} = \underline{\hspace{1cm}} \text{ L}$

k $9.15 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

l $3840 \text{ mm} = \underline{\hspace{1cm}} \text{ cm}$

2 Find the greatest common divisor (GCD) of each pair of numbers.

a 12 and 18

b 35 and 21

c 16 and 40

3 Complete each pair of equivalent fractions.

a $\frac{7}{10} = \frac{\square}{30}$

b $\frac{3}{5} = \frac{12}{\square}$

c $\frac{18}{45} = \frac{\square}{5}$

4 Find the lowest common multiple (LCM) of each pair of numbers.

a 2 and 3

b 4 and 8

c 5 and 3

5 Simplify each fraction.

a $\frac{18}{45}$

b $\frac{32}{64}$

c $\frac{80}{100}$

d $\frac{15}{36}$

6 Evaluate each expression.

a $\frac{2}{5} \times 20$

b $\frac{1}{3} \times 15$

c $\frac{3}{4} \times 24$

d $\frac{5}{6} \times 36$

e 7.6×100

f 5.4×10

g 0.39×100

h 25×10

Skillsheet

24-hour time

MAT08MGSS10039

7 Convert each time to 24-hour time.

a 7:15 a.m.

b 3:45 p.m.

c 8:50 p.m.

d 12:10 a.m.

8 Convert each time to 12-hour time.

a 0410

b 1105

c 1415

d 2335

11-01 Equivalent ratios

Puzzle sheet

Simplifying ratios

MAT08NAPS00025

A **ratio** consists of two or more numbers that compare the parts or shares of things of the same type, in the same units. For example, if a cake recipe uses sugar to flour in a ratio of 1 to 2, written '1 : 2,' it means that for every one part of sugar we need two parts of flour. Each number in a ratio is called a **term** of the ratio.

Equivalent ratios are *equal* ratios and can be found in a similar way to that used for finding equivalent fractions.

Summary

To find an **equivalent ratio**, multiply or divide each term by the same number.

Example 1

Complete each pair of equivalent ratios.

a $2 : 7 = \underline{\hspace{1cm}} : 35$

b $6 : 5 = 18 : \underline{\hspace{1cm}}$

c $12 : 8 = 3 : \underline{\hspace{1cm}}$

Solution

- a To find the missing term, look at the two known matching terms, 7 and 35.

7 is *multiplied* by 5 to give 35, so do the same thing to the 2.

$$2 \times 5 = 10$$

$$\begin{array}{c} \times 5 \\ \curvearrowright \\ 2 : 7 = 10 : 35 \\ \curvearrowleft \\ \times 5 \end{array}$$

- b The two known matching terms are 6 and 18.

6 is *multiplied* by 3 to give 18, so do the same thing to the 5.

$$5 \times 3 = 15$$

$$\begin{array}{c} \times 3 \\ \curvearrowright \\ 6 : 5 = 18 : 15 \\ \curvearrowleft \\ \times 3 \end{array}$$

- c The two known matching terms are 12 and 3.

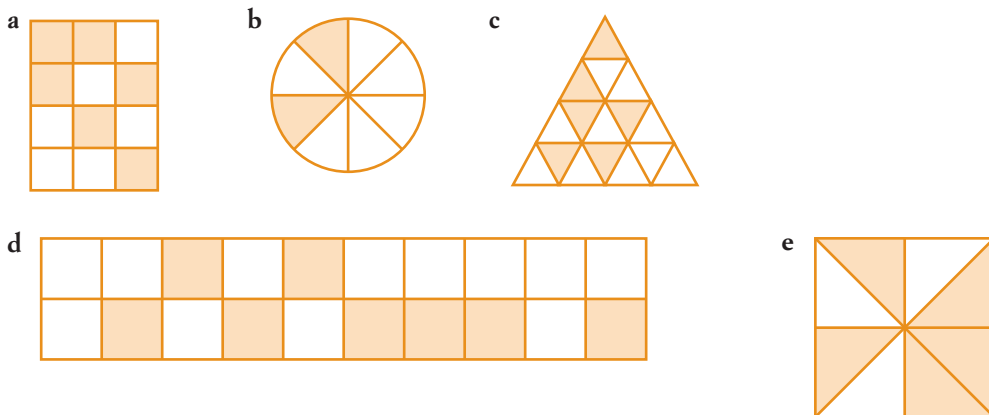
12 is *divided* by 4 to give 3, so do the same thing to the 8.

$$8 \div 4 = 2$$

$$\begin{array}{c} \div 4 \\ \curvearrowright \\ 12 : 8 = 3 : 2 \\ \curvearrowleft \\ \div 4 \end{array}$$

Exercise 11-01 Equivalent ratios

- 1 For each shape:
 - i write the ratio of shaded to unshaded parts
 - ii write an equivalent ratio to the ratio from i



See Example 1 2 Copy and complete each pair of equivalent ratios.

a $2 : 3 = 8 : \underline{\hspace{1cm}}$

b $1 : 5 = 2 : \underline{\hspace{1cm}}$

c $3 : 5 = \underline{\hspace{1cm}} : 15$

d $4 : 7 = \underline{\hspace{1cm}} : 35$

e $5 : 8 = 20 : \underline{\hspace{1cm}}$

f $7 : 12 = 49 : \underline{\hspace{1cm}}$

g $5 : 11 = \underline{\hspace{1cm}} : 66$

h $3 : 4 = \underline{\hspace{1cm}} : 100$

i $2 : 1 = 10 : \underline{\hspace{1cm}}$

j $\underline{\hspace{1cm}} : 9 = 20 : 36$

k $12 : \underline{\hspace{1cm}} = 3 : 1$

l $17 : 34 = \underline{\hspace{1cm}} : 2$

m $\underline{\hspace{1cm}} : 45 = 6 : 9$

n $24 : 12 = 4 : \underline{\hspace{1cm}}$

o $16 : \underline{\hspace{1cm}} = 2 : 5$

p $\underline{\hspace{1cm}} : 20 = 15 : 60$

q $24 : 20 = 6 : \underline{\hspace{1cm}}$

r $50 : 40 = \underline{\hspace{1cm}} : 20$

Worked solutions

Exercise 11-01

MAT08NAWS10077

3 Which of the following ratios is **not** equivalent to the ratio $32 : 48$? Select the correct answer **A, B, C** or **D**.

A $16 : 24$

B $4 : 6$

C $2 : 3$

D $6 : 8$

4 Which pair of ratios are **not** equivalent? Select the correct answer **A, B, C** or **D**.

A $12 : 21, 4 : 6$

B $15 : 10, 35 : 20$

C $32 : 8, 6 : 1$

D $48 : 60, 4 : 5$

5 Copy and complete each pair of equivalent ratios.

a $2 : 3 : 4 = 6 : \underline{\hspace{1cm}} : \underline{\hspace{1cm}}$

b $3 : 4 : 5 = \underline{\hspace{1cm}} : 20 : \underline{\hspace{1cm}}$

c $0.1 : 0.3 = 0.5 : \underline{\hspace{1cm}}$

d $0.7 : 0.4 = \underline{\hspace{1cm}} : 2$

e $\underline{\hspace{1cm}} : 4.5 = 7.8 : 13.5$

f $\frac{2}{3} = \frac{\underline{\hspace{1cm}}}{9}$

g $\frac{7}{8} = \frac{21}{\underline{\hspace{1cm}}}$

h $\frac{7}{20} = \frac{\underline{\hspace{1cm}}}{100}$

i $\frac{5}{3} = \frac{25}{\underline{\hspace{1cm}}}$

j $\frac{1}{2} : 3 = \underline{\hspace{1cm}} : 9$

k $9 : \frac{3}{7} = \underline{\hspace{1cm}} : 3$

Worksheet

Simplifying ratios

MAT08NAWK00046

Worksheet

Ratio calculations

MAT08NAWK10100

11-02 Simplifying ratios

Summary

- To **simplify a ratio**, keep dividing both terms by the same number, preferably a large number such as their greatest common divisor (GCD), until each term is as small as possible
- If all terms are even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

Example 2

Simplify each ratio.

a $25 : 40$

b $24 : 16$

c $18 : 30 : 15$

Solution

a $25 : 40 = \frac{25}{5} : \frac{40}{5}$
 $= 5 : 8$

Divide both terms by their GCD, 5.

OR enter $25 : 40$ as a fraction $\frac{25}{40}$ on the calculator:

25 $\text{a}^{\text{b/c}}$ 40 $=$

On some calculators, use the $\frac{\Box}{\Box}$ key instead of $\text{a}^{\text{b/c}}$

b $24 : 16 = \frac{24}{8} : \frac{16}{8}$
 $= 3 : 2$

Divide both terms by their GCD, 8.

OR enter $24 : 16$ as an improper fraction $\frac{24}{16}$ on the calculator: 24 $\text{a}^{\text{b/c}}$ 16 $=$

Then to change the mixed numeral answer to an

improper fraction, press: d/c $\left(\begin{array}{c} \text{SHIFT} \\ \text{2ndF} \end{array} \right)$ $\text{a}^{\text{b/c}}$

On some calculators, press $\frac{a}{c} \leftrightarrow \frac{d}{c}$ $\left(\begin{array}{c} \text{SHIFT} \\ \text{S} \leftrightarrow \text{D} \end{array} \right)$

c $18 : 30 : 15 = \frac{18}{3} : \frac{30}{3} : \frac{15}{3}$
 $= 6 : 10 : 5$

Divide all terms by their GCD, 3.

Because there are more than 2 terms in this ratio, the calculator cannot be used here

Example 3

Simplify each ratio.

a $\frac{3}{5} : \frac{1}{3}$

b $0.7 : 0.05$

Solution

If a ratio has terms that are fractions or decimals, it can be simplified by converting the terms to **whole numbers**.

- a For fractions, multiply both terms by a common multiple, preferably the **lowest common multiple** (LCM) of the denominators.

$\frac{3}{5} : \frac{1}{3} = \frac{3}{5} \times 15 : \frac{1}{3} \times 15$
 $= 9 : 5$

LCM of 5 and 3 is 15.

- b For decimals, multiply both terms by the appropriate power of ten. In this case, multiply by 100 (move the decimal place two places to the right).

$0.7 : 0.05 = 0.7 \times 100 : 0.05 \times 100$
 $= 70 : 5$
 $= 14 : 1$

Simplifying.

Example 4

Simplify the ratio of 2 hours to 1 day.

Solution

First, change the values to the same units.

$$1 \text{ day} = 24 \text{ hours}$$

$$2 \text{ hours} : 1 \text{ day} = 2 \text{ hours} : 24 \text{ hours}$$

$$= 2 : 24$$

$$= 1 : 12$$

Exercise 11-02 Simplifying ratios**See Example 2**

1 Simplify each ratio.

a $10 : 100$

b $12 : 24$

c $12 : 30$

d $35 : 49$

e $18 : 12$

f $56 : 24$

g $1000 : 100$

h $45 : 99$

i $87 : 87$

j $123 : 321$

k $51 : 17$

l $3 : 48$

m $8 : 12 : 20$

n $15 : 20 : 30$

o $27 : 9 : 36$

p $14 : 35 : 21 : 49$

See Example 3

2 Simplify each ratio.

a $\frac{1}{3} : \frac{2}{5}$

b $\frac{1}{4} : \frac{1}{3}$

c $\frac{3}{4} : \frac{2}{3}$

d $\frac{1}{2} : \frac{3}{8}$

e $\frac{2}{5} : \frac{3}{10}$

f $\frac{4}{5} : \frac{1}{2}$

g $\frac{5}{8} : \frac{1}{4}$

h $\frac{2}{3} : \frac{1}{2}$

i $\frac{3}{4} : \frac{7}{16}$

j $\frac{4}{5} : \frac{1}{2}$

k $\frac{5}{6} : \frac{2}{5}$

l $\frac{6}{5} : \frac{2}{3}$

m $0.4 : 0.7$

n $1.3 : 0.8$

o $0.5 : 0.3$

p $0.9 : 1.8$

q $0.6 : 0.8$

r $3.6 : 2.4$

s $0.05 : 0.2$

t $0.25 : 0.5$

u $0.375 : 0.25$

v $12 : 8.4$

w $2.4 : 1.2 : 3.6$

x $4.5 : 1 : 0.9$

See Example 4

3 Simplify each ratio.

a 50 cm to 2 m

b 300 g to 1.2 kg

c 5 days to 7 weeks

d 30 min to 2 hours

e 70 cents to \$2.10

f 2 years to 6 months

g 15 hours to 2 days

h 20 mm to 1 m

i 4 tonnes to 350 kg

j 25 min to 3 hours

k 18 m to 1 km

l 8 months to 4 years

m 2 days to 8 hours

n 75 cents to \$5

o \$2.70 : \$12

4 On a farm there are 200 orange and mandarin trees in total. If there are 120 orange trees, what is the ratio of orange to mandarin trees?



5 In a class of 28 students, there are 16 girls. Find the ratio of:

a girls to boys

b boys to girls

c girls to students in the class

6 A store has 30 gas heaters and 20 electric heaters in its warehouse. Find the ratio of:

- a gas heaters to electric heaters b gas heaters to all heaters
c all heaters to electric heaters

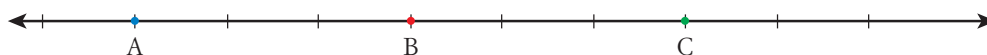
7 A man earns \$75 000 a year and spends \$64 000 a year. Find the ratio of:

- a his earnings to expenses b his savings to earnings

8 Zoe buys goods for \$320 and sells them for \$380. Find the ratio of:

- a the cost price to the selling price b the selling price to the cost price
c the profit to the selling price d the selling price to the profit

9 The line below is divided into units of length as shown.



Find each ratio of lengths.

- a $AB : BC$ b $AC : AB$ c $BC : AC$ d $AC : BC$

10 A vending machine is filled with bottles and cans. The ratio of the number of bottles to the total number of contents is 3 : 8.

- a What fraction of the contents are cans?
b What is the ratio of the number of cans to the number of bottles?

11 0.5 m^3 of cement is added to $\frac{3}{8} \text{ m}^3$ of metal to make a mixture. What is the ratio of cement to metal? Select the correct answer A, B, C or D.

- A 4 : 3 B 7 : 8 C 3 : 4 D 3 : 16

12 When comparing the rate at which babies are born in different countries (that have different population sizes), we use a base number of 1000. The annual birth rate in Australia is approximately 12 per 1000 people (a ratio of 12 : 1000). The rate in India is approximately 22 per 1000 people, or 22 : 1000. Express each ratio in simplest form.

11-03 Ratio problems

Problems involving ratios can be solved using **equivalent ratios** or the **unitary method**. With the unitary method, we find the size of **one part** first.

Example 5

In an English class the ratio of boys to girls is 5 : 6. If there are 15 boys in the class, how many girls are there?

Solution

Method 1: Equivalent ratios

Write the problem as a pair of equivalent ratios.

$$\text{Boys : girls} = 5 : 6 = 15 : \underline{\quad}$$

$\begin{array}{c} \times 3 \\ \curvearrowright \\ \times 3 \end{array}$

15 boys

Worked solutions

Exercise 11-02

MAT08NAWS10078

Worksheet

Ratio applications

MAT08NAWK00047

Homework sheet

Ratios 1

MAT08NAHS10012

Number of girls = $6 \times 3 = 18$

There are 18 girls in the class.

Method 2: Unitary method

Boys : girls = 5 : 6

5 parts (boys) = 15

1 part = $15 \div 5 = 3$

6 parts (girls) = $6 \times 3 = 18$

There are 18 girls in the class.

← 'Unitary' means 'one'

Finding one part first.

Example 6

To make concrete, a builder mixes sand and cement in the ratio 5 : 4. If a mix of concrete contains 20 kg of cement, find:

- the amount of sand in the mix
- the total mass of the mix.



Solution

a Method 1: Equivalent ratios

$$\text{Sand : cement} = 5 : 4 = \text{---} : 20$$

$\begin{array}{c} \times 5 \\ \curvearrowright \\ \times 5 \end{array}$

$$\text{Amount of sand} = 5 \times 5 = 25 \text{ kg}$$

Method 2: Unitary method

Sand : cement = 5 : 4

4 parts (cement) = 20 kg

1 part = $20 \div 4 = 5 \text{ kg}$

5 parts (sand) = $5 \times 5 = 25 \text{ kg}$

- Total mass = 20 kg + 25 kg = 45 kg
So the total mixture was 45 kg.

20 kg of cement.

Cement and sand.

Exercise 11-03 Ratio problems

- Alison and Elena buy a length of material and divide it between them in the ratio 2 : 3. If Alison has 3.6 m, what length of material does Elena have? Select the correct answer **A**, **B**, **C** or **D**.
A 1.8 m **B** 2.4 m **C** 5.4 m **D** 9 m
- A tiler uses 4 green tiles to every 3 white ones. How many white tiles are used if 100 green tiles are used?
- When making concrete, sand and cement are mixed in the ratio 4 : 1. If 140 kg of cement has been delivered, what mass of sand is needed?
- Two lengths of timber are in the ratio 4 : 7. The longer length is 56 cm. What is the shorter length?
- In a school, the ratio of teachers to students is 1 : 18. If the college has 80 teachers, how many students are there?
- The speed of two boats is in the ratio 7 : 4. The speed of the slower boat is 10 km/h. Find the speed of the faster boat.

See Example 5

See Example 6



Worked solutions

Exercise 11-03

MAT08NAWS10079

- In a rectangle, the ratio of the width to the length is 5 : 12. The length is 48 cm.
a Find the width of the rectangle. **b** Find the perimeter of the rectangle.
- An alloy contains copper and iron in the ratio 2 : 5. A quantity of alloy contains 20 kg of iron. What mass of copper does it contain?
- Ali, Bree and Felicity share a Lotto prize in the ratio 10 : 8 : 7. If Bree received \$72,
a how much did the other two receive? **b** what was the total prize money shared?
- In a triangle, the lengths of the sides are in the ratio 3 : 4 : 5. If the longest side is 30 cm long, find the perimeter of the triangle.
- The masses of two packets of detergent are in the ratio 3 : 10.
a If the lighter packet has a mass of 1.5 kg, what is the mass of the larger one?
b If the heavier packet costs \$12.50 and the lighter packet costs \$3.90, which packet is the cheaper per kilogram and by how much?
- Enzo's Produce Store buys fruit and vegetables in the ratio 5 : 7. The mass of fruit ordered is 8.5 tonnes. What is the total mass of produce ordered?



- Simon and Joshua's heights are in the ratio 8 : 9. If Joshua is 1.71 m tall, how tall is Simon?
- In an outback mining town, the ratio of women to men is 2 : 5. If there are 240 women, how many men and women are there in the town altogether? Select **A**, **B**, **C** or **D**.
A 840 **B** 600 **C** 96 **D** 1680

Worked solutions

Exercise 11-03

MAT08NAWS10079

- 15 To make Superglue, the contents of Tube A and Tube B are mixed in the ratio 3 : 1.
- If 15 mL of Tube A is used, how much of Tube B is needed?
 - How much glue is made altogether if 15 mL of Tube B is used?

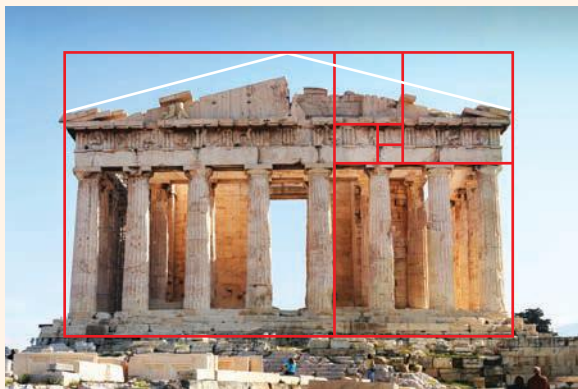
Just for the record

The golden ratio

Ancient Greek mathematicians and artists discovered a 'perfect' rectangle where the ratio of its length to width followed this rule:

$$\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}.$$

The exact value of the ratio is the surd $\frac{\sqrt{5}-1}{2}$, which is equal to 1.618033989 ... This ratio is called the **golden ratio** and a rectangle with these dimensions is called a **golden rectangle**. The ancient Greeks used the golden ratio in art and architecture. Many buildings used golden rectangles as it was thought to be the most pleasing of all rectangles. The Parthenon in Athens was built to the dimensions of the Golden Rectangle.



The golden ratio also appears in nature, such as in the contours of Nautilus shells.

- Find another application of the golden ratio or golden rectangle.
- Find the special relationship between the golden ratio and its reciprocal.

11-04 Scale maps and plans

Scale maps and plans are a special application of ratios used to represent real locations and buildings. Lengths and distances on the scale diagrams are in the same ratio as the real lengths and distances.

Summary

The scale ratio on a scale diagram is written in the form
Scaled length : real length
 where **scaled length** is the length on the diagram.

For example, a scale ratio of 1 : 100 means that the real lengths are 100 times larger than the lengths on the diagram.

Map scales

Map scales are often expressed in the same form as ratios. A scale of 1 cm : 1 km means that 1 cm on the map represents an actual distance of 1 km.

Worksheet

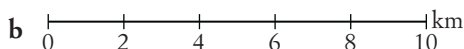
Map of Adelaide

MAT08NAWK10103

Example 7

Simplify each map scale.

a 1 cm : 1 km



Solution

a 1 cm : 1 km = 1 cm : 1000 m

1 km = 1000 m

= 1 cm : 100 000 cm

1 m = 100 cm

= 1 : 100 000

b The length of the segment from 0 to 10 on the diagram is 5 cm.

So 5 cm on the map represents 10 km of actual distance.

Scale = 5 cm : 10 km

= 5 cm : 10 × 1000 × 100 cm

Convert 10 km to cm.

= 5 cm : 1 000 000 cm

= 1 : 200 000

Simplifying the ratio.

Example 8

A map has a scale of 1 : 25 000.

a What is the actual distance if the scaled distance is 4 cm?

b What is the scaled distance if the actual distance is 3.5 km?

Solution

a Scaled distance = 4 cm

Actual distance = 4 × 25 000 cm

= 100 000 cm

= 1000 m

1 m = 100 cm

= 1 km

1 km = 1000 m

b Actual distance = 3.5 km

= 3500 m

1 km = 1000 m

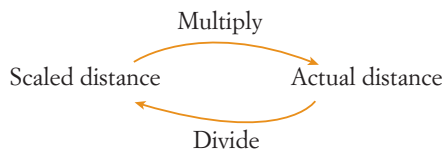
= 350 000 cm

1 m = 100 cm

Scaled distance = 350 000 cm ÷ 25 000

= 14 cm

Scale plans



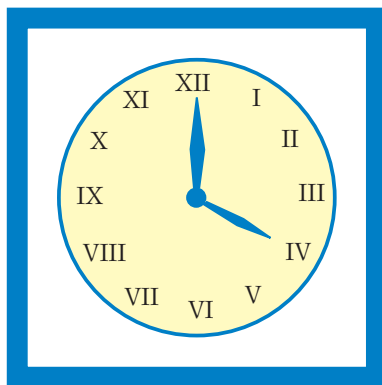
Example 9

Worksheet

Scale drawings

MAT08NAWK10102

This diagram of a clock is drawn to a scale of 1 : 6. Measure its length and calculate its actual length.



Scale 1 : 6

Solution

Scaled length = 5 cm

by measurement

$$\begin{aligned}\text{Actual length} &= 5 \text{ cm} \times 6 \\ &= 30 \text{ cm}\end{aligned}$$

Example 10

This drawing of a screw is drawn to a scale of 5 : 1. Find its actual length.



Scale 5 : 1

Solution

A scale of 5 : 1 means that the real screw is five times **smaller** than the one drawn.

Scaled length = 4 cm

by measurement

$$\begin{aligned}\text{Actual length} &= 4 \text{ cm} \div 5 \\ &= 0.8 \text{ cm}\end{aligned}$$

Exercise 11-04 Scale maps and plans

1 Simplify each map scale.

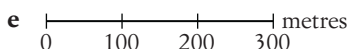
See Example 7

a $1 \text{ cm} : 5 \text{ km}$

b $1 \text{ mm} : 1 \text{ m}$

c $1 \text{ cm} : 500 \text{ km}$

d $1 \text{ cm} : 25 \text{ km}$



2 A map has a scale of $1 : 50\,000$. Which of the following distances in kilometres is represented by 64 mm on the map? Select the correct answer **A**, **B**, **C** or **D**.

See Example 8

A 0.32 km

B 3.2 km

C 32 km

D 320 km

3 A street directory uses a scale of $1 \text{ cm} : 200 \text{ m}$.

a Simplify this ratio.

b Find the actual distance, in kilometres, represented by each scaled distance.

i 7 cm

ii 9.5 cm

iii 12.4 cm

c Find the scaled distance, in centimetres, used to represent each actual distance.

i 18 km

ii 1500 m

iii 9.6 km

4 On a map using a scale of $1 : 10\,000\,000$, the Nile (world's longest river) would be the length of an average shoe lace (about 66.7 cm). How many kilometres long is the Nile River?

5 The town of Gilgandra is 66 km north of Dubbo. On a map with a scale of $1 : 100\,000$, what is the scaled distance between the two towns?

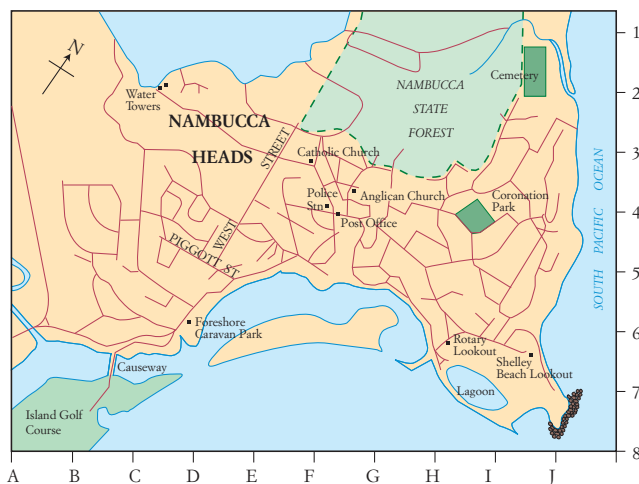
6 Lord Howe Island is 2.8 km wide. How long would its scaled width be on a map with a scale of $1 : 50\,000$?

7 This map of Nambucca Heads has a scale of $1 : 40\,000$.

Worked solutions

Exercise 11-04

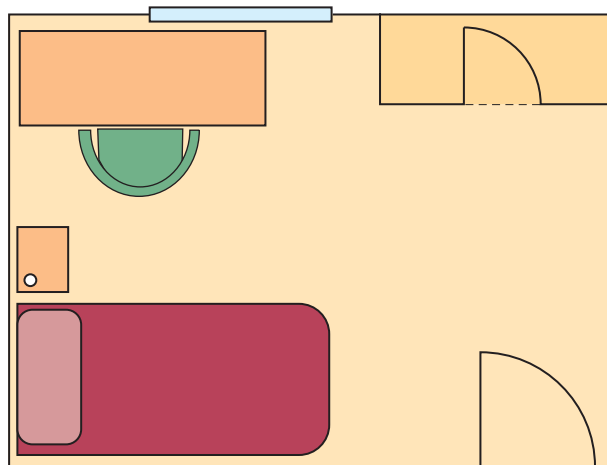
MAT08NAWS10080



- a Find the distance between:
- i the Water Towers (C2) and the Catholic Church (F3)
 - ii the Anglican Church (G4) and Coronation Park (I4)
 - iii Rotary Lookout (H6) and Shelley Beach Lookout (J6)
 - iv the post office (F4) and the Foreshore Caravan Park
- b Find the length of:
- i West Street (E4)
 - ii Piggott Street (D5)
- c How long is the lagoon (I7)?
- d What are the dimensions of the cemetery (J2)?
- e How long is the causeway leading to the Island Golf Course (B7)?
- f To train for a fun run, Tegan decides to run 8 km three times a week. What distance will this be on the map? Outline a possible course for her training run, starting and finishing at the Foreshore Caravan Park (D6).

See Example 9

- 8 This is a scale plan of a bedroom.

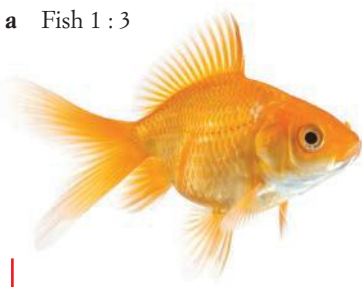


Scale 1 : 50

By measurement and calculation, find the actual:

- a length of the bedroom
 - b width of the doorway
 - c length of the bed
 - d length of the window
 - e length of the table
 - f area of the bedroom
- 9 Measure the length of each scaled-down image below, then use the scale ratio to calculate its actual length.

- a Fish 1 : 3



Length

- b House
1 : 300



Length

c Pen 1 : 4



d Tennis racquet 1 : 16



- 10 An electronics engineer designs a mobile phone SIM card using a diagram with a scale of 100 : 1. If the scaled drawing is 80 cm long, what is its actual length in millimetres? Select the correct answer **A**, **B**, **C** or **D**.

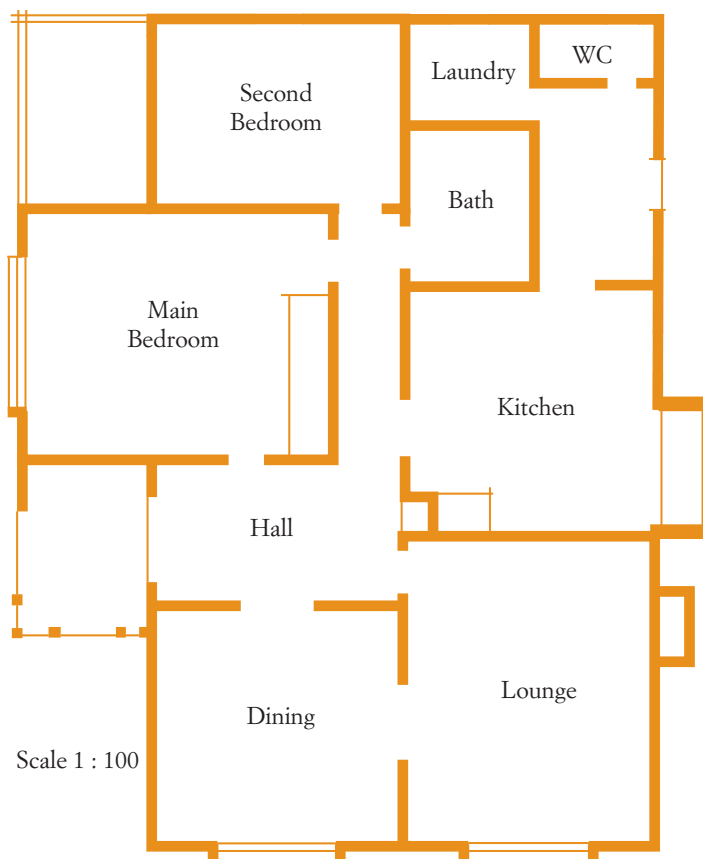
A 80

B 8

C 0.8

D 0.08

- 11 This house plan is drawn to a scale of 1 : 100.



Measure and calculate:

a the length of the main bedroom	b the length of the window in that room
c the length of the laundry	d the area of the bathroom
e the longer side of the lounge room	f the area of the dining room

See Example 10

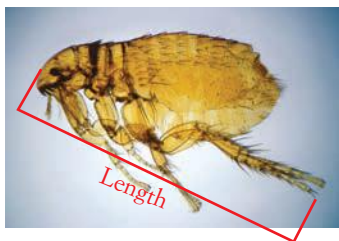
Worked solutions

Exercise 11-04

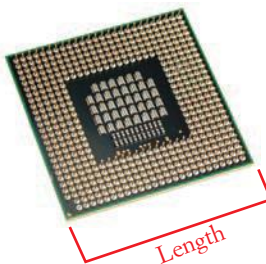
MAT08NAWS10080

- 12 Measure the length of each magnified image below, then use the scale ratio to calculate its actual length.

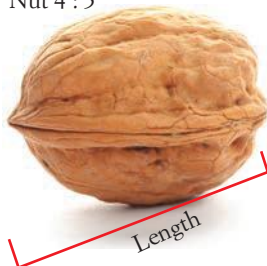
a Flea 100 : 1



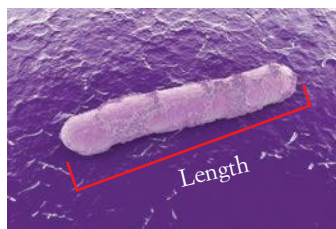
b Microchip 3 : 1



c Nut 4 : 3



d Bacterium 100 : 1



Investigation: Scale drawings

- 1 Measure the dimensions of your bedroom and the dimensions of the items of furniture in it.
- 2 Using a scale of 1 : 100, draw a scale diagram of your bedroom on graph paper.
- 3 Draw scale diagrams of each piece of furniture on another piece of graph paper, then cut them out.
- 4 Rearrange the furniture in your bedroom on the scale diagram to see which arrangement you prefer best.

Mental skills 11

Maths without calculators

24-hour time

24-hour time	12-hour time	24-hour time	12-hour time
0000	12:00 a.m. (midnight)	1200	12:00 p.m. (midday)
0100	1:00 a.m.	1300	1:00 p.m.
0200	2:00 a.m.	1400	2:00 p.m.
0300	3:00 a.m.	1500	3:00 p.m.
0400	4:00 a.m.	1600	4:00 p.m.
0500	5:00 a.m.	1700	5:00 p.m.
0600	6:00 a.m.	1800	6:00 p.m.
0700	7:00 a.m.	1900	7:00 p.m.
0800	8:00 a.m.	2000	8:00 p.m.

24-hour time	12-hour time	24-hour time	12-hour time
0900	9:00 a.m.	2100	9:00 p.m.
1000	10:00 a.m.	2200	10:00 p.m.
1100	11:00 a.m.	2300	11:00 p.m.

To convert from 24-hour time to 12-hour time:

- if it begins with '00', then it is the 12:00 a.m. (midnight) hour
- if it begins with '12', then it is the 12:00 p.m. (midday) hour
- if it is less than 1200, then it is a.m. (morning)
- if it is 1300 or more, then it is p.m. (afternoon/evening) time: subtract 12 from the hour.

1 Study each example.

a Convert 1850 to 12-hour time.

1800 > 1300, so it is p.m. time, so subtract 12 from the hour.

$$18 - 12 = 6$$

$$1850 = 6:50 \text{ p.m.}$$

b Convert 0430 to 12-hour time.

0430 < 1200, so it is a.m. time.

$$0430 = 4:30 \text{ a.m.}$$

c Convert 0015 to 12-hour time.

0015 begins with 00, so it is 12:00 a.m. time

$$0015 = 12:15 \text{ a.m.}$$

2 Now convert each time to 12-hour time.

a 0845

b 1320

c 1750

d 0017

e 2105

f 1832

g 1115

h 0238

i 1440

j 0320

k 1655

l 2331

m 0108

n 1018

o 2000

p 0643

To convert from 12-hour time to 24-hour time:

- if it is the 12:00 a.m. (midnight) hour, begin with '00'
- if it is 'a.m.' time or the 12:00 p.m. (midday) hour, write as is but make sure the hour has two digits (for example, 02, 09).
- if it is 1:00 p.m. or later, then add 12 to the hour.

3 Study each example.

a Convert 4:10 a.m. to 24-hour time.

It is 'a.m.' time, so write as a 4-digit number and rename 4 as 04.

$$4:10 \text{ a.m.} = 0410$$

b Convert 4:10 p.m. to 24-hour time.

It is after 1 p.m., so add 12 to the hour: $4 + 12 = 16$

$$4:10 \text{ p.m.} = 1610$$

c Convert 12:47 a.m. to 24-hour time.

It is in the 12:00 a.m. (midnight) hour, so change the 12 to 00.

$$12:47 \text{ a.m.} = 0047$$

4 Now convert each time to 24-hour time.

a 6:35 p.m.

b 8:05 a.m.

c 11:45 a.m.

d 11:20 p.m.

e 2:21 a.m.

f 12:30 p.m.

g 3:48 p.m.

h 7:11 p.m.

i 9:08 a.m.

j 9:50 p.m.

k 12:42 a.m.

l 7:39 a.m.

m 1:59 a.m.

n 10:18 p.m.

o 10:46 a.m.

p 5:23 p.m.

Homework sheet

Ratios 2

MAT08NAHS10013

11-05 Dividing a quantity in a given ratio

Elyse and Henry won a cash prize of \$420 for winning an art competition but instead of dividing the money evenly (\$210 each), they decide to divide it in the ratio 3 : 4, so that Elyse receives 3 parts while Henry receives 4 parts. Henry receives more because he did more of the work and paid more when buying the materials for the artwork.

Problems involving dividing a quantity in a given ratio can be solved using the **unitary method** or **fraction method**.

Example 11

Divide a cash prize of \$420 between Elyse and Henry in the ratio 3 : 4.

Solution

Method 1: Unitary method

Total number of parts = $3 + 4 = 7$

7 parts = \$420

1 part = $\$420 \div 7$

= \$60

Elyse's share (3 parts) = $3 \times \$60$

= \$180

Henry's share (4 parts) = $4 \times \$60$

= \$240

$\therefore$ Elyse receives \$180 and Henry receives \$240.

Method 2: Fraction method

Total number of parts = $3 + 4 = 7$

Elyse's share (3 parts) = $\frac{3}{7} \times \$420$

= \$180

Henry's share (4 parts) = $\frac{4}{7} \times \$420$

= \$240

Check: $\$180 + \$240 = \$420$

Example 12

Nick, Ian and Rob agreed to divide the profits of their removalist business in the ratio 2 : 3 : 5. If their profit this year was \$45 000, find the size of each person's share of the profits.

Solution

Method 1: Unitary method

Total number of parts = $2 + 3 + 5 = 10$

10 parts = \$45 000

1 part = $\$45\ 000 \div 10$

= \$4500

Nick's share = $2 \times \$4500$

= \$9000

Ian's share = $3 \times \$4500$

= \$13 500

Method 2: Fraction method

Total number of parts = $2 + 3 + 5 = 10$

Nick's share = $\frac{2}{10} \times \$45\ 000$

= \$9000

Ian's share = $\frac{3}{10} \times \$45\ 000$

= \$13 500

Rob's share = $\frac{5}{10} \times \$45\ 000$

= \$22 500

Video tutorial

Dividing a quantity in a given ratio

MAT08NAVT10020

Animated example

Shares in ratios

MAT08NAAE00011

Technology worksheet

Excel: Applying ratios

MAT08NACT00009

Technology

Excel: Applying ratios

MAT08NACT00023

$$\text{Rob's share} = 5 \times \$4500$$

$$= \$22\,500$$

$\therefore$ Nick, Ian and Rob receive \$9000,
\$13 500 and \$22 500 respectively.

$$\text{Check: } \$9000 + \$13\,500 + \$22\,500 = \$45\,000$$

Exercise 11-05 Dividing a quantity in a given ratio

- Find the total number of parts if the ratio is:
 a $2 : 7$ b $4 : 1$ c $3 : 4$ d $2 : 5 : 6$
- Divide \$500 in the ratio:
 a $4 : 1$ b $7 : 3$
- Phuong, Janelle and Ahmet bought a Lotto ticket for \$12. They contributed \$3, \$3 and \$6 respectively to the purchase price. They won \$4000 and agree to split the winnings in the same ratio.
 a Simplify the ratio $3 : 3 : 6$.
 b How much prize money does each person get?
- Brad and Angelina share the weekly rent of \$450 in the ratio $3 : 2$. What is Angelina's share?

See Example 11



- Divide 450 kg in the ratio:
 a $4 : 5$ b $3 : 2$
- Divide 720 cm in the ratio:
 a $1 : 3 : 5$ b $5 : 3 : 4$
- Company directors Judy, Gas and Robert share the company profits in the ratio $5 : 3 : 3$. Which of the following is the amount that Judy receives in a year when profits are \$121 000? Select the correct answer **A**, **B**, **C** or **D**.
A \$11 000 **B** \$24 000 **C** \$33 000 **D** \$55 000
- In Year 8, the ratio of boys to girls is $4 : 5$. If there are 225 students in Year 8, find how many girls there are.
- In Year 9, the ratio of boys to girls is $3 : 2$. If there are 125 students in Year 9, find how many more boys there are than girls.
- Adam needs to make 800 g of short-crust pastry. Flour and butter are needed in the ratio $3 : 1$. How much flour is needed?

See Example 12

Worked solutions

Exercise 11-05

MAT08NAWS10081

- 11 A company posts 1386 letters in a week. The ratio of local to overseas letters is 2 : 7. How many overseas letters are sent in a week?
- 12 A truck carries fruit and vegetable boxes in the ratio 5 : 7. If it carries a total mass of 7.5 tonnes, what mass of vegetables does it carry?
- 13 An alloy of mass 176 kg is made from copper and zinc in the ratio 5 : 6. Find the mass of copper in the alloy.
- 14 At a school, the ratio of students who speak a second language to students who speak only English is 5 : 8. If there are 923 students at the school, how many students speak only English?
- 15 A 20 m cable is cut into three sections in the ratio 2 : 3 : 5. Find the length of each section.
- 16 When making mortar, sand and concrete is mixed in the ratio 6 : 1. If we need 280 kg of mortar, how much sand will we need?
- 17 In a study of 225 000 people, it was found that the ratio of right-handed people to left-handed people was 13 : 2.
- How many left-handed people were there?
 - How many more right-handed people than left-handed people were there?
- 18 Angus earns twice as much as Catriona. If the sum of their wages is \$210 000, how much does each earn?

Worked solutions

Exercise 11-05

MAT08NAWS10081

Worksheet

Rates

MAT08NAWK00044

11-06 Rates

While a ratio compares two or more quantities measured in the same units, a **rate** compares two quantities measured in **different units**.

A rate shows how one quantity changes with another quantity. We write a rate using a '/' symbol in the form 'something *per* something else'. For example:

- the price of petrol is stated in cents per litre, or c/L
- your heart beats at a rate measured in beats per minute, or beats/min
- the speed of a car is measured in kilometres per hour, or km/h.

The word 'per' means 'for each' so we express a rate '*per* single unit'. For instance, travelling at a rate of 50 km/h means travelling 50 km in *each* hour.

Example 13

Write each statement as a simplified rate.

- A factory produces 87 cars in 3 hours
- Ham costs \$50 for 8 kg

Solution

- The production rate would be expressed in cars per hour.

$$\begin{aligned}\text{Production rate} &= \frac{87 \text{ cars}}{3 \text{ hours}} \\ &= 29 \text{ cars/hour}\end{aligned}$$

We can write a rate as a fraction: divide the number of cars by the number of hours

- b The cost would be expressed in dollars per kilogram.

$$\begin{aligned}\text{Cost} &= \frac{\$50}{8 \text{ kg}} \\ &= \$6.25/\text{kg}\end{aligned}$$

Divide the number of dollars
by the number of kilograms

Exercise 11-06 Rates

- Write the units suitable for each rate below, in the form ____/____.

a typing speed	b heartbeat rate
c cost of a mobile phone call	d cost of bananas
e a person's wage	f cost of petrol
g population growth	h the cost of water
i population density	
- Write each statement as a simplified rate.

a 51 sheep in 3 hours	b \$10.75 for 2.5 kg
c 208 students for 8 teachers	d 136 points in 4 games
e 546 words in 6 minutes	f 34 articles in 4 hours
g 72 cars in 14 days	h 5040 boxes in 8 hours
i 259 metres in 7 seconds	j 46 000 bottles in 50 hours
k 7944 revolutions in 6 minutes	l \$175 for 5 hours
m 448 km in 8 hours	n \$16.50 for 6 kg
o 114 runs in 24 overs	p 243 km using 30 litres
q \$126 for 12 hours	r 2520 kg for 60 hectares
- The cost of sending a 5.5 kg parcel to Malaysia is \$88. What is the postage rate? Select the correct answer **A**, **B**, **C** or **D**.
A \$0.34/kg **B** \$16/kg **C** \$0.07/kg **D** \$484/kg
- In your own words, explain what is meant by each rate.

a a speed of 100 km/h
b a traffic flow of 150 cars/h
c petrol consumption of 10.3 L/100 km
d a farmer keeping 60 sheep/hectare
- A lift should carry no more than 1600 kg or 20 people. What is this weight allowance, in kg/person?
- A pulp mill clears 27 000 hectares of forest in 15 years. At what rate in hectares/year does the mill clear the forest?
- The cost of 53 litres of petrol is \$73.67. Express this cost in c/L.
- A complaints hotline took 2190 calls in one year. Calculate the number of calls per month.

See Example 13

11-07 Best buys

Often it is difficult to decide whether you are getting the best value for money. Shoppers compare both the price and the size of items when they look for the **best buy** ('best value for money'). The biggest container does not always provide the best value. The best buy can be found by comparing the **unit cost** (cost of one item or unit) of each brand or size. A unit could be one gram or one millilitre.

Summary

Unit cost = cost $\div$ number of items or units

If you look carefully at price tags on supermarket shelves, you will notice unit prices displayed in small print under the main prices.

Example 14

Which brand of baked beans is the better buy?



Solution

Calculating the unit cost (cost per gram) for each brand:

Bean There = $\$4.60 \div 500 = \$0.0092 / \text{g}$

Mr Beanz = $\$1.65 \div 200 = \$0.00825 / \text{g}$

Mr Beanz is the better buy.

It has the lower unit cost.

Exercise 11-07 Best buys

See Example 14

- Find the better buy in each pair of items.
 - A 2 kg box of sultanas for \$8.85 or a 1 kg box for \$4.65
 - Six DVDs for \$15.50 or 8 DVDs for \$20
 - 45 g chips for \$2.10 or 150 g for \$6.50
 - Two tubs of yoghurt for \$1.24 or 7 tubs for \$4.30
 - 3 kg of corn flour for \$5.85 or 500 g of corn flour for \$1
 - 600 mL bottle of fruit juice for \$3.20 or a 2.25 L carton for \$8.05
- Michael purchased 400 g of soccerball ham for \$3.59, while Kim purchased 300 g for \$2.54. Who got the better buy?

- 3 Which size of Fruit Bix cereal is the best buy?



- 4 Happy Tam cat food can be purchased in four different packages.

Pack 1: 24×250 g cans for \$39.99

Pack 2: 12×300 g cans for \$24.99

Pack 3: 9×400 g cans for \$25.99

Pack 4: 6×1 kg can for \$38.50

Which of the following shows the packs ordered from best to worst buy? Select the correct answer **A**, **B**, **C** or **D**.

A 4 2 1 3

B 3 2 1 4

C 4 1 2 3

D 2 1 3 4

- 5 For each item, find which size is the best value for money.

a



b



c



d



- 6 For each item, which size gives the best value for money?

		Small	Medium	Large
a	Washing powder	450 g for \$3.99	600 g for \$5.15	1 kg for \$8.95
b	Margarine	350 g for \$4.15	500 g for \$5.99	750 g for \$8.50
c	Ice cream	500 mL for \$1.39	750 mL for \$2.30	2 L for \$5.99
d	Peas	400 g for \$2.05	500 g for \$2.49	600 g for \$2.50

Worked solutions

Exercise 11-07

MAT08NAWS10082

Just for the record

In 2009, the Australian Government made it compulsory for supermarkets to display unit prices when selling certain grocery items. Unit pricing means displaying the price of goods per unit of measure:

- volume: per 100 millilitres
- weight: per 100 grams
- length: per metre
- area: per square metre
- for 40 or fewer items: per item
- for over 40 items: per 100 items

For example:

MUSHROOM SOUP \$2.38ea

350g

||||||| || |||||

\$0.68/100g

MUSHROOM SOUP \$3.20ea

500g

||||||| || |||||

\$0.64/100g

Which size of mushroom soup is the better buy?



Worksheet

Rate problems

MAT08NAWK10104

Puzzle sheet

Rates problems

MAT08NAPS00027

Worksheet

Ratios and rates review

MAT08NAWK00049

11-08 Rate problems

Problems involving rates can usually be solved by multiplying or dividing. The following strategy may help.

- Write the units of the rate x/y as a fraction: $\frac{x}{y}$
- To find the quantity in the numerator, x , **multiply** by the rate.
- To find the quantity in the denominator, y , **divide** by the rate.

Example 15

Ben types 55 words per minute. How many words can he type in 20 minutes?

Solution

The units of the rate expressed as a fraction is $\frac{\text{words}}{\text{min}}$.

To find the number of **words** (the numerator), **multiply** by the rate.

$$\begin{aligned}\text{Number of words} &= 20 \times 55 \\ &= 1100\end{aligned}$$

← No. of minutes $\times$ words typed per minute

Example 16

A factory makes pens at the rate of 50 pens per minute.

- How many hours and minutes will it take to produce 10 000 pens?
- How many pens are produced in an 8-hour day at the factory?

Solution

The units of the rate expressed as a fraction are $\frac{\text{pens}}{\text{min}}$.

- To find the number of **minutes** (the denominator), **divide** by the rate.

$$\begin{aligned}\text{Time taken} &= 10\,000 \div 50 \\ &= 200 \text{ minutes} \\ &= \frac{200}{60} \text{ hours} \\ &= 3 \text{ h } 20 \text{ min}\end{aligned}$$

How many times 50 divides into 10 000

- To find the number of **pens** (the numerator), **multiply** by the rate.

$$\begin{aligned}\text{No. of pens in one hour} &= 60 \times 50 \\ &= 3000 \\ \text{No. of pens in 8 hours} &= 8 \times 3000 \\ &= 24\,000\end{aligned}$$

$$1 \text{ h} = 60 \text{ min}$$

$$\text{No. of mins} \times \text{pens per min}$$

Video tutorial

Rate problems

MAT08NAVT10021

Exercise 11-08 Rate problems

- Marisol is paid \$17.80 per hour. How much will she earn if she works 38 hours in a week?
- A batsman scores at a rate of 32 runs per innings. How many runs will he score in 5 innings?
- A car uses 14 litres of petrol to travel 147 kilometres. How far can it travel on:
 - one litre of petrol?
 - 20 litres of petrol?
- A farmer can graze 16 sheep per hectare. If he has 21 hectares set aside for sheep, how many sheep can he graze?
- A tap drips water at a rate of 18 mL/h. How much water would be wasted in one day?
- An aeroplane can carry a total of 450 passengers per flight. How many flights will it take to carry 2 700 passengers?
- The Wong family needs to buy new carpet for the family home. The carpet costs \$125/m and the carpet layers charge \$480 to lay it. How much will it cost to carpet the house if the Wongs need 28 metres of carpet?
- A farmer uses fertiliser at a rate of 28 kg/ha. How many hectares can she cover if she has 200 kg of fertiliser?
- Tatiana is paid \$17.50/hour to babysit. Find:
 - how much she earns for babysitting for 4 hours
 - how long she babysat to earn \$43.75.

See Example 15

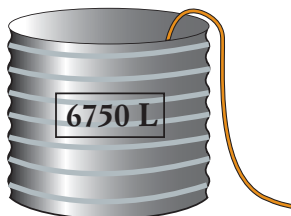
See Example 16

Worked solutions

Exercise 11-08

MAT08NAWS10083

- 10 An aircraft travelled at 900 km/h. How far did it travel in 20 minutes?
- 11 a Petrol costs \$1.35/L. If Rocky had \$20 in his pocket, how much petrol could he buy (correct to the nearest 0.1 litre)?
 b If Rocky's car can travel 9 km on one litre of petrol, how far can he travel on the petrol he bought with \$20?
- 12 Smokers lose approximately 3% of their lung capacity per year. If Phil with 100% lung capacity starts smoking at 20 years of age, at what age will he have 25% lung capacity? Select the correct answer A, B, C or D.
 A 75 B 28 C 45 D 33
- 13 The temperature in Mittagong at 7:00 a.m. is 8°C. If the temperature rises at a rate of 3°C/h, find:
 a the temperature at 10 a.m. and at 2:00 p.m.
 b the time at which the temperature will be 23°C.
- 14 David and Bobby take turns mowing a rectangular lawn 60 m long and 40 m wide.
 a Find the area of the lawn.
 b When David cuts the grass, he does it in one hour using an old lawn mower. What is his mowing rate in m^2/min ?
 c Bobby uses a new lawn mower and it takes him 40 minutes. What is his mowing rate in m^2/min ?
 d How long will it take them to mow the lawn if they mowed together?
- 15 Jane is filling this tank with a pipe that pumps water at 1.25 litres per second.



How long will it take to fill the tank:

- a in seconds? b in minutes? c in hours?

- 16 Kim and Sonya started their holiday on a full tank of petrol. The reading on the odometer (in km) was

0	3	4	5	6	8

.

When the tank became empty, the reading was

0	3	5	2	1	8

.

Petrol cost \$1.54/litre and they needed to pay \$73.92 to fill the tank.

- a How many kilometres did they travel on a full tank of petrol?
 b How much petrol is there in a full tank?
 c How far can their car travel on 1 L of petrol? Answer to the nearest 0.1 km.

Worksheet

What's my speed?

MAT08NAWK10105

11-09 Speed

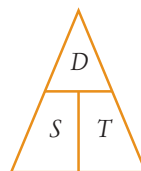
Speed is a rate that compares the distance travelled with the time taken. Average speed is calculated by dividing the distance travelled by the time taken.

Summary

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Speed can be measured in kilometres/hour (km/h) or metres/second (m/s).

The relationship between distance, speed and time can be remembered using this triangle.



If we cover the quantity we are looking for, the rest of the diagram tells us what to do.

- To find the **speed**, cover the S and we are left with $\frac{D}{T}$, so speed = distance $\div$ time
- To find the **distance**, cover the D and we are left with $S \times T$, so distance = speed $\times$ time
- To find the **time**, cover the T and we are left with $\frac{D}{S}$, so time = distance $\div$ speed

Example 17

A bullet train travels 320 km in 1 hour 20 minutes.

- Calculate its average speed.
- How long will it take to travel 1000 km?
- How far will it travel in 5 hours?

Solution

$$\begin{aligned} \text{a Average speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{320 \text{ km}}{1 \text{ h } 20 \text{ min}} \\ &= \frac{320 \text{ km}}{1 \frac{1}{3} \text{ h}} \\ &= 240 \text{ km/h} \end{aligned}$$

$$20 \text{ min} = \frac{20}{60} \text{ h} = \frac{1}{3} \text{ h}$$

$$\begin{aligned} \text{b Time} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{1000}{240} \text{ h} \\ &= 4 \frac{1}{6} \text{ h} \\ &= 4 \text{ h } 10 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{c Distance} &= \text{speed} \times \text{time} \\ &= 240 \times 5 \\ &= 1200 \text{ km} \end{aligned}$$

By the speed triangle, or because we divide by the rate to find the h in $\frac{\text{km}}{\text{h}}$.

Press $\circ \div \circ$ or **DMS** on the calculator.

By the speed triangle, or because we multiply by the rate to find the km in $\frac{\text{km}}{\text{h}}$.

Homework sheet

Rates

MAT08NAHS10014

Puzzle sheet

Speed

MAT08NAPS00024

Worksheet

Speed

MAT08NAWK00045

Animated example

Rates: Triangle method

MAT08NAEE00010

Exercise 11-09 Speed

See Example 17

Quiz

Applying ratios and rates

MAT08NAQZ00007



- 1 Find the average speed in km/h for each statement.
 - a A horse rider travels a distance of 15 km in 3 hours.
 - b A jet travels 2400 km in 5 hours.
 - c A bushwalker walks 25 km in 5 hours.
 - d A cyclist rides 85 km in 5 hours.
 - e A motorbike travels 250 km in 2.5 hours.
 - f A car travels 280 km in 3.5 hours.
 - g A truck travels 390 km in 3 hours and 15 minutes.
 - h A car travels 90 km in 45 minutes.
 - i A motorbike on the race track travels 250 km in 1 hour and 15 minutes.
 - j A boat travels 15 km in 1 hour and 40 minutes.
 - k An athlete runs 200 m in 20 seconds.
 - l A kangaroo travels 1.5 km in 15 minutes.
- 2 Find the average speed in m/s for each statement.
 - a An athlete runs 200 m in 20 seconds.
 - b A swimmer swims 100 m in 64 seconds.
 - c A bird flies 348 m in 6 seconds.
 - d A cyclist travels 960 m in 2 minutes.
 - e A swimmer swims 1500 m in 16 minutes 40 seconds.
- 3 A boat travels 24 km in 90 minutes. Which of the following is its average speed? Select the correct answer **A**, **B**, **C** or **D**.

A 12 km/h	B 16 km/h	C 18 km/h	D 36 km/h
- 4 Peter walked to school in 15 minutes, a distance of 1500 metres. Find his speed in:
 - a metres per minute
 - b kilometres per hour
- 5 Mrs Hassam lives 20 kilometres from the nearest railway station and it takes her 30 minutes to drive there in her car. Find her average speed in kilometres per hour.
- 6 A racing car travels at an average speed of 210 km/h for 2 hours and 20 minutes. How far did it travel? Select the correct answer **A**, **B**, **C** or **D**.

