nelson maths.

11

General Mathematics

Units **1** & **2**

Lani Killalea
Tracey MacBeth-Dunn
Anna Wethereld

Series editor
Stephen Swift

QLD

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11

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LEARNING DISCOVERY

COMMUNICATIONS NETWORKS

Imagine a map of all the fibre optic pipes crisscrossing a city. Communications networks can be represented by matrices that help us understand how these pipes are connected and how to find the best routes for sending messages.

Contributing authors: Ross Brodie, Regina Edwards, Stephen Swift

QLD

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Preface

Nelson Maths 11–12 QLD is a new QCE mathematics series that is backed by research into the science of learning. The design and structure of the series has been informed by teacher advice and evidence-based pedagogy, with the focus on preparing QCE students for their exams and maximising their learning achievement.

- Using **backwards learning design**, this series has been built by analysing past QCE exam questions and ensuring that all theory and examples are precisely mapped to the QCAA syllabus and exams.
- To reduce the **cognitive load** for 21st century learners, explanations are clear and concise, using the technique of **chunking** text with accompanying diagrams and infographics.
- The exercise structure of **Recap**, **Mastery** and **Exam practice** leads students from procedural fluency to **higher-order thinking** using the learning techniques of **interleaving**.
- **Exam practice** includes exam-style questions and in the Year 12 book past QCE exam questions with **examiner advice and feedback**.
- The cumulative structure of Exercise **Recaps** and **Cumulative examinations** in every chapter is built on the learning and memory techniques of **spacing** and **retrieval**, as recommended by the new syllabus.

About the authors

Lani Killalea has taught Mathematics and Technology for over 20 years and was Head of Department and Deputy Principal in Brisbane and regional schools. She teaches at Aspley State High School, was a district panellist for Brisbane North and has taught a range of subjects from learning support to senior courses.

Tracey MacBeth-Dunn has taught Mathematics for over 30 years, including experience as Head of Mathematics and external exam marker for General Mathematics. She teaches at Canterbury College in Waterford and was a lead author on the Australian Curriculum series *Nelson Senior Maths 11–12 General Mathematics*.

Anna Wethereld has been a leader in schools, curriculum, assessment and teacher training for over 25 years, including at Brisbane State High School, QCAA and Queensland College of Teachers. She was a panellist for Mathematics C (Specialist Maths) for Brisbane North, a Head of Mathematics and Deputy Principal. As the state's Mathematics Curriculum Manager at NESA, Anna led Mathematics K–12 in NSW and coordinated the development of its new Years 11–12 syllabuses.

Series editor **Stephen Swift** taught Mathematics, Science and Computing in urban and regional schools, and was Head of Mathematics at Wellington Point State High School in Brisbane. He was the lead author of *Nelson QMaths 7–12* and *Nelson Senior Maths 11–12* (Australian Curriculum).

Contributing authors

Sharon Kirkby wrote many of the worksheets.
Deborah Da Cruz, Scott Smith, John Drake, Katie Jackson and **Joanne Magner** created the video tutorials.
Joel Speranza created the *Exam question analysis* videos.
Kahlia Dreyer and **Trisha Goss** wrote the topic tests.
With thanks to **Jeremy Hunter** for reviewing the text.

Syllabus reference grid

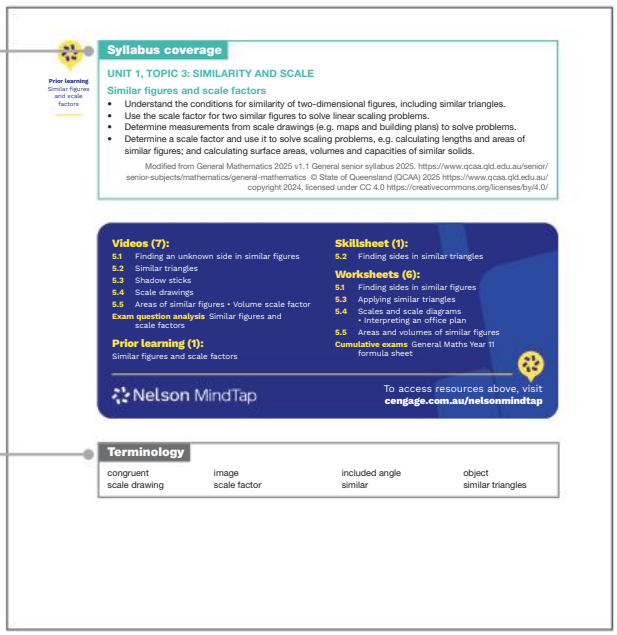
Topics and subtopics	*Nelson Maths 11* *General Mathematics QLD* chapter	
UNIT 1: MONEY, MEASUREMENT, ALGEBRA AND LINEAR EQUATIONS		
1 Consumer arithmetic		
Applications of rates, percentages and use of spreadsheets	1	Incomes and budgets
	3	Prices and shares
2 Shape and measurement		
Pythagoras' theorem	2	Length, area and volume
Mensuration	2	Length, area and volume
3 Similarity and scale		
Similar figures and scale factors	5	Similar figures and scale factors
4 Algebra		
Linear and non-linear relationships	4	Linear and non-linear relationships
5 Linear equations and their graphs		
Linear equations	6	Linear equations and graphs
Straight-line graphs	6	Linear equations and graphs
UNIT 2: APPLICATIONS OF LINEAR EQUATIONS AND TRIGONOMETRY, MATRICES AND UNIVARIATE DATA ANALYSIS		
1 Applications of linear equations and their graphs		
Simultaneous linear equations and their applications	7	Simultaneous equations and piecewise graphs
Piecewise linear graphs and step graphs	7	Simultaneous equations and piecewise graphs
2 Applications of trigonometry		
Applications of trigonometry	8	Right-angled trigonometry
	11	The sine and cosine rules
3 Matrices		
Matrices and matrix arithmetic	9	Matrices
4 Univariate data analysis 1		
Making sense of data relating to a single statistical variable	10	Analysing data
5 Univariate data analysis 2		
Comparing data for a single numerical variable across two or more groups	12	Comparing data

About this book

At the beginning of each chapter

Syllabus coverage and extracts are shown at the front of the chapter, along with a listing of **Nelson MindTap** chapter resources.

Terminology previews the keywords and phrases from within the chapter.



Prior learning
Similar figures and scale factors

Syllabus coverage

UNIT 1, TOPIC 3: SIMILARITY AND SCALE

Similar figures and scale factors

- Understand the conditions for similarity of two-dimensional figures, including similar triangles.
- Use the scale factor for two similar figures to solve linear scaling problems.
- Determine measurements from scale drawings (e.g. maps and building plans) to solve problems.
- Determine a scale factor and use it to solve scaling problems, e.g. calculating lengths and areas of similar figures; and calculating surface areas, volumes and capacities of similar solids.

Modified from General Mathematics 2025 v1.1 General senior syllabus 2025. https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/general-mathematics © State of Queensland (QCAA) 2025 https://www.qcaa.qld.edu.au/ copyright 2024, licensed under CC 4.0 https://creativecommons.org/licenses/by/4.0/

Videos (7):
5.1 Finding an unknown side in similar figures
5.2 Similar triangles
5.3 Shadow sticks
5.4 Scale drawings
5.5 Areas of similar figures • Volume scale factor
Exam question analysis Similar figures and scale factors

Prior learning (1):
Similar figures and scale factors

Skillsheet (1):
5.2 Finding sides in similar triangles

Worksheets (6):
5.1 Finding sides in similar figures
5.3 Applying similar triangles
5.4 Scales and scale diagrams • Interpreting an office plan
5.5 Areas and volumes of similar figures
Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap
To access resources above, visit cengage.com.au/nelsonmindtap

Terminology

congruent, scale drawing, image, scale factor, included angle, similar, object, similar triangles

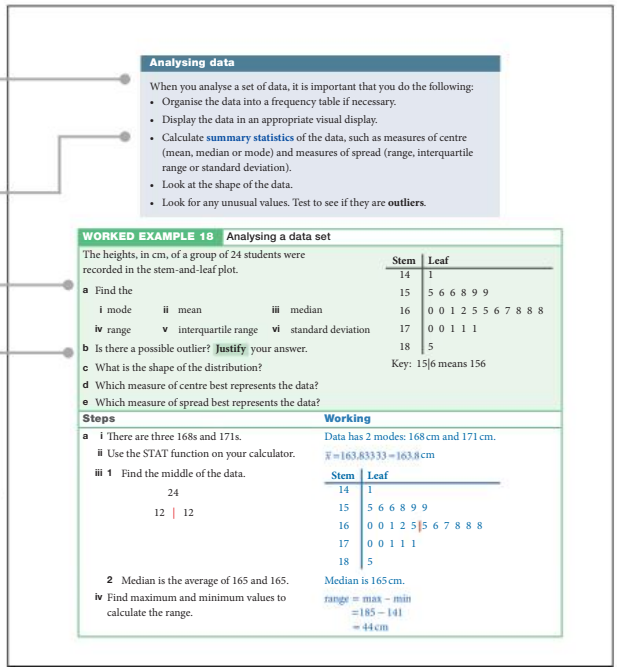
In each chapter

Important facts and formulas are highlighted in a shaded box.

Important words and phrases are printed in blue and listed in the **Glossary and index** at the back of the book.

Worked examples are explained clearly step-by-step, with the mathematical working shown on the right-hand side.

Cognitive verbs are highlighted in light green. See QCAA's glossary of cognitive verbs for more verbs and details.



Analysing data

When you analyse a set of data, it is important that you do the following:
- Organise the data into a frequency table if necessary.
- Display the data in an appropriate visual display.
- Calculate **summary statistics** of the data, such as measures of centre (mean, median or mode) and measures of spread (range, interquartile range or standard deviation).
- Look at the shape of the data.
- Look for any unusual values. Test to see if they are **outliers**.

WORKED EXAMPLE 18 Analysing a data set

The heights, in cm, of a group of 24 students were recorded in the stem-and-leaf plot.

Stem	Leaf
14	1
15	5 6 6 8 9 9
16	0 0 1 2 5 5 6 7 8 8 8
17	0 0 1 1 1
18	5

Key: 15|6 means 156

a Find the
i mode ii mean iii median
iv range v interquartile range vi standard deviation
b Is there a possible outlier? **Justify** your answer.
c What is the shape of the distribution?
d Which measure of centre best represents the data?
e Which measure of spread best represents the data?

Steps	Working
a i There are three 168s and 171s.	Data has 2 modes: 168 cm and 171 cm.
ii Use the STAT function on your calculator.	$\bar{x} = 163.83333 \approx 163.8$ cm
iii 1 Find the middle of the data. 24 12 | 12	Stem | Leaf: 14 | 1; 15 | 5 6 6 8 9 9; 16 | 0 0 1 2 5 | 5 6 7 8 8 8; 17 | 0 0 1 1 1; 18 | 5
2 Median is the average of 165 and 165.	Median is 165 cm.
iv Find maximum and minimum values to calculate the range.	range = max − min = 185 − 141 = 44 cm

Exam hacks highlight valuable exam hints and common student errors.

It is law in Australia for sellers to display the unit costs of food; however, many supermarkets show unit cost only in quite small print on the shelf label.

Exam hack

Make sure you compare measurements in the same units to calculate the best value.

EXERCISE 3.1 Comparing prices ANSWERS p. 398

Mastery

1 WORKED EXAMPLE 1 For each of the following, determine
   i the unit cost correct to 2 decimal places.
   ii which size is the better value.
   a Children's toothpaste costs $1.71 for 45 g or $2.49 for 110 g
   b Caster sugar is $1.86 for 500 g or $2.64 for 1 kg
   c A 300 mL bottle of sauce for $1.56 or a 500 mL bottle for $2.75
   d A 10-pack of long-life milk for $23.70 or a 12-pack for $27.60

In each chapter exercise, **Recap** questions revise skills from the previous exercise (useful for lesson starters), **Mastery** questions provide skill practice linked to worked examples, while **Exam practice** applies learned skills to exam-style problems (including past QCE questions in the Year 12 book), including multiple choice, short response, problem-solving, and simple and complex questions.

EXERCISE 1.3 Wages and salaries ANSWERS p. 395

Recap

1 Which of the following does not represent a rate?
   A 100% B 60 km/h C 3 people D 6 joules per second

2 A garden is watered for half an hour using tank water. If the rate of water released from the tank is 12 L per minute, how much water is used to water the garden?

Mastery

3 WORKED EXAMPLE 9 Anders earns a salary of $126 400. Calculate their salary, to the nearest dollar, as
   a a weekly rate b a fortnightly rate c a monthly rate

4 Santi's base pay rate as an electrician is $94 624 per year. What is her fortnightly income?

5 WORKED EXAMPLE 10 Robert is paid $24.10/h for a 38-hour week. How much would he earn as an annual income?

6 Jenny is a first-year apprentice hairdresser who earns $27.78/h for a 38-hour week. What is her pay as an annual amount?

7 If working a 38-hour week, determine which is the better pay for each pair.
   a $36.61/h or $73 000 per year? b $105 280 per year or $2104.40/week?
   c $44.01/h or $3018/fortnight? d $84 000 per year or $42.90/h?

8 WORKED EXAMPLE 11 Tyson receives a weekly salary of $1050. He is to receive a salary increase equal to the rate of inflation, which is 3.5%. What will his new weekly salary be?

9 Ewan earns $34.92/h and works 38 hours per week. How much does he receive fortnightly?
   A $1326.96 B $2653.92 C $1323.28 D $2646.56

10 Jezza earns $1126.70 for a 38-hour week. What is his wage rate?
   A $42.81/h B $28.17/h C $29.65/h D $30.60/h

11 Zarah's annual salary is $86 425. What does she earn each fortnight?
   A $7202.08 B $3324.04 C $1728.50 D $1662.02

12 For a 38-hour week, which of these is the best pay?
   A $32.45/hour B $1202.10/week C $5420.90/month D $64 980.20/year

Exam practice

13 (3 marks) Jordan earns $32.27 per hour working at a take-away store.
   a Last week, Jordan worked 12 hours. How much did she earn? *[1 mark]*
   b Yesterday, Jordan earned $177.49. How many hours did she work? *[2 marks]*

14 (3 marks) CF If Ebony is paid $7016 per month and Shaye earns $3352 per fortnight, who earns more?

15 (4 marks) CU Kristy earns $1890 per week. She calculated her monthly amount by multiplying by 4 and then multiplying that by 12 to get her annual salary. Determine the reasonableness of Kristy's calculation and explain whether her approach was correct.

Complex familiar CF and complex unfamiliar CU questions are clearly tagged and colour-coded.

Technology promotes ICT in the classroom, using spreadsheets and the internet.

TECHNOLOGY Simple interest spreadsheet

You can make a spreadsheet for simple interest calculations.
Put the following headings in the cells indicated.

A3: Principal	A4: Interest rate %	A5: Term	A6: Payments/year
D3: Loan	D4: Total	D5: Instalment	D7: Investment
D8: Lump sum	D9: Regular payment		

You can use cell formatting, colours and column width to make it look professional.
Put in, say, 20 000 for the principal, 6.8 for the interest rate, 4 for the term and 12 for the payments/year in cells B3 to B6.
The total to pay back for a loan is the principal + interest, $P + Pin$.
This translates to the formula **=B3+B3*B4/100*B5**. Put this formula in cell E4.
Don't forget the equals sign to make it a formula.
To get the instalments, divide by the total number of payments, **B5*B6**.
Type the formula **=E4/(B5*B6)** in cell E5.
You need the brackets to make sure the multiplication is calculated first.
An investment can give either a lump sum at the end or a regular payment.
For the lump sum, put the formula **=B3+B3*B4/100*B5** in cell E8.
For the regular payment, put the formula **=B3*B4/(100*B6)** in cell E9.
Try your spreadsheet with different problems.
A completed 'Simple interest' spreadsheet can be downloaded from Nelson MindTap.

Investigations explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks.

Investigation Wage rates

Use the **Fair Work** website www.fairwork.gov.au to investigate the wage rates for a 17-year-old in the following jobs:

- kitchen assistant
- apprentice sheet-metal worker
- childcare assistant
- retail assistant
- apprentice hairdresser
- labourer
- apprentice carpenter
- nursing aide

Compare casual and permanent rates and find out the differences in the employee conditions.

At the end of each chapter

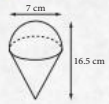
Exam question analysis leads students through a QCE exam-style question that exemplifies the chapter, discusses how to approach the question, provides advice on interpreting the question, highlights keywords, describes common student errors and presents a fully worked solution and marking key.

Each exam question analysis is accompanied by a video walkthrough explained by Joel Speranza.

EXAM QUESTION ANALYSIS

Exam-style question (5 marks)

Calculate the surface area of the ice-cream cone, including the ice cream. Leave your answer in whole centimetres.



Reading the question

- The question is asking for a surface area calculation of a composite 3D shape.
- There are 2 shapes in the diagram: cone and hemisphere.
- Two measurements have been given.
- Answer will need to be rounded.

Thinking about the question

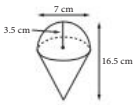
- As the diagram gives a diameter, a radius will need to be found. This can be used for both shapes.
- The radius represents the height of the hemisphere. This will need to be subtracted from the whole height.
- The hemisphere will require the sphere formula to be halved.
- As the 2 objects are on top of each other, neither will need a base.

Worked solution (✓ = 1 mark)

radius of hemisphere and cone $= \frac{1}{2} \times 7$
$= 3.5$ cm

height of cone $= 16.5 - 3.5$
$= \mathbf{13\ cm}$ ✓

curved surface area of hemisphere $= \frac{1}{2} \times 4\pi r^2$
$= 2 \times \pi \times 3.5^2$
$= \mathbf{76.9690\ldots\ cm^2}$ ✓



Chapter summary for easy reference.

1 Chapter summary

Percentages

- **Percentage** means 'per 100', and is a number expressed as a fraction of 100.
- A percentage increase or decrease in a quantity is always expressed as a percentage of the original value.

$$\text{percentage increase} = \frac{\text{amount of increase}}{\text{original value}} \times 100\%$$

$$\text{percentage decrease} = \frac{\text{amount of decrease}}{\text{original value}} \times 100\%$$

- When increasing or decreasing a quantity by a percentage, calculate it 'out of' the original value.
- To increase something by x%, multiply by $(100 + x)$%.
- To reduce something by x%, multiply by $(100 - x)$%.

Rates

- A **rate** compares quantities or variables that have different units.
- A rate is calculated per unit or item.
- For 2 quantities, quantity 1 (y) and quantity 2 (x):

$$\text{rate} = \frac{\text{quantity 1}}{\text{quantity 2}} \quad \text{or} \quad \text{quantity 1} = \text{rate} \times \text{quantity 2}$$

$$r = \frac{y}{x} \qquad y = rx$$

- An employee working hours at unconventional times can be paid a **penalty rate**, such as **time-and-a-half** and **double time**.
- **Allowances** are additional payments paid to employees for doing certain tasks or working in unfavourable conditions.

Commission, piecework and royalties

- **Commission** workers earn a proportion of the value of sales or services. It is usually a percentage of the value of an item sold.
- Some commission workers also receive a small **retainer**, which is paid regularly regardless of sales.
- **Piecework** payments are made for each item or task completed.
- Some businesses pay a piecework bonus or incentive in addition to wages.
- **Royalty** payments are made for the use of intellectual property, such as songs or books.

Government allowances and pensions

- **Youth Allowance**, **Austudy** and **ABSTUDY** are paid to eligible Australian apprentices or full-time students.
- **Age Pension** and **Disability Support Pension** are

Cumulative examinations 1 and **2** are mini exams based on the format of the QCE examinations 1 and 2, with about 50% of questions focusing on the chapter in which they appear and 50% on previous chapters.

Cumulative examination 1 contains simple familiar questions in multiple choice and short response format: 30 minutes, 18 marks.

Cumulative examination 2 contains complex familiar and complex unfamiliar questions in short response format: 30 minutes, 12 marks.

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions 5 marks

1 An increase in an amount by 42% is equivalent to

A 42% **B** 0.42 **C** 1.42 **D** 1.042

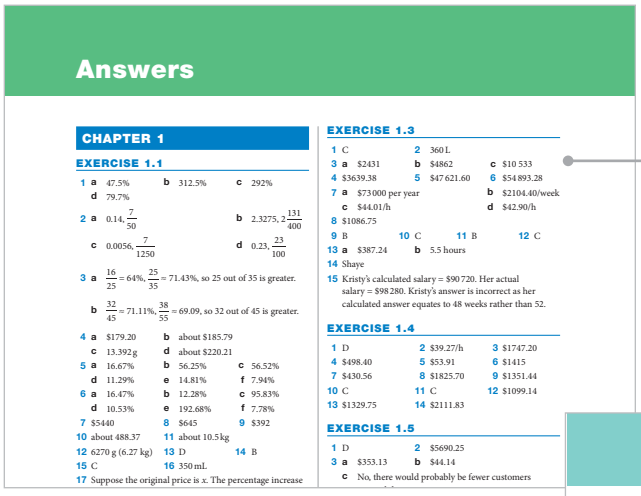
2 Alex measures his speed while riding his e-bike at 30 km/h. What is his speed in m/s?

A 500 **B** 108 **C** 1.8 **D** 8.33

3 Silvia receives a salary of $105 440. How much does she receive each week?

A $2027.69 **B** 8786.67 **C** $2196.67 **D** $4055.38

At the end of the book



Answers

CHAPTER 1

EXERCISE 1.1

1 **a** 47.5% **b** 312.5% **c** 292% **d** 79.7%

2 **a** 0.14, $\frac{7}{50}$ **b** 2.3275, $2\frac{131}{400}$ **c** 0.0056, $\frac{7}{1250}$ **d** 0.23, $\frac{23}{100}$

3 **a** $\frac{16}{25} = 64\%$, $\frac{25}{35} = 71.43\%$, so 25 out of 35 is greater.

b $\frac{32}{45} \approx 71.11\%$, $\frac{38}{55} = 69.09$, so 32 out of 45 is greater.

4 **a** \$179.20 **b** about \$185.79 **c** 13.392 g **d** about \$220.21

5 **a** 16.67% **b** 56.25% **c** 56.52% **d** 11.29% **e** 14.81% **f** 7.94%

6 **a** 16.47% **b** 12.28% **c** 95.83% **d** 10.53% **e** 192.68% **f** 7.78%

7 \$5440 **8** \$645 **9** \$392 **10** about 488.37 **11** about 10.5 kg **12** 6270 g (6.27 kg) **13** D **14** B **15** C **16** 350 mL

17 Suppose the original price is x. The percentage increase

EXERCISE 1.3

1 C **2** 360 L **3** **a** \$2431 **b** \$4862 **c** \$10 533 **4** \$3639.38 **5** \$47 621.60 **6** \$54 893.28 **7** **a** \$73 000 per year **b** \$2104.40/week **c** \$44.01/h **d** \$42.90/h **8** \$1086.75 **9** B **10** C **11** B **12** C **13** **a** \$387.24 **b** 5.5 hours **14** Shaye

15 Kristy's calculated salary = \$90 720. Her actual salary = \$98 280. Kristy's answer is incorrect as her calculated answer equates to 48 weeks rather than 52.

EXERCISE 1.4

1 D **2** \$39.27/h **3** \$1747.20 **4** \$498.40 **5** \$53.91 **6** \$1415 **7** \$430.56 **8** \$1825.70 **9** \$1351.44 **10** C **11** C **12** \$1099.14 **13** \$1329.75 **14** \$2111.83

EXERCISE 1.5

1 D **2** \$5690.25 **3** **a** \$353.13 **b** \$44.14 **c** No, there would probably be fewer customers

Answers (with worked solutions provided on Nelson MindTap for teachers to allocate to students).

Glossary and index

ABSTUDY Government allowance paid to eligible Aboriginal apprentices and students. (p. 19)

adjacency matrix A matrix that shows direct connections between points in a network. (p. 278)

adjacent side The side that is next to a given angle in a right-angled triangle, which, with the hypotenuse, forms the arms of the angle. (p. 235)

Age Pension A pension paid to eligible Australians who are aged over 67 and not working. (p. 20)

algebraic expression (or **expression**) Variables and numbers connected by operations (p. 110)

angle of depression The angle formed by looking down from a horizontal line. (p. 245)

angle of elevation The angle formed by looking up from a horizontal line. (p. 245)

annual Each year; in a year. (p. 9)

annulus The shape that is the area between two circles with the same centre. (p. 51)

apex The vertex opposite the base of a cone or a pyramid. (p. 59)

arc length The length of a part of a curve, measured along the curve. (p. 46)

area The size of a plane or curved surface, measured in square units. (p. 43)

assets test An assessment of the value of a person's possessions (for example, property, car), to work out the amount of an allowance or pension the person will be paid. (p. 20) *See also* **income test**.

budget Lists of income and expenses. A *balanced* budget has equal totals of income and expenses. (p. 22)

buying price *See* **cost price**.

capacity Another name for volume, used for the volumes of containers for liquids. (p. 66)

Cartesian plane or **number plane** A grid system referenced by intersecting horizontal and vertical lines called the x-axis and y-axis respectively. The axes have scales starting from 0 at the intersection and a point on the plane is described by 2 coordinates from the x- and y-axes. (p. 178)

casual worker An employee whose hours of work may vary from day to day or week to week. (p. 12)

categorical data A statistical variable with data that are categories (names), not numbers. (p. 289) *See also* **numerical variable, nominal variable, ordinal variable**.

chart A graph or display of data. (p. 291) *See also* **pie graph, dot plot, stem-and-leaf plot, bar chart, column chart, histogram**.

circumference The perimeter or boundary of a circle. (p. 44)

class centre The middle of a class. (p. 297)

class interval, class A range of data values in a frequency table of grouped data, such as 5–9 or 9.5–19.5. (p. 297)

coefficient The number that precedes a variable in an algebraic term. For example, the coefficient of $5x$ is 5. (p. 110)

column chart A graph with vertical bars with heights proportional to values. (p. 291) *See also* **bar chart**.

A combined **Glossary and index**.

Nelson MindTap

Nelson MindTap is an online learning space that provides students with interactive learning and assessment experiences. Margin links in the student book signpost multimedia student resources found on Nelson MindTap*.

Videos

Skillsheets

Worksheets

For students:

- **Engage** with the **online eBook** by adding notes, highlights, bookmarks, and using the **Search** and **Read Aloud** (in Australian voice) functions.
- **Watch videos** featuring expert teacher advice to unpack worked examples and deepen your understanding.
- **Revise** using pre-chapter **prior learning sheets, skillsheets** and **worksheets** to practise your skills and build your confidence.
- **Navigate** your own learning path, accessing the content and support as you need it.

For teachers:

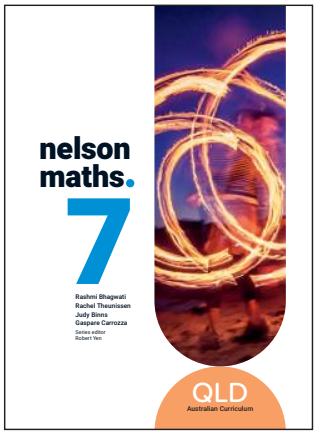
- **Access topic tests, teaching plans** and **worked solutions** to each exercise set.
- **Use course customisation** to tailor content and assign tasks to groups of students or the whole class.
- **Integrate** content directly within your school's LMS for ease of access.
- Help build your students' exam readiness with **Cognero Assess** – a test bank containing hundreds of questions and answers to create, assign or export formative and summative tests.

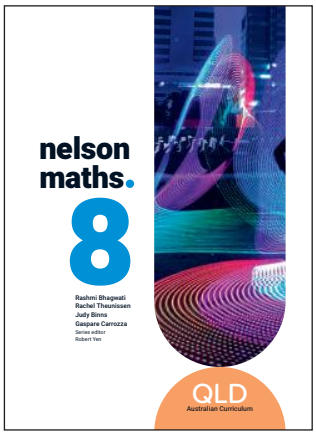
* Complimentary access to these resources is only available to schools that use this book as part of a class set, book hire or booklist. Not available for single purchases. Alternatively, teacher access can be purchased as a separate subscription for schools. Contact your Cengage Learning Consultant for information about access and conditions.

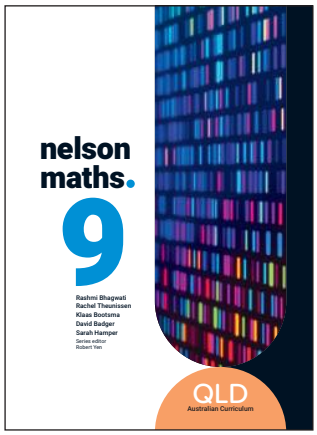
Security & privacy:

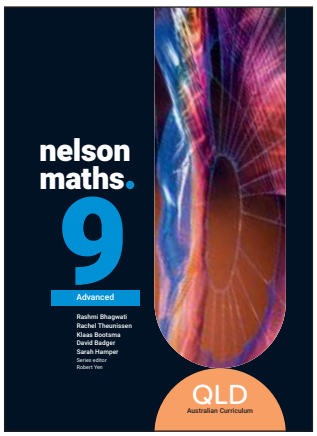
Nelson MindTap joined the Safer Technologies 4 Schools (ST4S) Product Badge Program in 2024. The annual ST4S assessment is part of our commitment to supporting the online security and safety of students and schools. Learn more at st4s.edu.au

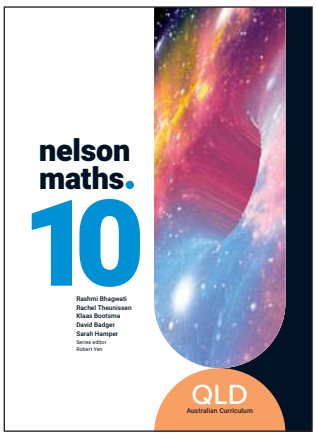
Nelson Maths 7–12 QLD series

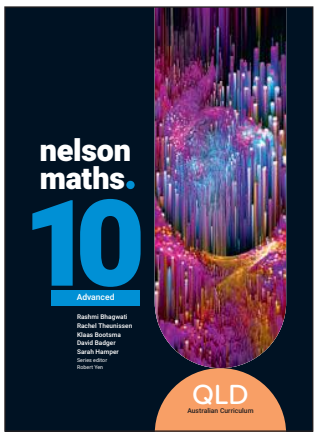


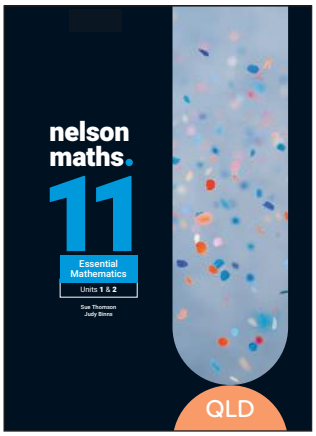


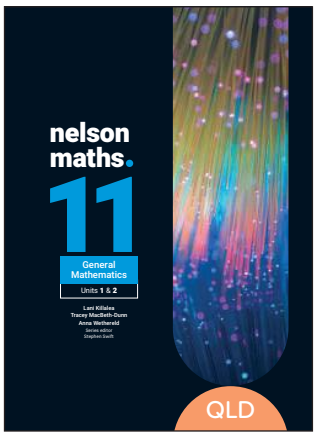


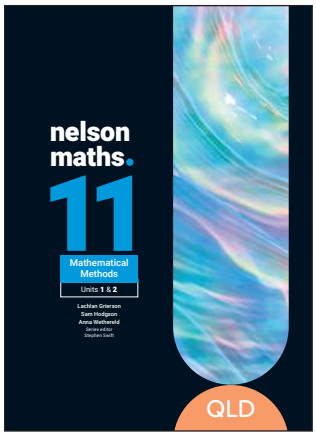


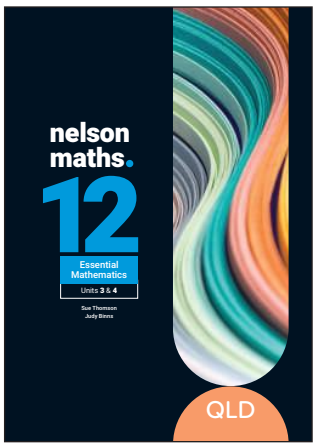


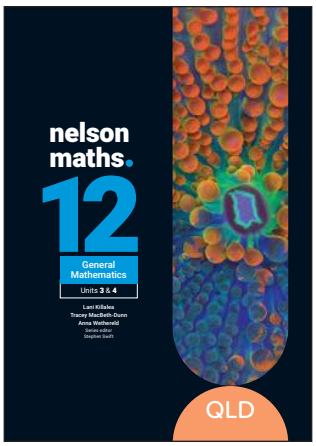


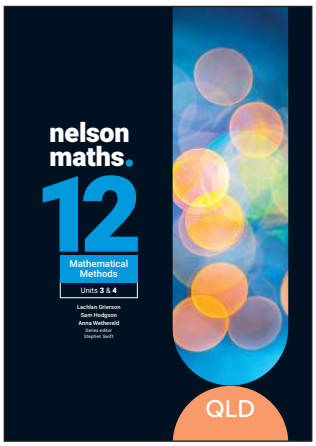












List of symbols and abbreviations

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\pm$	plus or minus
π	pi ≈ 3.14
$\equiv$	is congruent to
$\|\|\|$	is similar to
$\ldots$	and so on
$^\circ$	degree
$\angle$	angle
$\triangle$	triangle
$\|\|$	is parallel to
$\perp$	is perpendicular to
$\therefore$	therefore
$\sqrt{\ }$	square root, radical sign
S	surface area
S 37° W	a compass bearing
017°	a true bearing
AUD	Australian dollar
GST	goods and services tax
P/E	price–earnings ratio
$\bar{A}$, A'	not A, the complement of A
S	sample space
$n(A)$	the number of elements in A
$P(A)$	the probability of A
LHS	left-hand side
RHS	right-hand side
m	gradient of a straight line
c	y-intercept of a straight line
(x, y)	Cartesian coordinates, ordered pair
AA, AAA	angle–angle test
RHS	right angle–hypotenuse-side test
SAS	side–angle–side test
SSS	side–side–side test
%	percentage
A	matrix A
a_{34}	element in 3rd row and 4th column of matrix A
I	identity matrix
cos	cosine ratio
sin	sine ratio
tan	tangent ratio
$\bar{x}$	mean
$\sum$	the sum of
Q_0	minimum value
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
Q_4	maximum value
IQR	interquartile range
SD, s_x	standard deviation
σ	standard deviation of a population

List of mathematical verbs

A glossary of 'doing words' commonly found in mathematics problems

Cognitive verbs are highlighted in light green. See QCAA's glossary of cognitive verbs for more details.

analyse: study and state in detail the relationship of parts of a situation

apply: use knowledge or a procedure in a given situation

calculate: find a numerical value

comment: express an opinion or judgement about a statement or calculation

communicate: transmit information to others

compare: state similarities and differences and their significance

consider: take into account; think carefully

construct: draw an accurate diagram or logically arrange items or ideas

convert: change from one form to another

classify: identify and sort into categories

deduce: use logic and reason to prove something

define: give the meaning of or identify in exact terms

demonstrate: show to be correct

describe: state the features of a situation, object, pattern, event, etc.

determine: find the answer or make a decision using evidence

discuss: give reasons based on evidence for and against a position or proposal

establish: introduce and develop a result

evaluate: find the value of or state the application, strengths and limitations of a solution

examine: investigate the details and assumptions of a situation

expand: remove brackets or change a product to an extended sum of terms

explain: state the meaning in logical detail

explore: examine or state the details and assumptions of a situation

factorise: convert to factors or change a sum of terms to a product of factors

formulate: give mathematical expression to an idea or situation

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: state the type, name or distinguishing feature of an item or situation

interpret: state a conclusion or trend from given information

investigate: establish facts, trends or conclusions from collected information

isolate a variable: express a formula or equation with one variable only, usually on the left-hand side (LHS)

justify: give reasons or evidence for an answer or conclusion

modify: change to accommodate different information

obtain: find an answer or conclusion

predict: use given information or results to guess what will happen under different conditions

prove: use logical steps to establish the truth of a result, statement or assumption

recall: remember (and state)

recognise: use knowledge to identify features of a situation

show that: in questions where the answer is given, use mathematical reasoning to prove that the answer is true

simplify: reduce the size of numbers in a fraction, or reduce the size of an algebraic expression

sketch: draw a diagram that shows the general shape and includes relevant features

solve: find the answer or explanation for a problem, particularly the values of variables in equations or inequalities

substitute: replace a variable by a number to calculate an answer

test: check that a statement or result is correct

translate (to mathematical form): express a situation as mathematical relationships

verify: check a solution or result, usually referring to the given situation

write/state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step.)

CHAPTER

1 INCOMES AND BUDGETS

Syllabus coverage

Nelson MindTap chapter resources

Terminology

1.1 Percentage problems
- Percentages
- Percentage increase and decrease

1.2 Rate problems

1.3 Wages and salaries
- Wages and inflation

1.4 Overtime, penalty rates and allowances
- Overtime and penalty rates
- Allowances and bonuses

1.5 Commission, piecework and royalties
- Commission
- Piecework
- Royalties

1.6 Government allowances and pensions
- Youth Allowance
- Austudy
- ABSTUDY
- Age Pension
- Disability Support Pension

1.7 Budgets

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

Prior learning
Incomes and budgets

Syllabus coverage

UNIT 1, TOPIC 1: CONSUMER ARITHMETIC

Applications of rates, percentages and use of spreadsheets

- Understand the meaning of rates and percentages.
- Calculate weekly, fortnightly or monthly wages from an annual salary, and wages from an hourly rate, including situations involving overtime and other allowances and earnings based on commission or piecework.
- Calculate income support payments based on government allowances and pensions.
- Prepare a personal budget for a given income, taking into account fixed and discretionary spending.
- Use a spreadsheet to display examples of the above computations when multiple or repeated computations are required, e.g. preparing a wage sheet displaying the weekly earnings of workers in an organisation, preparing a budget, investigating the potential cost of owning and operating a car over a year.

Modified from General Mathematics 2025 v1.1 General senior syllabus 2025 https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/general-mathematics © State of Queensland (QCAA) 2025 https://www.qcaa.qld.edu.au/copyright 2024, licensed under CC 4.0 https://creativecommons.org/licenses/by/4.0/

Videos (7):

1.1 Converting fractions and decimals to percentages • Percentage increase and decrease
1.2 Rate problems
1.4 Overtime
1.5 Commission • Commission involving sliding scales
Exam question analysis Incomes and budgets

Prior learning (1):

1.1 Incomes and budgets

Skillsheets (2):

1.1 Mental percentages • Percentage calculations

Worksheets (14):

1.1 Percentage shortcuts • Percentages without calculators • Percentage problems • Working with percentages
1.2 Rapid rates
1.3 Wages and salaries • Wage sheet
1.4 S-mart payroll
1.5 Pay day
1.6 Earning money
1.7 Personal budgets • Budget grid • Budgeting scenarios
Cumulative exams General Maths Year 11 formula sheet

Puzzles (6):

1.1 Percentage power • Discounts code puzzle
1.2 Rate problems • Rate skills
1.5 Commission and piecework • Income - commission and piecework

Spreadsheets (2):

1.3 Wage sheet
1.7 Personal budget

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

ABSTUDY	Age Pension	Austudy	budget
casual	commission	consumer price index (CPI)	Disability Support Pension
discretionary expenses	double time	fixed expenses	full-time
inflation	overtime	part-time	penalty rate
percentage	piecework	rate	retainer
royalty	salary	time-and-a-half	wage
Youth Allowance			

1.1 Percentage problems

Percentages

Percentage means 'per 100', and is a number expressed as a fraction of 100.

When you say 100% of something, it means all of it.

Converting to and from percentages

- To convert a number to a percentage, multiply it by 100%.
- To convert a percentage to a number, divide it by 100%.

WORKED EXAMPLE 1 Converting numbers to percentages

Convert each number to a percentage.

a 1.9 **b** $\frac{36}{48}$ **c** $4\frac{2}{9}$

Steps	Working
a Multiply by 100%.	$1.9 \times 100\% = 190\%$
b Multiply by 100%.	$\frac{36}{48} \times 100\% = 75\%$
c Multiply by 100%.	$4\frac{2}{9} \times 100\% = 422\frac{2}{9}\%$

WORKED EXAMPLE 2 Converting percentages to decimals and fractions

Convert each percentage to a decimal and fraction.

a 165% **b** $5\frac{1}{4}\%$

Steps	Working
a **1** To convert to a decimal, divide by 100%.	$165\% = 165 \div 100$ $= 1.65$
2 To convert to a fraction, write it 'over' 100.	$165\% = \frac{165}{100}$
3 Simplify if possible.	$= 1\frac{13}{20}$
b **1** Use your calculator to divide by 100%.	$5\frac{1}{4}\% = 5\frac{1}{4} \div 100$ $= \frac{21}{400}$
2 Use the calculator's fraction-to-decimal conversion key.	$= 0.0525$
3 Reuse above working for fraction answer.	$5\frac{1}{4}\% = \frac{21}{400}$

Videos
Converting fractions and decimals to percentages
Percentage increase and decrease

Skillsheets
Mental percentages
Percentage calculations

Worksheets
Percentage shortcuts
Percentages without calculators
Percentage problems
Working with percentages

Puzzles
Percentage power
Discounts code puzzle

WORKED EXAMPLE 3 Calculating and comparing using percentages

Tammy scored 46 out of 75 for an English test and 27 out of 40 for a Maths test.
Determine in which test she did better.

Steps	Working
1 Change the English mark to a percentage.	English % $= \frac{46}{75} \times 100\%$ $= 61\frac{1}{3}\%$
2 Change the Maths mark to a percentage.	Maths % $= \frac{27}{40} \times 100\%$ $= 67\frac{1}{2}\%$
3 Compare the answers.	Tammy did better in Maths.

Percentage increase and decrease

- A **percentage increase** is the increase in a quantity expressed as a percentage of the original value.
- A **percentage decrease** is the decrease in a quantity expressed as a percentage of the original value.

When increasing or decreasing by a percentage, calculate it 'out of' the original value.

Percentage increase and decrease

- percentage increase $= \frac{\text{amount of increase}}{\text{original value}} \times 100\%$
- percentage decrease $= \frac{\text{amount of decrease}}{\text{original value}} \times 100\%$
- To increase something by x%, you multiply by $(100 + x)$%.
- To reduce something by x%, you multiply by $(100 - x)$%.

WORKED EXAMPLE 4 Calculating percentage increase or decrease

Determine the percentage increase or decrease.

a The price of a carton of eggs changes from \$5.00 to \$5.20.

b The number of people who attended a course last week was 24 and this week was 21.

Steps	Working
a 1 Calculate the amount of increase.	amount of increase = \$5.20 − \$5.00 = \$0.20
2 Substitute into the formula: percentage increase $= \frac{\text{amount of increase}}{\text{original value}} \times 100\%$	percentage increase $= \frac{0.20}{5.00} \times 100\%$ $= 4\%$
b 1 Calculate the amount of decrease.	amount of decrease = 24 − 21 = 3
2 Substitute into the formula: percentage decrease $= \frac{\text{amount of decrease}}{\text{original value}} \times 100\%$	percentage decrease $= \frac{3}{24} \times 100\%$ $= 12.5\%$

WORKED EXAMPLE 5 Increasing and decreasing by a percentage

a **Calculate** an increase of 6.3% to $483.

b The population of a town decreased by 30% to 5320 over 3 years. What was the decrease in population?

Steps	Working
a 1 The original amount is 100%, so an increase of 6.3% gives 106.3%.	price increase = (100 + 6.3)% = 106.3%
2 Write the percentage as a decimal.	106.3% = 1.063
3 Multiply by the original amount.	483 × 1.063 = 513.429
4 State the answer, rounding appropriately.	The increased amount is $513.43.
b 1 A decrease of 30% gives 70% of the original population.	70% (or 0.7) of original population = 5320
2 Calculate the original population.	100% of original population = 5320 ÷ 0.7 = 7600
3 Find the decrease.	decrease in population = 7600 – 5320 = 2280
4 State the answer.	The population decreased by 2280 people.

Exam hack

See if you can verify your answer by testing it back into the original problem.

In part **b** above, check that 2280 out of 7600 is 30%.

EXERCISE 1.1 Percentage problems

ANSWERS p. 395

Mastery

1 WORKED EXAMPLE 1 Convert each number to a percentage, correct to one decimal place.

a $\frac{19}{40}$ **b** 3.125 **c** $2\frac{23}{25}$ **d** 0.797

2 WORKED EXAMPLE 2 Convert each percentage to a decimal and fraction.

a 14% **b** 232.75% **c** 0.56% **d** 23%

3 WORKED EXAMPLE 3 For each pair of exam scores, determine which is greater.

a 16 out of 25 or 25 out of 35

b 32 out of 45 or 38 out of 55

4 Evaluate

a 32% of $560 **b** 210% of $88.47

c 5.4% of 248 g **d** 3.43% of $6420

5 WORKED EXAMPLE 4 Find the percentage increase or decrease shown for each pair of values. Round to 2 decimal places where necessary.

a $84 to $98 **b** 128 mL to 200 mL

c 230 to 100 **d** 62 g to 69 g

e $135 to $115 **f** 63 L to 58 L

6 Find each increase or decrease as a percentage of the original value. Round to 2 decimal places where necessary.

a 28 less gives 142 **b** 14 L extra gives 128 L

c 23 more gives 47 **d** $20 less is $170

e 79 more produces a total of 120 **f** 35 mg reduction gives 415 mg

7 WORKED EXAMPLE 5 What was the original amount if a 15% increase gives a final amount of $6256?

8 What was the original amount if a 20% decrease gives a final amount of $516?

9 What original amount gives $490 after an increase of 25%?

10 $420 was remaining after a decrease of 14%. What was the original amount?

11 The mass of some clothes was increased by 24% by the water they retained from the washing machine. Before hanging them out to dry, their mass was 13 kg. What would their mass be when they are dry?

12 Adding 270 g of dye to a molten mixture increased the mass of recycled plastic by 4.5%. What was the final mass?

Exam practice

13 Express $\frac{17}{40}$ as a percentage.

A 34% **B** 33% **C** 23.5% **D** 42.5%

14 26% of $320 is

A $81.25 **B** $83.20 **C** $123.31 **D** $142.60

15 A sofa priced at $785.99 is reduced by 35% at the end-of-year sale. Its new price is

A $275.10 **B** $589.49 **C** $510.89 **D** $432.29

16 (2 marks) After 12% evaporation, there was 308 mL of solvent left. How much was there originally?

17 (3 marks) CF An increase of $16 is followed by a decrease of $16. Show that the percentage changes are different.

18 (3 marks) CU A shop decreases prices by $30 on all pairs of jeans at a sale. A pair were originally $$x$. What percentage increase will return them to the original price? Write the percentage as an expression involving x.

1.2 Rate problems

Video Rate problems

Worksheet Rapid rates

Puzzles Rate problems

Rate skills

A **rate** compares quantities with different units. Examples of rates include speed (km/h or m/s), energy consumption per hour (kW/h) and cost per unit ($/kg).

A rate is calculated per unit or item and can be found by dividing the first quantity by the second quantity. The order of the division is shown by the rate units.

Rates

For 2 quantities, quantity 1 (y) and quantity 2 (x), the rate (r) is given by:

$$\text{rate} = \frac{\text{quantity 1}}{\text{quantity 2}} \quad \text{or} \quad \text{quantity 1} = \text{rate} \times \text{quantity 2}$$

$$r = \frac{y}{x} \qquad y = rx$$

WORKED EXAMPLE 6 Calculating rates

a Sahel works as a server at a local restaurant and is paid \$63 for a 4-hour shift.
Calculate his hourly rate of pay.

b Marshall runs 6.2 km in 21 minutes. What is his running rate in kilometres per hour, correct to 2 decimal places?

Steps	Working
a **1** Identify the variables and their units.	\$63, 4 hours
2 Identify the required rate.	$r = \$/\text{h}$
3 Write the rate as a fraction of the 2 variables and evaluate.	$r = \frac{y}{x}$ $= \frac{63}{4}$ $= 15.75$
4 State the answer, including the units.	Sahel is paid \$15.75/h.
b **1** Identify the variables and their units.	6.2 km, 21 minutes
2 Identify the required rate.	$r = \text{km/h}$
3 Convert minutes to hours.	$\frac{21 \text{ minutes}}{60 \text{ minutes}} = 0.35 \text{ hours}$
4 Write the rate as a fraction of the 2 variables and evaluate.	$r = \frac{y}{x}$ $= \frac{6.2}{0.35}$ $= 17.714\,28\ldots$
5 State the answer, rounding to 2 decimal places.	Marshall's running rate is 17.71 km/h.

WORKED EXAMPLE 7 Making comparisons using rates

A journalist wants to **determine** which of 2 inner city suburbs is more heavily populated.
Balmain in Sydney covers an area of 1.55 km^2 and has an estimated population of 10 349 people.
Teneriffe in Brisbane covers an area of 0.9 km^2 and has an estimated population of 5520 people.
Which suburb has the greater population density?

Steps	Working
1 Identify the variables and their units.	population of people, km^2
2 Identify the required rate.	$r = \text{people/km}^2$
3 Evaluate the population density for each suburb.	$r = \frac{y}{x}$ Balmain $= \frac{10\,349}{1.55}$ ≈ 6677 people per km^2 Teneriffe $= \frac{5520}{0.9}$ ≈ 6133 people per km^2
4 Answer the question asked.	Balmain has the greater population density.

WORKED EXAMPLE 8 Determining quantities using rates

Cherries currently cost \$18.90 per kilogram. **Calculate** how many kilograms you could buy for \$35.

Steps	Working
1 Identify the variables and their units.	\$, kg
2 Identify the rate equation.	$r = \text{unit cost (\$/kg)} = \dfrac{\text{total cost (\$)}}{\text{mass (kg)}}$
3 Rearrange to isolate the required quantity and evaluate.	$\text{mass} = \dfrac{\text{total cost}}{\text{unit price}}$ $= \dfrac{35}{18.90}$ $= 1.85185\ldots$
4 State the answer, rounding appropriately and including the units.	1.85 kg could be purchased for \$35.

EXERCISE 1.2 Rate problems

ANSWERS p. 395

Recap

1 Convert 1.65 to a percentage.

A 16.5% **B** 165% **C** $1\frac{13}{20}\%$ **D** 65%

2 Find the percentage increase from Eric's first javelin throw of 24.78 m to his second of 30.24 m.

Mastery

3 WORKED EXAMPLE 6 Express each rate in the units shown.

a 500 g of minced meat costs \$8.50 (\$/kg)

b A hockey player scored 65 points in 15 games (points/game)

c A bird flies 12.8 km in 40 minutes (km/h)

d A 4 L tin of gloss paint covers 50 m^2 (m^2/L)

e A ball hit by a baseball player travels 100 m in 10 seconds (km/h)

f The density of a given substance is 100 g per cm^3 (kg/m^3)

4 Convert the following rates to the units indicated. Give answers correct to 2 decimal places where necessary.

a 36 km/h to m/s

b 46 cents per gram to \$/kg

c 20 cm per 10 minutes to cm/s

d \$8.65 per kilogram to cents per gram

5 Jodie works as an apprentice and is paid \$591.80 for 36 hours. Determine her rate of pay.

6 WORKED EXAMPLE 7 A farmer harvested 754 bags of cotton from a 65-hectare paddock and 892 bags from an 88-hectare paddock. Which paddock produced the higher yield (rate) of cotton?

7 Nathan scored an average of 15.8 points per game in 10 games and Jarome scored an average of 8 points per game in 18 games. Who scored more points?

8 WORKED EXAMPLE 8 For a 4-hour trip, a car has an average speed of 88 km/h. How far has it travelled?

Exam practice

9 200 mL of a child's medicine cost \$25. What is this cost per dollar?

A 12.5 mL/\$ **B** 8 mL/\$ **C** 0.125 L/\$ **D** 0.08 L/\$

10 60 km/h is approximately

A 17 m/s **B** 1 km/s **C** 17 m/min **D** 1 m/s

11 (3 marks) CF A certain type of gum tree grows at a rate of 5 cm per month for the first 2 years of its life. Explain whether it will grow to 1.07 m tall within that timeframe.

12 (5 marks) CU Alexis is tracking her fitness. She uses her smart watch to count her heart beats, H, while swimming for 10 minutes (600 seconds). The total heartbeats at 3 different times are shown in the table below.

t (seconds)	20	60	600
H	20	80	1700

Calculate Alexis' average heartbeat rate in beats/min in each of the 3 sessions and interpret how it changes with exercise.

1.3 Wages and salaries

Worksheets
Wages and salaries
Wage sheet

Spreadsheet
Wage sheet

A **wage** is the amount paid to an employee for each hour worked. The wage rate is usually stated as a payment per hour, such as \$26.30/h.

A **salary** is a fixed amount paid to an employee, usually an annual (yearly) figure. It is paid as equal weekly, fortnightly or monthly payments.

Many employees work between 36 and 40 hours full-time. Wage-earners receive overtime pay for additional hours worked. Salaried employees do not normally receive overtime payments.

Converting 1 year into months, fortnights or weeks

- 1 year = 12 months
- 1 year = 26 fortnights
- 1 year = 52 weeks

WORKED EXAMPLE 9 Converting an annual salary into monthly or weekly pay

Stuart's annual salary is \$89 130. **Calculate**

a his monthly pay

b his weekly pay.

Steps	Working
a There are 12 months in a year.	monthly pay = \$89 130 ÷ 12 = \$7427.50
b There are 52 weeks in a year.	weekly pay = \$89 130 ÷ 52 ≈ \$1714.04

Exam hack

One month is not equal to 4 weeks. When calculating monthly pay, divide the annual salary by 12. When calculating weekly pay, divide the annual salary by 52.

WORKED EXAMPLE 10 Calculating a wage

Sally is paid at a rate of $1525.32 for a 38-hour week.

a What is Sally's hourly rate?

b What would Sally's wage be if she got the same rate of pay for a 40-hour week?

Steps	Working
a Calculate the hourly rate.	hourly rate = 1525.32 ÷ 38 = $40.14 Sally's hourly rate is $40.14.
b Calculate the weekly wage.	weekly wage = 40.14 × 40 = 1605.60 Sally's pay for a 40-hour week at a rate of $40.14/h would be $1605.60.

Investigation Wage rates

Use the **Fair Work** website www.fairwork.gov.au to investigate the wage rates for a 17-year-old in the following jobs:

- kitchen assistant
- apprentice sheet-metal worker
- childcare assistant
- retail assistant
- apprentice hairdresser
- labourer
- apprentice carpenter
- nursing aide

Compare casual and permanent rates and find out the differences in the employee conditions.

Wages and inflation

Inflation is a measure of how prices increase over time. It is measured as the rate of change in prices and is often written as a percentage for a year.

In Australia, this inflation percentage is called the **consumer price index (CPI)**.

As prices increase, the value of money (or purchasing power) decreases. For example, a cup of coffee in Queensland cost around $4 in 2019, $5 in 2024 and will increase to $8 one day.

Wages, salaries, pensions and other payments are often increased to counterbalance the decrease in the value of money.

WORKED EXAMPLE 11 Calculating a wage increase in line with inflation

Kai's fortnightly salary is $4320. The rate of inflation in a particular year is 4.3%.
Calculate what Kai's fortnightly salary must be at the end of the year to account for inflation.

Steps	Working
1 The change in Kai's salary will be an increase due to inflation.	increase = (100 + 4.3)% = 104.3%
2 Write the percentage as a decimal.	104.3% = 1.043
3 Multiply by the original salary.	4320 × 1.043 = 4505.76
4 State the answer.	Kai's fortnightly salary at the end of the year will be $4505.76.

EXERCISE 1.3 Wages and salaries

ANSWERS p. 395

Recap

1 Which of the following does not represent a rate?

A 100% B 60 km/h C 3 people D 6 joules per second

2 A garden is watered for half an hour using tank water. If the rate of water released from the tank is 12 L per minute, how much water is used to water the garden?

Mastery

3 WORKED EXAMPLE 9 Anders earns a salary of $126 400.
Calculate their salary, to the nearest dollar, as

a a weekly rate b a fortnightly rate c a monthly rate

4 Santi's base pay rate as an electrician is $94 624 per year. What is her fortnightly income?

5 WORKED EXAMPLE 10 Robert is paid $24.10/h for a 38-hour week.
How much would he earn as an annual income?

6 Jenny is a first-year apprentice hairdresser who earns $27.78/h for a 38-hour week.
What is her pay as an annual amount?

7 If working a 38-hour week, determine which is the better pay for each pair.

a $36.61/h or $73 000 per year? b $105 280 per year or $2104.40/week?

c $44.01/h or $3018/fortnight? d $84 000 per year or $42.90/h?

8 WORKED EXAMPLE 11 Tyson receives a weekly salary of $1050. He is to receive a salary increase equal to the rate of inflation, which is 3.5%. What will his new weekly salary be?

9 Ewan earns $34.92/h and works 38 hours per week. How much does he receive fortnightly?

A $1326.96 B $2653.92 C $1323.28 D $2646.56

10 Jezza earns $1126.70 for a 38-hour week. What is his wage rate?

A $42.81/h B $28.17/h C $29.65/h D $30.60/h

11 Zarah's annual salary is $86 425. What does she earn each fortnight?

A $7202.08 B $3324.04 C $1728.50 D $1662.02

12 For a 38-hour week, which of these is the best pay?

A $32.45/hour B $1202.10/week C $5420.90/month D $64 980.20/year

Exam practice

13 (3 marks) Jordan earns $32.27 per hour working at a take-away store.

a Last week, Jordan worked 12 hours. How much did she earn? *[1 mark]*

b Yesterday, Jordan earned $177.49. How many hours did she work? *[2 marks]*

14 (3 marks) CF If Ebony is paid $7016 per month and Shaye earns $3352 per fortnight, who earns more?

15 (4 marks) CU Kristy earns $1890 per week. She calculated her monthly amount by multiplying by 4 and then multiplying that by 12 to get her annual salary. Determine the reasonableness of Kristy's calculation and explain whether her approach was correct.

Video
Overtime

Worksheet
S-mart payroll

1.4 Overtime, penalty rates and allowances

Overtime and penalty rates

Overtime is working beyond normal hours of work and is paid at a higher rate of pay.

If you work at unconventional times, such as weekends, public holidays or nights, you may also be paid at a higher rate. This is called a **penalty rate**.

Overtime and penalty rates only apply if you work for a wage.

Overtime and penalty rates

- **time-and-a-half**: 1.5 times (150%) the normal pay rate
- **double time**: 2 times (200%) the normal pay rate

WORKED EXAMPLE 12 Calculating wages involving overtime

Felix works in a childcare centre and is paid $29.50 per hour for an ordinary 38-hour work week.

Calculate Felix's wage in a week in which he works his normal hours, 4 hours at time-and-a-half and 1 hour at double time.

Steps	Working
1 Work out ordinary pay.	ordinary pay = $29.50 × 38 = $1121
2 Work out pay at time-and-a-half.	time-and-a-half pay = $29.50 × 4 × 1.5 = $177
3 Work out pay at double time.	double time pay = $29.50 × 1 × 2 = $59
4 Calculate the total wage.	Felix's total wage = $1121 + $177 + $59 = $1357

For workplaces with weekend work, the penalty rate for Saturday work is often 1.25 times the normal rate and Sunday work may be 1.75 times the normal rate.

Casual workers have no guarantee of future employment. Their hours of work can vary from day to day or week to week. Casual workers usually earn a higher hourly rate than permanent employees. Permanent employees may be **part-time** or **full-time**.

Investigation Casualisation of the Australian workforce

It is said that part-time and casual work has increased in Australia.

Find out how the Australian Bureau of Statistics (ABS) defines casual work.

- How does the ABS decide whether someone is employed or unemployed?
- How does the ABS define part-time and full-time work?
- What proportion of workers are currently casual?
- What proportion are currently part-time?
- What is underemployment?
- What proportion of people are underemployed?
- Are the claims of newspapers and internet news sites about part-time and casual work supported by ABS figures?

Find information about the rights of part-time and casual workers in your local area.

WORKED EXAMPLE 13 Calculating wages for casual employees

Shiro, a permanent delivery driver, is paid $1385.86 for a normal 38-hour week. The penalty rates are time-and-a-half for Saturday work, double time for Sunday work, double time-and-a-half for public holidays and triple time for Good Friday or Christmas Day.
Kym, a casual driver, is paid an extra 25% of the base rate for ordinary or penalty rates.
A permanent driver is paid at time-and-a-half for the first 2 hours of overtime and double time thereafter.
A casual driver receives an extra 10% of the base rate for overtime.

a What pay rate does Shiro earn for overtime?

b What pay rate does Shiro earn for working on Anzac Day?

c What is the ordinary pay rate for Kym?

d What pay rate does Kym earn for overtime?

Steps	Working
a 1 Work out the permanent hourly rate.	hourly rate $= \frac{1385.86}{38}$ $= \$36.47/\text{h}$
2 Work out the overtime rates.	overtime rate up to 2 h (1.5×) = 1.5 × $36.47/h = $54.71/h overtime rate after 2 h (2×) = 2 × $36.47/h = $72.94/h
3 State the answer.	Shiro's overtime rate is $54.71/h up to 2 h and $72.94/h thereafter.
b Work out the rate for Anzac Day, which is a public holiday.	Shiro's public holiday rate (2.5×) = 2.5 × $36.47/h = $91.18/h
c Work out the casual rate.	percentage of permanent rate = 100 + 25 = 125% Kym's casual rate = 1.25 × $36.47/h = $45.59/h
d 1 Add 10% to overtime rates.	percentage of permanent rate = 125 + 10 = 135% casual overtime rate = 1.35 × $45.59/h = $61.55/h
2 State the answer.	Kym's overtime rate is $61.55/h.

Allowances and bonuses

Allowances are additional payments paid to employees for doing certain tasks, working in unfavourable conditions or working in certain locations. They might be paid for expenses incurred as part of their job.

Common allowances include:

- tools and equipment
- uniforms and special clothing
- travel expenses
- phones and technology
- cars and phone expenses.

Some companies pay employees bonuses (extra pay) for doing good work. This is a performance incentive for meeting work targets and deadlines.

WORKED EXAMPLE 14 Calculating wages including allowances

Jacob works in a fly-in-fly-out (FIFO) job. His base salary is $125 622. He receives a site allowance of $20 000 p.a., a shift allowance of $12 000 p.a., and a tools and equipment allowance of $550 p.a. **Calculate** Jacob's weekly pay.

Steps	Working
1 Work out the total salary, including all allowances.	total salary = 125 622 + 20 000 + 12 000 + 550 = 156 172
2 Convert to weekly pay.	weekly pay = 156 172 ÷ 52 = $3003.307 69…
3 State the answer, rounding as appropriate.	Jacob's weekly pay is $3003.31.

EXERCISE 1.4 Overtime, penalty rates and allowances

ANSWERS p. 395

Recap

1 Chris receives a salary of $105 321. How much does he receive each fortnight?

A $2194.19 **B** $8776.75 **C** $2025.40 **D** $4050.81

2 Scott earns $85 771 per annum and works an average of 42 hours a week. What is his average hourly rate?

Mastery

3 WORKED EXAMPLE 12 Ollie earns $36.40 an hour as a steward. In any given week, he works a normal 38-hour week and 5 additional hours overtime paid as double time. What is his weekly wage, correct to 2 decimal places?

4 Deb has a casual job paying $35.60 per hour. One week she works 8 hours at her normal rate and 4 hours at time-and-a-half. What is her wage for this week, correct to 2 decimal places?

5 If Tai works 4 hours at double time and earns $431.28, what is his normal hourly rate?

6 Joti earns normal rates on weekdays and time-and-a-half on weekends. His normal wage is $28.30 an hour. What is his weekly wage in a week in which he works 38 hours during the week and 8 hours over the weekend?

7 WORKED EXAMPLE 13 Sarah works as a casual employee at a local store. The normal rate for a permanent employee is $31.20 per hour. As a casual employee, she earns 15% more than the base rate of normal rates and penalty rates. How much does Sarah earn if she works 12 hours during normal hours?

8 WORKED EXAMPLE 14 Michael works for an energy company. He is paid $42.65 per hour for a 38-hour work week. When he is required to work at heights above 14 m, he is paid a height allowance of $8.20 per hour. What would Michael earn in a normal week when 25 hours are worked at a height above 14 m?

9 Amaty works as a crime scene cleaner and is paid $32.24 per hour for a 36-hour working week. When they work with toxic cleaning substances, they are paid an allowance of $5.30 per hour. What is Amaty's salary in a week in which they use toxic cleaning substances all week?

Exam practice

10 Ibrahim earns $1240.70 per week without overtime. On days he works overtime, he earns time-and-a-half for the first 2 hours and double time thereafter. On Monday he worked one hour overtime, on Wednesday 3 hours overtime and on Thursday another $2\frac{1}{2}$ hours overtime. How much did he earn for the week?

A $1452.93 **B** $1559.04 **C** $1583.53 **D** $1632.50

11 Jayne receives $28.42/h plus an allowance of $2.34/h for dirty work. How much does she receive in a 38-hour week when she is doing dirty work for 2 days out of 5?

A $1082.30 **B** $1097.44 **C** $1115.53 **D** $1133.31

12 (3 marks) As a casual server, Fran earns 20% more than permanent servers. The permanent rate is $26.17/h. Calculate Fran's earnings for a 35-hour working week in normal hours.

13 (4 marks) CF Niles works a 36-hour working week for a set wage of $1063.80, plus time-and-a-half for any overtime. Determine his wage in a week in which he worked 42 hours.

14 (4 marks) CU Max works in a laboratory and is paid $35.81 per hour for a 37-hour working week. The first 4 hours overtime he works each week is paid at time-and-a-half, with the rest paid at double time. He also earns a hazards allowance of $4.65 per hour. Calculate Max's earnings for a week in which he works 46 hours.

1.5 Commission, piecework and royalties

Commission

Salespeople are often paid by **commission**. It is a percentage of the value of sales or services provided.

Most commission workers earn a fixed amount per work period (e.g. weekly, fortnightly or monthly) in addition to their commission. This is called a **retainer**. A retainer does not depend on sales or services provided.

Video Commission

Worksheet Pay day

Puzzles Commission and piecework

Income – commission and piecework

WORKED EXAMPLE 15 Calculating income including commission

Serena works for a real estate agency that charges the seller 2.5% commission. Serena is paid a $2500 monthly retainer plus 65% share of the agency's commission, less an administration fee of $939. How much does she make in a month in which she sells a house for $720 000?

Steps	Working
1 Calculate the agency's commission.	Agency's commission = 0.025 × $720 000 = $18 000
2 Calculate Serena's commission.	Serena's commission = 0.65 × $18 000 = $11 700
3 Add in the retainer and subtract the administration fee.	Serena's pay = $11 700 + $2500 – $939 = $13 261
4 State the answer.	Serena receives $13 261 for that month.

WORKED EXAMPLE 16 Calculating commission involving sliding scales

Andy works as a used car salesman and earns commission on his sales at the following rates:

- 15% on the first $30 000 of sales
- 10% on the next $20 000 of sales
- 8% on the remainder of sales.

This month, Andy made sales totalling $82 000. What is his commission?

Steps	Working
1 Break the sales $82 000 into its parts: $30 000, $20 000 and $32 000. Then calculate the commission on each.	commission on first $30 000 = 0.15 × $30 000 = $4500 commission on next $20 000 = 0.1 × $20 000 = $2000 commission on the last $32 000 = 0.08 × $32 000 = $2560
2 Calculate the total commission.	total commission = $4500 + $2000 + $2560 = $9060
3 State the answer.	Andy receives a commission of $9060.

Video Commission involving sliding scales

Piecework

Piecework is a payment method in which a person is paid for each item made or task completed. The amount for each item or action is called the **piece rate**.

WORKED EXAMPLE 17 Calculating income from piecework

Ingrid charges \$4 per business shirt and \$3.20 per t-shirt or polo.
How much will she earn for ironing 12 business shirts, 3 t-shirts and 6 polos?

Steps	Working
1 Break the work into the 2 categories, then calculate each amount earnt.	business shirts = 12 × \$4 = \$48 t-shirts or polos = 9 × \$3.20 = \$28.80
2 Calculate the total pay.	total earnt = \$48 + \$28.80 = \$76.80
3 State the answer.	Ingrid earns \$76.80.

Royalties

A **royalty** is a payment made for the use of intellectual property, such as songs, books, patents or other types of assets. Royalties are generally calculated as a percentage of the revenue or profit. Types of royalties include:

- book royalties: the author(s) receive a percentage of the sales revenue for each book sold.
- music royalties: musicians earn royalties when their music is played, sold, streamed, or used in various media.

A royalty gives creators money for their creations or inventions when other people use them.

WORKED EXAMPLE 18 Calculating income from royalties

Heidi is an emerging musician and is paid a royalty of 15% of the value of each song when it is streamed. What royalty would Heidi receive if her newest song was streamed 315 170 times at an average price of \$0.02?

Steps	Working
1 Write the details of the royalty.	15%, 315 170 streams at \$0.02
2 Calculate the total sales price.	total sales = 315 170 × \$0.02 = \$6303.40
3 Calculate the royalty.	royalty = 0.15 × \$6303.40 = \$945.51
4 State the answer.	Heidi earns a royalty of \$945.51.

EXERCISE 1.5 Commission, piecework and royalties

ANSWERS p. 395

Recap

1 Michelle is paid a normal hourly rate of \$25.60 per hour. What is her pay on a day in which she works 8 hours at normal rate and 4 hours at double time?

A \$204.80 **B** \$358.40 **C** \$307.20 **D** \$409.60

2 Cynthia's boss said that staff had exceeded all requirements this year, so the company is giving employees bonuses of $7\frac{1}{2}$% of their annual salary. Cynthia's annual salary is \$75 870. What is her bonus?

Mastery

3 WORKED EXAMPLE 15 Li works on 12.5% commission in a department store. After working an 8-hour shift on a Saturday, he sold $2825 worth of stock.

a What does he earn for the shift?

b What is Li's hourly rate?

c Would you expect him to earn this rate throughout the week?

4 WORKED EXAMPLE 16 A real estate agent works on a commission-only basis. For each property, the commission is separately calculated as 0.45% of the value of the property up to $400 000 and 0.15% of the remainder. In one month the agent sold properties for $1 055 000, $1 678 000 and $988 000. What is her income for that month?

5 WORKED EXAMPLE 17 Ainslie got a job picking apples in Stanthorpe in February. She was paid $65 a bin. With experience and practice, she was able to pick about 5 bins in an 8-hour day, 5 days a week. Calculate her hourly rate and her earnings for 6 weeks' work.

6 WORKED EXAMPLE 18 Calculate the royalties on each sale.

a 4751 books sold at $55.20 with a 9% royalty payment

b 18 432 song download purchases sold at $1.19 with a 12% royalty payment

c 7715 game apps sold at $3.60 with a 7.5% royalty payment

7 Aziz sells videos at a weekend market. He pays $5 per video for stock and sells them for $12 each. He has to pay a fee of $80 per day for a stall at the market. The market opens at 8 am and closes at 3 pm. It takes Aziz half an hour to set up before the market opens and half an hour to pack up afterwards. In an average week, he sells 75 videos over the weekend.

Calculate how much money he takes, how much he gets after expenses and his hourly rate.

8 Candace works on a commission of 14% plus a retainer of $300 per week selling cosmetics. In one week, she worked for 45 hours and sold $5940 worth of cosmetics.

a Calculate her earnings and hourly rate for the week.

b Calculate the value of cosmetics she sold in a week in which she earned $940.

9 A pizza parlour in Toowoomba pays $8.20 per delivery. One night, Peter delivered 34 pizzas between 5:30 pm and 11:30 pm, but also travelled 240 km in his car.

a How much did he get from the pizza parlour?

b His car uses 9.2 L/100 km and petrol cost him 210.9 c/L. How much did he pay for petrol?

c What were Peter's real earnings and hourly rate? Explain your reasoning.

Exam practice

10 Isla works on a commission of 12.5% of sales. What does she earn for selling cosmetics worth $49, $38.99, $125.99 and $32.60 in one hour?

A $30.82 **B** $67.50 **C** $29.59 **D** $33.07

11 Oskar works in quality control inspecting every 25th item from a production line. He earns $1.25 for every plastic bottle and $1.36 for every plant pot he inspects. How much does he earn for inspecting 28 bottles and 42 pots?

A $97.64 **B** $122.36 **C** $63.89 **D** $92.12

12 Tash works as a dental equipment salesperson. She receives a commission of $2370 on sales of $15 800. What is Tash's commission rate?

A 0.015% **B** 15% **C** 0.15% **D** 20%

13 (3 marks) CF Kim uses an embroidery machine to embroider patterns and personalised designs onto different items such as shirts, shoes and skirts. She is paid $35 for every 100 standard images she embroiders. If a product requires additional embroidery elements, Kim is paid an additional $5 for every 100 items. How much will Kim earn if she embroiders 280 items that each require 2 additional elements?

14 (4 marks) CF Jake sells whitegoods. He has the option of working for a straight commission of 7% or for a commission of 5.5% and a retainer of $170/week. What value of whitegoods would he need to sell in a week to be better off on the straight commission?

15 (5 marks) CU Charles is paid $2.70 per parcel delivered. Working 5 days per week, he starts at 4:30 am, sorting and packing his van to start delivering from 7 am to 2:30 pm. He delivers an average of 145 parcels a day. He estimates his van costs $180 a week to run and $300 a week to pay off. Determine Charles' average hourly rate of pay, taking into account the running costs of his van.

Worksheet
Earning money

1.6 Government allowances and pensions

Some people are unable to work or can only work limited hours. They may get payments, pensions and/or allowances from the Government. To receive these benefits, they must satisfy certain requirements, such as age, income, disability, and/or asset tests.

Current payment rates can be found at the **Services Australia** website at www. servicesaustralia.gov.au.

If you're studying, training or doing an Australian Apprenticeship, you may be eligible for a government payment or allowance.

Youth Allowance

Youth Allowance is paid to full-time students or Australian Apprentices who are:

- 18 to 24 and studying full time
- 16 to 24 and doing a full-time Australian Apprenticeship
- 16 to 17 and independent or needing to live away from home to study
- 16 to 17, studying full time and have completed Year 12 or equivalent.

The size of payments is based on the following circumstances:

Your situation	Your maximum fortnightly payment from 1 January 2024
Single, no children, younger than 18, and live at your parent's home	$395.30
Single, no children, younger than 18, living away from your parent's home to study, train or look for work	$639.00
Single, no children, 18 or older and live at your parent's home	$455.20
Single, no children, 18 or older and need to live away from your parent's home	$639.00
Single, with children	$806.00
A couple, with no children	$639.00
A couple, with children	$691.80

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/how-much-youth-allowance-for-students-and-apprentices-you-can-get?context=43916#payment-rates. Creative Commons Attribution 3.0 Australia Licence, https://creativecommons.org/licenses/by/3.0/au/

Your income and your parents' income will be taken into account when calculating how much Youth Allowance you will receive. Parents' combined income over $62 634 reduces the payment by $0.20 for every dollar over this amount.

If you earn over $509 (gross income) a fortnight, your allowance rate will decrease by $0.50 for every dollar over $509, and by $50.50 plus $0.60 for every dollar over $610.

WORKED EXAMPLE 19 Youth Allowance

Risto is eligible for Youth Allowance. **Determine** how much he will receive over a 12-month period if he is over 18, single with no children, and living at home while studying.

Steps	Working
1 Identify the quantity to be found.	yearly Youth Allowance
2 Identify the relevant allowance amount.	fortnightly payment = \$455.20
3 Convert to an annual amount.	annual amount = 455.20 × 26 = \$11 835.20
4 State the answer.	Risto will receive \$11 835.20 in Youth Allowance over a 12-month period.

Austudy

Austudy is financial assistance offered by the government if you are:

- 25 or older
- a full-time student in an approved course or Australian Apprenticeship
- under the **income test** limits.

Your situation	Your maximum fortnightly payment from 1 January 2024
Single, no children	\$639.00
Single, with children	\$806.00
A couple, no children	\$639.00
A couple, with children	\$691.80

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/how-much-austudy-you-can-get?context=22441.
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If you earn an income, the same income test requirements apply as for Youth Allowance eligibility.

WORKED EXAMPLE 20 Calculating Austudy

Will and Grace are full-time students living as a couple and have 2 children.
Calculate their annual combined Austudy income.

Steps	Working
1 Identify the quantity to be found.	yearly Austudy allowance
2 Identify the relevant allowance amount.	fortnightly payment = \$691.80
3 Convert to an annual amount.	annual amount = 691.80 × 26 = \$17 986.80 combined amount = \$17 986.80 × 2 = \$35 973.60
4 State the answer.	Will and Grace will receive a combined annual total of \$35 973.60 in Austudy Allowance.

ABSTUDY

ABSTUDY is a group of payments for Aboriginal or Torres Strait Islander (ATSI) students or apprentices and can help with:

- school fees
- boarding fees
- living costs
- travel costs
- buying school materials.

Age Pension

You may be eligible for the **Age Pension** if you:

- are over the pension age (currently 67 years)
- are under the income **assets test** limits
- have been an Australian resident for at least 10 years.

Per fortnight	Single	Couple each	Couple combined	Couple apart due to ill health
Maximum basic rate	$1020.60	$769.30	$1538.60	$1020.60
Maximum Pension Supplement	$81.60	$61.50	$123.00	$81.60
Energy Supplement	$14.10	$10.60	$21.20	$14.10
Total	$1116.30	$841.40	$1682.80	$1116.30

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/how-much-age-pension-you-can-get?context=22526.
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There are income rules as shown below:

Single person	
Income per fortnight	Amount your pension will reduce by
Up to $204 (free area)	$0
Over $204	50 cents for each dollar over $204

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/income-test-for-age-pension?context=22526.
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Couple living together or apart due to ill health Different rates apply for partners getting a payment other than a pension	
Combined income per fortnight	Amount each member of the couple's pension will reduce by
Up to $360 (free area)	$0
Over $360	25 cents for each dollar over $360

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/income-test-for-age-pension?context=22526.
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WORKED EXAMPLE 21 Age Pension

How much does an aged couple living together receive in total in a year if they have a combined income from investments of $700/fortnight and are eligible for the maximum basic rate?

Steps	Working
1 Identify the quantity to be found.	Age Pension for a couple with an income
2 Identify any income considerations and pension reduction amount.	Combined income is over $360. pension reduction = (700 – 360) × 0.25 = $85
3 Identify the relevant allowance amount and adjust by the reduction amount.	combined rate = 1538.60 – 85 = $1453.60/fortnight
4 Convert to an annual amount.	annual amount = 1453.60 × 26 = $37 793.60
5 Find the annual investment income.	investment income = $700 × 26 = $18 200
6 Find the total amount.	total amount = $37 793.60 + $18 200 = $55 993.60
7 State the answer.	The couple will receive $55 993.60 in combined income for the year.

Disability Support Pension

The **Disability Support Pension** provides financial assistance if you have a physical, intellectual or psychiatric condition that is likely to persist for more than 2 years and limits your ability to work.

If you are under 21, the following payments may apply:

Your situation	Maximum rate per fortnight
Single, younger than 18, and dependent	$548.80
Single, younger than 18, and independent	$792.50
Single, 18 to 20, and dependent	$608.70
Single, 18 to 20, and independent	$792.50
A couple, and younger than 21	$792.50

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/payment-rates-for-disability-support-pension?context=22276.
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If you are 21 or older, with or without children, or younger than 21 with a child in your care, the following payments may apply:

Pension rates per fortnight	Single	Couple each	Couple combined	Couple each separated due to ill health
Maximum basic rate	$ 1020.60	$769.30	$1538.60	$ 1020.60
Maximum Pension Supplement	$81.60	$61.50	$123.00	$81.60
Energy Supplement	$14.10	$10.60	$21.20	$14.10
TOTAL	$1116.30	$841.40	$1682.80	$1116.30

Website: servicesaustralia.com.au https://www.servicesaustralia.gov.au/payment-rates-for-disability-support-pension?context=22276.
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If you earn an income, the same income test rates apply as for the Age Pension.

WORKED EXAMPLE 22 Disability Support Pension

Rahman is 19 years old and living independently. **Calculate** how much more he will receive each fortnight from his Disability Support Pension after he turns 21 if he receives the maximum basic rate.

Steps	Working
1 Identify the quantities to be found.	Disability Support Pension – single, 18 to 20, and independent Disability Support Pension – maximum basic rate
2 Identify the relevant pension amounts and find the difference.	difference = $1020.60 – $792.50 = $228.10
3 State the answer.	Rahman will receive $228.10 more a fortnight.

EXERCISE 1.6 Government allowances and pensions

ANSWERS p. 395

Recap

1 Suzanne is a manufacturer's representative who works on a 15% commission with a retainer of $250 per week. One week she persuades 40 shops to buy a complete display of hand tools at $145 each. How much does she earn for that week?

2 Sven earns a royalty of 15% on sales of his children's book. If there were $12 430 in sales last year, what did Sven make in royalties?

A $12 430 **B** $2486 **C** $10 565.50 **D** $1864.50

Mastery

3 WORKED EXAMPLE 19 Carole is 18 years old, living at home and attends university. She works 4 nights a week at a local pizza shop and makes $66 a night. How much Youth Allowance can she receive per fortnight?

4 WORKED EXAMPLE 20 Sue Ellen is in her last year of a certificate III course in office management. She works part-time as a receptionist and gets paid $490 a fortnight. She is single and has just turned 26. How much does she receive in Austudy per fortnight?

5 WORKED EXAMPLE 21 Coola is 70 years old and is on the maximum basic pension. She gets an income of $250 a week from the widows' benefit of her husband's superannuation. How much does she receive from her pension and how much does she receive altogether in a year?

6 WORKED EXAMPLE 22 Merv is 23 years old and receives the single maximum basic disability pension. He also receives $230 a week income from a trust fund. How much does he receive from his pension and what is his total income for the year?

Exam practice

7 Jesinta is single, aged 22, on a Disability Support Pension and entitled to the Energy supplement. How much does she get each fortnight?

A $1020.60 **B** $1116.30 **C** $1034.70 **D** $1101.60

8 Bill and Mari are in their seventies, both widowed and on the basic Age Pension. When they marry and move in together, by how much does their combined income per fortnight decrease?

A $518.00 **B** $502.60 **C** $251.30 **D** $769.30

9 Lidia is 19 years old and has moved away from home to study at James Cook University. Her parents have a combined income of $73 200. How much does she receive in Youth Allowance per fortnight?

A $455.20 **B** $639.00 **C** $418.68 **D** $557.72

10 (3 marks) CF Danielle and Sam are friends and are both receiving Austudy. They were living at home when they decided to rent a unit together. They moved into their new place at the start of the second year of university. They are both 22 and have incomes of $210 and $260 a week respectively from part-time jobs.

Did their income change after they moved into the unit together? Justify your answer.

Worksheets
Personal budgets
Budget grid
Budgeting scenarios

1.7 Budgets

Managing your finances is an important life skill. One way to do this is to create a **budget**.
A budget involves balancing your estimate of expected income and expenses.
Your income is the money you earn, and expenses are the things you spend money on.

Expenses and budgets

- **Fixed expenses** are essential living costs, must be paid and are less easily varied. Such expenses include rent or loan repayments, food and electricity.
- **Discretionary expenses** are non-essential living costs such as entertainment, clothing or eating out.
- A budget is a list of income and a list of expenses. Expenses usually include money allocated for savings and emergencies.
- A budget where income and expenses are equal is called a *balanced* budget.

Creating a budget

There are some simple things you can do to help prepare and use a budget.

1. Plan your budget over a time period that suits your lifestyle or purpose. This could be a week, a fortnight or a month and may match your pay cycle.
2. Identify all income streams for that time period, such as work, investments and any allowances.

3. List your expenses:
   - identify your fixed expenses that must be paid every week or month, like rent, transport, groceries and electricity.
   - identify your discretionary expenses – things you would like to buy or spend money on, such as nights out or entertainment.

It may be helpful to organise your annual expenses into categories and save money for these each time period. For example, if your car registration costs are approximately $1000 and you are creating a fortnightly budget, you should save $1000 ÷ 26 ≈ $39 each fortnight for your car registration.

4. Consider any goals that you want to save for, such as buying a car or going on a holiday.
5. Calculate the total income and total expenses.
6. Balance your budget. This means you are not spending more than you have.

When you make a budget, overestimate your spending by allowing 4 weeks in a month, 12 weeks in a quarter and 50 weeks in a year when you divide expenses to make weekly amounts. This gives you a margin for price changes.

WORKED EXAMPLE 23 Creating a personal budget

Karen earns $546 a week after tax. She rents a furnished unit with a friend and her share of the rent is $350 a fortnight. She spends about $170 a week on food, household items and toiletries, including takeaway meals. She catches the train to work each weekday, costing $4.50 a day. The power (electricity and gas) bills total about $280 a quarter and she pays half of this. Karen estimates that she spends $210 a month on clothes. She spends about $120 a month on her prepaid mobile and $30 a week on entertainment.

a **Construct** a weekly budget and work out how much Karen could save in a year.

b **Consider** where she could make savings.

Steps	Working
a **1** Identify the time period for the budget.	weekly
2 List income and expenses. Calculate weekly costs if the item is given in a different time unit or is to be shared. Rent = 350 ÷ 2 = 175 Transport = 5 × 4.50 = 22.50 Power = 280 ÷ 2 ÷ 12 = 11.67 Clothes = 210 ÷ 4 = 52.50 Phone = 120 ÷ 4 = 30	

Weekly income		Weekly expenses	
Wage	$546	Rent	$175
		Food, etc.	$170
		Transport	$22.50
		Power	$11.67
		Clothes	$52.50
		Mobile	$30
		Entertainment	$30
Total income	$546	Total expenses	$491.67

3 Balance the budget.	balance = income – expenses = $546 – $491.67 = $54.33
4 Determine potential saving for the year.	savings = $54.33 × 52 = $2825.16
5 Answer the question.	If Karen's estimates are correct, she could save $54.33 per week, which would amount to $2825.16 a year.
b Consider her expenses. Make suggestions.	She might be able to cut spending on food by buying fewer takeaways. She could cut her mobile use by topping up smaller amounts and rationing her use.

WORKED EXAMPLE 24 Creating a budget for a specific item or event

Antonia is going for a 7-day sailing trip around the Whitsundays with 4 friends. The yacht costs $700 a day to hire, and she estimates it will use about 100 L of diesel at $1.90/L. Food for the week will probably be about $900. It will cost about $50 each to travel to and from the port. Antonia also wants to include a contingency budget of 10% of the total expenses for unexpected costs that may arise. **Construct** a budget and find the cost per person.

Steps	Working
1 Identify the time period for the budget.	7 days = one week
2 As this budget is for an event, you do not need to include the income section. List the expenses. Calculate weekly item costs. Yacht hire = 700 × 7 = 4900 Fuel = 100 × 1.9 = 190 Transport = 50 × 4 = 200	

Expenses	
Yacht hire	$4900
Fuel	$190
Food	$900
Transport	$200
Contingency	
Total expenses	

3 Calculate the 10% contingency on the expenses.	total expenses = $4900 + $190 + $900 + $200 = $6190 contingency = 0.1 × $6190 = $619
4 Complete the budget table and calculate the total expenses.	

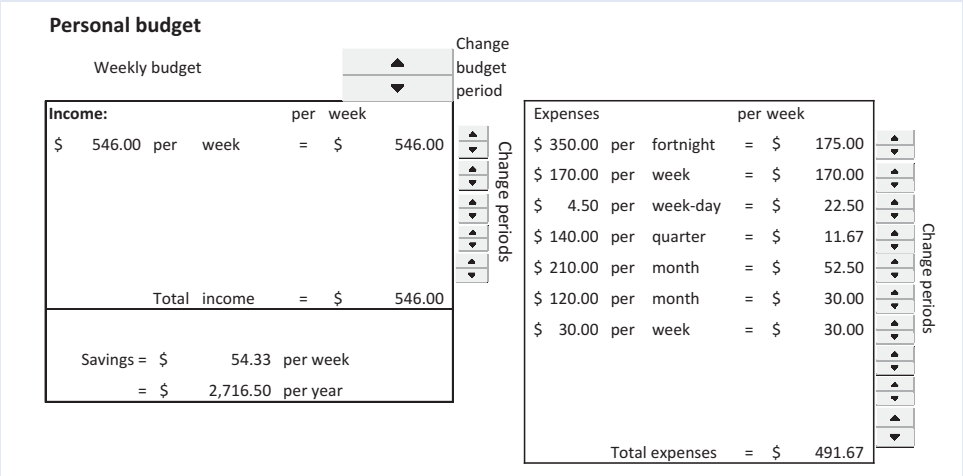
Expenses	
Yacht hire	$4900
Fuel	$190
Food	$900
Transport	$200
Contingency	$619
Total expenses	$6809

5 Determine the cost per person.	cost per person = $6809 ÷ 4 = $1702.25
6 Answer the question.	Based on Antonia's estimates, the cost per person for the trip should be about $1702.25.

TECHNOLOGY Budget spreadsheet

Spreadsheet Personal budget

You can work out weekly, fortnightly or monthly budgets using the 'Personal budget' spreadsheet on Nelson MindTap.
The 'spinners' change between weekday, weekly, fortnightly, monthly, quarterly and annually for the budget, income and expenses.
Enter the information and the spreadsheet automatically works out the budget.
The snapshot shown is Karen's weekly budget from Worked example 23.



Investigation Moving out

Can you afford to move away from home when you start work?

- Research how much it costs to rent a flat or part of a share-house in your area.
- Write down lists of what you spend money on including haircuts, phone charges, toiletries, train and bus fares, snacks, shoes, sports fees, home entertainment and movies.
- Use the lists to estimate your living costs.
- Estimate the cost of your food, including all meals (items such as breakfast cereal, fruit, vegetables, bread and cooking ingredients). Don't forget takeaway food and drinks.
- Talk to other people to investigate the cost of household bills, such as power, gas and water, and other household costs like cleaning materials.
- Convert your listed expenses to weekly or fortnightly amounts and work out a budget.

Check the income you can get and decide if you could afford to move out.

