nelson maths.

12

Essential Mathematics

Units **3** & **4**

Sue Thomson
Judy Binns

QLD

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LEARNING DISCOVERY

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Nelson

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Judy Binns
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Contents

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Preface

Nelson Maths 11–12 QLD is a new QCE mathematics series that is backed by research into the science of learning. The design and structure of the series has been informed by teacher advice and evidence-based pedagogy, with the focus on preparing QCE students for their exams and maximising their learning achievement.

- All theory, examples and practical applications are precisely mapped to the QCAA syllabus, including coverage of **problem-solving** and **complex** questions.
- To reduce the **cognitive load** for 21st century learners, explanations are clear and concise, using the technique of **chunking** text with accompanying diagrams and infographics.
- The cumulative structure of the **4 Practice sets** is built on the learning and memory techniques of **spacing** and **retrieval**, as recommended by the new syllabus.

This book caters for students heading towards the workforce or further training after school, and teaches useful mathematical skills and applications for life. The author team are experts in teaching and writing for senior practical mathematics courses, and have published for both state and Australian curriculums.

About the authors

Sue Thomson and **Judy Binns** have had long careers in creating mathematics texts for Queensland, NSW, Victoria and Western Australia, writing *Nelson QMaths Essential Mathematics 11–12*, *New Century Maths 11–12 Mathematics Standard 1*, *Nelson VICmaths Foundation Mathematics 11–12* and *Nelson Senior Maths 11–12 Mathematics Essentials* respectively.

Sue Thomson is an innovative teacher and educational leader. She was an examination writer, assessor, marker and curriculum writer, and a councillor for the Australian Association of Mathematics Teachers (AAMT). Sue is a prolific and successful author with an interest in language development, financial literacy and making mathematics accessible to all.

Judy Binns has taught and led mathematics in urban and rural schools. She has an interest in motivating students with learning difficulties, and wide experience in teaching senior practical mathematics courses. Judy also co-authored *Nelson Maths 7–8*.

Contributing authors

Georege Dimitriadis wrote the worked solutions.
Deborah Van Hoek wrote many of the *Nelson MindTap* worksheets.
Rashmi Bhagwati, **Sue Thomson** and **Judy Binns** wrote the topic tests.
John Drake, **Katie Jackson** and **Joanne Magner** created the video tutorials.

Syllabus reference grid

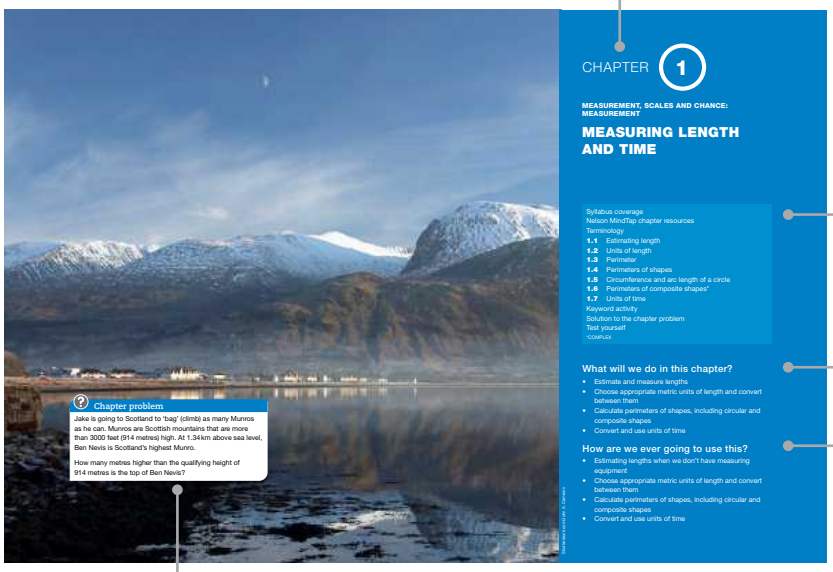
Topics and subtopics	*Nelson Maths 12 Essential Mathematics QLD* chapter	
UNIT 3: MEASUREMENT, SCALES AND CHANCE		
1 Measurement		
Converting units of measure	1	Measuring length and time
	4	On the surface
	7	Turn up the volume
Geometry	3	The shape of our world
Linear measure	1	Measuring length and time
Area measure	4	On the surface
Volume and capacity	7	Turn up the volume
2 Scales, plans and models		
Interpret scale drawings	5	From paper to reality
Creating scale drawings	5	From paper to reality
Right-angled triangles	8	So, you've got a right angle
3 Probability and relative frequencies		
Simulations	6	Will it happen?
Simple probabilities	9	Taking chances
UNIT 4: GRAPHS, DATA AND LOANS		
1 Bivariate graphs		
Cartesian plane	13	Graphing lines
Bivariate scatterplots	14	Scattering the data
Line of best fit	14	Scattering the data
	16	Fitting the data
2 Summarising and comparing data		
Summarising and interpreting data	10	It's better than average
Comparing datasets	12	Comparing data
3 Loans and compound interest		
Compound interest	11	Investing money
Reducing balance loans	15	Borrowing money

Note: The Fundamental topic, Calculations, is first covered in Chapter 2 *What's the price?*, then integrated into all other chapters.

About this book

At the beginning of each chapter

Each chapter begins on a double-page spread.



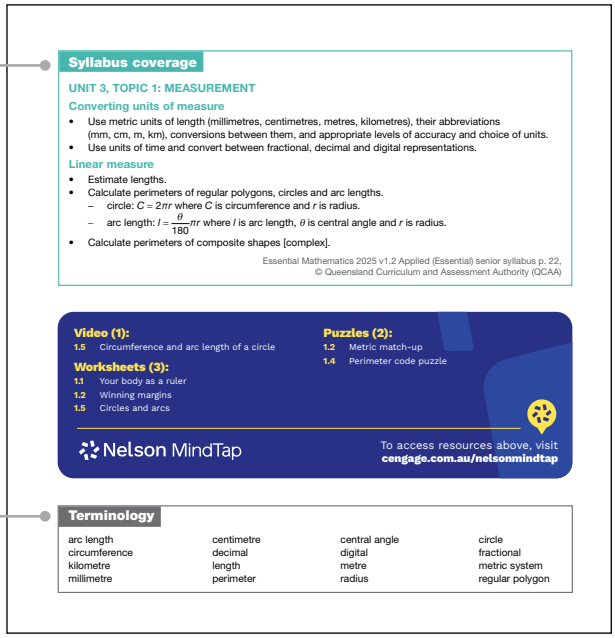
Chapter table of contents includes COMPLEX syllabus content marked by *.

What will we do in this chapter? list of outcomes.

How are we ever going to use this? list of applications.

Chapter problem to be solved.

Syllabus coverage and extracts are shown at the front of the chapter, along with a listing of **Nelson MindTap chapter resources**.



Terminology previews the keywords and phrases from within the chapter.

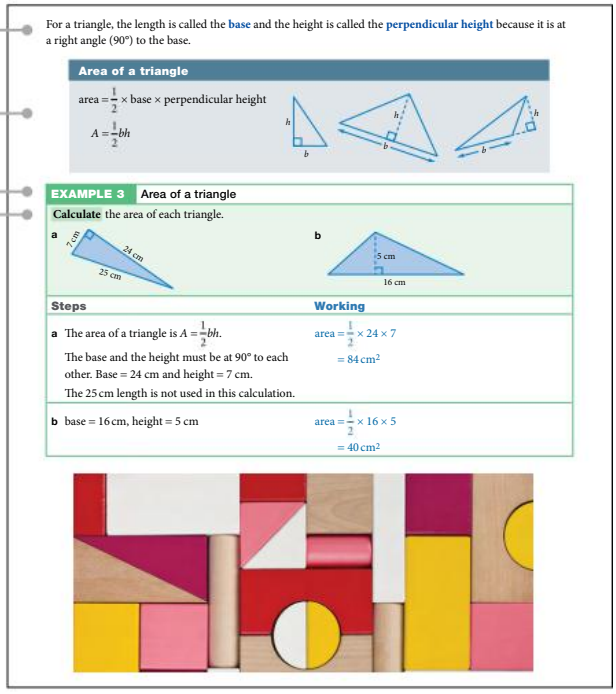
In each chapter

Important words and phrases are printed in blue and listed in the **Glossary and index** at the back of the book.

Important facts and formulas are highlighted in a shaded box.

Examples are explained clearly step-by-step, with the mathematical working shown on the right-hand side.

Cognitive verbs are highlighted in light green. See QCAA's glossary of cognitive verbs for more verbs and details.



For a triangle, the length is called the **base** and the height is called the **perpendicular height** because it is at a right angle (90°) to the base.

Area of a triangle

area $= \frac{1}{2} \times$ base $\times$ perpendicular height

$A = \frac{1}{2}bh$

EXAMPLE 3 Area of a triangle

Calculate the area of each triangle.

a 7 cm, 24 cm, 25 cm

b 5 cm, 16 cm

Steps	Working
a The area of a triangle is $A = \frac{1}{2}bh$. The base and the height must be at 90° to each other. Base = 24 cm and height = 7 cm. The 25 cm length is not used in this calculation.	area $= \frac{1}{2} \times 24 \times 7$ $= 84\,\text{cm}^2$
b base = 16 cm, height = 5 cm	area $= \frac{1}{2} \times 16 \times 5$ $= 40\,\text{cm}^2$

Graded exercises are linked to the worked examples and include exam-style problems and realistic applications.

EXAMPLE 12 Converting time units 3

Express 1.7 months in months and days, assuming that an 'average month' is 30 days.

Steps	Working
Change 0.7 months into days. Multiply 0.7 by 30.	$0.7 \times 30 = 21$ days
Write the answer.	1.7 months is 1 month and 21 days.

4 Express each time in hours and minutes.
a 100 minutes **b** 495 minutes

5 Express as days, hours and minutes. Write your answer correct to the nearest minute.
a 5.36 days **b** $11\frac{7}{10}$ days

6 EXAMPLE 12 Express each time in months and days, assuming there are 30 days to the month. Write your answer correct to the nearest day.
a 9.8 months **b** 5.24 months **c** 8.3 months **d** $9\frac{4}{5}$ months

7 What time is it 8.6 hours after 10:35 am? PS

8 It takes 687 Earth days for Mars to complete one orbit of the Sun. Express this time in Earth years, months and days, assuming 30 days in a month. Answer to the nearest Earth day. PS

Keyword activity

COMPLETE THE BLANKS ANSWERS p. 432

Copy and complete this summary of the chapter.

Throughout history, people have used different ways to measure items. In the 18th century, the French developed a measuring system called the **1**_____ system. In this system, the metre is the basic unit for measuring **2**_____. For larger distances, we use **3**_____, which are equivalent to 1000 metres. To measure small lengths, we use millimetres, which are one- **4**_____ of a metre.

We use length units when we measure the distance around the outside of a shape, which is called its **5**_____. The distance around the outside of a circle is called its **6**_____. To calculate this and the arc length of a circle, we use formulas involving the number π (pi).

Problem-solving PS and complex C questions are clearly tagged and colour-coded.

Investigations and **practical activities** explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks.

Technology promotes ICT in the classroom, using spreadsheets and the internet.

Practical activity Area of a triangle

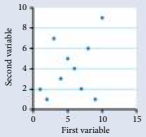
In this activity you are going to investigate the relationship between the area of a triangle and the area of a rectangle. You will need a pair of scissors and the 'Areas of triangles' worksheet, which can be downloaded from Nelson MindTap.

What you have to do

1 Cut out the pair of triangles in Triangle set A.
2 Put the triangles together to make a rectangle.
3 Determine the relationship between the length and height of the rectangle and the triangles.
4 Cut out the pair of triangles in Triangle set B.
5 Keep one triangle whole and cut the other triangle along the height into 2 smaller triangles.
6 Arrange the first triangle and the 2 pieces of the second triangle to make a rectangle.
7 Determine the relationship between the length and height of the rectangle and the triangles.

TECHNOLOGY Changing correlations

For this table of values and scatterplot, the correlation coefficient for the variables is 0.366. Download the spreadsheet 'Changing correlations' from Nelson MindTap.

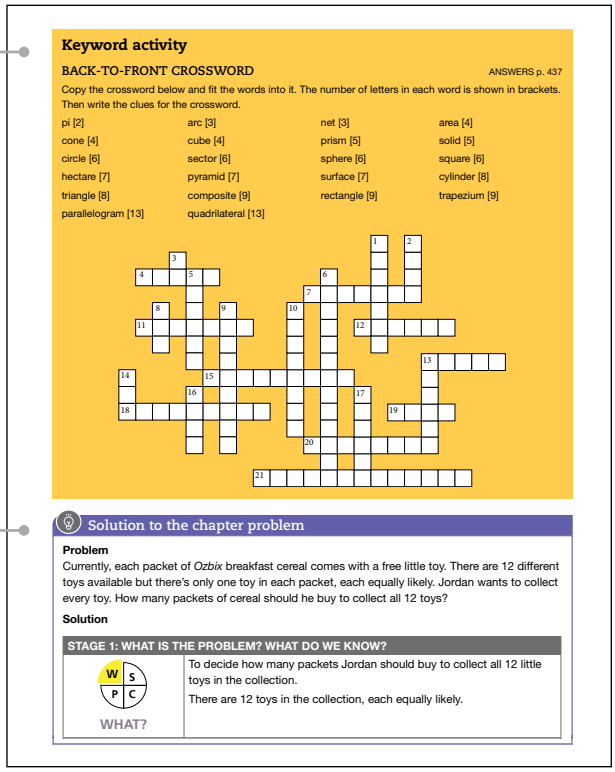


First variable	1	2	3	4	5	6	7	8	9	10
Second variable	2	1	7	3	5	4	2	6	1	9

The spreadsheet contains the data in the above table and calculates the correlation coefficient. Change the values in the Second variable row to produce a set of data whose correlation coefficient is:
a 0.9 **b** −0.3 **c** close to 0.

At the end of each chapter

Keyword activity focuses on the mathematical language and terminology learned in the chapter.



Keyword activity

BACK-TO-FRONT CROSSWORD ANSWERS p. 437

Copy the crossword below and fit the words into it. The number of letters in each word is shown in brackets. Then write the clues for the crossword.

pi [2]	arc [3]	net [3]	area [4]
cone [4]	cube [4]	prism [5]	solid [5]
circle [6]	sector [6]	sphere [6]	square [6]
hectare [7]	pyramid [7]	surface [7]	cylinder [8]
triangle [8]	composite [9]	rectangle [9]	trapezium [9]
parallelogram [13]	quadrilateral [13]		

Solution to the chapter problem revisits the problem introduced at the start of the chapter and presents a solution using 4 stages: WHAT?, SOLVE, CHECK and PRESENT.

Solution to the chapter problem

Problem

Currently, each packet of *Ozbix* breakfast cereal comes with a free little toy. There are 12 different toys available but there's only one toy in each packet, each equally likely. Jordan wants to collect every toy. How many packets of cereal should he buy to collect all 12 toys?

Solution

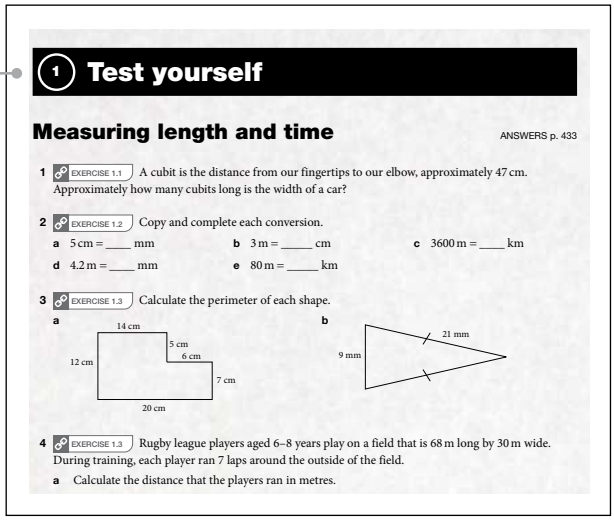
STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

To decide how many packets Jordan should buy to collect all 12 little toys in the collection.

There are 12 toys in the collection, each equally likely.

Test yourself contains revision questions linked to the relevant exercise set.



1 Test yourself

Measuring length and time ANSWERS p. 433

1 EXERCISE 1.1 A cubit is the distance from our fingertips to our elbow, approximately 47 cm. Approximately how many cubits long is the width of a car?

2 EXERCISE 1.2 Copy and complete each conversion.

a 5 cm = ____ mm b 3 m = ____ cm c 3600 m = ____ km

d 4.2 m = ____ mm e 80 m = ____ km

3 EXERCISE 1.3 Calculate the perimeter of each shape.

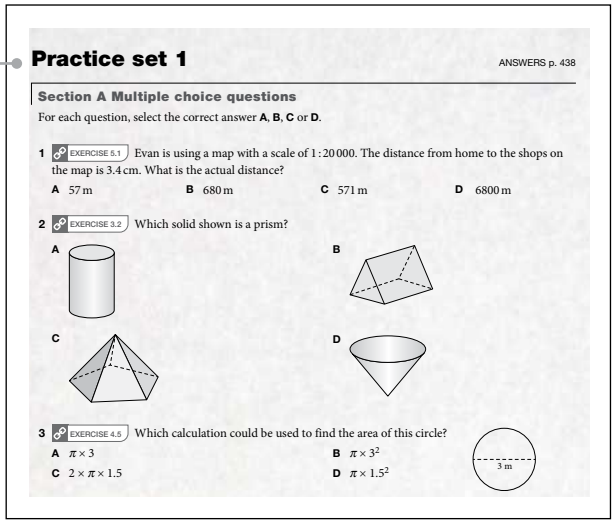
a

b

4 EXERCISE 1.3 Rugby league players aged 6–8 years play on a field that is 68 m long by 30 m wide. During training, each player ran 7 laps around the outside of the field.

a Calculate the distance that the players ran in metres.

Practice sets revise the skills and knowledge of previous chapters and include multiple choice and short answer questions.



Practice set 1 ANSWERS p. 438

Section A Multiple choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 5.1 Evan is using a map with a scale of 1 : 20 000. The distance from home to the shops on the map is 3.4 cm. What is the actual distance?

A 57 m B 680 m C 571 m D 6800 m

2 EXERCISE 3.2 Which solid shown is a prism?

A

B

C

D

3 EXERCISE 4.5 Which calculation could be used to find the area of this circle?

A $\pi \times 3$ B $\pi \times 3^2$

C $2 \times \pi \times 1.5$ D $\pi \times 1.5^2$

At the end of the book

Answers

CHAPTER 1

EXERCISE 1.1

1 240 cm
2 **a** 20 cm **b** 120 cm **c** 6 cm
d 63 cm **e** 73.5 cm **f** 550 cm
3 **a** 140 cm **b** Teacher to check.
4 705 cm
5 **a** length = 2820 cm, width = 940 cm, height = 1410 cm
b length = 28.2 m, width = 9.4 m, height = 14.1 m
6 14.4 m

EXERCISE 1.2

1 **a** 1000, × **b** 10, + **c** 100, +
d 1000, + **e** 100, × **f** 1000, +
g 10, ×
2 **a** 30 **b** 500 **c** 0.4 **d** 2000
e 3 **f** 2 **g** 0.5 **h** 0.25
i 0.6 **j** 600 **k** 4.5 **l** 800
m 0.8 **n** 9000 **o** 0.09 **p** 6500
3 **a** 640 m **b** 64 000 cm
4 **a** 850 m **b** 0.85 km
5 **a** 45 000 cm **b** 4.8 km
c 1 300 000 mm **d** 0.4175 km
6 6 km

EXERCISE 1.4

1 **a** 86 cm **b** 21 mm **c** 12 m **d** 32 cm
2 **a** 45 mm **b** 31.5 cm **c** 240 m **d** 18.4 km
3 38.4 m
4 98.4 mm
5 **a** 520 cm **b** 83.3 mm
6 6 m
7 **a** rectangle **b** 800 mm
c 950 mm **d** 1250 mm
e The length of the join between the blue and white sections isn't part of the flags perimeter, but it is included in the perimeter of each separate section.

EXERCISE 1.5

1 **a** 94.2 cm **b** 56.5 mm
c 19.5 km **d** 55.0 m
2 **a** 15.1 mm **b** 7.0 mm **c** 66.8 cm
3 **a** 203 cm **b** 44 mm
4 40 200 km
5 **a** Teacher to check. **b** 99.4 mm
c The arcs between the vertices of the coin are longer than the straight edges.
6 25
7 **a** 36 880 km **b** 1740 km
c 2500 km; the road doesn't follow a straight line.

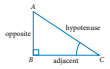
Answers (with worked solutions provided on Nelson MindTap for teachers to allocate to students).

Glossary and index

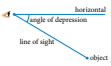
2D shapes Flat shapes that have 2 dimensions: length and width. (p. 51)

3D shapes Solid shapes that have 3 dimensions: length, width and depth. (p. 51)

adjacent side The side next to a given angle in a triangle leading to the right angle. In the diagram, the side BC is adjacent to angle C. (p. 213)



angle of depression When an observer looks at an object that is lower; the angle that the eye turns down from the horizontal. (p. 219)



angle of elevation When an observer looks at an object that is higher; the angle that the eye turns up from the horizontal. (p. 219)

area The amount of surface occupied by a flat shape, measured in square units. (p. 75)

bivariate data Data that relates 2 variables measured on the same group, e.g., height and weight of students. (p. 369)

box plot A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers. (p. 323)

capacity Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 181)

Cartesian plane Another name for the number plane. (p. 349)

centi- One-hundredth $\left(\frac{1}{100}\right)$. (p. 8)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.14159…), d is the **diameter** and r is the **radius**. (p. 18)

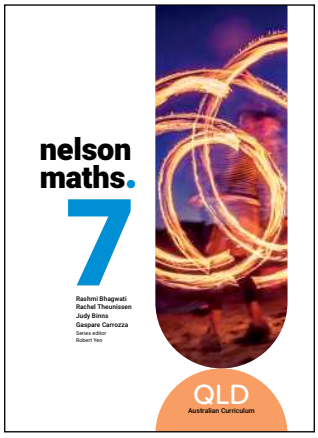
cluster A group of data scores that are bunched or close together. (p. 280)

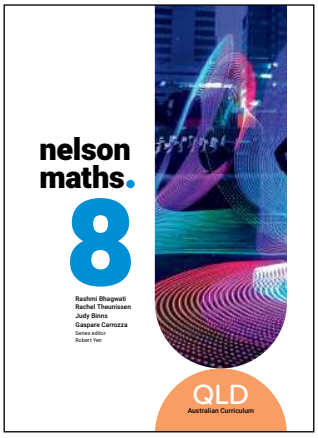
compound interest Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**. (p. 300)

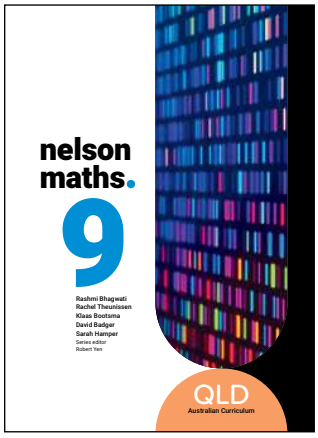
compounding period How often interest is calculated when using compound interest, for example, monthly,

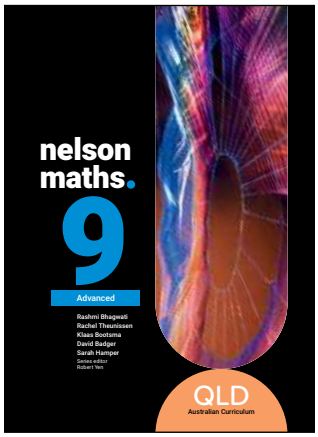
A combined **glossary and index**.

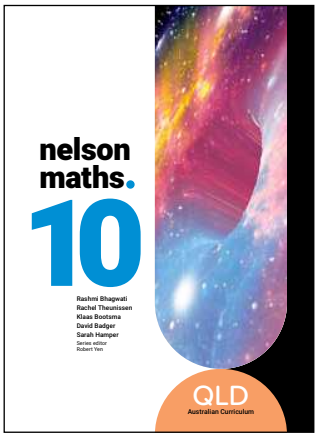
Nelson Maths 7–12 QLD series

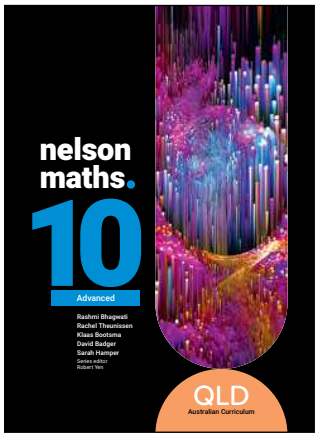


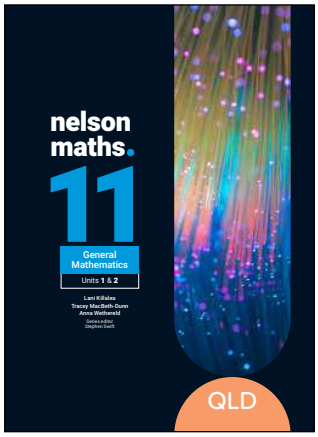


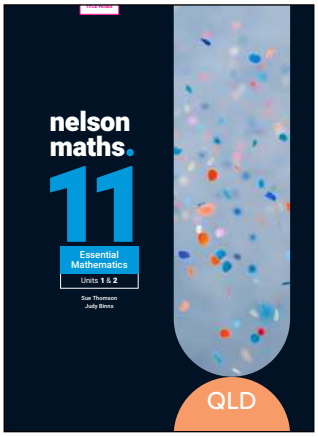


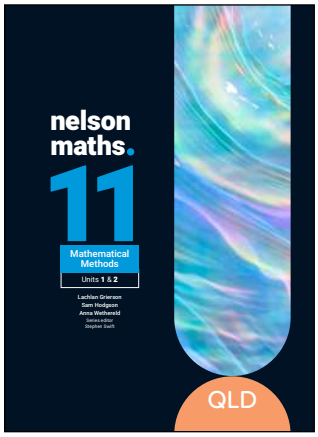


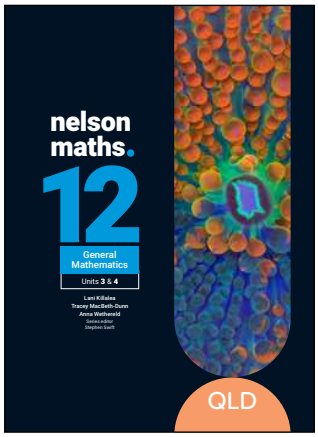


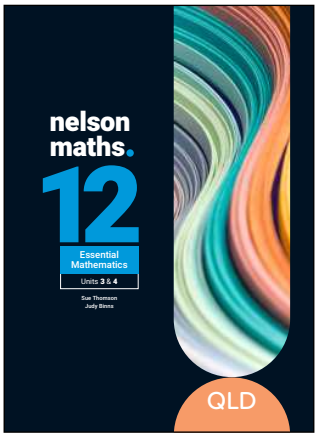


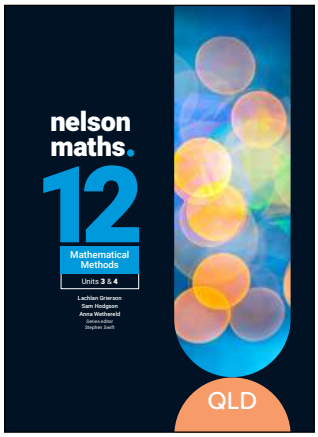












List of mathematical verbs

A glossary of 'doing words' commonly found in mathematics problems

Cognitive verbs are highlighted in light green. See QCAA's glossary of cognitive verbs for more verbs and details.

analyse: study and state in detail the relationship of parts of a situation

apply: use knowledge or a procedure in a given situation

calculate: find a numerical value

classify: identify and sort into categories

comment: express an opinion or judgement about a statement or calculation

communicate: transmit information to others

compare: state similarities and differences and their significance

consider: take into account; think carefully

construct: draw an accurate diagram or logically arrange items or ideas

convert: change from one form to another

deduce: use logic and reason to prove something

define: give the meaning of or identify in exact terms

demonstrate: show to be correct

describe: state the features of a situation, object, pattern, event, etc.

determine: find the answer or make a decision using evidence

discuss: give reasons based on evidence for and against a position or proposal

evaluate: find the value of or state the application, strengths and limitations of a solution

examine: investigate the details and assumptions of a situation

explain: state the meaning in logical detail

explore: examine or state the details and assumptions of a situation

identify: state the type, name or distinguishing feature of an item or situation

interpret: state a conclusion or trend from given information

investigate: establish facts, trends or conclusions from collected information

justify: give reasons or evidence for an answer or conclusion

modify: change to accommodate different information

predict: use given information or results to guess what will happen under different conditions

recall: remember (and state)

recognise: use knowledge to identify features of a situation

simplify: reduce the size of numbers in a fraction, or reduce the size of an algebraic expression

sketch: draw a diagram that shows the general shape and includes relevant features

solve: find the answer or explanation for a problem, particularly the values of variables in equations or inequalities

substitute: replace a variable by a number to calculate an answer

test: check that a statement or result is correct

verify: check a solution or result, usually referring to the given situation

write/state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

Chapter problem

Jake is going to Scotland to 'bag' (climb) as many Munros as he can. Munros are Scottish mountains that are more than 3000 feet (914 metres) high. At 1.34 km above sea level, Ben Nevis is Scotland's highest Munro.

How many metres higher than the qualifying height of 914 metres is the top of Ben Nevis?

John A. Cameron/Shutterstock.com

CHAPTER 1

MEASUREMENT, SCALES AND CHANCE: MEASUREMENT

MEASURING LENGTH AND TIME

What will we do in this chapter?

- Estimate and measure lengths
- Choose appropriate metric units of length and convert between them
- Calculate perimeters of shapes, including circular and composite shapes
- Convert and use units of time

How are we ever going to use this?

- Estimating lengths when we don't have measuring equipment
- Choose appropriate metric units of length and convert between them
- Calculate perimeters of shapes, including circular and composite shapes
- Convert and use units of time

9780170484664

Syllabus coverage

UNIT 3, TOPIC 1: MEASUREMENT

Converting units of measure

- Use metric units of length (millimetres, centimetres, metres, kilometres), their abbreviations (mm, cm, m, km), conversions between them, and appropriate levels of accuracy and choice of units.
- Use units of time and convert between fractional, decimal and digital representations.

Linear measure

- Estimate lengths.
- Calculate perimeters of regular polygons, circles and arc lengths.
  - circle: $C = 2\pi r$ where C is circumference and r is radius.
  - arc length: $l = \frac{\theta}{180}\pi r$ where l is arc length, θ is central angle and r is radius.
- Calculate perimeters of composite shapes [complex].

Video (1):

1.5 Circumference and arc length of a circle

Worksheets (3):

1.1 Your body as a ruler

1.2 Winning margins

1.5 Circles and arcs

Puzzles (2):

1.2 Metric match-up

1.4 Perimeter code puzzle

Terminology

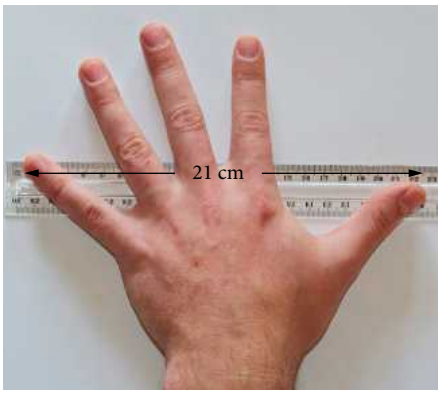
arc length	centimetre	central angle	circle
circumference	decimal	digital	fractional
kilometre	length	metre	metric system
millimetre	perimeter	radius	regular polygon

1.1 Estimating length

Most of us don't walk around with a tape measure or ruler in our pocket, but we often need to know how long an object is. There are a few tricks we can use to closely estimate a length. Our body parts can act as rulers for us; they're convenient and we always have them with us! If you record or memorise the length of your little finger, the width of your palm, the length of your shoe and the length of your stride, you can use them to estimate lengths.

The photos below show some useful lengths. Your own measurements will be slightly different from these.

iStock.com/Hanis



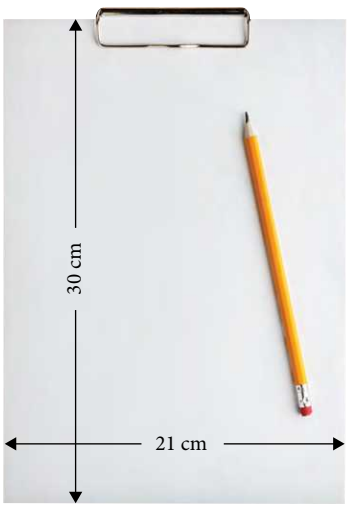
Scout Kozakiewicz

Altanaka/Shutterstock.com

Altanaka/Shutterstock.com

Reserve Bank of Australia

Reserve Bank of Australia



Boltenkoff/Shutterstock.com

EXAMPLE 1 Estimating length

Melanie used her shoe to estimate the length of a table. She counted that it was 5 shoes long. Approximately how long is the table?

Steps	Working
According to the photo, a shoe is approximately 30 cm long, so multiply the number of shoes by 30.	$5 \times 30 = 150$ cm
Write your answer.	The table is approximately 150 cm long.

EXERCISE 1.1 Estimating length

ANSWERS p. 432

1 EXAMPLE 1 Emma's verandah is 8 shoe lengths wide. Approximately how many centimetres wide is Emma's verandah?

2 Find the approximate length of each item in centimetres.

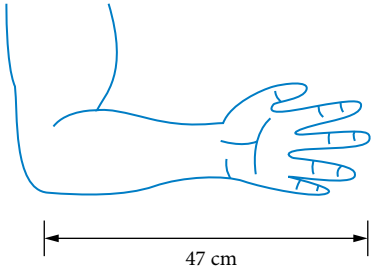
	Item	Length
a	Children's story book	4 little fingers
b	Length of a work bench	8 $50 notes
c	Height of a cake	2 thumbs
d	Height of a skateboard jump	3 widths of an A4 piece of paper
e	Child's height	$3\frac{1}{2}$ hand spans
f	The distance across the room	11 paces

3 a This horse is 14 hand widths high. Calculate the height of the horse in cm.

b Why do you think we measure a horse's height as it is shown in the photo and not as the distance of the top of its ears to the ground?

Lenkadan/Shutterstock.com

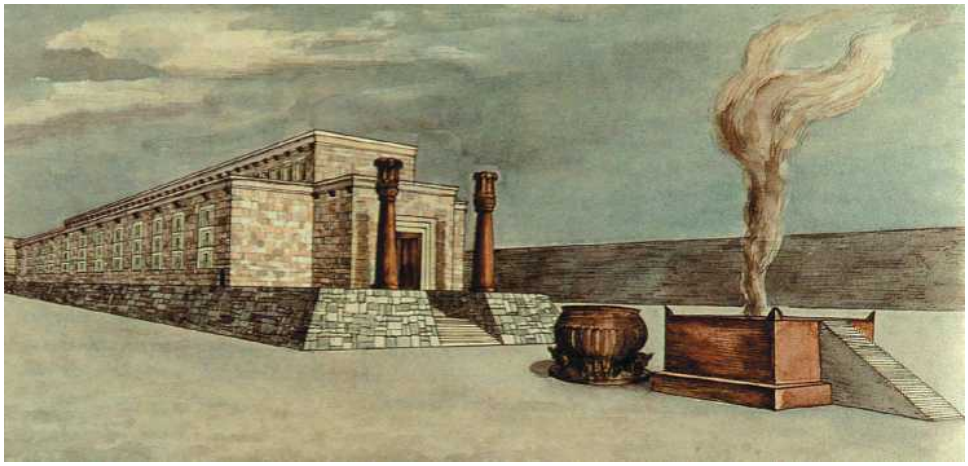
4 A **cubit** is the distance from the tip of our fingers to our elbow. Noah measured a piece of rope as 15 cubits long. Calculate the length of the rope in cm.



5 The Bible includes the measurements of King Solomon's Temple, which is thought to have been built around 1000 BCE. The temple's length was 60 cubits, its width 20 cubits, and its height 30 cubits.

a Calculate the length, width and height of King Solomon's Temple in centimetres.

b Divide each measurement by 100 to convert them to metres.



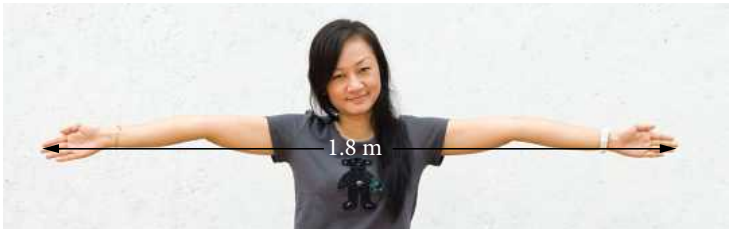
Zev Radovan/Alamy Stock Photo

6 A **fathom** is an old English measurement that we can use to help us approximate the length of fabric or fencing wire. It was also used in the imperial system to measure the depth of water (equal to 6 feet). A fathom is the distance from the fingertips on one hand to the fingertips on the other when the arms are stretched wide. A fathom is 1.8 metres long.

Alexis measured that she has 8 fathoms of fabric. How many metres of fabric does she have?

Practical activity Your body as a ruler

To complete this investigation, each member of your group needs a ruler and a copy of the table below, which can be downloaded from Nelson MindTap as the worksheet 'Your body as a ruler'.



AmpYang Images/Shutterstock.com

The length of a 'fathom'.

Part 1

Measure each body part and record the length in the table.

Name:		
Body part	**Approximation use**	**Length**
Top section of thumb	Small lengths	
Little finger	Small lengths	
Hand span	Short lengths	
Foot (shoe)	Short distances on the ground	
Pace	Walking distances	
Fathom	Lengths of fabric or rope	

Worksheet
Your body as a ruler

Part 2

Use your body measurements and a calculation to approximate each length.

1 The length of your classroom

2 The width of a page from this book

3 The length of your calculator

4 The length of a pen

5 The height of your desk above the floor

6 The width of a window in your classroom

7 The distance from your classroom door to the school canteen

8 The height of the classroom door above the floor

Part 3

Use a ruler or tape measure to check the accuracy of your approximations.

How good is your body as a ruler?

Puzzle
Metric match-up

1.2 Units of length

Using body parts to estimate lengths is convenient, but it's not very accurate because measurements vary from person to person. In the 1790s, the French Academy of Sciences developed a standardised measurement system called the **metric system**.

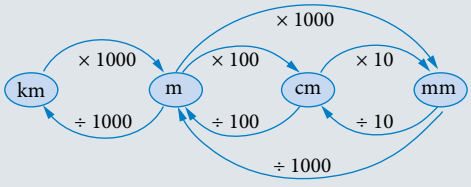
In the metric system, the **metre** is the basic unit for measuring length. Originally 1 metre was defined to be one ten-millionth of the distance from the Equator to the North Pole at sea level.

Unit	Relationships	How we use it
kilometre (km)	1 km = 1000 m	Used to measure long distances For example, the distance between cities or countries.
metre (m)	1 m = 100 cm 1 m = 1000 mm	Used to measure medium lengths For example, the length of a sporting field or a room's width.
centimetre (cm)	1 cm = 10 mm	Used to measure small lengths For example, the dimensions of a brick.
millimetre (mm)		Used to measure very small lengths For example, the length of an insect or the width of your toenail.

Converting units of length

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.



EXAMPLE 2 Converting length

Convert:

a 30 m to cm **b** 780 m to km **c** 65 cm to mm

Steps	Working
a There are 100 cm in 1 metre. The conversion factor is 100. Changing from m to cm is changing to a smaller unit. We need to multiply by the conversion factor.	30 m = 30 × 100 cm = 3000 cm
b There are 1000 m in 1 km. Changing from m to km is changing to a bigger unit. We need to divide by the conversion factor.	780 m = 780 ÷ 1000 km = 0.78 km
c There are 10 mm in 1 cm. Changing from cm to mm is changing to a smaller unit. We need to multiply by the conversion factor.	65 cm = 65 × 10 mm = 650 mm

EXERCISE 1.2 Units of length

ANSWERS p. 432

1 In each part, state the conversion factor and whether you have to multiply or divide.

a km to m **b** mm to cm **c** cm to m **d** m to km

e m to cm **f** m to km **g** cm to mm

2 EXAMPLE 2 Copy and complete each conversion.

a 3 cm = ___ mm **b** 5 m = ___ cm **c** 400 m = ___ km

d 2 km = ___ m **e** 30 mm = ___ cm **f** 200 cm = ___ m

g 500 mm = ___ m **h** 250 m = ___ km **i** 60 cm = ___ m

j 60 cm = ___ mm **k** 4500 m = ___ km **l** 0.8 km = ___ m

m 8 mm = ___ cm **n** 90 m = ___ cm **o** 90 m = ___ km

p 6.5 m = ___ mm

3 Follow each step to convert 0.64 km into cm.

a Change 0.64 km into m.

b Change your answer to part **a** into cm.

4 Follow each step to convert 85 000 cm into km.

a Convert 85 000 cm into m.

b Change your answer to part **a** into km.

5 Copy and complete each conversion.

a 0.45 km = ___ cm **b** 4 800 000 mm = ___ km

c 1.3 km = ___ mm **d** 41 750 cm = ___ km

6 Express 5970 m in kilometres, correct to the nearest km.

7 Change 75 640 mm to metres, correct to the nearest m.

8 Every morning, Devi goes to the pool to train. This morning, Devi completed 78 laps at training. Each lap was 50 m long.

a How many metres did Devi swim this morning?

b How many kilometres did Devi swim at training this morning?

c Devi's coach wants her to swim 4.5 km tomorrow. How many laps of the 50 m pool will Devi have to complete to swim 4.5 km?

9 In one layer of a coil there are 165 wire turns. Each wire turn is 12.95 cm long. Calculate the total length of the wire in metres. Express your answer correct to the nearest metre. PS

10 Melissa is making 75 cm-long elastic bandages from a bulk roll of bandage 120 m long. How many bandages will she be able to make? PS

11 When Brent went canoeing, he paddled down 3 sections of 540-metre rapids and 2 sections of 860-metre rapids.

a Calculate the total length of the rapids in metres.

b Express the distance in kilometres.

12 In first-class competitions, cricket pitches are 22.6 m long. During a test match, Yan ran the length of the pitch 137 times.

a Calculate the length she ran in metres.

b Calculate the length she ran in kilometres, correct to 2 decimal places.

13 Ian has a lawn and garden maintenance business. He takes all of his green waste to be recycled at the tip. Last week he drove to the tip 6 times and each return trip was 26.8 km. Ian claims 75c for each km he travels as a business expense. Calculate the amount he will claim for last week's trips to the tip. PS

14 What unit (mm, cm, m or km) would you choose to measure:

a the length of a newborn baby?

b the height of a mobile phone tower?

c the distance from Noosa to Gympie?

d the width of your ring finger?

e the depth of the water where you go snorkelling?

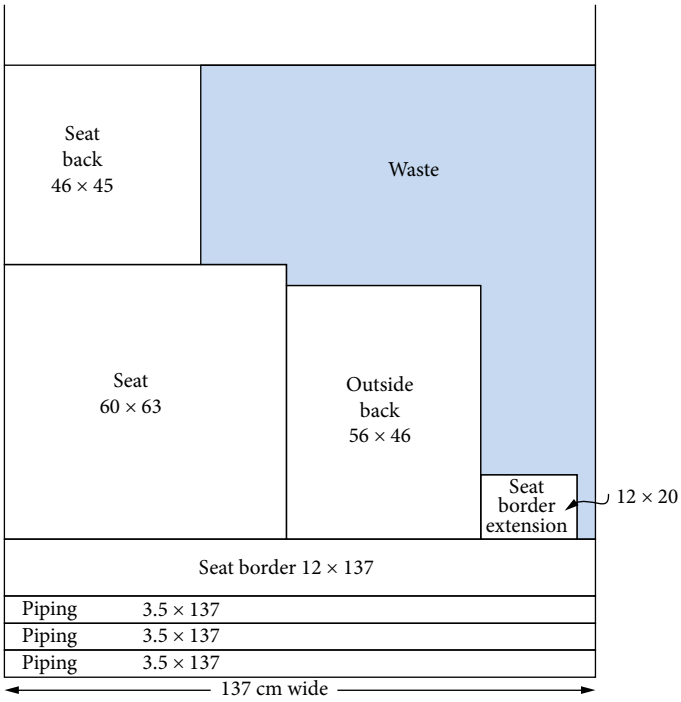
f the diagonal of your TV screen?

iStock.com/Elixirpix

15 Luke is an upholstery apprentice. To calculate the amount of material he needs to cover a chair, he lays the pattern out and draws a cover plan. A cover plan helps minimise waste and costs, as upholstery materials can be very expensive!

On the cover plan shown, all measurements are in cm, the first dimension is the length, and the second dimension is the width.

The material is 137 cm wide and costs $165 per metre.



a Calculate the length of material that Luke needs for the chair.

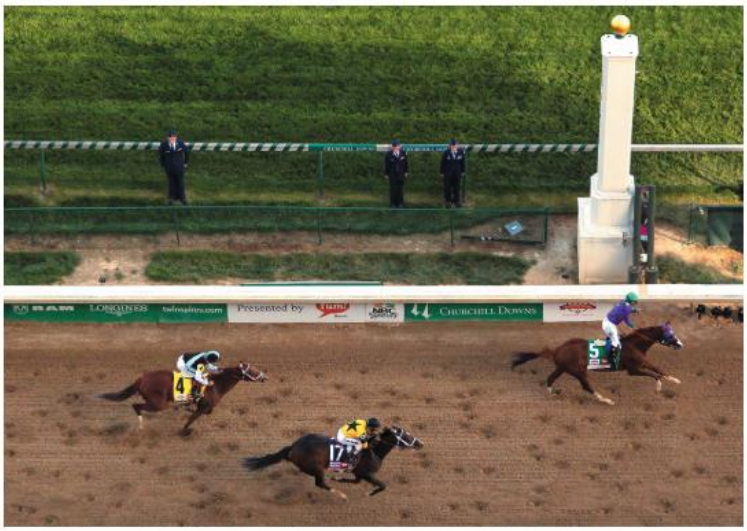
b This chair is one of 6 dining room chairs. Explain why Luke won't need 6 times the amount of material from part **a** to cover all 6 chairs.

c How much money will Luke save on the material cost by ordering the minimum amount he requires rather than 6 times the material required for 1 chair? PS

Worksheet
Winning margins

Problem solving | Winning margins

In horseracing, the winning margin is the distance between the winning horse and the horse that comes second, described using words like 'by a nose' and 'short half-head'. These words are descriptive rather than accurate.



Jamie Squire/Getty Images Sport/Getty Images

Winning margin	Description	Metric equivalent
nose	The smallest winning margin, under 3 inches 1 inch = 2.5 cm	
head	The length of a horse's head Approximately $\frac{1}{8}$ of a horse's length of 2.4 m	
neck	The length of a horse's neck Approximately $\frac{1}{4}$ of a horse's length	
$\frac{1}{2}$ length	$\frac{1}{2}$ the length of a horse	
$\frac{3}{4}$ length	$\frac{3}{4}$ the length of a horse	
length	The length of a horse	
2 lengths	Double the length of a horse	
3 lengths	3 times the length of a horse	
4 lengths	4 times the length of a horse	

Look at the photo of the finish line of a horserace. Describe the winning margin using metric and traditional racing terms.
This exercise can be downloaded from Nelson MindTap as the worksheet 'Winning margins'.

Problem solving — Minimising material requirements

A customer wants Luke to cover a bedroom chair in expensive material. The material is 137 cm wide.
This table shows the number and size of the rectangular pattern pieces for the job.

Number of pieces	Description	Length (cm)	Width (cm)
1	Seat	62	65
3	Seat border	12	65
2	Seat border piping	70	3.5
2	Inside back sides	80	27
1	Inside centre back	80	30
2	Inside back piping	80	3.5
1	Outside back	56	48
2	Outside back piping	56	3.5

Construct a cover plan (see the last question in the previous exercise on page 11) to determine the minimum amount of material Luke will need to complete the job.

Group activity — Accuracy in measuring lengths

In this activity you will investigate how accurately each member of your group can measure lengths. Each group will need a ruler, tape measure and trundle wheel.

What you have to do

1. Choose 3 suitable distances in your school environment; for example, the length of the school verandah, or the distance from your classroom doorway to the nearest tree.
2. Each member of the group measures the lengths as accurately as possible.
3. Compare your group's measurements. At what level of accuracy are the measurements the same?

Chapter problem

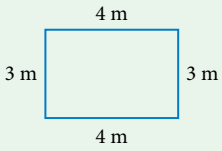
You've covered the skills required to solve the chapter problem. Can you solve it now?

1.3 Perimeter

When we measure the distance around the outside of a shape, we are measuring its **perimeter**.

EXAMPLE 3 Perimeter

To decorate her baby's bedroom, Menhal is applying a wallpaper **frieze** around the middle of its walls. The diagram shows the dimensions of the room.



The frieze is available in 5 m and 10 m rolls.

Calculate the number of rolls of frieze Menhal will need.

Elizabeth Whiting & Associates/Alamy Stock Photo

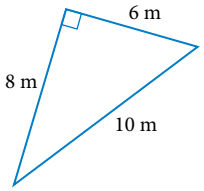
Steps	Working
Calculate the perimeter of the room by adding up the lengths of all its sides.	perimeter = 3 + 4 + 3 + 4 = 14 m
Then compare the length that Menhal needs to the lengths of the rolls.	Menhal needs to buy one 10 m roll and one 5 m roll (or three 5 m rolls). (She will have 1 m left over.)

EXERCISE 1.3 Perimeter

ANSWERS p. 432

1 EXAMPLE 3 Calculate the perimeter of each shape.

a



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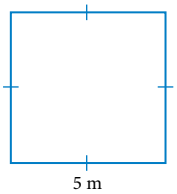

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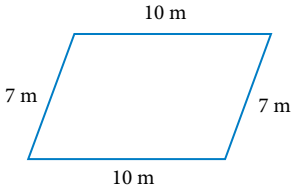
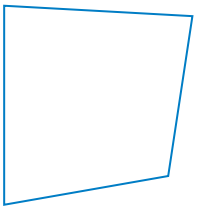


2 The sides of a **rectangle** are 4 m and 11 m long. Calculate its perimeter.

3 Measure the sides of each quadrilateral accurately, then calculate its perimeter.

a



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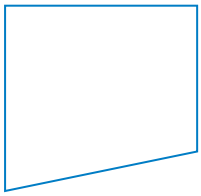


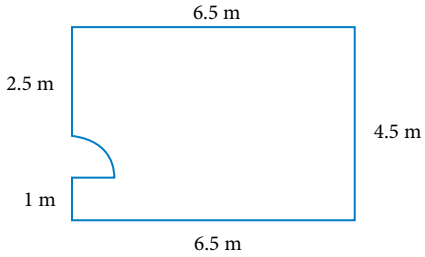
4 The perimeter of a rectangle is 36 m. What could the lengths of its sides be? Suggest 2 possible sets of sides.

5 Kylie needs to replace the fence around her pool and BBQ area. In her **scale drawing**, 1 cm represents 1 m.

a What is the perimeter of Kylie's scale drawing?

b Kylie is going to put a fence around the perimeter with a 90 cm wide gate in it. How long will the fence be in metres?

6 Hiroshi is replacing the **skirting boards** around the floor in his TV room. PS



Skirting boards are the wooden boards that run around the room along the base of the walls.

a Calculate the length of the skirting boards that Hiroshi will need.

b The skirting boards cost $30 per metre. How much will it cost Hiroshi to buy the skirting boards?

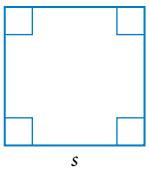
1.4 Perimeters of shapes

Puzzle
Perimeter code puzzle

Sometimes we can calculate perimeters more quickly using a **formula**.

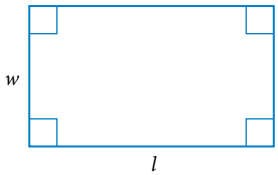
The perimeter of a square can be found by adding all of the sides:

$$\text{Perimeter} = s + s + s + s$$
$$= 4s$$



The perimeter of a rectangle can be found by adding all of the sides:

$$\text{Perimeter} = l + w + l + w$$
$$= 2l + 2w \text{ or } 2(l + w)$$



Perimeters of squares and rectangles

The **perimeter of a square** is

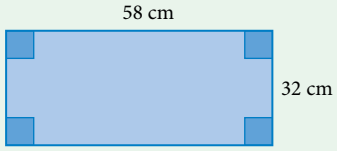
$P = 4s$, where s is the length of one side.

The **perimeter of a rectangle** is

$P = 2l + 2w$ or $P = 2(l + w)$, where l is length and w is width.

EXAMPLE 4 Perimeter of a rectangle

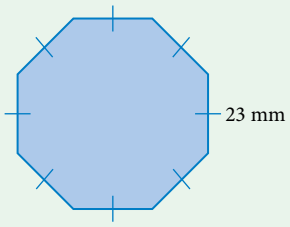
Use a formula to **calculate** the perimeter of this rectangle.



Steps	Working
Using $P = 2l + 2w$, where $l = 58$ and $w = 32$	$P = 2 \times 58 + 2 \times 32$ $= 116 + 64$ $= 180$
OR using the formula, $P = 2(l + w)$	$P = 2 \times (58 + 32)$ $= 2 \times 90$ $= 180$
Write the answer, including units.	The perimeter is 180 cm.

EXAMPLE 5 Perimeter of a regular polygon

Calculate the perimeter of this regular octagon.



This is a **regular** octagon because all 8 sides have the same length and all the angles are equal.

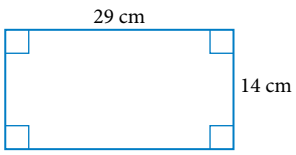
Steps	Working
As all sides of the regular octagon are the same length, we can simply multiply 23 mm by 8.	perimeter $= 8 \times 23$ $= 184$
Write the answer, including units.	The perimeter is 184 mm.

EXERCISE 1.4 Perimeters of shapes

ANSWERS p. 432

1 EXAMPLE 4 Calculate the perimeter of each shape.

a



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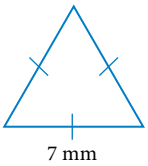

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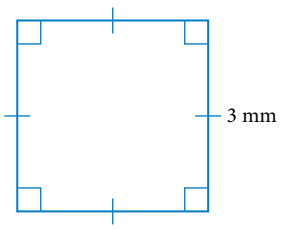
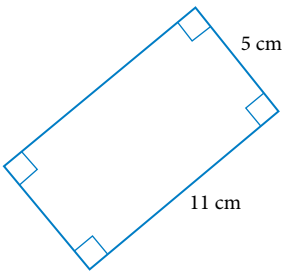
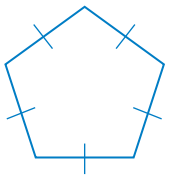


d



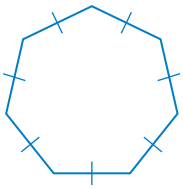
2 EXAMPLE 5 Calculate the perimeter of each regular polygon.

a



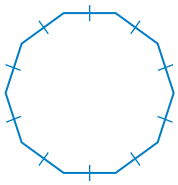
9 mm

b



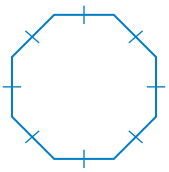
4.5 cm

c



24 m

d



2.3 km

3 The Australian flag that flies above Parliament House in Canberra is 12.8 m long and 6.4 m high. Calculate the perimeter of this flag.

6.4 m

12.8 m

Peter Probst/Shutterstock.com

4 Our 50c coin is in the shape of a regular dodecagon, a 12-sided polygon. Each side is 8.2 mm long. Calculate the perimeter of a 50c coin.

GOLFX/Shutterstock.com

5 Calculate the perimeter of each regular polygon.

a

65 cm

Zoart Studio/Shutterstock.com

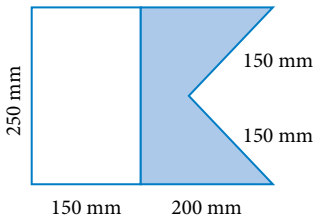
b

11.9 mm

Starsphinx/Shutterstock.com

6 The perimeter of a 10-sided regular polygon is 60 m. How long is each side?

7 Maritime rules require dive boats to fly a blue and white flag when there are divers in the water. The flag tells other boat drivers to go slowly and stay away from the divers.



a What geometrical shape is the white section of the flag?

b What is the perimeter of the white section of the flag?

c Calculate the perimeter of the blue section of the flag.

d What is the perimeter of the whole flag?

e Why isn't the perimeter of the whole flag the same amount as adding the perimeter of the white section and the blue section together? PS

Worksheet
Circles and arcs

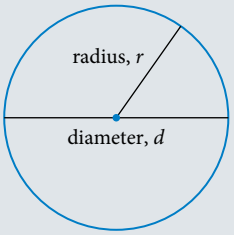
1.5 Circumference and arc length of a circle

The perimeter of a circle is called its **circumference**.

Circumference of a circle

$$C = 2\pi r$$

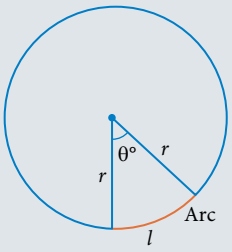
where r is the **radius** of the circle.



Arc length of a circle

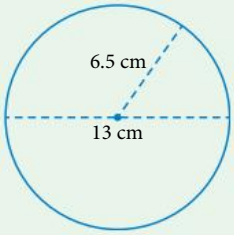
$$l = \frac{\theta}{180}\pi r$$

where θ is the central angle.



EXAMPLE 6 Circumference of a circle

Calculate the circumference of this circle.



Steps	Working
Use the formula $C = 2\pi r$, where $r = 6.5$ cm.	$C = 2 \times \pi \times 6.5$ $= 40.8407\ldots$
Write the answer. The values in the question had one decimal place so use one decimal place in your answer.	The circumference of the circle is 40.8 cm.

EXAMPLE 7 The length of an arc

Calculate the length of arc AB, correct to the nearest mm.

Steps	Working
Arc length: $l = \frac{\theta}{180}\pi r$, where $\theta = 125°$ and $r = 8$ mm.	arc $AB = \frac{125}{180} \times \pi \times 8$ $= 17.453\ldots$
Write the answer.	The length of arc AB is approximately 17 mm.

Video
Circumference and arc length of a circle

EXERCISE 1.5 Circumference and arc length of a circle

ANSWERS p. 432

1 EXAMPLE 6 Calculate, correct to one decimal place, the circumference of a circle with:

a radius 15 cm

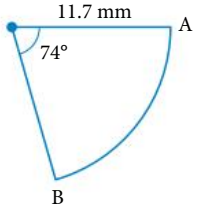
b diameter 18 mm

c radius 3.1 km

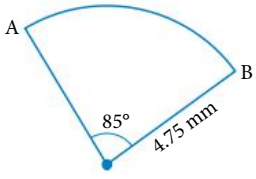
d diameter 17.5 m

2 EXAMPLE 7 Calculate, correct to one decimal place, the length of the arc AB in each diagram.

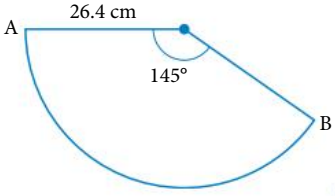
a



b

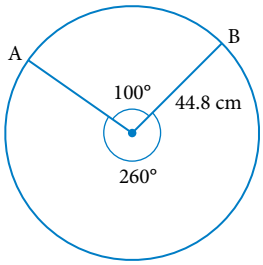


c

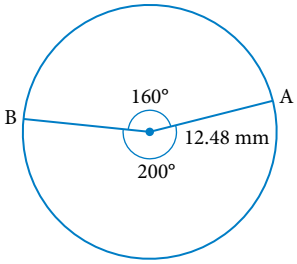


3 Calculate the length of the larger arc AB in each diagram, correct to the nearest whole number.

a



b



4 The equatorial radius of the Earth, the length from the centre of the Earth to the equator, is about 6400 km. The equator is the circumference of the Earth. Calculate the length of the equator, correct to the nearest 100 km.

Nerthuz/Shutterstock.com

5 In Exercise 1.4, Question **4**, you calculated the perimeter of a 50c coin. Now, imagine that there is a circle around the coin. PS

GOLFX/Shutterstock.com

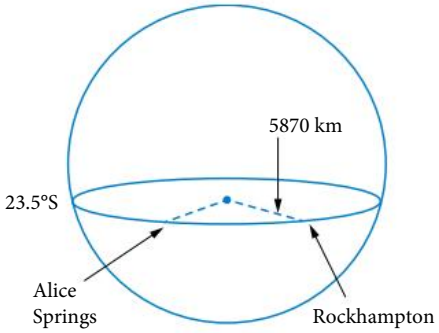
a Which do you think will be larger: the circumference of the circle or the perimeter of the coin?

b The diameter of the circle is 31.65 mm. Calculate the circumference, correct to one decimal place.

c Each side of the coin is 8.2 mm long. Was your guess in part **a** correct? Why is one of these measures bigger than the other?

6 Sam has a circular garden with a diameter of 2.4 m in her front yard. She is going to plant flowers every 30 cm around the circumference. How many plants will she need? PS

7 Rockhampton and Alice Springs both lie on the 23.5° S parallel of latitude. The radius of the 23.5° S parallel is 5870 km. PS



a Calculate the entire length of the 23.5° S parallel of latitude, correct to the nearest 10 km.

b The angle at the centre of the 23.5° circle made by joining the positions for Alice Springs and Rockhampton is 17°. Calculate the distance along the 23.5° S parallel from Rockhampton to Alice Springs. Answer correct to the nearest 10 km.

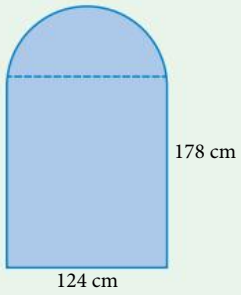
c Use Google Maps or a similar website to find the distance from Rockhampton to Alice Springs by car. Suggest a reason why this distance is more than the answer to part **b**.

1.6 Perimeters of composite shapes*

Most things in real life aren't just one shape. Often, items are made from a combination of shapes.

EXAMPLE 8 Perimeter of composite shapes

The window in Bella's lounge room is in the shape of a semicircle on top of a rectangle, as shown. **Calculate** the perimeter of Bella's window in metres, correct to 2 decimal places.



Steps	Working
The perimeter is made up of half the circumference of a circle and 3 sides of the rectangle.	
First, calculate half the circumference: $\frac{1}{2} \times 2\pi r$ Don't round until the last step.	half the circumference $= \frac{1}{2} \times 2 \times \pi \times 62$ $= 194.7787\ldots$
3 sides of the rectangle.	rectangle $= 178 + 124 + 178$ $= 480$
Calculate the total perimeter.	total perimeter $= 194.7787\ldots + 480$ $= 674.7787\ldots$ cm
Convert to metres.	$= 6.747\,787\ldots$ m
Write the answer.	The perimeter of Bella's window is 6.75 m.

EXAMPLE 9 Perimeter of a sector of a circle

This logo's shape is $\frac{3}{4}$ of a circle with a radius of 8 mm.

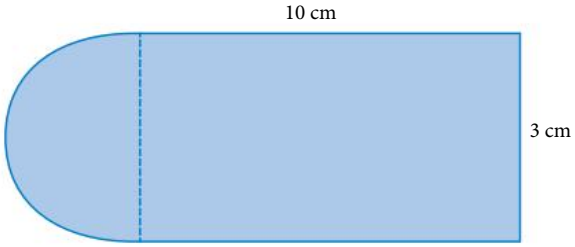
Calculate its perimeter, correct to one decimal place.

Steps	Working
The perimeter is $\frac{3}{4}$ of the circumference plus 2 radii.	perimeter $= \left(\frac{3}{4} \times 2 \times \pi \times 8\right) + (2 \times 8)$
Radii is the plural form of radius. One radius, 2 radii.	$= 53.6991\ldots$
Write the answer.	The logo's perimeter is approximately 53.7 mm.

EXERCISE 1.6 Perimeters of composite shapes*

ANSWERS p. 432

1 EXAMPLE 8 This diagram shows the shape of a tool that scrapes paint off glass. Calculate the perimeter of the tool, correct to one decimal place.



2 The Sunshine Coast Lions run around the outside of a circular field as part of their training. The diameter of the field is 180 m.

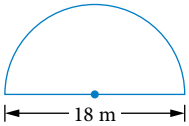
a How far do they run (to the nearest metre) when they complete 8 laps of the field?

b The coach wants the team to run 2 km. How many laps of the field will the team have to run to complete 2 km? Answer to one decimal place.

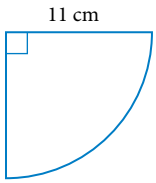
Vadymvdrobot/Dreamstime LLC

3 EXAMPLE 9 Calculate, correct to one decimal place, the perimeter of each shape.

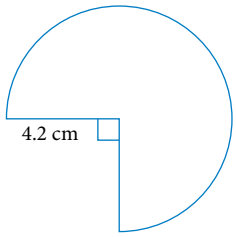
a



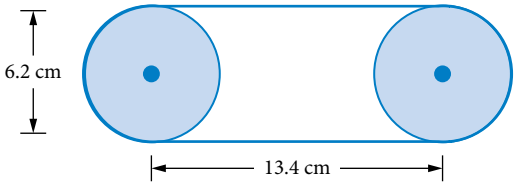
b



c

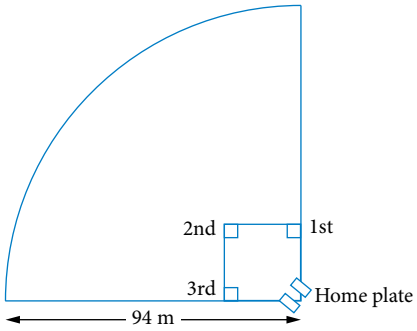


4 The diagram shows a belt around a pair of pulleys. Calculate the length of the belt, correct to one decimal place.

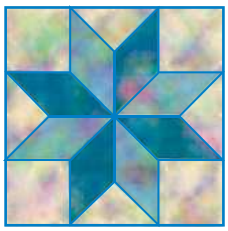


5 A traditional softball field is in the shape of a quadrant. What is the perimeter of the field? Answer to the nearest metre.

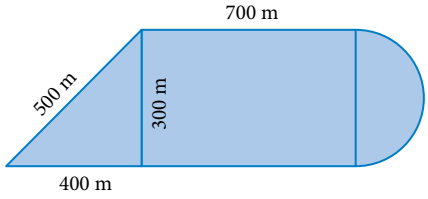
A quadrant is a quarter of a circle.



6 Rachel is making a 12-star quilt with a star pattern as shown. Each edge of the star is 6 cm long. She is going to put a trim around the outside of all 12 stars in the quilt. How many metres of trim will she use?



7 The diagram shows a walking circuit in a national park. The paths surround a shape that includes a triangle, a rectangle and a semicircle. Calculate the length of the circuit, correct to the nearest metre.



1.7 Units of time

Have you ever wondered why there are 60 minutes in one hour and 24 hours in one day?
The ancient Egyptians divided both the day and the night into 12 equal sections:
12 hours from midnight to midday and another 12 hours from midday to midnight.

The idea of 60 minutes in one hour came from the Babylonians. They thought the number 60 was special and used a base 60 number system. For example, there are 60° in each angle of an equilateral triangle and 1° of angle size is divided into 60 minutes.

Unit	Relationships
month	1 month = 31 days for January, March, May, July, August, October, December 1 month = 30 days for April, June, September, November 1 month = 28 days for February 1 month = 29 days for February during leap years
week	1 week = 7 days
day (d)	1 d = 24 h
hour (h)	1 h = 60 min 1 h = 3600 s
minute (min)	1 min = 60 s
second (s)	

EXAMPLE 10 Converting time units 1

Convert:

a $\frac{1}{5}$ hours into minutes

b $7\frac{3}{4}$ days into days and hours

Steps	Working
a Multiply hours by 60 to convert into minutes.	$\frac{1}{5} \times 60 = 12$ $\frac{1}{5}$ of an hour is 12 minutes.
b Multiply the fraction part of the days by 24 to convert it to hours.	$\frac{3}{4} \times 24 = 18$ $7\frac{3}{4}$ days is 7 days and 18 hours.

EXAMPLE 11 Converting time units 2

Convert:

a 3.45 hours into hours and minutes

b 380 minutes into hours and minutes

c 6.4 days into days and hours

d Express 5 hours and 20 minutes as hours.

Steps	Working
a Your calculator has a useful button for changing 3.45 hours into minutes. Enter 3.45, then press °‘“.	3.45 hours = 3 hours and 27 minutes
Alternatively, to change 0.45 hours into minutes, multiply 60 minutes (1 hour) by 0.45.	0.45 × 60 = 27 3.45 hours = 3 hours and 27 minutes
b To change 380 minutes into hours, divide by 60. On your calculator, press °‘“.	380 ÷ 60 = 6.333 3333…. 6 hours 20 minutes
Alternatively, subtract the hours to leave just the decimal part on your screen, then multiply by 60.	0.333 333… × 60 = 20 380 minutes = 6 hours and 20 minutes.
c There are 24 hours in a day. To change 6.4 days into days and hours, multiply the 0.4 by 24.	0.4 × 24 = 9.6 6.4 days = 6 days and 9.6 hours
Then convert the hours into hours and minutes by multiplying the decimal part by 60.	0.6 hours = 0.6 × 60 = 36 6.4 days = 6 days 9 hours and 36 minutes
d To enter 5 hours and 20 minutes into your calculator, press 5 °‘“ 20 °‘“ then press the equals button. The S⇔D will toggle between different ways to express 5 hours and 20 minutes.	5 hours and 20 minutes = $\frac{16}{3}$ hours or 5.333 333… hours

EXAMPLE 12 Converting time units 3

Express 1.7 months in months and days, assuming that an ‘average month’ is 30 days.

Steps	Working
Change 0.7 months into days. Multiply 0.7 by 30.	0.7 × 30 = 21 days
Write the answer.	1.7 months is 1 month and 21 days.

EXERCISE 1.7 Units of time

ANSWERS p. 432

1 EXAMPLE 10 Convert each part of an hour to minutes.

a $\frac{1}{2}$ hour **b** $\frac{3}{5}$ hour **c** $\frac{7}{10}$ hour

2 Express each amount in days and hours.

a $8\frac{5}{12}$ days **b** $4\frac{3}{4}$ days

3 EXAMPLE 11 Express each time in hours and minutes.

a 4.5 hours **b** 5.8 hours **c** 7.4 hours

d $8\frac{1}{4}$ hours **e** $11\frac{5}{8}$ hours

4 Express each time in hours and minutes.

a 100 minutes **b** 495 minutes

5 Express as days, hours and minutes. Write your answer correct to the nearest minute.

a 5.36 days **b** $11\frac{7}{10}$ days

6 EXAMPLE 12 Express each time in months and days, assuming there are 30 days to the month. Write your answer correct to the nearest day.

a 9.8 months **b** 5.24 months **c** 8.3 months **d** $9\frac{4}{5}$ months

7 What time is it 8.6 hours after 10:35 am? PS

8 It takes 687 Earth days for Mars to complete one orbit of the Sun. Express this time in Earth years, months and days, assuming 30 days in a month. Answer to the nearest Earth day. PS

Keyword activity

COMPLETE THE BLANKS

ANSWERS p. 432

Copy and complete this summary of the chapter.

Throughout history, people have used different ways to measure items. In the 18th century, the French developed a measuring system called the **1** _____ system. In this system, the metre is the basic unit for measuring **2**_____. For larger distances, we use **3**_____, which are equivalent to 1000 metres. To measure small lengths, we use millimetres, which are one- **4**_____ of a metre.

We use length units when we measure the distance around the outside of a shape, which is called its **5**_____. The distance around the outside of a circle is called its **6**_____. To calculate this and the arc length of a circle, we use formulas involving the number π (pi).

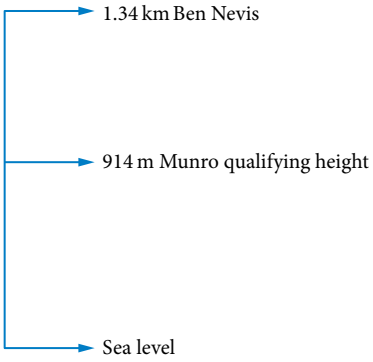
Calculations involving time are more complicated because there are 60 **7**_____ in a minute, 60 minutes in an **8**_____, 24 hours in a day and 7 days in a **9**_____.

Solution to the chapter problem

Problem

Jake is going to Scotland to 'bag' (climb) as many Munros as he can. Munros are Scottish mountains that are more than 3000 feet (914 metres) high. At 1.34 km above sea level, Ben Nevis is Scotland's highest Munro. How many metres higher than the qualifying height of 914 metres is the top of Ben Nevis?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To work out how much higher 1.34 km is than 914 m.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	Draw a diagram and use the word clue: How much higher → subtract.  Convert 1.34 km to metres to compare with 914 m. 1.34 km = 1.34 × 1000 m = 1340 m height difference = 1340 – 914 = 426 m
STAGE 3: CHECK THE SOLUTION	
 CHECK	Ben Nevis is a very high mountain. The answer is the size we expect.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	The top of Ben Nevis is 426 m higher than the qualifying height for a Munro.

1 Test yourself

Measuring length and time

ANSWERS p. 433

1 EXERCISE 1.1 A cubit is the distance from our fingertips to our elbow, approximately 47 cm. Approximately how many cubits long is the width of a car?

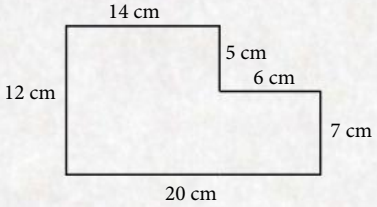
2 EXERCISE 1.2 Copy and complete each conversion.

a 5 cm = ____ mm **b** 3 m = _____ cm **c** 3600 m = ____ km

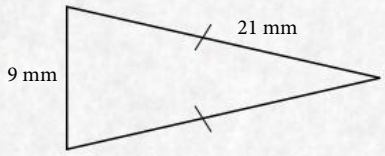
d 4.2 m = ____ mm **e** 80 m = _____ km

3 EXERCISE 1.3 Calculate the perimeter of each shape.

a



b



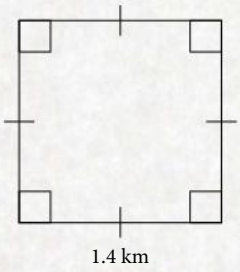
4 EXERCISE 1.3 Rugby league players aged 6–8 years play on a field that is 68 m long by 30 m wide. During training, each player ran 7 laps around the outside of the field.

a Calculate the distance that the players ran in metres.

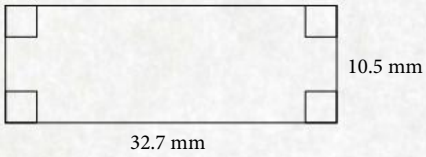
b Convert your answer to part **a** to kilometres.

5 EXERCISE 1.4 Calculate the perimeter of each shape.

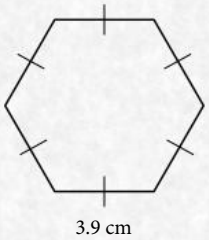
a



b



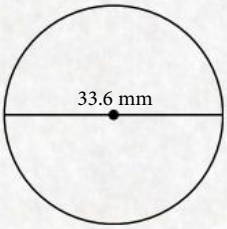
c



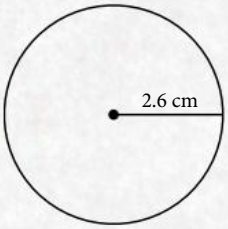
6 EXERCISE 1.5 Calculate the circumference of each circle, correct to one decimal place.

a

b

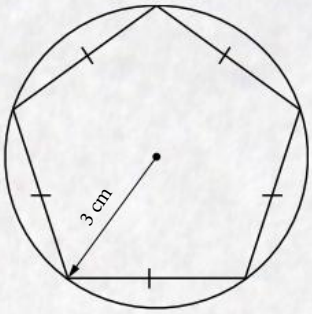


c



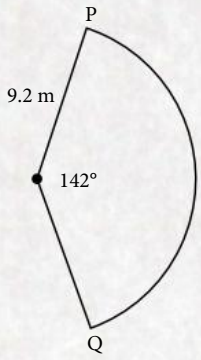
9780170484664

7 EXERCISES 1.4, 1.5 This circle of radius 3 cm passes through the vertices of the regular pentagon. The sides of the pentagon are 3.527 cm long. How much longer is the circumference of the circle than the perimeter of the pentagon, correct to 2 decimal places?



8 EXERCISE 1.5 Calculate the length of the arc PQ, correct to one decimal place.

a



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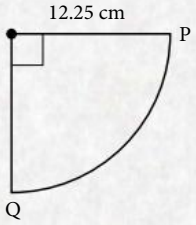

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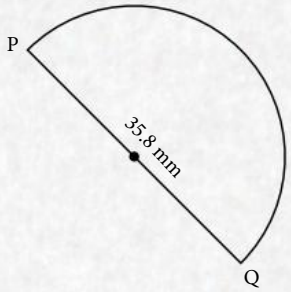
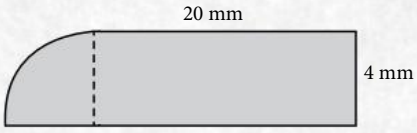
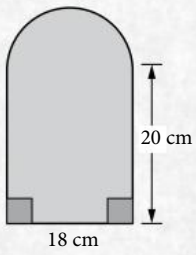


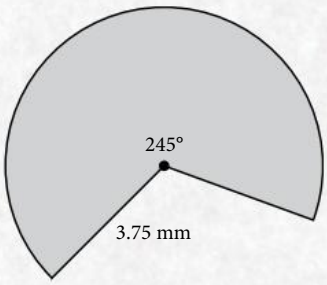
9 EXERCISE 1.6 What is the perimeter of this shape made from a quarter of a circle and a rectangle? Answer correct to one decimal place.



10 EXERCISE 1.6 This shape is made from a semicircle and a rectangle. Calculate its perimeter, correct to the nearest centimetre.



11 EXERCISE 1.6 Calculate the perimeter of this shape, correct to 2 decimal places.



12 EXERCISE 1.7 Express 3.85 days in days, hours and minutes.

13 EXERCISE 1.7 Assuming there are 30 days in a month, express 15.7 months in months and days.

Chapter problem

Damian makes wooden bowls to sell at the local craft markets. He wants to receive $72 for each bowl made. The craft markets take 25% of the sale price. At what price should he sell each bowl so that he can receive $72?

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CHAPTER 2

MEASUREMENT, SCALES AND CHANCE: CALCULATIONS

WHAT'S THE PRICE?

What will we do in this chapter?

- Calculate with numbers, including using correct order of operations
- Estimate and check the reasonableness of answers
- Understand place value after the decimal point in decimals
- Round answers to decimal places
- Solve practical problems involving numbers

How are we ever going to use this?

- Pricing items for sale
- Ordering supplies
- Shopping
- Budgeting
- Every day

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Syllabus coverage

UNIT 3, FUNDAMENTAL TOPIC: CALCULATIONS

Calculations

- Solve practical problems requiring basic number operations.
- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.
- Use leading-digit approximation to obtain estimates of calculations.
- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.
- Apply approximation strategies for calculations.

Terminology

decimal places	estimate	leading-digit approximation	order of operations
percentage decrease	percentage increase	reasonableness	round down
round up	strategies		

2.1 Knitted handbags

Ella makes woollen handbags. She knits them, then shrinks them in hot water in a washing machine to make a very firm stiff fabric. This process is called **felting**, which reduces the size of the wool by 25 to 40%. You might be surprised by the amount of maths involved in designing Ella's handbags!

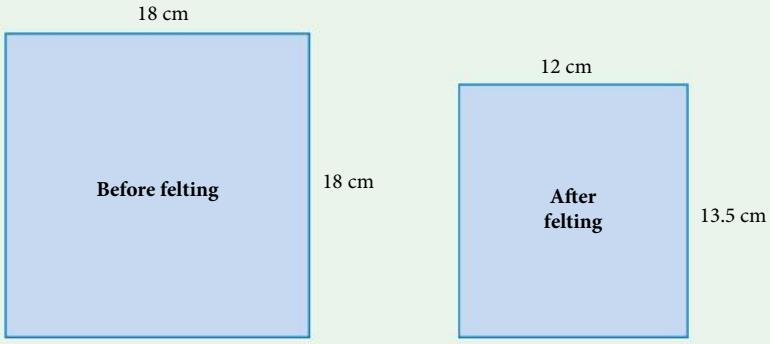
The steps in designing such a handbag include:

- testing the wool to make sure it felts and how much it will shrink
- deciding the dimensions of the completed bag
- calculating the number of stitches.

Puzzles
Number review puzzle
Percentages review puzzle

EXAMPLE 1 Percentage decrease

Ella made a test sample of the wool she planned to use. The sample was an 18 cm square, and after felting it was 12 cm wide and 13.5 cm long.



Calculate the percentage decrease in the width and length of the sample.

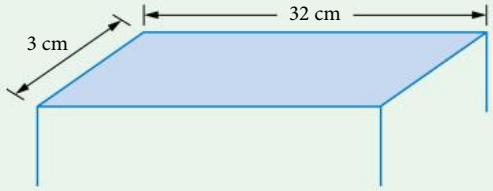
Steps	Working
The width reduced from 18 cm to 12 cm, which is 6 cm. percentage decrease $= \frac{\text{decrease}}{\text{original}} \times 100\%$.	percentage decrease $= \frac{6}{18} \times 100\%$ $= 33.333\ldots\%$ $\approx 33.3\%$ This is a decrease of $\frac{1}{3}$.
The length reduced from 18 cm to 13.5 cm, which is 4.5 cm.	percentage decrease $= \frac{4.5}{18} \times 100\%$ $= 25\%$ This is a decrease of $\frac{1}{4}$.
Write the answer.	The percentage decrease in the width was 33.3% and in the length was 25%.

EXAMPLE 2 Using a formula

Ella worked out a formula for calculating the number of stitches needed for a handbag, for each dimension (width, depth, height):

$$\text{number of stitches} = \frac{23}{17} \times \text{dimension}$$

The top of Ella's handbag is shown here (width 32 cm, depth 3 cm).



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Use the formula to calculate the number of stitches Ella needs for:

a the width of the handbag

b the depth of the handbag

c the perimeter of the top of the handbag.

Steps	Working
a Substitute 32 into the formula to calculate the stitches for the width. Round to the nearest whole number because parts of a stitch aren't possible!	stitches for the width $= \frac{23}{17} \times 32$ $= 43.2941\ldots$ ≈ 43
b Substitute 3 into the formula for the depth.	stitches for the depth $= \frac{23}{17} \times 3$ $= 4.0588\ldots$ ≈ 4
c Total number of stitches for the perimeter is double the stitches for the width plus double the stitches for the depth. Remember to use the correct **order of operations**!	stitches for the perimeter $= 2 \times 43 + 2 \times 4$ $= 94$

EXAMPLE 3 Finding the amount before a percentage change

The wool that Ella uses for her handbags shrinks 25% on the length during felting. Ella wants to make a handbag that is 36 cm long. **Calculate** the length she should make the bag *before* she felts it.

Steps	Working
Find the original length. The bag's length shrinks by 25%. After felting it will be 75% of the original length.	75% of the original length = 36 cm
Determine 100% by first finding 1% by dividing by 75.	1% of the original length = 36 cm ÷ 75 = 0.48 cm
Find the whole original length by multiplying by 100%.	original length = 0.48 cm × 100 = 48 cm
Check the answer!	0.75 × 48 = 36. Correct!
Write the answer.	If Ella wants the felted length to be 36 cm, she will need to make the original length 48 cm.

Bag before felting, 48 cm wide

Bag after felting, 36 cm wide

EXAMPLE 4 Leading-digit approximation

Ella is calculating 48.32 × 216.5. Estimate the answer she should get.

Steps	Working
Round the values to the first digit.	48.32 → 50
This is called **leading-digit approximation**.	216.5 → 200
Approximate the answer using the rounded values.	48.32 × 216.5 ≈ 50 × 200 = 10 000
Write the conclusion.	48.32 × 216.5 ≈ 10 000 (The actual value is 10 461.28.)

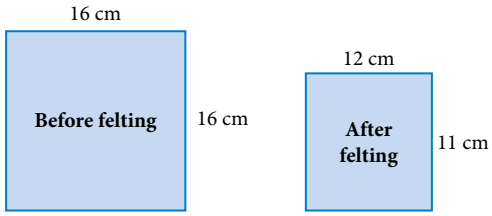
EXERCISE 2.1 Knitted handbags

ANSWERS p. 433

1 Round each number of stitches to the nearest whole number.

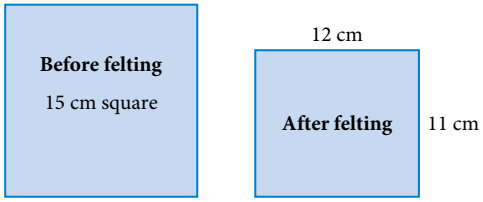
a 39.17 **b** 72.78 **c** 67.55

2 EXAMPLE 1 This diagram shows a test sample of the wool used for making a handbag before and after felting. Calculate the percentage decrease in the width and length of the sample.

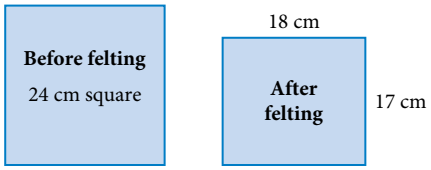


3 Only wool that shrinks at least 25% in both width and length during felting is suitable for making bags. Determine whether each brand of wool shown is suitable for bag making. Justify each answer.

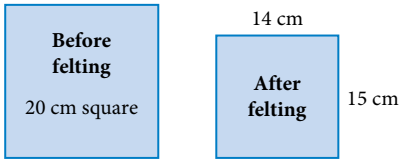
a Comfort brand



b Cozy brand



c San Francisco brand



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4 EXAMPLE 2 For the top of each handbag shown below, use the formula:

$$\text{number of stitches} = \frac{23}{17} \times \text{dimension}$$

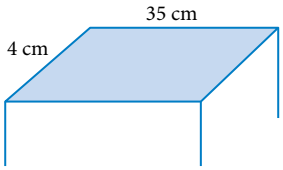
to calculate the number of stitches needed for:

i the width of the handbag

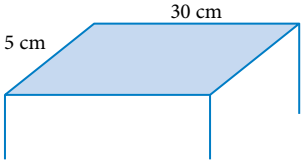
ii the depth of the handbag

iii the perimeter of the top of the handbag.

a



b



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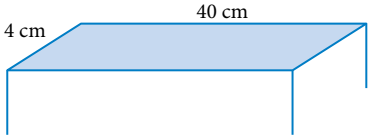
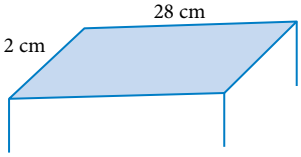


d



5 The length of Ella's handbags is reduced by 25% during felting.

a How much will a bag that is 36 cm long shrink during the felting process?

b Calculate the felted length of a bag that was 52 cm long before felting.

6 EXAMPLE 3 How long will each bag need to be before felting to create these felted lengths? Answer correct to one decimal place as required.

a 24 cm

b 33 cm

c 40 cm

d 45 cm

7 EXAMPLE 4 Complete the missing values in the table to estimate the value of the calculation using leading-digit approximation. The first row has been completed as an example.

	Calculation	Rounding to the leading digits	Estimated answer	Actual answer
	23.5×38.4	20×40	800	902.4
a	312×18.9			
b	$795 + 103$			
c	$1928 - 522$			
d	4829×10.4			
e	$\$822.60 \times 9.7$			

8 **a** Sketch your own rectangular handbag, including the dimensions of the width, depth and length.

b Calculate the number of stitches you will need to make your bag.

c Determine the length you will need to knit your bag before felting to make the bag the length you want. PS

2.2 Egg baskets

This basket for chocolate eggs requires a repeated knitted pattern of stitches.

EXAMPLE 5 Pattern repeats and decimal remainders

The pattern for Ella's knitted egg baskets has 6 stitches in the repeat and requires an additional 4 stitches. To make one basket, Ella needs about 57 stitches.

a How many pattern repeats fit in 57 stitches?

b In this context, **explain** what the 0.5 in the number of pattern repeats represents.

c **Calculate** the number of stitches Ella will need to make this item.

Steps	Working
a Divide the total number of stitches by the number of stitches in the pattern repeat.	number of pattern repeats $57 \div 6$ $= 9.5$ repeats
b It doesn't mean that there are 5 stitches left over! Calculate the number of stitches it represents by multiplying by the number of stitches in the pattern repeat.	leftover stitches $= 0.5 \times 6$ $= 3$
c Ella will need 9 pattern repeats and an extra 4 stitches. Remember to do the multiplication before the addition.	stitches $= 9 \times 6 + 4$ $= 58$

EXERCISE 2.2 Egg baskets

ANSWERS p. 433

1 EXAMPLE 5 How many pattern repeats can be completed in the following number of stitches?

	Number of stitches available	Number of stitches in the pattern repeat
a	52	4
b	75	5
c	42	7

2 How many leftover stitches does the number after the decimal point represent?

	Pattern repeats	Number of stitches in the repeat
a	8.75	4
b	11.2	5
c	9.875	8
d	11.67	12

3 Complete the table.

	Pattern	Number of repeats required	Number of stitches required
a	6 stitches in the repeat plus 2	8	
b	5 stitches in the repeat plus 4	9	
c	8 stitches in the repeat plus 6	7	
d	9 stitches in the repeat plus 4	11	
e	4 stitches in the repeat plus 2		50
f	6 stitches in the repeat plus 4		58

4 Ella calculated that she needs about 75 stitches, but she also needs a number that is a multiple of 6 plus 4. How many stitches does she need?

5 To make the bottom of a basket sit flat, the starting number of stitches in an egg basket must be a multiple of 8 plus 4. Which of the following number of stitches is suitable for an egg basket?

44 54 64 68 36 48

6 Ella wants to make a basket to suit an egg with a maximum circumference of 27 cm. PS

a Use the formula:

$$\text{Number of stitches} = \frac{23}{17} \times \text{length required}$$

to calculate the number of stitches that will make the circumference 27 cm. Answer correct to one decimal place.

b The number of stitches must be a multiple of 8 plus 4. What is the smallest number of stitches bigger than your answer to part **a** that is a multiple of 8 plus 4?

7 Design your own basket. PS

- Measure the circumference or perimeter of the object you want to place in the basket.
- Follow the pattern guidelines in question **6** to determine the number of stitches you will need to make your basket.

2.3 What is the cost?

When Ella makes handbags, she uses the following supplies:

- wool
- fabric for the lining
- zippers
- handles
- buttons to close the front flap

She obtains most of the supplies in bulk online.

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EXAMPLE 6 Calculating costs

Ella makes her own zippers so that she can make them any length she chooses. She has a large roll of bulk zipper and a packet of zipper slides.

A zipper roll costs $62.70 and contains 200 m of zipper. The slides cost $14.45 for a pack of 100.
Determine how much it costs Ella to make a 25 cm zipper to sew into a bag. Answer to the nearest 0.1 cent.

Steps	Working
Ella needs 25 cm of zipper and 1 slide.	
To find the cost of a zipper, first calculate the zipper cost per metre.	cost per metre = $62.70 ÷ 200 = $0.3135
Ella needs 25 cm = 0.25 m. Multiply the cost per metre by 0.25.	zipper cost = 0.25 × $0.3135 = $0.078 375
Find the cost of one slide.	slide cost = $14.45 ÷ 100 = $0.1445
Calculate the total cost.	total cost = $0.078 375 + $0.1445 = $0.222 875 ≈ 22.3 cents
Write the answer.	It costs Ella 22.3c to make a 25 cm zipper.

EXERCISE 2.3 What is the cost?

ANSWERS p. 433

1 Ella visited the local craft store to buy some supplies. The store was selling balls of wool for $5 each and buttons for 55c each. The normal price was $6 per ball and 75c per button. Ella bought 8 balls of wool and 6 buttons.

a Explain how you know that the value of the expression $8 \times 5 + 6 \times 0.55$ represents the total cost of the purchases in dollars.

b Remembering to use the correct order of operations, evaluate $8 \times 5 + 6 \times 0.55$ to calculate the total cost.

c Write an expression for the *normal* price of the items Ella bought in dollars.

d According to the rules for the order of operations, what part of the expression you wrote in part **c** do you have to do first?

e How much did Ella save by buying the wool and buttons on sale?

2 Match each expression in **a** to **d** with the cost descriptions in **A** to **D**, then use the correct order of operations to calculate the value. Use the costs in question **1**. All expressions are in dollars.

a	$8 \times 0.55 + 4 \times 5$	**A**	The change from \$20 when Ella bought 8 full price buttons
b	$20 \times (6 - 5)$	**B**	The cost of 8 discounted buttons and 4 discounted balls of wool
c	$20 - 8 \times 0.75$	**C**	The difference in the price of 2 balls of wool and 2 buttons when they are on sale compared to their normal price
d	$2 \times (6 + 0.75) - 2 \times (5 + 0.55)$	**D**	The amount Ella saved when she bought 20 balls of wool on sale compared to the normal price

3 EXAMPLE 6 Use the price of zippers and slides in Example 6 to calculate the cost of making each length of zipper, correct to the nearest cent.

a 40 cm zipper **b** 15 cm zipper **c** 24 cm zipper

Worksheet
Order of operations

4 Handles are an expensive part of bags but the more you buy in bulk, the cheaper they are. This table shows the price of pairs of leather handles. Postage for up to 50 pairs of handles is \$9.

Quantity	Price per pair
1 pair	\$19.95
5 to 10 pairs	\$16.25
More than 10 pairs	\$15.00

a Calculate the price of buying 2 pairs of handles, including postage.

b What is the cost per pair of handles, including postage, when Ella orders 2 pairs at the same time?

c How much cheaper per pair is it to order 12 pairs of handles at the same time (including postage) than ordering 2 pairs? PS

5 This table shows the materials Ella used when she was making a special-order bag.

Item	Cost
6 balls of wool	\$5.99 per ball
1 button	49c
1 pair of handles	\$19.95 plus \$9 postage
2 zippers	24.5c each
40 cm of lining fabric	\$6 per m

a Without using a calculator, and using leading-digit approximation, estimate the total cost of the materials.

b Use a calculator to check the accuracy of your estimate.

c Will Ella make a profit if she sells the completed bag for \$65? Justify your answer.

Skillsheets
Mental calculations
Percentage calculations

Worksheets
Percentage shortcuts
Percentages without calculators
Profit and loss

2.4 What is the price?

Members of the local creative community can sell items they make through the community shop. The shop charges different percentages depending on the type of item and the value. This table shows the percentages.

Type of item	Percentage of selling price
Items with a selling price over $400	5%
Quilts	10%
Wood items	20%
Handmade clothing and accessories, including handbags	25%
All other items	30%

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EXAMPLE 7 Selling price 1

Ivan wants to sell a leather document case he made. The case cost him $52 to make and he wants to make $32 profit. **Determine** the price he should sell the case for at the community shop.

Steps	Working
Add the cost price and the profit to calculate the amount Ivan wants to receive from the sale.	Ivan's share = $52 + $32 = $84
From the table, the shop takes 30%.	
Ivan will get 70%.	70% of price = $84
Determine 100%. First, find 10% of the price by dividing by 7.	10% of price = $84 ÷ 7 = $12
Then, find 100% by multiplying by 10.	selling price = $12 × 10
Alternatively, divide by 70 to find 1%, then multiply by 100 to find 100%	= $120
Check by calculating 70%.	70% × $120 = $84, correct.
Write the answer.	Ivan should charge $120 for the document case.

EXAMPLE 8 Selling price 2

Sophia wants to receive $570 from the sale of her large oil painting. What price should she put on it?

Steps	Working
Because the price is over $400, the shop will charge 5% for the sale. Sophia will receive 95% of the sale.	95% of the price = $570
Divide by 95 to calculate 1%.	1% of the price = $570 ÷ 95 = $6
Multiply by 100 to find the whole price.	whole price = $6 × 100 = $600
Check the solution.	95% × $600 = $570, correct
Write the answer.	Sophia should price the painting at $600.

EXERCISE 2.4 What is the price?

ANSWERS p. 433

Use the table on the previous page to answer the following questions.

1 How much will the community shop receive from the sale of:

- **a** a quilt priced at $320?
- **b** a pottery set of plates selling for $420?
- **c** a hand-knitted jumper on sale for $276?
- **d** some wooden serving bowls selling for $180?

2 Calculate the amount the crafts person will receive from selling:

- **a** an oil painting priced at $820
- **b** a leather handbag priced at $80
- **c** a hand-painted silk scarf priced at $36
- **d** a carved bone ornament with a marked price of $70.

A scarf is a fashion accessory.

3 Sarina has a painting she plans to sell for around $400. How much more will she receive from the sale if she prices it at $405 compared to $399? PS

4 Terri spent $86 on wool and buttons to make a hand-crocheted coat. She sold the coat through the community shop for $100. How much profit did she make?

5 EXAMPLES 7, 8 Calculate the selling price of each item in the table.

	Item	How much the crafts person wants to receive
a	Baby's cot quilt	$63
b	Set of wooden spoons	$30
c	Hand-knitted jumper	$120
d	Stained glass window	$456
e	Blanket for a horse	$210

6 It cost Ella $42 for the materials to make one of her handbags and she wants to sell it at the shop, making a profit of $30. What selling price should she put on the bag?

7 A mystery item sold in the shop for $125. The person who made the item received $100 from the sale. What could the item have been? PS

8 When Ella sold a large handbag for $132, she made a profit of $47. How much did the materials cost to make this handbag? PS

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Keyword activity

DEFINITIONS

Write a sentence to explain the meaning of each phrase.

1 multiple of 7

2 round to the nearest whole number

3 order of operations

4 formula

5 percentage decrease

6 leading-digit approximation

Solution to the chapter problem

Problem

Damian makes wooden bowls to sell at the local craft markets. He wants to receive $72 for each bowl made. The craft markets take 25% of the sale price.
At what price should he sell each bowl so that he can receive $72?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
W S P C WHAT?	To calculate the price Damian should charge for the bowl. He wants to receive $72. The community centre takes 25% of the sale price.
STAGE 2: SOLVE THE PROBLEM	
W S P C SOLVE	Damian will receive 100% − 25% = 75% of the sale price. 75% of the sale price = $72 Divide by 75 to get 1%. 1% of the sale price = $72 ÷ 75 = $0.96 Multiply by 100 to get 100%. Sale price = $0.96 × 100 = $96 Damian should charge $96 for the bowl.
STAGE 3: CHECK THE SOLUTION	
W S P C CHECK	75% of $96 = $72. $96 is correct.
STAGE 4: PRESENT THE SOLUTION	
W S P C PRESENT	Damian should charge $96 for the bowl if he wants to receive $72.

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2 Test yourself

What's the price?

ANSWERS p. 433

1 EXERCISE 2.1 Round:

a 73.42 to the nearest whole number

b 624.625 to one decimal place

c 1.449 to one decimal place

d 13.836 to 2 decimal places

2 EXERCISE 2.1 What length is 25% shorter than 36 cm?

3 EXERCISE 2.1 Use leading-digit approximation to estimate the value of each expression.

a 79.32×104 **b** $9724 - 398$ **c** $\$495 \times 21$

4 EXERCISE 2.1 What starting length do I need to have 45 cm after a 25% decrease?

5 EXERCISE 2.2 Ella is designing a handbag that is 41 cm wide and 8 cm deep. Use the formula:

$$\text{number of stitches required} = \frac{92}{66} \times \text{length required}$$

to calculate the number of stitches required for:

a the width of the handbag

b the depth of the handbag

c the perimeter of the handbag

6 EXERCISE 2.2 Pedro needs to order 22 clips, but they are only available in multiples of 8. How many will Pedro need to order?

7 EXERCISE 2.2 Which number is NOT a multiple of 6?

18 54 28 42

8 EXERCISE 2.2 Jamiela is using a pattern with a 4-stitch pattern repeat. She calculated that she needs 21.5 pattern repeats. How many stitches does the 0.5 represent?

9 EXERCISE 2.2 Round up 49 to the next multiple of 6.

10 EXERCISE 2.3 Cameron is going to order 10 zippers priced at $2.99 each. Without using a calculator, estimate the total cost.

11 EXERCISE 2.3 If Cameron orders 12 or more zippers, the price per zipper drops to $2.20. How much cheaper is it to order 12 zippers than 10?

12 EXERCISE 2.3 Calculate the value of each expression.

a $14 + 3 \times 5$ **b** $(27 - 9) \div (5 - 3)$ **c** 2×4^2

13 EXERCISE 2.4 The community craft centre charges 15% of the selling price of all items in the store.

a Calculate the amount the centre will receive from selling an item for $60.

b How much will the creator of the $60 item receive?

14 EXERCISE 2.4 Thomas made a wooden chopping board that he sold in his local community market. The board cost him $24 to make and he sold it for $40, but the market charged 20% of the price for the sale.

a Did Thomas make a profit or a loss?

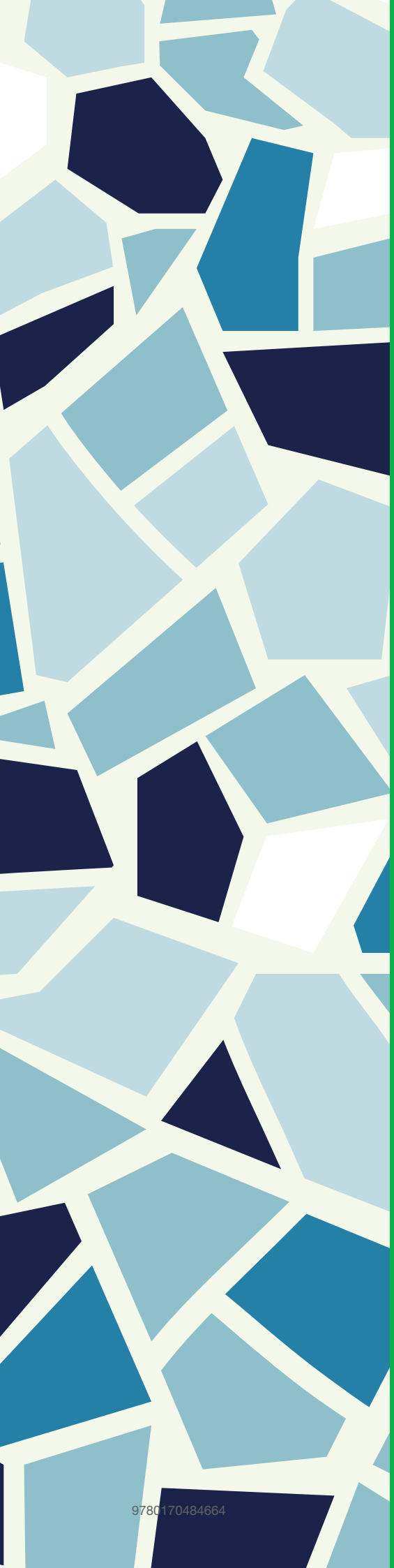
b How much was his profit or loss?

15 EXERCISE 2.4 Thomas is wondering what price to put on a serving platter he made. He wants to receive $60 for the platter after the market takes their 20%. What price should he put on the platter?

Chapter problem

Andrew and Robyn are renovating their bathroom. They want to have one feature wall with tiles that are NOT square. What shapes could the tiles be? Create some designs for their feature wall.

9780170484664



CHAPTER 3

MEASUREMENT, SCALES AND CHANCE: MEASUREMENT

THE SHAPE OF OUR WORLD

What will we do in this chapter?

- Draw and recognise the properties of 2D shapes and 3D solids
- Interpret 2D representations of 3D objects

How are we ever going to use this?

- Whenever we use shapes or solids in design
- To make models of 3D shapes
- To sketch 3D solids from different views
- In a career where design is an important element, for example, graphic design

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Syllabus coverage

UNIT 3, TOPIC 1: MEASUREMENT

Geometry

- Recognise the properties of standard polygons, e.g. number of vertices, straight edges and angles.
- Recognise the properties of prisms and pyramids, e.g. number of vertices, straight edges and flat faces.
- Interpret different forms of two-dimensional representations of three-dimensional objects, including nets of prisms and pyramids [complex].

Worksheets (8):

3.1 2D shapes
3.2 Sorting solids • 3D solids • Euler's rule • Enlarging a logo
3.3 Sketching solids • Isometric dot paper
3.4 Making solids
Test yourself Isometric dot paper

Puzzle (1):

3.1 Polygon puzzle

Terminology

2D	3D	cross-section	net
polygon	prism	pyramid	quadrilateral
solid	triangle		

3.1 2D shapes

2D shapes are flat and have 2 dimensions: length and width.

3D shapes are solids and have 3 dimensions: length, width and depth.

2D shapes surround us every day. The most common 2D shapes are triangles and quadrilaterals.

EXERCISE 3.1 2D shapes

ANSWERS p. 433

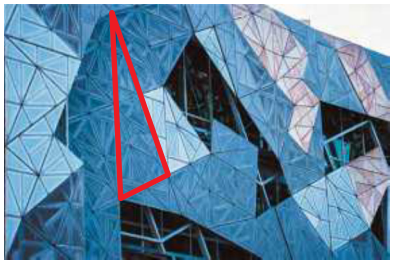
This exercise can be downloaded from Nelson MindTap as the worksheet '2D shapes'.

1 Triangles

Use the word list to complete the following paragraphs. You can use words from the word list more than once. You may like to cut out some triangles and use them to help you fill in the blanks.

Martin Lee/Alamy Stock Photo

Sueddeutsche Zeitung Photo/Alamy Stock Photo



Arnab Maity Photography/ Shutterstock.com

Word list: angle, angles, equilateral, scalene, sides, two, three.

All triangles have _____ sides and _____ vertices. We name triangles according to the lengths of their _____. A triangle with 3 sides the same length is called an _____ triangle. This triangle also has 3 equal _____. Each _____ is equal to 60°. An isosceles triangle has _____ sides and _____ angles equal. A _____ triangle has no sides the same length.

Mirek Hejnicki/Shutterstock.com

Alexander Chaikin/ Shutterstock.com

Jurij Maity/Shutterstock.com

The surface of this side of the pyramid is an equilateral triangle.

Word list: acute, obtuse, right, 180°

We can also name triangles according to the types of angles in them. A triangle with 3 angles less than 90° is called an _____-angled triangle. A triangle with one angle between 90° and 180° is called an _____-angled triangle. A right-angled triangle has one _____ angle.

The 3 angles of a triangle add up to _____.

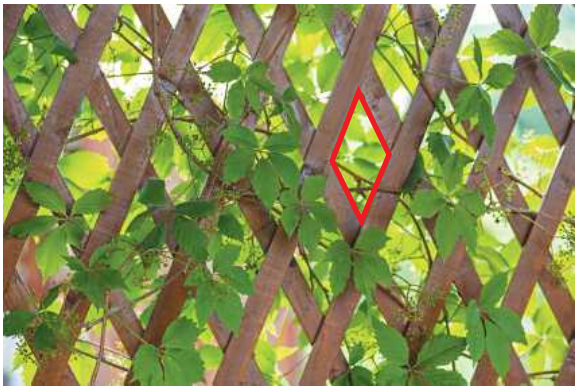
Puzzle
Polygon puzzle

Worksheet
2D shapes

2 The parallelogram and rhombus

Use the word list to complete the following paragraphs. You can use words from the word list more than once. You may like to cut out a parallelogram and a rhombus and use them to help you fill in the blanks.

Christian Mueller/Shutterstock.com



Pavel L Photo and Video/Shutterstock.com

Word list: bisect, equal, four, opposite, parallel, 90°, rectangle, square, 360°.

Each of these shapes has ____ sides and _____ vertices. A parallelogram has both pairs of _____ sides equal in length. Each pair of opposite sides is also _____. The opposite pairs of angles are _____. The diagonals of a parallelogram cut each other in half. We say they _____ each other.

A rhombus has all of the features of a parallelogram. In addition, all four sides of the rhombus are _____. The diagonals bisect each other at an angle of _____.

A parallelogram looks like a _____ pushed over and the rhombus looks like a _____ pushed over.

The 4 angles in each of these quadrilaterals add up to _____.

3 The rectangle and square

Use the word list to complete the following paragraphs. You can use words from the word list more than once. You may like to cut out a rectangle and a square and use them to help you fill in the blanks.

Buro Pavel/Shutterstock.com

Fad82/Shutterstock.com

Word list: diagonals, equal, parallelogram, rhombus, 90°

A rectangle has all the features of a _____. However, in a rectangle all the angles are _____ and the diagonals are _____ lengths.

A square has all the features of a _____. Also, the sides are all _____. The angles are all _____. The _____ are the same length.

4 Other quadrilaterals

Use the word list to complete the following paragraphs. You can use words from the word list more than once.

RTimages/Shutterstock.com

Hurst Photo/Shutterstock.com

Word list: diagonals, equal, parallel, quadrilateral, 90°, 360°.

Any shape with 4 sides is called a _____. The trapezium is a quadrilateral with one pair of opposite sides _____. A kite is a bit different. The 'top' pair of sides are _____ and the 'bottom' pair of sides are also _____. The angles at the 'sides' are _____. The _____ intersect at an angle of _____.

In all quadrilaterals the 4 angles add up to _____.

5 More polygons

Use the word list to complete the following paragraphs. You can use words from the word list more than once.

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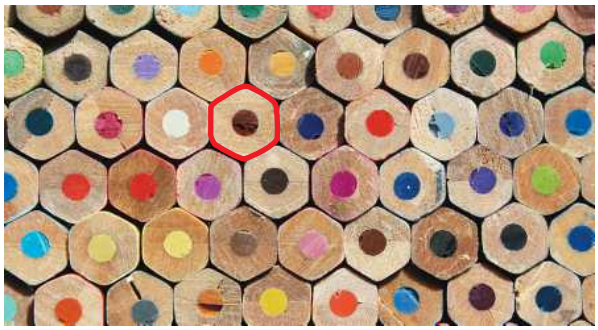
Irin-k/Shutterstock.com

Koosen/Shutterstock.com

AAP Image/www.spaceimaging.com



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Word list: decagon, hexagon, number, octagon, pentagon, quadrilaterals, regular.

A polygon is a shape with straight sides. We use the _____ of sides they have to name each polygon. We have already looked at triangles and _____ in detail. We have the following names for polygons with more than 4 sides:

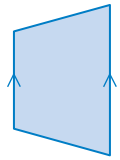
Name	Number of sides
	5
	6
Heptagon	7
	8
Nonagon	9
	10
Undecagon	11
Dodecagon	12

A rhombus has 4 equal sides but **not** 4 equal angles, so it is **not** a **regular polygon**.

When all the sides and angles of a polygon are equal, it is called a _____ polygon.
The equilateral triangle and the square are _____ polygons.

6 Name each shape.

a



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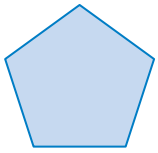

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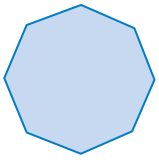

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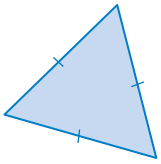

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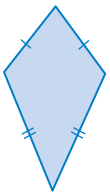

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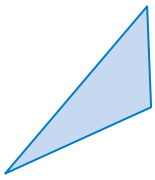

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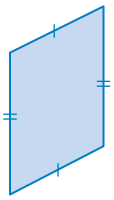

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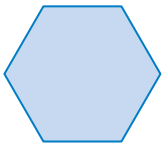

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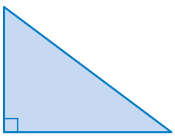

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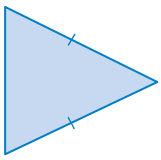

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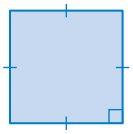

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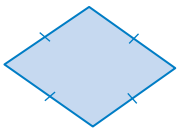


7 Is each statement TRUE or FALSE? PS

a An isosceles triangle must be acute-angled.

b A square is a regular quadrilateral.

c An obtuse-angled triangle must also have acute angles.

d A rhombus is a special parallelogram.

e An acute-angled triangle cannot be isosceles.

f A rectangle is a special trapezium.

g An equilateral triangle must be acute-angled.

h A trapezium must have 2 equal sides.

Investigation | Road signs

Road signs in Australia are based on a number of geometrical shapes. Use the internet to research the following questions. Google Images may be helpful.

1 The regular octagon is used for only one sign. What sign is it? Draw or copy an example.
2 What signs use the equilateral triangle? Draw or copy an example.
3 Many signs are rectangular. Name some of the signs that use rectangles and draw or copy some examples.
4 Warning signs are usually black on yellow. Why are those colours used? What shape are they? Draw or copy an example.
5 Some signs have the shape on the right. What sort of shape is it? Draw or copy an example.

Present your findings using a video or presentation software.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

3.2 3D solids

Worksheets
Sorting solids
3D solids

We live in a 3D world. Everywhere around us we see 3D solids – buildings, cars, supermarket items.

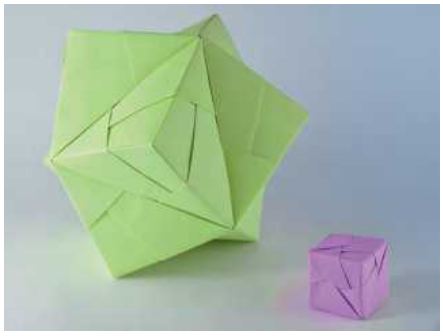
EXERCISE 3.2 3D solids

ANSWERS p. 434

This exercise can be downloaded from Nelson MindTap as the worksheet '3D solids'.

Use the word list to complete each paragraph.

1 Polyhedrons



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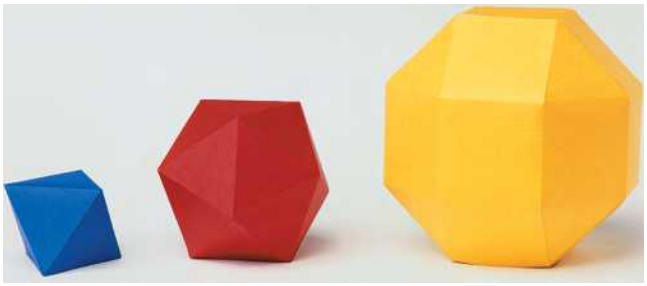


Dorling Kindersley ltd/Alamy Stock Photo

Word list

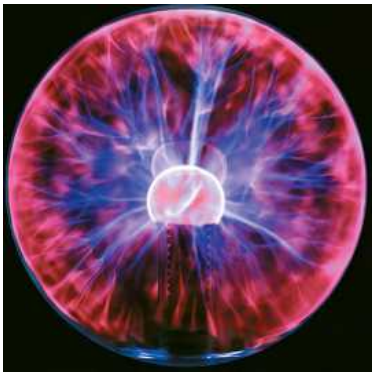
edge	flat	hexahedron	pentahedron
prism	pyramid	rectangular	vertex

Solids can have flat faces and curved faces. Solids that only have _____ faces are called polyhedrons. Some of the common polyhedrons are the _____ prism, the triangular _____ and the square _____. Polyhedrons have similar names to polygons. We call a solid with 5 faces a _____; a solid with 6 faces is called a _____. When the faces of a polyhedron meet, they form an _____. When 3 or more edges meet, they form a _____.

2 Solids with curved surfaces

iStock.com/Enviromantic

David Moore/Alamy Stock Photo



WHITE RABBIT83/Shutterstock.com

Word list

circle cone curved flat rectangle sphere

Some solids are not polyhedrons because they have _____ faces. A solid that has only one curved face is the _____. The cylinder has 2 _____ faces and one _____ face. When we flatten the curved face, it is a _____. A _____ has one flat face and one curved face. When we flatten the curved section, it is a sector of a _____.

3 Copy and complete this table.

Solid	Number of faces	Shapes of faces	Number of identical faces
Cube			
Cylinder			
Square pyramid			
Triangular prism			
Rectangular prism			
Cone			
Triangular pyramid			

4 Prisms and pyramids

Prisms and pyramids are special types of polyhedrons.

Paop/Dreamstime LLC

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Tobias Arhelger/Shutterstock.com

Word list

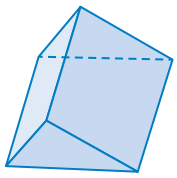
bottom	cross-section	end	pointed	rectangle
shape	size	square	triangle	

A prism has the same _____ from one end to the other. Both ends are the same _____. Prisms take their names from the shape at each _____. A rectangular prism has a _____ at each end and a triangular prism has a _____ at each end.

A pyramid has a _____ top, called the apex. The shape at the _____ of the pyramid gives the pyramid its name. A square pyramid has a _____ at the bottom. A pyramid's _____ is NOT the same from the bottom to the top. Each cross-section is the same shape but not the same _____.

5 Name each solid.

a



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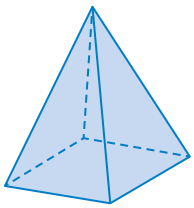

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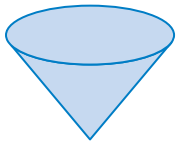

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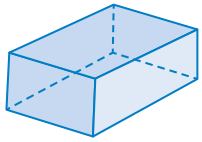

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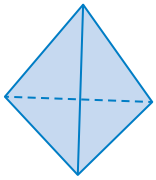

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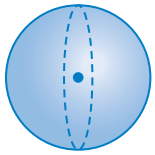

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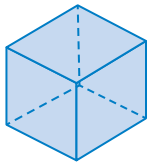


6 Name each solid shown in these pictures.

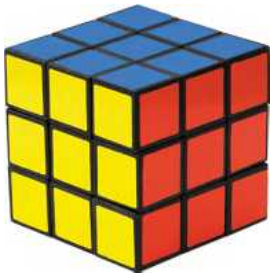
a

Martin Lee/Alamy Stock Photo

b

M. Unal Ozmen/Shutterstock.com

c



PeterVrabel/Shutterstock.com

d

Greg Pease/Getty Images

e

SUJIN SAENSING/Shutterstock.com

f

PeterG/Shutterstock.com

7 For all polyhedrons, there is a relationship between the number of faces, vertices and edges. This was discovered by the Swiss mathematician Leonard Euler.

