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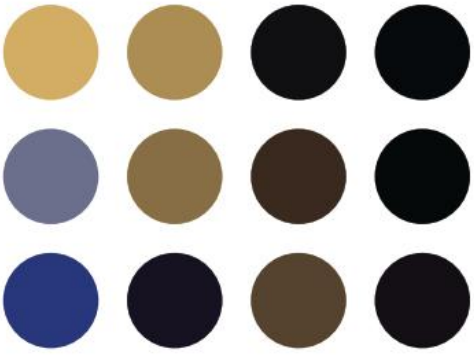

MATHEMATICS GENERAL 2

HSC Course

NEW CENTURY MATHS

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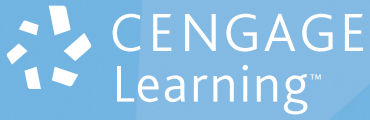

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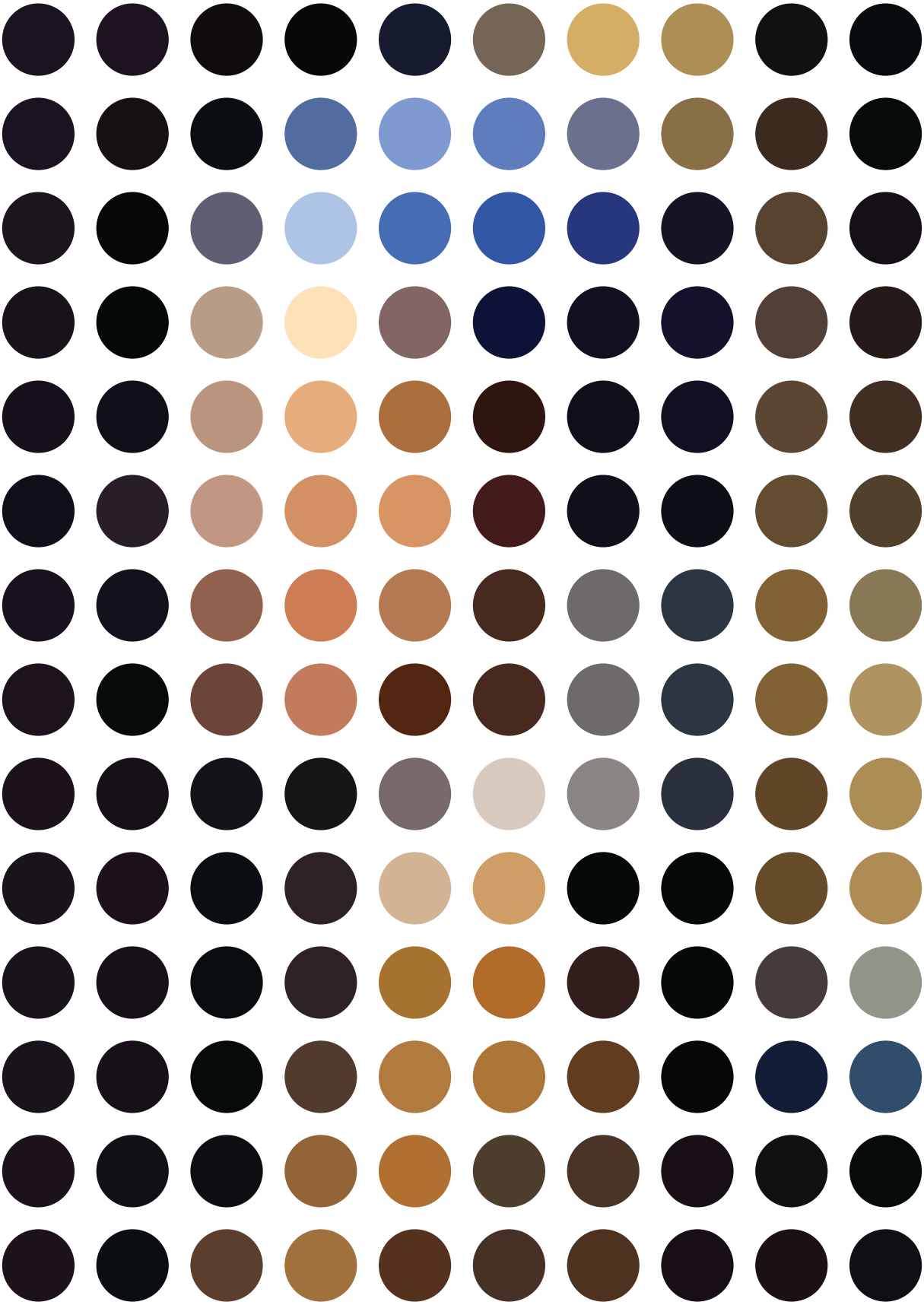
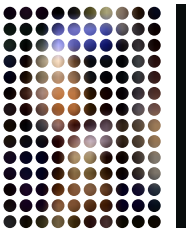


Paintings throughout history rely on strict mathematical foundations. They paved the way in perspective and grid structures, highlighting the thought process and intricate construction behind art. This extreme pointillism illustration has now taken the paintings one step further remixing and reducing Vermeer's *Girl with a Pearl Earring* to a simple mathematical colour dot equation. *Girl with a Pearl Earring* for the Twitter generation.



Left: Girl with a Pearl Earring remixed, © Gary Andrew Clarke

Right: Girl with a Pearl Earring, c.1665–6 (oil on canvas), Jan Vermeer (1632–75) The Bridgeman Art Library/Mauritshuis, The Hague

THIRD EDITION

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MATHEMATICS GENERAL 2

HSC Course

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CONTENTS

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PREFACE

First published in 2000, *New Century Maths 12 Mathematics General 2 HSC course* has been rewritten for the new Mathematics General syllabus (2012). In this book, teachers will find those familiar features that have made *New Century Maths* a leading mathematics series, such as clear worked examples, graded exercises, syllabus codes and references, Investigation, Technology, SkillCheck pre-chapter exercises, worked solutions to exercises, Topic overview with mind maps, glossaries, worksheets and topic tests.

The Mathematics General 2 course is designed for students who will work or study in fields that require a good mathematical or statistical background. We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant and meaningful examples of mathematics being used in society and industry. This 3rd edition also heralds the new interactive *NelsonNetBook* version of the text, and the *NelsonNet* student and teacher websites of print and multimedia resources.

We thank our families for their continued support and patience, and Alan Stewart and Anna Pang for swiftly transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing twenty-first century mathematics in the classroom and beyond.

ABOUT THE AUTHORS

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC examiner and has worked on the HSC Advice Line. Klaas has been the lead author of the *New Century Maths 9–10* series since 1995, including new editions for the Australian Curriculum in NSW.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. Her expertise is in using modelling, problem solving and ICT for the effective learning of mathematics, and she has presented workshops both nationally and for MANSW. Sarah is a co-author of the new editions of *New Century Maths 9–10* for the Australian Curriculum in NSW.

Margaret Willard has extensive experience writing units of work designed for distance education and was Manager at TAFE's distance education unit, OTEN (Open Training and Education Network). She has served on the executive board of MANSW (the Mathematical Association of NSW), managed its post-secondary programs and has presented at both MANSW and TAFE conferences.

Robert Yen has taught at Hurlstone Agricultural, Ambarvale and Eagle Vale high schools in southwest Sydney. He has co-edited *Reflections*, the MANSW journal, and written the General Mathematics HSC study guides for *The Sydney Morning Herald*. Robert has been writing for *New Century Maths* since 1995 and now works for Cengage Learning as a series editor.

Contributing authors

Deborah Smith wrote and edited many of the *NelsonNet* print resources (blackline masters) and teaches at Bulli High School.

Trisha Goss wrote the topic tests.

Natalie Caruso wrote the worked solutions to all exercise sets.

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SYLLABUS REFERENCE GRID

Strand and substrand	New Century Maths 12 Mathematics General 2 HSC Course Chapter
Financial mathematics	
FM4 Credit and borrowing	1 Loans and annuities
FM5 Annuities and loan repayments	1 Loans and annuities
Data and Statistics	
DS4 Interpreting sets of data	4 Statistical distributions
DS5 The normal distribution	11 Sampling and the normal distribution
DS6 Sampling and populations	11 Sampling and the normal distribution
Measurement	
MM4 Further applications of area and volume	3 Area and volume
MM5 Applications of trigonometry	5 The sine and cosine rules
MM6 Spherical geometry	9 Geometry of the Earth
Probability	
PB2 Multistage events and applications of probability	7 Probability
Algebra and Modelling	
AM3 Further algebraic skills and techniques	2 Equations and linear functions
AM4 Modelling linear relationships	2 Equations and linear functions
AM5 Modelling non-linear relationships	10 Non-linear functions
Mathematics and Health	
FSHe1 Body measurements	8 Health and medicine
FSHe2 Medication	8 Health and medicine
FSHe2 Life expectancy	8 Health and medicine
Mathematics and Resources	
FSRe1 Water availability and usage	6 Water usage
FSRe2 Dams, land and catchment areas	6 Water usage
FSRe3 Energy and sustainability	12 Energy and sustainability

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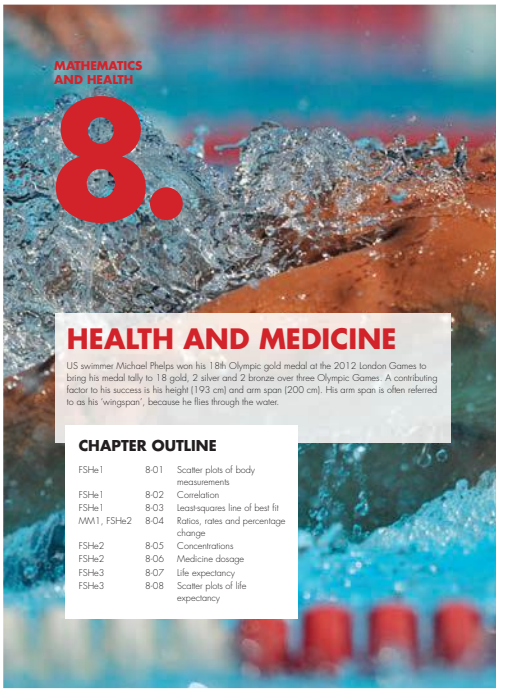
USER'S GUIDE



AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread with a **Chapter outline** showing the content of the chapter with syllabus codes, a list of content descriptions, and a **Terminology** word list.
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter.

TERMINOLOGY

biometric data
concentration
dosage
life expectancy
microgram
negative
regression
strong
bivariate data
correlation
drip rate
line of fit
milligram
positive
scatter plot
trendline
causality
correlation coefficient (*r*)
leastsquares
linear
moderate
predict
solution
weak

Skillcheck

Assignment 8

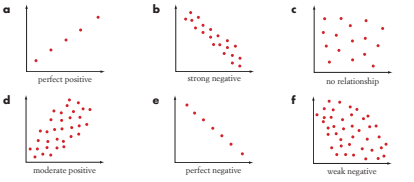
1 Convert:
a 500 g to kg **b** 3 000 000 mg to g **c** 684 g to mg
2 If $y = 0.85x + 22.5$, find the value of:
a y when $x = 180$ **b** x when $y = 35.25$
3 Find the gradient of the line that passes through (–1, 5) and (4, 8).
4 Increase $600 by 5%, then decrease the result by 5%.
5 If 15 out of 130 females smoke, what percentage of females do not smoke? Answer correct to three significant figures.
6 A 1 L salt solution contains water and 50 mL of salt. Find the ratio of:
a salt to water **b** salt to solution.

8-01 Scatter plots of body measurements

Body measurement

A page of scatter plots

A **scatter plot** or **scattergram** is a graph of points on the number plane. Each point represents the values of two different variables, so scatter plots represent **bivariate data** (data relating to two variables). By observing the pattern of the dots in a scatter plot, we can find the direction and strength of the relationship represented. The closer the dots, the stronger the relationship.



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EXAMPLE 11

A drug solution for a child contains the drug in the concentration 50 µg/mL of solution.

a How many milligrams of the drug are needed to make 100 mL of the solution?

b How many litres of the solution contain 2 g of the drug?

Solution

a Concentration = 50 µg/mL

Amount of drug in 100 mL = 100 × 50 µg

= 5 000 µg

= 5 mg **1 mg = 1 000 µg**

b Concentration = 50 µg/mL

= 0.000 05 g/mL **1 g = 1 000 000 µg**

Volume containing 2 g = 2 ÷ 0.000 05 mL

= 40 000 mL

= 40 L **1 L = 1 000 mL**

Exercise 8-05 Concentrations

Example 10

1 Which of the following is equal to 200 milligrams? Select **A**, **B**, **C** or **D**.

A 0.2 g **B** 200 000 µg **C** 0.002 kg **D** 0.2 µg

2 Convert:

a 50 mg to µg **b** 80 kg to g **c** 100 g to mg

d 650 mL to L **e** 75 mg to g **f** 10.5 L to mL

3 A nurse described the concentration of a drug as 5 mg/mL. Describe what this means.

4 Which of these drug concentrations is the strongest (or most concentrated)? Select **A**, **B**, **C** or **D**.

A 2 mg/mL **B** 1 mg/10 mL **C** 5 g/L **D** 0.5 mg/10 mL

5 A solution of 400 mL of disinfectant contains 24 g of alcohol. Express the concentration in g/L.

Example 11

6 A solution for a sick child contains a drug with a concentration of 8 mg/mL.

a How many grams of the drug are needed to make 500 mL of the solution?

b How many litres of the solution would 10 g of the drug make?

7 A sheep solution contains 5 g of cortisone per 20 mL of solution. If 350 sheep need 2.5 g of cortisone each, how many litres of sheep solution are needed?

8 An antibiotic in a syrup has a concentration of 25 mg/mL. Calculate the:

a number of millilitres of syrup that contain 200 mg of antibiotic

b number of grams of antibiotic in 1.5 L of syrup.

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JUST FOR THE RECORD

Quit for life

The World Health Organisation estimates that more than 1 billion people in the world are regular smokers, with 80% of smokers living in low- and middle-income countries. Each pack of cigarettes a person smokes reduces his or her life by 28 minutes and the life expectancy of a typical smoker is reduced by 25 years.

During the 1960s, there was an increase in deaths from lung cancer and cardiovascular disease. This was attributed to the increase in cigarette smoking. Public health campaigns (such as grim warnings and images on cigarette packs), public health regulations (such as bans on smoking in public places) and taxes on tobacco have all had an effect on the reduction in smoking rates. Australia introduced plain packaging for cigarettes in 2012 with the aim of reducing the number of smokers from around 15% of the population to 10% by 2018.

How does plain packaging of cigarettes deter smoking?

TECHNOLOGY: LIFE EXPECTANCY CALCULATORS

1 Search on the Internet for a life expectancy calculator like the one below. It calculates life expectancy based on age and country. It does not account for sex, smoking status, height/weight, education, exercise, medical conditions, diseases or injuries.

Calculate your Life Expectancy

Select Your Age* 17

Select Your Country* Australia

You are expected to live up to the age of 81.72. You have another 65 years to enjoy life. So live happily.

2 Search on the Internet for another life expectancy calculator that requires you to answer more questions relating to your lifestyle and family so that it determines a more accurate life expectancy value. Compare the answer given by this calculator to the answer given by the calculator you found in Question **1**.

3 A calculator that shows how many years cigarette smoking can take out of your lifespan can be found at the website www.uwhealth.org, clicking on **Health information**, then **Interactive tools** under General health information, then **How does smoking affect your lifespan?**

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IN EACH CHAPTER

- Important facts and formulas are highlighted in a **Summary** box.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.
- Worked solutions to all exercise questions are provided on the *NelsonNet* teacher website.
- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.
- **Technology** promotes ICT in the classroom, using spreadsheets, graphics calculators and the Internet.
- **Just for the record** contains interesting facts and applications of the mathematics learnt in the chapter.

7 The heights and weights of eight teenagers were recorded.

Height (*h* cm)	155	168	162	175	163	178	180	165
Weight (*w* kg)	50	73	55	84	62	76	80	59

a Draw a scatter plot for the data.

b Describe the pattern of the data.

c Describe the relationship between the student's weight and height.

8 A paleontologist measured the thigh bone (femur) and upper arm bone (humerus) of some fossils. The results are shown in the table.

Femur (*f* cm)	60	73	56	50	67	62	72	59
Humerus (*h* cm)	70	78	59	40	70	61	71	58

a Plot the data on a scatter plot.

b Describe the relationship between the lengths of the two bones.

c Do you think that there is a linear relationship between femur and humerus lengths in humans? Justify your answer.

INVESTIGATION

ANCIENT BODY MEASUREMENTS

Shutterstock.com/Antonio Gutierrez

In ancient and medieval times, people used parts of the body to measure lengths.

A **palm** or **handbreadth** was the width of your four fingers pressed together and is about 7.5 cm.

A **hand span** was the distance between thumb and little finger when the fingers are spread and is about 22.5 cm.

A **foot** was the length of King Henry I's foot, defined as 12 inches or about 30.5 cm. An average person's foot today is 23.5 cm.

A **cubit** was the length of the forearm, from elbow to tip of middle finger. Since people from ancient cultures differed in size, the cubit ranged from 44 cm to 52 cm. Today, the average adult cubit is 46 cm.

1 Show that:

a 1 cubit = 6 palms **b** 1 cubit = 2 spans **c** 1 span = 3 palms

2 Noah's ark was described to be 300 cubits long, 50 cubits wide and 30 cubits high. Convert these measurements to:

a spans **b** palms **c** feet **d** metres

3 Collect measurements for the cubit, foot, palm and span from 10–12 people.

a Find the average for each measurement and compare with the values given above.

b Select any two measurements, graph them on a scatter plot and determine whether there is a relationship between them.

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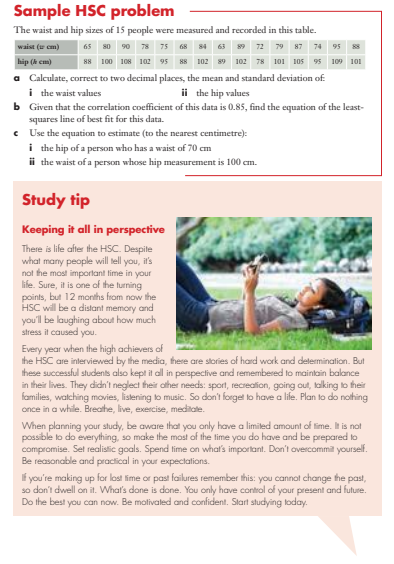


Sample HSC problem

The waist and hip sizes of 15 people were measured and recorded in this table.

waist (x cm)	65	80	90	78	75	68	84	63	89	72	79	87	74	95	88
hip (h cm)	88	100	108	102	95	88	102	89	102	78	101	105	95	109	101

a Calculate, correct to two decimal places, the mean and standard deviation of:
i the waist values **ii** the hip values

b Given that the correlation coefficient of this data is 0.85, find the equation of the least-squares line of best fit for this data.

c Use the equation to estimate (to the nearest centimetre):
i the hip of a person who has a waist of 70 cm
ii the waist of a person whose hip measurement is 100 cm.

Study tip

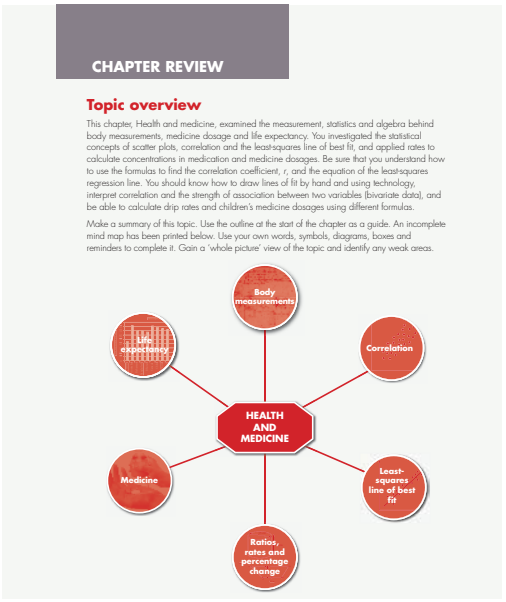
Keeping it all in perspective

There is life after the HSC. Despite what many people will tell you, it's not the most important time in your life. Sure, it is one of the turning points, but 12 months from now the HSC will be a distant memory and you'll be laughing about how much stress it caused you.

Every year when the high achievers of the HSC are interviewed by the media, there are stories of hard work and determination. But these successful students also kept it all in perspective and remembered to maintain balance in their lives. They didn't neglect their other needs: sport, recreation, going out, talking to their families, watching movies, listening to music. So don't forget to have a life. Plan to do nothing once in a while. Breathe, live, exercise, meditate.

When planning your study, be aware that you only have a limited amount of time. It is not possible to do everything, so make the most of the time you do have and be prepared to compromise. Set realistic goals. Spend time on what's important. Don't overcommit yourself. Be reasonable and practical in your expectations.

If you're making up for lost time or past failures remember this: you cannot change the past, so don't dwell on it. What's done is done. You only have control of your present and future. Do the best you can now. Be motivated and confident. Start studying today.



AT THE END OF EACH CHAPTER

- **Sample HSC problem** demonstrates a typical HSC question using content learnt from the chapter.
- **Study tip** provides useful advice for senior students in revising coursework and preparing for exams.
- **Topic overview** concludes the chapter and includes a mind map exercise.
- **Revision** is a set of review exercises that are linked to the relevant exercise set.
- **Practice papers** after every three chapters revise the skills and knowledge of those chapters (the *NelsonNet* teacher website also contains a Practice HSC Exam and three Mini HSC exams).

AT THE END OF THE BOOK

- **Instructional glossary** describes mathematical 'doing' words and HSC-exam-style keywords and verbs.
- **Mathematical glossary** is a comprehensive dictionary of course terminology.
- All terms printed in **red** in this book appear in the glossary.
- **Answers** and **Index** are included.

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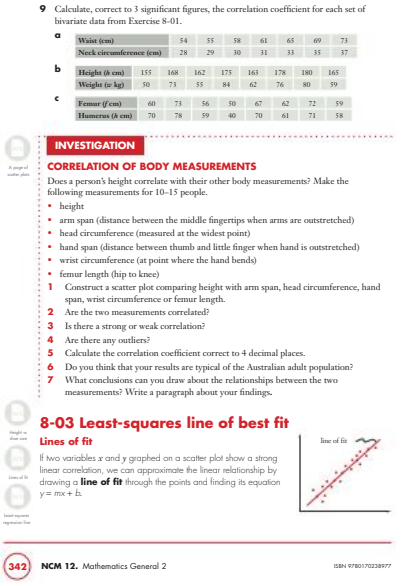

9 Calculate, correct to 3 significant figures, the correlation coefficient for each set of bivariate data from Exercise 8-01.

a

Waist (cm)	54	55	58	61	65	69	73
Neck circumference (cm)	28	29	30	31	33	35	37

b

Height (h cm)	155	168	162	175	163	178	180	165
Weight (w kg)	50	73	55	84	62	76	80	59

c

Femur (f cm)	60	73	56	50	67	62	72	59
Humerus (h cm)	70	78	59	40	70	61	71	58

INVESTIGATION

CORRELATION OF BODY MEASUREMENTS

Does a person's height correlate with their other body measurements? Make the following measurements for 10-15 people.

- height
- arm span (distance between the middle fingertips when arms are outstretched)
- head circumference (measured at the widest point)
- hand span (distance between thumb and little finger when hand is outstretched)
- wrist circumference (at point where the hand bends)
- femur length (hip to knee)

1 Construct a scatter plot comparing height with arm span, head circumference, hand span, wrist circumference or femur length.
2 Are the two measurements correlated?
3 Is there a strong or weak correlation?
4 Are there any outliers?
5 Calculate the correlation coefficient correct to 4 decimal places.
6 Do you think that your results are typical of the Australian adult population?
7 What conclusions can you draw about the relationships between the two measurements? Write a paragraph about your findings.

8-03 Least-squares line of best fit

Lines of fit

If two variables x and y graphed on a scatter plot show a strong linear correlation, we can approximate the linear relationship by drawing a **line of fit** through the points and finding its equation $y = mx + b$.

line of fit

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SUMMARY

A line of fit:

- represents most or all of the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible

EXAMPLE 5

The heights and arm spans of 10 male swimmers are shown in the table (from Example 1).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Graph the data on a scatter plot and draw a line of fit to represent all data points.
b Find the equation of the line of fit.
c Use the graph and the equation to estimate the arm span of a male swimmer who is 188 cm tall.
d Use the graph and the equation to estimate the height of a male swimmer with an arm span of 195 cm.
e Is it possible to predict the arm span of a female swimmer who is 165 cm tall?

Solution

a Use a transparent ruler to draw a line of fit. Points not on the line should be roughly equally-spaced from the line.

Height v Arm span

Your line of fit may differ from the one shown here but should look similar.

b The equation of the line is of the form $s = mh + b$.

To find the gradient m, choose two points from the line, say (192, 194) and (199, 202).

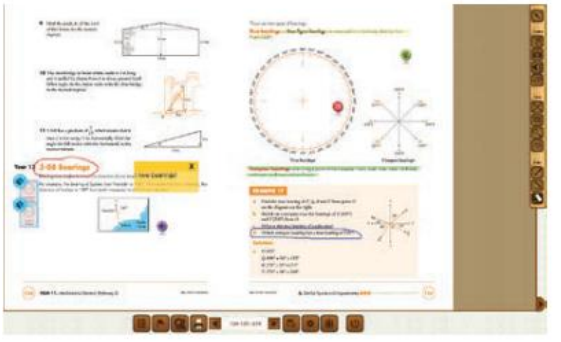
$$m = \frac{202 - 194}{199 - 192}$$
$$= \frac{8}{7}$$
$$= 1.1428\ldots$$
$$\approx 1.14$$

Rounding to two decimal places.

$\therefore s = 1.14h + b$

To find the vertical intercept b, substitute one point on the line, say (199, 202) again.

ISBN 9780170238977 8. Health and medicine 343



NELSONNET STUDENT WEBSITE

- Margin icons link to print (PDF) and multimedia resources found on the *NelsonNet* student website **www.nelsonnet.com.au**. These include:

  Worksheet Skillsheet

Greedy pig game Ratios

- **Worksheets** that are write-in enabled PDFs
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **ExamView quizzes:** interactive and self-marking.

NELSONNET TEACHER WEBSITE

The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

- **Teaching program,** in Microsoft Word and PDF formats
- **Topic tests,** in Microsoft Word and PDF formats
- **Worked solutions** to each exercise set
- **Chapter PDFs** of the textbook
- **ExamView** exam-writing software and questionbanks
- **Resource Finder:** search engine for *NelsonNet* resources.

NELSONNETBOOK

NelsonNetBook is the digital interactive version of this book found on *NelsonNet*.

- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students.

1.

LOANS AND ANNUITIES

Since the Global Financial Crisis (GFC) of 2008, people's ideas and expectations about saving for the future have changed significantly. Dramatic changes in the share market take longer to recover from, and there is a greater risk associated with investing in shares when their outcomes are so uncertain. Recently, **annuities** have again become popular, especially for retirees, as they are low-risk and involve investing money to provide a series of guaranteed, regular payments over a set number of years (or even for life).

CHAPTER OUTLINE

IN THIS CHAPTER YOU WILL:

- apply the simple interest formula $I = Prn$ to flat rate loans to calculate interest, repayments and total amount repaid
- apply the simple interest formula to problems involving term payments
- use a table to calculate and examine the progress of a reducing balance loan
- use the compound interest formula $FV = PV(1 + r)^n$ to calculate future value, present value and compound interest earned
- calculate the costs involved in credit card purchases, interest and fees
- use a future value table to calculate the future value of an annuity (FVA), the interest earned and the contribution per period
- use a present value table to calculate the present value of an annuity (PVA), the interest earned and the contribution per period
- use a present value table to calculate loan repayments and the total amount repaid
- use a loan repayment table to calculate repayments, total amount and interest paid for a reducing balance loan
- calculate the fees and charges associated with a home loan
- compare different types of home loans using graphs and calculations, including different payment options

TERMINOLOGY

annuity
balance
cash advance
compound interest
contribution
credit card
deferred payment
deposit
fee
flat interest
flat rate loan
future value
future value of an annuity (FVA)
hire purchase
interest-free period
loan repayment table
period
present value
present value of an annuity (PVA)
principal
reducing balance loan
repayment
term
term payments

Assignment 1

Percentage calculations

Mental percentages

SkillCheck

1 How many:

a fortnights in 18 months?
b months in 25 years?
c weeks in 20 years?
d quarters in $6\frac{1}{2}$ years?

2 Complete each blank.

a 12% p.a. = _____% per quarter
b 8% p.a. = _____% per month

3 Evaluate each expression correct to two decimal places.

a $1000(1 + 0.03)^4$
b $\dfrac{4000}{(1+0.008)^{10}}$

4 Find the simple interest earned when \$5000 is invested for 8 years at 6.4% p.a.

5 If \$2500 is invested at 5% p.a. for 3 years, compounded yearly, find correct to the nearest cent:

a the final amount of the investment
b the compound interest earned.

6 If \$3000 is invested at 0.72% per month for 1 year, compounded monthly, find correct to the nearest cent the compound interest earned.

Simple interest

Flat rate loan repayments: Spreadsheet

1-01 Flat rate loans

A **flat rate loan** is one where simple (flat) interest is charged on the principal (loan amount) for the term of the loan. Personal loans used for purchasing goods, such as cars or computers, are examples of flat rate loans and usually have short terms of 1 to 5 years. Fees and charges apply and can be added to the loan or paid upfront.

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SUMMARY

The simple interest formula

$$I = Prn$$

where I = interest

P = principal or initial amount

r = interest rate per period, expressed as a decimal

n = number of periods.

EXAMPLE 1

Billy borrowed \$16 500 for 4 years at 9% p.a. flat interest to buy a new motorbike.

a How much interest does Billy have to pay?

b What amount does he repay altogether?

c What is his monthly repayment?

Solution

a $P = \$16\,500$, $r = 0.09$, $n = 4$

$I = Prn$

$= \$16\,500 \times 0.09 \times 4$

$= \$5940$

Billy pays \$5940 in interest.

b Amount to repay $= \$16\,500 + \$5940$ **principal + interest**

$= \$22\,440$

c Monthly repayment $= \$22\,440 \div 48$ **4 years = 4 × 12 months = 48 months**

$= \$467.50$

EXAMPLE 2

Lauren took out a personal loan for \$8200 over 3 years at 12% p.a. flat interest to buy a jet ski. The bank charges were stamp duty \$39.50, loan insurance \$67.50 and establishment fee \$130, and Lauren added these charges to the principal.

a Find her fortnightly repayment.

b Find the interest saved if she paid the charges upfront.

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Solution

a Total amount borrowed = \$8200 + \$39.50 + \$67.50 + \$130
= \$8437

Interest is paid on the total amount borrowed, including bank charges.

$P = \$8437$, $r = 0.12$, $n = 3$

$I = Prn$
$= \$8437 \times 0.12 \times 3$
$= \$3037.32$

Total amount to repay = \$8437 + \$3037.32
= \$11 474.32

principal + interest

Fortnightly repayment = \$11 474.32 ÷ 78
= \$147.10666…
≈ \$147.11

3 years = 3 × 26 fortnights
= 78 fortnights
Always round up for repayments.

b $P = \$8200$, $r = 0.12$, $n = 3$

$I = Prn$
$= \$8200 \times 0.12 \times 3$
$= \$2952$

Interest saved = \$3037.32 – \$2952
= \$85.32

Example 1

Exercise 1-01 Flat rate loans

1 Jake borrowed \$12 000 at 10% p.a. flat interest over 5 years to buy a second-hand car.

a What total amount must he repay?

b What is his monthly repayment?

2 Jack invests \$3000 for 1 year and 8 months. The flat rate of interest is calculated at 8% per annum. What is the total value of the investment at the end of this period? Select the correct answer **A**, **B**, **C** or **D**.

A \$6400 **B** \$3400 **C** \$3432 **D** \$3420

3 Georgia took a holiday loan of \$13 500 at 15% p.a. flat interest to be repaid in fortnightly repayments over 3 years.

a How much will her holiday cost altogether?

b How much is each fortnightly repayment?

c How much would she save if the interest rate was reduced to 14% p.a.?

4 Ziad obtained a \$2900 loan at flat interest of 11.2% p.a. to buy a guitar. He paid monthly repayments over 2 years.

a How much interest did he pay?

b What was the monthly repayment?

c How much less would he repay per month if the interest rate was reduced by 0.5%?

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5 Mai bought a new washing machine and dryer and borrowed $2300 to help with the purchase. If the simple interest rate is 14.6% p.a. and the term of the loan is 18 months, find her fortnightly repayment. Select **A**, **B**, **C** or **D**.

A $71.90 **B** $154.99 **C** $71.89 **D** $107.84

Example 2

6 Santo bought a bed for $2850 and paid it off weekly over 18 months at 18.55% p.a. flat interest. If the stamp duty of $29, establishment fee of $125 and loan protection insurance of $45 were included in the amount borrowed, find:

a the amount borrowed

b the weekly repayment, correct to the nearest cent

c the amount of interest saved if the charges are paid up front.

7 Natalie borrows $14 500 over 3 years at 13.5% p.a. flat interest to help towards her wedding. If she adds the $37.50 stamp duty and $115 per year loan insurance to the amount borrowed, how much does she repay per month?

8 The NCM Bank advertised the following flat interest rates per annum for secured loans.

Loan security	Amount of loan	
	Greater than $20 000	$20 000 or less
New motor vehicle	9.4%	9.4%
Used motor vehicle	9.9%	10.75%
New or used motorbike, caravan or boat	10.49%	10.75%

Ali borrowed $22 500 over 5 years for a new car.

Bryce borrowed $9000 over 20 months for a second-hand motorbike.

Chloe borrowed $24 000 over $4\frac{1}{2}$ years for a new boat.

For each person find:

a the amount of interest charged

b their monthly repayment.

9 The Family Credit Union advertised the following personal loans.

Loan type	Terms and conditions	Interest rate (p.a.)
New car loan	Loan availability—up to $40 000 Maximum term—10 years	9.25%
New computer loan	Loan availability—$2000 to $40 000 Maximum term—5 years	7.25%
Other types	Loan availability—$2000 to $40 000 Maximum term—4 years	8.5%

Calculate the repayment for each personal loan.

a $28 400 for a new car to be repaid quarterly over 3 years.

b $32 000 for home improvements to be repaid fortnightly over 4 years.

c $2000 for a new computer to be repaid monthly over 18 months.

TECHNOLOGY: FLAT RATE LOAN REPAYMENTS

1 Create the spreadsheet shown to calculate repayments per period for a flat rate loan. Enter appropriate formulas for cells B6 to B8.

2 Show that the monthly repayment required to pay off a loan of $10 000 in 4 years at 9% p.a. interest is $283.34.

3 Change the values in your spreadsheet (cells B1 to B4) to calculate:

a the monthly repayment needed to pay off a $6500 loan in 3 years at 6.5% p.a. interest

b the fortnightly repayment required to pay off a loan of $4750 in 18 months at 8% p.a. interest.

	A	B	C	D
1	Principal	$10,000		
2	Interest rate (% p.a.)	9		
3	Term of loan (years)	4		
4	No. of periods per year	12		
5				
6	Total Interest			
7	Total amount to repay			
8	Repayment per period			
9				

4 Suppose you want to buy a car for $25 000. Find the current flat interest rate for a car loan plus any fees and charges. If you can afford to repay $200 per month, would you be able to pay the car off in 5 years?

Term payments: Spreadsheet

1-02 Term payments

Term payments means 'paying off' an expensive item over time, sometimes after a **deposit** is made. If the customer does not keep up with the regular payments, higher interest may be charged or the item may be repossessed (taken back).

A term payments plan is sometimes called **hire purchase** because the customer actually hires the item until it is completely paid off.

EXAMPLE 3

A refrigerator is advertised for $1200 cash or on terms at 10% deposit and $57 per month for 2 years. If buying on terms:

a how much more will the refrigerator cost than if it was bought for cash?

b what is the percentage flat rate of interest charged per annum, correct to one decimal place?

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Solution

a Total cost on terms $= 10\% \times \$1200 + \$57 \times 24$ **2 years = 24 months**

$= \$1488$

Difference between terms and cash $= \$1488 - \$1200$

$= \$288$ **Interest paid**

b Balance after 10% deposit made $= \$1200 - 10\% \times \$1200$

$= \$1080$

$P = \$1080$, $I = \$288$, $n = 2$

$I = Prn$

$\$288 = \$1080 \times r \times 2$

$= \$2160r$

$r = \dfrac{\$288}{\$2160}$

$= 0.13333\ldots$

$= 13.333\ldots\ \%$

$\approx 13.3\%$

The flat rate of interest charged is 13.3% p.a.

EXAMPLE 4

'Deferred' means 'delayed'

With a **deferred payment plan**, you do not make any repayments until a later date, such as after three years.

Two competing retail stores offered different deferred payment plans to customers wanting to buy a dishwasher for $1350.

'Unpaid balance' and 'outstanding amounts' both mean the remaining part of the loan still owing.

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a Which plan should Peter choose if he wants more time to save up enough to pay off the dishwasher before any interest is charged?

b Souki chose the Krazy Kitchens plan and had paid $800 when the 300 days were up. How much does she owe now?

Solution

a Awesome Appliances, because its deferred payment period of 2 years is longer than Krazy Kitchens' period of 300 days.

b Outstanding amount = $1350 − $800
= $550

The outstanding amount is charged interest from the date of purchase.

$I = \$550 \times 0.0011 \times 300$
$= \$181.50$

P = $550, r = 0.0011 per day, n = 300 days

Total owing = $550 + $181.50
= $731.50

Example 3

Exercise 1-02 Term payments

1 Del and Barry bought an old cottage for $11 000 for their farm. They paid 10% deposit and monthly repayments of $360 for 4 years. How much interest did they pay by buying on terms? Select **A**, **B**, **C** or **D**.

A $18 380 **B** $17 280 **C** $7380 **D** $6280

2 Scott purchased an engagement ring for $7560, paying $100 deposit and 7.2% p.a. interest for 18 months. What was the total amount paid for the ring? Select **A**, **B**, **C** or **D**.

A $8365.68 **B** $8265.68 **C** $7460 **D** $8056.80

3 David and Sarah bought a second-hand motorhome for $19 500 to travel around Australia. There were two options for paying for the motor home.

Plan A: 20% deposit and 36 monthly instalments of $595

Plan B: No deposit and monthly instalments over 4 years at 8.9% p.a.

By calculating and comparing, find which plan:

a charges more interest

b has lower monthly instalments.

4 *Autocar* is a car dealership selling new cars on terms.

AUTOCAR WEEKLY SPECIALS

Price includes registration costs, stamp duty and delivery for approved customers. Price shown for base model and includes standard features only.

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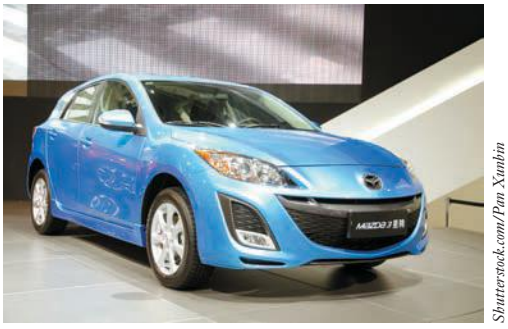

Shutterstock.com/Pan Xunbin

Mazda 4-door

$15 790 cash or $500 deposit and $92 per week over 5 years

Shutterstock.com/Jose Gil

Toyota Hybrid

$26 990 cash or 10% deposit and monthly payments at 11.9% p.a. over 3 years

iStockphoto/delectus

Peugeot 3-door hatch

$29 990 cash or no deposit and fortnightly repayments at 8.7% p.a. over 2 years.

a How much do you pay for the Toyota Hybrid on *Autocar's* terms?

b How much would you save on the Peugeot 3-door hatch by paying cash rather than term payments?

c What is the flat rate of interest per annum, correct to one decimal place, charged on the Mazda 4-door if making term payments?

d Give a possible reason *Autocar* charges different rates of interest for different cars.

5 The Rosa Finance company had the following agreement for purchasing a yacht.

Find:

a the total cost of the yacht, including charges and allowing for trade-in if paying by cash

b the monthly repayment if paying by hire purchase (excluding stamp duty and registration).

Rosa finance

Hire Purchase agreement

Cash price of yacht	$47 500
Deposit	$6500
Trade-in allowance on old yacht	$5400
Stamp duty	$475
Registration	$1203

Flat interest rate of 15.8% p.a.
Equal monthly repayments over 5 years

Rosa finance

6 Two finance companies offered deferred payment plans and charged interest of 0.2% per day on outstanding amounts from the purchase date.

Local Loans: 10% deposit then nothing to pay for 12 months

Feelgood Finance: Buy now and pay nothing for 100 days

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a Jayden bought a $1600 desk through Local Loans. Find:

i the amount owing 12 months after purchase

ii the amount owing after 396 days if only the deposit had been paid.

b Adison bought a computer for $1350 using Feelgood Finance. Calculate the amount if he repaid the debt in full after:

i 90 days

ii 101 days

c Write down one positive and one negative feature of each plan.

Example 4

7 Billy Joe bought a drum set for $1600 and was deciding between buying on terms or opting for a deferred payment plan.

How much will he pay for the drum set if he:

a buys on terms?

b uses the deferred payment plan and pays the amount owing 2 years after purchase?

c uses the deferred payment plan, pays $800 deposit upfront and fees only for 2 years, and the rest after 2 years 6 months?

8 Ishmail bought a car navigation system (GPS) on terms. How much would he save by paying cash?

9 Amrika bought outdoor furniture that was priced at $3299. She paid a deposit of $900 and took out a loan for the balance that was paid off in 24 monthly instalments of $130.18. What flat interest rate per annum was charged on her loan? Select **A**, **B**, **C** or **D**.

A 11% **B** 15% **C** 28% **D** 47%

10 Write down one advantage and one disadvantage of:

a paying cash for goods

b buying on terms

c using a deferred payment plan.

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1-03 Reducing balance loans

Reducing balance loan grids

Reducing balance loan: Spreadsheet

A **reducing balance loan** is less expensive than a flat rate loan because interest is calculated on the **balance owing**, the remaining amount of the loan still unpaid, rather than on the original principal borrowed. You pay less interest on a reducing balance loan because the interest decreases as the balance owing decreases.

You can save interest and pay off a reducing balance loan more quickly by:

- making repayments more frequently, for example, weekly or fortnightly rather than monthly
- making extra payments of any size from time to time, for example, if you receive a bonus or pay rise at work
- increasing the size of each repayment (even by just a few dollars)

EXAMPLE 5

Claire borrowed $395 000 to buy a studio apartment at 8% p.a. reducible interest. She made monthly payments of $3400. The table shows the progress of the loan for the first 2 months.

Reducing balance loan for Claire's studio apartment				
Amount borrowed	$395 000			
Interest rate (% p.a.)	8			
Monthly repayment (R)	$3400			
Month (n)	**Principal (P)**	**Interest (I)**	**Amount owing before repayment ($P + I$)**	**Balance ($P + I - R$)**
1	$395 000	$2633.34	$397 633.34	$394 233.34
2	$394 233.34	$2628.23	$396 861.57	$393 461.57

a Show how the interest of $2633.34 for the first month was calculated.

b Add rows showing the progress of the loan for the next 3 months.

c Find the principal at the start of the sixth month.

d How much has Claire paid off the principal after 5 months?

e Calculate how much interest was paid in the first 5 months.

f How much interest will Claire pay in 5 months on a loan of $395 000 at a flat interest rate of 8% p.a.?

Solution

a Amount owing (principal) at the start of the first month = $395 000

$$\text{Interest for 1st month} = \$395\,000 \times \frac{0.08}{12}$$
$$= \$2633.3333\ldots$$
$$\approx \$2633.34$$

Round up for loan interest.

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b

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment ($P + I$)	Balance ($P + I - R$)
1	\$395 000	\$2633.34	\$397 633.34	\$394 233.34
2	\$394 233.34	\$2628.23	\$396 861.57	\$393 461.57
3	\$393 461.57	\$2623.08	\$396 084.65	\$392 684.65
4	\$392 684.65	\$2617.90	\$395 302.55	\$391 902.55
5	\$391 902.55	\$2612.69	\$394 515.24	\$391 115.24
		Total: \$13 115.24		

c Principal at start of sixth month is \$391 115.24. **Same as the balance at the end of the fifth month.**

d Amount paid off principal after 5 months = \$395 000 – \$391 115.24
= \$3884.76

e Interest paid in first 5 months = \$13 115.24 **Total of the interest column.**

f Flat interest for 5 months = \$2633.34 × 5 **(Interest for first month) × 5**
= \$13 166.70 **Higher than reducing balance loan interest.**

Example 5

Exercise 1-03 Reducing balance loans

1 From the table in Example **5** above, what was the amount that had been paid off the principal after four months? Select **A**, **B**, **C** or **D**.

A \$2617.90 **B** \$392 684.65 **C** \$3097.45 **D** \$391 902.55

2 **a** Add four rows to the table in Example **5** to show the progress of the loan for months 6 to 9 (details for the fifth month are shown to start you off).

Month (n)	Principal (P)	Interest (I)	Amount owing before repayment ($P + I$)	Balance ($P + I - R$)
5	\$391 902.55	\$2612.69	\$394 515.24	\$391 115.24
6				
7				
8				
9				

b What is the principal at the start of the ninth month?

c What amount had been paid off the principal after nine months?

d What total interest was charged for the first 9 months?

3 Cate borrows \$370 000 to buy a house. Interest is charged at 6.4% per annum, compounded monthly. How much does she owe at the end of the first month, after she has made a \$2100 repayment? Select **A**, **B**, **C** or **D**.

A \$365 927 **B** \$369 873 **C** \$371 962 **D** \$391 580

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4 For a reducing balance loan of $180 000 at 9% p.a. over 12 years, with fortnightly repayments of $932.75, find:

a the total amount to repay

b the total interest payable

c the equivalent flat rate of interest per annum, correct to one decimal place

d the interest payable on a loan of $180 000 over 12 years at a flat interest rate of 9% p.a.

5 Brooke borrowed $450 000 to purchase her first home. The bank lent her the money at 10% p.a. reducible interest and fortnightly repayments of $1960.

Fortnights (n)	Principal ($\$P$)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$450 000	$1730.77	$451 730.77	$449 770.77
2	$449 770.77	$1729.89	$451 500.66	$449 540.66
3				
4				
5				
6				

a Copy and complete the table above showing the progress of Brooke's loan for the first 6 fortnights.

b How much had she paid off the principal after 6 repayments?

c How much interest did she pay in the first 12 weeks?

6 Goran borrows $16 000 for his upcoming wedding, taking out a personal loan that has the following conditions.

- Extra repayments can be made at no extra cost
- Loan approval fee: $150 (to be paid upfront when loan is established)
- Loan administration fee: $30 (charged after each 3 months of the loan)
- Interest rate: 13.9% p.a.

His monthly repayments (R) are $436 over 4 years.

a Calculate the monthly interest rate as a decimal correct to 6 decimal places.

b The table below shows the progress of Goran's loan. Show that the interest paid in the second month is $182.42.

Month	P	I	$P + I$	$P + I - R$
1	$16 000	$185.33	$16 185.33	$15 749.33
2	$15 749.33			
3				

c Copy and complete this table.

d After 3 months Goran is charged a fee. What is the fee and how much will it be?

e How much in interest charges will Goran have paid after 3 months?

7 Kim and Rick borrow $17 800 to finance an overseas holiday. They will repay their loan over 7 years at a rate of 12.5 % p.a., with fortnightly repayments of $147.

a Calculate the fortnightly interest rate as a decimal correct to 5 decimal places.

b Copy and complete the table below for the first 2 fortnights' repayments.

Fortnight	P	I	$P+I$	$P+I-R$
1				
2				

c How much will Kim and Rick pay in interest after 2 repayments have been made?

d How much is still owing on the loan after two fortnights?

TECHNOLOGY: REDUCING BALANCE LOANS ON A SPREADSHEET

The spreadsheet shows the progress of a $200 000 loan at 8% p.a. reducible interest and monthly repayments of $1500. Enter the labels, values and formulas shown. **Fill Down** for columns C, D and E until the amount owing in column E is negative, at row 338.

	A	B	C	D	E
1	Principal (P)	200000			
2	Interest rate (% p.a.)	8			
3	Repayment per period (R)	1500			
4	No. periods per year	12			
5					
6					
7	**Period no.**	**P**	**I**	**P + I**	**P + I - R**
8	1	=B1	=B8*$B$2/100/$B$4	=B8+C8	=D8-$B$3
9	=A8+1	=E8			
10	=A9+1	=E9			

The details for the **first 3 months** and the **final month** of the loan are shown.

7	Period no.	P	I	P + I	P + I - R
8	1	$200,000.00	$1,333.33	$201,333.33	$199,833.33
9	2	199,833.33	$1,332.22	$201,165.56	$199,665.56
10	3	199,665.56	$1,331.10	$200,996.66	$199,496.66
338	331	$1,015.93	$6.77	$1,022.71	-$477.29

Create a graph, using **Insert** and **Scatter with Lines and Markers** to draw a line graph showing the progress of the loan.

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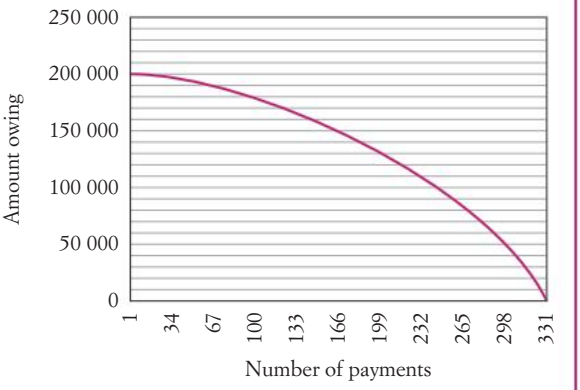
1 Adjust your spreadsheet to show that:

- **a** the complete term of this loan is 27 years 7 months
- **b** the final repayment needed to reduce the balance to zero (and pay off the loan) is $1022.71
- **c** if the monthly repayment is doubled to $3000, the loan will be repaid in 7 years 5 months.



2 **a** Convert a monthly repayment of $1500 to a fortnightly repayment.

b Find the interest saved by making fortnightly repayments instead of monthly repayments.

3 Use the spreadsheet to find:

- **a** the term for a $500 000 home loan at 6.5% p.a. reducible interest and monthly repayments of $3650
- **b** the total interest on a travel loan of $40 000 at 8.3% p.a. reducible interest and fortnightly repayments of $680
- **c** the final payment on a home improvement loan of $150 000 at 9.35% p.a. reducible interest and monthly repayments of $1740.

1-04 Compound interest

WS Compound interest

Compound interest is interest that is added to the **principal** and reinvested, so the amount of interest increases. The investor not only earns interest on the principal but also earns 'interest on the interest'.

You should already be familiar with the compound interest formula $A = P(1 + r)^n$, where A is the final amount and P is the principal. In the financial world, the principal is called the **present value** (PV) and the final amount is called the **future value** (FV) or **compounded value**, so now we can write the compound interest formulas as shown on the next page.

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SUMMARY

Compound interest formulas

$FV = PV(1 + r)^n$ and $I = FV - PV$ ← Interest = Future Value – Present Value

where: FV = future value (final amount)

PV = present value (principal, initial value)

n = number of compounding periods

r = interest rate per compounding period, expressed as a decimal

I = compound interest earned

EXAMPLE 8

$8500 is invested for 4 years at 7.5% p.a., compounded monthly. Calculate the interest earned.

Solution

Because interest is compounded monthly, r and n must be expressed in months:

$PV = \$8500, r = \frac{0.075}{12} = 0.00625$ per month, $n = 4 \times 12 = 48$ months

$FV = PV(1 + r)^n$

$= \$8500(1 + 0.006\,25)^{48}$

$= \$8500(1.006\,25)^{48}$

$= \$11\,463.092\ldots$

$\approx \$11\,463.09$

Round down for investment interest.

$I = FV - PV$

Interest = Future value – Present value

$= \$11\,463.09 - \$8500$

$= \$2963.09$

Interest earned = $2963.09

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EXAMPLE 9

What present value must be invested for 8 years at 9% p.a. compounded quarterly to grow to a future value of $10 000?

Solution

$FV = \$10\,000$, $r = \dfrac{0.09}{4} = 0.0225$ per quarter, $n = 8 \times 4 = 32$ quarters

$$FV = PV(1 + r)^n$$
$$\$10\,000 = PV\,(1 + 0.0225)^{32}$$
$$\$10\,000 = PV\,(1.0225)^{32}$$
$$PV = \frac{\$10\,000}{(1.0225)^{32}}$$
$$= \$4906.523\ldots$$
$$\approx \$4906.53$$

Round up for present value.

A present value of $4906.53 must be invested.

Exercise 1-04 Compound interest

Example 8

1 What is the future value when $27 000 is invested for 16 years at 7% p.a.? Select **A**, **B**, **C** or **D**.

A $52 708 **B** $30 240 **C** $57 240 **D** $79 708

2 A house was purchased in Sydney in 1987 for $86 200. Assume that the value of the house has increased by 3% per annum since then. Which expression gives the future value of the house in 2015? Select **A**, **B**, **C** or **D**.

A $86\,200(1 + 0.03)^{28}$ **B** $86\,200(1 + 0.3)^{28}$

C $86\,200 \times 28 \times 1.03$ **D** $86\,200 \times 28 \times 1.3$

3 For each investment, calculate:

i the future value (FV)

ii the compound interest (I) earned.

a $134 000 for 12 years at 6.5% p.a.

b $5000 for 5 years at 9% p.a. compounded monthly

c $14 000 for 8 years at 8% p.a. compounded quarterly

d $18 200 for 12 years at 8.3% p.a. compounded half-yearly

e $34 300 for 20 years at 7.8% p.a. compounded weekly.

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Example 9

4 What present value must be invested to grow to $50 000 at:

a 7.5% p.a. for 4 years?

b 9.2% p.a. compounded monthly for 8 years?

c 10% p.a. compounded quarterly for 2 years?

5 If a present value of $5000 is invested for 4 years, which is the best interest rate? Select **A**, **B**, **C** or **D**.

A 6% p.a. compound interest

B 6.2% p.a. simple interest

C 5.8% p.a. compounded quarterly

D 5.2% p.a. compounded monthly

6 A lottery prize of $250 000 is invested in a 13-month term deposit at 10% p.a. compounded monthly. How much is the investment worth at the end of the 13 months?

7 Jarryd's inheritance grew to a future value of $31 737.48 when it was invested at 8% p.a. for 6 years. What was the present value of his inheritance?

8 Michaela opened an account with $1000 and forgot about the account for 20 years. Interest of 5% p.a. was paid every six months. Had her money doubled in 20 years? Show all working to justify your answer.

TECHNOLOGY: INVESTMENTS COMPOUNDED MONTHLY, WEEKLY AND QUARTERLY

In this activity, we will use a spreadsheet to compare the amount of compound interest earned on a present value of $1500 for different compounding periods. Given an interest rate of 4.8% p.a. over 2 years, we will calculate the total interest if compounded monthly (columns A to D), weekly (columns G to J) and quarterly (columns L to O).

1 Enter the following information into a new spreadsheet, but in cells B6, H6 and M6 respectively, enter the formulas **=B4/12, =B4/52** and **=B4/4** to calculate the interest rates shown there. Note that cell B4 contains the value 0.048 in percentage format.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	**Compound Interest Comparison**														
2															
3	**Investment**	$1,500													
4	**Interest rate**	4.80%													
5	Months in 2 years	24					Weeks in 2 years	104				Quarters in 2 years	8		
6	**Monthly Interest Rate**	0.004					**Weekly Interest Rate**	0.000923077				**Quarterly Interest Rate**	0.012		
7	**Compounded monthly for 2 years**	**PV**	**FV**	**I**			**Compounded Weekly for 2 years**	**PV**	**FV**	**I**		**Compounded Quarterly**	**PV**	**FV**	**I**

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2 For **interest compounded monthly**, enter the following formulas in row 8 ($B$6 is an absolute cell reference and fixes the cell value so it will not change when **Fill Down** is used).

	Compounded monthly for 2 years	PV	FV	I
7				
8	1	-B3	-B8*(1+$B$6)	-C8-B8

3 In cell A9, enter the formula =**A8+1** and **Fill Down** to row 31 (24 months). In cell B9 enter =**C8** and **Fill Down** to row 31. Select cells C8 and D8 and **Fill Down** to row 31.

4 In cell C32, type **Total Interest Earned** and in cell D32, enter =**sum(D8:D31)**.

5 For **interest compounded weekly**, enter the following formulas in row 8.

Compounded Weekly for 2 years	PV	FV	I
1	=B3	=H8*(1+$H$6)	=I8-H8

6 In cell G9, enter =**G8+1** and **Fill Down** to row 111 (for 104 weeks). In cell H9 enter =**I8** and **Fill Down** to row 111. Select cells I8 and J8 and **Fill Down** to row 111.

7 In cell I112, type **Total Interest Earned** and in cell J112, enter =**sum(J8:J111)**.

8 For **interest compounded quarterly**, enter the following formulas in row 8.

Compounded Quarterly for 2 years	PV	FV	I
1	=B3	=M8*(1+$M$6)	=N8-M8

9 In cell L9, enter =**L8+1** and **Fill Down** to row 15 (for 8 quarters). In cell M9 enter =**N8** and **Fill Down** to row 15. Select cells N8 and O8 and **Fill Down** to row 15.

10 In cell N16, type **Total Interest Earned** and in cell O16, enter =**sum(O8:O15)**.

11 Compare your answers in cells **D32**, **J112** and **O16**. Which compounding period earned the most interest? Why?

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Credit card statement

1-05 Credit cards

Using a credit card is like taking out a short-term loan. You use the card to purchase items and pay for them later when you receive a monthly statement. The interest charged can be flat or compound and is charged daily on each item from the date of purchase.

Features of credit cards

- More convenient than carrying cash, useful for purchasing items online, by phone or by post
- Monthly statement provides record of spending.
- Many have an interest-free period of 55 days to repay the amount owing in full, otherwise interest is calculated from the date of each purchase.
- Interest-free period does not apply to cash advances (withdrawals) made on the card, which are charged a higher interest rate than purchases.
- A high credit limit such as $5000 can be both an advantage and a disadvantage.

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EXAMPLE 10

Mitchell's credit card has a flat interest rate of 14% p.a. and no interest-free period. He uses the card to make the following purchases for the period 1 August to 31 August.

2 August	Phone recharge	$85
16 August	Jeans	$99
29 August	Movies	$22

If Mitchell pays his account in full on 3 September, how much does he pay?

Solution

We need to calculate the interest for each purchase separately.

Daily interest rate $= \frac{0.14}{365}$ **1 year = 365 days**

Purchase amount	No. of days' interest* (from date of purchase to 3 Sep.)	Interest
$85	$31 - 2 + 3 = 32$	$\$85 \times \frac{0.14}{365} \times 32 = \$1.04328\ldots$
$99	$31 - 16 + 3 = 18$	$\$99 \times \frac{0.14}{365} \times 18 = \$0.68350\ldots$
$22	$31 - 29 + 3 = 5$	$\$22 \times \frac{0.14}{365} \times 5 = \$0.04219\ldots$
Total = $206		Total = $1.76897… ≈ $1.77

Do not round interest values until we calculate the total.

Total purchases = $206

Total interest = $1.77

Total payment = $206 + $1.77
= $207.77

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EXAMPLE 11

Miss Spent's credit card has an interest-free period of 55 days, with interest on purchases compounded daily if the amount owing is not paid in full by the due date. Her monthly statement is shown below.

FIRSTBANK

Statement Period	14 December 2014 to 13 January 2015
Account No	2483 0222 1223 6517
Credit Limit	$5000
Available Credit	$3824.38

Miss Spent
16 Dollar Lane
Decimal Point 2178

Account Summary

Balance from previous statement	**$816.97**
Payment and other credits	$500.00CR
Purchases, cash advances	$791.00
Interest and other charges	$67.65
Closing Balance	**$1175.62**
Minimum payment required	$60.00
Due date	7 February 2015

Your Transaction Record

Date	Reference	Details	Amount
15 Dec 2014	3146532	Payment-Thank you	$500.00CR
15 Dec 2014		Interest	$8.65
18 Dec 2014	4367549	'Fresh at the Bay' Restaurant	$85.60
21 Dec 2014	3020542	ATM Withdrawal	$100.00
21 Dec 2014	3020542	ATM Fee	$4.00
31 Dec 2014	5746380	Ballet tickets	$265.50
2 Jan 2015		Annual Fee	$55.00
7 Jan 2015	2345698	Fabulous Fashions	$256.50
12 Jan 2015	6348921	Sam's Hair and Beauty	$83.40

Annual compound interest rate 22.484%
Daily compound interest rate 0.0616%

a Show how the closing balance of $1175.62 was calculated.

b What is the credit limit on this card?

c For this period, what was the total amount of:

i cash advances? **ii** purchases? **iii** interest and other charges?

d How much was paid by Miss Spent since the previous statement and on what date?

e When does the interest-free period start and end?

f What interest will be charged if this account is paid in full on 6 February?

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Solution

a Closing balance = \$816.97 – \$500 + \$791 + \$67.65
= \$1175.62

b \$5000

c **i** Cash advances = \$100 **ATM withdrawals.**

ii Purchases = \$791 – \$100
= \$691

iii Interest and other charges = \$67.65

d \$500 on 15 December 2014.

e From 14 December 2014 to 7 February 2015. **The start of the statement period until the due date.**

f As 6 February falls within the interest-free period, interest is only charged on the cash advance, from 21 December to 6 February (47 days).
Using the compound interest formula on $P = \$100$, $r = 0.000\,616$, $n = 47$:
$A = \$100(1 + 0.000\,616)^{47}$
$= \$102.9366\ldots$
Interest = \$102.9366… – \$100
= \$2.9366…
= \$2.94

Exercise 1-05 Credit cards

1 Roberto is charged 0.04% compound interest per day on his credit card purchase of \$1200. Find the amount of interest owing after 30 days. Select **A**, **B**, **C** or **D**.

A \$14.40 **B** \$14.48 **C** \$4.80 **D** \$12.00

2 Karin has a credit card that charges simple interest of 22.5% p.a. and has no interest-free period. She bought shoes for \$235 on 21 December and paid her account on 30 December. Find the amount she paid. Select **A**, **B**, **C** or **D**.

A \$235 **B** \$287.88 **C** \$236.30 **D** \$1.30

Example 10

3 Eamon has a credit card with no interest-free period and a simple interest rate of 0.0453% per day. The purchases below were made in November. If Eamon pays his account in full on 1 December, how much does he pay?

4 November	Swim trunks	\$45.95
18 November	Swim goggles	\$88.00
25 November	Wetsuit	\$275.00

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4 NCM Bank credit cards have no interest-free period. For the credit card statement below, find for the due date:

a the number of days interest charged on the football tickets

b the total amount due.

Miss Alannah Sheppard **Due date: 1 April**		**NCM BANK** **Credit card statement**	
Previous balance $876.15	**Payment** $876.15	**Total purchases**	**Interest** .....
Date	**Purchases**	**Amount**	
15 March	Coffee maker	$465.95	
20 March	Football tickets	$139.37	
28 March	Car tyres	$446.79	
Annual simple interest rate: 17%			

Example 11

5 A credit card has an interest-free period of 55 days. One monthly statement is shown.

Statement period: 1 June 2016 to 30 June 2016 Mr C Evans 1 Mountain Avenue Tournay 2126	Account number Balance from previous statement Payment and other credits Purchases Interest and other charges Closing balance	543210 $3256.43 $3000.00 $2195.48 $3.84 $2455.75
Annual compound interest rate 18.25% Daily compound interest rate 0.05%	Minimum payment required Due date	$212 25 July 2016
Date	**Purchases**	**Amount**
15 June 2016	Payment – thank you	$3000.00 CR
15 June 2016	Interest	$3.84
18 June 2016	Bike repairs	$1346.58
27 June 2016	Cycling outfit	$665.50
27 June 2016	Bicycle helmet	$183.40

a Show how the closing balance of $2455.75 was calculated.

b How much was paid by Mr Evans and on what date?

c When does the interest-free period start and end?

d What compound interest will be charged on the three purchases if this account is paid in full on:

i 20 July? **ii** 30 July?

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6 This is the July statement for Claire's credit card account.

STATEMENT

PRL BANK

Ms C. Howe

Statement date: 28 August 2016 Credit limit: $5000

Previous balance	Payments	Purchases	Interest
$305.60	$305.60	$886.50	?
Date	**Purchases**	**Amount**	**Closing balance**
25 July	Cengage Car Insurance	$886.50	?

Annual percentage rate: 21% Daily percentage rate: 0.0575%

Note: Simple interest is charged on amounts from (and including) the purchase date up to (and including) the statement date (shown at the top of the statement).

Minimum payment due: $50 or 5% of closing balance, whichever is larger.

What is the minimum payment due on this account?

A $16.43 **B** $40.83 **C** $45.22 **D** $50

7 David and Sarah bought a dining table and chairs for $2100 on 11 July using a credit card. Simple interest was charged at a rate of 20.35% per annum for purchases on the credit card. No other purchases were made on this credit card account. There was no interest-free period. The period over which interest was charged included the purchase date and the payment date. What amount was paid when Sarah paid the account in full on 19 August?

A $2135.12 **B** $2137.47 **C** $2145.66 **D** $2146.83

INVESTIGATION

COMPARING CREDIT CARDS

The best credit card is the one that matches your spending and repayment habits. If you plan to use the card for small or regular purchases and pay the total amount each month, choose a card with an interest-free period and a low annual fee. If, however, you plan to use the card to purchase many items and only pay off part of the debt each month, a low interest rate is better.

The rates, fees and interest-free days of five credit cards are shown in the table on the next page.

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Card	Interest rate p.a. on purchases	Interest rate p.a. on cash advances	Annual fee	Interest-free days
ANZ First Visa	19.39%	20.99%	$30	44
ANZ Low rate MasterCard	13.14%	21.49%	$58	55
Commonwealth Student MasterCard	19.99%	21.49%	$0	55
St George Vertigo MasterCard	13.24%	21.49%	$55	55
Westpac No annual fee MasterCard	14.49%	19.59%	$0	55

westpac.com.au, commbank.com.au, anz.com, stgeorge.com.au, November 2012

Select the best credit card for each person, giving reasons for your choice.

a Cody uses his credit card frequently but always pays off the total amount due on time.

b Erin likes to spend big on her card, and only makes the minimum payment each month.

c Mikhail often uses his card to make cash advances.

1-06 Future value of an annuity (FVA)

An **annuity** is a financial plan involving regular equal **contributions** (payments) to an account earning compound interest. Each contribution is made at the *end* of each compounding period. **Superannuation** is an example of an annuity, in which part of your pay is deposited regularly into a fund to provide future finance for your retirement years.

WS Annuity grids

WS Annuity: Spreadsheet

WS Annuities: Graphics calculator

EXAMPLE 12

The table shows the progress of an annuity over 5 years with a contribution of $600 per year made at the *end* of each year and an interest rate of 8% p.a.

Year	Balance at start of year, P	Interest, I	Contribution at end of year, a	Balance at end of year, $P + I + a$
1	$0	$0	$600	$600
2	$600	$48	$600	$1248
3	$1248	$99.84	$600	**A**
4	**B**	**C**	$600	$2703.67
5	$2703.67	$216.29	$600	$3519.96

a Find, to the nearest cent, the value of **A**, **B** and **C**, showing all calculations.

b What will be the balance at the end of the 6th year?

c If the contribution is doubled to $1200, what is the balance at the end of the 4th year?

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Solution

a Balance at end of 3rd year: **A** = \$1248 + \$99.84 + \$600 = \$1947.84

Balance at start of 4th year: **B** = \$1947.84 (same as **A**)

Interest earned in 4th year: **C** = $0.08 \times \$1947.84 = \$155.83$

Check: Balance at end of 4th year = \$1947.84 + \$155.83 + \$600
= \$2703.67

b Balance at end of 6th year = $\$3519.96 + 0.08 \times \$3519.96 + \$600$
= \$4401.56

c Balance at end of 4th year when contribution is \$600: \$2703.67 (from table)

Balance at end of 4th year when contribution is \$1200 = $\$2703.67 \times 2$
= \$5407.34

Future value table

An annuity earns more interest and grows faster than a principal that is simply invested into an account, because you keep making regular, equal payments into the account.

We know that **future value** is the final amount of an investment.

The **future value of an annuity** (FVA) is the total value of the investment at the end of the term, after the final contribution is made. It is the sum of all contributions plus interest.

The table below gives the future value (in dollars) of an annuity with a contribution of \$1 per period. The values are rounded to four decimal places.

Future value interest factors
(Future value of an annuity with a contribution of \$1 at the end of each period)

Period	Interest rate per period								
	1%	2%	3%	4%	5%	6%	8%	10%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0800	2.1000	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2464	3.3100	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.5061	4.6410	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.8666	6.1051	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.3359	7.7156	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.9228	9.4872	10.0890
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.6366	11.4359	12.2997
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	12.4876	13.5795	14.7757
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	14.4866	15.9374	17.5487
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	16.6455	18.5312	20.6546
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	18.9771	21.3843	24.1331

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EXAMPLE 13

Dominic opens an account that earns interest of 12% p.a. compounded monthly. He contributes $100 at the end of each month for one year.

a Use the table on page 29 to calculate the future value of Dominic's annuity.

b How much interest will he earn on his investment?

Solution

a Interest rate per month = 12% ÷ 12 = 1%

No. of months = 12

Future value of $1 per month for 12 months = $12.6825 **From table.**

Future value of $100 per month for 12 months = $12.6825 × 100
= $1268.25

b Interest earned = $1268.25 − 12 × $100 **Future value – total contributions**
= $68.25

EXAMPLE 14

Use the table on page 29 to find the contribution per period of an annuity that will give a future value of:

a $15 000 if the annuity earns 6% p.a. for 3 years

b $2600 if the annuity earns 10% p.a. compounded half-yearly for 4 years.

Solution

a FV = $15 000, Interest rate = 6% p.a., No. of periods = 3 years

Future value of $1 per month at 6% for 3 years = $3.1836 **From table.**

Let $\$a$ be the contribution amount that will give a future value of $15 000.

Future value of $\$a$ per month at 6% for 3 years = $3.1836 × a
= $15 000

So a = $15 000 ÷ $3.1836
= 4711.647…
≈ 4711.65 **Round up for contribution.**

The contribution will be $4711.65 per year.

b FV = $2600, Interest rate = 10% ÷ 2 = 5% (per half-year), No. of periods = 4 × 2 = 8 half-years

Future value of $1 per half-year at 5% per half-year for 8 half-years = $9.5491 **From table.**

As with the compound interest formula, the interest rate and number of periods must be expressed with the same time units.

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$\$2600 = \$9.5491 \times a$

So $a = \$2600 \div \$9.5491$
$= 272.276\ldots$
≈ 272.28

The contribution will be \$272.28 per half-year.

Exercise 1-06 Future value of an annuity

Example 12

1 The table shows the progress of an annuity with a contribution of \$5000 per year made at the *end* of each year for 5 years.

Year	Balance at start of year, P	Interest, I	Contribution, a	Balance at end of year, $P + I + a$
1	\$0	\$0	\$5000	\$5 000
2	\$5 000	\$400	\$5000	\$10 400
3	\$10 400	\$832	\$5000	\$16 232
4	\$16 232	X	\$5000	\$22 530.56
5				

a What is the annual interest rate?

b How much interest (X) was earned in the 4th year?

c Complete the table to find the amount of the annuity at the end of 5 years.

Example 13

2 Use the table on page 29 to find the future value of an annuity (FVA) with a contribution of:

a \$8000 per year for 5 years at 10% p.a.

b \$600 per quarter for 9 months at 8% p.a. compounded quarterly

c \$5400 per half-year for 5 years at 6% p.a. compounded half-yearly

d \$900 per month for 10 months at 12% p.a. compounded monthly.

3 Cesar's Truck Repairs has a plant replacement fund and deposits \$5000 on 30 June and 31 December each year. If the account earns interest of 8% p.a. compounded half-yearly, how much will be in the account at the end of 5 years?

4 Tom deposits \$660 at the end of each month for 12 months into an account earning 12% p.a. compounded monthly. Immediately after making the last payment, he withdraws all his money to buy a boat that costs \$9300. Does he have enough in his account to buy the boat?

Example 14

5 Find the contribution per period (a) of an annuity whose future value (FV) is:

a \$10 000, at 8% p.a. for 3 years

b \$10 000, at 8% p.a. compounded quarterly for 3 years

c \$26 500, at 10% p.a. compounded half-yearly for 6 years

d \$9800, at 1% per month for 3 months.

6 Katarina would like to buy a house in 4.5 years time and requires a deposit of \$40 000. What contribution should she make every six months into an account earning interest of 6% p.a. compounded half-yearly to reach this amount?

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7 Darren deposits an amount at the end of each year into a superannuation fund paying 8% p.a. If he plans to have $400 000 after 11 years, what is his regular deposit?

8 John and Nancy start a trust fund for their grandaughter, Samantha, on her 8th birthday to finance her university education. What amount should they deposit each birthday in an account earning 10% p.a. so that she will have $50 000 on her 18th birthday?

9 Selina invested $7600 at 8% p.a. compounded quarterly for 3 years.

a What is the final amount of her investment? Answer to the nearest dollar.

b Would she have achieved a better financial result if she had chosen an annuity of $760 per quarter for 3 years at the same interest rate? Justify your answer.

10 The shop owners in a small shopping centre estimate that $50 000 will be needed in 2 years time to repaint and retile the common areas. They plan to make regular payments in to a sinking fund earning 8% p.a. compounded quarterly.

a How much must be deposited into the sinking fund at the end of each quarter?

b How much interest will the sinking fund earn?

c How much would be in the sinking fund if they deposited $4000 per quarter for 2 years at the same interest rate?

11 Bianca is saving to go on holidays. She wants $6000 in twelve months so she can go on a cruise. How much must she invest at the end of each quarter at 4% p.a. compounded quarterly to achieve this?

12 The Ahmeds plan to buy a house in 3 years time and will need a deposit of $60 000. If they deposit $4000 at the end of each quarter into an account earning 16% p.a. compounded quarterly, will they have the deposit in 3 years time?

INVESTIGATION

A FORMULA FOR THE FUTURE VALUE OF AN ANNUITY

The values in the future value table are calculated using a special formula for the future value of an annuity:

$$FVA = a\left\{\frac{(1+r)^n - 1}{r}\right\}$$

where: a = contribution per period, paid at the end of the period

r = interest rate per compounding period, expressed as a decimal

n = number of compounding periods

This formula is NOT part of the Mathematics General 2 course and does not have to be learnt. Note that there are some similarities between this formula and the compound interest formula.

1 Brad and Kara open an account earning interest of 10% p.a. They deposit $10 000 at the end of each year for 4 years.

a Use the future value table to show that the future value of their annuity is $46 410.

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b Use the *FVA* formula to show that the future value of their annuity is $46 410.

c Show that they will earn $6410 in interest.

2 According to the future value table, the future value of an annuity with a contribution of $1 per period, at an interest rate of 3% per period over 7 years is $7.6625. Use the *FVA* formula to show that this value is correct.

3 A **sinking fund** is an annuity where the future value has been decided and you pay regular contributions into the fund to reach this target. For example, in an apartment block, each unit owner pays into a sinking fund so money is available for maintenance and repairs.

The owners of a block of units estimate that $20 000 will be needed in 4 years time to repaint the building. They open a sinking fund with interest of 6% p.a. compounded quarterly. Use the *FVA* formula to show that $1115.31 must be deposited each quarter into the sinking fund to reach this target.

JUST FOR THE RECORD

Who wants to be a millionaire?

Did you know that if you open an account at age 20 and save $100 per week at an interest rate of 10% p.a. compounded weekly, you will be a millionaire just after your 50th birthday?

How long will it take you to become a millionaire if you start saving $100 per week now at the current interest rate?

iStockphoto/pictafolio

1-07 Present value of an annuity (PVA)

We know that **present value** means the principal or initial amount of an investment.

The **present value of an annuity** (PVA) is the single sum of money (or principal) you could invest today at the same compound interest rate as the annuity, to produce the same future value over the same period of time.

Annuities: Graphics calculator

Loan repayments: Graphics calculator

Present value table

The table below gives the present value (in dollars) of an annuity with a contribution of $1 per period. The values are rounded to four decimal places.

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Present value interest factors (Present value of an annuity with a contribution of $1 at the end of each period)									
Period	Interest rate per period								
	1%	2%	3%	4%	5%	6%	8%	10%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9259	0.9091	0.8929
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.7833	1.7355	1.6901
3	2.941	2.8839	2.8286	2.7751	2.7232	2.6730	2.5771	2.4869	2.4018
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3121	3.1699	3.0373
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	3.9927	3.7908	3.6048
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.6229	4.3553	4.1114
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.2064	4.8684	4.5638
8	7.6517	7.3255	**7.0197**	6.7327	6.4632	6.2098	5.7466	5.3349	4.9676
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.2469	5.7590	5.3282
10	9.4713	8.9826	8.5302	**8.1109**	7.7217	7.3601	6.7101	6.1446	5.6502
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.1390	6.4951	5.9377
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.5361	6.8137	6.1944

EXAMPLE 15

Patrick is saving for a holiday and deposits $1200 into an account at the end of every 6 months for 5 years. The account pays 8% p.a. compounded half-yearly. Use the above table to find the single amount he could invest now to produce the same result at the same interest rate over the same period.

Solution

Interest rate per half-year = 8% ÷ 2 = 4%

No. of half-years = 5 × 2 = 10

Present value of $1 per half-year at 4% for 10 half-years = 8.1109 **From table.**

Present value of $1200 per half-year at 4% for 10 half-years = $8.1109 × 1200

= $9733.08

Patrick could invest a single principal of $9733.08 instead to achieve the same result as the annuity.

Loan repayments

A series of loan repayments is an annuity where the amount borrowed is the **present value** of the series of repayments and the repayments are the **contributions**. For reducing balance loans, the present value table can be used to calculate the size of the loan repayments.

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EXAMPLE 16

Andrea borrowed $3000 at 12% p.a. repayable quarterly over 2 years to pay for a holiday to celebrate the end of university.

a How much will she repay per quarter?

b How much interest will Andrea pay over the 2-year loan?

Solution

a PV = $3000, Interest rate = 12% ÷ 4 = 3% per quarter
No. of periods = 2 × 4 = 8 quarters

Present value of $1 per quarter at 3% for 8 quarters = $7.0197 **From table.**

Let $\$a$ be the contribution (repayment) that will give a present value of $3000.
Present value of $\$a$ per half-year at 3% for 8 quarters = $7.0197 × a
= $3000

So a = $3000 ÷ $7.0197
= 423.368…
≈ 423.37 **Round up for contribution.**

The repayment will be $423.37 per quarter.

b Interest paid = 8 × $427.37 − $3000 **Total repayments − principal**
= $418.96

Exercise 1-07 Present value of an annuity

Example 15

1 Use the table on page 34 to find the present value of an annuity (PVA) with a contribution of:

a $760 per year for 10 years at 4% p.a.

b $400 per month for 8 months at 24% p.a. compounded monthly

c $1600 per half-year at 16% p.a. compounded half-yearly for $5\frac{1}{2}$ years

d $1140 per year at 6% p.a. for 4 years.

2 Mr and Mrs Murphy are saving for new electrical appliances for their kitchen renovation and deposit $800 into an account at the end of every 6 months for 2 years. The account pays 6% p.a. compounded half-yearly. Find the single amount they could invest now to produce the same result at the same interest rate over the same period.

3 What is the present value of an annuity in which $1000 is invested at the end of each year for 5 years and interest of 8% p.a. is paid annually? Select **A**, **B**, **C** or **D**.

A $11 574 **B** $3993 **C** $8620 **D** $357

4 Angela deposits $200 at the end of each quarter for 2 years into an account earning 16% p.a. compounded quarterly. What single sum of money invested now at the same rate would achieve the same financial result?

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5 Lulu invests $500 every 6 months in an account earning 6% p.a. compounded half-yearly. Find, correct to the nearest cent, how much Lulu:

a will have at the end of 5 years, using the future value table on page 29

b could invest now to grow to the same amount after 5 years at the same interest rate.

Example 16

6 Find the regular repayment (a) for a loan with principal:

a $7400, at 2% per month for 9 months

b $38 000, at 4% per quarter for 2 years

c $29 500, at 16% p.a. compounded half-yearly over $3\frac{1}{2}$ years.

7 Ajay plans to borrow $20 000 from the credit union at 8% p.a. to landscape his backyard. He wants to repay it in equal quarterly repayments over 3 years.

a How much does Ajay repay each quarter?

b Calculate the total amount Ajay will repay to the credit union over the 3 years.

8 Jacqui is saving for her wedding and reception. She borrowed $16 000 at 12% p.a. repayable quarterly over 3 years.

a How much will she repay per quarter?

b How much interest will Jacqui pay over the 3-year loan?

9 James has just turned 17 and plans to buy a car when he turns 20. He has a part-time job and wants to deposit the same amount each quarter into an account earning 12% p.a. compounded quarterly. What amount:

a should he deposit at the end of each quarter to have $12 000 in 3 years time?

b could be invested now to grow to $12 000 in 3 years at 12% p.a. compounded quarterly?

10 Jennifer borrows $25 000 and pays it back in regular instalments at the end of each half-year for 5 years. If interest is 20% p.a. compounded half-yearly, find:

a the amount of each instalment

b how much she repays altogether.

11 Peter used his Lotto winnings to buy an annuity from the Goodbuy Insurance Company. The annuity of $10 000 per year for 9 years is at an interest rate of 10% p.a. compounded annually. How much was Peter's Lotto win?

12 Callum is a DJ and took out a loan to buy some new stereo equipment. If he paid back the loan at $225 per month over 10 months and the interest rate was 12% p.a. compounded monthly:

a how much did he pay back altogether?

b how much did he borrow?

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INVESTIGATION

A FORMULA FOR THE PRESENT VALUE OF AN ANNUITY

The values in the future value table are calculated using a special formula for the future value of an annuity:

$$PVA = a\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

where: a = contribution per period, paid at the end of the period

r = interest rate per compounding period, expressed as a decimal

n = number of compounding periods

This formula is NOT part of the Mathematics General 2 course and does not have to be learned.

Joel deposits $400 at the end of each month for 6 months in an account earning 12% p.a. compounded monthly. Show that the sum of money that Joel could invest now to achieve the same future value as the annuity is $2318.20, using:

a the present value table

b the *PVA* formula

INVESTIGATION

SUPERANNUATION PAYMENTS

With superannuation, you make regular contributions *to* an annuity during your working life to provide funds for your retirement. When you retire, you can receive your superannuation as a lump sum or reinvest it into another annuity that pays you in regular instalments. When receiving payments *from* an annuity, the present value is the lump sum that must be invested to provide those regular payments over time.

Suppose that when you retire you want to receive annual payments of $50 000 for 30 years. What lump sum do you need to invest now to provide this? Find an online calculator like the one below to compute what you need to have a lump sum of $471 345.72.

Inputs			
Annual Payout at end of year:	$	50,000.00	
Growth Rate:		10	%
Years to Pay Out:		30	
Results			
Present Value (=Starting Principal):	$	471,345.72	

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1 Show that if Kevin contributes $100 per week for 15 years to a superannuation fund earning 10.4% p.a. compounded weekly, he will have $187 571 when he retires.

2 Kevin uses his lump sum to purchase an annuity at 12% p.a. compounded monthly, giving him a series of equal payments each month for 20 years. How much is each payment?

Loan repayments table

1-08 Loan repayment tables

Published loan repayment tables allow us to easily calculate repayments for reducing balance loans.

EXAMPLE 17

This table gives the monthly repayments for a reducing balance loan of $1000 for various terms and interest rates.

Monthly repayments per $1000 borrowed

Interest rate (% p.a.)	Term (years)					
	5	10	15	20	25	30
5	$18.87	$10.61	$7.91	$6.60	$5.85	$5.37
6	$19.33	$11.10	$8.44	$7.16	$6.44	$6.00
7	$19.80	$11.61	$8.99	$7.75	$7.07	$6.65
8	$20.28	$12.13	$9.56	$8.36	$7.72	$7.34
9	$20.76	$12.67	$10.14	$9.00	$8.39	$8.05
10	$21.25	$13.22	$10.75	$9.65	$9.09	$8.78

The Dixons borrowed $570 000 at 7% p.a. reducible interest for a term of 20 years. Use the table to find:

a their monthly repayment

b the total amount repaid

c the interest paid

d how much more interest they would pay if the loan was over 25 years instead of 20 years.

Solution

a Term = 20 years, interest rate = 7% p.a.

Monthly repayment for $1000 = $7.75 **From table.**

Monthly repayment for $570 000 = $7.75 × 570

= $4417.50

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b Total amount repaid = $4417.50 × 12 × 20
= $1 060 200

Monthly repayments over 20 years.

c Interest = $1 060 200 – $570 000
= $490 200

Total amount repaid – principal.

d Term = 25 years, interest rate = 7% p.a.

Monthly repayment for $1000 = $7.07

Monthly repayment for $570 000 = $7.07 × 570
= $4029.90

Total amount repaid = $4029.90 × 12 × 25
= $1 208 970

Monthly repayments over 25 years.

Interest paid = $1 208 970 – $570 000
= $638 970

Total amount repaid – principal.

Difference in interest paid = $638 970 – $490 200
= $148 770

Exercise 1-08 Loan repayment tables

Use the table from the previous page to answer questions **1**, **2** and **3**.

Example 17

1 Find the monthly repayment on a $450 000 loan at 9% p.a. for 15 years. Select **A**, **B**, **C** or **D**.

A $5701.50 **B** $4563 **C** $4837.50 **D** $4050

2 Find the interest paid on a $500 000 loan at 6% p.a. over 30 years. Select **A**, **B**, **C** or **D**.

A $580 000 **B** $277 600 **C** $1 080 000 **D** $777 600

3 For each reducing balance loan shown below, calculate:

i the monthly repayment

ii the total amount repaid

iii the total interest paid

a $380 000 at 7% over 15 years

b $925 000 at 9% over 25 years

c $1 230 000 at 6% over 20 years

d $697 000 at 10% over 10 years

4 This table gives the monthly loan repayments required to pay off various loans in 10 years.

Monthly loan repayments—10-year term

Principal borrowed	Interest rate			
	6% p.a.	7% p.a.	8% p.a.	10% p.a.
$5000	$55.52	$58.06	$60.67	$66.08
$20 000	$222.05	$232.22	$242.66	$262.31
$50 000	$555.11	$580.55	$606.64	$660.76
$100 000	$1110.21	$1161.09	$1213.28	$1321.51

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For each loan with 10-year terms, calculate the amount of interest paid.

a $20 000 at 8% p.a. **b** $100 000 at 6% p.a.

c $5000 at 7% p.a. **d** $50 000 at 10% p.a.

5 **a** If loan **c**'s interest rate from question **4** dropped by 1%, how much less would the monthly repayment be?

b How much more interest would you pay if loan **a**'s interest rate increased by 2%?

6 The table below shows monthly repayments for loans over terms from 15 to 20 years.

Monthly loan repayments

Principal	Term of loan (years)					
	15	16	17	18	19	20
$50 000	$492.37	$477.25	$464.15	$452.73	$442.73	$433.92
$60 000	$590.85	$572.70	$556.98	$543.28	$531.27	$520.70
$70 000	$689.32	$668.15	$649.81	$633.83	$619.82	$607.48
$80 000	$787.80	$763.60	$742.64	$724.37	$708.36	$694.26
$90 000	$886.27	$859.05	$835.47	$814.92	$796.91	$781.05
$100 000	$984.74	$954.50	$928.30	$905.46	$885.45	$867.83

a How much would you repay per year on a loan of:

i $60 000 for 19 years?

ii $80 000 for 17 years?

b Give **two** ways of finding the monthly repayment for a loan of $180 000 for 16 years.

c What is the interest saved by borrowing $300 000 over 15 years instead of 20 years?

7 The table shows the monthly and fortnightly repayments for some reducing balance loans over different terms.

	Repayment amounts					
	1 year		18 months		2 years	
Loan amount	monthly	fortnightly	monthly	fortnightly	monthly	fortnightly
$10 000	$885	$408	$606	$280	$467	$215
$20 000	$1770	$815	$1213	$559	$935	$430
$30 000	$2655	$1223	$1819	$838	$1402	$646
$40 000	$3540	$1630	$1869	$1117	$2425	$861

a What is the total amount to repay if you have a loan of $30 000 over 2 years with monthly repayments?

b How much interest do you save if you choose fortnightly repayments for the loan in part **a**?

c Jamal took a $40 000 loan over 1 year while Nadine took a $40 000 loan over 2 years. If each made monthly repayments, what was the difference in the total amount repaid?

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1-09 Repaying a home loan

EXAMPLE 18

Tanya applied for a $495 000 home loan to purchase a city apartment. There are a number of additional costs associated with taking out the loan.

Registration of mortgage	$102
Registration of transfer	$204
Home loan establishment fee	$600
Rate lock (fixed interest rate) fee	$750
Late payment charge	$20 per late payment
Loan service fee	$10 per month

This fee makes the interest rate on Tanya's loan stay the same, unlike a variable interest rate where the rate can change often.

a Tanya must also pay stamp duty (state government tax) of $29 240. If she is not locking her interest rate, what is the total amount in charges she must pay upfront?

b Calculate, correct to one decimal place, Tanya's upfront charges as a percentage of her home loan.

c After paying the upfront charges, how much did Tanya pay in fees in the first year if she did not make any late repayments?

Solution

a Upfront charges = $29 240 + $102 + $204 + $600
= $30 146

b $\frac{\$30\ 146}{\$495\ 000} \times 100\% = 6.0901...$
$\approx 6.1\%$

c Loan service fee = $10 × 12
= $120

EXAMPLE 19

Jennifer takes out a personal loan to install a swimming pool costing $48 700 at her house. She has two options:

- **Option 1:** Fortnightly repayments of $882.65 at 12.6% p.a. reducible interest
- **Option 2:** Monthly repayments of $1632.10 at 12.6% p.a. reducible interest

The two graphs below show the progress of the loan under each option.

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a Approximately how many years and months will it take for Jennifer to repay the loan if she makes her repayments:

i fortnightly?

ii monthly?

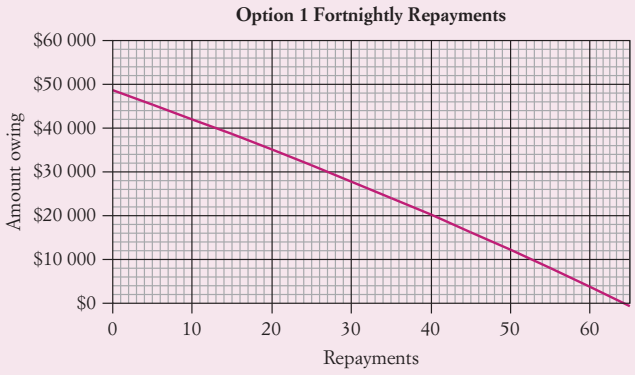
b How much time would she save by paying off the loan fortnightly instead of monthly?

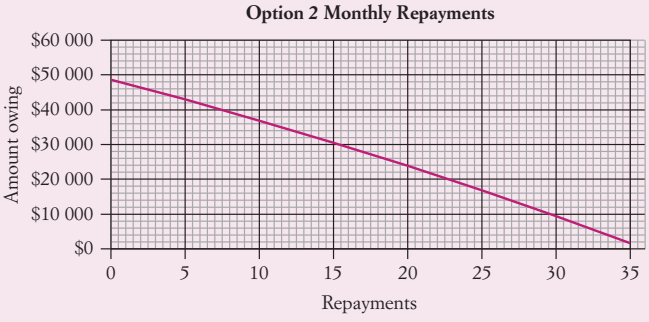
c Calculate the total of Jennifer's repayments if she pays them:

i fortnightly

ii monthly

d How much interest would she save by paying off the loan fortnightly instead of monthly?





Solution

a Reading from the graphs:

i Approximately 64 fortnights $= 64 \times 2$ weeks
$= 128$ weeks
$= 128 \div 52$ years
$= 2.4615\ldots$ years
$= 2$ years $+ 0.461\ldots \times 12$ months
$= 2$ years $+ 5.5384\ldots$ months
≈ 2 years 6 months

ii Approximately 36 months $= 36 \div 12$ years
$= 3$ years

b Time saved paying fortnightly $= 3$ years $-$ 2 years and 6 months
$= 6$ months

c **i** Total fortnightly repayments $= \$882.65 \times 64$
$= \$56\ 489$

ii Total monthly repayments $= \$1632.10 \times 36$
$= \$58\ 764.96$

The difference in interest paid is the same as the difference in total repayments.

d Interest saved paying fortnightly $= \$58\ 764.96 - \$56\ 489$
$= \$2275.96$

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Exercise 1-09 Repaying a home loan

Example 18

Questions **1** to **3** refer to this table of United Banking loan interest rates and fees.

Home loan	Car loan	Personal loan
Interest rate: 6.8% p.a. Registration of mortgage: $102 Registration of transfer: $204 Loan establishment fee: $600 Rate lock fee: $750 Late payment charge: $20 Loan service fee: $10 per month	Interest rate: 10.25% p.a. Loan establishment fee: $295 Early termination fee: • $750 if original loan is $12 000 or less • 6.25% of original loan if over $12 000 Late payment charge: $40	Interest rate: 14.50% p.a. Loan approval fee: $150 Rate lock fee: $750 Late payment charge: $20 Loan service fee: $30 per quarter

1 Alia borrowed $9000 to buy a second-hand car.

- **a** What interest rate was she charged?
- **b** What does she pay to establish the loan?
- **c** What will she be charged if she terminates the loan early?

2 Vaesna establishes a home loan of $500 000.

- **a** What interest rate is he charged?
- **b** What is the total amount in charges that Vaesna must pay upfront?
- **c** He made late home loan repayments 2 months in a row. How much did his late payment and service charges amount to?

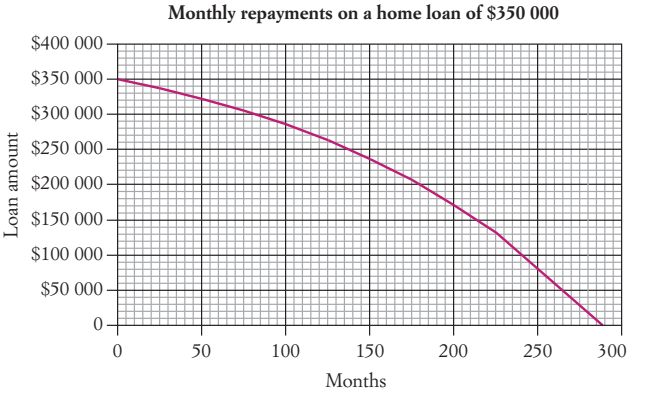
3 Elizabeth borrows $4700 to fill her new studio apartment with furniture and electrical goods.

- **a** What interest rate is she charged?
- **b** How much did she pay when her loan was approved?
- **c** In the first year of her loan she made one late payment and had to pay administration charges. How much did Elizabeth pay?

Example 19

4 The graph below shows a home loan of $350 000 repaid with monthly repayments.

- **a** Estimate from the graph how many years and months it took to pay off this loan.
- **b** Estimate how many years and months it took for the home loan to reduce to $250 000.
- **c** How much money was still owing after 20 years?



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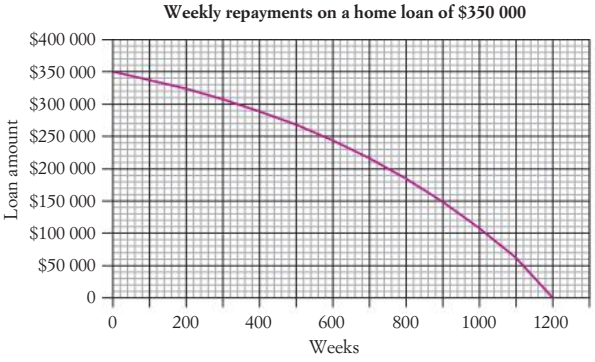
d If the loan was repaid with monthly repayments of $2490, how much was repaid in total?

e Calculate how much interest was paid.

f The graph here shows the same loan of $350 000 repaid with weekly repayments. Estimate how many years and months it took to pay off this loan.



g Estimate how many years and months it took for the home loan to reduce to $300 000.

h How much of the loan was still owing after 15 years?

i If the loan was repaid with weekly repayments of $580, how much was repaid in total?

j Calculate how much interest was paid.

k How much time would be saved by paying off the loan weekly instead of monthly?

l How much interest would be saved by paying off the loan weekly instead of monthly?

5 The graph shows the progress of a $400 000 loan over a term of 25 years.

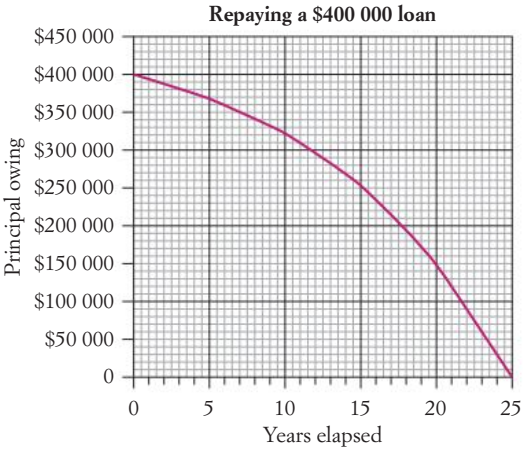
a Find, to the nearest $10 000, the principal owing after:

i 5 years **ii** 15 years

b In what year was:

i a principal of $300 000 owing?

ii the principal half-paid off?



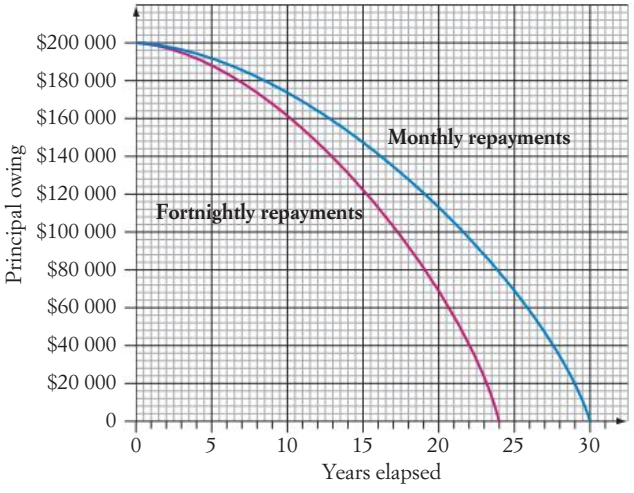
6 The graph shows the progress of a $200 000 loan at 6.7% p.a. reducible interest over 30 years for monthly and fortnightly repayments.

a How much sooner can you repay the loan by making fortnightly repayments?

b After how many years was a principal of $100 000 owing if repayments were made:

i monthly?

ii fortnightly?



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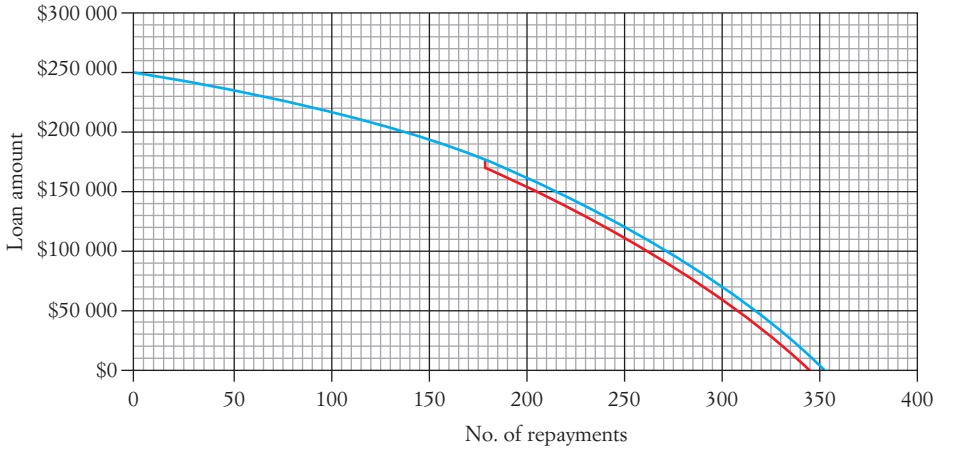
c How much of the amount borrowed was still owing after:

i 10 years with fortnightly repayments?

ii 20 years with monthly repayments?

7 The blue graph below shows the progress of the Hayes family's home loan, at monthly repayments of $1497.



a What amount did the Hayes family borrow?

b How much will they owe after 150 months?

c How many years and months will it take to pay off the loan?

d In total, how much will they repay?

e Calculate how much interest they will pay.

f After 15 years (180 months) into the loan, the family made one extra payment of $10 000 off the loan. It changed the length of the loan, as shown by the red graph and the 'dip' at 180 months. How many years and months will it take to pay off the loan now?

g How much will the Hayes family repay now, including the $10 000?

h How much interest will they pay now?

i How much time was saved on the loan after paying the extra $10 000?

j How much interest was saved on the loan after paying the extra $10 000?

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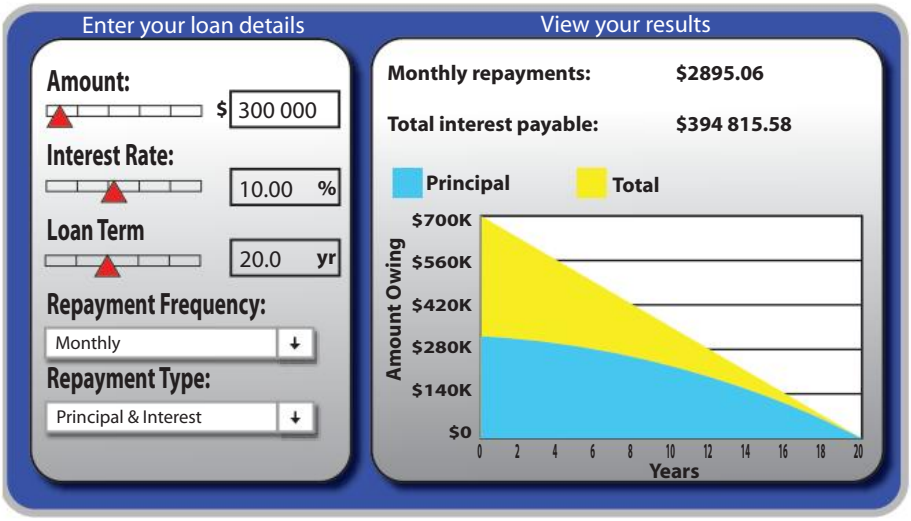
INVESTIGATION

CHANGING VARIABLES ON HOME LOANS



Use the Internet to find an online loan calculator to see the effect of:

- higher or lower interest rates on total interest paid
- repaying fortnightly or weekly rather than monthly
- changing the term of the loan on the size of the repayments
- increasing the size of the repayment on the time for the loan to be repaid

The graph above shows the progress of a reducing balance loan over time, with total amount owing (principal + interest) shown in yellow and the amount of principal owing shown in blue.

Suppose you borrowed $400 000 to buy an apartment at 8% p.a. reducible interest to be repaid monthly over 25 years. Use your loan calculator to find:

a the size of each monthly repayment

b how increasing the monthly payment by $50 affects the term of the loan

c how much sooner you would repay the loan if you made repayments fortnightly instead of monthly

d how much sooner you would repay the loan if you made an extra payment of $10 000 after 10 years

e the amount paid in stamp duty (use a stamp duty calculator).

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Sample HSC problem

This table is used to calculate monthly loan repayments on reducing balance loans.

Monthly repayments per $1000 borrowed

Interest rate (% p.a.)	Term (years)				
	5	10	15	20	25
5	$18.87	$10.61	$7.91	$6.60	$5.85
5.5	$19.10	$10.85	$8.17	$6.88	$6.14
6	$19.33	$11.10	$8.44	$7.16	$6.44
6.5	$19.57	$11.35	$8.71	$7.46	$6.75

James borrowed $320 000 at 5.5% p.a. for 10 years.

a What is his monthly loan repayment?

b What total amount does he repay?

c How much interest does he pay?

d What is the equivalent flat interest rate per annum?

Study tip

Switch off your phone and computer

It is very easy to achieve nothing in three hours of 'study time' if much of it is spent on tidying up your desk, chatting with friends on the phone or social networks, having excessive fridge and TV breaks, or playing games on the computer. If you are serious about studying, you need to program into your study routine blocks of time when you cannot be interrupted by phone calls, messages or friendly visits. This is especially important when you have a big task at hand, such as preparing for an exam or completing a major assignment. Establish an arrangement with your family so that they know not to disturb you during your study time.

Put your phone and other digital devices away. Don't study in front of the TV or computer. Time passes very quickly if you do. Either study *or* watch TV *or* use the computer, but don't try to multi-task. You can listen to music or have the radio on as long as it doesn't affect your concentration. As for those fun things you want to do? Do them during a study break as a reward for completing a significant amount of work.

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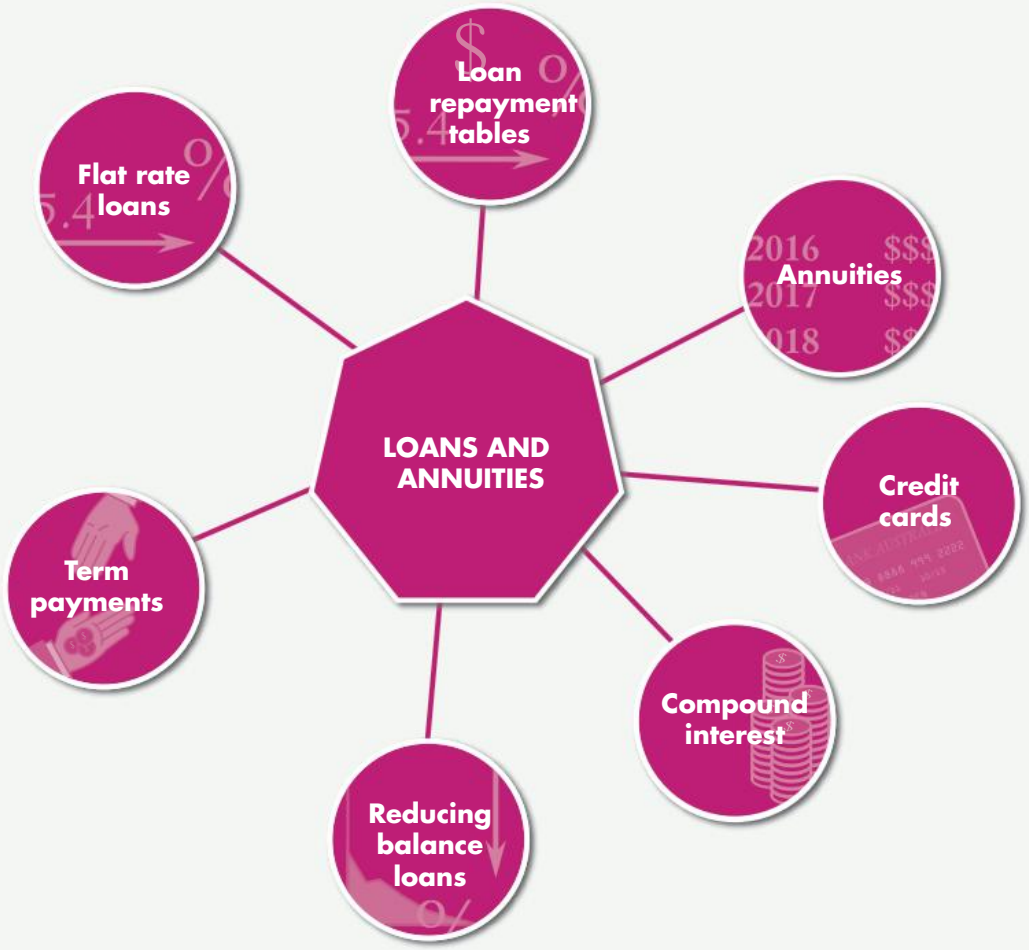
CHAPTER REVIEW

Topic overview

This chapter, 'Loans and annuities', examined the financial mathematics of credit, loans and annuities. You should be able to calculate the interest and repayments involved in flat rate loans, term payments, reducing balance loans and credit cards, and use tables and graphs to track the progress of a reducing balance loan. You used tables to calculate future and present values of annuities, and loan repayments. You should understand the financial jargon of this topic, and be able to make informed decisions on different loan and investment options.

Make a summary of this topic. Use the outline at the start of this chapter as a guide.
An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

1 Casey bought a motorcycle for $15 000 and paid 10% deposit. He took a flat rate loan out at 12% p.a. for the remainder plus the additional charges of $100 delivery, $35 stamp duty, $250 registration, third-party insurance and $80 per year loan insurance. If he repaid monthly instalments over 4 years, find:

Exercise 1-01

a the total amount borrowed

b the amount of interest charged

c the monthly instalment.

2 Scott bought a new dishwasher and refrigerator and borrowed $1900 to help with the purchase. If the flat interest rate is 12.7% p.a. and the term of the loan is 2 years, find his monthly repayment. Select **A**, **B**, **C** or **D**.

Exercise 1-01

A $198.55 **B** $178.44 **C** $99.28 **D** $89.22

3 Juliet bought a new bedroom suite from Fancy Furniture Discounts. The cash price was $8000 but Juliet chose to buy on terms paying 10% deposit plus fortnightly repayments of $238 for 18 months.

Exercise 1-02

a How much did she actually pay for the bedroom suite?

b How much interest did she pay?

c What was the flat rate of interest charged (correct to one decimal place) per annum?

4 Su-Yin wants to buy new kitchen appliances for $10 000. If she chose to buy on terms, how much per month would she pay on the following hire purchase terms?

Exercise 1-02

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Exercise 1-03

5 Thomas borrowed $18 000 for a new bathroom at 9% p.a. reducible interest. He made monthly payments of $420.

a Draw up a table showing the progress of the loan in the first 4 months, with columns as shown below, where $R = \$420$.

Month	P	I	$P + I$	$P + I - R$
1	$18 000			
2				
3				
4				

b How much had Thomas paid off the principal after his fourth payment?

c How much interest did he pay in the first four months?

d How much interest did he save in the first four months compared with a flat interest rate of 9% p.a.?

Exercise 1-04

6 Beatrice invested $9500 for 6 months at 4.75% p.a. compounded monthly. How much interest did she earn? Select **A**, **B**, **C** or **D**.

A $2491.66 **B** $227.87 **C** $3050.12 **D** $109.12

Exercise 1-04

7 Eddie invested his inheritance in an account earning 8% p.a. compounded monthly. His investment grew to $10 000 in 8 years. How much did he inherit?

Exercise 1-05

8 Minjee has a credit card with no interest-free period. Her monthly statement for October is shown.

Ms Minjee Lee Payment due: 1 November 2014		**GOLF BANK** **Credit Card statement**	
Previous balance $1325.68	**Payment** $1325.68	**Total purchases**	**Interest** .....
Date	**Purchases**	**Amount**	
15 October	Golf lesson	$325.50	
20 October	Golf shoes	$295.00	
28 October	Massage	$137.00	
Annual simple interest rate: 16.38% Daily percentage rate: 0.0449%			

If Minjee pays her account in full on 5 November, how much interest does she pay?

Use the future value of an annuity (FVA) table on page 29 to answer questions **9** and **10**.

Exercise 1-06

9 Alaska puts $5000 in an account at the end of each year for 12 years.

a What is the future value of her annuity if the interest rate is 8% p.a?

b How much interest will she earn?

Exercise 1-06

10 Irven contributes $16 000 per quarter for 1 year into an annuity that earns interest at 12% p.a. compounded quarterly. Calculate the future value of Irven's annuity.

Use the present value of an annuity (PVA) table on page 34 to answer questions **11** and **12**.

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11 Greta deposits $2000 at the end of each month for 9 months into an account earning 12% p.a. compounded monthly. What single amount could she invest now to produce the same financial result over the same period at the same interest rate?

Exercise 1-07

12 Find the regular repayment (a) for a loan with principal:

Exercise 1-07

a $13 500, at 2% per month for 6 months

b $45 000, at 6% per quarter for $2\frac{1}{2}$ years

13 This table gives the monthly car loan repayments for different loan amounts and terms.

Exercise 1-08

Monthly car loan repayments

Loan amount	Term of loan (months)			
	18	24	36	48
$10 000	$619	$480	$342	$273
$12 000	$743	$576	$410	$328
$14 000	$867	$672	$478	$383
$16 000	$991	$768	$547	$437
$18 000	$1114	$864	$615	$492
$20 000	$1238	$960	$684	$547

Consider a loan of $20 000 over 4 years. Use the table to calculate:

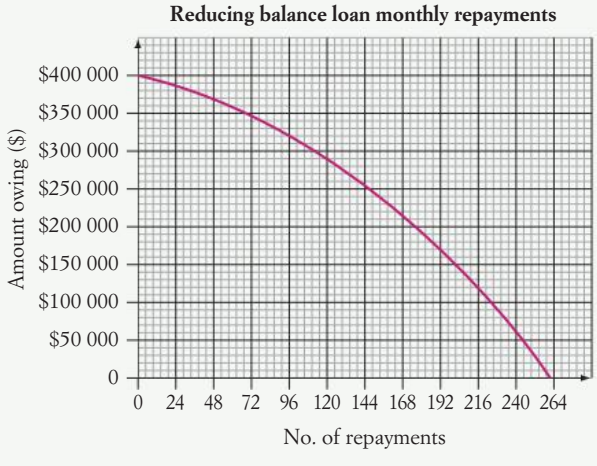
a the monthly repayment

b the total amount repaid

c the total interest paid

14 The graph below shows the progress of Ante's home loan, in which he made monthly repayments of $3000.

Exercise 1-09



a How much money did Ante borrow?

b What was the amount owing after 10 years?

c How many years and months did Ante take to pay off the loan?

d What was the time taken for the loan to be half-paid?

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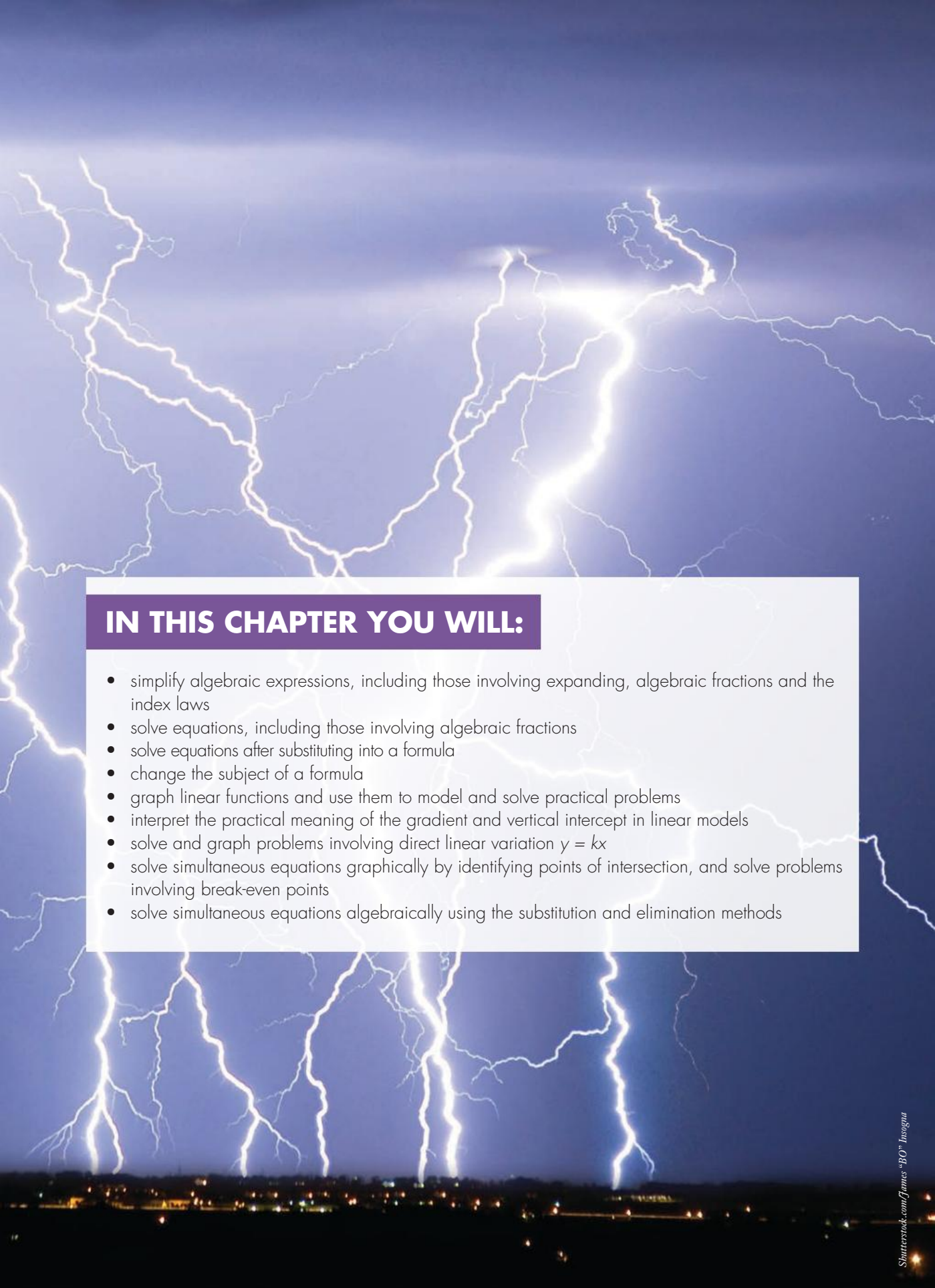
ALGEBRA AND MODELLING

2.

EQUATIONS AND LINEAR FUNCTIONS

Many relationships in real life, where one quantity or variable directly affects another, can be described by a table of values, a graph or an algebraic formula. For example, the temperature of air varies with the height above sea level and the time between seeing lightning and hearing thunder varies directly with the distance from the storm. When the relationships are modelled using a linear function, the gradient and *y*-intercept have special significance.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- simplify algebraic expressions, including those involving expanding, algebraic fractions and the index laws
- solve equations, including those involving algebraic fractions
- solve equations after substituting into a formula
- change the subject of a formula
- graph linear functions and use them to model and solve practical problems
- interpret the practical meaning of the gradient and vertical intercept in linear models
- solve and graph problems involving direct linear variation $y = kx$
- solve simultaneous equations graphically by identifying points of intersection, and solve problems involving break-even points
- solve simultaneous equations algebraically using the substitution and elimination methods

TERMINOLOGY

break-even point
constant term
elimination method
gradient
index notation
point of intersection
simultaneous equations
varies
coefficient
dependent variable
expand
independent variable
like terms
power
subject of a formula
vertical intercept
constant of variation
direct linear variation
formula
index laws
linear function
proportional
substitution method
y-intercept

Assignment 2

Algebra review

SkillCheck

1 Simplify each expression.

a $4m + 7 - 3m + 9$ **b** $6n \times 5mn$ **c** $\frac{12hp}{20p}$ **d** $2x + 5a - 6a + x$

2 Expand each expression.

a $3(2r - 5)$ **b** $-6(y + 7)$ **c** $-2(8 - k)$

3 Evaluate $c^2 + c - 2$ if:

a $c = 7$ **b** $c = -3$ **c** $c = 10$

4 Solve each equation.

a $3x + 17 = 38$ **b** $8d - 4 = 3d - 9$ **c** $20 - 2w = -16$

5 For the linear function $y = 3x + 1$, what is:

a the dependent variable? **b** the gradient?

c the *y*-intercept? **d** the value of x when $y = -14$?

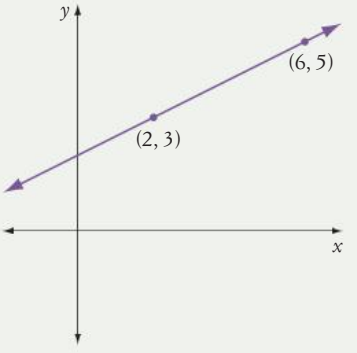
6 Graph $y = 3x + 1$ and $y = 5 - x$ on the same number plane and write the coordinates of their point of intersection.

7 Find the linear function for this table of values.

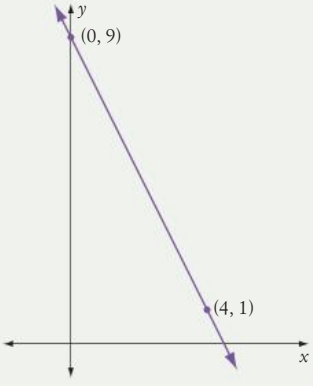
x	1	2	3	4	5
y	–2	1	4	7	10

8 Find the gradient and equation of each line below.

a



b



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2-01 Algebraic expressions

An **algebraic expression** is made up of terms involving variables and numerals. For example, $8t^3 + 5t^2 - 1$ has three terms: $8t^3$, $5t^2$ and -1.

EXAMPLE 1

Simplify each expression.

a $15m - 2d - 8m + 6d$ **b** $3d + 8d^2 - 3 - d$ **c** $18mp^2 \div 45m^2p$

Solution

a $15m - 2d - 8m + 6d = 15m - 8m - 2d + 6d$

$= 7m + 4d$

Collecting like terms $15m$ and $-8m$, $-2d$ and $6d$.

b $3d + 8d^2 - 3 - d = 8d^2 + 3d - d - 3$

$= 8d^2 + 2d - 3$

c $18mp^2 \div 45m^2p = \dfrac{18mp^2}{45m^2p}$

$= \dfrac{18}{45} \times \dfrac{m}{m^2} \times \dfrac{p^2}{p}$

$= \dfrac{2}{5} \times \dfrac{1}{m} \times \dfrac{p}{1}$

$= \dfrac{2p}{5m}$

Dividing the numbers and variables separately.

SUMMARY

- When **adding or subtracting algebraic terms,** only 'like terms' can be added or subtracted
- When **multiplying or dividing algebraic terms,** multiply or divide the numbers and variables separately.
- When **expanding an algebraic expression** with brackets, multiply each term inside the brackets by the term outside.

  $a(b + c) = ab + ac$

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EXAMPLE 2

Expand and simplify each expression.

a $3(6m + 7) - 2(m^2 + 5)$ **b** $r(4r - 10) - (4r - 10)$

Solution

a $3(6m + 7) - 2(m^2 + 5) = 18m + 21 - 2m^2 - 10$
$= -2m^2 + 18m + 11$

Expanding.
Collecting like terms 21 and –10.

b $r(4r - 10) - (4r - 10) = 4r^2 - 10r - 4r + 10$
$= 4r^2 - 14r + 10$

–(4r – 10) means –1(4r – 10).

Example 1

Exercise 2-01 Algebraic expressions

1 Simplify each expression.

a $x^3 + 2x^2 + 5x^2 - x^3$ **b** $4m + 7 + 18m - 5$
c $3bd + 4d + d^2 - 5d$ **d** $6c - 6 - c + 20$
e $2r + r + 2r^2 - r$ **f** $7tu + t^2 - 8t^2 + tu$
g $-3e + 9 + e + e^2$ **h** $5p + 4p - 9p^2 + 7p^2$
i $2xy + 3yz + 4xy - 8yz$ **j** $7k + 11y - 20 + 3k - 6$
k $16j - 3jk - 5j - 4jk$ **l** $8u^2 + 6 - 10 - 5u^2$

2 Simplify each expression.

a $3m^2 \times 2mn$ **b** $-5p \times (-6p)$ **c** $(-2rt^2)^2$
d $16w^5y^2 \div 8wy$ **e** $10u^2 \times \frac{3u}{2}$ **f** $\frac{18k^2y}{3ky}$
g $4p^2 \div 10p$ **h** $6r \times (-3r^2)$ **i** $(3x^2)^3$
j $5u \times u$ **k** $5u \div u$ **l** $\frac{6de^2}{14d^2e^2}$

Example 2

3 Expand and simplify each expression.

a $3(k + 4) + 4(k + 4)$ **b** $2(d + 1) - 2(d - 1)$ **c** $5(r + 3) + 4(r + 10)$
d $8(m + 5) - 3(m - 5)$ **e** $2(5a + 6) + (a - 1)$ **f** $3(2t + 1) - (4t + 8)$
g $x(x + 7) - 5(x - 3)$ **h** $5u(2 - u) + u(6u + 3)$ **i** $4(4m + 1) - 3(3m - 2)$
j $8(5 - 2r) + r(3r + 1)$ **k** $6(2d - 3) + (2d - 3)$ **l** $f(5f + 5) + 5(f + 5)$

4 Which of the following is the simplified expression for $4(2m + 7) - 5(2m + 3)$?
Select **A**, **B**, **C** or **D**.

A $-2m + 43$ **B** $-2m + 25$ **C** $-2m + 13$ **D** $-2m + 23$

5 Which of the following is the simplified expression for $\frac{-24a^2b^2}{-16ab^2}$? Select **A**, **B**, **C** or **D**.

A $\frac{3a}{2}$ **B** $\frac{3ab}{2}$ **C** $-\frac{3a}{2b}$ **D** $-\frac{3ab}{2}$

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6 Which of the following is the simplified expression for $10x(x-1)+x(5x+4)$? Select **A**, **B**, **C** or **D**.

A $15x^2+5$ **B** $15x^2-6x$ **C** $15x^2-14x$ **D** $15x^2+6x$

7 Simplify each expression.

a $-3b^2\times\frac{4bd}{6}$ **b** $(-4ab)^2$ **c** $5x+4xz-4x+6$

d $4ay+y-ay+y$ **e** $8kp^2\div 12k$ **f** $4de\times 5d^2e$

g $\frac{-10ut}{4t^2}$ **h** $6jr+2r-4jr-5r$ **i** $6wz\times\frac{5w^2}{2z}$

j $8tu+4u^2-u^2-8t$ **k** $8a^2y\times\frac{1}{2}y^2$ **l** $(5f^3)^2$

2-02 Algebraic fractions

Operations with algebraic fractions follow the same rules as numerical fractions.

WS Algebraic fractions

SUMMARY

When **adding and subtracting fractions**, first convert them so that they have the same denominators.

EXAMPLE 3

Simplify each expression.

a $\frac{k}{8}+\frac{k}{3}$ **b** $\frac{7x}{10}-\frac{2x}{3}$ **c** $\frac{4d}{5}+\frac{3d}{4}$

Solution

a $\frac{k}{8}+\frac{k}{3}=\frac{3\times k}{3\times 8}+\frac{8\times k}{8\times 3}$ **Common denominator is $8\times 3=24$**

$=\frac{3k}{24}+\frac{8k}{24}$

$=\frac{11k}{24}$

b $\frac{7x}{10}-\frac{2x}{3}=\frac{3\times 7x}{30}-\frac{10\times 2x}{30}$

$=\frac{21x}{30}-\frac{20x}{30}$

$=\frac{x}{30}$

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c $\frac{4d}{5}+\frac{3d}{4}=\frac{4\times 4d}{20}+\frac{5\times 3d}{20}$

$=\frac{16d}{20}+\frac{15d}{20}$

$=\frac{31d}{20}$

SUMMARY

When **multiplying fractions**, cancel any common factors, then multiply numerators and multiply denominators.

When **dividing by a fraction** $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.

EXAMPLE 4

Simplify each expression.

a $\frac{4dg}{5m}\times\frac{m}{12g}$ **b** $\frac{kw}{8}\div\frac{kd}{6}$

Solution

a $\frac{4dg}{5m}\times\frac{m}{12g}=\frac{{}^{1}\cancel{4}\,dg}{5\cancel{m}}\times\frac{\cancel{m}}{{}_{3}\cancel{12}\,g}$

$=\frac{d}{15}$

b $\frac{kw}{8}\div\frac{kd}{6}=\frac{kw}{8}\times\frac{6}{kd}$ **Multiply by the reciprocal.**

$=\frac{\cancel{k}w}{{}^{4}\cancel{8}}\times\frac{\cancel{6}^{3}}{\cancel{k}d}$

$=\frac{3w}{4d}$

Example 3

Exercise 2-02 Algebraic fractions

1 Simplify each expression.

a $\frac{d}{6}+\frac{d}{2}$ **b** $\frac{p}{3}-\frac{p}{5}$ **c** $\frac{2w}{7}-\frac{w}{3}$

d $\frac{3x}{5}+\frac{7x}{8}$ **e** $\frac{2d}{3}+\frac{m}{2}$ **f** $\frac{5c}{4}+\frac{7x}{12}$

g $\frac{7a}{9}-\frac{4a}{5}$ **h** $\frac{9x}{10}-\frac{2x}{3}$ **i** $\frac{8k}{5}-\frac{k}{3}$

j $\frac{7c}{9}-\frac{2x}{5}$ **k** $\frac{4h}{3}+\frac{w}{5}$ **l** $\frac{3f}{4}+\frac{f}{12}$

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Example 4

2 Simplify each expression.

a $\frac{d}{5} \times \frac{m}{3}$ **b** $\frac{2k}{5} \div \frac{3}{w}$ **c** $\frac{4h}{5} \div \frac{h}{10}$

d $\frac{15gh}{4} \times \frac{2h}{3g}$ **e** $\frac{3ph}{5} \div \frac{2p}{3}$ **f** $\frac{4ay}{15} \times \frac{3x}{8y}$

g $\frac{3mn}{10k} \div \frac{12mp}{5}$ **h** $\frac{abd}{12} \times \frac{4b}{3ad}$ **i** $10m \div \frac{5m}{2}$

j $\frac{5wm}{6} \times \frac{8w}{15mn}$ **k** $\frac{10abc}{3} \times \frac{6}{5bc}$ **l** $\frac{8k}{5} \div \frac{32k}{15d}$

2-03 Index laws

Index notation is the use of **powers** to write repeated multiplication of a number.

For example, in $5^3 = 5 \times 5 \times 5$, 5 is called the **base** and 3 is called the **index**, **power** or **exponent**.

SUMMARY

Index laws and properties

When multiplying terms with powers that have the same base, add the powers:

$$a^m \times a^n = a^{m+n}$$

When dividing terms with powers that have the same base, subtract the powers:

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$$

When raising a term with a power to another power, multiply the powers:

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Any number raised to a power of zero is equal to 1:

$$a^0 = 1$$

EXAMPLE 5

Simplify each expression.

a $5w^3 \times 4w^7$ **b** $12k^6 \div 4k^2$ **c** $(3m^5)^2$

d $8y^5 \times \frac{1}{2}y^3$ **e** $\frac{25k^2d}{15kd^3}$ **f** $4x^0 + 7$

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Solution

a $5w^3 \times 4w^7 = 5 \times 4 \times w^3 \times w^7$
$= 20w^{3+7}$
$= 20w^{10}$

b $12k^6 \div 4k^2 = \frac{{}^{3}\cancel{12}k^6}{{}_{1}\cancel{4}k^2}$
$= 3k^{6-2}$
$= 3k^4$

c $(3m^5)^2 = 3^2 \times (m^5)^2$
$= 9 \times m^{5 \times 2}$
$= 9m^{10}$

You can raise each part of the term to the power separately

d $8y^5 \times \frac{1}{2}y^3 = 8 \times \frac{1}{2} \times y^5 \times y^3$
$= 4y^8$

e $\frac{25k^2d}{15kd^3} = \frac{25}{15} \times \frac{k^2}{k} \times \frac{d}{d^3}$
$= \frac{5}{3} \times \frac{k}{1} \times \frac{1}{d^2}$
$= \frac{5k}{3d^2}$

f $4x^0 + 7 = 4 \times 1 + 7$
$= 11$

Example 5

Exercise 2-03 Index laws

1 Simplify each expression.

a $m^4 \times m^5$	**b** $y^5 \div y^3$	**c** $(w^3)^4$
d $3p^4 \times 2p$	**d** $(4b^2)^3$	**f** $15x^6 \div 3x$
g $24d^5 \div 8d^3$	**h** $(g^3w^2)^5$	**i** $(2m^5)^5$
j $6d^4 \times 5d^3$	**k** $28y^4w^3 \div 7yw^2$	**l** $(4k^3p^2)^4$
m $b^5 \times b^3 \div b^6$	**n** $(a^3)^4 \times (a^5)^2$	**o** $(x^2y^3)^4 \div y^7$

2 Simplify each expression.

a $3m^2 \times 2mn$	**b** $-5p \times (-6p)$	**c** $(-2rt^2)^2$
d $5k^0 + 4$	**e** $3q^0 - 2q^0$	**f** $16w^5y^2 \div 8wy$
g $-10u^2 \times \frac{3u}{2}$	**h** $6 - h^0$	**i** $\frac{18k^2y}{3ky}$
j $4p^2 \div 10p$	**k** $6r \times (-3r^2)$	**l** $(3x^2y^5)^3$
m $5u \times u^0$	**n** $15u \div u$	**o** $\frac{14m^3e^2}{10me^2}$

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p $12m^0 - (3m^0)^2$ **q** $15w^5z^3 \div 18w^3z^5$ **r** $3x^5 \times \frac{1}{9}x^2$

s $7 - 8a^0$ **t** $\frac{8d^3n^4}{6dn^5}$ **u** $(4g^0)^2 \times 3g^5$

3 Which of the following is the simplified expression for $\frac{14k}{5r} \times 10r^2$? Select **A**, **B**, **C** or **D**.

A $\frac{14k}{5r}$ **B** $8kr^2$ **C** $8kr^3$ **D** $28kr$

2-04 Solving equations

Equations code puzzle

An **equation** contains an algebraic expression and an equals (=) sign, for example, $6 - 2y = 3y + 16$. An equation is **solved** when a value of the variable (for example, y) is found that makes the equation true.

Solving equations by balancing

SUMMARY

To solve an equation:

- keep the equation balanced by doing the same operation to both sides
- aim to have the variable (for example, x) on one side of the equation and a number on the other side, for example, $x = 6$.

EXAMPLE 6

Solve each equation.

a $3(4k - 7) = 23$ **b** $\frac{m}{4} - 5 = 3m + 8$

c $\frac{2d}{3} + \frac{d}{5} = -4$ **d** $\frac{4}{5y} = 7$

Solution

a

$$3(4k - 7) = 23$$
$$12k - 21 = 23$$
$$12k - 21 + 21 = 23 + 21$$
$$12k = 44$$
$$\frac{12k}{12} = \frac{44}{12}$$
$$k = 3\tfrac{2}{3}$$

Expand the LHS first.

Add 21 to both sides.

Divide both sides by 12.

Checking:

$\text{LHS} = 3\left(4\left[3\tfrac{2}{3}\right] - 7\right) = 3\left(7\tfrac{2}{3}\right) = 23 = \text{RHS}$

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b $\frac{m}{4} - 5 = 3m + 8$

$\frac{m}{4} - 5 + 5 = 3m + 8 + 5$ **Add 5 to both sides.**

$\frac{m}{4} = 3m + 13$

$\frac{m}{4} \times 4 = 3m \times 4 + 13 \times 4$ **Multiply both sides (each term) by 4.**

$m = 12m + 52$

$m - 12m = 12m - 12m + 52$ **Subtract 12*m* from both sides.**

$-11m = 52$

$\frac{-11m}{-11} = \frac{52}{-11}$ **Divide both sides by –11.**

$m = -4\frac{8}{11}$

c $\frac{2d}{3} + \frac{d}{5} = -8$

$15\left(\frac{2d}{3}\right) + 15\left(\frac{d}{5}\right) = 15(-8)$ **Multiply each term (both sides) by 15 (a common multiple of 3 and 5).**

$10d + 3d = -120$

$13d = -120$

$\frac{13d}{13} = \frac{-120}{13}$ **Divide both sides by 13.**

$d = -9\frac{3}{13}$

d $\frac{4}{5y} = 7$

$\frac{4}{5y} \times 5y = 7 \times 5y$ **Multiply both sides by 5*y*.**

$4 = 35y$

$35y = 4$ **Swap sides so that y is on the left.**

$\frac{35y}{35} = \frac{4}{35}$ **Divide both sides by 35.**

$y = \frac{4}{35}$

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Exercise 2-04 Solving equations

Example 6

1 Which of the following is the solution to the equation $6(2 - 3w) = -24$? Select **A**, **B**, **C** or **D**.

A $w = -6$ **B** $w = -2$ **C** $w = 2$ **D** $w = 6$

2 Solve each equation.

a $4m + 12 = 23$ **b** $15 - 7k = 6$ **c** $5w - 8 = 2w$

d $26 + 8g = 15$ **e** $\frac{k}{4} + 5 = 3$ **f** $\frac{d-6}{5} = 2$

g $3(p - 4) = -27$ **h** $4x + 4 = x - 11$ **i** $12d = 8 + 8d$

j $\frac{m}{2} + 5 = 4m - 7$ **k** $12 - x = \frac{x}{5} + 2$ **l** $\frac{m}{3} + \frac{m}{5} = 8$

m $\frac{d}{2} - \frac{d}{5} = 4$ **n** $\frac{y}{2} + 3 = \frac{y}{5} - 1$ **o** $\frac{4}{5} - \frac{n}{3} = 2$

3 In which line was an error made in solving this equation?

$8c + 16 = c - 12$ Line 1

$9c + 16 = -12$ Line 2

$9c = -28$ Line 3

$c = \frac{-28}{9}$ Line 4

$c = -3\frac{1}{9}$ Line 5

Select **A**, **B**, **C** or **D**.

A Line 2 **B** Line 3 **C** Line 4 **D** Line 5

4 Solve each equation.

a $\frac{5h-2}{2} = 29$ **b** $\frac{3r}{7} = 4$ **c** $-4(2k + 1) = 0$

d $\frac{7m}{10} = 3$ **e** $\frac{5-9x}{8} = 4$ **f** $8(4 - 2c) = -16$

g $10y - 8 = 46 - 2y$ **h** $5(d + 5) = 2(2d + 18)$ **i** $\frac{2a-8}{5} = 2a$

j $\frac{4}{d} = 5$ **k** $10 = \frac{3}{2k}$ **l** $\frac{8}{x} + 4 = 7$

m $3 = 5 + \frac{7}{m}$ **n** $\frac{5}{h} - 13 = 3$ **o** $4 = 1 - \frac{3}{y}$

5 Which of the following is the solution to the equation $\frac{5}{p} + \frac{1}{4} = 10$? Select **A**, **B**, **C** or **D**.

A $p = 9\frac{3}{4}$ **B** $p = \frac{3}{4}$ **C** $p = \frac{4}{15}$ **D** $p = \frac{20}{39}$

2-05 Formulas and equations

Sometimes, after we substitute values into a formula, the result is an equation that must be solved.

EXAMPLE 7

The mean, M, of three numbers x, y and z is given by the formula $M = \frac{x+y+z}{3}$. If three numbers have a mean of 22 and two of the numbers are 25 and 26, what is the third number?

Solution

Here $M = 22$, $x = 25$ and $y = 26$.

$$M = \frac{x+y+z}{3}$$

$$22 = \frac{25+26+z}{3}$$

$$22 = \frac{51+z}{3}$$

$$\frac{51+z}{3} = 22$$

$$\frac{51+z}{3} \times 3 = 22 \times 3$$

$$51 + z = 66$$

$$51 + z - 51 = 66 - 51$$

$$z = 15$$

The third number is 15.

Example 7

Exercise 2-05 Formulas and equations

1 The formula for converting Fahrenheit (°F) temperatures to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert 38°C to °F.

2 The number of days fresh milk will keep if stored at T°C is $D = \frac{6}{T+1}$. If a carton of fresh milk lasted 4 days, at what temperature was it stored?

3 The volume of a cylinder is $V = \pi r^2 h$, where r is the base radius and h is its height. If a cylinder has a volume of 904.78 cm^3 and a radius of 8 cm, find its height to the nearest centimetre.

4 The formula for converting speeds from k km/h to M metres per second is $M = \frac{5k}{18}$. Convert 12 m/s to km/h.

5 The distance d km that an observer can see to the horizon from a height of h m is given by $d = 8\sqrt{\frac{h}{5}}$. Which of the following heights allows you to see the horizon at a distance of 100 km? Select **A**, **B**, **C** or **D**.

A 358 m **B** 390 m **C** 625 m **D** 781 m

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6 The simple interest earned when $\$P$ is invested at a rate of r per period for n periods is $I = Prn$, where r is a decimal. For how long must a principal of \$2300 be invested at 6.5% p.a. to earn simple interest of \$1196?

7 The average speed, S, of a moving object is $S = \frac{d}{t}$, where d is the distance travelled and t is the time taken.

a Find the distance travelled by a car in 3 hours if its average speed is 90.2 km/h.

b Find the time taken (to the nearest minute) for a cyclist to travel 250 km if his average speed is 17.5 km/h.

8 Young's rule for calculating the medicine dosage for infants is $D = \frac{yA}{y+12}$, where y is the age of the infant in years and A is the adult dosage. How old is Libby if her medicine dosage is 0.6 mL and the adult dosage is 15 mL?

9 The formula for converting Australian dollars (\$A) to New Zealand dollars (\$NZ) is NZ = 1.2362A. Convert the following \$NZ amounts to \$A, correct to the nearest cent.

a \$NZ20 **b** \$NZ38 **c** \$NZ100

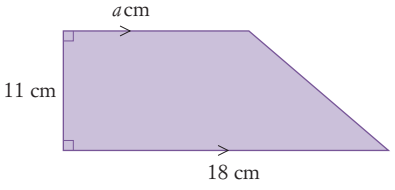
10 The body mass index (BMI) of an adult is $B = \frac{m}{h^2}$, where m is the mass in kilograms and h is the height in metres. If Kate is 1.76 m tall and has a BMI of 24.2, calculate her mass to the nearest kg.

11 The blood alcohol content (BAC) of a person who consumes alcoholic drinks is BAC = $0.0012na$ where n is the number of drinks taken and a is the amount of alcohol in each drink. If Mark drank 4 glasses of wine, each containing 150 mL, and registered a BAC level of 0.0792:

a how many millilitres of alcohol were present in each glass?

b what percentage of the wine was alcohol?

12 The area of a trapezium is given by the formula $A = \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is the distance between them. If the area of the trapezium is 176 cm^2, find the length of the side marked a.



13 The volume (in cubic centimetres) of wood in a tree is $V = 0.4724d^2h + 9.86$, where d is the diameter of the trunk and h is the height of the lowest branch. All dimensions are in centimetres. Calculate, correct to the nearest cm, the height of the lowest branch in a tree whose trunk diameter is 55 cm and whose volume is 400 000 cm^3.

14 The air temperature, T°C, outside an aeroplane is given by the formula $T = 15 - 0.006h$, where h is the height of the plane in metres above sea level. Calculate, correct to the nearest metre, the height of a plane if the outside temperature is –39°C.

15 The density ρ kg/m^3 of a substance can be calculated using the formula $\rho = \frac{m}{V}$, where m is its mass (in kg) and V is its volume (m^3). Calculate correct to two significant figures the volume of a cork block if its density is 240 kg/m^3 and its mass is 20 kg.

16 A formula used to find the total resistance in an electrical circuit is $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}$. Find the value of R_1 if R_{total} is 15 and R_2 is 20.

17 Newton's second law of motion is $a = \frac{F}{m}$, where a is the acceleration in m/s^2, F is the force in newtons, N, and m is the mass in kg. What is the mass of an object if a force of 40 N results in an acceleration of 18 m/s^2?

18 The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$. If a rectangular prism with surface area 132 cm^2 has length 7 cm and width 3 cm, find its height.

JUST FOR THE RECORD

The tallest building in the world

In 2010, the Burj Khalifa Tower in Dubai, in the United Arab Emirates, became the world's tallest building at 830 m high with a record-breaking 163 floors.

Some other tall landmarks are:

CN Tower, Toronto	553 m
Taipei 101 tower, Taipei	508 m
Petronas Towers, Kuala Lumpur	452 m
Empire State Building, New York	381 m
Q1 Tower, Gold Coast	321 m
Eiffel Tower, Paris	320 m
Sydney Tower	305 m
Leaning Tower of Pisa	55 m

Shutterstock.com/Corvm Bogicevic

Use the formula $d = 8\sqrt{\frac{h}{5}}$ to calculate the distance that can be seen to the horizon from the top of the Burj Khalifa Tower.

2-06 Changing the subject of a formula

In the formula $v = u + at$, v is called the **subject of the formula** because it is on the LHS. To change the subject of a formula to another variable, use the same rules as for solving an equation. The answer is not a number but an algebraic expression (another formula).

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EXAMPLE 7

If a moving object has initial speed u m/s and acceleration a m/s^2, its final speed v m/s is given by the formula $v = u + at$. Change the subject of this formula to:

a u **b** t

Solution

a

$v = u + at$

$u + at = v$ — **Swap sides so new subject *u* appears on the LHS.**

$u = v - at$ — **Subtract *at* from both sides.**

b

$v = u + at$

$u + at = v$ — **Swap sides so new subject *t* appears on the LHS.**

$at = v - u$ — **Subtract *u* from both sides.**

$t = \dfrac{v - u}{a}$ — **Divide both sides by *a*.**

EXAMPLE 8

The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$

a Make h the subject of the formula.

b Make r the subject of the formula.

Solution

a

$V = \dfrac{1}{3}\pi r^2 h$

$\dfrac{1}{3}\pi r^2 h = V$ — **Swap sides so new subject *h* appears on the LHS.**

$\pi r^2 h = 3V$ — **Multiply both sides by 3 to remove the fraction.**

$h = \dfrac{3V}{\pi r^2}$ — **Divide both sides by πr^2.**

b

$V = \dfrac{1}{3}\pi r^2 h$

$\dfrac{1}{3}\pi r^2 h = V$ — **Swap sides so *r* appears on the LHS.**

$\pi r^2 h = 3V$ — **Multiply both sides by 3.**

$r^2 = \dfrac{3V}{\pi h}$ — **Divide both sides by πh.**

$r = \pm\sqrt{\dfrac{3V}{\pi h}}$ — **Take the square root of both sides.**

Exercise 2-06 Changing the subject of a formula

1 Make x the subject of each formula.

a $y = 2x + 4$ **b** $T = 3x - 7$ **c** $d = \frac{x+1}{3}$

d $p = 18 - x$ **e** $k = 4x + r$ **f** $C = n + \frac{x}{2}$

g $S = 10bx$ **h** $V = \frac{x-5}{6}$ **i** $z = 12 - ax$

2 If an object travels a distance, d, in time, t, its average speed, S, is given by the formula $S = \frac{d}{t}$. Change the subject of the formula:

a to d **b** to t

3 The angle sum of a shape with n sides is $A°$, where $A = 180(n - 2)$.

a Use the formula to find the angle sum of a shape with 12 sides.

b Make n the subject of the formula.

c If the angle sum of a polygon is 1260°, how many sides does it have?

4 The volume of a pyramid has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Which of the following is the correct formula for A?

Select **A**, **B**, **C** or **D**.

A $A = \frac{1}{3}Vh$ **B** $A = \frac{3}{Vh}$ **C** $A = 3Vh$ **D** $A = \frac{3V}{h}$

5 The area of a trapezium has the formula $A = \frac{1}{2}(x + y)h$. Change the subject of the formula:

a to h **b** to y

Example 8

6 Change the subject of each formula to the variable shown in brackets:

a $A = \pi r^2$ $[r]$ **b** $y = mx + b$ $[m]$

c $v^2 = u^2 + 2as$ $[s]$ **d** $z = \frac{x-m}{s}$ $[x]$

e $E = mc^2$ $[c]$ **f** $K = \frac{1}{2}mv^2$ $[v]$

g $c^2 = a^2 + b^2$ $[b]$ **h** $x = \sqrt{6y + 3}$ $[y]$

i $A = \frac{1}{2}bh$ $[h]$ **j** $T = 2\pi\sqrt{\frac{l}{g}}$ $[l]$

k $V = IR - E$ $[R]$ **l** $e = ir + \frac{Q}{C}$ $[Q]$

m $v = \sqrt{\frac{mr}{\pi}}$ $[r]$ **n** $s = ut + \frac{1}{2}at^2$ $[a]$

o $V = \frac{4}{3}\pi r^3$ $[r]$ **p** $K = \frac{m+d}{w}$ $[d]$

7 The body-mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is the adult's mass in kilograms and h is their height in metres. Change the subject of the formula to h and hence find, correct to two decimal places, the height of a person with a body mass index of 25 and a mass of 60 kg.

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8 The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of its circular base and h is its perpendicular height. Show that this formula can be rewritten as $h = \frac{3V}{\pi r^2}$ and hence find (to the nearest centimetre) the height of a cone with volume 3539.53 cm^3 and base radius 13 cm.

9 Make F the subject of the formula $C = \frac{5}{9}(F - 32)$ and hence find F when:

a $C = 40$ **b** $C = 100$

10 The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$, where l metres is the length of the swing and g m/s^2 is the gravitational acceleration.

Make l the subject of the formula and then find, correct to the nearest 0.1 m, the value of l if $g = 9.8$ and it takes 3 seconds to swing back and forth once.

INVESTIGATION

GETTING THE RIGHT FORMULA

Match each formula to its correct description. (Some descriptions are used twice.)

Formulas

1 $I = Prn$

2 $c^2 = a^2 + b^2$

3 $s = \frac{d}{t}$

4 $\bar{x} = \frac{\Sigma x}{n}$

5 $P(\bar{E}) = 1 - P(E)$

6 $A = \pi r^2$

7 $C = 2\pi r$

8 $y = kx$

9 $y = mx + b$

10 $A = \frac{1}{2}(a + b)h$

11 $V = Ah$

12 $FV = PV(1 + r)^n$

13 $I = FV - PV$

14 $\text{IQR} = Q_3 - Q_1$

15 $A = \frac{1}{2}xy$

16 $A = s^2$

17 $C = \pi d$

18 $S = V_0(1 - r)^n$

19 $A = lb$

20 $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

21 $S = V_0 - Dn$

22 $V = \pi r^2 h$

23 $A = \frac{1}{2}bh$

24 $P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

25 $\frac{\text{absolute error}}{\text{measurement}} \times 100\%$

26 $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

Descriptions

A Area of a trapezium

B Area of a circle

C Probability of a complementary event

D Mean of a data set

E Volume of a prism

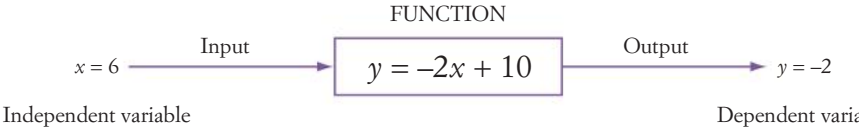
F Volume of a cylinder

G Pythagoras' theorem
H Circumference of a circle
I Probability of an event
J Interquartile range of a data set
K Linear variation
L Area of a square
M Area of a rectangle
N Speed
O Area of a triangle
P Gradient of a line
Q Simple interest
R Compound interest earned
S Declining balance depreciation
T Area of a rhombus
U Straight-line depreciation
V Equation of a line
W Percentage error
X Future value of an investment after compound interest

2-07 Linear functions

Graphing linear equations

A **function** is a relationship between two variables, like a 'number machine' that converts an input value, x, into an output value, y.

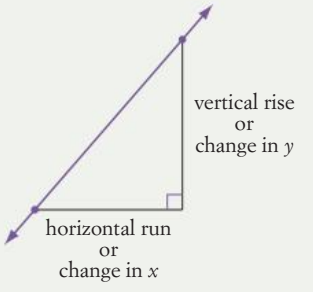


$y = -2x + 10$ is a **linear function** because its graph is a straight line. y is the **dependent variable** because the value of y depends on the value of x, while x is the **independent variable**.

SUMMARY

- A linear function has the form $y = mx + b$ and its graph is a straight line.
- The **gradient**, m, is the rate of change of y relative to x.

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)} = \frac{\text{change in } y}{\text{change in } x}$$



- The **y-intercept** or **vertical intercept**, b, is the value of y when $x = 0$.

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In the linear function $y = -2x + 10$, the number in front of x, **−2**, is called the **coefficient** of x, while the number on its own without an x, **10**, is called the **constant term**. For the graph of $y = -2x + 10$, −2 is the **gradient** and 10 is the ***y*-intercept**.

EXAMPLE 9

Graphing lines: Graphics calculator

a Complete this table of values for the linear function $y = 2x - 1$, then graph $y = 2x - 1$ on a number plane.

x	−2	−1	0	1	2
y					

b Find the gradient and y-intercept of the line.

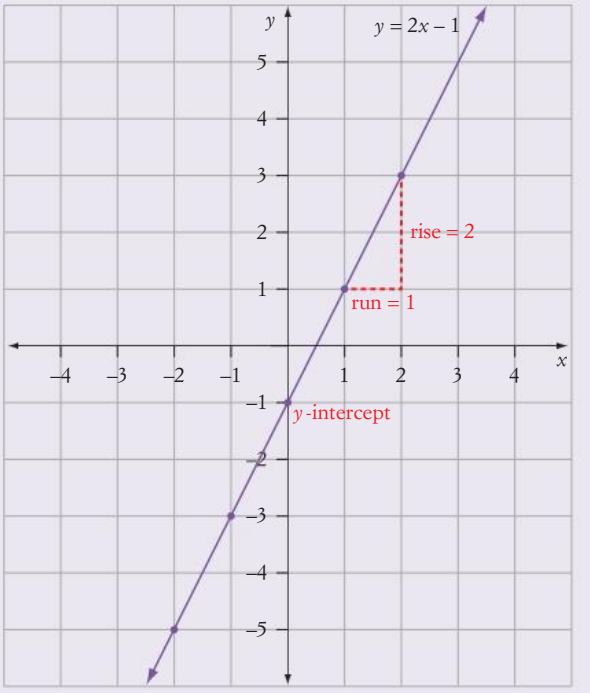
c How are the gradient and y-intercept of the line related to the equation $y = 2x - 1$?

Solution

a Substituting x-values into $y = 2x - 1$ to find y:

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

Graphing the points (–2, – 5), (– 1, – 3), (0, – 1), (1, 1), (2, 3):



b From the graph, gradient $(m) = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$, and the y-intercept is –1.

Note also that in the table of values, as x increases by 1, y increases by 2 (the gradient) and when $x = 0$, $y = -1$ (the y-intercept).

c The gradient 2 is the coefficient of x in the equation $y = 2x - 1$ and the y-intercept –1 is the constant term.

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Finding the equation of a line

EXAMPLE 10

What is the linear function for this table of values?

x	–4	–1	5	10
y	–23	–8	22	47

Solution

The equation has the form $y = mx + b$. Select any two points to calculate the gradient, say (–1, –8) and (–4, –23):

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{-8-(-23)}{-1-(-4)} \quad \text{using the points}$$

$$= \frac{15}{3}$$

$$= 5$$

$\therefore$ The equation has the form $y = 5x + b$.

'$\therefore$' is the mathematical symbol for 'therefore'

To find b, substitute any point, say (10, 47):

$47 = 5(10) + b$

$47 = 50 + b$

$-3 = b$

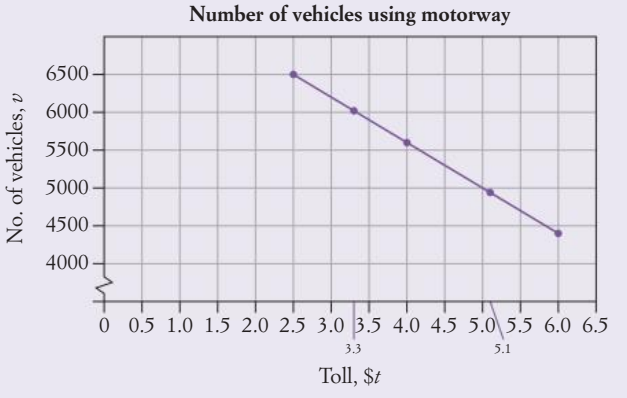
$b = -3$

$\therefore$ The equation is $y = 5x - 3$

EXAMPLE 11

A traffic researcher found a linear relationship between the toll charge and the number of vehicles using a motorway each day.

Toll, \$$t$	2.5	3.3	4	5.1	6
No. of vehicles, v	6500	6020	5600	4940	4400



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a What is the independent variable?

b Find the linear function of the form $v = mt + b$.

c What is the vertical intercept and what does it represent?

d Use the function to predict the daily number of vehicles using the motorway when the toll is \$4.80.

e What will happen to the number of vehicles using the motorway if the toll *increases* by \$1.50?

Solution

a The independent variable is t, the toll. **v depends on t.**

b First, find m:

$$m = \frac{\text{change in } v}{\text{change in } t}$$

$$= \frac{4400 - 5600}{6 - 4}$$ **Using points (4, 5600) and (6, 4400).**

$$= \frac{-1200}{2}$$

$$= -600$$

$\therefore v = -600t + b$

Second, find b:

$6500 = -600(2.5) + b$ **Using point (2.5, 6500).**

$= -1500 + b$

$b = 8000$

$\therefore v = -600t + 8000$

c The vertical intercept is 8000. This represents the number of vehicles using the motorway if $t = 0$, that is, if there is no toll.

d Substitute $t = 4.8$ into $v = -600t + 8000$:

$v = -600(4.8) + 8000$

$= 5120$

5120 vehicles will use the tollway when the toll is \$4.80.

e The gradient $m = -600$ means that for every \$1 increase in the toll, the number of vehicles using the motorway will decrease by 600. So if the toll increases by \$1.50:

Number of vehicles will decrease by $1.5 \times 600 = 900$.

Alternative method

Suppose that the toll increases by \$1.50 from \$4 to \$5.50.

When $t = 4$, $v = 5600$ (from table).

When $t = 5.5$,

$v = -600(5.5) + 8000$

$= 4700$

Number of vehicles will decrease by $5600 - 4700 = 900$.

Exercise 2-07 Linear functions

1 Which of these is a linear function? Select **A**, **B**, **C** or **D**.

A $y = -\frac{1}{x}$ **B** $y = 12 - x$ **C** $y = 2x^2 + 9$ **D** $y = 2^x$

Example 9

2 **a** For the linear function $y = 3x + 2$, what is:

i the independent variable? **ii** the dependent variable?

b Graph $y = 3x + 2$ on a number plane after completing this table of values.

x	–2	–1	0	1	2
y					

c Find the gradient and y-intercept of the line.

d How are the gradient and y-intercept of the line related to the equation $y = 3x + 2$?

3 **a** Graph $y = 4 - 3x$ on a number plane after completing this table of values.

x	–2	–1	0	1	2
y					

b Find the gradient and y-intercept of the line.

c Explain how the gradient and y-intercept are related to the equations of the line.

4 For each linear function, write down its gradient and y-intercept.

a $y = -3x + 7$ **b** $y = 5 - x$ **c** $y = \frac{1}{2}x - 3$

5 Graph each linear function shown in question **4** above on a separate number plane.

Example 10

6 Which is the linear function for this table of values? Select **A**, **B**, **C** or **D**.

x	4	7	12	20
y	10	31	66	122

A $y = \frac{1}{7}x - 18$ **B** $y = \frac{1}{7}x + 10$ **C** $y = 7x - 1$ **D** $y = 7x - 18$

7 Find the linear function for each table of values.

a

k	6	10	13	15
c	152	120	96	80

b

t	1	3	6	14
v	9	21	39	87

c

n	3	8	20	35
p	141	126	90	45

d

x	2	5	11	18
t	–2	13	43	78

Example 11

8 The table below shows the quarterly cost of water usage for different volumes of water.

Volume, v (kL)	16	25	48	60	81	99
Cost, C (\$)	168.24	183	220.72	240.40	274.84	304.36

a What is the independent variable and what does it represent?

b Find a linear function for C in terms of v.

c What does the gradient represent?

d What does the vertical intercept represent?

e What is the cost of using 54 kL of water?

f Find the volume of water usage that costs \$265.