EXERCISE 1.7 Budgets

ANSWERS p. 396

Recap

1 Sam is aged $17\frac{1}{2}$ and studying teaching. She lives at home with her parents who are both unemployed. How much Youth Allowance will Sam be paid per fortnight?

2 Jack is 27 and has just started studying Geoscience at university. He is a single parent and earns less than \$509 per fortnight in his tutoring job. How much Austudy will he receive in a year?

Mastery

3 WORKED EXAMPLE 23 Ayla is sharing a small furnished flat with her best friend. She earns \$510 a week after tax and the flat costs them \$410 a fortnight, which includes power. Ayla hardly ever eats out, so she spends only \$95 a week on food and household necessities. She pays \$24 a week for the bus to and from work. She spends \$200 a month on her mobile account and \$105 a week on nights out and entertainment. Ayla spends about \$1900 a year on clothes and is trying to save up for an overseas trip. Work out her budget and how much she could save in a year.

4 Justin earns \$1380 a fortnight after tax. He shares a flat with his best mate, Hao. The flat costs \$450 a week and the shared power bill is usually about \$120 for a month. Justin and Hao equally share the cost of groceries and other household items. This usually comes to about \$190 a week. Justin likes to eat takeaway food; he also likes to party. He estimates that he spends about \$150 a week on takeaway food and entertainment. Justin is paying off his motorbike at \$180 a month and spends about \$30 a fortnight to run it. He estimates that he spends about \$1400 a year on clothes and \$100 a month on his mobile account. Work out Justin's budget and how he could save for the deposit on a car.

5 WORKED EXAMPLE 24 A couple plans to fly to Townsville from Brisbane for a 7-day holiday on Magnetic Island. The return airfare is \$449 per person, and airport transfers to and from Townsville and Magnetic Island are \$165 each. The tariff for their room is \$285 a night, which includes breakfast. They think that lunch and dinner will be a total of about \$65 a day each and that spending on other purchases might be about \$25 each a day.

Work out a budget for the couple's trip.

6 Sandy shares a furnished house with 2 of her friends. She earns \$1360 a fortnight after tax and the house costs them \$870 a fortnight, which includes electricity charges. All 3 are vegetarians who mostly eat at home, so they only spend about \$90 each a week on food and household items. Sandy spends about \$180 a month on her mobile and allows herself \$80 a week for entertainment and incidentals. Sandy walks to the bus stop and pays \$32 a week on fares to and from work. She knows from experience that she spends about \$2100 a year on clothes and shoes. Sandy is trying to save up for a car. Work out her budget and how much she could save in a year.

Exam practice

7 (2 marks) Copy and complete the following table to determine Tiana's monthly savings.

Annual salary	Investment income (monthly)	Fixed expenses (monthly)	Discretionary expenses (monthly)	Monthly savings
$ 95 800	$545	$3740	$2531	

8 (4 marks) CF A 5-day Year 8 camp is planned. It is anticipated that 50 girls and 41 boys will attend. The camp will be held at a campsite that costs $100 per day for each cabin. The 5 teachers will be accommodated in one cabin, and the students will use separate cabins for boys and girls. There are 6 bunks in each cabin. The catering is $122 per person. Buses seating 55 people will be hired for the trip, at a cost of $482 each way for each bus.

The cost of the camp is paid by students.

Construct a budget to find the cost per student, rounding up to the nearest $5.
Consider if this is enough to allow for contingencies and justify your answer.

9 (4 marks) CU A club is selling sausages to raise funds. The treasurer gets the following prices from a wholesaler:

Sausages: $60/100

Bread rolls: $55/100

Tomato sauce: $10.80 for a 4 L bottle

The treasurer thinks that most people will have 20 mL of sauce on a sausage. Work out the cost of 1000 sausages and the profit made if they sell 950 at $4 each.

EXAM QUESTION ANALYSIS

Exam-style question (4 marks)

A real estate agent sells a house for $475 500. They receive 1% commission on the first $250 000 of a sale, 1.5% on the next $200 000 of the sale and 2% on the remaining balance. The agent receives a base salary of $1800 per fortnight. Calculate how much the agent will be paid if this is the only house they sell in the fortnight.

Reading the question

- The question is about a house sale and a real estate agent's commission.
- The pay is broken up into commission percentages and retainer (base salary).
- The question is asking for the fortnightly pay if the $475 500 house sells.

Thinking about the question

- It would be easiest to break the component parts into separate working sections and add them all at the end.
- There are 3 levels of commission: 1%, 1.5% and 2%. The $475 500 will need to be broken up into 3 parts.
- Percentages can be written in decimal form.

Worked solution (✓ = 1 mark)

1% commission = 250 000 × 0.01

= **$2500** ✓

1.5% commission = 200 000 × 0.015

= **$3000** ✓

2% amount = (475 500 − 450 000) × 0.02

= **$510** ✓

$250 000 + $200 000 = $450 000

fortnightly pay = 2500 + 3000 + 510 + 1800

= $7810

The agent will receive **$7810** pay for the fortnight. ✓

1.7

Video Exam question analysis: Incomes and budgets

1 Chapter summary

Percentages

- **Percentage** means 'per 100', and is a number expressed as a fraction of 100.
- A percentage increase or decrease in a quantity is always expressed as a percentage of the original value.

$$\text{percentage increase} = \frac{\text{amount of increase}}{\text{original value}} \times 100\%$$

$$\text{percentage decrease} = \frac{\text{amount of decrease}}{\text{original value}} \times 100\%$$

- When increasing or decreasing a quantity by a percentage, calculate it 'out of' the original value.
- To increase something by x%, multiply by $(100 + x)$%.
- To reduce something by x%, multiply by $(100 - x)$%.

Rates

- A **rate** compares quantities or variables that have different units.
- A rate is calculated per unit or item.
- For 2 quantities, quantity 1 (y) and quantity 2 (x):

$$\text{rate} = \frac{\text{quantity 1}}{\text{quantity 2}} \quad \text{or} \quad \text{quantity 1} = \text{rate} \times \text{quantity 2}$$

$$r = \frac{y}{x} \qquad\qquad y = rx$$

Wages and salaries

- A **wage** is the amount paid to an employee for each hour worked and usually stated as a payment per hour, such as \$26.30/h.
- A **salary** is a fixed amount paid, usually on a yearly basis, such as \$75 291/year.

  1 year = 12 months

  1 year = 26 fortnights

  1 year = 52 weeks

Overtime, penalty rates and allowances

- **Overtime** is working beyond normal hours of work and is paid at a higher rate of pay.
- An employee working hours at unconventional times can be paid a **penalty rate**, such as **time-and-a-half** and **double time**.
- **Allowances** are additional payments paid to employees for doing certain tasks or working in unfavourable conditions.

Commission, piecework and royalties

- **Commission** workers earn a proportion of the value of sales or services. It is usually a percentage of the value of an item sold.
- Some commission workers also receive a small **retainer**, which is paid regularly regardless of sales.
- **Piecework** payments are made for each item or task completed.
- Some businesses pay a piecework bonus or incentive in addition to wages.
- **Royalty** payments are made for the use of intellectual property, such as songs or books.

Government allowances and pensions

- **Youth Allowance**, **Austudy** and **ABSTUDY** are paid to eligible Australian apprentices or full-time students.
- **Age Pension** and **Disability Support Pension** are paid to eligible people who are over the pension age, or are unable to, or have limited ability to, work because of a disability.

Budgets

- **Income** is money received.
- **Expenses** are money spent.
- **Fixed expenses** are items you must pay.
- **Discretionary expenses** are things you choose to pay.
- A **budget** is a complete list of income and expenses over the same period. A balanced budget has equal income and expenses.
- Contingencies are costs that arise unexpectedly.

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

Worksheet
General Maths Year 11 formula sheet

1 An increase in an amount by 42% is equivalent to

A 42% **B** 0.42 **C** 1.42 **D** 1.042

2 Alex measures his speed while riding his e-bike at 30 km/h. What is his speed in m/s?

A 500 **B** 108 **C** 1.8 **D** 8.33

3 Silvia receives a salary of $105 440. How much does she receive each week?

A $2027.69 **B** 8786.67 **C** $2196.67 **D** $4055.38

4 Billy is paid $30.60/h during a normal work week and time-and-a-half on Saturday. How much does Billy earn if he works 8 hours on both Thursday and Friday and 6 hours on Saturday?

A $612.00 **B** $673.20 **C** $765.00 **D** $1009.80

5 A fitness salesman sells $50 000 worth of gym equipment and receives $12 000 for his services. What is the term used to describe the money he earns?

A salary **B** commission **C** royalty **D** wage

Section 2 4 short response questions

13 marks

6 (3 marks) Erica is paid $19.32 per hour at her new job, plus a meal allowance of $25.40 per day. If Erica works 9 hours per day for a full 5-day work week, how much will she be paid in total?

7 (2 marks) Lee does quality control for an importer of radio-controlled toy cars. He gets paid $3.90 for every car he inspects. One day he checked 43 cars in 6 hours. How much did he receive, and what was this as an hourly rate?

8 (3 marks) Jess is eligible for the maximum fortnightly Youth Allowance of $455.20, but she realises it will be impacted by her parents' combined income of $78 990. Jess knows that the maximum annual Youth Allowance is reduced by $1 for every $5 that a youth's parents' income exceeds $62 634. Calculate Jess' Youth Allowance after adjusting for her parents' income.

9 (5 marks) Samar has a net income of $665 per week. He has created the following weekly budget to help manage his finances.

Expenses	
Rent	$155
Petrol	$40
Car registration	$15
Car maintenance	$10
Insurances	$30
Food	$95
Power	**A**
Phone and internet	$90
Entertainment	$70
Clothes	$80
Savings	$50
Total	**$665**

a How much does his power cost (A)? *[1 mark]*

b Samar has been advised by his insurer that he has been undercharged and next month will need to pay the balance owing. For the last 3 weeks he has put aside his savings, as well as his insurance money to pay the bill. How much money has Samar put aside to pay the bill? *[1 mark]*

c It turns out the bill for insurance is $650. It is due in 2 weeks. Samar realises he doesn't have enough put aside. Determine if he can reallocate money in his budget to have enough money to pay the bill. Justify your answers with suitable calculations and reasons. *[3 marks]*

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions

12 marks

1 (4 marks) Charlie has been offered a new job as a salesman and given 2 options for calculating his income:

Method 1: 26% commission on all sales only.

Method 2: 9% commission on all sales plus $550 per week.

Based on Charlie's research of the company, the average sales total per employee per month is $16 550.

If Charlie is to select his income method based on the average sales total, state which method he should choose. Justify your answer with appropriate calculations.

Worksheet
General Maths Year 11 formula sheet

2 (4 marks) Mia is paid an hourly rate of $18 on normal weekdays. On Saturday she earns time-and-a half, and on Sunday double time.

Her timesheet for last week is shown.

	Start	Finish	Unpaid break
Thursday	9:00 am	1:30 pm	30 minutes
Saturday	9:00 am	4:00 pm	1 hour
Sunday	9:00 am	3:00 pm	1 hour

Mia decides she doesn't like working on weekends but wants to keep her weekly wage the same. How many extra hours will she need to work on weekdays to be able to achieve this?

3 (4 marks) Phong is preparing his annual budget spreadsheet.

This year he expects to earn:

- $160 every week coaching swimming at the local pool
- 5% interest from an investment of $4000.

His planned expenses are:

- $25 per week on lunches
- $36 per week on public transport
- $60 per month on entertainment.

Any remaining income, Phong intends to save. His partially completed budget spreadsheet is shown.

	A	B	C	D	E
1	Phongs Annual Budget				
2					
3	Income			Expenses	
4	Wage	$8,320		Lunches	$1,300
5	Interest on investment			Transport	
6				Entertainment	
7					
8					

If Phong started saving at the beginning of the year, will he have enough saved by the end of the year for a $5500 deposit on a car? If not, determine how long it will take him. Justify your answer with appropriate calculations.

CHAPTER

2 LENGTH, AREA AND VOLUME

Syllabus coverage

Nelson MindTap chapter resources

Terminology

2.1 Pythagoras' theorem problems
- Pythagoras' theorem
- Pythagoras' theorem in two dimensions
- Pythagoras' theorem in three dimensions

2.2 Perimeter and area
- Units of length and area
- Perimeter and area of standard shapes
- Arcs and sectors

2.3 Perimeter and area of composite shapes

2.4 Surface area

2.5 Volume
- Units of volume and capacity

2.6 Surface area and volume of composite shapes

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

9780170484749

Syllabus coverage

UNIT 1, TOPIC 2: SHAPE AND MEASUREMENT

Pythagoras' theorem

- Understand and use Pythagoras' theorem to solve practical problems in two dimensions and simple applications in three dimensions.
  - $c^2 = a^2 + b^2$ where c is length of the hypotenuse and a and b are lengths of the two perpendicular sides.

Mensuration

- Calculate perimeters, P, of standard two-dimensional objects in practical situations, including circles, sectors, triangles, rectangles, trapeziums, parallelograms and composites.
  - circle: $C = 2\pi r$ where C is circumference and r is radius
  - sector: $P = 2r + \frac{\theta}{180}\pi r$ where θ is central angle and r is radius
- Calculate areas, A, of standard two-dimensional objects in practical situations, including circles, sectors of circles, triangles, rectangles, parallelograms, trapeziums and composites.
  - circle: $A = \pi r^2$ where r is radius
  - sector: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius
  - triangle: $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height
  - parallelogram: $A = bh$ where b is base length and h is perpendicular height
  - trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel lengths and h is perpendicular height
- Calculate surface areas, S, of standard three-dimensional objects in practical situations, including rectangular prisms, cylinders, pyramids, cones, spheres and composites.
  - cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height
  - cone: $S = \pi rs + \pi r^2$ where r is radius and s is slant height
  - sphere: $S = 4\pi r^2$ where r is radius
- Calculate volumes, V, and capacities of standard three-dimensional objects in practical situations, including rectangular prisms, cylinders, pyramids, cones, spheres and composites.
  - prism: $V = Ah$ where A is base area and h is perpendicular height
  - cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height
  - pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height
  - cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height
  - sphere: $V = \frac{4}{3}\pi r^3$ where r is radius
- Solve practical problems involving shape and measurement.

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Prior learning
Length, area and volume

Videos (13):

2.2, 2.3 Perimeter and area of a sector 1
- Perimeter and area of a sector 2

2.4 Surface area of a prism
- Surface area of a cylinder
- Surface area of a pyramid
- Surface area of a cone and sphere

2.5 Volume and capacity units
- Volumes of prisms and cylinders
- Capacity of a cylinder
- Volume of a pyramid
- Volume of a sphere

2.6 Volumes of composite shapes

Exam question analysis Length, area and volume

Prior learning (1):

2.1 Length, area and volume

Skillsheets (3):

2.1 Pythagoras' theorem

2.4, 2.5 Solid shapes

2.6 Volumes of composite shapes

Worksheets (20):

2.1 Pythagoras' theorem time trial
- Pythagorean triads
- Applications of Pythagoras' theorem
- Pythagoras' problems

2.2 Length, area and volume conversions
- A page of circular shapes

2.3 Area ID • A page of circular shapes
- Composite areas • Areas of composite shapes
- Applications of area
- Measurement in the home

2.4 Nets of solids • A page of solid shapes
- Surface area of solids • Surface area

2.5 Length, area and volume conversions
- Estimating area and volume
- Volumes of water

2.6 A page of solid shapes • Volumes of solids

Cumulative exams General Maths Year 11 formula sheet

Puzzles (4):

2.1 Pythagorean two-step problems

2.4 Surface area riddle • Formula matching game

Chapter summary Length, area and volume find-a-word

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

annulus	apex	arc length	area
base	capacity	circumference	cone
cross-section	cylinder	face	hemisphere
hypotenuse	net	perimeter	prism
pyramid	Pythagoras' theorem	sector	sphere
surface area	three-dimensional (3D)	vertex/vertices	volume

2.1 Pythagoras' theorem problems

Pythagoras' theorem

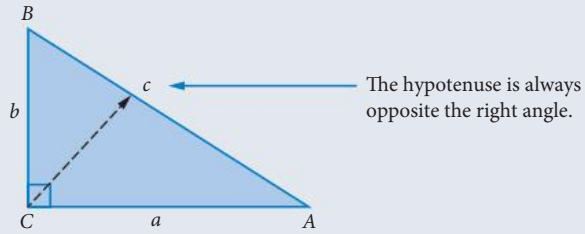
Pythagoras' theorem is a rule used to calculate the length of a side in a **right-angled triangle**, where 2 side lengths are known.

Pythagoras' theorem

In a right-angled triangle

$$c^2 = a^2 + b^2$$

where c is the length of the **hypotenuse** and a and b are the lengths of the other 2 sides. The hypotenuse is the longest side.



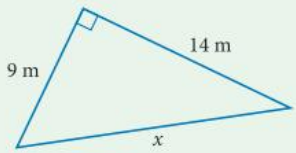
The naming convention for triangles is to use capital letters for corners (**vertices**).

Use the same lower case letters for the *opposite* sides.

WORKED EXAMPLE 1 Finding unknown lengths in right-angled triangles

Find the unknown length in each triangle.

a



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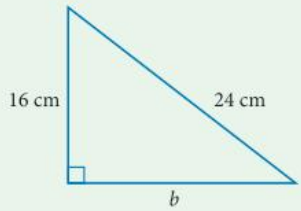


Steps	Working
a **1** Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
2 x is the hypotenuse.	$x^2 = 9^2 + 14^2$
3 Calculate the sum.	$= 277$
4 Find the square root.	$x = \sqrt{277}$
5 Use $\sqrt{\ }$ on your calculator.	$= 16.6433\ldots$
6 Round off.	≈ 16.6
7 Write the answer in a sentence.	The unknown length is about 16.6 m.
b **1** Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
2 Substitute in the values, where 24 is the hypotenuse.	$24^2 = 16^2 + b^2$
3 Work out the squares.	$576 = 256 + b^2$
4 Rearrange and evaluate.	$b^2 = 576 - 256$
	$= 320$
5 Find the square root.	$b = \sqrt{320}$
6 Use your calculator.	$= 17.8885\ldots$
7 Round off.	≈ 17.9
8 Write the answer.	The third side is about 17.9 cm.

Skillsheet
Pythagoras' theorem

Worksheets
Pythagoras' theorem time trial

Pythagorean triads

Applications of Pythagoras' theorem

Pythagoras' problems

Puzzle
Pythagorean two-step problems

Exam hack

Pythagorean triples (or triads) are integers that satisfy Pythagoras' theorem. They often appear in questions using the rule. Some of these are as follows:

3, 4, 5 $(3^2 + 4^2 = 5^2)$

5, 12, 13 $(5^2 + 12^2 = 13^2)$

8, 15, 17 $(64 + 225 = 289)$

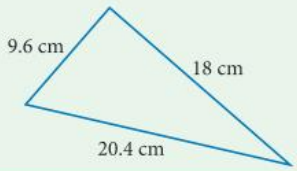
7, 24, 25 $(49 + 576 = 625)$

Multiples such as 6, 8, 10; 15, 20, 25; 10, 24, 25 are also Pythagorean triads.

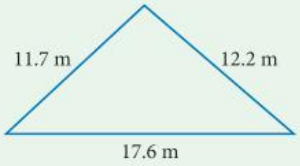
WORKED EXAMPLE 2 Identifying right-angled triangles

Determine if each triangle is a right-angled triangle.

a



b



Steps	Working
a 1 Does Pythagoras' theorem work?	Does $c^2 = a^2 + b^2$?
2 Find the square of the longest side.	$20.4^2 = 416.16$
3 Find the sum of the squares of the shorter sides.	$9.6^2 + 18^2 = 92.16 + 324$ $= 416.16$
4 Compare the results.	$20.4^2 = 9.6^2 + 18^2$
5 Write the answer.	This satisfies Pythagoras' theorem, so it is a right-angled triangle.
b 1 Find the square of the longest side.	$17.6^2 = 309.76$
2 Find the sum of the squares of the shorter sides.	$11.7^2 + 12.2^2 = 136.89 + 148.84$ $= 285.73$
3 Compare the results.	$17.6^2 \neq 11.7^2 + 12.2^2$
4 Write the answer.	The triangle is not a right-angled triangle.

Pythagoras' theorem in two dimensions

Pythagoras' theorem can be used to solve many practical problems.

Identify the right-angled triangle and the hypotenuse first in a practical problem. Then, write a variable for the unknown side.

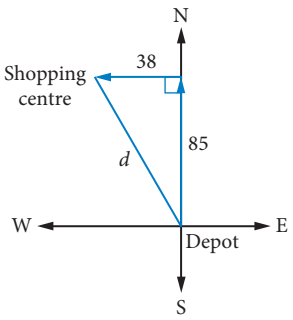
Exam hack

1. Use or draw a diagram with all the information about the triangle.
2. Show the right angle, which will help you identify what you need to find.

Your diagram does not need to show all the details described in the problem.

WORKED EXAMPLE 3 Applying Pythagoras' theorem

A truck leaves a country depot and travels directly north for a distance of 85 km. The driver then turns due west and travels for another 38 km to a shopping centre. How far is the shopping centre from the depot?

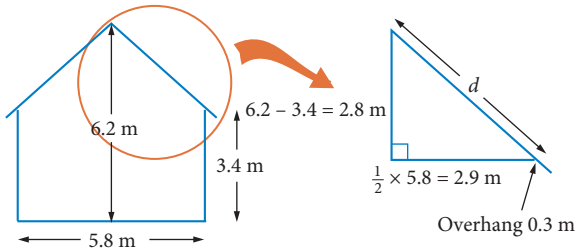
Steps	Working
1 Draw a diagram to represent the information. Draw the journey north first, then draw the journey due west. Join the positions of the depot and shopping centre with a straight line and label it d.	
2 Apply Pythagoras' theorem.	$d^2 = 85^2 + 38^2$
3 Evaluate.	$= 8669$
4 Take square roots of both sides.	$d = \sqrt{8669}$
5 Evaluate.	$= 93.107\ldots$
6 Round off and state the result.	The shopping centre is about 93.1 km from the depot.

WORKED EXAMPLE 4 Pythagoras' theorem in practical situations

The front of a shed is 5.8 m wide. The sides are 3.4 m high and the highest point of the roof is 6.2 m above its base. The roof of the shed is made from sheets of iron that overhang by 30 cm on both sides.

How long are the sheets of iron?

Eric Backman/Shutterstock.com

Steps	Working
1 Draw a diagram and show the dimensions. Change the overhang to the same units as the other dimensions (m).	
2 Apply Pythagoras' theorem to the triangle.	$d^2 = 2.9^2 + 2.8^2$
3 Evaluate.	$= 8.41 + 7.84$ $= 16.25$
4 Find the square root.	$d = 4.0311\ldots$
5 The overhang needs to be added.	sheet length $= 4.0311\ldots + 0.3$ m $= 4.3311\ldots$
6 Round off appropriately and state the result.	The roof sheets are about 4.331 m long.

Exam hack

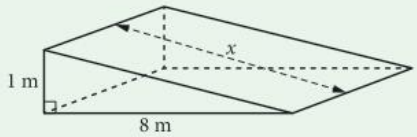
For construction, you usually round to the nearest mm.

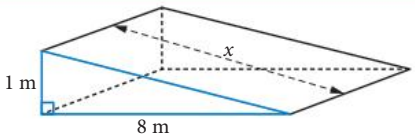
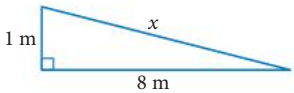
Pythagoras' theorem in three dimensions

In **three-dimensional** (solid) shapes, identify the face that has the right-angled triangle first.

WORKED EXAMPLE 5 Pythagoras' theorem in 3D 1

A new ramp is being built at the front of a building to improve accessibility. What is its slant length, x, correct to 2 decimal places?



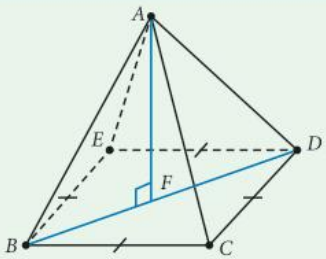
Steps	Working
1 Identify the right-angled triangle that includes the unknown.	
2 Draw the triangle, labelling the known and unknown side lengths.	
3 Use Pythagoras' theorem to find x.	$x^2 = 1^2 + 8^2$ $x^2 = 65$ $x = \sqrt{65}$ $x = 8.0622...$ m
4 Write your answer in the required units and round to the required degree of accuracy.	The slant length of the ramp is 8.06 m.

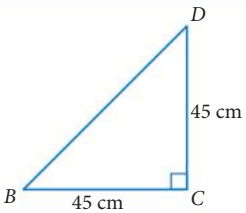
WORKED EXAMPLE 6 Pythagoras' theorem in 3D 2

The square pyramid shown has a base length of 45 cm and a slant edge of 60 cm.

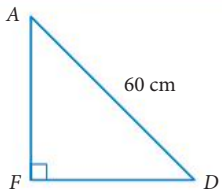
a Find the length BD, correct to 2 decimal places.

b Find the perpendicular height AF, correct to one decimal place.



Steps	Working
a 1 Identify the base of the pyramid as having the right-angled triangle containing BD. Draw the triangle, labelling the known and unknown lengths.	
2 Use Pythagoras' theorem to find the unknown length BD.	$BD^2 = 45^2 + 45^2$ $BD^2 = 4050$ $BD = \sqrt{4050} = 63.639\,61...$
3 Write your answer in the required units and round to the required degree of accuracy.	The length of BD is 63.64 cm.

b 1 Draw the relevant right-angled triangle containing AF, labelling the known and unknown lengths.



2 Find FD which is half of BD.

Use the unrounded value in your calculator for BD from part **a**.

$$FD = \frac{BD}{2} = \frac{63.639\ldots}{2} = 31.819\ldots$$

3 Use Pythagoras' theorem to find AF. Use the calculator value.

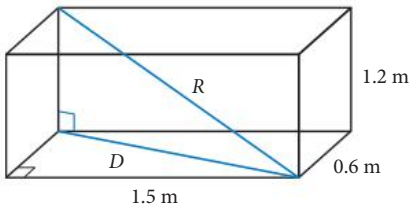
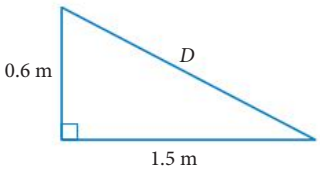
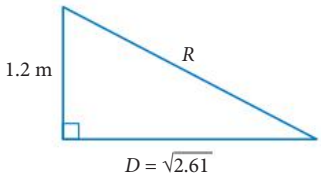
$$60^2 = AF^2 + (31.819\ldots)^2$$
$$AF^2 = 60^2 - (31.819\ldots)^2 = 2587.5$$
$$AF = \sqrt{2587.5} = 50.867\,47\ldots$$

4 Write your answer.

The length of AF is 50.9 cm.

WORKED EXAMPLE 7 Pythagoras' theorem in 3D 3

A hardware supplier ships specialty metal rods to building sites. The largest rectangular crate available has the dimensions 1.5 m × 0.6 m × 1.2 m. What is the maximum length of rod that can be shipped in this crate?

Steps	Working
1 Draw a diagram of the crate with the rod, R, in it, labelling all known dimensions. Draw a right-angled triangle containing the rod, R. This is marked in blue, where the height is 1.2 m, and the length of side D is unknown.	
2 Identify a second right-angled triangle containing D. This is in the base of the crate. Label known and unknown lengths.	
3 Use Pythagoras' theorem to find D. Leave the value for D as a square root as it will be squared again in step 5.	$D^2 = 0.6^2 + 1.5^2$ $D^2 = 2.61$ $D = \sqrt{2.61}$
4 Draw the right-angled triangle containing the rod, R.	
5 Use Pythagoras' theorem to find R. Use the value of D^2 from step 3.	$R^2 = D^2 + 1.2^2$ $R^2 = 2.61 + 1.2^2$ $R^2 = 4.05$ $R = \sqrt{4.05}$ $= 2.0124\ldots$ m
6 Answer the question.	The maximum length of the specialty metal rod that can fit in the crate is 2.012 m long.

Exam hack

Think ahead in your working. Leaving D in square root form will make later work easier and more accurate.

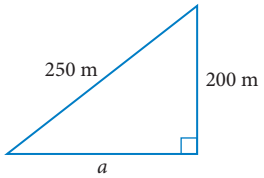
EXERCISE 2.1 Pythagoras' theorem problems

ANSWER p. 396

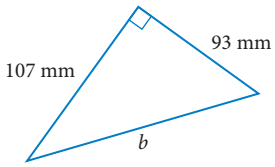
Mastery

1 WORKED EXAMPLE 1 Find the value of the unknown side in each triangle.

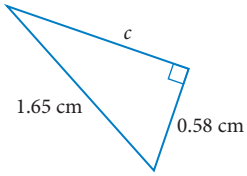
a



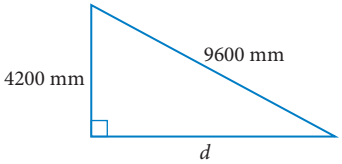
b



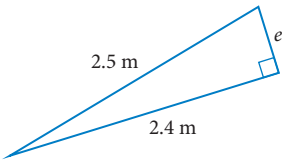
c



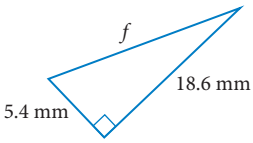
d



e

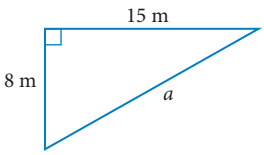


f

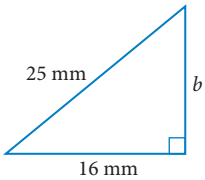


2 Use Pythagoras' theorem to find the unknown side in each triangle.

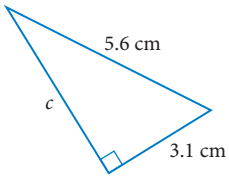
a



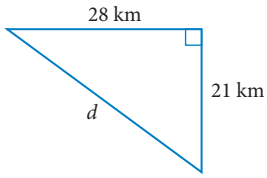
b



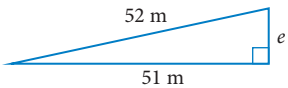
c



d



e



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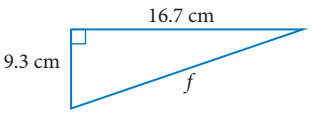
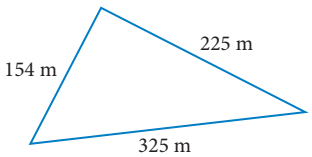
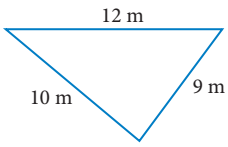


3 WORKED EXAMPLE 2 Determine whether each triangle is a right-angled triangle.

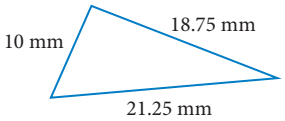
a



b

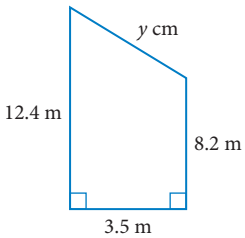


c

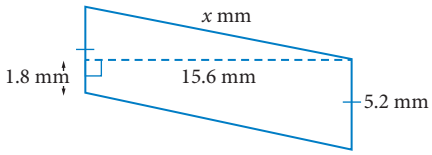


4 Calculate the unknown side in each figure.

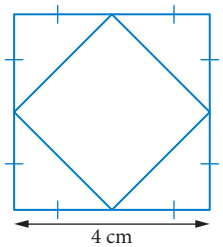
a



b

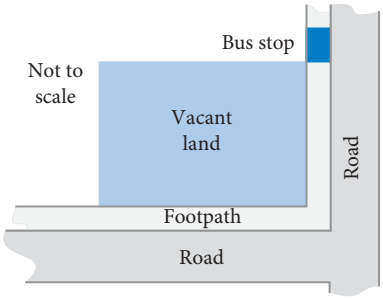


5 Use Pythagoras' theorem to determine the side length of the smaller square and hence calculate the area of the smaller square.



6 WORKED EXAMPLE 3 Melinda leaves home on her morning jog. She travels for 850 m due north from her home and then jogs due east for 620 m, before turning south and continuing for 1.2 km. How far, to the nearest metre, is Melinda from her home?

7 WORKED EXAMPLE 4 A vacant corner block of land measures 32 m by 22 m. Instead of going around the footpath, many people walk diagonally across the unfenced block to reach a bus stop. How much further will they need to walk when the block is fenced?

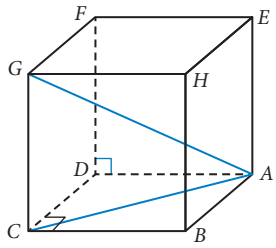


8 WORKED EXAMPLE 5 Aaron wants to use a cylindrical tin with a pop lid as his pencil case. How tall must the tin be if it is to fit Aaron's longest art pencil, which is 17.5 cm long, given the tin's base diameter is 6.75 cm?

9 WORKED EXAMPLES 6, 7 This cube has a side length of 6 cm. Find the length of

a the base diagonal AC, to one decimal place

b the diagonal AG, to 2 decimal places.



Exam practice

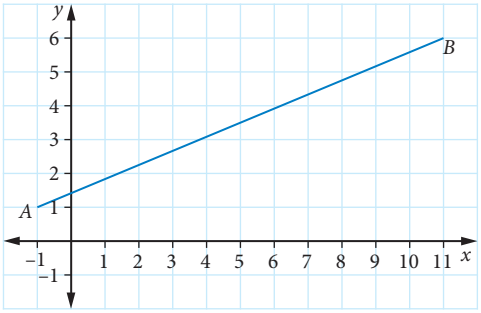
10 The length of AB is

A 5

B 10

C 11

D 13



11 (1 mark) A wall frame 3500 mm long and 2100 mm high is to have a diagonal brace fitted. How long must the brace be?

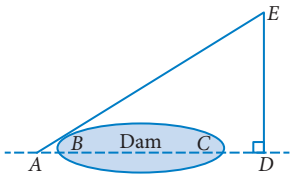
12 (2 marks) CF A farmer wants to know the length of a small dam on their property. They take the following measurements:

$AB = 15$ m

$CD = 35$ m

$AE = 395$ m

$DE = 224$ m

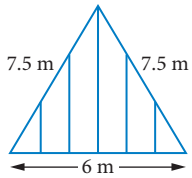


What is the length of the dam from B to C?

13 (3 marks) CF A rectangular room 8 m long, 4 m wide and 3 m high is to be used for a home theatre. The screen is set up at the centre of a wall that is 4 m wide. The projector is to be mounted on the ceiling facing the screen.

If the position of the projector must be 7 m in a straight line from the centre of the screen, determine how far back horizontally from the screen it must be installed.

14 (3 marks) CU Amira is building a roof for a new garage. The roof has a gable end in the shape of an isosceles triangle, with a base of 6 m and sloping sides of 7.5 m. She decides to place 5 evenly-spaced vertical pieces of timber as supports for the gable, as shown. Determine the total length of support timber needed.



15 (3 marks) CU The Louvre pyramid in Paris is in the shape of a square pyramid. The square base has a diagonal length of 48.01 m and the pyramid reaches a height of 21.6 m. What is its slant edge length?

Alexander Kagan/Unsplash.com

2.2 Perimeter and area

Units of length and area

The common units used for measuring length are kilometres (km), metres (m), centimetres (cm) and millimetres (mm).

Common units for area are square kilometres (km^2), hectares (ha), square metres (m^2), square centimetres (cm^2) and square millimetres (mm^2).

A **square metre** is the amount of space in a square of length 1 m.

A **square kilometre** is the amount of space in a square of length 1 km.

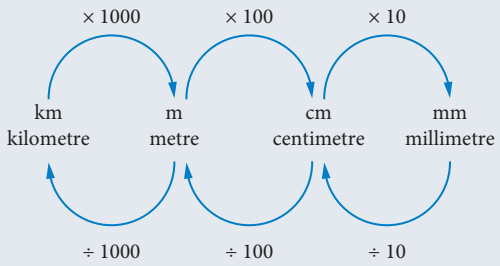
A **hectare** is the amount of space in a square of length 100 m, so it is equal to $100^2 = 10\,000\ m^2$.

Worksheet
Length, area and volume conversions

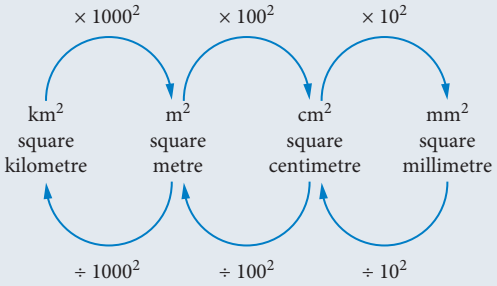
Metric units of length and area

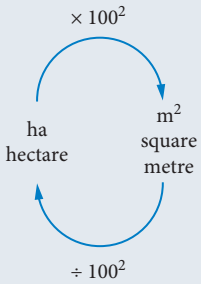
- To convert from a larger unit to a smaller unit, use multiplication (×).
- To convert from a smaller unit to a larger unit, use division (÷).

Length units



Area units (square units)





WORKED EXAMPLE 8 Converting units of length and area

Convert each unit.

a 4.2 m to cm

b 250 000 mm to km

c 120 000 cm^2 to m^2

d 2.34 km^2 to ha

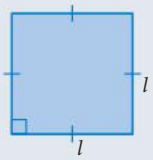
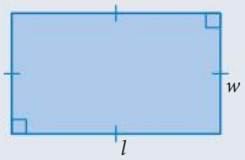
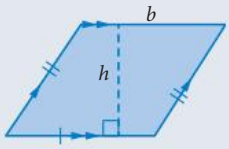
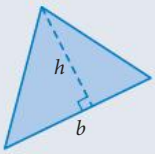
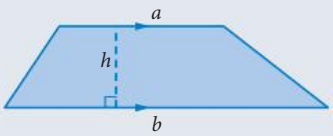
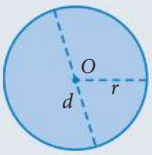
Steps	Working
a Identify the conversion. $\times 100$ m metre → cm centimetre	$4.2\,m = 4.2 \times 100\,cm$ $= 420\,cm$
b Identify the conversion. km kilometre ← m metre ← cm centimetre ← mm millimetre $\div 1000$ $\div 100$ $\div 10$	$250\,000\,mm = 250\,000 \div 10 \div 100 \div 1000\,km$ $= 0.25\,km$
c Identify the conversion. m^2 square metre ← cm^2 square centimetre $\div 100^2$	$120\,000\,cm^2 = 120\,000 \div 100^2\,m^2$ $= 12\,m^2$
d Identify the conversion. $\times 1000^2$ km^2 square kilometre → m^2 square metre $\times 100^2$ ha hectare → m^2 square metre	$1\,km^2 = 1000^2\,m^2 = 1\,000\,000\,m^2$ $1\,ha = 100^2\,m^2 = 10\,000\,m^2$ So, $1\,km^2 = \dfrac{1\,000\,000}{10\,000} = 100\,ha$. $2.34\,km^2 = 2.34 \times 100\,ha$ $= 234\,ha$

Perimeter and area of standard shapes

The **perimeter** of a shape is the total length around its outside edge.

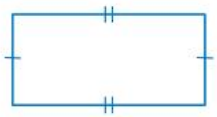
The **area** of a shape is the total amount of surface inside the shape.

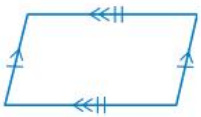
Perimeter and area formulas

Shape	Perimeter	Area
Square 	$P = 4l$	$A = l^2$
Rectangle 	$P = 2l + 2w$ $= 2(l + w)$	$A = lw$
Parallelogram 		$A = bh$
Triangle 		$A = \frac{1}{2}bh$
Trapezium 		$A = \frac{1}{2}(a + b)h$
Circle 	$C = 2\pi r$ $C = \pi d$ The perimeter of a circle is called the **circumference**.	$A = \pi r^2$

Exam hack

When you draw or use diagrams, show equal side lengths with dashes. Show parallel sides using arrows.





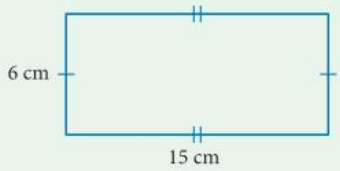
WORKED EXAMPLE 9 Finding perimeters and areas

For each shape, **calculate**

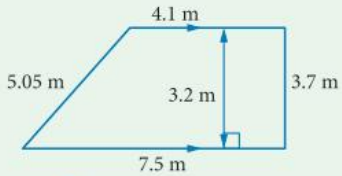
i its perimeter

ii its area

a

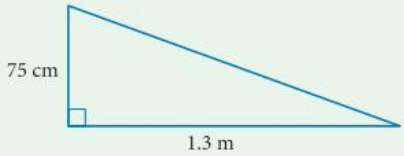


b



c

d



Steps	Working
a i 1 Identify the shape and state the formula.	rectangle
2 Calculate the perimeter.	$P = 2(l + w)$ $= 2(6 + 15)$ $= 42\text{ cm}$
ii 1 State the area formula.	$A = lw$
2 Calculate the area.	$= 15 \times 6$ $= 90\text{ cm}^2$
b i 1 Identify the shape.	trapezium
2 Calculate the perimeter.	$P = 5.05 + 7.5 + 3.7 + 4.1$ $= 20.35\text{ m}$
ii 1 State the area formula.	$A = \frac{1}{2}(a + b)h$
2 Calculate the area.	$= \frac{1}{2}(4.1 + 7.5) \times 3.2$ $= 18.56\text{ m}^2$
c i 1 Identify the shape and state the formula.	circle
2 Calculate the perimeter.	$C = 2\pi r$ $= 2\pi \times 25$ $= 157.08\text{ mm}$
ii 1 State the area formula.	$A = \pi r^2$
2 Calculate the area.	$= \pi \times (25)^2$ $= 1963.5\text{ mm}^2$
d i 1 Make all the units the same – convert to m.	$75\text{ cm} = 0.75\text{ m}$
2 Find the unknown side length using Pythagoras' theorem.	$c^2 = 0.75^2 + 1.3^2$ $= 2.2525$ $c = \sqrt{2.2525}$ $c \approx 1.5\text{ m}$
3 Calculate the perimeter.	$P = 0.75 + 1.3 + 1.5$ $= 3.55\text{ m}$
ii 1 State the area formula.	$A = \frac{1}{2}bh$
2 Calculate the area.	$= \frac{1}{2} \times 1.3 \times 0.75$ $= 0.49\text{ m}^2$

Investigation Paper sizes

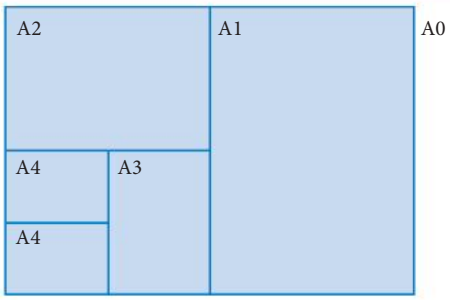
Paper comes in sizes A0, A1, A2, A3, A4 and so on. All sizes are generated from the A0 size, which is a rectangular shape with an area of $1\,\text{m}^2$.

The sides of the rectangle are in the ratio $\sqrt{2}:1$.

The sizes A1, A2 and so on are formed by halving the longer side.

1 Determine the dimensions of a sheet of A0 paper.

2 Now use the dimensions that you have just worked out to calculate the dimensions of a sheet of A4 paper. Measure a sheet of A4 paper and compare its dimensions with those you have calculated.



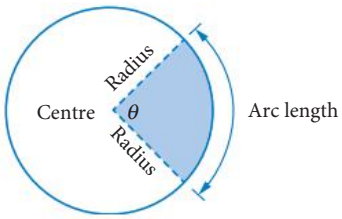
Arcs and sectors

Videos
Perimeter and area of a sector 1
Perimeter and area of a sector 2

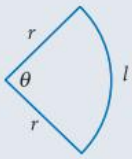
Worksheet
A page of circular shapes

An arc is the length of a section of the circumference of a circle. It is between 2 radii, r. You can calculate the **arc length** using the angle between them.

A **sector** is a fraction of a circle, $\frac{\theta}{360^\circ}$, formed by 2 radii and the arc between them.



Arcs and sectors



Arc length

$$l = \frac{\theta}{180}\pi r$$

Sector area

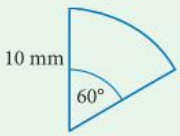
$$A = \frac{\theta}{360}\pi r^2$$

WORKED EXAMPLE 10 Arc length, perimeter and area of a sector

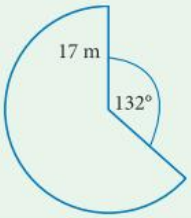
Calculate each measurement correct to 2 decimal places for each sector.

i arc length **ii** perimeter **iii** area

a



b



Steps		Working
a i 1	State the known values.	$\theta = 60^\circ$, $r = 10$
2	Use the arc length formula to find the arc length.	$l = \frac{\theta}{180}\pi r$ $= \frac{60}{180}\pi \times 10$ $= 10.471\,97\ldots$
	Round to the required units.	arc length $= 10.47$ mm
ii 1	Add the 2 radii and the arc length to find the perimeter.	$P = 2r + l$ $= 2 \times 10 + 10.471\,97\ldots$ $= 30.471\,97\ldots$
2	Round to the required units.	sector perimeter $= 30.47$ mm

iii 1	State the known values.	$\theta = 60^\circ, r = 10$
2	Use the sector area formula to find the area.	$A = \frac{\theta}{360}\pi r^2$ $= \frac{60}{360}\pi \times 10^2$ $= 52.359\,87\ldots$
3	Round to the required units.	sector area $= 52.36\,\text{mm}^2$
b i 1	State the known values.	$r = 17$
	Calculate the sector angle.	$\theta = 360^\circ - 132^\circ$ $= 228^\circ$
2	Use the arc length formula to find the arc length.	$l = \frac{\theta}{180}\pi r$ $= \frac{228}{180}\pi \times 17$ $= 67.6489\ldots$
3	Round to the required units.	arc length $= 67.65\,\text{m}$
ii 1	Add the 2 radii and the arc length to find the perimeter.	$P = 2r + l$ $= 2 \times 17 + 67.6489\ldots$ $= 101.64896\ldots$
2	Round to the required units.	sector perimeter $= 101.65\,\text{m}$
iii 1	State the known values.	$\theta = 228^\circ, r = 17$
2	Use the sector area formula to find the area.	$A = \frac{\theta}{360}\pi r^2$ $= \frac{228}{360}\pi \times 17^2$ $= 575.016\,17\ldots$
3	Round to the required units.	sector area $= 575.02\,\text{m}^2$

EXERCISE 2.2 Perimeter and area

ANSWER p. 396

Recap

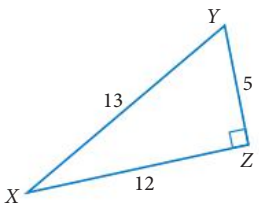
1 Which of the following is correct for $\triangle XYZ$?

A $x^2 = y^2 + z^2$

B $13^2 = y^2 + x^2$

C $y^2 = x^2 + z^2$

D $12^2 = 5^2 + z^2$



2 The side measurements of 4 different triangles are given below. Which one is a right-angled triangle?

A 4, 5, 12 **B** 10, 10, 15 **C** 5, 12, 13 **D** 9, 12, 18

Mastery

3 WORKED EXAMPLE 8 Change each measurement to the unit indicated.

a $2500\,\text{cm} \rightarrow \text{m}$ **b** $0.000\,24\,\text{m}^2 \rightarrow \text{mm}^2$ **c** $5800\,\text{m}^2 \rightarrow \text{km}^2$

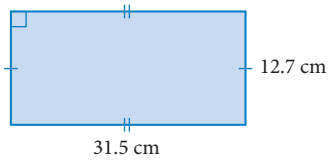
d $2.1\,\text{km} \rightarrow \text{mm}$ **e** $45\,721\,\text{mm} \rightarrow \text{m}$ **f** $0.0021\,\text{ha} \rightarrow \text{m}^2$

g $0.036\,\text{km}^2 \rightarrow \text{m}^2$ **h** $56.8\,\text{mm}^2 \rightarrow \text{cm}^2$ **i** $6\,720\,000\,\text{mm}^2 \rightarrow \text{m}^2$

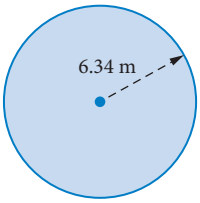
4 WORKED EXAMPLE 9 For each shape, calculate

i its perimeter **ii** its area

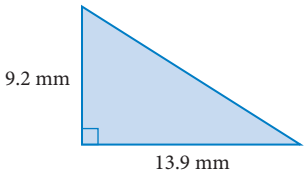
a



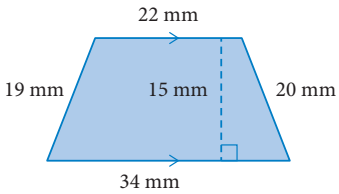
b



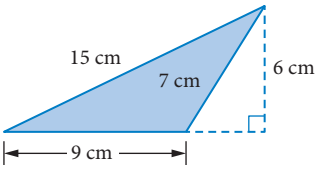
c



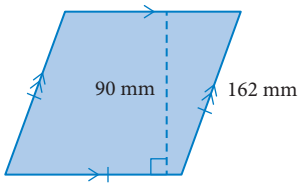
d



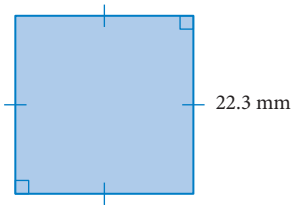
e



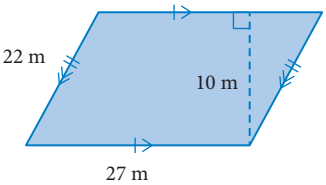
f



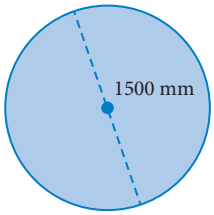
g



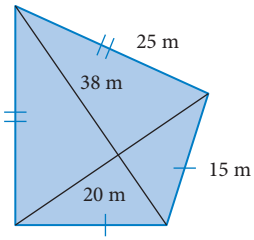
h



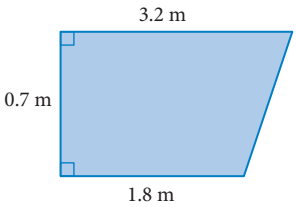
i



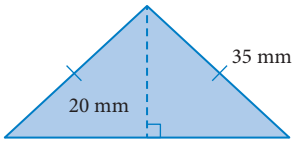
j



k



l



5 Find the area of each object described.

a Triangular scarf with base 1.5 m and height 0.8 m

b Circular lid of radius 25 mm

c Triangular wall tile with base 25 cm and height 12 cm

d Trapezoidal table top with parallel sides 1.2 m and 1.8 m and height 80 cm

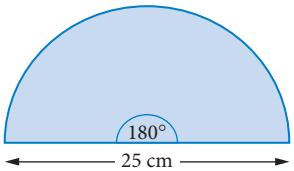
e Compact disc of diameter 12 cm

f Circular tablecloth of radius 1.5 m

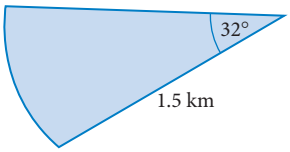
6 WORKED EXAMPLE 10 For each of the following sectors, calculate to one decimal place, the

i arc length **ii** perimeter **iii** area

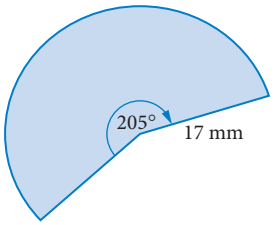
a



b



c



7 A circular plate has an area of 707 cm^2. What is its radius?

8 A square piece of metal has an area of 110.25 cm^2. What are its dimensions?

9 A triangle has an area of 208.32 cm^2. If the base of the triangle is 9.3 cm long, what is its perpendicular height?

10 The floor of a room measures 3540 mm by 5650 mm. How many square metres of carpet are required to cover the floor? (Give your answer correct to one decimal place.)

Exam practice

11 To convert 320.8 mm^2 to cm^2, you need to

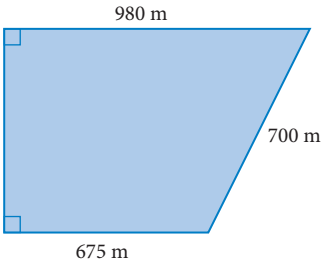
A multiply by 100

B move the decimal point 2 places to the right

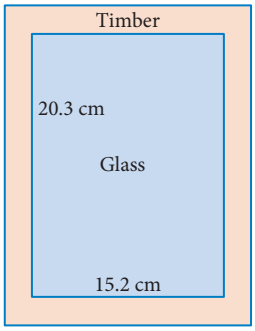
C divide by 100

D move the decimal point one place to the left

12 (3 marks) CF A farmer has a large field in the shape of a trapezium, as shown. The farmer is preparing the field for a crop of maize and requires 43 kg of fertiliser per ha. How much fertiliser must the farmer buy?



13 (3 marks) CU A photo frame consists of a rectangular piece of glass surrounded by a timber edge. The glass is 15.2 cm by 20.3 cm and the frame is 1.8 cm wide. Calculate the area of the timber frame.



14 (5 marks) CU A fence is to be erected around a block of land that has a slanting front boundary and parallel side boundaries of 27 m and 42 m. The rear boundary is 20 m long and is at right angles to the side boundaries. The posts are to be a maximum of 2 m apart, there are 2 railings, and 100 mm palings are to be spaced 20 mm apart. The posts cost \$12.50 each, the railings cost \$5.40/m and the palings cost \$1.40 each. A 2.5 m gate costing \$130 is to be placed at the front.
Determine the cost of materials to complete the job.

Videos
Perimeter and area of a sector 1

Perimeter and area of a sector 2

Worksheets
Area ID

A page of circular shapes

Composite areas

Areas of composite shapes

Applications of area

Measurement in the home

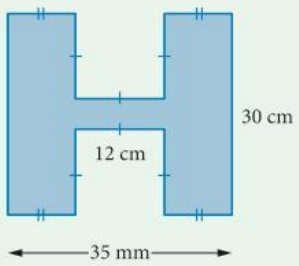
2.3 Perimeter and area of composite shapes

Shapes that are made by combining 2 or more shapes together are called **composite shapes**.

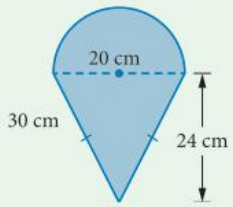
WORKED EXAMPLE 11 Perimeter and area of composite shapes 1

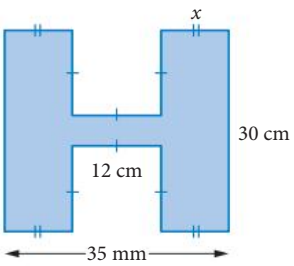
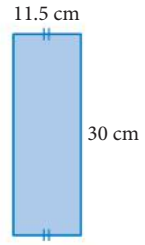
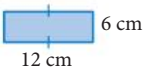
For each shape, **calculate** correct to 2 decimal places where necessary

a **i** its perimeter



b **ii** its area

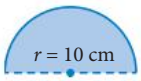


Steps	Working
a i 1 Identify the shapes.	The shape consists of 2 equivalent rectangles joined by a smaller rectangle.
2 Find the perimeter.	P = sum of the sides $= 2 \times 35 + 2 \times 30 + 4 \times 12$ $= 178$
Alternatively: **1** Find all unknown lengths.	 $x = (35 - 12) \div 2 = 11.5$
2 Find the perimeter.	$P = 4 \times 11.5 + 2 \times 30 + 6 \times 12$ $= 178$
3 Write the answer in the required units.	The perimeter of the shape is 178 cm.
ii 1 Find the area of the larger equivalent rectangles.	$A = lw$ $= 30 \times 11.5$ $= 345\text{ cm}^2$ $2A = 690\text{ cm}^2$ 
2 Find the area of the smaller rectangle.	$A = 12 \times 6$ $= 72\text{ cm}^2$ 
3 Calculate the combined shape area.	area of shape $= 690 + 72$ $= 762\text{ cm}^2$
4 Write the answer in the required units.	The area of the shape is 762 cm^2.

b i 1 Identify the shapes.

The shape is a semicircle joined to a triangle.

2 Find the perimeter of the semicircle.



$P = \frac{1}{2} 2\pi r$

$= \pi \times 10$

$= 31.41$ cm

3 Find the total perimeter.

total perimeter $= P_{\text{semicircle}}$ + sides

$= 31.41 + 2 \times 30$

$= 91.41$

4 Write the answer in the required units.

The perimeter of the shape is 91.41 cm.

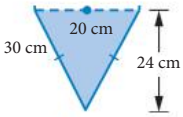
ii 1 Find the area of the semicircle.

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi (10)^2$

$= 157.0796\ldots$

2 Find the area of the triangle.



$A = \frac{1}{2} bh$

$= \frac{1}{2} \times 20 \times 24$

$= 240$

3 Calculate the area of the combined shape.

area of shape $= 157.0796\ldots + 240$

$= 397.0796\ldots$

4 Round off to the required level of accuracy and write the answer in the required units.

The area of the shape is about 397.08 cm^2.

Annulus

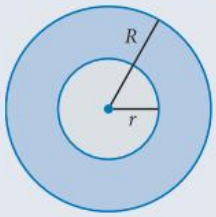
An **annulus** is the area between 2 circles with the same centre.

perimeter = circumference of outer circle + circumference of inner circle

$P = 2\pi R + 2\pi r$

annulus area = area of outer circle − area of inner circle

$A = \pi R^2 - \pi r^2$

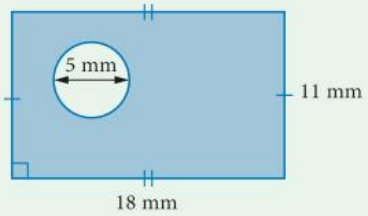


Some composite shapes are formed by removing a shape from another shape.

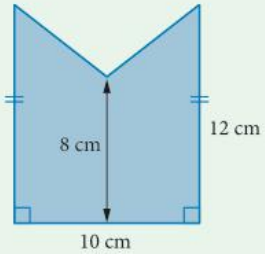
WORKED EXAMPLE 12 Perimeter and area of composite shapes 2

Find the shaded area of each composite shape. State your answers to one decimal place, where required.

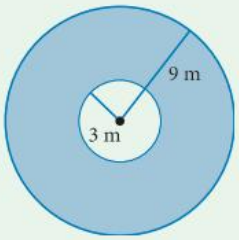
a

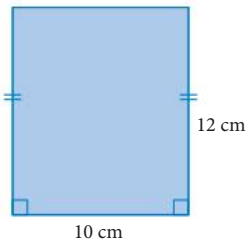
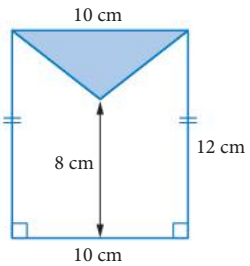


b



c



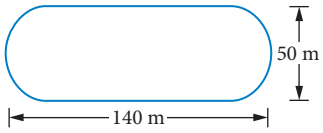
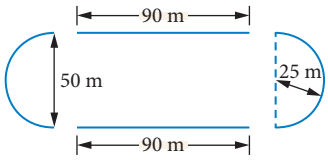
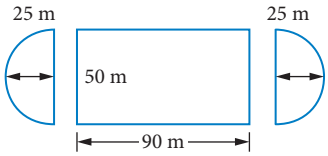
Steps	Working
a 1 Identify the shapes.	The shaded region is a rectangle with a circular unshaded region.
2 Find the area of the rectangle.	$A = lw$ $= 18 \times 11$ $= 198\,\text{mm}^2$
3 Find the area of the circle.	$A = \pi r^2$ $= \pi(2.5)^2$ $= 19.634\,95\ldots$
4 Find the area of the composite shape by subtracting the empty space shape.	area of shape $= 198 - 19.634\,95\ldots$ $= 178.365\,04\ldots$
5 Round off to the required level of accuracy and write the answer in the required units.	The area of the composite shape is $178.4\,\text{mm}^2$.
b 1 Identify the shapes. *Note:* This could also be solved by adding the area of 2 trapeziums.	The shaded region is a rectangle with a triangular unshaded region.
2 Find the area of the rectangle.	$A = lw$ $= 12 \times 10$ $= 120\,\text{cm}^2$ 
3 Find the area of the triangle.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 4$ $= 20\,\text{cm}^2$ 
4 Find the area of the composite shape by subtracting the area of the triangle.	area of shape $= 120 - 20$ $= 100$
5 Write the answer in the required units.	The area of the composite shape is $100\,\text{cm}^2$.
c 1 Identify the shape.	The shape is an annulus.
2 Find the area of the annulus.	$A = \pi R^2 - \pi r^2$ $= \pi(9)^2 - \pi(3)^2$ $= 226.194\,67\ldots$
3 Round off to the required level of accuracy and write the answer in the required units.	The area of the annulus is $226.2\,\text{m}^2$.

WORKED EXAMPLE 13 Applying perimeter and area formulas

A local council is renovating a sports ground which is 140 m long and 50 m wide and has semicircular ends.

a The council is enclosing the grounds with fencing all the way around with 4 gates, each 2 m wide. The gates cost \$200 each and the fencing costs \$55/m. Find the cost of fencing the sports ground.

b The grounds require new grass which costs \$110 per square metre. Find the cost of the new grass.

Steps	Working
a 1 Sketch the shape, include all important information.	
2 Identify each perimeter section.	
3 Calculate the circular length at each end. *Hint:* As each end is a semicircle, added together they form a complete circle.	$C = 2\pi r$ $= 2\pi \times 25$ $= 157.079\,63\ldots$
4 Calculate the perimeter – this represents the whole length around the grounds.	$P = 2 \times 90 + 157.079\,63\ldots$ $= 337.079\,63\ldots$
5 Remove the length taken up by the 4 gates, each 2 m in size.	fence length $= 337.079\,63\ldots - 4 \times 2$ $= 329.079\,63\ldots$
6 Find the total cost.	total cost = fence cost + gate cost $= 55 \times 329.079\,63\ldots + 4 \times 200$ $= 18\,099.38 + 800$ $= 18\,899.38$
7 Round off and write the answer.	The cost of fencing the grounds is about \$18 899.38.
b 1 Identify each area section.	
2 Calculate the area of the 2 semicircles.	$A = \pi r^2$ $= \pi(25)^2$ $= 1963.495\,40\ldots$
3 Calculate the area of the rectangle.	$A = lw$ $= 90 \times 50$ $= 4500$
4 Find the total area.	area of the grounds $= 4500 + 1963.495\,40\ldots$ $= 6463.495\,408\ldots$
5 Calculate the total cost.	grass cost $= 6463.495\,408\ldots \times 110$ $= 710\,984.4949$
6 Round off and write the answer.	The cost of the grass is \$710 984.50.

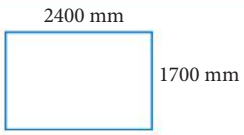
You may use perimeter and area to work out paint or tile quantities.

For practical jobs, round up to the nearest sensible amount, like a half-tile or full tin of paint. You also need to allow for wastage of about 5%. There are always mistakes or accidents.

WORKED EXAMPLE 14 Using area in practical jobs

Calculate the number of 200 mm × 200 mm tiles with a 5 mm grout joint needed to tile the floor of a kitchen measuring 2400 mm by 1700 mm.
Allow for 5% wastage.

Bernard Hermant/Unsplash.com

Steps	Working
1 Identify the key measurements and draw a floor plan.	
2 Work out the tile size, including the 5 mm of grout.	tile width = 200 + 5 = 205 mm
3 Work out the number of tiles required for the length of the kitchen.	number for 2400 mm side = 2400 ÷ 205 = 11.7073…
4 Round up to the nearest $\frac{1}{2}$ or full tile.	≈ 12 tiles
5 Work out the number of tiles required for the width of the kitchen.	number for 1700 mm side = 1700 ÷ 205 = 8.2926…
6 Round up to the nearest $\frac{1}{2}$ or full tile.	≈ 8.5 tiles
7 Work out the total number of tiles.	number of tiles = 12 × 8.5 = 102 tiles
8 Add 5% wastage.	total with wastage = 105% of 102 tiles = 1.05 × 102 = 107.1 tiles
9 Round off and write the answer.	107 tiles are needed for the kitchen.

EXERCISE 2.3 Perimeter and area of composite shapes

ANSWER p. 397

2.3

Recap

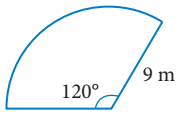
1 What is the area of the sector?

A $84.8\,m^2$

B $18.8\,m^2$

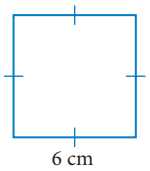
C $9.4\,m^2$

D $36.8\,m^2$

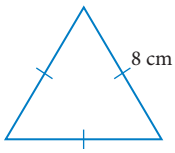


2 Which shape has the largest perimeter?

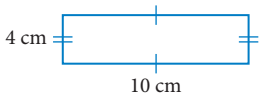
A



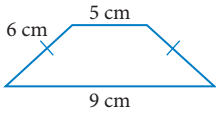
B



C



D

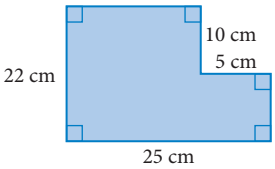


Mastery

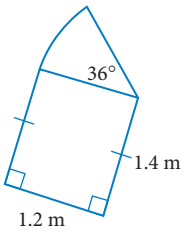
3 WORKED EXAMPLE 11 For each shape, calculate

i its perimeter **ii** its area

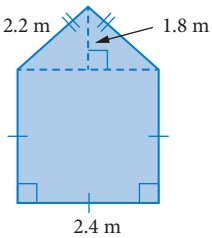
a



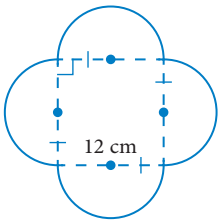
b



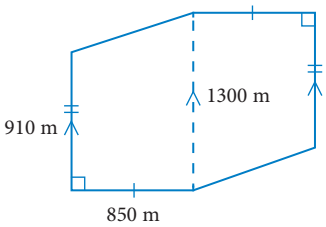
c



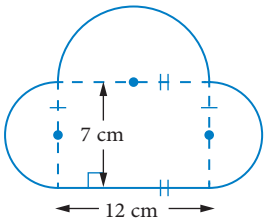
d



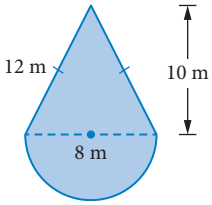
e



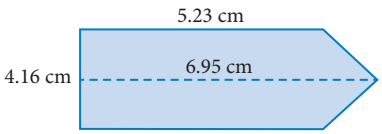
f



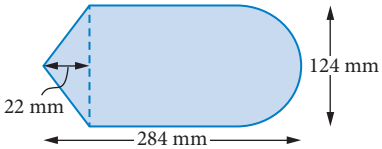
g



h

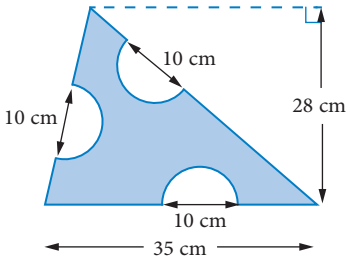


i

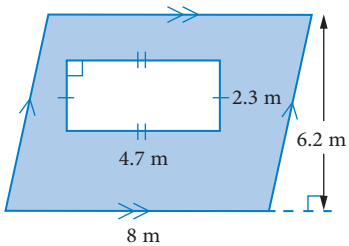


4 WORKED EXAMPLE 12 For each shape, calculate the shaded area.

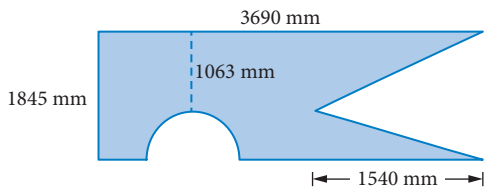
a



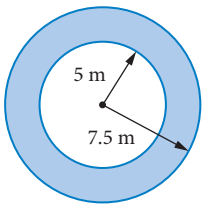
b



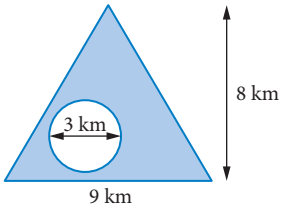
c



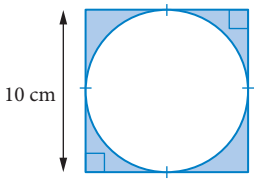
d



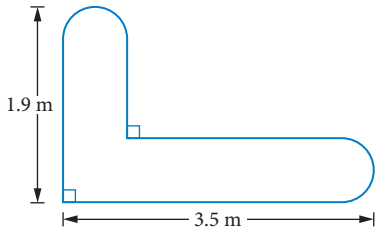
e



f



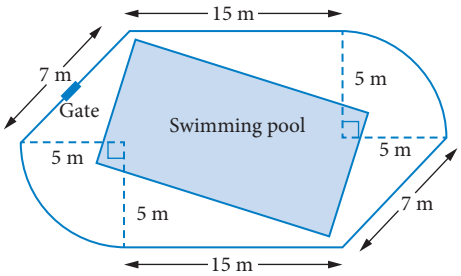
5 WORKED EXAMPLE 13 The L-shaped kitchen bench with semicircular ends shown here has been made from chipboard veneer. The bench is 600 mm wide. It must have an edging strip of veneer attached all the way along the edge to finish the job. What length of edging veneer is required?



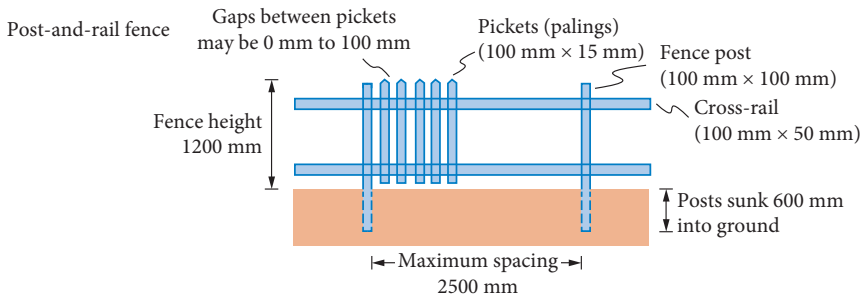
6 A swimming pool is set in a wooden deck, as shown. A pool fence is to be erected around the border, with a 1200 mm wide gate. The cost of materials for a 1200 mm high fence is \$42/m, and the gate costs \$120. Installation costs are \$10/m for the fence and gate.

a Calculate the cost of materials to complete the job.

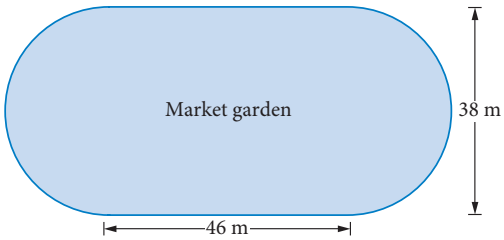
b Find the cost of installing the fence and gate.



7 The market garden plot shown below is to be enclosed with the wooden post-and-rail fence, as illustrated.



Market garden plot



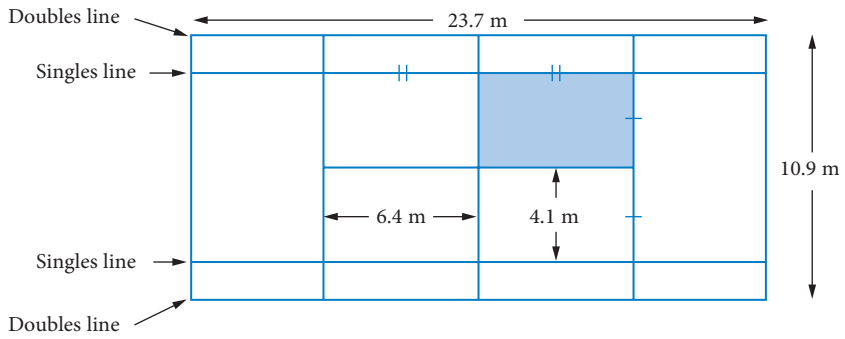
a Determine the number of posts and the length of cross-rails needed to complete the task.

b Calculate the minimum number of 100 mm × 15 mm palings required if the maximum spacing desired is 75 mm.

c Work out the cost of the materials if posts cost \$14.50 each, rails cost \$3.40/m and palings cost \$2.10 each.

8 A circular horse-racing track is 700 m across the inside and 15 m wide. The track has to be re-fenced on both sides with post-and-rail fencing at a cost of \$34/m for materials and labour. Find the cost of erecting the fences.

9 A diagram of a full-sized tennis court is shown below.



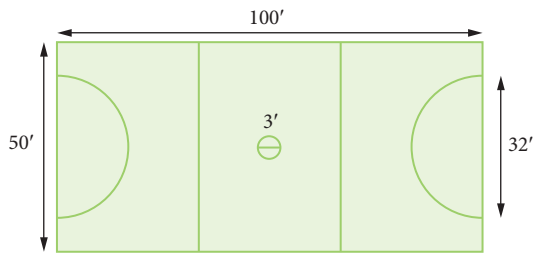
a What area (shaded) does a player have to serve into?

b Find the total area for play in a singles match.

c Find the extra area that is available for a doubles match.

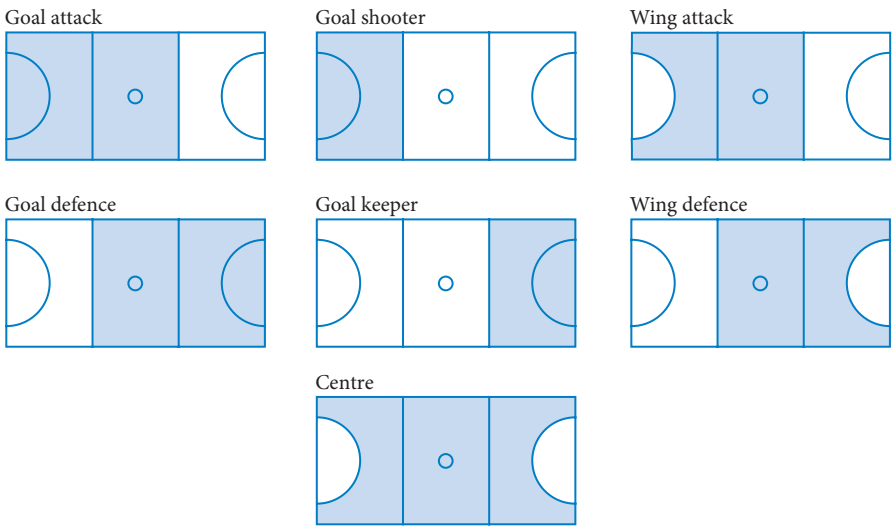
10 The minute hand of a clock is 10 cm long. It is 3 cm longer than the hour hand. In one full revolution, how much more of the face is covered by the minute hand than by the hour hand?

11 A netball court is shown here, with measurements in feet. The court is divided into 3 equal sections, which are referred to as 'thirds'.

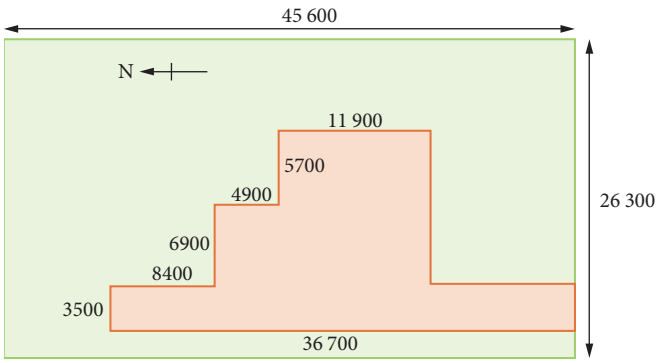


a Convert the dimensions to metric measurements using 1 foot (1′) ≈ 0.3048 m.

b Each player can go only into certain parts of the court, as shown by the shaded areas below. Calculate the area that each player may enter and express it as a percentage of the whole court area.



12 If A-grade turf costs \$12.00/m^2, find the cost of turfing the yard shown below, assuming that the green-shaded area is to be turfed. All measurements are given in millimetres.



13 WORKED EXAMPLE 14 An ensuite floor with dimensions 3200 mm × 3600 mm is to be tiled with 230 mm square ceramic tiles. Allow for 5 mm grout and wastage of 5%. The tiles cost \$4.80 each, glue costs \$40/L and grout costs \$44.50 a bag. A litre of glue will cover 2 m^2, and a bag of grout is enough for 5 m^2 of tiles.

a How many tiles are needed?

b How much glue is needed?

c How much grout is needed?

d What is the total cost?

Exam practice

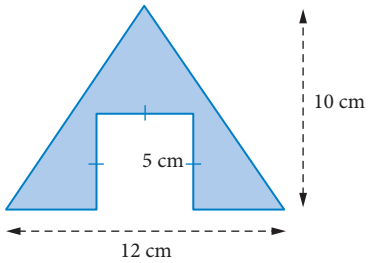
14 The shaded area in the diagram in square centimetres is

A 35

B 45

C 60

D 85



15 (2 marks) CF Five squares of side 32 cm are cut from a sheet of vinyl 1.02 m by 67 cm. What area of vinyl remains?

16 (3 marks) CU A netball is 230 mm in diameter and the hoop is 380 mm in diameter. Determine the percentage of the hoop that is occupied when a goal is scored.

17 (5 marks) CU Blake wants to paint a bedroom which has the dimensions 2400 mm × 3520 mm × 2400 mm ($l \times w \times h$). They plan to do 2 coats of paint on the walls and know that the paint covers 14 m^2/L. Allowing for 5% waste, how many litres of paint must Blake buy to do the job if paint is only sold in 4 L and 1 L tins?

Exam hack

Always check the number of marks awarded for each question or part of a question. If it's worth 2 or more marks, make sure you show your full working.

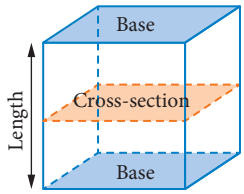
2.4 Surface area

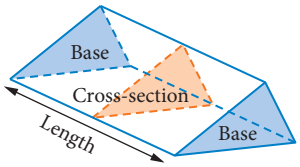
The **surface area** of a solid (3D) shape is the area of all its **faces** added together.

Drawing or visualising the **net** of a solid object can help to identify all the object's faces and calculate the surface area of the object. A net is a flat (2D) shape that can be folded up to form a solid.

A **cross-section** is a flat 'slice' of a solid shape cut parallel to its **base**.

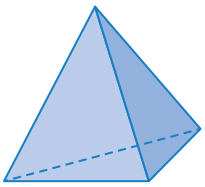
A **prism** has the same cross-section all the way along its length.



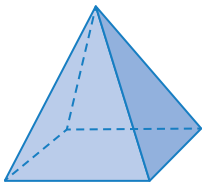


A **pyramid** is a solid made up of a base and triangular faces that meet at an **apex**. Pyramids get their name from the shape of their base.

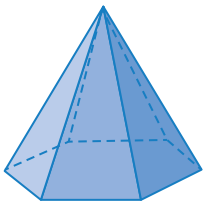
Triangular pyramid



Square pyramid



Hexagonal pyramid

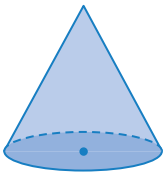


A **cone** is a solid with a circular base and an apex.

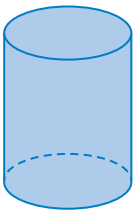
A **cylinder** is an object that has a constant circular cross-section along its full length and a curved face. In cylinders, the length is often referred to as the height.

A **sphere** is the shape of a ball and has one curved face. A **hemisphere** is half a sphere.

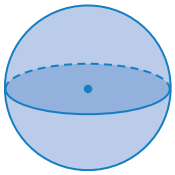
Cone



Cylinder



Sphere



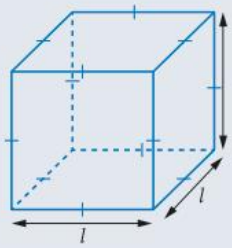
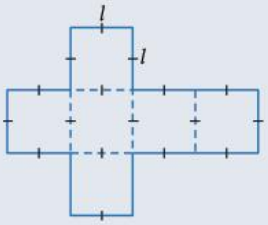
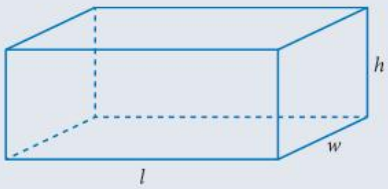
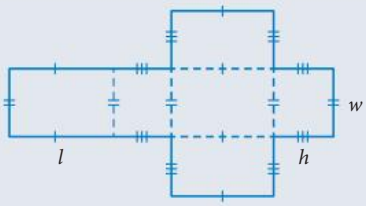
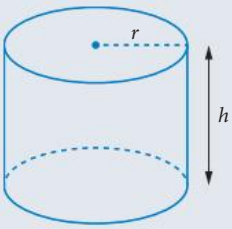
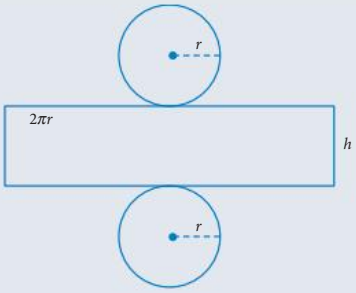
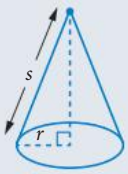
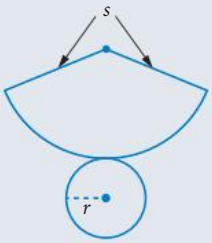
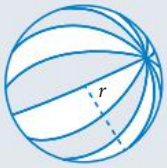
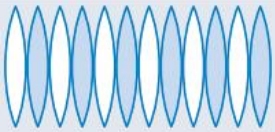
Videos
Surface area of a prism
Surface area of a cylinder
Surface area of a pyramid
Surface area of a cone and sphere

Skillsheet
Solid shapes

Worksheets
Nets of solids
A page of solid shapes
Surface area of solids
Surface area

Puzzles
Surface area riddle
Formula matching game

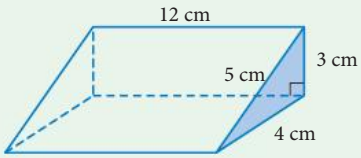
Surface area formulas

Object	Net	Surface area
Cube 	 6 squares	$S = 6l^2$
Rectangular prism 	 6 rectangles	$S = 2lw + 2lh + 2wh$
Cylinder 	 A rectangle and 2 circles	$S = 2\pi rh + 2\pi r^2$
Cone 	 A sector and a circle	$S = \pi rs + \pi r^2$
Sphere 	 The net is not useful to help calculate the surface area	$S = 4\pi r^2$

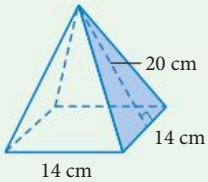
WORKED EXAMPLE 15 Using surface area formulas

Calculate the surface area of each solid.

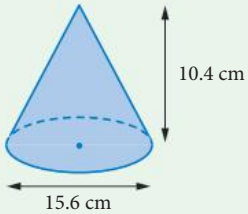
a

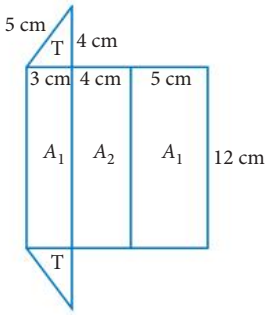


b



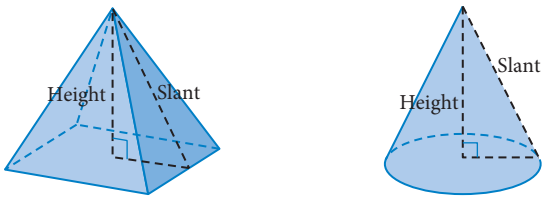
c



Steps	Working
a 1 Identify the object and the shapes that make up the net.	triangular prism = 2 triangles and 3 rectangles
Drawing a net can help to identify all the faces required for the surface area.	
2 Calculate the area of the triangle, T.	$T = \frac{1}{2}bh$ $= \frac{1}{2} \times 4 \times 3$ $= 6$
3 Find the various areas and add them together.	$S = A_1 + A_2 + A_3 + 2T$ $= 3 \times 12 + 4 \times 12 + 5 \times 12 + 2 \times 6$ $= 36 + 48 + 60 + 12$ $= 156$
4 Write the answer in the required units.	The surface area is $156\,\text{cm}^2$.
b 1 Identify the object and the shapes that make up the net.	square pyramid = 1 square and 4 triangles
2 Find the various areas and add them together.	$S = 14^2 + 4 \times \frac{1}{2} \times 14 \times 20$ $= 756$
3 Write the answer in the required units.	The surface area is $756\,\text{cm}^2$.
c 1 Identify the object and the shapes that make up the net.	cone = 1 circle and a sector of a circle
2 Use the surface area formula, calculating any missing values.	$r = \frac{15.6}{2}$ $= 7.8$ $s^2 = a^2 + b^2$ $= 7.8^2 + 10.4^2$ $= \sqrt{169}$ $= 13$ $S = \pi rs + \pi r^2$ $= \pi \times 3.9 \times 13 + \pi \times 3.9^2$ $= 207.06237\ldots$
3 Write the answer in the required units. Round if necessary.	The surface area is $207\,\text{cm}^2$.

Exam hack

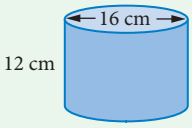
You can use the slant length of a pyramid or cone to find the height or vice versa using Pythagoras' theorem.



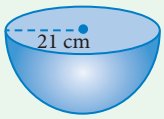
WORKED EXAMPLE 16 Applying surface area formulas

Calculate the surface area of each solid. Round to the nearest whole number.

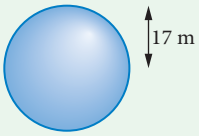
a an open topped tin

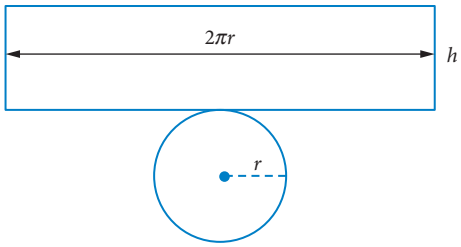


b a hemispherical bowl with a lid



c a glass sphere



Steps	Working
a 1 Identify the object and the shapes that make up the net. *Note:* This is open topped so you only need one circle for the base.	cylinder = rectangle + circle 
2 Use the surface area formula, calculating any missing values.	$r = 8$ cm $S = 2\pi rh + \pi r^2$ $= 2\pi \times 8 \times 12 + \pi \times 8^2$ $= 804.2477\ldots$
3 Write the answer in the required units, rounding if necessary.	The surface area is about 804 cm^2.
b 1 Identify the object and the shapes that make up the net.	hemisphere = half sphere + circle
2 Use the surface area formula or find the various areas and add them together.	$S = 2\pi r^2 + \pi r^2$ $= 3\pi r^2$ $= 3 \times \pi \times 21^2$ $= 4156.32\ldots$
3 Write the answer in the required units, rounding if necessary.	The surface area is about 4156 cm^2.
c 1 Identify the object.	sphere
2 Use the surface area formula, calculating any missing values.	$S = 4\pi r^2$ $= 4 \times \pi \times 17^2$ $= 3631.68\ldots$
3 Write the answer in the required units, rounding if necessary.	The surface area is about 3632 m^2.

EXERCISE 2.4 Surface area

ANSWER p. 397

Recap

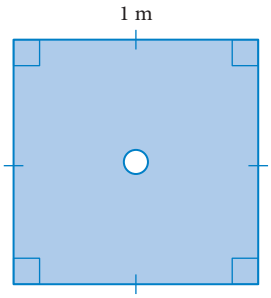
1 A stove has one circular burner. This burner has a radius of 11 cm. The area of the top of this burner, in square centimetres, is closest to

A 35 **B** 69 **C** 95 **D** 380

2 The diagram shows the area of a school shower cubicle. The drain in the centre of the floor is circular with a diameter of 10 cm. What is the area of the shower cubicle, excluding the drain?

A 10 000 cm^2 **B** 9921 cm^2

C 9686 cm^2 **D** 9969 cm^2

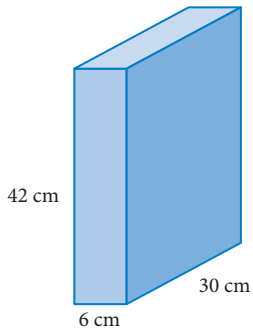


Mastery

3 The surface area of this rectangular prism is closest to

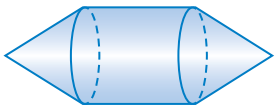
A 1250 cm^2 **B** 3400 cm^2

C 1700 cm^2 **D** 2400 cm^2



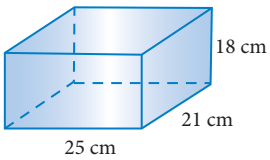
4 How many surfaces does this solid have?