A 90 km	B 294 km	C 462 km	D 490 km

Worked solutions

Exercise 11-09

MAT08NAWS10084

- 7 Find the distance travelled for each statement.
 - a A car travels for 8 hours at an average speed of 90 km/h.
 - b A bushwalker travels for 7 hours and 30 minutes at an average speed of 4 km/h.
 - c A truck travels for 14 hours at an average speed of 95 km/h.
- 8 Majid can walk at a speed of 6 km/h. How far can he walk in 30 minutes?
- 9 A bus travels at 75 km/h for 5 hours, and then travels a further distance of 200 km in 4 hours. Find the average speed for the whole journey, correct to one decimal place.
- 10 A car travels for 3 hours at 60 km/h and then travels a further distance of 100 km at 50 km/h. What is the average speed for the trip?
- 11 How long does it take a motor scooter to travel 60 km at a speed of 40 km/h?
- 12 There are other units for measuring speed, such as the **knot** and the **mach**. Find what these special terms mean and where they are used.
- 13 Kieran swims at a speed of 1.85 m/s. What would be his time (to two decimal places) for a 100 metre sprint?
- 14 Sue runs 200 metres in 24 seconds. How far will she run in one minute at the same rate?
- 15 A flywheel rotates at a rate of 2400 revolutions per minute. How many revolutions does it make in 30 seconds?

Worked solutions

Exercise 11-09

MAT08NAWS10084

Technology Travelling distances and times

Visit a map website such as WhereIs or Google Maps for this investigation to examine travelling distances and times between two locations.

Harry lives near the Wyong Hill Reserve in NSW and is travelling to the Tuggerah shopping centre to see a movie.

- 1 Use the website to locate both Wyong Hill Reserve and Tuggerah shopping centre, then find the travelling distance and time for Harry's trip if he is going by car.
- 2 Find the travelling distance and time for Harry's trip if he is travelling on foot (walking).
- 3 How much further would you have to travel if you were going from the Wyong Hill Reserve to the shops by car instead of walking there?
- 4 Would you recommend that Harry travel by car or on foot for the trip? Why?
- 5 a Calculate the average speed of the car for the trip.
b Why do you think this average speed is slower than the normal speed limit of 60 km/h?
- 6 Harry is meeting Alinta at Tuggerah to see a movie. Alinta lives in Berkeley Vale. Find the travelling distance and time for Alinta's trip if she is travelling by car.
- 7 If Alinta's actual driving time was 5 minutes, what was her average speed?
- 8 If Harry's actual speed was 80 km/h, how many minutes and seconds will his trip take?
- 9 So will Harry or Alinta arrive at Tuggerah first? By how minutes and seconds?

Puzzle sheet

Jane's diary

MAT08NAPS10030

Worksheet

Travel graph stories

MAT08NAWK10106

Worksheet

The hare and the
tortoise

MAT08NAWK00048

11-10 Travel graphs

A **travel graph** is a line graph that describes a journey, by comparing distance with time (on the vertical and horizontal axes respectively). The slope or steepness of the graph indicates the speed.

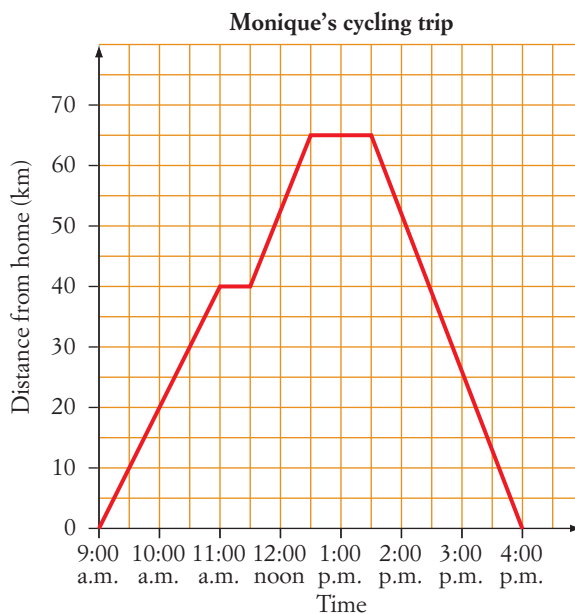
Summary

On a **travel graph**:

- a horizontal (flat) section on the graph indicates a stop
- the steeper the line, the greater the speed (more distance covered in less time)
- a section going down, towards the right, indicates a change in direction or that the traveller is returning towards the start.

Example 18

This graph shows Monique's cycling trip.



- At what time did Monique leave home?
- When was Monique's first stop? How far from home was she?
- Find her average speed over the first two hours.
- What time was it when Monique began her journey home?
- How far did she travel all together?
- Find her average speed during the trip home.
- For how long did Monique stop altogether during the trip?

Solution

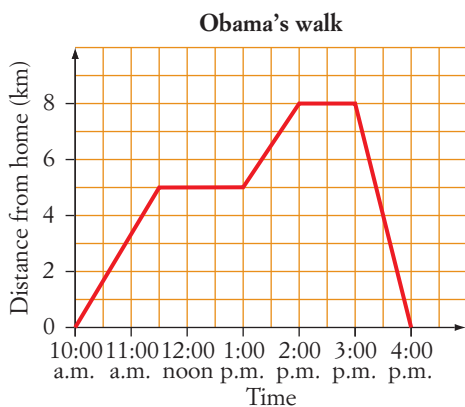
- a Monique left home at 9 a.m. At the start of the graph, when distance = 0.
- b Monique first stopped at 11 a.m., 40 km from home. This is where the graph is flat.
- c Distance = 40 km, time = 2 hours
Average speed = $\frac{40 \text{ km}}{2 \text{ h}} = 20 \text{ km/h}$
- d Monique started returning home at 1:30 p.m. This is where the graph points downward.
- e Monique travelled 65 km, then returned home.
Total distance = $2 \times 65 \text{ km} = 130 \text{ km}$
- f Distance = 65 km, time = $2\frac{1}{2}$ hours
Average speed = $\frac{65 \text{ km}}{2\frac{1}{2} \text{ h}} = 26 \text{ km/h}$
- g First stop: $\frac{1}{2}$ hour. Second stop: 1 hour
Total stopping time = $\frac{1}{2} + 1 = 1\frac{1}{2}$ hours

Where the graph is flat, the distance from home does not change, which means that Monique has stopped

Exercise 11-10 Travel graphs

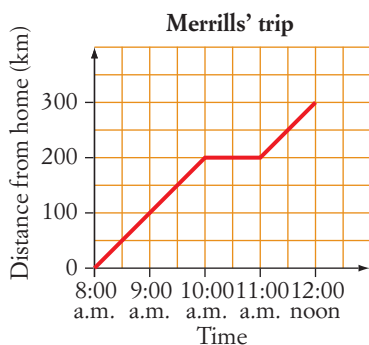
- 1 This travel graph shows Obama's return trip to his friend Julia's house.

See Example 18



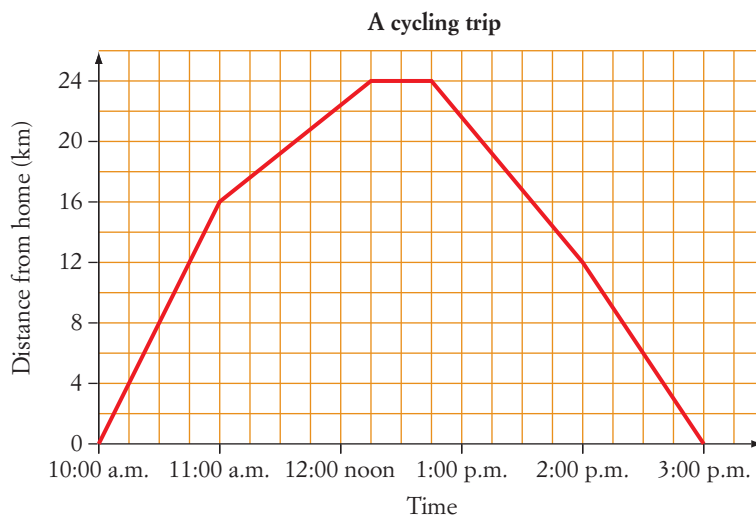
- a How long did it take Obama to walk to Julia's house?
- b How far is Julia's house from Obama's house?
- c Find Obama's average speed:
i before his first stop ii after his first stop
- d How is a higher speed shown on the graph?
- e What was the total distance that Obama travelled?
- f Between what times does Obama stop on his walk?
- g When did Obama start his trip home?
- h How long did it take him to walk home?

- 2 Merrill drives from Sydney to Canberra, stopping to visit friends in Goulburn.

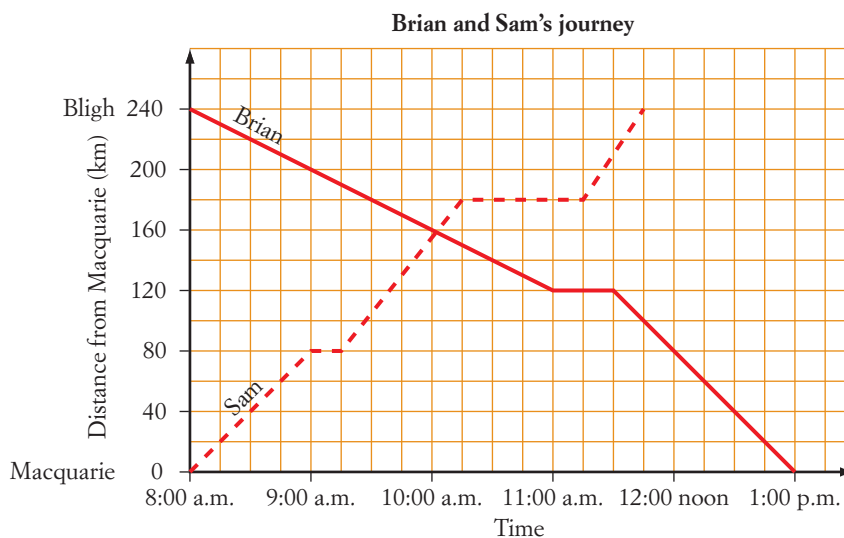


- a How far is Canberra from Sydney?
- b How long does the trip take?
- c For how long does Merrill stop at Goulburn?
- d Find her average speed for the journey (excluding stops).
- e How far is it from Goulburn to Canberra?
- f When does Merrill travel faster: before or after Goulburn?

- 3 This graph shows a cyclist's day trip.



- a At what time did the speed of the cyclist
 - i increase?
 - ii decrease?
 - b When did the cyclist start to return home?
 - c How far did the cyclist travel altogether on this day?
 - d How long did the cyclist spend 'on the road'?
 - e Find the cyclist's average speed for:
 - i the first hour
 - ii 11:00 a.m. to 12:15 p.m.
 - iii 12:45 p.m. to 2:00 p.m.
 - iv the entire day
- 4 Brian travels from Bligh to Macquarie, while Sam travels from Macquarie to Bligh.



- a How far is it between the two towns?
- b Who is travelling faster? How can you tell?

Worked solutions

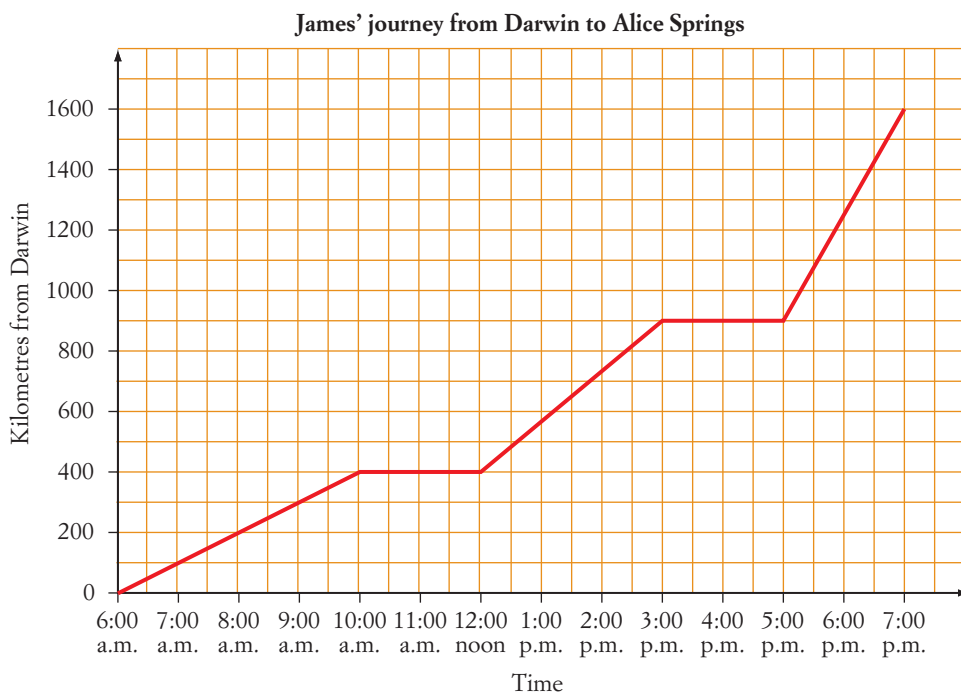
Exercise 11-10

MAT08NAWS10085

- c How far is Brian from Macquarie at 11 a.m.?
- d How far is Sam from Macquarie at 11 a.m.?
- e At what time do Brian and Sam pass each other? How far are they from Macquarie when they pass?
- f Is Sam travelling faster before 9 a.m. or after 9:15 a.m.? How does the graph show this?
- g Calculate Brian's average speed before he stops.
- h For how long did Sam stop altogether on the trip? Select the correct answer **A, B, C or D.**

A 60 minutes **B** 75 minutes **C** 5 minutes **D** 90 minutes

- 5 James travelled from Darwin to Alice Springs as shown in the graph below.

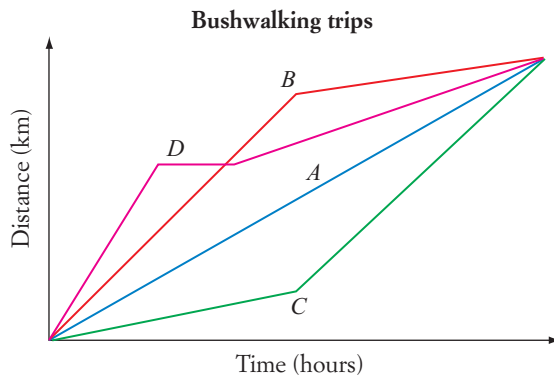


- a How far is Alice Springs from Darwin?
- b When did James arrive in Alice Springs?
- c What was James' average speed in the last two hours?
- d Which of the following statements is **false**? Select the correct answer **A, B, C or D.**
 - A** James stopped for 4 hours altogether.
 - B** James was travelling fastest in the last two hours.
 - C** James' speed for the first 4 hours was 100 km/h.
 - D** James' average speed for the whole trip was 120 km/h.
- e Samantha leaves Darwin one hour after James and travels at a constant speed of 125 km/h towards Alice Springs. Copy the graph and add Samantha's journey to it.
- f Find the approximate time when Samantha overtakes James.

- 6 This travel graph shows Marnie's cycling journey. Write a story about her ride, based on the information in the graph.



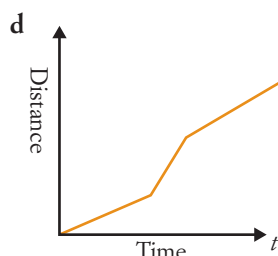
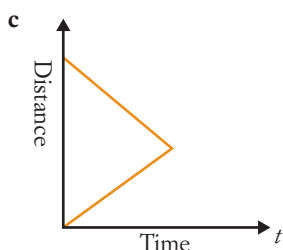
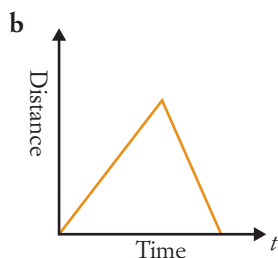
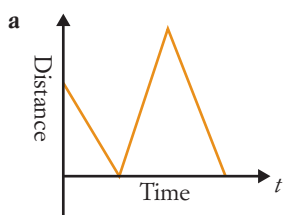
- 7 Match each bushwalker described below with the correct travel graph.



- a Santosh maintained the same speed all day.
- b Michelle was fast at first, but slowed down.
- c Tahlia was the fastest, but then got slower after stopping for lunch.
- d Jim was slow at first, but picked up speed.



- 8 Which one of the graphs below could **not** be a travel graph? Select the correct answer **A**, **B**, **C** or **D**. Why not?



11-11 Sketching informal graphs

Line graphs are useful because they provide a visual picture of the relationship between two variables, for example, distance and time. For many situations or events, the rates of change are not constant, so graphs of these situations or events are more likely to be informal and involve **curves** rather than straight lines.

Example 19

Draw a graph to represent a bath's water level over time.

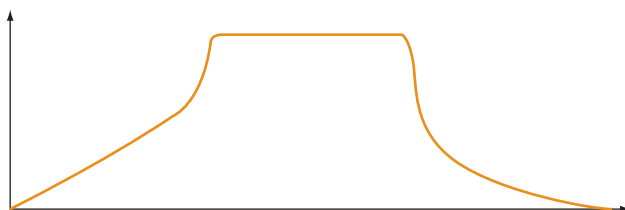
- Harry switches on the tap and the water level gradually rises.
- Harry switches off the tap, gets into the bath and the water level quickly rises.
- Harry stays in the bath for a while before climbing out, reducing the water level again.
- Harry unplugs the bath and it drains completely: the water level decreasing quickly at first, then slowly towards the end.

Solution

The horizontal axis should represent Time.

The vertical axis should represent Height of water.

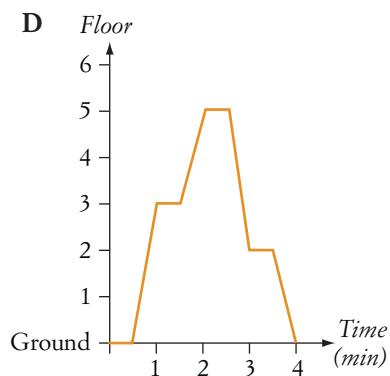
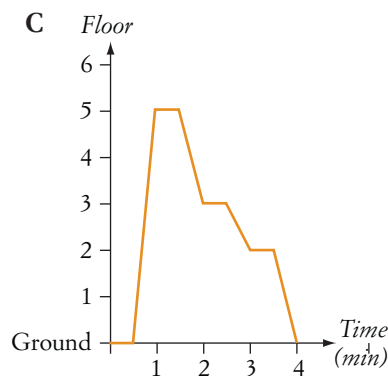
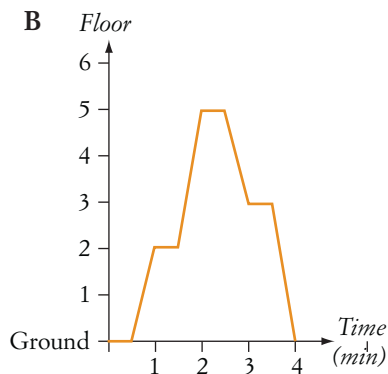
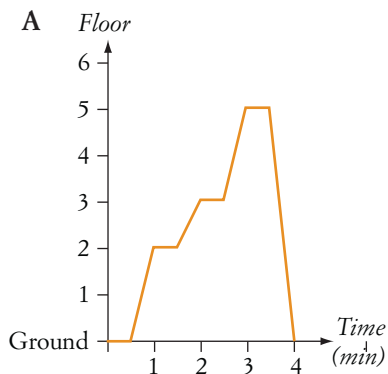
Bath's water level



Exercise 11-11 Sketching informal graphs

See Example 19

- 1 The lift in a six-storey building started at the ground floor, went to the second floor, then the fifth floor, then the third floor, before returning to ground floor. Which of the following graphs represents this situation? Select the correct answer **A**, **B**, **C** or **D**.

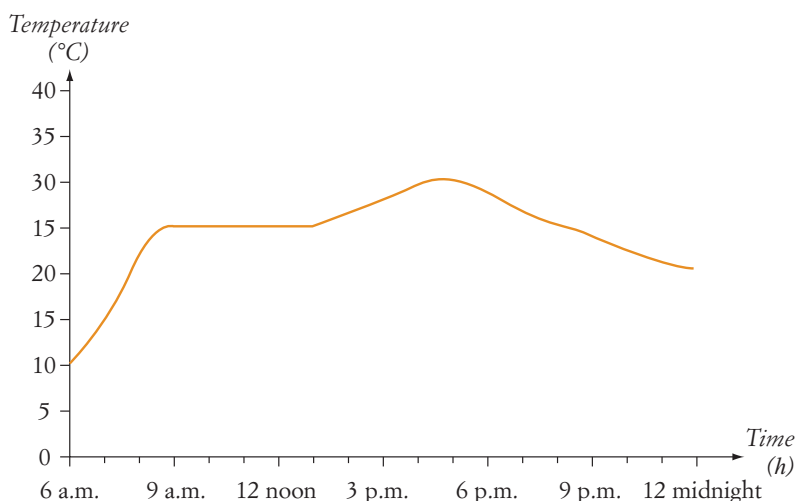


- 2 A triathlete has the following training program for a marathon.

- Start at 5 a.m., run 12 km in 2 hours, rest $\frac{1}{2}$ hour.
- Run a further 8 km in the next hour, rest $\frac{1}{2}$ hour.
- Pick up bike and cycle home, arriving at 10 a.m.

Draw the graph of this training session.

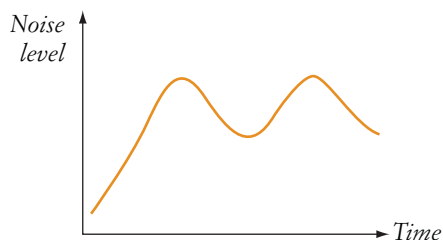
- 3 Make up a story to describe the changes in temperature over a day as shown on this graph.



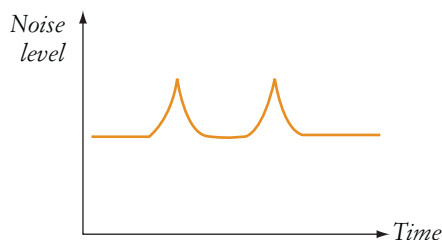
- 4 Match each Year 8 class to the graph of its noise level.

- 8 Green is constantly noisy.
- 8 Yellow is quiet until the teacher leaves the room.
- 8 Brown is regularly told to be quiet by their teacher.
- 8 Red just gets louder and louder.

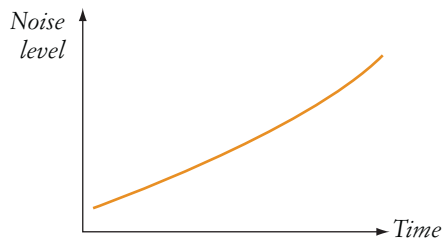
a



b



c



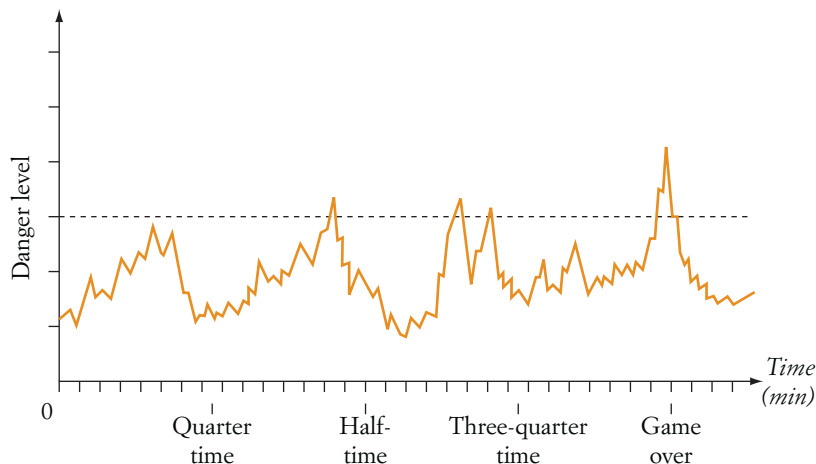
d



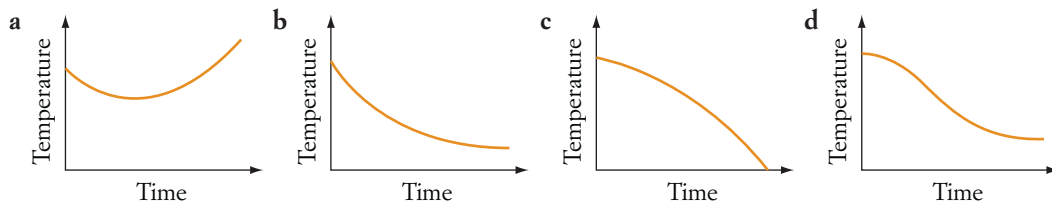
- 5 Sketch a graph of the noise level of this classroom.

- The students enter the classroom and the noise level increases.
- They settle down and work quietly.
- There are small group discussions and then the teacher talks.
- Towards the end of the lesson the noise level increases.
- Then the teacher speaks about the homework for the night and the class is dismissed.

- 6 Grandad likes watching basketball but during a match his level of excitement (measured by his heartbeat rate) can reach dangerous levels, as shown by the graph below.



- Describe Grandad's excitement level in the first quarter.
 - How many times does he become too excited during the match?
 - What happens to his excitement level after the match?
 - Describe what might have happened in the final quarter to make him so excited.
- 7 A cup of tea sits on the kitchen bench cooling. At first it loses heat quickly but as time passes it loses heat more slowly until it reaches room temperature. Which graph below best illustrates this? Select the correct answer **A**, **B**, **C** or **D**.



- 8 Sketch a graph to represent each situation.
- The temperature in your town over a 24-hour period.
 - The volume of petrol in a car. It is filled up, then runs for a while, stops for some time and then continues the trip.
 - The height of a plane travelling from Sydney to London with a refuelling stop in Dubai.
 - The number of cans of drink in a vending machine if it is filled in the morning and in the afternoon.
 - Your hunger level over a day during your waking hours.

11-12 Time differences

Worksheet

Time calculations

MAT08MGWK10107

Video tutorial

Time differences

MAT08MGVT10022

Homework sheet

Travel graphs and time

MAT08NAHS10015

Skillsheet

Units of time

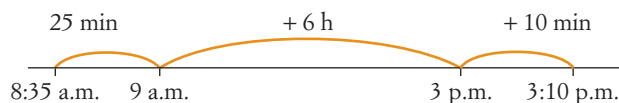
MAT08MGSS10038

Example 20

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

Solution

Method 1



From 8:35 a.m. to 9:00 a.m. = 25 minutes

From 9:00 a.m. to 3:00 p.m. = 6 hours

From 3:00 p.m. to 3:10 p.m. = 10 minutes

$$\begin{aligned}\text{Total time difference} &= 6 \text{ h} + 25 \text{ min} + 10 \text{ min} \\ &= 6 \text{ h } 35 \text{ min}\end{aligned}$$

Method 2

Convert to 24-hour time first, then use the calculator's **0° ' "** or **DMS** key to subtract the times.

Time difference = 3:10 p.m. - 8:35 a.m.

$$\begin{aligned}&= 1510 - 0835 \\ &= 6 \text{ h } 35 \text{ min}\end{aligned}$$

$$15 \text{ } 0^{\circ} ' " 10 \text{ } 0^{\circ} ' " - 8 \text{ } 0^{\circ} ' " 35 \text{ } 0^{\circ} ' " =$$

Example 21

Find $7 \text{ h } 5 \text{ min} - 3 \text{ h } 24 \text{ min}$.

Solution

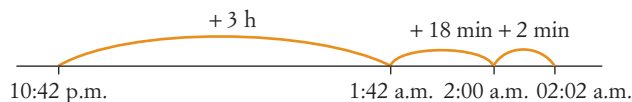
$$\begin{aligned}7 \text{ h } 5 \text{ min} - 3 \text{ h } 24 \text{ min} &= (7 - 3) \text{ h} + (5 - 24) \text{ min} \\ &= 4 \text{ h} + (-19) \text{ min} \\ &= 3 \text{ h } (60 - 19) \text{ min} \\ &= 3 \text{ h } 41 \text{ min}\end{aligned}$$

OR On a calculator, enter $7 \text{ } 0^{\circ} ' " 5 \text{ } 0^{\circ} ' " - 3 \text{ } 0^{\circ} ' " 24 \text{ } 0^{\circ} ' " =$

Example 22

What is the time 3 hours 20 minutes after 10:42 p.m.?

Solution



3 hours after 10:42 p.m. is 1:42 a.m.

20 minutes after 1:42 a.m. is 2:02 a.m.

Exercise 11-12 Time differences

See Example 20

- 1 What is the difference in time between 11:42 a.m. and 2:13 p.m.? Select the correct answer **A**, **B**, **C** or **D**.

A 2 h 31 min

B 3 h 55 min

C 7 h 29 min

D 11 h 55 min

- 2 Calculate the time difference between each pair of times.

a 6:15 p.m. and 8:10 p.m.

b 11:16 a.m. and 12:06 p.m.

c 4:10 a.m. and 8:55 a.m.

d 11:25 p.m. and 3:20 a.m.

e 0725 and 1310

f 2120 and 0815

g 4:10 a.m. and 12:15 p.m.

h 0940 and 1310

i 1245 and 1725

See Example 21

- 3 Simplify each expression.

a 2 h 15 min + 4 h 32 min

b 3 h 25 min + 8 h 27 min

c 6 h 42 min - 3 h 13 min

d 12 h 37 min - 5 h 6 min

e 7 h 12 min + 5 h 18 min

f 1 h 42 min + 6 h 27 min

g 15 h 57 min - 9 h 48 min

h 6 h 2 min - 4 h 17 min

i 9 h 37 min + 2 h 52 min

j 4 h 49 min + 7 h 18 min

k 8 h 18 min - 3 h 27 min

l 5 h 31 min - 3 h 48 min

- 4 A film starts at 3:14 p.m. and ends at 5:09 p.m. How long is the film?

See Example 22

- 5 What time will it be:

a 6 hours after 2:00 p.m.?

b 3 hours after 10:00 a.m.?

c 20 minutes after 7:15 p.m.?

d 2 hours 32 minutes after 10:45 a.m.?

e 3 hours 29 minutes after 10:35 p.m.?

f 5 hours after 9:32 a.m.?

g 9 hours after 6:17 p.m.?

h 55 minutes after 3:30 p.m.?

i $4\frac{1}{4}$ hours after 4:30 a.m.?

j 5 hours and 25 minutes after 9:45 a.m.?

- 6 A car rally began at 8:20 a.m. Here are some of the cars and the times they ran. Write the cars in their order of finishing and the time each crossed the finishing line.

Toyota 6:21 (6 h 21 min)

Volvo 5:23

Ford 5:44

Nissan 6:01

Subaru 5:59

Peugeot 5:42

Worked solutions

Exercise 11-12

MAT08MGWS10086

- 7 This timetable is part of a schedule for a bus travelling between Sydney and Wagga Wagga.

Sydney to Wagga Wagga		Wagga Wagga to Sydney	
Sydney	2:30 p.m.	Wagga Wagga	7:15 a.m.
Strathfield	3:00 p.m.	Gundagai	8:25 a.m.
Yagoona	3:20 p.m.	Jugiong	8:54 a.m.
Liverpool	3:45 p.m.	Yass	9:41 a.m.
Mittagong	4:40 p.m.	Goulburn*	10:41 a.m.
Goulburn*	5:40 p.m.	Mittagong	12:10 p.m.
Yass	7:10 p.m.	Liverpool	1:05 p.m.
Jugiong	7:55 p.m.	Yagoona	1:20 p.m.
Gundagai	8:20 p.m.	Strathfield	1:35 p.m.
Wagga Wagga	9:30 p.m.	Sydney	2:05 p.m.

*30 minute meal stop at Goulburn

- How long does the trip from Sydney to Wagga Wagga take?
 - How long would the trip take without a meal break?
 - Ali joins the return bus at Jugiong and gets off at Liverpool. How long is his trip?
 - Find the time taken from Liverpool to Sydney, and from Sydney to Liverpool on the return trip. Suggest a reason for the difference.
 - At what time does the bus from Wagga Wagga arrive in Goulburn?
 - Where is the return bus at 1:35 p.m.?
 - Chris is waiting at Yagoona at 2:45 p.m. for the bus to Mittagong. How long will he have to wait before the bus arrives and when will he reach Mittagong?
- 8 This is part of the ferry timetable for Woy Woy to Empire Bay.

MONDAY TO FRIDAY					
FERRY DEPARTS FROM WOY WOY					
DEPARTS	SARATOGA	DAVISTOWN			ARRIVES
Woy Woy	Veterans Hall	Lintern St.	Central (RSL)	Pine Av.	Empire Bay
6:35am	6:45am	6:50am	7:00am	-	-
7:45am	7:55am	8:00am	8:10am	8:10am	8:15am
9:00am	9:10am	9:15am	9:25am	9:25am	9:30am
10:45am	10:55am	11:00am	11:10am	11:10am	11:15am
12:30pm	12:40pm	12:45pm	12:55pm	12:55pm	1:00pm
1:50pm	2:00pm	2:05pm	2:15pm	2:15pm	2:20pm
3:30pm	3:40pm	3:45pm	-	-	-
4:50pm	5:00pm	5:05pm	5:15pm	-	-
5:50pm	6:00pm	6:05pm	6:15pm	-	-
6:50pm	7:00pm	7:05pm	7:15pm	7:15pm	7:20pm

- How many trips from Woy Woy arrive at Empire Bay each weekday?
- If George catches the 7:45 a.m. ferry from Woy Woy, at what time will he arrive at Empire Bay?
- If Sally catches the 2:05 p.m. ferry from Lintern St, where will she be at 2:15 p.m.?
- Joe needs to be at Central (RSL) before 10:00 a.m. What is the latest ferry he can catch from Saratoga to be there on time?

- e How long does it take to travel from Saratoga to Empire Bay?
- f At what time does the last bus leave Woy Woy in the evening?
- g Suppose a new ferry service departs Woy Woy at 11:15 a.m. What time should it arrive at Pine Avenue?

NSW

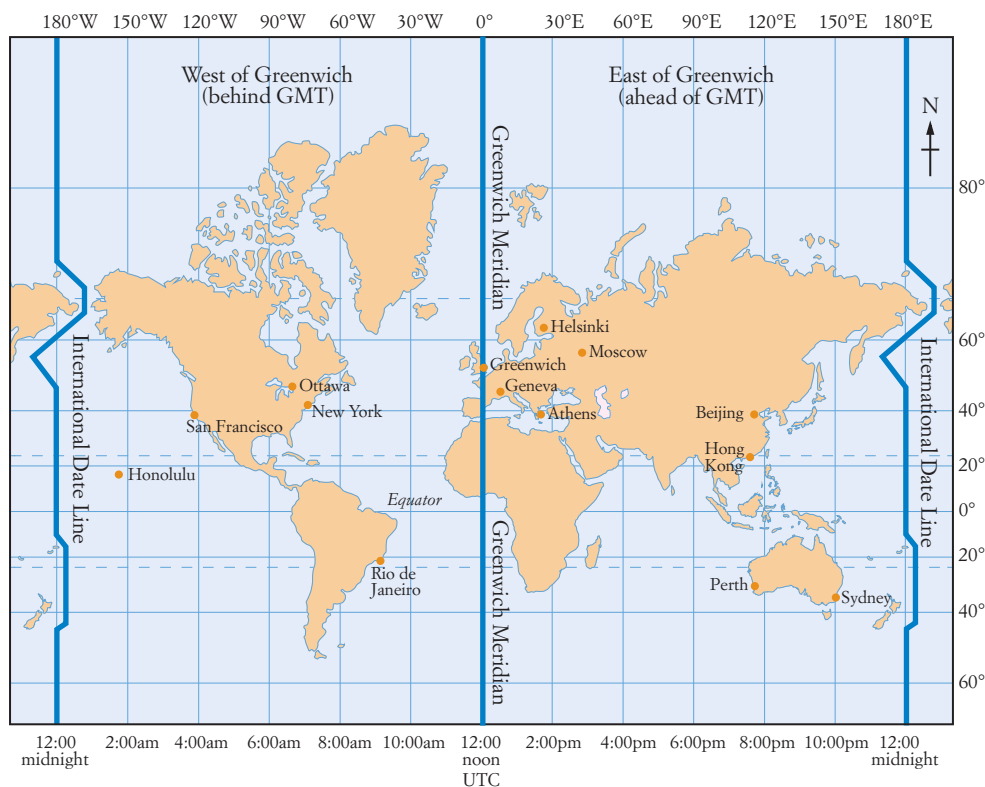
11-13 International time zones

Worksheet

World time zones

MAT08MGWK10108

The world is divided into 24 hourly time zones. Time is the same throughout each zone. The centre of each time zone is a meridian of longitude (an imaginary line running from the North Pole to the South Pole). Each hourly time zone covers 15° of longitude.



The map above shows how times around the world are related. All time is measured in relation to the time at the Greenwich Observatory (in London), either ahead or behind UTC (Coordinated Universal Time), also known as Greenwich Mean Time (GMT). Greenwich is pronounced 'Grennitch'. Time zones in Australia are ahead of UTC because Australia is east of Greenwich. Time zones in the USA are behind UTC because the USA is west of Greenwich.

Exercise 11-13 International time zones

- Use the map above to determine whether each city is ahead of or behind UTC.

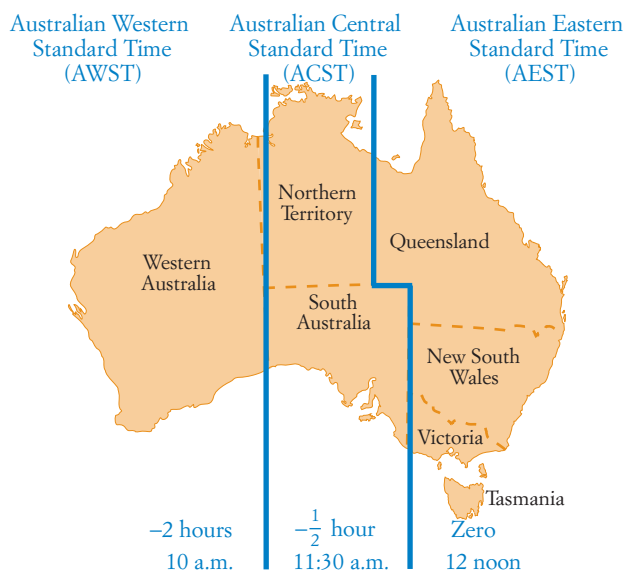
a Sydney	b Auckland	c Rio de Janeiro	d Perth
e Beijing	f Honolulu	g Moscow	h Athens
i Hong Kong	j Helsinki	k New York	l Ottawa
- Find the time in each city when it is noon in Greenwich.

a Sydney	b Perth	c New York	d Beijing
e San Francisco	f Honolulu	g Moscow	h Geneva
- What is the time difference between:

a Sydney and Perth?	b Sydney and Beijing?
c Sydney and Honolulu?	d Sydney and Moscow?
e Sydney and New York?	f Perth and Beijing?
g San Francisco and New York?	h Honolulu and Moscow?
i Geneva and Perth?	j San Francisco and Geneva?
- If it is 2:00 p.m. in Sydney, what is the time in:

a Greenwich?	b Perth?	c New York?	d Beijing?
e San Francisco?	f Honolulu?	g Moscow?	h Geneva?
- A cricket match being played in India is telecast live at 7:00 p.m. Sydney time. What is the local time of the cricket match if Sydney's time is $4\frac{1}{2}$ hours ahead of India's?
- Simone, in Newcastle (NSW), wants to use the Internet to chat with her cousin Zac in Vancouver, Canada. The time in Vancouver is 18 hours behind the time in Newcastle. At what time should Simone log on to the Internet to catch Zac when it is 3:00 p.m. in Vancouver?
- Brisbane is 2 hours behind Auckland. A plane leaves Auckland at midday and takes 3 hours to fly to Brisbane. What is the local time in Brisbane when the plane lands? Select the correct answer **A, B, C or D**.

A 11 a.m.	B 1 p.m.	C 3 p.m.	D 5 p.m.
- Find out what happens if you cross the International Date Line (IDL). Why isn't the IDL straight?
- This map shows the three time zones for Australia.



Worked solutions

Exercise 11-13

MAT08MGWS10087

State whether each city is ahead of, behind or has the same time as Adelaide (SA).

a Sydney	b Melbourne	c Darwin
d Perth	e Mt Isa (Qld)	f Geraldton (WA)
g Cobar (NSW)	h Ceduna (SA)	i Cairns (Qld)

10 What is the time difference between:

a Sydney and Adelaide?	b Melbourne and Perth?	c Adelaide and Melbourne?
d Hobart and Darwin?	e Canberra and Perth?	f Brisbane and Canberra?

11 If it is 11 p.m. in Sydney, what time is it in:

a Melbourne?	b Adelaide?	c Perth?
d Darwin?	e Hobart?	f Canberra?

12 If it is 11:30 p.m. in Adelaide, what time is it in:

a Melbourne?	b Sydney?	c Perth?
d Darwin?	e Hobart?	f Brisbane?

Worked solutions

Exercise 11-13

MAT08MGWS10087

- 13 a** Joe flies from Sydney to Perth, taking 4 hours. If he leaves Sydney at 2 p.m., at what time does he land in Perth? Give your answer as Perth local time.
- b** When Joe flies home, he leaves Perth at 9 a.m. At what time does he land in Sydney? Give your answer as Sydney local time.
- 14 a** Find out when daylight saving begins and ends.
- b** Why do we have daylight saving?
- c** How does daylight saving affect the different time zones?
- d** If it is 12:30 p.m. in Western Australia (not on daylight saving), what time is it in New South Wales on Eastern Standard Daylight Saving Time?

Investigation: World trip

Plan a trip around the world with at least three stopovers (for example, Tokyo, Hanoi, Cairo). Use airline timetables so you can give details of departures, arrivals and the length of each flight. Does it matter if you head east or west when you start? What affect does the International Date Line have on your trip?

Power plus

- 1 Australia's annual birth rate in 2011 was approximately 12 per 1000. Given that its population then was 22 750 000, approximately how many babies were born in Australia in 2011?
- 2 **a** Singapore has a population of 5 180 000. The population density of Singapore is 7358 persons/km². What is the area of this very crowded country?
b Australia's population density is only 3.0 persons/km². The area of Australia is 7 682 300 km². If Australia was populated as densely as Singapore, what would its population be?
- 3 A spider moves at 1 cm/s. If the spider is in the back left-hand corner of your classroom, find how long (in minutes) it will take to reach:
a the nearest person **b** you **c** the teacher's desk
- 4 Convert each speed to km/h. (Round your answers to two decimal places.)



- a** An African cheetah runs at 27 m/s.
b A German peregrine falcon dives at 97 m/s.
c A Tanzanian snake travels at 3.3 m/s.
d A racing cyclist rides at 23 m/s.
- 5 Work out a formula for converting speed expressed as x m/s to y km/h.

Chapter 11 review

Language of maths

Puzzle sheet

Ratios, rates and time
find-a-word

MAT08NAPS10031

best buy
daylight saving
distance
divide
equivalent
International
Date Line

km/h
map
per (/)
plan
rate
ratio

scaled length
scale ratio
simplify
speed
stationary
term

time zone
travel graph
unitary method
unit cost
UTC (Coordinated
Universal Time)

- 1 When comparing the prices and sizes of items to find the **best buy**, do we look for the highest or lowest **unit cost**?
- 2 What type of measurement compares quantities of different types, expressed using two units?
- 3 What is meant by the **scale ratio** of a map or plan?
- 4 What does an **average speed** of 65 km/h actually mean?
- 5 How is a change in speed shown on a travel graph?
- 6 In which country is the UTC or GMT time zone?

Topic overview

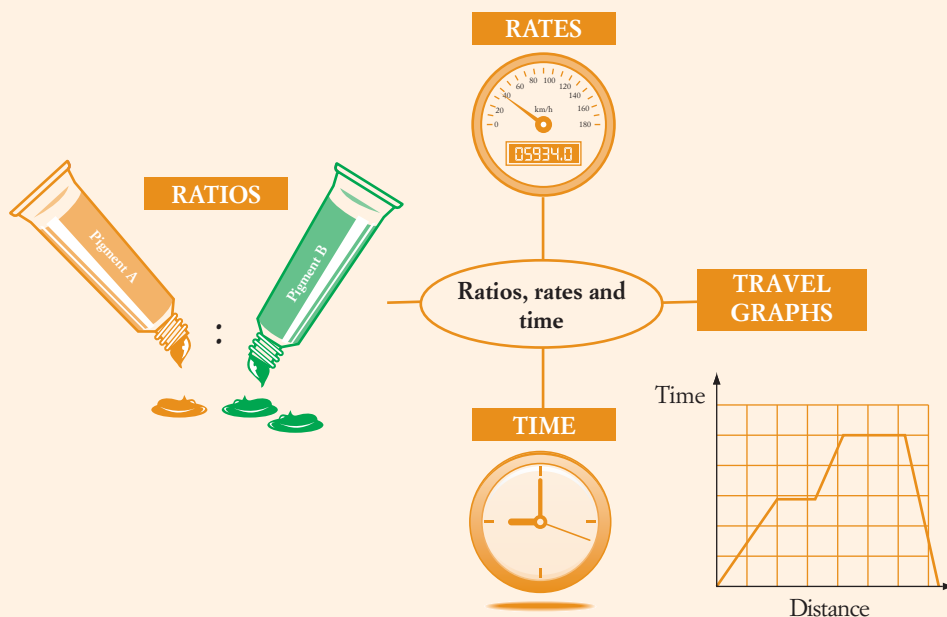
Worksheet

Mind map: Ratios, rates
and time

MAT08NAWK10109

- Give examples of places in which ratios, rates and time are used.
- What did you learn in this chapter?
- What did you find most difficult about this topic? Discuss any problems with your teacher or with a friend.

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



- 1 Copy and complete each pair of equivalent ratios.

See Exercise 11-01

a $1 : 3 = 4 : \underline{\hspace{1cm}}$

b $2 : 5 = 6 : \underline{\hspace{1cm}}$

c $3 : 7 = \underline{\hspace{1cm}} : 21$

d $4 : 9 = \underline{\hspace{1cm}} : 45$

e $0.5 : 0.8 = 2 : \underline{\hspace{1cm}}$

f $3 : 4 : 5 = \underline{\hspace{1cm}} : 20 : \underline{\hspace{1cm}}$

- 2 Simplify each ratio.

See Exercise 11-02

a $12 : 21$

b $25 : 75$

c $6 : 36$

d $25 : 45$

e $18 : 6 : 24$

f $5 : 25 : 100$

g $\frac{1}{3} : \frac{1}{2}$

h $\frac{3}{4} : \frac{9}{16}$

i $1\frac{1}{2} : 2\frac{1}{2}$

j $0.98 : 0.245$

k $1.5 : 6$

l $91 : 5.6$

m $5 \text{ km} : 200 \text{ metres}$

n $20 \text{ kg} : 3600 \text{ g}$

o $\$25 : \$4.25$

p $18 \text{ months} : 4 \text{ years}$

q $10 \text{ days} : 5 \text{ weeks}$

r $8 \text{ h} : 3 \text{ days}$

- 3 a The ratio of girls to boys in Year 8 is $4 : 3$. If there are 75 boys in Year 8, find how many girls there are.

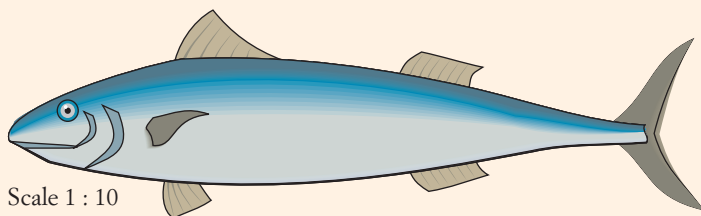
See Exercise 11-03

- b Donald and Evelyn invested in a business in the ratio $5 : 7$. If Evelyn invested \$63 000, how much did Donald invest?

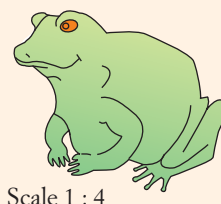
- 4 Measure the length of each scale drawing below, and then use the ratio to work out the actual length of the object shown.

See Exercise 11-04

- a Fish



- b Frog



- 5 On a tourist map of Sydney, the scale is given by $\overbrace{\hspace{2cm}}^{0 \hspace{1cm} 500 \text{ m.}}$

See Exercise 11-04

- a Write this scale as a simplified ratio.

- b Find the actual distance between the following places given the scaled distance.

i Circular Quay station to the Opera House (2.5 cm)

ii Pyrmont Bridge to Parliament House (4.4 cm)

- c Find the scaled distance between the following places given the actual distance.

i Art Gallery of NSW to Sydney Tower (875 m)

ii Circular Quay station to Central station (2.5 km)

- 6 Ellen and Portia share the weekly rent of \$420 on their apartment in the ratio $4 : 3$. How much does each woman pay?

See Exercise 11-05

- 7 Write each statement as a rate in simplified form.

See Exercise 11-06

a \$10.50 for 3 kg

b 220 km in 2 hours

c \$56.40 for 4 hours

d 260 runs in 50 overs

Chapter 11 revision

See Exercise 11-07

- 8 For each pair, determine which is the better buy.
- A 1 kg box of biscuits for \$5.60 or a 700 g carton for \$4.00.
 - A 350 g can of soup for \$4.20 or a 550 g can for \$7.15.
 - A 1.5 L bottle of lemonade for \$2.45 or a 375 mL can for \$0.70.
 - 3×40 g chocolate bars for \$2.36 or a 500g chocolate bar for \$9.

See Exercise 11-08

- 9
- Mince is \$3.99/kg. How much does 5 kg of mince cost?
 - Dean earns \$18.70/h. How much is she paid for 38 hours work?
 - How many litres of petrol can you buy with \$40 if petrol costs \$1.35/L? Answer to the nearest 0.1 L.
 - Fertiliser is used at 20 kg/ha. How many hectares can be covered with 144 kg of fertiliser?

See Exercise 11-09

- 10 Find the speed, in km/h, of a cyclist who travels a distance of 45 km in 2 hours and 15 minutes.

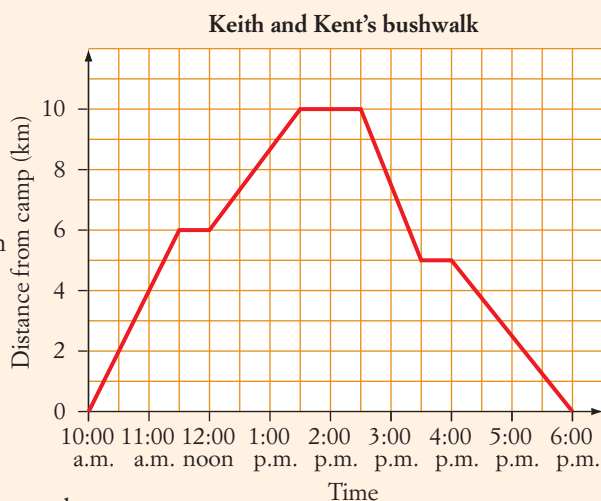
See Exercise 11-09

- 11 Find the distance covered by a truck that travels for 5 hours and 30 minutes at an average speed of 45 km/h.

See Exercise 11-10

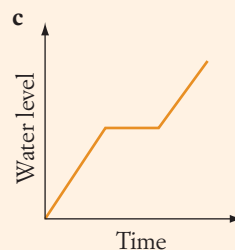
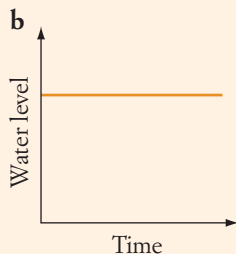
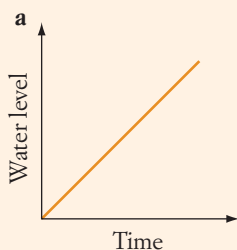
- 12 Keith and Kent decided to go bushwalking. This travel graph shows their walk.

- How far did they travel from camp?
- How many stops did they make?
- Find their average speed between their first and second stops.
- How long did it take them to return home?
- Between what times were they walking fastest?

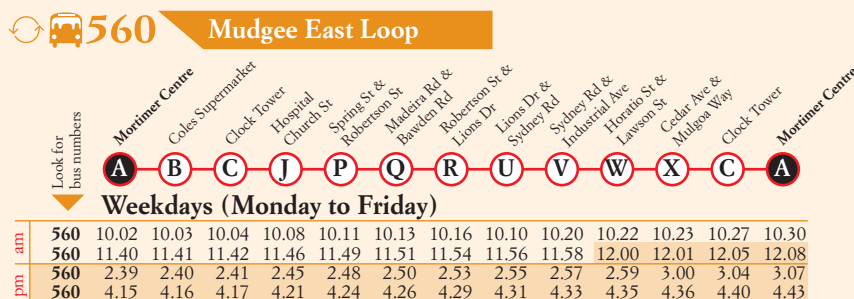


See Exercise 11-11

- 13 Match each description to its correct graph.
- The water level of the bath stayed constant at 50 cm.
 - The bath was filled at a steady rate. The tap was turned off to let the hot bathwater cool. The rest of the bath was filled with cold water.
 - The bath was filled with warm water at a steady rate.



- 14 Draw a graph to represent the following story. 'The weight of the baby elephant increased quickly at first but then slowed down to a steady rate.' See Exercise 11-11
- 15 What is the time:
 a 3 hours and 20 minutes after 8:05 p.m.? b 4 hours and 45 minutes before 11:15 p.m.?
See Exercise 11-12
- 16 Calculate the time difference between each pair of times.
 a 10:10 a.m. and 6:40 p.m. b 12:05 p.m. and 3:20 a.m.
 c 1230 and 2320
See Exercise 11-12
- 17 A section of a bus timetable is shown below. See Exercise 11-12



- a What do you think the shaded area represents?
 b How many bus services run each day?
 c How long is the journey from the Mortimer Centre to Robertson St?
 d Tom catches the bus from Spring St at 2:48 p.m. Where does the bus stop after 11 minutes?
 e Mary needs to meet a friend at the Clock Tower at 3:10 p.m. Which is the latest bus she can catch from Madeira Rd to be there on time?
- 18 Use the world map on page 454 to find the time difference between:
 a Sydney and Athens b Sydney and Hong Kong
 c Perth and Honolulu d San Francisco and Helsinki
See Exercise 11-13
- 19 An international hockey match in Holland is played at 3:30 p.m. local time. At what time should Shegufta wake up to watch the game live on TV in Sydney if Sydney's time is 8 hours ahead of Holland's? See Exercise 11-13

Mixed revision 4

See Exercise 10-02

1 Solve each equation.

a $2a + 7 = 9$

b $3m - 2 = 7$

c $3x + 5 = -10$

d $\frac{t}{3} - 5 = 1$

e $\frac{k}{7} + 1 = 12$

f $8 - 2x = 4$

f $\frac{2x}{3} = 6$

g $\frac{5x}{4} = -10$

h $\frac{a+3}{2} = 9$

See Exercise 11-02

2 Simplify each ratio.

a $2 : 6$

b $96 : 34$

c $2 \text{ m to } 15 \text{ cm}$

d $10 \text{ minutes to } 2 \text{ hours}$

e $\frac{2}{5} : \frac{1}{3}$

f $0.7 : 2.8$

See Exercise 11-07

3 For each pair of sizes, find the better buy.

a Chips: 300g for \$5.20 or 400g for \$7.20

b Pasta: 1.5 kg for \$3.30 or 750g for \$1.80

See Exercise 12-01

4 Complete this table of values using the formula $y = 2x + 9$.

x	-1	0	1	2	3
y					

See Exercise 10-03

5 Solve each equation.

a $5q - 10 = 2q + 8$

b $4x + 5 = 8x - 7$

See Exercise 10-04

6 Solve each equation.

a $2(y + 3) = 10$

b $5(d - 4) = 2(d + 5)$

See Exercise 11-12

7 How many hours and minutes is it from:

a 4:30 a.m. to 9:55 a.m.?

b 1505 to 2210?

See Exercise 11-03

8 The ratio of boys to girls at a conference is 4 : 3. If there are 15 girls, find the number of boys.

See Exercise 11-04

9 a If 5 cm on a map represents 1 km in actual distance, what length would be used to represent 5 km?

b Using a scale of 1 : 500, what actual length, in metres, would a scaled length of 4 cm represent?

See Exercise 10-05

10 Five bricks have the same mass as one brick and 4 kg. What is the mass of one brick? Use an equation to solve this problem.