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9 This conversion graph shows the linear relationship between the Celsius and Fahrenheit scales for measuring temperature.

a Find a formula for F in terms of C.

b What is the independent variable?

c What is the vertical intercept and what does it represent?

d Use the formula to convert:

i 12°C to °F

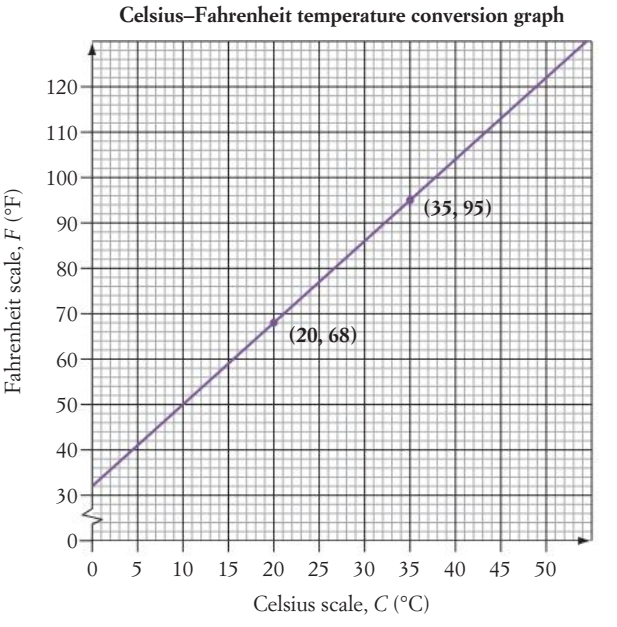
ii 86°F to °C

e Use the graph to convert:

i 12°C to °F

ii 86°F to °C

f If C increases by 5°, what does F increase by?



10 Lizzie recorded the value of her computer every 6 months. She graphed this data and drew a 'line of fit' as shown below to estimate the values.

a Find the equation of Lizzie's line of fit.

b Use the equation to calculate the original value of her computer.

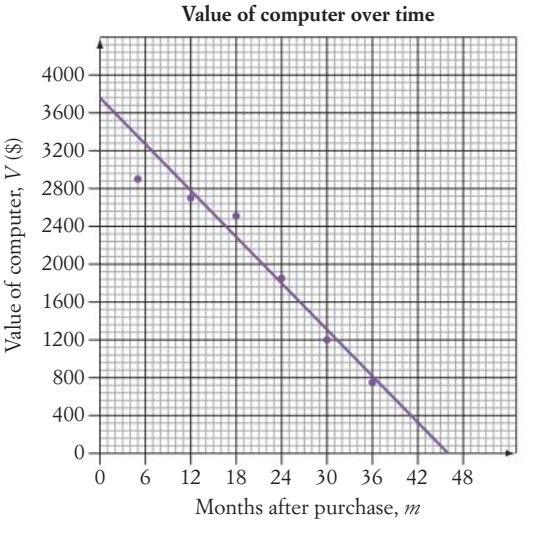
c What is the gradient of the line and what does it represent?

d Calculate the value of her computer after:

i 20 months **ii** $3\frac{1}{2}$ years

e When will the value of the computer be zero? Answer in years and months.

f What are the limitations of this model?



11 A shoe's length (in inches) has a linear relationship with its shoe size.

Shoe size, s	2	5	7	8	12
Length, L inches	9	10	$10\frac{2}{3}$	11	$12\frac{1}{3}$

a What is the independent variable?

b Find the linear function for L in terms of s.

c What is the length of a shoe of size $7\frac{1}{2}$?

d What size is a shoe of length 13 inches?

12 The weekly cost \$$C$ of manufacturing notebook computers at a factory is shown in the table below, where n is the number of computers produced.

No. of computers, n	50	85	120	136	157
Weekly cost, \$$C$	21 900	31 350	40 800	45 120	50 790

a Find the linear function for C.

b What is the cost of producing 200 notebook computers?

c How many computers can be produced for \$60 000?

d The factory made 44 *more* computers this week than last week. By how much did the cost increase from last week?

2-08 Direct linear variation

SUMMARY

If y varies as x, or y is directly proportional to x, then $y = kx$, where k is a constant. k is called the **constant of variation** or **constant of proportionality**.

A direct linear relationship exists between x and y. If x increases (or decreases), y increases (or decreases). If x is doubled (or halved), y is doubled (or halved).

EXAMPLE 12

On a road trip, the total distance travelled, D km, varies directly as the time, t hours.

Time, t (hours)	0	2	3	5	8
Distance travelled, D (km)	0	160	240	400	640

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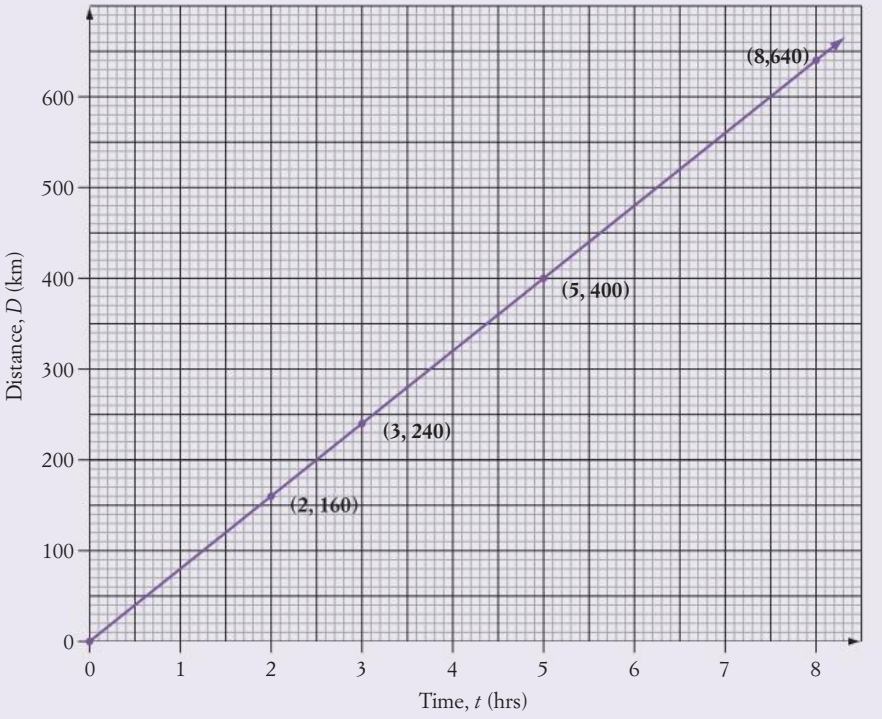
a Graph the relationship between D and t.

b Find the equation for D in terms of t.

c Find the time it takes to travel 550 km, in hours and minutes correct to the nearest minute.

Solution

a



b As D varies directly as t, the variation equation is of the form $D = kt$.

To find k, substitute a point from the table, say (3, 240):

$$D = kt$$
$$240 = k\,(3)$$
$$k = \frac{240}{3}$$
$$= 80$$
$$\therefore D = 80t$$

c When $D = 550$:

$$550 = 80t$$
$$t = \frac{550}{80}$$
$$= 6.875 \text{ h}$$
$$= 6 \text{ h } 52 \text{ min } 30 \text{ s}$$
$$\approx 6 \text{ h } 53 \text{ min}$$

The time it takes to travel 550 km is 6 hours 53 minutes.

EXAMPLE 13

The mass M (in grams) of petrol varies as its volume V (in litres). The mass of 60 L of petrol is 42 kg.

a Find the variation equation relating M to V.

b What is the mass of 35 L of petrol?

c What is the volume (correct to the nearest litre) of petrol that has a mass of 80 kg?

Solution

a As M varies directly as V, the variation equation is $M = kV$.

Substitute $M = 42$ and $V = 60$ to find k.

$$42 = k(60)$$
$$k = \frac{42}{60}$$
$$= 0.7$$
$$\therefore M = 0.7V$$

b When $V = 35$:

$$M = 0.7(35)$$
$$= 24.5$$

35 L of petrol has a mass of 24.5 kg.

c When $M = 80$:

$$80 = 0.7V$$
$$V = \frac{80}{0.7}$$
$$= 114.2857\ldots$$
$$\approx 114$$

80 kg of petrol has a volume of 114 L.

SUMMARY

To solve a linear variation problem:

Step 1: Identify the two variables (say x and y) and form a variation equation $y = kx$.

Step 2: Substitute values for x and y to find k, the constant of variation.

Step 3: Rewrite $y = kx$ using the value of k.

Step 4: Substitute a value for x or y into $y = kx$ to solve the problem.

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Exercise 2-08 Direct linear variation

Example 12

1 The table below shows the weights of astronauts on Earth and on the Moon.

Weight on Earth, E (kg)	55	60	65	72	75	85
Weight on Moon, M (kg)	9.1	9.9	10.7	11.9	12.4	14.1

a Graph the relationship between M and E.

b What is the vertical intercept?

c Find the gradient of the line.

d Find the equation for M in terms of E.

e If Sally weighs 8.6 kg on the Moon, what is her weight on Earth?

2 This table shows the fuel used when travelling certain distances by a small car.

Fuel used, f (L)	3	12	15	18	30	40
Distance travelled, D (km)	50	200	250	300	500	660

a Graph the relationship between D and f.

b Find the equation for D in terms of f.

c Find the amount of fuel used when travelling a distance of 680 km.

3 The mass, M (g) of a type of steel is given for different volumes, V (cm^3) in the table below.

Volume, V (cm^3)	3	8	11	25	35	60
Mass, M (g)	23.4	62.4	85.8	195	273	468

a Find the equation for M in terms of V.

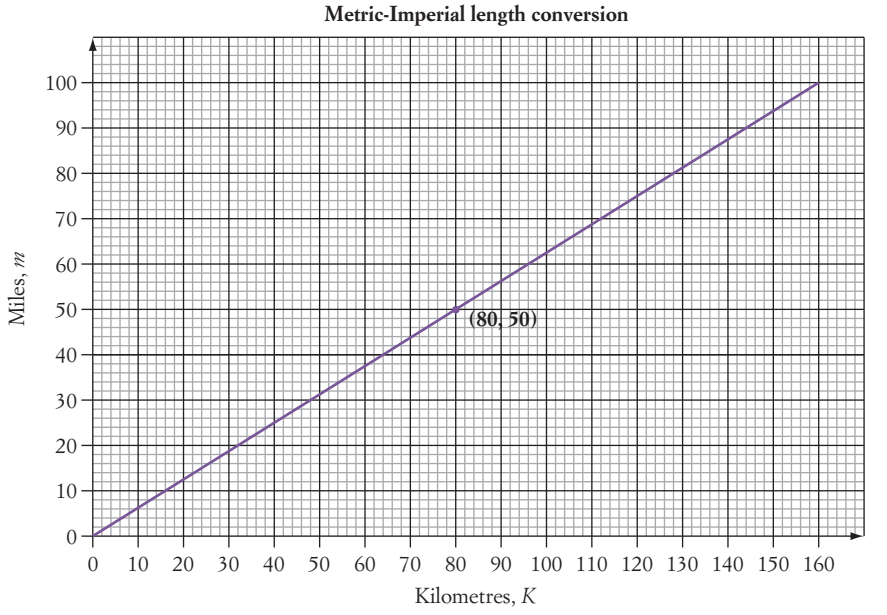
b Find the mass of a piece of steel if it has a volume of 180 cm^3.

c Convert 1 m^3 to cubic centimetres (cm^3).

d If a piece of steel has a volume of 1 m^3, calculate its mass in kilograms.

Example 13

4 This graph converts kilometres (a metric unit for length) to miles (an imperial unit of length).



a What is the equation for m in terms of K?

b Use the equation to convert 115 km to miles.

c Convert 160 miles to kilometres.

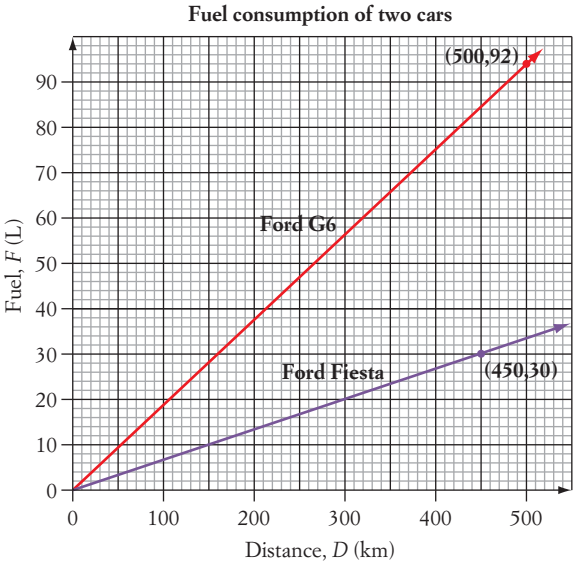
5 The graph below shows the fuel consumption of two models of Ford cars: Fiesta and G6.

a Find the equation for F in terms of D for each car.

b Calculate how much more fuel is used by a Ford G6 than a Ford Fiesta over a distance of:

i 100 km **ii** 280 km

c How much further can a Ford Fiesta be driven than a Ford G6 on 50 L of fuel?



6 The download time, T seconds, of a computer file is directly proportional to the size of the file, S kB. On Jai's computer, a file of 1800 kB requires 36 seconds to download.

a Find the variation equation relating T to S.

b How long it will take Jai's computer to download a 3000 kB file?

c If a file took 80 seconds to download, what was its size?

7 During a thunderstorm, the time between seeing lightning and hearing thunder varies as the distance of the observer from the centre of the storm. If thunder is heard 8 seconds after the lightning for a storm that is 42 km away, calculate the distance of a storm when thunder is heard 12 seconds after the lightning. Select **A**, **B**, **C** or **D**.

A 2 km **B** 60 km **C** 63 km **D** 180 km

8 The increase in pressure experienced by a scuba diver is directly proportional to her depth under the water. The increase in pressure at depth 40 m is 235 kPa (kilopascals).

a Find the variation equation for pressure in terms of depth.

b What increase in pressure is experienced at a depth of 60 m?

c At what depth, to the nearest metre, is the increase in pressure 830 kPa?

9 The cost of running a car varies directly with the distance travelled. The cost is $355.20 for a distance of 500 km. If the car travels 15 000 km in one year, what is the cost of running the car for a year?

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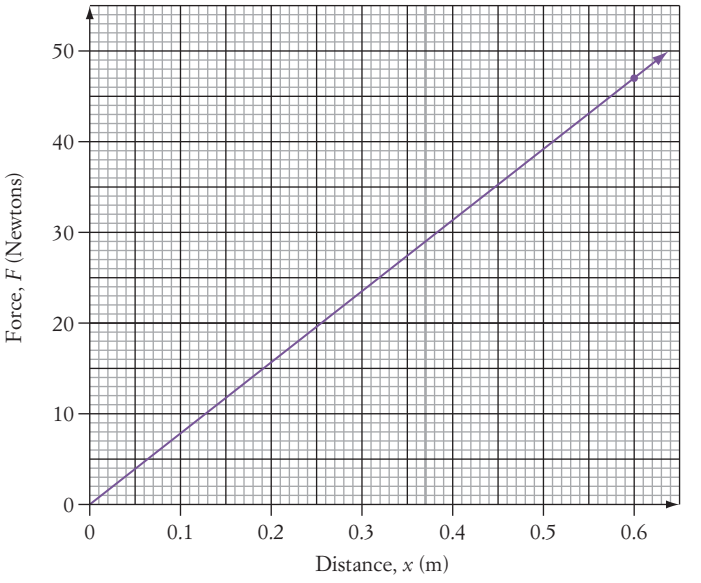
10 This graph shows the force, F (in newtons, N), exerted by a spring when it is stretched to a distance x (in metres), by a mass of 2 kg.



a Find the equation for F in terms of x.

b Calculate how much force is required to stretch the spring:

i 0.35 m

ii 0.52 m

c A force of 30 N is applied to the spring. How far is the spring stretched? Answer to the nearest 0.01 m.

11 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, which of the following is the amount of time it would take for the temperature to drop 10°C? Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

12 At 3:30 p.m. one day, the shadows of different objects were measured, and they were found to be directly proportional to their heights. A tree of height 250 cm had a shadow of 325 cm.

a Calculate the length of the shadow of a lamppost of height 320 cm.

b What is the height of a letterbox, correct to the nearest centimetre, if its shadow is 1.11 m long?

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2-09 Intersection of lines

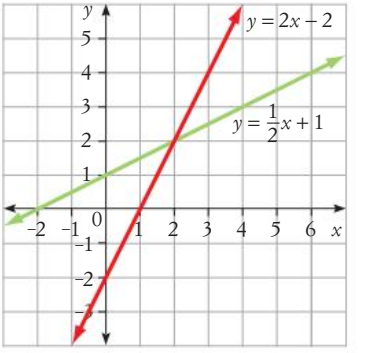
Intersection of lines

A pair of equations, such as $y = \frac{1}{2}x + 1$ and $y = 2x - 2$, can be solved together to find the values of x and y that satisfy *both* equations. Because they are solved at the same time, they are called **simultaneous equations**.

Graphing lines: Graphic calculator

If simultaneous equations are graphed on the number plane, their solution is the coordinates of the point where their graphs intersect. For example, the graphs of $y = \frac{1}{2}x + 1$ and $y = 2x - 2$ cross at the point (2, 2), so the solution to the simultaneous equations is $x = 2$, $y = 2$.

Simultaneous equations can be used to solve problems involving linear modelling.



EXAMPLE 14

Ally is organising an outdoor morning tea to raise money for charity. The cost of hiring a marquee is \$116 and the catering is \$6.20 per person. To cover these costs, Ally is charging each person \$12. The cost and revenue functions are represented by the formulas:

Cost: $C = 6.2n + 116$

Revenue: $R = 12n$

where C and R are in dollars and n is the number of people.

a Graph both functions on the same set of axes for $n = 0$ to 50.

b Use the graph to find the cost (to the nearest \$10) of the morning tea for:

i 12 people

ii 30 people

c Why would it be unwise to run a morning tea for 12 people?

d For what value of n does the cost equal the revenue?

e Why is the point of intersection of the two lines called the '**break-even**' point? What happens for values of n above this point?

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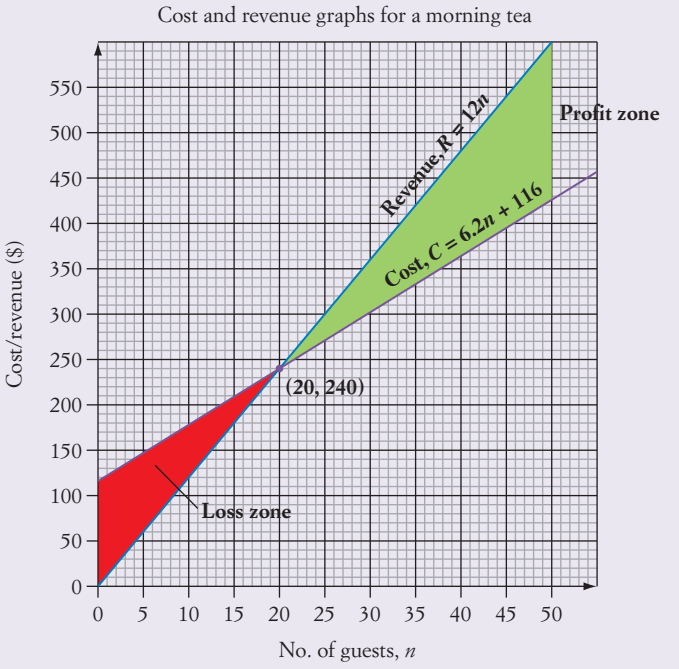
Solution

a



b From the graph:
i \$190 **ii** \$300

c The revenue would be less than the cost, so Ally would lose money.

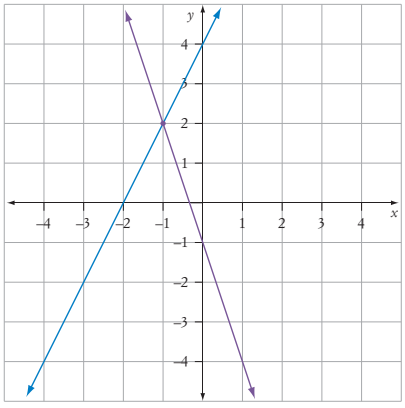
d $n = 20$ $(C = R = \$240)$

e When $n = 20$, the revenue covers the cost exactly, so Ally breaks even.
For values of n above 20, the revenue is more than the cost so Ally would make a profit.

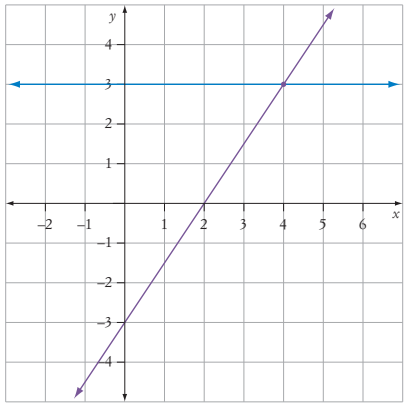
Exercise 2-09 Intersection of lines

1 For each graph, find the equations of both lines and the coordinates of their point of intersection.

a



b



Example 14

2 Solve this pair of simultaneous equations graphically: $y = 2x - 1$ and $y = -x + 5$.

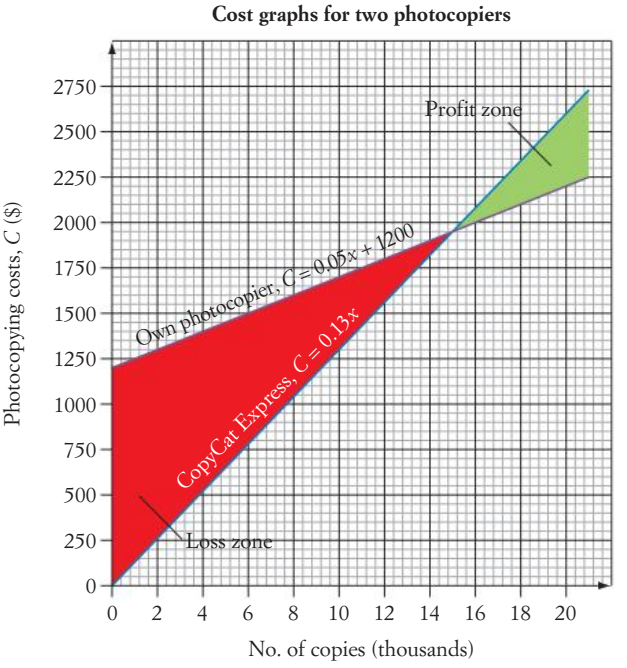
3 At CopyCat Express, the charge for photocopying is 13 cents per sheet. Klaas is investigating the purchase of his own photocopier for $1200, which brings the cost down to 5 cents per sheet. The two cost functions are:

CopyCat Express: $C = 0.13x$

Own photocopier: $C = 0.05x + 1200$

where C is in dollars and x represents the number of copies made.

The graph of the functions is shown below.



a Use the graph to determine which option is cheaper for making 5000 copies.

b Use the formulas to find the cost of making 5000 copies for each option.

c Why is the 'Own photocopier' option more expensive for smaller values of x?

d What is the value of x at the break-even point?

e What happens when x exceeds this break-even value?

4 The monthly cost of purchasing n new DVDs for a video store is $C = 16n + 576$ while the monthly income earned from renting out those new DVDs is $I = 48n$, where C and I are in dollars.

a Graph both functions for values of n from 0 to 40 and find the break-even point.

b What would happen if the store bought fewer DVDs than the break-even value of n?

c Calculate the monthly cost of purchasing 35 DVDs and the income earned from these.

d According to the formula, what monthly income is earned from each new DVD?

e What is the vertical intercept of the Cost graph and what does it represent?

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5 Dave claims that a quick 'rule-of-thumb' for converting Celsius temperatures to Fahrenheit is 'double it and add 30'.

a Graph Dave's rule and the actual conversion formula for values of C from 0 to 100.

Dave's rule: $F = 2C + 30$

Actual conversion formula: $F = \frac{9}{5}C + 32$

where C represents the temperature in °C and F represents the temperature in °F.

b For what value of C is Dave's rule exactly equal to the actual conversion formula?

c For what values of C does Dave's rule give answers that are too high?

d For what values of C is the difference between the two formulas not more than 5°F?

e Is Dave's 'rule-of-thumb' a good one? Justify your answer.

6 Jenni is comparing two Internet cafes. QuickByte has a monthly membership fee of $8 and charges 60 cents per hour of Internet use. Game Hunters charges $1 per hour of Internet use but has no monthly access fee. The two costs are represented by the formulas:

QuickByte: $C = 0.6t + 8$

Game Hunters: $C = t$

where C is in dollars and t is the number of hours of Internet use.

a Graph both formulas for values of t from 0 to 50 and find the break-even point.

b If Jenni uses an Internet cafe for 24 hours per month, which cafe is better for her? Why?

c Calculate the cost of 24 hours Internet use for each cafe.

d Compare the advantages of each cafe, taking the break-even point into account.

7 The trip charges for two taxi companies are shown below.

Burntrubber Taxis: $C = 0.9k + 4.6$ ($4.60 plus 90 c/km)

Whiteknuckle Cabs: $C = 1.2k + 2.5$ ($2.50 plus $1.20/km)

where C is in dollars and k represents kilometres travelled.

a Graph both formulas for values of k from 0 to 11 and find their point of intersection.

b What does the point of intersection represent?

c Write two advantages of using each taxi company.

d Jake travels to and from work each day by taxi, a distance of 11 km each way. Determine which is the better taxi for him to use, and calculate the saving he would make taking it over 5 days.

Graphing lines: graphics calculator

TECHNOLOGY: GRAPHING LINEAR FUNCTIONS

Linear functions can be graphed using a graphics calculator or graphing software.
Learn to:

- graph a line by entering its equation (**graph** function)
- graph a line by entering the coordinates of two points on the line (**plot** function)
- use the **trace** and **zoom** functions to find points on a line, including the x- and y-intercepts
- solve a pair of simultaneous linear equations graphically using the **trace** and **zoom** functions to locate the point of intersection.

Simultaneous equations

2-10 Simultaneous equations

Solving simultaneous equations using graphs can be time-consuming and inaccurate. A faster and more formal approach is to solve them algebraically, using either the **substitution method** or the **elimination method**.

The substitution method

With the **substitution method**, make x or y the subject of one equation and then **substitute** it into the other equation.

EXAMPLE 15

Solve each pair of simultaneous equations.

a $y = x + 5$ and $y = 5x - 3$ **b** $5x + 3y = 18$ and $y = 5 - 2x$

Solution

a Label each equation:

$y = x + 5$ (1)

$y = 5x - 3$ (2)

Use equation (1) to substitute for y in equation (2).

$$x + 5 = 5x - 3$$
$$x + 5 - 5x = 5x - 3 - 5x$$
$$-4x + 5 = -3$$
$$-4x + 5 - 5 = -3 - 5$$
$$-4x = -8$$
$$\frac{-4x}{-4} = \frac{-8}{-4}$$
$$x = 2$$

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Now substitute $x = 2$ in equation (1) [or (2)] to find y:

$y = x + 5$ (1)

$y = 2 + 5$

$\therefore y = 7$

$\therefore$ The solution is $x = 2$ and $y = 7$.

Check by substitution that $x = 2$ and $y = 7$ satisfies both equations (1) and (2)

b $5x + 3y = 18$ (1)

$y = 5 - x$ (2)

Since y is the subject in (2), substitute it into equation (1) to give an equation using x only.

$5x + 3(5 - x) = 18$

$5x + 15 - 3x = 18$

$2x + 15 = 18$

$2x = 3$

$x = \frac{3}{2}$

$= 1\frac{1}{2}$

Now substitute $x = 1\frac{1}{2}$ into equation (2) to find y:

$y = 5 - 1\frac{1}{2} = 3\frac{1}{2}$

$\therefore$ The solution is $x = 1\frac{1}{2}$ and $y = 3\frac{1}{2}$.

Check by substitution that $x = 1\frac{1}{2}$ and $y = 3\frac{1}{2}$ satisfies both equations (1) and (2).

The elimination method

With the **elimination method**, add or subtract the equations to **eliminate** (delete) one of the variables.

EXAMPLE 16

Solve each pair of simultaneous equations.

a $2x + 5y = 12$ and $3x - 5y = 8$ **b** $3x + 2y = 10$ and $3x - 4y = 2$

Solution

a $2x + 5y = 12$ (1)

$3x - 5y = 8$ (2)

Since there are the same number of ys in both equations and they are opposite in sign ($5y$ and $-5y$), add equations (1) and (2) to eliminate y.

$(2x + 3x) + (5y - 5y) = 12 + 8$ [(1) + (2)]

$5x = 20$

$\frac{5x}{5} = \frac{20}{5}$

$x = 4$

Substitute $x = 4$ into equation (1) [or (2)] to find the value of y:

$2(4) + 5y = 12$

$8 + 5y = 12$

$5y = 4$

$y = \frac{4}{5}$

The solution is $x = 4$ and $y = \frac{4}{5}$.

Check by substitution that $x = 4$ and $y = \frac{4}{5}$ satisfies both equations (1) and (2).

b $3x + 2y = 10$ (1)

$3x - 4y = 4$ (2)

Since there are the same number of xs in both equations and they have the *same* sign ($3x$ and $3x$), subtract equation (2) from (1) to eliminate x.

$(3x - 3x) + (2y - [-4y]) = 10 - 4$ [(1) – (2)]

$6y = 6$

$y = \frac{6}{6}$

$= 1$

Substitute $y = 1$ into equation (1) [or (2)] to find the value of y:

$3x + 2(1) = 10$

$3x + 2 = 10$

$3x = 8$

$x = \frac{8}{3}$

$= 2\frac{2}{3}$

The solution is $x = 2\frac{2}{3}$ and $y = 1$.

Using substitution, check that the solution $x = 2\frac{2}{3}$ and $y = 1$ satisfies both equations (1) and (2).

Example 15

Exercise 2-10 Simultaneous equations

1 Solve each pair of simultaneous equations using the substitution method.

a $y = 2x$
$y = 5x - 6$

b $x = 3y - 2$
$x = y + 2$

c $3m + 4k = 8$
$m = 2k$

d $d = 4a - 1$
$2a + 3d = 11$

e $4w - 2p = 5$
$w = 2p + 3$

f $n = 3x + 5$
$n = 5x - 3$

Example 16

2 Solve each pair of simultaneous equations using the elimination method.

a $2m + 3n = 10$
$5m - 3n = 4$

b $5y + 4k = 12$
$5y + 2k = 2$

c $4x - y = 15$
$x + y = 1$

d $w - 2g = 10$
$3w + 2g = 2$

e $2m + 3p = 14$
$2m - p = 10$

f $6g + 3h = 16$
$4g + 3h = 10$

g $7x - 4y = 20$
$5x + 4y = 4$

h $4a + c = 8$
$a - c = 5$

i $3k + 5e = 16$
$2k - 5e = 5$

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3 Sarah and Alla both sell kitchenware and earn commissions. Sarah's commission is $450 plus 4% of her sales while Alla's commission is 24% of her sales. These rates can be expressed by the formulas:

Sarah's commission $C = 0.04n + 450$

Alla's commission $C = 0.24n$

where n represents the value of sales in dollars.

a Use the substitution method to solve the pair of simultaneous equations.

b For what range of sales does Alla earn more than Sarah?

4 A catering company charges $22 per person for a buffet lunch. The cost to the company for the lunch is $180 plus $4 per person. These rates can be expressed by the formulas:

Revenue $D = 22N$

Cost $D = 4N + 180$

where $D is the revenue and cost, and N is the number of people attending the lunch.

a Solve the pair of simultaneous equations to find the number of people required to break even.

b If 60 people attend this lunch, how much profit will the company make?

Sample HSC problem

Objects of different masses, k kg, are tied to a spring and the length of the spring, L cm, is measured. The results are shown in the table.

Mass, k kg	8	15	21	29	30	34
Length of spring, L cm	74.4	108	136.8	175.2	180	199.2

a If $L = mk + 36$, find the value of m.

b What is the vertical intercept of the graph of the above formula and what does it represent?

c How much does the spring's length increase for each new kilogram of mass added?

d Calculate the mass of an object that will make the spring 156 cm long.

Study tip

Learn from your mistakes

'Any man can make mistakes, but only an idiot persists in his error.'

Cicero (106–43 BCE), Roman statesman

Life is a series of successes and failures. While the successes can be enjoyable and uplifting, there are many lessons to be learnt from the failures. If you perform badly in an exam, learn from the experience. You can't change the past but you can use its mistakes to improve your future.

Don't throw away your old exams. Use them to find out where and how you went wrong. Identify your areas of weakness and make an effort to study them in more detail. Work on your exam technique and try redoing exams when you study.

Your teachers are a valuable resource. They know the HSC course best as well as your strengths and weaknesses. Also use your friends and classmates. Study in groups if you find it easier. Ask an older student for help.

Use the library, Maths faculty or a bookstore to find study guides, summaries and past exams. Visit websites such as the Board of Studies, HSC Online and the Mathematical Association of NSW.

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CHAPTER REVIEW

Topic overview

This chapter, Equations and linear functions, revised and extended algebra concepts introduced in the Preliminary Course. New theory included algebraic fractions, index laws, changing the subject of a formula and simultaneous equations, so be sure to include them in your topic summary. By now you should be competent in simplifying algebraic expressions, solving a variety of equations, graphing linear functions and locating points of intersection, and solving problems involving linear modelling and direct linear variation.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been included below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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Revision

Exercise 2-01

1 Simplify each expression.

a $3f^2 \times \dfrac{2fh}{15h}$ **b** $(4b^3)^2$ **c** $2xy + 4y - 2y - 2xy$

d $2m^2n \div 4m$ **e** $2f + 4f^2 + 3f - f^2$ **f** $6gh \times 4gh$

Exercise 2-01

2 Expand and simplify each expression.

a $3(d + 4) + 8(d - 1)$ **b** $z(4 + y) - y(4 - z)$

c $\pi r(r + s) + \pi r^2$ **d** $4(3m + y) - 9(10m + 7y)$

Exercise 2-02

3 Simplify each expression.

a $\dfrac{y}{2} + \dfrac{y}{5}$ **b** $\dfrac{4m}{7} - \dfrac{m}{3}$ **c** $\dfrac{2k}{5} + \dfrac{7k}{8}$

d $\dfrac{4y}{3} \times \dfrac{d}{5}$ **e** $\dfrac{5k}{2} \div \dfrac{k}{10}$ **f** $\dfrac{14mn}{5} \times \dfrac{3}{7m}$

Exercise 2-03

4 Simplify each expression.

a $(4y^2m)^3$ **b** $15m^6 \div (-3m)^4$ **c** $a^7 \times a^3 \div a^5$

d $(c^2d^5)^4 \div d^7$ **e** $7m^0 - 5$ **f** $3d^2 \times (-2d^5)$

g $\dfrac{20m^3n^5}{12mn^3}$ **h** $6y^3d^4 \times \dfrac{1}{9}y^4$ **i** $30k^6x^7 \div 6kx^4$

Exercise 2-04

5 Solve each equation.

a $10k - 17 = 4k + 16$ **b** $7(2x + 12) = 5x$ **c** $\dfrac{4p + 6}{3} = -10$

d $\dfrac{5}{m} = 8$ **e** $10 + \dfrac{3}{y} = 6$ **f** $\dfrac{d}{3} + 5 = 3d - 4$

g $\dfrac{a}{4} + \dfrac{a}{3} = 8$ **h** $\dfrac{3k - 2}{5} = 4$ **i** $\dfrac{h}{3} - \dfrac{h}{5} = -2$

Exercise 2-05

6 Eve's height, h cm, over her first 20 years can be modelled by the formula $h = 5.5a + 68$, where a is her age in years. At what age (to the nearest year) did Eve's height reach 155 cm?

Exercise 2-06

7 Change the subject of each formula to the variable indicated in brackets.

a $P = 110 + \dfrac{y}{2}$ (y) **b** $V = \dfrac{1}{2}hd^2 + 9.9$ (d) **c** $V = IR - E$ (I)

Exercise 2-07

8 Find the linear function for this table of values.

x	12	20	36	44	60
y	15	17	21	23	27

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9 The number of bacteria in a sample of milk is checked at the start of each day.

Exercise 2-07

Day, d	0	1	2	3	4	5
Bacteria per mL, B	3000	15 727	28 454	41 181	53 908	66 635

a Find the linear function for B in terms of d.

b If this linear function was graphed, what would be the gradient of the line and what does it represent?

c Calculate the amount of bacteria per millilitre on the seventh day.

d Milk is unfit for use when the number of bacteria exceeds 100 000/mL. At the start of which day will this occur?

10 The fuel consumption, F L/100 km, of a certain make of car, varies as the mass, M kg, of the car. One such car with a mass of 1200 kg has a fuel consumption of 13.2 L/100 km.

Exercise 2-08

a What is the fuel consumption of a car that weighs 870 kg?

b What is the mass, to the nearest kilogram, of a car that has a fuel consumption of 11.9 L/100 km?

11 The cost of manufacturing netbook computers is $C = 270p + 8400$, where p is the number of computers made and C is the cost in dollars. The revenue from selling these computers is $R = 410p$, where R is the revenue (sales) in dollars.

Exercise 2-09

a Graph both functions on the same number plane for values of p from 0 to 100.

b What are the values of p, C and R at the break-even point?

c Calculate the cost and revenue from producing 50 netbook computers.

d What would happen if the company made 50 netbook computers?

e What is the vertical intercept of the cost graph and what does it represent?

f How much does one netbook computer sell for?

12 Solve each pair of simultaneous equations.

Exercise 2-10

a $y = 3x - 4$
$y = x + 5$

b $2w + 5p = 15$
$2w - p = 3$

c $3k + 4m = 9$
$2k - 4m = 6$

MEASUREMENT

3.

AREA AND VOLUME

Australia has a land area of about 7 600 000 km^2 while New Zealand's land area is only about 270 000 km^2, making Australia over 28 times the size of New Zealand. However, when you compare their coastlines (perimeters), Australia's is not even twice as long as New Zealand's. How are areas and perimeters of countries calculated? After all, they are not regular shapes so we can't use the usual formulas.

CHAPTER OUTLINE

IN THIS CHAPTER YOU WILL:

- calculate and compare the percentage error of a measurement
- find the areas of composite shapes
- find the length of an arc and the area of a sector and annulus
- use Simpson's rule to approximate the area of an irregular shape
- find volumes and surface areas of prisms, cylinders and spheres
- find volumes of pyramids and cones
- find volumes and surface areas of composite solids
- calculate error in area and volume calculations

TERMINOLOGY

annulus
arc length
closed
composite
concentric circles
cone
cylinder
hemisphere
open
percentage error
prism
pyramid
quadrant
sector
Simpson's rule
sphere
surface area
two applications

Assignment 3

SkillCheck

1 How many significant figures does each measurement have?

a 6.73 cm **b** 12 mL

c 68.0 g **d** 12 000 km (to nearest 100 km)

2 Write each measurement correct to two significant figures.

a 24.67 m **b** 3.69 kg

c 248 670 000 km **d** 0.05249 cm

3 Copy and complete each statement.

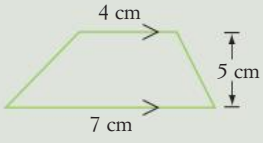
a 1 m^2 = _____ cm^2 **b** 1 m^3 = _____ cm^3 **c** 1 ha = _____ m^2

d 1 cm^2 = _____ mm^2 **e** 1 cm^3 = _____ mL **f** 1 m^3 = _____ L

4 Find the area of each shape.

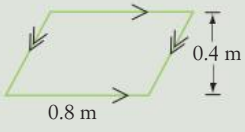
a

b

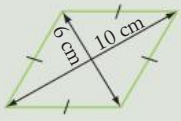


c

d



e



5 **a** If $A = \pi(R^2 - r^2)$, find the value of A, to the nearest whole number, if $R = 6$, $r = 4$.

b If $l = \frac{\theta}{360} \times 2\pi r$, find l, correct to two decimal places, for $r = 14$, $\theta = 135$.

c If $A = \frac{\theta}{360} \times \pi r^2$, find A, correct to two significant figures, when $\theta = 112$, $r = 7.4$.

6 A mosaic floor contains 250 000 tiles, each 1 cm by 1 cm. What area in square metres is covered by tiles?

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3-01 Percentage error

Any measurement has an accuracy equal to $\pm\frac{1}{2}$ of the smallest unit marked on the scale of the measuring device. This accuracy is sometimes called the **absolute error**. For example:

- a measured length of 26 m on a trundle wheel with a 1 m scale has an absolute error of ±0.5 m and the true length is in the range 26 m ± 0.5 m
- a measured volume of 35.8 L on a petrol bowser measuring to the nearest 0.1 L has an accuracy of ±0.05 L and the actual volume is in the range 35.8 L ± 0.05 L

The **percentage error** of a measurement is the absolute error as a percentage of the measurement. For example, an error of ±0.5 m in a measurement of 1000 m is relatively small compared to an error of 0.5 m in a measurement of 10 m.

SUMMARY

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

The smaller the percentage error, the more accurate the measurement.

EXAMPLE 1

The length of a basketball court is measured as 26 m (to the nearest metre).

The amount of petrol pumped into a car is recorded as 35.8 L.

a For each measurement, calculate:

i the range of possible values of the measurement

ii the percentage error, correct to two significant figures.

b Which measurement is more accurate?

Solution

a i Length of basketball court:

Absolute error = ±0.5 × 1 m ± $\frac{1}{2}$ of the smallest unit

= ±0.5 m

Range of possible values:

= 26 m ± 0.5 m

= 25.5 m to 26.5 m

Amount of petrol:

Absolute error = ±0.5 × 0.1 L ± $\frac{1}{2}$ of the smallest unit

= ±0.05 L

Range of possible values

= 35.8 L ± 0.05 L

= 35.75 L to 35.85 L

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ii Length of basketball court:

$$\text{Percentage error} = \frac{\pm 0.5}{26} \times 100\%$$
$$= \pm 1.9230...$$
$$\approx \pm 1.9\%$$

Amount of petrol:

$$\text{Percentage error} = \frac{\pm 0.05}{35.8} \times 100\%$$
$$= \pm 0.13966...$$
$$\approx \pm 0.14\%$$

b The petrol measurement is more accurate because it has the smaller percentage error.

Exercise 3-01 Percentage error

1 What is the smallest unit on a scale used to record a measured mass of 35.41 g? Select **A**, **B**, **C** or **D**.

A 1 g **B** 0.01 g **C** 0.005 g **D** 0.1 g

2 What is the absolute error of a measured length of 5.1 m? Select **A**, **B**, **C** or **D**.

A ±0.5 m **B** ±0.01 m **C** ±0.05 m **D** ±1 m

Example 1

3 A village square has sides of length 14.8 m.

a Between what two values could the actual length of the sides lie?

b What is the absolute error of this measurement?

c Calculate, to one decimal place, the percentage error of this measurement.

4 A rectangle measures 10.3 m by 6.7 m.

a Between what two values does its length lie?

b Between what two values does its width lie?

5 What is the percentage error of a measured length of 42.8 L? Select **A**, **B**, **C** or **D**.

A ±0.1% **B** ±0.2% **C** ±0.12% **D** ±0.05%

6 For each measurement, find:

i the absolute error

ii the percentage error, correct to the same number of significant figures as in the measurement.

a a distance of 6452 km (to the nearest kilometre)

b a perimeter of 6.9 m

c a length of 8000 m (to the nearest 100 m)

d a mass of 39.85 mg

e a capacity of 245 ML

f a measured height of 165 cm

7 Which of the measurements in question **6** is the most accurate? Why?

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8 A planet has a diameter of 12 000 km, correct to the nearest 1000 km.

- **a** How many significant figures are in the measured diameter?
- **b** What is the absolute error of the measured diameter?
- **c** Calculate the percentage error, correct to two decimal places.

9 Find, correct to two significant figures, the percentage error of each measurement.

a A cube with a 3 cm edge. **b** A cube with a 3.0 cm edge.

10 The Moon has a radius of 1740 km (to the nearest 10 km). Find, correct to three significant figures, the percentage error of this radius.

11 A rug measures 300 cm by 200 cm, where measurements are to the nearest 10 cm.

- **a** How many significant figures are in these measurements?
- **b** Calculate the percentage error for the measurement 200 cm.

3-02 Areas of composite shapes

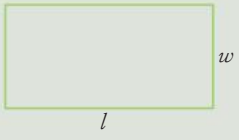
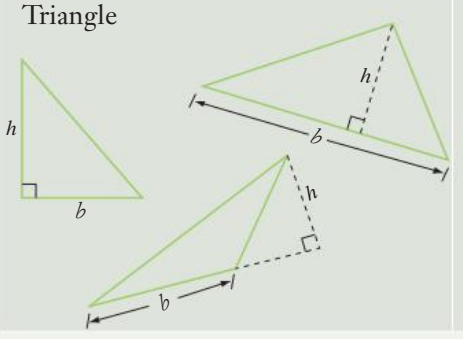
A composite shape is made up of two or more basic shapes. You should learn the area formulas of the basic shapes given in the table.

Areas of composite shapes

Offset surveys

SUMMARY

Area formulas

Shape	Area	Formula
Rectangle 	A = length × width	$A = lw$
Triangle 	$A = \frac{1}{2} \times$ base × perpendicular height	$A = \frac{1}{2}bh$

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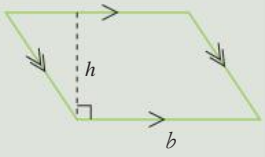
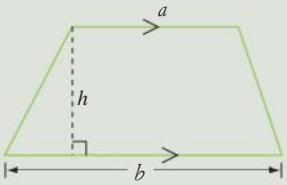
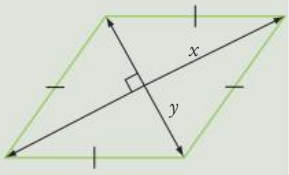
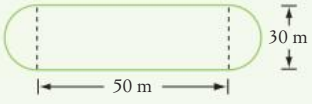
Parallelogram		A = base × perpendicular height	$A = bh$
Trapezium		$A = \frac{1}{2} \times$ sum of parallel sides × perpendicular distance between sides	$A = \frac{1}{2}(a + b)h$
Rhombus		$A = \frac{1}{2} \times$ product of diagonals	$A = \frac{1}{2}xy$
Circle		$A = \pi \times (\text{radius})^2$	$A = \pi r^2$

EXAMPLE 2

All sports fields in Battenball are in the shape of a rectangle with a semicircle on each end as shown.



a What is the area of this sports field, correct to the nearest square metre?

b If turf costs $10.50 per square metre, what would it cost to cover this field with turf?

Solution

a Area of field = area of rectangle + area of circle

$= 50 \times 30 + \pi \times 15^2$

$= 2206.858...$

$\approx 2207 \text{ m}^2$

2 semicircles = 1 circle

Radius of circle $= \frac{1}{2} \times 30 = 15$ **m**

b Cost of turf $= 2207 \times \$10.50$

$= \$23\ 173.50$

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EXAMPLE 3

A surveyor made a notebook entry to sketch a building block, with measurements shown in metres.

		71	
	28	29	
		22	45
	10	8	
		0	

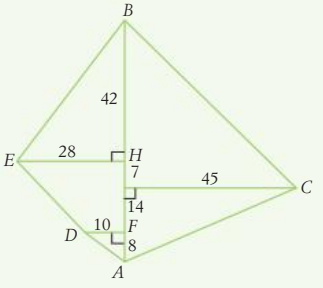
a Draw a diagram of the building block.

b Find the area of the block, correct to two significant figures.

c A fence is to be constructed along the side AC. Find its length to the nearest metre.

Solution

a



b Area $= \triangle BAC + \triangle BHE + \triangle AFD +$ trapezium $EHFD$ **3 triangles and one trapezium.**

$$= \frac{1}{2}\times 71\times 45 + \frac{1}{2}\times 42\times 28 + \frac{1}{2}\times 8\times 10 + \frac{1}{2}\times (10+28)\times 21$$

$$= 2624.5$$

$$\approx 2600 \text{ m}^2$$

c $AC^2 = 22^2 + 45^2$ **Using Pythagoras' theorem on $\triangle CGA$.**

$$= 2509$$

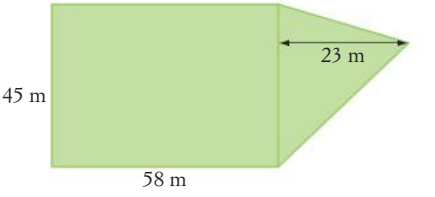
$$AC = \sqrt{2509}$$

$$= 50.0899\ldots$$

$$\approx 50 \text{ m}$$

Exercise 3-02 Areas of composite shapes

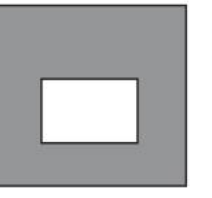
1 Find the area of the park shown. Select **A**, **B**, **C** or **D**.



A 3127.5 m^2 **B** 3944 m^2

C 3645 m^2 **D** 1972 m^2

2 A square sheet of metal of side 2.4 m has a rectangular hole cut out of it.



a Find the shaded area if the hole has dimensions 1.1 m by 0.6 m.

b Find the cost of engraving this area at $26.50 per square metre.

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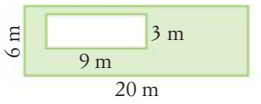
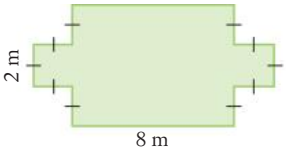
3 Find each shaded area, correct to the nearest square unit where necessary.

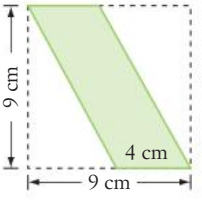
a



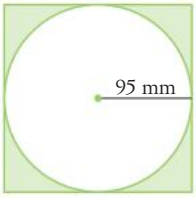
b



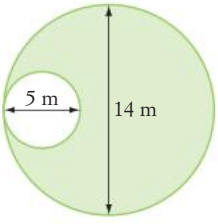
c



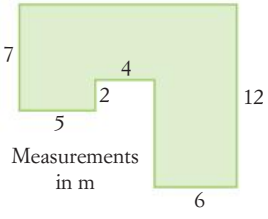
d



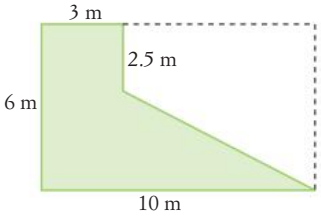
e



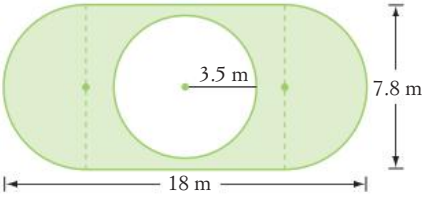
f



g

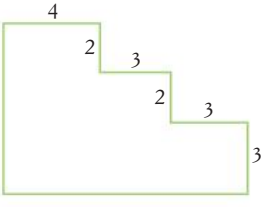


h

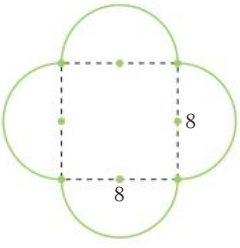


4 Find the area of each shape, correct to the nearest square metre where necessary. All measurements are in metres.

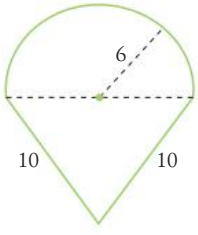
a



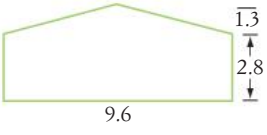
b



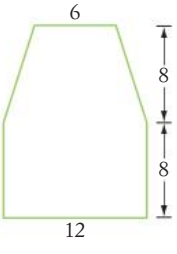
c



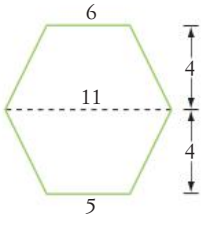
d



e



f



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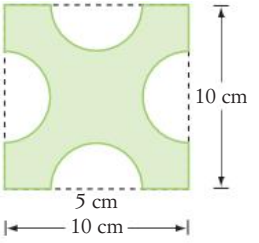
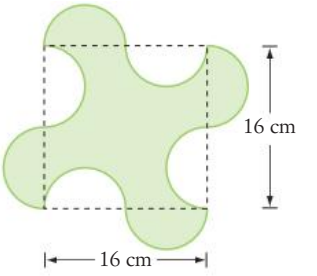
5 **a** Find the area of each plastic throwing toy, correct to one decimal place.

i

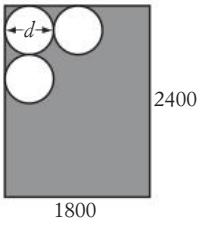


ii



b Can you see an easy way of determining the area of the toy in part **ii** above?

6 A factory buys sheets of tin measuring 2400 mm by 1800 mm. Circles of diameter d mm are cut from each sheet.

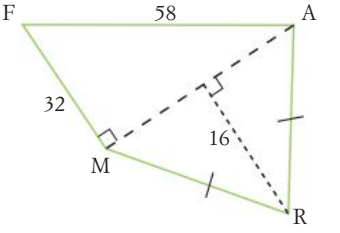


a What is the area in square metres of a sheet of tin?

b If 12 whole circles can be cut from one sheet of tin, what is the value of d?

c What area of tin is left when all 12 circles are removed? Answer in square metres to two significant figures.

7 The diagram shows the shape of a farm, where measurements are in metres.



a Find the area of the farm to the nearest m^2.

b The farm is to be fenced around its perimeter. What length of fence is needed? Answer to the nearest metre.

8 For each notebook entry of a block of land, sketch the block and find its area to the nearest square metre. All measurements are in metres.

Example 3

a

	48	
14	36	
	15	21
	0	

b

	140	
37	98	
	52	58
16	30	
	0	

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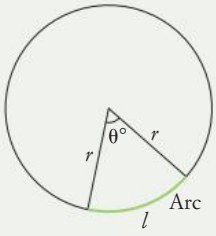
3-03 Circular shapes

Length of an arc

An **arc** is a part of the circumference of a circle. Its length (l) is a fraction of the circumference ($2\pi r$) of the circle. There are 360° in a circle.

SUMMARY

$l = \frac{\theta}{360} \times 2\pi r$ where θ° is the size of the central angle.



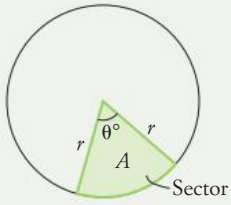
Area of a sector

A **sector** is a region of a circle bounded by an arc and two radii. Its area (A) is a fraction of the area (πr^2) of the circle.

'radii' is the plural of 'radius': one radius, two radii.

SUMMARY

$A = \frac{\theta}{360} \times \pi r^2$, where θ° is the size of the central angle.



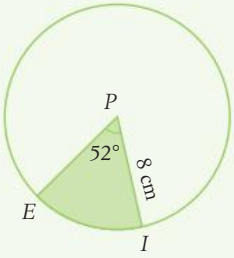
Note: If θ = 90°, the sector is called a **quadrant** (quarter of a circle) and if θ = 180°, the sector is called a **semicircle** (half of a circle).

EXAMPLE 4

Find, correct to two significant figures:

a the length of arc EI

b the area of sector PIE.



Solution

a
$$l = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{52}{360} \times 2 \times \pi \times 8$$
$$= 7.2605...$$
$$\approx 7.3 \text{ cm}$$

Length of arc EI is 7.3 cm.

b
$$A = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{52}{360} \times \pi \times 8^2$$
$$= 29.0422...$$
$$\approx 29 \text{ cm}^2$$

Area of sector PIE is 29 cm^2.

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Area of an annulus

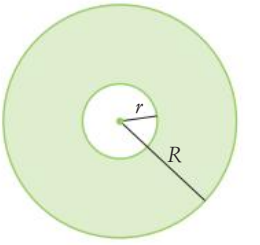
An **annulus** is the ring-like shape between two **concentric circles** of different radii.

Concentric means 'having the same centre'.

Area of annulus = area of large circle − area of small circle

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$



SUMMARY

$A = \pi(R^2 - r^2)$

where R = radius of outer circle, r = radius of inner circle

EXAMPLE 5

Find the area of this annulus, correct to one decimal place.

Solution

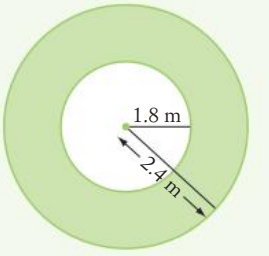
$A = \pi(R^2 - r^2)$

$= \pi(2.4^2 - 1.8^2)$

$= 7.9168\ldots$

$\approx 7.9 \text{ m}^2$

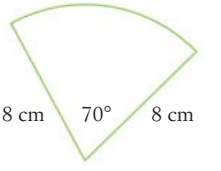
Area of the annulus is 7.9 m².



Exercise 3-03 Circular shapes

Example 4

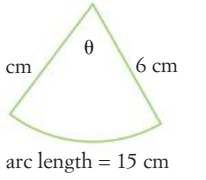
1 Find the area of this sector correct to one decimal place.
Select **A**, **B**, **C** or **D**.



A 3.09 cm² **B** 9.7 cm²

C 39.1 cm² **D** 9.8 cm²

2 This sector has an arc length of 15 cm.
What is the size of θ to the nearest degree?



A 72° **B** 48°

C 53° **D** 143°

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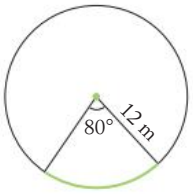
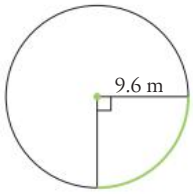
3 Find the length of each arc marked, correct to one decimal place.

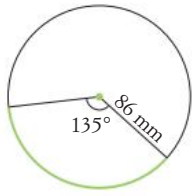
a



b

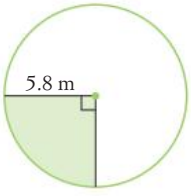


c

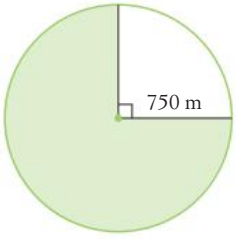


4 Find the area of each shaded sector, correct to the nearest square unit.

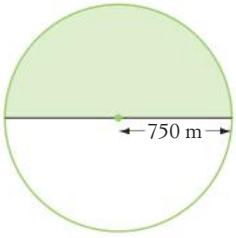
a



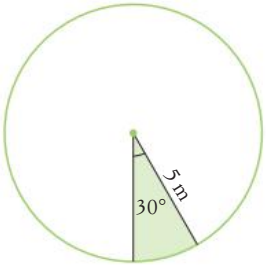
b



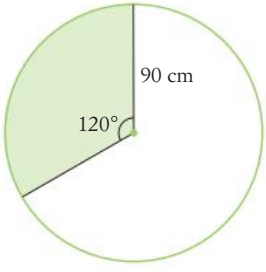
c



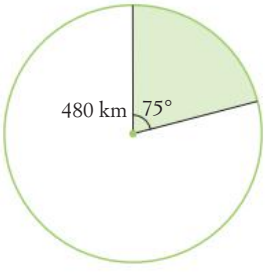
d



e



f



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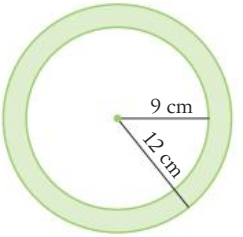
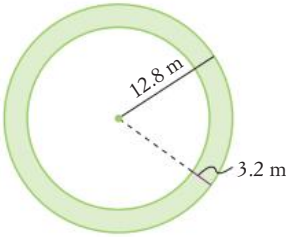
Example 5

5 Find the area of each annulus, correct to three significant figures.

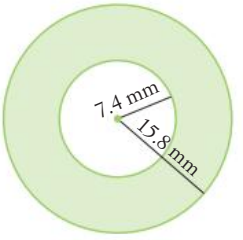
a



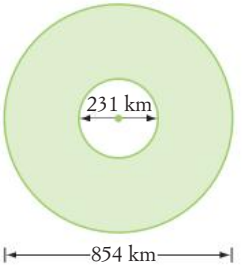
b



c



d

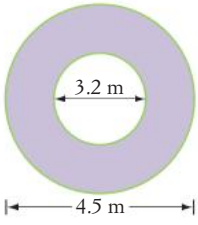


6 A pizza has diameter 30 cm and is cut into eight equal pieces.

a What is the area of each piece? Answer to the nearest square centimetre.

b What is the length of the crust on each piece of pizza? Answer correct to the nearest centimetre.

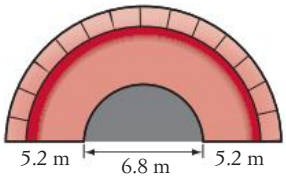
7 A large stormwater pipe has an outer diameter of 4.5 m and an inner diameter of 3.2 m. Its cross-section is shown here.



a What is the shape of the cross-section?

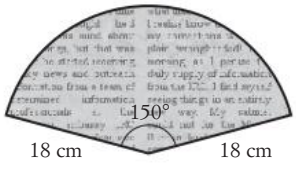
b What is the area of this cross-section, correct to one decimal place?

8 **a** The entrance to a tunnel is shown. What is its area, correct to the nearest square metre?



b How much will it cost to paint this area if paint is \$152 per 10 L can and one can covers 30 m^2? (*Note:* This paint can only be bought in 10 L cans.)

9 At the animal park, newspaper is cut in the shape of a sector and rolled to hold animal food. For one piece of newspaper find its



a area, correct to the nearest square centimetre

b perimeter, correct to the nearest centimetre.

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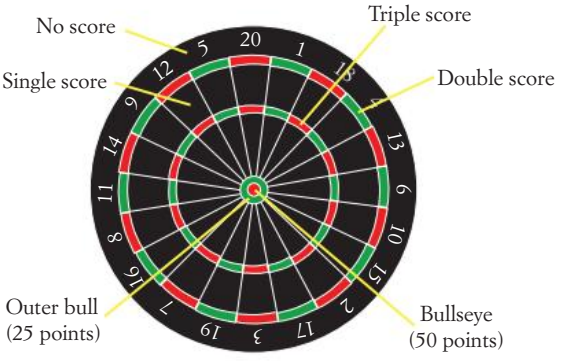
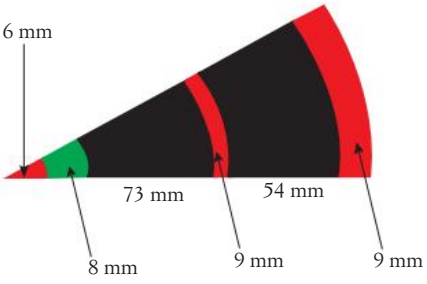
10 This metal washer has an inner radius of 2.5 mm and the area of its front face is 140 mm^2. What is its outer radius, correct to two significant figures?

INVESTIGATION

DARTBOARD AREAS

A dartboard is made up of 20 equal sectors, one for each of the numbers 1 to 20. The measurements for one such sector are shown below on the right.





Calculate, to the nearest square millimetre, the following areas on the dartboard.

a the total scoring area (excluding the no-score area)

b the bullseye **c** the outer bull (an annulus)

d double 2 **e** triple 20 **f** score of 7

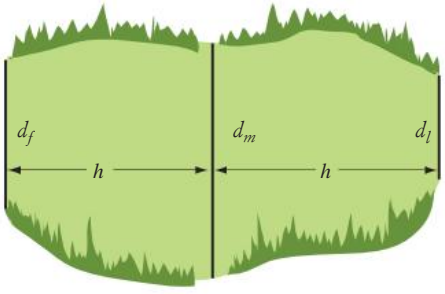
Simpson's rule

3-04 Simpson's rule

Simpson's rule is an approximation method for calculating the area of an irregular shape.

Divide the shape into two strips of equal width, h, and take three vertical measurements, d_f, d_m and d_l, as shown.

The approximate area is found by this formula:



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SUMMARY

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

where h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

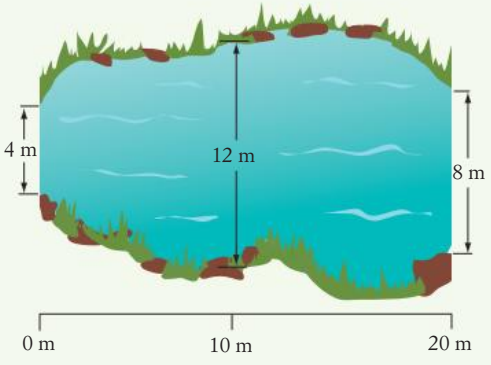
d_l = last measurement

or in words:

$$\text{Area} \approx \frac{\text{width of strip}}{3} \text{ (first measurement + 4 × middle measurement + last measurement)}$$

EXAMPLE 6

A surveyor took three vertical measurements across a pond at 10 m intervals. Use Simpson's rule to find the approximate area of the pond.



Solution

$h = 10$, $d_f = 4$, $d_m = 12$, $d_l = 8$

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

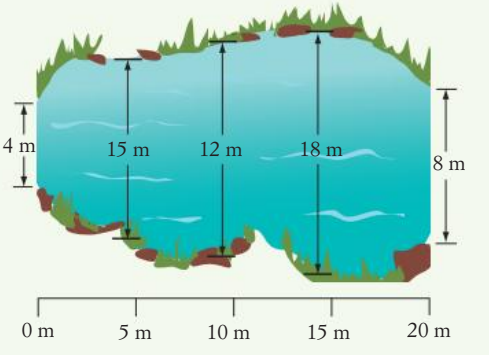
$$= \frac{10}{3}(4 + 4 \times 12 + 8)$$

$$= 200 \text{ m}^2$$

The approximate area of the pond is 200 m^2.

EXAMPLE 7

The surveyor then took five vertical measurements at 5 m intervals across the same pond from Example **6**. Use two applications of Simpson's rule to find a better approximation for the area of the pond.



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Solution

Apply Simpson's rule twice, once on each 'half' of the pond.

Using the first three measurements,

$d_f = 4,\ d_m = 15,\ d_l = 12$

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$
$$= \frac{5}{3}(4 + 4 \times 15 + 12)$$
$$= 126.6666\ldots$$

Using the last three measurements,

$d_f = 12,\ d_m = 18,\ d_l = 8$

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$
$$= \frac{5}{3}(12 + 4 \times 18 + 8)$$
$$= 153.3333\ldots$$

Do not round these answers yet

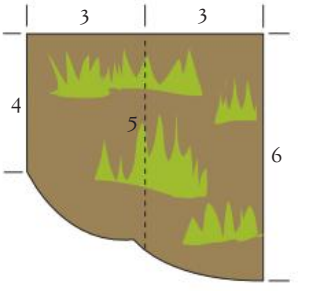
Total area = 126.6666… + 153.3333…

= 280 m²

A better approximation for the area of the pond is 280 m².

Example 6

Exercise 3-04 Simpson's rule



1 A garden has the following measurements in metres. Use Simpson's rule to find its area to the nearest square metre. Select **A**, **B**, **C** or **D**.

A 15

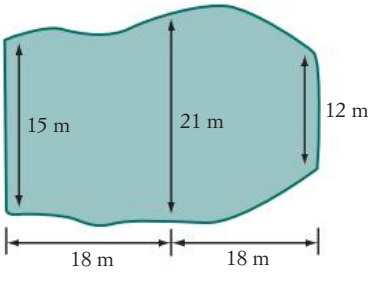
B 45

C 30

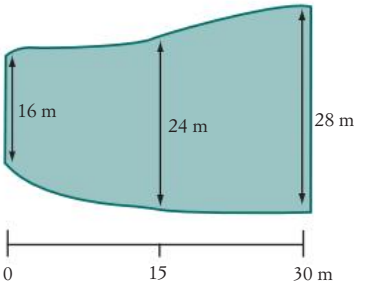
D 90

2 Use Simpson's rule to approximate each shaded area.

a



b



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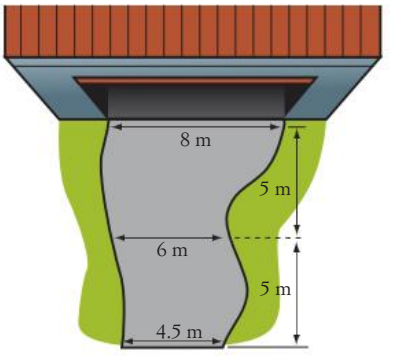
3 To calculate an approximate area for her driveway, Olivia took three measurements at 5 m intervals.

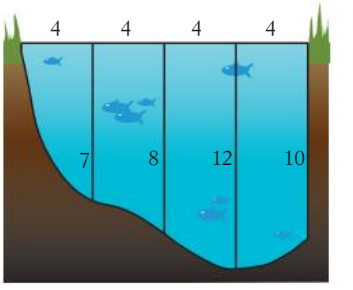


a Use Simpson's rule to find the area of the driveway, correct to the nearest m^2.

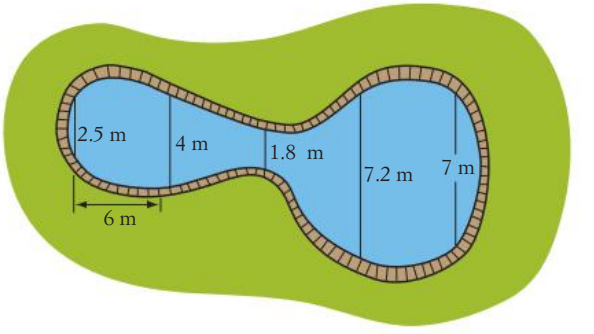
b What is the cost of sealing the driveway if sealer costs \$45/L and 1 L covers 12 m^2?

4 The cross-section of a river is shown, with measurements in metres. Use two applications of Simpson's rule to approximate the area of the cross-section.

Example 7



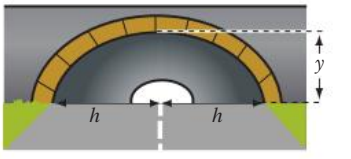
5 The shape of a new swimming pool is marked out on the lawn and five vertical measurements are taken at 6 m intervals.



a Use Simpson's rule to find the area of the pool.

b What volume of soil, correct to two significant figures, will need to be removed if the pool is to be 1.2 m deep?

6 A tunnel has a cross-section as shown.



Use Simpson's rule to find a formula for the cross-sectional area. Select **A**, **B**, **C** or **D**.

A hy **B** $4hy$

C $\frac{hy}{3}$ **D** $\frac{4hy}{3}$

7 Five measurements in metres were taken across a lake at 12 m intervals as shown.

x (m)	0	12	24	36	48
y (m)	5	17	23	15	20

a Draw a neat sketch of the lake showing the five measurements.

b Use two applications of Simpson's rule to find the approximate area of the lake.

c What is the capacity of the lake in megalitres if it has a constant depth of 20 m?

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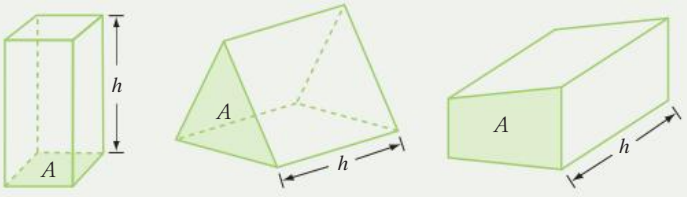
3-05 Volume and surface area of a prism

Volume of a prism

SUMMARY

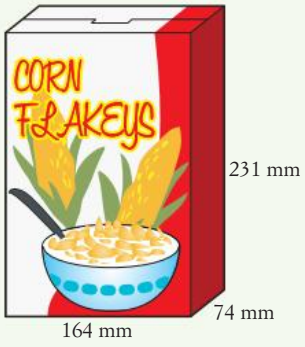
Volume = Area of base × height

$V = Ah$



EXAMPLE 8

Find, correct to the nearest cubic centimetre, the volume of this cereal box.



Solution

Convert all measurements to centimetres first.

164 mm = 16.4 cm, 74 mm = 7.4 cm,

231 mm = 23.1 cm

$V = Ah$

$= (16.4 \times 7.4) \times 23.1$

$= 2803.416$

≈ 2803

The volume of the box is 2803 cm^3.

Volume is in cubic units.

Surface area of a prism

SUMMARY

The **surface area of a solid** is the sum of the areas of its faces. The solid can be closed or open.

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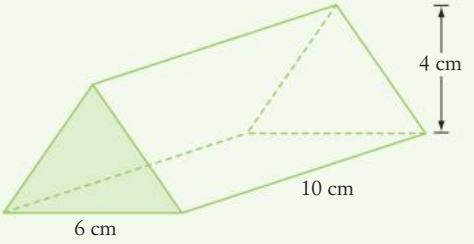
A closed rectangular prism has 6 faces.

An open triangular prism with no end faces has 3 faces.

EXAMPLE 9

This triangular prism is open at one end. Calculate its surface area.

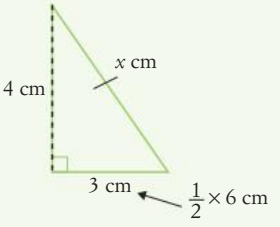
'Surface area' here means the external (outside) surface area.



Solution

The open prism has one triangular face and three rectangular faces.

To find the areas of the faces, we must first find the slant height (x cm) of the triangle using Pythagoras' theorem.



$$x^2 = 3^2 + 4^2$$
$$= 25$$
$$x = \sqrt{25}$$
$$= 5$$

Surface area = area of triangle + area of base rectangle + 2 × area of side rectangles

$$= \frac{1}{2} \times 6 \times 4 + (10 \times 6) + 2 \times (10 \times 5)$$
$$= 172$$

The surface area of this triangular prism is 172 cm^2.

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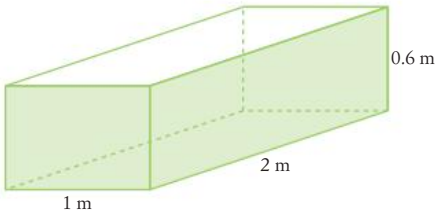
Example 8

Exercise 3-05 Volume and surface area of a prism

1 Find the volume of this prism.
Select **A**, **B**, **C** or **D**.



A 1.2 m^3

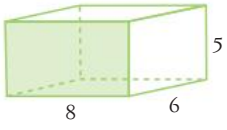
B 7.6 m^3

C 7.2 m^3

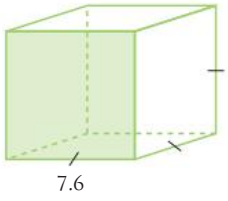
D 2.4 m^3

2 Find the volume of each prism. All measurements are in metres.

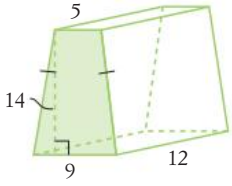
a



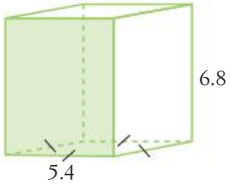
b



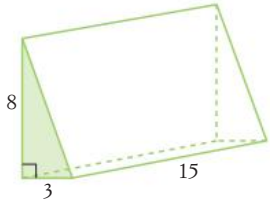
c



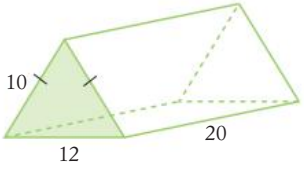
d



e



f



Example 9

3 Find the surface area of the prism in question **1** if it is open at the top. Select **A**, **B**, **C** or **D**.

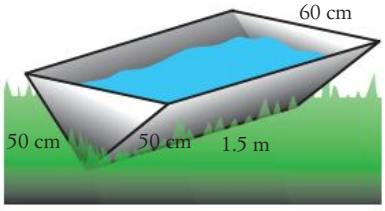
A 172 m^2 **B** 3.6 m^2 **C** 5.6 m^2 **D** 6.6 m^2

4 Calculate the surface area of the closed cereal box in Example **8** on page 112.
Select **A**, **B**, **C** or **D**.

A 1342 cm^2 **B** 938 cm^2 **C** 188 cm^2 **D** 2803 cm^2

5 Find the surface area of each prism in question **2**, correct to the nearest square metre.
All prisms are closed and all measurements are in metres.

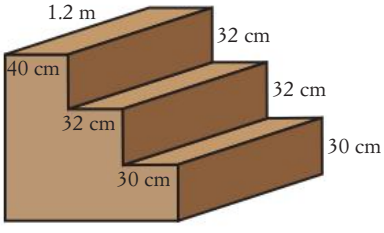
6 An open water trough has dimensions as shown.
What is its external surface area, correct to two significant figures?



7 If the cross-section of a prism has area, A, and perimeter, P, and the length of the prism is h, then its surface area can be calculated using the formula $S = 2A + Ph$. The $2A$ represents the area of the two identical end faces, while Ph is the total area of the rectangular side faces, which combine to make one big rectangle.

a A portable set of stairs has dimensions as shown. Use the formula to calculate its surface area in square metres.

b Calculate its volume in cubic metres.

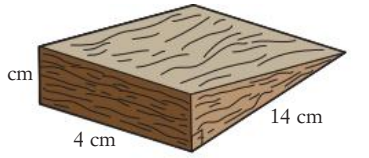


ISBN 9780170238977

8 A timber wedge is used to prop open a door.

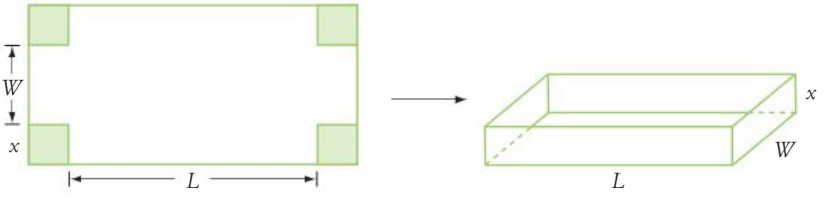
a Calculate the volume of the wedge.

b Kobi applies varnish to a box of wedges using 250 mL pots, each covering 1 m^2 of surface area. How many wedges can he varnish with 5 pots?



TECHNOLOGY: VOLUME AND SURFACE AREA OF A BOX

1. Cut out a sheet of cardboard 30 cm by 20 cm.
2. Cut a 2 cm square from each corner of the sheet and fold the large sheet to make an open box.
3. Calculate the volume and external surface area of the box.
4. Flatten the box and cut a larger 5 cm square from each corner and make a new box.
5. What is the volume and surface area of the new box?
6. Create the spreadsheet below to investigate the volume and surface area of the boxes made using squares of length $x = 0$ to 10 cm.



Enter the formulas shown in row 7 and A8. Copy the formulas down to row 17 to get the values shown.

	A	B	C	D	E	F
1	Volume and surface area of an open box					
2						
3	length of sheet	30				
4	width of sheet	20				
5						
6	length of square x	box length L	box width W	box height x	VOLUME LWx	SURFACE AREA $LW + 2Lx + 2Wx$
7	**0**	**=$B$3–2*A7**	**=$B$4–2*A7**	**=A7**	**=B7*C7*D7**	**=B7*C7+2*B7*D7+2*C7*D7**
8	=A9+1					
9						
17	10	10	0	10	0	200

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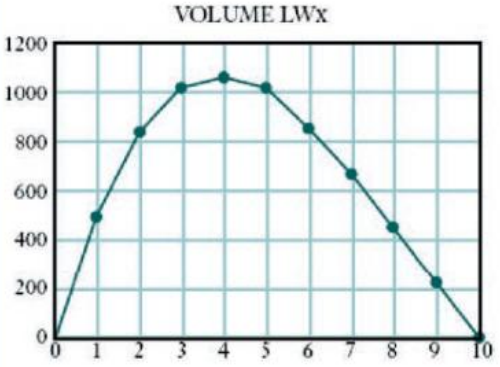
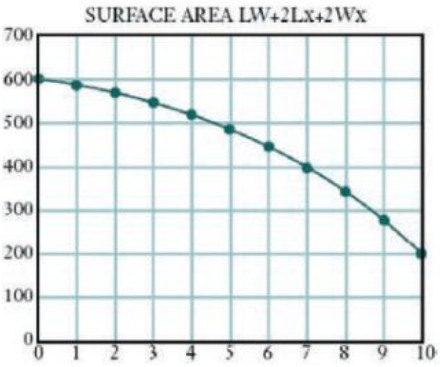
7 Draw line graphs for the volume and surface area using the Chart option.





8 What is the side length of the square that gives the largest:

a volume? **b** surface area?

9 What is the largest:

a volume? **b** surface area?

Measurement in the home

3-06 Volume and surface area of a cylinder

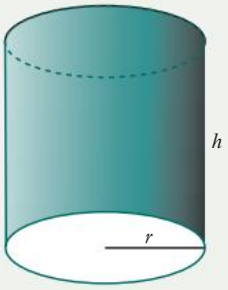
Volume of a cylinder

SUMMARY

$$V = \pi r^2 h$$

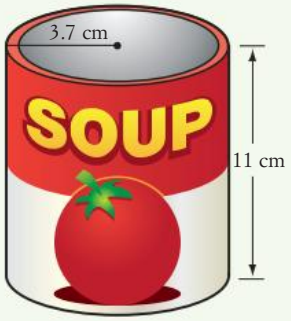
where r = radius of circular base

h = perpendicular height



EXAMPLE 10

A can of soup has base radius 3.7 cm and height 11 cm. What is its volume correct to the nearest millilitre?



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Solution

$V = \pi r^2 h$

$= \pi \times 3.7^2 \times 11$

$= 473.0924\ldots \text{ cm}^3$

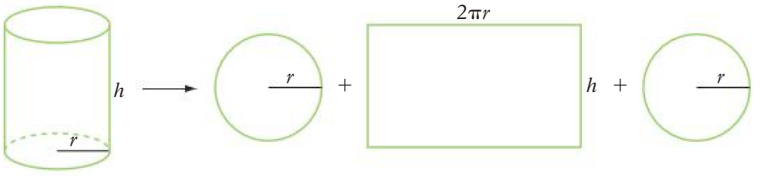
$= 473.0924\ldots \text{ mL}$ $\quad 1 \text{ cm}^3 = \text{mL}$

$\approx 473 \text{ mL}$

Volume is 473 cm^3.

Surface area of a cylinder

A closed cylinder has two circular ends and a curved surface. The curved surface is a rectangle whose length is equal to the circumference ($2\pi r$) of the circular end.



Surface area of a closed cylinder = area of rectangle + area of 2 circles

$= 2\pi r \times h + 2 \times \pi r^2$

$= 2\pi rh + 2\pi r^2$

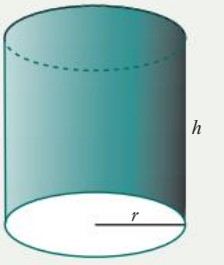
SUMMARY

Surface area of a closed cylinder

$A = 2\pi rh + 2\pi r^2$

where r = radius of circular base

h = perpendicular height

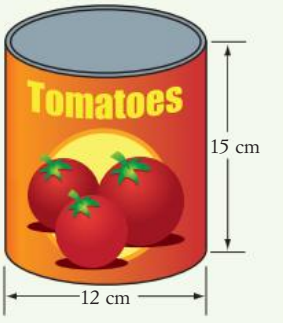


EXAMPLE 11

A can of tomatoes has base diameter 12 cm and height 15 cm. Calculate to the nearest square centimetre:

a the surface area of the closed can

b the area of the label wrapped around the can if there is a 2 cm overlap.



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Solution

a $A = 2\pi rh + 2\pi r^2$

$= 2 \times \pi \times 6 \times 15 + 2 \times \pi \times 6^2$

$= 791.6813\ldots$

$\approx 792\ \text{cm}^2$

The surface area of the can is $792\ \text{cm}^2$.

$r = \frac{1}{2} \times 12\ \text{cm} = 6\ \text{cm}$

b Length of label $= 2\pi r + 2$, height of label $= h$

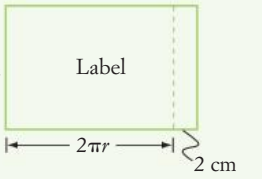
Area of label $= (2\pi r + 2) \times h$

$= (2 \times \pi \times 6 + 2) \times 15$

$= 595.4866\ldots$

$\approx 595\ \text{cm}^2$

The area of the label is $595\ \text{cm}^2$.

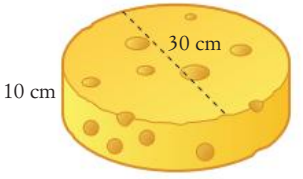


Example 10

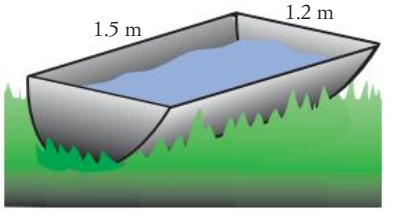
Exercise 3-06 Volume and surface area of a cylinder

1 Find, correct to one decimal place, the volume of each solid.

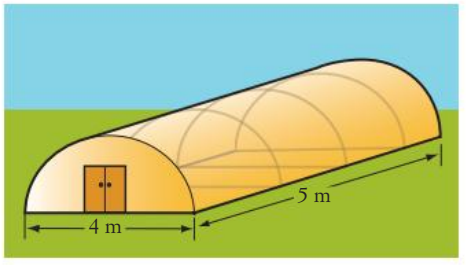
a a wheel of Swiss cheese



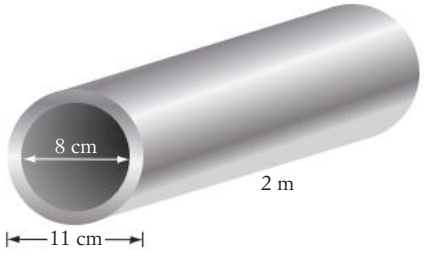
b a water trough in the shape of a half-cylinder



c a hothouse



d a piece of plumber's pipe



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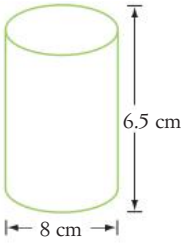
2 A V6 Holden car has six cylinders. Each cylinder has a bore (diameter) of 8 cm and a stroke (height) of 6.5 cm.



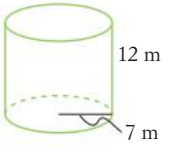
a Find the volume of one cylinder, correct to the nearest cm^3.

b Find the engine capacity (sum of the capacities of the six cylinders) of the car to the nearest litre.

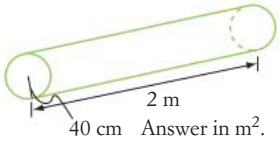
Example 11

3 Find, correct to two significant figures, the surface area of each cylinder.

a

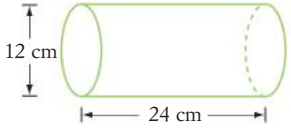


b

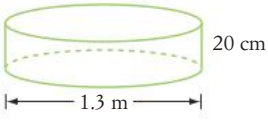


Answer in m^2.

c

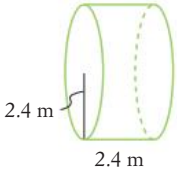


d



Answer in m^2.

e



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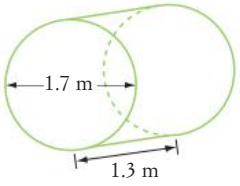
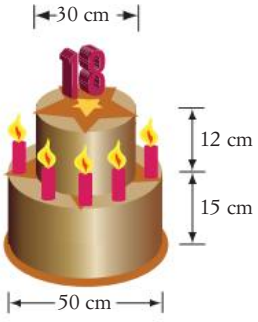


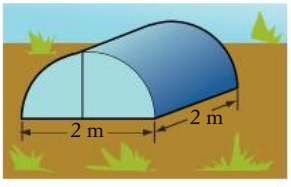
4 A two-tier cake is shown in the diagram. All surfaces of the cake except the base are iced. Find, correct to the nearest square centimetre, the area that is iced.



5 A dome tent is in the shape of a half-cylinder. The floor area is 2 m by 2 m. Calculate, correct to one decimal place:

a its volume

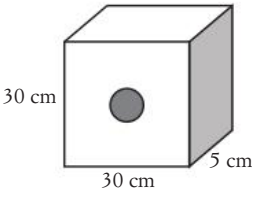
b its surface area including the floor.



6 A concrete block is a square prism with a cylindrical hole of diameter 10 cm drilled through it.

a What is the volume of the concrete block to the nearest cm^3?

b What percentage (correct to one decimal place) of concrete was removed to make the hole?



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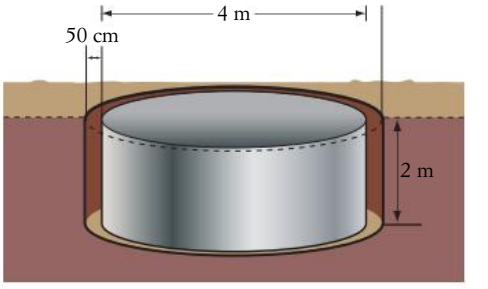
7 A cylindrical tank with diameter 4 m is placed in a 2 m deep circular hole so that there is a gap of 50 cm between the side of the tank and the hole. The top of the tank is level with the ground.



a What volume of dirt was removed to make the hole? Answer to the nearest m^3.

b What is the capacity of the tank, to the nearest litre?

8 A cylinder has a base radius of 5 cm. If the radius is doubled, what happens to the volume? Select **A**, **B**, **C** or **D**.

A volume doubles

B volume triples

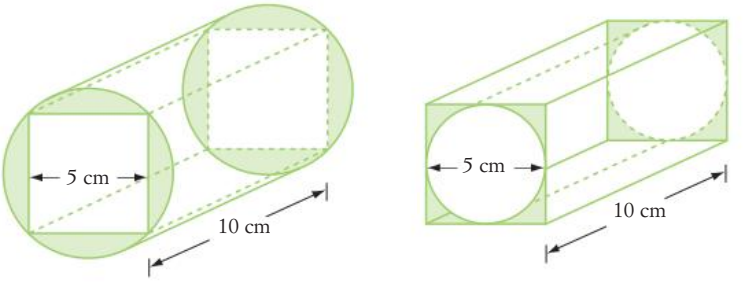
C volume is 4 times larger

D volume is halved

INVESTIGATION

SQUARE OR ROUND PEG?

Have you ever heard the saying that 'you can't put a square peg in a round hole'?



1 Compare the volumes occupied by a square peg of side 5 cm in a round hole of depth 10 cm and a round peg of diameter 5 cm in a square hole of depth 10 cm.

2 Which peg occupies more space?

3 What percentage of the hole does each peg occupy?

4 Is it better to put a square peg in a round hole or a round peg in a square hole? Justify your answer.

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3-07 Volume and surface area of a sphere

A sphere is a perfectly round solid and every point on its surface is the same distance from its centre. This distance is its radius and twice the radius is the diameter.

SUMMARY

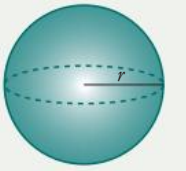
Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

where r = radius of the sphere

Surface area of a sphere

$$A = 4\pi r^2$$

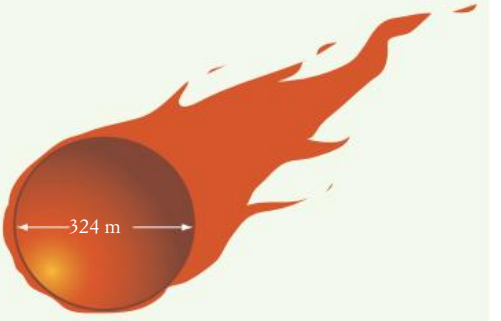


EXAMPLE 12

An asteroid is in the shape of a sphere with diameter 324 m.

a What is its volume in scientific notation, correct to three significant figures?

b What is its surface area, correct to the nearest hectare?



Solution

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 162^3$ $\quad r = \frac{1}{2} \times 324 \text{ km} = 162 \text{ km}$

$= 17\,808\,758.84$

$\approx 1.78 \times 10^7 \text{ m}^3$

The volume of the asteroid is approximately 1.78×10^7 m^3.

b $A = 4\pi r^2$

$= 4 \times \pi \times 162^2$

$= 329\,791.8304 \text{ m}^2$

$= 32.9791\ldots \text{ ha}$ $\quad 1 \text{ ha} = 10\,000 \text{ m}^2$

$\approx 33 \text{ ha}$

The surface area of the asteroid is approximately 33 ha.

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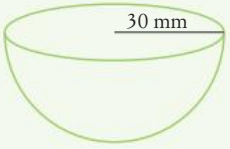
EXAMPLE 13

Find the surface area of a closed hemisphere with radius 30 mm.
Give your answer correct to the nearest 10 mm^2.



Solution

Area of curved surface $= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Area of circular top $= \pi r^2$

Surface area of closed hemisphere $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 30^2$

$= 8482.3001\ldots$

≈ 8480 mm^2

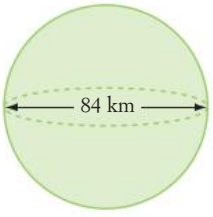
The surface area of the hemisphere is approximately 8480 mm^2.

Example 12

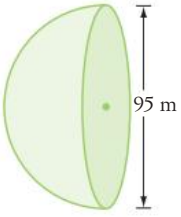
Exercise 3-07 Volume and surface area of a sphere

1 Find the volume of each solid, correct to three significant figures.

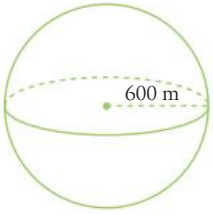
a



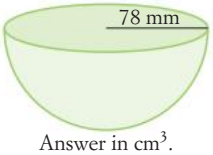
b



c

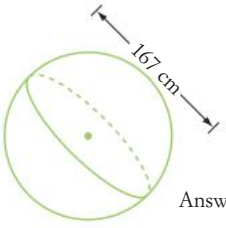


d



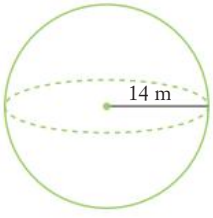
Answer in cm^3.

e



Answer in m^3.

f



2 The Earth is a sphere with a diameter of approximately 12 683 km. Four-fifths of the Earth's surface is covered by water.

a Calculate in scientific notation, correct to four significant figures:

i the number of square kilometres of the Earth's surface that are not covered by water

ii the volume of the Earth in cubic kilometres.

b If the mass of the Earth is 5.974×10^{24} tonnes, what is the mass per cubic kilometre (t/km^3)?

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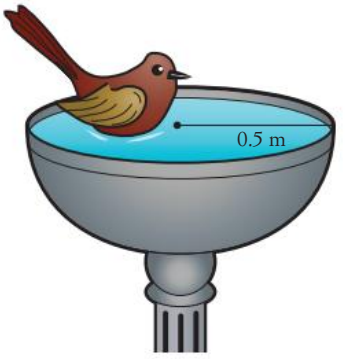
Example 13

3 A hemispherical metal bird bath has an internal radius of 0.5 m.

a What is its capacity in litres?

b What is the internal surface area?

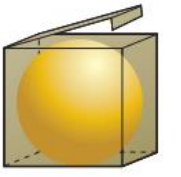


4 Christmas baubles are spheres with a diameter of 6 cm. They are packed in individual boxes so they just touch each side of the box.

a What is the volume of each bauble, to the nearest cm^3?

b What volume of air, to the nearest cm^3, is in a box containing one bauble?

c Each bauble is covered in fabric costing \$15 per m^2. How much does it cost to cover 100 baubles?



5 The volume of a sphere is 400 cm^3. What is its radius in cm, correct to one decimal place? Select **A**, **B**, **C** or **D**.

A 4.6 **B** 3.2 **C** 5.6 **D** 7.3

6 A mirror ball in the shape of a sphere of radius 0.28 m hangs over the dance floor at a disco. The surface is covered with 1 cm^2 mirror tiles.

a What is the volume of the mirror ball in cm^3, correct to three significant figures?

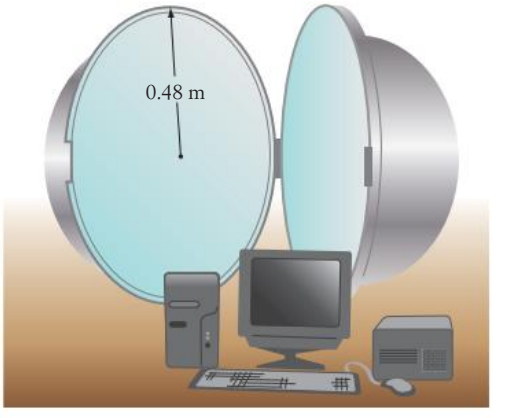
b How many whole mirror tiles cover the surface of the ball?

7 A very powerful computer-operated light is in the shape of two hemispheres hinged together.
The curved surface of the light is solid steel and the front of each hemisphere is special glass.
If the radius of the light is 0.48 m, find, correct to two significant figures:

a the area of steel covering the light

b the area of glass

c the volume of the light.



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JUST FOR THE RECORD

Circles and spheres in sport

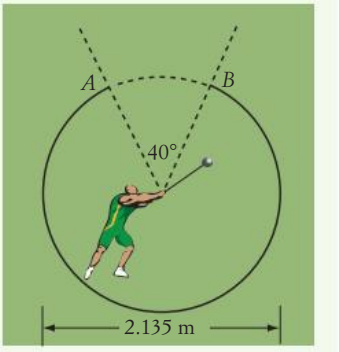
The shotput and hammer throw are two athletics field events requiring the participant to start in a throwing circle of diameter 2.135 m and launch into a 40° sector.

The **shot** is a spherical iron ball weighing at least 7 kg for a man and 4 kg for a woman. The shot is 'put' (not thrown) from the throwing circle into the marked sector.

The **hammer** has a spherical head attached to a handle by a length of steel about 1 m long and is launched from the throwing circle using a circular wind-up action.

The shot and hammer can be launched from any position in the circle but must land within the marked sector outside the circle.

What is the length of arc *AB*?



The hammer throw

3-08 Volumes of pyramids and cones

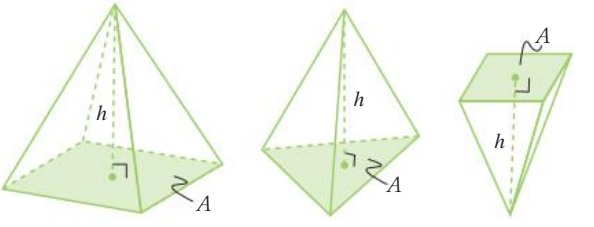
A page of solid shapes

Volumes of solids

Measurement formulas chart

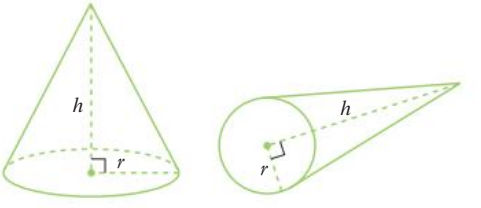
Formula matching game

The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height.



A cone is like a pyramid with a 'circular base'.

The volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same circular base and height.



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SUMMARY

Volume of a pyramid

$$V = \frac{1}{3}Ah$$

where

A = area of base

h = perpendicular height

Volume of a cone

$$V = \frac{1}{3}Ah$$

$$= \frac{1}{3}\pi r^2 h$$

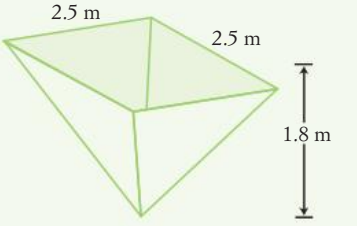
where

r = radius of circular base

h = perpendicular height

EXAMPLE 14

This grain bin has the shape of an open square pyramid. How many cubic metres of grain will it hold?



Solution

$$V = \frac{1}{3}Ah$$

$$= \frac{1}{3} \times 2.5^2 \times 1.8$$

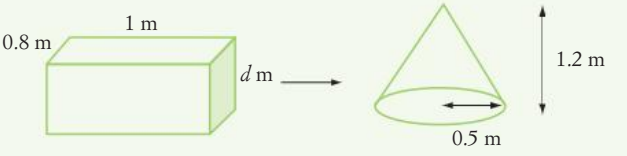
$$= 3.75 \text{ m}^3$$

The bin will hold 3.75 m^3 of grain.

The base is a square of side 2.5.

EXAMPLE 15

Billy the plumber dug a rectangular trench, 1 m by 0.8 m, to lay gas pipes. He ended up with a conical pile of dirt, 1.2 m high with a base radius of 0.5 m.



a How many square metres of dirt, correct to three decimal places, did Billy dig out?

b What was the depth of the hole, correct to one decimal place?

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Solution

a $V = \frac{1}{3}\pi r^2 h$ **Formula for the volume of a cone.**

$= \frac{1}{3} \times \pi \times 0.5^2 \times 1.2$

$= 0.31415\ldots$

$\approx 0.314\ \text{m}^3$

Billy dug out 0.3 m^3 of dirt.

b Let the depth of the hole be d m.

$V = Ah$ **Formula for the volume of a prism.**

$= 1 \times 0.8 \times d$

$= 0.8d$

But the volume is also 0.314 m^3 from **a**.

$0.314 = 0.8d$

$0.8d = 0.314$

$d = \frac{0.314}{0.8}$

$= 0.3925$

≈ 0.4 m

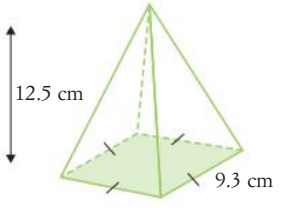
The depth of the hole was 0.4 m.

Example 14

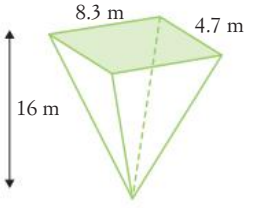
Exercise 3-08 Volumes of pyramids and cones

1 Find the volume of each pyramid, correct to the nearest whole number.

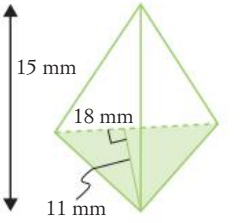
a



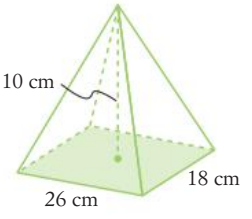
b



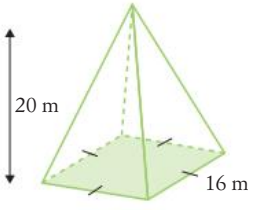
c



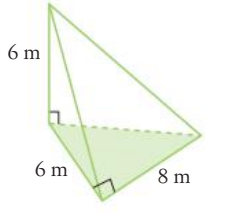
d



e



f



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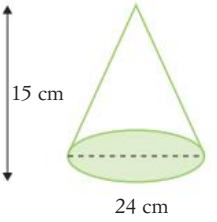
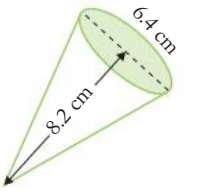
Example 15

2 Find the volume of each cone, correct to two decimal places.

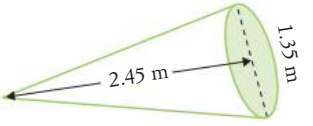
a



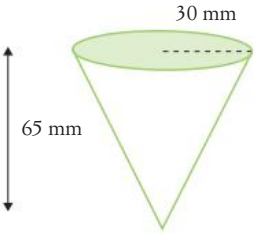
b



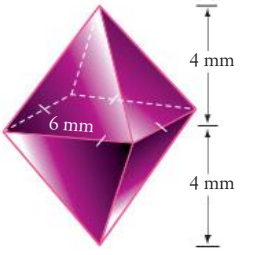
c



d



3 This crystal is in the shape of an octahedron. Calculate its volume.



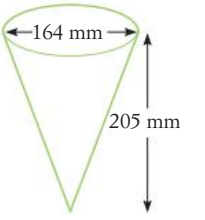
4 A large paper cup in the shape of a cone is used to hold popcorn.

It has a base diameter of 164 mm and a height of 205 mm.

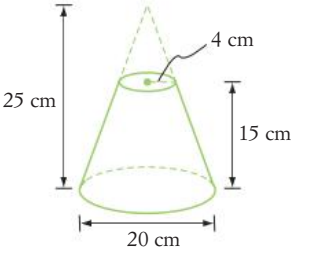
Calculate, correct to three significant figures, the volume of the cone:

a in cubic centimetres

b in litres.



5 A cone of height 25 cm and base diameter 20 cm has its top section removed to leave a frustum of height 15 cm. Find the volume of this frustum.



6 The largest pyramid in Egypt is the Great Pyramid of Cheops (or Khufu). It has a square base of side 230 m and a perpendicular height of 137 m. What is its volume correct to 3 significant figures?

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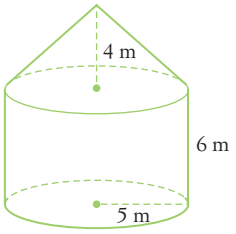
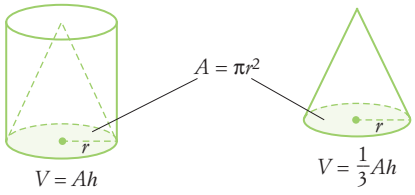
7 A wheat silo is made up of a cylinder with a cone on top. What volume of wheat will it hold when full? Answer to the nearest cubic metre.



INVESTIGATION

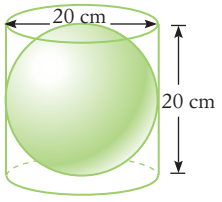
CYLINDERS, CONES AND SPHERES

1 The volume of the cone is $\frac{1}{3}$ the volume of the cylinder.



Take a cylindrical can and remove the top. Make a paper cone with the same base and height as your can. Fill the cone with sand or water and tip them into the can. Repeat this two more times. The can should now be full of sand.

2 The sphere on the right fits snugly into a cylinder of diameter and height both 20 cm.



a Calculate the volume of the sphere and the cylinder. Do not round your answers.

b What fraction of the cylinder's volume is taken up by the sphere?

c Is this fraction the same for any sphere and cylinder with equal diameters? Experiment with different diameters.

d Is there any way of proving this result?

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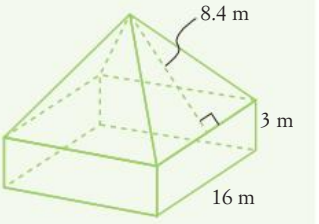
3-09 Volume and surface area of composite solids

A composite solid is made of two or more solids.

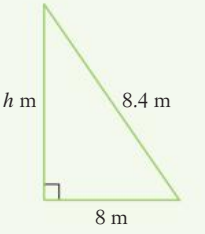
EXAMPLE 16

Callum's family hired this marquee for his 18th birthday party. Find its volume correct to the nearest cubic metre.



Solution

The marquee is in the shape of a square prism with a square pyramid on top. To calculate the volume of the pyramid, we must first find its perpendicular height using Pythagoras' theorem. Let this height be h m.



$h^2 = 8.4^2 - 8^2$

$= 6.56$

$h = \sqrt{6.56}$

$= 2.561249\ldots$ **Do not round yet.**

Volume of pyramid $= \frac{1}{3} \times 16^2 \times 2.561249\ldots$ $V = \frac{1}{3}Ah$

$= 218.5599\ldots \text{ m}^3$ **Do not round yet.**

Volume of square prism $= 16^2 \times 3$ $V = Ah$

$= 768 \text{ m}^3$

Total volume $= 218.5599\ldots + 768$

$= 986.5599\ldots$

$\approx 987 \text{ m}^3$ **Rounding at the end.**

The volume of the marquee is 987 m^3.

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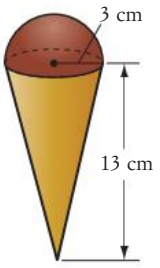
Exercise 3-09 Volume and surface area of composite solids

Example 16

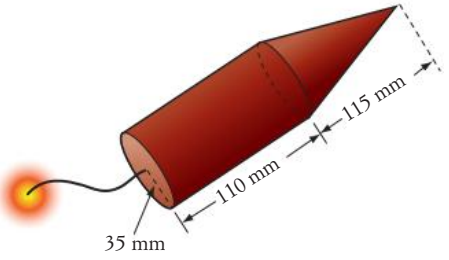
1 A choctop cone is a favourite with movie-goers. The cone is full of ice-cream and has a scoop of ice-cream on top in the shape of a hemisphere that is covered in chocolate. Find, correct to one decimal place:



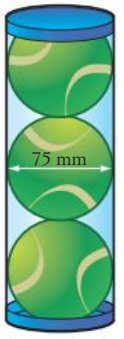
a the total volume of ice-cream

b the surface area that is covered with chocolate.

2 At a fireworks display, rockets packed completely with gunpowder were fired into the air. What volume of gunpowder was contained in each rocket? Answer to the nearest 1 cm^3.



3 A cylindrical can contains three tightly packed tennis balls, each with a diameter of 75 mm.



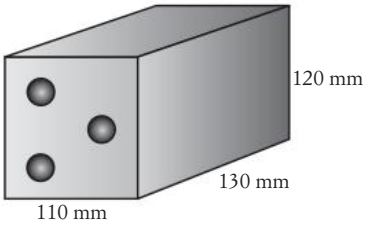
a What is the volume of the can (to the nearest cubic millimetre)?

b What percentage of the can contains air? Answer correct to one decimal place.

c If the three balls were packed tightly into a square prism instead of a cylinder, what would be the volume of the prism?

d Why do manufacturers pack tennis balls into cylindrical cans? Justify your answer.

4 A metal-fabricating company makes a machine part by drilling three holes of diameter 20 mm through a block of metal.

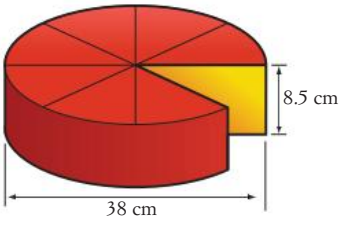


a What is the volume of metal before drilling?

b What is the volume of metal after drilling? Answer to the nearest cm^3.

c What percentage (correct to the nearest whole number) of metal was removed when the holes were drilled?

5 This cheese wheel of diameter 38 cm has one sector removed. Find, correct to two significant figures:



a the volume of the cheese wheel

b the surface area of the missing section [Hint: surfaces are 2 sectors and 3 rectangles].

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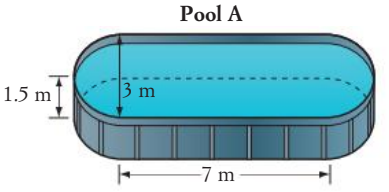
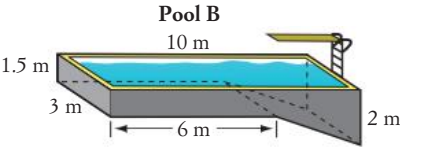
6 Two neighbours are arguing over whose swimming pool holds more water.



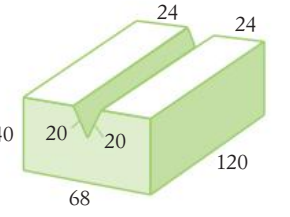


a What is the volume of water in each pool, correct to the nearest litre?

b Which pool holds more water, and by how much?

c If the base and sides of pool B are to be tiled, what area is to be tiled? Answer correct to the nearest square metre.

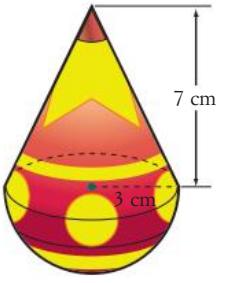
7 This is a woodworker's bench, with measurements in centimetres. Find:



a its volume in cubic metres, correct to two significant figures

b its total surface area in square centimetres.

8 Max received this spinning top for Christmas. Find its volume correct to the nearest cm^3.



INVESTIGATION

VOLUME AND PACKAGING

1 **a** Design a cardboard package to hold six golf balls (each with a diameter of 37 mm) so that the minimum amount of cardboard is used.

b Would the same packaging be suitable to hold:

i 6 billiard balls? **ii** 6 ping pong balls? Why?

2 **a** Design a cardboard carton to hold 12 cans of soup:

ii in one layer **ii** in two or more layers.

b Which carton uses the least amount of cardboard?

c Which carton is the most appropriate for this product? Why?

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3-10 Error in area and volume calculations

Errors in measurements will cause further errors in any area or volume calculation. As a general rule, a calculated area or volume can only be as accurate as the *least* accurate measurement used in the calculation, that is, the measurement with the fewest significant figures.

EXAMPLE 17

Shutterstock.com/Mark Snelson

Marty is laying some floorboards for a living room with a rectangular floor of dimensions 5.2 m by 3.5 m.

a What is the smallest possible area of the living room?

b What is the largest possible area of the living room?

c Calculate the area of the living room to an appropriate number of significant figures.

Solution

a The dimensions of the floor are given to the nearest 0.1 m, so the absolute error is $\pm 0.5 \times 0.1 \text{ m} = \pm 0.05 \text{ m}$

Shortest possible length = 5.2 m – 0.05 m = 5.15 m

Shortest possible width = 3.5 m – 0.05 m = 3.45 m

$\therefore$ Smallest possible area = $5.15 \times 3.45 = 17.7675 \text{ m}^2$

b Longest possible length = 5.2 m + 0.05 m = 5.25 m

Longest possible width = 3.5 m + 0.05 m = 3.55 m

$\therefore$ Largest possible area = $5.25 \times 3.55 = 18.6375 \text{ m}^2$

c Calculated area = 5.2×3.5 **Correct to 2 significant figures.**

$= 18.2 \text{ m}^2$

$\approx 18 \text{ m}^2$ ← Round to two significant figures because 5.2 m by 3.5 m are both measured to two significant figures.

Example 17

Exercise 3-10 Error in area and volume calculations

1 A village square has sides of length 14.8 m.

a Find its largest possible area.

b Find its smallest possible area.

c Calculate the area of the village square correct to an appropriate number of significant figures.

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2 A rectangular hall measures 10.3 m by 6.7 m and is about to get new bamboo flooring.

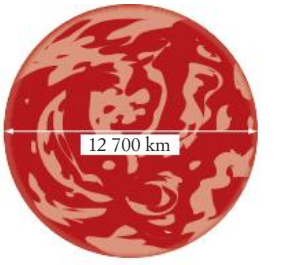
- **a** Between what two values does the floor area lie?
- **b** Which measurement, 10.3 m or 6.7 m, has fewer significant figures?
- **c** Calculate the area of the floor, correct to an appropriate number of significant figures.

3 A rhombus has diagonals of length 8.4 cm and 9.2 cm.

- **a** What are the largest and smallest possible areas?
- **b** What value should be recorded as the calculated area?

4 A planet has diameter 12 700 km, correct to the nearest 100 km.

- **a** Find the upper and lower limits of the planet's volume, correct to 4 significant figures.
- **b** What should be the recorded volume of the planet?



5 The edge of a cube is 3.4 cm.

- **a** Between what two values does the surface area lie?
- **b** What value should you record for the cube's surface area?
- **c** Between what two values does the volume lie?
- **d** What value should you record for the cube's volume?

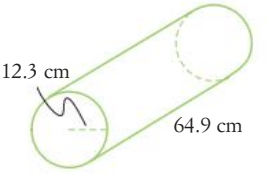
6 The Moon has a radius of 1740 km (to the nearest 10 km).

- **a** What are the largest and smallest possible surface areas? Answer correct to the nearest km^2.
- **b** What is the calculated surface area of the Moon?

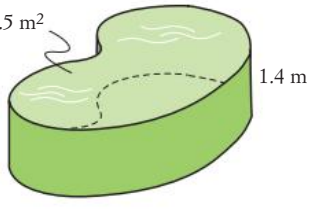
7 Find the volume of each solid, rounded to an appropriate number of significant figures.

a

b



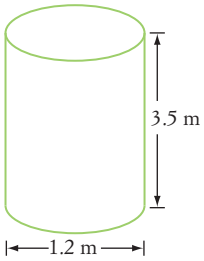
c

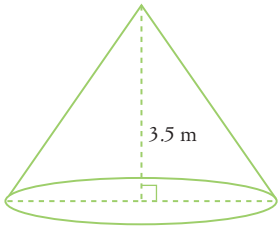


8 Find the surface area of the cylinder in question **7b**, rounded to an appropriate number of significant figures.

Sample HSC problem

A cylindrical hole 3.5 m deep and 1.2 m wide is dug for a well. The dirt removed to make the hole is put into a conical heap with the same height as the well.





Find, correct to two significant figures:

a the surface area of the hole, including the base

b the capacity of the hole, in litres

c the diameter of the base of the conical heap of dirt.

Study tip

Multiple-choice questions

- Multiple-choice questions test precise knowledge—one or two of the answers are usually close, but there is only one correct answer.
- Multiple-choice questions are not necessarily easy, even if they are usually worth only one mark each—do not rush through them.
- There is no easy way of choosing the correct answer—you will need to carefully work out the answer and check that it matches one of the answers given.
- You don't have to **show** working, but you will need to **do** working to find the correct answer.
- If you are not sure of the answer, make an educated guess.
- Often it is possible to eliminate one or two of the options and narrow your choice.
- Do not leave a multiple-choice question blank—even if you guess, you have a one-in-four chance of achieving the correct answer.
- Don't cross out or change an answer impulsively—you may have been right the first time!

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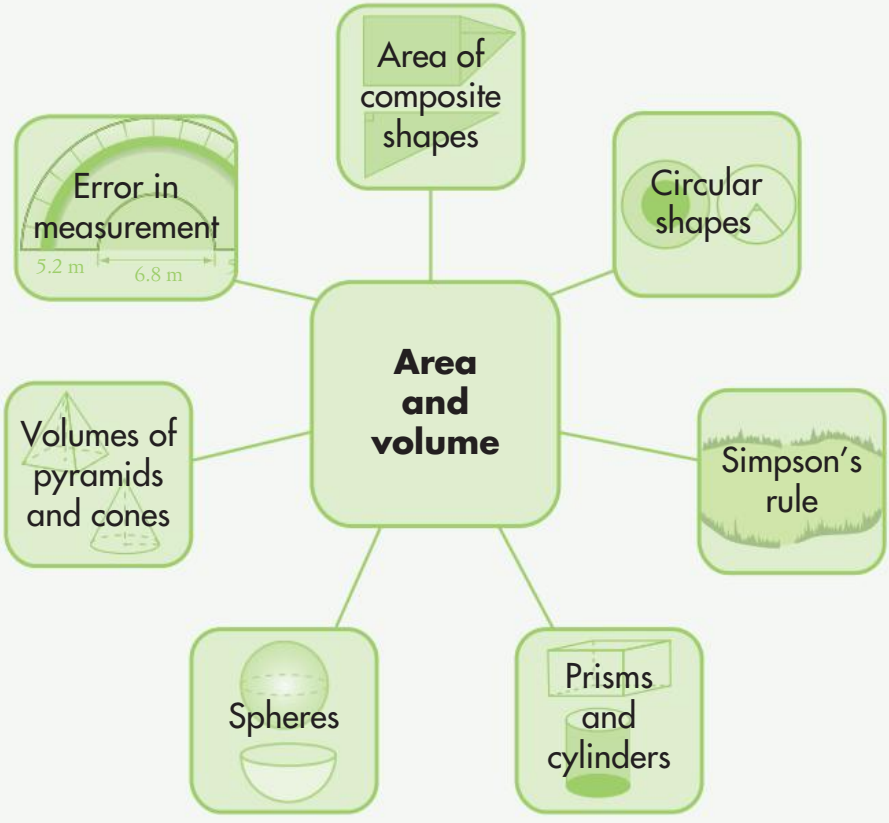
CHAPTER REVIEW

Topic overview

This chapter, Area and volume, builds on the measurement work covered in the Preliminary course. You examined the areas, surface areas and volumes of more complex and composite shapes—sectors, annuluses, prisms, cylinders and spheres—and the volumes of pyramids and cones. Simpson's rule for finding an approximate area was applied to irregular shapes using one or two applications of the formula (using 3 or 5 values). Percentage error and errors in area and volume calculations were also covered. There are many new formulas in this chapter, so be sure to include them in your topic summary along with appropriate diagrams and examples. Aim to become competent at calculating the surface area and volume of solid shapes, including composite solids.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 3-01 Exercise 3-10

1 The side of a square courtyard is measured to be 5.8 m.

a Find the absolute error of this measurement.

b Find, correct to two decimal places, the percentage error of this measurement.

c Between what two values does the actual area of the courtyard lie?

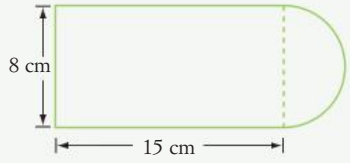
d Calculate the area of the courtyard, correct to an appropriate number of significant figures.

e The courtyard is concreted to a depth of 5 cm. Calculate the volume of concrete used, in cubic metres.

Exercise 3-02

2 Find the area of each shape, correct to two significant figures.

a



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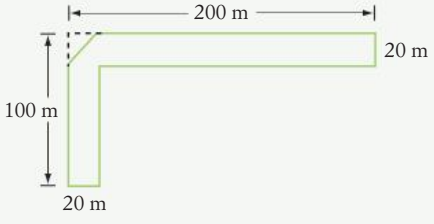


Exercise 3-03

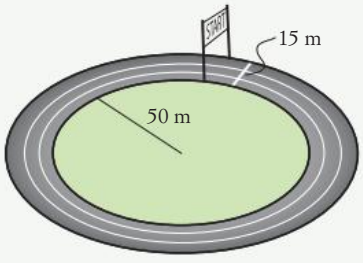
3 Calculate, correct to one decimal place,

a the area of the sector

b the length of the arc.

Exercise 3-03

4 An athletics field consists of a grass area in the shape of a circle, surrounded by a running track 15 m wide. The grass area has a radius of 50 m.

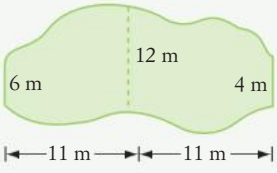


a What is the cost of fertilising the grass area at \$0.95/m^2?

b A special rubber compound is to be laid on the running track. What is the area of the track, correct to two significant figures?

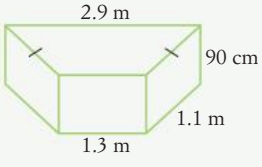
Exercise 3-04

5 Use Simpson's rule to find the area of this irregular field, correct to one decimal place.



Exercise 3-05

6 Blocks of concrete are used in the construction of a sports stadium. One of the blocks is shown in the diagram. Calculate, correct to one decimal place,



a its surface area **b** its volume.

Exercise 3-06

7 A large stainless steel cylinder for storing wine is 7 m high and has a diameter of 2.8 m. How many full 750 mL bottles of wine can be filled from this container?

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Exercise 3-07

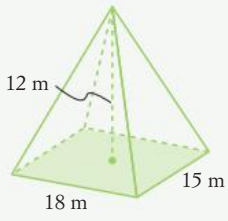
8 A Geography classroom has a world globe of diameter 40 cm. Find, correct to two decimal places,

i its surface area **ii** its volume.

Exercise 3-08

9 Find the volume of each solid, correct to two significant figures.

a



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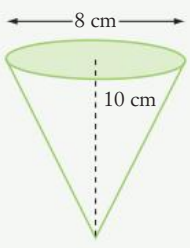
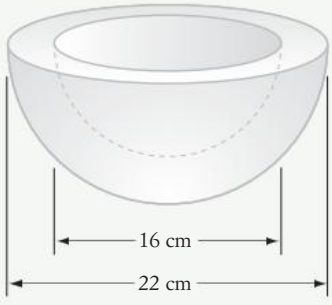


Exercise 3-09

10 Rosemary is having a party and decides to make an 'ice bowl' to hold punch. She freezes water in the space between two hemispherical bowls of diameter 16 cm and 22 cm. Find, correct to two decimal places,

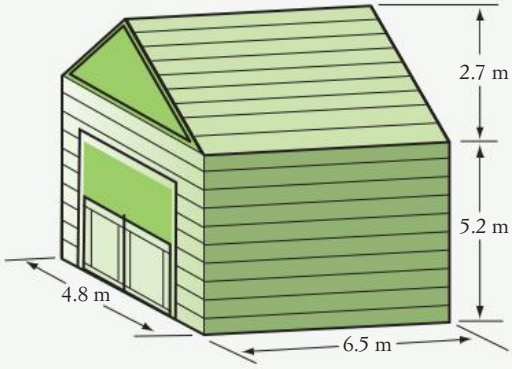
a the volume of ice used to make the bowl

b how many litres of punch the 'ice bowl' will hold.



Exercise 3-09

11 Find the volume of this barn.

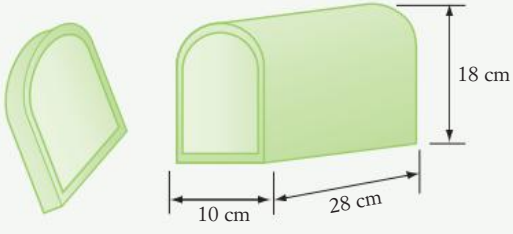


Exercise 3-09

12 A 'hi-top' loaf of bread is cut into thick slices in the shape of a rectangle combined with a semicircle. Find, correct to two significant figures,

a the volume of the loaf

b the length of crust on one slice of bread.



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Practice paper 1

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Exercise 2-01

Question 1

Simplify $4ab + b^2 - 3ab - 4b^2$.

(A) $-2ab^3$

(B) $ab - 3b^2$

(C) $7ab - 3b^2$

(D) $ab - 5b^2$

Exercise 1-01

Question 2

Meg borrowed \$7500 for 3 years at 9.5% p.a. flat interest to buy a jet ski. What amount does she repay altogether?

(A) \$2137.50

(B) \$2250

(C) \$9637.50

(D) \$9750

Exercise 2-03

Question 3

Simplify $4t^5 \times \frac{1}{2}t^3$.

(A) $2t^{15}$

(B) $8t^8$

(C) $2t^8$

(D) $8t^{15}$

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Question 4

Exercise 1-06

This table shows the future value of an annuity with a contribution of $1 at the end of each period.

Period	Interest rate per period			
	3%	5%	8%	12%
5	5.3091	5.5256	5.8666	6.3528
8	8.8923	9.5491	10.6366	12.2997
10	11.4639	12.5779	14.4866	17.5487
12	14.1920	15.9171	18.9771	24.1331

Use the table to calculate the future value of an annuity with a contribution of $8000 per year at 5% p.a. for 8 years.

(A) $95 491

(B) $76 392.80

(C) $112 000

(D) $46 392.80

Question 5

Exercise 3-01

A bridge length was measured to be 35.6 m. What is the percentage error of this measurement, correct to the same number of significant figures as in the measurement?

(A) ±0.3%

(B) ±0.140%

(C) ±0.14%

(D) ±0.28%

Question 6

Exercise 2-04

Solve $5 - 2x = -6$.

(A) $x = -5\frac{1}{2}$

(B) $x = 5\frac{1}{2}$

(C) $x = -\frac{1}{2}$

(D) $x = \frac{1}{2}$

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Exercise 2-08

Question 7

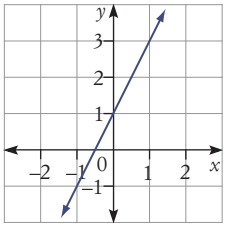
R is directly proportional to x. When $x = 12$, $R = 30$. What is the value of R when $x = 5$?

(A) 20 (B) 12 (C) 12.5 (D) 2.5

Exercise 2-07

Question 8

What is the equation of this line?



(A) $y = -3x + 1$

(B) $y = 3x + 1$

(C) $y = -2x + 1$

(D) $y = 2x + 1$

Exercise 1-05

Question 9

Calculate the compound interest earned when \$8600 is invested at 5% p.a. compounded monthly for 3 years.

(A) \$1355.58

(B) \$1388.66

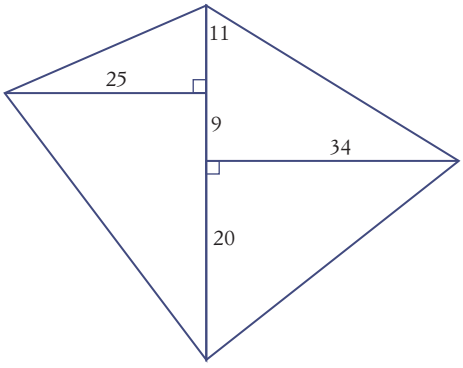
(C) \$8707.95

(D) \$9955.58

Exercise 3-02

Question 10

Calculate the area of this field in square metres. Dimensions are in metres.



(A) 2360 m^2

(B) 590 m^2

(C) 1180 m^2

(D) 915m^2

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Section 2

3 questions: 10 marks each

Question 11

Marks

Exercise 1-03

(a) This table shows the progress of a $70 000 reducing balance loan at 11% p.a. interest. The monthly repayment is $840.

Loan table				
Amount borrowed	$70 000.00			
Interest rate (% p.a.)	11			
Monthly repayment (R)	$840			
Month (n)	Principal (P)	Interest (I)	$P + I$	$P + I - R$
1	$70 000.00	$641.67	$70 641.67	$69 801.67
2	$69 801.67	$639.85	$70 441.52	$69 601.52
3	$69 601.52	$638.01	$70 239.53	$69 399.53
4	$69 399.53	$636.16	$70 035.69	$69 195.69
5	$69 195.69	$634.29	$69 829.98	$68 989.98

(i) Show how the interest of $636.16 for the fourth month is calculated. **1**

(ii) How much of the loan is left after the fourth month? **1**

(iii) What is the total interest paid in the first 4 months? **1**

(iv) How much will be owing at the end of the sixth month? **2**

Exercise 3-08

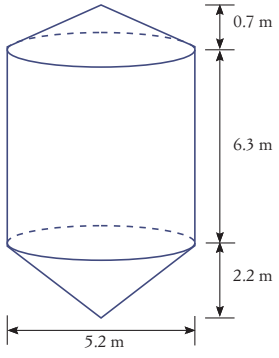
(b) Kay has a credit card account with no interest-free period and an interest rate of 13% p.a. She makes the following purchases for the period 1 January to 31 January. **3**

8 Jan	Popcorn maker	$63.75
11 Jan	Books	$49.90
15 Jan	Petrol	$24.50
21 Jan	Desk lamp	$37.45
28 Jan	Phone bill	$45.35

Calculate the interest charged on purchasing the popcorn maker that is due on 31 January. **2**

Exercise 3-09

(c) This grain hopper is made up of a cylinder and two cones. It has a diameter of 5.2 m. Calculate the volume of the grain hopper, correct to two significant figures. **3**



[Volume of a cone $V = \frac{1}{3}\pi r^2 h$]

[Volume of a cylinder $V = \pi r^2 h$]

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Exercise 2-06

Exercise 3-10

Exercise 1-01

Exercise 3-04

Question 12 **Marks**

(a) Make b the subject of the formula: $A = \frac{1}{2}bh$ **1**

(b) A horseyard is a square with sides of length 35 m.

(i) What is the absolute error of this measurement? **1**

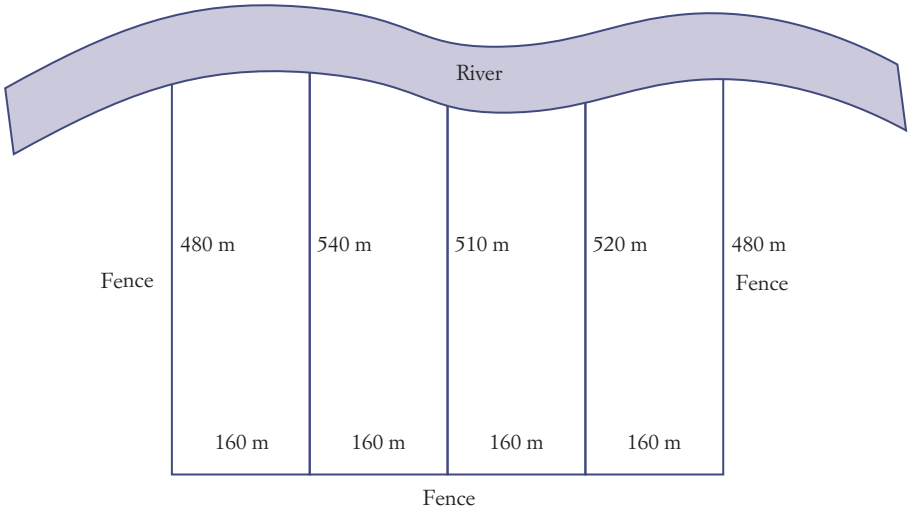
(ii) Between what two values does the area of the horseyard lie? **2**

(c) Chrissie borrowed \$12 000 to buy a car using a flat rate loan at 8.75% p.a. interest over 4 years.

(i) Calculate the total interest paid. **1**

(ii) Calculate Chrissie's monthly repayment to the nearest cent. **2**

(d) The diagram below shows a block of land bounded by a river and three fences.



Use two applications of Simpson's rule to find the approximate area of the block, correct to the nearest square metre.

$$[A = \frac{h}{3}(d_f + 4d_m + d_l)]$$ **3**

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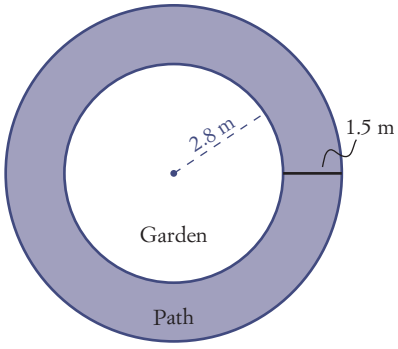
Question 13 **Marks**

Exercise 3-03

(a) A path 1.5 m wide surrounds a circular garden of radius 2.8 m, so it has the shape of an annulus.



(i) What is the area of the path in square metres, correct to one decimal place. **1**

(ii) How many whole cubic metres of soil do you need to buy to fill the garden to a depth of 80 cm? **2**

Exercise 2-09

(b) The cost of manufacturing yo-yos is given by the function $C = 3n + 90$ where n is the number of yo-yos made and C is the cost in dollars. The revenue from selling yo-yos is given by the function $R = 6n$ where n is the number of yo-yos sold and R is the sales total in dollars.

(i) Graph both functions on the same set of axes for values of n from 0 to 50. **1**

(ii) Write the coordinates of the point of intersection of the two graphs. This is called the **break-even point**. **1**

(iii) Calculate the cost of producing 25 yo-yos. **1**

(iv) State whether the point where $n = 25$ is above or below the break-even point and describe what would happen if the factory produced 25 yo-yos. **1**

Exercise 2-10

(c) Solve these simultaneous equations:

$x + 3y = 14$

$x - y = 10$ **3**

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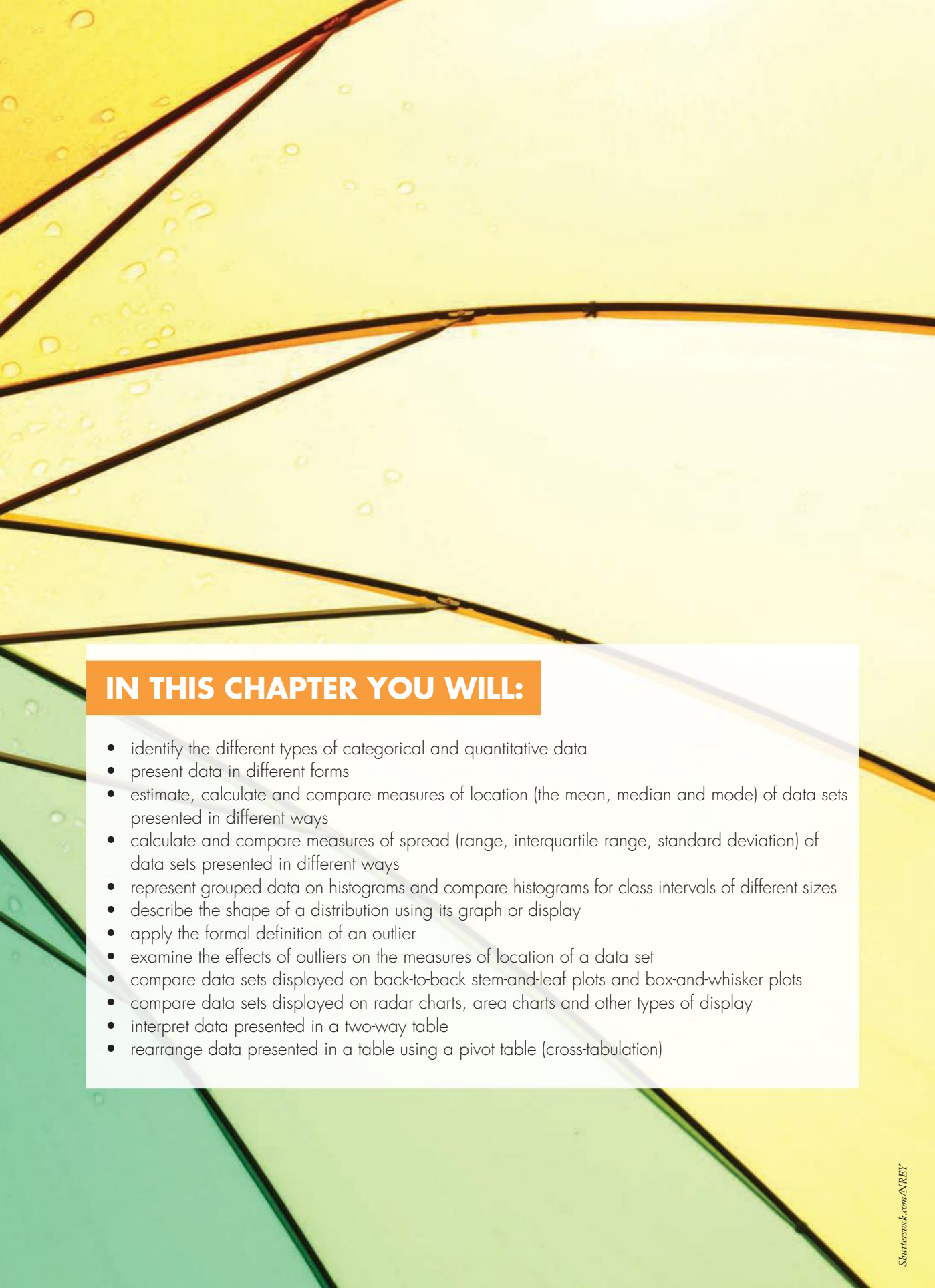
4.

STATISTICAL DISTRIBUTIONS

Does Melbourne have more rainy days than Sydney? Do a greater number of people use the Internet more in the city than in the country? How old are the people of Australia, and how has the distribution of age groups changed over the past century? These questions can be answered by comparing data sets, in particular, looking at the shapes of their graphs and the measures of location and spread.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- identify the different types of categorical and quantitative data
- present data in different forms
- estimate, calculate and compare measures of location (the mean, median and mode) of data sets presented in different ways
- calculate and compare measures of spread (range, interquartile range, standard deviation) of data sets presented in different ways
- represent grouped data on histograms and compare histograms for class intervals of different sizes
- describe the shape of a distribution using its graph or display
- apply the formal definition of an outlier
- examine the effects of outliers on the measures of location of a data set
- compare data sets displayed on back-to-back stem-and-leaf plots and box-and-whisker plots
- compare data sets displayed on radar charts, area charts and other types of display
- interpret data presented in a two-way table
- rearrange data presented in a table using a pivot table (cross-tabulation)

Statistics crossword

TERMINOLOGY

area chart
clustered column graph
measure of location
mode
pivot table
sample
smoothness
symmetry
box-and-whisker plot
interquartile range
measure of spread
negatively skewed
positively skewed
sample mean
standard deviation
tail
cluster
mean
median
outlier
radar chart
skewness
standardised score
two-way table

Assignment 4

Statistics review

Statistical measures

SkillCheck

1 For each data set below calculate:

i the mean **ii** the median **iii** the mode

a 2 3 4 6 7 **b** 3 5 6 7 7 8 9 12 **c** 1 2 2 3 4 4 5 **d** 8 3 5 2 0 7 9 5 4

2 For each data set below calculate:

i the range **ii** the interquartile range **iii** the standard deviation

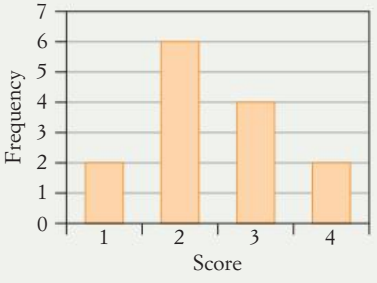
a 3 4 6 7 8 9 **b** 7 2 8 3 5 9 **c** 7 7 8 9 10 11 12 **d** 3 4 5 7 7 8 8 8

3 Find the mean of each data set.

a

x	f
10	9
11	8
12	3
13	7

b



4 Find the mode of each data set.

a

b

3	1 2 4 4 5
4	0 3
5	3 5 5 5 7
6	8 9
7	3 3

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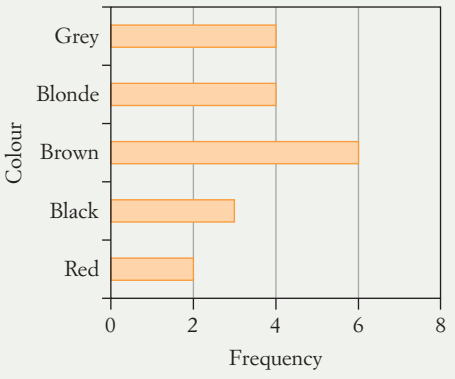
c



d

Make of car	Frequency
Holden	56
Ford	47
VW	24
Subaru	32
Toyota	47

4-01 Collecting and presenting data

Types of data

- **Categorical data** are represented by categories, usually in words or symbols and can be nominal or ordinal
  - **nominal data** is categorical data that cannot be ordered, such as the type of pet you own
  - **ordinal data** is categorical data that can be ordered, such as shirt size – small, medium, large
- **Quantitative data** are represented by quantities or numbers and can be **discrete** or **continuous**
  - **discrete data** can be counted and are distinct, separate values, such as the number of computers in schools
  - **continuous data** are measured on a smooth scale, such as the weights of footballers

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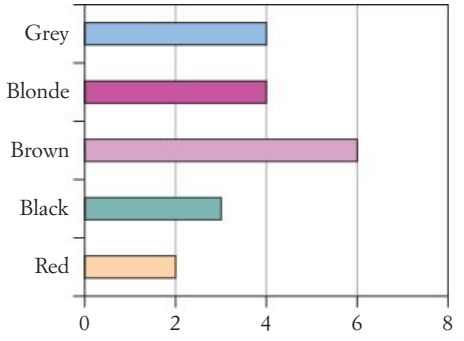
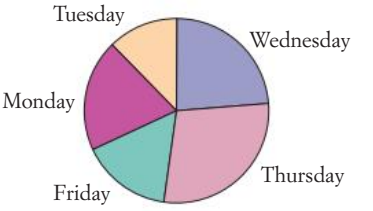
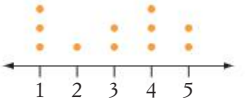
Presenting data

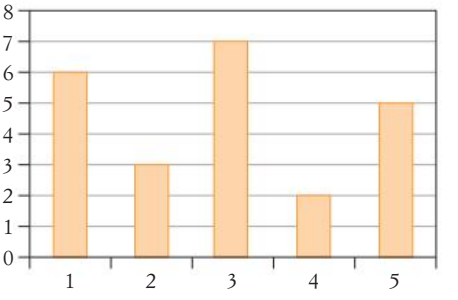
Categorical data is best displayed in a **column graph**, **divided bar graph** or **sector graph**.





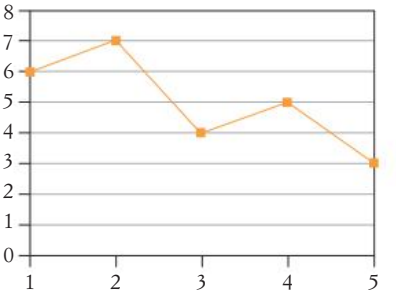
Quantitative discrete data is best displayed in a **column graph**. For small data sets, use a **dot plot** or **stem-and-leaf plot**.

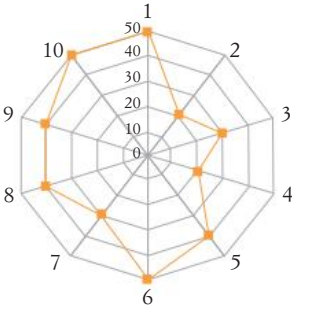




Stem	Leaf
0	3 5
1	2 6 7
2	0 3 3
3	1 9
4	2 3

Quantitative continuous data is best displayed on a **line graph** or a **radar chart**.





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EXAMPLE 1

For each data set described below, state:

- i the type of data: categorical nominal, categorical ordinal, quantitative discrete or quantitative continuous
- ii a graph that could be used to display the data.

a The numbers of people attending different sporting events at the Olympic Games.

b The types of breakfast cereal on sale in supermarkets.

c The body temperature of a hospital patient over a 24-hour period.

Solution

a Olympic Games crowd sizes

- i Quantitative discrete (the data are distinct whole numbers)
- ii Column graph (each column represents the number attending a particular sporting event)

b Breakfast cereals

- i Categorical nominal (brands of breakfast cereal are words)
- ii Sector graph or divided bar graph

c Body temperatures of a patient

- i Quantitative continuous (temperature is represented by numbers measured on a smooth scale)
- ii Line graph or radar chart

Exercise 4-01 Collecting and presenting data

Example 1

1 What type of data are the heights of players in a softball team? Select **A**, **B**, **C** or **D**.

A categorical nominal **B** categorical ordinal

C quantitative discrete **D** quantitative continuous

2 State whether each of the following data are categorical nominal (CN), categorical ordinal (CO), quantitative discrete (QD) or quantitative continuous (QC).

a temperature of water in a swimming pool

b star rating of a washing machine

c number of people who voted for The Greens in the last election

d response time when a patient's reflexes are tested

e breeds of dog

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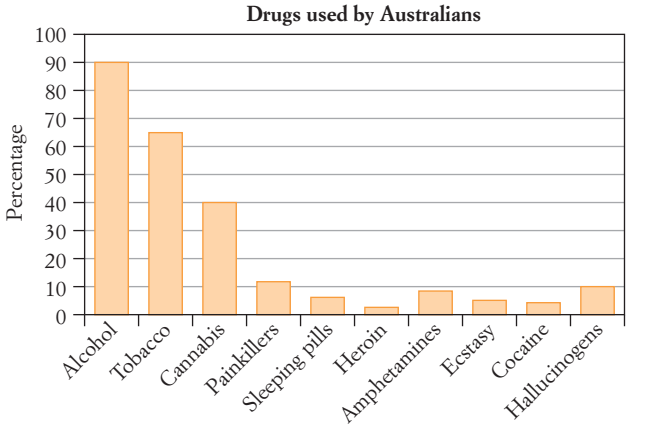
3 For each graph:

i state the type of display **ii** describe the type of data displayed

iii comment briefly on the strengths and weaknesses of the display.

a



b

4 The following results are the number of hours spent studying per week by HSC students in the week before their exams.

7	6	10	14	20	40	34	10	35	38
3	12	15	27	26	42	35	38	40	6
23	43	12	46	40	38	39	30	28	32
40	41	39	22	27	26	25	32	33	35
38	6	10	14	38	43	29	31	32	46
31	34	21	18	12	23	14	6	9	23

a Organise the data into a grouped frequency distribution table using the class intervals 0–9, 10–19, ..., 40–49.

b Construct a frequency histogram for this data.

c Now organise the data into a grouped frequency distribution table using the intervals 0–5, 6–11, 12–17, 18–23, ..., 42–47 and construct a frequency histogram.

d Compare the histograms for the different class intervals. Comment on the strength and weakness of each display.

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5 The blood alcohol content (BAC) of 50 drivers who tested positive when breathalysed are as follows.

0.001	0.030	0.015	0.002	0.008	0.002	0.014	0.040	0.031	0.028
0.011	0.003	0.051	0.042	0.005	0.006	0.008	0.021	0.036	0.041
0.033	0.011	0.002	0.036	0.054	0.037	0.008	0.024	0.049	0.060
0.055	0.058	0.003	0.017	0.044	0.059	0.023	0.064	0.030	0.035
0.065	0.004	0.028	0.031	0.009	0.010	0.020	0.052	0.036	0.058

a Organise the data into a grouped frequency distribution using the class intervals $0.00 - < 0.01$, $0.01 - < 0.02$, $0.02 - < 0.03$, $0.03 - < 0.04$, $0.04 - < 0.05$, $0.05 - < 0.06$, $0.06 - < 0.07$.

b Construct a frequency histogram for this data.

c A zero BAC applies to all learner drivers, Provisional 1 and Provisional 2 drivers and all visiting drivers holding an equivalent licence.

BAC under 0.02 applies to drivers of vehicles of gross mass greater than 13.9 tonnes, drivers of vehicles carrying dangerous goods and to drivers of public vehicles (taxi, bus).

A BAC under 0.05 applies to all other licences not subject to a zero or 0.02 BAC limit.

i How many drivers were under the 0.02 limit?

ii What percentage of drivers are definitely 'over the limit'?

iii What percentage of drivers could be 'over the limit'? Give reasons.

iv 30% of the drivers who had a BAC reading of under 0.02 were learner drivers or provisional drivers. How many will lose their licence?

d Tori decides to use the class intervals $0.00 - < 0.02$, $0.02 - < 0.05$, $0.05 - > 0.05$.

i Why do you think Tori organised the data in this manner?

ii Give one advantage and one disadvantage of Tori's way of organising the data.

4-02 Measures of location

A **measure of location** is an **average** used to indicate the centre of a **data set**. The mean or median best describe the centre of **quantitative** data whereas the mode is the only central measure of **categorical** data.

Statistical measures

SUMMARY

- **Mean**, $\bar{x} = \dfrac{\text{sum of scores}}{\text{number of scores}} = \dfrac{\Sigma x}{n}$ or $\dfrac{\Sigma fx}{\Sigma f}$
- **Median** = the middle score (or average of the two middle scores) when scores are arranged in order
- **Mode** = the most common score or category. A data set can have more than one mode or no mode at all.

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EXAMPLE 2

For each data set below, find:

i the mean (correct to one decimal place) **ii** the median **iii** the mode

a

Score (x)	Frequency (f)
2	2
3	4
4	1
5	3
6	5

b

Stem	Leaf
0	3 5
1	2 6 7
2	0 3 3
3	1 9
4	2 3

Solution

a The data set has 15 scores.

Sum of f column.

i $$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{2\times2 + 3\times4 + 4\times1 + 5\times3 + 6\times5}{15}$$
$$= 4.333\ldots$$
$$\approx 4.3$$

ii The scores arranged in order are:

2 2 3 3 3 3 4 5 5 5 6 6 6 6 6

Median

Median $(Q_2) = 5$

The middle (8th) score.

iii Mode = 6

The most common score.

b The data set has 12 scores.

i $$\text{Mean } \bar{x} = \frac{3+5+12+16+\ldots+42+43}{12}$$
$$= \frac{274}{12}$$
$$= 22.8333\ldots$$
$$\approx 22.8$$

ii The scores arranged in order are:

3 5 12 16 17 20 23 23 31 39 42 43

$$\text{Median} = \frac{20+23}{2}$$
$$= 21.5$$

The average of the 6th and 7th scores.

iii Mode = 23

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Exercise 4-02 Measures of location

Example 2

1 The test scores of a sample of Biology students are shown in the stem-and-leaf plot. What is the median of the data? Select **A**, **B**, **C** or **D**.

Stem	Leaf
5	0 1 2
6	3 4
7	1 5
8	6
9	1 3

A 7 **B** 64
C 71 **D** 67.5

2 The hair colour of 75 people was recorded.

Hair colour	No. of people
Brown	16
Blonde	25
Black	14
Red	7
Other	13

a What is the modal hair colour?

b Why is the mode the only valid measure of location here?

3 The households on a street were surveyed on the number of computers owned, and the results are shown in the frequency table. What is the mean number of computers owned per household on that street? Select **A**, **B**, **C** or **D**.

No. of computers	Frequency
2	2
3	4
4	1
5	3
6	5

A 4.3 **B** 4.0
C 3.3 **D** 3.0

4 Calculate the mean, median and mode of each data set.

a Cost of two concert tickets

$300, $400, $600, $440, $300, $700, $250, $580, $260

b Masses of large rocks

165 kg, 146 kg, 178 kg, 190 kg, 158 kg, 147 kg, 201 kg, 194 kg

5 Joshua swam 1 km each morning for 10 days to prepare for a swimming carnival. His times (in minutes) were:

28 24 22 24 25 24 26 26 24 27

a What is his median swim time?

b What is his mean swim time?

6 The ages of adults and children at a playground are shown below.

Adults:	26	22	34	39	21	33	24	42		
Children:	6	8	11	9	10	8	11	12	6	7

a What is the median adult age?

b What is the mean adult age?

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7 Thirty households were surveyed on the number of cars they owned. The results are shown in the table. Find:

a the mean number of cars per household

b the median number of cars per household.

Number of cars	Frequency
0	2
1	11
2	12
3	3
4	2

8 The table shows the marks obtained by students in a quiz.

a How many students completed the quiz?

b Copy and complete the frequency table and then calculate the mean.

c Find the median.

Mark in quiz (x)	Frequency (f)	fx
3	2	
4	4	
5	6	
6	9	
7	14	
8	9	
9	5	
10	1	

9 Students in Year 10–12 who had casual work were asked how many hours they worked per fortnight. The results are presented in the grouped frequency table below.

Hours worked per fortnight	Class centres (x)	Frequency (f)	fx
2 – < 4		8	
4 – < 6		11	
6 – < 8		15	
8 – < 10		21	
10 – < 12		17	
12 – < 14		12	
14 – < 16		8	
16 – < 18		7	
18 – < 20		1	

a Copy and complete the frequency distribution table.

b How many students were surveyed?

c Find the mean number of hours worked.

d What is the median number of hours worked by students?

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10 The table shows the number of goals per game scored by the Newcastle United Jets football team in the 2011–12 season.

Goals scored per game	Cumulative frequency
0	6
1	17
2	20
3	24
4	24
5	25

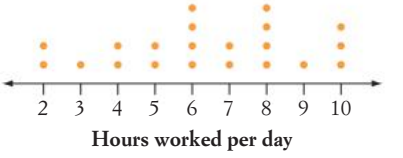
a How many games were played?

b In how many games did the Newcastle United Jets score 2 goals?

c What is the median number of goals per game?

d Calculate the mean number of goals scored per game.

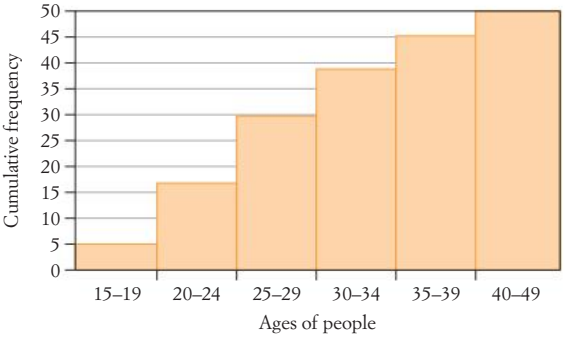
11 The number of hours per day that Michelle worked for three weeks before Christmas are shown in the dot plot below.



a Find the median.

b Calculate the mean number of hours worked per day by Michelle.

12 The ages of people attending the Keep Fit and Stay Fit centre from 6 a.m. to 8 a.m. on a Saturday morning are shown in this cumulative frequency histogram.



a How many people went to the centre from 6 a.m. to 8 a.m.?

b How many people aged 30–34 attended the centre on a Saturday morning?

c What was the median age of the people attending the fitness centre?

d What is the modal age of people attending?

4-03 Measures of spread

A **measure of spread** is used to indicate how spread out a data set is.

Statistical calculations

Statistical measures

SUMMARY

- **Range** = highest score – lowest score
- **Interquartile range (IQR)**
  = upper quartile – lower quartile
  = $Q_3 - Q_1$

  This is the range of the middle 50% of scores.
- **Standard deviation**, σ_n, is found using a calculator's statistics mode

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The **standard deviation** is the best measure of spread because its value, like the mean, depends on every score in the data set. The larger the standard deviation, the more spread out the scores are.

EXAMPLE 3

For each data set below, find:

i the range **ii** the interquartile range

a

Score (x)	Frequency (f)
15	1
16	5
17	4
18	3
19	2

b

Stem	Leaf
0	4 5 8 9
1	2 2 5
2	0 3 8 8 8
3	1 6 7 9
4	0 5

Solution

a The data set has 15 scores.

The scores arranged in order are:

15 16 16 16 16 16 17 17 17 17 18 18 18 19 19

Q_1 Q_2 Q_3

i Range = 19 – 15 = 4

ii Interquartile range = $Q_3 - Q_1$
= 18 – 16
= 2

b The data set has 18 scores.

Stem	Leaf
0	4 5 8 9
1	2 2 5
2	0 3 8 8 8
3	1 6 7 9
4	0 5

Q_1, Q_2, Q_3

i Range = 45 – 4 = 41

ii Interquartile range = $Q_3 - Q_1$
= 36 – 12
= 24

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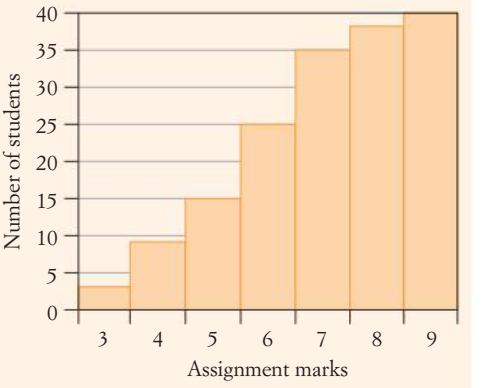
EXAMPLE 4

The results of assignments completed by Year 12 students at Baramvale High are shown in the cumulative frequency histogram.



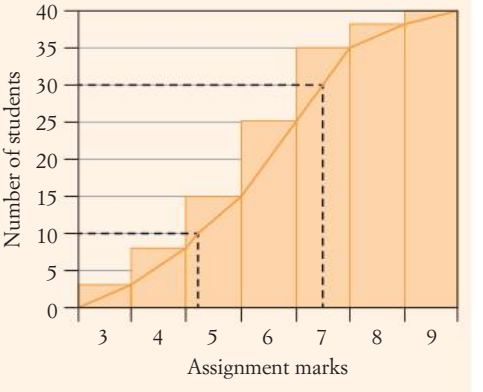
a How many students completed the assignment?

b What was the range?

c Find the interquartile range.

Solution

a 40 students completed the assignment.



b Range $= 9 - 3 = 6$

c Interquartile range $= Q_3 - Q_1$

$= 7 - 5$

$= 2$

Mean and standard deviation from a calculator

EXAMPLE 5

Statistics mode: Graphics calculator

The net weekly wages of 8 labourers are: \$730 \$490 \$600 \$440 \$490 \$370 \$700 \$580

Use your calculator to show that:

a the mean wage is \$550

b the standard deviation is \$117.26.

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Solution

Follow the instructions for the statistics mode (SD or STAT) of your calculator as shown in the tables below.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR This also clears the statistical memory.	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 730 = 490 =, etc. to enter in column AC to leave table	730 M+ 490 M+, etc.
Calculate the mean. ($\bar{x}$ = 550)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Calculate the standard deviation. (σ_n = 117.26)	SHIFT 1 Var $x\sigma_n$ =	RCL σx
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and pressing CLEAR. STAT EDIT
Clear the statistical memory.	With the cursor in List 1 column EXIT F6 DEL-A Yes	With the cursor on L1 CLEAR ENTER
Enter data.	730 EXE 490 EXE etc, in List 1 F6 CALC SET to set (if required) 1Var Xlist: List 1 1Var Freq: 1 EXE	370 ENTER 490 ENTER etc. in List 1
Calculate the mean ($\bar{x}$ = 550) and standard deviation (σ_n = 117.26).	1VAR to calculate $\bar{x}$ and $x\sigma_n$ (scroll down for more statistics)	STAT CALC 1-Var Stats ENTER to calculate $\bar{x}$ and σx (scroll down for more statistics)

EXAMPLE 6

Twenty echidnas from Booderee National Park were tagged and returned to their habitat. Rangers later captured several samples of 10 echidnas over a 2-month period and recorded the number tagged in each sample.

Find, correct to two decimal places,

a the mean number of tagged echidnas per sample

b the standard deviation of tagged echidnas.

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No. of echidnas tagged per sample	Frequency
0	8
1	11
2	5
3	4
4	2
5	1

Solution

For data presented in a frequency table, follow the instructions for your calculator as shown in the tables below for the following answers:

a Mean $\bar{x} \approx 1.48$ **b** Standard deviation $\sigma_n \approx 1.34$

Note: You can also find the mean by adding an '*fx*' column to the table, calculating the totals of the Frequency and *fx* columns, and dividing them using the formula $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 0 = 1 =, etc. to enter in x column 8 = 11 =, etc. to enter in FREQ column AC to leave table	0 2ndF STO 8 M+ 1 2ndF STO 11 M+, etc.
Calculate the mean ($\bar{x}$ = 1.4838...)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Calculate the standard deviation (σ_n = 1.34091...)	SHIFT 1 Var $x\sigma_n$ =	RCL σx

Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and CLEAR, STAT EDIT
Clear statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2 column	With cursor on L1 CLEAR ENTER Repeat for L2
Enter scores in List 1.	0 EXE 1 EXE, etc.	0 CLEAR 1 ENTER, etc.