A 2 **B** 3 **C** 4 **D** 5

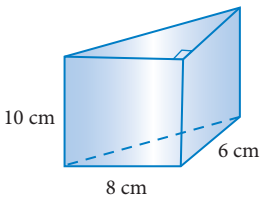


5 WORKED EXAMPLE 15 Calculate the surface area of each solid.

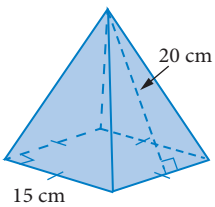
a



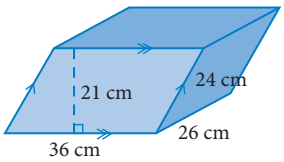
b



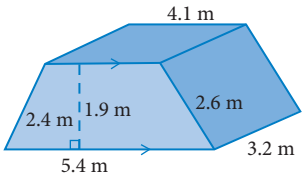
c



d



e



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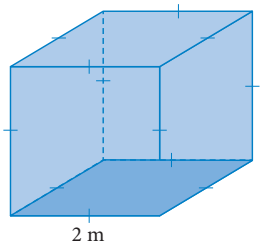
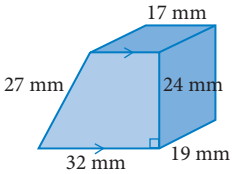
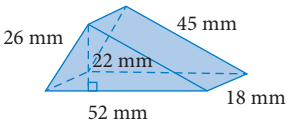


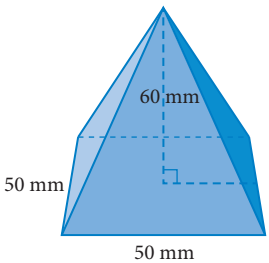
g



h

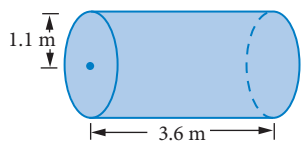


i



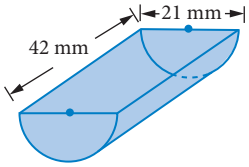
6 WORKED EXAMPLE 16 Calculate the surface area of each solid, correct to one decimal place.

a



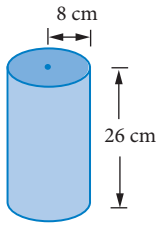
Closed cylinder

b



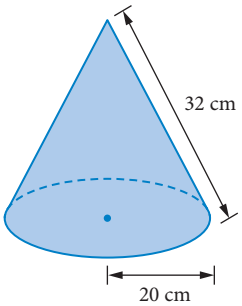
Open half-cylinder

c

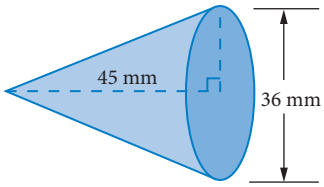


Cylinder (open both ends)

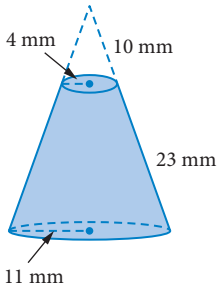
d



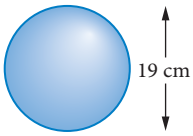
e



f



g



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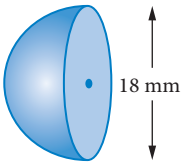
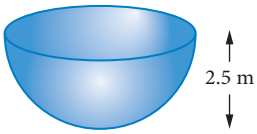


Closed

i

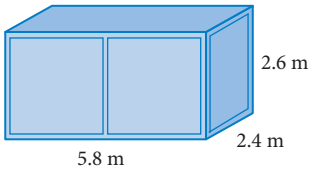


Open

7 The outside of this shipping container needs to be painted all over.

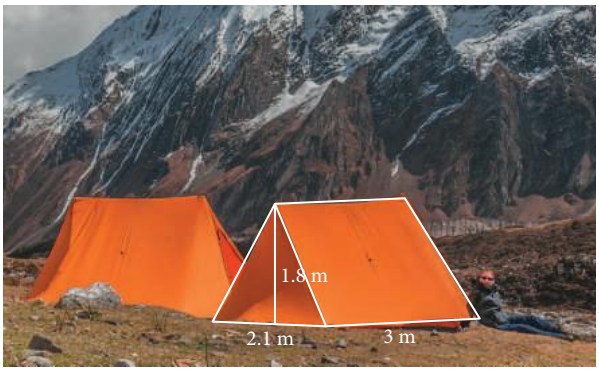
a Find the total area to be painted.

b Find the cost of painting the container if the paint only comes in 4 L tins that cost \$52, each litre of paint covers 8 m^2, and labour costs are \$7.50/m^2.



8 A cereal box is 8 cm wide, 25 cm long and 20 cm high. How many square metres of cardboard is needed to make 5000 boxes, allowing 15% extra for wastage and joins?

9 A tent with a floor is in the shape of a triangular prism 3 m long, 1.8 m high and 2.1 m wide. What area of cloth is needed to make the tent if no allowance is made for windows, door flaps and joins?

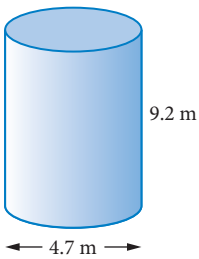


MoLarjung/Shutterstock.com

10 This silo is constructed from an aluminium alloy. It has a top and a base. The material costs \$8.50/m^2 and a 5% allowance is needed for wastage. Labour costs for the construction of the silo are \$45.50/h.

a How much metal is needed?

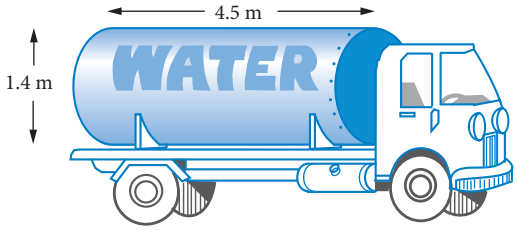
b What is the total material and labour costs of constructing the silo if it takes 20 hours?



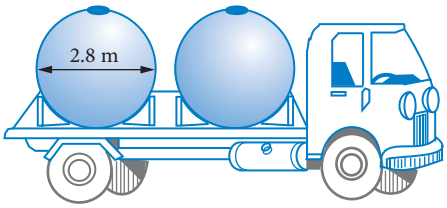
11 The water tank on this truck needs repainting. Find, correct to one decimal place

a the area to be painted (including the ends)

b the number of litres of paint needed if the coverage is $9\,m^2/L$.



12 This truck carries liquid oxygen in 2 identical spherical containers. Find the total surface area of the spheres.



Exam practice

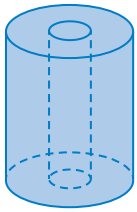
13 How many surfaces does this solid have?

A 2

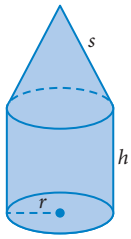
B 3

C 4

D 5

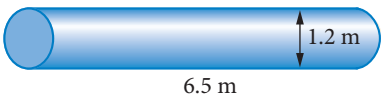


14 (2 marks) Write an expression to find the surface area of this object.

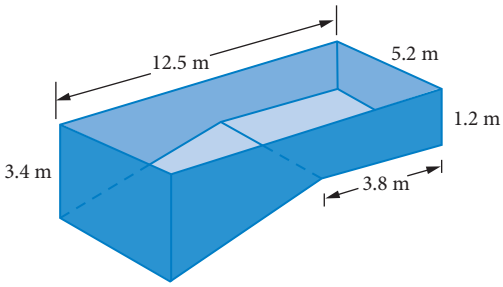


15 (4 marks) CF A rectangular sheet of steel is to be rolled and welded to form the pipe shown.

Find the cost of constructing the pipe if $1.35\,m^2$ must be allowed for overlap, and materials and labour costs $\$75/m^2$.



16 (4 marks) CU A newly-constructed swimming pool is shown. The interior surface requires one coat of special sealant that costs \$78.50/L. If each litre covers $8\,m^2$, calculate the cost of sealant required.



17 (4 marks) CU A cannery has ordered tomato juice cans that are 20 cm high and 8 cm in diameter. The labels are to be printed and cut from 50 m rolls of 40 cm wide paper. Each label requires a 1-cm overlap when placed on the can. Determine how many labels can be cut from each roll.

Worksheets
Length, area and volume conversions

Estimating area and volume

Volumes of water

2.5 Volume

Units of volume and capacity

The **volume** of a solid is the amount of space it occupies.

Volume is measured in cubic units.

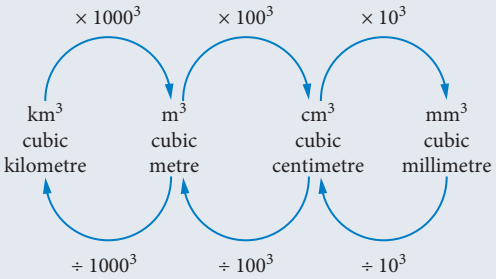
The **capacity** of a solid is the amount of fluid the solid could hold.

Capacity can be measured in units based on litres.

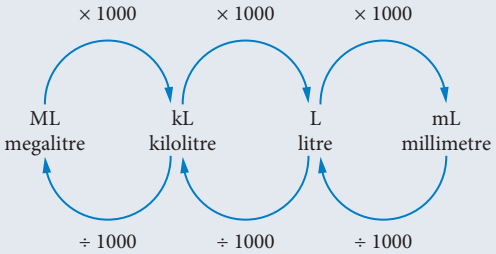
Metric units of volume and capacity

- To convert from a larger unit to a smaller unit, use multiplication (×).
- To convert from a smaller unit to a larger unit, use division (÷).

Volume units (cubic units)



Capacity units



You may need to convert between volume and capacity.

Volume and capacity conversions

$1\text{ cm}^3 = 1\text{ mL}$
$1000\text{ cm}^3 = 1\text{ L}$
$1\text{ m}^3 = 1\text{ kL}$

Video
Volume and capacity units

WORKED EXAMPLE 17 Converting units of volume and capacity

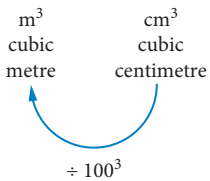
Convert each measurement to the unit indicated.

a $120\,000\text{ cm}^3$ to m^3

b 2.34 m^3 to L

c 83 cm^3 to mL

d 3.6 L to cm^3

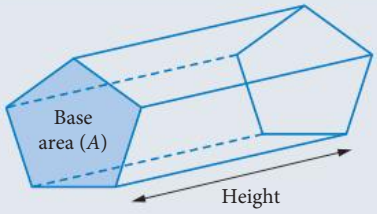
Steps	Working
a Identify the conversion.  	$120\,000\text{ cm}^3 = 120\,000 \div 100^3$ $= 0.12\text{ m}^3$
b Identify the conversion.	$1\text{ m}^3 = 1\text{ kL} = 1000\text{ L}$ $2.34\text{ m}^3 = 2.34 \times 1000$ $= 2340\text{ L}$
c Identify the conversion.	$1\text{ cm}^3 = 1\text{ mL}$ $83\text{ cm}^3 = 83\text{ mL}$
d Identify the conversion.	$1\text{ L} = 1000\text{ cm}^3$ $3.6\text{ L} = 3.6 \times 1000$ $= 3600\text{ cm}^3$

Volumes of prisms and cylinders

Prisms and cylinders have the same cross-section from one end of the object to the other.

Volumes of prisms and cylinders = base area × height

$$V = A \times h$$



Volume of pyramids, cones and spheres

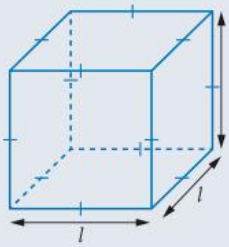
Volume of pyramids and cones $= \frac{1}{3}Ah$, where A is the base area.

Volume of a sphere $= \frac{4}{3}\pi r^3$

Many objects have formulas to calculate their volumes.

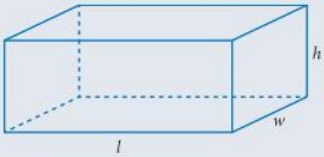
Volumes of specific solid shapes

Cube



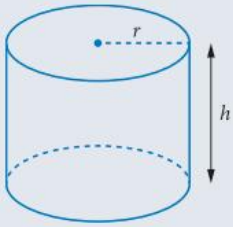
$V = l^3$

Rectangular prism



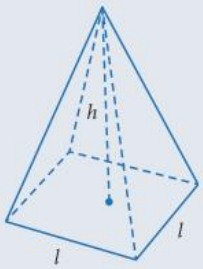
$V = lwh$

Cylinder



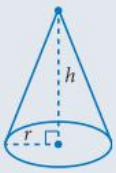
$V = \pi r^2 h$

Square pyramid



$V = \frac{1}{3}l^2 h$

Cone



$V = \frac{1}{3}\pi r^2 h$

Sphere

$V = \frac{4}{3}\pi r^3$

Videos
Volumes of prisms and cylinders
Capacity of a cylinder
Volume of a pyramid
Volume of a sphere

Skillsheet
Solid shapes

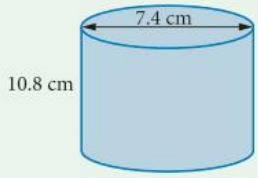
WORKED EXAMPLE 18 Applying volume formulas

For each solid, **calculate** correct to 2 decimal places

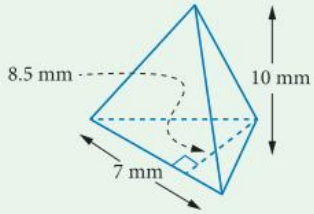
i its volume

ii its capacity

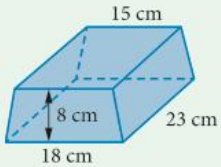
a



b



c



Steps		Working
a i 1	Identify the object.	cylinder
2	Use the volume formula, calculating any missing values.	$V = \pi r^2 h$ $= \pi\left(\frac{7.4}{2}\right)^2 \times 10.8$ $= 464.4907\ldots$
3	Write the answer in the required units, rounding if necessary.	The volume is 464.49 cm^3.
ii 1	Identify the conversion and convert the volume to capacity.	$1\ cm^3 = 1\ mL$ $464.4907\ldots\ cm^3 = 464.4907\ldots\ mL$
2	Write the answer in the required units, rounding if necessary.	The capacity is 464.49 mL.
b i 1	Identify the object.	triangular pyramid
2	Use the volume formula, calculating any missing values.	$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times \frac{1}{2} \times 7 \times 8.5 \times 10$ $= 99.1666\ldots$
3	Write the answer in the required units, rounding if necessary.	The volume is 99.17 mm^3.
ii 1	Identify the conversion and convert the volume to capacity. You need to convert: $mm^3 \rightarrow cm^3$ and then $cm^3 \rightarrow mL$	$99.1666\ldots\ mm^3 = 99.1666\ldots \div 10^3$ $= 0.099\,16\ldots\ cm^3$ $1\ cm^3 = 1\ mL$ $0.099\,16\ldots cm^3 = 0.099\,16\ldots mL$
2	Write the answer in the required units, rounding if necessary.	The capacity is 0.10 mL.
c i 1	Identify the object.	trapezoidal prism
2	Use the formula $V = Ah$ where A is the area of a trapezium.	$V = Ah$ $= \frac{1}{2} \times (15 + 18) \times 8 \times 23$ $= 3036$
3	Write the answer in the required units, rounding if necessary.	The volume is 3036 cm^3.
ii 1	Identify the conversion and convert the volume to capacity.	$1\ cm^3 = 1\ mL$ $3036\ cm^3 = 3036\ mL$
2	Write the answer in the required units, rounding if necessary.	The capacity is 3036 mL.

EXERCISE 2.5 Volume

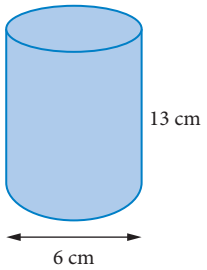
ANSWER p. 397

Recap

1 The surface area of this closed cylinder is closest to

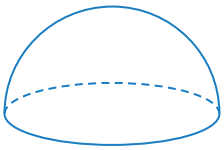
A $245\,cm^2$ **B** $57\,cm^2$

C $150\,cm^2$ **D** $300\,cm^2$

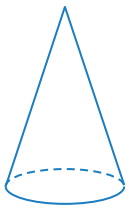


2 Which of these objects is a prism?

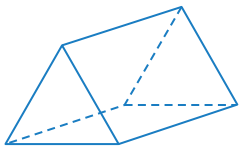
A



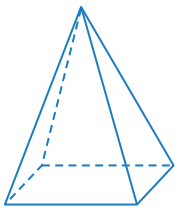
B



C



D



Mastery

3 The most appropriate unit for the volume of a backyard swimming pool is

A cm^3 **B** m^3 **C** ML **D** mm^3

4 What would be the most appropriate unit for the volume of each object?

a a can of soft drink **b** a tablespoon **c** a shipping container

d Wivenhoe dam **e** an engine's cylinders **f** a laundry tub

g a dessert spoon **h** a bath **i** a house

j a car tyre

5 WORKED EXAMPLE 17 Convert each measurement to the unit indicated.

a $26\,m^3 \rightarrow cm^3$ **b** $320\,000\,cm^3 \rightarrow m^3$ **c** $2.8 \times 10^{11}\,cm^3 \rightarrow ML$

d $800\,mL \rightarrow L$ **e** $56\,kL \rightarrow m^3$

6 Convert each measurement to the unit indicated.

a 2400 mL to L **b** 3.2 L to cm^3 **c** 7.5 L to mL

d 730 mL to L **e** 41 cm^3 to mL **f** 8.2 m^3 to L

g 432 mL to cm^3 **h** 3.24 L to m^3 **i** 2320 cm^3 to L

7 WORKED EXAMPLE 18 Find the volume of a cylinder with

a radius 17 cm, height 22 cm **b** diameter 20 mm, height 15 mm

c diameter 2.4 m, height 1.9 m **d** radius 8.3 cm, height 5.2 cm

8 Find the volume of these shapes.

a Conical pile of sugar with a height of 3.4 m and a radius of 4.8 m

b Square pyramid with a base measuring 13 cm by 13 cm and a height of 11 cm

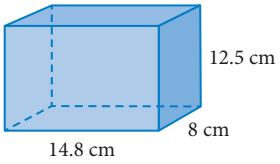
c Rectangular-based pyramid with a base measuring 2.1 m by 1.7 m and a height of 3.3 m

d Conical funnel with a height of 26 cm and a diameter of 18 cm

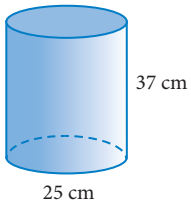
e Rectangular-based pyramid with a base measuring 42 mm by 21 mm and a height of 11 mm

9 Find the volume of each solid.

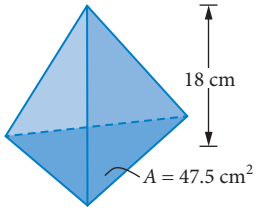
a



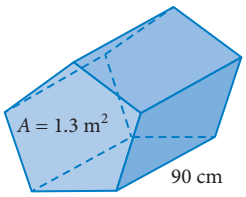
b



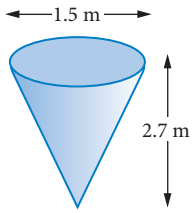
c



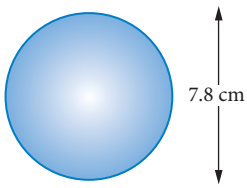
d



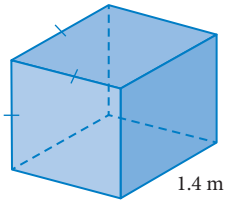
e



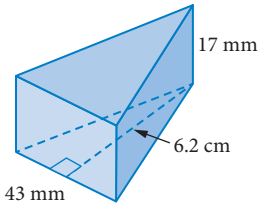
f



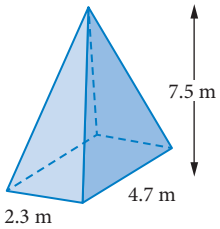
g



h



i



10 A house brick measures 230 mm × 110 mm × 76 mm. What is its volume in

a cubic millimetres?

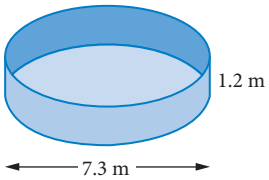
b cubic centimetres?

Exam practice

11 (2 marks) An engineering company wants to build a truck body that will hold 18 m^3 of material. The truck body will be in the shape of a rectangular prism. If the dimensions of the base are 1.8 m by 4.8 m, how high will the sides need to be?

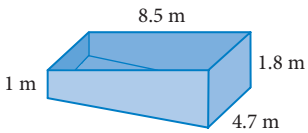
12 (4 marks) CU For each swimming pool shown, determine the time taken to fill the pool at a rate of 8 L/minute.

a



[2 marks]

b



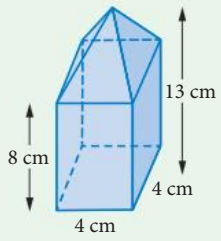
[2 marks]

2.6 Surface area and volume of composite shapes

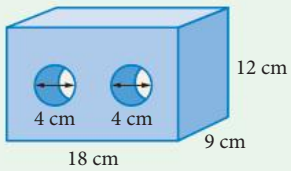
WORKED EXAMPLE 19 Volumes of composite shapes

Calculate the volume of each composite solid, rounding your answer to the nearest unit.

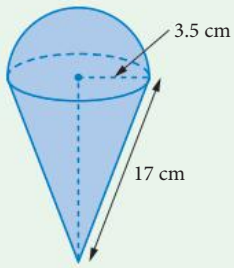
a



b



c



Steps	Working
a 1 Identify the parts of the object.	rectangular prism and square pyramid combined
2 Find the volume of the prism.	$V = lwh$ $= 4 \times 4 \times 8$ $= 128\text{ cm}^3$
3 Find the volume of the pyramid, calculating any missing values.	$V = \frac{1}{3}l^2h$ $= \frac{1}{3} \times 4^2 \times 5$ $\quad h = 13 - 8 = 5$ $= 26.6666\ldots\text{ cm}^3$
4 Calculate the total volume by adding the volumes.	volume of shape $= 128 + 26.6666\ldots$ $= 154.6666\ldots$
5 Write the answer in the required units, rounding if necessary.	The volume of the object is about 155 cm^3.
b 1 Identify the parts of the object.	rectangular prism with 2 cylinders removed
2 Find the volume of the prism.	$V = lwh$ $= 18 \times 9 \times 12$ $= 1944\text{ cm}^3$
3 Find the volume of the cylinders, calculating any missing values.	$V = \pi r^2 h$ $= \pi \times 2^2 \times 9$ $= 113.0973\ldots\text{ cm}^3$
4 Calculate the total volume by subtracting the volumes.	volume $= 1944 - 2 \times 113.0973\ldots$ $= 1717.8053\ldots\text{ cm}^3$
5 Write the answer in the required units, rounding if necessary.	The volume of the object is about 1718 cm^3.

Video Volumes of composite shapes

Worksheets A page of solid shapes

Volumes of solids

c	**1** Identify the parts of the object.	A cone and a hemisphere combined.
	2 Find the volume of the cone, calculating any missing values.	Find the height, h, of the cone using Pythagoras' theorem: $17^2 = h^2 - 3.5^2$ $h = \sqrt{17^2 - 3.5^2}$ $= 16.636\ldots$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi \times 3.5^2 \times 16.636\ldots$ $= 213.407\ldots$
	3 Find the volume of the hemisphere, calculating any missing values.	$V = \frac{4}{3}\pi r^3 \div 2$ $= \frac{4}{3}\pi \times 3.5^3 \div 2$ $= 89.797\ldots$
	4 Calculate the total volume by adding the volumes.	volume of shape $= 213.407\ldots + 89.797\ldots$ $= 303.204\ldots$
	5 Write the answer in the required units, rounding if necessary.	The volume of the object is about $303\,\text{cm}^3$.

EXERCISE 2.6 Surface area and volume of composite shapes

ANSWER p. 397

Recap

1 5.73 L is the same as

A $5730\,\text{mm}^3$ **B** 573 mL **C** $0.0573\,\text{m}^3$ **D** 0.0573 kL

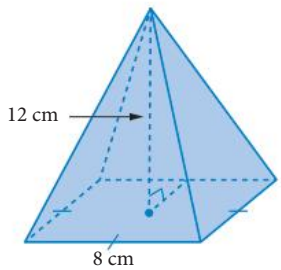
2 Which calculation would give the volume for the pyramid shown?

A $V = \frac{1}{3} \times 8^2 \times 12$

B $V = \frac{1}{3} \times 8 \times 12^2$

C $V = \frac{1}{3} \times 4^2 \times 12$

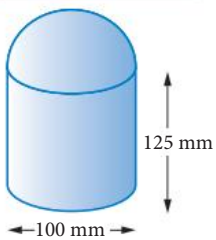
D $V = \frac{1}{2} \times 8^2 \times 12$



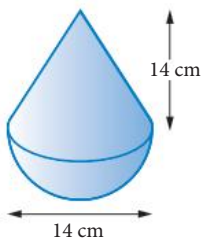
Mastery

3 WORKED EXAMPLE 19 Calculate the volume of each solid, correct to the nearest whole unit.

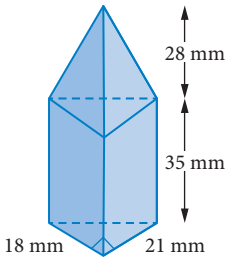
a



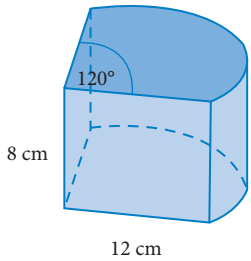
b



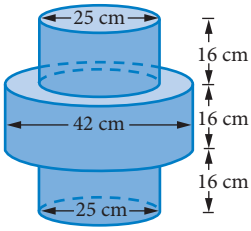
c



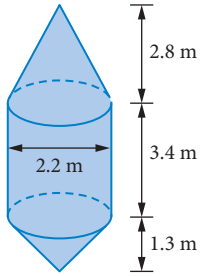
d



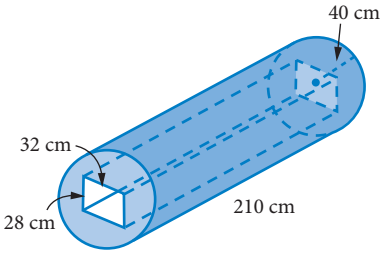
e



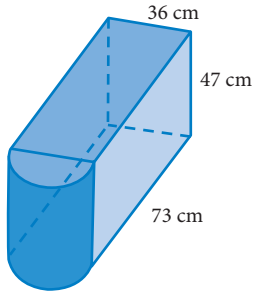
f



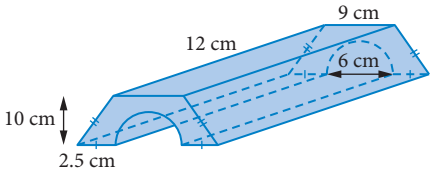
g



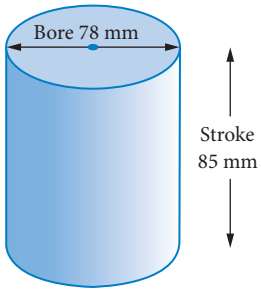
h



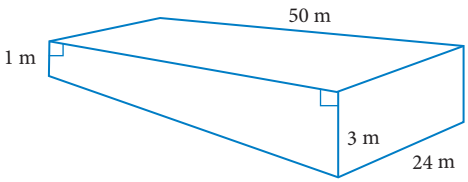
i



4 The engine capacity of a motor vehicle is the total capacity of the cylinders in the engine. It is usually stated in cubic centimetres or litres. Calculate the engine capacity of a 4-cylinder car if each cylinder has a bore (diameter) of 78 mm and a stroke (length) of 85 mm. Answer in litres.



5 a Calculate the volume of this swimming pool in cubic metres.



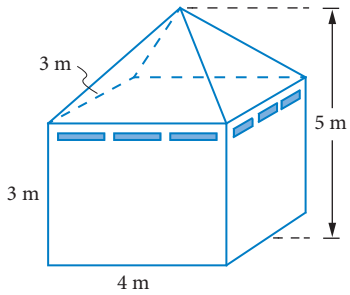
b What is the capacity of the pool in kilolitres?

c The pool is filled using a pipe that pumps water at 4.5 L per second. How long will this take, correct to the nearest hour?

6 This is a tent used for the fumigation of imported goods suspected of carrying pests from overseas.

a Calculate the volume of the tent.

b The gas used for fumigation is dense and it pushes out the air through special vents when the tent is prepared for use. If the gas can be delivered at a rate of $0.035\,m^3/s$, how long will the tent take to fill?

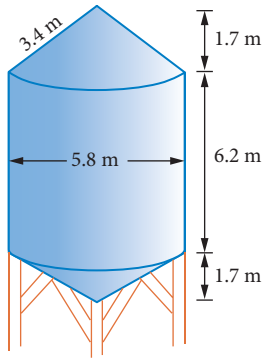


7 This grain silo is made up of a cylinder and 2 identical shallow cones.

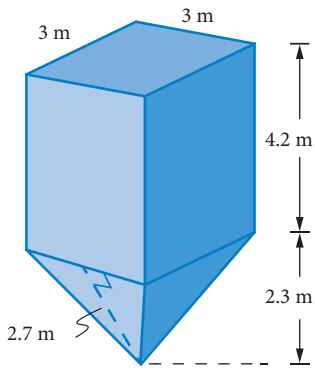
a How many cubic metres of barley can this silo hold?

b Some barley is removed from the full silo and the level of grain drops by 2 m. What percentage of the grain has been removed?

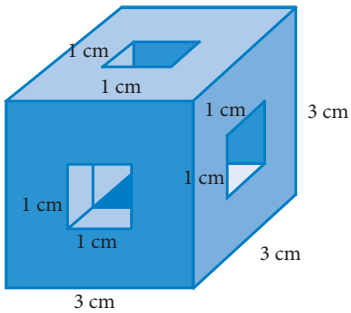
c A farmer wishes to build a silo to these dimensions. How many square metres of sheet metal are required if 5% must be allowed for overlaps and wastage?



8 This grain hopper stores wheat. The outside of the hopper has to be painted with 2 coats of paint. How much will the painting cost if a contractor will do the job for $\$34/m^2$?



9 This cubic block has holes that pass all the way through it. Each hole is $1\,cm \times 1\,cm \times 3\,cm$. Find the total surface area (internal and external) of the block.

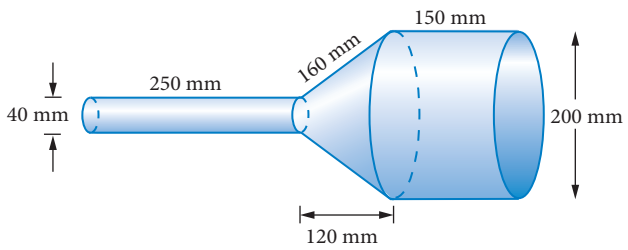


Exam practice

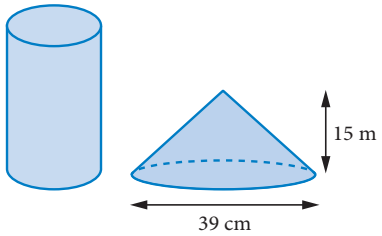
10 (3 marks) CF The Great Pyramid of Giza in Egypt has been reduced in size from its original dimensions by weather. It is known that it originally had a volume of 2.57 million m^3 and a square base of side length 230 m.

Its volume has been reduced by about 6% over the years. Assuming that the base is still a 230 m square, what is its height now?

11 (3 marks) CU This water funnel is to be constructed of copper sheeting. What area of sheeting will be required, if 5% must be allowed for seams and overlap?



12 (3 marks) CU When this cylindrical silo filled with sugar was emptied, the sugar formed a conical pile 15 m high with a base diameter of 39 m. Calculate the height of the silo if it has a radius of 8.5 m.

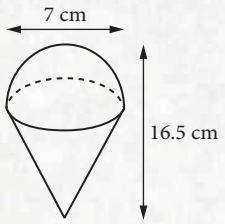


Video Exam question analysis: Length, area and volume

EXAM QUESTION ANALYSIS

Exam-style question (5 marks)

Calculate the surface area of the ice-cream cone, including the ice cream.
Leave your answer in whole centimetres.



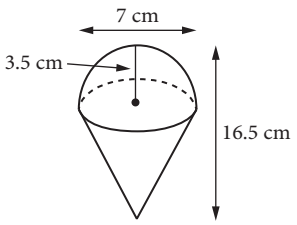
Reading the question

- The question is asking for a surface area calculation of a composite 3D shape.
- There are 2 shapes in the diagram: cone and hemisphere.
- Two measurements have been given.
- Answer will need to be rounded.

Thinking about the question

- As the diagram gives a diameter, a radius will need to be found. This can be used for both shapes.
- The radius represents the height of the hemisphere. This will need to be subtracted from the whole height.
- The hemisphere will require the sphere formula to be halved.
- As the 2 objects are on top of each other, neither will need a base.

Worked solution (✓ = 1 mark)



radius of hemisphere and cone $= \frac{1}{2} \times 7$

$= 3.5$ cm

height of cone $= 16.5 - 3.5$

$= \mathbf{13\ cm}$ ✓

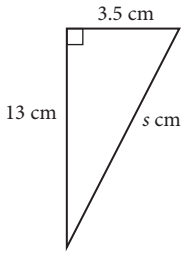
curved surface area of hemisphere $= \frac{1}{2} \times 4\pi r^2$

$= 2 \times \pi \times 3.5^2$

$= \mathbf{76.9690\ldots\ cm^2}$ ✓

curved surface area of cone $= \pi rs$

Find the slant height s of the cone using Pythagoras' theorem:



$s^2 = 13^2 + 3.5^2$

$= 181.25$

$s = \sqrt{181.25}$

$= \mathbf{13.4629\ldots}$ ✓

curved surface area of cone $= \pi rs$

$= \pi \times 3.5 \times 13.4629\ldots$

$= \mathbf{148.0324\ldots\ cm^2}$ ✓

total surface area $= 76.9690\ldots + 148.0324\ldots$

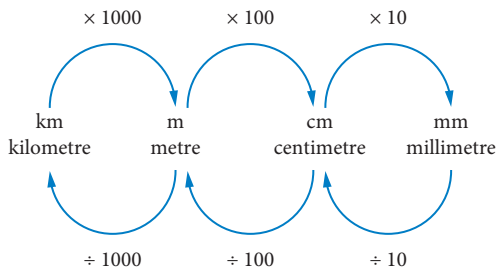
$= 225.0014\ldots$

$\approx \mathbf{225\ cm^2}$ ✓

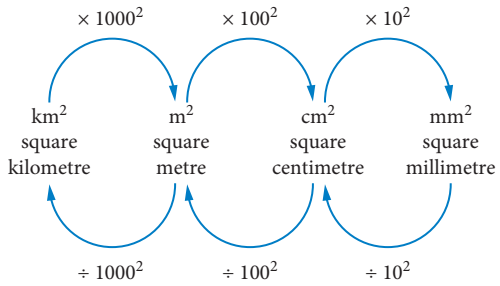
2 Chapter summary

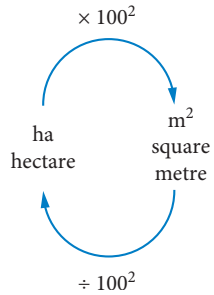
Converting units of measurement

Length

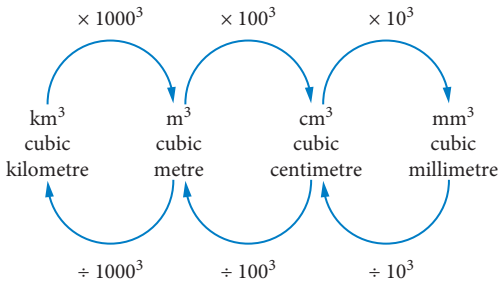


Area

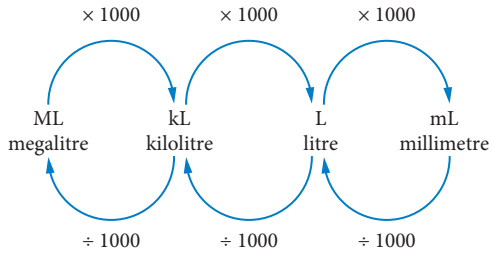




Volume

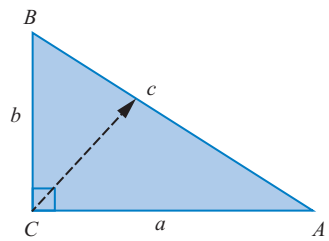


Capacity



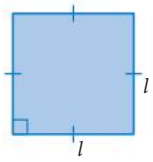
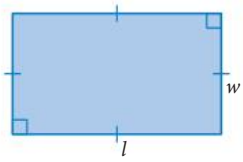
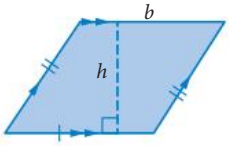
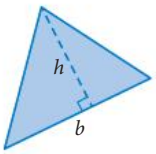
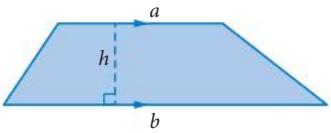
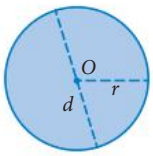
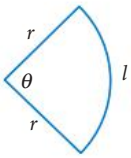
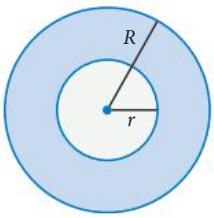
Pythagoras' theorem

- $c^2 = a^2 + b^2$ where c is the **hypotenuse**.



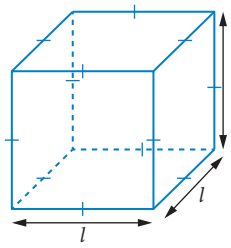
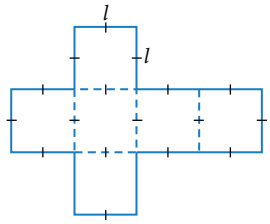
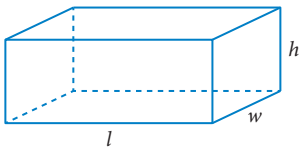
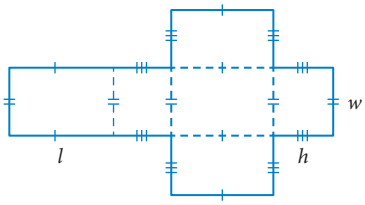
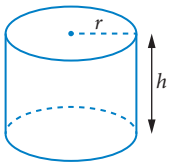
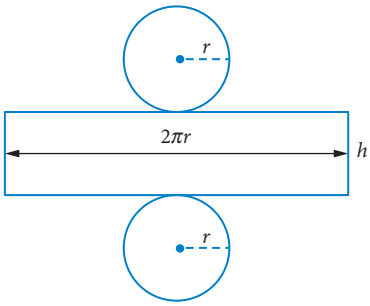
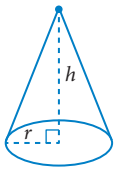
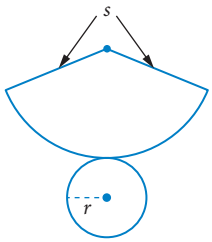
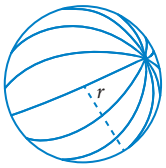
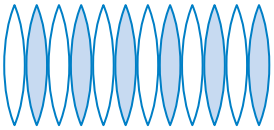
Puzzle
Length, area and volume find-a-word

Perimeter and area

Square  $P = 4l$ $A = l^2$	**Rectangle**  $P = 2l + 2w$ $= 2(l + w)$ $A = lw$	**Parallelogram**  $A = bh$
Triangle  $A = \frac{1}{2}bh$	**Trapezium**  $A = \frac{1}{2}(a+b)h$	**Circle**  $C = 2\pi r$ $C = \pi d$ $A = \pi r^2$
 $l = \frac{\theta}{180}\pi r$ $A = \frac{\theta}{360}\pi r^2$	**Annulus**  $P = 2\pi R + 2\pi r$ $A = \pi R^2 - \pi r^2$	

Surface area

- The **surface area** of a solid is the area of all its **faces** added together.
- Drawing or visualising the **net** of a solid helps to identify all of the faces.

Object	Net	Surface area
Square prism (cube) 	 6 squares	$S = 6l^2$
Rectangular prism 	 6 rectangles	$S = 2lw + 2lh + 2wh$ $= 2(lw + lh + wh)$
Cylinder 	 A rectangle and 2 circles	$S = 2\pi rh + 2\pi r^2$
Cone 	 A sector and a circle	$S = \pi rs + \pi r^2$
Sphere 		$S = 4\pi r^2$

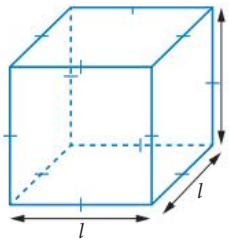
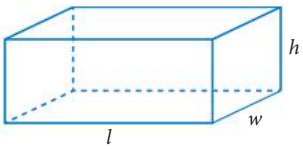
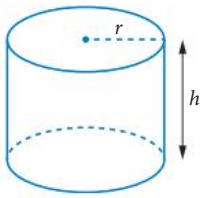
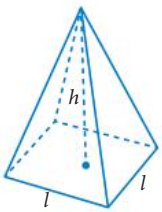
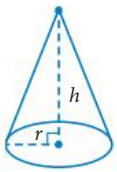
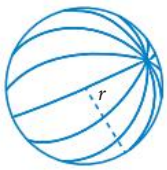
Volume

- The **volume** of a solid is the amount of space it occupies.
- The **capacity** of a solid is the amount of fluid it could hold.
- Volume and capacity

  $1\text{ cm}^3 = 1\text{ mL}$

  $1000\text{ cm}^3 = 1\text{ L}$

  $1\text{ m}^3 = 1\text{ kL}$
- Volume of prisms and cylinders = base area × height = Ah
- Volume of pyramids and cones = $\frac{1}{3}$ × base area × height = $\frac{1}{3}Ah$

Cube  $V = l^3$	**Rectangular prism**  $V = lwh$	**Cylinder**  $V = \pi r^2 h$
Square pyramid  $V = \frac{1}{3}l^2h$	**Cone**  $V = \frac{1}{3}\pi r^2 h$	**Sphere**  $V = \frac{4}{3}\pi r^3$

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

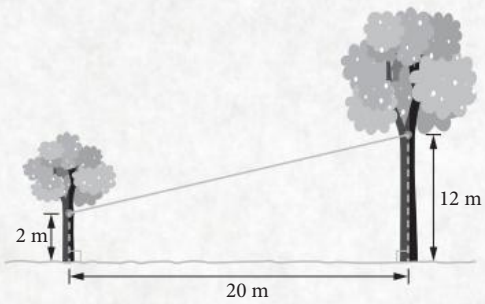
Total: 18 marks

Section 1 5 multiple choice questions

5 marks

Worksheet
General Maths Year 11 formula sheet

1 A flying fox is being set up between 2 big trees. The 2 trees are on level ground 20 metres apart and are joined by a cable. This cable is attached 12 metres above the ground on one tree and 2 metres above the ground on the other. What is the length of cable between the 2 trees, correct to 2 decimal places?

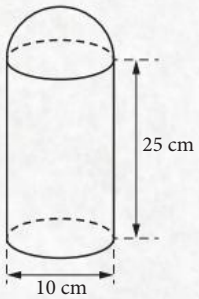


A 17.32 m **B** 22.36 m **C** 23.32 m **D** 20.10 m

2 During the recent rain event, a 1.5-hectare sporting park was covered to a depth of 17 cm. How many kilolitres of water covered the park? (1 ha = 10 000 m^2)

A 2 550 000 kL **B** 255 000 kL **C** 2550 kL **D** 2.55 kL

3 The solid below is made up of a hemisphere on top of a cylinder.



What is the total surface area of the solid, correct to the nearest square centimetre?

A 1021 cm^2 **B** 1100 cm^2 **C** 2513 cm^2 **D** 2827 cm^2

4 In one year, a population increased by 20%, and then the next year, it decreased by 10%. What was the percentage increase over the 2 years?

A 30% **B** 10% **C** 15% **D** 8%

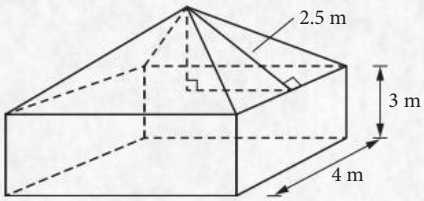
5 Ashton is 17 years old and works in his parents' store, earning $22.75 per hour on normal workdays. On Sunday he is paid double time. How much does Ashton earn if he works 6 hours on Friday and 3 hours on Sunday?

A 204.75 **B** $273 **C** $238.88 **D** $264

Section 2 4 short response questions

13 marks

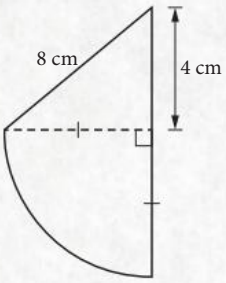
6 (3 marks) The roof of a square cubby house is in the shape of a square pyramid with identical triangular faces. The sides of the cubby house are rectangles and there is no floor. The dimensions of the cubby house are shown on the diagram.



a Sketch a net of the cubby house. *[1 mark]*

b What is the surface area of the cubby house? *[2 marks]*

7 (4 marks) A piece of a child's puzzle is shown below. It is made up of a right-angled triangle and a quadrant of a circle. What is the perimeter of the puzzle piece, correct to 2 decimal places?



8 (3 marks) Zain worked 24 hours in the last week and was paid $643.50. He worked 20 hours during the normal work week and 4 hours overtime, which was paid at time-and-a half. What is Zain's normal hour rate?

9 (3 marks) A real estate agent earns a commission for selling a house. They are paid 3% for the first $750 000 of the sale price and 1.5% for any amount over $750 000. How much would they earn if they sold a house for $1 000 000?

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions

13 marks

1 (3 marks) Hetty wants to buy a new car. A car brochure shows the following fuel consumption rates.

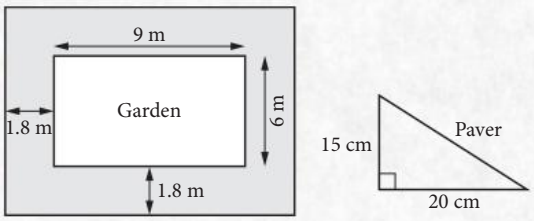
	Petrol	Electric
Fuel consumption	7.7 L/100 km	40 kWh/100 km

Hetty knows in an average year, she drives approximately 15 000 km.
She looks up the following fuel costs: Petrol = \$2.25/L and electricity = \$ 0.35/kWh.

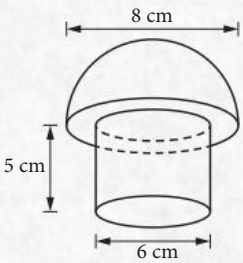
How much cheaper would the electric model of the car be to run in a year, compared with the petrol model?

Worksheet
General Maths Year 11 formula sheet

2 (5 marks) A path 1.8 m wide is to be built around a rectangular garden that is 9 m long by 6 m wide. The path is to be covered in triangular pavers that have side lengths of 20 cm and 15 cm as shown. The pavers are being laid without any gaps or overlaps. How many pavers are needed for the path?



3 (5 marks) Bethany is making small mushroom paper weights to sell at markets. The base is a cylinder with a height of 5 cm and a diameter of 6 cm. The top section is a hemisphere with a diameter of 8 cm. The mushrooms are to be made out of resin which costs \$0.32/cm^3.
How much does it cost Bethany to make 5 paper weights for her friends?



CHAPTER

3 PRICES AND SHARES

Syllabus coverage

Nelson MindTap chapter resources

Terminology

3.1 Comparing prices

3.2 Percentage applications

Mark-ups and discounts

GST

Profit and loss

3.3 Simple interest

3.4 Currency exchange

3.5 Shares and dividends

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

9780170484749

Syllabus coverage

UNIT 1, TOPIC 1: CONSUMER ARITHMETIC

Applications of rates, percentages and use of spreadsheets

- Compare prices and values using the unit cost method.
- Apply percentage increase or decrease in various contexts, e.g. inflation of costs and wages, percentage mark-ups and discounts, percentage profit and loss, GST, simple interest.
  - $I = Pin$ where I is simple interest, P is principal, i is interest rate per year and n is number of years
- Use currency exchange rates to convert between the Australian dollar and foreign currencies.
- Calculate the dividend paid on a portfolio of shares, given the dividend yield or dividend paid per share, and compare share values by calculating a price-to-earnings (P/E) ratio.
  - $\text{dividend yield} = \dfrac{\text{dividend}}{\text{share price}} \times 100$
  - $\text{P/E ratio} = \dfrac{\text{market price per share}}{\text{annual earnings per share}}$
- Use a spreadsheet to display examples of the above computations when multiple or repeated computations are required, e.g. preparing a wage sheet displaying the weekly earnings of workers in an organisation, preparing a budget, investigating the potential cost of owning and operating a car over a year.

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3.1

Prior learning
Prices and shares

Videos (5):
3.1 Unit pricing
3.2 GST
3.3 Simple interest formula • Simple interest rate
Exam question analysis Prices and shares

Prior learning (1):
3.1 Prices and shares

Worksheets (12):
3.2 Repeated percentage change • Profit and loss
3.3 Simple interest 1 • Simple interest 2 • Simple interest – Buying on terms
3.4 Currency exchange • Currency rate exchange • Currency conversion graph
3.5 Shares and dividends • Share tables • Share graphs
Cumulative exams General Maths Year 11 formula sheet

Puzzle (1):
3.1 Best buys riddle

Spreadsheet (1):
3.3 Simple interest

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

buying price	cost price	currency	discount
dividend	dividend yield	exchange rate	face value
goods and services tax (GST)	loss	market price	mark-up
per annum (p.a.)	price-to-earnings (P/E) ratio	principal	profit
selling price	share	shareholder	simple interest
unit cost			

Video
Unit pricing

Puzzle
Best buys riddle

3.1 Comparing prices

Generally, you want to get the best deal or value for money when you go shopping or buy something online. However, not all products are sold in the same packet size or in the same quantity.

How do you know what is the best value? You can compare prices by using the **unit cost** method. The unit cost is the cost per unit of an item such as \$/kg, \$/L. The unit cost method involves comparing the prices of the same amount of the same product to work out which is the **best value for money**.

Comparing prices

To compare prices, consider the

- unit cost: the cost of 1 item, 1 litre, 1 kilogram, etc
- quantity: how much do you actually need?
- quality: is the quality acceptable?

WORKED EXAMPLE 1 The unit cost method

You need to buy shampoo. Your favourite brand comes in 3 different sizes: 300 mL for \$7.00, 400 mL for \$10.50 and 600 mL for \$19.00. Which size of shampoo is the best value?

Steps	Working
1 Write the quantities for each size.	300 mL for \$7.00 400 mL for \$10.50 600 mL for \$19.00
2 Choose a convenient unit for each.	$300\text{ mL} = \frac{300}{100} = 3$ units of 100 mL $400\text{ mL} = \frac{400}{100} = 4$ units of 100 mL $600\text{ mL} = \frac{600}{100} = 6$ units of 100 mL
3 Calculate the unit prices.	\$7.00 ÷ 3 = \$2.33/100 mL \$10.50 ÷ 4 = \$2.63/100 mL \$19.00 ÷ 6 = \$3.16/100 mL
4 Write the answer.	The best buy is 300 mL for \$7.00.

In the above example, it doesn't matter which size of shampoo you buy as it is likely you will use it all because Shampoo doesn't go off over time. Sometimes, you need to also consider the unit price and quantity.

WORKED EXAMPLE 2 The unit cost method and quantity

Julie lives by herself and only uses milk in her 2 nightly cups of tea. She uses an average of 70 mL of milk per cup of tea. She can buy 3 L of milk for \$4.50, 2 L for \$3.10, 1 L for \$1.60 or 600 mL for \$1.30. Which size of milk should she buy?

Steps	Working
1 Write the quantities for each size in the same unit (litres) and calculate the unit prices.	3 L for \$4.50 $\rightarrow \frac{4.50}{3} = \$1.50/\text{L}$ 2 L for \$3.10 $\rightarrow \frac{3.10}{2} = \$1.55/\text{L}$ 1 L for \$1.60 $\rightarrow \frac{1.6}{1} = \$1.60/\text{L}$ 600 mL for \$1.30 $\rightarrow \frac{1.30}{0.6} = \$2.17/\text{L}$
2 Consider the quality of the item.	Milk goes off 4–5 days after opening.
3 Estimate the quantity required.	8 to 10 cups × 70 mL/cup = 560 to 700 mL
4 Consider the unit prices *and* quantity.	Although 3 L for \$4.50 is the best value, it would go off before Julie would use it all. 1 L for \$1.60 would last 5 days but she would waste at least 300 mL. 600 mL for \$1.30 would last 4 days, so she would have to get more after 4 days.
5 Compare the costs over a common period. $4 \times 5 = 20$	For 20 days using 1 L, $4 \times \$1.60 = \$6.40$. For 20 days using 600 mL, $5 \times \$1.30 = \$6.50$.
6 Write the answer.	Julie should buy the 1 L bottle of milk.

It is law in Australia for sellers to display the unit costs of food; however, many supermarkets show unit cost only in quite small print on the shelf label.

Exam hack

Make sure you compare measurements in the same units to calculate the best value.

EXERCISE 3.1 Comparing prices

ANSWERS p. 398

Mastery

1 WORKED EXAMPLE 1 For each of the following, determine

- **i** the unit cost correct to 2 decimal places.
- **ii** which size is the better value.

a Children's toothpaste costs \$1.71 for 45 g or \$2.49 for 110 g

b Caster sugar is \$1.86 for 500 g or \$2.64 for 1 kg

c A 300 mL bottle of sauce for \$1.56 or a 500 mL bottle for \$2.75

d A 10-pack of long-life milk for \$23.70 or a 12-pack for \$27.60

2 WORKED EXAMPLE 2 Collette needs 3 tablespoons (90 g) of tomato paste for a Madras curry. She can buy 200 g sachets for \$2.00 each, 140 g tubs for \$1.80 each or a 500 g bottle for \$3.25. She uses tomato paste about once a week. Which should she buy?

3 Gary lives on his own. He can buy 1.1 kg of diced beef for \$17.60, 750 g of round steak for \$11.24, 900 g of chuck steak for \$12.60 or 550 g of gravy beef for \$7.98. All are suitable for the stew he wants to make. Which is the best buy?

Exam practice

4 Which is the best buy of Vegemite for a family?

A 560 g jar for \$8.80

B 145 g tube for \$4.46

C 380 g jar for \$6.60

D 150 g jar for \$3.58

5 (2 marks) CF A big Swedish store sells 3 types of wrapping paper:

- Glittrande Gåvor (Glittering Gifts) – 5 m of wrapping paper for \$8
- Vintermagi (Winter Magic) – contains 10 m of wrapping paper for \$15
- Pastell Drömmar (Pastel Dreams) – 20 m of wrapping paper for \$20

List the wrapping papers in order of best to least value buy.

Worksheets
Repeated percentage change
Profit and loss

3.2 Percentage applications

Mark-ups and discounts

When the price of goods or services is increased by an amount or percentage, it's called a **mark-up**.

When the price of goods or services is decreased by an amount or percentage, it's called a **discount**. If it is expressed as a percentage, it is called a percentage discount.

Mark-ups and discounts

Mark-ups	Discounts
If an item is marked up by r%: mark-up = original price $\times \frac{r}{100}$ new price = original price + mark-up $= \text{original price} \times \frac{(100+r)}{100}$	If an item is discounted by r%: discount = original price $\times \frac{r}{100}$ new price = original price – discount $= \text{original price} \times \frac{(100-r)}{100}$
percentage mark-up (r) $= \frac{\text{mark-up}}{\text{original price}} \times 100$	percentage discount (r) $= \frac{\text{discount}}{\text{original price}} \times 100$
original price = new price $\times \frac{100}{(100+r)}$	original price = new price $\times \frac{100}{(100-r)}$

WORKED EXAMPLE 3 Calculating mark-up and discount

a A bakery puts a mark-up of 65% on a loaf of bread. If the bread costs on average \$0.94 to make, **calculate** the mark-up.

b A television is priced at \$675. In a flash sale, it is discounted by 10%. Find the discount.

Steps	Working
a 1 Identify key information.	original price = \$0.94, r = 65%
2 Write the formula, substitute the values into the formula and evaluate.	mark-up = original price $\times \frac{r}{100}$ $= 0.94 \times \frac{65}{100}$ $= 0.611$
3 Answer the question, rounding where appropriate.	The mark-up to customers is \$0.61.
b 1 Identify key information.	original price = \$675, r = 10%
2 Use the formula.	discount = original price $\times \frac{r}{100}$ $= 675 \times \frac{10}{100}$ $= 67.5$
3 Answer the question, rounding where appropriate.	The discount is \$67.50.

WORKED EXAMPLE 4 Calculating new prices

a A computer costs the retailer \$460. The retailer marks the computer up by 45%. What is the selling price of the device?

b A shoulder bag is priced at \$120 and in an end-of-season sale is discounted by 30%. What is the new selling price?

Steps	Working
a 1 Identify the key information.	original price = \$460, r = 45%
2 Write the formula, substitute the values into the formula and evaluate.	new price = original price $\times \frac{(100 + r)}{100}$ $= 460 \times \frac{145}{100}$ $= \$667$
3 Answer the question, rounding where appropriate.	The selling price of the computer is \$667.
b 1 Identify key information.	original price = \$120, r = 30%
2 Use the formula.	new price = original price $\times \frac{(100 - r)}{100}$ $= 120 \times \frac{70}{100}$ $= \$84$
3 Answer the question, rounding where appropriate.	The new selling price after the discount is \$84.

WORKED EXAMPLE 5 Percentage mark-ups or discounts

The initial cost of a bathrobe is \$60.

a What percentage mark-up is applied to the bathrobe if the cost is increased to \$76?

b What percentage discount is applied to the bathrobe if the cost is reduced to \$42?

Steps	Working
a **1** Identify key information.	original price = \$60, new price = \$76, mark-up = \$16
2 Write the formula, substitute the values into the formula and evaluate.	$r = \frac{\text{mark-up}}{\text{original price}} \times 100$ $= \frac{16}{60} \times 100$ $= 26.666\,66\ldots\%$
3 Answer the question, rounding where appropriate.	The percentage mark-up is 26.67%.
b **1** Identify key information.	original price = \$60, new price = \$42, discount = \$18
2 Use the formula.	$r = \frac{\text{discount}}{\text{original price}} \times 100$ $= \frac{18}{60} \times 100$ $= 30\%$
3 Answer the question, rounding where appropriate.	The percentage discount is 30%.

WORKED EXAMPLE 6 The original price from a mark-up or discounted price

Stefano bought a pair of sunglasses in "30% off everything" sale for \$35.
How much were the sunglasses originally?

Steps	Working
1 Identify key information.	new price = \$35, $r = 30\%$
2 Use the formula.	original price = new price $\times \frac{100}{(100 - r)}$ $= 35 \times \frac{100}{(100 - 30)}$ $= 35 \times \frac{100}{70}$ $= \$50$
3 Answer the question.	The sunglasses were originally \$50 before the sale.

Exam hack

Check your answer by applying the percentage discount or percentage mark-up to the value. In Worked example 6, 30% of \$50 = \$15, giving \$50 – \$15 = \$35. OK!

GST

The **goods and services tax (GST)** is currently a 10% tax on most sales and services in Australia. It is collected at the point of sale as part of the total price of the item.

An easy way to think about GST is that it is simply a mark-up where:

- the original price = price without the GST
- the new price = price with the GST
- the change in price = the GST amount
- $r = 10\%$.

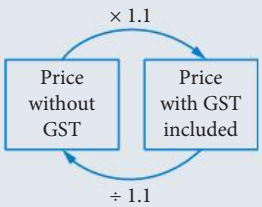
GST calculations

$r = 10\% = 0.1$

GST = price without GST × 0.1

GST = price with GST – price without GST

price with GST = price without GST × 1.1



WORKED EXAMPLE 7 Calculating GST

a If the cost of electricity this quarter to your house is $315.50, how much GST will be added to the bill?

b The selling price of a car is $59 620 including GST. What is the price of the car without GST?

Steps	Working
a **1** Use the GST formula and evaluate to the nearest cent.	GST = price without GST × 0.1 = 315.50 × 0.1 = $31.55
2 Answer the question.	The GST is $31.55.
b **1** Use the formula and evaluate.	price without GST = price with GST ÷ 1.1 = 59 620 ÷ 1.1 = $54 200
2 Answer the question.	The price of the car without GST is $54 200.

WORKED EXAMPLE 8 Exploring the effect of price change on GST payable

At a sale, a department store discounts electrical goods by 25%. A refrigerator has a marked price of $682.

a What is the discounted price?

b What is the change in the amount of GST included in the price?

Steps	Working
a **1** Identify key information.	marked-up price = $682, $r = 25\%$
2 Use the formula.	new price = original price × $\frac{(100 - r)}{100}$ $= 682 \times \frac{75}{100}$ = $511.50
3 Answer the question.	The new selling price after the discount is $511.50.

Video GST

b	**1** Find the GST on the marked price.	price without GST = price with GST ÷ 1.1 = 682 ÷ 1.1 = \$620 GST = price with GST – price without GST = 682 – 620 = \$62 You can also divide the GST-inclusive price by 11 to find the GST: \$682 ÷ 11 = \$62. The GST on the marked price is \$62.
	2 Find the GST on the discounted price.	discounted price without GST = price with GST ÷ 1.1 = 511.5 ÷ 1.1 = \$465 GST = price with GST – price without GST = 511.50 – 465 = \$46.50 The GST on the discounted price is \$46.50.
	3 Find the change in GST.	change in GST = 62 – 46.50 = \$15.50
	4 Answer the question.	The change in GST is \$15.50.

Profit and loss

All businesses work on the basic principle of **profit** and **loss**.

Profit and loss are usually expressed in absolute terms (the amount of money) or percentage terms (the amount of money as a percentage of the income).

Profit and loss

Cost price (or **buying price**) is the amount of money paid for an item by a business.
Selling price is the amount of money an item is sold for by a business.
A profit is made when items are sold for more than they cost to produce.
A loss is made when items are sold for less than what they cost to produce.

Profit	Loss
profit = selling price – cost price	loss = cost price – selling price
profit percentage = $\frac{\text{profit}}{\text{cost price}} \times 100$	loss percentage = $\frac{\text{loss}}{\text{cost price}} \times 100$
selling price = cost price × $\frac{100 + P\%}{100}$	selling price = cost price × $\frac{100 - L\%}{100}$

WORKED EXAMPLE 9 Finding profit and loss percentages

A market stall holder bought 120 vintage vinyl records for \$666. She priced them at \$12 each, but at 2 pm she still had 30 left. She dropped the price to \$5 each and sold the rest before the market closed at 3 pm. She did not have to pay GST.

a What percentage profit did she make on the records she sold for \$12 each?

b What percentage loss did she make on the records she sold for \$5 each?

c The stall cost her \$50 for the day. What percentage profit did she make overall?

Steps	Working
a 1 Find the cost price of a record.	cost price $= \frac{666}{120}$ $= \$5.55$
2 Find the profit.	profit = selling price − cost price $= 12 - 5.55$ $= \$6.45$
3 Find the percentage profit.	profit percentage $= \frac{\text{profit}}{\text{cost price}} \times 100$ $= \frac{6.45}{5.55} \times 100$ $= 116.21621\ldots$ $\approx 116.2\%$
b 1 Calculate the loss.	loss = cost price − selling price $= \$5.55 - \$5$ $= \$0.55$
2 Find the percentage loss.	percentage loss $= \frac{\text{loss}}{\text{cost price}} \times 100$ $= \frac{0.55}{5.50} \times 100$ $\approx 9.9\%$
c 1 Find the total sales.	total sales $= 90 \times \$12 + 30 \times \$5$ $= \$1230$
2 Find the total cost.	total cost $= \$666 + \$50$ $= \$716$
3 Find the net profit.	net profit $= \$1230 - \$716$ $= \$514$ Net profit means with all costs taken out.
4 Find the net profit percentage.	net profit percentage $= \frac{\text{net profit}}{\text{total cost}} \times 100\%$ $= \frac{514}{716} \times 100\%$ $\approx 71.8\%$
5 Write the answer.	The overall percentage profit is 71.8%.

EXERCISE 3.2 Percentage applications

ANSWERS p. 398

Recap

1 The unit cost (per gram) of a 150-gram tube of tomato paste sold at a supermarket for \$4.10 is

A \$27.33 **B** \$0.27 **C** \$0.03 **D** \$0.02

2 Adam is very health conscious. Which is the better buy for him to put on his bread?

A Unsalted butter \$4.00 for 500 g

B Organic spreadable butter \$6.00 for 200 g

C Normal butter \$1.85 for 250 g

D Olive oil spread \$3.50 for 500 g

Mastery

Ignore GST in these questions unless otherwise stated.

3 WORKED EXAMPLES 3, 4 Complete the following table.

	Item	Cost price	Percentage mark-up	Marked price
a	Table	800	75%	
b	Software bundle		90%	\$570
c	Bottle of soft drink	\$3.00		\$4.50

4 WORKED EXAMPLE 5 A furniture store sells a three-piece lounge suite for \$5000, or the chairs and sofa separately at \$1400 for each chair and \$2800 for the sofa. The cost price of the chairs was \$760 and the cost price of the sofa was \$1240. Calculate the percentage mark-up on the

a lounge suite as a set **b** sofa sold separately.

5 WORKED EXAMPLE 6 A food truck works on a mark-up of 180%. What is the cost price of each item?

a Spring roll sold for \$3.20 **b** Cod sold for \$6.20 a piece

6 After a fire, smoke-damaged goods are sold at a discount of 15% to clear stock.

a What is the discount price of a tracksuit with an original price of \$175?

b A jacket is marked down to \$238. What was the original price?

7 A hardware store has a '$12\frac{1}{2}$% off' day when everything in the shop is discounted by $12\frac{1}{2}$%. The garden section is discounting wheelbarrows by \$10 on top of the $12\frac{1}{2}$%.

a What are the final price and the discount on a wheelbarrow marked at \$72?

b What is the overall percentage discount?

8 WORKED EXAMPLES 7, 8 A furniture shop paid \$120 for a coffee table and sold it at a mark-up of 65%.

a How much GST does the customer pay?

b What is the final price of the coffee table?

c How much GST was included in the price paid by the shop?

9 WORKED EXAMPLE 9 A new car dealer values a trade-in at \$8000 to help clinch the sale of a new car, but can only get \$7500 for it from a wholesale dealer. What percentage loss does the car dealer make on the trade-in?

10 An agricultural supplier works on a percentage profit of 28%. Drums of liquid fertiliser are sold for $582.40 each. What was the cost price?

11 A second-hand dealer bought the following items: fishing reel ($15), video camera ($220), jewellery box ($35), filing cabinet ($80) and printer ($340). The fishing reel sold the next day for $20, the video camera sold for $290 and the filing cabinet sold for $130. The jewellery box and printer were in the window for several weeks, and eventually sold for $25 and $255 respectively. Calculate the percentage profit or loss on each item.

Exam practice

12 (4 marks) Mead Murphy, an antique dealer, made a profit of $171 when he sold an antique bisque doll. His percentage profit was 57%. What did he pay for the doll? What was the sale price?

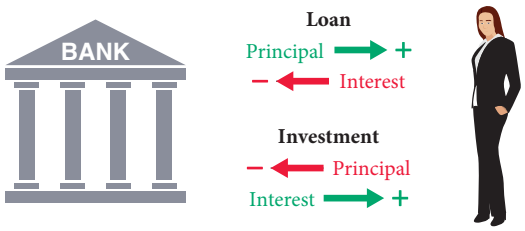
13 (3 marks) CF Discounts of 10% and 20% are applied successively. Explain why the total discount is not 30%.

14 (4 marks) CU Peter made 15% profit when he sold some pine bookcases he had made, taking all costs into account. If he considered only the cost of the wood, it would have been a 55% profit. The sale price was $372 for each bookcase. Calculate how much the wood for one bookcase cost.

3.3 Simple interest

You have to pay to borrow money and you get paid when you invest money. The money paid in both cases is called **interest**. It is paid for the use of money.

The amount that is borrowed or invested is called the **principal**.



Simple interest (also called **flat-rate** interest) is a fixed percentage of the amount borrowed or invested. It is calculated on the original amount. The amount is calculated for each year. This is called **per annum (p.a.)**.

Simple interest

$$I = Pin$$

where I = total interest
P = principal
i = **interest rate** as a decimal per year
n = period of time for the investment or loan in years.

$$A = P + I$$

where A = total value after n years.

Videos
Simple interest formula
Simple interest rate

Worksheets
Simple interest 1
Simple interest 2
Simple interest – Buying on terms

WORKED EXAMPLE 10 Calculating simple interest

Fina invests \$7015 for 2 years into a term deposit account that returns 3% p.a. simple interest. How much will Fina's investment be worth at the end of this period?

Steps	Working
1 Identify the key information.	$I = ?$, $P = \$7015$, $i = 3\% = 0.03$, $n = 2$
2 Calculate the interest.	$I = Pin$ $= \$7015 \times 0.03 \times 2$ $= \$420.90$
3 Calculate the total value of the investment.	$A = P + I$ $= \$7015 + \$420.90$ $= \$7435.90$
4 Answer the question, rounding as appropriate.	The value of Fina's investment will be \$7435.90.

WORKED EXAMPLE 11 Finding *P*, *i* or *n* for simple interest

a An investment of \$15 000 earns \$2000 over 3 years. What is the simple interest rate?

b An investment of \$5000 earns 5.5% p.a. simple interest. How long until it is worth \$6000?

Steps	Working
a 1 Identify the key information.	$I = \$2000$, $P = \$15\,000$, $i = ?\% = 0.03$, $n = 3$
2 Substitute into the formula and solve for *i*.	$I = Pin$ $2000 = \$15\,000 \times i \times 3$ $2000 = 45\,000 \times i$ $\frac{2000}{45\,000} = i$ $0.044\,44\ldots = i$
3 Answer the question, rounding as appropriate.	The simple interest rate is 4.44% p.a.
b 1 Identify the key information.	$I = 6000 - 5000 = 1000$, $P = \$5000$, $i = 5.5\% = 0.055$, $n = ?$
2 Calculate the interest.	$I = Pin$ $1000 = 5000 \times 0.055 \times n$ $1000 = 275 \times n$ $\frac{1000}{275} = n$ $3.636\,36\ldots = n$
3 Answer the question, rounding as appropriate.	The investment will need 3.7 years to grow by \$1000.

Note: the *n* value was rounded up to get *at least* \$1000.

WORKED EXAMPLE 12 Calculating simple interest over different time periods

Annabelle invested \$75 000 in a 30-day investment at 6.4% p.a. simple interest. What is the value of her investment at the end of the term?

Steps	Working
1 Calculate the interest.	$I = Pin$ $= \$75\,000 \times 0.064 \times \frac{30}{365}$ $= \$394.52$
2 Calculate the total value of the investment.	$A = P + I$ $= \$75\,000 + \$394.52$ $= \$75\,394.52$
3 Answer the question, rounding as appropriate.	The value of Annabelle's investment will be \$75 394.52.

TECHNOLOGY Simple interest spreadsheet

You can make a spreadsheet for simple interest calculations.
Put the following headings in the cells indicated.

A3: Principal	A4: Interest rate %	A5: Term	A6: Payments/year
D3: Loan	D4: Total	D5: Instalment	D7: Investment
D8: Lump sum	D9: Regular payment		

You can use cell formatting, colours and column width to make it look professional.
Put in, say, 20 000 for the principal, 6.8 for the interest rate, 4 for the term and 12 for the payments/year in cells B3 to B6.
The total to pay back for a loan is the principal + interest, $P + Pin$.
This translates to the formula **=B3+B3*B4/100*B5**. Put this formula in cell E4.
Don't forget the equals sign to make it a formula.
To get the instalments, divide by the total number of payments, **B5*B6**.
Type the formula **=E4/(B5*B6)** in cell E5.
You need the brackets to make sure the multiplication is calculated first.
An investment can give either a lump sum at the end or a regular payment.
For the lump sum, put the formula **=B3+B3*B4/100*B5** in cell E8.
For the regular payment, put the formula **=B3*B4/(100*B6)** in cell E9.
Try your spreadsheet with different problems.
A completed 'Simple interest' spreadsheet can be downloaded from Nelson MindTap.

	A	B	C	D	E
1	Simple interest				
2					
3	Principal	20000		Loan	
4	Interest rate %	6.8		Total	25440
5	Term	4		Instalment	530
6	Payments/year	12			
7				Investment	
8				Lump sum	25440
9				Regular payment	113.33333

Spreadsheet
Simple interest

EXERCISE 3.3 Simple interest

ANSWERS p. 398

Recap

1 A local handyman offers a 12% discount for customers who pay in cash. How much would a customer pay if they paid their bill of $534 in cash?

A $469.92 **B** $64.08 **C** $598.08 **D** $522

2 GST is applied to a jumper with a marked price of $130. The new price of the jumper is

A $118.18 **B** $143 **C** $144.30 **D** $115.70

Mastery

3 WORKED EXAMPLE 10 Calculate the simple interest charged on each loan.

a $500 at 12% p.a. for 3 years

b $3200 at 15% p.a. for 8 months

c $4500 at 8% p.a. for 4 years

4 WORKED EXAMPLE 11 Determine the simple interest rate p.a. for each investment.

a $5000 amounted to $5750 after 2 years.

b $800 amounted to $1008 after 4 years.

c $9000 amounted to $10 312.50 after 2 years and 4 months.

5 WORKED EXAMPLE 12 What is the value of each investment at simple interest at the end of the term?

a $2100 at 17.5% p.a. for 9 months

b $7000 at 11% p.a. for 2.5 years

c $9000 at 13.9% p.a. for 4 years

6 The simple interest on a loan of $500 over 3 years is $225. What is the interest rate p.a.?

7 Julia borrowed $7500 at 8.3% p.a. simple interest for 5 years. Calculate the interest and fortnightly payment.

8 Elena borrowed $8000 at 13.8% p.a. simple interest over 3 years. How much does she pay back each fortnight?

9 Phan can invest in a fixed-interest account that earns 6.4% p.a. interest. How much must she invest to make $200 a week in interest?

10 An executive on a salary of $2400 a week wants to live on the simple interest to the amount of 60% of her salary when she retires. She can earn 6% p.a. interest. How much must she invest to reach her goal?

Exam practice

11 How much interest is earned if $3500 is invested for 1 year at a simple interest rate of 5% per annum?

A $350 B $3675 C $3325 D $175

12 (3 marks) CF A company borrowed $200 000 on the short-term money market for 60 days. The company had to repay $207 000. What interest rate p.a. did the company pay?

13 (4 marks) CU Carmen can invest $6000 in 2 ways. She can invest it at 12% p.a. simple interest for 2 years, or she can invest $2000 at 13% p.a. and the rest at 11.6% p.a. Calculate the return from each investment option and state which investment would give the greater return.

Worksheets
Currency exchange
Currency rate exchange
Currency conversion graph

3.4 Currency exchange

When you travel overseas or shop online from companies outside of Australia, our Australian dollar is converted to the **currency** of the country you are in or buying from. For example, if you travel to or purchase from the US, Britain or Japan, the Australian dollar is converted to US dollars, British pounds or Japanese yen respectively.

Exchange rates state what one currency, such as the Australian dollar (AUD), is worth in another currency.

The value of the Australian dollar changes, like other prices. It depends on the demand.

One Australian dollar (A$1) was worth the following amounts in some countries in 2024.

Country	Name and symbol	Code	Units per AUD
Canada	Dollar, C$	CAD	0.8805
China	Renminbi, Yuan	RMB, CNY	4.6930
Europe (most of)	Euro, €	EUR	0.6051
India	Rupees, ₹	INR	55.11
Japan	Yen, ¥	JPY	96.86
New Zealand	Dollar, NZ$	NZD	1.0672
South Africa	Rand, R	ZAR	12.32
United Arab Emirates	Dirham, د.إ	AED	2.3908
United Kingdom	Pound, £	GBP	0.5165
USA	Dollar, US$	USD	0.6523

For example, the exchange rate per AUD for the Indian rupee (INR) is 55.11, which means that one A$1 will be exchanged for (or equivalent to) 55.11 rupee: A$1 = 55.11 INR

Twenty Australian dollars would be exchanged for 20 times this amount: A$20 = 1102.20 INR.

3.4

Exchange rate calculations

To convert *from Australian dollars* (AUD) to a foreign currency:

foreign currency = AUD × exchange rate

To convert *to Australian dollars* (AUD) from a foreign currency:

$$\text{AUD} = \frac{\text{foreign currency}}{\text{exchange rate}}$$

When you exchange money, it can be actual money or a transfer between accounts.

Financial institutions do not keep foreign cash for transfers, so these cost less than cash.

WORKED EXAMPLE 13 Converting currency

Using the exchange rate 1 AUD = 0.5165 GBP to convert

a $575 Australian dollars to pounds

b £1230 to Australian dollars.

Steps	Working
a 1 Write the exchange rate.	1 AUD = £0.5165
2 Convert by using: foreign currency = AUD × exchange rate	$575 × 0.5165 = 296.9875
3 Answer the question, rounding as appropriate.	A$575 is equal to £296.99.
b 1 Convert by using: $\text{AUD} = \frac{\text{foreign currency}}{\text{exchange rate}}$	$\text{AUD} = \frac{1230}{0.5165}$ = $2381.413 359
2 Answer the question, rounding as appropriate.	£1230 is equal to A$2381.41.

EXERCISE 3.4 Currency exchange

ANSWERS p. 398

Recap

1 What is the interest rate, per annum, for a deposit of $2000 that earns interest of $75 over a period of 6 months?

A 3.75% **B** 1.875% **C** 7.5% **D** 0.13%

2 When the simple interest formula is transposed to make i the subject

A $i = \frac{I}{Pn}$ **B** $i = PIn$ **C** $i = \frac{100I}{Pn}$ **D** $i = n$

Mastery

Unless otherwise stated, use the table of exchange rates on page 98 for this exercise.

3 WORKED EXAMPLE 13 Copy and complete each statement.

a A$800 = US$_____ **b** A$1460 = US$_____

c A$_____ = US$3421 **d** A$_____ = US$550

4 Convert each amount into the currency indicated in the brackets. Round your answer to 2 decimal places.

a $430 AUD (NZ$)

b $5000 AUD (rupee)

c €1059 (AUD)

d د.إ12 000 (AUD)

5 Tamomi is on an exchange trip from South Africa to Australia and staying with a family in Brisbane. She has R60 400 to spend. How much is this equal to in Australian dollars?

Exam practice

6 (2 marks) When Yoshi came back from Japan, he had 40 000 yen left. How much would he get for it in AUD?

7 (2 marks) CF Cara thought that a sewing machine advertised in a British magazine was especially cheap at £800. What would she pay in Australian currency?

8 (2 marks) CU How much would 500 USD be worth in Japanese yen?

Worksheets
Shares and dividends
Share tables
Share graphs

3.5 Shares and dividends

You may invest your money in shares. A **share** is a unit of ownership in a company.

Shares can be bought and sold on the share market or the stock exchange. When a company first floats shares, the price the shares are sold for is called the **face value** of the share. The price shares are later sold for is called the **market price** or simply the share price.

The values of the shares go up and down depending on the profits and losses of the company.

All shares in a company are equal and as a **shareholder** you are entitled to a share of the company's profits.

Your share of the profit is called a **dividend**. There are 2 common ways the amount of the dividend is determined:

- as the number of dollars each share receives
- as a percentage of the current price of the shares, called the **dividend yield**.

To invest in shares, you should consider the **price-to-earnings (P/E) ratio**. The P/E ratio indicates how much shares cost per dollar of the profits earned (also called earnings). The higher the ratio, the more you will have to invest for each dollar or profit. The lower the ratio, the less you need to invest for each dollar of profit, which is better for you!

If the market value of a share is $30 and earnings are $6, the P/E ratio is 30 ÷ 6 = 5. This is as if you pay $5 for every $1 in earnings. Across all shares, a P/E ratio of about 15 is normal. It varies between sectors so you should only compare P/E ratios of companies in similar industries.

Dividend yield and P/E ratio

$$\text{dividend yield} = \frac{\text{dividend}}{\text{share price}} \times 100\%$$

$$\text{price-to-earnings (P/E) ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$$

WORKED EXAMPLE 14 Dividends and percentage profit

a Nicole owns 500 shares in Company Y. If there are a total of 8000 shares in the company, what percentage of the company does Nicole own?

b Company Y decides to raise \$100 000 by selling a further 12 000 new shares. What is the face value of these new shares?

c The company paid a dividend of \$2.30 per share. How much did Nicole receive?

d Nicole bought her shares for \$5.63 and now sells them for \$9.77. What percentage profit did she make (ignore any fees she may have had to pay)?

Steps	Working
a 1 Convert 500 out of 8000 to a percentage.	percentage ownership $= \frac{500}{8000} \times 100$ $= 6.25\%$
2 Answer the question.	Nicole owns a 6.25% share in the company.
b 1 Divide the total money to be raised by the number of shares to be sold.	face value $= \frac{100\,000}{12\,000}$ $=$ \$8.33/share
2 Answer the question.	The new shares will be initially sold for \$8.33.
c 1 Multiply the number of shares owned by the dividend to be paid.	amount received $= 500 \times 2.30$ $= 1150$
2 Answer the question.	Nicole will be paid \$1150.
d 1 Work out Nicole's profit for selling a share.	profit = sale price − original cost $= 9.77 - 5.63$ $=$ \$4.14
2 Use the formula: $\frac{\text{profit}}{\text{original}} \times 100$	% profit $= \frac{4.14}{5.63} \times 100$ $= 73.534\,63\ldots$
3 Answer the question.	Nicole's percentage profit is 73.53%.

WORKED EXAMPLE 15 Dividends and dividend yield

a The current market price of a certain share is \$15.60 and the company recently paid a dividend of \$3.50 per share. What was the dividend yield, correct to 2 decimal places?

b If Yui owns 120 000 shares worth \$0.45 per share and the dividend yield is 4%, what would be the total dividend Yui would receive?

Steps	Working
a Use the formula for dividend yield, substitute in the values and evaluate.	dividend yield $= \frac{3.50}{15.60} \times 100\%$ $\approx 22.44\%$
b 1 Find the total value of the shares.	total value $= 120\,000 \times 0.45$ $=$ \$54 000
2 Use the formula for dividend yield, substitute in the values, rearrange and evaluate.	dividend yield $= \frac{\text{dividend}}{\text{share price}} \times 100\%$ dividend $= \frac{4}{100} \times 54\,000$ $= 2160$
3 Answer the question.	The dividend Yui would receive is \$2160.

WORKED EXAMPLE 16 Calculating the P/E ratio

Anwar bought 5000 shares for \$48 000 with a dividend yield of 3%. The company pays 25% of its earnings each year in dividends and the earnings per share are \$1.16. **Determine** the

a market value of the shares

b dividend per share

c P/E ratio, correct to 2 decimal places.

Steps	Working
a Divide the cost Anwar paid for the shares by the number of shares bought.	market price $= \frac{48\,000}{5000}$ $= \$9.60$
b Use the dividend yield to calculate the dividend per share.	dividend per share $= \$9.60 \times \frac{3}{100}$ $= \$0.29$
c Use the formula, substitute and evaluate.	P/E ratio $= \frac{\text{market price per share}}{\text{annual earnings per share}}$ $= \frac{9.60}{1.16}$ $= 8.28$

EXERCISE 3.5 Shares and dividends

ANSWERS p. 398

Recap

1 Greg buys a game online for US\$90. If the exchange rate is A\$1 for US\$0.6523, calculate how much he pays in Australian dollars, to the nearest cent.

A \$137.97 **B** \$58.70 **C** \$75.89 **D** \$150.44

2 How much would a computer priced at A\$980 cost in the UK if A\$1 = 0.5165(GBP)?

Mastery

3 WORKED EXAMPLE 14 Wally owns 800 of the 100 000 shares available in the XMPI Company.

a What percentage of the company does Wally own?

b If the XMPI company has earnings of \$5 000 000, how much of these earnings does Wally have a claim to?

4 Flight by Night is an investment company with 100 000 shares available. Phil owns 9% of them.

a How many shares does Phil own?

b If the company has annual earnings of \$5 million dollars, how much is Phil entitled to?

c How many more shares would Phil need to buy to get an annual earnings share of \$500 000?

5 A company wishes to raise \$10 million by selling 500 000 new shares. Calculate the value of each share.

6 Alice owns 600 shares in UXW Co. The current market price of the UXW Co. shares is \$6.90 per share and the company has declared a dividend of \$0.35 per share.

a How much does Alice receive in dividends in total?

b What is the percentage dividend yield for this share, correct to one decimal place?

7 WORKED EXAMPLE 15 What is the dividend yield for shares with a share price of \$17.70 and paying a dividend of 90 cents?

8 Calculate the total dividend to Vee, who owns 1800 shares at \$23.20 per share, if the dividend yield is 4.6%.

9 WORKED EXAMPLE 16 Shares in Another Chance Pty Ltd have a market value of \$35.80, and the company has annual earnings of \$5.67 per share. Shares in Because I Can Pty Ltd have a market value of \$10.10 and the company has annual earnings of \$0.90 per share for the same 12-month period.

a What is the price-to-earnings (P/E) ratio for each company for the given time period?

b Which shares have been a better investment?

Exam practice

10 (2 marks) CF The price-to-earnings (P/E) ratio of a company this financial report is 18. If the company has 600 000 shares and declared annual earnings of \$5 000 000, what is the current price of the shares?

EXAM QUESTION ANALYSIS

Exam-style question (5 marks)

After their last financial report, the Retro Industries Ltd had a price-to-earnings (P/E) ratio of 21. If the annual earnings declared were \$10 000 000 and there were 2 500 000 shares, determine the current price of the shares.

Video
Exam question analysis: Prices and shares

Reading the question

- The question is about P/E ratios and share prices.
- P/E ratio, annual earnings and number of shares has been given.

Thinking about the question

- P/E ratio formula will need to be used.
- P/E ratio has been given, so another variable in the formula is missing.
- The answer needs to be the current share price, or the market price per share in the formula.

Worked Solution (✓ = 1 mark)

annual earnings per share = 10 000 000 ÷ 2 500 000 = **4** ✓

$$\text{P/E ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}} \checkmark$$

P/E ratio = 21, let x represent market price per share.

$$21 = \frac{x}{4} \checkmark$$

$$21 \times 4 = x$$

$$\mathbf{\$84 = x} \checkmark$$

The current market price is **\$84**. ✓

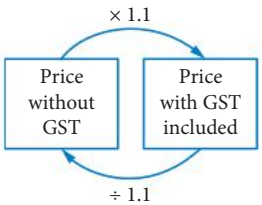
3 Chapter summary

Comparing prices

- **Unit price** is the cost of 1 item, 1 kg, 1 litre, etc.
- When comparing prices, consider the:
  - unit cost: the cost of 1 item, 1 litre, 1 kilogram, etc.
  - quantity: how much do you actually need?
  - quality: is the quality acceptable?
- An item of acceptable quality at the lowest unit price is the best value to a shopper.

Percentage applications

- **Cost price** (or **buying price**) is the amount of money paid for an item by a shop.
- **Mark-up** is the amount added to the cost price of an item for sale.
- **Marked price** (or **selling price**) = cost price + mark-up
- A **discount** is an amount taken off the marked price.
- In Australia, the **goods and services tax (GST)** is added to the price of an item or service and paid to the government by the seller. It is currently 10% of the cost price and must be included in retail prices by shops.



- A **profit** is made when items are sold for more than they cost to produce.

  profit = selling price – cost price
- A **loss** is made when goods are sold for less than what they cost to produce.

  loss = cost price – selling price
- **Percentage** mark-ups, discounts, profits and losses are usually 'out of' the cost price.

Simple interest

- **Simple interest** is also called flat-rate interest.
- For simple interest,

$$I = Pin$$

where I = total interest

P = principal

i = interest rate as a decimal per year

n = period of time for the investment or loan in years.

$$A = P + I$$

where A = total value after n years.

Currency exchange

- The money of a country is called its **currency**.
- To convert *from* Australian dollars (AUD) to a foreign currency:

  foreign currency = AUD × exchange rate
- To convert *to* Australian dollars (AUD) from a foreign currency:

$$\text{AUD} = \frac{\text{foreign currency}}{\text{exchange rate}}$$

Shares and dividends

- **share:** part of the ownership of a company
- **market price:** the current price that shares are sold for on the stock exchange
- **face value:** the original price of a share
- **dividend:** the amount paid for each share from company earnings.
- dividend yield $= \dfrac{\text{dividend}}{\text{share price}} \times 100\%$
- price-to-earnings (P/E) ratio

$$= \frac{\text{market price per share}}{\text{annual earnings per share}}$$

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Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

Worksheet General Maths Year 11 formula sheet

1 Which of the following packets of flour is the best buy?

A \$2.10 for 500 g **B** \$3.35 for 750 g **C** \$4.71 for 1.2 kg **D** \$7.92 for 2 kg

2 An item was discounted by 20% and then a further discount of 30% was applied to the reduced price. What was the total percentage discount?

A 35% **B** 44% **C** 50% **D** 55%

3 Jayda invested \$4000 for 1 year and 8 months in a short-term deposit account earning 5% simple interest per annum. How much is her investment worth at the end of this time, correct to the nearest dollar?

A \$4333 **B** \$4300 **C** \$8000 **D** \$4360

4 If a circle has a diameter of 12 cm, what is its circumference?

A 24π **B** 12π **C** 6π **D** 3π

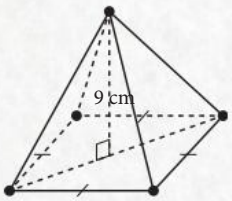
5 A square pyramid has a perpendicular height of 9 cm and a volume of 243 cm^3. What is the side length of the square base?

A 81

B 27

C 3

D 9



Section 2 4 short response questions

13 marks

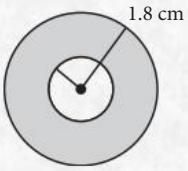
6 (2 marks) QMart makes a profit of 33% on all sales. If the cost price of a bedroom suite was \$580, for what price should QMart sell it?

7 (3 marks) Liz has 2000 shares in Awesome Investments. The current price of the shares is \$25.10, and the company declares a dividend of \$1.13 per share.

a How much does Liz receive in dividends in total? *[1 mark]*

b What is the dividend yield for this share, correct to one decimal place? *[2 marks]*

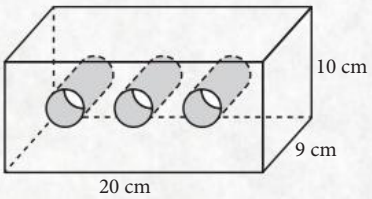
8 (3 marks) A washer is being designed in the shape of an annulus with an outer radius of 1.8 cm. The area of the face of the washer is to be 9.49 cm^2. What is the size of the inner radius of the washer, correct to 2 decimal places?