See Exercise 11-09

11 a A train travels 264 km in 8 hours. What is its average speed?

b A bus travels at 45 km/h for 5 hours. How far does it travel?

See Exercise 12-02

12 Find the formula for this table of values.

x	2	3	4	5	6	7
y	6	9	12	15	18	21

- 13 Here are the first three shapes in a pattern made of toothpicks.

See Exercise 12-03



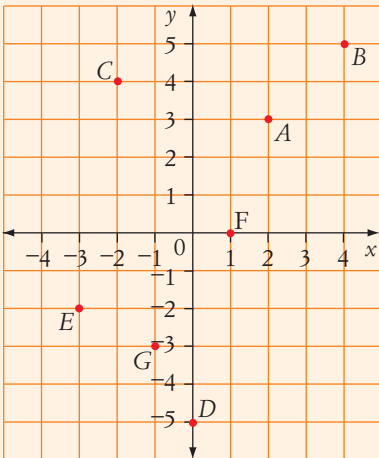
- a Copy and complete this table.

Number of triangles t	1	2	3	4	5	8
Number of toothpicks, p	3					

- b Write the rule for the pattern in words.
 c Write the rule as a formula.
 d How many toothpicks are needed to build 20 triangles?

- 14 Write the coordinates of each point marked on the number plane.

See Exercise 12-04



- 15 Graph each linear equation on a number plane after completing the table of values.

See Exercise 12-06

- a $y = x + 3$

x	-1	0	1	2
y				

- b $y = 4x - 1$

x	-1	0	1	2
y				

- 16 Using the formula $A = \frac{1}{2}bh$, find h when $A = 63$ and $b = 10.5$.

See Exercise 10-05

- 17 A truck is loaded with fruit and vegetables in the ratio 5 : 9. The total weight is 4.2 tonnes. What is the weight of the fruit carried in the truck?

See Exercise 11-05

- 18 In 20 hours, a train travels 800 km. How far would it travel in 11 hours at the same rate?

See Exercise 11-08

- 19 Test whether the point (3, 7) lies on the line with the equation:

See Exercise 12-06

a $y = 2x + 4$

b $y = 3x - 2$

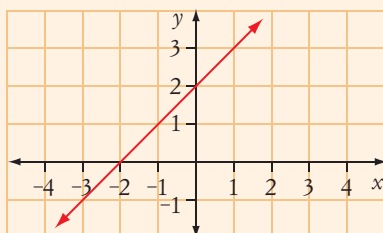
Mixed revision 4

See Exercise 11-08 **20** Water flows into a tank at a rate of 15 litres per hour. How long will it take to fill a tank that holds 750 litres?

See Exercise 12-07 **21** Graph the line $y = 2$ on a number plane.

See Exercise 12-09 **22** Solve the equation $2x - 3 = 5$ graphically.

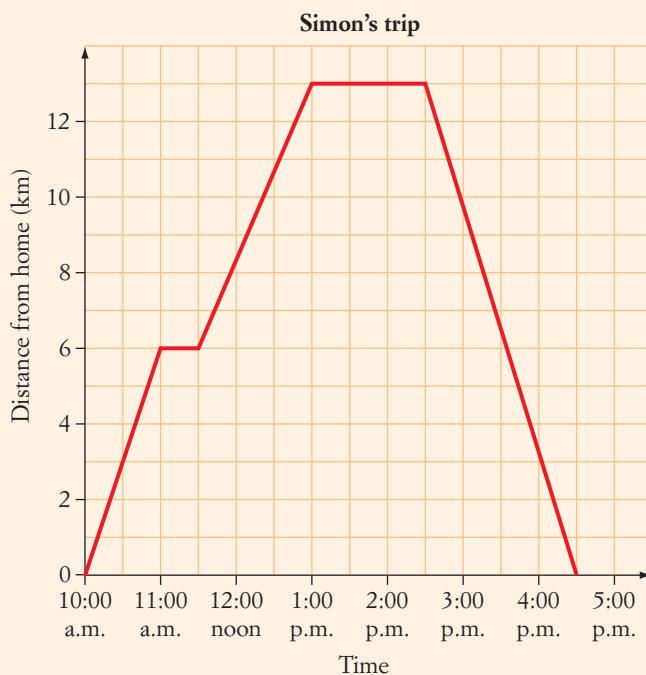
See Exercise 12-07 **23** Find the equation of this line.



See Exercise 11-13 **24** The time in Sydney is 10 hours ahead of UTC time in London. What is the time in Sydney when it is 2 p.m. in London?

See Exercise 12-10 **25** Graph the lines $y = 2x$ and $y = x + 2$ on the same number plane and find their point of intersection.

See Exercise 11-10 **26** Simon's cycling trip is described by the travel graph below.



- When, and for how long, did Simon stop for lunch?
- How far did Simon cycle in total?
- What was Simon's speed on the return journey home?

See Exercise 12-10 **27** Solve the equation $3x + 2 = x - 1$ graphically.

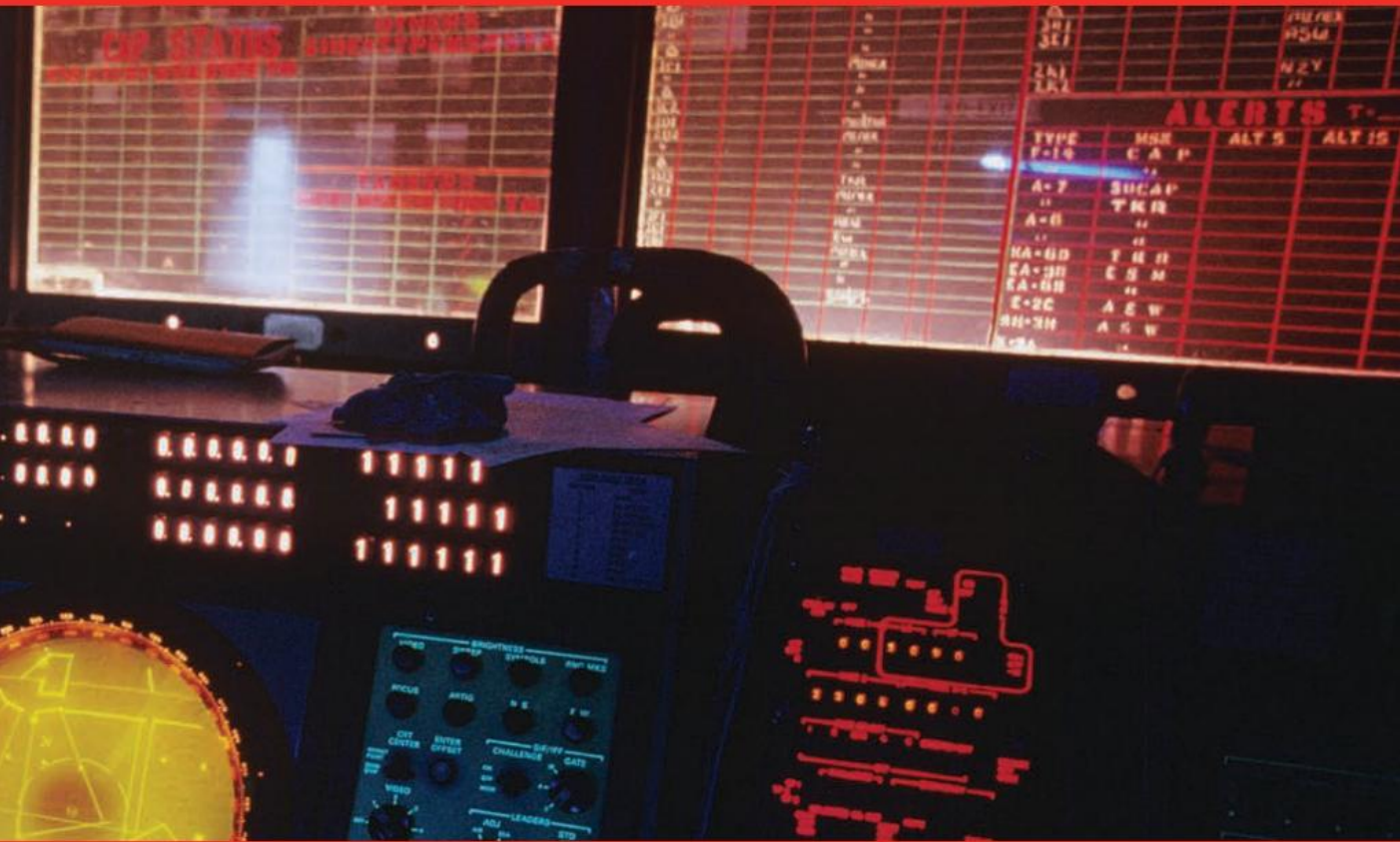


12

Number and algebra

Graphing linear equations

The famous French philosopher and mathematician René Descartes (1596–1650) is famous for linking algebra and geometry for the first time. He showed that many relationships and formulas could be represented by simple graphs, one of which is the straight line. There are numerous real-life situations where linear relationships are formed, such as predicting the location of an aeroplane at a particular time.



Chapter outline

	Proficiency strands				
12-01 Tables of values	U	F			
12-02 Finding the rule	U	F		R	C
12-03 Finding rules for number patterns		F	PS	R	C
12-04 The number plane	U	F			C
12-05 Graphing number patterns	U	F		R	C
12-06 Graphing linear equations	U	F		R	C
12-07 Finding the equation of a line	U	F		R	C
12-08 Comparing linear and non-linear equations	U	F	PS	R	C
12-09 Solving linear equations graphically	U	F		R	C
12-10 Intersecting lines	U	F		R	C

Wordbank

consecutive numbers Numbers that follow each other in order, for example, 2, 3, 4

constant term The term in an equation that is a number only and does not contain a variable, for example, the 5 in $y = 2x + 5$

horizontal Going across, sideways, flat

linear Involving a line

linear equation A formula whose graph is a straight line

number plane A coordinate grid system based on the x - and y -axes, also called a Cartesian plane

table of values A table of ordered pairs of numbers, usually following a formula and which can be graphed on a number plane

y -intercept The y -value at which a line crosses the y -axis on the number plane

In this chapter you will:

- form a table of values for a linear relationship by substituting a set of appropriate values for either of the pronumerals and graph the number pairs on the Cartesian plane
- recognise a given number or geometric pattern, complete a table of values, describe the pattern in words and algebraic symbols, and represent that relationship on a number grid
- given coordinates, plot points on the Cartesian plane and find coordinates for a given point
- plot linear relationships on the Cartesian plane with and without the use of digital technologies
- derive a rule for a set of points that has been graphed on a number plane
- graph more than one line on the same set of axes using ICT and compare the graphs to describe similarities and differences, for example, parallel, pass through the same point
- use ICT to graph linear and simple non-linear relationships such as $y = x^2$
- solve linear equations using graphical techniques and verify solutions by substitution
- graph two intersecting lines on the same set of axes and read off the point of intersection

SkillCheck

Worksheet

StartUp assignment 12

MAT08NAWK10110

1 For each equation, find the missing number.

a $2 \times 3 + \underline{\quad} = 10$

b $2 \times (-1) + \underline{\quad} = 3$

c $3 \times 1 - \underline{\quad} = 1$

d $-2 \times 2 + \underline{\quad} = -2$

e $4 \times 3 - \underline{\quad} = 8$

f $-1 \times (-1) - \underline{\quad} = -4$

2 Evaluate each expression.

a $3x - 2$ if $x = 2$

b $\frac{1}{2}x + 3$ if $x = 6$

c $2x - 1$ if $x = -1$

d $-x + 1$ if $x = 0$

e $-2x - 1$ if $x = 1$

f $10 - x$ if $x = 3$

3 Solve each equation.

a $3x - 2 = 7$

b $-2x - 1 = 3$

c $-x + 1 = 4$

d $2x - 1 = -5$

e $3x + 2 = x - 1$

f $x - 2 = 4 - x$

Worksheet

Tables of values

MAT08NAWK00025

12-01 Tables of values

Example 1

Complete this table of values using the formula $v = 2u + 1$.

u	-2	-1	0	2	6	9
v						

Solution

Substitute each value of u in the table into the formula $v = 2u + 1$ to find the value of v .

- When $u = -2$, $v = 2 \times (-2) + 1 = -3$
- When $u = -1$, $v = 2 \times (-1) + 1 = -1$

- When $u = 0$, $v = 2 \times 0 + 1 = 1$
- When $u = 2$, $v = 2 \times 2 + 1 = 5$
- When $u = 6$, $v = 2 \times 6 + 1 = 13$
- When $u = 9$, $v = 2 \times 9 + 1 = 19$

Completing the table, we have:

u	-2	-1	0	1	2	6	9
v	-3	-1	1	3	5	13	19

Exercise 12-01 Tables of values

- 1 Complete each table of values given the formula.

a $y = x - 1$

x	3	7	-1	-8	4	5
y					3	

b $q = 3p$

p	8	-3	2	5	10	-4
q				15		

c $k = h \div 2$

h	12	8	-2	-6	0	7
k	6					

d $y = x + 7$

x	-2	11	7	0	13	-9
y		18				

e $b = 2a$

a	8	-1	-5	0	4	2
b			-10			

f $y = x - 3$

x	10	-4	7	5	-8	11
y	7					

- 2 Given the rule $y = -x - 4$, what is the value of y when $x = -3$? Select the correct answer A, B, C or D.

A -1

B -7

C 7

D 12

- 3 Complete each table of values given the formula.

a $p = 5m - 1$

m	3	10	1	4	8	-6
p		49				

b $q = 3p - 2$

p	4	-1	7	10	2	-6
q			19			

c $t = 4r + 6$

r	2	0	-3	9	-5	4
t	14					

d $z = 5y + 2$

y	4	8	7	-5	6	-2
z		42				

e $y = 2x - 3$

x	0	1	2	3	4	5
y						

f $b = \frac{1}{2}a - 2$

a	-2	-1	0	1	2	3
b						

g $k = 3h + 5$

h	-1	0	1	2	3	4
k						

h $y = 1 - x$

x	-2	-1	0	1	2	3
y						

See Example 1

Worked solutions

Exercise 12-01

MAT08NAWS10088

Worked solutions

Exercise 12-01

MAT08NAWS10088

- 4 Which table of values matches the formula
- $y = 3x - 1$
- ? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

A

x	1	2	3	4	5
y	2	3	4	5	6

B

x	1	2	3	4	5
y	2	5	8	11	14

C

x	1	2	3	4	5
y	4	7	10	13	16

D

x	1	2	3	4	5
y	0	3	6	9	12

- 5 Which table of values follows the rule
- $c = \frac{h+2}{2}$
- ? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

A

h	0	2	4
c	1	3	5

B

h	0	2	4
c	1	2	5

C

h	0	2	4
c	0	1	3

D

h	0	2	4
c	1	2	3

Worksheet

Finding the rule

MAT08NAWK10112

12-02 Finding the rule

We can often find the formula for a table of values. If the values in the top row are **consecutive** (increase by 1 each time), look for a pattern in the values in the bottom row.

Example 2

Find the formula for each table of values.

a

p	2	3	4	6	7	8
q	1	2	3	5	6	7

b

m	-2	-1	0	1	2	3
n	-6	-3	0	3	6	9

c

r	1	2	3	4	5	6
t	1	3	5	7	9	11

d

d	1	2	3	4	5	6
e	8	12	16	20	24	28

Solution

- a What has been done to each value in the top row to get the value in the bottom row?

p	2	3	4	6	7	8
q	1	2	3	5	6	7

The pattern is:

$$2 - 1 = 1$$

$$3 - 1 = 2$$

$$4 - 1 = 3, \text{ and so on.}$$

The formula is $q = p - 1$.

b

<i>m</i>	-2	-1	0	1	2	3
<i>n</i>	-6	-3	0	3	6	9

The pattern is:

$$3 \times (-2) = -6$$

$$3 \times (-1) = -3$$

$$3 \times 0 = 0, \text{ and so on.}$$

The formula is $n = 3m$.

- c The formula for this table of values is not so obvious. It involves **two** operations: a multiplication and either an addition or a subtraction.

<i>r</i>	1	2	3	4	5	6
<i>t</i>	1	3	5	7	9	11

$\underbrace{\quad\quad}_2 \quad \underbrace{\quad\quad}_2 \quad \underbrace{\quad\quad}_2 \quad \underbrace{\quad\quad}_2 \quad \underbrace{\quad\quad}_2$

If the values in the top row are consecutive, the bottom row helps us find the **multiplier**.

The values in the bottom row go up by 2 each time, so the **multiplier is 2**. This means the formula must have $2 \times r$ in it.

$$2 \times 1 - 1 = 1$$

$$2 \times 2 - 1 = 3$$

$$2 \times 3 - 1 = 5, \text{ and so on.}$$

The formula is $t = 2r - 1$.

d

<i>d</i>	1	2	3	4	5	6
<i>e</i>	8	12	16	20	24	28

$\underbrace{\quad\quad}_4 \quad \underbrace{\quad\quad}_4 \quad \underbrace{\quad\quad}_4 \quad \underbrace{\quad\quad}_4 \quad \underbrace{\quad\quad}_4$

The values in the top row are **consecutive**, the values in the bottom row go up by 4, so the **multiplier is 4**, and the equation is of the form $e = 4d$ ____.

Choose any ordered pair from the table to find the missing number, say (2, 12).

When $d = 2$, $e = 4 \times 2 + 4 = 12$, so the missing number is 4.

So the equation is $e = 4d + 4$.

(Checking that this is also true for another ordered pair (1, 8). When $d = 1$, $e = 4 \times 1 + 4 = 8$)

Exercise 12-02 Finding the rule

- 1 Complete the formula for each table of values.

a $y = \_\_\_ x$

<i>x</i>	0	1	2	3
<i>y</i>	0	3	6	9

c $y = x + \_\_\_\_\_\_$

<i>x</i>	-1	0	1	2
<i>y</i>	3	4	5	6

b $y = \_\_\_ x$

<i>x</i>	0	1	2	3
<i>y</i>	0	-2	-4	-6

d $y = x - \_\_\_\_\_\_$

<i>x</i>	0	1	2	3
<i>y</i>	-4	-3	-2	-1

See Example 2

e $y = 10 - \underline{\hspace{2cm}} x$

x	-1	0	1	2
y	13	10	7	4

g $k = \underline{\hspace{2cm}}$

p	1	2	3	4
k	5	6	7	8

i $j = \underline{\hspace{2cm}}$

h	4	5	9	11
j	8	10	18	22

f $y = x \div \underline{\hspace{2cm}}$

x	0	1	2	3
y	0	$\frac{1}{2}$	1	$1\frac{1}{2}$

h $y = \underline{\hspace{2cm}}$

x	-1	0	1	2
y	-4	-3	-2	-1

j $p = \underline{\hspace{2cm}}$

m	0	5	30	45
p	0	1	6	9

Worked solutions

Exercise 12-02

MAT08NAWS10089

- 2 Find the formula for each table, then complete the last two columns.

a

f	1	2	3	4	7	8
h	1	4	7	10		

b

m	1	2	3	4	6	9
p	2	7	12	17		

c

m	0	1	2	3	5	8
b	3	6	9	12		

d

h	3	4	5	6	9	11
k	8	10	12	14		

e

r	0	1	2	3	4	7
s	1	4	7	10		

f

a	2	3	4	5	10	11
b	2	4	6	8		

g

m	0	1	2	3	5	6
n	5	8	11	14		

h

c	3	4	5	6	9	10
d	31	41	51	61		

i

w	3	4	5	6	7	8
x	14	19	24	29		

j

y	5	6	7	8	9	10
z	4	6	8	10		

k

a	1	2	3	4	7	9
m	5	9	13	17		

l

z	4	5	6	7	9	12
t	1	3	5	7		

- 3 What is the formula for this table? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

p	3	4	5	6
m	8	11	14	17

A $m = 2p + 1$

B $m = 3p - 1$

C $m = 2p + 4$

D $m = 3p + 1$

- 4 What is the formula for this table? Select the correct answer
- A**
- ,
- B**
- ,
- C**
- or
- D**
- .

x	1	2	3	4	5
y	-5	-7	-9	-11	-13

A $y = -2x - 5$

B $y = 2x - 5$

C $y = -2x - 3$

D $y = 2x - 3$

5 Find the formula for each table, then complete the table.

a

x	0	1	2	3	9
y	-1	-3	-5	-7	

b

A	0	1	2	3	10
C	0	0.5	1	1.5	

c

s	-1	0	1	2	7
t	18	13	8	3	

d

t	-1	0	1	2	5
v	30	20	10		

e

x	0	1	2	3	6
D	-0.5	-1	-1.5		

12-03 Finding rules for number patterns

Example 3

For this number pattern: 2, 5, 8, 11, 14, ...

a find the 8th term

b find a formula for the n th term, T .

Solution

a This number pattern is increasing by 3 each time. By continuing the pattern:

2, 5, 8, 11, 14, 17, 20, 23

8th term = $20 + 3 = 23$

b Write the number pattern as a table of values.

Term of pattern, n	1	2	3	4	5
Number, T	2	5	8	11	14

The number pattern goes up by 3 each time, so the multiplier for the formula is 3, and the equation is of the form $T = 3n$ ____.

Choose any ordered pair from the table to find the missing number, say (1, 2).

When $n = 1$, $T = 3 \times 1$ -1 = 2, so the missing number is -1.

$$3 \times 1 - 1 = 2$$

$$3 \times 2 - 1 = 5$$

$$3 \times 3 - 1 = 8 \text{ and so on}$$

So the general formula for the n th term is $T = 3n - 1$.

Worksheet

Patterns and rules

MAT08NAWK10113

Homework sheet

Tables of values

MAT08NAHS10020

4



arm length = 1

arm length = 2

arm length = 3

Arm length, a	1	2	3	4	5	6
Number of tiles, t						

- ## Solution

- a**
- | | | | | | | |
|----------------------|---|---|---|---|----|----|
| Arm length, a | 1 | 2 | 3 | 4 | 5 | 6 |
| Number of tiles, t | 3 | 5 | 7 | 9 | 11 | 13 |

- b** The number of tiles increases by 2 each time. This means that the multiplier is 2, and the equation is of the form $t = 2a$ ____.
- Choose (1, 3) from the table to find the missing number.
- When $a = 1$, $t = 2 \times 1 + \underline{1} = 3$.
- In words, the rule is: The number of tiles is 2 times the arm length plus 1.
- c** As a formula, the rule is $t = 2a + 1$.
- d i** When $a = 18$, $t = 2 \times 18 + 1 = 37$
- 37 tiles are needed for a design with an arm length of 18.
- ii** When $a = 60$, $t = 2 \times 60 + 1 = 121$
- 121 tiles are needed for a design with an arm length of 60.

Exercise 12-03 Finding rules for number patterns

- 1 For each number pattern:

- i** find the 8th term
ii find a formula for the n th term, T
iii use the formula to find the 8th term and the 20th term.
- | | |
|---------------------------------|---------------------------------|
| a 1, 3, 5, 7, 9, ... | b 2, 4, 6, 8, 10, ... |
| c 8, 10, 12, 14, 16, ... | d 7, 10, 13, 16, 19, ... |

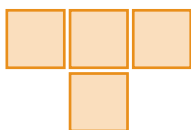
e 7, 15, 23, 31, 39, ...

f 2, 7, 12, 17, 22, ...

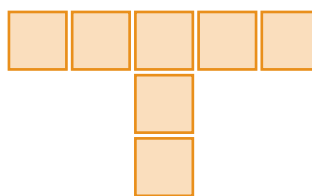
g 19, 17, 15, 13, 11, ...

h 1, 4, 9, 16, 25, ...

- 2 a Here are the first two T-shapes in a geometrical pattern. Draw the next three T-shapes.



arm length = 1



arm length = 2

- b Copy and complete the table.

Arm length, a	1	2	3	4	5	8	11
Number of tiles, t	4						

- c Write the rule for the pattern in words.

- d Write the rule as a formula.

- e How many tiles are needed to build a T-shape with arm length of:

i 15?

ii 33?

- 3 a Here are the first two flight shapes, made from toothpicks. Draw the next three.



flight shape 1



flight shape 2

- b Copy and complete the table.

Number of flight shape, f	1	2	3	4	6	9	10
Number of toothpicks, t	7						

- c Write the rule for the pattern in words.

- d Write the rule as a formula.

- e How many toothpicks are needed to build:

i flight shape 25?

ii flight shape 40?

- 4 a The first two rockets are shown below. Draw the next three rockets.



stage 1



stage 2

- b Copy and complete the table.

Number of stage, s	1	2	3	4	5	8	11
Number of toothpicks, t							

- c Write the rule for the pattern in words.

- d Write the rule as a formula.

- e How many toothpicks are needed to build:

i stage 12?

ii stage 45?

See Example 4

Worked solutions

Exercise 12-03

MAT08NAWS10090

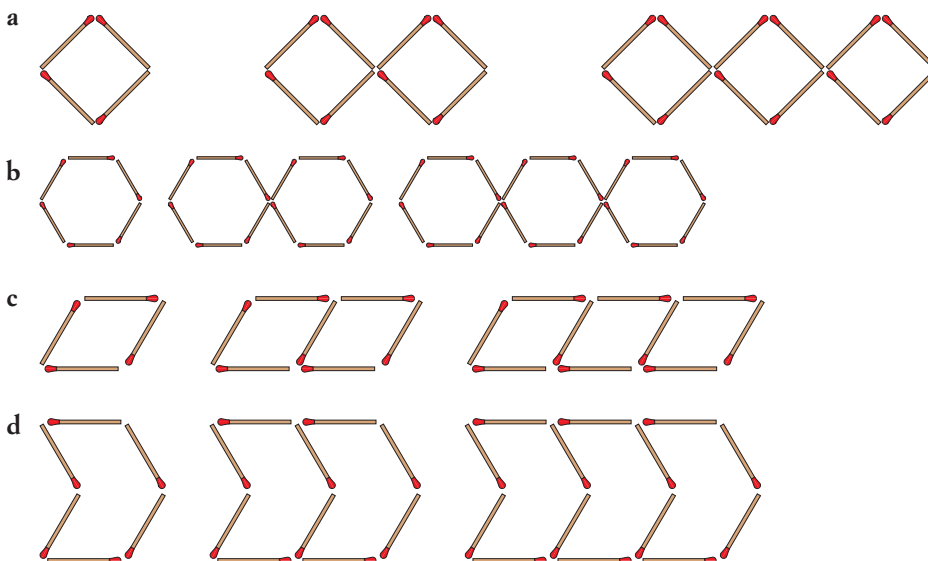
5 For each toothpick pattern below:

- draw the next three terms
- copy and complete this table

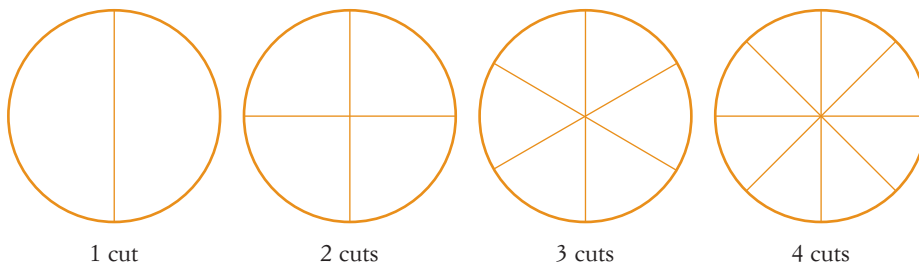
Number of shapes (n)	1	2	3	4	5
Number of toothpicks (T)					

iii find the formula for the pattern

iv find the number of toothpicks required to make 12 shapes.

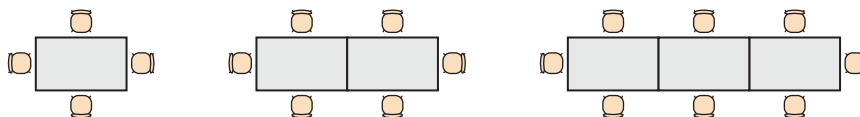


6 The pattern below is formed by cutting a cake c times to make p pieces. Which of the following is the formula that best describes the pattern? Select the correct answer **A**, **B**, **C** or **D**.



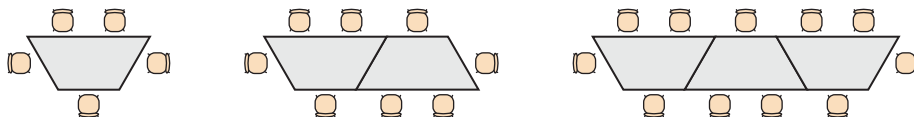
- A** $p = 2c$ **B** $c = 2p$ **C** $p = c + 2$ **D** $c = p + 2$

7 In a restaurant, chairs are placed around rectangular tables as shown below.



- Find the number of chairs required for:
 - 2 tables
 - 3 tables
 - 4 tables
 - 12 tables
- Find a formula that relates the number of chairs, C , to the number of tables, T .
- How many chairs would be needed for a row of 50 tables?
- A booking is made at the restaurant for 30 people. How many tables are needed?

- 8 Students arrange themselves around desks ready for group work in the classroom. The seating arrangement for n students at d desks is shown below.

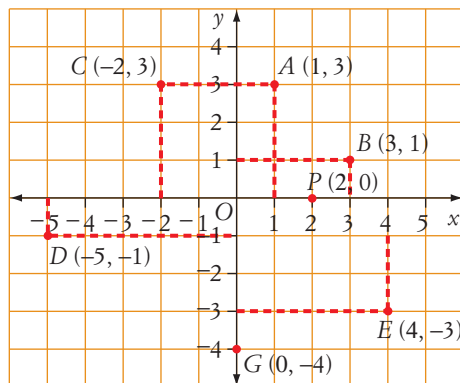


- Find the number of students who can be seated around a row of:
 - 2 desks
 - 3 desks
 - 4 desks
 - 10 desks
- Find a formula that relates the number of students, n , to the number of desks, d .
- Find the number of students that could be seated around 18 desks.
- Class 8Y has 29 students. How many desks are needed for them?

12-04 The number plane

A **number plane** is a grid made from a horizontal number line called the **x -axis** and a vertical number line called the **y -axis**. The number plane is also called a **Cartesian plane**, named after René Descartes (pronounced 'Ren-ay Day-cart'), a French mathematician and philosopher who developed the idea.

Points on the number plane are located using a pair of coordinates or **ordered pair**. Note that order is important with coordinates: $A(1, 3)$ is not the same as $B(3, 1)$.



Summary

In any ordered pair, the first number is called the **x -coordinate** and the second number is called the **y -coordinate**.

The point $(0, 0)$ is called the **origin**.

To locate a point, always start from $(0, 0)$: the x -coordinate tells you how far to move *across*, the y -coordinate tells you how far to move *up* or *down*.

The number plane is divided evenly into four regions called **quadrants**.

Skillsheet

The number plane

MAT08NASS10040

Worksheet

The number plane

MAT08NAWK10114

Worksheet

Number plane review

MAT08NAWK10115

Worksheet

Number plane grid paper

MAT08NAWK10120

Worksheet

A page of number planes

MAT08NAWK10121

Puzzle sheet

Coordinates code puzzle

MAT08NAPS00013

Worksheet

Plotting points and lines

MAT08NAWK00032

Technology worksheet

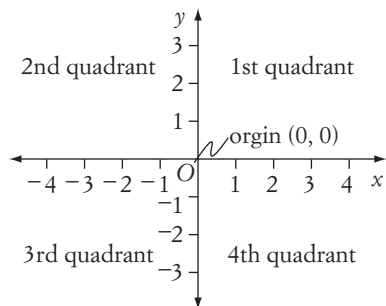
Excel: Plotting points

MAT08NACT00006

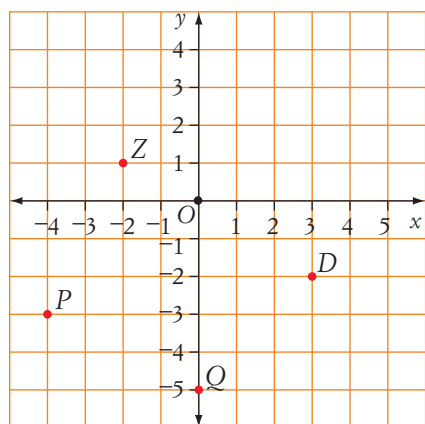
Technology

Excel: Points plotter

MAT08NACT00020

**Example 5**

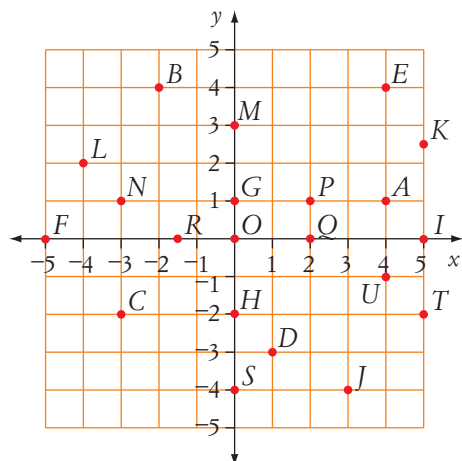
- Write the coordinates of point D and state what quadrant it is in.
- Write the point with coordinates $(-4, -3)$ and state what quadrant it is in.
- Write the coordinates of point Q .

**Solution**

- $D(3, -2)$ is in the 4th quadrant.
- $(-4, -3)$ is the point P , which is in the 3rd quadrant.
- Point Q is 0 on the x -axis and -5 on the y -axis, so its coordinates are $(0, -5)$.

Exercise 12-04 The number plane

Questions 1 to 5 refer to this diagram.



See Example 5

- Name the point with each pair of coordinates.
a (2, 1) **b** (-3, 1) **c** (5, $2\frac{1}{2}$) **d** (5, -2) **e** (0, 3)
f (0, -4) **g** (2, 0) **h** ($-1\frac{1}{2}$, 0) **i** (-4, 2) **j** (4, -1)
- Write the coordinates of each point.
a A **b** B **c** C **d** D **e** E
f F **g** G **h** H **i** I **j** J
- In which quadrant does each of these points lie?
a N **b** T **c** E **d** L **e** C
- List all of the points that are on the x -axis and write their coordinates.
- List all of the points that are on the y -axis and write their coordinates.
- If a point has a positive x -coordinate and a negative y -coordinate, which quadrant is it in? Select the correct answer **A**, **B**, **C** or **D**.
A 1st quadrant **B** 2nd quadrant **C** 3rd quadrant **D** 4th quadrant
- Determine the quadrant in which each point lies.
a (3, -5) **b** (-2, -4) **c** (-8, 1) **d** (5, 4)
- Draw a number plane, with both axes extending from -8 to 8, or use the link to print the Plane grid.
 - Mark these points.
 A(3, 0) B(-2, 2) C(4, 5) D($3\frac{1}{2}$, -5) E(-1, 7)
 F(0, -4) G(-6, 0) H($\frac{1}{2}$, $-7\frac{1}{2}$) I(-2, 6) J(7, 0)
- Draw a number plane, with both axes extending from -3 to 3, or use the link to print the Plane grid.
 - Mark these points.
 A(0, 1), B(2, -1), C(-1, 2), D(-2, 3), E(3, -2) F(1, 0).
 - What do you notice about the points? Check this with your ruler
 - Points that lie on the same straight line are said to be **collinear**. Write down the coordinates of any four points on a number plane that are collinear.

Worked solutions

Exercise 12-04

MAT08NAWS10091

Worksheet

Number plane grid
paper

MAT08NAWK10120

Worksheet

A page of number
planes

MAT08NAWK10121

Just for the record

Latitude and longitude

A position on the Earth's surface is indicated by a coordinate system of **latitude** and **longitude**. Latitude measures how many degrees north or south places are in relation to the Equator, an imaginary line that runs across and around the Earth and divides it in half. Longitude measures how far east or west places are from the Greenwich meridian, an imaginary line that runs down the Earth from the North Pole to the South Pole, passing through Greenwich in London, England. Every place on Earth can be located by its latitude and longitude coordinates. For example, the position coordinates of Canberra are 35°S 149°E. When you look up a place in an atlas or GPS navigator, it tells you its latitude and longitude coordinates.

Find out the latitude and longitude of your town or city.



Mental skills 12

Maths without calculators

The unitary method with percentages

The unitary method is used when you are only given a *percentage* of an amount and you need to find the amount. It is called the unitary method because we find 1% of the amount first, then multiply that by 100 to find the whole (100%).

1 Study each example.

a If 8% of a number is 24, what is the number?

8% of the number = 24

$\therefore$ 1% of the number = $24 \div 8 = 3$

$\therefore$ 100% of the number = $3 \times 100 = 300$.

The number is 300. *Check:* $8\% \times 300 = 24$

b If 15% of an amount is \$90, what is the whole amount?

15% of the amount = \$90

$\therefore$ 1% of the amount = $\$90 \div 15 = \$6$

$\therefore$ 100% of the amount = $\$6 \times 100 = \$600$.

The amount is \$600. *Check:* $15\% \times \$600 = \$90$

2 Find the whole amount if:

a 5% of the amount is \$35

c 20% of the amount is 80

e 90% of the amount is \$270

g 40% of the amount is 100

i 25% of the amount is \$75

b 11% of the amount is \$88

d 6% of the amount is 42

f 15% of the amount is \$60

h 120% of the amount is \$360

j 8% of the amount is 40

Homework sheet

Graphing tables of values

MAT08NAHS10021

Worksheet

Number plane grid paper

MAT08NAWK10120

Worksheet

A page of number planes

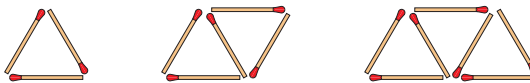
MAT08NAWK10121

12-05 Graphing number patterns

We can graph number and geometrical patterns on a number plane.

Example 6

Consider this pattern of toothpicks used to make triangles.



a Copy and complete the table of values for this pattern.

No. of triangles, x	1	2	3	4	5
No. of toothpicks, y					

b Find the formula for this table of values.

c Graph this table of values.

Solution

a

No. of triangles, x	1	2	3	4	5
No. of toothpicks, y	3	5	7	9	11

- b Bottom row of y -values increase by 2, so the multiplier in the formula is 2, and the equation is of the form $y = 2x$ ____.

Choose (2, 5) from the table to find the missing number.

When $x = 2$, $y = 2 \times 2 + 1 = 5$.

The formula is $y = 2x + 1$.

- c Each column in the table forms an ordered pair.

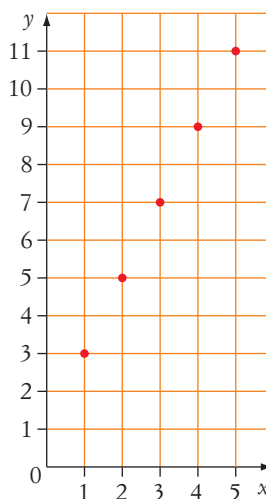
No. of triangles, x	1	2	3	4	5
No. of toothpicks, y	3	5	7	9	11

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 (1, 3) (2, 5) (3, 7) (4, 9) (5, 11)

Graph each ordered pair on a number plane.

Note that the points are **collinear**, that is, they lie on a straight line. When this happens, we say that the relationship or pattern is **linear**.

- **linear** = involving a line
- **collinear points** = points that lie on a straight line



Exercise 12-05 Graphing number patterns

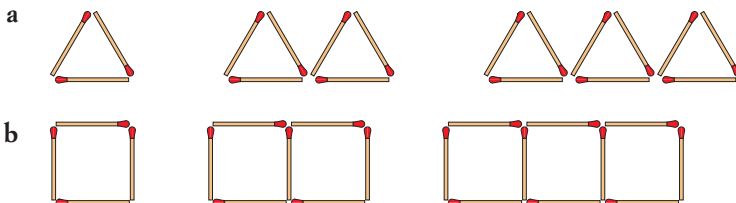
- 1 For each pattern below:

- i copy and complete this table

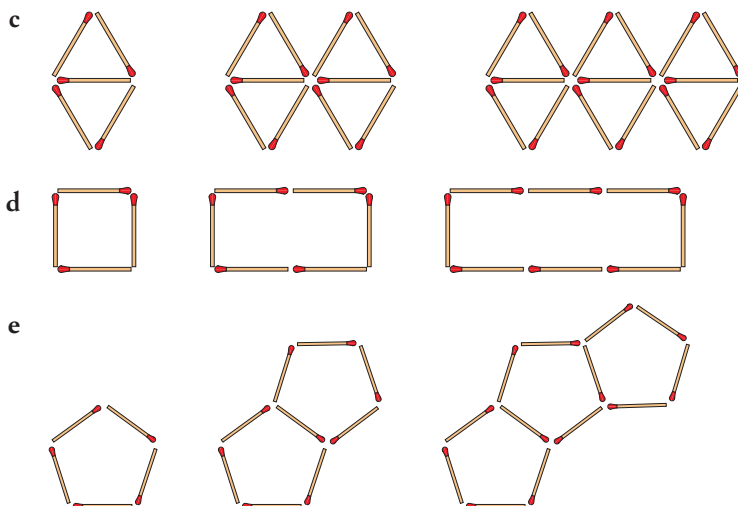
No. of shapes, x	1	2	3	4	5
No. of toothpicks, y					

- ii find the formula for the table of values

- iii graph the table of values.



See Example 6



Worked solutions

Exercise 12-05

MAT08NAWS10092

- 2 Does each graph in question 1 show a linear relationship?
- 3 The following table shows the height of a tree as it ages. Graph this table of values and determine whether there is a linear pattern between the age and height of the tree.

Age, x (years)	5	10	15	20	25
Height, y (metres)	10	12.5	17.5	21	26

- 4 Cans in supermarkets are often stacked neatly in triangles. Consider the relationship between the number of cans on the bottom row, x and the total number of cans, y .



- a Copy and complete this table.

Length of triangle, x	1	2	3	4	5	6	7
No. of cans, y							

- b Graph this table on a number plane.
- c Is the relationship linear?
- d Which formula matches the table of values? Select the correct answer A, B or C.

A $y = x(x - 1)$ B $y = \frac{x+1}{2}$ C $y = \frac{x(x+1)}{2}$

12-06 Graphing linear equations

A **linear equation** is an equation or formula whose graph on a number plane is a **straight line**.

Summary

To graph a linear equation:

- complete a table of values
- graph the table of values on a number plane
- rule a line through the points and label the line with the equation.

Example 7

Graph $y = x + 1$ on a number plane.

Solution

Complete a table of values. Choose x -values close to 0 for easy calculation and graphing.

x	-2	-1	0	1	2
y	-1	0	1	2	3

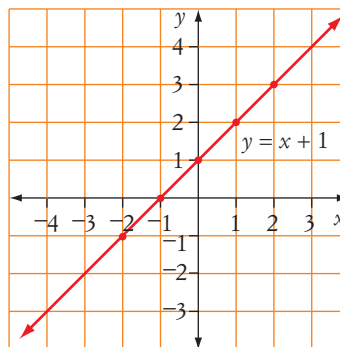
Graph the table of values on a number plane.

Rule a straight line through the points, place arrows at each end, and label the line with its equation.

Note:

- Every pair of values that follows the linear equation $y = x + 1$ lies on this line, for example, $(3, 4)$, $(-4, -3)$, $(1\frac{1}{2}, 2\frac{1}{2})$, $(5, 6)$, $(100, 101)$, $(-\frac{1}{2}, \frac{1}{2})$.
- Every point on the line follows the linear equation $y = x + 1$.
- There are an infinite number of points that satisfy the rule.

For these reasons, the line is infinite so we must draw arrows at each end.



Worksheet

Graphing linear equations

MAT08NAWK00029

Puzzle sheet

Matching linear equations

MAT08NAPS00011

Worksheet

Number plane grid paper

MAT08NAWK10120

Puzzle sheet

Graphing functions

MAT08NAPS00014

Technology worksheet

Excel: Graphing straight lines: Finding intercepts

MAT08NACT00007

Technology

Excel: Linear functions plotter

MAT08NACT00021

Video tutorial

Graphing linear equations

MAT08NAVT10023

Video tutorial

Linear functions

MAT08NAVT00004

Example 8

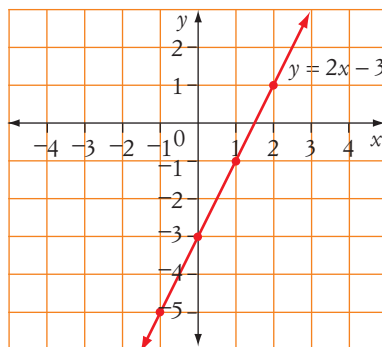
- Graph $y = 2x - 3$ on a number plane.
- Test whether the point $(5, 8)$ lies on the line $y = 2x - 3$.

Solution

a

x	-1	0	1	2
y	-5	-3	-1	1

- b We could extend the line $y = 2x - 3$ to see if $(5, 8)$ lies on it, but a simpler way is to check whether $(5, 8)$ follows or **satisfies** the formula $y = 2x - 3$. If it does, then when $x = 5$, $y = 8$. When $x = 5$, $y = 2 \times 5 - 3 = 7 \neq 8$. $\therefore (5, 8)$ does not lie on the line. (In fact, $(5, 7)$ does)



Exercise 12-06 Graphing linear equations

See Example 7

- 1 For each linear equation, copy and complete the table of values and graph the equation on a number plane.

a $y = x + 3$

x	-1	0	1	2
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

x	-1	0	1	2
y				

d $y = \frac{x}{2}$

x	-1	0	1	2
y				

e $y = 6 - x$

x	0	1	2	3
y				

f $y = x$

x	-1	0	1	2
y				

See Example 8

- 2 Graph each linear equation on a number plane.

a $y = 2x + 1$

b $y = 2x - 1$

c $y = 3x$

d $y = 3x - 2$

e $y = 2x + 4$

f $y = 3x - 5$

- 3 Look at the graphs you drew in question 2. Why are they called **increasing graphs**?

- 4 Graph each linear equation on a number plane.

a $y = -x$

b $y = -x + 5$

c $y = 4 - x$

d $y = -x - 3$

e $y = 1 - 3x$

f $y = -2x$

- 5 Look at the graphs you drew in question 4. Why are they called **decreasing graphs**?

- 6 By looking at each linear equation, predict whether its graph will be increasing or decreasing.

a $y = 5x - 4$

b $y = x + 9$

c $y = 3 - x$

d $y = 2x$

e $y = -x + 5$

f $y = -3x - 1$

- 7 Test whether each of the following points lies on the given line by:

i substituting its coordinates into the linear equation

ii examining the graphs of the lines you drew in questions 1 and 2

a $(4, 7)$ $y = x + 3$

b $(-1, -1)$ $y = x - 2$

c $(5, 1)$ $y = 6 - x$

d $(-3, -5)$ $y = 2x + 1$

e $(4, 10)$ $y = 3x - 2$

f $(-2, 1)$ $y = 2x + 4$

- 8 On which of the following lines does the point $(1, 3)$ lie? Select the correct answer A, B, C or D.

A $y = -x + 3$

B $y = x + 4$

C $y = 2 - 2x$

D $y = 2x + 1$

Worksheet

Number plane grid paper

MAT08NAWK10120

Worksheet

A page of number planes

MAT08NAWK10121

Worked solutions

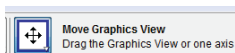
Exercise 12-06

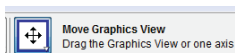
MAT08NAWS10093

Technology Graphing linear equations

Using GeoGebra

- Under **View**, click on **Axes**, **Grid** and **Spreadsheet**.
- In cell A1 of the spreadsheet, enter $y = 2x - 3$, then press Enter.
- In the **Graphics View**, you will see the graph of $y = 2x - 3$. Right-click on the line, choose **Object Properties**, and change the colour of the line.



- Click  and use your graph to find the value of y when:

a $x = 5$ **b** $x = -6$ **c** $x = -1.5$

- Use your graph to find the value of x when:

a $y = 11$ **b** $y = -9$ **c** $y = 14$

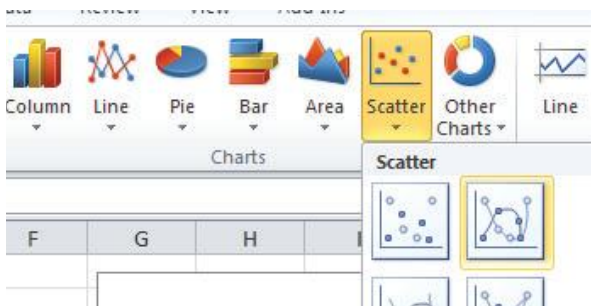
Using a spreadsheet

- Enter these values into a spreadsheet.

	A	B
1	x	y
2	-4	
3	-3	
4	-2	
5	-1	
6	0	
7	1	
8	2	
9	3	
10	4	
11	5	
12		

- In cell B2, enter the formula $=2*A2-3$ to evaluate $2x - 3$ and use **Fill Down** to copy the formula down to cell B11.

- Highlight columns A and B, then click **Insert** and **Scatter** (with **Smooth Lines and Markers**).



- Click the bottom-right corner of the graph border to enlarge the graph.

- Use your graph to predict the value of y when:

a $x = 5$ **b** $x = -6$ **c** $x = -1.5$

- Use your graph to predict the value of x when:

a $y = 11$ **b** $y = -9$ **c** $y = 14$

Investigation: Special lines

- 1 Draw a number plane with both axes extending from -4 to 4 and graph each table of values on it.

a

x	-1	0	2	3
y	2	2	2	2

b

x	-2	0	1	3
y	-1	-1	-1	-1

c

x	2	-1	0	-4
y	1	1	1	1

d

x	-2	3	0	4
y	-3	-3	-3	-3

- 2 **a** What do all of the lines in question 1 have in common?
b What do all of the tables of values in question 1 have in common?
c The formula for the table of values and line in question 1 **a** is $y = 2$. It does not have an x in it because the value of y does not depend on x but is always equal to 2. What are the formulas for the other three lines in question 1?
- 3 Draw another number plane and graph each table of values on it.
- a**
- | | | | | |
|-----|------|------|-----|-----|
| x | 1 | 1 | 1 | 1 |
| y | -3 | -2 | 0 | 2 |
- b**
- | | | | | |
|-----|------|------|------|------|
| x | -2 | -2 | -2 | -2 |
| y | -2 | 0 | 1 | 3 |
- c**
- | | | | | |
|-----|------|-----|------|-----|
| x | 3 | 3 | 3 | 3 |
| y | -3 | 2 | -2 | 0 |
- d**
- | | | | | |
|-----|------|------|------|------|
| x | -3 | -3 | -3 | -3 |
| y | -4 | -1 | 2 | 4 |
- 4 **a** What do all of the lines in question 3 have in common?
b What do all of the tables of values in question 3 have in common?
c The formula for the table of values and line in question 3 **a** is $x = 1$. What are the formulas for the other three lines in question 3?

Worksheet

Finding linear equations

MAT08NAWK00030

Puzzle sheet

Matching linear
equations

MAT08NAPS00011

Video tutorial

Finding the equation of
a line

MAT08NAVT10024

Technology

GeoGebra: Equation of
a line

MAT08NATC00006

Quiz

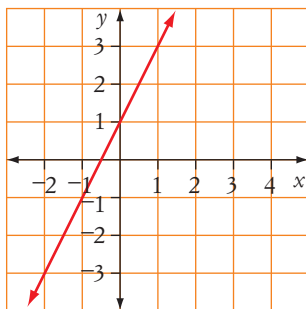
Functions and graphs

MAT08NAQZ00004

12-07 Finding the equation of a line

Example 9

Find the equation of this line.



Solution

First draw a table of values using the coordinates of the points on the line, with consecutive x -values.

x	-2	-1	0	1
y	-3	-1	1	3

$\xrightarrow{+2}$ $\xrightarrow{+2}$ $\xrightarrow{+2}$

The y -values go up by 2, so the multiplier is 2, and the equation is of the form $y = 2x$ ____.

Choose the point (1, 3) to find the missing number.

When $x = 1$, $y = 2 \times 1 + 1 = 3$.

So the equation is $y = 2x + 1$. (Check that this is also true for (0, 1)).

Horizontal and vertical lines

A **horizontal** line is flat and runs across (sideways). The word 'horizontal' comes from 'horizon,' the flat line that is the edge of the Earth as far as your eye can see.

A **vertical** line runs up and down, at right angles to a horizontal line. The word 'vertical' means upright. For example, vertical blinds are made up of columns of slats.

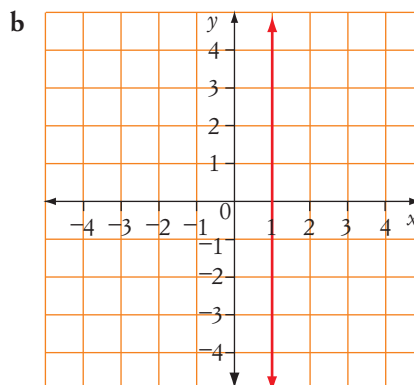
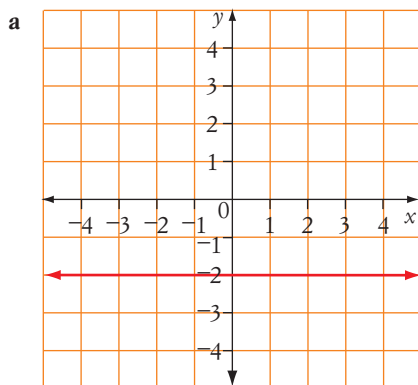
Technology worksheet

GeoGebra: Horizontal and vertical lines

MAT08NACT10010

Example 10

Find the equation of each line.



Solution

a This is a **horizontal** line.

x	-1	0	1	2
y	-2	-2	-2	-2

y is always equal to -2 .

So the equation is $y = -2$.

b This is a **vertical** line.

x	1	1	1	1
y	0	1	2	3

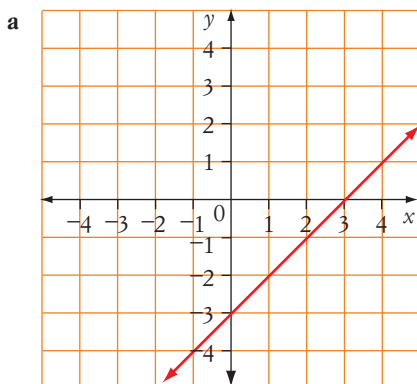
x is always equal to 1.

So the equation is $x = 1$.

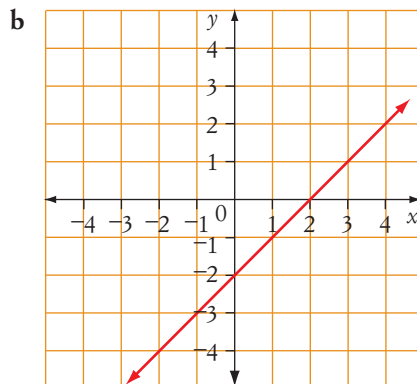
Summary

The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).

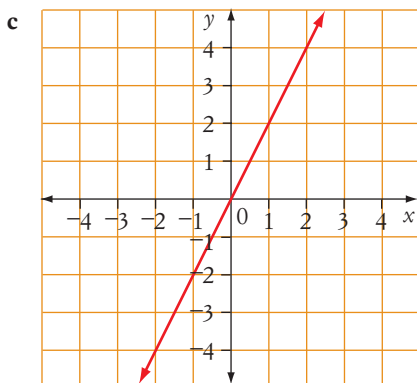
The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).

Exercise 12-07 Finding the equation of a line**See Example 9** 1 For each graph below, complete a table of values and use it to find the equation of the line.