Worksheet
Euler's rule

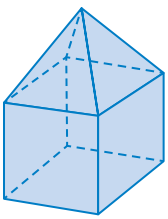
a Complete this table by counting the faces, vertices and edges. Use the diagrams in previous questions to help you.

Solid	Number of Faces (F)	Number of Vertices (V)	Number of edges (E)	F + V
Square pyramid				
Rectangular prism				
Triangular prism				
Triangular pyramid				
Cube				
Trapezoidal prism				
Hexagonal prism				
Square pyramid				
Pentagonal pyramid				

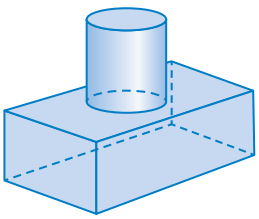
b Describe in your own words the relationship between the number of faces, vertices and edges. C PS

8 Each composite solid below is made up of 2 basic solids. Name the 2 solids that have been combined.

a



b



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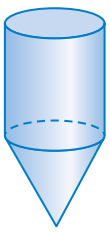

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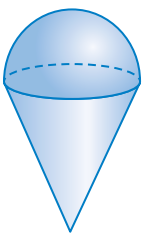

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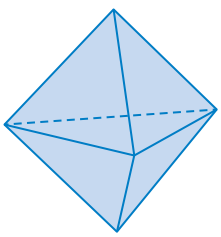
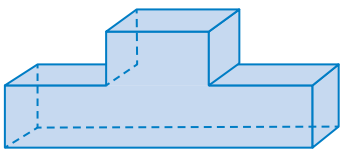


f



9 Is each statement TRUE or FALSE? PS

a A triangular prism has a triangle for its cross-section.

b A cylinder is a polyhedron.

c A rectangular prism is a hexahedron.

d A cone has 2 faces.

e A cube and a square pyramid each has 6 faces.

f A pyramid has the same cross-section from bottom to top.

Investigation — Logos and buildings

1 Find 5 examples of company logos.
- Present each logo and state what shapes have been used to create the logo.

2 Find 5 examples of buildings or bridges or sculptures.
- Present each example and state what solids have been used.
- Identify what shapes are on the surface of the solids you have chosen.

Present your findings as a poster or using presentation software.

Worksheet
Enlarging a logo

Investigation — Stacking and packaging

1 Look around your classroom. Why are bricks the shape of rectangular prisms? Why is the room a rectangular prism? What shape are most books? Why do you think this is?

2 Imagine you are in a supermarket. What solids are used for packaging? Why do you think manufacturers use different shapes? How are items stacked on the shelves? What part does packaging play in this?

3 Research the hexagon shape bees use to make honeycomb and why they use it.

StudioSmart/Shutterstock.com

4 Find other 3D structures in your environment. What solid shapes have been used? Why might this be so? If you can't think of any, use the internet to find some examples of different structures.

Worksheets
Sketching solids

Isometric dot paper

3.3 Drawing solids

Solid shapes can be difficult to draw when you are completing design work. This exercise will give you some ideas on how to sketch some common solids.

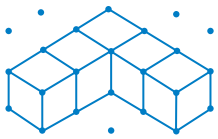
EXERCISE 3.3 Drawing solids

ANSWERS p. 434

1 Copy each diagram on isometric dot paper, which can be downloaded from Nelson MindTap as the worksheet 'Isometric dot paper'.

a

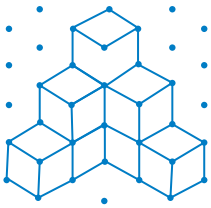
b



c

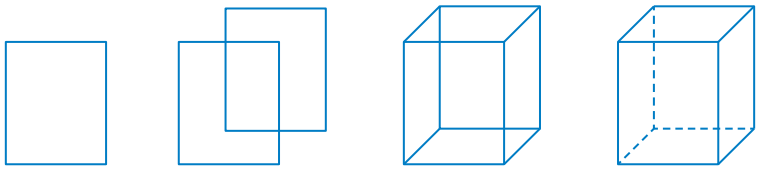
d

e



f

2 Sketch a rectangular prism using these diagrams and instructions.



a Draw a rectangle that will be the front face of the prism.

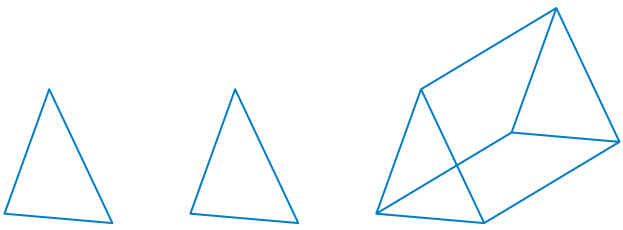
b Draw another rectangle the same size but position it slightly above and to the right of the first rectangle.

c Join the matching vertices (corners).

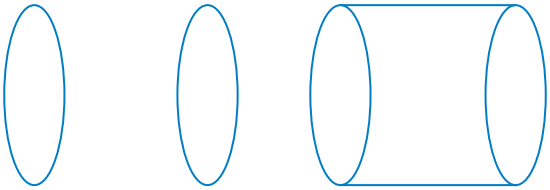
d Make the outside edges of the prism darker and use lighter or dotted lines for the inside edges.

3 Draw each solid using the method shown in Question **2**.

a triangular prism



b cylinder



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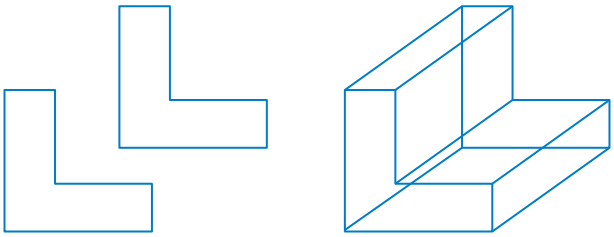
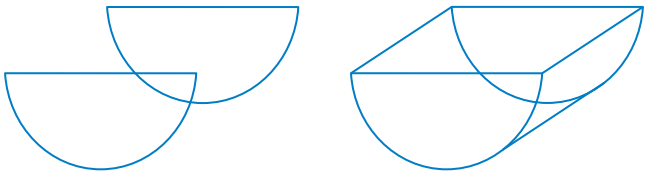
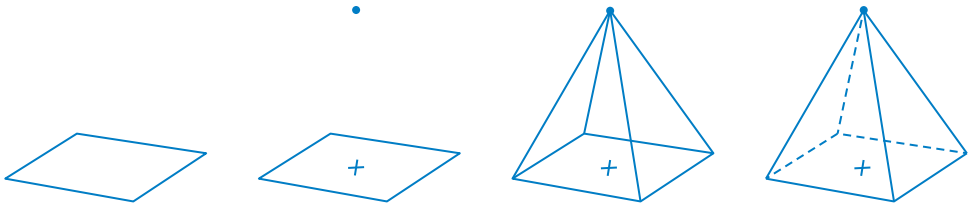
c L-shaped prism



d half-cylinder



4 Sketch a rectangular pyramid using these diagrams and instructions.



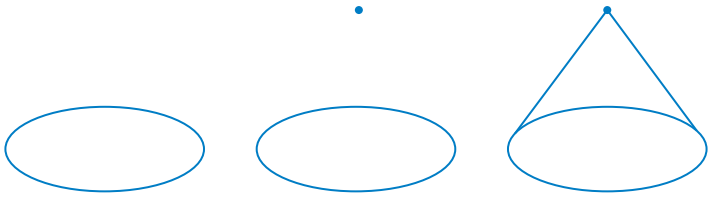
a Draw a parallelogram for the rectangular base.

b Determine the centre of the parallelogram (where the diagonals cross) and place a dot above this position: this dot will be the top of the pyramid (called the apex).

c Join the 4 vertices of the parallelogram to the dot.

d Make the outside edges of the pyramid darker and the inside edges lighter or dotted.

5 Sketch a cone using these diagrams and instructions.



a Draw an oval for the circular base.

b Draw a dot above the centre of the oval.

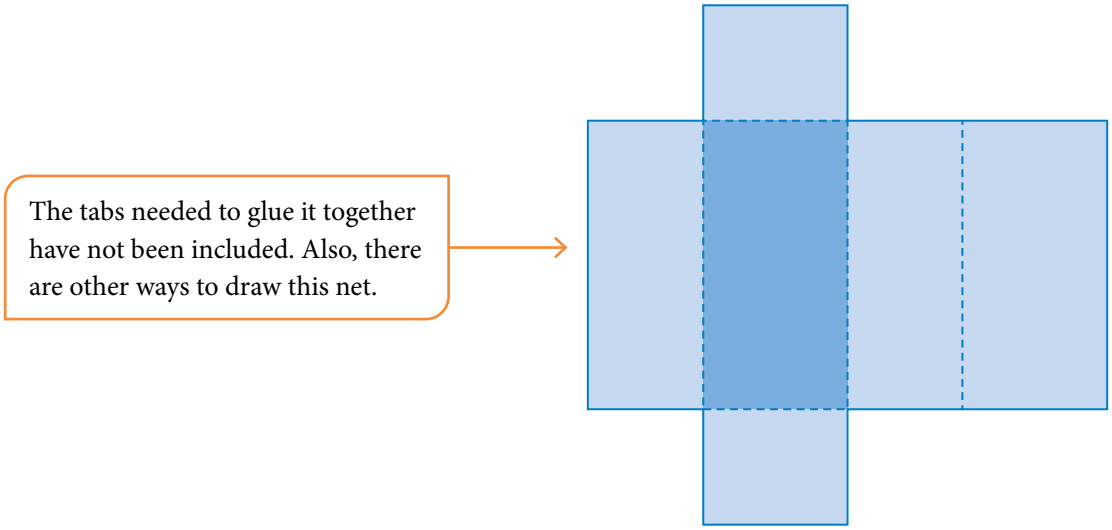
c Join the dot to the oval.

Worksheet
Making solids

3.4 From 3D to 2D*

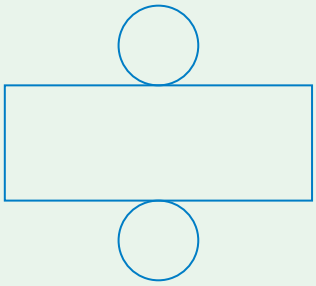
Nets of solids

A cardboard box starts out as a flat piece of cardboard. We cut out a particular shape and fold it to make the box. The 2D shape we cut out is called the **net** of a solid. For the box, the net would look like this:



EXAMPLE 1 Net of a solid

Identify the solid shape this net would make.



Steps	Working
The rectangle would curve around to match the circles.	This is the net of a cylinder.

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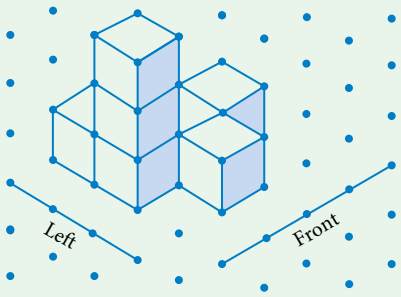
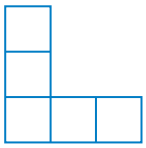
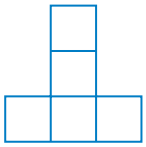
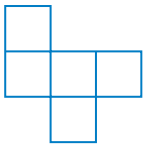
Different views of solids

You can also draw a solid from different perspectives (points of view). You can draw what you would see when looking from the front or the top or one of the sides.

EXAMPLE 2 Views of solids

For this solid, draw the front, left and top views.



Steps	Working
Imagine you are standing in front of the solid – this is what you would see.	 Front view
Imagine you are standing on the left of the solid – this is what you would see.	 Left view
Looking down at the solid from above – this is what you would see.	 Top view

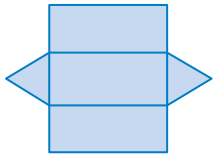
If you have trouble visualising these solids, make them out of centicubes and move them around.

EXERCISE 3.4 From 3D to 2D*

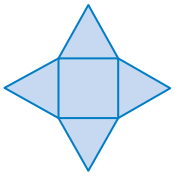
ANSWERS p. 434

1 EXAMPLE 1 Match each net in **a** to **f** with the solid it makes in **A** to **F**. Write the name of each solid.

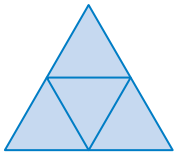
a



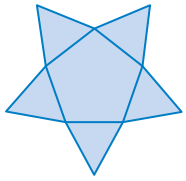
b



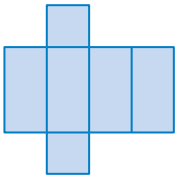
c



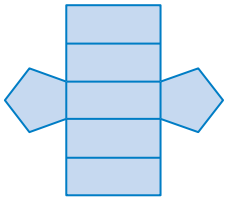
d



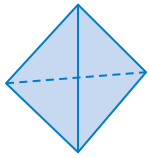
e



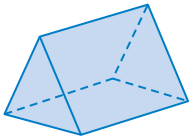
f



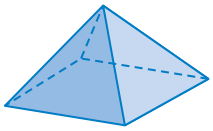
A



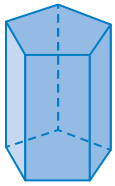
B



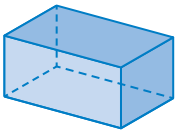
C



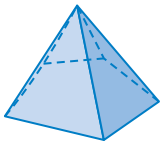
D



E

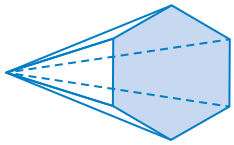


F

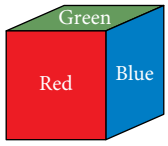


2 **a** What is the name of this solid?

b Draw the net of this solid.

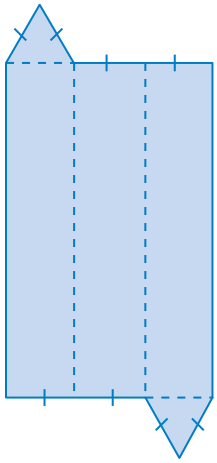


3 Draw the net of this cube, showing the correct positions of the 3 coloured faces shown.

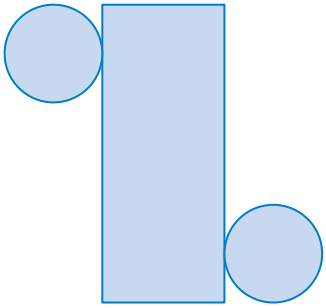


4 For each net, state what solid it would make.

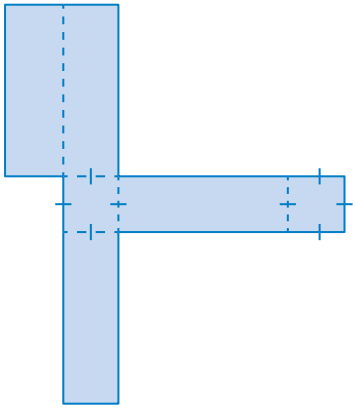
a



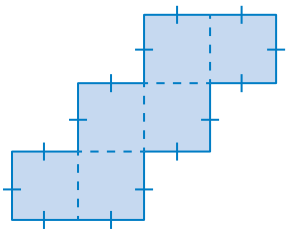
b



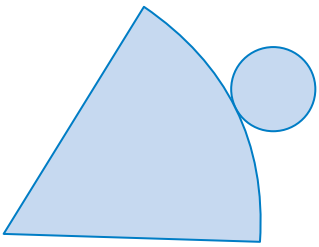
c



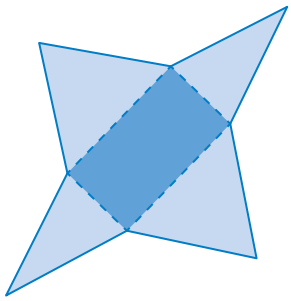
d



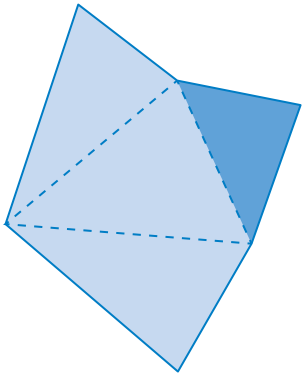
e



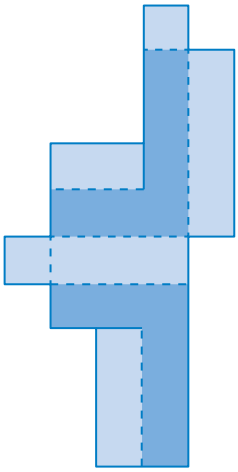
f



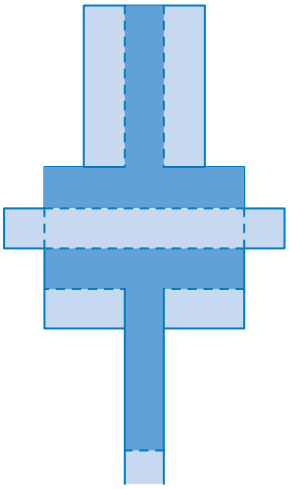
g



h

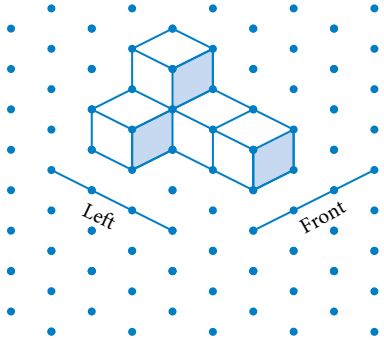


i



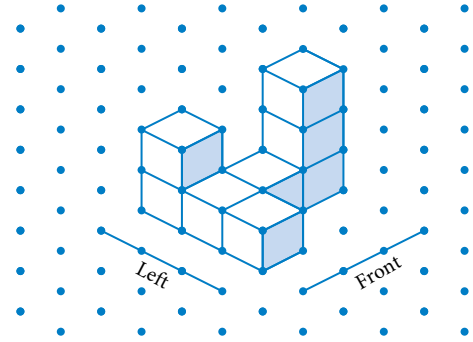
5 EXAMPLE 2 For each solid, draw each view stated.

a



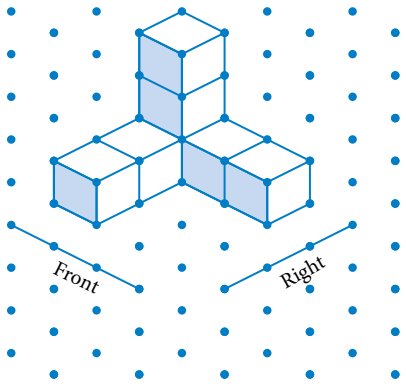
i front view
ii left view
iii top view

b



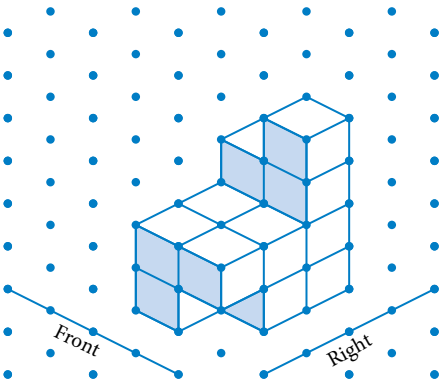
i left view
ii back view
iii top view

c



i front view
ii right view
iii top view

d



i back view
ii right view
iii top view

6 Draw the top and front view of each object. PS

a

Maxx-Studio/ Shutterstock.com

b

Vereshchagin Dmitry/ Shutterstock.com

c

Olga Kovalenko/ Shutterstock.com

d

Africa Studio/Shutterstock.com

7 Sketch the front, left and right view of this children's cubby house. PS

© Sue Thomson

8 Sketch the front, left and 'bird's eye' view of this small country church. PS

'Bird's eye view' means the top view.

© Sue Thomson

Keyword activity

DEFINITION MATCH

ANSWERS p. 435

Match each word with its definition

Words		Definitions	
1	net	**A**	A solid with all flat faces
2	polyhedron	**B**	Describes an object that has length, width and height
3	prism	**C**	A solid that comes to a point
4	pyramid	**D**	Describes an object that has length and width only
5	quadrilateral	**E**	A solid whose cross-section is the same from end to end
6	3D	**F**	A 4-sided shape, where all sides are straight
7	triangle	**G**	What you get when you “flatten” a solid into its faces
8	2D	**H**	A 3-sided shape, where all sides are straight

Solution to the chapter problem

Problem

Andrew and Robyn are renovating their bathroom. They want to have one feature wall with tiles that are NOT square. What shapes could the tiles be? Create some designs for their feature wall.

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

- Work out what shapes can be used
- Draw some designs

STAGE 2: SOLVE THE PROBLEM

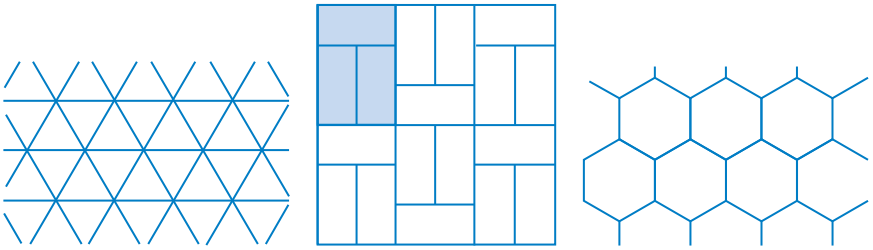
SOLVE

What shapes?

The shapes will need to fit together to cover the wall.

We could use triangles, rectangles, parallelograms, other quadrilaterals or even hexagons. However tiles usually come as triangles, rectangles, squares or hexagons. So we would probably use a triangular, rectangular or hexagonal tile for the feature wall.

Designs



Many other designs are possible if we mix the shapes together.

Andrew and Robyn could search online to find a design they like.

STAGE 3: CHECK THE SOLUTION

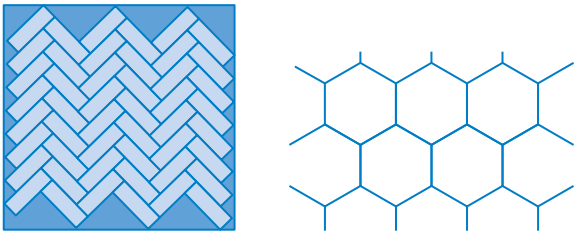
CHECK

We have answered both parts of the question and we have used realistic shapes for our tile design.

STAGE 4: PRESENT THE SOLUTION

PRESENT

Andrew and Robyn would probably use tiles that are rectangular or hexagonal. These are 2 possible designs:



3 Test yourself

The shape of our world

ANSWERS p. 435

1 EXERCISE 3.1 Draw a neat sketch of each 2D shape.

a triangle that is right-angled and isosceles

b hexagon

c quadrilateral with 4 right angles

d triangle that is scalene and obtuse-angled

e quadrilateral whose diagonals cross at right angles

f octagon

2 EXERCISE 3.1 What polygon am I? There may be more than one answer.

a I have 3 sides and all of my angles are equal.

b I am a quadrilateral with both pairs of opposite sides parallel.

c I have 5 sides.

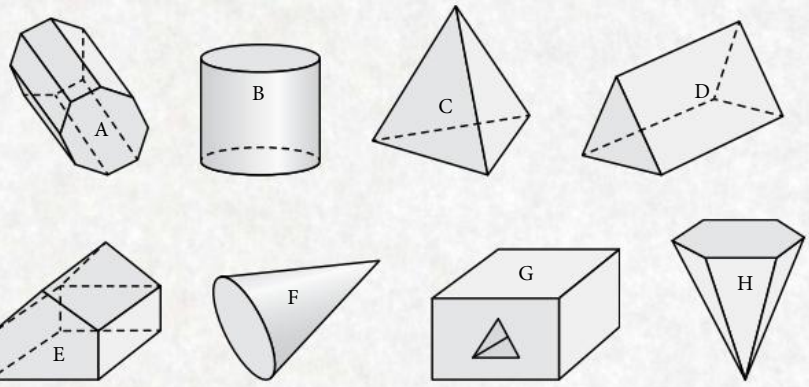
d I have 4 sides and my diagonals bisect one another.

e I am a quadrilateral with one pair of parallel sides.

f I have 3 sides. My angles are 60°, 80° and 40°.

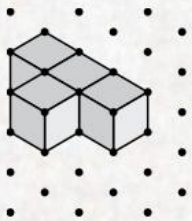
3 EXERCISE 3.2 Which of these shapes are:

a prisms? **b** pyramids? **c** solids with curved faces?

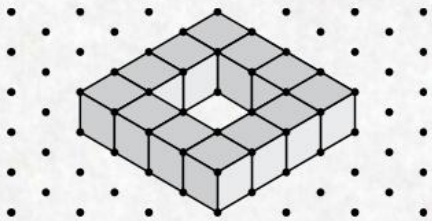


4 EXERCISE 3.3 Copy each diagram on isometric dot paper.

a



b



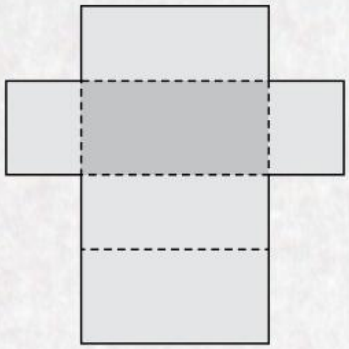
Worksheet
Isometric dot paper

5 EXERCISE 3.3 Draw each solid using the method shown in Exercise 3.3 Question **2**, p. 60.

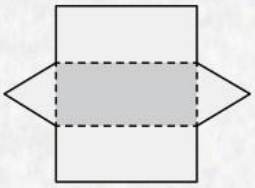
a rectangular prism **b** cylinder

6 EXERCISE 3.4 Name the solid that can be made from each net.

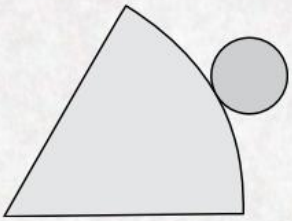
a



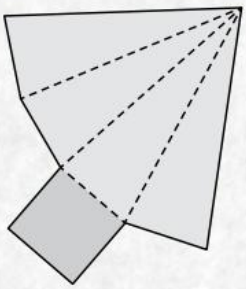
b



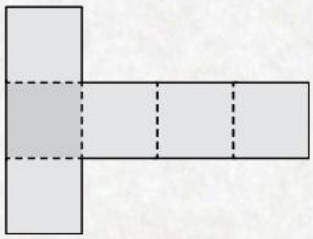
c



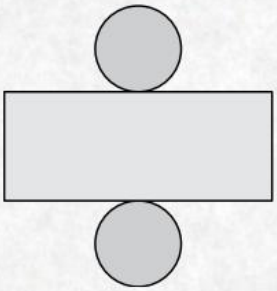
d



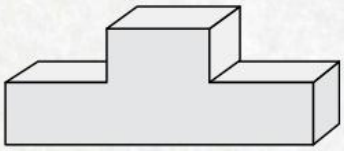
e



f



7 EXERCISE 3.4 This diagram shows a podium for awarding medals to competitors at a sports tournament.



Front

For the podium, draw:

a the right view **b** the front view **c** the top view

Chapter problem

Jess is making gift boxes to sell.
She plans to make 3 types of boxes:

- **Cube:** 15 cm long
- **Rectangular prism:** 20 cm by 10 cm by 8 cm
- **Cylinder:** radius 4.5 cm and height 34 cm

Should she charge the same for each gift box?

Give reasons for your answer.

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CHAPTER 4

MEASUREMENT, SCALES AND CHANCE: MEASUREMENT

ON THE SURFACE

What will we do in this chapter?

- Estimate and measure area
- Choose appropriate metric units for area and convert between them
- Use formulas to calculate area, including composite shapes
- Calculate the surface area of prisms, pyramids, cylinders, cones, spheres and composite solids

How are we ever going to use this?

- When we are doing craft and need to work out the amount of materials required
- If we are covering cushions or making quilts
- When we calculate the quantities of tiles, paint and carpet for building or renovating a home or office
- Many trades require knowledge of area and surface area

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Syllabus coverage

UNIT 3, TOPIC 1: MEASUREMENT

Converting units of measure

- Use metric units of area (square millimetres, square centimetres, square metres, square kilometres, hectares), their abbreviations (mm^2, cm^2, m^2, km^2, ha) conversions between them, and appropriate choices of units.

Area measure

- Estimate areas of different shapes
- Calculate areas of standard shapes, triangles, parallelograms and circles.
  - Triangle $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height
  - Parallelogram: $A = bh$ where b is base length and h is perpendicular height
  - Circle: $A = \pi r^2$ where r is radius
- Calculate areas of trapeziums and sectors [complex].
  - trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are the parallel lengths and h is perpendicular height.
  - sector: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius.
- Calculate areas of composite figures by decomposing them into standard shapes [complex].
- Calculate surface areas of prisms and cylinders [complex].
  - cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height.
- Calculate surface areas of pyramids and cones [complex].
  - cone: $S = \pi rs + \pi r^2$ where r is radius and s is slant height.
- Calculate surface areas of composite shapes and spheres [complex].
  - sphere: $S = 4\pi r^2$ where r is radius.

Terminology

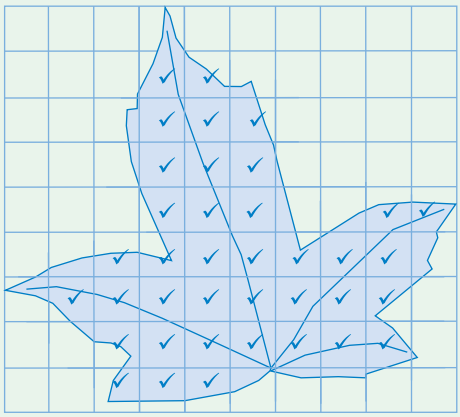
area
base
central angle
circle
composite
cone
cylinder
hemisphere
parallel
parallelogram
perpendicular height
prism
pyramid
radius
regular
sector
slant height
sphere
surface area
trapezium
triangle

4.1 Estimating area

The **area** of a shape is the amount of surface enclosed by the shape. Area is measured in square units. The number of small squares required to cover the shape is a measure of the shape's area.

EXAMPLE 1 Estimating area

Estimate the area of this leaf, which has been drawn on a 1 cm grid.



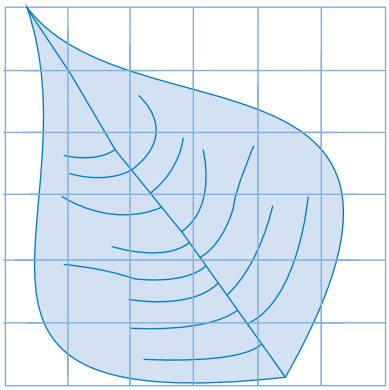
Steps	Working
Count how many squares it takes to cover the leaf. Count a square only if more than half of the square is on the leaf, and put a tick in it. The unit of area will be square cm or cm^2.	38 squares have ticks in them. The area of the leaf is about 38 cm^2.

EXERCISE 4.1 Estimating area

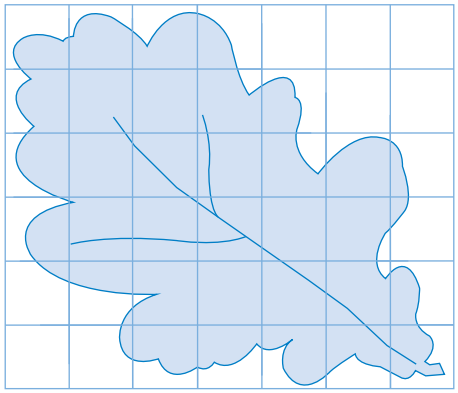
ANSWERS p. 435

1 EXAMPLE 1 Estimate the area of each leaf.

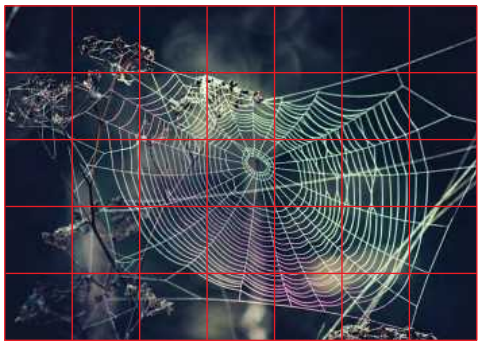
a



b

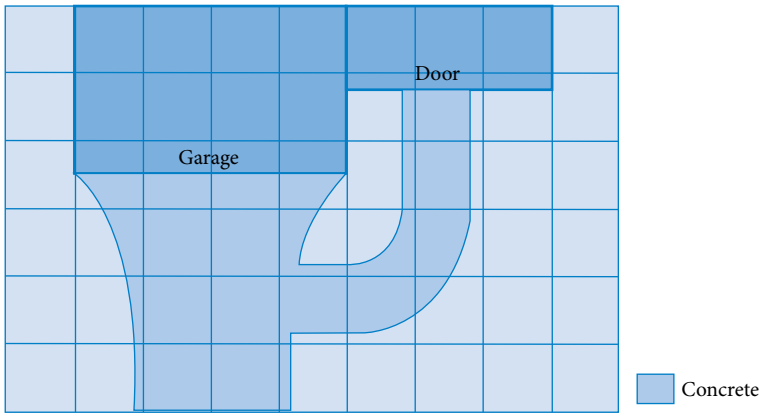


2 Many spiders make a new web every night. Each web takes about an hour to make. Estimate the area covered by the spider's web. Each square has sides of 1 cm.



SIMEON PHOTOS/Shutterstock.com

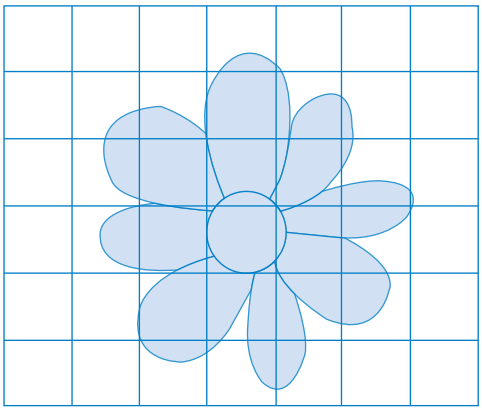
3 Jon is having the driveway and path at his new house covered in patterned concrete.
The concrete company will charge $65 per square metre. Each square on the plan represents 1 m^2.



a Estimate the area of the driveway and path.

b Approximately how much will the concrete company charge Jon?

4 Estimate the area of this flower. Each square represents 1 cm^2.



5 Find some leaves in your school grounds and estimate their area. What type of leaf has the largest area?

Worksheet
Areas of triangles

Practical activity Area of a triangle

In this activity you are going to investigate the relationship between the area of a triangle and the area of a rectangle.
You will need a pair of scissors and the 'Areas of triangles' worksheet, which can be downloaded from Nelson MindTap.

What you have to do

1 Cut out the pair of triangles in Triangle set A.

2 Put the triangles together to make a rectangle.

3 Determine the relationship between the length and height of the rectangle and the triangles.

4 Cut out the pair of triangles in Triangle set B.

5 Keep one triangle whole and cut the other triangle along the height into 2 smaller triangles.

6 Arrange the first triangle and the 2 pieces of the second triangle to make a rectangle.

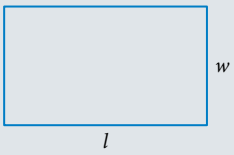
7 Determine the relationship between the length and height of the rectangle and the triangles.

4.2 Areas of rectangles and triangles

Area of a rectangle

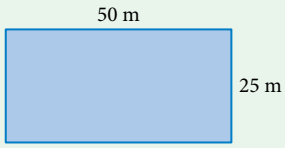
area = length × width

$A = l \times w$

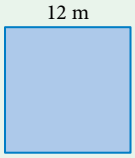


EXAMPLE 2 Area of rectangles and squares

a The floor of an Olympic swimming pool is 50 m long and 25 m wide. **Calculate** the area of the floor.



b Gymnastic floor competitions are held in a square with sides 12 m long. **Determine** the area of the square.

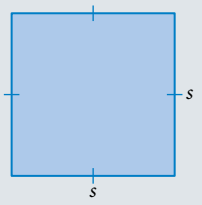


Steps	Working
a Multiply the length by the width. The units in the question are metres, so the answer for area is in square metres (m^2).	area = $l \times w$ = 50×25 = $1250\ m^2$
b The length and the width are both 12 m. To calculate the area, multiply 12 by 12.	area = $l \times w$ = 12×12 = $144\ m^2$

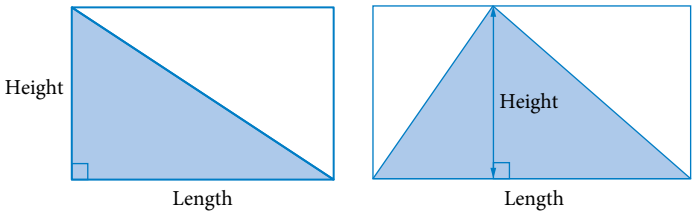
Area of a square

area = side × side

$A = s^2$



In the practical activity, you saw that 2 triangles can be placed together to make a rectangle. This means that the area of the triangle is half the area of the rectangle.

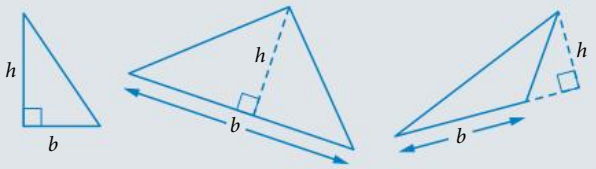


For a triangle, the length is called the **base** and the height is called the **perpendicular height** because it is at a right angle (90°) to the base.

Area of a triangle

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

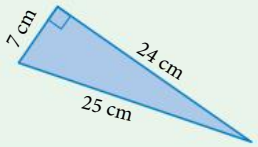
$$A = \frac{1}{2}bh$$



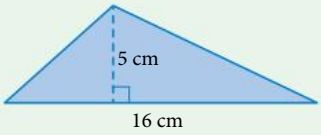
EXAMPLE 3 Area of a triangle

Calculate the area of each triangle.

a



b



Steps	Working
a The area of a triangle is $A = \frac{1}{2}bh$. The base and the height must be at 90° to each other. Base = 24 cm and height = 7 cm. The 25 cm length is not used in this calculation.	$\text{area} = \frac{1}{2} \times 24 \times 7$ $= 84\,\text{cm}^2$
b base = 16 cm, height = 5 cm	$\text{area} = \frac{1}{2} \times 16 \times 5$ $= 40\,\text{cm}^2$

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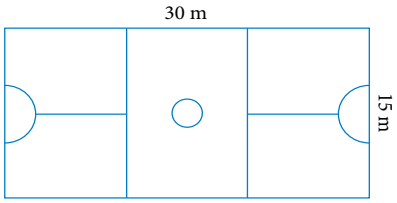
EXERCISE 4.2 Areas of rectangles and triangles

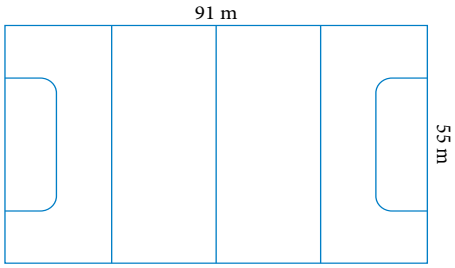
ANSWERS p. 435

1 EXAMPLE 2 Calculate the area of each court or field used for sport.

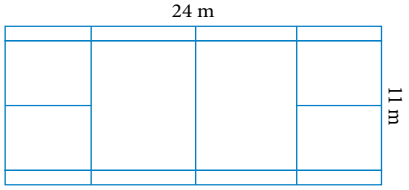
a Netball



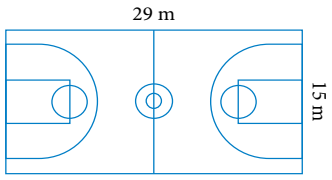
b Hockey



c Tennis

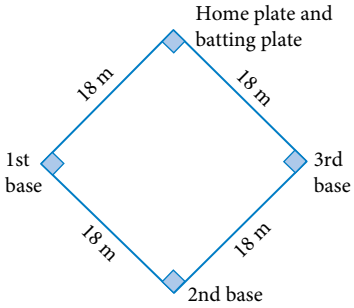


d Basketball



2 The cricket pitch in the school practice nets is 20 m long and 3.4 m wide. Calculate the area of the cricket pitch.

3 The infield of a junior softball ground is in the shape of a square with sides 18 m long. PS

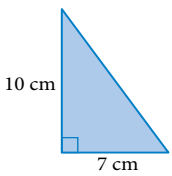


a What is the area of the infield?

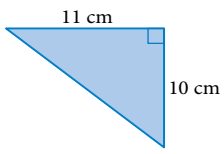
b A batter can score a home run if she hits the ball a long way and then runs through all the bases and back to the home plate. Kate hit a home run. How far did she run?

4 EXAMPLE 3 Calculate the area of each triangle.

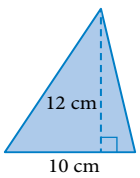
a



b



c



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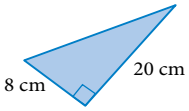

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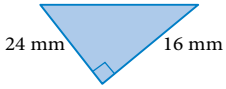

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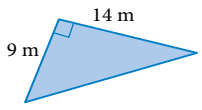
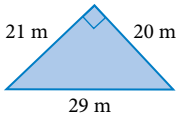


5 **a** Which 2 sides of the triangle on the right do you use in the formula $A = \frac{1}{2}bh$ to calculate the area of the triangle?



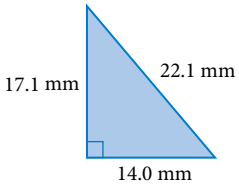
b What is the area of the triangle?

6 Nabil's driveway is in the shape of a rectangle, 15 m long and with an area of 45 m^2.
How wide is the driveway?

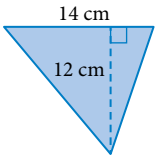
7 Calculate the area of each triangle.

> Make sure to use the 2 dimensions that are at right angles to each other.

a



b



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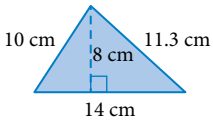
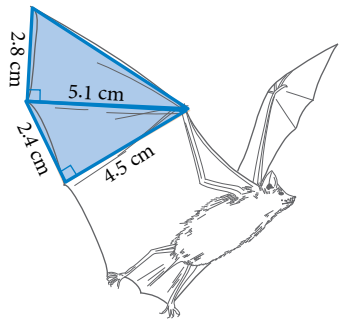


8 There are 81 known species of bats in Australia, and more than 1100 worldwide. The end sections of their wings are triangular. Calculate the area of each triangular section in this bat's wing.



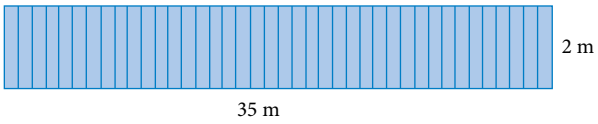
9 The area of a rectangle is 36 cm^2. PS

a What could the dimensions of the rectangle be? Give 2 possible sets of values.

b Calculate the perimeter of the rectangle for your suggested values in part **a**.

c What is the smallest the perimeter could be?

10 The diagram shows the dimensions of a fence that Dean is going to paint.
One litre of paint covers 12 m^2. PS



a Does Dean need to calculate the area or perimeter of the fence to work out how much paint he needs?

b How many litres of paint will Dean need to paint *both sides* of the fence?
Answer correct to the nearest tenth of a litre.

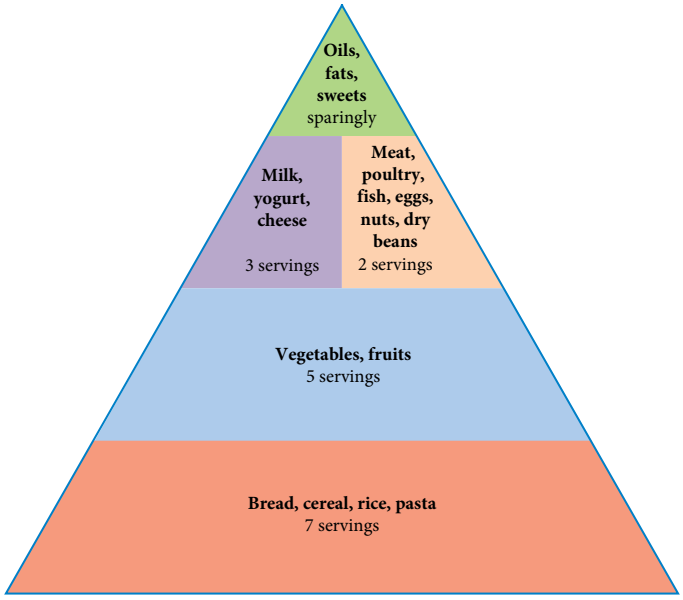
c How many 4 L tins of paint will he need to buy?

11 **a** Estimate the area of this stamp.

b Use a ruler to measure the stamp and then calculate the stamp's area.
How close was your approximation?

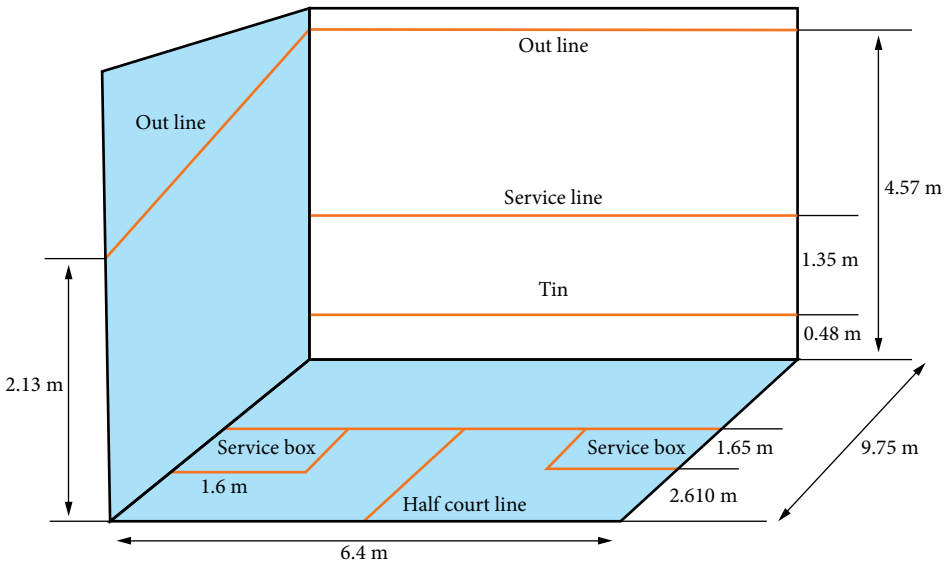
12 The height of a windsurfer's triangular sail is 3.1 m and its width is 1.9 m.
Calculate, correct to one decimal place, the area of the sail.

13 The healthy food triangle shows the proportions of different types of foods recommended for a healthy diet. Use a ruler and make any necessary measurements, then determine the area of the healthy food triangle.



14 Draw 2 possible right-angled triangles that each has an area of 24 cm^2, showing values for the base and height of the triangle. PS

Problem solving The lines on a squash court



The lines on the 2 side walls, the floor and the back wall of a squash court need painting. The lines will be 10 cm wide. One litre of line paint covers 11 m^2 of area.

What you have to do

1 Find the total length of the lines to be painted. The 'out lines' on the side walls are each 10.05 m long.

2 Find the area of the lines that has to be painted.

3 Find the amount of paint required.

Worksheet
Painting the fence

4.3 Renovating Grant's house

Grant is very excited. He has just bought his first house and is planning to do some renovations. As you work through this exercise, you will see how common perimeter and area calculations are in everyday life.

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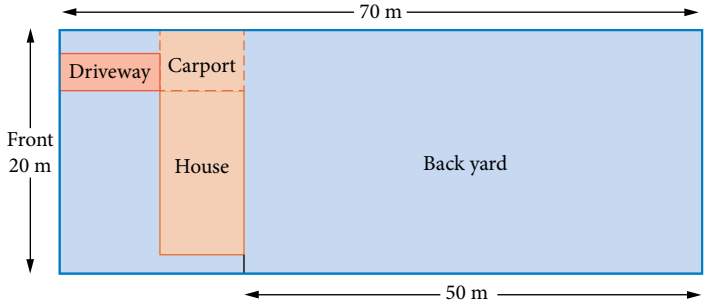
EXERCISE 4.3 Renovating Grant's house

ANSWERS p. 435

1 Grant's house is shown on this diagram.



a The block of land is 20 m wide and 70 m long. What is the area of the block of land?

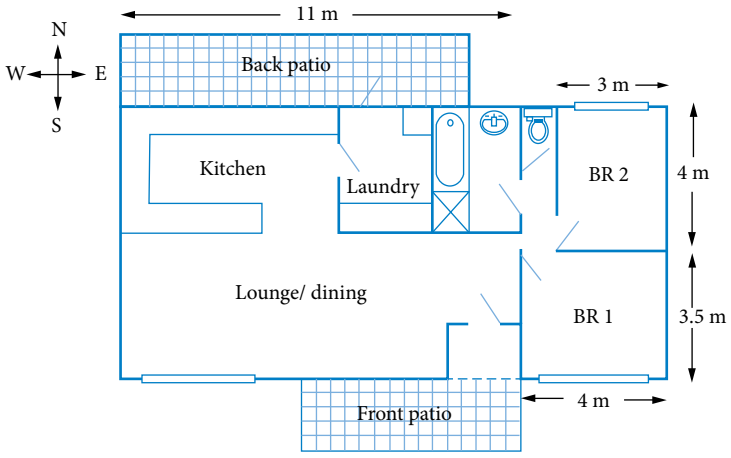
b The land is valued at $95 per square metre. Calculate the value of the block of land.

c The concrete in the driveway needs replacing. It is 3 m wide and 11 m long. Calculate its area.

d Grant would like to replace the concrete with paving tiles. The tiles cost $26 per square metre. How much will the tiles cost?

e The grass in the backyard needs fertilising. One bag of fertiliser covers 150 m^2. Will one bag be enough for the backyard? Give a reason for your answer.

2 The diagram shows the floor plan of Grant's house.



a How many bedrooms are in the house?

b How many doors are in the house?

c What are the dimensions of bedroom 2?

d When you're standing in the lounge room, looking out the window, in what compass direction (north, south, east or west) are you facing?

3 The skirting boards in bedroom 1 need replacing.

a Calculate the perimeter of bedroom

b The door is 1 m wide. How many metres of skirting board will be required?

> Skirting boards are the wooden boards that run around the room along the base of the walls.

4 When it rains, the gutter across the front of the house leaks.

a How long is the gutter across the front of the house?

b New guttering comes in 5 m lengths. How many lengths of gutter will Grant need to buy to replace the leaking gutter?

5 Grant is going to paint the walls in the lounge/dining room. He calculated that he needs to paint 84 m^2 of wall.

a How many square metres will the paint in one 4 L tin cover?

b Grant will need to apply two coats of paint. How many litres of paint will he need?

c If Grant buys the paint in 4 L tins, how many tins of paint will he need?

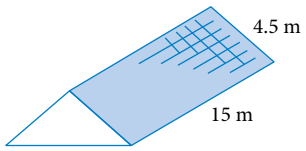
d The same paint is available in 4 L and 10 L tins. The 4 L tin costs $64 and the 10 L tin costs $110. What is the cheapest way for Grant to buy the paint that he needs?

6 Grant is going to put new carpet on the floors in the bedrooms. Carpet is sold in rolls 3.6 m wide.

a Explain why Grant will need 4 m of carpet for bedroom 1 and another 4 m for bedroom 2, if he lays the carpet without any joins.

b Including underlay and laying costs, the carpet costs $180 per metre. Calculate the cost of the carpet for the bedrooms.

7 The roof tiles need high-pressure steam cleaning. The cleaning company charges $13 per square metre. The diagram shows the dimensions of the roof.



a What shape are the 2 roof sections?

b What area is covered by the roof tiles?

c How much will the roof cleaning cost?

8 The towel rail in the bathroom is broken. The space for the rail is 86 cm long. Grant can buy towel rails that are 1 m long or 75 cm long.

a How much longer is the 1 m rail than the space in the bathroom?

b Do you think Grant should cut the longer rail or use the shorter rail? Give a reason for your answer.

9 Grant plans to put mirror tiles on one wall in the lounge/dining room to make the room look bigger. He plans to cover a square with area 3 m by 3 m. The tiles are 30 cm by 30 cm and they cost $5.40 each. Calculate the cost of the tiles.

10 Grant wants to buy a new bed that is 220 cm wide and 180 cm long. Is this size bed suitable for the house? Explain your answer. PS

Worksheet
Australian areas

4.4 Units of area

Area is measured in square units, which are based on the length units.

Area unit	The size of a square of length:	Approximately the size of:
square millimetre (mm^2)	1 mm (actual size: ▪)	
square centimetre (cm^2)	1 cm (actual size: ■)	a fingernail
square metre (m^2)	1 m	the floor of a large shower recess
hectare (ha)	100 m	the area bounded by an athletics track, or an international rugby pitch
square kilometre (km^2)	1 km	a theme park

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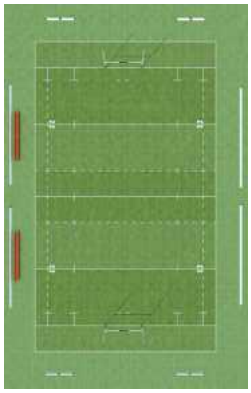
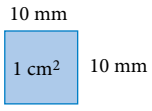
Foamfoto/Shutterstock.com

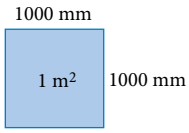


Archideaphoto/Alamy Stock Photo

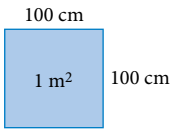
$1\text{ cm} = 10\text{ mm}$
$1\text{ cm}^2 = 10 \times 10\text{ mm}^2$
$= 100\text{ mm}^2$



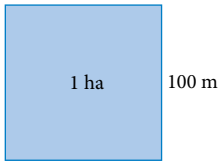
$1\text{ m} = 1000\text{ mm}$
$1\text{ m}^2 = 1000 \times 1000\text{ mm}^2$
$= 1\,000\,000\text{ mm}^2$



$1\text{ m} = 100\text{ cm}$
$1\text{ m}^2 = 100 \times 100\text{ cm}^2$
$= 10\,000\text{ cm}^2$



$1\text{ ha} = 100 \times 100\text{ m}^2$
$= 10\,000\text{ m}^2$



Similarly, $1\text{ km}^2 = 1\text{ km} \times 1\text{ km} = 1000\text{ m} \times 1000\text{ m} = 1\,000\,000\text{ m}^2$.

When converting area units, we have to convert the length unit twice. One conversion is for the length and the other is for the width. We *square* the simple linear conversion factor to get the area conversion factor.

For example, to change from m to cm, multiply by 100,

but to change from m^2 to cm^2, multiply by $100^2 = 10\,000$.

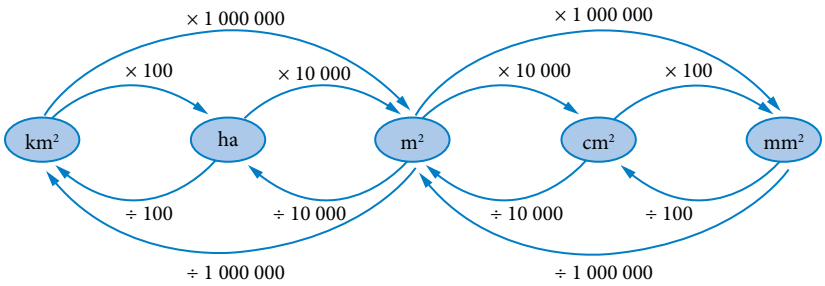
Units of area

$1\text{ cm}^2 = 10^2\text{ mm}^2 = 100\text{ mm}^2$
$1\text{ m}^2 = 100^2\text{ cm}^2 = 10\,000\text{ cm}^2$
$1\text{ m}^2 = 1000^2\text{ mm}^2 = 1\,000\,000\text{ mm}^2$
$1\text{ ha} = 100^2\text{ m}^2 = 10\,000\text{ m}^2$
$1\text{ km}^2 = 1000^2\text{ m}^2 = 1\,000\,000\text{ m}^2$

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.

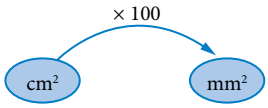
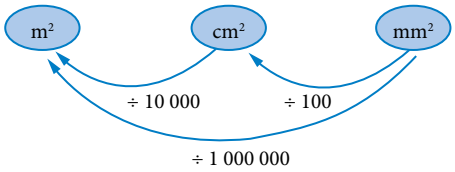
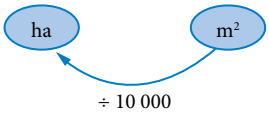
This diagram shows how to convert between units of area.



EXAMPLE 4 Converting between area units

Convert:

a 3 cm² to mm² **b** 4000 mm² to m² **c** 81 000 m² to ha

Steps	Working
a cm² to mm², large to small unit: × 100 	3 cm² = 3 × 100 mm² = 300 mm²
b mm² to m², small to large unit: ÷ 1 000 000 	4000 mm² = 4000 ÷ 1 000 000 m² = 0.004 m²
c m² to ha, small to large unit: ÷ 10 000 	81 000 m² = 81 000 ÷ 10 000 ha = 8.1 ha

EXERCISE 4.4 Units of area

ANSWERS p. 435

1 Select the best unit (km², ha, m², cm² or mm²) for measuring each area.

a a farm **b** the floor of a classroom **c** a shirt

d a football field **e** a sheet of paper **f** your eardrum

g Australia **h** a butterfly **i** K'gari (Fraser Island)

2 EXAMPLE 4 Copy and complete each conversion.

a 7.9 cm² = _____ mm² **b** 1 500 000 mm² = _____ m²

c 690 mm² = _____ cm² **d** 76 000 000 m² = _____ km²

e 865 000 cm² = _____ m² **f** 12 ha = _____ m²

g 0.32 km² = _____ m² **h** 4.5 m² = _____ cm²

i 0.75 m² = _____ mm² **j** 19 000 m² = _____ ha

3 Arrange these areas from smallest to largest:
$6.5\,m^2$, $25\,050\,000\,mm^2$ and $114\,000\,cm^2$.

To compare sizes, the measurements need to be in the same units.

4 Arrange these areas from largest to smallest: $990\,mm^2$, $54\,cm^2$ and $0.000\,032\,m^2$.

5 The centre of Adelaide is approximately a square bounded by streets called North, South, East and West Terraces. Its area is about $3\,521\,000\,m^2$. Express the area of this square in:

a hectares

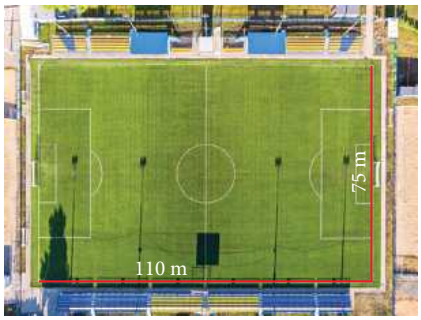
b square kilometres

6 **a** Calculate the area of this rugby league field in square metres.

b How much smaller than 1 ha is the area of this field?

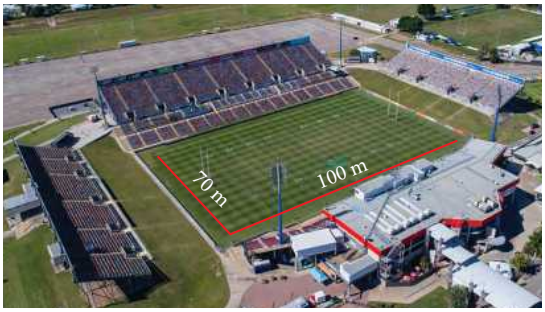
Hemis/Alamy Stock Photo

7 Which field below has an area closer to 1 ha? Justify your answer.



iStock.com/Mr_Twister

Soccer field



iStock.com/Cameron Laird Photography

Rugby union field

8 A large bushfire is burning out of control. Overnight it destroyed 24 000 ha of bush.

a Convert 2400 ha to m^2.

b Convert your answer from part **a** to km^2.

c The burnt bush is in the shape of a rectangle. What could the dimensions of the burnt area be?

9 Rahim is going to tile his verandah. The square tiles are 30 cm long.

a Calculate the area of one tile in cm^2.

b How many tiles cover 1 m^2? (answer to the nearest whole tile)

c Rahim's verandah is a rectangle 3 m wide by 8.4 m long. How many tiles will he need to cover the verandah?

d To allow for cutting and breakage, Rahim is going to order 5% more than the minimum number of tiles he requires. How many tiles should he order?

10 The area of the United Kingdom is 241 540 km^2 and the area of Queensland is 1 853 000 km^2. How many times bigger is Queensland than the United Kingdom? Express your answer correct to 2 decimal places. PS

11 Square miles are larger than square kilometres. 1 $mile^2 = 2.59\ km^2$

a The area of the surface of the Earth is 196 940 000 $miles^2$. Calculate the area of the surface of the Earth in km^2.

b The Earth's seas and oceans cover an area of 361 132 000 km^2. Express this amount in square miles.

Investigation Estimating area

It takes practice to become good at estimating. Complete these practical estimation activities to develop your group's estimation skills.

To complete these activities, each group will need:

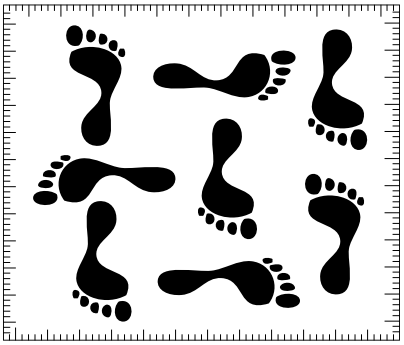
- 4 metre rulers or measuring tapes
- paper, pencils and scissors

Make a copy of this table:

Estimating area					
Names	Group members' estimates				Real measurement
Footprints in a square metre					
Handprints in a square metre					
Area of a car number plate in square centimetres					

Activity 1: Footprints in a square metre

Imagine you are going to cover a square metre with your footprints. How many footprints will fit inside 1 m^2 without any overlaps? Record your group's estimates on the record sheet.



How good was your estimate?

- Make a template of your foot. Put a piece of paper under your foot and trace around the outside. Use scissors to cut out the template. Make several templates.
- Place four 1-metre rulers on the floor to outline 1 m^2.
- Systematically place your foot template in the 1 m^2 and count the number required to cover the square.

Activity 2: Handprints in a square metre

Now imagine that you are going to cover a square metre with your handprints without any overlaps. How many handprints will you need?

Use a similar method to that for checking footprints in a square metre to check the accuracy of your group's handprint estimates.

Activity 3: The area of a car number plate

Estimate the area covered by a standard car number plate.

To check your estimate, measure the length and the height of an appropriate number plate, then use the formula Area = $l \times w$ to calculate the area.

Ingehogenbijl/Shutterstock.com

Activity 4: Laying grass

Keira is planning to use rolls of grass to cover her backyard that is 15.4 m wide by 12 m long. Each roll of grass is 50 cm wide and 2 m long.

a Draw a diagram to represent Keira's backyard.

b Calculate the area of Keira's backyard and the area covered by one roll of grass.

c How many rolls of grass will Keira need to buy to cover her backyard?

For part **c**, assume we can only buy whole rolls of grass.

d On the diagram you drew in part **a**, decide whether it will be better to lay the grass in rows or columns or both. Find the best way.

e Describe the size and shape of the pieces of grass that will be left over after Keira has finished covering her backyard.

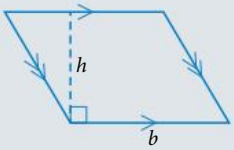
4.5 Areas of shapes

Areas

Area of a parallelogram

area = base (b) × perpendicular height (h)

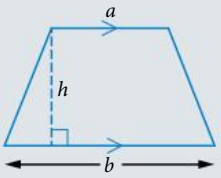
$A = bh$



Area of a trapezium

area = $\frac{1}{2}$ × sum of parallel sides × perpendicular height (h)

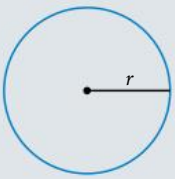
$A = \frac{1}{2}(a + b)h$



Area of a circle

area = $\pi \times (\text{radius})^2$

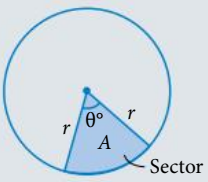
$A = \pi r^2$



Area of a sector*

area = $\frac{\text{central angle}}{360} \times \pi \times (\text{radius})^2$

$A = \frac{\theta}{360} \times \pi r^2$

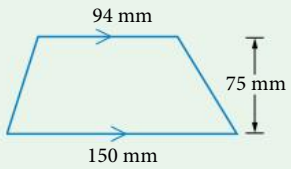


The sector area is a fraction of the area of the circle, and $\frac{\theta}{360}$ is the fraction because there are 360° in a circle.

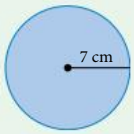
EXAMPLE 5 Area of trapeziums, circles and sectors

Calculate the area of each shape, correct to 2 decimal places where necessary.

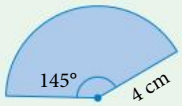
a



b



c



Steps	Working
a For a trapezium, $A = \frac{1}{2}(a+b)h$ $a = 94$, $b = 150$, $h = 75$	area $= \frac{1}{2} \times (94 + 150) \times 75$ $= 9150\,\text{mm}^2$
b For a circle, $A = \pi r^2$. $r = 7$	area $= \pi \times 7^2$ $= 153.9380\ldots$ $\approx 153.94\,\text{cm}^2$
c For a sector, $A = \frac{\theta}{360} \times \pi r^2$. $\theta = 145$, $r = 4$	area $= \frac{145}{360} \times \pi \times 4^2$ $= 20.2458\ldots$ $\approx 20.25\,\text{cm}^2$

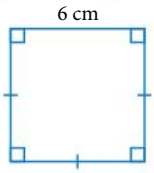
Video
Areas of trapeziums, circles and sectors

EXERCISE 4.5 Areas of shapes

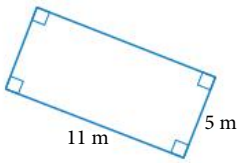
ANSWERS p. 436

1 EXAMPLE 5 Calculate the area of each quadrilateral.

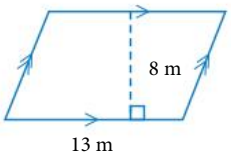
a



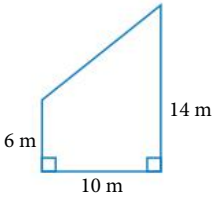
b



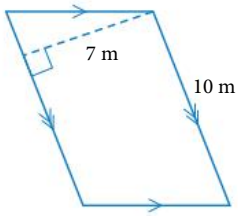
c



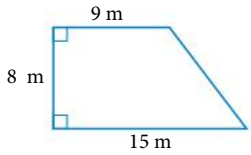
d



e

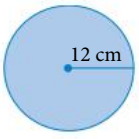


f

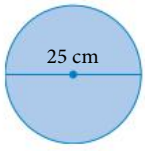


2 Find the area of each circle or sector, correct to 2 decimal places.

a



b



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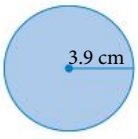

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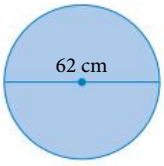

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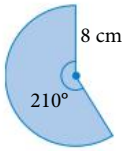

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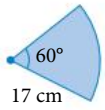

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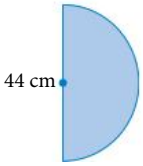

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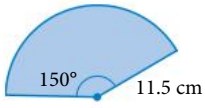
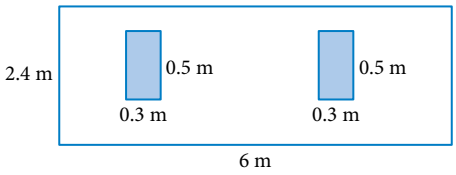


3 Phuong is a bricklayer. She uses 65 bricks to build 1 m² of wall.

a How many square metres of wall can Phuong build with 715 bricks?

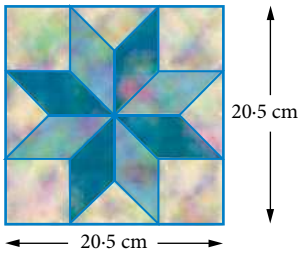
b Phuong is going to build this brick fence leaving 2 holes as requested by the customer.

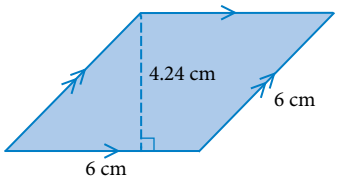
How many square metres of brickwork are in the fence?



c How many bricks will Phuong need to build the fence?

4 Rachel uses parallelograms to make a star pattern on a quilt. The star pattern is shown on the left, and one of the parallelograms shown on the right. PS





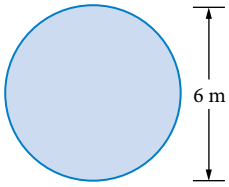
a Calculate the area of one parallelogram.

b What is the area covered by one star?

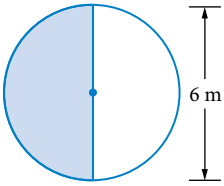
c How much of the area of the square is **not** covered by the star?

5 Calculate the area of each shaded region, correct to one decimal place.

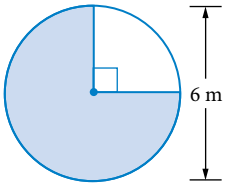
a



b



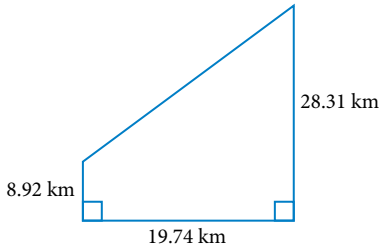
c



6 a Use leading-digit approximation to estimate the area of this trapezium.

b Calculate the area of the trapezium in km², correct to one decimal place.

c Calculate the percentage error involved in the leading-digit approximation. Answer to the nearest whole percentage.



7 A pizza is 30 cm in diameter.

a What is the area of the pizza, correct to 2 decimal places?

b The pizza is cut into 8 equal pieces. What is the area of each piece?

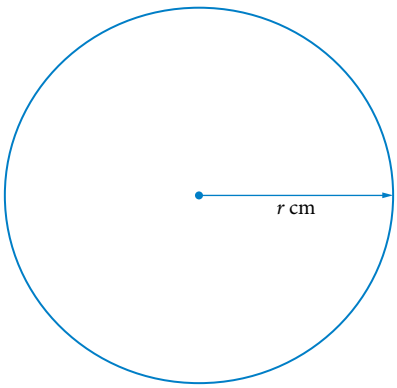
c Corey eats 3 slices and Anya eats 5 slices. How many more square centimetres of pizza does Anya eat?

Tobik/Shutterstock.com

8 During World War II in the 1940s, army barracks were built with a semicircular end with diameter 7 m. Find the area of tin used for one end of the building, correct to 2 decimal places.

Paul Briden/Shutterstock.com

9 This circle has an area of 36 cm^2. Find its radius, r cm, correct to 2 decimal places. PS



Worksheet Composite areas

Puzzle Area code puzzle

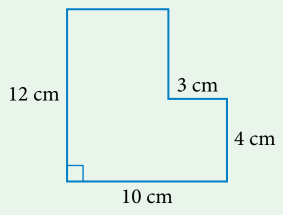
4.6 Areas of composite shapes*

Composite shapes are made up of smaller simpler shapes. We can find the areas of composite shapes by adding or subtracting the areas of simpler shapes.

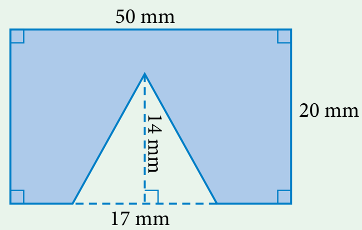
EXAMPLE 6 Areas of composite shapes 1

Determine the area of each shape.

a

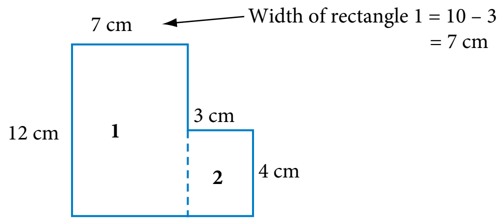


b



Steps | **Working**

a Divide the shape into 2 rectangles as shown.



Calculate the area of each rectangle and add them together.

> There is more than one way to divide this shape. All methods result in the same answer.

area of rectangle 1 $= 7 \times 12$
$= 84\text{ cm}^2$

area of rectangle 2 $= 3 \times 4$
$= 12\text{ cm}^2$

total area $= 84 + 12$
$= 96\text{ cm}^2$

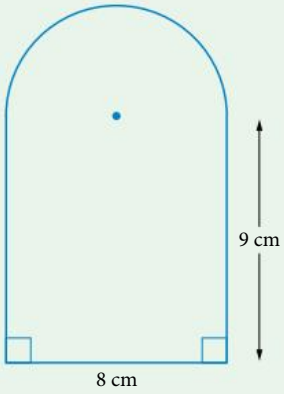
b Find the area of the rectangle and subtract the area of the triangle.

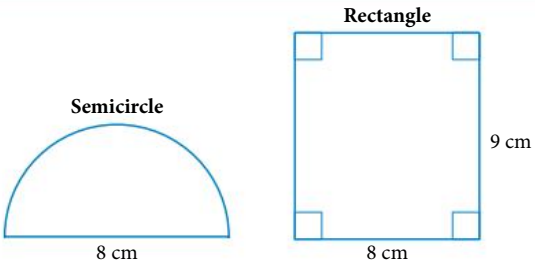
> Sometimes, it is easier to find the area by subtraction.

area $= 50 \times 20 - \frac{1}{2} \times 17 \times 14$
$= 881\text{ mm}^2$

EXAMPLE 7 Areas of composite shapes 2

Calculate the area of this shape, correct to 2 decimal places.



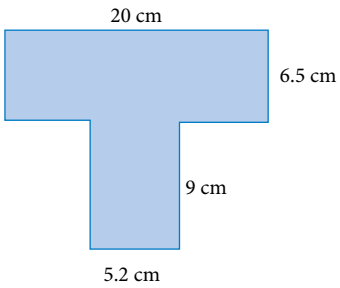
Steps	Working
This shape is a semicircle and a rectangle.	
Find the area of the semicircle first.	radius, $r = \frac{1}{2} \times 8 = 4$ area of semicircle $= \frac{1}{2} \times \pi r^2$ $= \frac{1}{2} \times \pi \times 4^2$ $= 25.1327 \ldots$
Find the area of the rectangle.	area of rectangle $= lw$ $= 8 \times 9$ $= 72$
Add both areas together.	total area $= 25.1327 \ldots + 72$ $= 97.1327 \ldots$ $\approx 97.13 \text{ cm}^2$

EXERCISE 4.6 Areas of composite shapes*

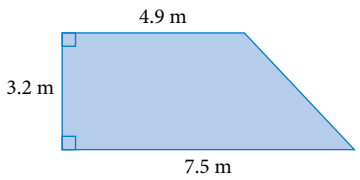
ANSWERS p. 436

1 EXAMPLE 6 Find the area of each shape, correct to 2 decimal places where necessary.

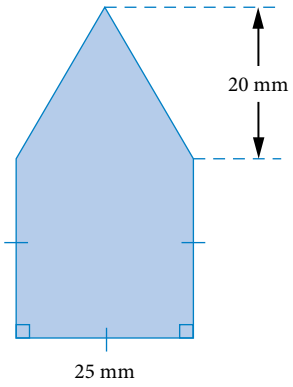
a



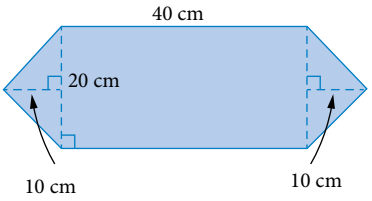
b



c

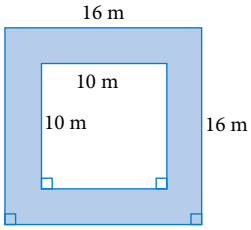


d

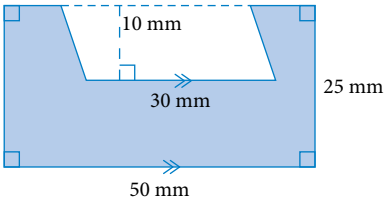


2 Calculate the area of each shaded region.

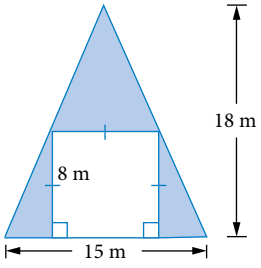
a



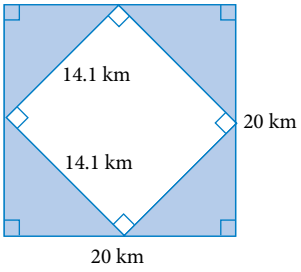
b



c

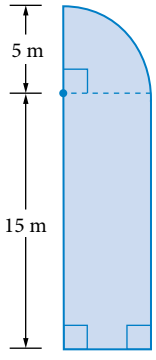


d

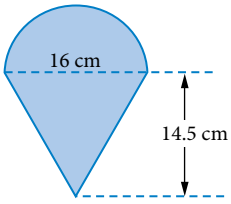


3 EXAMPLE 7 Calculate the area of each shaded region, correct to 2 decimal places.

a

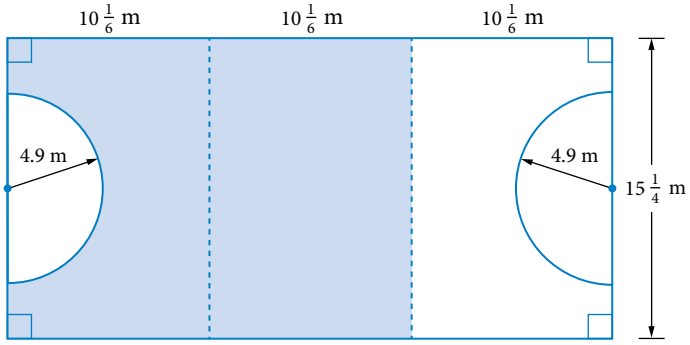


b



9780170484664

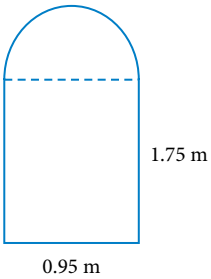
4 The diagram shows a netball court.



a Calculate, correct to 2 decimal places, the area of one of the goal semicircles.

b Calculate, correct to one decimal place, the size of the shaded region where the 'wing attack' player can play.

5 Susan and Avery are putting 3 arch windows in the front room of their house. The diagram shows one of the windows with its dimensions.

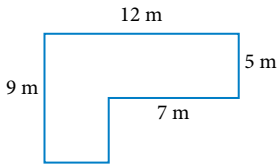


a Find the area of one arch window, correct to 2 decimal places.

b Find the area of all 3 arch windows.

c 10% of the area is NOT glass. Find the area of glass used.

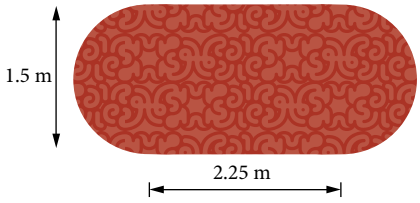
6 A children's playground has an L-shape as shown.



a What is the area of the playground?

b The playground is to be covered with woodchips for safety. How much will the woodchips cost if they are sold for $7.20 per square metre?

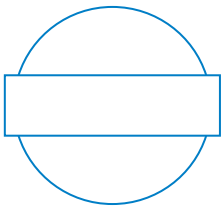
7 Casey buys a new rug for their living room. The rug is composed of one rectangle and 2 semicircles as shown.



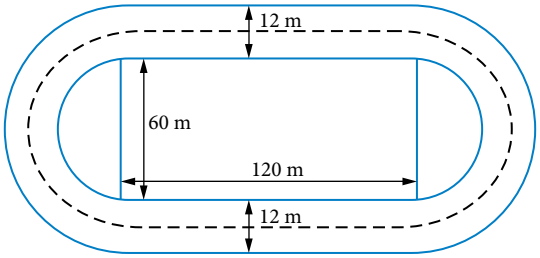
a Calculate the area of the rug, correct to 2 decimal places.

b Casey's living room floor is a rectangle 3 m by 2.5 m. What area of the floor will NOT be covered by the rug?

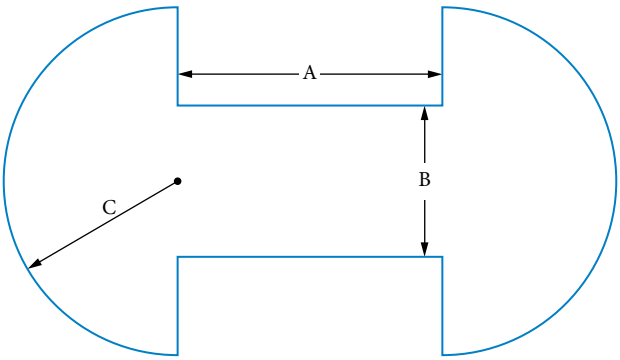
8 This badge is made up of a metal disc overlaid with a rectangle. The metal disc has a radius of 3.5 cm. The rectangle is 1 cm longer than the diameter of the circle and 1.5 cm wide. Calculate the total area of the metal in the badge, correct to 2 decimal places. **PS**



9 Gotham City has a multi-purpose football field surrounded by an athletics track as shown. Calculate the area of the athletics track, correct to 2 decimal places. **PS**



10 The scale drawing shows the whale viewing platform at Outlook Point. It has a semi-circular section at both ends and a rectangle in the middle. **PS**



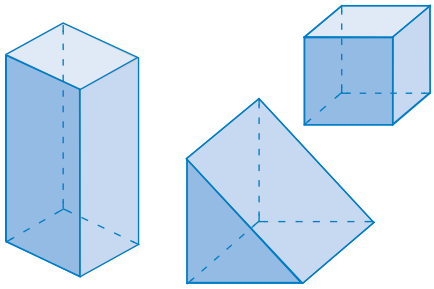
Scale of 1 : 210

a Determine the real-life lengths of A, B and C in metres to one decimal place.

b The platform needs a coat of paint. 1 tin of paint can cover 22 m^2. Kurt is going to buy the paint. How many tins will he need to buy?

4.7 Surface areas of prisms*

A **prism** is a solid shape that has identical ends. Some examples are shown: a rectangular prism, **triangular prism** and cube. All side faces are rectangles.



When we want to make boxes for packaging, we need to know the area of the faces of the box. This is called **surface area** because it is the sum of the areas of all the surfaces of the box. The easiest way to calculate the surface area of an object is to draw its **net**. We learned about the nets of solids in Chapter 3, *The shape of our world*. We calculate the area of each shape on the net individually and then add them together to find the total surface area.

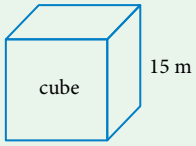
Video
Surface area of a prism

Worksheet
Nets of solids

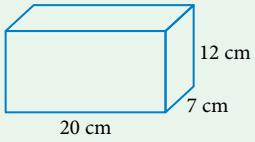
EXAMPLE 8 Surface area of a prism

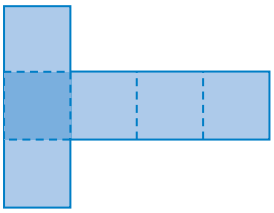
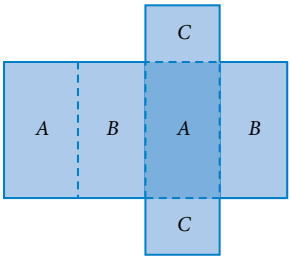
Calculate the surface area of each prism.

a



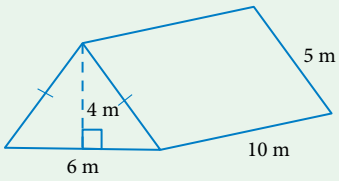
b

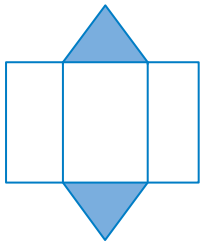


Steps	Working
a A cube has 6 identical faces, all squares.	
This is the net.	
The surface area is 6 times the area of each square.	total surface area $= 6 \times 15^2$ $= 1350\ \text{m}^2$
b This is the net of a rectangular prism. Notice that the rectangles are in matching pairs.	
The ends (C) are the same rectangles.	area of ends $= 2 \times (7 \times 12)$ $= 168\ \text{cm}^2$
The top and bottom faces (A) are the same size.	area of top/bottom $= 2 \times (20 \times 7)$ $= 280\ \text{cm}^2$
The front and back faces (B) are the same size.	area of front/back $= 2 \times (20 \times 12)$ $= 480\ \text{cm}^2$
Add the areas together.	total surface area $= 168 + 280 + 480$ $= 928\ \text{cm}^2$

EXAMPLE 9 Surface area of a triangular prism

Find the surface area of this triangular prism.



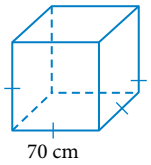
Steps	Working
This is the net of the prism. There are 3 rectangles and 2 identical triangles.	
Find the area of the base rectangle.	area of base $= 6 \times 10$ $= 60\,\text{m}^2$
Find the area of the two side rectangles.	area of side faces $= 2 \times (5 \times 10)$ $= 100\,\text{m}^2$
Find the area of the 2 triangles.	area of 2 triangles $= 2 \times \left(\frac{1}{2} \times 6 \times 4\right)$ $= 24\,\text{m}^2$
Add the areas together.	total surface area $= 60 + 100 + 24$ $= 184\,\text{m}^2$

EXERCISE 4.7 Surface areas of prisms*

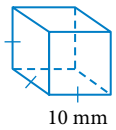
ANSWERS p. 436

1 EXAMPLE 8 Find the surface area of each cube.

a



b



2 Charlotte has a plastic storage cube that is 35 cm long with no top. Calculate its external surface area. (Ignore the holes for the handles.)

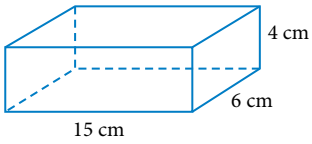
Stephanie Jackson - Home and office collection/Alamy Stock Photo

3 This metal cube sculpture has a side length of 450 cm. The surface is to be covered with a weather-resistant finish. How many square metres need to be covered?

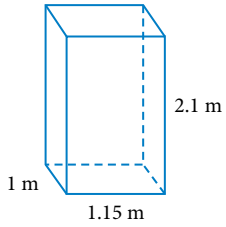
Holly Vegter/Shutterstock.com

4 Find the surface area of each prism.

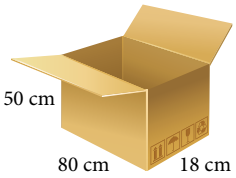
a



b



5 Yusuf's new TV was packed in this box. Find the surface area of the box.



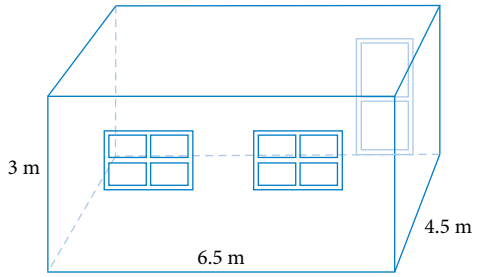
6 Alexa and Jai built a new living room.

a Calculate the area of the walls and ceiling to be painted. Subtract 6 m^2 for the windows and door.

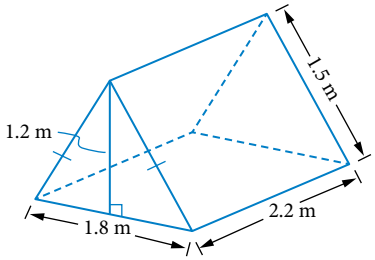
b If paint costs $23 for a 4-litre can that covers 10 m^2, calculate how much it would cost to paint the room with 2 coats.

c Calculate the area of the floor to be covered with wood parquet.

d If wood parquet costs $15.70 per square metre, how much would it cost to cover the floor? Answer to the nearest 10 dollars.



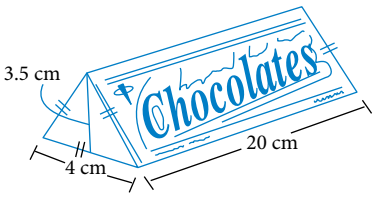
7 EXAMPLE 9 Tim's company makes camping equipment, including this tent. What amount of material is required to make the tent (including the floor)? PS



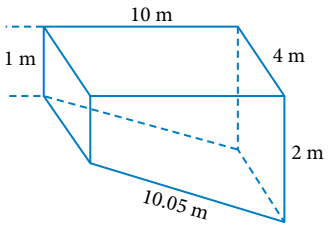
8 The diagram shows a chocolate box in the shape of a prism. The ends of the box are equilateral triangles.

a Calculate the surface area of the box.

b The manufacturer allows an extra 10% of the surface area for tabs and wastage. How much cardboard is required for one box?



9 Harish and Rachna have a backyard pool shaped as shown. It needs to be repainted. There are tiles around the top edge to a depth of 40 cm. Calculate the area to be painted. PS
Remember to convert 40 cm to m.

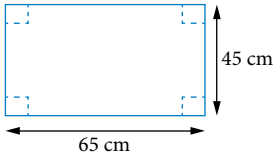


10 A metal tray is made by taking a rectangular piece of metal and cutting squares from the corners. The edges are then bent upwards and welded at the corners to form the tray. PS

a This piece of metal has squares of side length 10 cm cut out of each corner. What are the dimensions of the tray that can be formed?

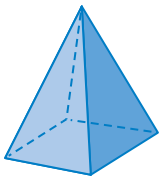
b The bottom of the tray is to be lined with material. What area of material is required?

c The sides and the outside, including the base, are to be enamelled. What area is to be enamelled?



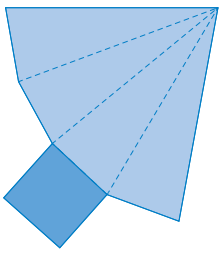
4.8 Surface areas of pyramids*

A **pyramid** has a triangle, quadrilateral or other polygon as its base, and all side faces are triangles that meet at a point called the **apex**. We name pyramids by the shape of their base. For example, this is a square pyramid.

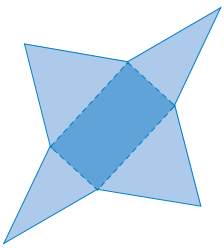


The nets of some pyramids are shown below: these will help us find their surface areas.

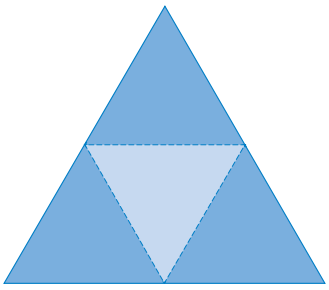
Square pyramid



Rectangular pyramid



Triangular pyramid

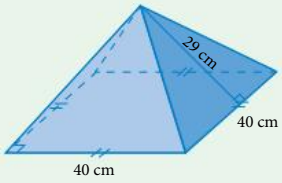


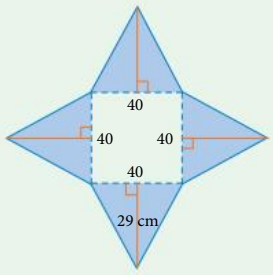
Surface area of a pyramid

Surface area of a pyramid = area of base + area of all triangular side faces

EXAMPLE 10 Surface area of a square pyramid

A square pyramid and its net are shown. **Calculate** the surface area of the pyramid.

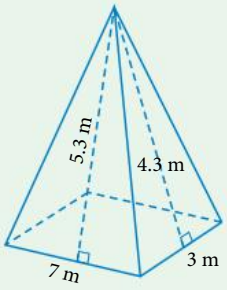




Steps	Working
Calculate the area of the square base.	area $= 40^2$ $= 1600\,\text{cm}^2$
Calculate the area of the 4 triangles. They are all the same because the base is a square.	area $= 4 \times \left(\frac{1}{2} \times 40 \times 29\right)$ $= 2320\,\text{cm}^2$
Add the areas together.	total surface area $= 1600 + 2320$ $= 3920\,\text{cm}^2$

EXAMPLE 11 Surface area of a rectangular pyramid

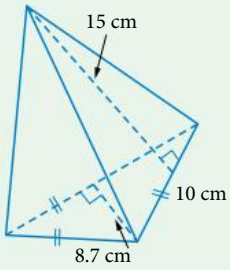
Find the surface area of this rectangular pyramid.



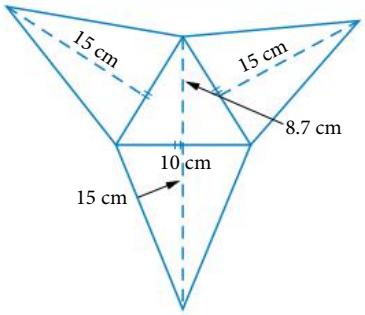
Steps	Working
Find the area of the rectangular base.	area $= 7 \times 3$ $= 21\,\text{m}^2$
Opposite triangles are identical. Find the area of the left/right triangles.	area $= 2 \times \left(\frac{1}{2} \times 3 \times 4.3\right)$ $= 12.9\,\text{m}^2$
Find the area of the front/back triangles.	area $= 2 \times \left(\frac{1}{2} \times 7 \times 5.3\right)$ $= 37.1\,\text{m}^2$
Add the areas together.	total surface area $= 21 + 12.9 + 37.1$ $= 71\,\text{m}^2$

EXAMPLE 12 Surface area of a triangular pyramid

Determine the surface area of this triangular pyramid.



Note that its base is an equilateral triangle.

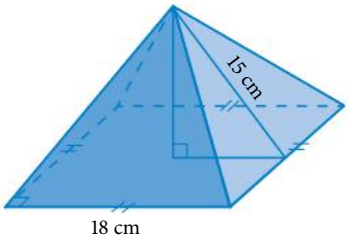
Steps	Working
The net of this pyramid is shown: the base is an equilateral triangle and the 3 side faces are identical triangles.	
Find the area of the 3 side faces.	$\text{area} = 3 \times \left(\frac{1}{2} \times 10 \times 15\right)$ $= 225\ \text{cm}^2$
Find the area of the base triangle.	$\text{area} = \frac{1}{2} \times 10 \times 8.7$ $= 43.5\ \text{cm}^2$
Add the areas together.	$\text{total surface area} = 225 + 43.5$ $= 268.5\ \text{cm}^2$

EXERCISE 4.8 Surface areas of pyramids*

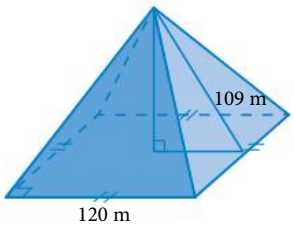
ANSWERS p. 436

1 EXAMPLE 10 Find the surface area of each square pyramid.

a



b



2 Andrea is making a stained glass lampshade in the shape of a square pyramid. The bottom edges are 20 cm long and the slant height of the triangles is 24 cm. Find the total area of glass that Andrea will need to make the lampshade.

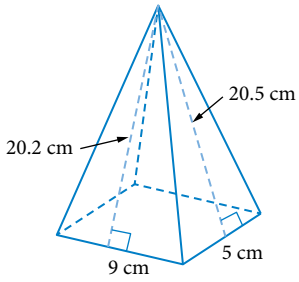
Note: A lampshade does not have a base.

Photo courtesy of Sue Thomson

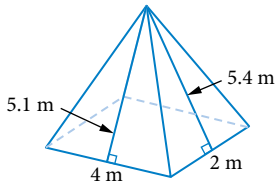
3 The Great Pyramid in Egypt is a square pyramid with the sides of the base 230 m and the slant height of the sides 300 m. Calculate the surface area of the Great Pyramid, including the base.

4 EXAMPLE 11 Calculate the surface area of each rectangular pyramid.

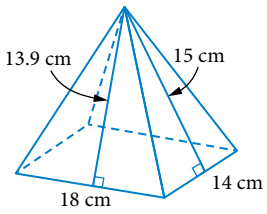
a



b

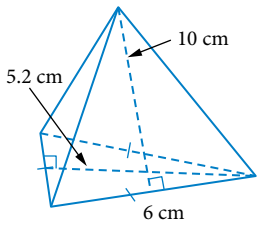


5 Harry believes his seedlings grow best when they are planted in glass pyramids. He uses a glass cover in the shape of a rectangular pyramid that fits over a glass base 18 cm by 14 cm. Calculate the total surface area of the pyramid and the glass base.

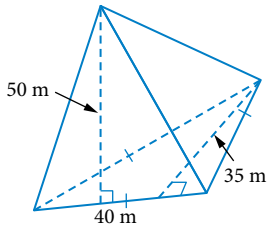


6 EXAMPLE 12 Calculate the surface area of each triangular pyramid.

a



b



7 Min wins the state public speaking competition and receives a crystal glass trophy in the shape of a triangular pyramid. The base is an equilateral triangle with sides 4 cm and perpendicular height 3.5 cm. The slant height of the sides of the trophy is 22 cm. Calculate the surface area of the glass.

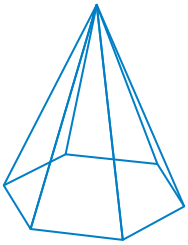
8 The entrance to the Louvre museum in Paris is a large square pyramid made of glass. The base of the pyramid is 35 m long. The slant height of the sides of the pyramid is 27.8 m. PS

Ttstudio/Shutterstock.com

a Calculate the area of the 4 side faces of the pyramid.

b The side faces are made out of glass pieces in the shape of a rhombus. There are approximately 640 of these on the 4 sides. What is the approximate area of one glass piece?

9 This is a hexagonal pyramid box made to hold chocolates. The base has sides of 8 cm and its area is 83 cm^2. The slant height of the triangles is 15 cm. Calculate the surface area of this chocolate box. PS



Hexagonal pyramid

10 The roof of one section of Sue's house is a square pyramid. The base of each section of roof is 10 m and the slant height is 5.3 m. At the top, a square pyramid has been sliced off. The removed section has a square base 1.8 m long and a slant height of 1.9 m.

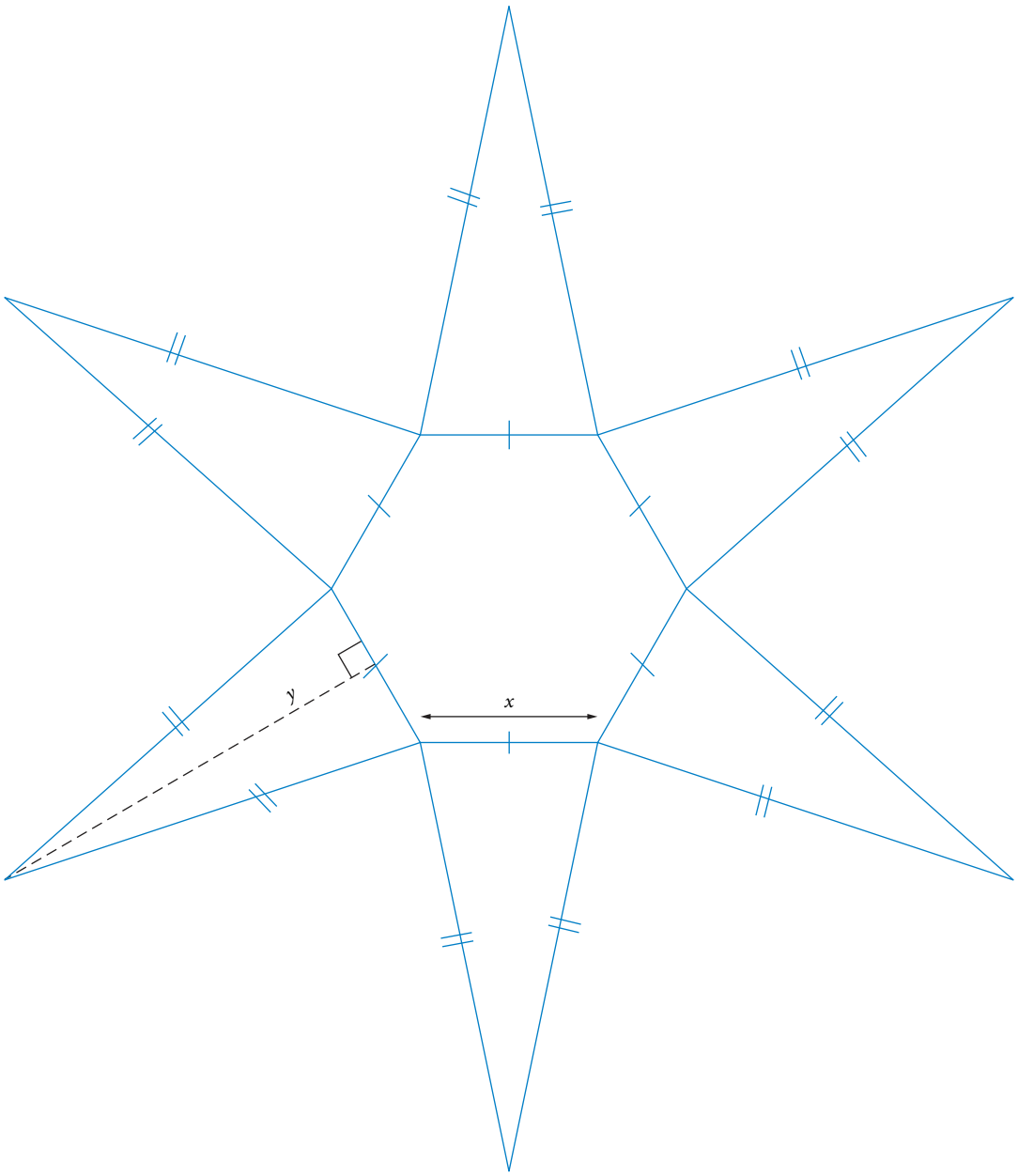
Photo courtesy of Sue Thomson

a What was the surface area of the roof *before* the top section was removed?

b What is the surface area of the section of roof that has been removed?

c What is the surface area of the roof that Sue has to cover with roof tiles?

11 Samara is making a cardboard template to make gift boxes in the shape of hexagonal pyramids. The diagram shows a scale drawing of the net of the boxes. The scale is 1 : 4.



Scale of 1 : 4

The table shows the lengths in the scale diagram.

Dimension	Length in the scale drawing
x	2.5 cm
y	6 cm

a Use the formula for the area of a regular hexagon, $A = \frac{3 \times \sqrt{3}}{2} \times s^2$ where s is the length of the sides, to show that the area of the hexagonal base on the template is 260 cm^2 correct to the nearest cm^2.

b Calculate the total area of cardboard in the template.

c 100 cm^2 of the cardboard Samantha is using weighs 0.65 g. Calculate the weight of the template in grams.

Investigation | Finding pyramid buildings

Search the internet for buildings or structures that are pyramids.

Write down the locations of at least 10 pyramid buildings or structures. Include pictures where possible.

Choose one of the buildings or structures you have found and find its dimensions.

Calculate the surface area of the building or structure.

4.9 Surface area of cones*

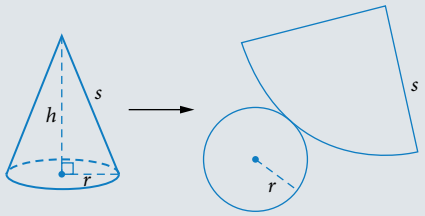
The surface area of a cone is in 2 sections, the curved section and the circular end.

S = curved section + circle

Surface area of a cone

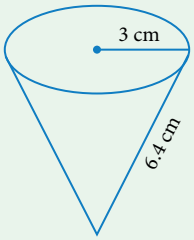
$$S = \pi rs + \pi r^2$$

where r is radius of a circle and s is slant height.



EXAMPLE 13 Surface area of a cone

Calculate the surface area of the curved section, the top and the total surface area of this cone in square centimetres, correct to one decimal place.

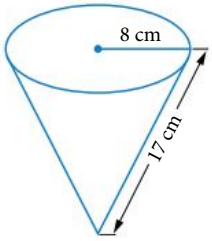


Steps	Working
In the formula the radius, $r = 3$ cm, and the slant height, $s = 6.4$ cm.	area of the curved section $= \pi rs$
The curved section of the formula is πrs	$= \pi \times 3 \times 6.4$
Substitute the values into πrs.	$\approx 60.3\,\text{cm}^2$
The area of the top is a circle, the πr^2 section in the formula.	area of the top $= \pi r^2$
	$= \pi \times 3^2$
	$\approx 28.3\,\text{cm}^2$
To find the total surface area, use the whole formula.	$S = \pi rs + \pi r^2$
	$= \pi \times 3 \times 6.4 + \pi \times 3^2$
	$\approx 88.6\,\text{cm}^2$

EXERCISE 4.9 Surface area of cones*

ANSWERS p. 436

1 EXAMPLE 13 Calculate, in mm^2 correct to one decimal place:



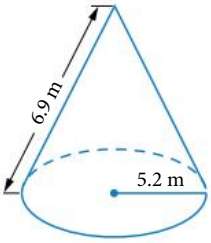
a the surface area of the curved section of this cone

b the area of the top

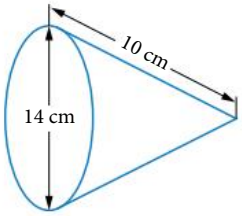
c the total surface area of the cone.

2 Calculate the surface area of each cone, correct to one decimal place.

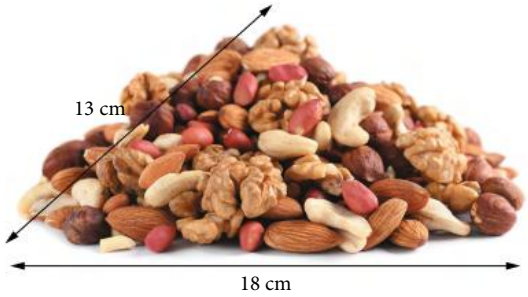
a



b



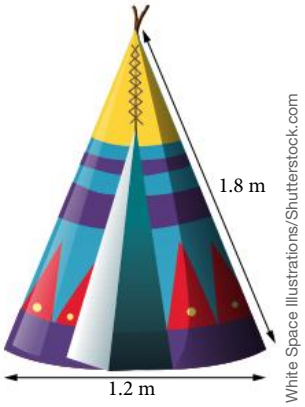
3 This pile of nuts is in the shape of a cone. What area of nuts is exposed to the air? Answer to the nearest cm^2.



Studio Mars/Shutterstock.com

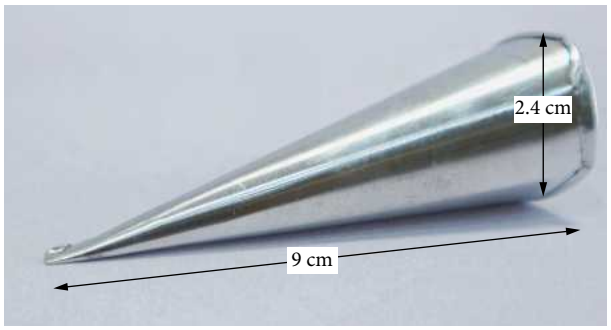
Hint: What is the curved surface area of the cone without the circular base?

4 Jamie likes to sleep inside a toy tent in his bedroom. The base and the side are made from fabric. Calculate the area, in m^2 to 2 decimal places, of fabric used to make the tent.



White Space Illustrations/Shutterstock.com

5 Emily uses metal molds when she makes Danish pastries. The molds are hollow with no end. Calculate the external surface area of the mold correct to the nearest cm^2.



Yellyana/Shutterstock.com


Sandipras/Shutterstock.com

6 Filomena makes and sells waffle cone ice-creams. She makes 8 cm radius circular waffles on a waffle maker. While the circular waffles are still warm, she wraps them around a waffle mold, overlapping parts of the circles to make a cone. The mold's slant height is 16 cm and the radius of the top is 3 cm. PS

Irishasel/Shutterstock.com

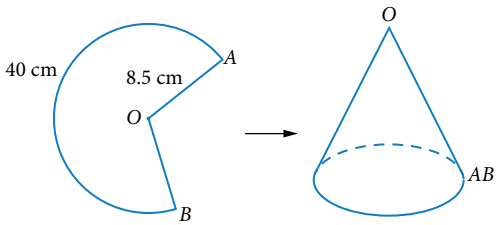

iStock.com/Bdspn

a Calculate the area of the circular waffle. Answer to the nearest cm^2.

b What is the surface area of the curved section of the cone mold? Answer to the nearest cm^2.

c Approximately how many square cm of overlap are created when the circle is rolled into a cone?

7 To make a funnel Tom uses a sector of a circle, as shown, and joins OA to OB. OA = 8.5 cm and the length of the arc of the sector is 40 cm. C PS



a What is the slant height of the funnel?

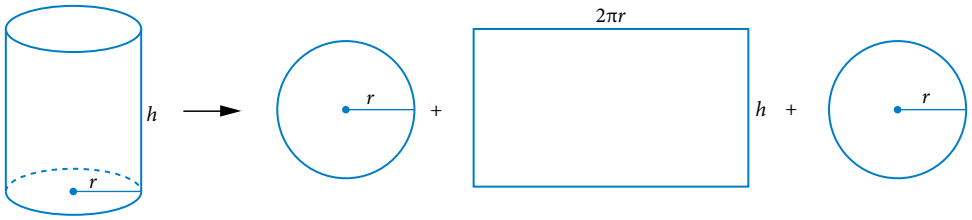
b Explain how you know that the circumference of the circle at the end of the cone assembled is 40 cm.

c Use the formula $r = \frac{C}{2\pi}$ to calculate the radius of the circle. Answer to one decimal place.

d Calculate the area of the curved section of the funnel correct to the nearest cm^2.

4.10 Surface areas of cylinders and spheres*

If we flatten out a **cylinder**, we get this net of 2 circles and a rectangle.



Note: The length of the rectangle is the circumference of the circle ($2\pi r$).

Surface area of a cylinder = area of 2 circles + area of rectangle

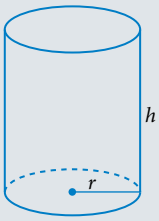
$= 2 \times \pi r^2 + 2\pi r \times h$ Circumference $= 2\pi r$

$= 2\pi r^2 + 2\pi rh$

Surface area of a cylinder

Surface area = area of rectangle + area of 2 circles

$S = 2\pi rh + 2\pi r^2$



There is a simple formula for the surface area of a **sphere**.

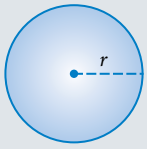
Surface area of a sphere $= 4 \times \pi \times (\text{radius})^2$ Like 4 times the area of a circle

$= 4\pi r^2$

Surface area of a sphere

surface area $= 4 \times \pi \times (\text{radius})^2$

$S = 4\pi r^2$

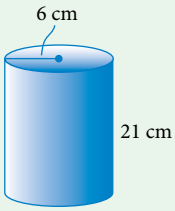


Worksheets
A page of solid shapes
Nets of solids

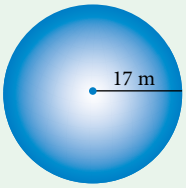
EXAMPLE 14 Surface area of a cylinder and sphere

Determine the surface area of each solid, correct to 2 decimal places.

a



b



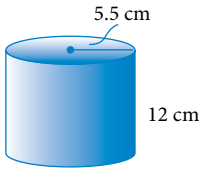
Steps	Working
a Use the formula for the surface area of a closed cylinder with $r = 6$ and $h = 21$.	$S = 2\pi rh + 2\pi r^2$ $= 2 \times \pi \times 6 \times 21 + 2 \times \pi \times 6^2$ $= 1017.8760\ldots$ $= 1017.88\,\text{cm}$
b Use the formula for the surface area of a sphere with $r = 17$.	$S = 4 \times \pi \times r^2$ $= 4 \times \pi \times 17^2$ $= 3631.6811\ldots$ $\approx 3631.68\,\text{m}^2$

EXERCISE 4.10 Surface areas of cylinders and spheres*

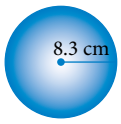
ANSWERS p. 436

1 EXAMPLE 14 Find the surface area of each solid, correct to 2 decimal places.

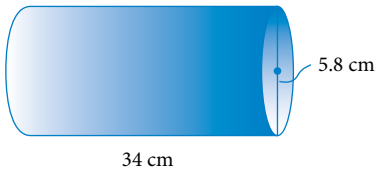
a



b



c



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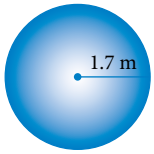
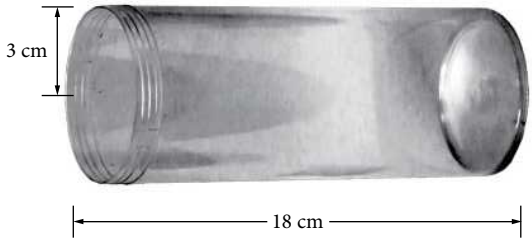


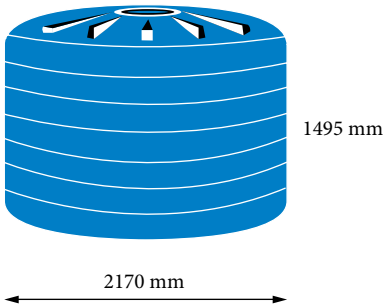
2 A can of pineapple rings has height 11 cm and radius 5 cm. A label is to be wrapped around the curved surface. Calculate the area of the label, correct to 2 decimal places.

3 Calculate, correct to one decimal place, the area of recycled plastic used to make this food container. Note that it has an open end.



4 A tennis ball has a diameter of 7 cm. Calculate the surface area of the tennis ball, correct to the nearest cm^2.

5 How much plastic would be needed to make this water tank? Answer correct to the nearest square metre.



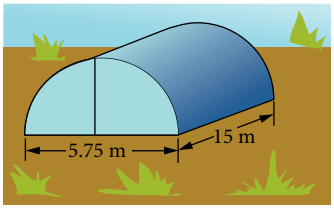
6 The Earth is approximately a sphere with a radius of about 6400 km.

a Calculate the surface area of the Earth, correct to the nearest 10 square km.

b Approximately 70% of the Earth's surface is water. What area of the Earth's surface is covered with water?

c What area of the Earth's surface is land?

7 A hydroponics shed is made in the shape of half of a cylinder. Find the area of heavy plastic needed for the shed, correct to the nearest square metre. Do not include the floor. PS



8 A disco mirror ball is 33 cm in diameter. It is covered with small mirror tiles that have an area of 1.44 cm^2 each. PS

a Find the surface area of the sphere, correct to 2 decimal places.

b How many tiles are used to cover the mirror ball?

9 Yara bought a tube for sending posters through the mail. It is a cardboard cylinder with plastic plugs at each end. The tube is 1.1 m long and has a radius of 2.5 cm. PS

Damon Davison/Shutterstock.com

a Calculate the amount of cardboard used in creating the tube, correct to the nearest square centimetre.

b Calculate the total area of the plastic ends, correct to 2 decimal places.

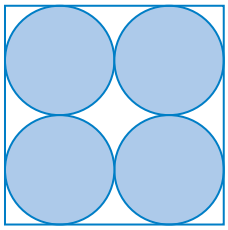
Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

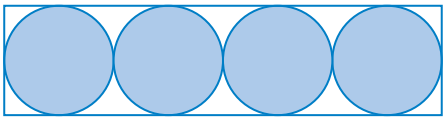
Investigation — Christmas bauble boxes

Irena has been asked to design a new cardboard box for a set of 4 spherical Christmas baubles with a diameter of 6.5 cm. She presents 2 designs:

An open square box



An open rectangular box



An open box has no top.

1. Calculate the area of cardboard required for the open square box. Allow for an extra centimetre on each dimension (length, width, height) to have space for interior packaging. Add 10% extra area for overlapping sections to glue the box together.
2. Calculate the area of cardboard required for the open rectangular box. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
3. Is there another possible design for holding the 4 baubles? If so, provide a sketch and calculate the area of cardboard required for it. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
4. Which design would you recommend? Justify your decision.


iStock.com/Yasinguneysu

4.11 Surface areas of composite solids*

When we were calculating the area of some shapes, we had to separate the shape into several smaller shapes. We can use the same technique when we are calculating the surface areas of composite solids.

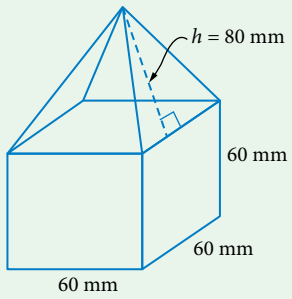
Surface area

To find the surface area of a solid:

1. calculate the area of each face
2. add the areas together to determine the total surface area.

EXAMPLE 15 Surface area of a composite solid

Calculate the surface area of this composite solid made from a square pyramid and a cube.

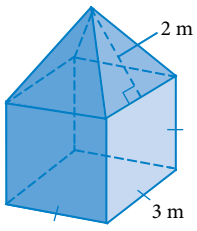


Steps	Working
This solid has 4 triangular faces and 5 square faces. Find the area of the 4 triangles.	area of triangular faces $= 4 \times \frac{1}{2} \times 60 \times 80$ $= 9600\,\text{mm}^2$
Find the area of the 5 squares.	area of square faces $= 5 \times 60 \times 60$ $= 18\,000\,\text{mm}^2$
Add the areas together.	total surface area $= 9600 + 18\,000$ $= 27\,600\,\text{mm}^2$

EXERCISE 4.11 Surface areas of composite solids*

ANSWERS p. 436

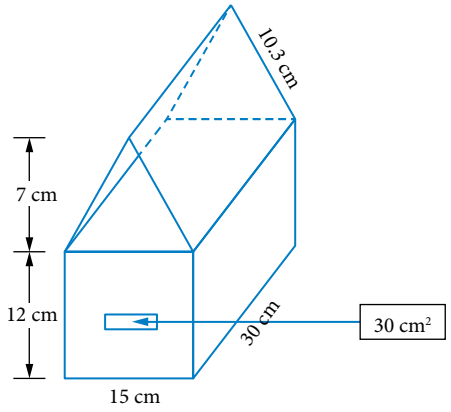
1 EXAMPLE 15 Manuel is building a greenhouse, as shown in the diagram. Calculate the surface area of his greenhouse, *not including the floor*.