9 (5 marks) A house brick is made of a special type of clay and has dimensions 20 cm × 9 cm × 10 cm.

a What is the total volume of the clay brick? *[1 mark]*

Three identical cylindrical holes each with a radius of 1.5 cm are to be cut through the brick as shown.



b After the cylindrical holes have been cut out of the brick, what is the volume of the brick? Round your answer to the nearest cm^3. *[3 marks]*

c What percentage of clay was removed to make the cylindrical holes in the brick? Round your answer to one decimal place. *[1 mark]*

Cumulative examination 2

Complex familiar and unfamiliar

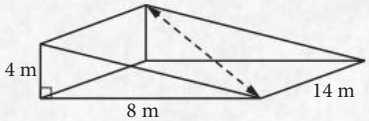
Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions

12 marks

1 (4 marks) A ramp is being constructed for a BMX competition. It has a rectangular base measuring 14 m by 8 m and is 4 m high at the point at which it connects to the start platform. Decorative tape is to be laid diagonally across the ramp. Determine the length of decorative tape needed.



Worksheet
General Maths Year 11 formula sheet

2 (4 marks) John has $10 000 to invest and is interested in 2 different investment products:

Product 1: 6% p.a. compounding annually

Product 2: 6.3% p.a. simple interest.

If John plans to invest in one of these accounts for 2 years, which would provide him with the best return? Justify your answer with appropriate calculations.

3 (4 marks) Cheyanne has found a pair of shoes to purchase online from a store in Switzerland. The shoes cost 147.66 CHF (Swiss francs). She knows that:

- one Australian dollar (AUD) = €0.6051 (EUR)
- one Swiss franc (CHF) = €1.062 69 (EUR)

Cheyanne's bank charges a foreign currency fee equivalent to 2.5% of the Australian dollar value of the purchase. Determine the total amount in AUD Cheyanne will pay for the shoes.

CHAPTER

4 LINEAR AND NON-LINEAR RELATIONSHIPS

Syllabus coverage
Nelson MindTap chapter resources
Terminology
4.1 Linear and non-linear expressions
4.2 Formulas
4.3 Transposing formulas and equations
4.4 Tables of values
Exam question analysis
Chapter summary
Cumulative examination 1
Cumulative examination 2

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Syllabus coverage

UNIT 1, TOPIC 4: ALGEBRA

Linear and non-linear relationships

- Substitute numerical values into linear and simple non-linear algebraic expressions, and evaluate.
- Find the value of a pronumeral in linear and simple non-linear equations given the values of the other pronumerals, transposing equations where necessary.
- Use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities.

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Videos (5):

4.2 Formulas • Formulas and equations

4.3 Changing the subject of a formula 1 • Changing the subject of a formula 2

Exam question analysis Linear and non-linear relationships

Prior learning (1):

4.1 Linear and non-linear relationships

Skillsheet (1):

4.1 Algebra using diagrams

Worksheets (7):

4.1 Expanding algebra • Algebra review • Collecting like terms

4.2 Working with formulas

4.3 Subject of a formula • Changing the subject of a formula

Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Prior learning
Linear and non-linear relationships

Terminology

algebraic expression	coefficient	constant (term)	dependent variable
equation	formula	independent variable	inverse operation
like terms	linear equation	linear expression	non-linear equation
non-linear expression	pronumeral	subject (of a formula)	term
transpose	variable		

Skillsheet
Algebra using diagrams

Worksheets
Expanding algebra

Algebra review

Collecting like terms

4.1 Linear and non-linear expressions

Terms and coefficients

- A **variable** (or **pronumeral**) is a symbol, often a letter, that stands for a number.
- A **constant** is a pure number.
- An **algebraic expression** (or **expression**) has variables and numbers connected by operations.
  For example, $5a + 4bc - 8d^2 + 12$ is an algebraic expression, where 12 is the constant.
- A **term** is part of the expression separated by + or – from the rest.
  For example, $5a + 4bc - 8d^2 + 12$ has 4 terms.
- Terms that have the same variables and powers are called **like terms**.
  For example, $4b^2$ and $10b^2$.
- A **coefficient** is a constant multiplied by a variable or variables.
  For example, in $5a + 4bc - 8d^2 + 12$, the coefficient of bc is 4.

WORKED EXAMPLE 1 Simplifying expressions

Simplify each expression.

a $8a + 4b - 5a + 7b + c$ **b** $3x^2 + 4xy - 7xx + 6$ **c** $2m \times 3n \times 6p$

Steps	Working
a **1** Group like terms.	$8a + 4b - 5a + 7b + c = 8a - 5a + 4b + 7b + c$
2 Add/subtract like terms.	$= 3a + 11b + c$
b **1** Group like terms.	$3x^2 + 4xy - 7xx + 6 = 3x^2 - 7x^2 + 4xy + 6$
2 Add/subtract like terms.	$= -4x^2 + 4xy + 6$
c **1** Multiply variables and coefficients separately.	$2m \times 3n \times 6p = 2 \times 3 \times 6 \times m \times n \times p$
2 Simplify.	$= 36mnp$

Linear equations and expressions

An **equation** has algebraic expressions and numbers on each side of an equals sign.

The equals sign makes it an equation instead of an expression.

$2a + 8$ is an expression and $2a + 8 = 10$ is an equation.

Linear expressions and **linear equations** have variables with a power of 1.

For example, both $2a + 8$ and $2a + 8 = 10$ are linear as the power of the variable a is 1.

However, $2d^2 - 7$ and $\sqrt{H} + 12 = J$ are not linear. The powers of d and H are not 1.

Graphs of linear equations are **straight lines**.

Non-linear expressions and **non-linear equations** have variables with a power not equal to 1. Their graphs are not straight lines but curves.

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WORKED EXAMPLE 2 Identifying linear expressions

Identify each expression as linear or non-linear.

a $3x - 5$ **b** $\frac{3u - 7}{5}$ **c** πr^2 **d** $2(m - 3)$ **e** $\frac{9 - 2x}{x}$

Steps	Working
a **1** Identify the variables.	The variables are y and x.
2 Write the power of the variable.	The power of x is 1.
3 The variable has a power of 1.	The equation is linear.
b **1** Write the expression as a difference of terms.	$\frac{3u - 7}{5} = \frac{3u}{5} - \frac{7}{5}$
2 Write the power of the variable.	The power of u is 1.
3 The variable has a power of 1.	The expression is linear.
c **1** Identify the variables.	The variables are A and r.
2 Write the power of the variable.	The power of r is 2.
3 The variable has a power not equal to 1.	The equation is non-linear.
d **1** Expand the expression.	$2(m - 3) = 2m - 6$
2 Write the power of the variable.	The power of m is 1.
3 The variable has a power of 1.	The expression is linear.
e **1** Write the expression as a difference of terms.	$\frac{9 - 2x}{x} = \frac{9}{x} - \frac{2x}{x}$
2 Simplify and write the equation with powers.	$= 9x^{-1} - 2$
3 Write the power of the variable.	The power of x is -1.
4 The variable has a power not equal to 1.	The expression is non-linear.

A graph can be used to determine whether a set of data or a relationship is linear.

Consider the sequence of squares formed using matches shown below.

1 square 2 squares 3 squares 4 squares ...

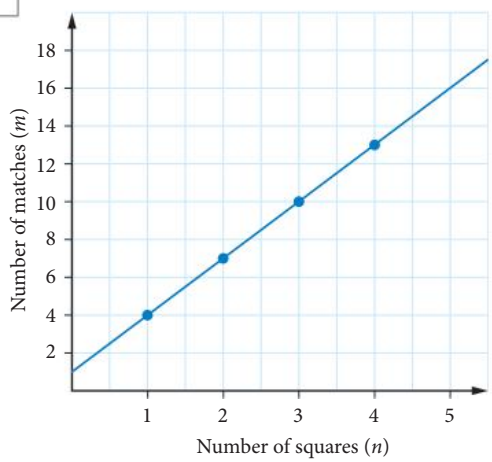
This sequence can be summarised in a table.

Number of squares (n)	1	2	3	4
Number of matches (m)	4	7	10	13

The number of matches m is dependent on how many squares n are to be created. In this situation, n is the **independent variable**. m is the **dependent variable** because its value depends on the value of n.

The graph has the independent variable on the horizontal axis and the dependent variable on the vertical axis.

The points are on a straight line, so it is a linear relationship.



WORKED EXAMPLE 3 Identifying linear relationships graphically

Determine whether each relationship is linear or non-linear by graphing it.

a $3x - 5$

b

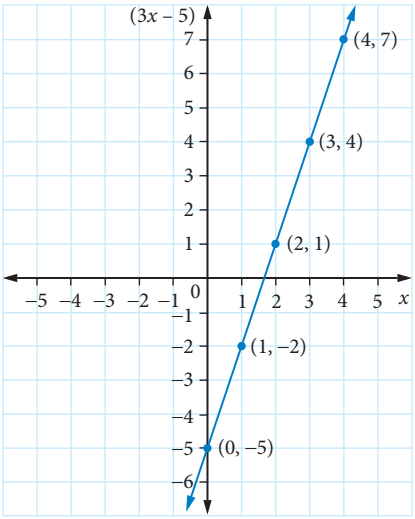
x	0	1	2	3	4	5
y	0	3	4	3	0	−5

Steps	Working
a **1** Create a table of values for x and $3x - 5$. x is the variable and $3x - 5$ is the expression.	

x	0	1	2	3	4
$3x - 5$	−5	−2	1	4	7

2 Plot the points and join them.

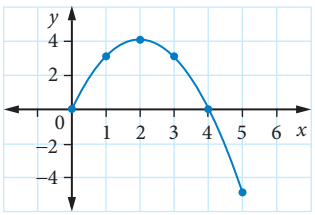
The independent variable is plotted on the horizontal axis.



3 Identify the relationship.

The graph is a straight line, so the expression is linear.

b **1** Plot the points and join them.



2 Identify the relationship.

The graph is not a straight line, so the expression is non-linear.

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In a linear relationship, as x increases by 1 or another constant amount, y increases by a constant amount. For a table of values, this means that the y values have the same difference between successive terms. This is called a **common difference**.

4.1

WORKED EXAMPLE 4 Identifying linear relationships using tables of values

Identify if each table of values shows a linear relationship by determining if there is a common difference.

a

Variable	1	2	3
Expression	100	95	90

b

x	–2	2	6	10
y	2	4	8	16

c

x	3	4	5
y	13	19	25

Steps		Working
a	**1** Check the increase of the independent variable.	The variable goes up by 1.
	2 Work out the difference between successive terms.	difference 1 to 2 = 100 – 95 = 5 difference 2 to 3 = 95 – 90 = 5 There is a common difference of 5.
	3 Identify the relationship.	The relationship is linear.
b	**1** Check that the x values increase by a constant amount.	x consistently goes up by 4.
	2 Work out the difference between successive y values. Two different differences are enough to show non-linearity.	difference –2 to 2 = 4 – 2 = 2 difference 2 to 6 = 8 – 4 = 4 There is no common difference.
	3 Identify the relationship.	The relationship is non-linear.
c	**1** Check that the x values increase by a constant amount.	x consistently goes up by 1.
	2 Work out the difference between successive y values.	difference 3 to 4 = 19 – 13 = 6 difference 4 to 5 = 25 – 19 = 6 There is a common difference of 6.
	3 Identify the relationship.	The relationship is linear.

TECHNOLOGY Non-linear expressions

The age and length of dugongs in Moreton Bay are measured in the field. The results are shown here.

Age (years)	1.0	1.5	2.5	4.0	5.0	7.0	8.0	9.5	12.0	13.0	13.0	14.5	15.5	16.5	20.0	22.5	29.0	31.5
Length (m)	1.8	1.9	2.0	2.3	2.3	2.4	2.5	2.4	2.5	2.5	2.5	2.6	2.7	2.6	2.7	2.7	2.7	2.8

1. Open a new spreadsheet and enter the headings 'Age' and 'Length' in cells A1 and B1 respectively.
2. Enter the ages shown here into cells A2–A19 and then enter the lengths into cells B2–B19.
3. Select the area A1:B19, click on the **Insert** tab, and then click on the **Charts** menu.
4. Select **Marked Scatter** to draw a scatterplot of the data.
5. Next, add a line of best fit for the data. The line of best fit is also called a trendline. Right click on any data point and select **Add Trendline…**. The Linear trendline will be selected, so press **OK**.
6. Try different types of trendlines until you find one that best fits the data.
7. Is the data linear or non-linear?

EXERCISE 4.1 Linear and non-linear expressions

ANSWERS p. 398

Mastery

1 WORKED EXAMPLE 1 Simplify each expression.

a $10g - 8g + 2h + 10$

b $8m - 4 - 2m + 5n$

c $3(p - 5) + p - 4$

d $2x^2 + 10 - 7x^2 - 8y + 3xy - 7$

e $-4 + 7q^2 + 3pq - 2q^2$

f $a - 5 + a + 10b + 4a - 3ab$

g $8m(2m + 1) - 2m(5m + 2 + 2m - 5)$

h $3(-7y + 5 + 8y) - 4(3 + y + 5)$

2 Simplify each expression.

a $a \times 4b^2 \times 2c$

b $3d \times d^2g^2 \times (-5)$

c $24xy \div 16x^2$

d $7y \times 2y^2 \times (-x^4) \times (-3xy)$

e $\dfrac{34x^4y^3}{28x^2y^2z}$

f $4p \times 6q^4 \times 2p \times p^3q$

g $36ab^2 \div (-12a^3b)$

h $\dfrac{38h^5j^3k^2}{42h^2jk^3}$

3 WORKED EXAMPLE 2 Classify each expression as linear or non-linear.

a $8a + 2$

b $3y - 2y + 4$

c $5x(2x - 7)$

d $\dfrac{1}{3}(2n - 7)$

e $\dfrac{8y - 2y^2}{6}$

f $4g + 3g - 9$

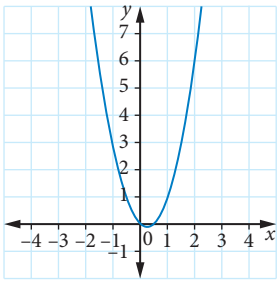
g $\dfrac{5s - 3t}{t^2}$

h $7c - 2\sqrt{c}$

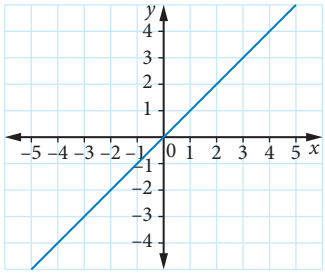
i $(4 + 3n)(n - 1)$

4 WORKED EXAMPLE 3 State whether each graph represents a non-linear expression.

a



b



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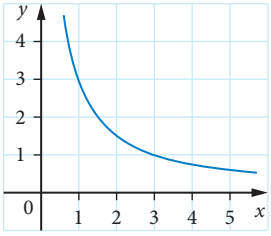

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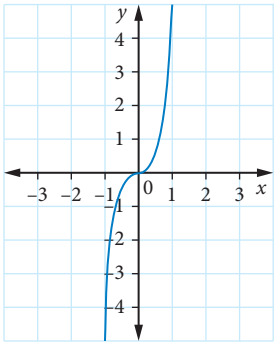

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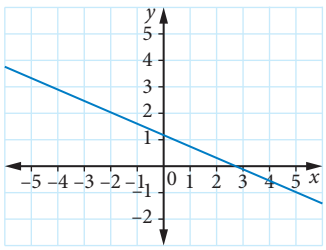

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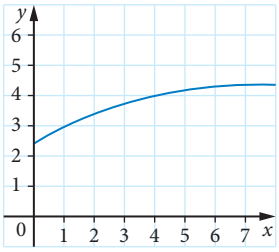

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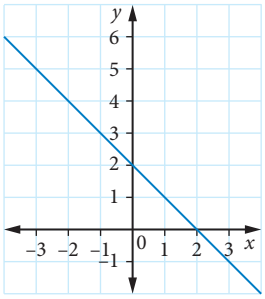

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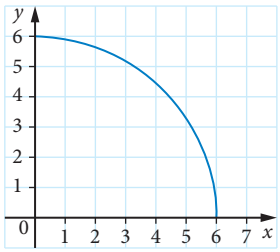

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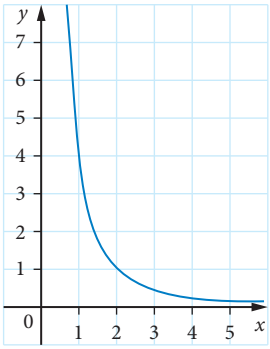

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5 Draw a graph of each expression to determine whether the expression is linear or non-linear.

a $7 - 2x$ **b** $9 - x^2$ **c** $x^3 + 2$

d $\frac{5x}{4} + 3$ **e** $2 + \sqrt{x}$ **f** $3(2x - 1)$

6 WORKED EXAMPLE 4 Determine whether each table of values represents an expression that is linear or non-linear.

a

Variable	1	2	3	4
Expression	3	13	23	33

b

Variable	0	5	10	15
Expression	20	16	14	6

c

Variable	−3	−2	−1	0
Expression	9	4	1	0

d

Variable	5	6	7	8
Expression	1	2	3	4

e

Variable	0	2	4	6
Expression	0	2	8	16

f

Variable	1	4	7	10
Expression	1	16	49	100

g

Variable	2	4	6	8
Expression	50	35	20	5

h

Variable	0.5	1	1.5	2
Expression	13	26	39	52

7 TECHNOLOGY: NON-LINEAR EXPRESSIONS The data below shows the English and Maths scores for a group of students.

English score	50	59	78	60	93	55	90	65	57	81	48	54	56	43
Maths score	52	56	71	61	90	60	86	63	56	82	50	53	52	40

a Enter the data into a spreadsheet and add a trendline to determine if there is a linear relationship between the 2 scores.

b Predict the Maths score for a student whose English score is 70.

8 A wildlife park has a breeding program for lions. The staff involved in the program know that the minimum daily calorie intake for lion cubs changes as they get older. The table shows how it changes.

Use a spreadsheet to determine whether the relationship between minimum daily calorie intake and the age of the lion cubs is linear or non-linear.

Age (weeks)	Calorie intake
1	750
2	1100
3	1250
4	1450
5	1500

Exam practice

9 Which of the following expressions is not linear?

A $3x - \frac{5x}{3}$ **B** $2(3a + 1)$ **C** $2y - 5x + 9$ **D** $6p + 2\sqrt{p}$

10 (2 marks) CF Determine whether the area of a circle is a linear or non-linear function of its radius. Explain your reasoning.

11 (2 marks) CU The values of a variable and corresponding values of an expression are shown in the table. Determine whether the expression is linear or non-linear.

Variable	−2	0	3	4
Expression	8	2	−7	−10

Videos
Formulas

Formulas and equations

Worksheet
Working with formulas

4.2 Formulas

A **formula** has a **subject** if it has a single variable on the left-hand side.

In the formula for speed, $s = \frac{d}{t}$, s is the subject of the formula.

The value of the subject of a formula can be worked out by substitution.

WORKED EXAMPLE 5 Using a linear formula

The cost to hire an e-scooter is \$1 to unlock the scooter and \$0.45 per minute to scoot, which is given by the rule

$$C = 0.45t + 1$$

where C is the cost in dollars and t is the time in minutes.

How much will it cost to hire a scooter for 25 minutes?

Steps	Working
1 Write the formula.	$C = 0.45t + 1$
2 Substitute in known variables and evaluate.	$= 0.45 \times 25 + 1$ $= \$12.25$
3 State the answer.	It costs \$12.25 to hire the e-scooter for 25 minutes.

Exam hack
Always show the substitution step in your working.

WORKED EXAMPLE 6 Using a non-linear formula

The formula for the displacement (s) of an object is given by

$$s = ut + \frac{1}{2}at^2$$

where u is the initial velocity, a is the acceleration and t is the time.

Find the value of s when $u = 26$, $a = -5$ and $t = 4$.

Steps	Working
1 Write the formula.	$s = ut + \frac{1}{2}at^2$
2 Substitute in known variables and evaluate.	$= 26 \times 4 + \frac{1}{2} \times (-5) \times 4^2$
3 State the answer.	The displacement is 64.

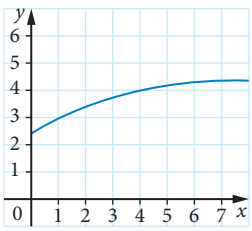
EXERCISE 4.2 Formulas

ANSWERS p. 399

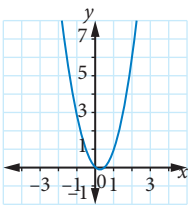
Recap

1 Which graph shows a linear relationship?

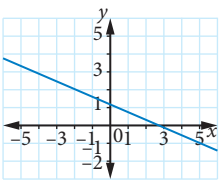
A



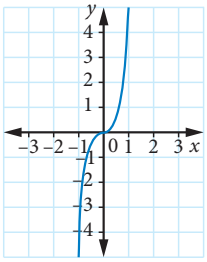
B



C



D



2 Which expression is non-linear?

A $2y + 3z$ **B** $g\left(\frac{7}{g} + 2\right)$ **C** $4w - 3w^2$ **D** $\frac{12a - 6}{5}$

4.2

Mastery

3 WORKED EXAMPLE 5 If $y = mx + c$, find y when

a $m = 2, x = -3, c = -5$ **b** $m = -2, x = -3, c = 4$ **c** $m = -8, x = 0, c = 14$

4 WORKED EXAMPLE 6 Find the value of A in each formula when $x = 3$, $y = -2$ and $z = 5$.

a $A = 3x - 4y$ **b** $A = 5(x + 2y)$ **c** $A = x(y + z)^2$

d $A = 3xz - 2y$ **e** $A = y^2 - x^2$ **f** $A = y^2 - z^2$

5 If $a = 2.4$, $b = -3.5$ and $c = 1.2$, find Y when

a $Y = a - b + c$ **b** $Y = -abc$ **c** $Y = a^2 + b^2$

d $Y = 4ac - 3b^2$ **e** $Y = \frac{a + 6b}{c}$ **f** $Y = \frac{7(3a - 4bc)}{12b}$

6 Copy and complete each table given the rule.

a $y = 5x - 3$

x	0	1	2
y			

b $C = -2D + 7$

C	−1	4	5
D			

7 A car travels a distance, d km, in t hours at an average speed of v km/h where $d = v \times t$. Find the distance travelled by the car if it travels at a speed of 105 km/h for 3 hours.

8 The area of a trapezium is given by the formula $A = \frac{1}{2}(a + b)h$. Find the area when

a $h = 2, a = 3, b = 6$ **b** $h = 4.2, a = 6.8, b = 5.3$

9 The cost of hiring a reception hall is given by the formula $C = 85t + 2500$, where C is the total cost in dollars and t is the number of hours the hall is hired for. What is the cost of hiring the reception hall for 8 hours?

10 The compound interest formula is

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

where

FV = future value of the investment or loan plus interest

PV = initial amount invested or initial loan amount

r = annual interest rate expressed as a decimal

n = number of times that interest is compounded per year

t = number of years the amount is invested or borrowed

Use the formula to calculate the amount to which

a \$6000 will grow if it is invested at 5% p.a. for 7 years compounded annually

b \$10 000 will grow if it is invested at 7% p.a. for 9 years compounded monthly

c \$8000 will grow if it is invested at 8% p.a. for 5 years compounded daily.

11 The surface area of a cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$, where r is the radius of the circular base and h is the height of the cylinder.

a If the height of a cylinder is 15 cm and the radius is 3 cm, what is its surface area to 2 decimal places?

b If the height is tripled, what is the surface area of the cylinder to 2 decimal places?

12 The current, I amperes, that flows through a circuit of resistance, R ohms, is given by $I = \frac{240}{R}$.

What is the current that flows through the circuit if the resistance is 200 ohms?

Exam practice

13 (1 mark) Consider the equation $q = \frac{4P}{2P + 5}$. Find the value of q when $P = 5$.

14 (2 marks) CF The density of a body, D g/cm^3, is related to its volume, V cm^3, and its mass, M g, by the formula

$$D = \frac{M}{V}$$

Find the volume of a block of steel with density 9.6 g/cm^3 and mass 2.4 kg.

15 (2 marks) CF Two consecutive numbers add up to 51. What are the two numbers?

16 (4 marks) CU The area, A, of an equilateral triangle drawn on the surface of a balloon is given by

$$A = kr^2$$

where r is the radius of the balloon.

A second balloon is inflated to twice the diameter of the first balloon and another triangle is drawn. The new triangle has an area 3 times the area of the original triangle.
How does the value of k for the first balloon compare with the value of k for the second?

Worksheets
Subject of a formula

Changing the subject of a formula

4.3 Transposing formulas and equations

Formulas can be rearranged to make a different variable the subject. This is called **transposing** the formula or equation or changing the subject of the formula.

Inverse operations are used to 'undo' the original operation.

Inverse operations

Operation	Inverse operation
+	−
−	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

Whatever you do to one side of the formula or equation must be done to the other side to maintain **balance**.

When you transpose a formula, work in the reverse order to the usual order of operations.

WORKED EXAMPLE 7 Changing the subject 1

Make a the subject of each formula.

a $ad + c = p$ **b** $a(p + q) = k$ **c** $5a - m = 2a + 3n$

Steps	Working
a **1** Write the formula.	$ad + c = p$
2 Undo the $+ c$ first. Subtract c from both sides.	$ad + c - c = p - c$
3 Simplify.	$ad = p - c$
4 Undo the $\times d$. Divide both sides by d.	$\frac{ad}{d} = \frac{(p - c)}{d}$
5 Simplify the LHS.	$a = \frac{(p - c)}{d}$
b **1** Write the formula.	$a(p + q) = k$
2 Undo the $\times (p + q)$ by dividing by $(p + q)$. Simplify if needed.	$a = \frac{k}{(p + q)}$
c **1** Write the formula.	$5a - m = 2a + 3n$
2 Group the terms containing a on the LHS by subtracting $2a$.	$5a - m - 2a = 3n$ $3a - m = 3n$
3 Undo the $- m$ by adding m.	$3a = 3n + m$
4 Undo the $\times 3$ by dividing by 3.	$a = \frac{3n - m}{3}$

Videos
Changing the subject of a formula 1
Changing the subject of a formula 2

WORKED EXAMPLE 8 Changing the subject 2

Make x the subject of each formula.

a $6(y + 1) = 7(x - 2)$ **b** $3y = \sqrt{7x + 5}$

Steps	Working
a **1** Write the equation.	$6(y + 1) = 7(x - 2)$
2 Reverse the equation to get x on the LHS.	$7(x - 2) = 6(y + 1)$
3 Undo the $\times 7$ by dividing by 7.	$x - 2 = \frac{6(y + 1)}{7}$
4 Undo the $- 2$ by adding 2.	$x = \frac{6(y + 1)}{7} + 2$
b **1** Write the equation.	$3y = \sqrt{7x + 5}$
2 Reverse the equation.	$\sqrt{7x + 5} = 3y$
3 Square both sides.	$7x + 5 = (3y)^2$
4 Simplify.	$7x + 5 = 9y^2$
5 Undo the $+ 5$ by subtracting 5.	$7x = 9y^2 - 5$
6 Undo the $\times 7$ by diving by 7.	$x = \frac{9y^2 - 5}{7}$

WORKED EXAMPLE 9 Transposing and evaluating formulas

Make m the subject of each formula and **calculate** the value of m when $n = 8$ and $p = 12$.

a $m^2 + n = p$

b $\frac{2n}{3m} - 4 = p$

Steps			Working
a	**1**	Write the formula.	$m^2 + n = p$
	2	Undo the $+ n$.	$m^2 = p - n$
	3	Undo the square by taking the positive and negative square root of both sides.	$m = \pm\sqrt{p - n}$
	4	Substitute for $n = 8$ and $p = 12$.	$m = \pm\sqrt{12 - 8}$
	5	Simplify and evaluate.	$= \pm\sqrt{4}$ $= \pm 2$
b	**1**	Write the formula.	$\frac{2n}{3m} - 4 = p$
	2	Undo the $- 4$.	$\frac{2n}{3m} = p + 4$
	3	Invert both sides. Remember, $a = \frac{a}{1}$.	$\frac{3m}{2n} = \frac{1}{p + 4}$
	4	Undo the $\div 2n$.	$3m = \frac{1}{p + 4} \times 2n$
	5	Simplify.	$3m = \frac{2n}{p + 4}$
	6	Undo the $\times 3$.	$m = \frac{2n}{3(p + 4)}$
	7	Substitute for $n = 8$ and $p = 12$.	$m = \frac{2 \times 8}{3(12 + 4)}$
	8	Simplify and evaluate.	$= \frac{16}{3 \times 16}$ $= \frac{1}{3}$

EXERCISE 4.3 Transposing formulas and equations

ANSWERS p. 400

Recap

1 The subject of the formula $A = 2\pi r(r + h)$ is

A h **B** 2 **C** r **D** A

2 The value of the expression $u(v - u)$ when $u = -2$ and $v = 3$ is

A -2 **B** -10 **C** 10 **D** 3

Mastery

3 WORKED EXAMPLE 7 Transpose each formula to make the variable shown in square brackets the subject.

a $v = u(2 - 3at)$ $[t]$

b $G = k(I - ct)$ $[c]$

c $m = \frac{b - a}{c - a}$ $[b]$

d $a = \frac{u + at}{s}$ $[u]$

e $\frac{b}{a} = \frac{c}{d}$ $[d]$

f $kn = d - h$ $[k]$

4 Transpose each formula so that the variable shown in square brackets is the subject.

a $(R - a)^2 + c = d^2$ [R]

b $v = \sqrt{\dfrac{a-b}{c}}$ [a]

c $y = 4ab^2c^2$ [b]

d $d = \sqrt{bc} - 5$ [c]

e $m = \dfrac{a}{b} - \dfrac{1}{c}$ [c]

f $\dfrac{1}{f} = \dfrac{1}{u} - \dfrac{1}{v}$ [v]

g $V = \dfrac{4}{3}\pi r^3$ [r]

h $3 = \sqrt{(x-2)^2 + (y-1)^2}$ [y]

5 WORKED EXAMPLE 8 Isolate x on the LHS of each equation.

a $y = 2x - 7$

b $y + 5 = 3x - 8$

c $4(y + 3) = 2(5 - x)$

d $7(x - 5) = 2x + 3y$

e $3(x + 6) = 7(2x - y)$

f $3(7x - y) + 8 = 6(2y - 5x)$

6 Transpose each equation to isolate y on the LHS.

a $x = 3y - 6x^2 + 8$

b $5x = y^2 - 12$

c $5x - \sqrt{2y} - 3 = 0$

d $3xy - 7 = 0$

e $2x - \dfrac{3}{y} = 5$

f $7x + 2 = \sqrt{3 - 2y}$

7 WORKED EXAMPLE 9 Transpose each formula so that A is the subject and then find the value of A if $x = 3$, $y = 2$ and $z = 5$.

a $x = A + z$

b $15x = Az^2$

c $z = \sqrt{6xy} + 3A$

8 Transpose each formula as required and then find the value of

a u if $v = u + at$ and $v = 12$, $a = 2$ and $t = 3$

b m if $k = m(1 - ct)$ and $k = 28$, $c = 4$ and $t = 2$

c E if $I = \dfrac{E - V}{R}$ and $I = 4.2$, $V = 7$ and $R = 2.5$

d h if $d = 8\sqrt{\dfrac{h}{a}}$ and $d = 24$ and $a = 5$

e t if $s = \dfrac{(u + v)t}{2}$ and $s = 15$, $u = 3$ and $v = 5$

f r if $p = ghr + B$ and $p = 980$, $g = 9.8$, $h = 15$ and $B = 24.5$.

9 Dan jumped out of an aircraft to begin skydiving. Without a parachute, it would take him 24.74 s to hit the ground, accelerating at $g = 9.8\ \text{m/s}^2$. Use the formula $s = gt^2$ to determine from what height, s, Dan jumped if t is the time in seconds it would take him to reach the ground under gravity.

Exam practice

10 If $H = 6J - R$ is transposed to make J the subject then

A $J = \dfrac{H + R}{6}$

B $J = \dfrac{H}{6} - R$

C $J = \dfrac{H}{6} + R$

D $J = \dfrac{H - R}{6}$

11 (4 marks) CF The time, T s, for a pendulum to complete one swing is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $g = 9.8\ \text{m/s}^2$ and l = length of the pendulum in metres.

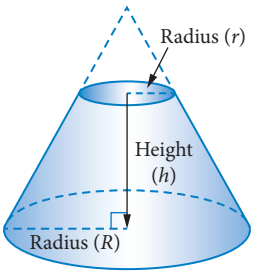
Find the length of a pendulum (nearest cm) that will make the time for a swing 4.5 seconds.

12 (3 marks) CF In an NRL game, a try, T, is worth 4 points and a conversion, C, is worth 2 points. If there are no field goals or penalties in a game, the total number of points scored, P, can be found by $P = 4T + 2C$. In a game in which there were no penalties or field goals, the Bayside Fish scored a total of 30 points, which included 3 conversions. How many tries did they score in this game?

13 (4 marks) CU A frustum is the part of a cone that remains when it is cut parallel to its base.

The volume of the frustum is one-third of the product of:

- the vertical distance, h, between the base and the cut end
- π
- the sum of the flat circular areas and the square root of the product of these areas.



Find a formula for this volume involving r, R and h.

4.4 Tables of values

A table of values can be used to evaluate a formula or equation for multiple values.

A table of values can be used to see the relationship between input values and output values.

WORKED EXAMPLE 10 A table of values

Consider the formula for the area of a circle, $A = 2\pi r^2$. **Construct** a table of values for the area of the circle for radius lengths of 1, 2, 3 and 4 cm. Round the areas to 2 decimal places.

Steps	Working
1 Substitute and evaluate the area formula for the different values of r.	When $r = 1$, $A = 2\pi(1)^2 = 6.28$ When $r = 2$, $A = 2\pi(2)^2 = 25.13$ When $r = 3$, $A = 2\pi(3)^2 = 56.55$ When $r = 4$, $A = 2\pi(4)^2 = 100.53$
2 Construct the table of values.	

r	1	2	3	4
A	6.28	25.13	56.55	100.53

A two-way table can be used for a formula or equation with *2* variables.

The table below is for values of F for the formula $F = ma$, for different values of m and a.

For example, when $m = 2$ and $a = 10$, $F = 2 \times 10 = 20$.

F		a				
		0	5	10	15	20
m	1					
	2			20		
	3					

WORKED EXAMPLE 11 Constructing a two-way table

Apply the formula $A = \frac{B+2}{C}$ to complete this table of values.

A		C: −3	3	9	15
B	1				$\frac{1}{5}$
	2	$-1\frac{1}{3}$			
	3		$1\frac{2}{3}$		

Steps

Substitute and evaluate the formula for A for the different values of B and C to complete the table. The value in the shaded box has been calculated as $A = \frac{3+2}{9} = \frac{5}{9}$.

Working

A		C: −3	3	9	15
B	1	-1	1	$\frac{1}{3}$	$\frac{1}{5}$
	2	$-1\frac{1}{3}$	$1\frac{1}{3}$	$\frac{4}{9}$	$\frac{4}{15}$
	3	$-1\frac{2}{3}$	$1\frac{2}{3}$	$\frac{5}{9}$	$\frac{1}{3}$

WORKED EXAMPLE 12 Finding unknown values in a table

A two-way table for the equation $y = 2x + 5z$ is shown below.

a What are the values of A and B in the table?

b **Describe** the pattern for y when $x = z$.

y		z: 0	1	2	3
x	1	2	7	12	17
	2	4	A	14	19
	3	6	11	16	B

Steps	Working
a **1** Locate A in the table and the corresponding values for x and z.	$y = A$ when $x = 2$ and $z = 1$.
2 Determine the value of y for these values.	$A = 2(2) + 5(1)$ $= 9$
3 Locate B in the table and the corresponding values for x and z.	$y = B$ when $x = 3$ and $z = 3$.
4 Determine the value of y for these values.	$B = 2(3) + 5(3)$ $= 21$
b **1** Locate where the values of x and z are equal (for example, $x = 1$, $y = 1$) and identify the corresponding values for y.	The values of y when $x = z$ are 7, 14, 21.
2 Describe the pattern you observe.	Each time x and z increase by 1, the value of y increases by 7.

WORKED EXAMPLE 13 Using a spreadsheet for a table of values

The volume of a sphere is given by the equation $V = \frac{4}{3}\pi r^3$.

a Use a spreadsheet to construct a table to show the volumes of spheres where $r = 1, 2, 3 \ldots 10$.

b What is the volume of a sphere with a radius of 15 cm?

Steps	Working
a Type 'Radius' in cell A1 and 'Volume' in cell B1. Enter '1' into A2 and '2' into A3. Highlight A2:A3 and drag the bottom right-hand corner of the block down to A11. This fills cells A4 to A11 with values that increase by 1 each time. Enter the equation for the volume of a sphere into cell B2 as follows: **=(4/3)*PI()*POWER(A2,3)** This equation uses the value in cell A2 raised to the power of 3. Hold the bottom right-hand corner of B2 and drag the block down to B11. This copies the equation into each cell.	
b **1** Enter '15' into A12. Click on B11 and then drag the bottom right-hand corner to B12.	
2 State the result.	The volume of a sphere with a radius of 15 cm is about 14 137.17 cm^3.

	A	B
1	Radius	Volume
2	1	4.1887902
3	2	33.5103216
4	3	113.097336
5	4	268.082573
6	5	523.598776
7	6	904.778684
8	7	1436.75504
9	8	2144.66058
10	9	3053.62806
11	10	4188.7902

	A	B
1	Radius	Volume
2	1	4.1887902
3	2	33.5103216
4	3	113.097336
5	4	268.082573
6	5	523.598776
7	6	904.778684
8	7	1436.75504
9	8	2144.66058
10	9	3053.62806
11	10	4188.7902
12	15	14137.1669
13		
14		

Investigation Body mass index (BMI)

The body mass index (BMI) is a method used to estimate your total amount of body fat. It is calculated by dividing your mass (m) in kilograms by your height (h) in metres squared. In practice, a person's weight is used rather than their mass. The equation for BMI is $B = \frac{m}{h^2}$.

BMI is a simple indicator of body fat rather than a true measure. It is a useful indicator because it relies solely on height and weight and does not require any other equipment. This means that people can have their BMI routinely measured and calculated with reasonable accuracy.

Studies have shown that BMI levels are good indicators of future health risks. This means that BMI is a screening tool to decide if your weight might be putting you at risk for a variety of health problems.

There are limitations to the usefulness of BMI because it is a measure of excess weight rather than excess body fat.
So, BMI does not distinguish between excess fat, muscle or bone.

Differences in the values of BMI between people of the same age and sex are attributed to body fat. BMI does not take into account the facts that older adults, women and body builders tend to have more body fat than other people for an equivalent BMI.

BMI	Weight status
Below 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

The table shows how BMI can be interpreted.

1 Construct a spreadsheet to calculate the BMI of Renata who is 168 cm tall and weighs 64 kg.
Type 'm' in cell A1, 'h' in cell B1 and 'B' in cell C1.

Enter '64' into A2 and '1.68' into A3.
Enter the equation for BMI into cell C2 as follows:
=A2/POWER(B2,2)
This equation uses the value in cell B2 raised to the power of 2.
Renata has a BMI of 22.7.

2 Extend the spreadsheet to calculate the BMI for each person listed in the below table.

Person	Weight (kg)	Height (cm)
Adel	73	157
Shani	69	169
Lindell	52	149
Kai	77	171
Riley	54	174
Lucca	68	162
Madison	75	168
Lola	77	153
Matteo	81	181
Mia	92	177

3 Use the BMI measure to determine the weight status (underweight, normal, overweight or obese) of each of the people in the table above.

WORKED EXAMPLE 14 A two-way table spreadsheet 1

The equation used to calculate the body mass index (BMI) of a person h metres tall weighing m kg is $B = \frac{m}{h^2}$.

a Using a spreadsheet, construct a table of values for the BMI using values of m in intervals of 10 from 40 kg to 100 kg and values for h in intervals of 0.25 m from 1 m to 2 m.

b Using the table of values, find the BMI of a person who weighs 70 kg and has a height of 175 cm.

Steps	Working

a 1 Set up a table with m as the row variable and h as the column variable.

	A	B	C	D	E	F	G
	BMI				Height (m)		
			1	1.25	1.5	1.75	2
	Mass (m)	40					
		50					
		60					
		70					
		80					
		90					
		100					

2 In cell C3, type the formula **$B3/(C$2)^2**

It is important that you put the $ sign in. The '$B3' tells Excel to use values from column B and 'C$2' tells it to use values from row 2.

C3 fx =$B3/(C$2)^2

	A	B	C	D	E	F	G
1	BMI				Height (m)		
2			1	1.25	1.5	1.75	2
3	Mass (m)	40	40				
4		50					
5		60					
6		70					
7		80					
8		90					
9		100					

3 Copy cell C3 and paste it over the whole range required.

You can change the format of the answers by highlighting the whole range and changing it to the number format and then selecting the number of decimal places you want.

	A	B	C	D	E	F	G
	BMI				Height (m)		
			1	1.25	1.5	1.75	2
	Mass (m)	40	40				
		50					
		60					
		70					
		80					
		90					
		100					

	A	B	C	D	E	F	G
1	BMI				Height (m)		
2			1	1.25	1.5	1.75	2
3	Mass (m)	40	40.00	25.60	17.78	13.06	10.00
4		50	50.00	32.00	22.22	16.33	12.50
5		60	60.00	38.40	26.67	19.59	15.00
6		70	70.00	44.80	31.11	22.86	17.50
7		80	80.00	51.20	35.56	26.12	20.00
8		90	90.00	57.60	40.00	29.39	22.50
9		100	100.00	64.00	44.44	32.65	25.00

	A	B	C	D	E	F	G
1	BMI				Height (m)		
2			1	1.25	1.5	1.75	2
3	Mass (m)	40	40.00	25.60	17.78	13.06	10.00
4		50	50.00	32.00	22.22	16.33	12.50
5		60	60.00	38.40	26.67	19.59	15.00
6		70	70.00	44.80	31.11	22.86	17.50
7		80	80.00	51.20	35.56	26.12	20.00
8		90	90.00	57.60	40.00	29.39	22.50
9		100	100.00	64.00	44.44	32.65	25.00

b Examine the values in the table to identify the BMI of a person with a mass of 70 kg who is 1.75 m tall.

The BMI of the person is 22.86.

WORKED EXAMPLE 15 A two-way table spreadsheet 2

A person who is 176 cm tall wishes to achieve a BMI of 24. Use a spreadsheet to determine the weight (correct to one decimal place) they should reach.

Steps | **Working**

1 Construct a spreadsheet to calculate BMI values. Allocate both the variables and expression to separate columns: *m* in column 1, *h* in column 2 and BMI in column 3.

	A	B	C
1	m	h	BMI
2			
3			

2 Identify the range of values to explore for each variable: *m* from 60 to 80 and *h*, in this case, is consistent at 176.
Type the formula for the BMI into cell C2: **=A2/(B2)^2**. Copy it down the table of values.

C2 fx =A2/(B2)^2

	A	B	C	D
1	m	h	BMI	
2	60	1.76	19.3698347	
3	61	1.76	19.6926653	
4	62	1.76	20.0154959	
5	63	1.76	20.3383264	
6	64	1.76	20.661157	
7	65	1.76	20.9839876	
8	66	1.76	21.3068182	
9	67	1.76	21.6296488	
10	68	1.76	21.9524793	
11	69	1.76	22.2753099	
12	70	1.76	22.5981405	
13	71	1.76	22.9209711	
14	72	1.76	23.2438017	
15	73	1.76	23.5666322	
16	74	1.76	23.8894628	
17	75	1.76	24.2122934	
18	76	1.76	24.535124	
19	77	1.76	24.8579545	
20	78	1.76	25.1807851	
21	79	1.76	25.5036157	
22	80	1.76	25.8264463	

3 Examine the values in the table.

The weight required is between 74 kg and 75 kg.

4 You need to explore this new range, so enter '74' into cell A24 and '74.1' into A25. Highlight cells A24:A25 and then place the cursor on the bottom right-hand corner of the highlighted block and drag the cursor down to cell A34. This gives a range of weights from 74 kg to 75 kg.

Copy columns B and C into this range.

24	74	1.76	23.8894628
25	74.1	1.76	23.9217459
26	74.2	1.76	23.9540289
27	74.3	1.76	23.986312
28	74.4	1.76	24.018595
29	74.5	1.76	24.0508781
30	74.6	1.76	24.0831612
31	74.7	1.76	24.1154442
32	74.8	1.76	24.1477273
33	74.9	1.76	24.1800103
34	75	1.76	24.2122934

5 Examine the values in the table.

The weight required is between 74.3 kg and 74.4 kg.

6 Repeat the process: in cell A36 enter '74.31', and '74.32' into cell A37. Highlight A36:A37 and then place the cursor on the bottom right-hand corner of the block and drag the cursor down to cell A45. This gives a range of weights from 74.31 kg to 74.40 kg.

Copy columns B and C into this range.

36	74.31	1.76	23.9895403
37	74.32	1.76	23.9927686
38	74.33	1.76	23.9959969
39	74.34	1.76	23.9992252
40	74.35	1.76	24.0024535
41	74.36	1.76	24.0056818
42	74.37	1.76	24.0089101
43	74.38	1.76	24.0121384
44	74.39	1.76	24.0153667
45	74.4	1.76	24.018595

7 Examine the values in the table.

The weight required is closer to 74.3 kg than 74.4 kg.

8 State the result.

A person who is 176 cm should weigh approximately 74.3 kg to achieve a BMI of 24.

EXERCISE 4.4 Tables of values

ANSWERS p. 400

Recap

1 If $a = 2$, $b = 4$, $c = 7$ and $d = 10$, $bd - ac =$

A 54 **B** 26 **C** −62 **D** 62

2 Make h the subject of $B = \dfrac{m}{h^2}$.

Mastery

3 WORKED EXAMPLE 10 The circumference of a circle is given by $C = 2\pi r$, where r is the radius. Copy and complete the table of values, correct to 2 decimal places, for the given values of r.

r (cm)	0.5	1	1.5	2
C (cm)				

4 The formula $F = \frac{9}{5}C + 32$ is used to convert degrees Celsius (C) to degrees Fahrenheit (F). Copy and complete the table of values for the given values for C, correct to 2 decimal places where appropriate.

C	−5	10	25	30
F				

5 WORKED EXAMPLES 11, 12 Consider the formula $G = \frac{H + 5}{K}$. Copy and complete the table of values for the given values of H and K.

G		K			
		1	2	3	4
H	6			$3\frac{2}{3}$	
	12	17			
	18				

6 Consider the formula $F = ma$. Copy and complete the table of values for the given values of m and a.

F		a			
		5	10	15	20
m	20				
	30				
	50				

7 WORKED EXAMPLE 13 The cost to hire a surfboard is given by $C = 20h + 15$, where C is the cost per hour and h is the time the board is hired for in hours.

The spreadsheet shows the hire cost for a number of different hire times.

	A	B
1	h	C
2	0.5	25
3	1	35
4	2	55
5	3	75
6	5	115
7	8	175

a Write the formula used to calculate the value in cell B2.

b What is the cost to hire the surfboard for 5 hours?

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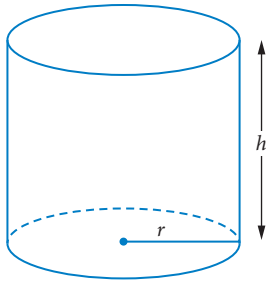
8 WORKED EXAMPLES 14, 15 The curved surface area of a cylinder is given by the formula $S = 2\pi rh$, where r is the radius and h is the height of the cylinder.



a Use a spreadsheet to construct a table to show the curved surface areas of cylinders that are 10 cm high for $r = 1, 2, 3 \ldots 15$.

b Construct a table to show the curved surface areas of cylinders with a radius of 10 cm for $h = 10, 11, 12 \ldots 20$.

c Use the spreadsheet to find the curved surface area for a cylinder that is 17 cm high with a radius of 7 cm.

9 Susan plans to invest \$1000 into an account paying 6.2% p.a. She wants to know how much interest she will earn after 1, 2, 3 … 10 years. Use the simple interest formula

$$I = Pin$$

where I is the interest, P is the amount invested over n years with an annual interest rate i expressed as a decimal to complete the following.

a Construct a table of values using a spreadsheet to show how much interest Susan will earn after $n = 1, 2, 3 \ldots 10$ years.

b How many years would it take for her initial investment in this account to double in size?

10 An arithmetic progression (AP) is a sequence of numbers in which each successive term differs from the term before by the same amount. For example:

$$3, 7, 11, 15, 19 \ldots$$

The first term is known as a and the common difference between terms is d.

So, for the AP shown, $a = 3$ and $d = 7 - 3 = 4$.

The general form can be found using the formula

$$a, a + d, a + 2d, a + 3d \ldots$$

If the number of terms in an AP is n, its last term (l) can be found using the formula

$$l = a + (n - 1)d$$

Use a spreadsheet to calculate the last term of each of the following APs.

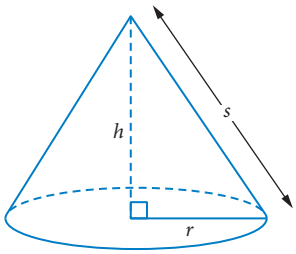
a 5, 11, 17, 23 … to 17 terms

b 111, 102, 93, 84 … to 10 terms

c 24, 32, 40, 48 … to 25 terms

11 The surface area of a cone with a base is given by

$$S = \pi rs + \pi r^2$$



a Use a spreadsheet to construct a table to calculate the surface area of a cone with values for the radius in intervals of 0.1 from 1 m to 2 m and values for the slant height $s = 1.0, 1.1, 1.2 \ldots 1.5$ m.

b Use the table of values to calculate the slant height (correct to 2 decimal places) of a cone with radius 1.2 m and surface area of 12.10 m^2.

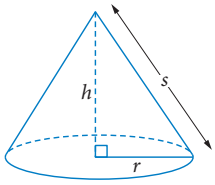
Exam practice

12 (2 marks) The distance d (in km) travelled by a hiker at any time t (in hours) is given by the formula $d = 5.6t$. Complete the table of values for the given values for t.

t	1	2	3	4
d				

13 (4 marks) **CF** The volume (V) of a cone is given by

$$V = \frac{1}{3}\pi r^2 h$$



Calculate the height (correct to one decimal place) of a cone with a radius of 10 cm and volume of 1500 cm^3.

14 (5 marks) **CU** A chef is designing a new sauce for a special dish. The new sauce is a combination of 2 sauces he already used on other meals. The taste of the sauce is important but so is its cost. The cost depends on the amount of each existing sauce used. Sauce A costs 5 cents per tablespoon and sauce B costs 8 cents per tablespoon. One tablespoon equals 15 mL.

The cost, \$$C$, per serve of the new sauce is given by the formula $C = \frac{5A + 7B}{100}$.

The table shows the cost of different combinations of each sauce, which taste equally good according to the chef. The values of X and Y are not shown.

What is the cheapest combination of sauce A and sauce B to make at least 300 mL of new sauce?

C		B			
		7	12	15	20
A	5	X	1.09	1.3	1.65
	10	0.99	1.34	1.55	1.9
	25	1.74	2.09	Y	2.65

Video Exam question analysis: Linear and non-linear relationships

EXAM QUESTION ANALYSIS

Exam-style question (3 marks)

The cost to rent a houseboat is given by the formula $C = 230d + 2.1f$, where d = the number of days rented and f = litres of fuel used. Fifi has \$1200 to spend on a houseboat rental for her 3-day long weekend. Determine how much fuel she can use during the trip.

Reading the question

- The question contains a linear equation in formula form.
- Variables are given.
- There is a limit on the number of days and the amount Fifi is willing to spend.
- Fuel amount (f) is the variable that needs to be found.

Thinking about the question

- Fuel (f) is the variable to be made the subject of the formula. Transposing the formula should occur first.
- Two values have been given for variables d and C.
- A worded statement needs to be given at the end.

Worked Solution (✓ = 1 mark)

Transpose the formula:

$$\mathbf{C = 230d + 2.1f} \checkmark$$

$$C - 230d = 2.1f$$

$$\mathbf{\frac{C - 230d}{2.1} = f} \checkmark$$

Substitute values into the formula:

$$f = \frac{C - 230d}{2.1}$$

$$f = \frac{1200 - 230 \times 3}{2.1}$$

$$f = 242.857\ldots$$

Fifi will be able to use approximately **243 L** of fuel during her trip. ✓

4 Chapter summary

Linear and non-linear expressions

- An **algebraic expression** is a collection of variables and constants connected by **operations** including +, –, × and ÷.
- **Constant terms** contain only numbers.
- A constant that is multiplied by a variable is called the **coefficient** of the variable.
- Only **like terms** may be added or subtracted.
- A **linear expression** can be identified in a number of ways:
  - graph is a straight line
  - highest power of variable/s is 1
  - rate of change is constant
- Expressions that are not linear are called **non-linear expressions**.
- Equations that are not linear are called **non-linear equations**.
- When graphing, the **independent variable** is represented on the horizontal axis and the **dependent variable** is represented on the vertical axis.

Equations and formulas

- The **subject** of a formula is the variable by itself on the left-hand side.
- A formula or equation can be manipulated or **transposed** using **inverse operations**.
- An inverse operation has the effect of undoing the original operation.

Operation	Inverse operation
+	–
–	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

Tables of values

Tables are often used to present and summarise data because it makes it easier to see trends or relationships.

If a formula or equation involves 2 variables, then a two-way table can be used to represent the data.

(Subject of the formula)		Variable 2				
Variable 1						

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 Which formula is correct when S is the subject, given $Q = 2\pi\left(3p + \frac{S}{4}\right)$?

A $S = \frac{Q}{2\pi} - 3P$ **B** $S = \frac{2Q}{\pi} - 12P$ **C** $S = \frac{Q}{8\pi} + 12P$ **D** $S = \frac{2Q}{3\pi} + 4P$

2 Leanne buys a computer monitor for \$476 including GST.
How much is the GST part of the price?

A \$47.60 **B** \$428.40 **C** \$43.27 **D** \$432.73

3 Evaluate the expression $\frac{2a+b}{4}$ when $a = 63$ and $b = 42$.

A 26.25 **B** 21 **C** 36.75 **D** 42

4 Dani bought 200 shares for \$1. They are now worth \$1.50. Dani received a dividend of \$0.10 per share.
What is the dividend yield?

A 6.67% **B** 10% **C** \$20 **D** 15%

5 Which expressions are non-linear?

I $5d + 7$

II $3y(2y - 5)$

III $4x - x + 6$

IV $3x - 3\sqrt{x}$

A II only **B** III only **C** II and IV **D** II and III

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Section 2 4 short response questions

13 marks

6 (4 marks) Phillipa is organising the local Sunday market. The venue has capacity for a maximum of 200 stalls. From past experience, Phillipa knows that if she charges C dollars to rent a stall then the number of stalls, S, that will be rented is given by $S = 200 - 5C$.

a Copy and complete the table. *[1 mark]*

C	5	15	30
S			

b Is the formula for the number of stalls linear? Justify your answer. *[1 mark]*

c What would be the cost to rent a stall if 100 stalls are rented? *[1 mark]*

d Does it make sense for Phillipa to charge \$40 per stall? Justify your answer. *[1 mark]*

7 (3 marks) The surface area of a sphere is given by

$$S = 4\pi r^2$$

where r is the radius of the sphere.

a Transpose the formula to make r the subject. *[1 mark]*

b Find the radius of the sphere when the surface area is 324π cm^2. *[2 marks]*

8 (3 marks) Kate bought some furniture on an 'easy-finance' plan when she moved away from home. The furniture cost \$3410, and she had to pay it off at \$95.75 a fortnight over 2 years.

a How much did the furniture actually cost her? *[1 mark]*

b What simple interest rate was she charged p.a., to the nearest percentage? *[2 marks]*

9 (3 marks) Abel is preparing for a party and comparing the prices of different chocolates.
Chocolicious comes in a 300 g bag costing \$8.50.
Devilishly Delicious has a bulk bag costing \$24.20 for 1.5 kg.

a Which chocolate is the better value for money? Justify your answer with appropriate calculations. *[2 marks]*

b Change the price of the more expensive chocolate to match the unit price of the other chocolate. *[1 mark]*

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions (12 marks)

1 (4 marks) Briana decided to simplify the equation $4(2y + 3) - 2(y + 1) = x$, before finding the value when $x = -2$. Her working is shown below. Briana realises she has made a mistake when she tries to verify her answer. To help Briana, identify in which line she has made the error, briefly explain the issue, then write the correct solution.

Line 1	$4(2y + 3) - 2(y + 1) = x$
Line 2	$8y + 12 - 2y + 2 = x$
Line 3	$6y + 14 = x$
Line 4	$6y + 14 = -2$
Line 5	$6y = -16$
Line 6	$y = -\frac{8}{3}$
Line 7	Check: $4\left(2\left(-\frac{8}{3}\right) + 3\right) - 2\left(-\frac{8}{3} + 1\right) = x$
Line 8	$-6 \neq -2$

2 (4 marks) Isla bought some shares for \$25 000. The next year, her dividend was the equivalent of 7% interest. Find her total dividend and the P/E ratio, correct to 2 decimal places.

3 (4 marks) Your body mass index (BMI) is a measure of your weight, m, in kilograms, relative to your height, h, in metres. It is calculated using the formula $B = \frac{m}{h^2}$.

A person is considered 'healthy' if their BMI is in the range 18.5 to 24.9. By first completing the table of values, determine the percentage of BMIs that would be considered unhealthy in the given sample. Round your answers to one decimal place.

BMI		h (m)		
		1.64	1.85	1.93
m (kg)	56			
	79			
	91			

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CHAPTER

5 SIMILAR FIGURES AND SCALE FACTORS

Syllabus coverage
Nelson MindTap chapter resources
Terminology
5.1 Similar figures
5.2 Similar triangles
5.3 Applications of similar triangles
5.4 Scale drawings
5.5 Areas and volumes of similar figures
Exam question analysis
Chapter summary
Cumulative exam 1
Cumulative exam 2

Prior learning
Similar figures and scale factors

Syllabus coverage

UNIT 1, TOPIC 3: SIMILARITY AND SCALE

Similar figures and scale factors

- Understand the conditions for similarity of two-dimensional figures, including similar triangles.
- Use the scale factor for two similar figures to solve linear scaling problems.
- Determine measurements from scale drawings (e.g. maps and building plans) to solve problems.
- Determine a scale factor and use it to solve scaling problems, e.g. calculating lengths and areas of similar figures; and calculating surface areas, volumes and capacities of similar solids.

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Videos (7):

5.1 Finding an unknown side in similar figures
5.2 Similar triangles
5.3 Shadow sticks
5.4 Scale drawings
5.5 Areas of similar figures • Volume scale factor
Exam question analysis Similar figures and scale factors

Prior learning (1):

Similar figures and scale factors

Skillsheet (1):

5.2 Finding sides in similar triangles

Worksheets (6):

5.1 Finding sides in similar figures
5.3 Applying similar triangles
5.4 Scales and scale diagrams • Interpreting an office plan
5.5 Areas and volumes of similar figures
Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

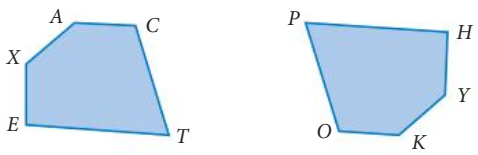
Terminology

congruent	image	included angle	object
scale drawing	scale factor	similar	similar triangles

5.1 Similar figures

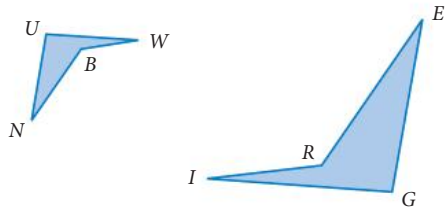
When an **object** is enlarged or reduced, the **image** is the same shape but a different size.

Congruent figures are exact copies of each other.
They are exactly the same shape and size.
The symbol for congruency is ≡.



ACTEX ≡ *KOPHY*

Similar figures are exactly the same shape but not necessarily the same size.
The symbol for similarity is |||.

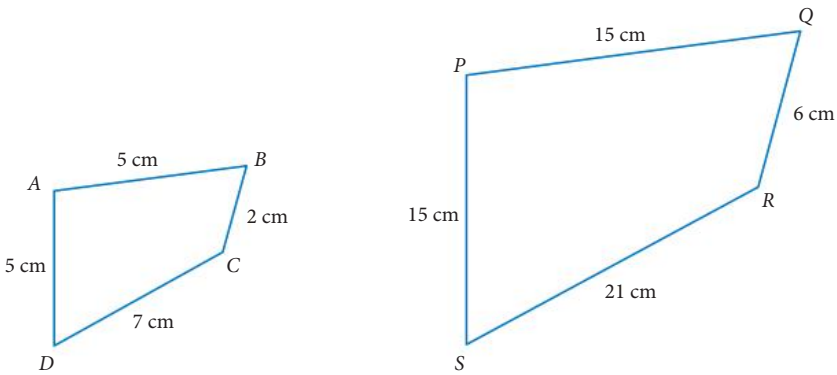


UWBN ||| *GIRE*

Video
Finding an unknown side in similar figures

Worksheet
Finding sides in similar figures

Corresponding vertices are named in the same order for similar or congruent figures (as above).
Similar figures are *enlargements or reductions* of each other.
The quadrilaterals below are similar: *ABCD* ||| *PQRS*.



The corresponding angles are equal, so $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle D = \angle S$.
The sides opposite corresponding angles are corresponding sides.
The ratios of corresponding sides are equal:

$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RS}{CD} = \frac{SP}{DA} = 3$$

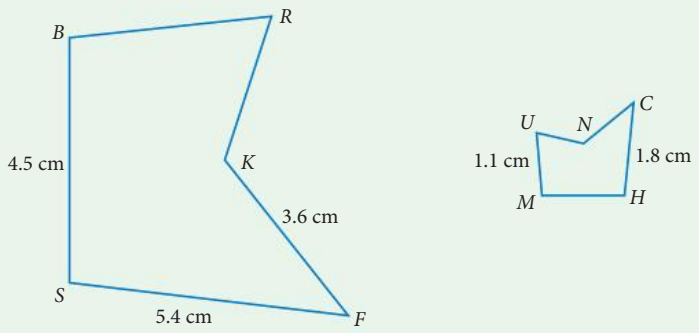
This ratio is the **scale factor**.

Properties of similar figures

- Corresponding angles are equal.
- Ratios of each pair of corresponding sides are equal.
- Scale factor = $\dfrac{\text{image length}}{\text{original length}}$
- Original objects are either enlarged or reduced to get the image of the object.

WORKED EXAMPLE 1 Similar figures

a **Calculate** the scale factor between the 2 objects to determine if the object has been reduced or enlarged.



b Write a statement of similarity for these similar figures.

c **Calculate** the values of *BR* and *NC*.

Steps	Working
a **1** Identify corresponding sides where lengths are known.	$HC \parallel\!\mid SF$ (image $\parallel\!\mid$ original)
2 Use the rule for scale factor.	Scale factor $= \dfrac{HC}{SF}$
3 Substitute.	$= \dfrac{1.8}{5.4}$ $= \dfrac{1}{3}$
4 State the result.	The scale factor is $\dfrac{1}{3}$, therefore the image has been reduced.
b **1** Identify corresponding vertices.	$\angle B = \angle M$ $\angle R = \angle U$ $\angle K = \angle N$ $\angle F = \angle C$ $\angle S = \angle H$
2 Write the similarity statement with corresponding vertices in the same order.	$RKFSB \parallel\!\mid UNCHM$
c **1** Write the scale factor rule for *BR*.	$\dfrac{MU}{BR} = \dfrac{1}{3}$
2 Solve for *BR*.	$BR = 3 \times MU$ $= 3 \times 1.1$ $= 3.3$
3 Write the scale factor rule for *NC*.	$\dfrac{NC}{KF} = \dfrac{1}{3}$
4 Solve for *NC*.	$NC = \dfrac{1}{3} \times KF$ $= \dfrac{1}{3} \times 3.6$ $= 1.2$
5 Write the answer.	$BR = 3.3$ cm and $NC = 1.2$ cm.

Exam hack

Make sure you are consistent with the order of corresponding vertices in your answer.

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EXERCISE 5.1 Similar figures

ANSWERS p. 401

Mastery

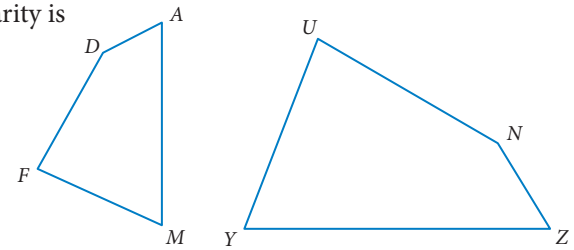
1 These 2 shapes are similar. The statement of similarity is

A *FDAM* ||| *YZNU*

B *DAMF* ||| *NZYU*

C *MDFA* ||| *NZYU*

D *DAMF* ||| *YNUZ*



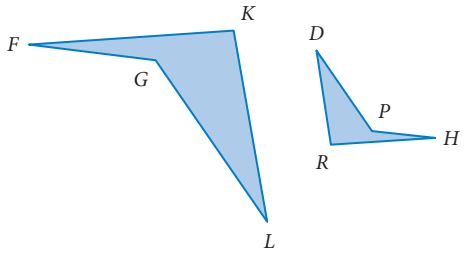
2 These 2 shapes are similar. The statement of similarity is

A *GFKL* ||| *RHDP*

B *KLFG* ||| *RDPH*

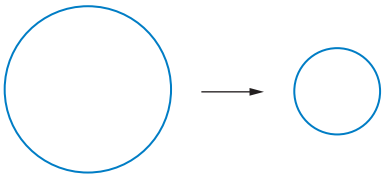
C *GLKF* ||| *PHRD*

D *FKLG* ||| *HRDP*

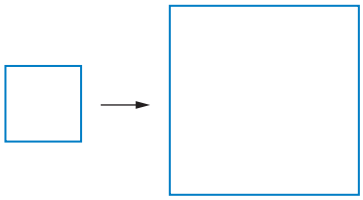


3 Each pair of shapes below is similar. Use a ruler to find the scale factor.

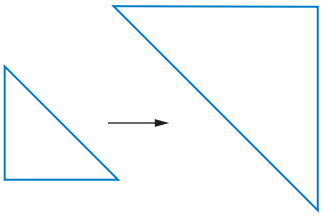
a



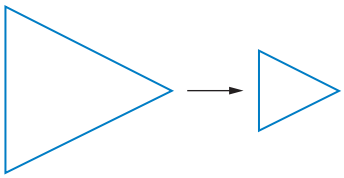
b



c



d

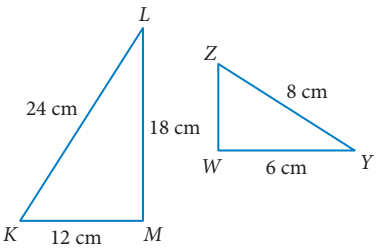


4 WORKED EXAMPLE 1 For each pair of similar figures

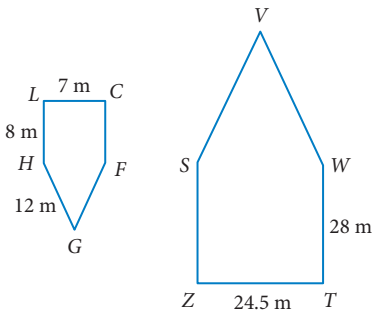
i write a statement of similarity

ii calculate the scale factor.

a



b



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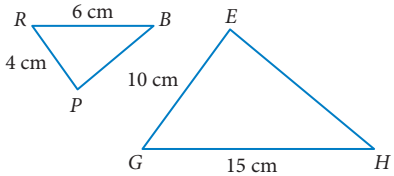

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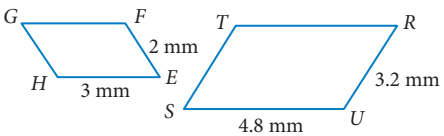

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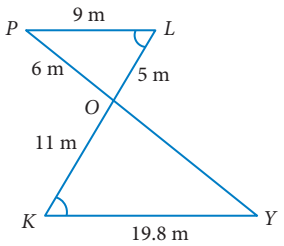

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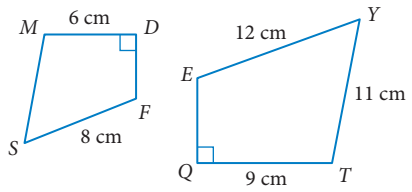
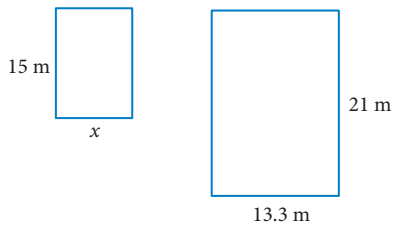


5 Calculate the value of x in each pair of similar figures.

a



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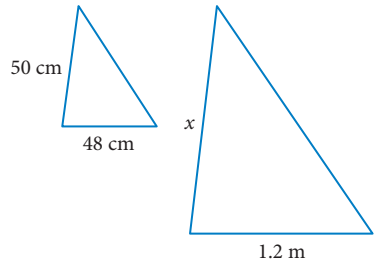

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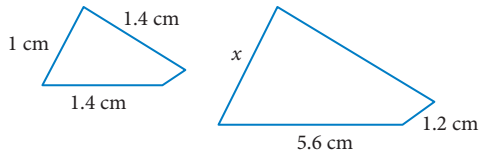

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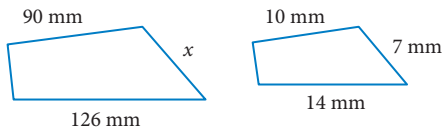

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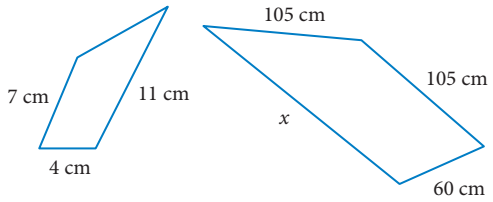

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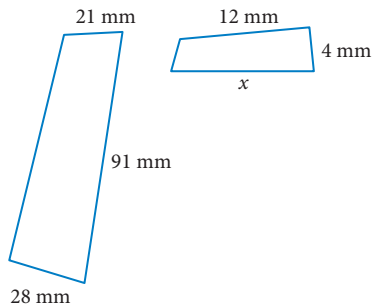


6 Is each statement true (T) or false (F)?

a All squares are similar.

b Corresponding sides of similar figures are in the same ratio.

c All rhombuses are similar.

d All circles are similar.

e Corresponding angles of similar figures are equal.

f All parallelograms are similar.

g Any 2 isosceles triangles are always equal.

h All equilateral triangles are similar.

i All hexagons are similar.

j Since the angles of 2 rectangles are equal, they must be similar.

k All right-angled triangles are similar.

l All regular octagons are similar.

m If 2 triangles have 2 pairs of corresponding angles equal, they must be similar.

7 A photograph measures 105 mm wide by 60 mm high. An enlargement has a height of 20 cm. What is the width?

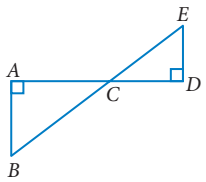
8 **a** Explain why 2 parallelograms with corresponding angles that are equal are not necessarily similar.

b Draw a pair of parallelograms with equal corresponding angles that are similar.

c Draw a pair of parallelograms with equal corresponding angles that are not similar.

Exam Practice

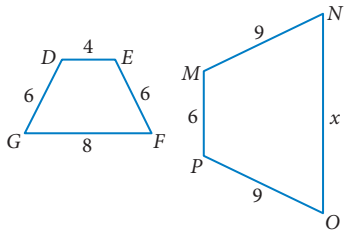
9 (2 marks) Identify corresponding sides and vertices.



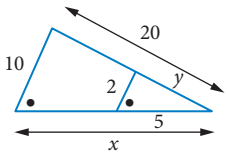
10 (2 marks) Calculate the value of x in this scale factor ratio.

$$\frac{30}{5} = \frac{18}{x} = \frac{12}{2}$$

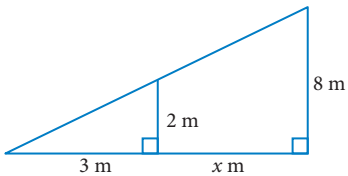
11 (2 marks) Find the value of x in this pair of similar figures.



12 (3 marks) CF These triangles are similar. Find the value of each pronumeral.



13 (3 marks) CU Determine the length of x.



Skillsheet
Finding sides in similar triangles

5.2 Similar triangles

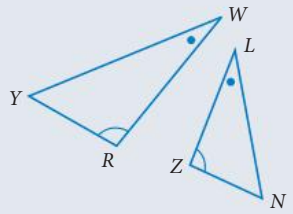
There are 4 tests for similar triangles.

Tests for similar triangles

Angle-angle-angle (AAA)

Two pairs of corresponding angles equal make the third pair equal, as the sum of the angles in each triangle is 180°.

So, $\triangle YWR \parallel\!| \triangle NLZ$ by AAA.



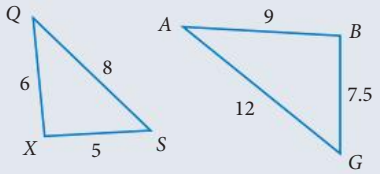
Side-side-side (SSS)

All corresponding sides of the triangles are in the same ratio.

$$\frac{BG}{XS} = \frac{AG}{QS} = \frac{AB}{QX}$$

$$\frac{7.5}{5} = \frac{12}{8} = \frac{9}{6} = 1.5$$

So, $\triangle QSX \parallel\!| \triangle AGB$ by SSS.



Side-angle-side (SAS)

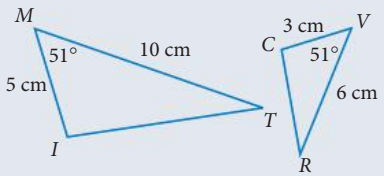
Two corresponding sides of the triangles are in the same ratio and the **included** (enclosed) **angles** are equal.

$$\frac{CV}{IM} = \frac{VR}{MT}$$

$$\frac{3}{5} = \frac{6}{10} = 0.6$$

$\angle TMI = \angle RVC = 51°$

So, $\triangle TMI \parallel\!| \triangle RVC$ by SAS.



Right angle-hypotenuse-side (RHS)

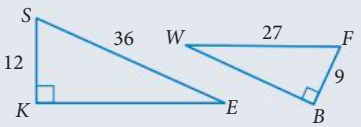
The hypotenuses and one pair of corresponding sides in right-angled triangles are in the same ratio.

$$\frac{WF}{SE} = \frac{FB}{SK}$$

$$\frac{27}{36} = \frac{9}{12} = 0.75$$

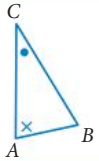
$\angle SKE = \angle FBW = 90°$

So, $\triangle SKE \parallel\!| \triangle FBW$ by RHS.



Exam hack

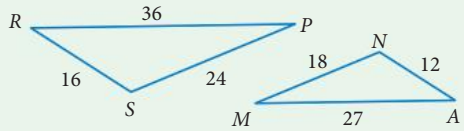
Formal triangle similarity proofs should list all 3 vertices that make the angle, so write $\angle ACB$ instead of just $\angle C$, unless it is obvious which angle is $\angle C$.



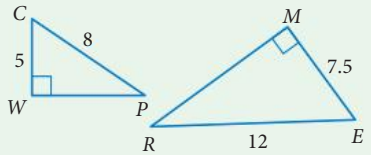
WORKED EXAMPLE 2 Triangle similarity

Determine if the triangles in each pair are similar.

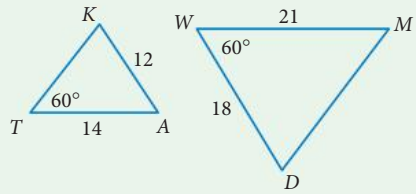
a



b

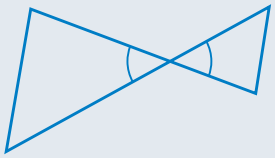
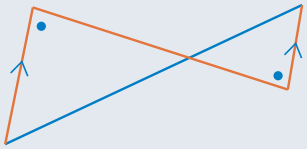
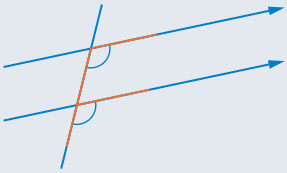
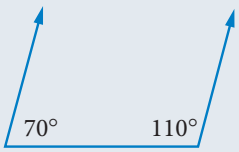
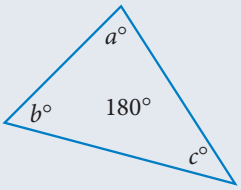
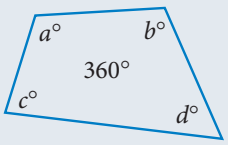


c



Steps	Working
a 1 All side lengths are known, so test using SSS. Find ratios of corresponding sides.	$\frac{MA}{PR} = \frac{27}{36} = \frac{3}{4}$ $\frac{AN}{RS} = \frac{12}{16} = \frac{3}{4}$ $\frac{MN}{PS} = \frac{18}{24} = \frac{3}{4}$
2 The ratios are equal. State the result.	$\triangle RPS \parallel\!\mid \triangle AMN$ (SSS)
b 1 The RHS test could be used.	$\angle CWP = \angle EMR = 90°$
2 Find the ratio of the hypotenuses.	$\frac{RE}{PC} = \frac{12}{8} = \frac{3}{2}$
3 Find the ratio of the corresponding sides.	$\frac{ME}{WC} = \frac{7.5}{5} = \frac{3}{2}$
4 The hypotenuses and a corresponding side of each right-angled triangle have the same ratio. State the result.	$\triangle CPW \parallel\!\mid \triangle ERM$ (RHS)
c 1 Two sides and an angle in each triangle are known, so the SAS test could be used. No other test for similar triangles can be used.	$\angle T = \angle W$, but $\angle T$ is *not* included (enclosed) between the 2 known sides.
2 State the result.	The 2 triangles do not satisfy the SAS test. They are not necessarily similar.

Angle rules

Vertically opposite angles are equal.	
Alternate angles on parallel lines are equal.	
Corresponding angles on parallel lines are equal.	
Co-interior angles on parallel lines add to 180°.	
The angle sum of a triangle is 180°. $a + b + c = 180°$	
The angle sum of a quadrilateral is 360°. $a + b + c + d = 360°$	

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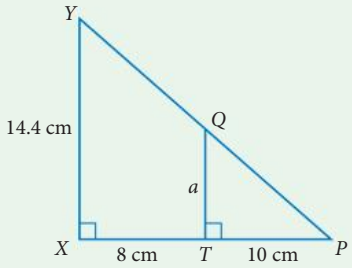
WORKED EXAMPLE 3 Using AAA

Prove that the 2 triangles are similar and calculate the value of a.



Steps	Working
1 Identify the triangles.	The triangles are $\triangle YPX$ and $\triangle QPT$.
2 Look for the rule to use.	Corresponding right angles and shared angle P, so AAA.
3 Write the proof for the AAA rule.	$\angle YXP = \angle QTP$ both marked as 90° **(A)** $\angle P$ common to both triangles **(A)** $\angle XYP = \angle TQP$ (sum of angles = 180°) **(A)**
4 State the result.	$\triangle YPX \,\|\|\| \, \triangle QPT$ (AAA)
5 The triangles are similar, so corresponding side ratios are equal.	$\frac{QT}{YX} = \frac{PT}{PX}$
6 Substitute for known values.	$\frac{a}{14.4} = \frac{10}{(10+8)}$
7 Evaluate.	$18a = 144$ $a = \frac{144}{18}$ $a = 8$

Video
Similar triangles

EXERCISE 5.2 Similar triangles

ANSWERS p. 402

Recap

1 When an image is reduced in size, the shape is considered similar if:

A all sides are the same size

B corresponding sides share a common scale factor

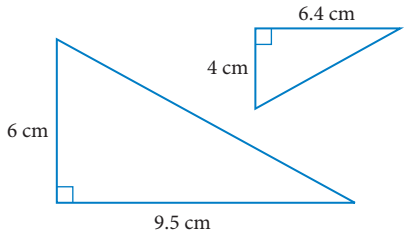
C all internal angles are reduced

D corresponding sides and angles are the same

2 Determine the scale factor for this pair of similar figures to find the value of x.

Mastery

3 Which similarity test should be used to show that these 2 triangles are similar?



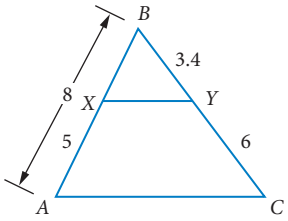
A AAA

B RHS

C SAS

D SSS

4 Which similarity test should be used to show that $\triangle XYB$ and $\triangle ACB$ are similar?



A SAS

B SSS

C RHS

D AAA

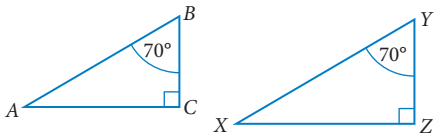
5 **a** Draw $\triangle DAF$ with $\angle D = 70°$, $\angle F = 50°$ and $DF = 60$ mm.

b Now, draw $\triangle KUT$ with $\angle K = 70°$, $\angle T = 50°$ and $KT = 90$ mm.

c Is $\triangle DAF$ similar to $\triangle KUT$? Why?

6 WORKED EXAMPLE 2 Test each pair of triangles for similarity. If they are similar, write a statement of similarity and state the test used.

a



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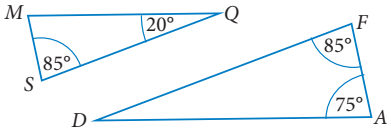
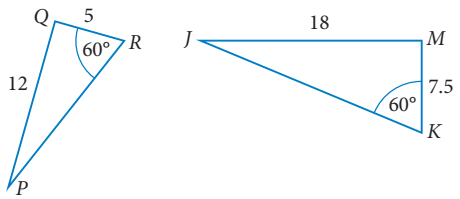


c



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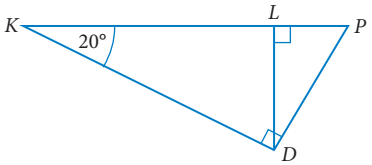

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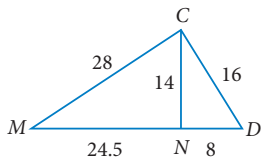

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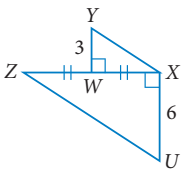
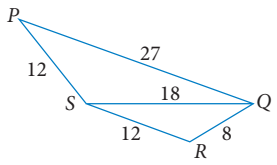
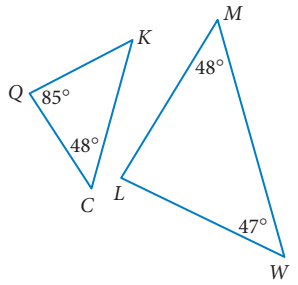


g



7 Is each pair of triangles similar? If they are similar, write a statement of similarity and state the test used.

a



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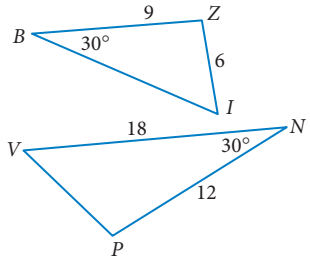
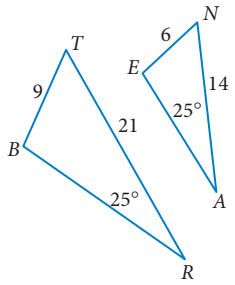
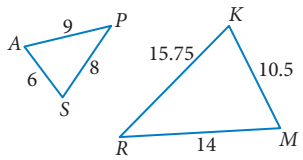


c



d



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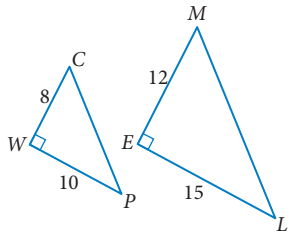

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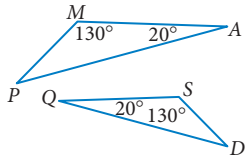

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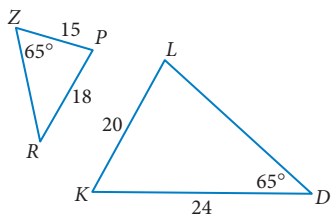

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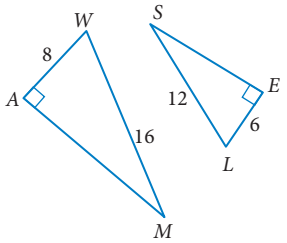

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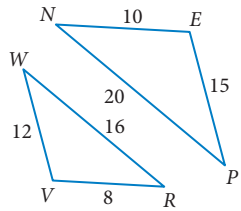
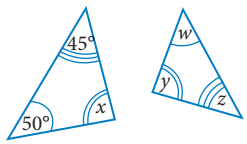


8 WORKED EXAMPLE 3 Find the value of the unknowns in each pair of similar figures.

a



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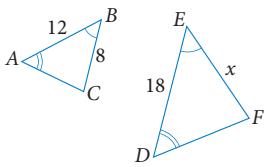

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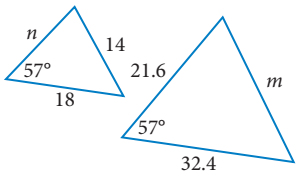

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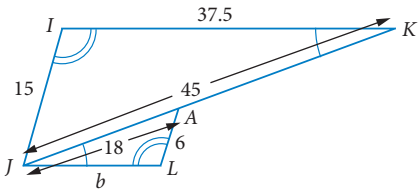

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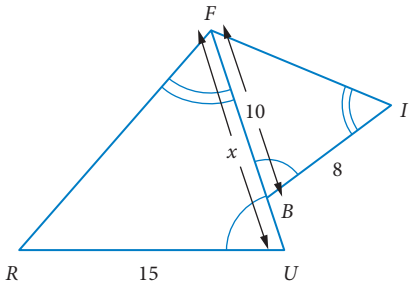

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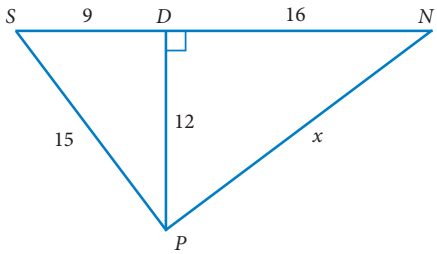
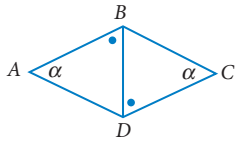


Exam practice

9 Which similarity test should be used to show that $\triangle ABD$ and $\triangle CDB$ are similar?

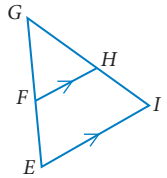


A AAA **B** SSS **C** RHS **D** SAS

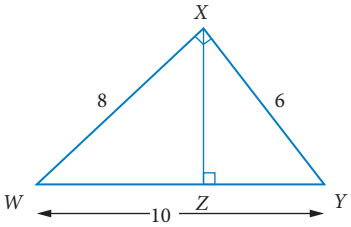
10 State the test that best proves similarity of right-angled triangles, where two pairs of short corresponding sides share a common ratio.

A AAA **B** SSS **C** RHS **D** SAS

11 (2 marks) Prove that $\angle GFH \,|||\, \angle GEI$.



12 (2 marks) Show that triangles WXZ, WXY and XYZ are all similar.



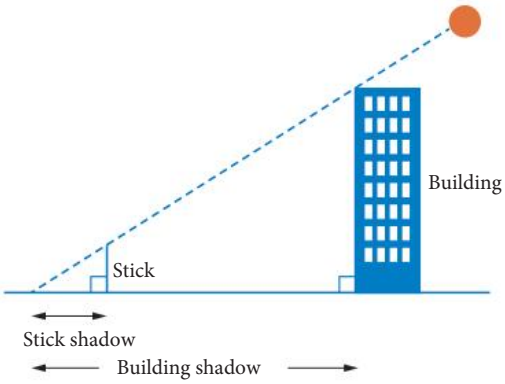
13 (3 marks) CF A right-angled triangle has sides that measure 14 cm, 48 cm and 50 cm. A similar triangle has a hypotenuse that measures 20 cm. Find the length of the shortest side.

5.3 Applications of similar triangles

The application of similar triangles can determine heights of tall objects, such as trees and buildings, using the shadows cast by sticks. In 240 BCE, the Greek astronomer Eratosthenes used shadows to measure the size of Earth.

The shadows must be measured at the same time to make the triangles similar.

Always sketch diagrams or use a given diagram. Add extra labels to reduce the chance of making mistakes.



Video Shadow sticks

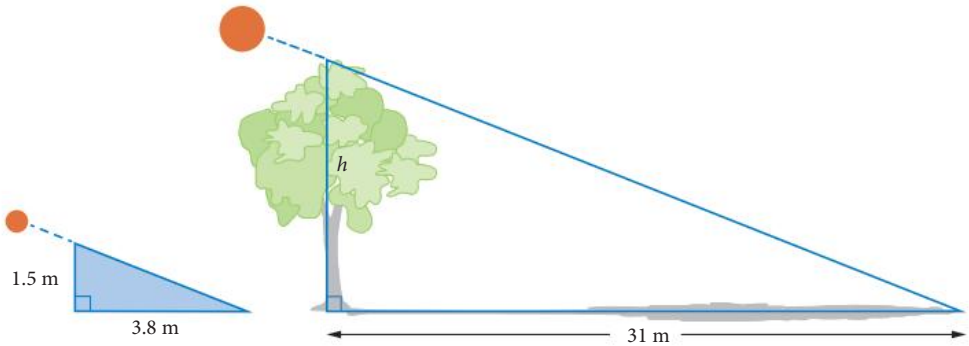
Worksheet Applying similar triangles

WORKED EXAMPLE 4 Shadow sticks

A stick of height 1.5 metres casts a shadow of length 3.8 metres. At the same time, a tree casts a shadow 31 metres long. What is the height of the tree?

Kemphoto artistic works/Alamy Stock Photo

Steps	Working
1 Draw a diagram to show the information.	