Enter frequencies in List 2.	8 EXE 11 EXE, etc. CALC SET to set (if required) 1Var Xlist: List 1 1Var Freq: List 2 EXE	8 ENTER 11 ENTER, etc.
Calculate the mean ($\bar{x}$ = 1.4838…) and standard deviation (σ_n = 1.3409…)	1VAR to calculate $\bar{x}$ and $x\sigma_n$ (scroll down for more statistics)	STAT CALC 1-Var Stats and type 'L1, L2' by pressing 2nd 1 , 2nd 2 ENTER to calculate $\bar{x}$ and σx (scroll down for more statistics)

Exercise 4-03 Measures of spread

Example 3

1 For each data set find:

i the range **ii** the interquartile range

a 3 5 5 6 8 10 10 10 12 13 15 18

b 27 22 20 15 17 17 22 18 26 24 12 16 24 25

c 15 12 8 9 12 10 7 6 11 14 16

d 4 2 6 8 4 7 9 2 2 1 5 4 5 8 10

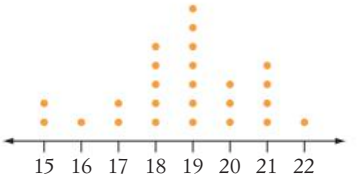
e

Score (x)	Frequency (f)
2	2
3	6
4	4
5	2
6	2

f

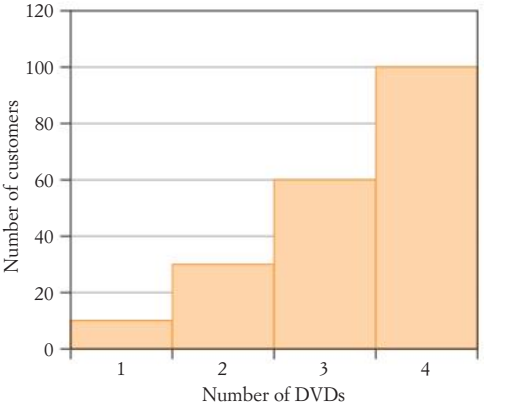
Stem	Leaf
2	0 3 4 5 8
3	3 4 6 8 9 9
4	1 2 7 8
5	3 5 6 7
6	2 7

g



Example 4

2 A video rental store surveyed a sample of customers on the number of DVDs they borrowed over a long weekend. The results are shown in this **cumulative** frequency histogram.



a How many customers were surveyed?

b What was the median number of DVDs borrowed?

c How many customers borrowed two DVDs?

d What was the mode?

e Find the interquartile range.

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3 Find the mean and standard deviation for each data set in question **1**.

Example 5

Example 6

4 The annual salaries of employees at Gen Y manufacturing company are shown in the table.

Annual salary (× $1000)	Number of employees
20 – < 30	10
30 – < 40	6
40 – < 50	4
50 – < 60	5
60 – < 70	3
70 – < 80	2

a How many people are employed at the company?

b Using **class centres** 25, 35, 45, …, find the estimated mean salary of the employees, to the nearest $100.

c What is the standard deviation, to the nearest $100?

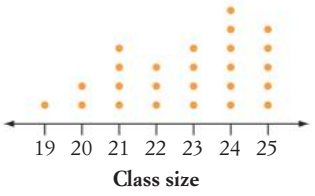
5 The numbers of stray dogs collected by the RSPCA in several suburbs are shown in the table.

No. of dogs	5	6	7	8	9	10
No. of suburbs	14	18	15	26	20	31

a What is the mean number of dogs collected per suburb?

b What is the standard deviation of the number of dogs? Answer correct to two decimal places.

6 The numbers of students in different Year 11 classes at Westvale High School are shown in the dot plot below.



a What is the mean number of students per Year 11 class?

b What is the population standard deviation, correct to one decimal place?

7 The 2011 monthly rainfall figures (in mm) for Picton, NSW are as follows.

53.4 22.2 179.5 29.8 30.4 78.6 39.8 31.8 37.8 49.6 168.4 56.4

a What is the median monthly rainfall?

b What is the mean monthly rainfall?

c What is the interquartile range of the monthly rainfall at Picton?

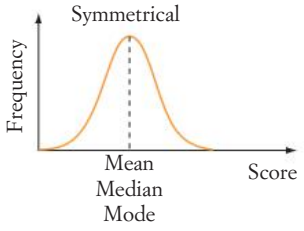
d What is the population standard deviation of the monthly rainfall?

e If you were asked to decide the most appropriate measures of location and spread for these figures, which two would you choose?

4-04 The shape of a distribution

The **shape** of a statistical **distribution** (data set) shows how the data is spread, and can be seen by drawing a curve around its graph or display.

A distribution is **symmetrical** if the data are balanced or evenly spread about the centre of the distribution, with the mean, median and mode being equal. One example of a symmetrical distribution is students' marks in an HSC examination.



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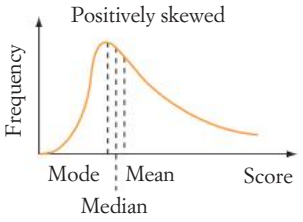
A distribution is **positively skewed** if its tail points to the right (the positive direction), because the mean is above the mode and median.

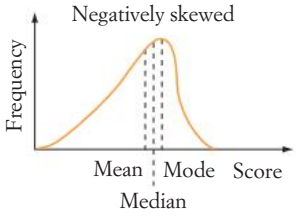
The word 'skewed' means twisted.

One example of a positively skewed distribution is house prices in a large country town.



If a distribution is **negatively skewed**, then its tail points to the left (the negative direction) because the mean is below the mode and median.

One example of a negatively skewed distribution is the heights of the players in a basketball team.



Peaks are the high points of the distribution and represent the more frequent scores. The highest peak is the mode.

Clusters are groups of scores that are bunched or close together.

EXAMPLE 7

For each distribution shown below:

i describe its shape

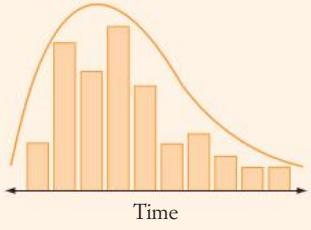
ii identify any clusters

iii state the number of peaks and modes.

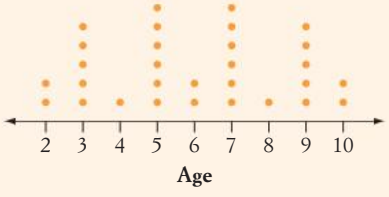
a Marks in a Japanese test

Stem	Leaf
3	1 2
4	3 5 9
5	0 2 6
6	4 5
7	3 5 6 7 7 8 9
8	0 2 4 6 6 8 8 9
9	1 2 4 8 9

b Amount of traffic on Sydney's roads



c Ages of children at a cinema



d Ages of people in a small coastal town

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Solution

a **i** negatively skewed (tail points towards lower scores)

ii clusters in the 70–90s

iii 3 peaks, 3 modes: 77, 86 and 88

b **i** positively skewed (tail points towards right)

ii clusters at earlier hours

iii 2 peaks, one mode

c **i** symmetrical

ii no clusters

iii 4 peaks, 2 modes: 5 and 7

d **i** positively skewed and smooth (tail points toward higher ages)

ii cluster from 15 to 29

iii one peak, one mode

Exercise 4-04 The shape of a distribution

Example 7

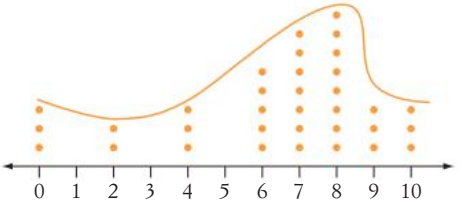
1 The dot plot below shows the judges' scores in a diving competition. Which of the following statements is true about the distribution? Select **A**, **B**, **C** or **D**.

A The data is positively skewed with a cluster around 6 to 8.

B The data is symmetrical with no modes.

C The data is negatively skewed with one mode.

D The data is positively skewed with a cluster around 0 to 4.

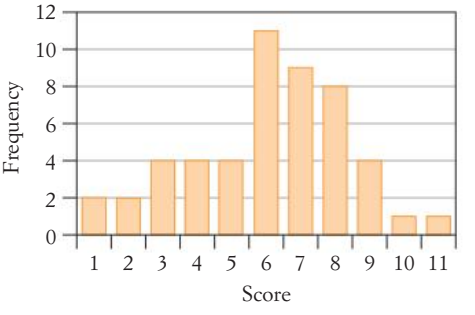


2 For each of the following distributions:

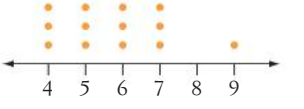
i describe its shape

ii state the number of peaks.

a



b



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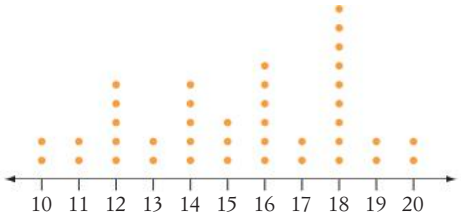
c

Stem	Leaf
1	3 4 6 6 6 7 8 9 9
2	0 7
3	1 2 2 5 7 8 8 9
4	0 2 3
5	2 9

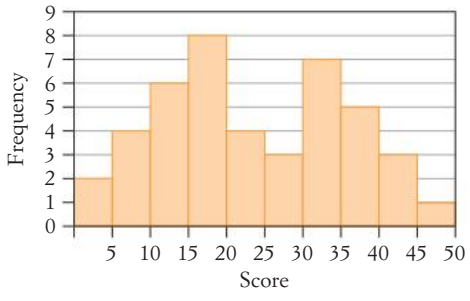
d



e

Stem	Leaf
4	1 3
5	5 5 6
6	0 3 5 5 6 8
7	2 6
8	5 5 8

f



3 This stem-and-leaf plot shows the number of mobile phones sold in January across various OzTel stores in Australia.

Stem	Leaf
2	2 6
3	0 1
4	4 8
5	2 6 9
6	1 3 4 5
7	0 2 3 4 4 5 5 7 7 7 8 9
8	3 5 7 7 8 8 8 8 9
9	2 8

a How many Oztel stores were surveyed?

b Describe the shape of the data.

c Where does the clustering occur?

d What is the mode?

4 The number of visits to the MyFace website was recorded between 1200 (noon) and 2100 (9 p.m.) one day.

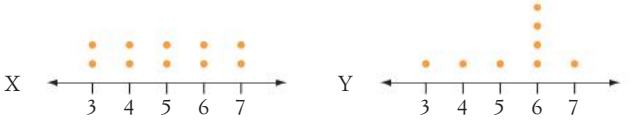
Hour	1201–1300	1301–1400	1401–1500	1501–1600	1601–1700	1701–1800	1801–1900	1901–2000	2001–2100
Hits	1300	800	400	2100	2500	4500	3900	5300	2300

a Draw a histogram to represent this data.

b Comment on the shape of your histogram, mentioning clusters and peaks.

c Suggest a possible reason for the skewness of this data.

5 Which of the following statements is true about the data sets below? Select **A**, **B**, **C** or **D**.



A Y is positively skewed.

B X does not have a mode.

C The mean of Y is 5.

D X and Y are both symmetrical.

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6 These are the ages of employees at the Berry Good Biscuit factory.

16 36 15 16 15 19 55 59 18 20 50 22 21 35 22 19 15 17 43 49

a Draw a stem-and-leaf plot for this data.

b Comment on the shape of the distribution, mentioning skewness, peaks and clusters.

7 This dot plot represents the number of accidents per month at a factory over a year.

a Comment on the shape of the dot plot.

b What is the mode?

c Calculate the mean (correct to one decimal place) and compare it to the mode.

4-05 The effect of outliers

An **outlier** is a very high or very low score in a data set that is clearly apart from the other scores. It can occur for a variety of reasons and should be investigated. If it was obtained as the result of an incorrect measurement, it should be excluded.

SUMMARY

An **outlier** is a score that is either:

- less than $Q_1 - 1.5 \times \text{IQR}$ or
- greater than $Q_3 + 1.5 \times \text{IQR}$

where Q_1 (or Q_L) is the lower quartile, Q_3 (or Q_U) is the upper quartile, and IQR is the interquartile range.

EXAMPLE 8

The following scores are marks obtained by students in a test.

11 8 12 12 15 13 10 25 12 11 7
10 13 16 10 12 16 11 12 16 17 20

Test which scores are outliers.

Solution

The scores arranged in order are:

7 8 10 10 10 11 11 11 12 12 12 12 12 13 13 15 16 16 16 17 20 25

Q_1 or Q_L Q_2 Q_3 or Q_U

$\text{IQR} = Q_3 - Q_1$
$= 16 - 11$
$= 5$

$\therefore 1.5 \times \text{IQR} = 1.5 \times 5$
$= 7.5$

$\therefore Q_1 - 1.5 \times \text{IQR} = 11 - 7.5$
$= 3.5$

$Q_3 + 1.5 \times \text{IQR} = 16 + 7.5$
$= 23.5$

$\therefore$ A score is an outlier if it is less than 3.5 or greater than 23.5.
$\therefore$ 25 is an outlier.

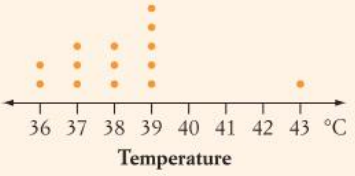
Outliers and measures of location

Outliers can affect the measures of location of a data set.

- The mean is most affected by outliers (because its value depends on every score).
- The median can be affected, but not by much.
- The mode is not affected at all.

EXAMPLE 9

The dot plot shows the temperatures of patients in a hospital ward.



a Calculate the mean, mode and median of this data set.

b What is the outlier temperature?

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the measures of location for the distribution.

Solution

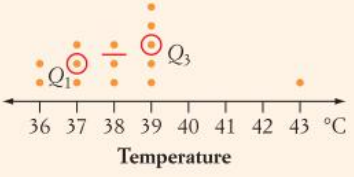
a $\text{Mean} = \dfrac{535}{14} \approx 38.2$

$\text{Mode} = 39$

$\text{Median} = \dfrac{38 + 38}{2} = 38$

b From the dot plot, $Q_1 = 37$, $Q_3 = 39$

$\text{IQR} = Q_3 - Q_1$
$= 39 - 37$
$= 2$



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$1.5 \times \text{IQR} = 1.5 \times 2$
$= 3$

If 43 is an outlier, it must be greater than $Q_3 + 1.5 \times \text{IQR}$.

$\therefore Q_3 + 1.5 \times \text{IQR} = 39 + 3$
$= 42$

$\therefore$ the outlier is 43.

c Mean $= \frac{492}{13} \approx 37.8$

Mode = 39

Median = 38

d The high outlier does not affect the mode and median but it increases the mean.

Exercise 4-05 The effect of outliers

Example 8

1 The following scores are the number of goals scored by a hockey team during a season.

3 2 0 0 1 2 3 2 4 8 2 3 5 2 1 3 4 4 2 3

a Find the interquartile range.

b Find the value of:

i $Q_1 - 1.5 \times \text{IQR}$ **ii** $Q_3 + 1.5 \times \text{IQR}$

c Is the score of 8 goals an outlier? Give reasons.

2 Determine whether the following data sets have outliers.

a 2 5 6 6 7 8 10 10 15

b 9 13 13 14 14 15 15 15 15 16 16 16 16 16 17 17 18

c

Stem	Leaf
1	2 9
2	0 3 4 4 8
3	4 5 6 7
4	1 4 9
5	0 2
6	8

d

Score (x)	Frequency (f)
4	3
5	12
6	4
7	3
8	0
9	1

Example 9

3 The employees at the Bread and Butter Cafe earned the following wages in a week.

\$450 \$520 \$610 \$230 \$900 \$420 \$590

a What is the mean wage?

b What is the median wage?

c Find the interquartile range.

d The manager's wage is an outlier. What is this wage and how do we verify that it is an outlier?

e If the manager's wage is not included, how does this affect the mean and median wage?

f If each employee receives a 10% pay rise, what will be the new mean and median wage? Is it 10% more than the old mean and median?

4 The number of cups of coffee drunk by a sample of HSC exam markers in one night is shown in the table.

Cups of coffee	No. of markers
2	1
3	4
4	5
5	9
6	0
7	0
8	1

a How many markers were surveyed?

b What is the outlier?

c What is the mean if the outlier:

i is included? **ii** is not included?

d If the outlier is included, what effect does this have on the mean number of cups of coffee that were drunk?

5 A group of friends goes to the movies. The ages of the group are

13 12 11 14 12 15 14 13.

If Sally brings her 5-year-old sister as well, which of the following will happen? Select **A**, **B**, **C** or **D**.

A The median age increases **B** The median age decreases

C The mean age increases **D** The mean age decreases

6 In a netball tournament of five matches, the points scored by three teams are:

The Wombats:	24	18	14	6	22
The Possums:	16	16	15	18	15
The Koalas:	36	8	14	16	12

a What are the mean and median scores for each team?

b Which team is the most consistent? Why?

c An error was made in the scoring for the Wombats – the score of 6 should have been 16. What are the new mean and median?

d Which team is most consistent now? Why?

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7 Pam and Percy sell copiers. The numbers of copiers that they sell each week are sorted in ascending order.

Pam:	1	2	3	3	5	6	7	8	12	25
Percy:	3	3	3	14	16	18	18	24	32	35

a What is the modal number of copiers sold by each person?

b What could you say about each person if you only knew the mode?

c What is the median number of copiers sold by each person?

d What is the mean number of copiers sold by each person?

e Which measure of location, mean, median or mode, is best for comparing their sales performances?

f Who is the better salesperson? Justify your answer.

8 This dot plot represents the number of accidents each month at a factory over a year.

a Calculate the mean, mode and median of this data set.

b What is the outlier number of accidents? Explain why.

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the three measures of location.

9 Rupert's bookstore employs the following people with annual wages as shown.

1 store manager	$73 800
2 cashiers	$34 200 each
2 part-time clerical staff	$28 500 each
3 salespeople	$46 500 each
2 part-time cleaners	$13 500 each

a Find the mean, median and modal annual salary for the 10 employees.

b Which measure of location would Rupert use to make the salaries appear higher? Why?

c Which measure of location best represents the average wage for an employee at Rupert's bookstore? Why?

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JUST FOR THE RECORD

The Challenger space shuttle disaster

In January 1986, an engineer working on the space shuttle program at NASA predicted that at low air temperatures, the potential for damage to the shuttle would be extremely high. For a temperature of 12°C, he calculated a damage index of 11. He compared this to data from previous flights (as shown in the table below) and recommended that the *Challenger* flight be delayed due to the low air temperature on the day.

Year	Data from previous flights				1986
Air temperature (°C)	26	14	19	23	12
Damage index	0	4	0	0	11

However, his advice was ignored and the outlier was not considered important enough to delay the flight. The *Challenger* exploded just after takeoff, killing all seven astronauts. Later it was found that two rubber O-rings had failed to seal a joint at low temperatures, causing the shuttle to disintegrate.

Give another example of when an outlier should not be ignored.

4-06 Comparing data sets using plots

Double stem-and-leaf plots

A double (or back-to-back) **stem-and-leaf plot** compares two data sets.

EXAMPLE 10

This double stem-and-leaf plot compares the amounts spent (in dollars) by a group of boys and girls visiting the Royal Easter show.

Boys		Girls
2 1	1	1 2 3 8 9
3 0	2	0 2 4 4 6 7 8
8 6 6 5 4 4 2 0	3	4
6 6 6 5 5 3	4	9

a What is the modal amount spent for the whole group?

b What is the mean amount, to the nearest dollar, spent by:

i boys? **ii** girls? **iii** the whole group?

c Did boys or girls have the greater range of amounts spent?

d Compare the shapes of the two data sets (boys and girls).

e On average, did boys or girls spend more? Justify your answer.

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Solution

a $46

b **i** $$\text{Mean for boys} = \frac{\$612}{18} = \$34$$

ii $$\text{Mean for girls} = \frac{\$327}{14} = \$23.35714\ldots \approx \$23$$

iii $$\text{Mean for group} = \frac{\$939}{32} = \$29.343775 \approx \$29$$

c Range for boys = $46 – $11 = $35 Range for girls = $49 – $11 = $38

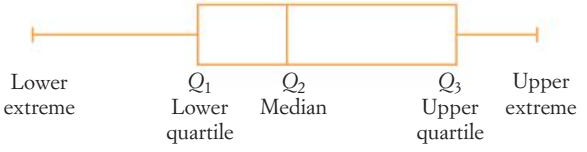
Girls had the greater range.

d The boys' spending was negatively skewed, while the girls' spending was positively skewed.

e The boys spent more, as shown by the greater mean of the boys, and in the shape of the stem-and-leaf plot.

Box-and-whisker plots

A **box-and-whisker plot** (or box plot) uses a **five-number summary** to show the spread of data.



The box contains the middle 50% of scores with each whisker representing 25% of the remaining scores.

Box-and-whisker plots

Box-and-whisker plots: Graphics calculator

EXAMPLE 11

Liz and George deliver pamphlets to letterboxes. The number of pamphlets delivered per hour over 12 hours is shown.

Liz:	24	25	26	27	28	28	31	32	32	32	35	35
George:	15	18	21	24	25	29	31	31	32	38	38	45

a Find the five-number summary for each data set and draw two box plots on the same axis.

b What are the interquartile ranges of pamphlets delivered by each person?

c What is the median number of pamphlets delivered by each person?

d Compare the two data sets, describing their shapes and measures of location and spread.

e Is Liz is a better worker than George? Give reasons for your answer.

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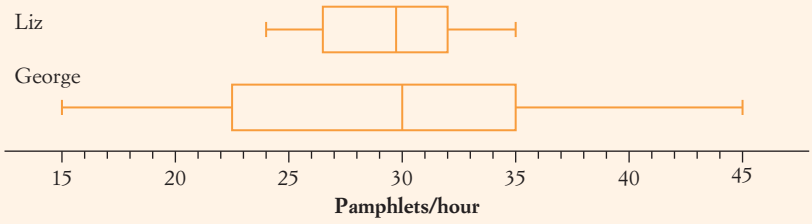
Solution

a Liz: 24 25 26 27 28 28 31 32 32 32 35 35

lower extreme $Q_1 = 26.5$ $Q_2 = 29.5$ $Q_3 = 32$ upper extreme

George: 15 18 21 24 25 29 31 31 32 38 38 45

lower extreme $Q_1 = 22.5$ $Q_2 = 30$ $Q_3 = 35$ upper extreme



b Interquartile range for Liz = 32 – 26.5 = 5.5
Interquartile range for George = 35 – 22.5 = 12.5

c Median for Liz = 29.5
Median for George = 30

d Both distributions are fairly symmetrical and have similar medians. The data set for Liz has a much smaller range and interquartile range so its spread is generally smaller.

e Liz is not necessarily the better worker because the medians of both data sets were similar. Liz is, however, more consistent as there is less spread in her delivery rate.

Example 10

Exercise 4-06 Comparing data sets using plots

1 A class was given two different Science tests and their marks are displayed in the double stem-and-leaf plot.

a Find the range, interquartile range and median for each test.

b Which Science test was easier? Justify your answer.

Test A		Test B
2	0	3
5	1	
5	2	
	3	8
8	4	5
	5	2 4 7
5	6	2
7	7	
0	8	
2	9	4

2 The weights (in kilograms) of two teams of players were recorded.

Netball players: 77 72 80 77 62 72 82 79 58 75

Hockey players: 81 86 64 74 92 75 73 81 64 52 82 79

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a Display the data in a back-to-back stem-and-leaf plot.

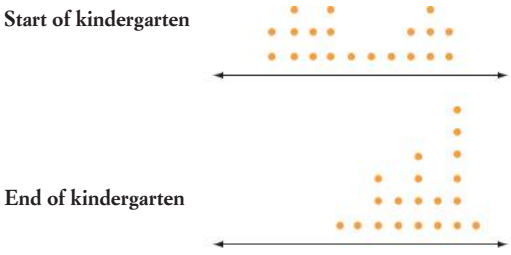
b Comment on the shape of each set of data, mentioning skewness, clusters and peaks.

c What is the mean weight of each team (correct to one decimal place)?

d What is the median weight of each team?

e Which is the better measure of location, the mean or median? State a reason for your answer.

3 The dot plots show the reading scores for a group of kindergarten students at the start and finish of the year.



Which of the following statements is true about the kindergarten students' reading scores at the end of the year? Select **A**, **B**, **C** or **D**.

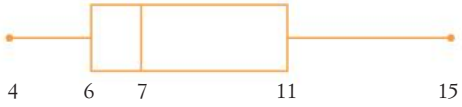
A The mean has increased and the standard deviation has decreased.

B The mean and standard deviation have both increased.

C The mean and standard deviation have both decreased.

D The mean has decreased and the standard deviation has increased.

4 What is the interquartile range of the data shown in this box-and-whisker plot? Select **A**, **B**, **C** or **D**.

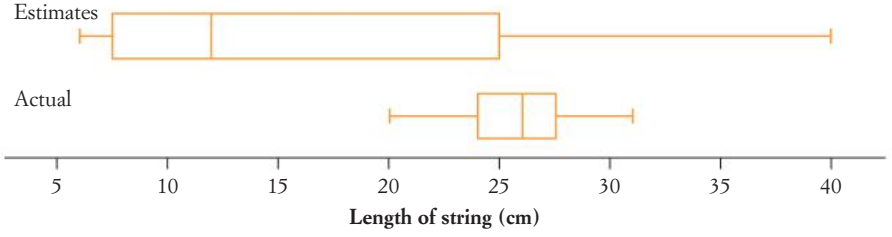


A 2 **B** 5

C 7 **D** 11

Example 11

5 Yashneel decided to investigate his theory that 'People always underestimate the length of a piece of string'. He asked students to estimate the lengths of several pieces of string. He then measured the actual lengths and displayed the results in box-and-whisker plots.



a Write down the median of:

i the estimated lengths **ii** the actual lengths.

b Compare the spreads of each data set.

c Do you agree with Yashneel's theory? Justify your answer.

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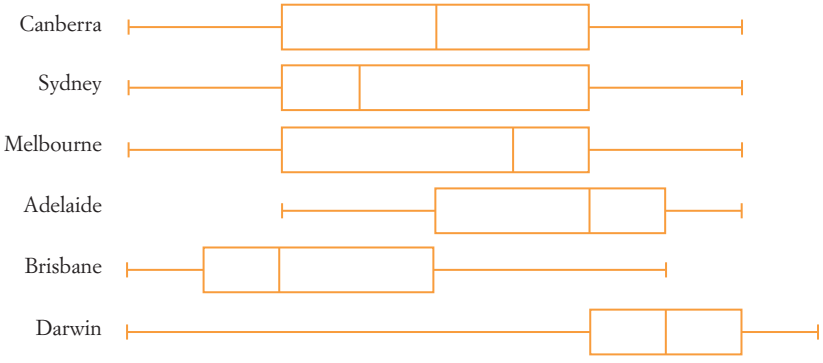
6 **a** Use the boys' and girls' data from the double stem-and-leaf plot in Example **10** on page 170 to create two five-number summaries and draw their box-and-whisker plots.

b What information is seen more easily in the box plots than in the stem-and-leaf plots?

7 The box-and-whisker plots below show the distribution of petrol prices in six Australian capital cities.



a Which city has:

i the greatest range of prices?

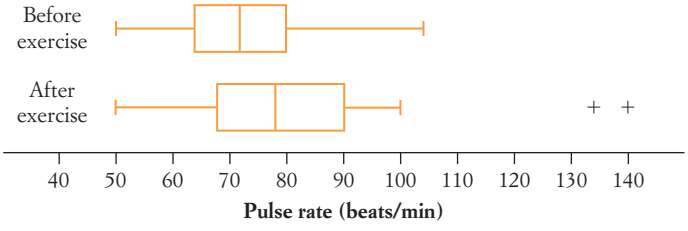
ii the lowest median price?

iii the smallest interquartile range of prices?

iv a symmetrical distribution of prices?

b Is petrol cheaper in Sydney than in Melbourne? How can you tell?

8 A group of 20 people had their pulse rates taken before and after an exercise class and the results were displayed. Two outlier pulse rates (shown as '+') were excluded from the 'After exercise' data.



a How many people had a pulse rate between 64 and 72 before the exercise class?

b The lower extreme 'before' and 'after' the class did not change. Give a possible reason for this.

c Give a possible reason for the two outlier pulse rates.

d What was the interquartile range of pulse rates after the class?

e By how much did the median pulse rate increase?

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9 Twelve people took part in the QUIT smoking program. The number of cigarettes smoked per day was recorded before the start of the program and 6 weeks later.

Before:	21	10	36	16	32	42	9	14	21	18	34	45
6 weeks later:	6	24	31	21	16	19	16	18	28	32	8	13

a Find the five-number summary for each data set and draw two box-and-whisker plots.

b What are the interquartile ranges of each data set?

c Is the QUIT program generally working for this group of people? Justify your answer.

10 This table shows the average number of rainy days per month for Sydney and Melbourne.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	12	12	13	12	12	12	10	10	10	12	11	12
Melbourne	8	7	9	12	14	14	15	16	15	14	12	11

Bureau of Meteorology

a Use a double stem-and-leaf plot to display the data.

b Draw box plots representing the data.

c Write down one fact that is clearly evident in each display.

d Does Melbourne generally have more rainy days than Sydney? Justify your answer.

TECHNOLOGY: BOX-AND-WHISKER PLOTS ON A GRAPHICS CALCULATOR

Box-and-whisker plots: graphics calculator

The statistics mode (STAT) of a graphics calculator can be used to draw and compare box-and-whisker plots. Use it to display the rainfall data from question **10** on the previous page. You **enter** the scores, **set** the graph type, **select** the graphs to turn on and then **draw** the box plots.

Follow the instructions below for a Casio graphics calculator.

1. Enter the 12 Sydney values in List 1 and the 12 Melbourne values in List 2.
2. Press **SET** and **GPH1** to set StatGraph1 to **Box (MedBox)**, **List 1**, **1**, **Off**.
3. Press **GPH2** to set StatGraph2 to **Box (MedBox)**, **List 2**, **1**, **Off**.
4. Press **SEL** to switch StatGraph1 and StatGraph2 to **On**.
5. **DRAW** the graphs and compare the two box plots.

Follow the instructions below for a Texas instruments graphics calculator.

1. Enter the 12 Sydney values in List 1 and the 12 Melbourne values in List 2.
2. Press **STAT PLOT**, select Plot1 and set the graph to **On**, **Box plot (diagram)**, **Xlist: L1**, **Freq: 1**.
3. Select Plot2 and set the graph to **On**, **Box plot (diagram)**, **Xlist: L2**, **Freq: 1**.
4. Press **Zoom** and select **9:ZoomStat** to draw the graphs and compare the two box plots.

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4-07 Comparing data sets using charts

Radar chart

Radar chart paper

Comparing city temperatures

Comparing word lengths

Investigating young drivers

A **radar chart** is a circular line graph used to show changes over a cycle. It can be used to compare two sets of data.

EXAMPLE 12

This radar chart shows air pollution levels at two different workplaces over 10 working days.

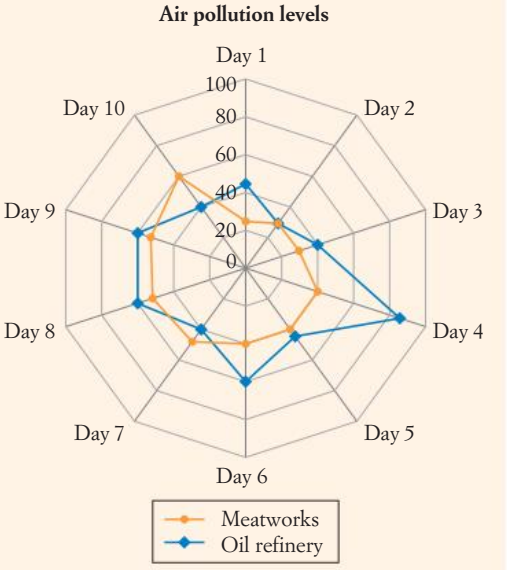
a On what days was the pollution level lowest at the oil refinery?

b What was the maximum pollution level at the meatworks?

c On what day was the pollution level at both workplaces equal?

d Which workplace had the higher overall pollution level? Justify your answer.

e Which workplace had more variation in pollution level? Justify your answer.



Solution

a Day 2 — **Point is closest to centre of graph.**

b 60 — **On Day 10.**

c Day 2 — **Where the graphs meet.**

d Oil refinery, because its graph is generally further from the centre than the meatworks graph.

e Oil refinery, because its graph was generally sharper and less rounded than the meatworks graph.

Area chart

Area charts

An **area chart** consists of layers or 'bands', each representing a data set over a period of time. It is used to compare two or more sets of data as well as to show the sum of data over the period of time.

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EXAMPLE 13

The table shows the numbers of males and females attending rugby matches in July from 2004 to 2009.

Year	2004	2005	2006	2007	2008	2009
Males (×1000)	36	24	21	33	38	41
Females (×1000)	11	9	15	12	13	19

a Draw an area chart to represent the data.

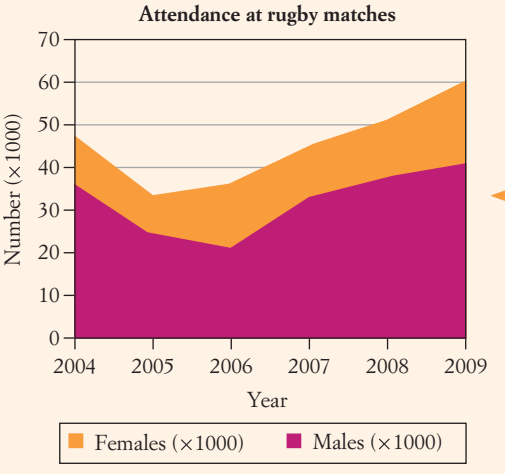
b In which year were overall attendances the lowest?

c Which year shows the greatest number of female attendances?

d Identify one difference between the male and female data and suggest a reason for this.

Solution

a Draw a line graph for male attendance using the values in the table and shade in the area beneath it. Draw a line graph for total attendance by *adding the values* for females to those of males. Shade in the area between the two lines using a different colour or pattern. This area represents the female attendance.



You can also use the chart option of your spreadsheet program to draw an area chart.

b 2005 **Where the overall graph is lowest.**

c 2009 **Where the 'females' band of the area graph is thickest.**

d More males than females attend matches, possibly as rugby is considered a man's sport.

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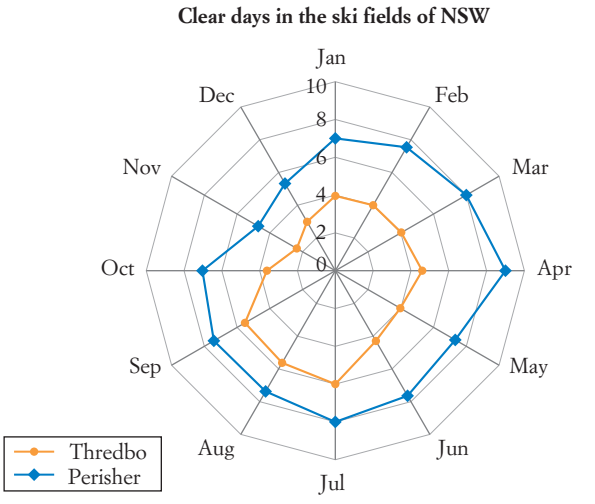
Example 12

Exercise 4-07 Comparing data sets using charts

1 The numbers of clear days each month over a year at Thredbo and Perisher ski fields in the Snowy Mountains are shown on the radar chart.



a How many clear days did Thredbo have in March?

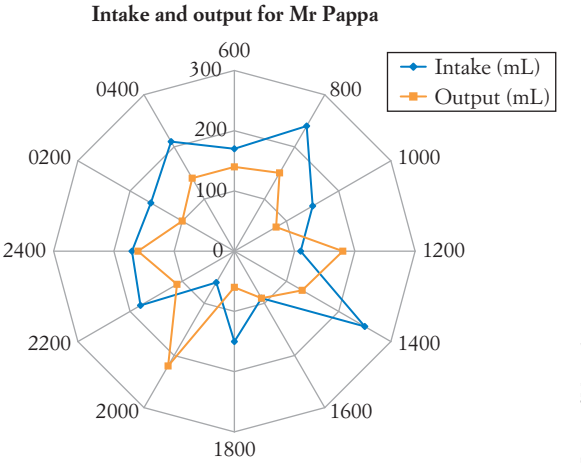
b What was the highest number of clear days at any ski field? When and where was this?

c How many days were not clear in Perisher in July?

d Which month showed the greatest difference in the number of clear days between both ski fields?

e Do you agree with the statement 'The weather is better for skiing at Perisher'? Justify your answer.

2 Mr Pappa was admitted to hospital with a suspected stomach ulcer. His fluid intake (e.g. water and medicine) and output (e.g. vomit and urine) over a 24-hour period are shown on the radar chart.



a At what time was his intake greatest?

b At what time was his output least?

c What was more consistent, his intake or his output?

d At what time(s) was his output greater than his intake?

e Write two observations from the radar chart about Mr Pappa's intake and output over the 24-hour period.

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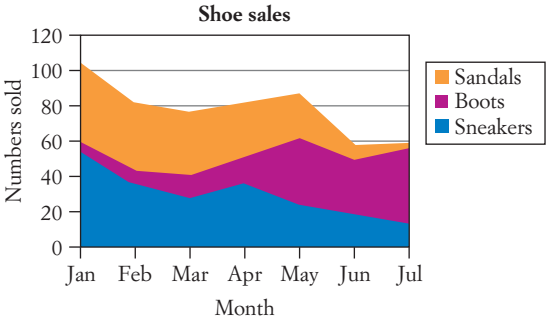
Example 13

3 This area chart shows the monthly sales of three types of shoes at the ShooBiz store over six months last year.



a From the chart, find the approximate number of:

i sandals sold in January

ii shoes sold in March

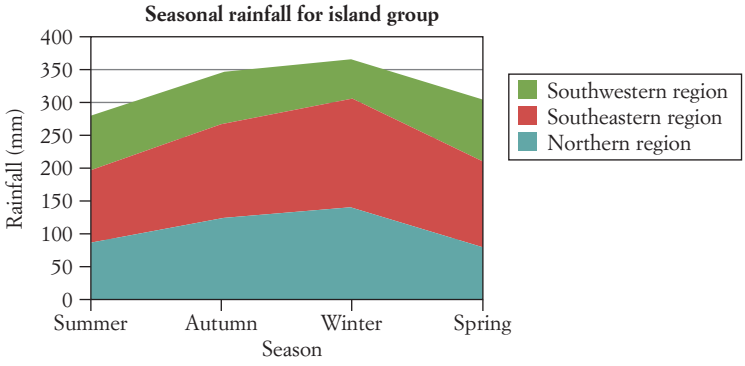
iii boots sold in July.

b In which month were the same number of sandals and sneakers sold?

c Suggest a reason why not many boots are sold in January.

d Identify one trend in this graph and suggest a reason for this.

4 The area chart shows the seasonal rainfall of an island group in the Pacific Ocean.



a What was the rainfall for the northern region in summer?

b What was the rainfall for the south-eastern region in spring?

c What was the total rainfall in autumn?

d Which region is the wettest? How is this shown in the graph?

e In which region is the rainfall about the same in each season?

5 Clark and Lois are freelance journalists writing articles for newspapers and magazines. Their monthly earnings last year are shown in the table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Clark's earnings ($)	370	240	530	570	780	1030	770	620	790	520	430	490
Lois' earnings ($)	150	420	480	530	850	1280	920	650	810	480	390	350

a Represent the data in a radar chart.

b Represent the data in an area chart.

c What information is best seen in the radar chart?

d Give one trend that is clearly seen in the area chart.

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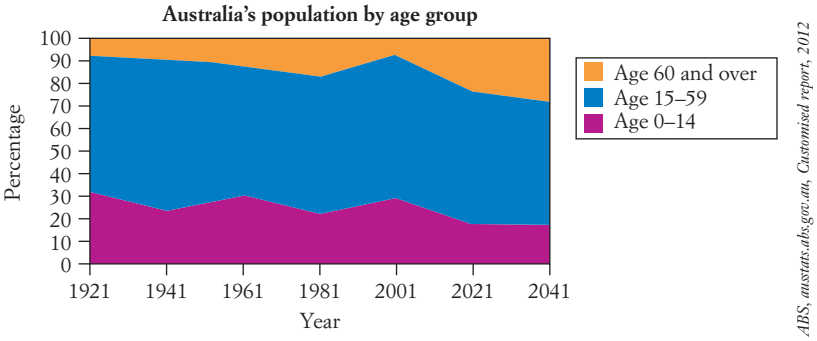
6 The area graph shows the make-up of Australia's population over 120 years.



a How do you think the data for 2021 and 2041 was obtained?

b In 1961, approximately what percentage of the population was between:

i 0 and 14? **ii** 15 and 59?

c Approximately what percentage of the population is expected to be 60 or over in 2021?

d In which year does the graph show that the percentage of 0–59 year olds is about 80%?

e Suggest a reason why the percentage of 0–14 year olds is expected to keep decreasing from 2001 to 2041.

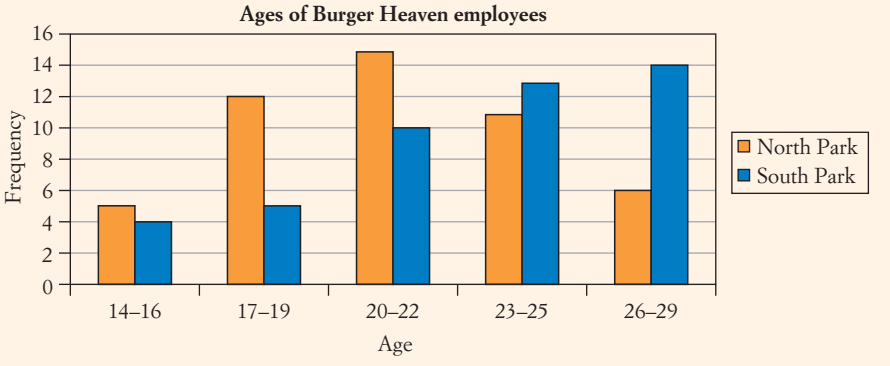
4-08 Multiple displays

Statistics allows us to compare two or more sets of data. A company's CEO compares this year's monthly sales figures with those of previous years; meteorologists compare the climate in two cities; and governments investigate the age groups of a city's population to build better facilities.

We have looked at displaying data sets in stem-and-leaf plots, box-and-whisker plots, radar charts, area charts and two-way tables. Data sets can also be compared in frequency tables, **clustered column graphs** and population pyramids.

EXAMPLE 14

The ages of the employees at Burger Heaven were compared across two stores: North Park and South Park. The results are shown in the clustered column graph below.



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a How many employees are there at the South Park store?

b What was the modal age group at the North Park store?

c Compare the shapes of the age distributions of both stores.

d Which store has the lower average age? Justify your answer.

Solution

a $4 + 5 + 10 + 13 + 14 = 46$ **Adding the South Park data.**

b 20–22 years **The age group with the tallest column.**

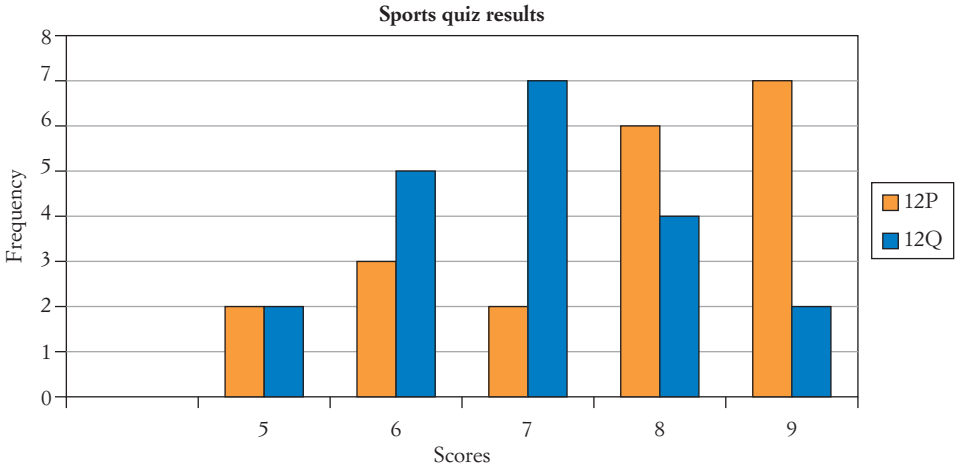
c The North Park ages are symmetrical while the South Park ages are negatively skewed, with a bigger representation of higher ages.

d North Park has the lower average age, because compared to South Park it has a larger number of younger employees. Its columns for the lower age groups are taller than those of South Park.

Exercise 4-08 Multiple displays

Example 14

1 Two Year 12 classes, 12P and 12Q, were given a sports quiz, and the students' scores are shown in the clustered column graph below.



Which of the following statements is true? Select **A**, **B**, **C** or **D**.

A 12P's scores are positively skewed with a higher mean than 12Q's scores.

B 12Q's scores are symmetrical with a lower mean than 12P's scores.

C 12P's scores are negatively skewed with a lower mean than 12Q's scores.

D 12Q's scores are symmetrical with the same mean as 12P's scores.

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2 The coach of two netball teams needs to decide which team should win the Most Consistent Team award. These are the scores for both teams over a season of matches.

The Birds	55	23	35	51	56	48	70	52	64	72
The Bees	18	41	23	46	48	24	56	27	36	48

a Display the data in a clustered column graph, using classes of 10–19, etc.

b Use your display to describe the shape and features of each data set.

c By finding suitable measures of location and spread, decide which team is more consistent. Justify your answer.

3 The workers in two regions were surveyed across different age groups to find out who belonged to a workers' union. The results are displayed in the back-to-back histogram below.

Union membership by age and region

a What percentage of 45–54 year olds belonged to a union in the eastern region?

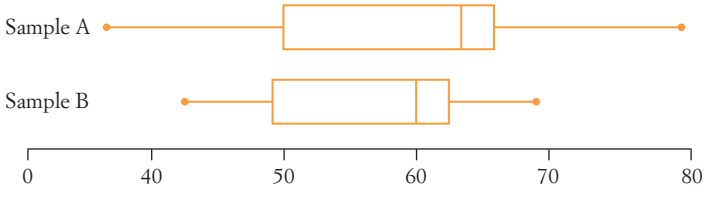
b Which age group had about the same percentage of union members across both regions?

c Compare the shapes of both distributions.

d Comment on the statement 'People in the eastern region are more likely to join a union'. Justify your answer.

4 Two samples of 10 people each were given stopwatches and asked to stop the watches when they thought 60 seconds had elapsed. The results, in seconds, for the 'estimated minute' are shown in the stem-and-leaf and box-and-whisker plots below.

Sample A		Sample B
4	3	
3	4	2 8 9
0	5	0 8
8 6 5 4 2	6	0 1 2 5 8
1	7	
0	8	



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a Find the median and interquartile range of each sample's estimates.

b Comment on the shape of each data set in terms of skewness and clusters.

c Which sample:

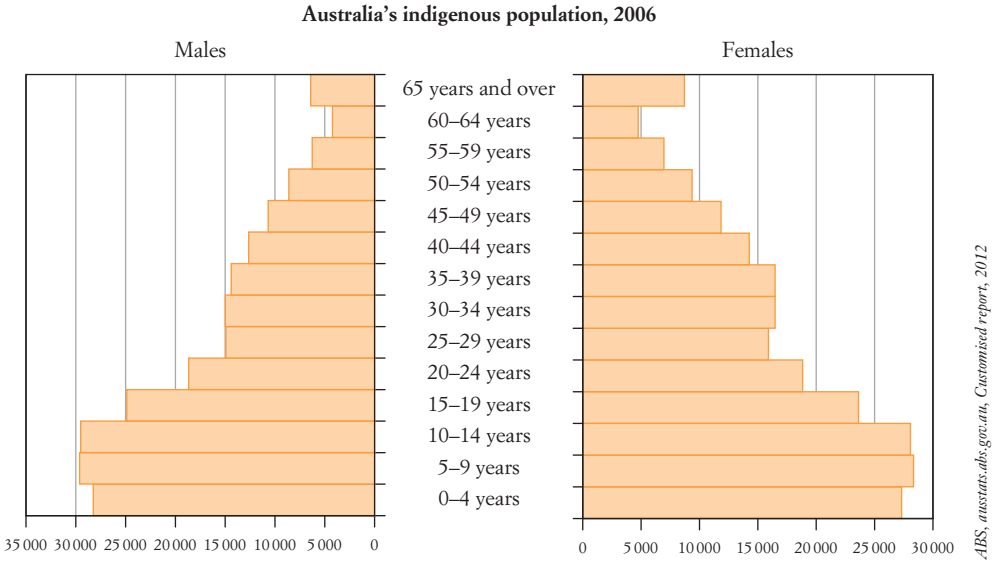
i gave the higher average estimates?

ii had a larger spread of estimates?

Justify your answer.

d Comment on the ability of each sample to estimate a minute.

5 This population pyramid shows the number of Indigenous persons in Australia in 2006.



a To the nearest 1000, find:

i the number of females in the 55–59 age group

ii the number of people in the 15–19 age group.

b In which age group were there about 5000 females?

c Suggest a reason why there are more people in the over-65 age group than in the 60–64 age group.

d Compare the shapes of both distributions.

e Comment on the statement that 'There were about the same number of indigenous males as females'.

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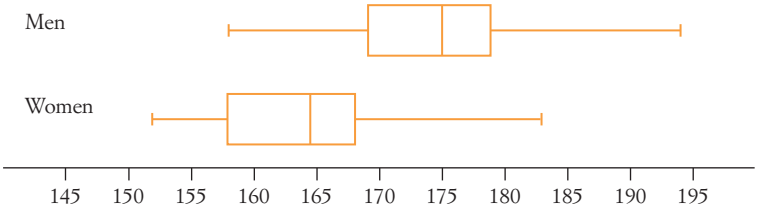
6 The heights, in centimetres, of a group of men and women are displayed in the two plots below.

Men		Women
8	15	2 4 4 5 6 8 8 9
9 7 7 5 2	16	0 2 3 3 4 5 5 5 5 6 7 8 8
9 9 8 8 6 5 5 4 4 4 2 1	17	2 3 4 4
8 6 3 2	18	3
4	19	



a Which group had the higher median?

b Which measure of spread was equal to 10 for both groups?

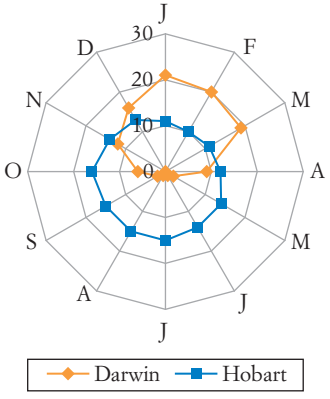
c Which group had the wider spread of heights?

d Comment on the skewness of each data set.

e Calculate the mean heights of men and women (correct to one decimal place).

f Which is the better measure of location for these data sets, the mean or the median? Why?

7 The average number of rainy days per month for Darwin and Hobart are displayed on the radar chart.



Bureau of Meteorology

a How many rainy days per month did Hobart have on average in August?

b In which month did Darwin have 12 rainy days?

c In which months did Darwin have more rainy days than Hobart?

d Estimate the range of rainy days for Hobart.

e Which city had a higher number of rainy days per month generally? Justify your answer.

f Which city had a greater range of rainy days per month generally? Justify your answer.

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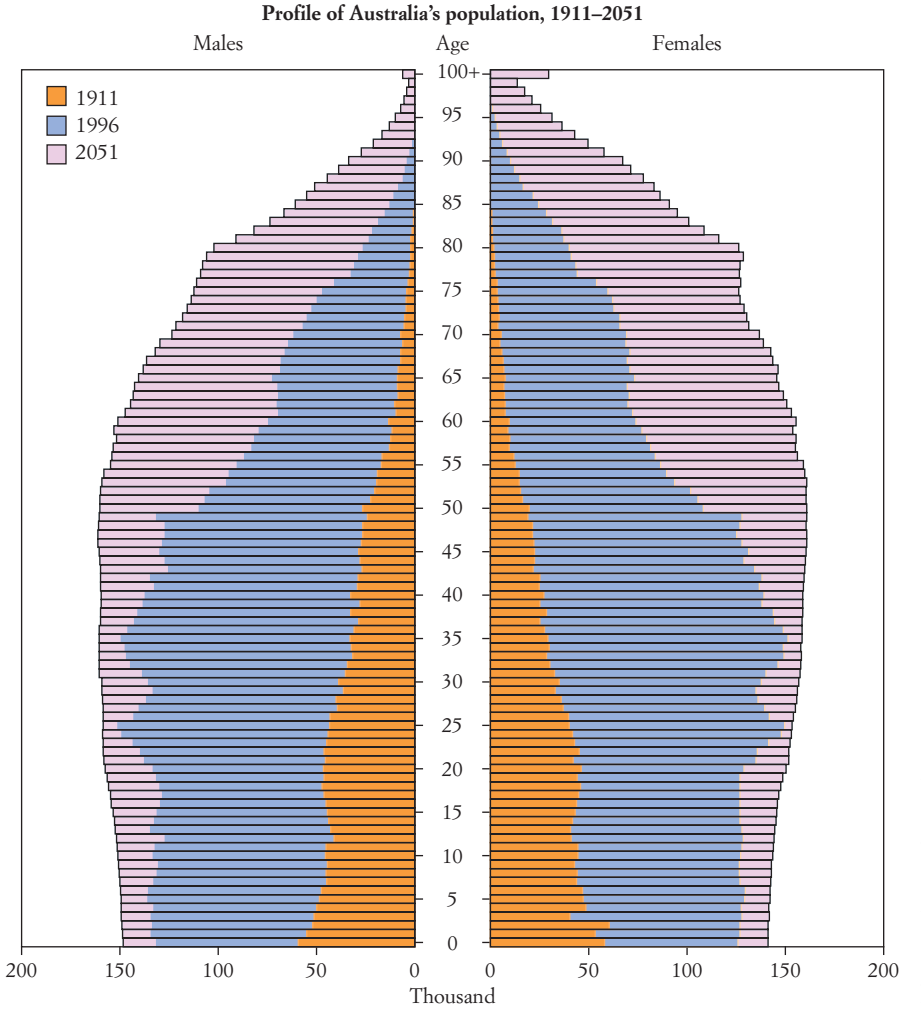
8 This population pyramid consists of three back-to-back histograms, showing the male and female Australian populations in 1911, 1996 and 2051 (projected).



a How many males were aged 20 in 1911?

b How many females were aged 35 in 1996?

c Approximate the increase in the numbers of males aged 60 from 1911 to 2051.

d Find one age group from 1996 where there were more females than males.

e Comment on the changes in the distribution of ages from 1911 to 1996.

f Write down one difference between the male and female populations in 2051.

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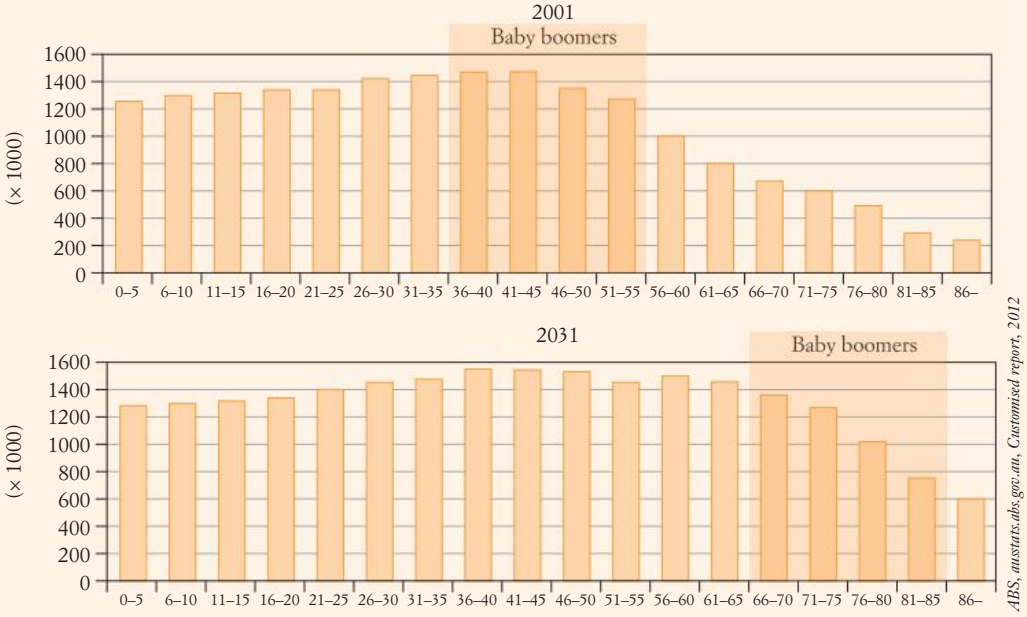
JUST FOR THE RECORD

Baby boomers

After World War II ended in 1945, there was a 'baby boom' in Australia. We regard the 'baby boomer generation' as those born between 1946 and 1965. The result of the large increase in births during this period affects Australia's population statistics as this generation ages.

The two graphs show the population of baby boomers moving from 2001 to 2031.



ABS, ausstats.abs.gov.au, Customised report, 2012

Compare the population distributions in 2001 and 2031.

How will the baby boomers affect the age distribution of the Australian population in 2031?

4-09 Two-way tables

A two-way table is a table of values used to compare two characteristics, each with two options.

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EXAMPLE 15

A large sample of adults was surveyed on their exercise habits and the results sorted by gender.

	Exercise	No exercise
Men	3028	1532
Women	1804	946

a How many men were surveyed?

b Calculate, correct to one decimal place, the percentage of:

i adults surveyed who were women

ii adults who exercised that were men.

c What fraction of the women surveyed exercised?

d Are men and women similar in the proportions of those who exercise? Justify your answer.

Solution

Add an extra row and column to the table for calculating totals.

	Exercise	No exercise	TOTAL
Men	3028	1532	4560
Women	1804	946	2750
TOTAL	4832	2478	7310

a 4560

b **i** $\frac{2750}{7310} \times 100\% \approx 37.6\%$ **Total women/total adults surveyed.**

ii $\frac{3028}{4832} \times 100\% \approx 62.7\%$ **Exercising men/total exercisers.**

c $\frac{1804}{2750} = \frac{82}{125}$ **Exercising women/total women.**

d Percentage of men who exercise
$= \frac{3028}{4560} \times 100\%$
$\approx 66.4\%$

Percentage of women who exercise
$= \frac{1804}{2750} \times 100\%$
$= 65.6\%$

As these percentages are similar, it can be concluded that similar proportions of men and women exercise.

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Exercise 4-09 Two-way tables

1 The results of a survey on employment are shown in the table. How many men were surveyed? Select **A**, **B**, **C** or **D**.

	Employed	Not employed
Women	58	32
Men	62	24

A 90 **B** 56
C 86 **D** 120

2 A group of women was surveyed on their interest in golf. Use the results in the table to calculate the percentage of right-handed women who play golf. Select **A**, **B**, **C** or **D**.

	Play golf	Do not play golf
Left-handed	34	59
Right-handed	83	85

A 49.40% **B** 31.80%
C 44.83% **D** 70.94%

Example 15

3 A sample of country and city people were surveyed on whether they had access to wireless Internet.

	Country	City
Wireless Internet	58	73
No wireless Internet	31	24

a How many country people were surveyed?
b What fraction of city people do not have wireless Internet?
c What percentage (to the nearest whole number) of the sample have wireless Internet?
d Compare the proportions of people in the country and city who have wireless Internet.

4 The population of Healthville was surveyed to find out who had private health insurance.

	Healthville City	Healthville South
Insurance	4563	4048
No insurance	5577	8602

a How many people live in Healthville City?
b How many people do not have health insurance?
c Find the percentage of people:
i in Healthville City who have health insurance
ii with health insurance who live in Healthville City.
d What fraction of people in Healthville South do not have health insurance?

5 Optel and Four are the only two mobile phone service providers in Dolphin Creek. The numbers of users were noted over 10 years.

	Optel	Four
2000	298 695	201 198
2010	414 428	276 216

a How many people in Dolphin Creek owned mobile phones in 2010?
b What percentage of people used Four as their provider in 2010?
c What percentage of people used Optel as their provider in 2000?

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d What happened to the number of Four users from 2000 to 2010?

e Comment on the statement 'Optel users doubled from the years 2000 to 2010'.

6 People in two age groups were asked for their opinions about changing the Australian flag. The results are displayed in the table.

	18–39	40–69
Change the flag	1430	590
Keep the flag	2100	1550

a How many people were surveyed?

b How many people wanted to keep the flag?

c What fraction of 40–69-year-olds wanted to keep the flag?

d What percentage of 18–39-year-olds wanted to change the flag?

e Which age group was more definite in its response? What was this response? Justify your answer.

7 A group of people was surveyed on their smoking habits.

	Female	Male
Smoker	4126	4502
Non-smoker	8634	9547

a How many male smokers were there?

b How many of the women surveyed did not smoke?

c Calculate, to one decimal place, the percentage of:

i non-smokers who were male **ii** females who were smokers.

d Write down one observation about people's smoking habits from this poll.

4-10 Pivot tables

A **pivot table** is a tool for sorting and summarising data presented in a table. It can sort, count and sum spreadsheet data and then rearrange it from a different viewpoint in a new table. The ability to 'rotate' the way in which data is presented gives the pivot table its name.

Comparing city temperatures

Comparing word lengths

Investigating young drivers

EXAMPLE 16

The table below shows the pulse rates of 30 people aged 18–20 who were asked to run on the spot for one minute. Pulse rates were taken before and after the exercise and the running was rated as low, medium and high.

Running rate	Pulse before	Pulse after	Gender	Weight
Medium	82	150	F	58
High	96	176	F	62
High	76	88	M	75
Low	78	141	M	74
Low	80	140	M	90
Low	66	89	F	60
Medium	70	95	F	41

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Running rate	Pulse before	Pulse after	Gender	Weight
High	56	110	M	63
High	64	126	M	70
Medium	68	84	F	51
Medium	65	82	M	75
Medium	47	136	F	77
High	50	90	M	87
Medium	88	150	M	60
Medium	80	146	M	70
Low	70	98	F	50
Medium	70	96	F	74
Medium	78	168	M	64
Medium	85	148	M	66
Medium	61	93	M	81
Medium	65	120	M	80
Medium	73	130	F	55
Low	72	160	F	63
High	50	83	M	82
Low	73	125	F	54
High	68	132	M	77
Medium	75	130	F	56
Medium	83	139	M	92
Low	92	120	F	61
High	52	89	M	78

Use a pivot table to report the data sorted by running rate, and for each running rate calculate the average of Pulse Before and Pulse by gender, as shown by this table.

Running rate	Average of Pulse Before	Average of Pulse After
High		
Female		
Male		
Low		
Female		
Male		
Medium		
Female		
Male		
Grand total		

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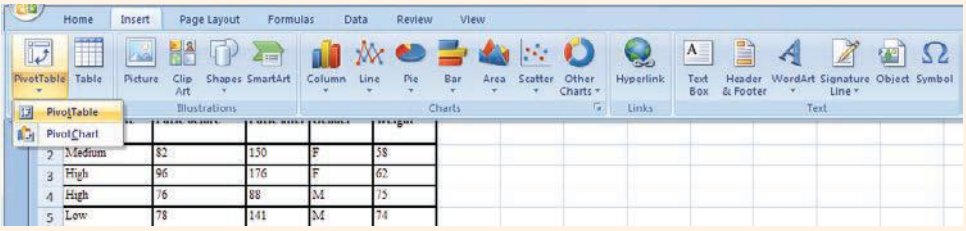
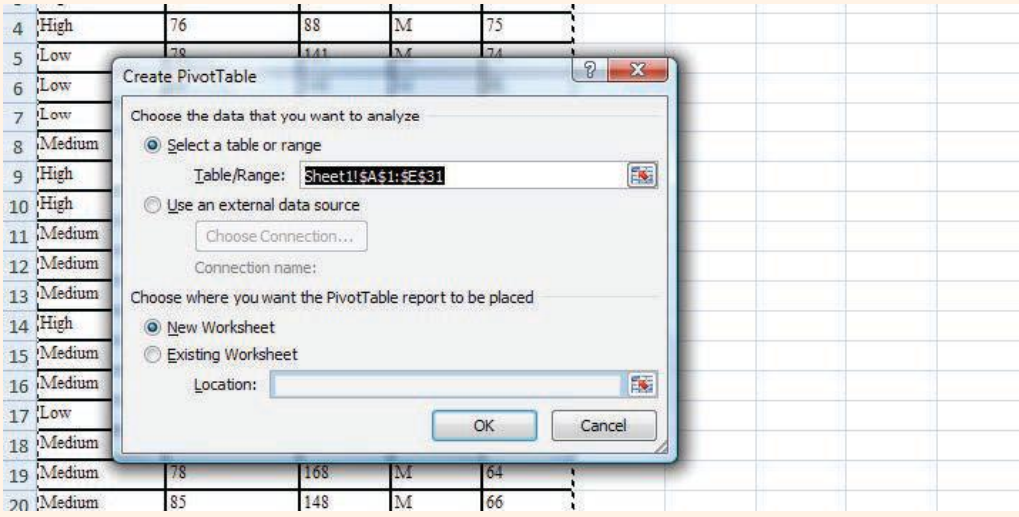
Solution

1 Enter the data in a spreadsheet. Click on a cell within the data. Then select **Insert** and **PivotTable**.

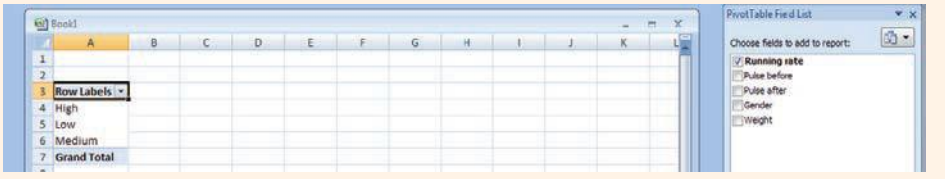


2 A **Create Pivot Table** dialog box will appear.



The data to be analysed has been selected. Choose **New Worksheet** for the **Pivot Table report** and **OK**.

3 The **PivotTable Field List** on the right is used to specify the layout of the pivot table, showing all the column headings from the spreadsheet (the source data). Below it is an area section that shows the four layout areas of the pivot table. Tick the checkbox **Running rate**. This field is then added to the 'Row' section of the pivot table.



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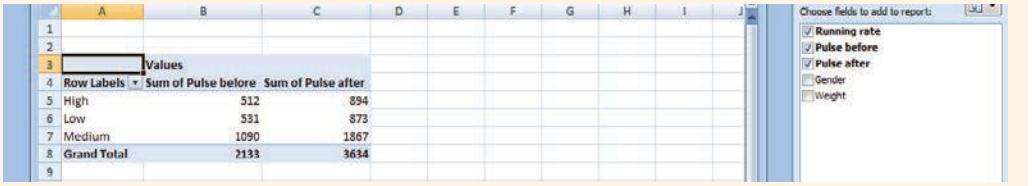
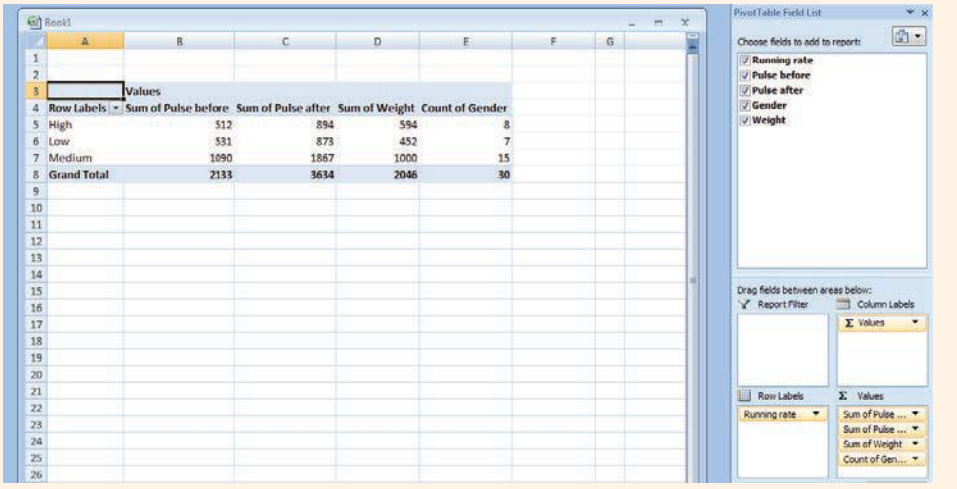
4 Now add value fields by ticking **Pulse before** and **Pulse after**.

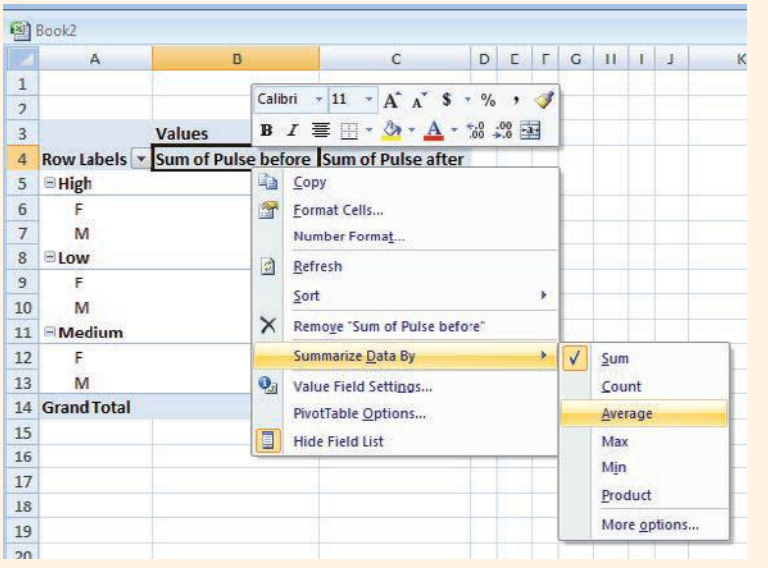


5 Add the Weight and Gender fields to the **Values** field.



6 At the bottom right area section, drag the **Gender** field from **Values** to **Row labels**.

7 Right-click on the **Sum of Pulse before** cell, select **Summarise data by** and **Average**.



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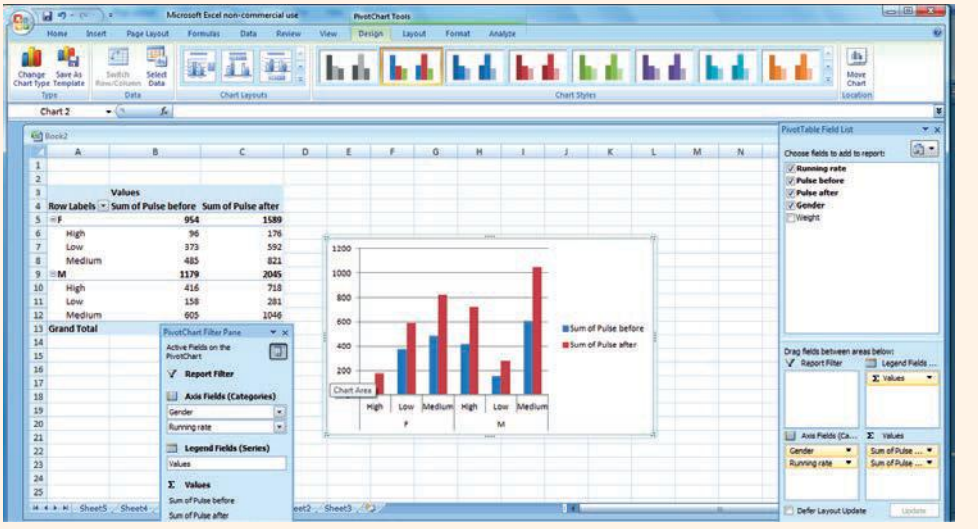
Do the same for **Sum of Pulse after**. To remove the Weight values from the table, untick it in the PivotTable field list. The completed pivot table is shown below.

Running rate	Average of Pulse Before	Average of Pulse After
High	**64**	**111.75**
F	96	176
M	59.42857143	102.5714286
Low	**75.85714286**	**124.7142857**
F	74.6	118.4
M	79	140.5
Medium	**72.66666667**	**124.4666667**
F	69.28571429	117.2857143
M	75.625	130.75
Grand total	**71.1**	**121.1333333**

8 Create a Pivot Chart by selecting **PivotTable tools**, **Pivot Chart** and **Column graph**.



The chart can be modified using the **Pivot Chart Filter Pane**.

9 Drag the fields **Running rate**, **Pulse before**, **Pulse after**, **Gender** and **Weight** between the areas to obtain different pivot table reports.

Example 16

Exercise 4-10 Pivot tables

1 The following data shows the heart rate of thirty-one people after they completed exercise by stepping up and down steps for one minute.

Stepping rate	Height of step	Heart rate
Moderate	Short	92
Fast	Short	120
Slow	Tall	85
Fast	Short	137
Fast	Short	146
Fast	Tall	150
Moderate	Tall	125
Moderate	Tall	130
Fast	Tall	142
Fast	Short	125
Slow	Tall	110
Slow	Short	84
Fast	Short	115
Moderate	Tall	121
Moderate	Short	105
Moderate	Short	123
Fast	Short	132
Fast	Tall	147
Moderate	Tall	138
Moderate	Tall	136
Moderate	Short	129
Moderate	Short	133
Slow	Short	104
Slow	Tall	122
Slow	Short	95
Fast	Short	153
Moderate	Tall	143
Moderate	Short	120
Slow	Tall	108
Fast	Tall	139
Slow	Tall	121

a Enter the data into an Excel spreadsheet and make up a pivot table by putting 'Height of step' in Row labels, 'Heart Rate' in Σ Values and Stepping rate' in Column labels.

b Use your pivot table to find the average heart rate and standard deviation:

i for Short Step and Moderate Stepping rate

ii for Tall Step and Fast Stepping rate.

c Make up a pivot chart to show the different heart rates for Short and Tall Step at the different Stepping rates. Print your results.

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2 The data below shows the revenue of sales of a company for four quarters over two years.

Year	Quarter	Month	Revenue ($)
2010	1	Jan	630 460
2010	1	Feb	515 000
2010	1	Mar	585 345
2010	2	Apr	521 678
2010	2	May	456 230
2010	2	Jun	396 020
2010	3	Jul	417 845
2010	3	Aug	524 670
2010	3	Sept	585 640
2010	4	Oct	547 650
2010	4	Nov	639 539
2010	4	Dec	652 680
2011	1	Jan	652 523
2011	1	Feb	567 320
2011	1	Mar	489 468
2011	2	Apr	545 358
2011	2	May	499 420
2011	2	Jun	406 220
2011	3	Jul	429 746
2011	3	Aug	535 820
2011	3	Sept	588 792
2011	4	Oct	567 456
2011	4	Nov	648 651
2011	4	Dec	664 265

a Make up a pivot table with the 'Quarter' field in the Row section, the Year field in the Column labels and the 'Revenue' field in Σ Values.

b What was the total revenue for:

i 2010 **ii** 2011?

c Which quarter had the highest revenue?

d Make a pivot chart. Use your pivot chart to describe any trends in Revenue over the two years.

e Move the 'Quarter' field to the Column labels (or Legend fields) and the 'Year' field to the Row labels (or Axis fields). How has the chart changed and what do you notice about Revenue from 2010 to 2011?

ISBN 9780170238977

3 The results of four assessment tasks in English, Maths, History and Senior Science for Roslyn, Ben, Parissa and Gobind are shown below.

Task	Student	Subject	Mark
1	Rosyln	English	61
1	Ben	English	55
1	Parissa	English	80
1	Gobind	English	72
1	Rosyln	Maths	68
1	Ben	Maths	74
1	Parissa	Maths	78
1	Gobind	Maths	69
1	Rosyln	History	84
1	Ben	History	83
1	Parissa	History	72
1	Gobind	History	88
1	Rosyln	Senior Science	58
1	Ben	Senior Science	89
1	Parissa	Senior Science	87
1	Gobind	Senior Science	80
2	Rosyln	English	63
2	Ben	English	74
2	Parissa	English	84
2	Gobind	English	76
2	Rosyln	Maths	55
2	Ben	Maths	70
2	Parissa	Maths	72
2	Gobind	Maths	79
2	Rosyln	History	80
2	Ben	History	88
2	Parissa	History	83
2	Gobind	History	79
2	Rosyln	Senior Science	68
2	Ben	Senior Science	93
2	Parissa	Senior Science	90
2	Gobind	Senior Science	85

Task	Student	Subject	Mark
3	Rosyln	English	60
3	Ben	English	68
3	Parissa	English	78
3	Gobind	English	73
3	Rosyln	Maths	70
3	Ben	Maths	80
3	Parissa	Maths	81
3	Gobind	Maths	85
3	Rosyln	History	87
3	Ben	History	79
3	Parissa	History	65
3	Gobind	History	61
3	Rosyln	Senior Science	72
3	Ben	Senior Science	86
3	Parissa	Senior Science	84
3	Gobind	Senior Science	88
4	Rosyln	English	56
4	Ben	English	58
4	Parissa	English	81
4	Gobind	English	52
4	Rosyln	Maths	72
4	Ben	Maths	89
4	Parissa	Maths	83
4	Gobind	Maths	90
4	Rosyln	History	79
4	Ben	History	84
4	Parissa	History	80
4	Gobind	History	75
4	Rosyln	Senior Science	66
4	Ben	Senior Science	74
4	Parissa	Senior Science	68
4	Gobind	Senior Science	78

a Make up a pivot table with Student and Task in the Row label, Subject in the Column label and mark in Σ Values.

b Use the pivot table to find

i the subject with the highest average mark

ii the student with the highest average mark.

c Mark up a pivot chart, moving your fields if necessary, to see any trends in the displayed data.

INVESTIGATION

SELF-SERVE SUPERMARKET CHECKOUTS

One of the main roles of a statistician is to critically analyse related data sets and report on the findings. Businesses often use the results of an analysis for promotional purposes and companies report to their shareholders.

To critically analyse data sets, you need to draw suitable displays, find measures of location and spread and write a report on the relationship between the data sets, mentioning skewness, clusters, peaks and outliers as well as noting any patterns or unusual features. From this report you can draw conclusions and make recommendations.

At a supermarket, the times (in seconds) taken to check a basket of 20 grocery items at 15 self-serve and 15 cashier-assisted checkouts were recorded.

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Self-serve	45	58	63	43	75	69	84	65	96	73	90	61	84	72	96
Cashier	95	105	82	110	125	148	136	137	86	99	145	119	101	97	124

Critically analyse the data and report back to the manager of the store on any benefits of self-serve checkouts.

Sample HSC problem

The ages of members of a yoga class at Husky Body Shop one morning are shown in the stem-and-leaf plot.

Women		Men
4	2	2 6
2	3	2 2 5 6 8
8 3	4	2 4 5 6
9 4 0 0	5	3
8 5 4 3 3	6	3 3 4

a Find the interquartile range of the ages of the women in the class.

b Find the median age of the men in the class.

c Compare the shapes of age distributions of the women and men.

d Which group was younger on average: women or men? Justify your answer.

Study tip

Finding the key of a topic and using mnemonics

When studying a novel in English class, you look for the underlying theme of the book. When studying a mathematics topic, you can also look for 'the key', the main idea that threads through the topic, the reason for learning it. Once you understand and appreciate the point of a topic, a lot of its concepts and ideas will fall into place.

Analyse the content of each topic/chapter for meaning and relevance. What is the key idea or theme? How and where will you use the mathematics? Use each chapter's introductory outline and review pages to help you. Ask yourself and your teacher questions about the topic. Have an opinion. Sound informed.

A mnemonic (pronounced 'ne-monic' with a silent first m) is a memory aid—a saying, rhyme or picture to help you remember a rule or list of ideas. You probably already know some mnemonics for spelling ('i before e except after c') and the number of days in each month ('Thirty days has September …').

To memorise a complex list or rule, make up a saying or picture that is easy to remember. For example, the initials for the definitions of basic trigonometry SOH CAH TOA can be remembered using the saying 'Some Old Hens Can Always Hide Their Old Age' or 'Sun Over Head Caused A Huge Tan On Arm'. The gradient formula $m = \frac{\text{rise}}{\text{run}}$ can be learnt by picturing yourself at sun*rise* waking up to go for a *run* (moving forward or across).

Be creative in your use of mnemonics. In fact, the sillier the phrase or picture, the better the mnemonic because absurd words and images are easier to recall.

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CHAPTER REVIEW

Topic overview

Statistics crossword

This chapter, Statistical distributions, revised and extended the statistics covered in the Preliminary Course. It compared two data sets in a variety of displays, including double stem-and-leaf plots, box plots, radar charts and area charts. You used measures of location and spread to compare data sets and interpreted information from different displays, including the shape of distributions. Be sure to include skewness, area charts, the effect of outliers and pivot tables in your summary as well as a glossary of statistical terms.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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Revision

Exercise 4-01

1 Classify the following data as categorical nominal (CN), categorical ordinal (CO), quantitative discrete (QD) or quantitative continuous (QC).

- **a** temperature of a hospital patient
- **b** amount of mail delivered to households in Campbelltown
- **c** annual water consumption in Sydney
- **d** type of public transport used to commute to work
- **e** number of first-year university students in Australia
- **f** most popular film released this year.

Exercise 4-02

2 The number of fires put out by fire brigades in country towns on one weekend are shown in the table.

No. of fires	1	2	3	4	5	6
No. of brigades	6	4	5	2	2	1

- **a** How many fire brigades were surveyed?
- **b** What was the mean number of fires put out?
- **c** What was the modal number of fires put out?
- **d** What was the standard deviation of the number of fires? Answer to two decimal places.

Exercise 4-03

3 A group of ten competitors entered the KAC cross-country ski race. Their times, to the nearest minute, are:

38 17 42 36 38 41 19 22 54 16

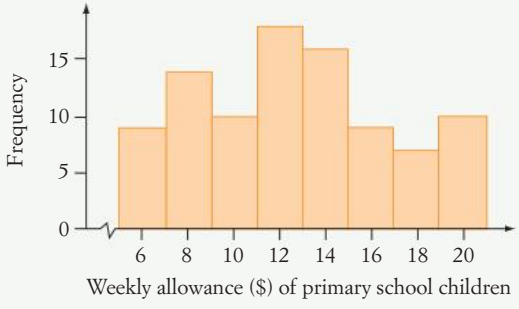
- **a** Find the mean time.
- **b** Find, correct to two significant figures, the population standard deviation of the times.
- **c** Draw an ordered stem-and-leaf plot for the data.
- **d** Find the median.
- **e** Find the interquartile range.

Exercise 4-04

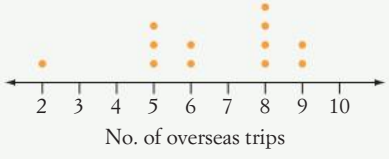
4 For each statistical display below:

- **i** find the mode and range of the data set
- **ii** describe the shape of the distribution, mentioning any clusters or outliers.

a



b



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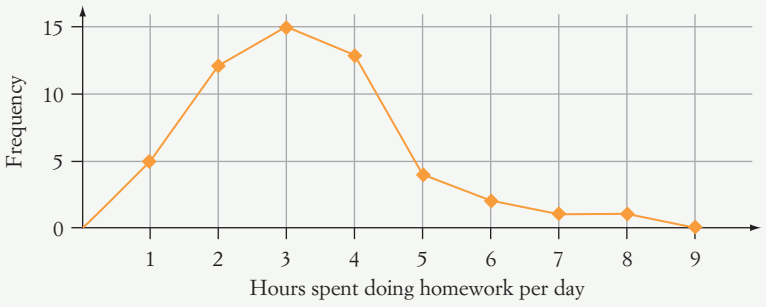
c



Exercise 4-05

5 Consider the set of scores:

3 4 5 5 8 9 12 15 18

a Find the mean.

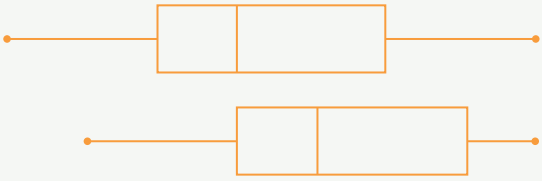
b Find the median.

c What is the effect on the mean and median if an outlier of 50 is added to this data set?

d Is the mean or median a better measure of location when there is an outlier in the data set?

Exercise 4-06

6 Which measure is the same for both of these box-and-whisker plots? Select **A**, **B**, **C** or **D**.



A the mean

B the range

C the median

D the interquartile range

Exercise 4-06

7 A factory produces small metal rods designed to have a mass of 50 g. Samples were taken from two different machines and a stem-and-leaf plot was drawn.

Machine A		Machine B
	4	4
9 9 9 9 9	4	8 9
3 2 2 1 1 0 0 0	5	0 0 0 0 0 1 1 1 2 3 3 3 4 4
8 8 7 6 6 5	5	5 6 7 7 8 9
0	6	1 2 3
	6	5

a Find the modal mass for each machine.

b Find the median and interquartile range for each machine.

c On average, which machine produces smaller rods?

d Find a five-number summary for each data set and construct box plots for the data.

e 'Machine B produces rods of a more consistent mass than Machine A'. Do you agree with this statement? Justify your answer.

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Exercise 4-06

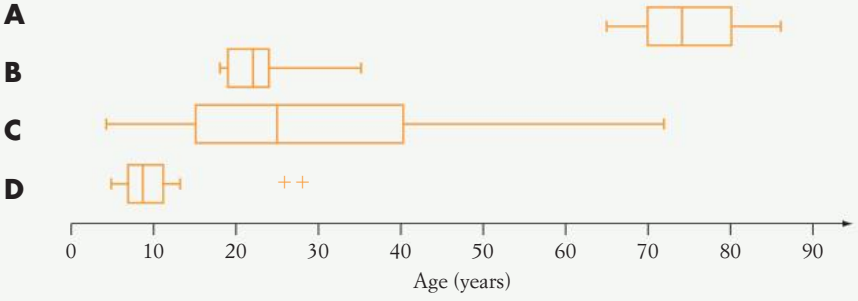
8 Match each sample of people to the correct box-and-whisker plot.

a Spectators at a football match

b Senior citizens on a bus trip

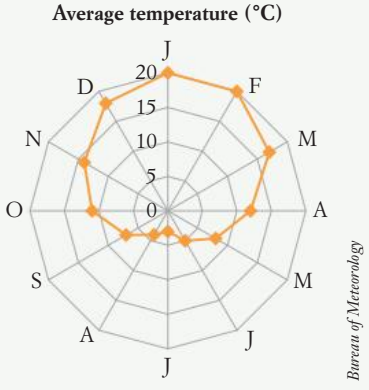
c Visitors to a dance party

d Primary school zoo excursion attendees



Exercise 4-07

9 Cabramurra is the highest town in Australia. The radar chart shows its average monthly temperature while the table shows its average monthly rainfall.



	J	F	M	A	M	J	J	A	S	O	N	D
Average rainfall (mm)	107	78	87	113	158	172	195	198	185	181	128	127

a What is the range of monthly rainfalls?

b What was the maximum temperature and when does this occur?

c Which month had an average temperature of over 15°C with an average rainfall below 80 mm?

d What trend can be seen in the monthly temperature figures over one year?

e If you were organising an outdoor fete in Cabramurra, which month would you choose? Justify your answer.

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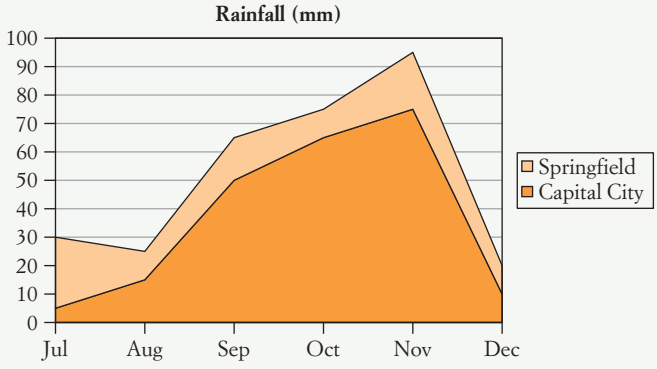
Exercise 4-07

10 The monthly rainfalls (in millimetres) for two cities are shown in the area chart.



a Write down two observations about the rainfall in the two areas.

b What is the approximate rainfall in October for Springfield?

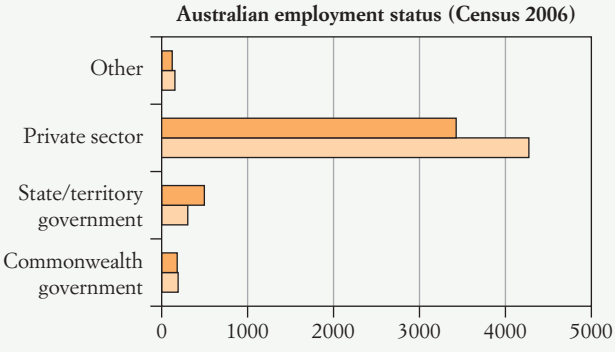
c What is the wettest month in Capital City?

d Find the combined amount of rain falling over both areas in September.

e Which area gets more rain? Justify your answer.

Exercise 4-08

11



ABS, ausstats.abs.gov.au, Customised report, 2012.

a What information is displayed in this clustered column graph?

b Is the data quantitative or categorical? Justify your answer.

c In which area or areas were more females employed than males in 2006?

d Which employment status showed the smallest difference between women and men?

e Comment briefly on the strengths and weaknesses of the display.

Exercise 4-09

12 This two-way table describes the employment status of a group of women and men.

	Women	Men
Casual work	542	438
Full-time work	269	602

a How many men were surveyed?

b Find, as a percentage correct to two decimal places,

i the percentage of women employed in full-time work

ii the percentage of the whole group employed in casual work.

c What fraction of casual workers are men?

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13 The data in the table shows the diagnosis and treatment for patients with eye allergies/ complaints.

Exercise 4-10

Month	Gender	Diagnosis	Treatment
Mar	F	Allergy	Eye Drops
Mar	M	Allergy	Eye Drops
Mar	M	Cataract	Surgery
Mar	F	Ptergyium	Eye Drops
Mar	F	Allergy	Eye Drops
Mar	F	Cataract	Surgery
Mar	F	Allergy	Eye Drops
Mar	F	Allergy	Eye Drops
Apr	F	Allergy	Eye Drops
Apr	M	Ptergyium	Eye Drops
Apr	M	Presbyopia	Reading glasses
Apr	F	Refractive Error	Reading glasses
Apr	M	Allergy	Eye Drops
Jun	M	Refractive Error	Reading glasses
Jun	M	Presbyopia	Reading glasses
Jun	F	Refractive Error	Reading glasses
Jun	F	Cataract	Surgery
Jun	M	Cataract	Surgery
Jun	F	Ptergyium	Eye Drops
Jun	F	Ptergyium	Eye Drops
Jun	M	Cataract	Surgery
Jun	M	Conjunctivitis	Eye Drops

a Make up a pivot table to show diagnosis in Row labels and Treatment in Σ Values.

b On your pivot table report include a pie chart showing the different eye complaints.

c Change your pivot table to the diagnosis and type of treatment.

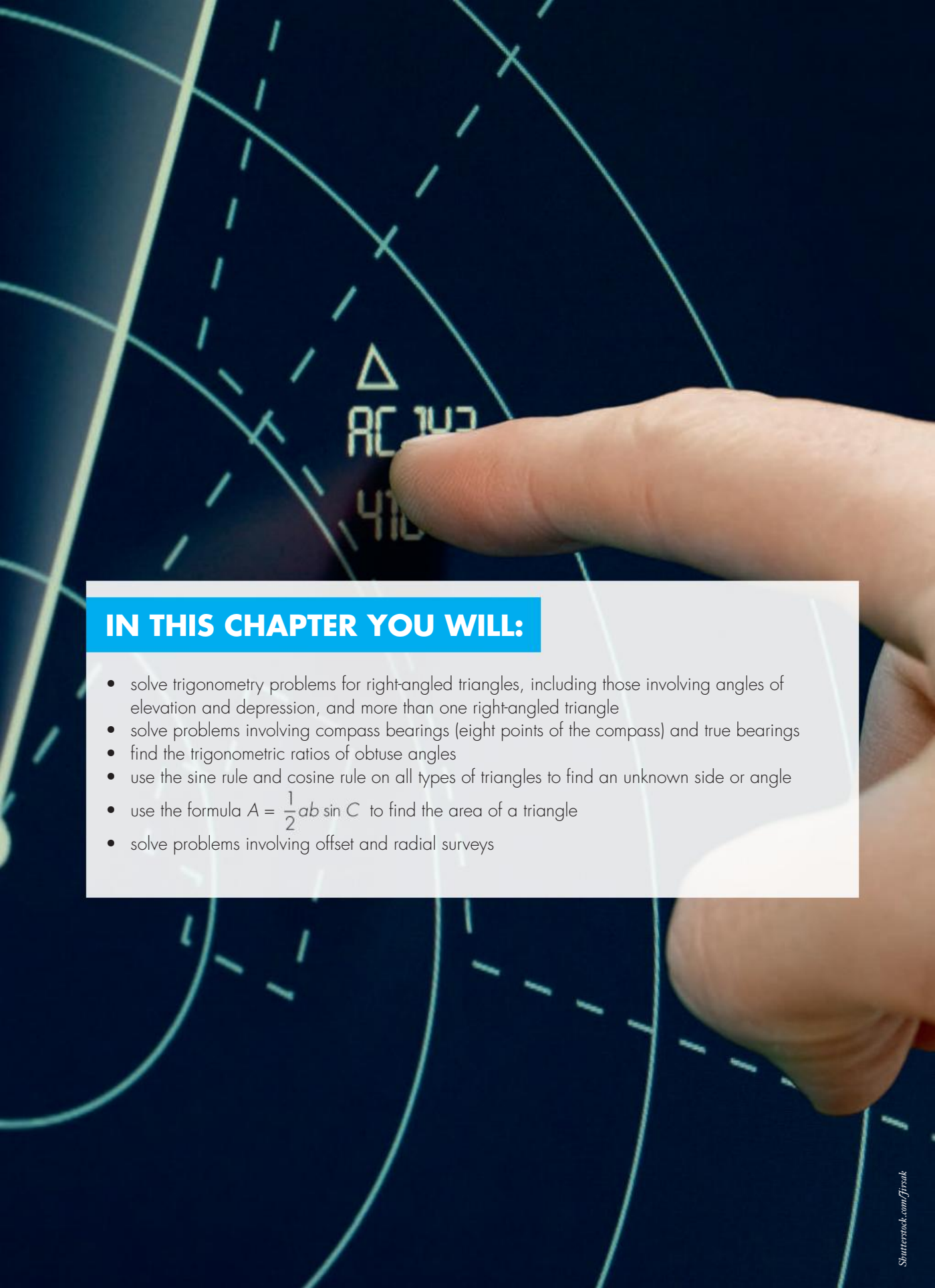
MEASUREMENT

5.

THE SINE AND COSINE RULES

An air traffic controller sees two planes on his radar screen. One is 16.4 km from the airport and the other is 7.5 km from the airport. If he knows that the angle between them is 136°, what is the distance between the planes?

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- solve trigonometry problems for right-angled triangles, including those involving angles of elevation and depression, and more than one right-angled triangle
- solve problems involving compass bearings (eight points of the compass) and true bearings
- find the trigonometric ratios of obtuse angles
- use the sine rule and cosine rule on all types of triangles to find an unknown side or angle
- use the formula $A = \frac{1}{2}ab \sin C$ to find the area of a triangle
- solve problems involving offset and radial surveys

TERMINOLOGY

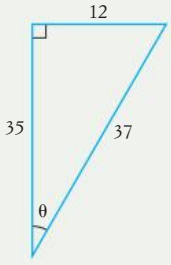
angle of depression
angle of elevation
bearings
compass bearings
compass radial survey
convention
cosine
cosine rule
included angle
non-right-angled
non-right-angled triangle
obtuse angle
offset survey
plane table
quadrant
radial survey
right-angled
sine
sine rule
tangent
three-figure bearings
trigonometric ratio
true bearings

Assignment 5

Finding sides in similar triangles

SkillCheck

1 For this diagram, write the fraction for:



a $\sin\theta$
b $\cos\theta$
c $\tan\theta$

2 Find the size of θ from question **1** to the nearest degree.

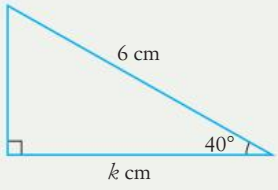
3 Convert 37.15° to degrees and minutes.

4 Evaluate, correct to two decimal places:

a $25 \sin 18°25'$ **b** $\dfrac{9}{\cos 32°}$ **c** $4^2 + 8^2 - 2 \times 4 \times 8 \cos 116°$

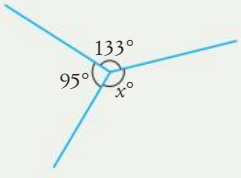
d $\dfrac{17 \sin 40°}{\sin 68°}$ **e** $\dfrac{6^2 + 7^2 - 11^2}{2 \times 6 \times 7}$ **f** $\dfrac{1}{2} \times 9 \times 5 \sin 38°$

5 Find k, correct to two decimal places.



6 Find the value of each pronumeral, giving reasons.

a



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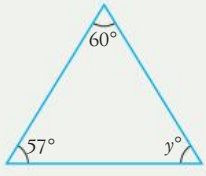

c

7 **a** Find A, to the nearest degree, if $\cos A = \dfrac{15}{17}$.

b Find B, to the nearest minute, if $\sin B = 0.7329$.

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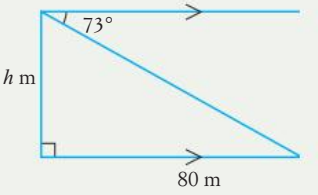
8 Find h, correct to one decimal place.



5-01 Right-angled triangle trigonometry

Trigonometry review

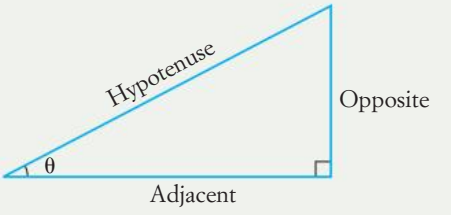
SUMMARY

For right-angled triangles:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad (\text{SOH})$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad (\text{CAH})$$

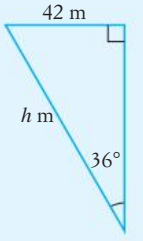
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad (\text{TOA})$$



Some students memorise the initials SOH-CAH-TOA (pronounced 'so car toe-ah'), while others remember a phrase like '**S**ome **O**ld **H**ens **C**an **A**lways **H**ide **T**heir **O**ld **A**ge'.

EXAMPLE 1

Find h correct to two decimal places.



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Solution

$$\sin 36° = \frac{42}{h}$$

opposite / hypotenuse

$$h = \frac{42}{\sin 36°}$$

If the unknown side is in the denominator, divide to find the answer. Make sure your calculator is in degrees (D) mode.

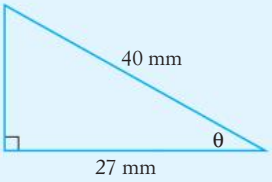
$$= 71.4546\ldots$$

$$\approx 71.45 \text{ m}$$

From the diagram, 71.45 m seems a reasonable answer.

EXAMPLE 2

Find θ, correct to the nearest minute.



Solution

$$\cos \theta = \frac{27}{40}$$

$\theta = 47.5458\ldots°$ **Calculator sequence:** SHIFT cos 27 $a^{b/c}$ 40 =

$\approx 47°32'45.06''$ **Calculator sequence:** o ' " or DMS

$\approx 47°33'$

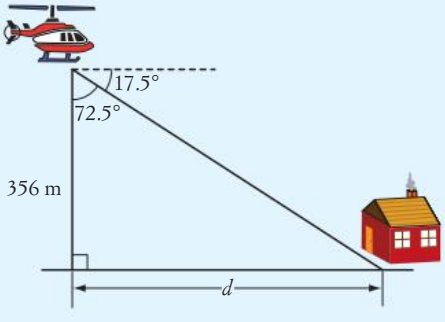
From the diagram, 47°33′ seems a reasonable answer.

EXAMPLE 3

The angle of depression of Katie's house from a helicopter 356 m above the ground is 17.5°. Calculate, correct to three significant figures, the ground distance between the helicopter and Katie's house.

Solution

Draw a diagram and let the ground distance be *d*. The angle of depression is 17.5°, so the adjacent angle inside the triangle is $90° - 17.5° = 72.5°$



$$\tan 72.5° = \frac{d}{356}$$

$$= 356 \times \tan 72.5°$$

$$= 1129.0877\ldots$$

$$\approx 1130 \text{ m}$$

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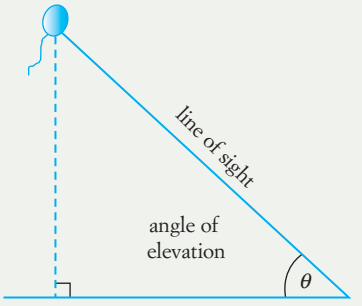
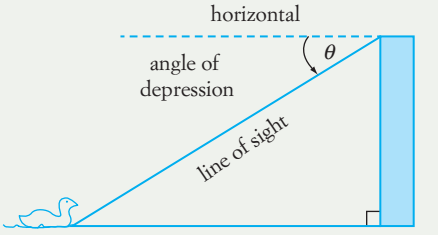
SUMMARY

- The **angle of elevation** is the angle *looking up*: if we feel **elevated**, things are 'looking up'.
- The **angle of depression** is the angle *looking down*: if we feel **depressed**, things are 'looking down'.





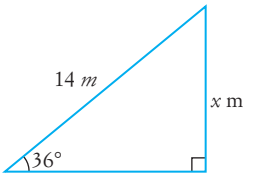
Angle of elevation and angle of depression problems usually involve the tan ratio.

Exercise 5-01 Right-angled triangle trigonometry

Example 1

1 Find the value of the pronumeral in each triangle. Give your answer to two decimal places.

a



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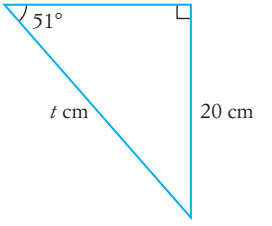

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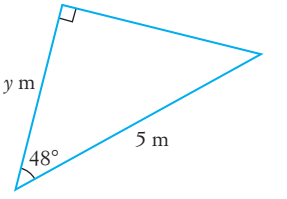
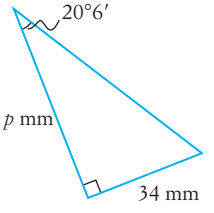
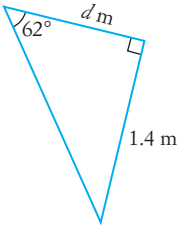


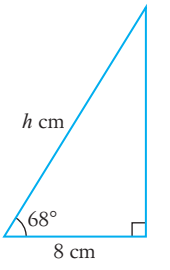
d



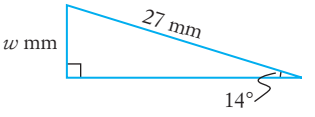
e



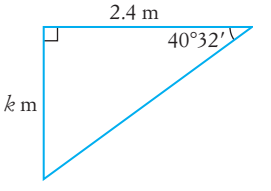
f



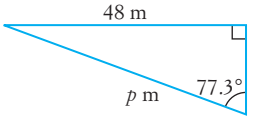
g



h



i



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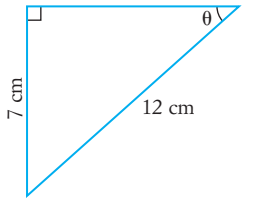
Example 2

2 Find θ to the nearest degree.

a



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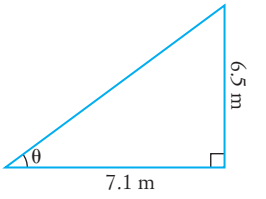

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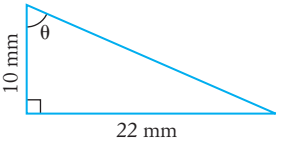
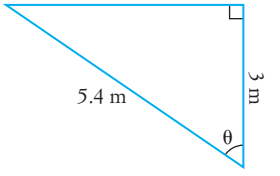
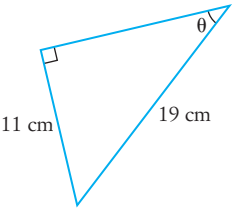


d



e



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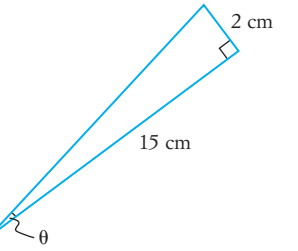
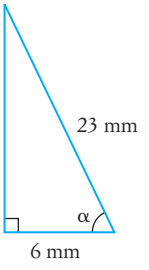
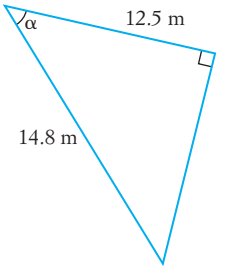


3 Find α to the nearest minute.

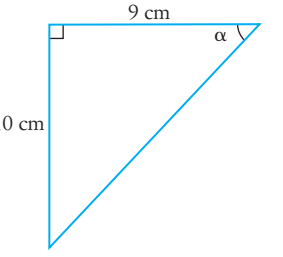
a



b



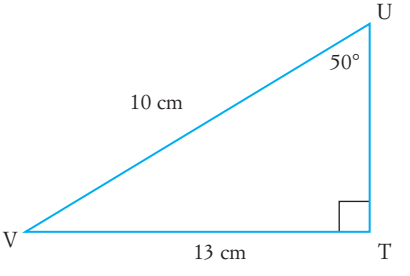
c



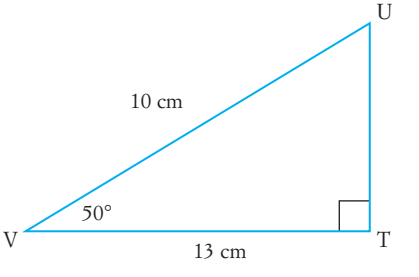
4 Mark the following information on a diagram: In $\triangle PQR$, $\angle P = 90°$, $\angle R = 76°10'$ and $PR = 4.1$ m.

5 In $\triangle TUV$, $\angle T$ is right-angled, $\angle U = 50°$, $UV = 13$ cm and $TV = 10$ cm. Which diagram fits this description? Select **A**, **B**, **C** or **D**.

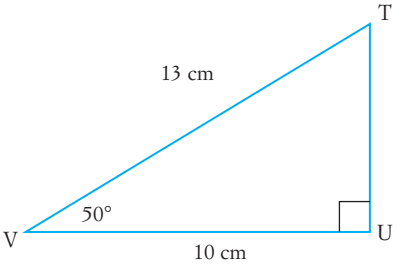
A



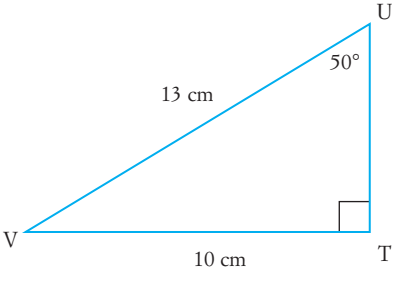
B



C



D



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6 A kite with a string of length 120 m is 96 m above the ground. Which of the following is the size of the angle that the string makes with the horizontal? Select **A**, **B**, **C** or **D**.

A 37° **B** 39° **C** 51° **D** 53°

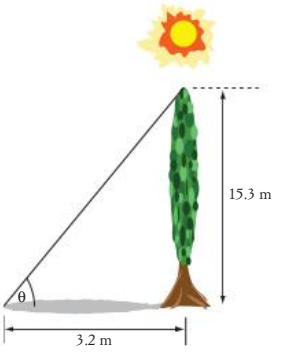
7 A 5.8 m ladder leaning against a wall makes an angle of 72° with the ground. Calculate, correct to one decimal place,

a the distance that the ladder reaches up the wall

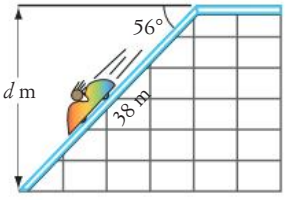
b the distance between the foot of the ladder and the base of the wall.

8 A tree of height 15.3 m casts a shadow of 3.2 m along the ground. Calculate the angle of elevation of the sun, θ, to the nearest minute.

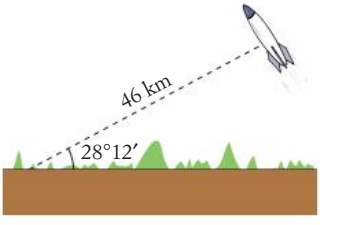
Example 3



9 A section of a rollercoaster ride has a 56° angle of depression. If the length of the section is 38 m, what is the length of its vertical drop, correct to one decimal place?



10 A radar station measures the angle of elevation of a missile to be 28°12′ and the line-of-sight distance is 46 km. What is the altitude (height) of the missile, correct to one decimal place?

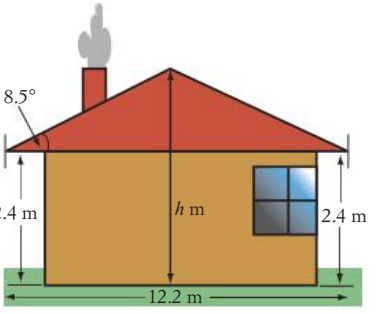


11 A lifeguard on top of an observation tower 12.5 m high notices a surfer in trouble at an angle of depression of 8°. How far, correct to one decimal place, is the surfer from the tower?

12 The roof of this house has a pitch of 8.5°. How high, h m, is the apex of the roof above the ground? Select **A**, **B**, **C** or **D**.

A 0.9 m **B** 3.3 m

C 3.6 m **D** 4.2 m



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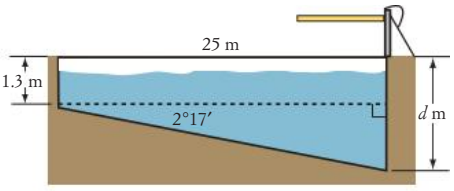
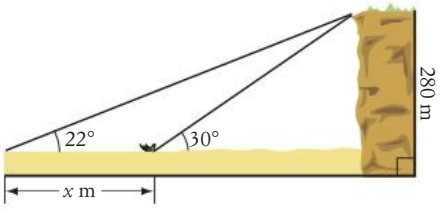
13 A boat is 420 m from the base of a cliff. The angle of depression of the boat from the top of the cliff is 28°. Find the height of the cliff to the nearest metre.

14 The floor of a swimming pool inclines at 2°17′ to the horizontal. The shallow end is 1.3 m deep and the pool is 25 m long. What is the depth, to the nearest 0.1 m, at the deep end?

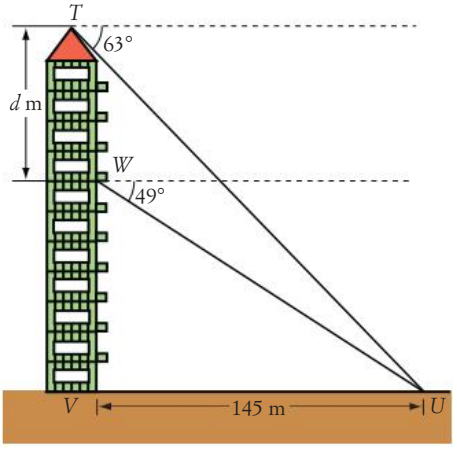


15 Jason finds that the angle of elevation of a cliff, 280 m above sea level, is 22°. After walking towards the cliff, he finds that the angle of elevation increases to 30°. How far did Jason walk? Answer correct to the nearest metre.

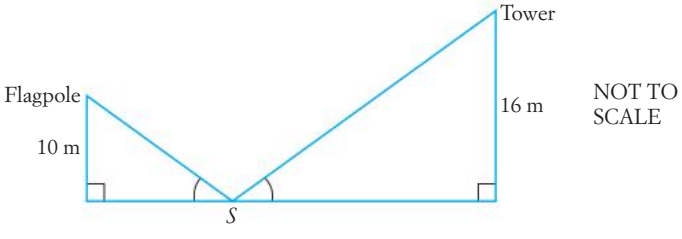


16 From the rooftop, T, of her apartment block, Rose observes the playground in the courtyard, U, at an angle of depression of 63°. From her apartment window, W, the angle of depression is 49°. The playground is 145 m from the apartment block. Show that the vertical distance between the rooftop and Rose's apartment is d metres, where

$$d = 145(\tan 63° - \tan 49°).$$



17 A point, S, lies between a flagpole, 10 metres high, and a tower, 16 metres tall. Sophie is standing between the flagpole and the tower. From Sophie's position, the angle of elevation to the top of the flagpole is 34° and to the top of the tower is 73°.



Calculate the direct distance between the base of the flagpole and the bottom of the tower.

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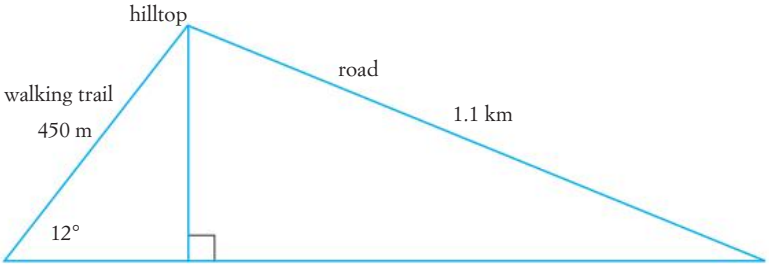
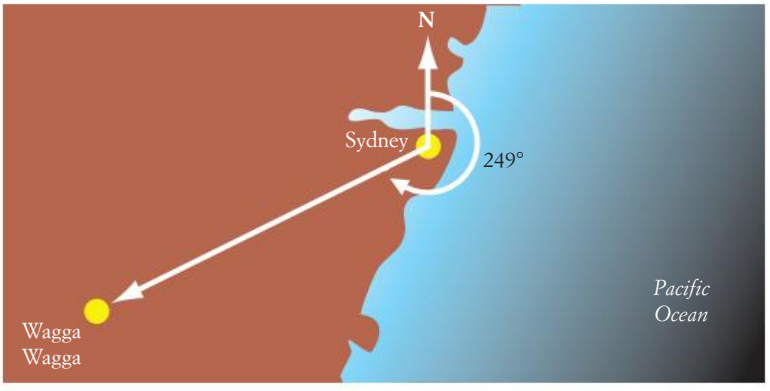
18 A hilltop is viewed from two different locations. The walking trail up the mountain is 450 m long at an angle of inclination of 12°. The bus travels on a road that is 1.1 km long on the opposite side of the mountain.



Calculate the angle of inclination of the road.

5-02 Bearings

Bearings use angles to show the direction of one location from a given point. For example, the **true bearing** of Wagga Wagga from Sydney is 249°, measured clockwise from north.



A page of bearings

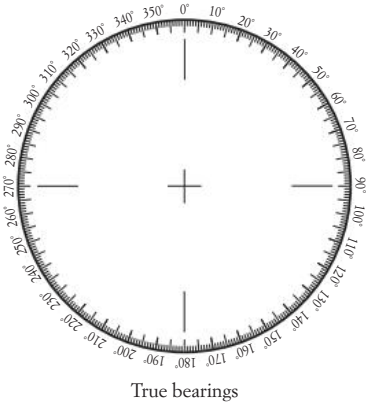
16 points of the compass

NSW map bearings

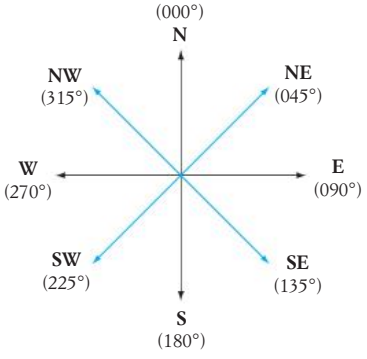
Bearings match-up

True bearings or **three-figure bearings** are angles between 000° and 360° measured in a clockwise direction from north.

Compass bearings refer to eight points of the compass: north, north-east, east, south-east, south, south-west, west and north-west.



True bearings



Compass bearings

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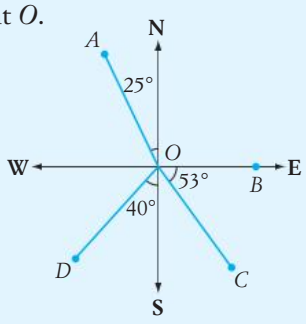
EXAMPLE 4

Write the true bearings of locations A, B, C and D from point O.



Solution

A: $360° - 25° = 335°$

B: $090°$

C: $90° + 53° = 143°$

D: $180° + 40° = 220°$

***B* is due east.**

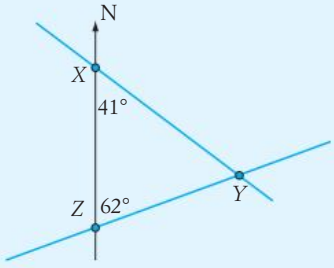
EXAMPLE 5

Find the true bearing:

a of Y from Z

b of Y from X

c from Y to X



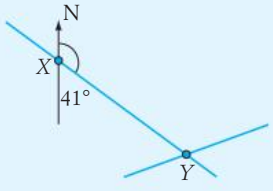
Solution

a The starting position is after the **from**, so start from Z and measure the angle to Y from north.

The true bearing of Y from Z is $062°$.

b The starting position is X, measure the angle to Y from north.

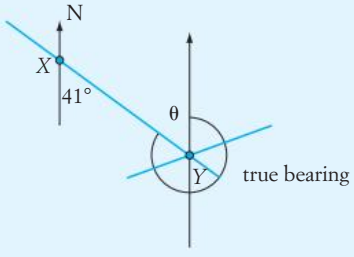
The true bearing of Y from X: $180° - 41° = 139°$



c The starting position is Y, measure the angle to X from north.

$\theta = 41°$ (alternate angles on parallel lines)

The true bearing of X from Y: $360° - 41° = 319°$



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EXAMPLE 6

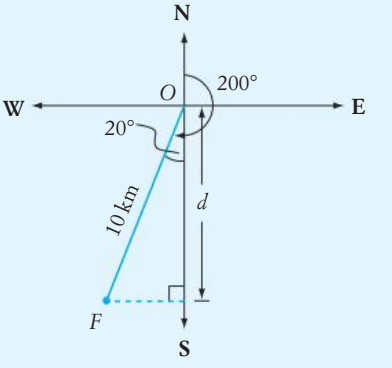
A triathlete runs 10 km on a bearing of 200° from the end of a cycle leg to the finish line.

a How far south does the triathlete run? Answer to the nearest kilometre.

b What is the bearing of the end of the cycle leg from the finish line?

Solution

Let O be the end of the cycle leg and the centre of the compass rose and F be the finish line.



a Let d be the distance the triathlete runs south.

Now $\angle FOS = 200° - 180° = 20°$

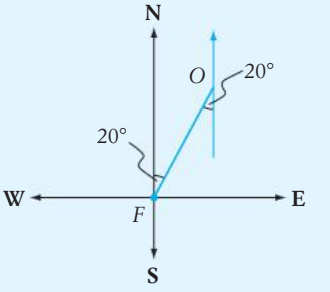
$$\cos 20° = \frac{d}{10}$$

$$d = 10 \cos 20°$$

$$= 9.3969\ldots$$

$$\approx 9 \text{ km}$$

b To find the bearing of O from F, draw another compass rose through F.



$\angle NFO = 20°$ **Alternate angles between parallel lines.**

The bearing of the end of the cycle leg from the finish line is 020°.

Take note where the bearing is *from*: this is where we draw the centre of our compass rose and *from* where we take the bearing.

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Exercise 5-02 Bearings

1 In which quadrant (NE, SE, SW, NW) does each bearing lie?

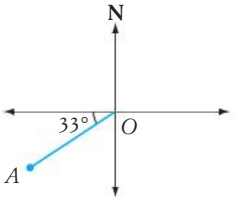
a 332° **b** 245° **c** 169°

d 007° **e** 218° **f** 095°

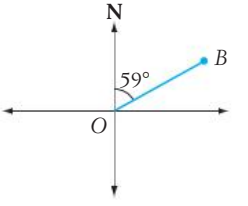
Example 4

2 Write the true bearing of each point from O.

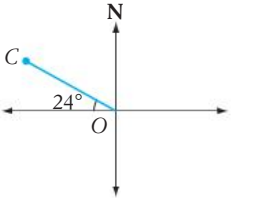
a



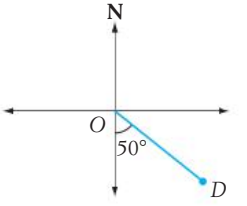
b



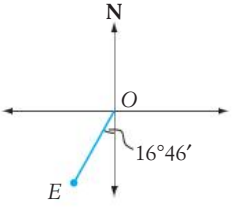
c



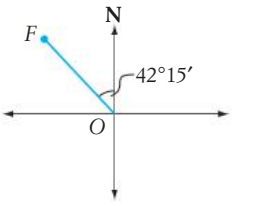
d



e



f



3 Sketch each of these bearings on a separate compass rose.

a 081° **b** 247° **c** 300°

d 107° **e** 180° **f** 282°

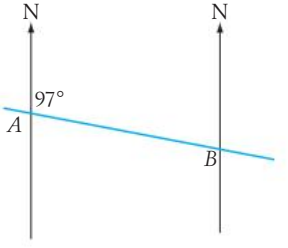
4 Write the true bearing of:

a NW **b** SW **c** NE

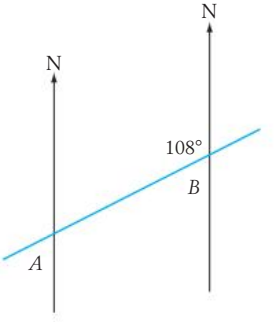
Example 5

5 Find the bearing of B from A.

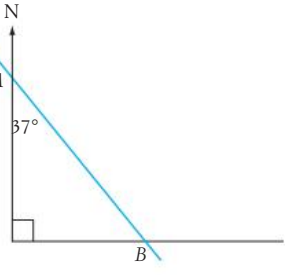
a



b

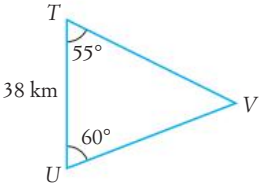


c

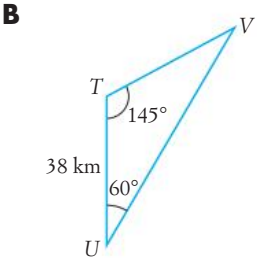


6 Tangerine (T) is 38 km due north of Upton (U). Vickery (V) is on a bearing of 145° from Tangerine and 060° from Upton. Which diagram represents this? Select **A**, **B** or **C**.

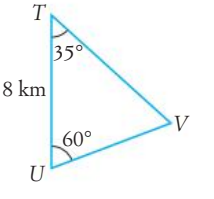
A



B



C



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Example 6

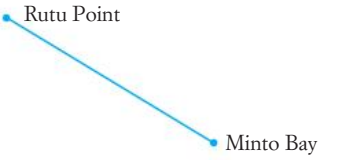
7 Amelia walked for 7.2 km on a bearing of 204° from base camp.

a How far west is she from camp? Answer correct to one decimal place.

b How far south is she from camp?

c What is the bearing of the camp from her current position?

8 Rutu Point is 6.4 km west and 4.5 km north of Minto Bay.



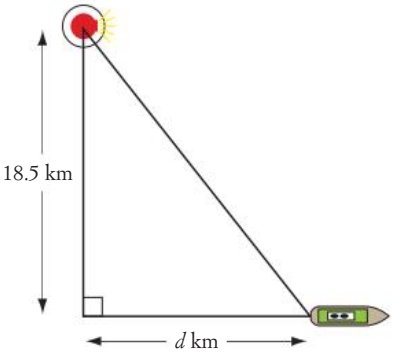
What is the bearing of Minto Bay from Rutu Point?
Select **A**, **B**, **C** or **D**.

A 125° **B** 145° **C** 305° **D** 325°

9 A ship's captain calculates that a lighthouse is 18.5 km due north of his ship. He sails due east for some time and calculates that the lighthouse is now on a bearing of 322° from the ship.

a Find the distance travelled by the ship, correct to one decimal place.

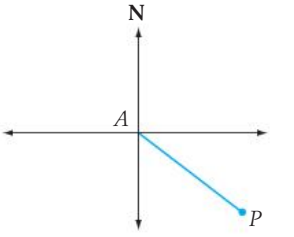
b What is the time taken to travel this distance if the ship's average speed is 48 km/h? Answer to the nearest minute.



10 A plane, P, takes off from an airport, A, at 10:15 a.m. and flies on a bearing of 150° at a speed of 700 km/h.

a How far due south of the airport, A, is the plane at 1:45 p.m.? Answer to the nearest kilometre.

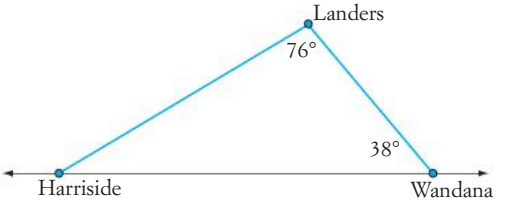
b What is the bearing of the airport from the plane then?



11 A hot air balloon travelled 11 km due south of Camden but drifted 1.1 km west. What was its true bearing from Camden, to the nearest degree?

12 Shania began a bushwalk and travelled 3 km due west. She then ran 1.5 km due north to the lookout. What was her true bearing from the lookout to her starting point for the bushwalk?

13 Three towns are shown in the diagram. What is the true bearing of Landers from Harriside?



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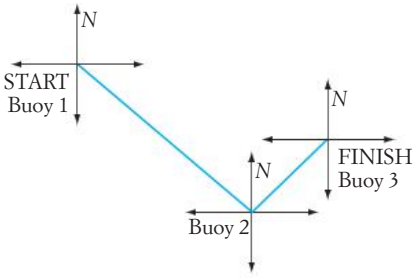
14 Jonas rode his kayak from buoy 1 on a bearing of 135° for 4.2 km then turned at buoy 2 on a bearing of 045° and continued for 2.8 km to the finish line at buoy 3. What is his true bearing from the finish line (buoy 3) to the start (buoy 1)?



JUST FOR THE RECORD

Why 360 degrees? Why 60 minutes?

In number, measurement and currency, we use a decimal or base 10 system. But in 2000 BC, the ancient Babylonians used a base 60 system. Why did they choose 60?

- 60 is divisible by 1, 2, 3, 4, 5, 6, 10, 15, 20, 30 and 60 whereas 10 is only divisible by 1, 2, 5 and 10.
- $60 \times 6 = 360$ is the Babylonian approximation for the number of days in a year, so they made one revolution equal to 360°, meaning that the Earth travelled 1° around the Sun each day of their year.

The Babylonian influence on our measurement system has lasted over 4000 years. Our units for measuring time and angles still follow a base 60 system.

- 1 hour = 60 minutes = 3600 seconds
- 1 month = 30 days
- 1 day = 24 hours
- 1 year = 12 months

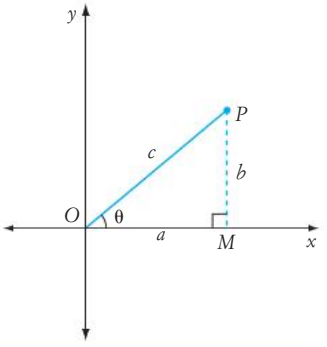
Many imperial units of measurement and currency use multiples of 6. Find out how many:

a inches in a foot **b** feet in a yard **c** feet in a fathom
d items in a dozen **e** items in a gross **f** pence in a shilling

WS
Trigonometry calculations

5-03 Trigonometry with obtuse angles

Trigonometry can be used with triangles that are not right-angled. To do this, we must define the sine, cosine and tangent ratios for angles greater than 90° and less than 180°—that is, **obtuse angles**.



In the first diagram, for the acute angle θ in $\triangle PMO$, we have:

$\sin\theta = \frac{b}{c}$, $\cos\theta = \frac{a}{c}$, $\tan\theta = \frac{b}{a}$

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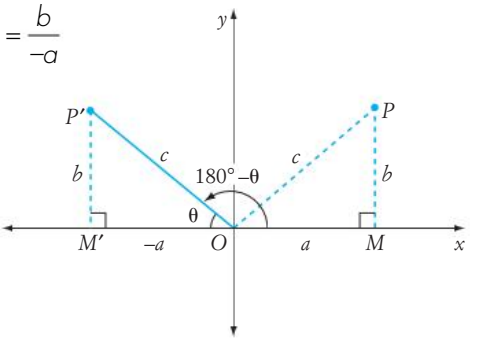
In the second diagram, $\triangle PMO$ has been reflected across the y-axis to create $\triangle P'M'O$, where $OM' = -a$ and $\angle P'OM = 180° - \theta$ is obtuse.

Now we can use $\triangle P'M'O$ to define the trigonometric ratios for obtuse $\angle P'OM$:

$$\sin(180° - \theta) = \frac{b}{c}, \cos(180° - \theta) = \frac{-a}{c}, \tan(180° - \theta) = \frac{b}{-a}$$

Comparing these with the ratios of sin θ, cos θ and tan θ, we have:



SUMMARY

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

$\tan(180° - \theta) = -\tan\theta$

θ and $(180° - \theta)$ are supplementary angles because they add to 180°.

For acute angles, sin, cos and tan are positive.

For obtuse angles:

- sin is positive
- cos and tan are negative.

EXAMPLE 7

Use a calculator to find, correct to four decimal places,

a sin 50° **b** sin 130° **c** cos 50°

d cos 130° **e** tan 50° **f** tan 130°

Solution

a $\sin 50° \approx 0.7660$ **b** $\sin 130° \approx 0.7660$ **sin 130° = sin 50°**

c $\cos 50° \approx 0.6428$ **d** $\cos 130° \approx -0.6428$ **cos 130° = –cos 50°**

e $\tan 50° \approx 1.1918$ **f** $\tan 130° \approx -1.1918$ **tan 130° = –tan 50°**

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EXAMPLE 8

Calculate θ, to the nearest degree, if θ is obtuse:

a $\sin\theta = 0.8081$ **b** $\cos\theta = -0.3576$ **c** $\tan\theta = -2.4192$

Solution

a $\sin\theta = 0.8081$

$\theta = 53.9107\ldots°$ **Calculator sequence:** SHIFT sin **0.8081** =

If sin θ is positive, θ can be acute or obtuse, but the calculator gives the acute angle only.

For obtuse θ:

$\theta = 180° - 53.9107\ldots°$ **The angle supplementary to 53.9107...°.**

$= 126.0892\ldots°$

$\approx 126°$

Checking: $\sin 126° = 0.8090\ldots \approx 0.8081$

b $\cos\theta = -0.3576$

$\theta = 110.9528\ldots°$ **Calculator sequence:** SHIFT sin (−) **0.3576** =

$\approx 111°$

Checking: $\cos 111° = -0.3583\ldots \approx 0.3576$

c $\tan\theta = -2.4192$

$\theta = -67.5417\ldots°$ **Calculator sequence:** SHIFT sin (−) **2.4192** =

If tan θ is negative, the calculator gives a negative answer for θ, which must be added to 180° to calculate the obtuse angle.

For obtuse θ:

$\theta = -67.5417\ldots° + 180°$ **The angle supplementary to 67.5417...°.**

$= 112.4582\ldots°$

$\approx 112°$

Checking: $\tan 112° = -2.4750\ldots$

≈ -2.4192

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Exercise 5-03 Trigonometry with obtuse angles

1 Positive or negative?

a $\sin\theta$ for obtuse θ **b** $\sin\theta$ for acute θ

c $\tan\theta$ for obtuse θ **d** $\cos\theta$ for obtuse θ

e $\cos\theta$ for acute θ **f** $\tan\theta$ for acute θ

2 Calculate each ratio, correct to four decimal places. Example 7

a $\cos 146°$ **b** $\tan 120°$ **c** $\sin 163.7°$

d $\tan 175°$ **e** $\cos 105°23'$ **f** $\sin 116.2°$

g $\cos 129°57'$ **h** $\sin 100.29°$ **i** $\tan 143°38'$

3 Is θ acute or obtuse or both?

a $\tan\theta$ is positive **b** $\cos\theta$ is negative **c** $\sin\theta$ is positive

d $\cos\theta$ is positive **e** $\tan\theta$ is negative

4 Find A, to the nearest minute, if A is acute.

a $\sin A = 0.8756$ **b** $\cos A = \frac{15}{17}$ **c** $\tan A = 2.0341$

d $\cos A = 0.2925$ **e** $\tan A = \frac{15}{4}$ **f** $\sin A = \frac{12}{18}$

g $\tan A = 0.8042$ **h** $\cos A = \frac{3}{10}$ **i** $\tan A = \frac{2}{14}$

5 Find θ, to the nearest degree, if θ is obtuse. Example 8

a $\cos\theta = -0.2566$ **b** $\tan\theta = -\frac{6}{11}$ **c** $\sin\theta = 0.7890$

d $\cos\theta = -\frac{3}{5}$ **e** $\sin\theta = \frac{1}{4}$ **f** $\tan\theta = -3.8522$

g $\sin\theta = \frac{4.7}{5}$ **h** $\cos\theta = -\frac{6}{15}$ **i** $\tan\theta = -\frac{7.3}{14.1}$

j $\tan\theta = -1.4285$ **k** $\cos\theta = -0.5$ **l** $\tan\theta = -1$

6 Evaluate these trigonometric expressions, correct to two decimal places.

a $\frac{14\sin 55°}{\sin 143°}$ **b** $5^2 + 7^2 - 2 \times 5 \times 7 \cos 80°$

c $4^2 + 11^2 - 2 \times 4 \times 11 \cos 132°10'$ **d** $\frac{12\sin 25°17'}{19}$

e $\frac{1}{2} \times 3 \times 10 \sin 151°$ **f** $\frac{6\sin 38.5°}{\sin 67.6°}$

g $\frac{6^2 + 5^2 - 12^2}{2 \times 6 \times 5}$ **h** $\frac{6.7\sin 118°}{13.2}$

i $\frac{1}{2} \times 6 \times 7 \sin 84°40'$ **j** $\frac{13^2 + 8^2 - 10^2}{2 \times 13 \times 8}$

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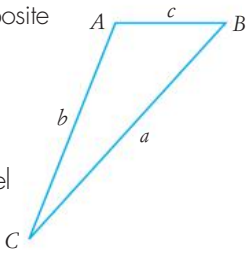
Discovering the sine rule

5-04 The sine rule

The **sine rule** is a relationship between the sides of a triangle and their opposite angles. Notice that in any triangle, the smallest angle is always opposite the shortest side ($\angle C$ and c in the diagram on the right) and the largest angle is always opposite the longest side ($\angle A$ and a in the diagram on the right).



If we label the angles of a triangle with capital letters A, B and C, then we label the sides opposite these angles with matching lower case letters a, b and c.

SUMMARY

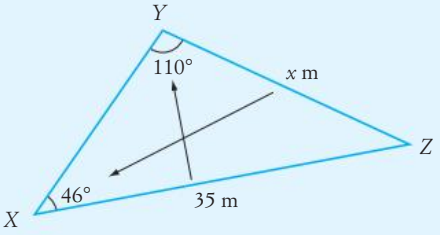
The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine rule is used with two sides of a triangle and the angles opposite these sides.

EXAMPLE 9

Calculate x, correct to three significant figures.



Solution

$$\frac{x}{\sin 46°} = \frac{35}{\sin 110°}$$

$\frac{x}{\sin X} = \frac{y}{\sin Y}$

Making x the subject.

$$x = \frac{35 \sin 46°}{\sin 110°}$$

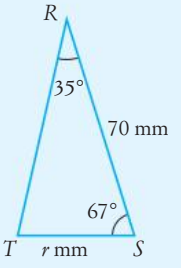
$$= 26.7926\ldots$$

$$\approx 26.8$$

From the diagram, an answer of 26.8 m looks reasonable.

EXAMPLE 10

In $\triangle RST$, $\angle R = 35°$, $\angle S = 67°$, $t = 70$ mm. Find r, correct to one decimal place.



Solution

$$\frac{r}{\sin 35°} = \frac{70}{\sin T°}$$

$\frac{r}{\sin R} = \frac{t}{\sin T}$

First find $\angle T$ using the angle sum of a triangle.

$\angle T = 180° - 35° - 67° = 78°$

$$\frac{r}{\sin\ 35°} = \frac{70}{\sin\ 78°}$$

$$r = \frac{70\ \sin\ 35°}{\sin\ 78°}$$

$$= 41.0473\ldots$$

$$\approx 41.0$$

From the diagram, an answer of 41.0 mm looks reasonable.

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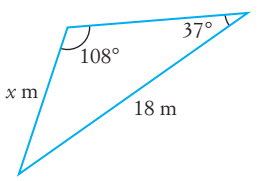
Exercise 5-04 The sine rule

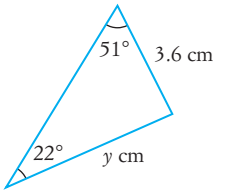
Example 9

1 Find the value of the pronumeral, correct to two decimal places.

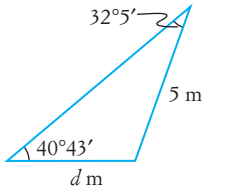
a



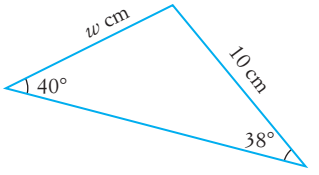
b



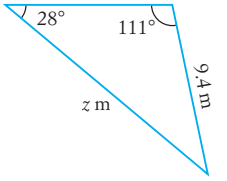
c



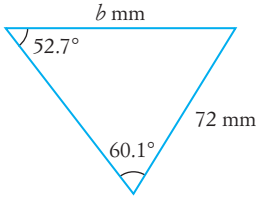
d



e



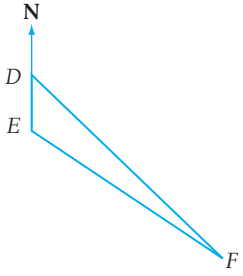
f



2 Doomben (D) is due north of Everett (E). Fiskell (F) is 18 km on a bearing of 133° from Doomben and on a bearing of 124° from Everett. Copy and complete the diagram, then calculate to the nearest 0.1 km the distance between:

a Everett and Fiskell

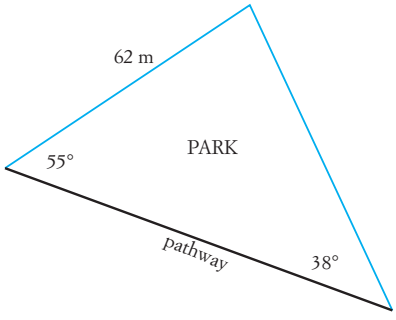
b Everett and Doomben.



Example 10

3 In $\triangle MTV$, $\angle M = 71°$, $\angle T = 46°$ and $TV = 18.3$ cm. Calculate the length of MV, correct to one decimal place.

4 A park is to have a pathway constructed along one boundary, as shown in the diagram below. Find the length of the path, correct to the nearest metre.



5 Ian sails on a bearing of 163° from A to B for 31 nautical miles. He then sails on a bearing of 195° to C, where he is directly south of A.

a Draw a triangle ABC showing this information.

b Explain why $\angle ABC = 148°$.

c Calculate how far south Ian has travelled, to the nearest nautical mile.

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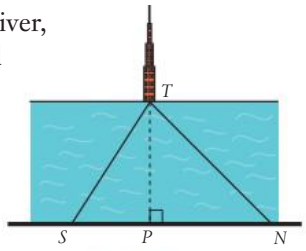
6 Susannah and Natalie, standing 230 m apart on one side of a river, observe a tower on the opposite side. Given that $\angle S = 78°$ and $\angle N = 66°$, calculate to the nearest metre:

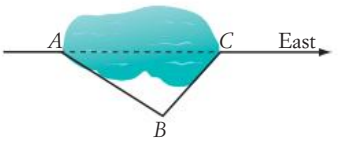


a the distance ST between Susannah and the tower

b the distance NT between Natalie and the tower

c the width TP of the river, using either ST or NT.

7 A surveyor travelling due east needs to make a detour on a bearing of 128° to avoid a flooded area. After detouring 8 km, he turns exactly north-east and travels until he joins the original line of travel.



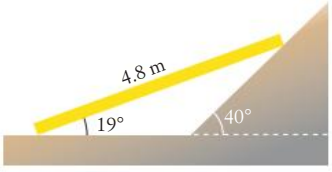
a Copy the diagram and mark on it all the information given.

b Find the size of each angle in $\triangle ABC$.

c Calculate the length of AC, correct to one decimal place.

8 A 4.8 m plank is placed between the ground and a ramp that is inclined at 40° to the ground.

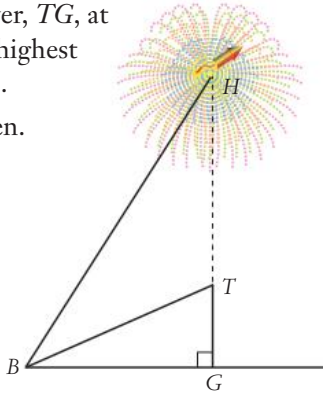
If the plank is inclined at 19° to the ground, how far up the ramp is the other end of the plank? Select **A**, **B**, **C** or **D**.



A 2.4 m **B** 4.1 m **C** 9.5 m **D** 10.6 m

9 Queenstown is 8.5 km due east of Pleasantville. The bearings of Rowley from Pleasantville and Queenstown are 037° and 324° respectively. Calculate the distance between Pleasantville and Rowley to the nearest 0.1 km.

10 Boun (B) sees fireworks being launched from the top of a tower, TG, at an angle of elevation of 38°. When the fireworks are at their highest point, H, 800 m above the tower, the angle of elevation is 80°.



a Copy the diagram and mark on it all the information given.

b Show that $\angle BTH = 128°$.

c Calculate the distance between Boun and the fireworks at H, to the nearest metre.

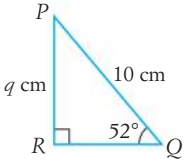
d Calculate the distance between Boun and the top, T, of the tower, correct to the nearest metre.

e Calculate the height of the tower, to the nearest metre.

INVESTIGATION

THE SINE RULE AND RIGHT-ANGLED TRIANGLES

Find q in the diagram, correct to two decimal places:



a using the sine ratio

b using the sine rule.

Why do both methods give the same answer?

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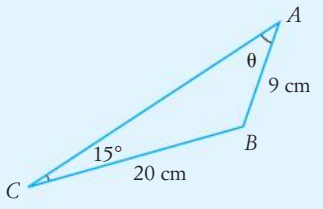
5-05 Using the sine rule to find an unknown angle

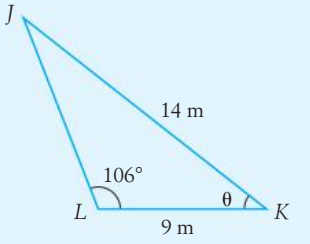
EXAMPLE 11

Find the value of θ, correct to the nearest minute, in each of these triangles.

a



b



Solution

We still look for two sides and the angles opposite them.

a

$$\frac{20}{\sin\theta} = \frac{9}{\sin 15°}$$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{\sin\theta}{20} = \frac{\sin 15°}{9}$$

Turn both fractions upside-down.

$$\sin\theta = \frac{20\sin 15°}{9}$$

Make sin θ the subject.

$$= 0.5751\ldots$$

$$\theta = 35.1103\ldots°$$

Calculator sequence: SHIFT sin ANS

$$\approx 35°6'37.36''$$

$$\approx 35°7'$$

b The side opposite θ is not known, so calculate $\angle J$ first.

$$\frac{9}{\sin J} = \frac{14}{\sin 106°}$$

$\frac{j}{\sin J} = \frac{l}{\sin L}$

$$\frac{\sin J}{9} = \frac{\sin 106°}{14}$$

$$\sin J = \frac{\sin 106°}{14}$$

$$= 0.6179\ldots$$

$$J = 38.1668\ldots°$$

Calculator sequence: SHIFT sin ANS

$$= 38°10'0.75''$$

$$\approx 38°10'$$

$$\therefore \theta = 180° - 106° - 38°10'$$

Angle sum of triangle.

$$= 35°50'$$

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Obtuse-angled answers

If sin θ is positive, θ could be acute *or* obtuse. This means that any sine rule problem involving an unknown angle could have two answers. For example, the answer to Example **10a** was 35°7′, but it also could be 180° – 35°7′ = 144°53′, an obtuse angle. You can usually tell from the diagram whether the unknown angle is acute or obtuse.

In this course we will assume the acute-angled answer only, unless the obtuse-angled answer is asked for (as in the next example).

EXAMPLE 12

Find θ, to the nearest degree, given that it is **obtuse**.

Solution

$$\frac{10}{\sin\theta} = \frac{5}{\sin 22°}$$

$$\frac{\sin\theta}{10} = \frac{\sin 22°}{5}$$

$$\sin\theta = \frac{10\sin 22°}{5}$$

$$= 0.7492\ldots$$

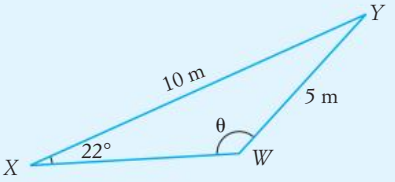
$$\theta = 48.5222\ldots°$$

$$\approx 49°$$

But θ is obtuse.

$$\therefore \theta \approx 180° - 49°$$

$$= 131°$$

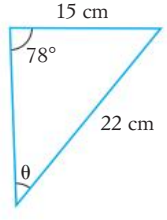


Example 11

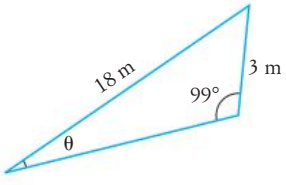
Exercise 5-05 Using the sine rule to find an unknown angle

1 Find θ in each triangle. Give your answer correct to the nearest minute.

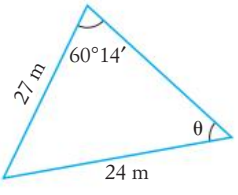
a



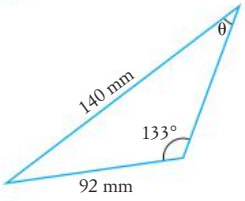
b



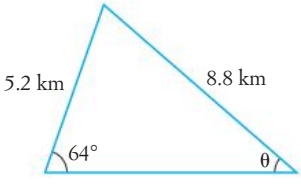
c



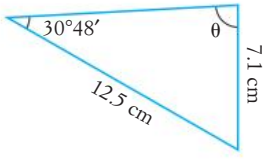
d



e



f



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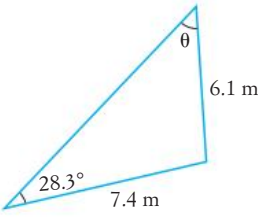
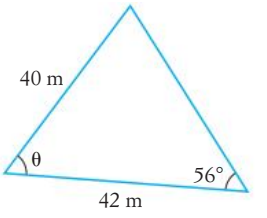
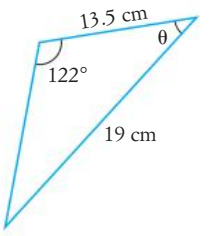
g



h



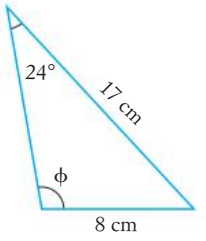
i



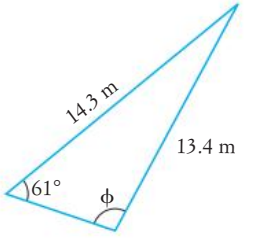
2 Find ϕ in each triangle, correct to the nearest degree, given that ϕ is obtuse.

Example 12

a



b



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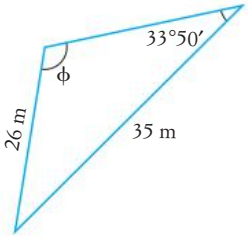


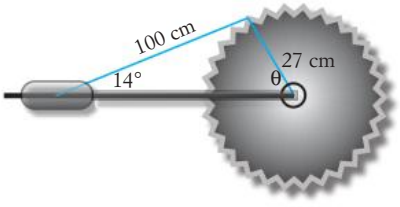
3 In $\triangle UWS$, $UW = 28$ mm, $WS = 40$ mm and $\angle S = 31°$. Calculate the size of $\angle U$, to the nearest minute, if it is:

a acute **b** obtuse.

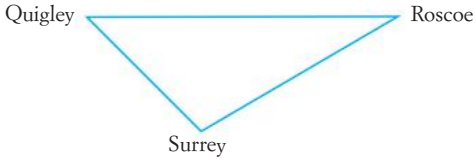
4 A surveyor positions three checkpoints M, N and P around a lake at a picnic ground. $MN = 728$ m, $MP = 638$ m and $\angle N = 57°$. Find the size of $\angle M$. Select **A**, **B**, **C** or **D**.

A 50° **B** 73° **C** 76° **D** 96°

5 The crank and connecting rod of an engine are 27 cm and 100 cm long respectively. What angle does the crank make with the horizontal when the angle made by the connecting rod is 14°? Answer to the nearest degree.



6 Surrey is 21 km on a bearing of 140° from Quigley. Roscoe is 31 km from Surrey and due east of Quigley. Find the bearing of Surrey from Roscoe to the nearest degree.



7 A bushwalker travels from A to B on a bearing of 345° for 8.5 km. She then turns and walks for 13.2 km to C, directly north of A. Show that the bearing of C from B is 010°.

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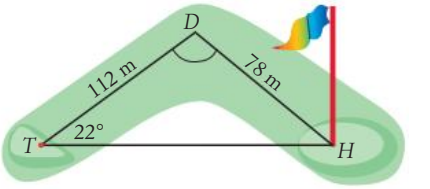
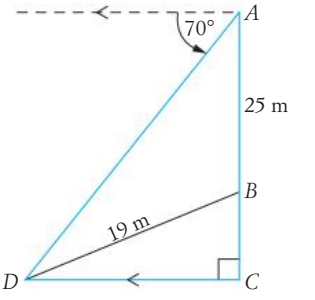
8 The 18th hole at a golf course has a tee at T, a hole at H and a dogleg at D. If $TD = 112$ m, $DH = 78$ m and $\angle HTD = 22°$, find $\angle TDH$ correct to the nearest degree.



9 James walked 2.8 km due west of home, then turned and walked on a bearing of 296° until he was 4.9 km from home. What is James's bearing from home? Select **A**, **B**, **C** or **D**.

A 011° **B** 015° **C** 281° **D** 349°

10 The angle of depression from A to D is 70°. The length of AB is 25 metres and the length of BD is 19 metres. Calculate the angle of elevation from D to B. Answer to the nearest degree.



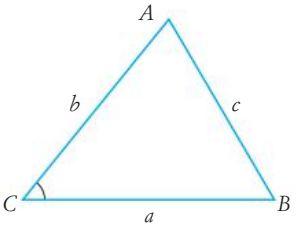
Finding an unknown side

5-06 The cosine rule

The **cosine rule** is a relationship between the three sides of a triangle and one of its angles.

The cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$ works for all triangles. ($\angle C$ can be any size)

The cosine rule is used with three sides and one angle of a triangle.



SUMMARY

The cosine rule

$c^2 = a^2 + b^2 - 2ab \cos C$

where c is the unknown side, a and b are the two known sides, and C is the angle opposite c.

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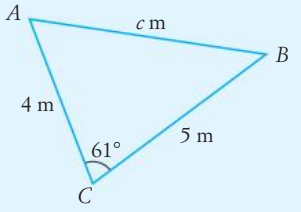
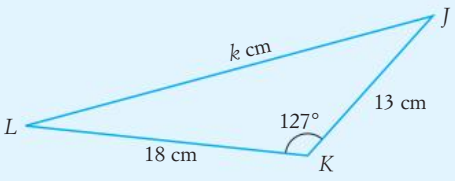
EXAMPLE 13

Find the value of the pronumeral in each triangle, correct to two decimal places.

a



b



Solution

Write the formula with the squared unknown side as the subject.

a $c^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \cos 61°$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$

$= 21.6076\ldots$

$c = \sqrt{21.6076}$ $\quad$ **Taking the square root.**

$= 4.6483\ldots$

≈ 4.65 ← From the diagram, an answer of 4.65 m looks reasonable.

b $k^2 = 18^2 + 13^2 - 2 \times 18 \times 13 \cos 127°$ $\quad k^2 = j^2 + l^2 - 2jl \cos K$

$= 774.6494\ldots$

$k = \sqrt{774.6494}$

$= 27.8325\ldots$

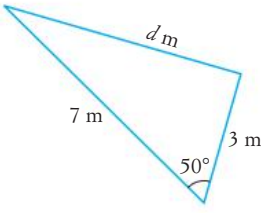
≈ 27.83 ← From the diagram, an answer of 27.83 cm looks reasonable.

Exercise 5-06 The cosine rule

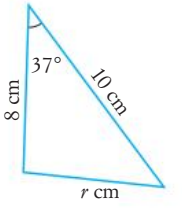
1 Find the length of the unknown side in each triangle, correct to two decimal places.

Example 13

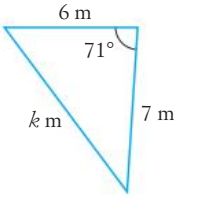
a



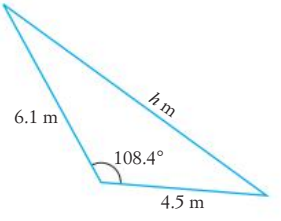
b



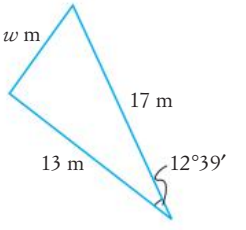
c



d



e



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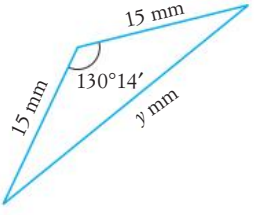

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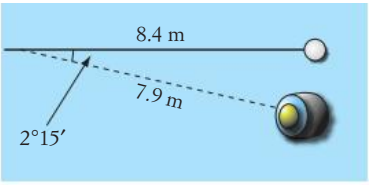
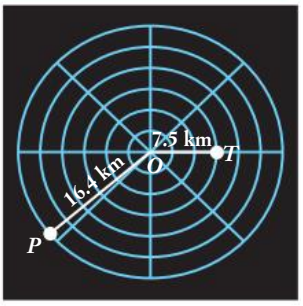
2 In a game of lawn bowls, Amie is aiming to hit the jack (target ball) 8.4 m away. If she bowls 2°15′ off-centre and her bowl travels 7.9 m, how far is her bowl from the jack? Answer correct to one decimal place.



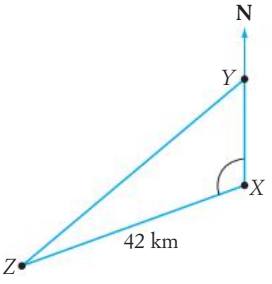
3 An air traffic controller looks on his radar screen and sees that plane P is 16.4 km from the airport, O, while plane T is 7.5 km from the airport. If he knows that the angle $\angle POT = 136°$, what is the distance between the planes? Answer correct to one decimal place.



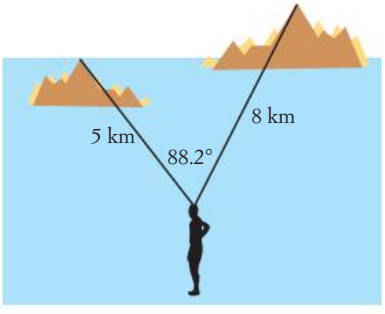
4 The bearing of Z from X is 230° and the distance of Z from X is 42 km.

a Explain why $\angle X$ is 130°.

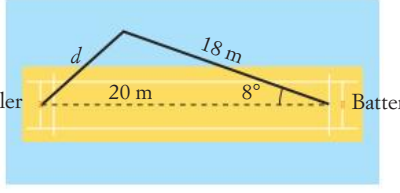
b If Y is due north of X and $XY = 23$ km, calculate the distance of Z from Y, correct to the nearest kilometre.



5 Angus observes the peaks of two hills from a plain. One is at a distance of 5 km and the other is at a distance of 8 km. If the angle between the lines of sight is 88.2°, calculate the distance between the peaks, correct to the nearest metre.



6 In a cricket match, the distance between the bowler and the batter was 20 m. During one bowl, the batter hit the ball at an angle of 8° to the line of the pitch and the bowler ran and caught the ball after it had travelled 18 m.



How far did the bowler run to catch the ball? Select **A**, **B**, **C** or **D**.

A 1.1 m **B** 2.0 m **C** 3.3 m **D** 4.0 m

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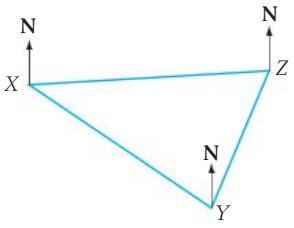
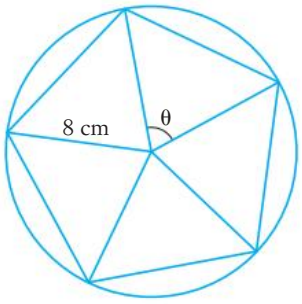
7 A yacht sails from X to Y on a bearing of 130° for 4.2 km. It then turns and travels to Z on a bearing of 025° for 2.9 km.



- **a** Copy the diagram and mark the given information on it.
- **b** Explain why $\angle XYZ = 75°$.
- **c** Calculate the distance XZ, correct to one decimal place.

8 A regular pentagon is inscribed in a circle of radius 8 cm.

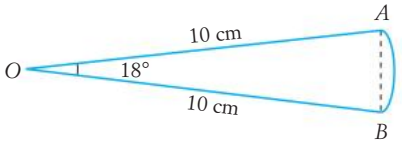


- **a** How many degrees are in a revolution?
- **b** What is the size of angle θ?
- **c** Calculate the perimeter of the pentagon, giving the answer correct to two decimal places.

9 Jonesville (J) is 84 km due north of Leeton (L). Kellyfield (K) is 60 km from Jonesville on a bearing of 148°.

- **a** Copy the diagram and clearly mark the above information on it.
- **b** Find the size of $\angle J$.
- **c** Calculate, to the nearest kilometre, the distance between Kellyfield and Leeton.

10 OAB is a sector of a circle with radius 10 cm and central angle 18°.

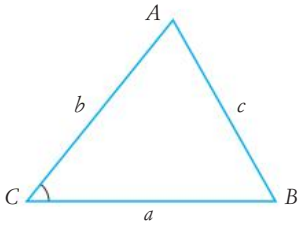


- **a** Use the arc length formula to calculate the length of arc AB, correct to two decimal places.
- **b** Use the cosine rule to calculate the length of interval AB, correct to two decimal places.
- **c** Which is longer—the arc AB or the interval AB—and by how much?

5-07 Using the cosine rule to find an unknown angle

Finding an unknown angle

The cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$ can be used to find an unknown angle if the lengths of all three sides of the triangle are known. We can rearrange the formula so that $\cos C$ is the subject.



SUMMARY

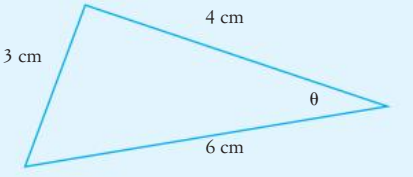
The cosine rule with cos C as the subject

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

where $\angle C$ is the unknown angle, a and b are the two sides adjacent to $\angle C$, and c is the opposite side.

EXAMPLE 14

Find θ to the nearest degree.



Solution

$$\cos \theta = \frac{4^2 + 6^2 - 3^2}{2 \times 4 \times 6} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{43}{48}$$

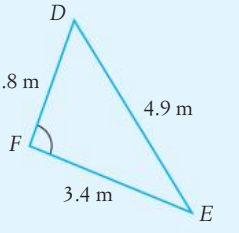
$$\theta = 26.3843\ldots°$$

$$\approx 26°$$

Calculator sequence: SHIFT cos 43 $a^b/_c$ 48 =

EXAMPLE 15

$\triangle DEF$ has sides $DE = 4.9$ m, $EF = 3.4$ m and $DF = 2.8$ m.
Find $\angle F$, to the nearest minute.



Solution

$$\cos F = \frac{3.4^2 + 2.8^2 - 4.9^2}{2 \times 3.4 \times 2.8} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{-4.61}{19.04}$$

$$F = 104.0118\ldots°$$

$$\approx 104°0'42.51''$$

$$\approx 104°1'$$

Calculator sequence: SHIFT cos ((−) 4.61 $a^b/_c$ 19.04) = .

cos F is negative so F is obtuse.