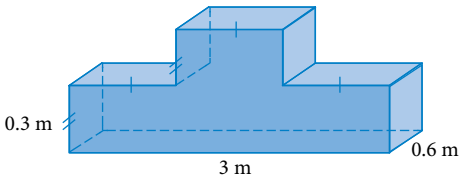
2 This metal water tank supplies water to steam trains. It has the shape of a square prism, where each side face is a rectangle 3.2 m wide by 1.4 m high. Find the area of metal in the water tank, including the base and the lid.

Desmond Kean/Shutterstock.com

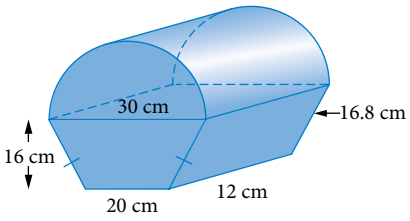
3 The lower section of Tyrone's metal letterbox is a rectangular prism and the upper section is a triangular prism. The upper section is open at both triangular ends to hold long, rolled-up articles. The rectangular slot is 30 cm^2. Calculate the amount of sheet metal in Tyrone's letterbox. **PS**



4 The local athletics club has a portable podium for medal presentations. Every surface, excluding the base, needs repainting. Calculate the surface area to be repainted. **PS**



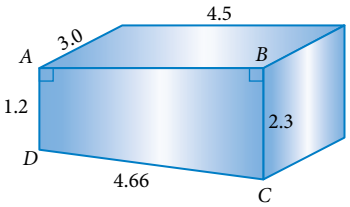
5 Anh is making a jewellery box shaped as shown. He is going to cover it with special material. **PS**



a Find the surface area of the jewellery box, correct to the nearest square centimetre.

b Anh allows an extra 10% of material for the edges and wastage. How much material should he buy, correct to the nearest square centimetre?

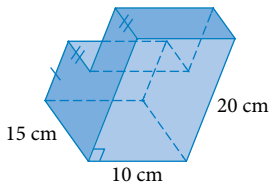
6 Bruno needs to resurface the interior of his pool. (All measurements are in metres.)



a Calculate the area of the trapezium *ABCD*.

b Calculate the total surface area of the 5 faces that Bruno needs to resurface, correct to 2 decimal places.

7 Ginny has made a pair of wooden bookends in an L-shape. She is going to stain the bookends before she uses them. Find the total surface area of both bookends.



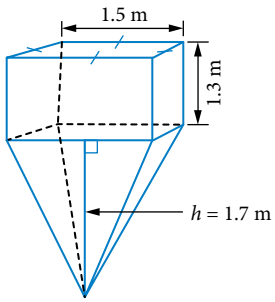
8 This truck delivers LPG to homes for use in cooking. The tank is comprised of a cylinder with a hemisphere at each end. PS

Justin Kase zninez/Alamy Stock Photo

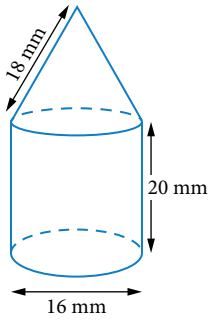
a Calculate the surface area of the LPG tank, correct to one decimal place.

b 1 L of paint covers 14 m^2 of the tank. How much paint will Bill need to cover the tank with 2 coats? Answer in litres, correct to one decimal place.

9 Nadia used sheet metal to make this hopper to use in the beer making process. The hopper is in the shape of a square pyramid under a square prism, without a top. How much sheet metal did Nadia use to make the hopper? PS



10 Rachel plans to make some novelty chocolate-dipped cakes for her daughter's birthday party. The cakes have a cylindrical base with a cone on top. Rachel will dip the cakes in chocolate to completely cover the cakes. The dimensions of the cakes are in the sketch.



Coffeeflavour/Shutterstock.com

a Calculate the total surface area of one cake. Answer to the nearest square mm.

b From past experience Rachel knows she will need 10 g of melted chocolate to cover 500 mm^2 of cake. What weight of chocolate will she need to make 24 of the novelty cakes? Answer correct to the nearest hundred grams.

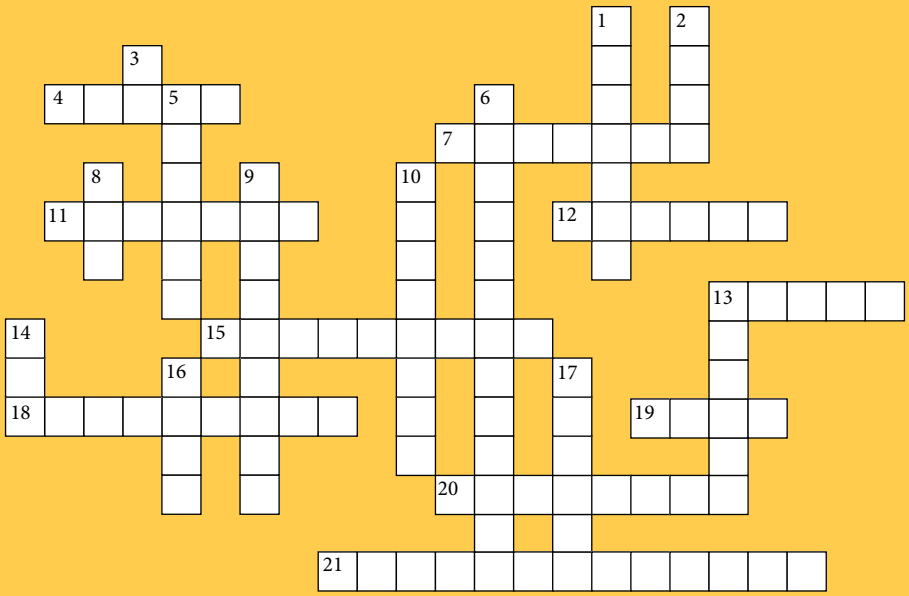
Keyword activity

BACK-TO-FRONT CROSSWORD

ANSWERS p. 437

Copy the crossword below and fit the words into it. The number of letters in each word is shown in brackets. Then write the clues for the crossword.

pi [2]	arc [3]	net [3]	area [4]
cone [4]	cube [4]	prism [5]	solid [5]
circle [6]	sector [6]	sphere [6]	square [6]
hectare [7]	pyramid [7]	surface [7]	cylinder [8]
triangle [8]	composite [9]	rectangle [9]	trapezium [9]
parallelogram [13]	quadrilateral [13]		



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Solution to the chapter problem

Problem

Jess is making gift boxes to sell. She plans to make 3 types of boxes:

- **Cube:** 15 cm long
- **Rectangular prism:** 20 cm by 10 cm by 8 cm
- **Cylinder:** radius 4.5 cm and height 34 cm

Should she charge the same for each gift box? Give reasons for your answer.

Solution

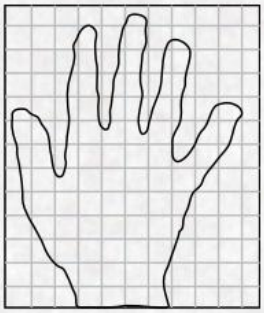
STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To work out whether Jess should charge the same amount for each box. The dimensions of the boxes are known and surface area will have to be worked out for each one.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	Compare the surface areas of the boxes to see if they are equal. **Cube:** surface area $= 6 \times$ (area of a square face) $= 6 \times 15^2$ $= 1350\ \text{cm}^2$ **Rectangular prism:** surface area $= 2 \times$ (base area) $+ 2 \times$ (front face area) $+ 2 \times$ (side face area) $= 2 \times (20 \times 10) + 2 \times (20 \times 8) + 2 \times (10 \times 8)$ $= 880\ \text{cm}^2$ **Cylinder:** surface area $= 2\pi r^2 + 2\pi rh$ $= 2 \times \pi \times (4.5)^2 + 2 \times \pi \times 4.5 \times 34$ $\approx 1089\ \text{cm}^2$ The boxes have different surface areas.
STAGE 3: CHECK THE SOLUTION	
 CHECK	The 3 surface areas are all around $1000\ \text{cm}^2$. The values are what we expect.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	Jess should not charge the same amount for each box because they use different amounts of materials. Compared to the price of the rectangular prism, she should charge more for the cylinder and even more for the cube.

4 Test yourself

On the surface

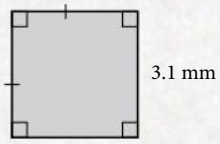
ANSWERS p. 437

1 EXERCISE 4.1 Estimate the area of Isabella's hand. Each square represents 1 cm².



2 EXERCISE 4.2 Calculate the area of each shape, correct to one decimal place.

a



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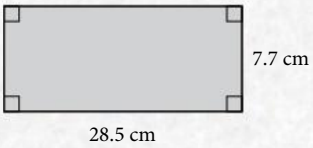
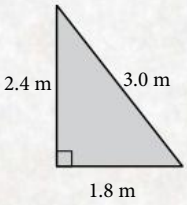


c



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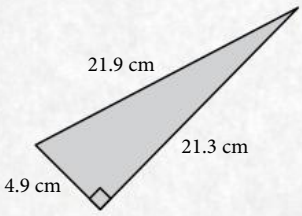

3 EXERCISE 4.3 Grant is going to buy tiles to cover his rectangular verandah, which is 11 m long by 2.5 m wide. How many square metres of tiles should he order, allowing an additional 10% for cutting and breakages? Answer correct to the nearest square metre.

4 EXERCISE 4.4 Copy and complete each conversion.

a 400 mm² = _____ cm²

b 5.4 cm² = _____ mm²

c 25 000 m² = _____ ha

d 5500 cm² = _____ m²

e 70 cm² = _____ mm²

f 6 m² = _____ cm²

5 EXERCISES 4.2, 4.5 Match each shape in **a** to **g** with the formula for its area in **A** to **G**.

a square **b** rectangle **c** circle **d** parallelogram

e trapezium **f** sector **g** triangle

A $A = \frac{1}{2}(a + b)h$ **B** $A = \pi r^2$ **C** $A = s^2$ **D** $A = \frac{\theta}{360}\pi r^2$

E $A = bh$ **F** $A = lw$ **G** $A = \frac{1}{2}bh$

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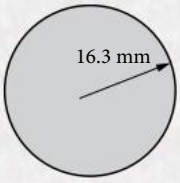
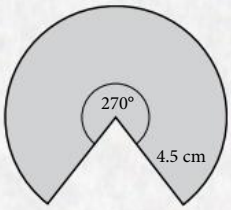
6 EXERCISE 4.5 Calculate the area of each shape, correct to one decimal place.

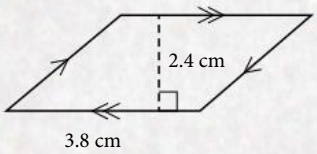
a



b



c



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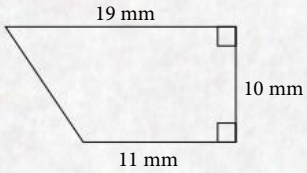
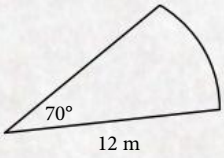
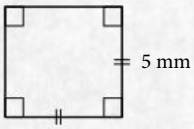


e

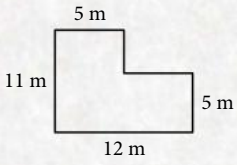


f



7 EXERCISE 4.6 Find the area of each shape.

a



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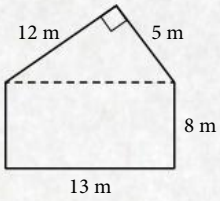

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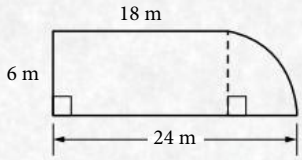
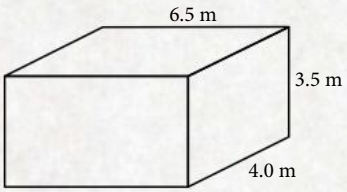
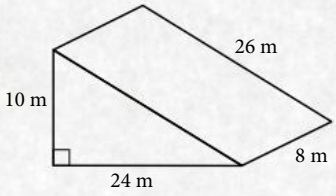


8 EXERCISE 4.7 Determine the surface area of each prism.

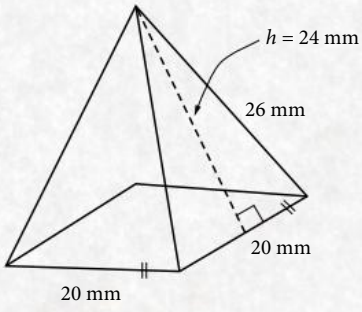
a



b

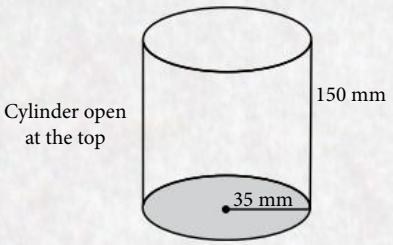


9 EXERCISE 4.8 Find the surface area of this square pyramid.

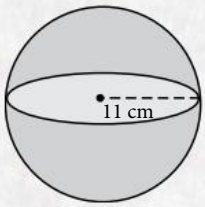


10 EXERCISES 4.9, 4.10 Calculate the surface area of each solid, correct to the nearest whole number.

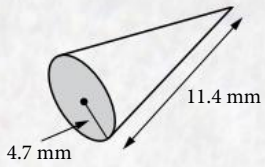
a



b

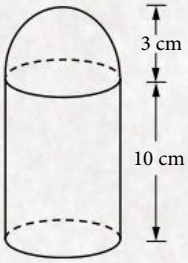


c



11 EXERCISE 4.11 Calculate the surface area of each solid.

a



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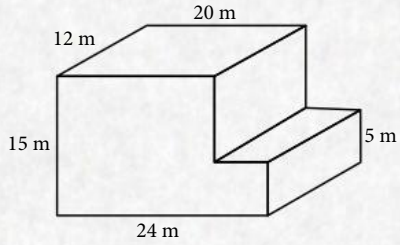

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Chapter problem

Francesca and Michael are building a house according to the plan shown on the next page. They need to order carpet for the living room and the 3 bedrooms.
How many square metres of carpet will they need?
Answer to the nearest square metre.

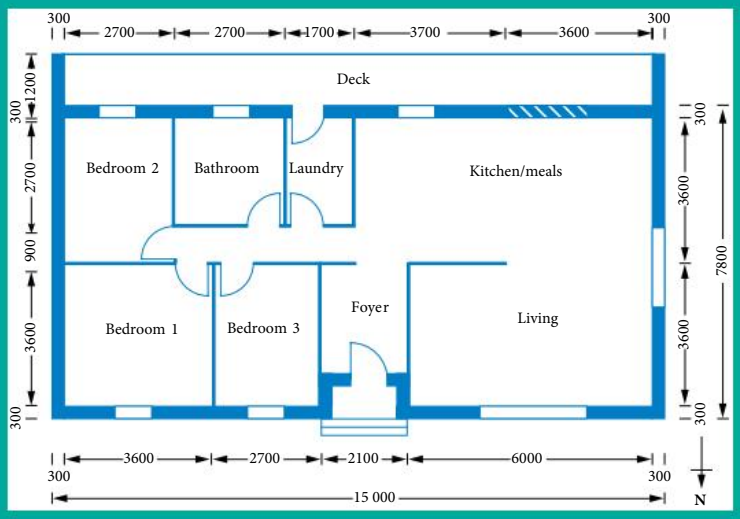
Nikonaft/Shutterstock.com

CHAPTER 5

MEASUREMENT, SCALES AND CHANCE: MEASUREMENT

FROM PAPER TO REALITY

*COMPLEX



What will we do in this chapter?

- Use scales and calculate with scales
- Interpret scale drawings, including commonly used symbols and abbreviations
- Find actual measurements from scale drawings, such as lengths, perimeters and areas
- Calculate quantities and costs for home renovations
- Create scale drawings

How are we ever going to use this?

- Determining lengths on house plans, maps and other scale drawings
- Building, renovating or buying a home
- Interpreting scale models and building plans
- Working in a building or painting trade

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Syllabus coverage

UNIT 3, TOPIC 2: SCALES, PLANS AND MODELS

Interpret scale drawings

- Interpret commonly used symbols and abbreviations in scale drawings.
- Find actual measurements from scale drawings, including lengths, perimeters and areas.
- Estimate and compare quantities, materials and costs using actual measurements from scale drawings [complex].

Creating scale drawings

- Understand and apply drawing conventions of scale drawings, including scales in ratio, clear indications of dimensions and clear labelling [complex].
- Construct scale drawings by hand and by using software packages [complex].

Video (1):

5.1 Map scales

Worksheets (2):

5.2 Scale drawings • Interpreting an office plan

5.4 Scale drawings

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

Terminology

dimensions	frieze	house plans	scale
scale drawing	skirting boards	symbols	

5.1 Scale drawings

A **scale drawing** is a reduced or enlarged version of a real object. The most common scale drawings are maps and house plans. By taking measurements on the scale drawing, we can calculate the size of objects in real life using the **scale** on the drawing.

Scale drawings

To calculate an actual (real-life) length on a scale drawing:

- measure the scaled length on the scale drawing
- multiply by the scale
- convert your answer to the required units if necessary.

To calculate a scaled length from a real-life measurement:

- divide the real-life measurement by the scale
- convert your answer to the required units if necessary.

Scales on a diagram are most commonly given as a statement, such as '1 cm represents 5 m', or as a ratio, such as '1 : 500'.

Video
Map scales

EXAMPLE 1 Using scales 1

Keiran is using a map with the scale: 1 cm represents 2 km.

a How far would he have to walk if the distance on the map is 6 cm?

b Keiran is planning a 25 km hike with friends. **Determine** how far this is on the map.

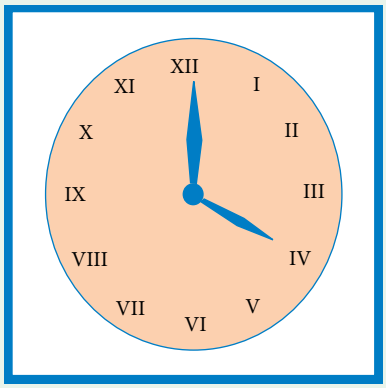
Steps	Working
a Multiply the scaled distance by the scale. Scaled distance = 6 cm.	actual distance = 6 × 2 = 12 km Keiran would have to walk 12 km.
b To calculate a scaled (map) distance, DIVIDE the actual distance (25 km) by the scale.	scaled distance = 25 ÷ 2 = 12.5 cm 25 km is 12.5 cm on the map.

DJgis/Shutterstock.com

EXAMPLE 2 Using scales 2

The diagram of a clock is drawn to a scale of 1 : 6.

Measure its scaled length and **calculate** its actual length.



Scale 1 : 6

Steps	Working
Measure the scaled length on the diagram.	scaled length = 5 cm
Multiply by the scale.	actual length = 5 × 6
	= 30 cm

EXERCISE 5.1 Scale drawings

ANSWERS p. 437

1 EXAMPLE 1 On a map with the scale '1 cm represents 100 km', the Nile River in Egypt is 67 cm long. How long is the Nile River in real life?

2 A street map uses a scale of '1 cm represents 200 m'.

a Find the actual distance, in kilometres, represented by each scaled distance.

i 7 cm **ii** 9.5 cm **iii** 12.4 cm

b Find the scaled distance, in centimetres, used to represent each actual distance.

i 7 km **ii** 1500 m **iii** 3.3 km

Remember: 1 km = 1000 m.

Check your units!

3 Lord Howe Island is 2.8 km long. A map of the region has a scale of '1 cm represents 0.5 km'. Calculate the length of the island on the map.

4 A map has a scale of 1 : 50 000. What distance is represented by 64 mm on the map? Give your answer in metres. C

5 The town of Hughenden is 216 km north of Winton. On a map with a scale of 1 : 2 600 000, what is the scaled distance between the 2 towns? Answer to the nearest millimetre.

6 The Dunn family are travelling around Queensland and they decide to stay in Longreach for a few days. They have a map of Longreach with a scale of 1 : 16 000. PS

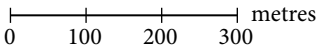
a The children decide to walk to the skate park. On the map, the distance from their motel to the park is 3 cm. How far is the walk from the motel to the skate park?

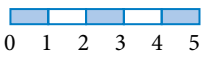
b Dad likes to do a 4 km run each day when he is on holiday. How far is this distance on the map?

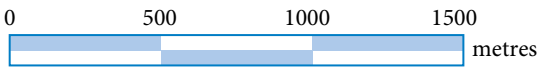
c The family walked from their motel to the Arts and Cultural Centre, down the main street to the railway station and back to the motel. This distance is 10.4 cm on the map.
How far have they walked? Round your answer to the nearest 100 m.

Geoff Marshall/Alamy Stock Photo

7 For each map scale, what length does 1 cm represent?

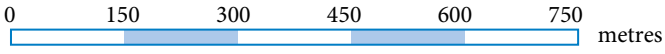
a


b


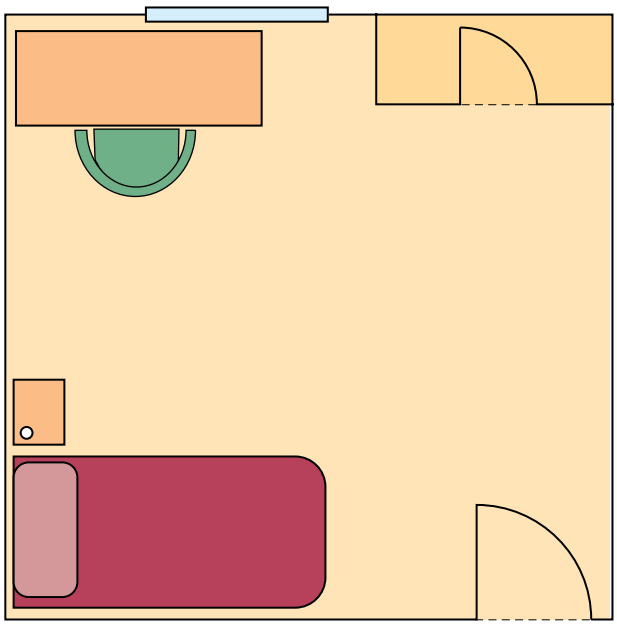
c


d


e


f


8 EXAMPLE 2 A scale plan of a bedroom is shown below.



Scale 1 : 50

By measurement and calculation, find the actual:

a length of the bedroom
b width of the doorway
c length of the bed
d length of the window
e length of the table
f width of the bed
g perimeter of the room
h floor area of the room

9 Measure the length of each scale drawing, then use the scale to calculate its actual length.

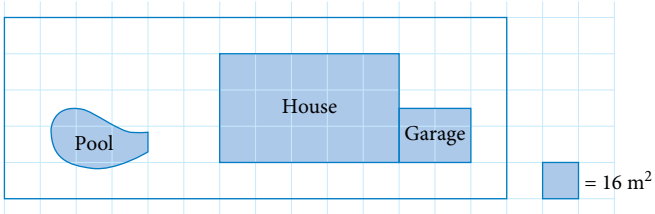
a Scale: 1 cm represents 0.7 m

Tamaguramo/Shutterstock.com

b Scale: 1 : 24

AlexMaster/Shutterstock.com

10 This scale diagram shows the position of the house, garage and the pool on a block of land.



a Determine the actual area of the house in square metres (m^2).
b Estimate the actual area of the pool in square metres (m^2).
c Determine the actual area of the block of land in square metres (m^2).
d What are the dimensions of the block of land in metres? C PS

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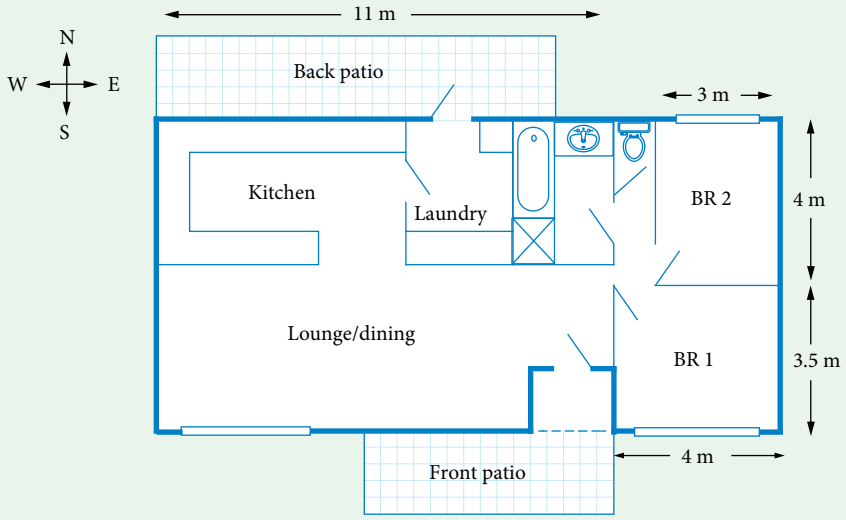
5.2 Reading house plans

Before any house or building can be built, a plan must be drawn up. House plans use many different symbols and abbreviations. They are either drawn to scale or have measurements written on them. Often, measurements are shown in **millimetres** to avoid the use of decimal points, which can lead to errors in printing and reading.

Worksheets
Scale drawings
Interpreting an office plan

EXAMPLE 3 Reading plans

This is the floor plan for Grant's house from Chapter 4, *On the surface*.



a What are the dimensions of Bedroom 1?

b What is the area of Bedroom 1?

c What is the length across the front of the house?

d What is the width of the house excluding the patios?

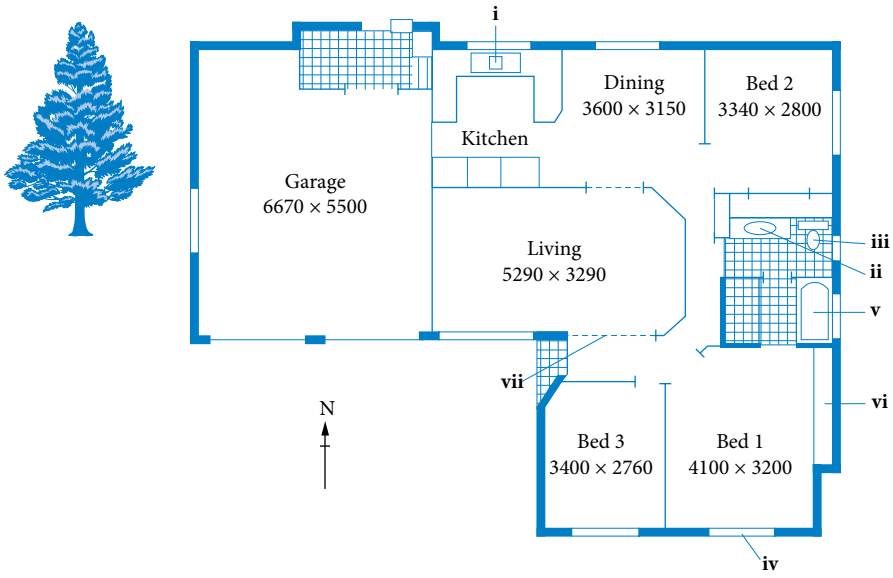
e By measuring the width of Bedroom 2, **determine** what scale has been used in drawing this plan.

Steps	Working
a Find bedroom 1 on the plan. The dimensions are the length and the width.	The dimensions of bedroom 1 are 4 m by 3.5 m
b Area = lw	area = 4×3.5 $= 14\text{ m}^2$
c Find the full length from the plan.	length = 11 + 4 = 15 m
d Find the full width from the plan.	width = 4 + 3.5 = 7.5 m
e Measure the width of Bedroom 2. 1.5 cm on the plan equals 3 m in real-life. Change m to cm.	width = 1.5 cm 1.5 cm : 3 m = 1.5 cm : 300 cm = 1.5 : 300 = 1 : 200 (dividing both by 1.5) The scale used is 1 : 200.

EXERCISE 5.2 Reading house plans

ANSWERS p. 437

1 This is the plan for Menhal's new house.



a What does each symbol used on the plan represent?

i  **ii**  **iii** 

iv **v**  **vi** 

b Three areas on the plan are shaded with this pattern.  What does this mean?

c How many bedrooms does the house have?

d Which bedrooms have built-in wardrobes?

e What are the dimensions of bedroom 1?

f Where is the laundry?

g Where is the linen press?

h How many bathrooms does the house have?

i How many toilets are there?

j How many doors lead into the bathroom?

k Which room has dimensions 5.29 m by 3.29 m?

l What are the dimensions of the garage?

m If Menhal looks out of the window of each room mentioned below, in which direction is she facing?

i living room **ii** dining room **iii** bedroom 1

vi bedroom 2 **v** garage

n In which direction do the garage doors face?

o If Menhal stands on the front doorstep, can she see into the bathroom?

p If Menhal stands inside the front doorway, list the 4 rooms she can see into.

q If Menhal is working in the kitchen, list the 3 rooms she can see into.

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2 a How many bedrooms does this house have?

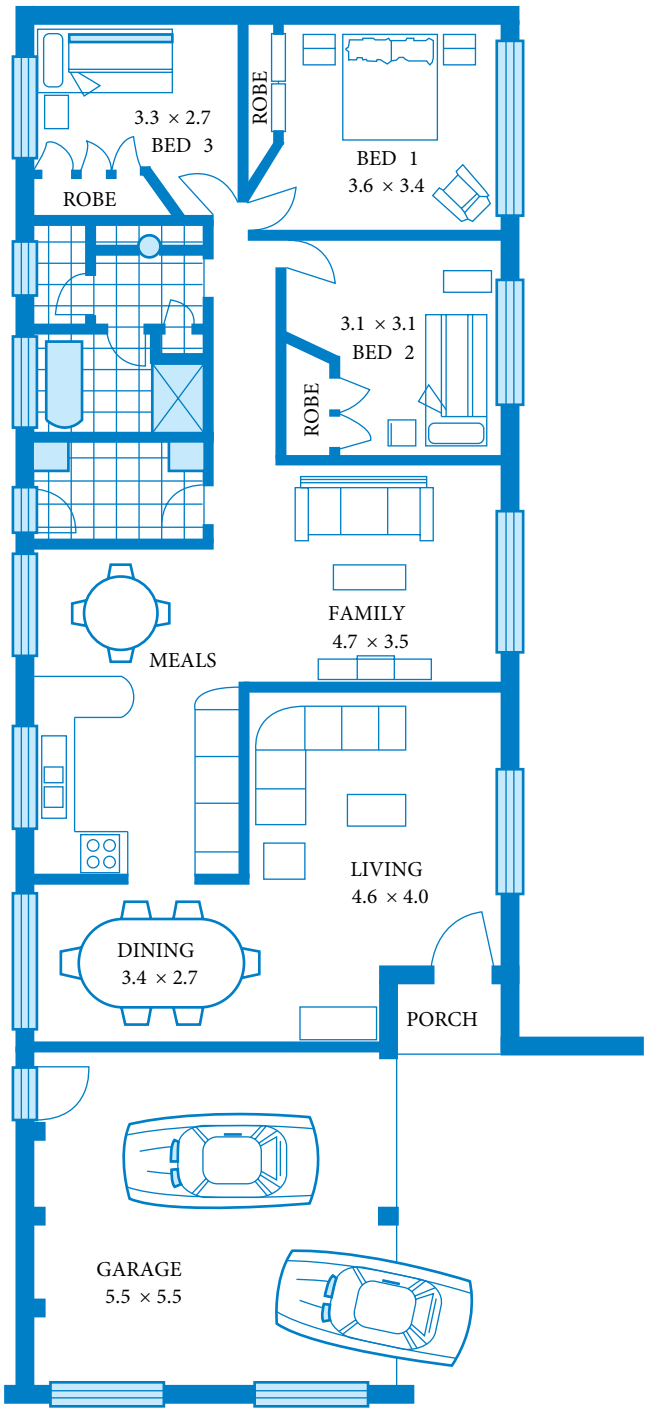
b How many of the bedrooms have built-in wardrobes?

c Is there an ensuite bathroom?

d What are the dimensions of the family room?

e Which bedroom has the largest floor area?

f The floor area of the house is 128 square metres. Calculate the cost of building this house at the rate of $2300 per square metre.



3 EXAMPLE 3 This house plan has a scale of 1 : 100.

Measure and calculate:

a the length of the main bedroom

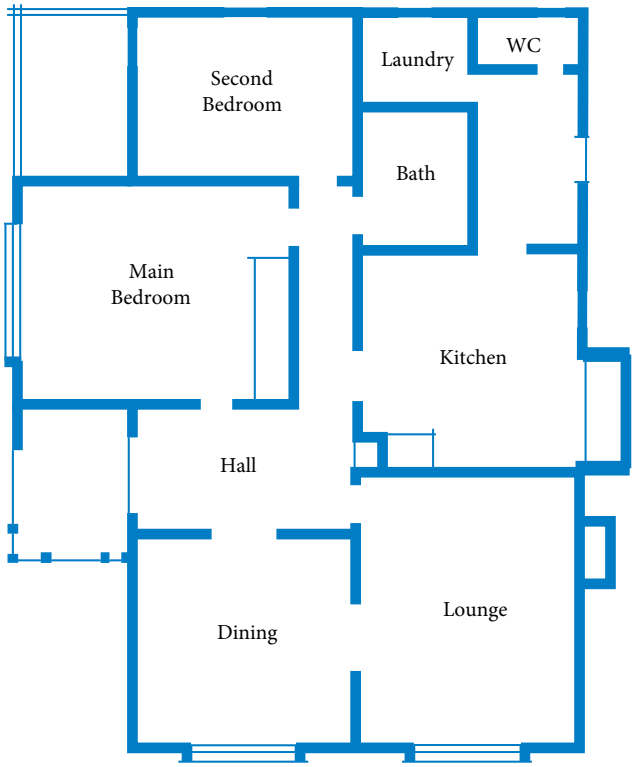
b the length of the window in that room

c the length of the laundry

d the area of the bathroom

e the longer side of the lounge room

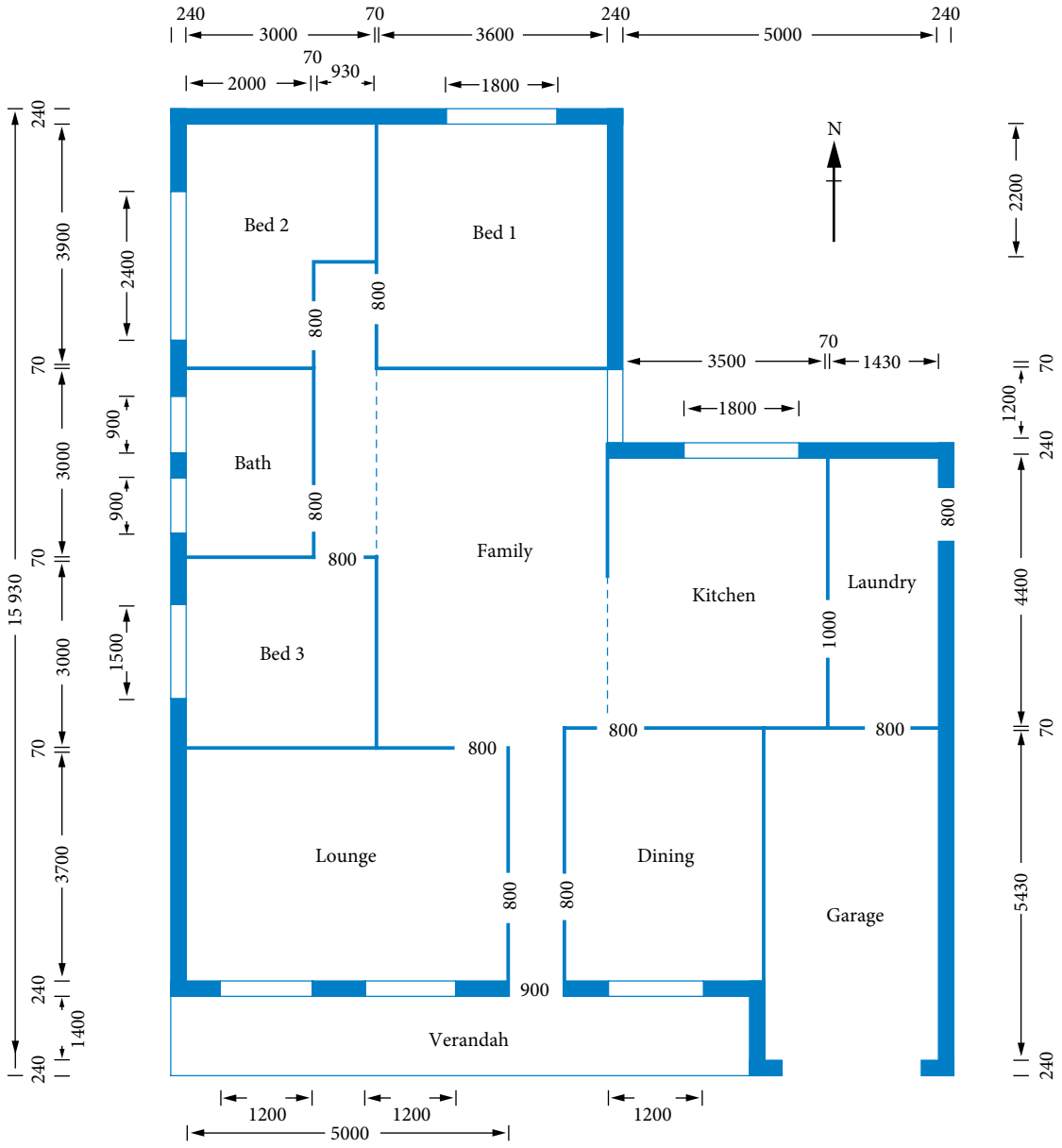
f the area of the dining room.



Scale 1 : 100

Fir Mamat/Alamy Stock Photo

4 Josie and Dan are buying the house with this floor plan.



a What is the thickness of the external walls of this house in mm?

b What is the thickness of the internal walls?

c What is the front width of the house in metres?

d What is the length of the left side of the house?

e What are the dimensions of the family room?

f By measuring the length of the lounge, determine what scale has been used in drawing this plan.

5 **a** All measurements on this plan are in metres. What are the dimensions of the family room? Express your answer in mm.

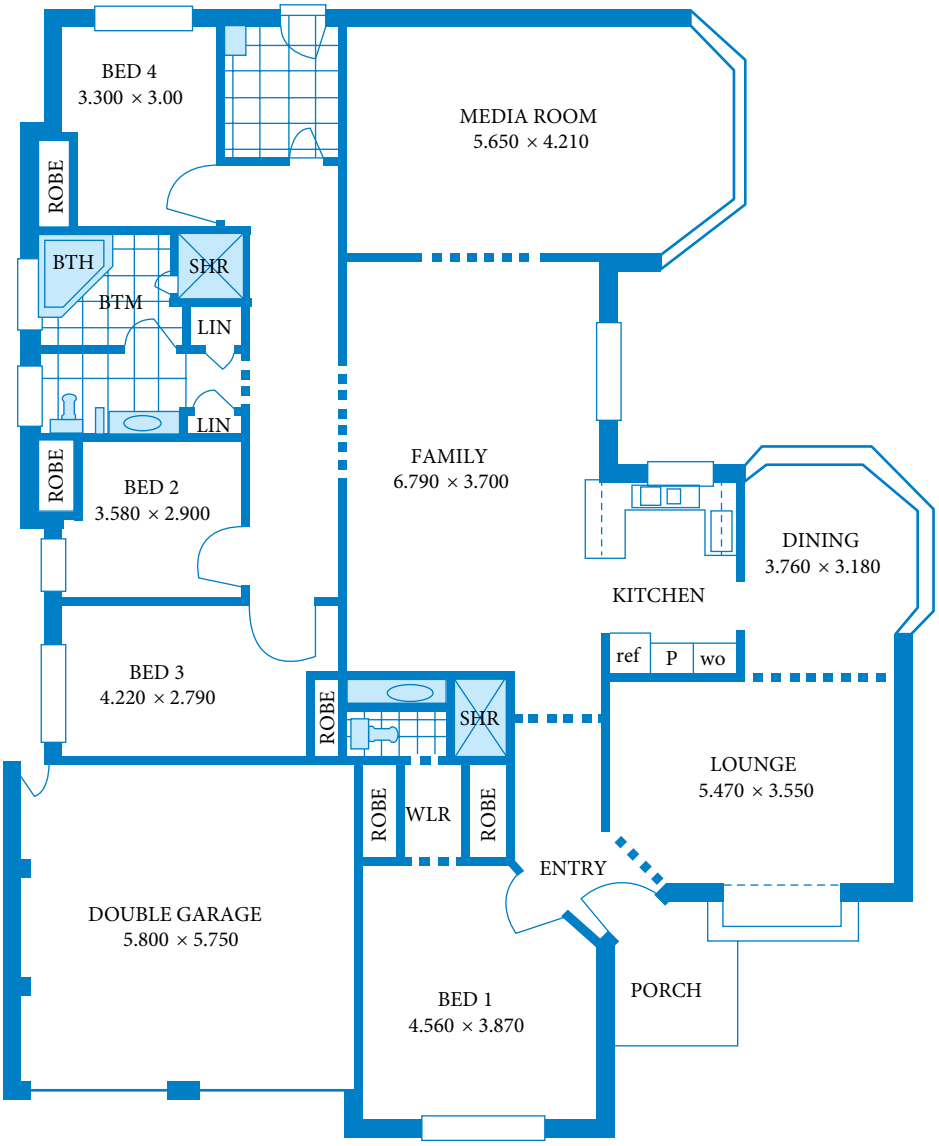
b How many toilets are there?

c Where are the 2 linen presses?

d What does 'SHR' stand for?

e Which bedroom has the smallest floor area?

f Which is bigger in area: the family room or the media room? By how many square metres? PS



Investigation My home

There are many websites that contain house floor plans. Search for 'project homes' on the internet and choose some floor plans for houses that you like.

For each plan:

- calculate the floor area of the whole house
- if possible, obtain an estimate for the cost of building the house
- calculate the cost per square metre.

Compare and contrast the features of each house.

Decide which house you prefer based on the information you have found above and present your findings.

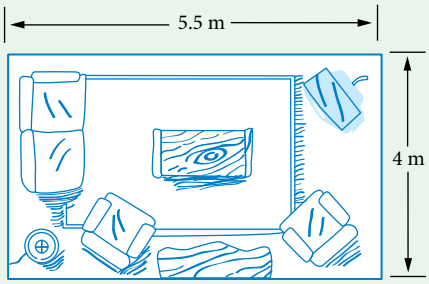
Lev Kropotov/Shutterstock.com

5.3 Renovate my house*

We use house plans to calculate the materials required for a job and to estimate costs.

EXAMPLE 4 Calculating costs

This is a diagram of Jackie's family room.



Jackie is laying cork tiles on the family room floor. The tiles cost $37.50 per square metre.

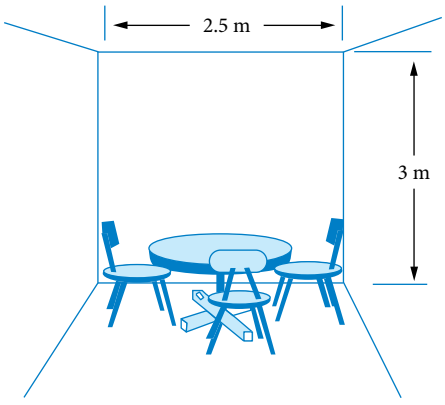
Calculate how much the cork tiles will cost in total.

Steps	Working
First, find the area of the floor.	area $= 5.5 \times 4$
	$= 22\,\text{m}^2$
Multiply the area by the price.	cost $= 22 \times \$37.50$
	$= \$825$

EXERCISE 5.3 Renovate my house*

ANSWERS p. 438

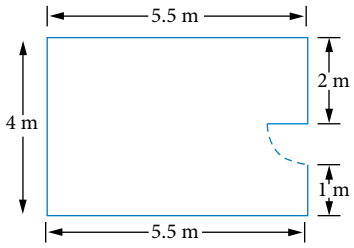
1 EXAMPLE 4 The dining room in Nicole's home unit is very small. To make it look bigger, she plans to cover one wall with mirror tiles. The wall she plans to cover is 3 metres high and 2.5 metres wide.



a What is the area of the wall?

b The mirror tiles Nicole has chosen cost $36.50 per square metre. What will be the total cost of the mirror tiles for the wall in Nicole's dining room?

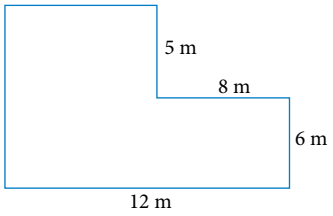
2 Luis is replacing the skirting boards in his lounge room.



a How many metres of skirting boards will he need?

b Simple skirting boards cost $17 per metre. Calculate the total cost of the skirting boards for the job.

3 The floor area of Jesse's new house is shown.



a What is the floor area of this house?

b Using a cost of $1900 per square metre, calculate the amount Affordable Homes charges to build this house.

Use the floor plan for Grant's house in Example 3 on page 129 to answer Questions **4** to **7**.

4 Grant is renovating Bedroom 2. The skirting boards need replacing.

a Calculate the perimeter of Bedroom 2.

b The door is 1 m wide. How many metres of skirting board are needed?

c Solid timber skirting boards cost approximately $75 per metre. What will be the cost of the skirting boards?

5 Grant is also replacing the carpet in Bedroom 2.

a How many square metres of carpet will he need?

b Carpet comes in rolls that are 3.6 m wide. What length of carpet will Grant need, rounded up to the nearest metre?

c The carpet he likes costs $129.95 a metre. How much will the carpet cost in total?

6 a Grant also needs to paint the ceiling in Bedroom 1. What is the area of the ceiling? PS

b The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2 of the ceiling.

i Calculate the number of litres of paint they require.

ii The ceiling paint costs $49.90 for a 4-litre can. Calculate the cost of paint for the ceiling.

7 a In what order should Grant do the renovations from Questions **4**, **5** and **6**? Give reasons for your answer. PS

b What is the approximate total cost of these renovations?

Use the floor plan for Josie and Dan's house on page 133 to answer Questions **8** to **11**.

8 Josie and Dan have decided to replace all the floor coverings before they move into the house. They are going to lay wood parquet flooring in the lounge room. The flooring costs $92 per square metre. How much will the flooring cost?

9 Other rooms need new carpet. Josie and Dan have chosen a carpet that is 3.66 m wide. How many metres of carpet are required for:

a bedroom 1? **b** bedroom 2? **c** bedroom 3?

10 The kitchen and laundry are to have new tiled floors. The tiles cost $41.10 per square metre, allowing for breakage and wastage. How much will it cost to buy tiles for the floor? Answer correct to the nearest dollar. PS

11 Josie and Dan are expecting a baby and they are going to make bedroom 2 the baby's room. They want to decorate the room with a frieze pattern around the walls. It will go above the door and the windows.

Elizabeth Whiting & Associates/Alamy Stock Photo

a How many metres long will the frieze be?

b Each roll of frieze is 5 m long. How many rolls of frieze will Josie and Dan need to buy?

12 Josie and Dan were surprised by the estimated cost of their renovations. Before signing off on the purchase of this house, they checked out the cost of building a project home of similar style, with the same floor area as this house. The cost of the project home was approximately $2600 per square metre. Calculate the estimated cost of building this project home.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Worksheet
Scale drawings

5.4 Constructing scale drawings*

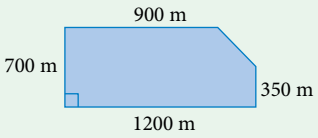
To make a scale drawing, start with a rough sketch, including the required measurements. Then choose a scale and draw the diagram accurately.

Lengths on scale drawings

To calculate a scaled length, divide the actual length by the scale.

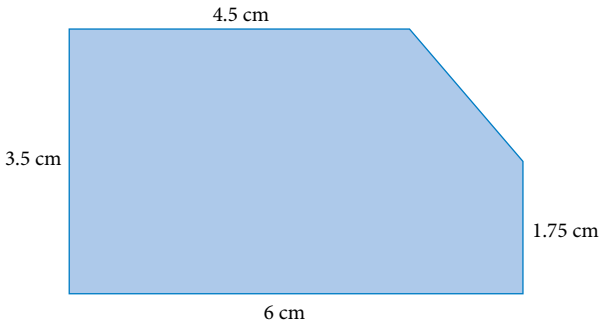
EXAMPLE 5 Creating scale drawings 1

This is a sketch of Farmer Freda's field.



a **Construct** a scale drawing of the field.

b What is the length of the unknown side of the field?

Steps	Working
a Choose a suitable scale – we want it to fit easily on the page.	Use the scale: 1 cm represents 200 m (which is the same as 1 : 20 000).
Calculate the scaled length for each measurement by dividing by the scale.	900 ÷ 200 = 4.5 cm 700 ÷ 200 = 3.5 cm 1200 ÷ 200 = 6 cm 350 ÷ 200 = 1.75 cm
Use these measurements to draw your scale drawing.	
b Measure the unknown length on the scale drawing and multiply by the scale.	measured length = 2.3 cm 2.3 × 200 = 460 m The unknown side is 460 m.

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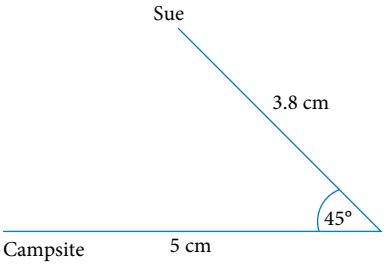
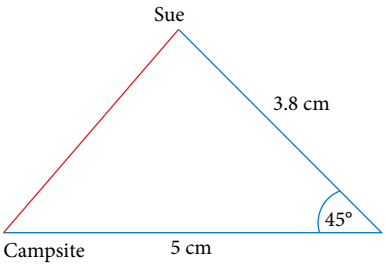
EXAMPLE 6 Creating scale drawings 2

Cleo is going on a bushwalk from her camping site. She walks 2.5 km due east and then 1.9 km northwest.

iStock.com/Pixdeluxe

a **Construct** a scale drawing of Cleo's walk.

b Use your scale drawing from part **a** to **determine** how far Cleo is from her campsite.

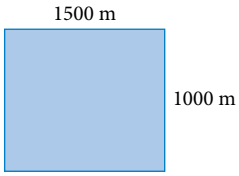
Steps	Working
a Choose a suitable scale.	Use the scale: '1 cm represents 0.5 km' (which is the same as 1 : 50 000).
Find the scaled measurements by dividing by the scale.	$2.5 \div 0.5 = 5$ cm $1.9 \div 0.5 = 3.8$ cm
Use a ruler and a protractor to construct the scale drawing. Northwest is at an angle of 45° (halfway) between west and north.	
b Join the starting point to the finishing point and measure this line.	
	The line is 3.5 cm long.
Find the actual distance by multiplying by the scale.	actual distance $= 3.5 \times 0.5$ $= 1.75$ km
	Cleo is 1.75 km from her campsite.

EXERCISE 5.4 Constructing scale drawings*

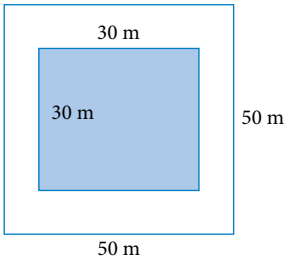
ANSWERS p. 438

You will need a ruler and a protractor to complete this exercise, or you can use drawing/designing software. In every question, make sure you state clearly what scale you have used.

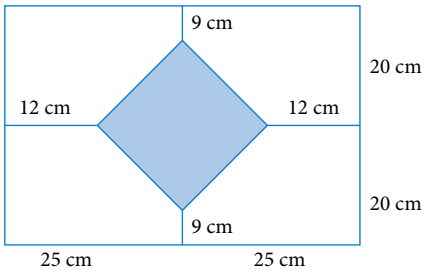
1 EXAMPLE 5 Make a scale drawing of this field.



2 Antje drew a diagram of a courtyard. Make a scale drawing of the courtyard.



3 Samantha designed this cutting plate. Make a scale drawing of her cutting plate.

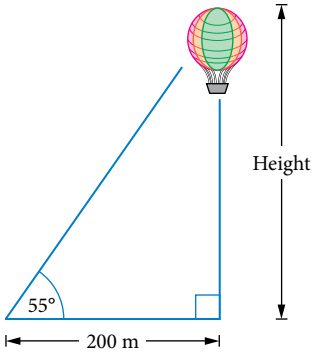


4 EXAMPLE 6 Hamish and Jacob are setting off on a hike. They walk 5 km due west of their starting point and then turn to walk 7 km south. They stop for lunch and then walk another 6 km northeast before stopping for afternoon tea.

a Make a scale drawing of Hamish and Jacob's walk.

b How far are they from their starting point?

5 Construct a scale drawing to calculate the actual height of the hot air balloon above the ground.



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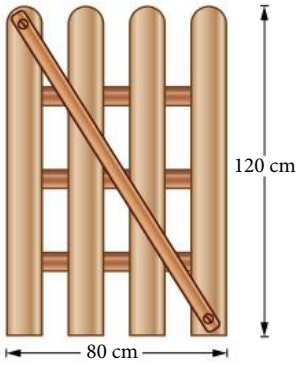
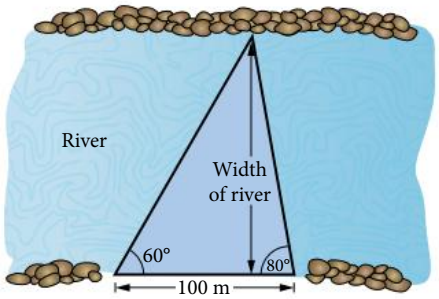
6 An old wooden gate 80 cm wide by 120 cm high needs a diagonal brace for support.



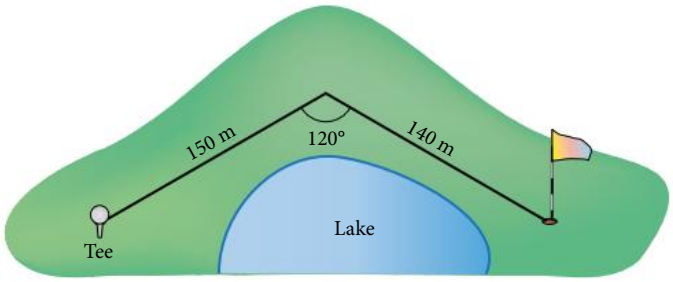
a Construct a scale drawing of the gate and find the actual length of the brace.

b Use Pythagoras' theorem to check the answer you obtained from the scale drawing. Answer correct to one decimal place. PS

7 Jodie wants to swim across the river. Draw this diagram to scale and calculate the width of the river. PS



8 A golf course has a large lake as an obstacle. Most golfers follow the dog leg around the lake. How far is it straight across the lake, from the tee to the hole? Use a scale drawing to answer this question.



9 Johann makes boxes for storing children's toys, as shown. C PS

Richard Peterson/Shutterstock.com

a The width of the box is 40 cm. Construct a scale diagram of the net of this box, using a scale of 1 : 20.

b Calculate the area of materials needed to construct one of these boxes. Answer in square metres.

Investigation — Make my own scale drawing

1 Choose something that is a large rectangle, for example, a paved area, a quadrangle, a brick wall, a whiteboard or the classroom floor.
2 Measure the length and width of the rectangle.
3 Make a scale drawing of your rectangle. Be sure to include the scale you used.
4 Measure the length and width of the school reception area or foyer.
5 There are plans to enlarge the length and width of the area in step 4 by 50%. What will be the new dimensions of this area?
6 Draw a scale diagram of the enlarged area. Add the position of furniture and any other items in the school entrance area. Show the scale you used.

Keyword activity

WORD MATCH

ANSWERS p. 438

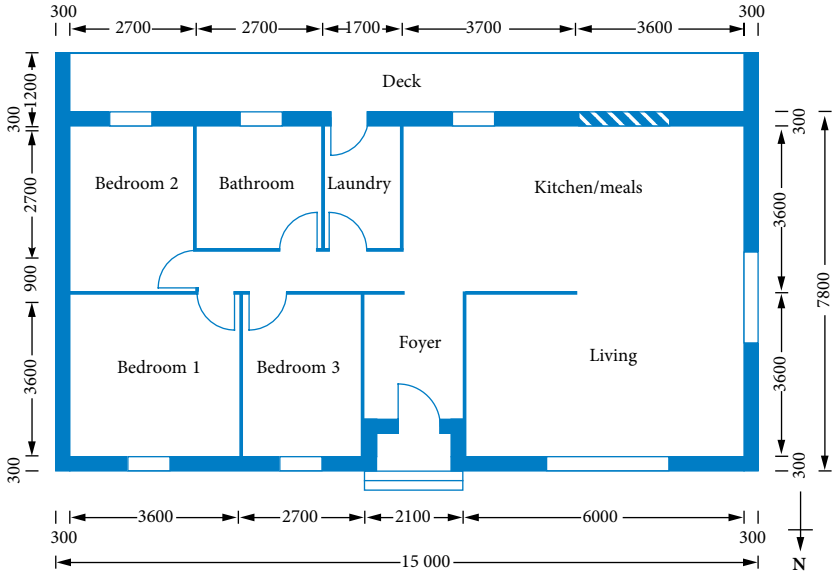
Match each word in the left column to its correct meaning in the right column.

Words		Meaning	
1	centimetre	A	A unit used to measure distances between towns
2	dimensions	B	Represents real objects that are too big to draw with the actual measurements
3	enlarge	C	The relationship between an actual object and its diagram
4	house plan	D	One-thousandth of a kilometre
5	kilometre	E	To make a drawing or object larger
6	metre	F	Labels that illustrate features on a house plan
7	millimetre	G	One-hundredth of a metre
8	reduce	H	A diagram showing rooms and measurements of a house
9	scale	I	The unit of length used in house plans
10	scale drawing	J	The length and width of a room
11	symbols	K	To make a drawing or object smaller

Solution to the chapter problem

Problem

Francesca and Michael are building a house according to the plan shown. They need to order carpet for the living room and the 3 bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.



Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

To find the amount of carpet needed for the 4 rooms.

We are given a floor plan with the dimensions of the rooms on it.

STAGE 2: SOLVE THE PROBLEM

SOLVE

First, read off the dimensions for each room.

Living room: 6000 mm × 3600 mm

Bedroom 1: 3600 mm × 3600 mm

Bedroom 2: 2700 mm × (2700 + 900) mm = 2700 mm × 3600 mm

Bedroom 3: 2700 mm × 3600 mm

As the problem asks for how many **square metres** of carpet are needed, measurements should be changed to metres by dividing by 1000 before we calculate the areas.

Living room: 6 m × 3.6 m Bedroom 1: 3.6 m × 3.6 m

Bedroom 2: 2.7 m × 3.6 m Bedroom 3: 2.7 m × 3.6 m

Area of living room = 6 m × 3.6 m

= 21.6 m^2

Area of bedroom 1 = 3.6 m × 3.6 m

= 12.96 m^2

Area of bedroom 2 = 2.7 m × 3.6 m

= 9.72 m^2

Area of bedroom 3 = 2.7 m × 3.6 m

= 9.72 m^2

Add the areas together:

Total area = 21.6 + 12.96 + 9.72 + 9.72

= 54 m^2

STAGE 3: CHECK THE SOLUTION

CHECK

The size of each room seems realistic, and we have used the correct area formula. The answer is about right.

STAGE 4: PRESENT THE SOLUTION

PRESENT

Francesca and Michael will need 54 m^2 of carpet.

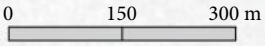
5 Test yourself

From paper to reality

ANSWERS p. 438

1 EXERCISE 5.1 Measure the height of the building in the photograph below and work out the actual height of the building. Scale: 1 : 250

Karamysh/Shutterstock.com

2 EXERCISE 5.1 On a tourist map of Brisbane, the scale is given by 

a Write this scale as a simplified ratio.

b Find the actual distance between the following places given the scaled distance:

i Parliament House to Queens Gardens (3.8 cm)

ii Roma St Station to Queen St Mall (6.7 cm)

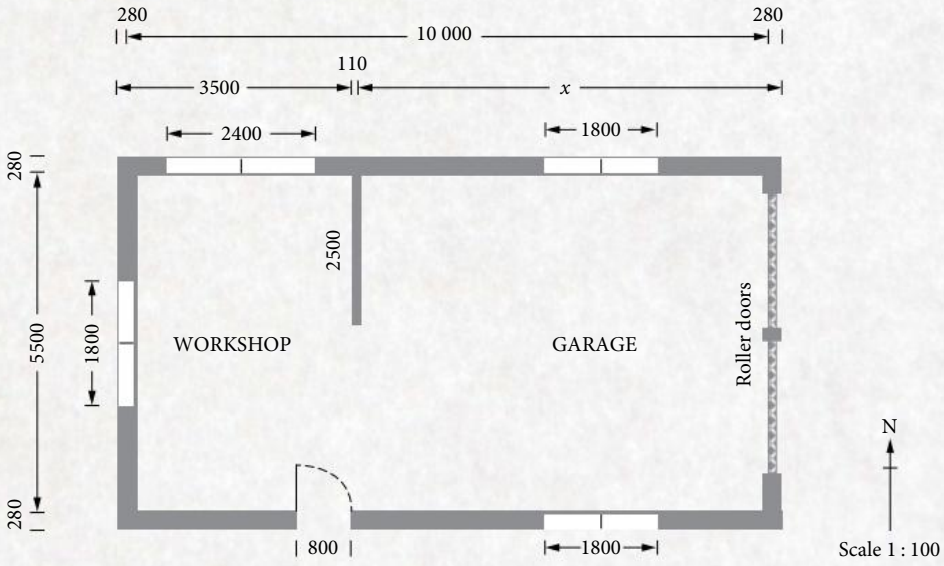
c Find the scaled distance between the following places given the actual distance:

i Botanic Gardens to Eagle St Pier (415 m)

ii St John's Cathedral to the Supreme Courts (1.1 km)

d The Kurilpa pedestrian bridge measures 2.6 cm on the map. How long is it?

3 EXERCISE 5.2 This is the plan for a workshop and garage. Measurements are in millimetres.



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a What is the width of the external walls of the building?

b What is the width of the wall between the workshop and the garage?

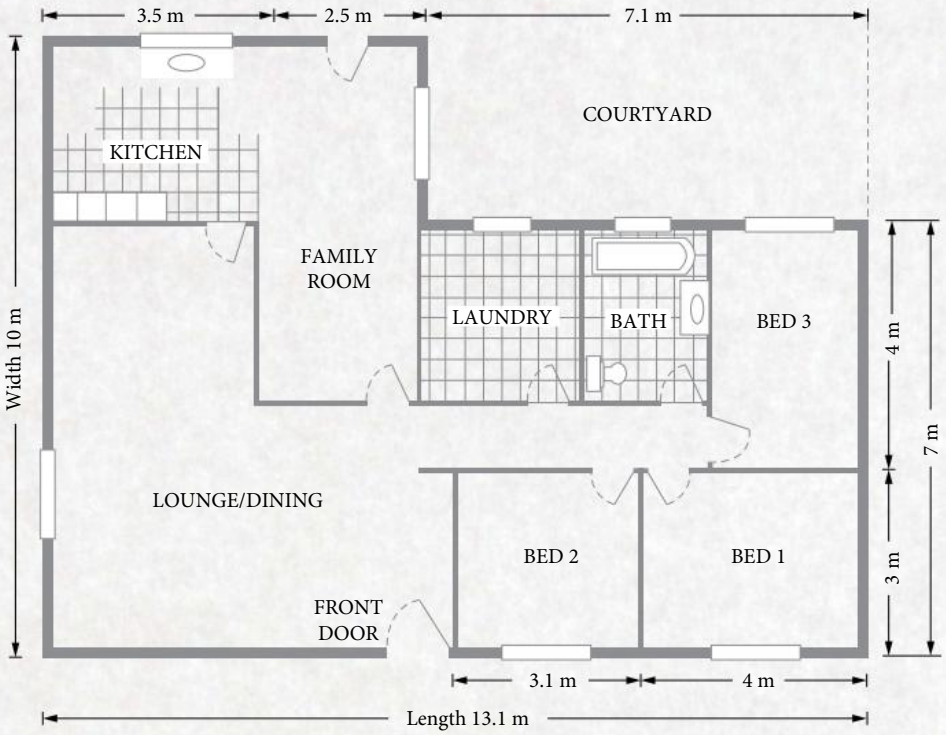
c Find the internal width of the building.

d What are the dimensions of the workshop?

e What are the dimensions of the garage?

f By measuring the length of a garage window, determine what scale has been used in drawing this plan.

4 EXERCISES 5.2, 5.3 This is the plan for the house that Sam and Nic are buying.



a How many doors are shown on this plan?

b What are the dimensions of the house?

c Calculate the width of the kitchen.

d **i** Find the floor area of the house.

ii An estimate for the cost of building a house is \$1344/m^2. Estimate the cost of building this house, correct to the nearest dollar.

e What percentage (correct to one decimal place) of the house's floor area is taken up by bedroom 1?

f A builder is going to tile the courtyard. What is the area of the courtyard?

g To allow for cutting and fitting, a builder always buys 10% more tiles than required to cover a floor area.

i How many square metres of tiles should the builder buy for the courtyard, including the extra 10%?

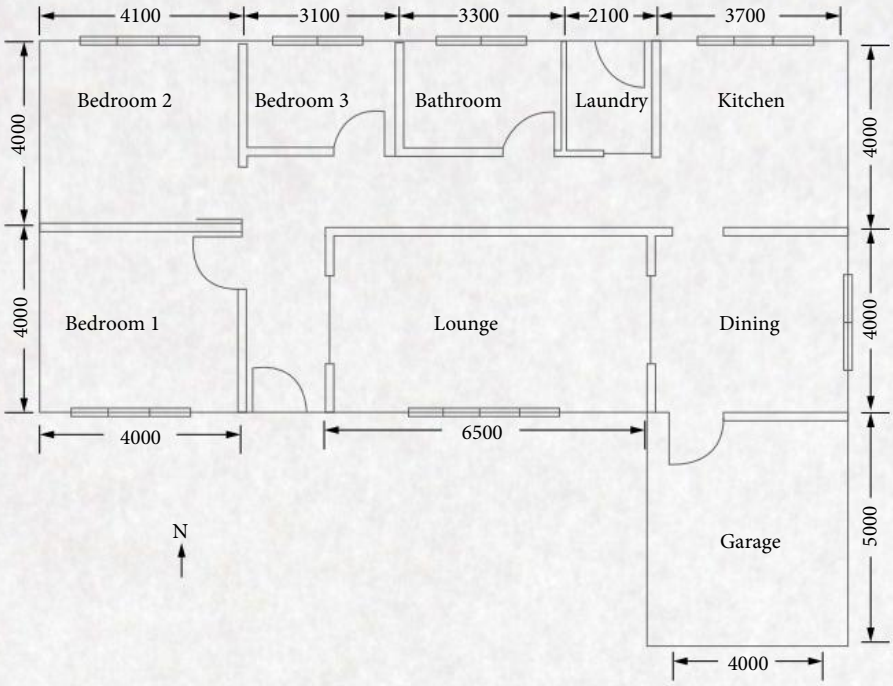
ii If each tile measures 32 cm by 32 cm, how many tiles are needed in total?

h The guttering across the front of the house needs replacing.

i How much guttering is needed?

ii The guttering company charges a fee of \$120 plus \$71.75 per metre to supply and install the guttering. How much will they charge to gutter the front of the house?

5 EXERCISE 5.3 This is the floor plan for Harry and Meghan's house. They have decided to renovate some areas of the house.



a Harry and Meghan are going to lay wood parquet flooring in the lounge room. The flooring costs $68 per square metre. How much will the flooring cost?

b They are going to replace the skirting boards in Bedroom 1. Assume the door is 820 mm wide.

i How many metres of skirting board are needed?

ii Skirting boards cost approximately $30 per metre. What will be the cost of the skirting boards?

c The tiles on the floor of the bathroom and laundry need replacing. The width of these rooms is 2300 mm.

i Calculate the total floor area of the bathroom and laundry.

ii Each tile is 200 mm by 200 mm. How many tiles are needed?

iii The tiles cost $41.10 per square metre, allowing for wastage. How much will the new tiles cost?

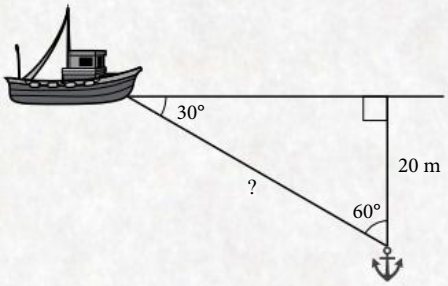
d The ceiling of the lounge room needs to be repainted.

i What is the area of the ceiling?

ii The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2. How many litres of paint will be required to do the ceiling?

iii Ceiling paint costs $34.90 for a 4-litre can. Estimate the cost of painting the ceiling.

6 EXERCISE 5.4 Neil and Ted are keen scuba divers. When they moor their boat, the angle between the anchor rope and the top of the water is 30°. Construct a scale drawing to calculate what length anchor rope they need if the water is 20 m deep.



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Practice set 1

ANSWERS p. 438

Section A Multiple choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

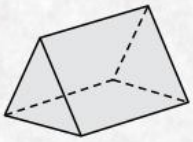
1 EXERCISE 5.1 Evan is using a map with a scale of 1 : 20 000. The distance from home to the shops on the map is 3.4 cm. What is the actual distance?

A 57 m **B** 680 m **C** 571 m **D** 6800 m

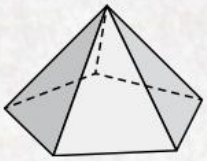
2 EXERCISE 3.2 Which solid shown is a prism?

A

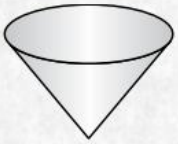
B



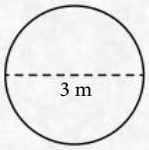
C



D



3 EXERCISE 4.5 Which calculation could be used to find the area of this circle?



A $\pi \times 3$ **B** $\pi \times 3^2$

C $2 \times \pi \times 1.5$ **D** $\pi \times 1.5^2$

4 EXERCISE 1.7 Express 10.7 days in days, hours and minutes.

A 10 days 16 hours 48 minutes **B** 10 days 16 hours 8 minutes

C 10 days 8 hours 40 minutes **D** 10 days 7 hours 48 minutes

5 EXERCISE 5.3 Jack and Jesinta decide to repaint their ceilings. The total area of their ceilings is 90 m^2 and they require 2 coats of paint. 1 L of paint covers 12 m^2. Paint costs $34.90 for a 4 L can.

How much does the paint cost to repaint the ceilings?

A $69.80 **B** $139.60 **C** $261.75 **D** $523.50

6 EXERCISE 4.2 Find the area of a rectangular window that is 2 metres long and 90 cm wide.

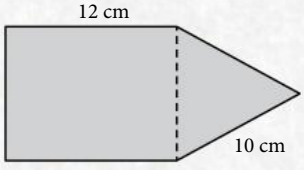
A 0.18 m^2 **B** 1.8 m^2 **C** 18 m^2 **D** 180 m^2

7 EXERCISE 3.1 I am a quadrilateral with equal opposite sides. My diagonals bisect each other and meet at right angles. What am I?

A parallelogram **B** trapezium

C rectangle (non-square) **D** rhombus

8 EXERCISE 1.6 This shape is made up of a rectangle and an equilateral triangle. Calculate its perimeter.



A 22 cm **B** 44 cm

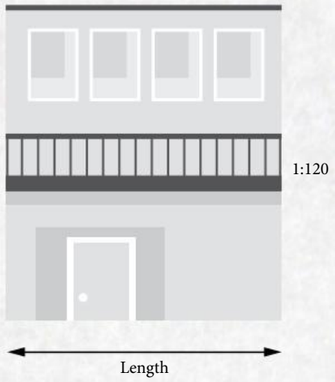
C 54 cm **D** 64 cm

Section B Short answer questions

1 EXERCISE 5.1 Measure the length of each scaled drawing, then use the scale ratio to calculate its actual length.

a

b



2 EXERCISE 1.2 Copy and complete each conversion.

a 6.4 m = _____ cm

b 240 mm = _____ cm

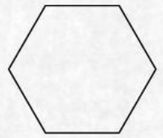
c 15.8 km = _____ m

d 9800 mm = _____ m

3 EXERCISE 3.3 Sketch a triangular prism.

4 EXERCISE 4.3 The front yard of a house is rectangular and measures 20 m by 8 m. The backyard is also rectangular and measures 35 m by 7.5 m. Calculate the cost of covering both yards with turf that costs $18.60/m^2.

5 EXERCISE 1.4 Miriam is creating a hexagonal brooch with equal sides of 1.2 cm. She is outlining it with silver wire. Calculate the length of wire needed.



6 EXERCISES 5.2, 5.3 Tim and Kim bought a house with the floor plan shown on page 131 in Exercise 5.2, Question **2**.

a What are the dimensions of the garage?

b Tim and Kim decide to paint the garage floor.

i How many square metres of paint will they need?

ii Paint costs $75/square metre. How much does the concrete cost?

c How many windows are there in this house?

d The carpet in Bedroom 1 needs to be replaced.

i How many square metres of carpet will they need?

ii Carpet costs $85/m^2. How much will it cost to replace the carpet?

e The ceilings in all 3 bedrooms need to be repainted with 2 coats of paint.

i How many square metres of ceiling need to be repainted?

ii One litre of ceiling paint covers 12 m^2. Superior white ceiling paint costs $49.90 for a 4 L tin. Calculate the cost of repainting the ceilings with 2 coats.

7 EXERCISE 3.1 Draw a neat diagram of each shape.

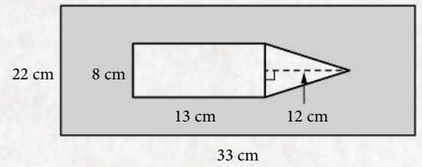
a An isosceles, obtuse-angled triangle

b A parallelogram with side lengths 4 cm and 2 cm

c A right-angled, scalene triangle

d A kite

8 EXERCISE 4.6 A metal cutting template is shown below.

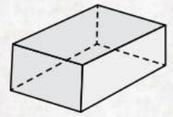


a Calculate the area of metal removed from the plate.

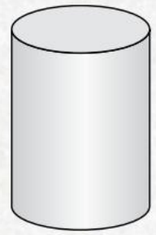
b Calculate the area of the remaining metal plate.

9 EXERCISE 3.4 Draw the net of each solid.

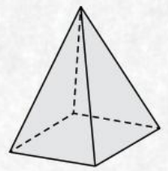
a



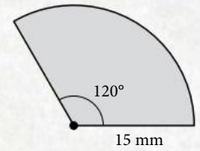
b



c

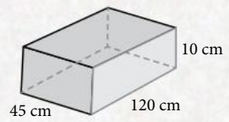


10 EXERCISE 1.5 Calculate the perimeter of this sector, correct to 2 decimal places.

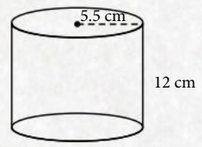


11 EXERCISES 4.7, 4.8, 4.9, 4.10 Find the surface area of each solid, correct to 2 decimal places where necessary.

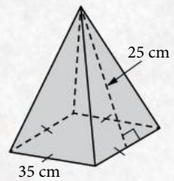
a



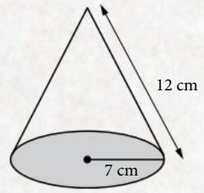
b



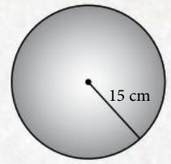
c



d



e



Chapter Problem

Currently, each packet of Ozbix breakfast cereal comes with a free little toy. There are 12 different toys available but there's only one toy in each packet, each equally likely. Jordan wants to collect every toy. How many packets of cereal should he buy to collect all 12 toys?

iStockphoto/PeopleImages

CHAPTER 6

MEASUREMENT, SCALES AND CHANCE: PROBABILITY AND RELATIVE FREQUENCIES

WILL IT HAPPEN?

What will we do in this chapter?

- Simulate chance situations with technology
- Use relative frequency to measure chance
- Discuss complications when simulating events

How are we ever going to use this?

- When deciding whether something is likely to happen
- Making predictions about chance events
- Knowing whether popular beliefs about chance events are true or false

9780170484664

Syllabus coverage

UNIT 3, TOPIC 3: PROBABILITY AND RELATIVE FREQUENCIES

Simulations

- Express probabilities formally using fractions, decimals, ratios and percentages.
- Perform simulations of probability experiments using technology.
- Recognise that the repetition of chance events is likely to produce different results.
- Identify relative frequency as probability.
- Identify factors that could complicate the simulation of real-world events [complex].

Worksheets (4):

6.2 Greedy pig game

6.3 A page of spinners • Coins probability • Dice probability

Spreadsheets (10):

6.1 Heads and tails

6.2 Rolling a die

6.3 Sports heroes • John Kerrich coin toss

6.4 50-year flood • Medical treatment • 4-child families • Sum of 2 dice • Freeway accidents

Solution to the chapter problem Breakfast cereals

Practice set 2 Medical treatment • Breakfast cereals

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

dice	die	experiment	prediction
probability	random	relative frequency	simulation

6.1 Heads or tails?

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Anyone who's been involved in playing or watching sport is familiar with tossing a coin. A coin may be tossed to decide things like:

- which team will kick off
- who will bat or serve first
- which goal a team will attack

A coin is tossed because it is believed to be a fair way to make decisions. Both players or teams have the same chance of winning, but sometimes there are claims that one team 'always loses the toss' or that the coin 'always comes up heads'. In this section you are going to investigate such claims. Fortunately, you don't have to spend a lot of time tossing coins and writing down the results. Spreadsheets can be used to simulate tossing a coin. Simulate means to pretend, imitate or model a real situation.

Relative frequency

The **probability** that an **event** will happen can be estimated by using the results of a simulation. The formula involves counting the number of times the event happened and writing it as a fraction over the total number of trials.

$$\text{relative frequency} = \frac{\text{number of times the event happens}}{\text{total number of trials}}$$

Relative frequency is sometimes called **experimental probability** because it is used to estimate the actual probability or **theoretical probability**.

EXAMPLE 1 Relative frequency

Toni tossed a coin 10 times and got 6 heads and 4 tails.

Calculate the relative frequency of getting a tail.

Steps	Working
There were 4 tails in 10 trials.	$P(\text{tail}) \frac{4}{10} = \frac{2}{5}$
relative frequency = $\frac{\text{number of times the event happens}}{\text{total number of trials}}$	P(tails) means 'probability of a tail'.
Write the answer.	In Toni's experiment, the relative frequency of getting a tail is $\frac{2}{5}$.
	Note: This is different to the theoretical probability that we will consider in Chapter 9, *Taking chances*.

Spreadsheet
Heads and tails

EXERCISE 6.1 Heads or tails?

ANSWERS p. 438

Download the 'Heads and tails' spreadsheet from Nelson MindTap. Use this spreadsheet to simulate tossing a coin repeatedly.

1 Run the simulation 10 times, concentrating only on the first 10 results highlighted in yellow.

2 Record the number of heads and tails each time.

3 Record the biggest number of the same (repeated) result, for example, 5 heads in a row or 3 tails in a row.

4 Make a note of any times the results alternate between heads and tails, that is, HTHTHTHTHT or THTHTHTHT.

5 Do heads always show exactly half the time?

6 In what percentage of the 10 trials were there exactly 5 heads and 5 tails?

7 What was the largest number of the same result in a row?

8 On how many occasions were the results alternating between heads and tails?

9 Run the spreadsheet simulation again. This time, concentrate on the bottom table that shows the percentage of heads. For how many coin tosses is the percentage of heads closest to 50%?

10 Which percentage value changes the most? Can you explain why?

11 Is it possible to tamper with a coin so that it doesn't show heads 50% of the time?

12 In recent years, it was discovered that some British coins were biased, that is, they didn't show heads 50% of the time. Search the internet to find out which coin it was.

13 Try this experiment: Stick a small amount of Blu Tack to one side of a coin. Toss the coin 20 times, recording the result each time. Which side showed more often - the side with or without the Blu Tack?

14 EXAMPLE 1 Run the spreadsheet simulation 3 more times, noting the number of heads that occurred in the first 10 tosses (the cells highlighted in yellow). Calculate the relative frequency involved each time.

15 Is the value of the relative frequency the same each time?

6.2 Rolling a die

'Dice' is the plural form of the word 'die'. When we have 2 or more, they are called dice, but if we only have one of them, it's a 'die'.
We are going to investigate what happens when we roll a normal, 6-sided die. We will use a spreadsheet simulation rather than do lots of die-rolling.

EXERCISE 6.2 Rolling a die

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Download the 'Rolling a die' spreadsheet from Nelson MindTap to complete this group activity.
The spreadsheet simulates rolling a die 24 times.

1 When we roll a normal 6-sided die, any one of the numbers from 1 to 6 can show.
Run the simulation once. Did each of the numbers from 1 to 6 occur the same number of times? Is this what you expected would happen? Why or why not?

2 Calculate the relative frequency for each number based on your simulation.

3 Will other groups in your class have the same probabilities? Give a reason for your answer.

4 Run the spreadsheet 4 times to simulate rolling a die 96 times. Calculate the total frequency for each of the numbers from 1 to 6.

5 Theoretically, in 96 rolls each number should show 16 times. How close did your results come to 16?

6 If you were rolling a real die and you put a small piece of Blu Tack on one side to make that side a little bit heavier, do you think that all the numbers would still show the same number of times? If not, which number would occur more frequently and which less frequently than the others? PS

7 Imagine you had a wooden die and you were able to use sandpaper to round one of the edges to make it smoother. What effect would this have on the frequency of the rolled numbers?

Worksheet
Greedy pig game

Spreadsheet
Rolling a die

TECHNOLOGY — Starting to play

You will need access to a random number generator to complete this activity. Suitable generators are available with a google search or an app on your phone.
Gamers in the board game Sam likes to play have to roll a 6 on a die before they can start. In the most recent game Sam had to roll the die 12 times before she rolled her first 6 and could start to play. She couldn't believe this was possible and she has asked you to explain!
The questions you are going to investigate are:

- How many rolls of a normal die are required to roll a 6?
- Will it always take this number of rolls? If not, within what range will the number of rolls usually lie?
- Determine a typical or average number of rolls required.

Working in pairs, generate random numbers from 1 to 6 to represent the rolls of a normal die. Record your results in a suitable table. Remember to record times when a previous number is repeated.

- Compare your results with other students in your class. Were they similar or different?
- Write a reply to Sam's complaints based on the results of your investigation.

Worksheets
A page of spinners

Coins probability

Dice probability

6.3 Relative frequency

Quite often, it is impossible to calculate the actual probability of an event. When unable to calculate a theoretical probability, using past records or perform an experiment can help to calculate the relative frequency.

$$\text{relative frequency of an event} = \frac{\text{number of times the event happens}}{\text{total number of trials}}$$

The **relative frequency** of an event is the frequency of the event as a fraction of the total frequency.

EXAMPLE 2 Relative frequency and probability

Emily is a park ranger. This table shows the data she recorded about kangaroo deaths in the park.

a **Calculate** the probability that the death of a kangaroo was caused by a motor vehicle. Express your answer as a decimal, correct to 3 decimal places.

b Use Emily's data to estimate the probability that a kangaroo in her area will die from old age. Express your answer correct to the nearest percentage.

Cause of death	Number of deaths
Hit by a motor vehicle	78
Shot	12
Caught in a fence or trap	11
Old age	25
Starvation	3
Other	7
No known cause	4
Total	**140**

Steps	Working
a Number of deaths due to motor vehicles = 78 Total = 140	$P(\text{killed by motor vehicle}) = \frac{78}{140}$ $= 0.557\,14\ldots$ $= 0.557$
b Number of deaths due to old age = 25 Total = 140	$P(\text{death from old age}) = \frac{25}{140} \times 100\%$ $= 17.8571\ldots\%$ $= 18\%$

iStock.com/Ross1248

EXAMPLE 3 Probability and relative frequency

Souraya counted the contents of 10 boxes of matches that were each labelled as containing 50 matches. Her results were:

53 49 50 48 52

51 50 49 50 51

a What is the relative frequency of a box containing exactly 50 matches?

b **Calculate** the probability that a box contains more than 50 matches.

Steps	Working
a Number containing 50 matches = 3 Total = 10	$P(50 \text{ matches}) = \frac{3}{10}$
b Number containing over 50 matches = 4 Total = 10	$P(\text{more than } 50 \text{ matches}) = \frac{4}{10}$ $= \frac{2}{5}$ The probability that a box contains more than 50 matches is $\frac{2}{5}$.

Usually, we write probabilities as fractions, but we can express probability using fractions, decimals, ratios and percentages.

EXAMPLE 4 Expressing a probability in different forms

There are 24 main meals on the menu at Ava's favourite restaurant. Only 5 of the meals are vegetarian. Ava is choosing a meal at random. What is the probability that she will choose a vegetarian meal? Answer as a fraction, decimal, ratio and percentage. Round the decimal and percentage answers to 2 decimal places.

Steps	Working
Write 5 out of 24 as a fraction.	$P(\text{vegetarian}) = \frac{5}{24}$
To convert to a decimal, either: Divide the numerator (top) by the denominator (bottom) OR Use the fraction to decimal toggle button on your calculator. S⇔D	$P(\text{vegetarian}) = 5 \div 24$ $= 0.208\,333\,33\ldots$ ≈ 0.21
Write the fraction in ratio form.	$P(\text{vegetarian}) = 5:24$
Multiply the fractional or decimal probability by 100% to convert to a percentage.	$P(\text{vegetarian}) = \frac{5}{24} \times 100\%$ $= 20.8333\ldots$ $\approx 20.83\%$

EXERCISE 6.3 Relative frequency

ANSWERS p. 438

1 EXAMPLE 4 Renee measured the tail lengths of a sample of adult quokkas on Rottnest Island, WA.

Tail length (cm)	Frequency
24	2
25	7
26	9
27	10
28	8
29	7
30	4
31	3
Total	**50**

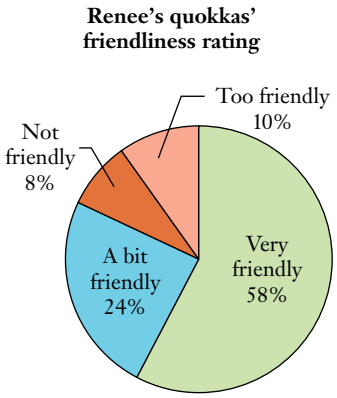
Jessika Knaupe/Shutterstock.com

a What is the probability that an adult quokka will have a 27 cm long tail? Answer as a fraction.

b Renee's wildlife manual says that adult quokka tails range from 25 to 30 cm long. What is the probability that an adult quokka will have a tail that is outside this range? Answer as a fraction, decimal, ratio and percentage.

c What is the most **likely** length for an adult quokka's tail?

2 Renee was surprised by how friendly the 50 quokkas were. She gave each animal a friendliness rating and displayed the data in a pie chart.



a Calculate the experimental probability that a randomly-selected quokka will be:

i not friendly (give your answer as a decimal)

ii very friendly or too friendly (give your answer as a fraction)

iii not friendly (give your answer as a ratio)

b What real-life situations could affect the probability that an individual quokka is friendly? C

3 In the past 12 months, when Jason visited his favourite restaurant, he noted that he had to wait for a table 8 times and he got a table straight away 16 times.

a How many times did Jason go to the restaurant?

b What is the relative frequency of 'not having to wait'?

c What is the experimental probability that next time Jason goes to the restaurant he will have to wait for a table?

4 Murphy's Law states that if anything can go wrong, then it will! Libby decided to test this theory by dropping a piece of toast and seeing whether it landed buttered-side up (good) or buttered-side down (bad). She performed 40 trials of her experiment, and her results were:

Buttered-side up: 5

Buttered-side down: 35

a What is the relative frequency of the toast landing buttered-side up?

b Use Libby's data to determine the probability that a dropped piece of toast will land buttered-side down. Answer as a decimal.

c Repeat Libby's experiment to determine the probability that the bread will land buttered-side up if it is knocked off the edge of a table.

Watch out! Make sure that you first cover the floor to avoid staining.

iStock.com/John shepherd

5 Mitchell visited the old whaling station in Albany, WA. He asked a random selection of people some questions about whaling and presented his results below.

Question	YES	NO
Do you agree with the international ban on killing whales?	19	1
Is it OK to make and sell souvenirs made from whale bones?	13	7
Are there too many whales in the oceans around WA?	2	18

a What is the probability that a person selected at random agrees with the international ban on killing whales?

b Calculate the probability that a person selected at random does not agree with making and selling souvenirs from whale bones. Express your answer as a decimal.

c What is the probability that a randomly selected person thinks there are too many whales in the oceans around WA?

d Predict the answers that a person who is a member of the 'Save our whales' group would give to each of Mitchell's questions.

e Suggest a factor that could create bias in Mitchell's results. C

6 Latu rolled a die 75 times and displayed the results in a table.

Number	Frequency
1	21
2	12
3	10
4	11
5	12
6	9

a What is the experimental probability of rolling 3 with this die?

b Copy and complete the table to show the experimental probability of rolling each number as a decimal, correct to 2 places.

Number on the die	1	2	3	4	5	6
Experimental probability			0.13			

c The theoretical probability of tossing each number on a normal die, to 2 decimal places, is 0.17. Latu believes his die is biased. Is he correct? Justify your answer.

Spreadsheet Sports heroes

7 Lauren loves to collect Sports Heroes cards. There are 10 different cards in the set and one of them is placed randomly in each packet of bubble gum. PS

a Predict the number of packets of gum that Lauren will need to buy to get all 10 cards.

b Download the 'Sports heroes' spreadsheet from Nelson MindTap.

c Run the spreadsheet 20 times to simulate buying 20 packets of gum. Record the number of packets required to get all 10 cards.

d Find the relative frequency of getting a full set of cards from fewer than 15 packets of gum.

e How is this situation more complicated in real-life? C

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Practical activity The game show problem

For this activity, each pair of students will need 3 cards from a deck of playing cards.

You are the contestant in a TV game show. Behind one of the 3 doors there is a car that you could win. The host asks you to choose a door and you choose Door 1. The host then opens *Door 3* to show you that the car isn't behind it. This means that the car is behind either Door 1 or Door 2. Should you stick with Door 1 or switch to Door 2?

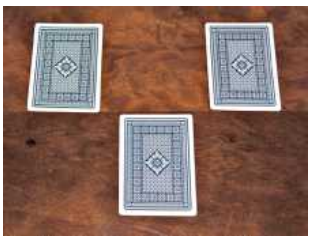
To simulate this problem, decide which of your 3 cards will be the 'CAR' card. Then you are going to work out the relative frequency that the car is behind the door that wasn't your first choice.

1 Decide who will be the game show host and who will be the contestant.

2 Copy this frequency table.

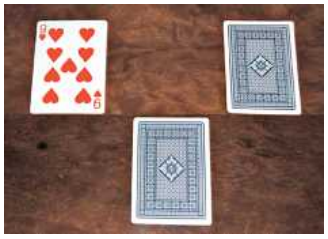
Outcome	Tally	Frequency
Car was first choice		
Car wasn't first choice		

3 The host shuffles the 3 cards, looks at them, then places them face down on the table.



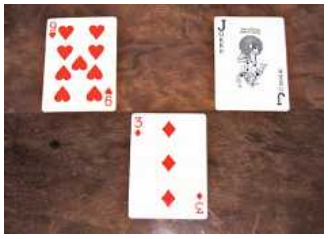
4 The contestant chooses one card and moves it down, as shown in the photo.

5 The host displays one of the 2 remaining cards that is *not* the car.



6 Turn over both the remaining cards. Record in your frequency table whether the car was the card first chosen by the contestant or whether it was the other card.

7 Perform the simulation at least 24 times.



8 Determine the relative frequency that the car is behind the door which wasn't the contestant's first choice.

9 Decide on the best strategy. Should the contestant stick with their original choice or switch?

Spreadsheet
John Kerrich coin toss

TECHNOLOGY — Prisoner of war John Kerrich

Download the spreadsheet 'John Kerrich coin toss' from Nelson MindTap to complete this investigation.

During the Second World War, John Kerrich, a British mathematician, spent his time as a prisoner of war conducting probability experiments. He tossed a coin 10 000 times and recorded 5067 heads. The spreadsheet simulates his experiment for only 400 tosses.

Run the simulation several times, then answer the following questions.

- What observations can you make about the percentage of heads as the number of tosses increases?
- For which number of tosses does the percentage of heads change the most?
- How does the difference between the number of heads and tails change as the number of tosses increases?

John Kerrich hypothesised that the greater the number the tosses, the closer the percentage of heads gets to 50%, but the difference between the number of heads and tails increases. How well does your data support his hypothesis?

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**John Kerrich coin toss**												
2									Press delete in the red square to generate a new set of data				
3													
4	Number of toss	Result	Head or Tail										
5	1	0	Head										
6	2	0	Head				**Heads**	**Tails**	**Percentage heads**	**Difference between number of heads and number of tails**			
7	3	1	Tail			Rolls 1 to 10	5	5	50	0			
8	4	1	Tail			Rolls 1 to 50	23	27	46	4			
9	5	1	Tail			Rolls 1 to 100	54	46	54	8			
10	6	0	Head			Rolls 1 to 200	103	97	51.5	6			
11	7	1	Tail			Rolls 1 to 300	153	147	51.00	6			
12	8	1	Tail			Rolls 1 to 400	198	202	49.5	4			
13	9	0	Head										
14	10	0	Head										
15	11	0	Head										
16	12	1	Tail										

Challenge!

Copy the data in row 404 down to row 504 to simulate 500 tosses, then extend the table to include 1 to 500 tosses. Deduce the formulas you'll need by looking at the formulas for the other rows.

6.4 Does bad luck happen often?

People often have the wrong idea about probability and chance. They may overestimate the chance of something good happening, for example, thinking it's easier to win lotto than it really is. They may also greatly underestimate the chance of something bad happening, such as being struck by lightning. The truth is that it is much more likely to be struck by lightning than to win a major prize in lotto.

EXERCISE 6.4 Does bad luck happen often? PS

ANSWERS p. 439

Work in groups to complete this exercise.

1 The 1-in-50-year flood

Download the '50-year flood' spreadsheet from Nelson MindTap.

Imagine you are buying a block of land that is in a 1-in-50-year flood zone. Will it flood very often?

A 1-in-50-year flood zone means that in every year, there is a probability of $\frac{1}{50}$ that there will be a major flood. On average, over the long run, this flood will happen once in every 50 years, but in the short run, there is no pattern to the frequency of floods.

Spreadsheet
50-year flood

a Run the flood simulation several times and make a note of the number of times there is a flood.

b How often does a flood happen? Run the simulation 5 times and make a note of the number of years between floods. Is there a pattern?

c Run the simulation again, looking for floods in 2 consecutive years (2 years in a row). How many simulations were required to get one with floods 2 years in a row?

d Do 1-in-50-year floods happen every 50 years?

e The real estate agent says to you: 'I know it's in a 1-in-50-year flood zone, but you don't have to worry. It flooded last year, so it will be 49 years before another flood.'

How would you answer him?

f What other factors are there, apart from rainfall, that contribute to flooding? C

g Can a location have more than one flood in a year? Research floods in Gympie in 1898.

h If a town is in a 1-in-50-year flood zone and it experiences a large flood, do you think the chance of experiencing another large flood a few weeks later if there is more heavy rain is greater or smaller than $\frac{1}{50}$? Give some real-world reasons to justify your answer.

Benjamin tomkinson/Alamy Stock Photo

2 The medical operation

Spreadsheet
Medical treatment

Download the 'Medical treatment' spreadsheet from Nelson MindTap.

Imagine you have a serious medical condition. It isn't life-threatening, but it's very painful and restricts a lot of what you can do each day. Your doctor says that there is an operation that will completely cure the condition, but there is a 1-in-20 (5%) chance that you could die during the operation. What would you do? Would you still have the operation?

a Suppose you say YES and decide to go through with the operation. Choose a number at random from 1 to 65 to represent your patient number and write it down.

b Run the spreadsheet several times. It simulates 65 patients having the operation. Record what happens to the patient with your number.

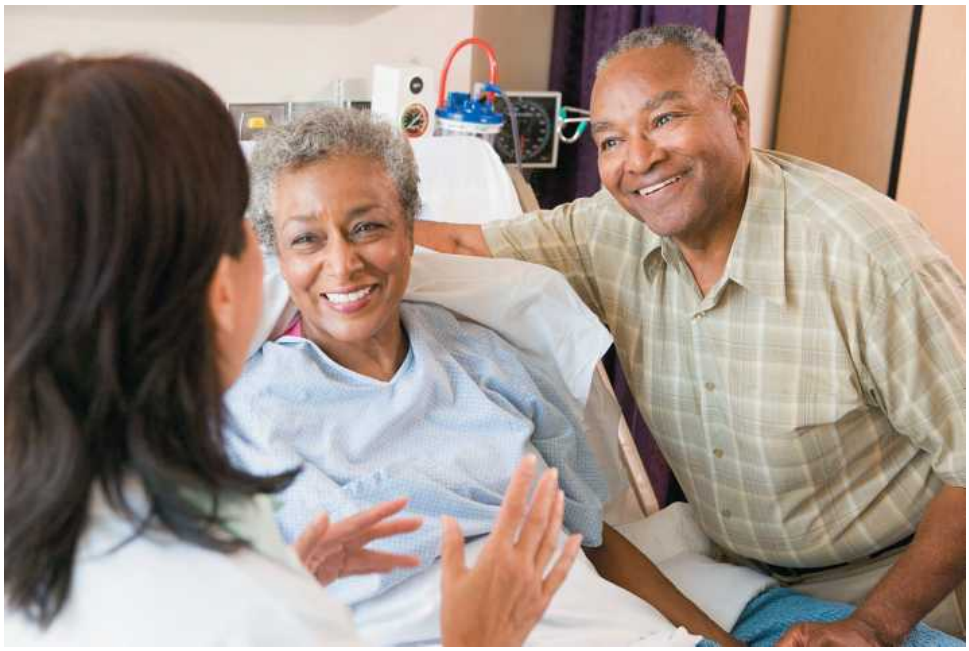
c Did you die in any of the simulations?

d Did anyone in your group die?

e A common belief is that something with a small probability isn't going to happen on the first trial. Run the simulation until the first patient dies. How many simulations did it take?

f Can something with a small probability happen in the first trial?

g What other real-life factors might influence the result of an operation? C



iStock.com/Monkeybusinessimages

3 Number of girls in a family

Spreadsheet
4-child families

Work in pairs and download a copy of the spreadsheet '4-child families' from Nelson MindTap to complete this activity. This spreadsheet simulates the sex of the children in 4-child families.

a Run the simulation 12 times, recording the number of girls in the family each time.

b What is the most common number of girls in a 4-child family?

c What is the relative frequency of the most common number of girls in a 4-child family?

d Melissa is the mother of 3 girls and she would like to have a son. She thinks that if she has another baby, it's more likely to be a boy because she has 3 girls already. Run the simulation again, looking for families with girls for the first 3 children and record the sex of the 4th child. Continue until you've got 12 results. Were there more boys than girls for the 4th child?

e Repeat the simulation, this time concentrating on families with boys for the first 3 children. Was a boy or a girl more common as the 4th child?

Kzenon/Alamy Stock Photo

f Are the same numbers of baby boys and girls born in Australia? Research the question and determine an experimental probability that an Australian baby will be a girl.

g Is it just a chance event that makes the numbers of boy and girl babies different? Research the reasons why different numbers of boys and girls are born in Australia.

4 The total rolled on 2 dice

You will need to work in pairs and download the 'Sum of 2 dice' spreadsheet from Nelson MindTap to complete this activity.

Spreadsheet
Sum of 2 dice

a If we roll a pair of dice and add the 2 numbers, the most common sum is 7. Run the simulation and check that this statement is true.

b Think about this challenge: Roll a sum of 7 twice before you roll a sum of 6 and a sum of 8 in any order. Which is more likely: 7 twice or a 6 and an 8? Write down your prediction.

c Run the simulation at least 12 times to see which occurs first.

d Were you right? How good is your prediction?

TECHNOLOGY Freeway accidents

Spreadsheet
Freeway accidents

Accidents are a serious problem on the M1, the freeway between Brisbane and Coolangatta. The probability that a severe accident (where a person is killed or seriously injured) on the M1 involves a truck or other heavy vehicle is 0.12.

To complete this activity, download the 'Freeway accidents' spreadsheet from Nelson MindTap. This spreadsheet simulates 50 accidents on the M1 and counts the number of accidents involving trucks and other heavy vehicles.

1 Over 50 accidents, what is the expected number of times that a truck is involved?

2 Run 40 simulations of 50 accidents using the spreadsheet and record the number of times a truck is involved.

3 Calculate the percentage of simulations in which 6 trucks were involved in accidents.

4 Calculate the percentage of simulations in which the number of accidents involving trucks was:

a 5, 6 or 7 **b** within the range 3 to 9

5 What conclusion could you make from this simulation?

Keyword activity

DEFINITIONS

ANSWERS p. 439

Write a sentence explaining the meaning of each of the following terms in this chapter.

die	dice	prediction
probability	relative frequency	simulation

Solution to the chapter problem

Problem

Currently, each packet of *Ozbix* breakfast cereal comes with a free little toy. There are 12 different toys available but there's only one toy in each packet, each equally likely. Jordan wants to collect every toy. How many packets of cereal should he buy to collect all 12 toys?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To decide how many packets Jordan should buy to collect all 12 little toys in the collection. There are 12 toys in the collection, each equally likely.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	Simulate this problem using the 'Breakfast cereals' spreadsheet, downloaded from Nelson MindTap. Run the simulation several times and record the smallest and largest number of packets Jordan needs to buy to collect all 12 toys. The answers will depend on the simulation. For example: Smallest number obtained in the simulation: 18 packets Largest number: 51 Jordan will need to buy from 18 to 51 packets.
STAGE 3: CHECK THE SOLUTION	
 CHECK	51 packets seem too many, but we know that our brains aren't good at estimating chance events. Run the simulation a few times more to check if it's right. ✓ Yes, correct
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	Because it's a chance event, an exact number of boxes Jordan will have to buy is not able to be calculated, but it will be somewhere between 18 and 51.

Note: This simulation assumes that the toys are placed in the boxes at random and in equal numbers. This may not be the case. The cereal company might make 1 or 2 toys more 'rare' and harder to get.

Spreadsheet
Breakfast cereals

6 Test yourself

Will it happen?

ANSWERS p. 439

1 EXERCISE 6.3 Matt wanted to determine the probability that a dropped thumbtack would land with its sharp point facing up. He dropped 30 thumbtacks.

TakeStockPhotography/Shutterstock.com

a What is the relative frequency that a thumbtack will land with the point facing up?

b Is the dropped thumbtack more likely to land with the point facing up or down? Give a reason for your answer.

2 EXERCISE 6.3 Copy and complete this table expressing probability values as fractions, decimals (to 2 decimal places where appropriate), percentages (to the nearest percentage) and ratios.

Fraction	Decimal	Percentage	Ratio
$\frac{7}{8}$			7:8
	0.15		
		28%	
$\frac{11}{15}$			

3 EXERCISE 6.4 Saskia tossed a coin 6 times and it showed heads 4 times. She claims that the coin is biased. Write a sentence to explain why Saskia's conclusion may be wrong.

4 EXERCISE 6.1 Describe an event that has a probability of $\frac{1}{2}$.

5 EXERCISE 6.4 Decide whether each statement is true or false. If they are false, correct them.

a If you toss a coin and get 5 heads in a row, then most likely the next toss will be a tail.

b The first 4 children in a family are all girls. If the mother has another baby, there's a 50% chance that it will be another girl.

Chapter problem

Large baobab trees found in the Kimberley region of Western Australia are a great emergency source of fresh drinking water. Up to 76% of the volume of the trunk is fresh water. The trunk of the tree in the photo is 3.2 m tall and its circumference is 7.5 m. Approximately how many litres of fresh water does the tree hold?

Nataliya Derkach/Shutterstock.com

CHAPTER 7

MEASUREMENT, SCALES AND CHANCE: MEASUREMENT

TURN UP THE VOLUME

What will we do in this chapter?

- Choose appropriate metric units of mass, volume and capacity and convert between them
- Estimate and measure mass, volume and capacity
- Calculate the volume and capacity of prisms, cylinders, cones, spheres and pyramids
- Measure the volume of food items and capacity of containers

How are we ever going to use this?

- When comparing the quantities inside different-sized food containers
- When identifying the quantities of ingredients in packaged foods
- When calculating the quantity of materials for a job, for example, the amount of soil or mulch needed in our garden
- Volume is a key component of important trades such as plumbing and landscape gardening

9780170484664

Syllabus coverage

UNIT 3, TOPIC 1: MEASUREMENT

Converting units of measurement

- Use metric units of volume (cubic millimetres, cubic centimetres, cubic metres, cubic kilometres), their abbreviations (mm^3, cm^3, m^3, km^3), conversions between them, and appropriate choices of units.
- Understand and use the relationship between volume and capacity, recognising that $1\,cm^3 = 1\,mL$ (millilitre), $1000\,cm^3 = 1\,L$ (litre), $1\,m^3 = 1\,kL$ (kilolitre), $1000\,kL = 1\,ML$ (megalitre).
- Use metric units of mass (milligrams, grams, kilograms, tonnes), their abbreviations (mg, g, kg, t), conversions between them and appropriate choices of units.

Volume and capacity

- Estimate volumes and capacities of various objects.
- Calculate volumes and capacities of prisms and cylinders.
  - prism: $V = Ah$ where A is base area and h is perpendicular height
  - cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height.
- Calculate volumes and capacities of pyramids and cones.
  - pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height
  - cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height.
- Calculate volumes and capacities of composite shapes and spheres [complex]
  - sphere: $V = \frac{4}{3}\pi r^3$ where r is radius.

Video (1):

7.3 Volume and capacity

Worksheets (9):

7.1 Mass time trial

7.3 Mass and capacity matchup • Mass, volume and capacity time trial

7.4 Converting volume units • Estimating area and volume

7.6 Measurement formulas chart
- Formula matching game
- A page of solid shapes
- Sweet areas and volumes

Puzzles (2):

7.6 Volume code puzzle
- Officer Cubic

Terminology

base area	capacity	cone	cubic
cylinder	kilogram	kilolitre	litre
mass	megalitre	millilitre	perpendicular height
prism	pyramid	radius	sphere
tonne	volume		

7.1 Measuring mass

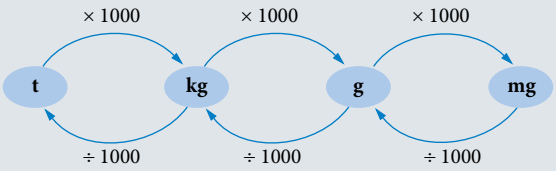
The **gram** is the basic unit for **mass** and all other mass units are based on the gram.

The **milligram** is used to measure small quantities, for example, the mass of medicine, vitamins, food and jewellery.

Units of mass

Unit	Relationships
milligram (mg)	
gram (g)	1 g = 1000 mg
kilogram (kg)	1 kg = 1000 g
tonne (t)	1 t = 1000 kg

To change from a small unit to a bigger unit, **divide** by the conversion factor.
To change from a big unit to a smaller unit, **multiply** by the conversion factor.



Worksheet
Mass time trial

EXAMPLE 1 Converting units of mass 1

Convert 750 kg to

a grams

b tonnes

Steps	Working
a There are 1000 g in 1 kg. Changing from kg to g is changing to a smaller unit, so multiply by the conversion factor.	750 kg = 750 × 1000 g = 750 000 g
b There are 1000 kg in 1 t. Changing from kg to t is changing to a larger unit, so divide by the conversion factor.	750 kg = 750 ÷ 1000 t = 0.75 t

Mark Humphreys/Shutterstock.com

Sometimes, it is necessary to convert between non-metric and metric units of mass.

EXAMPLE 2 Converting units of mass 2

Rachel is catching a flight from Mexico City to Monterey, California. She wants to take her baby stroller on the plane, but the airline only allows strollers with a mass of 20 pounds or less. Rachel's stroller has a mass of 8.5 kg. Will she be able to take the stroller on the flight? (1 kg = 2.2 pounds)

Tyler Olson/Shutterstock.com

Steps	Working
Convert 8.5 kg to pounds. 1 kg = 2.2 pounds. Changing from kg to pounds is changing to a smaller unit, so multiply by the conversion factor.	8.5 kg = 8.5 × 2.2 pounds = 18.7 pounds Rachel's stroller is 18.7 pounds, which is less than the airline's limit of 20 pounds. She can take it on the flight.
OR Convert 20 pounds to kg by dividing by 2.2.	20 pounds = 20 ÷ 2.2 kg ≈ 9.09 kg The airline's limit is 9.09 kg and Rachel's stroller is 8.5 kg, which is less than the limit. She can take it on the flight.

EXERCISE 7.1 Measuring mass

ANSWERS p. 439

1 EXAMPLE 1 Copy and complete each conversion.

a 3 kg = ___ g
b 12 t = ____ kg
c 1500 g = ___ kg
d 2400 kg = ___ t
e 850 kg = ____ g
f 900 g = ____ kg
g 2.5 g = _____ mg
h 500 mg = ___ g

2 A hospital pharmacist ordered 2000 tablets. Each tablet has a mass of 5 mg.

a Calculate the total mass of the tablets in mg.
b What is the total mass in grams?

3 Vitamin C powder contains 90% ascorbic acid and 10% calcium.

a What mass of calcium is in 40 milligrams of vitamin C?
b What mass of ascorbic acid is in 60 milligrams of vitamin C?
c Calculate the number of milligrams of ascorbic acid in 2.4 grams of vitamin C.

4 The gross mass of a bottle of 500 tablets is 155 g. The mass of the bottle only is 20 g.

gross mass = total mass including bottle
net mass = mass of tablets only

a Calculate the net mass of the tablets.

b What is the net mass of the tablets in mg?

c What is the mass of one tablet in mg?

5 How many 50 mg injections can a nurse make from a 1 g container of streptomycin medicine? PS

6 List 3 items whose mass you would measure in

a tonnes **b** kilograms **c** grams **d** milligrams

7 The size of precious stones is measured in carats. Erin's engagement ring contains a 1.8 carat diamond. What is the mass of the diamond in mg? (1 carat = 200 mg)

Varisastockphoto/Shutterstock.com

8 EXAMPLE 2 Nelsonlink Airlines has a carry-on luggage limit of 12 pounds. Karen's bag is 5 kg.

a Calculate the mass of Karen's bag in pounds. 1 kg = 2.2 pounds.

b Is Karen's bag light enough to take on the flight? Justify your answer.

9 Jettison Air has two sets of restrictions on the size of bags it allows on flights.

- The mass of the bag must be 50 pounds or less.
- The sum of the bag's dimensions (length + width + height) must be less than 62 inches.

Orlando's bag is 50 cm long, 19 cm high, 32 cm wide and has a mass of 24 kg.

Is Orlando's bag allowed on the flight? Justify your answer. PS

(Note: 1 kg = 2.2 pounds and 1 inch = 2.5 cm.)

10 A standard house brick has a mass of 2.7 kg.

a A pallet of bricks contains 500 bricks. Calculate the mass of one pallet of bricks.

b A truck carries 8 pallets of bricks. Calculate the weight of the bricks in tonnes.

Angurt/Shutterstock.com

11 In China, the mass of tea leaves is measured in 'jin'. One jin = 500 g. Calculate in grams the mass of a packet of tea that is 3.2 jin.

12 The mass of precious metals can be measured in troy ounces (1 troy ounce = 31.103 g). Gazi bought a 1 kg gold bar as an investment. How much was Gazi's gold bar worth on the day when gold was valued at $1331 per troy ounce? PS

Investigation — Worth your weight in gold

You need a set of bathroom scales.

Have you heard the expression 'You're worth your weight in gold'? In this investigation, you are going to calculate the monetary value of yourself, your friend or your backpack if the weight was pure gold.

What you have to do

1. Measure the mass of the person you are going to value, or your backpack, in kg.
2. Multiply the mass by 32.15 to convert the mass to troy ounces.
3. Use the internet to research today's price for 1 troy ounce of gold, for example, $5090.22 AUD (Australian dollars).
4. Multiply the mass in troy ounces by the price of 1 troy ounce of gold.

How much are you, your friend or your backpack worth?

Practical activity — Estimating mass

You will need a set of kitchen scales (for measuring small masses) and a set of bathroom scales (for measuring larger masses). You will also need some items to estimate and measure their mass.

This table shows some common items and their approximate mass.

Mass	Items with this approximate mass
1 mg	A grain of sand A tiny insect like a sandfly
1 g	A paperclip
2.5 g	A 5-cent coin
100 g	A smartphone without a case
1 kg	1 litre of water 5 medium-sized oranges
71 kg	An average Australian woman
87 kg	An average Australian man

iStock.com/Fotos-v

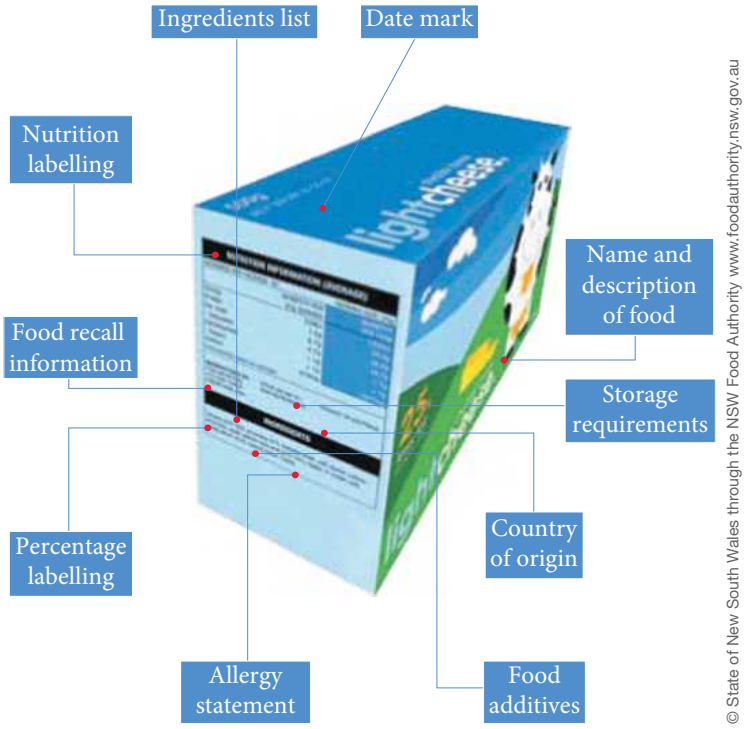
What you have to do

1. Choose an item, for example, a library card.
2. Select an item from the table that has a similar mass. For example, a library card is similar in mass to a 5c coin.
3. Compare the 2 items and estimate the mass of the unknown item. A library card is about twice as heavy as a 5c coin, so it should be about 5 g.
4. Use the scales to check the accuracy of your estimate.
5. Repeat the activity with 4 other items. Have your estimating skills improved with practice?

7.2 What's in our food?

Australian law requires food manufacturers to provide specific information in labels. All food packaging must contain the following information.

Name and description of food	Nutrition labelling
Country of origin	Ingredients list
Date mark	Allergy statement
Food additives	Food recall information
Storage requirements	Percentage labelling



All manufacturers are required to list the ingredients of their foods in descending order of mass.

EXAMPLE 3 Food labels

This label was found on a can of evaporated milk.

NUTRITIONAL INFORMATION

SERVINGS PER PACK: 3.75		SERVING SIZE: 100mL	
AVERAGE QUANTITY	PER SERVING	%DI* & RDI** PER SERVING	PER 100mL
ENERGY	410kJ	5%	410kJ
-CALORIES	98 Cal		98Cal
PROTEIN	8.0g	16%	8.0g
FAT, TOTAL	1.6g	2%	1.6g
-SATURATED	1.1g	5%	1.1g
CARBOHYDRATE	12.6g	4%	12.6g
-SUGARS	12.6g	14%	12.6g
SODIUM	100mg	4%	100mg
CALCIUM	240mg	30%**	240mg

*PERCENTAGE DAILY INTAKES ARE BASED ON AN AVERAGE ADULT DIET OF 8700KJ. YOUR DAILY INTAKE MAY BE HIGHER OR LOWER DEPENDING ON YOUR ENERY NEEDS.
**RECOMMENDED DIETARY INTAKE (AUST/NZ)

a How many mL does the can contain?

b According to the label, what is the recommended daily amount of fat?

c How many servings of the milk do we need to consume more than 1 g of calcium?

Steps	Working
a There are 3.75 servings per pack and each serving is 100 mL.	Amount in the can = 3.75 × 100 mL = 375 mL
b One serving contains 1.6 g of fat and this is 2% of the recommended daily amount. From this, we need to find 100% of the recommended daily amount of fat.	2% of daily amount = 1.6 g 1% of daily amount = 1.6 g ÷ 2 = 0.8 g 100% of daily amount = 0.8 g × 100 = 80 g The recommended daily amount of fat is 80 g.
c Each serving contains 240 mg of calcium. 1 g = 1000 mg. Divide 1000 by 240.	Number of servings = 1000 ÷ 240 ≈ 4.17 We need more than 4 servings. We need 5 servings to consume more than 1 g of calcium.

EXERCISE 7.2 What's in our food?

ANSWERS p. 439

1 EXAMPLE 3 This label is from a can of soup. Read the label to obtain the information to answer each question.

a Does the soup contain any artificial flavours?

b What are the code numbers of the food colours added to the soup?

c What vegetables are in the soup?

d How many servings are in the can?

e How many grams of protein are in a serving of this soup?

f Lycopene is an antioxidant that may be beneficial for health. It is the substance that gives the red colouring to tomatoes. The recommended daily intake of lycopene for women is 6 mg. How many servings of this soup does a woman need to consume in a day to obtain her recommended amount of lycopene?