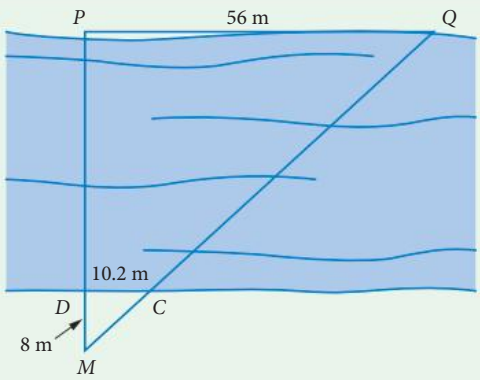
2 The triangles are similar.	The ratio of corresponding sides is equal.
3 Identify corresponding sides.	$\frac{h}{1.5} = \frac{31}{3.8}$
4 Solve for h.	$h = \frac{31}{3.8} \times 1.5$
5 Evaluate.	$= 12.2368\ldots$
6 Round and write the answer.	The tree is about 12.2 metres tall.

Exam hack

Do not forget to state the units on your final answer.

WORKED EXAMPLE 5 Triangulation

A surveyor makes a sketch of a river at the site where a bridge is to be constructed. The width of a river needs to be known. A surveyor places 2 pegs (P and Q) 56 metres apart on one side of the river, as shown.

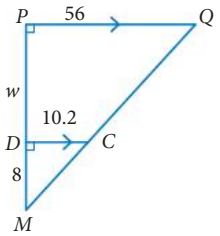


Another peg is placed perpendicular to P on the other side of the river at M.

A line of sight is taken from M to Q and a peg is placed at C. A final peg is placed at D so that $DC \parallel PQ$.

DC and DM are measured to be 10.2 m and 8 m, respectively.

Calculate the width of the river.

Steps	Working
1 Draw a diagram using the information provided. Let the width of the river be w.	
2 Use the AAA test.	$\triangle MDC \parallel\!\mid \triangle MPQ$ (AAA)
3 The ratio of corresponding sides is equal.	$\frac{PM}{DM} = \frac{QM}{CM}$
4 Substitute for known values.	$\frac{w+8}{8} = \frac{56}{10.2}$
5 Cross-multiply and simplify.	$10.2(w+8) = 56 \times 8$ $10.2w + 81.6 = 448$
6 Evaluate.	$w = 35.921\ldots$
7 Round off and state the result.	The river is about 35.9 metres wide.

Investigation Shadow sticks

A metre ruler makes an excellent shadow stick because the calculations are simplified by having the stick exactly 1 m high. In groups of 2 or 3, use a metre ruler and a long tape to measure the heights of buildings and trees around the school.

You will need to make sure the metre ruler is vertical when you are measuring its shadow. This can be done using a spirit level. If you don't have a spirit level, you can check that the stick is vertical by line of sight against something known to be vertical, such as the corner of a building or a pole.

Check your measurements by comparing the results of your group with the results of other groups.

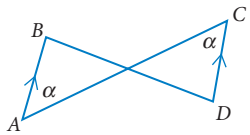
EXERCISE 5.3 Applications of similar triangles

ANSWERS p. 402

5.3

Recap

1 Which rule proves that $\angle ABD = \angle CDB$?



A Alternate angles on parallel lines

B Co-interior angles on parallel lines

C Corresponding angles on parallel lines

D Vertically opposite angles

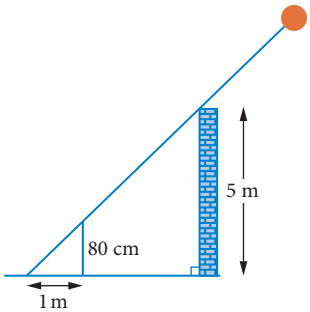
2 A pair of nested triangles share a common ratio of 1.5 for two corresponding sides. They have the same angle between the sides. The rule to prove these are similar is

A AAA **B** RHS **C** SAS **D** SSS

Mastery

3 WORKED EXAMPLE 4 A vertical 80 cm long stick is placed so that the end of its shadow matches the shadow cast by a wall 5 m high. The base of the stick is 1 m from the end of its shadow.

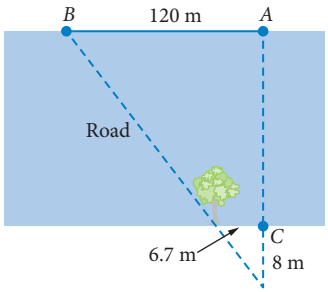
Calculate the length of the shadow of the wall.



4 WORKED EXAMPLE 5 The distance across a road cutting is to be calculated using triangulation. Sightings are taken of 2 points A and B on the opposite side of the road. They are 120 m apart.

The sides of the road cutting are parallel. Point C on the surveyor's side is directly opposite point A. The surveyor notices that a small tree on the edge of the road is opposite C. It just obscures point B. The tree is 6.7 m from C, measured parallel to the road.

What is the width of the road cutting?

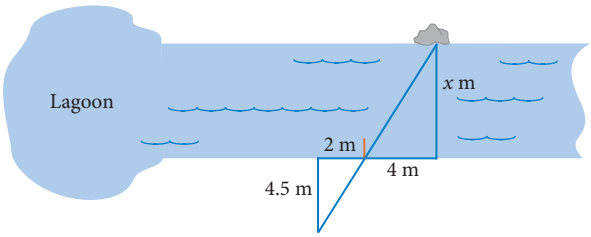


5 A group of students used a straight stick 1.4 m long as a shadow stick. The shadow of a tree was 44.1 m long. At the same time, the shadow of the stick was 1.8 m long.

How tall is the tree?

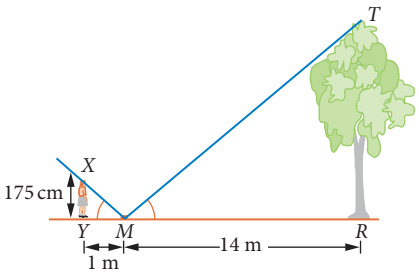
6 Water is rushing out of the long, narrow entrance to a lagoon at low tide. There is a large rock on the opposite bank of the entrance. A fisherman is directly opposite the rock. He walks 4 m along the bank of the entrance and puts a stick in the sand. After walking a further 2 m along the bank, he then must move 4.5 m away from the bank in order to line up the stick and rock.

How wide is the lagoon entrance?

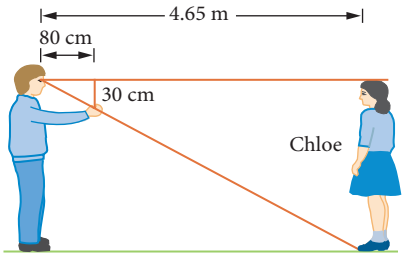


Exam practice

7 (2 marks) Madison measures the height of a tree (TR) by placing a mirror (M) on level ground and moving backwards in line with the tree until she can see the top of the tree in the mirror. Madison is standing at Y and her eyes are 175 cm above the ground. The mirror is 1 m from her and 14 m away from the tree. How tall is the tree?



8 (2 marks) Ryan is holding a 30 cm ruler 80 cm from his eyes, as shown. He finds that when Chloe is standing 4.65 m away from him on level ground, the top of her head lines up with the top of the ruler. How tall is Chloe?



9 (3 marks) CF The facade of a historic building was preserved when the building was demolished for a new shopping centre. In the morning, the sun shone through the facade, casting a shadow across the street. There were bright patches in the shadow from the windows of the facade. The whole shadow was 25 m long. The bright patch from a 1.8 m-high window was 2.4 m long.

How high was the facade?

Worksheets
Scales and scale diagrams

Interpreting an office plan

5.4 Scale drawings

Scale

A scale drawing is a diagram that represents a real object with a similar figure.
The **scale** (or **scale factor**) is a ratio.
Scale = scaled length : actual length

Building plans and maps are examples of **scale drawings**.

WORKED EXAMPLE 6 Changing a scale

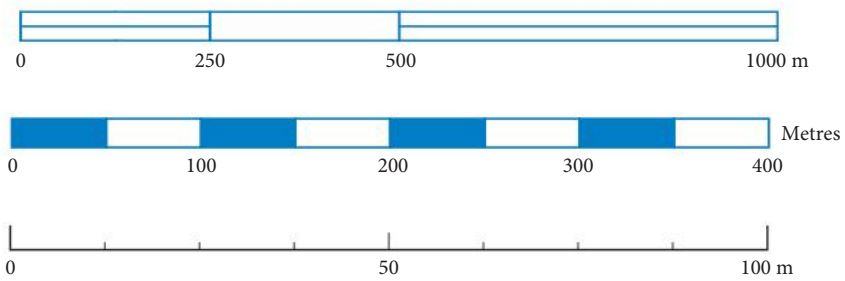
a Change a scale of 1 cm : 200 m to a ratio.

b Complete the following:

1 : 500 is the same as 1 cm : … m.

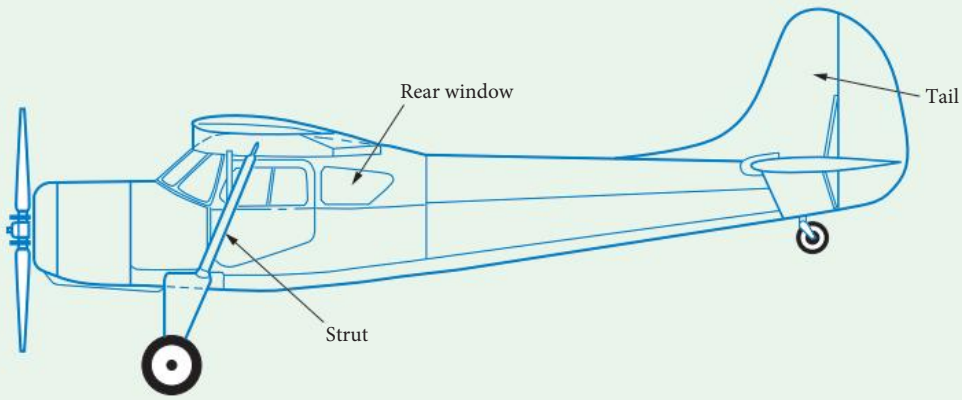
Steps	Working
a Make the units the same using 1 m = 100 cm.	1 cm : 200 m = 1 cm : 200 × 100 cm = 1 cm : 20 000 cm
Remove the units.	= 1 : 20 000
b The real object is 500 times bigger.	Real size for 1 cm : 500 cm
Change to metres.	500 cm = 5 m
Write the answer.	1 : 500 is the same as 1 cm : 5 m.

Sometimes, a scale is shown by a legend like these:



WORKED EXAMPLE 7 Scale drawing

The drawing shows a fixed-wing aircraft drawn to a scale of 1 : 125. Use a ruler to measure in mm when answering the questions.



a What is the length of the propeller?

b A line is drawn between the centres of the front and rear wheels on the drawing. If the actual distance is 10.88 m, how long should the line on the drawing be?

Steps	Working
a **1** Measure the propeller length on the drawing.	drawing length = 32 mm
2 Write the scale.	scale = 1 : 125
3 Multiply by the scale factor, 125.	actual length = 32 × 125 mm = 4000 mm
Every 1 mm on the drawing is the same as 125 mm in the actual measurement.	
4 Write the answer in metres.	The length of the propeller is 4 m.
b **1** Write the scale.	scale = 1 : 125
2 Name the unknown quantity.	drawing length = x
3 Write an equivalent ratio to the scale using the actual measurement and x.	1 : 125 = x : 10.88 m
4 Convert to cm.	1 : 125 = x : 1088 cm
5 Write the ratios as fractions.	$\frac{1}{125} = \frac{x}{1088}$
6 Reverse the equation and solve for x.	$x = \frac{1}{125} \times 1088$
7 Evaluate.	= 8.704
8 Round off and write the answer, including units.	The line on the drawing should be 8.7 cm long.

Video Scale drawings

Exam hack

Make a list at the side of different conversions for the same scale.

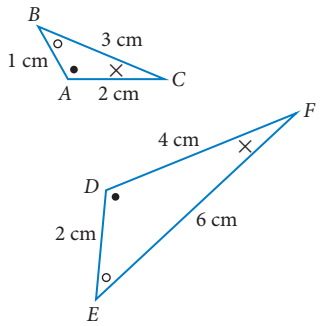
For example, 1 : 200, 1 mm : 200 mm, 1 mm : 20 cm, 1 mm : 0.2 m

EXERCISE 5.4 Scale drawings

ANSWERS p. 402

Recap

Refer to these triangles for Questions 1 and 2.



1 Determine if the triangles share a common ratio.

2 Write a proof to determine that $\triangle ABC \,|||\, \triangle DEF$.

Mastery

3 A scale model of the Eiffel Tower is 6 cm tall. The Eiffel Tower is 300 m tall.
What is the scale of the model?

A 1 : 50 **B** 1 : 250 **C** 1 : 500 **D** 1 : 5000

4 A map has a scale of 1 : 250 000. The distance between 2 cities on the map is 18 cm.
What is the actual distance between the cities?

A 7.2 km **B** 4.5 km **C** 45 km **D** 72 km

5 An architect's blueprint for a new house shows the back deck as 7 cm long.
The scale used to create the blueprint is 0.5 cm : 1 m. What is the actual length of the deck?

A 35 m **B** 14 m **C** 3.5 m **D** 140 m

6 WORKED EXAMPLE 6 Write each scale in ratio form.

a 1 cm : 1 m **b** 1 cm = 1 km **c** 1 mm = 1 m

d 1 mm : 10 m **e** 2 cm : 1 m **f** 5 mm = 1 km

7 Find each missing value.

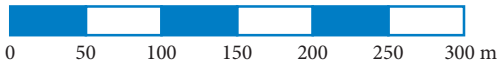
a 1 : 100 is the same as 1 cm : … m **b** 1 : 1000 is the same as 1 mm = … m

c 1 : 500 is the same as 1 cm = … m **d** 1 : 10 000 is the same as 1 cm : … m

e 1 : 650 000 is the same as 1 cm = … km **f** 1 : 50 000 is the same as 1 mm : … m

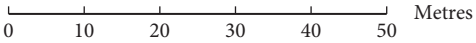
8 Write each scale as a ratio in simplest form.

a



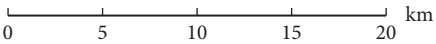
b

c



d

e



f

9 WORKED EXAMPLE 7 Use the aircraft plan in Worked example 7 to find the real measurements of the

a overall length of the aircraft

b length of the strut joining the wing and the forward wheel housing

c diameter of the forward wheel

d distance from the top of the tail to the bottom of the rear wheel

e width and height of the rear window.

10 Use the bathroom plan to find the real measurement (in mm) for the

a length of the bath

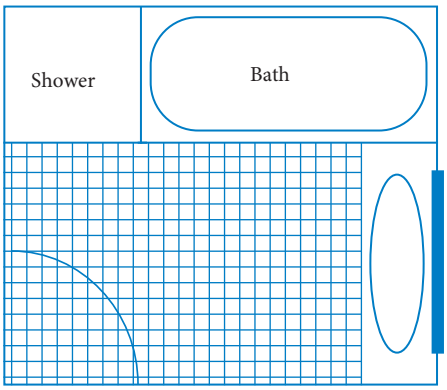
b width of the shower

c width of the door

d width of the window

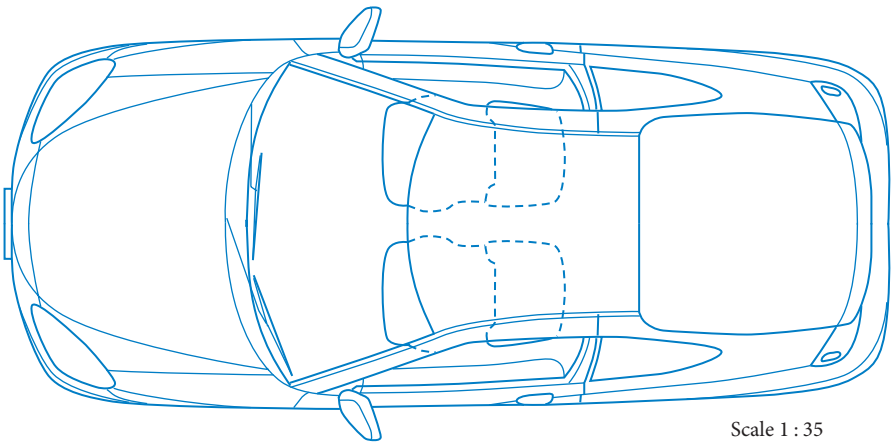
e length of the vanity unit

f width of the vanity unit.



Scale 1 : 50

11 This is a scale drawing of the top of a car.



Scale 1 : 35

In real measurements

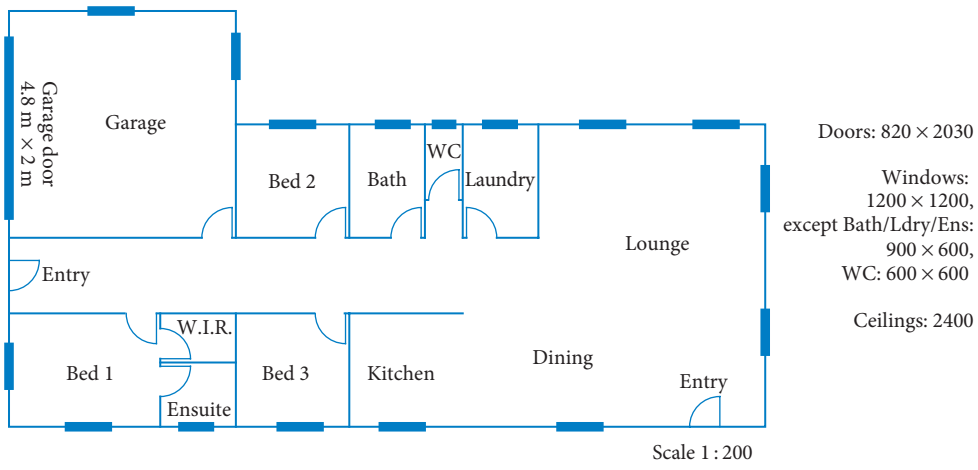
a how long is the car?

b how wide is the car body?

c how wide is the front car seat?

d what is the smallest distance between the seats?

Questions 12 to 14 refer to the house plan shown below.



12 Find the cost of painting the ceilings of the house and garage with 2 coats, given that the paint covers 11 m^2/L and only comes in 4 L tins costing \$78.60 each. Fully explain your reasoning.

13 **a** Find the dimensions of the garage.

b If the materials and labour have been quoted at \$88.70/m^2, how much will it cost to coat the floor of the garage with a seamless epoxy resin coating?

14 The cost of materials and labour for carpet is \$82.50/m^2. How much will it cost to carpet the lounge/dining/entry floor?

Exam practice

15 (2 marks) A map has a scale of 1 : 150 000.

a What actual distance is represented by 12 cm on the map? *[1 mark]*

b Calculate the distance on the map if the actual distance is 2.2 km. *[1 mark]*

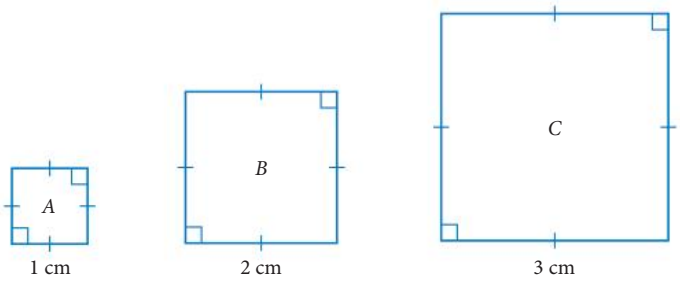
16 (2 marks) The distance between Brisbane and Mackay is 800 km. What distance would this be on a map drawn to a scale of 1 : 5 000 000?

17 (2 marks) The scale on a map is 1 : 25 000. The distance between 2 points on a map is 275 mm. What is the true distance between the points?

18 (3 marks) CU A flat roof is 6.9 m wide and 10 m long. Corrugated iron sheets are 2 m by 900 mm and must be overlapped by 8 cm at their sides and ends. How many sheets of corrugated iron are needed to cover the roof?

5.5 Areas and volumes of similar figures

Consider the 3 squares shown below.



Squares B and C are enlargements of square A, so the figures are similar.

When the linear scale factor (k) is 2, the area scale factor $(B:A) = \frac{4}{1} = 4\ (k^2)$.

When the linear scale factor (k) is 3, the area scale factor $(C:A) = \frac{9}{1} = 9\ (k^2)$.

Video Areas of similar figures

Worksheet Areas and volumes of similar figures

Area scale factors

$$\text{linear scale factor} = k = \frac{\text{length of image}}{\text{length of original}}$$

$$\text{area scale factor} = k^2 = \frac{\text{area of image}}{\text{area of original}}$$

WORKED EXAMPLE 8 Area scale factor

A photo has a length of 6 cm and an enlargement has a length of 15 cm.

a **Calculate** the area scale factor.

b If the smaller photo has an area of 24 cm^2, what is the area of the enlargement?

Auscape International Pty Ltd/Alamy Stock Photo

Steps	Working
a **1** Calculate the linear scale factor.	linear scale factor $= \frac{15}{6} = \frac{5}{2}$
2 Calculate the area scale factor.	area scale factor = (linear scale factor)^2
3 Substitute.	$= \left(\frac{5}{2}\right)^2$
4 Evaluate.	$= \frac{25}{4}$
5 State the result.	The area scale factor of the enlargement is $\frac{25}{4}$.
b **1** Calculate the area of the larger photo.	area of larger photo $= \frac{25}{4} \times 24$
2 Evaluate.	$= 150$
3 State the result.	The area of the enlargement is 150 cm^2.

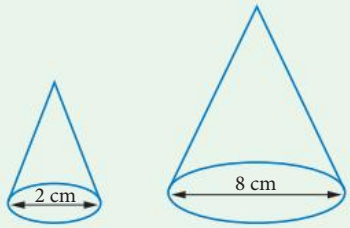
The rule for the scale factor for surface area is the same as for area.

WORKED EXAMPLE 9 Surface area scale factor

A cone has been enlarged to scale.

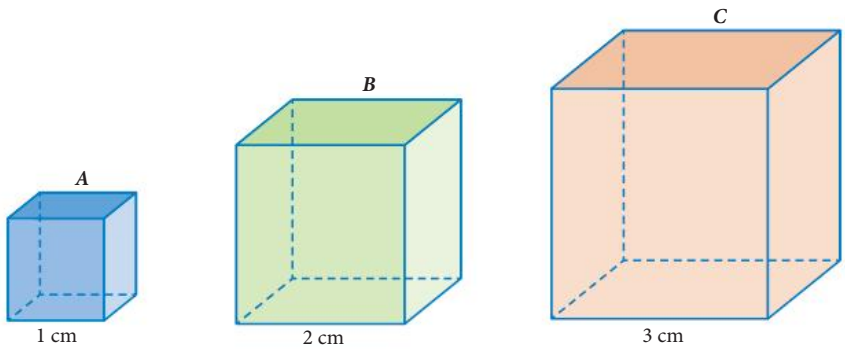
a **Calculate** the area scale factor.

b The original image has a total surface area of $13\,\text{cm}^2$. **Use** the scale factor to calculate the surface area of the enlarged image.



Steps	Working
a **1** Calculate the linear scale factor.	linear scale factor $= \frac{8}{2} = 4$
2 Calculate the area scale factor.	area scale factor $=$ (linear scale factor)^2 $= 4^2$ $= 16$
3 State the result.	The area scale factor is 16.
b **1** Calculate the surface area of the image shape.	$S = 13\,\text{cm}^2 \times 16$
2 Evaluate.	$= 208\,\text{cm}^2$
3 State the result.	The area of the enlargement is $208\,\text{cm}^2$.

Consider these 3 cubes.



Cubes B and C are enlargements of cube A, so the shapes are similar.
The volume of a cube is $V = s^3$, where s is the side length.

When the linear scale factor is 2, the volume scale factor $(B:A) = \frac{2^3}{1} = \frac{8}{1} = 8$.

When the linear scale factor is 3, the volume scale factor $(C:A) = \frac{3^3}{1} = \frac{27}{1} = 27$.

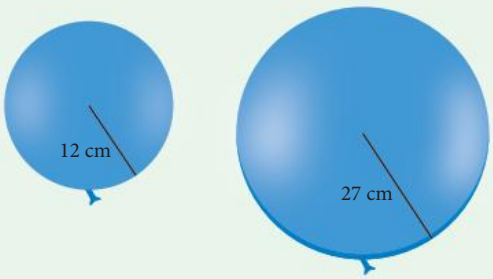
Volume scale factors

linear scale factor $= k$

volume scale factor $= \frac{\text{volume of image}}{\text{volume of original}} = k^3$

WORKED EXAMPLE 10 Volume scale factor

A spherical balloon has a radius of 12 cm. It is inflated until the radius reaches 27 cm.



a **Calculate** the linear scale factor.

b **Calculate** the volume scale factor.

c The capacity of the larger balloon is 82.5 L. What is the capacity of the smaller one?

Steps	Working
a Calculate the linear scale factor.	linear scale factor $= \frac{27}{12} = \frac{9}{4}$
b **1** Calculate the volume scale factor.	volume scale factor = (linear scale factor)^3
2 Substitute.	$\left(\frac{9}{4}\right)^3 = \frac{729}{64}$
c **1** Volume and capacity are the same.	volume of larger balloon = volume of smaller balloon × volume scale factor
2 Let the volume of the small balloon = x.	$82.5 = x \times \frac{729}{64}$
3 Solve for x.	$x = \frac{82.5 \times 64}{729}$ $= 7.2427\ldots$
4 State the result.	The capacity (volume) of the smaller balloon is about 7.2 L.

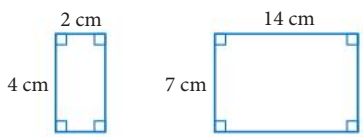
Video
Volume scale factor

EXERCISE 5.5 Areas and volumes of similar figures

ANSWERS p. 402

Recap

1 Give reasons why this pair of shapes are similar.



2 Using the scale 1 : 2000, calculate the actual measurement of the block length, if the plan length is 1 cm.

Mastery

3 WORKED EXAMPLE 8 A rectangle has a length of 4 cm and a similar rectangle has a length of 6 cm. The area scale factor is

A 1.5 **B** 3.375 **C** 4.90 **D** 2.25

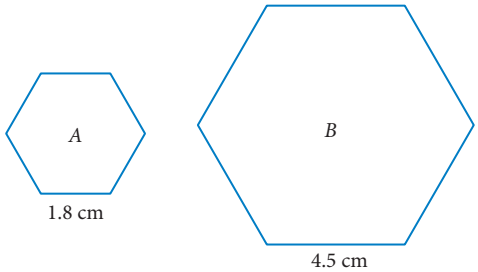
4 Shapes A and B are similar. If the area of A is $17\,cm^2$, then the area of B is

A $42.5\,cm^2$

B $106.25\,cm^2$

C $125.25\,cm^2$

D $68\,cm^2$



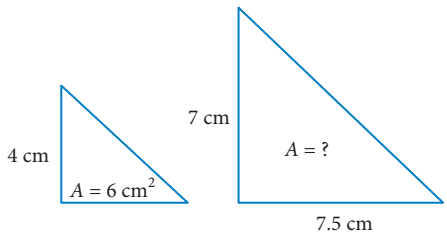
5 For each pair of similar shapes, calculate the

i linear scale factor

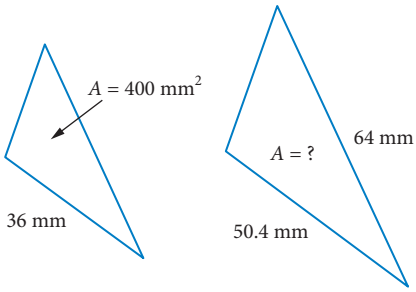
ii area scale factor

iii unknown area.

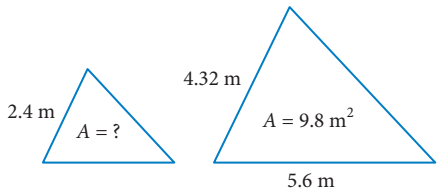
a



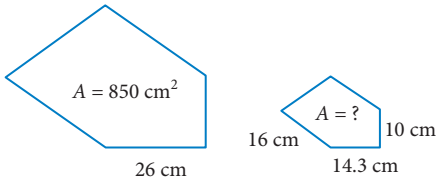
b



c



d



6 WORKED EXAMPLE 9 The architect's plans for a building extension show a rectangular verandah with a length of 7 cm and a width of 3.5 cm. The scale on the plans is shown as 1 : 250.

a Calculate the area scale factor.

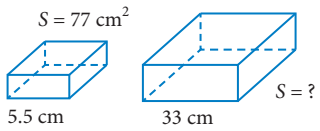
b Calculate the area of the verandah.

7 For each pair of similar solid shapes, calculate the

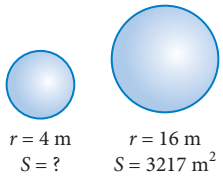
i area scale factor

ii unknown surface area

a



b



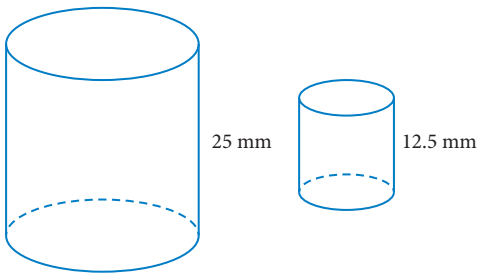
8 WORKED EXAMPLE 10 The 2 cylinders shown here are similar. The heights of the cylinders are shown. If the volume of the larger cylinder is $8\,cm^3$, then the volume of the smaller cylinder is

A $4\,cm^3$

B $10\,cm^3$

C $1\,cm^3$

D $64\,cm^3$



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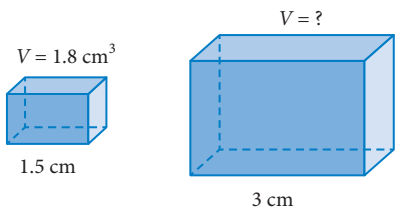
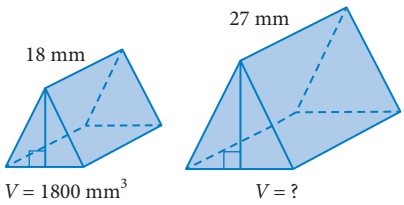
9 For each pair of similar 3D shapes, calculate the

- **i** linear scale factor
- **ii** volume scale factor
- **iii** unknown volume indicated.

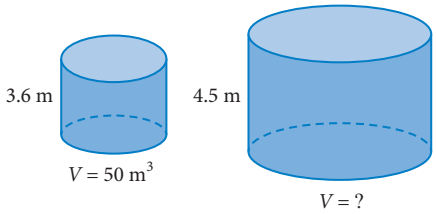
a



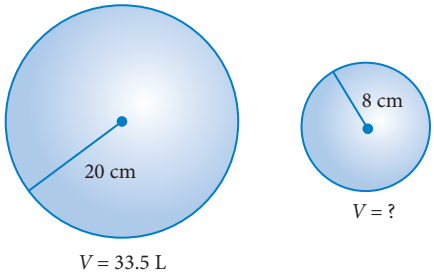
b



c

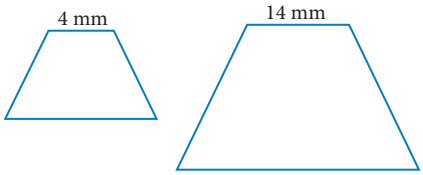


d



Exam practice

10 The area scale factor for the following shapes is

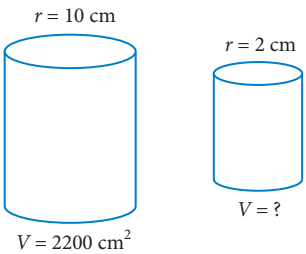


A 3.25 **B** 3.5 **C** 10.5 **D** 12.25

11 Two shapes have a linear scale with $k = 0.6$. Which of the following is the volume scale?

A 0.84 **B** 0.216 **C** 0.36 **D** 1.8

12 (3 marks) CF Calculate the volume of the following reduced cylinder.



13 (3 marks) CF Two similar-shaped drink bottles are made by a manufacturer. One has a volume of 600 mL and the other 1.25 L. Calculate the linear scale factor between 2 bottles.

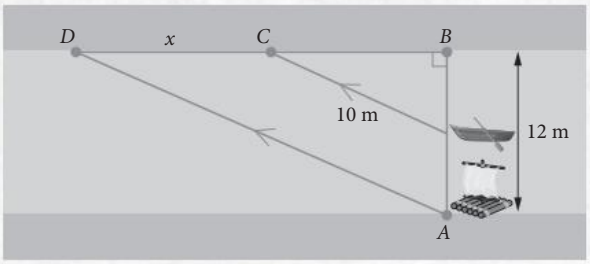
14 (3 marks) CU A model of an SUV has a scale of 1 : 15. If the surface area of the body of the SUV that is spray-painted is 6.74 m^2, what is the equivalent painted area on the model?

Video Exam question analysis: Similar figures and scale factors

EXAM QUESTION ANALYSIS

Exam-style question (4 marks)

The diagram below shows the drifting paths of a boat and raft down a river. The boat starts from a position halfway across the river and will drift to point C. The raft starts from point A and will drift to point D.



Using the diagram, calculate the distance between the boat and the raft once they reach the other side of the river at their finishing points.

Reading the question

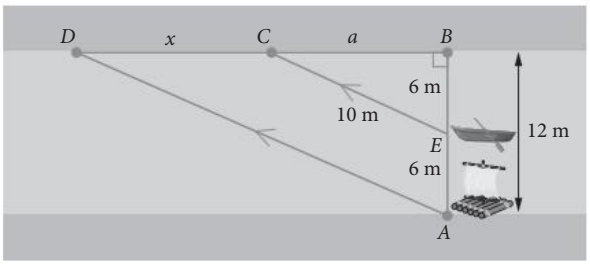
- The boat is starting from halfway across the river and will travel to C.
- The raft will travel from A to D.
- The aim is to find x, the distance between C and D.

Thinking about the question

- The image shows nested right-angled triangles. Similar triangles and a scale factor can be used.
- If the boat is halfway across the river, it is 6 m from the shore.
- The distance DB needs to be calculated. To do this, CB needs to be found.
- There isn't enough information for CB using scale factor. Use Pythagoras' theorem.

Worked solution (✓ = 1 mark)

Label the additional information onto the diagram.



Let $CB = a$:

$a^2 = c^2 - b^2$

$a^2 = 10^2 - 6^2 = 64$

$a = \sqrt{64} = 8$

$CB = \mathbf{8\,m}$ ✓

scale factor $= \dfrac{12}{6}$ Triangles ABD and EBC are similar.

$= \mathbf{2}$ ✓

Find DB using scale factor:

$DB = CB \times 2$

$= 8 \times 2$

$= \mathbf{16\,m}$ ✓

$DC = 16 - 8$

$= 8\,\text{m}$

The distance between the boat and raft will be **8 m** at the riverbank. ✓

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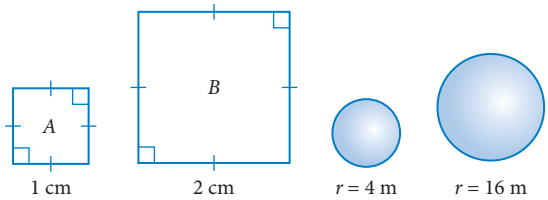
5 Chapter summary

- **Similar figures** are exactly the same shape but not necessarily the same size. The symbol for similarity is |||.



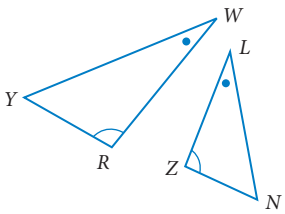
In similar figures:

- corresponding angles are equal
- the ratios of corresponding sides are equal
- **scale factor** = $\dfrac{\text{image length}}{\text{original length}}$

Tests for similar triangles

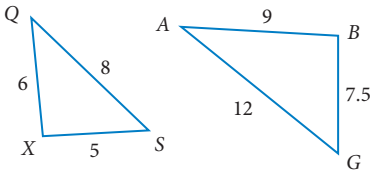
- **angle-angle-angle** (**AAA**) Two pairs of corresponding angles of the triangles are the same size.

AAA



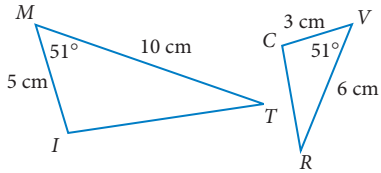
- **side-side-side** (**SSS**) All corresponding sides of the triangles are in the same ratio.

SSS



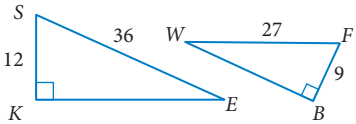
- **side-angle-side** (**SAS**) Two corresponding sides of the triangles are in the same ratio and the **included angles** are equal.

SAS



- **right angle-hypotenuse-side** (**RHS**) The hypotenuses and one pair of corresponding sides are in the same ratio in 2 right-angled triangles.

RHS



Scale drawings

- A **scale drawing** is a diagram that represents a real object.

  The **scale** of a drawing is written as a ratio: plan length : actual length.

Area and volumes of similar figures

- If the linear scale factor is k, the **area scale factor** is k^2 and the **volume scale factor** is k^3.

Worksheet
General Maths Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

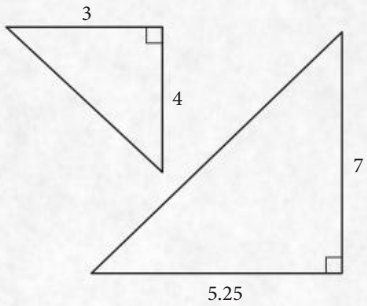
Section 2 (13 marks): 5 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 Which similarity test can prove that these triangles are similar?



A ASA **B** AAA

C RHS **D** SAS

2 The height of a model of the 74 m-high Story Bridge is 29.6 cm. The scale of the model is

A 1 : 25 **B** 1 : 2500 **C** 1 cm : 25 m **D** 1 : 250

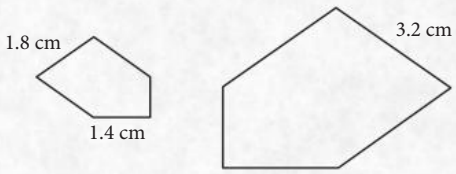
3 Amelia invested \$5600 in treasury bonds paying simple interest at 4.75% p.a. The payments are made every 6 months. How much will Amelia get in her next payment?

A \$133 **B** \$266 **C** \$532 **D** \$1596

4 Which equation states that 8 more than 3 times a number equals 32?

A $8x + 3 = 32$ **B** $3x + 8 = 32$ **C** $\frac{x}{3} + 8 = 32$ **D** $x + 24 = 32$

5 The 2 shapes shown here are similar. If the area of the smaller shape is 25 cm^2, then the area of the larger shape is closest to



A 50 cm^2 **B** 88 cm^2

C 79 cm^2 **D** 105 cm^2

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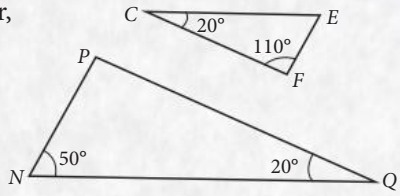
Section 2 5 short response questions

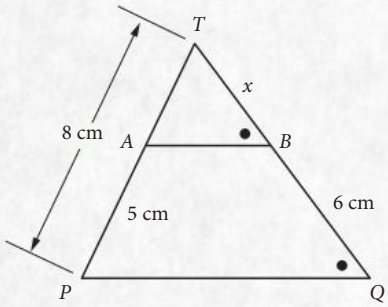
13 marks

6 (2 marks) Determine whether these 2 triangles are similar, giving a test for similarity if they are.



7 (2 marks) Jayden works a regular 36-hour week and is eligible for overtime calculated at time-and-a-half. He is paid $36.50 per hour and works an additional 6 hours overtime in the week. Calculate Jayden's total weekly pay.

8 (3 marks) Calculate the value of x in the diagram.



9 (2 marks) Find the surface area of a hemisphere with a diameter of 12 cm.

10 (4 marks) Felicity travels to Japan. She exchanges A$1200 to Japanese yen.

a How many yen will she have if the exchange rate is A$1 = ¥96? *[1 mark]*

b While in Japan, Felicity spends ¥86 400. How much of her A$1200 has been spent (based on the rate from part **a**)? *[1 mark]*

c On returning to Australia, the exchange rate changed to A$1 = ¥94.
How many Australian dollars will Felicity receive in exchange for the Yen she did not spend? *[2 marks]*

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

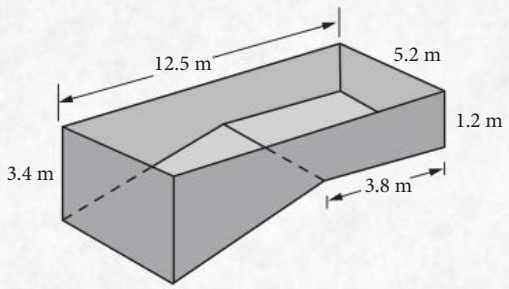
4 short response questions

12 marks

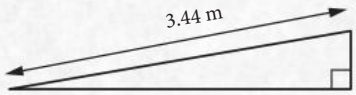
1 (2 marks) A spherical container has a radius of 4 m. What is the height of a rectangular container that is 7 m long and 5 m wide if the 2 containers have the same surface area? Give your answer correct to 2 decimal places.

2 (2 marks) During the sale season, a shop discounts a blender by 20%. As the blenders were not selling, the shop further discounts the blender by 40%. If the item is on sale for $45, what was the original price of the blender?

3 (4 marks) A newly constructed swimming pool is shown. If the price of water is $400 per megalitre, calculate how much it will cost to fill the pool.



4 (4 marks) The Australian standard for the slope (gradient) of a wheelchair ramp is 1 : 14 for ramps with lengths over 1900 mm. Determine the overall height of a 3.44 m ramp that is compliant with the Australian standard.



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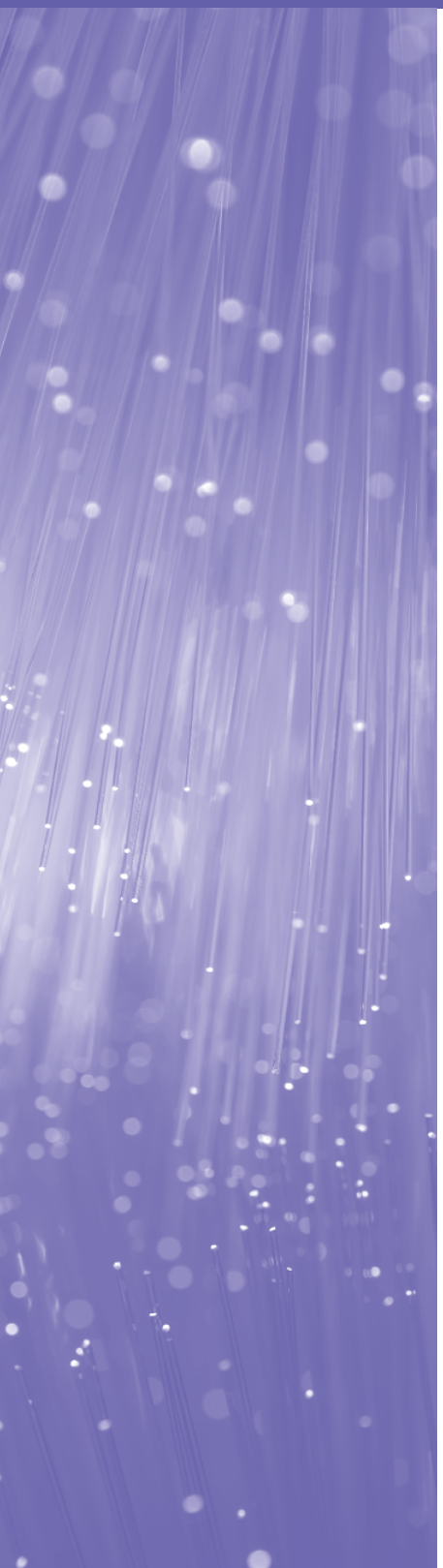
CHAPTER

6 LINEAR EQUATIONS AND GRAPHS



Syllabus coverage

Nelson MindTap chapter resources

Terminology

6.1 Equations

6.2 Equations with variables on both sides

6.3 Equation problems

6.4 Straight-line graphs

Linear functions

The slope of a line

6.5 Finding the equation of a line

6.6 Linear modelling

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

Prior learning
Linear equations and graphs

Syllabus coverage

UNIT 1, TOPIC 5: LINEAR EQUATIONS AND THEIR GRAPHS

Linear equations

- Solve linear equations, including equations with variables on both sides and equations with rational solutions.
- Develop a linear equation from a description in words.
- Solve practical problems involving linear equations.

Straight-line graphs

- Understand and use the slope–intercept form of a linear function, $y = mx + c$ where m is slope (gradient) and c is y-intercept.
- Construct a straight-line graph using a linear function of the form, $y = mx + c$ where m is slope (gradient) and c is y-intercept.
- Determine the slope (gradient), x-intercept and y-intercept of a straight line from both its equation and its graph.
- Interpret, in context, the slope (gradient) and intercept of a linear function used to model and analyse a practical situation.
- Construct and analyse a straight-line graph to model a given linear relationship, e.g. modelling the cost of filling a fuel tank of a car against the number of litres of petrol required.

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Videos (14):

6.2 Equations with variables on both sides
• Equations with brackets

6.4 Graphing linear equations
• Gradient and y-intercept of a line
• Graphing $y = mx + c$ • Intercepts of a line
• Testing if a point lies on a line

6.5 The gradient of a line
• Gradient (slope) of a line
• Finding the equation of a line
• The equation of a line through 2 points

6.6 Linear modelling • Applying linear functions

Exam question analysis Linear equations and graphs

Prior learning (1):

6.1 Linear equations and graphs

Skillsheets (4):

6.1 Solving equations by backtracking
• Solving equations by balancing
• Solving equations using diagrams

6.4 Graphing linear equations

Spreadsheet (1):

6.4 Graphing $y = mx + c$

Worksheets (19):

6.1 One- and two-step equations

6.3 Using equations to solve problems
• Word problems with equations

6.4 Graphing linear functions 1
• Graphing linear functions 2
• Drawing gradients • Linear graphs
• A page of lines • A page of number planes
• Number plane grid paper
• Straight-line equations

6.5 Finding the gradient between two points on a line • Equations of a line
• Gradient and y-intercept • $y = mx + c$
• Finding the equation of a line • A page of lines

6.6 Practical applications

Cumulative exams General Maths Year 11 formula sheet

Puzzles (8):

6.1 Substitution code puzzle

6.2 Equations with unknowns on both sides
• Equations riddle • Equations code puzzle
• Equations order activity

6.3 Writing and solving equations

6.5 Linear equations code puzzle
• Equation of a line

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

Terminology

Cartesian (number) plane	coordinates	dependent variable	equation
gradient	independent variable	inverse operation	linear equation
linear function	linear model	origin	quadrants
rise	run	slope	slope–intercept form
solution	solve	x-axis	x-intercept
y-axis	y-intercept		

6.1 Equations

An **equation** such as $3x - 4 = 10$ has an 'equal' sign. Solving an equation means finding a value for the variable that makes the statement true. This value is called the **solution** of the equation.

WORKED EXAMPLE 1 Checking an equation solution

Determine whether $x = 8$ is a solution to the equation $\frac{x-2}{3} = 3$.

Steps	Working
1 Substitute 8 for x in the LHS (left-hand side).	$\text{LHS} = \frac{x-2}{3}$ $= \frac{8-2}{3}$
2 Evaluate.	$= 2$ $\text{RHS} = 3$
3 Compare the LHS and RHS (right-hand side).	$\text{LHS} \neq \text{RHS}$
4 State the result.	$x = 8$ is not a solution.

Equations are solved using **inverse operations**. An inverse operation has the effect of undoing the original operation.

Whatever you do to one side of the equation must be done to the other side.

As discussed in Chapter 4, use inverse operations in the *reverse order* of operations performed on the variable in the equation.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

WORKED EXAMPLE 2 Solving simple equations

Solve each equation.

a $m - 53 = 46$ **b** $7b = 98$

Steps	Working
a 1 Write the equation.	$m - 53 = 46$
2 Undo '− 53' using the inverse operation (+ 53).	$m = 46 + 53$
3 Evaluate.	$m = 99$
b 1 Write the equation.	$7b = 98$
2 Undo '× 7' using the inverse operation (÷ 7).	$b = \frac{98}{7}$
3 Evaluate.	$b = 14$

$\frac{98}{7}$ is equal to $98 \div 7$

Skillsheets
Solving equations by backtracking
Solving equations by balancing
Solving equations using diagrams

Worksheet
One- and two-step equations

Puzzle
Substitution code puzzle

WORKED EXAMPLE 3 Equations with two operations

Solve each equation.

a $16 - 3n = 7$ **b** $\frac{4m}{3} = 16$

Steps	Working
a 1 Write the equation.	$16 - 3n = 7$
2 Undo the '+16' using the inverse operation (– 16).	$-3n = 7 - 16$
3 Evaluate.	$-3n = -9$
4 Undo the '× (–3)' using the inverse operation (÷ (– 3)).	$n = \frac{-9}{-3}$
5 Evaluate.	$n = 3$
b 1 Write the equation.	$\frac{4m}{3} = 16$
2 Undo the '÷ 3' using the inverse operation (× 3).	$4m = 16 \times 3$
3 Evaluate.	$4m = 48$
4 Undo the '× 4' using the inverse operation (÷ 4).	$m = \frac{48}{4}$
5 Evaluate.	$m = 12$

Exam hack

You can always check your solution by substituting it back into the equation.

Exam hack

Students often make the mistake of using inverse operations on one side of the equation only, not to both sides. Maintain **balance** on each side of the equal sign.

EXERCISE 6.1 Equations

ANSWERS p. 403

Mastery

1 WORKED EXAMPLE 1 For each equation, determine whether $x = 12$ is a solution.

a $x + 8 = 20$ **b** $4x - 11 = 37$ **c** $4x - x = 36$

d $3(x - 5) = 24$ **e** $9 - \frac{2x}{3} = 1$ **f** $2(x + 2) = 4(x - 8)$

2 WORKED EXAMPLE 2 Solve each equation.

a $x - 21 = 13$ **b** $a + 22 = 46$ **c** $5u = 45$

d $\frac{d}{7} = 12$ **e** $f - 28 = 13$ **f** $k + 46 = 17$

g $-8e = 112$ **h** $\frac{y}{11} = 13$ **i** $h - 17 = -28$

3 Solve each equation.

a $8 - k = 8$ **b** $10.5 = 8.4 - x$ **c** $6 - t = 0$

d $9 - g = 6$ **e** $5 - p = 9$ **f** $121 - u = 98$

g $-y + 4 = 7$ **h** $-22 - n = -8$ **i** $3 - a = 1.7$

4 WORKED EXAMPLE 3 Solve each equation.

a $3z - 5 = 16$ **b** $5x + 22 = 7$ **c** $4y - 9 = 7$

d $8a + 17 = 57$ **e** $13 + 5b = 28$ **f** $17 + 9c = 8$

g $3.2p - 9.3 = 13.1$ **h** $-7 = 2u + 5$ **i** $0.8q - 1.7 = 3.1$

j $\frac{m}{4} - 6 = -3$ **k** $\frac{t}{7} = \frac{12}{28}$ **l** $-\frac{4x}{5} = -12$

Exam practice

5 If 5 is subtracted from the product of 2 and p, the result is 13. The equation that represents this statement is

A $5 - 2 + p = 13$ **B** $5 - 2p = 13$ **C** $2 - 5p = 13$ **D** $2p - 5 = 13$

6 An equilateral triangle with a perimeter of 36 cm, has side dimensions of $2x$.
The value of x is

A 3 **B** 6 **C** 30 **D** 42

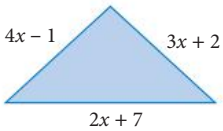
7 (6 marks) Solve each equation.

a $4y + 2 = 34$ *[2 marks]*

b $\frac{x}{15} + 3 = 48$ *[2 marks]*

c $\frac{-12 + d}{11} = -3$ *[2 marks]*

8 (4 marks) CF This triangle has a perimeter of 35 m.
Determine which type of triangle it is.



9 (3 marks) CF The perimeter of a rectangle is 360 m. Its length is 40 m more than its width.
Use an equation to find the width.

6.2 Equations with variables on both sides

To solve equations that have variables on both sides, move all the terms with variables to one side of the equation and all the numbers to the other side. Remember to use inverse operations.
For equations with brackets, expand all expressions with brackets first.

WORKED EXAMPLE 4 Variables on both sides

Solve each equation.

a $8m = 5m + 21$ **b** $11 - 6u = 3u - 16$

Steps	Working
a 1 Write the equation.	$8m = 5m + 21$
2 Move variables to the LHS by subtracting $5m$ from both sides.	$8m - 5m = 21$
3 Simplify.	$3m = 21$
4 Undo the '× 3'.	$m = \frac{21}{3}$
5 Evaluate.	$m = 7$
b 1 Write the equation.	$11 - 6u = 3u - 16$
2 Move variables to the RHS (so they are positive).	$11 = 3u - 16 + 6u$
3 Simplify.	$11 = 9u - 16$
4 Undo the '– 16'.	$11 + 16 = 9u$
5 Simplify and swap sides so that u is on the LHS.	$9u = 27$
6 Undo the '× 9'.	$u = \frac{27}{9}$
7 Evaluate.	$u = 3$

Videos
Equations with variables on both sides
Equations with brackets

Puzzles
Equations with unknowns on both sides
Equations riddle
Equations code puzzle
Equations order activity

WORKED EXAMPLE 5 Equations with brackets

Solve the equation $4(y + 4) = 2(3y - 4)$.

Steps	Working
1 Write the equation.	$4(y + 4) = 2(3y - 4)$
2 Expand the brackets.	$4y + 16 = 6y - 8$
3 Move variables to the RHS (so they are positive).	$16 = 6y - 8 - 4y$
4 Simplify.	$16 = 2y - 8$
5 Undo the '–8'.	$16 + 8 = 2y$
6 Simplify and swap sides.	$2y = 24$
7 Undo the '× 2'.	$y = \frac{24}{2}$
8 Evaluate.	$y = 12$

EXERCISE 6.2 Equations with variables on both sides

ANSWERS p. 403

Recap

1 Determine whether $f = 12$ is a solution of $3f - 11 = 25$.

2 Solve each equation.

a $3x + 54 = 15$ **b** $-22 + 4m = 6$

Mastery

3 a WORKED EXAMPLE 4 To solve the equation $7x + 3 = -9 - 4x$, the first step is to

A add $7x$ to both sides **B** add 3 to both sides

C add $4x$ to both sides **D** subtract $4x$ from both sides

b To solve the equation $9x - 5 = 7x + 6$, the first step is to

A subtract $7x$ from both sides **B** add $7x$ to both sides

C subtract 5 from both sides **D** add 6 to both sides

4 Solve each equation.

a $7g = 4g - 9$ **b** $7x = -x + 16$ **c** $12s = 7s - 25$

d $21 + 5u = -2u$ **e** $5w = 2w - 36$ **f** $13q - 28 = -q$

g $9x = -2x - 33$ **h** $17y = 12y + 20$ **i** $5r - 15 = 2r$

5 Solve each equation.

a $3b + 4 = 9 - 5b$ **b** $5x + 13 = 3 - 2x$ **c** $3w - 7 = 4 - 5w$

d $7a - 6 = 4a + 9$ **e** $6y - 11 = 5y + 8$ **f** $5n + 2 = 6n + 1$

g $6h + 5 = h - 8$ **h** $4u + 7 = 11u + 12$ **i** $3d - 15 = d - 8$

6 WORKED EXAMPLE 5 Solve each equation.

a $5(b - 2) = 3b + 7$ **b** $5(y + 1) = y - 13$ **c** $3(m - 8) = 5m$

d $4(r + 7) = r$ **e** $3(z - 5) = 9 - 2z$ **f** $6u - 7 = 3(2 - u)$

g $-3(x - 4) = 7x$ **h** $-4(3d - 1) = 5d$ **i** $2m - 3 = 5(m - 3) - 4m$

7 Solve each equation.

a $3(e + 2) = 4(e - 2)$ **b** $6(y - 1) = 4(y + 5)$ **c** $5(p - 3) = 2(p - 3)$

d $5(x + 6) = 4(2 - 3x)$ **e** $2(n - 12) = 3(2n - 3)$ **f** $4(k + 5) = 2(3k + 7)$

g $2(3w + 7) = 4(w + 2)$ **h** $5(q - 2) = 3(2q + 5) - 5$ **i** $2 - (3t + 1) = 6(t + 2)$

Exam practice

8 (4 marks) True or false?

a $w = 8$ is a solution of $3w - 6 = 2(w + 1)$. *[1 mark]*

b $x = -2$ is a solution of $-2(2x - 3) = 3(x + 5)$. *[1 mark]*

c $y = 3$ is a solution of $\frac{4y}{3} - 5 = 3y$. *[1 mark]*

d $z = -4$ is a solution of $\frac{5z}{4} - 3 = 4 + 3z$. *[1 mark]*

9 (3 marks) CF A student's incorrect attempt to solve an equation is shown below.

$$3(x - 4) = 7x + 4$$

$3x - 12 = 7x + 4$	Line 1
$4x - 12 = 4$	Line 2
$4x = 16$	Line 3
$x = 4$	Line 4

Identify the line where the error has been made, and explain the error.

10 (3 marks) CF A student's incorrect attempt to solve an equation is shown below.

$$2(x + 3) = 7 - 3(x - 2)$$

$2x + 6 = 7 - 3x - 6$	Line 1
$2x = -3x - 5$	Line 2
$5x = -5$	Line 3
$x = -1$	Line 4

Identify the line where the error has been made, and explain the error.

Worksheets
Using equations to solve problems

Word problems with equations

Puzzle
Writing and solving equations

6.3 Equation problems

This table shows words and phrases that indicate the operations of +, –, × and ÷.

Operation	Word/phrase
addition	add, added to, the sum of, more than, increased by, the total of, in all, exceeds by, plus
subtraction	subtract, subtract from, difference, less, less than, fewer than, decreased by, diminished by, take away, reduced by, exceeds, minus
multiplication	multiply, times, the product of, multiplied by, times as much, of
division	divide, divides, divided by, the ratio of, the quotient of, equal shares of, per

WORKED EXAMPLE 6 Worded equations

Write an equation for each statement, using x to **represent** the unknown.

a When 12 is added to a number, the result is 21.

b 5 times the sum of a number and 3 is reduced by 4 and the result is 17.

c When 2 times a number is divided by 15, the result is 3 more than the number.

Steps	Working
a 'Added to' means '+'.	$x + 12 = 21$
b 1 Begin with the operation most closely associated with the variable. 'The sum of a number and 3' means '$x + 3$'.	$x + 3$
2 '5 times' means '× 5'.	$5(x + 3)$
3 'Reduced by 4' means '– 4'.	$5(x + 3) - 4$
4 'Result is 17' means '= 17'.	$5(x + 3) - 4 = 17$
c 1 Begin with the operation most closely associated with the variable. '2 times' means '× 2'.	$2x$
2 'Is divided by 15' means '÷ 15'.	$\frac{2x}{15}$
3 '3 more than the number' means '$x + 3$'.	$\frac{2x}{15} = x + 3$

Solving equation problems

1. Read the problem, identify the unknown quantity and define the variable (such as x).
2. Highlight keywords and write their mathematical meaning.
3. Draw a diagram if you think it will help.
4. Use the information given to write an equation.
5. Solve the problem and state the solution in words.
6. Check the solution in the original problem.

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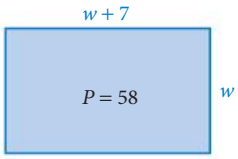
WORKED EXAMPLE 7 Solving simple word problems

The product of 6 and 5 more than a number is 18. **Identify** the number.

Steps	Working
1 Define the unknown quantity.	Let the number be n.
2 'Product' means '×'; '5 more than a number' means '$n + 5$'.	$6(n + 5) = 18$
3 Undo the '× 6'.	$n + 5 = \frac{18}{6}$
4 Simplify.	$n + 5 = 3$
5 Undo the '+ 5'.	$n = 3 - 5$
6 Simplify.	$n = -2$
7 Check the result.	LHS $= 6(-2 + 5) = 18$ RHS $= 18$ LHS = RHS ✓
8 Answer the word problem in words.	The number is −2.

WORKED EXAMPLE 8 Solving word problems

A rectangular shed is 7 m longer than it is wide. Its perimeter is 58 m. **Determine** its width.

Steps	Working
1 Draw a diagram. The perimeter is 58 m.	
2 Define the variable.	Let the width be w.
3 Write the length in terms of w.	The length must be $w + 7$.
4 Write the formula for the perimeter of a rectangle.	$P = 2(l + w)$
5 Substitute the values shown on the diagram.	$58 = 2[(w + 7) + w]$
6 Simplify.	$58 = 2(2w + 7)$
7 Expand using the distributive law.	$58 = 4w + 14$
8 Undo '+14'.	$58 - 14 = 4w$
9 Simplify and swap sides.	$44 = 4w$
10 Undo '×4'.	$w = \frac{44}{4}$ $= 11$
11 Write the solution in words.	The width of the rectangle is 11 m.
12 Check the result.	$P = 2[(11 + 7) + 11]$ $= 58$ ✓

EXERCISE 6.3 Equation problems

ANSWERS p. 403

Recap

1 Is $t = 12$ a solution of this equation?

$$\frac{2t}{6} = 1 + (t - 3)$$

2 Solve each equation.

a $-3x + 54 = 15x$

b $6(n - 2) + 3 = 3(n + 2)$

Mastery

3 Which equation best matches each statement?

a Half a particular number is 17.

A $x = \frac{17}{2}$ **B** $2x = 17$ **C** $\frac{x}{2} = \frac{17}{2}$ **D** $\frac{x}{2} = 17$

b A particular number is divided by 3 and the result is decreased by 5 to give 12.

A $\frac{x}{3} - 5 = 12$ **B** $3x + 5 = 12$ **C** $5 - \frac{x}{3} = 12$ **D** $\frac{x}{3} + 5 = 12$

c When the sum of a number and 3 is multiplied by 5, the result is 7 more than twice the number.

A $5x + 3 = 7 + 2x$ **B** $5(x + 3) = 7 + 2x$

C $x + 3 \times 5 = 7 + 2x$ **D** $5x + 3 = 7 - 2x$

d 3 times the difference between a number and 2 is the same as 4 times the difference between twice the number and 7.

A $3(2 + x) = 4(7 + 2x)$ **B** $3 \times x - 2 = 4(2x + 7)$

C $3x - 2 = 4 \times 2x - 7$ **D** $3(x - 2) = 4(2x - 7)$

4 WORKED EXAMPLE 6 Write a linear equation for each statement, using n to represent the unknown.

a The product of 10 and 12 more than a number is equal to 180.

b The quotient of 14 and 7 less than a number is 2.

c When the sum of a number and 15 is halved and is then decreased by 5, the result is 3.

d If 4 times a number is increased by 17 and then doubled, the answer is –6.

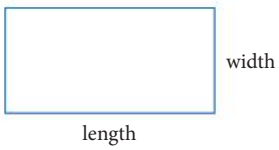
e 3 more than the product of 7 and 2 more than a number is 11.

f A number is increased by a third of itself and then reduced by 14, giving 9.

g When 10 times a number is reduced by 7 and then the result is doubled, the answer is –12.

h If two-thirds of a number is reduced by the product of 7 and the number, the result is 10.

5 Write an expression for the perimeter of each rectangle described in terms of its width (w).



a The length is twice the width.

b The length is 4 cm more than the width.

c The width is 3 m less than the length.

d The width is one-third of the length.

e The length is 4 mm longer than twice the width.

f The length is 5 cm shorter than 3 times the width.

g The width is 7 m shorter than one-quarter of the length.

h The length decreased by 12 mm is equal to the width.

6 WORKED EXAMPLE 7 For each situation

i write an equation

ii find the solution of the equation.

a If 7 is added to y, the result is 4.

b If 9 is added to m, the result is 2.

c If 31 is added to a, the result is 13.

d If 12 is added to x, the result is 8.

e If 3 is added to d, the result is -13.

f If y is subtracted from 26, the result is 39.

g If g is added to -12, the result is 4.

h If w is subtracted from 7, the result is -4.

7 WORKED EXAMPLE 8 Bosco scored 25 marks more in his History exam than he did in his Science exam. If he scored a total of 145 marks in both exams, how many marks did he score in the Science exam?

8 Maxine is 4 times as old as her son Juan, and the sum of their ages is 50. How old is Juan?

9 Two consecutive numbers add to give a total of 31. What are the numbers?

10 A triangle with a perimeter of 54 cm has 2 sides of length x cm and one side of length $x + 9$ cm. Find the length of each side.

Exam practice

11 (3 marks) Convert each statement into an equation.

a The quotient of a number and 5 is equal to 30. *[1 mark]*

b 4 times a number added to 10 is 26. *[1 mark]*

c 6 is subtracted from 3 times a number. The result multiplied by 3 gives 18. *[1 mark]*

12 (2 marks) How old am I if 400 reduced by 2 times my age is 244?

13 (2 marks) When 5 is subtracted from a number and the result is multiplied by 4, the answer is the same as when double the number is added to 10. Find the number.

14 (4 marks) CF Ewan is a tyre fitter. He is paid \$22.50 per hour, plus \$5 for each tyre that he fits. He earned \$360 for an 8-hour shift. Paige is another tyre fitter, She is paid \$14.50 per hour, plus \$5 for each tyre that she fits. How much would Paige earn for doing the same work as Ewan?

Video Graphing linear equations

Skillsheet Graphing linear equations

Worksheets Graphing linear functions 1

Graphing linear functions 2

Drawing gradients

Linear graphs

A page of lines

A page of number planes

Number plane grid paper

Straight-line equations

Spreadsheet Graphing $y = mx + c$

6.4 Straight-line graphs

Linear functions

A **linear function** or **linear equation** is the equation of a straight line on the number plane, also called the **Cartesian plane**.

The **slope–intercept form** (or **gradient–intercept form**) of a linear function is $y = mx + c$, where m is the **slope** (**gradient**) and c is the **y-intercept**.

A linear function or equation has a variable such as x, and has no powers or roots of x.

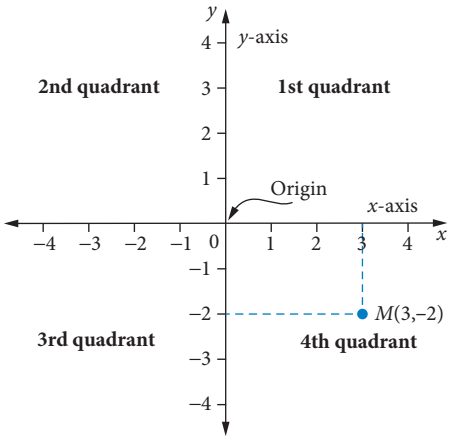
A Cartesian plane has a numbered horizontal **x-axis** and a numbered vertical **y-axis**.

The axes divide the plane into 4 **quadrants**.

Any point in the plane has a value on each **axis**. These 2 values are called **coordinates**. They are written within round brackets with the **x-coordinate** first and the **y-coordinate** second, (x, y).

Point M in the diagram has coordinates $(3, -2)$.

The **origin** is the point $(0, 0)$.



WORKED EXAMPLE 9 Plotting a straight line from a table of values

Draw the graph of the function $y = 3x - 5$ using a table of values for the values $0 \le x \le 4$.

Steps	Working
1 At least 3 points are preferred. Choose any 3 values of x from 0 to 4.	$y = 3x - 5$

x	0	2	4
y			

2 Calculate the y values using the rule $y = 3x - 5$.

For $x = 0$, $y = 3 \times 0 - 5 = -5$.

For $x = 2$, $y = 3 \times 2 - 5 = 1$.

For $x = 4$, $y = 3 \times 4 - 5 = 7$.

x	0	2	4
y	–5	1	7

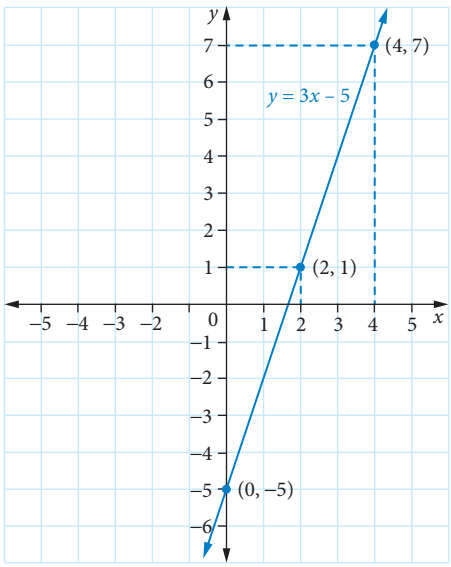
3 Write the ordered pairs for the function.

$(0, -5)$, $(2, 1)$ and $(4, 7)$

4 Draw a Cartesian plane, ensuring the axes extend far enough to fit all the points. Label and number the x- and y-axes.

Mark in the ordered pairs and join them with a line. The line should extend beyond the 2 end points, marked by arrowheads.

Write the equation next to the line.



9780170484749

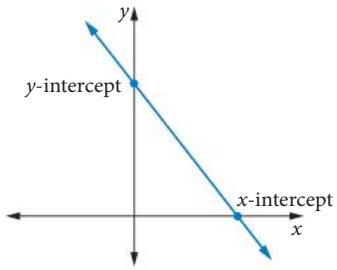
A straight line crosses the x-axis at the **x-intercept**.

It crosses the y-axis at the y-intercept.

The x-intercept is the x value of the function where $y = 0$.
The y-intercept is the y value where $x = 0$.

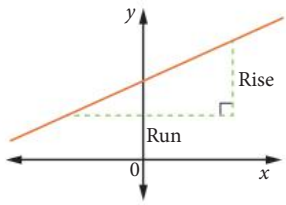
A linear function can be graphed using 3 different methods:

- table of values
- using the slope and y-intercept
- using the x- and y-intercepts.



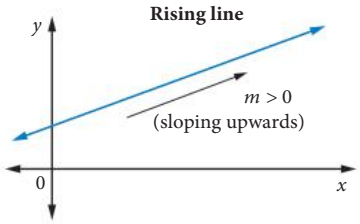
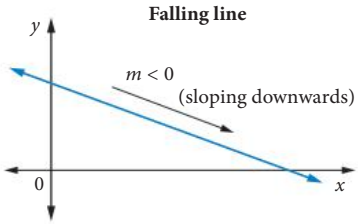
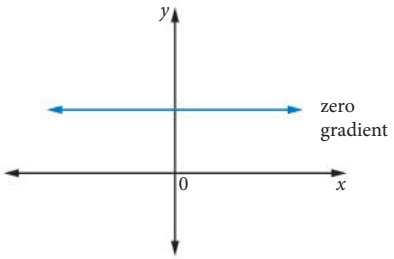
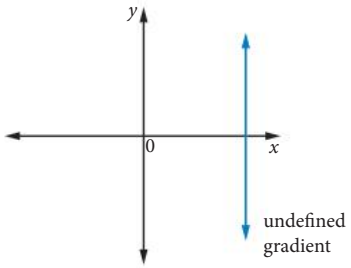
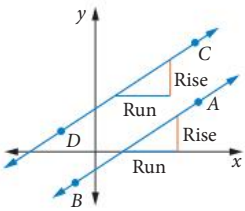
The slope of a line

The **slope** or **gradient** of a line measures how much the line goes up (the vertical rise) for each horizontal step of 1 unit. The gradient is represented by the symbol m and its value is a fraction given by $\dfrac{\text{vertical } \textbf{rise } (\uparrow)}{\text{horizontal } \textbf{run } (\rightarrow)}$.



The gradient of a line

$$\text{gradient } m = \frac{\text{rise}}{\text{run}}$$

Positive slope	Negative slope
Lines that slope upwards (rise) from left to right have a **positive** gradient (m). 	Lines that slope downwards (fall) from left to right have a **negative** gradient. 
Horizontal lines	**Vertical lines**
These lines have a rise of zero. This means a horizontal line has a **zero gradient**. So, $m = \dfrac{0}{\text{run}} = 0$. 	For a vertical line, the vertical run is 0. So, $m = \dfrac{\text{rise}}{0}$. A vertical line has an **undefined gradient**. 
Parallel lines	
If $AB \parallel CD$, $m_{AB} = m_{CD}$. The gradients of **parallel lines** are equal. 	

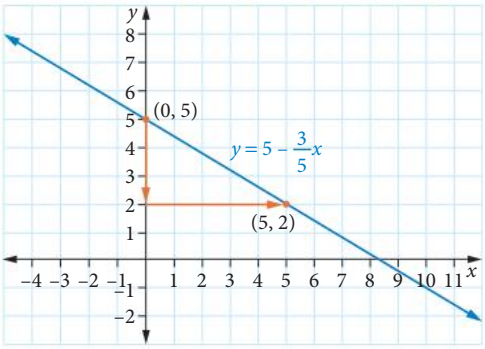
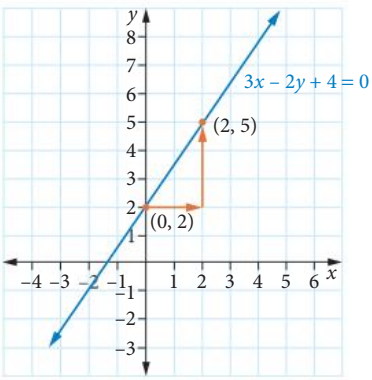
Videos
Gradient and y-intercept of a line

Graphing $y = mx + c$

WORKED EXAMPLE 10 Sketching a line from the slope (gradient) and y-intercept

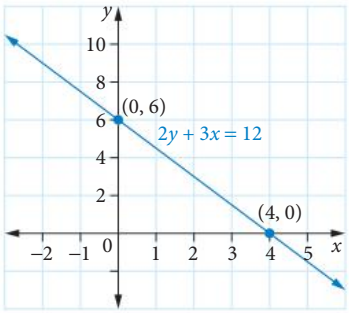
Sketch each line using the slope and y-intercept.

a $y = 5 - \frac{3}{5}x$ **b** $3x - 2y + 4 = 0$

Steps	Working
a 1 Write the equation.	$y = 5 - \frac{3}{5}x$
2 Write in the form $y = mx + c$.	$y = -\frac{3}{5}x + 5$
3 Identify the slope and y-intercept.	slope $(m) = -\frac{3}{5}$ y-intercept $(c) = 5$
4 Plot the y-intercept at $(0, 5)$. slope $= \frac{\text{rise}}{\text{run}} = \frac{-3}{5}$ A negative gradient means the line slopes down from left to right. Start at $(0, 5)$ and move 3 units *down* and 5 units *across* to the right. Plot the point at $(5, 2)$. Draw a straight line through the 2 points. Add arrowheads and label the graph.	
b 1 Write the equation.	$3x - 2y + 4 = 0$
2 Rearrange to get $y = mx + c$.	$3x + 4 = 2y$ $\frac{2y}{2} = \frac{3x}{2} + \frac{4}{2}$ $y = \frac{3}{2}x + 2$
3 Write the slope and y-intercept.	slope $(m) = \frac{3}{2}$ y-intercept $(c) = 2$
4 Plot the y-intercept $(0, 2)$. gradient $= \frac{\text{rise}}{\text{run}} = \frac{3}{2}$ A positive gradient means the line slopes up from left to right. Start at $(0, 2)$ and move 2 units *across* to the right and 3 units *up*. Plot the point at $(2, 5)$. Draw a straight line through the 2 points. Label the graph.	

WORKED EXAMPLE 11 Using intercepts to draw a straight line

Graph the function $2y + 3x = 12$ by finding its intercepts.

Steps	Working
1 Substitute $y = 0$ to find the x-intercept.	$2y + 3x = 12$ $2 \times 0 + 3x = 12$ $3x = 12$ $x = \frac{12}{3}$ $x = 4$
2 State the x-intercept.	The x-intercept is $(4, 0)$.
3 Substitute $x = 0$ to find the y-intercept.	$2y + 3 \times 0 = 12$ $2y = 12$ $y = \frac{12}{2}$ $y = 6$
4 State the y-intercept.	The y-intercept is $(0, 6)$.
5 Mark the intercepts and draw the line. Label the graph.	

Exam hack

Make sure you label the line with its equation.

Video
Intercepts of a line

Any point on the graph of the function satisfies the equation.

WORKED EXAMPLE 12 Checking points on a straight line

Test whether the point $A(-1, 2)$ lies on the graph of

a $y = 4x - 5$ **b** $3x - 2y + 7 = 0$

Steps	Working
a 1 Write the function.	$y = 4x - 5$
2 Substitute $(-1, 2)$ into the LHS.	LHS $= y = 2$
3 Substitute $(-1, 2)$ into the RHS.	RHS $= 4x - 5$ $= 4 \times (-1) - 5$ $= -9$
4 State the result.	LHS $\neq$ RHS, so A does not lie on the line.
b 1 Write the function.	$3x - 2y + 7 = 0$
2 Substitute $(-1, 2)$ into the LHS.	LHS $= 3 \times (-1) - 2 \times 2 + 7$ $= 0$ RHS $= 0$
3 State the result.	LHS = RHS, so A does lie on the line.

Video
Testing if a point lies on a line

EXERCISE 6.4 Straight-line graphs

ANSWERS p. 403

Recap

1 Solve $\frac{k-10}{2} = -7$.

2 Double a number added to 12 is 24. Find the number.

Mastery

3 State the quadrant in which each point is located.

a $(-3, -2)$ **b** $(9, 1)$ **c** $(5, -10)$

d $(18, 2)$ **e** $(-30, 45)$ **f** $(-13, -20)$

4 Copy and complete the table of values for each function.

a $y = -3x + 2$

x	0	2	5
y			

b $y = 4x + 1$

x	−3	0	3
y			

c $y = 5x - 12$

x	−2	0	2
y			

d $y = 7 - 3x$

x	−1	0	3
y			

5 WORKED EXAMPLE 9 Copy and complete the table for each function, then plot its graph on a separate Cartesian plane.

a $y = 2x + 5$

x	−3	0	3
y			

b $y = 4x - 11$

x	−1	2	4
y			

c $y = 8 - 5x$

x	−1	0	4
y			

d $y = -3x + 8$

x	−2	0	2
y			

6 Plot the graph of each function using a table of values for the values of x given.

a $y = 2x - 3$ $(-2 \le x \le 2)$ **b** $y = 4 - 3x$ $(-3 \le x \le 3)$

c $y = -4x + 8$ $(-2 \le x \le 4)$ **d** $y = 10 - 5x$ $(-3 \le x \le 2)$

e $y = -3x + 11$ $(-1 \le x \le 5)$ **f** $y = 9 - 4x$ $(-1 \le x \le 3)$

g $y = 6x - 5$ $(-2 \le x \le 3)$ **h** $y = -7x + 4$ $(-1 \le x \le 3)$

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7 WORKED EXAMPLE 10 Sketch each line using the slope and y-intercept.

a $y = 2x$ **b** $y = 5 - 3x$ **c** $y = -x + 1$

d $y = 4 - 7x$ **e** $y = x + 2$ **f** $y = -8x$

g $2x - y + 1 = 0$ **h** $7x + y - 14 = 0$ **i** $x - 3y + 6 = 0$

8 WORKED EXAMPLE 11 Sketch each line using its x- and y-intercepts.

a $2x + 5y = 5$ **b** $3x + 2y = 8$ **c** $4x - 3y = 9$

d $5x - 2y = 8$ **e** $3x + 4y = 8$ **f** $3y + 4x + 12 = 0$

g $5y - 8x - 10 = 0$ **h** $7x + 4y - 10 = 0$ **i** $12 + 4y - 5x = 0$

9 WORKED EXAMPLE 12 For each equation, state which point(s) lies on its graph.

a $3x + 2y + 8 = 0$	$(-4, 2)$	$(3, 4)$	$(-2, 1)$	$(2, -7)$	$(4, 0)$
b $y = x - 3$	$(-1, 2)$	$(1, -2)$	$(3, 0)$	$(-2, 5)$	$(-3, -6)$
c $y = 2x + 6$	$(0, -6)$	$(2, 8)$	$(1, 14)$	$(-2, 2)$	$(3, 12)$
d $y = -3x + 2$	$(1, 1)$	$(-1, 3)$	$(3, -7)$	$(0, -2)$	$(-4, 14)$
e $3x - y - 6 = 0$	$(-4, 6)$	$(0, -3)$	$(2, 0)$	$(10, -12)$	$(-4, -18)$

10 Which of these points lie on the line $y = 5 - 3x$?

A $(2, 1)$ **B** $(-1, 2)$ **C** $(5, 10)$ **D** $(-2, 11)$

11 Which of these points lies on the line $y - 4x = 7$?

A $(-1, 3)$ **B** $(2, 1)$ **C** $(3, -22)$ **D** $(-1, -3)$

12 Which of these points lies on the line $2y - 3x + 5 = 0$?

A $(5, 10)$ **B** $(-3, -7)$ **C** $(1, 1)$ **D** $(-5, 10)$

13 **a** On the same set of axes, plot the graphs of $y = x - 2$ and $y = -3x + 6$ for $0 \le x \le 3$.

b What are the coordinates of the point where the 2 graphs cross?

c Check whether this point satisfies $y = x - 2$.

d Check whether this point satisfies $y = -3x + 6$.

14 **a** Plot the graphs of $2x + y = 8$ and $3x - y = 7$ on the same set of axes for $0 \le x \le 5$.

b Use your graphs to find the coordinates of the point that satisfies both functions.

15 Lisa and Giang are positioned on the y-axis looking through separate telescopes at the same object. Lisa looks along the line $y + 2x = 6$, and Giang looks along the line $y + 4x = -3$. Use a Cartesian plane to find the distance between the telescopes.

Exam practice

16 (4 marks) State whether each equation is linear or non-linear.

a $3x = -6 + y$ *[1 mark]*

b $y = x^2$ *[1 mark]*

c $17y - 3x^2 = 45x$ *[1 mark]*

d $2x + 2y + 8 = 0$ *[1 mark]*

17 (1 mark) Change $4y - 12x = -16$ to slope–intercept form.

18 (2 marks) Determine if the point $(1, 6)$ is on the line $y = 2x + 4$.

19 (2 marks) The gradient of a line is 3 and the y-intercept is -5.

a Write the equation of the line. *[1 mark]*

b Sketch the line on the Cartesian plane. *[1 mark]*

20 (3 marks) CF The growth of grass that is not mown is monitored over a number of weeks and recorded in the below table of values.

Weeks (x)	0	1	2	3	4	5
Height (y)	1	3	5	7	9	11

Draw a graph of the growth. Find the likely height (in cm) of the grass after 7 weeks.

Videos
The gradient of a line

Gradient (slope) of a line

Finding the equation of a line

Worksheets
Finding the gradient between two points on a line

Equations of a line

Gradient and y-intercept

$y = mx + c$

Finding the equation of a line

A page of lines

Puzzles
Linear equations code puzzle

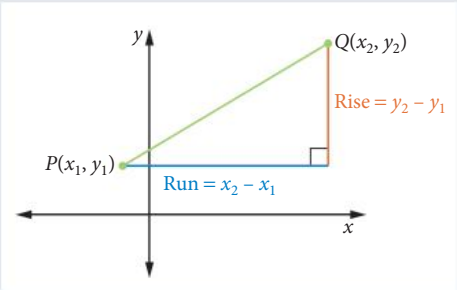
Equation of a line

6.5 Finding the equation of a line

The gradient formula

gradient $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$,

where (x_1, y_1) and (x_2, y_2) are points on the line.



WORKED EXAMPLE 13 The gradient formula

Calculate the gradient of the straight line that passes through the points $(-2, 5)$ and $(1, -7)$.

Steps	Working
1 Identify the 1st and 2nd points.	Let $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (1, -7)$.
2 Write the formula for the gradient.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
3 Substitute known values.	$= \dfrac{-7 - 5}{1 - (-2)}$
4 Simplify.	$= \dfrac{-12}{3}$ $= -4$
5 State the result.	The gradient of the line is -4.

Even if you swap the 1st and 2nd points, you will still get the right answer.

WORKED EXAMPLE 14 Finding the equation of a line

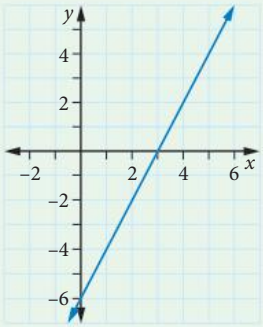
For each line:

i **state** whether the gradient is positive or negative

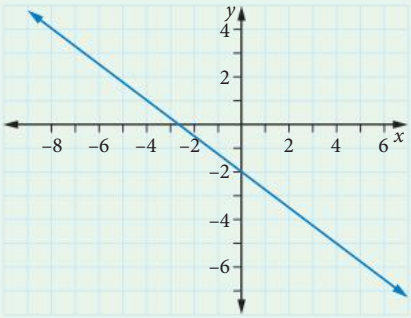
ii **calculate** the gradient

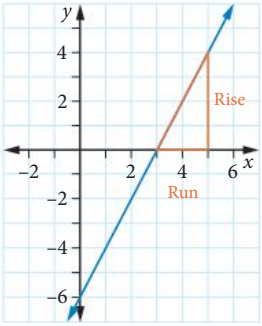
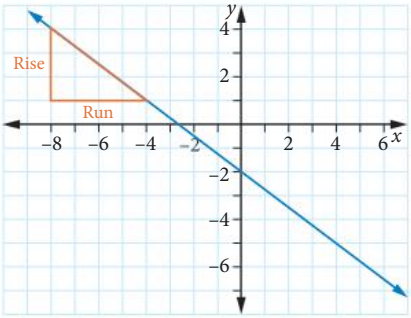
iii write the equation of the line.

a



b



Steps	Working
a i The line slopes upwards from left to right.	The gradient is positive.
ii **1** Choose 2 convenient points on the line and draw a right-angled triangle to show the rise and run.	
Select obvious points on the graph, do not estimate.	
2 Calculate the gradient using the formula.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{4}{2}$ $= 2$
	or by substituting (3, 0) and (5, 4) into the 2-point formula.
iii **1** Write the y-intercept.	$c = -6$
2 Use the equation $y = mx + c$.	The equation is $y = 2x - 6$.
b i The line slopes downwards from left to right.	The gradient is negative.
ii **1** Choose 2 points on the line and show the rise and run.	
2 Calculate the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= -\frac{3}{4}$
	or by substituting (−8, 4) and (−4, 1) into the 2-point formula.
iii **1** Write the y-intercept.	$c = -2$
2 Use the equation $y = mx + c$.	$y = -\frac{3}{4}x - 2$

If the slope and a point on a line are known, you can determine the equation.

Video
The equation of a line through 2 points

WORKED EXAMPLE 15 The equation of a line through two points

Determine the equation of the line passing through the points $(-1, 4)$ and $(5, 22)$.

Steps	Working
1 Identify the 1st and 2nd points.	Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (5, 22)$.
2 Calculate the gradient using the 2 points.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
3 Substitute values.	$= \dfrac{22 - 4}{5 - (-1)}$
4 Simplify.	$= \dfrac{18}{6}$ $= 3$
5 Substitute m and a point, say $(-1, 4)$, in $y = mx + c$.	$4 = 3 \times (-1) + c$
6 Solve for c to find the y-intercept.	$4 = -3 + c$ $c = 4 + 3$ $= 7$
7 Write the equation using m and c.	The equation of the line is $y = 3x + 7$.

EXERCISE 6.5 Finding the equation of a line

ANSWERS p. 407

Recap

1 Write the equation of each line in slope–intercept form.

a gradient = 2 and y-intercept = 4

b slope = –3 and y-intercept = 1

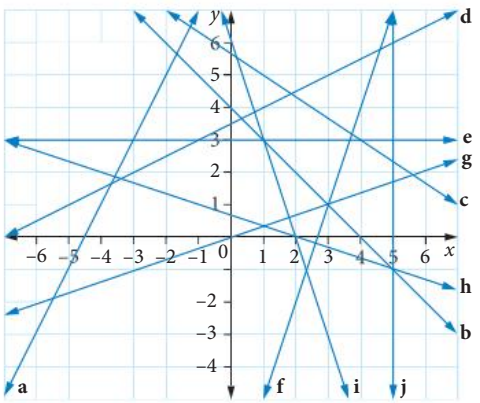
c y-intercept = –6 and slope = 5

d gradient = 3 and y-intercept = –2

2 Calculate the x- and y-intercepts of the line with equation $5x + 2y = 40$.

Mastery

3 State whether the gradient of each line shown is positive, negative or neither.
There are 10 lines, from **a** to **j**.