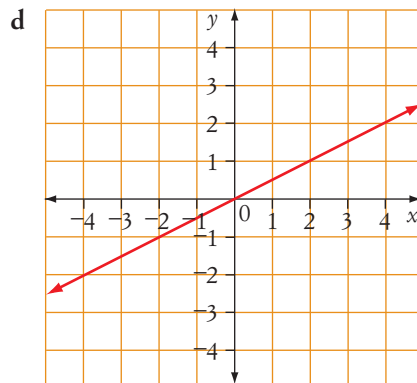
x	-1	0	1	2
y				



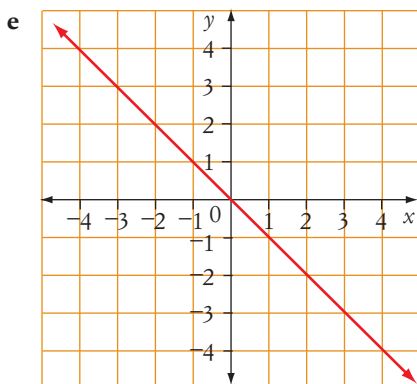
x	-2	-1	0	1
y				



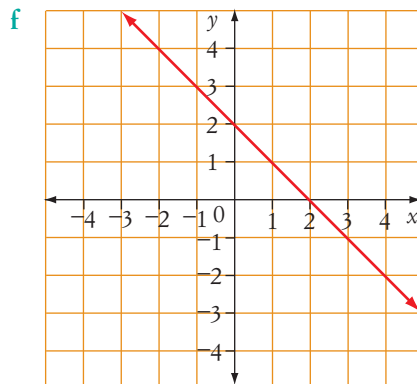
x	-1	0	1	2
y				



x	0	1	2	3
y				



x	-1	0	1	2
y				

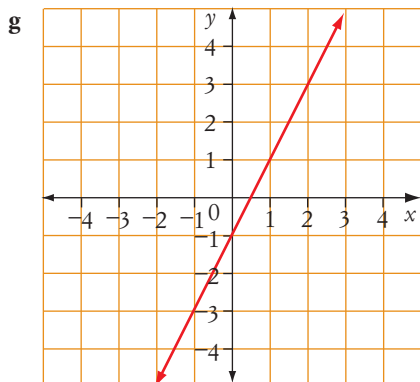


x	-3	-2	-1	0
y				

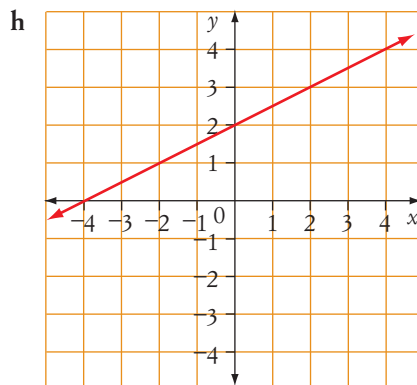
Worked solutions

Exercise 12-07

MAT08NAWS10094



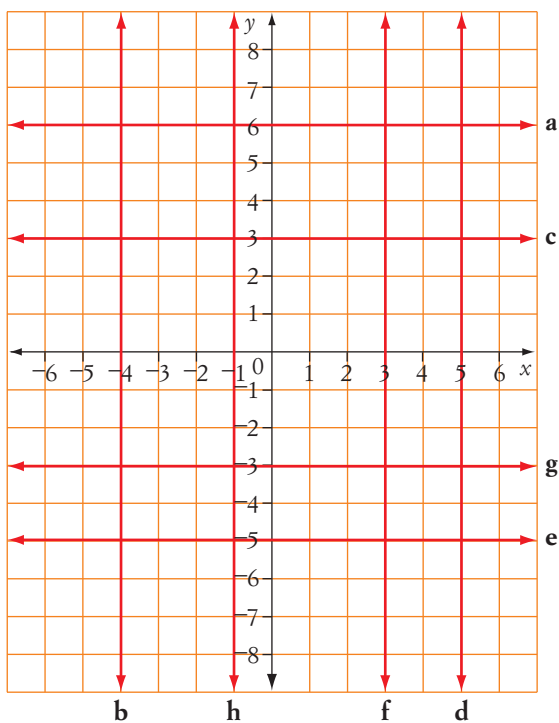
x	-1	0	1	2
y				



x	-2	-1	0	1
y				

2 Write down the equation of each line in the diagram below.

See Example 10



3 Graph each line on the same number plane.

a $x = 4$

b $x = -2$

c $y = 5$

d $y = -1$

e $x = 1$

f $y = -3$

g $y = 4$

h $x = -3$

Worked solutions

Exercise 12-07

MAT08NAWS10094

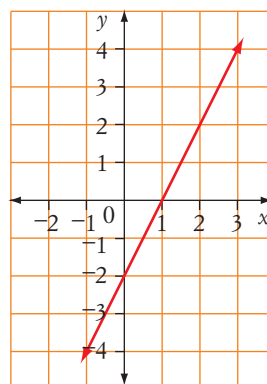
- 4 What is the equation of this line? Select the correct answer **A**, **B**, **C** or **D**.

A $y = -x + 2$

B $y = 2x - 2$

C $y = 2 - 2x$

D $y = x - 2$



Technology Comparing linear equations

Using GeoGebra

In this activity, we will use a spreadsheet to compare the graphs of $y = x$, $y = 2x$ and $y = 3x$.

- Under **View**, click on **Axes**, **Grid** and **Spreadsheet**.
- In the **Spreadsheet View**, enter the three linear equations as shown below to graph them.

Spreadsheet				
	A	B	C	D
1	$y = x$	$y = 2x$	$y = 3x$	
2				

- For each equation graphed, right-click on the line, click **Object Properties** and change the colour of the line.
- Compare the three lines. What is the same and what is different? Describe any patterns and important features. Write your answers on your spreadsheet or in your workbook.
- Enter three new sets of three linear equations in rows 1, 4 and 6 as shown.

Spreadsheet				
	A	B	C	
1	$y = -x$	$y = -2x$	$y = -5x$	
2				
3				
4	$y = x + 1$	$y = x + 2$	$y = x + 4$	
5				
6				
7	$y = -x + 1$	$y = -x + 2$	$y = -x - 4$	
8				
9				

You may wish to change the colour of each line to remember which equation is associated with each graph (as described in step 3 above).

- Compare the three lines of each set and describe the pattern and important features of each set.

Using a spreadsheet

In this activity, we will use a spreadsheet to compare the graphs of $y = x$, $y = 2x$ and $y = 3x$.

- 1 Set up this spreadsheet.

	A	B	C	D
1	x	$y = x$	$y = 2x$	$y = 3x$
2	-2			
3	-1			
4	0			
5	1			
6	2			
7				

- 2 In cell B2 enter **=A2** to calculate $y = x$, then use **Fill Down** to copy this formula down to B6.
- 3 In cell C2 enter **=2*A2** to calculate $y = 2x$, then **Fill Down** to C6.
- 4 In cell D2, write a formula to calculate $y = 3x$, then **Fill Down** to D6.
- 5 Highlight all cells. Click **Insert** and **Scatter (Smooth Lines and Markers)** to graph all three lines on the same axes.
- 6 Compare the three lines. What is the same and what is different? Describe any patterns and important features. Write your answers on your spreadsheet or in your workbook.
- 7 Set up a similar spreadsheet for each set of three linear equations shown below. Use appropriate formulas to complete each column and graph each set of three lines. Compare the three lines in each set and describe the pattern and important features of each set.

a

G	H	I	J
x	$y = -x$	$y = -2x$	$y = -5x$
-2			
-1			
0			
1			
2			

b

L	M	N	O
x	$y = x + 1$	$y = x + 2$	$y = x - 4$
-2			
-1			
0			
1			
2			

c

Q	R	S	T
x	$y = -x + 1$	$y = -x + 2$	$y = -x - 4$
-2			
-1			
0			
1			
2			

12-08

Comparing linear and non-linear equations

A **linear equation** such as $y = 2x + 1$ is made up of two terms:

- the **variable term**, or the term with the x
- the **constant term**, the number without the x .

The number in front of the x is called the **coefficient** of x . For example,

In the linear equation:

$$y = 2x + 1$$

The coefficient of x is 2. The constant term is 1.

NSW

Puzzle sheet

Matching linear equations

MAT08NAPS00011

Video tutorial

Linear functions

MAT08NAVT00004

Technology worksheet

Graphing straight lines:
Changing m- and c-values

MAT08NACT00008

Technology

Excel: Multiple line plotter

MAT08NACT00022

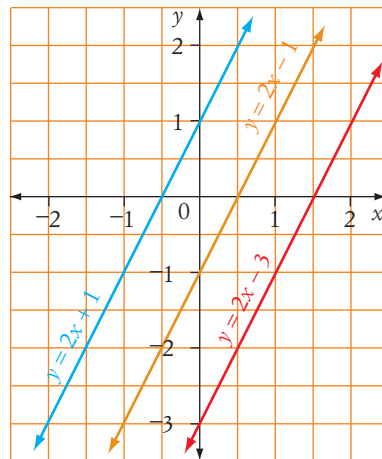
Example 11

The lines $y = 2x + 1$, $y = 2x - 1$ and $y = 2x - 3$ have been graphed below.

- How are the lines similar?
- How are the linear equations similar?
- The **y-intercept** of a line is the value at which the line crosses the y-axis. Find the y-intercept of each line in the diagram.
- How are the linear equations different?

Solution

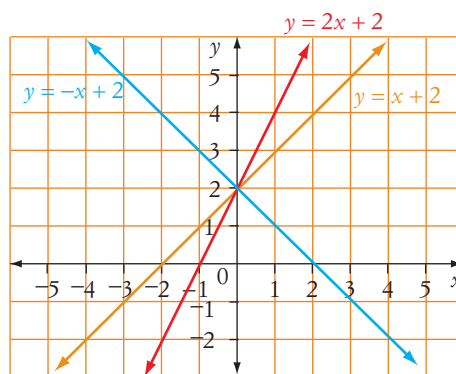
- The lines point in the same direction and have the same slope, they are parallel.
- The linear equations all have the same coefficient of x , 2.
- | | |
|----------------|------------------|
| $y = 2x + 1$: | y-intercept = 1 |
| $y = 2x - 1$: | y-intercept = -1 |
| $y = 2x - 3$: | y-intercept = -3 |
- The linear equations have different constant terms: 1, -1, -3.



Example 12

The lines $y = x + 2$, $y = -x + 2$ and $y = 2x + 2$ have been graphed below.

- Find the y-intercept of each line.
- How are the linear equations similar?
- How are the lines different?
- How are the linear equations different?
- How does the coefficient of x in a linear equation affect its graph?
- How does the constant term in a linear equation affect its graph?

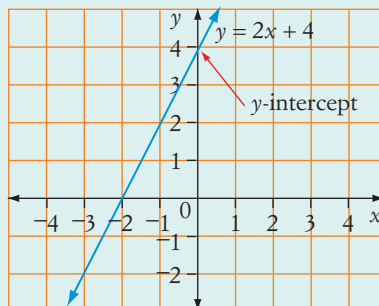


Solution

$y = -x + 2$:	y-intercept = 2
$y = 2x + 2$:	y-intercept = 2
$y = x + 2$:	y-intercept = 2
- The linear equations all have the same constant term, 2.
- The lines point in different directions: they are not parallel.
- The linear equations have different coefficients of x : -1, 2 and 1.
- The coefficient of x affects the direction of the graph.
- The constant term gives the y-intercept of the graph.

Summary

- In a linear equation, the **coefficient of x** gives the **direction** of the line.
- If two or more lines have the same coefficient of x in their equations, then they are parallel.
- The **constant term** gives the **y -intercept** of the line.



Exercise 12-08 Comparing linear and non-linear equations

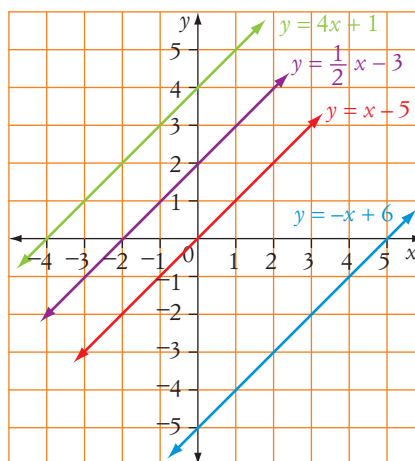
The exercise is best completed using *GeoGebra*, a spreadsheet or other graphing software.

- 1 For each linear equation given below, state the coefficient of x and the constant term.

a $y = 4x + 1$ b $y = \frac{1}{2}x - 3$ c $y = x - 5$ d $y = -x + 6$

- 2 The lines $y = x + 4$, $y = x + 2$, $y = x$ and $y = x - 5$ have been graphed below.

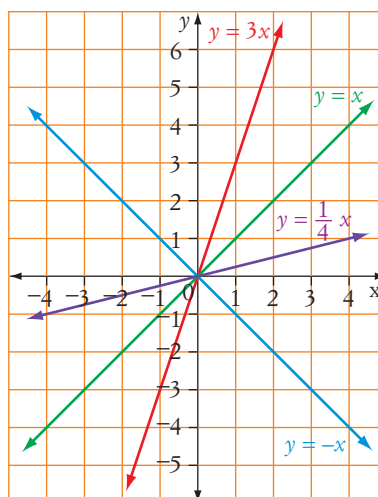
- How are the lines similar?
- How are the linear equations similar?
- How are the lines different?
- How are the linear equations different?



See Example 11

- 3 The lines $y = \frac{1}{4}x$, $y = x$, $y = 3x$ and $y = -x$ have been graphed below.

- How are the lines similar?
- How are the linear equations similar?
- How are the lines different?
- How are the linear equations different?
- Which line is the steepest? What is the coefficient of x in its equation?
- Which line is the least steep? What is the coefficient of x in its equation?
- How does the size of the coefficient of x in a linear equation affect the steepness of its line?



See Example 12

- 4 Graph $y = 3x$, $y = 3x - 1$ and $y = 3x + 2$ on the same number plane.
- What is the same about all three lines?
 - What is the same about all three linear equations?
 - What is different about the lines?
 - What is different about the linear equations?
- 5 Graph $y = x + 1$, $y = 3x + 1$ and $y = \frac{1}{2}x + 1$ on the same number plane.
- What is the same about all three lines?
 - What is the same about all three linear equations?
 - What is different about the lines?
 - What is different about the linear equations?
 - What is the coefficient of x in the equation with the steepest line?
- 6 Graph $y = -x - 2$, $y = -2x - 2$ and $y = -\frac{1}{2}x - 2$ on the same number plane.
- How are the lines similar?
 - How are the linear equations similar?
 - How are the lines different?
 - How are the lines different to the lines in question 5?
 - What is the coefficient of x in the equation with the least steepest line?
- 7 The graphs in question 5 are all increasing while the graphs in question 6 are all decreasing. What feature of their equations indicates this?
- 8 Which equation when graphed gives a decreasing line with a y -intercept of 5? Select the correct answer **A**, **B**, **C** or **D**.
- A** $y = 5x$ **B** $y = 5 - x$ **C** $y = 5x - 5$ **D** $y = -5x$
- 9 Copy and complete each statement.
- The equations of parallel lines have the same _____.
 - Linear equations with the same constant term have graphs that have the same _____.
 - Linear equations with a negative coefficient of x have graphs that are _____.
 - The equations of lines with the same y -intercept have the same _____.
 - The equations of lines that are decreasing have a _____ coefficient of x .

- 10 Graph each **non-linear equation** after completing the table of values given.

a $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

b $y = \sqrt{x}$

x	0	1	2	4	5	7	9
y							

Worked solutions

Exercise 12-08

MAT08NAWS10095

c $y = x^3$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y									

d $y = \frac{1}{x}$

x	0.5	1	1.5	2	2.5	3	3.5	4	4.5
y									

- 11 How are the non-linear equations (which have curved graphs) in question 10 different to the linear equations?

12-09 Solving linear equations graphically

Equations can be solved **algebraically** using the balancing and backtracking methods. However, we can also solve an equation **graphically** by first graphing it on the number plane.

Example 13

Solve the equation $2x - 3 = 5$ graphically.

Solution

Graph $y = 2x - 3$ on a number plane (the LHS of the equation).

x	-1	0	1	2
y	-5	-3	-1	1

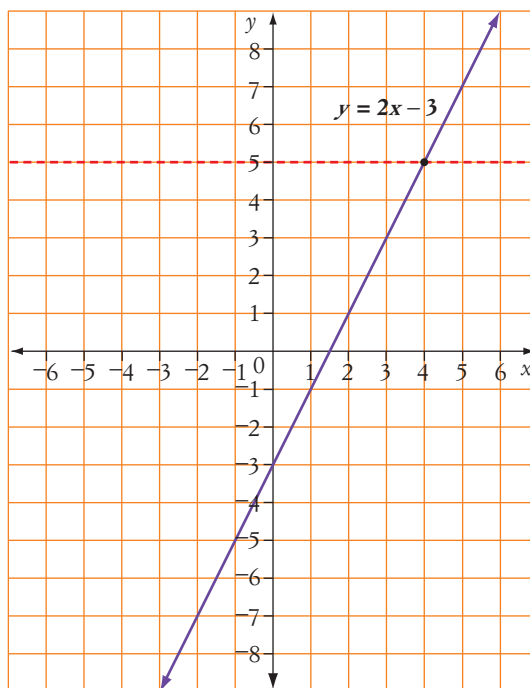
For each point in the table and on the line, the y -coordinate is the value of $2x - 3$. So to solve $2x - 3 = 5$, we need to find the point whose y -coordinate is 5.

Draw a dotted horizontal line at $y = 5$ (shown in red on the diagram) and read off the coordinates for the point on $y = 2x - 3$ crossed by this line.

This point is $(4, 5)$, which means that the solution to $2x - 3 = 5$ is $x = 4$.

Check: $2 \times 4 - 3 = 5$

LHS = left-hand side



Worksheet

Number plane grid
paper

MAT08NAWK10120

Worksheet

A page of number
planes

MAT08NAWK10121

Exercise 12-09 Solving linear equations graphically

Worksheet

Number plane grid paper

MAT08NAWK10120

See Example 13

Worksheet

A page of number planes

MAT08NAWK10121

Worked solutions

Exercise 12-09

MAT08NAWS10096

This exercise is best completed using *GeoGebra*, a graphics calculator or other graphing software (see the *Technology* section that follows for more instructions).

- 1 Use the graph in Example 13 above to solve each equation. Check your solutions.

a $2x - 3 = 3$ b $2x - 3 = -7$ c $2x - 3 = 0$

- 2 Graph $y = 2x + 1$ and use it to solve each equation graphically.

a $2x + 1 = 7$ b $2x + 1 = 10$ c $2x + 1 = -5$

- 3 Solve each equation in question 2 algebraically.

- 4 Graph $y = 2x - 1$ and use it to solve each equation graphically.

a $2x - 1 = -5$ b $2x - 1 = -1$ c $2x - 1 = 4$

- 5 Graph $y = 3x - 2$ and use it to solve each equation graphically.

a $3x - 2 = 7$ b $3x - 2 = -11$ c $3x - 2 = -2$

- 6 Graph $y = -x + 1$ and use it to solve each equation graphically.

a $-x + 1 = -3$ b $-x + 1 = 4$ c $-x + 1 = -1$

- 7 Graph $y = -2x - 1$ and use it to solve each equation graphically.

a $-2x - 1 = -9$ b $-2x - 1 = 0$ c $-2x - 1 = 3$

- 8 Graph $y = \frac{1}{2}x + 3$ and use it to solve each equation graphically.

a $\frac{1}{2}x + 3 = 5$ b $\frac{1}{2}x + 3 = 1$ c $\frac{1}{2}x + 3 = 2$

Technology

GeoGebra: Solving equations

MAT08NATC00010

Technology Solving linear equations graphically

In this activity we will use *GeoGebra* or a graphics calculator to solve $2x + 1 = 4$.

Using *GeoGebra*

- Click **View** and **Spreadsheet View**
- Enter the equations in cells A1 and A2 as shown.

	A	
1	$y = 2x + 1$	
2	$y = 4$	
3		

- Find the point of intersection of the two equations graphed. What is the value of x ? This is the solution to the equation $2x + 1 = 4$.

- 4 Enter the two new equations in cells A4 and A5 as shown. Read off the point of intersection from the graph. What is the value of x ?

	A
1	$y = 2x + 1$
2	$y = 4$
3	
4	$y = -3x$
5	$y = -6$
6	
7	

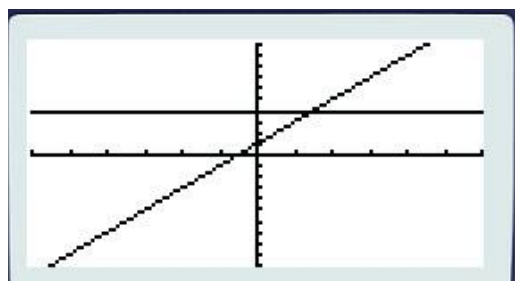
- 5 Use *GeoGebra* to solve some of the equations from Exercise 12-09 above.

Using a Casio graphics calculator

- Press **MENU**, select **GRAPH** and press **EXE** to display the **Graph Function** screen.
- Enter the two equations in Y1 and Y2 as shown. For x , use the **X,0,T** key and press **EXE** after each equation.



- Press **SHIFT** **F3** for **V-Window** (View window) to set up the scale of your x - and y -axes. Set Xmin to -6 , Xmax to 6 , scale to 1 , Ymin to -10 , Ymax to 10 and scale to 1 . Press **EXE**.
- Press **F6** for **Draw** or **EXE** to graph the two lines.



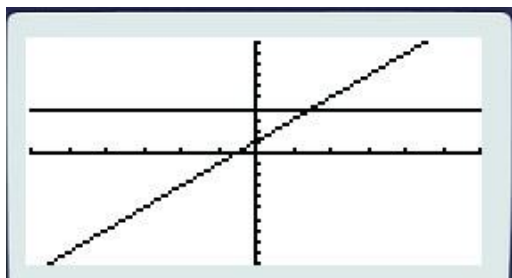
- Press **SHIFT** **F5** for **G-Solv** (Graph-solve) and **F5** for **ISCT** (point of intersection). The calculator should read $(1.5, 4)$ or $x = 1.5, y = 4$. This means that the solution to $2x + 1 = 4$ is $x = 1.5$.
- Press **EXIT** to return to the **Graph Function** screen.
- Solve each equation graphically:

a $-3x = -6$	b $2x + 5 = -1$	c $1 - 3x = 4$	d $4x + 6 = 8$
e $x = -x$	f $1 - 6x = 10$	g $4 + 3x = -2$	h $4 + x = 5 - x$

- 8 Now write 3 different equations of your own and use the calculator to solve them. Swap them with other students in your class and try to solve their equations using the calculator. Check your answers to make sure your entries in the calculator are correct.

Using a Texas Instruments (TI) calculator

- 1 Press **Y=** to display the **Graph Function** screen.
- 2 Enter at Y1 ' $2x + 1$ ' and at Y2 ' 4 '. For x , use the **X,θ,T,n** key and press **ENTER** after each equation.
- 3 Press **WINDOW** to set up the scale of your x - and y -axes. Set Xmin to -6 , Xmax to 6 , Xscale to 1 , Ymin to -10 , Ymax to 10 and Yscale to 1 . Press **ENTER**.
- 4 Press **GRAPH** to graph the two lines.



- 5 Press **2nd** **TRACE** for **CALC** (Calculate) and **5: intersect** for point of intersection. Press **ENTER** three times to locate $(1.5, 4)$ or $x = 1.5, y = 4$. This means that the solution to $2x + 1 = 4$ is $x = 1.5$.
- 6 Press **Y=** to return to the **Graph Function** screen and go to question 7 above.

Technology Intersecting lines

We will graph the two straight lines $y = x + 5$ and $y = 3 - x$ using a spreadsheet to find their point of intersection.

- 1 Set up this spreadsheet.

	A	B	C
1	x	$y = x + 5$	$y = 3 - x$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

- 2 In cell B2, enter $=A2+5$, then **Fill Down** to cell B10.
- 3 In cell C2, enter $=3-A2$, then **Fill Down** to cell C10.
- 4 Highlight all three columns and click **Insert** and **Scatter (Smooth Lines and Markers)**.

- From the graph, read the point of intersection of the two straight lines.
- Find the point of intersection for each pair of straight lines below, by setting up a separate spreadsheet for each pair of lines and using **Insert** and **Scatter (Smooth Lines and Markers)** (as above) to graph each pair of lines on the same set of axes.

a

	A	B	C
1	x	$y = 2x - 2$	$y = -x + 7$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

b

	A	B	C
1	x	$y = 3x - 6$	$y = -4x - 6$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

c

	A	B	C
1	x	$y = 10 - 2x$	$y = 2x$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

12-10 Intersecting lines

Example 14

Graph $y = 2x - 2$ and $y = x + 1$ on the same number plane and use the graphs to solve the equation $2x - 2 = x + 1$.

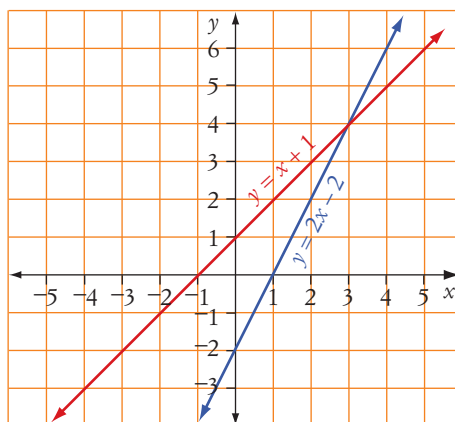
Solution

$$y = 2x - 2$$

x	-1	0	1	2
y	-4	-2	0	2

$$y = x + 1$$

x	-1	0	1	2
y	0	1	2	3



To solve $2x - 2 = x + 1$, we need to find the point on both lines where the y-coordinate is the same.

This is the point where the two lines intersect.

This point is (3, 4), which means that the solution to $2x - 2 = x + 1$ is $x = 3$.

Check: LHS = $2 \times 3 - 2 = 4$

RHS = $3 + 1 = 4$

LHS = RHS

Worksheet

Line intersections

MAT08NAWK00031

Homework sheet

Graphing linear equations

MAT08NAHS10022

Worksheet

A page of number planes

MAT08NAWK10121

Worksheet

Number plane grid paper

MAT08NAWK10120

Worksheet

A page of number planes

MAT08NAWK10121

Worksheet

Number plane grid paper

MAT08NAWK10120

Exercise 12-10 Intersecting lines

This exercise is best completed using *GeoGebra*, a graphics calculator or other graphing software (see the previous *Technology* sections for instructions).

- Graph each pair of equations and find their point of intersection.

a $y = x$ and $y = -x + 2$	b $y = x + 1$ and $y = 2x$
c $y = -x - 1$ and $y = x + 1$	d $y = \frac{x}{2} - 2$ and $y = -x + 1$
e $y = 2x + 1$ and $y = x + 3$	f $y = -2x - 1$ and $y = x + 5$
- What is the point of intersection of $y = x + 1$ and $y = -x + 5$? Select the correct answer **A**, **B**, **C** or **D**.
A (3, 2) **B** (2, 3) **C** (4, 1) **D** (4, 5)
- Solve each equation graphically.

a $2x - 1 = x + 2$	b $3x - 2 = 2x - 1$	c $x - 2 = 4 - x$
d $2x = x + 3$	e $3x + 2 = x - 1$	f $-2x + 3 = 2x + 1$
- Solve each of the equations in question 3 algebraically.
- These three linear equations form a triangle when graphed on the number plane.
 $y = x + 2$ $y = \frac{x}{2} - 1$ $y = -3x + 6$
 Find the coordinates of the three vertices of the triangle.

See Example 14

Worked solutions

Exercise 12-10

MAT08NAWS10097

Power plus

- Graph each equation accurately and state whether it is linear or non-linear.

a $y = x^3 - 2$	b $x + y = 5$	c $y = x^2 - 1$
d $y = 4 - x^2$	e $x - y = 4$	f $y = \frac{4}{x}$

Language of maths

coefficient	formula	number plane	term
collinear	graphically	origin	variable
consecutive	horizontal	quadrant	vertical
constant term	increasing	satisfy	x -axis
coordinates	intersection	solution	y -axis
decreasing	linear	table of values	y -intercept

Puzzle sheet

Linear equations
crossword

MAT08NAPS00012

- 1 What is the difference between **linear** and **collinear**?
- 2 In which quadrant of the number plane are the x - and y -coordinates of a point both negative?
- 3 What are **consecutive** numbers?
- 4 What is the 'answer' to an equation called?
- 5 What does it mean if the coordinates of a point **satisfy** the equation of a line?
- 6 For the linear equation $y = 4x + 1$, which is the **constant term**?

Topic overview

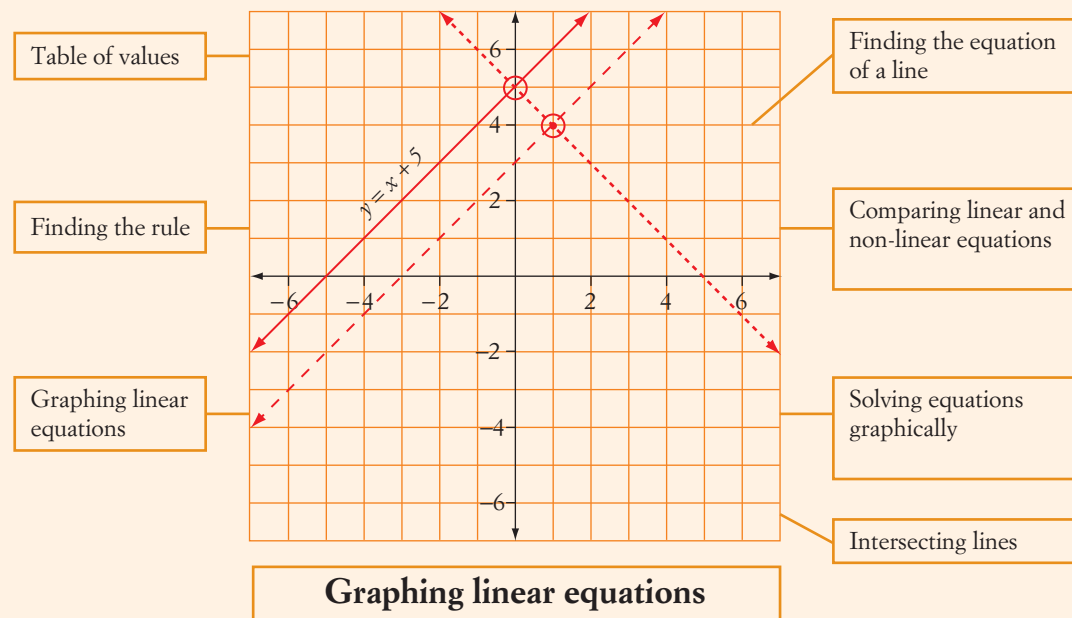
- Write in your own words what you have learnt about linear equations and their graphs.
- What parts of this topic did you have difficulty with? Discuss them with a friend.
- Where might you use the skills acquired in this chapter?

Worksheet

Mind map: Graphing
linear equations

MAT08NAWK10119

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 12 revision

See Exercise 12-01

- 1 Copy and complete each table of values.

a $y = x + 3$

x	-1	0	1	2	3
y					

b $q = 2p - 7$

p	-2	-1	0	1	2
q					

c $d = 5 - c$

c	-1	0	1	2	3
d					

See Exercise 12-02

- 2 Find the formula for each table of values.

a

x	-1	0	1	2	3
y	-3	-2	-1	0	1

b

m	1	2	3	4	5
p	2	5	8	11	14

c

m	0	1	2	3	4
n	10	8	6	4	2

See Exercise 12-03

- 3 For this number pattern: 6, 10, 14, 18, 22, ...

- a find the 7th term
b find a formula for the n th term, T .

See Exercise 12-03

- 4 This pattern of rhombuses is made up of toothpicks.



- a Copy and complete the table.

No. of rhombuses, x	1	2	3	4	5	8	10
No. of toothpicks, y	4						

- b Write the rule for the pattern in words.
c Write the rule as a formula.
d How many toothpicks are needed to build a pattern of:
i 12 rhombuses? ii 15 rhombuses?

See Exercise 12-04

- 5 Graph each of the following points on a number plane.

- a $P(1, -4)$ b $Q(-3, -1)$ c $R(-1, 2)$
d $S(0, 4)$ e $T(2\frac{1}{2}, -2)$ f $U(-2, 0)$

See Exercise 12-04

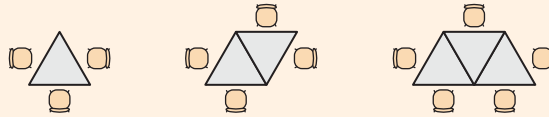
- 6 Which point from question 5:

- a is on the y -axis? b is in the 2nd quadrant?
c is on the x -axis? d is in the 4th quadrant?

- 7 a Copy and complete this table for the pattern of tables and chairs.

See Exercise 12-05

Number of tables (x)	1	2	3	4	5
Number of chairs (y)					

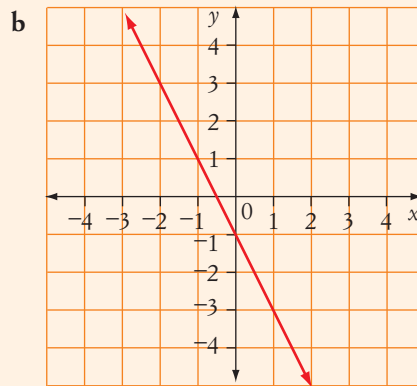
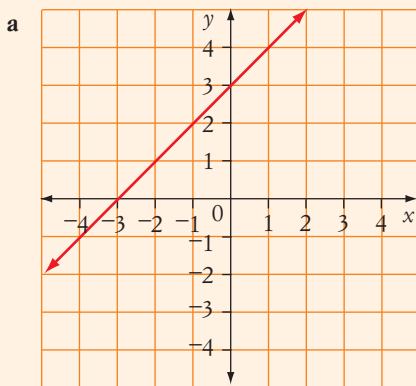


- b Find the formula for the linear relationship between the numbers of tables and the numbers of chairs.
 c Graph the table of values.
- 8 Graph each linear equation on a number plane after completing a table of values.
 a $y = 2x - 1$ b $y = -x + 3$ c $y = \frac{1}{2}x - 2$
- 9 Test whether each point lies on the line $y = 3x + 2$.
 a $(1, 2)$ b $(-1, 4)$ c $(-2, -4)$ d $(0, 2)$
- 10 Find the equation of each line.

See Exercise 12-06

See Exercise 12-06

See Exercise 12-07



- 11 Graph each linear equation on the same number plane.

See Exercise 12-07

- a $y = -3$ b $x = 4$
- 12 a Graph $y = -x + 2$, $y = x + 2$ and $y = 2x + 2$ on the same number plane.
 b What is the same about the three lines?
 c What is the same about the three linear equations?
 d What is different about the three lines?

See Exercise 12-08

- 13 Solve each equation graphically.

See Exercise 12-09

- a $3x - 4 = 5$ b $2x + 1 = -2$
- 14 Graph $y = 2x$ and $y = x - 2$ and find their point of intersection.

See Exercise 12-10

- 15 Solve the equation $3x - 3 = x + 3$:

See Exercise 12-10

- a graphically b algebraically

1 Evaluate each expression.

See Chapter 2

a $\sqrt{169}$

b $\sqrt{256}$

c $\sqrt{289}$

d $\sqrt[3]{27}$

e $\sqrt[3]{125}$

f $\sqrt[3]{5832}$

2 Use a factor tree to write 200 as a product of its prime factors.

See Chapter 2

3 Simplify each fraction.

See Chapter 6

a $\frac{3}{12}$

b $\frac{18}{24}$

c $\frac{35}{100}$

4 Convert each improper fraction into a simplified mixed numeral.

See Chapter 6

a $\frac{36}{5}$

b $\frac{67}{10}$

c $\frac{74}{12}$

5 If $m = 2$, $n = -1$ and $p = 3$, evaluate each expression.

See Chapter 3

a mnp

b $2p - m$

c $3m - n^2$

6 Simplify each expression.

See Chapter 3

a $3a \times 2r$

b $-7p - 2p + 6p$

c $7u \times 5u$

d $9x - 3 + 12 - 4x$

e $6p - 3q + 2p - 4q$

f $\frac{24ut}{4mt}$

7 Shane scores 32 points out of his team's score of 128. What percentage is this?

See Chapter 6

8 These are scores in a spelling test:

See Chapter 7

5 4 8 6 1 5 8 5 4 2 5.

Find:

a the mean (to one decimal place)

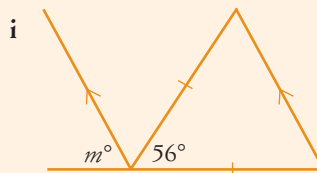
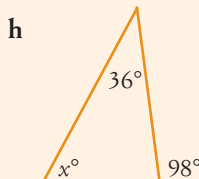
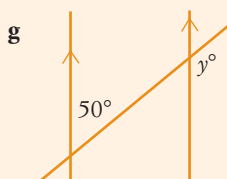
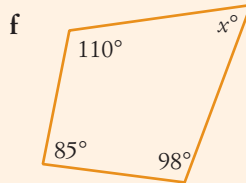
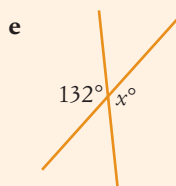
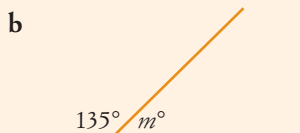
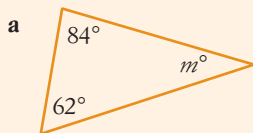
b the median

c the mode

d the range.

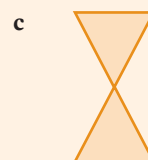
9 Find the value of each pronumeral, giving reasons.

See Chapter 4



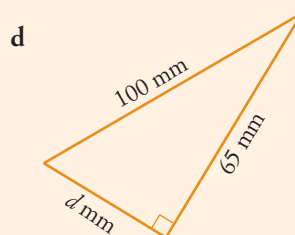
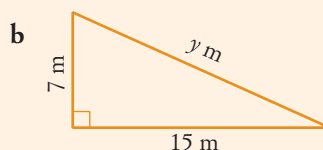
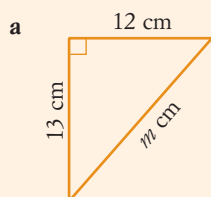
See Chapter 4

- 10 i Copy each shape and mark any axes of symmetry.
ii Decide whether each shape has rotational symmetry. If it does, state the order of rotational symmetry.



See Chapter 1

- 11 Find the value of each pronumeral, correct to one decimal place.



See Chapter 2

- 12 Use index notation to simplify each expression.

a $6^5 \div 6^3$

b $2^3 \times 2^4$

c $3^9 \div 3^7$

d 5×5^2

e $(2^3)^4$

f $(2^3 \times 5)^2$

g $3^2 \div 3^2$

h 6×4^0

See Chapter 2

- 13 Test whether 756 is divisible by:

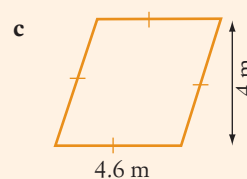
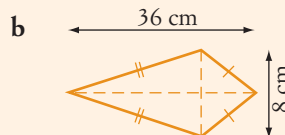
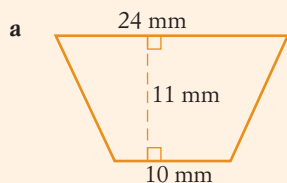
a 3

b 4

c 6

See Chapter 5

- 14 Find the area of each quadrilateral.

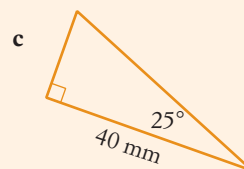
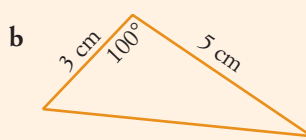
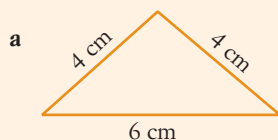


See Chapter 5

- 15 Classify each triangle below:

i by sides

ii by angles.



16 Copy and complete:

See Chapter 5

a $8 \text{ m}^2 = \text{ } \text{cm}^2$ **b** $4900 \text{ m}^2 = \text{ } \text{ha}$ **c** $650 \text{ m}^3 = \text{ } \text{cm}^3$

17 Evaluate each expression.

See Chapter 6

a $\frac{2}{3} + \frac{3}{4}$ **b** $\frac{2}{3} - \frac{1}{5}$ **c** $\frac{7}{10} \times \frac{2}{3}$ **d** $\frac{4}{9} \div \frac{5}{6}$
e $1\frac{3}{8} + \frac{1}{4}$ **f** $4 - \frac{5}{7}$ **g** $5 \div \frac{4}{9}$ **h** $3\frac{2}{5} \times 1\frac{4}{5}$

18 Convert 15% into a simple fraction.

See Chapter 6

19 Solve each equation.

See Chapter 10

a $5a - 13 = 12$ **b** $5d + 6 = 2d - 9$ **c** $4(2r + 1) = 28$

20 Classify each type of data as categorical (C), numerical discrete (ND) or numerical continuous (NC).

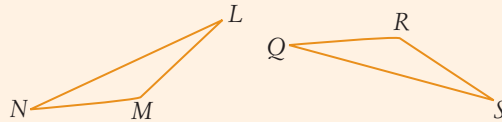
See Chapter 7

- a** number of taxis on the road **b** rating of film (1 star to 5 stars)
c weights of babies in a maternity ward **d** brands of TVs

21 These two triangles are congruent.

See Chapter 8

- a** Which side in $\triangle QRS$ matches side NM ?
b Which angle in $\triangle LMN$ matches $\angle S$?
c Copy and complete: $\triangle LMN \equiv \text{ } \text{ } \text{ }$



22 Gavin earns \$22.40 per hour. How much will he earn if he works 42 hours in a week?

See Chapter 11

23 Find the formula for each table of values.

See Chapter 12

a

x	4	5	6	7	8
y	16	19	22	25	28

b

x	3	3	3	3	3
y	-1	2	4	-2	0

24 Graph each linear equation on the same number plane.

See Chapter 12

a $y = 3$ **b** $x = -1$

25 Expand and simplify each expression.

See Chapter 3

a $3(a + 5)$ **b** $4(5 - 2k)$ **c** $3(2p - 5) - 6p$

26 Simplify each ratio.

See Chapter 11

a $42 : 49$ **b** $16 \text{ kg} : 800 \text{ g}$ **c** $10 \text{ mm} : 1 \text{ m}$

27 A machine makes transistors, of which 2 in 35 are faulty. If the machine has made 16 faulty transistors, how many good ones would you expect?

See Chapter 11

See Chapter 11

- 28** What time will it be:
a 4 hours 25 minutes after 2:15 p.m.?
b 6 hours 40 minutes before 8:35 a.m.?

See Chapter 12

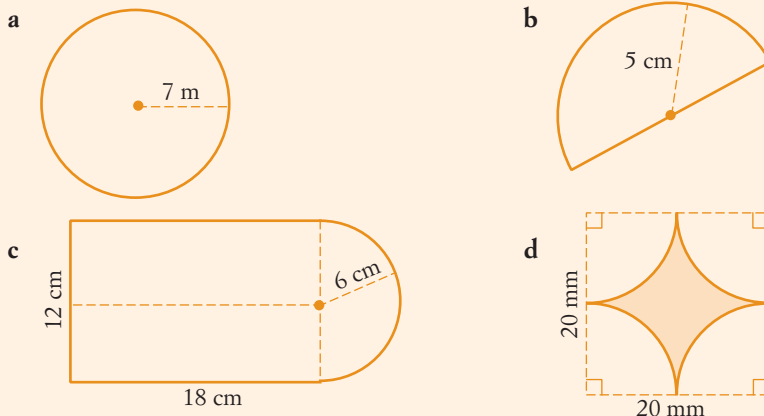
- 29** **a** Graph $y = 2x - 1$ on a number plane.
b Test whether (5, 9) lies on the line.
c Graph $y = x - 2$ on the same number plane.
d Solve the equation $2x - 1 = x - 2$ graphically.

See Chapter 4

- 30** Name the shape that matches each description.
a A quadrilateral with equal diagonals.
b A triangle with three equal sides.
c A quadrilateral with opposite sides equal and two axes of symmetry.
d A triangle with two equal angles.

See Chapter 5

- 31** For each shape, find correct to two decimal places:
i the perimeter **ii** find the area.



See Chapter 3

- 32** Factorise each expression.
a $2y + 8$ **b** $5a - 15ab$ **c** $-8ab + 2b$

See Chapter 6

- 33** A savings account pays 4% interest each year. If Haroon puts \$500 in his account, how much does he get in interest after one year?

See Chapter 9

- 34** A bag has 7 yellow counters, 3 red counters and 4 blue counters. If one counter is selected at random, what is the probability of selecting:
a a yellow counter? **b** a blue counter? **c** a counter that is not red?

See Chapter 11

- 35** In an orchard, the ratio of orange trees to lemon trees is 7 : 4. If there are 2800 lemon trees, how many orange trees are there?

See Chapter 11

- 36** A painter uses 8 litres of paint to cover 32 square metres of wall. What area can he cover with 20 litres of paint?

- 37 The ages of patients waiting at a doctor's surgery are shown on this stem-and-leaf plot.

Stem	Leaf
1	3 8
2	1 9
3	5
4	2
5	0 4 7
6	
7	3 5

See Chapter 7

Find:

- a the range
b the median
c the mean, correct to one decimal place.
- 38 a Draw a rectangle and its diagonals.
b Shade two pairs of congruent triangles.
c What does this show about the diagonals of a rectangle?
- 39 For each pair of congruent triangles in each diagram, state which test proves that they are congruent.

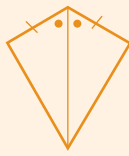
See Chapter 8

See Chapter 8

a



b



c



d



- 40 The scale of a plan is 1 : 2000. What distance in centimetres on the plan would represent a real distance of 50 m?

See Chapter 11

- 41 Which of the following is the best buy for strawberries?

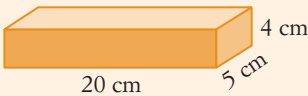
See Chapter 11

A 600 g for \$8.60 B 500 g for \$7.00 C 1 kg for \$15

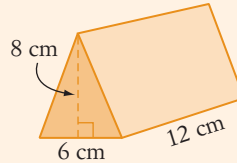
- 42 Find the volume of each solid. (For part f, give your answer to one decimal place).

See Chapter 5

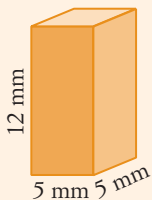
a



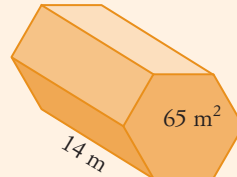
b



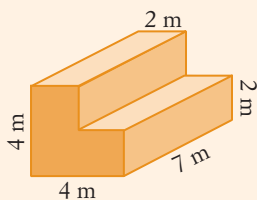
c



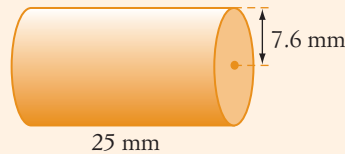
d



e



f



See Chapter 6

- 43** Anne earned \$42 a week for delivering papers. If she was given a 20% pay rise, how much does she earn now?

See Chapter 7

- 44 a** Complete a frequency table for this data, including an 'fx' column.

53	50	50	52	47	49	49	50	48	50
52	49	47	51	51	52	50	53	50	52
50	48	52	49	50	49	51	45	52	50
51	52	51	50	52	50	51	52	49	51

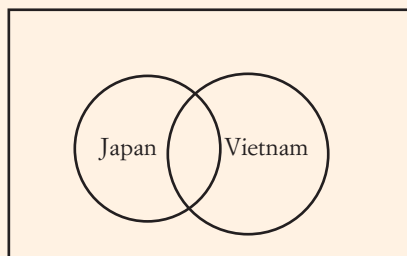
- b** Find the mean of this data.
c Show this data in a frequency histogram and polygon.

See Chapter 9

- 45** A survey was held to find whether of a sample of 30 Australians had travelled to Japan or Vietnam. The results are shown below.

Japan 9 Vietnam 18 Japan and Vietnam 5

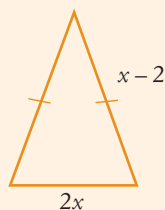
- a** Copy and complete the Venn diagram to show this information.



- b** How many people did not visit either country?
c How many people had visited Japan but not Vietnam?
d What is the probability that a person selected randomly from this sample has not been to Japan?

See Chapter 10

- 46** If the perimeter of this isosceles triangle is 24 cm, find the value of x .

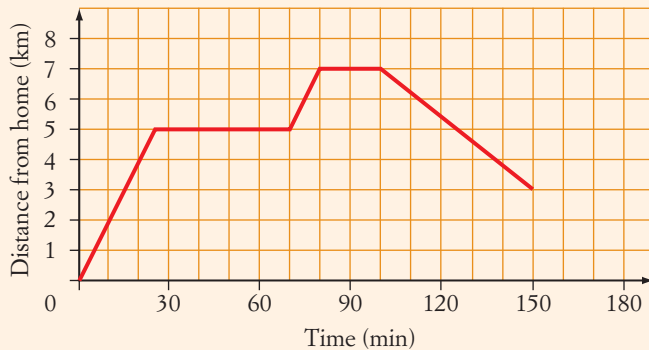


See Chapter 11

- 47** If Dino can run 400 metres in 66 seconds, calculate his speed in m/s, to one decimal place.

- 48 Mel went on a bike ride and her trip is shown in the travel graph.

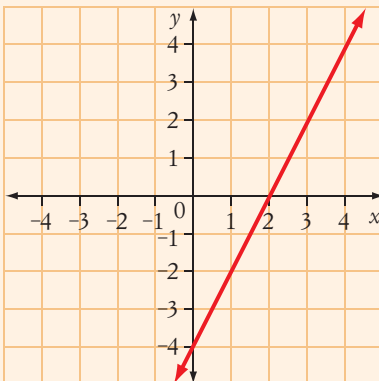
See Chapter 11



- How far from home was Mel after 20 minutes?
 - When and where was she furthest from home?
 - For how long did Mel rest on her second stop?
 - When Mel started travelling home, what was her average speed?
- 49 Solve the linear equation $4 - 2x = 6$ graphically.
- 50 Find the equation of this line.

See Chapter 12

See Chapter 12



Basic keys

Key	Use	Key	Use
+ , - , × , ÷	Basic operations	(-) or +/-	Enters negative numbers
=	Equals sign, gives the answer	$\frac{\Box}{\Box}$ or a^{b/c}	Enters fractions
.	Decimal point	()	Enters parentheses
DEL	Deletes previous entry	x²	Squares a number
ANS	Retrieves previous answer	√	Finds the square root of a number
	Allows us to move around the screen	x³	Cubes a number
MODE or SHIFT or 2ndF	Accesses other operations	$\sqrt[3]{\Box}$	Finds the cube root of a number

1 Basic operations

Example 1

Performing a calculation

Question	Calculator steps	Answer
$34 + 6 - 16$	34 + 6 - 16 =	24
7.4×9.1	7.4 × 9.1 =	67.34
$234 \div 1.5$	234 ÷ 1.5 =	156
$4569 \div (1.7 + 4.3)$	4569 ÷ (1.7 + 4.3) =	761.5

Example 2

Correcting a wrong entry

Calculating $34 + 6 - 16$ a Wrong entry: 34 **+** 8To delete the 8 and correct this, enter 34 **+** 8 **DEL** 6 **-** 16 **=**b Wrong entry: 34 **+** 6 **+** 16 **=**To correct this, press **←** repeatedly to go back to the second **+**, press **DEL** and then **-**, then press **=**

Calculator skills

Exercise 1

Basic operations

1 Calculate each expression.

- a** 362×14 **b** $26.3 - 11.8$ **c** $810 \div 18$ **d** 5.3×1.08
e $51.1 \div 14.6$ **f** $19.3 + 8.8 - 1.2$ **g** $2.4 \div 0.2 + 11$ **h** $5.6 \times (0.1 + 0.2)$
i $(56 - 24) \div (1.2 - 1)$ **j** $12.6 \times 3.8 - 18.4$
k $(16.07 + 9.02) - (38.3 - 5.126)$ **l** $(24.8 + 3.06 + 9.4) \div 3$

2 Bronte purchased a loaf of bread for \$2.75, a packet of cornflakes for \$4.55 and 3 kg of tomatoes at \$3.95 per kg.

- a** Find the total cost. **b** Find the change Bronte will receive from a \$20 note.

3 Mark started the month with \$1650 in his bank account. He made deposits of \$380 and \$725.50 and withdrew \$880. How much was left in Mark's account?

4 An unloaded truck has a mass of 6550 kg. It is loaded with eight boxes, each weighing 124 kg. What is the total mass of the truck and the boxes?

5 Salma needs to cut a 9.6 m length of rope into four equal pieces. Find the length of each piece.

6 Cameron mailed 25 Christmas cards to his friends. If the stamps were 45c each, how much did he spend on stamps?

7 An orchard has 1288 trees arranged in 56 rows. How many trees are in each row?

8 A train takes 15 hours to travel 1128 km. What is its average speed in km/h?

9 Chloe bought 60 L of petrol at 142.9c per litre. How much did she pay for the petrol?

10 Wombat High School has the following numbers of students in each Year level. The whole school is going on an excursion. If each bus can carry 55 people, how many buses will be needed?

Year 7	Year 8	Year 9	Year 10	Year 11	Year 12
122	128	123	131	117	94

11 The manager of a sports store bought 32 tennis racquets at \$79.60 each and sold them at \$120.80 each.

- a** Find the profit made on each tennis racquet. **b** Find the total profit.

12 A batsman scored the following runs over five cricket innings. Find the batsman's average score.

27 4 123 66 47

2 Integers

Example 3

Question	Calculator steps		Answer
	Casio	Sharp	
$-4 + 9$	$(-)$ 4 + 9 =	$+/-$ 4 + 9 =	5
$17 \times (-2)$	17 $\times$ $(-)$ 2 =	17 $\times$ $+/-$ 2 =	-34
$18 - (-3) \times 4$	18 - $(-)$ 3 $\times$ 4 =	18 - $+/-$ 3 $\times$ 4 =	30

Exercise 2 Integers

1 Calculate each expression.

a $-5 + 12$

b $-6 - 3$

c -3×4

d $16 \div (-2)$

e $12 - (-3)$

f $-19 + 4$

g $-25 \times (-3)$

h $-55 \div (-11)$

i $8 - 12 + (-1)$

j $-3 + 7 - 2 - (-5)$

k $6 \times (-2) \times 8$

l $-18 \div (-2) \times 5$

m $-3 \times 7 - 4$

n $(-3 + 9) \times 4$

o $80 - (-2) \times 8$

p $12 \times (-3) + 50$

q $64 \div (-8) \times 2$

r $27 - (-3) \times (-4)$

s $18 \times (-5 + 3)$

t $-14 - 3 \times (-8)$

3 Powers and roots

Check with your teacher to locate the following keys: x^2 x^3 $\sqrt{\quad}$ $\sqrt[3]{\quad}$

Example 4

Question	Calculator steps	Answer
16^2	16 x^2 =	256
11^3	11 x^3 =	1331
4^5	4 $x^\square$ (or x^y) 5 =	1024
$\sqrt{196}$	$\sqrt{\quad}$ 196 =	14
$\sqrt[3]{4913}$	$\sqrt[3]{\quad}$ 4913 =	17

Calculator skills

Exercise 3 Powers and roots

1 Calculate each expression.

a 14^2 b 23^2 c 6^3 d 1.2^3 e $(-13)^2$ f $(-7)^3$
 g 2^7 h 9^4 i $\sqrt{676}$ j $\sqrt{21\,904}$ k $\sqrt[3]{729}$ l $\sqrt{6^2 \times 7^2}$

2 Evaluate each expression correct to one decimal place.

a $\sqrt{24}$ b $\sqrt{101}$ c $\sqrt{11 \times 2}$ d $\sqrt[3]{100}$

3 Calculate each expression.

a $5^2 - 3^2$ b $16^2 - 10^2$ c $\sqrt{6} \times \sqrt{6}$ d $\sqrt{25 \times 4}$

4 Fractions

Some calculators have two ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into two blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). LINE mode is used in the following examples. Ask your teacher if you need help with the MATH mode.

Example 5

Converting fractions

Question	Calculator steps		Answer
	Casio	Sharp	
Simplify $\frac{16}{20}$	16  20 =	16  20 =	$\frac{4}{5}$
Change $1\frac{3}{4}$ to an improper fraction	1  3  4 =   ( )	1  3  4 = 2ndF 	$\frac{7}{4}$
Change $\frac{19}{4}$ to a mixed numeral	19  4 =	19  4 =	$4\frac{3}{4}$
Change $\frac{3}{4}$ to a decimal	3  4 =  or 3  4 =	3  4 =  or 3  4 =	0.75
Change 1.7 to a fraction	1.7 = 	1.7 = 	$1\frac{7}{10}$

 is a key that converts between standard fractions and decimals.