INGREDIENTS

Vegetables (45%) (Potatoes, Blue Peas, Onion, Tomatoes, Green Beans), Water, Marinated Angus Beef (16%) (Angus Beef, Marinade (Salt, Mineral Salts (450, 451), Dextrose Monohydrate, Thickeners (1422, 415), Maize Thickener (1422), Potato Starch, Yeast Extracts (from Barley). Natural Flavours, Salt, Soy Sauce (Contains Wheat), Pepper, Hydrolysed Vegetable Protein, Colours (150c, 160v), Herb Extract.

NUTRITION INFORMATION		
SERVINGS PER PACKAGE: 2 SERVING SIZE: 265G		
	AVG QUANTITY PER SERVING	AVG QUANTITY PER 100g
ENERGY	675kJ	255kJ
PROTEIN	13.5g	5.1g
FAT, TOTAL	3.2g	1.2g
- SATURATED	1.6g	0.6g
CARBOHYDRATE	18.3g	6.9g
- SUGARS	3.2g	1.2g
DIETARY FIBRE	2.7g	1.0g
SODIUM	760mg	290mg
LYCOPENE	1.2mg	0.4mg

No Artificial Flavours
No Preservatives

2 Katja is reading this label on a can of tuna.

TUNA
98% FAT FREE
Wild caught in the Western Central Pacific Ocean

TUNA - *spicy deli*

Ingredients: Purse-seine caught Skipjack Tuna *(Katsuwonus pelamis)* (56%), Water, Chilli Sauce [Water, Chilli (1.7%), Sugar, Garlic, Vinegar, Salt, Tapioca Thickener (1442)], Sugar, Onion, Whole Chilli (2.2%), Sunflower Oil, Salt Natural Colour (Paprika Extract), Traces of Wheat, Milk, Egg, Soy, Crustacea & Sesame.

PACKED FOR: E.M. FISH CO. AUST. LTD 80 DORCAS ST, SOUTH MELBOURNE, VICTORIA 3205, AUSTRALIA. FREECALL: 1800 037 000 (Aust. only) DISTRIBUTED IN NEW ZEALAND BY EM TUNAFISH LIMITED 456 GEORGE ST, HASTINGS 4122, NEW ZEALAND. FREECALL: 0800 653 000 (NZ only) For more information about sustainability, visit www.tuna.com.au MADE IN THAILAND FROM IMPORTED AND LOCAL INGREDIENTS.

NUTRITION INFORMATION SERVINGS PER PACKAGE: 1 SERVING SIZE: 95g	AVERAGE QUANTITY PER SERVING	AVERAGE QUANTITY PER 100g
ENERGY	380kJ	400kJ
PROTEIN	14.0g	14.7g
FAT, TOTAL	1.7g	1.8g
-SATURATED	0.4g	0.4g
-TRANS	0g	0g
-POLYUNSATURATED	0.8g	0.9g
-OMEGA-3	170mg	180mg
-EPA	30mg	30mg
-DHA	140mg	145mg
-MONOUNSATURATED	0.5g	0.5g
CARBOHYDRATE	4.7g	4.9g
-SUGARS	4.4g	4.6g
SODIUM	370mg	390mg

a Where was the tuna made?

b Does the tuna contain any sugar?

c How many grams of protein are contained in a serving?

d How many mg of saturated fat are in a serving?

e Katja is allergic to wheat. Can she eat the tuna?

3 The label shows nutritional information about a serving of Thai green curry chicken.

a How many kilojoules are in a serving?

b How much less than 1 g of sodium is included in a serving?

c What percentage of the fat in a serving is saturated fat?

d The meal may contain traces (tiny amounts) of five substances that might give some people an allergic reaction. What are they?

Ingredients when prepared as directed: Cooked Rice (50%) (Water, Milled Jasmine Rice, Rice Bran Oil), Sauce (Chicken (15%), Coconut Milk (12%), Water, Vegetables (6%) (Long Bean, Garlic, Shallot), **Fish Sauce** (**Anchovy**), Sugar, Chili, Lemongrass, Galangal, Basil, **Soybean Oil**, **Shrimp Paste** (**Crustacea**), Spices, Salt, Modified Maize Starch (1422).

Contains Crutacea, Fish, and Soybean Products.
Made on a production line that also produces foods containing: Peanuts, Tree Nuts, Sesame Seeds, Egg and Milk products.

NUTRITION INFORMATION		
SERVINGS PER PACKAGE: 1		SERVINGS SIZE: 320 g
AVG QTY PER SERVING		
ENERGY	1830 kJ (438 Cal)	571 kJ (137 Cal)
PROTEIN, TOTAL	15.4 g	4.8 g
-GLUTEN	0 mg	0 mg
FAT, TOTAL	10.2 g	3.2 g
-SATURATED	7.0 g	2.2 g
CARBOHYDRATE	68.5 g	21.4 g
-SUGARS	4.5 g	1.4 g
DIETARY FIBRE	2.9 g	0.9 g
SODIUM	830 mg	260 mg
POTASSIUM	290 mg	90 mg

4 The information in the table below appeared on a bottle of BBQ sauce.

Servings per packet: 12		Serving size 30 mL
	Per serving	Per 100 mL
Energy	273 kJ	911 kJ
Protein	0.3 g	1.1 g
Fat	0.1 g	0.2 g
Carbohydrate **– total** – sugars	 16.1 g 13.7 g	 53.6 g 45.5 g
Cholesterol	nil	nil
Sodium	300 mg	1000 mg
Potassium	36 mg	118 mg

a How many servings are in the bottle?

b How big is each serving?

c How much BBQ sauce does the bottle contain?

d Calculate the total energy contained in the bottle.

e How many grams of non-sugar carbohydrate are there in one serving?

f Is there more protein or sodium in a serving? Explain your answer.

g What percentage of the carbohydrate in the sauce is sugar?

h How much salt (sodium) is in the bottle? Answer in grams.

i The bottle contains *no cholesterol*, but this does not necessarily mean that the food is low in fat. How much fat is contained in the bottle?

5 This label on a milk drink includes information about the recommended daily allowances of vitamins and minerals.

	Per serving	Percentage of recommended daily allowance
Iron	2.7 mg	20%
Calcium	110 mg	50%
Vitamin B1	0.18 mg	20%
Vitamin B2	0.14 mg	25%
Sodium	49 mg	10%
Potassium	120 mg	10%

Matthew explained how he calculated the recommended daily allowance of iron correctly:
'I divided 2.7 by 20 to find out 1%, then I multiplied by 100 to get 100%'

Recommended daily allowance of iron $= 2.7 \div 20 \times 100$

$= 13.5$ mg

Use this method to calculate the recommended daily allowance of:

a calcium

b vitamin B2, correct to one decimal place

c potassium, in grams.

Investigation What's in my favourite food?

"You are what you eat."

Are you sure you know what you're eating?

What you have to do

- Find the nutritional information from the packets of 5 of your favourite packaged foods.
- List any food allergies included in the information.
- List any preservatives, artificial flavourings and colours in the food items.
- Record the name of any ingredient that surprises you.
- Calculate the total amount of fat and sugar in the package.
- Calculate the percentage of the mass of the contents that is protein, sugar and sodium.
- Healthy food has low amounts of fat, sugar, preservatives, sodium, artificial colours and flavourings and high amounts of protein. List the 5 food items in order from the most healthy to the least healthy.

7.3 Units of volume and capacity

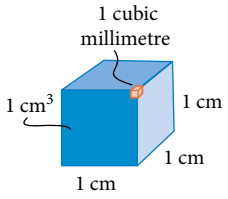
The amount of wheat a silo can hold, the quantity of sand in a truck and the amount of concrete required for a driveway are examples of **volume**.

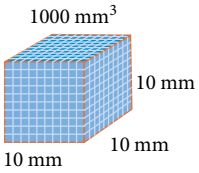
The volume of a solid is the amount of space occupied by the solid. Volume is measured in cubic units.

Sharomka/Shutterstock.com

Volume unit	The size of a cube of length:	Approximately the size of:
cubic millimetre (mm^3)	1 mm	a grain of raw sugar or rock salt
cubic centimetre (cm^3)	1 cm	a tooth or a pain relief tablet
cubic metre (m^3)	1 m	2 washing machines

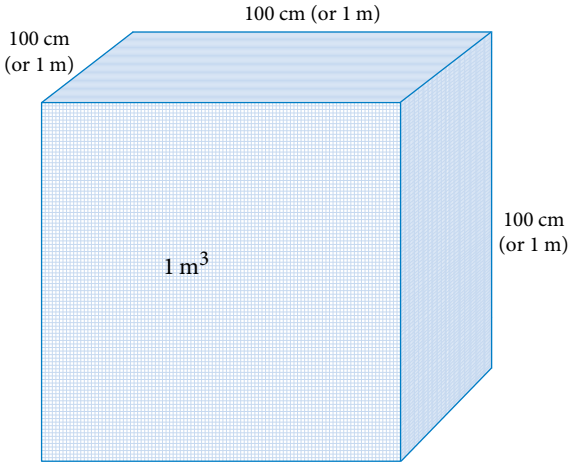
These diagrams compare a cubic millimetre with a cubic centimetre.





$1\ cm^3 = 10\ mm \times 10\ mm \times 10\ mm$
$= 1000\ mm^3$

This diagram compares a cubic centimetre with a **cubic metre**.



$1\ m^3 = 100\ cm \times 100\ cm \times 100\ cm$
$= 1\,000\,000\ cm^3$

When converting area units in Chapter 4, *On the surface*, you had to convert the length unit twice.
When converting volume units, you have to convert the length unit 3 times.
You *cube* the simple linear conversion factor to get the volume conversion factor.

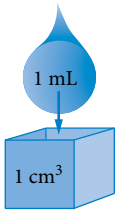
For example, to change from m to cm, multiply by 100,
but to change from m^3 to cm^3, multiply by $100^3 = 1\,000\,000$

Units of volume

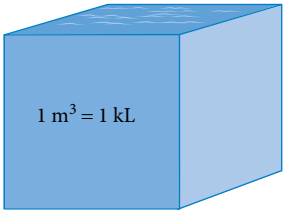
$1\text{ cm}^3 = 1000\text{ mm}^3$
$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$
To change from a small unit to a bigger unit, **divide** by the conversion factor.
To change from a big unit to a smaller unit, **multiply** by the conversion factor.

Volume measures the amount of space inside a container, while **capacity** measures the amount of liquid or gas a container will hold.

1 cm^3 holds 1 mL



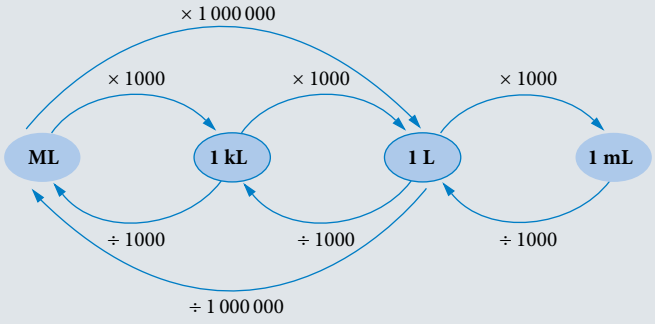
1 m^3 holds 1000 L or 1 kL



Capacity unit	Approximately the size of:
millilitre (mL)	a large drop of water
litre (L)	a tall carton of milk
kilolitre (kL)	a small rainwater tank
megalitre (ML)	half an Olympic-sized swimming pool

Units of capacity

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1000 kL
= 1 000 000 L



EXAMPLE 4 Converting volume units

Convert:

a 36 cm^3 to mm^3

b $84\,000\,000\text{ cm}^3$ to m^3

Steps	Working
a cm^3 to mm^3, large to small unit: $\times 10^3 = 1000$	$36\text{ cm}^3 = 36 \times 1000\text{ mm}^3$ $= 36\,000\text{ mm}^3$
b cm^3 to m^3, small to large unit: $\div 100^3 = 1\,000\,000$	$84\,000\,000\text{ cm}^3 = 84\,000\,000 \div 1\,000\,000\text{ m}^3$ $= 84\text{ m}^3$

Video
Volume and capacity

Worksheets
Mass and capacity matchup

Mass, volume and capacity time trial

EXAMPLE 5 Volume and capacity 1

Convert:

a 5 cm^3 to mL

b 1850 mL to litres

Steps	Working
a 1 cm^3 holds 1 mL. The number of cm^3 and mL are always the same.	5 cm^3 holds 5 mL
b mL to L: small to large unit: ÷ 1000.	1850 mL = 1850 ÷ 1000 L = 1.85 L

Video
Volume and capacity

EXAMPLE 6 Volume and capacity 2

The volume of a large fishpond is 3.4 m^3. How many litres of water does it hold?

Ron Zmiri/Shutterstock.com

Steps	Working
1 m^3 holds 1000 L. Multiply the number of m^3 by 1000.	3.4 m^3 holds 3.4 × 1000 L = 3400 L
Write your answer.	The fishpond holds 3400 L of water.

EXERCISE 7.3 Units of volume and capacity

ANSWERS p. 439

1 What unit would you use (m^3, cm^3 or mm^3) to measure the volume of each object?

a A bedroom
b A backpack
c A mobile phone
d A matchbox
e A concert hall
f A swimming pool
g Your calculator
h A car
i A driver's licence
j A glass of water
k A USB drive
l A box of laundry powder

2 Match the correct volume (**A** to **G**) with each of the items (**a** to **g**) listed.

a bottle of nail polish
b box of tissue
c glass of fruit juice
d bottle of lemonade
e classroom
f school hall
g box of cereal

A 200 m^3
B 3980 m^3
C 1250 cm^3
D 5000 cm^3
E 20 000 mm^3
F 250 cm^3
G 2200 cm^3

Hint: You may find it helpful to put the items in descending order of volume and then choose the measurements.

9780170484664

3 EXAMPLE 4 Copy and complete each conversion.

a $5000\,cm^3 =$ _____ mm^3
b $1.6\,m^3 =$ _____ cm^3
c $6000\,cm^3 =$ _____ m^3
d $4000\,mm^3 =$ _____ cm^3
e $160\,000\,cm^3 =$ _____ m^3
f $250\,mm^3 =$ _____ cm^3
g $0.18\,m^3 =$ _____ cm^3
h $0.12\,cm^3 =$ _____ mm^3
i $4\,m^3 =$ _____ mm^3
j $9\,600\,000\,mm^3 =$ _____ m^3

4 Match the correct capacity (**A** to **K**) with the items (**a** to **k**) listed.

a car petrol tank
b a cup of flour
c bathtub
d bucket of water
e can of drink
f glass of water
g Olympic swimming pool
h bottle of lemonade
i teaspoon
j water storage tank
k A water storage reservoir

A 200 mL
B 23 kL
C 5 mL
D 70 L
E 1250 mL
F 1.875 ML
G 250 mL
H 9 L
I 375 mL
J 180 L
K 500 ML

Hint: You may find it helpful to put the items in descending order of capacity and then choose the measurements.

5 EXAMPLE 5 Convert each measurement to mL.

a $8\,cm^3$ **b** $1500\,cm^3$ **c** $425\,cm^3$

6 Convert each measurement to litres.

a 2000 mL **b** 3500 mL **c** 250 mL

7 Arrange in ascending order: $42\,cm^3$, $4210\,mm^3$, $0.0042\,m^3$

8 **a** The volume of a large container is $5000\,cm^3$. How many millilitres does the container hold?
b How many litres will a container with a volume of $5000\,cm^3$ hold?

9 The volume of Marty's chest of drawers is $306\,000\,cm^3$. What is this in cubic metres?

10 Ruchi's lunchbox has a volume of $2520\,cm^3$. What is the volume of her lunchbox in cubic millimetres?

11 EXAMPLE 6 How many litres can a $2\,m^3$ container hold?

12 What is the volume in cubic centimetres of a 1.25-litre soft drink bottle?

13 Each can in a box of 24 cans of soft drink holds 375 mL. How many litres of soft drink are contained in the box? PS

14 A tap leaks 10 mL of water every 50 seconds. How much water will the tap lose in: PS

a 1 second? **b** 1 minute? **c** 3 hours? **d** 1 day?

15 Hans ordered 1 m³ of pine bark to mulch his vegetable garden. After he finished putting the mulch on his garden, he had 20% of the original amount left. How many cubic centimetres of mulch were left over? PS

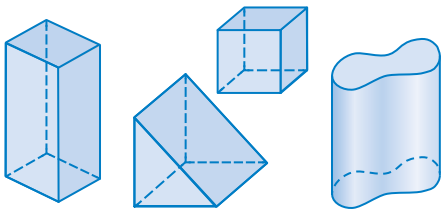
Alison Hancock/Shutterstock.com

Worksheet
Converting volume units

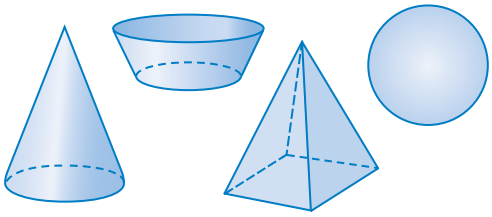
7.4 Volumes of prisms

These diagrams show some solids with identical ends and some solids that don't have identical ends.

Examples of solids with identical ends



Examples of solids that don't have identical ends



A solid with identical ends and flat sides is called a **prism**. The three prisms shown above on the left are a rectangular prism, triangular prism and cube. The ends of a prism are shapes with straight sides (rectangles, triangles or any other polygon) and is part of the name of the prism, for example, rectangular prism, triangular prism.

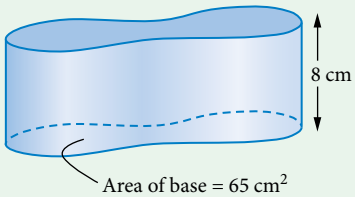
Volume of a prism or other solid with identical ends

$$V = A \times h$$

where A is the area of the end or base and h is the height.

EXAMPLE 7 Volume of a solid

The area of the base of this solid is 65 cm². What is the volume of the solid?



Steps	Working
Both ends of the solid are identical, so we can use the formula $V = A \times h$ $A = 65$ and $h = 8$	volume $= 65 \times 8$ $= 520$ cm³

9780170484664

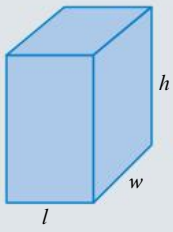
Volume of a rectangular prism

For a rectangular prism the base is a rectangle, so $A = l \times w$ and the formula becomes $V = l \times w \times h = lwh$.

Volume of a rectangular prism

$$V = lwh$$

where l = length, w = width, h = height.



Volume of a cube

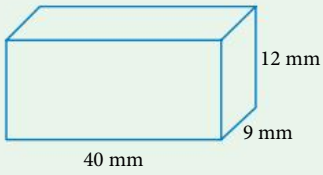
$$V = s^3$$

where s = side length.

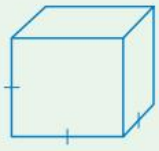
EXAMPLE 8 Volume of a prism

Find the volume of each prism.

a



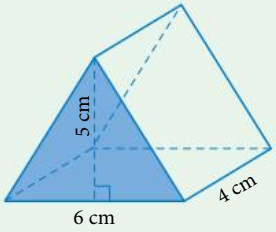
b



Steps	Working
a Use the formula $V = lwh$	$V = 40 \times 9 \times 12$ $= 4320\ \text{mm}^3$
b This is a cube, a special type of rectangular prism. $V = s^3$.	$V = 6.5 \times 6.5 \times 6.5$ $= 6.5^3$ $= 274.625\ \text{cm}^3$

EXAMPLE 9 Volume of a triangular prism

Find the volume of this triangular prism.



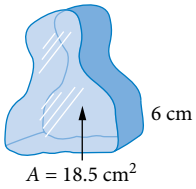
Steps	Working
For a triangular prism, $A = \frac{1}{2}bh$ $b = 6$, $h = 5$	$V = \left(\frac{1}{2} \times 6 \times 5\right) \times 4$ $= 60\ \text{cm}^3$

EXERCISE 7.4 Volumes of prisms

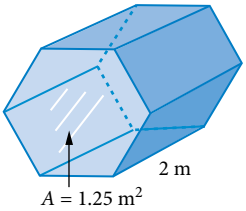
ANSWERS p. 439

1 EXAMPLE 7 Find the volume of each solid.

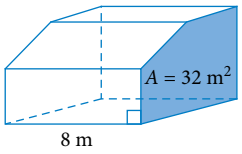
a



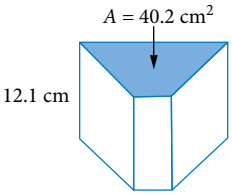
b



c

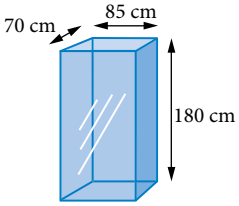


d

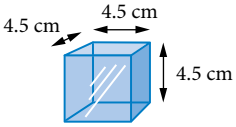


2 EXAMPLE 8 Find the volume of each prism.

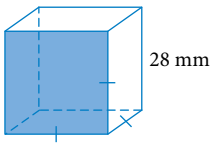
a



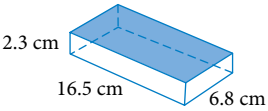
b



c

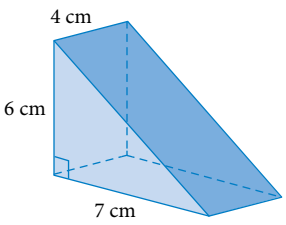


d

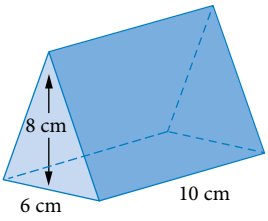


3 EXAMPLE 9 Find the volume of each triangular prism.

a



b



4 Concrete blocks with the shape of rectangular prisms are used to build houses.

a If each concrete block measures 45 cm by 17 cm by 21 cm, calculate the volume of one block.

b A wall is made from 80 concrete blocks. Calculate the volume of the wall.

5 These storage cubes have a side length of 35 cm.

a Calculate the volume of one cube.

b Inga has a stack of 11 cubes in her bedroom to store books. What volume of books can she store?

c Inga's books have an average volume of 1425 cm^3 each. Approximately how many books can she store?

Leszek Glasner/Shutterstock.com

6 Calculate the volume of each food box.

a

GS1 Smart Media

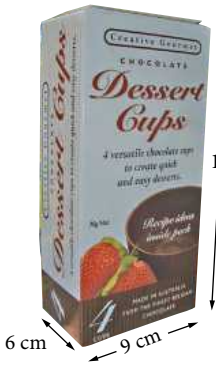
b

GS1 Smart Media

c

GS1 Smart Media

d



GS1 Smart Media

7 Michael is laying pavers for a new rectangular outdoor area, 4.2 m by 2.7 m. Each paver is 300 mm by 300 mm and 50 mm thick.

Remember to have all your measurements in the same units!

a How many pavers will he need?

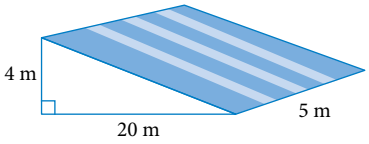
b Calculate the volume of one paver.

c Calculate the volume for the total number of pavers required for this job.

d Pavers come in boxes of 10. How many boxes will Michael need to purchase?

e Each box costs $56.70. How much will the pavers cost?

8 This concrete ramp was built at Hawk's Garden beach to give easy access to the beach. Calculate the volume of concrete required for this ramp.



9 Nazneen plans to install air-conditioning inside her house. The air conditioner is available in 4 sizes.

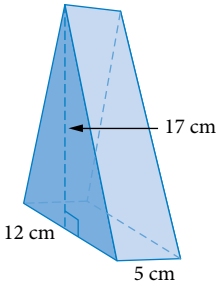
Air conditioner size	Volume of air in house
Small	210 m^3
Medium	350 m^3
Large	500 m^3
Extra large	720 m^3

Nazneen's house is in the shape of a rectangular prism, with the dimensions shown on the photograph.

© Sue Thomson

What size air conditioner will Nazneen need?

10 Rhianna won the 'Star Performance' award at her school concert. Find the volume of glass used to make this trophy.



11 The photograph shows the mini-truck Darren uses in his landscaping business.

Bax Walker/Alamy Stock Photo

a Write down the width, height and length of the tray in metres.

b Use a leading digit approximation to estimate the volume of the tray in m^3.

c Calculate the volume of the tray in m^3.

d Calculate the capacity of the tray in litres.

Remember: 1 m^3 holds 1000 L.

12 This inflatable children's pool contains water 20 cm deep. How many litres of water are in the pool? PS

Make sure you convert 20 cm to metres before you start the calculations!

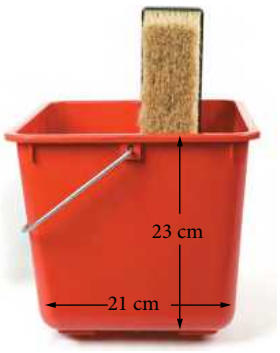
Smikeymikey1/123RFStock photo

13 This bucket has a square base with sides of 21 cm.

a The sides of the bucket are 23 cm high. Calculate the volume of the bucket in cm^3.

b How many whole litres of water can the bucket hold?

c One litre of water weighs 1 kg. Approximately how much will the bucket weigh when it is half-full of water?



Spinetta/Shutterstock.com

14 Zina's swimming pool is 5.8 m long and 3.2 m wide. Hot, dry winds from Central Australia caused 11 cm of water in the pool to evaporate.

a What solid shape could be used to represent the volume of water that evaporated from Zina's pool?

b How many cubic metres of water evaporated from the pool?

c How many litres of water are required to top up Zina's pool?

d The pump on Zina's water tank delivers 105 L per minute. For how long will she need to pump water from the tank into her pool to replace the evaporated water? Express your answer correct to the nearest minute.

Practical activity — Estimating volume

It takes practice to become good at estimating. Complete these practical group activities to develop your estimation skills.

To complete these activities, each group will need:

- 4 metre rulers or tape measures
- Paper, pencils and scissors

The worksheet 'Estimating area and volume', which can be downloaded from Nelson MindTap.

Worksheet
Estimating area and volume

Estimating volume		
Names	**Group members' estimates**	**Real measurement**
Volume of a car boot in cubic metres		
Volume of a car in cubic metres		

Activity 1: The volume of a car boot

Car boots come in different sizes. Before your group starts this activity, select an appropriate car.

Record the group's estimates for the volume of the car boot. Then measure the boot's width, depth, and height in metres, to calculate its volume in cubic metres (m^3).

Kilukilu/Shutterstock.com

Activity 2: The volume of a car

The volume of most cars can be modelled by using 2 or 3 rectangular prisms.

Record the group's estimates for the volume of the car. Then measure the width, depth, and height of each section of the car in metres. Calculate the volume of each section in cubic metres (m^3), then add them together to determine the total volume.

Vibrant Image Studio/Shutterstock.com

7.5 Packaging our food

EXERCISE 7.5 Packaging our food

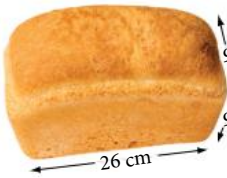
ANSWERS p. 440

1 The end of this loaf of bread is a rectangle 9 cm by 9.5 cm.

a What is the area of the end of the loaf of bread?

b The loaf is 26 cm long. Calculate the volume of the loaf of bread.

c The loaf contains 22 slices of bread. Calculate the average volume of a slice of bread, correct to the nearest cm^3.



Kzww/Shutterstock.com

2 The image below shows the contents of a box of breakfast cereal.

© Sue Thomson

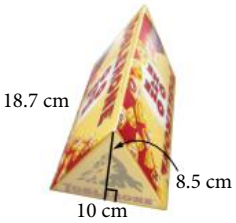
a Use the formula $V = lwh$ to calculate the volume of the box.

b What is the volume of the cereal contents?

c How many cubic centimetres of space are in the box?

d Approximately what percentage of the volume of the box is space?

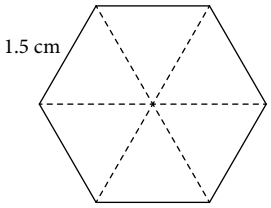
3 The formula for the volume of a prism is $V = Ah$, where A = the area of the end or base and h = the height. Find the volume of this chocolate box, correct to the nearest cubic centimetre.



© Sue Thomson

4 The formula $V = \dfrac{\pi \times h(b+t)^2}{4}$ gives the volume of a tapered, round food container. In the formula, b = the diameter of the circular base, t = the diameter of the circular top and h = the height.
Use the formula to calculate the volume of this yoghurt container.

5 Kris bought coffee beans in a container that has the shape of a hexagonal prism. The prism is 12 cm long and the base is a regular hexagon with sides 1.5 cm as shown.
Equilateral triangles make the base of the prism.



10 cm^3 of coffee has a mass of 4 grams. Determine the volume of the container and the weight of coffee beans that it can hold. **PS**

6 The oil in this bottle is 15 cm deep and the area of the bottom of the bottle is 35.5 cm^2.
Calculate the volume of oil in the bottle.

Investigation The space in a cereal box

You need 3 different-sized boxes of breakfast cereal and a ruler.

Manufacturers leave space in the boxes of cereal to help prevent the contents from being squashed or damaged. Your task is to determine whether there is a relationship between the volume of the box and the amount of space left in the box.

What you need to do

1. Measure each box and the unopened contents.
2. Determine the volume of each box and the amount of space left in each box.
3. Express the amount of space as a percentage of the size of the box. You can use the formula $\frac{\text{amount of space in cm}^3}{\text{volume of the box in cm}^3} \times 100\%$ to make this calculation.
4. Which size box contains the greatest percentage of space?

Investigation Designing a smaller can

Changing the dimensions of a package is one method that manufacturers use to disguise price rises. Keeping the price the same but reducing the size of the contents is equivalent to increasing the price.

The formula $V = 3.14r^2h$, where r = the radius of the base and h = the height, can be used to calculate the volume of this can of dog food.

This can has a base radius of 5 cm and a height of 10 cm and it contains 785 cm^3.

Design a can that looks almost the same size but contains about 20% less volume.

Hints

- How much is 20% less than 785 cm^3?
- Reduce the radius or the height, or both, by a small amount and calculate the volume. Is it in the range you want? If not, try some other values for the radius and height.

Dlibrary/Mars Petcare Australia

7.6 Volumes of cylinders, cones, pyramids and spheres

Volume of a cylinder

We can also use the formula $V = Ah$ to find the volume of a cylinder, because a cylinder has identical ends. For a cylinder, the base is a circle, with area $A = \pi r^2$.

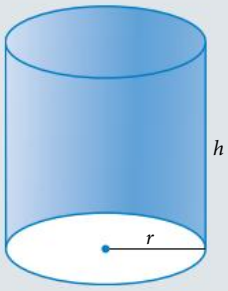
$V = Ah$

$= \pi r^2 \times h$

$= \pi r^2 h$

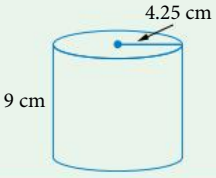
Volume of a cylinder

$V = \pi r^2 h$



EXAMPLE 10 Volume of a cylinder

Calculate the volume of this cylinder, correct to 2 decimal places.



Steps	Working
$V = \pi r^2 h$	$V = \pi \times 4.25^2 \times 9$
$r = 4.25$ and $h = 9$	$= 510.7051\ldots$
	$\approx 510.71\ \text{cm}^3$

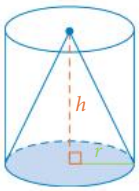
Volume of a cone

The volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same base and height.

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times \pi r^2 \times h$

$= \frac{1}{3} \times \pi r^2 h$



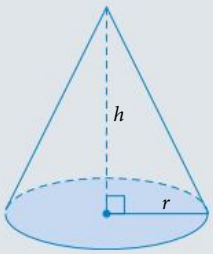
Worksheets
Measurement formulas chart
Formula matching game
A page of solid shapes
Sweet areas and volumes

Puzzles
Volume code puzzle
Officer Cubic

Volume of a cone

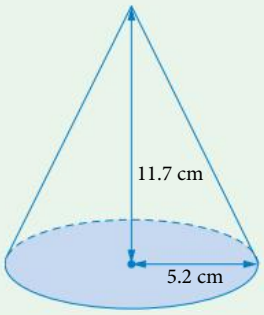
$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the perpendicular height.



EXAMPLE 11 Volume of a cone

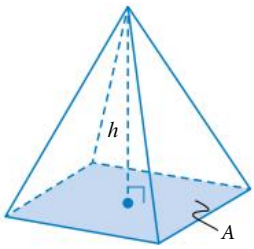
Calculate the volume of this cone, correct to one decimal place.

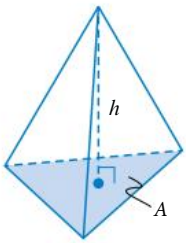


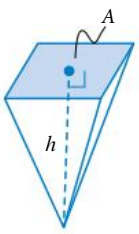
Steps	Working
Use the formula $V = \frac{1}{3}\pi r^2 h$ with $r = 5.2$ cm and $h = 11.7$ cm	$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3} \times \pi \times 5.2^2 \times 11.7$ $= 331.2997 \ldots$ The volume of the cone is 331.3 cm^3.

Volume of a pyramid

The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height.





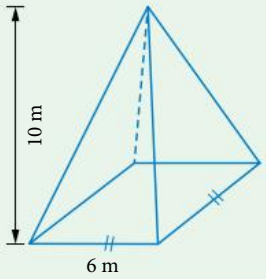


Volume of a pyramid

$$V = \frac{1}{3}Ah$$

EXAMPLE 12 Volume of a pyramid

Calculate the volume of this square pyramid.



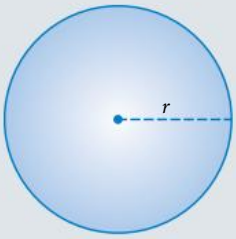
Steps	Working
$V = \frac{1}{3}Ah$	
The base, A, is a square.	$A = 6^2$
$A = s^2$, where $s = 6$.	$= 36\text{ m}^2$
$V = \frac{1}{3}Ah$, *where* $h = 10$.	$V = \frac{1}{3} \times 36 \times 10$
	$= 120\text{ m}^3$

Volume of a sphere*

Volume of a sphere

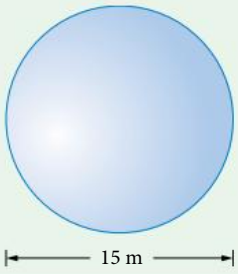
$$\text{Volume} = \frac{4}{3} \times \pi \times \text{radius}^3$$

$$V = \frac{4}{3}\pi r^3$$



EXAMPLE 13 Volume of a sphere

Calculate the volume of this sphere, correct to the nearest cubic metre.



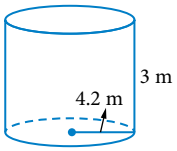
Steps	Working
The diameter is 15 m.	$r = \frac{1}{2} \times 15 = 7.5$
So the radius r is half of 15.	
$V = \frac{4}{3}\pi r^3$.	$V = \frac{4}{3} \times \pi \times 7.5^3$
	$= 1767.1458\ldots$
	$\approx 1767\text{ m}^3$

EXERCISE 7.6 Volumes of cylinders, cones, pyramids and spheres

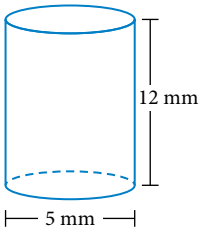
ANSWERS p. 440

1 EXAMPLES 10, 11 Calculate, correct to one decimal place, the volume of each solid.

a



b



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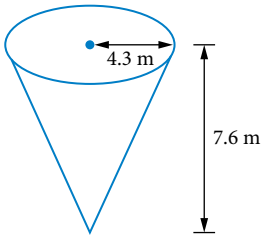

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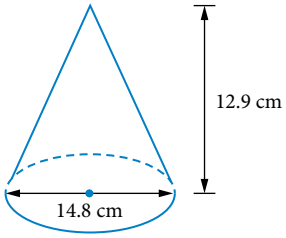
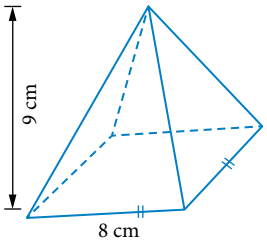
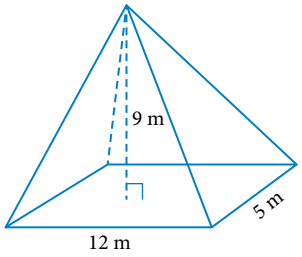


2 EXAMPLE 12 Calculate the volume of each pyramid.

a



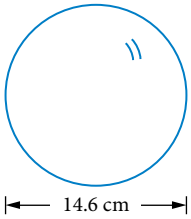
b



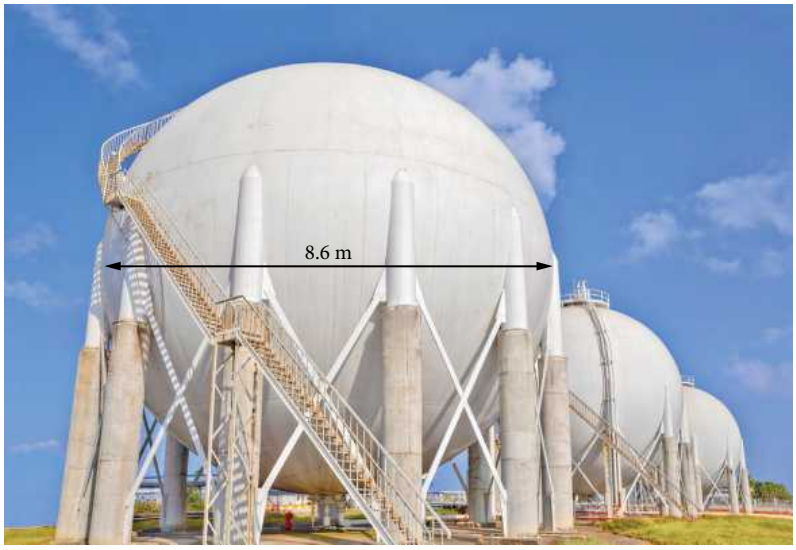
3 EXAMPLE 13 Calculate, correct to one decimal place, the volume of each sphere.

a

b



4 Large spherical tanks are used to hold gas. Calculate the volume of this tank, correct to the nearest m^3.



iStock.com/HAYKIRDI

5 At the Plain of Jars archaeological site in Laos, there are thousands of huge stone containers that are more than 2000 years old. The larger ones were probably used to store food. The container in the photo is 1.87 m tall, the outside diameter is 96 cm and the stone is 15 cm thick. PS

Migel/Shutterstock.com

a Explain how you know that the radius of the space in the middle of the container is 33 cm.

b Determine the capacity of the middle container in litres correct to the nearest L.

Be careful! Make sure all the measurements you use are in the same units.

6 The Pyramid of Khafre in Egypt is famous because the smooth surface covering the top is still there. The pyramid has a square base that is 215.3 m long and its height is 136 m.

Remember: $\frac{1}{3} = 0.333\,333\ldots$
The leading digit is 0.3.

RovingPhotogZA/Shutterstock.com

a Use the leading digit approximation technique to estimate the volume of the pyramid in m^3. Will this estimate be smaller, larger or the same as the exact value? Justify your answer.

b Calculate the volume of the Khafre pyramid. Express your answer correct to the nearest 100 m^3.

7 This Cambodian rubbish bin, in the shape of a sphere, is made from recycled truck tires. The diameter of the sphere is 82 cm. How many litres of rubbish can it hold? Answer to the nearest whole litre. PS

imageBROKER.com/Alamy Stock Photo

8 Kimberly has hats she plans to fill with novelty items for a children's party. The diameter of the base is 10 cm and the slant height is 13 cm.

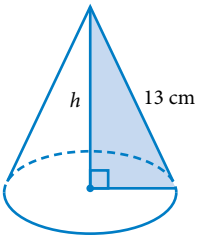
a What is the radius of the base?

b Use Pythagoras' theorem to calculate the perpendicular height of the cone.

c Calculate the volume of the cone correct to the nearest cm^3.

d How many square centimetres of cardboard were used to make the hat? PS

Fotoksa/Shutterstock.com



9 Do you remember Karen and her ice cream shop in Exercise 4.9? Karen fills her waffle cones with ice cream and puts a hemisphere of ice cream on the top. C PS

a Calculate the volume of ice cream in one of Karen's waffle cone ice creams.

b Karen buys her ice cream in 20 L containers. How many ice creams can she make from one 20-L container?

iStock.com/WS Studio

10 One of the old fuel tanks on the Space Shuttle contained liquid hydrogen. The tank is a cylinder, 21.2 m long and 8.4 m wide, with a hemisphere of radius 4.2 m at each end.
Calculate, correct to one decimal place, the volume of the tank in cubic metres. PS

ZUMA Press, Inc./Alamy Stock Photo

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Group activity PS — What size protein ball?

Lyn makes orange protein balls. The recipe suggests that 4 cm is the ideal diameter of the balls and that the ball will contain 1573 kilojoules. Lyn wants to make protein balls with fewer calories by reducing the ball's diameter.
She makes samples in 3 diameter sizes:

4 cm

3.5 cm

3 cm

As a group, determine the number of kilojoules in the 2 smaller size balls. Is there another size Lyn should consider?

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Problem solving — Packaging casks

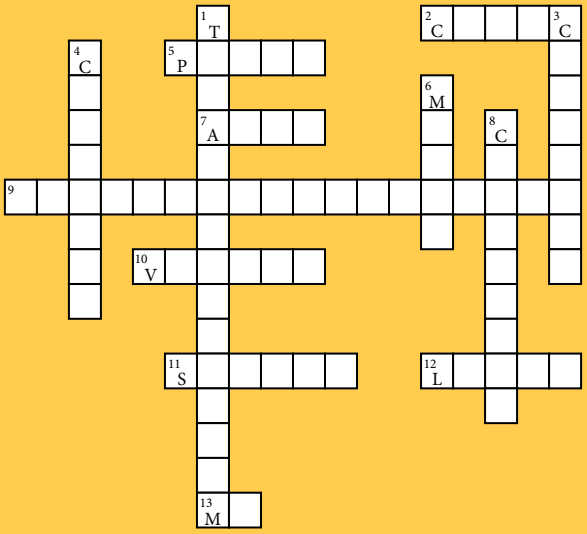
Gerry works in the delivery section of a supermarket. He has a 1 m^3 package of 5-litre casks of fruit juice.
As a group, determine the number of casks in the package.

Keyword activity

CROSSWORD

ANSWERS p. 440

Copy and complete this crossword, using the clues below.



Across

2 Volume can be measured in _____ metres.

5 A solid that has flat faces and identical cross-sections.

7 The amount of surface a flat shape covers.

9 A solid shape that has a rectangle for the base and comes to a point at the top (2 words).

10 The amount of space a solid takes up.

11 The shape of a ball.

12 A unit of capacity equal to 1000 cm^3.

13 Millimetre (abbreviation).

Down

1 A solid shape with identical triangles at each end. The other faces are rectangles. (2 words)

3 A solid shape with circular ends, the shape of a can.

4 The amount of liquid or gas a container can hold.

6 A standard unit of length.

8 A solid made from 2 or more solids.

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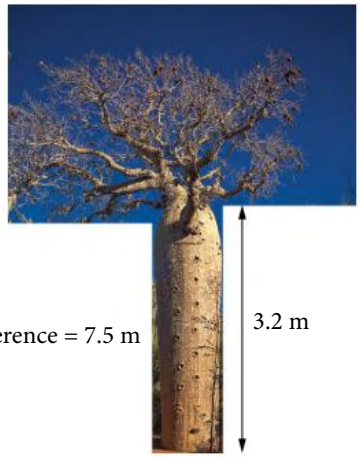
Solution to the chapter problem

Problem

Large baobab trees found in the Kimberley region of Western Australia are a great emergency source of fresh drinking water. Up to 76% of the volume of the trunk is fresh water. The trunk of the tree in the photo is 3.2 m tall and its circumference is 7.5 m. Approximately how many litres of fresh water does the tree hold?



Nataliya Derkach/Shutterstock.com

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

To find the number of litres of water the baobab tree could hold.

We know the tree is 3.2 m tall and has a circumference of 7.5 m.

STAGE 2: SOLVE THE PROBLEM

SOLVE

Calculate the volume of the cylinder using the formula $V = \pi r^2 h$.
We know $h = 3.2$ m. Need to calculate r from the circumference.

$$C = 2\pi r$$
$$7.5 = 2\pi r$$
$$r = \frac{7.5}{2\pi}$$
$$r = 1.193\ldots$$

Volume of trunk $= \pi r^2 h$
$= \pi \times 1.193\ldots^2 \times 3.2$ m
$= 14.323\ldots$ m^3

Volume of water = 76% of 14.323 ...
$= 10.886\ldots$ m^3
$= 10.886\ldots \times 1000$ L (1 m^3 = 1000 L)
$\approx 10\,886$ L

The tree could hold up to 10 886 litres of water.

STAGE 3: CHECK THE SOLUTION

CHECK

It's a lot of water but it's a big tree.
Check the thinking and calculations.
There are no obvious errors. The answer is probably right.

STAGE 4: PRESENT THE SOLUTION

PRESENT

The tree could hold up to 11 000 L of water.

The calculations do not take into account the thickness of the tree trunk, so the actual amount of water the tree can hold would be less than the calculated amount.

7 Test yourself

Turn up the volume

ANSWERS p. 440

1 EXERCISE 7.1 Copy and complete each conversion.

a 5 kg = _____ g

b 200 g = _____ kg

c 1.4 t = _____ kg

d 3.5 g = _____ mg

e 7500 kg = _____ t

2 EXERCISE 7.2 The label on a can of light coconut milk shows the following nutritional information.

Nutritional information Servings per packet: 3.85 Serving size: 70 mL			
	Average quantity per serving	Average quantity per 100 mL	Regular coconut milk per 100 mL
Energy	498 kJ	711 kJ	1025 kJ
Protein	1.4 g	_____ g	2.6 g
Fat – total saturated	11.7 g 10.4 g	16.7 g 14.8 g	25.05 g 22.2 g
Carbohydrate Sugars	2.2 g _____ g	3.1 g 1.7 g	3.9 g 1.6 g
Sodium	34 mg	49 mg	24 mg

a How many kJ are in one serving?

b How many mg of carbohydrates are in one serving?

c Use a calculation to show that regular coconut milk contains 50% more fat than light coconut milk.

d Explain how you know that 10 mL of light coconut milk contains 0.2 g of protein, then copy and complete the missing value for the protein in an average 100 mL.

e How many grams of sugar are in an average serve?

f Use a calculation to show that the can contains approximately 270 mL.

3 EXERCISE 7.3 Copy and complete each conversion.

a 5 cm^3 = _____ mm^3

b 2 m^3 = _____ cm^3

c 500 mm^3 = ____ cm^3

d 0.25 m^3 = _____ cm^3

e 24 000 cm^3 = ____ m^3

f 36 000 mm^3 = ____ cm^3

4 EXERCISE 7.3 Complete each statement.

a A container with a volume of 24 cm^3 holds _____ mL.

b The volume of a container with a capacity of 6 L is _____ cm^3.

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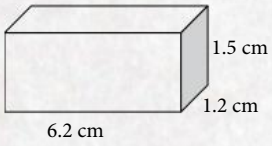
5 EXERCISE 7.3 The volume of a small wine barrel is 0.7 m³. How many litres does the barrel hold?

Eltoro69/Dreamstime LLC

6 EXERCISE 7.4 Calculate the volume of each solid.

a



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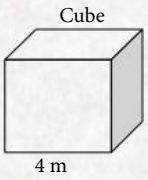

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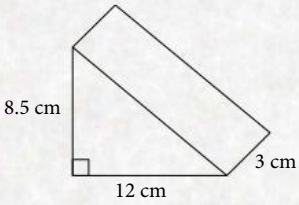

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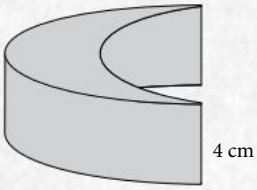


Area of base = 12 cm²

7 EXERCISE 7.5

© Sue Thomson

a Calculate the volume of the box containing the pancake mix.

b There is 7 cm of space in the top of the box to stop the contents being squashed. Calculate the volume of the actual pancake mix.

c What percentage of the volume of the box is pancake mix? Answer to the nearest whole percentage.

8 EXERCISES 7.5, 7.6

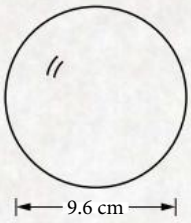
a Calculate the volume of the container of Mexican spice. Answer to the nearest cm^3.

b 90% of the volume of the container is spice. What is the volume of the spice in an unopened spice container?

c The mass of the contents is 50 g. Jess needs to add 12 cm^3 of spice to the Mexican tacos she is cooking. What is the weight of the spice she needs to add? Answer correct to the nearest gram. PS

9 EXERCISE 7.6 Calculate, correct to one decimal place, the volume of each solid.

a



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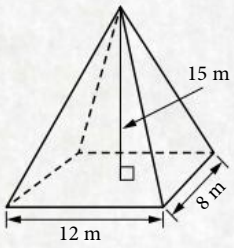

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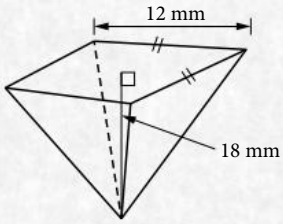

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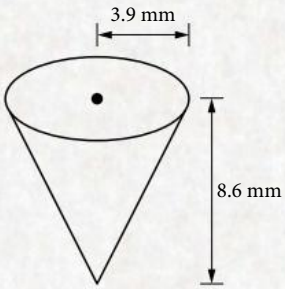
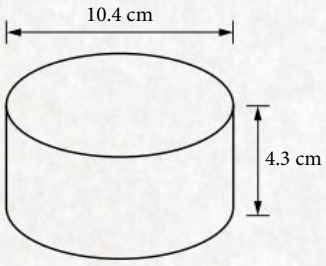


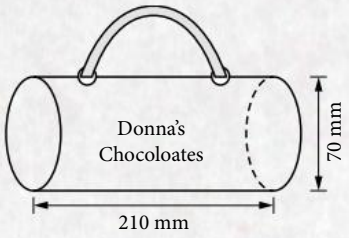
e



10 EXERCISE 7.6 This is the container Donna designed to hold the chocolates she makes and sells.

a Calculate correct to the nearest cubic centimetre the volume of the container.

b Express the capacity of the container in litres.



11 During the Secret War (1960–1975) the American CIA covertly dropped millions of bombs on Laos. The American Government of the time claimed to not know of the CIA's activities. The countryside is still damaged by large numbers of hemispherical craters caused by the bombs. The diameter of the bomb crater in the photograph is 18.5 m.

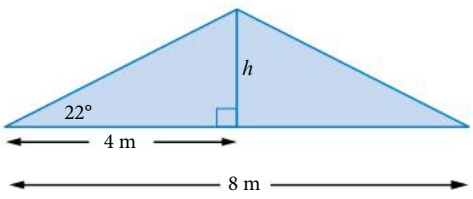
Matyas Rehak/Shutterstock.com

a Use the formula $C = 2\pi r$ to calculate the distance around the top of the crater.
Answer correct to the nearest metre.

b EXERCISE 7.6 How many cubic metres of rocks and soil are required to fill the hole?
Answer to the nearest whole number.

Chapter Problem

Chris is building a garage. The garage is 8 m wide and the pitch of the roof is 22°. How high does he need to make the truss, h, correct to 2 decimal places?



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CHAPTER 8

MEASUREMENT, SCALES AND CHANCE: SCALES, PLANS AND MODELS

SO, YOU'VE GOT A RIGHT ANGLE

What will we do in this chapter?

- Solve practical problems using Pythagoras' theorem
- Use trigonometry to calculate the lengths of sides in right-angled triangles, including the hypotenuse
- Use trigonometry to calculate the sizes of angles in right-angled triangles
- Use trigonometry to solve practical problems, including those involving angles of elevation and depression

How are we ever going to use this?

- Determine a length when we can't measure it
- Many people who work in trades, for example, carpenters, builders and land surveyors, use right-angled triangle calculations in their work

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Syllabus coverage

UNIT 3, TOPIC 2: SCALES, PLANS AND MODELS

Right-angled triangles

- Apply Pythagoras' theorem to solve problems for all side lengths where c is length of hypotenuse and a and b are lengths of the perpendicular sides, using the formulas:
  - $c^2 = a^2 + b^2$
  - $a^2 = c^2 - b^2$
  - $b^2 = c^2 - a^2$
- Apply the cosine, sine and tangent ratios to find unknown angles (θ) and sides [complex].
  - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
- Use the concepts of angle of elevation and angle of depression to solve practical problems [complex].

Videos (4):

8.5 Angles of elevation and depression • Angle of depression
8.6 Finding an unknown side
8.8 Finding the hypotenuse

Skillsheet (1):

8.1 Pythagoras' theorem

Worksheets (12):

8.1 Pythagoras' leopard • Pythagoras' problems • Pythagoras theorem time trial • Pythagorean two-step problems • Applications of Pythagoras' theorem
8.4 Using the tan ratio to calculate an angle
8.5 Angles of elevation and depression
8.6 Trigonometric ratios • Trigonometric calculations • Identifying the correct trigonometric ratio
8.8 Calculating lengths and angles • Mixed trig questions

Puzzles (3):

8.1 Pythagoras' puzzle
8.8 A triggy riddle
Keyword activity That's the right angle!

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

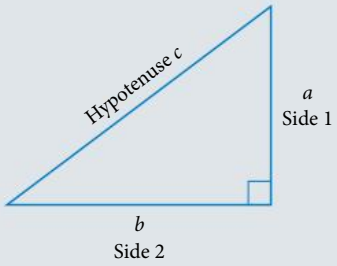
adjacent	angle	angle of depression	angle of elevation
cosine	hypotenuse	opposite	Pythagoras' theorem
right angle	right-angled triangle	side	sine
tangent	trigonometry		

8.1 Pythagoras' theorem

Pythagoras was an ancient Greek mathematician who lived from 580 to 500 BCE. **Pythagoras' theorem** is named after him, even though no one is sure whether it was Pythagoras himself or one of his followers who proved the theorem. What we do know is that mathematicians from different ancient civilisations knew about the theorem well before Pythagoras himself.

Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides.



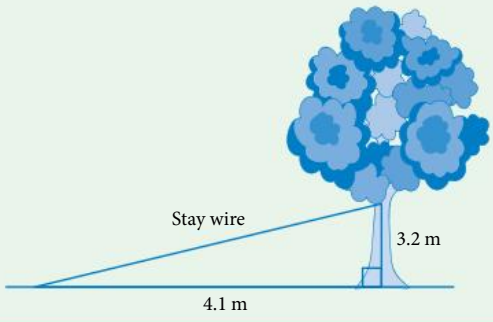
$$(\text{hypotenuse})^2 = (\text{side 1})^2 + (\text{side 2})^2$$
$$c^2 = a^2 + b^2$$

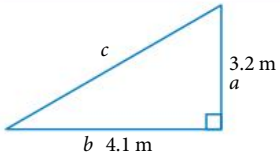
The **hypotenuse** is the longest side of a right-angled triangle. When we want to calculate either of the perpendicular sides, we can use either of the formulas:

$$a^2 = c^2 - b^2 \text{ or } b^2 = c^2 - a^2$$

EXAMPLE 1 Calculating the hypotenuse

Daniel needs to install a wire to support a tree damaged in a storm. He will attach the wire to the tree 3.2 m above the ground and anchor it 4.1 m from the base of the tree. He'll need to allow an extra 0.5 m of wire to wrap around the tree. Calculate the length of wire he will need.



Steps	Working
Label the hypotenuse as c and the perpendicular sides as a and b. It doesn't matter which sides you make a and b.	
Substitute the known lengths into Pythagoras' theorem and find c.	$c^2 = a^2 + b^2$ $c^2 = 3.2^2 + 4.1^2$ $c^2 = 27.05$ $c = \sqrt{27.05}$ $c = 5.2$ m to one decimal place
Daniel needs an additional 0.5 m.	$5.2 + 0.5 = 5.7$
State the answer.	Daniel requires 5.7 m of wire.

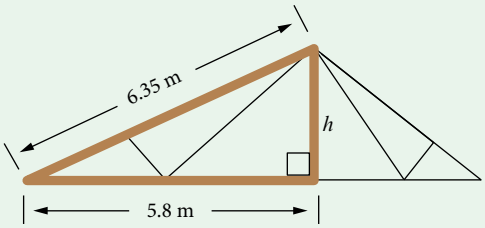
Skillsheet Pythagoras' theorem

Worksheets Pythagoras' leopard
Pythagoras' problems
Pythagoras theorem time trial
Pythagorean two-step problems
Applications of Pythagoras' theorem

Puzzle Pythagoras' puzzle

EXAMPLE 2 Calculating one of the shorter sides

Use Pythagoras' theorem to find the height (h) of this roof truss, correct to one decimal place.

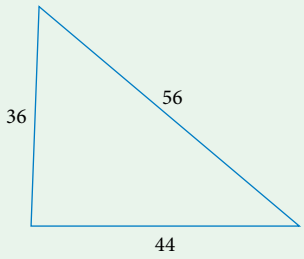


Steps	Working
The hypotenuse is 6.35 and one side is 5.8. Label the hypotenuse c, side 5.8 b, then use the form of Pythagoras' theorem: $a^2 = c^2 - b^2$	$a^2 = c^2 - b^2$ $h^2 = 6.35^2 - 5.8^2$ $= 6.6825$ $h = \sqrt{6.6825}$ $\approx 2.6\text{ m}$

We can also use Pythagoras' theorem to test whether a triangle contains a right angle. If the theorem works, there's a right angle. If the theorem doesn't work, the triangle isn't right-angled.

EXAMPLE 3 Testing for a right angle

Is this triangle right-angled?

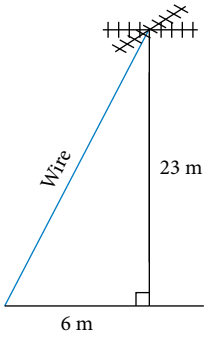


Steps	Working
We need to check whether $56^2 = 36^2 + 44^2$. If both sides of the equation are equal, then the triangle is right-angled.	$56^2 = 3136$ $36^2 + 44^2 = 3232 \neq 3136$ Both sides are not equal. Pythagoras' theorem doesn't work for these side lengths, so the triangle is not right-angled.

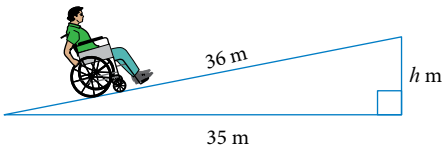
EXERCISE 8.1 Pythagoras' theorem

ANSWERS p. 440

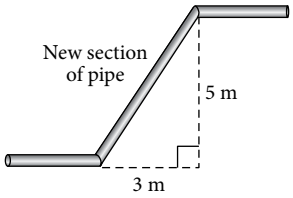
1 EXAMPLE 1 What length of wire is required to connect the top of a 23 m TV antenna to a hook 6 m from the base of the antenna? Answer correct to one decimal place.



2 EXAMPLE 2 This wheelchair ramp is 36 m long and covers a horizontal distance of 35 m. Calculate the rise, h m, of the ramp, correct to one decimal place.

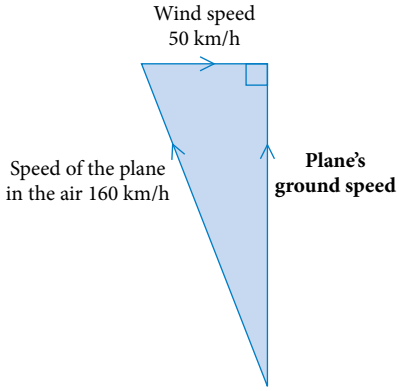


3 Tomasz installs a new section of pipe to join 2 existing pipes. Calculate the length of the new section of pipe. Express your answer in metres, correct to the nearest millimetre.



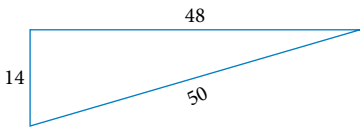
When a measurement is in metres, the nearest mm means 3 decimal places. The nearest cm is 2 decimal places.

4 Ellie is flying a small plane at 160 km/h against a 50 km/h wind, as shown in the diagram. Calculate the plane's ground speed, correct to the nearest km/h.



5 EXAMPLE 3 Use Pythagoras' theorem to test whether each triangle is right-angled.

a



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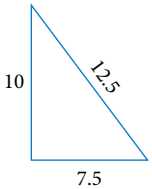
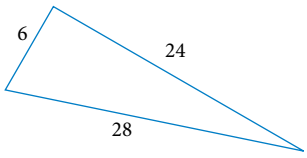
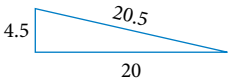


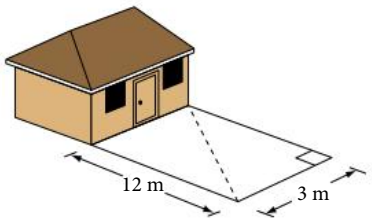
c



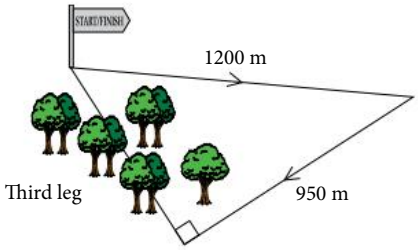
d



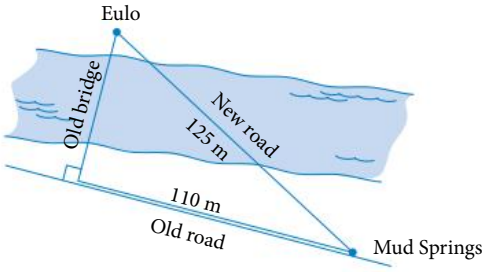
6 Mohammed is laying a concrete slab 12 m by 3 m in front of his shed. He uses Pythagoras' theorem to check that the corners of the slab are right angles. How long should the diagonal be?
Express your answer in metres, correct to the nearest centimetre.



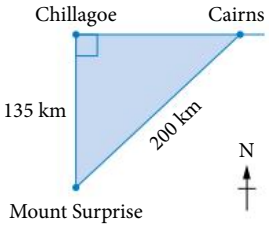
7 The school cross-country course is in the shape of a right-angled triangle. The first leg is 1200 m and the second leg is 950 m. The third leg, through thick scrub, is difficult to measure. Calculate its length, correct to the nearest metre.



8 Bella had to cross the river to get from Mud Springs to Eulo. The new road across the river was closed for repairs so she had to use the old bridge. How much farther did she have to travel using the old road and old bridge compared to the direct route across the new bridge?
Answer correct to one decimal place. **PS**



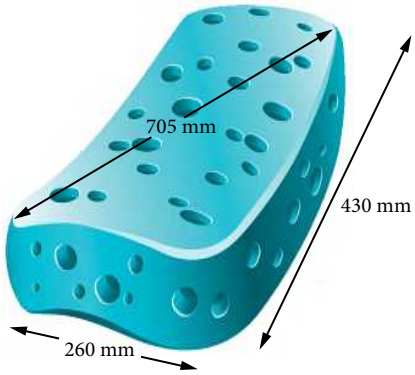
9 Joe is the pilot of a small plane. He planned to fly 200 km from Mount Surprise to Cairns. Because of poor weather conditions between Mount Surprise and Cairns, Joe flew 135 km due north to Chillagoe then turned due east and flew to Cairns. Calculate the distance from Chillagoe to Cairns, correct to the nearest kilometre. **PS**



10 The hypotenuse of a right-angled triangle is 24 m.
Draw 2 possible triangles, showing the lengths of the other 2 sides, correct to 2 decimal places. **PS**

Investigation | Cutting rectangles

Renée works in an upholstery business. One of her jobs is cutting out rectangular pieces of foam to make seat cushions. She always has a problem judging whether the cut foam is square (square means 'at right angles').



Renée thinks the foam in the diagram is square. Is she right?
Describe a process Renée could use to check whether her foam blocks are square.

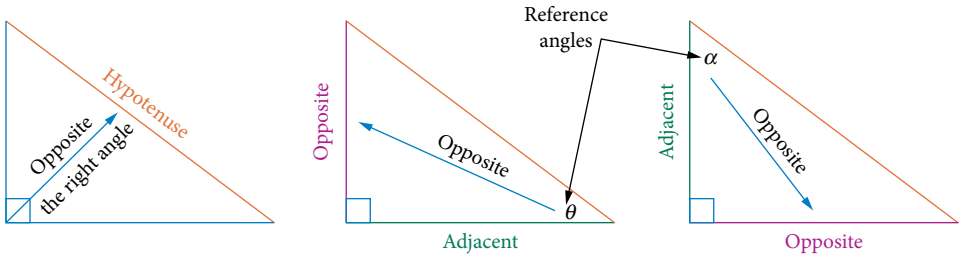
8.2 The sides of a right-angled triangle*

The word 'trigonometry' comes from two Greek words that mean 'the measurement of triangles'. **Trigonometry** is the branch of mathematics used to solve problems involving triangles.

In a right-angled triangle, the longest side is the side opposite the right angle, and it is called the **hypotenuse**. The names of the other 2 sides are determined by the reference angle.

The **opposite side** is the side facing the reference angle.

The **adjacent side** is next to the reference angle. Adjacent means 'next to'.

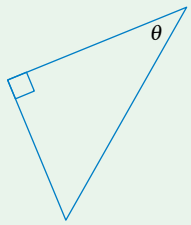


When naming the sides of a right-angled triangle, always start with the hypotenuse because its position never changes. Then determine the opposite and adjacent sides.

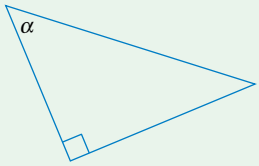
EXAMPLE 4 Adjacent and opposite sides

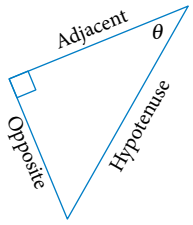
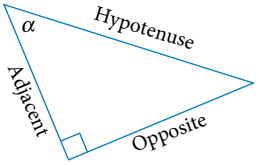
For each triangle, name its sides according to the marked angle.

a



b



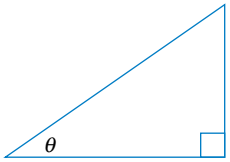
Steps	Working
a The hypotenuse is always the longest side, opposite the right angle. The opposite side is on the other side of the triangle from θ. The adjacent side joins θ to the right angle.	
b The hypotenuse is opposite the right angle. The opposite side is opposite α. The adjacent side joins α to the right angle.	

EXERCISE 8.2 The sides of a right-angled triangle*

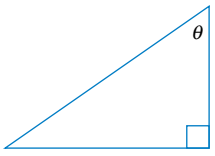
ANSWERS p. 440

1 EXAMPLE 4 Copy each triangle and name its sides according to the marked angle.

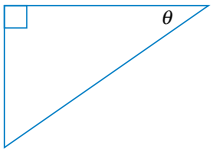
a



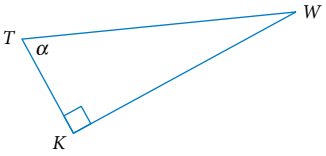
b



c



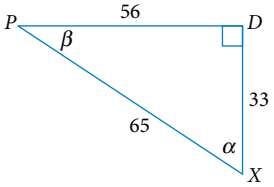
2 The sides of $\triangle TWK$ are TW, TK and WK. Which side is opposite angle α?



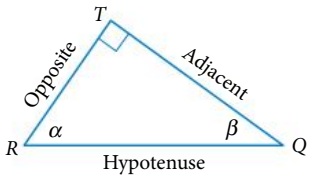
3 **a** How long is the hypotenuse in this triangle?

b How long is the opposite side to angle α?

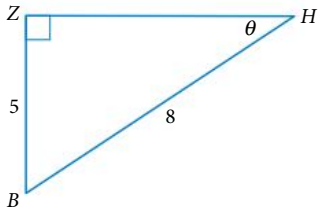
c How long is the adjacent side to angle β?



4 Which angle, α or β, was the reference angle for naming the sides of $\triangle RTQ$?

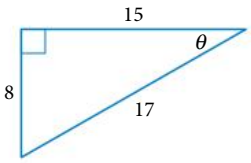


5 In $\triangle HZB$, the fraction $\frac{\text{opposite}}{\text{hypotenuse}}$ for θ is equal to $\frac{5}{8}$ because opposite = 5 and hypotenuse = 8.

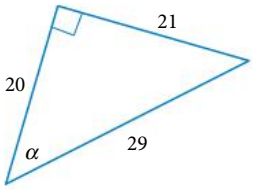


Determine the fraction $\frac{\text{opposite}}{\text{hypotenuse}}$ for the marked angle in each triangle.

a



b



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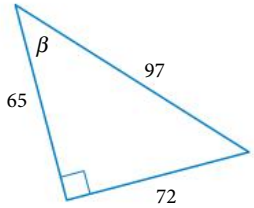


6 The side opposite angle β in a right-angled triangle is 3 cm long and the adjacent side is 5 cm long.

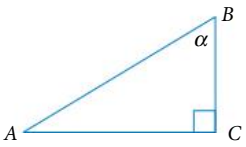
a Draw the triangle as accurately as possible.

b Measure the size of angle β, correct to the nearest degree.

7 **a** Which side is the hypotenuse in this triangle: AB, AC or BC?

b What is the opposite side to angle α?

c What is the adjacent side to angle α?



8.3 The tangent ratio*

The **tangent** ratio, abbreviated tan, is the ratio of the length of the opposite side to the length of the adjacent side for a reference angle in a right-angled triangle. It is the fraction $\frac{\text{opposite}}{\text{adjacent}}$.

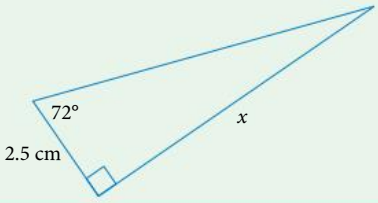
The tangent ratio

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The tangent ratio can be used to calculate the length of an unknown side in a right-angled triangle.

EXAMPLE 5 The tan ratio

Use the tangent ratio to find the length of x in the triangle. Express your answer correct to 2 decimal places.



Steps	Working
1 For 72° in the triangle, x is the opposite side and 2.5 cm is the adjacent side. Use the formula $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.	$\tan 72° = \frac{x}{2.5}$
2 Solve the equation. First, swap sides, then multiply both sides by 2.5.	$\frac{x}{2.5} = \tan 72°$ $\frac{x}{2.5} \times 2.5 = \tan 72° \times 2.5$ $x = 2.5 \tan 72°$
3 Enter 2.5 tan 72 = on your calculator.	$x = 7.694\,20$ ≈ 7.69 cm
From the diagram, $x \approx 7.69$ cm seems a reasonable answer.	

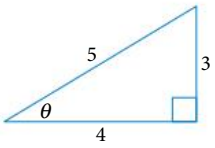
Make sure that your calculator is set in degrees mode, DEG or D.
If it is set to RAD or GRAD, your calculator will give you the wrong answer.
Ask your teacher for help if needed.

EXERCISE 8.3 The tangent ratio*

ANSWERS p. 441

1 For each triangle, write $\tan\theta$ as a fraction.

a



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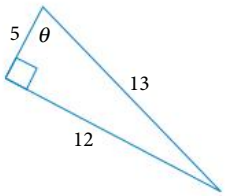
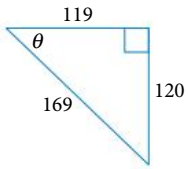


c



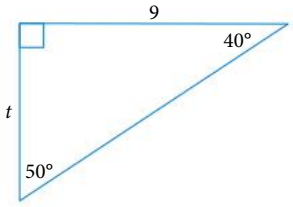
2 Accurately construct a triangle in which $\tan\theta = \frac{2}{3}$.

3 Copy and complete the working to calculate the value of t in the diagram. Express your answer correct to one decimal place.

$$\frac{t}{9} = \tan\square$$

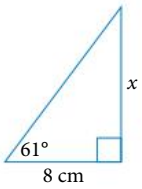
$$\square \times \frac{t}{9} = \tan\square \times \square$$

$$t \approx \square$$

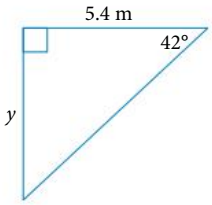


4 EXAMPLE 5 Find the value of each variable, correct to 2 decimal places.

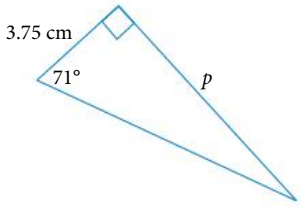
a



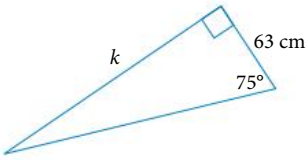
b



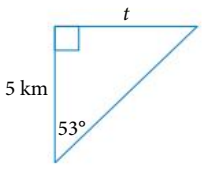
c



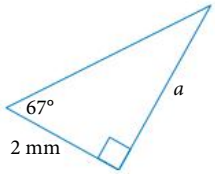
d



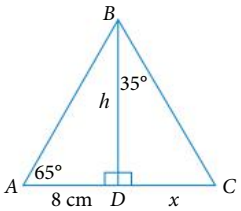
e



f



5 These 2 right-angled triangles share a side BD.



a Use $\triangle ABD$ to find h, correct to 2 decimal places.

b Use $\triangle BDC$ to find x, correct to one decimal place.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

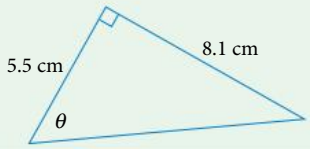
Worksheet
Using the tan ratio to calculate an angle

8.4 Using tan to find an angle*

We can use the formula $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to calculate angles, as well as sides in a right-angled triangle.

EXAMPLE 6 Calculating angles with tan

Calculate the size of angle θ, correct to the nearest degree.



Steps	Working
1 The opposite side is 8.1 cm and the adjacent side is 5.5 cm. Use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find the angle θ.	$\tan\theta = \frac{8.1}{5.5}$
2 To 'undo the tan', press SHIFT tan on your calculator. Enter SHIFT tan 8.1 ÷ 5.5 =.	$\theta = 55.8230$
3 Round to the nearest degree.	$= 56°$

From the diagram, 56° seems a reasonable answer.

EXERCISE 8.4 Using tan to find an angle*

ANSWERS p. 441

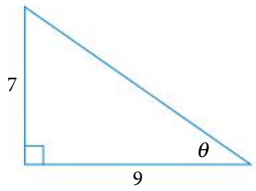
1 EXAMPLE 6 Use your calculator to find the value of θ, correct to the nearest degree.

a $\tan\theta = 0.86$ **b** $\tan\theta = 1.07$ **c** $\tan\theta = \frac{3}{4}$

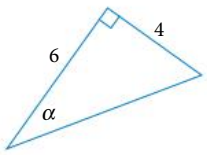
d $\tan\theta = \frac{3}{5}$ **e** $\tan\theta = \frac{11}{8}$ **f** $\tan\theta = 1\frac{1}{3}$

2 Use the tan ratio to find the size of each marked angle, correct to the nearest degree.

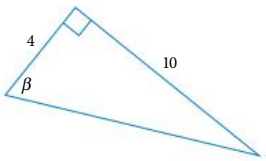
a



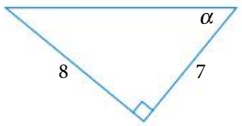
b



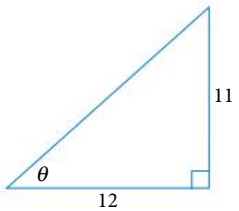
c



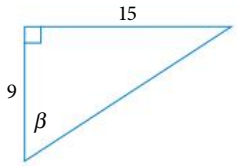
d



e



f



3 The shorter (non-hypotenuse) sides of a right-angled triangle are 4 cm and 6 cm long. Calculate the sizes of all the angles in the triangle, correct to the nearest degree. PS

4 The shorter sides in a right-angled triangle are both 5 cm long. PS

a Find the sizes of the angles in the triangle.

b Explain why tan 45° = 1.

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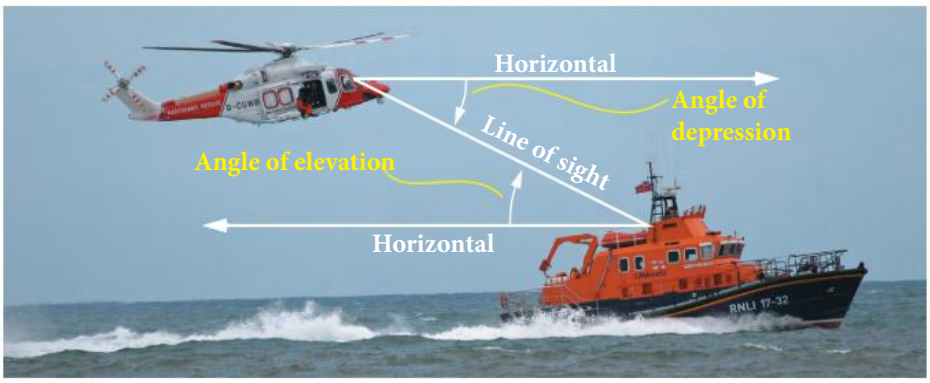
8.5 Angles of elevation and depression*

Angles of elevation and depression are used regularly to solve practical measurement problems.



Stephen How/Alamy Stock Photo

Angles of elevation and depression

The **angle of elevation** is the angle the eye turns **up** from the horizontal to look at an object in a higher position.

The **angle of depression** is the angle the eye turns **down** from the horizontal to look at an object in a lower position.

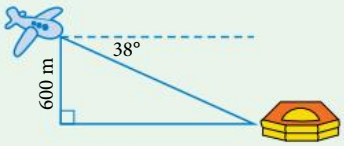
Videos
Angles of elevation and depression

Angle of depression

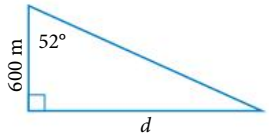
Worksheet
Angles of elevation and depression

EXAMPLE 7 Angle of depression

A pilot flying at a height of 600 m saw a life raft in the sea at an angle of depression of 38°.
Calculate the horizontal distance from the plane to the life raft, correct to the nearest metre.



It's a common error to write the angle of depression as the top angle inside the triangle. The angle of depression is outside the triangle.

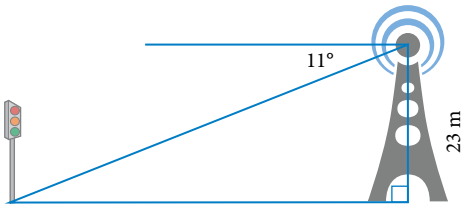
Steps	Working
Let d be the horizontal distance from the plane to the life raft.	
The angle of depression and the angle at the top of the triangle add to 90°.	
The top angle = 90° − 38° = 52°.	
Use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ because the opposite and the adjacent sides are involved.	$\tan 52° = \frac{d}{600}$
	$d = 600 \times \tan 52°$
	$= 767.9649\ldots$
Write the answer.	The horizontal distance from the plane to the life raft is 768 m.

EXERCISE 8.5 Angles of elevation and depression*

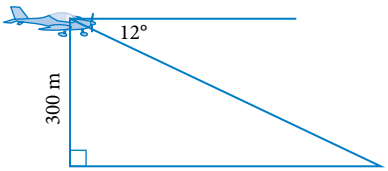
ANSWERS p. 441

In this exercise, express your answers correct to one decimal place (for lengths) or the nearest degree (for angles), unless instructed otherwise.

1 EXAMPLE 7 From the top of a mobile phone tower 23 m high, the angle of depression to the bottom of a set of traffic lights is 11°. How far is the set of traffic lights from the base of the mobile phone tower?

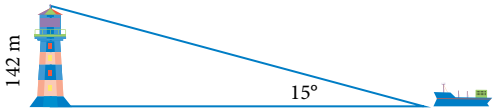


2 Maahi is flying an ultra-light aircraft at a height of 300 m. Her angle of depression to the landing strip is 12°.



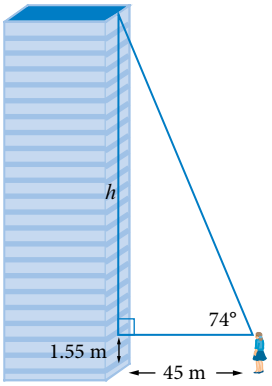
Calculate the horizontal distance from the plane to the landing strip.

3 A navigation chart shows that the top of a lighthouse is 142 m above sea level. A ship's navigator measured the angle of elevation to the top of the lighthouse as 15°.



How far is the ship from the base of the lighthouse?

4 Abrar is 45 m away from a tall office block. He measured the angle of elevation to the top of the office block as 74°.



a Calculate the height, h, correct to 3 decimal places.

b Abrar's eyes are 1.55 m above the ground. Calculate the height of the building.

5 When Mason was 810 m away from the centre of Uluru at ground level, he measured the angle of elevation to the top of the rock as 20°.

Aldo Manganaro/Shutterstock.com

a Draw a diagram to show Mason's measurements.

b Calculate, correct to the nearest metre, the height of Uluru using Mason's measurements.

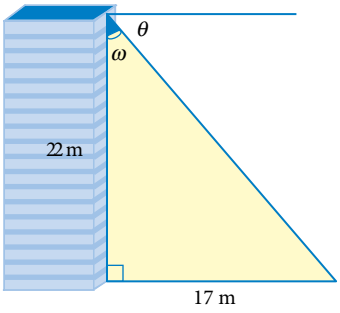
6 The third level of the Eiffel Tower in Paris is 276 m above the ground. When Elyse was standing on the third level of the tower, she measured the angle of depression to a group of her friends on the ground as 19°. How far were Elyse's friends from the base of the Eiffel Tower below her? PS

Beboy/Shutterstock.com

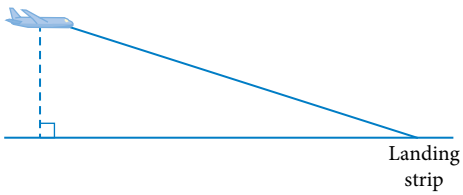
7 The front of an office block has security lights that come on when they detect movement. The lights are 22 m above the ground, and they light up a 17 m wide strip in front of the building.

a Calculate the size of angle ω.

b Determine the size of angle θ, the light's angle of depression.



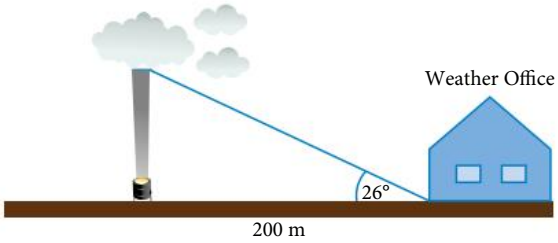
8 From a small plane flying at a height of 800 m, the angle of depression to the landing strip is 14°.



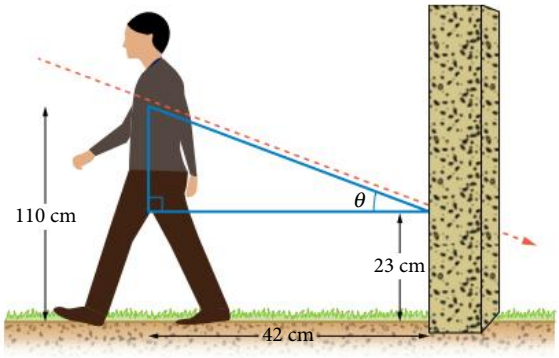
a Copy the diagram and show the position of the angle of depression.

b Calculate, correct to the nearest metre, the horizontal distance from the plane to the landing strip.

9 During World War II, searchlights were used with trigonometry to calculate the height of clouds. Calculate the height of the clouds in the diagram below when the angle of elevation to the clouds is 26°.



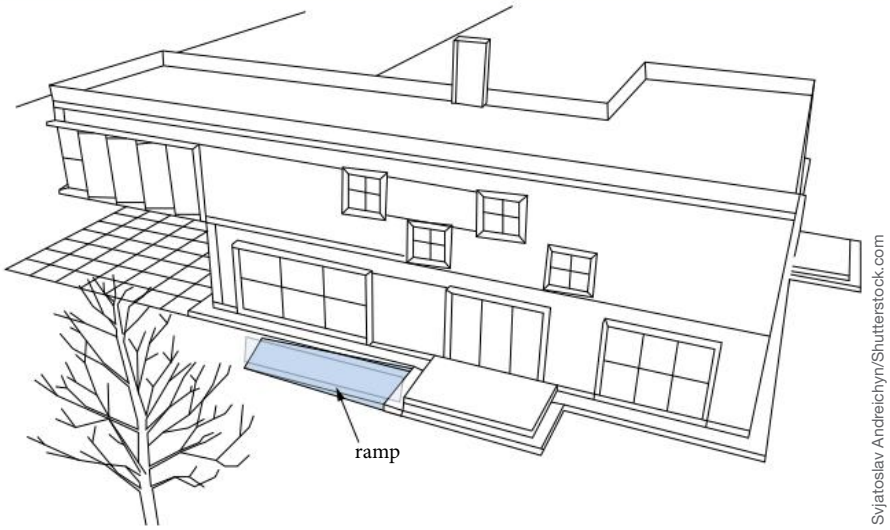
10 Forensic experts are investigating a shooting scene. A man was walking when a bullet flew past him 110 cm above the ground. The bullet then lodged in a wall 42 cm behind the man and 23 cm above the ground.



a Calculate the length of the side opposite angle θ by subtracting 23 from 110.

b Calculate the angle of elevation from the bullet to where the gun was fired.

Extended investigation Childcare access ramp



Svjatoslav Andreichyn/Shutterstock.com

The new community centre is installing a ramp to improve accessibility for people with disabilities, as well as to allow for easier access for families with strollers and prams. The ground in front of the centre is level but there are 2 steps up to the front door.

Your task is to research appropriate information, design a suitable ramp and produce a report detailing the results of your research, calculations and design. Your report should include:

- the usual height of 2 steps
- the recommended angle of inclination for a wheelchair, stroller or pram ramp
- a sketch showing your design
- the horizontal and actual lengths covered by the ramp you designed.

8.6 The sine and cosine ratios*

We use the **sine** and **cosine** ratios when calculations involve the hypotenuse.

The abbreviation for sine is **sin** and it is pronounced 'sign'.

The abbreviation for cosine is **cos**.

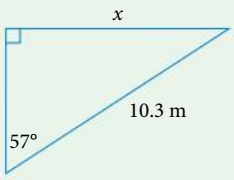
The sine and cosine ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

EXAMPLE 8 Using sine to calculate a side length

Calculate the value of x in this triangle, correct to one decimal place.

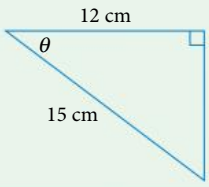


Steps	Working
x is the opposite side and 10.3 m is the hypotenuse. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin 57° = \frac{x}{10.3}$ $\frac{x}{10.3} = \sin 57°$
Multiply both sides by 10.3.	$\frac{x}{10.3} \times 10.3 = \sin 57° \times 10.3$
Evaluate x.	$x = 10.3 \sin 57°$ $= 8.6383$ ≈ 8.6 m

From the diagram, $x \approx 8.6$ m seems a reasonable answer.

EXAMPLE 9 Using cos to calculate an angle

Calculate the size of θ, correct to the nearest degree.



Steps	Working
For angle θ, 12 cm is the adjacent side, 15 cm is the hypotenuse. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos\theta = \frac{12}{15}$
Use the 'undo the cos' keys on a calculator: SHIFT cos 12 ÷ 15 =.	$\theta = 36.8698$ $\approx 37°$

From the diagram, $\theta = 37°$ seems a reasonable answer.

Video Finding an unknown side

Worksheets Trigonometric ratios

Trigonometric calculations

Identifying the correct trigonometric ratio

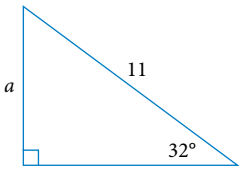
Finding an unknown angle

EXERCISE 8.6 The sine and cosine ratios*

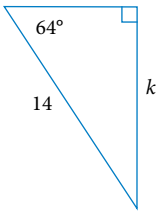
ANSWERS p. 441

1 EXAMPLE 8 Use the sin ratio to calculate the value of each variable, correct to one decimal place.

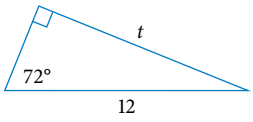
a



b

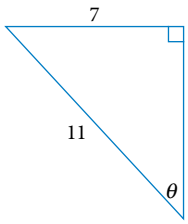


c

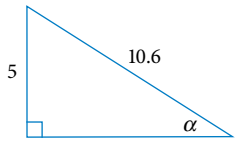


2 Use the sin ratio to determine the size of each marked angle, correct to the nearest degree.

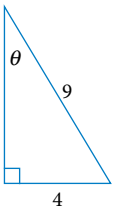
a



b

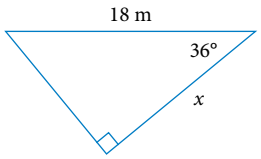


c

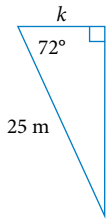


3 EXAMPLE 9 Use the cos ratio to determine the values of each variable, correct to the nearest metre.

a



b



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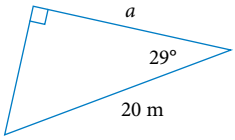
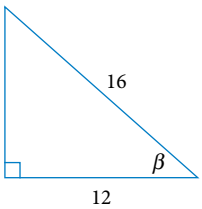


4 Calculate the value of each marked angle using the cos ratio, correct to the nearest degree.

a



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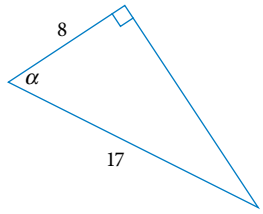

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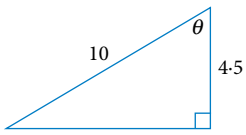
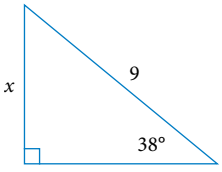


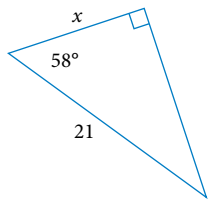
5 For each triangle, choose the sin or cos ratio, then calculate the value of x correct to one decimal place.

> When the side you know is the hypotenuse, you will need to use sin if the side with the variable is opposite and cos if it is adjacent.

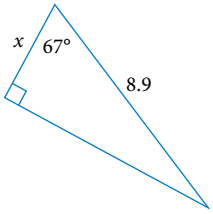
a



b



c



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8.7 Sine, cos or tan?*

One way to remember the trigonometry ratio formulas is with the phrase 'Only half an hour of algebra' and the trigonometry keys on your calculator in order:

Remembering the meaning of sin, cos and tan

Only Half	An Hour	Of Algebra
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$

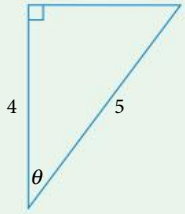
Some students like to say "SOH-CAH-TOA" to help them remember the formulas.

Choose the memory method that works for you.

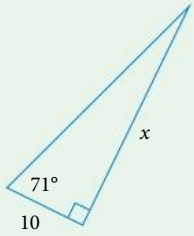
EXAMPLE 10 Choosing the right trig ratio to use

Which trigonometry ratio can we use to find the value of the variable in each triangle?

a



b

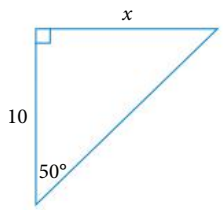


Steps	Working
a The marked sides are the adjacent, A, and the hypotenuse, H. '**A**n **h**our' is the middle section of the phrase, and the middle trig key on the calculator is cos.	The cos ratio can be used.
b The marked sides are the opposite, O, and the adjacent, A. '**O**f **a**lgebra' is the last section of the phrase, and the last trig key on the calculator is tan.	The tan ratio can be used.

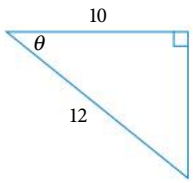
EXERCISE 8.7 Sine, cos or tan?* ANSWERS p. 441

1 EXAMPLE 10 In each triangle, which trigonometry ratio could be used to find the value of the variable?

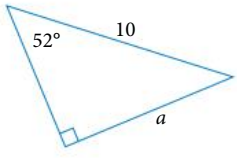
a



b

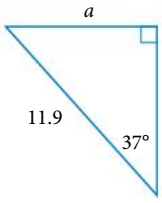


c

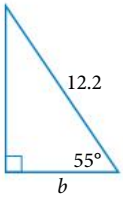


2 Find the value of each variable, correct to one decimal place.

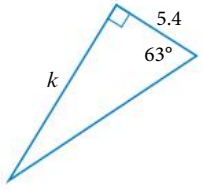
a



b

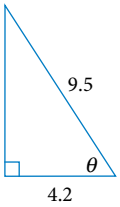


c

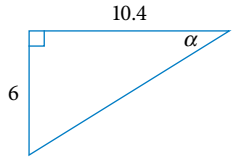


3 Determine the size of each marked angle, correct to the nearest degree.

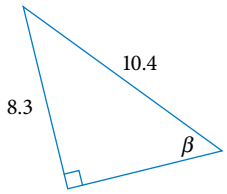
a



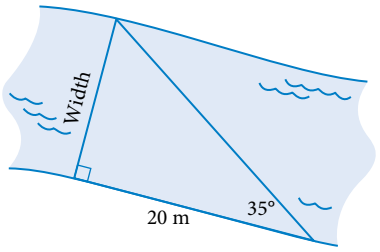
b



c

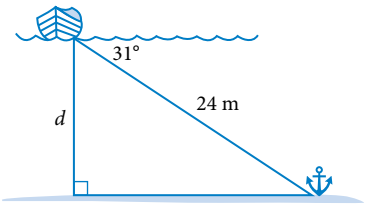


4 During a flood, Goran needed to work out the width of the river. The diagram shows the measurements he took.



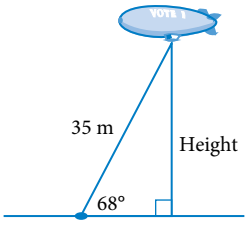
Use Goran's measurements to determine the width of the flooded river, correct to the nearest metre.

5 A boat's anchor chain is 24 m long, and it is making an angle of 31° with the top of the water.



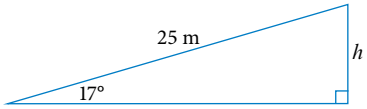
How deep, d m, is the water, correct to one decimal place? **PS**

6 An airship is anchored to the ground by a 35 m long rope. The angle between the rope and the ground is 68°.



How high is the airship above the ground, correct to one decimal place?

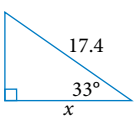
7 A skateboard ramp 25 m long leans at 17° to the horizontal.



How much higher is one end of the ramp than the other, correct to one decimal place? **PS**

8 Find the value of each variable, correct to one decimal place.

a



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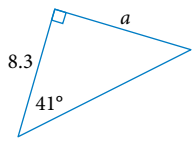
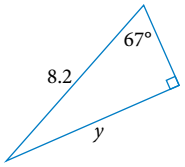
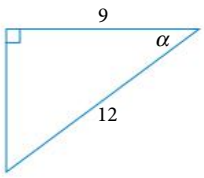


c



9 Determine the size of α, correct to the nearest degree.

a



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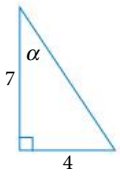

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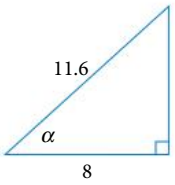
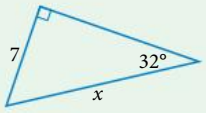


8.8 Finding the hypotenuse*

EXAMPLE 11 Calculating the hypotenuse

Calculate the length of the hypotenuse, correct to one decimal place.



Steps	Working
The sides involved are the opposite and the hypotenuse.	
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin 32° = \frac{7}{x}$
Take the reciprocal of both sides to put x on top.	$\frac{1}{\sin 32°} = \frac{x}{7}$
Swap sides to put x on the left.	$\frac{x}{7} = \frac{1}{\sin 32°}$
Multiply both sides of the equation by 7.	$\frac{x}{7} \times 7 = \frac{1}{\sin 32°} \times 7$
7 ÷ sin 32 =	$x = \frac{7}{\sin 32°}$
	$= 13.2095$
	≈ 13.2

From the diagram, $x \approx 13.2$ seems a reasonable answer.

Video
Finding the hypotenuse

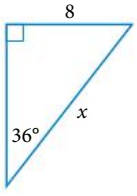
Worksheets
Finding an unknown side
Calculating lengths and angles
Mixed trig questions

Puzzle
A triggy riddle

EXERCISE 8.8 Finding the hypotenuse* ANSWERS p. 441

1 EXAMPLE 11 Calculate the length of the hypotenuse in each triangle, correct to one decimal place.

a



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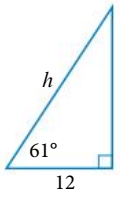

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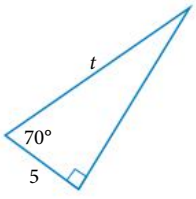
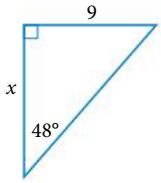
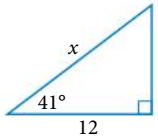


2 Determine, correct to one decimal place, the value of x in each triangle.

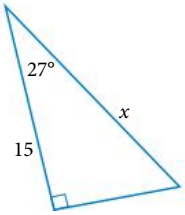
a



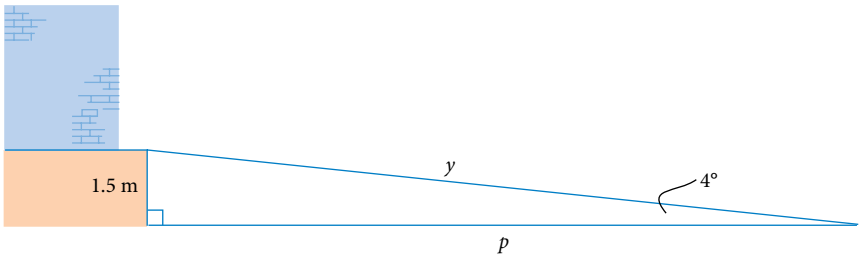
b



c



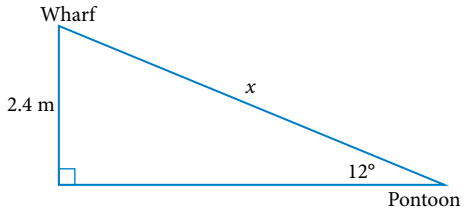
3 Scott is designing a wheelchair access ramp. The ramp will connect the car park with the building. The building is 1.5 m higher than the car park. The angle of inclination must not exceed 4°. PS



a Calculate the minimum horizontal distance, p, required for the ramp, correct to 2 decimal places.

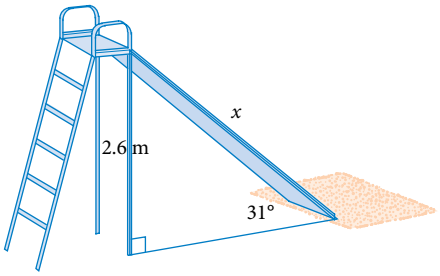
b How long is the corresponding ramp, y, correct to one decimal place?

4 At low tide, a wharf is 2.4 m above the water. The ramp joining the wharf to a floating pontoon makes an angle of 12° to the horizontal.



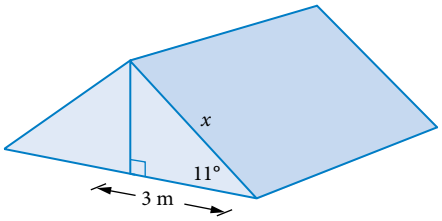
Calculate the length of the ramp, x, correct to one decimal place. PS

5 The council installs a children's slide in the park. The top of the side is 2.6 m high, and the slide makes an angle of 31° with the ground.



Calculate the length of the slide, x, correct to 2 decimal places.

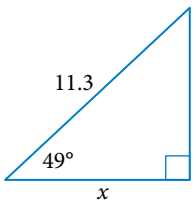
6 This diagram shows a shed roof.



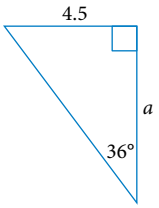
Calculate x, the length of each piece of iron sheeting required. Round your answer up to the nearest 10 cm. PS

7 Find the value of each variable, correct to one decimal place.

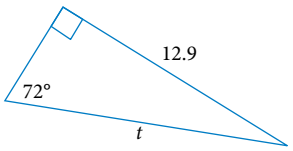
a



b



c

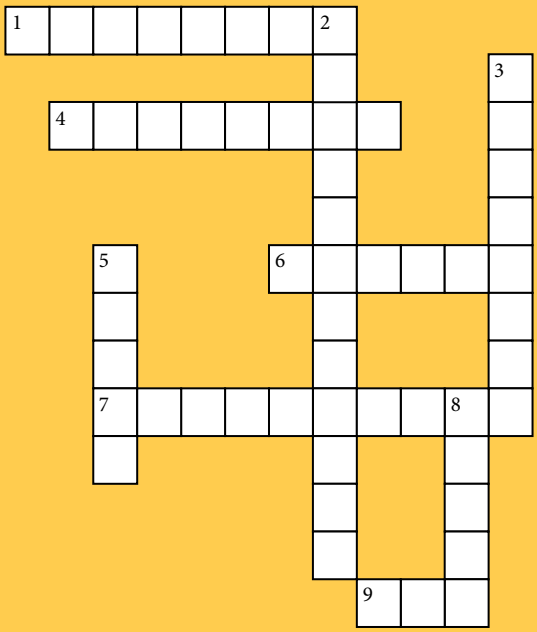


Keyword activity

TRIGONOMETRY CROSSWORD

ANSWERS p. 441

Copy the crossword and complete the trigonometry summary below to complete the crossword puzzle.



The word 'trigonometry' comes from two Greek words that combine to mean 'the measurement of triangles'. In modern times [2]_____ has many applications, including determining the size of [6]_____ and [8]_____ in right-angled triangles.

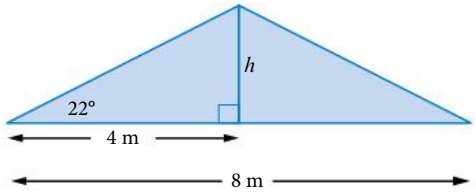
In a [5]____ angled triangle the side opposite the right angle is called the [7]_____, but the names of the other sides are determined by the reference angle. The [3]_____ side is opposite the reference angle and the [1]_____ side is next to the reference angle.

The ratio of the opposite side to the adjacent side is called the [4]_____ _____ (2 words).
The sin or the [9]_____ ratios are used when calculations involve the hypotenuse.

Puzzle
That's the right angle!

Solution to the chapter problem

Problem



Chris is building a garage. The garage is 8 m wide and the pitch of the roof is 22°.
How high does he need to make the truss, h, correct to 2 decimal places?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To work out the height of the truss, h. We know the angle 22° and 2 lengths, 4 m and 8 m.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	h is the opposite side to 22°, 4 m is the adjacent side. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan 22° = \frac{h}{4}$ $\frac{h}{4} = \tan 22°$ $h = \tan 22° \times 4$ $= 1.6161$ ≈ 1.62 m
STAGE 3: CHECK THE SOLUTION	
 CHECK	According to the diagram and considering the sizes of roofs, a truss height of 1.62 m sounds reasonable.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	Chris will need to make the truss 1.62 m high.

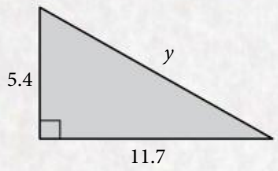
8 Test yourself

So, you've got a right angle

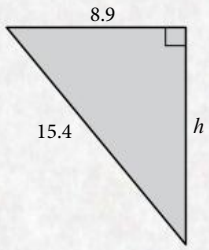
ANSWERS p. 441

1 EXERCISE 8.1 Find the value of each variable, correct to one decimal place.

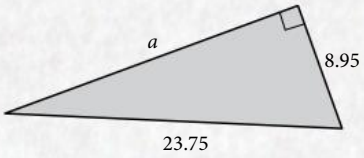
a



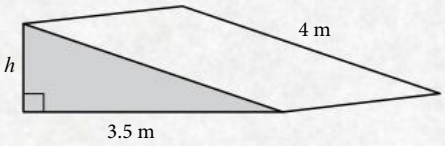
b



c



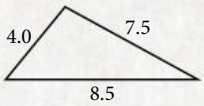
2 EXERCISE 8.1 Gianni works in a factory that makes solar panels. He is making a frame to hold a large panel on a roof and angle it correctly for the sun. The panel is 4 m long and the base of the frame is 3.5 m long.



How high, h, does Gianni need to make the frame (correct to one decimal place)?

3 EXERCISE 8.1 Use Pythagoras' theorem to test whether each triangle is right-angled.

a



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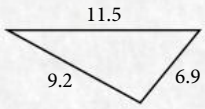
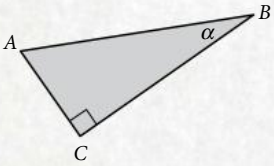


4 EXERCISE 8.2 This triangle has sides AB, AC and CB.



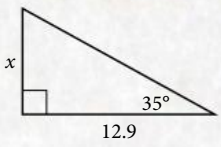
Name the side that is:

a the hypotenuse

b opposite to angle α

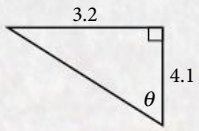
c adjacent to angle α

5 EXERCISE 8.3 Use the tan ratio to find the value of x, correct to one decimal place.

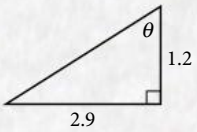


6 EXERCISE 8.4 Determine, correct to the nearest degree, the size of each angle θ.

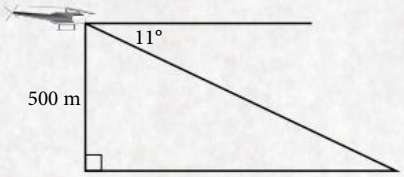
a



b

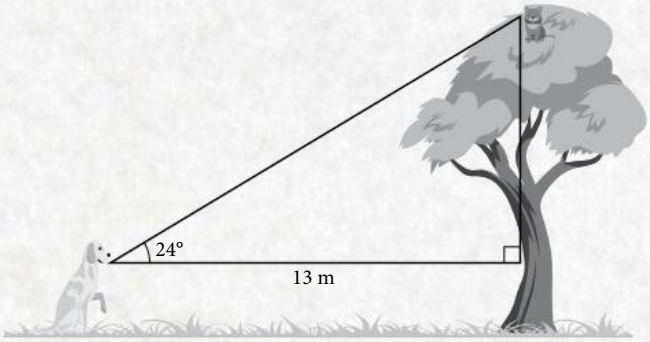


7 EXERCISE 8.5 State emergency workers are flying at a height of 500 m towards a mine disaster. The pilot can see the disaster site at an angle of depression of 11°.



Calculate the horizontal distance from the helicopter to the disaster, correct to the nearest 10 metres.

8 EXERCISE 8.5 Ginger the dog chased Oscar the cat up a tree. Ginger is 13 m from the base of the tree and her angle of elevation to Oscar is 24°. Ginger's eyes are 0.6 m above the ground. How high above the ground is Oscar, correct to one decimal place?

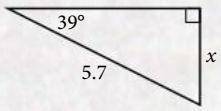


9 EXERCISE 8.6 When Sharyn skied 50 m down a slope, her height above sea level changed by 10 m. Calculate the size of angle θ, correct to the nearest degree.

Alexfe/Shutterstock.com

10 EXERCISE 8.7 Which trigonometry ratio (sin, cos or tan) can be used to calculate the value of each variable? Write the ratio, then calculate the value of the variable, correct to one decimal place for the side lengths, or to the nearest degree for the angles.

a



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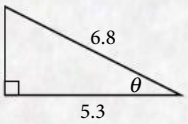

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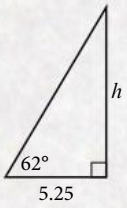
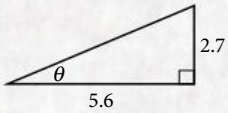
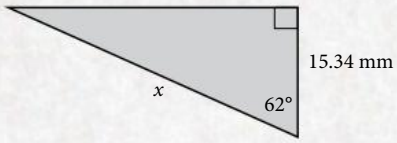


d



11 EXERCISE 8.8 Determine the value of x, correct to 2 decimal places.

a



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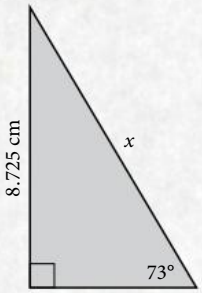

Chapter problem

Jade thinks that there is something wrong with her pair of dice. When she plays dice games, some numbers seem to come up more often than others on each die.
How can she test whether each die is fair and not 'loaded'?

Milanazavr/Shutterstock.com

CHAPTER 9

MEASUREMENT, SCALES AND CHANCE: PROBABILITY AND RELATIVE FREQUENCIES

TAKING CHANCES

What will we do in this chapter?

- Use the language of chance
- Calculate the probability of simple events
- Construct a sample space
- Use tables and tree diagrams to determine sample spaces and calculate probabilities

How are we ever going to use this?

- Determining the chance that something will or won't happen
- Making predictions about chance events
- Know whether common beliefs about chance are true
- Make sensible choices if we are going to take risks or gamble

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Syllabus coverage

UNIT 3, TOPIC 3: PROBABILITY AND RELATIVE FREQUENCIES

Simple probabilities

- Construct a sample space for an experiment.
- Use a sample space to determine the probability of outcomes for an experiment.
- Use arrays and tree diagrams to determine the outcomes and the probabilities for experiments.
- Model and solve problems involving probability experiments.

Videos (2):
9.3 Tables and tree diagrams
• Tree diagrams

Skillsheets (2):
9.1 The language of chance
• Sample space

Worksheets (5):
9.1 Games of chance
9.3, 9.4 Matching probabilities • Probability review

Keyword activity Probability of $\frac{1}{2}$
• Probability crossword

Puzzle (1):
9.3 Chance puzzle

Spreadsheet (1):
9.4, Test yourself Heads and tails

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

Terminology

array
biased die
certainty
die/dice
equally likely
experiment
impossible event
model
outcome
probability
sample space
theoretical probability
tree diagram

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9.1 Theoretical probability

Maya is hoping to score a 5 when she rolls a normal 6-sided die. When she rolls the die, the number showing on the top of the die could be 1, 2, 3, 4, 5 or 6, and each number is just as likely to show as any of the others. There is only one 5 from a possible 6 numbers. The probability or chance that she will roll a 5 is 1 number out of 6 numbers, or $\frac{1}{6}$.

Probability of an event

The probability of an event occurring, where all outcomes are **equally likely**, is given by the formula

$$P(\text{event}) = \frac{\text{number of ways the event can happen}}{\text{total number of possible outcomes}}$$

This can also be written:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Because probability is a fraction, the smallest value for probability is 0 and the largest value is 1.
An **impossible event** has a probability of 0, while a **certain event** has a probability of 1.
Unlikely events have a probability close to 0 while likely events have a probability close to 1.

A list of all the possible outcomes in a chance situation is called the **sample space**.

Skillsheets
The language of chance
Sample space

Worksheet
Games of chance

EXAMPLE 1 Sample space

An 8-sided die has the numbers from 1 to 8 written on its faces. Dean rolls this die.

Jellynewt/Shutterstock.com

a List the sample space for this situation.

b What is the probability that Dean rolls a number greater than 5?

Steps	Working
a The sample space is a list of all the possible outcomes. List the numbers from 1 to 8.	Sample space = 1, 2, 3, 4, 5, 6, 7, 8
b There are 3 numbers greater than 5 on the die: 6, 7 and 8. Write 3 on the top of the fraction. There are 8 possible numbers in the sample space. All of the numbers are equally likely, so use the probability formula. Write 8 in the bottom of the fraction.	$P(\text{greater than } 5) = \frac{3}{8}$ '3 chances out of 8'

P(greater than 5) is the shorthand way of writing 'the probability that it will be greater than 5".

EXERCISE 9.1 Theoretical probability

ANSWERS p. 441

1 EXAMPLE 1

a List the sample space for rolling a normal die.

b What is the probability of rolling a number less than 3 on a normal die?

Don't forget to simplify your fraction answer.

2 a List the sample space for tossing a coin.

b What is the probability that a coin will land showing a head? Express your answer as a fraction, decimal, percentage and ratio.

3 Peter has a paper bag containing 6 red discs and 4 blue discs. When he selects a disc at random from the bag, what is the probability that it will be red?

The word 'random' is very important. If Peter wasn't selecting at random, he would be able to deliberately pick a specific colour.

4 The game of pool is played on a table with 15 coloured balls numbered 1 to 15.

Samir is going to choose a ball at random and sink it. What is the probability that he will choose either the 6 or the 8? Answer as a fraction and a ratio.

Libor Fousek/Shutterstock.com

5 Daniela is going to choose a coloured chocolate at random from a packet of 50 chocolates. In the packet, there are 14 green, 10 orange, 20 blue and 6 red chocolates.

a What is the probability that Daniela will choose an orange chocolate? Answer as a fraction and a decimal.

b What is the probability that she will choose a chocolate that is not green? Answer as a fraction and a percentage.

c What is the most likely colour she will choose?

6 A letter is selected at random from the expression HAPPY HOLIDAY.

What is the probability that the letter is:

a an L?

b a Y?

c either a P or an H?

d a consonant?

The letters A, E, I, O and U are called vowels. The other letters of the alphabet are called consonants.

7 a What is the probability of randomly choosing E from the letters in the alphabet?

b Aaron is going to choose a letter at random from a page in a novel.

Explain why the probability that the letter will be an E is *not* $\frac{1}{26}$.

8 The tickets in a raffle are numbered from 1 to 200.

a Calculate the probability that the winning ticket will be number 157. Answer as a fraction and a decimal.

b What is the probability that the winning ticket will be greater than 150? Answer as a fraction and a percentage.

c What is the probability that the winning ticket will be an even number?

d What is the probability that the digit '1' will be part of the winning number? PS

Deborah Harmes/Alamy Stock Photo

9 In a bag there are 20 balls, coloured either red, blue or green. 6 of the balls are red.

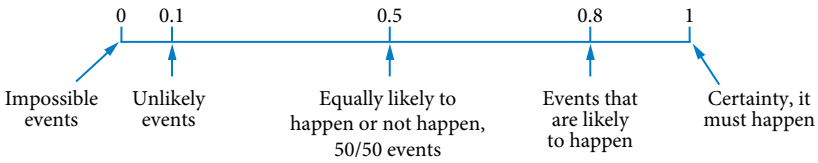
a What is the probability of selecting a red ball at random from the bag?

b The probability of selecting a blue ball from the bag is $\frac{2}{5}$. How many of the balls in the bag are blue?

c What is the probability of selecting a green ball from the bag? PS

Group activity — The probability scale

The value of a probability ranges from 0 to 1. The closer the probability of an event is to 1, the more likely it is to happen. This probability scale shows some probabilities and their meaning.



In your opinion, how likely is each event below? Discuss and decide where on the probability scale the event should be. Compare your group's opinion with other groups.

1 A coin will land with tails facing up.
2 A die will show a 5.
3 It will rain tonight.
4 You will come to school tomorrow.
5 A pink elephant will fly past the window in the next 5 minutes.
6 You will pass your driving test the first time you try.
7 You will roll a total from 2 to 12 on a pair of dice.
8 Prince William will be crowned King of the United Kingdom by the year 2035.
9 Queensland will switch to daylight saving next summer.
10 There will be a road accident in Brisbane today.
11 The price of petrol will fall to 85c/litre.
12 A baby boy will be born in Townsville Hospital this week.
13 Australia's prime minister will change this year.
14 The Australian cricket team will win the next Ashes series.
15 The Brisbane Broncos will win the next NRL competition.

9.2 Using tables to list outcomes

Listing all possible outcomes in a sample space can be difficult when probability problems become more complex. There are several systematic ways to work out the total number of possibilities. For more complicated situations like rolling 2 dice together, drawing a grid or table (array) is a useful method.

EXAMPLE 2 Arrays and sample space

Zoe tosses a coin and a die together.

Determine the probability that she tosses:

a a 5 and a head **b** a 5 or a head.

Steps **Working**

For the coin, there are 2 possible outcomes: heads or tails.

For the die, there are 6 possible outcomes: 1 to 6.

The table shows the sample space for a coin and a die tossed together.

		Die					
		1	2	3	4	5	6
Coin	Head	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	Tail	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

a The table shows that there are 12 possibilities.

Only one of the outcomes (H, 5) shows 5 and a head.

$P(\text{5 and a head}) = \frac{1}{12}$

b Count every outcome in the Head row and 5 column (but don't count H, 5 twice).

$P(\text{5 or a head}) = \frac{7}{12}$

EXERCISE 9.2 Using tables to list outcomes

ANSWERS p. 442

1 EXAMPLE 2 Ziad tosses a coin and an 8-sided die numbered 1 to 8.

a Copy and complete the table to show all possible outcomes.

		Die							
		1	2	3	4	5	6	7	8
Coin	Head								
	Tail								

b What is the probability that Ziad tosses:

i a 7 and a tail? **ii** a 7 or a tail?

iii a number less than 5 and a tail? **iv** a head and an odd number?

c Calculate the probability of each event.

i {head and 6} **ii** {tail and (1 or 2)}

2 Nami rolls a pair of dice and adds the 2 numbers that come up.

a Copy and complete this 2-dice grid to list all possible totals.

		2nd die					
	+	1	2	3	4	5	6
1st die	1			4			
	2						
	3				7		
	4						
	5	6				10	
	6						12

b How many outcomes are in the sample space?

c What is the probability of rolling a total of:

i 5? **ii** 8?

iii 10? **iv** 12?

d Which is more likely: a total of 9 or a total greater than 10?

e How many times more likely is a total of 7 than a total of 4?