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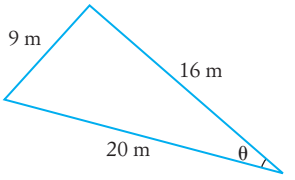
Exercise 5-07 Using the cosine rule to find an unknown angle

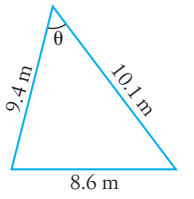
Example 14

1 Find θ, to the nearest degree, in each triangle.

a



b



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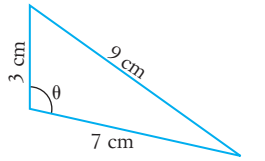

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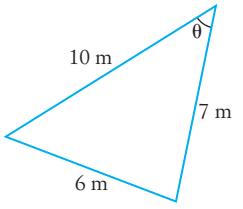

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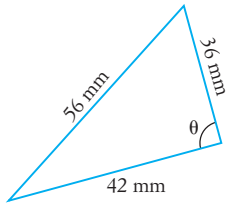

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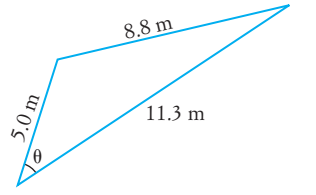
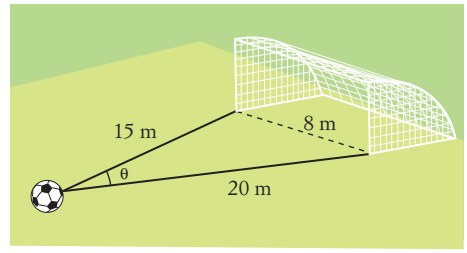


Example 15

2 In $\triangle GST$, $GS = ST = 8.4$ cm and $GT = 12.7$ cm.

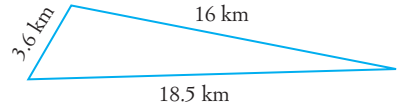
- **a** Calculate the size of $\angle S$, to the nearest degree.
- **b** Calculate the size of $\angle G$, to the nearest degree.
- **c** What type of triangle is $\triangle GST$?
- **d** What is the size of $\angle T$?

3 A soccer goal is 8 m wide. A player shoots for goal when he is 20 m from one post and 15 m from the other post. Within what angle must the shot be made to score a goal? Select **A**, **B**, **C** or **D**.



A 10° **B** 21°
C 57° **D** 102°

4 A triathlon has a triangular circuit with three legs: swimming 3.6 km, cycling 18.5 km and running 16 km. What is the size of the angle between the cycling and running legs? Answer to the nearest degree.



5 The two legs of a ladder are 2.1 m long and stand on the ground 0.9 m apart. Calculate, to the nearest minute:

- **a** the angle between the legs
- **b** the angle each leg makes with the ground.

6 A triangle has sides of length 8 cm, 15 cm and 17 cm. Find the size of the smallest angle (answer correct to the nearest minute).

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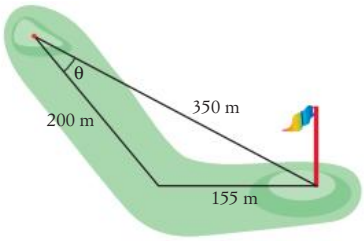
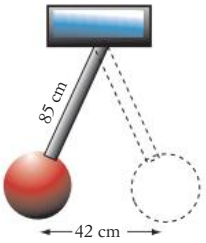
7 A golf hole is 350 m in a straight line from the tee. A ball is driven 200 m from the tee but it is off-line. The ball lands 155 m to the right of the hole. By what angle, to the nearest minute, is the shot off-line?



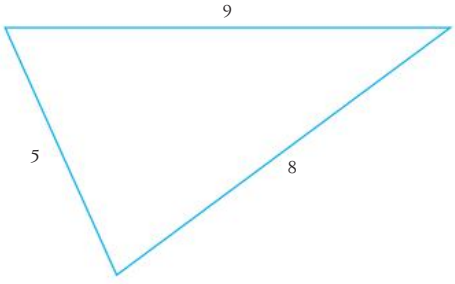
8 A pendulum of length 85 cm swings through a horizontal distance of 42 cm. What is the angle swept by the pendulum? Answer correct to the nearest degree.



9 A triangle has sides of length 8 cm, 10 cm and 16 cm. What is the size of its largest angle? Select **A**, **B**, **C** or **D**.

A 55° **B** 125° **C** 149° **D** 156°

10 What is the size of the smallest angle in this triangle?



5-08 Area of a triangle

The area of a triangle has the formula $A = \frac{1}{2}bh$, where b is the base length and h is the perpendicular height. However, there is another formula for the area of a triangle when you have been given the length of two sides and the angle between them.

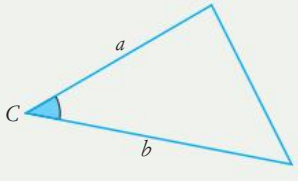
SUMMARY

Area of a triangle

In $\triangle ABC$,

$A = \frac{1}{2}ab \sin C$

where a and b are two sides and $\angle C$ is the included angle.



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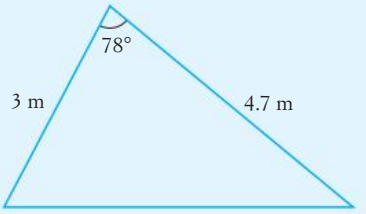
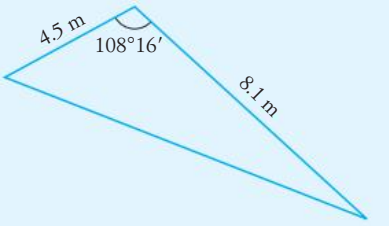
EXAMPLE 16

Calculate the area of each triangle, correct to two decimal places.

a



b



Solution

a $A = \frac{1}{2} \times 3 \times 4.7 \sin 78°$ $\quad A = \frac{1}{2}ab \sin C$

$= 6.8959\ldots$

$\approx 6.90 \text{ m}^2$

b $A = \frac{1}{2} \times 4.5 \times 8.1 \times \sin 108°16'$

$= 17.3066\ldots$

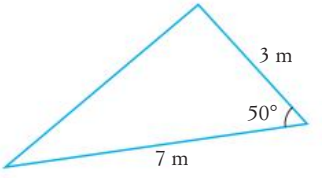
$\approx 17.31 \text{ m}^2$

Exercise 5-08 Area of a triangle

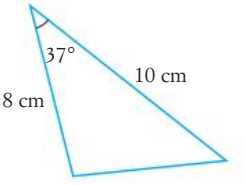
1 Calculate the area of each triangle, correct to two decimal places.

Example 16

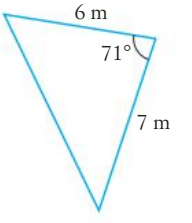
a



b



c



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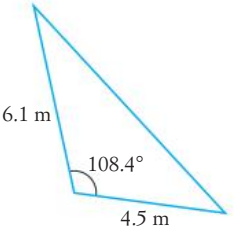
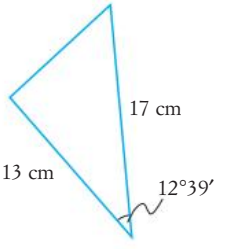


e



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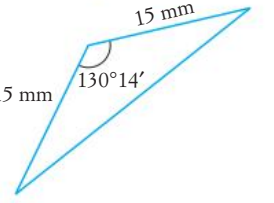

2 A parallelogram has sides of length 7 cm and 5.2 cm, and an included angle of 130°. Calculate its area, correct to one decimal place.

3 A kite has adjacent sides of length 4 cm and 7 cm. The longer diagonal makes angles of 45° and 23.8° with the sides of the kite. What is the area of the kite? Select **A**, **B**, **C** or **D**.

A 15.5 cm^2 **B** 16.5 cm^2 **C** 24.3 cm^2 **D** 26.1 cm^2

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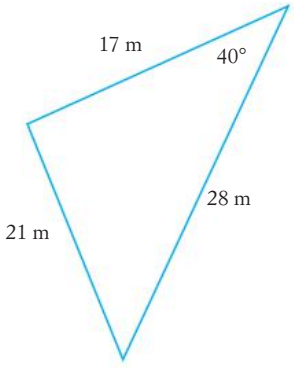
4 What is the area of this triangle, correct to the nearest square metre? Select **A**, **B**, **C** or **D**.

A 115 m²

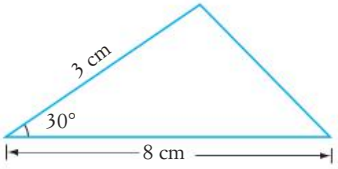
B 153 m²

C 163 m²

D 182 m²



5 Calculate the area of this triangle.

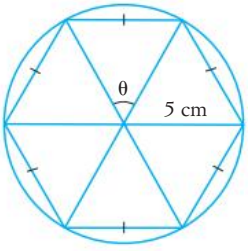


6 A regular hexagon is inscribed in a circle of radius 5 cm.

a What is the size of angle θ?

b Calculate the area of the hexagon, correct to two decimal places.

c What is the length of one side of this hexagon?



7 Calculate, correct to two decimal places, the area of an equilateral triangle with side 7 cm.

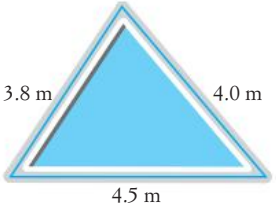
8 The area of an equilateral triangle with side s can be found using the formula $A = \frac{\sqrt{3}s^2}{4}$.

Use the formula to calculate the area of an equilateral triangle of side 7 cm and check that it agrees with the answers obtained in question **7**.

9 A traffic island has sides of length 3.8 m, 4.0 m and 4.5 m.

a Use the cosine rule to calculate, to the nearest minute, the size of one angle of the island.

b Hence, calculate the area of the island, correct to one decimal place.

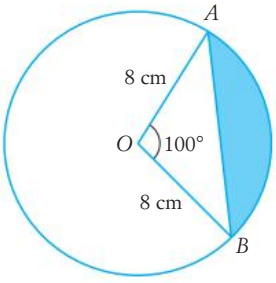


10 O is the centre of a circle with radius 8 cm.

a Calculate, to two decimal places, the area of sector OAB using the formula $A = \frac{\theta}{360}\pi r^2$.

b Calculate the area of $\triangle OAB$, correct to two decimal places.

c Hence, find the area of the shaded segment.



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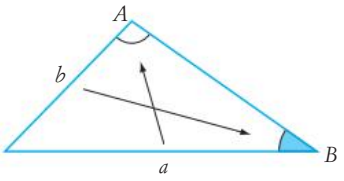
5-09 Applications of the sine and cosine rules

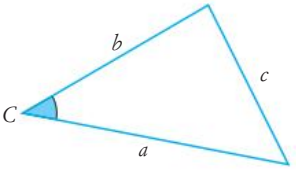
Finding an unknown side

Finding an unknown angle

The **sine rule** $\frac{a}{\sin A} = \frac{b}{\sin B}$ is used for triangle problems involving two sides and their opposite angles.

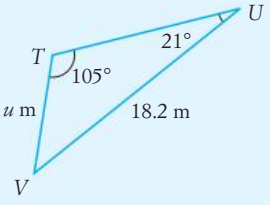


The **cosine rule** $c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ is used for triangle problems involving three sides and one angle.



EXAMPLE 17

Find u, correct to three significant figures.



Solution

The problem involves two sides and two angles: use the sine rule.

$$\frac{u}{\sin 21°} = \frac{18.2}{\sin 105°}$$

$$u = \frac{18.2 \sin 21°}{\sin 105°}$$

$$= 6.7523\ldots$$

$$\approx 6.75 \text{ m}$$

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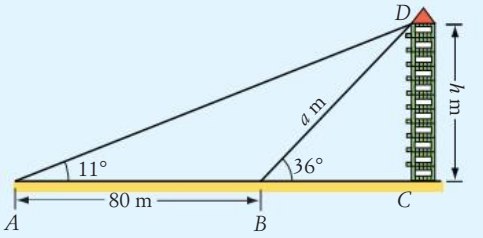
EXAMPLE 18

Kasun observes a tower at an angle of elevation of 11°. Walking 80 m towards the tower, he finds that the angle of elevation increases to 36°.



a Show that $a = \dfrac{80 \sin 11°}{\sin 25°}$.

b Hence evaluate the height, h metres, of the tower, correct to two significant figures.

Solution

a First find $\angle ADB$, then use the sine rule to find a.

$\angle ABD = 180° - 36° = 144°$ **Angles on a straight line.**

$\angle ADB = 180° - 11° - 144°$ **Angle sum of a triangle.**

$= 25°$

$$\frac{a}{\sin 11°} = \frac{80}{\sin 25°}$$

$$\therefore a = \frac{80 \sin 11°}{\sin 25°}$$

b Using the sine ratio (not the sine rule) in the right-angled triangle DCB:

$$\sin 36° = \frac{h}{a}$$

$$h = a \sin 36°$$

$$= \frac{80 \sin 11° \sin 36°}{\sin 25°}$$ **Using the answer from part a.**

$$= 21.2304\ldots$$

$$\approx 21$$

The height of the tower is 21 m.

Note: This is an example of a 'Show that…' question, where the answer to the question is given. 'Show that' questions are popular in HSC exams. Do these questions as you would any other question and aim to achieve the given answer at the end. However, you must not use the given answer in your working out.

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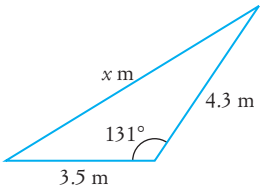
Exercise 5-09 Applications of the sine and cosine rules

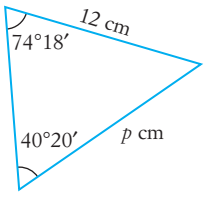
Example 17

1 Find the value of the pronumeral in each triangle, correct to two decimal places.

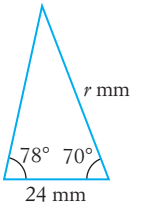
a



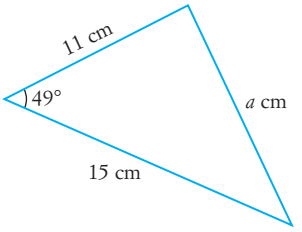
b



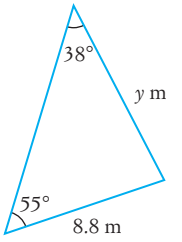
c



d



e



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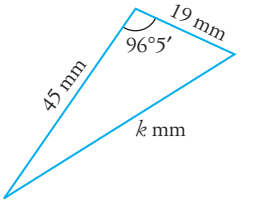
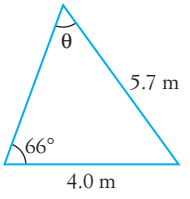
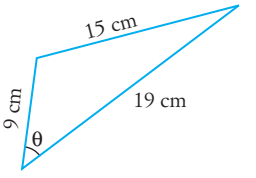


2 Find the value of θ to the nearest degree.

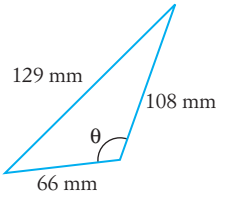
a



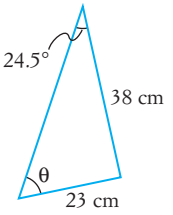
b



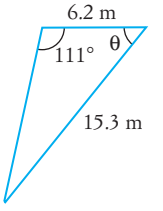
c



d



e



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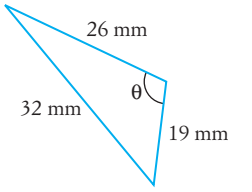
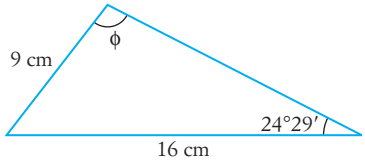


3 Find the size of ϕ, given that it is obtuse.
Select **A**, **B**, **C** or **D**.

A 47°27′ **B** 132°33′

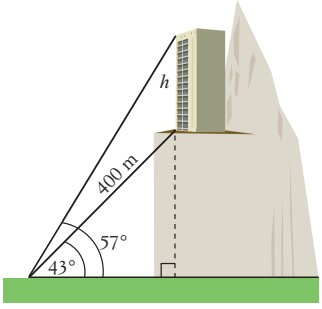
C 135°48′ **D** 166°31′



4 Rhys observes the top and bottom of a mountain resort to be at angles of elevation of 57° and 43° respectively. He is 400 m from the bottom of the resort.

a Copy this diagram and find the sizes of all angles.

b Find the height of the resort, correct to two significant figures.



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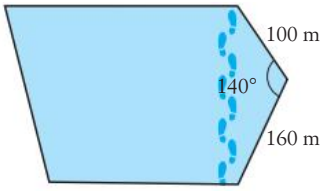
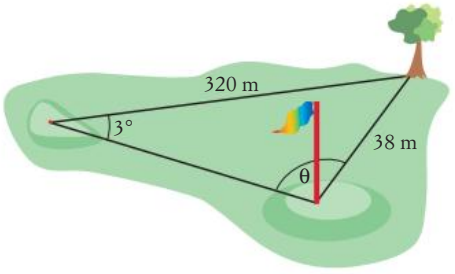
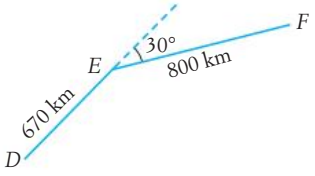
5 Instead of walking along two edges of a park, Megan takes a short cut through the park as shown. Calculate the distance saved (to the nearest metre) by taking the short cut.



6 A golfer drives a ball 320 m at an angle of 3° off-centre. The ball lands 38 m from the hole. Calculate the obtuse angle θ to the nearest minute.



7 A plane flies from Dunstan to Emmett, a distance of 670 km. It then turns 30° and flies another 800 km to Frankston. Calculate the distance between Dunstan and Frankston to the nearest kilometre.



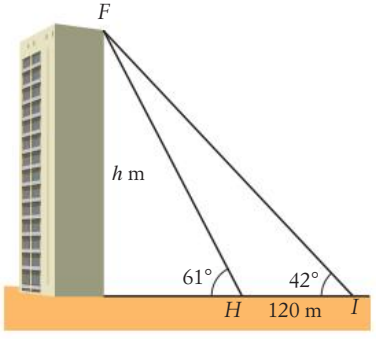
Example 18

8 At point I, Justine observes the top, F, of an office tower at an angle of elevation of 42°. She walks 120 m towards the tower to H where the angle of elevation increases to 61°.

a Explain why $\angle IFH = 19°$.

b Show that $FH = \dfrac{120 \sin 42°}{\sin 19°}$.

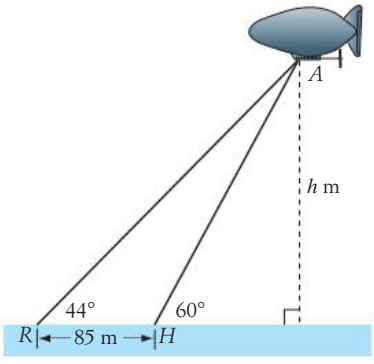
c Hence show that $h = \dfrac{120 \sin 42° \sin 61°}{\sin 19°}$ and evaluate h to the nearest metre.



9 Hayley (H) and Rebecca (R), standing 85 m apart, observe an airship (A) at angles of elevation of 60° and 44° respectively.

a Prove that the distance between Hayley and the airship is given by $HA = \dfrac{85 \sin 44°}{\sin 16°}$

b Hence, calculate the height of the airship, h metres, correct to two significant figures.



10 A parallelogram has adjacent sides of length 7 cm and 5.2 cm and an included angle of 130°. What is the length of its longer diagonal? Select **A**, **B**, **C** or **D**.

A 5.4 cm **B** 11.1 cm **C** 12.3 cm **D** 29.2 cm

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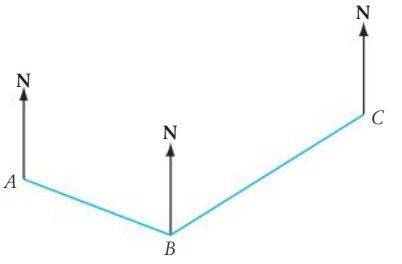
11 A yacht sails 7 km from A to B on a bearing of 124°. It then sails 10 km to C. The size of $\angle ABC$ is 116°.

a What is the bearing of C from B?

b Calculate the distance from A to C. Answer correct to the nearest kilometre.

c What is the bearing of A from C? Answer correct to the nearest degree.

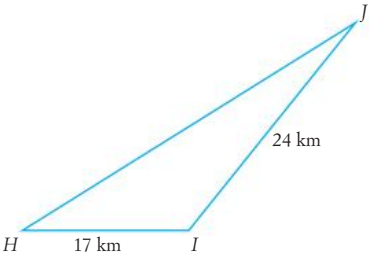


12 In the diagram on the right, Hillier (H), Innes (I) and Jokerby (J) represent the locations of three towns. Innes is due east of Hillier and the true bearing of Jokerby from Innes is 052°.

a Find the size of $\angle HIJ$.

b Find the distance from Hillier to Jokerby correct to one decimal place.

c Calculate the bearing of Innes from Jokerby.



5-10 Offset and radial surveys

Surveying involves setting out the positions of proposed construction or engineering works. It involves the measurement of the boundaries of a field and the calculation of perimeters and areas. There are a number of surveying techniques:

- **offset survey**, studied in the Preliminary Course
- Simpson's rule for approximating areas, studied in Chapter 3
- plane table radial survey
- compass radial survey
- triangulation.

When right-angled triangles are involved, surveyors use Pythagoras' theorem, the three trigonometric ratios and the area formula $A = \frac{1}{2}bh$.

When non-right-angled triangles are involved, surveyors use the sine and cosine rules and the area formula $A = \frac{1}{2}ab \sin C$.

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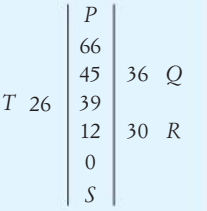
Offset survey

EXAMPLE 19

A surveyor drew this diagram in his notebook. All measurements are in metres.



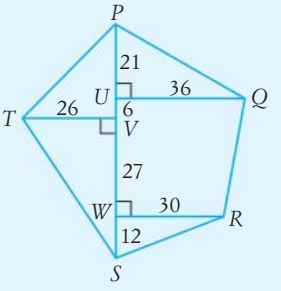
a Sketch a diagram of the field.

b Calculate the area of the field, correct to two significant figures.

c Calculate the length QR, correct to the nearest metre.

Solution

a



b The area of the field can be found by adding together the areas of three triangles and one trapezium.

Area of $\triangle PQU = \frac{1}{2} \times 36 \times 21 = 378$ m^2

Area of trapezium $UQRW = \frac{1}{2} \times (36 + 30) \times 33 = 1089$ m^2 ***UW* = 6 + 27 = 33**

Area of $\triangle RSW = \frac{1}{2} \times 30 \times 12 = 180$ m^2

Area of $\triangle PST = \frac{1}{2} \times 66 \times 26 = 858$ m^2 ***PS* = 66 from notebook diagram.**

Total area of field $= 378 + 1089 + 180 + 858$

$= 2505$

≈ 2500 m^2

c The length of QR can be found using Pythagoras' theorem.

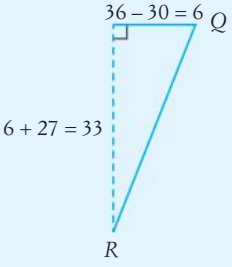
$QR^2 = 33^2 + 6^2$

$= 1125$

$QR = \sqrt{1125}$

$= 33.5410$

≈ 34 m



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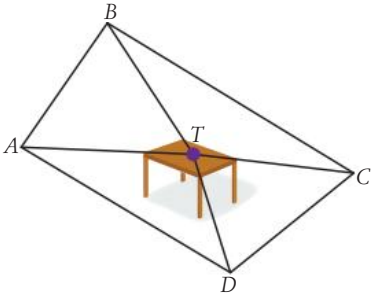
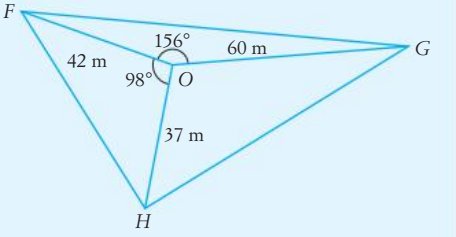
Plane table radial survey

In a **plane table radial survey**, a table is placed in the middle of the field and the angles and distances to each corner are measured and marked on a large sheet of paper pinned to the desk. The word 'radial' means coming out of a central point, like a radius or a ray.



EXAMPLE 20

This diagram shows the result for a radial survey of a field FGH.



a Find the size of $\angle HOG$.

b Find the perimeter of the field.

Solution

a $\angle HOG = 360° - 98° - 156° = 106°$ **Angles in a revolution.**

b Each side of field FGH can be calculated using the cosine rule.

$FG^2 = 42^2 + 60^2 - 2 \times 42 \times 60 \cos 156°$

$= 9968.2691\ldots$

$FG = 99.8412\ldots$ m **Do not round answers yet.**

$GH^2 = 37^2 + 60^2 - 2 \times 37 \times 60 \cos 106°$

$= 6192.8298\ldots$

$GH = 78.6945\ldots$ m

$HF^2 = 42^2 + 37^2 - 2 \times 42 \times 37 \cos 98°$

$= 3565.5499\ldots$

$HF = 59.7122\ldots$ m

Perimeter $= 99.8412\ldots + 78.6945\ldots + 59.7122\ldots$

$= 238.2479\ldots$

≈ 240 m

Round answer to two significant figures as measured values are correct to two significant figures.

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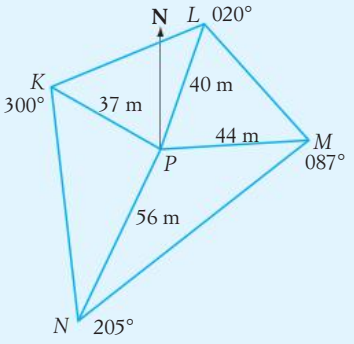
Compass radial survey

A **compass radial survey** is similar to a plane table survey except that the true bearing of each corner is measured by a compass placed in the middle of the field.

EXAMPLE 21

This diagram shows the result of a compass radial survey.



a Calculate the size of the four angles that meet at P.

b Hence calculate the area of the field $KLMN$.

Solution

a $\angle KPL = (360° - 300°) + 20° = 60° + 20° = 80°$

$\angle LPM = 87° - 20° = 67°$

$\angle MPN = 205° - 87° = 118°$

$\angle NPK = 300° - 205° = 95°$

b The area of field $KLMN$ is found by adding the areas of four triangles.

$$\text{Area } \triangle KPL = \frac{1}{2} \times 37 \times 40 \sin 80° = 728.7577\ldots \text{ m}^2$$

$$\text{Area } \triangle LPM = \frac{1}{2} \times 40 \times 44 \sin 67° = 810.0442\ldots \text{ m}^2$$

$$\text{Area } \triangle MPN = \frac{1}{2} \times 56 \times 44 \sin 118° = 1087.7914\ldots \text{ m}^2$$

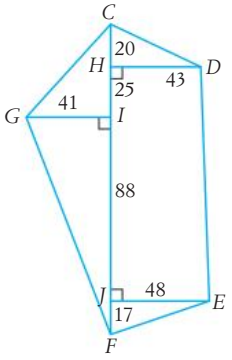
$$\text{Area } \triangle NPK = \frac{1}{2} \times 37 \times 56 \sin 95° = 1032.0577\ldots \text{ m}^2$$

$$\text{Area } KLMN = 728.7577\ldots + 810.0442\ldots + 1087.7914\ldots + 1032.0577\ldots$$
$$= 3658.6490\ldots$$
$$\approx 3700 \text{ m}^2$$

Round the answer to two significant figures as measured values are correct to two significant figures.

Exercise 5-10 Offset and radial surveys

1 Calculate the area of field $CDEFG$, expressing your answer correct to the nearest square metre.



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Example 19

2 This is a surveyor's notebook entry of a park *ABCD*.

a Sketch a diagram of this park and calculate its area, correct to the nearest square metre.

b Liam walks from *A* to *C* via *B*. What distance did he walk? Answer correct to the nearest 0.1 m.

		A	
		67	
D	26	41	
		25	22 *B*
		0	
		C	

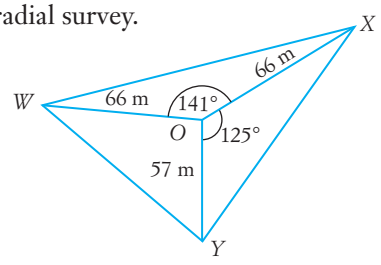
Example 20

3 This diagram illustrates the result of a plane table radial survey.

a Find the size of $\angle WOY$.

b Calculate the area of $\triangle WOY$, to the nearest square metre.

c Calculate the perimeter of the triangular field, *WXY*, correct to two significant figures.



Example 21

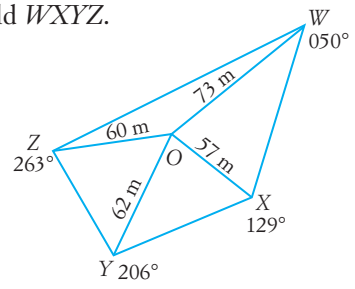
4 This diagram shows a compass radial survey of the field *WXYZ*.

a Find the size of $\angle ZOW$.

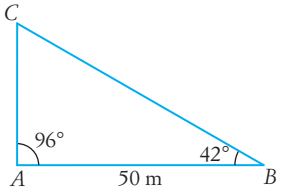
b Calculate the area of $\triangle ZOW$, correct to two significant figures.

c Find the size of $\angle YOX$.

d Calculate the length of *YX*, correct to two significant figures.



5 This figure shows an example of **triangulation**: surveying using angles of a triangle. A baseline, *AB*, is chosen and measured. A third point, *C*, is positioned and the angles from *A* and *B* are measured. If $\angle A = 96°$, $\angle B = 42°$ and $AB = 50$ m, calculate the lengths of *AC* and *CB*, correct to two decimal places.



6 This notebook entry is from an offset survey where measurements are in metres.

a Make a sketch of the field *KLMNP* and calculate its area.

b Calculate the length of *PN*, correct to one decimal place.

		K	
		57	
		36	35 *L*
P	38	30	
N	36	3	
		0	
		M	

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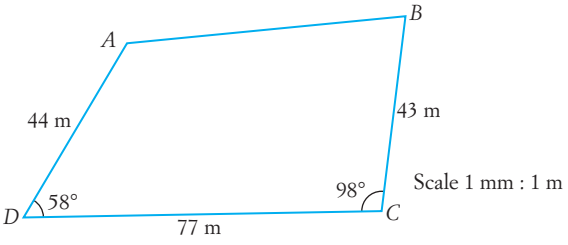
7 Accurately construct this scale diagram of the field *ABCD* on a sheet of paper.

Find the area of the field by measurement and calculation:

a using an offset survey along the traverse line *AC*

b using the formula $A = \frac{1}{2}ab \sin C$ in triangles *ADC* and *ABC*.

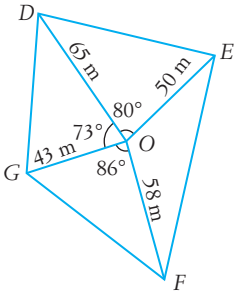
Write your answers correct to two significant figures.



8 This is a radial survey of a field *DEFG*.

a What is the size of $\angle EOF$?

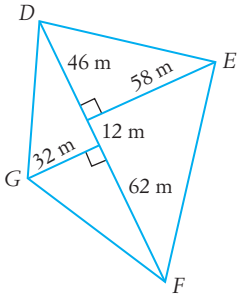
b Calculate the area of the field *DEFG*, to the nearest square metre.



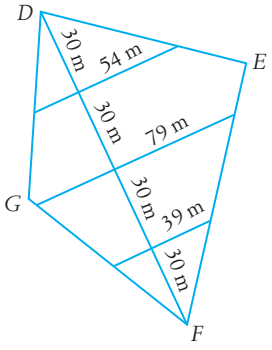
9 This is an offset survey of the same field *DEFG* from question **9**.

a What is the length of the traverse line *DF*?

b Calculate the area of the field *DEFG*.



10 In this survey of the same field *DEFG*, offsets are taken at 30 m intervals along *DF*. Use two applications of Simpson's rule $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$ to calculate the area of the field *DEFG*.



11 **a** Your answers for the area of the field *DEFG* from questions **8**, **9** and **10** are similar but not exactly the same. Why?

b Which of the three methods do you think is the most accurate? Why?

c Which method is the least accurate? Why?

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INVESTIGATION

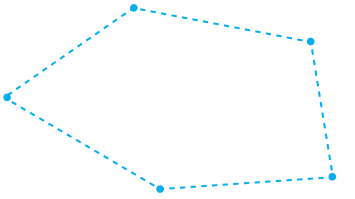
SURVEYING ON SCHOOL GROUNDS

Equipment: Traffic cone markers, teacher's protractor or navigational compass, teacher's set square or T-square, ruler, tape measure and/or trundle wheel, alidade, table, art paper, masking tape or Blu-Tack.

An alidade is a sighting device that follows the line of sight. It may be as simple as a small length of wood holding two nails.

a Stake out a field of five sides on school grounds, placing the cones on the corners.

b Select a surveying method to find the area and perimeter of the field: offset survey, plane table radial survey or compass radial survey.



c If using an offset survey, choose a traverse line and walk along it with a trundle wheel or tape measure. Measure a perpendicular offset to each corner of the field.

d If using a radial survey, set up a table in the middle of the field and tape a large sheet of blank paper to it. Place an alidade on the table. Use a protractor or navigational compass to measure the angles between each corner, then measure the distances to each corner.

e Make a scale drawing of the field and mark on it all relevant information.

f Use the scale drawing to measure and calculate the perimeter of the field.

g Use Pythagoras' theorem (offset survey) or the cosine rule (radial survey) to calculate the perimeter of the field. How does this answer compare to your scale drawing answer?

h Calculate the area of the field, to the nearest square metre.

i Choose another surveying method and compare your results.

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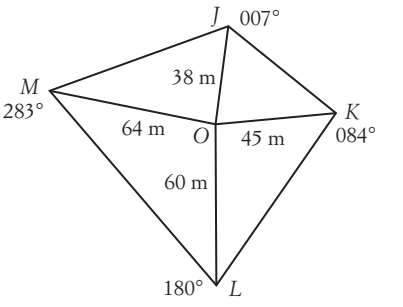
Sample HSC problem

A compass radial survey is conducted on the field $JKLM$.

a What is the size of $\angle MOL$?

b Calculate the distance between M and L, correct to the nearest metre.

c Calculate the area of the triangle MOL, correct to the nearest square metre.



Study tip

Practise with past exams

Why is practising with past exams such an effective study technique? Because it allows you to become familiar with the format, style and level of difficulty expected in an HSC exam, as well as the common topic areas tested. Knowing what to expect helps alleviate exam anxiety. Remember, mathematics is a subject in which the exam questions are fairly predictable. The people who write the exams are not going to ask many unusual questions. By the time you have worked through many past exams, this year's HSC paper won't seem that much different.

Don't throw your old exams away. Use them to identify your mistakes and weak areas for further study. Revising topics and then working on mixed questions is a great way to study maths. You might like to complete a past HSC exam under timed conditions to improve your exam technique.

You can find past HSC exam papers with solutions, practice exams and revision guides from bookstores, the library, the HSC Online website, your maths faculty or the Mathematical Association of NSW (MANSW). This textbook also contains chapter revision and practice papers.

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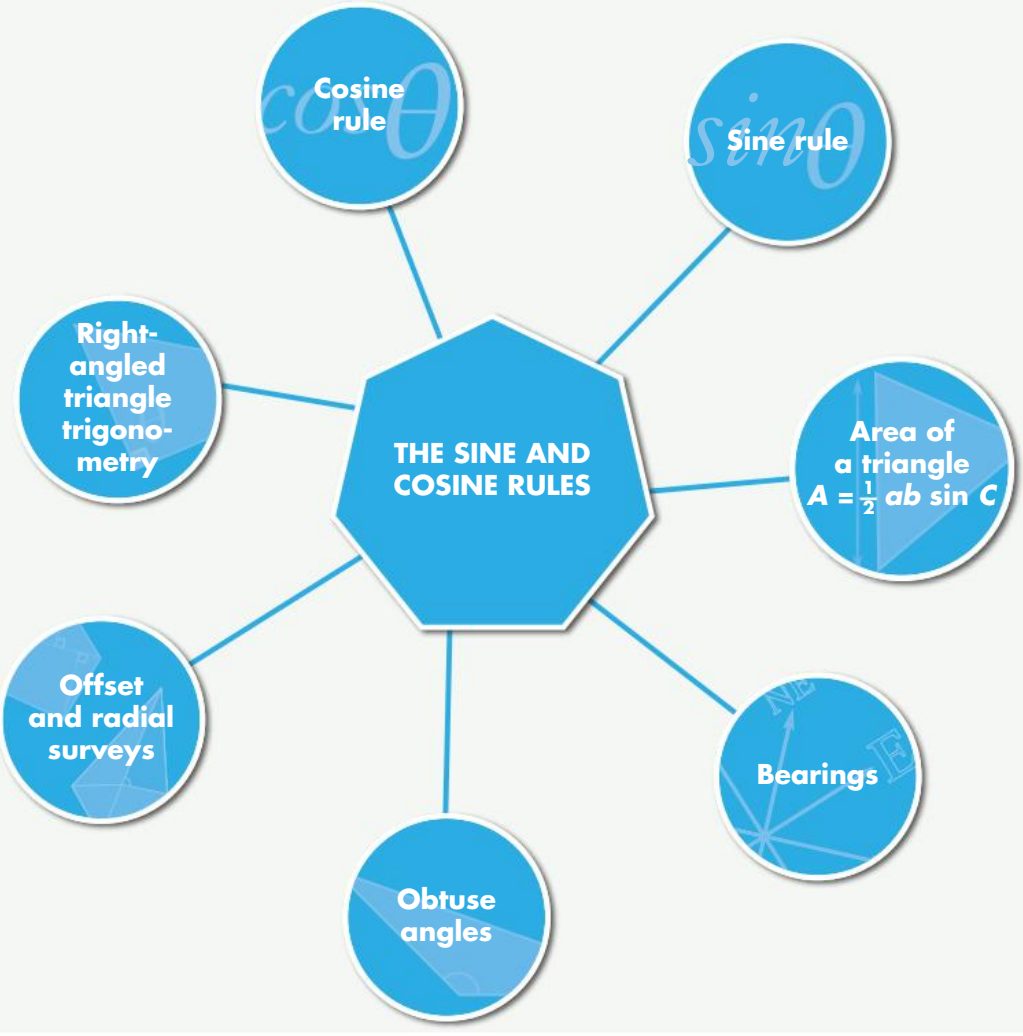
CHAPTER REVIEW

Topic overview

This chapter, The sine and cosine rules, extended trigonometry theory to non-right-angled triangles. There were three main formulas covered: the sine rule, the cosine rule, and the area of a triangle. Aim to become competent and confident in using these formulas, especially when applying them to practical problems. Be sure to differentiate between the basic sine, cosine and tangent ratios for right-angled triangles, and the sine and cosine rules for non-right-angled triangles. Also practise solving problems involving the use of bearings and angles of elevation and depression.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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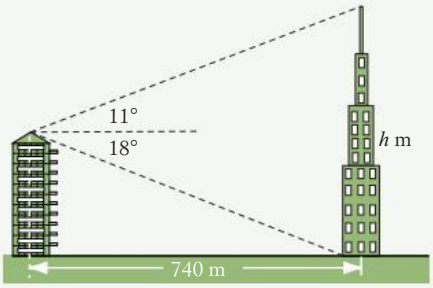
Revision

Exercise 5-01

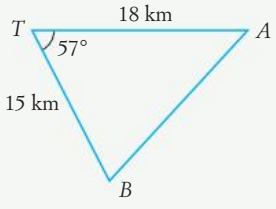
1 Micha stands on the rooftop of her apartment block and observes a tower 740 m away. The angle of elevation to the top of the tower is 11° and the angle of depression to the bottom is 18°. Calculate the height of the tower, to the nearest metre.



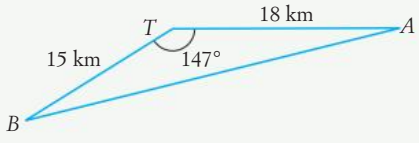
Exercise 5-02

2 Two cars left a town, T, on different roads. Car A travelled 18 km on a bearing of 088° while car B travelled 15 km on a bearing of 145°. Which diagram accurately shows this information? Select **A**, **B**, **C** or **D**.

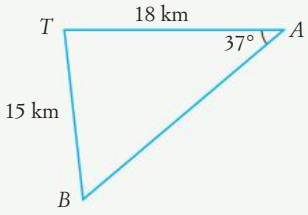
A



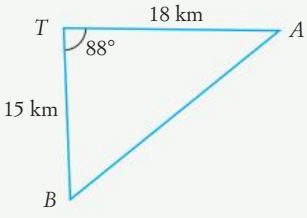
B



C



D



Exercise 5-02

3 Shelbyville is 4.2 km from Springfield on a bearing of 200°. How far south of Springfield is Shelbyville, correct to two decimal places?

Exercise 5-03

4 Is θ acute or obtuse (or both) when:

a cos θ is positive? **b** tan θ is negative? **c** sin θ is positive?

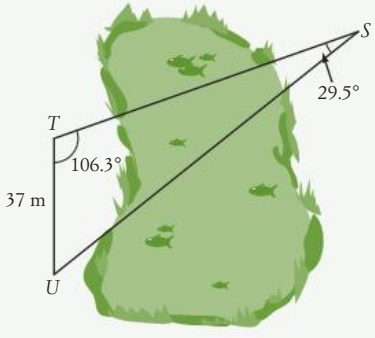
Exercise 5-03

5 Find θ, to the nearest degree, if θ is obtuse.

a cos θ = −0.8731 **b** sin θ = 0.2454 **c** tan θ = −0.9765

Exercise 5-04

6 Sophie (S) notices two trees, T and U, 37 m apart on the other side of a lake. If $\angle T = 106.3°$ and $\angle S = 29.5°$, calculate SU, the distance between Sophie and the second tree, correct to three significant figures.



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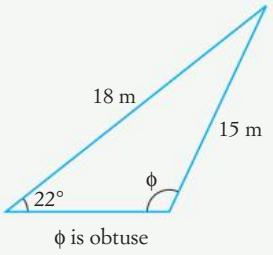
7 Find the size of ϕ in each triangle, correct to the nearest minute.

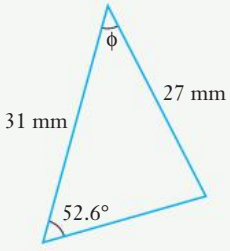
Exercise 5-05

a



ϕ is obtuse

b

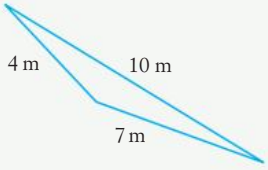


8 Two ships sail from a port. One sails 14 km due south while the other sails 17 km on a bearing of 120°. How far apart are the ships, correct to one decimal place?

Exercise 5-06

9 If the largest angle in this triangle is θ, which of these formulas is correct?

Exercise 5-07



A $\cos\theta = \dfrac{4^2+7^2-10^2}{2\times4\times7}$ **B** $\cos\theta = \dfrac{4^2+10^2-7^2}{2\times4\times10}$

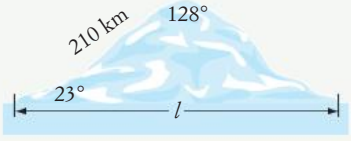
C $\cos\theta = \dfrac{7^2+4^2-10^2}{2\times7\times10}$ **D** $\cos\theta = \dfrac{10^2+7^2-4^2}{2\times7\times4}$

10 An isosceles triangle has two equal sides of length 5.1 cm and one side of length 8.3 cm. Calculate the size of one of the equal angles, correct to the nearest minute.

Exercise 5-07

11 The largest iceberg ever recorded was found in Antarctica. Its cross-section was roughly in the shape of a triangle, illustrated on the right. Calculate:

Exercise 5-08



a its length, l, correct to two significant figures

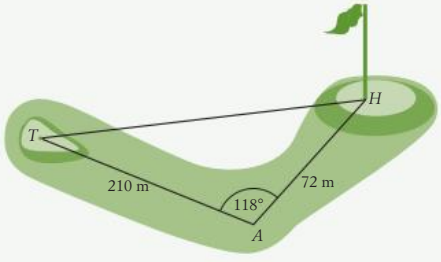
b the area of the cross-section, to the nearest square kilometre.

12 The towns of Laurieton, Merrylands and Nutbush are linked by three straight roads. Laurieton is 25 km from Merrylands and 20 km from Nutbush. Calculate, to the nearest degree, the size of the angle between the two roads coming out of Nutbush if the distance between Merrylands and Nutbush is 11 km.

Exercise 5-09

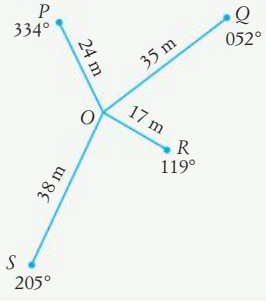
13 A golf hole at H has a dogleg at A. $TA = 210$ m, $AH = 72$ m, $\angle A = 118°$. How far is the tee, T, from the hole? Answer to the nearest metre.

Exercise 5-09



14 This diagram shows the result for a compass radial survey of a field $PQRS$.

Exercise 5-10



a Find the size of $\angle POQ$.

b Hence, calculate the length of PQ, to the nearest metre.

c Find the size of $\angle POS$.

d Hence, calculate the area of $\triangle POS$, to the nearest square metre.

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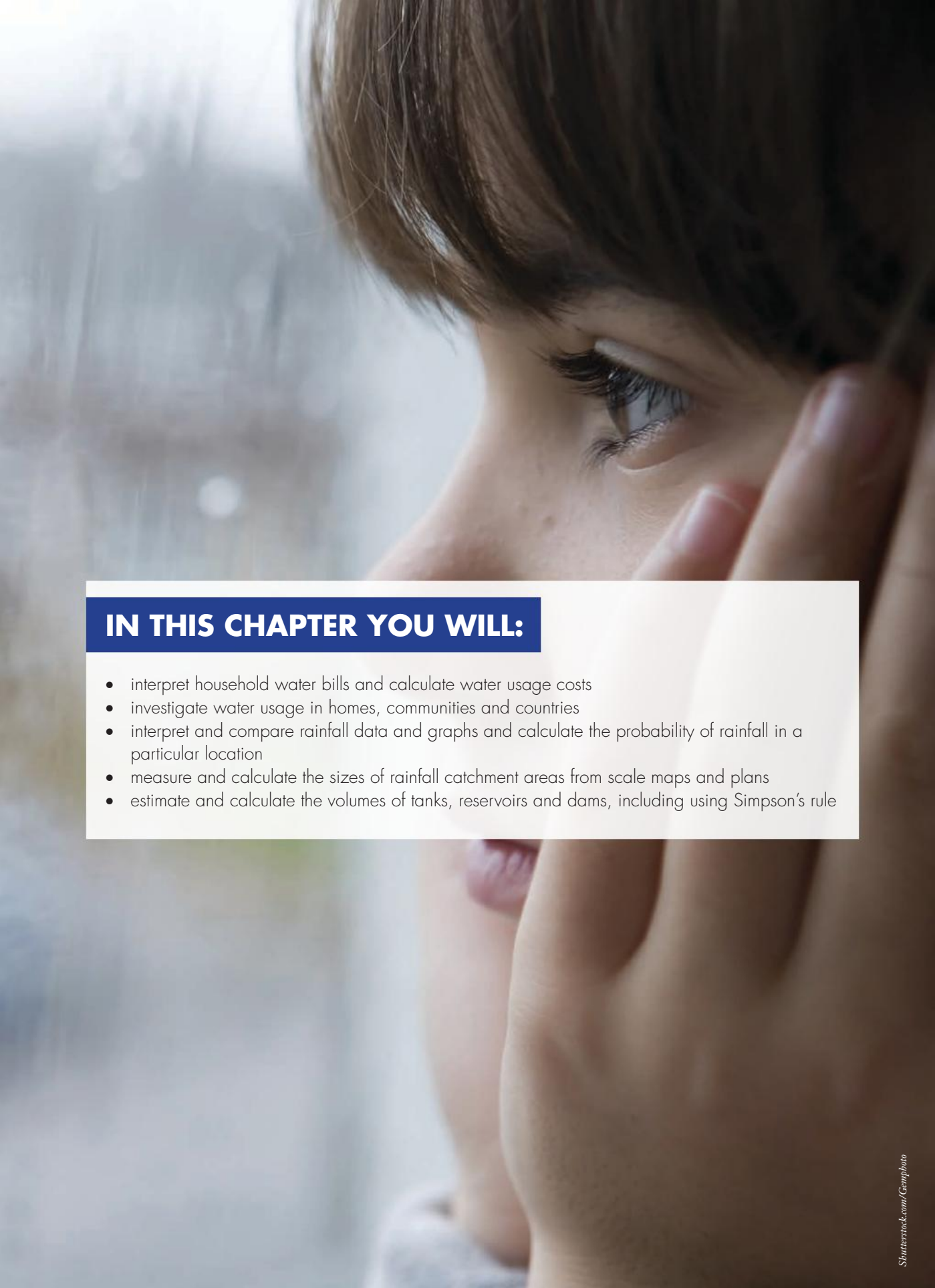
MATHEMATICS AND RESOURCES

6.

WATER USAGE

Water is an essential part of all life on Earth. In fact, approximately 70% of the Earth's surface is covered by water. It is important that we measure how much rainfall different areas receive, determine the amount of water held by our dams and record and analyse our water usage to ensure that there is enough water for people to access for everyday needs.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- interpret household water bills and calculate water usage costs
- investigate water usage in homes, communities and countries
- interpret and compare rainfall data and graphs and calculate the probability of rainfall in a particular location
- measure and calculate the sizes of rainfall catchment areas from scale maps and plans
- estimate and calculate the volumes of tanks, reservoirs and dams, including using Simpson's rule

TERMINOLOGY

approximate
area
capacity
catchment area
cubic metre
dam
gigalitre
kilolitre
land
litre
map
megalitre
perimeter
rainfall
reservoir
scale
sewerage
Simpson's rule
storage
tank
two applications

Assignment 6

SkillCheck

1 Simplify each ratio.

a 20 : 500 **b** 100 : 10 **c** 25 : 40 **d** 0.7 : 4.9

2 **a** During exercise a woman has a heart rate of 135 beats per minute. Convert this rate to beats/second.

b How many times would this woman's heart beat in half an hour?

3 The cost of 55 L of petrol is $78.65. Express this cost as a rate in cents per litre.

4 Convert:

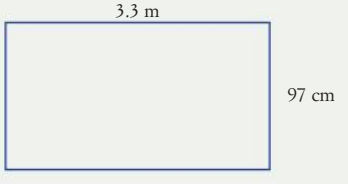
a 8.4 cm to mm **b** 9600 mm to metres

c 36 000 mL to litres **d** 2610 L to kilolitres

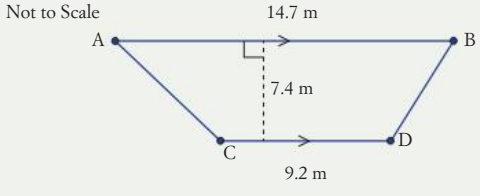
e 5.2 m^3 to L **f** 230 000 cm^2 to m^2

5 Find the area of each shape in square metres.

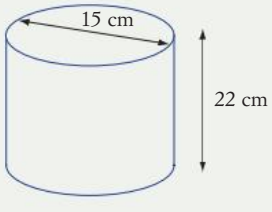
a



b



6 Calculate the volume of this cylinder correct to two decimal places.



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6-01 Water usage in the home

Your home has a **water meter** that measures the amount of water used in your home. A water bill is sent to your home every quarter listing the cost of the water service and sewerage (which are fixed), and the cost of the water used in your home (which is variable).

Water usage and costs

Water usage in your home

EXAMPLE 1

Yitong's water bill shows the following charges.

Water service	$41.39
Wastewater (sewerage)	$162.88
Water usage 33.5 kL @ $2.115 per kL	**A**
Amount due	**B**

Calculate Yitong's total water usage (**A**) and the total amount due (**B**).

Solution

$$\begin{aligned}\text{Total water usage} &= 33.5 \times \$2.115 \\ &= \$70.8525 \\ &\approx \$70.85\end{aligned}$$

$$\begin{aligned}\text{Total amount due} &= \$41.39 + \$162.88 + \$70.85 \\ &= \$275.12\end{aligned}$$

Exercise 6-01 Water usage in the home

Example 1

1 Anil's water bill shows the following charges.

Water service	$41.39
Wastewater (sewerage)	$162.88
Water usage 52.1 kL @ $2.115 per kL	**A**
Amount due	**B**

a Calculate Anil's total water usage (**A**).

b Find the total amount due (**B**).

2 For the month of June, a household used 425 L of water per day. How much water did they use for the entire month? Answer in kilolitres. Select **A**, **B**, **C** or **D**.

A 11.9 kL **B** 12.75 kL **C** 13.175 kL **D** 17 kL

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3 This is a water bill from Riverstone Water.

Riverstone WATER

General Enquiries 13 20 92
Telephone Payments (02) 9350 4455

Last bill	Credits	Balance	This bill	Total amount due
$125.30	$125.30	$0.00	$104.50	**$104.50**

Date of issue 12 May 2014

Please pay by **02/06/14**

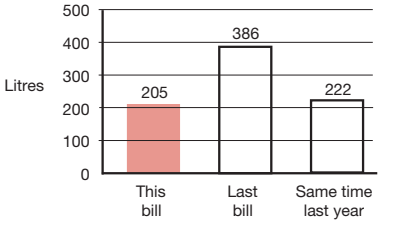
MR WALTER TANK
3/47 LAKESIDE DRIVE
RIVERSTONE NSW 2765

Account number **4402 4244 418**

Account for strata unit — 3/47 Lakeside Drive, Riverstone

Charges	**1 April 2014–30 June 2014**	$
Water service		20.00
Sewerage service		70.10
Usage charge	**10 Feb 2014–8 May 2014**	14.40
Water	18 kilolitres at 80.0 cents per kilolitre	*See over for details*

Your average daily usage — **Total amount due** **$104.50**



1000 litres = 1 kilolitre

Water Meter Details	*Date read*	*Reading*
This reading	8 May 2014	699 kilolitres
Last reading	10 Feb 2014	681 kilolitres

Total water used in 88 days was 18 kilolitres

a How many days are there from the bill's date of issue to the payment's due date?

b Is this bill paid monthly, bimonthly or quarterly?

c What are the fixed charges for, and how much are they in total?

d How many kilolitres of water were used in the period covered by this bill?

e Was more water used each day for this bill compared to the last bill? Give a possible reason for this.

f Why is it more useful to compare average daily usage with that of the same time last year?

g Verify the average daily usage of 205 L by dividing the volume of water used by the number of days covered by this bill.

h What is the cost per kilolitre for water usage?

i Verify the water usage charge of $14.40 using this rate.

j A backyard swimming pool contains 40 kL of water. Calculate the cost of filling this pool with water.

4 For the final quarter of the year (October to December), the Oliver family used 66.24 kL of water. On average, how many litres of water did they use each day? Select **A**, **B**, **C** or **D**.

A 585 **B** 690 **C** 720 **D** 736

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5 Kate's water bill shows that water usage is charged at $2.115/kL.

a If Kate used 24 kL of water, how much will she need to pay?

b Kate paid $32.57 in water usage charges on her latest bill. How many kilolitres of water did she use?

c Kate was also charged $41.39 for the water service and $162.88 for the wastewater (sewerage) service. What was the total of Kate's bill?

6 The Jager family has two adults and two teenagers. In one year, their water usage was as shown below.

Quarter	Date	Water usage (kL)
1	1 Jan 2015–31 Mar 2015	51.0
2	1 Apr 2012–30 Jun 2015	47.3
3	1 July 2015–30 Sept 2015	46.5
4	1 Oct 2015–31 Dec 2015	54.8

a What was the Jager family's total water usage for the year?

b On average, how much did they use per quarter?

c Calculate the average daily water use for 1 January to 31 March.

d The Jagers use their dishwasher twice a day, requiring 21 L of water each time. How much water did their dishwasher use over the 3rd quarter?

e Calculate what percentage of the 3rd quarter total water usage was for the dishwasher. Answer correct to one decimal place.

7 The Harris family are calculating their water usage cost for the quarter. The current reading is 8764 kL and the previous reading was 8112 kL. What was their water cost if water usage is charged at $2.115/kL? Select **A**, **B**, **C** or **D**.

A $652 **B** $985 **C** $1378.98 **D** $3569.57

8 Jeremy and Annie live in Homebush. They pay a recycled water charge of $9.37 per quarter. If they also use 12 kL of recycled water at $1.70/kL and 20.2 kL of drinking water at $2.115/kL, calculate their water usage charges for the quarter.

9 The Green family pay a recycled water charge of $11.58 per bill. On their latest bill they used 39 kL of recycled water at $1.70/kL. How much did they pay in total for recycled water? Select **A**, **B**, **C** or **D**.

A $66.30 **B** $77.88 **C** $385.32 **D** $517.92

10 The average daily water usage for two families, for the quarter 1 January–31 March, is shown below.

The Allaf family (4 people) - 560 L a day

The Johns family (6 people) - 710 L a day

For each family:

a calculate the water usage costs for the quarter if water costs $2.115 per kL and state which family pays more

b calculate the average daily water use per person and state which family uses more.

WS

Water usage in your home

JUST FOR THE RECORD

Water usage in the home

• Washing your hands/face	5 L
• Brushing your teeth (tap running)	5 L
• Brushing your teeth (tap not running)	1 L
• Cooking and making coffee/tea	8 L per day
• Flushing the toilet	9 L to 13 L
• Flushing the toilet (half flush)	4.5 L to 6 L
• Household tap	18 L per minute
• Washing the dishes (hand)	18 L
• Washing the dishes (dishwasher)	25 L per cycle
• Bath use	85 L to 150 L
• Shower (8 minutes)	80 L to 120 L
• Washing machine (front loading)	120 L per cycle
• Washing machine (top loading)	180 L per cycle
• Washing the car (with hose)	100 L to 300 L
• Garden sprinkler	1 kL to 1.5 kL per hour
• Garden hose	1.8 kL per hour
• Swimming pool (backyard)	20 kL to 55 kL

On average, a four-person Sydney house (with garden) uses 936 litres of water per day. Outside taps and toilet flushing account for half of this usage.

How much water does your household use each day? Find out by investigating your water bill.

INVESTIGATION

WATER USAGE IN YOUR HOME

Visit the Sydney Water website www.sydneywater.com.au to find out more about water bills and the costs of water usage, including recycled water and wastewater. Also investigate water efficiency in the home.

Over a one-week period, investigate the amount and cost of water used in your home, including showering and bathing, washing clothes, watering the garden, washing a car and using the toilet.

Put a bucket in your shower to measure how much water is used per minute, and time your showers. Find the amount of water that your kitchen sink, washing machine and toilet uses each time, and keep a tally of their weekly usage.

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1. Draw a graph comparing the amount of water used in different areas of your home.
2. Investigate the Water Efficient Targets on the Sydney Water website and compare your household use with the daily water consumption targets. Give an explanation of your findings.

Find a recent water bill for your home and interpret the different costs. Can you recommend any changes to your family to help save water in your home?

INVESTIGATION

WATER USAGE AROUND THE WORLD

The table shows the average daily water usage per person (in litres) for 16 countries.

1. Use a spreadsheet to graph this data on a column graph.
2. Why do you think USA and Australia use the most amount of water?
3. Investigate why India's water use is significantly lower than many Western countries.
4. Five of the countries listed use less than 100 L of water per person per day. Give two reasons why their daily water usage is so low.

US	575
Australia	493
Italy	386
Japan	374
Mexico	366
Spain	320
Norway	301
France	287
Brazil	187
UK	149
India	135
China	86
Kenya	46
Cambodia	15
Ethiopia	15
Haiti	15

www.data360.org

6-02 Rainfall

The amount of rain falling in a location is measured in **millimetres**. For example, a rainfall of 5 mm means that the amount of water falling would fill a container to a height of 5 mm.

Rainfall radar chart

EXAMPLE 2

The Gold Coast had the following daily rainfall (in millimetres) for 10 consecutive days in July.

2.6 10.2 5.6 24.4 23.6 0.6 0.0 0.6 1.2 7.0

a Find the mode.

b What is the range?

c Calculate the median and the mean.

d Explain why the mean and median values are so different.

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Solution

a Mode = 0.6 mm **Occurs twice**

b Range = 24.4 – 0
= 24.4

c For the median, first sort the 10 scores in order.
0.0 0.6 0.6 1.2 2.6 5.6 7.0 10.2 23.6 24.4
The median is the average of the 5th and 6th scores.

$$\text{Median} = \frac{2.6+5.6}{2}$$
$$= 4.1$$

$$\text{Mean} = \frac{75.8}{10}$$
$$= 7.58$$

sum of scores / number of scores

d The mean is affected by the outliers 23.6 and 24.4. This has significantly increased the value of the mean. The median is the middle score and is unaffected by outliers.

EXAMPLE 3

The table below shows the average number of rainy days per month in Hobart, Tasmania, during the months of December, January and February.

Month	Number of rainy days
December	7
January	6
February	5

Bureau of Meteorology

a Calculate the probability of rain for any particular day in:
i December **ii** February (assume 28 days in February).

b Calculate the probability of rain on two consecutive days in January.

c Calculate the probability of no rain over a weekend in February.

Solution

a **i** $P(\text{rain on a day in December}) = \frac{7}{31}$

ii $P(\text{rain on a day in February}) = \frac{5}{28}$

b $P(\text{two consecutive rainy days in January}) = \frac{6}{31} \times \frac{6}{31} = \frac{36}{961}$

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c $P(\text{non-rainy day in February}) = 1 - \frac{5}{28} = \frac{23}{28}$

$P(\text{no rain over a weekend in February}) = \frac{23}{28} \times \frac{23}{28} = \frac{529}{784}$

Exercise 6-02 Rainfall

1 For the first fortnight in July, Campbelltown had the following daily rainfall (in mm).

0 0 0.2 0 0 1.2 0 0.2 0 0

9.6 0.2 0.8 0.2

a What is the mode? **b** What is the range?

c What is the median? **d** What is the mean, correct to two decimal places?

e Based on this data, what was the probability of rain in July for Campbelltown? Select **A**, **B**, **C** or **D**.

A $\frac{5}{14}$ **B** $\frac{3}{7}$ **C** $\frac{1}{2}$ **D** $\frac{9}{14}$

Example 2

2 Nelson Bay had the following rainfall for 10 consecutive days in April 2012.

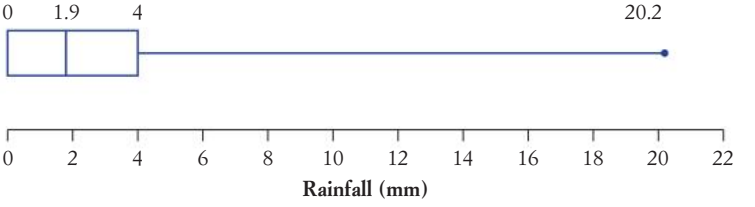
0.3 0 0 12.1 32.3 50.0 45.2 8.0 0.2 0.2

Bureau of Meteorology

a Find the mode. **b** Find the range. **c** Find the median.

d Find the mean. **e** Explain why the mean and median values are so different.

3 This box-and-whisker plot describes the rainfall figures for two weeks in February.



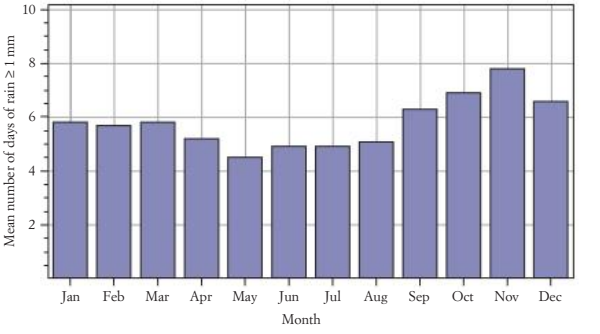
a Find the highest rainfall and the range. **b** Find the median.

c Calculate the interquartile range. **d** Describe how the data is spread.

4 The graph on the right from the Bureau of Meteorology shows the mean number of rain days in Cooma for each month.

a Which month has the most rain days?

b Which month has the least rain days?



Bureau of Meteorology

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c Which season has the least rain days?

d What is the probability that a particular month in Cooma will have more than 6 days of rain?

e What is the probability that a winter month in Cooma will have fewer than 6 days of rain?

5 This Bureau of Meteorology data shows the average number of days of rain in Katoomba.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
11.2	11.1	11.1	8.5	7.6	7.9	7.4	7.2	7.5	8.4	9.5	10.2

Bureau of Meteorology

a Use the data to construct a column graph showing the mean number of days of rain per month for Katoomba.

b Which month has the lowest number of days of rain?

c Which season has the highest number of days of rain?

d What is the probability that a particular month in Katoomba will have more than 10 days of rain on average?

e What is the probability that a **winter** month in Katoomba will have at most 8 days of rain on average?

6 The Bureau of Meteorology rainfall data (in mm) for Brisbane for each day in February 2011 is shown below.

3.6	3.8	2.0	0.2	2.2	7.0	1.6	6.4	14.2	0	1.2	0	0	0
0	21.0	2.2	19.0	0	0	0	68.8	0.2	3.6	0	0	0	0

Bureau of Meteorology

a What was the highest rainfall for the month?

b How many days had no rainfall?

c What percentage of days had no rainfall?

d What was the mean rainfall for the month, correct to two decimal places?

e In February 2012, there were 15 days without rain and the remaining days (note that it was a leap year) had the following rainfall (in mm).

0.4	3.2	0.2	32.2	1.2	14.8	1.6	20.0	0.2	5.2	10.4	58.4	14.0	0.4

i What was the highest rainfall for February 2012?

ii What percentage of days had no rainfall?

ii What was the mean rainfall for February 2012, correct to two decimal places?

iv Compare the February 2011 and 2012 data. What are the similarities and differences?

7 Use the Bureau of Meteorology website www.bom.gov.au/climate to find the climate statistics for Brisbane.

a Find 'mean rainfall' and 'number of days with rain' for February. How would you describe the historical rainfall patterns for February in Brisbane?

b Do your answers from question **6** reflect these patterns or were they unusual?

c Use 'number of days with rain' to calculate the probability of rain in February in Brisbane.

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8 This table describes the number of rain days in Mudgee in October, November and December of 2011.

Example 3

Month	No. of rain days
October	13
November	9
December	12

Calculate the probability of:

a a day in November having rain

b a day in December not having rain

c rain on two consecutive days in November

d no rain for a working week (Monday to Friday) in October

e rain on one or both days of a weekend in December.

9 The daily rainfall for Weipa in North Queensland for two weeks in March is shown below.

17.2, 19.0, 4.2, 4.6, 91.4, 28.0, 36.8, 112.4, 48.8, 29.6, 101.8, 23.4, 0.2, 0

a Calculate the range.

b Calculate the mean.

c Find the median.

d Find the standard deviation.

e Write down any outliers of this data set.

f What is the mode?

g Write a few sentences describing the rainfall for these two weeks in Weipa. Support your explanation with the statistics calculated above.

TECHNOLOGY: SUMMER RAINFALL IN MUDGEE

1 Use the Bureau of Meteorology website www.bom.gov.au to find the number of rain days in Mudgee for the past three summers.

2 Copy and complete the table below, changing the years to the past three years.

	Number of rain days		
	2012	2013	2014
December			
January			
February			
Total			

3 **a** Calculate the total number of rain days in Mudgee in those three summers.

b What is the total number of days in three summers?

c Calculate the probability of a day in summer having rain.

4 For Mudgee in summer, calculate the probability of:

a two rainy days in a row

b five rain days in a row

c a non-rainy day

d a week without rain.

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INVESTIGATION

COMPARING RAINFALL IN DIFFERENT AREAS

1 Use the Bureau of Meteorology website www.bom.gov.au to find rainfall data in your suburb or town and another suburb or town for last month.

2 Copy and complete this table with statistical calculations.

	Locations	
Mean		
Median		
Lowest score		
Highest score		
Range		
Standard deviation		
Interquartile range		

3 Construct a box-and-whisker plot for each location.

4 Use your statistical data and box-and-whisker plot to compare the rainfall in both locations. Comment on similarities and differences.

6-03 Water usage in the community

Australia is the world's driest inhabited continent, with water availability affected by drought, making water a valuable resource.

Exercise 6-03 Water usage in the community

1 The table below shows the annual per capita water consumption in Australian states and territories for 2008–09.

Per Capita Consumption (kL) 2008-09									
	Australia	**NSW**	**Vic.**	**Qld**	**SA**	**WA**	**Tas.**	**NT**	**ACT**
Per capita/Total water consumption (kL)	642	639	549	757	719	611	906	681	137
Per capita/Household water consumption (kL)	81	75	63	70	75	145	136	173	78

Australian Water Association

'Per capita' means 'per person'

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a Which state or territory had the highest total water consumption? What was the amount?

b Which state or territory had the lowest household water consumption? What was the amount?

c Which states and territories had a higher per capita total water consumption than the national per capita total water consumption? What do you think is the reason for this?

d Which states and territories had a higher per capita household water consumption than the national per capita total household consumption? What do you think is the reason for this?

e Construct a graph (on paper or using technology) comparing the household water consumption in each state and territory. Include a title and labels.

2 The water consumption by sector in Australia for 2008–09 is shown in the table below.

Sector	Water consumption (gigalitres)
Agriculture	6995
Forestry and Fishing	101
Mining	508
Manufacturing	677
Electricity and gas	328
Water supply	2396
Other industries	1327
Household	1768

Australian Water Association adapted from http://www.awa.asn.au/Water_Facts.aspx

a How many litres in a megalitre (ML)?

b If one gigalitre is 1000 ML, how many litres are there in a gigalitre (GL)?

c Calculate the total water consumption for Australia in 2008–09.

d What percentage (correct to one decimal place) of total water consumption does Agriculture represent?

e In 2004–05, the water consumption for Agriculture was 12 191 GL. How does this compare with the water consumption for Agriculture in 2008–09?

f Construct a sector graph (on paper or using technology) for this data. Include a title and labels.

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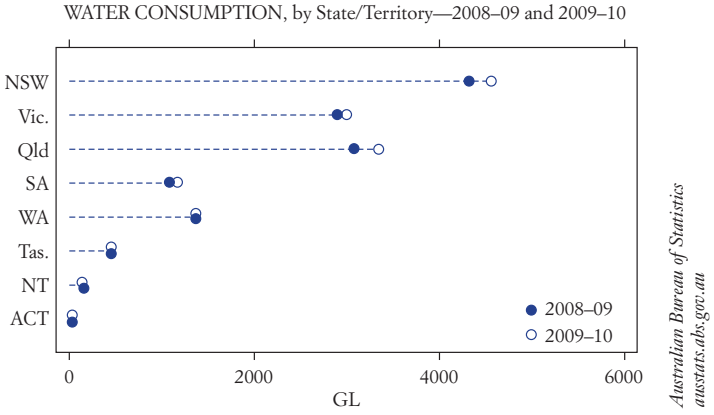
3 This graph compares water consumption in Australian states and territories in 2008–09 and 2009–10.



a Estimate the water consumption in 2009–10 for:

i Western Australia

ii Queensland

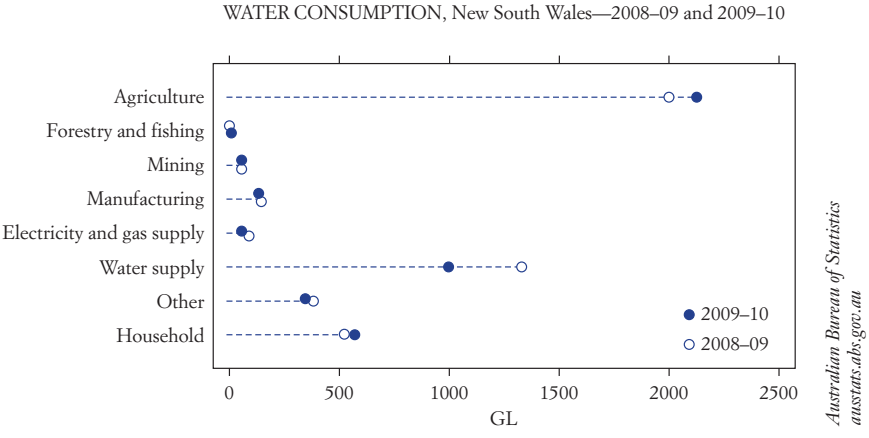
iii New South Wales

b Which states and territories used less water in 2009–10 than in 2008–09?

c Give two reasons why New South Wales, Victoria and Queensland have a significantly higher water consumption than the other states and territories.

d Estimate how many more gigalitres of water were consumed by New South Wales in 2009–10 than by Queensland.

4 The graphs below show the water consumption of two different states, NSW and WA, comparing 2009–10 with 2008–09.



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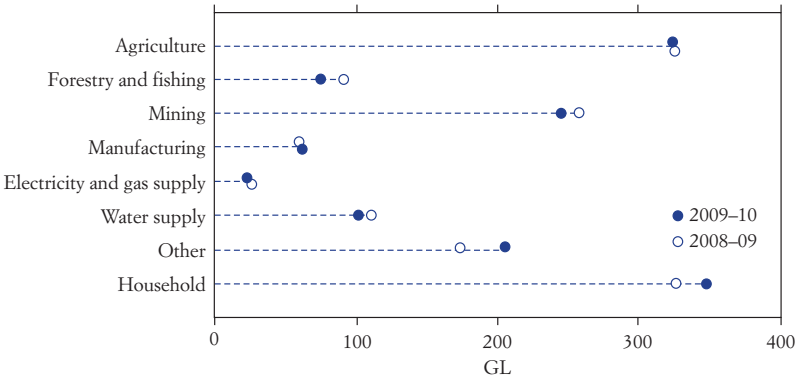
WATER CONSUMPTION, Western Australia—2008–09 and 2009–10



Australian Bureau of Statistics ausstats.abs.gov.au

a Look at the scale used on each graph. What do you notice?

b What are the three main uses of water in WA? How does this compare with the three main uses of water in NSW?

c NSW in 2009–10 consumed 236 GL less water than in 2008–09. If this represented a decrease of 5%, calculate how much water was consumed in NSW altogether.

d Households consumed 565 GL of the total water used in NSW. What percentage of total water consumption does this represent?

e Describe any similarities and differences between NSW and WA in the water usage for:

i mining **ii** households

f In WA the mining industry used 245 GL out of a total 1386 GL of water consumed in 2009–10. What percentage (correct to one decimal place) does this represent?

5 This table shows the monthly storage levels of Warragamba Dam in Sydney in 2011–2012, listing the percentage of the dam's capacity and the level of water below capacity in metres.

	A	B	C
1	**2011/ 2012 Warragamba Dam storage**		
2			
3	**Month**	**Available storage %**	**Storage level (m)**
4	Jul-11	78.1	-6.26
5	Aug-11	79.1	-5.95
6	Sep-11	79.3	-5.9
7	Oct-11	79.3	-5.9
8	Nov-11	78.8	-6.05
9	Dec-11	81.3	-5.3
10	Jan-12	80.8	-5.44
11	Feb-12	88.1	-3.31
12	Mar-12	96.8	-0.88
13	Apr-12	98.6	-0.37
14	May-12	96	-1.1
15	Jun-12	99.2	-0.18

Sydney Catchment Authority

a What was the available storage in October 2011?

b What is the level below capacity (in metres) in February 2012?

c What is the percentage decrease in the available storage of Warragamba Dam from October to November 2011?

d How much did the storage level rise at Warragamba between January and February 2012?

e In which month was the dam at its highest level?

f In which month was the dam at its lowest level?

g Construct a graph of the available storage of the dam over the 12 months.

h Describe the change in the available storage over the 12 months.

i Warragamba Dam holds 2 027 000 ML when full. How many megalitres of water did it hold in March 2012?

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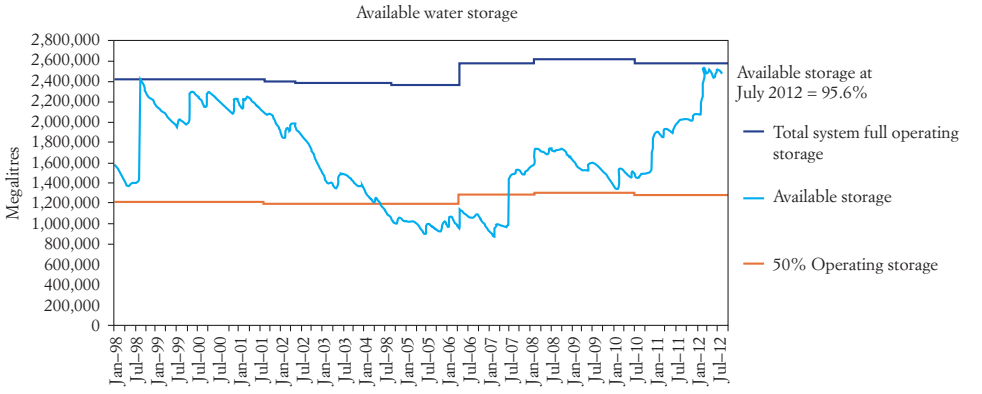
6 This graph shows the available water storage for NSW from January 1998 to July 2012.



Sydney Catchment Authority

a Estimate the available water storage in July 2011.

b Give another time between 1998 and 2012 when the available water storage was at the same level as July 2011.

c When was the available water storage at its lowest and what was this amount?

d Why do you think the water storage was so low at this time?

e How many megalitres was 'total system full operating storage' in July 2012?

f How many megalitres of water is considered to be 'half full' in January 2001?

g For how many years was the available water storage above 2 000 000 ML?

7 The table below shows the household costs of water usage in dollars per kilolitre in the urban areas of Australian states and territories for 2009–2010.

	NSW	Vic.	Qld	SA	WA	Tas.	NT	ACT
Expenditure $/kL	2.12	1.96	2.46	2.34	1.82	1.46	1.08	2.40

Australian Bureau of Statistics
ausstats.abs.gov.au

a Which state or territory has the highest household expenditure per kilolitre?

b Which state or territory has the lowest household expenditure per kilolitre?

c For the data in the table, find:

i the mean **ii** the range **iii** the median

iv the interquartile range **v** the standard deviation

d Construct a box-and-whisker plot of this data, on paper or using technology.

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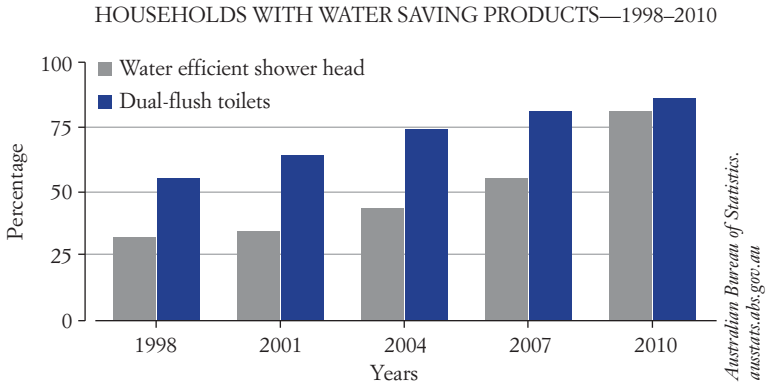
8 This graph shows the percentages of Australian bathrooms using water-efficient shower heads and dual-flush toilets from 1998 to 2010.



a Which is the more popular water-saving product?

b What percentage of Australian households had water-efficient shower heads in 2007?

c In which year did 63% of households have dual-flush toilets?

d In which year did the largest increase occur for water-efficient shower heads?

e Calculate the overall percentage increase in dual-flush toilets installed from 1998 to 2010.

f Briefly explain why there has been a significant increase in households installing water-saving products between 2001 and 2010.

INVESTIGATION

WATER COSTS: SYDNEY VS MELBOURNE

Investigate the costs of water usage in Sydney and Melbourne, or any two Australian cities, including your own. You may find the websites www.sydneywater.com.au and www.citywestwater.com.au useful. Investigate and explain any similarities and differences between the two cities, looking at the following water usage costs:

- water service charges
- cost of drinking water per kilolitre
- cost of recycled water
- wastewater (sewerage) charges
- stormwater charges

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Scale maps

6-04 Scale maps and plans

Length and area in relation to land, catchment areas and water storage can be calculated using scale diagrams.

EXAMPLE 4

Calculate the scale used on this photograph of an Olympic pool that has a standard length of 50 metres.

Alamy/Paul Kennedy

Solution

Scaled length = 2.5 cm **By measurement**

Scale = 2.5 cm : 50 m

= 2.5 cm : 5000 cm **1 m = 100 cm**

$= \frac{2.5}{2.5} : \frac{5000}{2.5}$

= 1 : 2000

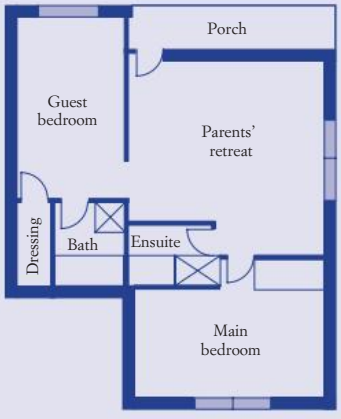
EXAMPLE 5

This is the floor plan of an extension to a house.

a Find the length across the guest bedroom.

b Calculate the area of the guest bedroom (excluding the dressing area).

New extension



Scale 1 : 200

Solution

a Scaled length of guest bedroom = 14 mm **By measurement**

Actual length of guest bedroom = 14 mm × 200 **Scale is 1 : 200**

= 2800 mm

= 2.8 m

b Scaled width of guest bedroom = 24 mm

Actual width of guest bedroom = 24 mm × 200

= 4800 mm

= 4.8 m

Area of guest bedroom = 2.8 × 4.8

= 13.44 m^2

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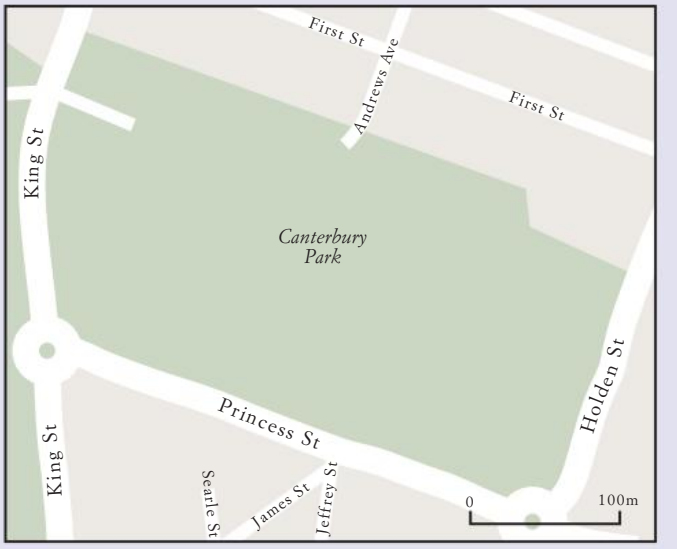
EXAMPLE 6

a What does 1 cm represent on this map of Canterbury Park?



b Estimate the perimeter of Canterbury Park.

c Estimate the area of Canterbury Park.

Solution

a Scale = 2 cm : 100 m

$= 1\,\text{cm} : 50\,\text{m}$

1 cm represents 50 m

b Using a ruler, measure the length of each boundary of the park. Assume that the park has the shape of a trapezium.

Top parallel side = 7.4 cm × 50 = 370 m

Right side = 3.1 cm × 50 = 155 m

Bottom parallel side = 6.3 cm × 50 = 315 m

Left side = 3.9 cm × 50 = 195 m

Perimeter = 370 + 155 + 315 + 195

= 1035 m

c Area $= \frac{1}{2}(370 + 315) \times 155$

$= 53\,087.5\ \text{m}^2$

Area of a trapezium formula.

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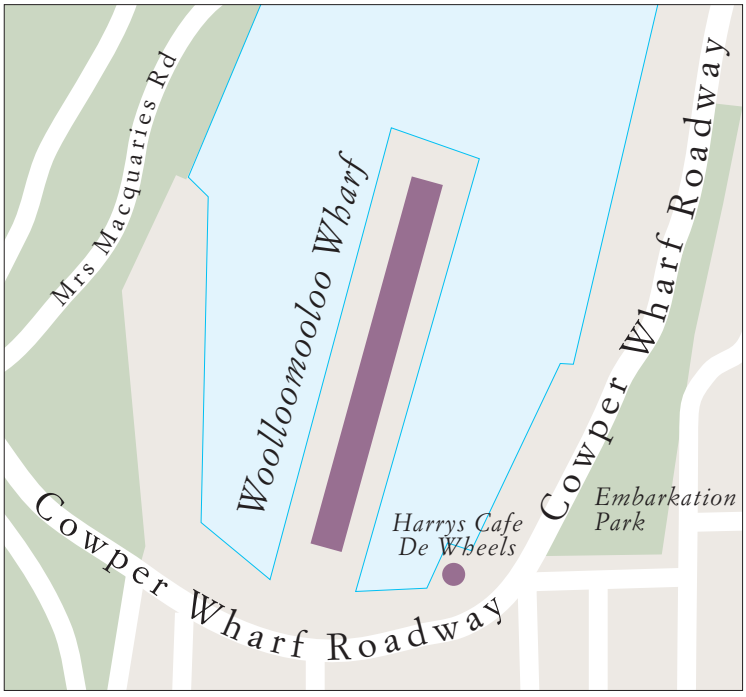
Example 4

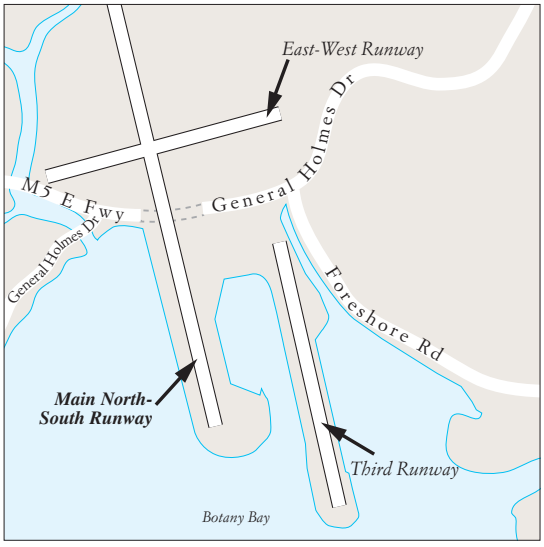
Exercise 6-04 Scale maps and plans

1 Measure each feature shown below to find the scale of each map in the form 1 : n, where n is rounded to the nearest whole number.

a The Woolloomooloo Wharf in Sydney is 410 metres long.



b The North-South runway at Sydney airport is 3960 m long.



c Conrod Straight at Mount Panorama in Bathurst is 1.916 km long.

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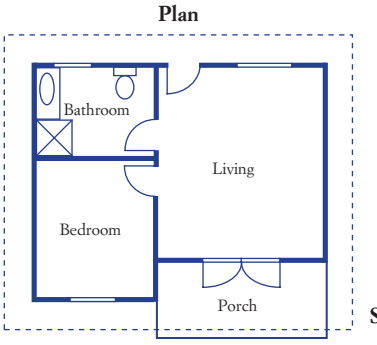
Example 5

2 This scale diagram shows a cabin in a national park. The dotted lines indicate how far the roof extends past the walls.



a What is the actual length of the porch? Select **A**, **B**, **C** or **D**.

A 2.2 m **B** 4.4 m

C 5.5 m **D** 11 m

b What is the actual perimeter of the living room?

c What is the length of the left side of the cabin? Select **A**, **B**, **C** or **D**.

A 1.6 m **B** 3.2 m **C** 6.4 m **D** 9.4 m

d What is the actual area of the bathroom?

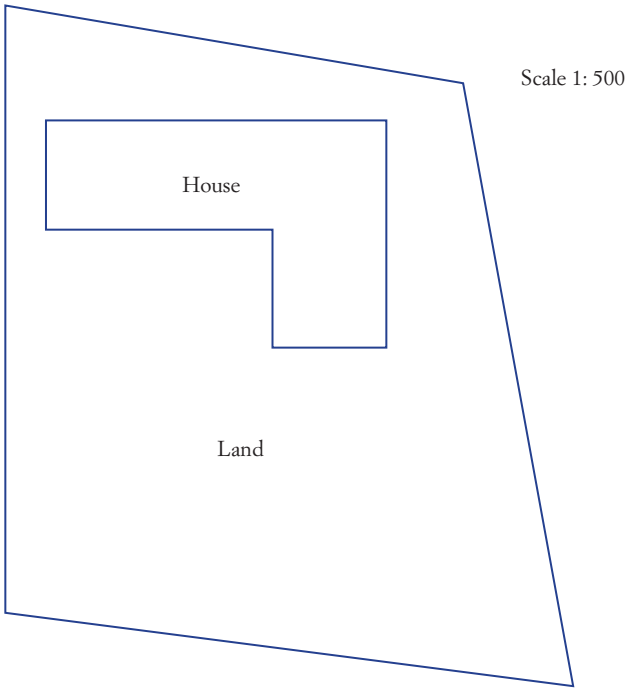
Example 6

3 For each map, use the scale to measure and calculate:

i the perimeter of the location to the nearest metre

ii the area of the location to the nearest square metre by approximating a trapezium

a a house block



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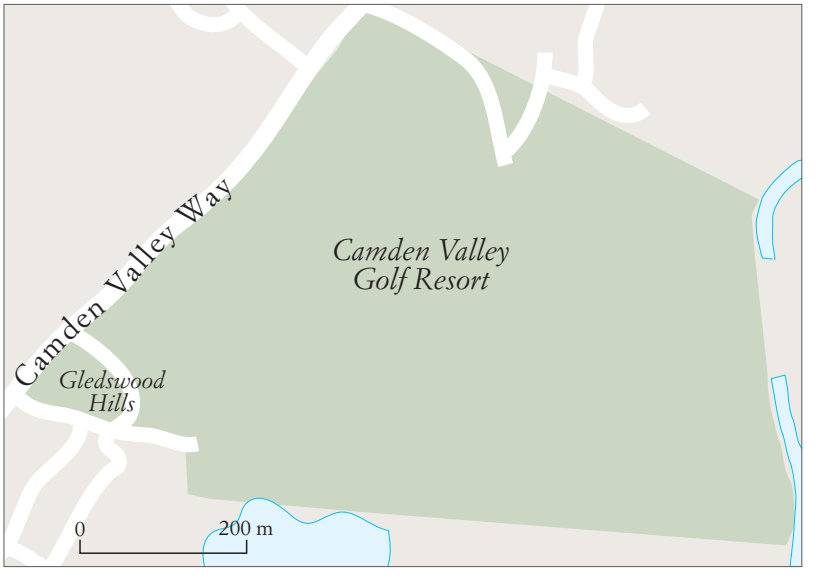
b Australian Reptile Park

c Camden Golf club



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Use either of the following to measure the length of each location given below.

i The Google Maps website (www.map.google.com) and use the online tools to measure the length.

ii Google Earth and from the menu choose **Tools** and **Ruler** and **Line** (settings should be in kilometres). Click on each end of the location to calculate its length.

- **a** Anzac Parade in Canberra (that leads to the Australian War Memorial)
- **b** the Sydney Harbour Bridge
- **c** Queen Victoria Building in George St, Sydney
- **d** a runway at Bankstown Airport or at your closest airport
- **e** the path travelled by the Bundeena-Cronulla ferry
- **f** the length of your street

INVESTIGATION

METHODS FOR ESTIMATING LAND AREAS

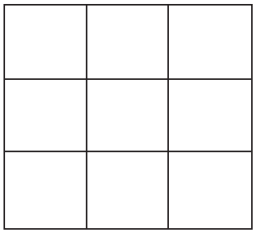
There are different methods for estimating areas of a block of land and catchment areas such as reservoirs and dams.

The Brickpit is a large pond at Sydney Olympic Park with a raised circular walkway for observing the endangered Green and Golden Bell Frog. We will use two approximation methods for calculating its area.

Grid Square Method

1 Search 'The Brickpit' at Google Maps www.maps.google.com and find its image in Satellite view.

2 Print out the image or copy-and-paste it into a word processor or graphics file. Create a square grid **like the one on the right** over the image using the scale in the bottom left corner. Each square on the right has an actual length of 100 m.



3 Estimate the area of the circle in grid squares: approximately 2.5.

Each grid square is 100 m × 100 m = 10 000 m^2

Area of the circle ≈ 10 000 × 2.5 = 25 000 m^2

Polygon method

1 **Fly To** 'The Brickpit' using **Google Earth** software and zoom in until the scale is 100 m.

2 Right-click **Places** (on left), select **Add** and **Folder**. Call it 'Brick Pit'.

3 Select **Add Polygon** (top of screen – tools) 

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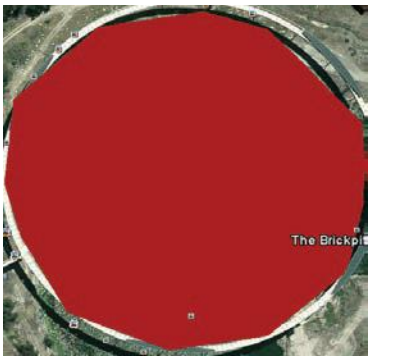
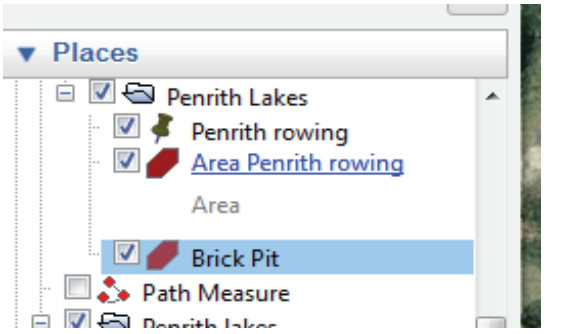
4 Click around the perimeter of the circle to create a polygon. The more 'clicks' or 'vertices' you create, the more accurate the area calculation will be.



5 We will copy the image to another website to calculate its area. In the left column, right-click on **Brick Pit** and **Copy**.



6 Visit www.earthpoint.us, select **Polygon Area** from the menu, and paste the image into the box that appears. Choose 'square meters' and **View on Webpage**. A box should appear underneath that shows the approximate area.

7 For each map below, use the Grid Square or Polygon method to estimate the area of the specified location.

a Sydney International Aquatic Centre, Homebush

b Rowing Centre, Penrith

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c East End Oval, Cessnock

d Jervis Bay

6-05 Volumes of tanks and dams

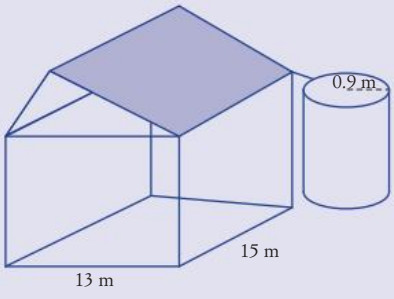
Water tanks

Volumes of water

EXAMPLE 7

Rainwater falling onto the right face of the roof of a building is collected into an empty cylindrical water tank.

The top view of the right face shows the aerial dimensions of the catchment area.



a After 20 mm of rainfall, how many litres of water are drained into the tank?

b If the radius of the water tank is 90 cm, what was the water level in the tank after 20 mm of rainfall, correct to the nearest 0.1 m?

Top view of right face

This is the right face's 'horizontal' area as seen from above, not its actual sloped surface area.

Solution

a Area of a rectangle $= 12 \times 15$
$= 180 \text{ m}^2$

Volume of rainfall $= 180 \text{ m}^2 \times 20 \text{ mm}$
$= 180 \text{ m}^2 \times 0.02 \text{ m}$
$= 3.6 \text{ m}^3$
$= 3.6 \times 1000 \text{ L}$
$= 3600 \text{ L}$

change 20 mm to m

$1 \text{ m}^3 = 1000 \text{ L}$

b Volume of a cylinder $= \pi r^2 h$

$V = 3.6 \text{ m}^3$ from **a** and $r = 90 \text{ cm} = 0.9 \text{ m}$

$3.6 = \pi \times 0.9^2 \times h$
$= 2.5446\ldots h$

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$$h = \frac{3.6}{2.5446...}$$
$$= 1.4147...$$
$$= 1.4 \text{ m}$$

The height of the water in the tank was 1.4 m.

SUMMARY

Simpson's rule for estimating area

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

where h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

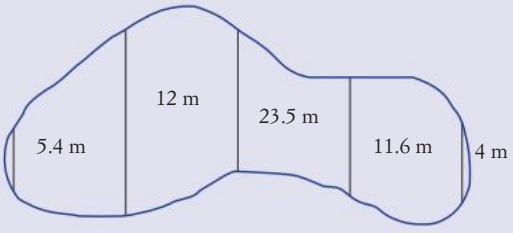
d_l = last measurement

or in words:

$$\text{Area} \approx \frac{\text{width of strip}}{3} (\text{first measurement} + 4 \times \text{middle measurement} + \text{last measurement})$$

EXAMPLE 8

The width of a lake is measured at 10 m intervals.



a Use two applications of Simpson's rule to estimate the area of the lake correct to one decimal place.

b Calculate the volume of water in the lake correct to 2 significant figures if it is 2.2 m deep.

Solution

a Using the first three measurements:

$d_f = 5.4$, $d_m = 12$, $d_l = 23.5$

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$
$$= \frac{10}{3}(5.4 + 4 \times 12 + 23.5)$$
$$= 256.333...$$

Using the last three measurements:

$d_f = 23.5$, $d_m = 11.6$, $d_l = 4$

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$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$
$$= \frac{10}{3}(23.5 + 4 \times 11.6 + 4)$$
$$= 246.333\ldots$$

Total area $= 256.333\ldots + 246.333\ldots$
$= 502.666\ldots$
$\approx 502.7\ \text{m}^2$

b $V = Ah$
$= 502.666\ldots \times 2.2$
$= 1105.866\ldots$
$\approx 1100\ \text{m}^3$

The volume of the lake is $1100\ \text{m}^3$.

SUMMARY

Simpson's rule for estimating volume

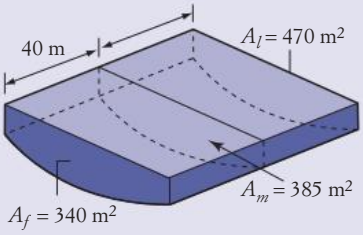
Given the three cross-sectional areas of an irregular solid at equal intervals:

$$V = \frac{h}{3} \times \left\{ \text{Area}_{\text{left end}} + 4 \times \text{Area}_{\text{middle}} + \text{Area}_{\text{right end}} \right\}$$
$$\approx \frac{h}{3}\left(A_f + 4A_m + A_l\right)$$

where h = distance between successive cross-sectional areas.

EXAMPLE 9

Estimate the capacity of this dam in megalitres (ML), correct to one decimal place.



Solution

$h = 40, A_f = 340, A_m = 385, A_l = 470$

$$V \approx \frac{h}{3}(A_f + 4A_f + A_l)$$
$$= \frac{40}{3}(340 + 4 \times 385 + 470)$$
$$= 31\,333.333\ldots\ \text{m}^3$$
$$= 31\,333.333\ldots\ \text{kL}$$
$$= 31\,333.333 \div 1000\ \text{ML}$$
$$= 31.333\,333\ldots\ \text{ML}$$
$$\approx 31.3\ \text{ML}$$

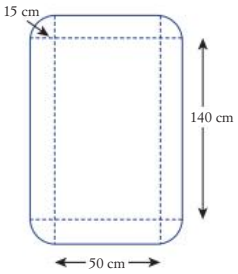
The capacity of the dam is 31.3 ML.

$1\ \text{m}^3 = 1\ \text{kL}$
$1\ \text{ML} = 1000\ \text{kL}$

Exercise 6-05 Volumes of tanks and dams

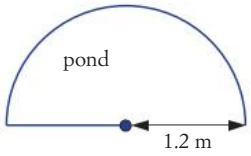
Example 10

1 Calculate the capacity of each tank or pool correct to the nearest 0.01 kL.



- **a** A cylindrical rainwater tank with diameter 9.36 m and height 2.18 m.
- **b** A rectangular swimming pool with dimensions 2.4 by 4.8 m filled to a depth of 1.2 m.
- **c** A water tank with a cross-section as shown on the right and a height of 1.6 m.

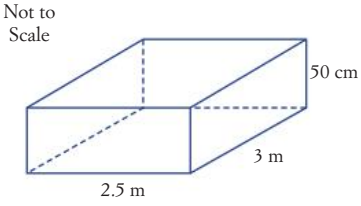
2 An artificial semi-circular pond has a radius of 1.2 m and a depth of 0.4 m. Calculate its volume correct to the nearest litre.



3 During a flood, 1.4 hectares of land was covered by water to a depth of 15 cm. How many kilolitres of water covered the land? Select **A, B, C** or **D**.

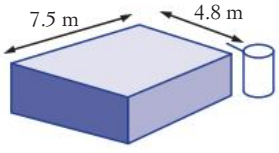
A 2.1 kL **B** 2 100 kL **C** 210 000 kL **D** 2 100 000 kL

4 Bernadette has installed two water bladders under her new house to collect rain. They are identical and in the shape of a rectangular prism as shown on the right.



- **a** Calculate the combined capacity of the water bladders, in litres.
- **b** In April, Bernadette's family uses an average of 570 L/day. If the water bladders are full, how long will it take to use all the water in the bladders (assuming no rainfall)? Answer to the nearest day.

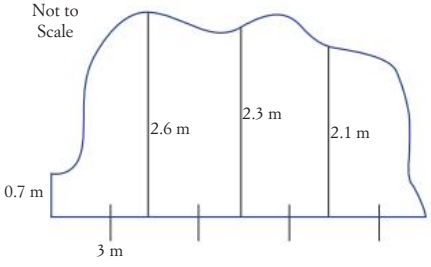
5 Rainwater falls onto the rectangular roof of a shed so that it drains into an empty cylindrical water tank of radius 60 cm and height 2.4 m. The aerial dimensions of the catchment area are 7.5 m by 4.8 m.



- **a** After 50 mm of rainfall, how many litres of water are drained into the tank?
- **b** What is the water level after 50 mm of rainfall?
- **c** How many millimetres of rain need to fall onto the roof to completely fill the tank?

Example 11

6 The diagram shows the outline of a lake in a botanical garden.



- **a** Use two applications of Simpson's rule to find the approximate surface area of the lake.
- **b** The average water depth in the lake is 1.6 metres. How many whole litres of water can the lake hold when full?

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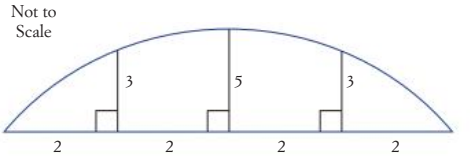
7 An artificial pond is designed for a new shopping centre.

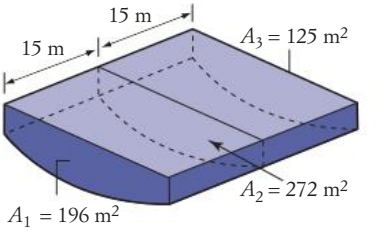


a Calculate the area of the cross-section using two applications of Simpson's rule.

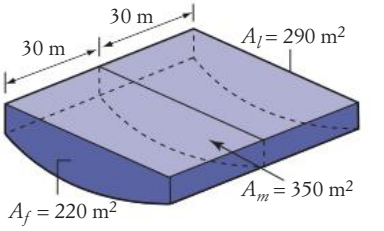
b If the pond is 2.5 m deep, calculate its capacity to the nearest litre.

Example 12

8 Calculate the approximate capacity of a small dam on a farmer's property in kL, given that the three cross-sectional areas on the right were measured 15 metres apart.



9 Calculate the capacity of this dam in megalitres.



JUST FOR THE RECORD

Sydney's Tank Stream

Newspix/Craig Greenhill

In 1788, Sydney was chosen by the first British settlers because of its deep harbour and its freshwater from the Tank Stream. Because Australia is a dry land, water was vital for the survival of the colony. The Tank Stream was originally discovered within thick bush and drained to Sydney Cove, now known as Circular Quay. The Tank Stream still flows beneath the city as part of its stormwater system.

Search for a map of the original path of the Tank Stream in Sydney.

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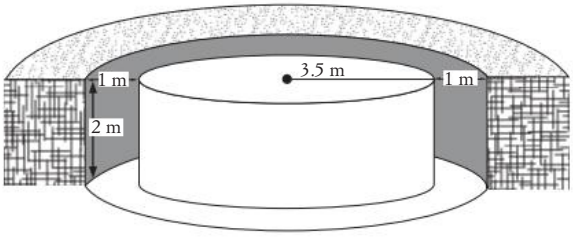
Sample HSC problem

A cylindrical water storage tank of height 2 metres is installed below ground level in a wildlife park. The tank is used to provide drinking water for the animals and birds.

The radius of the tank is 3.5 metres. The hole to be dug is 2 metres deep. When the water tank is placed in the hole there is a gap of 1 metre all the way around the outside of the tank.



a What volume of soil is removed from the hole when it is dug for the water tank to be installed? Answer to the nearest cubic metre.

b Calculate the capacity of the tank, correct to the nearest litre.

c A hose is connected to the tank to provide water to the animals at an average rate of 450 litres per hour throughout the day. If the tank is full, for how long can the hose be used before the tank is emptied? Answer in days and hours.

Study tip

Ten tips for tackling exams

1 Be prepared.
2 Find out about the format of the exam: the topics to be tested, time allowed, number and format of questions, number of marks per question, whether formulas will be supplied.
3 Use the reading time to browse through the exam to see the work that is ahead of you. Note the harder questions—you may need to spend more time on them.
4 Attempt every question.
5 Spend the first minute of each question planning and thinking before you start writing.
6 Write clearly, draw big diagrams, spread out your working and set it out neatly. Write down the page, not across.
7 Don't spend too much time on one question. Keep an eye on the time.
8 Make sure you have answered the question. Did you remember to round the answer and/or include units? Does the answer sound reasonable? Did you use all of the relevant information given?
9 If your working-out to a hard question is taking too long, then it's probably wrong. Don't get bogged down. If you're getting nowhere, retrace your steps, start again, or skip the question and return to it later with a clearer mind.
10 Once you have completed the exam, go over it again. Double-check your answers, especially the harder ones or those of which you're unsure.

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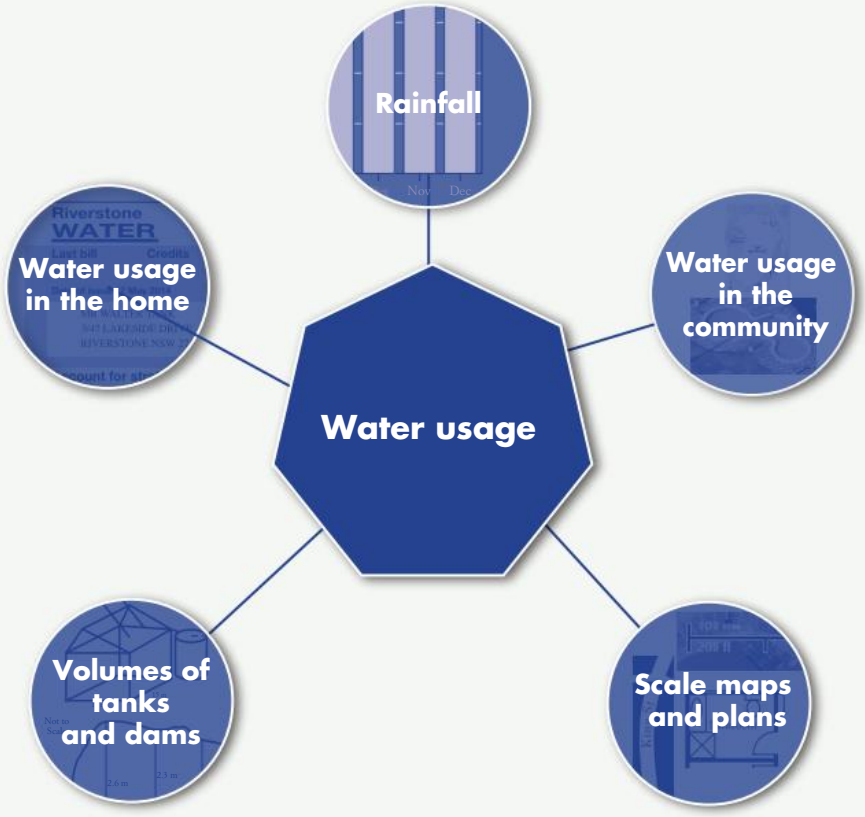
CHAPTER REVIEW

Topic overview

This chapter, Water usage, looked at the measurement and statistics of water usage in the home and the community, as well as investigating rainfall and water storage in tanks and dams. You should be able to interpret tables and graphs relating to water use and rainfall, measure and calculate perimeters and areas of catchment areas using scale maps and plans, and find the volumes of tanks and dams using formulas as well as Simpson's rule.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 6-01

1 Jed's water bill shows that water usage is charged at $2.115/kL.

a If Jed used 21.6 kL, how much will he need to pay?

b Jed was also charged $41.39 for the water service and $162.88 for the wastewater (sewerage) service. What was the total of Jed's bill?

Exercise 6-02

2 Newcastle had the following rainfall (in mm) for 12 consecutive days in July.

2.2 12.8 22.2 0.8 0.2 0 2.0 0 3.6 4.0 0 0

a Find the range of the rainfall.

b Find the median.

c Find the mean correct to one decimal place.

d Are there any outliers?

e Explain why the mean and median values are so different

Exercise 6-02

3 The number of days rainfall for Weipa, Queensland, in February, March and April 2012 is shown in the table.

Month	No. of rain days
February (29 days)	24
March	25
April	9

Bureau of Meteorology

Based on this data, calculate the probability of:

a a day in February having rain

b a day in April not having rain

c rain on three consecutive days in March

d no rain on a weekend in February

Exercise 6-03

4 The table below shows the monthly data on available storage and level of water below capacity for Nepean Dam in NSW for the second half of 2010 and 2011.

Nepean Dam		Storage Capacity = 69 810 ML				
2010	Storage Level (m)	Available Storage (%)		2011	Storage Level (m)	Available Storage (%)
July	–3.78	83.5		July	–3.21	85.7
August	–6.19	73.8		August	–3.2	85.8
September	–7.12	70.2		September	–4.09	82.1
October	–6.42	72.9		October	–5.63	75.9
November	–6.46	72.7		November	–6.7	71.8
December	–3.33	85.2		December	–6.26	73.5

Sydney Catchment Authority

a What is the storage level in July 2011? Is there more or less water compared to July 2010 and by how much?

b What is the percentage decrease in the available storage of Nepean Dam from September to October 2011?

c How much did the storage level rise at Nepean Dam between November and December 2010?

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d For the second half of 2010 in which month was its available storage at its:

i highest level? **ii** lowest level?

e Nepean Dam can hold 69 810 megalitres of water when full. How many megalitres of water (correct to one decimal place) did it hold in December 2010?

f Draw a line graph for available storage on paper or using technology.

Exercise 6-04

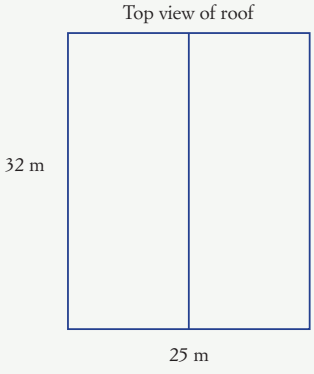
5 Use the given scale to estimate:

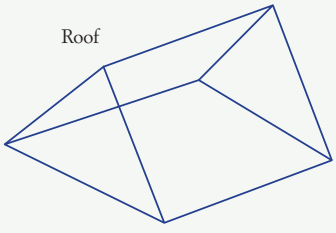
a to the nearest kilometre the perimeter of Lake Munmorah, NSW (string may be helpful)

b to the nearest square kilometre the area of the lake

Exercise 6-05

6 Rain falls on the gable roof as shown below. The roof sections are rectangular and the aerial dimensions of the catchment area are shown in the top view.





Calculate in litres the volume of water that was collected on this roof after 9 mm of rainfall.

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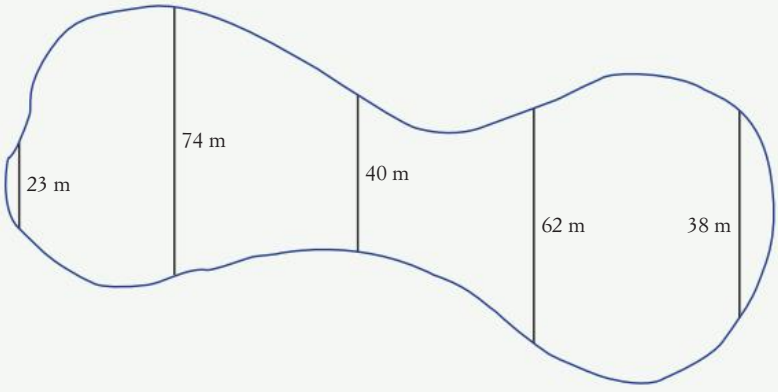
Exercise 6-05

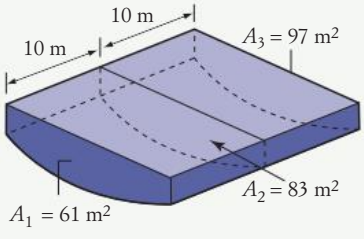
7 **a** Use two applications of Simpson's rule to estimate the area of this dam if the measurements shown were taken 30 m apart.



b What is the volume of water in the dam, correct to 2 significant figures, if the dam has an average depth of 26 m ?

Exercise 6-05

8 Calculate the approximate capacity of a reservoir in a national park, in kilolitres, given that the three cross-sectional areas shown in the diagram are measured 10 metres apart.



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Practice paper 2 ●●○○

Recommended time: One hour

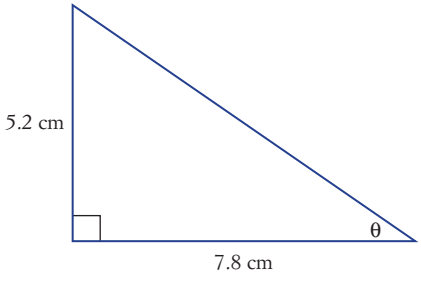
Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Exercise 5-01

Question 1

What is the size of angle θ, correct to the nearest minute, in the diagram below?



(A) 56°19′ (B) 41°49′

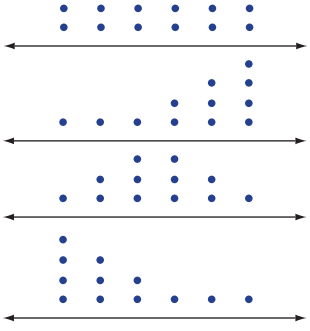
(C) 48°11′ (D) 33°41′

Exercise 4-04

Question 2

Which dot plot illustrates a distribution that is positively skewed?

(A)



(B)

(C)

(D)

Exercise 4-04

Question 3

The 6 employees at the Bug and Beef Cafe earn the following weekly wages:

$350 $420 $510 $130 $320 $460

The mean and standard deviation of this data set were calculated. If a wage of $720 is added to this data set, how will its mean and standard deviation be affected?

(A) The mean will increase but the standard deviation will decrease.

(B) The mean will increase but the standard deviation will stay the same.

(C) The mean and standard deviation will both increase.

(D) The mean and standard deviation will both decrease.

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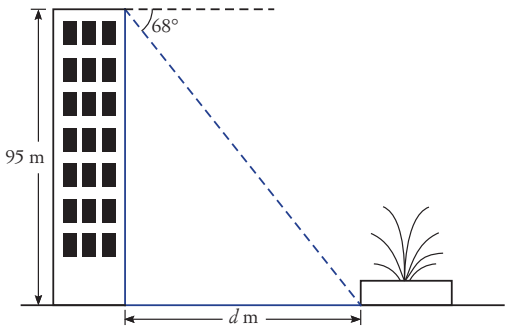
Exercise 5-01

Question 4

From the rooftop of her apartment building that is 95 m high, Jessie can see the park's fountain at an angle of depression of 68°. Calculate the distance between the fountain and the bottom of the apartment building.



(A) 38 m (B) 234 m
(C) 102 m (D) 254 m

Exercise 6-01

Question 5

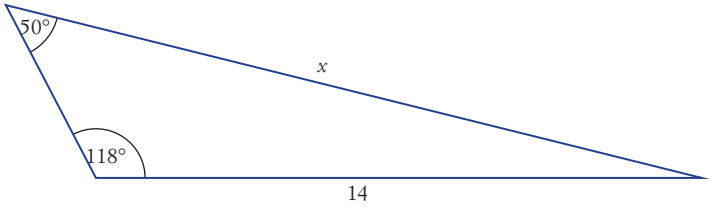
From 25 July to 24 October inclusive, the Haywood family used 42.7 kL of water. Calculate their daily water usage to the nearest litre.

(A) 516 L (B) 474 L (C) 464 L (D) 388 L

Exercise 5-04

Question 6

Which expression gives the correct value of x in this diagram?



(A) $x = \dfrac{14 \sin 50°}{\sin 118°}$

(B) $x = \dfrac{14 \sin 118°}{\sin 50°}$

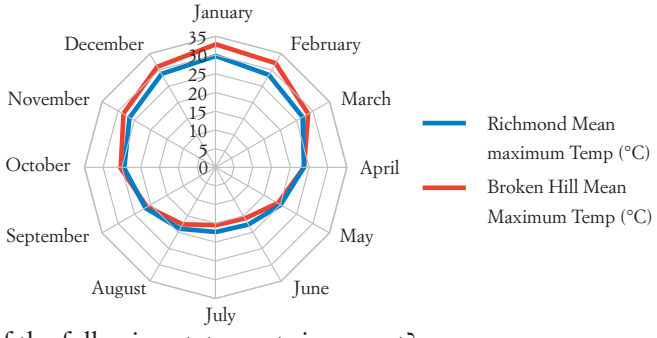
(C) $x = \dfrac{\sin 50°}{14 \sin 118°}$

(D) $x = \dfrac{\sin 118°}{14 \sin 50°}$

Exercise 4-07

Question 7

This radar chart compares the mean maximum monthly temperatures (in °C) in Richmond and Broken Hill.



Which one of the following statements is correct?

(A) During the winter months, it is colder in Richmond than in Broken Hill.

(B) The mean temperature in August for both towns is the same.

(C) In January, the mean temperature in Broken Hill is about 3°C hotter than that in Richmond.

(D) Broken Hill has a higher mean temperature than Richmond for 8 months of the year.

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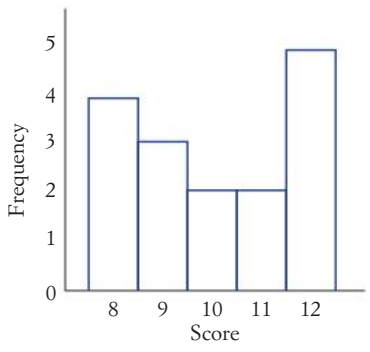
Exercise 4-04

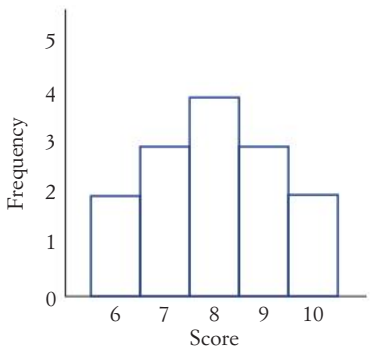
Question 8

The data sets, A and B, are displayed in these histograms.

A



B



Which one of these statements is true about the mode and range of the distributions?

(A) A has a higher mode and B has a higher range.

(B) A has a higher mode and the ranges are the same.

(C) The modes are the same and B has a higher range.

(D) The modes are the same and the ranges are the same.

Exercise 4-03

Question 9

A manufacturer took a random sample of 10 screws for quality testing. Their lengths in centimetres were measured to be:

2.00 1.99 1.98 2.01 2.01 1.97 2.03 1.98 2.01 2.00

For this sample, which one of these statements is false?

(A) The modal length is 2.01 cm.

(B) The mean length is 1.998 cm.

(C) The median length is 2.01 cm.

(D) The interquartile range is 0.03 cm.

Exercise 5-05

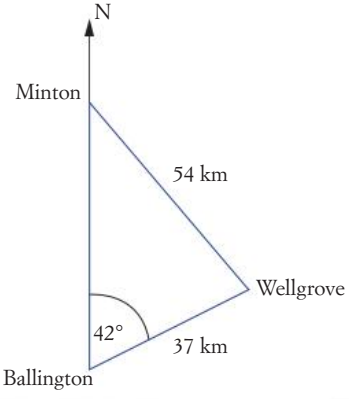
Question 10

Minton is due north of Ballington and 54 km from Wellgrove. The bearing of Wellgrove from Ballington is 42° and they are 37 km apart.

Find the true bearing of Wellgrove from Minton.

(A) 027° (B) 111°

(C) 146° (D) 153°



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Section 2

3 questions: 10 marks each

Exercise 6-03

Question 11 — Marks

(a) Jemima's water bill shows that water usage is charged at \$2.115/kL.

(i) On her latest bill, she paid \$37.86 in water usage charges. How many kilolitres of water did she use? Answer correct to 1 decimal place. **1**

(ii) Jemima was also charged \$41.39 for the water service, \$162.88 for the wastewater service and a recycled water charge of \$11.58. She also used 21 kL of recycled water at \$1.70/kL. What was the total of Jemima's water bill? **2**

Exercise 4-03

(b) A sample of households was surveyed on the number of tablet devices they owned, with the results shown in the frequency table.

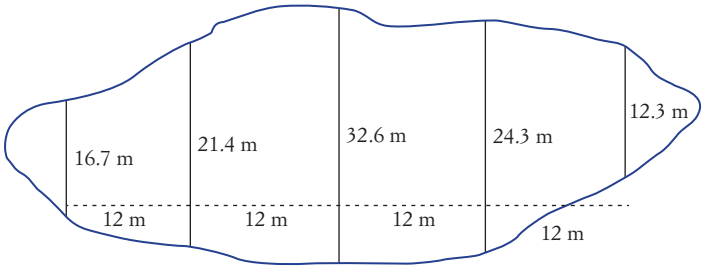
No. of tablets	Frequency
0	2
1	4
2	3
3	2
4	1
5	1

(i) Find the median of this sample. **1**

(ii) Find the standard deviation of this sample correct to one decimal place. **1**

Exercise 6-05

(c) A dam has five vertical measurements taken at 12 m intervals.



(i) Use two applications of Simpson's rule to approximate the area of the lake.

$[A = \frac{h}{3}(d_f + 4d_m + d_l)]$ **3**

(ii) If the dam is 5.4 m deep, calculate its capacity in kilolitres. **2**

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Question 12

Marks

Exercise 4-09

(a) The students at Port Jackson High School who used mobile phones were surveyed according to their gender and the type of phone plan they had. The results are shown in the table.

	Pre-paid plan	Post-paid plan	Total
Female students	165	140	305
Male students	151	96	247

(i) How many students at Port Jackson High School use mobile phones? **1**

(ii) What fraction of the students who use mobile phones are male? **1**

(iii) What fraction of female students who use mobile phones have a post-paid plan? **1**

(iv) What percentage of the students on a pre-paid plan are male? Answer correct to one decimal place. **2**

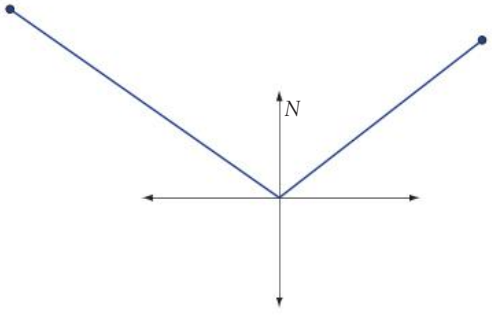
Exercise 6-05

(b) Rainwater is collected in a water tank from the roof of a block of classrooms at a primary school. The area of the roof is 300 m^2. During a day of heavy rainfall, 15 mm of rain falls on the roof and is collected in the water tank. How many litres of water were collected? **2**

Exercise 5-09

(c) Caitlin leaves town on a road with a bearing of 290° and travels 20 km. Bae leaves the same town on a road with a bearing of 040° and travels 18 km.

(i) The diagram below illustrates the situation. Copy and complete it, showing all given information. **1**



(ii) Calculate the distance between Caitlin and Bae, correct to the nearest 0.1 km. **2**

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Exercise 5-10

Question 13 **Marks**

(a) An offset survey on a field, *PARK*, was conducted and the following notebook entries were made by the surveyor. All measurements are in metres.

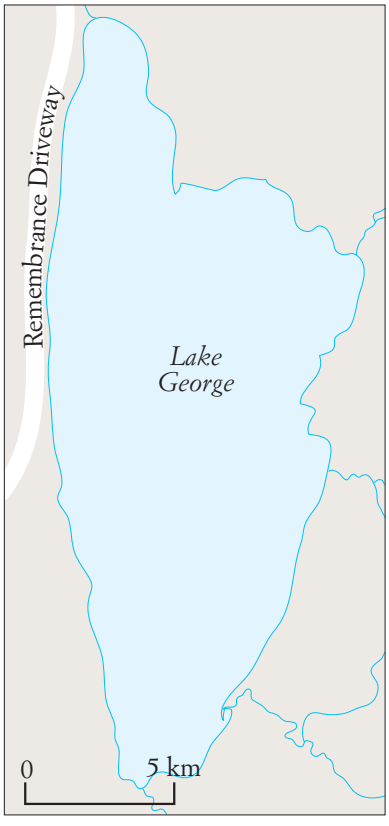
	A	
	144	
	81	51 *R*
P 25	32	
	0	
	K	

(i) Draw a diagram of the field, showing all measurements. **1**

(ii) Calculate the area of the field. **2**

Exercise 6-04

(b) Use the scale to estimate to the nearest kilometre the perimeter of Lake George, ACT. **2**



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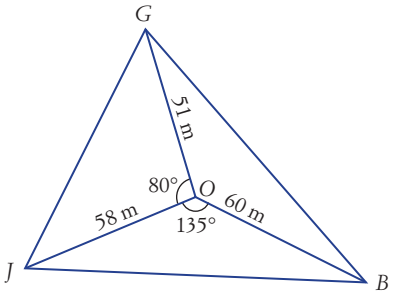
(c) The diagram illustrates the result of a plane table radial survey.



(i) Find the size of $\angle GOB$. **1**

(ii) Calculate the area of ΔGOB to the nearest square metre. **2**

(iii) Calculate the length of GB to the nearest metre. **2**

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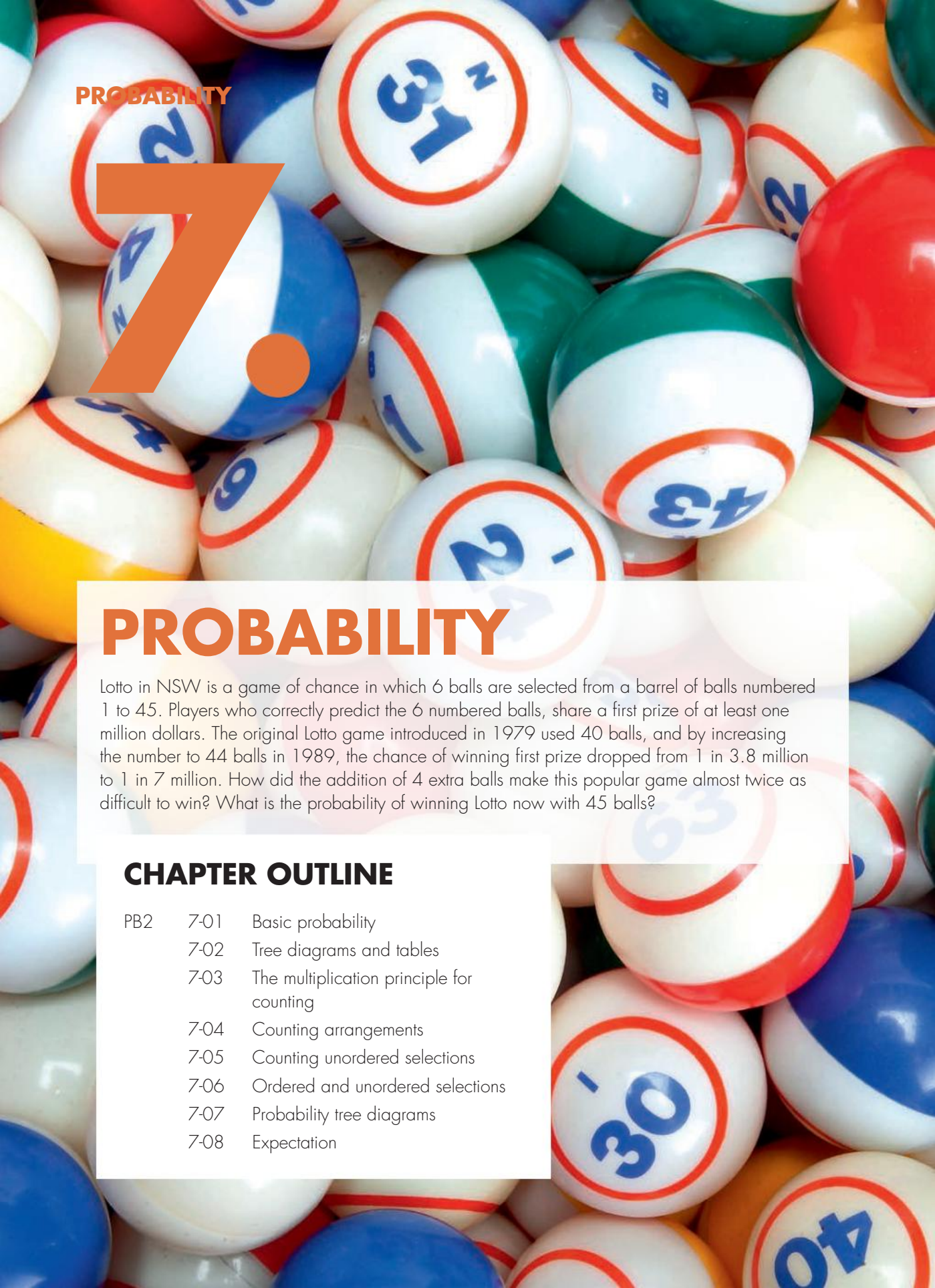
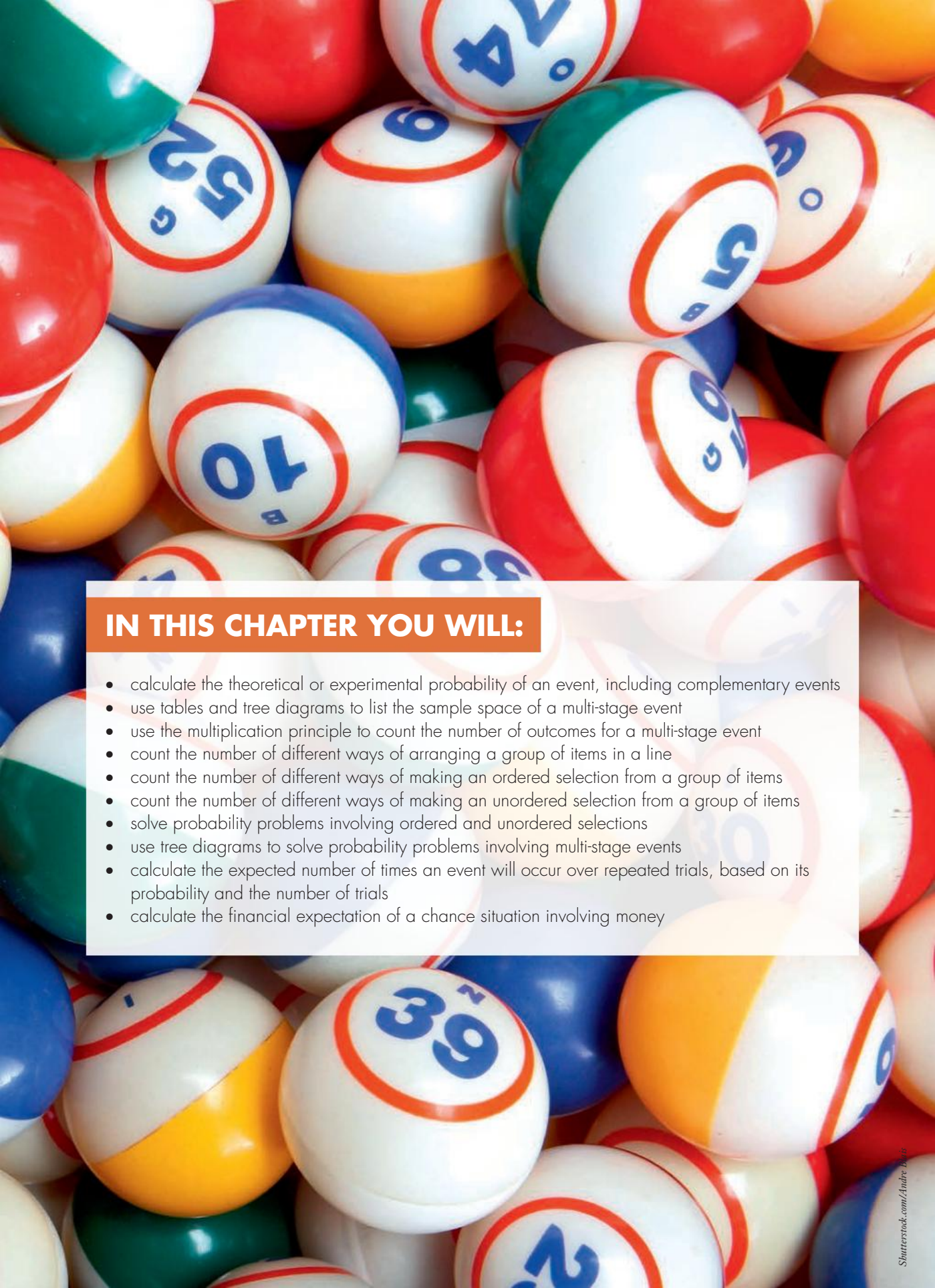
7.



PROBABILITY

Lotto in NSW is a game of chance in which 6 balls are selected from a barrel of balls numbered 1 to 45. Players who correctly predict the 6 numbered balls, share a first prize of at least one million dollars. The original Lotto game introduced in 1979 used 40 balls, and by increasing the number to 44 balls in 1989, the chance of winning first prize dropped from 1 in 3.8 million to 1 in 7 million. How did the addition of 4 extra balls make this popular game almost twice as difficult to win? What is the probability of winning Lotto now with 45 balls?

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- calculate the theoretical or experimental probability of an event, including complementary events
- use tables and tree diagrams to list the sample space of a multi-stage event
- use the multiplication principle to count the number of outcomes for a multi-stage event
- count the number of different ways of arranging a group of items in a line
- count the number of different ways of making an ordered selection from a group of items
- count the number of different ways of making an unordered selection from a group of items
- solve probability problems involving ordered and unordered selections
- use tree diagrams to solve probability problems involving multi-stage events
- calculate the expected number of times an event will occur over repeated trials, based on its probability and the number of trials
- calculate the financial expectation of a chance situation involving money

Probability crossword

TERMINOLOGY

arrangement
at least
complementary event
expected
experimental probability
event
financial expectation
multiplication principle
multi-stage event
ordered selection
outcome
probability
random
relative frequency
sample space
selection
simulation
test
theoretical probability
tree diagrams
trial
two-stage event
two-way table
unordered selection

Assignment 7

Probability review

SkillCheck

1 **a** What is the probability of a certain event?

b Express the probability of an event that has an even chance of occurring as a decimal.

2 A drawer contains 8 white socks, 2 black socks, 4 grey socks and 6 blue socks. If one sock is randomly selected, what is the probability that it is:

a white? **b** grey or white? **c** not blue? **d** yellow?

3 A person is chosen at random from the population. What is the probability that they were not born on a Saturday or Sunday?

4 Evaluate each expression.

a $\frac{7\times6\times5\times4}{4\times3\times2\times1}$ **b** $\frac{3}{7}\times\frac{4}{6}+\frac{4}{7}\times\frac{3}{6}$

c $3\times(0.7)^2\times0.3$ **d** $\$5\times\frac{1}{4}+\$3\times\frac{3}{5}-\$2\times\frac{3}{20}$

5 For families with three children, list all possible arrangements of boys and girls.

6 How many outcomes are possible when two dice are rolled together?

7 The password of a bank account is 4 capital letters followed by 2 numbers, for example, LOVE12. How many such passwords are possible?

8 The daily weather in Springfield for June is shown in the table.

Find the relative frequency of:

a a fine day **b** a day without rain.

Weather	No. of days
Fine	12
Rain	8
Cloudy	10

Matching probabilities

Greedy pig game

7-01 Basic probability

An **outcome** is a result of an experiment or game. When two coins are tossed, one possible outcome is HT (head-tail).

The **sample space** is the set of all possible outcomes. When two coins are tossed, the sample space has four outcomes: {HH, HT, TH, TT}.

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An **event** is a group of one or more outcomes. When two coins are tossed, the event of both coins being the same consists of two outcomes: {HH, TT}.

SUMMARY

- If all outcomes in a sample space are **equally likely**, the **theoretical probability** of an event occurring is:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A probability can be expressed as a fraction or decimal, ranging from 0 to 1, or as a percentage from 0 to 100%.
- A **certain event** must occur and has a probability of 1.
- An **impossible event** cannot occur and has a probability of 0.

Complementary events

If $P(E)$ is the probability of E occurring, then $P(\bar{E})$ is the probability of E *not* occurring. $\bar{E}$ is called the **complementary event** to E, and $P(E) + P(\bar{E}) = 1$.

SUMMARY

$P(\text{event does not occur}) = 1 - P(\text{event does occur})$

$P(\bar{E}) = 1 - P(E)$

EXAMPLE 1

A jar contains 12 red, 7 yellow, 8 white and 13 black jellybeans. If a jellybean is randomly selected from this jar, express as a decimal the probability that it is:

a green **b** black or white **c** not red **d** black, yellow or red.

Solution

Total number of jellybeans = 12 + 7 + 8 + 13 = 40

a $P(\text{green}) = \frac{0}{40} = 0$ **Impossible event: no green jellybeans.**

b $P(\text{black or white}) = \frac{13+8}{40}$

$= \frac{21}{40}$

$= 0.525$

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c $P(\text{not red}) = 1 - P(\text{red})$

$$= 1 - \frac{12}{40}$$
$$= \frac{7}{10}$$
$$= 0.7$$

d $P(\text{black, yellow or red}) = \frac{13+7+12}{40}$

$$= \frac{32}{40}$$
$$= 0.8$$

Alternatively:

$P(\text{black, yellow or red}) = 1 - P(\text{white})$ **Black, yellow or red = not white.**

$$= 1 - \frac{8}{40}$$
$$= \frac{4}{5}$$
$$= 0.8$$

Experimental probability

The **experimental probability** of an event occurring is a **relative frequency** based on past statistics or data from repeated trials of the same experiment or situation.

SUMMARY

Experimental probability of an event = relative frequency of the event

$$= \frac{\text{frequency of the event}}{\text{total frequency}}$$

EXAMPLE 2

A roulette wheel at a casino has 37 numbers: 0 to 36. The results of 250 spins of the wheel are shown in the table.

Outcome	Frequency
0	5
1–9	60
10–18	62
19–27	64
28–36	59

Calculate the following probabilities as percentages correct to one decimal place.

a The experimental probability of spinning a number from 19 to 27.

b The theoretical probability of spinning a number from 19 to 27.

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c The calculated (theoretical) probability of spinning 0.

d The experimental probability of spinning a number less than 10.

Solution

Total frequency = 5 + 60 + 62 + 64 + 59 = 250

a $P(19 \text{ to } 27) = \frac{64}{250} \times 100\%$ — From the table.

$= 25.6\%$

b $P(19 \text{ to } 27) = \frac{9}{37} \times 100\%$ — *P*(choosing 9 numbers from 37)

$= 24.324\ldots$

$\approx 24.3\%$

c $P(0) = \frac{1}{37} \times 100\%$ — *P*(choosing 1 number from 37)

$= 2.702\ldots$

$\approx 2.7\%$

d $P(<10) = \frac{5+60}{250} \times 100\%$ — From the table.

$= 26\%$

Exercise 7-01 Basic probability

Example 1

1 From the words NEW CENTURY, one letter is selected at random. Find the probability that it is:

a N **b** Y **c** a consonant

d E or U **e** not T **f** A

2 Paolo buys 8 tickets in a raffle where 1250 tickets are sold. Calculate, as a percentage, the probability that he wins first prize.

3 In a batch of 160 instant (scratch) lottery tickets, 35 contain a cash prize. What is the probability of winning a cash prize on an instant lottery ticket from this batch?

4 What is the probability of not being born on a Tuesday?

Example 2

5 A survey was taken of the vehicles passing an intersection over a 1-hour period. Calculate, correct to three decimal places, the probability that the next vehicle to pass the intersection is:

Vehicle	Frequency
Car	722
Truck	132
Semi-trailer	58
Motor bike	34
Bus	19

a a bus **b** a truck **c** not a car

d neither a truck nor a semi-trailer.

6 A traffic light is green for 60 seconds, amber for 4 seconds and red for 80 seconds. Calculate the percentage probability that the traffic light is red. Select **A**, **B**, **C** or **D**.

A 12.5% **B** 55.6% **C** 57.1% **D** 71.4%

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7 One coin has fallen out of a piggy bank containing four $2 coins, eight $1 coins, three 50-cent coins and six 20-cent coins. What is the probability that it is:

a a 20-cent coin? **b** a gold coin?

c not a $2 coin? **d** not a 20-cent coin?

8 A road survey shows the number of car accidents occurring on each day of the week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
No. of accidents	6	4	8	14	28	18	18

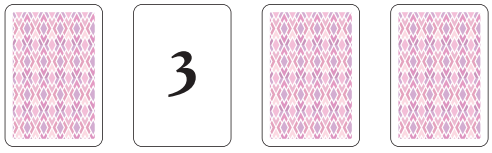
Calculate, as a percentage correct to three significant figures, the probability that a car accident occurs on:

a Monday **b** Friday

c a day other than Thursday **d** the weekend.

9 Jodie's English class has 22 students. If 4 students are chosen at random from this class to make a speech, what is the probability that Jodie will be chosen?

10 Four cards marked with the numbers 1, 2, 3 and 4 are placed face down on a table. One card is turned over as shown.



What is the probability that the next card turned over is an even number?
Select **A**, **B**, **C** or **D**.

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

11 In a football match, the Knights have a 42% chance of beating the Dragons while the Dragons have a 51% chance of beating the Knights.

a What other outcome is possible?

b What is the probability of this outcome?

12 A bag contains 16 red, 19 blue and 15 yellow balls only. If one ball is drawn out at random, what is the probability that it is:

a blue? **b** not yellow? **c** not green?

d neither red nor blue? **e** green? **f** white, red or yellow?

13 Give an example of an event that has a probability of:

a 1 **b** 0

14 The stand-by times of a batch of mobile phone batteries were tested and recorded. Calculate the percentage probability that a battery selected at random will have a stand-by time of:

a 70–74 hours **b** 60–69 hours

c 75 or more hours **d** 79 or fewer hours.

Stand-by time (hours)	Frequency
60–64	4
65–69	8
70–74	134
75–79	11
80–84	3

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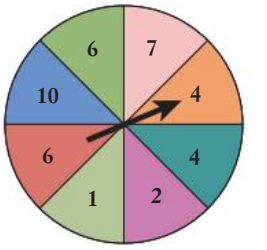
15 If this spinner is spun, what is the probability that the arrow will point to:



a an even number? **b** a number greater than 5?

c 5? **d** 6?

16 a What is the probability that a telephone number selected at random from the phone book ends in 9?

b Why isn't this the same as the probability that the number selected begins with 9?

17 A die is loaded (biased) so that the chance of rolling a 6 is double the chance of rolling any of the 5 other numbers. Find the probability of rolling:

a a 6 **b** a 5 or 6

c an odd number **d** a number less than 6.

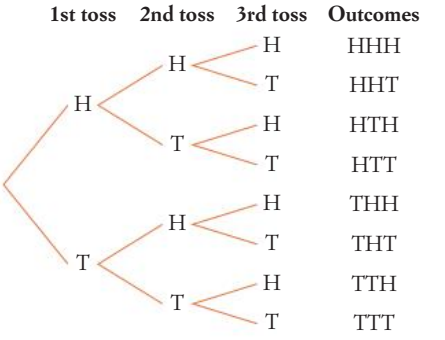
18 A box contains r red, w white and b brown socks. Write an algebraic expression for the probability of randomly selecting a sock that is:

a white **b** not red.

7-02 Tree diagrams and tables

A **multi-stage event** consists of two or more events occurring together, such as rolling two dice or tossing 3 coins. The sample space can be found using a list, table or tree diagram.

A **tree diagram** is a systematic way of listing all the possible outcomes in a multi-stage event. This tree diagram shows the possible outcomes when a coin is tossed three times.



EXAMPLE 3

The digits 7, 2, 3 and 6 are written on separate cards. Two cards are selected at random to form a 2-digit number.

a Use a tree diagram to list all possible outcomes.

b What is the probability that the number formed is divisible by 3?

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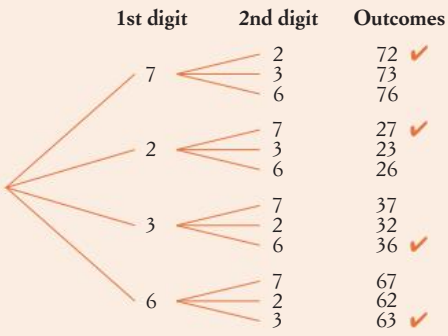
Solution

a



A digit cannot be used twice.

b There are 12 outcomes. Four outcomes are divisible by 3 (ticked above).

$P(\text{number divisible by 3}) = \frac{4}{12} = \frac{1}{3}$

For **two-stage** events, such as rolling two dice, a table is more convenient than a tree diagram for listing the sample space.

EXAMPLE 4

Two dice are rolled and their sum is calculated.

a Use a table to list all possible sums.

b What is the probability of rolling a sum of 10?

Solution

a

		2nd die					
	+	**1**	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	**10**
	5	6	7	8	9	**10**	11
	6	7	8	9	**10**	11	12

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b There are 36 possible outcomes.

$$P(\text{sum of } 10) = \frac{3}{36} \quad \text{3 ways of rolling a sum of 10.}$$
$$= \frac{1}{12}$$

Exercise 7-02 Tree diagrams and tables

Example 3

1 **a** A family has 4 children. Use a tree diagram to list the possible arrangements of boys and girls.

b What is the probability that the family consists of:

i 3 girls and 1 boy? **ii** 2 girls and 2 boys?

2 The numbers 2, 5, 6 and 9 are written on separate cards and placed in a box. Two cards are selected at random, one after the other, to form a 2-digit number. Draw a tree diagram to list all possible outcomes, then calculate the probability that the number formed is:

a 65 **b** odd **c** less than 59 **d** divisible by 5.

Example 4

3 In the dice game called Craps, two dice are rolled and their total calculated. A player wins if he rolls a sum of 7 or 11 and loses if he rolls a sum of 2 or 12. Calculate the probability of:

a a win **b** a loss **c** rolling a total greater than 9.

4 Three coins are tossed together. Find the probability of getting:

a exactly 1 head **b** 2 heads and 1 tail in that order

c 2 heads and 1 tail in any order **d** no heads.

5 Erin, Kristen, Shelley, Adelle and Bianca are in a basketball team. They need to select a captain and a vice-captain.

a Draw a tree diagram to determine how many pairings are possible.

b If every player is equally likely to be picked, what is the probability that:

i Kristen and Adelle fill the positions (in any order)?

ii Kristen is captain and Adelle is vice-captain?

iii Erin is captain?

6 Over the 4 days of the Easter long weekend (Friday to Monday), on any day the probability that it will rain is 50%.

a Use a tree diagram to list all possibilities of rain (R) and no rain ($\overline{R}$) over the 4 days.

b Find the probability that it will rain on:

i exactly 2 of the days **ii** at least 1 of the days.

7 A coin is tossed and a die rolled. Find the probability of getting:

a a head and an odd number **b** a tail and a number greater than 2.

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8 A, C and T are written on separate cards and drawn out of a box in random order. Calculate the probability that:

a the letters are drawn out in the order CAT

b A is the last letter drawn out

c T is not the last letter drawn out.

9 When Mr Faulds drives to work he passes through 4 sets of traffic lights. Each set has an equal chance of showing red or green (count amber as red).

a Use a tree diagram to count all possible arrangements of red and green lights.

b Calculate the probability that on his journey to work Mr Faulds faces:

i all green lights **ii** 2 red lights (in any order)

iii at least 1 red light **iv** 3 green lights.

10 Two dice are rolled and the positive difference between the two numbers is calculated.

a Use a table to find all 36 possible outcomes.

b What is the probability of rolling a difference of 2? Select **A**, **B**, **C** or **D**.

A $\frac{1}{36}$ **B** $\frac{1}{18}$ **C** $\frac{1}{9}$ **D** $\frac{2}{9}$

c What is the probability of rolling a difference that is greater than 2?

d Which difference has the highest probability? Select **A**, **B**, **C** or **D**.

A 0 **B** 1 **C** 2 **D** 3

11 Three digits are randomly chosen from 5, 9, 8 and 1 to make a three-digit number. What is the probability that this number is:

a greater than 500? **b** even? **c** divisible by 3?

INVESTIGATION

A HANDY ODDS-AND-EVENS GAME

Adam and Eve play a game in which each person holds out one hand at the same time, showing a number of fingers (1 to 5) to the other person. If the total number of fingers shown on both hands is even, Adam wins. If the total is odd, Eve wins.

1 Is this a fair game?

2 If you were Adam, how could you improve your chance of winning?

3 If you were Eve, how could you improve your chance of winning?

7-03 The multiplication principle for counting

For multi-stage events, it is often impractical to list all possible outcomes using a tree diagram as the total number of possibilities may be too large. Instead, we can use the multiplication principle to determine the number of all possible outcomes.

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EXAMPLE 5

How many different three-course dinners are possible from this menu?

Entrée	Main course	Dessert
Pumpkin soup	Cajun prawns	Pavlova
Calamari rings	Steak Diane	Black forest cake
Potato wedges	Roast lamb	Chocolate mousse
	Chicken Dijon	Mango and ice cream
	Grilled perch	

Solution

There are 3 possible entrées, 5 possible main courses, 4 possible desserts.

Number of possible three-course dinners $= 3 \times 5 \times 4 = 60$

SUMMARY

The multiplication principle for counting outcomes of multi-stage events.

- If A has a outcomes and B has b outcomes, then A and B together have $a \times b$ outcomes.
- More generally, if A has a outcomes, B has b outcomes, C has c outcomes, and so on, then A, B, C, ... together have $a \times b \times c \times \ldots$ outcomes.

EXAMPLE 6

A password is made up of 6 characters, each one a digit or a capital letter. One such password is D5RE21. How many different passwords are possible?

Solution

Number of possible characters $= 36$ **26 letters + 10 digits**

Number of possible passwords $= 36 \times 36 \times 36 \times 36 \times 36 \times 36$

$= 36^6$

$= 2\ 176\ 782\ 336$

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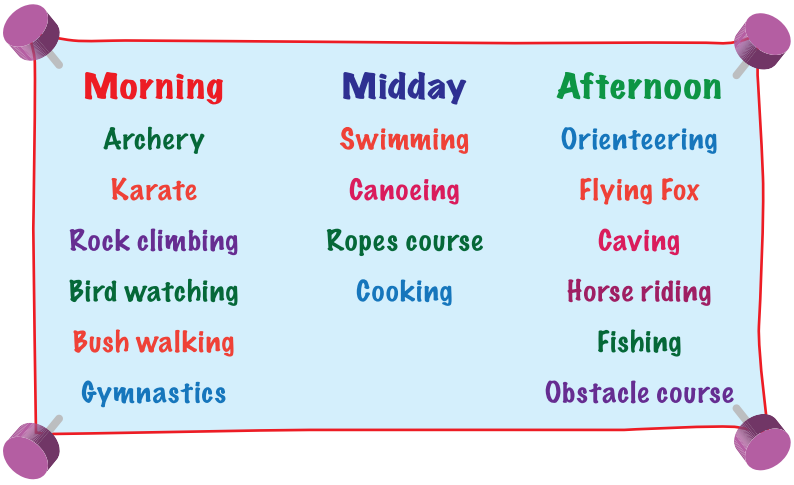
Example 5

Exercise 7-03 The multiplication principle

1 At a school camp, students choose one activity for each of the three sessions per day. How many different activity schedules are possible per day?



2 Anthony has 10 shirts, 6 pairs of pants and 4 pairs of shoes. In how many different ways can he dress himself?

3 A poker machine has 5 wheels. Each wheel can display 12 different pictures. How many arrangements of 5 pictures are possible on one line?

Example 6

4 A school needs to elect a boy captain and a girl captain. The boy candidates are Carlos, David, John, Martin and Peter and the girl candidates are Lydia, Vicki and Rosie.

a How many pairings of boy and girl captains are possible?

b List the possible pairings.

5 In a football-tipping competition, a player must pick the winners of 8 football matches each week. In each match, either of the two teams can win (assume no draws). How many possible outcomes are there for the 8 football matches?

6 Telephone numbers on the North Coast of NSW have eight digits. The first two digits must be 66. How many possible telephone numbers are there for this region?

7 In the game of Yahtzee, five dice are rolled together. How many different outcomes are possible?

8 FM radio stations in NSW have a call sign of '2' followed by three letters of the alphabet. One such call sign is 2DAY. How many different call signs are possible?

9 A new make of car has the following options.

Colour:	white, red, blue, green or silver
Transmission:	manual or automatic
Air conditioning:	yes or no
Airbags:	yes or no
Model:	sedan, hatchback or van

How many different arrangements of options are available?

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10 A bank account PIN (personal identification number) has four digits.

a How many PINs are possible?

b If letters of the alphabet are used instead of numbers, how many PINs are possible?

11 A coin is tossed 10 times. How many arrangements of heads and tails are possible?

12 An electronic security gate is opened using a six-character code where the first two characters are letters from A to F and the last four characters are digits from 0 to 9. One such code is DF1269. How many different codes are possible? Select **A**, **B**, **C** or **D**.

A 48 000 **B** 236 196 **C** 360 000 **D** 6 760 000

13 Ten friends—Raymond, Grace, Deborah, Harry, Nicole, Cindy, Jeff, Ivy, Adriana and William—go to a dance.

a How many boy–girl dance pairs are possible?

b List these possible pairs.

14 In the Braille alphabet (for the vision impaired), characters are represented by a cell of 1 to 6 raised dots printed in two columns. The cells for the letters Y, E and N are shown on the right.

Y E N

How many different cells are possible using the Braille system? (*Note*: A completely blank cell is allowed.)

15 In NSW, a black-on-white car number plate has 3 letters, 2 numbers and 1 letter, in that order. One such plate is BQZ 35N. How many of these number plates are possible?

16 In the late 1990s, telephone numbers in Australia changed from 7 digits to 8 digits.

a Why do you think this happened?

b How many *extra* phone numbers were possible by the addition of the extra digit?

JUST FOR THE RECORD

Choosing a good password

Did you know that one of the most-commonly used passwords is 'password1'?

It is important to use a password that is not easy for hackers to crack. Due to advances in technology, hackers can test over 6 million passwords per second.

- Don't choose actual words or names: hackers use dictionaries to generate passwords to log into your account illegally.
- Use a combination of numbers, lower case and capital letters, and symbols if they are allowed: this increases the number of possible passwords, making them harder to crack.
- Change your password every month, and don't use the same password for every account.
- Create a password with at least eight characters.
- One strategy is to choose a phrase or sentence to remember, then use the first letter of each word, and/or symbols or numbers to represent some words.

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There are websites with **password checkers** that test the strength of any password you type in. The higher the strength, the harder the password is to crack.

1 How many eight-character passwords are possible if a combination of numbers, lower case and capital letters are used?

2 Visit a website that checks password strength.

7-04 Counting arrangements

Ordered and unordered selections

EXAMPLE 7

Six friends—Rachel, Ross, Chandler, Monica, Phoebe and Joey—stand in line for a group photo. In how many ways can they be arranged?

Solution

Draw boxes to represent the six positions.

There are six choices for the first position, 5 choices for the second position, 4 choices for the third position, and so on. There is only one choice for the last position.

Number of possible arrangements $= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

Factorial notation, $x!$

$x!$ (read 'x factorial') is the product of all whole numbers from x down to 1.

For example:

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ '6 factorial'

$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ '9 factorial'

Your calculator has an $x!$ key. Enter 6 $x!$ to show that $6! = 720$.

On a graphics calculator, $x!$ is found by pressing OPTN for Casio or MATH for Texas Instruments, then selecting PROB or PRB (Probability).

SUMMARY

The number of ways in which n different items can be arranged is

$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$

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EXAMPLE 8

Santosh's office has 10 parking spaces for staff. In how many different ways can 10 cars be parked in these 10 spaces?

Solution

Draw boxes for the 10 parking spaces.

10	9	8	7	6	5	4	3	2	1

Number of possible ways $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 10!$

$= 3\ 628\ 800$

EXAMPLE 9

The 6 friends from Example **7** wish to have smaller group photos on a couch that seats only 4 people. How many arrangements are possible for a four-person photo?

Solution

Draw boxes for the four positions on the couch.

6	5	4	3

Number of possible arrangements $= 6 \times 5 \times 4 \times 3$

$= 360$

6 people, 4 positions:
4 terms in the multiplication

Arrangements are ordered selections

Examples **7**, **8** and **9** involve **arrangements** or **ordered selections** since items are arranged in a particular order.

EXAMPLE 10

A girls' school is electing a captain and a vice-captain. There are five candidates: Ang, Beth, Cassie, Dasha and Elena.

a How many possible pairings of captain/vice-captain are there?

b List the possible pairings.

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Solution

a

C	VC
5	4

Number of possible pairings = $5 \times 4 = 20$

5 people, 2 positions:
2 terms in the multiplication

b The 20 possible pairings (captain, vice-captain) are:

(A, B)	(A, C)	(A, D)	(A, E)	A = Ang
(B, A)	(B, C)	(B, D)	(B, E)	B = Beth
(C, A)	(C, B)	(C, D)	(C, E)	C = Cassie
(D, A)	(D, B)	(D, C)	(D, E)	D = Dasha
(E, A)	(E, B)	(E, C)	(E, D)	E = Elena

Example 7
Example 8
Example 9
Example 10

Exercise 7-04 Counting ordered selections

1 How many different ways are there of placing 4 letters into 4 envelopes (one letter per envelope)?

2 In how many different ways can the letters of the word CENTURY be arranged?

3 In how many different ways can Bret, Jemaine and Murray sit in a row? List the ways.

4 Five boys—Arden, Dominic, Mark, Ivan and Brad—run a 100 m race.

a How many 1st-2nd-3rd-4th-5th placings are possible?

b How many 1st-2nd-3rd placings are possible?

5 How many different two-digit numbers can be made from the digits 3, 8 and 1 if:

a the digits can be repeated? **b** the digits cannot be repeated?

List all possibilities in each case.

6 In horse racing, an Exacta bet is choosing the first two horses in a race in the correct order (1st-2nd).

a How many Exacta bets are possible for a 19-horse race?

b Benny has three favourite horses in the race: Jasmine's Joy, Mr Lucky and Big Shot. How many Exacta bets can he make using his three favourites? List the possibilities.

7 In how many ways can 12 rowers sit in 12 seats in a dragonboat? Select **A**, **B**, **C** or **D**.

A 144 **B** 4096 **C** 479 001 600 **D** 8.92×10^{12}

8 Each ticket to a concert has a three-letter code where the letters cannot be repeated. How many different tickets can be printed?

9 From a netball team of seven, a captain and vice-captain need to be selected. How many different pairings are possible?

10 Outside the town hall there are 3 flagpoles. The mayor's office owns 8 different flags. In how many ways can the flags be arranged on the 3 flagpoles (one flag per flagpole)?