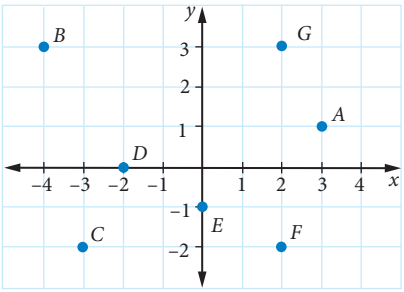


4 Use the diagram to calculate the gradient of the line that passes through points:

a A and B **b** B and D **c** D and C **d** G and F

e D and E **f** E and F **g** D and G **h** C and F

i A and D **j** C and A



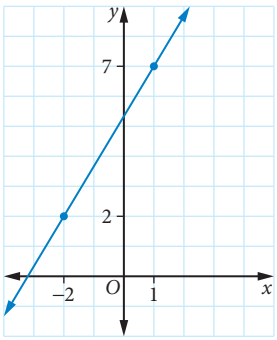
5 WORKED EXAMPLE 13 Calculate the gradient of the line that passes through each pair of points.

a (3, 5) and (5, 11) **b** (1, 6) and (4, 0) **c** (−1, −3) and (2, 3)

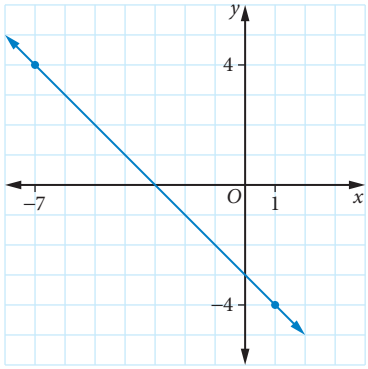
d (3, −6) and (4, 1) **e** (4, 2) and (−4, 1) **f** (0, 3) and (5, −7)

6 WORKED EXAMPLE 14 Find the equation of each line.

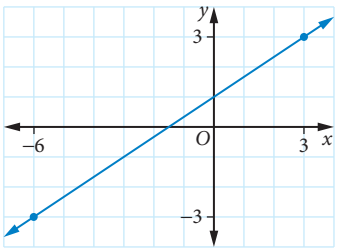
a



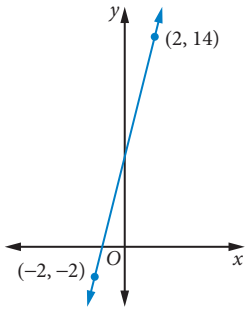
b



c

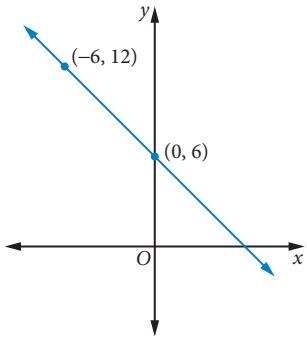


d



e

f



7 WORKED EXAMPLE 15 Find the equation of the line that passes through each pair of points.

a $(0, 12)$ and $(3, 24)$ **b** $(1, -7)$ and $(4, -13)$ **c** $(2, 6)$ and $(5, 15)$

d $(3, 3)$ and $(9, 7)$ **e** $(-4, 39)$ and $(1, 9)$ **f** $(2, -9)$ and $(8, -12)$

8 Find 2 points that have a straight line with a gradient of -2 passing through them.

9 Find the equation of the line that gives this table of values.

x	-1	0	1	2
y	-5	-1	3	7

10 A student showed the working on the right for the gradient of the line using the points $(-1, 4)$ and $(2, 2)$.

a Determine if the stated solution is correct.

b If the solution is not true, identify where an error was made.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-4}{2-1}$$

$$m = \frac{-2}{1}$$

Exam practice

11 Which equation is of a line that passes through the points $(1, -3)$ and $(3, 1)$?

A $y = 2x + 5$ **B** $y = 0.5x - 5$ **C** $y = -0.5x + 5$ **D** $y = 2x - 5$

12 Which equation is of a line that has zero gradient?

A $y = x$ **B** $y = 3$ **C** $x = 7$ **D** $y = 6x$

13 (1 mark) State the equation of a line with a gradient of 5 and a y-intercept of -3.

14 (1 mark) Find the equation of a line that passes through the origin and the point $(2, -6)$.

15 (2 marks) It costs \$10 for the initial hire fee and \$12 per hour of hire for an e-scooter. The table shows the cost for up to 4 hours.

Time t (hrs)	1	2	3	4
Cost C (\$)	22	34	46	58

Find the equation of the cost C for t hours.

16 (3 marks) CF A line passes through the point $(4, -2)$ and crosses the y-axis at $(0, 6)$. Where does it cross the x-axis?

6.6 Linear modelling

Many real-life situations have a linear relationship. When this happens, you can apply a **linear model** using a function of the form $y = mx + c$.
The variables could be different from x and y, such as C for cost, t for time and d for distance.

x is the **independent variable**. On a graph in the Cartesian plane, it goes on the horizontal axis.
y depends on x so it is the **dependent variable** and goes on the vertical axis.

WORKED EXAMPLE 16 Modelling growth

The height, h cm, of a seedling is given by $h = 2d + 10$, where d is the number of days since it was planted.

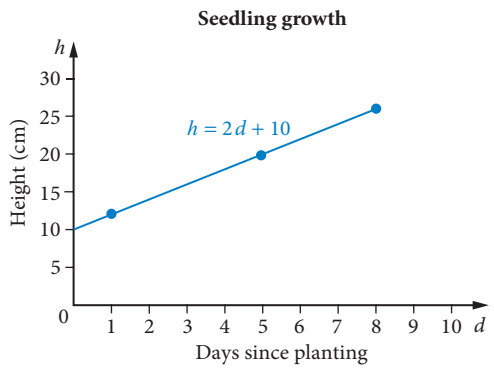
a Complete a table of values for the function for $0 \le d \le 8$.

b **Use** the table to draw a graph of the height function.

c What type of function is the height function?

d What do the values 2 and 10 in the height function represent?

In this example, d is the independent variable and h is the dependent variable.

Steps	Working
a **1** Complete a table of values for d and h using any 3 values of d from 0 to 8.	$h = 2d + 10$
2 State the ordered pairs.	$(1, 12)$, $(5, 20)$ and $(8, 26)$
b Graph the table of values. Label the axes and give the graph a title. Notice that the slope of the graph is positive. This means the function is rising. The height increases as the number of days increases.	
c The graph is a straight line.	The function $h = 2d + 10$ is linear.
d **1** Write the height function.	$h = 2d + 10$
2 Write the slope–intercept form of a linear function.	$y = mx + c$
3 Compare the functions.	$m = 2$ and $c = 10$
4 State the meanings of the coefficients.	2 is the gradient of the graph and represents the rate of growth in the height of the seedling of 2 cm per day. 10 is the h-intercept (or vertical intercept) and represents the starting height of the seedling (when first planted).

Videos
Linear modelling
Applying linear functions

Worksheet
Practical applications

WORKED EXAMPLE 17 Business models

A small business buys \$20 000 worth of computer equipment. For tax purposes, the value, \$$V$, of the equipment decreases (depreciates) over time according to the equation:

$$V = 20\,000 - 4000t$$

where t is the number of years since purchase.

a Complete a table of values for the value function for $0 \le t \le 6$.

b Use the table to draw a graph of the value function.

c When will the computer equipment be worthless?

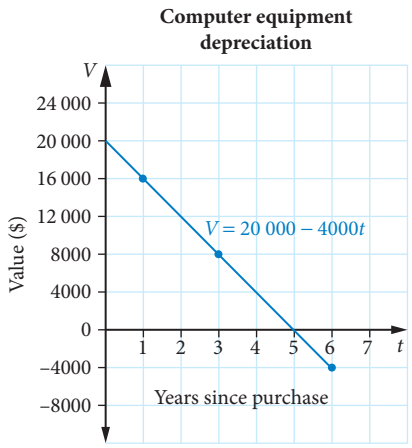
Steps	Working
a 1 Complete a table of values for t and V.	

t	1	3	6
V	16 000	8000	−4000

2 State the ordered pairs. (1, 16 000), (3, 8000) and (6, −4000).

b Graph the straight line.

Label the axes and add a title.



c 1 Find the value of t when $V = 0$ (the t-intercept). $t = 5$

2 State the result. The computer equipment will be worthless after 5 years.

EXERCISE 6.6 Linear modelling

ANSWERS p. 407

Recap

1 What is the gradient of the line passing through (5, 8) and (9, −2)?

2 Find the equation of the line connected by the points (4, 19) and (10, 64).

Mastery

3 The total cost (C) of producing a number of items (n) is given by the linear function $C = 15n + 25$.
What is the intercept on the vertical axis?

4 The cost (\$$C$) of running a function for a number of people (n) is given by the rule $C = 32n + 650$.
The gradient of the graph of the function is

A 650 **B** 32 **C** −32 **D** −650

5 WORKED EXAMPLE 16 The volume, V (in litres), of petrol remaining in a portable generator is given by

$$V = 8 - \frac{3}{2}h$$

where h is the number of hours after the generator starts with a full tank.

a Complete a table of values for the function for $0 \le h \le 6$.

b Use the table to draw a graph of the function.

c What type of function is it?

d Use the graph to find when there is 3 L of petrol left in the generator.

e How many litres of petrol are left in the generator after 2 hours?

f What does the value of the function at $h = 0$ represent?

g What does the value of the function where the graph crosses the h-axis represent?

6 The cost, C (in dollars), of hiring a clown for a birthday party is given by

$$C = 50 + 40h$$

where h is the number of hours for which the clown is hired.

a Draw a graph of the function for $0 \le h \le 8$.

b What type of function is it?

c Use the graph to find the cost of hiring the clown for 3 hours.

d The time may be charged for fractions of an hour. How long could the clown be hired for with $200?

e What does the value 50 in the cost function represent?

f What does the value 40 in the cost function represent?

7 WORKED EXAMPLE 17 A coffee wholesaler spends a maximum of $40 000 a week on coffee. He can buy coffee from Africa for $15/kg or from Papua New Guinea (PNG) for $9/kg.

a Write an equation for the number of kilograms of coffee from Africa (a) and PNG (p) that he can buy for $20 000.

b Complete a table of values for a and p and use it to draw the graph of the equation.

c If the supply of PNG coffee beans in one week is restricted to 800 kg, use the graph to work out how many kilograms of African coffee beans the wholesaler can buy.

8 A theatre is filled to capacity with 600 people. When the show finishes, people start leaving at a rate of 80 people per minute.

a Express P, the number of people in the theatre, as a function of t, the number of minutes after the show ends.

b Draw a graph of the function.

c Use the graph to find out how long after the show ends the theatre will still be half full.

Exam practice

9 (6 marks) A minibus can carry 20 passengers and a coach can carry 70 passengers. A total of 250 school students are to be taken on an excursion. Write the number of minibuses used as x and the number of coaches as y.

a How many students can be carried by x minibuses and y coaches? *[1 mark]*

b Write an equation for the number of minibuses and the number of coaches that could be used to carry 250 students. *[1 mark]*

c Draw the graph for this equation. *[2 marks]*

d On the same graph, graph the equations for transporting 150 students and 400 students. *[1 mark]*

e What do you notice about the graphs? *[1 mark]*

10 (3 marks) **CF** A weather balloon starts to sink at a rate of 80 m per minute (t) from a height (h) of 1 km. Use a rule to find how long it takes for the balloon to reach the ground.

11 (2 marks) **CF** Nadine is an electrician who charges \$50 per hour of labour, plus a callout fee of \$80 for each job. Her competitor, Josh, charges a \$50 callout fee, plus an additional \$55 per hour of labour. Who would be the most cost-efficient electrician for a 4-hour job?

Video Exam question analysis: Linear equations and graphs

EXAM QUESTION ANALYSIS

Exam-style question (7 marks)

Erika wants to save for a cruise holiday next year. She decides to use a savings account. Erika starts the account with \$300 and thinks she can put \$50 per week into the account.

Use an equation and graph to determine how long it will take Erika to save \$1200 for the cruise.

Explain how she could save the money in a shorter period of time.

Reading the question

- Highlight the important information in the question.
- She starts with \$300 and adds \$50 per week.
- It requires a linear graph to be drawn.
- It requires an explanation of how to save money faster.

Thinking about the question

- The equation and graph will be of the form $y = mx + c$.
- There is no restriction in the question as to which method to use to sketch the graph.
- The y-axis needs to go above \$1200. The scale on the graph needs to allow for this.
- The x-axis also needs to extend for enough weeks.

Worked solution (✓ = 1 mark)

Let y = total savings (S), x = number of weeks (t).

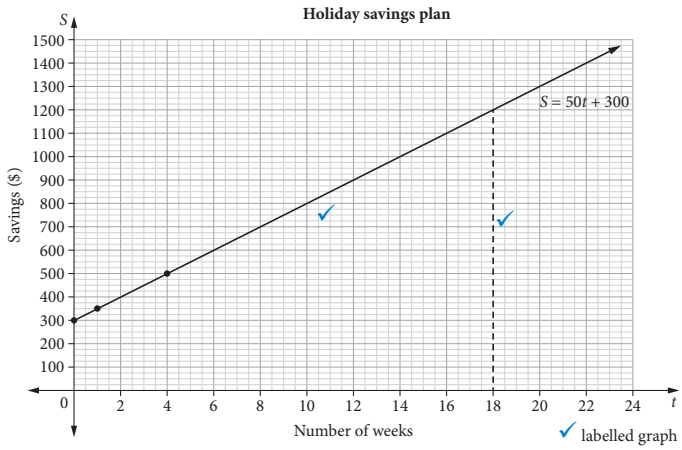
Substitute $c = 300$, $m = 50$ into the linear equation $y = mx + c$:

$\boldsymbol{S = 50t + 300}$ ✓

Use a table to graph.

t	0	1	4
S	300	350	500

✓



From the graph, it will take Erika **18 weeks** to save \$1200 for the cruise. ✓

Erika can shorten the savings time by **depositing more at the start or increasing the amount she saves each week.** ✓

6 Chapter summary

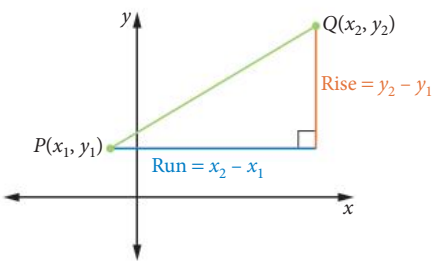
Equations

- An **equation** is a mathematical statement that 2 algebraic expressions are equal.
- **Solving** an equation means finding a value for the variable that makes the statement true.
- To solve an equation, you need to keep the equation **balanced** by doing the same thing to both sides of the equation.
- An **inverse operation** has the effect of undoing the original operation.

Operation	Inverse operation
+	–
–	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

Linear graphs

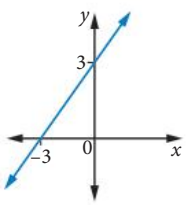
- A **linear function** is the equation of a line that can be written in the **slope–intercept form**. $\boldsymbol{y = mx + c}$, where m = slope and c = y-intercept
- The **slope** of a line is also called its **gradient** (m).
- gradient = $m = \dfrac{\text{rise}}{\text{run}}$



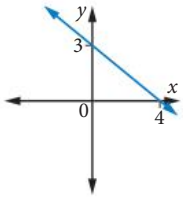
- If $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on a straight line:

  gradient of $PQ = m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$

- **Positive** lines slope upwards from left to right and have a positive gradient ($m > 0$).

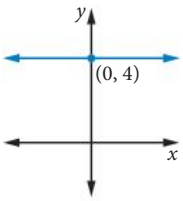


- **Negative** lines slope downwards from left to right and have a negative gradient ($m < 0$).

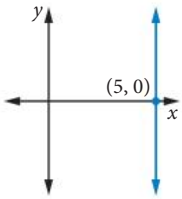


- The gradient of a horizontal line is 0.
- The gradient of a vertical line is **undefined**.

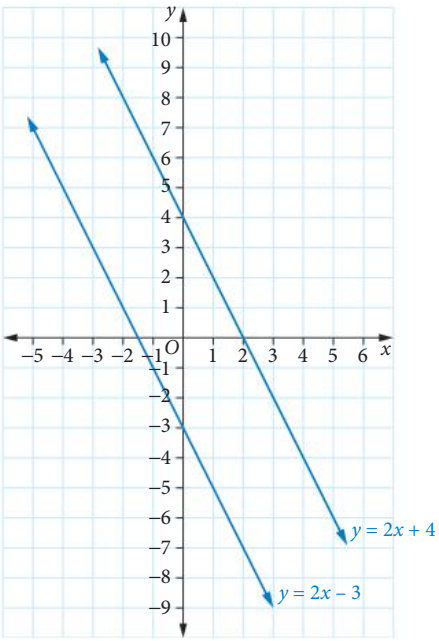
Horizontal (zero gradient)



Vertical (undefined gradient)



- The gradients of parallel lines are equal.



- The ***x*-intercept** is where $y = 0$ and the ***y*-intercept** is where $x = 0$.
- Relationships that can be represented by functions of the form $y = mx + c$ are called **linear models**.

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

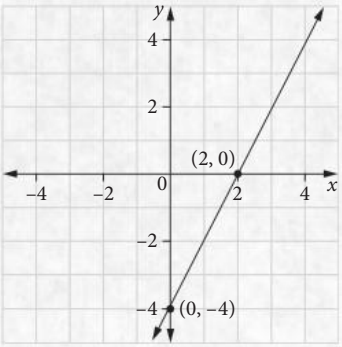
1 For this graph of $y = mx + c$

A $m = 2$ and $c = 4$

B $m = 1$ and $c = -4$

C $m = \frac{1}{2}$ and $c = -4$

D $m = 2$ and $c = -4$



2 285 500 cm^3 converted to m^3 is

A 0.2855 m^3 **B** 2.855 m^3 **C** 28.44 m^3 **D** 2855 m^3

3 A shareholder's share of a company's profits is called

A dividend **B** P/E ratio **C** brokerage **D** capital gains

4 A line passes through the origin and the point (3, −2). The gradient of this line is

A 3 **B** $-\frac{2}{3}$ **C** $-\frac{3}{2}$ **D** −2

5 The solution to $\frac{x-3}{4} + 6 = 7$ is

A $x = 5$ **B** $x = -7$ **C** $x = 8$ **D** $x = 7$

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Section 2 4 short response questions

13 marks

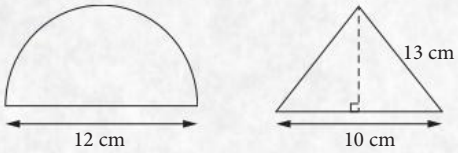
6 (2 marks) Find the x- and y-intercepts of the graph of $5x + 2y = 60$.

7 (3 marks) A company has shares with a market price of \$31.20 and paid a dividend yield of 10% last year.

a How much was the dividend per share? *[2 marks]*

b If an investor owned 350 shares, how much was their dividend payment? *[1 mark]*

8 (4 marks) Determine which of the 2 shapes below has the larger area.



9 (4 marks) The owner of a fast-food stall at a market has a total of \$200 to spend on hamburger patties and frankfurts for hot dogs each day. Each hamburger patty costs \$1.60 and each frankfurt costs 60c.

a Write an equation for the number of hamburger patties (h) and frankfurts (f) that the owner could buy. *[1 mark]*

b Complete a table of values for h and f and use it to draw a graph of the equation. *[2 marks]*

c The owner knows that 70 hamburgers are usually sold each day. How many frankfurts should be bought? *[1 mark]*

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

4 short response questions

12 marks

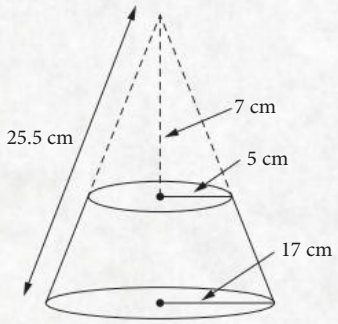
1 (3 marks) A leak has been discovered in a 600 L water tank. At 3 pm yesterday the tank was full. At 3 pm today the tank had 312 L left in it. Find a rule to calculate how much water will be in the tank at 9 am tomorrow morning.

2 (2 marks) $\frac{1}{x} = \frac{1}{m} + \frac{1}{n}$.

Find the value of x when $m = 2$ and $n = 8$.

3 (3 marks) Show that the points $(5, -4)$, $(-4, -1)$, $(-6, 6)$ and $(3, 3)$ are the vertices of a parallelogram.

4 (4 marks) Find the surface area of this truncated cone.



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CHAPTER

7 SIMULTANEOUS EQUATIONS AND PIECEWISE GRAPHS

Syllabus coverage
Nelson MindTap chapter resources
Terminology
7.1 Solving simultaneous equations graphically
7.2 The substitution method
7.3 The elimination method
7.4 Applications of simultaneous equations
7.5 Piecewise linear graphs
7.6 Step graphs
Exam question analysis
Chapter summary
Cumulative exam 1
Cumulative exam 2

Prior learning
Simultaneous equations and piecewise graphs

Syllabus coverage

UNIT 2, TOPIC 1: APPLICATIONS OF LINEAR EQUATIONS AND THEIR GRAPHS

Simultaneous linear equations and their applications

- Solve a pair of simultaneous linear equations, algebraically using substitution and elimination, and graphically.
- Solve practical problems involving simultaneous linear equations.

Piecewise linear graphs and step graphs

- Sketch piecewise linear graphs and step graphs.
- Interpret piecewise linear graphs and step graphs used to model practical situations.

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Videos (5):

7.2 Simultaneous equations 1 • Simultaneous equations 2 • The substitution method

7.3 Simultaneous equations 1 • Simultaneous equations 2 • The elimination method

Exam question analysis Simultaneous equations and piecewise graphs

Prior learning (1):

7.1 Simultaneous equations and piecewise graphs

Worksheets (8):

7.1 Sketching simultaneous equations • Intersection of lines

7.2 Substitution • Solving simultaneous equations • Simultaneous equations order activity

7.4 Simultaneous equations problems

7.6 Step graphs

Cumulative exams General Maths Year 11 formula sheet

Puzzle (1):

7.2 Simultaneous equations puzzle

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

continuous	discontinuous	elimination method
piecewise linear graph	point of intersection	simultaneous equations
simultaneous solution	step graph	substitution method

7.1 Solving simultaneous equations graphically

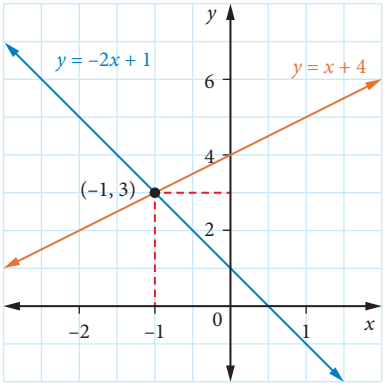
Look at the 2 linear functions that are graphed here. The graphs cross at the point $(-1, 3)$. This is called the **point of intersection**. Because the point of intersection lies on both the lines, it satisfies both functions.

Simultaneous equations are 2 or more equations with the same variables. Any solution must satisfy all the equations.

The solution of simultaneous equations in 2 variables is at the intersection of the lines.

It is called the **simultaneous solution** to the equations. It satisfies both equations *at the same time*.

Parallel lines do not intersect, so they do not have a simultaneous solution.



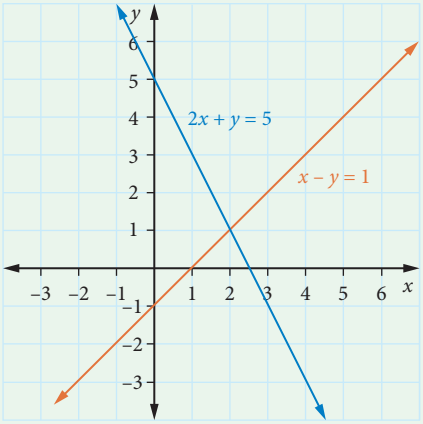
Worksheets
Sketching simultaneous equations

Intersection of lines

WORKED EXAMPLE 1 Simultaneous solutions from a graph

Use this graph to determine the simultaneous solution of the equations

$$x - y = 1$$
$$2x + y = 5$$



Steps	Working
1 Write the equations and number them.	$x - y = 1$ [1] $2x + y = 5$ [2]
2 Locate the point of intersection of the 2 lines. This gives the solution.	point of intersection = $(2, 1)$ So, $x = 2$ and $y = 1$.
3 Check the solution by substitution.	Check equation [1]: LHS $= x - y$ $= 2 - 1$ $= 1$ RHS $= 1$ LHS = RHS Check equation [2]: LHS $= 2x + y$ $= 2 \times 2 + 1$ $= 5$ RHS $= 5$ LHS = RHS
4 State the solution.	The simultaneous solution is $x = 2$ and $y = 1$.

WORKED EXAMPLE 2 Checking a simultaneous solution

Check whether the point (2, –1) satisfies this pair of simultaneous equations.

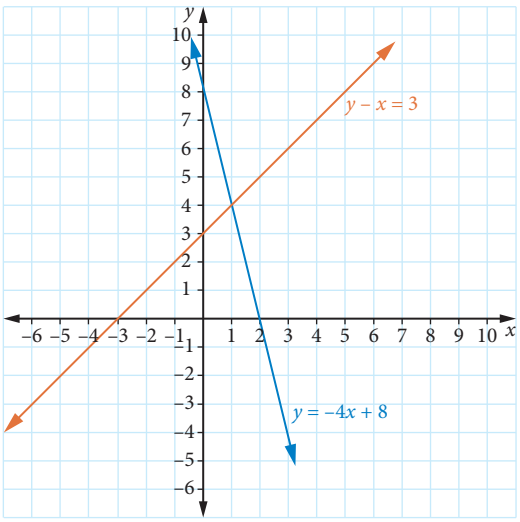
$$2x - 3y = 11$$
$$4y + x = -2$$

Steps	Working
1 Write the equations and number them.	$2x - 3y = 11$ [1] $4y + x = -2$ [2]
2 Check the solution by substitution.	Check equation [1]: $\text{LHS} = 2x - 3y$ $= 2 \times 2 - 3 \times (-1)$ $= 4 + 3$ $= 7$ $\text{RHS} = 11$
3 State the result.	LHS ≠ RHS so (2, –1) is not a solution of equation [1]. The first equation is *not satisfied*, so you can stop. (2, –1) is not the simultaneous solution.

WORKED EXAMPLE 3 Drawing graphs to solve simultaneous equations

Find the solution of these simultaneous equations by graphing.

$$y - x = 3$$
$$y = -4x + 8$$

Steps	Working
1 Write the equations and number them.	$y - x = 3$ [1] $y = -4x + 8$ [2]
2 To graph equation [1], find the y- and x-intercepts by substituting $x = 0$ and $y = 0$ respectively.	At $x = 0$, $y = 3$. At $y = 0$, $-x = 3$, so $x = -3$.
3 State the intercepts.	The x- and y-intercepts of equation [1] are –3 and 3 respectively.
4 Use the gradient and y-intercept method for equation [2].	$y = -4x + 8$ has gradient –4 and y-intercept 8.
5 Graph both equations on the same Cartesian plane.	
6 Use the graph to state the point of intersection.	The graphs intersect at (1, 4).
7 State the solution.	$x = 1$ and $y = 4$ is the solution.

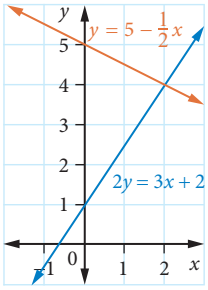
EXERCISE 7.1 Solving simultaneous equations graphically

ANSWERS p. 409

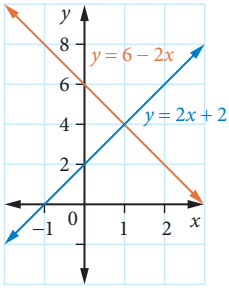
Mastery

1 WORKED EXAMPLE 1 Use the graph to find the simultaneous solution to each pair of equations.

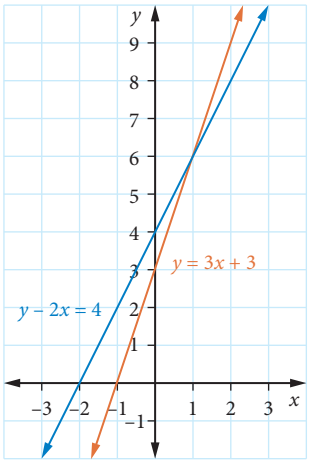
a $y = 5 - \frac{1}{2}x$ and $2y = 3x + 2$



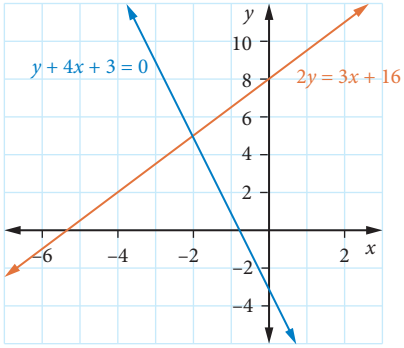
b $y = 6 - 2x$ and $y = 2x + 2$



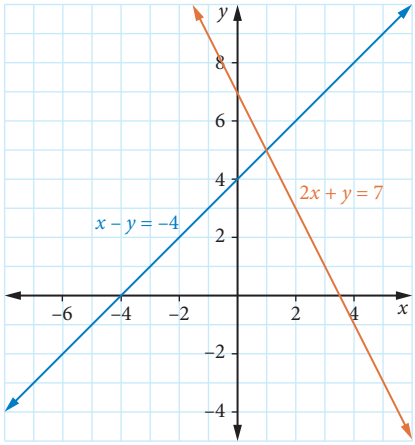
c $y = 3x + 3$ and $y - 2x = 4$



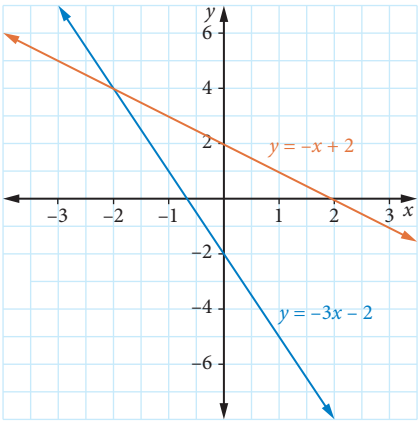
d $2y = 3x + 16$ and $y + 4x + 3 = 0$



e $x - y = -4$ and $2x + y = 7$



f $y = -x + 2$ and $y = -3x - 2$



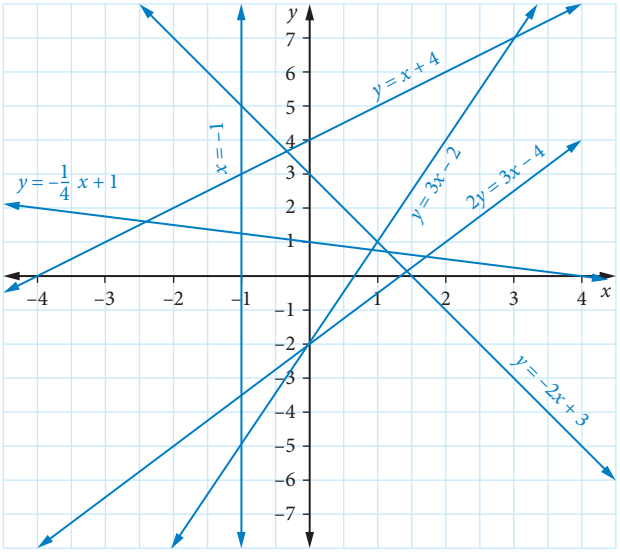
2 Use the graphs to find the simultaneous solution to these equations.

a $x = -1$ and $y = -2x + 3$

b $y = -2x + 3$ and $y = 3x - 2$

c $y = 3x - 2$ and $y = x + 4$

d $2y = 3x - 4$ and $y = 3x - 2$



3 Determine the simultaneous solution for $x - y = 5$ and $2x + y = -8$.

A $(-1, 6)$ **B** $(1, 6)$ **C** $(1, 5)$ **D** $(-1, -6)$

4 Determine the simultaneous solution for $3x + 4y = 27$ and $x + 2y = 11$.

A $(-5, 3)$ **B** $(5, 3)$ **C** $(-1, 6)$ **D** $(-5, -3)$

5 WORKED EXAMPLE 2 Check if the given coordinates satisfy the equations for each system of equations.

a $(-3, -1)$ $y = x + 2$ and $2y = 3x + 7$

b $(5, 1)$ $y = 5x - 24$ and $3y + 4x - 23 = 0$

c $(4, -2)$ $y = 6 - 2x$ and $x = 3y + 8$

d $(7, 5)$ $2y = 31 - 3x$ and $2x + 3y = 29$

e $(-2, -5)$ $3x - 2y - 4 = 0$ and $2x = 3y + 11$

f $(3, 7)$ $y = x + 4$ and $x = 16 - 2y$

g $(4, -3)$ $3x = y + 15$ and $4x + 7y + 5 = 0$

h $(6, -2)$ $x = 2 - 2y$ and $y = 16 - 3x$

i $(9, 1)$ $3y = 12 - x$ and $5x = 43 - 2y$

j $(2, 5)$ $x = 7 - y$ and $2x + 3y - 19 = 0$

6 WORKED EXAMPLE 3 Find the solution of each pair of simultaneous equations by graphing.

a $y = x$ and $y = 2x + 1$ **b** $y = x + 3$ and $y = 2x + 4$ **c** $y = -x + 2$ and $y = 2x - 1$

d $y = 2x - 7$ and $y = -x - 1$ **e** $y = -2x + 2$ and $y = 3x + 7$

7 Draw the graphs of each pair of simultaneous equations on the same set of axes. Then use the graphs to find the solution.

a $2x + y = 4$ and $x - y = 2$ **b** $3x - y = 7$ and $2x + y = 3$ **c** $10x - 5y = 50$ and $x + y = 2$

d $3x + 4y = -2$ and $x - 4y = -6$ **e** $4x + y = 23$ and $x + y = 8$

8 Solve each pair of simultaneous equations using a graphical method.

a $2x + 3y = 6$ and $2x - y = -10$

b $3x + y = 11$ and $4x - y = 3$

c $4x - 2y = -5$ and $x + 3y = 4$

d $2x + y = 7$ and $x - y + 4 = 0$

e $2y - 4x = 5$ and $4y + 2x = 5$

Exam practice

9 (3 marks) The heights of water (h cm) in 2 containers after a certain time (t seconds) are given by the following equations:

Container A: $h = 2t$

Container B: $h = 3t - 20$

Use a graphical method to find the time when the height of water in the 2 containers is the same.

10 (4 marks) Solve this system of 3 simultaneous equations graphically.

$2y = 3x + 16$

$y + 4x + 3 = 0$

$y + 2x = 1$

11 (4 marks) CF Line A is parallel to the line with equation $y + 2x = 5$ and passes through the point $(2, 3)$. Line B is parallel to the line with equation $x - y = 2$ and passes through the point $(-2, 2)$.

Draw the graphs of lines A and B and find their simultaneous solution.

12 (3 marks) CF At the beach, 2 hire companies offer surfboards for hire. Just Surf charges \$20 plus \$10 per hour to hire a surfboard and Surftime charges \$20 per hour. Using a graph, calculate how many hours a customer could hire a surfboard and be charged the same amount by either business.

7.2 The substitution method

Graphs do not always give an exact solution for simultaneous equations.

It is often more accurate and faster to use an **algebraic method**. There are 2 algebraic methods:

- the substitution method
- the elimination method.

The **substitution method** requires one of the equations to be substituted into the other one.

The substitution method for simultaneous equations

1. Number your equations.
2. Isolate a variable in one equation to express it as a formula.
3. Substitute into the other equation.
4. Solve the resulting equation.
5. Use the solution to find the value of the other variable.
6. Check that your solution satisfies both equations.

Videos
Simultaneous equations 1
Simultaneous equations 2

Worksheets
Substitution
Solving simultaneous equations
Simultaneous equations order activity

Puzzle
Simultaneous equations puzzle

WORKED EXAMPLE 4 The substitution method 1

Solve these simultaneous equations using the substitution method.

$$x + 2y = 23$$
$$y = x + 4$$

Steps	Working
1 Number the equations.	$x + 2y = 23$ [1]
[2] already has an isolated variable.	$y = x + 4$ [2]
2 Substitute for y from [2] into [1].	$x + 2(x + 4) = 23$
3 Expand and solve for x.	$x + 2x + 8 = 23$ $3x + 8 = 23$ $3x = 15$ $x = 5$
4 Substitute x into [2] to find y.	$y = x + 4$ [2] $y = 5 + 4$ $y = 9$
5 State the solution.	The simultaneous solution is $x = 5$ and $y = 9$.
6 Check in [1].	LHS $= x + 2y = 5 + 2 \times 9 = 23 =$ RHS ✓
7 Check in [2].	RHS $= x + 4 = 5 + 4 = 9 =$ LHS ✓

Exam hack

Check the simultaneous solution in *both* of the *original* equations.

Video The substitution method

WORKED EXAMPLE 5 The substitution method 2

Solve each system of simultaneous equations using the substitution method.

a $y = 2x - 6$
$y = 3x - 7$

b $2b - a = 3$
$a + 3b = 7$

Steps	Working
a 1 Number the equations.	$y = 2x - 6$ [1] $y = 3x - 7$ [2]
2 Both equations equal y. They can be equated.	$2x - 6 = 3x - 7$
3 Solve for x.	$-6 + 7 = 3x - 2x$ $x = 1$
4 Substitute $x = 1$ into [1] or [2].	$y = 2x - 6$ [1] $y = 2 \times 1 - 6$ $y = -4$
5 State the solution.	The simultaneous solution is $x = 1$ and $y = -4$.
6 Check in [1].	RHS $= 2x - 6 = 2 \times 1 - 6 = -4 =$ LHS ✓
7 Check in [2].	RHS $= 3x - 7 = 3 \times 1 - 7 = -4 =$ LHS ✓

b 1 Number the equations.	$2b - a = 3$ [1] $a + 3b = 7$ [2]	Both equations have a as a simple term.
2 Isolate a in both equations.	$2b - 3 = a$ [3] $a = 7 - 3b$ [4]	
3 Both equations equal a. They can be equated.	$2b - 3 = 7 - 3b$	
4 Solve for b.	$2b + 3b = 7 + 3$ $5b = 10$ $b = 2$	
5 Substitute $b = 2$ into [3] or [4] and solve.	$a = 7 - 3 \times 2$ [4] $a = 1$	
6 State the solution.	$a = 1$ and $b = 2$ is the simultaneous solution.	
7 Check in [1].	LHS $= 2b - a = 2 \times 2 - 1 = 3 =$ RHS ✓	
8 Check in [2].	LHS $= a + 3b = 1 + 3 \times 2 = 7 =$ RHS ✓	

WORKED EXAMPLE 6 The substitution method 3

Solve these simultaneous equations using the substitution method.

$$3x - 2y = -3.5$$

$$2x + 5y = 23$$

Steps	Working
1 Number the equations.	$3x - 2y = -3.5$ [1] $2x + 5y = 23$ [2]
2 $2x$ and $-2y$ are the simplest variable terms. Isolate x from [2].	$2x + 5y = 23$ [2] $2x = 23 - 5y$ $x = \dfrac{23 - 5y}{2}$ [3]
3 Substitute into equation [1]. You already used equation [2], so it *must* be [1].	$3\left(\dfrac{23 - 5y}{2}\right) - 2y = -3.5$
4 Multiply each term by the denominator, 2.	$2 \times 3\left(\dfrac{23 - 5y}{2}\right) - 2 \times 2y = 2 \times (-3.5)$
5 Cancel the 2s in the fraction term, simplify and solve.	$3(23 - 5y) - 4y = -7$ $69 - 15y - 4y = -7$ $-19y = -76$ $y = \dfrac{-76}{-19}$ $y = 4$
6 Substitute into [3] to find x.	$x = \dfrac{23 - 5 \times 4}{2}$ $= \dfrac{23 - 20}{2}$ $= \dfrac{3}{2}$ $= 1.5$
7 State the solution.	The solution is $x = 1.5$ and $y = 4$.

EXERCISE 7.2 The substitution method

ANSWERS p. 411

Recap

1 Transpose $2y - 6x = 8$ to make y the subject.

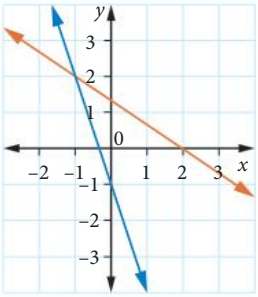
2 The simultaneous solution for the graph is

A $(2, -1)$

B $(-1, 2)$

C $(1, -2)$

D $(2, 1)$



Mastery

3 The solution of $y = 3x - 1$ and $y = x$ is

A $x = 1$ and $y = 1$ **B** $x = -1$ and $y = -1$ **C** $x = -\frac{1}{2}$ and $y = -\frac{1}{2}$ **D** $x = \frac{1}{2}$ and $y = \frac{1}{2}$

4 The simultaneous solution of $y - 4x = 10$ and $x + y = 0$ is

A $x = 1$ and $y = 1$ **B** $x = -2$ and $y = 2$ **C** $x = -\frac{1}{2}$ and $y = -\frac{1}{2}$ **D** $x = \frac{1}{2}$ and $y = \frac{1}{2}$

5 WORKED EXAMPLE 4 $p - 5q + 8 = 0$ [1] and $p + 3q = 16$ [2]

a Make p the subject of equation [1].

b Use substitution to solve these simultaneous equations.

6 Use the method of substitution to find simultaneous solutions for each pair of equations.

a $x = y + 3$
$x + 2y = 9$

b $y = 3x$
$2x - y = -6$

c $b = 2a - 3$
$a + 3b = 5$

d $c = 3d - 1$
$5c - 7d = 19$

e $c = 7 - 2d$
$3c - 2d = 9$

f $2m + 3n = -1$
$m + 4n = 2$

7 WORKED EXAMPLE 5 Find the simultaneous solutions for each pair of equations.

a $x + y = 5$
$x - y = 1$

b $2x + y = 12$
$3x - y = 13$

c $x + 2y = 6$
$x = 3y - 9$

d $3a + b = 8$
$b = a - 12$

e $c = d + 1$
$2c + d = 14$

f $x = 1 - y$
$3 = y + 2x$

8 WORKED EXAMPLE 6 Find the simultaneous solutions for each system of equations.

a $5m - 2n = 13$
$m + n = 4$

b $4a - b = 10$
$a + 3b = 9$

c $4m - n = 6$
$3m + 2n = -1$

d $5x + y = 12$
$3x + 2y = 3$

e $3x + 2y = 6$
$2x - 3y = 17$

f $5a + 2b = -8$
$a - 3b = -5$

Exam practice

9 (2 marks) A power bank costs \$6 less than a pair of earphones. Two power banks and 3 pairs of earphones cost a total of \$118. Write 2 equations for this situation and find the simultaneous solution using the substitution method.

10 (4 marks) The length of a rectangular room is 2 m longer than its width. The perimeter of the room is 68 m.

a Write an equation for the relationship between the length (l) and the width (w) of the room and another equation for the perimeter of the room. *[1 mark]*

b Use the method of substitution to find the length and width of the room. *[3 marks]*

11 (2 marks) Use the method of substitution to find a simultaneous solution to these equations.

$$ax + y = b$$
$$y = ax$$

12 (3 marks) CF A tool manufacturer calculates that its production costs of making tools, \$$C$, is given by the rule $C = n + 1500$, and its revenue from selling the tools, \$$R$, is given by the rule $R = 5n$, where n represents the number of tools produced. Determine the number of tools made when the costs \$$C$ are equal to the revenue \$$R$.

7.3 The elimination method

Videos
Simultaneous equations 1
Simultaneous equations 2

The **elimination method** also solves the simultaneous equations algebraically. This method is better for equations that are not in the form $y = mx + c$. It is easier to have the equations in the form $ax + by = c$, such as $2x + 3y = 1$.

The elimination method for simultaneous equations

1. Ensure both equations are in the form $ax + by = c$.
2. Check to see if there is a matching pair to eliminate.
3. If no matching pair exists, multiply one or both equations to create a matching pair to cancel.
4. Eliminate the matching pair by adding or subtracting the equations.
5. Solve to find the remaining variable.
6. Substitute into one of the original equations to find the other variable.
7. Check your solution satisfies both equations.

WORKED EXAMPLE 7 Elimination method with matching pairs

Solve these simultaneous equations by elimination.

$$2x + y = 12$$

$$3x - y = 13$$

Steps	Working
1 Label the 2 equations for reference in the working.	$2x + y = 12$ [1] $3x - y = 13$ [2]
2 The y is matching and opposite signs. Eliminate y by adding the 2 equations vertically.	$2x + \cancel{y} = 12$ $+\ 3x - \cancel{y} = 13$ $5x = 25$
3 Solve for x.	$x = \frac{25}{5}$ $x = 5$
4 Substitute $x = 5$ into either equation. Use [1] as it is easier to find y.	$2x + y = 12$ $2 \times 5 + y = 12$ $10 + y = 12$ $y = 12 - 10$ $y = 2$
5 State the solution.	The simultaneous solution is $x = 5, y = 2$ or $(5, 2)$

Video
The elimination method

WORKED EXAMPLE 8 Elimination method with one multiplication

Solve these simultaneous equations by elimination, by multiplying one equation.

$$5x + y = 12$$

$$3x + 2y = 3$$

Steps	Working
1 Label the 2 equations for reference in the working.	$5x + y = 12$ [1] $3x + 2y = 3$ [2]
2 Multiply equation [1] by 2 to create a matching pair.	$10x + 2y = 24$ [1] × 2 = [3]
3 The sign for the matching pair is the same sign, so subtract.	$10x + \cancel{2y} = 24$ [3] − [2] $-\ 3x + \cancel{2y} = 3$ $7x = 21$
4 Solve for x.	$x = \frac{21}{7}$ $x = 3$
5 Substitute $x = 3$ into either equation. Use [1] as it is easier to find y.	$5x + y = 12$ $5 \times 3 + y = 12$ $15 + y = 12$ $y = 12 - 15$ $y = -3$
6 State the solution.	The simultaneous solution is $x = 3, y = -3$

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WORKED EXAMPLE 9 Elimination method with two multiplications

Solve these simultaneous equations by elimination, by multiplying both equations.

$$2x - 4y = 13$$
$$3x - 5y = 17$$

Steps	Working
1 Label the 2 equations for reference in the working.	$2x - 4y = 13$ [1] $3x - 5y = 17$ [2]
2 Multiply equation [1] by 3 and equation [2] by 2 to create a matching pair ($6x$).	$6x - 12y = 39$ [1] × 3 = [3] $6x - 10y = 34$ [2] × 2 = [4]
3 Eliminate the x's by subtracting the 2 equations vertically.	$\cancel{6x} - 12y = 39$ [3] − [4] $-\ \cancel{6x} - 10y = 34$ $-2y = 5$
4 Solve for y.	$y = \frac{5}{-2}$ $y = -2.5$
5 Substitute $y = -2.5$ into either equation. Use [1] as it is easier to find x.	$2x - 4 \times (-2.5) = 13$ $2x + 10 = 13$ $2x = 13 - 10$ $x = \frac{3}{2}$ $x = 1.5$
6 State the solution.	The simultaneous solution is $x = 1.5, y = -2.5$

EXERCISE 7.3 The substitution method

ANSWERS p. 411

Recap

1 Determine whether the point (2, 4) is the solution to the simultaneous equations $x + 2y = 10$ and $3x - y = 2$.

2 Find the simultaneous solution for these equations.

$y = 3$

$2x - 4 = y$

Mastery

3 Decide whether you will need to add or subtract each of the matching pairs to eliminate them.

a $3y, 3y$ **b** $-6y, 6y$ **c** $13x, -13x$

d $24y, 24y$ **e** $-5x, 5x$ **f** $-x, x$

4 Calculate the new equation, if each equation is multiplied by the given factor.

a $x + y = -2$ (3) **b** $-2x - y = 5$ (4)

c $5x - 2y = 11$ (2) **d** $10x + 2y = 8$ (3)

5 WORKED EXAMPLE 7 Solve each set of simultaneous equations by elimination.

a $x + y = 6$
$x - y = 2$

b $2x + y = 10$
$x + y = 4$

c $10x + 2y = 28$
$x + 2y = 10$

d $3x + 2y = 8$
$3x - y = 5$

e $-3x + 2y = -4$
$5x - 2y = 8$

f $-2x + 5y = -3$
$4x - 5y = 21$

6 WORKED EXAMPLE 8 Solve each set of simultaneous equations by elimination.

a $3x + 4y = 7$
$x + y = 5$

b $2x + y = 10$
$3x - 2y = 8$

c $3x - 4y = 24$
$x - 2y = 10$

d $2x + 5y = 10$
$x - 3y = 5$

e $x + 3y = 9$
$4x - y = 10$

f $2x + 3y = -1$
$x + 4y = 2$

7 WORKED EXAMPLE 9 Solve each set of simultaneous equations by elimination.

a $4x - 3y = 0$
$3x + 4y = 25$

b $-7x + 3y = 22$
$3x - 2y = -11$

c $2x - 3y = -13$
$11x + 4y = 31$

d $5x - 2y = 17$
$4x + 3y = 9$

e $3x - 5y = -21$
$5x + 3y = -19$

f $5x + 4y = 16$
$3x + 7y = 5$

8 Two numbers are added to get 422 and subtracted to get 88. Find the two numbers.

Exam practice

9 (3 marks) Use the elimination method to find the simultaneous solution for $3x - 4y = 24$ and $x - 2y = 10$.

10 (3 marks) The difference of two numbers is 11. One of the numbers is 3 more than twice the other. Find the numbers.

11 (3 marks) CF 1500 people attended a children's concert. If the cost of a child's ticket was \$20 and the cost of an adult's ticket was \$35, how many adults attended if the total ticket sales were \$36 750?

12 (5 marks) CU A business wants to send 2 packages via courier. The courier company only allows packages up to 5 kg. The business fills 2 packages to weight capacity. The first box contains 20 calculators and 17 notepads, the second box, 35 calculators and 11 notepads. Find the weight of each calculator and notepad to the nearest gram.

Worksheet
Simultaneous equations problems

7.4 Applications of simultaneous equations

Solving problems with simultaneous equations

1. Read the information carefully and highlight the important information.
2. Choose variable names for the unknown quantities.
3. Use the information in the problem to form equations in terms of the variables.
4. Solve the equations using the most efficient method.
5. Check the solution to the problem.
6. Write a worded statement of the solution in relation to the problem.

WORKED EXAMPLE 10 Break-even point

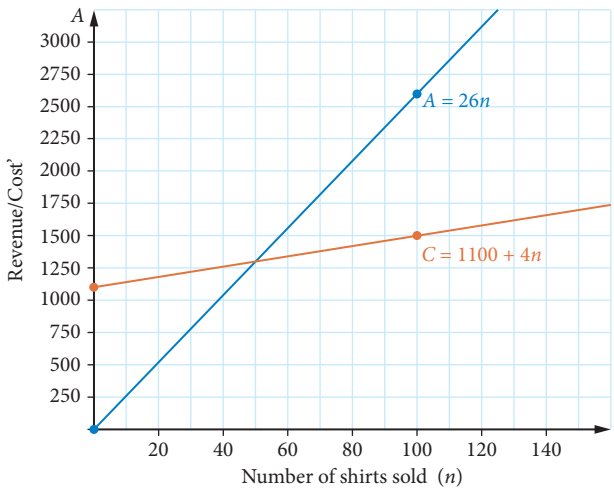
A manufacturer sells shirts for \$26. The amount received for selling n shirts is given by the equation $A = 26n$. The cost of making n shirts is given by the equation $C = 1100 + 4n$. The break-even point for the manufacturer is the value of n when the cost of production is the same as the revenue. The maximum production run is 300 shirts.

a Draw graphs of the equations on the same set of axes.

b **Use** the graph to find the break-even point.

c **Determine** the profit or loss, in dollars, for the following shirt orders:

i 30 shirts **ii** 110 shirts

Steps	Working
a 1 Write the equations and number them.	$A = 26n$ [1] $C = 1100 + 4n$ [2]
2 To find 2 points to graph [1], first choose some convenient n value for [1].	Let $n = 0$: $A = 26 \times 0 = 0$ $(0, 0)$ lies on $A = 26n$.
3 Select another n value for [1].	Let $n = 100$: $A = 26 \times 100 = 2600$ $(100, 2600)$ lies on $A = 26n$.
4 To find 2 points to graph [2], first choose some convenient n value for [2].	Let $n = 0$: $C = 1100 + 4 \times 0 = 1100$ $(0, 1100)$ lies on $C = 1100 + 4n$.
5 Select another n value for [2].	Let $n = 100$: $C = 1100 + 4 \times 100 = 1500$ $(100, 1500)$ lies on $C = 1100 + 4n$
6 Plot the points and join each pair with a straight line.	
b 1 The break-even point is where the graphs intersect.	point of intersection = $(50, 1300)$
2 State the result.	The break-even point is when 50 shirts are sold. The revenue and income are both \$1300 when 50 shirts are sold.

c i 1	Find the revenue when $n = 30$.	$A = 26 \times 30 = 780$
2	Find the production cost when $n = 30$.	$C = 1100 + 4 \times 30 = 1220$
3	Find the difference between revenue and production cost.	revenue – production cost $= 780 - 1220$ $= -440$
4	State the result. The negative sign means that revenue is less than costs, i.e. a loss is made.	When 30 shirts are sold, the manufacturer makes a loss of \$440.
ii 1	Find the revenue when $n = 110$.	$A = 26 \times 110 = 2860$
2	Find the production cost when $n = 110$.	$C = 1100 + 4 \times 110 = 1540$
3	Find the difference between revenue and production cost.	revenue – production cost $= 2860 - 1540$ $= 1320$
4	State the result. The positive value means that revenue is more than costs, i.e. a profit is made.	When 110 shirts are sold, the manufacturer makes a profit of \$1320.

WORKED EXAMPLE 11 Simultaneous equations problems 1

At a school play, adults were charged \$5 admission and children were charged \$2 admission.
A total of 400 people attended the play and the receipts were \$1100.

a **Formulate** 2 equations that represent this situation.

b **Solve** the simultaneous equations.

c How many adults attended the play?

Steps	Working
a 1 Assign the variables.	Let the number of adults $= a$ Let the number of children $= c$
2 There were 400 people altogether.	$a + c = 400$ [1]
3 There was \$1100 received.	$5a + 2c = 1100$ [2]
b 1 Isolate a in [1].	$a = 400 - c$
2 Substitute for a into [2].	$5(400 - c) + 2c = 1100$
3 Expand and solve.	$2000 - 5c + 2c = 1100$ $-3c = -900$ $c = 300$
4 Substitute 300 for c in [1].	$a + 300 = 400$ $a = 100$
5 Check in [1].	$100 + 300 = 400$ $400 = 400$ ✓
6 Check in [2].	$5 \times 100 + 2 \times 300 = 1100$ $500 + 600 = 1100$ $1100 = 1100$ ✓
7 State the solution.	The simultaneous solution is $a = 100$ and $c = 300$.
c State the answer.	There were 100 adults at the play.

WORKED EXAMPLE 12 Simultaneous equations problems 2

The length of a rectangle is 7 less than twice its width. The perimeter of the rectangle is 52 m. Find the length of the rectangle.

Steps	Working
1 Assign the variables.	Let length = l. Let width = w.
2 The length is 7 less than $2w$.	$l = 2w - 7$ [1]
3 The perimeter is 52 m.	$52 = 2(l + w)$ [2]
4 Substitute for l into [2]. Remember to use brackets.	$52 = 2[(2w - 7) + w]$ $52 = 2(3w - 7)$
Expand brackets and solve.	$52 = 6w - 14$ $66 = 6w$ $11 = w$ $w = 11$
5 Substitute 11 for w in [1].	$l = 2 \times 11 - 7 = 15$
6 Check in [1].	$15 = 2 \times 11 - 7$ $15 = 15$ ✓
7 Check in [2].	$52 = 2(15 + 11)$ $52 = 52$ ✓
8 State the solution.	The simultaneous solution is $w = 11$ and $l = 15$.
9 State the answer.	The length of the rectangle is 15 m.

EXERCISE 7.4 Applications of simultaneous equations

ANSWERS p. 411

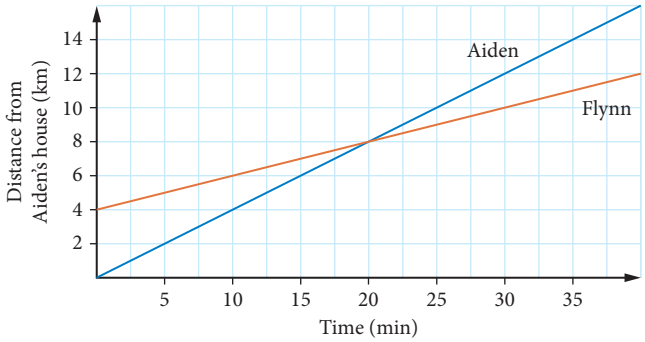
Recap

1 Find the value of y, given the 2 equations $x = 2y + 3$ and $x = 7$.

2 Find the x- and y-intercepts for $x + 2y = 4$.

Mastery

3 Aiden set off from his home and rode his bike at 24 km/h towards the sports ground. Flynn started running to the same sports ground at 12 km/h. Flynn began his journey from a place that was 4 km closer to the sports ground than Aiden's place. The travel graph shows their journeys.



a What are their relative positions 5 minutes after they start?

b What are their relative positions after 30 minutes?

c What is the significance of the point where the graphs intersect?

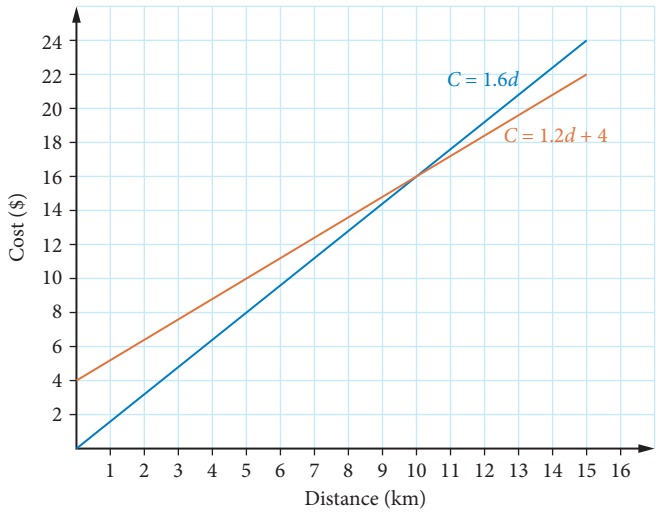
Use this information to answer Questions 4 and 5.

The Red Top taxi company charges a flagfall of \$4 plus \$1.20 per km, which can be modelled by the linear equation

$$C = 1.2d + 4$$

where C is the cost to travel d kilometres.

The Metro taxi company has a charging structure that gives the equation $C = 1.6d$.
The graph of these equations is shown.



4 For a journey of 15 km, which taxi company is cheaper and by how much?

A Red Top by \$4 **B** Metro by \$2 **C** Metro by \$4 **D** Red Top by \$2

5 What are the coordinates of the point when the cost of the journey is the same for both taxi companies?

A (15, 10) **B** (10, 16) **C** (0, 0) **D** (10, 15)

6 WORKED EXAMPLE 10 Stella sells handmade scented candles at a market for \$12.50. The amount she earns, A, for selling n candles is represented by the equation $A = 12.5n$. The cost of making n candles is represented by the equation $C = 84 + 5.5n$.

a Draw graphs of the equations on the same set of axes.

b Use the graph to find the break-even point.

c In the context of this problem, what does the break-even point mean?

d Determine the profit or loss, in dollars, if Stella sells 16 candles.

7 A number of adults (a) and children (c) attend a school fundraising concert. The number of people attending the concert is given by $a + c = 350$. The funds raised by the concert is given by the equation $25a + 15c = 6450$.

a Draw graphs of the equations on the same set of axes.

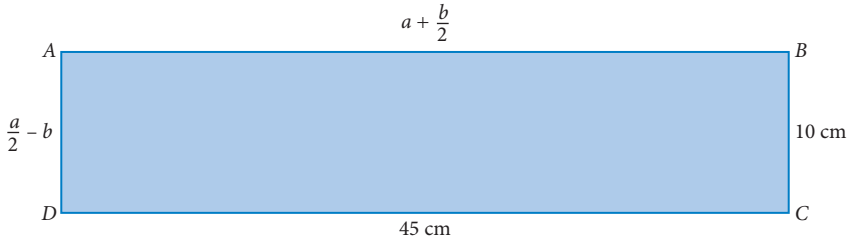
b Use the graph to find the total number of adults and children who attended the concert.

8 WORKED EXAMPLE 11 Mila bought 4 bagels and 3 croissants for \$21, and Ariel bought 2 bagels and 4 croissants for \$19.80.

a Letting b represent the cost of a bagel and c represent the cost of a croissant, write a pair of simultaneous equations to represent this information.

b Find the solution to the simultaneous equations developed in part **a**.

9 WORKED EXAMPLE 12 *ABCD* is a rectangle. Use this diagram to find the values of a and b.

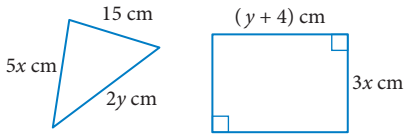


Exam practice

10 (3 marks) Two workers laid 250 bricks. If Sam had laid twice as many as he did and Peta had laid half as many as she did, there would have been 50 bricks left over. How many bricks did each lay?

11 (3 marks) I bought 3 trays of strawberries and 4 trays of cherries for \$36 at the local fruit shop. My friend bought 4 trays of strawberries and 3 trays of cherries for \$34. How much did we pay for each tray of strawberries and cherries?

12 (3 marks) The rectangle and triangle shown here have the same perimeter, 96 cm.



Determine the dimensions of the rectangle.

13 (3 marks) The total cost of tickets for 2 adults and 3 children to see a show was \$20. It cost \$31 for the tickets for a family group with 3 adults and 5 children. Find the cost of each type of ticket.

14 (3 marks) CF A fast-food stall owner at a football game knows that hot chips are 3 times as popular as meat pies and 4 times as popular as Chiko rolls. During half-time in the game, the stall served 209 people who each bought one item. How many serves of each food item were sold during this period?

15 (4 marks) CU Ross, Trish and Helen are comparing the number of hats that they own. Ross and Trish own the same number of hats, but Helen has fewer than either of them. Ross has 5 less than 6 times the number of Helen's hats, while Trish has one more than 3 times Helen's number of hats. How many hats does Helen have?

7.5 Piecewise linear graphs

Piecewise linear functions

A **piecewise linear function** is defined using 2 or more linear functions. The graph of a piecewise function is called a **piecewise linear graph** and consists of 2 or more non-overlapping line segments.

For example, a plumber charges \$100 for a house call, including the first hour of labour and then an additional hourly rate of \$60/h. This means the plumber charges you \$100 to come to your house and work for an hour and then an extra \$60 an hour for every hour after that.

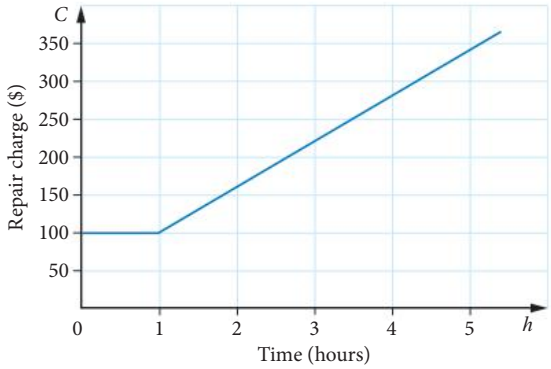
You can write the cost function, C, as

$$C = \begin{cases} 100 & \text{if} \quad 0 < h \le 1 \\ 100 + 60(h-1) & \text{if} \quad h > 1, \text{where } h = \text{hours worked.} \end{cases}$$

$0 < h \le 1$ means all values of h greater than zero up to *and including* 1.

The graph is shown.

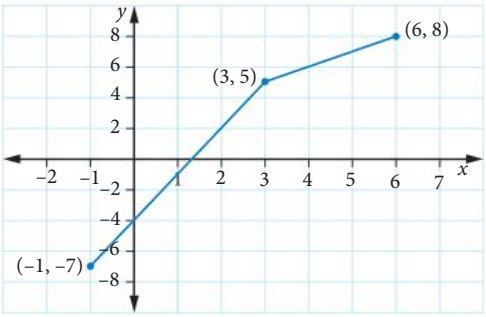
Only 2 points are needed to draw a linear function. This means you can draw each segment of a piecewise function using 2 points. Use the endpoints of each segment.



WORKED EXAMPLE 13 Continuous piecewise functions

Draw a graph of this piecewise function.

$$y = \begin{cases} 3x - 4 & \text{if} \quad -1 \le x < 3 \\ x + 2 & \text{if} \quad 3 \le x \le 6 \end{cases}$$

Steps	Working
1 Find the endpoints (−1) and (3) of the first segment.	When $x = -1$, $y = 3 \times (-1) - 4 = -7$ When $x = 3$, $y = 3 \times 3 - 4 = 5$
2 State the coordinates of the endpoints.	endpoints = $(-1, -7)$ and $(3, 5)$.
3 Find the endpoints of the second segment. (3) and (6)	When $x = 3$, $y = 3 + 2 = 5$ When $x = 6$, $y = 6 + 2 = 8$
4 State the coordinates of the endpoints.	endpoints = $(3, 5)$ and $(6, 8)$
5 Plot the endpoints and join them with a straight line. $(3, 5)$ is included as it is in the second segment.	

The previous example is called a **continuous** piecewise function because the graph has no holes or jumps.

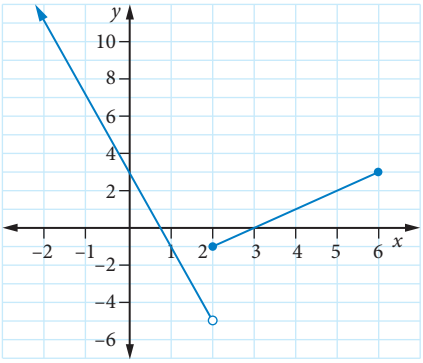
The next example is a **discontinuous** piecewise function.

7.5

WORKED EXAMPLE 14 Discontinuous piecewise functions

Draw a graph of the piecewise function.

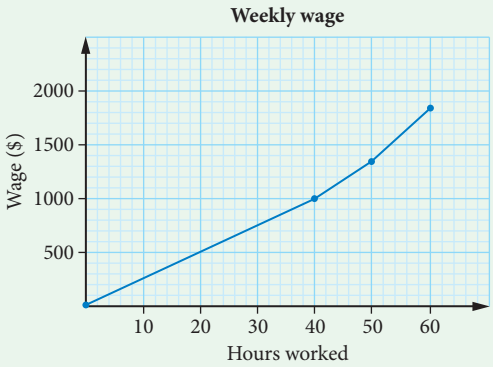
$$y = \begin{cases} 3 - 4x & \text{if} \quad x < 2 \\ x - 3 & \text{if} \quad 2 \le x \le 6 \end{cases}$$

Steps	Working
1 Choose a value of $x < 2$ for the first segment, such as $x = -1$.	$y = 3 - 4x \qquad x < 2$
Determine the upper boundary at $x = 2$.	When $x = -1$, $y = 3 - 4 \times (-1) = 7$
	When $x = 2$, $y = 3 - 4 \times 2 = -5$
2 State the coordinates of the two points on the first segment.	$(-1, 7)$ and $(2, -5)$, but $(2, -5)$ is not included
3 Find the endpoints of the second segment.	$y = x - 3 \qquad 2 \le x \le 6$ When $x = 2$, $y = 2 - 3 = -1$ When $x = 6$, $y = 6 - 3 = 3$
4 State the coordinates of the endpoints.	endpoints $= (2, -1)$ and $(6, 3)$
5 Plot the endpoints and join them with a straight line. Insert closed circles at $(2, -1)$ and $(6, 3)$ to indicate these points are included in the segment. Use an open circle at $(2, -5)$ to show it is not included.	

WORKED EXAMPLE 15 Interpreting piecewise graphs

Monique is a chef who normally works a 40-hour week. She is paid a penalty rate when she works more than 40 but less than 50 hours a week, and an even higher rate of pay when she works more than 50 but less than 60 hours a week.

The piecewise graph shows Monique's weekly wages for various hours worked.

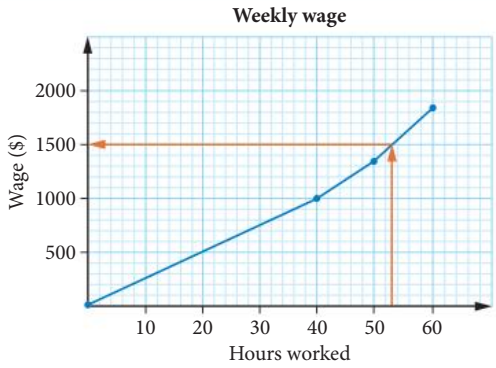
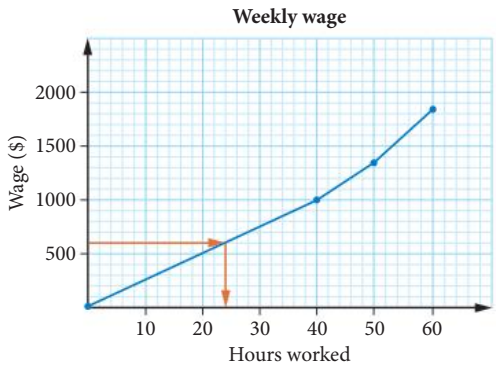


a **Describe** the graph and what it represents.

b **Use** the graph to find the wages earned when Monique works for 53 hours.

c **Use** the graph to find how many hours Monique worked if she earned $600 for the week.

d **Calculate** Monique's hourly rate of pay if she works less than 40 hours in a week.

Steps	Working
a Study the graph and comment on your observations.	There are 3 line segments in this piecewise graph. If h is the hours worked in a week, the first segment covers $0 < h \leq 40$, the second segment covers $40 < h \leq 50$ and the third segment covers $50 < h \leq 60$. The gradients of the segments increase as the hourly rate of pay increases. The gradient of the segment represents the hourly rate of pay for the number of hours covered by the segment.
b 1 Locate 53 hours on the horizontal axis. Draw a line vertically until it reaches the segment. Draw a line horizontally across to the vertical axis and read the value.	
2 State the result.	Monique earned $1500 for the week.
c 1 Locate $600 on the vertical axis. Draw a line horizontally until it reaches the segment. Draw a line vertically down to the horizontal axis and read the value.	
2 State the result.	Monique worked 24 hours in the week.
d 1 The hourly rate of pay is the same as the gradient for the segment. Calculate the gradient of the first segment by choosing any convenient point, e.g. (500, 20).	gradient $= \frac{\text{rise}}{\text{run}} = \frac{500}{20} = 25$
2 State the result.	The hourly rate of pay for working up to and including 40 hours a week is $25/hour.

EXERCISE 7.5 Piecewise linear graphs

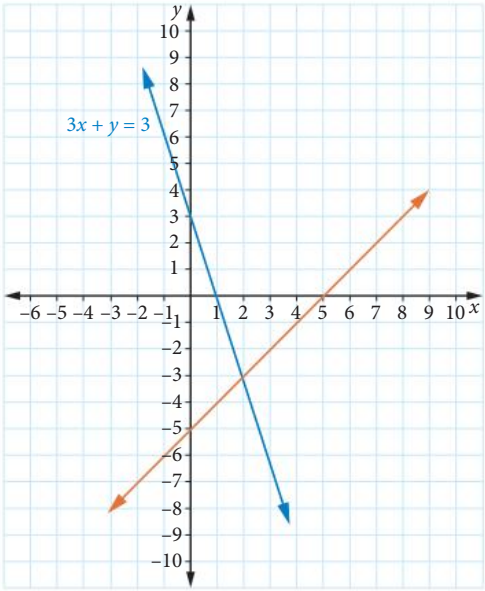
ANSWERS p. 411

Recap

1 The graph of 2 lines is shown.

a State the point of intersection.

b Determine the equation of the red line.



2 The most efficient algebraic method for solving a simultaneous equation where a matching pair already exists is the

A substitution method

B non-linear method

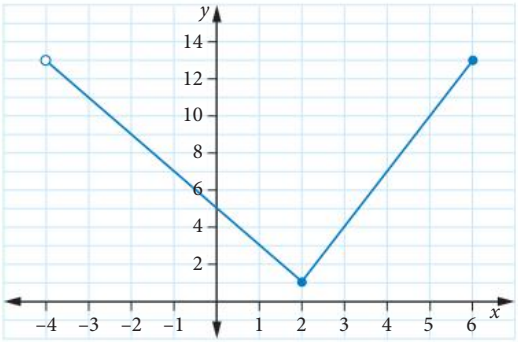
C elimination method

D function method

Mastery

Use the following piecewise function and its graph to answer Questions 3 and 4.

$$y = \begin{cases} 5 - 2x \text{ if } -4 < x \leq 2 \\ 3x - 2 \text{ if } 2 < x \leq 6 \end{cases}$$



3 For the segment $y = 5 - 2x$, which statement is true?

A The point $(-4, 13)$ is included, but the point $(2, 1)$ is not included.

B Neither the point $(-4, 13)$ nor the point $(2, 1)$ are included.

C Both the point $(-4, 13)$ and the point $(2, 1)$ are included.

D The point $(-4, 13)$ is not included, but the point $(2, 1)$ is included.

4 For the segment $y = 3x - 2$, which statement is true?

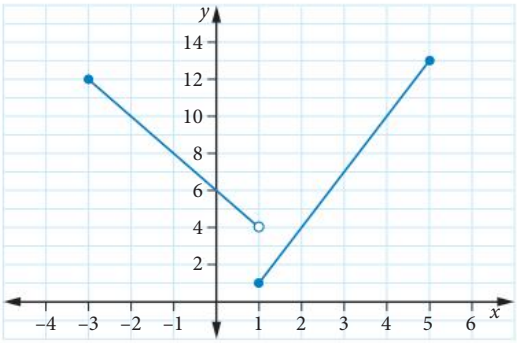
A Both the point $(2, 1)$ and the point $(6, 13)$ are included.

B The point $(2, 1)$ is not included but the point $(6, 13)$ is included.

C The point $(2, 1)$ is included but the point $(6, 13)$ is not included.

D Neither the point $(2, 1)$ nor the point $(6, 13)$ are included.

5 The graph of a piecewise function is shown. Which option correctly defines the function?



A $y = \begin{cases} 6 - 2x & \text{if} \quad -3 \le x < 1 \\ 3x - 2 & \text{if} \quad 1 \le x \le 5 \end{cases}$

B $y = \begin{cases} 6 - 2x & \text{if} \quad -3 < x < 1 \\ 3x - 2 & \text{if} \quad 1 \le x \le 5 \end{cases}$

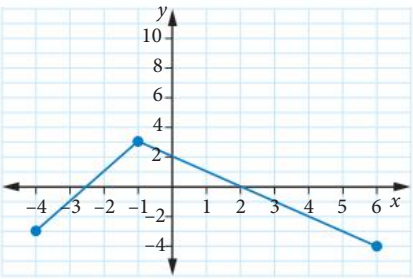
C $y = \begin{cases} 6 - 2x & \text{if} \quad -3 \le x \le 1 \\ 3x - 2 & \text{if} \quad 1 \le x \le 5 \end{cases}$

D $y = \begin{cases} 6 - 2x & \text{if} \quad -3 \le x < 1 \\ 3x - 2 & \text{if} \quad 1 \le x < 5 \end{cases}$

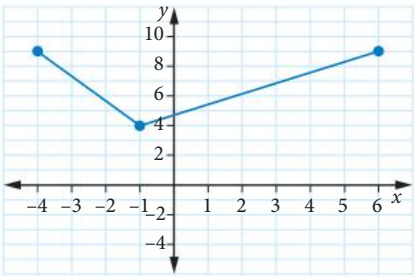
6 Which graph best represents the piecewise linear function?

$$y = \begin{cases} 2x + 5 & \text{if} \quad -4 \le x \le -1 \\ x + 4 & \text{if} \quad -1 < x \le 6 \end{cases}$$

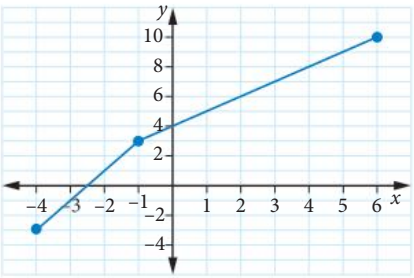
A



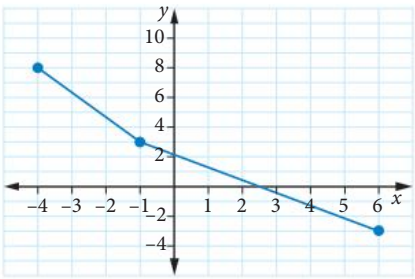
B



C



D



7 WORKED EXAMPLE 13 Draw a graph of each piecewise linear function.

a $y = \begin{cases} x & \text{if} \quad x < 3 \\ 3 & \text{if} \quad x \ge 3 \end{cases}$

b $y = \begin{cases} -x & \text{if} \quad x \le 1 \\ -1 & \text{if} \quad x > 1 \end{cases}$

c $y = \begin{cases} x + 5 & \text{if} \quad x < -2 \\ 3 & \text{if} \quad x \ge -2 \end{cases}$

d $y = \begin{cases} 1 - x & \text{if} \quad x \le 0 \\ x + 1 & \text{if} \quad x > 0 \end{cases}$

e $y = \begin{cases} x + 2 & \text{if} \quad x < 2 \\ 6 - x & \text{if} \quad x \ge 2 \end{cases}$

f $y = \begin{cases} 2x + 1 & \text{if} \quad x \le -2 \\ x - 1 & \text{if} \quad x > -2 \end{cases}$

8 WORKED EXAMPLE 14 Draw a graph of each piecewise linear function.

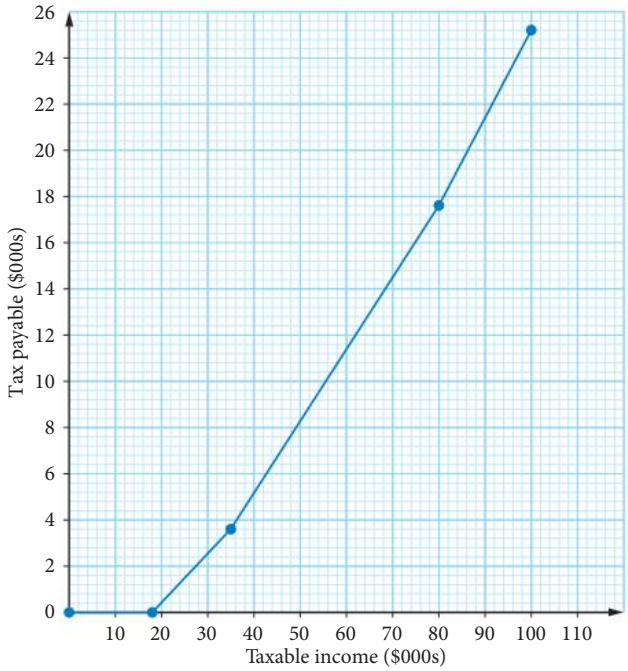
a $y = \begin{cases} x+2 & \text{if} \quad -2 \le x \le 2 \\ 2x+1 & \text{if} \quad 2 < x \le 5 \end{cases}$

b $y = \begin{cases} x+3 & \text{if} \quad -4 \le x < 0 \\ x & \text{if} \quad 0 \le x \le 4 \end{cases}$

c $y = \begin{cases} x-8 & \text{if} \quad -3 \le x \le 3 \\ \frac{1}{3}x-2 & \text{if} \quad 3 < x \le 9 \end{cases}$

d $y = \begin{cases} x+6 & \text{if} \quad -6 \le x < -3 \\ -\frac{2}{3}x-3 & \text{if} \quad -3 \le x \le 6 \end{cases}$

9 WORKED EXAMPLE 15 This piecewise linear graph shows the income tax payable on taxable incomes up to $100 000 in a previous year.



a Describe the major features of the graph.

b Which incomes attract the highest rate of taxation?

c Which incomes attract the lowest rate of taxation?

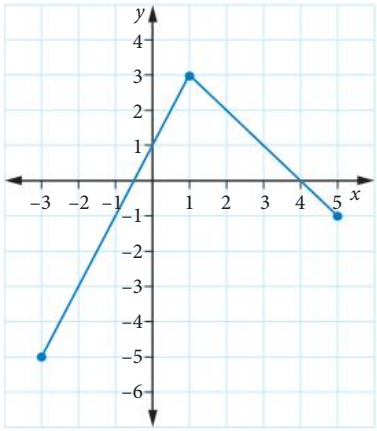
d Use the graph to find the tax payable on these taxable incomes.

i $15 000 **ii** $85 000 **iii** $60 000 **iv** $48 000

e Calculate, correct to 2 decimal places, the gradient of the line segment for incomes between $37 000 and $80 000?

Exam practice

10 (6 marks) The graph of a piecewise function is shown.



a State the domain (range of x values) for each segment. *[1 mark]*

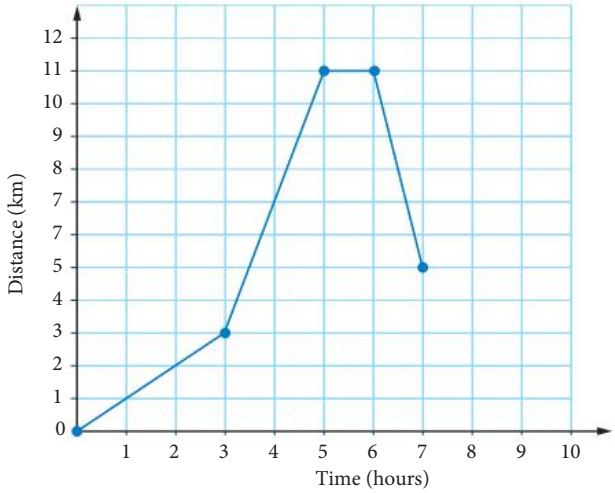
b Calculate the gradient and y-intercept of the segment on the left. *[1 mark]*

c Write the equation of the segment on the left. *[1 mark]*

d Write the equation of the segment on the right. *[1 mark]*

e Write the equation for the piecewise function. *[2 marks]*

11 (5 marks) The distance–time graph shows Edin's distance from home while he is driving around town during the day.



a What does the point at the origin refer to? *[1 mark]*

b How far was Edin from home after 4 hours? *[1 mark]*

c Explain what occurred between the 5th and 6th hour. *[1 mark]*

d During which part of the trip was Edin travelling the fastest? *[1 mark]*

e How many kilometres had Edin travelled in total after 7 hours? *[1 mark]*

12 (6 marks) CF Draw a graph of this piecewise linear function.

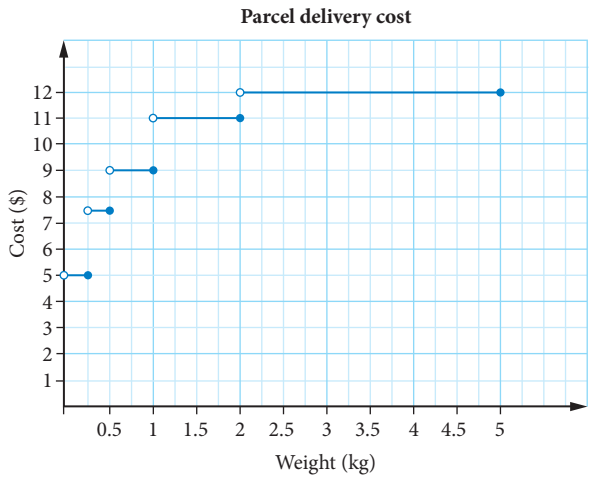
$$y = \begin{cases} -x-1 & \text{if} \quad -5 \le x \le -2 \\ x+1 & \text{if} \quad -2 < x < 1 \\ 1-x & \text{if} \quad 1 \le x < 4 \end{cases}$$

7.6 Step graphs

A small courier specialises in delivering parcels up to 5 kg in weight to buildings in the Brisbane central business district (CBD). Its charges are as follows:

Up to and including 250 g	$5.00
More than 250 g and up to and including 500 g	$7.50
More than 500 g and up to and including 1 kg	$9.00
More than 1 kg and up to and including 2 kg	$11.00
More than 2 kg and up to and including 5 kg	$12.00

This is a special type of piecewise graph called a **step graph** because it looks like steps.



WORKED EXAMPLE 16 Interpreting step graphs

Use the step graph above to answer the following questions.

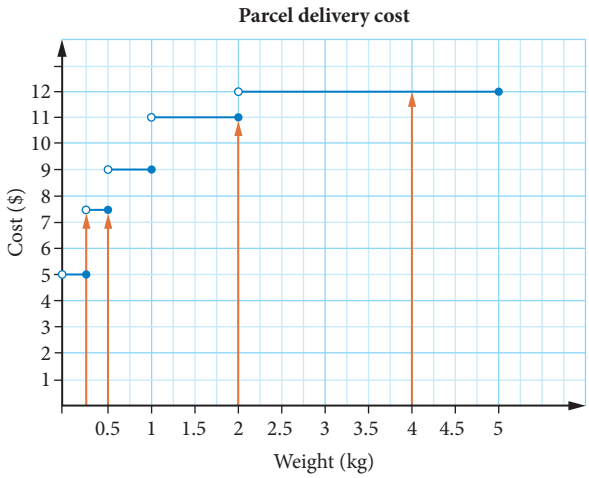
a How much would it cost to deliver a 4 kg parcel to the CBD?

b How much would it cost to deliver a 2 kg parcel?

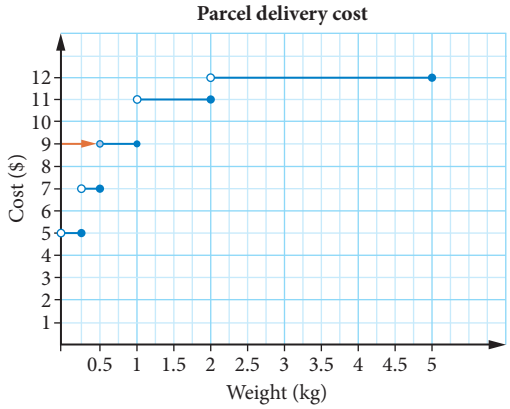
c How much would it cost to deliver a 500 g parcel?

d How much would it cost to deliver a 251 g parcel?

e If you only have $9 to spend, what are the range of weights the parcel could be?

Steps	Working
a 1 Draw a line vertically up from 4 kg until it crosses a step. Read the cost from the vertical axis.	
2 State the result.	A 4 kg parcel would cost $12 to be delivered.

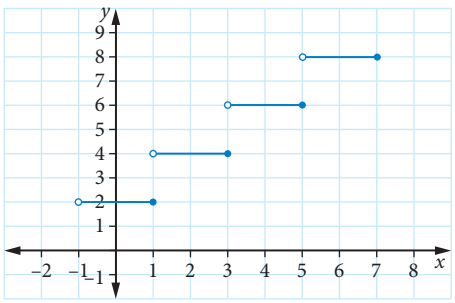
Worksheet
Step graphs

b Draw a line vertically up from 2 kg until it crosses a step. Read the cost from the vertical axis.	A 2 kg parcel would cost \$11 to be delivered.
c Draw a line vertically up from 500 g until it crosses a step.	A 500 g parcel would cost \$7.50 to be delivered.
d Draw a line vertically up from 251 g until it crosses a step.	A 251 g parcel would cost \$7.50 to be delivered.
e **1** Locate \$9 on the vertical axis. Locate the step that corresponds to \$9.	
2 State the result.	For \$9, parcels more than 500 g and up to and including 1 kg can be delivered.

WORKED EXAMPLE 17 Sketching step graphs

Graph the step function defined by the equation

$$y = \begin{cases} 2 & \text{if } -1 < x \leq 1 \\ 4 & \text{if } 1 < x \leq 3 \\ 6 & \text{if } 3 < x \leq 5 \\ 8 & \text{if } 5 < x \leq 7 \end{cases}$$

Steps	Working
1 The first step has $y = 2$ for x values between -1 and 1. Draw a horizontal line from -1 to 1 at $y = 2$. $x = -1$ is not included, so draw an open circle at $(-1, 2)$. $x = 1$ is included, so draw a closed circle at $(1, 2)$.	
2 Repeat the process for the other steps to complete the graph.	

EXERCISE 7.6 Step graphs

ANSWERS p. 413

7.6

Recap

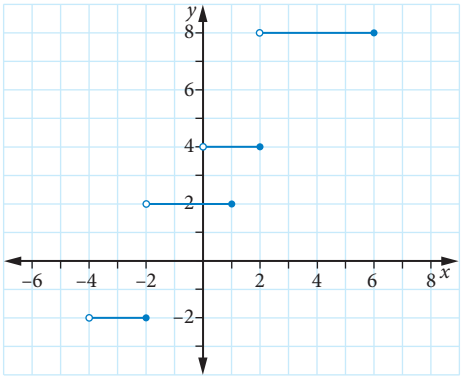
1 Determine the simultaneous solution for $y = x + 3$ and $y = -6x + 17$.

2 Find the x-intercept for the equation $2x + 5y = 10$.

Mastery

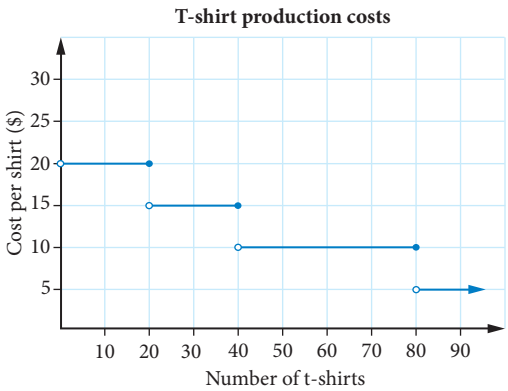
3 A student is supplied with a correct equation for a particular step function and asked to draw its graph. The graph produced by the student is shown.

List any errors that you think the student has made.



4 WORKED EXAMPLE 16 A company makes t-shirts for small and large retailers. The cost charged to manufacture a t-shirt varies depending on how many t-shirts are ordered.

The graph shows the cost per t-shirt paid by retailers.



a How much does each t-shirt cost if

i 15 are ordered?

ii 20 are ordered?

iii 21 are ordered?

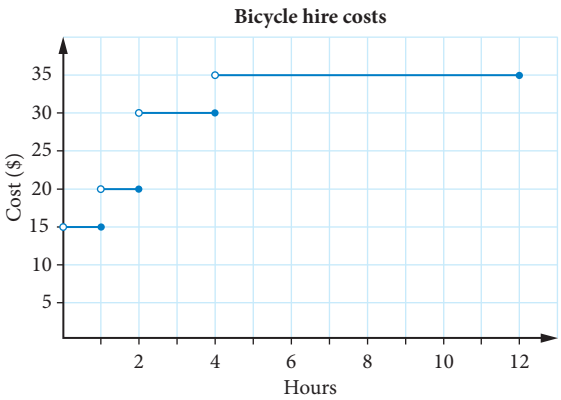
iv 62 are ordered?

b What is the total cost of 2 separate orders for 52 t-shirts and 25 t-shirts?

c How does the unit cost of a t-shirt change as the number of t-shirts manufactured increases? Why?

d How much does each t-shirt cost if 200 are ordered?