3 Felicity uses a pair of unusual dice in a board game she is designing.
She numbers one die 0, 1, 2, 3, 4 and 5, and the other die 1, 1, 3, 3, 4 and 6.
Players roll the dice and add the 2 numbers to determine their score.

a Copy and complete this grid for Felicity's dice.

+	0	1				
1						
1						
3						
3	3	4				
4						
6						

b What is the probability of rolling the following scores on Felicity's dice?

i 5 **ii** 10 **iii** 3 or 4

c What is the most likely score in Felicity's game?

iStock.com/Manfredxy

4 Stefan uses a normal pair of dice in his board game, but his rules require players to *subtract* the smaller number from the larger number.

a Copy the grid and show all the possible scores using Stefan's rules.

–	1	2	3	4	5	6
1						
2		0				
3					2	
4						
5						
6			3			

b What is the probability of scoring a 3?

c What is the probability of scoring a 6?

d What is the most likely score with Stefan's rules?

5 Julianne uses a normal pair of dice in her board game, but her rules require players to use just the *larger* number on the 2 dice as the score.

a Copy and complete the table for Julianne's dice.

	1	2	3	4	5	6
1				4		
2						
3						
4						
5					5	
6	6					

b What is the probability of scoring 3 with Julianne's rules?

c What is the most likely score?

d What score has a probability of $\frac{1}{4}$?

6 Christina made up an interesting special rule for a board game she is designing. Players roll a pair of dice, but they can choose to move:

- the sum of the 2 numbers on the dice, or
- either of the individual numbers showing.

Hayden is playing Christina's game and he needs 6 to win. What is the probability that Hayden can move 6 on his next roll of the dice?

Investigation The great cockroach race

A board game uses a pair of dice and features 4 racing cockroaches. The numbers on the faces of the dice are 0, 1, 2, 3, 4 and 5.

Zero to 5  Zania Studio/ Shutterstock.com								
6 to 11 								
12 to 18 								
19 to 25 								

Rules

- Roll the pair of dice and multiply the 2 resulting numbers together.
- The cockroach that has the product of the 2 numbers moves forward 1 square.
- The first cockroach to the end is the winner.

Do you think this game is fair? Does each cockroach have an equal chance of winning?

Give a reason for your opinion.

If you don't think it's fair, which cockroach has an unfair advantage?

Play the game several times.

Have you changed your opinion?

Explain why one cockroach has an unfair advantage.

Videos
Tables and tree diagrams

Tree diagrams

Worksheets
Matching probabilities

Probability review

Puzzle
Chance puzzle

9.3 Using tree diagrams

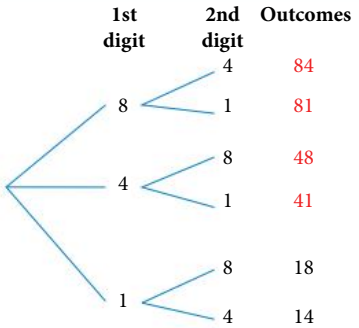
Tree diagrams are another way of listing all possible outcomes systematically. Tree diagrams start from a point on the left side of the page and grow sideways to the right. It's a good idea to leave space above the starting point so that the 'tree' has room to grow!

EXAMPLE 3 Tree diagrams

Caleb has these 3 cards. He chooses 2 cards at random to make a 2-digit number.

a How many different 2-digit numbers can he make?

b Calculate the probability that he makes a number greater than 40.

Steps	Working
Draw a tree diagram that lists all possible 2-digit numbers. For the first digit, Caleb can select 8, 4 or 1. For the second digit, he can choose one of the remaining 2 cards (he can't choose the same card twice).	
a Count the outcomes.	Caleb can make 6 different 2-digit numbers.
b 4 of the numbers listed (in red) are greater than 40.	$P(\text{number} > 40) = \frac{4}{6}$ $= \frac{2}{3}$

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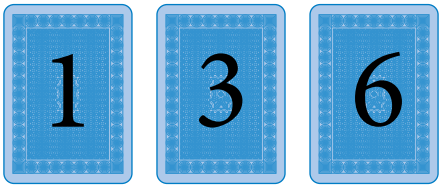
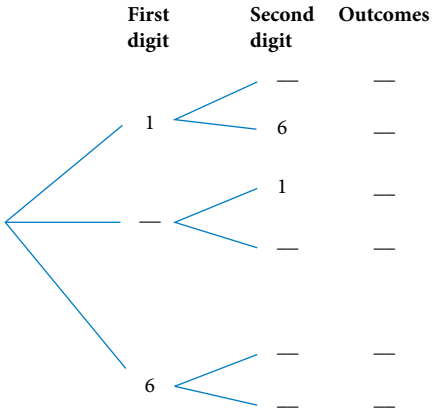
EXERCISE 9.3 Using tree diagrams

ANSWERS p. 442

1 EXAMPLE 3 Britney selects 2 cards at random from these 3 cards to make a 2-digit number.



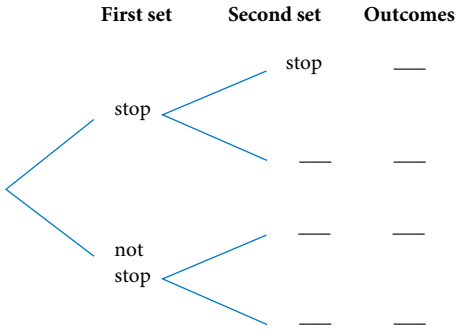
a Copy and complete the tree diagram to show all possible 2-digit numbers.



b What is the probability that Britney will make a number less than 35?

2 Every morning on her way to school, Gordana drives through 2 sets of traffic lights at which she is equally likely to have to stop or not stop.

a Copy and complete this tree diagram to show the possible light combinations.



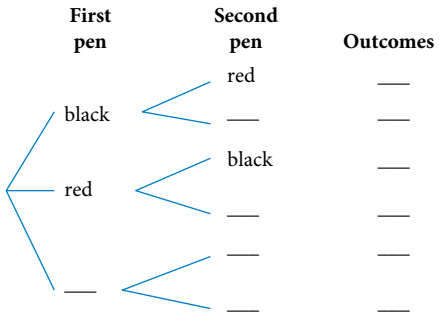
b What is the probability that Gordana will have to stop at 2 lights on her way to school tomorrow?

c Calculate the probability that she will have to stop at at least 1 set of traffic lights tomorrow morning.

3 Claire has a bag that contains 3 pens: 1 red, 1 blue and 1 black.

> When we choose a second item without putting the first item back, we call it 'without replacement'. If we put the first 1 back before we choose the second, it's called 'with replacement'.

a She selects a pen from the bag at random and then another without putting the first one back. Complete the tree diagram to show the possible colour combinations Claire could choose.



b What is the probability Claire chooses a red pen followed by a blue pen?

c What is the probability she chooses a black and a blue pen (in any order)?

d Construct another tree diagram to determine the sample space if Claire *replaces* the first pen before she selects the second.

e When Claire replaces the first pen, what's the probability that she will select the same pen twice?

4 A tennis squad has 3 boys, Mark, Wayne and Peter, and 3 girls, Sonia, Delta and Aniela. The coach chooses a boy and a girl from the squad to represent the school in a mixed doubles competition.

a Construct a tree diagram to determine all the possible mixed doubles pairs.

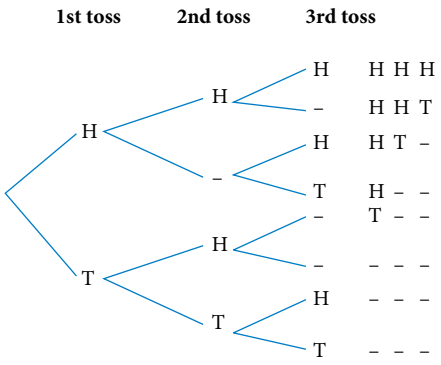
b What is the probability that Wayne and Delta will be selected to represent the school?

5 a Seth selects 2 cards at random from these 4 cards. Construct a tree diagram to list the sample space.

b How many different 2-card pairs can he select?

c What is the probability that Seth chooses 2 cards of the same colour?

6 a Copy and complete the tree diagram to show the possibilities when a coin is tossed 3 times.



b How many possibilities are there when a coin is tossed 3 times?

c Calculate the probabilities of these events when a coin is tossed 3 times.

i 3 heads

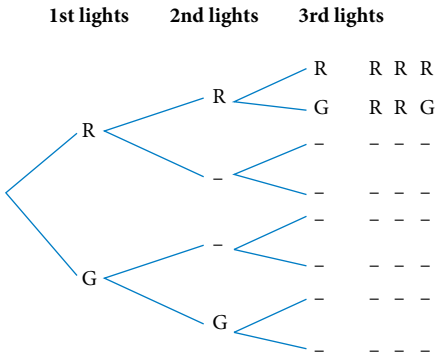
ii 2 heads then a tail

iii 2 heads and a tail in any order

iv At least 1 head

7 Every morning on his way to work, Kane drives through 3 sets of traffic lights which are equally likely to be red or green.

a Copy and complete this tree diagram to show all light combinations.



b On his next trip to work, determine the probability that Kane will get

i 3 red lights

ii 2 red and 1 green light in any order

iii a green light followed by 2 red lights

iv at least 1 green light

8 Cathy and Mark are planning to have 3 children.

a Assuming that the probability of a boy is equal to the probability of a girl, what is the probability that their first child will be a girl?

b Draw a tree diagram to calculate the probability that Cathy and Mark will have:

i 3 sons

ii 2 sons and then a daughter

iii exactly 1 daughter

iv at least 1 daughter

v 3 children of the same sex

9 The referee for a soccer match forgot to bring a coin to toss before the match. The captains agreed to decide who won the toss in this way:

When the referee says "Now!" the captain from each team will hold up any number of fingers and/or thumbs (that is, 1 to 5) on their right hand. If the sum of the fingers and thumbs is even, then Team A wins the toss. If the sum is odd, Team B wins the toss.

Is this a fair way to determine who wins the toss (that is, do both teams have an equal chance of winning)? Explain your answer. PS

Worksheets
Matching probabilities

Probability review

9.4 Comparing relative frequency and theoretical probability

Theoretical probability does not tell you what is definitely going to happen the next time a die is rolled. If the probability of rolling a 5 is $\frac{1}{6}$, it doesn't mean that in the next 60 rolls, a 5 will come up *exactly* 10 times. Theoretical probability tells you what will happen in the long run. Over many rolls, a 5 should come up in approximately $\frac{1}{6}$ of the rolls.

In Chapter 6, *Will it happen?*, you calculated experimental probabilities using relative frequency formula:

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

Now, let's compare theoretical probability with relative frequency.

EXAMPLE 4 Relative frequency and theoretical probability

Keenan rolled an 8-sided die 120 times and he recorded the results in a frequency table.

a **Determine** the relative frequency and theoretical probability of rolling a 6 on this die. Express the values as decimals, correct to 3 decimal places.

b **Comment** on the statement: 'According to the table, the law of averages says that the next roll will probably be a 4 because there haven't been enough 4s rolled so far.'

Number	Frequency
1	17
2	10
3	11
4	9
5	20
6	22
7	13
8	18

Steps	Working
a In the experiment, the number 6 occurred 22 times in 120 rolls.	$\text{relative frequency} = \frac{22}{120}$ $= 0.1833\ldots$ ≈ 0.183
Theoretically, there are 8 possible outcomes and the number 6 is one of them.	$\text{theoretical probability} = \frac{1}{8}$ $= 0.125$
b With each new roll, the probability of each number is the same, so 4 is not more likely.	The statement is incorrect. Each time the die is rolled, there is a $\frac{1}{8}$ chance that a 4 will be rolled.

A common mistake that gamblers make is thinking that if a number hasn't come up much in the past, it's got a higher chance of coming up next. The die doesn't have a memory of the numbers rolled in the past. Each number has the same chance.

EXERCISE 9.4 Comparing relative frequency and theoretical probability

ANSWERS p. 443

1 EXAMPLE 4

a What is the theoretical probability of getting a head when you toss a coin?

b Toss a coin 40 times and record the number of heads you get.

c Use the data you obtained in part **b** to determine the relative frequency of getting a head.

d Calculate the difference between the theoretical probability and the relative frequency.

e Josie has been playing a game that involves tossing a coin. The coin was tossed 10 times and heads came up only 3 times. Josie thinks that there is something wrong with the coin or someone is cheating.

i Download the **Heads and tails** spreadsheet from Nelson MindTap.

ii Run the simulation 40 times and calculate the relative frequency of getting 3 or fewer heads when you toss a coin 10 times.

iii Josie expected to get heads about half of the time. Run the simulation numerous times and concentrate on the percentage of heads. In which group, 10, 50, 100, 200 or 300 tosses, does the percentage of heads change by the biggest amount?

iv Write a sentence to explain how the percentage of heads changes as you increase the number of tosses.

v Are Josie's concerns about the coin, or the people she is playing with, justified? Explain your answer.

Spreadsheet
Heads and tails

2 When you roll a pair of dice, which event do you think is more likely to happen:

- a 1 or a 2 (or both) will show, or
- neither a 1 nor a 2 will show?

a Roll a pair of dice 40 times and record how many times a 1 or a 2 (or both) shows.

b Calculate the relative frequency that a 1 or a 2 (or both) will show.

c Repeat your experiment to check your results.

d This table lists all possible outcomes when a pair of dice is rolled. What is the theoretical probability of rolling 2 numbers that are the same?

e Determine the probability that at least one of the numbers showing will be a 1 or a 2.

f Why is it more likely that a 1 or a 2 will show than neither a 1 nor a 2 will show?

g Write a sentence to compare and contrast the relative frequency and theoretical probability of rolling a 1 or a 2 on a pair of dice.

	Die 2					
Die 1	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Investigation Fact or fallacy?

Scott is attempting to swim from Noosa to Hervey Bay. There are 2 possibilities: either he will swim the distance, or he won't. Therefore, the probability that Scott will swim the distance is $\frac{1}{2}$.

This claim is a fallacy (false statement) because the 2 outcomes, 'swimming the distance' and 'not swimming the distance' are NOT equally likely.

As a group activity, discuss each probability statement and decide whether they are fact (true) or fallacy (false).
Be ready to justify your group's opinion when other groups disagree!

1 I will either die when I'm 99 or I won't die when I'm 99. That's 1 possibility out of 2. The probability that I will die when I'm 99 is $\frac{1}{2}$.

2 The probability of throwing 2 heads on a pair of coins is $\frac{1}{3}$, because there are 3 possible outcomes: 2 heads, 2 tails, or a head and a tail.

3 There are 10 runners in a race.

a The name of each runner is on a separate piece of paper in a hat. The chance of picking the name of the winner out of the hat at random is $\frac{1}{10}$.

b Each runner in the race has a probability of $\frac{1}{10}$ of winning the race.

4 There is a 60% chance of rain on Saturday and a 40% chance of rain on Sunday. There is a 100% chance of rain on the weekend.

5 There is a mixture of red and blue balls in a bag. The probability of selecting a red ball at random from the bag is $\frac{1}{2}$.

6 Tony plays table tennis. He wins 3 out of 5 matches he plays, making the probability that he will win any match $\frac{3}{5}$. He is playing in a 5-match competition, and he has won the first 3 games. It is likely that he will lose the next 2 matches.

7 In Gold Lotto, players select 6 numbers from the numbers 1 to 45. The numbers 3, 11, 15, 16, 25 and 31 are more likely to be the 6 winning numbers than the numbers 1, 2, 3, 4, 5 and 6.

8 Mandy needs an operation to remove a dangerous melanoma from her back. Her doctor said that she has a 98% chance of surviving the surgery. Mandy could die during the surgery.

Worksheets
Probability of $\frac{1}{2}$.

Probability crossword

Keyword activity

PROBABILITY OF $\frac{1}{2}$

ANSWERS p. 443

What does a theoretical probability of $\frac{1}{2}$ really mean?

Group activity

The lines of text in the following paragraph are in the wrong order. Your challenge is to arrange the lines in the correct order. To make the task easier, print a copy of this activity from Nelson MindTap.

row then the next toss of the coin is more likely to be a head. Coins can't
happen. The theoretical probability of getting a head when a coin is tossed
of a head being $\frac{1}{2}$ does mean is that if a coin is tossed thousands of times, about
be a head. Neither does it mean that when a coin is tossed 100 times, it will
be a head 50 times. It also doesn't mean that if 8 tails are recorded in a
is $\frac{1}{2}$. This doesn't mean that every second time a coin is tossed, it will
what is going to happen.
half of the time it will land on heads but on no individual future occasion can you know
Theoretical probability is about the long-term chance that something will
remember what's happened in the past and the chance of getting a head in the
future doesn't change because of several tails in a row. What the probability

Solution to the chapter problem

Problem

Jade thinks that there is something wrong with her pair of dice. When she plays dice games, some numbers seem to come up more often than others on each die.
How can she test whether each die is fair and not 'loaded'?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To test whether the numbers on each die show as frequently as each other.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	On a normal, fair die each number shows approximately $\frac{1}{6}$ of the time. One die at a time, Jade should roll each die a large number of times, for example, 120, recording the number that it shows each time. Each number should show approximately $\frac{1}{6} \times 120 = 20$ times. If one number shows a lot more or a lot less than 20 times, for example, 29 or 8, then Jade should suspect that there is something wrong with the die. In this case, she should check the die again by rolling it another 120 times to see if the large or small amount is repeated.
STAGE 3: CHECK THE SOLUTION	
 CHECK	Jade should repeat the test a few times to make sure the results are consistent.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	Jade should roll each die 120 times and record the number of times each number appears. If each number appears approximately $\frac{1}{6} \times 120 = 20$ times, then the die is fair. If one or more numbers don't appear approximately 20 times, then she should repeat the test. If this keeps happening, then she can conclude that the die isn't fair.

9 Test yourself

Taking chances

ANSWERS p. 443

1 EXERCISE 9.1 Isabella is rolling a normal 6-sided die. List the sample space.

2 EXERCISE 9.1 I have an 8-sided die that has the numbers 0, 1, 2, 3, 4, 5, 6, and 7 on it. When I roll this die, what is the probability I will get:

- **a** a 4?
- **b** a number less than 3?
- **c** an odd number more than 2?
- **d** an 8?

3 EXERCISE 9.1 In a container there are 18 green, 12 orange, 10 yellow, 9 blue, 5 red and 8 brown chocolates. Helen selects a chocolate at random. What is the probability that the chocolate:

- **a** is green?
- **b** is red or blue?
- **c** is not orange or red?

4 EXERCISE 9.2 Dinesh rolls a 4-sided die numbered 1 to 4, and a normal 6-sided die together. He adds the 2 numbers shown on the dice.

- **a** Construct a table for the sample space and determine the total number of possibilities.
- **b** What is the probability of rolling a sum of 5?

5 EXERCISE 9.3 Suppose that the weather forecast for each day of the week is sunny, cloudy or raining, each being equally likely.

- **a** Use a tree diagram to show all 9 possible outcomes for the weather for Saturday and Sunday, the 2 days of the weekend.
- **b** Find the probability that:
  - **i** it rains on both days
  - **ii** the weather is the same on both days
  - **iii** it doesn't rain on the weekend
  - **iv** it is sunny on at least one of the days
  - **v** it is cloudy on one of the days and sunny on the other.

6 EXERCISE 9.3 A coin is tossed and a weekday selected at random.

- **a** Use a tree diagram or array to determine the sample space.
- **b** Calculate the probability of each event:
  - **i** {heads and Monday}
  - **ii** {tails and (Tuesday or Thursday)}
  - **iii** {heads and (not Friday)}

7 EXERCISE 9.3 Billy is investigating 'highs' and 'lows' when he rolls a standard die three times. He records a 'high' (H) if the number rolled is 4, 5 or 6 and a 'low' (L) when he rolls a 1, 2 or 3.

a Construct a tree diagram showing all H and L possibilities.

b List all possible outcomes from your tree diagram.

c Calculate the probability of rolling exactly 2 low (L) numbers.

8 EXERCISE 9.4

a Use the spreadsheet 'Heads and tails' from Nelson MindTap to simulate tossing a coin 25 times. According to this simulation, what is the relative frequency of obtaining a head?

b What is the theoretical probability of obtaining a head when you toss a coin?

c The answers to parts **a** and **b** aren't identical. How can you explain the difference?

9 EXERCISE 9.4 In the Davidson family, the probability that any child will be short-sighted is $\frac{1}{4}$. There are 15 children in the extended Davidson family and 7 of them are short-sighted.

a Using theoretical probability, how many of the 15 children would you expect to be short-sighted?

b How can you explain the difference between the expected frequency and the reality?

c Another baby is due to be born into the Davidson family. The family believes that this baby won't be short-sighted because they already have 7 short-sighted children. Is this thinking correct? Explain your answer.

Spreadsheet
Heads and tails

Spreadsheets
Medical treatment

Breakfast cereals

Practice set 2

ANSWERS p. 444

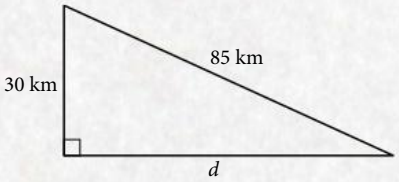
You will need the 'Medical treatment' spreadsheet from Exercise 6.4, p. 164, for Question **1** of Part B.
You will need the 'Breakfast cereals' spreadsheet from the Chapter 6 'Solution to the chapter problem', p. 166, for Question **7** of Part B.

You can download both spreadsheets from Nelson MindTap.

Section A Multiple choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 8.1 Calculate the length of side d, rounded to the nearest whole number.



A 55 km **B** 80 km **C** 90 km **D** 115 km

2 EXERCISE 6.3 Amila rolls a die repeatedly and records the frequency of each number rolled.

Number on the die	1	2	3	4	5	6
Frequency	18	15	15	19	16	13

What is the relative frequency of rolling an even number from this data?

A $\frac{49}{96}$ **B** $\frac{47}{96}$ **C** $\frac{47}{49}$ **D** $\frac{34}{62}$

3 EXERCISE 9.1 What is the probability that a student randomly chosen has a birthday in a month beginning with the letter J?

A $\frac{1}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{6}$ **D** $\frac{1}{12}$

4 EXERCISE 7.1 A tub of margarine weighs 500 g. Which mass below is lighter than the tub of margarine?

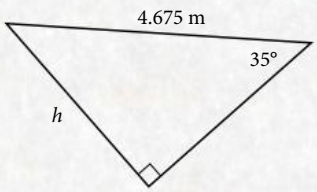
A 60 000 mg **B** 0.8 kg **C** 2.5 kg **D** 0.01 tonnes

5 EXERCISE 8.4 Determine the size of the angle θ, given $\tan\theta = 0.859$.

A 31° **B** 40° **C** 41° **D** 59°

6 EXERCISE 8.7 Calculate the length of the side marked h.

A 2.68 m **B** 3.27 m **C** 3.83 m **D** 4.68 m



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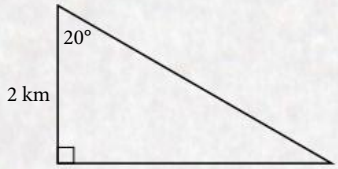
7 EXERCISE 7.3 The dimensions of a refrigerator are 150 cm (height), 60 cm (width) and 40 cm (depth). What is the capacity of the refrigerator in litres?

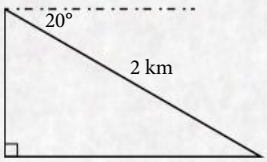
A 250 L **B** 300 L **C** 360 L **D** 430 L

8 EXERCISE 8.5 Minha is flying a glider at a height of 2 km. To land, she descends at an angle of depression of 20° to the ground. Which diagram illustrates this correctly?

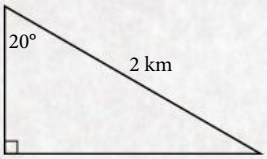
A



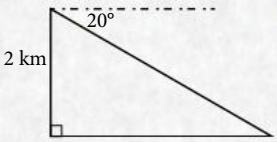
B



C



D



Section B Short answer questions

1 EXERCISE 6.4 For this question, you will need the 'Medical treatment' spreadsheet from Nelson MindTap.

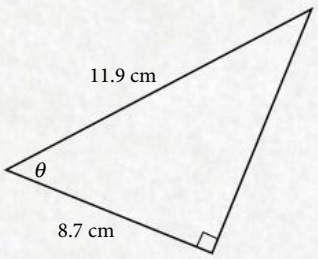
a Select a number at random from 1 to 65 to represent your patient number and write it down.

b Run the spreadsheet 20 times. Record what happens to the patient with your number.

c Did your patient die in any of the simulations?

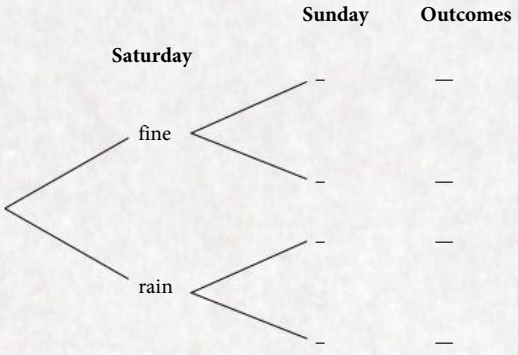
2 EXERCISE 7.2 A can of pineapple contains 14.4 g of sugar, which is 16% of the recommended daily allowance. Calculate the recommended daily allowance of sugar.

3 EXERCISE 8.7 Calculate the size of angle θ, correct to the nearest degree.



4 EXERCISE 9.3 The weather this weekend has an equal chance of being fine or rainy.

a Copy and complete this tree diagram to show the possible outcomes for the weather on Saturday and Sunday.



b What is the probability that it will be fine on both days?

5 EXERCISES 7.4, 7.5 A box of teabags is 17 cm long, 12.5 cm wide and 17.5 cm high.

a Calculate the volume of the box.

b How many boxes of teabags would fit into a crate 1.4 m long, 1.02 m wide and 1 m high?

6 EXERCISE 9.2 Two dice are rolled and the numbers are multiplied together.

a Copy and complete this table to show all possible products.

		1st die					
	×	1	2	3	4	5	6
2nd die	1	1	2	3			
	2	2	4				
	3	3	6				
	4			12			
	5						
	6						

b How many different products are possible?

c Which product is the most likely?

d Which product is the least likely?

e What is the probability of a product of:

i 6? **ii** 20? **iii** at least 20?

7 EXERCISE 6.3 Use the 'Breakfast cereals' spreadsheet from the Chapter 6 problem to answer these questions.

a How many boxes would you expect to have to buy to get all 12 toys?

b Run the simulation 20 times and record the number of boxes Jayden will have to buy to get all 12 toys.

c What was the lowest number of boxes?

d What was the largest number of boxes?

e How did your results compare with what you expected in part **a**?

8 EXERCISE 9.4 Haresh rolled a die 90 times and recorded the results shown below.

Number showing on die	1	2	3	4	5	6
Frequency	12	15	9	16	13	25

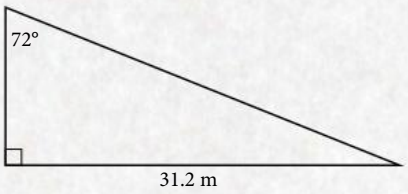
a What is the theoretical probability of rolling a 6? Answer as a decimal, correct to 3 decimal places.

b From the data in this experiment, what is the relative frequency of rolling a 6 with this die? Answer as a decimal, correct to 3 decimal places.

c Explain why Haresh's relative frequencies and theoretical probabilities aren't the same.

d Based on this data, if Haresh rolled the die 200 times, how many times would he expect the number 3 to come up?

9 EXERCISE 8.8 Calculate the length of the hypotenuse of this triangle, correct to one decimal place.



10 EXERCISE 6.3 Anton visited a wharf which tourists depart from to see the Great Barrier Reef. He asked a random selection of 20 people some questions about the Reef. These are his results.

Question	Yes	No
Do you agree that the Great Barrier Reef needs environmental protection?	17	3
Are there too many tourists visiting the Reef?	12	8
Is it OK to destroy parts of the Reef for economic benefits?	4	16

a What is the probability that a person selected at random agrees that there are too many tourists visiting the Reef?

b Calculate, as a decimal, the probability that a person selected at random does not agree with environmental protection for the Reef.

c Suggest a factor that could create bias in Anton's results.

11 EXERCISES 7.3, 7.6 A small water tank has the shape of a cylinder with a diameter of 720 mm and a height of 970 mm.

a Calculate the volume of the water tank in cubic metres, correct to 2 decimal places. Remember to change all measurements to metres first.

b What is the capacity of the water tank, correct to the nearest 10 litres?

12 EXERCISE 8.5 Linda is standing 750 m from the base of a tree. The angle of elevation to the top of the tree is 7°.

a Draw a diagram showing this information.

b Calculate the height of the tree, correct to one decimal place.

Chapter problem

Simone is a journalist with a local newspaper. She is writing an article about the prices of home units in the area. Simone has collected some information on sale prices in recent months:

$995 000	$896 000	$1 015 000	$1 079 000	$870 000	$869 000
$1 010 000	$1 019 000	$889 000	$975 000	$920 000	$1 040 000
$770 000	$959 000	$969 000	$1 425 000	$1 350 000	$1 095 000

What type of statistics should Simone use in her article?

What other information might her readers be interested in knowing?

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CHAPTER 10

GRAPHS, DATA AND LOANS: SUMMARISING AND COMPARING DATA

IT'S BETTER THAN AVERAGE

What will we do in this chapter?

- Calculate measures of central tendency of a set of data: the mean, the median and the mode
- Choose the best measure of central tendency to use in different situations
- Identify the effect of outliers on the mean and the median of a set of data
- Identify quartiles, deciles and percentiles in a set of data
- Describe the spread of a graphed set of data
- Calculate measures of spread of a set of data: the range, the interquartile range and the standard deviation
- Use a spreadsheet to calculate statistical measures
- Investigate misuse of data in the media

How are we ever going to use this?

- To calculate summary statistics to allow comparisons of data
- To understand the different 'averages' and know which is best in different situations
- To determine whether data has been analysed correctly in order to support opinions

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Syllabus coverage

UNIT 4, TOPIC 2: SUMMARISING AND COMPARING DATA

Summarising and interpreting data

- Identify mode from a dataset.
- Calculate measures of central tendency: median and mean ($\bar{x}$) from a dataset of n values
  - mean: $\bar{x} = \frac{\sum x}{n}$ where $\sum x$ = sum of all data values
- Investigate the suitability of measures of central tendency in various real-world contexts [complex].
- Investigate the effect of outliers on the mean and the median [complex].
- Calculate quartiles from a dataset.
- Interpret quartiles, deciles and percentiles from a graph [complex].
- Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers.
- Calculate and interpret statistical measures of spread, including the range, interquartile range and standard deviation [complex].
  - range = highest score – lowest score
  - IQR = $Q_3 - Q_1$
- Investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency and spread [complex].

Videos (2):

10.1 The mode, median and mean
10.3 Interquartile range

Worksheets (5):

10.1 Mean, median, mode 1
• Mean, median, mode 2
10.3 Interquartile range
10.5 Statistical calculations • Statistics review

Puzzle (1):

10.3 Above average code puzzle

Terminology

central tendency	decile	interquartile range	mean
median	mode	outlier	percentile
quartile	range	spread	standard deviation

10.1 What's the average?

Sets of **data** are often too big to understand just by looking at them. We need to analyse them and one way to do this is to find a typical or central value to represent all of the data. There are 3 types of **average** or **measures of central tendency**.

Measures of central tendency

- The **mode** is the most common or frequent data value(s).
- The **median** is the middle data value when the data values are placed in order from smallest to largest.
- The **mean** is calculated by adding all the data values and dividing by the number of values. This is what most people call the 'average'.

Video
The mode, median and mean

Worksheets
Mean, median, mode 1

Mean, median, mode 2

EXAMPLE 1 The mode, median and mean

The following data shows the daily maximum temperature (in °C) for 15 days on the Sunshine Coast in November.

22 25 24 35 22 25 24 20

23 29 33 32 30 25 21

For this set of data, **determine**:

a the mode **b** the median **c** the mean

Steps	Working
a 25 occurs 3 times, which is more than the number of times any other temperature occurs.	The mode is 25°C.
b Put the values in order.	20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 29, 30, 32, 33, 35
There are 15 values, so the middle one will be the 8th value.	20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 29, 30, 32, 33, 35
	The median is 25°C.
c Add all the values and divide by 15.	$\frac{390}{15} = 26$
	The mean is 26°C.

EXAMPLE 2 The mode(s)

Identify the mode for each set of data.

a 3, 3, 4, 5, 5, 6, 7, 9, 10 **b** 1, 2, 3, 5, 7, 10

Sometimes there is more than 1 mode and sometimes there is no mode at all!

Steps	Working
a Both 3 and 5 occur twice.	The modes are 3 and 5.
b All values occur only once.	There is no mode for this data.

EXAMPLE 3 The median

Determine the median for this set of data: 68 81 64 57 95 62

When you have an even number of values in the data, there are 2 middle values. To find the median, you average the 2 middle values.

Steps	Working
Arrange the values in order.	57 62 64 68 81 95
There are 2 middle values.	57 62 (64) (68) 81 95
Find the average of the 2 middle values by adding them and dividing by 2.	$\frac{64+68}{2} = 66$ The median is 66.

EXERCISE 10.1 What's the average?

ANSWERS p. 444

1 EXAMPLE 1 The scores in a class test were:

8 9 7 7 10 5 9 10 9 8 9 6 7

a Identify the mode for this data.

b Determine the median.

c Calculate the mean.

Remember to put the values in order.

2 EXAMPLE 2 What is the mode for each set of data?

a 6 9 2 1 2 9 2

b 67 43 89 45 54 86 45 76 54

c 12 3 6 5 8 2 7 1

d blue, green, yellow, green, blue, red, green, yellow, red, green, red, blue

e heart, spade, spade, diamond, spade, club, heart, heart, diamond, spade

3 EXAMPLE 3 Find the median for each set of data.

a 12 17 14 15 16 16 18

b 36 40 38 37 40 30

c 64.1 100 99.2 71.4 91.5 60.8 72.9

d 74 50 0 27 14 99 51 66

4 Last week a security company recorded the number of minutes it took one of their security guards, Leon, to respond to each alarm call. The results are shown below.

6 12 13 7 26 10 13 9

a How many alarm calls did Leon respond to last week?

b What is the mode of this data?

c What is Leon's median response time?

d Calculate his mean response time.

e Which of the 3 measures of central tendency most accurately describes the typical response time? Justify your answer. C

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5 This table shows the number of people who breached their bail conditions in a city over one year.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
19	17	14	12	13	14	12	14	10	11	14	16

For this set of data, find:

a the median

b the mode

c the mean (correct to one decimal place).

d In which months were the number of people breaching bail conditions the highest? Suggest a reason for this.

6 This is the data for the same offence of breaching bail conditions in a country region over one year.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
7	9	7	6	3	4	5	3	2	1	1	4

For this set of data, find:

a the median **b** the mode **c** the mean (correct to one decimal place)

7 **a** What are the differences in the statistics for questions **5** and **6**? Suggest a reason for these differences. PS

b What are the similarities in the statistics for questions **5** and **6**? Suggest a reason for these similarities.

8 Skye and her dad are having an argument about the number of phone calls Skye makes each night. The data are shown in the below table. PS

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of phone calls	9	3	6	2	13	1	1

a Find the mode, the median and the mean for this data.

b Which of these measures of central tendency could Dad use to demonstrate that Skye makes too many phone calls? Explain your answer.

c Which of these measures could Skye use to show that she doesn't make many calls? Explain your answer.

d Which of the 3 measures gives the best indication of the typical number of calls Skye makes each night? Justify your answer. C

9 Andrea asked 50 students how many text messages they sent within the last 2 hours.

No. of texts (Score)	Frequency	Score × frequency
0	8	$0 \times 8 = 0$
1	4	$1 \times 4 = 4$
2	10	
3	10	
4	15	
5	3	
Total	**50**	

a What is the mode for this data?

b Copy and complete the table.

c Use the formula: $\text{mean} = \dfrac{\text{total of } (\text{score} \times \text{frequency})}{\text{total of frequency}}$ to calculate the mean for this data.

You can also use your calculator or a spreadsheet to calculate the mean from a table.

10 Centrelink officers want to encourage jobseekers to keep applying for jobs. They recorded how many jobs each person applied for before they received their first job interview. PS

Score	Frequency
1	2
2	5
3	10
4	6
5	11
6	13
7	3

a How many people were surveyed?

b What is the mode for this data?

c Copy the table, add a 'score × frequency' column, then complete it.

d Calculate the mean for this data.

e Jon applied for 4 jobs and he hasn't got an interview yet. If you were his Centrelink case manager, what could you say to him to encourage him to keep applying for jobs? Mention the mean and the mode in your answer. C

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Investigation | Statistics in the media 1

We live in a world where news is available from many sources. In this investigation, you're going to evaluate the use of 'average', 'mean' and 'median' across the various news media.

Choose 3 different news sites. You could use the ABC news site, a local newspaper site, News.com, social media sites or any other news sites with which you're familiar with.

On each site:

Part A

- Search for any use of 'average'. Find one use of the word 'average' in a statistical sense in an article. Save it.
- Search for any use of 'mean'. Find one use of the word 'mean' in a statistical sense in an article. Save it.
- Search for any use of 'median'. Find one use of the word 'median' in a statistical sense in an article. Save it.

Part B

- For each article, decide if the mathematical word has been used appropriately.
- Look for further different uses of the word 'average' and compare them to the ones you found in Part A.
- Read a number of articles using the word average. Do they usually refer to the mean or the median?

Part C

- Find any articles that use these terms incorrectly and explain how they have misused these statistical terms.

Make a PowerPoint presentation of your examples and findings and share it with your class.

10.2 What's the outlier?*

An **outlier** is a data value that is very different from the rest of the data. It can be either much bigger than the other data values or much smaller than the other data values. An outlier can have a significant effect on the mean, but it has no impact on the median. In the following exercise, you will see the difference an outlier can make.

EXERCISE 10.2 What's the outlier?*

ANSWERS p. 445

1 This data shows the ages of members of the Binns and Thomson families.

Ages of the Binns family: 19 30 22 3 7 14 18 24 11

Ages of the Thomson family: 19 31 21 3 6 14 19 24 91

a Calculate the mean age of each family. Answer correct to one decimal place.

b What is the main difference between these 2 sets of data?

c What effect does the difference identified in part **b** have on the mean? C

2 Eleven houses have been sold in Keswick Street over the last 2 years. The selling prices are listed below:

$620 000	$625 000	$700 500	$738 000	$625 000	$1 800 000
$598 000	$612 000	$696 500	$720 000	$705 000	

a Find the median sale price for the houses.

b Find the mean sale price.

c Which measure of central tendency best describes the price of the houses in Keswick Street? Justify your answer. C

d Which price is the outlier in this data?

e Calculate the mean and the median of the remaining prices when this outlier is removed.

i Is the mean now closer to the median than previously?

ii Which of the measures was most affected by the outlier? C

3 Mark and Steve's batting scores for 6 innings of cricket are shown below.

Mark: 45 48 53 38 32 40 51

Steve: 23 57 6 125 65 5 37

a Calculate the mean score for each player. Answer correct to one decimal place.

b Which player is better if you use the mean?

c Find the median score for each player.

d Which player is better if you use the median?

e Which player would you rather have in your cricket team? Justify your answer. C

4 12 people work in a small business that sells electronics equipment. Their annual salaries are:

$71 000	$66 000	$64 000	$275 000
$72 000	$76 000	$70 000	$63 000
$67 000	$73 000	$61 000	$890 000

a Calculate the mean salary for this company. Answer to the nearest dollar.

b Find the median salary for this company.

c Which measure (mean or median) is the best reflection of a typical salary for this company? Justify your answer. C

d If you wanted to attract new employees to the company, which would you advertise as the average salary? Why? C

e Which values are outliers in this data?

f Calculate the mean salary of the remaining values if the outliers are removed.

Corepics VOF/Shutterstock.com

5 Darryl is a market gardener. In the first 5 years of his business, his annual profits were $32 000, $67 000, $71 000, $72 000 and $75 000.

a Find the mean and the median for this data.

b Which measure of central tendency gives the most accurate impression of Darryl's usual annual profit? Explain your answer. C

c In what situation might Darryl use the other measure to describe his usual annual profit?

d Which score is the outlier in this data?

e Calculate the mean and the median of the remaining scores if the outlier is removed.

iStock.com/Deimagine

6 An obstetrician specialises in providing care for expectant mothers and delivering their babies. The data below shows the number of caesarean deliveries performed by a sample of 20 Australian obstetricians in a 12-month period: PS

22	38	15	204	3	16	21	13	24	32
19	24	22	31	8	21	14	37	28	21

a What is the value of the outlier in this set of data?

b Some people would accuse a doctor who performed a lot of caesarean sections of doing unnecessary operations. Why might one obstetrician need to perform a lot more caesarean deliveries than all the other doctors?

c Calculate the mean and the median of the 20 values.

d Which measure best describes this data? Justify your answer. C

e Calculate the mean and the median without the outlier.

f What do your answers to parts **c** and **e** show? C

7 A property developer has 40 new apartments for sale. The 20 apartments on the first 5 floors are priced at $1 330 000 each. The 8 apartments on floors 6 and 7 are priced at $1 380 000, and the 8 apartments on floors 8 and 9 are priced at $1 425 000. The 3 apartments on the tenth floor are $1 835 000 each and the penthouse apartment on the top floor is priced at $2.7 million. PS

a Determine the median price of the apartments.

b Calculate the mean price of the apartments.

c When the developer is advertising the apartments for sale, which 'average' would the developer use? Explain your answer. C

d The developer will be speaking to potential investors in his company. What 'average' might he use to make his company look profitable? Explain your answer. C

e Which price(s) is an outlier in this data?

f Calculate the mean after removing the outlier(s).

John Gomez/Shutterstock.com

Chapter problem

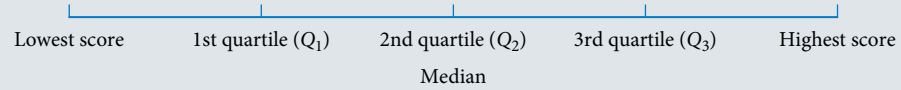
You've covered the skills required solve the chapter problem. Can you solve it now?

10.3 Range and interquartile range*

Sometimes is it useful to see how the data is spread out. The **range** and the **interquartile range** are both **measures of spread**.

Measures of spread

- range = highest data value – lowest data value
- **Quartiles** divide the data into 4 equal groups. 25% of the data is in each quartile.



- interquartile range = upper quartile (Q_3) – lower quartile (Q_1)
- 50% of the data lies within the interquartile range.

Video Interquartile range

Worksheet Interquartile range

Puzzle Above average code puzzle

EXAMPLE 4 Quartiles, range and IQR

The ages of the 23 people at a café are shown here.

33	23	28	36	27	15	32	18	13	13	38	38
27	7	34	27	12	26	33	21	24	39	20	

Determine:

a each quartile

b the range

c the interquartile range

Steps	Working
a Place the ages in ascending order and find the middle data value – it is the median but also it is the 2nd quartile. The median splits the data values into 2 halves. Find the middle of each half of the data values. These are the 1st and 3rd quartiles.	7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26, 27, 27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39 $Q_2 = 27$ 7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26 27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39 $Q_1 = 18$, $Q_3 = 33$ The quartiles are 18, 27 and 33.
b range = highest data value – lowest data value	range = 39 – 7 = 32
c interquartile range = upper quartile (Q_3) – lower quartile (Q_1)	interquartile range = 33 – 18 = 15

EXERCISE 10.3 Range and interquartile range* ANSWERS p. 445

Keep your answers to this exercise because you will need them again in Chapter 12, *Comparing data*.

1 EXAMPLE 4 The number of fish sold each day at lunchtime in a fish and chip shop during the month of August is shown below.

17 27 28 18 18 17 19 19 25 27 17 19 20 19 21 26
28 18 19 20 17 19 23 24 20 18 17 20 19 27 28

For this data, find:

a the range **b** each of the quartiles **c** the interquartile range

2 The following data shows the daily maximum temperatures (in °C) for 15 days in Cairns in July.

32 30 31 32 31 30 31 31 31 31 29 25 28 27 29

For this data, find:

a the range **b** each quartile **c** the interquartile range

3 The heights of 25 Year 11 students in centimetres were

151 167 181 172 179 155 159 162 169 174 178 180 158
166 171 168 157 160 175 172 150 169 163 170 176

Find:

a the range **b** each quartile **c** the interquartile range

If there is an even number of data values, then the quartile is the average of the 2 middle data values.

4 A Year 12 class was surveyed to find the number of hours each student spent on homework each week. These are the results:

4 5 7 8 3 6 9 9 8 4 7 5 3 3 4 9 3 4 7 8

Find

a the range **b** each quartile **c** the interquartile range

5 The following data is a record of the number of thefts from retail stores in 2 regions. PS

Inner City: 13, 13, 13, 12, 16, 25, 23, 20, 33, 25, 27, 25,
55, 20, 27, 33, 28, 26, 38, 24, 33, 55, 42, 48

Coastal: 62, 60, 52, 62, 52, 63, 60, 65, 74, 61, 36, 66
36, 69, 70, 47, 39, 64, 69, 55, 40, 60, 58, 52

a For each set of data, find

i the range **ii** each quartile **iii** the interquartile range

b Comment on the differences between these 2 sets of data.

Investigation | Deciles and percentiles

Parts of words give us hints about what they mean. Words like 'quad' and 'quart' have something to do with 4.
A quadrilateral is a shape with 4 sides, a quad bike has 4 wheels and when a mother has 4 babies at the same time, they are called quadruplets. A quarter is one of 4 equal parts, and when we divide an ordered group into 4 equal subgroups, we are making **quartile** groups.

iStock.com/Skynesher

Words that include 'deci' are about 10. In the decimal number system, there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. When numbers are sorted into large, ordered groups of 10 equal subgroups, they are called **decile** groups.

Words that have 'cent' in them are about 100, just as there are 100 cents in a dollar. Percent (%) is a value 'out of 100' and a century is 100 years or 100 runs in cricket. When numbers are sorted into large groups in order to make 100 equal subgroups, they are called **percentile** groups.

Now, imagine that all the students in your school were lined up on the school oval from the shortest to the tallest. Find out how many students there are in your school and then work through the following questions.

1 If the students were divided into 100 approximately equal groups, how many would be in each group?

2 Each of the 100 groups contains 1 percent of the whole school.
- The height of the tallest person in the shortest group is called the 1st percentile.
- The height of the tallest person in the second group is called the 2nd percentile.
- The height of the tallest person in the tallest group is called the 100th percentile.

Which group do you think you would be in?

3 If the students were divided into 10 approximately equal groups, how many would be in each group?

4 Each of the 10 groups is 10% of the whole group.
- The height of the tallest person in the shortest group is called the 1st decile.
- The height of the tallest person in the second group is called the 2nd decile.
- The height of the tallest person in the tallest group is called the 10th decile.

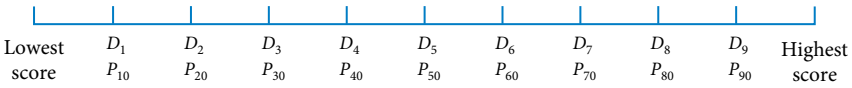
a Which group do you think you are in?
b In which groups would you expect most of the Year 8 students to fall?
c In which groups would you expect most Year 12 students to fall?
d How many percentile groups are in each decile group?
e How many percentile groups are in one quartile group?
f Why is the 10th percentile the same as the 1st decile?
g What percentile is the same as the 9th decile?

10.4 Deciles and percentiles*

As you discovered in the Investigation, another way of sorting large sets of data into groups is to divide it into decile or percentile groups. The data must first be arranged in ascending order. We will look at some simple examples.

Deciles: values that divide the data into 10 equal groups

Percentiles: values that divide the data into 100 equal groups



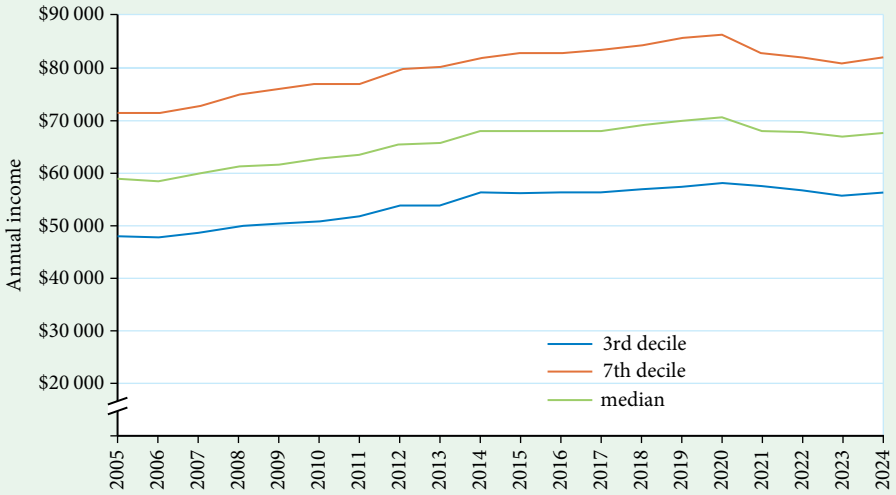
Deciles

While quartiles (Q_1, Q_2 and Q_3) separate data into quarters, **deciles** (D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8 and D_9) separate data into tenths. 'Deci-' means one-tenth.

- D_1 cuts off the lowest 10% of values.
- D_4 cuts off the lowest 40% of values.
- D_9 cuts off the lowest 90% of values (or the top 10% of values).

EXAMPLE 5 Reading decile graphs

This graph shows the changes in annual income over 20 years using the median and the 3rd and 7th deciles.



a **Identify** which decile is the median.

b In 2012, what income was on the 7th decile?

c Grant's annual income in 2018 was in the top 30% of the population. **Determine** his income.

d Between which 2 values were the middle 40% of incomes in 2005?

Steps	Working
a The median is the middle. Half of 10 is 5.	The median is the 5th decile.
b Read from the graph to the top line for 2012.	From the graph, approximately $80 000.
c Top 30% means above 70% or the 7th decile. Read from the graph to the top line for 2018.	From the graph, Grant's income was approximately $85 000.
d The middle 40% is between the 3rd and 7th deciles.	In 2005, • the 3rd decile was approximately $48 000 • the 7th decile was approximately $72 000 So, the middle 40% of incomes were between $48 000 and $72 000.

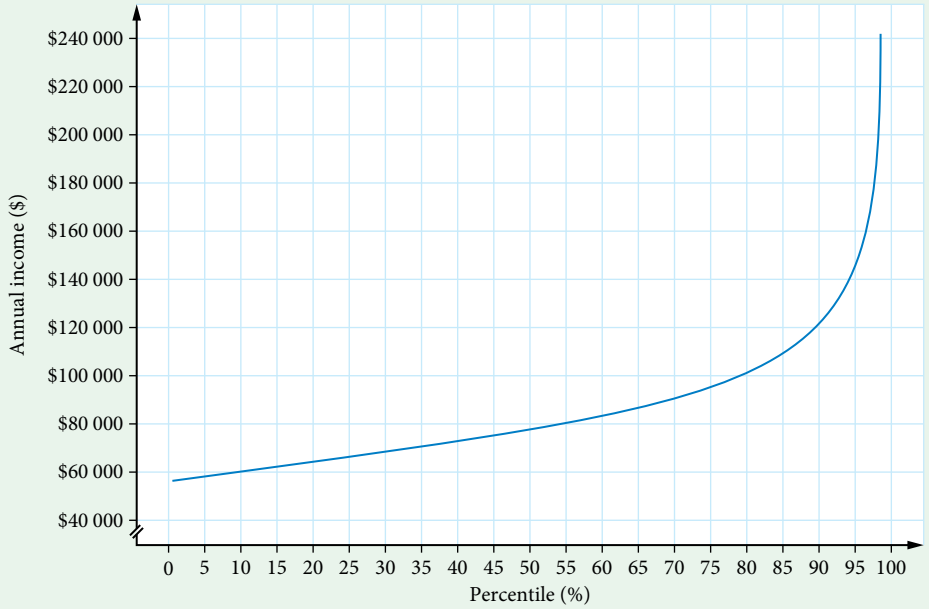
Percentiles

Percentiles (P_1, P_2, P_3, …, P_{99}) separate data into hundredths. 'Centi-' means one-hundredth.

- P_{24} cuts off the lowest 24% of values.
- P_{60} cuts off the lowest 60% of values.
- P_{87} cuts off the lowest 87% of values (or the top 13% of values).

EXAMPLE 6 Reading percentile graphs

This graph shows the annual income percentiles for a large population.



a **Identify** what income is at the 20th percentile.

b Georgina earns $80 000 per year. What percentile is this?

c **Calculate** the approximate difference in income between the 80th and 90th percentiles?

d The graph only goes up to the 99th percentile. **Explain** a possible reason for this.

Steps	Working
a Find the 20th percentile on the horizontal axis and read off the value on the vertical axis.	The income is about $62 000.
b Find $80 000 on the vertical axis and read off the value on the horizontal axis.	It is the 55th percentile.
c Find the incomes for the 80th and 90th percentiles. Difference means subtract.	80th percentile: $101 000 90th percentile: $121 000 difference: $121 000 − $101 000 = $20 000 The difference in income is approximately $20 000.
d The 100th percentile would have very high incomes.	Including very high incomes makes it difficult, or nearly impossible, to have an appropriate scale on the vertical axis.

EXERCISE 10.4 Deciles and percentiles*

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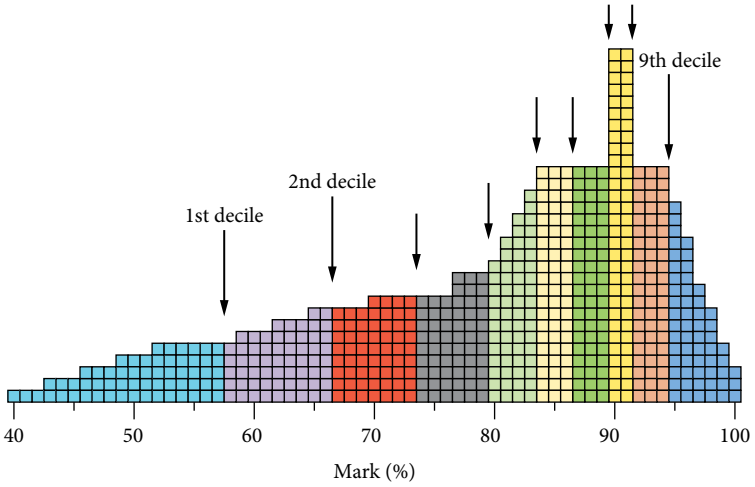
1 EXAMPLE 5 Use the graph in Example 5 to answer these questions.

- **a** In 2019, what income was at the 3rd decile?
- **b** In 2010, what income was at the 5th decile?
- **c** Indira has an annual income of $80 000. In what year is this at the 7th decile?
- **d** Between which 2 values were the middle 40% of incomes in 2024?
- **e** Harry earns $70 000 per year. In which 2 years did she drop out of the top 50% of the population for income?

2 EXAMPLE 6 Use the graph in Example 6 to answer these questions.

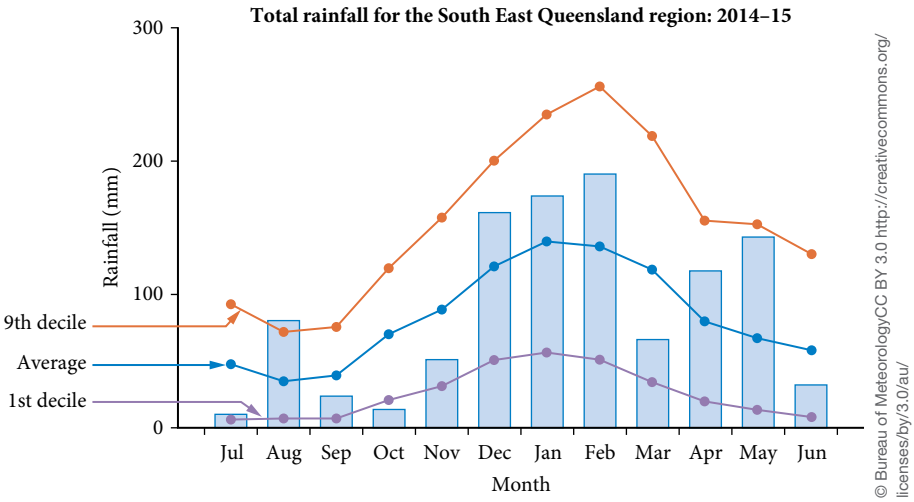
- **a** What income is at the 40th percentile?
- **b** Sue earns $80 000. What percentile is this?
- **c** What is the approximate difference in income between the 30th and 50th percentiles?
- **d** What is another name for the 50th percentile?
- **e** Approximately what percentage of the population earns more than $100 000?

3 This graph shows the results of a group of students on an exam out of 100.
The results have been divided into 10 groups.



- **a** What percentage of students scored 80 or more?
- **b** Find the value that separates the bottom 70% of students from the top 30%.
- **c** Krystal's score was on the second decile. What did she score in the exam?
- **d** Caitlyn scored 75. Between which 2 deciles did she score?
- **e** Do you think this exam was easy or difficult? Give reasons for your answer. C PS

4 This column graph shows the rainfall in South East Queensland for the 2014–2015 financial year, as compared to the line graphs showing long-term deciles and averages. PS



a For how many months of the year was the rainfall below average for the month?

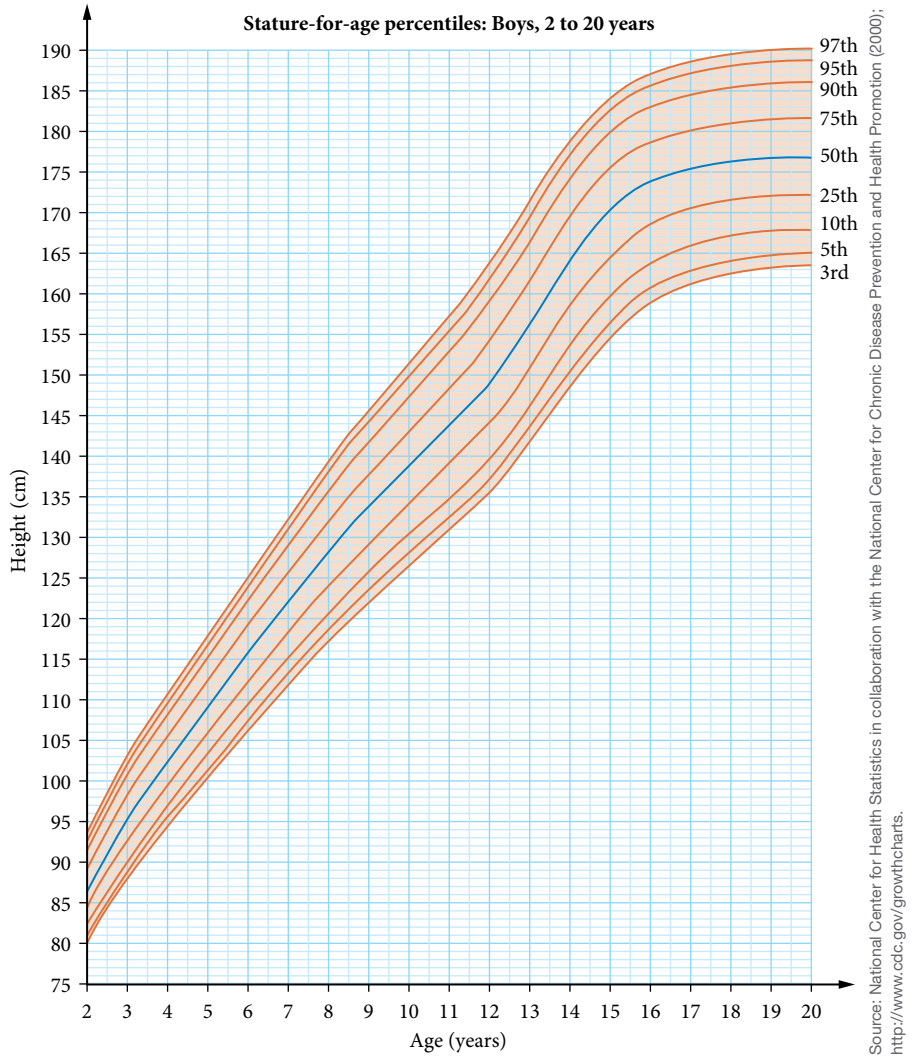
b State the month in which the rainfall was below the 1st decile.

c State the month in which the rainfall was above the 9th decile.

d How often was the rainfall between the long-term average and the 9th decile?

e Was the 2014–2015 financial year a good or poor year for rain? Give reasons for your answer. C PS

5 This percentiles chart shows the range of heights for boys aged 2 to 20 years. PS



a Adam is aged 9 and 129 cm tall. What percentage of boys his age are shorter than him?

b Justin is 11 years old and 155 cm tall. What percentage of boys his age are shorter than him?

c A boy's height usually follows the same percentile graph as he grows up. How tall should Justin be when he turns 18?

d Liong is 103 cm tall, which is at the 1st decile for boys his age. How old is Liong?

e Asam is 16 and his height is at the 3rd quartile.

i What is Asam's height now?

ii Predict what Asam's height will be when he turns 20 years old.

Worksheets
Statistical calculations

Statistics review

10.5 Standard deviation*

The **standard deviation** is another measure of spread, like the range and the interquartile range. It describes how far each value is from the mean. The bigger the standard deviation, the more spread out the values are.

The symbol for standard deviation is 'σ', which is the Greek letter 'sigma'. The formula for standard deviation is quite complicated, so you don't need to use it. Instead, you can use your calculator's statistics mode to calculate it.

EXAMPLE 7 Calculating mean and standard deviation 1

The net weekly wages of 8 casual workers are:

$730 $490 $600 $440 $490 $370 $700 $580

Use your calculator to find the mean and standard deviation for this data.

Steps	Working	
Follow the instructions for the statistics mode (SD or STAT) of your calculator:	Casio scientific	Sharp scientific
1 Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
2 Clear the statistical memory.	SHIFT 1 Edit, Del-A	2nd F DEL
3 Enter data.	SHIFT 1 Data to get table 730 = 490 =, etc. to enter in column AC to leave table	730 M+ 490 M+, etc.
4 Calculate the mean. ($\bar{x} = 550$) The symbol for mean is $\bar{x}$.	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
5 Calculate the standard deviation. ($\sigma = 117.260 \ldots$)	SHIFT 1 Var $x\sigma_n$ =	RCL σx
6 Return to normal (COMP) mode.	MODE COMP mean $\bar{x} = \$550$ standard deviation $\sigma \approx \$117.26$	MODE 0

EXAMPLE 8 Calculating mean and standard deviation 2

Twenty echidnas from Booderee National Park were tagged and returned to their habitat. Rangers later captured several samples of 10 echidnas and recorded the number tagged in each sample.

Echidnas tagged per sample	Frequency
0	8
1	11
2	5
3	4
4	2
5	1

Sean Barlow/Shutterstock.com

Calculate, correct to 2 decimal places:

a the mean number of tagged echidnas per sample

b the standard deviation of tagged echidnas.

Steps	Working	
For data presented in a frequency table, follow the instructions for your calculator as shown:	**Casio scientific**	**Sharp scientific**
1 Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
2 Clear the statistical memory.	SHIFT 1 Edit, Del-A	2nd F DEL
3 Enter data.	SHIFT 1 Data to get table 0 = 1 =, etc. to enter in x column 8 = 11 =, etc. to enter in FREQ column AC to leave table	0 2nd F STO 8 M+ 1 2nd F STO 11 M+ etc.
4 Calculate the mean ($\bar{x} = 1.4838$ ...)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
5 Calculate the standard deviation ($\sigma = 1.340$ ...)	SHIFT 1 Var $x\sigma_n$ =	RCL σx
	a mean $\bar{x} \approx 1.48$ **b** standard deviation $\sigma_n \approx 1.34$	

EXAMPLE 9 Comparing 2 data sets

Indoor cricket selectors are trying to choose between 2 pairs of players (Sanjeev/Angus vs Christian/Tyler) for the state team. Their scores (in runs) are:

Sanjeev/Angus	34	30	36	35	29	34
Christian/Tyler	41	26	37	35	25	34

Determine which pair is the more consistent.

Steps	Working
Use your calculator to find the standard deviation for each pair's scores.	Sanjeev/Angus: $\sigma \approx 2.58$ Christian/Tyler: $\sigma \approx 5.74$
The more consistent pair is the one with the smaller standard deviation.	Sanjeev/Angus is the more consistent pair of cricketers.

EXERCISE 10.5 Standard deviation*

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1 EXAMPLE 7 Find the mean and standard deviation of each set of data. Express your answers correct to 2 decimal places.

a 20, 24, 17, 21, 19, 26, 21, 23, 16

b 60, 51, 82, 65, 27, 38, 32, 64, 18, 20, 74

c 14, 13, 17, 14, 13, 16, 16, 17, 16, 12, 12

d 67, 58, 89, 72, 34, 45, 39, 71, 25, 27, 81

2 Martin is looking for a job in the construction industry. He collected information on the wages of apprentices at a large construction company. The following amounts are the weekly wages of 10 apprentices.

$542 $884 $774 $801 $758 $728 $601 $586 $675 $627

a Find the mean wage.

b Find the standard deviation of this data. Answer correct to 2 decimal place.

c Give a reason for the large standard deviation of this data.

3 Jana is looking for a unit to rent in Highgate Hill. She used a website to find the following data about the weekly rent for a number of units in Highgate Hill.

$400 $450 $600 $530 $529 $430 $430 $390

$550 $450 $350 $420 $540 $495 $480

a Find the mean rental price.

b Find the standard deviation for this data. Answer correct to 2 decimal places.

c Why can places in the same area have different weekly rents?

d If Jana can afford to pay $420 per week, how difficult will it be for her to find a place to rent?

4 EXAMPLE 8 Students' marks in a quiz out of 10 are shown in this frequency table.

Mark	Frequency
5	4
6	3
7	8
8	4
Total	**19**

Find, correct to one decimal place

a the mean **b** the standard deviation.

5 Andrea surveyed 50 students on how many text messages they sent within the last 2 hours.

Number of texts	Frequency
0	8
1	4
2	10
3	10
4	15
5	3
Total	**50**

For this data, find correct to 2 decimal places

a the mean **b** the standard deviation.

6 Centrelink officers recorded how many jobs each person applied for before they received their first job interview.

Number of jobs applied for	Frequency
1	2
2	5
3	10
4	6
5	11
6	13
7	3

For this data, find correct to 2 decimal places

a the mean

b the standard deviation.

7 EXAMPLE 9 Kate is looking for her ideal place to live. She likes a consistent, warm climate. She is considering 2 places: Port Paradise and Palm Tree Cove. This table shows the mean monthly temperatures (in °C) of each location.

	J	F	M	A	M	J	J	A	S	O	N	D
Port Paradise	30	28	26	25	24	24	24	24	26	27	28	30
Palm Tree Cove	32	34	28	26	25	22	20	21	25	29	31	32

a Calculate, correct to 2 decimal places, the standard deviations of the temperatures of each place.

b Use your answers to part **a** to advise Kate on which place would suit her better.

8 Brad, Aryn and Kim are keen computer gamers. Each week, they play 12 competition games against players of equal ability. The table shows the number of games each of them won during the last 6 weeks. PS

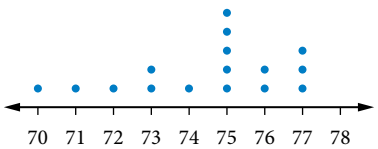
Brad	8	9	7	10	5	11
Aryn	2	4	6	3	7	5
Kim	10	5	8	3	9	4

a Calculate the mean number of games won out of 12 and the standard deviation of the weekly results for each player. Answer correct to one decimal place.

b Which player is the most consistent? Explain your answer.

c You have to select one player to compete at the state contest. Whom would you choose and why?

9 Paul's golf scores for 16 rounds of golf are shown on this dot plot. PS



a From smallest to largest, write out a list of Paul's scores for the 16 rounds of golf.

b Calculate the mean and the standard deviation for this data. Give your answers to 2 decimal places.

c Is Paul a consistent player? Explain your answer.

10 This stem-and-leaf plot shows Cherie's pulse rate for every 2 minutes while she was exercising on the cross-country machine at the gym. PS

Stem	Leaf
12	7 9
13	0 3 5
14	3 3 3 6
15	4 5 7

a Would you expect a large or a small standard deviation for this data? Explain your answer.

b Calculate the standard deviation for this data. Give your answer to 2 decimal places.

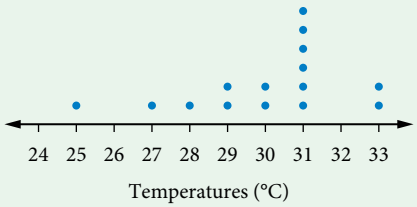
c Why do we NOT want a consistent pulse rate in this situation?

10.6 Describing the spread of data

Data shown on a **dot plot**, **stem plot** or histogram allows us to see the spread of the data. Data can be spread out or **clustered** around one place, or there might be gaps or outliers.

EXAMPLE 10 Spread on a dot plot

This dot plot shows the daily maximum temperatures (in °C) in Cairns over 15 days.
Comment on the spread of the data.



Steps	Working
Look for clusters.	These values are clustered around 31°C.
Look for gaps.	There are gaps at the bottom end of the data and at the top end.
The temperatures range from 25 to 33.	These temperatures are quite spread out.
Write a description.	The temperatures are quite spread out, but they are clustered around 31°C. There are gaps towards the bottom and top ends of the data.

EXERCISE 10.6 Describing the spread of data

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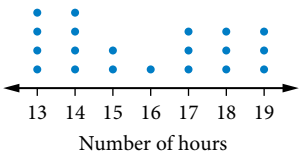
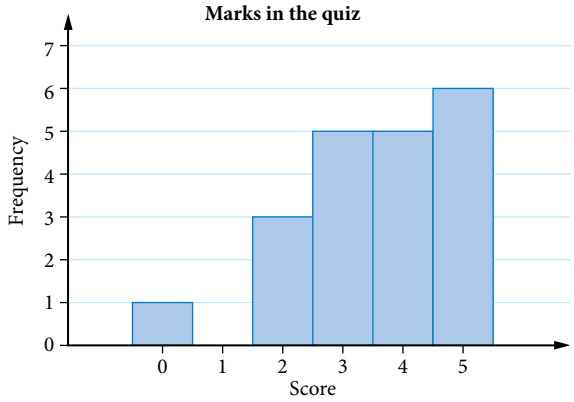
1 EXAMPLE 10 Ahmed surveyed his class on the number of hours each student spent on the internet each week. Comment on the spread of the values shown in this dot plot.



2 This stem plot shows the number of burglaries per month in Emu Springs over the last few years. Describe the spread of values shown in this data.

Stem	Leaf
3	7 7 8 8
4	1 3 4 4 5 5 6 6 8 9 9
5	1 1 1 2 2 3 3 4 4 5 5 5 8
6	0 2 3 6 6 7
7	0
8	
9	0

3 Tanika gave her class a short maths quiz of 5 questions. This frequency histogram shows the results.

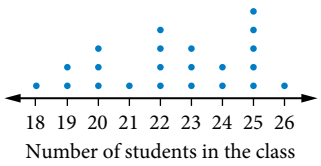


Comment on the spread of the values shown in this histogram.

4 Copy and complete the descriptions of the following sets of data using words from this list.

clustered	gaps	less dense
more dense	spread out	tightly packed

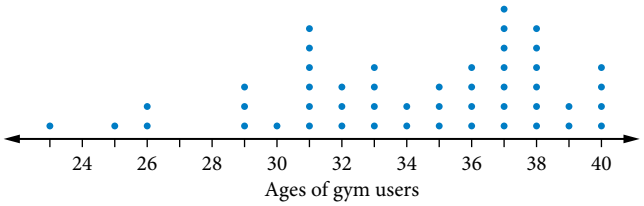
a This dot plot shows the number of students in each class at St Jude's Primary School.



Class sizes are _____ between 22 and 25.

The class sizes are _____ from 18 to 26.

b The ages of people exercising at a gym one weekend are shown in the dot plot below.



These ages show _____ between 26 and 29. They are ____ _____ between 36 and 38 but ____ _____ between 23 and 26. The values are _____ over a wide range.

c This stem plot shows the heights of students in centimetres in a fitness class.

These heights are _____ _____ in the 150s and 160s. They are _____ ___ from 137 to 177.

Stem	Leaf
13	7
14	0 1 3 5
15	3 4 5 7 7 7 8 9
16	2 2 3 6 6 8
17	0 1 5 5 7

d This stem plot shows the number of people who visited the information desk at Nelson Mall daily over a 3-week period.

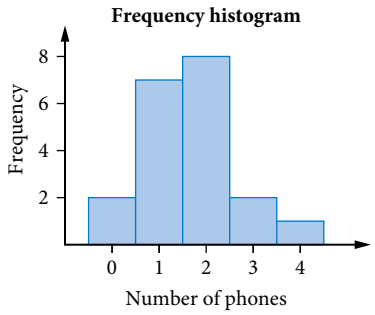
Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

These are evenly _____ ___ across the 80s, 90s, 100s and 110s. The data does not show any _____.

Kevinsuzhou/Dreamstime LLC

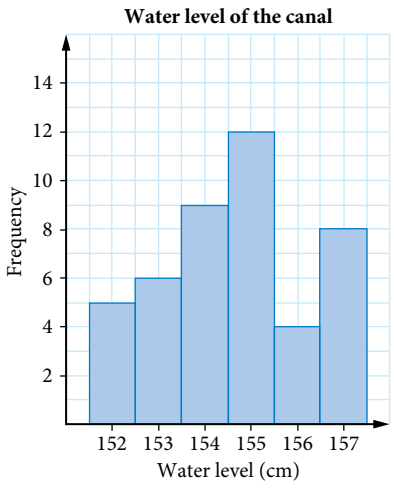
e This frequency histogram shows how many phones were owned in a sample of households.

The values are _____ at 1 and 2. The responses are not _____ ___ but _____ _____.



f This histogram shows the water level of a canal over 44 days.

The water level is _____ _____ from 152 to 157. There are no ____.



9780170484664

Investigation — Statistics in the media 2

In this Investigation, you're going to evaluate the use of the measures of spread across the various news media.

Use the articles you found in the investigation earlier in this chapter. If you didn't do the earlier investigation, choose 3 different news sites. You could use the ABC news site, a local newspaper site, News.com.au, social media sites or any other news sites with which you're familiar.

In the articles you found earlier, or on your chosen news sites:

1 Search for any use of the various words you have used for the spread of data in Exercises 10.3, 10.5 and 10.6 and record how they have been used.

2 Have you been able to find these words?

3 For each article, decide if the mathematical word has been used appropriately and explain how it has been used.

4 For any article that uses these terms incorrectly, explain how they have misused these statistical terms.

Make a PowerPoint presentation of your examples and findings and share it with your whole class.

Investigation — Tourism claims

The above advertisement appeared in a Victorian newspaper.

Work in a group to complete the following task.

1 Use the long-term climate data below to assess the claims in the article.

2 Construct a similar advertisement for a skiing trip, including data that shows the claims made in the advertisement may be misleading.

3 Prepare a 5-minute oral presentation for the class.

Maximum average daily June temperatures	Victoria	Hedon's Island
Minimum	0	18
First quartile	6	26
Median	9	30
Third quartile	12	32
Maximum	14	35

Average cloudy days in June	Victoria	Hedon's Island
Minimum	5	0
First quartile	10	1
Median	15	3
Third quartile	18	4
Maximum	24	6

Investigation | **Comparing the size of maths textbooks with English novels**

1 Ask your teacher to bring in a copy of every maths textbook they have and your English teacher to lend you a copy of every novel used in senior English classes.
2 Record the number of pages in each book for each of the 2 groups.
3 Calculate the mean and interquartile range for the number of pages in the maths textbooks.
4 Calculate the mean and interquartile range for the number of pages in the novels.
5 'Maths textbooks have more pages than English novels, but there is a greater variation in the number of pages in English novels.' Do you agree with this statement? Does your data support this? Justify your answer.

10.7 Statistics on a spreadsheet

We can calculate the mean, mode, median and other statistics using the statistical functions on a spreadsheet.

EXERCISE 10.7 Statistics on a spreadsheet

ANSWERS p. 446

1 a Enter into a spreadsheet the following data about the daily maximum temperatures in Alice Springs in one week.

	A	B	C	D
1	**Day**	**Temperature (°C)**		
2	Sunday	29		Mean
3	Monday	31		Mode
4	Tuesday	30		Median
5	Wednesday	33		Standard deviation
6	Thursday	29		Maximum
7	Friday	28		Minimum
8	Saturday	35		Range

b Copy each formula into the given cells.

> Sometimes if you type the first couple of letters, the spreadsheet will suggest the correct word.

Cell E2: =average(B2:B8)
Cell E3: =mode(B2:B8)
Cell E4: =median(B2:B8)
Cell E5: =stdev.p(B2:B8)
Cell E6: =max(B2:B8)
Cell E7: =min(B2:B8)
Cell E8: =E6–E7

c Save your results.

2 a Repeat question **1** for data for the town or city where you live. Go to the **Bureau of Meteorology** website www.bom.gov.au to find the data. PS

b Find data for 1 month instead of for 1 week. Repeat Question **1** for this new data. You will need to adjust the formulas you enter for the statistics.

3 a Enter into a spreadsheet the following data showing the monthly number of motor vehicle thefts in a capital city over 2 years.

	A	B	C	D	E
1	**Motor vehicle theft in a capital city**				
2					
3	Jan 2023	20		Jan 2024	15
4	Feb 2023	19		Feb 2024	19
5	Mar 2023	26		Mar 2024	17
6	Apr 2023	17		Apr 2024	21
7	May 2023	27		May 2024	20
8	Jun 2023	13		Jun 2024	18
9	Jul 2023	17		Jul 2024	13
10	Aug 2023	17		Aug 2024	13
11	Sep 2023	20		Sep 2024	14
12	Oct 2023	18		Oct 2024	14
13	Nov 2023	12		Nov 2024	18
14	Dec 2023	24		Dec 2024	20
15					
16	Mean			Mean	
17	Median			Median	
18	Mode			Mode	
19	Standard deviation			Standard deviation	

b Copy each formula into the given cells.

The data for 2024 has many modes, but the spreadsheet only lists one of them.

Cell B16: =average(B3:B14)

Cell B17: =median(B3:B14)

Cell B18: =mode(B3:B14)

Cell B19: =stdev.p(B3:B14)

Cell E16: =average(E3:E14)

Cell E17: =median(E3:E14)

Cell E18: =mode(E3:E14)

Cell E19: =stdev.p(E3:E14)

c Save your results.

d Comment on the differences between the 2 years.

4 Repeat Question **3** for Brisbane. You will need to use the internet to find the data. Try looking up 'crime statistics' and 'motor vehicle theft'.

5 a Create a new spreadsheet similar to that in Question **1** for the following data on the number of burgers sold between 11 am and 2 pm at Hungry Macs for one week. PS

Day of week	Burgers sold
Monday	60
Tuesday	71
Wednesday	63
Thursday	69
Friday	78
Saturday	90
Sunday	86

b Find the mean, the median and the standard deviation for this data and save your spreadsheet.

c Calculate the same statistics for the *weekdays* only.

d Describe the differences between the data for the whole week and the weekday-only data. Why do you think this is so?

6 This data shows the heights of 30 students' navels above the ground in centimetres. PS

107	101	100	81	97	98	68	96	104	93
90	94	95	86	92	81	85	98	88	90
99	81	100	81	90	104	100	93	98	113

a Create a spreadsheet similar to that in Question **1** for this data.

b Find the mean, the median, the mode and the standard deviation for this data and save your spreadsheet.

c Delete the outlier of 68 cm. Calculate the same statistics for the remaining data and save this new spreadsheet.

d Describe the differences between the 2 sets of data. Why do you think this is so?

Investigation Changing data

It is best to use a spreadsheet to complete this investigation.

1 For the set of data shown in the frequency table, find:

a the mean, correct to one decimal place

b the median

c the mode

d the standard deviation, correct to one decimal place.

Score	Frequency
5	11
6	13
7	10
8	8
9	3

2 For each challenge below, change the frequencies in the table without changing the total frequency of 45.

a Increase the mean without changing the mode.

b Decrease the mean without changing the mode.

c Make the median bigger by 1.

d Make the mean equal to 8 in two different ways.

e Make the mode 7 without changing the mean.

f Make the standard deviation as small as possible.

g Make the standard deviation as big as possible.

h Make the mean 7 and the standard deviation as close to 1 as possible.

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Keyword activity

CHAPTER SUMMARY

ANSWERS p. 446

average	interquartile range	median	mode
number	odd	order	outlier
percentiles	quartiles	range	spread
standard deviation	ten		

Copy and complete each sentence using a word from the list. When you have completed this, you will have a summary of the chapter.

1 The _____ is the most frequent data value.
2 Another word for the mean is the _____.
3 The mean is the sum of the data values divided by the _____ of values.
4 The _____ is the middle data value when they are arranged in order.
5 There's only one middle value when there is an _____ number of data values.
6 To find the median, you must first put the data values in _____.
7 A data value that is much different to the other values is called an _____.
8 _____ divide the data into 4 equal parts.
9 Deciles divide the data into _____ equal parts.
10 When a large amount of data is divided into 100 equal parts, they are called _____.
11 The standard deviation is a measure of _____.
12 The difference between the highest data value and the lowest data value is called the _____.
13 The difference between the upper quartile and the lower quartile is called the _____ _____.
14 A measure of spread that describes how far all data values are from the mean is called the _____ _____.

Solution to the chapter problem

Problem

Simone is a journalist with a local newspaper. She is writing an article about the prices of home units in the area. Simone has collected some information on sale prices in recent months:

$995 000	$896 000	$1 015 000	$1 079 000	$870 000	$869 000
$1 010 000	$1 019 000	$889 000	$975 000	$920 000	$1 040 000
$770 000	$959 000	$969 000	$1 425 000	$1 350 000	$1 095 000

What type of statistics should Simone use in her article?

What other information might her readers be interested in knowing?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

- To work out the statistics for the data – this includes the mean, the median and the mode.
- We can suggest other information that might be of interest to Simone's readers.

STAGE 2: SOLVE THE PROBLEM

SOLVE

People reading the article will be interested in knowing the lowest price ($770 000) and the highest price ($1 425 000) of home units sold recently in the area.

To calculate the average or typical price:

Mode: There is no mode as all the prices are different. Even if there were 2 prices the same, the mode is unlikely to reflect a typical price.

Mean: total of prices = $18 145 000

number of sales = 18

mean = $18 145 000 ÷ 18

≈ $1 008 056

Median: Put data values in order

$770 000	$869 000	$870 000	$889 000	$896 000	$920 000
$959 000	$969 000	$975 000	$995 000	$1 010 000	$1 015 000
$1 019 000	$1 040 000	$1 079 000	$1 095 000	$1 350 000	$1 425 000

$$\text{median} = \frac{975\,000 + 995\,000}{2}$$

$$= \$985\,000$$

In this case, the median is the best measure of central tendency as the mean has been increased by the outliers $1 350 000 and $1 425 000.

Simone's readers may also want to know how prices have changed over time.

She could include the change in prices over the last few years as a percentage.

She could also include a line graph showing changes in the median price over time.

STAGE 3: CHECK THE SOLUTION

CHECK

We have answered all parts of the problem.

STAGE 4: PRESENT THE SOLUTION

PRESENT

Simone should write an article giving the highest and lowest prices and the median price. She could illustrate it with a line graph showing the changes in the median price over recent times.

10 Test yourself

It's better than average

ANSWERS p. 446

1 EXERCISE 10.1 For each set of data, find

i the mode **ii** the median **iii** the mean

a Computer frauds per year: 808, 1126, 1003, 913, 300

b Student incomes ($): 32, 29, 41, 34, 29, 40, 40, 37, 39, 40, 33

c Temperatures at Cloncurry (°C): 25, 24, 23, 20, 16, 12, 11, 12, 17, 20, 23, 25

d Monthly rainfall at Tewantin (mm): 69, 95, 129, 143, 159, 151, 129, 158, 99, 65

2 EXERCISE 10.1 The following data gives the room occupancy rates for motels and hotels in Queensland per quarter (3 months) in recent times. These rates are given as percentages.

57 61 58 60 56 64 58 69 61 68

62 56 65 59 66 59 70 62 73

a Find the median for this data.

b What is the mode for this data?

c Calculate the mean for this data. Answer correct to one decimal place.

d Which of the 3 measures most accurately reflects the data? Justify your answer.

3 EXERCISE 10.2 For the computer fraud data in Question **1a** above:

a What is the outlier for this data?

b Calculate the mean of the data without the outlier.

c What effect does the outlier have on the mean calculated in Question **1**?

4 EXERCISE 10.2 For the rainfall data in Question **1d** above:

a What are the outliers for this data?

b Find the mean and the median for the data without the outliers included.

c Compare the mean with all data values included to the mean without the outliers included. What effect do the outliers have on the mean?

d Compare the median with all data values included to the median without the outliers included. What effect do the outliers have on the median?

5 EXERCISE 10.3 For each set of data in Question **1**, find

i the range

ii the lower and upper quartile

iii the interquartile range

6 EXERCISE 10.3 Using your answers to Question **5**:

a for which dataset is the range the better measure of spread?

b for which dataset is the interquartile range the better measure of spread?

7 EXERCISE 10.4 Refer to the graph in Exercise 10.4, Question **3** on page 274.

a What percentage of students scored more than 91?

b What is the 4th decile?

c Joanna scored 93%. Between which 2 deciles did she score?

8 EXERCISE 10.4 Refer to the graph in Exercise 10.4, Question **4** on page 275.

a For how many months of the year was the rainfall above average for the month?

b State the month(s) where the rainfall was between the 1st decile and the long-term average.

c What is the difference between the 1st and 9th deciles in February?

9 EXERCISE 10.5 Use your calculator to find, correct to 2 decimal places, the standard deviation of the temperatures at Cloncurry from Question **1c**.

10 EXERCISE 10.5 Carmelina recorded the number of drinks bought by people visiting her shop for one hour. The results are shown in the table.

Number of drinks	Frequency
0	7
1	5
2	13
3	4
4	1
5	2
6	1
Total	**33**

Find, correct to 2 decimal places

a the mean

b the standard deviation.

11 EXERCISE 10.5 Kobe is looking to move to the country and set up a market garden. He needs regular, consistent rainfall for his new business to be successful. He is considering 2 places. This table shows the mean monthly rainfall (in mm) of each place.

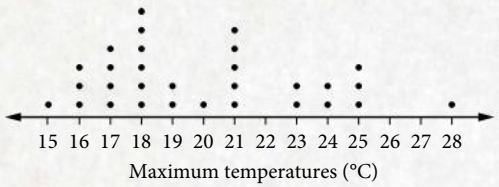
	J	F	M	A	M	J	J	A	S	O	N	D
Dalby	76	77	59	20	37	32	23	24	30	58	71	94
Clermont	117	114	75	38	34	34	25	19	20	35	57	92

a Calculate, correct to 2 decimal places, the mean and standard deviation of the rainfall in each place.

b Which place would you recommend to Kobe and why?

Dorothy Chiron/Shutterstock.com

12 EXERCISE 10.6 Describe the spread of data in this dot plot.



Chapter problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year. Melanie also invested $25 000 for 3 years, earning simple interest at 4.2% per year.

Whose investment will earn more interest?

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CHAPTER 11

GRAPHS, DATA AND LOANS: LOANS AND COMPOUND INTEREST

INVESTING MONEY

What will we do in this chapter?

- Calculate and compare simple and compound interest
- Use technology to calculate the future value of a compound interest investment
- Use technology to compare the progress of loans and investments, and the effects of changing interest rates and compounding periods

How are we ever going to use this?

- Determine the chance that an investment is safe
- Choose a safe investment that will give us a reasonable return
- Make calculations to check that we have received the correct investment returns
- Avoid falling for financial scams

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Syllabus coverage

UNIT 4: TOPIC 3: LOANS AND COMPOUND INTEREST

Compound interest

- Understand the concept of simple interest where I is simple interest, P is principal, i is interest rate per period and n is total number of periods to find unknown values using the formulas:
  - $I = Pin$
  - $P = \frac{I}{in}$
  - $i = \frac{I}{Pn}$
  - $n = \frac{I}{Pi}$
- Understand the concept of compound interest as a recurrence relation.
- Use an online calculator to determine the future value of a compound interest loan or investment.
- Calculate the future value of a compound interest loan or investment with annual periods using the formula $A = P(1 + i)^n$ where A is future value, P is principal, i is interest rate per annum and n is total number of years.
- Calculate the total interest paid or earned for compound interest loan or investment.
- Use a spreadsheet to determine the future value of a compound interest loan or investment and the total interest paid or earned [complex].
- Compare, numerically and graphically, the growth of simple interest and compound interest loans and investments [complex].
- Investigate the effect of the principal, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].

Terminology

annual	compound interest	future value	interest
interest rate	investment	loan	per annum (p.a.)
period	principal	recurrence relation	simple interest

11.1 Simple interest

When we invest money with **simple interest** or **flat rate interest**, the amount of **interest** earned is the same every year. The interest is a percentage of the **principal**, the original amount invested.

Simple interest formula

$$I = Pin$$

where I = the interest

P = principal (what you invest or borrow)

i = **interest rate** per time **period**, as a decimal

n = number of time periods.

We can also rearrange the simple interest formula with the principal, interest rate or number of time periods as the subject.

$$P = \frac{I}{in} \qquad i = \frac{I}{Pn} \qquad n = \frac{I}{Pi}$$

Video Simple interest

Worksheets Simple interest 1

What's the interest?

Simple interest riddle

Applications of simple interest

Puzzle Interesting puzzle

EXAMPLE 1 Simple interest

Chloe invested $6000 at 3.25% p.a. simple interest for 4 years.

a How much interest will she earn?

b How much will be in her account at the end of 4 years?

p.a. = **per annum** = per year

Steps	Working
a In the simple interest formula $I = Pin$, $P = 6000$, $i = 3.25\% = 0.0325$ and $n = 4$.	$I = 6000 \times 0.0325 \times 4$
	$= 780$
Write the answer.	Chloe will earn $780 in simple interest.
b Add the interest to the principal.	account total = $6000 + $780
	= $6780
Write the answer.	Chloe will have $6780 in her account.

EXAMPLE 2 Using other forms of the simple interest formula

Angie has $7000 to invest.

a At 5% p.a. simple interest, calculate the time Angie will need to invest the money to earn $1000 interest.

b What interest rate will she need to earn $900 interest in 2 years?

c Angie only wants to earn $600 interest at 4.9% p.a. interest in 2 years. How much does she need to invest?

Steps	Working
a $i = 0.05$, $I = \$1000$, $P = \$7000$	$n = \frac{I}{Pi}$
The interest rate is p.a., so the time will be in years.	$n = \frac{1000}{7000 \times 0.05}$
To calculate n, use the formula $n = \frac{I}{Pi}$.	$n = 2.857$ years or approximately 2 years 10 months.
	Angie needs to invest for 2 years 10 months.

b $P = \$7000$, $n = 2$, $I = \$900$ The time is years, so the interest rate will be p.a. To calculate i, use $i = \frac{I}{Pn}$.	$i = \frac{I}{Pn}$ $i = \frac{900}{7000 \times 2}$ $= 0.0643$ The interest rate needs to be 6.43% p.a.
c $I = \$600$, $i = 0.049$, $n = 2$ To calculate P use $P = \frac{I}{in}$.	$P = \frac{I}{in}$ $P = \frac{600}{0.049 \times 2}$ $= \$6122.45$ Angie needs to invest \$6122.45.

Sometimes money is not invested or borrowed for a whole number of years. We need to make sure the interest rate and the time period match.

EXAMPLE 3 Time in months, interest rate p.a.

Rosie invested \$9500 at 4.2% p.a. for 30 months. How much interest did she earn?

Steps	Working
The interest rate is per year, but the time is in months. We must change 30 months to years by dividing by 12.	time $= \frac{30}{12}$ years
$P = \$9500$, $i = 0.042$, $n = \frac{30}{12}$	$I = \$9500 \times 0.042 \times \frac{30}{12}$ $= \$997.50$
Write the answer.	Rosie earned \$997.50 in interest.

Converting months to years

- When the interest rate is p.a., the time must be in years.
- Divide the number of months by 12 to change the time into years.

EXAMPLE 4 Monthly interest rate

What percentage interest rate per month is equivalent to 5.52% p.a.?

Steps	Working
Divide an annual interest rate by 12 to change it to a monthly interest rate.	$5.52\% \div 12 = 0.46\%$
Write the answer.	5.52% p.a. is equivalent to 0.46% per month.

EXERCISE 11.1 Simple interest

ANSWERS p. 447

1 EXAMPLE 1 Use the simple interest formula $I = Pin$ to calculate the simple interest on each principal.

a \$900 at 3.2% p.a. for 4 years

b \$1560 at 3.9% p.a. for 4 years

c \$4500 at 5.5% p.a. for $3\frac{1}{2}$ years

d \$2750 at 5.1% p.a. for $2\frac{1}{2}$ years

2 EXAMPLE 2

a Use the formula $n = \frac{I}{Pi}$ to calculate the number of years Billy will need to invest \$10 000 to make \$4000 in simple interest at 6.5% p.a.

b What interest rate will Sammy need to earn \$500 in simple interest when she invests \$3900 for 2 years? Use the formula $i = \frac{I}{Pn}$.

c Use $P = \frac{I}{in}$ to calculate the principal required to earn \$875 in simple interest at 6.1% p.a. in 3 years.

3 EXAMPLE 3 What is the simple interest on each investment?

a \$840 at 3.5% p.a. for 18 months

b \$12 800 at 6.05% p.a. for 3 months

c \$2960 at $5\frac{1}{2}$% p.a. for 4 months

d \$880 at $6\frac{1}{2}$% p.a. for 1 month

4 EXAMPLE 4 What percentage interest per month is equivalent to 4.56% p.a.?

5 NMQ Credit Union offers investors 3.12% p.a. simple interest. Using 4 decimal places when required, express this rate of interest as a:

a monthly rate

b weekly rate

c 6-monthly rate

d daily rate

e fortnightly rate

f quarterly rate (3-monthly rate).

Divide by 52 for the weekly rate, divide by 2 for the 6-monthly rate and divide by 4 for the quarterly rate.

6 Nina borrowed \$3200 from a finance company for 2 years at 17% p.a. interest to buy some furniture for her home.

a How much interest did she have to pay?

b How much, including interest, did Nina have to repay the finance company?

7 Grandma is setting up a trust fund to provide for her granddaughter's education. She anticipates that the fund will earn 6.4% p.a. simple interest, and she wants the fund to generate \$10 000 in interest each year. How much will Grandma need to invest to meet her objective?

8 Deanna is saving the deposit for a unit. She has \$75 000 invested at 5.9% p.a. simple interest. How long will it take for the amount in the account to grow to \$100 000?
[Hint: How long will it take to earn \$25 000 interest?]

9 Tom is retired and uses his interest to help him pay his expenses. He needs \$35 000 p.a. to supplement his pension, and he has \$500 000 to invest. What is the minimum simple interest rate he needs?

10 Simone owed \$865 on her credit card. The credit card company charged her one month's interest at 22% p.a.

a How much interest was she charged?

b Calculate the total amount she had to repay the credit card company.

11 Eddie borrowed $55 000 from a finance company to set up an online business. He borrowed the money for 6 months and was charged 17.25% p.a. interest. How much did he have to repay the finance company, including interest? PS

12 Kelly made the mistake of signing a contract to buy a new house before she sold her old house. When it was time to pay for the new property, Kelly had to borrow $960 000 in bridging finance at 1.44% monthly.

a What daily interest rate is equivalent to 1.44% per month? Assume a month has 30 days.

b Kelly borrowed the money for 45 days. Calculate the amount of interest she had to pay.

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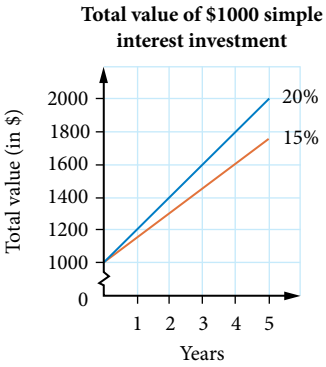
13 Last January, Jude received a text message from Darren, someone she knew briefly years ago. He told her that he was working for an investment company and knew of an investment opportunity that was paying 15% p.a. interest, paid every 6 months. Jude decided to invest $20 000 for 6 months in this business. PS

a How much interest will Jude's investment earn at 15% p.a. for one month?

b How much, including interest, should Jude receive when she withdraws her investment after 6 months?

c In July, Jude couldn't contact Darren. He had disappeared with her money. Why should Jude have been suspicious of the **investment opportunity**?

14 This graph shows the total value of a $1000 simple interest investment at 15% p.a. and 20% p.a. for 5 years.



a How much is the investment worth after 4 years at 15% p.a.?

b How much simple interest is earned in 3 years at 20% p.a.?

c How much more interest is earned in 5 years at 20% p.a. than at 15% p.a.?

d Both graphs start at 1000 on the vertical axis. What is the meaning of the 1000?

e How is the interest rate related to the steepness of the graph?

f Copy the graph and show the position of investments earning 5% p.a. and 10% p.a.

Investigation | Is my investment safe?

In this investigation, you will visit the **MoneySmart** website moneysmart.gov.au to find out about checking whether an investment is safe.

What you have to do:

1 Go to 'Scams and online safety', then the 'Investor alert list'.
2 Search the investor alert list for 'Thomas Moore Global'. Should you consider investing with this company? Explain.
3 In which country is Thomas Moore Global based?
4 List 4 different companies, each in a different country, with which you shouldn't invest.
5 Randomly choose 8 letters from the alphabet, for example, B, F, G, K, N, P, Q and Z.
6 Categorise each company that starts with the 8 letters according to the company's location, using this frequency table.

Geographical region	Tally	Frequency
Africa		
Asia		
Australia		
Europe		
Middle East, including United Arab Emirates		
South America		
United States		
Total		

7 What conclusion can you draw about the locations of these companies?
8 Read the section on the website about why overseas scammers target Australians, then write a paragraph describing your findings in this investigation. Include a sector graph to display your data visually.

Caimacanul/Shutterstock.com

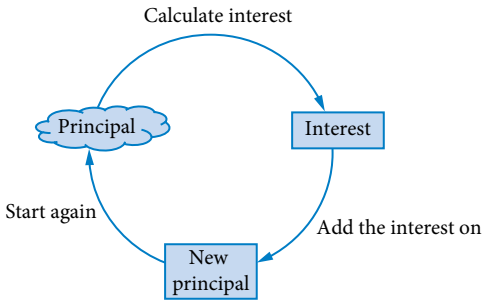
Weblink
MoneySmart

11.2 Compound interest

There's a well-known claim often quoted by financial investors:

'Compound interest is the eighth wonder of the world.
Those who understand it, earn it. Those who don't, pay it.'

With **compound interest**, you receive 'interest on your interest'. After interest is added to the investment, the next time interest is calculated, it will be based on this larger amount. The amount of interest grows or 'is **compounded**' as the size of the investment grows.



How does compound interest work?

Imagine that Sarina invested \$2000 at 5% p.a. interest compounded annually for 3 years. This is how her investment changed during the 3 years. Remember 5% is 0.05.

	Interest	Balance
End of the 1st year	$I = \$2000 \times 0.05 \times 1$ $= \$100$	$\$2000 + \$100 = \$2100$
End of the 2nd year	$I = \$2100 \times 0.05 \times 1$ $= \$105$	$\$2100 + \$105 = \$2205$
End of the 3rd year	$I = \$2205 \times 0.05 \times 1$ $= \$110.25$	$\$2205 + \$110.25 = \$2315.25$

Sarina invested \$2000 and at the end of 3 years the account contained \$2315.25. The increase is the interest. Sarina earned \$315.25 interest.

EXAMPLE 5 Compound interest as a recurrence relationship

Marco invested \$3000 at 5% p.a. interest compounded annually.

a Calculate the future value of Marco's investment at the end of 3 years.

b Calculate the total interest he earned.

Steps

a We need to calculate the interest for the first year and add it to the principal before we calculate the following year's interest.

Working

$I = Pin$, where $i = 5\% = 0.05$, $n = 1$ and P changes each year, but its initial value is \$3000.

	Interest	Balance
End of the 1st year	$I = \$3000 \times 0.05 \times 1$ $= \$150$	$\$3000 + \$150$ $= \$3150$
End of the 2nd year	$I = \$3150 \times 0.05 \times 1$ $= \$157.50$	$\$3150 + \$157.50$ $= \$3307.50$
End of the 3rd year	$I = \$3307.50 \times 0.05 \times 1$ $= \$165.38$	$\$3307.50 + \$165.38$ $= \$3472.88$

The future value of Marco's investment is \$3472.88.

Alternatively, calculate the value of 5% interest added to the principal by multiplying by 1.05; for example, $\$3000 \times 1.05 = \$3150$, giving you the answers in the Balance column straight away.

b total interest = final balance − original principal	total interest = \$3472.88 − \$3000 = \$472.88
OR Add the 3 interest amounts in the middle column of the table above.	total interest = \$150 + \$157.50 + \$165.38 = \$472.88
Write the answer.	The total interest earned over the 3 years is \$472.88.

A shortcut to calculating 5% interest and adding it to the principal is multiplying the principal by 1.05.

The value of Sarina's investment at the end of the 1st year, $A_1 = 1.05 \times 2000$.

The value of Sarina's investment at the end of the 2nd year, $A_2 = 1.05 \times A_1$

The value of Sarina's investment at the end of the 3rd year, $A_3 = 1.05 \times A_2$

Looking at the pattern, we can see that the value of Sarina's investment at the end of 16 years would be $A_{16} = 1.05 \times A_{15}$. This is called a recurrence formula, which is a mathematical summary of this process.

Compound interest recurrence formula

$$A_{n+1} = rA_n$$

In this formula, $r = (1 + \text{rate of interest}) = (1 + i)$ and A_0 is the principal or initial investment.

EXAMPLE 6 Compound interest recurrence formula

Use the compound interest recurrence formula to **calculate** the interest Alex will earn when she invests \$6000 at 4.5% p.a. annually compounding for 3 years.

Steps	Working
In the formula $A_{n+1} = rA_n$:	$A_1 = 1.045 \times \$6000$
A_0 = the original investment	$= \$6270$
$= \$6000$	$A_2 = 1.045 \times A_1$
r = 1 + the rate of interest, i	$= 1.045 \times \$6270$
$= 1 + 0.045$	$= \$6552.15$
$= 1.045$	$A_3 = 1.045 \times A_2$
	$= 1.045 \times \$6552.15$
	$= \$6847.00$
interest = final balance − principal	total interest = \$6847 − \$6000
Or, in this example, interest = $A_3 - A_0$.	= \$847
Write the answer.	The total interest earned over the 3 years is \$847.

i is the interest rate per time period as a decimal.

EXERCISE 11.2 Compound interest

ANSWERS p. 447

1 EXAMPLE 5 Yumi invested $8000 at 6% p.a. interest compounded annually for 3 years. Copy and complete this table to calculate the future value of her investment at the end of 3 years.

	Interest	Balance
End of the 1st year	$I = Pin$ $= \$8000 \times 0.06 \times 1$ $= \$\_\_\_\_\_$	$\$8000 + \$\_\_\_\_ = \$\_\_\_\_\_$
End of the 2nd year	$I = Pin$ $= \$\_\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_\_$	$\$\_\_\_\_ + \$\_\_\_\_ = \$\_\_\_\_\_$
End of the 3rd year	$I = Pin$ $= \$\_\_\_\_ \times 0.\_\_\_\_ \times 1$ $= \$\_\_\_\_\_$	$\$\_\_\_\_ + \$\_\_\_\_ = \$\_\_\_\_\_$

2 Jayden invested $12 000 for 3 years at 4% p.a. compounding yearly.

a How much interest did he earn in the first year?

b How much was in his account at the end of the first year?

c How much was in his account at the end of the second year?

d Calculate the future value of his investment at the end of 3 years.

e How much interest will Jayden earn during his 3-year investment?

3 Nhi invested $4000 at 3.2% p.a. for 2 years compounding annually.

a Calculate the future value of her investment at the end of the 2 years.

b How much interest will she earn during the 2 years?

c How much less interest would Nhi earn had it been simple interest rather than compound interest?

4 Lara has saved $12 000 from her weekend job that she plans to spend on an end of year holiday. She is going to invest the money for 3 months. Lara's investment is going to pay 0.7% per month, interest compounded monthly. PS

a Copy and complete the table.

	Interest	Balance
End of the 1st month	$I = Pin$ $= \$12\,000 \times 0.007 \times 1$ $= \$\_\_\_\_\_$	$\$12\,000 + \$\_\_\_\_\_ = \$\_\_\_\_$
End of the 2nd month	$I = Pin$ $= \$\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_$
End of the 3rd month	$I = Pin$ $= \$\_\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_$

b How much interest will Lara make during the 3-month investment?

5 EXAMPLE 6 Yasmin invested some money. This recurrence calculation shows the progress of her investment.

$A_1 = 1.07 \times \$600$ $A_2 = 1.07 \times \$642$ $A_3 = 1.07 \times \$686.94$

$= \$642$ $= \$686.94$ $= \$735.03$

a What was the value of Yasmin's initial investment?

b What was the value of her investment at the end of 3 years?

c How much interest did Yasmin earn for the 3-year investment?

d What annual rate of compounding interest did Yasmin's investment earn?

6 Jayden invested \$12 000 for 3 years at 4% p.a. compounding yearly.

a Explain how you know that $A_0 = \$12\,000$ and $r = 1.04$, in the compound interest recurrence formula $A_{n+1} = rA_n$

b Calculate the value of A_1.

c How much interest did he earn in the first year?

d Calculate the value of A_2 and A_3.

e What is the value of his investment at the end of 3 years?

f How much interest will Jayden earn during his 3-year investment?

7 Nhi invested \$4000 at 3.2% p.a. for 2 years compounding annually. Use the compound interest relationship $A_{n+1} = rA_n$ to answer the following questions.

a What is the value of A_0 and r?

b Calculate the value of A_1 and A_2.

c How much interest will Nhi earn during the 2 years?

d How much less interest would Nhi earn had it been simple interest rather than compound interest?

8 Zac has saved \$14 000 from his after-school job that he plans to spend on a car. He is going to invest the money for 3 months until he has his P-plates. Zac's investment is going to pay 0.7% per month, interest compounded monthly. PS

Russell Blake/Alamy Stock Photo

a In the formula $A_{n+1} = rA_n$, determine the value of A_0 and r.

b What are the values of A_1, A_2 and A_3?

c How much interest will Zac make during the 3-month investment?

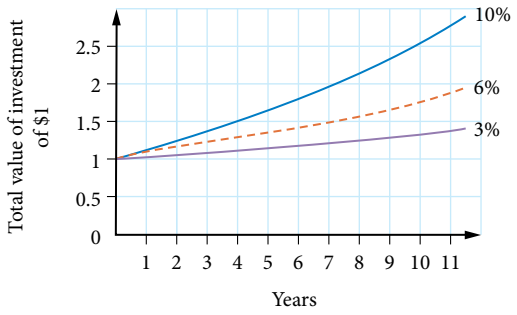
9 Last night Suresh received a phone call from an investment advisor he doesn't know telling him about an investment opportunity that is virtually risk-free. The offer is likely to be fully subscribed quickly, so he has to agree straight away. The minimum investment is \$10 000 and the interest rate is 15% p.a. compounding annually for 3 years.

a Using the recurrence formula $A_{n+1} = rA_n$, determine the values of r, A_0, A_1, A_2 and A_3.

b Calculate the amount of interest the advisor claims \$10 000 will receive in the next 3 years.

c What signs show that this opportunity is most probably a scam?

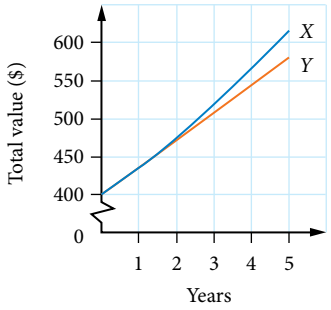
10 This graph shows the future value of a $1 compound investment at 3% p.a., 6% p.a. and 10% p.a.



a Describe how the shape of a compound interest graph is different to the shape of a simple interest graph.

b Approximately how long does it take for a $1 investment to double its value at 10% p.a. compound interest?

11 Nick and Adam invested the same amount of money at 9% p.a., but Nick's investment was 9% p.a. compounded annually while Adam's was 9% p.a. simple. The graphs show the future value of their investments after 5 years. **PS**



a How much was Nick and Adam's original investment?

b Match the graphs *X* and *Y* to Adam's and Nick's investment.

c After 5 years, state whose investment is higher and by how much.

Investigation | Is it a scam?

Technology and the internet have changed the way we do things and made life easier for us. But technology has also made life easier for criminals to use scams and financial fraud to rob us of our money.

1 Visit the **Scamwatch** website.

2 Investigate **Unexpected money** scams, found in the **Types of scams** tab. List 4 different reasons people fall for scams.

3 Prepare a list of answers you could give to possible scammers when they are trying to convince you to do something you don't want to do.


Antonio Guillem/Shutterstock.com

4 Investigate **What to do if you've been scammed**, found in the **Stop. Check. Project** tab. Describe what you should do if you have sent money to someone you think is a scammer.

Weblink
Scamwatch

11.3 The compound interest formula

The compound interest recurrence formula is useful for calculating the value of an investment, provided that you know the value of the account at the end of the previous interest period. If this information isn't known, it is a long and tedious method. Fortunately, there's another method!

Compound interest formula

$$A = P(1 + i)^n$$

where P = the principal

i = the interest rate per time period

n = the total number of time periods.

EXAMPLE 7 Compound interest formula

a How much will \$850 amount to in 3 years at 6.5% p.a. compounding annually?

b How much interest will be earned during the 3 years?

Steps	Working
a In the formula, $A = P(1 + i)^n$ $P = \$850$, $i = 0.065$, $n = 3$ and we need to calculate A.	$A = P(1 + i)^n$ $A = \$850 \times (1 + 0.065)^3$ $= \$1026.76$ At the end of 3 years, the \$850 will amount to \$1026.76.
b interest = final amount − principal	interest = \$1026.76 − \$850 = \$176.76 The investment will earn \$176.76 interest.

Wuttipong Boonleang/Shutterstock.com

EXAMPLE 8 Monthly compounding interest

Carl invested \$4800 at 9% p.a. monthly compounding interest for 2 years to help him save the deposit for a delivery truck. How much interest did he earn?

Steps	Working
1 In the compound interest formula P =\$4800, but we'll need monthly interest $i = 0.09 \div 12$, $i = 0.0075$ and n as the number of months. $n = 2 \times 12 = 24$	$A = P(1 + i)^n$ $A = \$4800 \times (1 + 0.0075)^{24}$ $= \$5742.78$
2 interest = final amount − principal	interest = \$5742.78 − \$4800 = \$942.78 Carl earned \$942.78 interest.

EXERCISE 11.3 The compound interest formula

ANSWERS p. 447

Use the compound interest formula to answer the questions in this exercise.

1 Use a calculator to evaluate each expression correct to 2 decimal places.

a $5629 \times (1 + 0.056)^7$ **b** $3350 \times (1.063)^5$ **c** $14\,670(1.08)^9$

2 EXAMPLE 7 Dimitri invested $970 at 6.8% p.a. annually compounding interest for 4 years.

a Write down the values of P, i and n for the compound interest formula.

b Calculate the value of Dimitri's investment at the end of 4 years.

c How much interest did Dimitri's investment earn?

3 Megan won $5260 in Lotto and she decided to invest it at 7.1% p.a. compounding annually for 3 years. How much will her investment be worth at the end of 3 years?

4 When Elliott was born his grandparents set up an investment account for him. They deposited $8000 in an account guaranteed to pay 6% p.a. annually compounding interest for 18 years.

a How much will be in Elliott's investment account when he turns 18 years old?

b How much interest will this investment earn?

5 Sam has $20 000 to invest and she is considering 2 different investments. The interest rate is 6% p.a. for both but one is simple interest and the other annually compounding. How much more interest will she earn over 5 years if she chooses the compound interest account?

6 EXAMPLE 8 Jenni invested $5000 at 7.2% p.a. monthly compounding interest for 6 years.

a In the formula $A = P(1 + i)^n$, explain why the value of $i = 0.006$ and $n = 72$.

b Calculate the value of Jenni's investment at the end of 6 years.

c How much interest will the investment earn?

7 Danielle's bank pays 7.8% p.a. compounding monthly on term deposits.

a Explain how you know that 7.8% p.a. is equivalent to 0.65% monthly.

b Danielle invested $1950 in a term deposit for 18 months. Calculate the value of her investment at the end of 18 months.

c How much interest did Danielle earn from this investment?

8 When Simon retired, he invested $185 000 from his superannuation funds at 8.16% p.a. monthly compounding interest for 5 years.

a What monthly interest rate is equivalent to 8.16% p.a.?

b For how many months did Simon invest the money?

c How much interest did Simon make from this investment?

9 Brin's trade union has its own credit union. It is currently offering term-deposit investors 7.8% p.a. compounding every 6 months for 7-year term deposits.

a What six-monthly interest rate is equivalent to 7.8% p.a.?

b Explain why $n = 14$ when interest is compounded every 6 months for 7 years.

c Brin has $8750 he plans to invest with the credit union. How much will his investment amount to in 7 years under the current term-deposit conditions?

10 Lara is considering 2 possible investments for the $40 000 she has saved from her part-time and after school job. She plans to invest the money for 4 years. PS

	Compounding period	Interest rate
Bank Investment	Monthly	6.9% p.a.
Credit union investment	Annually	7% p.a.

Which investment will give her the better return? Use calculations to justify your answer.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

11.4 Interest calculators*

We can use an online interest calculator to calculate compound interest, such as the one on **The Calculator Site** or **Money Smart** at moneysmart.gov.au.

Worksheets
Compounding periods: spreadsheet
Compound interest table

Spreadsheets
Different compounding periods
Interest calculator
Compound interest graph

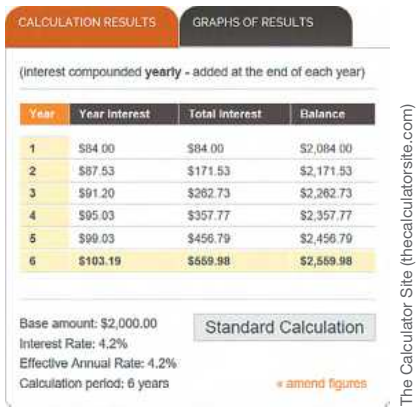
Weblink
MoneySmart

EXAMPLE 9 Online interest calculator

Jana invested $2000 in an account that earned 4.2% p.a. interest, compounded yearly for 6 years. Calculate the final value of her investment and the amount of interest she earned.

Steps | **Working**

Enter the principal, interest rate, period (6 years) and compound interval (yearly) into the compound interest calculator on **The Calculator Site** and clicking 'Calculate':

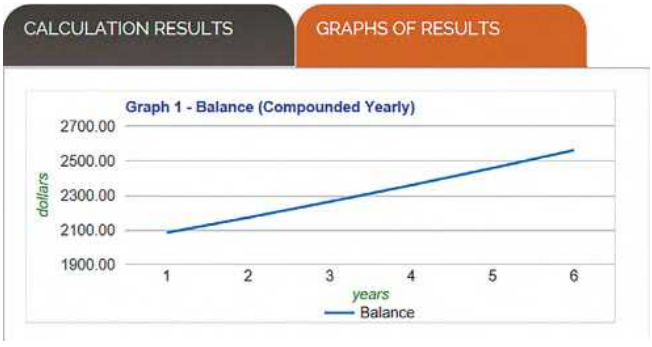


The Calculator Site (thecalculatorsite.com)

final value = $2559.98

total interest = $559.98

Click on the 'Graphs of results' tab to see a graph of the future value over the 6 years.



The Calculator Site (thecalculatorsite.com)

EXERCISE 11.4 Interest calculators* ANSWERS p. 448

Use an online compound interest calculator for each question.

1 EXAMPLE 9 Jayne invested $3000 for 7 years in annually compounding interest at 4.1% p.a.

- **a** Calculate the future value of her investment at the end of 7 years.
- **b** How much interest will the investment earn?
- **c** Use the online calculator to construct a graph showing the value of her investment during the 7 years.

2 Determine the future value of each investment.

	Principal	Interest rate p.a.	Term in years
a	$5500	2.9%	6
b	$1260	1.8%	4
c	$12 000	3.15%	3
d	$36 500	3.2%	8

3 Taj invested $46 000 at 2.85% p.a. annually compounding interest for 3 years. Determine the interest his investment earned each year and the total amount of interest.

4 When you lend someone money at compound interest, the calculations involved are exactly the same as when you invest in compound interest. Mr Kahill lent the soccer club $50 000 to use in upgrading club facilities. The loan was for 8 years at 5.6% p.a. annually compounding interest. The club agreed to repay the loan plus interest in one lump sum at the end of 8 years. PS

- **a** How much will the club have to repay at the end of the loan?
- **b** How much interest is the club being charged?
- **c** Construct a graph to display the amount the club owes Mr Kahill for the term of the loan.

5 Determine the future value of a $6400 loan annually compounding at 3.67% p.a. for 10 years, assuming no repayments are made. PS

6 Sandra has $20 000 to invest for 3 years. She is considering 2 different investments.

Option 1	Simple interest at 5.5% p.a.
Option 2	Compound interest at 4.5% p.a.

Which option do you recommend she take? Use figures to justify your answer.

11.5 Compound interest spreadsheets*

Spreadsheets are powerful tools to use for financial calculation. In this section, you will use a prepared spreadsheet, as well as make one of your own.

EXAMPLE 10 Compound interest spreadsheet

Download the 'Compound interest' spreadsheet from Nelson MindTap and use it to determine the amount of interest that a $1200 investment will earn at 12% p.a. interest compounded yearly for 8 years.