11 How many different three-digit numbers can be made from the digits 3, 8, 1 and 5 if:

a the digits can be repeated? **b** the digits cannot be repeated?

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12 There are 24 horses in a race.

a A trifecta is a bet on the first three horses in a race, in the correct order (1st-2nd-3rd). How many trifectas are possible for this race?

b A superfecta is a bet on the first six horses in a race, in the correct order (1st-2nd-3rd-4th-5th-6th). How many superfectas are possible for this race?

13 A school council needs to elect a president, secretary and treasurer from 13 members. How many results are possible if each member has an equal chance? Select **A**, **B**, **C** or **D**.

A 78 **B** 1716 **C** 2197 **D** 1 594 323

14 Three girls—Issy, Trish and Gabby—have a swimming race.

a How many possible 1st-2nd-3rd placings are there? List them.

b How many possible 1st-2nd placings are there? List them.

c How many possible 1st placings are there? List them.

15 There are four seats in Kramer's car: two at the front, two at the back.

a How many different ways are there of seating four people—Kramer, Jerry, George and Elaine—in the car? List them.

b List the different ways of filling the two back seats if Kramer drives.

16 a In how many different ways can 3 girls and 2 boys line up at the canteen?

b In how many ways can they line up if the 3 girls must be ahead of the 2 boys?

7-05 Counting unordered selections

Ordered and unordered selections

Suppose there are six tennis players—A, B, C, D, E and F. How many different doubles teams (two players) can be chosen from them? This is an example of an **unordered selection**, where AB and BA are the same team, so are AC and CA, and so on.

There are $6 \times 5 = 30$ *ordered selections* for the two positions:

AB	AC	AD	AE	AF
BA	BC	BD	BE	BF
CA	CB	CD	CE	CF
DA	DB	DC	DE	DF
EA	EB	EC	ED	EF
FA	FB	FC	FD	FE

Because the two players in each arrangement above can 'swap places' and it will still be the same team, we must divide the above answer by 2! or 2.

Number of possible doubles teams $= \dfrac{6 \times 5}{2 \times 1} = 15$

This is $\dfrac{\text{No. of arrangements of 2 players from 6 players}}{\text{No. of ways 2 players can arrange themselves}}$

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EXAMPLE 11

Three students are to be selected from a group of 8 to represent the school at a talented writers' day. How many combinations of 3 students are possible?

Solution

Number of possible combinations $= \dfrac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

This is $\dfrac{\text{No. of arrangements of 3 students from 8 students}}{\text{No. of ways 3 students can arrange themselves}}$

This is another example of an unordered selection. Once the three students are selected, it doesn't matter what order they are within the group.

SUMMARY

The number of **unordered selections** from n different items for r positions is

$$\frac{n \times (n-1) \times (n-2) \times \ldots}{r!}$$ (r terms)

For example, the number of unordered selections from 6 items for 4 positions is

$$\frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1}$$ (4 terms)

EXAMPLE 12

In Lotto, 6 balls are randomly selected from a barrel of balls numbered 1 to 45. How many different selections are possible?

Solution

There are 45 balls and 6 positions.

$$\text{Number of possible selections} = \frac{45 \times 44 \times 43 \times 42 \times 41 \times 40}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{45 \times 44 \times 43 \times 42 \times 41 \times 40}{6!}$$

$$= 8\,145\,060$$

So there is 1 chance in over 8 million of winning Lotto if you choose one set of 6 numbers.

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Exercise 7-05 Counting unordered selections

Example 11

1 Phillipa wants to study two languages at university. She can choose from Chinese, French, Spanish, Vietnamese and Polish.

a How many selections are possible? **b** List the selections.

2 Mrs Kapp asked her class of 24 students to work in groups of three. How many different groups are possible?

3 After winning a radio competition, Matthew can select 10 CDs from a collection of 24. How many possible selections of CDs can he make? Select **A**, **B**, **C** or **D**.

A 1 961 256 **B** 10^{24} **C** 7.1×10^{12} **D** 6.3×10^{13}

4 In horse racing, a quinella is a bet on the first 2 horses in a race in any order (1st-2nd or 2nd-1st). How many different quinellas are possible for a 22-horse race?

5 In court, a jury panel of 12 members is chosen from a group of 40 candidates. How many different panels are possible?

6 Nikita has 3 free concert tickets that she wants to give to three of her five friends.

a In how many ways can she give away these tickets?

b List the ways. (Let her friends be represented by A, B, C, D, E.)

Example 12

7 Before 1989, there were only 40 numbered balls in the NSW Lotto game. How many combinations of 6 balls were possible then?

8 Four students are to be selected from the student council of eight to represent the school at an Anzac Day ceremony. How many groups of 4 representatives are possible?

9 There are 4 members (A, B, C, D) in a debating team but only three of them speak. How many speaking teams of 3 are possible? List them.

10 A hospital's casualty section is supervised by a team of 5 nurses per shift. If there are 15 nurses available, how many different teams are possible for each shift?

11 Six boys are playing handball but only four can play on the court at a time. How many groupings of four are possible:

a if the order on the court is not important?

b if the order on the court (who is in which square) is important?

12 A pet shop has 5 kittens for sale. In how many different ways can Rebecca make a selection if she is allowed to purchase:

a 2 kittens? **b** 3 kittens?

13 Eighteen people go ten-pin bowling and organise themselves in teams of six per lane. How many teams are possible?

14 How many different hands of 5 cards can be selected from all the hearts in a normal deck of cards? Select **A**, **B**, **C** or **D**.

A 5 **B** 1287 **C** 154 440 **D** 2 598 960

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15 In an English exam, Walid needs to answer 3 questions from section A and 2 questions from section B.

- **a** How many possible selections of 3 questions can Walid make in section A if it contains 10 questions?
- **b** How many possible selections of 2 questions can he make in section B if it contains 8 questions?
- **c** In how many different ways can Walid answer this English exam?

16 A family has 6 children but only two are allowed to sit in the front of the family van.

- **a** How many ways are there of choosing 2 children from a group of six?
- **b** How many ways are there of choosing 4 children from a group of six?
- **c** What do you notice about your answers to parts **a** and **b**? Can you work out why?

INVESTIGATION

CORRECT OR A LOTTO RUBBISH?

Robert plays Lotto every week and bets the same six numbers: 1, 2, 3, 4, 5, 6. His mother tells him he's crazy because 'These numbers will never come up!' Is she right? What is Robert's strategy?

7-06 Ordered and unordered selections

It is important to distinguish between ordered and unordered selections.

Ordered selections (or **permutations**) are arrangements in which the order within the group is important. For example, ABC, ACB, BAC, etc. are *different*. Examples are:

- the first 3 places in a horse race
- electing a president, secretary and treasurer of a committee
- arranging photos on a page in a photo album.

Unordered selections (or **combinations**) are arrangements in which the order within the group is not important. For example, ABC, ACB, BAC, etc. are *the same*. Examples are:

- choosing 5 players for a basketball team
- selecting 6 numbers for a Lotto game
- selecting a sample of 20 items to be tested.

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EXAMPLE 13

Jack needs to visit four places today: the post office, the bank, the butcher and the doctor.

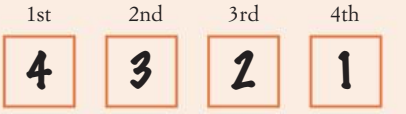
a In how many different ways can he visit these four places?

b In how many different ways can he visit these four places if he visits the doctor first?

Solution

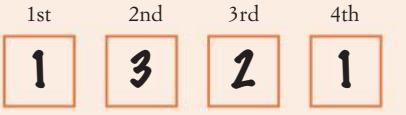
a Jack can visit the four places in any order so this is an ordered selection.

No. of ways $= 4 \times 3 \times 2 \times 1 = 24$



b If he visits the doctor first, there is only one possibility for 1st place.

No. of ways $= 1 \times 3 \times 2 \times 1 = 6$



EXAMPLE 14

Poker is a card game in which a player is dealt a 'hand' of 5 cards from a deck of 52 cards.

a How many different hands of 5 cards are possible?

b How many different hands of 5 hearts cards are possible?

c Hence find the probability (as a decimal correct to two significant figures) of being dealt a hand of 5 hearts cards.

Solution

This is an unordered selection since the order in which the cards are dealt is not important.

a Selecting 5 cards from 52 cards:

$$\text{Number of different hands} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}$$
$$= \frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$= 2\,598\,960$$

b Selecting 5 cards from 13 cards:

There are 13 hearts cards in a deck.

$$\text{Number of different hands of 5 heart cards} = \frac{13 \times 12 \times 11 \times 10 \times 9}{5 \times 4 \times 3 \times 2 \times 1}$$
$$= \frac{13 \times 12 \times 11 \times 10 \times 9}{5!}$$
$$= 1287$$

c $P(\text{5 hearts}) = \dfrac{1287}{2\,598\,000}$

$$= 0.0004953\ldots$$
$$\approx 0.00050$$

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Exercise 7-06 Ordered and unordered selections

1 State whether each situation describes an ordered or unordered selection.

- **a** Betting on the Melbourne Cup trifecta (the first three places).
- **b** The letters and numbers on a car number plate.
- **c** Selecting a volleyball team from 14 players.
- **d** Using a 6-digit code to open a suitcase.
- **e** Selecting a hand of 7 cards from a deck of 52 cards.
- **f** Choosing 3 friends to go to a concert with you.
- **g** Choosing a sample of 20 light globes to test.

Example 13

2 In an election, five candidates are listed at random on a ballot (voting) paper.

- **a** How many ways are there of listing the five candidates?
- **b** What is the probability that the candidates are listed in alphabetical order?
- **c** If Ms Ward is listed first, how many ways are there of listing the other four candidates?

Example 14

3 A bag contains 6 red marbles and 4 blue marbles. Four marbles are drawn from the bag.

- **a** How many different selections are possible?
- **b** How many different ways are there of selecting 4 red marbles from 6 red marbles?
- **c** What is the probability of selecting 4 red marbles from the bag?

4 From a group of 7 authors, three are selected to write a book. What is the probability that Klaas, David and Colin are selected?

5 **a** How many 4-letter arrangements can be made from the word COMPUTER?

- **b** What is the probability that an arrangement selected at random begins with the letter P?

6 Tom believes that in a 14-horse race, the first 3 placegetters must come from his list of five 'favourites'. A trifecta is a bet on the first three places of a horse race, in the correct order.

- **a** How many trifectas are possible from 14 horses?
- **b** If Tom wants to cover all possible trifectas of his five favourites, how many bets does he have to make?
- **c** If all 14 horses are equally likely to win the race, what is the probability that Tom will win the trifecta?

7 Two students are selected at random from a group of 5 girls and 4 boys.

- **a** How many different selections are possible?
- **b** How many selections of 2 girls are possible from 5 girls?
- **c** What is the probability that both students selected are girls?

8 Ristin has 4 ice gelato flavours in her fridge: chocolate, vanilla, mango and spearmint.

- **a** She wants to make a triple-decker gelato cone of 3 different flavours. How many triple-decker cones are possible?
- **b** What is the probability that a chocolate scoop will be on top?
- **c** Ed prefers his ice cream in a bowl, so the order within the bowl does not matter. Show that there are 4 possible combinations of 3 flavours and list them.

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9 Kara, Joshua and Alanna sit in a row. What is the probability that the two girls sit together? Select **A**, **B**, **C** or **D**.

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

10 a In Lotto, a player selects 6 numbers from 1 to 45. How many possible combinations are there?

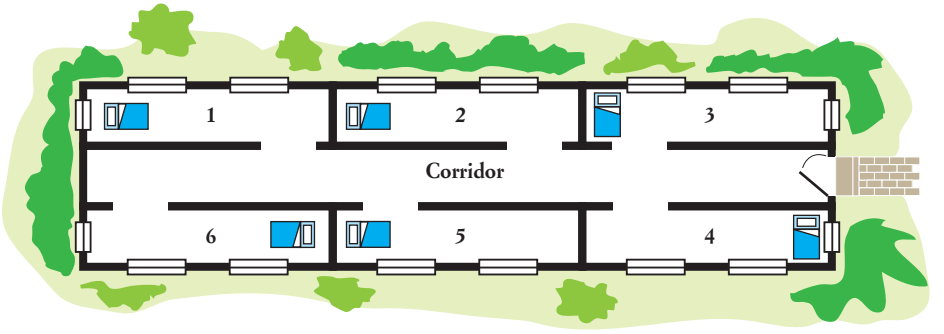
b In the Lotto Strike game, a player needs to select the first 4 numbers of a Lotto draw in the correct order. How many possible combinations are there?

c Which game is harder to win: Lotto or Lotto Strike? Justify your answer.

11 A bag contains ten \$2 coins, five \$1 coins and fifteen 10-cent coins. Mia randomly selects three coins from the bag. What is the probability that she selects three \$2 coins?

12 In poker, a 'royal flush' is getting 10, J, Q, K and A of the same suit. Show that there is 1 chance in 649 740 of achieving this.

13 Brad and Janet book into separate rooms on the top floor of a small hotel. There are 6 vacant rooms and they are randomly assigned a room each.



In how many ways can Brad and Janet be assigned their rooms? Select **A**, **B**, **C** or **D**.

A 12 **B** 15 **C** 30 **D** 360

14 The letters of the word MATHS are randomly arranged. Calculate the probability that:

a the letters are in alphabetical order

b the arrangement begins with M and ends with S

c the arrangement begins with M or S.

15 Five friends—Jack, Kevin, Lex, Marcel and Nick—play squash together.

a List the possible ways that two of them can be chosen to play each other for the first game.

b What is the probability that Nick plays in the first game?

c What is the probability that neither Kevin nor Lex plays in the first game?

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Tree diagrams

7-07 Probability tree diagrams

A **probability tree diagram** has the probability of each stage shown on the branches.

EXAMPLE 15

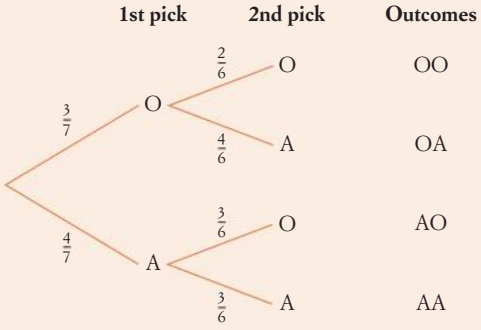
A fruit bowl contains 3 oranges and 4 apples. Prue selects 2 pieces of fruit at random from the bowl. Use a tree diagram to determine the probability that Prue selects:

a 2 oranges **b** an orange and an apple.

Solution

3 oranges, 4 apples: total 7

3O
4A



Suppose Prue picks an orange first.

- $P(\text{2nd pick is orange}) = \frac{2}{6}$ **6 pieces of fruit left: 2 are oranges**
- $P(\text{2nd pick is apple}) = \frac{4}{6}$ **6 pieces of fruit left: 4 are apples**

Suppose Prue picks an apple first:

- $P(\text{2nd pick is orange}) = \frac{3}{6}$ **6 pieces of fruit left: 3 are oranges**
- $P(\text{2nd pick is apple}) = \frac{3}{6}$ **6 pieces of fruit left: 3 are apples**

a $P(\text{OO}) = \frac{3}{7} \times \frac{2}{6}$ **Multiply probabilities on OO branches**

$= \frac{1}{7}$

b $P(\text{OA or AO}) = P(\text{OA}) + P(\text{AO})$ **Two ways of selecting an orange and apple**

$= \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6}$

$= \frac{4}{7}$

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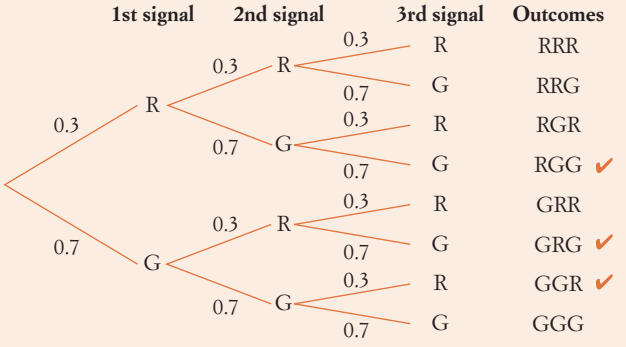
- A probability tree diagram lists all possible outcomes with the probabilities at each stage.
- Branches from the same point always have probabilities that add to 1, for example, $\frac{3}{7}+\frac{4}{7}$, $\frac{2}{6}+\frac{4}{6}$

EXAMPLE 16

To drive to work, Ms Dixon passes through 3 sets of traffic lights. The probability of a red signal (including amber) on each light is 0.3. Find the probability that on her way to work Ms Dixon gets:

a all green lights **b** two green lights **c** at least one green light



Solution

a $P(\text{GGG}) = 0.7 \times 0.7 \times 0.7$
$= 0.343$

b $P(\text{2 green}) = P(\text{RGG}) + P(\text{GRG}) + P(\text{GGR})$ **Ticked on the tree diagram.**
$= (0.3 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.7) + (0.7 \times 0.7 \times 0.3)$
$= 3 \times (0.3 \times 0.7 \times 0.7)$
$= 0.441$

c $P(\text{at least 1 green}) = 1 - P(\text{no green})$ **Use the complementary event rule.**
$= 1 - P(\text{RRR})$
$= 1 - (0.3 \times 0.3 \times 0.3)$
$= 0.973$

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SUMMARY

In a probability tree diagram:

- branches from the same point have probabilities that add to 1
- to calculate the probability of an outcome, *multiply* the probabilities along that branch
- to calculate the probability of two or more outcomes, *add* their calculated probabilities.
- $P(\text{at least one}) = 1 - P(\text{none})$

Exercise 7-07 Probability tree diagrams

Example 15

1 A jar contains 5 red lollies and 3 green lollies. Maya selects two lollies at random. Calculate the probability that they are of different colours. Select **A**, **B**, **C** or **D**.

A $\frac{13}{28}$ **B** $\frac{15}{32}$ **C** $\frac{15}{28}$ **D** $\frac{15}{56}$

2 Nikolai buys 2 tickets in a 50-ticket raffle with two prizes. What is the probability that he wins:

a 1st prize? **b** 1st and 2nd prizes?

c no prize? **d** at least 1 prize?

3 There are 10 batteries in a box and two are flat. Danielle takes 2 batteries out of the box at random. Calculate the probability that:

a both batteries are flat **b** only 1 of the batteries is flat.

4 A committee of 4 women and 3 men need to select a chairperson and a secretary. If every person is equally likely to be chosen, what is the probability that:

a both positions are filled by women?

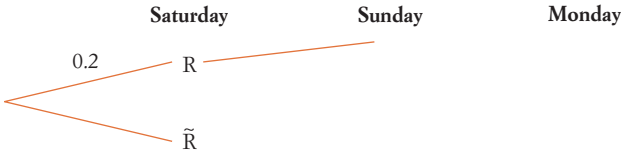
b both positions are filled by men?

c the chairperson is female and the secretary is male?

Example 16

5 For a long weekend (Saturday to Monday), the probability of rain on any day is 0.2.

a Copy and complete this tree diagram.



b Calculate the percentage probability that over the long weekend there

i is exactly 1 rainy day **ii** are no rainy days **iii** is at at least 1 rainy day.

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6 A die is rolled 3 times. What is the probability that 6 does not come up in any roll?

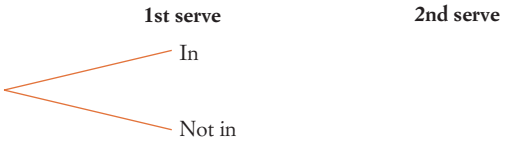
7 A biased coin comes up tails 63% of the time. If the coin is tossed 3 times, calculate the probability, correct to three decimal places, that:

a a tail comes up every time **b** a tail comes up twice.

8 Five cards are numbered 1, 2, 3, 4 and 5. If two cards are randomly selected, what is the probability of selecting:

a 2 even numbers? **b** 2 odd numbers?

9 A tennis player gets a second serve only if her first serve is not in. Jelena's first serve has a 0.78 probability of going in and her second serve has a 0.94 probability of going in.



a Copy and complete this tree diagram.

b A double fault occurs when both serves do not go in. What is the probability that Jelena serves a double fault?

c What is the probability that one of Jelena's serves goes in?

10 A student council has eight Year 10 students, six Year 11 students and four Year 12 students. Two students are selected from the council at random to represent the school at the mayor's lunch. Calculate the probability that:

a both students are from Year 10

b there is one student from each of Years 10 and 11

c at least one of the students is from Year 12

d each student is from a different Year.

11 Three people are selected at random. What is the probability that all were born in March? Select **A**, **B**, **C** or **D**.

A 0.0579% **B** 1.56% **C** 2.78% **D** 3.70%

12 In a town, 8% of people have a virus that can be detected by a medical test which gives a correct reading 90% of the time. If Sam is tested for the virus, what is the percentage probability that he:

a has the virus but it is not detected by the test?

b has the virus and it is detected?

c does not have the virus but it is falsely detected?

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INVESTIGATION

MORTALITY RATES

Insurance companies use **mortality rates** to calculate premiums for life insurance policies. Mortality rates are the probabilities that a person of a certain age will die that year. This table published in 2012 shows the mortality rates of Australians aged 0 to 100.

Age	Males	Females	Age	Males	Females
0	0.004 97	0.003 53	50	0.003 12	0.001 87
1	0.000 40	0.000 31	55	0.004 69	0.002 74
2	0.000 22	0.000 16	60	0.006 95	0.004 10
3	0.000 16	0.000 13	65	0.011 06	0.006 53
4	0.000 13	0.000 10	70	0.018 46	0.010 71
5	0.000 12	0.000 09	75	0.030 99	0.018 47
10	0.000 09	0.000 07	80	0.054 43	0.035 07
15	0.000 27	0.000 15	85	0.096 68	0.068 56
20	0.000 63	0.000 25	90	0.159 48	0.128 86
25	0.000 68	0.000 29	95	0.247 74	0.221 43
30	0.000 81	0.000 37	100	0.306 71	0.290 36
35	0.001 01	0.000 51			
40	0.001 34	0.000 78			
45	0.001 99	0.001 22			

Australian Bureau of Statistics. Life tables 2008 to 2010, mortality rate

a How do you think these figures were calculated?

b What patterns do you notice in the mortality rates as people get older? Why?

c Why do you think babies have a higher mortality rate?

d Why do you think there is a difference between male and female mortality rates?

e These mortality rates are lower than those in the year 2000, and this year's mortality rates will be lower than those in 2012. Why do you think this is so?

7-08 Expectation

Over repeated trials, the expected number of times a particular event will occur is found by multiplying its probability by the number of trials.

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SUMMARY

If the probability of an event is p and there are n trials, then the expected number of times the event will occur is $p \times n$.

EXAMPLE 17

A die is rolled 240 times. How many times would you expect a 3 or 5 to come up?

Solution

$$P(3 \text{ or } 5) = \frac{2}{6} = \frac{1}{3}$$

$$\text{Expected number of 3s or 5s} = \frac{1}{3} \times 240$$

$$= 80$$

Financial expectation

SUMMARY

The **financial expectation** of a game of chance is the average amount of money returned per game. It is calculated by multiplying each financial outcome by its probability and adding the results together.

EXAMPLE 18

Xanthe plays a game involving the tossing of 2 coins. She wins \$5 for 2 heads, \$1 for a head and a tail and \$0 for 2 tails. What is Xanthe's financial expectation from the game?

Solution

There are 4 possible outcomes: HH, HT, TH, TT.

$$P(\text{HH}) = \frac{1}{4}$$

$$P(\text{HT or TH}) = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{TT}) = \frac{1}{4}$$

$$\text{Financial expectation} = \$5 \times \frac{1}{4} + \$1 \times \frac{1}{2} + \$0 \times \frac{1}{2}$$

$$= \$1.75$$

Multiply each financial outcome by its probability and add the results.

This means that on average, Xanthe will win \$1.75 per game.

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EXAMPLE 19

Goran rolls a pair of dice in a game of chance that costs $1 per roll. The table lists the financial outcome for each event.

a Calculate the financial expectation for this game.

b On average, will he make a profit or a loss?

Event	Financial outcome
Double (both dice the same)	Win $2
Sum of 7	Win $3
Odd sum (except 7)	Win $1 (money back)
Even sum (except a double)	Lose $1

Solution

a There are 36 possible outcomes.

		2nd die					
	+	**1**	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(\text{double}) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{sum of 7}) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{odd sum except 7}) = \frac{12}{36} = \frac{1}{3}$$

$$P(\text{even sum except a double}) = \frac{12}{36} = \frac{1}{3}$$

$$\begin{aligned}\text{Financial expectation} &= \$2 \times \frac{1}{6} + \$3 \times \frac{1}{6} + \$1 \times \frac{1}{3} + (-\$1) \times \frac{1}{3} \\ &= \$0.8333\ldots \\ &\approx \$0.83\end{aligned}$$

b He will make a loss as the expected return (83 cents) is less than the cost of a game ($1).

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Exercise 7-08 Expectation

Example 17

1 The probability of a new light globe not working is 0.036. If 5821 light globes are produced by a factory in one day, how many of them can be expected not to be working?

2 On a TV game show, a contestant selects one of 9 panels which is turned around to reveal a prize. The panels contain 3 cash prizes, 2 holidays, 1 car and three other prizes. How often should a holiday be won over 260 shows? Select **A**, **B**, **C** or **D**.

A 29 **B** 47 **C** 58 **D** 65

3 One card is randomly selected from a normal deck of playing cards. If this experiment is performed 40 times, how many times should:

a a queen be selected? **b** a club be selected?

c an odd-numbered card be selected?

Example 18

4 A pair of coins is tossed 300 times. How often would you expect 2 heads to come up?

5 The probabilities of winning cash prizes on an instant lottery ticket are listed.

Prize	Probability
$2	0.17
$5	0.08
$50	0.008
$100	0.0047
$500	0.0001
$1000	0.000 05

a What is the financial expectation from one ticket?

b If Ailine buys 200 tickets at $2 each:

i how much does she spend?

ii how much can she expect to collect?

6 Jeff tosses two coins and wins $2 for 2 heads, $1 for 2 tails and nothing otherwise. Calculate his financial expectation.

7 You are given two options:

A a 100% chance of receiving $200 each week

B a 70% chance of receiving $500 and a 30% chance of losing $200 each week.

In the long run, which is the better option? Justify your answer.

8 Section 1 of a test has 20 multiple-choice questions, each with 4 possible answers A, B, C and D. Section 2 has 10 questions whose answers are either True or False. If every question in the test is randomly answered, what is the expected number of correct answers?

Example 19

9 Sonya rolls a die. If she rolls a 6 she wins $10, but if she rolls an odd number she loses $1. What is her financial expectation? Select **A**, **B**, **C** or **D**.

A $1.17 **B** $1.50 **C** $2.17 **D** $5.95

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10 Assume that the probability of a 20-year-old person in Australia dying this year is 5.5 in 10 000.

a Express this probability as a decimal.

b If there are 1 861 590 twenty-year-olds in Australia, how many are expected to die this year?

11 Marco bet $5 on each of these 5 horses in a race.

Horse	Probability of win	Payout from $5
Likely Lad	0.32	$11
Just a Minute	0.07	$68
Everyone's a Critic	0.11	$42
Sauerkraut	0.01	$480
Solar Eclipse	0.05	$96

a Calculate his financial expectation from this bet.

b On average, will Marco make a gain or a loss?

12 In Lotto, six numbers are chosen from 1 to 45. Last week, Lotto sold 22 million entries for a draw with a first prize of $3 million.

a What is the probability of winning first prize with a single entry?

b How much money did Lotto make if each entry costs 50 cents?

c How many first-prize winners should Lotto expect?

d What is each first-prize winner's share?

13 Out of 160 families that have 3 children, how many would you expect to have:

a 2 boys and 1 girl? **b** at least 1 girl?

14 The Getta Life insurance company has only 5 customers, whose details are listed below. The mortality rate is the probability of a person dying this year. The size of the policy is the amount that the company must pay the person's family if the person dies.

Name	Age	Mortality rate	Size of policy
Q Jumper	35	0.0011	$330 000
T Bagg	29	0.0004	$450 000
O Munni	51	0.0033	$235 000
L Liphant	64	0.0065	$200 000
C Shaw	44	0.0012	$270 000

a If each person pays a premium of $2500 each year, how much does the insurance company receive this year?

b What is the financial expectation of the company's payout this year?

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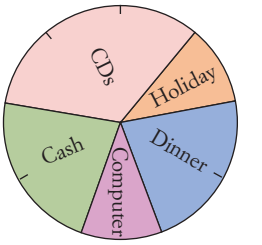
15 If this wheel is spun 60 times, how often can you expect:

a the holiday to come up?

b the CDs to come up?

c the dinner to not come up?



16 The Bulldogs have a 0.7 chance of winning a home game and a 0.41 chance of winning an away game. If they play 12 home games and 13 away games over a season, how many games are they expected to win?

INVESTIGATION

CASINOS AND WINNING STREAKS

a Is it fair that games of chance are operated so that a casino always wins in the long run? Examine this issue from different points of view.

b A coin is tossed over and over. It comes up heads 8 times in a row. On the very next toss, is a head more likely to come up again? Or a tail? Or does each outcome still have an equal chance?

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Sample HSC problem

Monique plays a game that costs $1 each time. She reaches into a box containing sixteen 20-cent coins, twenty-two $1 coins and twelve $2 coins, and randomly takes out 1 coin. She gets to keep that coin.

a What is the probability that Monique wins her $1 back?

b Calculate the financial expectation for this game.

c On average, will Monique make a gain or a loss on each game? How much will it be?

Study tip

Take advantage of 'down time'

At some times during the day, such as mid-afternoon or before dinner, your mind and body will be less motivated to work. Ever read a page of text and not taken anything in? When writers have 'writer's block' and run out of ideas, they do something different to take their minds off it. Take a break from your studies. Go for a walk or jog. Get some fresh air. Play with your dog. Have a glass of water. Stretch, meditate or exercise. Alternatively, use your 'down time' to complete some task that requires less effort, such as organising your desk or folders, catching up on missed work or doing some background reading. By changing your mood and environment, you can overcome boredom and fatigue and return to your studies with a 'second wind'.

Your time becomes more precious in Year 12, so take advantage of any spare time you have during the day, such as when commuting to school, waiting between appointments, or that half-hour before dinner. Use these pockets of 'waiting time' to complete little tasks. Revise your topic summaries on the train or outline a study plan just before dinner. This way, you can use your 'proper' study time for more in-depth homework and revision. Effective use of spare time today can save you much time and worry tomorrow.

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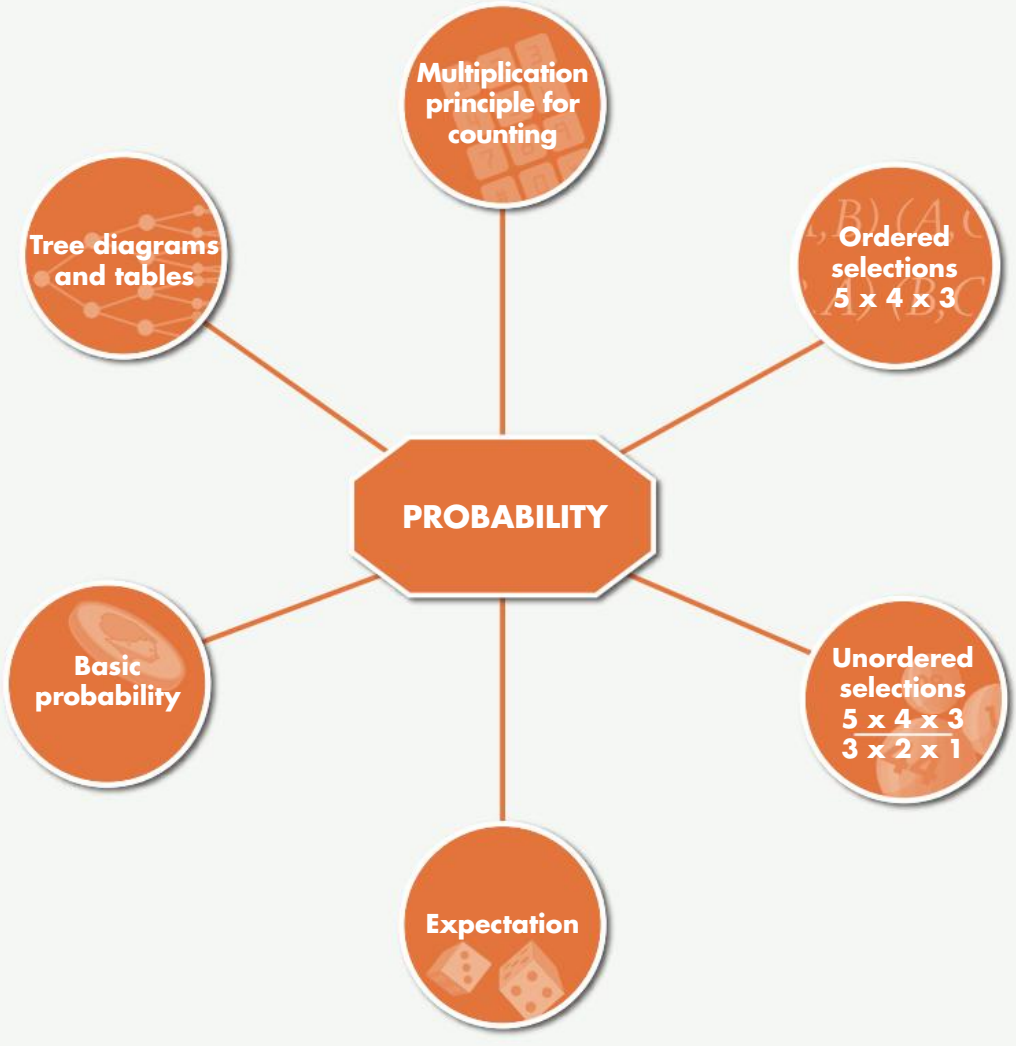
CHAPTER REVIEW

Topic overview

Probability crossword

This chapter, Probability, revised and extended probability theory from the Preliminary Course. Many complex ideas were covered—tree diagrams, ordered and unordered selections and expectation—so spend considerable time summarising these concepts. Aim to become competent at interpreting a probability problem and deciding on the best strategy for solving it: counting techniques, lists and tables, tree diagrams and probability rules. Many students have difficulty in constructing tree diagrams effectively, so practise drawing these regularly.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 7-01

1 An iceblock company is running a competition where the names of the prizes are printed on the iceblock sticks. The prizes are distributed in the quantities shown in the table. What is the probability of:

Prize	Frequency
Another iceblock	15 000
CD player	200
Mountain bike	150
Game system	60
Holiday	5
Car	1
No prize	104 584

a winning the car?

b winning another iceblock?

c winning a CD player or game system?

d not winning a mountain bike?

Exercise 7-02

2 Two dice are rolled. What is the probability of:

a getting a double (the same number on both dice)?

b a 6 appearing on at least one of the dice?

Exercise 7-02

3 What is the probability that in a family of 2 children:

a the older child is a girl?

b both are girls?

c there is 1 boy and 1 girl (in any order)?

d there is at least 1 boy?

Exercise 7-03

4 An office beverage maker provides the following options:

Beverage: Coffee or tea

Milk: Yes or no

Sugar: 0, 1 or 2 spoonfuls

How many different arrangements are possible?

Exercise 7-03

5 A standard number plate has 2 letters followed by 2 numbers and 2 letters. One such plate is RY 28 AU. How many different number plates are possible?

Exercise 7-04

6 In how many different ways can the coach of a cricket team organise the batting order of his 11 players?

Exercise 7-04

7 Five people run a race and prizes are awarded for 1st and 2nd places. How many possible pairings of 1st-2nd places are there?

Exercise 7-05

8 Pablo created 7 pieces of artwork and must select 2 of them for an exhibition. How many combinations are possible?

Exercise 7-05

9 In how many ways can a basketball coach select a team of 5 players from 10 students?

Exercise 7-06

10 A hand of 7 cards is dealt from a deck of 52 cards. How many different hands are possible?

Exercise 7-06

11 In Lotto, it is possible to buy a 'System entry' to bet on more than 6 numbers. For example, in a System 8 game, a player chooses 8 numbers and then all selections of 6 involving these 8 numbers are entered. How many selections are there in a System 8 entry?

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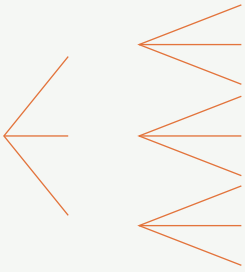
12 Which situation can be illustrated by this tree diagram?
Select **A**, **B** or **C**.



A Tossing a coin 3 times.

B Selecting 2 balls from a bag of red, blue and green balls.

C 3 students passing or failing an exam.

Exercise 7-07

13 Tegan buys 5 tickets in a raffle in which 80 tickets are sold. There are 3 prizes.
Use a tree diagram to find the probability that Tegan:

a wins all 3 prizes **b** wins at least 1 prize **c** wins exactly 1 prize.

Express your answers as percentages correct to two significant figures.

Exercise 7-07

14 In a group of 700 people, how many would you expect to have been born in October?

Exercise 7-08

15 Four hundred raffle tickets are sold. There are 3 cash prizes: $160, $120 and $80.
Calculate the financial expectation of one raffle ticket.

Exercise 7-08

16 A property owner is trying to sell his apartment. There are 4 possible outcomes as shown in the table on the right. Calculate his expected profit.

Outcome	Probability
Sell at a profit of $8000	26%
Sell at a profit of $2000	37%
Break even (no profit)	25%
Sell at a loss of $3000	12%

Exercise 7-08

Newspix/Dan Peled

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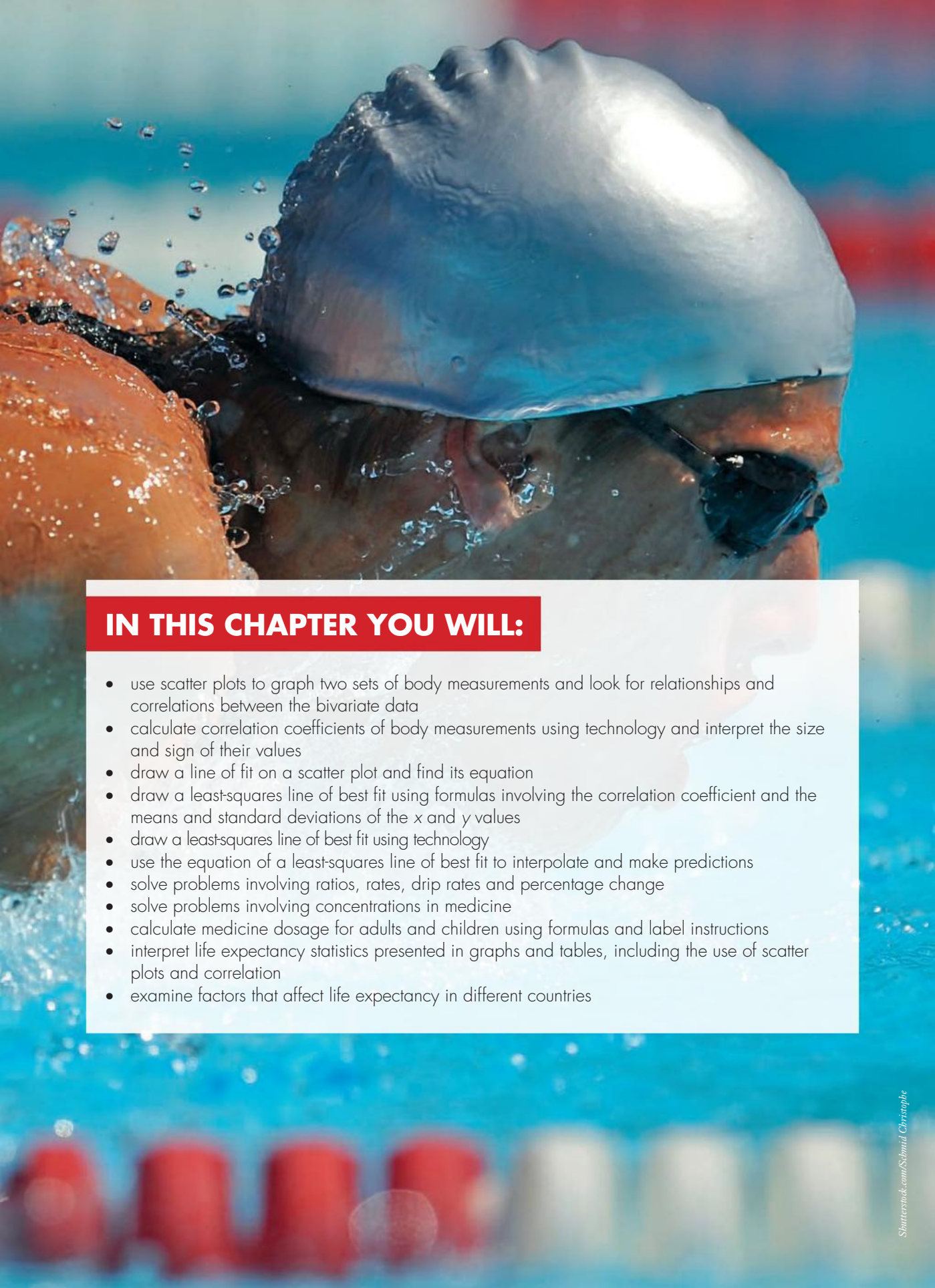
MATHEMATICS AND HEALTH

8.

HEALTH AND MEDICINE

US swimmer Michael Phelps won his 18th Olympic gold medal at the 2012 London Games to bring his medal tally to 18 gold, 2 silver and 2 bronze over three Olympic Games. A contributing factor to his success is his height (193 cm) and arm span (200 cm). His arm span is often referred to as his 'wingspan', because he flies through the water.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- use scatter plots to graph two sets of body measurements and look for relationships and correlations between the bivariate data
- calculate correlation coefficients of body measurements using technology and interpret the size and sign of their values
- draw a line of fit on a scatter plot and find its equation
- draw a least-squares line of best fit using formulas involving the correlation coefficient and the means and standard deviations of the x and y values
- draw a least-squares line of best fit using technology
- use the equation of a least-squares line of best fit to interpolate and make predictions
- solve problems involving ratios, rates, drip rates and percentage change
- solve problems involving concentrations in medicine
- calculate medicine dosage for adults and children using formulas and label instructions
- interpret life expectancy statistics presented in graphs and tables, including the use of scatter plots and correlation
- examine factors that affect life expectancy in different countries

TERMINOLOGY

biometric data
concentration
dosage
life expectancy
microgram
negative
regression
strong
bivariate data
correlation
drip rate
line of fit
milligram
positive
scatter plot
trendline
causality
correlation coefficient (r)
least-squares
linear
moderate
predict
solution
weak

Assignment 8

Skillcheck

1 Convert:
a 500 g to kg **b** 3 000 000 mg to g **c** 684 g to mg

2 If $y = 0.85x + 22.5$, find the value of:
a y when $x = 180$ **b** x when $y = 35.25$

3 Find the gradient of the line that passes through (–1, 5) and (4, 8).

4 Increase \$600 by 5%, then decrease the result by 5%.

5 If 15 out of 130 females smoke, what percentage of females do not smoke? Answer correct to three significant figures.

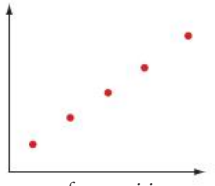
6 A 1 L salt solution contains water and 50 mL of salt. Find the ratio of:
a salt to water **b** salt to solution.

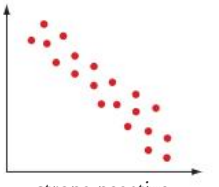
Body measurements

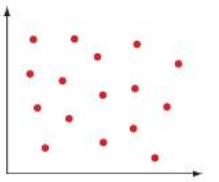
A page of scatter plots

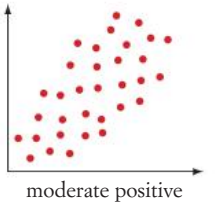
8-01 Scatter plots of body measurements

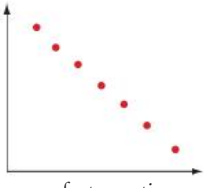
A **scatter plot** or **scattergram** is a graph of points on the number plane. Each point represents the values of two different variables, so scatter plots represent **bivariate data** (data relating to two variables). By observing the pattern of the dots in a scatter plot, we can find the direction and strength of the relationship represented. The closer the dots, the stronger the relationship.

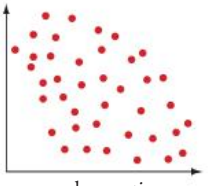
a

perfect positive

b

strong negative

c

no relationship

d

moderate positive

e

perfect negative

f

weak negative

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Data obtained by measuring physical characteristics, such as height, arm span and bone length are called **biometric data**. Body measurements usually have a moderate to strong linear relationship.

EXAMPLE 1

The heights and arm spans of 10 male swimmers are shown in the table.

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Plot the data on a scatter plot.

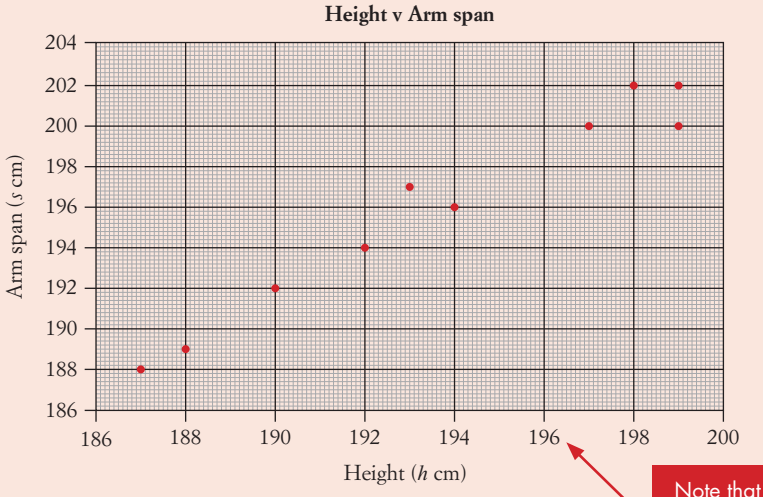
b Is the relationship between a person's height and arm span positive, negative or zero?

c Is the relationship strong, moderate or weak?

d Comment on the statement: 'For an individual, arm span and height are approximately the same'.

Solution

a



Note that for scatter plots, you do not need to start both axes at 0. Choose an appropriate start and end value for each axis to reflect the values of the given data.

b The relationship is positive. As height increases, arm span increases.

c The relationship is strong. Points are close together.

d True. According to the data (table and graph), arm span and height are approximately the same for an individual.

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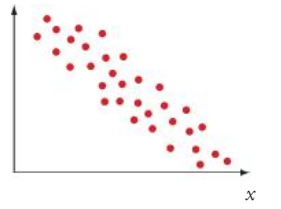
Exercise 8-01 Scatter plots of body measurements

In this exercise, draw scatter plots on graph paper or use a spreadsheet. We will use these scatter plots later in the chapter.

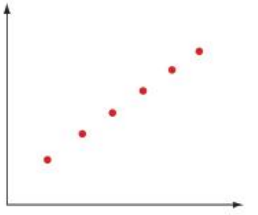
1 **a** What type of relationship is shown on this scatter plot?

b What is the missing word from this sentence?
'As the value of x increases, the value of y _____.'

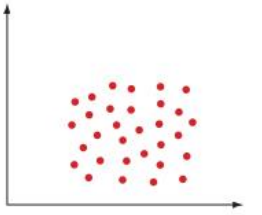


2 For each scatter plot, state whether a linear relationship exists and if it does, whether it is positive or negative, and whether it is strong, moderate or weak.

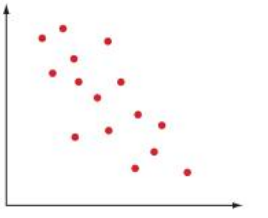
a



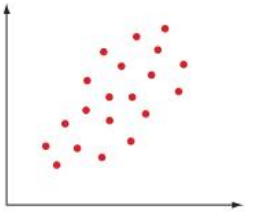
b



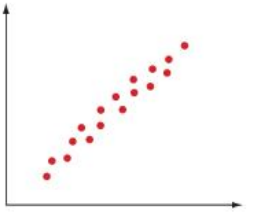
c



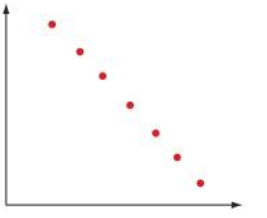
d



e

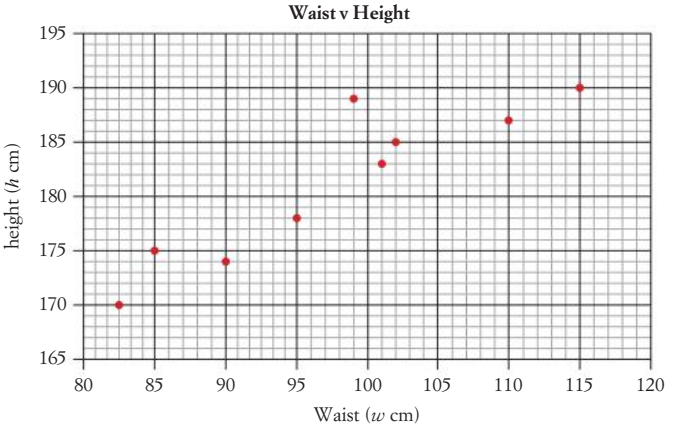


f



3 Which scatter plot in question **2** could represent the relationship between height and stride length of people?

4 The waist lengths and heights of 10 people were measured and the results graphed on a scatter plot.



a What is the waist circumference of the person who is 189 cm tall?

b How tall is the person with a waist circumference of 90 cm?

c Is the relationship between waist circumference and height linear?

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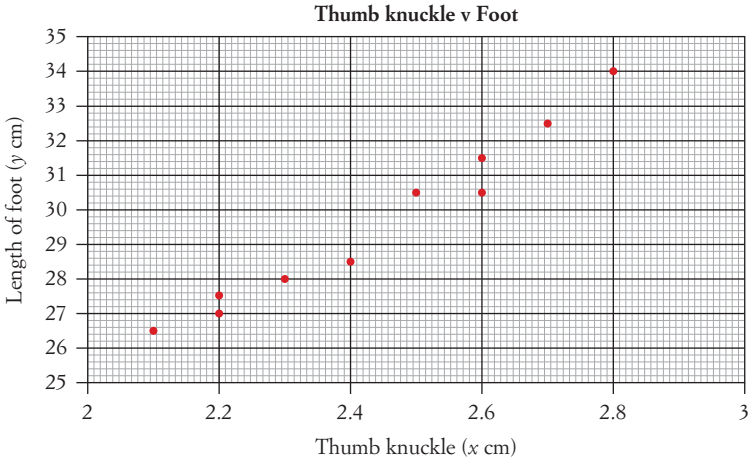
d What is the strength of the relationship? Justify your answer.

e Consider the statement: 'A person's waist circumference is approximately half of their height'. Does the data support this statement? Justify your answer.

5 Zach measured the width of his thumb knuckle and the length of his foot, and exclaimed to Jacinta that '12 of my thumb knuckles equal the length of my foot!' They tested this theory by taking their own measurements plus those of 8 friends and plotted the results.



a Describe the pattern of the data.

b Is the relationship between thumb knuckle width and foot length positive or negative?

c Is the relationship strong, moderate or weak?

d Do you agree with Zach's theory? Justify your answer.

Example 1

6 This is a table of typical body measurements, correct to the nearest centimetre, for girls aged 4–16 years.

Age (years)	4	6	8	10	12	14	16
Height	104	116	128	140	152	164	176
Chest circumference	58	60	64	68	76	84	92
Waist circumference	54	55	58	61	65	69	73
Hip circumference	61	64	69	74	82	90	98
Neck circumference	28	29	30	31	33	35	37
Outside leg (waist height)	59	68	77	86	94	102	110

a Plot the data for height and outside leg on a scatter plot.

b Describe the strength of the relationship between height and outside leg.

c Plot the data for height and hip circumference on a scatter plot.

d Describe the strength of the relationship between height and hip circumference.

e Plot the data for waist and neck circumference on a scatter plot.

f Describe the strength of the relationship between waist and neck circumference.

g Jenny measured her waist and neck and said 'my waist is double my neck measurement'. Do the data reflect this statement?

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7 The heights and weights of eight teenagers were recorded.

Height (h cm)	155	168	162	175	163	178	180	165
Weight (w kg)	50	73	55	84	62	76	80	59

a Draw a scatter plot for the data.

b Describe the pattern of the data.

c Describe the relationship between the student's weight and height.

8 A paleontologist measured the thigh bone (femur) and upper arm bone (humerus) of some fossils. The results are shown in the table.

Femur (f cm)	60	73	56	50	67	62	72	59
Humerus (h cm)	70	78	59	40	70	61	71	58

a Plot the data on a scatter plot.

b Describe the relationship between the lengths of the two bones.

c Do you think that there is a linear relationship between femur and humerus lengths in humans? Justify your answer.

INVESTIGATION

ANCIENT BODY MEASUREMENTS

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In ancient and medieval times, people used parts of the body to measure lengths.

A **palm** or **handbreadth** was the width of your four fingers pressed together and is about 7.5 cm.

A **hand span** was the distance between thumb and little finger when the fingers are spread and is about 22.5 cm.

A **foot** was the length of King Henry I's foot, defined as 12 inches or about 30.5 cm. An average person's foot today is 23.5 cm.

A **cubit** was the length of the forearm, from elbow to tip of middle finger. Since people from ancient cultures differed in size, the cubit ranged from 44 cm to 52 cm. Today, the average adult cubit is 46 cm.

1 Show that:

a 1 cubit = 6 palms **b** 1 cubit = 2 spans **c** 1 span = 3 palms

2 Noah's ark was described to be 300 cubits long, 50 cubits wide and 30 cubits high. Convert these measurements to:

a spans **b** palms **c** feet **d** metres

3 Collect measurements for the cubit, foot, palm and span from 10–12 people.

a Find the average for each measurement and compare with the values given above.

b Select any two measurements, graph them on a scatter plot and determine whether there is a relationship between them.

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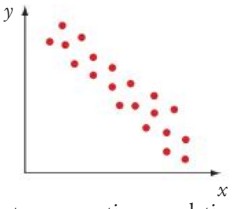
8-02 Correlation

If two variables are plotted on a scatter plot and the points have a linear relationship, we say that the two variables are **correlated**. **Correlation** is a measure of the strength of the linear relationship.

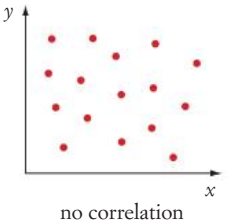
WS
Correlations matching game

A



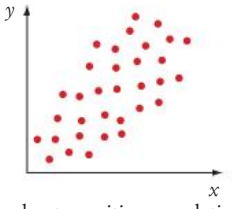
strong negative correlation

B



no correlation

C



moderate positive correlation

Correlation and causality

A correlation between two variables does not necessarily mean that one variable **causes** the other to occur. For example, a person's height and foot size have a high correlation (as height increases, foot size increases) but a change in height does not necessarily cause a change in foot size. External factors, such as age and growth rate are usually involved in strong but non-causal relationships.

EXAMPLE 2

Each pair of variables has a strong correlation. Determine whether each pair have a causal relationship, and for those that don't, suggest an external factor that may affect the correlation.

a height and weight of a person
b price of petrol and amount of petrol sold at a petrol station
c number of hot dogs and drinks sold at a football game
d amount of road traffic and driving time to school/work

Solution

a Not causal: affected by such factors as age, fat and muscle distribution.
b Causal: the higher the price, the lower the sales (negative correlation).
c Not causal: affected by number of people watching the game.
d Causal: the greater the traffic, the longer the driving time.

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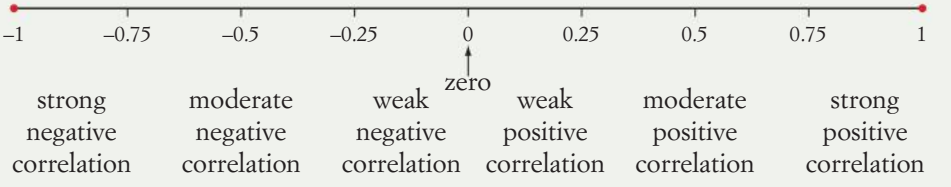
The correlation coefficient

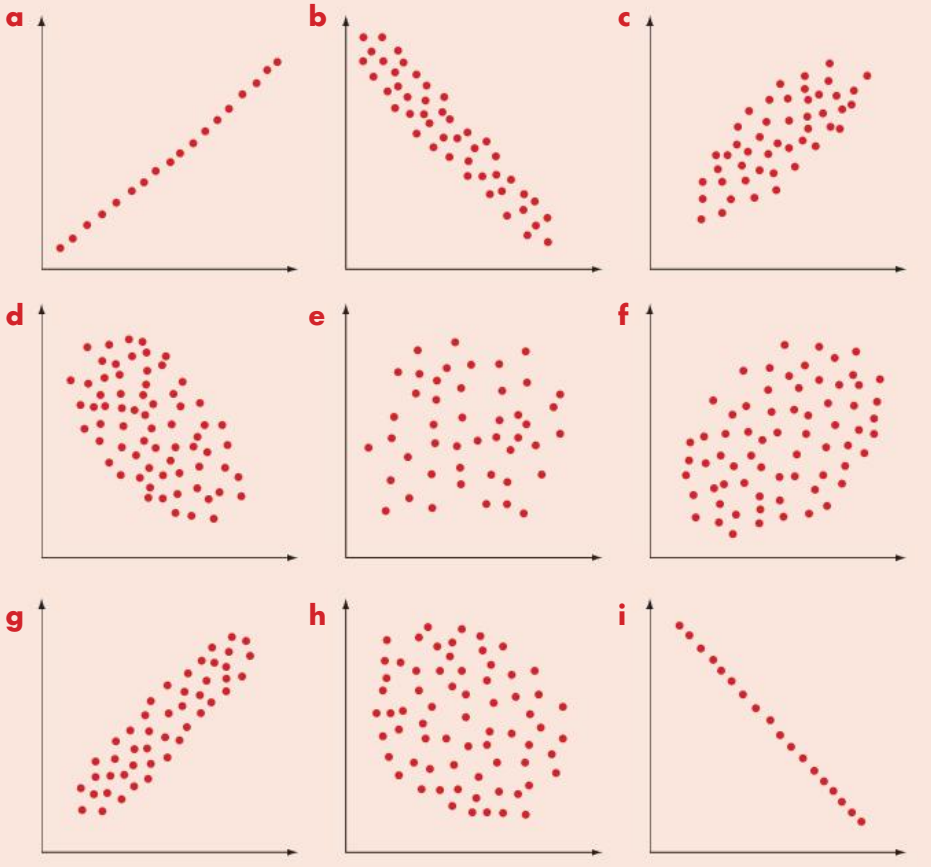
SUMMARY

The **correlation coefficient**, r, is a number between –1 and +1 that measures the correlation between two variables. This scale shows the meanings for the different values of r.



EXAMPLE 3

Match each scatter plot to a correlation coefficient below and describe the correlation.



$r = -1$ $r = -0.5$ $r = -0.2$ $r = 0$ $r = 0.3$

$r = 0.7$ $r = 0.8$ $r = -0.9$ $r = 1$

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Solution

a $r = 1$ perfect positive correlation
b $r = -0.9$ strong negative correlation
c $r = 0.7$ moderate positive correlation
d $r = -0.5$ moderate negative correlation
e $r = 0$ zero correlation
f $r = 0.3$ weak to moderate positive correlation
g $r = 0.8$ strong positive correlation
h $r = -0.2$ weak negative correlation
i $r = -1$ perfect negative correlation

Calculating the correlation coefficient

The correlation coefficient, r, can be calculated using a scientific calculator, graphics calculator, online calculator or spreadsheet. As with standard deviation, there is a complex formula for calculating the correlation coefficient but you are not required to use it.

EXAMPLE 4

The heights and arm spans of 10 male swimmers are shown in the table (from Example **1**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

Find the correlation coefficient, r, correct to 2 decimal places.

Solution

For bivariate data, follow the instructions for your calculator as shown in the tables below for a correlation coefficient of $r \approx 0.98$.

Operation	**Casio scientific**	**Sharp scientific**
Start statistics mode.	MODE STAT A+BX	MODE STAT LINE
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table Enter in X column 190 = 192 =, etc. enter in Y column 192 = 194 =, etc. AC to leave table	190 2ndF STO 192 M+ 192 2ndF STO 194 M+ etc.
Calculate the correlation coefficient ($r = 0.9805...$)	SHIFT 1 Reg r =	ALPHA r =

Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and CLEAR, then 2nd 0 and scroll down to select DiagnosticOn ENTER ENTER.
Clear statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2 column	STAT Edit … with cursor on L1 CLEAR ENTER Repeat for L2.
Enter x-scores in List 1.	190 EXE 192 EXE, etc.	190 CLEAR 192 ENTER, etc.
Enter y-scores in List 2.	192 EXE 194 EXE, etc.	192 ENTER 194 ENTER, etc.
Calculate the correlation coefficient ($r = 0.9805$ ...)	CALC REG X to see 'LinearReg' and the value of r.	STAT CALC LinReg(ax+b) ENTER to find value of r.

On a **spreadsheet**, enter the data into two rows as shown below, then enter the formula =PEARSON(B1:K1, B2:K2) into any blank cell to calculate the correlation coefficient $r = 0.9805$....

	A	B	C	D	E	F	G	H	I	J	K
1	**Height (cm)**	190	192	199	188	197	187	194	198	193	199
2	**Arm span (cm)**	192	194	202	189	200	188	196	202	197	200

The formal name for this linear correlation coefficient is **Pearson product-moment correlation coefficient**.

Exercise 8-02 Correlation

1 Describe in your own words:

a bivariate data **b** correlation **c** correlation coefficient

2 Which pair of variables has no correlation? Select **A**, **B**, **C** or **D**.

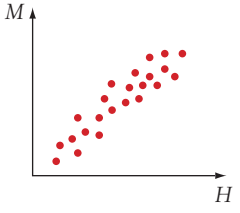
A car engine size and towing capacity

B height and ability to touch the ceiling

C number of pies eaten for lunch and handspan

D age and number of doctor visits

3 Which phrase describes the correlation between variables H and M on the scatter plot? Select **A**, **B**, **C** or **D**.



A moderate positive correlation

B weak negative correlation

C strong positive correlation

D moderate negative correlation.

4 Does each pair of variables have a positive, negative or zero correlation?

a number of alcoholic drinks consumed and reaction speed

b number of cigarettes smoked and probability of lung cancer

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c price of a packet of biscuits and the change from $10

d number of road accidents and number of ambulances on the road

e size of feet and swimming speed

f length of femur (thigh bone) and length of humerus (upper arm bone)

g tennis ability and time spent practising tennis shots

h time spent awake and time spent asleep

5 For each correlation coefficient value, draw a scatter plot that illustrates the correlation. Example 2

a 0.2 **b** 0.9 **c** –0.7

d 0.5 **e** –0.3 **f** 0

6 The following pairs of variables have a correlation. Which ones have a causal relationship? Example 3

a abdominal circumference and head circumference of a foetus

b height and arm span of nurses

c shoe size and age of children

d number of patients in a hospital emergency room and waiting time

e number of computers and number of TVs in a home

f home loan interest rate and size of monthly repayments

g driving speed of a vehicle and time taken to arrive home

h spelling test mark and English test mark

i driving speed of a vehicle and amount of fuel used

j temperature and ice cream sales

7 The systolic value (taken when the heart beats) of Victor's blood pressure was recorded every five years. Example 4

Age (*A* years)	15	20	25	30	35	40	45	50	55	60
Blood pressure (*B* mmHg)	117	120	121	122	123	125	127	129	131	134

Note: mmHg means millimetres of mercury

a Draw a scatter plot of the data.

b Describe any correlation between the two variables.

c Write an estimate for the correlation coefficient.

d Calculate the correlation coefficient, r, correct to 2 decimal places.

e How close was your estimate to the actual value?

8 A local gym recorded the age and pulse rate of 10 people before exercise.

Age (*A* years)	45	20	15	62	35	49	18	38	55	24
Pulse rate (*B* beats/min)	71	79	83	70	68	67	86	72	68	86

a Draw a scatter plot of the data.

b Describe any correlation between the two variables.

c Write an estimate for the correlation coefficient.

d Calculate the correlation coefficient, r, correct to 2 decimal places.

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9 Calculate, correct to 3 significant figures, the correlation coefficient for each set of bivariate data from Exercise 8-01.

a

Waist (cm)	54	55	58	61	65	69	73
Neck circumference (cm)	28	29	30	31	33	35	37

b

Height (h cm)	155	168	162	175	163	178	180	165
Weight (w kg)	50	73	55	84	62	76	80	59

c

Femur (f cm)	60	73	56	50	67	62	72	59
Humerus (h cm)	70	78	59	40	70	61	71	58

WS

A page of scatter plots

INVESTIGATION

CORRELATION OF BODY MEASUREMENTS

Does a person's height correlate with their other body measurements? Make the following measurements for 10–15 people.

- height
- arm span (distance between the middle fingertips when arms are outstretched)
- head circumference (measured at the widest point)
- hand span (distance between thumb and little finger when hand is outstretched)
- wrist circumference (at the point where the hand bends)
- femur length (hip to knee)

1 Construct a scatter plot comparing height with arm span, head circumference, hand span, wrist circumference or femur length.

2 Are the two measurements correlated?

3 Is there a strong or weak correlation?

4 Are there any outliers?

5 Calculate the correlation coefficient correct to 4 decimal places.

6 Do you think that your results are typical of the Australian adult population?

7 What conclusions can you draw about the relationships between the two measurements? Write a paragraph about your findings.

WS

Height vs shoe size

WS

Lines of fit

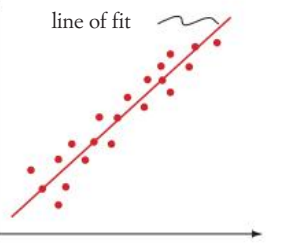
WS

Least-squares regression line

8-03 Least-squares line of best fit

Lines of fit

If two variables x and y graphed on a scatter plot show a strong linear correlation, we can approximate the linear relationship by drawing a **line of fit** through the points and finding its equation $y = mx + b$.



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SUMMARY

A line of fit:

- represents most or all of the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible

EXAMPLE 5

The heights and arm spans of 10 male swimmers are shown in the table (from Example **1**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Graph the data on a scatter plot and draw a line of fit to represent all data points.

b Find the equation of the line of fit.

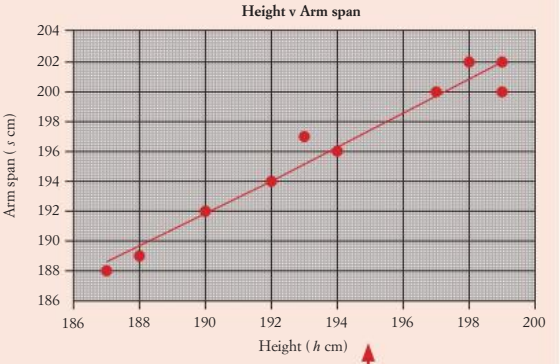
c Use the graph and the equation to estimate the arm span of a male swimmer who is 188 cm tall.

d Use the graph and the equation to estimate the height of a male swimmer with an arm span of 195 cm.

e Is it possible to predict the arm span of a female swimmer who is 165 cm tall?

Solution

a Use a transparent ruler to draw a line of fit. Points not on the line should be roughly equally-spaced from the line.



Your line of fit may differ from the one shown here but should look similar.

b The equation of the line is of the form $s = mh + b$.

To find the gradient m, choose two points from the line, say (192, 194) and (199, 202).

$$m = \frac{202 - 194}{199 - 192}$$
$$= \frac{8}{7}$$
$$= 1.1428\ldots$$
$$\approx 1.14$$

Rounding to two decimal places.

$\therefore\ s = 1.14h + b$

To find the vertical intercept b, substitute one point on the line, say (199, 202) again.

$$202 = 1.14(199) + b$$
$$= 226.86 + b$$
$$202 - 226.86 = b$$
$$b = -24.86$$
$$\therefore s = 1.14h - 24.86$$

c Reading from the graph, the arm span of a male swimmer who is 188 cm tall is 189 cm.

Substituting $h = 188$ into $s = 1.14h - 24.86$:

$$s = 1.14(188) - 24.86$$
$$= 189.46$$
$$\approx 189$$

The arm span of a male swimmer who is 188 cm tall is 189 cm.

d Reading from the graph, the height of a male swimmer with an arm span of 195 cm is 193 cm.

Substituting $s = 195$ into $s = 1.14h - 24.86$:

$$195 = 1.14h - 24.86$$
$$195 + 24.86 = 1.14h$$
$$1.14h = 219.86$$
$$h = \frac{219.86}{1.14}$$
$$= 192.8596\ldots$$
$$\approx 193$$

The height of a male swimmer with an arm span of 195 cm is 193 cm.

e No, you cannot predict the arm span of a female swimmer who is 165 cm tall. The data is about male swimmers only. Also, the value of 165 cm is outside the limits of the data here.

WS

Least-squares regression line

Least-squares regression line

A line of fit drawn on a scatter plot is called a **regression line**. The equation of this line represents all data points. Regress means to 'go back', so the points on the scatter plot are 'returning' to the regression line.

SUMMARY

The **least-squares line of best fit** (or **least-squares regression line**) is the line of fit that best represents all data points. Its gradient and y-intercept can be calculated using formulas involving the correlation coefficient r and the means and standard deviations of the x and y values.

The equation of the least-squares line of best fit is $y = mx + b$, where:

$$m = r \times \frac{\text{standard deviation of } y\text{-scores}}{\text{standard deviation of } x\text{-scores}} \left(\text{or } \frac{r\sigma_y}{\sigma_x} \right)$$

$b = \text{mean of } y\text{-scores} - m \times \text{mean of } x\text{-scores} = \bar{y} - m\bar{x}$

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If you measure the distance of each point from the line and then square them and add them (for example, $0.8^2 + 0.1^2 + 0^2 + \ldots$), the sum of the squared distances will be as small as possible, hence the name 'least-squares'.

EXAMPLE 6

The heights and arm spans of 10 male swimmers were found to have a correlation coefficient of 0.98 (from Example **4**).

Height (h cm)	190	192	199	188	197	187	194	198	193	199
Arm span (s cm)	192	194	202	189	200	188	196	202	197	200

a Calculate the mean and standard deviation for height and arm span values. Write standard deviation values correct to two decimal places.

b Find the equation of the least-squares line of best fit.

c Graph the equation of the least-squares line of best fit.

d Use the graph and the equation to estimate to the nearest centimetre:

i the arm span of a male swimmer 193 cm tall

ii the height of a male swimmer with an arm span of 202 cm.

e Write a sentence stating any similarities or differences between the line of fit drawn by eye in Example **5** and the least-squares line of best fit.

Solution

a

	Mean	Standard deviation
Height	$\bar{x} = 193.7$	$\sigma_x = 4.24$
Arm span	$\bar{y} = 196$	$\sigma_y = 4.88$

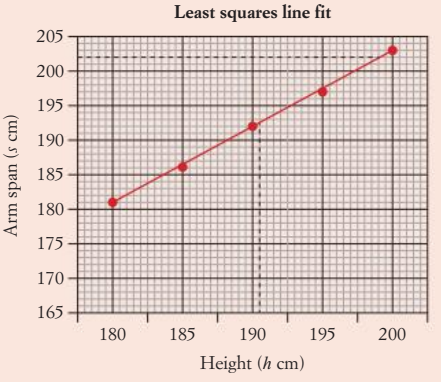
b
$$m = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$
$$= 0.98 \times \frac{4.88}{4.24}$$
$$= 1.1279\ldots$$
$$\approx 1.13$$
$$b = \bar{y} - m\bar{x}$$
$$= 196 - 1.13 \times 193.7$$
$$= -22.88$$

Equation of least-squares line is:

$s = 1.13h - 22.88$

c Complete a table of values for $s = 1.13h - 22.88$ and graph the line.

h	180	185	190	195	200
s	181	186	192	197	203



d From the graph, the arm span of a male swimmer 193 cm tall is 195 cm.

i From the equation, when $h = 193$,

$$\begin{aligned} s &= 1.13(193) - 22.88 \\ &= 195.21 \\ &\approx 195 \end{aligned}$$

The arm span of a male swimmer 193 cm tall is 195 cm.

ii From the graph, the height of a male swimmer with an arm span of 202 cm is 199 cm.

From the equation, when $s = 202$,

$$\begin{aligned} 202 &= 1.13h - 22.88 \\ 202 + 22.88 &= 1.13h \\ 1.13h &= 224.88 \\ h &= \frac{224.88}{1.13} \\ &= 199.0088... \\ &\approx 199 \end{aligned}$$

The height of a male swimmer with an arm span of 202 cm is 199 cm.

e The equation of the least-squares line of best fit ($s = 1.13h - 22.88$) is slightly different to the line of fit drawn by eye ($s = 1.14h - 24.86$) in Example **5** but the values are close.

Least-squares regression line: Graphics calculator

Least-squares regression line on a calculator

The equation of the least-squares line of best fit can also be found on a scientific or graphics calculator, and its line drawn using a graphics calculator or spreadsheet. See Example **4** on page 341 for the calculator steps involved, but change the last step to the following.

Operation	Casio scientific	Sharp scientific
On these calculators, the gradient is b and the y-intercept is a ($b = 1.127...$, $a = -22.329...$)	SHIFT 1 Reg b = SHIFT 1 Reg a =	ALPHA b = ALPHA a =

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Operation	Casio graphics	Texas Instruments graphics
On these calculators, the gradient is a and the y-intercept is b (a = 1.127..., b = –22.329…)	CALC REG X to see 'LinearReg' and the values of a and b	STAT CALC LinReg(ax+b) ENTER to find values of a and b.
To graph the least-squares line	EXIT EXIT EXIT GRPH SET and set StatGraph1 Graph Type to **Scatter** EXE GRPH1 and CALC X DRAW	STAT PLOT Plot 3 On selects the scatter plot STAT CALC LinReg(ax+b) ENTER GRAPH

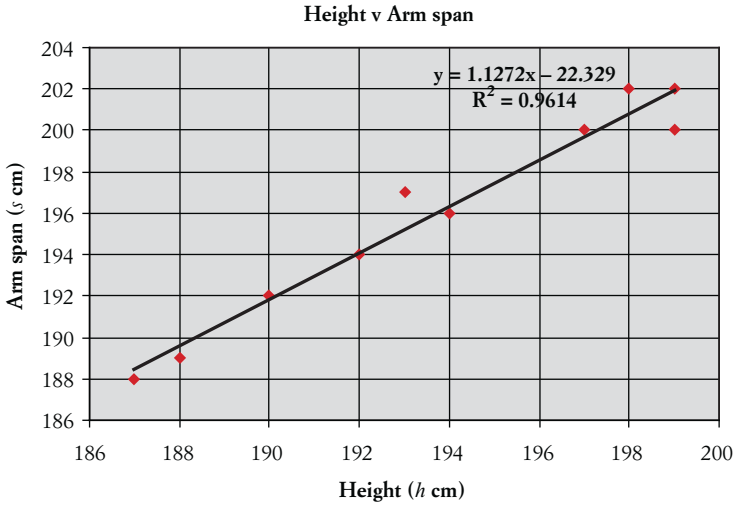
The calculators' equation for the least-squares line is $s = 1.127h - 22.329$, which is more exact than the equation $s = 1.13h - 22.88$ we found in Example **5** because we did some rounding in our calculations.

See the worksheet 'Linear regression: graphics calculator' for more details.

Least-squares regression line (trendline) on a spreadsheet

Follow these steps to graph a least-squares line of best fit on a spreadsheet.

1. Enter data into the spreadsheet.
2. Draw a scatter plot, add a title to the graph and the name of each axis.
3. From the toolbar, select **Chart** and **Add trendline**.
4. Click on the trendline and select **Format trendline**.
5. Select **options** and tick **Display equation on chart** and **Display R-squared value on chart** ($r = \sqrt{R^2}$)



$R^2 = 0.9614$ so the correlation coeficient $r = \sqrt{0.9614} = 0.9805\ldots \approx 0.98$

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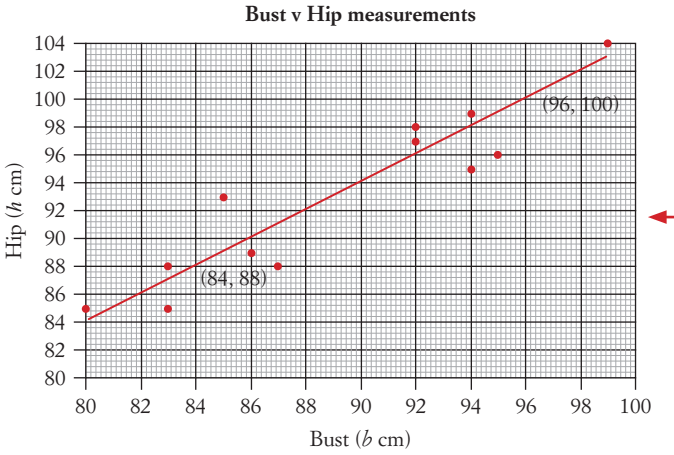
Example 5

Exercise 8-03 Least-squares line of best fit

1 The bust and hip measurements of 12 women were graphed on a scatter plot. A line of fit was then drawn by eye.

Bust (b cm)	83	95	99	85	94	80	87	94	92	86	83	92
Hip (h cm)	88	96	104	93	95	85	88	99	97	89	85	98



This line of fit does not pass through any points on the scatter plot but it does go through (84, 88) and (96, 100).

a Find the equation of the line of fit.

b Use the equation to predict the hip size of a woman with a bust size of 90 cm.

c Use the graph to estimate the bust size for a woman with a hip size of 100 cm.

d Is it possible to predict the hip size of a woman with a 110 cm bust? Why?

2 The wrists and sleeve lengths of 15 children were taken.

Wrist (w cm)	16	18	16	16	18	17	17	16	15.5	18.5	17	16.5	16	16	18
Sleeve length (s cm)	58	60	59	59	61	60	59.5	58	57.5	61.5	58.5	59	57.5	60	60.5

a Draw a scatter plot to represent the data.

b Draw a line of fit by eye.

c Use your graph to estimate:

i the wrist measurement of a student with a 60 cm sleeve length

ii the sleeve length of a student with a 15.6 cm wrist.

d Show that the equation of your line of fit is close to $s = w + 40$

e Use the equation of your line of fit to predict:

i the wrist measurement of a student with a 58.5 cm sleeve length

ii the sleeve length of a student with a 17.5 cm wrist.

Example 6

3 Fifteen people had their waist and chest measurements recorded. A scatter plot comparing waist and chest measurements is drawn.

Waist (w cm)	64	80	90	79	75	68	88	63	89	72	79	87	70	95	89
Chest (c cm)	84	93	103	94	90	85	102	84	100	73	96	100	88	105	89

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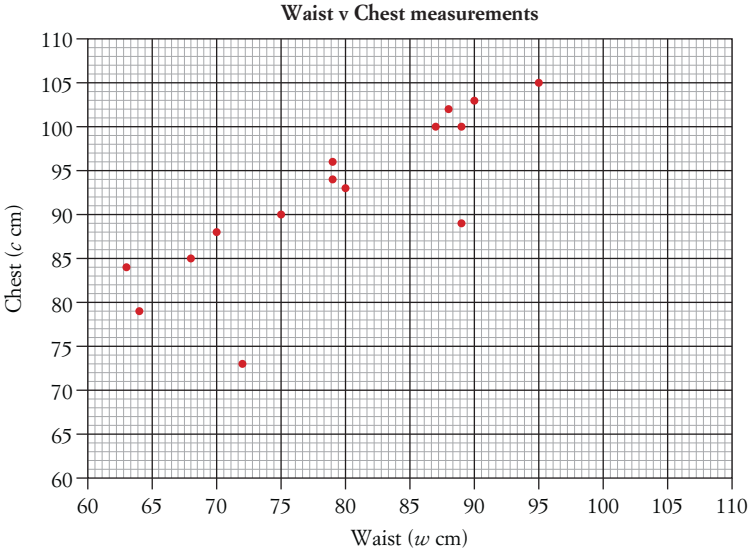

a Calculate correct to two decimal places:

i the correlation coefficient, r

ii the mean and standard deviation of the waist data

iii the mean and standard deviation of the chest data.

b Use the formulas to find the equation of the least-squares line of best fit.

c What are the coordinates of the two outliers?

4 **a** Use a graphics calculator or spreadsheet to graph the bust and hip measurements from question **1** on a scatter plot.

b Use the calculator or spreadsheet to graph the least-squares line of best fit.

c What is the equation of this line?

d How does this equation compare with the equation of the line of fit in question **1**?

e What is the correlation coefficient for this data? Is it a strong, moderate or weak correlation?

5 **a** Use a graphics calculator or spreadsheet to graph the wrist and sleeve length measurements from question **2** on a scatter plot.

b Use the calculator or spreadsheet to graph the least-squares line of best fit.

c What is the equation of this line?

d What is the correlation coefficient for this data? Is it a strong, moderate or weak correlation?

e Write a sentence describing any similarities and differences between the line of fit drawn in question **2** and this one.

6 Twelve students measured their height and stride (the biggest step they can take from a standing position measured between toe positions).

Height (h cm)	165	140	180	176	160	164	178	170	148	157	150	162
Length of stride (s cm)	105	85	111	104	98	95	108	102	92	97	88	100

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a Calculate the correlation coefficient, r, for the data correct to 3 decimal places.

b Calculate correct to two decimal places:

- **i** the mean and standard deviation for the height values
- **ii** the mean and standard deviation for the stride values.

c Find the equation of the least-squares line of best fit.

d Graph this line.

e Use the graph to find:

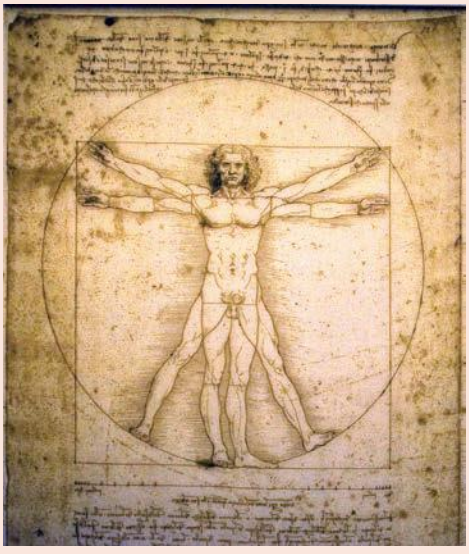
- **i** the length of stride of a student who is 167 cm tall
- **ii** the height of a student with a stride length of 90 cm.

f Use the equation to find:

- **i** the stride length of a student who is 167 cm tall
- **ii** the height of a student with a stride length of 90 cm.

JUST FOR THE RECORD

Vitruvian man



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The Italian artist and inventor Leonardo da Vinci lived from 1452 to 1519 during the Renaissance period. As well as painting the 'Mona Lisa', his great interest in anatomy and the work of the Roman architect Vitruvius led him to draw the 'Vitruvian Man' in 1487, an ink sketch of a male figure whose outstretched limbs touch the circumference of a circle and the edges of a square and whose navel is at the centre of the circle.

Da Vinci believed that each section of the human body is a percentage of the whole.

- height to head length = 8 : 1
- height to the distance from the elbow to the armpit = 8 : 1
- height to the distance from the elbow to the tip of the hand = 5 : 1
- height to arm span = 1 : 1
- height to hand span = 10 : 1
- height to the maximum width of the shoulders = 4 : 1
- height to length of foot = 7 : 1
- face length to ear length = 3 : 1

Choose two of these ratios and collect measurements to decide if the Vitruvian theory is still true today.

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8-04 Ratios, rates and percentage change

EXAMPLE 7

The human body is made up of bone, muscle and fat/organs in the ratio 3 : 7 : 10.

a Andrea has a mass of 65 kg. What is the mass of her bones?

b Ricky's body has 42 kg of fat and organs. What is his muscle mass?

Solution

Use the unitary method.

a Total number of parts = 3 + 7 + 10 = 20

20 parts = 65 kg

1 part = 65 ÷ 20

= 3.25 kg

3 parts = 3 × 3.25 kg **bone is 3 parts**

= 9.75 kg

Andrea has 9.75 kg of bones.

b 10 parts = 42 kg **fat and organs is 10 parts**

1 part = 42 ÷ 10

= 4.2 kg

7 parts = 7 × 4.2 kg **muscle is 7 parts**

= 29.4 kg

Ricky has 29.4 kg of muscle.

Drip rates

EXAMPLE 8

Mark grows medicinal herbs and uses a drip system to water the plants in his hothouse. Each plant needs 50 mL of water per day.

a How many litres of water are needed per day for 25 plants?

b If 1 mL of water contains 12 drops, how many drops are needed per plant per day?

c What is the drip rate (in drops per minute) per plant if the drip system runs for 4 hours per day?

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Solution

a Water needed per day for 25 plants $= 25 \times 50$ mL
$= 1250$ mL
$= 1.25$ L

b Number of drops per plant $= 50 \times 12$
$= 600$

50 mL per plant

c $\text{Drip rate per plant} = \dfrac{600 \text{ drops}}{240 \text{ min}}$
$= 2.5$ drops per minute

4 hours = 4 × 60 min
= 240 min

EXAMPLE 9

Chemist Megastore added 30% profit to a walking frame with a wholesale price of $200, but at a clearance sale it was discounted by 15%.

a What was the price of the walking frame in the clearance sale?

b What was the overall percentage profit made on the walking frame?

Solution

a Price $= \$200 \times 130\% \times 85\%$
$= \$200 \times 1.3 \times 0.85$
$= \$221$

100% + 30% increase = 130%
100% − 15% decrease = 85%

b Profit $= \$221 - \$200$
$= \$21$

$\text{Overall percentage profit} = \dfrac{21}{200} \times 100\% = 10.5\%$

OR: Overall change $= 1.3 \times 0.85$
$= 1.105$
$= 110.5\%$

Overall percentage profit $= 110.5\% - 100\%$
$= 10.5\%$

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Exercise 8-04 Ratios, rates and percentage change

Example 7

1 If a body contains bone, muscle and fat/organs in the ratio 3 : 7 : 10, calculate:

a the mass of muscle in a 98 kg body

b the mass of fat and organs if the mass of muscle is 35 kg.

2 Running uses 3500 kilojoules of energy per hour and walking uses 950 kilojoules per hour. If Imogen walked for 30 minutes while Vashty ran, how many more kilojoules did Vashty use than Imogen? Select **A**, **B**, **C** or **D**.

A 1275 **B** 1700 **C** 2125 **D** 2550

3 Blood consists of plasma and blood cells in the ratio 11 : 9. How much plasma is contained in 5 L of blood? Answer in millilitres.

4 Mae made punch for a party by mixing juice, lemonade and soda water in the ratio 2 : 7 : 4.

a How much punch will contain 700 mL of lemonade?

b How much juice is in 4 L of punch? Answer to the nearest mL.

5 A profit of $14 960 is divided between Ilhea and Robert in the ratio 9 : 7. What is Robert's share?

Example 8

6 A patient needs 200 millilitres of fluid per hour. Fluid is supplied via an intravenous drip at a constant rate.

a How many litres of fluid are needed per day for this patient?

b If 1 mL fluid contains 10 drops, how many drops are needed each hour?

c What is the drip rate (in drops per minute) if the drip runs for 4 hours?

7 A tap is dripping water at a constant rate of 40 drops per minute. If each drop contains 1.3 mL of water:

a how many litres of water will drip from the tap each day?

b what is the cost of the leaked water if water costs $1.55 per kL? Answer to the nearest cent.

8 A lone sailor crossed 1200 km of ocean in 8 days and 8 hours. What was her average speed?

9 Australia's death rate is 5.2 per 1000. If the population at the start of the year was 24 135 680, how many people died that year? Answer to the nearest whole number.

10 A nurse prepares an intravenous drip solution for a patient that contains 75 mg of sedative in every 10 mL of solution.

a If the patient needs 975 mg of sedative per day, how much solution should the nurse prepare to last one day?

b If each mL of solution contains 15 drops, how many drops are needed per hour if the drip runs all day?

c What is the drip rate (in drops per minute, correct to 2 decimal places)?

11 A hospital patient is fed a food solution via a drip that runs at 1 drop per second.

a If each drop contains 0.2 g of food, how many kilograms of food will the patient receive in a 4-hour period?

b What is the cost of one litre of food solution if it is 5.5c per mL?

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Example 9

12 What percentage change is equivalent to:

- **a** an increase of 10% followed by a decrease of 10%?
- **b** an increase of 2% followed by an increase of 3%?
- **c** a decrease of 8% followed by an increase of 10%?
- **d** a decrease of 5% followed by a decrease of 2%?

13 In preparation for the Olympic Games, Sally increased her muscle mass by 15%. Afterwards, her muscle mass decreased by 10%. What is the overall percentage change in her muscle mass?

14 Chemist Megastore adds 15% to the wholesale price of goods to calculate the selling price. In January, there is a '15% off' sale.

- **a** Calculate the discount price of a massage table with a wholesale price of $120.
- **b** What overall percentage loss does Chemist Megastore make on this massage table?
- **c** What is the wholesale price, correct to the nearest 5 cents, of a shower chair marked down to $85 in the January sale?

8-05 Concentrations

Pharmacists and nurses mix solutions that must contain just the right amount of a particular drug. The **concentration** of a drug is the amount of the drug compared to the whole solution, expressed as a rate in mass/volume. For example, a concentration of 50 mg/mL means 50 mg of drug in each mL of solution. The metric units used in concentrations are grams (g), milligrams (mg), micrograms (μg), litres (L) and millilitres (mL).

1 g = 1000 mg = 1000 000 μg 1 L = 1000 mL

1 mg = 1000 μg

EXAMPLE 10

Convert:

a 20 μg to mg **b** 8.5 L to mL **c** 14.7 g to mg

Solution

a 20 μg = 20 ÷ 1000 mg **1 mg = 1000 mg**
= 0.02 mg

b 8.5 L = 8.5 ÷ 1000 mL
= 8500 mL

c 14.7 g = 14.7 × 1000 mg **1 g = 1000 mg**
= 14 700 g

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EXAMPLE 11

A drug solution for a child contains the drug in the concentration 50 µg/mL of solution.

a How many milligrams of the drug are needed to make 100 mL of the solution?

b How many litres of the solution contain 2 g of the drug?

Solution

a Concentration = 50 µg/mL

Amount of drug in 100 mL = 100 × 50 µg

= 5 000 µg

= 5 mg **1 mg =1 000 mg**

b Concentration = 50 µg/mL

= 0.000 05 g/mL **1 g = 1 000 000 mg**

Volume containing 2 g = 2 ÷ 0.000 05 mL

= 40 000 mL

= 40 L **1 L = 1 000 mL**

Exercise 8-05 Concentrations

Example 10

1 Which of the following is equal to 200 milligrams? Select **A**, **B**, **C** or **D**.

A 0.2 g **B** 200 000 µg **C** 0.002 kg **D** 0.2 µg

2 Convert:

a 50 mg to µg **b** 80 kg to g **c** 100 g to mg

d 650 mL to L **e** 75 mg to g **f** 10.5 L to mL

3 A nurse described the concentration of a drug as 5 mg/mL. Describe what this means.

4 Which of these drug concentrations is the strongest (or most concentrated)? Select **A**, **B**, **C** or **D**.

A 2 mg/mL **B** 1 mg/10 mL **C** 5 g/L **D** 0.5 mg/10 mL

5 A solution of 400 mL of disinfectant contains 24 g of alcohol. Express the concentration in g/L.

Example 11

6 A solution for a sick child contains a drug with a concentration of 8 mg/mL.

a How many grams of the drug are needed to make 500 mL of the solution?

b How many litres of the solution would 10 g of the drug make?

7 A sheep solution contains 5 g of cortisone per 20 mL of solution. If 350 sheep need 2.5 g of cortisone each, how many litres of sheep solution are needed?

8 An antibiotic in a syrup has a concentration of 25 mg/mL. Calculate the:

a number of millilitres of syrup that contain 200 mg of antibiotic

b number of grams of antibiotic in 1.5 L of syrup.

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9 A vet used a vitamin supplement for cats, mixing it with water at a concentration of 10 g/L.

- **a** How many mg of vitamin supplement are needed for 200 mL of water?
- **b** How many mL of water are needed for 5 g of vitamin supplement?

10 A concentration of 'Painadull' is 1 mg/10 mL.

- **a** How many litres of the solution contain 5 g of Painadull?
- **b** How many grams of Painadull are needed to make one litre of the solution?

11 How many grams of drug are in a 1 L bottle of solution with a concentration of 1 g/20 mL? Select **A**, **B**, **C** or **D**.

A 20 **B** 500 **C** 50 **D** 200

12 Doctor Dan prescribed 2 mg of 'Shakestop' for a patient having a seizure. The nurse had a solution of 'Shakestop' with a concentration of 10 mg/50 mL. How many millilitres should the nurse give the patient?

13 Milo the cat takes 0.4 g of vitamins in 200 mL of milk to keep his fur shiny.

- **a** What is this rate in mg/mL?
- **b** How many mg of vitamins are in 120 mL of milk?

14 A premature baby is fed via a tube. A nurse makes a milk solution using 5 g of formula for each 50 mL of sterile water. The baby receives 100 mL of milk solution 6 times per day.

- **a** How much formula is used per day?
- **b** How many litres of milk solution can be made using a 1 kg tin of formula?
- **c** If each feed via the tube takes 20 minutes, what is the feed rate in mL/h?

15 A vet has recommended a vitamin supplement for Carmen's sick guinea pigs. It is to be mixed at a rate of 10 g/L in their drinking water.

- **a** How much of the vitamin supplement is required for the guinea pigs' 500 mL water bottle?
- **b** Carmen obtained the vitamin solution in a concentration of 240 g/300 mL from the chemist. How many millilitres of this solution should Carmen put in the 500 mL water bottle?

16 A patient is prescribed 4 g of pethidine per week. Each 10 mL liquid capsule has pethidine in the concentration of 20 mg/mL. How many capsules should be given to the patient per week?

Medicine dosage

8-06 Medicine dosage

Medicines are taken in many ways: in liquid, tablet or capsule form, or via an inhaler or intraveneous drip. When a doctor prescribes medicines for patients, factors such as the patient's age and weight are taken into account. **Medicine dosage** is the amount of medicine to be taken.

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EXAMPLE 12

Melanie bought a 30 g tube of 'Soothe' cream for sunburn, containing 10 mg/g of hydrocortisone acetate. She needs to take 50 mg of hydrocortisone acetate every 6 hours.

a What is the dosage frequency per day?

b What is the active ingredient and how many milligrams are contained in the 30 g tube?

c How many grams of cream should Melanie use per dose?

Solution

a The dosage frequency is every 6 hours, which is 4 times a day.

24 hours ÷ 6 hours

b The active ingredient is hydrocortisone acetate.

Concentration = 10 mg/g

Amount of hydrocortisone acetate in tube = 30×10 mg

$= 300$ mg

c Concentration = 10 mg/g

Required amount of cream = $50 \div 10$

$= 5$ g

50 mg of hydrocortisone acetate needed

Children's dosages

The correct dosage of a medicine for children and infants (babies) depends on the child's age, weight or body surface area. A child dosage is a certain fraction of the adult dosage and there are different models and formulas for calculating this.

Fried's rule for infants up to 24 months (2 years):

$$\text{Child dosage} = \frac{\text{age in months}}{150} \times \text{adult dosage}$$

Young's rule for infants up to 2 years:

$$\text{Child dosage} = \frac{\text{age in years}}{\text{age in years} + 12} \times \text{adult dosage}$$

Clark's rule for children 2 years and over

$$\text{Child dosage} = \frac{\text{weight in kg}}{70} \times \text{adult dosage}$$

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EXAMPLE 13

Ryan is 18 months old and weighs 13 kg. The adult dosage of a drug prescribed for him is 200 g.

Calculate Ryan's dosage using:

a Fried's rule **b** Young's rule

Solution

a $\text{Ryan's dosage} = \frac{18}{150} \times 200 \text{ g}$

$= 24 \text{ g}$

b $\text{Ryan's dosage} = \frac{1.5}{1.5 + 12} \times 200 \text{ g}$

$= 22.2222\ldots \text{ g}$

$\approx 22.2 \text{ g}$

Age = 18 months = 1.5 years

Exercise 8-06 Medicine dosage

1 Paula is aged 15 and weighs 50 kg. Her doctor prescribed 4 mg of a drug per kilogram of body weight to be taken each day. What will her daily dose be? Select **A**, **B**, **C** or **D**.

A 750 mg **B** 1300 mg **C** 190 mg **D** 200 mg

2 A 15 mL bottle of Betathroat mouthwash contains instructions to dilute 1 mL to 20 mL with water, gargle for 30 seconds and repeat every 4 hours until the bottle is finished.

a What is the amount of water per dose?

b What is the dosage frequency?

c How long will the bottle last?

Example 12

3 Anna had a sinus condition and bought a pack of 30 tablets from her local pharmacy. Each tablet contained 5 mg/g of phenylephrine hydrochloride. She takes 2 tablets every 4 hours until she finishes the pack.

a How many doses are in the pack?

b What is the dosage frequency per day?

c What is the active ingredient and how many milligrams does she take per day?

d How many grams of phenylephrine hydrochloride are in 100 g of tablets?

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4 These instructions were found on a pack of 24 tablets used to relieve pain and inflammation.

> DOSAGE:
> ADULTS AND CHILDREN OVER 12 YEARS:
> Take 1–2 tablets with fluid every 4–6 hours, as necessary.
> Do not take more than 6 tablets in 24 hours.
> CHILDREN 7 TO 12 YEARS:
> Take 1 tablet with fluid every 6–8 hours, as necessary. Do not take more than 4 tablets in 24 hours.
> Not recommended for use in children under 7 years of age.
>
> Do not exceed the recommended dose. Excessive use can be harmful. If you get an allergic reaction stop taking and see your doctor immediately.

a Costa has a severe headache. How many tablets should he take per dose to relieve the pain?

b How often should he take this dose?

c If Costa took 2 tablets at 4 p.m. on Thursday, then 2 tablets every 6 hours after, when does he take the last 2 tablets in the pack?

d Trizia is 10 years old and has a sore tooth. How many tablets should she take per dose and how often?

e Finn had an allergic reaction after taking a tablet. What should he do?

f Bethany is 6 years old. What is the recommended dose for her?

5 Li burned his arm on the stove and bought a 50 g tube of Paraderm Plus cream. The three active ingredients are chlorhexidine gluconate (1 mg/g), lignocaine hydrochloride (10 mg/g) and bufexamac (50 mg/g). He applied 2 g of cream every 3 hours.

a What is the total mass of the active ingredients in each gram of Paraderm Plus?

b What is the dosage frequency per day?

c How many grams of cream would contain 1 g of lignocaine hydrochloride?

d How many mg of chlorhexidine gluconate would be in a 30 g tube of the same cream?

Example 13

6 Mia is two years old and weighs 11 kg. A nurse calculates her medicine dosage given the adult dosage is 150 mg every 4 hours. Calculate Mia's daily dosage to the nearest mg using:

a Fried's rule **b** Young's rule

7 Ashleigh is 16 years old and weighs 50 kg. She takes a drug with a recommended adult dose of 420 mg every 6 hours. Using Clark's Rule, what would be Ashleigh's daily dosage? Select **A**, **B**, **C** or **D**.

A 1200 mg **B** 1800 mg **C** 300 mg **D** 336 mg

8 **Cowling's rule** for calculating a child's dosage is

$$\text{Child dosage} = \frac{\text{age in years} + 1}{24} \times \text{adult dosage}$$

If the adult daily dosage of an antibiotic is 100 mg, use Cowling's rule to find the daily dosage, correct to 1 decimal place, for:

a Brittany, aged 2 **b** Patrick, aged 6 **c** Mercedes, aged $10\frac{1}{2}$

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9 Elena is $1\frac{1}{2}$ years old. If the adult dosage of a drug is 1000 mg per day, find, correct to the nearest mg, the appropriate dosage for her using:

a Young's rule **b** Fried's rule

10 Sara was prescribed 500 mg of 'Nopain' per day, to be taken every 6 hours. The chemist gave her a bottle of solution containing a concentration of 100 mg/5 mL of 'Nopain'. How many millilitres of the solution should she take in each dose?

11 Each tablet in a pack of 21 antibiotic tablets contains 400 mg of metronidazole. A patient is prescribed 1000 mg of metronidazole per day. How many tablets is this?

12 A 20 kg toddler needs 40 mg/kg per day of amoxicillin to treat an infection. This medication comes in a solution with a concentration of 250 mg/5 mL. Use an appropriate formula to find how many millilitres of solution the toddler needs per day to get the correct dosage.

13 Vials of 'Nohurt' are dissolved in sterile water with a concentration of 1 g/50 mL.

a How many 100 mg vials will you need to make 1 litre of solution?

b How many millilitres of solution would thirty 100 mg vials make?

14 Tahir takes the drug 'Sleepeasy' in a solution with a concentration of 50 mg/ mL. He weighs 112 kg and needs 10 mg of 'Sleepeasy' per kilogram of body weight three times a day.

a How often should Tahir take the drug? After how many hours?

b How many millilitres of solution does he need per dose?

15 Soothapain tablets are for sinus and pain relief. The dosage instructions on a pack of 24 capsules are shown in the table.

<table>
<tr><td colspan="2">Each tablet contains
• Phenylephine Hydrochloride 5 mg
• Paracetamol 500 mg</td></tr>
<tr><td>Who can use this Soothapain?
Adults
Children 12 years and over</td><td>How many?
1–2 tablets</td></tr>
<tr><td>Use this Soothapain for
✓ sinus pain relief
✓ sinus headaches
✓ blocked or runny noses</td><td>How often?
Every 4 to 6 hours as necessary</td></tr>
<tr><td>Do not use
× if you have taken paracetamol
× for children under 12 years</td><td>Maximum in a day
8 tablets in 24 hours</td></tr>
<tr><td>Ask your doctor before use if you:
• have high blood pressure
• are taking antidepressants
• have heart problems
• are pregnant</td><td>When to stop
Adults: after a few days
Children: 48 hours, except on medical advice</td></tr>
</table>

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a What is the maximum number of capsules an adult should take

i per dose? **ii** per day?

b Kahlia is 10 years old. What is the recommended dosage for her?

c What is the total number of capsules a 13-year-old is allowed to take ?

d Guy is 42 and has high blood pressure. Can he take this medication?

e Larna is 27 and has a sinus headache. What is the maximum number of capsules she is allowed per day?

f Portia takes 2 tablets every 6 hours for 4 days. How many grams of paracetamol has she taken?

8-07 Life expectancy

Life expectancy is the average number of years of **life remaining** to a person at a specified age.

For example:

- a male aged 15 years with a life expectancy of 65.1 years is most likely to die at age 80.1 years
- a female aged 85 years with a life expectancy of 6.9 years is most likely to die at age 91.9 years

Life expectancy data is based on past statistics of people who have died. The table below shows the life expectancy of males and females in New South Wales during 2000–2010.

Life expectancy table for NSW

Life expectancy at exact age (years)	2000	2002	2004	2006	2008	2010
Male						
0	76.4	77.3	78.0	78.6	79.2	79.6
15	62.0	62.9	63.6	64.2	64.8	65.1
50	29.3	30.0	30.5	31.0	31.5	31.7
65	16.7	17.3	17.7	18.1	18.6	18.9
85	5.5	5.6	5.7	5.9	5.9	6.0
Female						
0	81.9	82.6	83.3	83.4	83.9	84.1
15	67.5	68.1	68.8	68.9	69.3	69.5
50	33.7	34.2	34.8	34.9	35.2	35.4
65	20.3	20.8	21.2	21.4	21.7	21.9
85	6.6	6.8	6.9	7.0	7.1	7.2

Australian Bureau of Statistics, ausstats.abs.gov.au. Customised report, 2012.

EXAMPLE 14

Use the life expectancy table above to find:

a the life expectancy for a boy born in 2002

b to what age a female aged 65 in 2008 is expected to live

c at what age you would expect to die if you were 15 years old in 2006 and male

d whether a 50-year-old male or a 50-year-old female in 2004 would live longer, and by how much

e the difference in the life expectancies at birth for two sisters born 8 years apart, the elder one in the year 2000.

Solution

a Life expectancy for a boy born in 2002 is 77.3 years.

b Expected age of death of a female aged 65 in 2008 is $65 + 21.7 = 86.7$ years

c Expected age of death of a male aged 15 in 2006 is $15 + 64.2 = 79.2$ years

d At age 50 in 2004,

Male life expectancy = 30.5

Female life expectancy = 34.8

So the female would live longer.

Difference in life expectancies is $34.8 - 30.5 = 4.3$ years

The female would live 4.3 years longer.

e Life expectancy of elder sister born in 2000 = 81.9 years

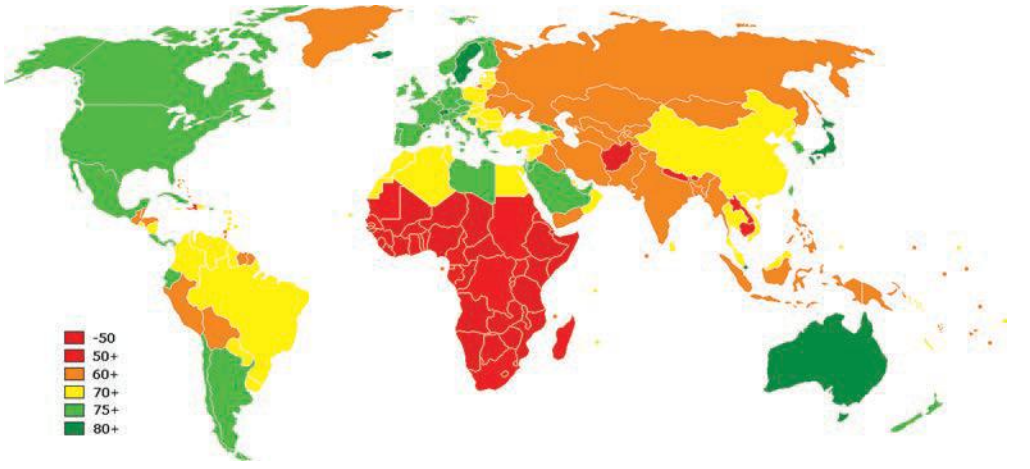
Life expectancy of younger sister born in 2008 = 83.9 years

Difference in life expectancies is $83.9 - 81.9 = 2$ years

Life expectancy around the world

A country with a good healthcare system and an abundance of food and shelter is likely to have a higher life expectancy. This colour-coded world map shows the life expectancy at birth in all countries. Australia is one of the few countries shown in dark green with a life expectancy at birth of 80 and over, whereas most of the African countries shown in red have a life expectancy below 60.

ISBN 9780170238977



English-Online
www.english-online.at/geography/life-expectancy/life-expectancy-where-people-live-longer.htm

The life expectancies at birth of males and females in the top 15 and bottom 15 countries of the world (for the years 2005–2010) are shown below.

Top 15			
Country	**Male**	**Country**	**Female**
Iceland	80	Japan	86
Japan	79	Hong Kong	85
Hong Kong	79	Australia	84
Australia	79	Switzerland	84
Switzerland	79	Spain	84
Sweden	79	France	84
Israel	79	San Marino	84
Macau	79	Iceland	83
Spain	78	Israel	83
Canada	78	Macau	83
Norway	78	Sweden	83
Italy	78	Canada	83
New Zealand	78	Italy	83
Singapore	78	Norway	83
Netherlands	78	Austria	83

Based on Australian Institute of Health and Welfare material. Source: United Nations, Department of Economic and Social Affairs, Population Division (2007),World Population Prospects: The 2006 Revision.

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Bottom 15			
Country	**Male**	**Country**	**Female**
Somalia	47	Malawi	48
Nigeria	46	DR Congo	48
DR Congo	45	Guinea-Bissau	48
Rwanda	45	Rwanda	48
Guinea-Bissau	45	Nigeria	47
Liberia	45	Liberia	47
Afghanistan	44	Central African Republic	46
Zimbabwe	44	Afghanistan	44
Central African Republic	43	Angola	44
Lesotho	43	Sierra Leone	44
Zambia	42	Zimbabwe	43
Mozambique	42	Lesotho	42
Angola	41	Mozambique	42
Sierra Leone	41	Zambia	42
Swaziland	40	Swaziland	39

Australian Institute of Health and Welfare
www.aihw.gov.au/life-expectancy-how-australia-compares/

EXAMPLE 15

Use the tables above to find:

a the life expectancy at birth for an Australian female
b the difference in life expectancies for a Swedish male and a Swedish female
c in which country are you most likely to live longest if you are male
d in which country are you most likely to live shortest if you are female
e the mean life expectancy across both sexes for the top 15 countries
f the modal life expectancy across both sexes for the bottom 15 countries

Solution

a The life expectancy at birth for an Australian female is 84 years.

b Difference in life expectancies for a Swedish male and female is $83 - 79 = 4$ years

c You are most likely to live longest if you are male and live in Iceland.

d You are most likely to live shortest if you are female and live in Malawi.

e Mean of top 15 countries $= \dfrac{\text{sum of ages}}{\text{number of ages}}$

$$= \frac{2434}{30}$$
$$= 81.1333...$$
$$\approx 81.1$$

f Mode of bottom 15 countries is 44. **The most frequent score.**

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Exercise 8-07 Life expectancy

1 The government spent $5 000 000 on a safety campaign to reduce pedestrian deaths. If the annual number of pedestrian deaths reduced from 12 000 to 9000 due to this campaign, how much was spent on each life saved?

2 A country has a population of 12 300 000. If 12% of the population smoked, how many were non-smokers?

3 Use the life expectancy table on page 363 to answer the following questions.

a Describe fully the type of person who has a life expectancy of:

i 83.4 years **ii** 62.9 years **iii** 18.1 years

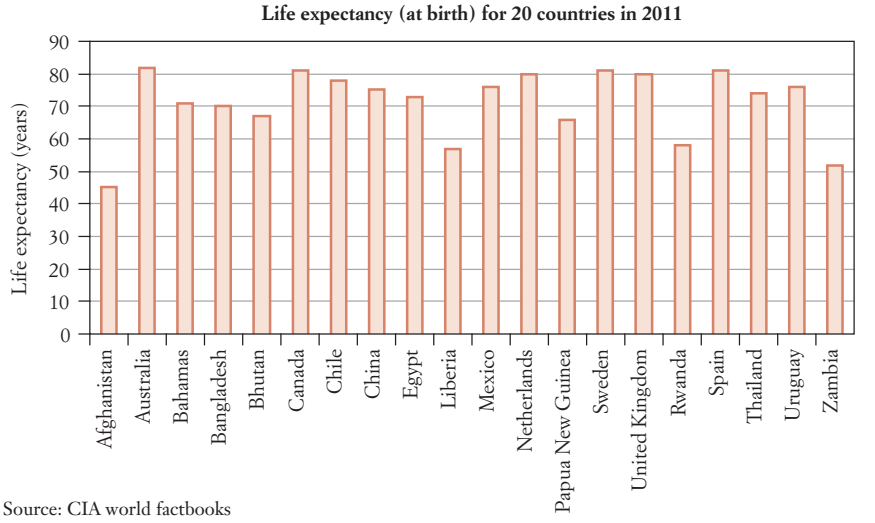
b Do men generally have a longer life expectancy than women? Why do you think this is so?

c In 2008, which female is more likely to live longer and by how much: a newborn baby or a 15-year-old girl? Why do you think this is so?

d Which 50-year-old male is more likely to live longer and by how much: one in 2002 or one in 2006? Why do you think this is so?

4 This graph shows the life expectancy at birth for 20 countries.

Example 14



Source: CIA world factbooks

a What is the life expectancy at birth of babies in China?

b Which country has:

i the highest life expectancy? **ii** the lowest life expectancy?

c Which country has a life expectancy of 52 years?

d The life expectancy at birth of a person in Afghanistan is 45 years. Give two possible reasons for this low value.

e Which countries have a life expectancy of:

i 80 or more years? **ii** between 50 and 70 years?

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Example 15

5 Use the life expectancy tables of countries on pages 365–366 to find:

- **a** the life expectancy at birth for a Japanese male
- **b** the difference in life expectancies for a Swaziland male and a Swaziland female
- **c** the country in which you are most likely to live longest if you are female
- **d** the country in which you are most likely to live shortest if you are male
- **e** the modal life expectancy for the top 15 countries
- **f** the median life expectancy for the females in the bottom 15 countries
- **g** the mean life expectancy for the females in the bottom 15 countries

6 This mortality table gives the probability (correct to 3 decimal places) of an Australian person dying at a specified age group (in 2009).

Age group	Probability of dying	Age group	Probability of dying	Age group	Probability of dying
<1	0.0042	35–39	0.0045	75–79	0.1826
1–4	0.0008	40–44	0.0058	80–84	0.2990
5–9	0.0005	45–49	0.0091	85–89	0.4585
10–14	0.0005	50–54	0.0135	90–94	0.6146
15–19	0.0018	55–59	0.0203	95–99	0.7416
20–24	0.0025	60–64	0.0328	100+	1.0000
25–29	0.0029	65–69	0.0662		
30–34	0.0034	70–74	0.1033		

© WHO. Life Expectancy: Life tables
http://apps.who.int/ghodata/?vid=60070

- **a** If Sharleen is 52, what is the probability of her dying?
- **b** If Dean is 47, what is the probability of him not dying?
- **c** What is the probability of an Australian dying aged 35–39:
  - **i** as a percentage?
  - **ii** as a fraction?
- **d** If there are 200 000 Australians aged 20–24, how many are expected to die?
- **e** At what age group does the probability of dying first exceed 0.01?
- **f** The probability values are generally low but relatively higher in the first four years of life. Why do you think this is so?
- **g** Why do the probability values increase with age?
- **h** Explain why the probability of dying at 100+ is 1.000.

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7 The clustered column graph below compares life expectancy at birth for males and females in Australian states and territories.

ABS Cat No. 3302.0

a What is the life expectancy for:

i a newborn girl in Queensland? **ii** a newborn boy in Western Australia?

b In which state or territory is the life expectancy for males 77 years?

c Which state or territory has the highest life expectancy for males? What is the value?

d Do women generally live longer than men? Is there a significant difference?

e Which state or territory has the largest gap between male and female life expectancies? What is this gap?

f What is the national average life expectancy for males?

g Which state or territory has the lowest life expectancy? Why do you think this is so?

h Which states and territories have a female life expectancy of more than 83 years?

ISBN 9780170238977

JUST FOR THE RECORD

Quit for life

The World Health Organisation estimates that more than 1 billion people in the world are regular smokers, with 80% of smokers living in low- and middle-income countries. Each pack of cigarettes a person smokes reduces his or her life by 28 minutes and the life expectancy of a typical smoker is reduced by 25 years.

During the 1960s, there was an increase in deaths from lung cancer and cardiovascular disease. This was attributed to the increase in cigarette smoking. Public health campaigns (such as grim warnings and images on cigarette packs), public health regulations (such as bans on smoking in public places) and taxes on tobacco have all had an effect on the reduction in smoking rates. Australia introduced plain packaging for cigarettes in 2012 with the aim of reducing the number of smokers from around 15% of the population to 10% by 2018.

How does plain packaging of cigarettes deter smoking?

TECHNOLOGY: LIFE EXPECTANCY CALCULATORS

1 Search on the Internet for a life expectancy calculator like the one below. It calculates life expectancy based on age and country. It does not account for sex, smoking status, height/weight, education, exercise, medical conditions, diseases or injuries.

You are expected to live up to the age of 81.72. You have another 65 years to enjoy life. So live happily.

2 Search on the Internet for another life expectancy calculator that requires you to answer more questions relating to your lifestyle and family so that it determines a more accurate life expectancy value. Compare the answer given by this calculator to the answer given by the calculator you found in question **1**.

3 A calculator that shows how many years cigarette smoking can take out of your lifespan can be found at the website **www.uwhealth.org**, clicking on **Health information**, then **Interactive tools** under General health information, then **How does smoking affect your lifespan?**

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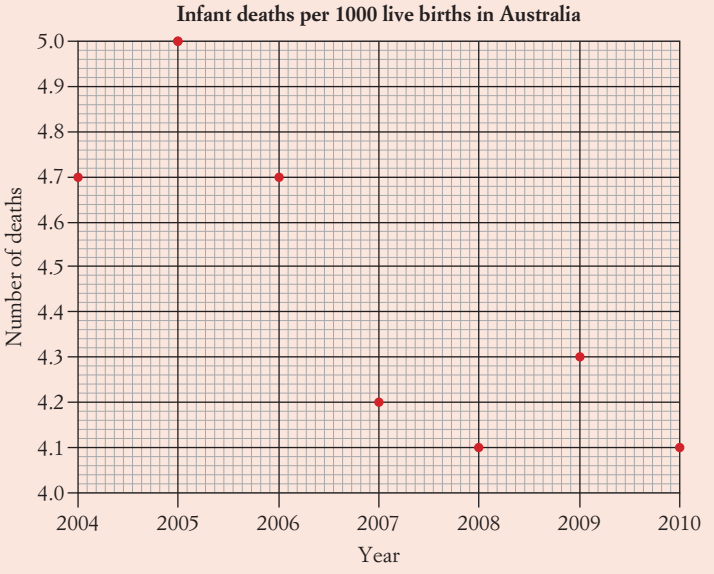
8-08 Scatter plots of life expectancy

EXAMPLE 16

This scatter plot shows the number of infant deaths per 1000 live births in Australia from 2004 to 2010. It has been drawn using the table of infant mortality data shown here.

Year	2004	2005	2006	2007	2008	2009	2010
Deaths per 1000	4.7	5.0	4.7	4.2	4.1	4.3	4.1

Based on data from Key tables from OECD—ISSN 2075-8480—© OECD 2012. www.oecd-ilibrary.org/social-issues-migration-health/infant-mortality_20758480-table9



a What percentage of babies died at birth in Australia in 2005?

b Use a graphics calculator or spreadsheet to construct a least-squares line of best fit for this data and find its equation in the form $y = mx + b$.

c Explain the meaning of the value for the gradient of the line.

d Calculate, correct to two decimal places, the correlation coefficient for this data and describe the correlation.

e Do you think the mortality rate will continue to decline? Justify your answer.

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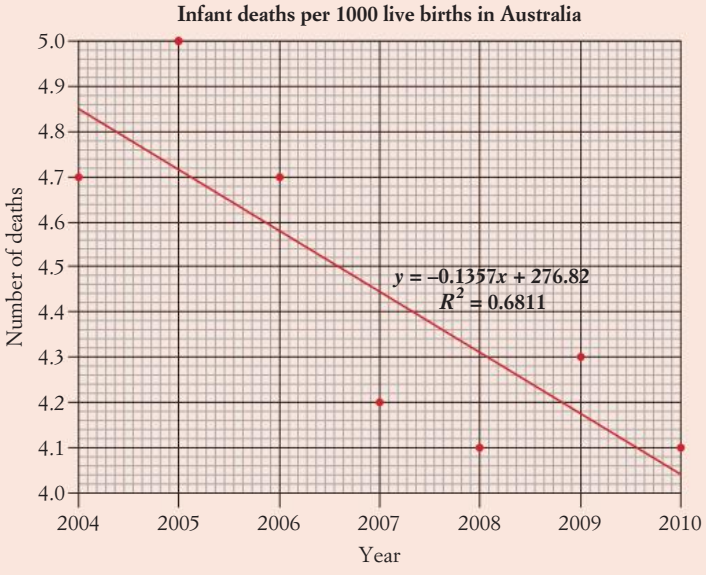
Solution

a Mortality rate in 2005 $= \dfrac{5}{1000} \times 100\%$

$= 0.5\%$

b The following trendline was created using a spreadsheet.



The least-squares line of best fit has the equation

$y = -0.1357x + 276.82$

c The gradient –0.1357 is the rate at which the mortality rate is decreasing each year.

d As $R^2 = 0.6811$, the correlation coefficient is

$r = \sqrt{0.6811}$

$= 0.825287...$

≈ 0.83

But the gradient and graph indicate that the correlation is negative, so $r \approx -0.83$.

The correlation is negative and strong.

e Yes, the mortality rate will continue to decline. Because of advancements in health and medicine, infants are less likely to die during childbirth.

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Exercise 8-08 Scatter plots of life expectancy

Example 16

1 This scatter plot shows the life expectancy at birth in Australia from 2004 to 2010. It has been drawn using the table of data shown here.

Year	2004	2005	2006	2007	2008	2009	2010
Life expectancy	80.5	80.9	81.1	81.3	81.5	81.6	81.8

Based on data from Key tables from OECD—ISSN 2075-8480—© OECD 2012
www.oecd-ilibrary.org/social-issues-migration-health/life-expectancy-at-birth-total-population_20758480-table8

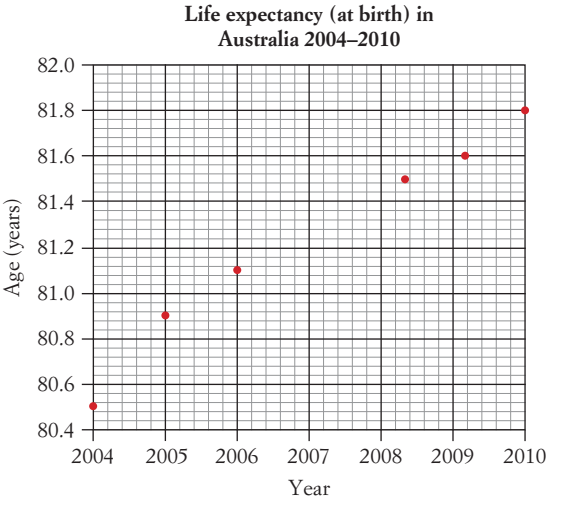
a Describe the correlation of this data.

b Estimate the correlation coefficient.

c Use a graphics calculator or spreadsheet to create a least-squares regression line for this data, and find its equation.

d Calculate the correlation coefficient correct to two decimal places.

e Explain the meaning of the value obtained for the gradient of the regression line.



2 A group of smokers and non-smokers have their ages and life expectancies recorded in the table.

Age (A years)	20	40	25	65	55	30	20	45	35	50	25	30	55	65	60
Life expectancy (E years)	48	24	32	6	8	40	52	9	25	10	44	29	19	1	1

a Draw a scatter plot for this data comparing age with life expectancy.

b If the smokers in the group have a lower life expectancy than the non-smokers, what is the most likely age for:

i the 30-year-old smoker to die? **ii** the 55-year-old non-smoker to die?

c What is the difference in life expectancies between the two 25-year-olds?

d Describe the correlation between age and life expectancy for this data.

e Calculate, correct to two decimal places, the correlation coefficient.

f Draw a line of fit by eye and show that its equation is close to $E = -A + 65$.

g Use the equation of the line to estimate the life expectancy of a 45-year-old and explain why this value differs from the one in the table.

h Use appropriate technology to find the equation of the least-squares regression line for this data and hence find the most likely age of death for a person who is 18 years old now.

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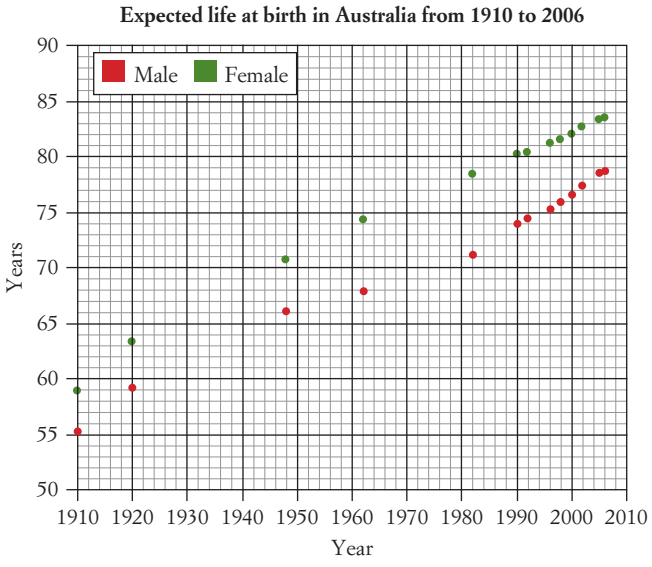
3 This scatter plot shows the life expectancy at birth for boys and girls in Australia from 1910 to 2006.



a Write one observation about the life expectancy for:

i females **ii** males

b When was the difference between the life expectancies for males and females:

i the least? **ii** the most?

c Describe the pattern of the life expectancies between 1910 and 2006.

d Copy the scatter plot and draw a line of fit by eye for the female data.

e What is the equation of your line of fit?

4 This table shows the ages and life expectancies for 12 indigenous Australians.

Age	69	65	70	68	77	62	77	75	68	64	65	72
Life expectancy	4	3	5	5	2	7	1	1	4	6	10	3

a Use a graphics calculator or spreadsheet to draw a scatter plot for this data.

b Use the scatter plot to describe the correlation of this data.

c Use a graphics calculator or spreadsheet to draw a least-squares regression line and find its equation.

d Calculate, correct to two decimal places, the correlation coefficient.

e Use the equation of the least-squares regression line to estimate the life expectancy of:

i Meg, who is 65 **ii** Joe, who is 71.

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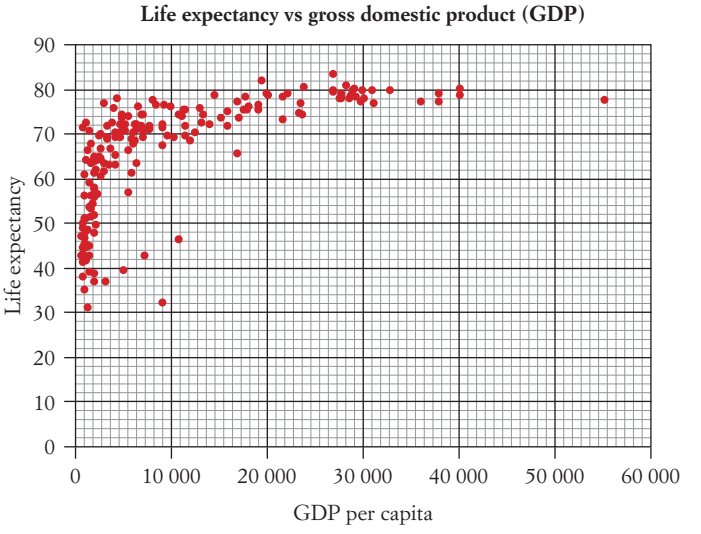
5 This scatter plot shows the life expectancy for countries compared to their gross domestic product (GDP) per person. The GDP is the total value of all goods and services produced by a country in a given year and is a good indicator of a country's standard of living, which correlates with life expectancy. The per capita amount is the country's GDP divided by its population.



Source: CIA Factbook

a Describe the life expectancy data when the GDP per capita is between $0 and $10 000.

b Explain what happens as the GDP capita increases above $10 000.

c One country produces about $55 000 GDP per capita. What is the life expectancy in this country?

d It is thought that above $25 000 GDP per capita, life expectancy ceases to increase. Do you agree with this statement? Justify your answer.

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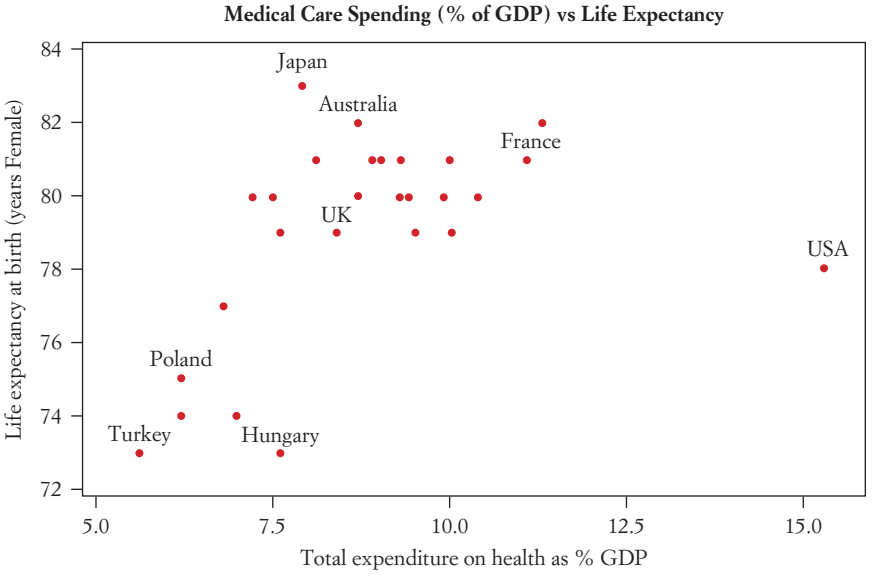
6 This scatter plot compares a country's percentage of GDP spent on healthcare with its life expectancy.



Based on data OECD Health Data 2012, accessed on 30 January 2013.
© OECD, OECD Health Data 2012. June 2012.

a What percentage of the GDP is spent on healthcare in:

i Hungary? **ii** Australia? **iii** France?

b What is the life expectancy in:

i Japan? **ii** UK? **iii** Poland?

c Which country spends the highest percentage of their GDP on healthcare and what is this percentage?

d Which country spends the lowest percentage of their GDP on healthcare and what is this percentage?

e Which country spends approximately 8.5% of their GDP on healthcare?

f Compare the spending on healthcare and the life expectancy in Hungary, UK and Japan.

g Is there a correlation between life expectancy and healthcare expenditure? Why?

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Sample HSC problem

The waist and hip sizes of 15 people were measured and recorded in this table.

waist (w cm)	65	80	90	78	75	68	84	63	89	72	79	87	74	95	88
hip (h cm)	88	100	108	102	95	88	102	89	102	78	101	105	95	109	101

a Calculate, correct to two decimal places, the mean and standard deviation of:

i the waist values **ii** the hip values

b Given that the correlation coefficient of this data is 0.85, find the equation of the least-squares line of best fit for this data.

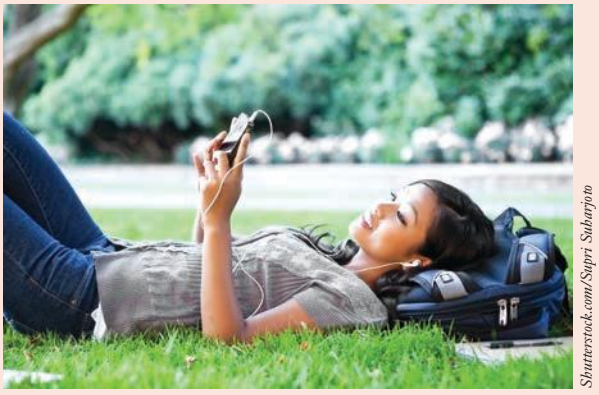
c Use the equation to estimate (to the nearest centimetre):

i the hip size of a person who has a waist of 70 cm

ii the waist size of a person whose hip measurement is 100 cm.

Study tip

Keeping it all in perspective



Shutterstock.com/Supri Suharjoto

There *is* life after the HSC. Despite what many people will tell you, it's not the most important time in your life. Sure, it is one of the turning points, but 12 months from now the HSC will be a distant memory and you'll be laughing about how much stress it caused you.

Every year when the high achievers of the HSC are interviewed by the media, there are stories of hard work and determination. But these successful students also kept it all in perspective and remembered to maintain balance in their lives. They didn't neglect their other needs: sport, recreation, going out, talking to their families, watching movies, listening to music. So don't forget to have a life. Plan to do nothing once in a while. Breathe, live, exercise, meditate.

When planning your study, be aware that you only have a limited amount of time. It is not possible to do everything, so make the most of the time you do have and be prepared to compromise. Set realistic goals. Spend time on what's important. Don't overcommit yourself. Be reasonable and practical in your expectations.

If you're making up for lost time or past failures remember this: you cannot change the past, so don't dwell on it. What's done is done. You only have control of your present and future. Do the best you can now. Be motivated and confident. Start studying today.

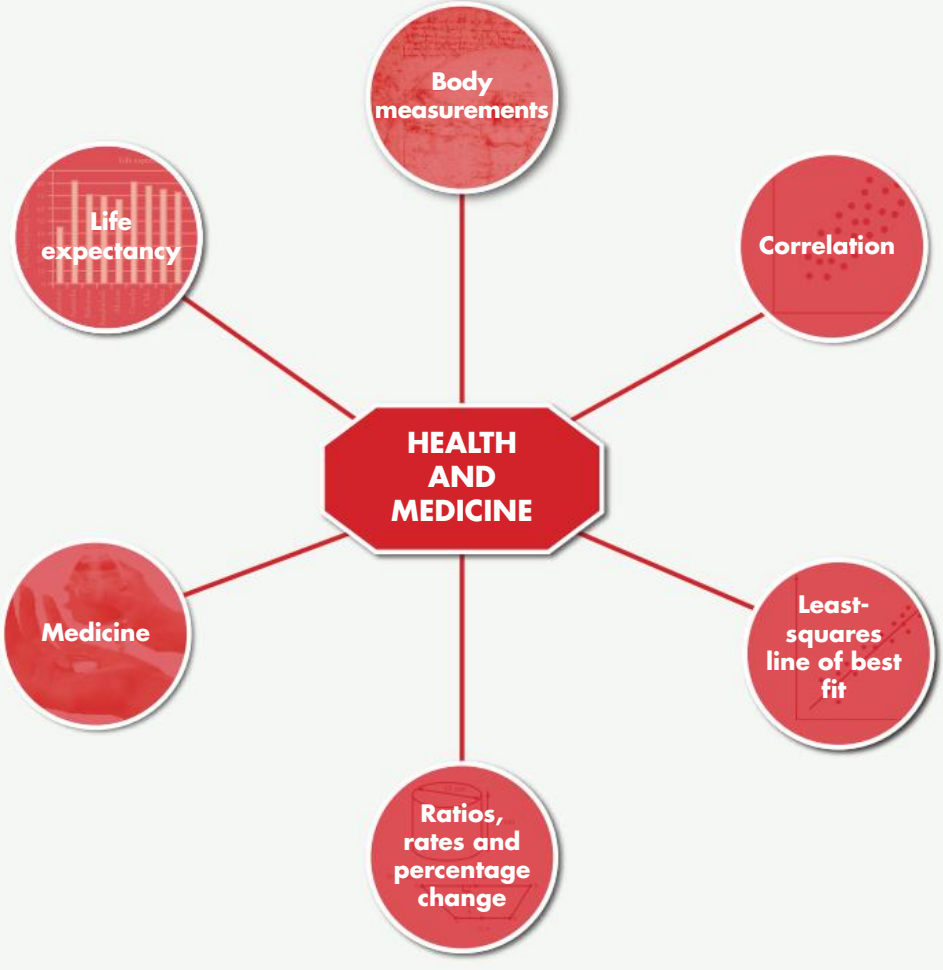
ISBN 9780170238977

CHAPTER REVIEW

Topic overview

This chapter, Health and medicine, examined the measurement, statistics and algebra behind body measurements, medicine dosage and life expectancy. You investigated the statistical concepts of scatter plots, correlation and the least-squares line of best fit, and applied rates to calculate concentrations in medication and medicine dosages. Be sure that you understand how to use the formulas to find the correlation coefficient, r, and the equation of the least-squares regression line. You should know how to draw lines of fit by hand and using technology, interpret correlation and the strength of association between two variables (bivariate data), and be able to calculate drip rates and children's medicine dosages using different formulas.

Make a summary of this topic. Use the outline at the start of the chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders to complete it. Gain a 'whole picture' view of the topic and identify any weak areas.



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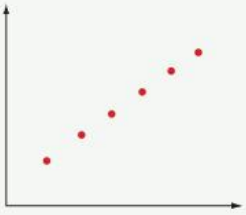
Revision

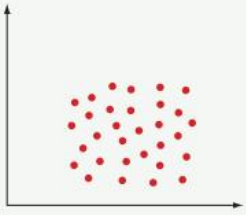
1 Describe the correlation of each scatter plot.

Exercise 8-01

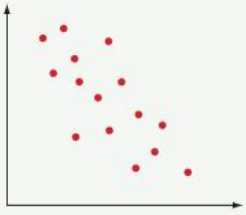
a



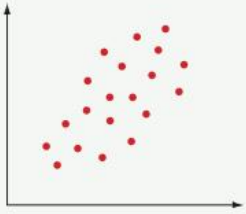
b



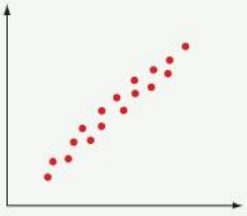
c



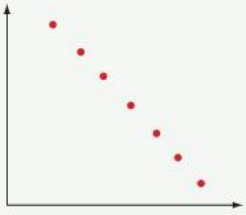
d



e



f



2 The heights and arm spans of 12 boys and 12 girls were measured in centimetres.

Exercise 8-01

Height	145	151	154	158	160	161	163	165	169	170	172	180
Boy arm span	146	150	153	158	159	160	164	163	170	167	170	178
Girl arm span	142	131	153	159	157	159	160	162	167	166	168	176

a Using graph paper or appropriate technology, draw a scatter plot comparing the heights of boys and their arm spans.

b Draw a similar scatter plot comparing the heights of girls and their arm spans.

c Describe the relationship between the boys' heights and arm spans.

d Is the relationship between height and arm span for girls stronger or weaker than that for boys? Justify your answer.

e There is an outlier in the girls' data. What is it and what does it represent ?

3 A local gym recorded the age and resting pulse rate of 10 people before exercise.

Exercise 8-02

Age	45	20	15	62	35	49	18	38	55	24
Pulse rate	71	79	83	70	68	67	86	72	68	86

a Draw a scatter plot of the data.

b Describe any correlation between the two variables.

c Write an estimate for the correlation coefficient.

d Calculate, correct to two decimal places, the correlation coefficient.

e How close was your estimate to the actual value?

4 For the pulse rate data in question **3**, find the equation of the least-squares line of best fit.

Exercise 8-03

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Exercise 8-04

5 Lara uses a drip system to water her medicinal plants. Each plant needs 35 mL of water per day.

a If 5 mL of water contains 50 drops, how many drops are needed per plant per day?

b How many litres of water are needed per day for 200 plants?

c What is the drip rate (in drops per hour) per plant if the drip system runs for 2 hours per day?

Exercise 8-05

6 **a** A painkiller has a concentration of 200 mg/5 mL. How many grams of painkiller are in a 1 L bottle of solution?

b If 1 L of solution contains 5 g of a drug, how many mg of the drug are in 750 mL of solution?

Exercise 8-05

7 10 L of solution contains 150 mL of pethidine.

a What is the concentration of pethidine as a rate in mL/L?

b How much pethidine is in 200 mL of solution?

c How much solution contains 60 mL of pethidine?

Exercise 8-06

8 Andy is 11 years old and weighs 38 kg. If the adult dosage of a drug is 500 mg per day, find the appropriate dosage, to the nearest mg/day, for Andy using:

a Young's rule: $\text{Child dosage} = \dfrac{\text{age in years}}{\text{age in years} + 12} \times \text{adult dosage}$

b Cowling's rule: $\text{Child dosage} = \dfrac{\text{age in years} + 1}{24} \times \text{adult dosage}$

Exercise 8-06

9 Clarise was prescribed 200 mg of 'Nomorehurt' per dose, to be taken every 4 hours. Her pharmacist only had a 50 mL bottle containing 100 mg/5 mL of 'Nomorehurt'. How many mL should she take:

a per dose? **b** per day?

iStockphoto/James Brey

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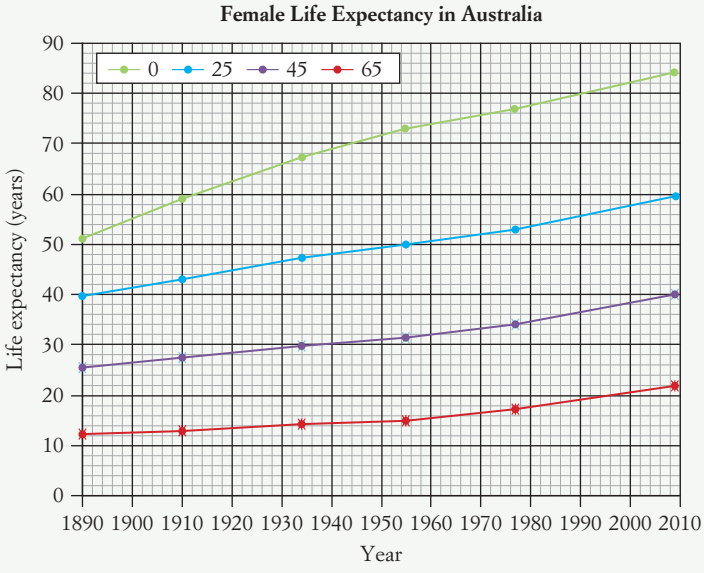
Exercise 8-07

10 These line graphs show the life expectancies for females of 4 different ages over a period of 120 years.



Australian Bureau of Statistics. ausstats.abs.gov.au

a Find the life expectancy of:

i a 25-year-old in 1976 **ii** a 45-year-old in 1910 **iii** a baby born in 2009

b At what age is a female most likely to die if she was:

i 65 in 1955? **ii** born in 1910? **iii** 25 in 1978?

c What is the difference in life expectancy between a 25-year-old in 1890 and a 25-year-old in 2009?

d Explain how the year in which you were born affects your life expectancy.

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9.

GEOMETRY OF THE EARTH

Did you know that the sun rises in Australia before most of the rest of the world, and that in the USA it is still 'yesterday'? The only places that are 'ahead' of us in time are the Pacific islands such as New Zealand, Fiji, Vanuatu and Tonga. To understand why, we must study the geometry and geography of the world, taking into account that the Earth is a sphere rather than a flat surface.

CHAPTER OUTLINE

IN THIS CHAPTER YOU WILL:

- use latitude and longitude to locate positions on the Earth's surface
- use the arc length formula to calculate the great circle distance between two places on the Earth's surface
- use the difference in longitude between two places on Earth to calculate the difference between their local times
- use international and Australian time zones to calculate the local time of a place
- solve travel problems involving time zones, including daylight saving and the International Date Line

TERMINOLOGY

Crossword of the Earth

angular distance
arc length
Coordinated Universal Time (UTC)
coordinates
daylight saving
Equator
great circle
Greenwich Meridian
hemisphere
International Date Line
kilometre
latitude
longitude
meridian
parallel
prime meridian
small circle
time zone

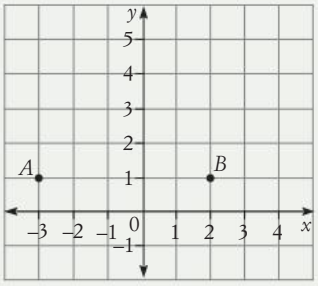
Assignment 9

SkillCheck

1 **a** Write the coordinates of points A and B.

b What is the length of the interval AB?

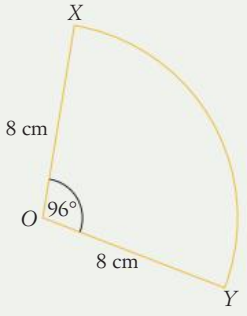
c Write the coordinates of point C if it is 3 units up and 4 units left from B.



2 **a** Plot the points $D(3, 5)$ and $E(3, 2)$ on a number plane.

b What is the length of the interval DE?

3 Find, correct to two decimal places, the length of the arc XY in the diagram.



4 A marathon runner travels 42 kilometres in $3\frac{1}{2}$ hours.

a Calculate his average speed.

b How long will it take him to travel 50 km at this speed? Answer in hours and minutes.

5 What is the time:

a 7 hours after 7:00 p.m.?

b 8 hours before 2:00 p.m.?

c 11 h 18 min after 1:15 p.m.?

d 7 h 36 min before 2:20 a.m.?

6 Use the o ' " key on your calculator to convert these times to hours and minutes.

a 22.7 hours

b 6.3 hours

7 Convert these times to hours and minutes.

a 482 minutes

b 271 minutes

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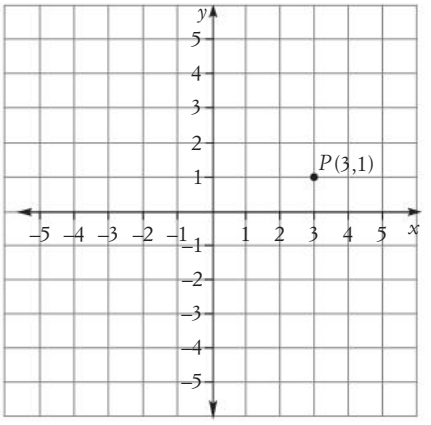
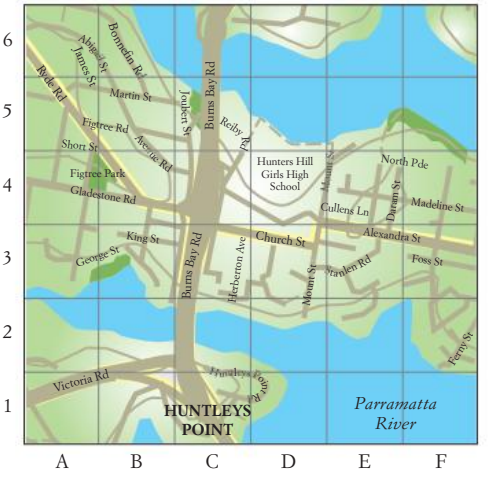
9-01 Latitude and longitude

When describing the location of a point on a number plane or map, we use a coordinate system involving ordered pairs (x, y).

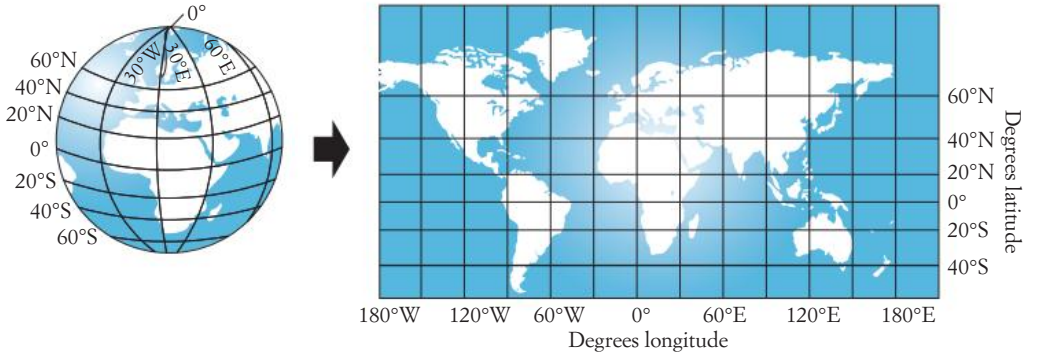


P has coordinates (3, 1) on this number plane.



Hunters Hill Girls High School has coordinates D4 here.

Positions on the Earth's surface are described by a coordinate system involving **latitude** and **longitude**. However, because the Earth is a sphere, we use a special grid of lines that run across and down a sphere. The diagrams below show this grid on a world globe and a flat world map.



Map of the world

Latitude and longitude

Australian coordinates

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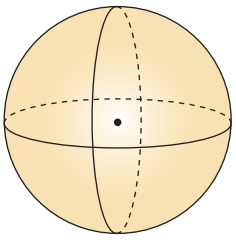
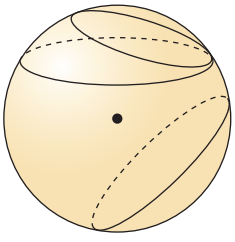
Great and small circles

If you cut a 'slice' through a sphere, its shape is a circle. A slice through the **centre** of a sphere is called a **great circle**, and its radius is the same as that of the sphere. Any other slice is called a **small circle**, because its radius is smaller than that of a great circle.



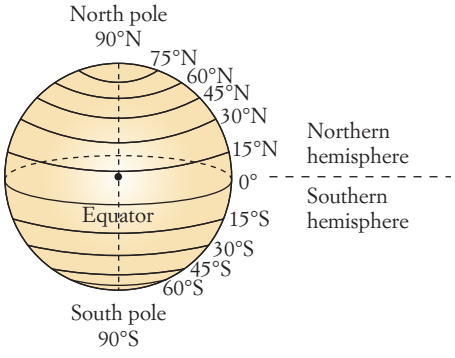
Great circles



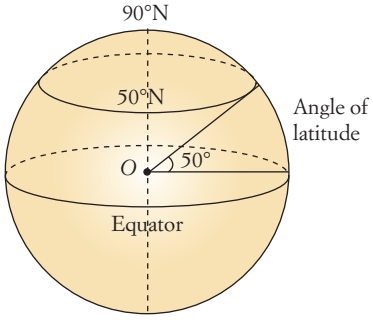
Small circles

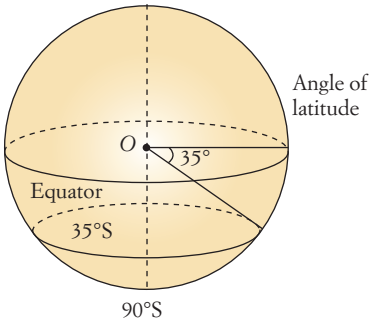
Parallels of latitude

Parallels of latitude are imaginary parallel circles that run around the Earth. The largest parallel of latitude is the **Equator**, 0°, which is a great circle. The other parallels of latitude are all small circles. The North Pole has a latitude 90° north (90°N). The South Pole has a latitude of 90°S.



The **angle of latitude** is the angle the parallel makes with the Equator at the centre, *O*, of the Earth. The diagrams below show the 50°N and 35°S parallels of latitude.





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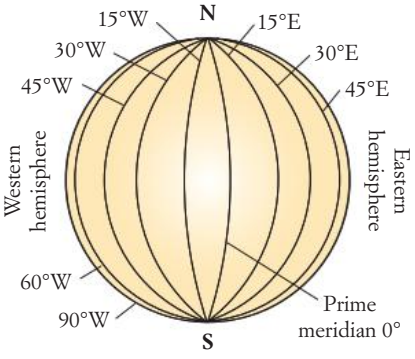
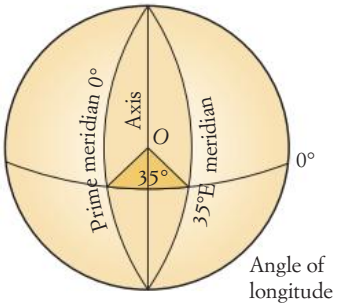
Meridians of longitude

Meridians of longitude are imaginary semicircles that run down the Earth. They are 'half' great circles that meet at the North and South Poles. The main meridian of longitude is the **prime meridian**, 0°. It is also called the **Greenwich meridian** since it runs through the Royal Observatory at Greenwich in London, England.

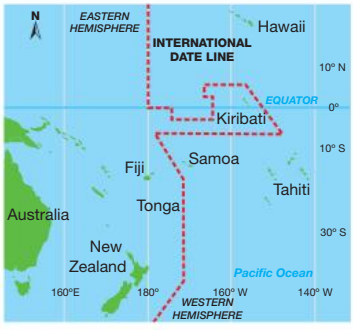
Greenwich is pronounced 'gren-itch'



The **angle of longitude** is the angle the meridian makes with the prime meridian at the centre, *O*, of the Earth. The diagram shows the 35°E meridian of longitude.

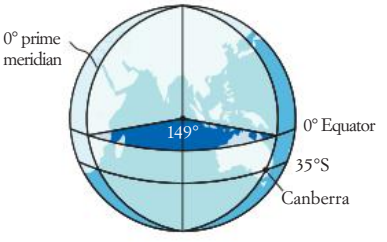


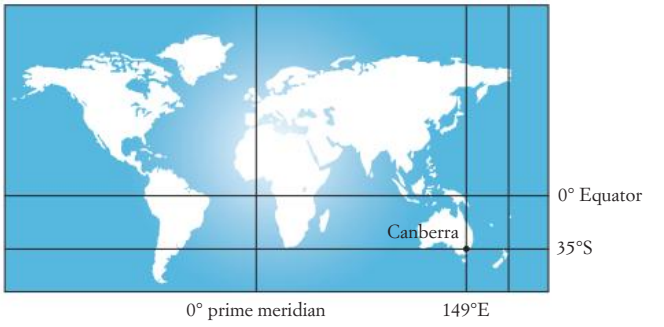
Meridians of longitude range from 180°E to 180°W. 180°E and 180°W are actually the same meridian, on the opposite side of the Earth to the prime meridian. It runs through the Pacific Ocean, east of Fiji.



Position coordinates

Locations on the Earth are described using latitude (°N or °S) and longitude (°E or °W). For example, Canberra has coordinates (35°S, 149°E), meaning it is 35° south of the Equator and 149° east of the prime meridian.





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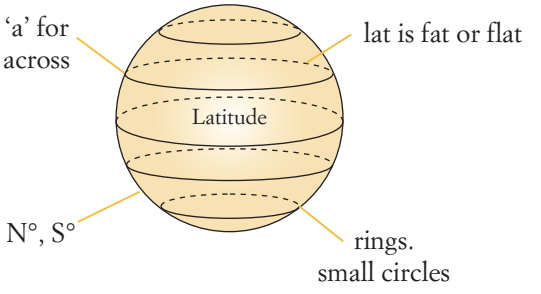
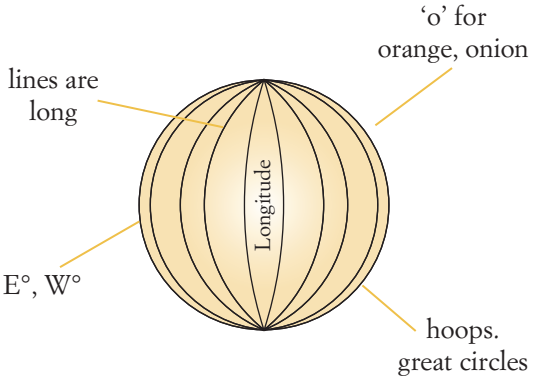
These diagrams show some mnemonics for understanding latitude and longitude.

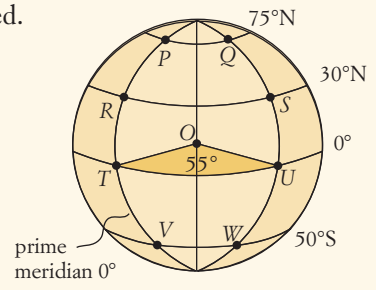




EXAMPLE 1

Match the following coordinates to the points illustrated.

a (50°S, 55°E) **b** (30°N, 55°E)
c (75°N, 0°E) **d** (0°, 0°)
e (0°, 55°E) **f** (75°N, 55°E)



Solution

Since $\angle TOU = 55°$, $QSUW$ is the 55°E meridian of longitude.

a (50°S, 55°E) is W **b** (30°N, 55°E) is S **c** (75°N, 0°E) is P
d (0°, 0°) is T **e** (0°, 55°E) is U **f** (75°N, 55°E) is Q

EXAMPLE 2

Sydney's coordinates are (34°S, 151°E) while Tokyo's are (35°N, 139°E).

a Find their difference in latitude.
b Find their difference in longitude.
c Which city is further west?

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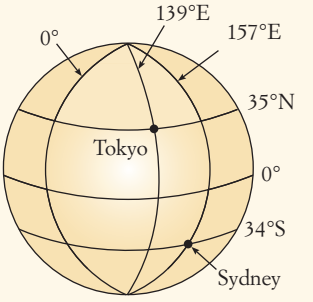
Solution

Draw a rough sketch to position the cities.



a Difference in latitude = 35° + 34° = 69° **Difference between 35°N and 34°S.**

b Difference in longitude = 151° − 139° = 12° **Difference between 151°E and 139°E.**

c Sydney is further east, so Tokyo is further west.

EXAMPLE 3

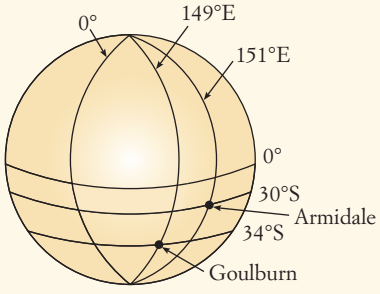
Goulburn has coordinates (34°S, 149°E). If Armidale is 4° north and 2° east of Goulburn, what are its coordinates?

Solution

Latitude = 34°S − 4° = 30°S

Longitude = 149°E + 2° = 151°E

The coordinates of Armidale are (30°S, 151°E).



Exercise 9-01 Latitude and longitude

Note: An atlas and/or a world globe may be helpful for this exercise.

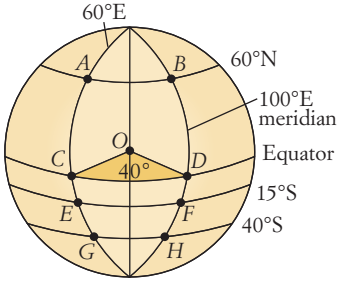
1 Match each set of coordinates to a point on the Earth.

a (0°, 100°E) **b** (40°S, 60°E)

c (60°N, 60°E) **d** (0°, 60°E)

e (15°S, 100°E) **f** (60°N, 100°E)

g (15°S, 60°E) **h** (40°S, 100°E)



Example 1

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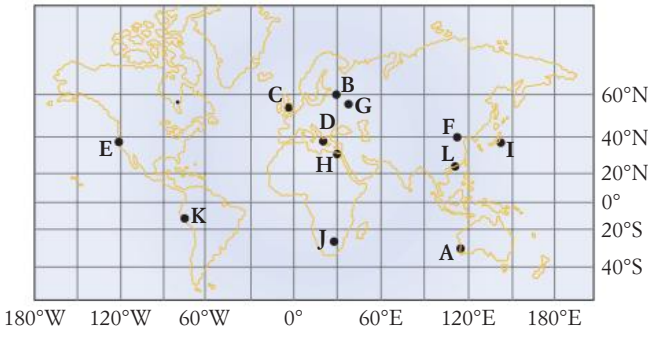
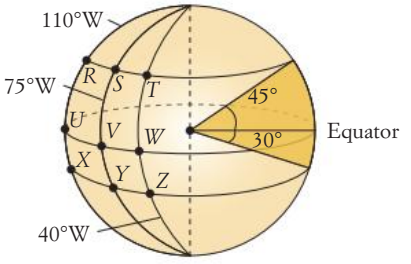
2 Match each city to a letter on the map below.

a Moscow (55°N, 37°E)
b Edinburgh (56°N, 3°W)
c Athens (38°N, 23°E)
d San Francisco (38°N, 122°W)
e Perth (32°S, 116°E)
f Beijing (40°N, 116°E)
g Alexandria (31°N, 30°E)
h Tokyo (35°N, 139°E)
i St Petersburg (60°N, 30°E)
j Johannesburg (26°S, 28°E)
k Lima (12°S, 77°W)
l Hong Kong (22°N, 114°E)



3 Write the coordinates of each of the points on the Earth.

a V
b T
c U
d Z
e R
f W
g X
h S



4 What can be found at the 90°S parallel of latitude?

5 Are these small circles or great circles?

a the 30°N parallel
b the prime meridian
c the 145°W meridian

6 Calculate the difference in latitude between each pair of cities, and state whether the second city listed is north or south of the first.

a Shanghai, China (31°N, 121°E) and New York, USA (40°N, 64°W)
b Nairobi, Kenya (1°S, 37°E) and Bangkok, Thailand (13°N, 100°E)
c Moscow, Russia (55°N, 37°E) and London, UK (51°N, 0°)
d Auckland, New Zealand (37°S, 174°E) and Canberra (35°S, 149°E)
e Melbourne (37°S, 145°E) and Cairo, Egypt (30°N, 31°E)
f Newcastle (33°S, 151°E) and West Wyalong (34°S, 147°E)

7 **a** Deniliquin is 1° due south of Hay. If Hay's coordinates are (34°S, 145°E), what are Deniliquin's coordinates? Select **A**, **B**, **C** or **D**.

A (34°S, 144°E)
B (35°S, 145°E)
C (33°S, 145°E)
D (34°S, 146°E)

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b Lord Howe Island is 6° due east of Forster. If Forster's coordinates are (32°S, 152°E) what are Lord Howe Island's coordinates? Select **A**, **B**, **C** or **D**.