Example 6

Operations with fractions

Question	Calculator steps		Answer
	Casio	Sharp	
$\frac{1}{2} + \frac{1}{3}$	1 $\frac{\square}{\square}$ 2 + 1 $\frac{\square}{\square}$ 3 =	1 $\text{a}^{\text{b/c}}$ 2 + 1 $\text{a}^{\text{b/c}}$ 3 =	$\frac{5}{6}$
$\frac{5}{7} - \frac{2}{3}$	5 $\frac{\square}{\square}$ 7 - 2 $\frac{\square}{\square}$ 3 =	5 $\text{a}^{\text{b/c}}$ 7 - 2 $\text{a}^{\text{b/c}}$ 3 =	$\frac{1}{21}$
$\frac{3}{5} \times \frac{2}{7}$	3 $\frac{\square}{\square}$ 5 $\times$ 2 $\frac{\square}{\square}$ 7 =	3 $\text{a}^{\text{b/c}}$ 5 $\times$ 2 $\text{a}^{\text{b/c}}$ 7 =	$\frac{6}{35}$
$\frac{4}{5} \div \frac{2}{3}$	4 $\frac{\square}{\square}$ 5 $\div$ 2 $\frac{\square}{\square}$ 3 =	4 $\text{a}^{\text{b/c}}$ 5 $\div$ 2 $\text{a}^{\text{b/c}}$ 3 =	$1\frac{1}{5}$

Example 7

Mixed operations with fractions

Question	Calculator steps	Answer
$\frac{13 + 7}{9 \times 2}$	(13 + 7) $\div$ (9 $\times$ 2) = or (13 + 7) $\frac{\square}{\square}$ (or $\text{a}^{\text{b/c}}$) (9 $\times$ 2) =	1.1111... $1\frac{1}{9}$

Exercise 4 Fractions

1 Simplify each fraction.

a $\frac{20}{32}$

b $\frac{40}{85}$

c $\frac{45}{72}$

d $\frac{112}{126}$

e $\frac{84}{100}$

f $\frac{48}{192}$

2 Convert each mixed numeral into an improper fraction.

a $1\frac{7}{8}$

b $5\frac{2}{9}$

c $10\frac{3}{4}$

3 Convert each improper fraction into a mixed numeral.

a $\frac{20}{9}$

b $\frac{44}{5}$

c $\frac{18}{8}$

4 Convert each fraction into a decimal.

a $\frac{1}{2}$

b $\frac{4}{5}$

c $\frac{7}{8}$

d $\frac{9}{25}$

e $\frac{34}{200}$

f $\frac{5}{9}$

5 Convert each decimal into a fraction in its simplest form.

a 0.35

b 2.04

c 0.001

d 0.365

e 1.06

f 0.325

Calculator skills

6 Simplify each expression.

a $\frac{3}{5} + \frac{1}{4}$

b $\frac{4}{7} + \frac{1}{2}$

c $\frac{2}{3} - \frac{3}{5}$

d $\frac{1}{3} \times \frac{4}{7}$

e $\frac{5}{7} \div \frac{15}{28}$

f $\frac{5}{6} - \frac{5}{8}$

g $\frac{9}{10} + \frac{2}{3}$

h $\frac{7}{8} \times \frac{5}{12}$

i $\frac{7}{8} \div \frac{1}{2}$

j $24 \div 1\frac{1}{4}$

k $5\frac{1}{4} + 1\frac{2}{5}$

l $9\frac{7}{8} - 1\frac{1}{3}$

m $5\frac{1}{4} \times 2\frac{3}{7}$

n $4\frac{2}{3} \div \frac{2}{3}$

o $\frac{8}{9} \div 6$

p $10\frac{1}{6} \times 1\frac{9}{10}$

7 Simplify each expression.

a $\frac{12 - 4 \times 2}{18 + 14}$

b $\frac{6^2}{100 \div 4 + 23}$

c $\frac{9 \times 9 - 2}{\sqrt{30} - 5}$

d $\frac{20 \div 2 + 8}{3 \times 9 \times 4}$

8 Mr Lanouri is at his office for $7\frac{1}{2}$ hours each day. He takes $\frac{3}{4}$ of an hour for lunch each day. How long does he work:

a in one day?

b in 5 days?

Instructional glossary

Mathematical 'doing' words

analyse To study something in great detail by breaking it into its parts.

bisect To cut in half.

calculate To find the value of a numerical expression. See also **evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or whole.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction into a decimal, or convert dollars to cents.

decrease To make smaller by subtracting or dividing.

estimate To make an educated guess about a number or answer; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2 , or evaluate $4x + 1$ when $x = 5$.

expand To remove the brackets in an algebraic expression; the opposite of **factorise**. Expanding $3(2y + 1)$ gives $6y + 3$.

factorise To take out the greatest common divisor (GCD) of an expression and insert brackets; the opposite of **expand**. Factorising $5x - 20$ gives $5(x - 4)$.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using previous answers or information supplied.

increase To make larger by adding or multiplying.

measure To find the size of something using an instrument. For example, to find a length using a ruler.

prove/show that In questions where the answer is given, to use mathematical reasoning to prove that the answer is true.

reduce (a fraction) to its lowest terms See **simplify (a fraction)**.

round (a number) To find the nearest approximation for a number. For example, round 4.3 to the nearest whole number (4), \$12.9598 to the nearest cent (\$12.96), 0.166 66 to three decimal places (0.167).

show working To show the steps you used to find an answer.

simplify To give the answer in its simplest, shortest, neatest form.

simplify (a fraction) To reduce the numerator and denominator of a fraction by dividing by their greatest common divisor (GCD). When the numerator and denominator are as small as possible, the fraction has been simplified or reduced to its lowest terms.

Glossaries

simplify (a ratio or rate) To reduce the terms or units of a ratio or rate by dividing by a common factor; similar to simplifying a fraction.

sketch To draw a rough diagram. Less accurate than to **construct**.

solve To find the value of the unknown variable in an equation.

substitute To replace a variable with a number, to evaluate an expression. For example, substituting $t = 5$ in the expression $t^2 + 6$ gives 31.

write correct to See **round (a number)**.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

Mathematical glossary

Some common symbols

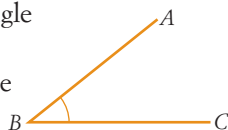
+	plus
−	minus
×	multiplied by (times)
÷	divided by
%	per cent
()	parentheses, round brackets
[]	(square) brackets
{ }	braces
=	is equal to
≈	is approximately equal to
≠	is not equal to
<	is less than
>	is greater than
≤	is less than or equal to
≥	is greater than or equal to
∴	therefore
−3	negative 3

$\sqrt{\quad}$	the square root of
$\sqrt[3]{\quad}$	the cube root of
$5 \overline{)855}$	855 divided by 5, or $855 \div 5$
$0.\dot{6}$	the recurring decimal 0.666 666 ...
π	pi, the decimal 3.14159 ...
$3x$	3 multiplied by x , $3 \times x$
$\frac{x}{2}$	x divided by 2, $x \div 2$
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\bar{x}$	the mean (average)
$\angle ABC$	angle ABC
$\triangle ABC$	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\equiv$	is congruent (identical) to
$P(E)$	The probability of an event, E

24-hour time Time of day written using four digits (instead of a.m. or p.m.) and the hours 0 to 23. For example, 1820 is the 24-hour time for 6:20 p.m.

A

acute angle A 'sharp' angle between 0° and 90° , for example, the marked angle in the diagram.

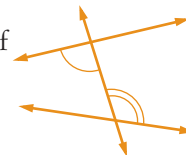


acute-angled triangle A triangle with all three angles acute.

algebraic expression A number written in algebraic form using variables, for example, $2xy + 4y - 5$.

algebraic term See **terms of an expression**.

alternate angles A pair of angles between two lines crossed by a transversal, on opposite sides of the transversal; for example, the two angles marked in the diagram.

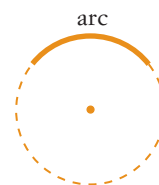


angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180° .

annulus A ring shape between two different-sized circles with the same centre.



arc Part of the circumference of a circle.



area The amount of surface enclosed by a shape, measured in square units.

Glossaries

ascending order Going up, increasing, from smallest to largest (1–2–3). The opposite of **descending order**.

at least Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, . . . , that is, '2 or more'.

average See **measure of location**.

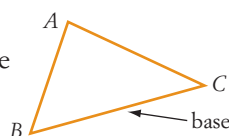
B

backtracking method A method of solving equations by 'undoing', or performing inverse (opposite) operations in reverse order.

balancing method A method of solving equations by performing inverse (opposite) operations on both sides.

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5 , the 3 is called the base.

base (of a shape) The bottom side of a flat shape such as a triangle.



best buy When comparing different brands or sizes of items during shopping, the best buy is the item with the lowest unit cost and is the best value for money.

bias In statistics, something that causes a sample to not truly represent the population.

braces/brackets See **grouping symbols**.

breadth Another word for 'width'; how wide or broad something is.

C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). See also **volume**.

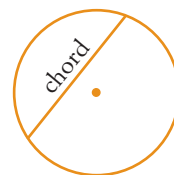
Cartesian plane Another name for **number plane**.

categorical data Non-numerical data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not numerical.

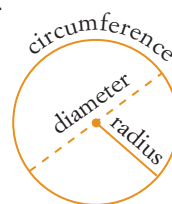
census A survey of the entire **population** of people or items, not just a survey of a **sample**.

certain Must happen; has a probability of 1 or 100%.

chord An interval joining two points on a circle.



circumference The perimeter of a circle, or the length of that perimeter. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius.



cluster A group of data scores that are bunched or close together.

coefficient The number 'in front of' a variable in an equation; a multiplier. For example, in $y = 3x - 7$, the coefficient of x is 3.

co-interior angles A pair of angles between two lines crossed by a transversal, on the same side of the transversal (co-interior means 'together inside'), for example, the two angles marked in the diagram.



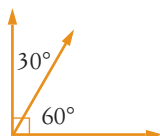
collinear points Points that lie in a straight line.

compasses Geometrical instrument for constructing circles and lines of equal length.



complementary

angles Two angles whose sum is 90° . The angles 30° and 60° are complementary.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

composite shape A shape made up of two or more basic shapes.

congruent Identical, exactly the same. The symbol ' $\equiv$ ' means 'is congruent to' or 'is identical to'.

congruent figures Identical figures, having the same shape and size.

congruence test One of four tests for proving that two triangles are congruent: SSS, SAS, AAS and RHS.

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10.

constant term The term in an equation that is a number only and does not contain a variable. For example, in $y = 3x - 7$, the constant term is -7 .

continuous data Numerical data that can be measured on a smooth scale without any gaps, that can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**.

convex quadrilateral A quadrilateral whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180° .



Convex



Non-convex

Coordinated Universal Time See UTC.

coordinates An ordered pair of numbers used to locate a point or position, for example, $A(2, 5)$ tells us that the point A is located 2 units to the right of and 5 units up from the origin.

corresponding angles A pair of angles in matching positions when two lines are crossed by a transversal. They are on the same side of the transversal and the lines, for example, the two angles marked on the diagram.



cost price The price it costs a retailer (shop) to buy (to resell). The cost price is usually less than the **selling price**.

cross-section A 'slice' of a solid cut across it rather than along it.



cube (of a number) The number raised to the power of 3. For example, 7 cubed $= 7^3 = 7 \times 7 \times 7 = 343$.

cube root (of a number) The value which, if cubed, gives the number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.

cubic metre A metric unit of volume, the volume of a 1 m cube.

cylinder A can-shaped solid with ends that are circles.



D

data Information, a collection of facts.

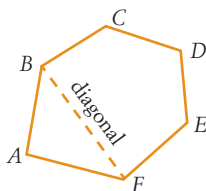
decimal places The places after the decimal point in a number. For example, 3.1416 has four decimal places.

Glossaries

denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3.

descending order Going down, decreasing, from largest to smallest (3–2–1). The opposite of **ascending order**.

diagonal An interval joining two non-adjacent vertices of a shape.



diameter An interval joining two points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**.



difference The result of a subtraction. The difference between 65 and 10 is 55.

discount The saving made between the original price of an item and the reduced price.

discrete data Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**.

distributive law A law of arithmetic that says that you can multiply by a number by splitting it into the sum or difference of two other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. More generally, $a \times (b + c) = a \times b + a \times c$ for any three numbers a , b and c .

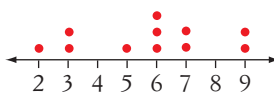
divided bar graph A rectangular graph which is divided into proportionately-sized sections to represent parts of a whole.



divisibility test A rule for testing whether a number is divisible by a specific value. For example, the divisibility test for 3 is to add the digits of the number and if the answer is a multiple of 3, then the number is divisible by 3.

divisor See **factor**.

dot plot A graph that uses dots above a number line to show the frequencies of data scores.



E

equation A mathematical statement that two quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$.

equilateral triangle A triangle with all three sides equal.



equivalent Having the same value. For example, 0.75 and $\frac{3}{4}$ are equivalent.

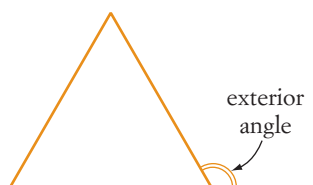
expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials.

experimental probability (or relative frequency) An estimate of theoretical probability; the relative frequency of an event in repeated trials of an experiment, found using the formula

$$P(E) = \frac{\text{number of times } E \text{ happened}}{\text{total number of trials}} \\ = \frac{\text{frequency of } E}{\text{total frequency}}$$

expression A mathematical description of one or more operations. For example, $2 \times 4 + 5$ and $3b - 7$ are mathematical expressions (but $3b - 7 = 5$ is an equation.) See also **algebraic expression**.

exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape.

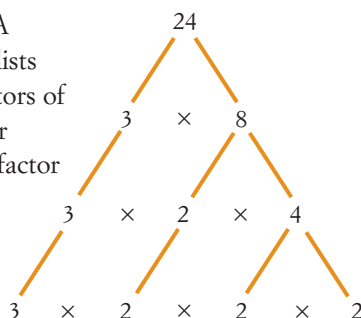


event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}.

F

factor (of a number) A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. Also called **divisor**.

factor tree A diagram that lists the prime factors of a number. For example, the factor tree for 24 is shown below.



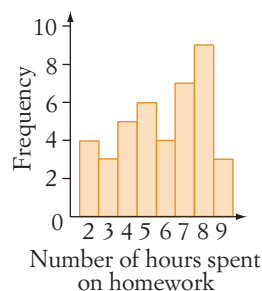
formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$.

fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

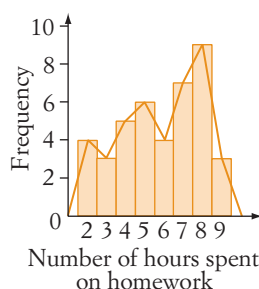
frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data.

frequency

histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings.



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the tops of the columns of a histogram. The graph looks like a mountain.



frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx .

G

goods and services tax (GST) A 10% tax added to the original price of an item or service.

greatest common divisor (GCD) See **highest common factor**.

Greenwich Mean Time (GMT) See **UTC**.

grouping symbols The collective name for parentheses (), (square) brackets [] and braces { }.

GST See **goods and services tax**.

Glossaries

H

hectare A large metric unit of area, equivalent to $10\,000\text{ m}^2$ or to the area of a square $100\text{ m} \times 100\text{ m}$.

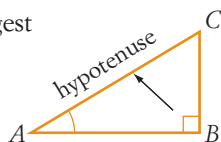
height How tall or high something is, the vertical distance between its top and its base.

highest common factor (HCF) Also called **greatest common divisor (GCD)**. The largest factor shared by two or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$.

horizontal Going across, sideways, flat.



hypotenuse The longest side of a right-angled triangle, opposite the right angle.



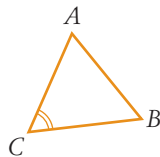
I

image A transformed shape after it has been translated, reflected or rotated.

impossible Cannot happen, no chance, has a probability of 0.

improper fraction A fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$.

included angle The angle between two given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$.



index (plural: **indices**, pronounced 'in-da-sees') See **power**.

index notation A way of writing repeated multiplication using indices (powers), in the form a^n . Index notation for $2 \times 2 \times 2 \times 2$ is 2^4 .

integer A number that is a positive or negative whole number or zero. The numbers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ are integers.

intersect To cross.

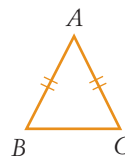
interval A section of a line with a definite length, such as AB below.



inverse operation The opposite or reverse operation, used when solving equation. For example, the inverse operation to adding is subtracting, the inverse operation to dividing is multiplying.

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction. In decimal form, its digits run endlessly without repeating.

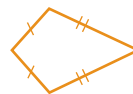
isosceles triangle A triangle with two equal sides.



K

kilo- A prefix meaning one thousand (1000), represented by the symbol k. For example, one kilogram is 1000 grams.

kite A quadrilateral with two pairs of equal adjacent sides.



L

LHS The left-hand side (of an equation).

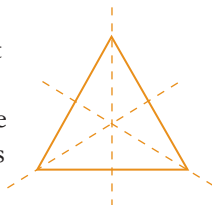
like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms.

likely A high chance it will happen, probable.

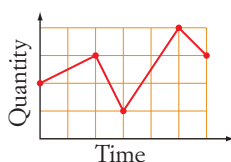
linear Involving a line.

linear equation A formula whose graph is a straight line.

line symmetry A plane shape has line symmetry if it can be folded so that one half fits exactly on top of the other half; that is, one half is the mirror image of the other. The line of fold is called the **axis of symmetry** (plural: **axes**). An equilateral triangle has three axes of symmetry. See also **rotational symmetry**.



line graph A graph made up of a line or several line intervals, often showing how a quantity is changing over time.



loss The money lost when selling an item at a lower price, when the selling price is less than the cost price. The opposite of **profit**.

lowest common multiple (LCM) The smallest multiple that is shared by two or more numbers. For example, the LCM of 4 and 10 is 20.

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the scores by the number of scores.

measure of location An average, middle or typical value of a set of data. The three measures of location are the **mean**, **median** and **mode**.

median The middle score when the scores are arranged in order. If the number of values is even, then the median is the average of the two middle values.

mega- A prefix meaning one million (1 000 000), represented by the symbol M. A megalitre is one million litres.

micro- A prefix meaning one-millionth, represented by the Greek letter μ . One microsecond is one-millionth of a second.

milli- A prefix meaning one-thousandth ($\frac{1}{1000}$), represented by the symbol m. One millimetre is one-thousandth of a metre.

mixed numeral A number written as a whole number and a fraction, for example $5\frac{3}{4}$.

mode The most common or frequent score(s) in a set of data.

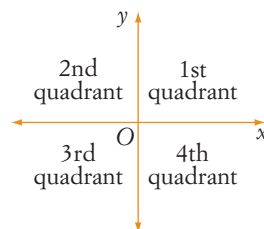
multiple (of a number) The product of the number and a whole number. For example, the multiples of 6 are 6, 12, 18, 24, ...

mutually exclusive events Events or categories that have no items in common.

N

negative number A number less than 0, written with a negative sign ($-$), for example, -3 , -10 and -7 .

number plane A coordinate grid system based on two number lines that cross at right angles: a horizontal line called the x -axis and a vertical line called the y -axis. Also called **Cartesian plane**.



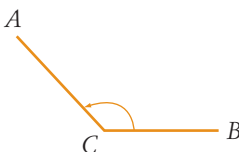
numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2.

numerical data Data that can be measured or counted using values, such as a person's height or number of goals scored. Data that is not **categorical**.

Glossaries

O

obtuse angle A 'wide' angle greater than 90° but less than 180° .



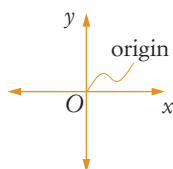
obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°).

order of operations The order in which a mixed expression such as $(17 - [3 + 1]) \times 2$ is evaluated. Work from left to right, first performing any operations in grouping symbols, then multiplication and division, and finally addition and subtraction.

order of rotational symmetry See **rotational symmetry**.

ordered pair A pair of numbers (x, y) that can be used as coordinates to plot a point on the number plane.

origin The point $O(0, 0)$ at the centre of the number plane, where the x -axis and y -axis cross.



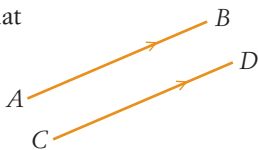
outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4.

outlier An extreme data value that is very different from the other values in a set.

P

positive number A number greater than 0, for example, 11, 6 and 4.

parallel lines Lines that point in the same direction and do not intersect.



$AB \parallel CD$ means 'AB is parallel to CD.'

parallelogram A quadrilateral in which the opposite sides are parallel.



parentheses (Pronounced 'pa-ren-th-sees'.) See **grouping symbols**.

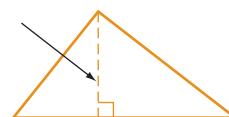
percentage A fraction with denominator 100 that is written in a special way. For example, 7% means $\frac{7}{100}$.

perimeter The distance around the outside of a shape. The sum of the lengths of its sides.

perpendicular

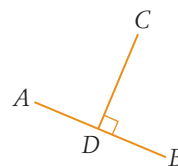
height The height of a shape, measured at right angles to the base.

perpendicular height



perpendicular lines

Lines that intersect to form a right angle. $AB \perp CD$ means AB is perpendicular to CD.



pi (π) A special number, approximately 3.1416, used in calculating circle measurements. Pi is a constant value found by dividing the circumference of any circle by the diameter.

polygon Any flat shape made up of straight sides.



population In statistics, all of the items being studied, the entire group.

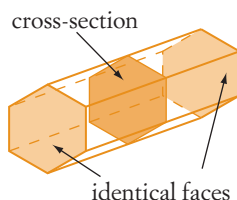
power (or index) The number of times a base is multiplied by itself. In 2^5 , the power is 5. Also called the **exponent**.

power of 10 The numbers 10, 100, 1000 and so on, formed by multiplying 10 by itself repeatedly.

profit The amount made when selling an item at a higher price, when the selling price is more than the cost price. The opposite of **loss**.

prime number A number that has only two factors, 1 and itself. For example, 2 and 7 are prime numbers, but 15 is not.

prism A solid shape with identical cross-sections with straight sides.



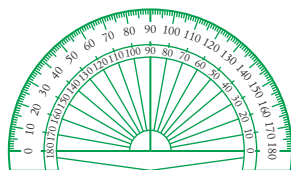
probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1.

product The result of multiplication. The product of 7 and 3 is 21.

pronumeral Another name for **variable**.

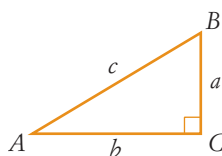
proper fraction A fraction whose numerator is less than its denominator, such as $\frac{3}{8}$.

protractor A geometrical instrument that measures the size of an angle in degrees.



Pythagoras' theorem

The formula $c^2 = a^2 + b^2$ for a right-angled triangle, where c is the length of the hypotenuse and a and b are the lengths of the other two shorter sides.



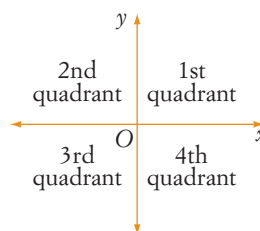
Pythagorean triad A set of three numbers that follow Pythagoras' theorem, such as 3, 4, 5.

Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle.



quadrant (of a number plane) A quarter of the number plane created by the x -axis and y -axis crossing at right angles.



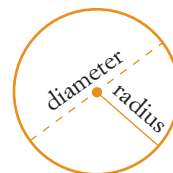
quadrilateral Any polygon with four sides.



quotient The result of a division. If 36 is divided by 3, the quotient is 12.

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**.



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely.

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population.

range In a set of data, the difference between the highest and lowest scores.

Glossaries

rate A relationship between two quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours).

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3: 40 (read '3 to 40').

reciprocal The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The product of any number and its reciprocal is 1.

rectangle A quadrilateral with four right angles.

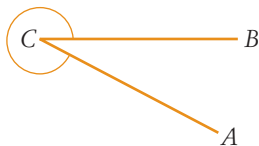


recurring (or repeating) decimal A decimal with one or more digits that repeat endlessly. For example 0.1666... is abbreviated as 0.1 $\dot{6}$.

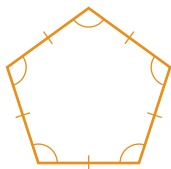
reflection The process of 'flipping' a shape across a line to give a mirror image that is back-to-front.



reflex angle A 'bent-back' angle greater than 180° but less than 360° .



regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has five equal sides and five equal angles.

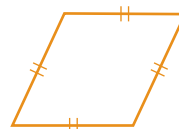


relative frequency The number of times an event or score occurred, written as a fraction of the total number of events or scores.

revolution An angle of 360° .

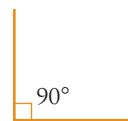


rhombus A quadrilateral with four equal sides.



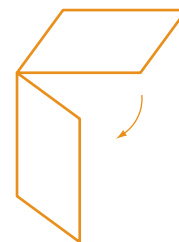
RHS The right-hand side (of an equation).

right angle A 90° angle, a 'square corner', a quarter-turn.

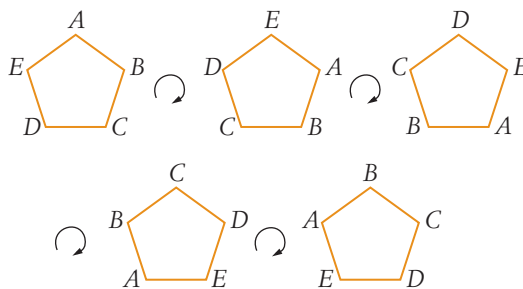


right-angled triangle A triangle with one 90° angle.

rotation The process of 'spinning' or turning a shape about or around a fixed point in a certain angle and direction (clockwise or anti-clockwise).



rotational symmetry A plane shape has rotational symmetry if it can be spun around its centre so that it fits onto itself again before making a complete revolution. The centre point is called the **centre of symmetry**. A regular pentagon has rotational symmetry, and because it can fit onto itself five times during one complete revolution we say that it has *rotational symmetry of order 5*. See also **line symmetry**.



S

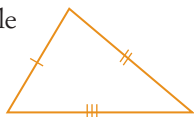
sample In statistics, a group of people or items selected from a population for study.

sample space In a probability situation, the set of all possible outcomes.

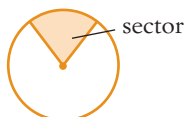
satisfy an equation When a variable (or variables) can be substituted into an equation to show that it is correct or true. For example, $x = 5$, $y = 8$ satisfies the equation $y = 3x - 7$.

scaled length The length on a scale drawing, map or plan, not the actual length.

scalene triangle A triangle with no equal sides.



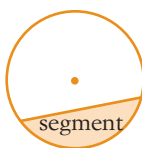
sector A region of a circle cut off by two radii, shaped like a slice of pizza.



sector graph Also called a pie chart, a circular graph that is divided into proportionately-sized sectors to represent parts of a whole.



segment A region of a circle cut off by a chord.



selling price The price at which an item is sold by the retailer (shop). The shop buys the item for the **cost price**.

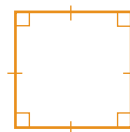
semicircle Half a circle.

solution In algebra, the value that makes an equation true.

speed A rate that compares distance travelled with time taken. Usually measured in kilometres per hour (km/h) or metres per second (m/s).

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

square A quadrilateral with four equal sides and four right angles.



square (of a number) The number multiplied by itself. For example, $7^2 = 7 \times 7 = 49$.

square root (of a number) The positive value which, if squared, gives the number. For example, $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$.

straight angle An angle of 180° .

stationary Not moving, stopped.

stem-and-leaf plot

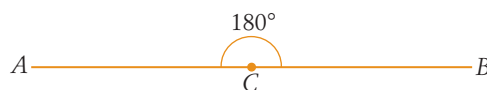
A 'number graph' that lists all the data scores in groups. Each score is split into a 'stem' and a 'leaf'.

This stem-and-leaf plot shows 12 test scores, from 42 to 82.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key: 5|8 stands for 58

straight angle An angle of 180° .

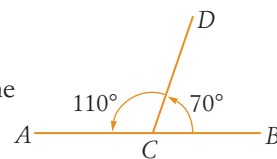


sum The result of an addition. The sum of 5 and 7 is 12.

superimpose To place one figure on top of another congruent figure so that sides and angles match.

supplementary

angles Two angles whose sum is 180° . The angles 70° and 110° are supplementary.



Glossaries

surd A square root (or other root) whose exact value cannot be found, such as $\sqrt{10}$ or $\sqrt[3]{7}$.

survey A study involving collecting information, facts and opinions.

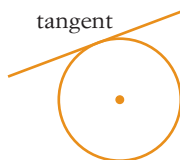
symmetry See **line symmetry** and **rotational symmetry**.

T

table of values A table of ordered pairs of numbers, usually following a formula and which can be graphed on a number plane. The table of values below is for the formula and linear equation $y = 2x - 3$.

x	-1	0	1	2
y	-5	-3	-1	1

tangent A line that touches a circle at one point. (A tangent cannot be inside a circle.)



terminating decimal A decimal whose digits do not repeat endlessly, for example, 0.125.

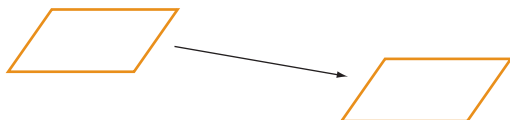
term (of a pattern) A number or diagram in a pattern.

term (of an expression) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has three terms: b^2 , $6b$ and -9 .

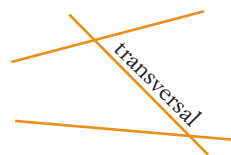
term (of a ratio) A number in a ratio, for example, in $14 : 9$, the terms are 14 and 9.

time zone A region of the world where all places experience the same time of day, for example, 10 a.m.

translation The process of 'sliding' a shape a certain distance and direction.



transversal A straight line that intersects two or more lines at different points.



trapezium A quadrilateral with one pair of opposite sides parallel.



travel graph A line graph that describes a journey and shows the distance travelled over time.

trial One go or run of a repeated probability experiment, for example, one roll of a die.

two-way table A table that shows the number of items belonging to overlapping categories.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

U

unit cost The cost of one item or unit, found by dividing the cost of the item by the number of items or units.

unitary method A method for finding a quantity by finding the size of one part or 1% first.

unknown Another name for **variable** because its value is usually not known, especially in an equation where the value can be found.

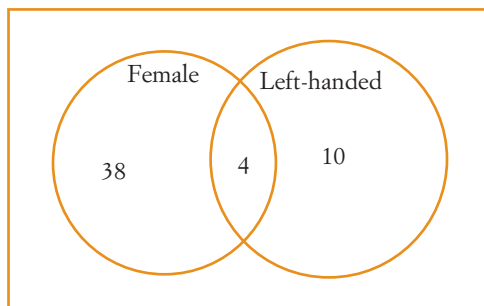
unlikely A low chance it will happen, improbable, probably won't happen.

UTC (Coordinated Universal Time) The time zone from which all time around the world is measured. Also called Greenwich Mean Time (GMT), it is the time measured at the Greenwich Observatory in London, England.

V

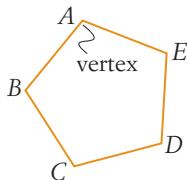
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**.

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories.



vertex (plural: **vertices**)

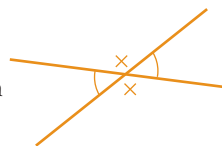
A corner of a shape or angle.



vertical Going up and down, at a right angle to the **horizontal**.

vertically opposite angles

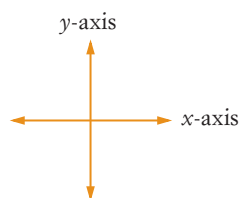
A pair of opposite and equal angles formed when two lines cross.



volume The amount of space occupied by a solid object, measured in cubic units.

X

x-axis The horizontal axis of a number plane (running across).

**Y**

y-axis The vertical axis of a number plane (running up and down).

y-intercept The y -value at which a line cuts the y -axis.

Chapter 1

SkillCheck

- 1 a 16 b 144 c 900 d 4.41
 e 106.09 f 25 g 169 h 18.92
 2 a 4 b 8 c 10 d 14
 e 11 f 7 g 1.1 h 1.5
 3 a 25 cm b 28 cm c 76 cm
 4 a 40 cm² b 16 cm² c 100 mm²
 5 a 10.3 b 4.7 c 7.7 d 0.9
 6 a 7.6 b 10.8 c 12.7

Exercise 1-01

- 1 a 14 b 30 c 8
 d 25 e 11 f 19
 2 a 3.46 b 6.71 c 31.64
 d 18.03 e 12.37 f 14.39
 g 9.90 h 29.80 i 4.90
 j 10.49 k 17.23 l 26.25
 3 a 10.91 b 8.60 c 12.53
 d 12.04 e 7.42 f 9.85
 g 4.46 h 10.29 i 31.49
 4 A 5 $\sqrt{98}$, $\sqrt{160}$, $\sqrt{52}$, $\sqrt{77}$, $\sqrt{18}$, $\sqrt{200}$

Exercise 1-02

- 1 a p b z c LN
 2 b 13 cm c 169 d 169 e 13, 169, 5, 12, 169
 3 B
 4 b

	Shorter sides		Hypotenuse	a^2	b^2	$a^2 + b^2$	c^2
	a	b	c				
a	6	8	10	36	64	$36 + 64$	100
b	2.5	6	6.5	6.25	36	$6.25 + 36$	42.25
c	1.5	2	2.5	2.25	4	$2.25 + 4$	6.25
d	4	7.5	8.5	16	56.25	$16 + 56.25$	72.25

5 Teacher to check.

Exercise 1-03

- 1 D
 2 a $p^2 = m^2 + n^2$ b $t^2 = r^2 + s^2$
 c $LK^2 = JL^2 + JK^2$ d $5^2 = d^2 + 3^2$
 e $12.5^2 = 10^2 + 7.5^2$ f $x^2 = 3^2 + 3^2$
 3 a $b = 10$ b $d = 25$ c $x = 15$
 4 a $m = \sqrt{865}$ b $b = \sqrt{52}$ c $x = \sqrt{569}$
 d $p = \sqrt{2}$ e $d = \sqrt{19.4}$
 5 a $m = 29.4$ b $b = 7.2$ c $x = 23.9$
 d $p = 1.4$ e $d = 4.4$
 6 D
 7 a $b = 7.81$ b $p = 7.94$ c $z = 3.02$
 8 a 40 b 39 c 41
 d 28 e 11 f 8
 g 45 h 6, 8 i 9, 12

Exercise 1-04

- 1 a $x = 12$ b $y = 16$ c $y = 8$
 d $a = 15$ e $d = 18$ f $x = 12$
 2 a $x = \sqrt{8}$ b $y = \sqrt{84}$ c $m = \sqrt{13 \ 321}$
 d $y = \sqrt{13}$ e $x = \sqrt{94 \ 176}$ f $x = \sqrt{128}$
 3 a $x = 2.8$ b $y = 9.2$ c $m = 115.4$
 d $y = 3.6$ e $x = 306.9$ f $x = 11.3$
 4 B
 5 a $k = 10.39$ b $q = 25.16$ c $BC = 28.28$
 6 Teacher to check

Mental skills 1

- 2 a 625 b 3025 c 2025 d 7225
 e 13 225 f 56.25 g 9025 h 38 025
 i 2.25 j 4225 k 24 025 l 60 025
 4 a 441 b 10 201 c 961 d 8281
 e 26.01 f 6561 g 3721 h 40 401
 i 1.21 j 16.81

Exercise 1-05

- 1 a $f = 85$ b $b = 4$ c $s = 55$
 d $r = 4.5$ e $l = 29$
 2 a $g = \sqrt{130}$ b $t = \sqrt{160}$ c $x = \sqrt{735}$
 3 a $q = 62.2$ b $a = 21.0$ c $p = 4.7$
 4 B
 5 a $i = 12.59$ b $y = 27.97$ c $v = 54.38$
 6 a $d = \sqrt{74}$ b $x = \sqrt{60}$ c $x = \sqrt{288}$
 d $p = \sqrt{85}$ e $x = \sqrt{164}$ f $p = \sqrt{560}$
 g $x = \sqrt{16.65}$ h $x = \sqrt{624}$ i $y = \sqrt{786.25}$

Exercise 1-06

- 1 a right-angled b right-angled
 c not right-angled d right-angled
 e not right-angled f right-angled
 g not right-angled h right-angled
 i not right-angled j right-angled
 k not right-angled l right-angled

Exercise 1-07

- 1 a, b, c, e, g, h are Pythagorean triads.
 2 D
 3-4 Teacher to check.
 5 a 12, 13
 b $a = 5$, $b = 12$, $c = 13$
 $a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169$
 $c^2 = 13^2 = 169$
 hence $a^2 + b^2 = c^2$, so it is a Pythagorean triad.

Answers

- 6 a 7, 24, 25
c 15, 112, 113
e 9, 40, 41
g 10, 49.5, 50.5
- b 11, 60, 61
d 4, 7.5, 8.5
f 19, 180, 181
h 51, 1300, 1301

Exercise 1-08

- 1 3.61 m 2 92.2 km 3 B
4 30.3 cm 5 $\sqrt{72}$ 6 4.8 m
7 a 48 cm b 31.38 cm c 170 cm
d 12.92 m e 28.94 m
8 $h = 10.39$ cm 9 6.32 m 10 16 m
11 a 192 cm^2 b 360 mm^2 c 84 mm^2
d 22 m^2 e 123.25 cm^2
12 38 m 13 23 cm 14 4.58 m or 458 cm

Power plus

- 1 a 22.6 b 10.6 c 27.7
d 5.4 e 10.3 f 2.8
2 a 16.2 m b 16.9 m
3 a 21.2 cm b 26.0 cm
4 18.4 cm

Chapter 1 revision

- 1 a 14.25 b 16.16 c 5.74 d 6.21
2 $\sqrt{104}, \sqrt{96}, \sqrt{12}, \sqrt{45}, \sqrt{88}$
3 a $BC, BC^2 = AB^2 + AC^2$ b $d, d^2 = e^2 + f^2$
c $s, s^2 = h^2 + k^2$
4 a $c = \sqrt{41}$ b $p = \sqrt{1325}$ c $x = \sqrt{914}$
5 a $m = 0.99$ b $f = 10.58$ c $BC = 4.87$
6 a $z = 145.0$ b $y = 7.0$ c $v = 42.6$
7 a right-angled b right-angled
c not right-angled
8 a, c, d are Pythagorean triads.
9 206 mm 10 a 9.4 m b 152.2 cm

Chapter 2

SkillCheck

- 1 a Missing numbers are -6, -4, -3, -1, 1, 2, 4, 6, 7, 9
b Missing numbers are -35, -30, -25, -15, -10, 5
2 -7, -5, -2, 0, 1, 6, 7, 11
3 12, 8, 5, 4, 2, -1, -3, -9
4 a 7 b 8 c 1
5 a $\frac{3}{100}$ b $\frac{7}{1000}$ c $\frac{49}{100}$ d $\frac{9}{10}$
6 a 3 b 1 c 2 d 5
7 a 1, 3, 5, 15 b 15, 30, 45, 60, 75
8 a prime b prime c composite d composite

Exercise 2-01

- 1 a 331 b 157 c 1587 d 255
e 421 f 413 g 368 h 734
i 612 j 35 k 276 l 47
2 a 119 b 112 c 233
d 95 e 250 f 242
3 A
4 a 36 b 54 c 24 d 62
e 28 f 43 g 81 h 146
i 36 j 18 k 68 l 14
m 43 n 172 o 26 p 99
5 a 233 b 1375 c 85
6 a 24 b 16 c 35 d 54
e 36 f 40 g 64 h 21
i 24 j 36 k 27 l 28
7 a 7 b 5 c 4 d 9
e 9 f 4 g 3 h 9
i 4 j 9 k 3 l 5
8 a 800 b 90 c 700 d 160
e 100 f 700 g 1500 h 240
i 180 j 320 k 1700 l 1400
9 a 160 b 3500 c 18 000 d 300
e 210 f 1800 g 36 000 h 2400
i 30 j 6 k 5 l 8
m 20 n 100 o 7 p 3
10 C
11 a 170 b 78 c 228 d 112
e 128 f 264 g 324 h 116
i 54 j 122 k 109 l 26
12 a 280 b 400 c 132 d 260
e 210 f 63 g 192 h 324
13 a 110 b 70 c 1400 d 540
e 1100 f 2600 g 16 h 18
i 8 j 40 k 82 l 14
14 1140 words
15 a 306 b 459 c 216 d 576
e 396 f 451 g 768 h 264
16 a 330 b 182 c 88 d 396
e 45 f 54 g 26 h 27
17 C
18 \$34

Exercise 2-02

- 1 a -2 b 5 c 2 d 6
e 4 f -3 g 4 h 0
i 6 j 3 k 2 l 0
m -5 n -3 o -1 p -8
2 a -2 b -6 c -7 d -4
e -8 f -10 g -6 h -12
i 12 j 5 k 12 l 18
m -1 n 3 o 0 p 1
3 9°C 4 C 5 30 cm

- 6 a -2 b 0 c -11 d 4
 e -13 f 6 g -1 h 2
 i -2 j -7 k -15 l -5

7 B

- 8 a $5 + (-9) = -4$ b $-2 + 3 = 1$
 c $-4 + (-5) = -9$ d $-4 + (-7) = -11$
 e $3 - 6 = -3$ f $-6 - 1 = -7$
 g $-2 - (-7) = 5$ h $-4 - 2 = -6$
 i $-8 - (-8) = 0$

9 a True

10 Echo Bay

- 12 a 0 b 2a c 2a d 0
 13 a + b E c + d -
 e E f - g + h E

Exercise 2-03

- 1 a -20 b -18 c 32 d -45
 e -70 f 30 g 9 h -24
 i -21 j 100 k -36 l 40
 m -44 n -60 o 36 p 63
 q -55 r 81 s -100 t 42
 2 a C b D c B
 3 a (-2) b (-4) c 5
 d -7 e -7 f -7

4 True 5 You get the negative of the integer squared.

6	$\times$	7	-5	-2	10
	-4	-28	20	8	-40
	-9	-63	45	18	-90
	5	35	-25	-10	50
	-6	-42	30	12	-60

- 7 a 28 b -90 c -40
 d 75 e 20 f -8
 8 a + b + c - d + e +
 9 Teacher to check.

Exercise 2-04

- 1 a -9 b 5 c -7 d -6
 e 5 f -7 g -5 h -2
 i 6 j -8 k -8 l 7
 m -5 n 20 o -9 p -6
 2 a A b B
 3 a -9 b -4 c 6 d -7
 e 4 f -4 g 9 h -7
 4 False
 5 a The answer is 1. b The answer is -1.
 6 a -2 b 3 c -2
 d 30 e -5 f -1
 g -20 h -2 i -70
 7 a (-6) b (-5) c 9
 d 32 e -24 f 100
 8 a a^2 b a^2 c 1 d -1

- 9 a + b + c -
 10 Teacher to check.

Mental skills 2A

- 2 a 3.5 b 2.4 c 0.12 d 0.36
 e 0.8 f 0.027 g 0.2 h 8.8
 i 0.24 j 0.012 k 1.8 l 0.028
 4 a 66.3 b 6630 c 6.63 d 0.663
 e 6.63 f 6630 g 0.663 h 663
 i 6630 j 66.3 k 0.663 l 0.0663

Exercise 2-05

- 1 a 5 b 30 c 22 d -32
 e 26 f 8 g -12 h 23
 i -1 j -5 k -19 l 0
 2 a -2 b 1 c 31 d 10
 e 28 f 3 g -42 h -8
 i 16 j 19 k 24 l 22
 3 a $\frac{1}{2}$ b 2 c $\frac{9}{10}$ d -6
 e -2 f $\frac{1}{4}$ g 1 h 3
 4 B
 5 a 8 b 5 c 3
 d (-2) e (-1) f -20
 g (-6) h 6 i 20
 6 a $8 - (3 + 7) = -2$
 b $(40 - 10) \times 5 = 150$
 c $27 \div (9 \div 3) = 9$
 d $8 + (4 - 3) \times 2 = 10$
 e $[8 + (4 - 3)] \times 2 = 18$
 f $(6 + 4) \times (-1) - 1 = -11$
 g $(13 + 3) \div (4 - 6) = -8$
 h $100 \div (10 + 10) + 5 = 10$
 i $100 \div (10 + 10 + 5) = 4$
 7 Teacher to check.

Exercise 2-06

- 1 B
 2 a 4.035, 4.05, 4.3, 4.35
 b 12.173, 17.12, 17.2103, 17.231
 c 14.08, 14.8, 14.801, 14.81
 d 0.032, 0.2, 0.295, 0.3
 3 a 1.621, 1.612, 1.61, 1.16
 b 3.305, 3.053, 3.05, 3.035
 4 a 3 b 2 c 1 d 0
 e 3 f 3 g 1 h 2
 5 a 3.9 b 4.1 c 0.3 d 7.3
 e 15.1 f 3.0 g 2.0 h 16.2
 6 a 68.91 b 107.06 c 3.60 d 4.71
 e 3.20 f 33.00 g 19.73 h 11.25

Answers

- 7 a 9.704 b 13.168 c 0.083 d 53.094
 8 a \$460.40 b \$460 c \$460.4
 d \$460.40 e \$460.395
 9 a 164.91 b 25.36 c 72.6 d 91.61
 e 15.7 f 77.27 g 18.44 h 2.68
 i 66.68 j 51.61 k 4.64 l 2.391
 10 C 11 7.6°C 12 0.8 m
 13 a 3.04 b 12.74 c 5.45 d 0.889
 e 4.326 f Teacher to check.
 14 a \$10.05 b \$9.95

Exercise 2-07

- 1 a 26 b 4.05 c 11.5 d 27.3
 e 0.21 f 0.088 g 0.02 h 1.26
 i 28.35 j 88.97 k 0.806 l 0.3672
 2 B 3 \$40.16
 4 a 6.2 b 16.3 c 2.61 d 0.139
 e 316.5 f 69.55 g 370 h 58.05
 i 0.58 j 5.2 k 170 l 1.2
 5 B
 6 a 43.44 m b 1.81 m
 7 a 5.91 b 14 c 5.66
 8 \$67.50
 9 a 8 b 2 c 0.04
 d 0.9 e 13.9 f 25.92
 10 16.2 km 11 \$14.10
 12 a 41.04 b 0.829 c 328.24
 d 4.7292 e 2.51706

Exercise 2-08

- 1 a 0.4 b 0.375 c 0.75 d 1
 e 0.5 f 0.75 g 0.6 h 0.25
 i 0.625 j 0.875 k 0.5 l 1
 2 They are equivalent fractions.
 3 a 0.6 b 4.27 c 0.28 d 3.83
 e 9.607 f 0.1519 g 0.052 h 12.234
 4 a 0.858333... b 0.1656565...
 c 0.272727... d 0.461538461538...
 5 a 0.1 b 0.16 c 0.83
 d 0.142857 e 0.6 f 0.285714
 g 0.2 h 0.428571 i 0.571428
 j 0.4 k 0.6 l 0.5
 m 0.857142 n 0.7 o 0.714285

- 6 0.142857, 0.285714, 0.428571, 0.571428,
 0.714285, 0.857142.
 7 a 3.6 b 1.71 c 531.18 d 3.26

Mental skills 2B

- 2 a 0.5 b 90 c 8 d 0.9
 e 90 f 0.7 g 8 h 30
 i 0.08 j 3.5 k 4 l 0.3
 4 a 16 b 160 c 1.6 d 1.6
 e 1.6 f 16 g 160 h 0.16
 i 0.016 j 160 k 0.16 l 0.0016

Exercise 2-09

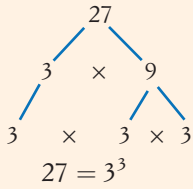
- 1 a 7^3 b 4^5 c 5^{10} d $(-8)^7$
 2 a 64 b 8 c -216 d 81
 e 7 f -32 g 1000 h 256
 i -1 771 561 j 18 954 k 972 l 2529
 m 39 n 4094
 3 a 3 b 3 c 2 d 6 e 3 f 5
 4 B 5 100^3 6 2^{10}
 7 a 36 b 36 c Yes, they both equal 36.
 8 a 400 b 400 c Yes
 9 a $3^2 \times 8^2$ b $a^2 \times b^2$
 10 a $6^2 \times 3^2 = 324$ b $2^2 \times 11^2 = 484$
 c $(2 \times 8)^2 = 2^2 \times 8^2 = 256$
 d $(3 \times 5)^2 = 3^2 \times 5^2 = 225$
 e $(10 \times 3)^2 = 10^2 \times 3^2 = 900$
 f $(4 \times 7)^2 = 4^2 \times 7^2 = 784$
 11 See table below.
 12 a 28 b 16 c 17 d 33
 e 2 f 7 g 13 h 6
 13 a 6.1 b 9.7 c 22.4 d 1.9
 e -7.9 f 44.7 g 1.0 h 10.3
 14 D 15 D 16 3 and 4
 17 a 5.7 b 10.2 c 8.2
 18 $\sqrt{6}$ $\sqrt{62}$ $\sqrt{14}$ $\sqrt[3]{181}$ $\sqrt[3]{49}$ $\sqrt[3]{674}$ $\sqrt[3]{1331}$
 19 a 10 b 10 c The answer is the same.
 20 a 9 b 9 c The answer is the same.
 21 a $\sqrt{16} \times \sqrt{9}$ b $\sqrt{a} \times \sqrt{b}$
 22 a $2 \times 4 = 8$ b $\sqrt{9} \times \sqrt{49} = 3 \times 7 = 21$
 c $\sqrt{36} \times \sqrt{4} = 6 \times 2 = 12$ d $\sqrt{81} \times \sqrt{25} = 9 \times 5 = 45$

Exercise 2-09

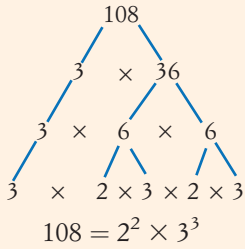
11	Number, x	1	2	3	4	5	6	7	8	9	10	11	12
	Number squared, x^2	1	4	9	16	25	36	49	64	81	100	121	144
	Number cubed, x^3	1	8	27	64	125	216	343	512	729	1000	1331	1728

Exercise 2-10

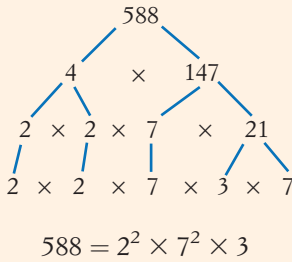
1 a



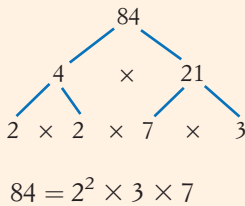
b



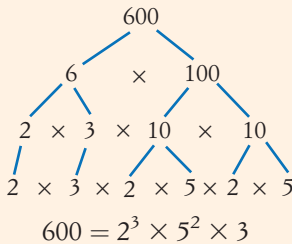
c



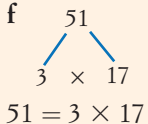
d



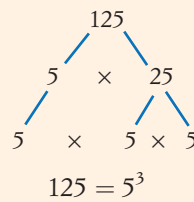
e



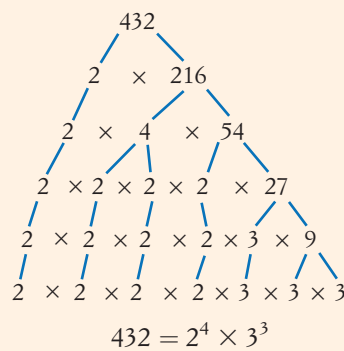
f



g



h



2 C

3 Teacher to check.

4 a 4	b 7	c 5	d 15
e 6	f 24	g 16	h 6
5 a 15	b 26	c 11	
6 a 72	b 12	c 110	d 28
e 30	f 24	g 45	h 110
7 a 90	b 72	c 80	
8 a 45	b 21	c 16	d 16 e 9 f 18

Exercise 2-11

1 a 6^5	b 4^7	c 2^{11}	d 8^{10}
e 11^8	f 7^{10}	g 5^7	h 6^{16}
i 2^9	j 5^9	k 2^{14}	l 3^{16}

2 D

3 a $5^6 = 15\,625$	b $7^8 = 5\,764\,801$
c $3^{13} = 1\,594\,323$	d $4^6 = 4096$
f $9^7 = 4\,782\,969$	g $6^9 = 10\,077\,696$
e $8^{10} = 1\,073\,741\,824$	f $7^4 = 2401$

4 a 2^4	b 3^2	c 7^6	d 2^5
e 5^2	f 10^4	g 11^7	h 6^7
i 2	j 16^6	k 35^{11}	l 20

5 D

6 a $2^4 = 16$	b $5^2 = 25$	c $4^3 = 64$
d $10^3 = 100\,000$	e $8^3 = 512$	f $6^8 = 1\,679\,616$
g 11	h $3^6 = 729$	

7	Number, n	n^2	n^3	n^4	n^5	n^6
	2	4	8	16	32	64
	3	9	27	81	243	729
	4	16	64	256	1024	4096
	5	25	125	625	3125	15 625

Answers

- 8 a 32 b 81 c 125 d 3125
e 4096 f 1944 g 64 h 3125
i 9 j 2 k 4 l 1
9 a 5^4 b 3^7 c 2^3
10 a a^{10} b x^6 c b^5 d y^5
e d^4 f x^2 g a h r^4

Exercise 2-12

- 1 a 2^{18} b 5^6 c 7^{20} d 8^{14}
e 3^{24} f 9^{24} g 6^{14} h 11^{25}
i 8^{30} j 4^{45} k $(-2)^{28}$ l $(-7)^{18}$
2 D
3 a $5^8 = 390\,625$ b $2^{15} = 32\,768$
c $3^{18} = 387\,420\,489$ d $6^{12} = 2\,176\,782\,336$
e $7^5 = 16\,807$ f $4^{12} = 16\,777\,216$
g $8^0 = 1$ h $4^9 = 262\,144$
4 a 1 b 1 c 1 d 1
e 1 f 1 g 1 h 6
i 1 j 7 k 1 l 3
m 1 n 2 o 6 p 9
5 B
6 1, 2, 8, 16, 64, 128, 256, 512, 2048, 4096
7 a 32 768 b 4096 c 16 d 512
e 1 f 256 g 1 h 128
8 a $(6^2)^4 = 6^8$ b $(5^3)^3 = 5^9$
c $(10^6)^3 = 10^{18}$, other possible answers
9 a a^8 b x^{36} c d^{10} d n^{18}
e 1 f 4 g 1 h 0

Power plus

- 1 a 9 b $2\sqrt{2}$ c $3\sqrt{3}$ d $5\sqrt{2}$
e $6\sqrt{2}$ f $3\sqrt{5}$ g $6\sqrt{5}$ h $8\sqrt{2}$
i $2\sqrt{7}$ j $10\sqrt{2}$ k $4\sqrt{3}$ l $7\sqrt{5}$
2 a 2^{-1} b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{2^2} = \frac{1}{4}$
3 a Yes b No c For odd values of n . d 0
4 a 24 000 b 455 000 c 0.0933
d 0.0638 e 8 700 000 f 0.005 82
g 0.126 h 2690 i 314 000 000
5 a 1.2×10^4 b 3.45×10^8 c 7×10^{-3}
d 4×10^3 e 5×10^{-4} f 4.1×10^{-4}
g 1.92×10^6 h 3.61×10^{-4} i 6.37×10^{-8}
6 Teacher to check.