5 The cost of hiring a bicycle is shown in the step graph.



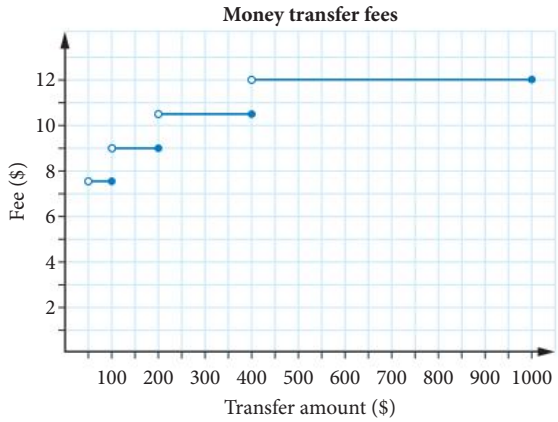
a What is the cost per hour when a bike is hired for 7 hours?

b What is the cost per hour when a bike is hired for 2 hours?

c What is the longest period a bicycle can be hired for $30?

d Three friends hire a bicycle each. One rides for half an hour, the second rides for 8 hours and the third for 4 hours. What is the total cost for the group to hire their bicycles?

6 The various fees charged to transfer money overseas are shown in the graph.



a How much does it cost to transfer $150?

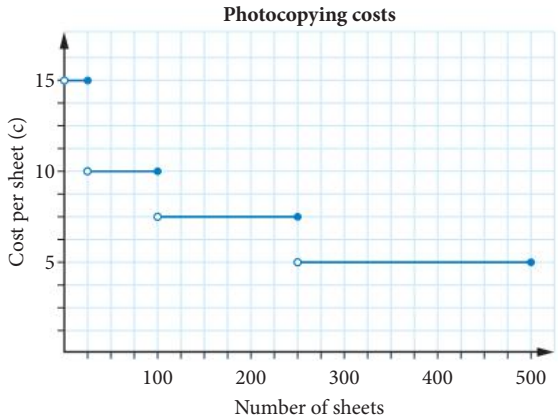
b How much does it cost to transfer $526?

c How much does it cost to transfer $201?

d How much does it cost to transfer $398?

e What is the maximum amount of money that can be transferred for $7.50?

7 The cost per sheet for photocopying charged by an office supplier is shown in the graph.



a How much does each copy cost if

i 75 are printed?

ii 10 are printed?

iii 421 are printed?

iv 250 are printed?

b What is the total cost of 2 separate print runs of 280 copies and 95 copies?

c How does the unit cost of copying change as the number of copies increases?

8 WORKED EXAMPLE 17 Graph each step function.

a $y = \begin{cases} -2 & \text{if} \quad -3 < x \le -1 \\ 1 & \text{if} \quad -1 < x \le 5 \\ 4 & \text{if} \quad 5 < x \le 9 \end{cases}$

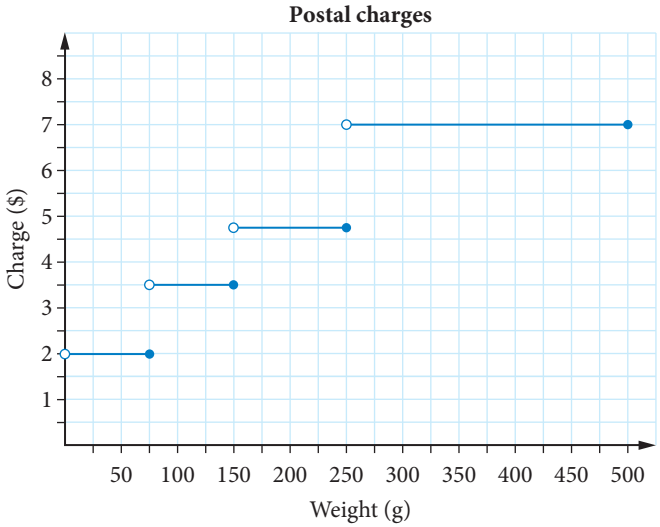
b $y = \begin{cases} 2 & \text{if} \quad 0 < x \le 10 \\ 4 & \text{if} \quad 10 < x \le 20 \\ 8 & \text{if} \quad 20 < x \le 30 \\ 16 & \text{if} \quad x > 30 \end{cases}$

c $C = \begin{cases} \$12 & \text{if} \quad 0 < x \le 8 \\ \$16 & \text{if} \quad 8 < x \le 20 \\ \$20 & \text{if} \quad 20 < x \le 32 \\ \$30 & \text{if} \quad 32 < x \le 50 \end{cases}$

d $y = \begin{cases} -3 & \text{if} \quad -3 \le x < -2 \\ -2 & \text{if} \quad -2 \le x < -1 \\ -1 & \text{if} \quad -1 \le x < 0 \\ 0 & \text{if} \quad 0 \le x < 1 \\ 1 & \text{if} \quad 1 \le x < 2 \\ 2 & \text{if} \quad 2 \le x < 3 \\ 3 & \text{if} \quad 3 \le x < 4 \end{cases}$

Exam practice

9 (3 marks) The step graph shows the postal charges for non-standard items up to 500 g in weight.



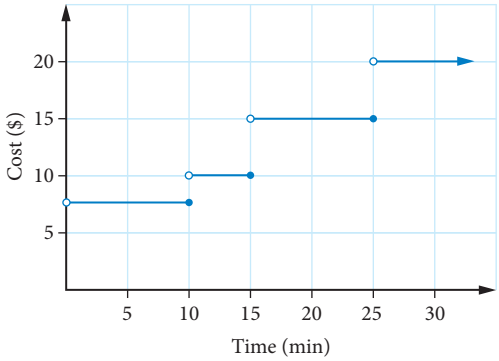
How much does it cost to post an item that weighs

a 110 g? *[1 mark]*

b 250 g? *[1 mark]*

c 425 g? *[1 mark]*

10 (4 marks) Use the step graph shown to develop an equation for the step function it represents.



11 (4 marks) CF A mobile gardener charges a call-out fee of $70 including the first hour of work. Labour is then charged at the rate of $30 per hour or part thereof for the next 2 hours and $25 per hour or part thereof for any additional hours.

Draw a step graph representing the fees charged by the gardener.

Video Exam question analysis: Simultaneous equations and piecewise graphs

EXAM QUESTION ANALYSIS

Exam-style question (5 marks)

The Griffin family purchased 5 burgers and 3 serves of hot chips for \$35.
The Simpson family bought 3 burgers and 4 serves of hot chips for \$28.70.
Determine the cost of each individual item.

Reading the question

- Two families buy burgers and hot chips.
- Griffin family – 5 burgers, 3 chips, Simpson family – 3 burgers, 4 chips.
- Find the cost of a burger and chips individually.

Thinking about the question

- Two equations with the same variables, therefore simultaneous.
- Let b = cost of burger and c = cost of chips for the equations.
- Solve for b and c.

Worked solution (✓ = 1 mark)

Let b = cost of one burger in dollars and c = cost of one serve of chips in dollars.

[1] Griffins $5b + 3c = 35$ $(\times 4)$ $20b + 12c = 140$

[2] Simpsons $3b + 4c = 28.70$ $(\times 3)$ ✓ $-\ 9b + 12c = 86.10$ ✓

$11b = 53.90$

$b = \dfrac{53.90}{11}$

$\boldsymbol{b = 4.90}$ ✓

Substitute b into [2]: $3b + 4c = 28.70$

$3 \times 4.90 + 4c = 28.70$

$14.70 + 4c = 28.70$

$4c = 28.70 - 14.70$

$c = \dfrac{14}{4}$

$\boldsymbol{c = 3.50}$ ✓

∴ burgers cost \$4.90 each and hot chips cost \$3.50 per serve. ✓

7 Chapter summary

Simultaneous equations may be solved graphically or algebraically. The point where 2 graphs intersect is called the **point of intersection**.
It indicates the simultaneous solution of the equations. Simultaneous equations whose graphs are parallel have no simultaneous solution.

The substitution method

1. Use one equation to express one variable in terms of the other.
2. Substitute this into the other equation.
3. Solve the resulting equation.
4. Use the solution to find the value of the other variable.
5. Check your solution satisfies both equations.

The elimination method

1. Ensure both equations are in the form $ax + by = c$.
2. Check if there is a matching pair to eliminate.
3. If no matching pair exists, multiply one or both equations to create a matching pair to cancel.
4. Eliminate the matching pair and find the remaining variable.
5. Use the first variable result to substitute into the original equations to find the other variable.
6. Check your solution satisfies both equations.

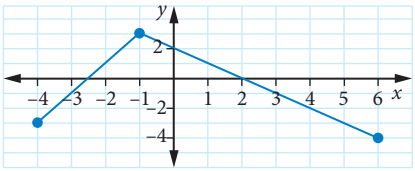
Applications of simultaneous equations

1. Read the information carefully.
2. Choose variable names for the unknown quantities.
3. Use the information in the problem to form equations in terms of the variables.
4. Solve the equations using the most convenient method.
5. Check the solution.
6. Write the solution in the same terms as the stated problem.

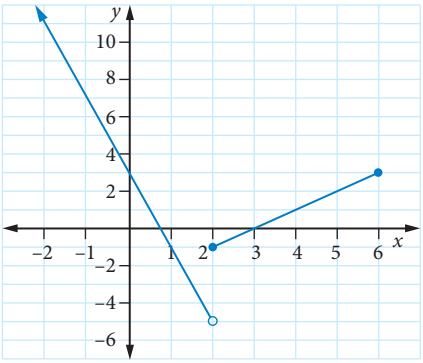
Piecewise linear graphs

- A **piecewise linear function** is defined using 2 or more linear functions. The graph of a piecewise function consists of 2 or more non-overlapping line segments.
- The ends of each segment in the graph of a piecewise function are labelled with a circle. An open circle indicates that the point is not included in the segment. A closed circle is used to indicate that the point is included in the segment.
- The graph of a **continuous** piecewise function has no holes or jumps. In the graph of a **discontinuous** piecewise function, the segments do not connect.

Continuous piecewise function



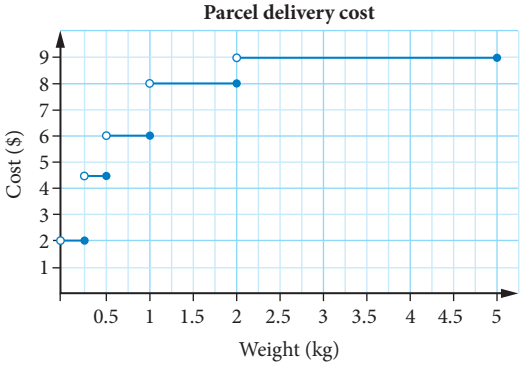
Discontinuous piecewise function



Step graphs

- A **step graph** is a special type of piecewise graph made up of horizontal segments. The end points of the segments have either open or closed circles. The graph looks like the steps of a staircase.

Parcel delivery cost



Worksheet
General Maths Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

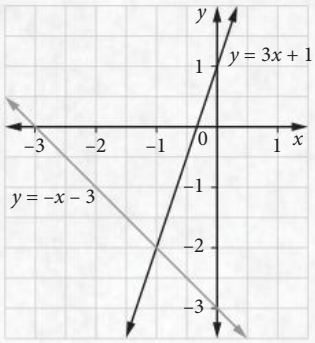
Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 Referring to the graph, the simultaneous solution for the equations $y = 3x + 1$ and $y = -x - 3$ is



A $(-2, -4)$

B $(-1, -2)$

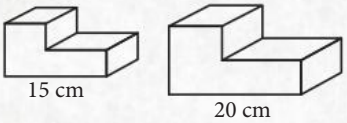
C $(-2, -1)$

D $(1, -3)$

2 What is the simultaneous solution for $y - 3x = 18$ and $7x - 3y = -44$?

A $(5, -3)$ **B** $(1, 21)$ **C** $(2, 24)$ **D** $(-5, 3)$

3 The 2 solids shown here are similar. If the volume of the smaller object is $150\,\text{cm}^3$, then the volume of the larger one is closest to



A $210\,\text{cm}^3$

B $360\,\text{cm}^3$

C $200\,\text{cm}^3$

D $250\,\text{cm}^3$

4 The most efficient method for solving the simultaneous equations $x = 2y - 8$ and $x = -y + 4$ is the

A substitution method **B** elimination method

C non-linear method **D** function method

5 A map has a scale of 1 : 125 000. If the distance between 2 cities on the map is 12 cm, the actual distance between them is

A 7.5 km **B** 150 km **C** 1500 m **D** 15 km

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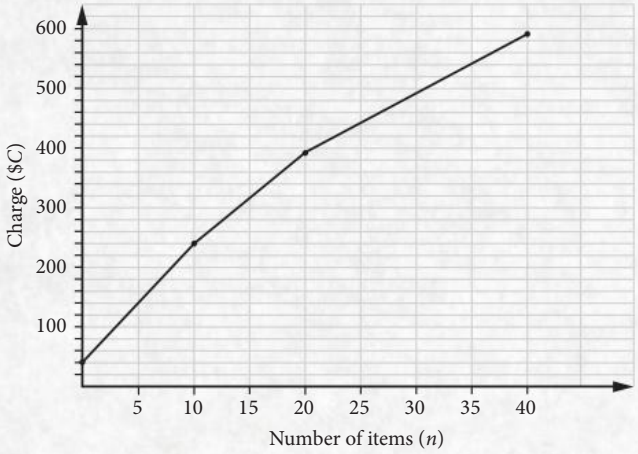
Section 2 4 short response questions

13 marks

6 (3 marks) Solve the simultaneous equations by substitution.

$$y = 2x - 4$$
$$x + y = 5$$

7 (3 marks) A silk screen shop specialises in printing images on t-shirts. The owner of the shop charges an initial fee to create the silk screen and then an additional charge to print the t-shirts. The graph represents the silk-screen shop's charges.



a What is the fee to set up the silk screen? *[1 mark]*

b How much would it cost for an order of 25 t-shirts? *[1 mark]*

c What is the cost per item for an order of 10 t-shirts? *[1 mark]*

8 (4 marks) Delivery drivers for a local business are paid a base pay of $750 per week and $2.50 per delivery. How much will a driver receive if they deliver in one week

a 800 packages? *[2 marks]*

b 690 packages? *[2 marks]*

9 (3 marks) A company announced a net yearly income of $18 750 000 at the end of the financial year. At the time of the announcement, the market price for shares in the company were $4.65.

a Determine the annual earnings per share if the company has 25 million shares in the market. *[1 mark]*

b Calculate the P/E ratio for this company. *[1 mark]*

c Describe how this P/E ratio would likely be interpreted by an investor. *[1 mark]*

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

4 short response questions

12 marks

1 (3 marks) A cyclist travels away from home at 10 km/h for 30 minutes. They meet a friend and stop for coffee for $1\frac{1}{2}$ hours. They then continue their journey away from home for an additional hour at 15 km/h. The cyclist returns home in 1 hour from this point.

Use this information to draw a piecewise graph for the distance from home (d) after (t) hours for the cyclist.

2 (2 marks) A mother is 3 times as old as her daughter, but in 10 years she will only be twice as old as her daughter. What are their present ages?

3 (4 marks) Two buildings beside each other cast shadows of different lengths in the early afternoon. The first building casts a shadow 1.8 m long. The length of the shadow of the second building cannot be measured because there are objects up against its base, but the edge of the shadow is marked. Later in the afternoon, the shadow of the first building is 3.9 m long, and the shadow of the second building is 1.2 m longer than before. The first building is 6.3 m high.

What is the height of the second building? Justify your answer.

4 (3 marks) Jess opens her piggy bank and discovers 27 coins. All the coins are 5 cent and 20 cent pieces. If the total value of the coins is \$3.75, how many of each coin does Jess have?

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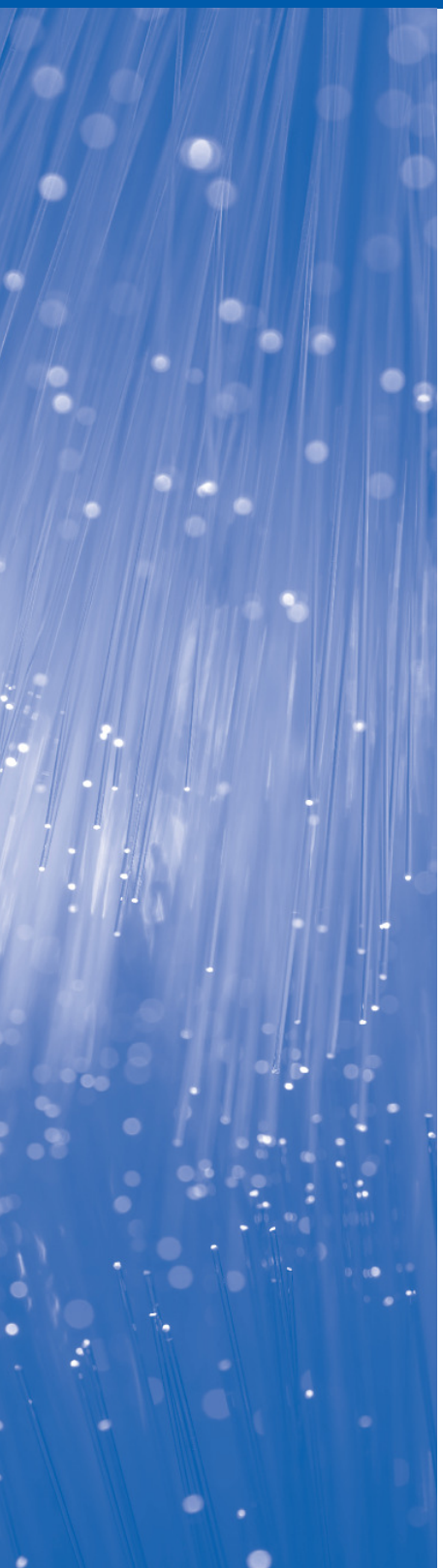
CHAPTER

8 RIGHT-ANGLED TRIGONOMETRY



Syllabus coverage

Nelson MindTap chapter resources

Terminology

8.1 The trigonometric ratios

8.2 Finding unknown sides

8.3 Finding unknown angles

8.4 Angles of elevation and depression

8.5 True bearings

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

Prior learning
Right-angled trigonometry

Syllabus coverage

UNIT 2, TOPIC 2: APPLICATIONS OF TRIGONOMETRY

Applications of trigonometry

- Understand and use the trigonometric ratios to find the size of an unknown angle, θ, or the length of an unknown side in a right-angled triangle.
  - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
- Solve two-dimensional practical problems involving the trigonometry of right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings.

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Videos (7):

8.2 Finding an unknown side
8.3 Finding an unknown angle
8.4 Angles of elevation and depression • Angle of depression 1
8.5 True bearings • Bearings
Exam question analysis Right-angled trigonometry

Prior learning (1):

8.1 Right-angled trigonometry

Worksheets (10):

8.1 Trigonometric ratios • Identifying the correct trigonometric ratio
8.3 Calculating lengths and angles • Trigonometry problems
8.5 Bearings 1 • Bearings 2 • Identifying bearings • Elevations and bearings • 16 points of the compass
Cumulative exams General Maths Year 11 formula sheet

Puzzles (2):

8.4 Angles of elevation and depression
Chapter summary Trigonometry crossword

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

adjacent side	angle of depression	angle of elevation	bearing
compass rose	cosine	hypotenuse	inverse trigonometric function
opposite side	right-angled triangle	sine	tangent
trigonometric ratio	trigonometry	true bearing	

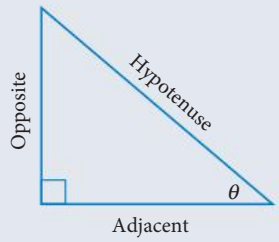
8.1 The trigonometric ratios

Trigonometry is the study of the relationship between the sides and angles of triangles. You can use it to calculate unknown lengths and angles.

Trigonometric ratios

- The 3 **trigonometric ratios** (**trigonometric functions**) are defined for a right-angled triangle. They are **sine** (sin), **cosine** (cos) and **tangent** (tan).

Ratio	Initials
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	SOH
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	CAH
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	TOA



- The **hypotenuse** is the longest side and is always opposite the right-angle.
- The **opposite side** is opposite the angle θ.
- The **adjacent side** is beside the angle θ, leading to the right angle.

Worksheets
Trigonometric ratios

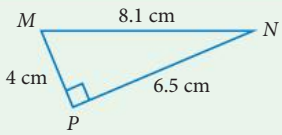
Identifying the correct trigonometric ratio

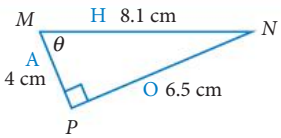
You can remember the 3 ratios as SOH-CAH-TOA (pronounced 'soh car toe-ah').

WORKED EXAMPLE 1 Finding trigonometric ratios

For this triangle, find these trigonometric ratios.

a tan M **b** cos M



Steps	Working
1 The angle we are looking at is M, so write θ on angle M. **2** Label the sides of the triangle with O, A and H to show opposite, adjacent and hypotenuse.	
a 1 Write the ratio: tangent uses opposite and adjacent.	$\tan M = \dfrac{\text{opposite}}{\text{adjacent}}$
2 The opposite side is 6.5 cm and the adjacent side is 4 cm.	$\tan M = \dfrac{6.5}{4}$
b 1 Write the ratio: cosine uses adjacent and hypotenuse.	$\cos M = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
2 The adjacent side is 4 cm and the hypotenuse is 8.1 cm.	$\cos M = \dfrac{4}{8.1}$

Exam hack

Make sure the calculator is set in degree mode.

Most scientific calculators will show a small DEG or D in the display if they are set in this mode.

If the display shows RAD (R) or GRAD (G), change it to degree mode to get the right answers.

WORKED EXAMPLE 2 Trigonometric ratios on your calculator

Use your calculator to find each value, correct to 4 decimal places.

a $\tan 34.8°$ **b** $6.1 \cos 47°$ **c** $\dfrac{210}{\sin 56°}$

Steps	Working
a 1 Use the TAN key.	tan $34.8 = 0.695\,018\ldots$
2 Round and write the answer.	≈ 0.6950
b 1 Use the COS key.	$6.1 \times$ cos $47 = 4.160\,189\ldots$
2 Round and write the answer.	≈ 4.1602
c 1 Use the fraction function on the calculator. Use the SIN key.	$\dfrac{210}{\sin 56°} = 253.305\,76\ldots$
2 Round and write the answer.	≈ 253.3058

The **inverse trigonometric functions** are used to find unknown angles.

On your calculator, the inverse functions are on top of the trigonometry keys.

To get $\tan^{-1}$, press 2nd F tan OR SHIFT tan (depending on your calculator).

WORKED EXAMPLE 3 Inverse trigonometric ratios on your calculator

Use your calculator to find $\sin^{-1}\left(\dfrac{4.4}{8}\right)$, correct to 2 decimal places.

Steps	Working
1 Use the $\sin^{-1}$ key by pressing 2nd F sin OR SHIFT sin	
2 Use the fraction button to enter $\dfrac{4.4}{8}$.	$\sin^{-1}\left(\dfrac{4.4}{8}\right) = 33.367\,01\ldots$
3 Round and write the answer.	$\approx 33.37°$

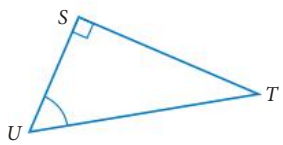
EXERCISE 8.1 The trigonometric ratios

ANSWERS p. 414

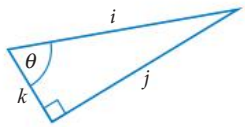
Mastery

1 Name the opposite side, the adjacent side and the hypotenuse for each marked angle below.

a



b



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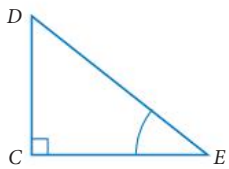

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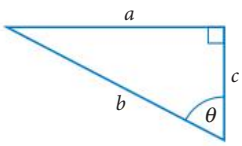

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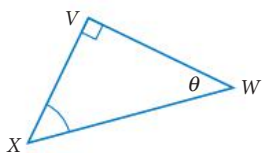
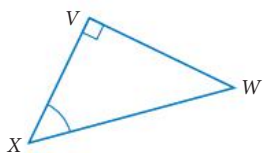
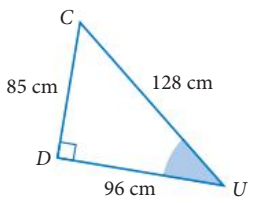


f



2 WORKED EXAMPLE 1 Write the sine, cosine and tangent ratio for each shaded angle.

a



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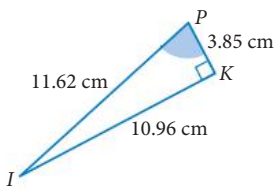

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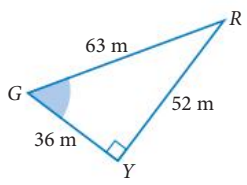


3 WORKED EXAMPLE 2 Calculate each expression, correct to 4 decimal places.

a $\cos 32°$ **b** $\sin 76.8°$ **c** $7 \times \tan 24°$

d $6.3 \cos 1.4°$ **e** $\dfrac{24}{\sin 84°}$ **f** $\dfrac{420}{\tan 64.1°}$

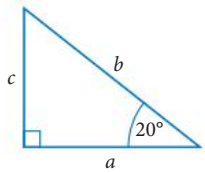
4 WORKED EXAMPLE 3 Calculate each expression, correct to 2 decimal places.

a $\sin^{-1}\left(\dfrac{4}{7}\right)$ **b** $\cos^{-1}\left(\dfrac{5.8}{11.2}\right)$ **c** $\tan^{-1} 4.6$

d $\cos^{-1} 0.85$ **e** $\sin^{-1}\left(\dfrac{1}{2}\right)$ **f** $\sin^{-1} 0.31415$

Exam practice

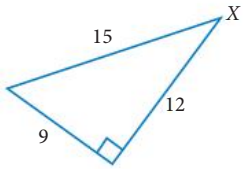
5 What is the correct expression for $\cos 20°$ in this triangle?



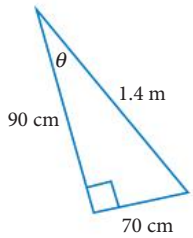
A $\dfrac{a}{b}$ **B** $\dfrac{a}{c}$

C $\dfrac{c}{b}$ **D** $\dfrac{c}{a}$

6 (2 marks) For this triangle, what is the value of $\sin X$ as a decimal?



7 (2 marks) Determine the value of $\cos\theta$, correct to 3 decimal places.



8 (2 marks) CF Determine the value of $\sin\theta$ if $\tan\theta = \dfrac{3}{4}$.

Video
Finding an unknown side

8.2 Finding unknown sides

A trigonometric ratio can be used to calculate the length of an unknown side in a right-angled triangle if one angle and one side are known.

Finding an unknown side

1. **Draw** a diagram, if not given.
2. **Label** the sides O, A, H.
3. Choose the **ratio** using SOH CAH TOA.
4. **Substitute** in the given values.
5. **Solve** for the unknown.
6. **Answer** the question.

When solving for an unknown:

If the unknown is on top of the ratio:

$$\cos 54° = \frac{y}{10}$$

then MULTIPLY

$$y = 10 \times \cos 54°$$

If the unknown is on the bottom of the ratio:

$$\cos 54° = \frac{10}{y}$$

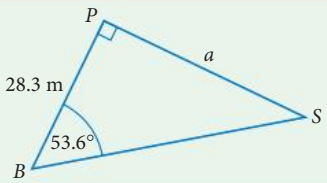
then SWITCH

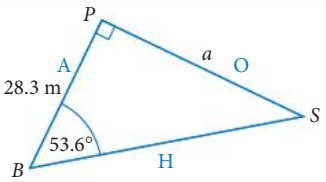
$$\cos 54° = \frac{10}{y}$$

$$y = \frac{10}{\cos 54°}$$

WORKED EXAMPLE 4 Finding an unknown side 1

Find the length of the unknown side, correct to one decimal place.



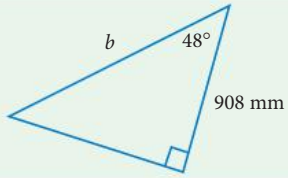
Steps	Working
1 Copy the diagram.	
2 Label the sides O, A, H.	
3 The unknown and known sides are O and A. Use tangent.	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
4 Substitute in the values.	$\tan 53.6° = \frac{a}{28.3}$
5 Solve for the unknown.	
a is on top, so MULTIPLY by 28.3.	$a = 28.3 \times \tan 53.6°$
Enter into your calculator.	$a = 38.3851\ldots$
Round to one decimal place.	$\approx 38.4\text{ m}$
6 Answer the question.	The length of a is 38.4 m.
Ensure that units are included.	

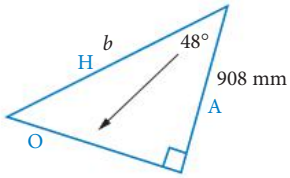
Exam hack

ALWAYS check your answer makes sense!

WORKED EXAMPLE 5 Finding an unknown side 2

Find the length of the unknown side, correct to the nearest mm.

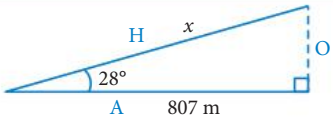


Steps	Working
1 Copy the diagram.	
2 Label the sides O, A, H.	
3 The sides A and H are the known and unknown respectively. Use cosine.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
4 Substitute in the values.	$\cos 48° = \dfrac{908}{b}$
5 Solve for the unknown. b is on the bottom, so SWITCH the b and the cos 48°.	$b = \dfrac{908}{\cos 48°}$
Enter into your calculator.	$b = 1356.984\ldots$
Round to nearest mm.	≈ 1357 mm
6 Answer the question.	The length of b is 1357 mm.

For practical applications of trigonometry, you need to interpret a worded problem. If there is no diagram, draw one that represents the situation. Remember you are looking for a right-angled triangle.

WORKED EXAMPLE 6 Applications of trigonometry

The slope of a suburban road up a steep hill is 28°. The survey map shows that the road covers a horizontal distance of 807 m to reach the top of the hill. **Determine** the actual length of the road surface, correct to the nearest metre.

Steps	Working
1 Draw a diagram. Mark on it the given information: horizontal distance = 807 m; slope angle = 28°; Unknown is road length = x	
2 Label the sides O, A, H.	
3 The sides A and H are the known and unknown sides respectively. Use cosine.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
4 Substitute in the values.	$\cos 28° = \dfrac{807}{x}$
5 x is on the bottom, so SWITCH the x and the cos 28°.	$x = \dfrac{807}{\cos 28°}$
Enter into your calculator.	$= 913.9840\ldots$
Round to the nearest metre.	≈ 914 m
6 Answer the question.	The actual length of road surface is 914 m.

EXERCISE 8.2 Finding unknown sides

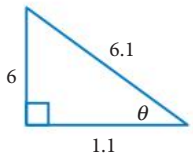
ANSWERS p. 414

Recap

1 Which ratio is correct for the triangle shown?

A $\cos\theta = \dfrac{6}{6.1}$ **B** $\sin\theta = \dfrac{6}{1.1}$

C $\sin\theta = \dfrac{6}{6.1}$ **D** $\tan\theta = \dfrac{1.1}{6}$



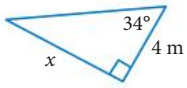
2 Calculate each expression, correct to 2 decimal places.

a $4\tan 57°$ **b** $\dfrac{230}{\sin 15.5°}$ **c** $\cos^{-1}\left(\dfrac{4}{5.5}\right)$

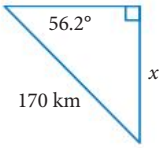
Mastery

3 WORKED EXAMPLE 4 Find the value of the pronumeral, correct to one decimal place.

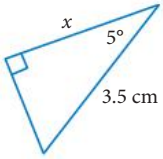
a



b

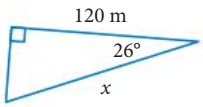


c

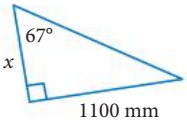


4 WORKED EXAMPLE 5 Find the value of the pronumeral, correct to one decimal place.

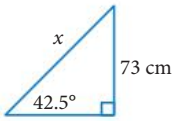
a



b

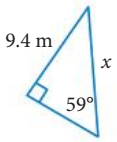


c

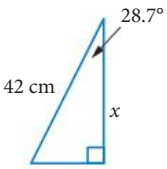


5 Find the length of the unknown side in each triangle, correct to 2 decimal places.

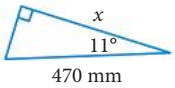
a



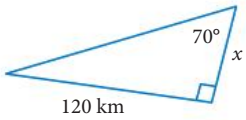
b



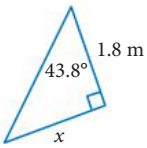
c



d



e



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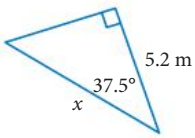
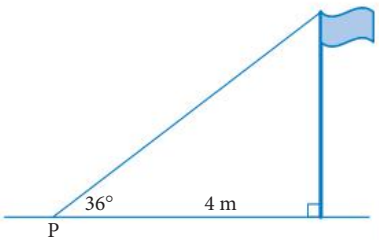
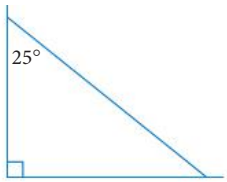


6 WORKED EXAMPLE 6 A flagpole is supported by a wire that runs from the top of the pole to a point, P, which is on the ground, 4 metres from the base of the pole. If the wire makes an angle of 36° with the ground, calculate the height of the flagpole, correct to one decimal place.

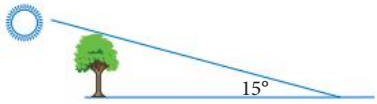


9780170484749

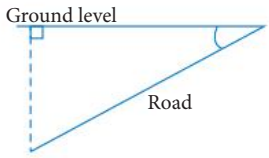
7 A ladder leans against a wall and makes an angle of 25° with the wall. If it reaches 4.5 metres up the wall, how long is the ladder? Answer correct to 2 decimal places.



8 A tree casts a shadow 20 m long. If the sun has an elevation of 15°, how tall is the tree? Answer correct to one decimal place.

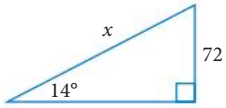


9 A road is constructed down into an open cut mine. The road is straight and is built with an angle of 8.4° with the horizontal. The road ends at 100 m below ground level. Calculate the length of the road, correct to the nearest metre.



Exam practice

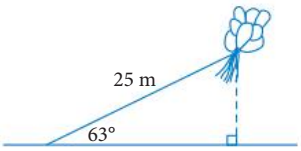
10 The value of x is given by



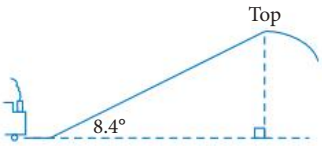
A $72 \cos 14°$ **B** $72 \times \sin 14°$

C $\dfrac{72}{\cos 14°}$ **D** $\dfrac{72}{\sin 14°}$

11 (3 marks) A bunch of helium balloons is tied to a weight that is sitting on level ground. There is a strong wind that blows the balloons so that the string is straight and makes an angle of 63° with the ground. If the string is 25 m long, determine how high the balloons are above the ground. Answer to one decimal place.



12 (3 marks) A train track runs from ground level straight up the sloping face of a hill. The top of the hill is 470 metres above ground level and the track is inclined at 8.4° to the horizontal.



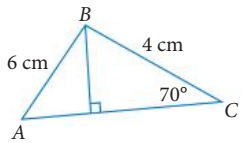
a Make a copy of the diagram and mark in the given information. Show the length of the train track as x. *[1 mark]*

b Calculate the length of the train track in kilometres, correct to one decimal place. *[2 marks]*

13 (3 marks) CF A wire 30 m long is used to hold a radio mast in place. The wire is at an angle of 72° to the ground and is tied 2 m from the top of the mast. Determine the height of the radio mast.

14 (3 marks) CF Ollie was out flying his kite in very strong winds. His kite is now stuck in the top of a tree, but the string is still pulled tight in his hand. He measures the distance to the base of the tree to be 24 m and the angle the string makes with the horizontal is 14°. If Ollie's hand is 1.2 m above the ground, calculate the height of the tree. Answer correct to the nearest centimetre.

15 (4 marks) CU Calculate the perimeter of $\triangle ABC$ from the diagram. Give your answer to one decimal place.



Video
Finding an unknown angle

Worksheets
Calculating lengths and angles

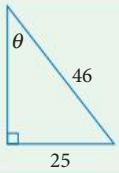
Trigonometry problems

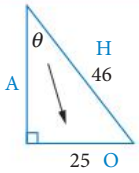
8.3 Finding unknown angles

If 2 sides of a right-angled triangle are known, you can use a calculator to find the angle corresponding to that ratio.

WORKED EXAMPLE 7 Finding an unknown angle

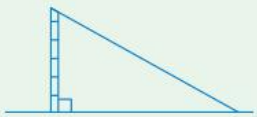
Find θ, correct to the nearest degree.

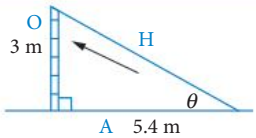


Steps	Working
1 Label the sides O, A, H.	
2 The sides O and H are known. Use sine.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
3 Substitute in the values.	$\sin\theta = \dfrac{25}{46}$
4 Solve for the unknown. To get θ on its own, we need to use the inverse function. Round to the nearest degree.	$\theta = \sin^{-1}\left(\dfrac{25}{46}\right)$ $= 32.92\ldots$ $\approx 33°$
5 Answer the question.	The value of θ is 33°.

WORKED EXAMPLE 8 Word problem

A slide has a 3-metre ladder attached, as shown in the diagram. If the base of the ladder is 5.4 metres from the bottom of the slide, what is the angle the slide makes with the ground? Answer correct to one decimal place.



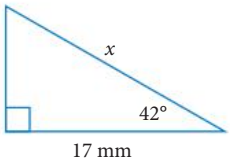
Steps	Working
1 Mark on the diagram the given information. The unknown is the angle the slide makes with the ground, θ.	
2 Label the sides O, A, H.	
3 The sides O and A are known. Use tan.	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
4 Substitute in the values.	$\tan\theta = \dfrac{3}{5.4}$
5 Solve for the unknown. Round to the nearest degree.	$\theta = \tan^{-1}\left(\dfrac{3}{5.4}\right)$ $= 29.054\ldots$ $\approx 29.1°$
6 Answer the question.	The angle the slide makes with the ground is 29.1°.

EXERCISE 8.3 Finding unknown angles

ANSWERS p. 414

Recap

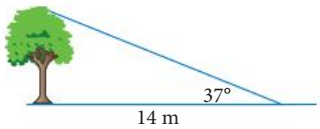
1 The value x can be found using



A $x = \dfrac{17}{\cos 42°}$ **B** $x = 17 \times \cos 42°$

C $x = \dfrac{\cos 42°}{17}$ **D** $x = 42 \times \cos 17$

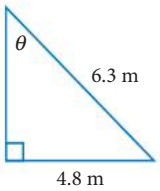
2 Calculate the height of the tree, correct to 2 decimal places.



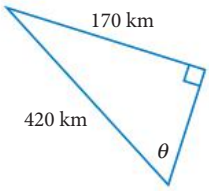
Mastery

3 WORKED EXAMPLE 7 Calculate the value of θ, correct to the nearest degree.

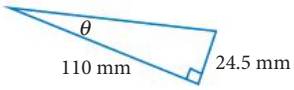
a



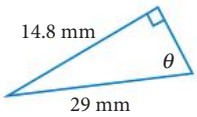
b



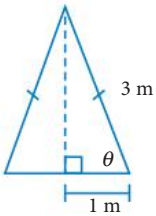
c



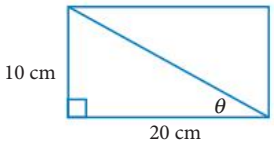
d



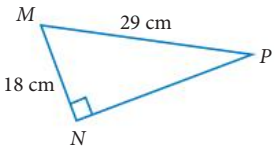
e



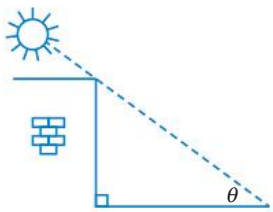
f



4 Calculate the size of the unknown angles in this triangle. Answer to the nearest degree.



5 WORKED EXAMPLE 8 A building that is 7.5 m tall casts a shadow 20 m long. Calculate the angle of the sun, θ, correct to one decimal place.



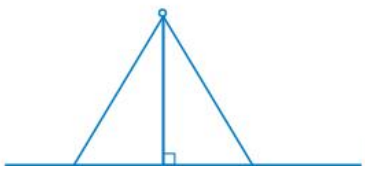
6 A ladder 5 m long has its end just resting on the top of a fence 3.7 m high. What angle does the ladder make with the ground? Answer to the nearest degree.

Exam practice

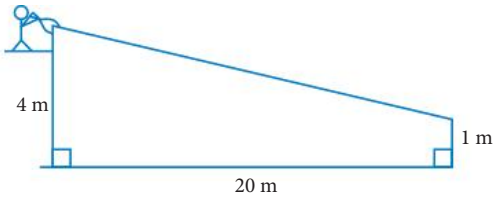
7 If $\tan\theta = 59.5$, what is the value of θ, correct to 2 decimal places?

A 1.69 **B** 1.70 **C** 89.03 **D** 89.04

8 (2 marks) A pole is supported by 2 wires that are each 8.2 m long. If the pole is 4.3 m high, what angle will the wires make with the ground? Answer correct to one decimal place.



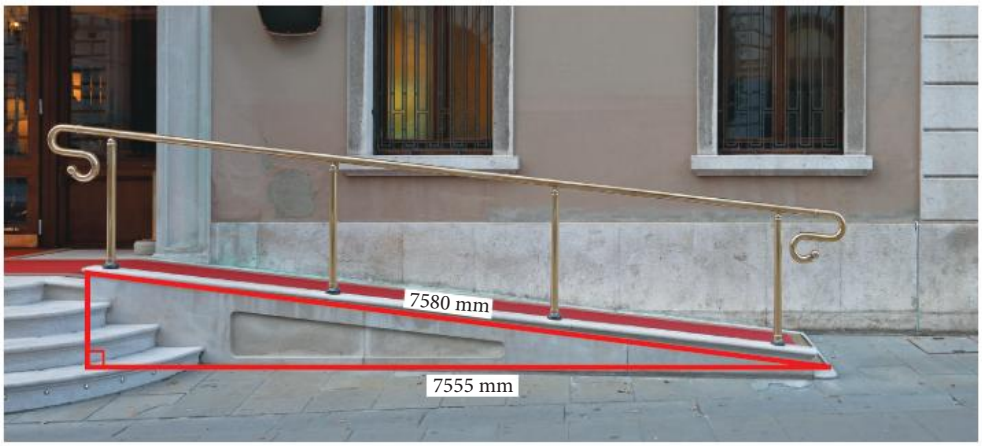
9 A flying fox is constructed in a children's playground. The raised end is 4 metres above the ground and the lower end is 1 m above the ground. The 2 ends are a horizontal distance of 20 metres apart.
Which one of the following expressions will give the angle that the flying fox makes with the vertical?



A $\tan\left(\frac{20}{3}\right)$ **B** $\tan\left(\frac{20}{4}\right)$ **C** $\tan^{-1}\left(\frac{20}{3}\right)$ **D** $\tan^{-1}\left(\frac{20}{4}\right)$

10 (3 marks) The start of a water slide is 30 m above the ground. The water slide is straight and is 54 m long. Determine the angle the water slide makes with the water, correct to the nearest degree.

11 (3 marks) CF A wheelchair ramp up to an entrance has a horizontal length of 7555 mm and a sloping length of 7580 mm. The maximum angle allowed for such ramps by Australian Standards is 4.76°.



Baloncici/Dreamstime LLC

Does the ramp meet the standard? Justify your answer.

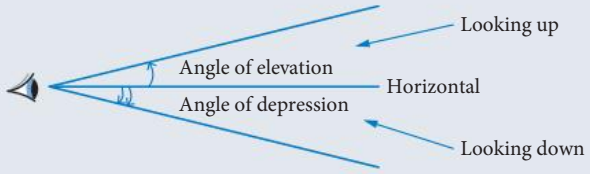
12 (3 marks) CU A clock's pendulum is 17 cm long and swings over a horizontal distance of 10 cm. Determine the angle that the pendulum makes with the vertical. Answer correct to one decimal place.

dyoma/Shutterstock.com

8.4 Angles of elevation and depression

Angles of elevation and depression

The **angle of elevation** is between the horizontal and a line sloping up. It is when you **look up**.
The **angle of depression** is between the horizontal and a line sloping down. It is when you **look down**.



Remember: you want a right-angled triangle in your diagram!

Video Angles of elevation and depression

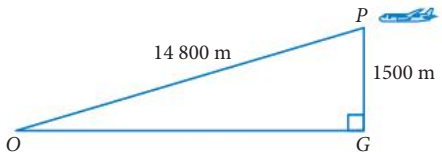
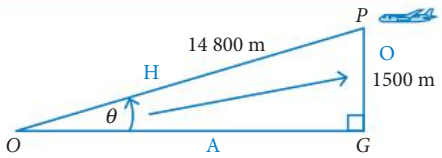
Puzzle Angles of elevation and depression

WORKED EXAMPLE 9 Angle of elevation

An observer spots a plane flying at a height of 1500 m. The plane is 14 800 m from him in a direct line.

a Draw a diagram and mark on it the given information.

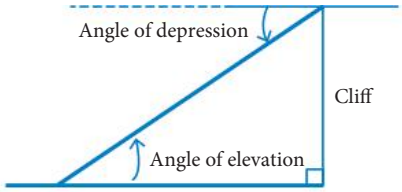
b What is the angle of elevation of the plane from the observer, correct to one decimal place?

Steps	Working
a Draw a diagram. Mark the observer (O) on the ground (G) and the plane (P) in the air. The height of the plane above ground is 1500 m. A right angle is formed between the ground and the vertical height. Distance from O to P = 14 800 m.	
b **1** Angle of elevation = θ (looking up)	
2 Label the sides O, A, H.	
3 The sides O and H are known. Use sine.	$\sin\theta = \frac{O}{H}$
4 Substitute in the values.	$\sin\theta = \frac{1500}{14\,800}$
5 Solve for the unknown.	$\theta = \sin^{-1}\left(\frac{1500}{14\,800}\right)$ $\theta = 5.816\ldots$
Round to one decimal place.	$\theta \approx 5.8°$
6 Answer the question.	The angle of elevation of the plane from the observer is 5.8°.

Video
Angle of depression 1

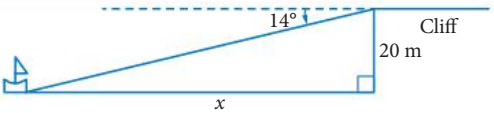
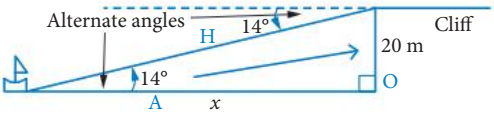
The angle of depression = angle of elevation as they are alternate (Z) angles.

This is very useful in questions using angles of depression.



WORKED EXAMPLE 10 Angle of depression

From the top of a cliff 20 m high, the angle of depression to a boat is 14°. How far is the boat from the base of the cliff? Answer correct to the nearest metre.

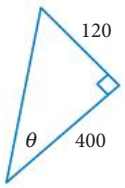
Steps	Working
1 Draw a diagram. Draw the cliff 20 m high. The water meets the cliff at a right angle. Draw the boat out at sea. The angle of depression is the angle you would look down to see the boat from the cliff. Label this angle 14°. Give the required distance a name.	 Let the distance from the base of the cliff to the boat = x.
2 Label the sides O, A, H. Using alternate angles, label the 14° angle in the triangle.	
3 The 20 m side and x are O and A. Use tangent.	$\tan\theta = \frac{O}{A}$
4 Substitute in the values.	$\tan 14° = \frac{20}{x}$
5 x is on the bottom, so SWITCH the x and tan (14°).	$x = \frac{20}{\tan 14°}$ $x = 80.215\ldots$
Round to the nearest metre.	$x \approx 80$ m
6 Answer the question.	The boat is 80 metres from the base of the cliff.

EXERCISE 8.4 Angles of elevation and depression

ANSWERS p. 414

Recap

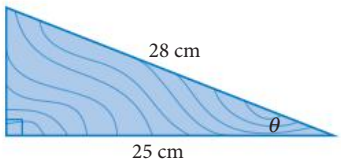
1 The expression that will give the value of θ is



A $\cos\left(\frac{120}{400}\right)$ **B** $\cos^{-1}\left(\frac{120}{400}\right)$

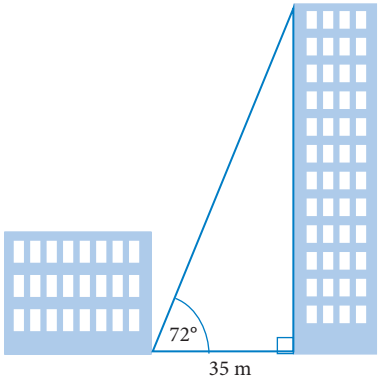
C $\tan\left(\frac{120}{400}\right)$ **D** $\tan^{-1}\left(\frac{120}{400}\right)$

2 A carpenter needs to cut a right-angled wedge out of timber. The wedge must be 25 cm long and have a sloping edge of 28 cm. Find the angle, θ, the carpenter should use to cut the timber, correct to one decimal place.

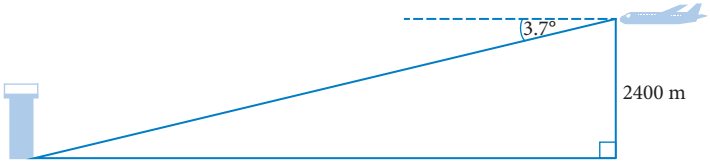


Mastery

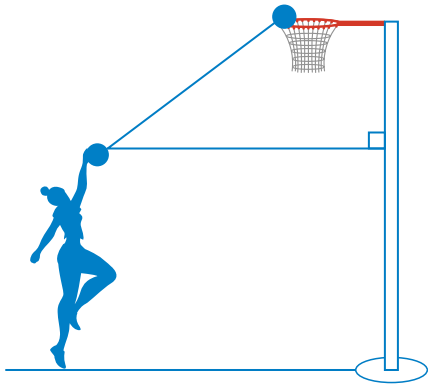
3 WORKED EXAMPLE 9 From ground level on the other side of the street, the angle of elevation to the top of a building is 72°. If the street is 35 m wide, find the height of the building, correct to the nearest metre.



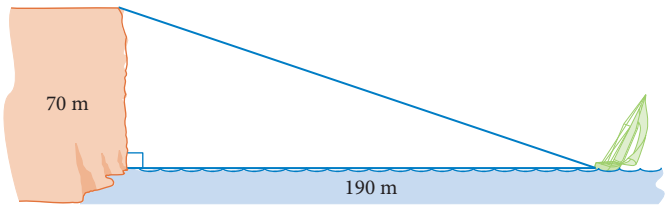
4 WORKED EXAMPLE 10 The pilot of a plane observes that the angle of depression to the base of the control tower at the airport is 3.7°. If the altimeter shows the plane is at a height of 2400 m, find the horizontal distance to the control tower.



5 Rebecca is playing netball. She is shooting for a goal towards a hoop 0.5 m in front of the goal post. The angle of elevation from her hands to the hoop is 38°. If Rebecca is 2.3 m from the base of the goal post, what is the distance from her hands to the hoop? Answer to the nearest centimetre.



6 A yacht is 190 m from the base of a 70 m cliff. What is the angle of elevation from the yacht to the top of the cliff? Answer to nearest degree.



7 An observer in a lighthouse that is 19 m high sees a boat at sea at an angle of depression of 18°. If the base of the lighthouse is at sea level, how far is the boat from the base of the lighthouse?

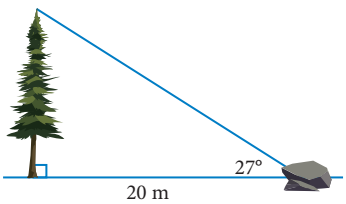
Exam practice

8 The angle of elevation of the top of a tree from a rock on the ground is 27°. The rock is 20 metres from the base of the tree.

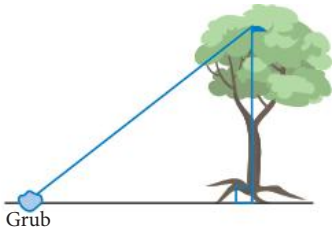
What is the height of the tree?

A 0.03 m **B** 9.83 m

C 39.25 m **D** 10.19 m



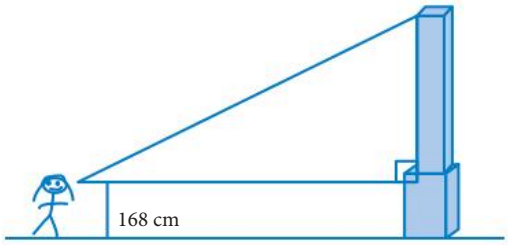
9 A bird in a tree observes a grub on the ground. The bird is 10.5 m above the ground and the distance between the bird and the grub is 15 m. Which expression gives the angle of depression of the grub from the bird?



A $\sin\left(\frac{10.5}{15}\right)$ **B** $\tan\left(\frac{10.5}{15}\right)$

C $\sin^{-1}\left(\frac{10.5}{15}\right)$ **D** $\tan^{-1}\left(\frac{10.5}{15}\right)$

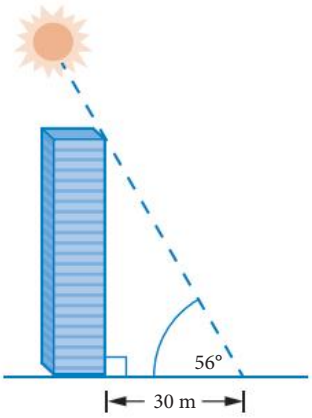
10 (3 marks) Emily is 168 cm tall and is standing 4 m from the base of a statue. She sees the top of the statue at an angle of elevation of 73.4°.



a Copy the diagram and mark on it the given information. *[1 mark]*

b Calculate the height of the statue above the ground. Answer correct to the nearest cm. *[2 marks]*

11 (4 marks) The shadow of a building is 30 m long when the angle of elevation of the sun is 56°.



a Calculate the height of the building. *[2 marks]*

b Determine the length of the shadow when the angle of elevation of the sun becomes 32°. *[2 marks]*

12 (4 marks) CF A lighthouse is standing on top of an 85 m cliff. A man standing at the top of the lighthouse spots a whale out to sea at an angle of depression of 17°. If the whale is 400 m out to sea, calculate the height of the lighthouse, correct to one decimal place.

13 (4 marks) CU A helicopter flying at a height of 1800 m is directly overhead. After 15 minutes, it is still 1800 m above the ground, but its elevation angle is 8°. Find the speed of the helicopter in km/h.

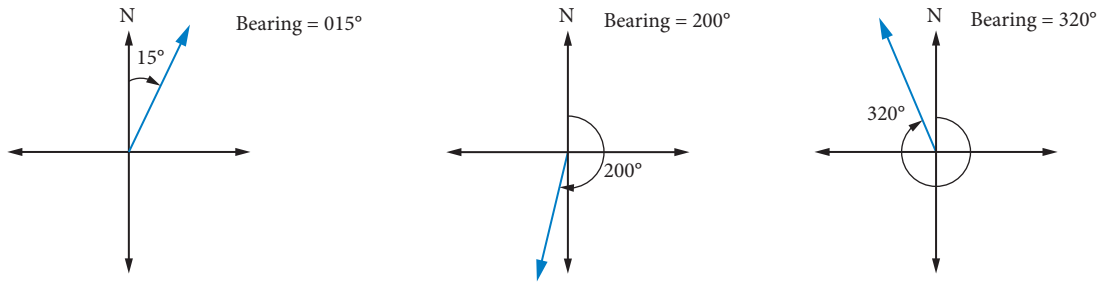
8.5 True bearings

Bearings use angles to show the direction of one location from a given point.

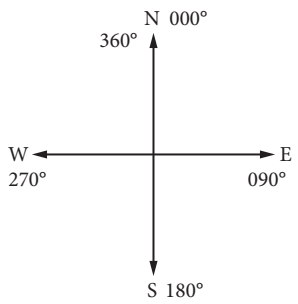
A **true bearing** is the angle measured in a clockwise direction from north.

It is always written as a 3-digit number between 000° and 360°.

For example:



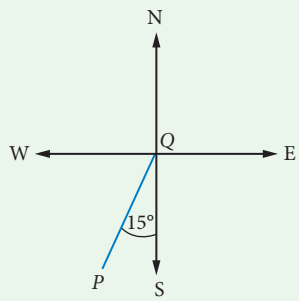
When working out bearings, it is useful to draw a simple **compass rose** and to remember the values shown at each of the 4 points.



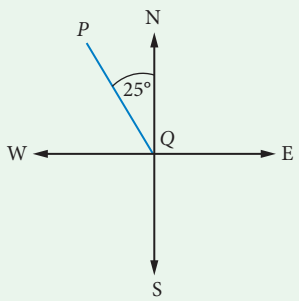
WORKED EXAMPLE 11 True bearings

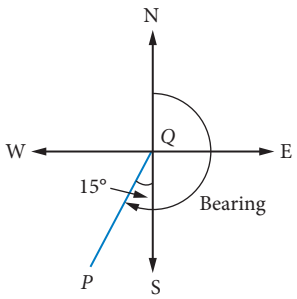
State the true bearing of P from Q.

a



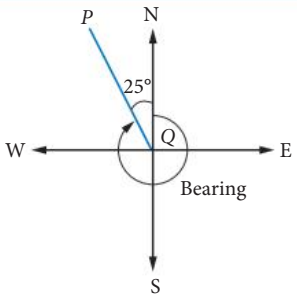
b



Steps	Working
a 1 Draw in the bearing from north in a clockwise direction. The word *from* tells you where the starting point is for your bearing.	
2 From north to south is 180° and add on the extra 15°.	bearing = 180° + 15° = 195°
3 Answer the question.	The true bearing of P from Q is 195°.

Worksheets
Bearings 1
Bearings 2
Identifying bearings
Elevations and bearings
16 points of the compass

b 1 Draw in the bearing from north in a clockwise direction.



2 A full circuit is 360° and then subtract the 25°. bearing = 360 – 25 = 335°

3 Answer the question. The true bearing of *P* from *Q* is 335°.

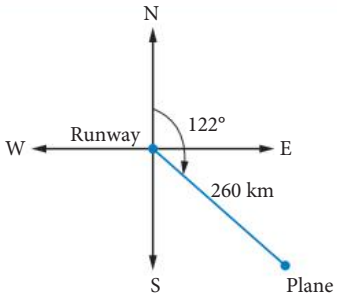
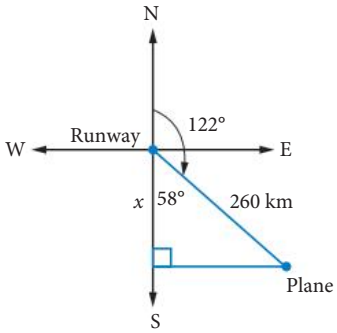
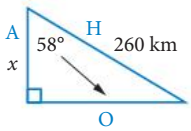
Videos
True bearings
Bearings

WORKED EXAMPLE 12 Bearing application

A plane leaves a runway and remains on a bearing of 122° for 260 km.

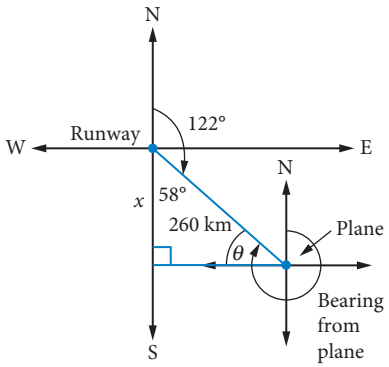
a How far south of the runway is the plane?

b What is the bearing of the runway from the plane?

Steps	Working
a 1 Draw a diagram. Start with the runway and draw in a compass rose. Draw in the position of the plane and mark in the bearing of 122°. The distance from the runway to the plane is 260 km.	
Now, look for the right-angled triangle.	180 – 122 = 58°
Draw the horizontal line in and mark the right angle. Calculate the angle in the triangle. Distance south = x	
2 Label the sides O, A, H.	
3 x and 260 km are A and H, so use cosine.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
4 Substitute in the values.	$\cos 58° = \dfrac{x}{260}$
5 Solve for the unknown.	$x = 260 \times \cos 58°$
x is on top, so MULTIPLY by 260.	$x = 137.779\ldots$
Round to one decimal place.	$x \approx 138$ km
6 Answer the question. Ensure that units are included!	The plane is 138 km south of the runway.

b 1 Start at the plane. Draw a compass rose.

Mark in the bearing from the plane to the runway from north in a clockwise direction.



2 To find the bearing, θ is needed.

Use angle sum of a triangle = 180°.

$\theta = 180° - 90° - 58°$

$\theta = 32°$

3 Bearing is $270 + \theta$.

bearing $= 270° + 32°$

$= 302°$

4 Answer the question.

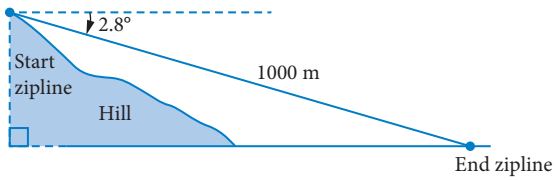
The bearing of the runway from the plane is 302°.

EXERCISE 8.5 True bearings

ANSWERS p. 414

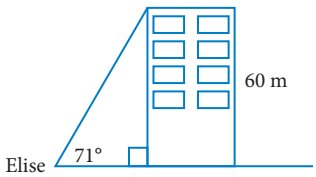
Recap

1 A zipline is 1000 m long. It runs from the top of a hill down to ground level. Calculate the height of the hill if the angle of depression of someone looking from the start of the zipline to the bottom is 2.8°.



A 998.8 m **B** 48.8 m **C** 48.9 m **D** 20 470.9 m

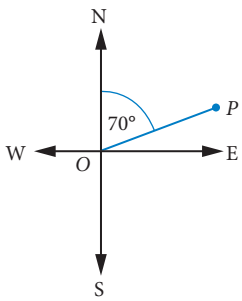
2 Elise was standing on level ground. She measured the angle of elevation from the ground to the top of a 60 m tall building to be 71°. Calculate how far Elise is from the base of the building, correct to the nearest metre.



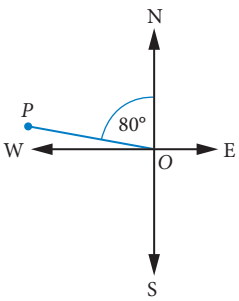
Mastery

3 WORKED EXAMPLE 11 For each diagram, state the true bearing of P from O.

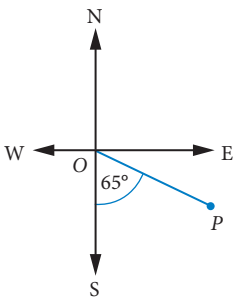
a



b



c



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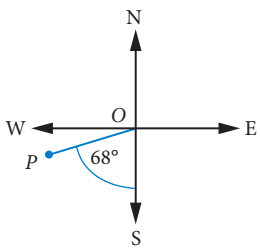
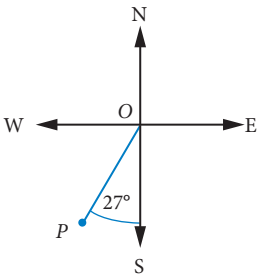
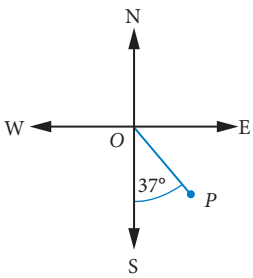


e



f



4 Sketch each of the following on a compass rose.

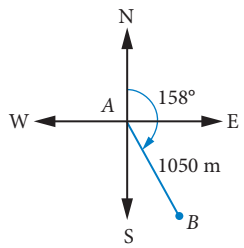
a 120° **b** 080° **c** 205°

d 315° **e** 210° **f** 045°

5 WORKED EXAMPLE 12 A hiker leaves a base camp (A) and remains on a bearing of 158° for 1050 m.

a How far east of the base camp is the hiker?

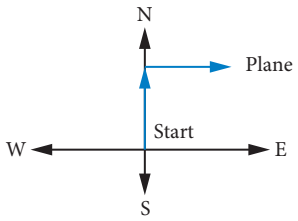
b What is the bearing of the base camp from the hiker?



6 A plane takes off and flies due north. It then turns and flies due east for 180 km. At this point it is on a bearing of 032° from its starting point.

a Copy the diagram and mark on it the given information.

b Calculate how far north the plane flew.
Answer correct to one decimal place.



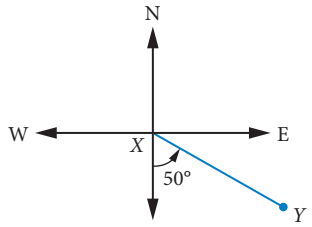
7 A ship travels 150 km on a bearing of 134°. How far south does it travel?

Exam practice

8 What is the bearing of X from Y?

A 050° **B** 130°

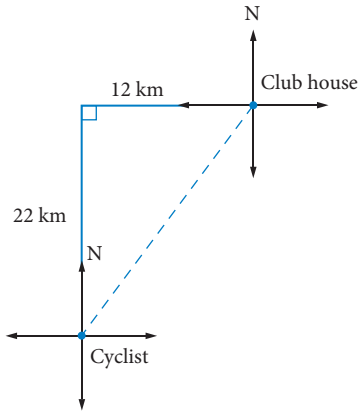
C 140° **D** 310°



9 (2 marks) A cyclist begins from a club house and travels due west for 12 km, then due south for 22 km.

a How far is the cyclist from the club house?
Answer to one decimal place. *[1 mark]*

b What is the bearing of the cyclist from the club house?
Answer to the nearest degree. *[1 mark]*



10 (3 marks) An aircraft is 370 km north of its original position. Its bearing is 342° from the original position. Find the distance actually travelled.

11 (3 marks) CF A charity race consists of 4 legs: from the starting point the race runs 6 km west, then 4 km north, then 2 km east and then return to the starting position.

Find the bearing of the final leg, correct to the nearest degree.

12 (4 marks) CU One town is directly north of another. From a point 20 km to the west of the road joining them, the towns are at bearings of 050° and 120°. How far apart are the towns?

EXAM QUESTION ANALYSIS

Exam-style question (4 marks)

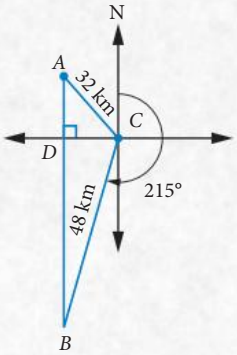
The diagram shows the location of A, B and C.

A is 32 km from C and lies due north of B.

B is on a bearing of 215° and 48 km from C.

a Show that the distance from C to D is 27.5 cm, correct to one decimal place. *[2 marks]*

b Determine the bearing of A from C, to the nearest degree. *[2 marks]*



Reading the question

- Part **a** is a 'show that' question.
  - Accuracy is to one decimal place.
- Part **b** asks for a bearing – a 3-digit number between 000° and 360°.
  - Accuracy is to the nearest degree.

Thinking about the question

Part **a**:

- Look for the right-angled triangle you use to find CD.
- Do you have an angle and a side in this triangle?

Part **b**:

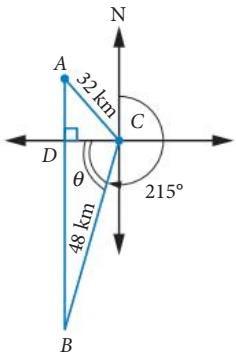
- Bearing of A *from* C means standing at C facing north. How far would you need to turn in a clockwise direction to look at A?
- What will you need to find that bearing?
- What triangle will you use?
- Remember that 'Show that' in part **a** implies you will use this value in your part **b** calculation.

Worked solution (✓ = 1 mark)

a From the bearing, you can find θ.

$\theta = 270° - 215°$

$= \mathbf{55°}$ ✓



This gives us an angle and a side in $\triangle BCD$.

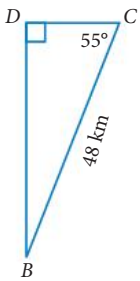
$\cos 55° = \dfrac{CD}{48}$

$CD = 48 \cos 55°$

$CD = 27.531\ldots$

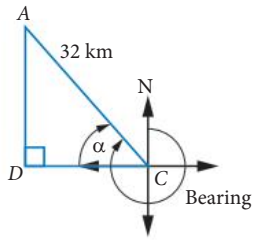
$\boldsymbol{CD \approx 27.5\text{ km}}$ ✓

$\therefore$ the distance from C to D is 27.5 km.



Video Exam question analysis: Right-angled trigonometry

b Bearing of A **from** C



$$\cos\alpha = \frac{CD}{32}$$

$$\cos\alpha = \frac{27.531\ldots}{32}$$

$$\alpha = \cos^{-1}\left(\frac{27.531\ldots}{32}\right)$$

$$\boldsymbol{\alpha = 30.64\ldots} \checkmark$$

$$\alpha \approx 31°$$

∴ bearing of A from C = 270° + 31° = 301° ✓

The bearing of A from C is 301°.

Exam hack

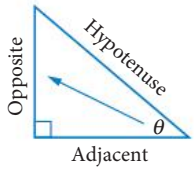
If an exam question has 2 parts and you can't do part **a** but could do part **b** if you knew the answer to part **a**, write x for the answer.
Then show the steps you would take in part **b** using the x in your working, or using the answer to part **a** if it is given, such as 27.5 in the above question.

The marker will award you marks for part **b**.

8 Chapter summary

Trigonometric ratios

Ratio	Initials
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	SOH
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	CAH
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	TOA



- Make sure your calculator is set in **degree mode**.

Finding an unknown side

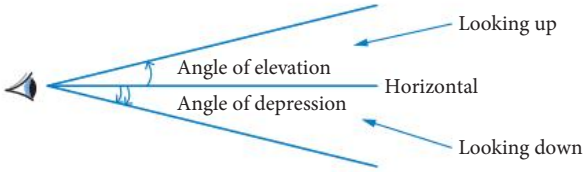
1. Draw a diagram, if not given.
2. Label the sides O, A, H.
3. Choose the ratio using SOH CAH TOA.
4. Substitute in the given values.
5. Solve for the unknown.
6. Answer the question.

If unknown is on top → MULTIPLY
$\cos 54° = \dfrac{y}{10}$ → $y = 10 \times \cos 54°$
if unknown is on bottom → SWITCH
$\cos 54° = \dfrac{10}{y}$ → $y = \dfrac{10}{\cos 54°}$

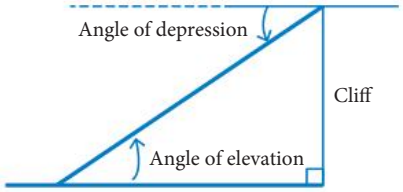
Finding an unknown angle

- If 2 sides of a right-angled triangle are known, you can use a calculator to find the angle.
- If $\sin\theta = a$, then $\theta = \sin^{-1} a$.

Angles of elevation and depression

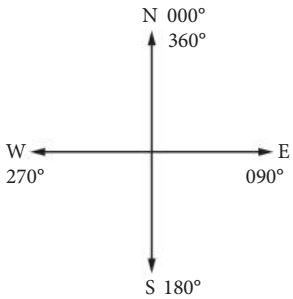


- The **angle of depression = angle of elevation** as they are alternate angles.



Bearings

- A **true bearing** is a 3-digit number between 000° and 360°. It is measured in a clockwise direction from north.
- When working out bearings, it is useful to draw a **compass rose** and to remember the values shown at each of the points.



Puzzle
Trigonometry crossword

Worksheet
General Maths Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

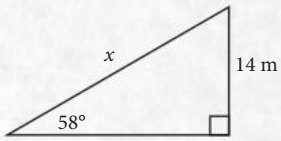
Section 2 (13 marks): 6 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 Which calculation gives the value of x?



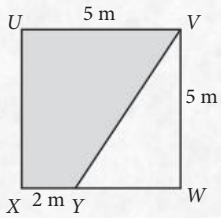
A $14 \cos 58°$ **B** $\dfrac{14}{\sin 58°}$

C $\dfrac{14}{\cos 58°}$ **D** $14 \sin 58°$

2 Omar invested \$7200 at a simple interest rate of 4.3% per annum. The total amount that Omar will have after 3 years is

A \$309.60 **B** \$928.80 **C** \$8128.80 **D** \$92 880

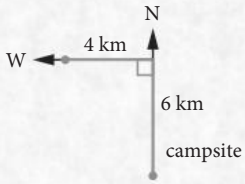
3 $UVWX$ is a square of side length 5 m, as shown. XY is 2 metres. The shaded area $UVYX$, in m^2, is



A 7.5 **B** 15

C 17.5 **D** 18

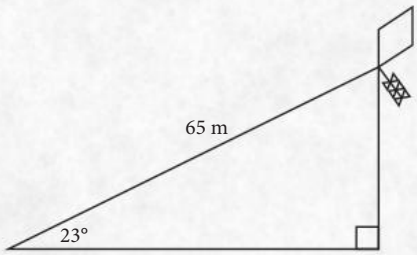
4 Matteo is at his campsite. He walks 6 km north and then turns and walks 4 km west. What is the bearing of his campsite from him?



A 034° **B** 056°

C 124° **D** 146°

5 The angle of elevation to a kite is 23°. The string is 65 m long. Find the height of the kite.



A 25.3 m **B** 25.4 m

C 27.5 m **D** 27.6 m

Section 2 6 short response questions

13 marks

6 (3 marks) Angelina bought 3500 shares in Mirage Limited for \$2.75 per share. Later in the year, dividends of 28.5 cents per share were paid.

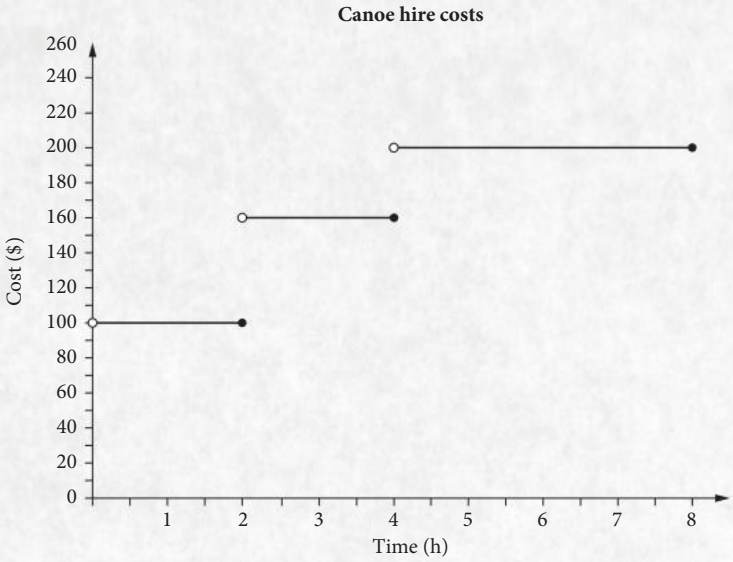
a How much did Angelina receive in dividends? *[1 mark]*

b What was the dividend yield? Answer correct to one decimal place. *[2 marks]*

7 (2 marks) Transpose this formula to make a the subject.

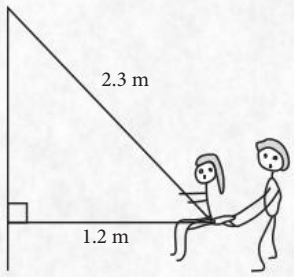
$$c^2 = a^2 + b^2$$

8 (2 marks) The cost of hiring a canoe is shown in this step graph.

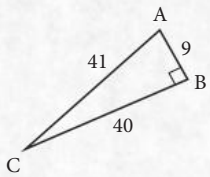


What is the difference in the hourly rate between hiring the canoe for 2 hours and hiring it for 4 hours?

9 (2 marks) Estelle is on a swing that has a chain 2.3 m long. Matthew pulls her back 1.2 m to start swinging. Calculate the angle that the swing makes with the vertical. Answer correct to one decimal place.



10 (2 marks) Find $\cos A$ for this triangle and then the size of angle A to the nearest degree.



11 (2 marks) The mast of a yacht is 5.4 m from the stern. A taut line from the top of the mast to the stern makes an angle of 74.8° with the deck. How high is the mast? Answer correct to 2 decimal places.

iStock.com/olgalngs

Worksheet
General Maths Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

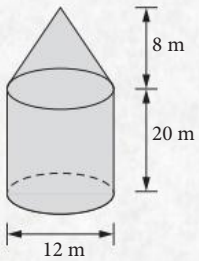
Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

4 short response questions

12 marks

1 (2 marks) As part of a town beautification, a mural is to be painted on a silo.

The silo is a cylinder with a cone on top. The dimensions are shown.



Calculate the area of the surface to be painted, correct to one decimal place.

2 (2 marks) The pilot of a plane saw an island at an angle of depression of 12°.
Her altimeter showed that the plane was at a height of 3500 m.

Calculate the horizontal distance to the island, to the nearest metre.

3 (4 marks) Chase saw a barn directly across the river from where he was standing. He walked in a straight line along the riverbank for 50 metres. He measured the angle from the riverbank to the barn to be 23°.

Chase then measured the angle of elevation to the top of the barn to be 8°.
Determine the height of the barn, correct to one decimal place.

4 (4 marks) Zoe, Skye and Amanda went to a theme park on the same day with their families.

Zoe's family paid $290. Zoe was with her mother and sister.

Skye's family were in a club that received a discount of 20% on their tickets, and they paid $396.
Skye's parents both came with their 3 children.

Amanda's mother came with her 5 children and 2 of her adult friends.

Children's tickets were bought for all the children and adult tickets for all the adults.

How much did Amanda's family and friends pay altogether for park admission?

Carefully justify your answer.

9780170484749

CHAPTER

9 MATRICES

Syllabus coverage
Nelson MindTap chapter resources
Terminology
9.1 Matrices
9.2 Types of matrices
9.3 Operations with matrices
9.4 Matrix multiplication
9.5 Powers of matrices
9.6 Applications of matrices
Exam question analysis
Chapter summary
Cumulative examination 1
Cumulative examination 2

Prior learning
Matrices

Syllabus coverage

UNIT 2, TOPIC 3: MATRICES

Matrices and matrix arithmetic

- Use matrices for storing and displaying information that can be presented in rows and columns, e.g. tables, databases, links in social or road networks.
- Recognise different types of matrices, including row matrix, column matrix, square matrix, zero matrix and identity matrix, and determine the size of the matrix.
- Perform matrix addition, subtraction and multiplication by a scalar.
- Perform matrix multiplication manually up to 3 × 3 matrices but not limited to square matrices.
- Determine the power of a matrix using technology with matrix arithmetic capabilities when appropriate.
- Use matrices, including matrix products and powers of matrices, to model and solve problems, e.g. costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person.

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Videos (3):

9.4 Matrix multiplication • Multiplying matrices

Exam question analysis Matrices

Prior learning (1):

9.1 Matrices

Worksheets (3):

9.3 Addition and subtraction of matrices

9.4 Matrix multiplication

Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

adjacency matrix	column matrix	communications network	defined
element	identity matrix	matrix/matrices	matrix product
order	power of a matrix	row matrix	scalar multiple
scalar multiplication	size	square matrix	zero matrix

9.1 Matrices

When analysing cost and pricing, organising communication between people and determining choices, a **matrix** may be used to display the data.

A matrix (plural **matrices**) is a table of numbers displayed in rows and columns, surrounded by brackets [ ]. A matrix is labelled by a capital letter, such as A.

Matrices

The **size** (or **dimensions**) **of a matrix** is written as the *number of rows* × *number of columns*.

$$A = \begin{array}{c} \begin{array}{cc} \text{Column 1} & \text{Column 2} \end{array} \\ \begin{bmatrix} -7 & 2 \\ 0 & 5 \\ -1 & 10 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \end{array}$$

For example, in matrix A above, there are 3 rows and 2 columns. The size of A is written as 3×2.

Each number in a matrix is called an **element**. Elements are identified by a lower case letter with the row and column as a subscript; for example, for matrix A above, $a_{31} = -1$ and $a_{12} = 2$.

The size of an $n \times n$ square matrix is just written as order n. A 3×3 matrix has order 3.

WORKED EXAMPLE 1 Characteristics of matrices

This table shows the results of the Group B qualifying rounds for the FIFA World Cup Football.

'Home' team	'Away' team	Result
Spain FIFA	Netherlands	1–5
Chile FIFA	Australia	3–1
Australia FIFA	Netherlands	2–3
Spain	Chile	0–2
Australia	Spain	0–3
Netherlands FIFA	Chile	2–0

a Show the Group B results as a matrix R in the order Australia, Chile, Netherlands, Spain.

b In your matrix from part **a**, what do the 0s in the diagonal show?

c In your matrix from part **a**, what do r_{13} and r_{31} show?

Steps	Working
a It is usual to put the goals scored in the rows with the opponents in the columns. Row 1 shows that Australia scored 1 goal against Chile, 2 against the Netherlands and 0 against Spain.	$R = \begin{array}{c} \\ \text{A} \\ \text{C} \\ \text{N} \\ \text{S} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{C} & \text{N} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 0 \\ 3 & 2 & 0 & 5 \\ 3 & 0 & 1 & 0 \end{bmatrix} \end{array}$
b The diagonals have the same country in the row and the column.	The 0s in the diagonal show that countries did not play themselves.
c The positions are for Australia vs Netherlands and Netherlands vs Australia.	$r_{13} = 2$ shows Australia scored 2 goals and $r_{31} = 3$ shows the Netherlands scored 3 goals in the match, so the Netherlands won.

WORKED EXAMPLE 2 Identifying elements

$$G = \begin{bmatrix} 2 & 3 & 11 & -3 & 9 \\ 5 & 0 & -4 & 8 & -1 \\ 7 & 1 & 6 & -5 & 4 \end{bmatrix}$$

a What is g_{23}? **b** What is g_{35}? **c** What is g_{42}?

Steps	Working
a It is in the 2nd row and 3rd column.	$g_{23} = -4$
b It is in the 3rd row and 5th column.	$g_{35} = 4$
c It is in the 4th row and 2nd column.	There is no 4th row so g_{42} does not exist.

EXERCISE 9.1 Matrices

ANSWERS p. 415

Mastery

1 State whether each is a matrix.

a $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & & \\ & 1 & \\ & & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

d $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & -2 \\ & 1 & 3 \end{bmatrix}$ **e** $\begin{bmatrix} \sqrt{3} & \frac{5}{6} \\ -2 & 7 \end{bmatrix}$ **f** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

2 WORKED EXAMPLE 1 $A = \begin{bmatrix} 1 & 4 & 3 & -2 \\ 9 & 0 & 6 & -1 \\ 7 & 2 & 8 & 5 \end{bmatrix}$

a What is the size of A?

b If possible, write the value of

i a_{12} **ii** a_{21} **iii** a_{34}

iv a_{22} **v** a_{32} **vi** a_{43}

3 WORKED EXAMPLE 2 A manufacturer builds desktop computers with a variety of central processor speeds and hard drive capacities. The result of a stock inventory is shown in the table.

	Processor (GHz)								
Hard drive (GB)	1.86	2.0	3.8	2.33	2.67	2.8	3.0	3.2	3.5
160	3	2	4	1	5	0	3	6	1
250	1	8	1	7	1	8	0	5	8
320	4	0	5	6	4	5	9	1	7
500	7	9	2	0	1	1	10	9	1

a Write the information in the table as a 4×9 matrix M.

b Write the value of

i m_{32} **ii** m_{21} **iii** m_{27}

Exam hack

Ensure you put square brackets around your matrix. Do not use gridlines.

4 This table shows the prices of timber from 4 different hardware suppliers.
All prices listed are per metre or per square metre (sheet).

Product	Jones & Co.	Brown Bros	Pine Hut	Allgoods
Hardwood framing	\$4.20	\$4.30	Cannot supply	\$4.40
Chipboard flooring	\$9.10	\$8.80	\$8.70	\$9.00
Structural plywood	\$6.20	\$6.40	\$6.50	\$6.10
Pine chamfer	\$3.80	\$3.90	\$3.60	\$3.70

a This table cannot be changed to a matrix. Why not?

b How can the table be changed so that it can be converted to a 4×4 matrix?

c Write the matrix T.

d What is the value of t_{24}?

5 A manufacturer of marbelite bathroomware uses the following quantities of materials (in kilograms) to make hand basins, shower bases, baths and spas.

Material	Hand basin	Shower base	Bath	Spa
Plaster filler	4	9	20	40
Epoxy resin	1	2	4	9
Catalyst	0.2	0.3	0.5	1

a Write the information as a matrix B with the materials in rows.

b What is the value of b_{23}?

c What is the value of b_{32}?

d What does the value of b_{22} represent?

6 An ecologist studied the diets of water beetles, mosquito larvae, fingerlings and adult fish in a freshwater pool. She found that mosquito larvae and fingerlings both ate microscopic algae and microscopic animals in the water, but that fingerlings also ate approximately 3 mosquito larvae each day. Water beetles ate about 5 mosquito larvae and 2 fingerlings each day, and adult fish ate about 6 fingerlings and 4 water beetles each day.

a Write the information as a 4×4 matrix P, showing the predation of species on each other.
Write rows and columns in the order: mosquito larvae, fingerlings, water beetles and adult fish.

b What is the significance of the row of 0s for mosquito larvae?

c Why does p_{33} have that value?

Exam practice

7 The table shows the number of Year 11 students enrolled in selected subjects over 4 years.

	2020	2021	2022	2023
Geography	50	52	61	68
Art	60	49	49	45
Legal Studies	38	40	35	36

Which matrix will best display this information?

A $\begin{bmatrix} 50 \\ 60 \\ 38 \end{bmatrix}$ **B** $\begin{bmatrix} 148 & 141 & 145 & 149 \end{bmatrix}$ **C** $\begin{bmatrix} 247 \\ 203 \\ 149 \end{bmatrix}$ **D** $\begin{bmatrix} 50 & 52 & 61 & 68 \\ 60 & 49 & 49 & 45 \\ 38 & 40 & 35 & 36 \end{bmatrix}$

8 (3 marks)

$$A = \begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} \qquad B = \begin{bmatrix} 12 & 7 & 0 \\ 1 & 1 & -5 \\ 2 & 10 & 9 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 6 & -5 \\ 2 & 0 & 0 \end{bmatrix}$$

a State the size of the matrix C. *[1 mark]*

b Identify the element *[2 marks]*

i b_{12}

ii a_{21}

9 (5 marks) On a particular trading day, the exchange rates for Australian dollars (AUD), United States dollars (USD), Great Britain pounds (GBP) and Japanese yen (JPY) were as follows:

1 AUD = 0.7324 USD,	0.5874 GBP,	86.24 JPY
1 USD = 1.3654 AUD,	0.8020 GBP,	117.75 JPY
1 GBP = 1.7024 AUD,	1.2469 USD,	146.82 JPY
1 JPY = 0.011 59 AUD,	0.008 93 USD,	0.006 811 GBP

a What is the equivalent of 1 AUD in USD? *[1 mark]*

b Put the information in a 4×4 matrix E in the order AUD, USD, GPB and JPY. *[2 marks]*

c What is the value of e_{23}? *[1 mark]*

d What is the value of e_{32}? *[1 mark]*

9.2 Types of matrices

Types of matrices

A **row matrix** has only one row.

$$\begin{bmatrix} 2 & 4 & 5 & 0 \end{bmatrix}$$

A **column matrix** has only one column.

$$\begin{bmatrix} 5 \\ -2 \\ 0.4 \end{bmatrix}$$

A **square matrix** has the same number of rows and columns.

$$\begin{bmatrix} 1 & -4 \\ 6 & 2 \end{bmatrix}$$

The size of this matrix is 2×2.

This matrix has order 2.

A **zero matrix** has all elements equal to 0.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

An **identity matrix** I is a square matrix with every element 0 except for the diagonal values, which are all 1s.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

WORKED EXAMPLE 3 Matrix type and size

Describe the type and size of each matrix.

a $\begin{bmatrix} -2 & 0.5 & 0 \\ 3 & 1 & 0 \\ 0 & 8 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Steps	Working
a The matrix has 3 rows and 3 columns.	It is a square matrix of order 3.
b Every element is zero.	It is a 2×3 zero matrix.
c It has 1s in the diagonal and 0s everywhere else.	It is the identity matrix of order 4.

EXERCISE 9.2 Types of matrices

ANSWERS p. 415

Recap

1 Name the size of matrix Y.

$$Y = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

2 State the element p_{13}.

$$P = \begin{bmatrix} 0 & -9 & -2 \\ 0 & 1 & 3 \\ 3 & -2 & 4 \end{bmatrix}$$

Mastery

3 WORKED EXAMPLE 3 Describe the type and size of each matrix.

a $\begin{bmatrix} 3 & 4 & 0 & 2 & 5 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ 3 \\ 0 \\ -4 \end{bmatrix}$

d $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **e** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

g $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **i** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

4 Construct the identity matrix I_5.

5 Construct a 5×2 zero matrix.

6 Construct a 2×7 zero matrix.

7 Use your own numbers to construct a square matrix of order 3.

8 The prices of Gold Class tickets at the cinema changes during the day. The table shows the prices by session.

Price of Gold Class tickets ($)

	9 am – 5 pm	5 pm – 11 pm
Adult	35	45
Concession	30	40

a Display this information in a matrix.

b State the order of the matrix.

c Describe which type of matrix this is.

Exam practice

9 (4 marks) State if each statement is true or false.

a A zero matrix must have a size such that the number of rows and columns are the same. *[1 mark]*

b A square matrix is made up of only zeros. *[1 mark]*

c If a matrix has size 1×3, it is a row matrix. *[1 mark]*

d An identity matrix must start with the number 1 in the first element. *[1 mark]*

10 Identify the identity matrix.

A $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **B** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **D** $\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$

11 (3 marks) The table displays the number of people who use different types of transport in the CBD on a weekend.