A (32°S, 146°E) **B** (38°S, 152°E) **C** (32°S, 158°E) **D** (26°S, 158°E)

8 Calculate the difference in longitude between each pair of cities, and state whether the second city listed is east or west of the first. Example 2

a Budapest, Hungary (47°N, 19°E) and Miami, USA (25°N, 80°W)

b Athens, Greece (38°N, 23°E) and Paris, France (49°N, 2°E)

c Havana, Cuba (23°N, 82°W) and Mexico City, Mexico (19°N, 99°W)

d Buenos Aires, Argentina (34°S, 58°W) and Johannesburg, South Africa (26°S, 28°E)

e Manila, Philippines (14°N, 121°E) and Port Moresby, Papua New Guinea (9°S, 147°E)

f Finley (35°S, 145°E) and Bourke (30°S, 146°E)

9 Broken Hill is 7° due west of Dubbo. If Dubbo's position is (32°S, 148°E), what is Broken Hill's position? Example 3

10 Ipswich is 2° north and 2° east of Moree (29°S, 150°E). What are Ipswich's coordinates?

11 Ballarat is 2° south and 6° west of Batemans Bay (35°S, 150°E). What are the coordinates of Ballarat? Select **A**, **B**, **C** or **D**.

A (33°S, 144°E) **B** (37°S, 156°E) **C** (33°S, 156°E) **D** (37°S, 144°E)

12 a In Australia, do Sydney and Tamworth lie on the same line of latitude or longitude?

b Do Athens, Greece and Sicily, Italy, lie on the same line of latitude or longitude?

13 Where on Earth is it possible to stand on every meridian of longitude?

INVESTIGATION

ATLAS ACTIVITIES

Use an atlas or the Internet to locate the positions of different places around the world. Some websites give the coordinates of any city you type in, for example, Google Earth lists the coordinates at the bottom of the page.

1 Use a world map or globe to estimate the coordinates of each city.

a Washington DC, USA **b** Singapore **c** Adelaide, Australia

d Dublin, Ireland **e** Lisbon, Portugal **f** Moscow, Russia

2 Find the actual coordinates of the cities from question **1**, and then list them in order from:

a west to east **b** north to south

3 Which city is located at:

a (19°N, 155°W)? **b** (14°N, 121°E)? **c** (34°S, 58°W)?

d (42°N, 12°E)? **e** (12°S, 131°E)? **f** (42°N, 87°W)?

4 How does a flat map distort the positions and sizes of places with high latitudes?

5 Investigate where these regions are in the world and how they got their names:

a the Middle East **b** the Orient **c** Western world

d the Antipodes **e** Ecuador **f** the West Indies

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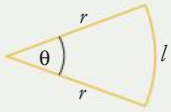
9-02 Great circle distances

In Chapter 3, we learnt the formula for the arc length of a circle.

SUMMARY

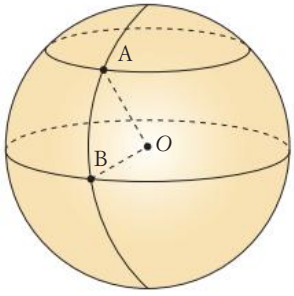
Arc length of a circle

$$l = \frac{\theta}{360} \times 2\pi r$$



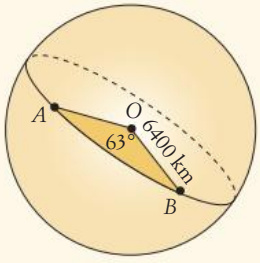
where θ is the number of degrees in the central angle, r is the radius and l is the length of the arc on the circumference.

On a flat surface, the shortest distance between two points is a straight line. Since the Earth's surface is curved, the shortest distance between A and B is the arc length AB of the great circle that passes through A and B. This is called the **great circle distance**, and the size of angle $\angle AOB$, where O is the centre of the Earth, is called the **angular distance**.



EXAMPLE 4

In the diagram on the right, A and B on the Earth's surface have an angular distance of 63°. Calculate the great circle distance between A and B, correct to the nearest kilometre. The radius of the Earth is 6400 km.



Solution

$$AB = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{63}{360} \times 2 \times \pi \times 6400$$
$$= 7037.1675\ldots$$
$$\approx 7037 \text{ km}$$

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EXAMPLE 5

Beijing, China and Perth, Australia have coordinates (40°N, 116°E) and (32°S, 116°E) respectively.

a What great circle joins Beijing and Perth?

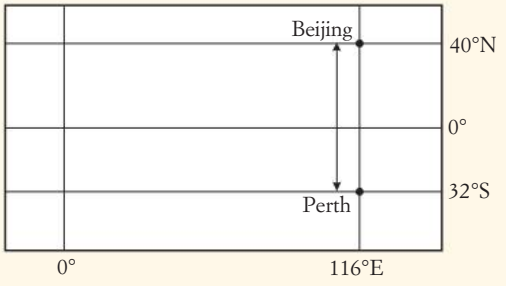
b What is the angular distance between these two cities?

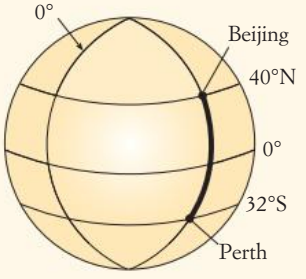
c Hence, calculate the shortest distance between Beijing and Perth, to the nearest kilometre, given that the Earth's radius is 6400 km.

Solution

a Draw either a rough flat map diagram or global diagram showing the positions of Beijing and Perth.

The great circle joining Beijing and Perth is the 116°E meridian of longitude.





b Angular distance = 40° + 32° = 72°

c Distance $= \frac{\theta}{360} \times 2\pi r$

$= \frac{72}{360} \times 2\pi \times 6400$

$= 8042.4772\ldots$

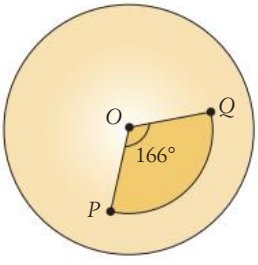
≈ 8042 km

SUMMARY

The radius of the Earth is approximately 6400 km.

Exercise 9-02 Great circle distances

1 In the diagram below, O is the centre of the Earth and the angular distance between P and Q on the Earth's surface is 166°. Calculate, correct to the nearest kilometre, the arc length PQ.



Example 4

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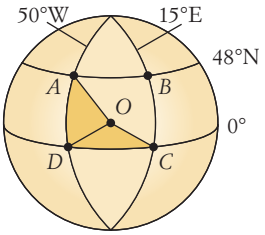
Example 5

2 

a What great circle passes through C and D?

b What is the angular distance between C and D?

c Calculate the shortest distance between C and D to the nearest kilometre.

d What great circle passes through A and D?

e Calculate the great circle distance between A and D to the nearest kilometre.

f What is the distance between B and C, to the nearest kilometre?

3 Paris, France has coordinates (49°N, 2°E). What is its distance from the Equator in kilometres? Select **A**, **B**, **C** or **D**.

A 223 **B** 980 **C** 5250 **D** 5473

4 Melbourne (37°S, 145°E) and Cairns (17°S, 145°E) lie on the same meridian of longitude.

a What is the angle made by these two cities at the centre of the Earth?

b Calculate the shortest distance between Melbourne and Cairns to the nearest kilometre.

5 The Antarctic Circle near the South Pole is the 66.5°S parallel of latitude. Calculate how many kilometres this circle is from the South Pole. Select **A**, **B**, **C** or **D**.

A 123 **B** 267 **C** 2625 **D** 7428

6 The Tropic of Capricorn is the 23.5°S parallel of latitude. Calculate, correct to the nearest kilometre, how far this circle is from:

a the North Pole **b** the Antarctic Circle.

7 Calculate, to the nearest kilometre, the **shortest** distance between each pair of cities.

a Osaka, Japan (34°N, 135°E) and Alice Springs (23°S, 135°E)

b Pretoria, South Africa (25°S, 28°E) and Minsk, Belorussia (54°N, 28°E)

c Kampala, Uganda (0°, 32°E) and Macapa, Brazil (0°, 51°W)

d Mawson, Antarctica (67°S, 63°E) and Chelyabinsk, Russia (54°N, 63°E)

e Port Moresby, Papua New Guinea (9°S, 147°E) and Nyngan (31°S, 147°E)

f Leeton (34°S, 146°E) and Townsville (19°S, 146°E).

8 If you dug a hole from Beijing, China (40°N, 116°E) through the centre of the Earth and came out on the other side, you would end up at point X, somewhere in Argentina.

a What would be the coordinates of the point X?

b How many kilometres are there between Beijing and point X:

i in a straight line through the Earth's centre?

ii along a great circle on the Earth's surface?

9 The great circle distance between Sydney and Honolulu, Hawaii is 4170 km. What angle is made at the centre of the Earth by these two cities? Answer to the nearest degree.

10 It is possible to fly from Perth (32°S, 115°E) to Tierra del Fuego (54°S, 65°W) on the southern tip of South America by going directly over the South Pole along a line of longitude. Calculate the great circle distance between Perth and Tierra del Fuego, correct to the nearest kilometre.

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TECHNOLOGY: LOCATIONS ON THE EARTH

Distance calculator websites

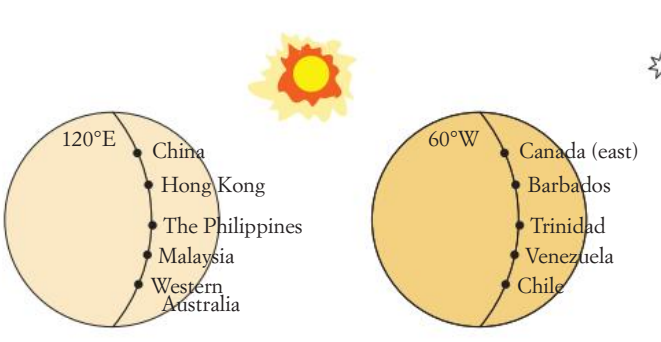
Use a distance calculator website to enter the names or coordinates of two places in the world, then find the shortest distance between them. Some computers have a distance calculator included in their system software.

Global positioning system

A global positioning system (GPS) or satellite navigator uses Earth satellites to calculate a person's location on Earth. Investigate how GPS works. Find out what an EPIRB is and how it is used in navigation.

9-03 Longitude and time differences

When the Sun shines directly on a meridian of longitude, it is 12 noon at all places along that meridian (meridian means 'midday' in Latin). On the other side of the world along the opposite meridian, it is midnight. For example, when it is noon in Western Australia on the 120°E meridian, it is midnight in Chile on the 60°W meridian.



Midday on the 120°E meridian

Midnight on the 60°W meridian

Since the Earth turns 360° in 24 hours, it turns 15° in 1 hour. For every 15° of longitude, the time difference is 1 hour and so for every 1° of longitude the time difference is 4 minutes.

SUMMARY

15° longitude = 1 hour time difference

1° longitude = 4 minutes time difference

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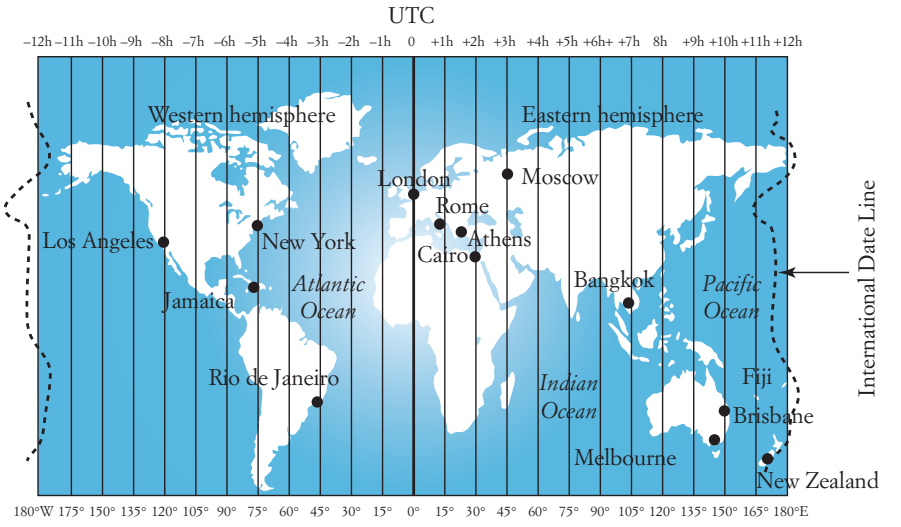
Local times around the world are measured relative to the time along the prime meridian 0°, called **Coordinated Universal Time** (**UTC**) or **Greenwich Mean Time** (**GMT**). Places east of the prime meridian are *ahead* of UTC, while places west are *behind* UTC.



EXAMPLE 9

The time in New York, USA is 5 hours behind UTC.

a What is the time in New York when it is 3 p.m. in London?

b What meridian of longitude passes through New York?

Solution

a 3 p.m. – 5 hours = 10 a.m.

b Difference in longitude = $5 \times 15°$ **1 hour difference = 15° longitude.**

$= 75°$

New York is *behind* UTC, so it is west of the prime meridian, with longitude 75°W.

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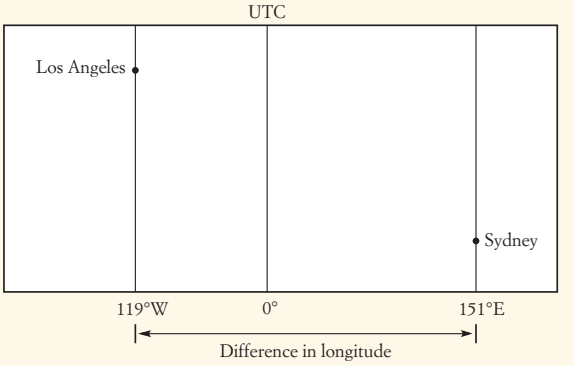
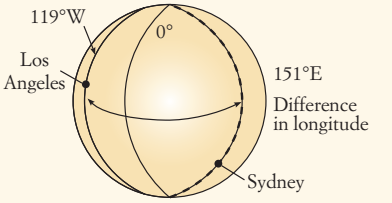
EXAMPLE 10

In Los Angeles, USA (119°W), a tennis match is being played at 2:00 p.m. on Tuesday. Dave is watching the match live on TV in Sydney (151°E). What is the day and time in Sydney?

Solution





Difference in longitude = 119° + 151° = 270°

Time difference = $\frac{270}{15}$ = 18 hours

Sydney is east of Los Angeles, so Sydney time is ahead of Los Angeles time.

Sydney time = 2:00 p.m. Tuesday + 18 hours = 8:00 a.m. Wednesday

EXAMPLE 11

When it is 9:00 p.m. in Mumbai, India (19°N, 73°E), what is the local time in Jakarta, Indonesia (6°S, 106°E)?

Solution

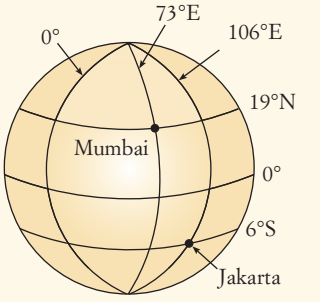
Difference in longitude = 106° − 73° = 33°

Time difference = $\frac{33}{15} = 2\frac{1}{5}$ h = 2 h 12 min

(or Time difference = 33 × 4 min = 132 min = 2 h 12 min)

Jakarta is east of Mumbai, so its time is ahead of Mumbai's.

Local time in Jakarta = 9:00 p.m. + 2 h 12 min = 11:12 p.m.



In Example **11**, the time in Jakarta will not really be 11:12 p.m. but will be 11:00 p.m. because the world is divided into hourly time zones. However, for the following exercise we shall **ignore time zones** and calculate 'local times', correct to the nearest minute.

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Exercise 9-03 Longitude and time differences

Example 9

1 The time in Tahiti, a Pacific island, is 10 hours behind UTC.

a What is the longitude of Tahiti?

b What is the time in London (UTC) when it is 6:30 a.m. in Tahiti?

c What is the time in Tahiti when it is 6:30 a.m. in London?

2 The time in Vienna, Austria (48°N, 16°E) is 3 hours behind the time in Tehran, Iran.

a Calculate the difference in longitude between the two cities.

b Hence, what meridian of longitude goes through Tehran?

c When it is 11:15 p.m. in Tehran, what is the time in Vienna?

Example 10

3 Hanoi, Vietnam lies on the 105°E meridian while Sydney lies on the 150°E meridian. When the time in Hanoi is 5:30 p.m., what is the time in:

a Sydney? **b** London?

4 What is the local time in Pretoria, South Africa (25°S, 30°E) when it is 10:45 p.m. in London? Select **A**, **B**, **C** or **D**.

A 8:45 p.m. **B** 9:05 p.m. **C** 12:25 a.m. **D** 12:45 a.m.

5 What is the time in London when it is 7:10 p.m. in Rio de Janeiro, Brazil (23°S, 43°W)?

6 When it is 2:30 p.m. in Alice Springs (134°E), what is UTC?

Example 11

7 Kalgoorlie, WA has coordinates (30°S, 121°E) while the Pacific island of Nauru has coordinates (0°, 166°E).

a Calculate the difference in longitude between these two places.

b Calculate the time difference between the two places.

c What is the time in Nauru when it is 9:45 a.m. in Kalgoorlie?

8 **a** Calculate the difference in longitude between New Delhi, India (28°N, 77°E) and Milan, Italy (41°N, 12°E).

b Calculate the time difference between New Delhi and Milan in hours and minutes.

c What is the local time in Milan when it is 5:00 p.m. in New Delhi?

9 **a** Calculate the time difference between Lisbon, Portugal (38°N, 9°W) and Taipei, Taiwan (25°N, 121°E).

b What is the local time in Taipei when it is 11:20 a.m. in Lisbon?

10 An ocean liner's local time is 5:45 p.m. when UTC is 8:05 a.m. What is its longitude position? Select **A**, **B**, **C** or **D**.

A 35°E **B** 35°W **C** 145°E **D** 145°W

11 Houston, USA has coordinates (29°N, 95°W) while Kabul, Afghanistan has coordinates (34°N, 70°E). If it is 6:20 p.m. Sunday in Houston, what is the time and day in Kabul?

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12 Guatemala is 6 hours behind UTC.

- **a** What meridian of longitude runs through Guatemala?
- **b** What is the time in the UK when it is 1:30 p.m. in Guatemala?

13 a When it is 7:30 p.m. Monday in Los Angeles, USA (34°N, 119°W), what is the time and day in Sydney (34°S, 151°E)?

- **b** A plane leaves Los Angeles at 7:30 p.m. Monday and flies $14\frac{1}{2}$ hours to get to Sydney. What is the local time in Sydney when it arrives?

14 When it is 7 a.m. in Broome, Western Australia, it is 11:40 p.m. the previous night, in Munich, Germany.

- **a** Calculate the difference in longitude between Broome and Munich.
- **b** Munich has longitude 12°E. Calculate the longitude of Broome.

15 When it is midday in Johannesburg, South Africa, it is 10:48 a.m. in Pisa, Italy.

- **a** Calculate the difference in longitude between Johannesburg and Pisa.
- **b** Pisa has longitude 10°E. Calculate the longitude of Johannesburg.

16 When it is 9:30 p.m. in Ottawa, Canada, it is 3:02 a.m. the next day in Bern, Switzerland.

- **a** What is the time difference between the 2 cities?
- **b** Calculate the difference in longitude between Ottawa and Bern.
- **c** Bern has longitude 7°E. Calculate the longitude of Ottawa.

JUST FOR THE RECORD

Daylight saving

In 1907, British builder William Willett noticed that the extra hours of daylight in the summer mornings were being 'wasted' because people were still asleep at this time. He proposed a scheme of putting clocks forward one hour so that people would wake up earlier and have more daylight hours in the afternoon for recreation.

Daylight saving was first trialled in Britain and Australia during the First World War as a means of conserving energy. In Australia, it was trialled again from 1971 and was formally adopted in NSW in 1976 when people voted 13 to 6 in favour of it. Queensland, Northern Territory and Western Australia voted against it and do not have daylight saving.

- **a** How does daylight saving reduce crime, pedestrian fatalities and energy costs?
- **b** Daylight saving is less popular in country areas. Why do you think this is so?
- **c** Some countries have daylight saving in winter. Why do you think this is so?

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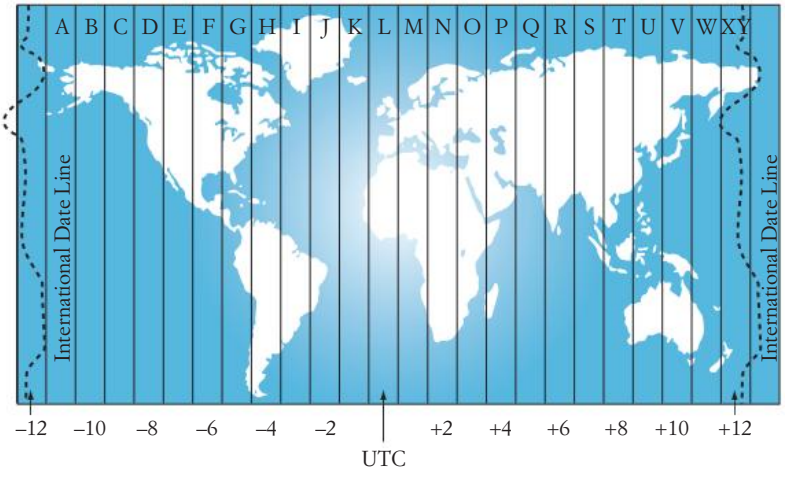
Table of time zones

Map of the world

9-04 International time zones

Since 1884, the world has been divided into standard time zones, with roughly one hour difference for every 15° of longitude. Some countries make variations for geographical and/or administrative reasons. For example, Central Australian Time is 9.5 hours (not 9 hours) ahead of UTC and China has only one time zone (UTC+8) even though it stretches across four. On the map below, places along the 105°E meridian (column S) are 7 hours ahead of UTC, while places along the 150°W meridian (column B) are 10 hours behind UTC.



This table lists the international time zones relative to UTC and **Australian Eastern Standard Time (AEST)**, the time zone for eastern Australia, covering Queensland, NSW, ACT, Victoria and Tasmania.

Hours from UTC	Hours from AEST	Places in this zone
–11	–21	American Samoa, Niue Island, Midway Island
–10	–20	Hawaii, Cook Islands, Tahiti
–9	–19	Alaska
–8	–18	USA/Canada Pacific time (west coast), Los Angeles, Vancouver
–7	–17	USA/Canada Mountain time, Mexico (west)
–6	–16	USA/Canada Central time (mid-west), Mexico (east), El Salvador, Nicaragua, Guatemala, Honduras, Belize
–5	–15	USA/Canada Eastern time (east coast), New York, Cuba, Peru
–4	–14	Canada Atlantic time, Chile, Barbados, Brazil (west), Bolivia
–3	–13	Argentina, Brazil (east), Uruguay, Greenland
–2	–12	South Sandwich Islands
–1	–11	Azores
0	–10	UK, Ireland, Iceland, Portugal, Ghana, Liberia, Mali, Morocco
+1	–9	Most of Europe, Algeria, Angola, Chad, Bosnia-Herzegovina, Croatia
+2	–8	Finland, Greece, Lebanon, Egypt, South Africa, Bulgaria, Cyprus
+3	–7	Russia (west), Saudi Arabia, Kenya, Madagascar, Iraq
+3.5	–6.5	Iran
+4	–6	Mauritius, United Arab Emirates, Armenia

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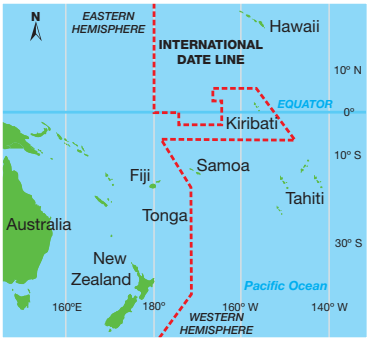
Hours from UTC	Hours from AEST	Places in this zone
+4.5	−5.5	Afghanistan
+5	−5	Pakistan, Maldives, Kazakhstan
+5.5	−4.5	India, Sri Lanka
+6	−4	Bangladesh, Bhutan, Kazakhstan, Kyrgystan
+6.5	−3.5	Myanmar (Burma), Cocos Islands
+7	−3	Vietnam, Cambodia, Thailand, Indonesia (west), Laos
+8	−2	Australian Western Standard Time (WA), Malaysia, China, the Philippines, Indonesia (central), Mongolia, Taiwan
+9	−1	Japan, North Korea, South Korea, Indonesia (east), East Timor
+9.5	−0.5	Australian Central Standard Time (NT, SA, Broken Hill)
+10	0	Australian Eastern Standard Time, Papua New Guinea, Guam
+10.5	+0.5	Lord Howe Island
+11	+1	Vanuatu, New Caledonia, Solomon Islands, Micronesia
+11.5	+1.5	Norfolk Island
+12	+2	New Zealand, Fiji, Nauru
+13	+3	Samoa, Tonga

The International Date Line

The **International Date Line** (**IDL**), on the opposite side of the world to the prime meridian, runs through the Pacific Ocean between Fiji and Hawaii. It is shown as the red dotted line on the map below, and is 'bent' to ensure that it does not run through any countries.



West of the IDL is the UTC+12 time zone including New Zealand, Fiji and Nauru, while east of the IDL is the UTC−11 time zone including American Samoa and Niue Island. So if you cross the IDL travelling east, today becomes yesterday (for example, Wednesday becomes Tuesday) and you experience the same day twice. The reverse occurs when you cross the IDL travelling west: today becomes tomorrow (for example, Sunday becomes Monday) and you 'lose' a day.

EXAMPLE 12

Habib is flying from New York, USA to Istanbul, Turkey to visit his grandfather. New York is 5 hours behind UTC while Turkey is 2 hours ahead of UTC.

a Habib wants to call his grandfather on the phone before he leaves New York. At what time should he call so that it is 6:00 p.m. in Turkey?

b Habib boards the plane at 1:45 p.m. New York time and the trip lasts 8.5 hours. What is the local time in Istanbul when he arrives?

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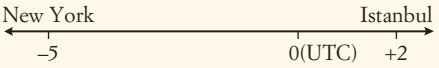
Solution

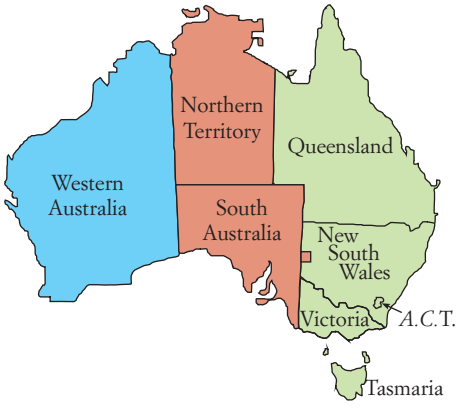


a Time difference = 5 + 2 = 7 hours
New York is 7 hours behind Istanbul.
New York time = 6:00 p.m. – 7 hours = 11:00 a.m.

b Arrival time = 1:45 p.m. + 8.5 hours **New York time.**
= 10:15 p.m.
= 10:15 p.m. + 7 hours **Istanbul time.**
= 5:15 a.m. (next day)

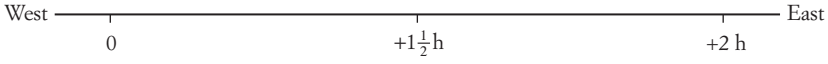
Australian standard time zones

There are three time zones in Australia (ignoring daylight saving), shown on the map below.



Australian Western Standard time (AWST) UTC+8	Australian Central Standard time (ACST) UTC+9.5	Australian Eastern Standard time (AEST) UTC+10

These time zones can also be shown on a timeline.



Western Standard Time	Central Standard Time	Eastern Standard Time
Perth	Adelaide	Sydney
	Alice Springs	Brisbane
	Darwin	Canberra
	Broken Hill (NSW)	Hobart

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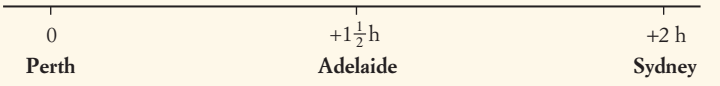
EXAMPLE 13

a Pete is in Perth and it is 11:30 a.m. He wants to ring his mother in Sydney. What time is it in Sydney?

b Kimberley has just finished work at 5 p.m. in Sydney. What time is it in Adelaide?

Solution

a



Moving from west (Perth) to east (Sydney) **ADD** 2 hours.

Time in Sydney = 11:30 a.m. + 2 hours = 1 p.m.

It is 1 p.m. in Sydney.

b Moving from east (Sydney) to central (Adelaide) **SUBTRACT** $\frac{1}{2}$ hour or 30 minutes.

Time in Adelaide = 5 p.m. – 30 min = 4:30 p.m.

It is 4:30 p.m. in Adelaide.

Daylight saving

In summer, many countries adopt **daylight saving time** or **summer time** and turn their clocks forward one hour to take advantage of the increased hours of sunlight. By 'losing' that hour at the start of daylight saving, people are actually working and sleeping 1 hour earlier and gaining maximum use of the daylight.

Daylight saving has been operating in Australia since 1971, although Queensland, Western Australia and the Northern Territory do not participate in the scheme. Each year, it runs from the first Sunday in October (mid-spring) to the last Sunday in April (mid-autumn).

SUMMARY

For daylight saving time, add one hour.

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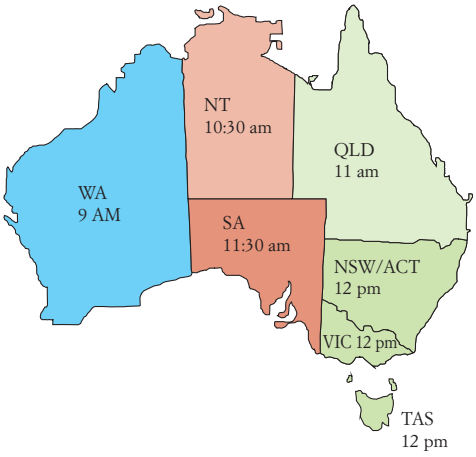
This map shows the local times around Australia during daylight saving when it is 12:00 midday in NSW.

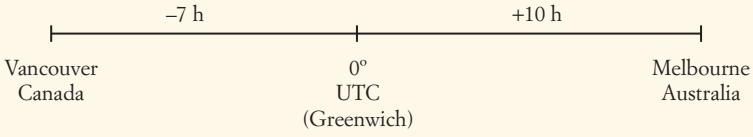


EXAMPLE 14

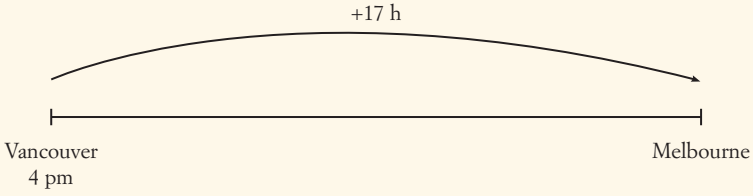
Simone, in Melbourne, wants to chat online with her cousin, Zac, in Vancouver, Canada. Canadian Pacific Standard Time is 8 hours behind UTC while Melbourne is 10 hours ahead of UTC. Daylight saving is operating in Canada. When should Simone logon to reach Zac at 4 p.m. Friday in Vancouver?

Solution

Vancouver time is UTC–8 but adding one hour for daylight saving gives UTC–7.



Time difference = 7 + 10 = 17 hours.



Melbourne is 17 hours ahead of Vancouver.

Logon time = 4:00 p.m. Friday + 17 hours

= 9:00 a.m. Saturday.

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Exercise 9-04 International time zones

Table of time zones

Example 14

Refer to the table on page 400 or the worksheet 'Table of time zones' for this exercise.

1 When it is 4:00 p.m. Saturday in Sydney, what is the time in:

a India? **b** Russia (west)? **c** Greece? **d** Tonga?

2 When it is 10:00 a.m. Tuesday in London, UK, what is the time in:

a Greenland? **b** Bosnia-Herzegovina? **c** Myanmar? **d** New Caledonia?

3 What is the time difference (in hours) between:

a Malaysia and New Zealand?

b Brazil (east) and Chile?

c Iceland and the Philippines?

d Algeria and Salt Lake City, USA (Mountain time)?

Example 13

4 The Australian Open Tennis competition is broadcast live from Melbourne at 8:30 p.m. local time. At what time is it seen in:

a Dallas, USA (Central time)? **b** Japan? **c** Sweden (Europe)?

5 It takes $8\frac{1}{2}$ hours for a plane to fly from Sydney to Hawaii. If Heath leaves Sydney at 1:30 p.m. Friday, what is the local time when he arrives in Hawaii? Select **A**, **B**, **C** or **D**.

A 2:00 p.m. Thursday

B 5:30 p.m. Thursday

C 2:00 a.m. Friday

D 10:00 p.m. Friday

6 If the time in Sydney is 6:00 p.m., what is the time in:

a Melbourne? **b** Hobart? **c** Alice Springs?

d Perth? **e** Adelaide? **f** Surfers Paradise?

7 Queensland, Northern Territory and Western Australia do not participate in daylight saving. If the daylight saving time in Sydney is 10:30 a.m., what is the time in:

a Melbourne? **b** Hobart? **c** Alice Springs?

d Perth? **e** Adelaide? **f** Surfers Paradise?

8 **a** What is the time in Santiago, Chile when it is 12:00 noon in Perth?

b What does this say about the positions of Perth and Santiago on the world globe?

9 A plane takes 12 hours to fly from Adelaide (UTC+9.5) to Cape Town, South Africa (UTC+2). If Shaunna leaves Adelaide at 10:00 a.m., what is the local time when she arrives in Cape Town?

10 Robert in Tasmania wants to call his brother in Scotland, UK when the time there is 4:30 p.m. daylight-saving time. At what time should Robert call? Select **A**, **B**, **C** or **D**.

A 1:30 a.m. **B** 2:30 a.m. **C** 3:30 a.m. **D** 5:30 a.m.

11 Indira flew from New Delhi, India to Sydney, leaving at 6:30 p.m. and arriving the next day at 12:00 p.m. local time. How long did her plane trip take?

12 The Superbowl is broadcast live from New York, USA (Eastern time) at 6:30 p.m. Monday. At what time and day is it seen in:

a Sydney? **b** Berlin, Germany (Europe)? **c** Dublin, Ireland?

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13 Wallaby Airlines flies from Adelaide to Mt Isa, Queensland, via Brisbane. The flight left Adelaide at 2:30 p.m. and arrived in Brisbane 2 h 40 min later.

The plane stayed in Brisbane for 35 minutes for refuelling and to take extra passengers on board. The flight from Brisbane to Mt Isa took 2 h 45 min. Find the local time when the plane arrived:

a in Brisbane **b** in Mt Isa

14 Laire boards a flight in Coffs Harbour at 9:50 a.m. Tuesday morning. The flight to Sydney takes 40 minutes. Laire then waits in Sydney for one hour and boards a flight to Perth, which takes 4 hours 25 minutes. What time is it in Perth when the plane lands?

15 A plane takes 2 hours and 45 minutes to fly from Wellington, New Zealand (UTC+12) to Sydney (UTC+10). If Paula leaves Wellington at 4:20 p.m. (daylight saving time) in September, what is the local time when she arrives in Sydney (standard time)?

16 A plane leaves Sydney at 1:00 p.m. Thursday and flies 20 hours to Paris, France. At what time does the plane arrive in Paris if daylight saving is operating in France but not in Sydney?

17 A plane leaves Hawaii on Monday at 12 noon and flies 6 hours west to Tokyo, Japan.

a When the plane crosses the International Date Line, what day does it become?

b What is the local time in Tokyo when the plane arrives?

18 Liz flew west from Buenos Aires, Argentina (UTC–3) to Melbourne (UTC+10). To her surprise, she discovered that when she arrived in Melbourne it was the same time, 7:30 p.m., as when she left Buenos Aires.

a How long did her plane journey take?

b Was it still the same day? If not, what day was it?

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INVESTIGATION

RESEARCHING TIME ZONES

a Why is UTC also known as GMT, Z time, Zulu time or Zebra time?

b How is time measured at the North and South Poles, where all the meridians of longitude meet?

c How is time measured in Australia's external territories, such as Norfolk Island, Lord Howe Island, Cocos Islands and Christmas Island (Indian Ocean)?

d Which country stretches across 11 time zones?

e Australia, USA and Canada are roughly the same length, but Australia has three time zones while USA and Canada have four and five time zones respectively. Why do you think this is so? (Hint: Look at the countries on a world globe or map.)

INVESTIGATION

PLAN AN OVERSEAS HOLIDAY

Use an atlas or a world globe to plan a trip to visit up to five different places in the world. Design an itinerary that lists the places you'll be visiting. You will need to take into account:

- distances between places
- mode of transport and travelling speeds: by air, land or sea
- arrival and departure times
- international time zones
- climate/seasonal conditions (these also affect the popularity of destinations).

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Sample HSC problem

Lillian is travelling by plane from Cape Town, South Africa (34°S, 18°E) to Stockholm, Sweden (59°N, 18°E) for a holiday.

a If the radius of the Earth is 6400 km, calculate, correct to the nearest kilometre, the distance between Cape Town and Stockholm.

b If the plane flies at an average speed of 820 km/h, how long, in hours and minutes, would a direct flight between Cape Town and Stockholm take?

c Lillian can only book a flight that takes her from Cape Town to Stockholm via London. After boarding the plane at Cape Town at 9:45 a.m. on Tuesday, Lillian takes 12.5 hours to fly to London, then after a 2-hour break she takes another 2 hours and 25 minutes to fly to Stockholm. Calculate the local time and day Lillian arrives in Stockholm if the time zones for Stockholm and Cape town are UTC+1 and UTC+2 respectively.

Study tip

Ten common errors made in HSC exams

1 Careless setting out of answers or 'bald answers' with no working
2 Careless errors in basic algebra
3 Not choosing the simplest and quickest method
4 Not reading or answering the whole question
5 Poorly drawn diagrams that are messy, small and without labels or detail
6 Not having a ruler: for drawing, measuring and graph reading
7 Rounding too early in a calculation, so that the final answer is inaccurate
8 Not checking whether a numerical answer makes sense or sounds reasonable
9 Difficulties in explaining answers in words: writing mini-essays rather than 1–2 sentences that are to the point
10 Not realising that the parts of a question are related and require an answer or hint from a previous part

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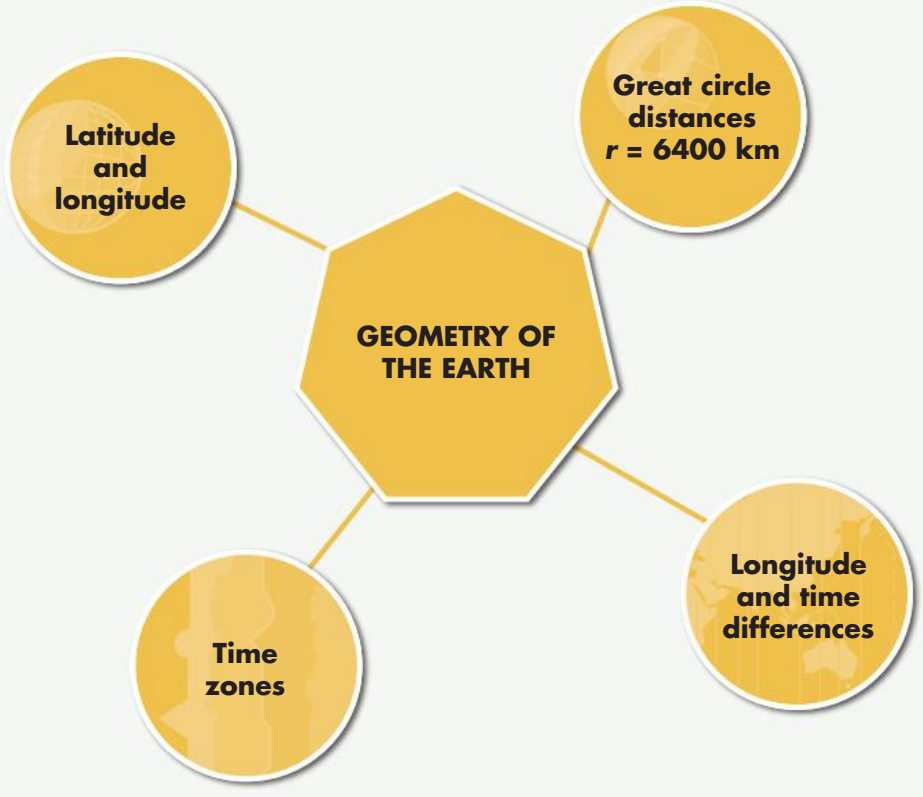
CHAPTER REVIEW

Topic overview

Crossword of the Earth

This short chapter, Geometry of the Earth, introduced entirely new concepts and skills related to the geography and measurement of the world. Make sure that all theory, rules, terminology and worked examples are included in your summary. This topic can be divided into three neat sections: latitude and longitude, calculating distances and calculating times. Students often experience difficulty in calculating differences in latitude, longitude and time, but such confusion can be overcome by achieving a deep and structured understanding of the topic.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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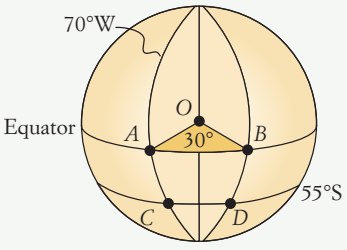
Revision

Exercise 9-01

1 What are the position coordinates of A, B, C and D on the Earth?



Exercise 9-01

2 Albury, NSW has coordinates (36°S, 147°E).

a What is the longitude of Albury?

b How many degrees is Albury north of the South Pole?

c Broome has coordinates (18°S, 122°E). What is the difference in longitude between Albury and Broome?

Exercise 9-01

3 Amsterdam, The Netherlands is 15° north and 122° west of Seoul, South Korea (37°N, 127°E). What are the coordinates of Amsterdam?

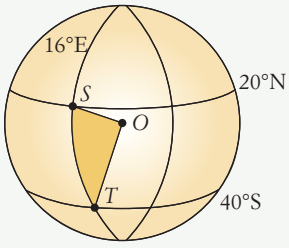
Exercise 9-02

4 Use the arc length formula to calculate the great circle distance, to the nearest kilometre, between Tokyo, Japan (35°N, 139°E) and Mt Isa (20°S, 139°E) (radius of the Earth is 6400 km).

Exercise 9-02

5 **a** What great circle runs through S and T?

b Calculate the shortest distance between S and T to the nearest kilometre.



Exercise 9-03

6 The time in Baghdad, Iraq (33°N, 44°E) is 7 hours ahead of the Falkland Islands.

a What line of longitude runs through the Falkland Islands?

b When it is 3:40 p.m. in the Falkland Islands, what is the time in Baghdad?

Exercise 9-03

7 **a** Calculate the difference in longitude between Naples, Italy (41°N, 14°E) and Bali, Indonesia (8°S, 115°E).

b Calculate the time difference between the two cities (ignoring time zones).

c What is the local time and day in Bali when it is 5:30 p.m. Thursday in Naples?

Exercise 9-04

8 The time in Anchorage, Alaska (USA) is 9 hours behind UTC while the time in Lusaka, Zambia is 2 hours ahead of UTC.

a When the time in Anchorage is 8:00 a.m., what is the time in London?

b When the time in Lusaka is 6:30 p.m., what is the time in Anchorage?

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9 What is the time in Brisbane (AEST) when it is:

a 4 p.m. in Perth? **b** 6 a.m. in Adelaide? **c** 2:30 a.m. in Hobart?

Exercise 9-04

10 A plane leaves Darwin on Thursday at 6:00 p.m. and travels east for 8 hours to reach Niue Island.

a When the plane crosses the International Date Line, what day does it become?

b If Darwin is UTC+9.5 and Niue Island is UTC−11, what is the time on Niue Island when the plane arrives there?

Exercise 9-04

11 Kerrie, living in Perth, wants to call her sister in Los Angeles, USA when the time there is 12 noon Sunday. At what time should she call if Perth is UTC+8 and Los Angeles is usually UTC−8 but the USA is on daylight saving time?

Exercise 9-04

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Practice paper 3

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each

Select the correct answer, A, B, C or D.

Exercise 8-04

Question 1

A 1 L solution contains 50 mL of salt. What is the ratio of salt to solution?

(A) 1 : 19 (B) 50 : 1 (C) 19 : 1 (D) 1 : 20

Exercise 9-01

Question 2

Port Moresby, Papua New Guinea has coordinates (9°S, 147°E). If Nyngan, NSW is due south of Port Moresby, which one of the following could be the coordinates of Nyngan?

(A) (4°S, 147°E) (B) (31°S, 147°E) (C) (9°S, 140°E) (D) (9°S, 151°E)

Exercise 7-08

Question 3

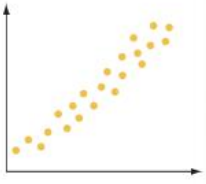
A coloured die has 2 red faces, 3 blue faces and 1 green face. If the die is rolled 100 times, how many times can the green face be expected to turn up?

(A) 10 (B) 17 (C) 33 (D) 60

Exercise 8-02

Question 4

Describe the correlation between the variables shown on the scatter plot.



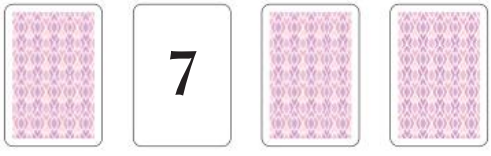
(A) weak negative correlation (B) strong positive correlation

(C) strong negative correlation (D) weak positive correlation

Exercise 7-01

Question 5

Four cards are marked with the numbers 7, 8, 9 and 10 and one card is turned over as shown.



What is the probability that the next card turned over is an even number?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

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Question 6

Exercise 9-03

Australian Eastern Standard Time (AEST) is 10 hours ahead of UTC. Calculate the local time in the United Kingdom when it is 6 p.m. in NSW and daylight saving is operating in the United Kingdom.

(A) 10 a.m. (B) 3 a.m. (C) 4 a.m. (D) 9 a.m.

Question 7

Exercise 7-04

There are 6 runners in a race. In how many different ways can the first 3 places be filled?

(A) 120 (B) 3 (C) 216 (D) 15

Question 8

Exercise 8-04

What percentage change is equivalent to an increase of 15% followed by a decrease of 15%?

(A) increase of 2.25% (B) decrease of 2.25%

(C) increase of 12.75% (D) decrease of 12.75%

Question 9

Exercise 9-02

Mt Isa, Queensland (20°S, 140°E) and Tokyo, Japan (35°N, 140°E) lie on the same meridian of longitude. Find the shortest distance between Mt Isa and Tokyo, given that the radius of the Earth is 6400 km. Give your answer correct to the nearest kilometre.

(A) 1676 km (B) 3072 km (C) 6144 km (D) 838 km

Question 10

Exercise 8-05

An antibiotic in a syrup has a concentration of 25 mg/mL. If Nathaniel needs 400 mg of the antibiotic, how many millilitres of the syrup should he take?

(A) 1.0 mL (B) 1.6 mL (C) 10 mL (D) 16 mL

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Section 2

3 questions: 10 marks each

Question 11 — Marks

Exercise 7-08

(a) Chris owns shares in Optel and his stockbroker advises him that there are 4 possible outcomes for his shares this week.

Outcome	Gain \$1120	Gain \$160	Lose \$480	Lose \$800
Probability	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{3}{20}$	$\frac{3}{40}$

Calculate Chris' financial expectation and state whether he can expect to gain or lose money. **2**

Exercise 7-06

(b) Jai, Marci and Robbie sit in a row for a photo. If every seating arrangement is equally likely, what is the probability that:

(i) Marci sits in the middle? **1**

(ii) Robert sits on one of the sides? **1**

(iii) Jai sits further left than Marci? **1**

Exercise 8-03

(c) Eight basketball players had their height and arm span measured. The results are shown below.

Height (x cm)	175	184	200	183	172	187	190	178
Arm span (y cm)	170	185	195	182	174	188	190	177

(i) Calculate, correct to 4 decimal places, the correlation coefficient, r, for this data. **1**

(ii) The gradient of the least-squares line of best fit for this data set can be found using the formula:

$$m = r \times \frac{\text{standard deviation of } y\text{-scores}}{\text{standard deviation of } x\text{-scores}}$$

Calculate this gradient, correct to 2 decimal places. Show your working. **2**

(iii) The y-intercept of the least-squares line of best fit for this data set can be found using the formula:

$$b = \text{mean of } y\text{-scores} - m \times \text{mean of } x\text{-scores}$$

Calculate this y-intercept, correct to 2 decimal places. Show your working. **2**

Question 12 — Marks

Exercise 9-02

(a) Norfolk Island (29°S, 168°E) is due north of Jackson Bay, New Zealand (43°S, 168°E).

(i) If the radius of the Earth is 6378 km, show that the distance between Norfolk Island and Jackson Bay is approximately 1558.44 km. **1**

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(ii) If a ship sails from Norfolk Island to Jackson Bay at an average speed of 55 km/h, how long will the journey take? Answer in days and hours, correct to the nearest hour. **1**

Exercise 8-06

(b) Amber is 11 years old and weighs 42 kg. She is prescribed a drug with a recommended adult dose of 350 mg three times a day. Use Clark's formula below to calculate her daily dosage.

$$\text{Child dosage} = \frac{\text{weight in kg}}{70} \times \text{adult dosage}$$ **2**

Exercise 9-03

(c) A plane flies east from Manila, the Philippines (14°N, 121°E) to Mexico City (19°N, 99°W). It flies for 19 hours, leaving Manila at 6:00 a.m. Tuesday.

(i) Which city is further south: Manila or Mexico City? **1**

(ii) After the plane crosses the International Date Line, what day will it be? **1**

(iii) If the Philippines is 8 hours ahead of UTC and Mexico City is 8 hours behind UTC, what is the time and day in Mexico City when the plane arrives? **2**

Exercise 7-06

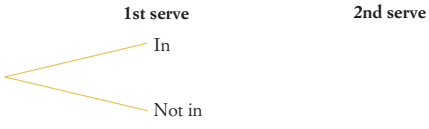
(d) At NCM bank, customers access their accounts using a 6-letter password code, for example, DKXREN. How many different passwords are possible? **2**

Question 13 — Marks

Exercise 7-07

(a) A tennis player only gets a second serve if the first serve is not in. The probability that Paige's first serve is in is 0.75 and the probability that her second serve is in is 0.92.

(i) Copy and complete this tree diagram. **2**



(ii) What is the probability that both of Paige's serves are not in? **1**

(iii) What is the probability that one of Paige's serves goes in? **1**

Exercise 8-08

(b) This table shows the age (A years) and life expectancy (E years) of a sample of 8 executives.

Age (A years)	20	25	30	20	45	35	25	30
Life expectancy (E years)	48	54	43	52	18	25	44	29

(i) Draw a scatter plot of this data. **2**

(ii) On your scatter plot, draw a line of fit by eye. **1**

(iii) Use your line of fit to estimate the age of an executive with a life expectancy of 20 years. **1**

(iv) Find the equation of your line of fit. **2**

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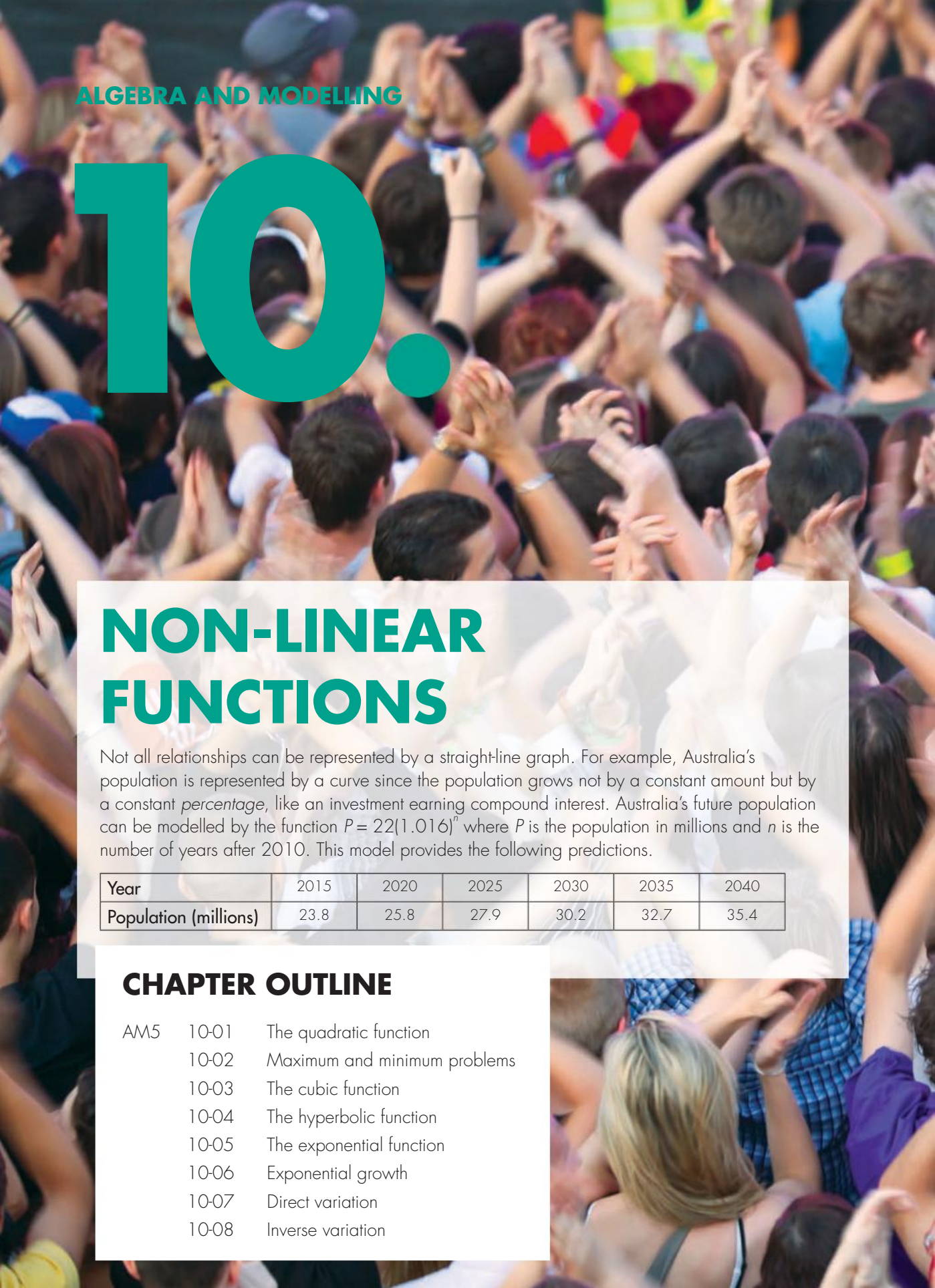


ALGEBRA AND MODELLING

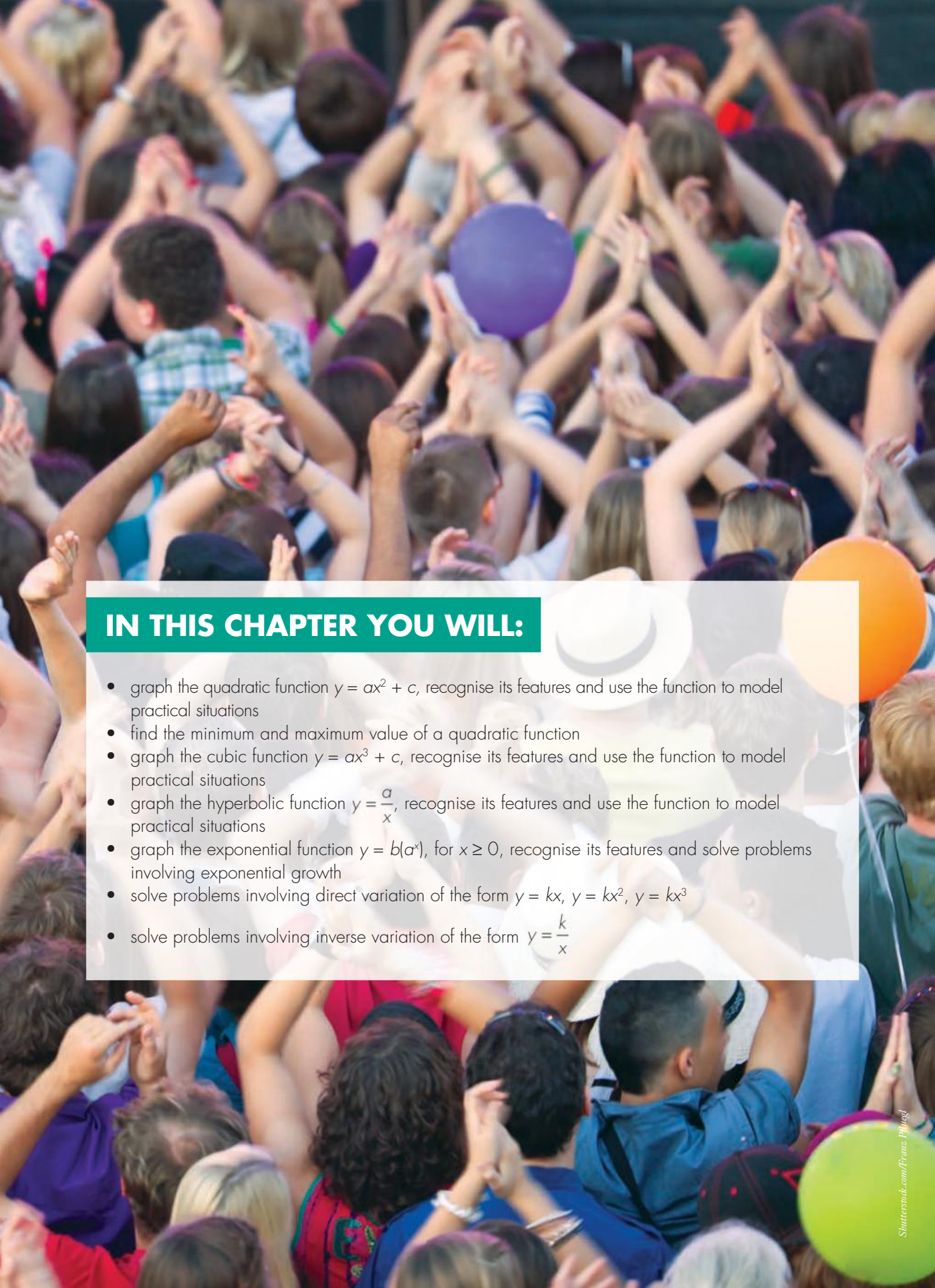
10.

NON-LINEAR FUNCTIONS

Not all relationships can be represented by a straight-line graph. For example, Australia's population is represented by a curve since the population grows not by a constant amount but by a constant *percentage*, like an investment earning compound interest. Australia's future population can be modelled by the function $P = 22(1.016)^n$ where P is the population in millions and n is the number of years after 2010. This model provides the following predictions.

Year	2015	2020	2025	2030	2035	2040
Population (millions)	23.8	25.8	27.9	30.2	32.7	35.4

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- graph the quadratic function $y = ax^2 + c$, recognise its features and use the function to model practical situations
- find the minimum and maximum value of a quadratic function
- graph the cubic function $y = ax^3 + c$, recognise its features and use the function to model practical situations
- graph the hyperbolic function $y = \frac{a}{x}$, recognise its features and use the function to model practical situations
- graph the exponential function $y = b(a^x)$, for $x \geq 0$, recognise its features and solve problems involving exponential growth
- solve problems involving direct variation of the form $y = kx$, $y = kx^2$, $y = kx^3$
- solve problems involving inverse variation of the form $y = \frac{k}{x}$

TERMINOLOGY

asymptote
cubic function
exponential growth
increasing
maximum
non-linear function
quadratic function
vertex
concave up/down
decreasing
hyperbola
initial value
minimum
parabola
varies as
vertical intercept
constant of variation
exponential function
hyperbolic function
limitation
model
proportional to
varies inversely as

Assignment 10

Tables of values

SkillCheck

1 Evaluate, correct to four decimal places:

a 1.4^6 **b** 5^{-4} **c** 6000×1.017^5

2 If $y = kx^2$, find k if $x = 4$ and $y = 10$.

3 If $p = \frac{k}{t}$, find k if $t = 16$ and $p = 32$.

4 If $x = 3$, evaluate:

a $2x^2 + 5$ **b** $4x^3$ **c** 5^x

d $-x^2 + 9$ **e** $\frac{-6}{x}$ **f** $3 \times (0.8)^x$

5 **a** Copy and complete this table for $y = x^2 + 2$.

x	-2	-1	0	1	2
y					

b Hence graph $y = x^2 + 2$ on a number plane.

6 The maximum distance, d metres, a ball covers if thrown at velocity v m/s is given by the formula $d = \frac{v^2}{9.8}$. Find:

a the maximum distance covered, correct to one decimal place, if the ball is thrown at 25 m/s

b the velocity at which a ball is thrown if it covers a maximum distance of 20 m.

Graphing quadratics

Graphing curves: Graphics calculator

Features of a parabola

10-01 The quadratic function

Linear functions are of the form $y = mx + b$ and their graphs are straight lines whereas **non-linear functions** take many different forms and their graphs are curves.

A **quadratic function** is a non-linear function where the highest power of the independent variable is 2. In this section, we will look at quadratic equations of the form $y = ax^2 + c$ where a and c are constants.

Here are some examples: $y = 5x^2$, $y = 2x^2 - 5$, $d = -\frac{1}{2}t^2 + 1$, $C = 7 - v^2$

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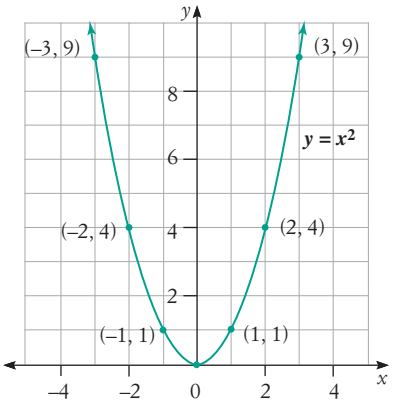
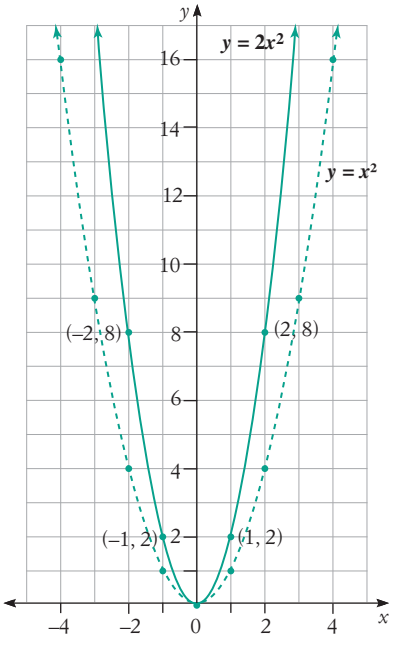
The shape of a parabola

The graph of a quadratic function is called a **parabola**. The simplest quadratic function is $y = x^2$ and its graph is drawn here using the table of values. The graph is symmetrical since $(-x)^2 = x^2$ for all values of x.

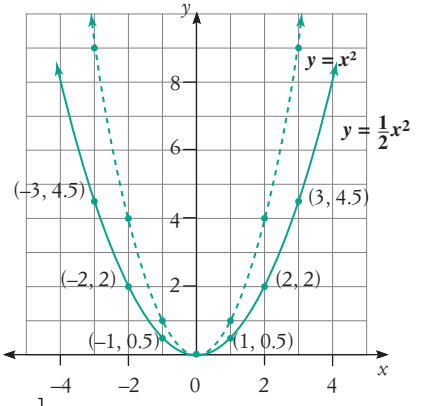
x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



Compare the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ below with the graph of $y = x^2$.



$y = 2x^2$ is narrower and steeper than $y = x^2$ as the y values are twice as large. It lies 'inside' the graph of $y = x^2$.



$y = \frac{1}{2}x^2$ is wider and flatter than $y = x^2$ as the y values are half as large. It lies 'outside' the graph of $y = x^2$.

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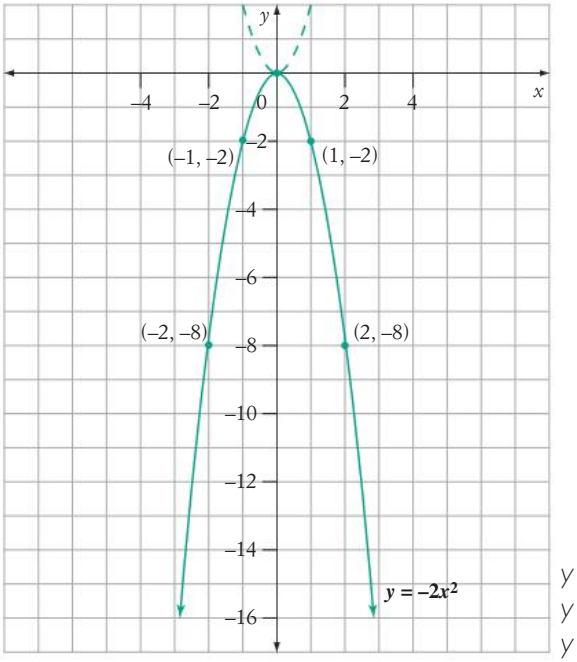
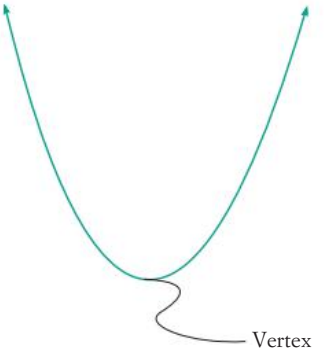


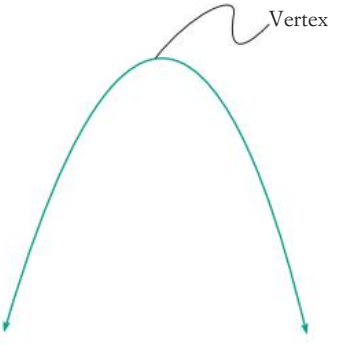
$y = -2x^2$ is exactly the same shape as $y = 2x^2$ but it is 'upside-down' as the y values are negative.

SUMMARY

- The graph of $y = ax^2 + c$ is a parabola with the y-axis as the axis of symmetry
- If a is positive, the parabola is **concave up** (shaped like a bowl)
- If a is negative, the parabola is **concave down** (like an upside-down bowl)
- The turning point of a parabola is called the **vertex**.



Concave up (a is positive)



Concave down (a is negative)

Positive = 'smiling' parabola
Negative = 'frowning' parabola

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EXAMPLE 1

a Draw the graph of $y = 2x^2 + 3$

b Write the coordinates of the vertex.

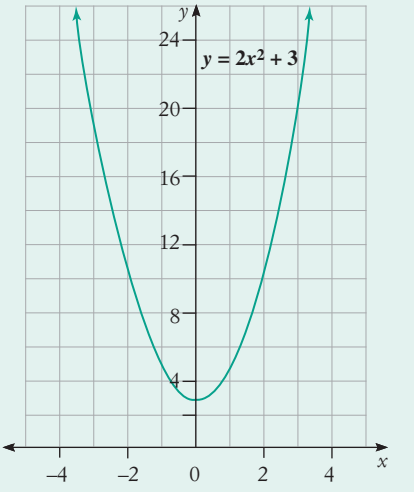
Solution

a $a = 2$ is positive so the parabola is concave up.

First complete a table of values.

x	−3	−2	−1	0	1	2	3
y	21	11	5	3	5	11	21

Then graph the table of values.



b The vertex is (0, 3).

SUMMARY

To graph $y = ax^2 + c$:

- determine the shape of the parabola by examining the size and sign of a
- complete a table of values
- find the vertex.

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Exercise 10-01 The quadratic function

1 Match each quadratic function with its graph below.

a $y = 2x^2 - 2$

b $y = -x^2 + 1$

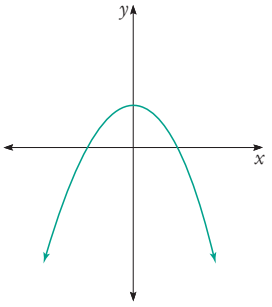
c $y = -7x^2$

d $y = x^2 - 1$

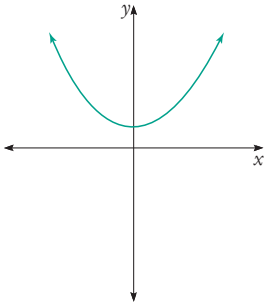
e $y = \frac{1}{2}x^2 + 1$

f $y = 2x^2$

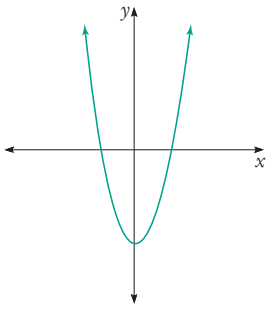
A



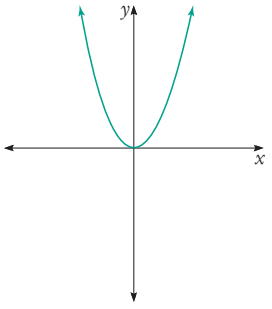
B



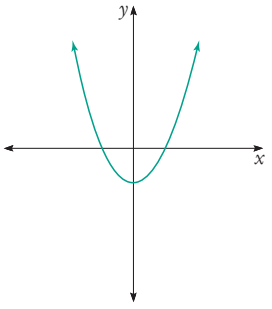
C



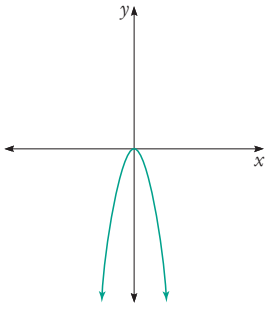
D



E



F



Example 1

2 Graph each quadratic function.

a $y = 2x^2 + 1$

b $y = \frac{1}{2}x^2$

c $y = -4x^2 + 4$

d $y = x^2 - 3$

e $P = n^2 + 8$

f $w = -2r^2 - 5$

3 A ball is thrown up in the air from the balcony of Ryan's apartment. Its height above ground level after t seconds is given by the quadratic function $h = -5t^2 + 20t + 25$, where h is the height in metres.

a What is the independent variable of this function?

b Graph $h = -5t^2 + 20t + 25$ for values of t from 0 to 6.

c What is the vertical intercept of this graph and what does it represent?

d Find the maximum height of the ball and the time when this occurs.

e When does the ball hit the ground?

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4 **a** Draw the graph of $y = x^2$ for x-values from -3 to $+3$.

b Construct a table of values for each of the following 4 quadratic functions.

$y = 3x^2$ $\quad y = -3x^2$ $\quad y = x^2 + 3$ $\quad y = 3x^2 + 3$

c Graph each quadratic function and say how each compares to the graph of $y = x^2$.

TECHNOLOGY: GRAPHING FUNCTIONS ON A GRAPHICS CALCULATOR

Graphing curves: graphics calculator

Use a graphics calculator, graphing software or a spreadsheet, graph the different functions studied in this chapter.

1 On a Casio calculator, press MENU and select **GRAPH**. On a TI calculator, press STAT PLOT and select **PlotsOff**, ENTER, Y= to turn off any statistical graphs.

2 Enter the function to be graphed at Y1 by pressing x, θ, T or x, T, θ, n, then EXE or ENTER.

3 To select the scale on a Casio, press V-Window and select the **STD** (standard) scale of -10 to $+10$ on both x- and y-axes, then EXIT. On a TI, press WINDOW, ZOOM and select **ZStandard**.

4 To graph the function, select **DRAW** on a Casio or GRAPH on a TI.

5 To zoom in or out of the graph, press ZOOM, select **Zoom IN** or **Zoom OUT**, position the cursor where you want to zoom and press EXE or ENTER.

6 Press TRACE and the left/right arrow keys to display the coordinates of points on the graph. Note that the exact coordinates of some points may not be shown due to the scale of the screen window.

For more detailed instructions on graphing functions on a graphics calculator, download the worksheet 'Graphing curves: graphics calculator'.

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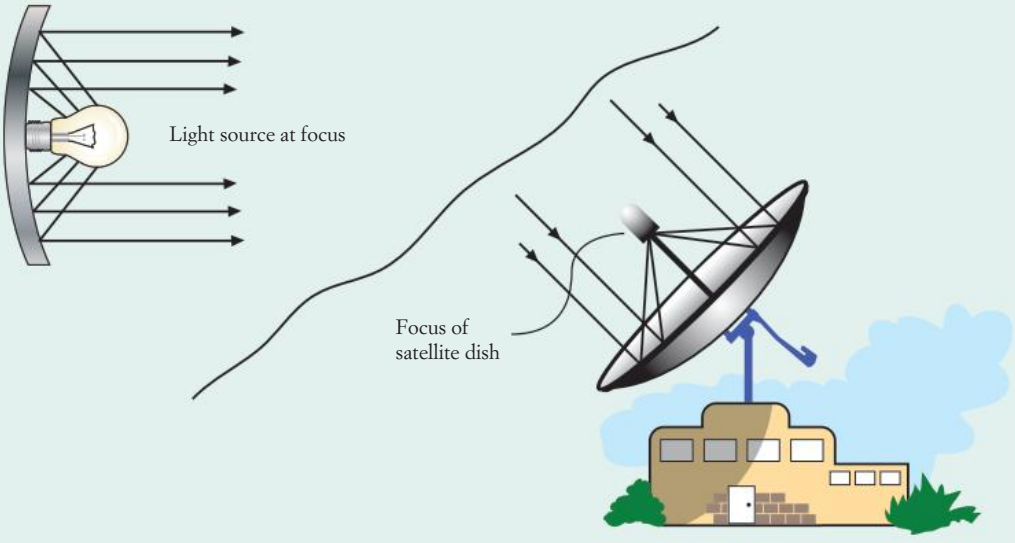
JUST FOR THE RECORD

The parabola in headlights and satellite dishes

The parabola is used in applications involving the reflection of light, sound and signals. For example, the shape of a car headlight or a satellite dish is a parabola.

In a parabolic car headlight, the light source is positioned at the **focus** of the parabola. Light from the focus bounces off the reflective surface and moves parallel to its axis of symmetry.



The reverse happens with a satellite dish. Signals arriving from space reflect off the dish and meet at the focus, where the signals are read.

Before every Olympic Games torch relay, the torch is lit by the rays of the Sun caught by a parabolic mirror at the Temple of Hera.

Where is the Temple of Hera?

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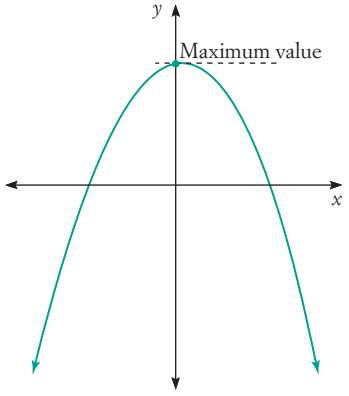
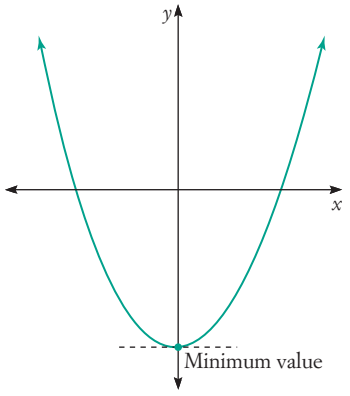
10-02 Maximum and minimum problems

The vertex of a parabola with an equation of the form $y = ax^2 + c$ is on the y-axis (so its x-coordinate is 0).

The maximum or minimum value of the quadratic function $y = ax^2 + c$ is the y-value of the vertex of the parabola.





EXAMPLE 2

For each quadratic function:

i sketch the parabola without plotting points, and mark the coordinates of the vertex

ii state whether the function has a maximum or minimum value and state this value.

a $y = -4x^2 + 5$ **b** $y = 6x^2 - 8$ **c** $y = 3x^2 + 4$

Solution

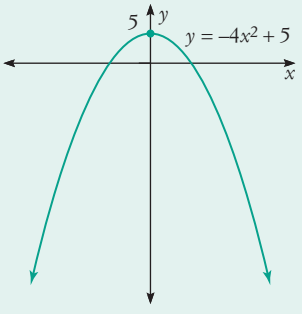
a **i** The parabola $y = -4x^2 + 5$ is concave down ($a = -4 < 0$)

Substitute $x = 0$ to find the vertex.

$y = -4(0^2) + 5$

$= 5$

The vertex is (0, 5).



ii $y = -4x^2 + 5$ has a maximum value of 5.

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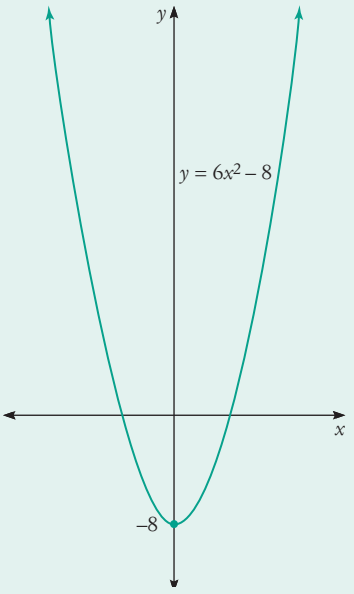
b **i** The parabola $y = 6x^2 - 8$ is concave up ($a = 6 > 0$)

$y = 6(0^2) - 8$

$= -8$

The vertex is $(0, -8)$.

ii $y = 6x^2 - 8$ has a minimum value of -8.



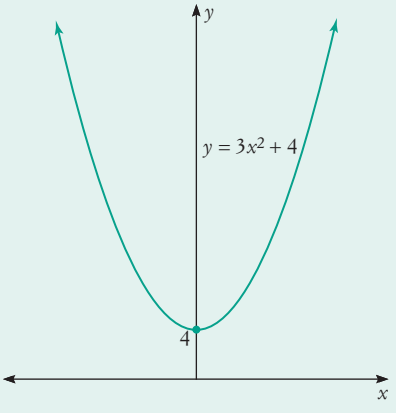
c **i** The parabola $y = 3x^2 + 4$ is concave up. **($a = 3 > 0$)**

$y = 3(0^2) + 4$

$= 4$

The vertex is $(0, 4)$.

ii $y = 3x^2 + 4$ has a minimum value of 4.



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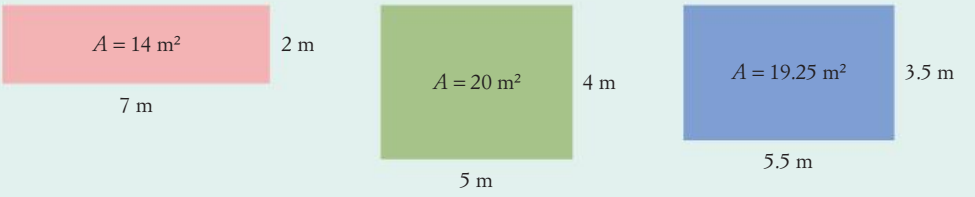
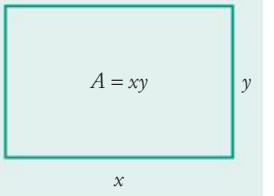
EXAMPLE 3

Farmer Brad has 18 m of fencing and wants to build a rectangular pig pen with the largest area possible. He sketches some possible pens using his 18 m of fencing.



Let x and y be the length and width of the pen, in metres.



a Explain why $2x + 2y = 18$.

b Hence show that $y = 9 - x$.

c Hence show that the area of the pen, A m^2, is given by the function $A = -x^2 + 9x$.

d Find the largest possible area of the pen.

e What are the dimensions of the pen that give this maximum area?

Solution

a Perimeter of pen $= x + y + x + y = 18$

$2x + 2y = 18$

b $2y = 18 - 2x$

$$y = \frac{18}{2} - \frac{2x}{2}$$

$$= 9 - x$$

To make y the subject divide each term by 2.

c $A = xy$

$= x(9 - x)$

$= 9x - x^2$

$= -x^2 + 9x$

Area = length × width.

Substitute $y = 9 - x$.

d Complete a table of values.

x	3	4	5	6	7
A	18	20	20	18	14

x is positive as it refers to a length.

The maximum value of A must occur at $x = 4.5$

Halfway between 4 and 5.

When $x = 4.5$, $A = -(4.5^2) + 9(4.5) = 20.25$

The largest possible area is 20.25 m^2.

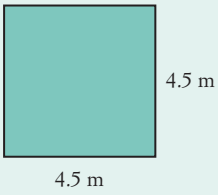
e When $x = 4.5$ m,

$y = 9 - x$

$= 9 - 4.5$

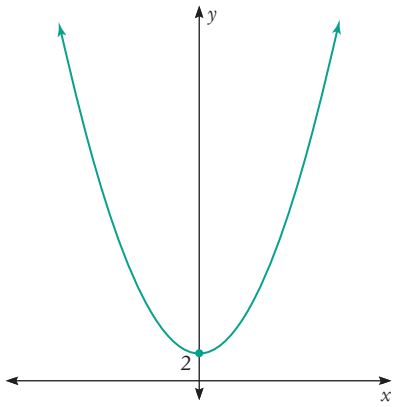
$= 4.5$

The pen with maximum area is a square, 4.5 m by 4.5 m.



Exercise 10-02 Maximum and minimum problems

1 What could be the equation of this parabola? Select **A**, **B**, **C** or **D**.



A $y = 2x^2$

B $y = x^2 + 2$

C $y = 2x^2 - 2$

D $y = -2x^2$

Example 2

2 **a** Draw a neat sketch of each parabola without plotting points, marking the coordinates of the vertex.

i $y = 4x^2 + 3$ **ii** $y = -2x^2 + 6$ **iii** $y = x^2 - 1$ **iv** $y = -3x^2 - 2$

b Say whether the quadratic expression has a maximum or minimum value and state this value.

3 A golf ball is hit into the air and its height, h metres, above the ground after time, t seconds, is given by the quadratic function $h = -5t^2 + 24t$.

a Find the height of the ball after 2 seconds.

b Show that the height of the ball after 1.8 and 3 seconds is the same. Why is this so?

c After how many seconds does the ball reach its maximum height? How did you find this value?

d Find the maximum height reached.

e Why is this quadratic model not useful for values of t greater than 5?

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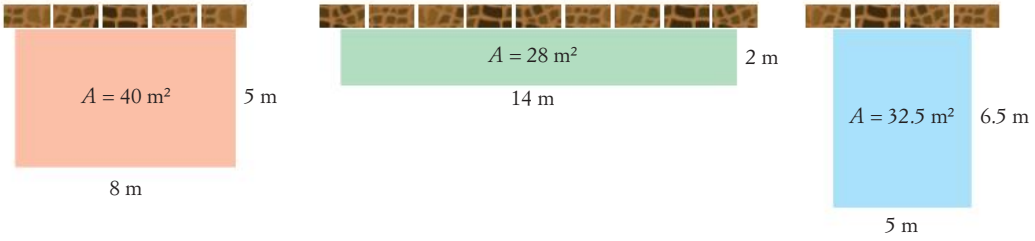
4 Madison has 18 m of wire fencing to border a new baby lamb enclosure. She wants to use an existing brick wall as one side and sketches three possible enclosures.

Example 3



a If x m and y m are the length and width of the enclosure, explain why $x + 2y = 18$.

b Hence show that $y = 9 - \frac{1}{2}x$.

c Hence show that the area, A m², of the enclosure is $A = -\frac{1}{2}x^2 + 9x$

d Find the maximum area of the enclosure and the dimensions that give this area.

5 At an air show, the height (in metres) of a plane in a power dive is given by $h = t^2 - 10t + 80$, where t is the time in seconds.

a Use a table of values to graph the function $h = t^2 - 10t + 80$ for values of t from 0 to 10.

b What are the coordinates of the vertex ?

c What is the minimum value of h and what does this represent?

d When is the lowest height of the plane reached?

e What is the height of the plane at the start of the power dive?

6 A sky rocket is sent up into the air at a fireworks display on New Year's Eve. It reaches a maximum height, fizzles out and the leftover pieces fall back down to Earth. The quadratic function $d = 4.9t^2$ gives the distance, d metres, that a piece falls in t seconds.

a What is the independent variable of this function?

b Copy and complete this table of values.

t	0	1	2	3	4	5	6	7	8	9	10
d											

c Graph this quadratic function.

d How far does a piece fall in:

i 3 seconds? **ii** 30 seconds?

e Looking at your answers to part **d**, why is this function is not valid for all values of t?

f Find, correct to the nearest 0.1 second, how long it takes for a piece to fall:

i 10 m **ii** 50 m

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7 The temperature, T°C, at a town n hours after 6:00 a.m., is given by the function $T = -\frac{1}{6}n^2 + 2n + 14$.

a Graph this function for values of n from 0 to 15.

b What was the temperature:

i at 6 a.m.? **ii** at 2 p.m.?

c What was the maximum temperature and when was it reached?

d Estimate to the nearest half-hour the times when the temperature reached 19°C. Why are there two answers?

e For how many hours (to the nearest whole number) was the temperature 19°C or higher?

f This quadratic model is only accurate for values of n from 0 to 13. Why do you think the model does not hold for values of n above 13?

INVESTIGATION

QUADRATIC HANDSHAKES

1 How many handshakes take place if three people meet and each person shakes hands with every other person?

2 How many handshakes take place if five people meet?

3 The formula for the number of handshakes when n people meet is $H = \frac{1}{2}n(n-1)$.

4 Use the formula to calculate the number of handshakes that take place when:

a 12 people meet **b** 22 people meet.

5 Use a guess-and-check method to find how many people meet if 666 handshakes take place.

Graphing cubics

10-03 The cubic function

A **cubic function** is a non-linear function where the highest power of the variable is 3. Its graph is called a **cubic curve**. In this section, we will look at cubic equations of the form $y = ax^3 + c$ where a and c are constants.

Here are some examples: $y = x^3$, $y = -x^3 + 1$, $d = \frac{1}{2}x^3$, $C = 4 - v^3$.

The shape of a cubic curve

Here is a table of values for $y = x^3$.

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

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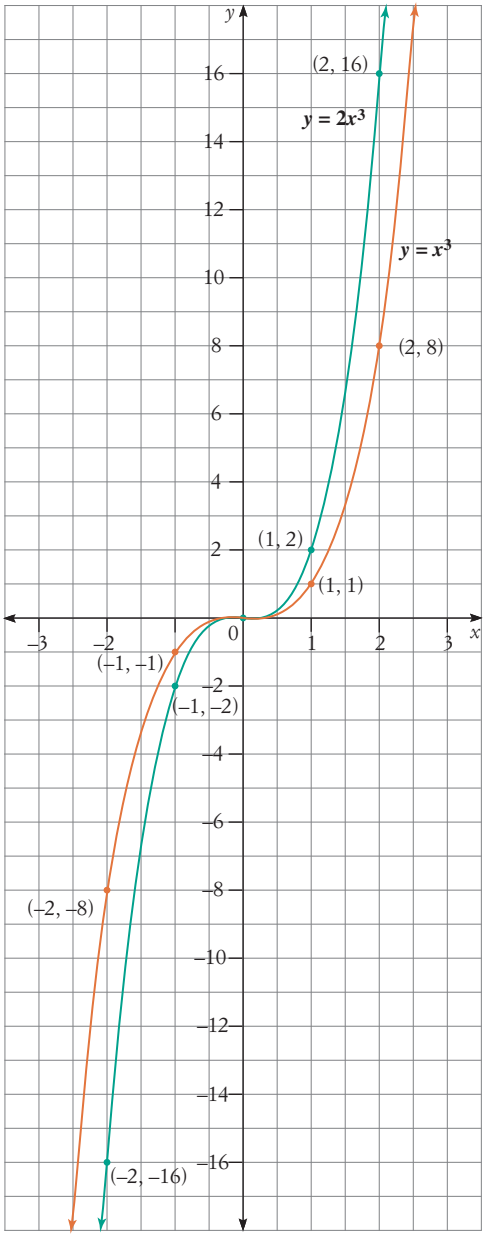
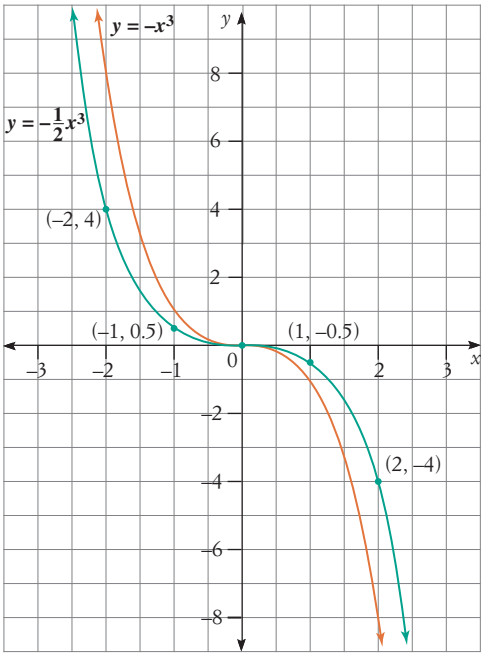
The graphs of $y = x^3$ and $y = 2x^3$ are shown below.



$y = 2x^3$ lies 'inside' and 'is steeper than' $y = x^3$.

The graphs of $y = -x^3$ and $y = -\frac{1}{2}x^3$ are shown below.



$y = -x^3$ is the mirror image of $y = x^3$.

$y = -\frac{1}{2}x^3$ lies 'outside' and 'is flatter than' $y = -x^3$.

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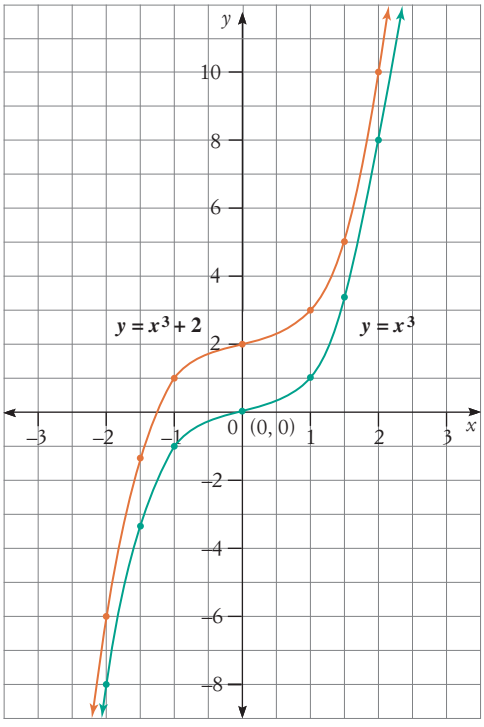
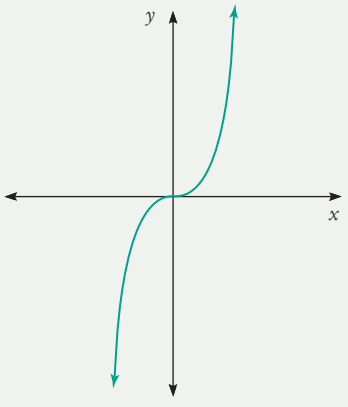
The graph of $y = x^3 + 2$ is shown below.



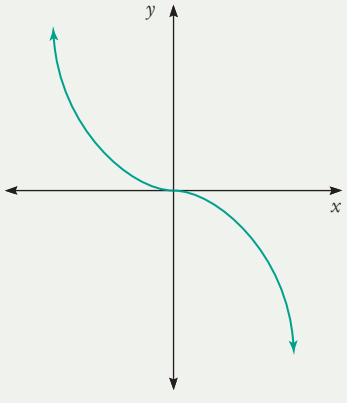
$y = x^3 + 2$ is exactly the same shape as $y = x^3$ but moved up 2 units.

SUMMARY

The cubic curve $y = ax^3$ has **point symmetry** around (0, 0): if you spin it upside down, it will fit onto itself.



If a is positive, the curve is **increasing**.



If a is negative, the curve is **decreasing**.

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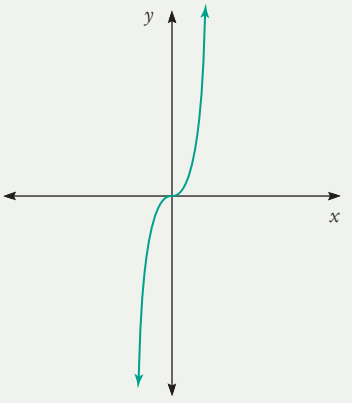
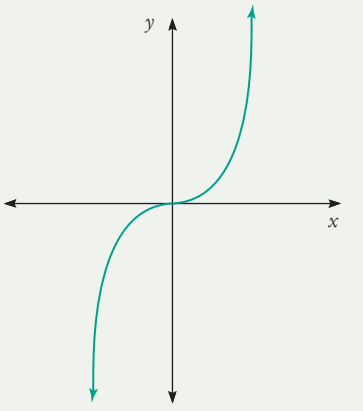


If a is large, the curve is **steeper**.



If a is small, the curve is **flatter**.

The cubic curve $y = ax^3 + c$ is the same shape as $y = ax^3$ but it is shifted c units up or down. It has **point symmetry** around $(0, c)$.

EXAMPLE 4

The volume of a sphere, V m^3, can be approximated by the cubic function $V = 4.19r^3$, where r m is the radius of the sphere.

a Graph this cubic function for values of r from 0 to 5.

b Use the graph to estimate the volume of a sphere with a radius of 4.6 m.

c Estimate the radius of the sphere with a volume of 140 m^3:

i from the graph **ii** from the equation

Solution

a Values of V, rounded to two decimal places.

r (m)	0	1	2	3	4	5
V (m^3)	0	4.19	33.52	113.13	268.16	523.75

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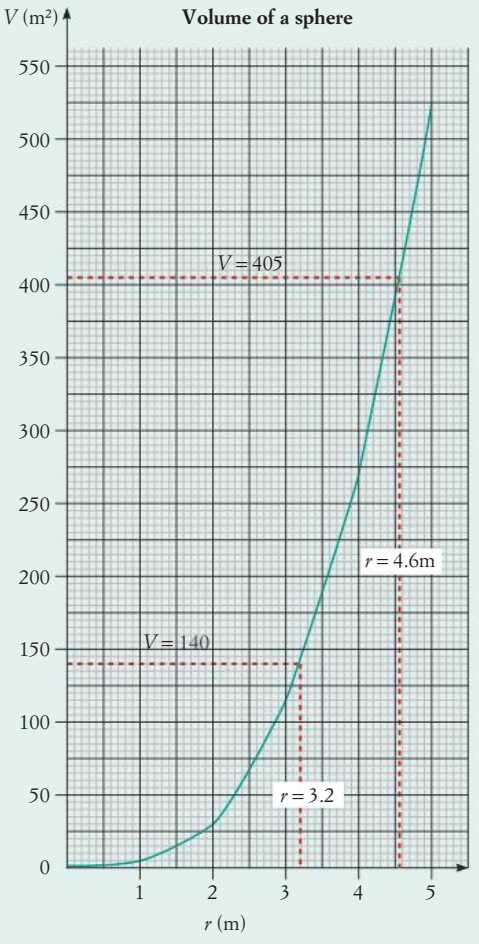
b When $r = 4.6$, $V \approx 405$ m^3

c **i** When $V = 140$, $r \approx 3.2$ m

ii When $V = 140$

$$140 = 4.19r^3$$
$$\frac{140}{4.19} = r^3$$
$$r^3 = 33.41288\ldots$$
$$r = \sqrt[3]{33.41288\ldots}$$
$$= 3.2208\ldots$$
$$\approx 3.2 \text{ m}$$



Exercise 10-03 The cubic function

1 Which equation represents an increasing cubic function? Select **A**, **B**, **C** or **D**.

A $y = \frac{1}{3}x^3$ **B** $y = -x^3 + 3$ **C** $y = -x^3 - 3$ **D** $y = -\frac{1}{3}x^3$

2 Which cubic curve is the steepest? Select **A**, **B**, **C** or **D**.

A $y = x^3$ **B** $y = \frac{1}{3}x^3$ **C** $y = 3x^3$ **D** $y = \frac{1}{2}x^3$

3 Which cubic curve passes through (0, 2)? Select **A**, **B**, **C** or **D**.

A $y = 2x^3$ **B** $y = -2x^3$ **C** $y = x^3 - 2$ **D** $y = x^3 + 2$

4 **a** What is the formula for the volume of a sphere?

b In Example **4** on page 433, is $V = 4.19r^3$ a good approximation for the volume of a sphere? Why?

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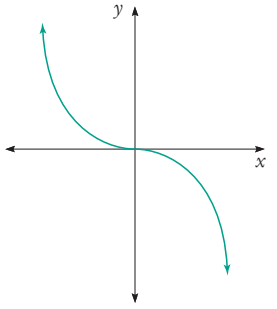
5 Match each cubic function with its graph below.

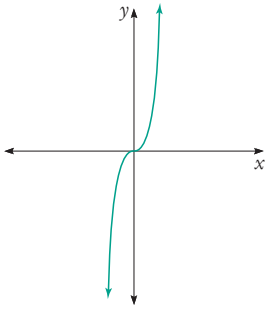
a $y = -x^3$ **b** $y = \frac{1}{4}x^3$ **c** $y = 4x^3$

d $y = x^3 - 1$ **e** $y = -\frac{1}{2}x^3$ **f** $y = x^3 + 1$

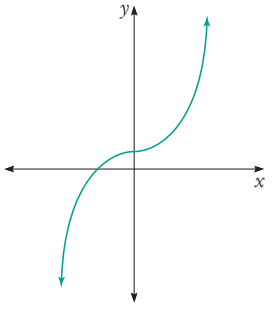
A



B



C



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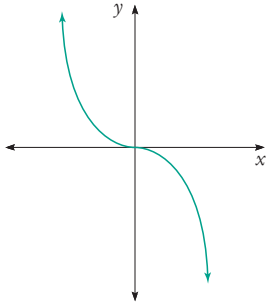

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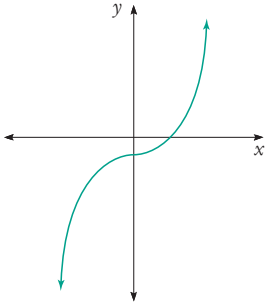

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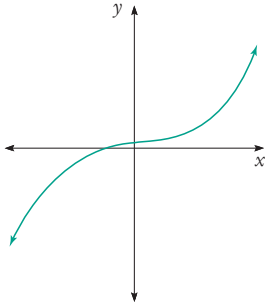

6 On the same number plane, graph $y = \frac{1}{3}x^3$ and $y = -3x^3$.

7 Cork is a fairly light material with a cubic centimetre weighing only 0.25 g. The mass, M g, of a cube of cork of length s cm is $M = \frac{1}{4}s^3$.

Example 4

a Copy and complete this table of values.

Length of cube, s cm	5	10	30	50	60	80
Mass of cube, M g						

b Calculate the mass in kilograms of a cork cube of length 45 cm. Select **A**, **B**, **C** or **D**.

A 0.011 25 kg **B** 11.25 kg **C** 22.781 25 kg **D** 22 781.25 kg

c Convert 93.312 kilograms to grams.

d Hence find the length of a cork cube that has a mass of 93.312 kg.

e Calculate the mass of a cubic metre of cork, in kilograms.

f Is it possible for a person to lift a cubic metre of cork? Justify your answer.

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8 The mass, M kg, of a child of height h cm, can be modelled by the function $M = \frac{1}{80\,000}h^3$.

a Graph this cubic function for values of h from 0 to 160.

b Use the graph to estimate the mass of a child of height:

i 90 cm **ii** 130 cm

c Use the function to calculate, to the nearest kilogram, the mass of a child of height:

i 90 cm **ii** 130 cm

d Use the graph to estimate the height of a child of mass 45 kg.

e Use the function to calculate, to the nearest metre, the height of a child of mass 45 kg.

f What are the limitations of this model? Why isn't it good for predicting the masses of babies or adults?

10-04 The hyperbolic function

A **hyperbolic function** is a non-linear function in which x is in the denominator.

The general form of a hyperbolic function is $y = \frac{k}{x}$. The graph of a hyperbolic function is called a **hyperbola** and it has two separate branches.

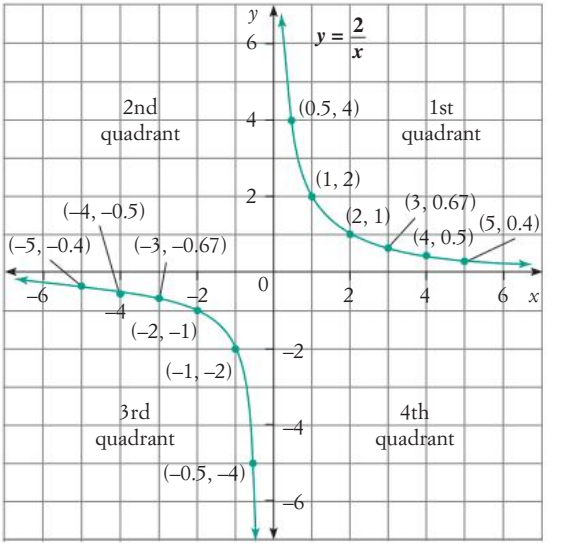
The shape of a hyperbola

A table of values and graph of $y = \frac{2}{x}$ are shown. y values are rounded to two decimal places.

x	−5	−4	−3	−2	−1	−0.5	−0.2	0	0.2	0.5	1	2	3	4	5
y	−0.4	−0.5	−0.67	−1	−2	−4	−10	—	10	4	2	1	0.67	0.5	0.4

You cannot divide any number by 0.

When $x = 0$, y has no value because $\frac{2}{0}$ is not defined. The two branches of the hyperbola get very close to the x- and y-axes but never touch them. This graph is in the 1st and 3rd quadrants.



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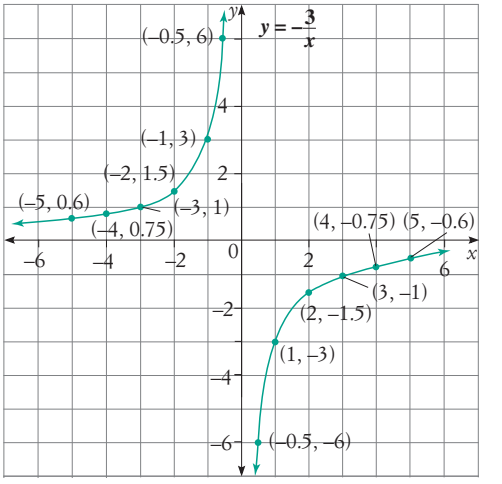
A table of values and graph of $y = -\frac{3}{x}$ are shown. The graph is in the 2nd and 4th quadrants.

x	−5	−4	−3	−2	−1	−0.5	−0.2	0	0.2	0.5	1	2	3	4	5
y	0.6	0.75	1	1.5	3	6	15	—	−15	−6	−3	−1.5	−1	−0.75	−0.6

This hyperbola is further away from the x- and y-axes than the graph of $y = \frac{2}{x}$ on the previous page.



SUMMARY

- The hyperbola $y = \frac{a}{x}$ has two separate branches in opposite quadrants of the number plane.
- It has **point symmetry** about (0, 0).
- The x- and y-axes are both **asymptotes**: the curve approaches them but never touches them.
- If a is positive, the hyperbola is **decreasing** and its branches are in the 1st and 3rd quadrants.
- If a is negative, the hyperbola is **increasing** and its branches are in the 2nd and 4th quadrants.
- The higher the value of a, the further the hyperbola is from the x- and y-axes.

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EXAMPLE 5

The time, T months, taken to pay off an interest-free loan is related to the size of each monthly repayment, $\$r$, by the hyperbolic function $T = \frac{1800}{r}$.

a Graph the function for values of r from 100 to 900.

b Use the graph to estimate the time to pay off the loan if the monthly repayment is \$250.

c Use the formula to find the monthly repayment required to pay off the loan in 5 months.

d Why is this model not useful for values of r greater than 1800?

Solution

a T-values are rounded to two decimal places.

r	100	200	300	400	500	600	700	800	900
T (months)	18	9	6	4.5	3.6	3	2.57	2.25	2

b From the graph, when $r = 250$, $T = 7$

Time taken to pay off the loan is 7 months.

c From the formula, when $T = 5$:

$$5 = \frac{1800}{r}$$

$$r = \frac{1800}{5}$$

$$= 360$$

The monthly repayment required is \$360.

d If $r > 1800$, $T < 1$. For example, if $r = 2000$, $T = 0.9$ months. It is not possible to pay off a loan in less than one month using monthly repayments.

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Exercise 10-04 The hyperbolic function

1 Which equation represents a decreasing hyperbolic function? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = \frac{2}{x}$ **C** $y = -\frac{x}{2}$ **D** $y = -\frac{2}{x}$

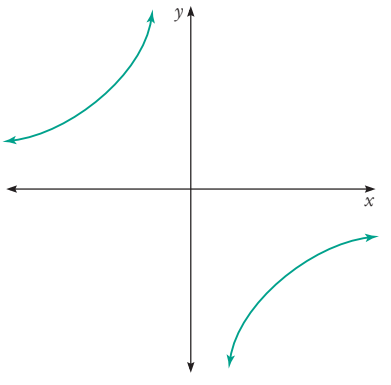
2 Which hyperbolic function's graph lies in the 2nd and 4th quadrants? Select **A**, **B**, **C** or **D**.

A $y = \frac{4}{x}$ **B** $y = \frac{1}{4x}$ **C** $y = -\frac{5}{x}$ **D** $y = \frac{3}{x}$

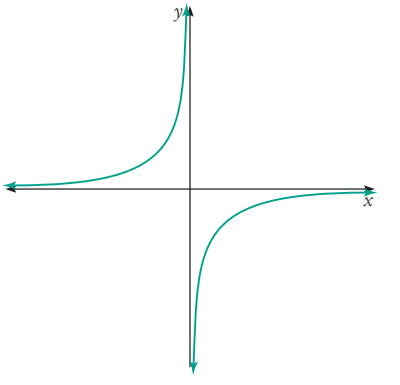
3 Match each hyperbolic function with its graph below.

a $y = \frac{3}{x}$ **b** $y = -\frac{1}{x}$ **c** $y = -\frac{4}{x}$ **d** $y = \frac{5}{x}$

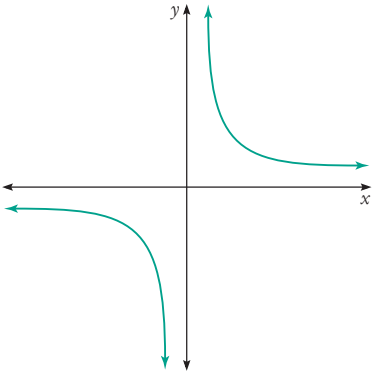
A



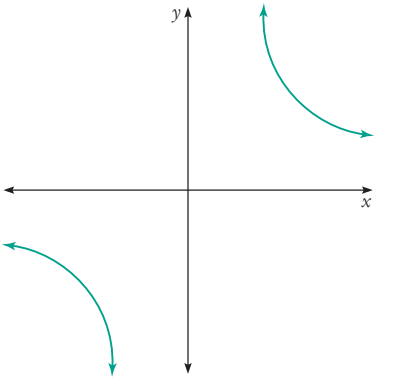
B



C



D



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Example 5

4 The time, T minutes, taken for Trish to drive to Nowra from her home depends on her average speed, s km/h, according to the function $T = \frac{4800}{s}$.

a What is the dependent variable?

b Graph T for values of s from 40 to 110.

c Calculate the time in minutes taken to drive to Nowra at a speed of 92 km/h:

i using the graph **ii** using the function.

d Use the graph to find the speed (to the nearest km/h) at which Trish must drive to reach Nowra in 70 minutes.

e How far is Nowra from Trish's house?

5 Nilshan, standing at the centre of a football field, notices that the length of his shadow decreases as he moves further away from the centre, according to the formula $h = \frac{56\,000}{d}$ where h cm is the length of his shadow and d cm is his distance from the centre.

a Copy and complete this table of values, rounding h to the nearest whole number if needed.

d (cm)	50	100	150	200	250	300	350	400
h (cm)								

b What is the length of Nilshan's shadow when he is 220 cm from the centre?

c How far is Nilshan standing from the centre if his shadow is 860 cm long?

Express all answers correct to the nearest centimetre.

iStockphoto/Susan Ridley

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10-05 The exponential function

An **exponential function** is a non-linear function where x is the power (or exponent). The general form of the exponential function is $y = b(a^x)$, where a and b are constants, and $a > 0$. The graph of an exponential function is called an **exponential curve**.

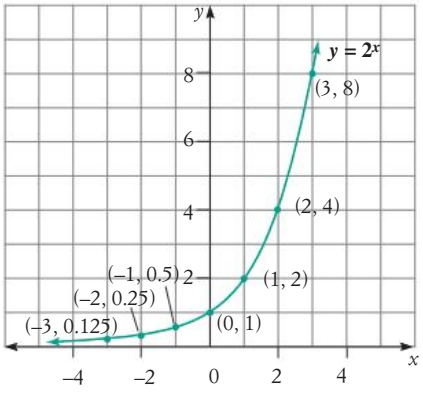
Graphing exponentials

Matching graphs

The shape of an exponential curve

A table of values and graph of $y = 2^x$ are shown.

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{8} = 0.125$	$\frac{1}{4} = 0.25$	$\frac{1}{2} = 0.5$	1	2	4	8

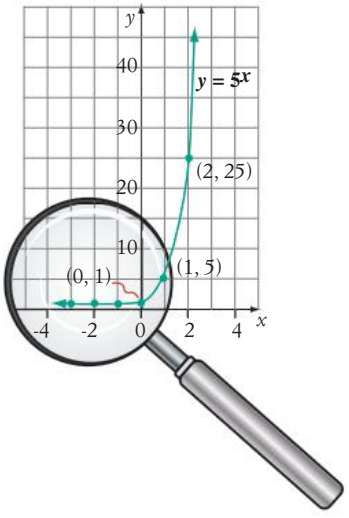


Note: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

The graph cuts the y-axis at (0, 1). As the x-values increase, the y-values are **multiplied by 2** (since $y = \mathbf{2}^x$) and the curve is increasing and becoming steeper. As the x-values decrease, the curve gets closer to the x-axis but never touches it.

A table of values and the graph of $y = 5^x$ are shown.

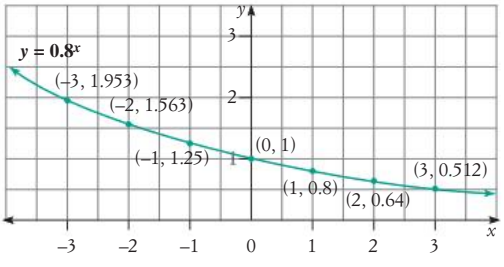
x	–3	–2	–1	0	1	2	3
y	$\frac{1}{125} = 0.008$	$\frac{1}{25} = 0.04$	$\frac{1}{5} = 0.2$	1	5	25	125



As the x-values increase, the y-values are **multiplied by 5** (since $y = \mathbf{5}^x$) so the curve increases and becomes even steeper than the graph of $y = 2^x$.

A table of values and the graph of $y = 0.8^x$ are shown. y-values are rounded to three decimal places.

x	–3	–2	–1	0	1	2	3
y	1.953	1.563	1.25	1	0.8	0.64	0.512



As the x-values increase, the y-values are multiplied by 0.8 so the curve decreases and becomes flatter and approaches the x-axis to the right. This is the opposite of the two previous graphs.

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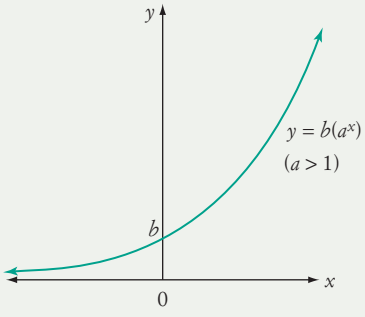
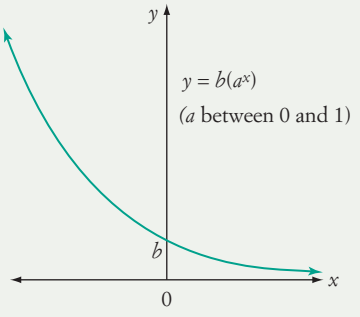
SUMMARY

- The **exponential curve** $y = b(a^x)$ is always above the x-axis.
- The x-axis is called an **asymptote**: the curve approaches it but never touches it.
- The y-intercept is b.
- If $a > 1$, the curve is **increasing**.
- If a is between 0 and 1, the curve is **decreasing**.





EXAMPLE 6

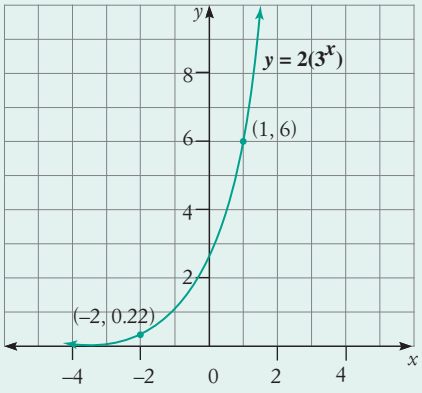
a Draw the graph of $y = 2(3^x)$.

b State the y-intercept and asymptote of the graph.

Solution

a Draw up a table of values: show y-values correct to 2 decimal places where needed.

x	−3	−2	−1	0	1	2	3
y	0.07	0.22	0.67	2	6	18	54



b The y-intercept is 2 and the asymptote is the x-axis.

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Exercise 10-05 The exponential function

1 Which equation represents an exponential function? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = 2^x$ **C** $y = \frac{2}{x}$ **D** $y = 2x$

2 Which equation represents a decreasing exponential function? Select **A**, **B**, **C** or **D**.

A $y = \left(\frac{1}{2}\right)^x$ **B** $y = 4(2^x)$ **C** $y = 2x^3$ **D** $y = 2(5^x)$

3 Which exponential graph has a y-intercept of 3? Select **A**, **B**, **C** or **D**.

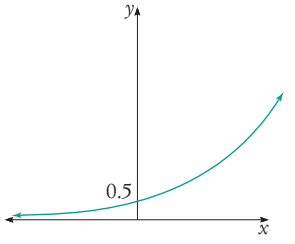
A $y = \left(\frac{1}{3}\right)^x$ **B** $y = 3(2^x)$ **C** $y = 2x^3$ **D** $y = 2(3^x)$

4 Match each exponential function to its graph below.

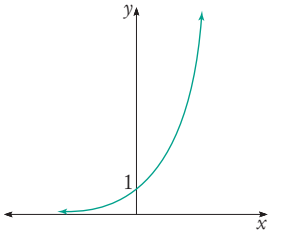
a $y = 2^x$ **b** $y = 10^x$ **c** $y = (0.1)^x$

d $y = 0.5(2^x)$ **e** $y = 8(2^x)$

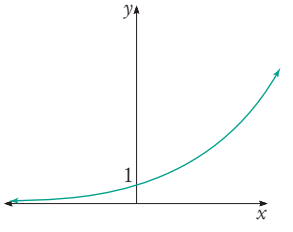
A



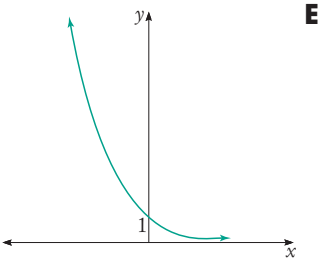
B



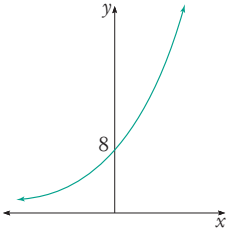
C



D



E



Example 6

5 **a** Complete the table of values and draw the graph of $y = 3^x$.

x	−3	−2	−1	0	1	2	3
y							

b State the y-intercept and asymptote of the graph.

6 **a** Draw the graph of $y = (0.6)^x$ **b** At what point does the graph cut the y-axis?

c Is this function increasing or decreasing?

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7 **a** Draw the graph of $y = 3(4^x)$ **b** What is the y-intercept?

c Explain why is there no x-intercept.

8 **a** Draw the graph of $y = 80(1.3)^x$ **b** What is the y-intercept?

c Is this function increasing or decreasing?

9 Which function has a graph that is decreasing? Select **A**, **B**, **C** or **D**.

A $y = 1.4^x$ **B** $y = \left(\frac{1}{4}\right)^x$ **C** $y = 0.4(1.1)^x$ **D** $y = \frac{1}{5}(2.5)^x$

10 A large sheet of paper is 0.1 mm thick. Suppose that it can be folded repeatedly. After 1 fold, the thickness of the sheet will double to 0.2 mm, after 2 folds to 0.4 mm, after 3 folds to 0.8 mm, and so on. After n folds, the sheet has a thickness of $T = 0.1(2^n)$, where T is the thickness in millimetres.

a Calculate the thickness of the sheet after:

i 5 folds **ii** 12 folds.

b Using a guess-and-check method, find the number of folds required for the sheet to be:

i 400 mm thick **ii** 1 m thick.

c What are the limitations of this model?

10-06 Exponential growth

Any quantity that increases according to the exponential function $y = b(a^x)$, where $a > 1$, is showing **exponential growth**. Something that grows exponentially increases slowly at first, then more quickly. Examples of exponential growth are:

- population size (people, animals, bacteria)
- an investment attracting compound interest
- a flu or computer virus
- the size of a bushfire.

EXAMPLE 7

The population of Busytown grows exponentially according to the function $P = 60\,000(1.08)^t$ where t is the number of years after 2010 and P is the population at the start of that year.

a What was Busytown's population in 2010?

b Calculate the population of Busytown in the year 2017 (to the nearest whole number).

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c Graph P for values of t from 0 to 8.

d Use a guess-and-check method to find when the population will be double that of 2010.

Solution

a When $t = 0$,

$P = 60\,000(1.08)^0$

$= 60\,000$

The population was 60 000 in 2010.

2010 is 0 years after 2010

$(1.08)^0 = 1$

b When $t = 7$,

$P = 60\,000(1.08)^7$

$= 102\,829.456\ldots$

$\approx 102\,829$

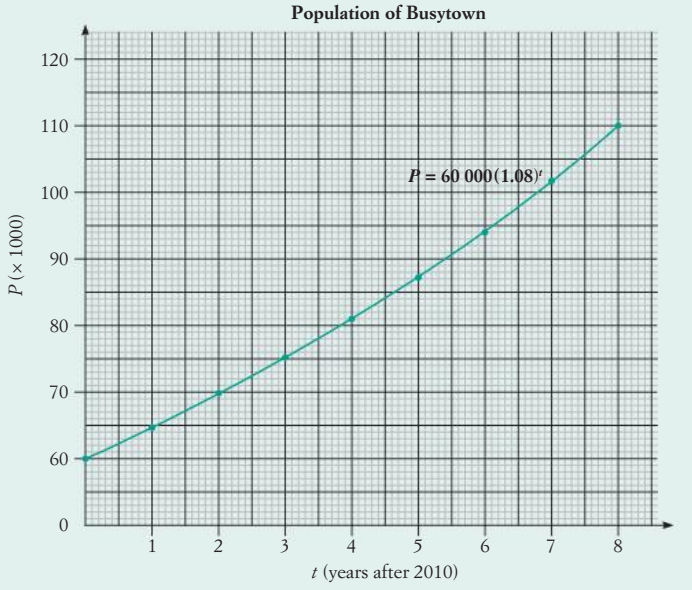
The population will be 102 829 in 2017.

2017 is 7 years after 2010

c

t	0	1	2	3	4	5	6	7	8
p	60 000	64 800	69 984	75 583	81 629	88 160	95 212	102 829	111 056

P-values are rounded to the nearest whole number.



d When $P = 2 \times 60\,000 = 120\,000$:

$120\,000 = 60\,000(1.08)^t$

$(1.08)^t = \dfrac{120\,000}{60\,000}$

$= 2$

Guess-and-check:
When $t = 9$, $(1.08)^9 = 1.9990\ldots$ **Too low but very close to 2.**
When $t = 10$, $(1.08)^{10} = 2.1589\ldots$ **Too high.**
The population will be double in early 2019 (9 years after 2010).

Note: When $t = 9.0065$, $(1.08)^{9.0065} \approx 2$

TECHNOLOGY: EXPONENTIAL GROWTH ON A CALCULATOR

In Example **7**, each new P-value is 1.08 times larger than the previous one so the population of Busytown is increasing by 8% each year. This can be keyed on the calculator like this:

Calculator keys	**Display**	
60 000 [=]	60000	$t = 0$
[×] 1.08 [=]	64800	$t = 1$
(Then keep pressing [=] repeatedly.)		
[=]	69984	$t = 2$
[=]	75582.72	$t = 3$
[=]	81629.3376	$t = 4$, etc.

Use your calculator to show that the population of Busytown in 2020 will be 129 535.

SUMMARY

For $y = b(a^x)$:

- exponential growth occurs when $a > 1$
- b is the initial value of y (when $x = 0$) and is the y-intercept of the exponential curve.

Exercise 10-06 Exponential growth

1 Which function shows exponential growth? Select **A**, **B**, **C** or **D**.

A $y = \frac{2}{x}$ **B** $y = 3(0.5)^x$ **C** $y = 7\left(\frac{2}{3}\right)^x$ **D** $y = 65(2.3)^x$

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Example 7

2 A bushfire is spreading through a forest and the amount of area burnt is modelled by the exponential function $A = 5(1.22)^h$ where A is in hectares and h is the time in hours.

a What was the initial area burnt?

b What area (to the nearest hectare) was burnt after 3.5 hours?

c After how many hours will 20 hectares of land be burnt?

d Graph the function for values of h from 0 to 6.

e Use your graph to determine:

i after how many hours 10 hectares of land will be burnt

ii the area burnt after $5\frac{1}{2}$ hours

f What are the limitations of this exponential model?

3 The compound interest formula is an example of exponential growth. If a principal of \$8000 is invested at 6.5% p.a. interest, the amount of the investment after n years is $A = 8000(1.065)^n$ where A is in dollars.

a What is the vertical intercept of the graph of the function and what does it represent?

b What is the amount of the investment after

i 1 year **ii** 5 years **iii** 10 years?

c How much interest has been earned after

i 1 year **ii** 5 years **iii** 10 years?

d Use a guess-and-check method to find the year in which the principal will be doubled.

4 The thickness, T cm, of a tree trunk grows according to the function $T = 16(1.11)^n$ where n is the time in years after the first measurement.

a What is the dependent variable of this function?

b Calculate, to the nearest centimetre, the thickness of the trunk:

i after 6 years **ii** when first measured.

c Calculate when the tree trunk will reach a thickness of 24.29 cm.

d Draw the graph of the function for values of n from 0 to 5.

5 Which of the following functions represents exponential growth? Select **A**, **B**, **C** or **D**.

A $y = 1000(0.015)^x$ **B** $y = 6\left(\frac{3}{4}\right)^x$ **C** $y = (0.7)^x$ **D** $y = 0.5(1.07)^x$

6 A scientist observed bacteria through a microscope and recorded the number of bacteria B at time t minutes. He recorded the population every 70 minutes and graphed his results as shown.

Time (t min)	0	70	140	210	280	350	420	490	560	630	700	770	840	910	980
No. bacteria (B)	100	200	400	800	1600	3200	6400	12 800	25 600	51 200	102 400	204 800	409 600	819 200	1 638 400

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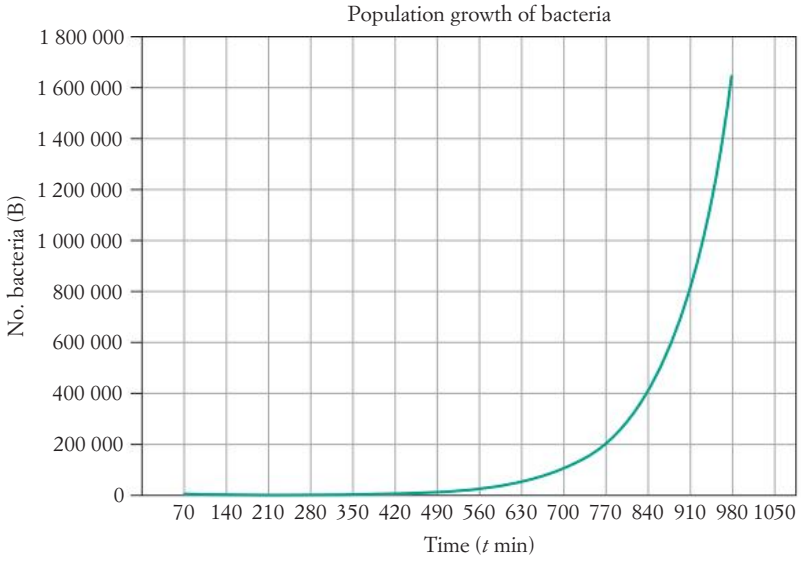

a How many bacteria are in the initial population?

b What happens in the first 490 minutes?

c Use the graph to estimate the number of bacteria after:

i 805 minutes **ii** 16 h

d Use the graph to determine after how many hours (to the nearest hour) the bacteria population is:

i 800 000 **ii** 1 500 000

e The scientist said 'the population is doubling every 70 minutes'. Do you agree with his findings? Justify your answer.

7 The number of germ cells in a particular culture is given by $N = 1000(6.9)^t$ where t is the time in minutes.

a Calculate the number of germ cells in the culture after:

i 10 minutes **ii** 1 hour

Answer in scientific notation correct to two significant figures.

b What was the initial number of germ cells?

c Estimate when the number of germ cells will pass 2 000 000.

8 The number of computers, C, affected by a virus after n hours is $C = 5(3.03)^n$.

a Is this an example of exponential growth?

b What is the vertical intercept and what does it represent?

c How many computers are affected by the virus after 8 hours?

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9 Australia's population can be modelled by the function $P = 22(1.016)^n$ where P is the population in millions and n is the number of years after the year 2010.

a Copy and complete this table of values, expressing P correct to one decimal place.

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
n (years)	0	5	10						
P (millions)									

b Graph P against n.

c Predict Australia's population in the year 2033 using:

i the graph **ii** the function

d Predict when Australia's population will reach 40 million using:

i the graph **ii** the function

10 A baby gorilla has a mass of M kg at age x months. The function $M = 1.4(1.3)^x$ can be used to calculate the mass of the gorilla up to 1 year of age.

a What is the mass of the gorilla at age:

i 3 months? **ii** 6 months? **iii** 1 year?

b Use a guess-and-check method to find the age of the gorilla with a mass of:

i 5 kg **ii** 25 kg

JUST FOR THE RECORD

Population clocks

A **population clock** is a counter that estimates the population of a country at any time using an exponential model. According to the population clocks at the Australian Bureau of Statistics website (www.abs.gov.au) and the US Bureau of Census website (www.census.gov), the populations of Australia and the world on 25 June 2013 were 23 094 509 and 7 093 894 440 respectively.

In 2013, Australia's overall population was increasing by 0.72 persons per minute and the world's population was increasing by 150 persons per minute. The world population is expected to reach 10 billion in 2100.

China has the highest population in the world and it is said that 'one in every five people in the world is Chinese'.

1 Find the current populations of Australia, China and the world, and decide if the 'one-in-five' claim is true.

2 Which other country has a population of over 1 billion?

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10-07 Direct variation

Direct variation occurs when two variables both change in the same direction. That is, as one increases so the other increases and as one decreases so the other decreases.

Some examples of direct variation are:

- fuel consumption and size of car: the bigger the car, the higher the fuel consumption
- stretch of a spring and the mass supported: the heavier the mass, the longer the stretch
- viewing range and height of tower: the higher up you go, the further you can see

Corbis/National Geographic Society/Tyrone Turner

SUMMARY

Variation	Equation
y varies as x	$y = kx$
y varies as the square of x	$y = kx^2$
y varies as the cube of x	$y = kx^3$

To solve a variation problem:

1. identify the two variables, say x and y, and form a variation equation involving a constant k
2. substitute given values of x and y to find k
3. rewrite the equation using the value of k
4. substitute a value for x or y into the equation to solve the problem.

Note: 'y is directly proportional to x' is another way of saying 'y varies directly as x'.

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EXAMPLE 8

The weight, M kg, of an astronaut on Mars varies directly with his weight, E kg, on Earth. A 72 kg person on Earth weighs 27.4 kg on Mars.

a Calculate how much a 60 kg person weighs on Mars.

b If an astronaut weighs 32 kg on Mars, what is his weight on Earth?

Give answers correct to 3 significant figures.

Solution

a $M = kE$

To find k, substitute $E = 72$, $M = 27.4$.

$27.4 = k \times 72$

$$k = \frac{27.4}{72}$$

$$= 0.380555\ldots$$

Do not round, leave value of k on your calculator display.

$\therefore M = 0.380555\ldots \times E$

When $E = 60$

$M = 0.380555\ldots \times 60$

Multiply the value of k on your calculator display by 60.

$= 22.8333\ldots$

≈ 22.8

A 60 kg person weighs 22.8 kg on Mars.

b When $M = 32$

$32 = 0.380555\ldots \times E$

$$E = \frac{32}{0.380555\ldots}$$

$= 84.08759\ldots$

≈ 84.1

The astronaut weighs 84.1 kg on Earth.

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EXAMPLE 9

The number of eggs, E, in a cake is proportional to the square of the diameter, d cm, of the cake. Two eggs are needed to make a cake of diameter 15 cm.

a How many eggs (to the nearest whole egg) are needed to make a cake of diameter 34 cm?

b What is the diameter of the cake that contains 6 eggs?

Solution

a The variation equation is $E = kd^2$ where k is the constant of variation.

Substitute $d = 15$ and $E = 2$ to find k.

$$2 = k \times 15^2$$

$$2 = k \times 225$$

$$k = \frac{2}{225}$$

easier to leave k as a fraction here

$$\therefore E = \frac{2}{225}d^2$$

When $d = 34$

$$E = \frac{2}{225} \times 34^2$$

$$= 10.27555...$$

$$\approx 10$$

So 10 eggs are needed for a cake of diameter 34 cm.

b When $E = 6$:

$$6 = \frac{2}{225} \times d^2$$

$$d^2 = \frac{6 \times 225}{2}$$

$$d^2 = 675$$

$$d = 25.9807\ldots$$

So 6 eggs will make a cake of diameter 26 cm.

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Exercise 10-07 Direct variation

1 Complete: With direct variation, when one variable increases, the other variable _____.

2 Which equation is an example of direct linear variation? Select **A**, **B**, **C** or **D**.

A $y = x^2$ **B** $E = kV$ **C** $A = \pi r^2$ **D** $y = \frac{2}{x}$

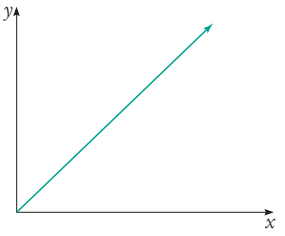
3 **a** If $y = kx^3$ and $y = 54$ when $x = 3$, what is the value of k?

b If $V = at^2$ and $V = 3500$ when $t = 24$, what is the value of a correct to 4 decimal places?

4 Match each variation equation with its graph below.

a $y = kx^2$ **b** $y = kx$ **c** $y = kx^3$

A



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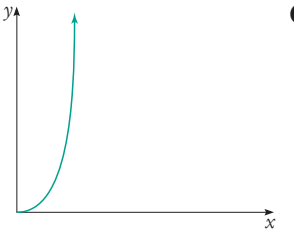
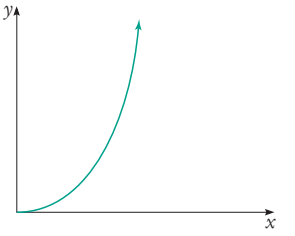


C



Example 8

5 The distance, D km, that can be travelled by Grace's car varies directly as the volume, V litres, of petrol in her car. Grace's car can travel 465 km on 48 L of petrol.

a Find the variation equation for D in terms of V.

b How far, to the nearest kilometre, can Grace's car travel on 56 L of petrol?

c How many litres of petrol does she use to travel 200 km? Answer correct to one decimal place.

Example 9

6 The braking distance, B metres, of a car is proportional to the square of its speed, s km/h. On a wet surface, a car travelling at 80 km/h covered 90 m before stopping.

a Find B in terms of s, writing the constant of variation as a fraction.

b What is the braking distance, to the nearest metre, of a car travelling at 70 km/h on the same surface?

c At what speed, to the nearest km/h, is a car travelling if it takes 120 m to stop on the same surface?

7 During a thunderstorm, the time between seeing lightning and hearing thunder varies as the distance of the observer from the centre of the storm. If thunder is heard 6 seconds after the lightning for a storm that is 35 km away, calculate the distance of a storm when thunder is heard 10 seconds after the lightning. Select **A**, **B**, **C** or **D**.

A 17 km **B** 58 km **C** 70 km **D** 210 km

8 The mass of a metal sphere varies directly as the cube of its diameter. A sphere with a diameter of 16 cm has a mass of 1536 g. Find the mass of a metal sphere with a diameter of 10 cm.

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9 The increase in pressure experienced by a scuba diver is directly proportional to his/her depth under the water. The increase in pressure at depth 40 m is 235 kPa (kilopascals).

a What increase in pressure is experienced at a depth of 32 m?

b At what depth, to the nearest metre, is the increase in pressure 735 kPa?

10 The maximum load, M kg, that can be suspended from a steel cable varies as the square of the thickness, t cm, of the cable. A cable 1.2 cm thick can carry a maximum load of 1440 kg. Calculate:

a the maximum load that can be carried by a cable of thickness 0.7 cm

b the thickness of a cable that can carry a maximum load of 3000 kg. Answer correct to one decimal place.

11 The cost, $\$C$ per km, of running a car varies as the cube of the speed, S km/h, of the car.

a Write the variation equation for C in terms of S and a constant of variation k.

b If it costs \$1.25 per km to run the car when it is travelling at a speed of 60 km/h, calculate k, in scientific notation correct to four significant figures.

c Hence calculate the cost of running the car when it is travelling at a speed of 85 km/h.

12 The distance an object falls under gravity varies as the square of the time taken. In 8 seconds, an object falls 313.6 metres. Calculate:

a the distance an object falls in 20 seconds

b the time it takes an object to fall 592.9 metres.

13 The fuel consumption, F L/100 km, of a certain type of car, varies as the mass, M kg, of the car. One such car has a mass of 1200 kg and a fuel consumption of 13.2 L/100 km.

a What is the fuel consumption of a car that weighs 870 kg?

b What is the mass, to the nearest kilogram, of a car that has a fuel consumption of 11.9 L/100 km?

14 Researchers discovered that the average queue length outside the rest rooms at a concert venue is proportional to the square of the average time spent by the users of the rest rooms. When the average time spent in the rest rooms is 113 seconds, the average queue length is 7.

a What is the average queue length if the average time spent in the rest room is 160 seconds?

b At a concert venue, there is a difference between the queue lengths for male and female rest rooms. Why do you think this is so?

15 The mass of an egg is proportional to the cube of its length. If an egg of length 6 cm has a mass of 54 g, what is the mass of an egg with a length of 8 cm? Select **A**, **B**, **C** or **D**.

A 24 g **B** 72 g **C** 80 g **D** 128 g

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10-08 Inverse variation

Variation problems

Inverse variation occurs when two variables change in different directions. That is, as one increases so the other decreases and as one decreases so the other increases.

Inverse means 'the opposite'.

Some examples of inverse variation are:

- time of journey and average speed: the faster you go, the less time it takes to get there
- air temperature and altitude of aeroplane: the higher up you fly, the lower the temperature
- speed of Internet downloads and number of users: the more users, the slower the download rate

SUMMARY

Inverse variation	Equation
y varies inversely as x.	$y = \frac{k}{x}$

Note: 'y is inversely proportional to x' is another way of saying 'y varies inversely as x'.

EXAMPLE 10

The blood alcohol content (BAC), B, of an adult male after drinking beer varies inversely as his weight, W kg. A 72 kg man has a BAC of 0.059 after drinking beer. Find the BAC, correct to three decimal places, of a 90 kg man who drinks the same amount as the 72 kg man.

Solution

The variation equation is $B = \frac{k}{W}$

This equation implies that the heavier the man, the lower his BAC.

Substitute $W = 72$ and $B = 0.059$ to find k.

$$0.059 = \frac{k}{72}$$
$$k = 0.059 \times 72$$
$$= 4.248$$
$$\therefore B = \frac{4.248}{W}$$

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When $W = 90$,

$$B = \frac{4.248}{90}$$
$$= 0.0472$$
$$\approx 0.047$$

The BAC of the 90 kg man is 0.047.

Exercise 10-08 Inverse variation

1 Complete: With inverse variation, when one variable increases, the other variable _____.

2 Which equation is an example of inverse variation? Select **A**, **B**, **C** or **D**.

A $y = \frac{x}{2}$ **B** $y = 2x^2$ **C** $y = \frac{1}{2}x$ **D** $y = \frac{2}{x}$

3 **a** If $y = \frac{a}{x}$ and $y = 12$ when $x = 5$, what is the value of a?

b If $V = \frac{k}{t}$ and $V = 7$ when $k = 56$, what is the value of t?

Example 10

4 The frequency, F beats per second, that a bird beats its wings varies inversely as the length, L cm, of its wings. A bird with wings of length 14 cm beats them at a frequency of 8 beats per second.

a Find the variation equation for F in terms of L.

b Calculate, correct to the nearest whole number, the wingbeat frequency for wings of length 18 cm.

c A bird beats its wings with a frequency of 4.5 beats per second. What is the length of its wings, correct to the nearest centimetre?

5 The temperature of the air is inversely proportional to the height above sea level. At 400 m above sea level, the temperature is 12°C.

a What is the temperature 1000 m above sea level?

b At what height, to the nearest metre, is the temperature 19°C?

6 The journey time varies inversely as the speed. At a speed of 60 km/h, it takes Harley 90 minutes to drive to Windsor. How fast does he need to go to reach Windsor in 50 minutes?

7 The amount of time it takes Sarah to move house is inversely proportional to the number of friends she has to help her. When she has 4 friends helping, the job takes $3\frac{3}{4}$ hours.

a How long will it take if she has 6 friends helping?

b How many friends must she have to help her to move house in 3 hours?

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8 The volume of gas in a balloon is inversely proportional to the pressure applied to compress it. Helium has a volume of 20 m^3 at a pressure of 9 kPa.

a What is the volume of helium at a pressure of 45 kPa?

b What pressure is necessary to compress the gas to 8 m^3?

9 Paul believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 16 people waiting, the train will arrive in 2.5 minutes.

a When will the train arrive if there are 5 people waiting?

b How many people are waiting at the station 10 minutes before the train arrives?

10 The average amount of time a teacher can spend helping each student during a lesson varies inversely with the number of students in her class. If there are 24 students in Mrs Ellsmore's class, she can spend an average of 1.9 minutes per student.

a Calculate the average time Mrs Ellsmore can spend with each student if there are 29 students in her class. Answer correct to the nearest 0.1 minute.

b Calculate the class size that would allow her to spend 2.3 minutes with each student.

11 The rate of vibration of a string varies inversely as its length. A string that is 8 cm long vibrates at 9375 Hz (hertz). What length of string will vibrate at 6250 Hz? Select **A**, **B**, **C** or **D**.

A 5 cm **B** 7 cm **C** 12 cm **D** 73 cm

12 The number of people who can attend a concert varies inversely with the amount of space allowed per person. If 3000 cm^3 of space is allowed per person, the concert hall can hold 14 000. How many people can be accommodated if 2200 cm^3 of space is allowed per person?

Shutterstock.com/Franz Pfluegl

Sample HSC problem

A certain strain of yeast cell grows exponentially according to the formula $N = 350(1.04)^t$ where N is the number of yeast cells and t is the time in minutes.

a Graph this function for values of t from 0 to 30.

b What is the vertical intercept and what does it represent?

c How many yeast cells are there after 80 minutes?

d Use the graph to estimate when the number of cells reaches 1000.

Study tip

Read the key words in an exam question

(* = Board of Studies definition)

- **Compare*:** show how things are similar or different
- **Describe*:** provide characteristics or features
- **Estimate:** make an educated guess for a number or measurement, find roughly or approximately
- **Evaluate/calculate:** find the value, the numerical answer.
- **Expand:** rewrite the algebraic expression without brackets, for example, $3(2b + 1) = 6b + 3$
- **Explain/Give a reason:** describe why or how
- **Hence:** find or prove a result using the answer(s) from the previous part of the question
- **Justify*:** support your argument or conclusion
- **Prove that/Show that/Deduce that:** in questions where the answer is given, prove that the result is true using formal mathematics and reasoning
- **Solve:** find the value of the unknown pronumeral in an equation
- **Write down/state:** write the answer, formula or result without working or explanation (usually in a one-mark question)

See the **Instructional glossary** on page 534 for more details.

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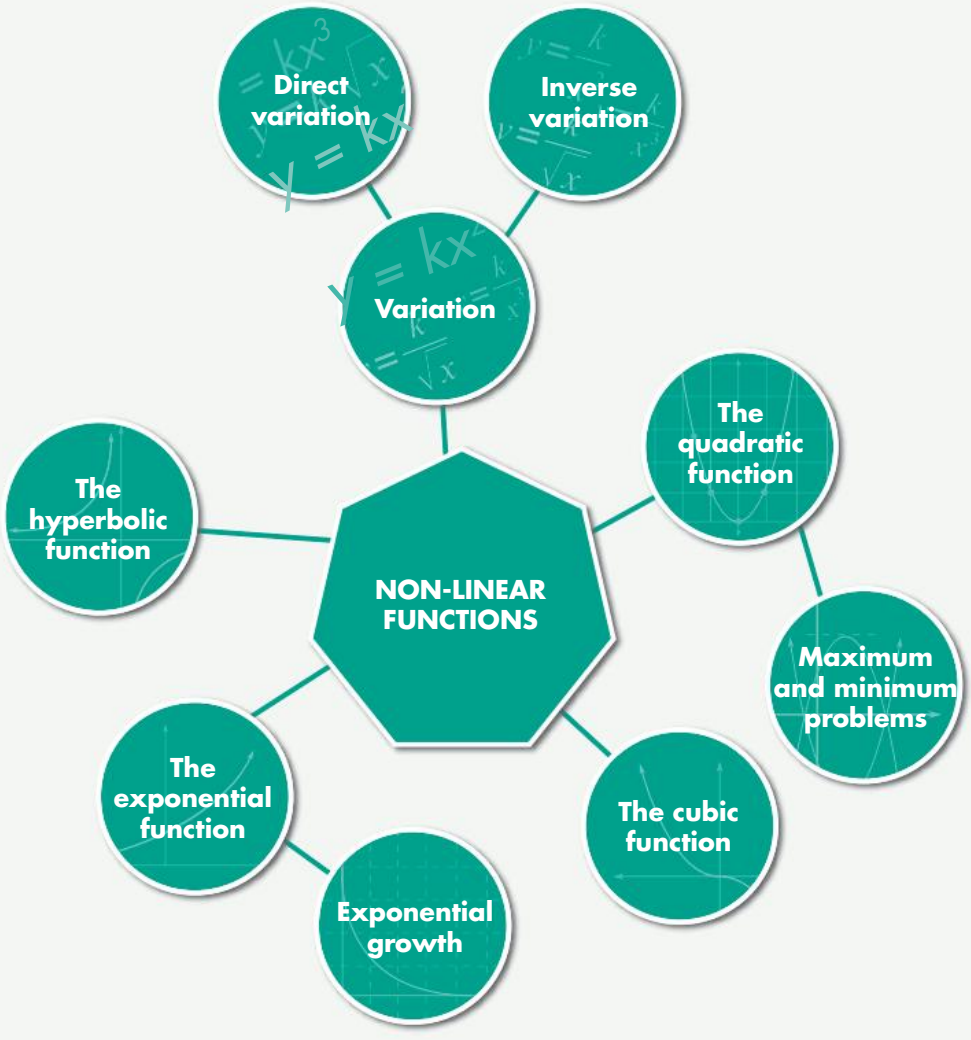
CHAPTER REVIEW

Topic overview

This chapter, Non-linear functions, examined four types of functions, their graphs and their use as algebraic models. Make sure that you know the name of each function, its equation and the properties of its graph, such as vertical intercepts and asymptotes. Practise applying the functions to practical problems such as population growth and the braking distance of a car. Different types of variation were also covered in this chapter, so revise the steps involved in solving a variation problem.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders to gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 10-01

1 **a** What type of function is $y = -x^2 + 4$?

b What is the name of its graph?

c Graph this function and write the coordinates of the vertex.

Exercise 10-02

2 The running cost, $\$C$ per hour, of a ship travelling at an average speed of s knots, is given by the function $C = 2s^2 - 128s + 2400$.

a Copy and complete this table of values.

s	22	26	30	34	38	42	46	50
C	552		360			552		

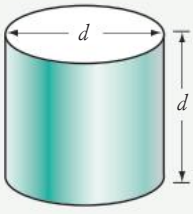
b Hence draw the graph for the function $C = 2s^2 - 128s + 2400$.

c Use your graph to estimate the average speeds that have a running cost of \$480 per hour.

d Find the minimum running cost and the speed that gives this cost.

Exercise 10-03

3 The volume of a cylinder whose height is the same as the diameter of its base is approximated by the function $V = 0.785d^3$ where V is the volume and d is the diameter.



a Calculate the volume of a cylinder with both diameter and height 11cm.

b Find the diameter of a cylinder with volume 215.58 cm^3. Answer correct to one decimal place.

Exercise 10-04

4 The number of days, D, taken to build a house is related to the number of builders, b, working on the job, according to the function $D = \frac{240}{b}$

a Graph this function for values of b from 2 to 10.

b How long, to the nearest day, will it take to build the house using 9 builders?

c How many builders are needed to build the house in 5 days?

d What are the limitations of this model?

Exercise 10-05

5 The population of a town is modelled by the function $P = 40\,000(1.03)^n$ where P is the population at the end of n years.

a What is the population of the town after 12 years?

b What was the initial population of the town?

c Determine in which year the initial population will double.

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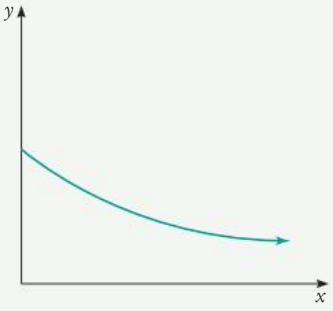
6 What could be the equation of this graph? Select **A**, **B**, **C** or **D**.



Exercise 10-06

A $y = 5(0.2)^x$

B $y = \frac{2}{x}$

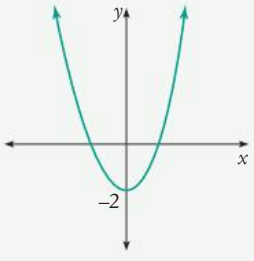
C $y = \frac{1}{2}x^3$

D $y = 5(2^x)$

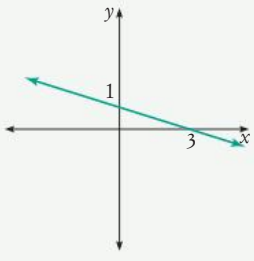
7 Match each graph with its function below.

Exercise 10-06

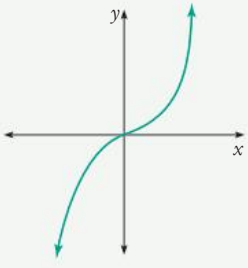
a



b



c



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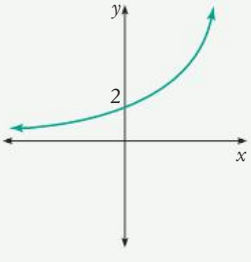
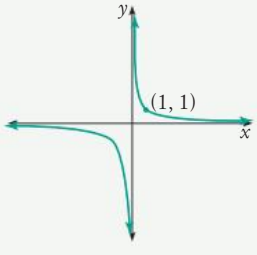


e



A $y = 2(2^x)$

B $y = -\frac{1}{3}x + 1$

C $y = \frac{1}{2}x^3$

D $y = x^2 - 2$

F $y = \frac{1}{x}$

8 The stretch of a spring varies as the mass of the load pulling it. A load of 24 kg causes the spring to stretch 10 cm. What is the mass of the load that will cause a stretch of 25 cm?

Exercise 10-07

9 At Frank's Pizzas, the cost of a pizza is proportional to the square of its diameter. A pizza with a diameter of 30 cm costs $6.80.

Exercise 10-07

a Calculate the cost of a pizza of diameter 45 cm.

b Calculate the diameter, to the nearest centimetre, of a pizza that costs $22.00.

10 The time taken, T hours, for a train trip between Sydney and Melbourne varies inversely as the average speed, S km/h, of the train. A trip takes 10 hours when the train is travelling at 80 km/h.

Exercise 10-08

a Graph T for values of S from 50 to 120.

b What type of graph is this?

c Calculate how long the trip will take travelling at 64 km/h.

d If the trip takes 12 hours, what is the speed of the train, correct to the nearest km/h?

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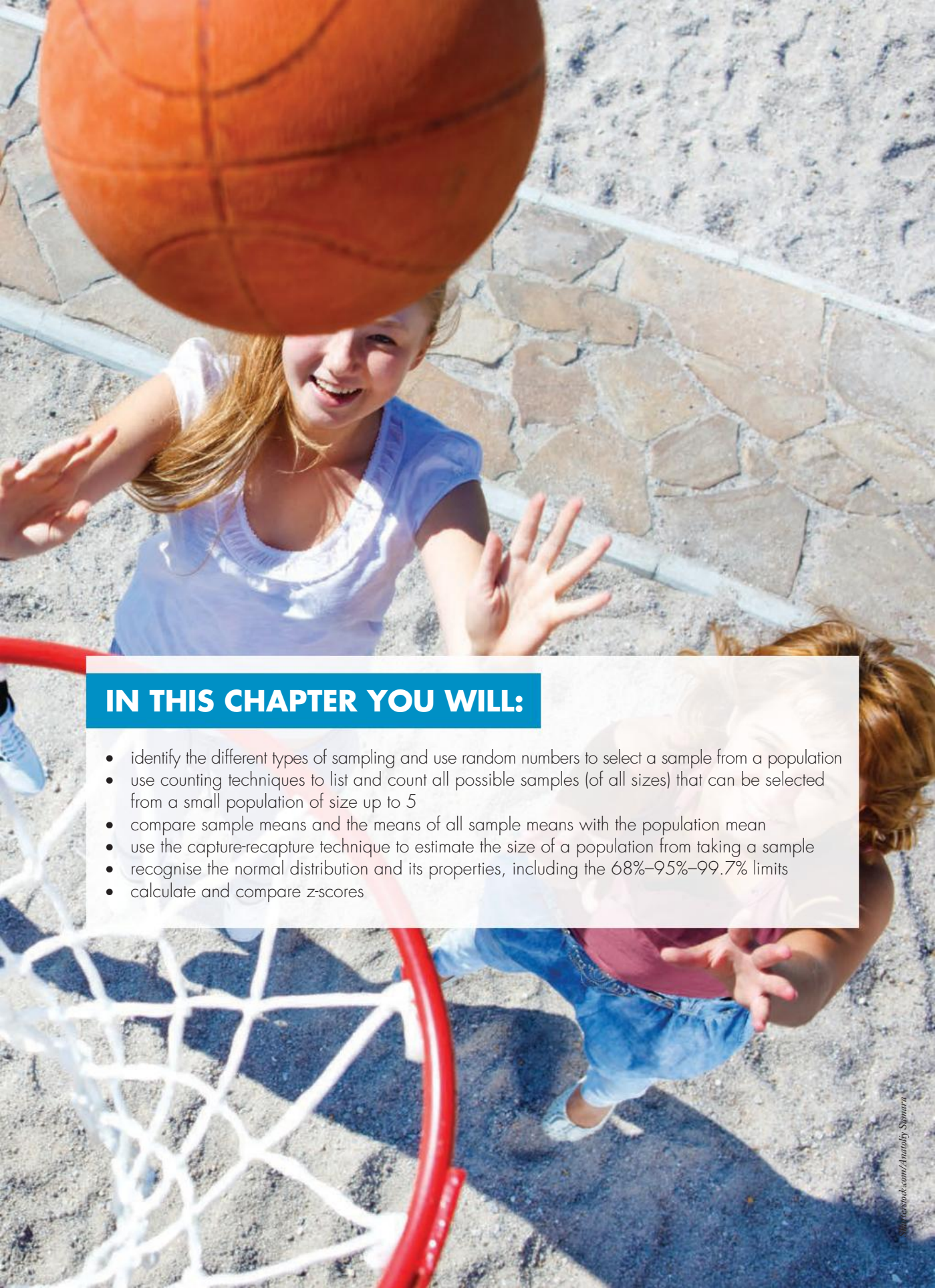
11.

SAMPLING AND THE NORMAL DISTRIBUTION

The mean height of Australian women is 161 cm, with a standard deviation of 7 cm. Australian basketball player Lauren Jackson is 195 cm tall, which makes her almost 5 standard deviations above the mean. How does this figure compare Lauren to all Australian women? What percentage of the adult female population have a height of 195 cm?

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- identify the different types of sampling and use random numbers to select a sample from a population
- use counting techniques to list and count all possible samples (of all sizes) that can be selected from a small population of size up to 5
- compare sample means and the means of all sample means with the population mean
- use the capture-recapture technique to estimate the size of a population from taking a sample
- recognise the normal distribution and its properties, including the 68%–95%–99.7% limits
- calculate and compare z-scores

TERMINOLOGY

capture-recapture technique
normal distribution
random sample
sample mean ($\bar{x}$)
standardised score
z-score
mean
population
raw score
sample size
stratified sample
normal curve
population mean (μ)
sample
standard deviation
systematic sample

Assignment 11

SkillCheck

1 Find, correct to the nearest whole number:

a 99.7% of 12 600 **b** 2.5% of 3200 **c** 0.3% of 6000.

2 Of 15 000 university students, 95% passed their final exam. How many passed?

3 A survey of 6000 people found that 68% did not smoke. How many people smoked?

4 In a region in India, 99.7% of days per year have a temperature greater than 25°C. How many days have a temperature less than or equal to 25°C? Answer to the nearest day.

5 Find the value of:

a $\frac{6^2+7^2-11^2}{2\times6\times7}$ **b**  **c** 

6 Given the formula $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ (SOH), $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ (CAH), find correct to one decimal place:

a z when $x = 170$, $\bar{x} = 160$, $s = 8$ **b** x when $z = 1.7$, $\bar{x} = 90$, $s = 12$

7 Solve each equation for P.

a $h = \frac{42}{\sin 36°}$ **b** $\cos\theta = \frac{27}{40}$ **c** $\tan 72.5° = \frac{d}{356}$

8 A team of two players is selected from a group of 6 tennis players. How many different selections are possible?

11-01 Sampling

In statistics, the **population** refers to the total number of items being studied, whether they are people, kangaroos in a national park, schools or cars. A **census** is a survey of the population that includes every member of it.

If the population is too large or too difficult to survey, a **sample** of items can be taken from the population and analysed. The results can then be used to provide estimates for population characteristics.

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Types of samples

There are three main types of samples.

- In a **random sample**, each item is equally likely to be chosen from the population, for example, selecting phone numbers for an opinion survey.
- In a **systematic sample**, items are chosen at regular intervals, for example, testing every tenth car on a production line.
- In a **stratified sample**, a random sample is taken from each stratum or layer of a population, for example, if a population contains 70% adults and 30% children, the stratified sample must contain 70% adults and 30% children.

EXAMPLE 1

What type of sample would you use to investigate the following?

a The amount of time spent on Facebook by Year 12 students.

b The quality of service in a major retail store.

c The types of books to buy for the school library.

Solution

a A random sample will provide a fair indication of the amount of time spent on Facebook by Year 12 students.

b A systematic sample where the first person is picked at random as they leave the store, then selecting other people at regular intervals as they leave the store.

c A stratified sample so that students from each year group are included.

Using random numbers to select a sample

Random numbers can be used to select people or items for a sample. For example, a calculator or spreadsheet can be set up to generate a random number from 1 to 100, and that number can be matched to the name of a person or item from a list. To generate such a number on:

Random numbers: Graphics calculator

- a **scientific calculator**, type RAN × 100 + 1 = and press = repeatedly for more random numbers
- a **graphics calculator**, enter 'int(ran#×100+1)' from the probability functions, and press EXE, then repeatedly press EXE for more random numbers
- a spreadsheet, type '=INT(RND()*100+1)' into a cell, and press the ENTER key, then the F9 key repeatedly for more random numbers.

Table of random numbers

You can also use tables of random numbers, such as the one below.

	1	2	3	4	5	6	7	8	9	10
1	99378	72295	32790	49470	70129	60957	89147	38411	60662	89310
2	40761	41158	33997	08086	42944	82276	63269	86322	96582	59070
3	40937	49212	34099	16256	74084	05344	04936	03735	09155	36258
4	72077	19973	79488	75988	05777	58054	71086	66554	57522	49524
5	91838	98434	46382	41956	72661	28166	43092	36864	65386	83180
6	69733	40493	04397	07142	71340	56201	71604	53662	35768	00034
7	70065	60572	99236	25352	07780	33474	77170	08116	29031	41870
8	34864	32214	99370	38275	22079	59257	92808	89221	60728	34427
9	28003	33903	90826	31071	78785	75383	65972	39658	04329	40765
10	40411	76275	19193	20175	61726	41647	37149	63330	85757	48347

To find a random number from 1 to 100, choose any starting point (say '60572' at row 7, column 2) and read off two-digit numbers by moving across: 60, 57, 29, 92, 36, etc.

Read '01' as 1, '02' as 2, etc.
Read '00' as 100.

EXAMPLE 2

A school is conducting a survey on TV viewing habits of Year 12 students and wants a random sample of 15 students from Year 12. The names of the students are as follows.

1	Abdou	**16**	Cruyff	**31**	Jacques	**46**	Martin	**61**	Ralston	**76**	Underwood
2	Adams	**17**	Curry	**32**	Johnson	**47**	McDonald R	**62**	Rapiti	**77**	Vu
3	Afif	**18**	Degning	**33**	Jones	**48**	McMahon	**63**	Rice D	**78**	Walker
4	Anderson	**19**	Elms	**34**	Kapoor	**49**	Mok	**64**	Rice A	**79**	White
5	Ashley	**20**	Everingham	**35**	Kaporis	**50**	Morris	**65**	Russell	**80**	Williams
6	Barry	**21**	Falzon	**36**	Kazi	**51**	Napier	**66**	Ryan	**81**	Wong
7	Bentley	**22**	Fisher	**37**	King	**52**	Nguyen T	**67**	Smith D	**82**	Woods
8	Best	**23**	Grasso A	**38**	Kini	**53**	Ngyuyen M	**68**	Smith J	**83**	Xien
9	Bradley	**24**	Grasso C	**39**	Knapenberger	**54**	Nori	**69**	Stephenson	**84**	Yee
10	Braithwaite	**25**	Grubb	**40**	Law	**55**	O'Brien	**70**	Taylor	**85**	Yen
11	Brown	**26**	Hancock	**41**	Lee	**56**	Ong	**71**	Thomas	**86**	Zammit
12	Budd	**27**	He	**42**	Ly	**57**	Parker	**72**	Thompson		
13	Cheng	**28**	Hoogeven	**43**	MacDonald	**58**	Pendleton	**73**	Thomson		
14	Close	**29**	Itatoka	**44**	Maloney	**59**	Praveen	**74**	Torning		
15	Cosgrove	**30**	Jackson	**45**	Mamun	**60**	Rahman	**75**	Trimarchi		

Use a calculator or spreadsheet to generate 15 random numbers from 1 to 86. Then use the numbers that you generated to select the 15 students for the sample.

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Solution

Use one of the methods described above to generate the 15 random numbers.
For example:

2, 68, 63, 47, 79, 73, 27, 30, 11, 56, 19, 2, 84, 38, 46, 42

Ignore any repeated numbers, such as this '2'

These numbers correspond to the following students chosen for the sample:

Adams, Smith J., Rice D., MacDonald, White, Thomson, He, Jackson, Brown, Ong, Elms, Yee, Kini, Martin, Ly.

Counting samples

EXAMPLE 3

A family has five children with the following ages: 1 4 6 10 12

a A researcher wants to test three of these children for reaction time. How many possible samples of size 3 can be chosen from the group of five children?

b List these samples.

Solution

a Number of possible samples = $\cos 20^\circ = \frac{d}{10}$

$= 10$

This is an unordered selection.

b The 10 possible samples (listed according to age) are:

1, 4, 6	1, 4, 10	1, 4, 12	1, 6, 10	1, 6, 12
1, 10, 12	4, 6, 10	4, 6, 12	4, 10, 12	6, 10, 12

Exercise 11-01 Sampling

Example 1

1 What is the best way of collecting data to find the three best-selling brands of chocolate? Select **A**, **B**, **C** or **D**.