Chapter 2 revision

- 1 a 1400 b 210 c 160 d 131
e 87 f 385 g 260 h 480
i 276 j 800 k 276 l 42
m 9 n 2800 o 333 p 90
q 48 r 725 s 160
2 a 116 b 94 c 22 d 93
3 a 2 b 7 c -11 d 0

- 4 a -14 b 24 c 49 d -80
5 a -9 b -4 c -12 d 3
6 a 57 b 29 c 3
d 62 e $\frac{1}{5}$ f 5
7 0.471, 0.47, 0.417, 0.147
8 a 0.5 b 13.11 c 98.427
9 a 30.5 b 19.72 c 10.4 d 3.3
e 0.923 f 6.25 g 2.3925 h 11.6
10 a 0.2 b 0.875 c 0.16
11 a 343 b 161 051 c 625
d 20 e 3 f -5
12 5.6
13 a 2, 2, 4, 10, 400 b 4, 2, 7, 14
14 a 6 b 140
15 $132 = 2^2 \times 3 \times 11$, $88 = 2^3 \times 11$,
HCF = $2 \times 2 \times 11 = 44$
16 $12 = 2^2 \times 3$, $45 = 3^2 \times 5$,
LCM = $2^2 \times 3 \times 3 \times 5 = 180$
17 $784 = 2^4 \times 7^2$, $\sqrt{784} = 2^2 \times 7 = 28$
18 a 4^7 b 6^{11} c 12^8 d 9^4
e 2^4 f 5^3 g 3 h 6^{12}
19 a 3^8 b 7^{10} c 6^9 d 3^{15}
20 a 1 b 1 c 4 d 8
e 1 f 1 g 1 h 0

Chapter 3

SkillCheck

- 1 a -1 b 4 c -10 d 5
e -18 f 3 g -13 h 32
i 36 j 8 k -1 l 18
2 a $7p$ b ab c k^2 d n^3
e $2mp$ f dkp g $6y$ h $3qt$
i $-d$ j $3q$ k $4a$ l x
3 a + b $\times$ c - d $\div$
e - f + g + h -
4 a 10 b 24 c 6 d 10
e 16 f 7 g 25 h 20
i 4 j 4 k 20 l 42
5 a 13 b 18 c 16 d 3 e 21 f 8

Exercise 3-01

- 1 a $5w$ b m^2 c $3abc$ d $6k$
e $\frac{m}{8}$ f $3f$ g $16m$ h $21wx$
i $9p$ j $4t^2$ k d l $16bcd$
m $\frac{26}{n}$ n $9h^2$ o $20x$ p a
q $8q$ r $12cn$ s $-6d^2r$ t $3a + 2c$
u a^3x^2
2 a $\frac{13}{2a}$ b $9 + 4m$ c $\frac{e-6}{5}$
d $4t - 8$ e $y^2 + z^2$ f $\frac{k}{9} - n$
g $\frac{p}{n+9}$ h $22 - \frac{e}{2}$ i $11 + 11u^2$
3 A 4 Teacher to check. 5 B

- 6 a $8 \times s \times t$ b $-2 \times y \times y$
 c $8 \div f$ d $r \times r \times t \times t$
 e $4 \times b \times b \times d$ f $5 \times x \times y - 2 \times a$
 g $(x-2) \div 3$ h $q \times q \times q$
 i $16 - 2 \times d \times d$ j $-4 \times d \div 3$
 k $h \times h \times i \times i$ l $4 \times j - 9 \div d$
- 7 D

Exercise 3-02

- 1 a $5n$ b $n+6$ c $\frac{n}{10}$ d $n+1$
 e $\frac{n}{2}$ f $n-20$ g n^2 h $20-n$
 i $n-1$ j $2n$ k $\frac{6}{n}$ l $n-8$
 m $-4n$ n $\sqrt{n}$
- 2 a $a+b+c$ b ab c $b-c$
 d $ac+b$ e $3b+a$ f $\frac{c}{b}$
 g $a-b$ h $5+bc$ i $b-c$
 j $\frac{ab}{c}$ k $\sqrt{a+c}$ l $ab-7c$
- 3 D
- 4 a $3y+8$ b $\frac{p}{5}-6$ c $4m+27$
 d $\frac{5k}{2}$ e $\frac{x}{6}+13$ f $\frac{b}{2}-2$
 g $\frac{p}{9}+10$ h $21n$
- 5 a x plus y minus z b 5 times K plus L
 c x times y minus 3 d W divided by 5
 e 3 times x , minus 1 f y minus x , then divided by 10
 g 2 times N , plus 21 h 7 minus e
- 6 a $t-b$ b $\frac{n}{5}$ c $y-3p$ d $2r+9$
 e $\frac{a}{12}$ f $4x$ g $\frac{160}{r}$ h $5x+3y$

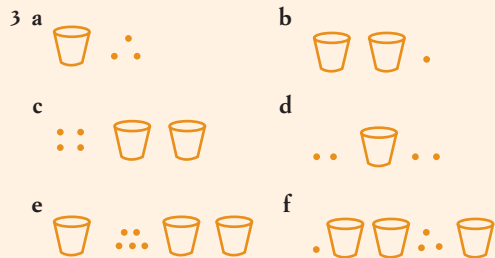
Exercise 3-03

- 1 C
- 2 a 8 b 12 c 4 d 10
 e 30 f 12 g 0 h 8
 i 5 j 19 k -3 l 3
- 3 a -150 b 25 c -27 d 2
 e -15 f 75 g -21 h 6
 i 18 j -10 k -90 l $\frac{1}{2}$
- 4 B
- 5 a -13 b -3 c 10 d -5
 e $-\frac{1}{4}$ f 1 g $\frac{1}{3}$ h 5
- 6 C 7 $A=42\text{ m}^2$ 8 \$212
 9 78.8°F 10 14 hours 11 \$5900

Exercise 3-04

- 1 a $x+2$ b $2x+3$
 c $2+2x$ d $3x+4$
 e $x+1+x+1$ or $2x+2$
 f $2+x+2+2x$ or $3x+4$
 g $1+x+3$ or $x+4$
 h $2x+2+2x+1$ or $4x+3$
 i $x+3+x$ or $2x+3$
 j $4x+5$

- 2 a $x+y+2$ b $2x+2y+1$ c $x+2y$
 d $2x+2y+3$ e $x+2y+2$ f $2x+3y+4$
 g $x+y$ h $2x+3y+1$



- 4 B
- 5 a D b A c F d B e C f E
 6 a D b F c A d B e C f E
 7 a $m+2p$ b $m+p$ c $3p+5$
 d $2m+2p+1$ e $m+2p$
- 8 C
- 9 a $4a+5d$ b $5h+9r$ c e
 d $4k-3$ e $2x+6z$ f $2p+s$
 g $4u+12$ h $4n-1$ i $3i$

Mental skills 3A

- 2 a 720 b 486 c 825 d 128
 e 560 f 216 g 189 h 432
 i 560 j 616 k 135 l 675

Exercise 3-05

- 1 a $4p, 2p, p$ b $m^2, 3m^2$ c $5ac, 7ca$
 d $vw, 2wv$ and $5v, 9v$ e $6pq, 3pq$ f $2d, 3d$
 g $4mn, 2nm, mn$ h $x^2y, 4x^2y$ i $5ba, ab$
- 2 C
- 3 a $11k$ b $7mn$ c $2xy$ d $9abc$
 e $6x^2$ f $5ef^2$ g $5d$ h $5mk$
 i $4xy$ j $-4de$ k $3k^2$ l $3w^2$
- 4 C
- 5 a $5k+4j$ b $9ab+2$ c $7m^2+2m$
 d $10s^2-7st$ e $10mn$ f $9k+3$
 g $2y^2$ h $12d-4e$ i $4-2x$
 j $8bg$ k $9-2g$ l $5a+2c$
- 6 a $3x-3y$ b $4ab+3bc$ c $9k^2+11$
 d $6bc-9a$ e x^2-7x+7 f $4p^2$
 g $12y+14$ h $7r+3s$ i $7b+6g$
 j $6x^2-13x+6$ k $52a$ l $7q-7$
 m $-6y$ n 12
- 7 C 8, 9 Teacher to check.

Answers

Exercise 3-06

- 1 D
 2 a 15y b 16m c 4km d 20t
 e 12f f 16xy g 30mp h 18bd
 i 12d² j 120mn k 48y² l 27r²
 m 72ab n x²y² o 42jk p 10r²s²t
 q 18ac²d r h²j³k² s 24p²q² t 6m²n²
 3 a -8bd b -24m² c -90kl d -60p
 e -63k f 75b² g -12m²n² h -12a²b²c
 i 40r² j -216d²e k 144x² l -40m²
 4 B
 5 a y⁴ b a³ c 2c² d t⁵
 e 7q³ f k³l² g m³n² h -5p⁴k³
 i -3y³ j -d²e²f k 4q²r⁴ l 11p
 m 35mn n a² o 2ry p -27p
 6 Teacher to check.

Exercise 3-07

- 1 A
 2 a 4m b 9e c -7p d 9x
 e 4w f 8 g -5 h $\frac{3}{4}$
 i 2p j -7 k $\frac{p}{5}$ l $-\frac{7n}{m}$
 3 a 2m² b -3e c -5p d 7x
 e -2w f 4m g -8ky h $\frac{3}{4}$
 i 5de j a k -8a l -2m
 4 D
 5 a -3 b -3 c -15y³ d 6e
 e 6x² f 8mn g 3 h 3a
 i $\frac{a}{5c}$ j 2d k $\frac{5p}{m}$ l 1
 m 8x n 9r + 14 o 18p
 6 Teacher to check.

Exercise 3-08

- 1 D
 2 a m⁷ b k⁹ c y¹⁰ d x⁸
 e n⁸ f q¹⁰ g b¹⁶ h s¹⁵
 i 42k⁶ j 20y⁴ k 3d⁹ l 55e⁴
 m h¹⁰n⁷ n -30c⁶d⁵ o f⁸g¹¹ p -10p⁷q²
 3 A
 4 a y b p³ c q³ d n⁵
 e k² f x² g 1 h m⁵
 i q j 9g⁴ k 3p⁴ l a⁵
 m 11xy n x³ o 7k³l⁵ p $\frac{5e^2}{7f}$
 5 B
 6 a y¹⁶ b x¹⁰ c y⁹ d m¹⁸
 e k²¹ f t⁴⁰ g 9m² h 8m¹²
 i x³y³ j p¹²q⁴ k 81h¹⁶ l k¹⁵z⁵
 m 16k⁴j⁴ n 16c⁶d² o 125x⁶y²⁴ p 64m²⁴p¹²
 7 a 1 b 1 c 1 d 5
 e 1 f 1 g 1 h h²
 i $\frac{b^6}{2}$ j 16x² k 1 l 13x
 m 3m + 1 n 3b² o 32 p $\frac{1}{2}$

Mental skills 3B

- 2 a 176 b 363 c 261 d 405
 e 682 f 707 g 1818 h 3564
 i 152 j 540 k 2142 l 588

Exercise 3-09

- 1 a 5m + 15 b 8m - 16 c 2j + 20
 d 3a + 6 e 4h - 28 f 6x + 6y
 g 36 - 9b h 11y - 66 i 8k + 20
 j 15m - 6 k 18x + 24 l 20a + 50b
 2 a -3n - 12 b -4v + 20 c -7d - 21
 d -10k + 20 e -2d - 2e f -25 + 5z
 g -6h - 6 h -m + 1 i -45 - 18x
 j -2f - 3 k -3u + 3v l -32 + 8k
 3 B
 4 a xy + 6x b pq - 2p c a² + 4a
 d 3b² - b e d² + 3de f 8m + 10n
 g 15k² - 10k h 21j² + 7ijk i 4x² + xy
 j -8p + 6q k k² - 12k l -4m + m²
 m 8n² + 20mn n 2x - 14x² o -2f² - 4f
 p -5j + 7k
 5 C 6-7 Teacher to check. 8 D
 9 a 6a + 26 b 6k + 6 c 3q - 4m
 d 6p - 22 e 4 - 2y f 17 - 7t
 g 12h + 5 h -8x + 8 i 18n + 4
 10 Teacher to check.
 11 a 5a + 14 b 8x + 4 c 17k + 18
 d 22t + 11 e n + 7 f 4d + 2
 g 22y + 23 h -2p - 7 i m² + 4m + 3
 j 7d + e k 6f² + 4 l -38e² + e
 12 a x + 5 b r - 3 c 8k - 3
 d 2a + 7 e y + 9 f 2t - 7

Exercise 3-10

- 1 a 1, 3, 5, 15 b 1, 2, 4, 8
 c 1, 17 d 1, 5, 25
 e 1, 5, 11, 55 f 1, 2, 3, 6, 7, 14, 21, 42
 2 a 3 b 2 c 6 d 4
 e 11 f 5 g 3 h 15
 3 Teacher to check. 4 C
 5 a 3x b 4j c 2x d 7k
 e 5 f 4m g 16c h de
 i 9 j 3 k 2m l 4y
 m a n 32 o 5w p 2xy
 q d r 12t
 6 C

Exercise 3-11

- 1 a 2 b 4 c x d y
 2 a x + 2 b p - 4 c 4k - 5 d j - 4k
 e x² + y² f 5 g 6 h 2

- i 2 j $3x + 1$ k $y + z$ l $b - c$
 m $q - 1$ n $b + d$ o $a + 2$ p h
 q n r y s k t f
- 3 a $4(x + 2)$ b $3(m + 2)$ c $5(a - 2)$
 d $5(3m - 2a)$ e $4(2x - y)$ f $4(3m - 4w)$
 g $h(k - 1)$ h $x(2y - m)$ i $p(12 + 5r)$
 j $x(x + 5)$ k $m(4 - m)$ l $y(y - 11)$
- 4 C
- 5 a $m + n$ b $2y - 1$ c $2p + 1$
 d $n - p$ e $y - 4$ f $q + r$
 g $7 - y$ h $4k + 1$ i $3x - 4z$
- 6 a $7a(b + 2)$ b $2x(1 - 3y)$ c $3m(1 - 3w)$
 d $5x(2y - 5w)$ e $4m(2a - 3p)$ f $3y(3x + 2m)$
 g $3x(1 + 3y)$ h $ab(1 + c)$ i $xy(z + a)$
 j $6m(3 - 4n)$ k $ac(2 + 3b)$ l $pq(r + m)$
 m $r(s + 7)$ n $4(s^2 + 4)$ o $p(p - 2)$
 p $2k(3k^2 - 1)$ q $j(3 + j)$ r $n(7 - n)$
 s $7e(1 + 3e)$ t $6q(1 - 2q)$ u $a^2(a^2 + 1)$
 v $xy(x + y)$ w $3m(mn + 3)$ x $3abc(4abc + 1)$

Exercise 3-12

- 1 a $k + 1$ b $n + p$ c $r - 2$ d $k - 9$
 e $-a$ f $-y$ g -6 h $-f$
- 2 a $-4(x - 3)$ b $-3(m + 3)$ c $-10(x - 2)$
 d $-5(3x - 2m)$ e $-2(3x + 4m)$ f $-5(x + 6)$
 g $-a(1 + b)$ h $-4(xyz - 2abc)$ i $-4(5 - x)$
 j $-d(x + y)$ k $-m(2 - 5p)$ l $-8(3x + 2y)$
- 3 B
- 4 a $-8x(2y + 3k)$ b $-4x(5 - 3y)$ c $-4m(2w - 3)$
 d $-5e(1 + 3f)$ e $-ab(1 + c)$ f $xy(a + w)$
 g $-6c(2 + 3w)$ h $-rt(2 - 7s)$ i $5h(4f + 1)$
 j $-19w(b + 2)$ k $-abc(1 - d)$ l $-abc(1 + d)$
- 5 a 10, 10 b Yes 6 A

Power plus

- 1 a $-4p - 9q$ b $-6m - 3n - 7mn$
 c $4j - k + 4$ d $5b^2$
 e $3x^2 - 6x$ f $5xy + 2yz + 4xz$
 g $-4d^2 + 4e^2 + 16$ h $13f^2 + 6f + 1$
- 2 a i $4x + 6$ ii $x^2 + 3x$
 b i $4p$ ii p^2
 c i $x + y + 16$ ii $\frac{xy}{2}$
 d i $2m + n + 8$ ii $\frac{n}{2}(2m + 3)$ or $mn + \frac{3m}{2}$
 e i $3x + 2y$ ii $x^2 + x$
 f i $2a + 2b$ ii $ad + bc - cd$
- 3 a $x^2 + 2x - 15$ b $2am - 3m$
 c $4y - 9$ d $6a - 2b + 14c$
 e $-5p^2 - 7pq + 10pr$ f $4k^2 - 5k - 5$
 g $27 - 20f + 30g$ h $-2x^2 + 7x + 115$
 i $x^2 + 8x + 15$ j $p^2 + 9p - 10$

- 4 a -2 b -24 c 10 d 9
 e 8 f 4 g 7 h 4
 i -1 j 3 k 1.2 l 0
- 5 a $p(9q + 5 - 2p)$ b $11(4k - 3l - m)$
 c $x(2 - 7x + 3y)$ d $(y + 2)(x + 3)$
 e $(b - 3)(a - 7)$ f $(k + 1)(4 - j)$
- 6 a m b $\frac{2n}{5}$ c k d $\frac{2d}{5}$
 e $\frac{3f}{2}$ f $\frac{7c}{10}$ g $\frac{6}{x}$ h $\frac{7b+5a}{ab}$

Chapter 3 revision

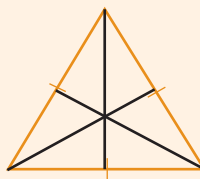
- 1 a $6p$ b $-r$ c $4dg$
 d $5t$ e $20m$ f $5k$
 g $6y^2$ h $\frac{16}{a}$ i $20cd$
- 2 a $c \times d$ b $3 \times y \times y$ c $w \div 6$
 d $c \times c + y$
- 3 a $y + 4$ b $6b - 3$ c $\frac{e}{q}$
- 4 a 3 b -10 c 19 d 24 e -15 f 29
- 5 a 12k b 17d c 17m
 d 18m e 3ac f 9cd
 g $10x + 10y$ h $15r + 15s$ i $28h + 7g$
- 6 a 10x b $14bd$ c $66m$ d $24pq$
 e $32w^2$ f $75j$ g $36m^2$ h $60p^3$
- 7 a 25x b $\frac{4b}{d}$ c 11m d 8rs
 e 5 f $7d^2$ g 7e h 13b
 i 2x j $4mn^2$ k 4e l 5ae
- 8 a $6n + 12$ b $-10b - 35$ c $9j^2 - 3j$
 d $2n + 18$ e $19v - 15$ f $9y - 18$
- 9 a 6 b 3p c 7
- 10 a $3(m + 3)$ b $5(h - 7)$ c $3a(2 + 3b)$
 d $2h(5 - 6m)$ e $xy(1 + z)$ f $d(c - e)$
 g $a(a - 6)$ h $8j(k + 3j)$
- 11 a $-2(1 + 2q)$ b $-4(f + 4)$ c $-3(4 - 5d)$
 d $-b(a - 1)$ e $-8b(2 + 3d)$ f $-k(4k + 1)$
 g $-x(x - 7)$ h $-hi(h + i)$

Mixed revision 1

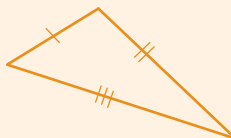
- 1 a 343 b 1225 c 256
 d 20 e 14 f 1.5
- 2 a 140 b 135 c 120
 d 480 e 180 f 15
- 3 a $x = 5$ b $y = 41$ c $x = 6.4$
 d $x = 24$ e $a = 17.7$ f $p = 33.5$
- 4 a 8 b -30 c -20 d 5
 e 28 f -6 g -2 h 2
 i -16 j -28 k 6 l -15
- 5 a 18 b 23 c -3 d -11 e 19
 f -99 g $4\frac{3}{4}$ h 8 i 2
- 6 a 13 b 9 c 10 d 10
 7 a $2k + 4j$ b $4p$ c $11p + 5q$ d $8x + 6y$
- 8 a -4.23 b 22.39 c 0.42 d 18.4
- 9 a 7.2 b 0.45 c 0.42 d 18.4

- 10 a $k + 5$ b $20k$ c $\frac{k}{2}$
 d $3k + 10$ e $k - 12$ f k^2
 11 a $10p$ b 6 c $-12x^2y$
 d $24ab$ e 2 f $2a$
 12 a $180 = 2^2 \times 3^2 \times 5$ b $875 = 5^3 \times 7$
 13 a 4 b 18
 14 a 2^7 b 7^{11} c 5^5 d 8
 e 3^8 f 11^{15} g 1 h 1
 15 a $15y^6$ b $5y^5$ c $7n^5$
 d m^{20} e $9k^4$ f $125m^9n^6$
 16 a $\sqrt{48}$ cm b 6.93 cm
 17 a 36 b 54
 18 a $4x + 8$ b $6m - 24$ c $-14k - 7$
 d $-3p - q$ e $6m + 4$ f $4p$
 19 a $2(m + 2)$ b $d(d - 1)$ c $6(p + 2q)$
 d $-12(x + 2)$ e $a(2b + 3)$ f $-2x(x - 3)$

d isosceles triangle



- 6 a 2 b 1 c 2 d 3
 7 D
 8



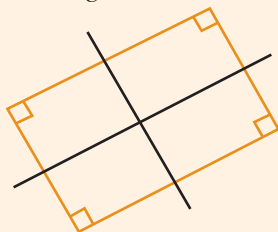
- 9 a b No



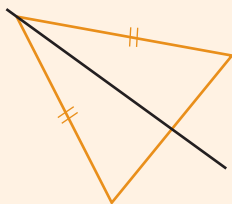
Chapter 4

SkillCheck

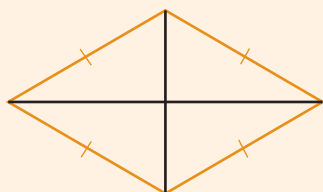
- 1 Teacher to check.
 2 a acute b obtuse c reflex d obtuse
 e right f revolution g reflex h straight
 3 a vertically opposite b right c reflex
 4 a $x = 60$ b $x = 123$ c $x = 162$
 5 a rectangle



b isosceles triangle



c rectangle



Exercise 4-01

- 1 a complementary b reflex
 c supplementary d obtuse
 e straight f vertically opposite
 g acute h adjacent
 i angles at a point
 2 a 74° b 57° c 19° d 88°
 3 a 155° b 31° c 73° d 132°
 4 D
 5 a $x = 70$ (angles in a right angle)
 b $a = 112$ (angles on a straight line)
 c $a = 110$ (vertically opposite angles),
 $b = 70$ (angles on a straight line)
 d $y = 285$ (angles at a point)
 e $a = 19$ (angles on a straight line)
 f $x = 53$ (adjacent angles)
 g $a = 52$ (angles on a straight line),
 $b = 128$ (vertically opposite angles),
 $c = 52$ (angles on a straight line)
 h $x = 165$ (angles at a point)
 i $x = 270$ (angles at a point)
 j $a = 45$ (vertically opposite angles),
 $b = 135$ (angles on a straight line),
 $c = 135$ (vertically opposite angles)
 k $k = 58$ (angles in a right angle)
 l $n = 70$ (angles on a straight line)
 m $w = 156$ (angles at a point)
 n $q = 62$ (angles on a straight line)
 o $y = 40$ (angles at a point)
 p $p = 45$ (equal angles in a right angle)
 q $p = 62$ (angles on a straight line)
 r $a = 23$ (adjacent angles)

- 6 a $m = 235$ (angles at a point)
 b $m = 270$ (angles at a point)
 c $a = 50$ (angles on a straight line)
 d $x = 170$ (angles at a point)
 e $a = 80$ (angles on a straight line),
 $b = 65$ (vertically opposite angles),
 $c = 115$ (angles on a straight line)
 f $a = 45$ (angles on a straight line)
 g $p = 90$ (angles on a straight line)
 h $m = 30$ (angles on a straight line)
- 7 C

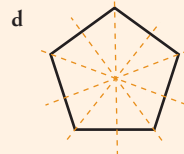
Exercise 4-02

- 1 a alternate b co-interior c corresponding
 d co-interior e alternate f co-interior
- 2 C
- 3 a e° b C c e° d f° e D
- 4 a $a = 70$ (alternate angles on parallel lines)
 b $d = 45$ (co-interior angles on parallel lines)
 c $a = 61$ (alternate angles on parallel lines)
 d $m = 120$ (corresponding angles on parallel lines)
 e $p = 89$ (co-interior angles on parallel lines)
 f $m = 112$ (co-interior angles on parallel lines)
- 5 a $a = 61$ (alternate angles on parallel lines),
 $b = 119$ (angles on a straight line)
 b $c = 70$ (co-interior angles on parallel lines),
 $d = 70$ (vertically opposite angles)
 c $a = 65$ (co-interior angles on parallel lines),
 $b = 115$ (alternate angles on parallel lines)
 d $g = 50$ (alternate angles on parallel lines),
 $h = 60$ (alternate angles on parallel lines),
 $m = 70$ (angles on a straight line)
 e $p = 100$ (angles on a straight line),
 $q = 80$ (alternate angles on parallel lines)
 f $y = 89$ (vertically opposite angles and corresponding angles on parallel lines)
 g $w = 108$ (co-interior angles on parallel lines),
 $x = 108$ (angles on a straight line),
 $y = 108$ (corresponding angles on parallel lines)
 h $a = 100$ (alternate angles on parallel lines [by extending the middle parallel line])
 i $n = 250$ (co-interior angles on parallel lines)
- 6 D
- 7 a Yes (alternate angles are equal)
 b Yes (corresponding angles are equal)
 c No (co-interior angles do **not** add to 180°)
 d Yes (co-interior angles add to 180°)

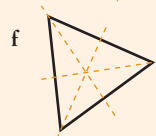
Exercise 4-03



c None

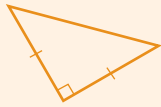
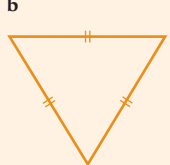
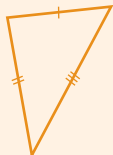
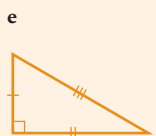
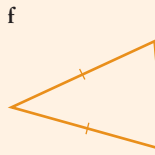


e None



- 2 a 2 b 8 c 1 d 2
 3 a Yes, 4 b Yes, 5 c Yes, 1 d Yes, 8
- 4 a 
 b 
- 5 Teacher to check. 6 Teacher to check.
- 7 a i 4 ii yes, 4 b i 6 ii yes, 6
 c i 1 ii no d i 6 ii yes, 6
 e i 5 ii yes, 5 f i no ii no
- 8 a Circle b Circle

Exercise 4-04

- 1 a scalene, obtuse-angled b equilateral, acute-angled
 c scalene, acute-angled d isosceles, right-angled
 e isosceles, obtuse-angled f scalene, obtuse-angled
 g scalene, right-angled h isosceles, acute-angled
 i isosceles, right-angled j isosceles, obtuse-angled
 k equilateral, acute-angled l scalene, right-angled
- 2 a  b  c 
 d  e  f 
- 3 A
- 4 No, all angles are 60° in equilateral triangles so they are not obtuse.
- 5 a b, d, e, h, i, j, k b b, k
- 6 a $x = 42$ (angles in an isosceles triangle)
 b $y = 7$ (angles in an isosceles triangle)
 c $x = 60$ (angles in an equilateral triangle)
 d $l = 4.8$ (angles in an isosceles triangle)
 e $a = 17.2$, $b = 17.2$ (angles in an equilateral triangle)
 f $r = 15$ (angles in an isosceles triangle)

Answers

7 No, the sum of all angles cannot be greater than 180° .

8 C

9	Equilateral triangle	3	3
	Isosceles triangle	1	0
	Scalene triangle	0	0

Mental skills 4A

- 2 a 70% b 66% c 45% d 88%
 e 75% f 75% g 80% h 80%
 i 55% j 35% k 30% l 80%
 m 90% n 45% o 52%
- 4 a 25% b 68% c 17% d 60%
 e 10% f 33.3% g 59% h 70.2%
 i 84% j 70% k 42.8% l 5.5%
 m 91% n 78.25% o 31.4%

Exercise 4-05

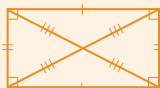
- 1 a convex b convex c non-convex
 d non-convex e convex f convex
- 2 a square b trapezium c kite d parallelogram
- 3 a square, rectangle
 b trapezium
 c square, rhombus
 d parallelogram, rectangle, square, rhombus
 e parallelogram, rectangle, square, rhombus
 f kite, square, rhombus
- 4 rhombus

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Rectangle	2	1
Parallelogram	0	0
Trapezium	0	0
Rhombus	0	1
Square	4	4
Kite	1	0

- 6 a trapezium b rhombus

7 B

8 a



b



d



- 9 a False b True c True d True
 e False f True g False

Exercise 4-06

- 1 a parallel
 b equal; angles; right angles
 c opposite; sides; equal; bisect
 d sides; sides; parallel; opposite; equal; right; bisect
 e equal; parallel; right angles; equal; bisect
 f equal; right angles; sides; equal; right angles
- 2 a Parallelogram, rectangle, rhombus, square
 b Square, rhombus, kite
 c Parallelogram, rectangle, rhombus, square
 d Trapezium
 e Parallelogram, rectangle, rhombus, square
 f Parallelogram, rectangle, rhombus, square
 g Parallelogram, rectangle, trapezium
 h Parallelogram, rectangle, square
 i Rectangle, square
 j Rhombus, square
- 3 $a + 70 = 180$ (co-interior angles on parallel lines);
 $a = 110$, $b = 70$ (opposite angles of a parallelogram)
- 4 a $m = 123$ b $a = 154$, $b = 26$
 c $x = 156$, $y = 156$ d $l = 45$
 e $a = 49$, $b = 90$ f $x = 87$, $y = 93$
 g $m = 75$, $n = 75$ h $a = 147$, $b = 33$
 i $a = 5$, $b = 7$ j $a = 4$, $c = 7$
 k $a = 70$, $b = 92$
- 5 B 6 D 7 A 8 A

Exercise 4-07

- 1 a $a = 45$ b $b = 60$ c $c = 50$
 d $d = 60$ e $e = 18$ f $f = 33$
 g $g = 120$ h $h = 80$ i $i = 45$
- 2 a $a = 127$ b $b = 140$ c $c = 60$
 d $d = 28$ e $e = 100$ f $f = 120$
 g $g = 50$ h $h = 136$ i $i = 120$
- 3 a $a = 89$ b $b = 62$ c $c = 109$
 d $d = 90$ e $e = 33$ f $f = 245$
 g $g = 90$ h $h = 90$ i $i = 140$
- 4 C 5 A 6 A 7 C

Exercise 4-08

- 1 a hexagon, irregular, convex
 b quadrilateral, irregular, convex
 c pentagon, irregular, non-convex
 d triangle, irregular, convex
 e decagon, regular, convex
 f heptagon, irregular, convex
 g octagon, regular, convex
 h nonagon, regular, non-convex

Polygon	Number of sides	Sum of angles inside polygon
hexagon	6	720°
heptagon	7	900°
octagon	8	1080°
nonagon	9	1260°
decagon	10	1440°

- 3 a 2340° b 3240° c 4140° d $17\ 640^\circ$
 4 a $a = 95$ b $b = 110$ c $d = 108$
 d $c = 160$ e $g = 140$ f $x = 90$
 g $e = 135$
 5 a 90° b 60° c 120° d 135°
 e 144° f 108° g 150°
 6 a 14 b 34 c 26 d 53 e 125

Mental skills 4B

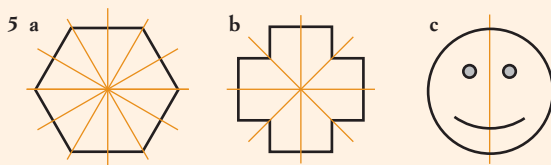
- 2 a $\frac{3}{4}$ b $\frac{7}{25}$ c $\frac{3}{10}$ d $\frac{7}{50}$ e $\frac{3}{50}$
 f $\frac{17}{20}$ g $\frac{8}{25}$ h $\frac{49}{100}$ i $\frac{14}{25}$ j $\frac{9}{10}$
 k $\frac{18}{25}$ l $\frac{13}{20}$ m $\frac{1}{5}$ n $\frac{6}{25}$ o $\frac{53}{100}$
 4 a $\frac{19}{25}$ b $\frac{1}{10}$ c $\frac{4}{5}$ d $\frac{9}{20}$ e $\frac{22}{25}$
 f $\frac{14}{25}$ g $\frac{3}{4}$ h $\frac{31}{100}$ i $\frac{17}{25}$ j $\frac{1}{20}$
 k $\frac{3}{5}$ l $\frac{27}{50}$ m $\frac{3}{50}$ n $\frac{49}{100}$ o $\frac{41}{50}$

Power plus

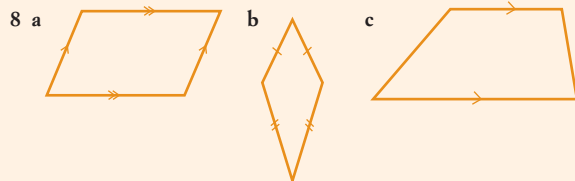
- 1 a $x = 121$ b $x = 88$ c $x = 96$
 d $x = 110$ e $x = 6$ f $x = 34$
 g $x = 67$ h $x = 155$ i $x = 117$
 j $x = 108$ k $x = 30$ l $x = 140$

Chapter 4 revision

- 1 a obtuse b acute c reflex
 2 a $x = 50$ b $a = 50, b = 130$ c $y = 5$
 d $n = 72$ e $w = 130$
 3 a corresponding b alternate c co-interior
 4 a $m = 100$ (alternate angles on parallel lines)
 b $x = 95$ (co-interior angles on parallel lines)
 c $x = 80$ (straight angle on parallel lines);
 $y = 80$ (corresponding angles on parallel lines)
 d $m = 88$ (alternate angles on parallel lines);
 $p = 88$ (corresponding angles on parallel lines)



- 6 a yes, 6 b yes, 4 c no
 7 a scalene, right-angled b scalene, obtuse-angled
 c isosceles, obtuse-angled d equilateral, acute-angled
 e isosceles, acute-angled f isosceles, acute-angled
 g isosceles, right-angled h scalene, obtuse-angled
 i isosceles, obtuse-angled



- 9 a rhombus, square b kite c rectangle, square
 d parallelogram, rhombus, rectangle, square
 e parallelogram, rhombus, rectangle, square
 10 a $a = 50$ b $a = 42$ c $y = 30$
 d $y = 53$ e $x = 34$ f $m = 106$
 11 a $m = 63$ b $x = 135$ c $y = 123$
 d $d = 70$ e $a = b = 115$ f $a = 56, b = 68$
 g $a = 48, b = 66$
 12 a $x = 42$ b $m = 66$ c $y = 130$

Chapter 5

SkillCheck

- 1 a 200 b 3.5 c 2.5
 d 4.6 e 4000 f 5.2
 g 0.6 h 8.2 i 400
 2 a 12 cm b 19.2 m c 86 m
 d 112 mm e 21 cm f 13 cm
 3 a 4 m^2 b 30 cm^2 c 56 m^2
 d 91.84 cm^2 e 58 mm^2 f 140 m^2

Exercise 5-01

- 1 a 84 mm b 33.2 cm c 69.3 mm
 d 158 cm^2 e 43.3 m f 12.4 km
 2 a 31.5 m b 34 m c 45 cm
 d 60 km e 66 cm f 64 m
 g 164 mm h 58 cm i 92 m
 3 a 12 cm b 34 m c 15 cm d 22 m
 4 A 5 7 m 6 4.5 m 7 4 cm
 8 a 120 mm b 180 mm c 200 mm d 174 mm
 9 16 cm 10 C 11 D

Exercise 5-02

- 1 a m^2 b cm^2 c m^2
 d km^2 e cm^2 f cm^2
 2 A

Answers

- 3 a 1 b 7.5 c 0.45
d 0.034 e 0.06 f 43.2
g 0.000 95 h 0.76 i 0.048
j 300 000 k 8.5 l 6 900 000
m 5400 n 7 000 000 o 4800
4 D 5 3 002 800 ha 6 1348.05 km²
7 a 0.875 km² b 87.5 ha
8 a 7 600 000 b 70 000 000 c 8 000 000
d 8.5 e 69 000 f 152 000
g 0.85 h 49
9 32.9% 10 a 8.81 km² b 6.2

Exercise 5-03

- 1 a 24 m² b 36 cm² c 48 cm²
d 3 cm² e 53.3 mm² f 37.21 m²
g 54.4 cm² h 85.56 km² i 340 mm²
j 6 m² k 84 cm² l 136 m²
2 a 10 800 cm² b 42 140 mm² c 547 200 m²
d 552 mm² e 243.6 mm² f 528 000 m²
3 D 4 5 cm 5 3 m 6 7 cm
7 Teacher to check. 8 60 m
9 a 15.6 cm² b 66.4 mm² c 8.4 m² d 30 km²

Exercise 5-04

- 1 a 150 m² b 51 m² c 88 m²
d 156 m² e 950 mm² f 36 cm²
g 189 m² h 1000 cm² i 71 m²
j 201.19 km² k 88 000 cm² l 280 mm²
m 20 m² 2 C 3 B

Mental skills 5A

- 2 a 0.7 b 0.6 c 0.07
d 0.45 e 0.98 f 0.4
4 a 0.61 b 0.04 c 0.9
d 0.38 e 0.271 f 0.007

Exercise 5-05

- 1 a 148 mm² b 1080 m² c 97.6 m²
d 33.02 mm² e 472 cm² f 30 cm²
g 862.5 mm² h 162 cm² i 680 cm²
2 a 8200 m² b 6560 m²
3 630 000 mm² 4 D
5 a 546 cm² b 1772 cm² 6 D

Exercise 5-06

- 1 a 1.5 cm² b 850 mm² c 8.16 m² d 210 cm²
2 a 240 m² b 80 cm² c 315 mm²
d 1344 m² e 1.8 m²
3 B

- 4 a 300 mm² b 172.5 mm² c 90 mm²
d 360 mm² e 120 mm² f 72 mm²
g 66.5 mm² h 510 mm² i 112 mm²

Exercise 5-07

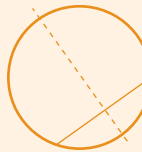
- 1 Teacher to check.
2 a diameter b segment c sector
d arc e radius f chord
g circumference h tangent
3 a radius b quadrant c tangent
d diameter e chord f arc
g sector h circumference i segment
4 B 5 B 6 Teacher to check.
7 a i 1 ii 0



- b i 1 ii 0



- c i 1 ii 0



- d i 2 ii 2



- e i 1 ii 0



- f i 1 ii 0



- 8 Teacher to check.

Exercise 5-08

- 1 a 12 cm b 24 m c 9 cm d 18 cm
2 a 12.57 cm b 25.13 m c 9.42 cm d 18.85 cm
3 a 4π cm b 8π m c 3π cm d 6π cm
4 a 36 m b 60 mm c 42 cm d 12 mm
5 a 37.7 m b 62.8 mm c 44.0 cm d 12.6 mm
6 a 12π m b 20π mm c 14π cm d 4π mm
7 4.40 m 8 a 188.5 cm b 478
9 40 023.9 km 10 100.5 mm 11 236 mm
12 a 6.28 cm b 21.99 cm c 18.85 cm d 31.42 cm
13 B

- 14 a 25.7 cm b 10.7 cm c 43.2 m
 d 142.8 mm e 22.3 cm f 384.1 cm
 g 100.7 mm h 18.0 cm i 57.1 cm
 j 40.2 m k 51.4 cm l 111.4 mm
- 15 a 210.5 m b $(154 + 18\pi)$ m
- 16 16 cm 17 Ali, 25.1 m

Mental skills 5B

- 2 a $\frac{3}{5}$ b $\frac{1}{4}$ c 0.105
 d 3.68 e $\frac{4}{10}$ f $\frac{5}{6}$
 g 2.817 h 0.503 i $\frac{1}{5}$
- 3 a 0.082, 0.81, 0.821 b 3.5, 3.51, 3.513, 3.55
 c $\frac{1}{6}, \frac{2}{5}, \frac{2}{3}$ d 0.007, 0.07, 0.7, 0.707
 e $\frac{2}{10}, \frac{3}{8}, \frac{1}{2}$ f $\frac{1}{4}, \frac{4}{10}, \frac{3}{5}$
- 5 a 0.25, $\frac{1}{5}, \frac{1}{6}, 16\%$ b 0.4, $\frac{1}{3}, 0.28, 27\%$
 c 50%, $\frac{1}{8}, 6\%, 0.05$ d $\frac{3}{4}, 0.639, 55\%, \frac{2}{5}$
 e 69%, $\frac{2}{3}, 0.609, 0.6$ f 22.5%, $\frac{2}{9}, 17\%, 0.105$

Exercise 5-09

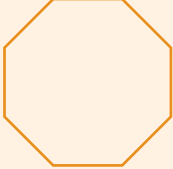
- 1 a 28.3 cm² b 452.4 mm² c 0.2 m²
 d 19.6 cm² e 3421.2 mm²
- 2 a 9π cm² b 144π mm² c 0.0625π m²
 d 6.25π cm² e 1089π mm²
- 3 a 13.85 cm² b 38.48 cm² c 78.54 cm²
- 4 a 4.41π cm² b 12.25π cm² c 25π cm²
- 5 539 m² 6 3690.67 cm² 7 196π cm²
- 8 D 9 6.2 m² 10 12 cm 11 A
- 12 a 12.6 cm² b 981.7 mm² c 5.3 cm²
 d 19.2 cm² e 235.6 mm² f 0.2 m²
 g 641.4 mm² h 351.9 m² i 1954.8 mm²
- 13 A
- 14 a 160.96 cm² b 221.54 m² c 85.81 mm²
 d 46.27 cm² e 21.46 cm² f 16.08 cm²
- 15 a i 153.9 m² ii 95.0 m² iii 58.9 m²
 b i 314.2 m² ii 254.5 m² iii 59.7 m²
- 16 a 9168 m² b 7326 m²
 c 1842 m² d \$44 668.50

Exercise 5-10

- 1 a m³ b cm³ c cm³ d m³ e cm³ f m³
- 2 C
- 3 a 5 000 000 b 1 600 000 c 2000
 d 4 000 000 000 e 0.006 f 8 200 000
 g 7 000 000 h 0.096 i 4
 j 0.16 k 0.25 l 12 000
 m 180 000 n 0.2
- 4 0.272 m³ 5 D 6 A
- 7 a E b G c F d C
 e A f B g D
- 8 2 460 000 mm³

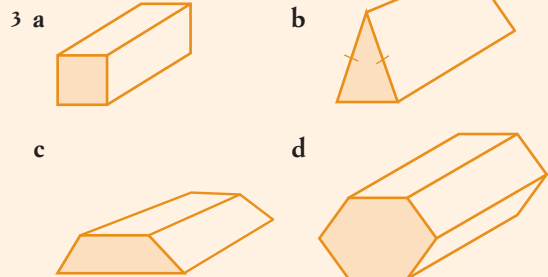
Exercise 5-11

- 1 a Yes b No c No d Yes e No f Yes

- 2 a i  ii octagonal prism

- b i  ii kite prism

- c i  ii pentagonal prism



- 4 a 210.8 m³ b 4.096 875 m³ c 486.42 cm³
 5 a 135 cm³ b 64 m³ c 6120 mm³
 d 1.728 m³ e 21 952 mm³ f 258.06 cm³
 6 a 90 cm³ b 15 400 mm³ c 7.8125 m³
 7 a 22 500 mm³ b 928 cm³
 c 13 200 cm³ d 288 cm³
- 8 a 0.306 m³ b 0.306 m³ Teacher to check.
- 9 A 10 5 m 11 7 m
- 12 a 220 cm³ b 11 220 mm³ c 24 000 m³
 d 29 450 mm³ e 82 m³
- 13 B 14 Teacher to check.

Exercise 5-12

- 1 C
- 2 a 1133.0 cm³ b 17 671.5 mm³ c 1.8 m³
 d 1606.0 cm³ e 17 583.1 cm³ f 301 592.9 mm³
 g 499.6 m³
- 3 a 4.25 cm b 4.75 cm c 678.58 cm³
- 4 B 5 8 cm
- 6 a 4021.2 m³ b 732 944.7 m³ c 1005.3 mm³
 d 786.1 m³ e 380.8 m³ f 3 685 088.2 mm³
- 7 4 m

Exercise 5-13

- 1 a mL b kL c L d mL e L f kL
 2 a 5 b 3400 c 1600
 d 4 e 2.98 f 7 100 000
 g 0.875 h 8200 i 800
 j 1.85 k 5400 l 900
 m 6000 n 35 o 1 200 000
 3 A 4 A
 5 a 121 500 cm³ b 121.5 L
 6 424 mL 7 1 800 000 L
 8 a 70 200 000 cm³ b 70 200 L
 9 a 24.7 L b 10
 10 Teacher to check. 11 1728 L
 12 a 1.32 m³ b 1320 L
 13 a 18.850 m³ b 18 849.6 L
 14 a 0.72 L b 17.28 L
 15 Teacher to check.

Power plus

- 1 a 35.4 cm b 1228.3 mm c 57.1 m
 2 a 7.9 m² b 14.4 m
 3 a 2.09 cm² c 1.73 cm² d 0.36 cm² e 2.81 cm²
 4 a 4.4 m² b 47.1 cm² c 39.3 m²

Chapter 5 revision

- 1 a 76 mm b 256 cm c 31.4 km
 2 a 2 b 70 000 c 0.8
 d 30 000 000 e 7 f 7.7
 g 0.0007 h 72 000 i 0.058
 3 a 56.7 m² b 51.6 mm² c 345 cm²
 d 49 km² e 69.3 m² f 775.5 cm²
 4 a 450 mm² b 133 cm² c 6.3 m² d 86 cm²
 5 a 288.51 mm² b 17.16 m² c 4126.5 cm²
 6 a 189 m² b 1560 mm² c 418 cm²
 7 a segment b radius c arc d centre
 e diameter f chord g sector h tangent
 8 a i 1 ii 0 b i 2 ii 2
 c i 1 ii 0 d i 1 ii 0
 9 a 12.6 cm b 62.8 mm c 44.0 cm d 88.0 cm
 10 a 21.42 cm b 40.85 cm c 26.85 cm d 56.55 cm
 11 a i 4 π cm² ii 12.6 cm²
 b i 100 π mm² ii 314.2 mm²
 c i 49 π cm² ii 153.9 cm²
 d i 196 π mm² ii 615.8 mm²
 12 a 28.27 cm² b 116.55 cm²
 c 37.70 cm² d 208.23 cm²
 13 a 8 200 000 b 3400 c 2
 d 1 000 000 000 e 8 000 000 000 f 45
 14 a 391.392 cm³ b 2200 mm³ c 672 m³
 d 35 200 mm³ e 25 350 cm³

- 15 a 452.39 mm³ b 543.43 m³ c 154 893.81 cm³
 16 a 3000 b 2.5 c 6500
 d 0.0072 e 0.12 f 0.035
 17 a 5.7 L b 4000 L c 2.5 L

Chapter 6

SkillCheck

- 1 a 3 b 8 c 20 d 4
 e 6 f 3 g 40 h 15
 2 a $\frac{17}{100}$ b $\frac{3}{100}$ c $\frac{51}{100}$ d $\frac{63}{100}$ e $\frac{89}{100}$
 3 a 70% b 5% c 61%
 d 29% e 88% f 93%
 4 a 13% b 73% c 8%
 5 a 87% b 27% c 92%
 6 65% 7 1, 2, 4, 5, 10, 20, 25, 50, 100
 8 a \$40 b 6 kg c \$5
 d 10 kg e \$3 f 2 kg

Exercise 6-01

- 1 a $1\frac{1}{2}$ b $3\frac{2}{5}$ c $2\frac{1}{4}$
 d $2\frac{2}{5}$ e 5 f $4\frac{3}{11}$
 2 a $\frac{7}{2}$ b $\frac{13}{3}$ c $\frac{23}{4}$ d $\frac{23}{5}$
 e $\frac{19}{5}$ f $\frac{65}{9}$ g $\frac{71}{7}$ h $\frac{93}{10}$
 3 Teacher to check. 4 Mixed numeral.
 5 a $\frac{1}{2}$ b $\frac{3}{4}$ c $\frac{3}{8}$ d $2\frac{7}{10}$ e $\frac{11}{15}$ f $\frac{5}{12}$
 6 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{6}{13}$ d $\frac{3}{4}$ e $\frac{3}{5}$ f $\frac{4}{7}$
 g $\frac{2}{3}$ h $\frac{3}{5}$ i $\frac{4}{7}$ j $\frac{4}{7}$ k $\frac{3}{5}$ l $\frac{9}{10}$
 7 A
 8 a $1\frac{1}{2}$ b 3 c $4\frac{1}{2}$ d $1\frac{4}{5}$

Exercise 6-02

- 1 a $\frac{4}{5}$ b $\frac{1}{2}$ c $\frac{5}{8}$ d $1\frac{3}{8}$
 2 a $\frac{7}{10}$ b $\frac{5}{21}$ c $\frac{17}{20}$ d $\frac{1}{4}$
 e $\frac{13}{24}$ f $\frac{1}{30}$ g $1\frac{5}{18}$ h $\frac{11}{24}$
 i $1\frac{1}{8}$ j $\frac{1}{7}$ k $3\frac{2}{9}$ l $2\frac{3}{5}$
 3 $\frac{17}{40}$ 4 D 5 a $\frac{5}{12}$ b $\frac{17}{30}$
 6 $1\frac{1}{2}$ 7 C 8 A
 9 a $1\frac{9}{10}$ b $3\frac{5}{8}$ c $16\frac{17}{24}$ d $5\frac{1}{45}$
 e $1\frac{21}{40}$ f $1\frac{19}{20}$ g $\frac{5}{6}$ h $1\frac{7}{10}$
 10 a 2, $2\frac{1}{2}$, $3\frac{1}{2}$, 1 b 1, $\frac{2}{6}$, $1\frac{1}{3}$, 0, $\frac{1}{2}$

Exercise 6-03

- 1 a $\frac{1}{3}$ b $\frac{10}{21}$ c $\frac{1}{64}$ d $\frac{3}{4}$
 e $\frac{9}{20}$ f $4\frac{1}{2}$ g $12\frac{7}{20}$ h $3\frac{1}{9}$
 2 D
 3 a 1 (or -1) b 0 c 1 d $\frac{1}{3}$

- 4 a $\frac{3}{14}$ b $\frac{8}{15}$ c $\frac{2}{3}$ d $1\frac{1}{2}$
 e 16 f 9 g $\frac{1}{8}$ h $\frac{1}{15}$
 i $3\frac{1}{2}$ j $3\frac{2}{11}$ k $2\frac{4}{5}$ l $3\frac{9}{22}$

- 5 $\frac{3}{32}$
 6 a $\frac{7}{8}$ b $\frac{2}{3}$ c $\frac{2}{21}$ d $\frac{7}{10}$

- 7 a Increase: dividing by a proper fraction is the same as multiplying by its reciprocal, an improper fraction, whose value is greater than 1, so the number will increase.
 b Increase: multiplying by an improper fraction, whose value is greater than 1, will make the number increase.