Steps

Insert $1200 for the principal and 12% for the interest.

	A	B	C	D
1	**Compound interest spreadsheet**			
2	Only enter data in cells shaded blue.			
3				
4	Principal	$1,200.00	Annual rate of interest as a percentage	12%
5				
6				
7		Account balance at the beginning of the year	Interest earned during the year	Account balance at the end of the year
8	Year 1	$1,200.00	$144.00	$1,344.00
9	Year 2	$1,344.00	$161.28	$1,505.28
10	Year 3	$1,505.28	$180.63	$1,685.91
11	Year 4	$1,685.91	$202.31	$1,888.22
12	Year 5	$1,888.22	$226.59	$2,114.81
13	Year 6	$2,114.81	$253.78	$2,368.59
14	Year 7	$2,368.59	$284.23	$2,652.82
15	Year 8	$2,652.82	$318.34	$2,971.16
16				

Working

future value after 8 years = $2971.16

interest = future value − principal

= $2971.16 − $1200

= $1771.16

EXAMPLE 11 Loan spreadsheet

David often lends money to members of his family, and he wants the principal and the interest repaid all at the end. Create a spreadsheet that he can use to calculate the total amount to be repaid at the end of the loan.

Steps

The spreadsheet needs to have places to enter the principal (C3) and the interest rate (C4).

Enter the interest rate as a decimal to simplify the formulas.

In B7 we want the amount loaned.
Formula for B7: =C3

D12

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the			
3			Amount loaned	
4			Interest rate as a decimal	
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7				
8				

We need 1 here as the starting year. The next year A8 = A7 + 1.

The amount owing at the beginning of the second year is the same as the amount owing at the end of the first year.
Formula for B8: =D7

Use *Pin* for the interest. P = B7, i = C4, n = 1, so we can leave n out. But we don't want the interest rate to change when we 'copy down', so we put $ signs in the formula for C7: =B7*$C$4

This is the amount at the beginning of the year, plus interest.
Formula for D7: =B7 + C7

Here is the spreadsheet with the formulas included.

D12

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the			
3			Amount loaned	
4			Interest rate as a decimal	
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	=C3	=B7*$C$4	=B7+C7
8	=A7+1	=D7		

Spreadsheets
Interest calculator
Compound interest graph
Compound interest

Enter 5000 in C3 and 0.06 in C4 for a principal of $5000 and an interest rate of 6% p.a., and use 'currency' format for all the cells to display money values. The final step is to highlight the last cell containing a formula in each column and fill it down.

You'll get something like this:

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the cells shaded green			
3		Amount loaned	$5,000.00	
4		Interest rate as a decimal	0.06	
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	$5,000.00	$300.00	$5,300.00
8	2	$5,300.00	$318.00	$5,618.00
9	3	$5,618.00	$337.08	$5,955.08
10	4	$5,955.08	$357.30	$6,312.38

Now the spreadsheet is ready for David to use.

RealPeopleStudio/Shutterstock.com

Spreadsheet
Compound interest

EXERCISE 11.5 Compound interest spreadsheets*

ANSWERS p. 448

Use the 'Compound interest' spreadsheet from Nelson MindTap for Questions **1** to **3**.

1 EXAMPLE 10 Use the spreadsheet to determine the future value of each investment.

	Principal	Interest rate p.a.	Term in years
a	$1000	5%	4
b	$2500	4.3%	6
c	$32 500	3.75%	8
d	$50 000	2.1%	5

2 Calculate the interest earned on each investment.

	Principal	Interest rate p.a.	Term in years
a	$3000	4%	5
b	$5250	3.25%	4
c	$9600	2.9%	8
d	$12 000	1.3%	6

3 Goran is saving to buy a car. He has saved $5460 from his part-time job that he plans to invest at 2.9% p.a. for 3 years.

a Calculate the future value of this investment.

b Goran hopes to have saved another $4000 when the investment matures. Calculate the total amount he will have to spend on the car.

4 EXAMPLE 11 Peta is creating a spreadsheet to show the future value of a compound interest investment. Download 'Peta's spreadsheet' from Nelson MindTap.

Spreadsheet
Peta's spreadsheet

	A	B	C	D
1	Peta's spreadsheet			
2	Only enter data in the cells shaded yellow.			
3				
4	Present value	$6,000.00		
5	Annual interest rate as a percentage	0.45		
6				
7	Year	Value at the beginning of the year	Interest	Value at the end of the year
8	1			
9				

a Write the formulas for cells B8, C8, D8 and A9, enter the formulas into the spreadsheet, then copy the formulas down to row 25.

b Peta meant to enter the decimal for 4.5% in B5 but she made a mistake. Correct her mistake.

c Use Peta's spreadsheet to determine the future value of a $24 000 investment at 4.5% annually compounding interest for 16 years.

d How much interest will this investment earn?

5 a Design your own spreadsheet to show the future value of a compound interest investment. Can you arrange it differently to the examples in this chapter? Add colour to make your spreadsheet interesting. PS

b Use your spreadsheet to determine the amount of time it will take for a $3000 investment to double in value (that is, become $6000) when the interest rate is 4% p.a.

11.6 Different compounding periods*

So far in this chapter you've mostly considered investments with interest compounding yearly, but interest can compound monthly, **quarterly**, daily or any time period. However, the calculations can be very repetitive and time-consuming. Fortunately, you can use technology to do the long calculations for you.

In this section, you will need access to an online calculator or download the 'Different compounding periods' spreadsheet from Nelson MindTap.

EXAMPLE 12 Different compounding periods

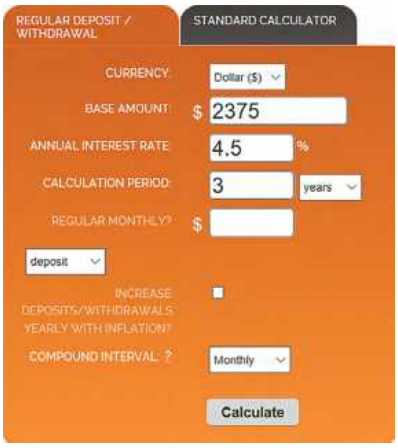
Mack invested $2375 for 3 years in an account that earned 4.5% p.a. interest, compounded monthly. Use technology to calculate the future value of his investment and the amount of interest he will earn.

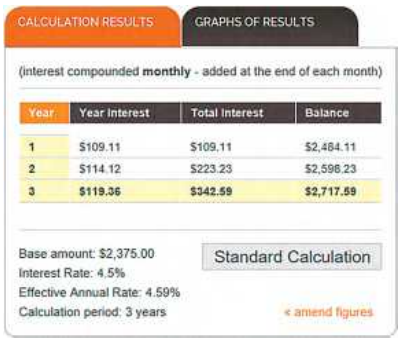
Steps | **Working**

Spreadsheet
Different compounding periods

Principal $2375, interest rate 4.5% p.a., period 3 years, compounded monthly.

Online calculator





The Calculator Site (thecalculatorsite.com)

future value = $2717.59

total interest = $342.59

'Different compounding periods' spreadsheet

As interest is compounded monthly, the number of periods per year is 12.

	A	B	C	D	E
1	Different compounding periods				
2					
3	How much compound interest will I earn?				
4	Only enter data in cells shaded in blue.				
5					
6	Principal	$2,375.00			
7	Annual rate of interest as a percentage	4.50%		Interest rate per compounding period, as a decimal	0.00375
8	Number of compounding periods per year	12		Number of compounding periods	36
9	Length of the investment in years	3		Final value of the investment	$2,717.59
10				Interest earned during the investment	$342.59
11					

© Microsoft Corporation

future value = $2717.59

total interest = $342.59

EXERCISE 11.6 Different compounding periods*

ANSWERS p. 448

1 EXAMPLE 12 Use the technology of your choice to answer these questions.
Determine the amount of interest that each investment will earn.

	Principal	Interest rate p.a.	Term	Compounding period
a	$10 000	6%	3 years	monthly
b	$2500	4.2%	4.5 years	quarterly
c	$1840	3.6%	5 years	half yearly
d	$3800	4.6%	2 years	daily

Quarterly means 4 times per year, that is, every 3 months.

2 Marissa won $5800 in lotto and invested it at 7.2% p.a. monthly compounding interest for 3 years.

a How much will Marissa's win grow to in 3 years?

b How much interest will Marissa's investment earn?

3 Brody saved $1660 for a holiday after he finishes Year 12. He invested it at 5% p.a. interest compounding monthly for 1 year. How much will be in his holiday account at the end of the year?

4 Jackie received a $2500 bonus as a result of increased company profits. She is going to invest it for 1 year at 6% p.a. How much more interest will she earn in monthly compounding compared to simple interest?

5 When Kristy was made redundant at work, she was given a lump sum payment of $40 000, which she invested for 5 years. How much more interest will she earn from daily compounding than annually compounding interest at 6% p.a.? PS

6 On his retirement, Yo-han received a lump sum superannuation payment of $450 000 and he is going to invest it for 2 years. He can invest it at 6.4% p.a. monthly compounding or 6.55% p.a. annually compounding interest. Which investment will give him the better return? Justify your answer. PS

iStock.com/XiXinXing

7 Lucy has $200 000 to invest to provide for her retirement. She considers 2 different investment options. PS

	Investment term	Fees
Finance company	5% p.a. compounding monthly	$10 per month
Managed funds	6% p.a. compounding daily	6.5% of the interest earned

Which option will give Lucy the higher return in 12 months? Explain your answer.

Investigation — Different compounding periods

In this investigation, you are going to determine how changing the compounding period (how often interest is compounded) affects the amount of interest earned.

What you have to do

1 Use either an online calculator or a spreadsheet to complete the missing values in the table.
Invest $10 000 for 4 years at 8% p.a.

Compounding period	Total interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

2 Write a sentence to describe the observations you've made.

3 Check the correctness of your observation by completing a table for another investment.
Invest $50 000 for 7 years at 12% p.a. compounding interest.

Compounding period	Total interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

4 Chris is going to invest $2000 for 3 years at 6% p.a. He can select either monthly compounding or fortnightly compounding interest. Which compounding period will give him the better return? Give a reason for your answer.

Worksheets
Simple vs compound interest: spreadsheet

Comparing interest rates

Spreadsheet
Comparing interest rates

TECHNOLOGY — Is compound interest always better than simple interest?

In this investigation, you are going to make some calculations to help you choose the investment with the better return.

What you have to do

1 Copy the tables in parts **A** to **C** below.

2 Use the technology of your choice to complete each tables. To make it easier, make the principal $1000 in every calculation.

3 Complete the class discussion questions after you have finished the calculations.

Which is better: simple or compound interest?

Part A

The interest rates and the terms are the same.

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 3% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Part B

The terms are the same, but the compound interest rate is higher than the simple interest rate.

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5.1% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3.75% p.a.	Term: 6 years Interest rate: 4% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 8.1% p.a.	Simple interest = Compound interest = Which investment is better?

Part C

The terms are the same, but the simple interest rate is higher than the compound interest rate.

Simple interest	Interested compounded annually	Simple interest
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 4.8% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 2.75% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 12% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 3 years Interest rate: 12% p.a.	Term: 3 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Class discussion questions

- When the interest rates and the terms are the same, does simple or compound interest give the better return?
- When the compound interest rate is higher than the simple interest rate, and the terms are the same, which type of investment produces the better return?
- When the simple interest rate is bigger than the compound rate, and the terms are the same, will one or the other type of interest always give the better return?

Keyword activity

DEFINITIONS MATCH

ANSWERS p. 448

Match the terms in the left column with their correct meanings in the right column.

	Word		Meaning
1	future value	**A**	An illegal method used to trick people out of their money.
2	compound interest	**B**	Occurring every year.
3	compounding period	**C**	The rate of return on an investment, usually expressed as a percentage per year.
4	scam	**D**	Interest that is calculated only on the original principal.
5	annual	**E**	The amount of time between compound interest calculations for an investment.
6	interest rate	**F**	The original amount invested or borrowed.
7	principal	**G**	The value of an investment over time.
8	superannuation	**H**	Interest that is calculated on the current value of an investment, including interest previously added to the principal.
9	simple interest	**I**	A regular saving fund to provide an income when you are retired.

Solution to the chapter problem

Problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested $25 000 for 3 years, earning simple interest at 4.2% per year.

Whose investment will earn more interest?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

We have to calculate whether Ryan or Melanie will earn the more interest.

We know:

	Principal	Term	Interest
Ryan	$25 000	3 years	4% compounded annually
Melanie	$25 000	3 years	4.2% simple interest

STAGE 2: SOLVE THE PROBLEM

SOLVE

Use the compound interest formula, $A = P(1 + i)^n$ with $P = \$25\,000$, $i = 0.04$ and $n = 3$.

$$A = 25\,000 \times (1 + 0.04)^3$$
$$= \$28\,121.60$$

Ryan's investment will be worth $28 121.60 at the end of 3 years.

Ryan's interest $= \$28\,121.60 - \$25\,000$
$= \$3121.60$

Use the simple interest formula to calculate Melanie's interest.

Interest $I = Pin$

$P = \$25\,000$, $i = 0.042$ and $n = 3$

Melanie's interest $= \$25\,000 \times 0.042 \times 3$
$= \$3150$

Compare the interest:

Melanie's investment will earn $28.40 more interest than Ryan's.

STAGE 3: CHECK THE SOLUTION

CHECK

Both values for interest are approximately the same, indicating that, probably, they are correct. When the principals, terms and rates of interest are the same, compound interest always pays more than simple interest. But in this case, the interest rates are not the same. Either simple or compound interest can produce the higher interest. It is likely that the solution is correct.

STAGE 4: PRESENT THE SOLUTION

PRESENT

Melanie's investment will pay $28.40 more than Ryan's investment.

11 Test yourself

Investing money

ANSWERS p. 448

1 EXERCISE 11.1 Calculate the simple interest earned on a $4000 investment at 4.1% p.a. for 8 years.

2 EXERCISE 11.1 Sam earned $195 in simple interest when she invested $1300 for 2 years. What was the rate of interest?

3 EXERCISE 11.1 How long will it take to earn $1000 simple interest at 7.2% p.a. on an $8000 investment?

4 EXERCISE 11.1 Ali invested $2400 for 3 months in an account paying simple interest at 6.4% p.a. How much interest did Ali's investment earn?

5 EXERCISE 11.2 Use the compound interest recurrence formula $A_{n+1} = rA_n$ to answer this question. Lan invested $5420 in annually compounding interest at 5% p.a.

a What is the value of A_0?

b Explain why the value of r is 1.05.

c Determine the value of A_1, A_2 and A_3.

d How much interest did Lan's investment earn in 3 years?

6 EXERCISE 11.3 Josephine invested $8400 at 4% p.a. compounded yearly.

a What values for P, i and n are required for the formula $A = P(1 + i)^n$ to calculate the value of Josephine's investment in 5 years' time?

b Calculate the value of Josephine's investment in 5 years' time.

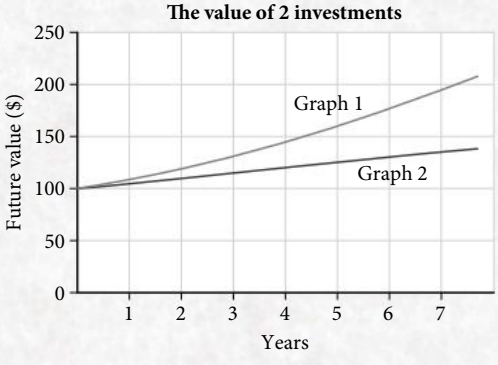
c How much interest will Josephine's investment earn?

7 EXERCISES 11.1, 11.3 Bev and Sally both invested $10 000 for 4 years. This table summarises their investments.

	Type of interest	Interest rate
Bev's investment	Simple	8.2% p.a.
Sally's investment	Compound	6.5% p.a.

Who earned the higher amount of interest and by how much?

8 EXERCISE 11.3 Jason drew a graph of 2 investments, shown on the right. One investment is paying simple interest and the other compound interest. Which graph is which?



9 EXERCISES 11.1, 11.4 Which investment has the better return for an $800 principal for a 6-year term?

Investment A: 3.4% p.a. simple interest

Investment B: 3.3% p.a. interest compounded yearly

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10 EXERCISE 11.4 Use an online calculator to determine the total amount of interest Pooja will earn when she invests $26 500 at 4.1% p.a. annually compounding interest for 7 years.

11 EXERCISE 11.5 Jill used this spreadsheet to calculate the future value and interest when she invested $60 000 at 8% p.a. with monthly compounding interest.

	A	B	C	D
1	**Jill's spreadsheet**			
2				
3				
4		**Initial investment C4**	$60,000.00	
5		**Annual interest rate as a decimal C5**	0.08	
6		**Monthly interest rate C6**	0.006666667	
7				
8				
9	**Number of months**	**Value of the investment at the beginning of the month**	**Monthly interest**	**Value of the investment at the end of the month**
10	1	$60,000.00	$400.00	$60,400.00
11	2	$60,400.00	$402.67	$60,802.67
12	3	$60,802.67	$405.35	$61,208.02
13	4	$61,208.02	$408.05	$61,616.07
14	5	$61,616.07	$410.77	$62,026.85
15	6	$62,026.85	$413.51	$62,440.36
16	7	$62,440.36	$416.27	$62,856.63
17	8	$62,856.63	$419.04	$63,275.67
18	9	$63,275.67	$421.84	$63,697.51
19	10	$63,697.51	$424.65	$64,122.16
20	11	$64,122.16	$427.48	$64,549.64
21	12	$64,549.64	$430.33	$64,979.97
22	13	$64,979.97	$433.20	$65,413.17
23	14	$65,413.17	$436.09	$65,849.26
24	15	$65,849.26	$439.00	$66,288.25
25	16	$66,288.25	$441.92	$66,730.17
26	17	$66,730.17	$444.87	$67,175.04
27	18	$67,175.04	$447.83	$67,622.88

a One of the formulas Jill used is =B10*$C$6. What does this formula calculate?

b Why did Jill use $ signs in the formula?

c How much interest did Jill's investment earn in the 14^{th} month?

d What formula could Jill use to make the spreadsheet calculate the total amount of interest earned in the first 12 months?

e What was Jill's investment worth at the end of 18 months?

f Construct a spreadsheet similar to Jill's if instead of compounding monthly, her investment compounds weekly.

g Use your spreadsheet to determine the extra interest Jill will earn in 6 months if the investment compounds weekly instead of monthly.

12 EXERCISE 11.6 Sam invested $3000 at 4.8% p.a. monthly compounding interest for 3 years.

a What was the monthly interest rate?

b Use the technology of your choice to determine how much interest Sam will earn.

13 EXERCISE 11.6 Pedro has $10 000 to invest for 6 years. Use the technology of your choice to determine which is the best investment:

- 4% p.a. monthly compounding
- 4.1% p.a. annually compounding
- 4.4% p.a. simple interest

Chapter problem

Ziad and Adrian are arguing about which local rugby league team is better. The points scored by their favourite teams in each match over the previous season are listed here:

Eagles: 20, 10, 40, 12, 17, 20, 22, 20, 34, 19, 36, 18, 24, 12, 38, 34, 24, 36, 32, 22, 6, 7, 38, 18

Cougars: 14, 18, 24, 39, 14, 4, 4, 14, 10, 13, 28, 22, 16, 18, 18, 12, 18, 28, 21, 6, 10, 18, 36, 12

a What is the best graph to use to show this data? Construct this graph.

b Which is the better team based on this data? Justify your answer.

c What additional data might you need to know to decide which is the better team?

Matt Jones/Shutterstock.com

CHAPTER 12

GRAPHS, DATA AND LOANS: SUMMARISING AND COMPARING DATA

COMPARING DATA

What will we do in this chapter?

- Calculate a five-number summary for a set of data and create a box plot for the data
- Compare 2 sets of data using back-to-back stem plots and parallel box plots
- Analyse and describe the shape of a set of data using words such as symmetrical, skewed and bimodal

How are we ever going to use this?

- When comparing products for sale
- When examining performances in a variety of sports
- To interpret data presented in the media

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Syllabus coverage

UNIT 4, TOPIC 2: SUMMARISING AND COMPARING DATA

Comparing datasets

- Complete a five-number summary for different datasets.
- Construct a box plot using a five-number summary.
- Compare parallel box plots and back-to-back stem plots for different datasets [complex].
- Compare the characteristics of the shape of histograms using symmetry, skewness and bimodality, where applicable [complex].

Videos (2):

12.1 Box-and-whisker plots
12.3 Double box plots

Worksheets (5):

12.2 Stem-and-leaf plots 2
12.3 Comparing word lengths • Investigating young drivers • Comparing sports scores • Comparing city temperatures

Terminology

back-to-back stem plot	bimodal	box plot	five-number summary
parallel box plots	quartiles	skewed	stem plot
symmetrical	uniform	unimodal	

12.1 Box plots

The 3 quartiles Q_1, Q_2 and Q_3 for a set of data and the lowest and highest values together make a **five-number summary**. This can then be graphed on a **box plot**, also called a **box-and-whisker plot**.

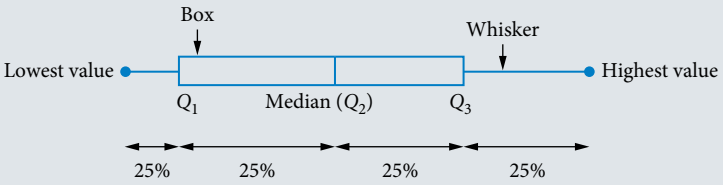
Video
Box-and-whisker plots

Five-number summary and box plot

The **five-number summary** for a set of data consists of:

- the lowest value
- the first or lower quartile, Q_1
- the second quartile, the median, Q_2
- the third or upper quartile, Q_3
- the highest value.

A **box plot** is a graph of a five-number summary.



EXAMPLE 1 Drawing box plots

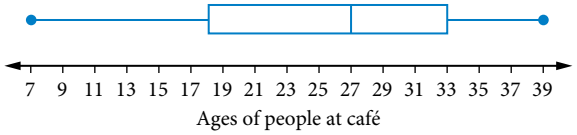
The ages of the 23 people at a café are shown here.

33 23 28 36 27 15 32 18 13 13 38 38
27 7 34 27 12 26 33 21 24 39 20

a **Determine** the five-number summary for this data.

b Draw a box plot for this data.

This data was also used in Example 4 of Chapter 10 on page 269.

Steps	Working
a Find the median: it is also the 2nd quartile.	7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26, 27, 27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39 $Q_2 = 27$
Find the middle of each half of the values. These are the 1st and 3rd quartiles.	7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26 27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39 $Q_1 = 18$, $Q_3 = 33$
For the five-number summary, include the lowest value (7) and the highest value (39).	The five-number summary is 7, 18, 27, 33, 39
b Draw the box plot with the box between 18 and 33, with a middle bar at 27, and the whiskers extending to 7 and 39.	 Ages of people at café

EXERCISE 12.1 Box plots

ANSWERS p. 448

In this exercise you will use your answers to Exercise 10.3 on page 270. If you can't find your own answers, then look up the answers at the back of the book.

1 EXAMPLE 1 Use your answers to Question **1** about the sale of fish in a fish-and-chip shop to complete the following questions.

a Copy and complete this table to record the five-number summary for this data.

Lowest value				Highest value
	18			28

b Draw a box plot for this data.

2 Use your answers to Question **2** about daily maximum temperatures in Cairns.

a Determine a five-number summary for this data.

b Draw a box plot for this data.

3 Use your answers to Question **3** about student heights to draw a box plot for this data.

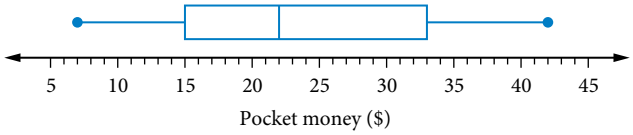
4 The monthly number of home burglaries in Emu Springs was recorded for 27 months.

21 25 17 23 16 21 41 22 25 20 22 11 20 12
13 12 6 12 10 19 30 22 21 14 34 33 24

a What is the five-number summary for this data?

b Draw a box plot for this data.

5 This box plot represents the amount of pocket money in dollars earned by a sample of 60 children.



a Determine the median.

b Calculate the range.

c How many children earned between:

i $33 and $42?

ii $15 and $42?

d Calculate the interquartile range.

6 This stem plot shows the daily number of students served at the school canteen over a 3-week period.

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

Key: 7|6 = 76

Fairfaxphotos/Janie Barrett Specialx 67530

a Determine the five-number summary for this data.

b Draw the box plot for this data.

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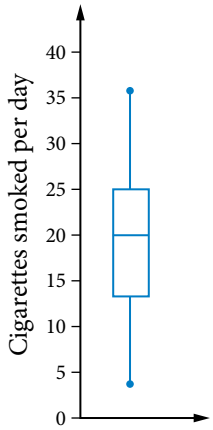
7 This box plot shows the number of cigarettes smoked per day by a sample of 60 smokers who are trying to quit.



- **a** What is the median number of cigarettes smoked per day?
- **b** What is the interquartile range?
- **c** What is the lowest value?
- **d** How many people smoked between 20 and 25 cigarettes per day?
- **e** How many people smoked fewer than 20 cigarettes per day?

8 **a** Use your answers to Question **5** about retail theft in Exercise 10.3 on page 270 to make a five-number summary for each set of data. PS

- **b** Draw a box plot for each set of data.
- **c** In which region do you think it would be safer to open a shop? Using the medians and the ranges, justify your answer.

12.2 Back-to-back stem plots*

Worksheet
Stem-and-leaf plots 2

You can use **back-to-back stem plots** (also known as **stem-and-leaf plots**) to compare 2 data sets.

EXAMPLE 2 Drawing back-to-back stem plots

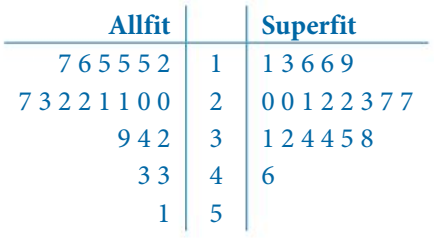
Madeline compares the ages of people attending 2 local gyms.

Allfit: 17, 22, 51, 12, 27, 43, 39, 20, 21, 15, 15, 43, 15, 20, 32, 21, 23, 16, 34, 22

Superfit: 19, 20, 32, 46, 27, 16, 11, 34, 38, 21, 13, 22, 34, 35, 23, 31, 20, 16, 22, 27


iStock.com/Lyndon Stratford

- **a** Draw a back-to-back stem plot for this data.
- **b** **Determine** the median age for each gym.
- **c** **Calculate** the range for each gym.
- **d** Are there any outliers in either gym? If so, state the outliers.
- **e** State one similarity between the 2 sets of data.
- **f** State one difference between the 2 sets of data.

Steps	Working
a The stem will be the tens digit and it is in the middle. The leaves will be the units digit for each number in the sets of data.	 Key: 2|1| = 12 |1|6 = 16
b Each gym has 20 values, so the middle values are the 10th and 11th values.	Allfit: middle values are 21 and 22. median for Allfit $= \frac{21+22}{2}$ $= 21.5$ Superfit: middle values are 22 and 23. median for Superfit $= \frac{22+23}{2}$ $= 22.5$
c range = highest value − lowest value	range for Allfit = 51 − 12 = 39 range for Superfit = 46 − 11 = 35
d An outlier is an extreme value.	51 is an outlier for Allfit.
e Look for similar things about the data.	The medians for both sets of data are quite close. Both sets of data are also clustered in the 20s.
f Look for different things about the data.	Allfit's ages are more spread out than Superfit's. Superfit's ages are spread evenly across the 10s, 20s and 30s, whereas Allfit's ages are clustered in the 10s and 20s.

EXERCISE 12.2 Back-to-back stem plots*

ANSWERS p. 449

1 EXAMPLE 2 These are the results of 2 Year 10 classes in their final mathematics exam.

10 Aqua: 84, 71, 79, 82, 78, 89, 71, 95, 93, 81, 85, 65, 70, 95, 91, 89, 89, 75, 62, 71, 69, 88, 94, 81, 85, 76, 80, 67, 60

10 Black: 88, 60, 66, 74, 42, 59, 61, 68, 54, 65, 56, 46, 65, 71, 57, 72, 40, 65, 41, 38, 35, 39, 51, 47, 39, 42, 39, 42

a Draw a back-to-back stem plot for this data.

b How many students in 10 Aqua?

c Find the median value for each class.

d Find the range for each class.

e Are there any outliers in either class? If so, state the outliers.

f If one class is the top class and one class is the middle class, which class is which? Justify your answer using your answers to parts **a** to **e**.

2 The school principal Mr Farley is concerned about absences in Year 11 and Year 12. The daily number of absentees over a 4-week period were:

Marmaduke St. John/Alamy Stock Photo

Year 11: 30, 15, 30, 23, 39, 20, 31, 42, 22, 41, 30, 25, 23, 30, 22, 30, 29, 15, 15, 44

Year 12: 20, 22, 12, 8, 19, 13, 14, 23, 7, 22, 18, 20, 13, 19, 21, 15, 24, 10, 26, 24

a Draw a back-to-back stem plot for this data.

b Find the median value for each year group.

c Find the range for each year group.

d Are there any outliers in either year group? If so, state the outliers.

e State one similarity between the 2 sets of data.

f State one difference between the 2 sets of data.

g Mr Farley believes there is more absenteeism in Year 11 than in Year 12.

i Does your analysis in parts **a** to **e** support this statement? Justify your answer.

ii What other information would we need to further evaluate this statement?

3 This back-to-back stem plot shows the heights in centimetres of boys and girls in a Year 12 class:

Boys		Girls
4 4 4 3	16	0 1 3 4 4
9 8	16	5 7
4 2 1 1	17	0 1 2 2 4
6 9 9	17	5 7
2 1 0	18	

Key: 3|16| = 163

|16|0 = 160

a Determine the five-number summary for each set of data.

b Draw the box plot for each set of data.

c Summarise the differences between the heights of boys and girls.

4 For a PE assignment, Kayne compares the heights of males and females in Year 12. The following data he collected are in centimetres. PS

Males: 178, 183, 167, 184, 181, 170, 190, 181, 181, 200, 183, 160, 165, 172, 178, 178, 168, 191, 181, 190, 180, 184, 180, 175, 170

Females: 178, 166, 166, 150, 168, 166, 163, 162, 167, 159, 157, 185, 176, 164, 165, 164, 160, 185, 176, 177, 171, 152, 173, 173

a Draw a back-to-back stem plot for this data. Use stems of 15, 16, …

b How many males were measured?

c How many females were measured?

d Determine each of the five-number summaries for the males and the females.

e State any outliers in each group.

f State one difference between the 2 sets of data.

g Write 2–3 sentences that Kayne could use as a conclusion for his assignment.

5 The daily maximum temperatures for Townsville, Qld and Kiama, NSW in the month of February are shown below. PS

Townsville: 32.1, 32.3, 33.1, 33.4, 31.8, 31.0, 33.7, 34.3, 32.7, 32.7, 31.0, 33.4, 33.6, 34.2, 32.3, 32.5, 31.9, 32.1, 32.8, 31.7, 29.3, 31.9, 31.2, 31.7, 31.2, 32.2, 29.5, 31.0

Kiama: 33.0, 27.9, 32.6, 27.6, 36.3, 29.6, 21.0, 23.0, 22.5, 25.0, 26.4, 22.1, 20.7, 22.3, 24.9, 25.6, 25.5, 24.5, 27.0, 32.5, 21.8, 20.3, 22.5, 24.7, 25.6, 25.9, 26.7, 25.4

a Draw a back-to-back stem plot for this data. Use stems 20, 21, 22, … up to 36.

b How many days are there in February?

c Find the median for each place.

d Find the range for each place.

e Are there any outliers in the data for either place? If so, state the outliers.

f State one difference between the 2 sets of data.

g Write 2–3 sentences comparing the temperatures in both places.

h Based on this data, which place would you prefer to live in? Justify your answer.

Autau/Shutterstock.com

Townsville

6 This back-to-back stem plot shows the number of points scored in each match by 2 basketball teams in a season.

Langley Lynx		Blakely Bears
6 6 5 4 3	4	4 9
8 8 3 0	5	2 3 3 6 8
8 8 6 6 3 1 1	6	5 6 8 9
7 4 3 0	7	0 0 1 3 6
6 6 5	8	2 5 7 7 9 9
2 2	9	0 3 4

Key: 3|4| = 43

|4|9 = 49

a How many matches were played in this season?

b Find the range for each team.

c Find the median for each team.

d Determine the five-number summary for each team.

e Draw a box plot for both teams.

f Comment on the similarities and differences of the points scored by the 2 teams.

g Which is the better-scoring team?

12.3 Parallel box plots*

In statistics, there are many opportunities to compare 2 sets of data.

We can compare sets of data by drawing **parallel box plots** (or **double box plots**) using a common scale.

EXAMPLE 3 Parallel box plots

The five-number summaries below describe the number of rainy days per month over 2 years for Sydney and Melbourne.

Sydney: 9, 11, 13, 14, 15

Melbourne: 7, 10, 14, 16, 19

a Draw parallel box plots for these summaries.

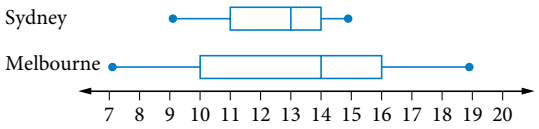
b Find the median for each city.

c What is the interquartile range for each city?

d Which city has more rainy days per month?

e If Corrina prefers a more consistent pattern of rainy days, which city would you recommend for her to live in? Justify your answer.

iStock.com/Katharina13

Steps	Working
a Use one scale for both box plots.	
b The median is the middle number in the five-number summary or the middle bar in the box plots.	median for Sydney = 13 median for Melbourne = 14
c interquartile range = $Q_3 - Q_1$ In a five-number summary, Q_1 is the 2nd value and Q_3 is the 4th value, or use the ends of the boxes in the box plots.	IQR for Sydney = 14 − 11 = 3 IQR for Melbourne = 16 − 10 = 6
d Compare the medians and boxes of the box plots.	Melbourne has more rainy days per month. Its median is higher and half its values are above 14, compared to one-quarter of values for Sydney.
e Consistent means the data is less spread out.	Sydney has the more consistent pattern of rainy days because its range and interquartile range are smaller than Melbourne's. This is shown by the shorter box plot.

Video Double box plots

Worksheets Comparing word lengths

Investigating young drivers

Comparing sports scores

Comparing city temperatures

EXERCISE 12.3 Parallel box plots* ANSWERS p. 450

1 EXAMPLE 3 Rigby and Alex are in different classes. The following five-number summaries are for yearly exams in each class.

Rigby's class: 48, 64, 75, 87, 96

Alex's class: 47, 57, 69, 80, 97

a Draw parallel box plots for these summaries.

b What is the median for:

i Rigby's class? **ii** Alex's class?

c What is the range for:

i Rigby's class? **ii** Alex's class?

d Both Rigby and Alex achieved 85 in the yearly exam. Who performed better in relation to their own class? Justify your answer.

e Which class generally performed better in the yearly exam? Justify your answer.

f Can we calculate the mean from the given information? Explain.

2 These are the waiting times in minutes for calls to customer service in 2 phone companies.

Chatphone: 10, 7, 6, 8, 7, 5, 6, 9, 7, 3, 8, 8, 9, 7, 9, 7, 9, 8

Oztel: 10, 5, 9, 9, 9, 10, 11, 9, 8, 7, 9, 7, 7, 6, 9, 8, 11, 11

Gorodenkoff/Shutterstock.com

a Find the five-number summary for each set of data.

b Construct parallel box plots for this data.

c What is the median for each company?

d Find the mode for each company.

e Calculate the interquartile range for each company.

f Chatphone claims that its waiting times are generally lower than those of Oztel. Is this correct? Justify your answer.

g Is this sufficient information to decide which company you would choose to have your mobile phone with? What other information would you need, if any?

3 The Mayor of Middleton claims that his town is safer for drivers than the nearby town of Blakewell. To test this, James measured the speed (in km/h) of a sample of 20 cars in each town. PS

Middleton: 60, 65, 70, 68, 62, 75, 80, 83, 82, 69, 73, 75, 85, 72, 67, 88, 90, 85, 72, 63

Blakewell: 76, 64, 58, 82, 72, 70, 68, 75, 63, 67, 74, 70, 79, 80, 73, 75, 71, 68, 72, 73

a Determine the five-number summary for each town's data.

b Construct parallel box plots for this data.

c What is the median for each town?

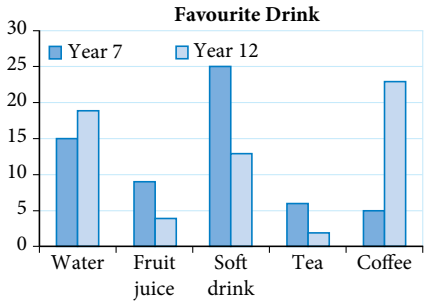
d Find the mode(s) for each town.

e Calculate the mean for each town.

f Determine the interquartile range for each town.

g Is the Mayor of Middleton correct? Justify your answer.

4 Yasmin surveyed a selection of Year 7 and Year 12 students at her school about their favourite drinks, and graphed the results on this clustered column graph.



a How many Year 7 students did Yasmin survey?

b How many Year 12 students did she survey?

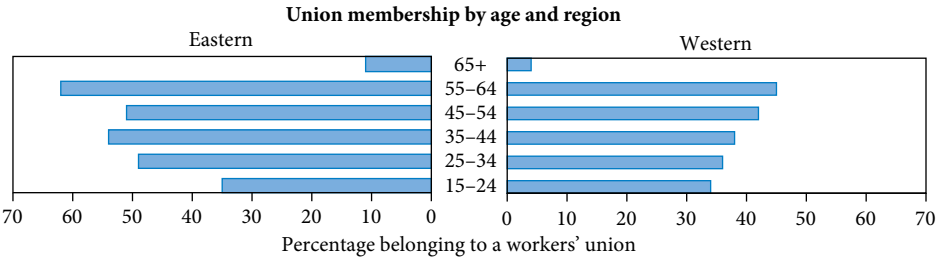
c What was the most popular drink in Year 7?

d What was the most popular drink in Year 12?

e Why do you think the most popular drink is different in Year 7 and Year 12?

Wavebreakmedia/Shutterstock.com

5 Workers in different age groups in 2 regions were surveyed to find out who belonged to a union. The results are displayed in a back-to-back histogram. PS



a What percentage of 45- to 54-year-olds in the eastern region belonged to a union?

b Which age group had about the same percentage of union members across both regions?

c Comment on the statement, 'People in the eastern region are more likely to join a union'. Justify your answer.

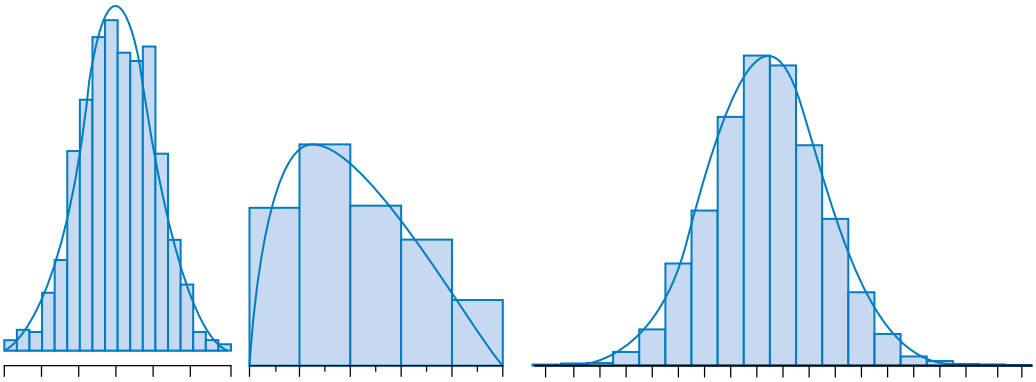
Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

12.4 The shape of a distribution*

When there is an overall pattern to the data in a histogram, you can draw a smooth curve around the histogram to represent the data. You can also draw a curve around dot plots and stem plots to see the shape of the data.

Here are 3 examples of smooth curves that represent the general **shape of a distribution**.



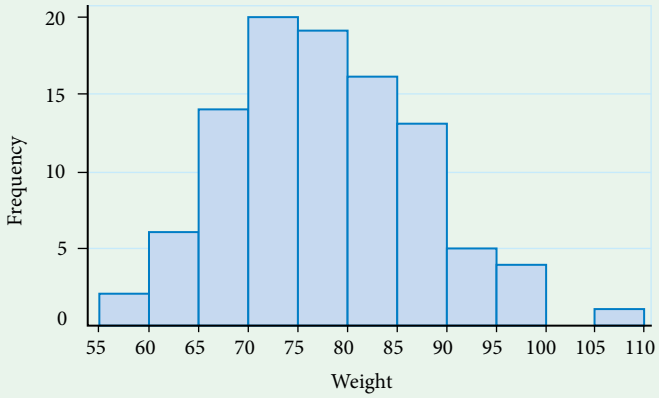
Symmetrical distributions

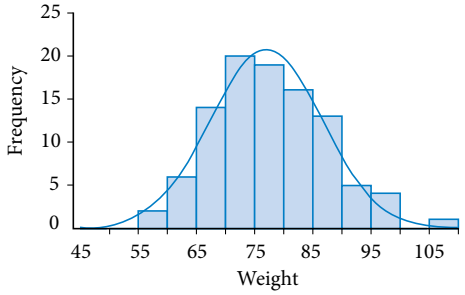
The first and third curves above show symmetrical distributions. One half is the mirror-image of the other half. You could fold it down the middle and the 2 sides would match.

A symmetrical curve could represent the weights of all high school students in Australia.

EXAMPLE 4 Shape of a histogram

Construct a smooth symmetrical curve over this histogram.



Steps	Working
Use the heights of the columns to form a smooth hump for the top of the curve. Then symmetrically flatten out the curve on both sides, slightly above the horizontal axis.	

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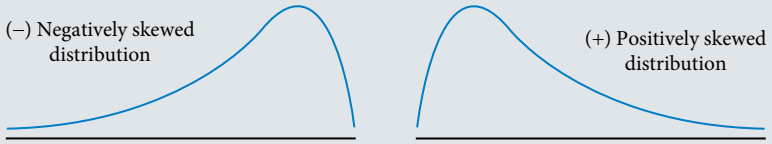
Histograms rarely have a perfect symmetrical shape. Sometimes they are pushed sideways or they have more than one peak (high point). Statisticians have names for the different types of curves and distributions.

Skewed distributions

When data is not symmetrical but pushed to one side, it is **skewed** (which means 'twisted'). We can identify the type of skew by looking at the 'tail' of the curve.

Skewed distributions

- When the tail is on the left, the data are negatively skewed.
- When the tail is on the right, the data are positively skewed.



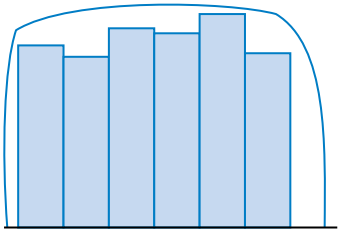
One way of remembering the direction of skewness is to note that on a number line, negative numbers are on the left and positive numbers are on the right.

A **negatively-skewed distribution** could represent the marks scored by students on an easy test. Most students achieved high marks while comparatively few scored low marks.

A **positively-skewed distribution** could represent house prices in a small country town. There are lots of moderate prices for the houses and comparatively few high prices for bigger houses and farms.

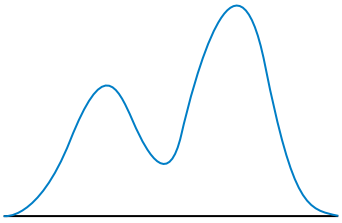
Uniform distributions

If we roll a normal die many times, each of the possible 6 numbers should appear roughly the same number of times, giving a **uniform distribution**. Its graph is close to one big rectangle. Uniform means 'the same' and in a uniform distribution, the values are evenly distributed.



Bimodal distributions

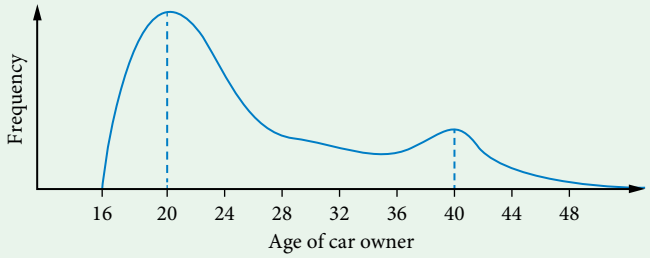
This graph represents the amount of traffic crossing a bridge in one day. There are 2 peaks on the curve, corresponding to the morning and afternoon busy times. Because it has 2 peaks, this curve represents a **bimodal distribution**.



When there is only one peak, it is called a **unimodal** distribution.

EXAMPLE 5 Shape of a distribution

This graph shows the number of car owners by age who claim on their car insurance.
Describe the shape of the distribution.



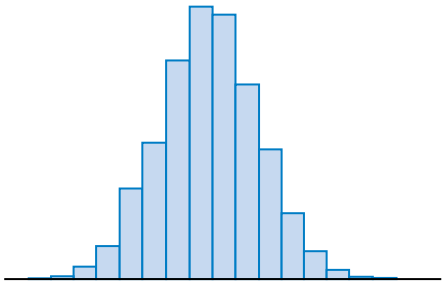
Steps	Working
The distribution is not symmetrical, so it is skewed. The tail points to the right, so it is positively skewed. There are peaks at age 20 and 40, so the distribution is also bimodal.	The distribution is positively skewed and bimodal.

EXERCISE 12.4 The shape of a distribution*

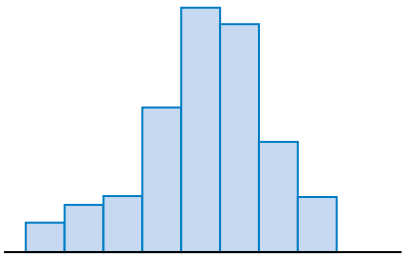
ANSWERS p. 450

1 EXAMPLE 4 Copy each histogram and sketch a smooth curve over each one. State whether the curve is symmetrical or not.

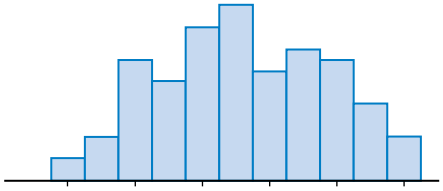
a



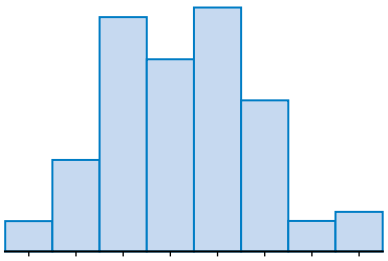
b



c

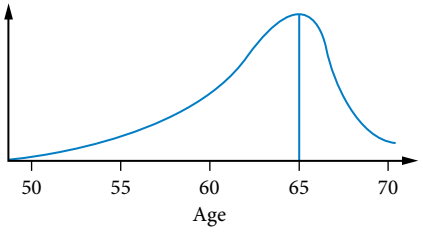


d

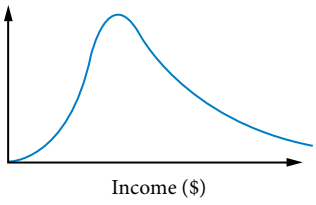


2 EXAMPLE 5 What type of distribution is shown in each graph? Choose from symmetrical, positively skewed, negatively skewed and bimodal.

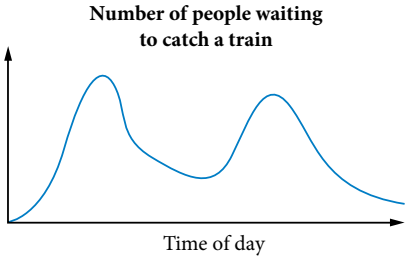
a **Age at retirement**



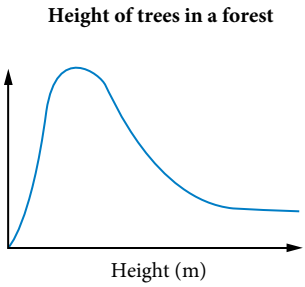
b **Australian family income**



c



d



3 Which graphs in Question **2** are unimodal (have one peak)?

4 Sketch a graph to represent each distribution described. Remember to label the axes.

a The age at which a person dies is negatively skewed, with a mode of 80 years.

b The percentage marks students scored on a difficult exam has a bimodal, positively skewed distribution. A small group scored 90%, but the majority scored around 40%.

c A bus carrying passengers to a concert includes a large group of dancers and a small group of rugby players. The distribution of the body mass (weights) of the passengers is bimodal.

d The heights of 2000 randomly-selected girls aged 17 are distributed symmetrically.

e When a die is rolled 100 times, the numbers that come up have a uniform distribution.

f The number of minutes visitors to a museum spend looking at a display that includes a 5-minute video is bimodal. Most visitors spend less than 1 minute looking at the display, but some also watch the entire video and view the display. Very few people look at the display between 1 to 5 minutes.

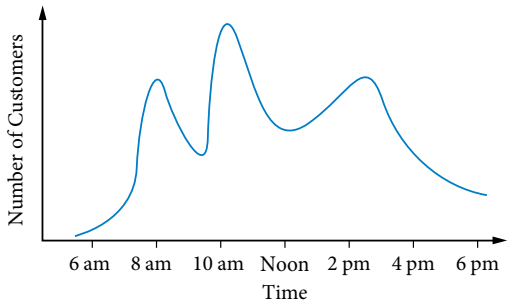
Evikka/Shutterstock.com

5 How can you tell that the data in this box-and-whisker plot is positively skewed?

6 Construct a box-and-whisker plot that shows data that are negatively skewed. PS

7 Construct a dot plot with scores from 1 to 8 that has a bimodal, symmetrical distribution. PS

8 This 'trimodal' distribution represents the number of customers at a city business at different times of the day.



a Why is it called a trimodal distribution?

b What type of business do you think it could be? Give a reason for your answer.

9 This stem plot shows the exam marks of a group of History students. PS

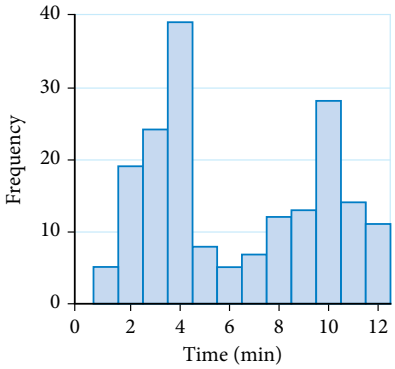
Stem	Leaf
5	3 6 7 8 9
6	2 5 4
7	1 2
8	2
9	0

Key: 5|3| = 53

a Describe the shape of the distribution.

b Change 4 of the exam marks to make the distribution more like a symmetrical distribution. For example, you could change the 59 to 79.

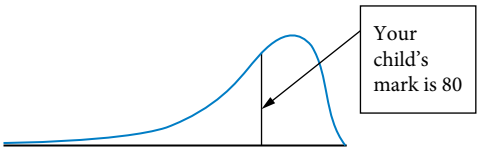
10 This histogram shows the amount of time customers at an appliance store had to wait for items to be brought in from the warehouse.



a Describe the shape of the distribution.

b Suggest a possible reason for the shape of the distribution.

11 Sometimes, test marks in student reports are presented as a graph showing the position of the mark relative to the distribution of the marks of all students. The graph shows Brock's test mark for mathematics compared to the rest of his class. **PS**



Test marks from 45 to 92

a Describe the shape of the distribution.

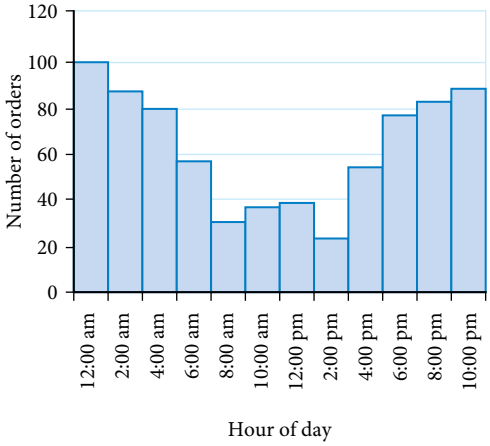
b Was the mathematics test easy or hard? Justify your answer.

c Estimate the mode mark.

d How did Brock perform in the test compared to the rest of the students?

Investigation Is it really bimodal?

This histogram shows the number of online orders a company receives per 2-hour interval over a 24-hour period. The data appears to be bimodal.



1 Construct another graph to display the same data, but start the horizontal axis at 12 pm.

2 Describe the shape of the distribution displayed on the graph you constructed.

3 When you are displaying cyclic data, for example hourly, daily, or monthly data, describe how you can construct a graph that makes the data appear to be bimodal when it is only unimodal.

Chuyuss/Shutterstock.com

Keyword activity

DEFINITIONS

ANSWERS p. 451

1 Describe a back-to-back stem plot and a parallel box plot in your own words. Draw a rough sketch of what they look like.

2 What is a five-number summary?

3 Match the terms **a–e** in the left column with their correct meanings **A–E** in the right column.

a	bimodal	**A**	A rectangular distribution where every value has a similar frequency.
b	skewed	**B**	A distribution with 2 peaks.
c	unimodal	**C**	The values that divide the data into 4 equal parts.
d	quartiles	**D**	A distribution where most values are to the left or right of centre and there is a tail on the side that doesn't have many values.
e	uniform	**E**	A distribution with 1 peak.

Solution to the chapter problem

Problem

Ziad and Adrian are arguing about which local rugby league team is better. The points scored by their favourite teams in each match over the season are listed here:

Eagles: 20, 10, 40, 12, 17, 20, 22, 20, 34, 19, 36, 18, 24, 12, 38, 34, 24, 36, 32, 22, 6, 7, 38, 18

Cougars: 14, 18, 24, 39, 14, 4, 4, 14, 10, 13, 28, 22, 16, 18, 18, 12, 18, 28, 21, 6, 10, 18, 36, 12

a What is the best graph to use to show this data? Construct this graph.

b Which is the better team based on this data? Justify your answer.

c What additional data might you need to know to decide which is the better team?

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?

WHAT?

- To decide on the best graph for this data and draw it.
- To decide which is the better team and justify our choice.
- To decide if any other information might be relevant in deciding the better team.
- We know the scores of both teams for each match of the season.

STAGE 2: SOLVE THE PROBLEM

SOLVE

a For this list of values, the best graph is a stem plot because it keeps all the data and gives the shape of the data.

Eagles		Cougars
7 6	0	4 4 6
9 8 8 7 2 2 0	1	0 0 2 2 3 4 4 4 6 8 8 8 8 8
4 4 2 2 0 0 0	2	1 2 4 8 8
8 8 6 6 4 4 2	3	6 9
0	4	

Key: 0\|1\| = 10

\|3\|6 = 36

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	b From the shape of the values of each team, it looks like the Eagles scored higher than the Cougars. Compare their medians and clusters. The median for the Eagles is 21. The median for the Cougars is 17. The values for the Cougars are clustered in the 10s. The values for the Eagles are spread evenly across the 10s, 20s and 30s. Based on this data, the better team is the Eagles. **c** You would need to know how many matches each team won. It would also help to know how many points were scored *against* each team.
STAGE 3: CHECK THE SOLUTION	
 CHECK	We have answered all parts of the question.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	**a** The best graph is a stem plot because it keeps all the data and gives you the shape of the data. **b** Based on the information given, the Eagles are the better team. **c** It would be useful to know how many games each team won and how many points were scored *against* each team.

12 Test yourself

Comparing data

ANSWERS p. 451

1 EXERCISE 12.1 The following are the Mathematics test results for Manuel's class.

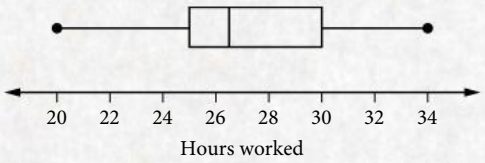
6 4 2 10 16 12 14 12 6

11 9 10 12 13 18 4 10 14

a What is the five-number summary for this data?

b Draw the box plot for this data.

2 EXERCISE 12.1 This box plot summarises the number of hours worked in one week by each employee of Café Coffee.



a What is the median for this data?

b Find the range.

c If there are 36 employees altogether, how many employees worked between 30 and 34 hours?

3 EXERCISE 12.2 The scores of 2 cricket teams in one season of the local competition are listed below:

The Bulls: 83 125 89 113 109 90 127 159 98 140 114 137

The Tigers: 130 144 104 72 139 133 109 97 138 147 126 139

a Draw a back-to-back ordered stem plot for this data.

b How many matches did each team play in one season?

c What is the median for each team?

d Are there any outliers in either set of data? If so, state the outliers.

e Which team is the better team? Justify your answer.

4 EXERCISE 12.3 These are the five-number summaries for the archery scores of Team Magenta and Team Blue.

Magenta: 1 5 7 8 10

Blue: 0 4 7 9 10

a Draw a parallel box plot for these five-number summaries.

b What is the median for each team?

c What is the interquartile range for each team?

d Shane in Team Blue and Adele in Team Magenta both scored 9. Who has performed better in relation to their own team? Justify your answer.

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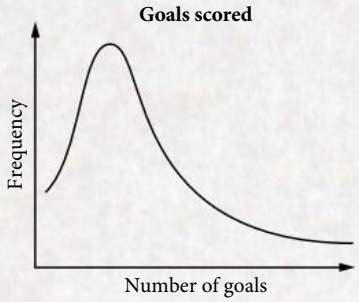
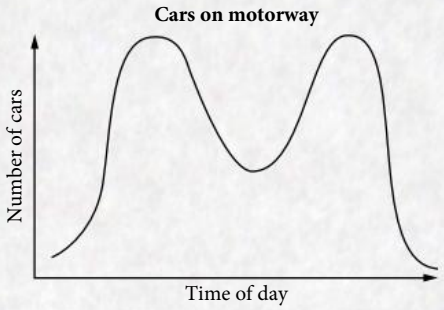
5 EXERCISE 12.4 What type of distribution is shown in each graph? Choose from symmetrical, positively skewed, negatively skewed, uniform and bimodal.

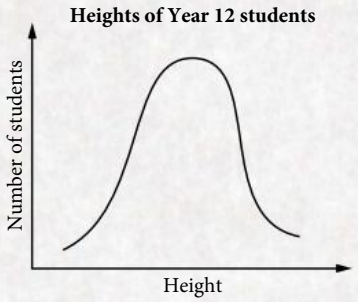
a



b



c



6 EXERCISE 12.4 Sketch a graph for each distribution described below. Remember to label the axes.

a The percentage marks students scored on an easy assessment task – the results are negatively skewed with most scoring between 80 and 90 percent.

b The masses of all Year 12 students in Queensland are distributed symmetrically.

c When a card is chosen randomly from a deck of cards 500 times, the suit of the card (diamonds, hearts, clubs, spades) has a uniform distribution.

Practice set 3

ANSWERS p. 452

You will need an online compound interest calculator for Question **6** of Section A and Questions **2** and **11** of Section B.

Section A Multiple choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

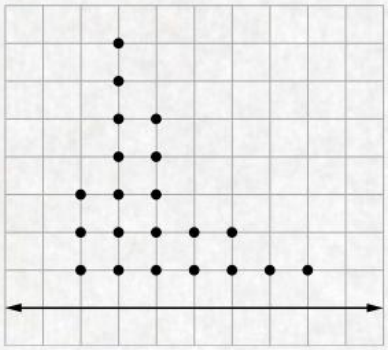
1 EXERCISE 11.1 Calculate the simple interest earned on $800 at 4% p.a. for 3 years.

A $12 **B** $24 **C** $32 **D** $96

2 EXERCISE 10.1 The favourite party food of a group of 3-year-old children was recorded. What is the only statistical measure that can be found for this data?

A mean **B** median **C** mode **D** range

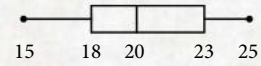
3 EXERCISE 12.4 Describe the shape of the data in this graph.



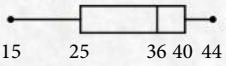
A symmetric **B** bimodal **C** negatively skewed **D** positively skewed

4 EXERCISE 12.1 Which box plot has an interquartile range of 10?

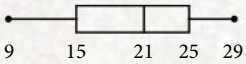
A



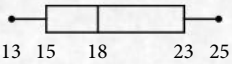
B



C



D



5 EXERCISE 10.5 Faizal records his golf scores for the last 10 weeks.

75 80 85 73 77 81 76 73 84 76

Calculate the standard deviation for the scores.

A 4.1 **B** 4.3 **C** 10 **D** 78

6 EXERCISE 11.3 Jane invests $4500 at 3.5% p.a. compounding annually for 8 years. Calculate the amount of interest she earns.

A $1260 **B** $1425.64 **C** $5925.64 **D** $9000

7 EXERCISE 10.3 Calculate the interquartile range for this set of data.

10 3 5 8 4 2 10

A 3 **B** 5 **C** 7 **D** 8

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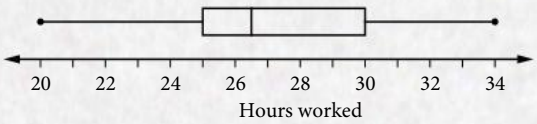
8 EXERCISE 11.2 What is the outlier for the data shown in this box plot?



A 34

B 20

C There isn't an outlier.

D An outlier cannot be conclusively determined from this particular box plot.

Section B Short answer questions

1 EXERCISE 10.2 This data set shows the number of goals scored in each game of the season by a local soccer team. Which score is an outlier?

2 1 0 0 7 2 3 2 4
3 2 9 1 2 1 2 4 3

2 EXERCISE 11.4 Damian invested his annual bonus of $5300 for 4 years at 2.7% p.a. compounding annually.

a Use an online compound interest calculator to calculate the future value of his investment at 4 years.

b How much interest will Damian's investment earn in 4 years?

3 EXERCISES 10.1, 10.3 This data shows the maximum temperature over 10 days in Roma.

23 14 20 21 9 20 16 19 18 17

For this data, find:

a the mean

b the mode

c the median

d the range

e the upper quartile

f the lower quartile

g the interquartile range.

4 EXERCISE 12.1 The heights in centimetres of the 23 students in a Year 12 Maths class are shown.

170, 165, 159, 167, 183, 174, 185, 174, 168, 152, 161, 163,
163, 176, 186, 169, 170, 149, 169, 172, 186, 151, 173

a Find the median height.

b Find the upper and lower quartiles of these heights.

c Calculate the interquartile range.

d State the five-number summary.

e Draw a box plot for this data.

5 EXERCISE 10.5 Evan recorded the number of magazines bought by customers at his newsagency for one hour. The results are shown in the table.

Number of magazines	Frequency
0	6
1	7
2	11
3	3
4	1
5	1
6	1
Total	**30**

For this data, calculate, correct to 2 decimal places:

a the mean

b the standard deviation.

6 EXERCISE 12.4 Sketch a graph for each distribution described below. Remember to label the axes.

a A die is rolled 300 times and the outcomes are uniformly distributed.

b The percentage marks students scored on an extremely difficult test – the results are positively skewed with most scoring between 30 and 40 per cent.

7 EXERCISE 11.2 Use the compound interest recurrence formula to calculate the interest Jaimie will earn when she invests $10 000 at 2.5% p.a. annually compounding for 3 years.

8 EXERCISE 12.2 Year 12 students completed 2 tests for their assessment. Both tests were marked out of 50. These are their marks.

English:	48	19	17	45	39	27	40	41	30	23
	38	32	30	27	31	34	36	20	25	22
	40	41	30	46	27	34	31	23	8	38
Maths:	39	30	20	47	35	35	27	36	34	44
	11	11	47	31	28	32	3	38	7	28
	29	21	32	46	19	50	31	49	17	23

a Draw a back-to-back stem plot for this data.

b Find the range for each test.

c Are there any outliers in either class? If so, state the outliers.

9 EXERCISE 12.3

a Find the five-number summary for each test in Question **8**.

b Draw a parallel box plot for these 2 sets of data.

10 EXERCISE 10.6 Draw:

a a dot plot with scores clustered around 23 to 25

b a histogram with 2 clusters and one gap

c a stem plot with scores clustered in the 20s, with 2 gaps.

11 EXERCISES 11.4, 11.6 Ozcan invests $8400 for 18 months at 2.4% p.a. compounding monthly. Use an online calculator to find the value of the investment at the end of 18 months.

12 EXERCISES 11.3, 11.4 Leanne is considering 2 possible investments for the $40 000 she has inherited from her aunt. She plans to invest the money for 6 years until she plans to travel overseas.

	Compounding period	Interest rate
Nelson Community Bank	Monthly	4.8% p.a.
Cengage Central Bank	Annually	5%

Which investment would you recommend? Use calculations to justify your answer.

Chapter problem

A letter of the alphabet can be formed by graphing these 4 lines on a number plane. What letter is it?

$y = 2x + 2$ $y = -2x + 2$ $y = 3x - 8$ $y = -3x - 8$

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CHAPTER 13

GRAPHS, DATA AND LOANS: BIVARIATE GRAPHS

GRAPHING LINES

What will we do in this chapter?

- Plot points on the Cartesian plane (number plane)
- Generate tables of values for linear functions
- Graph linear functions and apply them in real life

How are we ever going to use this?

- To represent relationships graphically
- To model real-world situations with algebra and a graph

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Syllabus coverage

UNIT 4, TOPIC 1: BIVARIATE GRAPHS

Cartesian plane

- Demonstrate familiarity with Cartesian coordinates in two dimensions by identifying and plotting points on the Cartesian plane.
- Generate a table of values for a given linear function, including for negative values of x.
- Graph a linear function from a table of values with pencil and paper and with graphing software.

Videos (3):

13.3 Graphing linear functions
• Gradient and y-intercept of a line

13.4 Linear modelling

Skillsheet (1):

13.3 Graphing linear equations

Worksheets (7):

13.1 Number plane grid paper

13.2 Formula frolics

13.3 Graphing points and lines • Graphing linear functions • A page of number planes • Drawing gradients • Gradient and y-intercept

Puzzles (2):

13.3 Graph gradients • Gradient goat

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

Terminology

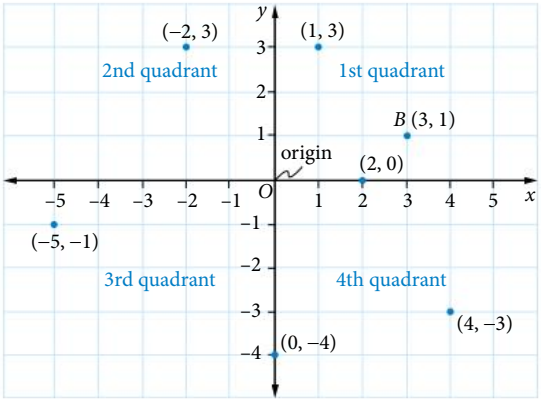
Cartesian plane	gradient	linear function	modelling
origin	quadrant	x-axis	y-axis
y-intercept			

13.1 The number plane

The **Cartesian plane**, another name for **number plane**, is a grid made from a horizontal number line called the ***x*-axis**, and a vertical number line called the ***y*-axis**, as shown. The centre of the number plane is called the **origin**.

Points on the number plane are described using a pair of numbers called **coordinates**, where the first number goes across from the origin and the second number goes up/down. For example, the point (3, 1) is 3 across and 1 up from the origin, labelled point *B* on the diagram. The origin has coordinates (0, 0).

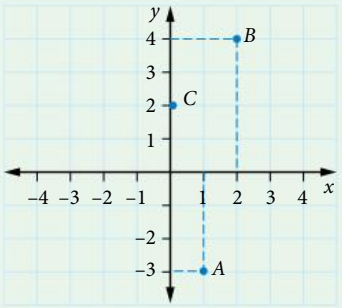
The number plane is divided into 4 regions called **quadrants**.

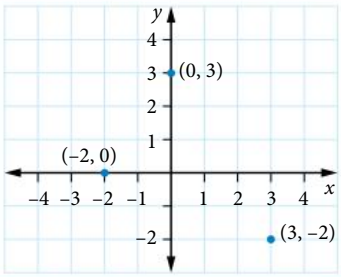


EXAMPLE 1 Coordinates on the number plane

a What are the coordinates of points *A*, *B* and *C* on the graph?

b Plot the points (3, −2), (0, 3) and (−2, 0) on a graph.



Steps	Working
a From the point, follow in a straight line to the *x*-axis to find the *x* value. From the point, follow in a straight line to the *y*-axis to find the *y* value.	Point *A* has coordinates (1, −3). *B* is (2, 4) and *C* is (0, 2).
b To plot (3, −2), find 3 on the *x*-axis, then go down 2. To plot (0, 3), find 0 on the *x*-axis, then go up 3. The point (0, 3) is on the *y*-axis. To plot (−2, 0), find −2 on the *x*-axis, then stay on the *x*-axis. The point (−2, 0) is on the *x*-axis.	

A set of points can be written in a table of values. The coordinates of each point are shown in a table column.

EXAMPLE 2 Plotting a table of values

Plot the points shown in this table of values.

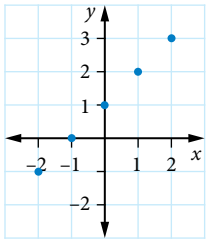
x	−2	−1	0	1	2
y	−1	0	1	2	3

Steps

From the table, the first point is $(-2, -1)$. The other points are $(-1, 0)$, $(0, 1)$, $(1, 2)$ and $(2, 3)$.

Remember: first number across, second number up/down.

Working



Memorise this: x is 'across' because x is 'a cross'.

EXERCISE 13.1 The number plane

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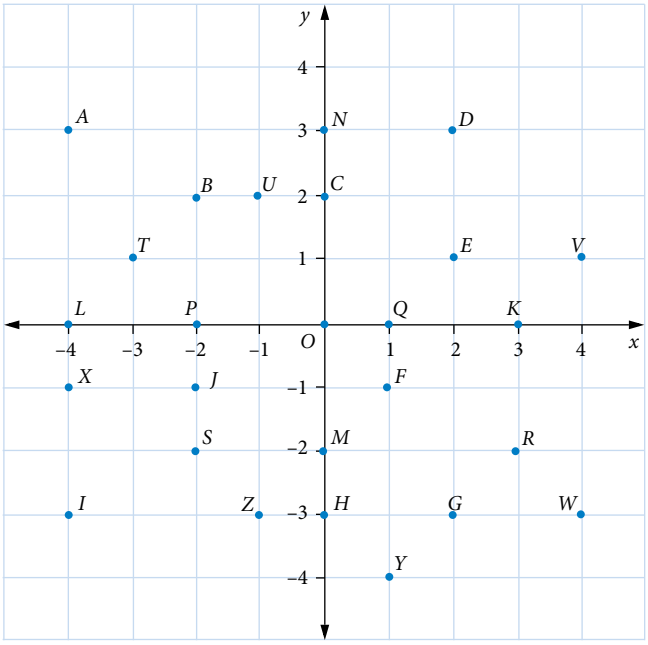
1 EXAMPLE 1 Write the coordinates of each of the points A to Z.



2 **a** Draw a Cartesian plane with both axes (lines) extending from −6 to 6.

b Plot these points on the number plane.

$A(3, 1)$	$B(-4, 3)$	$C(-3, 4)$	$D(-2, -2)$
$E(0, -2)$	$F(1, -5)$	$G(4, -4)$	$H(-3, 0)$
$I(-6, 0)$	$J(-1, -6)$	$K(-3, -4)$	$L(0, 5)$
$M(6, 2)$	$N(-3, 5)$	$P(5, -5)$	$Q(-1, 6)$

Worksheet
Number plane grid paper

3 List all of the points from Question **2** that are:

a in the 1st quadrant

b in the 2nd quadrant

c in the 3rd quadrant

d in the 4th quadrant

e on the border of 2 quadrants.

4 In which quadrant would you find each point?

a $(3, -5)$ **b** $(-2, -4)$ **c** $(-8, 1)$

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5 Copy this table and complete it by placing a + or a − sign in the blank spaces.

Points in:	*x*-coordinate	*y*-coordinate
1st quadrant	+	
2nd quadrant	−	
3rd quadrant		
4th quadrant		

6 EXAMPLE 2 Plot the points given in this table of values on a number plane.

x	0	1	2	3	4
y	−2	−1	0	1	2

You need to decide what numbers to use on the *x*- and *y*-axes. Look at the table you are given. Check what numbers you need on each axis.

7 For each table, plot the points on a number plane.

a

x	−6	−3	0	3	6
y	−2	−1	0	1	2

b

x	−5	−2	1	3	4
y	3	0	−3	−5	−6

c

x	4	2	0	−1	−2
y	0	2	4	5	6

d

x	−1	0	1	2	3
y	−2	0	2	4	6

Investigation — Changing points of view

In this investigation, we will see what happens when we change coordinates in different ways.

1 Draw a number plane with an *x*-axis from −15 to 15 and a *y*-axis from −20 to 20. Use 0.5 cm = 1 unit.

2 Plot these points and join them in the given order:
(3, 2) (11, 14) (9, 13) (10, 19) (5, 10) (6, 11) (3, 2)
You have drawn a lightning bolt in the first quadrant.

3 What happens to the lightning bolt if you multiply all the *x* values by −1? Rewrite the points with these new coordinates. Plot these points and join them as before. Describe in words what has happened to the lightning bolt.

4 Now investigate what happens if you multiply all the *y* values by −1. Rewrite the points, plot them and join them as before. Describe in words what has happened to the lightning bolt.

5 How would you place the lightning bolt in the other quadrant? Show that what you have said will actually work.

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Worksheet
Formula frolics

13.2 Tables of values

We can complete a table of values when we are given an equation (formula) relating the x and y values.

EXAMPLE 3 Completing tables of values

Complete each table of values given the equation.

a $y = 4x + 3$

x	−2	−1	0	1	2	3
y						

b $y = 10 - 2x$

x	−2	−1	0	1	2	3
y						

Steps	Working
a Substitute $x = -2$ in the equation and write the answer in the table under $x = -2$.	$y = 4 \times (-2) + 3$ $= -8 + 3$ $= -5$

x	−2	−1	0	1	2	3
y	−5					

Steps	Working
Repeat with the other 5 values of x in the table.	When $x = -1$, $y = 4 \times (-1) + 3 = -1$ When $x = 0$, $y = 4 \times (0) + 3 = 3$ When $x = 1$, $y = 4 \times (1) + 3 = 7$ When $x = 2$, $y = 4 \times (2) + 3 = 11$ When $x = 3$, $y = 4 \times (3) + 3 = 15$

x	−2	−1	0	1	2	3
y	−5	−1	3	7	11	15

Steps	Working
b Substitute $x = -2$ in the equation and write the answer in the table under $x = -2$.	$y = 10 - 2 \times (-2)$ $= 10 - (-4)$ $= 10 + 4$ $= 14$

x	−2	−1	0	1	2	3
y	14					

Steps	Working
Repeat with the other 5 values of x in the table.	When $x = -1$, $y = 10 - 2 \times (-1) = 12$ When $x = 0$, $y = 10 - 2 \times 0 = 10$ When $x = 1$, $y = 10 - 2 \times 1 = 8$ When $x = 2$, $y = 10 - 2 \times 2 = 6$ When $x = 3$, $y = 10 - 2 \times 3 = 4$

x	−2	−1	0	1	2	3
y	14	12	10	8	6	4

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EXERCISE 13.2 Tables of values

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1 EXAMPLE 3 Copy and complete each table of values for the given equation.

a $y = 2x + 3$

x	−5	−2	3	6	11
y					

b $y = 3x - 5$

x	4	2	5	7	−3
y					

c $y = \frac{1}{2}x$

x	−4	−2	3	6	9
y					

d $y = 20 - 2x$

x	−2	−1	4	7	10
y					

2 Copy and complete the table for the relationship $y = 12 - x$.

x	1	2	5	9	10
y					

3 Match each equation to its correct table of values.

a $y = 3x$ **b** $y = 3 + x$ **c** $y = x^2$

d $x \times y = 12$ **e** $y = 6 - x$ **f** $y = \frac{1}{2}x + 1$

A

x	1	2	3	4	6
y	1	4	9	16	36

B

x	1	2	−3	4	6
y	4	5	0	7	9

C

x	1	2	3	−4	6
y	12	6	4	−3	2

D

x	0	2	8	−4	6
y	1	2	5	−1	4

E

x	1	2	3	4	6
y	5	4	3	2	0

F

x	1	−2	3	4	−6
y	3	−6	9	12	−18

4 Find an equation for each table of values. PS

a

x	1	2	3	4	5
y	2	3	4	5	6

b

x	1	2	3	4	5
y	0.1	0.2	0.3	0.4	0.5

c

x	1	2	3	4	5
y	4	8	12	16	20

d

x	1	2	3	4	5
y	1	4	9	16	25

e

x	1	2	3	4	5
y	9	8	7	6	5

f

x	1	2	3	4	6
y	24	12	8	6	4

g

x	1	2	3	4	5
y	6	5	4	3	2

h

x	1	2	3	4	5
y	2	4	6	8	10

5 Construct your own table of values for the equation $x - y = 1$.

Skillsheet
Graphing linear equations

Worksheets
Graphing points and lines

Graphing linear functions

A page of number planes

13.3 Graphing linear functions

In Year 11, we used tables of values to graph straight lines.

The equation of a straight line is called a **linear function**.

Linear means 'of a line'.

Graphing linear functions

To graph a linear function:

1 Complete a table of values for the equation.

2 Plot the points from the table of values on a number plane.

3 Rule a straight line through the points.

4 Label the line with its equation.

You can choose any numbers you want for x, but you need to make sure the points will fit on your graph and be easy to calculate. It is easiest to choose whole numbers close to 0.

Video
Graphing linear functions

EXAMPLE 4 Graphing a straight line

Graph the linear function $y = x - 3$.

Steps	Working
Draw a table and choose some x values.	

x	−1	0	1	2	3	4
y						

Calculate the y values to complete the table.

$y = -1 - 3 = -4$

$y = 0 - 3 = -3$

$y = 1 - 3 = -2$

$y = 2 - 3 = -1$

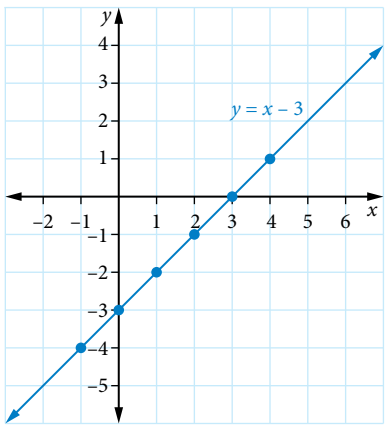
$y = 3 - 3 = 0$

$y = 4 - 3 = 1$

Do you notice a pattern with the y values?

x	−1	0	1	2	3	4
y	−4	−3	−2	−1	0	1

Draw a set of axes and plot the points. Rule a straight line through the points, place arrows at each end and label the line with its equation.



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Gradient and y-intercept of a line

In Year 11, Chapter 13, *Practical graphs*, we also learned about the gradient and y-intercept of a line, and how to find the gradient and y-intercept of a line from its equation.

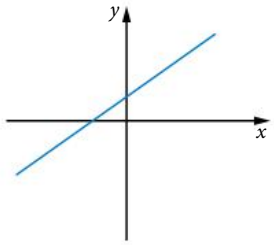
Gradient and y-intercept of a line

- The **gradient** of a line measures how steeply the line goes up or down.
- The ***y*-intercept** of a line is the value where the line crosses the y-axis.
- The equation of a line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept.
- If the equation of a line is $y = -2x + 1$, then its gradient is -2 and its y-intercept is 1.
- The gradient -2 is m, the number in front of the x in the equation.
- The y-intercept 1 is the constant (c, the number on its own) in the equation.

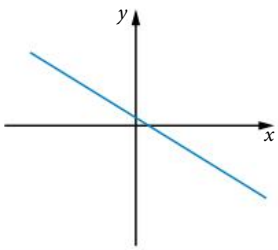
A **positive gradient** means the line goes up from left to right.

A **negative gradient** means the line goes down from left to right.

Positive gradient



Negative gradient



Video Gradient and y-intercept of a line

Worksheet Drawing gradients

Gradient and y-intercept

Puzzles Graph gradients

Gradient goat

EXAMPLE 5 Gradient and y-intercept

Determine the gradient and y-intercept of the line with equation:

a $y = 2x + 4$ **b** $y = -3x$ **c** $y = 7 - x$

Steps	Working
a The gradient is m, the number in front of the x.	$y = 2x + 4$
The y-intercept is c, the constant or the number on its own.	gradient $= 2$
	y-intercept $= 4$
b When there is no constant in the equation, the y-intercept is 0.	$y = -3x$
	gradient $= -3$
	y-intercept $= 0$
c $y = 7 - x$ can be rewritten as $y = -x + 7$	$y = 7 - x$
Remember: $-x = -1x$, so $m = -1$, $c = 7$.	gradient $= -1$
	y-intercept $= 7$

EXERCISE 13.3 Graphing linear functions

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1 EXAMPLE 4 Graph each linear function.

a $y = x + 2$ **b** $y = -x + 1$ **c** $y = 2x - 1$ **d** $y = 3x$

e $y = 2 - x$ **f** $y = \frac{x}{3}$ **g** $y = 1 - 3x$ **h** $y = \frac{x}{4} + 2$

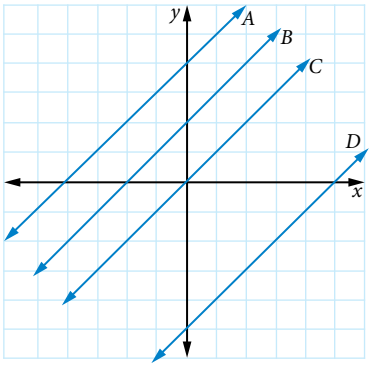
2 EXAMPLE 5 Find the gradient and y-intercept of the line with equation:

a $y = 2x + 4$ **b** $y = -4x - 5$ **c** $y = 4 - x$

d $y = x$ **e** $y = \frac{x}{5} - 3$ **f** $y = \frac{x}{2} + 5$

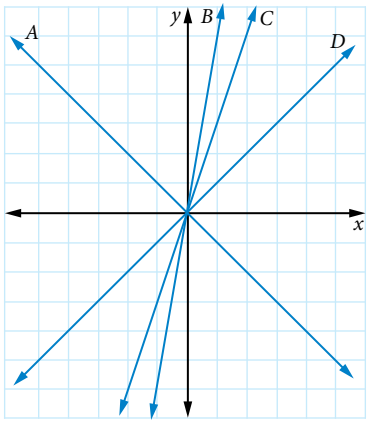
Remember: $\frac{x}{5} = \frac{1}{5}x$

3 Match each linear equation to its graph. PS



a $y = x$ **b** $y = x + 2$ **c** $y = x - 5$ **d** $y = x + 4$

4 Match each linear equation to its graph. PS



a $y = x$ **b** $y = 3x$ **c** $y = -x$ **d** $y = 6x$

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

TECHNOLOGY — Using technology to graph linear functions

To graph straight lines, we can use technology such as

- spreadsheets
- graphing software (GeoGebra or Desmos)
- graphing websites such as WolframAlpha

1 Graph the line $y = 2x - 3$ using:
a a spreadsheet **b** graphing software **c** a website
2 Which technology did you find the easiest to use? Why?
3 Were there any difficulties with any of the technologies? If so, what were they?
4 Use your preferred technology to graph a line that:
a has a positive gradient and y-intercept
b has a negative gradient
c goes through the origin
d goes through $(2, 3)$

Present your line graphs to the class.

13.4 Applying linear functions

Linear functions can be used to model many real-world situations, especially in business and finance, such as costs, profits and income. This is called **linear modelling**.

EXAMPLE 6 Linear modelling

Nabil's taxi charges \$4.30 flagfall and \$2.17 per kilometre.

a Write a linear function for the amount Nabil charges, \$$C$, when passengers travel d km.

b Graph this linear function.

c What is the gradient and vertical intercept of the graph and what do these values represent?

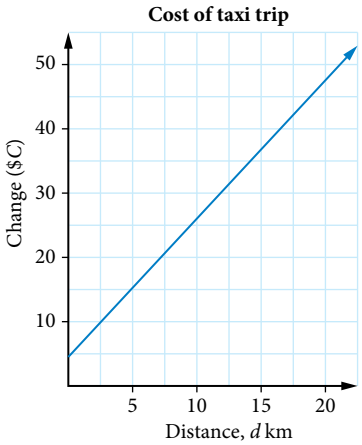
'Flagfall' is the initial charge, before any kilometres are travelled.

You say '**vertical intercept**' here, not 'y-intercept', because the equation involves d and C, not x and y.

Video
Linear modelling

Steps	Working
a Charge = flagfall + \$2.17 per km travelled	$C = 4.30 + 2.17 \times d$ $= 2.17d + 4.3$
b Complete a table of values, then graph the line with d on the horizontal axis.	$C = 2.17d + 4.3$
c Read off the gradient and vertical intercept from the equation.	For $C = 2.17d + 4.3$, the gradient is 2.17 and the vertical intercept is 4.3. The gradient is the charge per km (\$2.17). The vertical intercept is the flagfall (\$4.30).

Distance, d km	0	5	10	15	20
Charge, \$$C$	4.3	15.15	26	36.85	47.7



Jeff Greenberg/Universal Images Group/Getty Images

EXERCISE 13.4 Applying linear functions

ANSWERS p. 454

1 EXAMPLE 6 Hawkes Landscaping supplies garden soil. The charge is $60 plus $28 per tonne to deliver up to 25 tonnes of soil.

a Copy and complete this table of values.

Number of tonnes, n	0	1	2	3	4	5	10	15	20	25
Cost of soil, $\$C$										

b Write a linear function for the cost $\$C$ of n tonnes of soil.

c Graph this function for up to 25 tonnes of soil.

d What is the vertical intercept of the line?

e What physical quantity does the vertical intercept represent?

f What is the gradient of the line and what does it represent?

g Why do you think the company limits this pricing system to deliveries up to 25 tonnes?

h If you buy 40 tonnes of soil, you will need 2 deliveries. Calculate the cost of buying 40 tonnes of soil.

i Does it make any difference to the cost in part **h** if you have deliveries of 25 tonnes and 15 tonnes or 2 deliveries of 20 tonnes? Show calculations to support your answer.

2 Mira sells hot chips at her fast food store. Each day, the cooking oil costs $42 and each container of chips costs her 75c to make.

a Copy and complete this table.

Number of containers, n	0	10	20	30	40	50	60	100	200
Cost of making chips, $\$C$	42	49.5							

b Write a linear function to calculate the cost of making n containers of chips in one day.

c Use the function to calculate the cost of making 170 containers of chips.

d Yesterday, the cost of making chips was $149.25. How many containers of chips did Mira make?

e Construct a graph showing Mira's daily cost for producing n containers of chips.

f Find the vertical intercept and explain what this value represents.

g Find the gradient and explain what this value represents.

Peter Lopeman/Alamy Stock Photo

3 Zoe is a personal assistant who earns $20 per hour.

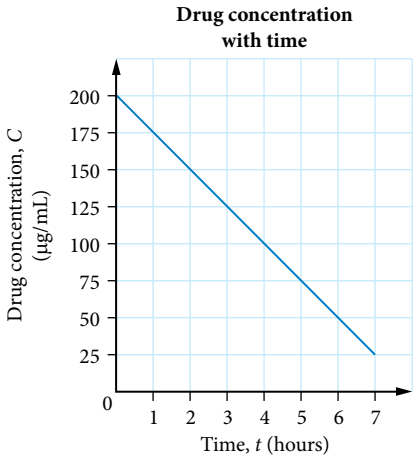
- **a** Explain why the equation $P = 20t$ represents the pay, $\$P$, Zoe receives for working t hours.
- **b** Draw a graph for up to 7 hours work.
- **c** What is the gradient of the line?
- **d** In this context, what does the gradient represent?
- **e** Explain why the vertical intercept is zero.

4 Mick is an electrician who charges $60 per hour.

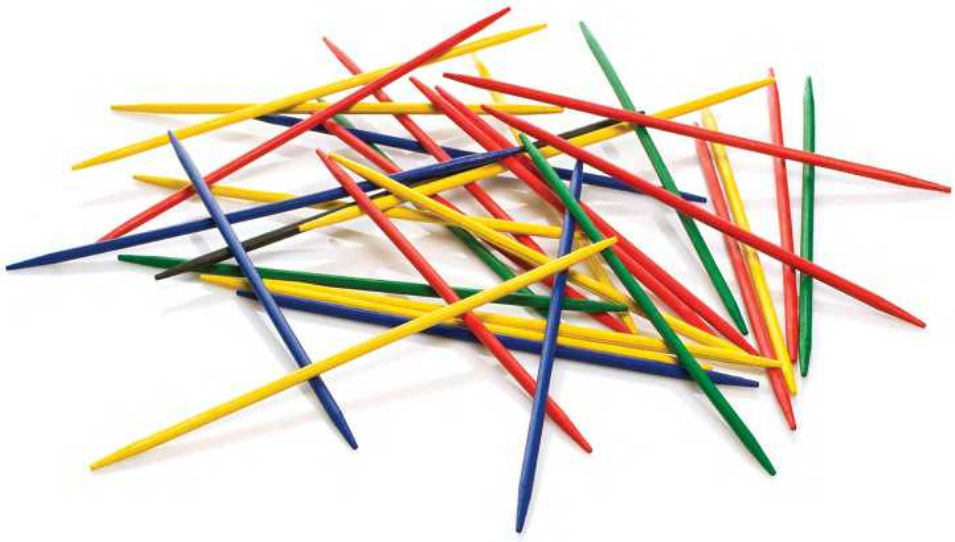
- **a** What is the equation for calculating Mick's charges?
- **b** Draw a graph for up to 5 hours work.
- **c** What is the gradient of this graph and what does it represent?

5 The concentration of a particular drug in a person's body decreases as time passes. This is represented by the graph shown, which has equation $C = -25t + 200$.

μg means micrograms or one-millionth of a gram



- **a** What is the gradient of the line?
- **b** What does the gradient represent?
- **c** Find the vertical intercept of the line.
- **d** What does the vertical intercept represent?
- **e** When will there be no drug remaining in the body?



Tikta Alik/Shutterstock.com

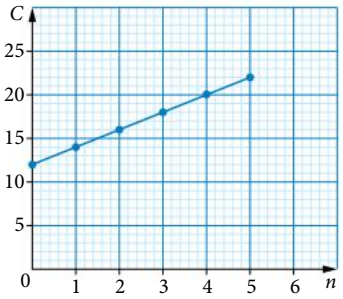
6 Match each linear function to its correct graph. PS

a $C = \frac{1}{2}n + 6$ **b** $C = 3n + 6$ **c** $C = \frac{1}{2}n + 12$

d $C = 2n + 12$ **e** $C = -\frac{1}{2}n + 12$ **f** $C = -2n + 12$

Remember: The gradient of a line is a measure of its steepness.

A



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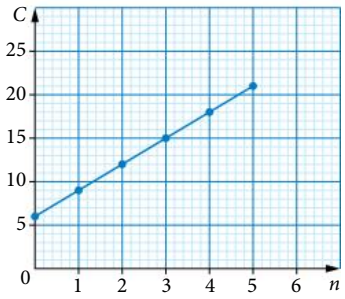
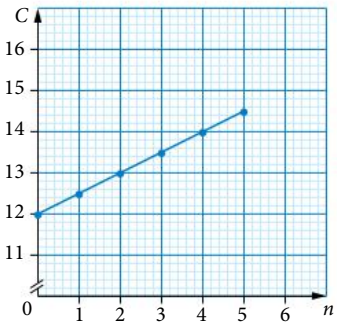


C



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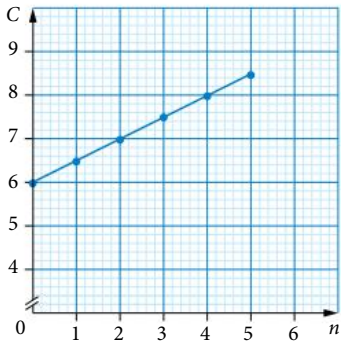
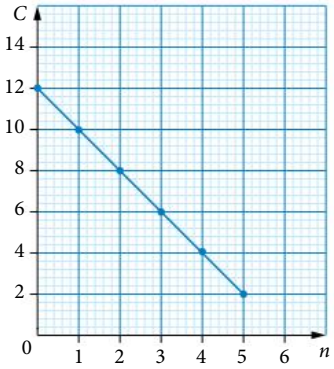


E



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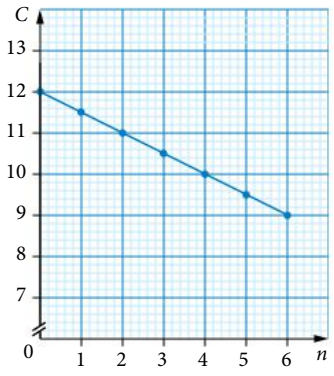

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Keyword activity

CHAPTER SUMMARY

ANSWERS p. 455

Use the listed words to copy and complete the summary of this chapter below.

axes	front	gradient	linear
ordered pair	own	origin	quadrants
table of values	*x*-axis	*y*-axis	*y*-intercept

The number plane is made up of two [1]_____, the [2]_____ and the [3]_____. The [4]_____ is where the two axes cross each other. The axes divide the number plane into four [5]_____. Points on the number plane are located by an [6]_____ _____.

When we are given an equation to graph on the number plane, we first need to complete a [7]_____ __ _____. We use this to plot the points. When the points form a straight line, we call the equation a [8]_____ function. The [9]_____ of a line measures how steeply the line goes up or down and the [10]_____ is where the line crosses the vertical axis.

We can find the gradient and the *y*-intercept from the equation of the line $y = mx + c$, such as $y = 2x + 7$. The gradient is the number in [11]_____ of the x. The *y*-intercept is the constant, the number on its [12]_____.

We often use linear functions to model practical situations.

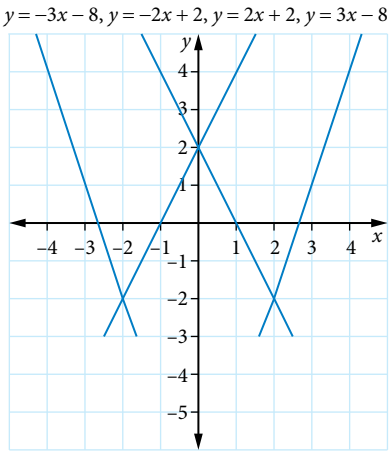
Solution to the chapter problem

Problem

A letter of the alphabet can be formed by graphing these 4 lines on a number plane.
What letter is it?

$y = 2x + 2$ $y = -2x + 2$ $y = 3x - 8$ $y = -3x - 8$

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To find the letter of the alphabet formed by 4 lines. We are given the equations of the lines.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	We need to graph all 4 lines on one number plane. We can use tables of values or technology.  We can then outline the letter formed. It is W.
STAGE 3: CHECK THE SOLUTION	
 CHECK	2 lines have a negative gradient and we have 2 lines going down from left to right. 2 lines have a positive gradient and we have 2 lines going up from left to right. The y-intercepts of the 4 lines are −8, 2, 2, −8. The lines have made a letter.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	The letter W is formed by the 4 lines.

13 Test yourself

Graphing lines

ANSWERS p. 455

1 EXERCISE 13.1 Plot each point on a number plane.

a $P(1,-4)$ **b** $Q(-3,-1)$ **c** $R(-1,2)$ **d** $S(1,3)$

e $T(0,4)$ **f** $U(2.5,-2)$ **g** $W(-2,0)$ **h** $Y(-3,1.5)$

2 EXERCISE 13.1 Which points from Question **1** are:

a on the y-axis? **b** in the 2nd quadrant?

c on the x-axis? **d** in the 4th quadrant?

3 EXERCISE 13.2 Copy and complete each table of values.

a $y = x + 3$

x	-1	0	1	2	3
y					

b $y = 2x - 7$

x	-2	-1	2	4	5
y					

c $y = 5 - x$

x	-3	-1	0	2	4
y					

4 EXERCISE 13.2 Find the equation for each table of values.

a

x	-1	0	1	2	3
y	-3	-2	-1	0	1

b

x	1	2	3	4	5
y	2	5	8	11	14

c

x	-2	-1	0	1	3
y	12	11	10	9	7

5 EXERCISE 13.3 Graph each linear function on a number plane.

a $y = 2x - 2$ **b** $y = -x + 3$ **c** $y = \frac{x}{2} - 2$

6 EXERCISE 13.3 Find the gradient and y-intercept of each linear function in Question **5**.

7 EXERCISE 13.4 Alyssa sells cups of coffee for \$4 each.

a Copy and complete this table of values.

Coffees sold, n	0	5	10	15	20
Sales, \$$S$					

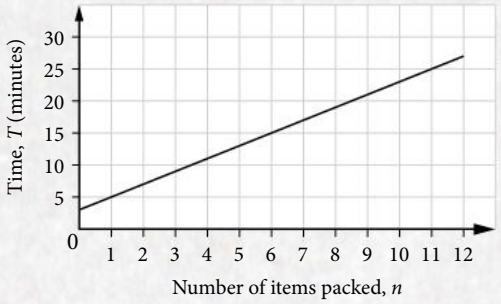
b Write an equation relating S, the dollars Alyssa receives, and n, the cups of coffee sold.

c Construct a graph of Alyssa's sales.

d What is the vertical intercept and what does this value represent?

e What is the gradient of the line and what does the gradient represent?

8 EXERCISE 13.4 Nikolai packs glass items into a cardboard box. He has to assemble the box before he can pack it. The graph shows the time, T minutes, it takes him to assemble one box and pack n glass items in it. The equation of the line is $T = 2n + 3$.



a How long does it take Nikolai to assemble a box before he starts to pack it?

b What is the gradient of the line?

c What physical amount does the gradient represent?

d Calculate the time it takes Nikolai to assemble a box and pack 15 glass items in it.

e How many glass items can Nikolai pack in a new box in 19 minutes?

Chapter problem

A consumer association investigated the quality and price of 8 pairs of gym shoes labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10. This table shows the results.

	A	B	C	D	E	F	G	H
Price	$320	$280	$260	$240	$180	$180	$80	$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?

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CHAPTER 14

GRAPHS, DATA AND LOANS: BIVARIATE GRAPHS

SCATTERING THE DATA

What will we do in this chapter?

- Draw scatterplots for bivariate data, for example, a person's height and shoe size
- Identify the dependent and independent variables in bivariate data
- Describe the association between the data as being positive or negative, linear or non-linear, strong, moderate or weak
- Consider whether one variable causes the other variable

How are we ever going to use this?

- When analysing data to determine whether 2 variables are related, such as the driving speed of a car and the amount of petrol used
- When using physical data to design personal training plans, such as those used by fitness industry professionals
- The police and security industry analyse data about criminals to predict and prevent crime

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Syllabus coverage

UNIT 4, TOPIC 1: BIVARIATE GRAPHS

Bivariate scatterplots

- Construct a scatterplot using a given dataset.
- Describe the patterns and features of bivariate data.
- Describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).

Line of best fit

- Identify dependent and independent variables.
- Distinguish between causality and correlation through examples [complex].

Worksheets (4):

14.1 A page of scatterplots • 2 mm grid paper

14.1, 14.2 Height vs shoe size • Body measurements

Spreadsheet (1):

14.4 Year 12 data

Puzzles (2):

14.4 Data crossword

Keyword activity Scattered find-a-word

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

association	bivariate	causal relationship	dependent variable
independent variable	scatterplot		

14.1 Scatterplots

Bivariate data is data with 2 variables. For example, you might collect data on the height and weight of people. Bivariate data can be graphed on a **scatterplot**.

One variable is the **independent variable** and is graphed on the **horizontal axis**.

The other variable is the **dependent variable** and is graphed on the **vertical axis**.

We can look at the scatterplot to see if it has any of the following features:

- there is a pattern to the points
- as one variable increases, the other variable increases (or decreases)
- there are groups of points (clusters)
- most of the points are together but a few are out on their own.

Worksheets
A page of scatterplots
Height vs shoe size
Body measurements

EXAMPLE 1 Drawing scatterplots

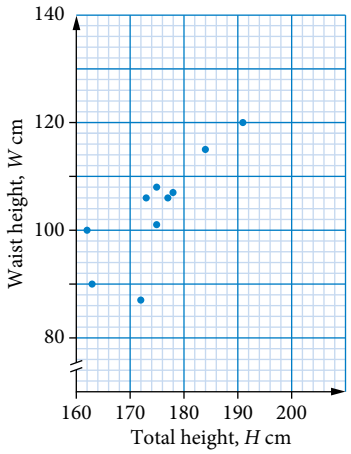
10 students had their heights and waist heights (above the ground) measured.

Total height, H cm	175	177	178	184	162	172	173	191	163	175
Waist height, W cm	101	106	107	115	100	87	106	120	90	108

a Graph this bivariate data in a scatterplot.

b Which is the independent variable, and which is the dependent variable?

c **Comment** on the features of the scatterplot.

Steps	Working
a The total height will go on the horizontal axis. The waist height will go on the vertical axis.	
b The independent variable is on the horizontal axis.	The independent variable is total height. The dependent variable is waist height.
c Describe any patterns.	As total height increases, waist height increases.

EXERCISE 14.1 Scatterplots

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Worksheet
2 mm grid paper

Graph paper is required for this exercise. Keep your scatterplots to use in later exercises and in Chapter 16, *Fitting the data*.

1 EXAMPLE 1 This table shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

a Graph this bivariate data in a scatterplot.

b Which is the independent variable, and which is the dependent variable?

c Comment on the features of the scatterplot.

2 Matthew asked a group of 11 students how many hours per week they spent playing sport and playing video games. He recorded the results in a table, shown below.

Sport	6	2	10	4	7	6	10	4	7	5	3
Video games	10	1	0	5	2	12	0	1	3	2	4

a Present this bivariate data in a scatterplot.

b Which is the independent variable, and which is the dependent variable?

c Comment on the features of the scatterplot.

3 Simone measured the heights and arm spans of a group of 10 senior students.

Height, H cm	170	195	181	181	166	200	163	162	183	167
Arm span, A cm	171	186	187	178	165	160	147	143	115	169

a Construct a scatterplot to show this bivariate data.

b Which is the independent variable, and which is the dependent variable?

c What does the scatterplot show?

4 This table shows the normal resting pulse of a sample of students and the time it takes each of them to swim 50 m.

Resting pulse (beats/min)	42	70	64	62	55	60	50	72	80
Swimming time (s)	30	48	50	43	40	45	36	49	59

a Construct a scatterplot for this set of data.

b Comment on the features of the scatterplot.

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5 This table shows the yearly sales of CD albums and digital albums in the early 2000s. The data is in thousands.

CD albums	46 174	49 818	44 045	38 659	39 529	33 114	30 223	27 356	14 226
Digital albums	91	418	788	1322	2279	3301	4818	6838	7377

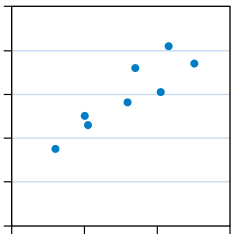
a Present this bivariate data in a scatterplot.

b Describe what the scatterplot shows.

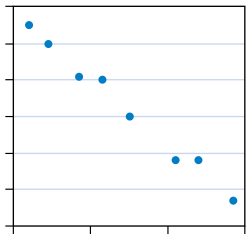
14.2 What is the relationship?

When bivariate data is graphed on a scatterplot, you can use it to see if there is a relationship between the 2 variables. The **association** between 2 variables should be considered in terms of **direction**, **shape** and **strength**.

The **direction** of the association can be **positive** or **negative**.

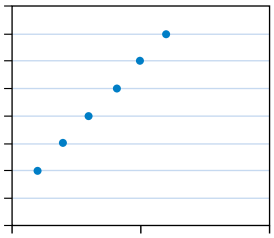


Positive (going up)

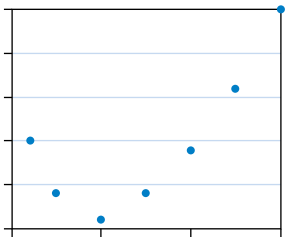


Negative (going down)

The **shape** of the association can be **linear** or **non-linear**.

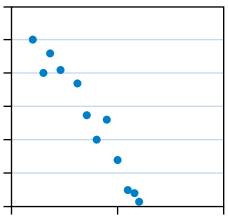


Linear pattern

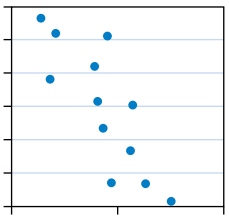


Non-linear pattern

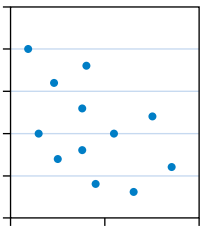
The **strength** of the association can be **strong**, **moderate** or **weak.**



Strong

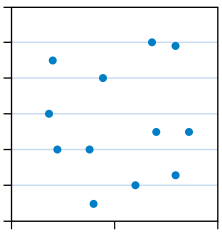


Moderate



Weak

Sometimes, the points have **no association**.

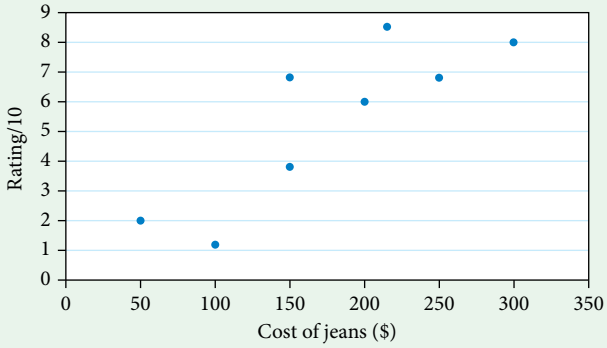


No association

Worksheets
Height vs shoe size
Body measurements

EXAMPLE 2 Describing the association

Describe the association between the variables shown in this bivariate scatterplot.



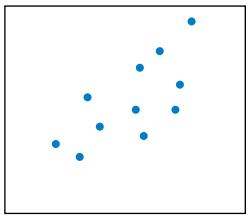
Steps	Working
The dots go up from left to right.	positive
The dots are close to forming a straight line.	linear
The points are spread out.	moderate
Write the answer.	This association is positive, linear and moderate.

EXERCISE 14.2 What is the relationship?

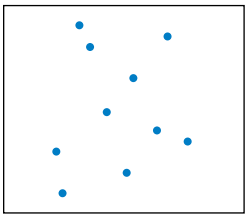
ANSWERS p. 456

1 EXAMPLE 2 Describe the association (direction, shape, strength) between the 2 variables shown in each scatterplot.

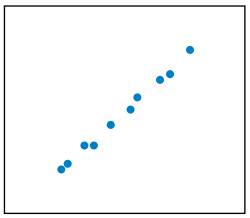
a



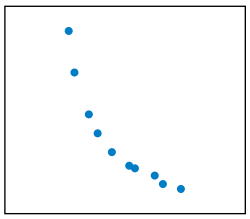
b



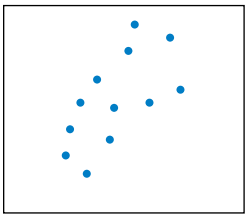
c



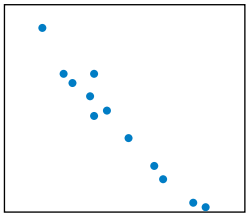
d



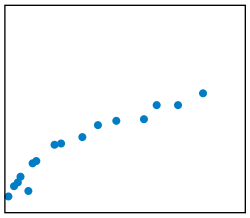
e



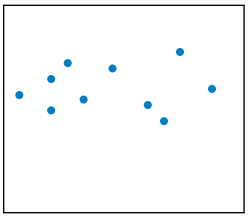
f



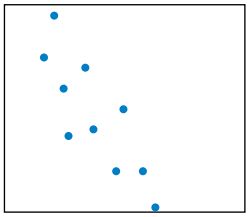
g



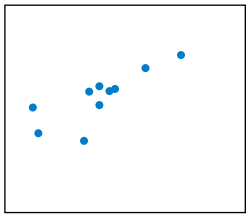
h



i



j



2 Look at the scatterplots you drew in Exercise 14.1. Describe the association of the variables in:

a Question **1** on heights

b Question **4** on pulse and time

c Question **5** on album sales

3 This table shows the ages of 10 boys and the time (in seconds) it took them to swim 50 metres.

Age (years)	6	15	7	8	7	9	14	12	10	11
Swim time (s)	76	32	56	49	54	45	36	41	38	36

a Construct a scatterplot to display the information.

b Is there a relationship between the boy's ages and the time it takes them to swim 50 m? Describe the association.

4 a For each student in your class, record the number of letters in their family name and the number of minutes they spend travelling to school (rounded to the nearest 10 minutes).

b Draw a scatterplot of the data with number of letters on the horizontal axis and travelling time on the vertical axis.

c Would you expect these 2 variables to have any association? Why or why not?

d Is there an association between these 2 variables? If so, describe it.

5 a Collect the following data for each student in your class.

- head circumference
- height

b Draw a scatterplot of the data with head circumference on the horizontal axis and height on the vertical axis.

c Is there a relationship between these 2 variables? If so, describe it.

6 Sketch a scatterplot to illustrate a set of data that has each type of association. PS

a weak, positive, linear association

b strong, negative, linear association

c strong, positive, non-linear association

d no association

Investigation Body circumferences

For each student in your class, record the following measurements correct to the nearest cm:

- wrist circumference
- neck circumference
- waist circumference

Construct scatterplots for:

- wrist circumference and neck circumference
- wrist circumference and waist circumference

Describe any relationships you can see in each scatterplot.

Keep the data and scatterplots for future investigations in Chapter 16.

14.3 Does one variable cause the other?*

Just because 2 variables have an association, it doesn't necessarily mean that one causes the other. At one time in the early 2000s, there was a strong positive association between the price of petrol and the Australian cricket team's run rate! The price of petrol and cricket run rates are unrelated; one couldn't possibly cause the other.

Where you find an association, you need to examine the variables and decide whether there is a **causal relationship** or not.

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Mitch Gunn/Shutterstock.com

EXAMPLE 3 Causal or not?

Each pair of variables below have a strong association. For each pair:

i decide if a change in one variable *causes* a change in the other variable

ii if there is no causal relationship, suggest other factors that might make the variables have an association.

a the height and weight of a person

b the price of petrol and the amount of petrol sold

Steps	Working
a A change in height does NOT cause a change in weight.	Not causal. Both height and weight are affected by age, diet and body shape.
b Yes, a change in petrol price causes a change in the amount of petrol purchased.	Causal. As the petrol price increases, petrol sales will decrease, because people wait for the price to come down again.

EXERCISE 14.3 Does one variable cause the other?* ANSWERS p. 456

1 EXAMPLE 3 Each pair of variables below have a strong association. For each pair:

- **i** determine if a change in one variable causes a change in the other variable
- **ii** if there is no causal relationship, suggest other factors that might make the variables have an association.

a the driving speed of a car and the amount of petrol used

b the length of the right foot and the length of the left foot of the same person

c the price of a particular brand of car and the number of cars sold

d the sale of hot chips and soft drinks at a football game

e the height and arm span of students

f kilojoules of energy consumed by a person and the weight gained

g number of rainy days in a month and the sales of umbrellas in that month

2 Write down 2 variables you would expect to have a positive linear relationship, where a change in one variable would cause a change in the other variable. PS

3 Write down 2 variables you would expect to have a positive linear relationship, where a change in one variable would NOT cause a change in the other variable. PS

4 Write down 2 variables you would expect to have a negative linear relationship, where a change in one variable would cause a change in the other variable. PS

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Ross Tomei/Dreamstime LLC

Spreadsheet
Year 12 data

Puzzle
Data crossword

14.4 Analysing data

Let's apply what you have learned about bivariate data to analyse data about a sample of Year 12 students.

EXERCISE 14.4 Analysing data

ANSWERS p. 457

This table gives information collected from 15 Year 12 students. You can download this table as a spreadsheet ('Year 12 data') from Nelson MindTap. You can also use the spreadsheet to draw scatterplots.

Student	Height (cm)	Arm span (cm)	Right foot length (cm)	Time to travel to school (mins)	Hours of homework per week	Resting pulse (beats per minute)	Hours watching TV per week	Number of siblings
Amy	176	175	26	7	6	64	2	4
Joe	178	175	23	40	6	66	10	2
Annika	151	150	30	10	23	95	4	1
Janine	168	175	25	15	7	76	14	2
Stephen	186	181	37	30	23	60	9	3
Thanh	187	187	28	5	2	74	5	2
Gillian	149	149	22	50	17	70	4	1
Vamsee	174	172	28	4	1	62	3	1
Lalaja	172	178	26	20	16	76	20	4
Darryl	177	177	23	15	10	64	10	3
Lyn	169	160	22	7	15	60	0	3
Jeremy	159	155	24	25	9	83	4	2
Ben	169	184	26	34	0	68	17	1
Abdul	163	159	24	20	3	77	9	2
Miriam	163	165	25	20	4	75	1	3

1 Use the data for arm span and the length of the right foot.

- **a** Draw a scatterplot for this data.
- **b** Name the independent and dependent variables.
- **c** Describe any features of the scatterplot.
- **d** Is there an association between these 2 variables? Describe it.
- **e** If there is an association, is it a causal relationship? If not, suggest other factors that might result in an association between these 2 variables.

2 Use the data for the number of siblings and time to travel to school.

- **a** Would you expect there to be an association between these 2 variables? Why or why not?
- **b** Construct a scatterplot for this data.
- **c** Describe any features of the scatterplot.

3 Choose 2 variables from the table that you would expect to have an association. PS

- **a** Draw a scatterplot for this data.
- **b** Name the independent and dependent variables.
- **c** What features does the scatterplot have?
- **d** Does the scatterplot show a relationship between these 2 variables? Describe it.
- **e** If there is a relationship, is it a causal relationship? If not, suggest other factors that might result in an association between these 2 variables.

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4 Choose 2 variables from the table that you would NOT expect to have an association. PS

a Draw a scatterplot for this data.

b Describe any features of the scatterplot.

c Is there an association between these 2 variables? Describe it.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 457

Use the list of words below to copy and complete a summary of the chapter.

association	bivariate	causes	dependent variable
independent variable	linear	moderate	negative
non-linear	positive	scatterplot	strong
weak			

In this chapter, we studied [1]_____ data, which is data with 2 variables. We learned how to graph this data on a [2]_____. On the graph, the variable on the horizontal axis is called the [3]_____ _____ and the variable on the vertical axis is called the [4]_____ _____. We can use the graph to decide whether there is an [5]_____ between the variables or not. We consider this relationship in terms of its:

Direction: whether it is [6]_____ or [7]_____.

Shape: is it [8]_____ or [9]_____?

Strength: is the relationship [10]_____, [11]_____ or [12]_____?

When we see a relationship, we considered whether one variable [13]_____ the other.

Solution to the chapter problem

Problem

A consumer association investigated the quality and price of 8 pairs of gym shoes, labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10.

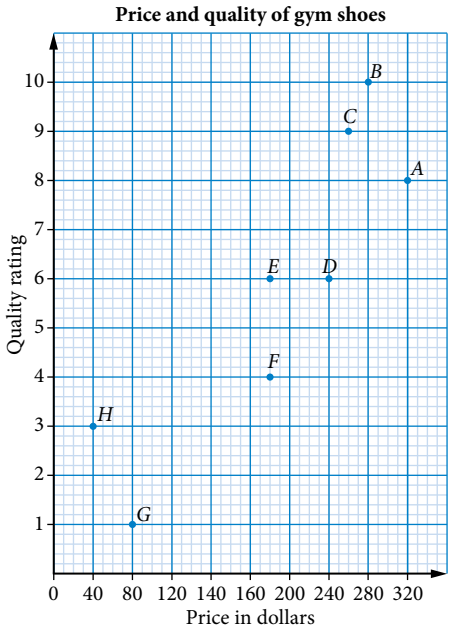
This table shows the results.

	A	B	C	D	E	F	G	H
Price	$320	$280	$260	$240	$180	$180	$80	$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?

Puzzle
Scattered
find-a-word

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To determine if there is a relationship between price and quality of gym shoes and the type of relationship. We know the data presented in the table.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	Draw a scatterplot to see if there is a relationship.  The scatterplot shows there is a relationship between the 2 variables: the more expensive the gym shoe, the better the quality. The relationship is strong, positive and linear. There would be a causal relationship. You would expect the more expensive shoes to use better materials and to be better constructed than the cheaper shoes.
STAGE 3: CHECK THE SOLUTION	
 CHECK	We have drawn the scatterplot correctly and our solution makes sense.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	This data shows a strong, positive, linear relationship between the price and quality of gym shoes: the more expensive the gym shoe, the better the quality.

14 Test yourself

Scattering the data

ANSWERS p. 457

1 EXERCISE 14.1 Eliza works in a coffee shop. She thinks there is a relationship between the daily average temperature and their hot chocolate drink sales.

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate sold	42	15	37	24	9	20	35	20	10	30	30	4

a Graph Eliza's bivariate data in a scatterplot.

b Which is the independent variable, and which is the dependent variable?

c Comment on the features of the scatterplot.

2 EXERCISE 14.2 Describe the association (direction, shape and strength) between the 2 variables shown in the scatterplot you drew in Question **1**.

3 EXERCISE 14.2 Sketch a scatterplot to illustrate bivariate data that have each type of association:

a weak, negative, linear association

b strong, positive, linear association

c no association

4 EXERCISE 14.3 Each pair of variables below has a strong association. Decide if the relationship is causal or suggest other factors that might link the variables.

a The leg and arm lengths of the same person

b The number of wins for a football team in a season and the sale of its merchandise

5 EXERCISE 14.4 This table comes from the larger table in Exercise 14.4 on page **376**.

Student	Hours of homework per week	Hours watching TV per week
Amy	6	2
Joe	6	10
Annika	23	4
Janine	7	14
Stephen	23	9
Thanh	2	5
Gillian	17	4
Vamsee	1	3
Lalaja	16	20
Darryl	10	10
Lyn	15	0
Jeremy	9	4
Ben	0	17
Abdul	3	9
Miriam	4	1

a Do you expect to find a strong association between these 2 variables? Why or why not?

b Draw a scatterplot for this data.

c Name the independent and dependent variables.

d Is there an association between these 2 variables? If so, describe it.