A census **B** survey **C** experiment **D** observation

2 Give two reasons for choosing a sample rather than a census.

3 What is meant by a **biased** sample?

4 Which type of random sample (simple, systematic or stratified) would best suit each situation?

a random breath testing **b** opinion poll on favourite movie

c taste-testing a new brand of soft drink

5 A survey about customer satisfaction is conducted by 'New Wheels' car magazine. A sample of 400 car buyers is to be selected from a group of 1500 Holden buyers, 1500 Ford buyers, 1500 Hyundai buyers and 1500 Toyota buyers. A random sample of 100 buyers from each brand was selected. Is this an example of a simple random sample? Give reasons.

Example 2

6 Select your own random sample of students for the TV viewing habits survey using the list of names given in Example **2**. Use a calculator or spreadsheet to generate 15 random numbers from 1 to 86 first.

7 Select another random sample of names for the survey in Example **2**, by generating 15 numbers from 1 to 86 from the table of random numbers on the previous page.

8 Write a numbered list of the students in your maths class. Suppose six students in your class need to be randomly selected for a survey about road safety. Use a calculator or spreadsheet to choose the students for the random sample.

Example 3

9 There are five women on the swimming squad: Lisa (L), Zoe (Z), Sarah (S), Trish (T) and Nina (N). Two of the swimmers are selected for drug testing.

a How many samples of size 2 can be chosen from this population?

b List all possible samples.

10 A playgroup has 10 children. A sample of two children is taken for a health check. How many samples can be chosen? Choose **A**, **B**, **C** or **D**.

A 35 **B** 45 **C** 70 **D** 90

11 A library has five branches: Central (C), Newhaven (N), Skyfall (S), Westvale (W) and Baramvale (B). A sample of branches needs to be chosen for IT training.

a How many possible samples are there if the sample size is:

i 4? **ii** 2? **iii** 3? **iv** 1? **v** 5?

b So how many possible samples from the five branches are there in total?

c List all the possible samples of size 4.

12 How many samples of any size are possible from a population of 3 items?

13 The eye colours and genders of a population of 30 students are as follows.

Student	Gender	Eye Colour		Student	Gender	Eye Colour
1	F	Grey		16	F	Green
2	F	Brown		17	F	Grey
3	M	Blue		18	M	Brown
4	F	Blue		19	M	Other
5	M	Green		20	M	Blue
6	F	Grey		21	F	Brown
7	F	Blue		22	F	Brown
8	F	Hazel		23	F	Grey
9	F	Blue		24	M	Blue
10	M	Hazel		25	F	Blue
11	M	Blue		26	M	Grey
12	F	Green		27	M	Hazel
13	M	Blue		28	F	Blue
14	M	Hazel		29	M	Grey
15	F	Brown		30	F	Blue

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a Calculate, correct to one decimal place, the percentage of the population who:

i are male **ii** are female **iii** have blue eyes

iv have brown eyes **v** have grey eyes **vi** have hazel eyes

b Select a random sample of size 10 from the population by generating 10 random numbers from 1 to 30 from the table of random numbers. Calculate, correct to one decimal place, the percentage of the sample who:

i are males **ii** have blue eyes **iii** have grey eyes

Compare the results of your sample with the population results.

c Select another random sample of size 8, using a calculator or spreadsheet. Calculate, correct to one decimal place, the percentage of the population who:

i are males **ii** have blue eyes **iii** have grey eyes

Compare the results of your sample with the population results and with the sample results of part **b**.

Comment on any similarities and/or differences.

14 a Count the number of male and female students in:

i your Maths class **ii** another Maths class **iii** in the combined classes

b What percentage in each class are:

i male? **ii** female?

c What percentage of the combined classes are:

i male? **ii** female?

d Which result would you use to estimate the percentage of males and females in Year 12 at your school?

e Find the number of males and females in Year 12 at your school and calculate the percentage of males and females.

f Compare the results in parts **b** and **c** with the result in part **e**. Does sample size have an effect in estimating the percentage of males and females in Year 11?

11-02 Comparing samples and populations

The statistics of a sample can be used to estimate the statistics of the population.

The symbol for the **population mean** is μ (the Greek letter 'mu') and the symbol for the **sample mean** is $\bar{x}$.

The symbol for the **mean of the sample means** is $\mu_{\bar{x}}$.

EXAMPLE 4

A population consists of the following scores: 0 2 4 6 8

a Find the mean μ of the population.

b List all possible samples of varying sizes from $n = 1$ to $n = 5$ from the population.

c Find the means ($\bar{x}$) of all the samples that have been listed.

d Find the mean of the sample means.

e How does the mean of the sample means compare to the mean of the population?

Solution

a $\mu = 4$

b The possible samples are:

$n = 1$: 0 2 4 6 8

$n = 2$: 0, 2 0, 4 0, 6 0, 8 2, 4 2, 6 2, 8 4, 6 4, 8 6, 8

$n = 3$: 0, 2, 4 0, 2, 6 0, 2, 8 0, 4, 6 0, 4, 8 0, 6, 8 2, 4, 6 2, 4, 8
2, 6, 8 4, 6, 8

$n = 4$: 0, 2, 4, 6 0, 2, 4, 8 0, 2, 6, 8 0, 4, 6, 8 2, 4, 6, 8

$n = 5$: 0, 2, 4, 6, 8

c The means of the samples are as follows:

$n = 1$: $\bar{x}$s are 0, 2, 4, 6, 8

$n = 2$: $\bar{x}$s are 1, 2, 3, 4, 3, 4, 5, 5, 6, 7

$n = 3$: $\bar{x}$s are 2 , , , 4, , 4, , , 6

$n = 4$: $\bar{x}$s are 3, 3.5, 4, 4.5, 5

$n = 5$: $\bar{x} = 4$

d The mean of all of the sample means is

$$= 6.7 \sin 118°$$
$$= 1$$
$$= 4$$

e The mean of the sample means ($\bar{x}$) is equal to the population mean (μ).

$\bar{x} = \mu = 4$.

SUMMARY

The mean of the distribution of all possible sample means ($\bar{x}$) is equal to the population mean (μ).

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Exercise 11-02 Comparing samples and populations

Example 4

1 A population consists of the following scores: 1 3 5 6 8

a Find the mean (μ) of the population.

b List all possible samples of varying sizes from $n = 1$ to $n = 5$ of the population.

c Find the means ($\bar{x}$) of all the samples that have been listed.

d Find the mean ($\bar{x}$) of the distribution of the sample means.

e Does the mean of the sample means equal the mean of the population?

2 The heights (in cm) of five basketball players from the Australian men's team are as follows:

190 203 209 211 212

a Find the mean (μ) of the population.

b List all the possible samples of varying sizes from $n = 1$ to $n = 5$ of the population.

c Find the mean ($\bar{x}$) of each sample and then calculate the mean ($\bar{x}$) of the sample means.

d Are the population mean and the mean of the sample means the same?

3 Consider the population consisting of the weights (in kg) of three students:

68 74 72

a What is the mean (μ) of the population?

b Samples are taken from this population. Find the number of samples of size:

i $n = 1$ **ii** $n = 2$ **iii** $n = 3$

c List all the possible samples of varying sizes from $n = 1$ to $n = 3$.

d Find, correct to two decimal places, the mean of each sample and then calculate the mean ($\bar{x}$) of the distribution of the sample means.

4 The heights in metres of the 12 players in the 2012 Australian women's Olympic basketball team are as follows.

No.	Player	Height	No.	Player	Height
4	O'Hea	1.85	10	Harrower	1.66
5	Richards	1.68	11	Summerton	1.91
6	Screen	1.80	12	Snell	1.82
7	Bishop	1.89	13	Jarry	1.85
8	Batkovic	1.95	14	Cambage	2.03
9	MacLeod	1.68	15	Jackson	1.95

a Find, correct to two decimal places, the mean height of the basketball players.

b Select a random sample of size $n = 3$ by either using the table of random numbers on page 466 or a calculator to generate random numbers from 1 to 15. Then use the list in the table.

c Repeat the process to obtain samples of:

i $n = 4$ **ii** $n = 5$

d Find the mean of each sample from parts **b** and **c**.

e How do the means of the samples compare with the mean of the population?

f Find the mean of the means of the three samples and compare it with the population mean.

5 Twenty young people, aged 17 to 18, had the reaction time (in seconds) for their dominant hand recorded. The results are shown in the table.

Student	Time	Student	Time	Student	Time
1	0.16	8	0.37	15	0.29
2	0.42	9	0.37	16	0.24
3	0.34	10	0.35	17	0.28
4	0.43	11	0.54	18	0.51
5	0.40	12	0.35	19	0.48
6	0.32	13	0.31	20	0.34
7	0.52	14	0.45		

a Find the mean reaction time of the population.

b Using your calculator or the table of random numbers on page 466, select four samples of size $n = 5$.

c Find the mean reaction times of these four samples and calculate the mean of the sample means.

d Compare your results with the population mean.

i Can the sample means be used for an estimate of the population mean?

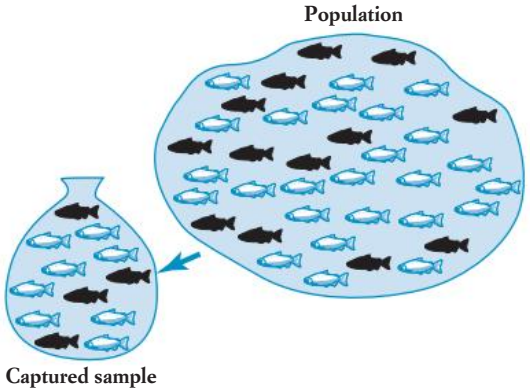
ii Is the mean of the sample means equal to the population mean?

e Repeat parts **b**, **c**, and **d** for four samples of size $n = 6$ and compare the results with those of sample size $n = 5$.

Capture - recapture cards

11-03 The capture-recapture technique

The **capture-recapture technique** is a way of estimating the size of a population. The technique was first applied to the study of fish and wildlife populations to monitor their size. It is used in the fishing industry to estimate fish populations in rivers, lakes and oceans and to monitor the number and size of each species that can be caught for commercial sale.



The capture-recapture technique involves capturing and tagging a sample of animals from the population, then releasing them back into their environment. After a period of time, another random sample is captured and the number of tagged animals counted. The size of the population is estimated using equivalent fractions, since both the captured and recaptured samples are representative of the population.

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EXAMPLE 5

A marine biologist needed to estimate the number of fish in a lake. She caught 120 fish, tagged them and returned them to the lake. Later, she caught 180 fish from the lake and found that 42 were tagged. Estimate the fish population of the lake.

Solution

Let P be the fish population of the lake.

In the (recaptured) sample, 42 out of 180 fish were tagged.

In the population, 120 out of P fish were tagged.

Compare the sample with the population, using equivalent fractions.

$$\frac{9}{\sin \mathcal{J}} = \frac{14}{\sin 106°}$$

$$\frac{j}{\sin J} = \frac{l}{\sin L}$$

Turn both fractions in the equation upside down, so that *P* is on top.

$$\frac{\sin \mathcal{J}}{\quad} = \frac{\sin 106°}{\quad}$$

Multiply both sides of the equation by 120.

$$\frac{10}{\sin \theta} = \frac{5}{\sin 22°}$$

There are about 514 fish in the lake.

Alternative method

Use this formula:

$$c = \sqrt{21.6076}$$

Note: The capture and recapture samples should be as large as possible. The larger the samples, the better they represent the population and the better the estimate will be.

Exercise 11-03 The capture-recapture technique

Example 5

1 From a lake, 100 rock lobsters were caught, tagged and returned. Six months later, a random sample of 80 lobsters was taken and 11 were found to be tagged. Which of the following values is the best estimate for the number of rock lobsters in the lake? Select **A**, **B**, **C** or **D**.

A 138 **B** 727 **C** 880 **D** 1375

2 From a forest, 30 deer were caught, tagged and released. Two months later, a sample of 160 deer was taken and 14 were found to be tagged. Estimate the deer population of the forest.

3 Tania randomly caught 200 blue starfish in a Fiji lagoon, then labelled and released them. Later that month, she collected 50 blue starfish from the same lagoon and eight were found to be labelled. How many blue starfish would you expect to find in this lagoon?

4 To estimate the kangaroo population in a forest, Casey tagged and released 160 kangaroos. Three weeks later, he took a sample of 120 kangaroos and found that 17 were tagged. Estimate the kangaroo population in the forest.

5 The fisheries department took 2000 freshwater salmon from a Tasmanian lake, tagged them and returned them. When a sample of 600 salmon was captured later, 12 were found to be tagged. Calculate the lake's salmon population.

6 To find the possum population in New Zealand's Otago province, 800 possums were caught and fitted with a microchip, then released. Some months later, 45 out of 640 possums were found to have a microchip. Estimate Otago's possum population correct to the nearest hundred.

7 Paula caught 15 grasshoppers in her backyard, then marked and released them. One week later she caught 65 grasshoppers and found that 20% were marked. Which of the following values is the best estimate for the number of grasshoppers in Paula's garden? Select **A**, **B**, **C** or **D**.

A 30 **B** 49 **C** 56 **D** 75

8 In the upper Cotter River region of the ACT, 1500 blackfish were caught, tagged and released. When 250 fish were later taken from the same region, 20 were found to be tagged. What was the estimated population of blackfish in the region?

9 To estimate the population of moose in Blackcomb Forest, a sample of 150 moose was captured, tagged and let loose. Two weeks later, another sample of moose was taken and 12% had a tag. Estimate the moose population of Blackcomb Forest:

a using the fraction for the recaptured sample

b using the unitary method.

10 An entomologist released a large number of butterflies into a forest after he had tagged 300 of them. When he later took a sample of 200, he found 15 of them had a tag. Approximately how many butterflies are there in the forest?

11 Fifteen wild rabbits were caught on a farm, marked and released. When 48 rabbits were recaptured, seven were found to be marked. Estimate the total number of rabbits on the farm.

12 Tim caught a large number of trout in Lake Eucumbene, and 15% were found to be tagged. If he had previously captured 4410 trout from the lake, and tagged and released them, estimate the number of trout in the lake:

a using equivalent fractions **b** using the unitary method.

13 Two hundred tiger snakes were captured on a property. They were marked with blue dye, and then released. When a new random sample of snakes was taken, 51 out of the 110 snakes captured had the blue mark. The local snake catcher estimated that there were about 400 tiger snakes on the property. Is he correct?

14 Bob caught 20 crocodiles from a river and 12 had a yellow tag. If 150 crocodiles were originally tagged, approximately how many crocodiles were in the river?

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INVESTIGATION

Sample size and estimating populations

Capture-recapture cards

Are large samples really better than small samples for estimating the size of a population?

We will use the 'capture-recapture' technique to estimate the number of fish in a lake.

You will need: Scissors, a coloured pen, and a paper bag or a cardboard box.

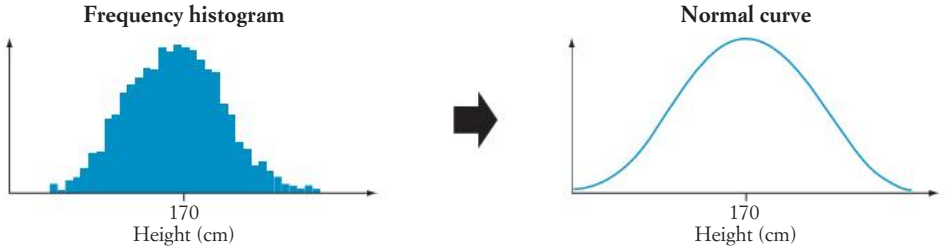
1. Work in groups of two to four students. Download the worksheet 'Capture-recapture cards' and print out fish picture cards or make them yourself. Photocopy the cards to make over 200 fish and cut them out. Alternatively, you may use toothpicks to represent the fish.
2. Ask your teacher, or another person from outside your group, to place most of the fish into the paper bag (or box), simulating the unknown number of fish in the lake. Make sure that you, and your group, cannot see how many fish are in the bag. Your task will be to use the 'capture-recapture' technique to estimate the fish population.
3. Take a random sample of 50 fish from the bag and 'tag' them all by marking them with a big cross. Return the fish to the bag and mix them up.
4. Copy this table.

Recapture sample size	Number tagged	Estimate of fish population
30		
60		
90		
120		
150		

5. Take a recapture sample of 30 fish from the bag and count how many are 'tagged'. Write this number in the correct cell in the middle column of the table and use the 'capture-recapture' method to calculate an estimate of the fish population. Write this estimate in the last column of the table.
6. Return the fish to the bag and take another recapture sample of 60 fish from the bag. Count how many are 'tagged' and calculate another estimate of the fish population. Write your results in the correct cell of the table.
7. Repeat Step **6**, using recapture samples of 90, 120 and 150 fish. Are all your estimates similar or different? Are your estimates getting closer to each other as you use larger samples?
8. To determine how good your estimates were, find the exact population by counting every fish in the bag.
9. How close were your estimates to the exact value? Did they improve as you increased the size of your recapture samples? Why or why not?

11-04 The normal distribution

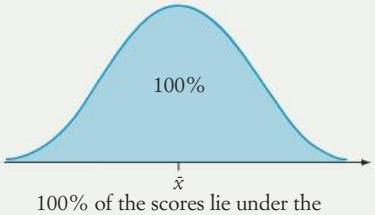
The graph of a **normal distribution** is the bell-shaped **normal curve**, which is smooth and symmetrical about its mean. The heights of a large **sample** of adults are represented by this frequency histogram with a mean of 170 cm. As the sample becomes larger and resembles the **population** of all adults, the histogram becomes closer to a normal curve.



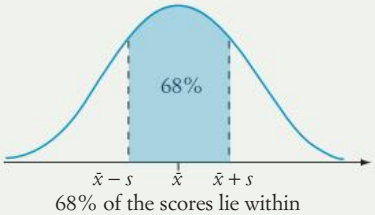
Many natural characteristics, such as height, blood pressure and skull width, are **normally distributed**. For any normal distribution, the mean, median and mode are equal.

SUMMARY

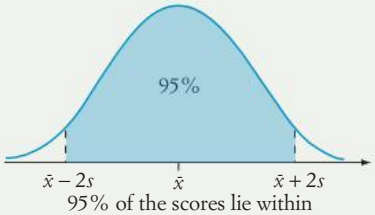
Percentages of scores under a normal curve



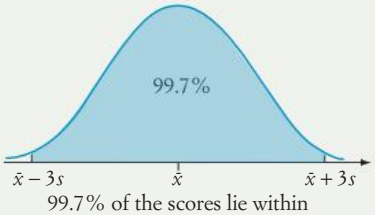
100% of the scores lie under the normal curve.



68% of the scores lie within 1 standard deviation of the mean.



95% of the scores lie within 2 standard deviations of the mean

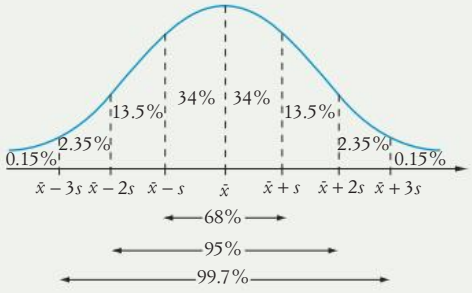


99.7% of the scores lie within 3 standard deviations of the mean.

The normal curve

This diagram shows the percentages under the normal curve in more detail. These can be calculated using the symmetry of the curve.

A full-page version of this diagram can be printed from the Worksheet 'The normal curve'.



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EXAMPLE 6

The heights of a large group of adults are normally distributed with a mean of 170 cm and a standard deviation of 8 cm.

a What percentage of heights:

i lie between 162 cm and 178 cm? **ii** are more than 178 cm?

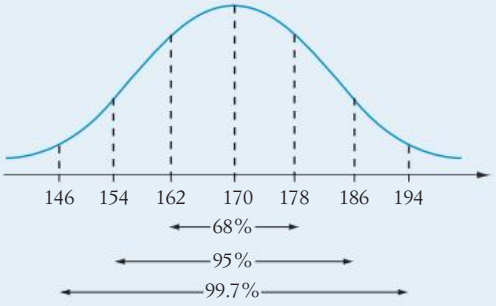
b If a random sample contains 6000 adults, how many have a height:

i between 146 cm and 194 cm? **ii** between 162 cm and 186 cm?

Solution

Mean $\bar{}$ = 170, standard deviation s = 8

Draw a diagram with values on the x-axis at different standard deviations from the mean.



a **i** 68%

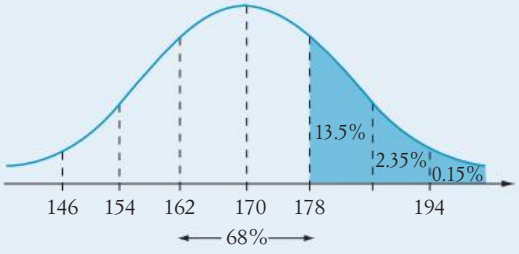
ii The required percentage is represented by the shaded area.

By symmetry of the curve, the percentage is

$$\cos\theta = \frac{4^2 + 6^2 - 3^2}{2 \times 4 \times 6}$$

Or 13.5% + 2.35% + 0.15% = 16%

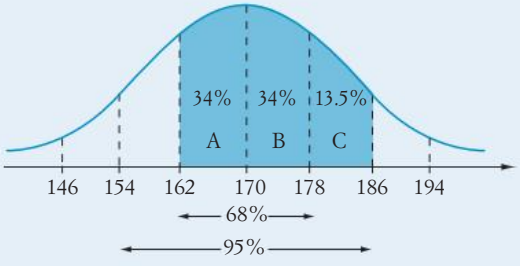
within one standard deviation of the mean



b **i** 99.7% × 6000 = 5982

ii The required percentage is represented by the shaded area A + B + C.

within 3 standard deviations of the mean



By symmetry, the percentage is

Area (A + B) and Area C

Or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area A and Area (B + C)

Or 34% + 34% + 13.5% = 81.5%

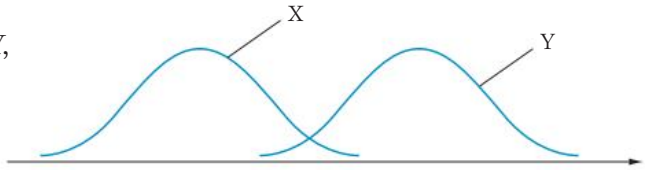
81.5% × 6000 = 4890

Area A + Area B + Area C

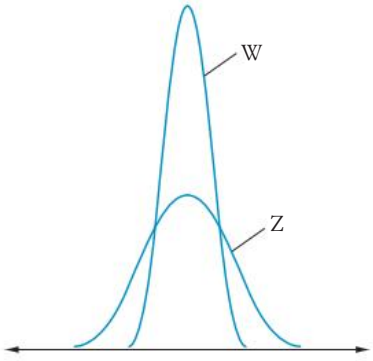
Exercise 11-04 The normal distribution

Hint: Draw a diagram.

1 **a** Which normal curve, X or Y, has the higher mean?



b Which normal curve, W or Z, has the higher standard deviation?



2 Roofing nails are produced by a machine and the lengths are normally distributed with a mean of 5.00 cm and a standard deviation of 0.01 cm. Complete the following.

a 68% of nails will have a length between _____ and _____ .

b 95% of nails will have a length between _____ and _____ .

c 99.7% of nails will have a length between _____ and _____ .

3 A factory makes circular brake discs with a mean diameter of 300 mm and a standard deviation of 2 mm. The diameters are normally distributed. If diameters more than 3 standard deviations from the mean are rejected, which of these diameters would be rejected? Select **A**, **B**, **C** or **D**.

A 295 mm **B** 307 mm **C** 306 mm **D** 296 mm

4 The nicotine content of a brand of cigarette is normally distributed with a mean of 14.5 mg and a standard deviation of 1.2 mg. What percentage of the nicotine contents are between 13.3 mg and 15.7 mg? Select **A**, **B**, **C** or **D**.

A 34% **B** 95% **C** 68% **D** 99.7%

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5 The expected life of light globes is normally distributed with a mean of 8000 hours.

a What is the standard deviation if 95% of globes are expected to last between 7700 and 8300 hours?

b Between what two values do 68% of the lifetimes lie?

c If 12 000 light globes are produced, how many are expected to last less than 7850 hours?

6 The skull widths of a group of adults are normally distributed with a standard deviation of 49 mm.

a What is the mean width if 2.5% of skull widths are less than 1297 mm?

b If there are 8000 adults in the group, how many have a skull width between 1297 mm and 1542 mm?

7 A machine produces steel pipes with a mean length of 148.4 mm and a standard deviation of 0.03 mm. The lengths are distributed normally.

a If pipes with a length more than 3 standard deviations from the mean are rejected, what percentage of pipes are rejected?

b One customer only accepts pipes with lengths between 148.34 mm to 148.43 mm. What percentage of pipes does she accept?

8 During pregnancy, the period from conception to delivery of a baby is called the gestation period. The gestation period for humans is approximately normally distributed with a mean of 266 days and a standard deviation of 16 days.

a Between what two values will 95% of gestation periods lie?

b What percentage of pregnancies will last more than 282 days?

c Is it possible to have a gestation period of 320 days? Justify your answer.

9 Packets of muesli have an advertised weight of 500 g. The actual weights were found to be normally distributed with a mean of 490 g and a standard deviation of 10 g.

a What percentage of packets would you expect to be below the advertised weight?

b Do you think this product is labelled fairly? Justify your answer.

10 Wedges of cheddar cheese are labelled as weighing 250 g. A survey found the weights to be normally distributed about a mean of 255 g with a standard deviation of 2.5 g.

a What percentage of wedges contained more than the labelled weight?

b If 6000 wedges were included in the survey, how many contained less than the labelled weight?

11 Packets of sugar are labelled as weighing 1 kg. When the weights of 400 packets were checked, they were found to be normally distributed with a mean of 1.015 kg and a standard deviation of 0.015 kg.

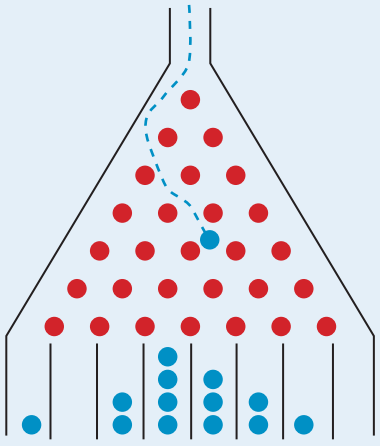
a What percentage of packets weighed less than the labelled weight?

b The packaging machine is to be reset so that the average weight of a packet is 1.030 kg. What percentage of packets will now weigh less than the labelled weight?

JUST FOR THE RECORD

Galton board

In 1890, English explorer and scientist Francis Galton designed a probability device called a Galton board, probability pinball, or bean machine. Balls fall through a system of pins into slots. Every time a ball hits a pin, it has an even chance of falling to the left or to the right of the pin. The piles of balls that accumulate in the slots at the bottom of the board form a histogram that is approximately a normal curve.



A demonstration of the Galton board can be viewed on www.youtube.com.

What is the probability that a ball lands in the first slot?

INVESTIGATION

IS THIS DATA NORMAL?

The stem-and-leaf plot displays the lifetimes, in hours, of two brands of batteries.

Betta Charge	Stem	Longa Life
5 5 4 2	10	3 4 5
8 7 7 7 7 7 7 4 4 3	11	2 3 3 4 4 5 6
9 9 8 8 7 6 6 6 5 4 4 0	12	1 2 2 3 3 4 5 5 7 9 9 9
8 8 8 7 7 7 6 5 4 3 1	13	0 2 3 3 4 4 4 5 6 6 8 8 9
9 8 8 8 5 5 2 2	14	1 2 2 3 5 5 6 7 8 8
7 7 5 1	15	0 3 3 4 6

1 Examine the data for Betta Charge batteries.

- **a** Find the mean, median and mode. Are they approximately equal?
- **b** Calculate the standard deviation.
- **c** Find the percentage of batteries with lifetimes within 1 standard deviation of the mean.
- **d** Find the percentage of batteries with lifetimes within 2 standard deviations of the mean.
- **e** Consider the percentages in parts **c** and **d** and decide if the data is normally distributed.

2 Are the lifetimes of Longa Life batteries normally distributed? Justify your answer.

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11-05 z-scores

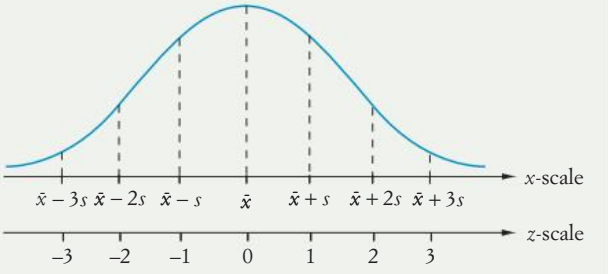
A ***z*-score** (**standardised score**) shows the position of a 'raw' score relative to the mean. For example, a z-score of 1.8 means a score is 1.8 standard deviations above the mean and a z-score of –0.5 means a score is half a standard deviation below the mean.

SUMMARY

The ***z*-score** is the number of standard deviations a score is from the mean.

$$\cos F = \frac{3.4^2 + 2.8^2 - 4.9^2}{2 \times 3.4 \times 2.8}$$

$$= \frac{-4.61}{19.04}$$



The distribution of z-scores is a normal distribution with a mean of 0 and a standard deviation of 1.

EXAMPLE 7

The measured amounts of milk in 600 mL cartons were found to be normally distributed with a mean of 600 mL and a standard deviation of 5 mL.

a Calculate the z-scores for the amounts:

i 610 mL **ii** 596 mL

b What amount of milk has a z-score of:

i 1.5? **ii** –2.6?

c What percentage of cartons have less than 590 mL of milk?

Solution

a $\bar{x} = 600$, $s = 5$, $A = \frac{1}{2}bh$

i $A = \frac{1}{2}ab \sin C$

$= 2$

ii

$= -0.8$

b **i**

$7.5 = x - 600$

$x = 607.5$ mL

ii

$-13 = x - 600$

$x = 587$ mL

c 590 mL is 2 standard deviations below the mean and has a z-score of –2.

Percentage of scores below $z = -2$ is

EXAMPLE 8

The net weights of packets of muffin mix have a normal distribution with a mean of 432 g and a standard deviation of 2 g.

a Complete the table.

Net weight (g)	428		432	434	
z-score		–1	0		2

b What percentage of packets will have a net weight:

i greater than 432 g? **ii** between 428 g and 434 g?

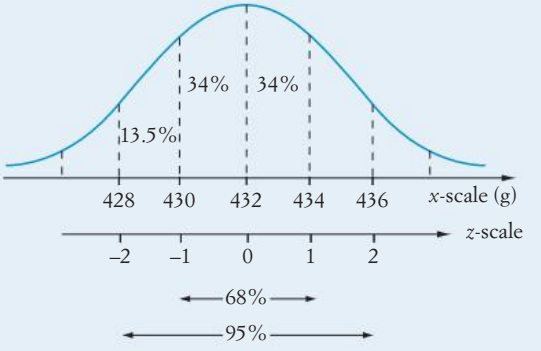
c If packets are labelled 'net weight 430 g', what percentage of packets will have a net weight less than 430 g?

Solution

a

Net weight (g)	428	430	432	434	436
z-score	–2	–1	0	1	2

b



i 50% **greater than $z = 0$**

ii **between $z = -2$ and $z = 1$**

Or 13.5% + 34% + 34% = 81.5%

c **less than $z = -1$**

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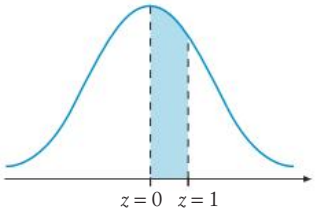
Exercise 11-05 z-scores

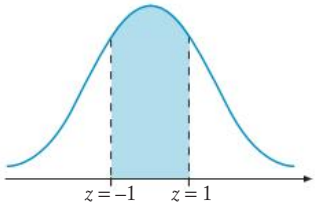
1 Explain the difference between a raw score and a z-score.

2 What proportion of scores is shaded in each of the following diagrams?

a



b



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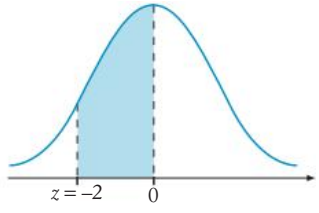
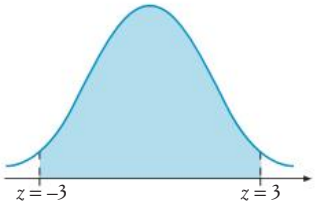


d



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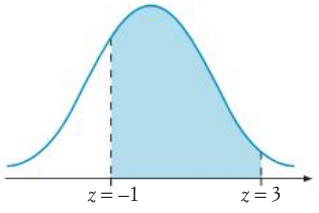

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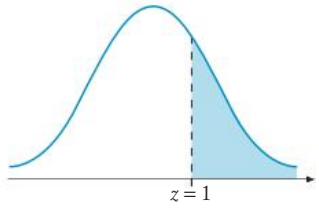

3 The ages of Bingo players are normally distributed. The mean age is 70 years and the standard deviation is 6 years.

Example 7

a If the ages are standardised, what is the z-score for each of the following players?

i Bill, 76 **ii** Mary, 52 **iii** Elsie, 91

b Esma's age has a z-score of 2.5. How old is she?

c What percentage of Bingo players are more than 82 years old?

d What percentage of players are between 64 and 88 years old?

4 In a test, the marks are normally distributed. The average mark is 70 and the standard deviation is 15. If Jordan has a z-score of 1.8, what is his actual mark? Select **A**, **B**, **C** or **D**.

A 97 **B** 85 **C** 27 **D** 87

5 Blood pressures are normally distributed with a mean of 130 and a standard deviation of 20. Sam has a z-score of –1 when his blood pressure is taken.

a Interpret this z-score in terms of the mean and standard deviation of blood pressures.

b What is Sam's blood pressure?

c If Zac has a blood pressure of 145, what is his z-score?

6 The heights of a group of preschool children are normally distributed with a mean of 90 cm and a standard deviation of 10 cm.

a Find the z-score, correct to one decimal place if necessary, for the following children.

i Jamal, 86 cm **ii** Maria, 1.25 m

b If Ann has a z-score of 2.2, how tall is she?

c Drew has a z-score of –1.4. How tall is he?

d What percentage of children are between 70 cm and 90 cm tall?

e What percentage of children are more than 1 m tall?

Example 8

7 Jelly beans are put into 100 g packets. The machine that performs this task is set to a mean mass of 100 g with a standard deviation of 2 g.

a Complete the table.

Mass (g)	94		98	100		104	
z-score		–2		0	1		3

b What percentage of packets will have a mass:

i more than 100 g? **ii** between 98 g and 102 g? **iii** less then 96 g?

c The masses of 2500 packets are noted. How many are expected to be greater than 104 g?

8 IQs (intelligence test scores) are normally distributed with a mean of 100 and a standard deviation of 16.

a What are the z-scores for IQs of:

i 100? **ii** 132?

b If 30 million people live in Australia, how many are expected to have an IQ above 132?

c To join the Four Sigma Club you need an IQ with a z-score of 4 or more.

i What is meant by a z-score of 4? **ii** What IQ is this?

9 The weights of flathead fish caught in Currumbene Creek are normally distributed with a mean of 1.2 kg and a standard deviation of 0.4 kg.

a Complete the table.

Weight (kg)		0.4		1.2		2.0	
z-score	–3		–1		1		3

b What percentage of flathead weigh between 0.4 kg and 1.6 kg?

c If 4800 flathead were caught one season, how many weighed over 2 kg?

d If flathead weighing less than 0.8 kg are returned to the creek, how many of the 4800 flathead would be returned?

10 Heidi sat for two Maths tests. The test results of Year 12 are normally distributed for both tests and the means and standard deviations are shown in the table.

	Trigonometry	Probability
Mean	64	59
Standard deviation	6.2	6.1

a If Heidi scored 65 in Trigonometry and 70 in Probability, what are her z-scores?

b If 56 students sat for Trigonometry, how many are expected to score less than 70.2?

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11-06 Comparing z-scores

Since z-scores do not depend on the value of the mean, we can use them to compare scores from different normal distributions.

z-scores

EXAMPLE 9

Ashleigh's marks for Maths, Science and History, as well as the mean and standard deviation of her class' marks for each subject, are given in the table.

Subject	Ashleigh's mark	Class mean	Class standard deviation
Maths	80	75	16
Science	60	54	12
History	70	78	10

a Find Ashleigh's z-score for each subject.

b In which subject did she perform best? Justify your answer.

Solution

a

Subject	Ashleigh's mark	Ashleigh's z-score
Maths	80	$C = \dfrac{a^2 + b^2 - c^2}{2ab}$
Science	60	$\dfrac{u}{\sin 21°} = \dfrac{18.2}{\sin 105°}$
History	70	$u = \dfrac{18.2 \sin 21°}{\sin 105°}$

b Science, because she had the highest z-score for this subject.

Exercise 11-06 Comparing z-scores

1 Ziad scored 78% on a Maths test and Nina scored 83% on an English test. Both students gained a z-score of 1.3. Who performed better? Select **A**, **B**, **C** or **D**.

A Nina

B Ziad

C both performed the same

D can't tell

2 Here are Jude's marks for her half-yearly exams, along with the class mean and standard deviation for each subject.

Subject	Mark	Mean	Standard deviation
English	65	75	14
Maths	72	65	10
Science	58	62	16
Art	78	85	8

a Find Jude's z-scores for each subject.

b Rank her subjects in order of her performance compared with the other students.

3 Test scores in three subjects are approximately normally distributed with means and standard deviations as shown. The raw scores for three students are given.

Subject	Peta	Mel	Joe	Mean	Standard deviation
Visual Arts	48	60	70	66	16
Legal Studies	67	82	74	68	12
Maths General 2	70	79	55	70	9

a Calculate the z-scores for each student.

b What is Mel's best subject?

c Which is better: Peta's mark in Maths General 2 or Joe's mark in Visual Arts?

d Who achieved the best result overall? What subject was this in?

4 Pattie and Jim each joined a Get Slim program. The weights of participants in each program are normally distributed.

	Mean weight	Standard deviation
Male group	93	7.5
Female group	83	6

a If Pattie weighs 86.4 kg and Jim weighs 97.2 kg, who is more overweight for their group? Justify your answer.

b If Jill joined the women's group and has a z-score of 2, how much does she weigh?

c Ty joined the men's group. If his z-score is 2, how much more than Jill does he weigh?

5 Annual rainfall figures are normally distributed. In Downtown, the annual rainfall distribution has a mean of 950 mm and a standard deviation of 420 mm. In Uptown, the mean annual rainfall is 1325 mm with a standard deviation of 238 mm. Which town is more likely to have an annual rainfall of 1100 mm? Justify your answer.

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6 The 2-year-old model of a particular car has an average value of $18 000 with a standard deviation of $1500. The 3-year-old model of the same car has an average value of $15 000 with a standard deviation of $1600. Which is the better car: a 2-year-old model with a value of $17 000 or a 3-year-old model with a value of $13 000?

INVESTIGATION

SCALING OF EXAM MARKS

In order to compare the results of different subjects, an examiner uses z-scores to scale raw exam marks to a common mean and standard deviation.

If Ashleigh's exam marks for each subject in Example **9** were scaled to a common mean of 65 and standard deviation of 12, her **scaled mark** in Maths would be:

$$a = \frac{80 \sin 11^\circ}{\sin 25^\circ}$$

$$\frac{a}{\sin 11^\circ} = \frac{80}{\sin 25^\circ}$$

$$3.75 = x - 65$$

$$x = 68.75$$

$$= 69$$

Show that Ashleigh's other scaled marks are 71 for Science and 55 for History.

Sample HSC problem

The HSC marks for Modern History follow a normal distribution with a mean of 65 and a standard deviation of 10.

a Haylee's mark in Modern History was 59. What was her z-score?

b What percentage of Modern History marks lie within the 55 to 65 range?

c Raad achieved a z-score of 2.3 for Modern History.

i Interpret this z-score in terms of the mean and the standard deviation.

ii What was Raad's actual mark in Modern History?

Study tip

When answering an exam question ...

- make sure that you show enough working
- note the number of marks allocated to the question: this indicates the amount of working or steps required
- choose the simplest and quickest method
- check that your answer sounds reasonable, especially if it involves money or measurement
- if it is a measurement question, don't forget the units (for example, mm, cm^2, hours, m^3)
- if it is a money question, don't forget the dollars and cents
- if it is an equation, don't forget the pronumeral: write '$x = 7$', not just '7'
- don't round answers too early—only round the final answer
- read the question again to make sure that you have actually answered it
- when writing worded answers, use correct terminology and write 1–2 sentences, not mini-essays
- use a ruler to draw large, neat graphs with labels and axes marked.

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CHAPTER REVIEW

Topic overview

This chapter, Sampling and the normal distribution, extended earlier work on sampling and the shape of a statistical distribution. You should be able to list all possible samples from a known small population, generate random numbers to create random samples, use the capture-recapture technique, and be able to draw conclusions about a population from a sample. We also examined the normal distribution and its properties and compared scores from different normal distributions. You should be able to quote the 68%–95%–99.7% limits of the normal curve, calculate *z*-scores and compare data using *z*-scores.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders to gain a 'whole picture' view of the topic and identify any weak areas.

Revision

Exercise 11-01

1 Bags of lollies are checked for weight. Every 100th bag of lollies is weighed.
Which type of sampling is this? Choose **A**, **B**, **C** or **D**.

A Stratified **B** Census **C** Systematic **D** Random

Exercise 11-01

2 Twenty people were surveyed about how they usually travelled to work. The results are as follows.

Person	Mode of travel	Person	Mode of travel
1	Walk	11	Bus
2	Bus	12	Walk
3	Bus	13	Bus
4	Car	14	Car
5	Car	15	Car
6	Car	16	Walk
7	Car	17	Bus
8	Bus	18	Car
9	Walk	19	Car
10	Bus	20	Bus

a What percentage of people travelled to work by:
i bus **ii** car **iii** walking?

b Use the table of random numbers on page 466 or your calculator to generate three random samples of size $n = 5$, $n = 6$, and $n = 8$

c For each sample collected in part **b**, find the percentage of people who travel by:
i bus **ii** car **iii** walking.

d Which sample best reflects the population of people who travel to work?

e Which mode of travel has not been included? Give reasons.

Exercise 11-02

3 The arm span (in cm) of five students in Year 9 are:
190 161 170 162 176

a List all possible samples for size $n = 1$ to $n = 2$ and calculate their means.

b Find the mean of the distribution of the sample means and verify that this is equal to the population mean.

Exercise 11-03

4 Anita had to estimate the number of kangaroos in a reserve using the 'capture-recapture' technique. She caught 40 kangaroos, tagged and then released them. A week later, she caught 100 kangaroos at random and found that 12 had been tagged. Estimate the number of kangaroos in the reserve.

Exercise 11-04

5 Cylindrical rods are produced by a machine and the diameters are normally distributed with a mean of 4.00 cm and a standard deviation of 0.02 cm. Complete the following.

a 68% of rods will have a diameter between _____ and _____ .

b 95% of rods will have a diameter between _____ and _____ .

c 99.7% of rods will have a diameter between _____ and _____ .

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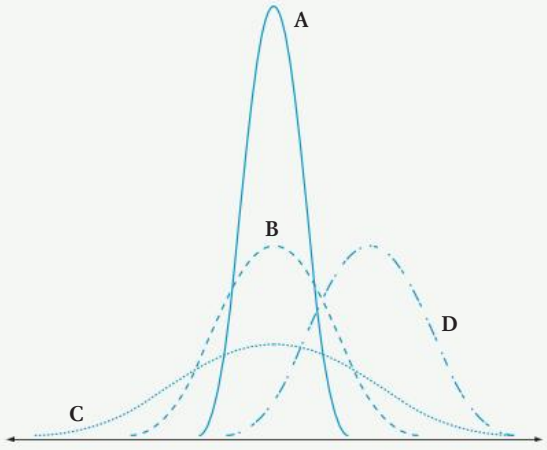
6 A machine produces bolts with a mean length of 75 mm and a standard deviation of 2 mm. The lengths are normally distributed. If lengths more than 3 standard deviations from the mean are rejected, which of these bolt lengths would be rejected? Select **A**, **B**, **C** or **D**.

Exercise 11-04

A 70 mm **B** 79 mm **C** 81 mm **D** 88 mm

7 The diagram shows curves representing four normal distributions: **A**, **B**, **C** and **D**.



Exercise 11-04

a Which distribution has the largest standard deviation?

b Which distribution has the smallest standard deviation?

c Which distributions have the same mean?

d Which distributions have the same standard deviation?

8 **a** Leonardo's z-score for his Art assessment task was 2.5. What does this mean?

Exercise 11-05

b If the task mean was 68 and Leonardo's raw score was 82, what was the task standard deviation?

c What was Michaelangelo's z-score if he scored 62 for the task?

d What was Raphael's score if his z-score was 0.8?

9 The time taken to repair computer components was found to be normally distributed with a mean of 8 hours and standard deviation of 2 hours.

Exercise 11-05

Time (h)			6	8		12	
z-score	–3	–2		0	1		3

a Complete the table.

b What percentage of repairs took:

i between 10 and 14 hours? **ii** less than 6 hours?

iii more than 12 hours? **iv** between 4 and 10 hours?

10 Tom's test scores with the class mean and standard deviation are shown in the table. The scores are normally distributed.

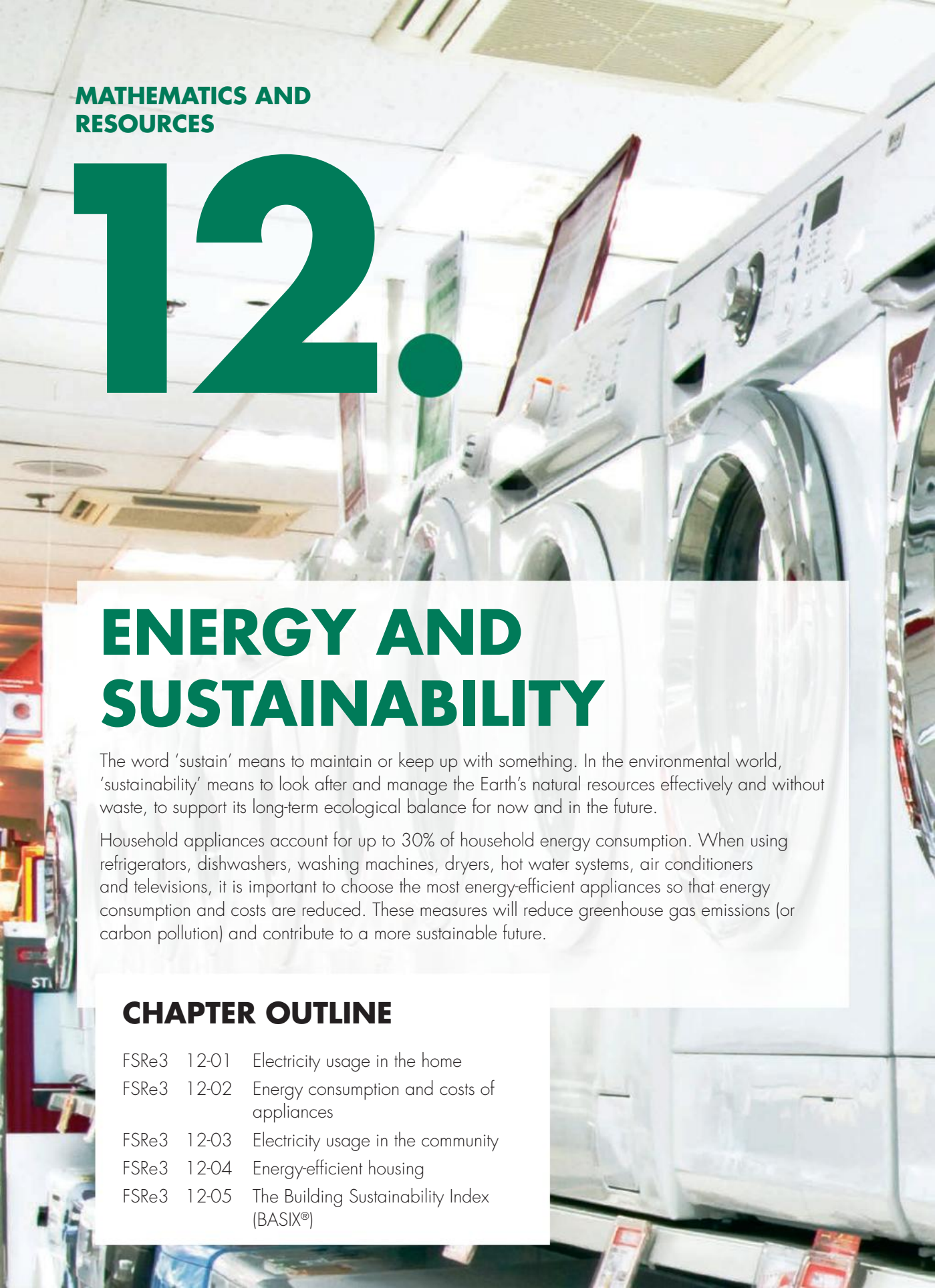
Exercise 11-06

Subject	Score	Class mean	Class standard deviation
Drama	80	75	16
Photography	60	54	12
Textiles	70	78	10

a Find Tom's z-score for each subject.

b In which subject did Tom perform best? Justify your answer.

c Show the position of Tom's z-scores on a standard normal curve.



MATHEMATICS AND RESOURCES

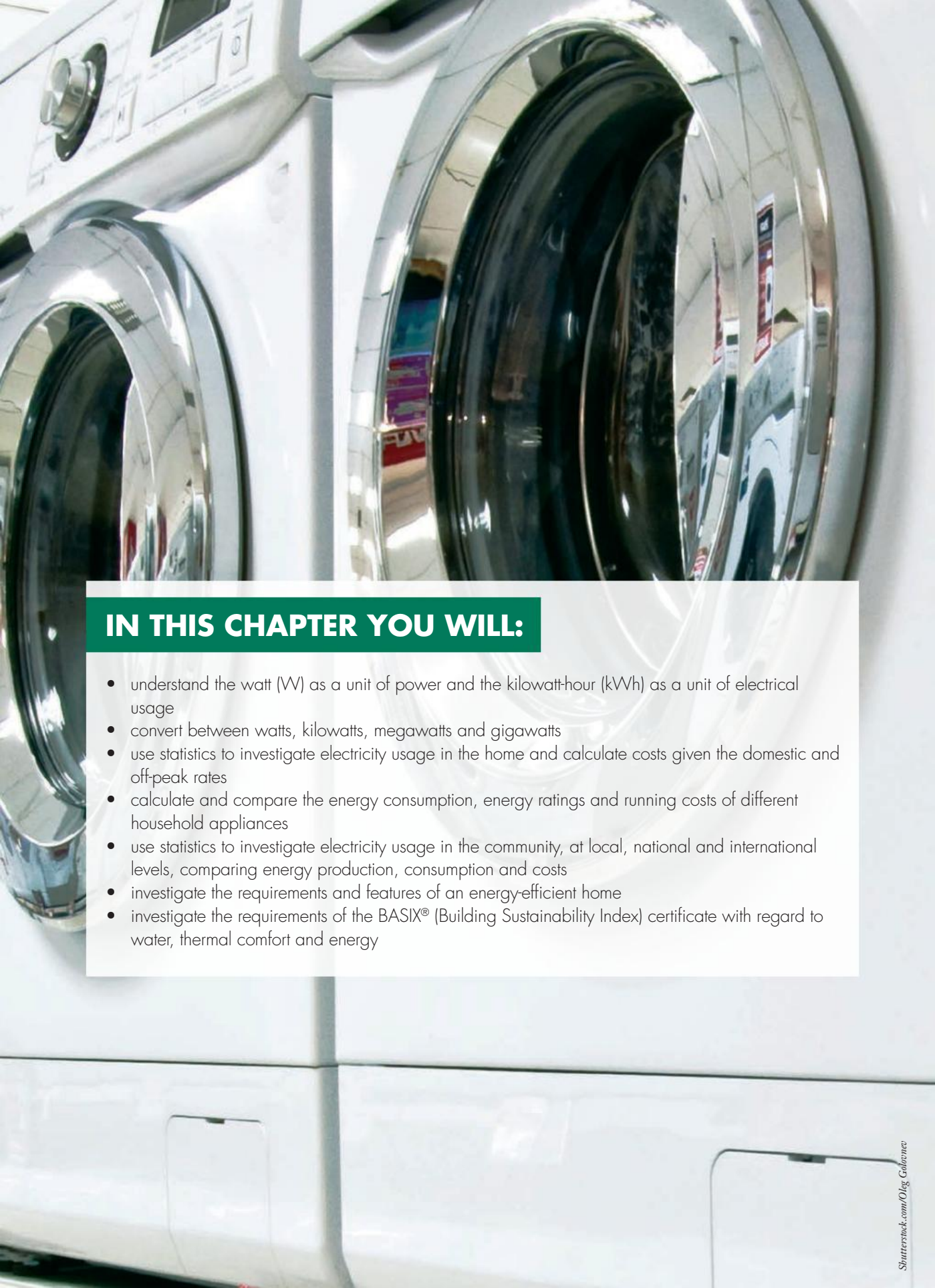
12.

ENERGY AND SUSTAINABILITY

The word 'sustain' means to maintain or keep up with something. In the environmental world, 'sustainability' means to look after and manage the Earth's natural resources effectively and without waste, to support its long-term ecological balance for now and in the future.

Household appliances account for up to 30% of household energy consumption. When using refrigerators, dishwashers, washing machines, dryers, hot water systems, air conditioners and televisions, it is important to choose the most energy-efficient appliances so that energy consumption and costs are reduced. These measures will reduce greenhouse gas emissions (or carbon pollution) and contribute to a more sustainable future.

CHAPTER OUTLINE



IN THIS CHAPTER YOU WILL:

- understand the watt (W) as a unit of power and the kilowatt-hour (kWh) as a unit of electrical usage
- convert between watts, kilowatts, megawatts and gigawatts
- use statistics to investigate electricity usage in the home and calculate costs given the domestic and off-peak rates
- calculate and compare the energy consumption, energy ratings and running costs of different household appliances
- use statistics to investigate electricity usage in the community, at local, national and international levels, comparing energy production, consumption and costs
- investigate the requirements and features of an energy-efficient home
- investigate the requirements of the BASIX® (Building Sustainability Index) certificate with regard to water, thermal comfort and energy

TERMINOLOGY

appliance
consumption
energy-efficient
incandescent light bulb
kilowatt-hour (kWh)
off-peak rate
standby power
thermal comfort
Building Sustainability Index (BASIX®)
domestic rate
energy rating
insulation
LED light
renewable energy
sustainability
watt (W)
compact fluorescent lamp (CFL)
downlight
gigawatt (GW)
kilowatt (kW)
megawatt (MW)
running cost
terawatt (TW)
watt-hour (Wh)

Assignment 12

SkillCheck

1 Calculate each amount correct to the nearest cent.

a $226 \times \$0.072\,800$ **b** $173.4 \times \$0.084\,700$ **c** $290.578 \times \$0.218\,500$

2 The quarterly electricity bill was \$423.85, before GST. Calculate the electricity bill after 10% GST is added, correct to the nearest cent.

3 How many days are there from 05/05/13 to 03/08/13?

4 The electricity account for the period 05/08/14 to 09/11/14 was \$468.24. What was the average daily cost of electricity usage?

5 Convert:

a 14 528 m to km **b** 645 ML to L

6 Write 241 500 000 in scientific notation.

Power problems

12-01 Electricity usage in the home

Units of power and energy usage

The **watt** (abbreviation **W**) is a unit of **power** and it is equal to one joule of energy per second.

Unit	Relationships
microwatt (μW)	$1\ \mu W = \frac{1}{1\,000\,000}\ W = 10^{-6}\ W$
milliwatt (mW)	$1\ mW = \frac{1}{1000}\ W = 10^{-3}\ W$
watt (W)	$1\ W = 1000\ mW = 1\,000\,000\ \mu W$
kilowatt (kW)	$1\ kW = 1000\ W$
megawatt (MW)	$1\ MW = 1000\ kW = 1\,000\,000\ W$ or $10^6\ W$
gigawatt (GW)	$1\ GW = 1000\ MW = 1\,000\,000\,000\ W$ or $10^9\ W$
terawatt (TW)	$1\ TW = 1000\ GW = 10^{12}\ W$
petawatt (PW)	$1\ PW = 1000\ TW = 10^{15}\ W$

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SUMMARY

Electrical energy usage is measured in **watt-hours (Wh)**, which is the amount of electrical energy used by a one-watt load (such as a light bulb) drawing power for one hour.

The electricity usage for households is measured in **kilowatt-hours (kWh)**, equal to 1000 watt-hours. The **cost** of electricity usage is given in **cents/kilowatt-hour** (c/kWh).

EXAMPLE 1

Convert:

a 8350 W to kW **b** 35 000 MW to W **c** 152 600 GW to kW

Give answers in scientific notation where necessary.

Solution

a $8350\text{ W} = 8350 \div 1000\text{ kW}$
$= 8.35\text{ kW}$
1 kW = 1000 W

b $35\,000\text{ MW} = 35\,000 \times 1000\,000\text{ W}$
$= 35\,000\,000\,000\text{ W}$
$= 3.5 \times 10^{10}\text{ W}$
1 MW = 1 000 000 W

c $152\,600\text{ GW} = 152\,600 \times 1\,000\text{ MW}$
$= 152\,600 \times 1000 \times 1000\text{ kW}$
$= 152\,600\,000\,000\text{ kW}$
$= 1.526 \times 10^{11}\text{ kW}$
1 GW = 1000 MW
1 MW = 1000 kW

Domestic and off-peak electricity

Domestic electricity refers to electricity that is used during the day.

Off-peak electricity refers to electricity that is used during the late evening and early morning, such as between 11 p.m. and 7 a.m., when the demand from households and businesses is much lower. Off-peak electricity is charged at a cheaper rate than domestic electricity, and is often used to heat water in homes.

EXAMPLE 2

Calculate the cost of electricity usage for a household where domestic usage is 954 kWh at \$0.284 500/kWh and off-peak usage is 748 kWh at \$0.099 370/kWh.

Solution

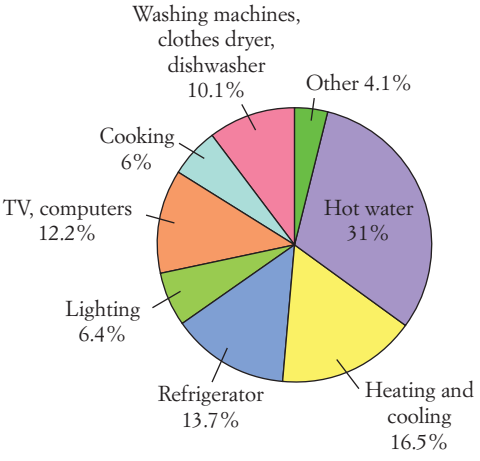
Cost of domestic usage = $954 \times \$0.284\,500$
$= \$271.413$
$\approx \$271.41$

Cost of off-peak usage = $748 \times \$0.099\,370$
$= \$74.328\,76$
$\approx \$74.33$

Total cost = \$271.41 + \$74.33
= \$345.74

Exercise 12-01 Electricity usage in the home

1 The sector graph shows the energy use for an average household in Sydney.



a What fraction of energy use is for cooking?

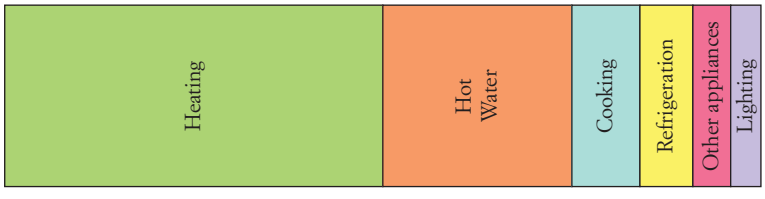
b What could be included in the 'Other' sector?

c If the household bill for 90 days is \$562.37, how much did 'TV, computers' cost?

d If lighting costs \$0.67 per day, what would the total electricity bill be for 90 days?

e Calculate, to the nearest degree, the angle size of the 'Hot water' sector.

2 The divided bar graph below shows the electricity use for an average Tasmanian household.



a What fraction of electricity is used for hot water?

b What percentage of electricity is used for heating?

c If the average Tasmanian household uses 10 000 kWh of electricity per year, how much electricity is for lighting?

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3 Express each measurement of power in scientific notation.

Example 1

a 5 700 000 MW **b** 3 000 000 kW **c** 4 845 000 000 GW

4 Which of the following is 6 840 000 MW expressed in scientific notation, correct to two significant figures? Select **A**, **B**, **C** or **D**.

A 6 800 000 MW **B** 6.9×10^6 MW **C** 6.84×10^6 MW **D** 6.8×10^6 MW

5 Convert:

a 5600 W to kW **b** 27 500 000 kW to MW **c** 3 495 000 mW to kW

6 How many kW are there in 8 450 000 W? Select **A**, **B**, **C** or **D**.

A 845 kW **B** 8.45 kW **C** 8450 kW **D** 84 500 kW

7 Convert 2.35 GW to:

a megawatts **b** kilowatts **c** watts

Example 2

8 This is an electricity bill from Lightning Energy.

Lightning Energy

Account Number: 71433973
Date Due: Friday 03 May 2015
Amount Due: $165.20

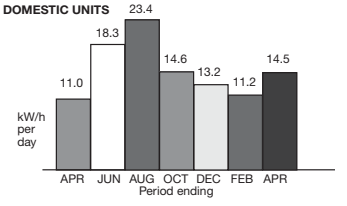
Ms B. SPARKS
13 AMPERE CRESCENT
LIGHTNING RIDGE, NSW 2834

ELECTRICITY ACCOUNT

Summary of Your Account

Description	Amount
Balance of last account	$148.55
Less payment(s) received to 13/03/15	$148.55 CR
Subtotal	$0.00
Amount of this account from 24/02/15 to 21/04/15	$165.20
Total Amount Due	**$165.20**

Your Electricity Usage



Never use more than one adaptor on the one power outlet. Better still use a power board to plug in additional low current applications as they have their own safety overload cut out switch.

Itemised Details

Meter information for period 24/02/15 to 21/04/15 — 56 days

Meter No.	This Read	Previous Read	Days	Usage (kWh)
0874221	72822	72009	56	813

Your Account Calculations

Pricing Option	Units Used by Rate	Unit Rate	Amount
DOMESTIC RATE	185	c24.950	$46.16
	628	c18.96	$119.07
		Total Electricity	**$165.23**

Plus Other Items

Rounded per Govt. Guideline	$0.03 CR
Total Amount of this Account	**$165.20**
Total Amount Due	$165.20

Your Itemised Payments

13/03/15	$148.55 CR
Total Payments	$148.55 CR

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a Is this bill paid monthly, two-monthly or quarterly?

b What was the average daily electricity usage over this period?

c From the graph, find the time of the year when electricity usage was the greatest. Why do you think this is so?

d How much electricity was used in the period covered by this bill?

e How many days of usage was covered by this bill?

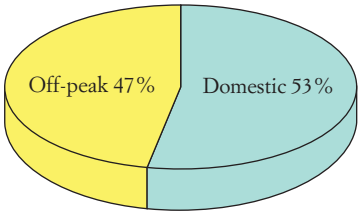
f When was the electricity meter read? What was the reading?

g Has the electricity bill increased or decreased since the last bill? By how much?

h Why do you think a rebate (discount) is 'available to eligible pensioners and users of approved life support machines'?

i There are two cost rates of electricity usage: one for the first 185 kWh, the other for the remainder. What are these rates and in what units should they be expressed?

9 The graph on the right shows the percentages of domestic and off-peak electricity usage for a Campbelltown household over a 96-day period.



a If the total usage was 3022 kWh, how many kilowatt hours was off-peak usage?

b Calculate the cost of:

i off-peak usage at \$0.092 800/kWh

ii domestic usage at \$0.265 800/kWh

c What was the average daily cost for domestic usage?

10 The following questions refer to the electricity bill shown on the next page.

a What was the average daily domestic electricity usage over the period of this bill?

b From the graph, find the time of the year when electricity usage was the greatest. Why do you think this is so?

c An extra fee is charged for both off-peak and domestic usage. What is this extra fee for and what is its new daily rate for off-peak?

d The cost of electricity increased over this billing period. How is this shown on the bill?

e Did off-peak and domestic usage rates increase at the same time? Give reasons for your answer.

f By what percentage did the cost for domestic usage increase?

g Find the percentage increase for off-peak usage charges. By how much was this increase greater than or less than the increase in domestic usage charges?

h By how much has electricity usage increased or decreased when compared to the same period the previous year?

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NCM Energy

Account Number: 56007899
Date Due: Thursday 23 August 2014
Amount Due: $215.59

MR B POWER
125 FARADAY DR
PICTON 2571

FOR ACCOUNT ENQUIRIES: PHONE 131 002
FOR 24HR EMERGENCY ASSISSTANCE: PHONE 131

ELECTRICITY ACCOUNT

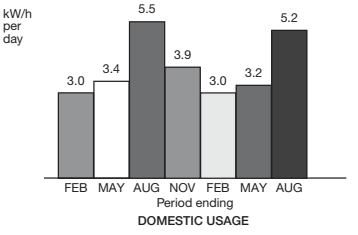
Issue Date: 03/08/2014

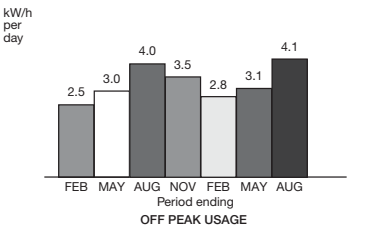
Summary of Your Account

Description		Amount
Balance of last account		$142.15
Less payments and rounding		$142.15 CR
Subtotal		$0.00
Account Summary – 05/05/14 to 03/08/14		
Electricity charges (see next page for details)*	$196.02	
Plus GST payable	$19.57	
Total amount of this account (including GST)		$215.59
Total Amount Due (including GST)		**$215.59**

*GST applies to this item

Your Electricity Usage





Itemised Details

Meter information for Period 05/05/14 to 03/08/14 – 90 days

Meter No.	Const	This Read	Previous Read	Days	Usage (kWh)
1234530	1	27812	27443	90	369
OFF PEAK – Total Usage					369
1234532	1	28559	28092	90	467
DOMESTIC – Total Usage					467

Your Account Calculations

Pricing Option	GST	Usage Applies	Rate	Amount
OFF PEAK				
Off Peak 1	Yes	229.600	$0.072800	$16.72
Off PeakAccess Chg $0.046000	Yes	56 days		$2.58
New Rate Period				
Off Peak 1	Yes	139.400	$0.084700	$11.81
Off PeakAccess Chg $0.047200	Yes	34 days		$1.60
DOMESTIC				
Domestic	Yes	290.578	$0.218500	$63.49
System Access Chg $0.598500	Yes	56 days		$33.52
New Rate Period				
Domestic	Yes	176.422	$0.242500	$42.79
System Access Chg $0.691400	Yes	34 days		$23.51
Electricity subject to GST				**$196.02**

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11 The pricing information for an electricity supplier is given below.

Tariff Type	
Off Peak (cents/kWh)	9.3170
Supply charge (cents/day)	5.1920
Domestic First 1750 kWh/qtr (cents/kWh)	26.6750
Balance (cents/kWh)	29.8210
Supply charge (cents/day)	76.0540

Calculate the electricity bill for a Penrith household with off-peak usage of 984 kWh and domestic usage of 1975 kWh for a 90-day period.

JUST FOR THE RECORD

Watts it all about?

Microwatts are used to measure power from solar cells used in calculators and watches.

A person's hearing aid uses less than one **milliwatt** while a laser pointer outputs about 5 milliwatts of light power.

A wind turbine has a power capacity of 1 to 3 **megawatts**, while a lightning strike may peak at a **terawatt**.

The total power of sunlight striking the Earth's atmosphere is 170–180 **petawatts**.

How much power is generated by a person climbing a flight of stairs?

A highly-trained athlete sprinting 100 metres? A car engine while cruising?

WS Electrical appliances

12-02 Energy consumption and costs of appliances

Heating and cooling make up almost a quarter of a home's energy use. The biggest users of energy in Australian homes are hot water systems. To reduce costs and improve efficiency you need to know how and where energy is used in the home.

EXAMPLE 3

How much energy is used (in kWh) when a 100 W light bulb is on for 5 hours?

Solution

Energy used in 5 h $= 100 \text{ W} \times 5 \text{ h}$

$= 500 \text{ Wh}$

$= 500 \div 1000 \text{ kWh}$ **1 kWh = 1000 Wh**

$= 0.5 \text{ kWh}$

ISBN 9780170238977

Running costs of electrical appliances

You can calculate the running cost of an electrical appliance by multiplying the power used (in kWh) by the cost of electricity.

Household energy costs

SUMMARY

Running cost = power used (kWh) × electricity cost ($/kWh)

EXAMPLE 4

Tamin takes about $1\frac{1}{2}$ hours per week to iron his clothes. If he uses a 2200 W iron and the cost of electricity is $0.230 200/kWh, calculate the cost of using the iron in one year.

Solution

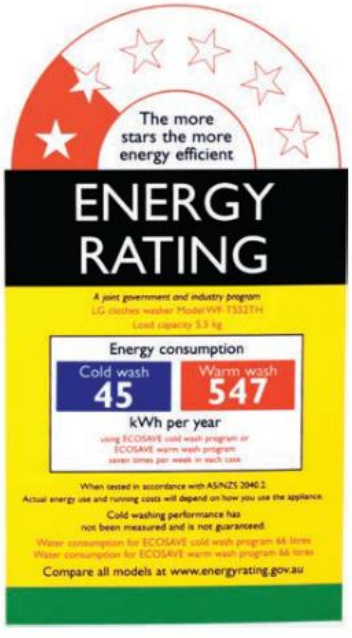
Power used in one year = 2200 W × 1.5 × 52
= 171 600 Wh
= 171.6 kWh

Annual cost = 171.6 × $0.230 200
= $39.502 32
≈ $39.50

Energy ratings

Energy ratings

All household appliances now display **Energy Rating Labels (ERL)** to help people choose energy-efficient appliances. These include televisions, washing machines, clothes dryers, refrigerators, freezers, dishwashers, air conditioners and heaters. These labels contain a star rating to show how efficient the appliance is – the more stars, the more efficient the estimated annual energy use in kWh per year. A standard label has 6 stars but for more efficient appliances there is a label that goes up to 10 stars. The label also lists an estimate of the power (in kWh) the appliance will use over a year.



Alamy/Mark Boulton

Hot water systems

Water heating can account for up to 30% of the energy used in a home. The types of hot water systems (storage and instantaneous) available are solar, heat pumps, natural gas and off-peak electric. Around half of Australia's eight million households use electric hot water systems, but these are no longer allowed in new homes as they produce three times the amount of greenhouse gases as low-emission systems like solar, heat pump and high efficiency gas.

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Lighting

7% of the electricity used in NSW homes is for lighting. The main types of light bulbs used in homes are incandescent (traditional-style), halogen (used in downlights), compact fluorescent lamp (CFL) and LED.

EXAMPLE 5

An incandescent 75W light bulb and the equivalent compact fluorescent 15W lamp are used for 5 hours a day. Compare the cost of using both bulbs over a year if electricity is charged at \$0.218 500/kWh.

Solution

Incandescent light bulb:

Electricity used in one year $= 75 \times 5 \times 365$
$= 136\,875$ Wh
$= 136.875$ kWh

Cost of electricity used $= 136.875 \times \$0.218\,500$
$= \$29.9071875$
$\approx \$29.91$

Compact fluorescent lamp:

Electricity used in one year $= 15 \times 5 \times 365$
$= 27\,375$ Wh
$= 27.375$ kWh

Cost of electricity used $= 27.375 \times \$0.218\,500$
$= \$5.981\,437\,5$
$\approx \$5.98$

Over a year, the cost of using the compact fluorescent lamp is approximately one-fifth of the cost of the incandescent light bulb.

Appliances

Washing machines, clothes dryers, fridges and freezers, televisions, dishwashers and other appliances can account for 25% of a household's electricity bill. Choosing energy-efficient products can reduce power use and running costs.

EXAMPLE 6

Calculate the lifetime running cost of the following televisions. Use 30.00 cents/kWh as an estimate for the cost of electricity and assume that the average lifetime of major appliances is 10 years.

a A Sony 138 cm, full HD LED LCD television with a 8-star label, energy consumption 319 kWh/yr.

b A Samsung, 140 cm, full HD LED LCD television with a 7-star label, energy consumption 388 kWh/yr.

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Solution

a Lifetime running cost $= 319 \times \$0.30 \times 10$
$= \$957$

b Lifetime running cost $= 388 \times \$0.30 \times 10$
$= \$1164$

Standby power

Many appliances such as sound systems, computers and cordless phones have a **standby mode** so that they can be turned on quickly. **Standby power** is the electricity used by appliances when they are turned off or waiting to be turned on by a remote control. Around 40% of household appliances use some standby power.

In Australia, the average standby power consumption is 92.2 W per household or 807 kWh per year, with an annual cost of $950 million (at an average cost of 15 cents/kWh). To save energy costs, you should aim to switch appliances completely off when not in use if possible, either by using the OFF switch or at the power point.

EXAMPLE 7

The power consumption of a cordless phone in standby mode is about 3W. What is the cost of the standby mode for the phone over a year if electricity is charged at the rate of 28.0500 cents/kWh?

Solution

Power consumption $= 3 \times 24 \times 365$ Wh
$= 26\,280$ Wh
$= 26.28$ kWh

Annual cost $= 26.28 \times \$0.280\,500$
$= \$7.371\,54$
$\approx \$7.37$

Exercise 12-02 Energy consumption and costs of appliances

Example 3

1 Calculate how much energy (in kWh) is consumed when each of the following light bulbs are used for 8 hours.

a a 100 W incandescent light bulb
b a 20 W compact fluorescent lamp
c a 12 W LED light bulb
d a 50 W halogen light bulb

2 How much energy is used if it takes 2 minutes and 36 seconds to boil 1 litre of water in a 2200 W kettle? Select **A**, **B**, **C** or **D**.

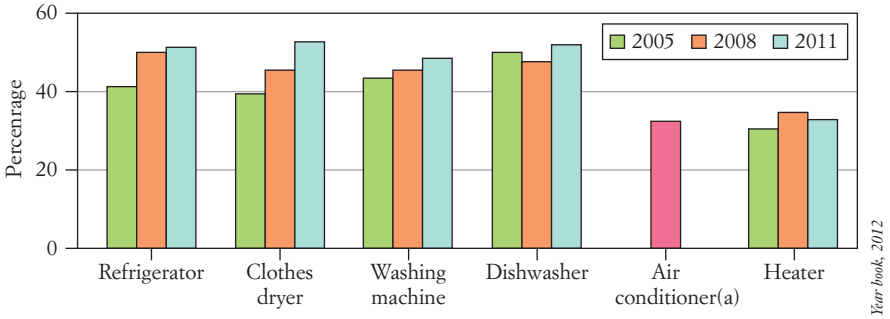
A 5.192 kWh **B** 0.5192 kWh **C** 0.095 33 kWh **D** 0.9533 kWh

3 How much energy is used over 5 hours by an air conditioner if the power input is 0.78 kW?

4 The Energy Rating on page 501 states that a certain washing machine that is used 365 times a year consumes 45 kWh of electricity per year for cold washes and 547 kWh per year for warm washes. Calculate, correct to 2 decimal places, how much energy is used on average for:

a a cold wash **b** a warm wash

5 The graph below shows the percentage of households that considered energy star rating as a factor when purchasing appliances.



(a) Portable air conditioner. No historical data for 2005 and 2008

ABS Year book, 2012

a In 2011, which appliances had more than half of households considering their energy star ratings?

b What percentage of households in 2011 considered energy star rating when purchasing a heater?

c Why do you think that clothes dryer purchasing showed the largest increase in households using the energy star rating as a factor?

d Why are more households considering energy star rating when purchasing appliances?

Example 4

6 Brian uses a 1500 W steam mop for 30 minutes a week to clean the tiled floors of his house.

a How many kWh of energy does the mop use over a year?

b If electricity is charged at the rate of 26.8400 c/kWh, calculate the cost of using the steam mop over a year.

7 The power input of a bar heater is 2400 W and is used on average for 5 hours a day during the winter quarter (92 days). Calculate the cost of using the heater during winter if power is charged at the rate of $0.288 200/kWh.

8 The power input of a top loader washing machine is 165 W. The machine is used three times a week and it takes 38 minutes to complete a cold water wash cycle. Electricity is charged at the rate of 29.8210 cents/kWh. What is the cost of using the washing machine in one year?

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9 A reverse cycle air conditioner is on for 260 hours a year for heating and 240 hours a year for cooling. Calculate the cost of running each brand of air conditioner for a year if the cost of electricity is 24.50 c/kWh.

a a Mitsubishi Electric – 2 star energy rating for cooling with power input 2.13 kW, $2\frac{1}{2}$ star energy rating for heating with power input 2.11 kW.

b a LG – 4 star energy rating for cooling with power input 0.56 kW, 4 star energy rating for heating with power input 0.73 kW.

c a Fujitsu – 4 star energy rating for cooling with power input 0.58 kW, 4 star energy rating for heating with power input 0.78 kW.

Example 5

10 A house has 18 lights which are used for an average of 4 hours per day. Electricity is charged at the rate of 24.45 cents/kWh. Calculate the cost of lighting for a year if:

a 75 W incandescent light bulbs are used

b 15 W compact fluorescent lamps are used

c 12 W LED lights are used

d 50 W halogen lights are used.

11 The table compares the cost of lighting over 15 years for different types of lights. The costs are calculated based on running one light for 6 hours per day with electricity charged at 28 cents/kWh. Copy and complete the table.

	LED	CFL	Halogen	Incandescent
Cost of lamp	$28	$8	$3.50	$2.50
Lamp wattage	12 W	15 W	50 W	75 W
Average life	40 000 h	10 000 h	4 000 h	1 000 h
Lamp cost over 15 years		$32		
Electricity cost over 15 years	$110.38			
Total cost				

Hint: Calculate how many hours a lamp is used over 15 years first, then how many lamps are used

How much money could be saved over 15 years by replacing:

a a 50 W halogen light by a 15 W CFL?

b a 75 W incandescent light by a 15 W CFL?

c a 15 W CFL light by a 12W LED light?

Example 6

12 The energy consumption for a 2-star top-loader washing machine is 123 kWh/year for a cold wash and 820 kWh per year for a warm wash. If electricity is charged at a rate of 29.82 cents/kWh, calculate the difference in cost between cold and warm washing for a year.

13 The Energy Rating label on a small bar refrigerator showed 1 star. The energy consumption was 282 kWh per year. Find the cost of running the bar fridge for one year if electricity costs 29.8210 cents/kWh.

14 a A 3-bedroom house has 18 lights. Compact fluorescent lights of 20 W are used and the lights are on for 6 hours per day. The cost of electricity is 30 cents/kWh. What is the cost of lighting for one year?

b If 12 W LED lights are used instead of CFLs, what is the cost of lighting for one year and how much will be saved?

15 Copy and complete the table below to calculate the cost of using household appliances over a quarter (91 days), using 30 cents/kWh as the cost of electricity.

Appliance	Average quarterly energy use	Cost/quarter
Refrigerator	130 kWh	
Freezer	70 kWh	
Washing machine	10 kWh (based on 4 cold washes/week)	
Dishwasher	92 kWh (for hot water wash, 7 times/week)	
Clothes dryer	110 kWh (based on 1 cycle/week)	
Television	171 kWh (based on television being on 10 h/day)	
Computer	216 kWh (based on computer being on 8 h/day)	

16 Assuming that the average lifetime of major appliances is 10 years, calculate the lifetime running cost of each appliance. Use 30 cents/kWh as an estimate for the cost of electricity.

a a Dishlex dishwasher, with a $2\frac{1}{2}$-star energy rating label and energy consumption of 383 kWh per year for a hot water cycle

b a Samsung refrigerator, with a 1-star energy rating label and energy consumption of 847 kWh per year

c a Fujitsu reverse cycle air conditioner, with a 4-star energy rating label for cooling with power input of 0.58 kW and a 4-star energy label for heating with power input 0.78 kW. The air conditioner is cooling for 250 hours per year and heating for 280 hours per year.

17 Which television has the lower running cost over 10 years and by how much? Use 30 cents/kWh for the cost of electricity.

- Samsung 152 cm full HD, plasma 3D TV with a $4\frac{1}{2}$ star energy rating label and energy consumption 837 kWh.
- Panasonic 152 cm full HD, plasma 3D TV with a $5\frac{1}{2}$ star energy rating label and energy consumption 661 kWh.

Example 7

18 A DVD recorder in standby mode uses 8 W. Find the cost of standby for the DVD recorder for one year if electricity is charged at the rate of 28.0500 cents/kWh.

19 The following table shows the average standby power use of some common household appliances.

Appliance	Power consumption	Running costs per year
Television	1.4 W	
Computer monitor	1.9 W	
Inkjet printer	3.8 W	
Microwave oven	3 W	
Dishwasher	2.7 W	
Sound system	4.2 W	

a Copy and complete the table if power is charged at a rate of 26.675 cents/kWh.

b Find the total cost of standby for the six appliances.

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20 Standby power can account for as much as 10% of the electricity used in a home. The average Sydney house uses 7654 kWh.

a How much energy is consumed in a year by appliances on standby in a Sydney home?

b Calculate the cost of standby power for one year if the cost of electricity is 26.6500 cents/kWh.

TECHNOLOGY: ENERGY CALCULATORS

The cost of appliances, heating and cooling a home, and lighting can be calculated using online energy calculators that can be found on the Internet. Search for an energy calculator such as this one at www.originenergy.com.au/calculator.

1. Use the energy calculator to find the yearly cost of running appliances such as washing machines, televisions, computers, refrigerators, freezers, dishwashers, kettles, sound systems and so on, as well as calculating the cost of lighting, heating and cooling a home.
2. Prepare a report, putting your information in a table so that the total energy use of your home can be calculated over a year.
3. Appliances can be compared in terms of their running costs for different numbers of years. One website that allows you to do this is www.energyrating.gov.au. Visit this website to 'Compare Products', select an appliance and compare the cost of 2–3 different brands, comparing the cost over 1 year and 10 years. Record or print your results.
4. Repeat this process for at least five other appliances that are used in the home.

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JUST FOR THE RECORD

Green hot water systems

Solar hot water systems cost more to install but can reduce electricity costs by 65 to 80%. All systems include an off-peak electric or gas booster.

Heat pumps extract heat from the air using a refrigerant gas and a compressor. They use about a third of the energy of an electric water heater.

Natural gas hot water systems have quicker heat recovery times and generally use a smaller storage tank than electric systems because the water is heated instantly.

Off-peak electric hot water systems have running costs that are similar to natural gas systems, but can only be used in storage systems of over 160 L capacity.

Find out what type of hot water system your home has.

12-03 Electricity usage in the community

Most electricity in Australia is generated from coal and gas, and renewable sources such as hydro, wind, solar and biomass. Renewable energy is environmentally-friendly because it uses resources that are in regular supply, unlike limited resources such as coal and oil that are mined. In 2009–10, 261 TWh of electricity were produced, of which 60.3 TWh were used in homes.

One **terawatt-hour (TWh)** is a billion kWh.

EXAMPLE 8

The table below shows the domestic electricity prices for a NSW supplier.

Electricity DOM (peak) Tariffs	Rates in cents no GST
Supply charge	67.18 c/day
Usage first 1755kWh/qtr	24.85 c/kWh
Remaining usage/qtr	27.71 c/kWh

Calculate the amount charged for a quarter (91 days) if the usage is 2142 kWh and 10% GST is charged.

Solution

Supply charges $= 91 \times \$0.6718$
$= \$61.1338$

Usage (first 1755kWh) $= 1755 \times \$0.2485$
$= \$436.1175$

Remaining usage $= (2142 - 1755) \times \$0.2771$
$= \$107.2377$

ISBN 9780170238977

Total charges before GST = \$604.489

Total charges after GST = \$604.489 × 1.1
= \$664.9379
≈ \$664.94

multiplying by 1.1 is the same as adding 10%

Exercise 12-03 Electricity usage in the community

1 This table shows the fuels used to generate electricity in Australia in 2008–2009. Display this data on a sector graph, on paper or using technology.

Fuel source	Total generation (TWh)
Black coal	143
Brown coal	57
Gas	39
Hydroelectricity	11.9
Wind	3.1
Other	6.1

2 This table shows the annual electricity generated (in TWh) by three countries.

Country	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Australia	198.2	198.2	210.3	225.3	237	225.3	244.2	244.2	239.9	232
Indonesia	95.8	95.8	110.2	120.2	123.4	125.7	142.4	134.4	129	141.2
Malaysia	63.5	68.4	68.4	73.6	72.7	96.0	96.0	99.3	93.8	93.8

Source: CIA World Factbooks

a Display this data on a multiple line graph, on paper or using technology.

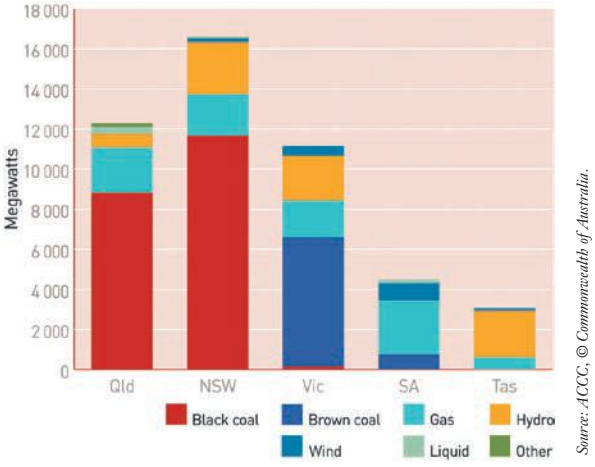
b How does the production of electricity in Australia compare with that in Indonesia and Malaysia?

c Give reasons as to why production is higher in Australia.

d Which country's electricity generation is growing the fastest?

Shutterstock.com/Dmitry Pistrov

3 The graph shows the electricity generation in five states.



Source: ACCC, © Commonwealth of Australia.

a What is the main fuel source for electricity generation in:

i NSW?

ii Victoria?

iii Tasmania?

b Estimate the percentage of electricity generated by black coal in:

i NSW **ii** Queensland

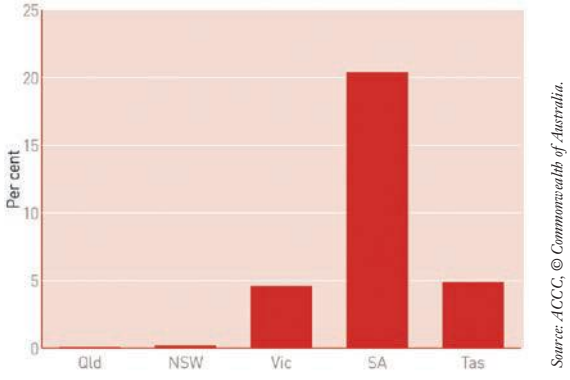
c Estimate the percentage of electricity generated by hydro in:

i NSW **ii** Queensland **iii** Tasmania

d What other fuel source could be included in 'other'?

4 This graph shows wind generation of electricity in 2009 as a percentage of generation capacity.

What percentage of electricity was generated by wind in:



Source: ACCC, © Commonwealth of Australia.

a NSW?

b Victoria?

c South Australia?

5 Wind energy supplies 6800 GWh of electricity annually, which is 2.4% of Australia's electricity needs. What is Australia's annual electricity need (to the nearest GWh)?

6 In many Australian households, solar power has become an important form of electricity. In 2011, solar photovoltaic (PV) panels were installed in half a million homes. If a house in Cronulla has a 3 kW solar PV that produces 11.7 kWh of electricity per day, and its annual household consumption is 8967 kWh:

a calculate how much electricity will be generated in the year

b find the percentage of the consumed household power that is generated by the PV system.

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7 A household in Castle Hill has a 2 kW solar panel system installed that produces 7.8 kWh of electricity per day. How much power will this generate in:

a one week? **b** one year?

8 This table shows the average household use of electricity in NSW local government areas in 2008–2011.

Local Government Area	Annual Household Use (kWh/yr)	Daily Household Use (kWh/d)
Baulkham Hills	10 429	28.6
Blacktown	7582	20.8
Blue Mountains	7358	20.2
Camden	8967	24.6
Campbelltown	7934	21.7
Fairfield	7254	19.9
Greater Lithgow	6413	17.6
Hawkesbury	9338	25.6
Holroyd	6582	18.0
Kiama	6565	18.0
Liverpool	7992	21.9
Parramatta	6642	18.2
Penrith	8811	24.1
Rylstone	6506	17.8
Shellharbour	6394	17.5
Shoalhaven	6072	16.6
Wingebaribee	7588	20.8
Wollondilly	9447	25.9
Wollongong	5918	16.2

Endeavour Energy

a Which area used:

i the most electricity? **ii** the least electricity?

b How much more electricity did households in Baulkham Hills use than households in Holroyd? Give your answer as a percentage correct to one decimal place.

c Give possible reasons for the difference between household energy use in Penrith and Wollongong.

d The annual household electricity use for Greater Lithgow and Rylstone is low when compared to Baulkham Hills, Camden, Penrith and Wollondilly. Give possible reasons.

Example 8

9 The tables below show the domestic electricity prices for suppliers in three states.

Network Area: NSW

Electricity DOM (peak) Tariffs	Rates in cents (no GST)
Supply charge	69.14 c/day
Usage first 1750 kWh/qtr	24.25 c/kWh
Remaining usage/qtr	27.11 c/kWh

Network Area: VIC

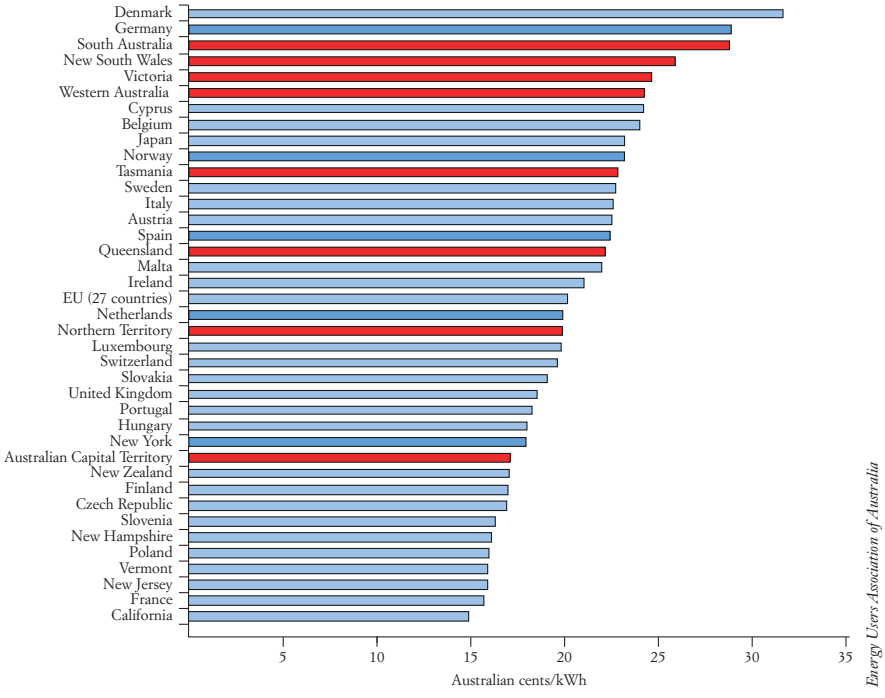
Electricity DOM (peak) Tariffs	Rates in cents (no GST)
Supply charge	70 c/day
Usage first 1020 kWh/qtr	18.3 c/kWh
Remaining usage/qtr	19.5 c/kWh

Network Area: QLD

Electricity DOM (peak) Tariffs	Rates in cents (no GST)
Supply charge	26.17 c/day
Usage	23.071 c/kWh

Compare the price of electricity for the three states by calculating the amount charged by each supplier for a quarter (91 days) if the usage is 2542 kWh and 10% GST is charged.

10 The graph below compares Australian electricity prices (in 2011) with prices in other countries and provinces. The Australian states and territories are shown in red.



Energy Users Association of Australia 'Electricity Prices in Australia: An International Comparison, CME, 2012' http://www.euaa.com.au/wp-content/uploads/2012/04/FINAL-INTERNATIONAL-PRICE-COMPARISON-FOR-PUBLIC-RELEASE-19-MARCH-2012.pdf

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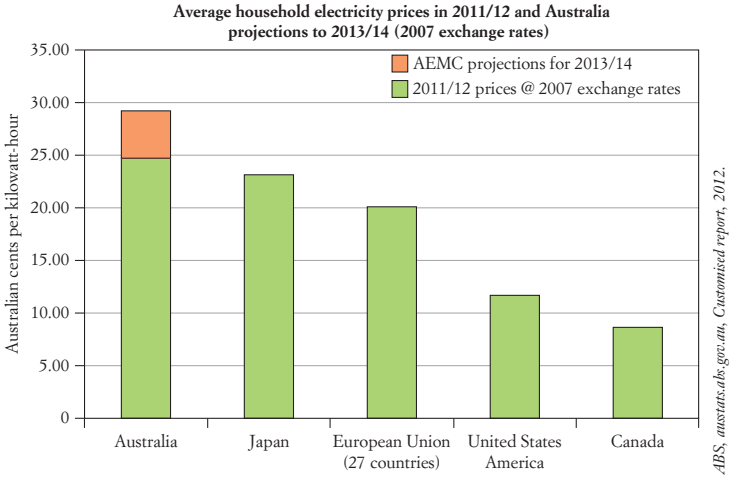
a How many Australian states are in the top ten for household electricity prices?

b What is the position of the Australian Capital Territory in the graph?

c Estimate how much higher electricity prices are in NSW per kWh compared to:

i the United Kingdom **ii** Spain **iii** California

11 The following graph compares international electricity prices for 2011–12.



a Estimate the cost of household electricity in Australia:

i in 2011–12 **ii** in 2013–2014

b How does the electricity cost in Australia compare with the cost in other countries?

c Calculate the difference in household electricity prices between the USA and Canada.

12 This table shows the average annual electricity consumption for different-sized households.

Number of people in the home	Electricity usage (kWh/yr)
1	4 900 – 6 600
2	6 600 – 8 800
3	7 600 – 10 100
4	8 700 – 11 200
5	12 200 – 16 300
6 or more	11 600 – 15 500
Average	6 500 – 8 700

Australian Bureau of Statistics.

a Explain why the usage doesn't double when the number of people double.

b Give possible reasons as to why electricity usage is lower for 6 or more people compared to 5 people.

INVESTIGATION

ENERGY CONSUMPTION IN YOUR HOME

1 Visit the government website **www.energymadeeasy.gov.au** and select 'Understand and compare your electricity usage' to calculate the average household electricity usage per day, per season and per year for:

a North Ryde **b** Bathurst

c Bondi Beach **d** where you live

2 Reducing electricity consumption can be achieved by understanding the running costs of your appliances. Saving energy fact sheets can be found at the website **www.originenergy.com.au** by searching for 'State fact sheets'.

a Use this site to obtain fact sheets for NSW, Victoria, Queensland and South Australia.

b Draw a clustered column graph from information on the fact sheets that can be used to compare electricity consumption between the states.

c What advice do the fact sheets give for reducing electricity consumption?

Another useful site is **www.savepower.nsw.gov.au**.

INVESTIGATION

RENEWABLE ENERGY

In 2012, renewable energy sources provided about 6% of NSW's total electricity usage. Visit the following websites to answer the questions below:

www.trade.nsw.gov.au/energy/sustainable/renewable

www.cleanenergycouncil.org.au

What are the renewable energy sources? In your investigation, include how much each source contributes to Australia's and NSW's total electricity output. Also obtain data on the global approach for each renewable source. Display your information in a table and/or graph.

Shutterstock.com/majciczka

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12-04 Energy-efficient housing

An energy-efficient home is designed to make the most of natural heating (the sun), cooling (shade and cooling breezes) and lighting. Using green design principles will reduce the need for appliances such as air conditioners and heaters and also reduce your impact on the environment. The following features should be considered when designing a 'clean and green' home.

- **Orientation**: living areas should face north to capture winter sun and the long axis of the house should run from east to west
- **Insulation**: roof insulation, light-coloured roofing and glazed windows reduce energy costs for heating and cooling
- **Windows/skylights**: full-length windows facing north allow winter sun to provide light and heat
- **Ventilation**: cross-ventilation allows air flow for cooling of the house
- **Landscaping/shading:** plants and trees can shade walls and windows
- **Building materials**: can absorb and store heat

Getty Images/Justin Horrocks

EXAMPLE 10

As a general rule, the total window area should be less than 25% of the floor area of a house. Give an estimate of the maximum window area of a rectangular house with dimensions 15 m by 7.8 m.

Solution

Floor area = 15 m × 7.8 m
= 117 m^2

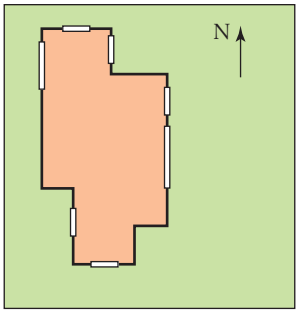
Window area = 25% × 117
= 29.25 m^2

The maximum window area is 29.25 m^2.

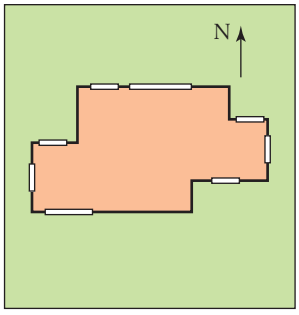
Exercise 12-04 Energy-efficient housing

1 Which site plan shows the house placed in the most energy-efficient way? Give reasons for your answer.

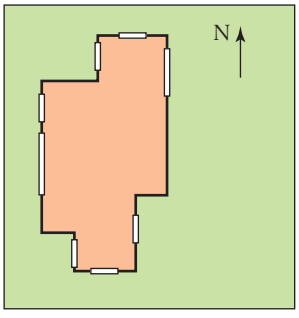
A



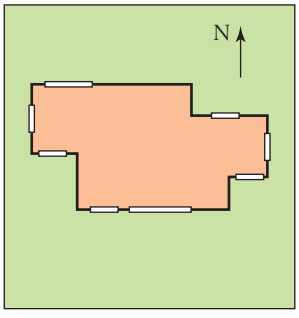
B



C



D



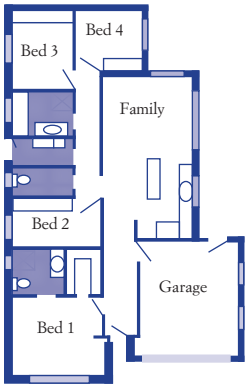
Example 10

2 **a** The plan of the house has been drawn to scale. If the length of the house is 20 m, calculate the scale used.

b Calculate the area of the house.

c The window and glazed door area is 30% of the floor area. Give an estimate of the window and glazed door area.

d Which side of the house should be facing North?



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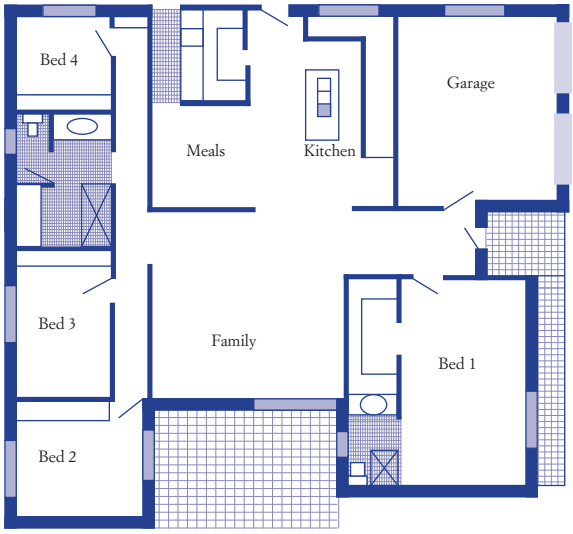
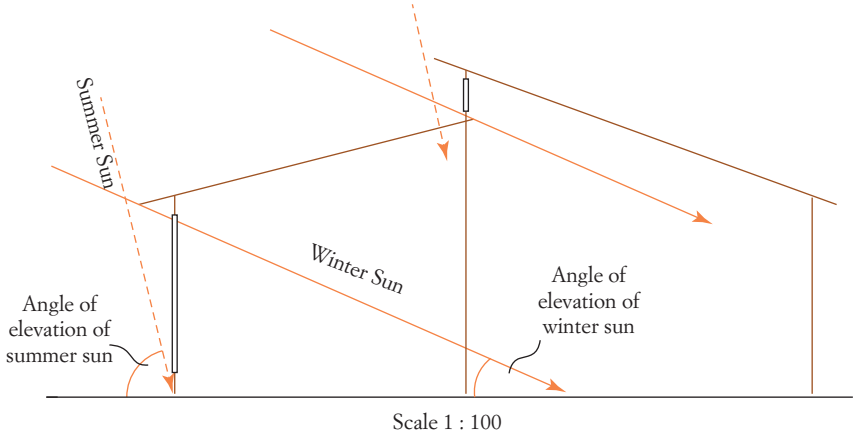
3 The house shown in the plan on the right will have the garage facing the street, which is in an easterly direction.

- **a** In what direction will the kitchen face?
- **b** In which direction will Bedrooms 2, 3 and 4 face?
- **c** Has the house been orientated in an energy-efficient way? Give reasons.
- **d** Does this house have cross-ventilation? Give reasons.



4 What is roof insulation and how does it reduce energy costs in the home in both summer and winter?

5 The diagram below is the side view of a house showing the angles of elevation for the winter and summer sun.



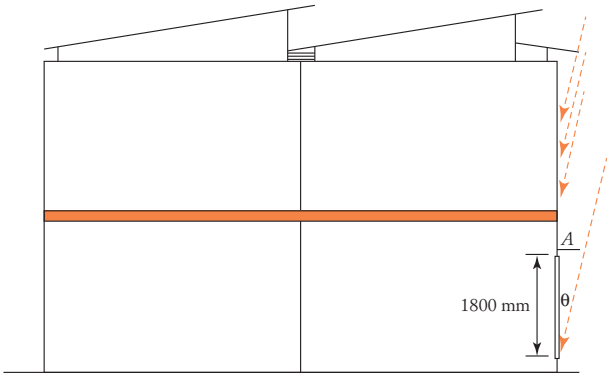
Scale 1 : 100

- **a** Use a protractor to find the angle of elevation of:
  - **i** the summer sun
  - **ii** the winter sun
- **b** Will the house benefit from the position of the window and skylight? Give reasons.

6 Name two ways of using green design principles to:

- **a** cool your home in summer
- **b** bring more light into your home in winter

7 The diagram below is the side view of a two-storey house facing north, showing the angle of elevation of the summer sun. A shade awning is to be placed at A, 150 mm above the head of the window.



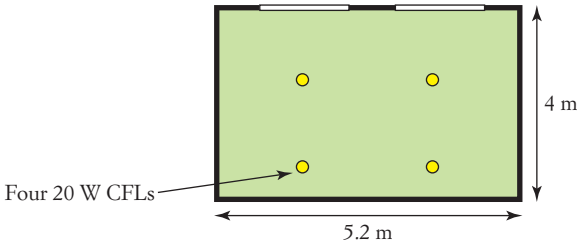
Scale 1 : 100

a Using the angle of elevation of the summer sun obtained in question **5**, find θ, the angle between the window and the rays of the summer sun.

b Use trigonometry to calculate the length of the shade awning needed so that the summer sun does not come through the downstairs window.

c The owner of the house does not want a shade awning for the upstairs window facing north. What needs to be done to reduce the amount of summer sun coming through this window?

8 The Building Code of Australia has introduced restrictions on the amount of electrical lighting in and around the home. The lamp power density limits are 5 W/m^2 for a house and 4W/m^2 for a balcony or verandah.



a Calculate the area of the room shown in the diagram.

b What is the lamp capacity (watts/lamp × number of lamps)?

c Calculate the lamp power density (lamp capacity ÷ area).

d Do the lights in the room comply with the limits?

9 Calculate the lamp power density for the room in question **8** if the following lights were used instead of the CFLs and state which lights comply with the 5 W/m^2 limit:

a two 100 W incandescent lights

b three 50 W halogen lights

c five 18 W LEDs

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INVESTIGATION

BUILDING AN ENERGY-EFFICIENT HOME

When building a new home, there are many things that can be done to reduce the amount of electricity used. For example, up to 35% of heat gain and loss from a house can be through a ceiling that is not insulated. Insulating the roof, walls and suspended floors will reduce energy use and cut electricity costs.

1 Investigate the following websites to find different ways that a house can be made to be more energy-efficient:

www.environment.nsw.gov.au

www.savepower.nsw.gov.au

www.yourhome.gov.au

2 What are 'ideal' settings for heating and for cooling?

3 How can landscaping help reduce your energy bill?

4 What type of lighting should be used in outdoor areas?

INVESTIGATION

ENERGY-EFFICIENT LIGHTS

Lighting takes up 10% of a household's electricity bill and about 25% of a business' electricity bill. The Australian government recognises that energy savings can be easily made by introducing compact fluorescent lamps (CFLs) and LED lights. As a result, the old incandescent light bulbs are being phased out.

1 The brightness of a light (light output) is measured in lumens or lux. Give a brief explanation of both.

2 Lights that have an efficiency level below 15 lumens/W are being phased out. What is the efficiency level of the following lights (you will need to find their light output in lm).

a Incandescent globes of:

i 100 W **ii** 75 W **iii** 60 W **iv** 40 W

b CFLs of:

i 20 W **ii** 15 W **iii** 11 W **iv** 8 W

c Halogen lights of:

i 72 W **ii** 53 W **iii** 42 W **iv** 28 W

d LED lights of:

i 7 W **ii** 12 W **iii** 18 W **iv** 25 W

3 What other measures is the Government taking to reduce home energy use? You may find the following websites useful:

www.yourhome.gov.au

www.climatechange.gov.au

www.environment.nsw.gov.au

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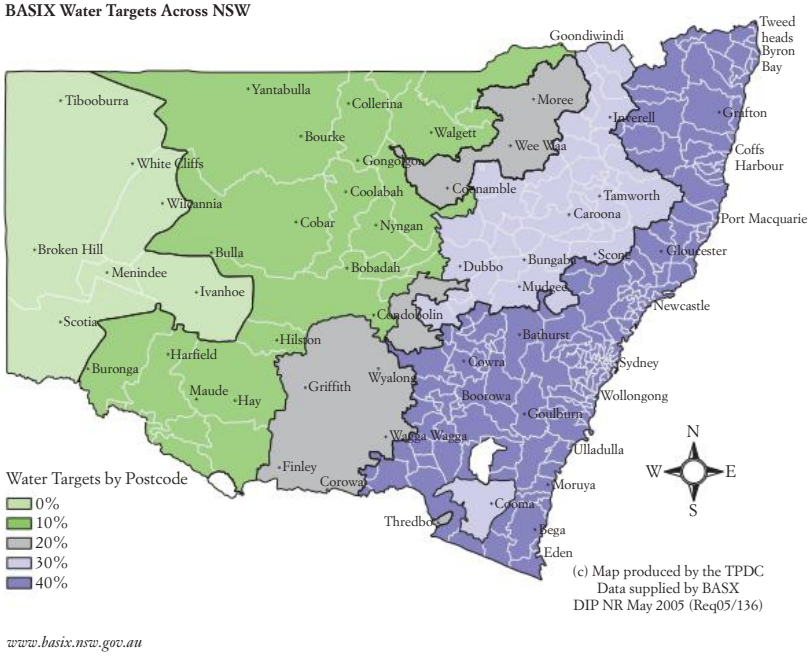
12-05 The Building Sustainability Index (BASIX®)

The **Building Sustainability Index (BASIX®)** (Note: BASIX is a registered trademark) was introduced by the NSW Government in 2004 to ensure that new homes are built to be more efficient in energy and water use. It is an online program that assesses a home design by examining features such as location, size, building materials, water source and fittings, hot water, cooling and heating systems. The design must meet energy and water reduction targets before a BASIX Certificate can be issued. Every development application for a new home must be submitted to the local council with a BASIX certificate.

The **BASIX tool** uses data from the design to calculate the anticipated water consumption and greenhouse gas emissions of the new home. It is divided into three sections: **water**, **thermal comfort** and **energy**.

BASIX water target

The water reduction target ranges from 0 to 40%, according to location, and a new home should not use more water than the state average.



www.basix.nsw.gov.au

Water reduction targets can be met by:

- installing a water tank for garden use and/or toilets and laundry use
- increasing the amount of roof area diverted to the water tank
- selecting star-rated shower heads, dual-flush toilets, water saving aerators and flow regulators in taps
- planting indigenous/low water use plant species

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BASIX thermal comfort target

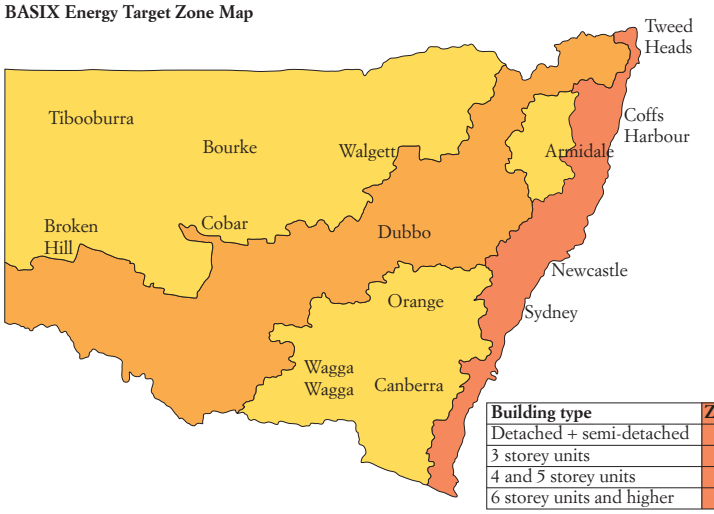
Thermal comfort refers to keeping the house cool in summer and warm in winter. A house will either pass or fail this section. The thermal comfort targets can be met by:

- insulating walls, ceiling roofs and suspended floors
- arrange to have glazing/windows facing north and reduce the amount facing west or south
- choosing a light/medium-coloured roof to reduce heat gain in summer
- designing eave overhangs and other external shading/awning to block the summer sun and allow the winter sun

BASIX energy target

The energy reduction target depends on building type and location. The average greenhouse reduction for building types in NSW is 40%. The map below shows the 3 BASIX energy zones.

BASIX Energy Target Zone Map



Building type	Zone 1	Zone 2	Zone 3
Detached + semi-detached	40	35	25
3 storey units	35	30	20
4 and 5 storey units	30	25	15
6 storey units and higher	20	15	5

www.basix.nsw.gov.au

The energy reduction target can be met by:

- choosing a solar, gas or an electric heat pump hot water system
- using ceiling fans
- installing a high efficiency air conditioning system for heating and cooling
- using energy efficient lights and making the most of natural lighting (skylights)
- using outdoor and indoor clothes drying lines

More information about the targets and BASIX benchmarks can be obtained from www.basix.nsw.gov.au.

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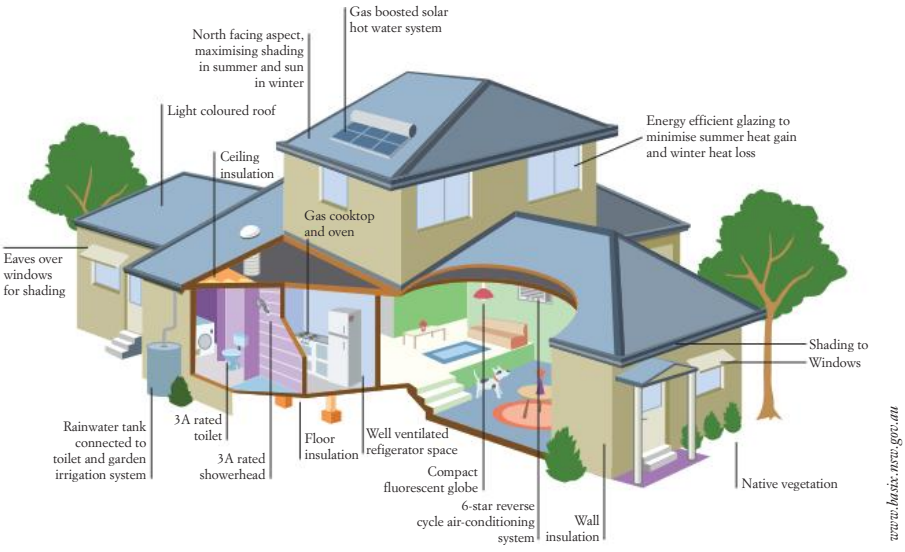
Exercise 12-05 The Building Sustainability Index (BASIX®)

1 The diagram below shows sustainable housing features.



List the sustainable features under the three BASIX® (Note: BASIX is a registered trademark) sections of Water, Thermal Comfort and Energy.

2 Use the map for BASIX Water Targets on page 520 to find the water reduction target for:

a Scone **b** Griffith **c** Sydney

3 Visit the BASIX website **www.basix.nsw.gov.au** to find the BASIX water reduction target for your home.

4 Use the BASIX Energy Target Zone map on page 521 to find the energy reduction target for your home.

5 Download a sample BASIX Certificate from the BASIX website.

a What units are used to measure the cooling and heating loads?

b For the Thermal Comfort section, what R-value is to be used for:

i external walls? **ii** ceiling and roof?

c Why do the ceiling and roof have a higher R-value than the external walls?

d What type of hot water system must be installed in a new home?

e What type of lighting is required?

f When heating and cooling, what energy rating is required?

6 This extract of a BASIX report shows information regarding the water commitments of a new house.

Water Commitments	Certifier check
Landscape	
The applicant must plant indigenous or low water use species of vegetation throughout 110 square metres of the site	

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Fixtures	
The applicant must install showerheads with a minimum rating of 3 star (> 4.5 but ≤ 6 L/min) in all showers	
The applicant must install taps with a minimum rating of 5 star in the kitchen	
The applicant must install basin taps with a minimum rating of 5 star in each bathroom	
Rainwater tank	
The applicant must install a rainwater tank of at least 3250 litres on the site	
The applicant must configure the rainwater tank to collect rain run-off from at least 203.36 square metres of the roof area of the development	
The applicant must connect the rainwater tank to at least one outdoor tap	

a What is the minimum star rating of:

i the showerheads? **ii** the basin taps?

b How many square metres of indigenous or low water use species of vegetation should be planted?

c If the site area is 509 m^2, what percentage of the site will have indigenous or low water use vegetation?

d How much roof area is required for run-off and what size of water tank must be installed?

e What other conditions apply to installing a water tank in order to meet the water commitments of BASIX?

7 This extract of a BASIX report shows information regarding the energy commitments of a new house.

Energy Commitments	Certifier check
Hot water	
The applicant must install the following hot water system in the development, or a system with a higher energy rating: solar hot water system with electric booster with a performance of 5 stars	
Cooling system	
The living areas must not incorporate any cooling system, or any ducting which is designed to accommodate a cooling system	
The bedrooms must not incorporate any cooling system, or any ducting which is designed to accommodate a cooling system	
Heating system	
The living areas must not incorporate any heating system, or any ducting which is designed to accommodate a heating system	
The bedrooms must not incorporate any heating system, or any ducting which is designed to accommodate a heating system	
Ventilation	
The applicant must install the following exhaust systems in the development:	
At least 1 bathroom: individual fan, ducted to façade or roof; operation control: manual switch on/off	
Kitchen: individual fan, ducted to façade or roof; operation control: manual switch on/off	
Laundry: individual fan, ducted to façade or roof; operation control: manual switch on/off	

Energy Commitments	Certifier check
Artificial lighting	
The applicant must ensure that the 'primary type of artificial lighting' is fluorescent or light emitting diode (LED) lighting in each of the following rooms, and where the word 'dedicated' appears, the fittings for those lights must only be capable of accepting fluorescent or light emitting diode (LED) lamps:	
– all bathrooms/toilets	
– the laundry	
Natural lighting	
The applicant must install a window and/or skylight in the kitchen of the dwelling for natural lighting	
The applicant must install a window and/or skylight in 4 bathrooms(s)/toilet(s) for natural lighting	

www.basix.nsw.gov.au

a What type of hot water system must be installed?

b What rooms must include an exhaust fan?

c What other design features could be used for ventilation that are not mentioned in the report?

d What type of lights are required in all bathrooms and toilets?

e What is the difference between artificial and natural lighting?

f What is a skylight and in which rooms is it appropriate to install one?

WS
Designing a sustainable house

8 Read this checklist and list the features on the list that your home includes.

Water	Yes	No
Star rated showerheads	·	·
Dual flush toilets	·	·
Water saving aerators and flow regulators	·	·
Water tank	·	·
Indigenous/low water use plant species (if Yes, indicate area in m^2)	·	·
Thermal comfort	Yes	No
Insulation in walls	·	·
Insulation in ceilings/roof	·	·
Light coloured roof	·	·
Eaves/overhangs/pergolas/awnings	·	·
Windows facing north (indicate percentage of windows space)	·	·
Performance glazing	·	·
Ventilation – exhaust fans (indicate where – kitchen/bathroom/laundry)	·	·
Energy	Yes	No
Hot water system (indicate type and star rating)	·	·
Ceiling fans	·	·
Air conditioning (if Yes, indicate star rating)	·	·

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Energy efficient lighting (if Yes, indicate type – Halogen/CFLs/LEDs)		
Skylights		
Indoor/Outdoor clothes drying lines		
Pool/Spa pump timer/cover		

9 Using a suitable scale, draw a site plan showing the house in which you live. Your diagram should include:

- an arrow showing North
- the dimensions of the block and the house
- gardens and large trees
- any water tanks

a Calculate the area of the block of land and the area taken up by the house.

b Calculate the area of the gardens and lawns.

c What percentage of the block is taken up by gardens (low water use only) and large trees?

INVESTIGATION

USING BASIX®

1 Visit the BASIX website at **www.basix.gov.au**. Go to Tutorials and complete all sections – the **Overview**, **Register your details**, **Login to BASIX** and **Sample projects**.

2 Go back to the original webpage and go to **Register**. For user type, select **Education**. When you have completed registration, login and use the BASIX assessment tool. You will need information such as Project address, Project type (separate dwelling house, site details).

INVESTIGATION

NABERS HOME RATINGS

NABERS (National Australian Built Environmental Rating System) measures the environmental performance of Australian buildings. The NABERS Home Rating rates the water and energy use of a home from 0 to 5 stars, where 2.5 stars is the measure of an 'average' environmental performance. Household information, such as the postcode, number of occupants, water bills, electricity and gas bills and any other energy sources are used to determine how your home's water and energy use compares to others.

1 Visit the website **www.nabers.com.au** and use the NABERS rating calculator to rate your home. You will need to have water and energy (electricity, gas) bills for the last 12 months. Once you have finished, print your NABERS HOME Energy and Water rating.

2 Briefly describe the information on the Energy and Water rating for your house and compare with the ratings obtained by other students in your class.

3 Who manages NABERS?

Sample HSC problem

Aditi is looking at buying a new television. She has three options.

A An LG 139 cm full HD, LED LCD TV with a 6-star energy rating label and energy consumption of 500 kWh, costing $3699.

B A Samsung 140 cm full HD, LED LCD TV with a 7-star energy rating label and energy consumption of 388 kWh, costing $3399.

C A Sony 138 cm full HD, LED LCD TV with an 8-star energy rating label and energy consumption of 319 kWh, costing $3099.

Calculate the total cost of each television, including the running cost over 10 years. Use 30 cents/kWh as an estimate for the cost of electricity.

Which television should Aditi buy?

Study tip

Ten more exam tips

1 Bring all of your equipment to the exam: pens, ruler, protractor, compasses, calculator (check that it works).

2 Don't worry if you feel nervous before the exam—this is normal and helps you perform better. However, being too casual or too anxious can be harmful to your performance.

3 Write in black or blue, not red. Use pencil only for diagrams and constructions.

4 Read each question and identify what needs to be found.

5 You don't need to be writing all of the time. What you are writing may be wrong and a waste of time. Spend some time thinking and considering the best approach.

6 Make sure that your answer sounds reasonable and realistic, especially if it involves money or measurement.

7 If you make a mistake, cross it out with a neat line. Don't scribble over it completely or use correction fluid (it is both time-consuming and messy). You may still get marks for it if it is correct.

8 Don't cross out or change an answer rashly. You may have been correct the first time.

9 Don't round your answer in the middle of a calculation. Round your answer at the end only.

10 Don't be afraid to write words and sentences in your answers, but don't use abbreviations that you've just made up.

Good luck!

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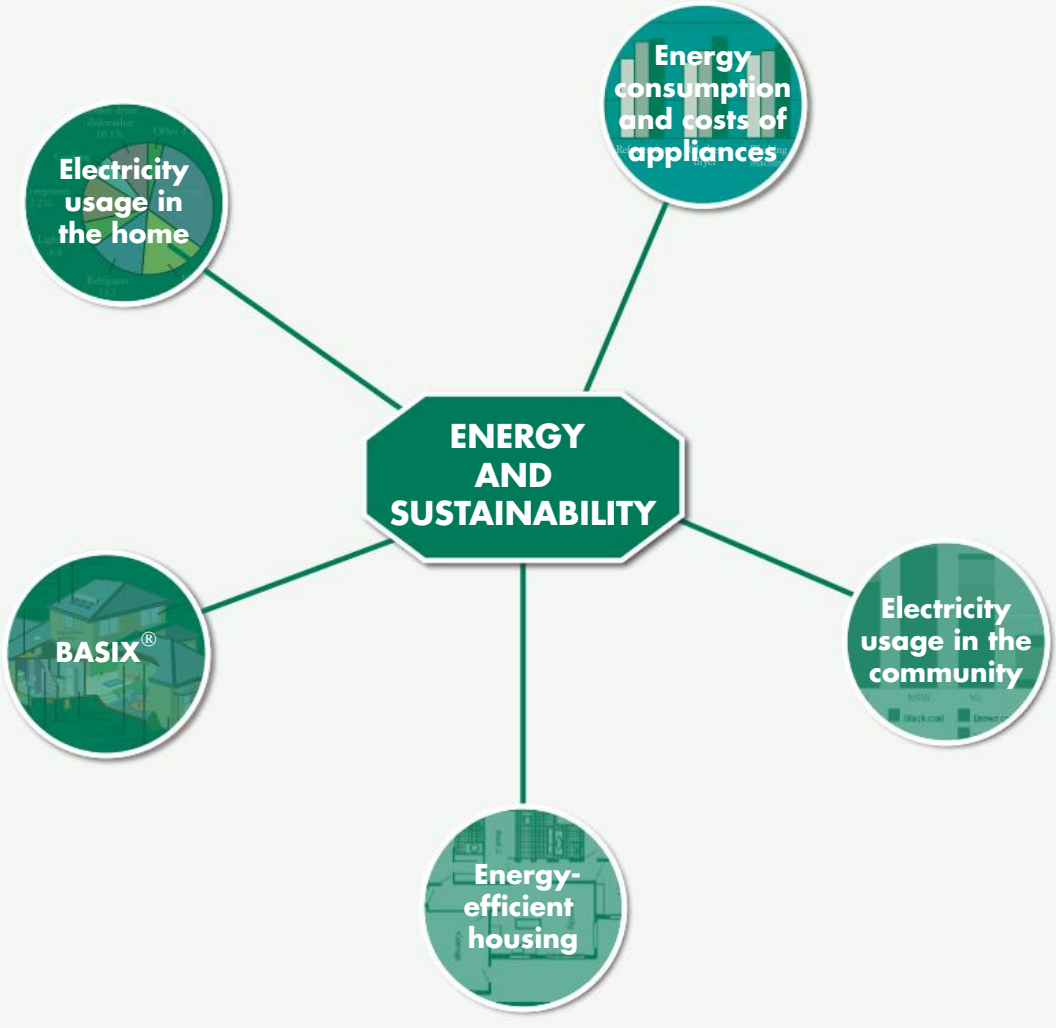
CHAPTER REVIEW

Topic overview

This chapter, Energy and Sustainability, looked at the comparison and calculation of household electricity usage and costs. You should have an understanding of the units related to power and be able to interpret electricity bills. You have learnt to calculate the costs of running different household appliances over a period of time and should be able to interpret their energy ratings and compare their costs. Statistics on electricity production, consumption and costs were examined on a local, state, national and international level. Finally, energy-efficient housing and the requirements of the Building Sustainability Index (BASIX®) were investigated.

Make a summary of this topic. Use the outline at the start of the topic as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders to gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 12-01

1 Convert:

a 15 600 kW to MW **b** 5.17 GW to kW **c** 45 000 000 mW to kW

Exercise 12-01

2 The pricing information for an electricity supplier is given below.

Tariff Type	
Off Peak (cents/kWh)	9.3170
Supply charge (cents/day)	5.1920
Domestic First 1750 kWh/qtr (cents/kWh)	26.6750
Balance (cents/kWh)	29.8210
Supply charge (cents/day)	76.0540

Calculate the quarterly (92 days) electricity bill for a Bowral household with off-peak usage of 1087 kWh and domestic usage of 2107 kWh.

Exercise 12-01

3 How much power is used by a cordless phone on standby mode over a year if its power input on standby is 3W?

Exercise 12-02

4 A 2400 W kettle is used for an average of 15 minutes per day. If the cost of electricity is 24.25 cents/kWh, how much will it cost to use the kettle over one year?

Exercise 12-02

5 Calculate the cost of using each of the following lights for a year if they are on for 5 hours per day and electricity is charged at a rate of $0.2614/kWh.

a a 15 W CFL **b** a 13 W LED **c** a 53 W halogen light

Exercise 12-02

6 The energy consumption for a 4-star 8 kg top-loader washing machine is 85 kWh/year for a cold wash and 360 kWh per year for a warm wash. If electricity is charged at a rate of 25.82 cents/kWh, calculate how much cheaper it is to wash in cold water than in warm.

Exercise 12-02

7 The energy consumption for a 3-star dishwasher is 362 kWh for a hot water wash, seven times a week. Electricity is charged at the rate of 28.15 cents/kWh. What is the cost of using the dishwasher over a year?

Exercise 12-03

8 A household in Sydney has solar PV cells installed on their roof. If they have a 1.5 kW system that produces 5.85 kWh of electricity daily, how much electricity will be generated in one year?

Exercise 12-03

9 The average daily household electricity use in Hawkesbury is 25.6 kWh/day while in Lithgow it is 17.6 kWh/day. Give one possible reason for this difference in electricity usage.

Exercise 12-04

10 **a** Name one type of home insulation and explain how this reduces energy costs.

b Why should an energy-efficient house face north?

Exercise 12-05

11 **a** What does BASIX® stand for?

b Name two strategies for reducing household water use.

c Name two strategies for improving thermal comfort in the home.

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Practice paper 4

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Exercise 10-04

Question 1

Which one of the following is a hyperbolic function?

(A) $y = 4^x$ (B) $y = 4x^3$ (C) $y = \left(\frac{1}{4}\right)^x$ (D) $y = \frac{4}{x}$

Exercise 12-01

Question 2

How much energy does a 18 W compact fluorescent lamp use if it is on for 6 hours per day for 1 year?

(A) 6.57 kWh (B) 10.80 kWh (C) 39.42 kWh (D) 584.0 kWh

Exercise 11-04

Question 3

The masses of a population of adults are normally distributed with a mean of 75 kg and a standard deviation of 11 kg. What percentage of the population has a mass between 64 kg and 97 kg?

(A) 83.35% (B) 81.5% (C) 68% (D) 57.5%

Exercise 12-02

Question 4

If the cost of electricity is 24.25 cents/kWh, calculate the cost of 780 kWh.

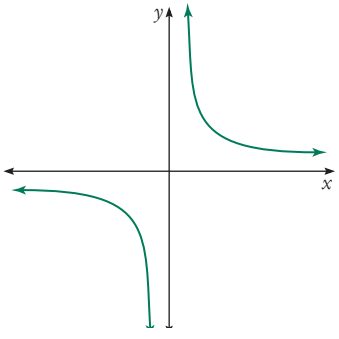
(A) \$18.92 (B) \$189.15 (C) \$18.72 (D) \$190.00

Exercise 10-01

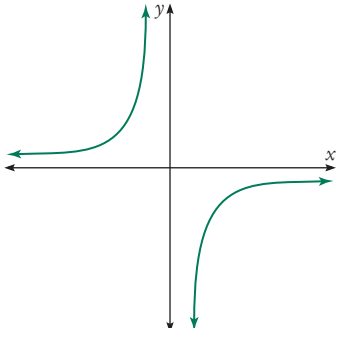
Question 5

Which one of these graphs could be the graph of $y = -\frac{2}{x}$?

(A)



(B)



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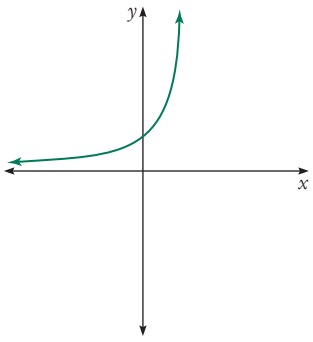
(C)



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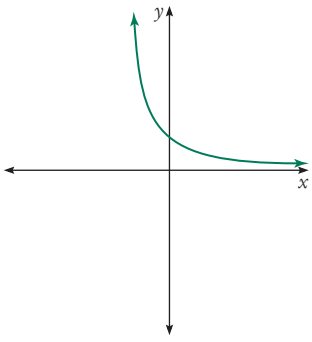

Exercise 10-08

Question 6

The time taken to complete a journey varies inversely with the speed of a car. A car takes 4 hours to complete a journey at a speed of 80 km/h. How long would the journey take if the car was travelling at 60 km/h?

(A) 3 hours (B) 4 hours 15 min

(C) 5 hours 20 min (D) 5 hours 33 min

Exercise 12-05

Question 7

The heights of a population of hockey players are normally distributed with a mean of 165 cm and a standard deviation of 12. If Jordan is a hockey player whose height has a z-score of 1.6, what is her height?

(A) 179 cm (B) 184 cm (C) 191 cm (D) 192 cm

Exercise 12-02

Question 8

The Energy Rating label of a reverse-cycle air conditioner shows the following.

COOLING	
CAPACITY OUTPUT 4.45 kW	POWER INPUT 1.25 kW
HEATING	
CAPACITY OUTPUT 4.75 kW	POWER INPUT 1.23 kW

Alamy/Global Warming Images/Ashley Cooper

The air conditioner is on for 5 hours per day for 115 days during winter. How much energy is used by the air conditioner in winter?

(A) 707 kWh (B) 719 kWh (C) 2559 kWh (D) 2731 kW

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Exercise 11-03

Question 9

Tamin scored 81% on a Maths test and Emma scored 72% on an English test, with both of them achieving a z-score of 1.6. Who performed better in her subject compared to the other students?

(A) Tamin (B) Emma

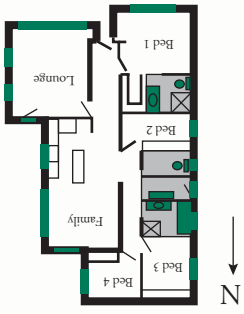
(C) Both performed the same (D) Unable to tell

Exercise 12-04

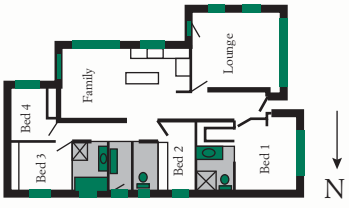
Question 10

Which diagram shows the most energy-efficient way to position a house on a block of land?

(A)



(B)



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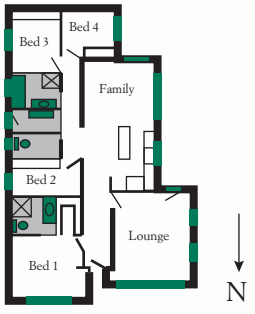

(D)

Section 2

3 questions: 10 marks each

Exercise 10-02

Question 11 Marks

(a) A ball is thrown up in the air from a cliff and its height above ground level after t seconds is given by the quadratic equation $h = -5t^2 + 20t + 60$ where h is the height in metres.

(i) Graph $h = -5t^2 + 20t + 60$ for values of t from 0 to 6. **2**

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(ii) Find the maximum height of the ball and the time when this occurs. **2**

(iii) What is the vertical intercept and what does it represent? **1**

Exercise 10-05

(b) A biologist studying a new strain of bacteria models its growth by the formula $N = 80\ (1.4)^t$ where N is the number of bacteria and t is the time in hours.

(i) How many bacteria are present after 5 hours? **1**

(ii) Show that the original number of bacteria present was 80. **1**

(iii) By what percentage is the bacteria increasing per hour? **1**

Exercise 11-03

(c) The capture-recapture technique was used to estimate the number of fish in a lake.

250 fish were caught and tagged and then released. A month later, 300 fish were caught at random and of these, 17 had tags.

(i) Estimate the population of fish in the lake. **1**

(ii) The estimated population of fish was 8% more than three years ago. What was the estimated population three years ago? **1**

Question 12 **Marks**

Exercise 10-04

(a) The distance (D) that an object falls under gravity varies directly as the square of the time (t) it falls. A skydiver falls 122.5 m in 5 seconds.

(i) Find the variation equation for D in terms of t. **1**

(ii) How far would a skydiver fall in 15 seconds? **1**

(iii) How long would it take for a skydiver to fall 2000 m? **1**

Exercise 11-02

(b) The masses (in kg) of 3 students are as follows:

61 75 68

(i) In how many ways can a sample of size $n = 2$ be selected from this group? **1**

(ii) List all possible samples of varying sizes from $n = 1$ to $n = 3$. **1**

(iii) Find the means of all the samples that have been listed and calculate the mean ($\mu_{\bar{x}}$) of the sample means. **2**

(iv) Find the population mean (μ). Is it equal to $\mu_{\bar{x}}$?

Exercise 12-03

(c) This table shows the pricing information for an electricity supplier.

Electricity (peak) Tariffs	Rates in cents no GST
Supply charge	67.18 c/day
Usage first 1755kWh/qtr	24.85 c/kWh
Remaining usage/qtr	27.71 c/kWh

Calculate the amount charged by the supplier for a quarter (92 days) if the usage is 2875 kWh and 10% GST is charged. **2**

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Question 13 **Marks**

Exercise 12-05

(a) A household in Sydney has a 1.5 kW solar panel system installed.

(i) How much power (to the nearest whole kWh) will this generate in one year if the average daily production of the solar panel system is 5.85 kWh? **1**

(ii) The average annual household consumption is 7988 kWh. What percentage (correct to one decimal place) of the household consumption is generated by the solar panel system? **1**

Exercise 12-01

(b) Convert 8.51 GW to kW. **1**

Exercise 12-02

(c) A three-bedroom house has 20 lights, all of which are compact fluorescent lights (CFLs) of 18 W each. The lights are on for an average of 6 hours per day and the cost of electricity is 24.85 cents/kWh.

(i) What is the cost of lighting for one year? **2**

(ii) If 12 W LED lights are used instead of the CFLs, how much is saved in the cost of lighting over one year? **2**

Exercise 11-02

(d) Claudia's marks for English, Maths and Geography, as well as the mean and standard deviation of the marks for each class, are given in the table.

Subject	Claudia's mark	Class mean	Class standard deviation
English	78	70	5
Maths	83	72	9
Geography	68	60	4

(i) Find Claudia's z-score for each subject. **1**

(ii) In which subject did she perform best? Justify your answer. **1**

(iii) The test results are normally distributed and there are 45 students in the Geography class. How many students performed better than Claudia in Geography? **1**

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GLOSSARY

Mathematical 'doing' words

calculate To find the value of a numerical expression. See also **evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or finished.

compare To show how things are similar or different.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction to a decimal, or convert dollars to cents.

decrease To make smaller.

describe To state characteristics or features.

estimate To make an educated guess for a number or measurement; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2, or evaluate $4x + 1$ when $x = 5$.

expand To remove the grouping symbols (brackets) in an algebraic expression. Expanding $3(2y + 1)$ gives $6y + 3$.

explain To describe why or how.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using answers found previously.

increase To make larger.

interpret To find meaning in the mathematical result.

justify To support your argument or conclusion.

measure To determine the size of something. For example, to determine length using a ruler.

prove/show that In questions where the answer is given, to use mathematical methods or reasoning to prove that the answer is true.

simplify To give the answer in its simplest, shortest, neatest form.

sketch To draw a rough diagram. Less accurate than to **construct**.

solve To find the value of the unknown pronumeral in an equation or inequality.

substitute To replace a variable with a number and evaluate.

verify To show that a statement or result is correct, usually by substituting back.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

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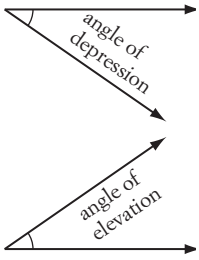
+	plus	$\sqrt{\ }$	the square root of
−	minus	$\sqrt[3]{\ }$	the cube root of
×	multiplied by (times)	$P(E)$	the probability of event E occurring
÷	divided by	$P(\bar{E})$	the probability of event E not occurring
%	per cent	$\angle ABC$	angle ABC
( )	parentheses, round brackets	ΔABC	triangle ABC
[ ]	(square) brackets	-3	negative 3
{ }	braces	$0.\dot{6}$	the recurring decimal 0.666 666…
=	is equal to	$3(4)$	3 multiplied by 4, 3×4
≈	is approximately equal to	$3x$	3 multiplied by x, $3 \times x$
≠	is not equal to	$\frac{x}{2}$	x divided by 2, $x \div 2$
<	is less than	x^2	x squared, $x \times x$
>	is greater than	x^3	x cubed, $x \times x \times x$
≤	is less than or equal to	$\bar{x}$	the mean (average)
≥	is greater than or equal to	π	pi (3.141 59…)
∴	therefore	θ	theta
		α	alpha
		μ	micro- (one millionth)

A

absolute error The maximum possible error for a measurement, equal to $\pm\frac{1}{2}$ of the smallest unit marked on the scale of the measuring device.

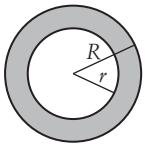
angle of depression The angle between the horizontal and the line of sight, looking down.

angle of elevation The angle between the horizontal and the line of sight, looking up.



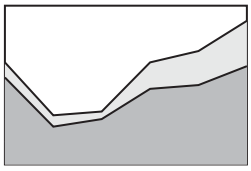
annuity An investment involving regular contributions (payments) into an account earning compound interest, for example, a superannuation fund.

annulus Ring-shaped area between two concentric circles of different radii.



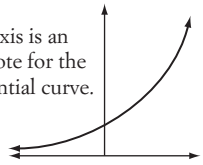
arc Part of the circumference of a circle.

area chart A graph consisting of different 'areas' or 'bands' each representing a data set over a period of time.



arrangement See **ordered selection**.

asymptote Line that a curve gets very close to but never actually touches.



at least A number equal to or greater than the given value (for example, 'at least 2' means 2 or more).

Australian Eastern Standard Time (AEST) Standard time zone for the eastern states of Australia.

B

balance owing The amount remaining or owing in a loan.

base (in index notation) A number being raised to a power. For example, in 2^5, the base is 2.

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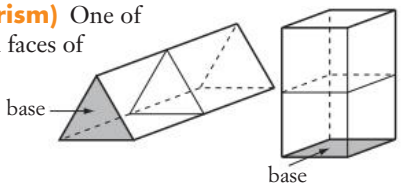
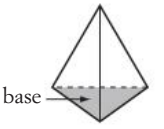
base (of a prism) One of the parallel end faces of a prism.



base (of a pyramid) The plane figure on which a pyramid rests.



BASIX® (Building Sustainability Index) A government standard for energy-efficiency housing, including requirements for environmentally-friendly water use, thermal comfort and energy use.

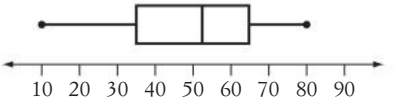
bearing See **compass bearing** and **true bearing**.

bias In statistics, an unwanted influence that stops a sample from being representative of a population.

biometric data Data related to body measurements, such as head circumference and arm span.

bivariate data Data involving two variables that can be graphed on a scatterplot.

box-and-whisker plot (or box plot) A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers.



break-even point The point or value at which a business stops making a loss and starts making a profit.

Building Sustainability Index (BASIX®) See **BASIX®.**

buying on terms See **term payments**.

C

capacity Maximum volume of liquid that can be held by a container, usually measured in millilitres (mL), litres (L) or kilolitres (kL).

capture-recapture technique A method of estimating the size of an animal population by capturing, tagging and releasing a sample of animals, then recapturing a new sample and counting the number of animals tagged.

cash advance A withdrawal of cash made on a credit card, which does not have an interest-free period, and is usually charged at a higher interest rate than on purchases.

categorical data Information or data represented as a category rather than a number (for example, makes of car, or the colours of eyes). Differs from **quantitative data**.

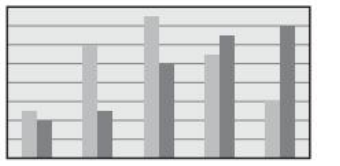
changing the subject of a formula Rearranging a formula so that a different variable is the subject. For example, the subject of the formula $v = u + at$ is v, but the subject can be changed to u by rewriting the formula as $u = v - at$.

class centre The centre of a class interval. For example, the class centre of the class interval 10–19 is 14.5.

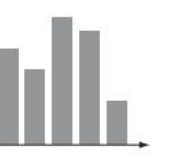
class interval In statistics, when there are many data scores, they can be grouped into class intervals. For example, ages of people could be grouped into class intervals of 1–10, 11–20, 21–30, and so on.

cluster A group of data scores that are bunched or close together.

clustered column graph A column graph that compares the data of two or more categories.

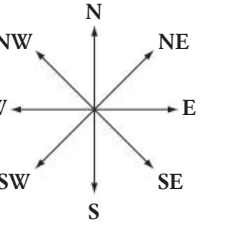


column graph A graph used to display categorical or quantitative discrete data.



coefficient A number in front of a variable in a function, for example, the coefficient of x in $y = -2x + 10$ is -2.

compass bearing A direction shown by points on a compass, such as north (N), south (S), east (E), west (W), north-east (NE), north-west (NW), south-east (SE), south-west (SW).



compass radial survey Survey where a compass is placed in the middle of a field and the true bearing and distance to each corner are measured. See also **plane table radial survey**.

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complementary event All the outcomes that are not the event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

compound interest Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**.

compounding period How often interest is calculated (for example, daily, monthly or yearly).

concentration The proportion of a drug in a solution, expressed as a rate in mass per volume, for example, 5 mg/mL.

constant of variation (or constant of proportionality) For example, if y varies as the square of x, the variation equation is $y = kx^2$ and the constant of variation is k because its value is fixed (constant, does not change).

constant term A number on its own in a function, for example, the constant term in $y = -2x + 10$ is 10.

continuous data Quantitative (numerical) data that can be measured on a smooth scale of values (without 'gaps'), such as the heights of people. Differs from **discrete data**.

contribution to an annuity Regular payment made to an annuity.

Coordinated Universal Time (UTC) See **UTC**.

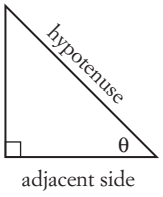
correlation The strength of the relationship between two variables: can be positive or negative, strong (high) or weak (low).

correlation coefficient A variable, r, that measures the correlation of two variables, and whose value is between -1 and 1.

cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **sine** and **tangent**.

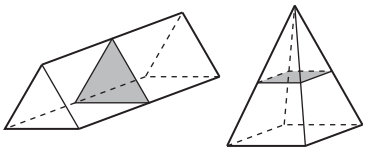


cosine rule Trigonometric formula used for triangle problems involving 3 sides and 1 angle.

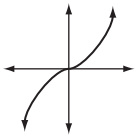
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

cross-section of a solid A plane shape formed when a solid is cut parallel to an end.

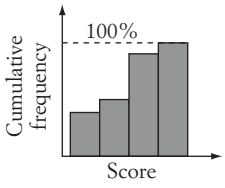


cubic function A function of the form $y = ax^3 + c$.

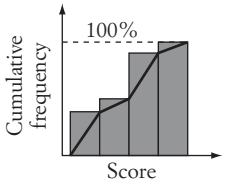


cumulative frequency A running total of frequencies.

cumulative frequency histogram A histogram where the height of each column represents the cumulative frequency at each score.



cumulative frequency polygon (or ogive) A line graph formed by joining the top right-hand corners of the columns of the cumulative frequency histogram.



D

data set Collection or group of data scores.

daylight saving Scheme where clocks are turned forward an hour to take advantage of increased hours of daylight during summer.

dam A large body of water held back by a barrier, often to store water for usage.

dependent variable The variable in a function that depends on another variable for its value. For example, if $y = 3x + 5$, y is the dependent variable because its value depends on the value of x. Differs from **independent variable**.

deposit When purchasing an item using term payments, a single large payment that is made first, for example, 10% of the purchase price.

direct linear variation (or direct proportion) The relationship between two variables (say x and y) by an equation of the form $y = kx$, where k is the constant of variation. Differs from **inverse variation**.

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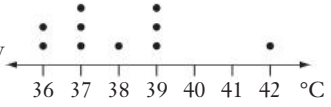
discrete data Quantitative (numerical) data that can be counted and whose values are separate and distinct, such as the number of pets owned, or the numbers of people in families. Differs from **continuous data**.

distribution (or frequency distribution) The way the scores of a data set are arranged, especially when graphed. See also **Shape of a distribution.**

divided bar graph A graph representing the parts of a whole by a rectangle divided into proportionately sized sections.

dosage A prescribed amount of medicine to be taken.

dot plot A graph that uses dots to show frequencies of data scores.



E

elimination method An algebraic way of solving simultaneous equations in which the equations are added or subtracted so that one of the variables is eliminated.

Energy Rating Label (ERL) A label on an electrical appliance showing the star rating of how energy-efficient it is (the more stars, the better), including the estimated number of kilowatt hours used per year (the lower, the better).

Equator The 0° parallel of latitude, the great circle running around the middle of the Earth. The latitude of any point on the Earth's surface is measured north or south of the Equator.

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}.

experimental probability An estimate of the theoretical or calculated probability; the relative frequency of an event in repeated trials of an experiment.

exponential curve A graph of an **exponential function**.

extrapolate To estimate the value of a quantity that is outside the given range of values. Different from **interpolate**.

extremes (upper and lower) The highest and lowest scores in a data set.

F

financial expectation In a game of chance, the expected or average amount of money returned per game, calculated by multiplying every financial outcome by its probability and adding the results together.

five-number summary Lower extreme, lower quartile, median, upper quartile and upper extreme of a data set. These five numbers are used to draw a **box-and-whisker plot**.

flat rate interest See **simple interest**.

flat rate loan Loan where **simple interest** is charged on the amount borrowed for the term of the loan. See also **reducing balance loan**.

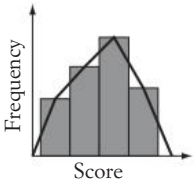
fortnightly Every two weeks.

frequency The number of times a score or group of scores occurs in a data set.

frequency distribution See **distribution.**

frequency histogram A column graph that shows the frequencies of scores.

frequency polygon A line graph formed by joining the midpoints of the tops of the columns of a frequency histogram.



function The relationship between two variables, for example, $y = 3x - 5$.

future value The amount of a growing investment at a specific time.

future value of an annuity (FVA) The final amount or total value of the annuity at the end of the term. Differs from **present value of an annuity**.

G

giga- (symbol G) A prefix meaning 'one billion'.

gigalitre (GL) 1000 ML or 1 000 000 000 L.

gigawatt (GW) 1000 MW or 1 000 000 000 W.

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gradient (symbol *m*) A measure of the steepness of a line; also the rate of change of y relative to x.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

great circle Circle formed when a slice is taken through the centre of the Earth. The Equator and all meridians of longitude are great circles.

Greenwich Mean Time (GMT) See **UTC**.

Greenwich meridian See **Prime meridian**.

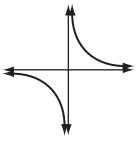
H

hectare A land measure equal to the area of a square 100 m by 100 m. 1 ha = 10 000 m^2.

hemisphere Half a sphere, or half of the Earth, for example, the southern hemisphere.

hire purchase See **term payments**.

hyperbola The graph of a hyperbolic function. It has two branches.



hyperbolic function A function of the form $y = \frac{a}{x}$. Its graph is a **hyperbola**.

I

independent variable A variable in a function whose values do not depend on any other variable. For example, if $A = \pi r^2$, r is the independent variable. This differs from the **dependent variable**.

index laws Rules for simplifying algebraic expressions involving powers of the same base, for example, $a^m \div a^n = a^{m-n}$.

index notation A way of writing repeated multiplication using powers. For example, $7 \times 7 \times 7 = 7^3$.

instalment See **repayment**.

interest Money earned on an investment or paid on a loan.

interest-free period A period of time when interest is not charged on an amount borrowed, used with term payments and credit card accounts.

International Date Line The imaginary line that runs through the Pacific Ocean and is approximately the 180° meridian of longitude. A day is either gained or lost when this line is crossed.

interpolate To estimate the value of a quantity that is within the given range of values. Differs from **extrapolate**.

interquartile range (IQR) The difference between the upper and lower quartiles of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data.

inverse variation (or inverse proportion) Relationship between two variables (say x and y) by an equation of the form $y = \frac{k}{x}$, where k is the constant of variation. Differs from **direct variation**.

K

kilo- (symbol k) A prefix meaning 'one thousand'. For example, a kilolitre is 1 000 litres.

kilowatt (kW) A unit of power equal to 1000 watts.

kilowatt-hour (kWh) A unit of electrical energy equivalent to that used by one kilowatt of power in one hour.

L

latitude The angular distance north or south of the Equator of a point on the Earth's surface; the size of the angle made between the point and the Equator at the Earth's centre.

least-squares line of best fit The line of best fit on a scatterplot where distances of each point from the line is as small as possible.

life expectancy The average number of years of life remaining to a person at a specified age, or the average number of years of life of a person at birth.

line of fit (or regression line) A straight line that represents a set of points obtained through experiment or observation.

linear function A function of the form $y = mx + b$, whose graph is a straight line.

linear modelling Using a linear function to approximate a real-life situation.

longitude The angular distance east or west of the prime meridian of a point on the Earth's surface; the size of the angle made between the point and the prime meridian at the Earth's centre.

lump sum A single amount of money, usually large, as opposed to a series of payments. Used to describe money paid into a loan or annuity, or a superannuation payout to a person on retirement.

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M

mean The average of a set of scores.

$$\text{mean (or } \bar{x}) = \frac{\text{sum of scores}}{\text{number of scores}} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$$

measure of location A statistical value, such as the mean, median or mode, which describes the centre or average of a set of data.

measure of spread A statistical value, such as the range, interquartile range or standard deviation, which describes the spread of a set of data.

median The middle score of a data set when scores are arranged in ascending order. If there are two middle scores, the median is the average of the two.

median class The class interval that contains the median score.

meridian of longitude Great semicircle running down the Earth's surface from the North Pole to the South Pole, measured east or west of the prime meridian (0° longitude).

mega- (symbol M) A prefix meaning 'one million'. For example, a megatonne is 1 000 000 tonnes.

megalitre (ML) 1000 kL or 1 000 000 L

megawatt (MW) 1000 kW or 1 000 000 W

micro- (symbol μ) A prefix meaning 'one millionth'. For example, a micrometre is one-millionth of a metre.

microgram (μg) One-millionth of a gram. 1 g = 1 000 000 μg.

milligram (mg) One-thousandth of a gram. 1 g = 1000 mg.

minute (angle measure) A measure of angle size. $\frac{1}{60}$ of a degree. 1° = 60′.

minute (time) A measure of time. $\frac{1}{60}$ of an hour. 1 h = 60 min.

modal class The class interval with the highest frequency.

mode The most common or frequent score(s) in a set of data.

modelling Using mathematics to describe a real-life pattern or relationship.

more than A number greater than the given value. For example 'more than 2' means any number above 2 (not including 2).

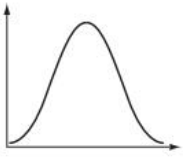
multi-stage event An event that involves two or more outcomes occurring together. For example a two-stage event that involves tossing a coin and rolling a die together could be tossing tails and rolling an odd number.

N

nominal data Categorical data that cannot be ordered, for example, colour of eyes.

non-linear function A function that is not of the linear form $y = mx + b$. Its graph is a curve and can take many forms.

normal distribution A statistical distribution that is symmetrical about its mean, whose graph is the bell-shaped **normal curve**. The mean, median and mode are all equal, represented by the peak of the curve.



notebook entry A diagram of measurements drawn by a surveyor in an **offset survey**.

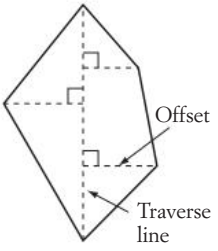
	40	
24	32	
	25	16
	0	

numerical data See **quantitative data**.

O

obtuse angle An angle greater than 90° and less than 180°.

offset survey (or traverse survey) A method of measuring lengths in an irregularly-shaped field, using offsets to each corner of the field from a traverse line.



ogive See **cumulative frequency polygon**.

ordered selection A selection of items where the order is important, for example, choosing a captain and vice-captain of a cricket team. Different from **unordered selection**.

ordinal data Categorical data that can be ordered, for example, level of swimming class.

outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4.

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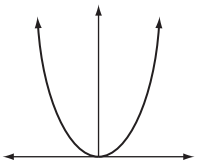
outlier An extreme (high or low) score in a data set that is much different than the other scores, either less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

P

parabola U-shaped graph of a quadratic function. See also **quadratic function**.



parallel of latitude Small circle running around the Earth's surface, parallel to and measured north or south of the Equator (0° latitude).

peak A high point of a distribution, and the highest peak shows the mode.

per annum (p.a.) Per year, annually.

percentage error The maximum possible error for a measurement, expressed as a percentage.

period Amount of time (for example, month, week, year).

pie chart See **sector graph**.

pivot table A tool for organising data in a table in different ways, summarised by different categories.

plane table radial survey Survey where a table is placed in the middle of a field and the angle and distance to each corner are measured. See also **compass radial survey**.

point symmetry A type of rotational symmetry in which a figure will map onto itself if turned 180°. A cubic curve has point symmetry; if you spin it upside down, it will fit onto itself.

population All of the items under investigation. See also **sample**.

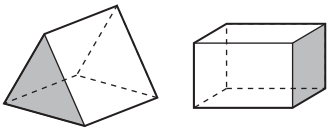
present value See **principal**.

present value of an annuity (PVA) The single sum of money that, if invested today at the same compound interest rate as the annuity, would produce the same future value (final amount) over the same period of time. See also **future value of an annuity**.

prime meridian (or Greenwich meridian) The 0° meridian of longitude that passes through Greenwich in London. The longitude of any point on the Earth's surface is measured east or west of this meridian.

principal (or present value) The original amount of money invested or borrowed.

prism A solid with flat faces and a uniform cross-section.

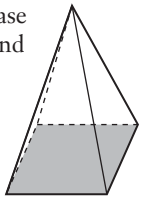


probability A measure of how likely an event E is to occur, written $P(E)$. Its value ranges from 0 to 1.

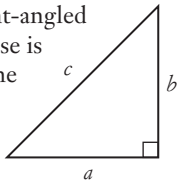
probability tree diagram A tree diagram in which probabilities are listed on the branches of every stage.

proportional to In the same ratio as. If y is proportional to x, we say $y = kx$ and x and y have a direct linear variation. See **direct variation**.

pyramid A solid with a polygon as a base (for example, a rectangle, or a triangle) and triangular side faces that meet at a point (called the apex).

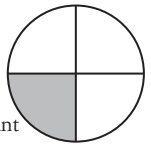


Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. $c^2 = a^2 + b^2$



Q

quadrant Quarter of a circle.



quadratic function A function of the form $y = ax^2 + bx + c$. Its graph is a **parabola**.

quantitative data (or numerical data) Data that can be counted or measured. It can be discrete or continuous. See also **categorical data**.

quarterly Four times a year; every three months.

quartiles Values that divide a data set into 4 equal parts when the scores are arranged in order. The 1st quartile (Q_1) is the lower quartile, the 2nd quartile (Q_2) is the median, the 3rd quartile (Q_3) is the upper quartile.

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quartiles (upper and lower) The upper quartile is the third quartile (Q_3) that cuts off the top 25% of scores in a data set, and the lower quartile is the 1st quartile (Q_1) that cuts off the bottom 25% of scores.

R

radar chart A display of data on a circular grid (like a spider's web) that shows changes over a certain period or cycle.

rainfall The amount of rain falling on a particular location, measured in millimetres.

random sample A sample for which every member of a population has an equal chance of selection.

range The difference between the highest score and lowest score in a data set.

raw score A score before it has been standardised into a z-score. See also **z-score**.

reducing balance loan A loan in which interest is charged on the amount owing each period and not on the principal borrowed. See also **flat rate loan**.

regression line See **line of fit**.

relative frequency The number of times an event or score occurs, written as a fraction of the total frequency. In probability, it is used to approximate the theoretical probability of an event.

$$\text{Relative frequency} = \frac{\text{frequency}}{\text{total frequency}}$$

repayment (or instalment) Amount paid regularly to pay off a loan.

reservoir A large area where rainwater is collected to provide the water supply for a community.

S

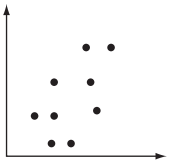
sample A group of items selected from a population for statistical study.

sample size The number of items in a sample.

sample space In probability, the set of all possible outcomes of a situation or experiment.

scale (on a map or diagram) The ratio of scaled length to actual length, for example, a scale of 1 : 500 means that lengths represented on the map or diagram are actually 500 times larger in real life.

scatterplot A graph of points on a number plane showing a relationship between two variables.

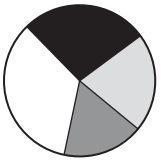


scientific notation A way of writing very large or very small numbers. For example, 98 000 000 = 9.8×10^7.

second A measure of time or angle size, $\frac{1}{60}$ of a minute.

sector Part of a circle bounded by an arc and two radii.

sector graph (or pie chart) A graph representing the parts of a whole population using a circle divided into proportionately-sized sectors.



sewerage The system of removing waste water from a home or building.

shape of a distribution The way the data in a frequency distribution is spread, can be symmetrical, positively skewed or negatively skewed.

significant figures Meaningful digits in a numeral that tell 'how many'. For example, 98 000 000 has two significant figures: 9 and 8.

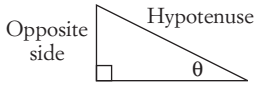
simple interest (or flat rate interest) Interest calculated only on the original principal invested or borrowed. Different from **compound interest**.

Simpson's rule Formula for finding the approximate area of an irregular-shaped block.

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

simultaneous equations Two or more equations that must be solved together so that the solution satisfies all equations. For example, the simultaneous equations $y = 3x - 5$ and $y = 2x$ have the solution $x = 5$, $y = 10$.

sine A ratio in a right-angled triangle:



$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **cosine** and **tangent**.

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sine rule Trigonometric formula used for triangle problems involving 2 sides and the 2 angles opposite them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

skewed The shape of a statistical distribution when most of the data scores are either low (positively skewed) or high (negatively skewed). The tail indicates the direction of the skew.

Negatively skewed Positively skewed

small circle A slice through a sphere that does not pass through the centre. It has a smaller radius than a **great circle**.

standard deviation (symbol σ_n) A measure of spread that describes the distribution of scores about the mean, whose value depends on every score in the data set.

standardised score See **z-score**.

statistics Collection, tabulation and displaying of data for the purpose of analysis.

stem-and-leaf plot A 'number graph' that lists all the data scores, with the first digit shown in the stem column. This stem-and-leaf plot lists 12 scores from 42 to 82.

4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

5 | 8 means 58

stratified sample A sample consisting of percentages of items from the 'strata' or 'layers' of a population. For example, a stratified sample from a population of 35% children and 65% adults should contain 35% children and 65% adults.

subject of a formula See **changing the subject of a formula.**

substitution method An algebraic way of solving simultaneous equations in which one variable is made the subject of one equation, then that equation is substituted into the other equation.

summary statistic A calculated value that represents or summarises a set of data (for example, the mean or the standard deviation).

superannuation fund A person's retirement fund that grows with regular contributions (payments) and compound interest, a type of annuity.

survey (land) To measure lengths and angles on a field and perform calculations on these measurements.

survey (statistical) To gather information for statistical purposes.

sustainability The practice of maintaining the Earth's natural resources effectively and without waste, to support its long-term ecological balance.

symmetrical distribution See **normal distribution**.