Exercise 6-04

- 1 a $\frac{3}{5}$ b $\frac{3}{4}$ c $\frac{31}{100}$ d $\frac{2}{25}$
 e $\frac{3}{10}$ f $\frac{17}{20}$ g $\frac{99}{100}$ h $\frac{3}{100}$
 i $1\frac{3}{5}$ j $1\frac{7}{20}$ k $\frac{1}{4}$ l $2\frac{1}{2}$
 2 C
 3 a 0.18 b 0.82 c 0.02 d 0.5
 e 1.2 f 0.511 g 0.79 h 0.125
 i 0.163 j 0.04 k 0.187 l 0.0525
 4 a 17% b 70% c 26%
 d 55% e 62.5% f 96%
 g $66\frac{2}{3}\%$ h 125% i 140%
 j 67.5% k 6.25% l $44\frac{4}{5}\%$
 5 a 38% b 55% c 96% d 62.5%
 e 8% f 5.4% g 60% h 0.3%
 i 190% j 40.5% k 126% l 11.4%
 6 a $\frac{13}{20}$, 65% b $\frac{3}{5}$, 60% c $\frac{1}{5}$, 0.2
 d $\frac{21}{25}$, 0.84 e 0.5, 50% f 0.125, 12.5%
 g $\frac{9}{25}$, 0.36 h 0.625, 62.5% i $\frac{73}{100}$, 73%
 j $0.\dot{3}$, $33\frac{1}{3}\%$ k $\frac{2}{3}$, 0.6
 7 a 75% b $\frac{4}{25}$ c $\frac{2}{3}$
 d 18% e $\frac{1}{4}$ f 55%
 g $\frac{19}{20}$ h 60% i $\frac{1}{6}$
 8 a 0.75, 78%, $\frac{4}{5}$, $\frac{9}{11}$ b 22%, $\frac{1}{4}$, 0.29, $\frac{7}{20}$
 c 57%, $\frac{3}{5}$, 0.605, 0.62
 9 C
 10 a 0.47, $\frac{9}{20}$, 43%, $\frac{2}{5}$ b 0.88, 86%, $\frac{21}{25}$, 0.08
 c $\frac{19}{20}$, 91%, 0.905, $\frac{9}{10}$

Mental skills 6A

- 2 a 19 b \$7.50 c 87.5 d \$20.20
 e \$3.76 f 40 g \$0.925 h 89.6
 i \$270 j \$0.38 k \$152.76 l \$7.25
 m 315.4 n \$1.07 o 42.6 p \$2431.76
 4 a 10 b 124 c \$490 d \$1.72
 e 14.4 f \$2540 g 78 h \$1.16
 i \$9 j \$16.80 k \$920 l 64
 6 a 100 b \$0.60 c 2.5 d \$1.35
 e \$1.84 f \$4.20 g 40 h 6.5
 i \$0.48 j \$6.90 k \$3.60 l 42

Exercise 6-05

- 1 a 24 b 7 c 4
 d 10 e 42 f 10
 g 750 m h 8 h i 400 mL
 j 250 kg k 10 months l 35 min
 2 C
 3 a \$55 b \$5 c 19.5 L
 d 126 cm e \$67.50 f 285 g
 g 473.76 m h 286.72 km i 152.29 mL
 j \$15.62 k \$89.68 l 45.539 kg
 m 104.5 n 453 o \$2613
 p 126 m q 595 g r \$460
 4 0.675 kg 5 a \$105 b \$315 6 234
 7 a 96 cm b 350 kg
 c 144 min d 75 mL
 e 108 kg or 108 000 g f 900 m
 g 84 h h 30 min
 i 27 000 mg or 27 g
 8 63 9 \$0.86 10 273 11 D
 12 17 13 24 318 14 1400
 15 a \$90 b \$500

Exercise 6-06

- 1 a $\frac{1}{2}$ b $\frac{19}{25}$ c $\frac{87}{100}$
 d $\frac{2}{3}$ e $\frac{1}{4}$ f $\frac{3}{8}$
 2 a 50% b 76% c 87%
 d $66\frac{2}{3}\%$ e 25% f $37\frac{1}{2}\%$
 3 62.5% 4 a $\frac{2}{5}$ b 36%
 5 No 6 a $\frac{172}{189}$ b 9%
 7 a 32% b $\frac{9}{67}$ c 13.4%
 8 5%
 9 a i \$169 ii \$205 iii \$249
 b i 11.8% ii 12.7% iii 12.0%
 c ii \$179 oven
 10 A
 11 a $\frac{1}{12}$ b $\frac{1}{4}$ c $\frac{7}{10}$ d $\frac{23}{100}$
 e $\frac{1}{8}$ f $\frac{1}{6}$ g $\frac{3}{20}$ h $\frac{19}{70}$
 i $\frac{1}{8}$ j $\frac{3}{8}$ k $\frac{8}{15}$ l $\frac{4}{35}$
 12 a $8\frac{1}{3}\%$ b 25% c 70% d 23%
 e $12\frac{1}{2}\%$ f $1616\frac{2}{3}\%$ g 15% h $2727\frac{1}{7}\%$
 i $12\frac{1}{2}\%$ j $37\frac{1}{2}\%$ k $53\frac{1}{3}\%$ l $11\frac{3}{7}\%$
 13 C
 14 a Broncos 124.9%, Bulldogs 112.0%, Cowboys 84.7%,
 Dragons 81.3%, Eels 115.6%, Knights 86.9%,
 Panthers 76.1%, Raiders 86.9%, Roosters 61.8%,
 Sea Eagles 184.7%, Sharks 117.6%, Storm 230.9%,
 Tigers 77.6%, Titans 79.9%, Warriors 151.5%
 b Storm, Sea Eagles, Broncos, Storm

Answers

Exercise 6-07

- 1 a \$157.50 b 480 c 73.2 km
d \$2650 e 152 kg f 13.3 L
2 a \$176 b 102 c 101.2 kg
d \$243.75 e 1240 L f \$1522.90
3 \$232.20 4 a \$405 b \$3105
5 \$1024.10 6 171.36 cm 7 D
8 \$18 691.50 9 \$145.20 10 798
11 \$1 160 000 12 B 13 \$80.10
14 a \$540 b \$378 c \$92 d 19.6%
15 650 16 \$423.30 17 510
18 Less than the original price, \$673.20

Exercise 6-08

- 1 a 8 kg b 45 mm c 6 L d \$250
2 a 18 m b 9 g c 3 h
d 1 L e \$20 f 12 km
g 5 mL h 6 days i 75 kg
3 a 6 t b 4 h c 30 sheep d \$9
4 a 7 b \$40 c 48 min
d \$3.60 e 12 cm f 80 mL
5 B
6 a \$9 b 5 kg c 20 kL
7 a \$100 b 154 c \$200
d 42 e \$270 f 54
8 a \$279 b 96 c \$70
d 57 e \$300 f 500

Mental skills 6B

- 2 a \$308 b 60 c 30 d \$450
e 96 f \$90 g \$60.50 h 150
i 210 j \$84 k \$300 l 98
4 a \$360 b 40 c 88 d \$22.50
e 450 f \$63 g \$80 h 50
i \$50.40 j \$87.50 k 225 l \$105

Exercise 6-09

- 1 a \$700 b \$560 c \$4600
d \$1900 e 400 cm f 280 kg
g 3 m h 150 min i \$400
j \$360 k 400 kg l 110 mm
2 \$1200 3 79 800 4 \$82 176
5 \$15 250 6 D 7 24 000
8 \$3000 9 \$300 10 \$435

Exercise 6-10

- 1 a i \$18 700 ii Loss
b i \$543 000 ii Profit
c i \$799 ii Loss
d i \$6800 ii Profit

- 2 a i \$17 ii 20%
b i \$185 ii 44.6%
c i \$1.45 ii 52.7%
3 a i \$5 ii 26.3%
b i \$1250 ii 46.3%
c i \$21 ii 26.6%
4 a \$1000 b 40%
5 a \$1800 b 10.6%
6 a \$234 b \$71.40 c \$468
d \$1267 e \$6.25 f \$16.20
7 23.31%
8 44.4%
9 a \$2090, \$22 990 b \$181, \$1991
c \$0.30, \$3.30 d \$14.50, \$159.50
e \$18, \$198 f \$2.70, \$29.70
10 a \$72, \$720 b \$18, \$180 c \$84, \$840
d \$1.70, \$17 e \$3.15, \$31.50 f \$1.52, \$15.20

Exercise 6-11

- 1 27 2 81.25% 3 80% 4 \$41.25
5 60% 6 234 points 7 348 8 37.5%
9 40% 10 12.5% 11 \$8625 12 11.4%
13 \$172.50 14 C

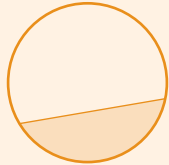
Power plus

- 1 \$96.80 2 a \$1627.50 b \$1765.84
3 a \$499 200 b \$519 168 c \$583 993.39
4 a Less b 20%
5 a \$187 000 b \$158 950 c \$97 615.17
6 a Less b 50%

Chapter 6 revision

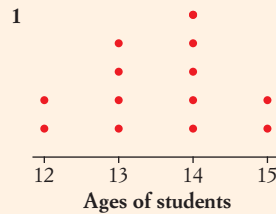
- 1 a $\frac{3}{4}$ b $\frac{7}{12}$ c $\frac{7}{9}$
2 a $3\frac{3}{4}$ b $4\frac{2}{5}$ c $4\frac{1}{3}$
3 a $\frac{9}{2}$ b $\frac{11}{3}$ c $\frac{34}{5}$
4 $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{1}{2}$
5 a $\frac{11}{15}$ b $\frac{1}{14}$ c $1\frac{5}{12}$
d $6\frac{7}{10}$ e $6\frac{1}{6}$ f $2\frac{7}{8}$
6 a $\frac{7}{18}$ b $\frac{8}{15}$ c $\frac{15}{28}$
d $\frac{27}{40}$ e $\frac{2}{5}$ f $3\frac{7}{12}$
7 a $\frac{27}{50}$ b $\frac{29}{100}$ c $\frac{13}{20}$ d $1\frac{57}{100}$
8 a 0.12 b 0.162 c 0.02 d 0.054
9 a 87.5% b 24% c $22\frac{2}{9}\%$ d 82.5%
10 a 56% b 61.3% c 70% d 4.8%
11 0.7, $0.725\frac{3}{4}$, 77%
12 a 6 b 16 c \$242
d 8 months e 108 L f 210 cm
13 a $\frac{17}{20}$ b 85%
14 48.1% 15 \$327 16 \$67 275
17 a 11 t b 12 cm c 9 min
18 \$147 111.11 19 20%
20 a \$8.70 b \$87.00 21 \$42.50

Mixed revision 2

- 1 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{3}{5}$
 2 a $y = 65$ b $k = 135, l = 45$
 c $x = 313$ d $q = 62$
 3 a 60 cm^2 b 110 cm^2 c 60 cm^2 d 880 cm^2
 4 a  b 
 c  d 
 5 a i scalene ii right-angled
 b i scalene ii acute-angled
 c i isosceles ii acute-angled
 d i equilateral ii acute-angled
 6 a $m = 60$ b $x = 50$ c $k = 65$ d $m = 60$
 7 a $\frac{3}{2}$ b $\frac{7}{4}$ c $\frac{22}{5}$
 8 a i $\frac{1}{4}$ ii 0.25 b i $\frac{2}{5}$ ii 0.4
 c i $\frac{17}{20}$ ii 0.85 d i $\frac{1}{8}$ ii 0.125
 9 a $d = 78$ b $f = 112$ c $r = 72$
 10 a 34% b 75% c 21.5% d 62.5%
 11 a parallelogram b rectangle c square d kite
 12 a 248.62 m^2 b 22.68 cm^2 c 90.2 m^2
 13 a $5\frac{1}{2}$ b $6\frac{1}{3}$ c $3\frac{1}{2}$
 14 a $1\frac{1}{5}$ b $\frac{11}{15}$ c $\frac{7}{20}$ d $4\frac{1}{10}$
 e $1\frac{1}{3}$ f $\frac{3}{10}$ g $\frac{3}{5}$ h $1\frac{3}{5}$
 15 a 137 m^2 b 112.5 cm^2
 16 a 31.42 cm b 21.99 cm
 17 a 78.54 cm^2 b 38.48 cm^2
 18 a $x = 130$ b $m = 50$ c $n = 35$
 d $p = 105$ e $k = 40$ f $x = 90$
 19 \$2.30 20 26.928 L
 21 a 4752 cm^3 b 640 m^3
 22 a $36\,000 \text{ cm}^2$ b 8.75 cm^2
 23 $\frac{3}{5}$, 63%, 0.67, $\frac{6}{7}$
 24 a true b false c true d false
 25 a \$0.30 b 528 kg c \$4.80 d 15 min
 26 a 40% b 35% c 50%
 27 3281
 28 a 15.42 cm b 91.42 mm c 80.55 cm
 29 a 14.14 cm^2 b 557.08 mm^2 c 339.29 cm^2
 30 a i 540° ii 108°
 b i 900° ii 128°
 c i 1440° ii 144°
 31 \$680 32 a \$499 b 45.4%
 33 a 62.1 m^2 b 55.2 m^2
 34 a 942.5 cm^3 b $14\,726.2 \text{ mm}^3$

Chapter 7

SkillCheck



2 a

Stem	Leaf
3	0
4	7 8 8 9
5	4 9 9
6	3 4 8 8 8

b 30

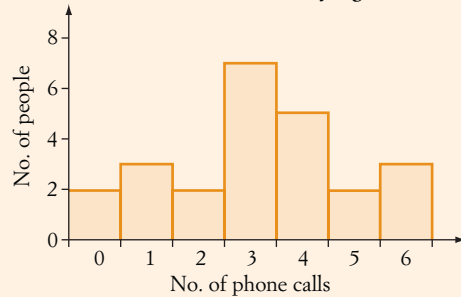
3 a 53

b 10

c 240

4 a

Calls made on Saturday night



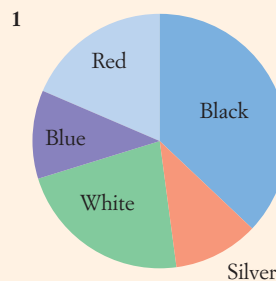
b 24

c 3

d 0, 2 and 5

e 6

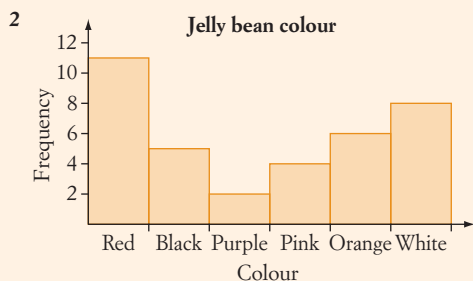
Exercise 7-01



Day of sport training

Sector graph with angles: Mon = 36° , Tues = 156° , Wed = 48° , Thurs = 84° , Fri = 36°

Answers



3 Hours of sleep

6	7	8	9	10	11

6 = 0.75 cm, 7 = 2.25 cm, 8 = 2.75 cm,
9 = 1.5 cm, 10 = 1.25 cm, 11 = 0.5 cm

4 D

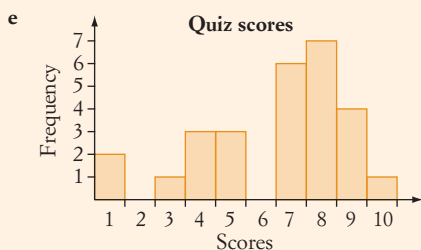
5 a

Score	Tally	Frequency
1		2
2		0
3		1
4		3
5		3
6		0
7		6
8		7
9		4
10		1
Total		27

b 10

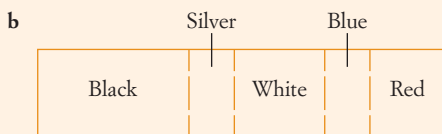
c 3

d 8



6 a

Colour	Frequency
Black (B)	10
Silver (S)	3
White (W)	6
Blue (U)	3
Red (R)	5
Total	27



Black

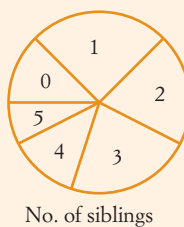
7 a

No. of siblings	Tally	Frequency
0		5
1		10
2		8
3		9
4		5
5		3
Total		40

b 5

c 1

d Sector graph with angles: 0 = 45°, 1 = 90°,
2 = 72°, 3 = 81°, 4 = 45°, 5 = 27°



Exercise 7-02

- 1 a N b C c N d N
e N f C g N h N
2 a D b C c D d D
e C f D g C h D
3 D
4 a C b C c ND d C
e ND f NC g ND h C

Exercise 7-03

- 1 a 1.58 b 1 and 2
2 a 6.75, 6 b 4.71, 2 c 4.875, 1 and 7
d 70.44, 96 e 47.25, no mode
3 See answers to question 2.
4 a Hyundai b spade c Korean
5 Data are categorical, not numerical.
6 a 7.71 b 7.0
c Mean, every score is involved in calculating it.
7 C
8 a Carrozza 26.33, Binns 33.33
b Binns family has higher scores, outlier of 93.
c Binns family has higher mean.
9 a 8Y 6.1; 8Z 6.1
b 8Z's marks are more spread out.
c 4
10 2940 11 a \$1812.50 b \$2750
12 Teacher to check.

Exercise 7-04

- 1 C
2 a i 8 ii 7 b i 47.5 ii 10

- c i 12 ii 4 d i 51 ii 99
e i 57 ii 0 f i 82.35 ii 37.4
- 3 a \$1.43 b median \$19.50, mode -\$25
c No. His winnings are small and his loss big. Not worth the effort.
- 4 a no mode b \$686 000
c \$771 845.45 d Teacher to check.
- 5 a Aldo 168.1, Peter 175.7
b Aldo 35, Peter 6
c 194
d i Aldo, as he is capable of a higher score.
ii Peter is more consistent.
- 6 C
- 7 a Daniel 29.68, Cristian 29.08
b Daniel 5, Cristian 4.5
c Cristian, he is consistent and fast.
- 8 6, 7, 8 or 9 9 Teacher to check.

Mental skills 7

- 2 a 88 b 167 c 314 d 85
e 129 f 121 g \$11.60 h \$6.75
i \$4.70 j \$30.05 k \$7.10 l \$32.90

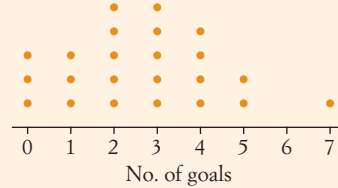
Exercise 7-05

- 1 a 34 b 2 c 2.1 d 2 e 8
2 a 24 b 11 c 8.8 d 5 e 9
- 3 a
- | Judge's score, x | Frequency, f | fx |
|--------------------|----------------|------------|
| 1 | 2 | 2 |
| 3 | 1 | 3 |
| 4 | 3 | 12 |
| 5 | 3 | 15 |
| 7 | 6 | 42 |
| 8 | 7 | 56 |
| 9 | 4 | 36 |
| 10 | 1 | 10 |
| Total | 27 | 176 |
- b 8 c 6.5 d 7 e 9
4 a 5 b 2 c 2.5 d 2.8
5 a 37 b 1.47 c 0.1 d 1.48 e 1.48
6 A
7 a 7 b 3 c 3 d 3.4
8 a 12 b 15 c 14.5

Exercise 7-06

- 1 a i 4 ii 11 iii 10 iv 9.8
b i 4 ii 24 iii 22 iv 22.3
c i 6 ii 36 iii 37 iv 37.6
d i 0.3 ii 1.8 iii 1.8 iv 1.8

2 a



- b 22 c 7 d 2.5 e 2.6
3 a 1 b 8 c 25 d 22.4 e 73% f C
4 a
-
- b 14 c 6 d 2 e 2.1 f 15
5 a

8 Hamper quiz marks



8 Yen quiz marks



- b 7 c 5 d 9

e

x	f	fx
0	1	0
1	1	1
2	2	4
3	2	6
4	1	4
5	4	20
6	3	18
7	3	21
8	1	8
9	1	9
Total	19	91

$$\bar{x} \approx 4.8$$

- f 8 Yen, marks clustered around higher scores
7 to 10.
- 6 a i 39 ii 33 iii 33 iv 30.1
b i 26 ii 94 iii 88 iv 86.3
c i 40 ii 131 iii 124.5 iv 121.45
d i 44 ii 17 iii 17 iv 22.1

7 a

Stem	Leaf
7	6
8	
9	2 5 7 8
10	1 1 5 5 6 8
11	2 4 7
12	4

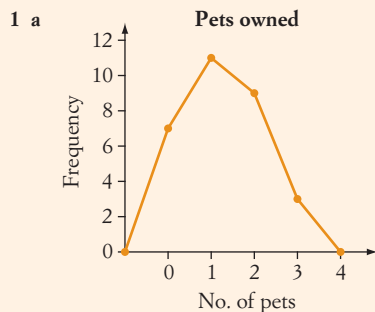
- b 76 c 101, 105 d 124
e 9, 10 f 66.7%

8 a

Stem	Leaf
13	7
14	5
15	4 5 6 7 8 8 9
16	0 0 1 3 3 4 4 5 6 7 7 7 9
17	1 5

- b 16 c 24 d 2
e 38 f 162
9 a 19 b 2
c Fuller's Ridge Rd, times are clustered around the lower scores.
d D
10 a 16 b 8 c 30
d 9, 13 and 30 e 30 f Narrabri Lions
g Wee Waa Reds, it has a range of 28 compared to Narrabri Lion's 26.

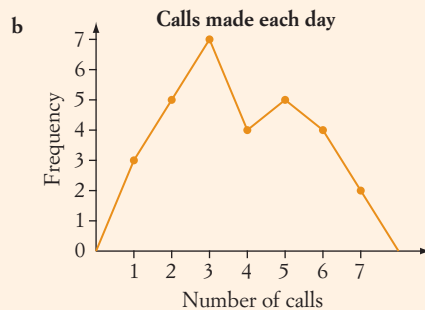
Exercise 7-07



- b 30 c 7 d 30% e 1 f 1.3

2 a

x	f
1	3
2	5
3	7
4	4
5	5
6	4
7	2
	30



- c 7 d 113 e 8 f 3 g 3.8
3 a 25 b 1 c 1 d 4

e

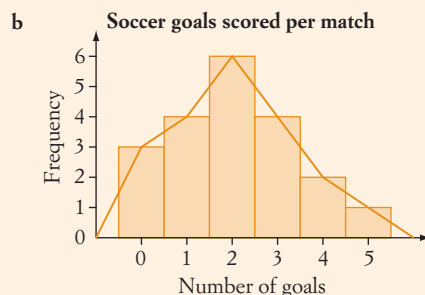
x	f	fx
1	13	13
2	5	10
3	3	9
4	1	4
5	3	15
	25	51

$$\bar{x} = 2.04$$

- f 1

4 a

x	f
0	3
1	4
2	6
3	4
4	2
5	1
	20



- c 20 d 5 e 2
f 2.05 g 7 h 15%
5 a 5, 6, 9, 12, 4, 8, 44 b 4 c 5
d 155 e 154.64 f 11
g 155 h 24

Exercise 7-08

- 1 a sample b census c sample d sample
e sample f census g census h census

- 2 a sample, survey
c census, observation
e sample, survey
g sample, measurement
- b sample, survey
d census, survey
f census, measurement
h census, survey
- 3 Teacher to check.
- 4 a Not random, students volunteered themselves for the survey rather than be chosen.
b Random.
c Not random, many people do not have their phone numbers listed in the book.
d Random.
e Not random, station focuses on rock music only, and listeners call in to vote rather than be randomly selected.
- 5 Teacher to check.

Exercise 7-11

- 1 a i 37 ii 49 iii 52.5 iv 55.3
b Mean, because there are no outliers.
- 2 a i 24 ii 32 iii 37 iv 36.4
b i increase ii no change
iii decrease iv decrease
- 3 a 30 b B
c Grades are categorical data, not quantitative (numerical).
- 4 a i \$427 000 ii \$91 500 iii \$135 000
b There is no mode, and the mean is affected by the outlier of \$484 000.
c Mean \$103 000, median \$90 000, yes.
- 5 a i 192.4 cm ii 193 cm
iii no mode iv 23
b i decrease ii no change
iii no change iv no change
- 6 a D b Sunjeet 32.8, Chris 33.7
c i Sunjeet ii Chris
d Sunjeet: his scores are consistently higher.

7 a

Forbes		Cobar
9 6 2 0	3	0 1 3 6 7
4	4	1 7 9 9
8 5 4	5	1 1 8
9 5 3 1	6	0 5
6 5 1	7	2 3 7 9
4 0 0	8	8
1 0	9	7

b

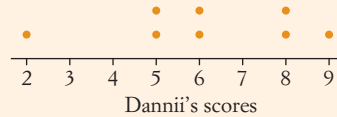
	Mean	Mode	Median	Range
Forbes	62.65	80	64	61
Cobar	56.2	49, 51	51	67

- c Forbes, it has a higher mean, mode and median.
d 97, mean

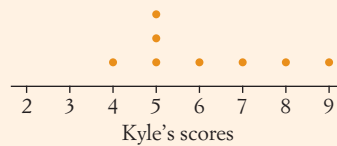
8 a

	Mean	Mode	Median	Range
Christina	20.9	19, 25	21.5	31
Mary	21.9	17, 20	20	21

- b Mary, lower range
d Mary, higher mean
- c Mary
e 4
- f i no ii yes



9 a



- b Dannii: range 7, median 6; Kyle: range 5, median 5.5
c Other than the outlier of 2, Dannii allocates higher marks, while Kyle's marks are spread out.

Power plus

- 1 a 74.2 b 77.2
c Increased by 3. d Does the same to the mean.
- 2 92% 3 No
- 4 a 40 b, c Teacher to check.
- 5 Yes. The sum of the four sisters' wages is \$8000, so each of the other two sisters earn much more.

Chapter 7 revision

1 a

x	f
1	2
2	4
3	7
4	5
5	1
6	1
	20

- b 3 c 5 d 10%
- e Sector graph with angles: 1 = 36°, 2 = 72°, 3 = 126°, 4 = 90°, 5 = 18°, 6 = 18°



No. of computers owned

- 2 a NC b C c ND
d NC e C f NC
- 3 a i 8.78 ii 6 b i 3.5 ii 1
c i 22.6 ii no mode

Answers

4 a i 10 ii 6 b i 3.5 ii 7

c i 23 ii 5

5 a 0, 7, 12, 12, 8, 5, 6, totals 24, 50

b 24 c 6 d 1

e 2 f 2.08

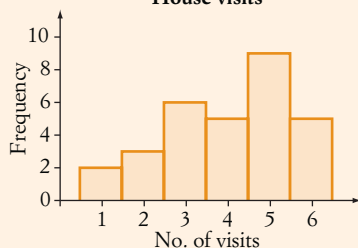
6 a i 4 ii 35 iii 33 iv 34.6

b i 35 ii 144 iii 148 iv 140.25

7 a

x	f
1	2
2	3
3	6
4	5
5	9
6	5
	30

b House visits



c 14 d 5 e 4

8 a census b sample c sample d census
9, 10 Teacher to check.

11 a Too broad, 'What is the area of your house in square metres?'

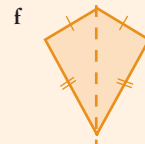
b Suggestive/biased towards republican answer, 'Should Australia become a republic?'

12 a 37 b 36 c 37.9

d mean e 11

f i increase ii no change

iii no change iv no change



2 a rectangle, square

b parallelogram, rectangle, rhombus, square

c parallelogram, rectangle, rhombus, square

d trapezium

e rhombus, square

f kite

3 a translation

b reflection or rotation

c rotation

d reflection

e rotation

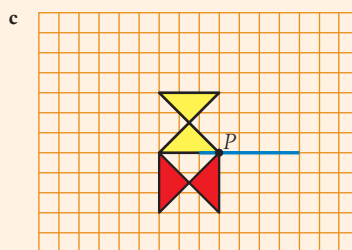
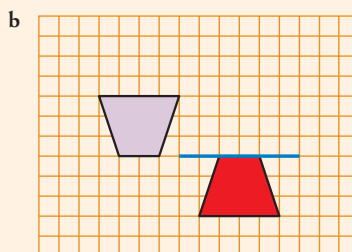
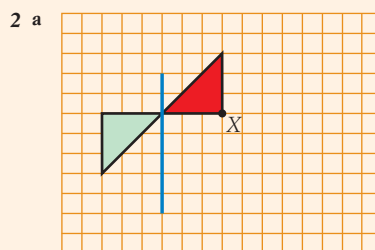
f translation

Exercise 8-01

1 a reflection or rotation, translation

b rotation, translation

c rotation, reflection



3 a i rotation, rotation

ii translation

b i reflection, reflection

ii rotation

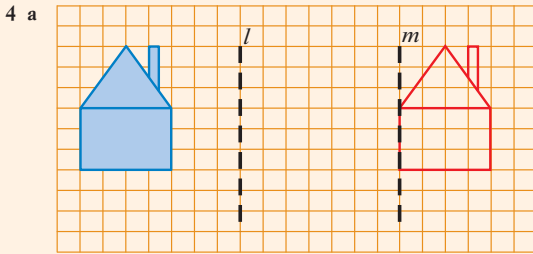
c i rotation, reflection

ii reflection

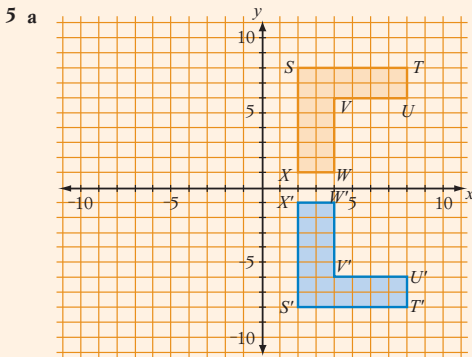
Chapter 8

SkillCheck





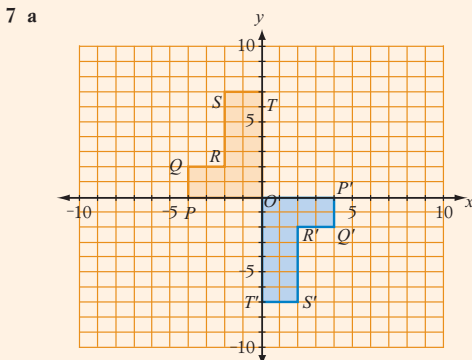
b translation



- b $S(2, 8) \rightarrow S'(2, -8)$
 $T(8, 8) \rightarrow T'(8, -8)$
 $U(8, 6) \rightarrow U'(8, -6)$
 $V(4, 6) \rightarrow V'(4, -6)$
 $W(4, 1) \rightarrow W'(4, -1)$
 $X(2, 1) \rightarrow X'(2, -1)$
 The x-coordinate of each vertex does not change while the y-coordinate changes sign.

6 a Rotation 180° clockwise about $S(0, 0)$

- b $T(3, 0) \rightarrow T'(-3, 0)$
 $U(3, -4) \rightarrow U'(-3, 4)$
 $V(5, -4) \rightarrow V'(-5, 4)$
 $W(5, 0) \rightarrow W'(-5, 0)$
 $X(8, 0) \rightarrow X'(-8, 0)$
 $Y(8, -7) \rightarrow Y'(-8, 7)$
 $Z(0, -7) \rightarrow Z'(0, 7)$



- b $Q(-4, 2) \rightarrow Q'(4, -2)$; The x- and y-coordinates both change sign.

8 a Translation 8 units right and 2 units down.

- b $W(-6, 8) \rightarrow W'(2, 6)$; $X(-2, 1) \rightarrow X'(6, -1)$;
 $Y(-6, 1) \rightarrow Y'(2, -1)$

The x-coordinate of each vertex increases by 8 units while the y-coordinate decreases by 2 units.

Exercise 8-02

1 A and H, B and K, E and L, F and P

2 a D and J b C c H

3 Other answers possible.

- a $ABCD \equiv JKLI$ b $\triangle MNO \equiv \triangle PRQ$
 c $NOPQ \equiv RSTU$ d $\triangle ACB \equiv \triangle EDF$
 e $WXYZ \equiv KLMJ$ f $JKLMNO \equiv STUPQR$

- 4 a $\angle F, DE$ b $\angle P, RQ$
 c $\angle Q, OP$ d $\angle R, WV$
 e $\angle IHG, EF$
 f $\angle QRS$ (or $\angle PON$), ON (or RS)
 g $\angle EDH$ (or $\angle FGH$), NJ h $\angle YXW, ZU$

5 A

- 6 a i $AB = DE, BC = EF, AC = DF$
 ii $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
 b i $LM = RQ, MN = QP, LN = RP$
 ii $\angle L = \angle R, \angle M = \angle Q, \angle N = \angle P$
 c i $AB = DF, AC = DE, CB = EF$
 ii $\angle A = \angle D, \angle B = \angle F, \angle C = \angle E$
 d i $PQ = CD, QR = DE, PR = CE$
 ii $\angle P = \angle C, \angle Q = \angle D, \angle R = \angle E$



Exercise 8-03

1 Teacher to check.

- 2 a $\triangle MNO$ b 60° 3 a $\triangle HIJ$ b 2 cm
 4 a different b congruent c different
 d different e different f congruent
 g congruent

Exercise 8-04

- 1 a SAS b AAS c AAS d SSS
 e RHS f SAS g AAS h SSS
 i RHS j SAS

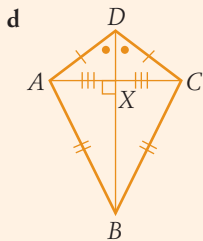
- 2 a B b $\triangle ABC \equiv \triangle DEF$
 3 a $p = 2, n = 20$ b $x = 15, y = 3$
 c $x = 58$ d $x = 4, y = 60$
 4 a AAS, $x = 6$ b AAS, $x = 6$
 5 a Vertically opposite angles.
 b SAS c $\triangle PQR \equiv \triangle STR$
 d $y = 28$ e $\angle S$
 f Alternate angles are parallel.

Mental skills 8A

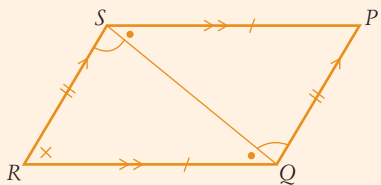
- 2 a 12:25 p.m. b 1:10 a.m. c 10:50 p.m.
 d 10:55 p.m. e 0610 f 0010
 g 9:10 a.m. h 3:15 a.m. i 1100
 j 2305 k 12:20 a.m. l 11:35 a.m.
 4 a 6:05 p.m. b 6:40 a.m. c 12:10 p.m.
 d 2:50 a.m. e 1245 f 0355
 g 10:50 p.m. h 12:15 p.m. i 1545
 j 0400

Exercise 8-05

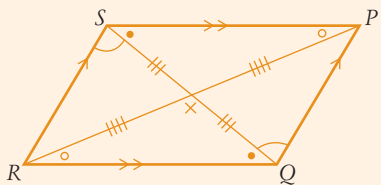
- 1 a SAS b $\angle C$ c $\angle CDB$ e SAS
 f XC g $\angle DXC$ h 90°



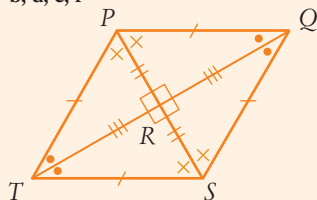
- i The diagonals intersect at right angles.
 2 a alternate angles on parallel lines
 b $\angle QSR$ c AAS d RQ e SR
 f opposite sides are equal g $\angle R$
 h opposite angles are equal



- 3 a $\angle XRQ$ b AAS c XQ
 d XR e Diagonals bisect each other.

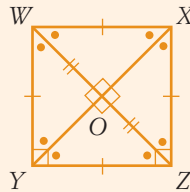


- 4 a SSS
 b 90° , 4 equal angles at a point (in a revolution) add up to 360° .
 b, d, e, f



- c The diagonals intersect at right angles.
 g The diagonals bisect each other.

- 5 a SAS b XZ
 c Diagonals are equal. e SSS
 f 90° g Diagonals bisect at right angles.
 h 45°



- i Diagonals bisect the angles of a square.

Exercise 8-06

- 6 c SAS d $\angle B$ e 90° f equal

Exercise 8-07

- 1 Teacher to check.

- 2 a



- b trapezium

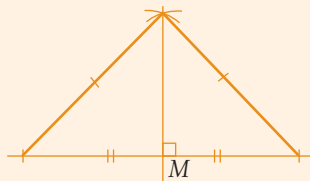
- 3 a



- b parallelogram

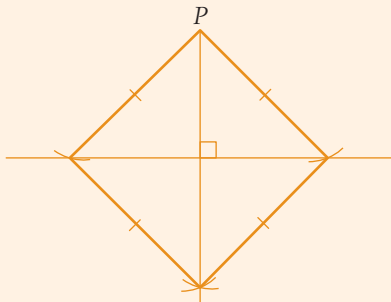
- 4-6 Teacher to check.

- 7 c SSS



- d Equal matching angles on a straight line add up to 180° , so each angle at M must be 90° .

8 b

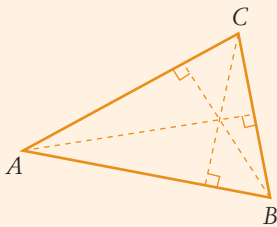


c Diagonals cross at right angles.

9 d rhombus

10 d square

11 b



c They all intersect at the same point.

Mental skills 8B

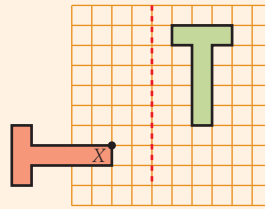
2 a 8 h 30 min	b 5 h 40 min	c 3 h 25 min
d 8 h 15 min	e 11 h 25 min	f 1 h 40 min
g 5 h 10 min	h 5 h 45 min	i 7 h 55 min
j 7 h 40 min		

Power plus

- 1 a WX ; XY , equal sides of a kite; WY
 b AC , $\triangle ABC$ is isosceles; DC , the midpoint of BC ; common; SSS
 c JK ; opposite sides of a parallelogram; GK , opposite sides of a parallelogram; GKI , opposite angles of a parallelogram; GHI , JKG , SAS
- 2 a all angles 90° b all sides equal
 c all sides equal
- 3 a parallelogram, rectangle, rhombus, square
 b kite, rhombus, square
 c rectangle, square
 d rectangle, square
 e parallelogram, rectangle, rhombus, square
 f parallelogram, rectangle, rhombus, square

Chapter 8 revision

1



2 a Reflection across the y -axis.

b $C(5, -4) \rightarrow (-5, -4)$, the x -coordinate changes sign while the y -coordinate stays the same.

3 a $AB = UT$ or $BC = TS$ or $AC = US$

b $\angle A = \angle U$ or $\angle B = \angle T$ or $\angle C = \angle S$

c $\triangle ABC \equiv \triangle UTS$

a $DE = WX$ or $FE = YX$ or $DF = WV$

b $\angle D = \angle W$ or $\angle E = \angle X$ or $\angle F = \angle V$

c $\triangle DEF \equiv \triangle WXV$

a $GH = OM$ or $IH = NM$ or $GI = ON$

b $\angle G = \angle O$ or $\angle H = \angle M$ or $\angle I = \angle N$

c $\triangle GHI \equiv \triangle OMN$

a $JK = RQ$ or $KL = QP$ or $JL = RP$

b $\angle J = \angle R$ or $\angle K = \angle Q$ or $\angle L = \angle P$

c $\triangle JKL \equiv \triangle RQP$

4 b no

5 a SAS

b RHS

c AAS

6 a SSS

b Four equal matching angles at R add up to 360° , so each angle is 90° .

c $\angle PTR$, $\angle STR$, $\angle SQR$

d i false

ii true

iii true

8-9 Teacher to check.

Chapter 9

SkillCheck

1 a 0.85	b 0.375	c 0.02	d 0.3
2 a {head, tail}		b {1, 2, 3, 4, 5, 6}	
c {female, male}		d {red, amber, green}	
e {win, lose, draw}		f {pass, fail}	
3 a 70%	b 56%	c $66\frac{2}{3}\%$	d 24%
4 a $\frac{2}{3}$	b $\frac{3}{5}$	c $\frac{5}{8}$	
5 C			
6 a $\frac{7}{25}$	b $\frac{1}{50}$	c $\frac{16}{25}$	d $\frac{4}{5}$
7 a more	b less	c less	
d less	e less	f more	

Exercise 9-01

1 a i {red, blue}	ii $\frac{1}{2}$
b i {red, blue, yellow, green}	ii $\frac{1}{4}$
c i {red, blue}	ii $\frac{1}{2}$
d i {red, white}	ii $\frac{1}{5}$

Answers

- e i {red, white, black, green} ii $\frac{1}{4}$
 f i {red, green, yellow} ii $\frac{1}{2}$
 2 a 2, equally likely b 3, not equally likely
 c 26, not equally likely d 2, equally likely
 e 10, equally likely f 2, not equally likely
 3 a {1, 3, 5} b {A, E, I, O, U}
 c {5, 6, 7, 8, 9} d {April, August}
 e {NSW, Qld, SA, Tas., Vic., WA}
 f {Years 7, 8, 9, 10, 11, 12}
 4 a $\frac{2}{20} = \frac{1}{10}$ b $\frac{3}{20} = 0.15$ c $\frac{11}{20} = 55\%$
 d $\frac{15}{20} = \frac{3}{4}$ e $\frac{7}{20} = 0.35$ f $\frac{13}{20} = 0.65$
 5 a $\frac{3}{16}$ b $\frac{8}{16} = \frac{1}{2}$ c $\frac{14}{16} = 0.875$ d $\frac{10}{16} = 62.5\%$
 6 C
 7 a {win, lose, draw}
 b No, teams have different abilities.
 8 D
 9 a 3 b {5, 6, 7, 8}
 c

Outcome	5	6	7	8
Probability as a fraction	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
Probability as a decimal	0.25	0.3	0.16	0.25
Probability as a percentage	25%	33 $\frac{1}{3}\%$	16 $\frac{2}{3}\%$	25%

- 10 C
 11 a C b A c E d H
 e D f G g B h F
 12 a 52 b Yes c 4
 d 13 e 2
 f i $\frac{1}{2}$ ii $\frac{1}{4}$ iii $\frac{1}{52}$
 iv $\frac{1}{13}$ v $\frac{5}{13}$ vi $\frac{3}{26}$
 13 a low b 9 14 Teacher to check.
 15 a 1 b 5 c 9 d 10

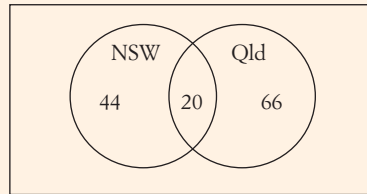
Exercise 9-02

- 1 a tossing heads
 b sunny day tomorrow
 c selecting a brown chocolate
 d rolling 1, 2, 3, 4 or 5
 e losing or having a draw (not winning)
 f selecting a diamonds, clubs or spades card (not selecting hearts)
 g born in spring, summer or autumn (not being born in winter)
 h being 15 or over
 2 a $\frac{1}{6}$ b $\frac{5}{6}$ c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{5}{6}$ f $\frac{2}{3}$
 3 a $\frac{1}{3}$ b $\frac{2}{3}$ c $\frac{5}{6}$ d 1
 4 a $\frac{45}{56}$ b D 5 a $\frac{1}{4}$ b $\frac{3}{4}$
 6 0.8 7 $\frac{199}{200}$
 8 a $\frac{3}{26}$ b $\frac{23}{26}$ c 1 d $\frac{11}{26}$ e $\frac{9}{13}$ f $\frac{1}{2}$
 9 a $\frac{12}{13}$ b 39% c 0.85
 10 a 0.65 b 0.35
 c Red 16, Blue 14, White 10 d 0
 11 a $\frac{3}{4}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d 0

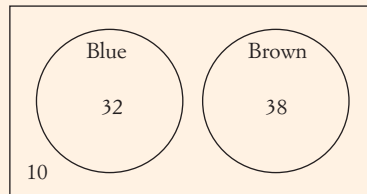
- 12 15% 13 C 14 a $\frac{9}{16}$ b $\frac{7}{16}$
 15 a $\frac{10}{11}$ b $\frac{7}{11}$ c $\frac{9}{11}$ d $\frac{8}{11}$
 16 a $\frac{3}{5}$ b $\frac{3}{5}$ c $\frac{4}{5}$ d $\frac{3}{5}$

Exercise 9-03

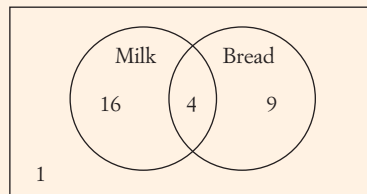
- 1 a 50 b 21 c 42
 d 33 e 8 f 9
 2 a Yes, mutually exclusive b 6
 c 40 d 46 e 0
 3 a No b 52 c 4
 d i 0.808 ii 0.731 iii 1 iv 0.192
 e right-handed male
 4 a 440 b 368 c 15 d A
 5 a



- b 130 c 110 d $\frac{22}{65}$ e 0
 6 a



- b Yes, cannot be both blue-eyed and brown-eyed.
 c 80 d 70 e 0 f 52.5%
 7 a



- b 4 c 16
 d i $\frac{3}{10}$ ii $\frac{1}{3}$ iii 29 iv 25

Exercise 9-04

- 1 a 27 b 18 c 2 d $\frac{12}{27}$
 e i $\frac{5}{27}$ ii $\frac{9}{27} = \frac{1}{3}$ c 96
 2 a 336 b 192 c 96
 d i $\frac{11}{21}$ ii $\frac{4}{21}$ iii $\frac{17}{21}$
 3 a 56; 101, 146; 101, 243 c 56
 b 146 d i $\frac{101}{344}$ ii $\frac{36}{43}$ iii $\frac{187}{344}$ iv $\frac{243}{344}$
 4 a 125; 70, 110; 165, 310 b 165
 c i $\frac{11}{31}$ ii $\frac{24}{31}$ iii $\frac{25}{62}$
 d D

5

	Swimming	Not swimming	
Bowling	11	20	31
Not bowling	34	5	39
	45	25	70

6 a

	GPS navigation	No GPS navigation	
4-doors	25	42	67
Not 4-doors	3	12	15
	28	54	82

b i $\frac{14}{41}$ ii $\frac{67}{82}$ iii $\frac{25}{82}$ iv $\frac{27}{41}$

7 a

	Needs glasses	Does not need glasses	
Male	22	60	82
Female	13	85	98
	35	145	180

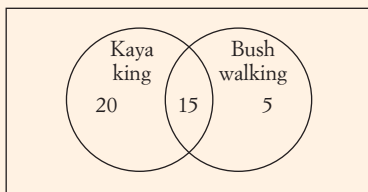
b i $\frac{49}{90}$ ii $\frac{7}{36}$ iii $\frac{13}{180}$ iv $\frac{167}{180}$ v $\frac{2}{3}$ vi $\frac{107}{180}$

Mental skills 9

- 2 a 16 000 b 210 c 9900
 d 600 e 16 000 f 400
 g 490 h 52 i 1500
 j 1360 k 195 l 17 600
- 4 a 8 b 50 c 8 d 22
 e 4 f 50 g 9 h 9.3
 i 8.5 j 4.23 k 9.6 l 40.8

Exercise 9-05

- 1 a $\frac{17}{28}$ b $\frac{3}{14}$ c $\frac{27}{28}$
 2 a {U, E, C, S}
 b No, some occur more than others.
 c i $\frac{3}{7}$ ii $\frac{2}{7}$ iii $\frac{5}{7}$ iv 0
 3 a {black, red, blue}
 b i $\frac{1}{3}$ ii $\frac{1}{3}$ iii 0
 c choosing a black or blue sock, $\frac{2}{3}$
 4 a 20% b 40% c 80% d 100%
 5 D
 6 a $\frac{1}{100}$ b $\frac{3}{5}$ c $\frac{1}{2}$ d $\frac{17}{20}$
 7 $\frac{1}{2}$
 8 a $\frac{8}{15}$ b $\frac{2}{5}$ c $\frac{7}{15}$ d $\frac{2}{5}$
 9 a $\frac{17}{30}$ b $\frac{1}{10}$ c $\frac{2}{3}$ d $\frac{13}{30}$
 10 a



b i $\frac{3}{8}$ ii $\frac{1}{2}$

11 0.9

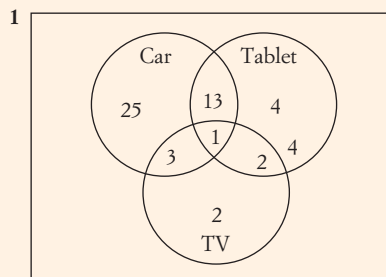
12 a 232; 140; 228, 272, 500

b i $\frac{57}{125}$ ii $\frac{7}{25}$ iii $\frac{1}{5}$
 iv $\frac{32}{125}$ v $\frac{23}{25}$ vi $\frac{83}{125}$

Exercise 9-06

- 1 a $\frac{1}{4}$ b 125
 2 Teacher to check.
 3 a $\frac{1}{2}$ b 40, less than c $\frac{11}{20}$ d $\frac{1}{5}$
 e 16, the observed frequency was less than expected f $\frac{11}{80}$
 4 a 50 b $\frac{8}{25}$ c $\frac{9}{50}$
 d i 400 ii 820
 5 c i $\frac{1}{6}$ ii $\frac{2}{3}$ iii $\frac{1}{2}$
 d i 12 ii 48 iii 36
 g i 167
 6 46%
 7 a Yes
 b Yes, 6 comes up much more often.
 c $13\frac{3}{4}\%$ d 11
 8 a i 0.06 ii 0.22 iii 0.44
 iv 0.76 v 0.34 vi 0.68
 b i 8 ii 5, 9 iii 2
 iv 10 v 10 vi 6, 7

Power plus



2 a

		First die					
	×	1	2	3	4	5	6
Second die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

- b 18 c They occur different numbers of times
 d 6, 12 e 1, 36
 f i $\frac{1}{9}$ ii $\frac{1}{18}$ iii $\frac{2}{9}$
 g 4

3 No, the probabilities of the three events are not equal.

Answers

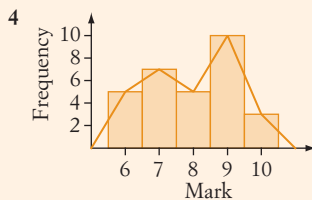
- 4 a {Paul and Summet, Paul and Tash, Paul and Nadine, Summet and Tash, Summet and Nadine, Tash and Nadine}
b $\frac{1}{2}$

Chapter 9 revision

- 1 a $\frac{1}{6}$ b $\frac{2}{3}$ c $\frac{1}{2}$ d $\frac{1}{3}$
2 a 5, 6, 7, 8
b i 25% ii 100% iii 25% iv 0
3 a Choosing a card that is not an Ace, $\frac{12}{13}$
b Rolling a number that is not a factor of 6, $\frac{1}{3}$
c Buying a ticket that does not win, $\frac{999}{1000}$
4 26%
5 a $\frac{6}{25}$ b $\frac{17}{40}$ c $\frac{67}{200}$ d $\frac{133}{200}$
6 a 73 b 16 c 50
d i $\frac{44}{73}$ ii $\frac{66}{73}$ iii $\frac{7}{73}$
7 a 70 b 43 c 63
d i $\frac{1}{10}$ ii $\frac{1}{2}$ iii $\frac{11}{35}$ iv $\frac{27}{70}$
8
- | | Tall | Not tall | |
|-----------|------|----------|----|
| Blond | 3 | 5 | 8 |
| Not blond | 10 | 12 | 22 |
| | 13 | 17 | 30 |
- 9 a 0.125 b 0.25 c 0.75 d 0.625
10 a $31\frac{1}{4}\%$ b $33\frac{1}{3}\%$ c 25

Mixed revision 3

- 1 A and J, C and F, D and G
2 a 25% b 75% c 75%
3 a
- | Mark | Tally | Frequency |
|------|-------|-----------|
| 6 | | 5 |
| 7 | | 7 |
| 8 | | 5 |
| 9 | | 10 |
| 10 | | 3 |
- b 30



- 5 a $\triangle ABC \equiv \triangle GHI$
b $AB = GH$ or $BC = HI$ or $AC = GI$,
 $\angle A = \angle G$ or $\angle B = \angle H$ or $\angle C = \angle I$

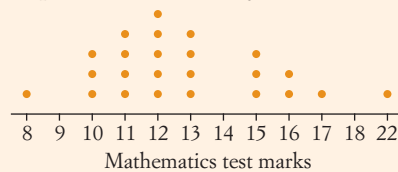
6 a

Score	Frequency
1	4
2	9
3	8
4	3
5	10
6	6
Total	40

b Result of roll of die

1	2	3	4	5	6

- 7 a numerical b categorical c categorical
8 a continuous b continuous c discrete
9 a $\frac{1}{2}$ b $\frac{5}{6}$ c $\frac{1}{3}$
10



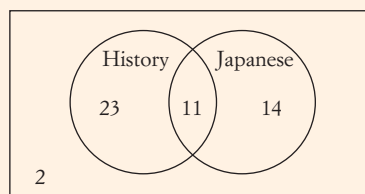
- a 12 b 12 c 22 d 10-13
11 a
-

- 12 a 92.5 b 90
13 a 92 b 8
14 $\frac{7}{10}$

15 a

x	f	fx
28	6	168
29	1	29
30	6	180
31	2	62
32	3	96
33	2	66
34	4	136
	24	737

- b $\frac{7}{24}$
c i 30.7 ii 30 iii 28, 30 iv 6
16 a



- b 14 c $\frac{1}{25}$

17 a SSS, SAS, AAS, RHS

b AAS

18 a

	Plays week end sport	Does not play weekend sport	
Male	49	23	72
Female	39	45	84
	88	68	156

b 156

c 88

d i $\frac{7}{13}$

ii $\frac{31}{78}$

19 Teacher to check.

20 a

Maths		Science
3	2	4
5 5	3	
8 3	4	0 0 3
8 8 2	5	4
	6	0 2 9
	7	3
7	8	8
6	9	7

b

	Median	Range
Maths	50	73
Science	57	64

c Maths, lower median.

21 a $\frac{3}{64}$

b $\frac{5}{16}$

c $\frac{55}{64}$

d $\frac{37}{64}$

Chapter 10

SkillCheck

- 1 a 7 e 6 i 10 2 a -3 e 3 i -10 3 a 17 4 a -6 5 a n 6 a $2m + 6$ c $4k + 28$ e $6a - 18$ g $10q - 30$ 7 a subtracting c dividing
- b 7 f 23 j 5 b -5 f -11 j -8 b 17 c 7 b 11x b $3x - 6$ d $5d - 5$ f $4b + 32$ h $45j + 9$ b multiplying d adding
- c 8 g 5 k 8 c -12 g 36 k -9 c 4 c $4r$ b $3x - 6$ d $5d - 5$ f $4b + 32$ h $45j + 9$ b multiplying d adding
- d 27 h 3 l 20 d -2 h 3 l -72 d 4 d -14 d $8t$

Exercise 10-01

- 1 a $x = 7$ d $k = 27$ g $n = -13$ j $g = 15.5$ b $y = 23$ e $k = -4$ h $p = -20$ k $a = 10\frac{1}{4}$ c $m = 12$ f $s = -2$ i $m = -8$ l $k = -7.7$

2 a $t = 51$

d $j = 74$

g $x = 33$

j $w = 7$

m $y = 20.9$

p $s = 6\frac{5}{6}$

3 a $x = 4$

d $b = 9$

g $p = 5$

j $q = -6$

m $p = 9\frac{1}{4}$

p $w = -1\frac{1}{3}$

4 a $s = 15$

d $w = 58$

g $r = -15$

j $t = 1\frac{1}{2}$

5 C

b $w = 11$

e $q = 29$

h $g = 11$

k $m = -15$

n $n = 8$

b $g = 4$

e $d = 9$

h $m = 13$

k $c = -18$

n $r = 2\frac{1}{4}$

b $m = 42$

e $a = -12$

h $m = -7$

k $n = 16$

6 Teacher to check.

c $a = 7$

f $f = 110$

i $j = -2$

l $d = 0$

o $b = 1\frac{1}{20}$

c $y = 9$

f $c = 9$

i $x = -5$

l $x = 8$

o $q = -3\frac{1}{3}$

c $d = 120$

f $f = -18$

i $x = 48$

l $x = -\frac{1}{2}$

Exercise 10-02

- 1 a $m = 6$ d $w = 6$ g $x = 7$ j $p = 8$ 2 a $m = 18$ d $k = -27$ g $n = 65$ j $x = 18$ 3 D 4 a $x = 6$ d $N = 5$ g $x = -4$ j $x = -24$ 5 C 6 a $x = 5$ d $N = 23$ g $k = -5$ 7 Teacher to check.
- b $x = 6$ e $h = -3$ h $a = -1$ k $y = 10$ b $c = 15$ e $h = 20$ h $t = 12$ k $v = 90$ b $x = 9$ e $N = 21$ h $x = -10$ k $x = -1\frac{1}{4}$ b $x = 9$ e $N = 22$ h $m = 31$
- c $k = 8$ f $d = -2\frac{3}{4}$ i $c = 2\frac{1}{2}$ l $r = 15\frac{1}{2}$ c $d = 42$ f $x = -24$ i $a = -32$ l $b = 21$ c $x = 12$ f $B = -4$ i $m = 4\frac{2}{5}$ l $x = 15$ c $N = 3$ f $x = -5$

Exercise 10-03

- 1 a $a = 6$ d $p = 3$ g $y = 0$ 2 D 3 a $a = 3$ d $p = 1$ g $x = \frac{2}{3}$ 4 D 5 a $d = -2$ d $x = -3$ g $t = 1\frac{1}{2}$ 6 Teacher to check.
- b $k = 4$ e $n = 3$ h $a = 7$ b $x = 2\frac{2}{3}$ e $m = 5\frac{1}{4}$ h $r = -1$ b $k = \frac{2}{3}$ e $k = 2\frac{1}{5}$ h $j = -8\frac{1}{2}$ c $x = 3$ f $q = 6$ i $r = 6$ c $d = 1$ f $x = 5$ i $y = 4\frac{1}{2}$ c $p = -2$ f $m = 5\frac{1}{2}$ i $q = -1$

Exercise 10-04

- 1 a $x = 1$ b $m = 4$ c $d = 6$
 d $p = 4$ e $j = 3$ f $d = 5$
 g $x = 9$ h $x = 8$ i $x = 3$
 j $x = 8$ k $k = 12$ l $q = 20$
 2 a $x = 3$ b $p = -4$ c $q = -3$
 d $h = 11$ e $r = -8$ f $e = 11$
 g $z = 1$ h $y = -7$ i $a = 6$
 j $d = -7$ k $k = 9$ l $c = 10$
 3 B
 4 a $x = -1$ b $r = 1$ c $p = -7$
 d $x = -2$ e $p = 5$ f $e = 6$
 g $y = 1$ h $a = 4$ i $w = -6$
 5 A
 6 a $x = 13$ b $x = 6$ c $k = -18$
 d $y = 2$ e $v = 4$ f $x = 11$
 g $p = 4$ h $s = 7$ i $d = -22$
 7 Teacher to check.

Mental skills 10

- 2 a 160 b 70 c 240 d 900
 e 2600 f 900 g 140 h 300
 i 180 j 770 k 18 l 34
 m 46 n 26 o 18 p 12
 q 40 r 8 s 14 t 24

Exercise 10-05

- 1 a $x = \pm 9$ b $x = \pm 12$ c $x = \pm 1$
 d $x = \pm 13$ e $m = \pm 71$ f $u = \pm 43$
 2 a $x \approx \pm 3.61$ b $x \approx \pm 7.34$ c $x \approx \pm 9.38$
 d $t \approx \pm 11.36$ e $h \approx \pm 30.76$ f $z \approx \pm 22.96$
 3 a $x = \pm\sqrt{41}$ b $x = \pm\sqrt{30}$
 c $x = \pm\sqrt{48}$ or $\pm 4\sqrt{3}$ d $a = \pm\sqrt{126}$ or $\pm 3\sqrt{14}$
 e $p = \pm\sqrt{75}$ or $\pm 5\sqrt{3}$ f $b = \pm\sqrt{509}$
 4 $x^2 = 0$, with solution $x = 0$
 5 It is not possible to find the square root of a negative number.
 6 For example, $x^2 = -1$ and $x^2 = -12$
 7 a positive b negative c zero

Exercise 10-06

- 1 a \$88 b 90 cents c 33 d 16
 2 a D, 102 b B, 49 c D, 56
 d B, 588 e C, \$8974
 3 a 75 b 49 c 27 d 16
 e 12 f 18 g 15 h 6
 4 a 86 b 8 c 100 kg
 d 6 e 27 f 16 cm
 5 a 77°F b $42\frac{25}{9}^\circ\text{C}$
 6 a \$600 b 122
 7 a \$13.90 b 28
 8 a $x = 8\frac{1}{2}$ b 17 cm c $6\frac{1}{2}$ cm
 9 a \$75 b 267
 10 $x = 7$

Power plus

- 1 a $x = 3$ b $x = 2$ c $x = 0$
 d $x = -1$ e $x = 2\frac{1}{7}$ f $x = 3\frac{1}{11}$
 g $x = 20$ h $q = 26\frac{2}{3}$ j $x = 30$
 2 a $x = 7$ b $x = 5$ c $x = 65$
 3 a $x = 5$ or -5 b $x = 4$ or -4 c $x = 6$ or -6
 d $x = 2$ or -2 e $x = -7$ or -1 f $x = 3$ or -5
 4 $\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$, 84 years

Chapter 10 revision

- 1 a $k = 7$ b $x = 11$ c $a = 14$
 d $a = 33$ e $x = 4$ f $f = 12$
 g $m = 28$ h $x = 24$ i $w = -6$
 j $k = -2$ k $m = 8$ l $x = -36$
 2 a $p = 5$ b $m = -3$ c $x = 15$
 d $h = 77$ e $n = -8$ f $a = 60$
 g $y = 10$ h $x = 10$ i $n = 25$
 j $x = -6$ k $k = 26$ l $d = -29$
 3 a $x = 1$ b $u = 3$ c $h = 3$
 d $v = -3$ e $x = 2\frac{3}{5}$ f $t = 3$
 4 a $n = 6$ b $x = 4$ c $x = 2$
 d $x = 7$ e $y = 4\frac{2}{3}$ f $d = 6$
 g $r = 21$ h $a = -2$ i $n = 3$
 5 a 5 b 3 6 a B b $n = 6$
 7 $x = 3$ 8 a 1260° b 8

Chapter 11

SkillCheck

- 1 a 200 b 180 c 3 d 2500
 e 3.8 f 0.18 g 180 h 85
 i 8 j 7.5 k 9150 l 384
 2 a 6 b 7 c 8
 3 a 21 b 20 c 2
 4 a 6 b 8 c 15
 5 a $\frac{2}{5}$ b $\frac{1}{2}$ c $\frac{4}{5}$ d $\frac{5}{12}$
 6 a 8 b 5 c 18 d 30
 e 760 f 54 g 39 h 250
 7 a 0715 b 1545 c 2050 d 0010
 8 a 4:10 a.m. b 11:05 a.m. c 2:15 p.m.
 d 11:35 p.m.