Chapter problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money if she takes the loan for 15 years or 30 years? What is the difference in amount paid?

b Suggest a reason Lee might decide to take the loan over 30 years.

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CHAPTER 15

GRAPHS, DATA AND LOANS:
LOANS AND COMPOUND INTEREST

BORROWING MONEY

What will we do in this chapter?

- Use technology to investigate the progress of a loan
- Investigate how the interest rate and repayment size affect the time taken to repay a loan and the total interest paid

How are we ever going to use this?

- To compare loans
- To calculate the total amount we will repay for a loan
- To make decisions about the best loans for our individual circumstances

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Syllabus coverage

UNIT 4, TOPIC 3: LOANS AND COMPOUND INTEREST

Reducing balance loans

- Understand that reducing balance loans are compound interest loans with periodic repayments.
- Use a calculator or an online calculator to model a reducing balance loan with annual repayments.
- Use a spreadsheet to model a reducing balance loan with non-annual repayments. [complex].
- Investigate the effect of the repayment amount, the interest rate and the number of compounding periods on the time taken to repay a loan [complex].

Video (1):
15.1 Reducing balance loans

Worksheets (2):
15.2 Reducing balance loan: spreadsheet
15.4 Can you afford to buy a home?

Spreadsheets (3):
15.2 Reducible loans • Gaynor's loan
Test yourself Reducible interest
Practice set 4 Reducible loans

Weblink (1):
15.3, 15.4 MoneySmart

Terminology

annual	compound interest	compounding periods	interest rate
loan	model	non-annual	periodic
reducing balance	repay	repayments	spreadsheet

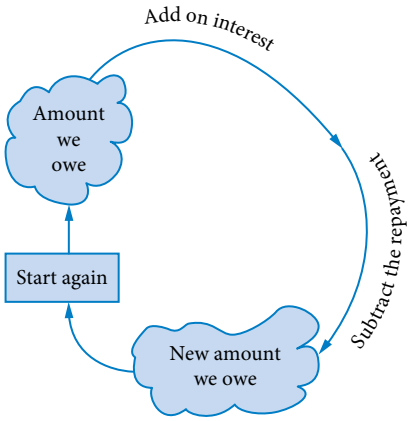
15.1 Reducing balance loans

When you borrow money from a financial institution such as a bank, credit union or finance company, you are charged **interest**. The interest rate depends on the type of financial institution and the reason you are borrowing the money. Wise consumers shop around for the best value before they commit themselves.

There are 2 types of interest on loans.

With **simple interest**, also known as **flat rate interest**, you pay the same amount of interest throughout the loan, regardless of the amount you owe.

With **reducible interest**, you only pay interest on the amount of money still owing. As the amount you owe decreases, the interest decreases.



This table shows the progress of a **reducing balance loan**. The principal is $15 000, borrowed at 9% p.a. monthly reducing interest, with monthly **repayments** of $800. All amounts are rounded to the nearest cent.

1st month's interest
0.09 ÷ 12 × $15 000

$15 000 + $112.50

$15 112.50 − $800

Reducing balance loan
Amount borrowed: $15 000
Interest rate: 9% p.a. monthly reducible
Monthly repayment (R): $800

Month	Principal (P)	Interest (I)	Principal + Interest ($P + I$)	Amount owing ($P + I - R$)
1st	$15 000	$112.50	$15 112.50	$14 312.50
2nd	$14 312.50	$107.34	$14 419.84	$13 619.84
3rd	$13 619.84	$102.15	$13 721.99	$12 921.99
4th	$12 921.99	$96.91	$13 018.91	$12 218.91

$14 419.84 − $800

The principal at the start of the 2nd row is the same as the amount at the end of the 1st row.

2nd month's interest
0.09 ÷ 12 × $14 312.50

$14 312.50 + $107.34

Video
Reducing balance loans

EXAMPLE 1 Reducing balance loan

When Ethan borrowed $10 000 from the bank to buy a small car, the bank charged 9% p.a. reducible interest and his monthly repayments were $240. **Determine** the values of **a** to **h** in the table, then **calculate** the amount Ethan will owe after he has made 3 repayments, and the total interest he will pay in the first 3 months. Round all values to the nearest cent.

Ethan's reducing balance car loan				
Amount borrowed: $10 000				
Interest rate: 9% p.a. monthly reducible				
Monthly repayments: $240				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$10 000	$75	$10 075	$9835
2	a	b	c	d
3	e	f	g	h

Steps	Working
a The principal for the 2nd month is the same as the amount owing at the end of the 1st month.	$9835
b Calculate 1 month's interest at 9% p.a. on the amount Ethan owes.	$9835 × 0.09 ÷ 12 = $73.76
c $P + I$ is the sum of **a** and **b**.	$9835 + $73.76 = $9908.76
d The amount owing at the end of the 2nd month is **c** minus the monthly repayment of $240.	$9908.76 − $240 = $9668.76
e The principal for the 3rd month is the same as the amount owing at the end of the 2nd month.	$9668.76
f Calculate 1 month's interest on $9668.76.	$9668.76 × 0.09 ÷ 12 = $72.52
g $P + I$ is the sum of **e** and **f**.	$9668.78 + $72.52 = $9741.28
h The amount owing at the end of the 3rd month is **g** minus $240.	$9741.28 − $240 = $9501.28

Ethan's reducing balance car loan				
Amount borrowed: $10 000				
Interest rate: 9% p.a. monthly reducing				
Monthly repayments: $240				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1st	$10 000	$75	$10 075	$9835
2nd	$9835	$73.76	$9908.76	$9668.76
3rd	$9668.76	$72.52	$9471.28	$9501.28

The amount Ethan still owes after he has made 3 repayments is the amount owing after the 3rd month. $9501.28

The interest Ethan has paid is the sum of the values in the interest column.

interest = $75 + $73.76 + $72.52
= $221.28

EXERCISE 15.1 Reducing balance loans

ANSWERS p. 457

1 EXAMPLE 1 Omar borrowed $7000 from the bank at 9% p.a. monthly reducible interest to buy his first car. His monthly instalments are $320.

a Use a calculation to show that Omar will pay $52.50 in interest in the first month.

b Use another calculation to show that Omar will owe $7052.50 at the end of the first month immediately before he makes his first repayment.

c Explain how you know that Omar will owe $6732.50 immediately after he makes his first repayment.

d Find the values of **A** to **H** in Omar's repayment table.

Omar's reducing balance car loan				
Amount borrowed: $7000				
Interest rate: 9% p.a. monthly reducible				
Monthly repayments: $320				
Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$7000	$52.50	$7052.50	$6732.50
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

e How much will Omar owe immediately after he makes his 3rd repayment?

f How much interest will Omar pay in the first 3 months of the loan?

g How much less interest will he pay in the 4th month than in the 1st month?

h Why is the amount of interest getting smaller each month?

Nestor Rizhniak/Shutterstock.com

2 a Copy and complete the first 4 lines of Vicky's personal loan.

Vicky's reducing balance loan				
Amount borrowed: $16 000				
Interest rate: 7.2% p.a. monthly redicible				
Monthly repayments: $400				
Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$16 000	$96	$16 096	$15 696
2				
3				
4				

b How much will Vicky owe immediately after she has made her 4th repayment?

c What percentage of Vicky's 4th repayment was interest? Give your answer correct to one decimal place.

3 Bianca borrowed $18 000 at 8% p.a. reducible interest, calculated every 6 months, to expand her business. She agreed to repay the loan over 3 years with half-yearly repayments of $3434. PS

a Use a calculation to show that Bianca will pay $720 interest in her first half-year.

b Find the values of **A** to **E** in the table.

Bianca's business loan				
Amount borrowed: $18 000				
Interest rate: 8% p.a. reducible				
Half-yearly repayments: $3434				
Half-years	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$18 000	$720	$18 720	$15 286
2	**A**	**B**	$15 897.44	**C**
3	$12 463.44	**D**	$12 961.98	$9527.98
4	$9527.98	$381.12	$9909.10	$6475.10
5	$6475.10	$259.00	**E**	$3300.10
6	$3300.10	$132.00	$3432.10	0

c After how long will Bianca have repaid more than half the loan?

4 Jackson borrowed $12 800 at 7.56% p.a. monthly reducible interest to buy a boat.

a Copy the table below and complete the first 3 rows.

Jackson's loan				
Amount borrowed: $12 800				
Interest rate: 7.56% p.a. monthly reducible				
Monthly repayments: $900				
Months	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$12 800			
2				
3				

b Calculate the total amount Jackson will repay in the first 3 months.

c How much will Jackson repay off the principal in the first 3 months of the loan?

d How much interest will he pay in the first 3 months?

5 a Create a spreadsheet to calculate the progress of a $10 000 loan at 6% p.a. monthly reducible interest with repayments of $260 per month. Use this spreadsheet outline to get started. PS

	A	B	C	D	E
1	**Months**	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
2	1	$10 000			
3	2				
4	3				
5	4				
6	5				

b Use your spreadsheet to calculate the amount still owing on the loan immediately after the 5th repayment.

15.2 Loan spreadsheets*

Home loans are an example of a reducing balance loan that can last for a long time, for example, 30 years. Before calculators were available, banks had to calculate loan and interest amounts using pen and paper! The table for a 30-year loan would require 360 rows, one for each month.

Today, however, we have technology such as spreadsheets to do the 'number-crunching' involved.

Worksheet Reducing balance loan: spreadsheet

Spreadsheet Reducible loans

EXAMPLE 2 Spreadsheets and reducible loans

To answer this question, download the 'Reducible loans' spreadsheet from Nelson MindTap.

Jordan is borrowing $20 000 at 6% p.a. reducible interest and his monthly repayments are $810.

a How long will it take Jordan to repay the loan?

b How much interest will Jordan pay?

Steps **Working**

In the blue cells, enter 20 000 for the loan amount, 0.06 for the interest rate and 810 for his monthly repayment.

	A	B	C	D	E	F
1	**Reducible loans**					
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.					
3						
4	Loan amount	$20,000.00				
5	Interest rate as a decimal	0.06				
6	Monthly repayment	$810.00				
7						
8		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment	
9	1st month	$20,000.00	$100.00	$20,100.00	$19,290.00	
10	2nd month	$19,290.00	$96.45	$19,386.45	$18,576.45	
11	3rd month	$18,576.45	$92.88	$18,669.33	$17,859.33	
12	4th month	$17,859.33	$89.30	$17,948.63	$17,138.63	
13	5th month	$17,138.63	$85.69	$17,224.32	$16,414.32	
14	6th month	$16,414.32	$82.07	$16,496.39	$15,686.39	
15	7th month	$15,686.39	$78.43	$15,764.83	$14,954.83	
16	8th month	$14,954.83	$74.77	$15,029.60	$14,219.60	
17	9th month	$14,219.60	$71.10	$14,290.70	$13,480.70	
18	10th month	$13,480.70	$67.40	$13,548.10	$12,738.10	
19	11th month	$12,738.10	$63.69	$12,801.79	$11,991.79	
20	12th month	$11,991.79	$59.96	$12,051.75	$11,241.75	
21	13th month	$11,241.75	$56.21	$11,297.96	$10,487.96	
22	14th month	$10,487.96	$52.44	$10,540.40	$9,730.40	
23	15th month	$9,730.40	$48.65	$9,779.05	$8,969.05	
24	16th month	$8,969.05	$44.85	$9,013.90	$8,203.90	
25	17th month	$8,203.90	$41.02	$8,244.92	$7,434.92	
26	18th month	$7,434.92	$37.17	$7,472.09	$6,662.09	
27	19th month	$6,662.09	$33.31	$6,695.40	$5,885.40	
28	20th month	$5,885.40	$29.43	$5,914.83	$5,104.83	
29	21st month	$5,104.83	$25.52	$5,130.35	$4,320.35	
30	22nd month	$4,320.35	$21.60	$4,341.95	$3,531.95	
31	23rd month	$3,531.95	$17.66	$3,549.61	$2,739.61	
32	24th month	$2,739.61	$13.70	$2,753.31	$1,943.31	
33	25th month	$1,943.31	$9.72	$1,953.03	$1,143.03	
34	26th month	$1,143.03	$5.72	$1,148.74	$338.74	
35	27th month	$338.74	$1.69	$340.44	-$469.56	
36	28th month	-$469.56	-$2.35	-$471.91	-$1,281.91	
37	29th month	-$1,281.91	-$6.41	-$1,288.32	-$2,098.32	
38	30th month	-$2,098.32	-$10.49	-$2,108.81	-$2,918.81	
39	31st month	-$2,918.81	-$14.59	-$2,933.41	-$3,743.41	
40	32nd month	-$3,743.41	-$18.72	-$3,762.12	-$4,572.12	
41	33rd month	-$4,572.12	-$22.86	-$4,594.98	-$5,404.98	
42	34th month	-$5,404.98	-$27.02	-$5,432.01	-$6,242.01	
43	35th month	-$6,242.01	-$31.21	-$6,273.22	-$7,083.22	
44	36th month	-$7,083.22	-$35.42	-$7,118.63	-$7,928.63	

a Look at the values in the last column, E. When the amount owing becomes negative, the loan has been paid off. The first negative amount owing is in cell E35, which corresponds to the 27th payment month.

Jordan will repay the loan after 27 months (which is 2 years and 3 months).

Jordan will make 26 monthly payments of $990, but his final payment only needs to be $340.44.

b Add all the values in the interest column up to the 27th month. Entering a formula into the spreadsheet will be the easiest way. The formula is =SUM(C9:C35).

Jordan will pay $1400.44 interest.

Spreadsheet
Reducible loans

EXERCISE 15.2 Loan spreadsheets*

ANSWERS p. 457

Use the 'Reducible loans' spreadsheet from Nelson MindTap for this exercise.

1 EXAMPLE 2 Samantha borrowed $5000 at 7% p.a. reducible interest to buy a motor scooter. Her monthly repayments are $350.

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- **a** How much interest will Samantha pay in the first month?
- **b** How much of Samantha's second repayment will be interest?
- **c** How much will Samantha owe immediately after she has made her 12th repayment?
- **d** How long will it take Samantha to repay the loan?
- **e** What is the value of Samantha's last repayment?
- **f** How much interest will Samantha pay on the loan?
- **g** Calculate the total amount Samantha will repay.

2 Trinh borrowed $6000 at 4.25% p.a. reducible interest to buy some new computer equipment. Each month Trinh repays $360.

- **a** How long will it take Trinh to repay the loan?
- **b** How much interest will he pay?
- **c** How much more than he borrowed will Trinh repay?

3 Jacinta's grandmother lends her $7000 interest-free to buy her first car, provided that she repays her $400 each month.

- **a** How long will it take Jacinta to repay the interest-free loan?
- **b** If she had to borrow the money from the bank, Jacinta would be charged 9% p.a. reducible interest. How much interest is she saving with her grandmother's interest-free loan?

4 Ashok wants to borrow $9000. The bank has offered him a reducible interest loan at 7.4% p.a. and suggested that he repay $400 each month. PS

- **a** How long will it take Ashok to repay the loan with monthly repayments of $400?
- **b** Ashok thinks he can afford to repay $550 per month. How long will it take him to repay the loan at $550 per month?
- **c** Will Ashok save any money by paying the loan off in a shorter period of time? Justify your answer.
- **d** What general conclusion can you make about the advantage of repaying a reducible loan quickly?
- **e** Is the same conclusion true for flat-rate loans? Explain your answer.

5 **a** On the 'Reducible loans' spreadsheet, what formulas are in cells B9, C9, D9 and E9?

- **b** Why do the formulas in C9 and E9 contain $ signs, but the formulas in B9 and D9 don't?

6 Lily made a spreadsheet of her reducing balance loan.

	A	B	C	D	E	F
1	**Reducible loans**					
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.					
3						
4	**Loan amount**	$4,000.00				
5	**Interest rate as a decimal**	0.06				
6	**Monthly repayment**	$120.00				
7						
8		**Amount owing at the beginning of the month**	**Interest charge for the month**	**Amount owing plus interest**	**Amount owing after the repayment**	
9	1st month	$4,000.00	$20.00	$4,020.00	$3,900.00	
10	2nd month	$3,900.00	$19.50	$3,919.50	$3,799.50	
11	3rd month	$3,799.50	$19.00	$3,818.50	$3,698.50	
12	4th month	$3,698.50	$18.49	$3,716.99	$3,596.99	
13	5th month	$3,596.99	$17.98	$3,614.97	$3,494.97	
14	6th month	$3,494.97	$17.47	$3,512.45	$3,392.45	
15	7th month	$3,392.45	$16.96	$3,409.41	$3,289.41	
16	8th month	$3,289.41	$16.45	$3,305.86	$3,185.86	
17	9th month	$3,185.86	$15.93	$3,201.79	$3,081.79	
18	10th month	$3,081.79	$15.41	$3,097.20	$2,977.20	
19	11th month	$2,977.20	$14.89	$2,992.08	$2,872.08	
20	12th month	$2,872.08	$14.36	$2,886.44	$2,766.44	
21	13th month	$2,766.44	$13.83	$2,780.28	$2,660.28	
22	14th month	$2,660.28	$13.30	$2,673.58	$2,553.58	
23	15th month	$2,553.58	$12.77	$2,566.35	$2,446.35	
24	16th month	$2,446.35	$12.23	$2,458.58	$2,338.58	
25	17th month	$2,338.58	$11.69	$2,350.27	$2,230.27	
26	18th month	$2,230.27	$11.15	$2,241.42	$2,121.42	
27	19th month	$2,121.42	$10.61	$2,132.03	$2,012.03	
28	20th month	$2,012.03	$10.06	$2,022.09	$1,902.09	
29	21st month	$1,902.09	$9.51	$1,911.60	$1,791.60	
30	22nd month	$1,791.60	$8.96	$1,800.56	$1,680.56	
31	23rd month	$1,680.56	$8.40	$1,688.96	$1,568.96	
32	24th month	$1,568.96	$7.84	$1,576.80	$1,456.80	
33	25th month	$1,456.80	$7.28	$1,464.09	$1,344.09	
34	26th month	$1,344.09	$6.72	$1,350.81	$1,230.81	
35	27th month	$1,230.81	$6.15	$1,236.96	$1,116.96	
36	28th month	$1,116.96	$5.58	$1,122.55	$1,002.55	
37	29th month	$1,002.55	$5.01	$1,007.56	$887.56	
38	30th month	$887.56	$4.44	$892.00	$772.00	
39	31st month	$772.00	$3.86	$775.86	$655.86	
40	32nd month	$655.86	$3.28	$659.14	$539.14	
41	33rd month	$539.14	$2.70	$541.83	$421.83	
42	34th month	$421.83	$2.11	$423.94	$303.94	
43	35th month	$303.94	$1.52	$305.46	$185.46	
44	36th month	$185.46	$0.93	$186.39	$66.39	

What formulas did Lily enter in cells B9, C9, D9, E9 and B10?

7 Gabriel borrowed $30 000 from the bank to buy a car. The bank is charging him 7.5% p.a. monthly reducible finance, and each month Gabriel repays $950. PS

Use a spreadsheet to determine the number of monthly repayments Gabriel will make and the total amount of interest he will pay on the loan.

8 Monique borrowed $6400 at 5.8% p.a. monthly reducing interest to go on a holiday to Japan. Her monthly repayments are $520.

a How long will it take Monique to repay the loan?

b How much interest will Monique pay?

9 Use the spreadsheet to investigate how increasing the interest rate affects the time taken to repay a loan if the monthly repayment stays the same. Copy and complete this statement: C

When interest rates go up, it takes _____ time to repay the loan and you pay _____ interest.

10 Use the spreadsheet to investigate how increasing the monthly repayment amount affects the time taken to repay a loan if the interest rate stays the same. Copy and complete this statement: C

If we increase our monthly repayments, it takes _____ time to repay the loan and you pay _____ interest.

11 Gaynor borrowed $90 000 to set up a business. She is required to make *quarterly repayments* at 8.4% p.a. reducible interest. Gaynor decided to make $10 000 repayments each quarter. Follow the steps to construct your own spreadsheet or download a partially constructed spreadsheet from Nelson MindTap to investigate the progress of Gaynor's loan. C

Spreadsheet
Gaynor's loan

	A	B	C	D	E
1	**Gaynor's business loan**				
2					
3	Amount borrowed				
4	Interest rate				
5	Quarterly repayment				
6					
7					
8	**Quarters**	**Amount owing at the beginning of the quarter**	**Interest for the quarter**	**Amount owing + interest**	**Amount owing after the quarterly repayment**
9	1				
10	2				
11	3				
12	4				
13	5				
14	6				
15	7				
16	8				
17	9				
18	10				
19	11				
20	12				
21	13				
22	14				
23	15				
24	16				
25	17				
26	18				
27	19				
28	20				

a Describe how you could calculate quarterly interest.

b Construct a formula you could use in cell C9.

c What formulas will you need in cells D9, E9 and B10?

d Enter the formulas and copy down.

> If the numbers in the spreadsheet are surprisingly large, check whether you've left out the $ in the formula for the interest or amount owing after the quarterly repayment.

e Calculate the total amount Gaynor will repay for the $90 000 loan.

f How long will it take to repay the loan?

g How much interest will she pay?

h Before she started to repay the loan, Gaynor realised that she couldn't afford to repay $10 000 each quarter, so she reduced her repayments to $7500 per quarter. What affect will the reduced repayments have on the total amount Gaynor repays and the total interest involved? C PS

15.3 Online loan calculators

Most people use online calculators to help them investigate and manage reducing balance loans. Find an online loan calculator to use in this section, for example, the mortgage calculator on the **MoneySmart** website.

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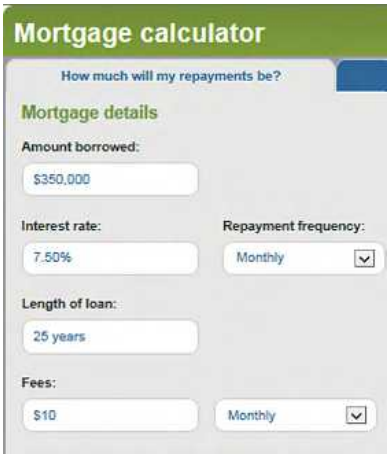
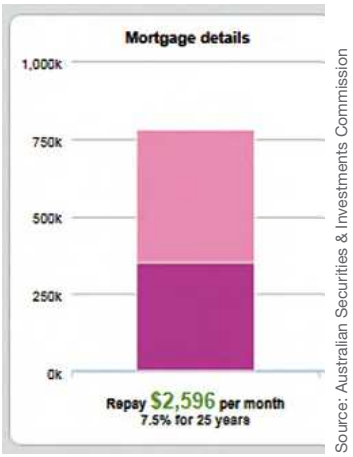
EXAMPLE 3 Online loan calculators

Elle borrows $350 000 to buy an apartment to live in. She will repay the loan in equal monthly instalments over 25 years at 7.5% p.a. monthly reducible interest. In addition, she will be charged a $10 monthly account-keeping fee.

a How much are Elle's monthly repayments?

b How much will Elle pay in interest and fees?

Steps	Working
a Enter the values into the mortgage calculator.	 Source: Australian Securities & Investments Commission  Source: Australian Securities & Investments Commission
	Elle's monthly repayments will be $2596.
b She will make monthly repayments for 25 years.	number of months = 25 × 12 = 300
Find the total repaid.	total repaid = repayment × number of months in 25 years = $2596 × 300 = $778 800
Calculate the interest and fees.	interest and fees = total repaid − amount borrowed = $778 800 − $350 000 = $428 800
Write your answer.	Elle will pay $428 800 in interest and fees.

If the online calculator's answer is a little different to yours, that's because it shows the repayment amount ($2566) **correct to the nearest dollar**, but when it does its calculations, it uses a more precise value.

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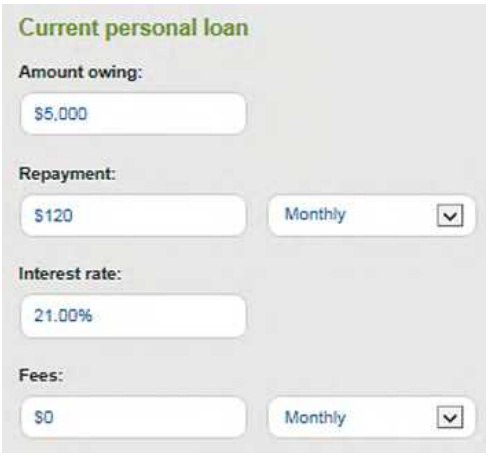
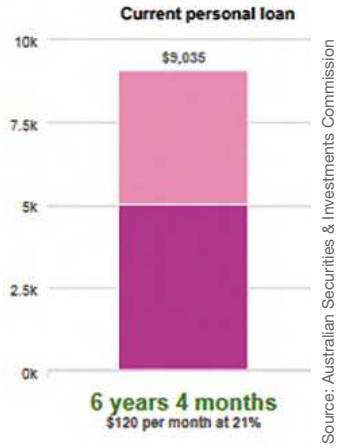
EXAMPLE 4 Personal loans

Owen overspent on his credit card and bought $5000 worth of items with it. His credit card charges 21% p.a. monthly reducible interest, but he can only afford to repay $120 per month.

a How long will it take Owen to repay the $5000 credit card bill if he makes $120 repayments each month?

b How much will it cost Owen to pay off his $5000 credit card bill?

Steps	Working
a Enter the values into the personal loan calculator on MoneySmart. Use the calculator in the tab 'How can I repay my loan sooner'. Set fees to $0.	 Source: Australian Securities & Investments Commission  Source: Australian Securities & Investments Commission The calculator says that it will take Owen 6 years and 4 months to pay off the loan.
b Calculate the number of repayments.	number of repayments = 6 × 12 + 4 = 76
Calculate the total repaid.	total repaid = repayment × number of months = $120 × 76 = $9120
Write your answer.	It will cost Owen $9120 to pay off his $5000 credit card bill.

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EXERCISE 15.3 Online loan calculators

ANSWERS p. 458

You will need access to online calculators for this exercise.
Use the personal loan calculator to answer Questions **1** to **3**.

1 EXAMPLE 3 Claire is borrowing $15 000 at 9.8% p.a. monthly reducible interest to start a business. She plans to repay the loan in monthly repayments over 5 years.

a How much are her monthly repayments?

b Calculate the total amount Claire will pay in the monthly instalments.

c How much interest will Claire pay during the 5 years?

2 Juan wants to borrow $40 000 to buy an SUV. He can get the money from a finance company at 17% p.a. reducible interest.

a Calculate the monthly repayments if Juan takes the loan over 15 years.

b How much interest will he pay if he takes the loan over 15 years?

c Explain why 15 years isn't a suitable term for a car loan.

d If you were Juan, what would you do?

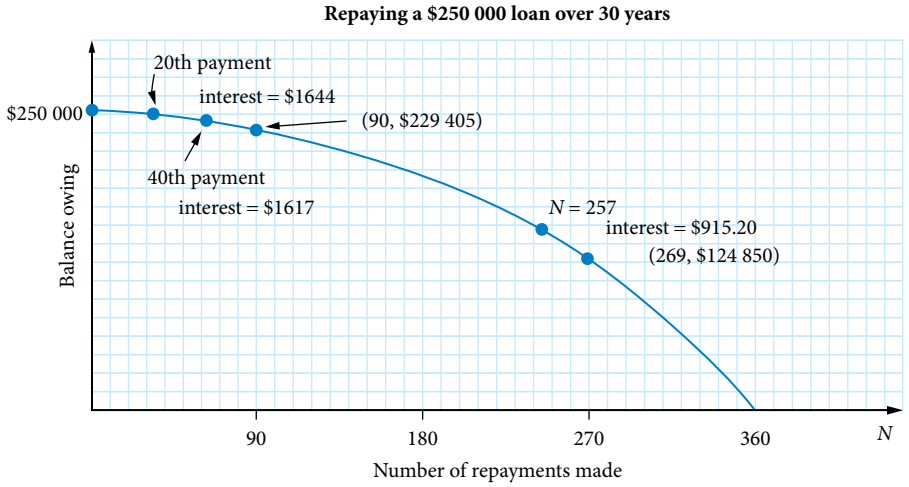
3 EXAMPLE 4 Hoa wants to borrow $10 000 to go on a trip to Canada. Her bank will lend her the money at 8.75% p.a. reducible finance with no monthly account fees. Hoa can afford to repay $650 per month.

a Use the 'How can I repay my loan sooner' tab on the personal loan calculator to calculate the number of monthly repayments Hoa will make to repay the loan.

b How much interest will Hoa pay?

Kavram/Shutterstock.com

4 When Chris bought his townhouse, he borrowed $250 000 at 8% p.a. monthly reducing interest over 30 years and his monthly repayments were $1834.40. The graph shows the balance he owed on the loan after N months.



a The graph shows that in the 20th month, the interest paid was $1644 and the monthly repayment was $1834.40. How much of the repayment was not interest but a decrease in the loan?

b The graph shows that in the 40th month, the interest paid was $1617. For a reducing balance loan, why is the interest in the 40th repayment less than the interest in the 20th repayment?

c Use the graph to determine the amount that Chris still owed on the loan after 90 months.

d Calculate the total monthly repayments Chris made during the 90 months.

e In the 257th month, how much of the repayment:

i was interest? **ii** was a decrease in the loan?

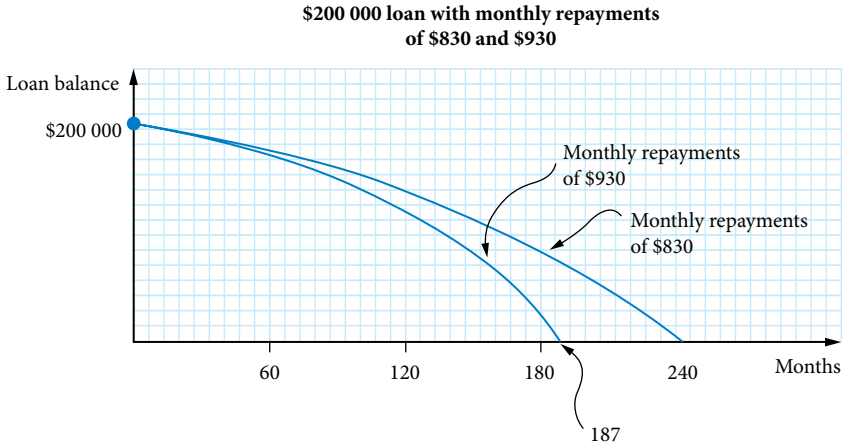
f What percentage of the loan is still owing after 269 months?

g How many years and months is 269 months?

h It took Chris 269 months to repay the first half of the loan. How long did it take him to repay the second half of the loan?

i For a reducing balance loan, when do we pay the most interest: at the beginning of the loan or at the end of the loan?

5 When Lani borrowed $200 000 to buy an apartment, the bank told her the repayments would be $830 per month. Lani decided that she could afford to repay $930 per month. The graph shows the balance of a $200 000 loan with monthly repayments of $830 and $930.



a How long will Lani take to repay the loan if she makes monthly repayments of:

i $830? **ii** $930?

b How many months will Lani save by making monthly repayments of $930?

c How much money will Lani save in repayments by making monthly repayments of $930?

d Why do you think some people may choose to make the smaller monthly repayments even though they will have to pay more in the long run?

Use the mortgage calculator to answer the remaining questions in this exercise.

6 Bree borrowed $260 000 at 6.75% p.a. over 15 years.

a How much were her monthly repayments?

b Interest rates fell to 6.5% p.a. How much cheaper were Bree's monthly repayments after the fall in interest rates?

7 When interest rates go up, the loan repayments increase. When Jai borrowed $170 000, the interest rate was 6.75% p.a. and he took the loan over 10 years.

a How much was Jai's monthly repayment when he took out the loan?

b By how much did Jai's monthly repayments increase when the interest rate went up to 7% p.a.?

c How much more did Jai have to repay each year at 7% p.a. compared to the original annual amount?

8 Zhi is buying a house. He is going to borrow $240 000 at 8% p.a. He is trying to decide whether to take the loan over 20 or 30 years.

a How much more are the monthly repayments over 20 years than over 30 years?

b How much more interest will he pay if he takes the loan over 30 years than over 20 years?

c If you were Zhi, would you take the loan over 20 or 30 years? Give reasons.

9 During the 1980s, interest rates rose as high as 18% p.a. How much more were the monthly repayments on a $150 000 loan over 20 years in the 1980s compared to the same loan at 5.7% p.a. in the late 2010s?

10 Sara is deciding between 2 different home loans. She is borrowing $140 000 over 25 years. Sara summarised the terms and conditions of the 2 loans in a table. PS

	Interest rate p.a.	Loan establishment fee	Annual loan fee	Mortgage discharge fee
Big bank	7.1%	$320	$248	$228
Small mortgage company	7.0%	$598	$76	$314

a Calculate the value of the monthly repayments for each loan.

b Copy and complete this table to help Sara determine the better loan.

	Big bank	Small mortgage company
Loan establishment fee		
Mortgage discharge fee		
Total annual loan fee over 25 years		
Total monthly repayments		
Total cost of the loan		

c Which 2 features of a loan – the interest rate, loan establishment fee, annual loan fee or mortgage discharge fee – most influence the total cost of a loan?

d Which of the 2 loans do you recommend Sara take? Why?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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15.4 Investigating loans*

When you borrow a large amount of money, does a change in the interest rate affect how long it takes to pay off the loan? Does increasing the monthly repayment amount save any money in the long run? In the following exercise, you are going to investigate these 2 questions yourself. You will need to visit the **MoneySmart** website moneysmart.gov.au and search for **Mortgage calculators**. Select **How can I repay my loan sooner?**

EXERCISE 15.4 Investigating loans*

ANSWERS p. 458

Work in groups for each investigation.

Changes in interest rates

1 Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $200 000		Monthly repayment $1500 Monthly fee $10	
Interest rate	Time to repay the loan	Total repaid	Interest
5.00%	16 years 6 months	$295 582	$95 582
5.5%			
5.75%			
6.00%			
7.00%			

2 Change the amount borrowed, then copy and complete this table.

Amount borrowed $		Monthly repayment $1500 Monthly fee $10	
Interest rate	Time to repay the loan	Total repaid	Interest
5.00%			
5.5%			
5.75%			
6.00%			
7.00%			

3 What happens to the amount of interest you pay on a $150 000 loan with monthly repayments of $1000 if the interest rate drops from 4% p.a. to 3.75% p.a.?

4 Write a sentence to describe how changes in interest rates affect the length of time it takes to repay a loan and the total interest involved. Assume the monthly repayment remains the same.

Changes in the monthly repayments

5 Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $200 000		Interest rate 6% p.a. Monthly fee $10	
Monthly repayment	Time to repay the loan	Total repaid	Interest
$1600	16 years 7 months	$318 030	$118 030
$1400			
$1250			
$2000			
$2400			

6 How does increasing the monthly repayment amount affect the total interest paid? What effect does decreasing the monthly repayments have?

7 How does increasing the monthly repayment amount affect the time required to pay off the loan? What effect does decreasing the monthly repayments have?

8 Does doubling the monthly repayments halve the time it takes to pay off the loan?

9 Suppose you are going to borrow money to buy a place to live in. What advice can you give yourself about interest rates and repayments?

TECHNOLOGY Making smart repayments

Let's examine the effects of making loan repayments more frequently and increasing the size of each repayment. Visit the **MoneySmart** website moneysmart.gov.au and search for **Mortgage calculators**. Select **How can I repay my home sooner?**

Weblink MoneySmart

1 Declan has a $250 000 loan at 8% p.a. and he can afford to repay $3000 per month or $1500 per fortnight. Does paying off a home loan fortnightly instead of monthly make any difference?
Use the online calculator to help you copy and complete the missing values in the table.

	Monthly repayments	Fortnightly repayments
Value of the repayment	$3000	$1500
Time to pay off the loan		
Total repayments		
Total interest		

2 Repeat Question **1** for a $300 000 loan at 12% p.a. with a monthly repayment of $5000 and a fortnightly repayment of $2500.

3 Investigate some other loan amounts (principals), but make sure the monthly repayment is twice the value of the fortnightly repayment.

4 What conclusions can you make? Does repaying half the monthly repayment each fortnight make any difference to the loan?

5 Madison borrowed $250 000 at 8% p.a. with a monthly repayment of $2000. Select the tab **How can I repay my home loan sooner?** Use the calculator to find the term (length of time) of Madison's loan. Record the total amount and the interest paid by Madison.

6 Does increasing the monthly repayments by $20 make any difference? Change the repayment from $2000 to $2020. Record the new term of the loan, as well as the new total amount and interest paid.

7 Repeat Question **6** for a larger monthly repayment.

8 What conclusions can you make? Does repaying a larger amount make much difference to the loan?

Worksheet
Can you afford to buy a home?

Investigation — Can you afford to buy a home?

Could you buy a home at age 25? Copy the table below or download 'Can you afford to buy a home?' from Nelson MindTap. Complete it by answering the questions on the next page.

Name					
1	Future job				
2	Annual gross pay				
3	Monthly gross pay				
4	Maximum monthly repayment				
5	Savings interest rate				
6	Deposit				
7	Mortgage interest rate				
		15 yrs	20 yrs	25 yrs	30 yrs
8	Maximum loan amount				
9	Monthly repayment				
10	Total repayments				
11	Total repayments if paid fortnightly				
12	The amount you can afford to spend on a property				
13	A property you can afford in a suitable location. Paste photo here.	**Address**			
		Price			
14	Net monthly income after deducting income tax and mortgage repayments.				
15	Five more expenses from my net monthly income.				
16	Strategies for making buying property more affordable.				

1 Write the type of career you plan to have when you are 25, for example, panel beater, vet nurse, childcare worker.

2 Research the annual gross pay you will receive in this occupation. Do not include any overtime.

3 Calculate your monthly gross pay.

4 Calculate 30% of your monthly pay. This is the maximum amount your monthly repayment can be.

5 Research the current savings interest rate.

6 Suppose you are going to save 30% of your income each month for 5 years at the current savings interest rate. Use the MoneySmart **Monthly savings calculator** to determine your savings after 5 years. This amount is your **deposit**.

7 Research the current mortgage interest rate.

8 Use the moneysmart.gov.au **How much can I borrow?** mortgage calculator to determine the maximum amount you can borrow over 15, 20, 25 and 30 years.

9 Record the monthly repayments in row 9 of the table.

10 Record the total you will repay in row 10 of the table.

Weblink
MoneySmart

11 Use the online calculator to determine the total you will repay if you make half the monthly repayment each fortnight. Record these amounts in row 11 of the table.

12 Calculate the amount you can afford to spend on a property by adding the amount you can borrow to the deposit you have saved.

13 Visit a real estate website and search for a property you can afford to buy in a suitable location. Paste a photograph of the property in the table. Record the address of the property and the selling price.

14 Assume that you will pay 25% of your gross pay in income tax. Subtract this tax and your monthly mortgage repayment from your gross pay. This amount represents the net pay you will have left each month. Write it in line 14 of the table.

15 List 5 different things you will have to pay out of the amount remaining in line 15.

16 Buying your first home is financially challenging. In your group, discuss strategies you could use to make it easier. Record the strategies in line 16 of the table.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 458

Use the terms in this list to copy and complete the chapter summary below.

credit union	decreases	flat	fortnightly	huge	interest
minimum	money	real estate	reducible	repayment	same

When you borrow money from a bank, [1]_____ or finance company, you have to pay [2]_____. Often finance companies charge simple or [3]_____ rate interest. When you borrow money at simple interest, you pay the [4]_____ amount of interest every year, based on the principal borrowed. Reducing balance loans charge [5]_____ interest, where the amount of interest [6]_____ because it is based on the loan amount still owing.

When you borrow a lot of money over a long time, for example, when you buy [7]_____, the total amount you repay can be [8]_____. Even small changes in interest rates can make a big difference to the size of each [9]_____. If you can pay more than the [10]_____ amount required each month, or repay half the monthly repayment [11]_____, you can save a lot of [12]_____ in interest.

iStock.com/Ginevre

Solution to the chapter problem

Problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money if she takes the loan for 15 years or 30 years? What is the difference in the amount paid?

b Suggest a reason Lee might decide to take the loan over 30 years.

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	To work out whether the total amount Lee will repay is higher if she takes the loan over 15 years or 30 years and what the difference is. To suggest a reason, Lee might decide to take the loan over 30 years. We know the interest rate is 7.8% p.a. reducible and the monthly repayments over 15 years are $2265.95 and over 30 years are $1727.70.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	**a** total amount repaid = monthly repayment × number of months number of months = number of years × 12 **Loan over 15 years** total amount repaid = $2265.95 × 15 × 12 = $407 871 **Loan over 30 years** total amount repaid = $1727.70 × 30 × 12 = $621 972 The total amount paid is higher for the 30-year loan. difference = $621 972 − $407 871 = $214 101 **b** Even though Lee will have to repay $214 101 more, she might choose the 30-year loan because she can't afford the monthly repayments of $2265.95 for the 15-year loan.
STAGE 3: CHECK THE SOLUTION	
 CHECK	We know that the longer we take to repay a loan, the more we must repay. The answer for the 30-year loan is much larger than for the 15-year loan.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	**a** Lee will pay more money if she takes the loan for 30 years. The difference is $214 101 when compared with the 15-year loan. **b** Lee might choose the 30-year loan because she can't afford the monthly repayments of $2265.95 for the 15-year loan.

15 Test yourself

Borrowing money

ANSWERS p. 458

1 EXERCISE 15.1 Duschinka borrowed $12 000 to buy some equipment for her photography business. The bank charges Duschinka 7.2% p.a. monthly reducible interest with monthly repayments of $370.

a Find the values of **A** to **H** in Duschinka's loan table.

Duschinka's business loan				
Amount borrowed: $12 000				
Interest rate: 7.2% p.a. monthly reducible				
Monthly repayments: $370				
Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	$12 000	$72	$12 072	$11 702
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

b How much will Duschinka owe immediately after she makes her 3rd repayment?

c How much interest will Duschinka pay in the first 3 months?

2 EXERCISE 15.2 Use the 'Reducible loan' spreadsheet from Nelson MindTap for this question.

Selena is borrowing $28 000 at 8% p.a. reducible interest to buy a new ute and her monthly repayments are $950.

Spreadsheet
Reducible interest

a How long will it take Selena to repay the loan?

b How much interest will she pay over the course of the loan?

3 EXERCISE 15.3 Izak is borrowing $270 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 20 years at 7.2% p.a. monthly reducible interest. In addition, he will be charged a $10 monthly account-keeping fee. Use an online calculator for the following questions.

a How much are Izak's monthly repayments?

b How much will he pay in interest and fees?

c Izak can afford to repay $2400 per month. How long will it take Izak to repay the loan if he pays $2400 per month?

d How much will Izak save in interest if he pays $2400 per month?

4 EXERCISE 15.4 C

a How does an increase in interest rates affect the value of the repayments on a loan?

b How does an increase in the size of each loan repayment affect the time it takes to pay off the loan and the total amount paid on the loan?

Chapter problem

Kaylene is a competitive runner. She studies the past winning times for the women's Olympic 200 m race and is surprised by the rate at which the times have improved. The women's winning times are slower than those of the men in their 200 m event, but the women seem to be catching up. She wondered whether women would ever run the event in the same or faster time than the men.

This graph shows the gold medal times for men and women's 200 m track events at the Olympic Games since 1948.

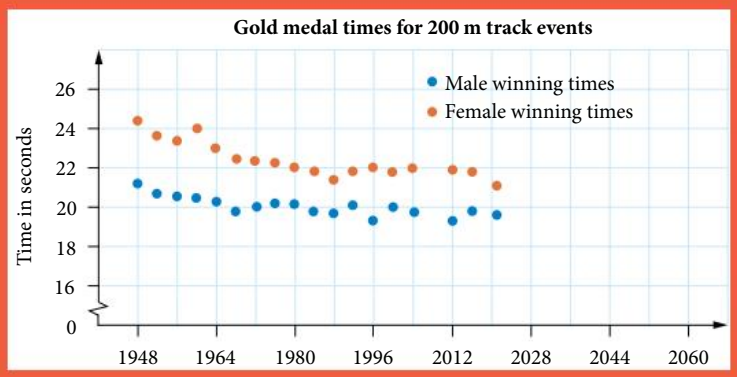
Will the women ever run the 200 m race faster than the men?

CHAPTER 16

GRAPHS, DATA AND LOANS: BIVARIATE GRAPHS

FITTING THE DATA

*COMPLEX



What will we do in this chapter?

- Draw a line of best fit by eye and using technology
- Understand correlation and use technology to calculate the correlation coefficient
- Use a line of best fit to make predictions, both by interpolation and extrapolation

How are we ever going to use this?

- When making predictions based on collected data
- When looking for relationships in data we have collected

Real Sports Photos/Shutterstock.com

9780170484664

Syllabus coverage

UNIT 4, TOPIC 1: BIVARIATE GRAPHS

Line of best fit

- Identify dependent and independent variables.
- Draw a line of best fit by eye.
- Use technology to determine the equation of the line of best fit in the form $y = mx + c$ where m is slope (gradient) and c is y-intercept [complex].
- Interpret the effect of the parameters m and c from the equation of the line of best fit in the form $y = mx + c$ [complex].
- Use technology to calculate the correlation coefficient (an indicator of the strength of linear association) [complex].
- Use the line of best fit to make predictions, both by interpolation and extrapolation [complex].
- Recognise the dangers of extrapolation [complex].

Terminology

correlation
correlation coefficient
extrapolation
gradient
interpolation
line of best fit
prediction
relationship
scatterplot
y-intercept

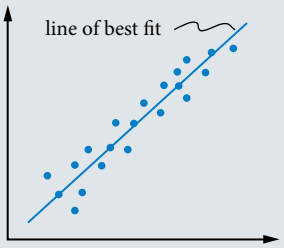
16.1 Drawing a line of best fit

In Chapter 14, *Scattering the data*, you drew scatterplots for bivariate data. If the data shows a strong linear association, you can approximate the linear relationship by drawing a **line of best fit** through the points.

Line of best fit

A line of best fit:

- represents most or all of the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible.



EXAMPLE 1 Line of best fit by eye

This table from Exercise 14.1, Question **1** on page 370 shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

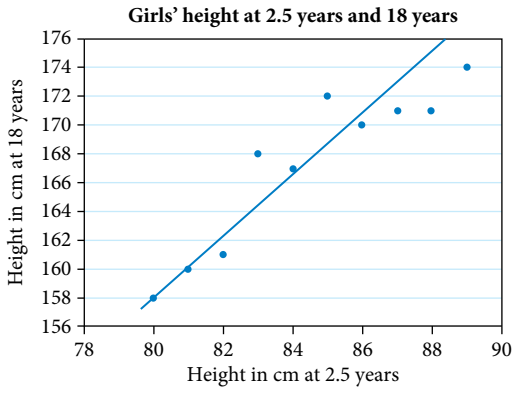
Graph this bivariate data on a scatterplot and draw a line of best fit for it.

Steps

Graph the data first.

Then draw a line through the middle of the points with roughly the same number of points above and below the line.

Working



You can find the equation of the line of best fit using technology.

EXAMPLE 2 Equation of the line of best fit*

a For the height data in Example 1, **determine** the equation of the line of best fit:

i using a scientific calculator **ii** using a spreadsheet

b Using the equation of the line, **identify**:

i the gradient **ii** the *y*-intercept

Steps **Working**

a i

Operation	Casio scientific	Sharp scientific
1 Start statistics mode.	MODE STAT *A+BX*	MODE STAT LINE
2 Clear the statistical memory.	SHIFT 1 Edit, Del-*A*	2nd F DEL
3 Enter data.	SHIFT 1 Data to get table Enter in *X* column 87 = 83 =, etc. Enter in *Y* column 171 = 168 =, etc. AC to leave table	87 2nd F STO 83 M+ 171 2nd F STO 168 M+ etc.
4 On these calculators, the gradient is *b* and the *y*-intercept is *a* (*b* = 1.709 ..., *a* = 22.781 …)	SHIFT 1 Reg *b* = SHIFT 1 Reg *a* =	ALPHA *b* = ALPHA *a* =

Another name for the line of best fit is 'regression line', and 'Reg' on the calculator stands for regression. The calculator expresses the equation of the line in the form $y = bx + a$.

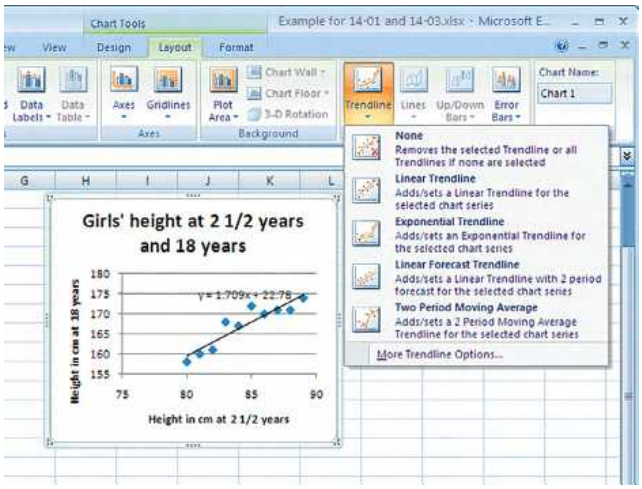
So, the line of best fit using the calculator has the equation $y = 1.709x + 22.78$.

You need to remember that on the calculator, *a* is the *y*-intercept and *b* is the gradient, not the same as $y = mx + c$.

ii Enter the table of values into a spreadsheet, then follow these instructions:

1 Graph the data on a scatterplot.

2 Select the graph and select **Trendline** from the **Layout** menu.



With a spreadsheet, you don't need to remember how to write the equation. It is given on the spreadsheet.

3 Select **Linear Trendline** and the line will appear on the graph.

4 Select **More Trendline Options** from the **Trendline** menu, then select **Display equation on the chart**.

The spreadsheet equation for the line of best fit is $y = 1.709x + 22.78$.

Note: The calculator and spreadsheet use a formula to calculate the line of best fit, but when you draw a line of fit by hand, you have to position the line 'by eye'. It's unlikely that you will position the line in exactly the same place that the technology does. This means that your line will be a little different from the line drawn using technology.

b i The gradient of the line is the number in front of the x in the equation.

The gradient of the line of best fit is 1.709.

ii The y-intercept of the line is the constant (the number on its own) in the equation.

The y-intercept of the line of best fit is 22.78.

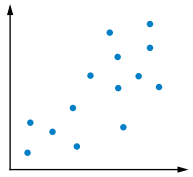
EXERCISE 16.1 Drawing a line of best fit

ANSWERS p. 459

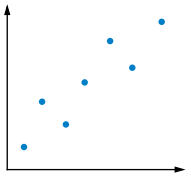
Keep your answers to this set of exercises. You will be using them again in Exercise 16.3.

1 EXAMPLE 1 Copy or print each scatterplot and draw a line of best fit. You can print a copy of the graphs from the worksheet 'Scatterplots' from Nelson MindTap.

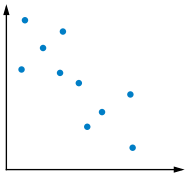
a



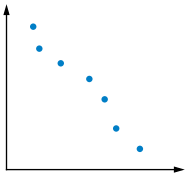
b



c



d



Worksheet
Scatterplots

2 This table shows the birth rate and female life expectancy for a number of countries. The birth rate is the number of births per year per 1000 of the population. Life expectancy is measured in years.

Country	Birth rate	Female life expectancy
Australia	13.3	85.5
Brazil	14.7	79.8
Canada	11.2	84.6
Fiji	21.4	76
Germany	8.1	83.5
Iraq	31.0	72
Kenya	36.1	61
Laos	28.0	69.5
Nepal	24.3	70
Niger	50.0	57
Rwanda	42.1	61

a Draw a scatterplot of female life expenctancy against birth rate.

b Draw in a line of best fit for this data.

3 This table shows the height above sea level and the average annual rainfall for some places in Australia.

City	Height above sea level (m)	Mean annual rainfall (mm)
Alice Springs	581	282
Ballarat	432	694
Hobart	24	616
Kalgoorlie	387	266
Mount Isa	365	463
Norfolk Island	73	1017
Perth	15	736
Winton	188	382

a Draw a scatterplot of mean annual rainfall against height above sea level.

b Draw a line of best fit for this data.

4 EXAMPLE 2 C

a Use technology to find the equation of the line of best fit for the birth rate and life expectancy data from Question **2**.

b Using this equation, state:

i the gradient of the line of best fit

ii the y-intercept of the line of best fit.

5 a Use technology to find the equation of the line of best fit for the height and rainfall data from Question **3**. C

b Using this equation, state:

i the gradient of the line of best fit

ii the y-intercept of the line of best fit. C

Fairfaxphotos/Wayne Taylor

6 This table shows information about the amounts of energy, carbohydrate and fat contained in 100 g of some takeaway foods.

Food	Energy (kilojoules)	Carbohydrate (grams)	Fat (grams)
Hamburger	1030	26.6	9.1
Cheeseburger	1070	23.9	11.2
Chicken burger	921	20.2	10.1
Fish burger	988	24.5	10.2
Grilled chicken wrap	771	13.4	10.2
Chicken salad	325	5.5	3.4
Egg and bacon wrap	767	15.3	9.1
Hash browns	1150	26.4	17.2
Chips	1480	39.7	19.0

a Draw a scatterplot of carbohydrate against energy.

b Draw a line of best fit for this data.

c Use technology to find the equation of the line of best fit. C

d Using this equation, state:

i the gradient of the line of best fit

ii the y-intercept of the line of best fit. C

e Draw a scatterplot of fat against energy and a line of best fit for this data.

f Use technology to find the equation of the line of best fit. C

g Using this equation, state:

i the gradient of the line of best fit

ii the y-intercept of the line of best fit. C

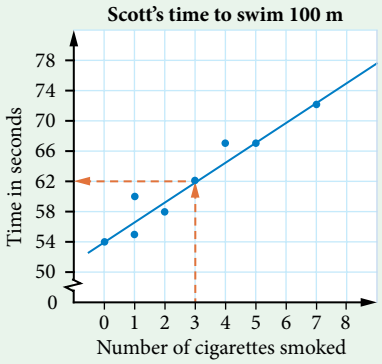
h State one similarity between these 2 lines of best fit.

16.2 Using a line of best fit*

Lines of best fit show the association between variables and you can use them to make predictions within the range of the data. This is called **interpolation**.

EXAMPLE 3 Interpolation

Scott is a competitive swimmer. He noticed that his times for the 100 m freestyle event are slower when he smokes cigarettes in the days before the event. He displayed his times for his last 8 events and the numbers of cigarettes he smoked in the 3 days before on a scatterplot. He drew a line of best fit through the data.



Master1305/Shutterstock.com

a **Identify** the independent variable.

b **Identify** the dependent variable.

c **Describe** what the line of best fit shows about the relationship between these 2 variables.

d **Use** the line of best fit to predict Scott's time for the 100 m freestyle when he has smoked 3 cigarettes in the 3 days before the event.

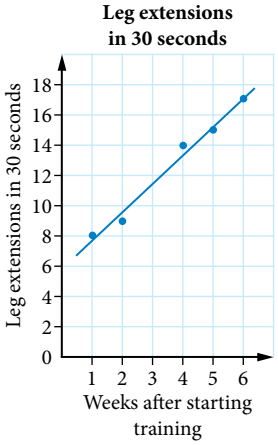
Steps	Working
a Independent variable is on the horizontal axis.	The independent variable is the number of cigarettes smoked.
b Dependent variable is on the vertical axis.	The dependent variable is the swimming time.
c Describe the relationship.	As the number of cigarettes smoked increases, Scott's time for the 100 m increases.
d Find 3 on the horizontal axis, go up to the line and then across to the time.	Scott's time will be approximately 62 seconds.

Video
Line of best fit

Worksheet
Lines of fit

EXERCISE 16.2 Using the line of best fit* ANSWERS p. 459

1 EXAMPLE 3 Anita is trying to strengthen her quadriceps muscles. This graph shows the number of leg extensions she was able to complete in 30 seconds each week after she started training.



Dusan Petkovic/Shutterstock.com

a What is the independent variable?

b What is the dependent variable?

c What does the line of best fit show about the relationship between these 2 variables?

d Anita forgot to record the number of leg extensions she could do in week 3. Use the line of best fit to predict this number.

2 Humanitarian agencies send people to assist when a natural disaster occurs. This scatterplot shows the number of medical staff one small agency sent to assist after earthquakes and floods occurred in different parts of the world, and the estimated number of victims in each disaster.

a What is the independent variable?

b What is the dependent variable?

c Use the line of best fit to estimate the number of medical staff the agency will send to a disaster with an estimated 200 victims.

d The agency sent 180 medical staff to assist in a disaster. Approximately how many victims were affected by the disaster?

e Describe the relationship between the number of medical staff and the number of victims.

f Kyle noted that the greater the number of medical staff, the more victims there were requiring help. He argued that if the agency decreased the number of medical staff, there would be less people needing help. What is wrong with Kyle's argument?

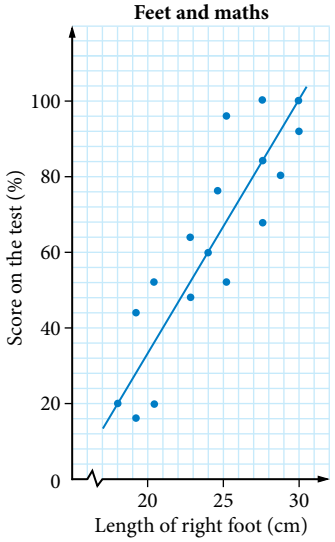
Stringer/Getty Images News/Getty Images

3 Billy announced, 'People with big feet are better at maths than people with small feet'. He had given a maths test to a large sample of students in the school library and measured the length of each person's right foot. He displayed the results on this scatterplot.

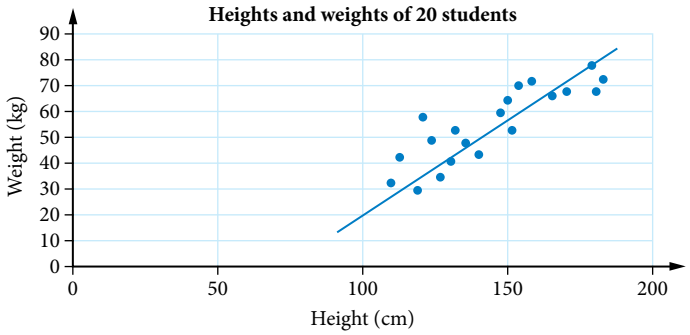
a According to Billy's line of best fit, what is the right foot length of a person who scored 58 on the maths test?

b Describe the relationship between the length of the right foot and the score on the test as shown by this line of best fit.

c Big feet don't cause high maths scores. How can Billy's results be explained?



4 This graph shows the scatterplot for the heights and weights of 20 students with a line of best fit drawn on it.

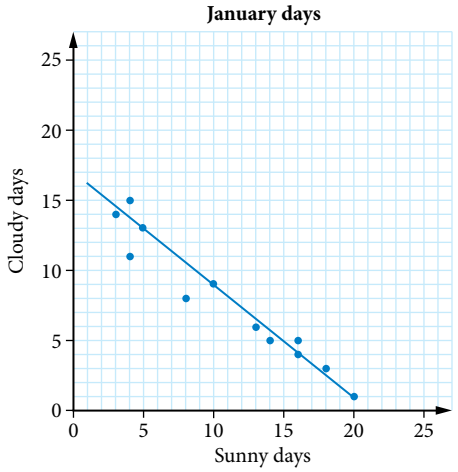


a Describe the relationship between the height and weight of students as shown by this line of best fit.

b Use the line of best fit to estimate the weight of a student who is 150 cm tall.

c Does an increase in height *cause* an increase in weight? Why or why not?

5 Yasmina graphed the number of sunny days in January against the number of dry cloudy days in January for 12 different Australian places. She then drew a line of best fit on her scatterplot.



a Use the line of best fit to estimate how many dry cloudy days there would be if there were 7 sunny days.

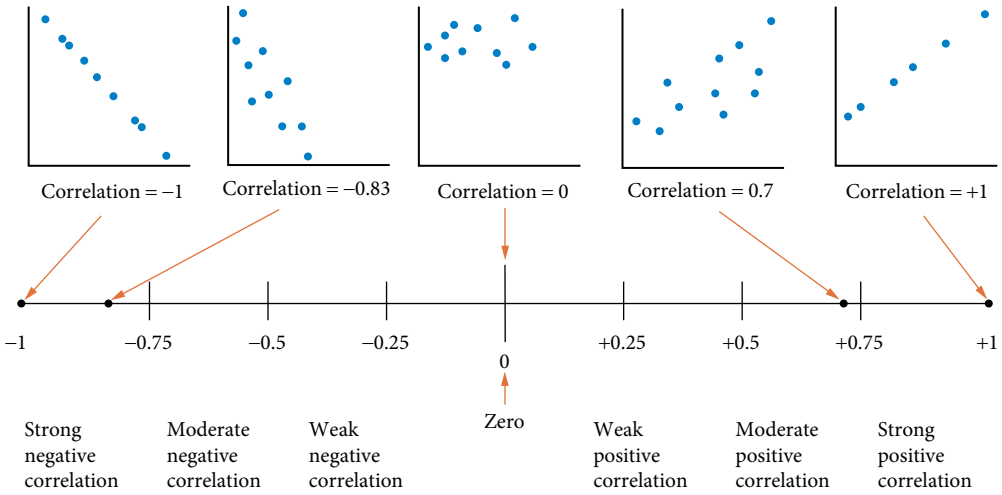
b Describe the relationship between the number of sunny days and the number of cloudy days in January.

c Suggest a reason for this relationship.

16.3 Correlation*

You learned about the **association** between 2 variables in Chapter 14, *Scattering the data*. The association can be positive or negative, linear or non-linear, strong, moderate or weak.

Correlation is a measure of how strongly 2 variables are related to each other. You can measure the strength of the relationship by calculating a value called the **correlation coefficient**. The correlation coefficient can have values from −1 to 1.



- The more the data points lie in a straight line and the stronger the association, the closer the correlation is to 1 or −1.
- The more spread out the data and the weaker the association, the closer the correlation is to 0.
- A positive association gives a positive value for the correlation coefficient.
- A negative association gives a negative value for the correlation coefficient.

You can calculate the correlation coefficient using an online calculator, scientific calculator or spreadsheet. As for standard deviation, there is a complex formula for calculating the correlation coefficient, but you are not required to use it.

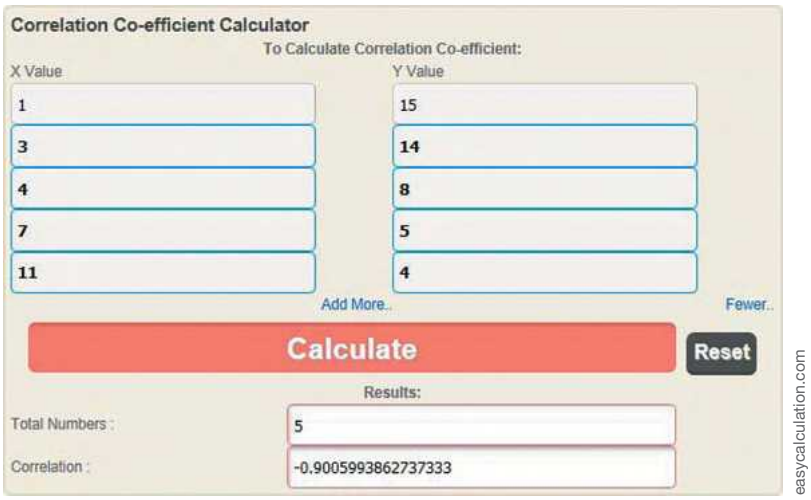
EXAMPLE 4 Calculating correlation

This table shows the number of stuffed toys owned by a group of children of different ages.
Calculate the correlation coefficient for the data.

Age	1	3	4	7	11
Number of stuffed toys owned	15	14	8	5	4

Method 1: Online calculator

Search the internet for 'correlation coefficient calculator' or use the **EasyCalculation** website.



Enter each pair of values, then click 'Calculate' to display the correlation coefficient as –0.9005...

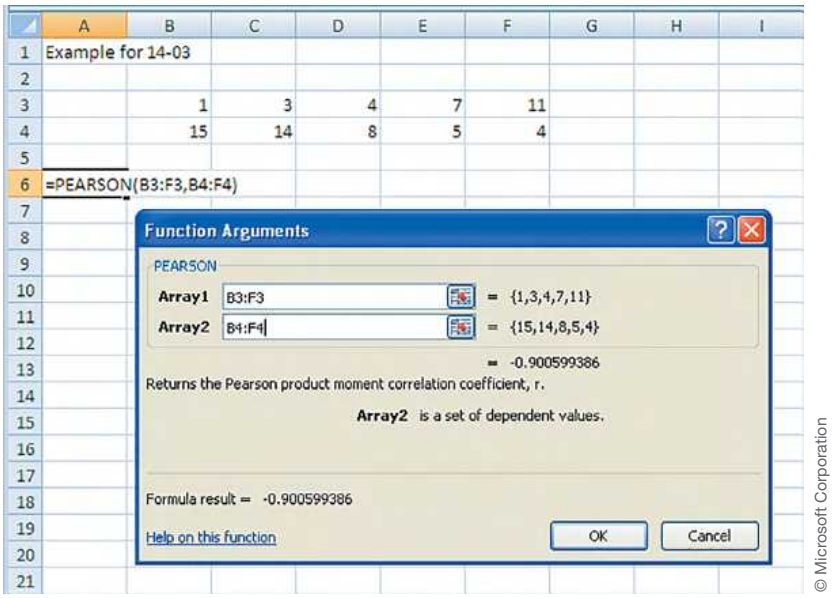
Method 2: Scientific calculator

Operation	Casio scientific	Sharp scientific
1 Start statistics mode.	MODE STAT *A*+*BX*	MODE STAT LINE
2 Clear the statistical memory.	SHIFT 1 Edit, Del-*A*	2nd F DEL
3 Enter data.	SHIFT 1 Data to get table Enter in *X* column 1 = 3 =, etc. Enter in *Y* column 15 = 14 =, etc. AC to leave table	1 2nd F STO 3 M+ 15 2nd F STO 14 M+ etc.
4 Calculate the correlation coefficient ($r = -0.9005 \ldots$)	SHIFT 1 Reg *r* =	ALPHA *r* =

Weblink Easy Calculation

Method 3: Spreadsheet

Type the data into the spreadsheet. The example shows the first row of data in cells B3 to F3 and the second set in B4 to F4.



2 Select the cell where you want to display the correlation coefficient and select 'Insert function' from the tool bar. In the **Search for a function** box, type PEARSON, then let the spreadsheet help you complete the formula to calculate the correlation coefficient, −0.9005…

> The correlation coefficient is also called the Pearson correlation coefficient, which is why the spreadsheet calls the formula PEARSON.

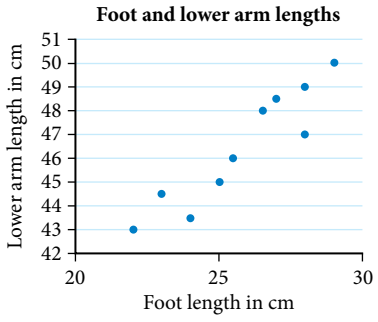
EXAMPLE 5 Using correlation

A sample of 10 people had their left foot length and left lower arm length (from their elbow to the tip of the middle finger) measured.

Left foot length (cm)	27	22	23	24	25	25.5	28	26.5	28	29
Left lower arm length (cm)	48.5	43	44.5	43.5	45	46	47	48	49	50

a **Construct** a scatterplot of this data.

b **Calculate**, correct to 2 decimal places, the correlation coefficient and describe the correlation in words.

Steps	Working
a Foot length is on the horizontal axis and arm length is on the vertical axis.	
b Calculate the correlation coefficient using one of the three methods shown in Example 4.	The correlation coefficient is 0.93. The data shows a strong, positive linear relationship between the length of the left foot and the length of the lower left arm.

EXERCISE 16.3 Correlation*

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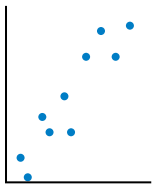
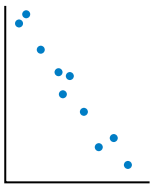
Keep your answers to this exercise for the next exercise, 16.4.

1 Match each scatterplot to its correlation coefficient.

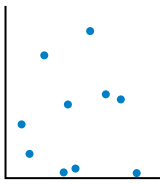
a



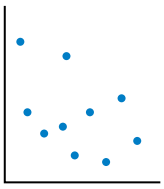
b



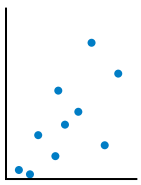
c



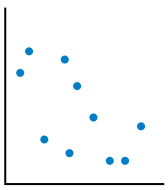
d



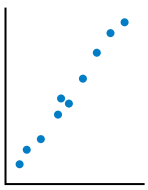
e



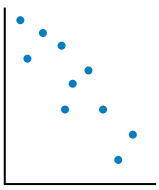
f



g



h



Correlation coefficients

A	−0.623	**B**	0.008	**C**	−0.977	**D**	−0.870
E	0.923	**F**	0.995	**G**	−0.441	**H**	0.681

2 EXAMPLE 4 Eliza works in a coffee shop. She thinks there is a relationship between the daily temperature and hot chocolate drink sales. This data comes from Test yourself 14, Question **1**.

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate drinks sold	42	15	37	24	9	20	35	20	10	30	30	4

a Construct a scatterplot to illustrate Eliza's information. If you completed Test yourself 14, you can use the scatterplot you graphed there.

b Calculate, correct to 2 decimal places, the correlation between temperature and the number of hot chocolate drinks sold.

c Is Eliza correct in thinking there is a relationship between temperature and high chocolate sales? Use your answers from parts **a** and **b** to justify your response.

d Is this a causal relationship? Does a change in one variable cause the change in the other variable?

3 Calculate the correlation coefficient for the following data in Exercise 16.1 on pages 407–408:

a Question **2** between birth rate and female life expectancy

b Question **3** between height above sea level and mean annual rainfall

c Question **6** between energy and carbohydrate content

d Question **6** between energy and fat content

e Question **6** between carbohydrate content and fat content

4 EXAMPLE 5 Vilas wanted to investigate whether the latitude of a city is related to its temperature.

City	Latitude (°S)	Mean January temperature (°C)
Adelaide	35	30
Alice Springs	23	40
Brisbane	27	31
Darwin	12	33
Hobart	43	24
Melbourne	38	27
Norfolk Island	29	25
Perth	32	32
Townsville	19	33

a Draw a scatterplot showing Vilas' data.

b Calculate the correlation coefficient for this data.

c Do you think these 2 variables are closely related? Why or why not?

d What other factors would influence the mean January temperature in a city?

5 In Exercise 14.1, Question **5** on page 371, you looked at the sales of CD and digital albums over 9 years.

a Calculate the correlation coefficient for this data.

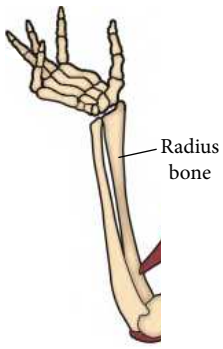
b What does this tell you about the relationship between these 2 variables?

c Is this a causal relationship? Does a change in one variable cause the change in the other variable?

6 Forensic scientists use formulas to predict a person's height from the length of the radius bone in the lower arm. All measurements are in centimetres. PS

male height = $6.650r + 80.405$

female height = $3.876r + 73.502$



a Copy and complete this table after measuring the radius bone of the members in your group.

Initials of person							
Length of radius bone (cm)							
Predicted height using the formula (cm)							
Measured height (cm)							

b Construct separate scatterplots for males and females, comparing predicted heights to measured heights.

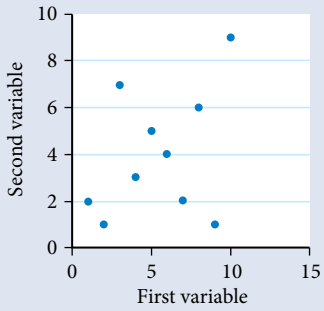
c Calculate the correlation coefficient.

d Describe the correlation.

e Suggest a reason the predicted heights may not be very accurate.

TECHNOLOGY | Changing correlations

For this table of values and scatterplot, the correlation coefficient for the variables is 0.366.
Download the spreadsheet 'Changing correlations' from Nelson MindTap.



First variable	1	2	3	4	5	6	7	8	9	10
Second variable	2	1	7	3	5	4	2	6	1	9

The spreadsheet contains the data in the above table and calculates the correlation coefficient.
Change the values in the Second variable row to produce a set of data whose correlation coefficient is:

a 0.9 **b** −0.3 **c** close to 0.

Spreadsheet
Changing correlations

16.4 Interpolation and extrapolation*

A line of best fit helps you to make **predictions** as you did in some of the questions in Exercise 16.2.
You learned that when making a prediction on information that lies *within* the range of the data, this is called **interpolation**. If you make a prediction on information that lies *outside* the range of the data, this is called **extrapolation**.

However, you need to be careful when you are making predictions from sets of data.

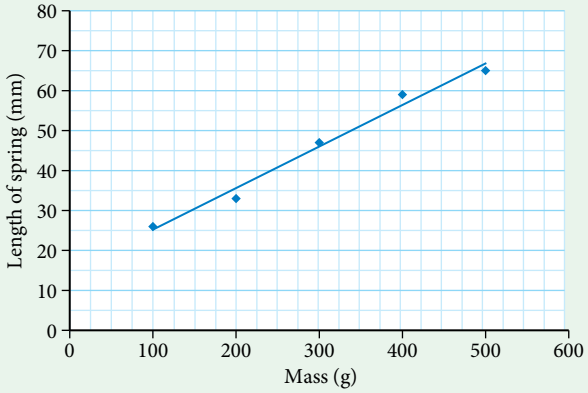
- Predictions within the range of the data can be unreliable if the association is weak. Predictions are more reliable if the association is strong and linear.
- Predictions that go outside the range of the data can be very unreliable because you can't be sure that the linear relationship continues beyond the data.

EXAMPLE 6 | Making predictions

In a science experiment, Clare and Dilani attached different weights to a spring. They recorded the length of the spring for each weight.

Mass (g)	100	200	300	400	500
Length of spring (mm)	26	33	47	59	65

They graphed the results on the scatterplot below and drew a line of best fit.



a **Use** the line to find:

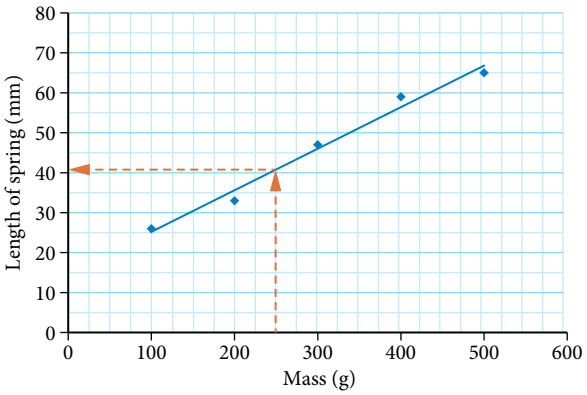
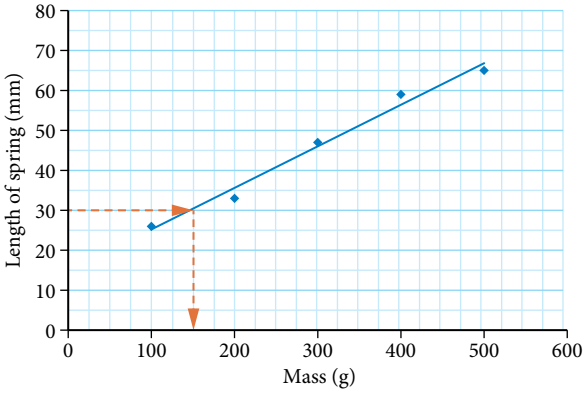
i the length of the spring for a mass of 250 g

ii the mass when the length of the spring is 30 mm.

b **Comment** on how reliable these predictions are.

c The equation of the line is $L = 0.104m + 14.8$. **Apply** the equation to **predict** the length of the spring when the attached mass is 800 g.

d **Comment** on the reliability of this prediction.

Steps	Working
a **i** Find 250 g on the horizontal axis, go up to the graph and across to length.	The length of the spring is approximately 41 mm. 
ii Find 30 mm on the vertical axis, go across to the line and down to mass.	The mass is approximately 150 g. 
b These predictions are within the given data.	These predictions are reasonably reliable as the association is strong and linear.
c Substitute $m = 800$ into the formula	$L = 0.104m + 14.8$ $= 0.104 \times 800 + 14.8$ $= 98$ The length of the spring with 800 g attached is approximately 98 mm.
d A mass of 800 g is outside the range of the data shown on the graph, so it is an example of **extrapolation**.	The prediction is not necessarily reliable as you don't know if the relationship continues to be linear. It is also possible that at some point the spring might break.

The strength of the correlation also indicates whether predictions on the data will be reliable. The closer the coefficient is to –1 or 1, the more reliable the prediction will be.

EXAMPLE 7 Reliability of predictions

This student data comes from Chapter 14, *Scattering the data*, Example **1** on page 369.

Total height, H cm	175	177	178	184	162	172	173	191	163	175
Waist height, W cm	101	106	107	115	100	87	106	120	90	108

a **Calculate** the correlation coefficient for this data.

b If you used this data to predict the waist height of a person 170 cm tall, how reliable would it be?

c If you used this data to predict the waist height of a person 130 cm tall, how reliable would it be?

Steps	Working
a Calculate the correlation coefficient using technology as shown on page 413–414.	$r = 0.81$
b The heights in the table range from 162 to 191 cm, so 170 cm is within the data range: this is interpolation.	As the correlation coefficient is quite close to 1 and 170 cm is within the data range, this prediction would be reasonably reliable.
c 130 cm is outside this data range: this is extrapolation.	The prediction may not be reliable as 130 cm is well outside the given data.

EXERCISE 16.4 Interpolation and extrapolation*

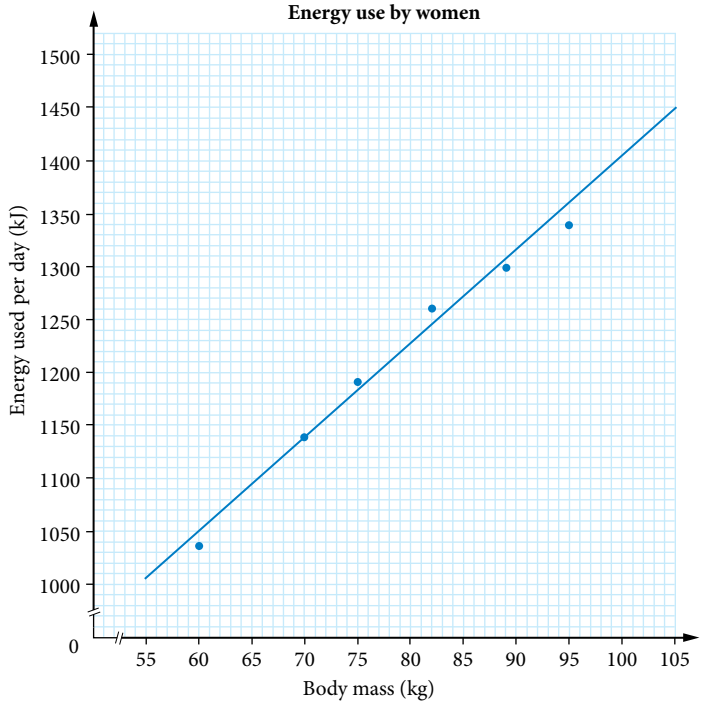
ANSWERS p. 460

This exercise requires some of your answers from Exercises 16.1 and 16.3.

1 EXAMPLE 6 Jasmine recorded the body mass of a sample of women at the gym and the amount of energy they use per day.

Body mass (kg)	60	70	75	82	89	95
Energy used (kJ)	1040	1140	1190	1260	1300	1340

She graphed the data on a scatterplot and drew a line of best fit.



a Use the line of best fit to find:

i the daily amount of energy used by a woman of mass 65 kg

ii the body mass of a woman who uses 1450 kJ of energy per day.

b How reliable are these predictions? Justify your answer.

c The equation of the line is $E = 16.9B - 143$, where B is the body mass and E is the energy used. Use it to predict the energy used by a woman of mass 120 kg.

d Would you expect this situation to continue to be a linear relationship beyond the data points? Why or why not?

2 In Exercise 16.1, Question **2** you drew a scatterplot on the birth rate (B) and female life expectancy (E) in a number of countries. A line of best fit for this data has equation $E = -0.725B + 91.2$.

a Predict the female life expectancy in a country where the birth rate is 40.

b Determine the birth rate in a country where female life expectancy is 65 years.

c How reliable are these predictions? Justify your answer.

d What would be the female life expectancy in a country where the birth rate is 60?

e How reliable do you think estimates outside this data would be? Justify your answer.

3 In Exercise 16.1, Question **3**, you drew a scatterplot for the height above sea level (H) and the mean annual rainfall (R) for a number of cities. A line of best fit for this data has equation $R = -0.769H + 755.5$. PS

a Predict the mean annual rainfall for a city 300 m above sea level.

b Determine the height above sea level for a city that receives 400 mm of rain per year.

c How reliable are these predictions? Justify your answer.

d Calculate the mean annual rainfall for a city 800 m above sea level.

e Predict the mean annual rainfall for a city 1000 m above sea level. Is this possible? What is wrong with this prediction?

4 EXAMPLE 7 Mr Armstrong, the Science teacher, sets up an experiment to measure the pressure of a gas at different temperatures. This table shows the results:

Temperature (°C)	10	20	30	40	50	60	70	80	90
Pressure (g/cm³)	27.9	30	29.8	32.1	31.9	34.1	33.8	34.8	36.6

a Calculate the correlation coefficient for this data.

b Mr Armstrong used this data to estimate the pressure at 56°C. How reliable is this estimate?

c Mr Armstrong used this data to predict the pressure at −10°C. How reliable is this estimate?

5 Ms Cranston has a set of results for her class on 2 Maths tests.

Student	Bill	Ruth	Mary	Ella	Greg	Jim	Meg	Tara	Bob	Clem	Bree	Amy
Algebra test	60	38	65	??	75	48	67	23	82	16	92	80
Data test	35	21	47	31	56	40	54	11	59	20	62	??

Ella and Amy missed a test and Ms Cranston wants to give them an estimate.

a Draw a scatterplot for this data (except for Ella and Amy) and draw a line of best fit.

b Predict the results that Ella and Amy might have received in the test they missed.

c Calculate the correlation coefficient for this data.

d Given this correlation coefficient, how reliable do you think these predictions are? Justify your answer.

6 Ryan investigated the heights (x cm) of a group of his friends and their hand spans (y cm, the maximum distance y between the tips of their thumb and little finger). **PS**

Height (x cm)	170	178	160	183	168	145	155
Hand span (y cm)	20	21	19	22	20	17	19

a Construct a scatterplot for the data and draw a line of best fit on the plot.

b A line of best fit has equation $y = 0.12x - 0.02$. Use the equation to predict the hand span of Izak, who is 165 cm tall. Answer correct to the nearest centimetre.

c Robert Wadlow was the tallest man in the world. His height was 272 cm (see photo below of his statue at a London museum). Use the equation of the line of fit to predict his hand span, correct to the nearest centimetre, and explain why this measurement is unlikely to be correct.

Gary Wilkinson/Alamy Stock Photo

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

Extended Investigation

Part A

Body circumferences

This section is the same as the Investigation in Chapter 14, *Scattering the data,* on page 373. If you completed that Investigation, you could skip Part A and use your work from that Investigation. Otherwise, you need to complete Part A.

For each student in your class, record the following measurements correct to the nearest cm:

- wrist circumference
- neck circumference
- waist circumference.

You can use a spreadsheet for this Investigation if you want.

Construct scatterplots for:

- wrist circumference and neck circumference
- wrist circumference and waist circumference.

Describe any relationships you can see in each scatterplot.

Part B

For each scatterplot in Part A:

1 add a line of best fit

2 find the equation of the line of best fit

3 calculate the correlation coefficient for the data

4 describe the association between the two variables.

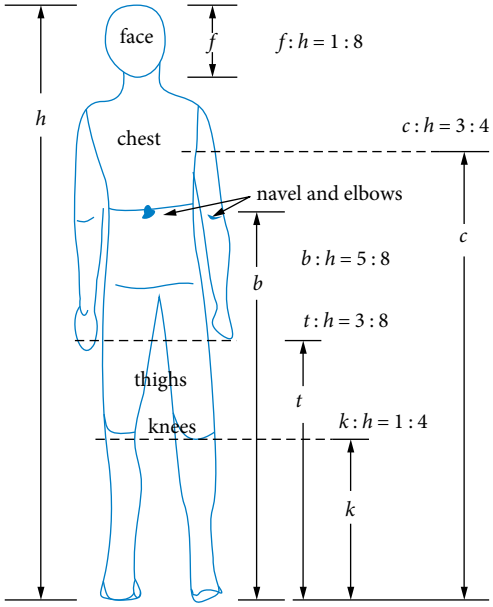
Make up 2 questions, one that requires interpolation and one that requires extrapolation, for each scatterplot.

Get another student to answer these questions and record their answers.

How reliable are these answers?

Part C

In Year 11, Chapter 5, *Colourful ratios*, you looked at body part ratios.



In this part of the Investigation, you will use the measurements of students in your class to see how accurate these ratios are.

1 Choose one of the ratios to investigate, for example, $k : h = 1 : 4$.

2 Record the measurements you have chosen for each student in your class, correct to the nearest cm.

3 Construct a scatterplot for your data.

4 Add a line of best fit to your scatterplot and find its equation.

5 Calculate the correlation coefficient for the data.

6 Describe the association between the 2 variables you have chosen.

Write a report on your findings and comment on whether your data agrees with the given ratio.

Keyword activity

CHAPTER SUMMARY

ANSWERS p. 460

Use this list of words to complete this summary of the chapter below.

bivariate	correlation	correlation coefficient	equation
extrapolation	interpolation	line of best fit	predictions
scatterplot	technology		

In this chapter, I have continued my study of [1]_____ data. I graphed such data on a [2]_____ and drew a [3]_____ _____ _____ _____ through them 'by eye'. I can also use [4]_____ to graph the line and find its [5]_____.

Two variables can have a strong association between them, also called the [6]_____. I can also enter a table of values into a calculator or spreadsheet to calculate the [7]_____ _____.

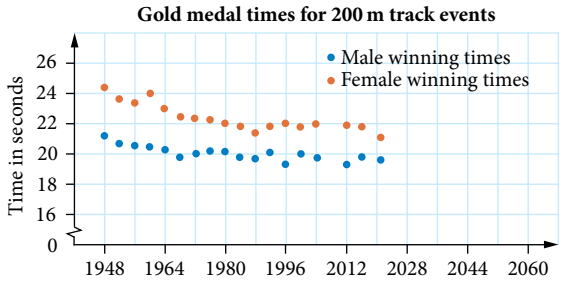
I can use both the lines of best fit and the correlation coefficient to help me make [8]_____ from the data. When I make estimates from within the data range, this is called [9]_____ and it is reliable if there is a high correlation between the data. When I make estimates outside the range of the data, this is called [10]_____ and can often be unreliable or impossible.

Solution to the chapter problem

Problem

Kaylene is a competitive runner. She studies the past winning times for the women's Olympic 200 m race, and is surprised by the rate at which the times have improved. The women's winning times are slower than those of the men in their 200 m event, but the women seem to be catching up. She wondered whether women would ever run the event in the same or faster time than the men.

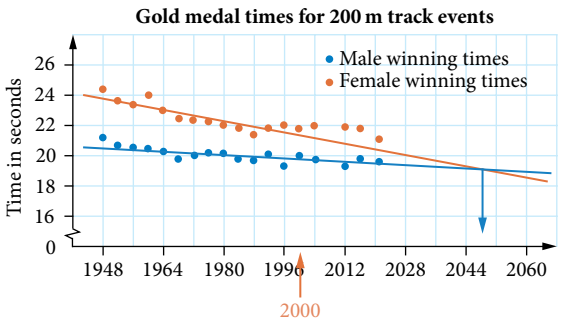
This graph shows the gold medal times for men and women's 200 m track events at the Olympic Games since 1948.



Will the women ever run the 200 m race faster than the men?

Jacob Lund/Shutterstock.com

Solution

STAGE 1: WHAT IS THE PROBLEM? WHAT DO WE KNOW?	
 WHAT?	• In the future, will women run 200 m faster than men? • We are given a graph of gold medal times for men and women's 200 m track events in the Olypmic Games since 1948.
STAGE 2: SOLVE THE PROBLEM	
 SOLVE	Add a line of best fit for men and for women. These lines intersect at around 2048.  If the linear model is valid, men and women will run the 200 m event in the same time in 2048 and after that women will be faster.
STAGE 3: CHECK THE SOLUTION	
 CHECK	This is unlikely. The year 2048 is a long way beyond the data, and the data cannot be linear forever. If the linear graph continues, there will come a year when the gold medal time is 0 or negative! It is more likely that both sets of times will level off.
STAGE 4: PRESENT THE SOLUTION	
 PRESENT	It is unlikely that, in the future, women will run faster than men over 200 m. It is unreliable to make a prediction based on a linear relationship that far outside the range of the data.

16 Test yourself

Fitting the data

ANSWERS p. 460

1 EXERCISE 16.1 A medical team is developing a new pain management treatment. In trials, they record the dosage given and the duration of relief.

Dosage (mL)	6	2	4	8	2	8	6	10
Duration of relief (h)	14	9	12	24	5	18	16	22

a Graph this bivariate data in a scatterplot and draw a line of best fit for this data.

b Use technology to find the equation of the line of best fit.

2 EXERCISE 16.2 For the data from Question **1**:

a what is the independent variable?

b what is the dependent variable?

c how many hours of relief could be expected if the dosage given was 5 mL?

d predict the dosage given if the relief lasted 10 hours.

e describe the relationship between the dosage given and the duration of the relief.

3 EXERCISE 16.3 Sketch a scatterplot to illustrate a set of data that has:

a a weak, positive correlation

b a strong, negative correlation

c no correlation.

4 EXERCISE 16.3 This data from Chapter 14, *Scattering the data,* compares the hours of homework and the hours of watching TV per week for a group of 15 students.

Student	Hours of homework per week	Hours watching TV per week
Amy	6	2
Joe	6	10
Annika	23	4
Janine	7	14
Stephen	23	9
Thanh	2	5
Gillian	17	4
Vamsee	1	3
Lalaja	16	20
Darryl	10	10
Lyn	15	0
Jeremy	9	4
Ben	0	17
Abdul	3	9
Miriam	4	1

a Calculate the correlation coefficient for this data.

b Do you think these 2 variables are closely related? Why or why not?

5 EXERCISE 16.4 A group of 12 students measured their heights and length of stride in centimetres. A stride is the biggest step a person can take from a standing position, measured from toe to toe.

Height (cm)	165	140	180	176	160	164	178	170	148	157	150	162
Length of stride (cm)	105	85	111	104	98	95	108	102	92	97	88	100

a Draw a scatterplot for this data and draw a line of best fit.

b Calculate the correlation coefficient for this data.

c Use your line of best fit to predict:

i the length of stride of Natalie who is 155 cm tall

ii the height of Emir who has a stride length of 90 cm.

d How reliable are these predictions? Justify your answer.

e Use technology to find the equation of the line of best fit.

f Use the equation to predict:

i the length of stride of Zhang, who is 200 cm tall

ii the height of Martika, who has a stride length of 80 cm.

g How reliable are these predictions? Justify your answer.

Practice set 4

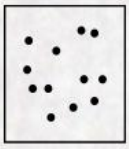
ANSWERS p. 461

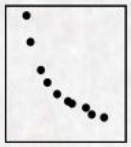
You need to download from Nelson MindTap the 'Reducible loans' spreadsheet for Question **1** of Section B.

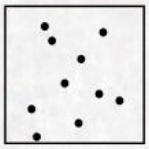
Section A Multiple choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 EXERCISE 16.1 On which of the diagrams shown would we be able to draw a line of best fit?

A 

B 

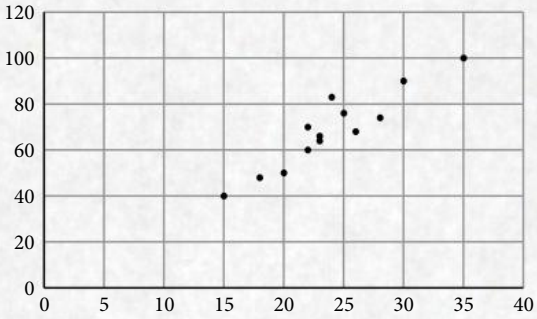
C 

D 

2 EXERCISE 13.2 Given the equation $y = -x - 4$, what is the value of y when $x = -3$?

A -1 **B** -7 **C** 7 **D** 12

3 EXERCISE 14.1 Which statement below is true for this scatterplot?



A As one variable increases, the other variable increases.

B As one variable increases, the other variable decreases.

C There are groups of points.

D Most of the points are together, but a few are out on their own.

4 EXERCISE 15.1 Nina is repaying a loan. The progress of her loan is shown below.

Nina's loan				
Amount borrowed: \$12 000				
Interest rate: 6% p.a. reducible				
Half-yearly repayments: \$1070				
Half-year	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	\$12 000	\$360	\$12 360	\$11 290
2	\$11 290	\$338.70	\$11 628.70	\$10 558.70
3	\$10 558.70	\$316.76	\$10 875.46	\$9 805.46
4	\$9 805.46	\$294.16	\$10 099.62	\$9 029.62
5	\$9 029.62	\$270.89	\$9 300.51	\$8 230.51
6	\$8 230.51	\$246.92	\$8 477.43	\$7 407.43

How much does Nina owe immediately after her 3rd repayment?

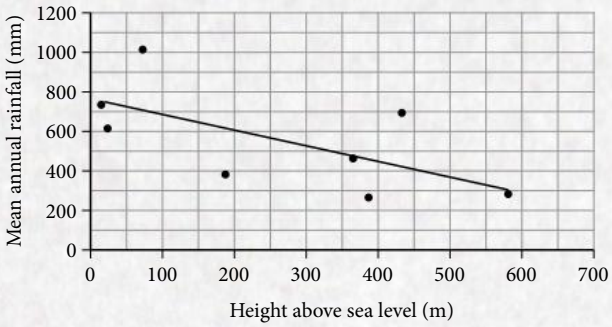
A \$10 875.46 **B** \$10 558.70 **C** \$9805.46 **D** \$9029.62

5 EXERCISE 13.3 When graphed, which linear equation below gives a line with a gradient of 3?

A $y = x + 3$ **B** $y = 3 - x$ **C** $y = 3x$ **D** $y = 3$

Spreadsheet
Reducible loans

6 EXERCISE 16.4 This scatterplot and line of best fit show the relationship between the height above sea level and the mean annual rainfall for some Australian towns.

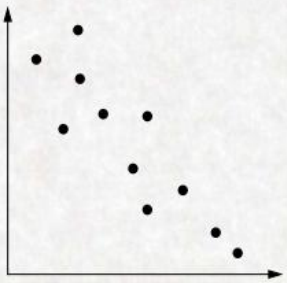


Use the line of best fit to predict the mean annual rainfall for a town with a height above sea level of 400 m, and the height above sea level of a town with a mean annual rainfall of 600 mm.

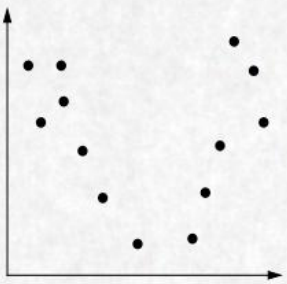
A 450 mm, 220 m
B 490 mm, 250 m
C 450 mm, 250 m
D 490 mm, 220 m

7 EXERCISE 14.2 Which scatterplot shows an association that is moderate, linear and positive?

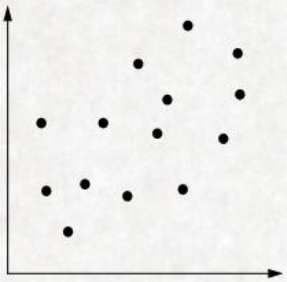
A



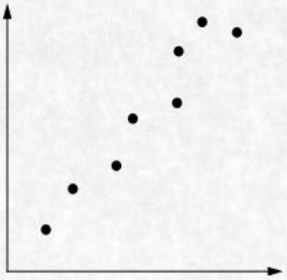
B



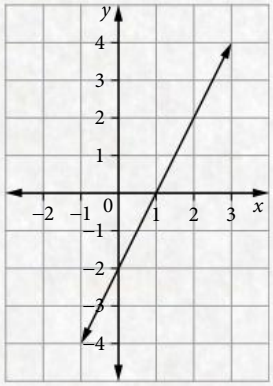
C



D



8 EXERCISE 13.2 What is the equation of this line?



A $y = -x + 2$
B $y = x - 2$
C $y = 2 - 2x$
D $y = 2x - 2$

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Section B Short answer questions

1 EXERCISE 15.3 You need the 'Reducible loans' spreadsheet from Nelson MindTap for this question.
Anton borrowed $13 000 at 5% p.a. reducible interest to buy a car. His monthly repayments are $470.

a How much interest will Anton pay in the first month?

b How much will he owe immediately after the 15th month?

c How long will it take Anton to repay the loan?

d What is the value of his last repayment?

e How much interest will Antony pay on the loan?

2 EXERCISE 13.2 Copy and complete each table of values for the given equation.

a $y = 4x - 3$

x	−4	−2	0	4	8
y					

b $x + y = 9$

x	−3	−1	0	2	5
y					

3 EXERCISES 14.1, 14.2 Soon Yi collected some data from students at her school about how many hours per week they spent playing sport and how many hours per week they spent watching TV. This table shows each student's response.

Sport (hours)	5	6	17	9	3	13	17	5	4	0	10
TV (hours)	8	6	1	2	10	0	2	9	9	10	2

a Present this bivariate data in a scatterplot.

b Comment on the features of the scatterplot.

c Describe the association between the variables.

4 EXERCISE 13.3 Graph each linear function.

a $y = x - 3$ **b** $y = 3 - x$

5 EXERCISE 15.4 For this question, use an appropriate online calculator such as the mortgage calculator at moneysmart.gov.au.

Joshua borrows $450 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 30 years at 5.5% p.a. reducible interest. He will also be charged a $5 monthly account-keeping fee.

a How much are Joshua's monthly repayments?

b How much will Joshua pay in interest and fees?

6 EXERCISES 16.1, 16.2 In question **3**, Soon Yi surveyed students at her school on how many hours per week they spent playing sport and how many hours per week they spent watching TV.

Sport	5	6	17	9	3	13	17	5	4	0	10
TV	8	6	1	2	10	0	2	9	9	10	2

a Using the scatterplot you drew in question **3**, draw a line of best fit for this data.

b What does the line of best fit show about the relationship between these 2 variables?

c Use the line of best fit to predict how many hours of TV a person who plays 8 hours of sport would watch.

d Use the line of best fit to predict how many hours of sport a person who watches 3 hours of TV would play.

7 EXERCISES 16.3, 16.4

a Calculate the correlation coefficient for the sport vs TV data in question **6**, correct to 3 decimal places.

b Do you think these 2 variables are closely related? Why or why not?

c Are the predictions made in questions **6 c** and **d** reliable? Why or why not?

8 EXERCISE 14.3 Each pair of variables below have a strong association. For each pair:

i decide if a change in one variable causes a change in the other variable

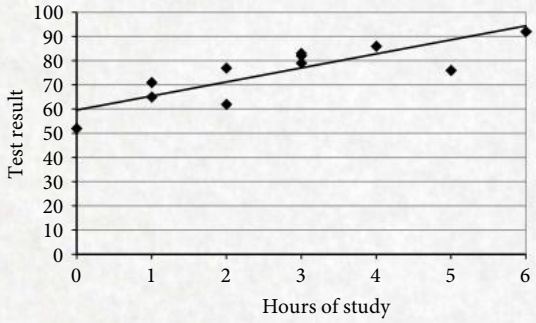
ii if there is no causal relationship, suggest other factors that might result in the variables having an association.

a The length of the right arm and the length of the left arm of the same person

b The price of a brand of sports shoes and the number of pairs sold

c The hours of exercise per week for a person and the person's fitness level

9 EXERCISE 16.4 Harry conducted a survey of 11 students where he asked each student how many hours they studied and their test result. This scatterplot show the results of his survey with a line of best fit for the data. The correlation coefficient for this data is 0.816.



a Use the line of best fit to predict:

i the test result for a student who studied for 3.5 hours

ii the hours of study for a student who scored 75 in the test

b How reliable are these predictions?

c Would you expect this situation to continue to be a linear relationship beyond the data points? Why or why not?

10 EXERCISE 14.4 This table compares the heights and resting pulses of a sample of 15 students.

a Would you expect there to be an association between these 2 variables? Why or why not?

b Construct a scatterplot for this data.

c Describe any features of the scatterplot.

Height (cm)	Resting pulse (beats per minute)
176	64
178	66
151	95
168	76
186	60
187	74
149	70
174	62
172	76
177	64
169	60
159	83
169	68
163	77
163	75

11 EXERCISE 15.1 Rose and Ian take out a personal loan to pay for an overseas holiday. They borrow $18 000 at 8.4% p.a. with monthly repayments of $475. Copy and complete this table, showing the first 4 months of their loan.

Rose and Ian's loan				
Amount borrowed: $18 000				
Interest rate: 8.4% p.a. monthly reducible				
Monthly repayments: $475				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$18 000	$126	$18 126	$17 651
2				
3				
4				

12 EXERCISE 13.4 Year 12 is running a fundraising sausage sizzle at the athletics carnival. It will cost them $32 for the gas bottle for the barbecue and each sausage in bread will cost $0.90 to make.

a Copy and complete this table.

Number of sausages in bread, n	0	10	20	30	40	50	100	150	200
Cost, $C	32	41							

b Write a linear function to calculate the cost, C, of making n sausages in bread.

c Construct a graph showing the cost of making n sausages in bread.

d What is the vertical intercept of this line? What does this value represent?

e What is the gradient of this line? What does the gradient represent?

Answers

CHAPTER 1

EXERCISE 1.1

1 240 cm

2 **a** 20 cm **b** 120 cm **c** 6 cm
d 63 cm **e** 73.5 cm **f** 550 cm

3 **a** 140 cm **b** Teacher to check.

4 705 cm

5 **a** length = 2820 cm, width = 940 cm, height = 1410 cm
b length = 28.2 m, width = 9.4 m, height = 14.1 m

6 14.4 m

EXERCISE 1.2

1 **a** 1000, × **b** 10, ÷ **c** 100, ÷
d 1000, ÷ **e** 100, × **f** 1000, ÷
g 10, ×

2 **a** 30 **b** 500 **c** 0.4 **d** 2000
e 3 **f** 2 **g** 0.5 **h** 0.25
i 0.6 **j** 600 **k** 4.5 **l** 800
m 0.8 **n** 9000 **o** 0.09 **p** 6500

3 **a** 640 m **b** 64 000 cm

4 **a** 850 m **b** 0.85 km

5 **a** 45 000 cm **b** 4.8 km
c 1 300 000 mm **d** 0.4175 km

6 6 km

7 76 m

8 **a** 3900 m **b** 3.9 km **c** 90 laps

9 21 m

10 160

11 **a** 3340 m **b** 3.34 km

12 **a** 3096.2 m **b** 3.10 km

13 $120.60

14 **a** cm **b** m **c** km
d mm **e** m **f** cm

15 **a** 128.5 cm
b Luke will need less than 6 × 128.5 cm because he can cut 4 seat backs from the waste material of 2 chairs.
c Luke will save $303.60.

EXERCISE 1.3

1 **a** 24 m **b** 20 m **c** 34 m

2 30 m

3 **a** 9.0 cm **b** 9.2 cm

4 Any 2 lengths that add to 18 m, e.g. 12 m by 6 m, or 9 cm by 9 cm

5 **a** 8.4 cm **b** 7.5 m

6 **a** 21 m **b** $630

EXERCISE 1.4

1 **a** 86 cm **b** 21 mm **c** 12 m **d** 32 cm

2 **a** 45 mm **b** 31.5 cm **c** 240 m **d** 18.4 km

3 38.4 m

4 98.4 mm

5 **a** 520 cm **b** 83.3 mm

6 6 m

7 **a** rectangle **b** 800 mm
c 950 mm **d** 1250 mm
e The length of the join between the blue and white sections isn't part of the flags perimeter, but it is included in the perimeter of each separate section.

EXERCISE 1.5

1 **a** 94.2 cm **b** 56.5 mm
c 19.5 km **d** 55.0 m

2 **a** 15.1 mm **b** 7.0 mm **c** 66.8 cm

3 **a** 203 cm **b** 44 mm

4 40 200 km

5 **a** Teacher to check. **b** 99.4 mm
c The arcs between the vertices of the coin are longer than the straight edges.

6 25

7 **a** 36 880 km **b** 1740 km
c 2500 km; the road doesn't follow a straight line.

EXERCISE 1.6

1 27.7 cm

2 **a** 4524 m **b** 3.6 laps

3 **a** 46.3 m **b** 39.3 cm **c** 28.2 cm

4 46.3 cm **5** 336 m

6 11.52 m **7** 2771 m

EXERCISE 1.7

1 **a** 30 min **b** 36 min **c** 42 min

2 **a** 8 days 10 hours **b** 4 days 18 hours

3 **a** 4 h 30 min **b** 5 h 48 min
c 7 h 24 min **d** 8 h 15 min
e 11 h $37\frac{1}{2}$ min

4 **a** 1 h 40 min **b** 8 h 15 min

5 **a** 5 days 8 h 38 min **b** 11 days 16 h 48 min

6 **a** 9 months 24 days **b** 5 months 7 days
c 8 months 9 days **d** 9 months 24 days

7 7:11 pm

8 1 year 10 months 18 days

KEYWORD ACTIVITY

1 metric **2** length **3** kilometres
4 thousandth **5** perimeter **6** circumference
7 seconds **8** hour **9** week

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TEST YOURSELF

1 4
2 **a** 50 **b** 300 **c** 3.6
d 4200 **e** 0.08
3 **a** 64 cm **b** 51 mm
4 **a** 1372 m **b** 1.372 km
5 **a** 5.6 km **b** 86.4 mm **c** 23.4 cm
6 **a** 179.1 km **b** 105.6 mm **c** 16.3 cm
7 1.21 cm
8 **a** 22.8 m **b** 19.2 cm **c** 56.2 mm
9 54.3 mm **10** 86 cm
11 23.54 mm **12** 3 days 20 h 24 min
13 15 months 21 days

CHAPTER 2

EXERCISE 2.1

1 **a** 39 **b** 73 **c** 68
2 width 25%, length 31.25%
3 **a** No, the width only shrinks 20%
b Yes, width shrinks 25%, length 29%
c Yes, width shrinks 30%, length 25%
4 **a** **i** 47 **ii** 5 **iii** 104
b **i** 41 **ii** 7 **iii** 96
c **i** 54 **ii** 5 **iii** 118
d **i** 38 **ii** 3 **iii** 82
5 **a** 9 cm **b** 39 cm
6 **a** 32 cm **b** 44 cm **c** 53.3 cm **d** 60 cm
7 **a**

312 × 18.9	300 × 20	6000	5896.8

b

795 + 103	800 + 100	900	898

c

1928 − 522	2000 − 500	1500	1406

d

4829 ÷ 10.4	5000 ÷ 10	500	464.3 (1 dp)

e

$822.6 × 9.7	$800 × 10	$8000	$7979.22

8 Teacher to check.

EXERCISE 2.2

1 **a** 13 **b** 15 **c** 6
2 **a** 3 **b** 1 **c** 7 **d** 8
3 **a** 50 **b** 49 **c** 62
d 103 **e** 12 **f** 9
4 76 **5** 44, 68, 36
6 **a** 36.5 **b** 44
7 Teacher to check.

EXERCISE 2.3

1 **a** One ball of wool costs $5, so 8 balls of wool cost $(8 × 5).
One button costs 55c ($0.55), so 6 buttons cost $(6 × 0.55).
Total cost is $(8 × 5 + 6 × 0.55).
b $43.30
c 8 × 6 + 6 × 0.75
d multiplication
e $9.20
2 **a** B $24.40 **b** D $20
c A $14 **d** C $2.40
3 **a** 27c **b** 19c **c** 22c
4 **a** $48.90 **b** $24.45 **c** $8.70
5 **a** approximately $68
b $68.27
c No, it cost her more than $65. She will make a loss.

EXERCISE 2.4

1 **a** $32 **b** $21 **c** $69 **d** $36
2 **a** $779 **b** $60 **c** $27 **d** $49
3 $105.45
4 No, she made an $11 loss
5 **a** $70 **b** $37.50 **c** $160
d $480 **e** $300
6 $96
7 wood item
8 $52

KEYWORD ACTIVITY

Teacher to check.

TEST YOURSELF

1 **a** 73 **b** 624.6 **c** 1.4 **d** 13.84
2 27 cm
3 **a** 8000 **b** 9600 **c** $10 000
4 60 cm
5 **a** 57 **b** 11 **c** 136
6 24 **7** 28 **8** 2
9 54 **10** $30 **11** $3.50
12 **a** 29 **b** 9 **c** 32
13 **a** $9 **b** $51
14 **a** profit **b** $8
15 $75

CHAPTER 3

EXERCISE 3.1

1 three, three, sides, equilateral, angles, angle, two, two, scalene, acute, obtuse, right, 180°
2 four, four, opposite, parallel, equal, bisect, equal, 90°, rectangle, square, 360°
3 parallelogram, 90°, equal, rhombus, equal, 90°, diagonals
4 quadrilateral, parallel, equal, equal, equal, diagonals, 90°, 360°
5 number, quadrilaterals, pentagon, hexagon, octagon, decagon, regular, regular
6 **a** trapezium **b** pentagon
c octagon **d** equilateral triangle
e kite **f** scalene triangle
g parallelogram **h** hexagon
i right-angled triangle **j** isosceles triangle
k square **l** rhombus
7 **a** F **b** T **c** T **d** T
e F **f** F **g** T **h** F

EXERCISE 3.2

1 flat, rectangular, prism, pyramid, pentahedron, hexahedron, edge, vertex

2 curved, sphere, flat, curved, rectangle, cone, circle

3

Solid	Number of faces	Shapes of faces	Number of identical faces
Cube	6	squares	6
Cylinder	3	2 circles and a cylinder	2
Square pyramid	5	square and 4 triangles	4
Triangular prism	5	2 triangles and 3 rectangles	2
Rectangular prism	6	rectangles	3 pairs of 2
Cone	2	circle and a sector	0
Triangular pyramid	4	triangles	0

4 cross-section, shape, end, rectangle, triangle, pointed, bottom, square, cross-section, size

5 **a** triangular prism **b** cylinder
c square pyramid **d** cone
e rectangular prism **f** triangular pyramid
g sphere **h** cube

6 **a** triangular prism **b** cone, hemisphere
c cube **d** pentagonal prism
e sphere
f square or rectangular pyramid

7 **a**

Solid	Number of Faces (F)	Number of Vertices (V)	Number of edges (E)	F + V
Square pyramid	5	5	8	10
Rectangular prism	6	8	12	14
Triangular prism	5	6	9	11
Triangular pyramid	4	4	6	8
Cube	6	8	12	14
Trapezoidal prism	6	8	12	14
Hexagonal prism	8	12	18	20
Square pyramid	5	5	8	10
Pentagonal pyramid	6	6	10	12

b The number of faces plus the number of vertices is 2 more than the number of edges.

8 **a** cube, square pyramid
b rectangular prism, cylinder
c cylinder, cone
d cone, hemisphere
e 2 triangular pyramids
f 2 rectangular prisms

9 **a** T **b** F **c** T
d T **e** F **f** F

EXERCISE 3.3

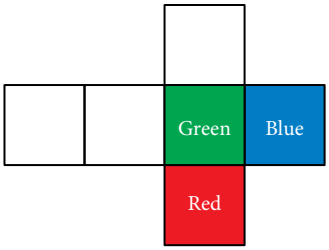
Teacher to check.

EXERCISE 3.4

1 **a-B, b-C, c-A, d-F, e-E, f-D**

2 **a** hexagonal pyramid **b** Teacher to check.

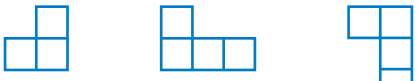
3



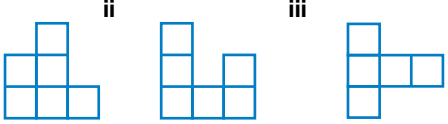
Other answers possible.

4 **a** triangular prism **b** cylinder
c rectangular prism **d** cube
e cone **f** rectangular pyramid
g triangular pyramid **h** L-shaped prism
i T-shaped prism

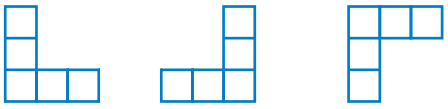
5 **a** **i** **ii** **iii**



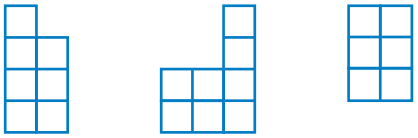
b **i** **ii** **iii**



c **i** **ii** **iii**

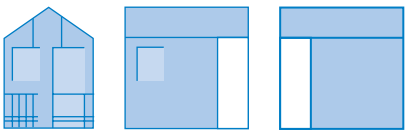


d **i** **ii** **iii**



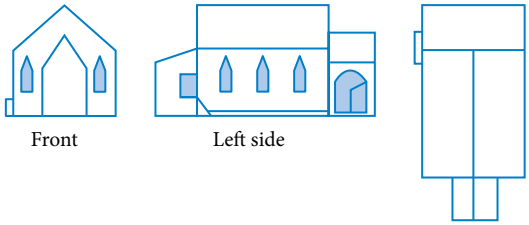
6 Teacher to check.

7



Front Left side Right side

8



KEYWORD ACTIVITY

1 G	**2** A	**3** E	**4** C
5 F	**6** B	**7** H	**8** D

TEST YOURSELF

1 Teacher to check.

2 **a** equilateral triangle
b parallelogram, rectangle, square, rhombus
c pentagon
d parallelogram, rectangle, square, rhombus
e trapezium
f scalene triangle, acute-angled triangle

3 **a** A, D, E, G **b** C, H **c** B, F

4 Teacher to check.

5 Teacher to check.

6 **a** rectangular prism **b** triangular prism
c cone **d** square pyramid
e cube **f** cylinder

7 Teacher to check.

CHAPTER 4

EXERCISE 4.1

1 **a** about 21 cm^2 **b** about 27 cm^2

2 about 25 cm^2

3 **a** about 16 m^2 **b** about $1040

4 about 12 cm^2

5 Teacher to check.

EXERCISE 4.2

1 **a** 450 m^2 **b** 5005 m^2
c 264 m^2 **d** 435 m^2

2 68 m^2

3 **a** 324 m^2 **b** 72 m

4 **a** 35 cm^2 **b** 55 cm^2 **c** 60 cm^2
d 80 cm^2 **e** 192 mm^2 **f** 63 m^2

5 **a** 21 m and 20 m **b** 210 m^2

6 3 m

7 **a** 119.7 mm^2 **b** 84 cm^2 **c** 56 cm^2

8 7.14 cm^2 and 5.4 cm^2

9 **a** Any pair of numbers that multiply to give 36; for example, 4×9, 3×12, 6×6.
b Teacher to check.
c 24 cm

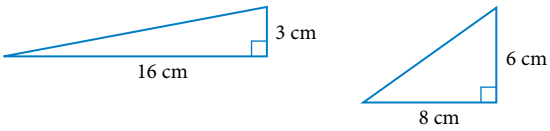
10 **a** area **b** 11.7 **c** 3

11 **a** Teacher to check.
b 15 cm^2

12 2.9 m^2

13 approximately 34.3 cm^2

14 Many answers are possible, e.g.



EXERCISE 4.3

1 **a** 1400 m^2 **b** $133 000
c 33 m^2 **d** $858
e No, the area of the backyard is 1000 m^2 and one bag will cover only 150 m^2.

2 **a** 2 bedrooms **b** 7 doors
c $4 m \times 3 m$ **d** south

3 **a** 15 m **b** 14 m

4 **a** 15 m **b** 3 lengths

5 **a** 48 m^2 **b** 14 L **c** 4 tins
d One 10 L tin and one 4 L tin, for $174.

6 **a** In bedroom 1, the 3.6 m wide carpet will be just bigger than 3.5 m. He will need the other dimension of the room for the length. This is 4 m. In bedroom 2, the 3.6 m width will fit the 3 m across with a bit to spare. It will need to be 4 m long.
b $1440

7 **a** rectangles **b** 135 m^2 **c** $1755

8 **a** 14 cm
b It will depend on how the rail is attached to the wall. The longer rail will provide more space to hang towels.