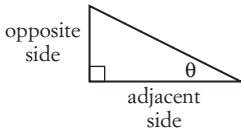
systematic sample A sample chosen by using a set pattern (for example, choosing every 10th number in a phone book).

T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. See also **cosine** and **sine**.



tera- (symbol T) A prefix meaning 'one trillion'. For example, a terawatt is 1 000 000 000 000 watts.

term of a loan The amount of time it takes a loan to be completely paid off.

term payments Paying off a loan plus interest by regular repayments. Also called **buying on terms** or **hire purchase**.

thermal comfort Living in an environment where the temperature is at a comfortable level, keeping the home cool in summer and warm in winter.

three-figure bearing See **true bearing**.

time zone A zone of the world in which the time is the same for all places.

traverse survey See **offset survey**.

tree diagram A diagram for listing all the possible outcomes of a multi-stage experiment such as tossing three coins together. See also **probability tree diagram**.

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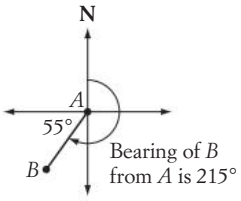
trial One of the attempts in a repeated situation, experiment or test.

trigonometry The branch of mathematics dealing with the relationships between the sides and angles of triangles.

true bearing (or three-figure bearing) A direction measured as an angle clockwise from due north, from 000° to 360°.



U

unitary method A way of calculating an amount by first finding one unit then multiplying.

unordered selection A selection of items where the order is not important, for example, choosing 3 students to represent a school. Different from **ordered selection**.

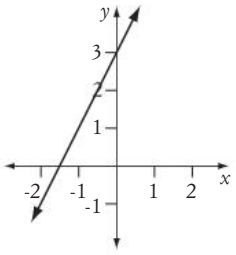
UTC (Coordinated Universal Time) or Greenwich Mean Time (GMT) Local time at the prime meridian (0° longitude) time zone, from which other times in the world are measured.

V

variable A pronumeral that can take a range of values (that is, it is not a **constant**).

vertex The turning point of a **parabola**.

vertical intercept (or y-intercept) The value at which a straight line graph cuts the vertical axis. For example, the vertical intercept of this graph is 3.



volume Amount of space occupied by a solid, measured in cubic units.

W

watt A unit of power equal to one joule of energy per second.

Y

***y*-intercept** See **vertical intercept**.

Z

***z*-score (or standardised score)** A calculated statistic that shows the number of standard deviations a **raw score** is above or below the mean. For example, a z-score of –2.3 means the raw score is 2.3 standard deviations below the mean.

$$z = \frac{x - \bar{x}}{s}$$

See also **raw score**.

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Notes

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ANSWERS

Chapter 1

SkillCheck

1 **a** 39 **b** 300
c 1040 **d** 26
2 **a** 3 **b** $0.\dot{6}$
3 **a** 1125.51 **b** 3693.64
4 $2560
5 **a** $2894.06 **b** $394.06
6 $269.71

Exercise 1-01

1 **a** $18 000 **b** $300 **2** B
3 **a** $19 575 **b** $250.97 **c** $405
4 **a** $649.60 **b** $147.90 **c** $1.21
5 C
6 **a** $3049 **b** $49.97 **c** $55.37
7 $580.84
8 Ali: **a** $10 575 **b** $551.25
Bryce: **a** $1612.50 **b** $530.63
Chloe: **a** $11 329.20 **b** $654.25
9 **a** $3023.42 **b** $412.31 **c** $123.20

Exercise 1-02

1 C **2** A
3 **a** Plan B **b** Plan B
4 **a** $35 661.89 **b** $5218.26 **c** 11.3%
d Popularity of car, excess stock is cheaper, price reductions on old stock due to new models being released.
5 **a** $43 778 **b** $1062.07
6 **a** **i** $2491.20 **ii** $2580.48
b **i** $1593 **ii** $1622.70
c Local Loans – Positive: 12 months to save up balance
– Negative: interest charged per day on amount owing
Feelgood Finance – Positive: 100 days to save up price of computer
– Negative: more interest, paid as no deposit
7 **a** $1950 **b** $1694.80 **c** $2198.50
8 $388.42 **9** B
10 **a** No interest, must have money.
b Don't need cash upfront, interest may be charged.
c Have time to save up, high interest charged from day of purchase if not repaid on time.

Exercise 1-03

1 C
2 **a**

Month	P	I	$P + I$	$P + I - R$
6	$391 115.24	$2607.43	$393 722.67	$390 322.67
7	$390 322.67	$2602.15	$392 924.82	$389 524.82
8	$389 524.82	$2596.83	$392 121.65	$388 721.65
9	$388 721.65	$2591.48	$391 313.13	$387 913.13

b $388 721.65 **c** $7086.87 **d** $23 513.13
3 B
4 **a** $291 018 **b** $111 018 **c** 5.1% p.a.
d $194 400
5 **a**

Month	P	I	$P + I$	$P + I - R$
1	$450 000	$1730.77	$451 730.77	$449 770.77
2	$449 770.77	$1729.89	$451 500.66	$449 540.66
3	$449 540.66	$1729.00	$451 269.66	$449 309.66
4	$449 309.66	$1728.11	$451 037.77	$449 077.77
5	$449 077.77	$1727.22	$450 804.99	$448 844.99
6	$448 844.99	$1726.33	$450 571.32	$448 611.32

b $1388.68 **c** $10 371.32
6 **a** 0.011 583 **b** $0.011\,583 \times \$15\,479.33 = \$182.42$
c

Month	P	I	$P + I$	$P + I - R$
1	$16 000	$185.33	$16 185.33	$15 749.33
2	$15 749.33	$182.42	$15 931.75	$15 495.75
3	$15 495.75	$179.49	$15 675.24	$15 239.24

d $30 **e** $547.24

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7 **a** 0.00481

b

Month	P	I	$P + I$	$P + I - R$
1	$17 800	$85.62	$17 885.62	$17 738.62
2	$17 738.62	$85.32	$17 823.94	$17 676.94

c $170.94 **d** $17 676.94

Exercise 1-04

1 D

2 A

3 **a** **i** $285 298.90 **ii** $151 298.90

b **i** $7828.41 **ii** $2828.41

c **i** $26 383.57 **ii** $12 383.57

d **i** $48 294.09 **ii** $30 094.09

e **i** $163 036.89 **ii** $128 736.89

4 **a** $37 440.03 **b** $24 018.64

c $41 037.33

5 A

6 $278 479.75 **7** $20 000

8 Yes $1000 \times 1.025^{40} = \$2685.06$

Exercise 1-05

1 B **2** C **3** $410.78

4 **a** 12 days **b** $1057.41

5 **a** $2455.75 **b** $3000 on 15 June

c 1 June to 25 July

d **i** $0 **ii** $42.69

6 D **7** D

Exercise 1-06

1 **a** 8% **b** $1298.56

c

Year	P	I	a	$P + I + a$
5	$22 530.56	$1802.44	$5000	$29 333

2 **a** $48 840.80 **b** $1836.24

c $61 905.06 **d** $9415.98

3 $60 030.50 **4** No, short by $929.55.

5 **a** $3080.34 **b** $745.60

c $1664.88 **d** $3234.22

6 $3937.36 **7** $24 030.52 **8** $3137.27

9 **a** $9639

b Yes, she would have $10 193.20.

10 **a** $5825.47 **b** $3396.24 **c** $34 332

11 $1477.69 **12** Yes, they will have $60 103.20.

Exercise 1-07

1 **a** $6164.28 **b** $2930.20

c $11 422.40 **d** $3950.21

2 $2973.68 **3** B **4** $1346.54

5 **a** $5731.95 **b** $4265.11

6 **a** $906.62 **b** $5644.10 **c** $5666.10

7 **a** $1891.20 **b** $22 694.40

8 **a** $1607.39 **b** $3288.68

9 **a** $845.55 **b** $8416.56

10 **a** $4068.61 **b** $40 686.10

11 $57 590

12 **a** $2250 **b** $2131.04

Exercise 1-08

1 B **2** A

3 **a** **i** $3416.20 **ii** $614 916

iii $234 916

b **i** $7760.75 **ii** $2 328 225

iii $1 403 225

c **i** $8806.80 **ii** $2 113 632

iii $883 632

d **i** $9214.34 **ii** $1 105 720.80

iii $408 720.80

4 **a** $9119.20 **b** $33 225.20

c $1967.20 **d** $29 291.20

5 **a** $2.54 **b** $2358 more interest

6 **a** **i** $6375.24 **ii** $8911.68

b Use $100 000 + $80 000 or $90 000 + $90 000. Answer: $1718.10

c Save $93 078.

7 **a** $33 648 **b** save $56 **c** $15 720

Exercise 1-09

1 **a** 10.25% p.a. **b** $295 **c** $750

2 **a** 6.8% p.a. **b** $906 **c** $60

3 **a** 14.50% p.a. **b** $150 **c** $140

4 **a** about 24 years

b approx. 11 years 3 months

c $100 000

d $717 120 **e** $367 120

f 23 years 3 months

g 6 years 4 months **h** $190 000

i $696 000 **j** $346 000

k 9 months **l** $21 120

5 **a** **i** $370 000 **ii** $250 000

b **i** 12 years **ii** 18 years

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6 **a** 6 years

b **i** 22.5 years

ii 17 years

c **i** \$160 000 **ii** \$112 000

7 **a** \$250 000 **b** \$195 000

c 29 years 4 months **d** \$526 944

e \$276 944 **f** 28 years 9 months

g \$526 465 **h** \$266 465

i 7 months

j \$10 479

Sample HSC problem

a \$3472 **b** \$416 640

c \$96 640 **d** 3.02% p.a.

Revision

1 **a** \$14 205 **b** \$6818.40 **c** \$437.99

2 C

3 **a** \$10 082 **b** \$2082 **c** 19.3% p.a.

4 \$297

5 **a**

Month	P	I	$P + I$	$P + I - R$
1	\$18 000	\$135	\$18 135	\$17 715
2	\$17 715	\$132.86	\$17 847.86	\$17 427.86
3	\$17 427.86	\$130.71	\$17 558.57	\$17 138.57
4	\$17 138.57	\$128.54	\$17 267.11	\$16 847.11

b \$1152.89 **c** \$527.11 **d** \$12.89

6 B **7** \$5284.14 **8** \$5.68

9 **a** \$94 885.50 **b** \$34 885.50

10 \$66 937.60

11 \$17 132

12 **a** \$2410.11 **b** \$6114.05

13 **a** \$547 **b** \$26 256 **c** \$6256

14 **a** \$400 000 **b** \$290 000

c 21 years 8 months

d 174 months or 14.5 years

Chapter 2

SkillCheck

1 **a** $m + 16$ **b** $30mn^2$ **c** $\frac{3h}{5}$

d $3x - a$

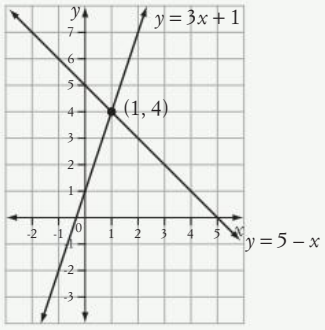
2 **a** $6r - 15$ **b** $-6y - 42$ **c** $-16 + 2k$

3 **a** 54 **b** 4 **c** 108

4 **a** $x = 7$ **b** $d = -1$ **c** $w = 18$

5 **a** y **b** 3 **c** 1 **d** $x = -5$

6



7 $y = 3x - 5$

8 **a** Gradient $= \frac{1}{2}$, equation $y = \frac{1}{2}x + 2$

b Gradient $= -2$, equation $y = -2x + 9$

Exercise 2-01

1 **a** $7x^2$ **b** $22m + 2$ **c** $3bd + d^2 - d$

d $5c + 14$ **e** $2r^2 + 2r$ **f** $-7t^2 + 8tu$

g $e^2 - 2e + 9$ **h** $-2p^2 + 9p$ **i** $6xy - 5yz$

j $10k + 11y - 26$ **k** $-7jk + 11j$

l $3u^2 - 4$

2 **a** $6m^3n$ **b** $30p^2$ **c** $4r^2t^4$ **d** $2w^4y$

e $15u^3$ **f** $6k$ **g** $\frac{2p}{5}$ **h** $-18r^3$

i $27x^6$ **j** $5u^2$ **k** 5 **l** $\frac{3}{7d}$

3 **a** $7k + 28$ **b** 4 **c** $9r + 55$

d $5m + 55$ **e** $11a + 11$ **f** $2t - 5$

g $x^2 + 2x + 15$ **h** $u^2 + 13u$ **i** $7m + 10$

j $3r^2 - 15r + 40$ **k** $14d - 21$

l $5f^2 + 10f + 25$

4 C **5** A **6** B

7 **a** $-2b^3d$ **b** $16a^2b^2$ **c** $4xz + x + 6$

d $3ay + 2y$ **e** $\frac{2p^2}{3}$ **f** $20d^3e^2$

g $\frac{-5u}{2t}$ **h** $2jr - 3r$ **i** $15w^3$

j $8tu + 3u^2 - 8t$ **k** $4a^2y^3$ **l** $25f^6$

Exercise 2-02

1 **a** $\frac{2d}{3}$ **b** $\frac{2p}{15}$ **c** $-\frac{w}{21}$ **d** $\frac{59x}{40}$

e $\frac{4d + 3m}{6}$ **f** $\frac{15c + 7x}{12}$ **g** $-\frac{a}{45}$

h $\frac{7x}{30}$ **i** $\frac{19k}{15}$ **j** $\frac{35c - 18x}{45}$

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k $\frac{20h+3w}{15}$ **l** $\frac{5f}{6}$

2 **a** $\frac{dm}{15}$ **b** $\frac{2kw}{15}$ **c** 8 **d** $\frac{5h^2}{2}$

e $\frac{9h}{10}$ **f** $\frac{ax}{10}$ **g** $\frac{n}{8kp}$ **h** $\frac{b^2}{9}$

i 4 **j** $\frac{4w^2}{9n}$ **k** $4a$ **l** $\frac{3d}{4}$

Exercise 2-03

1 **a** m^9 **b** y^2 **c** w^{12} **d** $6p^5$

e $64b^6$ **f** $5x^5$ **g** $3d^2$ **h** $g^{15}w^{10}$

i $32m^{25}$ **j** $30d^7$ **k** $4y^3w$ **l** $256k^{12}p^8$

m b^2 **n** a^{22} **o** x^8y^5

2 **a** $6m^3n$ **b** $30p^2$ **c** $4r^2t^4$

d 9 **e** 1 **f** $2w^4y$

g $-15u^3$ **h** 5 **i** $6k$

j $\frac{2p}{5}$ **k** $-18r^3$ **l** $27x^6y^{15}$

m $5u$ **n** 15 **o** $\frac{7m^2}{5}$

p 3 **q** $\frac{5w^2}{6z^2}$ **r** $\frac{1}{3}x^7$

s -1 **t** $\frac{4d^2}{3n}$ **u** $48g^5$

3 D

Exercise 2-04

1 C

2 **a** $m=2\frac{3}{4}$ **b** $k=1\frac{2}{7}$ **c** $w=2\frac{2}{3}$

d $g=-1\frac{3}{8}$ **e** $k=-8$ **f** $d=16$

g $p=-5$ **h** $x=-5$ **i** $d=2$

j $m=3\frac{3}{7}$ **k** $x=8\frac{1}{3}$ **l** $m=15$

m $d=13\frac{1}{3}$ **n** $y=-13\frac{1}{3}$ **o** $n=-3\frac{3}{5}$

3 A

4 **a** $h=12$ **b** $r=9\frac{1}{3}$ **c** $k=-\frac{1}{2}$

d $m=4\frac{2}{7}$ **e** $x=-3$ **f** $c=3$

g $y=4\frac{1}{2}$ **h** $d=11$ **i** $a=-1$

j $d=\frac{4}{5}$ **k** $k=\frac{3}{20}$ **l** $x=2\frac{2}{3}$

m $m=-3\frac{1}{2}$ **n** $h=\frac{5}{16}$ **o** $y=-1$

5 D

Exercise 2-05

1 100.4° F **2** 0.5° C **3** 4.5 cm

4 43.2 km/h **5** D **6** 8 years

7 **a** 270.6 km **b** 14 h 17 min

8 0.5 years

9 **a** \$A16.18 **b** \$A30.74 **c** \$A80.89

10 75 kg

11 **a** 16.5 mL **b** 11%

12 14 cm

13 280 cm **14** 9000 m **15** 0.083 m^3

16 $R_1 = 60$ **17** 2.2 kg **18** 4.5 cm

Exercise 2-06

1 **a** $x=\frac{y-4}{2}$ **b** $k=\frac{T+7}{3}$

c $x=3d-1$ **d** $x=18-p$

e $x=\frac{k-r}{4}$ **f** $x=2(C-n)$

g $x=\frac{S}{10b}$ **h** $x=6V+5$

i $x\approx\frac{12-z}{a}$

2 **a** $d=St$ **b** $t=\frac{d}{S}$

3 **a** 1800° **b** $n=\frac{A}{180}+2$ or $n=\frac{A+360}{180}$

c 9

4 D

5 **a** $h=\frac{2A}{x+y}$

b $y=\frac{2A}{h}-x$ or $y=\frac{2A-hx}{h}$

6 **a** $r=\pm\sqrt{\frac{A}{\pi}}$ **b** $m=\frac{y-b}{x}$

c $s=\frac{v^2-u^2}{2a}$ **d** $x=sz+m$

e $c=\pm\sqrt{\frac{E}{m}}$ **f** $v=\pm\sqrt{\frac{2K}{m}}$

g $b=\pm\sqrt{c^2-a^2}$ **h** $y=\frac{x^2-3}{6}$

i $h=\frac{2A}{b}$ **j** $l=\frac{T^2g}{4\pi^2}$

k $R=\frac{V+E}{I}$ **l** $Q=C(e-ir)$

m $r=\frac{v^2\pi}{m}$ **n** $a=\frac{2(s-ut)}{t^2}$

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o $r = \sqrt[3]{\frac{3V}{4\pi}}$ **p** $d = Kw - m$

7 $h = \sqrt{\frac{m}{B}}$, 1.55 m **8** 20 cm

9 $F = \frac{9C}{5} + 32$ or $F = \frac{9C + 160}{5}$

a 104° F **b** 212° F

10 $l = \frac{T^2 g}{4\pi^2}$, $l = 2.2$ m

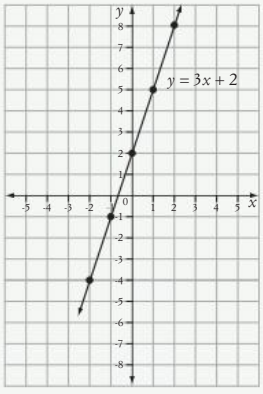
Exercise 2-07

1 B

2 **a** **i** x **ii** y

b

x	–2	–1	0	1	2
y	–4	–1	2	5	8

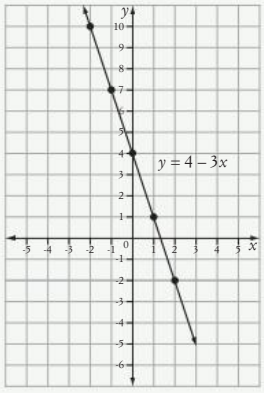


c Gradient = 3, y-intercept = 2

d The gradient 3 is the coefficient of x in the equation $y = 3x + 2$ and the y intercept 2 is the constant term.

3 **a**

x	–2	–1	0	1	2
y	10	7	4	1	–2



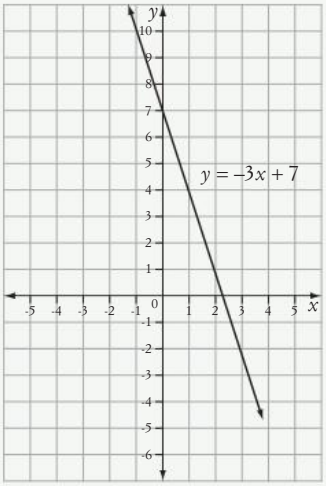
b Gradient = –3, y-intercept = 4

c The gradient –3 is the coefficient of x in the equation $y = 4 - 3x$ and the y intercept 4 is the constant term.

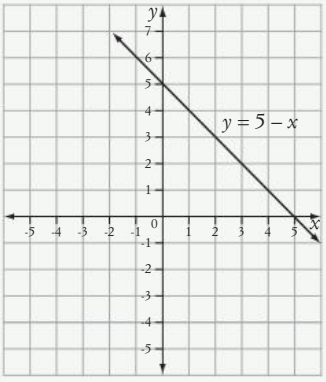
4 **a** $m = -3$, $b = 7$ **b** $m = -1$, $b = 5$

c $m = \frac{1}{2}$, $b = -3$

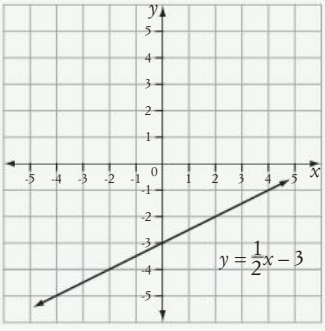
5 **a**



b



c



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6 D

7 **a** $c = -8k + 200$ **b** $v = 6t + 3$

c $p = -3n + 150$ **d** $t = 5x - 12$

8 **a** v, volume of water **b** $C = 1.64v + 142$

c Cost in dollars per kilolitre of water.

d Fixed cost of water usage (even when no water is used).

e \$230.56 **f** 75 kL

9 **a** $F = 1.8C + 32$ **b** C

c 32, the °F equivalent to 0°C.

d **i** 53.6°F **ii** 30°C

e **i** 53°F **ii** 30°C **f** 9°

10 **a** $V = -82m + 3760$ (answers may vary)

b \$3760

c −82, decrease in value of computer in dollars per month.

d **i** \$2120 **ii** \$316

e 3 years 10 months

f The model only works until 3 years 10 months. After that, the value of the computer should stay at \$0, not negative amounts.

11 **a** s **b** $L = \frac{1}{3}s + 8\frac{1}{3}$

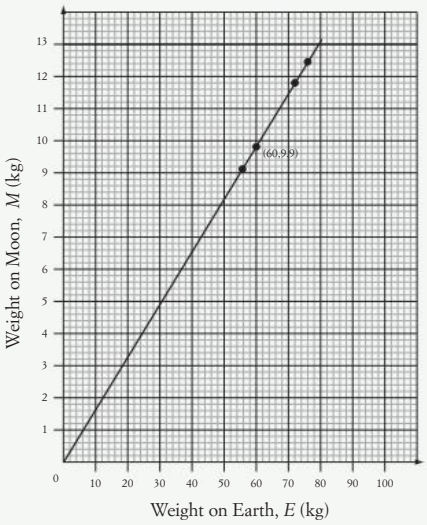
c $10.8\dot{3} = 10\frac{5}{6}$ inches **d** 14

12 **a** $C = 270n + 8400$ **b** \$62 400

c 191 **d** \$11 880

Exercise 2-08

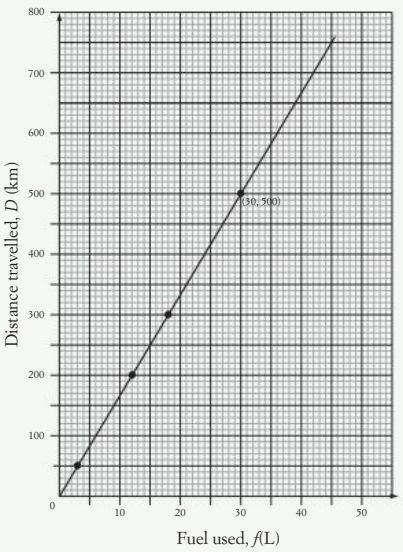
1 **a**



b 0 **c** 0.165

d $M = 0.165E$ **e** 52.1 kg

2 **a**



b $D = \frac{50}{3}f$ **c** 40.8 L

3 **a** $M = 7.8V$ **b** 1404 g

c 1000 000 cm^3 **d** 7800 kg

4 **a** $m = \frac{5}{8}K$ **b** 71.875 miles **c** 256 km

5 **a** Ford G6: $F = 0.184D$;
Ford Fiesta: $F = 0.067D$

b **i** 11.7 L **ii** 32.8 L

c 475 km

6 **a** $T = \frac{1}{50}S$ **b** 60 s **c** 4000 kB

7 C

8 **a** $P = 5.875D$

b 352.5 kPa **c** 141 m

9 \$10 656

10 **a** $F = \frac{235}{3}x$

b **i** 27.42 N **ii** 40.73 N

c 0.38 m

11 A

12 **a** 416 cm **b** 0.85 m

Exercise 2-09

1 **a** $y = 2x + 4$, $y = -3x - 1$, $(-1, 2)$

b $y = 1.5x - 3$, $y = 3$, $(4, 3)$

2 $(2, 3)$

3 **a** CopyCat Express

b CopyCat Express \$650, own photocopier \$1450.

c Initial cost of buying copier = \$1200.

d 15 000

e Cheaper to use own photocopier.

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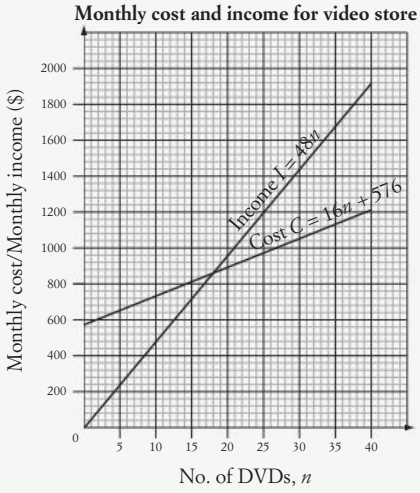
4 **a**



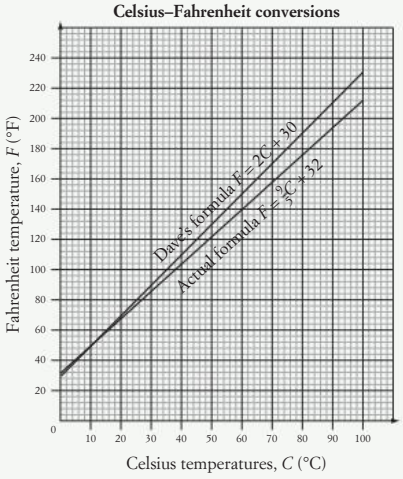
Break-even point is (18, 864)

b Make a loss: monthly cost would be more than monthly income.

c $1136, $1680 **d** $48

e 576, the fixed cost in dollars of buying new DVDs.

5 **a**

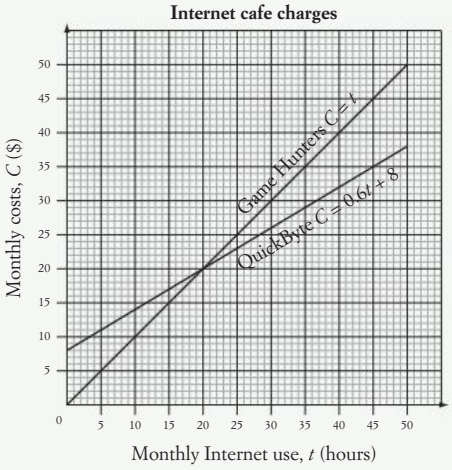


b $C = 10$ **c** $C > 10$

d From $C = 0$ to $C = 35$

e Yes, for values below 35°C.

6 **a**



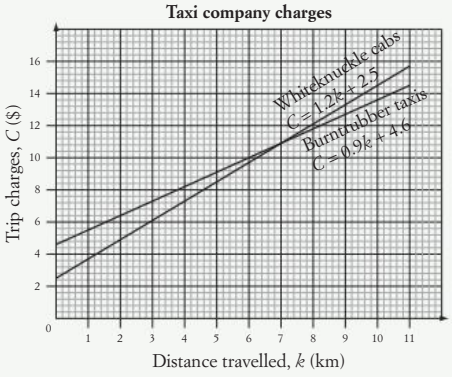
Break-even point is (20, 20)

b QuickByte, cheaper

c QuickByte $22.40, Game Hunters $24.

d Game Hunters is cheaper for users who access the Internet for less than 20 hours per month. QuickByte is cheaper for users who access the Internet for more than 20 hours per month.

7 **a**



Point of intersection is (7, 10.9)

b The distance travelled, 7 km, for which both companies charge the same, $10.90.

c Whiteknuckle is cheaper for trips under 7 km, Burntrubber is cheaper for trips over 7 km.

d Burntrubber, $12

Exercise 2-10

1 **a** $x = 2, y = 4$ **b** $x = 4, y = 2$

c $k = 0.8, m = 1.6$

d $a = 1, d = 3$ **e** $p = -1\frac{1}{6}, w = \frac{2}{3}$

f $x = 4, n = 17$

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2 **a** $m = 2, n = 2$ **b** $k = 5, y = -1\frac{3}{5}$

c $x = 3\frac{1}{5}, y = -2\frac{1}{5}$ **d** $w = 3, g = -3\frac{1}{2}$

e $p = 1, n = 5\frac{1}{2}$ **f** $g = 3, h = -\frac{2}{3}$

g $x = 2, y = -1\frac{1}{2}$ **h** $a = 2\frac{3}{5}, c = -2\frac{2}{5}$

i $k = 4\frac{1}{5}, e = \frac{17}{25}$

3 **a** $n = \$2250$, $C = \$540$ **b** $n > \$2250$

4 **a** 10 people **b** \$900

Sample HSC problem

a $m = 4.8$

b 36, the length of the spring in centimeters when there is no mass tied to it.

c 4.8 cm **d** 25 kg

Revision

1 **a** $\frac{2f^3}{5}$ **b** $16b^6$ **c** $2y$

d $\frac{mn}{2}$ **e** $5f + 3f^2$ **f** $24g^2h^2$

2 **a** $11d + 4$ **b** $4z - 4y + 2yz$

c $2\pi r^2 + \pi rs$ **d** $-78m - 59y$

3 **a** $\frac{7y}{10}$ **b** $\frac{5m}{21}$ **c** $\frac{51k}{40}$

d $\frac{4dy}{15}$ **e** 25 **f** $\frac{6n}{5}$

4 **a** $64y^6m^3$ **b** $\frac{5m^2}{27}$ **c** a^5

d c^8d^{13} **e** 2 **f** $-6d^7$

g $\frac{5m^2n^2}{3}$ **h** $\frac{2}{3}y^7d^4$ **i** $5k^5x^3$

5 **a** $k = 5\frac{1}{2}$ **b** $x = -9\frac{1}{3}$ **c** $p = -9$

d $m = \frac{5}{8}$ **e** $y = -\frac{3}{4}$ **f** $d = 3\frac{3}{8}$

g $a = 13\frac{5}{7}$ **h** $k = 7\frac{1}{3}$ **i** $h = -15$

6 16 years

7 **a** $y = 2P - 220$ **b** $d = \pm\sqrt{\frac{2V - 19.8}{h}}$

c $I = \frac{V + E}{R}$

8 $y = \frac{1}{4}x + 12$

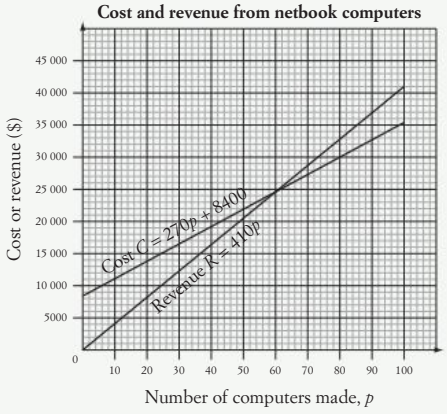
9 **a** $B = 12\,727d + 3000$ (answers may vary)

b Gradient = 12 727 and it represents the increase in bacteria per mL per day.

c 92 089 **d** 8th day

10 **a** 9.57 L/100 km **b** 1082 kg

11



b $p = 60$, $C = R = 24\,600$

c Cost \$21 900, revenue \$20 500.

d It would make a loss.

e 8400, the fixed initial cost in dollars of manufacturing computers.

f \$410

12 **a** $x = 4\frac{1}{2}, y = 9\frac{1}{2}$ **b** $p = 2, w = 2\frac{1}{2}$

c $k = 3, m = 0$

Chapter 3

SkillCheck

1 **a** 3 **b** 2 **c** 3 **d** 3

2 **a** 25 m **b** 3.7 kg

c 250 000 000 km **d** 0.052 cm

3 **a** 10 000 **b** 1 000 000

c 10 000 **d** 100

e 1 **f** 1000

4 **a** 28.3 cm^2 **b** 27.5 cm^2 **c** 3632 m^2

d 0.32 m^2 **e** 30 cm^2

5 **a** 63 **b** 32.99 **c** 54

6 25 m^2

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Exercise 3-01

1 B **2** C

3 **a** 14.75 m and 14.85 m

b ± 0.05 m **c** ± 0.3%

4 **a** 10.25 m and 10.35 m

b 6.65 m and 6.75 m

5 C

6 **a** **i** ± 0.5 km **ii** ± 0.007 750%

b **i** ± 0.05 m **ii** ± 0.72%

c **i** ± 50 m **ii** ± 0.63%

d **i** ± 0.005 mg **ii** ± 0.012 55%

e **i** ± 0.5 ML **ii** ± 0.204%

f **i** ± 0.5 cm **ii** ± 0.303%

7 **a** is most accurate: has the smallest percentage error

8 **a** 2 **b** ± 500 km **c** ± 4.17%

9 **a** ± 17% **b** ± 1.7%

10 ± 0.287%

11 **a** 2 **b** ± 2.5%

Exercise 3-02

1 A

2 **a** 5.1 m^2 **b** \$135.15

3 **a** 93 m^2 **b** 56 m^2 **c** 36 cm^2

d 7747 mm^2 **e** 134 m^2 **f** 127 m^2

g 30 m^2 **h** 89 m^2

4 **a** 52 m^2 **b** 165 m^2 **c** 105 m^2

d 33 m^2 **e** 168 m^2 **f** 66 m^2

5 **a** **i** 60.7 cm^2 **ii** 256 cm^2

6 **a** 4.32 m^2 **b** 600 mm **c** 0.93 m^2

7 **a** 1161 m^2 **b** 148 m

8 **a** 840 m^2 **b** 6879 m^2

Exercise 3-03

1 C **2** D

3 **a** 16.8 m **b** 15.1 m **c** 202.6 mm

4 **a** 26 m^2 **b** 1 325 359 m^2

c 883 573 m^2 **d** 7 m^2 **e** 8482 cm^2

f 150 796 km^2

5 **a** 198 cm^2 **b** 225 m^2 **c** 612 mm^2

d 531 000 km^2

6 **a** 88 cm^2 **b** 12 cm

7 **a** annulus **b** 7.9 m^2

8 **a** 98 m^2 **b** $4 \times \$152 = \$608$

9 **a** 424 cm^2 **b** 83 cm

10 7.1 mm

Exercise 3-04

1 C

2 **a** 666 m^2 **b** 700 m^2

3 **a** 61 m^2 **b** \$228.75

4 136 m^2

5 **a** 115.8 m^2 **b** 140 m^3

6 D

7 **b** 796 m^2 **c** 15.92 ML

Exercise 3-05

1 A

2 **a** 240 m^3 **b** 439 m^3 **c** 1176 m^3

d 198 m^3 **e** 180 m^3 **f** 960 m^3

3 C **4** A

5 **a** 236 m^2 **b** 347 m^2 **c** 703 m^2

d 205 m^2 **e** 317 m^2 **f** 736 m^2

6 1.7m^2

7 $A = 0.6644$ m^2, $P = 3.92$ m, $h = 1.2$ m

a 6 m^2 **b** 0.80 m^3 correct to 2 sig. figs.

8 **a** 84 cm^3 **b** 298 wedges

Exercise 3-06

1 **a** 7068.6 cm^3 **b** 0.8 m^3

c 31.4 m^3 **d** 8953.5 cm^3

2 **a** 327 cm^3 **b** 2 L

3 **a** 840 m^2 **b** 6.0 m^2 **c** 1100 cm^2

d 3.5 m^2 **e** 72 m^2 **f** 11 m^2

4 5451 cm^2

5 **a** 3.1 m^3 **b** 13.4 m^2

6 **a** 4107 cm^3 **b** 8.7%

7 **a** 39 m^3 **b** 25 133 L

8 C

Exercise 3-07

1 **a** 310 000 km^3 **b** 224 000 m^3

c 9.05×10^8 m^3 **d** 994 cm^3

e 2.44 m^3 **f** 11 500 m^3

2 **a** **i** 1.011×10^8 km^2 **ii** 1.068×10^{12} km^3

b 5.594×10^{12} t/km^3

3 **a** 262 L **b** 1.57 m^2

4 **a** 113 cm^3 **b** 103 cm^3 **c** \$16.96

5 A

6 **a** 92 000 cm^3 **b** 9852

7 **a** 2.9 m^2 **b** 1.4 m^2

c 0.46 m^3 Note: Light is a whole sphere but split in half.

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Exercise 3-08

1 **a** 360 cm^3 **b** 208 m^3 **c** 495 mm^3
d 1560 cm^3 **e** 1707 m^3 **f** 48 m^3
2 **a** 2261.95 cm^3 **b** 351.72 cm^3
c 1.17 cm^3 **d** 61 261.06 mm^3
3 96 mm^3
4 **a** 1440 cm^3 **b** 1.44 L
5 2450 cm^3 **6** 2 420 000 m^3 **7** 576 m^3

Exercise 3-09

1 **a** 179.1 cm^3 **b** 56.5 cm^2
2 571 cm^3
3 **a** 994 020 mm^3
b 33.3% **c** 1 265 625 mm^3
d Most efficient packaging as it has the least amount of air.
4 **a** 1 716 000 mm^3 **b** 1593 cm^3 **c** 7%
5 **a** 8400 cm^3 **b** 730 cm^2
6 **a** 42103 L; 48 000 L
b second pool by 5897 L **c** 73 m^2
7 **a** 0.31 m^3 **b** 33 414 cm^2
8 123 cm^3

Exercise 3-10

1 **a** 220.5225 m^2 **b** 217.5625 m^2
c 219 m^2
2 **a** 68.1625 m^2 and 69.8625 m^2
b 6.7 m [2 sig. figs] **c** 69 m^2
3 **a** 39.081 25 m^2 and 38.201 25 m^2 **b** 39 m^2
4 **a** 1.060×10^{12} km^3, 1.085×10^{12} km^3
b 1.07×10^{12} km^3
5 **a** 67.335 cm^2 and 71.415 cm^2 **b** 69 cm^2
c 37.595 375 cm^3 and 41.063 625 cm^3 **d** 39 cm^3
6 **a** 38 264 913 km^2, 37 827 603 km^2
b 3.80×10^7 km^2
7 **a** 6600 cm^3 **b** 30 800 cm^3 **c** 16 m^3
8 5970 cm^2

Sample HSC problem

a 14 m^2 **b** 4000 L **c** 2.1 m

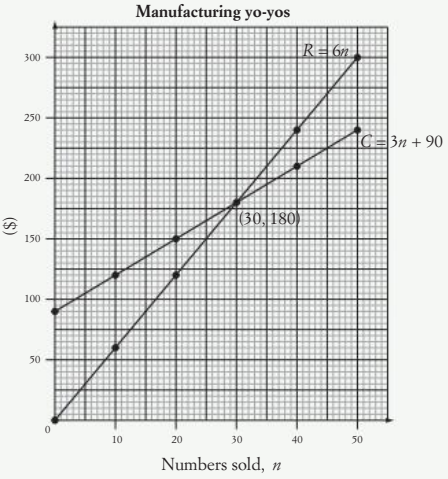
Revision

1 **a** ± 0.05 m **b** ± 0.86%
c 33.0625 m^2 and 34.2225 m^2
d 34 m^2 **e** 1.7 m^3
2 **a** 150 cm^2 **b** 5400 m^2
3 **a** 646.3 cm^2 **b** 28.1 cm
4 **a** $7461.28 **b** 5400 m^2
5 212.7 m^2
6 **a** 8.9 m^2 **b** 1.4 m^3
7 57 470 bottles
8 **a** 5026.55 cm^2 **b** 33 510.32 cm^3
9 **a** 1100 m^3 **b** 170 cm^3
10 **a** 1715 cm^3 **b** 1.1 L
11 204 m^3
12 **a** 4700 cm^3 **b** 52 cm

Practice paper 1

1 B **2** C **3** C
4 B **5** B **6** B
7 C **8** D **9** B
10 C
11 **a** **i** $\$69\,399.53 \times \frac{0.11}{12} \approx \$636.16$
ii $69 195.69
iii $2555.69 **iv** $68 782.39
b $0.52 **c** 150 m^3

12 **a** $b = \frac{2A}{h}$ **b** **i** ±0.5 m
ii 1190.25 m^2 and 1260.25 m^2
c **i** $4200 **ii** $337.50
d 331 733 m^2
13 **a** **i** 33.5 m^2 **ii** 20 m^3
b **i**



ii (30, 180) **iii** $165
iv The point is below the break-even point and the factory would lose money if they made 25 yo-yos.
c $x = 11, y = 1$

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Chapter 4

SkillCheck

1 **a** 4.4; 4; no mode **b** 7.125; 7; 7
c 3; 3; 2 and 4 **d** 4.8; 5; 5

2 **a** 6; 4; 2.115 **b** 7; 5; 2.5604
c 5; 4; 1.807 **d** 5; 3.5; 1.854

3 **a** 11.296 **b** 2.429

4 **a** no mode **b** 55
c brown **d** Holden

Exercise 4-01

1 D

2 **a** QC **b** CO **c** QD
d QC **e** CN

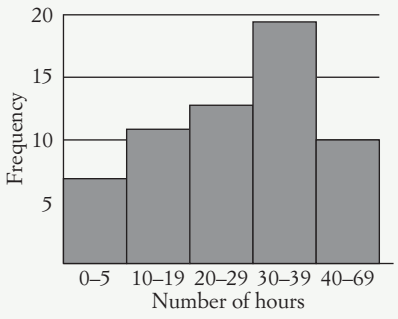
3 **a** column graph; categorical nominal data (drugs used); strength: clear, easy to read; weakness: exact percentages hard to read

b line graph; quantitative continuous data; strength: easy to see; weakness: exact ages not shown

4 **a**

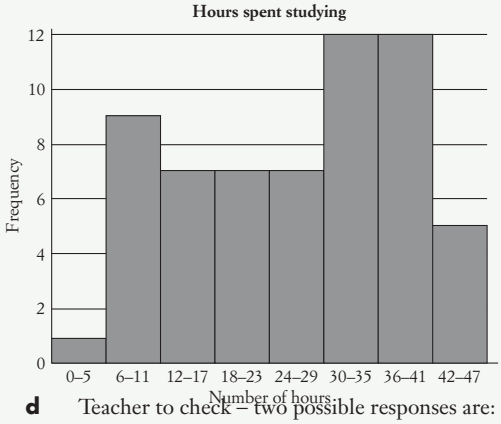
Number of hours	Frequency
0–9	7
10–19	11
20–29	13
30–39	19
40–49	10

b



c

Number of hours	Frequency
0–5	1
6–11	9
12–17	7
18–23	7
24–29	7
30–35	12
36–41	12
42–47	5



d Teacher to check – two possible responses are:

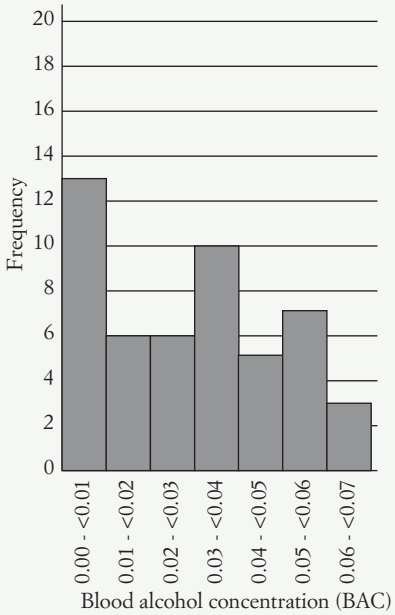
Smaller class intervals gives better indication of how many students spent different amounts of time studying.

The histogram with larger class intervals shows that the trend in the time that numbers of students studied for gradually increased up to 30–39 (negative skew).

5 **a**

BAC	Frequency
0.00 – < 0.01	13
0.01 – < 0.02	6
0.02 – < 0.03	6
0.03 – < 0.04	10
0.04 – < 0.05	5
0.05 – < 0.06	7
0.06 – < 0.07	3

b



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c **i** 19 **ii** 20%

iii 100% if all drivers were either learners, P1 or P2.

62% if all drivers were learners, P1, P2, drivers of large vehicles, dangerous goods, and public vehicles.

20% if all drivers not in the above categories

iv 6

d **i** Tori organised the drivers in terms of the BAC categories for different types of drivers

ii Advantage – gives numbers of drivers in each category of BAC.

Disadvantage – don't know the actual number of say P1, P2 drivers in the each category.

Exercise 4-02

1 D

2 **a** blonde

b categorical data; can't find mean or median

3 A

4 **a** \$425.56, \$400, \$300

b 172.375 kg, 171.5 kg, no mode

5 **a** 24.5 min **b** 25 min

6 **a** 29.5 **b** 30.125

7 **a** 1.73 **b** 2

8 **a** 50

b 6, 16, 30, 54, 98, 72, 45, 10 $\bar{x} = 6.62$

c 7

9 **a**

Hours worked per fortnight	Class centres (x)	Frequency (f)	fx
2 – < 4	3	8	24
4 – < 6	5	11	55
6 – < 8	7	15	105
8 – < 10	9	21	189
10 – < 12	11	17	187
12 – < 14	13	12	156
14 – < 16	15	8	120
16 – < 18	17	7	119
18 – < 20	19	1	19

b 100 students **c** $\bar{x} = 9.74$ hours

d 8 – < 10 hours

10 **a** 25 **b** 3 **c** 1 **d** 1.36

11 **a** 6 hours/day

b 6.38 hours or 6 h 23 min

12 **a** 50 **b** 9

c 25 – 29 or 27 or 28 **d** 25 – 29

Exercise 4-03

1 **a** **i** 15 **ii** 7

b **i** 15 **ii** 7

c **i** 10 **ii** 6

d **i** 9 **ii** 6

e **i** 4 **ii** 1.5

f **i** 47 **ii** 23.5

g **i** 7 **ii** 2

2 **a** 100 **b** 3 **c** 20

d 4 **e** 2

3 **a** 9.58, 4.271 **b** 20.36, 4.433

c 10.91, 3.118 **d** 5.13, 2.705

e 3.75, 1.199 **f** 41.29, 13.195

g 18.76, 1.795

4 **a** 30 **b** \$42 000 **c** \$16 400

5 **a** 7.9112 **b** 1.69

6 **a** 22.8 **b** 1.8

7 **a** 44.7 **b** 64.8

c 36.4 **d** 50.97

e The median and the interquartile range.

Exercise 4-04

1 C

2 **a** negative skew; 1 peak

b positive skew; 4 peaks

c positive skew; 2 peaks

d negative skew; 4 peaks

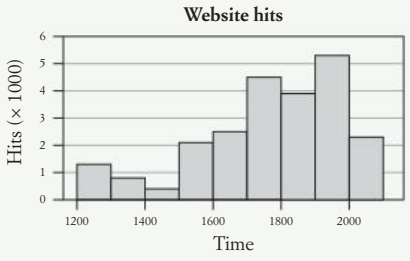
e symmetrical; 1 peak

f 2 peaks

3 **a** 36 **b** negatively skewed

c 70s and 80s **d** 88

4 **a**



b negatively skewed, cluster from 1700 to 2000, peak 1900–2000

c Most visits to Internet sites occur after work hours.

5 B

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6 **a**

Stem	Leaf
1	5 5 5 6 6 7 8 9 9
2	0 1 2 2
3	5 6
4	3 9
5	0 5 9

b positive skewed; cluster 15–19 years; peak mode of 15

7 **a** positively skewed

b 3

c 2.9 accidents (slightly less than mode)

Exercise 4-05

1 **a** IQR = 1.5

b **i** –0.25 **ii** 5.75

c Yes, since 8 is greater than 5.75 (= $Q_3 + 1.5 \times$ IQR).

2 **a** No, since outliers must be less than –1.25 or greater than 16.75.

b Yes, 9 is an outlier since it is less than 11 (= $Q_1 - 1.5 \times$ IQR).

c No, since outliers must be less than –11 and greater than 81.

d 9 is the outlier since it is greater than 7.5.

3 **a** \$531.43 **b** \$520

c IQR = 610 – 420 = 190

d \$900, $Q_3 + 1.5 \times$ IQR = 610 + 1.5 × 190 = \$895, so \$900 is an outlier.

e Mean and median would be lower (\$470, \$485).

f \$584.57, \$572, Yes

4 **a** 20 **b** 8 since 8 > 5 + 1.5 × 1.5

c **i** 4.35 **ii** 4.16

d The mean increases.

5 D

6 **a** Wombats: $\bar{x}$ = 16.8 points; median = 18 points

Possums: $\bar{x}$ = 16 points; median = 16 points

Koalas: $\bar{x}$ = 17.2 points; median = 14 points

b Possums, as mean and median are the same.

c $\bar{x}$ = 18.8 points; median = 18 points

d Still the Possums with the Wombats a close second.

7 **a** Pam 3 copiers; Percy 3 copiers

b They are equally good salespeople.

c Pam 5.5 copiers; Percy 17 copiers

d Pam 7.2 copiers; Percy 16.6 copiers

e Median as Pam has an outlier of 25.

f Percy as his median sales are much higher than Pam's.

8 **a** 2.92, 3, 3

b 9 since 9 > 4 + 1.5 × 3 = 8.5 ($Q_3 + 1.5 \times$ IQR)

c 2.36, 3, 3

d Increases the mean, no effect on the mode, median

9 **a** mean = \$36 570; median = \$34 200; mode = \$46 500

b mode **c** median, since it is not affected by the outlier of \$73 800

Exercises 4-06

1 **a** A: 90, 58.5, 56.5 B: 91, 18, 53

b Test B as it has a smaller interquartile range

2 **a**

Netball		Hockey
8	5	2
2	6	4 4
9 7 7 5 2 2	7	3 4 5 9
2 0	8	1 1 2 6
	9	2

b Netball: negative skew, cluster in 70s; Hockey: negative skew, cluster in 70s and 80s.

c 73.4, 75.25 **d** 76, 77

e Median, since data is skewed.

3 A

4 B

5 **a** **i** 12 cm **ii** 26 cm

b Estimates: range = 34 cm, IQR = 17.5 cm; Actual: range = 11 cm, IQR = 3.5 cm

c Definitely agree; the boxplot for estimates has a much greater spread as well as a much lower median, even though some people did overestimate.

6 **a** Boys: 11 30 35.5 45 46

Girls: 11 18 23 27 49

b Spread of scores, median, low, Q_1, Q_2, Q_3, Upper

7 **a** **i** Darwin **ii** Brisbane

iii Darwin **iv** Canberra

b Sydney (lower median price)

8 **a** 5

b Those with the lowest pulse did not exercise hard enough and their pulse stayed the same; or someone who had a higher pulse due to rushing to class could have reduced their pulse rate by not working hard in the class.

c Two people exercised very hard and got their pulse rates up very high.

d 22 beats/min **e** 6 beats/min

9 **a** 5 number summaries are:

Before: 9 15 21 35 45

6 weeks later: 6 14.5 18.5 26 32

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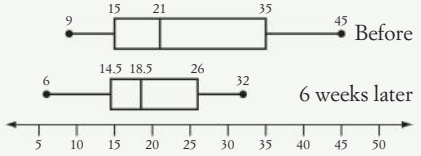


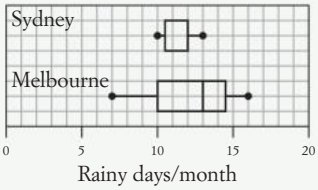
b 20, 11.5

c Yes. Interquartile range and median decreased.

10 a

Sydney		Melbourne
	0	7 8 9
3 2 2 2 2 2 2 2 1 0 0 0	1	1 2 2 4 4 4 5 5 6

b



c Stem-and-leaf plot: Sydney values cluster around 12 days.

Box plots: Sydney has a much smaller spread than Melbourne; median and upper quartile for Sydney are equal.

d Disagree with 'more'; the median number of rainy days per month is only 1 day greater for Melbourne (13) than for Sydney (12).

Exercise 4-07

1 a 4 days **b** 9 days at Perisher; April

c 23 days **d** April

e Not necessarily; although Perisher has the greater number of clear days, more information (such as snow conditions, temperature) is needed before deciding this.

2 a 1400 **b** 1800

c output **d** 1200, 2000

e Output generally less than intake; peak in output at 2000.

3 a i 45 **ii** 75 **iii** 40

b April or May **c** Too hot for boots.

d Peak in May (buying shoes for winter).

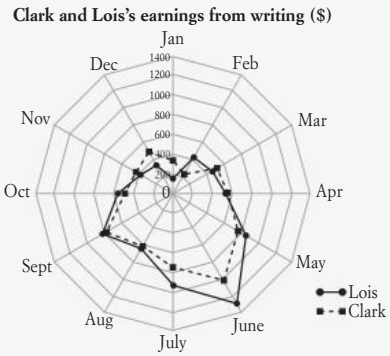
4 a about 90 mm

b about 120 mm **c** about 350 mm

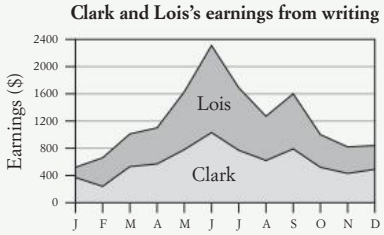
d Southeastern region; The widest 'band'.

e Southwestern region.

5 a



b



c Trends in earnings over the 12-month period.

d Highest earnings in June.

6 a estimated (or projected)

b i 30% **ii** 55%

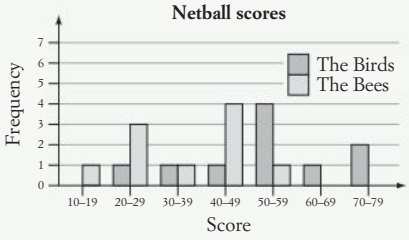
c 25% **d** 2021

e Age 60+ will increase from 2001 to 2041 and percentage of 0–14-year-olds will keep getting smaller as fewer babies are expected to be born.

Exercise 4-08

1 B

2 a



b Birds: negatively skewed data; peak 50–59.

Bees: two humped distribution; two peaks 20–29 and 40–49

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c Birds: $\bar{x} = 52.6$; $\sigma_n \approx 14.3$

Bees: $\bar{x} = 36.7$; $\sigma_n \approx 12.4$

The Birds have a higher mean and a slightly higher standard deviation.

The Bees have a smaller mean but a smaller standard deviation so are less spread out.

Neither team stands out as being more consistent than the other, but due to the lower standard deviation the Bees are a little more consistent.

3 **a** 52% **b** 15–24

c The number of persons belonging to a workers' union increases with age but drops off at 65+ as most people have retired from work by then.

d Yes, higher frequency in each age group

4 **a** A: 64.5, 18 B: 59, 13

b A is more spread out; A and B are roughly symmetrical.

c **i** A (median larger)

ii A (larger range and interquartile range)

d Group B was better

5 **a** **i** 7000 **ii** 49 000 **b** 60–64

c covers more years 65 → 100+

d roughly the same shape, positively skewed

e there are a few less males than females

6 **a** Men **b** Interquartile range

c Men; have the higher range

d Men not skewed, women positively skewed.

e Mean (men) = 175.0 cm; Mean (women) = 164.0 cm

f No real difference between mean and median.

7 **a** 15 **b** November

c Dec, Jan, Feb, Mar

d 6 **e** Hobart **f** Darwin

8 **a** 50 000 **b** about 150 000 **c** 140 000

d 92

e More than doubled over 30 years.

f More females older than 80 years.

Exercise 4-09

1 C **2** A

3 **a** 89 **b** $\frac{24}{97}$

c 70% **d** Country 65% to City 75%

4 **a** 10 140 **b** 14 179

c **i** 45% **ii** 53% **d** $\frac{17}{25}$

5 **a** 690 644 **b** 40%

c 59.75% **d** increased

e only increased by one third

6 **a** 5670 **b** 3650 **c** $\frac{155}{214}$

d 40.5% **e** 40–69 to keep the flag

7 **a** 4502 **b** 8634

c **i** 52.5% **ii** 32.3%

d About one-third of all people surveyed smoked; approx. equal number of males and females smoke.

Exercise 4-10

1 **a**

	A	B	C	D	E
1					
2					
3	Sum of Heart Rate	Column Labels			
4	Row Labels	Fast	Moderate	Slow	Grand Total
5	Short	928	702	283	1913
6	Tall	578	793	546	1917
7	Grand Total	1506	1495	829	3830
8					

b **i** $\bar{x} = 117$, $\sigma_n = 14.224$

ii $\bar{x} = 144.5$, $\sigma_n = 4.272$

c Teacher to check.

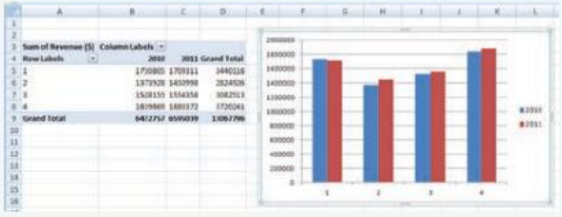
2 **a**

	A	B	C	D
1				
2				
3	Sum of Revenue ($)	Column Labels		
4	Row Labels	2010	2011	Grand Total
5	1	1730805	1709311	3440116
6	2	1373928	1450998	2824926
7	3	1528155	1554358	3082513
8	4	1839869	1880372	3720241
9	Grand Total	6472757	6595039	13067796
10				

b **i** $6 472 757 **ii** 6 595 039

c 4th quarter

d



Apart from the first quarter, there has been a slight increase in revenue from 2010 to 2011.

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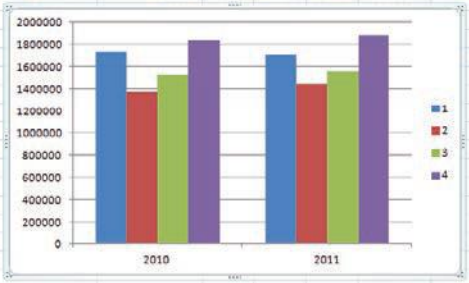
e

Sum of Revenue ($)	Column Labels				
Row Labels	1	2	3	4	Grand Total
2010	1730805	1373928	1528155	1839869	6472757
2011	1709311	1450998	1554358	1880372	6595039
Grand Total	3440116	2824926	3082513	3720241	13067796



The revenue is grouped in years. There has been a slight increase in revenue from 2010 to 2011 but the distribution of revenue over the four quarters is similar from 2010 to 2011.

3 a

Sum of Mark	Column Labels				
Row Labels	English	History	Maths	Senior Science	Grand Total
Ben	255	334	313	342	1244
1	55	83	74	89	301
2	74	88	70	93	325
3	68	79	80	86	313
4	58	84	89	74	305
Gobind	273	303	323	331	1230
1	72	88	69	80	309
2	76	79	79	85	319
3	73	61	85	88	307
4	52	75	90	78	295
Parissa	323	300	314	329	1266
1	80	72	78	87	317
2	84	83	72	90	329
3	78	65	81	84	308
4	81	80	83	68	312
Roslyn	240	330	265	264	1099
1	61	84	68	58	271
2	63	80	55	68	266
3	60	87	70	72	289
4	56	79	72	66	273
Grand Total	1091	1267	1215	1266	4839

b i History **ii** Gobind

c Teacher to check

Sample HSC problem

a 18 **b** 42

c Women: negative skew; Men: positive skew

d Men, since there were only 4 out of 15 in their 50s or 60s.

Revision

1 a QC **b** QD **c** QC
d CN **e** QD **f** CN

2 a 20 **b** 2.65 **c** 1
d 1.49

3 a 32.3 **b** 12

c

Stem	Leaf
1	6 7 9
2	2
3	6 8 8
4	1 2
5	4

d 37 **e** 22

4 a i 14, 12
ii symmetrical, cluster at 12–14

b i 8, 7
ii negatively skewed

c i 3, 7
ii positively skewed, cluster at 2 to 4

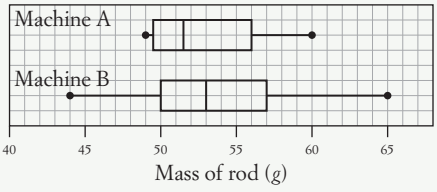
5 a 8.8 **b** 8
c mean and median are higher
d median

6 D

7 a A: 49 B: 50

b A: 51.5, 6.5 B: 53, 7 **c** A

d



e Disagree; machine B data is clustered in the 50s but has a bigger spread, thus it is not as consistent as machine A.

8 a C **b** A **c** B **d** D

9 a 120 **b** 20 in January and February
c February **d** high to low to high
e March, low rainfall and not too hot.

10 a Springfield has a lower rainfall except in July. Capital City has the most rain in November.
b 10 mm **c** November
d 65 mm
e Capital City as it has the largest area.

11 a Areas in which people worked in 2006.
b Categorical since this data is not numerical.
c State/Territory government
d Commonwealth government
e Strength: easy to see differences; weakness: exact numbers cannot be determined from graph.

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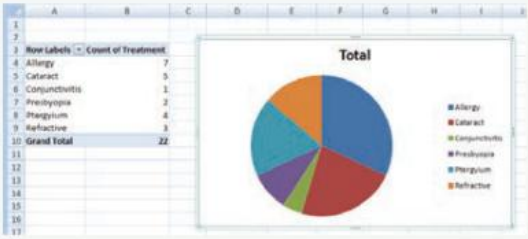
12 **a** 1040

b **i** 33.17% **ii** 52.94%

c $\frac{219}{490}$

13 **a, b**



c Teacher to check.

Chapter 5

SkillCheck

1 **a** $\frac{12}{37}$ **b** $\frac{35}{37}$ **c** $\frac{12}{35}$

2 19° **3** 37°9′

4 **a** 7.90 **b** 10.61 **c** 108.06

d 11.79 **e** −0.43 **f** 13.85

5 $k = 4.60$ cm

6 **a** $x = 132$ (∠'s at a point)

b $y = 63$ (∠ sum Δ)

c $z = 132$ (supplementary ∠'s)

7 **a** 28° **b** 47°8′

8 261.7 m

Exercise 5-01

1 **a** 8.23 m **b** 25.74 cm **c** 3.35 m

d 92.91 mm **e** 0.74 m **f** 21.36 cm

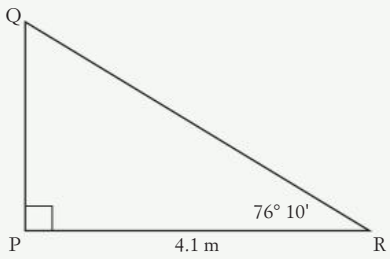
g 6.53 mm **h** 2.05 m **i** 49.20 m

2 **a** 36° **b** 42° **c** 66°

d 56° **e** 35° **f** 8°

3 **a** 74°53′ **b** 32°22′ **c** 48°1′

4



5 D **6** D

7 **a** 5.5 m **b** 1.8 m

8 78°11′ **9** 31.5 m

10 21.7 km **11** 88.9 m

12 B **13** 223 m **14** 2.3 m

15 208 m

16 $\tan 63° = \frac{VT}{145}$, $VT = 145 \tan 63°$; $\tan 49° = \frac{WV}{145}$,

$VT = 145 \tan 49°$

$d = VT - VW = 145 \tan 63° - 145 \tan 49°$

$d = 145(\tan 63° - \tan 49°)$

17 26.78 m **18** 4.9°

Exercise 5-02

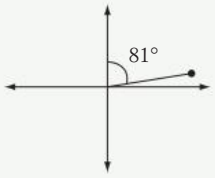
1 **a** NW **b** SW **c** SE

d NE **e** SW **f** SE

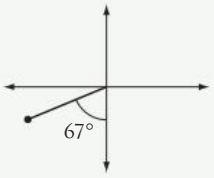
2 **a** 237° **b** 059° **c** 294°

d 130° **e** 196°46′ **f** 317°45′

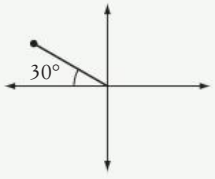
3 **a**



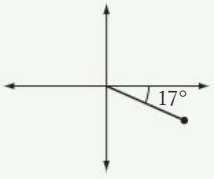
b



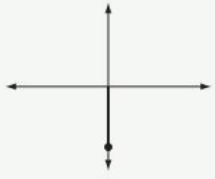
c



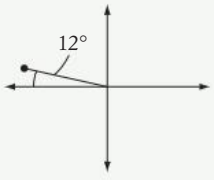
d



e



f



4 **a** 315° **b** 225° **c** 045°

5 **a** 097° **b** 072° **c** 143°

6 C

7 **a** 2.9 km **b** 6.6 km **c** 024°

8 A

9 **a** 14.5 km **b** 18 min

10 **a** 2122 km **b** 330°

11 186°

12 117° **13** 024° **14** 281°

Exercise 5-03

1 **a** positive **b** positive **c** negative

d negative **e** positive **f** positive

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2 **a** −0.8290 **b** −1.7321 **c** 0.2807
d −0.0875 **e** −0.2653 **f** 0.8973
g −0.6421 **h** 0.9839 **i** −0.7364

3 **a** acute **b** obtuse **c** both
d acute **e** obtuse

4 **a** 61°7′ **b** 28°4′ **c** 63°49′
d 73°0′ **e** 75°4′ **f** 41°49′
g 38°48′ **h** 72°33′ **i** 8°8′

5 **a** 105° **b** 151° **c** 128°
d 127° **e** 166° **f** 105°
g 110° **h** 114° **i** 153°
j 125° **k** 120° **l** 135°

6 **a** 19.06 **b** 61.84 **c** 196.07
d 0.27 **e** 7.27 **f** 4.04
g −1.38 **h** 0.45 **i** 20.91
j 0.64

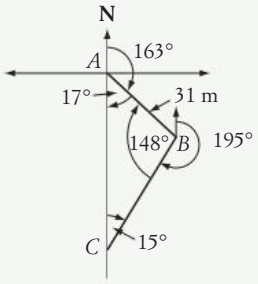
Exercise 5-04

1 **a** 11.39 m **b** 7.47 cm **c** 4.07 m
d 9.58 cm **e** 18.69 m **f** 78.46 mm

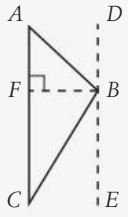
2 **a** 16.0 km **b** 3.4 km

3 13.9 cm **4** 101 m

5 **a**



b



$\angle A = 180° - 163° = 17°$ (angles on a straight line)

$\therefore \angle ABF = 180° - 90° - 17°$
$= 73°$

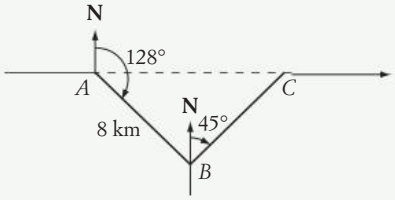
$\angle FBC = 270° - 195° = 75°$

$\therefore \angle ABC = 73° + 75°$
$= 148°$

c 63 nm

6 **a** 357 m **b** 383 m **c** 350 m

7 **a**

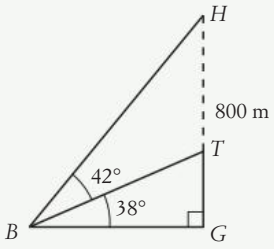


b $\angle A = 38°$, $\angle B = 97°$, $\angle C = 45°$ **c** 11.2 km

8 A

9 7.2 km

10 **a**



b $\angle BTH = 38° + 90°$
$= 128°$ (exterior angle of ΔTGB)
or $\angle BTG = 180° - 90° - 38°$
$= 52°$ (angle sum of ΔTGB)
$\angle BTH = 180° - 52°$
$= 128°$ (angles on a straight line)

c 942 m **d** 208 m **e** 128 m

Exercise 5-05

1 **a** 41°50′ **b** 9°28′ **c** 77°34′
d 28°43′ **e** 32°5′ **f** 64°21′
g 35°6′ **h** 63°29′ **i** 20°57′

2 **a** 120° **b** 111° **c** 131°

3 **a** 47°22′ **b** 132°38′

4 A **5** 64° **6** 239°

7 Teacher to check. **8** 125°

9 C **10** 43°

Exercise 5-06

1 **a** 5.57 m **b** 6.02 cm **c** 7.59 m
d 8.65 m **e** 5.17 m **f** 27.21 mm

2 0.6 m

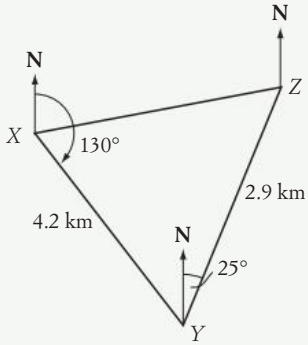
3 22.4 km

4 **a** 360 − 230 = 130° **b** 59 km

5 9300 m

6 C

7 **a**



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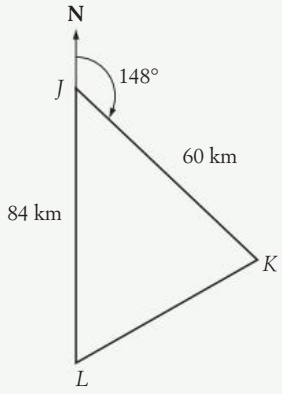
b $180° - 130° = 50°$
(co-interior angles between parallel lines)
$50° + 25° = 75°$

c 4.4 km

8 **a** 360° **b** 72° **c** 47.02 cm

9 **a**



b 32° **c** 46 km

10 **a** 3.14 cm **b** 3.13 cm

c arc length AB by 0.01 cm

Exercise 5-07

1 **a** 26° **b** 52° **c** 123°
d 36° **e** 91° **f** 48°

2 **a** 98° **b** 41°
c isosceles **d** 41°

3 B

4 9°

5 **a** 24°45′ **b** 77°38′

6 28°4′

7 8°28′

8 29°

9 B

10 Smallest ∠ opposite smallest side (length 5) 33°33′.

Exercise 5-08

1 **a** 8.04 m² **b** 24.07 cm² **c** 19.86 m²
d 13.02 m² **e** 24.20 cm² **f** 85.88 mm²

2 27.9 cm²

3 D **4** B **5** 6 cm²

6 **a** 60° **b** 64.95 cm² **c** 5 cm

7 21.22 cm²

8 21.22 cm²

9 **a** 70°25′ or 56°52′ or 52°43′ **b** 7.2 m²

10 **a** 55.85 cm² **b** 31.51 cm² **c** 24.34 cm²

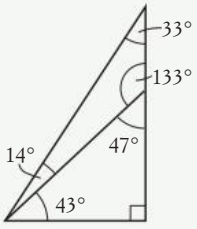
Exercise 5-09

1 **a** 7.11 m **b** 17.85 cm **c** 44.30 mm
d 11.38 cm **e** 11.71 m **f** 50.67 mm

2 **a** 40° **b** 51° **c** 92°
d 43° **e** 47° **f** 89°

3 B

4 **a**



b 180 m

5 15 m

6 153°51′

7 1420 km

8 **a** $\angle IFH + 42° = 61°$ (exterior angle of ΔFIH)
$\angle IFH = 19°$
or $\angle FHI = 180° - 61°$
$= 119°$ (angles on a straight line)
$\angle IFH = 180° - 119° - 42°$
$= 19°$ (angle sum of ΔFIH)

b $\dfrac{FH}{\sin 42°} = \dfrac{120}{\sin 19°}$

$FH = \dfrac{120 \sin 42°}{\sin 19°}$

c $\sin 61° = \dfrac{h}{FH}$

$h = FH \times \sin 61°$

$= \dfrac{120 \sin 42°}{\sin 19°} \times \sin 61°$

$h = 216$ m

9 **a** $\angle RHA = 180° - 60° = 120°$
$\angle RAH = 180° - 44° - 120° = 16°$

$\dfrac{AH}{\sin 44°} = \dfrac{85}{\sin 16°}$

$AH = \dfrac{85 \sin 44°}{\sin 16°}$

b 190 m

10 B

11 **a** 060° **b** 15 km **c** 266°

12 **a** 142° **b** 38.8 km **c** 232°

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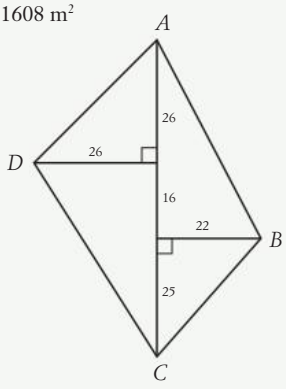
Exercise 5-10

1 9055 m²

2 **a** 1608 m²



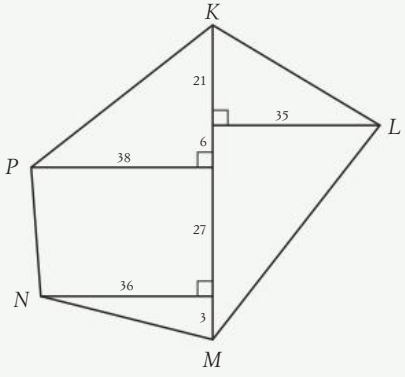
b 80.7 m

3 **a** 94° **b** 1876 m² **c** 320 m

4 **a** 147° **b** 1200 m² **c** 77°
d 74 m

5 $AC = 50$ m, $CB = 74.31$ m

6 **a**



Area = 2563.5 m²

b 27.1 m

7 2700 m²

8 **a** 121° **b** 5424 m²

9 **a** 120 m **b** 5400 m²

10 5300 m²

11 **a** Differences in the accuracy of surveying methods and measurements.

b Radial or offset survey; Simpson's rule is only an approximation.

c Simpson's rule is an approximation method.

Sample HSC problem

a 103° **b** 97 m **c** 1871 m²

Revision

1 384 m **2** A **3** 3.95 km

4 **a** acute **b** obtuse **c** both

5 **a** 151° **b** 166° **c** 136°

6 72.1 m

7 **a** 153°17′ **b** 61°36′

8 15.7 km **9** A **10** 35°32′

11 **a** 340 km **b** 14 004 km²

12 104° **13** 252 m

14 **a** 78° **b** 38 m
c 129° **d** 354 m²

Chapter 6

SkillCheck

1 **a** 1 : 25 **b** 10 : 1
c 5 : 8 **d** 1 : 7

2 **a** 2.25 **b** 4050

3 143 c/L

4 **a** 84 mm **b** 9.6 m **c** 36 L
d 2.61 kL **e** 5200 L **f** 23 m²

5 **a** 3.201 m² **b** 88.43 m²

6 3887.72 cm³

Exercise 6-01

1 **a** \$110.19 **b** \$314.46

2 B

3 **a** 21 days **b** quarterly
c water and sewerage service, \$90.10
d 18 kL
e No, in cooler months fewer showers, water garden less.
f Similar usage so can compare savings made for conditions.
g $\frac{18000}{19+30+8+31} = 204.545... = 205\,\text{L}$
h 80 c/kL
i $18 \times 0.8 = \$14.40$ **j** \$32

4 C

5 **a** \$50.76 **b** 15.4 kL **c** \$236.84

6 **a** 199.6 kL **b** 49.9 kL **c** 567 L/day
d 3.864 kL **e** 8.3%

7 C **8** \$72.49 **9** B

10 **a** Allaf: \$106.60, Johns: \$135.15, Johns pay more
b Allaf: 140 L/day Johns: 118.3 L/day Allaf's use more per person per day.

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Exercise 6-02

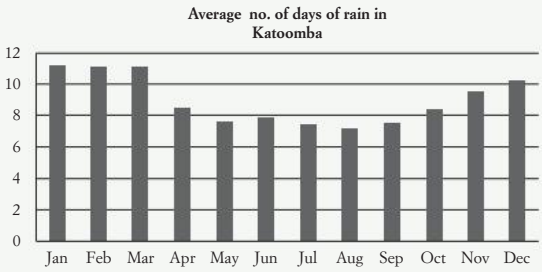
1 **a** 0 mm **b** 9.6 mm **c** 0.1 mm
d 0.89 mm **e** C

2 **a** 0, 0.2 **b** 50 **c** 4.15
d 14.83
e The mean is affected by outliers, increasing its value.

3 **a** 20.2 mm **b** 1.9 mm **c** 4 mm
d 50% of rainfall between 0 and 1.9 mm, 50% between 1.9 and 20.2 mm.

4 **a** November **b** May **c** Winter
d $\frac{1}{3}$ **e** 1

5 **a**



b August **c** Summer
d $\frac{1}{3}$ **e** 1

6 **a** 68.8 mm **b** 12 days
c 42.9% **d** 5.61 mm
e **i** 58.4 mm **ii** 51.7% **iii** 5.59 mm
iv They both contain an outlier of similar value. Mean rainfall is very similar, slightly more days without rain in 2012.

7 Teacher to check.

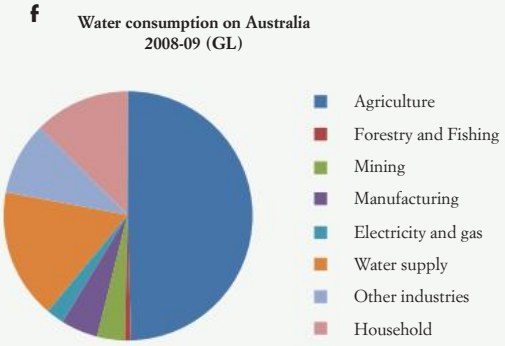
8 **a** $\frac{3}{10}$ **b** $\frac{19}{31}$ **c** $\frac{9}{100}$
d 0.066 **e** 0.624

9 **a** 112.4 mm **b** 36.96 mm **c** 25.7 mm
d 36.7 mm **e** 91.4, 101.8, 112.4 mm
f no mode
g There was a wide spread (range and standard deviation) due to the three high outliers. The mean was higher than the median due to the outliers.

Exercise 6-03

1 **a** Tasmania 906 kL **b** Victoria 63 kL
c QLD, SA, TAS, NT possibly due to higher non-household (agriculture and industrial) water usage.
d WA, TAS, NT. TAS possibly due to greater availability of water. WA and NT hotter, drier climate, uses more water.
e Teacher to check.

2 **a** 1 000 000 L **b** 1000 000 000 L (10^9 L)
c 14 100 GL **d** 49.6%
e Nearly double that of 2008–09
f



3 **a** **i** 1800 GL **ii** 3200 GL **iii** 4300 GL
b Northern Territory
c higher population, more industries using water
d 1100 GL

4 **a** WA scale is much smaller.
b WA: Household, Agriculture, Mining; NSW: Agriculture, Water Supply, Household.
c 4720 GL **d** 12%
e **i** More water used in WA than NSW (120 GL NSW, 260 GL WA).
ii Household water consumption much higher in NSW due to larger urban population.
f 17.7%

5 **a** 79.3% **b** 3.31 m
c 0.5% **d** 2.13 m
e June 2012 **f** July 2011
g Teacher to check.
h Increase in storage level, almost to capacity.
i 1 962 136 ML

6 **a** 2 000 000 ML
b July '02, January '02, July '99 or July '98
c Jan 2007, 900 000 ML
d drought **e** 2 600 000 ML
f 1 200 000 ML **g** 5 years

7 **a** QLD **b** NT
c **i** \$1.96 **ii** \$1.38 **iii** \$2.04
iv \$0.73 **v** \$0.46

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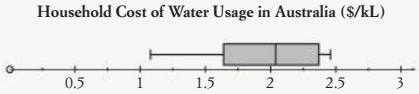
d



8 **a** dual flush toilets **b** 55%
c 2001 **d** 2007 to 2010
e 30%
f Drought conditions, education about saving water.

Exercise 6-04 (approx. answers)

1 **a** 1 : 6613 **b** 1 : 69 474
c 1 : 30 903
2 **a** B **b** 18.8 m
c C **d** 7.68 m²
3 **a** **i** 150 m **ii** 1380 m²
b **i** 1047 m
ii 67 457 m²
c **i** 2384 m **ii** 400 988 m²
4 Teacher to check.

Exercise 6-05

1 **a** 150.00 kL **b** 13.82 kL **c** 2.15 kL
2 905 L **3** B
4 **a** 7500 L **b** 13 days
5 **a** 1800 L **b** 1.6 m **c** 75.4 mm
6 **a** 24.1 m² **b** 38 560 L
7 **a** $22\frac{2}{3}$ m² **b** 56 667 L
8 7045 kL
9 Capacity of the dam ≈ 19.1 ML

Sample HSC problem

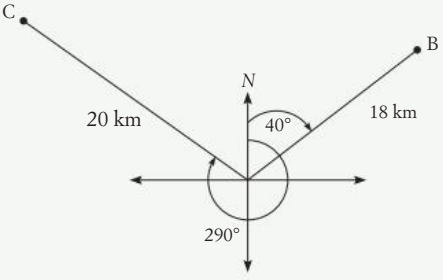
a 127.234… = 128 m³ (round up)
b 76 969 L **c** 7 days 3 hours

Revision

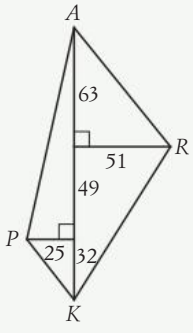
1 **a** \$45.68 **b** \$249.95
2 **a** 22.2 mm **b** 1.4 mm **c** 4.0 mm
d 12.8 and 22.2 mm
e mean affected by 2 outliers
3 **a** $\frac{24}{29} \approx 0.8128$ **b** $\frac{7}{10}$
c $\frac{15\,625}{29\,791} \approx 0.524$ **d** $\frac{25}{841} \approx 0.030$
4 **a** –3.21 m, more by 0.57 m
b 6.2% **c** 3.13 m
d **i** December **ii** September
e 59 478.1 ML **f** Teacher to check.
5 **a** 13 km **b** 7 km²
6 7200 L
7 **a** 6850 m² **b** 180 000 m³
8 $1633\frac{1}{3}$ kL

Practice paper 2

1 D **2** D **3** C
4 A **5** C
6 B **7** C **8** B
9 C **10** D
11 **a** **i** 17.9 kL **ii** \$289.41
b **i** 2 **ii** 1.4
c **i** 1108 m² **ii** 5983.2 kL
12 **a** **i** 552 **ii** $\frac{247}{552}$
iii $\frac{28}{61}$ **iv** 47.8%
b 4500 L
c **i**



ii 31.1 km
13 **a** **i**



ii 5472 m²
b 61 km
c **i** 145° **ii** 878 m² **iii** 106 m

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Chapter 7

SkillCheck

1 **a** 1 **b** 0.5

2 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{7}{10}$ **d** 0

3 $\frac{5}{7}$

4 **a** 35 **b** $\frac{4}{7}$ **c** 0.441 **d** \$2.75

5 BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

6 36

7 45 697 600

8 **a** $\frac{2}{5}$ **b** $\frac{11}{15}$

Exercise 7-01

1 **a** $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{7}{10}$ **d** $\frac{3}{10}$

e $\frac{9}{10}$ **f** 0

2 0.64% **3** $\frac{7}{32}$ **4** $\frac{6}{7}$

5 **a** 0.020 **b** 0.137 **c** 0.252 **d** 0.803

6 B

7 **a** $\frac{2}{7}$ **b** $\frac{4}{7}$ **c** $\frac{17}{21}$ **d** $\frac{5}{7}$

8 **a** 6.25% **b** 29.2% **c** 85.4% **d** 37.5%

9 $\frac{2}{11}$ **10** D **11** **a** a draw **b** 7%

12 **a** $\frac{19}{50}$ **b** $\frac{7}{10}$ **c** 1

d $\frac{3}{10}$ **e** 0 **f** $\frac{31}{50}$

13 **a** the sun rose this morning (certain event)

b a person is born on 31 February (impossible event)

14 **a** 83.75% **b** 7.5%

c 8.75% **d** 98.125%

15 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** 0 **d** $\frac{1}{4}$

16 **a** $\frac{1}{10}$

b Phone numbers from the same region begin with the same number (all Sydney phone numbers begin with 8 or 9)

17 **a** $\frac{2}{7}$ **b** $\frac{3}{7}$ **c** $\frac{3}{7}$ **d** $\frac{5}{7}$

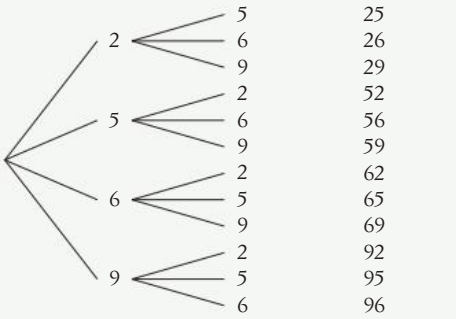
18 **a** $\frac{w}{r+w+b}$ **b** $\frac{w+b}{r+w+b}$

Exercise 7-02

1 **a** BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG

b **i** $\frac{1}{4}$ **ii** $\frac{3}{8}$

2

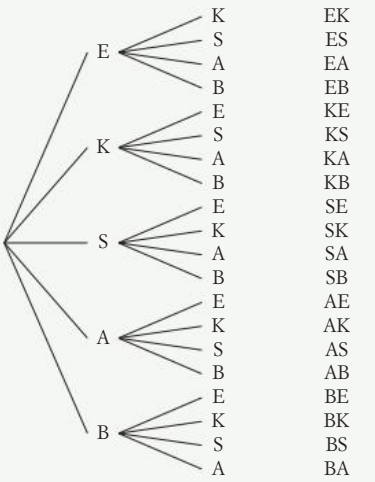


a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{5}{12}$ **d** $\frac{1}{4}$

3 **a** $\frac{2}{9}$ **b** $\frac{1}{18}$ **c** $\frac{1}{6}$

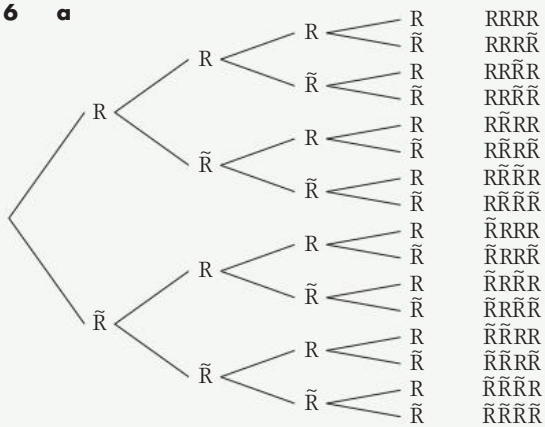
4 **a** $\frac{3}{8}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{1}{8}$

5 **a**



b **i** $\frac{1}{10}$ **ii** $\frac{1}{20}$ **iii** $\frac{1}{5}$

6 **a**



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b **i** $\frac{3}{8}$ **ii** $\frac{15}{16}$

7 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$

8 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$

9 **a** Same as the tree diagram in question **6a** except replace $\overline{R}$ with G.

b **i** $\frac{1}{16}$ **ii** $\frac{3}{8}$

iii $\frac{15}{16}$ **iv** $\frac{1}{4}$

10 **a**

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b D **c** $\frac{1}{3}$ **d** B

11 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

Exercise 7-03

1 144

2 240

3 248 832

4 **a** 15

b CL, CV, CR, DL, DV, DR, JL, JV, JR, ML, MV, MR, PL, PV, PR

5 256

6 1 000 000 **7** 7776 **8** 17 576

9 120

10 **a** 10 000 **b** 456 976

11 1024 **12** C

13 **a** 24

b RG, RD, RN, RC, RI, RA, HG, HD, HN, HC, HI, HA, JG, JD, JN, JC, JI, JA, WG, WD, WN, WC, WI, WA

14 64

15 45 697 600

16 **a** Population increased, more phones and phone numbers needed.

b 90 000 000

Exercise 7-04

1 24 **2** 5040

3 6; BJM, BMJ, JBM, JMB, MBJ, MJB

4 **a** 120 **b** 60

5 **a** 9; 11, 13, 18, 31, 33, 38, 81, 83, 88

b 6; 13, 18, 31, 38, 81, 83

6 **a** 342 **b** 6; JM, JB, MJ, MB, BJ, BM

7 C **8** 15 600 **9** 42

10 336

11 **a** 64 **b** 24

12 **a** 12 144 **b** 96 909 120

13 B

14 **a** 6; ITG, IGT, TIG, TGI, GIT, GTI

b 6; IT, IG, TI, TG, GI, GT **c** 3; I, T, G

15 **a** 24; KJGE, KJEG, KGJE, KGEJ, KEJG, KEGJ, JKGE, JKEG, JGKE, JGEK, JEKG, JEGK, GEKJ, GEJK, GKJE, GKEJ, GJKE, GJEK, EGKJ, EGJK, EKJG, EKGJ, EJKG, EJGK

b JG, JE, GJ, GE, EJ, EG

16 **a** 120 **b** 12

Exercise 7-05

1 **a** 10

b CF, CS, CV, CP, FS, FV, FP, SV, SP, VP

2 2024 **3** A **4** 231

5 5 586 853 480

6 **a** 10

b ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

7 3 838 380 **8** 70

9 4; ABC, ABD, ACD, BCD

10 3003

11 **a** 15 **b** 360

12 **a** 10 **b** 10

13 18 564 **14** B

15 **a** 120 **b** 28 **c** 3360

16 **a** 15 **b** 15

c Same answers, because the number of ways of choosing 2 children for the front is the same as the number of ways of choosing 4 children for the back.

Exercise 7-06

1 **a** ordered **b** ordered

c unordered **d** ordered

e unordered **f** unordered **g** unordered

2 **a** 120 **b** $\frac{1}{120}$ **c** 24

3 **a** 210 **b** 15 **c** $\frac{1}{14}$

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4 $\frac{1}{35}$

5 **a** 1680 **b** $\frac{1}{8}$

6 **a** 2184 **b** 60 **c** $\frac{5}{182}$

7 **a** 36 **b** 10 **c** $\frac{5}{18}$

8 **a** 24 **b** $\frac{1}{4}$

c 4; CVM, CVS, CMS, VMS

9 D

10 **a** 8 145 060 **b** 3 575 880

c Lotto, has more combinations.

11 $\frac{6}{203}$

13 C

14 **a** $\frac{1}{120}$ **b** $\frac{1}{20}$ **c** $\frac{2}{5}$

15 **a** JK, JL, JM, JN, KL, KM, KN, LM, LN, MN

b $\frac{2}{5}$ **c** $\frac{3}{10}$

Exercise 7-07

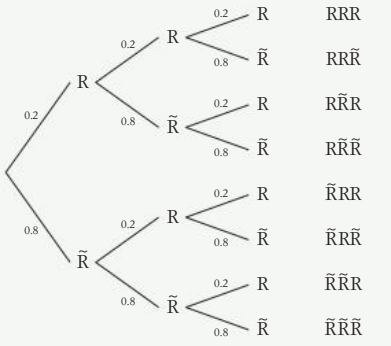
1 C

2 **a** $\frac{1}{25}$ **b** $\frac{1}{1225}$ **c** $\frac{1128}{1225}$ **d** $\frac{97}{1225}$

3 **a** $\frac{1}{45}$ **b** $\frac{16}{45}$

4 **a** $\frac{2}{7}$ **b** $\frac{1}{7}$ **c** $\frac{2}{7}$

5 **a**

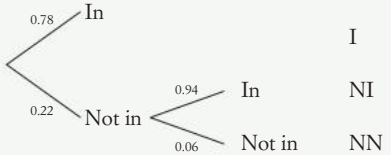


b **i** 38.4% **ii** 51.2% **iii** 48.8%

6 $\frac{125}{216}$ **7** **a** 0.250 **b** 0.441

8 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$

9 **a**



b 0.0132 **c** 0.9868

10 **a** $\frac{28}{153}$ **b** $\frac{16}{51}$ **c** $\frac{62}{153}$ **d** $\frac{104}{153}$

11 A

12 **a** 0.8% **b** 7.2% **c** 9.2%

Exercise 7-08

1 210

2 C

3 **a** 3 **b** 10 **c** 12

4 75

5 **a** \$1.71

b **i** \$400 **ii** \$342

6 \$0.75

7 B, with a financial expectation of \$290

8 10

9 A

10 **a** 0.000 55 **b** 1024

11 **a** \$22.50 **b** Loss, because he bet \$25

12 **a** $\frac{1}{8\,145\,060}$ **b** \$11 000 000

c 3 **d** \$1 000 000

13 **a** 60 **b** 140

14 **a** \$12 500 **b** \$2942.50

15 **a** 7 **b** 20 **c** 47

16 14

Sample HSC problem

a $\frac{11}{25}$ **b** \$0.984 **c** loss, \$0.016

Revision

1 **a** $\frac{1}{120000}$ **b** $\frac{1}{8}$ **c** $\frac{13}{6000}$ **d** $\frac{799}{800}$

2 **a** $\frac{1}{6}$ **b** $\frac{11}{36}$

3 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{3}{4}$

4 12

5 45 697 600 **6** 39 916 800 **7** 20

8 21 **9** 252 **10** 133 784 560

11 28

12 B

13 **a** 0.012% **b** 18% **c** 17%

14 58

15 \$0.90 **16** \$2460

Chapter 8

SkillCheck

1 **a** 0.5 kg **b** 3000 g **c** 684 000 mg

2 **a** 175.5 **b** 15

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3 $\frac{3}{5}$

4 $598.50 **5** 88.5%

7 **a** 1 : 19 **b** 1 : 20

Exercise 8-01

1 **a** strong negative **b** decreases

2 All have a linear relationship except **b**.

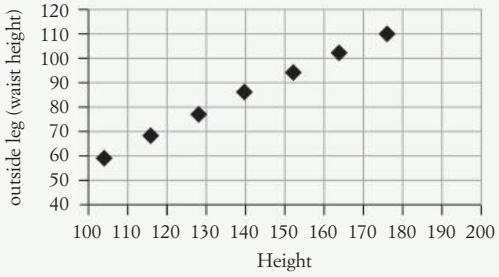
i **a** perfect positive
b no relationship
c weak negative
d weak positive
e strong positive
f perfect negative

3 **e**

4 **a** 99 cm **b** 174 cm **c** yes
d Strong positive as points are in a positive direction and close together.
e No, waist seems to be slightly more than half the height (about $\frac{3}{5}$)

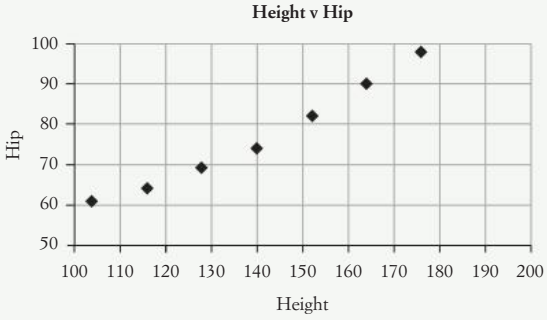
5 **a** linear **b** positive **c** strong
d Yes, each foot measure is roughly 12 times the corresponding knuckle measure.

6 **a**



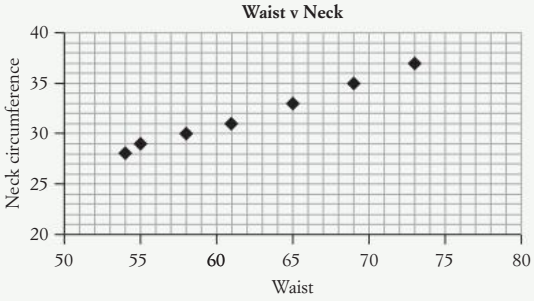
b strong or perfect positive

c



d strong positive

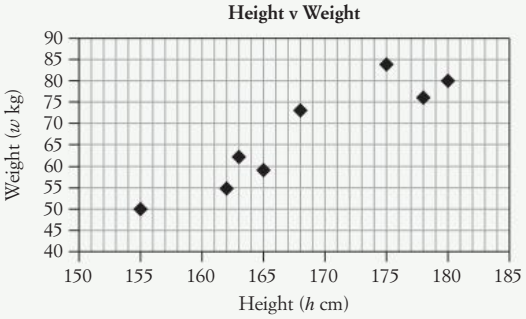
e



f strong positive

g yes

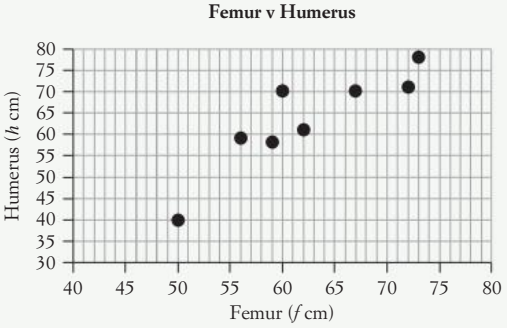
7 **a**



b strong positive

c As the height increases, the weight tends to increase.

8 **a**



b There is a strong positive relationship between the lengths of the two bones.

c Yes, by observing humans you can see that the femur and humerus are roughly the same size.

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Exercise 8-02

1 **a** Data relating to 2 variables.

b The strength of a relationship between 2 variables.

c A value between –1 and +1 describing the strength of the relationship between 2 variables.

2 C

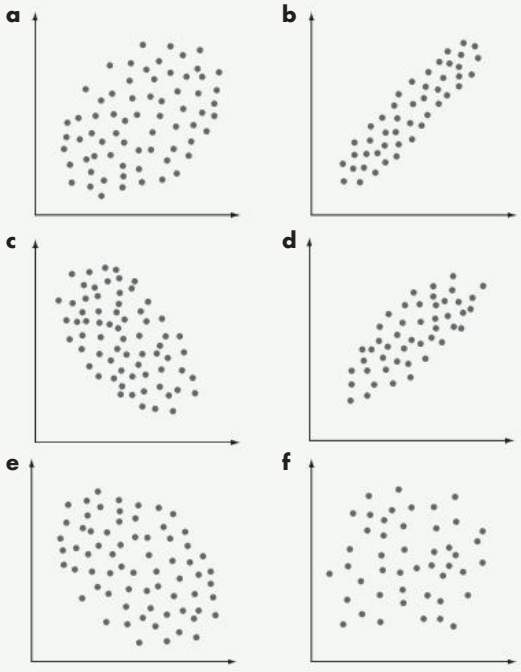
3 C

4 **a** negative **b** positive **c** negative

d positive **e** zero **f** positive

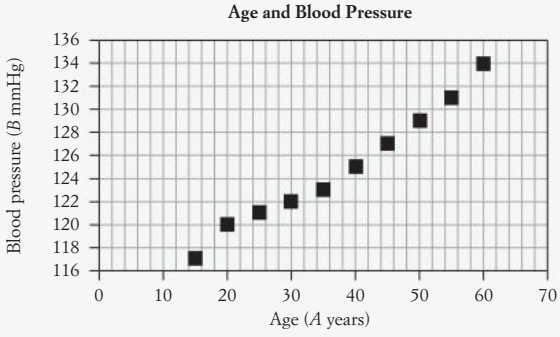
g positive **h** negative

5 **a**



6 causal: d, f, g, i, j

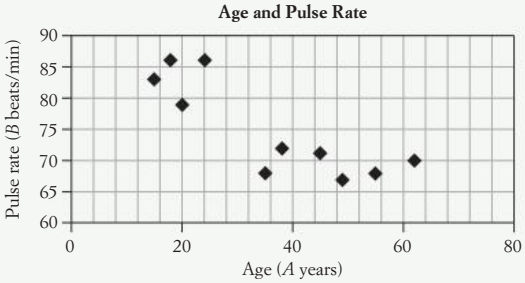
7 **a**



b strong positive

c 0.9 **d** 0.99

8 **a**



b moderate or strong negative

c $r = -0.8$ **d** $r = -0.84$

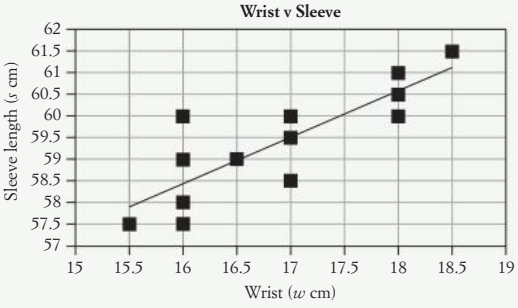
9 **a** $r = 0.997$ **b** $r = 0.925$ **c** $r = 0.894$

Exercise 8-03

1 **a** $h = b + 4$ **b** 94 cm **c** 96 cm

d No, this is outside the range of values.

2 **a, b**



c **i** 17.5 cm **ii** 58 cm

d Equation of line shown in part **b** is $s = w + 42.5$ (answers will vary)

e Using $s = w + 42.5$:

i $w = 17.5$ cm **ii** $s = 58.1$ cm

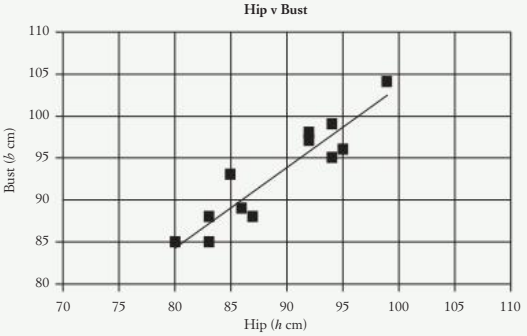
3 **a** **i** 0.81 **ii** 79.2, 9.90

iii 92.4, 8.58

b $c = 0.70\,w + 36.96$

c (72,73) and (89, 89)

4 **a, b**



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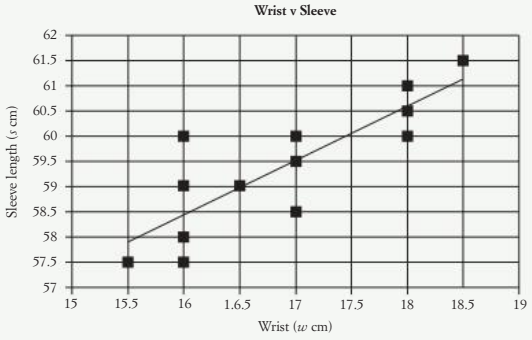
c $h = 0.9512\,b + 8.271$

d The equations are very similar: gradients are 1 and 0.95 and vertical intercepts are 4 and 8.27.

e $r = 0.9283$, strong positive correlation.

5 a, b



c $s = 1.0773\,w + 41.204$

d $r = 0.8368$, a strong positive correlation.

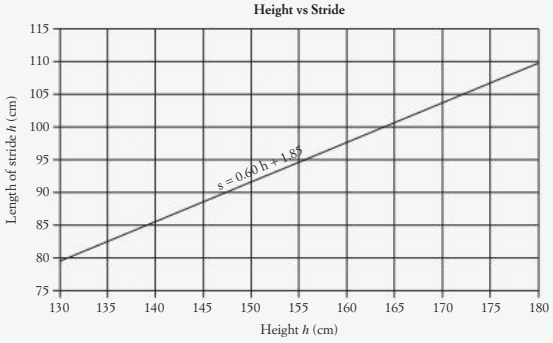
e The line of fit from question 2 had equation $s = w + 42.5$, which is close to that of the line shown here.

6 a $r = 0.942$

b **i** 162.5, 11.91 **ii** 98.75, 7.54

c $s = 0.60\,h + 1.85$

d



e, f **i** 102 cm **ii** 147 cm

Exercise 8-04

1 **a** 34.3 kg **b** 50 kg

2 A

3 2750 mL

4 **a** 1300 mL **b** 615 mL

5 $6545

6 **a** 4.8 L **b** 2000

c $33\frac{1}{3}$ drops/min

7 **a** 74.88 L **b** 12 c

8 6 km/h

9 125 506

10 **a** 130 mL **b** 81.25

c 1.35 drops/min

11 **a** 2.88 kg **b** $55

12 **a** decrease of 1%

b increase of 5.06%

c increase of 1.2%

d decrease of 6.9%

13 increase of 3.5%

14 **a** $117.30

b 2.25% **c** $86.95

Exercise 8-05

1 A

2 **a** 50 000 µg **b** 80 000 g **c** 100 000 mg

d 0.65 L **e** 0.075 g **f** 10 500 mL

3 5 mg of drug in every 1 mL of solution.

4 C

5 60 g/L

6 **a** 4 g **b** 1.25 L

7 3.5 L

8 **a** 8 mL **b** 37.5 g

9 **a** 2000 mg **b** 500 mL

10 **a** 50 L **b** 0.1 g

11 C

12 10 mL

13 **a** 2 mg/mL **b** 240 mg

14 **a** 60 g **b** 10 L **c** 300 mL/h

15 **a** 5 g **b** 6.25 mL

16 20

Exercise 8-06

1 D

2 **a** 19 mL **b** every 4 hours

c 15 doses (56 hours)

3 **a** 15 **b** every 4 hours

c phenylephrine hydrochloride 60 mg **d** 0.5 g

4 **a** 1–2 tablets **b** every 4–6 hours, as necessary

c Sunday 10 a.m.

d 1 tablet every 6–8 hours

e stop tablets and see a doctor. **f** none

5 **a** 61 mg **b** 3 hourly **c** 100 g

d 30 mg

6 **a** 144 mg **b** 129 mg

7 A

8 **a** 12.5 mg **b** 29.2 mg **c** 47.9 mg

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9 **a** 111 mg **b** 120 mg

10 25 mL

11 2.5 tablets

12 16 mL

13 **a** 200 vials **b** 150 mL

14 **a** every 8 hours, 3 times per day

b 22.4 mL

15 **a** **i** 2 **ii** 8

b nil **c** 16

d Maybe, but he should see his doctor first.

e 8

f 32 tablets = 16 g

Exercise 8-07

1 $1666.67 **2** 10 824 000

3 **a** **i** A female born in 2006 is expected to live until she is 83.4 years old.

ii A male who was 15 in 2002 is expected to live another 62.9 years, that is, until he is 77.9 years old.

iii A male who was 65 in 2006 is expected to live another 18.1 years, until he is 83.1 years old.

b No. Perhaps women look after their health better or men work and play harder.

c The 15-year-old. A female born in 2008 is expected to live for 79.2 years and a female who was 15 in 2008 is expected to live until she is 15 + 64.8 = 79.8 years. The slightly longer lifespan for the 15-year-old is because she has already survived for 15 years.

d A male who was 65 in 2006 is expected to live until he is 83.1 years and a male who was 15 in 2006 is expected to live until he is 79.2 years old. The longer life for the 65-year-old is because he has already survived for 65 years.

4 **a** 75 years

b **i** Australia **ii** Afghanistan

c Zambia

d War, lack of healthcare, lack of nutrition, lack of proper housing.

e **i** Australia, Canada, Netherlands, Sweden, UK, Spain

ii Bangladesh, Bhutan, Liberia, PNG, Rwanda, Zambia

5 **a** 79 years **b** 1 year **c** Japan

d Swaziland **e** 83 **f** 44 years

g 44.8 years

6 **a** 0.0135 **b** 0.9909

c **i** 0.45% **ii** $\frac{9}{2000}$

d 500 **e** 50–54

f More chance of dying from a fatal childhood illness, young children are more vulnerable to disease.

g Because as a person becomes older, the person's chances of dying at that age increase.

h You are certain to die at age 100 or over.

7 **a** **i** 83.5 years **ii** 79 years

b Tasmania **c** ACT about 80 years

d Yes, yes

e Northern Territory, about 6 years

f 78.8 years

g Northern Territory, high indigenous population with a lower life expectancy.

h All except Tasmania and the Northern Territory.

Exercise 8-08

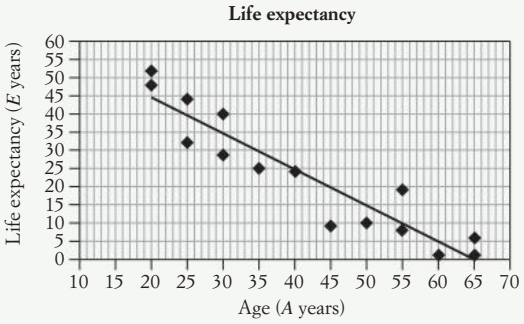
1 **a** strong positive **b** 0.98

c **ii** $y = 0.2x - 327.3$

d 0.98

e Life expectancy at birth increases by 0.2 years each year.

2 **a**



b **i** 59 years **ii** 74 years

c about 12 years

d strong negative **e** –0.94

f The equation of the line through (20, 44) and (50, 14) is $E = -A + 64$, which is very close to $E = -A + 65$

g 19 years. In data, the 45-year-old is a smoker with a life expectancy lower than that for the average 45-year-old represented by this equation.

h $E = -1.0002A + 64.541$; 64.5 years

3 **a** **i** Females have a higher life expectancy than males.

ii Over the years, the life expectancy for males has increased, with the biggest jump between 1910 and 1920.

b **i** 1910 **ii** 1982

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c Increases from 1910, flattens around 1950–60s then increases more rapidly after 1990.

e approximately $y = 0.24x - 400$

4 **b** strong negative **c** $y = -0.4x + 32$

d –0.78

e **i** 6 years **ii** 3.6 years

5 **a** non-linear

b Life expectancy varies mostly between 70 and 80 years.

c about 78

d Agree, as the points in the scatterplot show roughly the same life expectancy (around 80 years).

6 **a** **i** 7.5% **ii** about 9%

iii about 11%

b **i** 83 **ii** 79 **iii** 75

c USA, about 15.5%

d Turkey, about 6% **e** UK

f Hungary, UK and Japan spend between 7.5 and 8.5% but Japan has a higher life expectancy (83) than UK (79) or Hungary (73).

g Yes. Spending more on healthcare is related to higher life expectancy. The scatter plot shows a positive correlation.

Sample HSC problem

a **i** 79.13, 9.35 **ii** 97.53, 8.31

b $h = 0.76w + 37.39$

c **i** 90.6 cm **ii** 82.4 cm

Revision

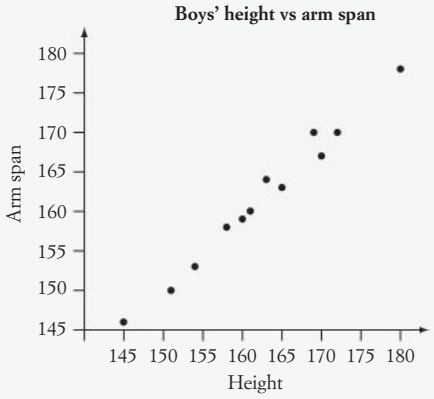
1 **a** perfect positive **b** no correlation

c weak to moderate negative

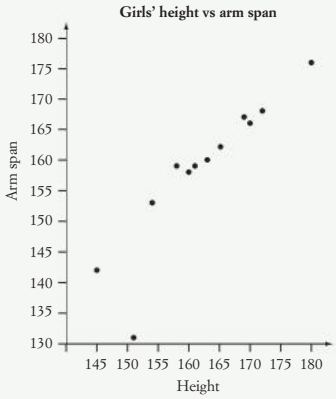
d weak to moderate positive

e strong positive **f** perfect negative

2 **a**



b

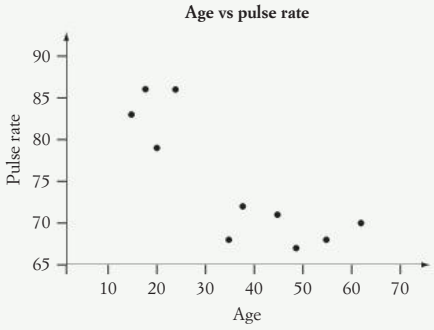


c strong positive

d weaker (correlation for boys is almost perfectly linear)

e (151, 131), a girl with a height of 151 cm and an arm span of 131 cm.

3 **a**



b strong negative **c** –0.8 **d** –0.84

4 $p = -0.39a + 89.16$

5 **a** 350 drops **b** 7 L **c** 175 drops/h

6 **a** 40 g **b** 3750 mg

7 **a** 15 mL/L **b** 3 mL **c** 4 L

8 **a** 239 mg/day **b** 250 mg/day

9 **a** 10 mL

b 60 mL

10 **a** **i** 52 **ii** 28 **iii** 84

b **i** 80 **ii** 59 **iii** 77

c 20 years

d Life expectancy has increased over the years.

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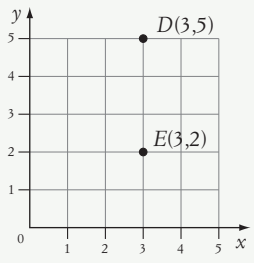
Chapter 9

SkillCheck

1 **a** $A(-3, 1)$, $B(2, 1)$ **b** 5 units **c** $C(-2, 4)$

2 **a**



b 3 units

3 13.40 cm

4 **a** 12 km/h **b** 4 h 10 min

5 **a** 2:00 a.m. **b** 6:00 a.m.
c 12:33 a.m. **d** 6:44 p.m.

6 **a** 22 h 42 min **b** 6 h 18 min

7 **a** 8 h 2 min **b** 4 h 31 min

Exercise 9-01

1 **a** D **b** G **c** A **d** C
e F **f** B **g** E **h** H

2 **a** G **b** C **c** D **d** E
e A **f** F **g** H **h** I
i B **j** J **k** K **l** L

3 **a** (0°, 75°W) **b** (45°N, 40°W)
c (0°, 110°W) **d** (30°S, 40°W)
e (45°N, 110°W) **f** (0°, 40°W)
g (30°S, 110°W) **h** (45°N, 75°W)

4 The South Pole.

5 **a** small **b** great **c** great

6 **a** 9°, north **b** 14°, north **c** 4°, south
d 2°, north **e** 67°, north **f** 1°, south

7 **a** B **b** C

8 **a** 99°, west **b** 21°, west **c** 17°, west
d 86°, east **e** 26°, east **f** 1°, east

9 (32°S, 141°E)

10 (27°S, 152°E) **11** D

12 **a** longitude **b** latitude

13 North and South Poles

Exercise 9-02

1 18 542 km

2 **a** Equator **b** 65°
c 7261 km **d** 50°W meridian
e 5362 km **f** 5362 km

3 D

4 **a** 20° **b** 2234 km

5 C

6 **a** 12 678 km **b** 4803 km

7 **a** 6367 km **b** 8824 km **c** 9271 km
d 13 516 km **e** 2457 km **f** 1676 km

8 **a** (40°S, 64°W)
b **i** 12 800 km **ii** 20 106 km

9 37°

10 10 500 km

Exercise 9-03

1 **a** 150°W **b** 4:30 p.m. **c** 8:30 p.m.

2 **a** 45° **b** 61°E **c** 8:15 p.m.

3 **a** 8:30 p.m. **b** 10:30 a.m.

4 D

5 10:02 p.m.

6 5:34 a.m.

7 **a** 45° **b** 3 h **c** 12:45 p.m.

8 **a** 65° **b** 4 h 20 min **c** 12:40 p.m.

9 **a** 8 h 40 m **b** 8:00 p.m.

10 C

11 5:20 a.m Monday.

12 **a** 90°W **b** 7:30 p.m.

13 **a** 1:30 p.m. Tuesday
b 4:00 a.m. Wednesday

14 **a** 110° **b** 122°E

15 **a** 18° **b** 28°E

16 **a** 5 h 32 min **b** 83° **c** 76°W

Exercise 9-04

1 **a** 11:30 a.m. **b** 9:00 a.m.
c 8:00 a.m. **d** 7:00 p.m.

2 **a** 7:00 a.m. **b** 11:00 a.m.
c 4:30 p.m. **d** 9:00 p.m.

3 **a** 4 h **b** 1 h
c 8 h **d** 8 h

4 **a** 4:30 a.m. **b** 7:30 p.m. **c** 11:30 a.m.

5 C

6 **a** 6:00 p.m. **b** 6:00 p.m. **c** 5:30 p.m.
d 4:00 p.m. **e** 5:30 p.m. **f** 6:00 p.m.

7 **a** 10:30 a.m. **b** 10:30 a.m. **c** 9:00 a.m.
d 7:30 a.m. **e** 10:00 a.m. **f** 9:30 a.m.

8 **a** 12 midnight **b** directly opposite sides

9 2:30 p.m.

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10 A **11** 13 h

12 **a** 9:30 a.m. Tuesday

b 12:30 a.m. Tuesday

c 11:30 p.m. Monday

13 **a** 5:40 p.m. **b** 9:00 p.m.

14 1:55 p.m.

15 4:05 p.m.

16 1:00 a.m. Friday

17 **a** Tuesday **b** 1:00 p.m.

18 **a** 11 h **b** No, the next day

Sample HSC problem

a 10 388 km **b** 12 h 40 min

c 1:40 a.m. Wednesday

Revision

1 **a** A (0°, 70°W), B (0°, 40°W), C (55°S, 70°W), D (55°S, 40°W)

2 **a** 147°E **b** 54° **c** 25°

3 (52°N, 5°E)

4 6144 km

5 **a** 16°E meridian

b 6702 km

6 **a** 61°W **b** 10:40 p.m.

7 **a** 101° **b** 6 h 44 min

c 12:14 a.m. Friday

8 **a** 5:00 p.m. **b** 7:30 a.m.

9 **a** 6:00 p.m. **b** 6:30 a.m. **c** 2:30 a.m.

10 **a** Wednesday **b** 5:30 a.m. Wednesday

11 3:00 a.m. Monday

Practice paper 3

1 D **2** B **3** B **4** B

5 C **6** D **7** A **8** B

9 C **10** D

11 **a** gain $352

b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{2}$

c **i** 0.9545 **ii** 0.91

iii 15.53

12 **a** **ii** 1 day and 4 hours

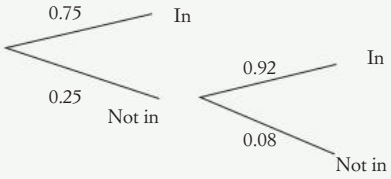
b 630 mg

c **i** Manila **ii** Monday

iii 9:00 a.m. Tuesday

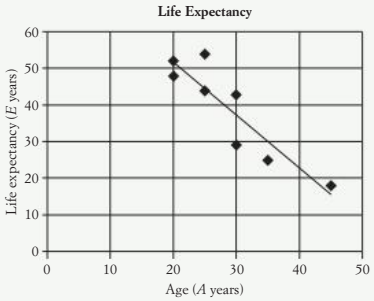
d $26^6 = 308\,915\,776$

13 **a** **i**



ii 0.02 **iii** 0.98

b **i, ii**



iii Teacher to check, approximately 43.

iv Teacher to check, approximately $E = -1.4A + 80$.

Chapter 10

SkillCheck

1 **a** 7.5295 **b** 0.0016 **c** 6527.6373

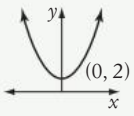
2 $k = \frac{5}{8}$ or 0.625 **3** 512

4 **a** 23 **b** 108 **c** 125

d 0 **e** –2 **f** 1.536

5

x	–2	–1	0	1	2
y	6	3	2	3	6



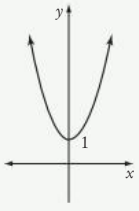
6 **a** 63.8 m **b** 14 m/s

Exercise 10-01

1 **a** C **b** A **c** F

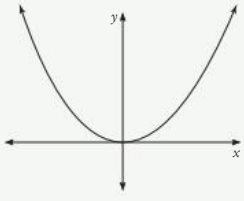
d E **e** B **f** D

2 **a**



$y = 2x^2 + 1$

b



$y = \frac{1}{2}x^2$

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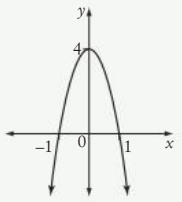
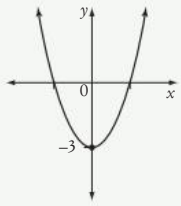
c



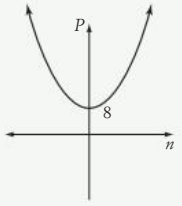
$y = -4x^2 + 4$

d



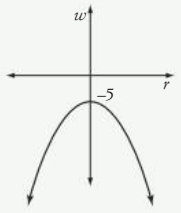
$y = x^2 - 3$

e



$P = n^2 + 8$

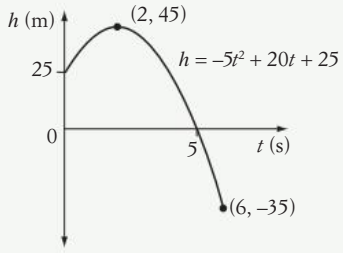
f



$w = -2r^2 - 5$

3 **a** t

b

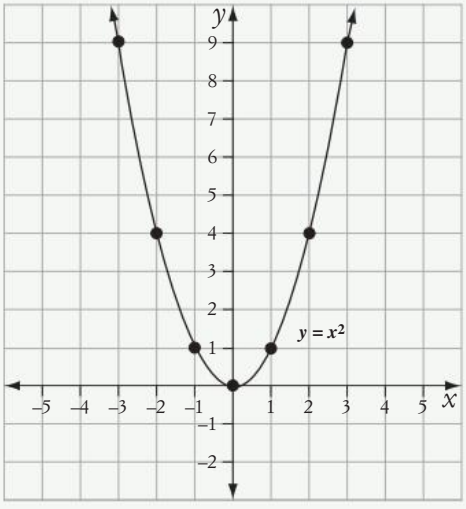


c 25, height of balcony in metres

d 45 m, 2 s **e** 5 s

4 **a** $y = x^2$

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



b **i** $y = 3x^2$

x	–3	–2	–1	0	1	2	3
y	27	12	3	0	3	12	27

ii $y = -3x^2$

x	–3	–2	–1	0	1	2	3
y	–27	–12	–3	0	–3	–12	–27

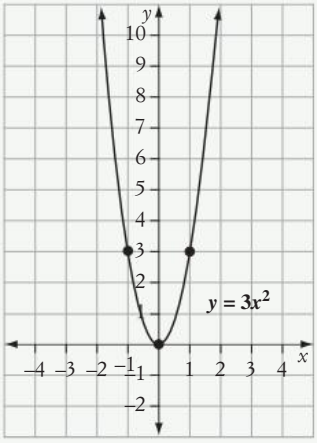
iii $y = x^2 + 3$

x	–3	–2	–1	0	1	2	3
y	12	7	4	3	4	7	12

iv $y = 3x^2 + 3$

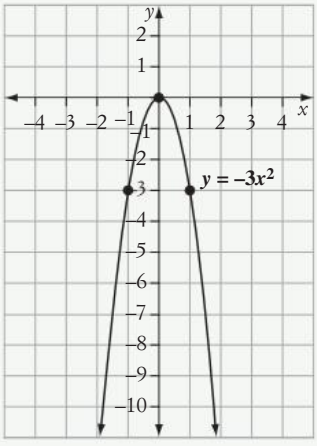
x	–3	–2	–1	0	1	2	3
y	30	15	6	3	6	15	30

c **i** $y = 3x^2$



$y = 3x^2$ is narrower and steeper than $y = x^2$

ii $y = -3x^2$



$y = -3x^2$ is narrower and steeper than $y = x^2$ and upside down.

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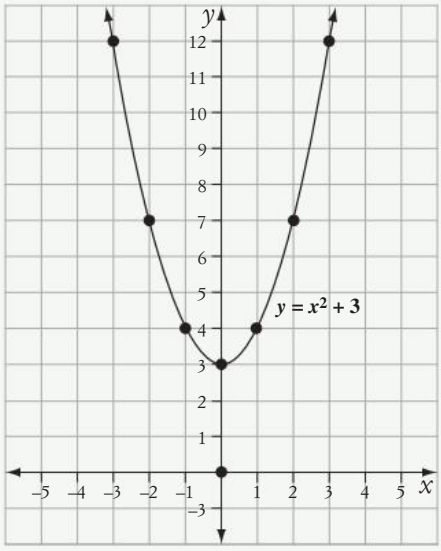
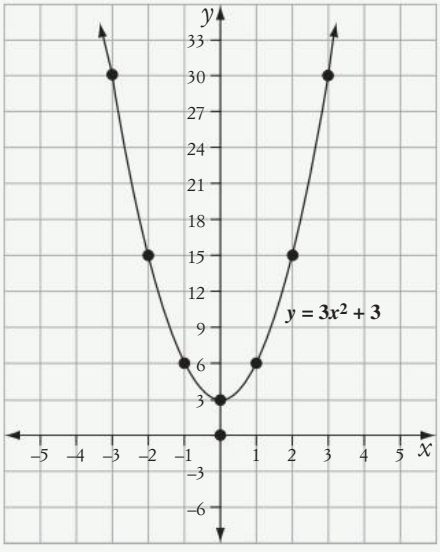
iii $y = x^2 + 3$



$y = x^2 + 3$ is the same shape as $y = x^2$ but is moved up 3 units, vertex is (0, 3).

iv $y = 3x^2 + 3$

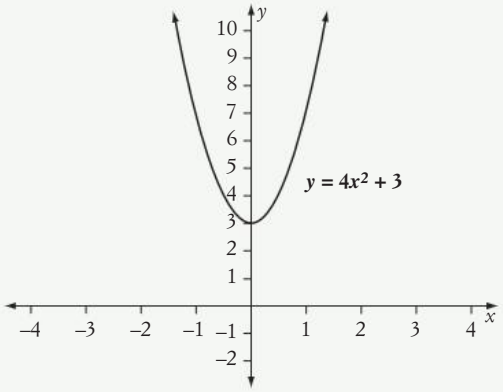


$y = 3x^2 + 3$ is narrower and steeper than $y = x^2$ and moved up 3 units, vertex is (0, 3).

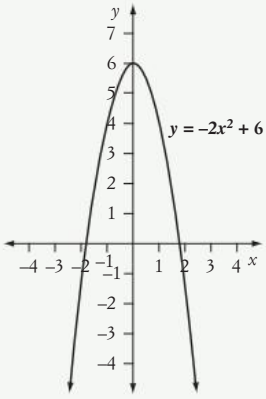
Exercise 10-02

1 B

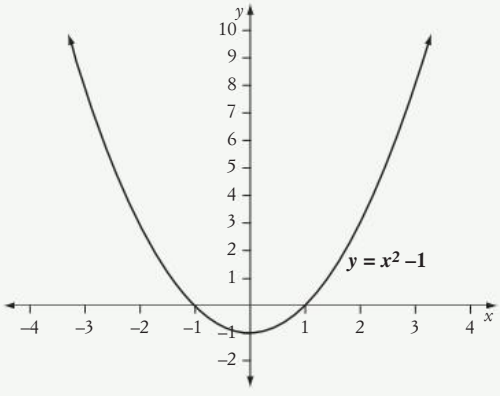
2 **i** $y = 4x^2 + 3$ minimum value of 3



ii $y = -2x^2 + 6$ maximum value of 6



iii $y = x^2 - 1$ minimum value of -1



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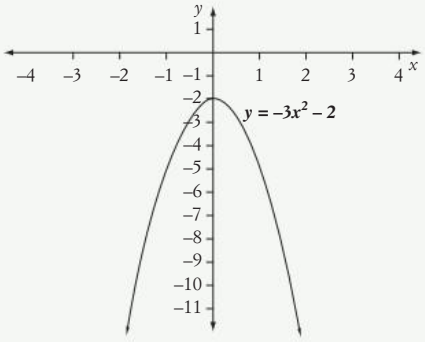
iv $y = -3x^2 - 2$ maximum value of -2



3 **a** 28 m

b Going up at 1.8 s, then coming down at 3 s.

c 2.4 s (halfway between 1.8s and 3s)

d 28.8 m

e h is negative and height cannot be negative.

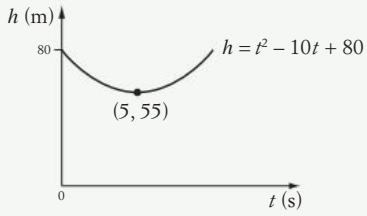
4 **a** Perimeter $= x + y + y = 18$ so $x + 2y = 18$

b Since $2y = 18 - x$, $y = 9 - \frac{1}{2}x$

c $A = xy = x(9 - \frac{1}{2}x) = -\frac{1}{2}x^2 + 9x$

d 40.5 m², 9 m by 4.5 m

5 **a**



b (5, 55)

c 55 m, the lowest height of the plane during the dive.

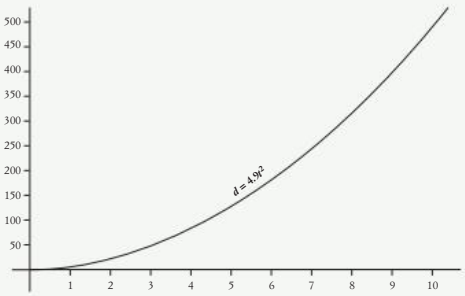
d at 5 s **e** 80 m

6 **a** t

b

t	0	1	2	3	4	5	6	7	8	9	10
d	0	4.9	19.6	44.1	78.4	122.5	176.4	240.1	313.6	396.9	490

c

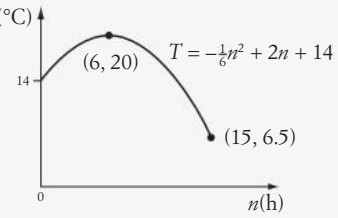


d **i** 44.1 m **ii** 4410 m

e The distance, d metres, cannot keep increasing infinitely. The piece would have to hit the ground eventually. A distance of 4410 m or 4.41 km is unrealistic for 30 seconds.

f **i** 1.4 s **ii** 3.2 s

7 **a**



b **i** 14°C **ii** $19\frac{1}{3}$°C

c 20°C at 12 noon.

d 9:30 a.m. and 2:30 p.m.; temperature rising at 9:30 a.m., temperature falling at 2:30 p.m.

e 5 hours

f Temperature will not continue to fall indefinitely after 13 hours but will eventually rise (and fall) again.

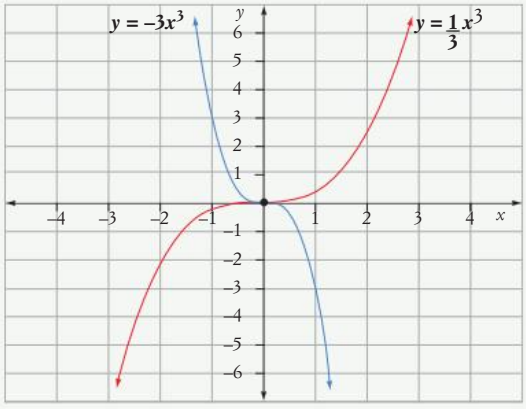
Exercise 10-03

1 A **2** C **3** D

4 **a** $V = \frac{4}{3}\pi r^3$ **b** yes, $\frac{4}{3} \times \pi \approx 4.19$

5 **a** D **b** F **c** B
d E **e** A **f** C

6



7 **a**

s cm	5	10	30	50	60	80
M g	31.25	250	6750	31 250	54 000	128 000

b C **c** 93 312 g **d** 72 cm

e 250 kg **f** No, too heavy

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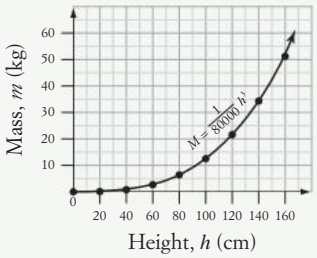
8 **a**

h cm	0	20	40	60	80	100	120	140	160
M kg	0	0.1	0.8	2.7	6.4	12.5	21.6	34.3	51.2



b, c **i** 9 kg **ii** 27 kg

d 153 cm **e** 153 cm

f A child's mass is not directly related to height; tall children are not necessarily heavy; babies weigh more than what the graph shows; adults have different shapes, sizes and masses.

Exercise 10-04

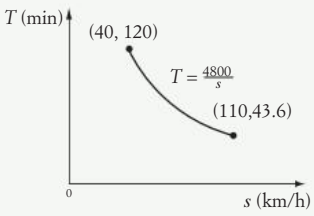
1 B

2 C

3 **a** C **b** B **c** A **d** D

4 **a** T

b



c **i** 52 min **ii** 52 min

d 69 km/h **e** 80 km

5 **a**

d	50	100	150	200	250	300	350	400
h	1120	560	373	280	224	187	160	140

b 255 cm **c** 65 cm

Exercise 10-05

1 B

2 A

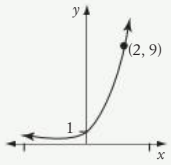
3 B

4 **a** C **b** B **c** D

d A **e** E

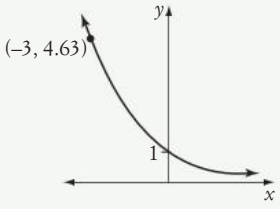
5 **a** $y = 3^x$

x	−3	−2	−1	0	1	2	3
y	0.04	0.11	0.33	1	3	9	27

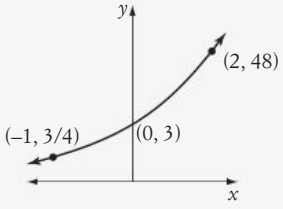


b y-intercept is 1. The asymptote is $y = 0$ (the x-axis).

6 **a** $y = (0.6)^x$ **b** (0, 1) **c** decreasing



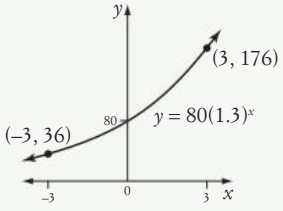
7 **a** $y = 3(4)^x$



b 3

c Because y can never equal zero, the graph does not cross the x-axis.

8 **a** $y = 80(1.3)^x$



b 80 **c** increasing

9 B

10 **a** **i** 3.2 mm **ii** 409.6 mm

b **i** 12 **ii** 14

c You cannot keep folding the sheet of paper, even after a few folds.

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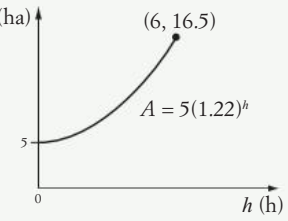
Exercise 10-06

1 D

2 **a** 5 ha **b** 10 ha **c** 7 h

d



e **i** 3.5 h **ii** 15 ha

f Assumes area burnt will continue growing quickly and never stop.

3 **a** 8000, the initial investment of $8000.

b **i** $8520 **ii** $10 960.69

iii $15 017.10

c **i** $520 **ii** $2960.69

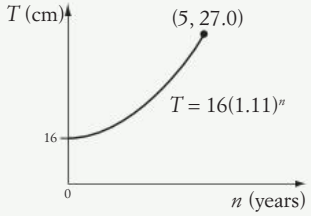
iii $7017.10

d After 11 years.

4 **a** T **b** **i** 30 cm **ii** 16 cm

c 4 years **d** $T = 16(1.11)^n$

n	0	1	2	3	4	5
T	16	17.8	19.7	21.9	24.3	27.0



5 D

6 **a** 100

b 100 bacteria multiply to 12 800.

c **i** 300 000 **ii** 1 400 000

d **i** 15 h **ii** 16 h

e Yes, approximately doubling every 70 minutes.

7 **a** **i** 2.4×10^{11} **ii** 2.1×10^{53}

b 1000 **c** After 3.9 minutes.

8 **a** yes

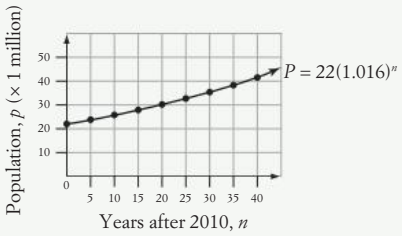
b 5, the initial number of computers affected.

c 35 523

9 **a**

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
n (years)	0	5	10	15	20	25	30	35	40
P (millions)	22	23.8	25.8	27.9	30.2	32.7	35.4	38.3	41.5

b



c **i** 32 million

ii 31.7 million

d **i** 38 years (2048)

ii 38 years (2048)

10 **a** **i** 3.08 kg **ii** 6.76 kg

iii 32.62 kg

b **i** 4.85 months

ii 11 months

Exercise 10-07

1 increases

2 B

3 **a** $k = 2$ **b** $a = 6.0764$

4 **a** C **b** A **c** B

5 **a** $D = 9.6875$ V

b 543 km **c** 20.6 L

6 **a** $B = \frac{9s^2}{640}$ **b** 69 m **c** 92 km/h

7 B

8 375 g

9 **a** 188 kPa **b** 125 m

10 **a** 490 kg **b** 1.7 cm

11 **a** $C = kS^3$ **b** 5.787×10^{-6} **c** $3.55

12 **a** 1960 m **b** 11 s

13 **a** 9.57 L/100 km

b 1082 kg

14 **a** 14

b There are longer queues for female rest rooms because females spend longer time in them and male rest rooms can accommodate more users.

15 D

Exercise 10-08

1 decreases

2 D

3 **a** $a = 60$ **b** $t = 8$

4 **a** $F = \frac{112}{L}$

b 6 beats/s

c 25 cm

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5 **a** 4.8°C **b** 253 m

6 108 km/h

7 **a** 2.5 h **b** 5

8 **a** 4 m³ **b** 22.5 kPa

9 **a** 8 min **b** 4

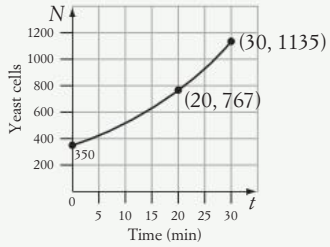
10 **a** 1.6 min **b** 20 students

11 C

12 19 091

Sample HSC problem

a



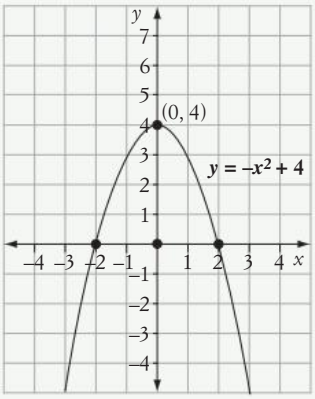
b 350, initial number of yeast cells

c 8067 **d** 27 min

Revision

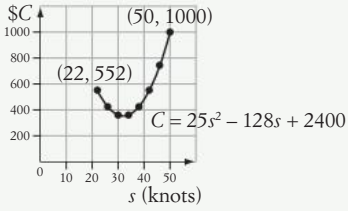
1 **a** quadratic **b** parabola

c $y = -x^2 + 4$



2 **a** 424, 360, 424, 744, 1000

b

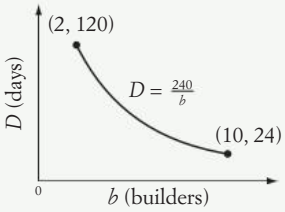


c 24 knots, 40 knots

d $352, 32 knots

3 **a** 1044.835 cm³ **b** 6.5 cm

4 **a**



b 27 days **c** 48

d Assumes the greater the number of builders, the fewer number of days. Eventually, too many builders will get in each others' way and slow down the completion time.

5 **a** 57 030 **b** 40 000 **c** 23 years

6 A

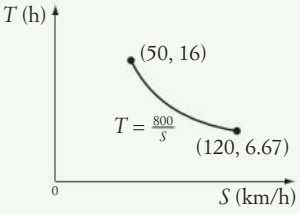
7 **a** D **b** B **c** C

d A **e** F

8 60 kg

9 **a** $15.30 **b** 54 cm

10 **a**



b hyperbolic **c** 12.5 h **d** 67 km/h

Chapter 11

SkillCheck

1 **a** 12 562 **b** 80 **c** 18

2 14 250 **3** 1920 **4** 1 day

5 **a** 13.5 **b** 97.35 **c** 81.5

6 **a** 1.3 **b** 110.4

7 **a** 77 **b** 136.4 **c** 51.2

8 15

Exercise 11-01

1 B

2 The population is too big or the cost may be too high.

3 Sample that is not randomly selected.

4 **a** systematic

b, c simple or stratified

5 No, because every sample of 400 does not have the same chance of being selected as 100 have to each come from Holden, Ford, Hyundai and Toyota.

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9 **a** 10

b LZ, LS, LT, LN, ZS, ZT, ZN, ST, SN, TN

10 B

11 **a** **i** 5 **ii** 10 **iii** 10 **iv** 5 **v** 1

b 31

c CNSW, CNSB, CNWB, CSWB, NSWB

12 7

13 **a** **i** 43.3% **ii** 56.7% **iii** 36.7% **iv** 16.7% **v** 20% **vi** 13.3%

Exercise 11 -02

1 **a** $\mu = 4.6$

b $n = 1$: 1 3 5 6 8

$n = 2$: 1, 3 1, 5 1, 6 1, 8 3, 5 3, 6 3, 8 5, 6 5, 8 6, 8

$n = 3$: 1, 3, 5 1, 3, 6 1, 3, 8 1, 5, 6 1, 5, 8 1, 6, 8 3, 5, 6 3, 5, 8 3, 6, 8 5, 6, 8

$n = 4$: 1, 3, 5, 6 1, 3, 5, 8 1, 3, 6, 8 1, 5, 6, 8 3, 5, 6, 8

$n = 5$: 1, 3, 5, 6, 8

c $n = 1$: $\bar{x}$'s are 1 3 5 6 8

$n = 2$: $\bar{x}$'s are 2 3 3.5 4.5 4 4.5 5.5 5.5 6.5 7

$n = 3$: $\bar{x}$'s are 3 $3\frac{1}{3}$ 4 4 $4\frac{2}{3}$ 5 $4\frac{2}{3}$ $5\frac{1}{3}$ $5\frac{2}{3}$ $6\frac{1}{3}$

$n = 4$: $\bar{x}$'s are 3.75 4.25 4.5 5 5.5

$n = 5$: $\bar{x}$'s are 4.6

d $\mu_{\bar{x}} = (1 + 3 + 5 + \ldots. + 5 + 5.5 + 4.6) \div 31 = 4.6$

e Yes

2 **a** $\mu = 205$ cm

b $n = 1$: 190 203 209 211 212

$n = 2$: 190, 203 190, 209 190, 211 190, 212 203, 209 203, 211 203, 212 209, 211 209, 212 211, 212

$n = 3$: 190, 203, 209 190, 203, 211 190, 203, 212 190, 209, 211 190, 209, 212 190, 211, 212 203, 209, 211 203, 209, 212 203, 211, 212 209, 211, 212

$n = 4$: 190, 203, 209, 211 190, 203, 209, 212 190, 209, 211, 212 190, 203, 211, 212 203, 209, 211, 212

$n = 5$: 190, 203, 209, 211, 212

c $n = 1$: $\bar{x}$'s are 190 203 209 211 212

$n = 2$: $\bar{x}$'s are 196.5 199.5 200.5 201 206 207 207.5 210 210.5 211.5

$n = 3$: $\bar{x}$'s are $200\frac{2}{3}$ $201\frac{1}{3}$ $201\frac{2}{3}$ $203\frac{1}{3}$ $203\frac{2}{3}$ $204\frac{1}{3}$ $207\frac{2}{3}$ 208 $208\frac{2}{3}$ $210\frac{2}{3}$

$n = 4$: $\bar{x}$'s are 203.25 203.5 205.5 204 208.75

$n = 5$: $\bar{x}$'s are 205

$\mu_{\bar{x}} = (190 + 203 + 209 + \ldots. + 204 + 208.75 + 205) \div 31 = 205$

3 **a** $\mu = 71\frac{1}{3}$

b **i** 3 **ii** 3 **iii** 1

c $n = 1$: 68 74 72

$n = 2$: 68, 74 68, 72 74, 72

$n = 3$: 68, 74, 72

d $n = 1$: $\bar{x}$'s are 68 74 72

$n = 2$: $\bar{x}$'s are 71 70 73

$n = 3$: $\bar{x} = 71\frac{1}{3}$

$\mu_{\bar{x}} = (68 + 74 + 72 + \ldots. + 70 + 73 + 71\frac{1}{3}) \div 7 = 71\frac{1}{3}$

4 **a** 1.84 m

b Teacher to check. Example: $n = 3$: 1.80, 1.68, 1.95

c Teacher to check. Example for $n = 4$: 1.68, 1.91, 1.66, 2.03

$n = 5$: 1.82, 1.95, 1.89, 1.95, 1.85

d Teacher to check. The means for the samples in **b**, **c** above are 1.81, 1.82, 1.89.

e Two of the above means are below the population mean and one is above.

f The mean of the three sample means is 1.84, which is equal to the population mean.

5 **a** $\mu = 0.3735$

b Teacher to check. Four possible samples of size $n = 5$ are:

0.31, 0.52, 0.34, 0.24, 0.43

0.35, 0.31, 0.32, 0.16, 0.45

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0.42, 0.31, 0.52, 0.43, 0.48

0.35, 0.31, 0.37, 0.16, 0.51

c The means for the 4 samples are: 0.368, 0.318, 0.432, 0.34

The mean of the sample means is 0.3645.

d **i** no

ii The mean of the sample means is not equal to the population mean but is very close to it.

e Teacher to check. Four possible samples of size $n = 6$ are:

0.24, 0.31, 0.45, 0.52, 0.35, 0.40

0.24, 0.51, 0.54, 0.48, 0.40, 0.42

0.35, 0.54, 0.28, 0.48, 0.34, 0.37

0.32, 0.35, 0.31, 0.34, 0.24, 0.35

The sample means are: 0.3783, 0.4317, 0.393, 0.3183

The mean of the sample means is 0.3803, which is not equal to the population mean but compares favourably to it.

Exercise 11-03

1 B **2** 343 **3** 1250

4 1129

5 100 000

6 11 400

7 D

8 18 750

9 **a**, **b** 1250

10 4000

11 103

12 **a**, **b** 29 400

13 Yes (the estimate is 431).

14 250

Exercise 11-04

1 **a** Y **b** Z

2 **a** 4.99 and 5.01

b 4.98 and 5.02 **c** 4.97 and 5.03

3 B **4** C

5 **a** 150 hours **b** 7850 and 8150 hours

c 1920

6 **a** 1395 **b** 7788

7 **a** 0.3% **b** 81.5%

8 **a** 234 and 298 days

b 16%

c Possible but unlikely as 320 is more than 3 standard deviations from mean.

9 **a** 84%

b No. Only 16% of packets are equal to or over the advertised weight.

10 **a** 97.5% **b** 150

11 **a** 16% **b** 2.5%

Exercise 11-05

1 Raw score is the actual value and z-score shows the number of standard deviations the value is above or below the mean.

2 **a** 34% **b** 68% **c** 47.5%

d 99.7% **e** 83.85% **f** 16%

3 **a** **i** 1 **ii** –3 **iii** 3.5

b 85 **c** 2.5% **d** 83.85%

4 A

5 **a** $z = -1$ means that Sam's blood pressure is one standard deviation below the mean.

b BP is 110 **c** 0.75

6 **a** **i** –0.4 **ii** 3.5

b 112 cm **c** 76 cm

d 47.5% **e** 16%

7 **a**

Mass	94	96	98	100	102	104	106
z-score	–3	–2	–1	0	1	2	3

b **i** 50% **ii** 68%

iii 2.5% **c** 62.5

8 **a** **i** 0 **ii** 2

b 750 000

c **i** 4 standard deviations above mean

ii 164

9 **a**

Weight	0	0.4	0.8	1.2	1.6	2.0	2.4
z-score	–3	–2	–1	0	1	2	3

b 81.5% **c** 120 **d** 768

10 **a** 0.16, 1.8 **b** 47

Exercise 11-06

1 C

2 **a** English –0.71; Maths 0.7; Science –0.25; Art –0.875

b Maths, Science, English, Art

3 **a**

Subject	Peta	Mel	Joe
Visual Arts	–1.125	–0.375	0.25
Legal Studies	–0.083	1.17	0.5
Maths General	0	1	–1.67

b Legal Studies

c Joe's mark in Visual Arts

d Mel in Legal Studies

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4 **a** z-scores are Pattie 0.57 and Jim 0.56. Pattie is more overweight as her z-score is higher.

b 95 kg **c** 13 kg

5 Downtown (z-score is 0.357 whereas for Uptown z-score is –0.945).

6 2-year model (z-score is –0.67 whereas 3-year model z-score is –1.25).

Sample HSC problem

a –0.6 **b** 34%

c **i** Raad's mark is 2.3 standard deviations above the mean.

ii 88

Revision

1 C

2 **a** **i** 40% **ii** 40% **iii** 20%

b – d Teacher to check.

e Rail – survey may have been taken in an area where rail services are not available.

3 **a** $n = 1$: 190 161 170 162 176

$n = 2$: 190, 161 190, 170 190, 162

190, 176 161, 170 161, 162 161, 176

170, 162 170, 176 162, 176

The means are:

190	161	170	162	176
175.5	180	176	183	165.5
161.5	168.5	166	173	169

b $\mu_{\bar{x}} = 171.8$, which is equal to the population mean, $\mu(= 171.8)$

4 333

5 **a** 3.98 and 4.02

b 3.96 and 4.04

c 3.94 and 4.06

6 D

7 **a** C **b** A **c** A, B, C

d B, D

8 **a** 2.5 standard deviations above the mean.

b 5.6 **c** –1.07 **d** 72

9 **a**

Time	2	4	6	8	10	12	14
z-score	–3	–2	–1	0	1	2	3

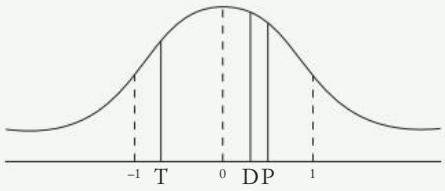
b **i** 15.85% **ii** 16%

iii 2.5% **iv** 81.5%

10 **a** Drama 0.3125, Photography 0.5, Textiles –0.8

b Photography. It has largest z-score.

c



Chapter 12

SkillCheck

1 **a** \$16.45 **b** \$14.69 **c** \$63.49

2 \$466.24

3 90 days

4 \$468.24 ÷ 96 = \$4.88

5 **a** 14.528 km **b** 645 000 000 L

6 2.415×10^8

Exercise 12-01

1 **a** $\frac{3}{50}$ **b** Standby power **c** \$68.61

d \$942.19 **e** 112°

2 **a** $\frac{1}{4}$ **b** 50% **c** 400 kWh

3 **a** 5.7×10^6 MW

b 3×10^6 kW

c 4.845×10^9 GW

4 D

5 **a** 5.6 kW **b** 27 500 MW

c 3.495 kW

6 C

7 **a** 2350 MW

b 2 350 000 kW or 2.35×10^6 kW

c 2 350 000 000 W or 2.35×10^9 W

8 **a** bimonthly **b** 14.5 kWh

c July/August – winter, more electricity used for heating.

d 813 kWh **e** 56 days

f 21 April, 72 822 kWh

g Increased, \$16.65

h Pensioners are low income earners. Life support machines use a lot of electricity but are essential.

i 24.95 cents/kWh for the first 185 kWh; 18.96 cents/kWh for the rest.

9 **a** 1420.34 kWh

b **i** \$131.81 **ii** \$425.72

c \$4.43

10 **a** 5.2 kWh/day

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b June to August – winter, when electricity is used for heating.

c System access charge. $0.0472/day for Off Peak (new rate).

d By the new rate period.

e Yes, the old rate for both applied for 56 days and the new rate also applied for both Off Peak and Domestic.

f 10.98% **g** 16.35%, 5.37%

h Domestic: decreased by 0.3kWh/day; Off Peak: increased by 0.1 kWh/day.

11 $698.71 (GST inclusive)

Exercise 12-02

1 **a** 0.8 kWh **b** 0.16 kWh

c 0.096 kWh **d** 0.4 kWh

2 C **3** 3.9 kWh

4 **a** 0.12 kWh

b 1.50 kWh

5 **a** Refrigerator, clothes dryer, dishwasher.

b 33%

c Clothes dryers use a lot of power.

d Increasing electricity prices

6 **a** 39 kWh **b** $10.47

7 $318.17 **8** $4.86

9 **a** $259.65 **b** $79.43 **c** $83.79

10 **a** $481.91 **b** $96.38 **c** $77.11

d $321.27

11

	LED	CFL	Halogen	Incandescent
Cost of lamp	$28	$8	$3.50	$2.50
Lamp wattage	12 W	15 W	50 W	75 W
Average life	40 000 h	10 000 h	4 000 h	1 000 h
Lamp cost over 15 years	$28	$32	$31.50	$82.50
Electricity cost over 15 years	$110.38	$137.97	$459.90	$689.85
Total cost	$138.38	$169.97	$491.40	$772.35

a $321.43 **b** $602.38 **c** $31.59

12 $207.85

13 $84.10

14 **a** $236.52 **b** $141.91, $94.61

15 $39, $21, $3, $27.60, $33, $51.30, $64.80

16 **a** $1149 **b** $2541 **c** $1090.20

17 Panasonic, by $528 **18** $19.66

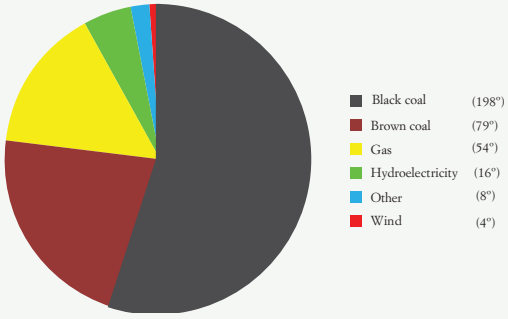
19 **a** $3.27, $4.44, $8.88, $7.01, $6.31, $9.81

b $39.72

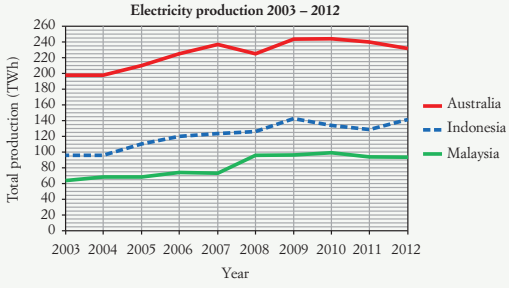
20 **a** 765.4 kWh **b** $203.98

Exercise 12-03

1 Fuels used to generate electricity in Australia in 2008–2009



2 **a**



b Australia's production is double that of Indonesia's and more than double that of Malaysia.

c Australia's population is more developed and households may have more items/appliances (pools, air-conditioners, dishwashers, clothes dryers, heaters etc) that use electricity.

d Indonesia

3 **a** **i** black coal **ii** brown coal **iii** hydro

b **i** 70–72% **ii** 70–72%

c **i** 14–15% **ii** 5% **iii** 75%

d solar

4 **a** 0.2% **b** 4.5% **c** 20.2%

5 283 333 GWh

6 **a** 4270.5 kWh **b** 47.6%

7 **a** 54.6 kWh **b** 2847 kWh

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8 **a** **i** Baulkham Hills

ii Wollongong

b 58.4%

c More energy used for heating in winter and for cooling in summer.

d More energy spent on heating, pool pumps, cooling.

9 NSW: $772.20 VIC: $601.87 QLD: $671.31

10 **a** 4 **b** 29th

c **i** 8c **ii** 4c **iii** 11c

11 **a** **i** 25 cents/kWh

ii 29 cents/kWh

b In 2011, Australian cost (just below 25c/kWh) is just above the Japanese cost (23c/kWh). Australia's cost is 5 cents dearer than the European Union, 13 cents dearer than the US and 16 cents dearer than Canada.

c Canada's cost of electricity is about 3c/kWh cheaper than the US cost.

12 **a** Heating/cooling a house is not dependent on the number of people.

b People may be sharing rooms when there are 6 or more people in a household.

Exercise 12-04

1 B, Most windows face North.

2 **a** 1 cm : 4 m or 1 : 400

b 190 m^2

c 57 m^2 **d** The family room/garage side.

3 **a** North **b** West

c No, as most windows face west.

d There is some cross ventilation from the kitchen through to the family room.

4 Roof insulation is a material (such as batts, foil-backed blankets, loose fill, etc.) placed in the ceiling to reduce temperature flow. This will reduce heat coming in during summer and reduce heat going out in winter.

5 **a** **i** 76° **ii** 24°

b Yes, as they don't allow for much summer sun to come through and do allow winter sun to come through.

6 **a** Tall trees situated south and south-west to provide shade in summer and provide wind break in cooler months.

b Don't plant tall trees on the north or north-east side of the house, since they will block the sun in winter when the sun is low.

7 **a** 14° **b** 486 mm

c Extend the eaves of the house on that side.

8 **a** 20.8 m^2 **b** 80 W **c** 3.85 W/m^2

d Yes, the lamp power density is less than 5 W/m^2.

9 **a** 9.6 W/m^2 **b** 7.2 W/m^2 **c** 4.3 W/m^2

The 5 LED lights comply.

Exercise 12-05

1 Water: native vegetation, 3A rated toilet, 3A rated shower heads, rain water tank connected to toilet and garden irrigation system.

Thermal Comfort: light covered roof, ceiling/floor/wall insulation, eaves and shading for windows, energy efficient glazing, north-facing aspect.

Energy: gas-boosted solar hot water system, 6-star reverse cycle air conditioner, compact fluorescent globes, north-facing aspect

2 **a** 30% **b** 20% **c** 40%

5 Teacher to check – these requirements may change due to changes in Government policy with regard to the targets in BASIX.

a MJ/m^2

b **i** 1.67 **ii** 3 (teacher to check)

c More direct sunlight on roof.

d Gas instantaneous or solar heating with electric or gas booster.

e CFL's or LED's

f 4 star or higher

6 **a** **i** 3-star **ii** 5-star

b 110 m^2 **c** 21.6%

d 203.36 m^2, 3250 L

e Rainwater tank must be connected to at least one outside tap.

7 **a** Solar hot water system with electric booster with a 5-star rating.

b bathroom, kitchen, laundry

c Location of windows and doors to allow cross ventilation.

d CFLs and LEDs

e Artificial lighting (such as CFLs, LEDs) use electricity, natural lighting allows sunlight to come through windows and skylights.

f A skylight is a 'window' in the ceiling. Bathroom, toilet, kitchen.

Sample HSC problem

A $5199 **B** $4563 **C** $4056

The Sony is the best buy.

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Revision

1 **a** 15.6 MW **b** 5 170 000 kW **c** 45 kW

2 $749.30

3 26.28 kWh **4** $53.11

5 **a** $7.16 **b** $6.20 **c** $25.28

6 $71.01 **7** $101.90 **8** 2135 kWh

9 People in Hawkesbury use air conditioners for heating/cooling, and may have pools while people in Lithgow may have alternatives to heating such as slow combustion heaters, more solar hot water heating.

10 **a** Batts/foil-backed blankets in the ceiling to stop heat coming in during summer, keeping the house cooler.

b To reduce heat from sun entering the house in summer due to high elevation of sun and to increase heat coming in the house from the sun in winter when elevation of the sun is lowest.

11 **a** Building Sustainability Index

b Teacher to check – using 3-star or more shower heads, rainwater tank with at least one outside tap, native vegetation.

c Teacher to check – insulation in floor, walls, ceiling, eaves and shading over windows.

Practice paper 4

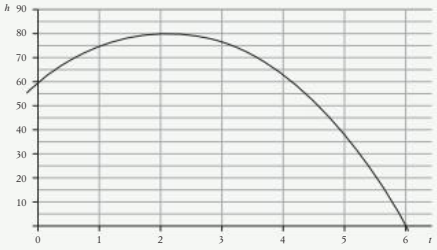
1 D **2** C **3** B

4 B **5** B **6** C

7 B **8** A **9** C

10 D

11 **a**



ii 80 m after 2 seconds

iii 60, the starting height of the ball or the cliff in metres

b **i** 430

ii When $t = 0$, $N = 80 \times (1.4)^0 = 80$

iii 40%

c **i** 4412 **ii** 4085

12 **a** **i** $D = 4.9t^2$ **ii** 1102.5 m

iii 20.2 s

b **i** 3

ii (61), (75), (68), (61, 75), (61, 68), (75, 68), (61, 75, 68)

iii 61, 75, 68, 68, 64.5, 71.5, 68, $\mu_{\tilde{x}} = 68$

iv $\mu = 68$, yes

c $889.11

13 **a** **i** 2135 kWh **ii** 26.7%

b 8 510 000 kW

c **i** $195.92 **iii** $65.31

d **i** English 1.60, Maths 1.22, Geography 2

ii Geography, highest z-score

iii 1 student

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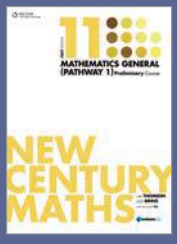

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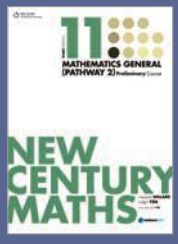

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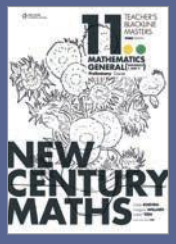

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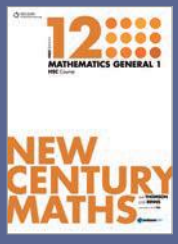

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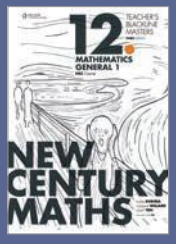

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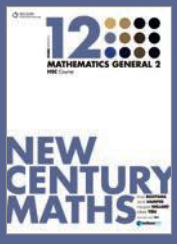

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