Exercise 11-01

- 1 (i only, ii Teacher to check.)
 a 6 : 6 or 1 : 1 b 2 : 6 or 1 : 3 c 6 : 10 or 3 : 5
 d 8 : 12 or 2 : 3 e 5 : 3
 2 a 12 b 10 c 9 d 20 e 32
 f 84 g 30 h 75 i 5 j 5
 k 4 l 1 m 30 n 2 o 40
 p 5 q 5 r 25
 3 D 4 A
 5 a 6 : 9 : 12 b 15 : 20 : 25 c 1.5
 d 3.5 e 2.6 f 6
 g 24 h 35 i 15
 j $1\frac{1}{2}$ k 63

Exercise 11-02

- 1 a 1:10 d 5:7 g 10:1 j 41:107 m 2:3:5 p 2:5:3:7
b 1:2 e 3:2 h 5:11 k 3:1 n 3:4:6
c 2:5 f 7:3 i 1:1 l 1:16 o 3:1:4
- 2 a 5:6 e 4:3 i 12:7 m 4:7 q 3:4 u 3:2
b 3:4 f 8:5 j 8:5 n 13:8 r 3:2 v 10:7
c 9:8 g 5:2 k 25:12 o 5:3 s 1:4 w 2:1:3 x 45:10:9
d 4:3 h 4:3 l 9:5 p 1:2 t 1:2
- 3 a 1:4 e 1:3 i 80:7 m 6:1
b 1:4 f 4:1 j 5:36 n 3:20
c 5:49 g 5:16 k 9:500 o 9:40
d 1:4 h 1:50 l 1:6
- 4 3:2
5 a 4:3 b 3:4 c 4:7
6 a 3:2 b 3:5 c 5:2
7 a 75:64 b 11:75
8 a 16:19 b 19:16 c 3:19 d 19:3
9 a 1:1 b 2:1 c 1:2 d 2:1
10 a $\frac{5}{8}$ b 5:3 11 A
12 Australia 3:250, India 11:500

Exercise 11-03

- 1 C 2 75 3 560 kg 4 32 cm
5 1440 6 17.5 km/h
7 a 20 cm b 136 cm 8 8 kg
9 a Ali \$90, Felicity \$63 b \$225
10 72 cm
11 a 5 kg b Heavier packet by 10 c/kg
12 20.4 t 13 1.52 m 14 A
15 a 5 mL b 60 mL

Exercise 11-04

- 1 a 1:500 000 d 1:10 000 i 1:50 000
b 1:1000 g 1:25 000 j 1:10 000
c 1:50 000 000 h 1:40 000
- 2 B
3 a 1:20 000 b i 1.4 km ii 1.9 km iii 2.48 km
c i 90 cm ii 7.5 cm iii 48 cm
4 6670 km 5 66 cm 6 5.6 cm
7 a i 0.96 km ii 0.6 km iii 0.44 km iv 1.0 km
b i 0.88 km ii 0.44 km c 0.36 km
d 120 m by 280 m e 160 m f 20 cm
- 8 a 4 m d 1.2 m
b 75 cm e 1.6 m
c 2.05 m f $4\text{ m} \times 3\text{ m} = 12\text{ m}^2$
9 a 13.5 cm b 7 m c 16 cm d 64 cm
10 B

- 11 a 5 m c 2 m e 5 m
b 2.5 m d $2\text{ m} \times 2.5\text{ m} = 5\text{ m}^2$ f $4\text{ m} \times 3.8\text{ m} = 15.2\text{ m}^2$
- 12 a 0.4 mm c 2.4 cm
b 8 mm d 0.32 mm

Mental skills 11

- 2 a 8:45 a.m. d 12:17 a.m. g 11:15 a.m. j 3:20 a.m. m 1:08 a.m. p 6:43 a.m.
b 1:20 p.m. e 9:05 p.m. h 2:38 a.m. k 4:55 p.m. n 10:18 a.m.
c 5:50 p.m. f 6:32 p.m. i 2:40 p.m. l 11:31 p.m. o 8:00 p.m.
- 4 a 1835 e 0221 i 0908 m 0159
b 0805 f 1230 j 2150 n 2218
c 1145 g 1548 k 0042 o 1046
d 2320 h 1911 l 0739 p 1723

Exercise 11-05

- 1 a 9 b 5 c 7 d 11 e 13 f 6
2 a \$400, \$100 b \$350, \$150
3 a 1:1:2 b Phuong \$1000, Janelle \$1000, Ahmet \$2000
4 \$180
5 a 200 kg, 250 kg b 270 kg, 180 kg
6 a 80 cm, 240 cm, 400 cm b 300 cm, 180 cm, 240 cm
7 D 8 125 9 25
10 600 g 11 1078 12 4.375 t
13 80 kg 14 568
15 4 m, 6 m, 10 m 16 240 kg
17 a 30 000 b 165 000
18 \$140 000, \$70 000

Exercise 11-06

- 1 a words/min d \$/kg g persons/year
b beats/min e \$/h h c/kL
c c/min f c/L i persons/km²
- 2 a 17 sheep/h c 26 students/teacher e 91 words/min g 5.1 cars/day i 37 m/s k 1324 revolutions/min m 56 km/h o 4.75 runs/over q \$10.50/h
b \$4.30/kg d 34 points/game f 8.5 articles/h h 630 boxes/h j 920 bottles/h l \$35/h n \$2.75/kg p 8.1 km/L r 42 kg/ha
- 3 B
4 a Can travel 100 km in one hour at this speed.
b 150 cars will pass in an hour at this rate.
c 10.3 L will be used over a distance of 100 km.
d For each hectare of land, there are 60 sheep.

Answers

- 5 80 kg/person
7 139 c/L
- 6 1800 ha/year
8 182.5

Exercise 11-07

- 1 a 2 kg box b 8 DVDs c 150 g
d 7 tubs e 3 kg f 2.25 L carton
2 300 g 3 750 g box 4 C
5 a 375 g b 2 L c 735 g d 115 g
6 a median b large c small d large

Exercise 11-08

- 1 \$676.40 2 160 runs
3 a 10.5 km b 210 km 4 336
5 432 mL 6 6 7 \$3980
8 $7\frac{1}{2}$ ha or 7.1 ha 9 a \$70 b 2.5 h
10 300 km 11 a 14.8 L b 133.2 km
12 C 13 a 17°C , 29°C b 12 p.m.
14 a 2400 m^2 b $40\text{ m}^2/\text{min}$
c $60\text{ m}^2/\text{min}$ d 24 min (at $100\text{ m}^2/\text{min}$)
15 a 5400 s b 90 min c 1.5 hr
16 a 650 km b 48 L c 13.5 km

Exercise 11-09

- 1 a 5 km/h b 480 km/h c 5 km/h
d 17 km/h e 100 km/h f 80 km/h
g 120 km/h h 120 km/h i 200 km/h
j 9 km/h k 10 m/s
l 0.1 km/min or 100 m/min
2 a 10 m/s b 1.5625 m/s c 58 m/s
d 8 m/s e 1.5 m/s
3 B 4 a 100 m/min b 6 km/h
5 40 km/h 6 D
7 a 720 km b 30 km c 1330 km
8 3 km 9 63.9 km/h
10 56 km/h 11 1.5 h
12 Teacher to check. 13 54.05 s
14 500 m 15 1200

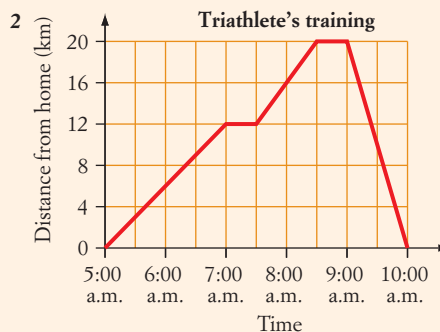
Exercise 11-10

- 1 a 4 h b 8 km
c i $3\frac{1}{3}$ km/h ii 3 km/h
d steeper graph e 16 km
f 11:30 a.m. to 1 p.m., 2 p.m. to 3 p.m.
g 3 p.m. h 1 h
2 a 300 km b 4 h
c 1 h d 100 km/h
e 100 km f Neither: the same speed.
3 a i 2 p.m. ii 11 a.m.
b 12:45 p.m. c 48 km d 4.5 h
e i 16 km/h ii 6.4 km/h
iii 9.6 km/h iv 9.6 km/h

- 4 a 240 km
b Sam, steeper graph, ending journey earlier.
c 120 km d 180 km
e 10 a.m., 160 km
f After 9:15 a.m., graph is steeper.
g 40 km/h h B
5 a 1600 km b 7 p.m. c 350 km/h d D
e See figure on next page top.
f 10:10 a.m.
6 Teacher to check.
7 a A b B c D d C
8 C, not possible to be at two different places/distances at the same time.

Exercise 11-11

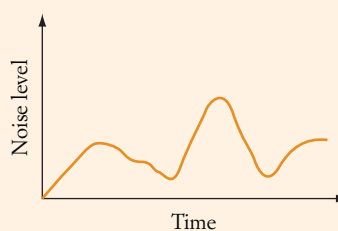
- 1 B



- 3 Teacher to check.

- 4 a 8 Brown b 8 Yellow
c 8 Red d 8 Green

- 5



- 6 a Grew steadily, then dropped.
b 4
c Returns to normal.
d His team was losing and it was a close match.
7 B
8 Teacher to check.

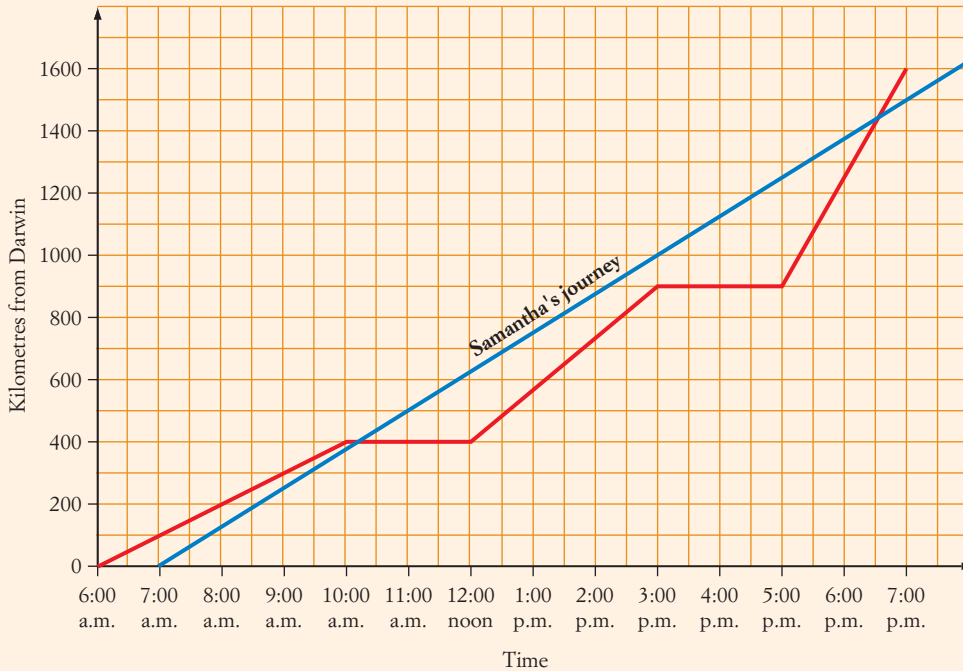
Exercise 11-12

- 1 A
2 a 1 h 55 min b 50 min c 4 h 45 min
d 3 h 55 min e 5 h 45 min f 10 h 55 min
g 8 h 5 min h 3 h 30 min i 4 h 40 min
3 a 6 h 47 min b 11 h 52 min c 3 h 29 min
d 7 h 31 min e 12 h 30 min f 8 h 9 min
g 6 h 9 min h 1 h 45 min i 12 h 29 min
j 12 h 7 min k 4 h 51 min l 1 h 43 min

Exercise 11-10

5 e

James' journey from Darwin to Alice Springs



4 1 h 55 min

- 5 a 8 p.m. b 1 p.m. c 7:35 p.m.
 d 1:17 p.m. e 2:04 a.m. f 2:32 p.m.
 g 3:17 a.m. h 4:25 p.m. i 8:45 a.m.
 j 3:10 p.m.

6 Volvo 1:43 p.m., Peugeot 2:02 p.m., Ford 2:04 p.m., Subaru 2:19 p.m., Nissan 2:21 p.m., Toyota 2:41 p.m.

- 7 a 7 h b 6 h 30 c 4 h 11 min
 d 1 h, 1 h 15 min, faster trip due to less traffic earlier in the day.

e 10:41 a.m. f Strathfield
 g 35 min, 4:40 p.m.

- 8 a 6 b 8:15 a.m.
 c Central (RSL) or Pine Ave.
 d 9:10 a.m. e 20 min
 f 6:50 p.m. g 11:40 a.m.

5 2:30 p.m.

8 Teacher to check.

- 9 a ahead
 d behind
 g ahead

10 a $\frac{1}{2}$ h b 2 h

11 a 11 p.m.

d 10:30 p.m.

12 a 12 midnight

d 11:30 p.m.

13 a 4 p.m.

6 9 a.m.

- b ahead
 e ahead
 h same

c $\frac{1}{2}$ h d $\frac{1}{2}$ h

b 10:30 p.m.

e 11 p.m.

b 12 midnight

e 12 midnight

b 3 p.m.

7 B

- c same
 f behind
 i ahead

e 2 h f 0 h

c 9 p.m.

f 11 p.m.

c 10 p.m.

f 12 midnight

14 d 3:30 p.m.

Power plus

1 273 000

2 a 704 km²

b 56 526 363 400

3 Teacher to check.

4 a 97.2 km/h

b 349.2 km/h

c 11.88 km/h

d 82.8 km/h

5 $y = 3.6x$

Exercise 11-13

- 1 a ahead b ahead c behind d ahead
 e ahead f behind g ahead h ahead
 i ahead j ahead k behind l behind

2 a 10 p.m. b 8 p.m. c 7 a.m. d 8 p.m.
 e 4 a.m. f 2 a.m. g 3 p.m. h 1 p.m.

3 a 2 h b 2 h c 20 h d 7 h
 e 15 h f 0 h g 3 h h 13 h
 i 7 h j 9 h

4 a 4 a.m. b 12 noon c 11 p.m. d 12 noon
 e 8 p.m. f 6 p.m. g 7 a.m. h 5 a.m.

Chapter 11 revision

1 a 4:12

b 6:15

c 9:21

d 20:45

e 2:32

f 15:20:25

2 a 4:7

b 1:3

c 1:6

d 5:9

e 3:1:4

f 1:5:20

g 2:3

h 4:3

i 3:5

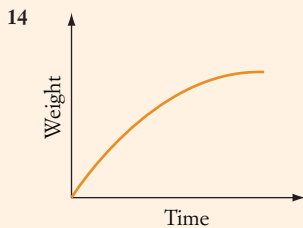
j 4:1

k 1:4

l 65:4

Answers

- m 25 : 1 n 50 : 9 o 100 : 17 p 3 : 8
 q 2 : 7 r 1 : 9
- 3 a 100 b \$45 000
 4 a 91 cm b 11.2 cm
 5 a 1 : 25 000 ii 1.1 km or 1100 m
 b i 625 m ii 10 cm
 c i 3.5 cm
 6 Ellen \$240, Portia \$180
 7 a \$3.50/kg b 110 km.h
 c \$14.10/h d 5.2 runs/over
 8 a 1 kg box b 350 g can
 c 1.5 L bottle d 500 g bar
 9 a \$19.95 b \$710.60 c 29.6 L d 7.2 ha
 10 20 km/h ii 247.5 km
 12 a 10 km b 3 c $2\frac{2}{3}$ km/h
 d $3\frac{1}{2}$ h e 2:30 p.m. to 3:30 p.m.
 13 a B b C c A



- 15 a 11:25 p.m. b 6:30 p.m.
 16 a 8 h 30 min b 15 h 15 min c 10 h 50 min
 17 a p.m. time b 4 c 9 min
 d Horatio St e 2:50 p.m.
 18 a 8 h b 2 h c 18 h d 10 h
 19 11:30 p.m.

Chapter 12

SkillCheck

- 1 a 4 b 5 c 2 d 2 e 4 f 5
 2 a 4 b 6 c -3 d 1 e -3 f 7
 3 a $x = 3$ b $x = -2$ c $x = -3$
 d $x = -2$ e $x = -1\frac{1}{2}$ f $x = 3$

Exercise 12-01

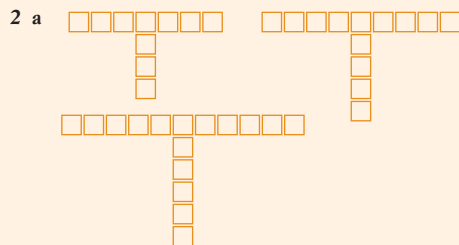
- 1 a 2, 6, -2, -9, 4 b 24, -9, 6, 30, -12
 c 4, -1, -3, 0, $3\frac{1}{2}$ d 5, 14, 7, 20, -2
 e 16, -2, 0, 8, 4 f -7, 4, 2, -11, 8
 2 A
 3 a 14, 4, 19, 39, -31 b 10, -5, 28, 4, -20
 c 6, -6, 42, -14, 22 d 22, 37, -23, 32, -8
 e -3, -1, 1, 3, 5, 7 f -3, -2 $\frac{1}{2}$, -2, -1 $\frac{1}{2}$, -1, - $\frac{1}{2}$
 g 2, 5, 8, 11, 14, 17 h 3, 2, 1, 0, -1, -2
 4 B 5 D

Exercise 12-02

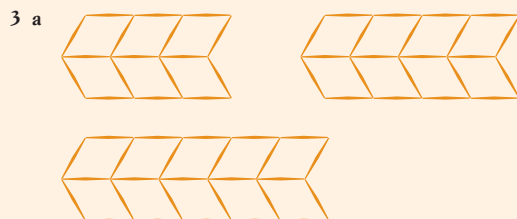
- 1 a $y = 3x$ b $y = -2x$ c $y = x + 4$
 d $y = x - 4$ e $y = 10 - 3x$ f $y = x \div 2$
 g $k = p + 4$ h $y = x - 3$ i $j = 2h$
 j $p = m \div 5$
 2 a $h = 3x - 2, 19, 22$ b $p = 5m - 3, 27, 42$
 c $b = 3m + 3, 18, 27$ d $k = 2h + 2, 20, 24$
 e $s = 3r + 1, 13, 22$ f $b = 2a - 2, 18, 20$
 g $n = 3m + 5, 20, 23$ h $d = 10c + 1, 91, 101$
 i $x = 5w - 1, 34, 39$ j $z = 2y - 6, 12, 14$
 k $m = 4a + 1, 29, 37$ l $t = 2z - 7, 11, 17$
 3 B 4 C
 5 a $y = -2x - 1, -19$ b $c = \frac{1}{2}A, 5$
 c $t = -5s + 13, -28$ d $v = -10t + 20, 0, -30$
 e $D = -\frac{1}{2}x - \frac{1}{2}, -2, -3\frac{1}{2}$

Exercise 12-03

- 1 a i 15 ii $T = 2n - 1$ iii 15, 39
 b i 16 ii $T = 2n$ iii 16, 40
 c i 22 ii $T = 2n + 6$ iii 22, 46
 d i 28 ii $T = 3n + 4$ iii 28, 64
 e i 63 ii $T = 8n - 1$ iii 63, 159
 f i 37 ii $T = 5n - 3$ iii 37, 97
 g i 5 ii $T = -2n + 21$ iii 5, -19
 h i 64 ii $T = n^2$ iii 64, 400

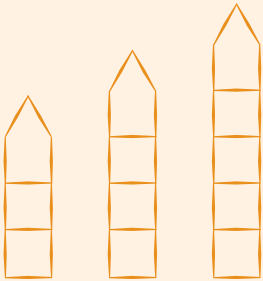


- b 7, 10, 13, 16, 25, 34
 c The number of tiles equals 3 times the arm length plus 1.
 d $t = 3a + 1$
 e i 46 ii 100



- b 12, 17, 22, 32, 47, 52
 c The number of toothpicks equals 5 times the number of the flight shape plus 2.
 d $t = 5f + 2$
 e i 127 ii 202

4 a



b 5, 8, 11, 14, 17, 26, 35

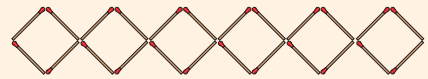
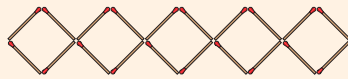
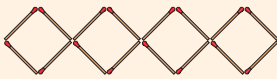
c The number of toothpicks equals 3 times the number of stages plus 2.

d $t = 3s + 2$

e i 38

ii 137

5 a i



ii 4, 8, 12, 16, 20

iii $T = 4n$

iv 48

b i



ii 6, 12, 18, 24, 30

iii $T = 6n$

iv 72

c i



ii 4, 7, 10, 13, 16

iii $T = 3n + 1$

iv 37

d i



ii 6, 10, 14, 18, 22

iii $T = 4n + 2$

iv 50

6 A

7 a i 6

ii 8

iii 10

iv 26

b $C = 2T + 2$

c 102

d 14

8 a i 8

ii 11

iii 14

iv 32

b $n = 3d + 2$

c 56

d 9

Exercise 12-04

1 a P

b N

c K

d T

e M

f S

g Q

h R

i L

j U

2 a (4, 1)

b (-2, 4)

c (-3, -2)

d (1, -3)

e (4, 4)

f (-5, 0)

g (0, 1)

h (0, -2)

i (5, 0)

j (3, -4)

3 a 2nd

b 4th

c 1st

d 2nd

e 3rd

4 $F(-5, 0)$, $R(-1\frac{1}{2}, 0)$, $O(0, 0)$, $Q(2, 0)$, $I(5, 0)$

5 $M(0, 3)$, $G(0, 1)$, $O(0, 0)$, $H(0, -2)$, $S(0, -4)$

6 D

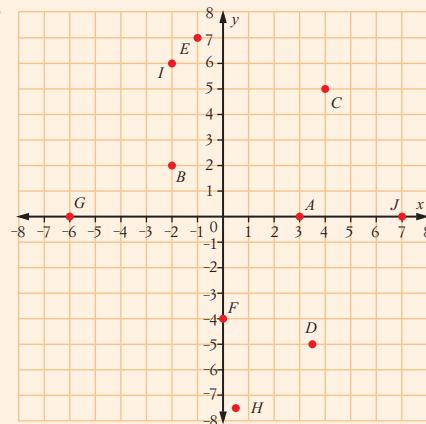
7 a 4th

b 3rd

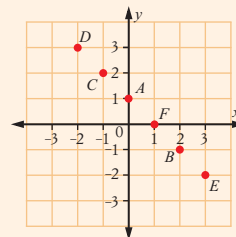
c 2nd

d 1st

8



9 b



c They all lie on the same line.

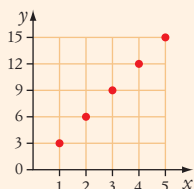
d Teacher to check.

Mental skills 12

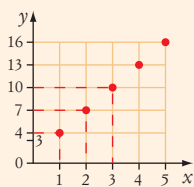
- 2 a \$700 b \$800 c 400 d 700
 e \$300 f \$400 g 250 h \$300
 i \$300 j 500

Exercise 12-05

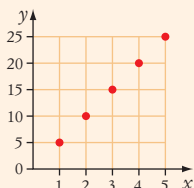
- 1 a i 3, 6, 9, 12, 15 ii $y = 3x$
 iii



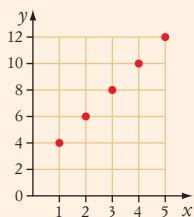
- b i 4, 7, 10, 13, 16 ii $y = 3x + 1$
 iii



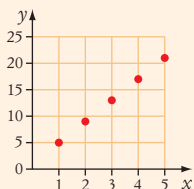
- c i 5, 10, 15, 20, 25 ii $y = 5x$
 iii



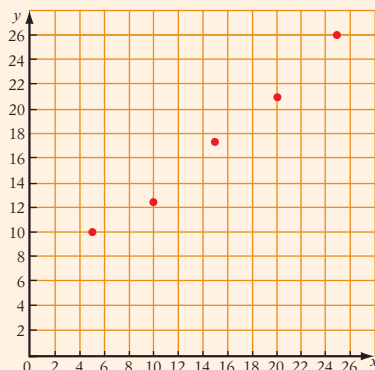
- d i 4, 6, 8, 10, 12 ii $y = 2x + 2$
 iii



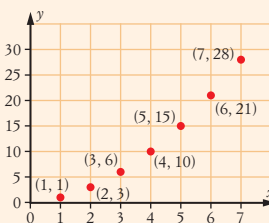
- e i 5, 9, 13, 17, 21 ii $y = 4x + 1$
 iii



- 3 There is a rough linear pattern but it is not perfect.



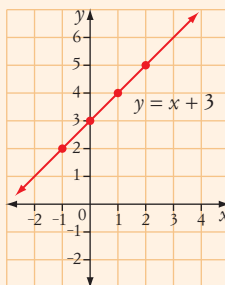
- 4 a 1, 3, 6, 10, 15, 21, 28
 b



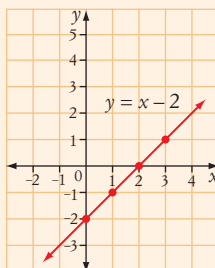
- c No d C

Exercise 12-06

- 1 a 2, 3, 4, 5

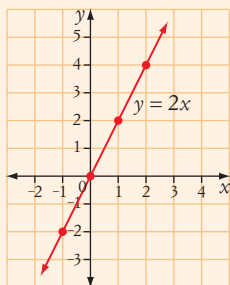


- b -2, -1, 0, 1

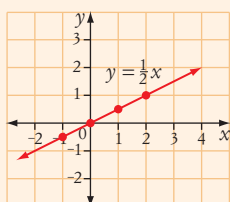


- 2 Yes

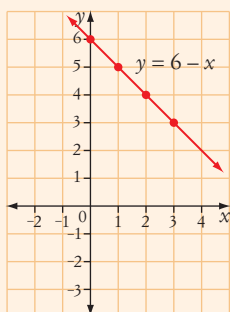
c -2, 0, 2, 4



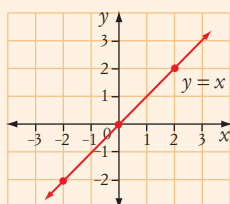
d $-\frac{1}{2}, 0, \frac{1}{2}, 1$



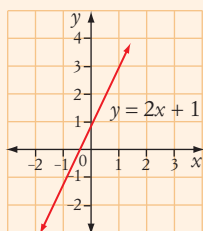
e 6, 5, 4, 3



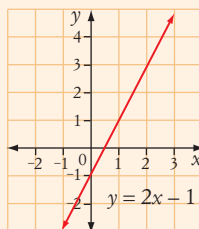
f -1, 0, 1, 2



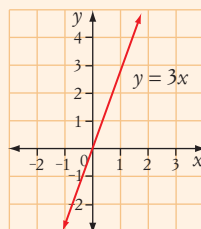
2 a



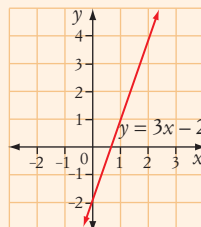
b



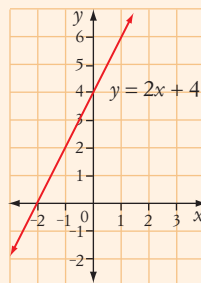
c



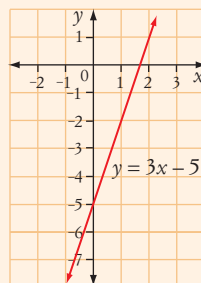
d



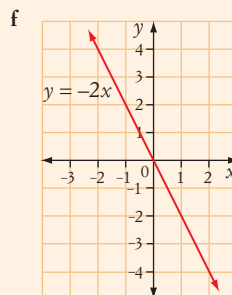
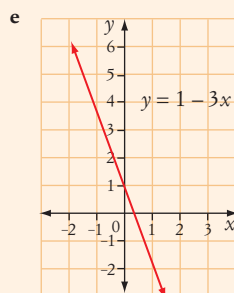
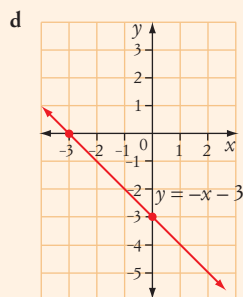
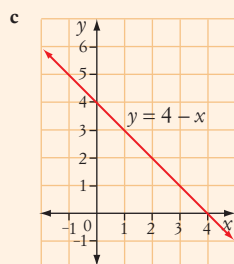
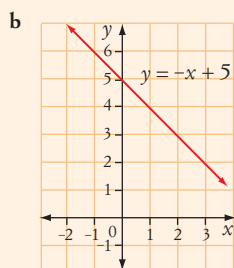
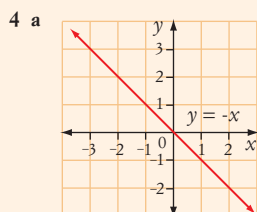
e



f



3 From left to right, the line is rising, going up.

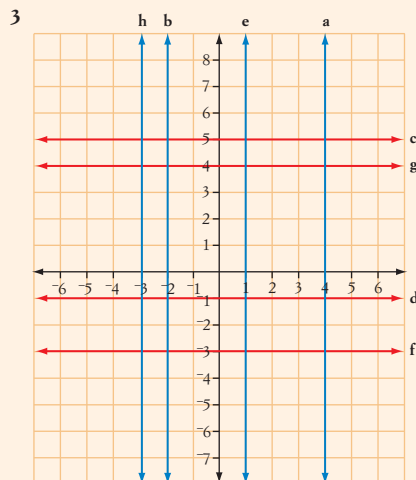


5 From left to right, the line is falling, going down.

- 6 a increasing b increasing c decreasing
 d increasing e decreasing f decreasing
 7 a yes b no c yes
 d yes e yes f no
 8 D

Exercise 12-07

- 1 a $-4, -3, -2, -1, y = x - 3$
 b $-4, -3, -2, -1, y = x - 2$
 c $-2, 0, 2, 4, y = 2x$ d $0, \frac{1}{2}, 1, 1\frac{1}{2}, y = \frac{1}{2}x$
 e $1, 0, -1, -2, y = -x$ f $5, 4, 3, 2, y = 2 - x$
 g $-3, -1, 1, 3, y = 2x - 1$ h $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, y = \frac{1}{2}x + 2$
 2 a $y = 6$ b $x = -4$ c $y = 3$
 d $x = 5$ e $y = -5$ f $x = 3$
 g $y = -3$ h $x = -1$

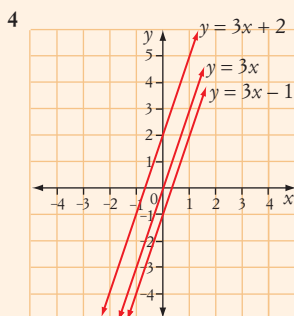


4 B

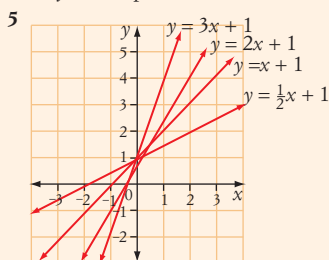
Exercise 12-08

1 a 4, 1 b $\frac{1}{2}$, -3 c 1, -5 d -1, 6

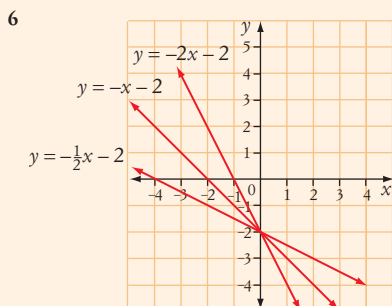
- 2 a point in the same direction, parallel
 b coefficient of x is 1
 c different y -intercepts
 d different constant terms
- 3 a all pass through (0, 0), y -intercept of 0
 b constant term of 0
 c point in different directions
 d different coefficients of x
 e $y = 3x$, 3 f $y = \frac{1}{4}x$, $\frac{1}{4}$
 g The greater the coefficient, the steeper the line.



- a parallel b coefficient of x is 3
 c y -intercepts d constant terms



- a y -intercept of 1
 b constant term of 1
 c different steepnesses, directions
 d coefficients of x
 e 3



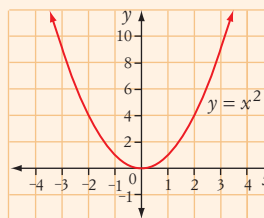
- a y -intercept of -2
 b constant term of -2
 c different steepnesses, directions
 d decreasing lines, pointing downwards
 e $-\frac{1}{2}$

7 coefficients of x

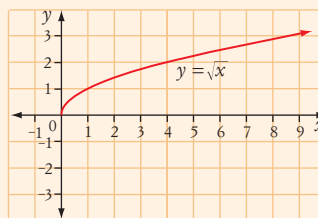
8 B

- 9 a steepness b y -intercept c decreasing
 d constant term e negative

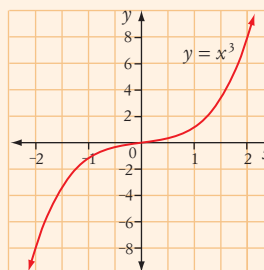
10 a 9, 4, 1, 0, 1, 4, 9



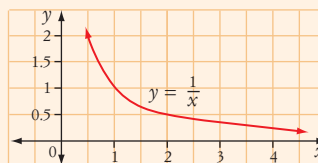
- b 0, 1, 1.41, 2, 2.24, 2.65, 3
 $y = \sqrt{x}$



- c -8, -3.375, -1, -0.125, 0, 0.125, 1, 3.375, 8



- d 2, 1, 0.6, 0.5, 0.4, 0.3, 0.29, 0.25, 0.2



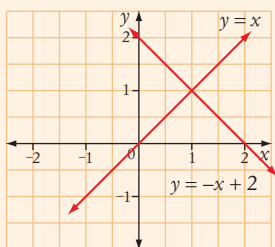
- 11 x is raised to a power, or square root of x , or appears in the denominator of a fraction.

Exercise 12-09

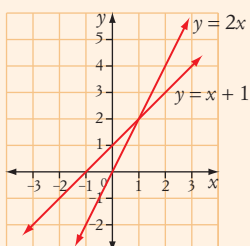
1 a $x = 3$	b $x = -2$	c $x = 1\frac{1}{2}$
2, 3 a $x = 3$	b $x = 4\frac{1}{2}$	c $x = -3$
4 a $x = -2$	b $x = 0$	c $x = 2\frac{1}{2}$
5 a $x = 3$	b $x = -3$	c $x = 0$
6 a $x = 4$	b $x = -3$	c $x = 2$
7 a $x = 4$	b $x = -\frac{1}{2}$	c $x = -2$
8 a $x = 4$	b $x = -4$	c $x = -2$

Exercise 12-10

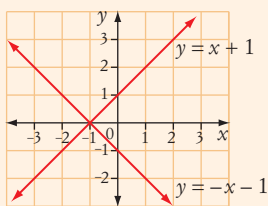
- 1 a (1, 1)



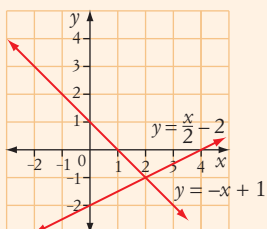
- b (1, 2)



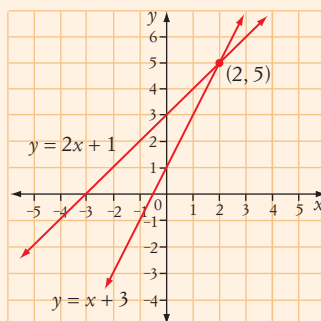
- c (-1, 0)



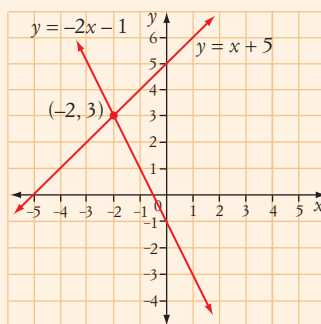
- d (2, -1)



- e (2, 5)

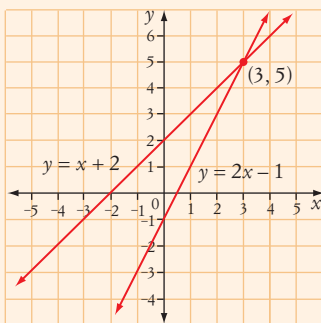


- f (-2, 3)

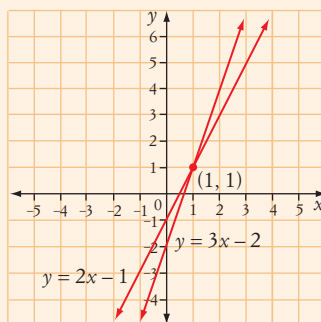


2 B

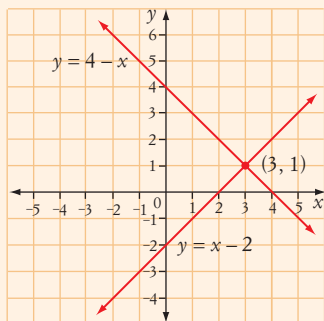
- 3, 4 a $x = 3$



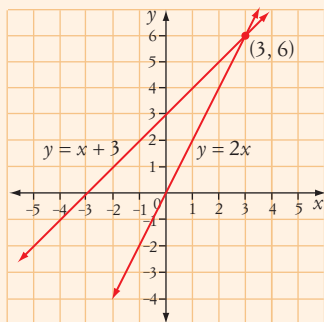
- b $x = 1$



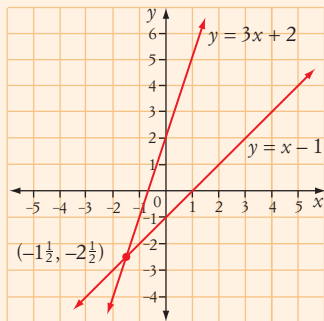
c $x = 3$



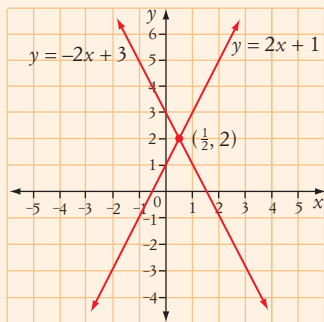
d $x = 1$



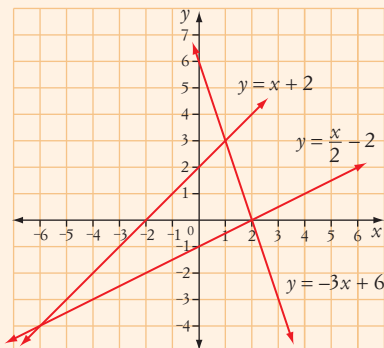
e $x = -1\frac{1}{2}$



f $x = \frac{1}{2}$

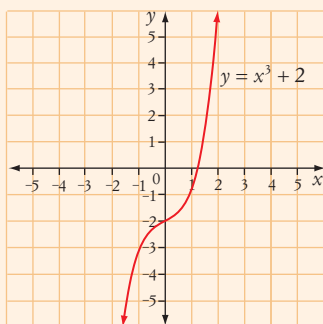


5 (1, 3), (2, 0), (-6, -4)

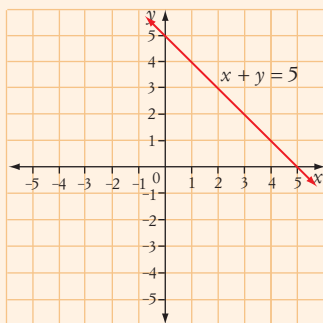


Power plus

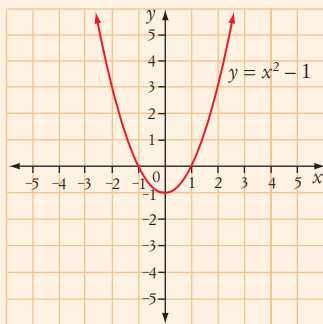
1 a nonlinear



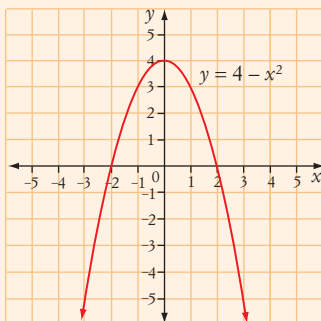
b linear



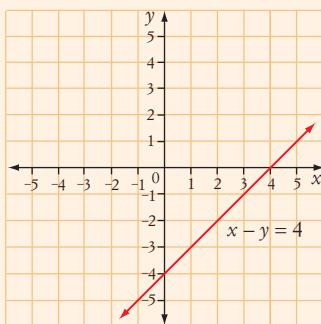
c nonlinear



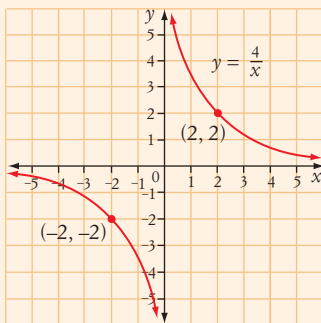
d nonlinear



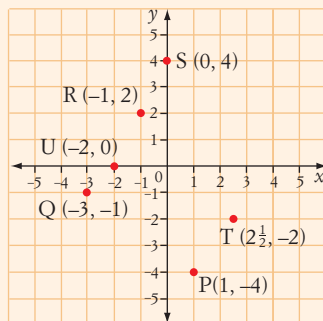
e linear



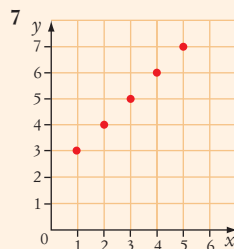
f nonlinear



5



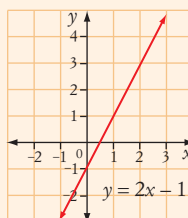
6 a S b R c U d T and P



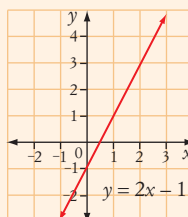
a 3, 4, 5, 6, 7

b $y = x + 2$

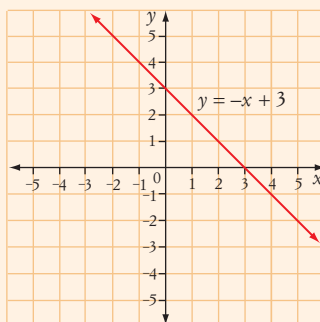
c



8 a

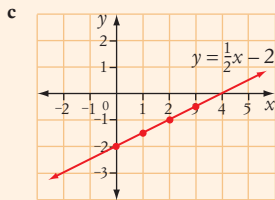


b

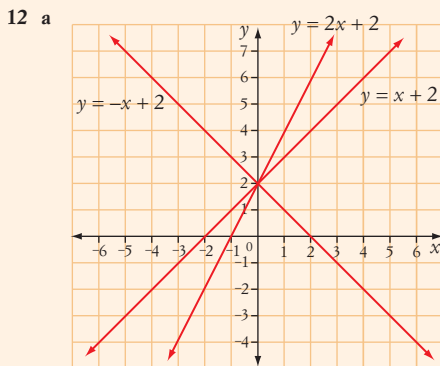
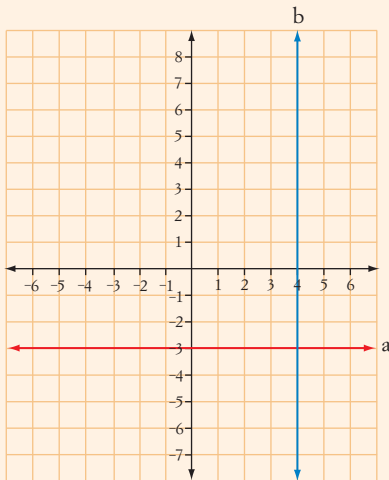


Chapter 12 revision

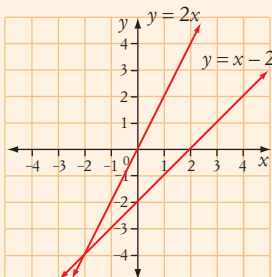
- 1 a 2, 3, 4, 5, 6 b -11, -9, -7, -5, -3
- c 6, 5, 4, 3, 2
- 2 a $y = x - 2$ b $p = 3m - 1$
- c $n = -2m + 10$
- 3 a 30 b $T = 4n + 2$
- 4 a 4, 7, 10, 13, 16, 25, 31
- b Number of toothpicks equals 3 times the number of rhombuses plus 1.
- c $y = 3x + 1$
- d i 37 ii 46



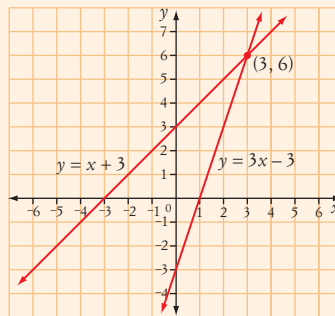
- 9 a No b No c Yes d Yes
 10 a $y = x + 3$ b $y = -2x - 1$
 11



- b y-intercept of 2
 c constant term of 2
 d different steepnesses/directions
 13 a $x = 3$ b $x = -1\frac{1}{2}$
 14 $(-2, -4)$

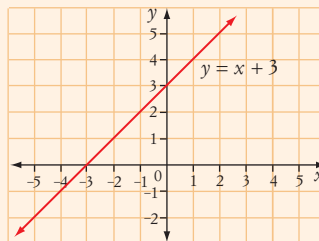


15 $x = 3, y = 6$

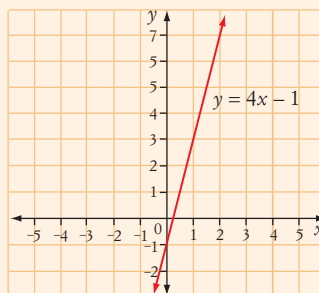


Mixed revision 4

- 1 a $a = 1$ b $m = 3$ c $x = -5$
 d $t = 18$ e $k = 77$ f $x = 2$
 g $x = 9$ h $x = -8$ i $a = 15$
 2 a 1 : 3 b 48 : 17 c 40 : 3
 d 1 : 12 e 6 : 5 f 1 : 4
 3 a 300 g for \$5.20 b 1.5 kg for \$3.30
 4 7, 9, 11, 13, 15
 5 a $q = 6$ b $x = 3$
 6 a $y = 2$ b $d = 10$
 7 a 5 h 25 min b 7 h 5 min 8 20
 9 a 25 cm b 20 m 10 1 kg
 11 a 33 km/h b 225 km 12 $y = 3x$
 13 a 5, 7, 8, 11, 17
 b The number of toothpicks is equal to the double the number of triangles plus 1.
 c $p = 2t + 1$ d 41
 14 $A(2, 3), B(4, 5), C(-2, 4), D(0, -5), E(-3, -2), F(1, 0), G(-1, -3)$.
 15 a 2, 3, 4, 5

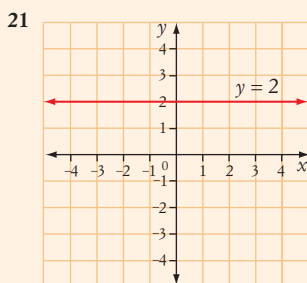


b $-5, -1, 3, 7$

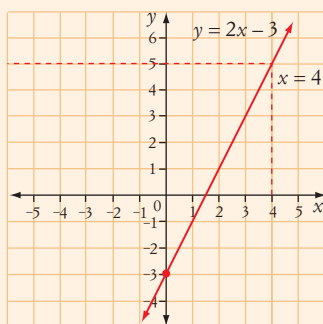


Answers

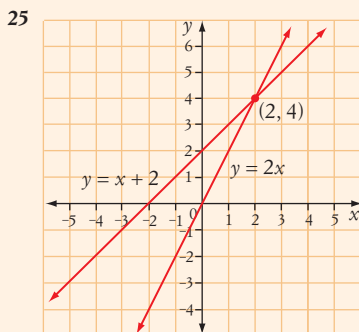
- 16 $h = 12$ 17 1.5 t 18 440 km
 19 a No b Yes 20 50 h



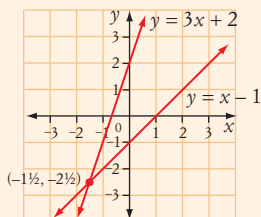
22 $x = 4$



23 $y = x + 2$ 24 12 midnight



- 26 a 1 p.m., for 1.5 hours b 26 km c 6.5 km/h
 27 $x = -1\frac{1}{2}$



General Revision

- 1 a 13 b 16 c 17
 d 3 e 5 f 18

2 $200 = 2^3 \times 5^2$

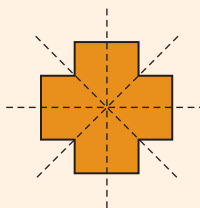
- 3 a $\frac{1}{4}$ b $\frac{3}{4}$ c $\frac{7}{20}$
 4 a $7\frac{1}{5}$ b $6\frac{7}{10}$ c $6\frac{1}{6}$
 5 a -6 b 4 c 5
 6 a $6ar$ b $-3p$ c $35u^2$
 d $5x + 9$ e $8p - 7q$ f $\frac{6u}{m}$

7 25%

- 8 a 4.8 b 5 c 5 d 7

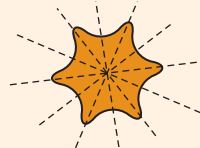
- 9 a $m = 34$, angle sum of a triangle
 b $m = 45$, angles on a straight line
 c $y = 68$, angle sum of a triangle
 d $m = 100$, isosceles triangle, exterior angle of a triangle or angles on a straight line
 e $x = 132$, vertically opposite angles
 f $x = 67$, angle sum of a quadrilateral
 g $y = 130$
 h $x = 62$, exterior angle of a triangle
 i $m = 62$, isosceles triangle, angles on a straight line or corresponding angles on parallel lines

10 a



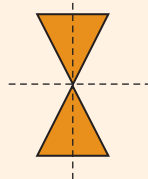
Rotational symmetry of order 4.

b



Rotational symmetry of order 6.

c



Rotational symmetry of order 2.

- 11 a $m = 17.7$ b $y = 16.6$ c $p = 25.5$ d $d = 76.0$
 12 a 6^2 b 2^7 c 3^2 d 5^3
 e 2^{12} f $2^6 \times 5^2$ g $3^0 = 1$ h 6
 13 a yes b yes c yes
 14 a 187 mm^2 b 144 cm^2 c 18.4 m^2
 15 a i isosceles ii acute-angled
 b i scalene ii obtuse-angled
 c i scalene ii right-angled
 16 a 80 000 b 0.49 c 650 000 000
 17 a $1\frac{5}{12}$ b $\frac{7}{15}$ c $\frac{7}{15}$ d $\frac{8}{15}$
 e $1\frac{5}{8}$ f $3\frac{2}{7}$ g $11\frac{1}{4}$ h $6\frac{3}{25}$

18 $\frac{3}{20}$

19 a $a = 5$

b $d = -5$

c $r = 3$

20 a QD

b QD

c QC

d C

21 a QR

b $\angle L$

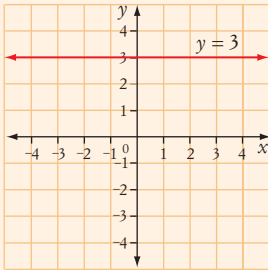
c $\triangle SRQ$

22 \$940.80

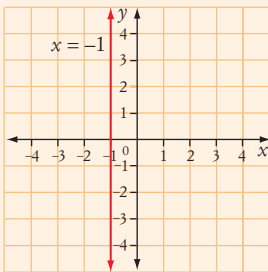
23 a $y = 3x + 4$

b $x = 3$

24 a



b



25 a $3a + 15$

b $20 - 8k$

c -15

26 a $6 : 7$

b $20 : 1$

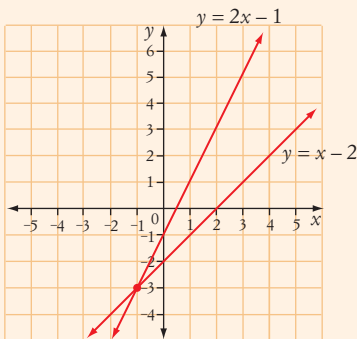
c $1 : 100$

27 280

28 a 6:40 p.m.

b 1:55 a.m.

29 a, c



b yes

d $x = -1$

30 a rectangle (or square)

b equilateral triangle

c rectangle (or rhombus)

d isosceles triangle

31 a i 43.98 m

ii 153.94 m²

b i 25.71 cm

ii 39.27 cm²

c i 66.85 cm

ii 272.55 cm²

d i 62.83 mm

ii 85.84 mm²

32 a $2(y + 4)$

b $5a(1 - 3b)$

c $-2b(4a - 1)$

33 \$20

34 a $\frac{1}{2}$

b $\frac{2}{7}$

c $\frac{11}{14}$

35 4900

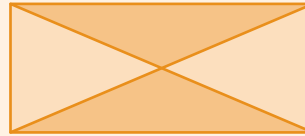
36 80 m²

37 a 62

b 42

c 42.5

38 a, b



c Diagonals are equal and bisect each other.

39 a RHS

b SAS

c SSS

d AAS

40 2.5 cm

41 B

42 a 400 cm³

b 288 cm³

c 300 mm³

d 910 m³

e 84 m³

f 4536.5 mm³

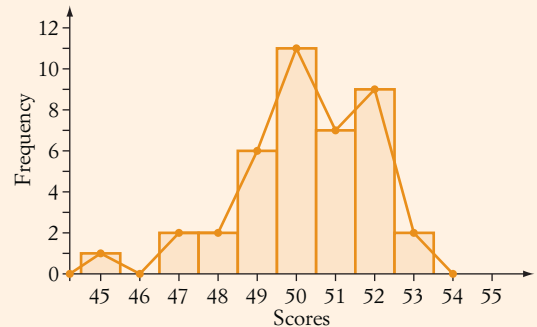
43 \$50.40

44 a

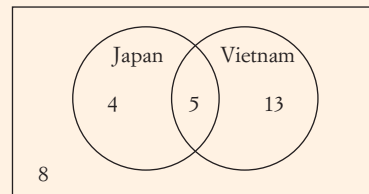
Score	Tally	f	f_x
45		1	45
47		2	94
48		2	96
49		6	294
50		11	550
51		7	357
52		9	468
53		2	106
Total		40	2010

b 50.25

c



45 a



b 8

c 4

d $\frac{7}{10}$

46 $x = 7$

47 6.1 m/s

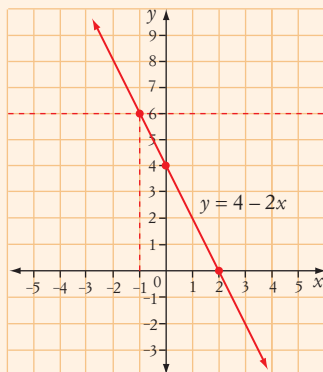
48 a 4 km

b 80 min, 7 km

c 20 min

d 8 km/h

49 $x = -1$



50 $y = 2x - 4$

Appendix 1 Calculator skills

Exercise A-01

1 a 5068	b 14.5	c 45
d 5.724	e 3.5	f 26.9
g 23	h 1.68	i 160
j 29.48	k -8.084	l 12.42
2 a \$19.15		b 85 cents
3 \$1875.50	4 7542 kg	5 2.4 m
6 \$11.25	7 23 trees	8 75.2 km/h
9 \$85.74	10 13 buses	
11 a \$41.20		b \$1318.40
12 53.4		

Exercise A-02

1 a 7	b -9	c -12
d -8	e 15	f -15
g 75	h 5	i -5
j 7	k -96	l 45
m -25	n 24	o 96
p 14	q -16	r 15
s -36	t 10	

Exercise A-03

1 a 196	b 529	c 216
d 1.728	e 169	f 343
g 128	h 6561	i 26
j 148	k 9	l 42
2 a 4.9	b 10.0	c 4.7
3 a 16	b 156	c 6
		d 4.6
		d 10

Exercise A-04

1 a $\frac{5}{8}$	b $\frac{8}{17}$	c $\frac{5}{8}$
d $\frac{8}{9}$	e $\frac{21}{25}$	f $\frac{1}{4}$
2 a $\frac{15}{8}$	b $\frac{47}{9}$	c $\frac{43}{4}$
3 a $2\frac{2}{9}$	b $8\frac{4}{5}$	c $2\frac{1}{4}$
4 a 0.5	b 0.8	c 0.875
d 0.36	e 0.17	f 0.5
5 a $\frac{7}{20}$	b $2\frac{1}{25}$	c $\frac{1}{1000}$
d $\frac{73}{200}$	e $1\frac{3}{50}$	f $\frac{13}{40}$
6 a $\frac{17}{20}$	b $1\frac{1}{14}$	c $\frac{1}{15}$
d $\frac{4}{21}$		
e $1\frac{1}{3}$	f $\frac{5}{24}$	g $1\frac{17}{30}$
h $\frac{35}{96}$		
i $1\frac{3}{4}$	j $19\frac{1}{5}$	k $6\frac{13}{20}$
l $8\frac{13}{24}$		
m $12\frac{3}{4}$	n 7	o $\frac{4}{27}$
p $19\frac{19}{60}$		
7 a $\frac{1}{8}$		b $\frac{3}{4}$
c $15\frac{4}{5}$ or 15.8		d $\frac{1}{6}$ or $0.1\dot{6}$
8 a $6\frac{3}{4}$ hours		b $33\frac{3}{4}$ hours

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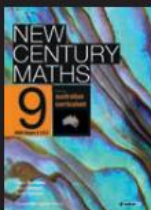
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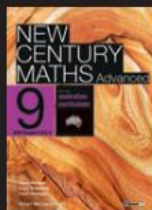
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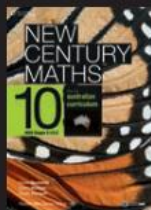
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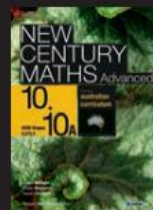
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