9 $540 (100 tiles)

10 The bed will fit in the bedrooms, but there won't be much spare space for a wardrobe or set of drawers in bedroom 2.

EXERCISE 4.4

1 **a** hectares **b** square metres
c square centimetres **d** square metres
e square centimetres **f** square millimetres
g square kilometres **h** square millimetres
i square kilometres

2 **a** 790 **b** 1.5 **c** 6.9
d 76 **e** 86.5 **f** 120 000
g 320 000 **h** 45 000 **i** 750 000
j 1.9

3 6.5 m^2, 114 000 cm^2, 25 050 000 mm^2

4 54 cm^2, 990 mm^2, 0.000 032 m^2

5 **a** 352.1 **b** 3.521

6 **a** 6800 m^2 **b** 3200 m^2

7 Soccer field. It is 1750 m^2 smaller than 1 ha. A rugby field is 3000 m^2 smaller than 1 ha.

8 **a** 24 000 000 **b** 24
c Many answers possible, e.g. 6 km by 4 km.
9 **a** 900 cm^2 **b** 11.11 **c** 280 **d** 294
10 7.67
11 **a** 510 074 600 **b** 139 433 205 $miles^2$

EXERCISE 4.5

1 **a** 36 cm^2 **b** 55 m^2 **c** 104 m^2
d 100 m^2 **e** 70 m^2 **f** 96 m^2
2 **a** 452.39 cm^2 **b** 490.87 cm^2
c 47.78 cm^2 **d** 3019.07 cm^2
e 117.29 cm^2 **f** 151.32 cm^2
g 760.27 cm^2 **h** 173.11 cm^2
3 **a** 11 m^2 **b** 14.1 m^2 **c** 917 bricks
4 **a** 25.44 cm^2 **b** 203.52 cm^2 **c** 216.73 cm^2
5 **a** 28.3 m^2 **b** 14.1 m^2 **c** 21.2 m^2
6 **a** 390 km^2 **b** 367.5 km^2 **c** 6%
7 **a** 706.86 cm^2 **b** 88.36 cm^2 **c** 176.71 cm^2
8 19.24 m^2
9 3.39 cm

EXERCISE 4.6

1 **a** 176.8 cm^2 **b** 19.84 m^2
c 875 mm^2 **d** 1000 cm^2
2 **a** 156 m^2 **b** 950 mm^2
c 71 m^2 **d** 201.19 km^2
3 **a** 94.63 m^2 **b** 216.53 cm^2
4 **a** 37.71 m^2 **b** 272.4 m^2
5 **a** 2.02 m^2 **b** 6.06 m^2 **c** 5.45 m^2
6 **a** 80 m^2 **b** $576
7 **a** 5.14 m^2 **b** 2.36 m^2
8 50.48 cm^2
9 5594.34 m^2
10 **a** A = 7.4 m B = 4.2 m C = 4.8 m
b (Area = 104 m^2, 4.7 tins of paint required)
Kurt will need to buy 5 tins of paint.

EXERCISE 4.7

1 **a** 29 400 cm^2 **b** 600 mm^2
2 6125 cm^2
3 121.5 m^2
4 **a** 348 cm^2 **b** 11.33 cm^2
5 12 680 cm^2
6 **a** 89.25 m^2 **b** $414
c 29.25 m^2 **d** $460
7 12.72 m^2
8 **a** 254 cm^2 **b** 279.4 cm^2
9 71 m^2
10 **a** 45 cm by 25 cm by 10 cm **b** 1125 cm^2
c 3925 cm^2

EXERCISE 4.8

1 **a** 864 cm^2 **b** 40 560 m^2
2 960 cm^2
3 190 900 m^2
4 **a** 329.3 cm^2 **b** 39.2 m^2
5 712.2 cm^2
6 **a** 105.6 cm^2 **b** 3700 m^2
7 139 cm^2
8 **a** 1946 m^2 **b** about 3 m^2
9 443 cm^2
10 **a** 106 m^2 **b** 6.84 m^2 **c** 99.16 m^2
11 **a** 260 cm^2 **b** 980 cm^2 **c** 6.37 g

EXERCISE 4.9

1 **a** 42 725.7 mm^2 **b** 20 106.2 mm^2 **c** 62 831.9 mm^2
2 **a** 197.7 m^2 **b** 373.8 cm^2
3 368 cm^2 **4** 4.52 m^2 **5** 34 cm^2
6 **a** 201 cm^2 **b** 151 cm^2 **c** 50 cm^2
7 **a** 8.5 cm **b** The arc *AB* is 40 cm long.
c 6.4 cm **d** 171 cm^2

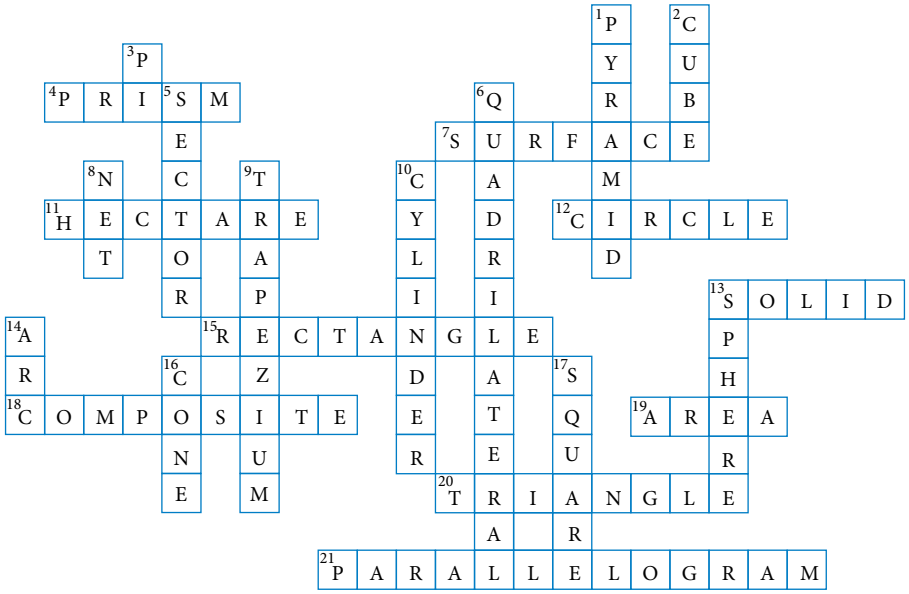
EXERCISE 4.10

1 **a** 604.76 cm^2 **b** 865.70 cm^2
c 672.36 cm^2 **d** 36.32 m^2
2 345.56 cm^2 **3** 367.6 cm^2
4 154 cm^2 **5** 18 m^2
6 **a** 514 718 540 km^2 **b** 360 302 978 km^2
c 154 415 560 km^2
7 161 m^2
8 **a** 3421.19 cm^2 **b** 2376 tiles
9 **a** 1728 cm^2 **b** 39.27 cm^2

EXERCISE 4.11

1 48 m^2 **2** 38.4 m^2
3 2118 cm^2 **4** 4.92 m^2
5 **a** 2716 cm^2 **b** 2988 cm^2
6 **a** 7.875 m^2 **b** 40.23 m^2
7 2300 cm^2
8 **a** 31.4 m^2 **b** 4.5 L
9 12.9 m^2
10 **a** 1659 mm^2 **b** 800 g

KEYWORD ACTIVITY



TEST YOURSELF

1 approximately 66 cm^2

2 **a** 9.6 mm^2 **b** 219.5 cm^2
c 2.2 m^2 **d** 52.2 cm^2

3 30 m^2

4 **a** 4 **b** 540 **c** 2.5
d 0.55 **e** 7000 **f** 60 000

5 **a** C **b** F **c** B **d** E
e A **f** D **g** G

6 **a** 834.7 mm^2 **b** 47.7 cm^2 **c** 9.12 cm^2
d 150.0 mm^2 **e** 88.0 m^2 **f** 25.0 mm^2

7 **a** 90 m^2 **b** 134 m^2 **c** 136.3 mm^2

8 **a** 125.5 m^2 **b** 720 m^2

9 1360 mm^2

10 **a** 36 835 mm^2 **b** 1521 cm^2 **c** 238 mm^2

11 **a** 273.32 cm^2 **b** 1576 m^2

CHAPTER 5

EXERCISE 5.1

1 6700 km

2 **a** **i** 1.4 km **ii** 1.9 km **iii** 2.48 km
b **i** 35 cm **ii** 7.5 cm **iii** 16.5 cm

3 5.6 cm

4 3200 m

5 83 mm

6 **a** 480 m **b** 25 cm **c** 1664 ≈ 1700 m

7 **a** 100 m **b** 2 km
c 250 m **d** 0.4 km or 400 m
e 0.5 km or 500 m **f** 100 m

8 **a** 4 m **b** 75 cm **c** 2 m
d 1.2 m **e** 1.6 m **f** 1 m
g 16 m **h** 16 m^2

9 **a** 4.2 m **b** 96 cm

10 **a** 240 m^2 **b** Teacher to check.
c 1120 m^2 **d** 56 m by 20 m

EXERCISE 5.2

1 **a** **i** kitchen sink **ii** vanity (or wash basin)
iii toilet **iv** window
v bath **vi** built-in wardrobe
b They are all tiled. **c** 3 **d** 1 and 2
e 4100 mm by 3200 mm
f in the back of the garage
g near the bathroom
h 1 **i** 1 **j** 2
k living room
l 6670 mm × 5500 mm
m **i** south **ii** north **iii** south
iv east **v** west
n south
o no
p Living room, Dining room, Bedroom 1, Bedroom 3
q Dining room, Bedroom 2, Living room

2 **a** 3 **b** 3
c no **d** 4.7 m by 3.5 m
e Bed 1 **f** $294 400

3 **a** 3.5 m **b** 1.8 m
c 1.4 m **d** 1.4 m × 1.8 m = 2.52 m^2
e 3.5 m **f** 2.8 m × 2.7 m = 7.56 m^2

4 **a** 240 mm **b** 70 mm
c 12.39 m **d** 14.29 m
e 3.6 m by 6.07 m **f** 1 : 111

5 **a** 6790 mm by 3700 mm
b 2 **c** outside the bathroom
d shower **e** bedroom 4
f family room = 25.123 m^2, media room = 23.787 m^2. Family room is bigger by 1.336 m^2

EXERCISE 5.3

1 **a** $7.5\,m^2$ **b** $273.75

2 **a** 18 m **b** $306

3 **a** $92\,m^2$ **b** $174 800

4 **a** 14 m **b** 13 m **c** $975

5 **a** $12\,m^2$ **b** 4 m **c** $519.80

6 **a** $14\,m^2$

b **i** $2\frac{1}{3}\,L \approx 3\,L$ **ii** $49.90

7 **a** painting, skirting boards, carpet

b $765

8 $1702

9 **a** 3.9 m **b** 3.9 m **c** 3 m

10 $892

11 **a** 13.8 **b** 3

12 approximately $192 400

EXERCISE 5.4

1-3 Teacher to check.

4 **a** Teacher to check. **b** 2.9 km

5 Teacher to check diagram, 285 m.

6 **a** Teacher to check. **b** 144.2 cm

7 132 m

8 252 m

9 **a** Teacher to check. **b** $1.04\,m^2$

KEYWORD ACTIVITY

1 G **2** J **3** E **4** H

5 A **6** D **7** I **8** K

9 C **10** B **11** F

TEST YOURSELF

1 10.75 m

2 **a** 1:100

b **i** 380 m **ii** 670 m

c **i** 4.15 cm **ii** 11 cm

d 260 m

3 **a** 280 mm

b 110 mm

c 5500 mm

d 3500 mm by 5500 mm

e 6390 mm by 5500 mm

f 1:120

4 **a** 9

b Length 13.1 m, Width 10 m

c 3 m

d **i** $109.7\,m^2$ **ii** $147 437 approximately

e 10.9%

f $21.3\,m^2$

g **i** $23.43\,m^2 \approx 24\,m^2$ **ii** 229 tiles

h **i** 13.1 m **ii** $1059.93

5 **a** $1768

b **i** 15.18 m **ii** $455.40

c **i** $12.42\,m^2$ **ii** $310.5 \approx 311$ tiles

iii $510.46 \approx $511

d **i** $26\,m^2$ **ii** $4\frac{1}{3}\,L$

iii $69.80 (2 cans required)

6 Teacher to check diagram, 40 m.

PRACTICE SET 1

Section A

1 B **2** B **3** D **4** A

5 B **6** B **7** D **8** C

Section B

1 **a** 300 cm = 3 m

b 225 cm = 2.25 m

2 **a** 640 **b** 24

c 15 800 **d** 9.8

3 Teacher to check.

4 $7858.50 **5** 7.2 cm

6 **a** 5.5 m by 5.5 m

b **i** $30.25\,m^2$ **ii** $2268.75

c 12

d **i** $12.24\,m^2$ **ii** $1040.40

e **i** $30.76\,m^2$ **ii** $99.80

7 Teacher to check.

8 **a** $152\,cm^2$ **b** $574\,cm^2$

9 Teacher to check.

10 61.42 mm

11 **a** $14 100\,cm^2$ **b** $604.76\,cm^2$

c $2975\,cm^2$ **d** $417.83\,cm^2$

e $2827.43\,cm^2$

CHAPTER 6

EXERCISES 6.1 AND 6.2

Teacher to discuss with the class.

EXERCISE 6.3

1 **a** $\frac{1}{5}$ **b** $\frac{1}{10}$, 0.1, 1:10, 10%

c 27 cm

2 **a** **i** 0.08 **ii** $\frac{17}{25}$ **iii** 2:25

b Many answers possible, for example, quokka's age, whether they've been fed by visitors to the island

3 **a** 24 **b** $\frac{16}{24}$ **c** $\frac{1}{3}$

4 **a** $\frac{5}{40}$ **b** 0.875

c Teacher to check.

5 **a** $\frac{19}{20}$ **b** 0.35 **c** $\frac{1}{10}$

d yes, no, no

e Special interest groups who have non-representative views.

9780170484664

6 **a** $\frac{10}{75} = \frac{2}{15}$

b 0.28, 0.16, 0.13, 0.15, 0.16, 0.12

c It could be biased. Number 1 appears to occur nearly twice as often as it should. More trials are required before a conclusion can be reached.

7 Teacher to check.

EXERCISE 6.4

Teacher to discuss answers with the class.

KEYWORD ACTIVITY

Teacher to check.

TEST YOURSELF

1 **a** $\frac{3}{10}$

b point down, higher relative frequency than pointing up

2

Fraction	Decimal	Percentage	Ratio
$\frac{7}{8}$	0.875	87.5%	7:8
$\frac{3}{20}$	0.15	15%	3:20
$\frac{7}{25}$	0.28	28%	7:25
$\frac{11}{15}$	0.73	73%	11:15

3 It is quite possible for a normal coin to show heads 4 out of 6 times in the short-run. It is only in the long-run that the percentage of heads approaches 50%.

4 tossing a coin, birth of boys or girls, guessing the right answer on a true/false question

5 **a** false; the coin doesn't have a memory. The next toss could result in either a head or a tail.

b true

CHAPTER 7

EXERCISE 7.1

1 **a** 3000 **b** 12 000 **c** 1.5 **d** 2.4
e 850 000 **f** 0.9 **g** 2500 **h** 0.5

2 **a** 10 000 **b** 10

3 **a** 4 **b** 54 **c** 2160

4 **a** 135 g **b** 135 000 **c** 270 mg

5 20

6 Teacher to check.

7 360 mg

8 **a** 11

b Yes, its mass is less than 12 pounds.

9 Sum of the dimensions = 40.4 inches, mass = 52.8 pounds. The bag is small enough, but it is too heavy.

10 **a** 1350 kg **b** 10.8 t

11 1600 g

12 It weighs 32.15 troy ounces and is worth approximately $42 793.

EXERCISE 7.2

1 **a** no **b** 150c, 160v
c potatoes, blue peas, onion, tomatoes, green peas
d 2 **e** 13.5 g **f** 5

2 **a** Thailand **b** yes **c** 14.0 g
d 400 mg **e** no

3 **a** 1830 kJ **b** 170 mg **c** 68.6%
d peanuts, tree nuts, sesame seeds, egg, milk

4 **a** 12 **b** 30 mL **c** 360 mL
d 3276 kJ **e** 2.4 g
f There is the same quantity of both.
g 85% **h** 3.6 g **i** 1.2 g

5 **a** 220 mg **b** 0.6 mg **c** 1.2 g

EXERCISE 7.3

1 **a** m^3 **b** cm^3 **c** cm^3
d mm^3 **e** m^3 **f** m^3
g cm^3 **h** m^3 **i** mm^3
j cm^3 **k** mm^3 **l** cm^3

2 **a** E **b** G **c** F **d** C
e A **f** B **g** D

3 **a** 5 000 000 **b** 1 600 000
c 0.006 **d** 4
e 0.16 **f** 0.25
g 180 000 **h** 120
i 4 000 000 000 **j** 0.0096

4 **a** D **b** G **c** J **d** H
e I **f** A **g** F **h** E
i C **j** B **k** K

5 **a** 8 mL **b** 1500 mL **c** 425 mL

6 **a** 2 L **b** 3.5 L **c** 0.25 L

7 4210 mm^3, 42 cm^3, 0.0042 m^3

8 **a** 5000 mL **b** 5L

9 0.306 m^3 **10** 2 520 000 mm^3

11 2000L **12** 1250 cm^3

13 9 L

14 **a** 0.2 mL **b** 12 mL
c 2160 mL **d** 17.28 L

15 200 000 cm^3

EXERCISE 7.4

1 **a** 111 cm^3 **b** 2.5 m^3
c 256 m^3 **d** 486.42 cm^3

2 **a** 1 071 000 cm^3 **b** 91.125 cm^3
c 21 952 mm^3 **d** 258.06 cm^3

3 **a** 84 cm^3 **b** 240 cm^3

4 **a** 16 065 cm^3 **b** 1 285 200 cm^3

5 **a** 42 875 cm^3 **b** 471 625 cm^3
c approximately 330 books

6 **a** 648 cm^3 **b** 3420 cm^3
c 1068.375 cm^3 **d** 1026 cm^3
7 **a** 126 pavers **b** 4 500 000 mm^3
c 567 000 000 mm^3 **d** 13 boxes
e $737.10
8 200 m^3
9 Volume = 420 m^3, so Nazneen requires a large air-conditioner.
10 510 cm^3
11 **a** 1 m by 0.9 m by 0.35 m **b** 0.4 m^3
c 0.315 m^3 **d** 315 L
12 504 L
13 **a** 10 143 cm^3 **b** 10 L **c** 5 kg
14 **a** rectangular prism **b** 2.042 m^3
c 2042 L **d** 19 minutes

EXERCISE 7.5

1 **a** 85.5 cm^2 **b** 2223 cm^3 **c** 101 cm^3
2 **a** 1725 cm^3 **b** 1050 cm^3
c 675 cm^3 **d** 39%
3 795 cm^3
4 791.7 cm^3
5 Volume is approximately 70 cm^3 and it holds approximately 28 g of coffee beans.
6 532.5 cm^3

EXERCISE 7.6

1 **a** 166.3 m^3 **b** 235.6 mm^3
c 147.2 m^3 **d** 739.7 cm^2
2 **a** 192 cm^3 **b** 180 m^3
3 **a** 212.2 mm^3 **b** 1629.5 cm^3
4 333 m^3
5 **a** $(96 - 2 \times 15) \div 2 = 33$ **b** 640 L
6 **a** 1 200 000 m^3, smaller because all the approximations were smaller than the original amounts
b 2 101 400 m^3
7 289 L
8 **a** 5 cm **b** 12 cm
c 314 cm^3 **d** 204 cm^2
9 **a** 200 cm^3 **b** 100
10 1485.2 m^3

KEYWORD ACTIVITY

ACROSS

2 CUBIC **5** PRISM
7 AREA **9** RECTANGULAR PYRAMID
10 VOLUME **11** SPHERE
12 LITRE **13** MM

DOWN

1 TRIANGULAR PRISM
3 CYLINDER **4** CAPACITY
6 METRE **8** COMPOSITE

TEST YOURSELF

1 **a** 5000 **b** 0.2 **c** 1400
d 3500 **e** 7.5
2 **a** 498 kJ **b** 220 mg
c Several answers possible, examples: $16.7 \times 0.5 = 8.35$ and $16.7 + 8.35 = 25.05$ or $25.05 \div 16.7 = 1.5$, $1\frac{1}{2}$ times bigger, thus increased by half or 50%.
d $1.4 \div 7 = 0.2$ g
e 1.19g
f $3.85 \times 70 = 269.5$
3 **a** 5000 **b** 2 000 000 **c** 0.5
d 250 000 **e** 0.024 **f** 36
4 **a** 24 **b** 6000 cm^3
5 700 L
6 **a** 11.16 cm^3 **b** 64 m^3
c 153 cm^3 **d** 48 cm^3
7 **a** 936 cm^3 **b** 572 cm^3 **c** 61%
8 **a** 136 cm^3 **b** 122 cm^3 **c** 29 g
9 **a** 463.2 cm^3 **b** 480.0 m^3 **c** 864.0 mm^3
d 137.0 mm^3 **e** 365.3 cm^3
10 **a** 808 cm^3 **b** 0.808 L
11 **a** 58 m **b** 1658 m^3

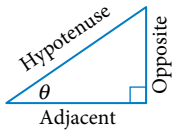
CHAPTER 8

EXERCISE 8.1

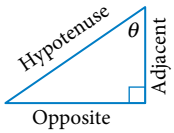
1 23.8 m **2** 8.4 m
3 5.831 m **4** 152 km/h
5 **a** yes **b** yes **c** no **d** yes
6 12.37 m **7** 733 m
8 44.4 m **9** 148 km
10 Teacher to check.

EXERCISE 8.2

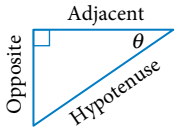
1 **a**



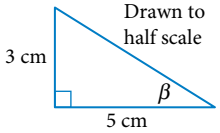
b



c



2 *WK*
3 **a** 65 **b** 56 **c** 56
4 β
5 **a** $\frac{8}{17}$ **b** $\frac{21}{29}$ **c** $\frac{72}{97}$
6 **a**



b 31°
7 **a** *AB* **b** *AC* **c** *BC*

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EXERCISE 8.3

1 a $\frac{3}{4}$ **b** $\frac{12}{5}$ **c** $\frac{120}{119}$

2 Teacher to check.

3 40°; 9, 40°, 9; 7.6

4 a 14.43 cm **b** 4.86 m **c** 10.89 cm
d 235.12 cm **e** 6.64 km **f** 4.71 mm

5 a 17.16 cm **b** 12.01 cm

EXERCISE 8.4

1 a 41° **b** 47° **c** 37°
d 31° **e** 54° **f** 53°

2 a 38° **b** 34° **c** 68°
d 49° **e** 43° **f** 59°

3 34°, 56° and 90°

4 a 45°, 45° and 90°

b The opposite and adjacent sides have the same length. When the numerator (top) of a fraction and the denominator (bottom) are the same, the fraction is 1.

EXERCISE 8.5

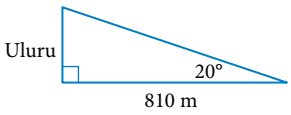
1 118.3 m

2 1411.4 m

3 530.0 m

4 a 156.934 m **b** 158.5 m

5 a

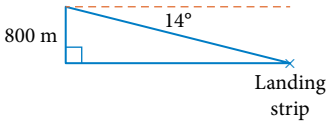


b 295 m

6 801.6 m

7 a 38° **b** 52°

8 a



b 3209 m

9 97.5 m

10 a 87 cm **b** 64°

EXERCISE 8.6

1 a 5.8 **b** 12.6 **c** 11.4

2 a 40° **b** 28° **c** 26°

3 a 15 m **b** 8 m **c** 17 m

4 a 41° **b** 62° **c** 63°

5 a 5.5 **b** 11.1 **c** 3.5

EXERCISE 8.7

1 a tan **b** cos **c** sin

2 a 7.2 **b** 7.0 **c** 10.6

3 a 64° **b** 30° **c** 53°

4 14 m **5** 12.4 m **6** 32.5 m **7** 7.3 m

8 a 14.6 **b** 7.2 **c** 7.5

9 a 41° **b** 30° **c** 46°

EXERCISE 8.8

1 a 13.6 **b** 24.8 **c** 14.6

2 a 8.1 **b** 15.9 **c** 16.8

3 a 21.45 m **b** 21.5 m

4 11.5 m **5** 5.05 m **6** 310 cm

7 a 7.4 **b** 6.2 **c** 13.6

KEYWORD ACTIVITY

ACROSS

1 ADJACENT **4** TAN RATIO
6 ANGLES **7** HYPOTENUSE
9 COS

DOWN

2 TRIGONOMETRY **3** OPPOSITE
5 RIGHT **8** SIDES

TEST YOURSELF

1 a 12.9 **b** 12.6 **c** 22.0

2 1.9 m

3 Yes, both contain right angles. Pythagoras' theorem works in both triangles.

4 a *AB* **b** *AC* **c** *BC*

5 9.0

6 a 38° **b** 68°

7 2570 m **8** 6.4 m **9** 12°

10 a sin, $x = 3.6$ **b** cos, $\theta = 39°$
c tan, $h = 9.9$ **d** tan, $\theta = 26°$

11 a 32.68 mm **b** 9.12 cm

CHAPTER 9

EXERCISE 9.1

1 a 1, 2, 3, 4, 5, 6 **b** $\frac{2}{6} = \frac{1}{3}$

2 a head, tail **b** $\frac{1}{2}$, 0.5, 50%, 1 : 2

3 $\frac{6}{10} = \frac{3}{5}$

4 $\frac{2}{15}$, 2 : 15

5 a $\frac{10}{50} = \frac{1}{5}$, 0.2 **b** $\frac{36}{50} = \frac{18}{25}$, 72% **c** blue

6 a $\frac{1}{12}$ **b** $\frac{2}{12} = \frac{1}{6}$
c $\frac{4}{12} = \frac{1}{3}$ **d** $\frac{8}{12} = \frac{2}{3}$

7 a $\frac{1}{26}$

b The letter E is used more frequently than other letters, e.g. Z.

8 a $\frac{1}{200}$, 0.005 **b** $\frac{50}{200} = \frac{1}{4}$, 25%
c $\frac{1}{2}$ **d** $\frac{119}{200}$

9 a $\frac{6}{20} = \frac{3}{10}$ **b** 8 **c** $\frac{6}{20} = \frac{3}{10}$

ANSWERS

EXERCISE 9.2

1 a

	1	2	3	4	5	6	7	8
Head	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6	H, 7	H, 8
Tail	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6	T, 7	T, 8

b **i** $\frac{1}{16}$ **ii** $\frac{9}{16}$ **iii** $\frac{4}{16} = \frac{1}{4}$ **iv** $\frac{4}{16} = \frac{1}{4}$

c **i** $\frac{1}{16}$ **ii** $\frac{2}{16} = \frac{1}{8}$

2 a

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36

c **i** $\frac{4}{36} = \frac{1}{9}$ **ii** $\frac{5}{36}$ **iii** $\frac{3}{36} = \frac{1}{12}$ **iv** $\frac{1}{36}$

d total of 9 **e** twice as likely

3 a

+	0	1	2	3	4	5
1	1	2	3	4	5	6
1	1	2	3	4	5	6
3	3	4	5	6	7	8
3	3	4	5	6	7	8
4	4	5	6	7	8	9
6	6	7	8	9	10	11

b **i** $\frac{5}{36}$ **ii** $\frac{1}{36}$ **iii** $\frac{9}{36} = \frac{1}{4}$

c 6

4 a

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b $\frac{6}{36} = \frac{1}{6}$ **c** 0 **d** 1

5 a

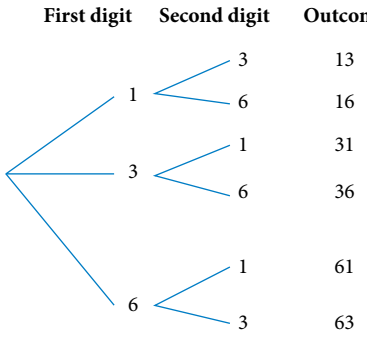
	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

b $\frac{5}{36}$ **c** 6 **d** 5

6 $\frac{16}{36} = \frac{4}{9}$

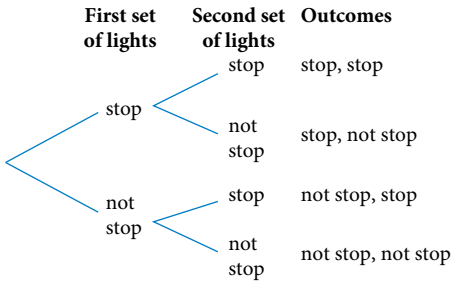
EXERCISE 9.3

1 a



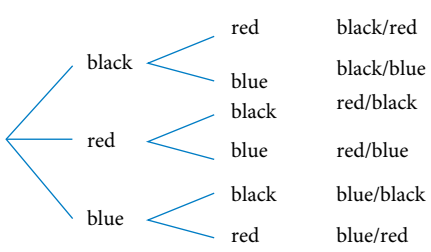
b $\frac{3}{6} = \frac{1}{2}$

2 a



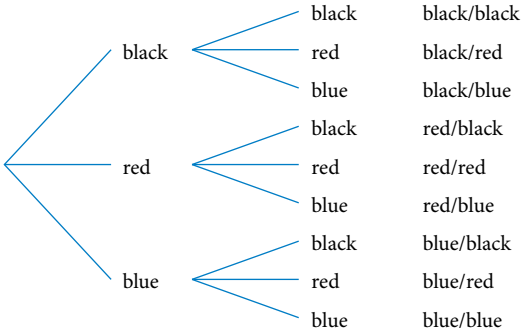
b $\frac{1}{4}$ **c** $\frac{3}{4}$

3 a



b $\frac{1}{6}$ **c** $\frac{2}{6} = \frac{1}{3}$

d



e $\frac{3}{9} = \frac{1}{3}$

4 a Teacher to check. **b** $\frac{1}{9}$

5 a Teacher to check.

b 6 **c** $\frac{4}{12} = \frac{1}{3}$

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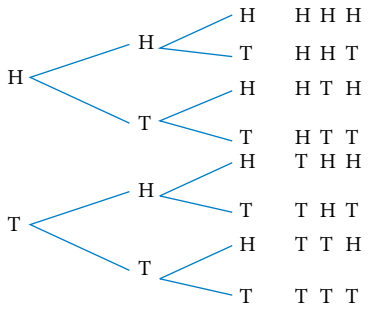
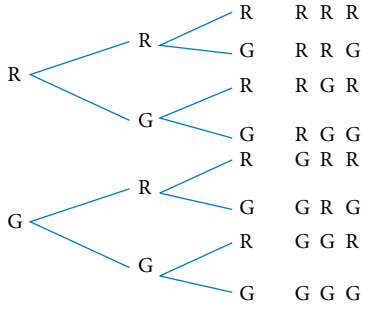
6 a



b 8

c **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{7}{8}$

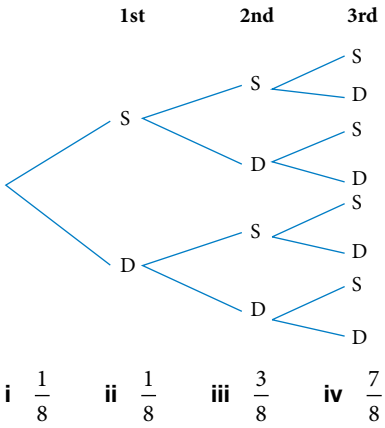
7 a



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

8 a $\frac{1}{2}$

b



i $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{7}{8}$ **v** $\frac{1}{4}$

9 $\{\text{odd}\} = \frac{12}{25}$, $\{\text{even}\} = \frac{13}{25}$, not fair because 'even' is more likely.

EXERCISE 9.4

Teacher to discuss answers with the class.

KEYWORD ACTIVITY

Theoretical probability is about the long-term chance that something will happen. The theoretical probability of getting a head when a coin is tossed is $\frac{1}{2}$. This doesn't mean that every second time a coin is tossed, it will be a head. Neither does it mean that when a coin is tossed 100 times, it will be a head 50 times. It also doesn't mean that if 8 tails are recorded in a row then the next toss of the coin is more likely to be a head. Coins can't remember what's happened in the past and the chance of getting a head in the future doesn't change because of several tails in a row. What the probability of a head being $\frac{1}{2}$ does mean is that if a coin is tossed thousands of times, about half of the time it will land on heads but on no individual future occasion can you know what is going to happen.

TEST YOURSELF

1 1, 2, 3, 4, 5, 6

2 a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{3}{8}$ **d** 0

3 a $\frac{9}{31}$ **b** $\frac{7}{31}$ **c** $\frac{45}{62}$

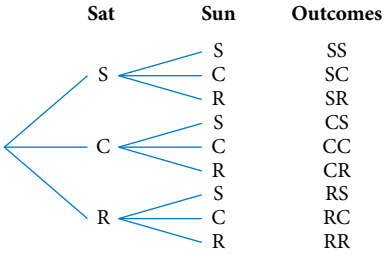
4 a

	Die 2					
Die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

There are 24 possible outcomes.

b $\frac{1}{6}$

5 a

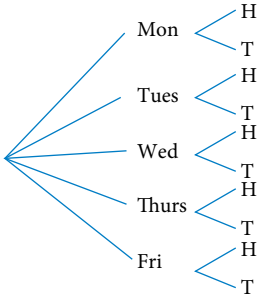


b **i** $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{4}{9}$ **iv** $\frac{5}{9}$ **v** $\frac{2}{9}$

6 a

	Weekday				
Coin	Monday	Tuesday	Wednesday	Thursday	Friday
	H, Mon	H, Tues	H, Wed	H, Thurs	H, Fri
	T, Mon	T, Tues	T, Wed	T, Thurs	T, Fri

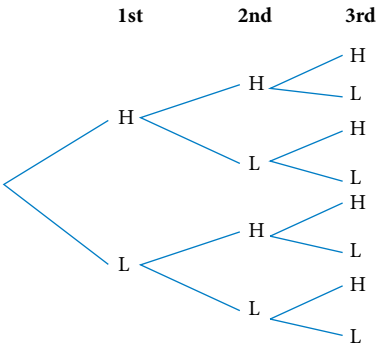
OR



b **i** $\frac{1}{10}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{5}$

ANSWERS

7 a



b HHH, HHL, HLH, HLL, LHL, LHH, LLH, LLL

c $\frac{3}{8}$

8 a Teacher to check. **b** $\frac{1}{2}$

c Theoretical probability is only a long-term average. It doesn't predict what happens in each trial of a chance experiment.

9 a about 4

b Probability is only a long-term average. It can't predict accurately for small events.

c No, each child has the same chance.

PRACTICE SET 2

Section A

1 B **2** B **3** B **4** A

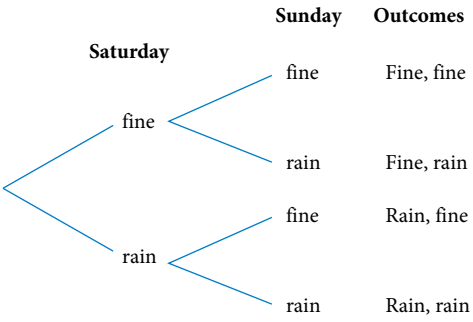
5 C **6** A **7** C **8** D

Section B

1 Teacher to check.

2 90 g **3** 43°

4 a



b $\frac{1}{4}$

5 a 3718.75 cm^3 **b** 384

6 a

	×	1st Die 1	2	3	4	5	6
2nd die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

b 18 **c** 6, 12 **d** 1, 36

e i $\frac{1}{9}$ **ii** $\frac{1}{18}$ **iii** $\frac{2}{9}$

7 Teacher to check.

8 a 0.167 **b** 0.278

c Teacher to check. **d** 20

9 32.8 m

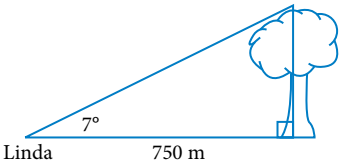
10 a $\frac{12}{20} = \frac{3}{5}$

b 0.15

c Anton is only asking people who are on their way to see the reef. Others answers possible.

11 a 0.39 m^3 **b** 390 L

12 a



b 92.1 m

CHAPTER 10

EXERCISE 10.1

1 a 9 **b** 8 **c** 8

2 a 2 **b** 45, 54 **c** no mode

d green **e** spade

3 a 16 **b** 37.5

c 72.9 **d** 50.5

4 a 8 **b** 13

c 11 **d** 12

e Teacher to check.

5 a 14 **b** 14 **c** 13.8

d January, February, December; Teacher to check.

6 a 4 **b** 1, 3, 4, 7 **c** 4.3

7 Teacher to check.

8 a mode = 1, median = 3, mean = 5

b mean, it is the highest

c mode, it is the smallest

d Teacher to check.

9 a 4

b

Number of texts (score)	Frequency	Score × frequency
0	8	0
1	4	4
2	10	20
3	10	30
4	15	60
5	3	15
Total	**50**	**129**

c 2.58

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10 **a** 50 **b** 6

c

Score	Frequency	Score × frequency
1	2	2
2	5	10
3	10	30
4	6	24
5	11	55
6	13	78
7	3	21
Total	**50**	**220**

d 4.4 **e** Teacher to check.

EXERCISE 10.2

1 **a** Binns ≈ 16.4, Thomson ≈ 25.3
b the last age
c The much larger 91 makes the mean higher.

2 **a** $696 500 **b** $767 273
c Median, the outlier will have an effect on the mean.
d $1 800 000
e mean $664 000, median $660 750
i yes, much closer
ii mean – it has come down by $103 273

3 **a** Mark ≈ 43.9, Steve ≈ 45.4
b Steve
c Mark = 45, Steve = 37
d Mark
e Teacher to check.

4 **a** $154 000
b $70 500
c median, 10 out of 12 scores are close to the median
d mean, it's higher
e $275 000, $890 000
f $68 300

5 **a** mean = $63 400, median = $71 000
b median, 4 out of 5 years are close to this value
c wanting to minimise tax paid
d $32 000
e mean = $71 250, median = $71 500

6 **a** 204 deliveries
b This doctor could be a specialist in difficult pregnancies and deliveries.
c mean = 30.65, median = 21.5
d median, more values are closer to it
e mean = 21.5, median = 21
f The outlier has a big effect on the mean, but only a very small effect on the median.

7 **a** $1 355 000
b $1 431 125
c median, it is lower
d mean, it is higher
e $1 835 000, $2 700 000
f $1362 222

EXERCISE 10.3

1 **a** range = 11
b $Q_1 = 18$, $Q_2 = 19$, $Q_3 = 25$
c 7

2 **a** range = 7
b $Q_1 = 29$, $Q_2 = 31$, $Q_3 = 31$
c 2

3 **a** range = 31
b $Q_1 = 159.5$, $Q_2 = 169$, $Q_3 = 174.5$
c 15

4 **a** range = 6
b $Q_1 = 4$, $Q_2 = 5.5$, $Q_3 = 8$
c 4

5 **a** Inner City
i 43
ii $Q_1 = 20$, $Q_2 = 25.5$, $Q_3 = 33$
iii 13
Coastal
i 38
ii $Q_1 = 52$, $Q_2 = 60$, $Q_3 = 64.5$
iii 12.5
b Coastal is much higher, but the interquartile ranges are much the same.

EXERCISE 10.4

1 **a** $58 000 **b** $62 000
c 2012 **d** $56 000 to $82 000
e never

2 **a** $72 000 **b** 55th percentile
c $11 000 **d** median
e 22%

3 **a** 60% **b** 90
c 66 or 67 **d** 3rd and 4th decile
e Easy, since the median score is 83 and more students scored at the higher end.

4 **a** 6 **b** October
c August **d** 5 times
e Poor, as the below average months are very poor. Other answers possible.

5 **a** 25% **b** 95%
c 187–188 cm **d** 5 years old
e **i** 178 cm or 179 cm
ii 182 cm or 183 cm

EXERCISE 10.5

1 **a** mean = 20.78, $\sigma = 3.05$
b mean = 48.27, $\sigma = 21.34$
c mean = 14.55, $\sigma = 1.83$
d mean = 55.27, $\sigma = 21.34$

2 **a** mean = $697.60 **b** $\sigma =$ $ 103.60$
c Teacher to check.

3 **a** $469.60 **b** $\sigma =$ 67.43
c Teacher to check.
d Quite difficult, as the mean is higher than $420. Other answers possible.

4 **a** $\bar{x} = 6.6$ **b** $\sigma = 1.0$

5 **a** $\bar{x} = 2.58$ **b** $\sigma = 1.52$

6 **a** $\bar{x} = 4.40$ **b** $\sigma = 1.61$

7 **a** Port Paradise: $\sigma = 2.17$, Palm Tree Cove: $\sigma = 4.46$

b Port Paradise, as the temperatures are more consistent (smaller standard deviation).

8 **a** Brad: mean = 8.3, $\sigma = 2.0$, Aryn: mean = 4.5, $\sigma = 1.7$, Kim: mean = 6.5, $\sigma = 2.6$

b Aryn, as he has the lowest standard deviation.

c Brad, as he has the highest mean and a fairly low standard deviation.

9 **a** 70, 71, 72, 73, 73, 74, 75, 75, 75, 75, 75, 76, 76, 77, 77, 77

b mean = 74.44, $\sigma = 2.06$

c Paul is a consistent player as his scores show a small standard deviation.

10 **a** You would expect a large standard deviation as the data is spread from 127 to 157.

b $\sigma = 10.08$

c Teacher to check.

EXERCISE 10.6

1 The number of hours is evenly spread over a small range.

2 The scores are clustered in the 40s and 50s and tightly packed. There is a gap at the top.

3 The test results are tightly packed with a gap at the bottom.

4 **a** clustered, spread out

b gaps, more dense, less dense, spread out

c more dense, spread out

d spread out, gaps

e clustered, spread out, tightly packed

f tightly packed, gaps

Other answers are possible.

EXERCISE 10.7

1 mean = 30.71, mode = 29, median = 30, standard deviation = 2.31, maximum = 35, minimum = 28, range = 7

2 Teacher to check.

3 **b**

	2023	2024
Mean	19.17	16.83
Median	18.5	17.5
Mode	17	20
SD	4.45	2.79

d Theft was higher in 2023, and the number of thefts were spread over a wider range in 2023. Other answers are possible.

4 Teacher to check.

5 **a** Teacher to check.

b mean = 73.9, median = 71, $\sigma = 10.5$

c mean = 68.2, median = 69, $\sigma = 6.3$

d Teacher to check.

6 **b** mean = 93.4, median = 94.5, mode = 81, $\sigma = 9.22$

c mean = 94.3, median = 95, mode = 81, $\sigma = 8.06$

d Without the outlier, the spread is reduced and the mean increases. Other answers possible.

KEYWORD ACTIVITY

1 mode 2 average 3 number

4 median 5 odd 6 order

7 outlier 8 quartiles 9 ten

10 percentiles 11 spread 12 range

13 interquartile range

14 standard deviation

TEST YOURSELF

1 **a** **i** no mode **ii** 913 **iii** 830

b **i** \$40 **ii** \$37 **iii** \$35.82

c **i** 12°C, 20°C, 23°C, 25°C

ii 20°C

iii 19°C

d **i** 129 mm **ii** 129 mm **iii** 119.7 mm

2 **a** 61%

b 56%, 58%, 59%, 61%, 62%

c 62.3%

d Modes are not useful. Mean or median are a good reflection of the data.

3 **a** 300 **b** 962.5 **c** It lowers the mean.

4 **a** 65, 69

b mean = 132.9 mm, median = 136 mm

c decrease the mean

d decrease the median

5 **a** **i** 826 **ii** 554, 1064.5 **iii** 510.5

b **i** \$12 **ii** \$32, \$40 **iii** \$8

c **i** 14°C **ii** 14°C, 23.5°C **iii** 9.5°C

d **i** 94 mm **ii** 95 mm, 151 mm

iii 56 mm

6 **a** student income, temperature (no outliers)

b computer fraud, rainfall (outliers)

7 **a** 30% **b** 79

c 8th and 9th decile

8 **a** 6

b July, September, November, March and June

c about 210 mm

9 $\sigma = 5.05$°C

10 **a** $\bar{x} = 1.91$ **b** $\sigma = 1.50$

11 **a** Dalby: $\bar{x} = 50.08$ mm, $\sigma = 24.37$ mm; Clermont: $\bar{x} = 55.00$ mm, $\sigma = 34.27$ mm

b Dalby, as the rainfall is more consistent there (smaller standard deviation).

12 Teacher to check.

CHAPTER 11

EXERCISE 11.1

1 **a** $115.20 **b** $243.36
c $866.25 **d** $350.63

2 **a** 6.15 years **b** 6.41% p.a.
c $4781.42

3 **a** $44.10 **b** $193.60
c $54.27 **d** $4.77

4 0.38%

5 **a** 0.26% **b** 0.06% **c** 1.56%
d 0.0085% **e** 0.12% **f** 0.78%

6 **a** $1088 **b** $4288

7 $156 250 **8** 5.65 years **9** 7%

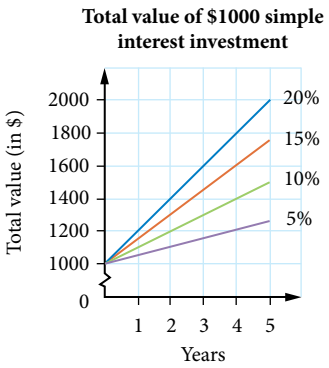
10 **a** $15.86 **b** $880.86

11 $59 743.75

12 **a** 0.048% **b** $20 736

13 **a** $250 **b** $21 500
c The interest rate was too high and the information was sent by text from someone she didn't know well.

14 **a** $1600 **b** $600 **c** $250
d the principal (starting amounts)
e The larger the rate of interest, the steeper the graph.
f



EXERCISE 11.2

1

	Interest	Balance
End of the 1st year	$I = Pin$ $= \$8000 \times 0.06 \times 1$ $= \$480$	$\$8000 + \$480$ $= \$8480$
End of the 2nd year	$I = Pin$ $= \$8480 \times 0.06 \times 1$ $= \$508.80$	$\$8480 + \$508.80$ $= \$8988.80$
End of the 3rd year	$I = Pin$ $= \$8988.80 \times 0.06 \times 1$ $= \$539.33$	$\$8988.80 + \$539.33$ $= \$9528.13$

2 **a** $480 **b** $12 480 **c** $12 979.20
d $13 498.37 **e** $1498.37

3 **a** $4260.10 **b** $260.10 **c** $4.10

4 **a**

	Interest	Balance
End of the 1st month	$I = Pin$ $= \$12\,000 \times 0.007 \times 1$ $= \$84$	$\$12\,000 + \$84$ $= \$12\,084$
End of the 2nd month	$I = Pin$ $= \$12\,084 \times 0.007 \times 1$ $= \$84.59$	$\$12\,084 + \$84.59$ $= \$12\,168.59$
End of the 3rd month	$I = Pin$ $= \$12\,168.59 \times 0.007 \times 1$ $= \$85.18$	$\$12\,168.59$ $+ \$85.18$ $= \$12\,253.77$

b $253.77

5 **a** $600 **b** $735.03
c $135.03 **d** 7%

6 **a** A_0 = principal, $r = 1 +$ the rate of interest
b $12 480
c $480
d $A_2 = \$12\,979.20$, $A_3 = \$13\,498.37$
e $13 498.37
f $1498.37

7 **a** $A_0 = \$4000$, $r = 1.032$
b $A_1 = \$4128$, $A_2 = \$4260.10$
c $260.10
d $4.10

8 **a** $A_0 = \$14\,000$, $r = 1.007$
b $A_1 = \$14\,098$, $A_2 = \$14\,196.69$, $A_3 = \$14\,296.06$
c $296.06

9 **a** $A_0 = \$10\,000$, $A_1 = \$11\,500$, $A_2 = \$13\,225$, $A_3 = \$15\,208.75$
b $5208.75
c High interest rate, unknown 'investment advisor', pressure to act quickly

10 **a** Simple interest graphs are straight lines. Compound interest graphs curve up (exponential).
b 7 years

11 **a** $400
b *X* compound (Nick), *Y* simple (Adam)
c Nick's investment is worth $35.45 more.

EXERCISE 11.3

1 **a** $8242.87 **b** $4546.86 **c** $29 325.40

2 **a** $P = \$970$, $i = 0.068$, $n = 4$
b $1261.99 **c** $291.99

3 $6461.81

4 **a** $22 834.71 **b** $14 834.71

5 $764.51

6 **a** $0.072\,12 = 0.006$, $n = 6 \times 12 = 72$
b $7691.74 **c** $2691.74

7 **a** $7.8 \div 12 = 0.65$
b $2191.20 **c** $241.20

8 **a** 0.68% **b** 60 **c** $92 820.41

9 **a** 3.9%
b $7 \times 2 = 14$
c $14 949.47

10

	Final amount	Interest
Bank	$52 672.26	$12 672.26
Credit union	$52 431.84	$12 431.84

Despite a lower interest rate, the bank investment generates an extra $240.41 interest.

EXERCISE 11.4

1 a $3974.44 **b** $974.44
c Teacher to check.

2 a $6529.12 **b** $1353.20
c $13 170.10 **d** $46 960.25

3 Year 1: $1311, Year 2: $1348.36, Year 3: $1386.79, Total: $4046.16

4 a $77 318.13 **b** $27 318.13
c Teacher to check.

5 $9177.22

6 The future values are: Option 1 $23 300, Option 2 $22 823.32.
Option 1 is the better one if it is an investment because she ends up with more money.

EXERCISE 11.5

1 a $1215.61 **b** $3218.44
c $43 630.30 **d** $55 475.18

2 a $649.96 **b** $716.50
c $2466.86 **d** $966.95

3 a $5948.93 **b** $9948.93

4 a B8: =B4 C8: = B8*$B$5 D8: = B8+C8 A9: = A8+1 B9: =D8
b 0.045 **c** $48 536.88
d $24 536.88

5 a Teacher to check. **b** approximately 18 years

EXERCISE 11.6

1 a $1966.81 **b** $517.13
c $359.36 **d** $366.16

2 a $7193.75 **b** $1393.75

3 $1744.93 **4** $4.19 **5** $464

6 The monthly compounding gives the better return (final values of $511 274.92 compared to $510 880.61).

7 The managed fund gives the better return (final values of $211 533.75 compared to $210 133.50).

KEYWORD ACTIVITY

1 G **2** H **3** E
4 A **5** B **6** C
7 F **8** I **9** D

TEST YOURSELF

1 $1312 **2** 7.5% p.a.
3 1.74 years **4** $38.40

5 a $5420
b $r = 1 + i$, $r = 1 + 0.05$
c $A_1 = \$5691$, $A_2 = \$5975.55$, $A_3 = \$6274.33$
d $854.33

6 a $P = \$8400$, $i = 0.04$, $n = 5$
b $10 219.88
c $1819.88

7

	Final amount	Interest
Bev	$13 280	$3280
Sally	$12 864.66	$2864.66

Bev's investment earned $415.34 more interest.

8 Graph 1, compound interest
Graph 2, simple interest

9 Investment B is better ($172.06 compared to $163.20).

10 $8607.59

11 a monthly interest for month 1
b to keep the cell reference the same when she copies down
c $436.09
d either =SUM(C10:C21) or =D21 – C4
e $67 622.88
f

	A	B	C	D
1	Jill's spreadsheet			
2				
3				
4		Initial investment C4	$60,000.00	
5		Annual interest rate as a decimal C5	0.08	
6		Monthly interest rate C6	0.006666667	
7		Weekly interest rate	0.001538462	
8				
9	Number of weeks	Value of the investment at the beginning of the week	Weekly interest	Value of the investment at the end of the week
10	1	$60,000.00	$92.31	$60,092.31
11	2	$60,092.31	$92.45	$60,184.76
12	3	$60,184.76	$92.59	$60,277.35
13	4	$60,277.35	$92.73	$60,370.08
14	5	$60,370.08	$92.88	$60,462.96
15	6	$60,462.96	$93.02	$60,555.98
16	7	$60,555.98	$93.16	$60,649.14
17	8	$60,649.14	$93.31	$60,742.45
18	9	$60,742.45	$93.45	$60,835.90
19	10	$60,835.90	$93.59	$60,929.49
20	11	$60,929.49	$93.74	$61,023.23
21	12	$61,023.23	$93.88	$61,117.11
22	13	$61,117.11	$94.03	$61,211.14
23	14	$61,211.14	$94.17	$61,305.31
24	15	$61,305.31	$94.32	$61,399.63
25	16	$61,399.63	$94.46	$61,494.09
26	17	$61,494.09	$94.61	$61,588.69
27	18	$61,588.69	$94.75	$61,683.45
28	19	$61,683.45	$94.90	$61,778.34
29	20	$61,778.34	$95.04	$61,873.39
30	21	$61,873.39	$95.19	$61,968.58
31	22	$61,968.58	$95.34	$62,063.91
32	23	$62,063.91	$95.48	$62,159.40
33	24	$62,159.40	$95.63	$62,255.03
34	25	$62,255.03	$95.78	$62,350.80
35	26	$62,350.80	$95.92	$62,446.73

g $6.37

12 a 0.4% **b** $463.66

13 The annual compound interest ($12 726.37 compared to $12 707.42 and $12 640).

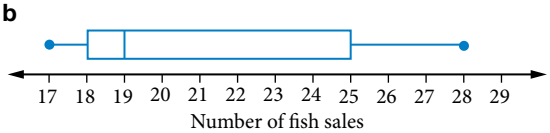
CHAPTER 12

EXERCISE 12.1

1 a

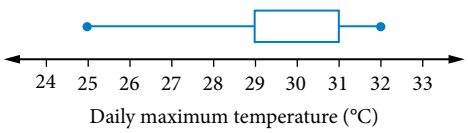
Lowest value	Q_1	Median	Q_3	Highest value
17	18	19	25	28

b

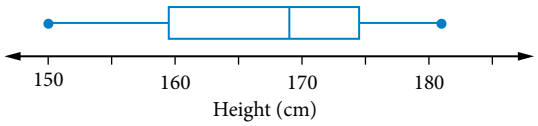


2 a 25, 29, 31, 31, 32

b

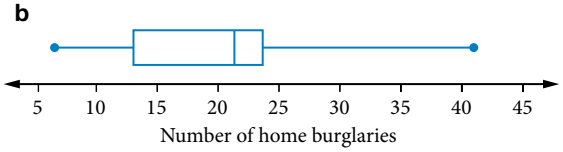


3



4 a 6, 13, 21, 24, 41

b



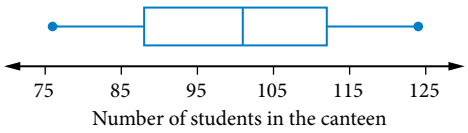
5 a 22 **b** 35

c **i** 15 **ii** 45

d 18

6 a 76, 88, 101, 112, 124

b



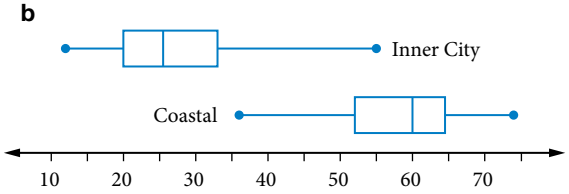
7 a 20 **b** 12 **c** 4

d 15 **e** 30

8 a Inner City: 12, 20, 25.5, 33, 55

Coastal: 36, 52, 60, 64.5, 74

b



c Inner City, lower risk of theft. "The median is 35 lower for Inner city even though the range is higher".

EXERCISE 12.2

1 a

Class A		Class B
	3	5 8 9 9 9
	4	0 1 2 2 2 6 7
	5	1 4 6 7 9
9 7 5 2 0	6	0 1 5 5 5 6 8
9 8 6 5 1 1 1 0	7	1 2 4
9 9 9 8 5 5 4 2 1 1 0	8	8
5 5 4 3 1	9	

Key: 0|6| = 60

|3|5 = 35

b 29 **c** A: 81, B: 55

d A: 35, B: 53 **e** an outlier in B (88)

f A top, B middle; A has a higher median.

2 a

Year 11		Year 12
	0	7 8
5 5 5	1	0 2 3 3 4 5 8 9 9
9 5 3 3 2 2 0	2	0 0 1 2 2 3 4 4 6
9 1 0 0 0 0 0	3	
4 2 1	4	

Key: 5|1| = 15

|1|0 = 10

b Year 11: 29.5, Year 12: 19

c Year 11: 29, Year 12: 19

d no outliers

e clustered in the 20s

f medians are very different

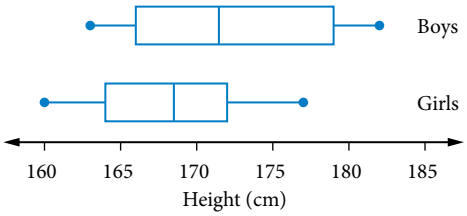
g **i** Yes. Year 11 have a higher median and a greater range.

ii Number of students in each year and compare using percentages.

3 a Boys: 163, 166, 171.5, 179, 182

Girls: 160, 164, 168.5, 172, 177

b



c Teacher to check.

4 a

Male		Female
	15	0 2 7 9
8 7 5 0	16	0 2 3 4 4 5 6 6 6 7 8
8 8 8 5 2 0 0	17	1 3 3 6 6 7 8
4 4 3 3 1 1 1 1 0 0	18	5 5
1 0 0	19	
0	20	

Key: 0|16| = 160

|15|0 = 150

b 25 **c** 24

d Males: 160, 171, 180, 183.5, 200

Females: 150, 162.5, 166, 174.5, 185

e 200 is an outlier in the male group.

f Male median is higher than the female median.

g Teacher to check.

ANSWERS

5 a

Townsville		Kiama
	20	3 7
	21	0 8
	22	1 3 5 5
	23	0
	24	5 7 9
	25	0 4 5 6 6 9
	26	4 7
	27	0 6 9
	28	
5 3	29	6
	30	
9 9 8 7 7 2 2 0 0 0	31	
8 7 7 5 3 3 2 1 1	32	5 6
7 6 4 4 1	33	0
3 2	34	
	35	
	36	3

Key: 3|29| = 293
|20|3 = 203

b 28

c Townsville: 32.15°C, Kiama: 25.45°C

d Townsville: 5°C, Kiama: 16°C

e Townsville: 29.3°C, 29.5°C; Kiama: 36.3°C, 33.0°C, 32.6°C, 32.5°C

f Townsville temperatures are close together, Kiama temperatures are widely spread out.

g Teacher to check.

h Teacher to check.

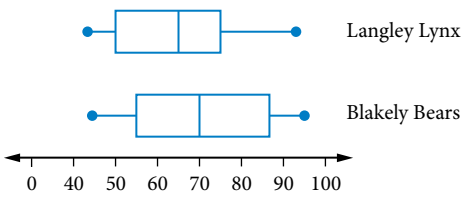
6 a 25

b Langley Lynx: 49, Blakely Bears: 50

c Langley Lynx: 66, Blakely Bears: 70

d Langley Lynx: 43, 51.5, 66, 75.5, 92
Blakely Bears: 44, 57, 70, 87, 94

e

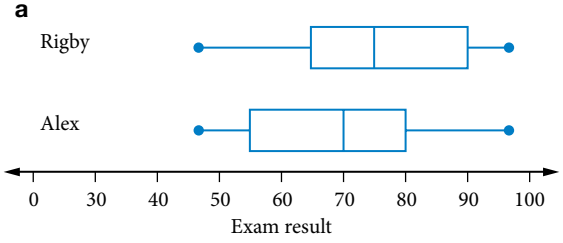


f Teacher to check.

g Blakely Bears

EXERCISE 12.3

1 a



b **i** 75 **ii** 69

c **i** 48 **ii** 50

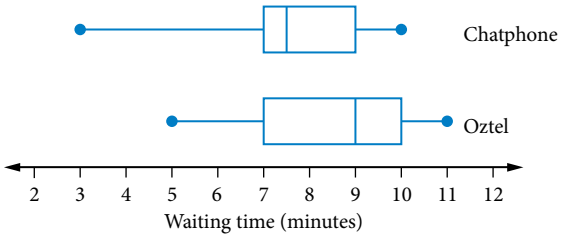
d Alex – his mark is 16% above the median, while Rigby's is only 10% above the median.

e Rigby's class – more than 50% of the scores are above the median for Alex's class.

f No, we don't have any scores or the numbers of students in each class.

2 a Chatphone: 3, 7, 7.5, 9, 10
Oztel: 5, 7, 9, 10, 11

b



c Chatphone 7.5; Oztel 9

d Chatphone 7; Oztel 9

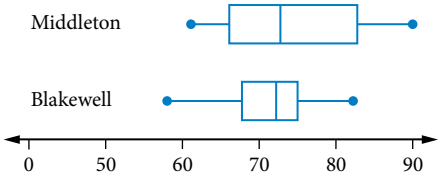
e Chatphone 2; Oztel 3

f Yes, Chatphone's median is 1.5 minutes less than Oztel and 75% of Chatphone's calls wait less than 50% of Oztel calls.

g Teacher to check.

3 a Middleton: 60, 67.5, 72.5, 82.5, 90;
Blakewell: 58, 68, 72, 75, 82

b



c Middleton 72.5 km/h; Blakewell 72 km/h

d Middleton 72, 75, 85; Blakewell 68, 70, 72, 73, 75

e Middleton 74.2 km/h; Blakewell 71.5 km/h

f Middleton 15 km/h; Blakewell 7 km/h

g No, the mean in Blakewell is lower and 25% of the sample in Middleton were faster than the entire Blakewell sample.

4 a 60 **b** 61 **c** soft drink

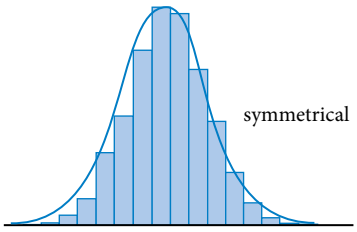
d coffee **e** Teacher to check.

5 a 52% **b** 15–24

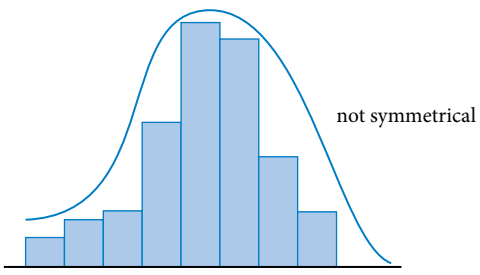
c Yes, higher frequency for the eastern region in each age group.

EXERCISE 12.4

1 a

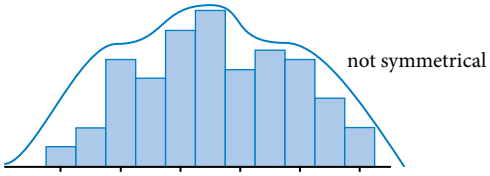


b

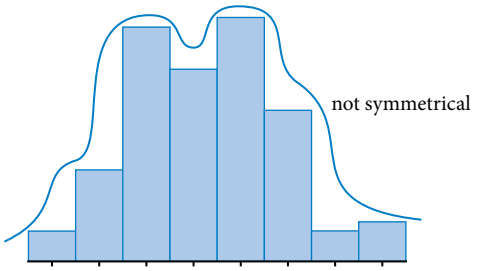


9780170484664

c



d

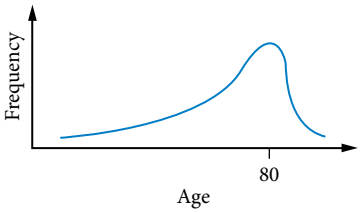


2 **a** negatively skewed
b positively skewed
c bimodal
d positively skewed

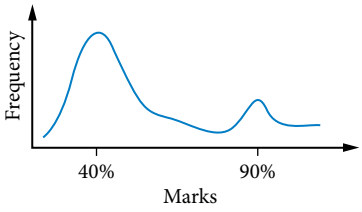
3 **a**, **b** and **d**

4 Many answers are possible. Teacher to discuss.

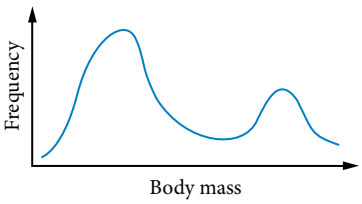
a



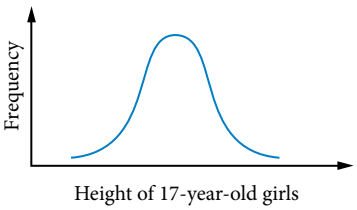
b



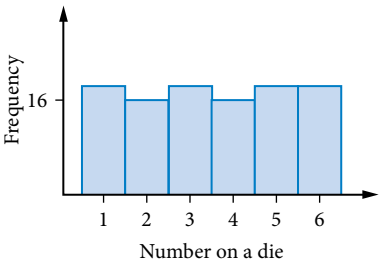
c



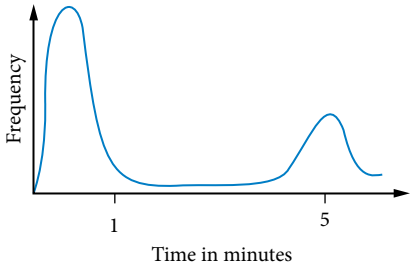
d



e

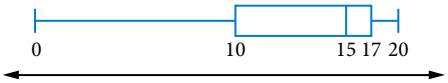


f



5 The tail extends on the right to 29, and half of the data lies between 5 and 9.

6 Teacher to discuss. The diagram shows a sample answer.



7 Many answers possible. Teacher to check.

8 **a** has 3 peaks
b Many possible answers. It could be a coffee shop. Peak times correspond to people going to work, plus morning and afternoon tea times.

9 **a** positively skewed
b Many answers are possible. For example, change 56, 57, 58, 59 to 86, 87, 78, 79 respectively.

10 **a** bimodal
b Many possible reasons, including the 3 given below.
- There are two warehouse employees and one works faster than the other.
- The information is from two different times, for example, weekdays and weekends. The weekend employees are only part-time and can't locate items as quickly as the full-time employees.
- Some items are more difficult to access than others.

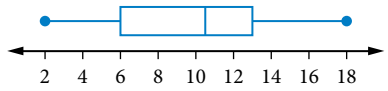
11 **a** negatively skewed
b easy, most people got high marks
c around 85
d average to just below average, but below the mode

KEYWORD ACTIVITY

a B **b** D **c** E **d** C **e** A

TEST YOURSELF

1 **a** 2, 6, 10.5, 13, 18
b



2 **a** median = 26.5 **b** 14
c 9 employees

3 a

Bulls		Tigers
	7	2
9 3	8	
8 0	9	7
9	10	4 9
4 3	11	
7 5	12	6
7	13	0 3 8 9 9
0	14	4 7
9	15	

Key: 3|8| = 83
|12|6 = 126

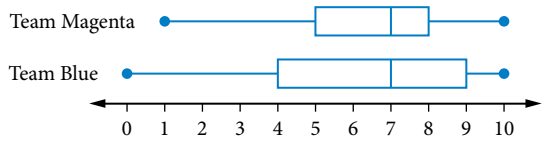
b 12

c Bulls: 113.5 Tigers: 131.5

d Bulls: 159 Tigers: 72

e Tigers – higher median and scores clustered in the 130s.

4 a



b Median is 7 for both teams.

c Team Magenta: 3, Team Blue: 5

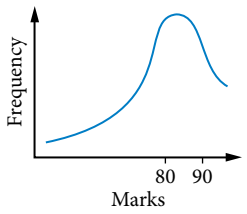
d Adele – she is in the top 25% of her team, while Shane is at the 75% mark in his team.

5 a positively skewed

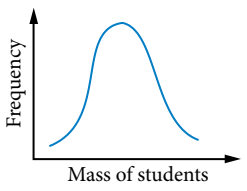
b bimodal

c symmetrical

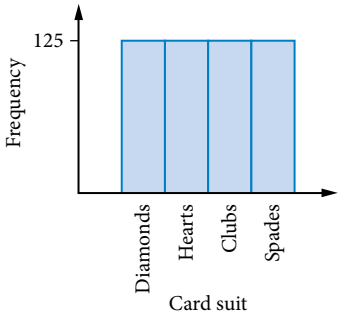
6 a



b



c



PRACTICE SET 3

Section A

1 D **2** C **3** D **4** C

5 A **6** B **7** C **8** D

Section B

1 7, 9

2 a $5896 **b** $596

3 a 17.7 **b** 20 **c** 18.5 **d** 14

e 20 **f** 16 **g** 4

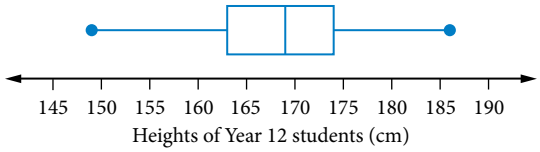
4 a 169 cm

b $Q_L = 163$ cm, $Q_U = 174$ cm

c 11

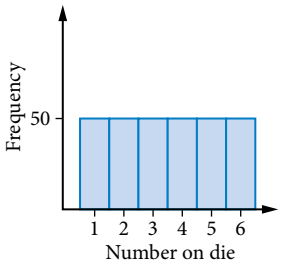
d 149, 163, 169, 174, 186

e

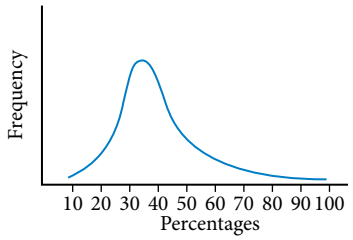


5 a 1.77 **b** 1.43

6 a



b



Other answers possible.

7 $768.91

8 a

English test		Maths test
8	0	3 7
9 7	1	1 1 7 9
7 7 7 5 3 3 2 0	2	0 1 3 7 8 8 9
9 8 8 6 4 4 2 1 1 0 0 0	3	0 0 1 1 2 2 4 5 5 6 8 9
8 6 5 1 1 0 0	4	4 6 7 7 9

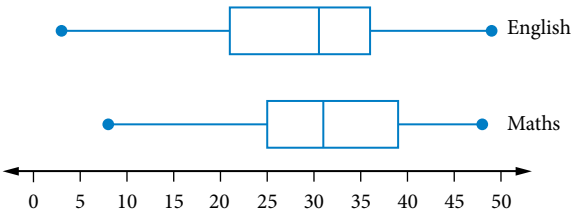
Key: 8|4| = 48
|2|0 = 20

b English: 40, Maths: 46

c There are no outliers for either data set.

9 a English: 8, 25, 31, 39, 48
Maths: 3, 21, 31, 38, 49

b



10 Teacher to check.

11 $8707.60

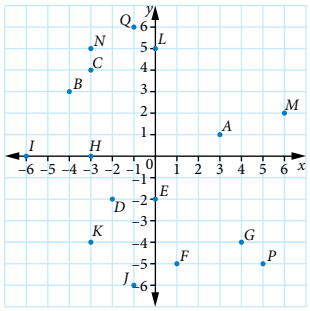
12 Cengage Central Bank, as it earns $284.18 more interest.

CHAPTER 13

EXERCISE 13.1

1 $A(-4,3)$, $B(-2,2)$, $C(0,2)$, $D(2,3)$, $E(2,1)$, $F(1,-1)$, $G(2,-3)$, $H(0,-3)$, $I(-4,-3)$, $J(-2,-1)$, $K(3,0)$, $L(-4,0)$, $M(0,-2)$, $N(0,3)$, $O(0,0)$, $P(-2,0)$, $Q(1,0)$, $R(3,-2)$, $S(-2,-2)$, $T(-3,1)$, $U(-1,2)$, $V(4,1)$, $W(4,-3)$, $X(-4,-1)$, $Y(1,-4)$, $Z(-1,-3)$

2



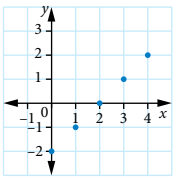
3 **a** *A*, *M* **b** *B*, *C*, *N*, *Q* **c** *D*, *K*, *J*
d *F*, *G*, *P* **e** *E*, *I*, *H*, *L*

4 **a** 4th **b** 3rd **c** 2nd

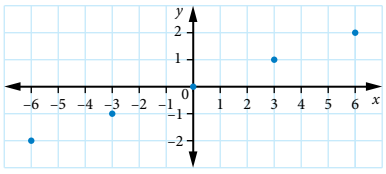
5

Points in:	*x*–coordinate	*y*–coordinate
1st quadrant	+	+
2nd quadrant	–	+
3rd quadrant	–	–
4th quadrant	+	–

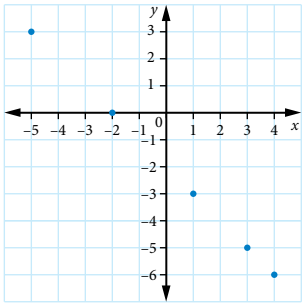
6



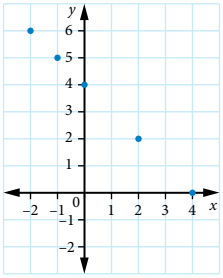
7 **a**



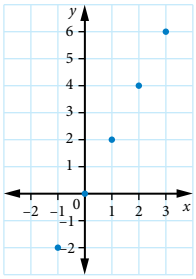
b



c



d



EXERCISE 13.2

1 **a**

x	–5	–2	3	6	11
y	–7	–1	9	15	25

b

x	–4	–2	3	6	9
y	–2	–1	1.5	3	4.5

c

x	4	2	5	7	–3
y	7	1	10	16	–14

d

x	–2	–1	4	7	10
y	24	22	12	6	0

2

x	1	2	5	9	10
y	11	10	7	3	2

3 **a** F **b** B **c** A
d C **e** E **f** D

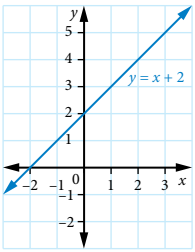
4 Other answers possible.
a $y = x + 1$ **b** $y = x \div 10$
c $y = 4x$ **d** $y = x^2$
e $x + y = 10$ **f** $x \times y = 24$
g $x + y = 7$ **h** $y = 2x$

5 Other answers possible.

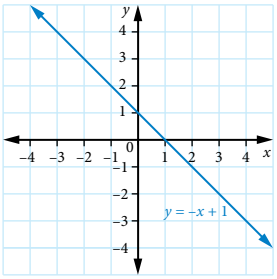
x	5	6	7	8	9
y	4	5	6	7	8

EXERCISE 13.3

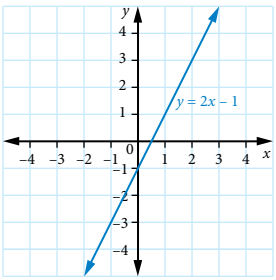
1 **a**



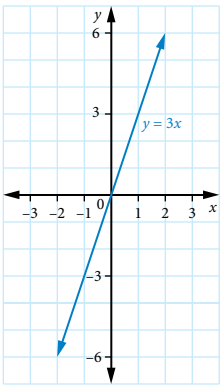
b



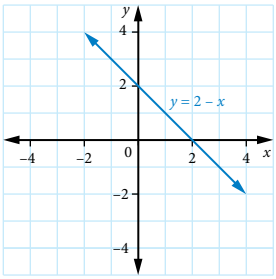
c



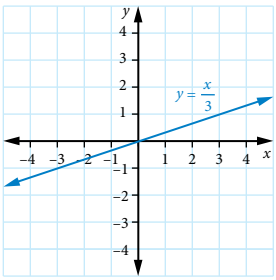
d



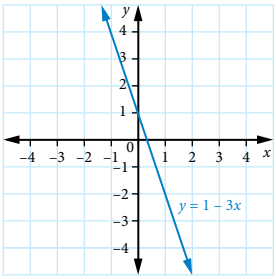
e



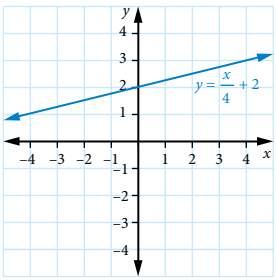
f



g



h



2 a gradient 2, y–intercept 4

b gradient -4, y–intercept -5

c gradient -1, y–intercept 4

d gradient 1, y–intercept 0

e gradient $\frac{1}{5}$, y–intercept -3

f gradient $\frac{1}{2}$, y–intercept 5

3 a *C* **b** *B* **c** *D* **d** *A*

4 a *D* **b** *C* **c** *A* **d** *B*

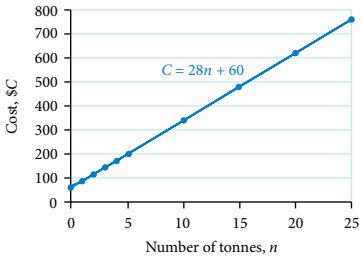
EXERCISE 13.4

1 a

Number of tonnes, n	0	1	2	3	4	5	10	15	20	25
Cost of soil, $C	60	88	116	144	172	200	340	480	620	760

b $C = 28n + 60$

c



d 60

e \$60 initial fee for the delivery.

f 28, the additional cost per ton of soil.

g It's the maximum amount the truck can carry.

h \$1240

i No, it doesn't make a difference.

$480 + 760 = 1240$ and $2 \times 620 = 1240$

2 a

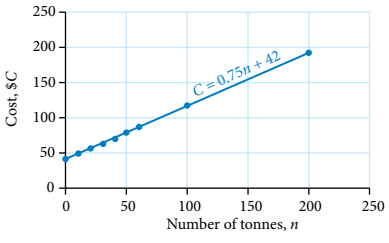
Number of containers, n	0	10	20	30	40	50	60	100	200
Cost of making chips, $C	42	49.50	57	64.50	72	79.50	87	117	192

b $C = 0.75n + 42$

c \$169.50

d 143

e

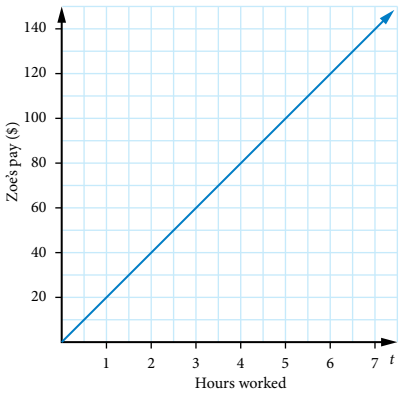


f 42, the initial cost of the oil.

g 0.75, the additional cost of each container of chips.

3 a For each hour, Zoe is paid \$20, so her pay is 20 times the number of hours she works.

b

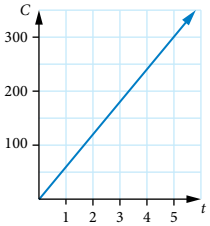


c 20 **d** Zoe's hourly pay

e If Zoe doesn't work, she is not paid anything.

4 a $C = 60t$

b



c gradient = 60; it represents Mick's hourly charge

5 a −25

b The decrease in drug concentration per hour. It goes down 25 µg/mL every hour.

c 200

d The initial concentration of the drug: 200 µg/mL

e 8 hours

6 a D **b** B **c** C

d A **e** F **f** E

KEYWORD ACTIVITY

1 axes **2** x–axis

3 y–axis **4** origin

5 quadrants **6** ordered pair

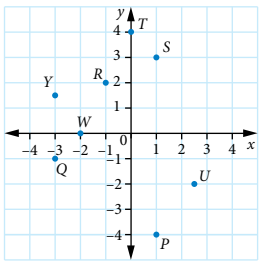
7 table of values **8** linear

9 gradient **10** y–intercept

11 front **12** own

TEST YOURSELF

1



2 a T **b** R, Y **c** W **d** P, U

3 a

x	−1	0	1	2	3
y	2	3	4	5	6

b

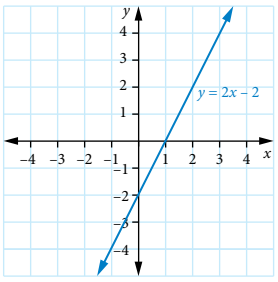
x	−2	−1	2	4	5
y	−11	−9	−3	1	3

c

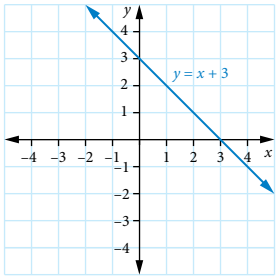
x	−3	−1	0	2	4
y	8	6	5	3	1

4 a $y = x - 2$ **b** $y = 3x - 1$ **c** $y = 10 - x$

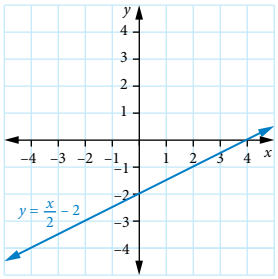
5 a



b



c



6 a gradient 2, y–intercept −2

b gradient −1, y–intercept 3

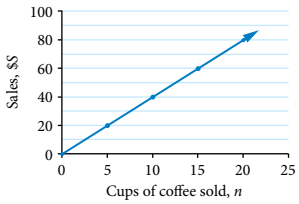
c gradient $\frac{1}{2}$, y–intercept −2

7 a

Coffees sold, n	0	5	10	15	20
Sales, $S	0	20	40	60	80

b $S = 4n$

c



d 0, Liza receives no money if she sells no coffees

e 4, the price of each coffee

8 a 3 minutes **b** 2

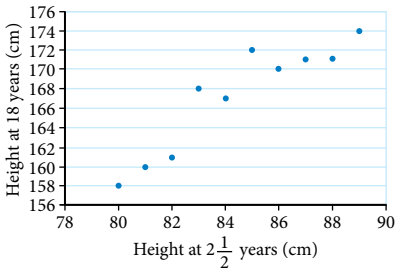
c Time to pack each item: 2 minutes

d 33 minutes **e** 8 items

CHAPTER 14

EXERCISE 14.1

1 a

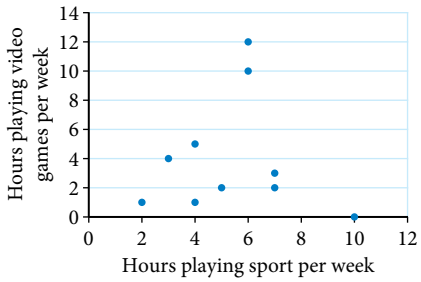


b Height at $2\frac{1}{2}$ years is the independent variable.

Height at 18 years is the dependent variable.

c As height at $2\frac{1}{2}$ years increases, the height at 18 years increases.

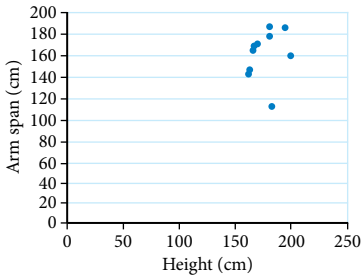
2 a



b 'Hours playing sport per week' is the independent variable and 'Hours playing video games per week' is the dependent variable.

c There is no pattern to this data.

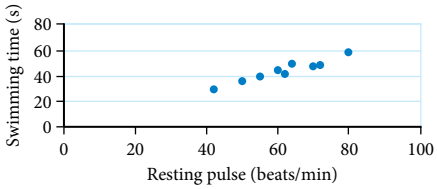
3 a



b 'Height' is the independent variable and 'Arm span' is the dependent variable.

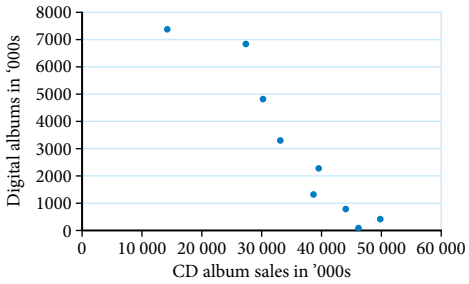
c There is one outlier. The other points show that as height increases, the arm span increases.

4 a



b As resting pulse increases, swimming time increases.

5 a



b As CD album sales increase, digital album sales decrease.

EXERCISE 14.2

1 a positive, linear, moderate

b no association

c positive, linear, strong

d negative, non-linear, strong

e positive, linear, weak

f negative, linear, strong

g positive, non-linear, strong

h no association

i negative, linear, moderate

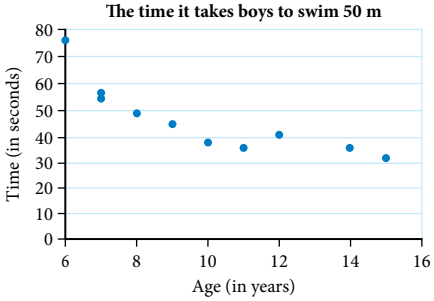
j positive, linear, moderate

2 a positive, linear, moderate

b positive, linear, strong

c negative, linear, moderate

3 a



b Yes, there is a strong, negative, linear correlation.

4–6 Teacher to check.

EXERCISE 14.3

1 a causal

b not causal, age causes the length of feet to increase

c causal

d not causal, number of spectators at the game influences these

e not causal, age causes these two to be linked

f causal

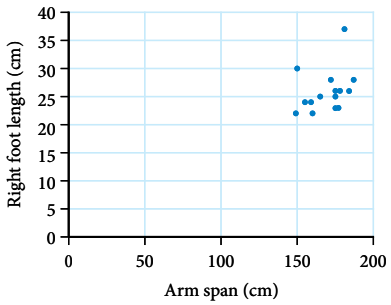
g causal

2–4 Teacher to check.

9780170484664

EXERCISE 14.4

1 a



b Independent variable: Arm span; dependent variable: Right foot length

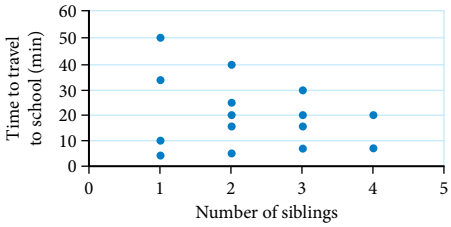
c As arm span increases, right foot length increases.

d positive, linear, moderate

e No, both are linked to age and body shape.

2 a No, teacher to check reasons.

b



c The scatterplot shows no relationship between the 2 variables.

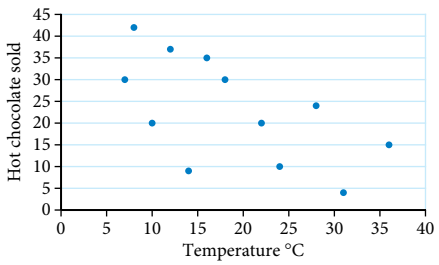
3–4 Teacher to check.

KEYWORD ACTIVITY

1 bivariate
2 scatterplot
3 independent variable
4 dependent variable
5 association
6 positive
7 negative
8 linear
9 non-linear
10 weak
11 moderate
12 strong
13 causes

TEST YOURSELF

1 a



b Independent variable: temperature; dependent variable: hot chocolate sold

c Teacher to check.

2 The association is negative, linear and weak.

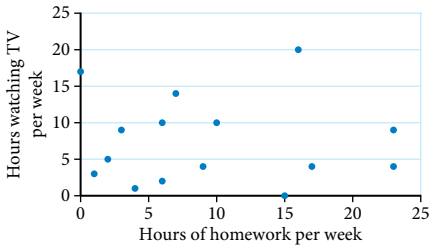
3 Teacher to check.

4 a Not causal; these are linked by age and body type.

b Causal; the more the team wins, the more likely fans will buy their merchandise.

5 a Teacher to check.

b



c Independent variable: hours of homework per week; dependent variable: hours watching TV per week

d There is no association between these 2 variables.

CHAPTER 15

EXERCISE 15.1

1 a $7000 × 0.09 ÷ 12 = $52.50

b $7000 + $52.50 = $7052.50

c $7052.50 − $320 = $6732.50

d **A** $6732.50 **B** $50.49
C $6782.99 **D** $6462.99
E $6462.99 **F** $48.47
G $6511.47 **H** $6196.47

e $6191.47 **f** $151.46

g $6.06 **h** He owes less.

2 a

$15 696	$94.18	$15 790.18	$15 390.18
$15 390.18	$92.34	$15 482.52	$15 082.52
$15 082.52	$90.50	$15 173.01	$14 773.01

b $14 773.01 **c** 22.6%

3 a $720; see worked solutions.

b **A** $15 286 **B** $611.44
C $12 463.44 **D** $498.54
E $6734.10

c 2 years

4 a

1	$12 800	$80.64	$12 880.64	$11 980.64
2	$11 980.64	$75.48	$12 056.12	$11 156.12
3	$11 156.12	$70.28	$11 226.40	$10 326.40

b $2700 **c** $2473.60 **d** $226.40

5 a

	A	B	C	D	E
1	Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	1	10 000	=B2*0.06/12	=B2+C2	=D2−260
3	2	=E2	Copy down	Copy down	Copy down
4	3	Copy down			

b $8939.35

EXERCISE 15.2

1 a $29.17 **b** $27.30 **c** $1024.05
d 15 months **e** $335.93 **f** $235.93
g $5235.93

2 a 18 months **b** $195.45 **c** $195.45

3 **a** 18 months **b** $532.31

4 **a** 25 months **b** 18 months

c Yes, he will save $202.26.

d The quicker you repay a reducing balance loan, the less interest you will pay.

e No, the interest is a fixed amount.

5 **a** =B4, =B9*$B$5/12, =B9+C9, =D9−$B$6

b The dollar signs keep the interest rate and the repayment amount the same when we copy down.

6 =B4, =B9*$B$5/12, =B9+C9, =D9−$B$6, =E9

7 The diagram shows the final lines of the spreadsheet.

	A	B	C	D	E
37	31st month	$ 4,925.75	$ 30.79	$ 4,956.54	$ 4,006.54
38	32nd month	$ 4,006.54	$ 25.04	$ 4,031.58	$ 3,081.58
39	33rd month	$ 3,081.58	$ 19.26	$ 3,100.84	$ 2,150.84
40	34th month	$ 2,150.84	$ 13.44	$ 2,164.28	$ 1,214.28
41	35th month	$ 1,214.28	$ 7.59	$ 1,221.87	$ 271.87
42	36th month	$ 271.87	$ 1.70	$ 273.57	-$ 676.43
43					
44					
45		Sum of Interest = SUM(C9:C44)			
46		$ 3,523.57			
47					

The spreadsheet shows that it will take 36 months for Gabriel to repay the loan. The interest is in column C. The formula =SUM(C9:C44) will produce the sum of the interest.

Gabriel will make 36 repayments and pay a total of $3523.57 in interest.

8 **a** 13 months **b** $214.45

9 more, more

10 less, less

11 **a** Several answers possible, for example, $10 000 × 0.084 ÷ 4.

b =B9*$B$4/4

c D9: =B9+C9, E9: =D9−$B$5, B10 = E9

e $100 807.30

f 11 quarters = 2 years 9 months

g $10 807.30

h She will repay $104 784.49, including interest of $14 784.49. By reducing the quarterly repayments, Gaynor will increase the length of time it will take her to repay the loan, and she will repay more, which includes more interest.

EXERCISE 15.3

1 **a** $317 **b** $19 020 **c** $4020

2 **a** $616

b $70 808

c The car will be worn out before it is paid for.

d Save up for the car or buy a cheaper car.

3 **a** 17 months **b** $645

4 **a** $190.40

b He owes less, and reducible interest is calculated on the amount owing.

c $229 405

d $165 096

e **i** $915.20 **ii** $919.20

f 49.9%

g 22 years 5 months

h 91 months (or 7 years 7 months)

i At the beginning, when the loan amount is at its highest.

5 **a** **i** 240 months **ii** 187 months

b 53 **c** $25 290

d They can't afford to pay the higher amount of monthly repayments or they don't want to.

6 **a** $2301 **b** $36

7 **a** $1952 **b** $22 **c** $264

8 **a** $246 **b** $152 280

c If I could afford the extra $246 per month, I would pay it. If I can't afford the whole $246/month, I would pay the extra that I can afford.

9 $1266. Payments in the 1980s were more than double the late 2010s value.

10 **a** $998, $989

b

	Big bank	Small mortgage company
Loan establishment fee	$320	$598
Mortgage discharge fee	$228	$314
Total annual loan fee over 25 years	$6200	$1900
Total monthly repayments	$299 400	$296 700
Total cost of the loan	$306 148	$299 512

c the interest rate and the annual loan fee

d The small mortgage company, because she will save $6636.

EXERCISE 15.4

Teacher to discuss answers with the class.

KEYWORD ACTIVITY

1 credit union	**2** interest	**3** flat
4 same	**5** reducible	**6** decreases
7 real estate	**8** huge	**9** repayment
10 minimum	**11** fortnightly	**12** money

TEST YOURSELF

1 **a** **A** $11 702 **B** $70.21

C $11 772.21 **D** $11 402.21

E $11 402.21 **F** $68.41

G $11 470.63 **H** $11 100.63

b $11 100.63 **c** $210.62

2 **a** 2 years 9 months **b** $3278.34

3 **a** $2136 **b** $242 602

c 15 years 10 months **d** $58 181

4 **a** Increases the value of the repayments.

b Decreases the time and the total amount.

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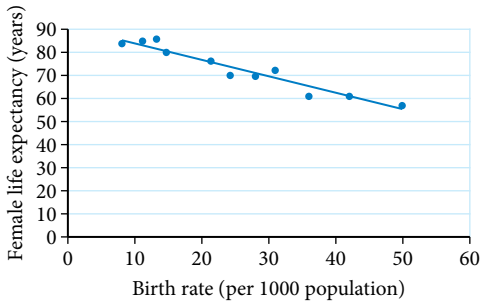
CHAPTER 16

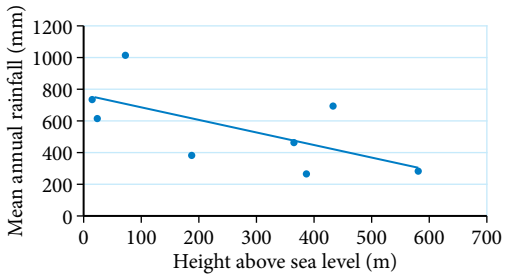
EXERCISE 16.1

1 Teacher to check.

2



3



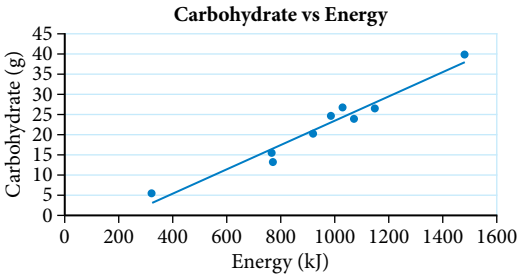
4 a $y = -0.7248x + 91.18$, other answers are possible

b **i** −0.7248 **ii** 91.18

5 a $y = -0.7691x + 755.52$, other answers are possible

b **i** −0.7691 **ii** 755.52

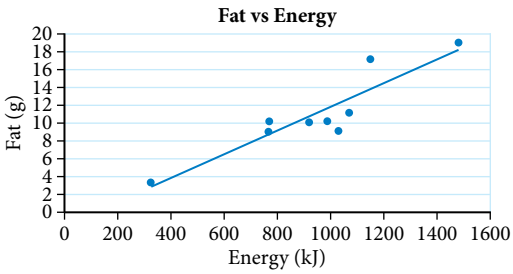
6 a, b



c $y = 0.0301x - 6.7032$, other answers possible

d **i** 0.0301 **ii** −6.7032

e



f $y = 0.0132x - 1.4528$, other answers possible

g **i** 0.0132 **ii** −1.4528

h They both have a positive association. Other answers possible.

EXERCISE 16.2

1 a weeks after starting training

b leg extensions in 30 seconds

c The longer Anita has trained, the more leg extensions she can do in 30 seconds.

d 11 or 12

2 a number of victims

b number of medical staff

c about 40 staff

d about 750 victims

e The greater the number of victims, the greater the number of medical staff sent.

f The agency calculated the number of medical staff required based on the number of victims, not the reverse as Kyle's statement implies.

3 a about 23.5 cm

b The longer the right foot, the higher the test score.

c The age of the students is relevant. Younger students who haven't learned as much maths generally have smaller feet and would score lower on the test.

4 a As the height of students increases, weight of students increases.

b about 55 kg

c no; teacher to check reasoning

5 a about 10 or 11

b As the number of sunny days increases, the number of dry cloudy days decreases.

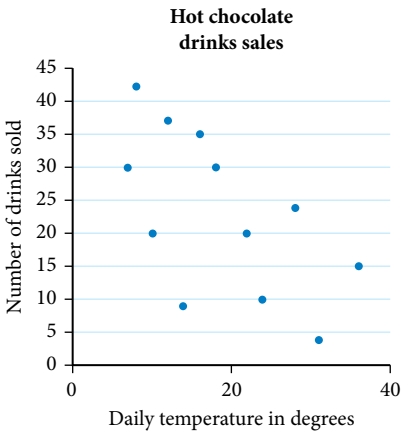
c The total number of sunny, dry cloudy and wet days must be the same as the number of days in the month. As the number of sunny days increases, the number of dry cloudy or wet days must decrease to keep the total the same as the number of days in the month.

EXERCISE 16.3

1 a E **b** C **c** B **d** G

e H **f** A **g** F **h** D

2 a



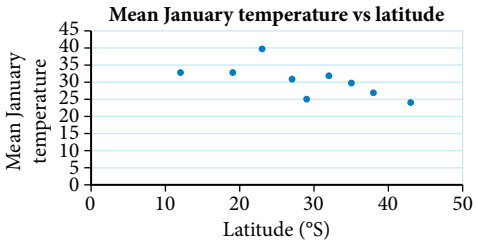
b −0.61

c Yes, the scatterplot shows a moderate negative linear association and the correlation coefficient confirms this.

d Yes, when it is colder, people are more likely to feel like having a hot drink.

3 **a** −0.9662 **b** −0.6332 **c** 0.9779
d 0.9095 **e** 0.8520

4 **a**



b −0.6522

c No, the correlation coefficient is only in the moderate range.

d Teacher to check.

5 **a** −0.9484

b They are closely related. As digital sales increase, CD sales decrease.

c Yes, if more people buy digital albums, then fewer people will buy CD albums.

6 Teacher to check.

EXERCISE 16.4

1 **a** **i** 1090 kJ **ii** 105 kg

b Reliable, as the correlation is strong.

c 1885 kJ

d Teacher to check.

2 **a** about 62 years

b about 36 births

c Reliable, as the correlation is strong.

d about 48 years

e Teacher to check.

3 **a** about 525 mm

b about 460 m

c Not very reliable, as the correlation is weak.

d about 140 mm

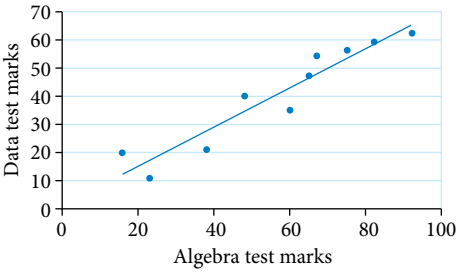
e −14 mm, not possible as you can't have negative rainfall

4 **a** 0.975 989

b Very reliable, as the correlation coefficient is close to 1.

c May not be reliable as −10°C is outside the data range.

5 **a**

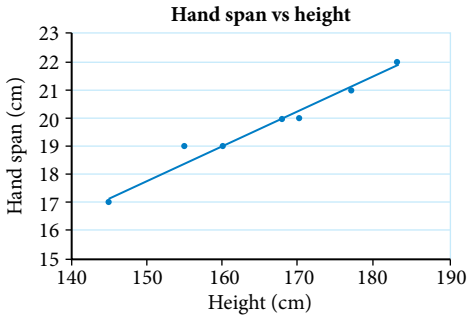


b Ella 43 in Algebra test, Amy 57 in Data test.

c 0.949 98

d Very reliable, as the correlation coefficient is close to 1.

6 **a**



b about 20 cm

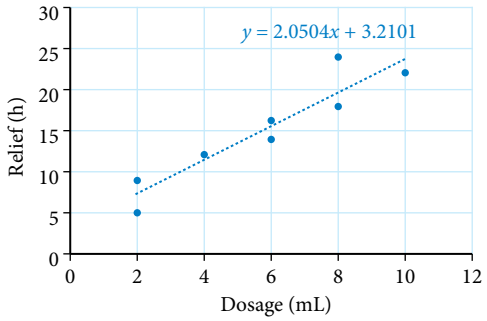
c It's about 33 cm. The value is very likely to be wrong because Robert Wadlow's height is much greater than the heights used in the scatterplot.

KEYWORD ACTIVITY

1 bivariate **2** scatterplot
3 line of best fit **4** technology
5 equation **6** correlation
7 correlation coefficient **8** predictions
9 interpolation **10** extrapolation

TEST YOURSELF

1 **a**



b $y = 2.0504x + 3.2101$

2 **a** dosage

b duration of relief

c From formula: 13.4621 hours or from graph: about 13 hours.

d From formula: 3.31 mL or from graph: about 3 mL.

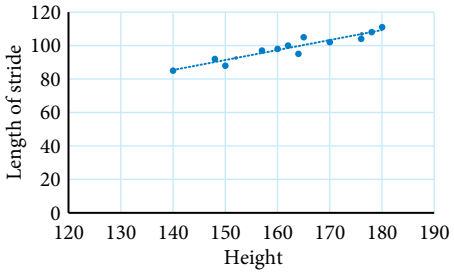
e As the dosage increases, the duration of relief increases.

3 Teacher to check.

4 **a** −0.0365

b No, they are not closely related, as the correlation coefficient is close to 0.

5 **a**



b 0.9421

c **i** 94 cm **ii** 147 cm

d Very reliable as the correlation coefficient is close to 1.

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e $y = 0.5963x + 1.8511$

f **i** 121 cm **ii** 131 cm

g These predictions are outside the original data, so may be unreliable. We don't know if the linear relationship continues.

PRACTICE SET 4

Section A

1 D **2** A **3** A **4** C

5 C **6** A **7** D **8** D

Section B

1 **a** $54.17 **b** $6577.25 **c** 30 months

d $211.15 **e** $841.14

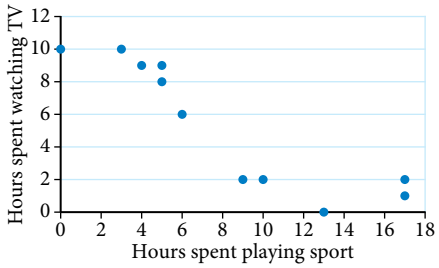
2 **a**

x	−4	−2	0	4	8
y	−19	−11	−3	13	29

b

x	−3	−1	0	2	5
y	12	10	9	7	4

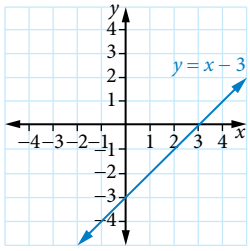
3 **a**



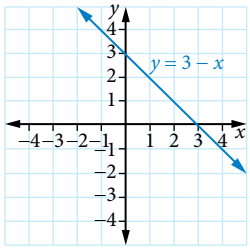
b As the hours playing sport increases, the hours watching TV decreases.

c The association is negative, linear and moderate.

4 **a**

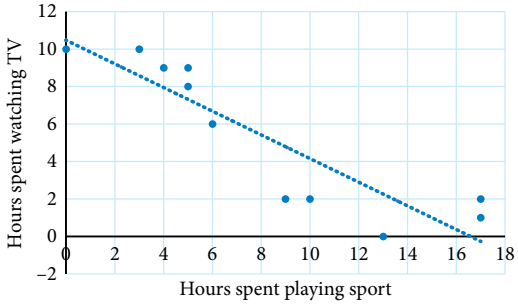


b



5 **a** $2560 **b** $471 600

6 **a**



b It is negative, linear and moderate.

c Approximately 5.5 hours of watching TV

d Approximately 12 hours playing sport

7 **a** −0.896

b Fairly closely related, as the correlation coefficient is close to −1.

c Reasonably reliable, as the correlation coefficient is close to −1.

8 **a** **i** not causal

ii age links these two measurements

b causal

c usually causal

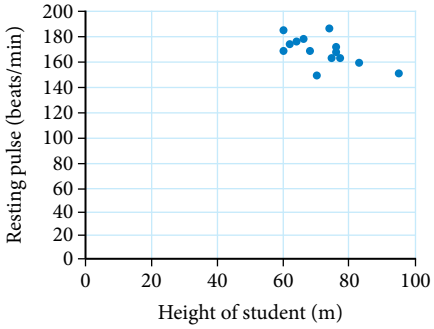
9 **a** **i** 80 **ii** 2.5 hours

b Reasonably reliable, as the correlation coefficient is close to 1.

c Teacher to check.

10 **a** Teacher to check.

b



c Most of the points are together, but a few are out on their own.

11 Month 2 $17 651, $123.56, $17 774.56, $17 299.56

3 $17 299.56, $121.10, $17 420.66, $16 945.66

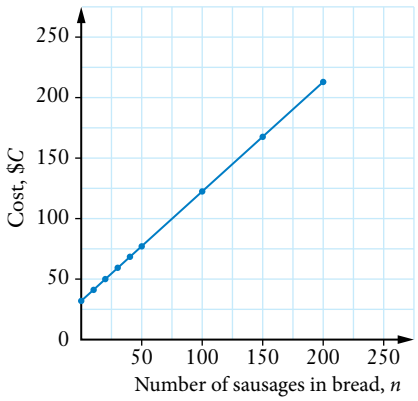
4 $16 945.66, $118.62, $17 064.28, $16 589.28

12 **a**

Number of sausages in bread, n	0	10	20	30	40	50	100	150	200
Cost, \$C	32	41	50	59	68	77	122	167	212

b $C = 32 + 0.9n$

c



d 32, initial cost of gas bottle

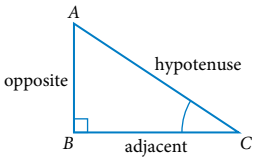
e 0.9, represents the cost per sausage in bread

Glossary and index

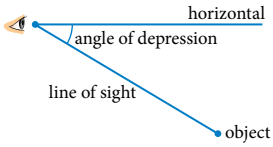
2D shapes Flat shapes that have 2 dimensions: length and width. (p. 51)

3D shapes Solid shapes that have 3 dimensions: length, width and depth. (p. 51)

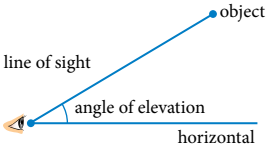
adjacent side The side next to a given angle in a triangle leading to the right angle. In the diagram, the side *BC* is adjacent to angle *C*. (p. 213)



angle of depression When an observer looks at an object that is lower; the angle that the eye turns down from the horizontal. (p. 219)



angle of elevation When an observer looks at an object that is higher; the angle that the eye turns up from the horizontal. (p. 219)



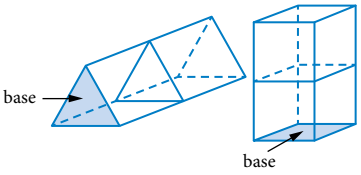
area The amount of surface occupied by a flat shape, measured in square units. (p. 75)

association A statistical term referring to the relationship between 2 variables. (p. 371)

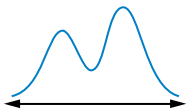
back-to-back stem plot Two stem plots in the same diagram. (p. 325)

Allfit		Superfit
7 6 5 5 5 2	1	1 3 6 6 9
7 3 2 2 1 1 0 0	2	0 0 1 2 2 3 7 7
9 4 2	3	1 2 4 4 5 8
3 3	4	6
1	5	

base (of a prism) One of the parallel end faces of a prism. (p. 78)

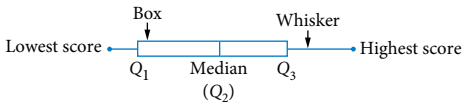


bimodal distribution A statistical distribution with 2 peaks. (p. 333)



bivariate data Data that relates 2 variables measured on the same group, e.g., height and weight of students. (p. 369)

box plot A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers. (p. 323)

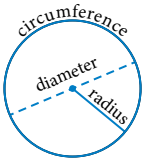


capacity Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 181)

Cartesian plane Another name for the number plane. (p. 349)

centi- One-hundredth $\left(\frac{1}{100}\right)$. (p. 8)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.141 59…), d is the **diameter** and r is the **radius**. (p. 18)



cluster A group of data scores that are bunched or close together. (p. 280)

compound interest Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**. (p. 300)

compounding period How often interest is calculated when using compound interest, for example, monthly, quarterly or yearly. (p. 307)

coordinates A pair of numbers that give the location of a point on the number plane. Also called an **ordered pair**. (p. 349)

correlation The strength of the relationship between 2 variables, can be positive or negative, strong or weak. (p. 412)

correlation coefficient A value between −1 and 1 that represents the correlation between 2 variables. (p. 412)

cosine A trigonometry ratio in a right-angled triangle:

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, where θ is an angle. (p. 223)

See also **sine** and **tangent**.

cubic metre (m^3) A unit of volume equal to the volume of a cube of length 1 m (the size of 2 washing machines). $1\,m^3 = 1\,000\,000\,cm^3$. (p. 180)

data Observations or facts which, when collected, organised and evaluated, become information. (p. 261)

deciles Values that divide a data set into 10 equal parts when the scores are arranged in order. (p. 271)
See also **percentiles** and **quartiles**.

dependent variable In statistics, a variable whose value depends on another variable. It is represented on the vertical axis of a scatterplot. (p. 369)

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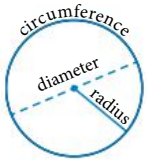
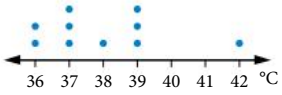
diameter The length of the interval passing through the centre of a circle and joining 2 points on the **circumference** of the circle. The diameter is double the **radius**. (p. 18)



dot plot A graph that uses dots to show frequencies of data scores. (p. 280)



double box plot *See* **parallel box plot**.

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 153)

extrapolation A modelling situation where predictions are made outside the range of the original data. (p. 417)
See also **interpolation**.

five-number summary These five values for a set of data: the lowest value, the lower quartile (Q_1), the median (Q_2), the upper quartile (Q_3), and the highest value. These are used to construct a **box-and-whisker plot**. (p. 323)

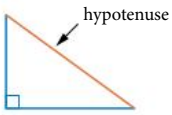
flat rate interest *See* **simple interest**.

formula A rule written as an algebraic equation, using variables. For example, the formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 15)

gradient Slope of a line whose value is $\frac{\text{rise}}{\text{run}}$. (p. 355)

hectare A large unit for measuring area, equal to 10 000 square metres. (p. 84)

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle. (p. 213)



independent variable In statistics, a variable whose value does not depend on another variable. It is represented on the horizontal axis of a scatterplot. (p. 369)

instalment *See* **repayment**.

interest Money earned on an investment, or money paid to a financial institution for borrowing. (pp. 295, 383)

interest rate The percentage of the investment or loan on which interest is calculated. (p. 295)

interpolation A modelling situation where predictions are made within the range of the original data. (p. 409)
See also **extrapolation**.

interquartile range (IQR) The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data. (p. 269)

kilo- One thousand. (p. 8)

kilogram (kg) 1000 grams. (p. 171)

leading-digit approximation Estimating the answer to a calculation by rounding each value in the calculation to the first digit; for example, 37×944 is approximately $40 \times 900 = 36\,000$. (p. 35)

likely Probably will happen; having a probability above $\frac{1}{2}$. (p. 158)

line of best fit A straight line that represents a set of points on a scatterplot, obtained through experiment or observation. (p. 405)

linear A word used to describe something to do with a line. (p. 354)

linear function An equation whose graph is a straight line. (p. 354)

linear modelling Using a linear function to approximate a real-life situation. (p. 357)

litre (L) A unit for measuring capacity, equal to 1000 mL. The size of a tall carton of milk. (p. 181)

mass A measure of size or weight in units such as grams, kilograms or tonnes. (p. 171)

mean The average of a set of numerical data, calculated by adding all the data values and dividing by the number of values. (p. 261)

measure of central tendency A statistical value, such as the **mean**, **median** or **mode**, that describes the centre or average of a set of data. (p. 261)

measure of spread A statistical value, such as the **range**, **interquartile range** or **standard deviation**, that describes the spread of a set of data. (p. 269)

median The middle data value of a set of numerical data, or the average of the 2 middle data values when scores are arranged in order. (p. 261)

mega- One million. (p. 181)

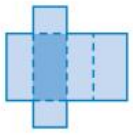
metric system A measurement system based on powers of 10. (p. 8)

milli- One-thousandth $\left(\frac{1}{1000}\right)$. (p. 8)

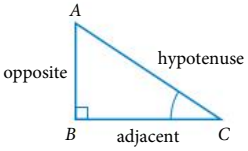
milligram (mg) One-thousandth $\left(\frac{1}{1000}\right)$ of a gram, a unit of mass. (p. 171)

mode The most common data value(s) in a set of data. (p. 261)

net (of a solid) The faces of a solid shape laid out flat. For example, this is the net of a rectangular prism. (p. 62)



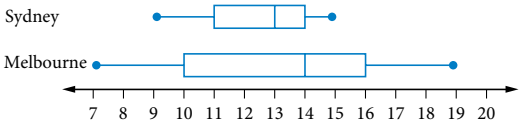
opposite side The side facing a given angle in a right-angled triangle. In the diagram, the side AB is opposite to angle C. (p. 213)



order of operations The order in which expressions with 2 or more operations are calculated: brackets, indices (powers), multiply and divide from left to right, add and subtract from left to right. This can be remembered using the initials BIDMAS. (p. 34)

outlier An extreme (high or low) data value in a data set that is very different from the other data values. It affects the mean, but not the mode or median. (p. 265)

parallel box plots (or **double box plots**) Two box plots shown on the same scale. (p. 329)



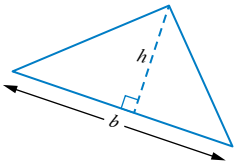
per annum (p.a.) Per year. (p. 295)

percentiles Values that divide a data set into 100 equal parts when the data are arranged in order. (p. 271)
See also **deciles** and **quartiles**.

perimeter The distance around the outside of a shape. (p. 14)

period Amount of time (for example: month, week, year) used in interest calculations. (p. 295)

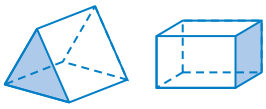
perpendicular height The height of a shape that is at right angles (90°) to the base. For example, h is the perpendicular height of this triangle. (p. 78)



polygon Any flat shape with straight sides, such as a rectangle or octagon. (p. 16) *See also* **regular polygon**.

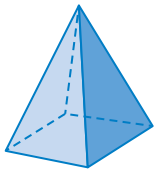
principal The original amount of money invested or borrowed, upon which interest is calculated. (p. 295)

prism A solid shape with flat sides that has the same shape at both ends. A triangular prism and square prism are shown below. (pp. 97, 184)



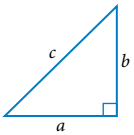
probability A measure of how likely an event E is to occur, written $P(\text{E})$. Its value ranges from 0 to 1. (p. 153)

pyramid A solid shape that has a polygon as its base, with side faces that are triangles meeting at a point called the apex. This is a diagram of a square pyramid. (p. 100)



Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides. (p. 209)

$c^2 = a^2 + b^2$



quarterly Every 3 months (quarter of a year). (p. 312)

quartiles Values that divide a set of data into 4 equal parts when the data are arranged in order. The 1st quartile (Q_1) is the lower quartile, the 2nd quartile (Q_2) is the median and the 3rd quartile (Q_3) is the upper quartile. (p. 269)
See also **deciles**, **interquartile range**, **percentiles**.

radius (plural: radii) The length of the interval joining the centre of a circle to the circumference. The radius is half of the **diameter**. (p. 18)

range For a set of data,
range = highest data value − lowest data value. (p. 269)

reducible interest Interest that is charged on the amount still owing on the loan; the type of interest charged on a reducing balance loan. (p. 383)

reducing balance loan A loan where the interest charged is calculated on the balance owing on the loan after each repayment. (p. 383)

regular polygon A polygon whose sides are the same length and angles are the same size. For example, a regular pentagon has 5 equal sides. (pp. 16, 54)

relative frequency The number of times an event or score occurs, written as a fraction of the total number of events or scores. (p. 153)

repayment (or **instalment**) Amount paid regularly to pay off a loan. (p. 383)

sample space A list of all the possible outcomes in chance situations. For example, when rolling a die, the sample space is 1, 2, 3, 4, 5, 6. (p. 237)

scale (on a map or diagram) The ratio of scaled length to actual length, for example, a scale of 1 : 500 means that lengths represented on the map or diagram are actually 500 times larger in real life. (p. 125)

scale drawing A drawing of an object, usually smaller, whose lengths are in the same ratio as the actual lengths of the object. (pp. 15, 125)

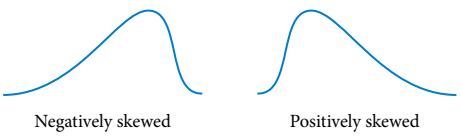
scatterplot A graph of points on a number plane showing a relationship between 2 variables. (p. 369)

shape of a distribution The way the data in a frequency distribution is spread; can be symmetrical, positively skewed or negatively skewed. (p. 332)

simple interest (or **flat rate interest**) Interest earned or charged only on the original amount of money (principal) invested or borrowed, different from compound interest. (pp. 295, 383)

sine A trigonometry ratio in a right-angled triangle:
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, where θ is an angle. (p. 223)
See also **cosine** and **tangent**.

skewed The shape of a statistical distribution when most of the data values are either low (positively skewed) or high (negatively skewed). The tail indicates the direction of the skew. (p. 333)



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square metre (m^2) A unit of area equal to the area of a square of length 1 m (the size of a large shower floor). $1\,m^2 = 10\,000\,cm^2$. (p. 84)

standard deviation (symbol σ) A statistical measure of the spread of a set of data values. (p. 276)

stem plot (or **stem-and-leaf plot**) A 'number graph' that lists all the data values, in groups. This stem plot shows 12 test scores, from 42 to 82. (pp. 280, 325)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key: 4|2 = 42

surface area The total area of all the faces of a solid shape. (p. 97)

tangent A trigonometry ratio in a right-angled triangle: $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$, where θ is an angle. (p. 216)
See also **sine** and **cosine**.

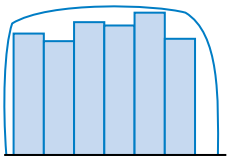
tonne (t) A unit of mass equal to 1000 kilograms. (p. 171)

tree diagram A diagram for listing all the possible outcomes of a multi-stage experiment, such as tossing 3 coins together. (p. 244)

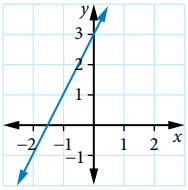
triangular prism A solid shape with a constant, triangular cross-section. (p. 97) *See also* **prism**.

trigonometry The study of the measurement of sides and angles in triangles. (p. 213)

uniform distribution A statistical distribution that is fairly constant and flat and is shaped like a rectangle. (p. 333)



vertical intercept (or ***y*-intercept**) The value at which a straight line graph cuts the vertical axis. For example, the vertical intercept of this graph is 3. (p. 357)



volume The amount of space occupied by a solid, measured in cubic units. (p. 180)

***x*-axis** The axis or number line going across (horizontally) on a number plane. (p. 349)

***y*-axis** The axis or number line going up and down (vertically) on a number plane. (p. 349)

***y*-intercept** *See* **vertical intercept**.