Number of people by transport type

Type of Transport	Saturday	Sunday
Bus	8950	3890
Train	6500	3950
Car	25 700	34 500
Bike	1590	2340
Scooter	850	720

a Transfer this information into a matrix, T. *[1 mark]*

b Create a row matrix to determine the method of transport used the most over the weekend. *[2 marks]*

9.3 Operations with matrices

Matrix operations

- Matrices must be the **same size** to be added or subtracted.
- To **add matrices**, add the corresponding elements.
- To **subtract matrices**, subtract the corresponding elements.
- To multiply a matrix by a number (called a **scalar multiple**), multiply every element by the number. This is called **scalar multiplication**.

$$A = \begin{bmatrix} 3 & 1 \\ 3 & 0 \\ 5 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -3 \\ 4 & 4 \\ 5 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 1 \\ 3 & 0 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 4 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 3+0 & 1+(-3) \\ 3+4 & 0+4 \\ 5+5 & -2+2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 7 & 4 \\ 10 & 0 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3 & 1 \\ 3 & 0 \\ 5 & -2 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 4 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 3-0 & 1-(-3) \\ 3-4 & 0-4 \\ 5-5 & -2-2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & -4 \\ 0 & -4 \end{bmatrix}$$

Worksheet Addition and subtraction of matrices

WORKED EXAMPLE 4 Adding and subtracting matrices

Given $A = \begin{bmatrix} -2 & 3 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & -2 \\ 1 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$,

find

a $A + C$ **b** $B - C$

Steps	Working
a Add corresponding elements.	$A + C = \begin{bmatrix} -2 & 3 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$ $= \begin{bmatrix} 4 & -1 \\ 1 & -2 \\ 6 & 2 \end{bmatrix}$
b 1 Subtract corresponding elements.	$B - C = \begin{bmatrix} 2 & 3 \\ 0 & -2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$
2 State the result.	$= \begin{bmatrix} -4 & 7 \\ -2 & 1 \\ -2 & -5 \end{bmatrix}$

WORKED EXAMPLE 5 Scalar multiplication

Given $D = \begin{bmatrix} 3 & -3 & 2 & -3 \\ -2 & 5 & -5 & -4 \end{bmatrix}$ and $E = \begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix}$, find

a $-3E$ **b** $2E - 4D$

Steps	Working
a Multiply every element by −3.	$-3E = -3\begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix}$ $= \begin{bmatrix} -3 & 18 & -12 & -18 \\ 6 & -15 & -12 & 24 \end{bmatrix}$
b 1 Display the operation.	$2E - 4D = 2\begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix} - 4\begin{bmatrix} 3 & -3 & 2 & -3 \\ -2 & 5 & -5 & -4 \end{bmatrix}$
2 Do the scalar multiples and state the result.	$= \begin{bmatrix} 2 & -12 & 8 & 12 \\ -4 & 10 & 8 & -16 \end{bmatrix} + \begin{bmatrix} -12 & 12 & -8 & 12 \\ 8 & -20 & 20 & 16 \end{bmatrix}$ $= \begin{bmatrix} -10 & 0 & 0 & 24 \\ 4 & -10 & 28 & 0 \end{bmatrix}$

Exam hack

Changing a subtraction to the addition of the opposite can help you avoid errors.

Exam hack

Always check the sizes of matrices before you begin an operations question. It will save you time.

EXERCISE 9.3 Operations with matrices

ANSWERS p. 415

Recap

1 The matrix $\begin{bmatrix} 18 & 27 & 24 \end{bmatrix}$ is an example of a

A row matrix **B** square matrix **C** column matrix **D** identity matrix

2 Identify the elements from the matrix $Y = \begin{bmatrix} 120 & 205 & 175 \\ 110 & 200 & 180 \end{bmatrix}$.

a y_{22} **b** y_{13} **c** y_{11}

Mastery

3 WORKED EXAMPLES 4, 5 Given $A = \begin{bmatrix} 2 & 0 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 9 & -3 \end{bmatrix}$, $C = \begin{bmatrix} -6 & 4 \\ 8 & -10 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & -7 \\ -3 & 1 \end{bmatrix}$, find

a $A + B$ **b** $C + B$ **c** $B + D$ **d** $C + D$

e $-B$ **f** $C - A$ **g** $B - D$ **h** $C - B$

i $A + A + A$ **j** $3A$ **k** $5B$ **l** $4C$

m $\frac{1}{2}C$ **n** $\frac{1}{3}B$ **o** $0.4B$ **p** $5A - B$

4 Given $A = \begin{bmatrix} 4 & 3 & 1 \\ 0 & 3 & 6 \\ 5 & 0 & -1 \\ 0 & 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -4 & -3 \\ 2 & -3 & 1 \\ 4 & 2 & 4 \\ -4 & -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 2 & 4 \\ 0 & 6 & 2 \\ 7 & 0 & 0 \end{bmatrix}$,

find

a $A + B$ **b** $A + C$ **c** $C + B$ **d** $B + A$

e $-B$ **f** $C - A$ **g** $A - C$ **h** $B - C$

i $B - A$ **j** $A - B$ **k** $4A$ **l** $3B$

m $-2C$ **n** $0.5A$ **o** $-\frac{3}{4}B$ **p** $4C - A$

5 Given $X = \begin{bmatrix} 3 & 0 & -1 & 2 \\ 5 & -4 & 3 & 1 \\ -1 & 2 & 0 & 4 \end{bmatrix}$, $Y = \begin{bmatrix} 1 & 4 & 3 & -5 \\ -3 & 2 & -1 & -6 \\ 0 & 5 & -2 & 3 \end{bmatrix}$, and $Z = \begin{bmatrix} 6 & -5 & 4 & -3 \\ -1 & 8 & -9 & 2 \\ 4 & 7 & -2 & 5 \end{bmatrix}$,

find

a $X + Y$ **b** $Z + Y$ **c** $Y + Y$ **d** $2Y$

e $-Z$ **f** $-Y + X$ **g** $X - Y$ **h** $Z - Y$

i $5X$ **j** $-3Z$ **k** $4Y$ **l** $-5X$

m $\frac{1}{2}Z$ **n** $0.1Y$ **o** $\frac{1}{3}Z$ **p** $4X - 3Z$

6 Find x in each equation.

a $\begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} + \begin{bmatrix} 3 & x \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -1 & x \end{bmatrix}$

b $\begin{bmatrix} 4 & 3 & 4 \\ -2 & 0 & x \end{bmatrix} + \begin{bmatrix} -1 & x^2 & -4 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

c $\begin{bmatrix} x^2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -x^3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -8 \end{bmatrix}$

7 Find A if $\begin{bmatrix} -1 & 2 \\ -4 & 7 \end{bmatrix} + A = \begin{bmatrix} 2 & 4 \\ -1 & 7 \end{bmatrix}$.

8 Find M if $M + \begin{bmatrix} 5 & -2 \\ 10 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}$.

Exam practice

9 The result of $2 \times \begin{bmatrix} 5 & 2 \\ 3 & -3 \\ 0 & 1 \end{bmatrix}$ is

A $\begin{bmatrix} 10 & 2 \\ 6 & -6 \\ 0 & 2 \end{bmatrix}$ **B** $\begin{bmatrix} 10 & 4 \\ 6 & -6 \\ 0 & 2 \end{bmatrix}$ **C** $\begin{bmatrix} 10 & 4 \\ 6 & 6 \\ 0 & 2 \end{bmatrix}$ **D** $\begin{bmatrix} 10 & 4 \\ 6 & -6 \\ 2 & 0 \end{bmatrix}$

10 (2 marks) CF

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 9 & 0 \\ 1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 5 \\ 7 & -1 \\ -2 & 8 \end{bmatrix}$ $D = \begin{bmatrix} 0 & -1 \\ 4 & 4 \\ 2 & 10 \end{bmatrix}$

Calculate

a $3B - 2A + B$ *[1 mark]*

b $3C + 2D - C$ *[1 mark]*

11 (5 marks) A bank has 3 branches in Cairns: City, Earlville and Stratford.
There are 4 types of accounts: Low fee, Business, Savings and Student/Pensioner.
The numbers of each type of account at the beginning of a year are shown in the table.

Branch	Low fee	Business	Savings	Student/Pensioner
City	300	400	200	150
Earlville	400	220	300	200
Stratford	250	150	400	150

The numbers of closures of accounts in that year were:

Branch	Low fee	Business	Savings	Student/Pensioner
City	30	70	20	20
Earlville	20	30	40	30
Stratford	40	10	50	10

The numbers of new accounts opened in that year were:

Branch	Low fee	Business	Savings	Student/Pensioner
City	50	100	10	15
Earlville	15	50	30	10
Stratford	30	25	20	40

Use a matrix to show

a the number of accounts at the beginning of the year. *[1 mark]*

b the number of closures in the year. *[1 mark]*

c the number of accounts opened in the year. *[1 mark]*

d an evaluated matrix expression for the number of accounts at the end of the year. *[2 marks]*

12 (6 marks) CU The wholesale prices (in \$) of some caulking compounds are shown by the below matrix. The first column represents the prices of 100 mL tubes, the second column shows the prices of 250 mL tubes and the third column shows the prices of 400 mL mastic gun cartridges.

$$W = \begin{array}{r} \\ \text{Bathroom sealant} \\ \text{Gutter \& downpipe sealant} \\ \text{Masonry sealant} \\ \text{Flexible gap filler} \end{array} \begin{array}{c} \begin{array}{ccc} 100\text{ ml} & 250\text{ ml} & 400\text{ ml} \end{array} \\ \begin{bmatrix} 2.40 & 4.90 & 8.40 \\ 4.20 & 7.90 & 11.80 \\ 3.80 & 8.40 & 11.00 \\ 5.90 & 11.80 & 14.40 \end{bmatrix} \end{array}$$

The retailer applies a mark-up of 70% and must also charge 10% GST to the customer.
Calculate the final prices for the customer.

9.4 Matrix multiplication

You now know how to multiply a matrix by a scalar (number).

Now, you will learn to multiply a matrix by another matrix.

Calculate the total sales for each store from these matrices.

Units sold

$$U = \begin{array}{c} \\ \text{Shop A} \\ \text{Shop B} \\ \text{Shop C} \end{array} \begin{array}{c} \begin{array}{cc} \text{Magnums} & \text{Calippos} \end{array} \\ \begin{bmatrix} 25 & 49 \\ 36 & 51 \\ 28 & 60 \end{bmatrix} \end{array}$$

Price per unit

$$P = \begin{array}{c} \text{Magnums} \\ \text{Calippos} \end{array} \begin{bmatrix} \$4.80 \\ \$3.20 \end{bmatrix}$$

To get the total for each store, multiply the price of each ice-cream by the number sold.

$$\begin{array}{c} \text{Shop A} \\ \text{Shop B} \\ \text{Shop C} \end{array} \begin{bmatrix} 25 \times 4.8 + 49 \times 3.2 \\ 36 \times 4.8 + 51 \times 3.2 \\ 28 \times 4.8 + 60 \times 3.2 \end{bmatrix} \text{ This can be written as } U \times P = \begin{bmatrix} 276.8 \\ 336 \\ 326.4 \end{bmatrix}.$$

When multiplying 2 matrices, the *number of columns* in the *first* matrix must be the same as the *number of rows* in the *second* matrix.

Video Matrix multiplication

Worksheet Matrix multiplication

Multiplying matrices

- For matrices to be multiplied, the number of columns in the first matrix must be the same as the number of rows in the second.
- The **matrix product** AB is defined if A has m columns and B has m rows so they can be multiplied together.
- Multiply the elements in each row of the first matrix by the elements in each column of the second matrix, then add the answers.
- Write the result in the same row and column of the answer matrix; e.g., the result of multiplying row 2 by column 3 goes into row 2, column 3 of the answer matrix.
- The product of an $n \times m$ matrix A and an $m \times p$ matrix B is an $n \times p$ matrix C.

WORKED EXAMPLE 6 Defining matrices

Determine if the matrices can be multiplied and state the final product size.

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$$

a AB **b** BA

Steps	Working
a 1 State sizes of each matrix.	$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix}$ Size $= 3 \times 2$
	$B = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$ Size $= 2 \times 4$
2 Work out the size of the answer.	$A \times B$
Check that the number of columns in A is the same as the number of rows in B.	$3 \times \boxed{2 \quad 2} \times 4$
	AB is defined. The size will be 3×4.

b 1 State the size of each matrix.	$B \times A$ $2 \times \boxed{4 \quad 3} \times 2$
2 Write your conclusion.	The number of columns in the first is different from the number of rows in the second. The matrices cannot be multiplied. BA is not defined.

Exam hack

Note the product size. This is handy to check your answer is correct.

Video
Multiplying matrices

WORKED EXAMPLE 7 Multiplying matrices

$$C = \begin{bmatrix} 2 & 2 \\ -1 & 2 \\ 0 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$$

Find CD.

Steps	Working
1 State the size of each matrix.	$C = \begin{bmatrix} 2 & 2 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$ Size $= 3 \times 2$ $D = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$ Size $= 2 \times 2$
2 Define the matrix product.	$C \times D$ $3 \times \boxed{2 \quad 2} \times 2$ CD is defined. Product size will be 3×2.
3 Multiply the matrices.	$\begin{bmatrix} 2 & 2 \\ -1 & 2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$ $= \begin{bmatrix} 2\times1 + 2\times4, & 2\times3 + 2\times1 \\ -1\times1 + 2\times4, & -1\times3 + 2\times1 \\ 0\times1 + 5\times4, & 0\times3 + 5\times1 \end{bmatrix}$ $= \begin{bmatrix} 10 & 8 \\ 7 & -1 \\ 20 & 5 \end{bmatrix}$

Multiplying with an identity matrix

When a matrix is multiplied by the identity matrix of an appropriate size, the result will be the same as the original matrix.
For example,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -10 & 0 \\ 5 & 3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 5 & 3 \\ -6 & 4 \end{bmatrix}$$

EXERCISE 9.4 Matrix multiplication

ANSWERS p. 417

Recap

Use these matrices to answer Questions 1 and 2.

$A = \begin{bmatrix} 3 & 8 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 4 & 0 \\ -3 & 5 & 1 \end{bmatrix}$

1 Describe the type and size of

a matrix C **b** matrix A

2 Find $2B + D$.

Mastery

3 WORKED EXAMPLE 6

$A = \begin{bmatrix} 9 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 & 0 \\ 4 & -1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 3 & 2 \end{bmatrix}$

Using the listed matrices, state whether each product is defined (state yes or no).

a AB **b** BC **c** CD

d CB **e** AC **f** CA

4 WORKED EXAMPLE 7 Calculate each product.

a $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$

d $\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 5 & 1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix}$ **f** $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$

g $\begin{bmatrix} 3 & 4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$ **h** $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ **i** $\begin{bmatrix} 5 & -1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$

j $\begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 0 & 1 \end{bmatrix}$ **k** $\begin{bmatrix} \frac{1}{3} & 1 \\ 1 & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 3 \end{bmatrix}$ **l** $\begin{bmatrix} 6 & 12 \\ 6 & 18 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{6} \end{bmatrix}$

5 Calculate $\begin{bmatrix} 3 & 6 \\ 10 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 10 & 5 \end{bmatrix}$.

What is unusual about these products?

6 Calculate $\begin{bmatrix} 3 & 1 & 3 \\ 1 & 11 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 & -2 \\ 1 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 & -2 \\ 1 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 3 \\ 1 & 11 & 1 \\ 3 & 1 & 3 \end{bmatrix}$.

What is unusual about these products?

7 Calculate $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Is this what you expected?

8 Explain why the product of $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ is not possible.

9 Multiply $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by $\begin{bmatrix} 7 & 1 \\ 6 & 6 \end{bmatrix}$. What do you find?

10 Multiply $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by $\begin{bmatrix} 0 & 2 & 5 & 1 \\ -3 & 1 & 2 & 1 \end{bmatrix}$. What do you find?

Exam practice

Use these matrices to answer Questions 11 and 12.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 6 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ $\quad C = \begin{bmatrix} 10 & 6 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$

11 (4 marks) Determine if each operation can occur.

a AD *[1 mark]*

b DB *[1 mark]*

c DA *[1 mark]*

d $BC - A$ *[1 mark]*

12 (5 marks) Find

a BC *[2 marks]*

b $CA + C$ *[3 marks]*

13 (5 marks) A kitchenware chain has 5 stores ordering stock from a central warehouse. The orders of cutlery sets for a catalogue special are as follows.

Setting	Brisbane	Logan	Chermside	Cairns	Gold Coast
6 place	12	15	10	12	16
8 place	10	8	10	12	6
10 place	8	4	10	4	12

a Write the stock orders as a 5×3 matrix. *[1 mark]*

b The prices of the sets are \$117.80, \$155 and \$190 respectively.
Write the prices as a column matrix. *[1 mark]*

c Multiply the matrices to obtain the invoice amounts for each store. *[3 marks]*

9.5 Powers of matrices

Powers of matrices

For the square matrix A, A^n, where n is a positive integer, is called the **power of a matrix**. Only square matrices can have powers.

$A^2 = A \times A$ and $A^3 = A \times A \times A$ are called the square and cube of A respectively.

WORKED EXAMPLE 8 Powers of a matrix

Given $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$, find

a A^2 **b** A^3

Steps	Working
a 1 Write as a multiplication.	$A^2 = A \times A$
2 Substitute A.	$= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$
3 Perform the multiplication.	$= \begin{bmatrix} 2 \times 2 + -1 \times 3, & 2 \times -1 + -1 \times 5 \\ 3 \times 2 + 5 \times 3, & 3 \times -1 + 5 \times 5 \end{bmatrix}$
	$= \begin{bmatrix} 1 & -7 \\ 21 & 22 \end{bmatrix}$
b 1 Write as a multiplication.	$A^3 = A \times A \times A$
2 Substitute A and $A \times A$. (use the solution from A^2)	$= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -7 \\ 21 & 22 \end{bmatrix}$
3 Perform the multiplication.	$= \begin{bmatrix} 2 \times 1 + -1 \times 21, & 2 \times -7 + -1 \times 22 \\ 3 \times 1 + 5 \times 21, & 3 \times -7 + 5 \times 22 \end{bmatrix}$
	$= \begin{bmatrix} -19 & -36 \\ 108 & 89 \end{bmatrix}$

Exam hack

Make sure you give yourself plenty of horizontal space on the paper for working out the answer.

WORKED EXAMPLE 9 Square of a 3 × 3 matrix

For $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$, find A^2.

Steps	Working
1 Write as a multiplication.	$A^2 = A \times A$
2 Substitute A.	$= \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$
3 Perform the multiplication.	$= \begin{bmatrix} 1\times1+-2\times0+0\times2, & 1\times-2+-2\times1+0\times0, & 1\times0+-2\times3+0\times1 \\ 0\times1+1\times0+3\times2, & 0\times-2+1\times1+3\times0, & 0\times0+1\times3+3\times1 \\ 2\times1+0\times0+1\times2, & 2\times-2+0\times1+1\times0, & 2\times0+0\times3+1\times1 \end{bmatrix}$
	$= \begin{bmatrix} 1 & -4 & -6 \\ 6 & 1 & 6 \\ 4 & -4 & 1 \end{bmatrix}$

Investigation Powers that give zero matrices

The matrix $\begin{bmatrix} -4 & 2 \\ -8 & 4 \end{bmatrix}$ to any power will always give the 2 × 2 zero matrix.

1. Is it possible to have a power of a non-zero matrix that gives the 3 × 3 zero matrix?
2. Can you find at least one example?
3. What about 4 × 4 matrices?
4. Why is it not possible to have a power of a matrix that gives the 2 × 3 zero matrix?

EXERCISE 9.5 Powers of matrices

ANSWERS p. 417

Recap

1 Determine whether each product can be defined.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 10 & 6 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$

a AB **b** AD **c** DC **d** DB

2 State the type of each matrix A to D.

a $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $B = \begin{bmatrix} 11 & 9 & 7 \end{bmatrix}$ **c** $C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ **d** $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Mastery

3 WORKED EXAMPLE 8 Find the square of each matrix.

a $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 1 \\ 3 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

d $\begin{bmatrix} -4 & 3 \\ 2 & -2 \end{bmatrix}$ **e** $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

4 WORKED EXAMPLE 9 Find the cube of each matrix.

a $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 4 \\ -3 & -1 \end{bmatrix}$ **c** $\begin{bmatrix} -3 & 6 \\ 1 & 4 \end{bmatrix}$

d $\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 2 \\ -6 & -6 \end{bmatrix}$

5 Find the fourth power of each matrix.

a $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$

d $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ **e** $\begin{bmatrix} 6 & -7 \\ -5 & 2 \end{bmatrix}$

6 For $A = \begin{bmatrix} 12 & -9 \\ 16 & -12 \end{bmatrix}$, find

a A^2 **b** A^3 **c** A^4

Comment on your answers.

7 Find the square of each matrix.

a $\begin{bmatrix} 3 & 0 & -3 \\ 5 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & -3 & 0 \\ 6 & 6 & 0 \\ 3 & -9 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & -2 & 1 \\ 3 & -2 & 1 \\ 3 & -2 & 1 \end{bmatrix}$ **d** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

8 Find the cube of each matrix.

a $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Exam practice

9 (3 marks) If $A = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$, find A^2.

10 (3 marks) CF Comment on your results for the square, cube and fourth powers of this matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

11 (3 marks) CF Find the square, cube and fourth powers of the matrix $M = \begin{bmatrix} 9 & -4 \\ 18 & -8 \end{bmatrix}$, and comment on your results.

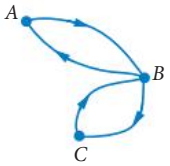
9.6 Applications of matrices

A **network** shows connections between points and can be represented by a matrix with '1's showing connections and '0's indicating there are no connections.

This is called an **adjacency matrix**. It shows the points that are next to each other (adjacent) in the network.

A **communications network** shows communication links.

The network below shows the official communications in an office between the manager, Alice, and the deputy managers, Bella and Callum. It is represented by the adjacency matrix on its right. The diagonal has all zeroes because you do not officially communicate with yourself.

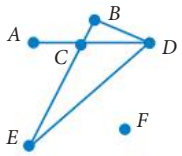


$$C = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

The square of a communications network shows the number of ways that each person can communicate to another in the network via a third person.

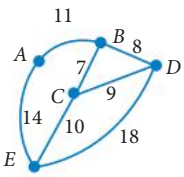
Two different kinds of networks are shown below and are represented by the matrices on their right.

Connection network



$$N = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Road network



$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 11 & 0 & 0 & 14 \\ 11 & 0 & 7 & 8 & 0 \\ 0 & 7 & 0 & 9 & 10 \\ 0 & 8 & 9 & 0 & 18 \\ 14 & 0 & 10 & 18 & 0 \end{bmatrix} \end{array}$$

WORKED EXAMPLE 10 Communication matrices

Find the number of ways that a person in this network can communicate to another person via a third person

$$C = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Steps	Working
1 Square the adjacency matrix.	$C^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $C^2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
2 Interpreting the matrix. Where 2 different people have a number other than zero, they may communicate via a third person.	$C^2 = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$ The diagonal values for XX, YY, ZZ are redundant in this matrix.
3 Write the answer.	There is 1 third party link between X and Z.

Exam hack

Don't be afraid to change a row matrix to a column matrix or a column matrix to a row matrix to change the size so that you can multiply matrices in application questions. This is a trick to use in complex questions.

EXERCISE 9.6 Applications of matrices

ANSWERS p. 418

Recap

1 Explain what 'defining a matrix product' means.

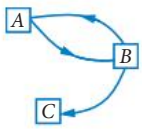
2 Square $\begin{bmatrix} 3 & 1 \\ 5 & 9 \end{bmatrix}$.

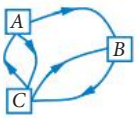
Mastery

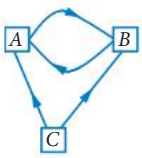
3 Which matrix could be a one-step communication network? (There may be more than one.)

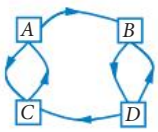
A $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **B** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

4 Construct a matrix showing the links of communication for the following networks.

a


b


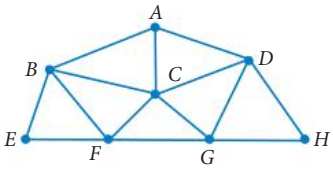
c


d


5 The communication network between people in a factory is shown.

a Write the adjacency matrix for the network.

b Find the number of ways that B can communicate with G via a third person.



6 WORKED EXAMPLE 10 The communications matrix for Pranav, Quentin, Robert and Sally is

given by $C = \begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$.

Find the number of ways that

a Quentin can communicate with Robert via a third person.

b Pranav can communicate with Sally via a third person.

c Robert can communicate with Pranav via a third person.

7 A toy store needs to mark-up the price of purchased goods by 75% before selling them to consumers. Listed below are the wholesale price of the products.

	Toys for 0–5 years	Toys for 5+ years
Board games	\$10.60	\$25.00
Soft toys	\$8.80	\$17.20
Art sets	\$5.40	\$12.80
Figurines	\$10.00	\$16.40

a Represent these costs in a matrix C.

b Use scalar multiplication to mark-up the prices for sale.

c Create a matrix showing the profit margin of each item.

8 These matrices show the daily sales of popular gelato flavours and the price of each size of serving for a gelato shop.

$$P = \begin{array}{c} \begin{array}{ccc} \text{Small} & \text{Medium} & \text{Large} \end{array} \\ \begin{bmatrix} \$4 & \$7 & \$9 \end{bmatrix} \end{array} \quad S = \begin{array}{c} \\ \text{Choc} \\ \text{Mint} \\ \text{Lemon} \end{array} \begin{array}{c} \begin{array}{ccc} \text{S} & \text{M} & \text{L} \end{array} \\ \begin{bmatrix} 30 & 41 & 28 \\ 15 & 25 & 10 \\ 27 & 10 & 5 \end{bmatrix} \end{array}$$

Calculate the daily takings for the business.

Exam practice

9 (4 marks) CF Maths students decide to analyse the number of wheels and seats on the bicycles and scooters at the school. The first matrix displays the number of bicycles and scooters at each campus in the school. The second displays the number of wheels and seats each one has.

$$\begin{array}{c} \\ \text{Junior} \\ \text{Senior} \end{array} \begin{array}{c} \begin{array}{cc} \text{Bicycle} & \text{Scooter} \end{array} \\ \begin{bmatrix} 20 & 8 \\ 32 & 17 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Bicycle} \\ \text{Scooter} \end{array} \begin{array}{c} \begin{array}{cc} \text{Wheels} & \text{Seats} \end{array} \\ \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} \end{array}$$

Determine how many wheels and seats are found at each campus.

10 (5 marks) An international airline packages tours to sell through its own and other agents. Tours are priced according to the principal destination, number of nights' accommodation and the number of stopovers. Agents add their own commission onto the total package prices, but the airline's agents work on a commission of 35%. The packages most commonly sold are:

- British Explorer – includes 2 stopovers and 14 days accommodation
- European Wanderer – includes 4 stopovers and 21 days accommodation
- East Coast US Explorer – includes 2 stopovers and 14 days accommodation
- West Coast US Explorer – includes 3 stopovers and 14 days accommodation
- East Coast US Wanderer – includes 5 stopovers and 28 days accommodation
- West Coast US Wanderer – includes 4 stopovers and 21 days accommodation

Principal destination	Accommodation	Stopovers
London: $3000	$240/day	Singapore, New Delhi, Rome, Frankfurt, Athens, Paris $400 each
Los Angeles: $2800	$220/day	Honolulu, San Francisco, Las Vegas, Seattle, Vancouver $440 each
New York: $3200	$210/day	Honolulu, Los Angeles, Chicago, St. Louis, Kansas City, Toronto, Quebec $500 each

a Calculate the cost of each tour and write it as a column matrix. *[2 marks]*

b Use a scalar multiple to find the commission for each tour charged by the airline's agents. *[1 mark]*

c Use matrix addition or another scalar multiple to find the cost to the customer of each tour. *[2 marks]*

11 (7 marks) CF Use your answers to question **10** to answer this question.

Three agents forward the following bookings to the airline.

Agent	BE	EW	ECE	WCE	ECW	WCW
A	2	1	0	3	2	1
B	4	2	2	4	1	2
C	3	1	5	3	3	2

Calculate the commission that each agent will receive.

12 (6 marks) CU A smallgoods manufacturer produces 3 types of salami – Italian, Hungarian and Austrian – in mild and hot versions. The prices of each type are shown in the table.

Type of salami	Italian	Hungarian	Austrian
Mild	$12.50	$15.00	$17.50
Hot	$13.20	$16.40	$18.00

The orders from 3 delicatessens are as follows:

George's: 3 MI, 1 HI, 2 MH, 3 HH, 1 MA, 0 HA

Frank's: 2 MI, 2 HI, 3 MH, 1 HH, 3 MA, 2 MA

Maria's: 4 MI, 5 HI, 0 MH, 1 HH, 0 MA, 2 HA

Calculate the price of each deli's order.

Video Exam question analysis: Matrices

EXAM QUESTION ANALYSIS

Exam-style question (4 marks)

A retailer is comparing the weekly sales of 2 brands of sunglasses at 3 of their outlet stores.
The following matrices show the number of sales of Boldeyes and Sunray sunglasses at each store, as well as the price of each brand.

$$S = \begin{array}{r} \\ \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{array} \begin{array}{c} \begin{array}{cc} \text{Boldeyes} & \text{Sunray} \end{array} \\ \begin{bmatrix} 30 & 90 \\ 22 & 100 \\ 28 & 60 \end{bmatrix} \end{array} \qquad P = \begin{array}{r} \\ \text{Boldeyes} \\ \text{Sunray} \end{array} \begin{array}{c} \text{Unit price \$} \\ \begin{bmatrix} 27 \\ 65 \end{bmatrix} \end{array}$$

Calculate the weekly sales of sunglasses at each store.

Reading the question

- 2 matrices, 3 stores
- Numbers of sales are given, as well as the unit price of each brand.
- Find the weekly sales for each store.

Thinking about the question

- The sales numbers need to be multiplied to the price to get the total sales.
- Matrices must be multiplied.
- Find the size of each matrix.
- Define the multiplication.
- Check that the product size matches the answer.

Worked solution (✓ = 1 mark)

Sizes of matrices

$$\begin{array}{ccccc} & S & & P & \\ 3 \times & \boxed{2 \quad 2} & \times & 1 & \checkmark \end{array}$$

Product matrix is 3×1.

Multiplication SP

$$S = \begin{bmatrix} 30 & 90 \\ 22 & 100 \\ 28 & 60 \end{bmatrix} \qquad P = \begin{bmatrix} 27 \\ 65 \end{bmatrix}$$

$$SP = \begin{bmatrix} 30 \times 27 + 90 \times 65 \\ 22 \times 27 + 100 \times 65 \\ 28 \times 27 + 60 \times 65 \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} 6660 \\ 7094 \\ 4656 \end{bmatrix} \checkmark$$

Sales of sunglasses for each store were:

Store 1 = \$6660

Store 2 = \$7094

Store 3 = \$4656 ✓

9 Chapter summary

Matrices

- The **size** of a matrix is written as the number of rows × the number of columns.

  For example, $A = \begin{bmatrix} 2.4 & 0 & 6 & \frac{1}{2} \\ 0 & -5.3 & 4 & -5 \end{bmatrix}$

  is a 2 × 4 ('2 by 4') matrix.
- The **elements** of a matrix are identified by a lower case letter with the row and column as a subscript; for example, for matrix A above, $a_{24} = -5$ and $a_{13} = 6$.

Types of matrices

Type of matrix	Example
A **row matrix** has only one row.	$\begin{bmatrix} 2 & 4 & 5 & 0 \end{bmatrix}$
A **column matrix** has only one column.	$\begin{bmatrix} 5 \\ -2 \\ 0.4 \end{bmatrix}$
A **square matrix** has the same number of rows and columns.	$\begin{bmatrix} 1 & -4 \\ 6 & 2 \end{bmatrix}$ This matrix has order 2.
A **zero matrix** has all elements equal to 0.	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
An **identity matrix**, I, is a square matrix with every element 0 except for the diagonal values, which are all 1s.	$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operations with matrices

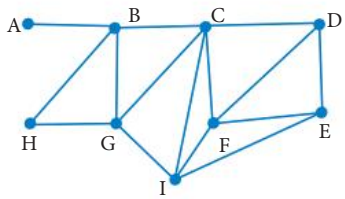
- Matrices must be the same size to be added or subtracted.
- To add matrices, add the corresponding elements.
- To subtract matrices, subtract the corresponding elements.
- To multiply a matrix by a number, multiply every element by the number. This is called **scalar multiplication**.
- A product AB is defined if A has m columns and B has m rows so they can be multiplied together.

  e.g. row × column row × column

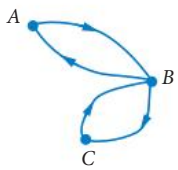
  3 × $\boxed{2 \quad 2}$ × 1
- The **product** of an $n \times m$ matrix A and an $m \times p$ matrix B is an $n \times p$ matrix C.
- For the square matrix A, A^n for a positive integer is called the **power of a matrix.**
- A^2 and A^3 are called the square and cube of A respectively.
- Only square matrices can have powers.

Matrix applications

- A **network** shows connections between points.



An **adjacency matrix** shows a network with 1s for connections and 0s for no connection. They can have numbers to represent distances instead of 1s.



$$C = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

A **communications network** shows communication links. It can be shown by an adjacency matrix. The square of a communications network shows the number of ways that each person can communicate to another via a third person.

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 1

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 4 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 Matrices $M = \begin{bmatrix} 7 & 5 & 0 \\ 1 & 5 & 0 \\ 3 & -3 & -2 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$ are what type of matrices respectively?

A identity and row
B square and column
C identity and column
D square and row

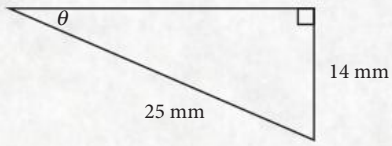
2 What is the simultaneous solution for $3x = y + 5$ and $y = -8$?

A $x = 1, y = -8$
B $x = -1, y = -8$
C $x = 1, y = 8$
D $x = -8, y = -1$

3 The element c_{16} in $C = \begin{bmatrix} 3 & 4 & 14 & 0 & 19 & 5 & 19 \\ 15 & 9 & 7 & 9 & 0 & 1 & 25 \end{bmatrix}$ is

A 3
B 0
C 5
D 1

4 Which formula is needed to find the angle, θ, in this triangle?



A $\theta = \sin^{-1}\left(\frac{14}{25}\right)$
B $\theta = \cos^{-1}\left(\frac{14}{25}\right)$
C $\theta = \sin^{-1}\left(\frac{25}{14}\right)$
D $\theta = \cos^{-1}\left(\frac{25}{14}\right)$

5 Calculate A^2 if $A = \begin{bmatrix} 2 & 9 \\ -3 & 5 \end{bmatrix}$.

A $A^2 = \begin{bmatrix} -23 & 58 \\ -21 & -2 \end{bmatrix}$
B $A^2 = \begin{bmatrix} -23 & 63 \\ -21 & 52 \end{bmatrix}$
C $A^2 = \begin{bmatrix} 23 & 63 \\ -21 & -2 \end{bmatrix}$
D $A^2 = \begin{bmatrix} -23 & 63 \\ -21 & -2 \end{bmatrix}$

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Section 2 4 short response questions

13 marks

6 (3 marks) $A = \begin{bmatrix} 2 & 3 & 10 & 7 \\ -2 & 0 & 6 & -6 \\ 1 & -5 & 5 & 4 \end{bmatrix}$. Write down the value of

a a_{21} *[1 mark]*

b a_{34} *[1 mark]*

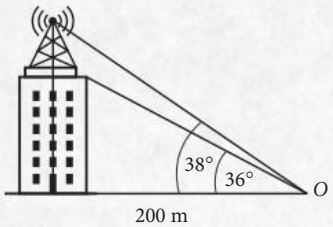
c a_{23} *[1 mark]*

7 (3 marks) Perform the operation $2A - B + C$ for these matrices.

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 2 \\ 7 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

8 (3 marks) Find the simultaneous solution for $3(2 + y) = x$ and $2(y - 1) = x$.

9 (4 marks) From a point O, which is 200 m from a building, the angles of elevation to the bottom and top of a transmitter tower are 36° and 38° respectively.
Find the height of the tower, correct to one decimal place.



Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

4 short response questions

12 marks

1 (3 marks) Given $3\begin{bmatrix} x & 4 \\ 1 & -2 \end{bmatrix} + 2\begin{bmatrix} 6 & -3 \\ 2 & y \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 7 & -5 \end{bmatrix}$, find x and y.

2 (3 marks) A car travelled 25 km west and then travelled 12 km on a bearing of 310°. How far is the car from its starting point, correct to the nearest kilometre?

3 (3 marks) Solve this pair of simultaneous equations.

$$\frac{2}{m} + \frac{3}{n} = 20 \quad \text{and} \quad \frac{2}{m} + \frac{2}{n} = 16$$

4 (3 marks) Given the following matrix sizes, determine whether it is possible to calculate the value of the expression $CB + A + DB$. Justify your answer, showing all calculations.

$A = 2 \times 2$ $B = 3 \times 2$ $C = 2 \times 3$ $D = 1 \times 3$

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CHAPTER

10 ANALYSING DATA

Syllabus coverage

Nelson MindTap chapter resources

Terminology

10.1 Categorical and numerical variables

10.2 Displaying data

Frequency tables

Dot plots

Pie charts

Bar/Column charts

10.3 Stem-and-leaf plots and histograms

Stem-and-leaf plots

Histograms

10.4 The mean, median and mode

10.5 The range and interquartile range

10.6 The standard deviation

10.7 The shape of a distribution

Exam question analysis

Chapter summary

Cumulative examination 1

Cumulative examination 2

Prior learning
Analysing data

Syllabus coverage

UNIT 2, TOPIC 4: UNIVARIATE DATA ANALYSIS 1

Making sense of data relating to a single statistical variable

- Understand the meaning of univariate data.
- Classify a statistical variable as categorical or numerical.
- Classify a categorical variable as ordinal or nominal and use tables and pie, bar and column charts to organise and display the data, e.g. ordinal: income level (high, medium, low); nominal: place of birth (Australia, overseas).
- Classify a numerical variable as discrete or continuous, e.g. discrete: the number of people in a room; continuous: the temperature in degrees Celsius.
- Select, construct and justify an appropriate graphical display to describe the distribution of a numerical dataset, including dot plot, stem-and-leaf plot, column chart and histogram.
- Describe a graphical display in terms of the number of modes, shape (symmetric versus positively or negatively skewed), measures of centre and spread, and outliers, and interpret this information in the context of the data.
- Understand and calculate the mean, median, mode, range and interquartile range (IQR) of a dataset, with and without technology.
  - mean: $\bar{x} = \dfrac{\sum x}{n}$
  - median: $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ data value
- Understand and calculate the (sample) standard deviation, s_x, of a dataset, using technology only.
- Use statistics as measures of centre and spread of a data distribution, being aware of their limitations.

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Videos (11):

10.1 Categorical and numerical data • Types of data
10.3 Frequency histogram • Stem-and-leaf plots
10.4 Statistics from a frequency table • Stem-and-leaf plots 2
10.5 Interquartile range 1 • Interquartile range 2
10.7 The shape of a frequency distribution • Outliers
Exam question analysis Analysing data

Prior learning (1):

10.1 Analysing data

Skillsheets (3):

10.4 Statistical measures • Mean, median, mode on a calculator
10.6 Standard deviation on a calculator

Worksheets (17):

10.2 Every picture tells a story • Frequency tables • Frequency distribution tables • Australian statistics
10.3 Histograms • Stem-and-leaf plots
10.4 Mean, median, mode and range • Mode, median and mean • Measures of central tendency
10.5 Interquartile range • Statistics mode: Graphics calculator
10.6 Standard deviation • Statistical calculations • Statistics review • Calculating and interpreting summary statistics
10.7 Shapes of distributions
Cumulative exams General Maths Year 11 formula sheet

Puzzles (4):

10.1 Statistical data match-up
10.4 Statistical match-up
10.5 Statistical measures puzzle
Chapter summary Data and statistics crossword

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

categorical	class interval	continuous	discrete
distribution	dot plot	histogram	interquartile range (IQR)
mean	measure of centre	measure of spread	median
mode	negatively skewed	nominal	numerical
ordinal	outlier	positively skewed	quartile
range	standard deviation	stem-and-leaf plot	symmetrical

10.1 Categorical and numerical variables

Data is information that , when collected and organised, can be analysed and used to make decisions.

Univariate data is about a single variable. For example, a person's height or the number of pets they have.

Videos
Categorical and numerical data
Types of data

Puzzle
Statistical data match-up

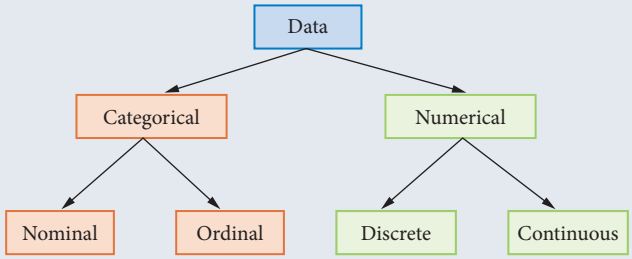
Types of data

There are two kinds of **categorical data**:

- **Nominal data** has no order.
- **Ordinal data** has an order; for example, A, B, C or first, second, third and fourth preference.

Numerical data are numbers. There are also 2 kinds:

- **Discrete data** has only particular values; for example, goals scored or number of people.
- **Continuous data** can have any values (often between certain limits); for example, length of a football field or your height.



Don't think that just because there are numbers, the data must be numerical. Placings in a race are 1st, 2nd, 3rd … last; but these are really names. It doesn't make sense to add them or find an average, so the data is not numerical.

If it makes sense to add the data or find an average, the data is numerical. If it doesn't, then the data is categorical.

WORKED EXAMPLE 1 Types of data

Jessica is 16 years old, 169 cm tall, has blond hair and 3 living grandparents. She rates rock music as 'like' on a scale from 'dislike strongly' to 'like strongly'.

Classify each of her stated attributes as nominal, ordinal, discrete or continuous.

Steps	Working
1 She is 16. Age has been rounded down to the nearest year but can take any value.	Age is a continuous, numerical variable.
2 Height can be measured with a ruler or tape.	Height is a continuous, numerical variable.
3 Hair colours are categories that don't have an order.	Hair colour is a nominal, categorical variable.
4 She can only have 0, 1, 2, 3 or 4 living grandparents.	Number of living grandparents is a discrete, numerical variable.
5 The 'dislike–like' scale has an order.	Music preference is an ordinal, categorical variable.

EXERCISE 10.1 Categorical and numerical variables

ANSWERS p. 419

Mastery

1 WORKED EXAMPLE 1 Classify each data item.

a Placings in the final of the 100 m race

b Ages of girls in a volleyball team

c Distances to shops from your school

d Costs of restaurant meals

e Types of trees native to your area

f Shoe sizes of Year 11 students at Ipswich High

g Teams in the National Basketball League

h Hair colour of students in your class

i Birth weights of babies born at Mater Hospital

j Weekly incomes of doctors in Rockhampton

2 Sean looked at data that showed the house number that people lived at. He said that this was numerical, discrete data. Is he correct? Justify your answer.

Exam practice

3 Martika had her students rate how much they loved maths. She asked them to rate it from 1 for dislike, up to 5 for love. What type of data is this?

A nominal **B** ordinal **C** discrete **D** continuous

4 (2 marks) A fruit shop has 3 different grades of tomatoes for sale at prices of \$2.99/kg, \$4.99/kg and \$6.99/kg. Peter said that the average price of tomatoes sold must be \$4.99/kg. State whether you agree or disagree with Peter and give reasons for your answer.

5 (2 marks) Maddie said that the distance for a javelin throw is a discrete variable because it is always measured to the nearest 2 cm. Jess disagreed with Maddie, saying that the distance for a throw is continuous. Explain who you think is correct, carefully justifying your answer.

6 (4 marks) The table below shows the ages of students in a Year 11 class.

Age	15	16	17	18
Number students	4	15	3	1

a What kind of variable is student age? Give reasons for your answer. *[2 marks]*

b What kind of variable is the number of students? Justify your answer. *[2 marks]*

7 (2 marks) CF The travel destinations of people leaving Brisbane International Airport were coded as follows:

New Zealand = 1, Asia = 2, Pacific Islands = 3, America = 4, Africa = 5, Europe = 6

The average for 50 people was worked out to be 2.2, and this was given as evidence that the most common destination was Asia. Explain what is wrong with this conclusion.

10.2 Displaying data

Once the data has been collected, it needs to be organised and displayed so that you can see patterns and trends. Categorical data is organised using a **frequency table**. It is often displayed using **dot plots**, **pie charts** and **bar chart** or **column charts**.

Frequency tables

- Tally marks are used to help in counting.
- The number of times each score appears is its **frequency**.

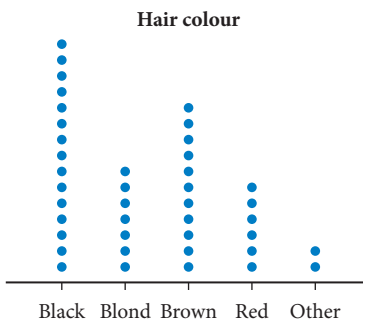
Data categories

Hair colour	Tally	Frequency
Black	𝍸 𝍸 𝍸	15
Blond	𝍸 ||	7
Brown	𝍸 𝍸 |	11
Red	𝍸 |	6
Other	||	2
Total		**41**

There were 41 answers given.

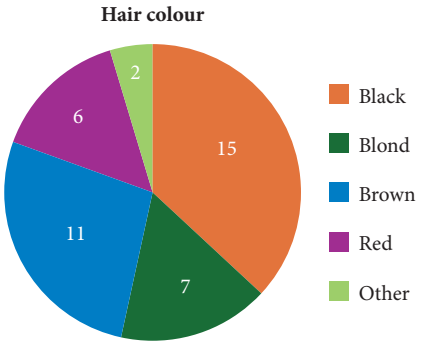
Dot plots

- A dot plot is useful as long as the data is not too spread out or the dataset is not too large.
- A dot plot can be used for categorical and discrete data.



Pie charts

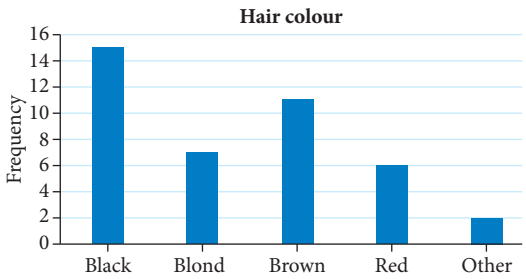
- A circle is divided into sectors so that the angle shows the frequency.
- A pie chart is only suitable for categorical data.
- The angle for each value is calculated as $\frac{\text{frequency}}{\text{total}} \times 360°$.

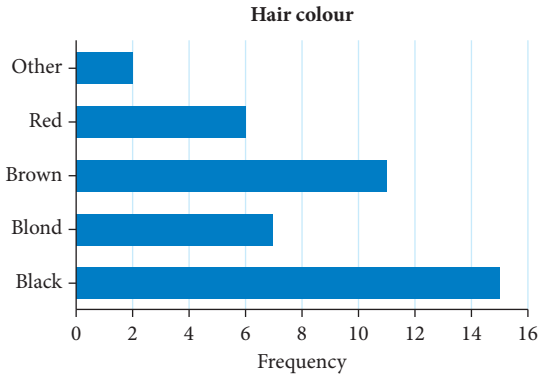


Worksheets
Every picture tells a story
Frequency tables
Frequency distribution tables
Australian statistics

Bar/column charts

- The height/length of each bar or column shows the frequency.
- These charts can be used for categorical and discrete data.
- When vertical bars are used, it is called a column chart.





WORKED EXAMPLE 2 Frequency table and dot plot

The sizes, in mm, of a mixed packet of O-rings are listed.

18	16	20	15	16	10	15	16	10	20
18	18	14	10	15	12	12	15	10	20
16	20	10	20	14	15	14	15	12	20
15	15	18	10	12	20	12	12	10	12

a Complete a frequency table of the sizes.

b Make a dot plot of the sizes.

Steps

a **1** The O-rings range in size from 10 to 20 mm.

2 Draw a table with columns:
Size of O-rings (mm), Tally, Frequency

3 Complete the tally column.

4 Fill in the frequency column and the total.

5 Do a quick check that the total is the same as the number of data values.

Working

Size of O-rings (mm)	Tally	Frequency
10	𝍸 ||	7
11		0
12	𝍸 ||	7
13		0
14	|||	3
15	𝍸 |||	8
16	||||	4
17		0
18	||||	4
19		0
20	𝍸 ||	7
Total		40

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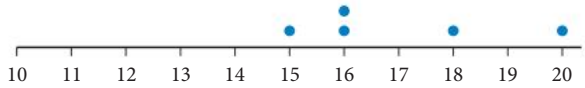
b **1** The lowest value is 10 and the highest is 20.

Draw the scale.

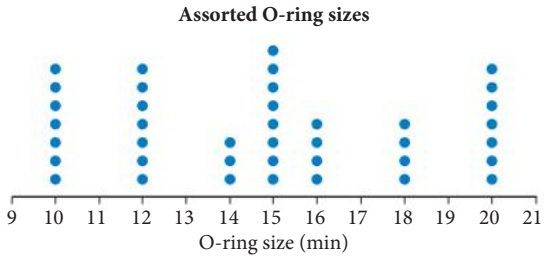
2 Put a dot above the first values: 18, 16, 20 and 15.

Put an evenly spaced dot over the next number, 16.



3 Continue with the rest of the numbers to complete the dot plot.

Add a frequency scale, title and scale names.



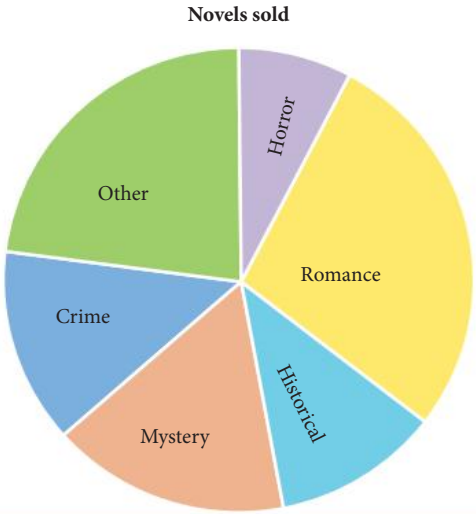
> You can use a dot plot to work out frequencies and the shape of the distribution at the same time.

WORKED EXAMPLE 3 Pie chart

Draw a pie chart of the following sales of novels.

Type	Horror	Romance	Historical	Mystery	Crime	Other
Sales	4097	14 875	6172	8942	7241	12 320

Steps	Working
1 Find the total sales.	total = 4097 + 14 875 + 6172 + 8942 + 7241 + 12 320 = 53 647
2 Write each category as a fraction of the total. Then multiply the fraction by 360° to find the angle in the pie chart.	Horror $= \frac{4097}{53\,647} \times 360°$ = 27° Romance $= \frac{14\,875}{53\,647} \times 360°$ = 100°
3 Do a quick check that the angles add to 360°. (Remember that if answers are rounded, they may be just above or below.)	Historical $= \frac{6172}{53\,647} \times 360°$ = 41° Mystery $= \frac{8942}{53\,647} \times 360°$ = 60° Crime $= \frac{7241}{53\,647} \times 360°$ = 49° Other $= \frac{12\,320}{53\,647} \times 360°$ = 83°
4 Draw the graph. MUST have: – title – label for each sector	



WORKED EXAMPLE 4 Column chart

Forty people on Simpson St gave the following ratings of a restaurant in Mt Isa.

Rating	Very poor	Poor	Average	Good	Very good
Number of people	5	8	11	10	6

a Draw a column chart to show the ratings.

b What percentage of people gave the restaurant an above average rating?

Steps	Working
a 1 Decide on the scale. Need to go to biggest value on y-axis.	y-axis needs to go to 11, so go up by 2s.
2 Draw the axes. Use a ruler! Be sure to include: – the title: Restaurant ratings – the axes labels: Rating and Frequency – an appropriate scale: categories on x-axis and up to 11 on y-axis.	
3 Draw the column chart. Leave a gap between columns. A bar chart has the categories on the vertical axis and the frequency on the horizontal axis.	
b 1 Above average means good or very good. Write as a fraction and then multiply by 100 to calculate a percentage.	above average $= 10 + 6 = 16$ $\therefore \frac{16}{40} \times 100 = 40\%$
2 Answer the question.	40% of people gave the restaurant an above average rating.

EXERCISE 10.2 Displaying data

ANSWERS p. 419

Recap

1 Samples were taken from different waterways to look at what marine life was present. Data relating to the following variables was taken from the samples:

- name of species
- colour
- body length (in millimetres)
- body weight (in milligrams)

The number of these variables that are discrete numerical variables is

A 0 **B** 1 **C** 2 **D** 3

2 What data type would the numbers on the Matildas' soccer jerseys be classified as? Justify your answer.

Mastery

10.2

3 WORKED EXAMPLE 2 Jeremy recorded the types of vehicles that went past the school over a short period on a tally chart, shown below.

Cars	Motorbikes	Trucks	Taxis	Other
𝍸 𝍸 𝍸 𝍸	𝍸 ||||	𝍸 𝍸 𝍸	𝍸 𝍸	𝍸 𝍸 𝍸
𝍸 𝍸 𝍸 𝍸		𝍸 𝍸 𝍸		𝍸 𝍸 𝍸
𝍸 𝍸 𝍸 𝍸				𝍸 𝍸 ||
𝍸 𝍸 𝍸 𝍸				
𝍸 𝍸 |||				

Complete a frequency table of his observations.

4 Make a dot plot of this set of data.

82	86	84	86	83	86	83	86	83	87	82
87	82	85	84	85	85	85	84	87	81	86
84	86	79	85	83	88	84	87	82	86	81
88	82	88	84	88	83	86	83	84	84	85

5 WORKED EXAMPLE 3 Draw a pie chart showing the origins of overseas-born Australians:

England	961 570
India	845 800
China	655 760
New Zealand	598 090
The Philippines	361 860
Vietnam	298 960
Other	4 453 600

Source: Australian Bureau of Statistics. (June, 2023). *Australia's Population by Country of Birth*. ABS. https://www.abs.gov.au/statistics/people/population/australias-population-country-birth/latest-release.

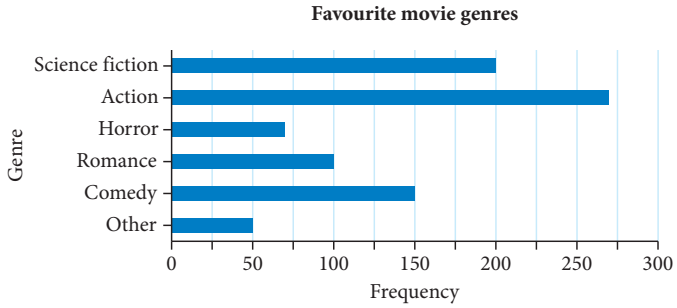
6 WORKED EXAMPLE 4 Draw a column chart of the ratings by people for Sucralose.

Rating	1	2	3	4	5
Frequency	8	17	6	3	6

7 Draw a bar chart of the accommodation ratings shown below.

1 star: 5 2 stars: 15 3 stars: 55 4 stars: 48 5 stars: 32

8 This graph shows the results of a survey about favourite movie genres.



a How many people prefer horror movies?

b How many people favour comedy movies?

c What is the most popular movie genre?

d How many people were surveyed altogether?

e What percentage prefer action?

Exam practice

9 Students were surveyed on their method of transport to school. The results are shown in the frequency table.

The percentage of students who walk or ride a bike is closest to

A 8 **B** 12

C 70 **D** 88

Method of getting to school	Frequency
Bus	280
Train	75
Car	170
Bike	20
Walk	50
Other	5
Total	**600**

10 (6 marks) Students did a survey of the eye colour of Year 11 General Maths students. The results were:

blue	brown	hazel	brown	brown	blue	brown	green	brown	blue	grey	brown
blue	brown	blue	hazel	blue	brown	hazel	brown	green	brown	brown	hazel
green	hazel	blue	blue	hazel	blue	blue	brown	blue	brown	hazel	brown
hazel	blue	green	blue	hazel	grey	brown	hazel	blue	brown	blue	brown
brown	green	brown	brown	blue	brown	blue	brown	green	brown	hazel	brown

a Complete a frequency table. *[2 marks]*

b Determine the percentage of students surveyed who have blue eyes. *[2 marks]*

c Construct a column chart representing the eye colour of the Year 11 General Maths students. *[2 marks]*

11 (5 marks)

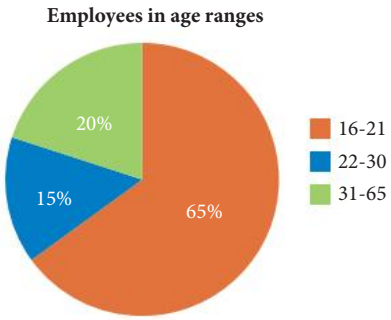
a Make a dot plot of these ages of people at a party. *[2 marks]*

17	17	16	20	14	18	15	18	16	18	17	18	15
17	15	18	17	19	16	17	15	18	14	18	16	18

b What is the most common age? *[1 mark]*

c What percentage of people at the party are under the age of 18? *[2 marks]*

12 (6 marks) This pie graph shows the proportion of workers in 3 age brackets in a large retail organisation.



a Juan said that the sector angle for the 31–65 year-old range will be 20°. Is he correct? Justify your decision. *[2 marks]*

b If the total number of employees is 1420, determine the number aged 31–65. *[2 marks]*

c If there were 480 employees aged 22–30, determine the number aged 16–21. *[2 marks]*

10.3 Stem-and-leaf plots and histograms

Worksheets
Histograms
Stem-and-leaf plots

Continuous numerical data is treated differently from categorical or discrete data.

A frequency table for **grouped data** has equal class intervals for the data such as 10–19, 20–29, etc.

35–<40 means any value 35 or more but less than 40.

Ages	Class centre	Tally	Frequency
35–<40	37.5	𝍸	5
40–<45	42.5	𝍸 ||	7
45–<50	47.5	𝍸	5
50–<55	52.5	||||	4
55–<60	57.5	||	2
Total			23

- The data is grouped into **class intervals**. There should be NO overlap.
- The intervals must all be the same size.
- There should only be 5–10 groups.
- The **class centre** is found by averaging the lowest and highest value of the interval, for example, $\frac{35+40}{2} = 37.5$. This will be used later to represent the group.

Continuous data is shown using **stem-and-leaf plots** and **histograms**.

Stem-and-leaf plots

- A stem-and-leaf plot:
  - visually displays the shape of the data
  - has the advantage that the actual data values can be seen
  - has the data ordered from smallest to largest.
- The 'stem' is the leading digit and the 'leaf' is the last digit. So, 13.2 would have a stem of 13 and a leaf of 2. The key would be: 13|2 means 13.2.
- There is always a title and a key. This makes clear what the value of the data is.
- If there is a lot of data, the stem may be split in half.

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4 6 7 8 9 9
5	1 2 2 3 5 7

Key: 3|5 means 35

As there are only 3 groups, this would be better represented with split stems.

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4
4	6 7 8 9 9
5	1 2 2 3
5	5 7

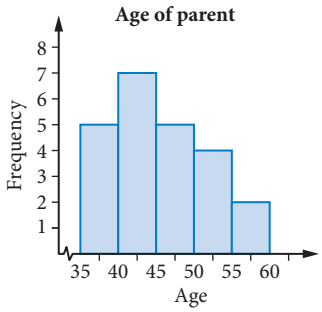
Key: 3|5 means 35

This is 40 to 44.

This is 45 to 49.

Histograms

- A histogram:
  - is like a column chart but does not have spaces between the columns.
  - should always leave a half-column gap at the beginning.
- The left and right edges of the columns are at the class boundaries, and the columns are centred on the class centres.



Video
Frequency histogram

WORKED EXAMPLE 5 Grouped frequency table and histogram

The speeds (in km/h) of motorists in an 80 km/h zone were recorded by police as follows.

74.6	75.6	67.8	80.1	69.6	95.4	72.3	82.9	69.5	83.7	72.7
84	74.3	89.9	71.9	76.5	71.2	79.8	63.5	78.1	68.6	75.7
72.3	75.5	72.4	78.6	61.1	75.5	71.8	81.1	69	81.7	74.7
76.8	67	80.9	65.6	81.8	70.1	83	64.6	87.7	66.9	74.5

a Make a frequency table of the speeds.

b How many people were speeding?

c Draw a histogram of the speeds.

61–<66 means 61 up to but not including 66.

Steps | **Working**

a 1 The lowest speed was from 61.1 km/h to 95.4 km/h.

Classes of width 5 give a reasonable number of classes.

The maximum legal speed is 80 km/h, so makes sense to make a class interval from 81–<86. Follow this through the table.

2 Complete the tally column.

3 Complete the table.

Speed	Class centre	Tally	Frequency
61–<66	63.5	||||	4
66–<71	68.5	卌 |||	8
71–<76	73.5	卌 卌 卌	15
76–<81	78.5	卌 ||	7
81–<86	82.5	卌 ||	7
86–<91	87.5	||	2
91–<96	92.5	|	1
Total			**44**

b 1 Speeding is over 80km/h.

$7 + 2 + 1 = 10$

2 Answer the question.

∴ 10 people were speeding according to these results.

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c **1** The data ranges from 61.1 to 95.4.

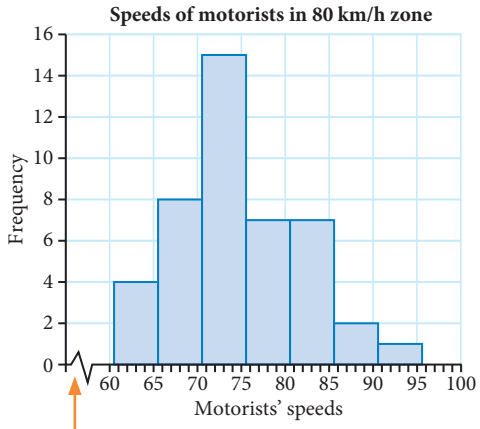
2 Draw the axes, using a ruler. Include:

- a title: Speeds of motorists in 80 km/h zone
- labels on the axes: Motorists' speeds and Frequency

Appropriate scales are from 60 to 100 on x-axis and up to 15 on y-axis.

3 Draw the histogram.

Remember, you must leave a half column gap at the beginning.



If not starting a graph at 0, then use this symbol to show a 'break' in the axis.

WORKED EXAMPLE 6 Stem-and-leaf plots

Complete a stem-and-leaf plot of these ages of the mothers of some Year 11 students.

38	43	35	52	55	57	47	49	39	44	46	43
48	44	40	51	53	36	42	49	52	39	44	52

Steps | **Working**

1 The data ranges from 35 to 57.

So as data is in 30s, 40s and 50s – the first digits are 3, 4 and 5.

Put these in the stem on the left.

Then put the second digits of the first few values.

Stem	Leaf
3	8 5
4	3
5	2

2 Continue with the rest of the values, leaving spaces between the leaves.

Stem	Leaf
3	8 5 9 6 9
4	3 7 9 4 6 3 8 4 0 2 9 4
5	2 5 7 1 3 2 2

3 When you have added all the data, rearrange the leaves in order.

Add a title and key.

Ensure that the numbers are neatly lined up under each other.

Ages of mothers of Year 11 students

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4 6 7 8 9 9
5	1 2 2 2 3 5 7

Key: 4|3 = 43

Video Stem-and-leaf plots

EXERCISE 10.3 Stem-and-leaf plots and histograms

ANSWERS p. 420

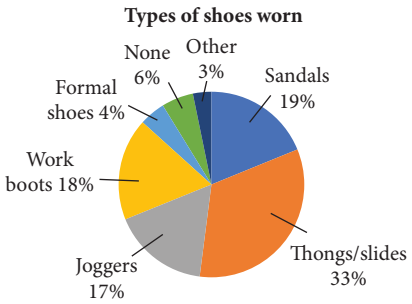
Recap

1 Jenny wrote down the type of shoes worn by each person who walked past her in 15 minutes. She represented the results in this pie chart.

Jenny recorded the results for a total of 90 people. How many of them were wearing work boots?

A 16 **B** 5

C 18 **D** 20



2 A Year 11 class with 30 students listed the number of siblings (brothers and sisters) they each had. The results are shown below.

3	1	0	4	3	2	0	2	1	1
6	2	2	1	0	0	2	1	1	4
2	0	1	2	3	0	0	1	2	3

Construct a frequency table for this data.

Mastery

3 WORKED EXAMPLE 5 Construct a grouped frequency table for the following test scores, using intervals of size 10.

36 47 42 44 38 32 28 11 23 48 33 24 7 21 46 45

4 WORKED EXAMPLE 6 The marks awarded to a group of Year 11 students are listed below.

36	45	78	78	45	80	34	46	58	55
40	56	60	75	90	52	88	68	78	80
64	96	74	71	67	82	78	92	94	78

a Between what values do the marks lie?

b Make a stem-and-leaf plot of these marks.

Exam hack

It is usually easier to do a quick stem-and-leaf plot and then re-draw it neatly with the leaves in order from smallest to largest.

5 Draw a histogram of this set of grouped data.

Heights	130–<140	140–<150	150–<160	160–<170	170–<180	180–<190	190–<200
Frequency	1	0	12	18	11	7	1

6 The numbers of songs that Year 11 students had on their personal music players (including mobiles) are listed below.

75	403	131	320	194	202	154	306	152	226	173	320
185	317	53	224	101	280	173	321	158	371	114	282
115	214	163	497	173	259	136	283	135	291	193	208
130	217	119	494	93	223	173	328	170	263	156	420

a What class intervals would you choose for this data? Explain your reasoning.

b Make a frequency table.

c Draw a histogram.

d How would this data be classified?

e What could you say about the number of songs people had?

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Exam practice

Use this stem-and-leaf plot to answer Questions 7 and 8.

The stem-and leaf plot shows the marks out of 100 on a maths test.

Stem	Leaf
3	4
4	9
5	3 6 7 7
6	0 1 1 2 4 4 7 7 7
7	3 5 8
8	2
9	6 8

Key: 5|3 means 53

7 How many people completed the test?

A 7 **B** 21 **C** 28 **D** 98

8 What was the most common score?

A 6 **B** 7 **C** 60 **D** 67

9 (6 marks) The weights, to the nearest kg, of 40 bungee jumpers were recorded:

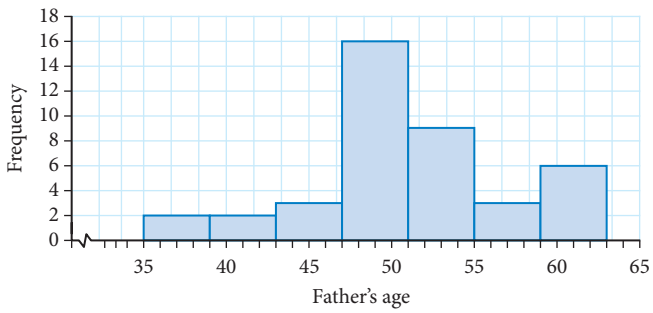
41	58	63	37	49	58	71	33	85	58
73	81	46	55	38	80	48	50	62	59
63	44	77	62	58	73	62	75	52	60
69	61	55	47	76	42	66	70	73	62

a Complete a frequency table. *[2 marks]*

b Draw a frequency histogram. *[2 marks]*

c From 200 bungee jumpers, how many would you expect to weigh 70 kg or more? Justify your answer. *[2 marks]*

10 (7 marks) The following histogram shows the age of the fathers of students attending a school.



a What type of data would you classify the fathers' ages as? Explain. *[1 mark]*

b The first interval is 35–<39. Describe what this means. *[1 mark]*

c What was the most common age group? *[1 mark]*

d What is the class centre of 43–<47? *[1 mark]*

e How many fathers' ages were recorded? *[1 mark]*

f Complete a frequency table of the fathers' ages. *[2 marks]*

11 (9 marks) Paula conducted a survey on the amount of money ($) people had spent on take-away food last week, rounded to the nearest dollar. The results are shown in the frequency table.

Money spent	Frequency
0–<10	2
10–<20	4
20–<30	5
30–<40	6
40–<50	10
50–<60	0
60–<70	2
70–<80	1

a This data is numerical. Is it discrete or continuous? Justify your answer. *[1 mark]*

b Construct a histogram to represent this data. *[2 marks]*

c What can you say about the maximum amount of money spent on take-away food last week? Explain. *[2 marks]*

d Paula wanted to draw a stem-and-leaf plot, so she went to the original data.

The results are shown.

48	15	74	0	25	12	65	35	40	28
15	37	25	30	45	40	31	42	48	6
18	22	49	69	42	28	30	40	36	41

Construct a stem-and-leaf plot. *[2 marks]*

e What can this stem-and-leaf plot tell you that the histogram cannot? Explain. *[2 marks]*

12 (2 marks) CU Data was collected on the length of fish caught during a fishing trip. The following frequency table and stem-and-leaf plot represent this information.

Stem	Leaf
1	4 5
2	0 3 3 7 8
3	2 4 6 6 6 9
4	0 0 1 3
5	0
6	8

Key: 1|4 means 14

Length of fish	Frequency
10–<20	2
20–<30	5
30–<40	6
40–<50	10
50–<60	0
60–<70	1

Which of these gives you the most information? Justify your answer.

10.4 The mean, median and mode

You often want a 'typical' value to represent a set of data.
The 3 typical values are the **mean**, **median** and **mode**. They are called the **measures of centre**.

Measure of centre

Mean ($\bar{x}$)

- The mean is the average of the data values.

$$\text{mean} = \frac{\text{sum of all the values}}{\text{number of values}} = \frac{\Sigma x}{n}$$

Σ means 'the sum of'
x means the individual data values
n is the total number of values

- When the data is represented as a frequency table, the mean can be found by adding a $f \times x$ column to the table and then using: $\text{mean} = \dfrac{\text{total of } f \times x \text{ column}}{\text{total of } f \text{ column}} = \dfrac{\Sigma fx}{\Sigma f}$.
- The mean of grouped data is calculated using class centres, so it only gives an *estimate* of the mean.

Median

- It is the middle value when all data values are written in order from smallest to largest.
- If there is an even number of values, then it is the average of the middle 2.
- In general, the median is the $\left(\dfrac{n+1}{2}\right)$th data value, provided the values are in order.
- If there is a large number of values in a frequency table, it may be necessary to look at the cumulative frequency to find the median value.
- The cumulative frequency is a progressive total of the frequencies.

Mode

- It is the most common data value – or category, the one with the highest frequency.
- There can be more than one mode.
- For grouped data, the **modal class** is the group that has the highest frequency.

Videos
Statistics from a frequency table
Stem-and-leaf plots 2

Skillsheet
Statistical measures

Worksheets
Mean, median, mode and range
Mode, median and mean
Measures of central tendency

Puzzle
Statistical match-up

WORKED EXAMPLE 7 Calculating the mean, median and mode

Find the mean, median and mode of this set of data.

9 6 7 14 3 5 14 5 8 5 12 8

Steps	Working
1 $\text{mean} = \dfrac{\text{sum of all the values}}{\text{number of values}}$	$\text{mean} = \dfrac{9+6+7+14+3+5+14+5+8+5+12+8}{12}$ $= \dfrac{96}{12}$ $= 8$
2 There are 12 data values: $n = 12$. Place the data in order and use $\left(\dfrac{n+1}{2}\right)$th. To find the average, add both values and divide by 2.	median: $\dfrac{12+1}{2} = 6.5$th data value So, the median is the average of the 6th and 7th values. 3 5 5 5 6 7 8 8 9 12 14 14 $\text{median} = \dfrac{7+8}{2} = 7.5$
3 There are more 5s than any other value.	The mode is 5.
4 Answer the question.	The mean is 8, the median is 7.5 and the mode is 5.

Skillsheet
Mean, median, mode on a calculator

You can use your calculator's statistical (STAT) functions to find the mean, median and mode of a set of data. Download detailed instructions from *Nelson MindTap*.

WORKED EXAMPLE 8 Calculating the mean, median, mode from a frequency table

A sample of people at a club in the city on Saturday night gave their ages as follows.

Age	18	19	20	21	22	23	24	25	26
Frequency	32	35	38	33	25	25	17	8	4

a Find the mean, median and mode.

b State the typical age. **Justify** your decision.

Steps | **Working**

a 1 Rewrite as a vertical table with $f \times x$ column.

The *cf* column will be explained when calculating the median.

Age (x)	f	$f \times x$	cf
18	32	$32 \times 18 = 576$	32
19	35	665	$32 + 35 = 67$
20	38	760	$67 + 38 = 105$
21	33	693	$105 + 33 = 138$
22	25	550	
23	25	575	
24	17	408	
25	8	200	
26	4	104	
Total	217	4531	

$$\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{4531}{217} = 20.88\ldots$$

2 There are 217 data values: $n = 217$.

With the data in order, use $\left(\frac{n+1}{2}\right)$th.

median: $\frac{217+1}{2} = 109$th data value

To find the 109th value, complete a cumulative frequency (*cf*) column. This is where the frequencies are added up.

So, in the row of $x = 21$, the 106th to the 138th values are all 21s; this includes the 109th (middle) value, the median.

median = 21

3 The highest frequency is 38: age of 20.

mode = 20

4 Answer the question.

The mean is 20.9, the median is 21 and the mode is 20.

b Refer to the data in part **a** and write your response.

The typical age is 21. Both the mean and the median show that this is the age that represents the centre of the data.

WORKED EXAMPLE 9 Calculating the mean, median, mode from a stem-and-leaf plot

The monthly rainfall, in mm, is shown in the stem-and-leaf plot below.

Stem	Leaf
0	0 0 0 1 1 4 4 5 9
1	0 2 2 2 2 3 3 7 8
2	2 4 6 6 6 6 9
3	0 0 1 3
4	0
5	8

Key: 2|6 means 26

Calculate the mean, median and mode.

Steps	Working
1 Find the mean using $\text{mean} = \dfrac{\text{sum of all the values}}{\text{number of values}}$ $n = 31$	$\text{mean} = \dfrac{0+0+0+1+1+4+4+5+9+10+12+\cdots+33+58}{31}$ $= \dfrac{544}{31}$ $= 17.54\ldots$ ≈ 17.5
2 There are 31 data values: $n = 31$. The median is the 16th value.	median: $\dfrac{31+1}{2} = 16\text{th data value}$

Stem	Leaf
0	0 0 0 1 1 4 4 5 9
1	0 2 2 2 2 3 3 7 8
2	2 4 6 6 6 6 9
3	0 0 1 3
4	0
5	8

median = 13

3 Find the mode(s): There are four 12s and four 26s.	There are 2 modes: 12 and 26
4 Answer the question.	The mean is 15.9, the median is 13 and the modes are 12 and 26.

EXERCISE 10.4 The mean, median and mode

ANSWERS p. 421

Recap

1 What is the largest data value in the stem-and-leaf plot?

A 17.3

B 20.9

C 59

D 209

Stem	Leaf
15	4
16	
17	1 3 3 3 5 7 9
18	4 6
19	0 1 1 4
20	5 9

Key: 18|6 means 18.6

2 Data was collected on the weights of dogs, in kg, who went to the local vet in a week. The data was recorded and placed in a grouped frequency table. Explain what the interval 5–<10 means.

Mastery

3 WORKED EXAMPLE 7 Find the mean, median and mode of each set of data.

a 28, 25, 37, 24, 33, 28, 24, 28, 35, 25, 21, 27, 22

b 3, 6, 8, 10, 5, 4, 6, 8, 8, 3, 4, 6

4 Find the mean, median and mode of each set of data.

a Shoe sizes of the Brisbane Bullets basketball players:

11 15 14 12 14 13 14

b Average speed (km/h) for the lead car in the first 10 laps of a Formula One race:

205 212 208 212 217 220 215 225 210 217

5 WORKED EXAMPLE 8 Find the mean, median and mode of each set of data.

a

Adult teeth lost	Frequency
3	2
4	4
5	6
6	10
7	12
8	8
9	6
10	5
11	2
12	1
13	3
Total	**59**

b

Age for first job	Frequency
15	5
16	9
17	8
18	12
19	15
20	9
21	6
22	4
23	1
Total	**69**

6 Find the mean, median class and modal class of each set of data.

a

Room width	Frequency
1.5–<2	2
2–<2.5	1
2.5–<3	3
3–<3.5	8
3.5–<4	6
4–<4.5	1
4.5–<5	2
5–<5.5	4
5.5–<6	5
6–<6.5	2
Total	**34**

b

Height	Frequency
140–<145	1
145–<150	3
150–<155	3
155–<160	5
160–<165	12
165–<170	10
170–<175	12
175–<180	8
180–<185	11
185–<190	5
Total	**70**

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7 WORKED EXAMPLE 9 Find the mean, median and mode of each set of data.

a

Stem	Leaf
3	7 7 8
4	0 1 4 6 6 6 7 9
5	2 2 3 7
6	1 5
7	
8	3

Key: 6|5 means 65

b

Stem	Leaf
12	3 3 5 7 8 9 9 9 9
13	0 0 2 5 8
14	2 2 6
15	1 7
16	
17	8

Key: 12|9 means 129

Exam practice

8 Stefan's maths teacher gives the class a quick quiz every week. The following frequency table shows his results.

Score	Frequency
1	1
2	2
3	4
4	6
5	1

Which expression calculates Stefan's mean score?

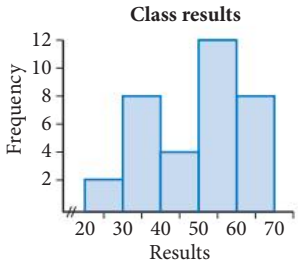
A $\frac{1+4+12+24+5}{14}$

B $\frac{1+4+12+24+5}{5}$

C $\frac{1+2+3+4+5}{14}$

D $\frac{1+2+3+4+5}{5}$

9 The histogram below shows the results of a class in an exam.



Which of the following statements is true?

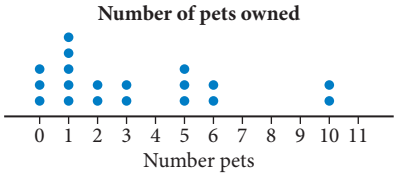
A The mode is 12.

B The median class is 50–<60.

C The highest score was 70%.

D All of the above.

10 (5 marks) The dot plot below shows the number of pets owned by students in a class.



a Find the mode, mean and median number of pets owned. *[3 marks]*

b Which of the measures of centre is the best representation for the measure of the centre? Justify your answer. *[2 marks]*

11 (5 marks) The monthly incomes of the employees of a manufacturer are shown in the table.

Income ($)	Frequency
2500–2999	12
3000–3499	29
3500–3999	44
4000–4499	39
4500–4999	38
5000–5499	25
5500–5999	17
6000–6499	9
6500–6999	5
7000–7499	2
7500–7999	1

a Find the mean, median class and modal class for the employees' incomes. *[3 marks]*

b What is the typical employee's income? Justify your answer. *[2 marks]*

12 (2 marks) CF Prisha worked out that in the last 10 weeks, the mean number of songs she had downloaded was 74 songs per week. If she downloaded 120 this week, what is her new mean?

13 (3 marks) CU The number of cars parked on a side street to a major shopping centre was recorded every hour for 8 trading hours over 5 weekdays. The results are shown below.

15	20	18	19	20	19	18	15	18	20
12	18	21	17	19	14	16	20	17	18
17	19	15	20	16	19	18	20	19	17
20	18	20	15	18	12	19	18	17	19

What was the typical number of cars parked on the side street over this period? Justify your decision.

10.5 The range and interquartile range

A key feature to look at when analysing data is the **measure of spread** of the data.

Two ways of measuring this is the **range** and the **interquartile range (IQR)**.

Range

- range = maximum − minimum
- The more spread out the data, the higher the range.
- It takes all values into account so it can be affected by **outliers**.

Interquartile range (IQR)

- Data is separated into quarters, with the boundary values Q_1, Q_2, Q_3, called **quartiles**.

25%	25%	25%	25%
Minimum	Q_1	Q_2	Q_3 **Maximum**
	Lower quartile	Median	**Upper quartile**

- 25% of the data lie below the lower quartile or first quartile Q_1
- 50% of the data lie below the median or second quartile Q_2
- 75% of the data lie below the upper quartile or third quartile Q_3
- If a quartile is between 2 values, take their average like you do for the median.
- The IQR is the range of the middle 50% of the data.

$$\text{IQR} = Q_3 - Q_1$$

- Because of this, the IQR is not affected by outliers.

Videos
Interquartile range 1
Interquartile range 2

Worksheets
Interquartile range
Statistics mode: Graphics calculator

Puzzle
Statistical measures puzzle

WORKED EXAMPLE 10 Range and interquartile range

Find the range and interquartile range of

12 17 6 9 3 3 7 8 10 19 10 12 11

Steps	Working
1 Look for maximum and minimum values.	range = maximum − minimum = 19 − 3 = 16
2 Arrange values in order.	3 3 6 7 8 9 10 10 11 12 12 17 19
3 There are 13 data values. Find the median: As $n = 13$, there will be 6 data values before the median and 6 data values after.	3 3 6 7 8 9 10 10 11 12 12 17 19 6 6 median
4 Find the middle of each half and find the quartiles. If a quartile is between 2 values, take the average of the 2.	3 3 6 | 7 8 9 10 10 11 12 | 12 17 19 3 3 3 3 $Q_1 = \frac{6+7}{2} = 6.5$ $Q_3 = \frac{12+12}{2} = 12$
5 Find the interquartile range.	IQR = $Q_3 - Q_1$ = 12 − 6.5 = 5.5
6 Answer the question.	The range is 16 and the interquartile range is 5.5.

WORKED EXAMPLE 11 Range and interquartile range from frequency table

Find the range and interquartile range for the following data set.

Score	Frequency
9	1
10	3
11	5
12	8
13	10
14	1
15	2

Steps	Working
1 Look for maximum and minimum values. The maximum value is 15 and the minimum value is 9.	range = maximum − minimum = 15 − 9 = 6
2 A cumulative frequency (cf) column added to the table will make this easier. At 9 → total is 1. At 10 → total is 4. So, 2nd, 3rd, 4th data values were 10. At 11 → total is 9. So, 5th, 6th …. up to 9th data values were 11. etc. We see the total at the bottom. There are 30 data values.	
3 Split the data in half. The middle data value is the median. As $n = 30$, the median is the 16th value (15 in each half). Split the 15s in half. 30 15 | 15 7 | 7 | 7 | 7 Q_1 ↑ Q_3 median The middle of the lower half is Q_1 = 8th value and the upper half is Q_3 = 23rd value.	Q_1 is the 8th data value = 11 Q_3 is the 23rd data value = 13
4 Find the interquartile range.	IQR = $Q_3 - Q_1$ = 13 − 11 = 2
5 Answer the question.	The range is 6 and the interquartile range is 2.

Score	Frequency	cf	
9	1	1	
10	3	4	
11	5	9	← Q_1
12	8	17	
13	10	27	← Q_3
14	1	28	
15	2	30	

WORKED EXAMPLE 12 Range and interquartile range from a stem-and-leaf plot

A manufacturer recorded the number of items produced on one machine each hour in a 24-hour period.

Stem	Leaf
19	6 7 9
20	0 0 2 5 5 8
21	1 4 4 6
22	0 0 1 3 7 7 7 8
23	4 8
24	7

Key: 19|6 means 196

a **Calculate** the range.

b **Calculate** the interquartile range.

Steps	Working
a 1 Look for maximum and minimum values. Use the key.	range = maximum − minimum = 247 − 196 = 51
2 Answer the question.	The range is 51.
b 1 Order the values, if required. This is an ordered stem-and-leaf plot so already in order.	
2 There are 24 data values. Mark the median. 24 12 | 12 Draw a line that is after the 12th data value.	Stem | Leaf 19 | 6 7 9 20 | 0 0 2 5 5 8 21 | 1 4 4 | 6 22 | 0 0 1 3 7 7 7 8 23 | 4 8 24 | 7
3 Find the middle of each half and find the quartiles. 24 12 | 12 6 | 6 | 6 | 6 Q_1 Q_3	Stem | Leaf 19 | 6 7 9 Q_1 20 | 0 0 2 | 5 5 8 21 | 1 4 4 | 6 22 | 0 0 1 3 7 | 7 7 8 23 | 4 8 Q_3 24 | 7
4 Q_1 and Q_3 are found by averaging the 2 values the lines are between.	$Q_1 = \frac{202 + 205}{2} = 203.5$ $\quad Q_3 = \frac{227 + 227}{2} = 227$
5 Find the interquartile range.	IQR = $Q_3 - Q_1$ = 227 − 203.5 = 23.5
6 Answer the question.	The interquartile range is 23.5.

EXERCISE 10.5 The range and interquartile range

ANSWERS p. 421

Recap

1 Which statement is **true** for the following data set?

3 5 4 2 8 6 4 3 7 4 3 3 7 8

A range is 5 **B** mode is 3 **C** median is 3.5 **D** mean is 67

2 The heights of workers in an office were recorded. The results are shown.

Height (cm)	Frequency
140–<145	1
145–<150	0
150–<155	2
155–<160	5
160–<165	7
165–<170	8
170–<175	8
175–<180	3
180–<185	0
185–<190	1
Total	**35**

a Find the estimated mean, median class and modal class of the heights.

b What could you report to the manager about the heights of the workers?

Mastery

3 WORKED EXAMPLE 10 Find the range and interquartile range for each set of data.

a 4, 2, 5, 2, 7, 5, 8, 8, 3, 5, 8, 3, 5, 9

b 18, 15, 14, 12, 13, 17, 18, 16, 14, 16, 17, 19, 12, 16, 16, 19

c 24, 26, 29, 33, 39, 42, 41, 27, 36, 34, 28, 27, 24, 27, 44

4 Find the range and interquartile range for each set of data.

a 28, 25, 37, 24, 33, 28, 24, 28, 35, 25, 21, 27, 22

b 3, 6, 8, 10, 5, 4, 6, 8, 8, 3, 4, 6

c 4, 7, 5, 9, 6, 8, 6, 7

5 WORKED EXAMPLE 11 Find the range and interquartile range for each set of data.

a

Score	Frequency
0	2
1	5
2	5
3	7
4	7
5	6
6	6
7	3
8	2
9	1

b

Score	Frequency
25	3
26	6
27	10
28	16
29	22
30	20
31	17
32	6
33	4

6 WORKED EXAMPLE 12 Find the range and interquartile range for each set of data.

a

Stem	Leaf
0	3 3 4 5 5 7 8 9
1	2 4 4 5
2	3 4 8 9 9
3	0 0 2
4	3 3 3 5
5	9

Key: 2|3 means 23

b

Stem	Leaf
12	8
13	3 4 6 6 8
14	0 1 2 2 5 7 9
15	3 3 3
16	
17	8

Key: 12|8 means 12.8

Exam practice

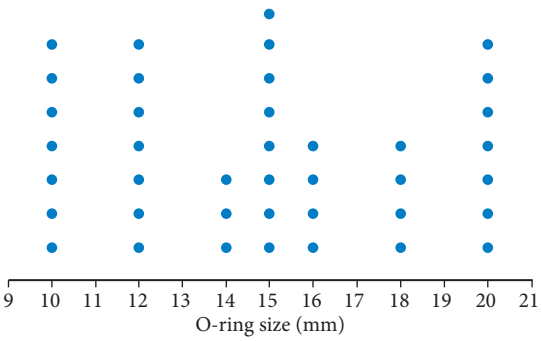
7 The interquartile range for this data is

A 12

B 10

C 6

D 15



8 (4 marks) The council ran a free sports day in the school holidays. The ages of the participants are shown in the frequency table.

Score	Frequency
10	6
11	10
12	12
13	13
14	6
15	2
16	0
17	0
18	1

a Calculate the range. *[1 mark]*

b Calculate the interquartile range. *[1 mark]*

c The company running the sports day wrote a report for the council. Should they use the range or interquartile range to represent the spread of the data? Justify your answer. *[2 marks]*

9 (4 marks) CF The times (in seconds) taken for 26 job applicants to complete a manual dexterity test are shown.

14 11 10 12 19 14 5 17 12 10 11 19 15

19 10 21 14 27 16 23 15 14 18 20 17 19

Within how many seconds would the middle half of the applicants complete the test? Justify your answer.

10 (3 marks) CU Tena and Juan are employed as quality controllers at a factory where computer motherboards are assembled. The number of motherboards that they each inspect each hour over a 9-hour period are shown below.

Tena: 30, 31, 29, 30, 29, 23, 33, 35, 30

Juan: 28, 30, 26, 12, 29, 31, 35, 48, 31

Who is the more consistent worker? Explain your reasoning.

Skillsheet
Standard deviation on a calculator

Worksheets
Standard deviation

Statistical calculations

Statistics review

Calculating and interpreting summary statistics

10.6 The standard deviation

The range and interquartile range use only 2 values of a dataset to measure spread. It is better to use all the data to measure the spread.

Standard deviation, s_x

- The **standard deviation** measures the spread of data around the mean and uses all the data values.
- The STAT function on a scientific calculator can find the standard deviation, as well as the mean.
- The lower the standard deviation, the more consistent the data values are. The higher the standard deviation, the more spread out the data.

There are 2 types of standard deviation: sample (s_x) and population σ. The sample standard deviation is used in this book and course.

Measures of spread

- The best measure of spread to use depends on the situation.
- The standard deviation is the best measure of spread as it uses every data value, BUT this means it can be affected by outliers.
- The interquartile range only looks at the middle 50% of the data, so it is not affected by outliers.
- The range is easy to calculate but only uses 2 data values. This limits the information about spread within the data set. It can be affected by outliers.

WORKED EXAMPLE 13 Standard deviation and mean

Find, correct to one decimal place, the mean and standard deviation of

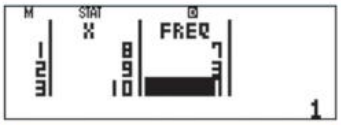
87, 69, 95, 73, 88, 47, 95, 63, 91, 66, 59, 70, 67, 71, 57, 82, 65, 84, 69, 83

Steps	Working
1 Enter the data using the STAT mode of your calculator.	
2 Then find the mean, $\bar{x}$ and s_x.	$\bar{x} = 74.05$ $s_x = 13.40\ldots$
3 Round to one decimal place.	$\bar{x} \approx 74.1$ $s_x \approx 13.4$

WORKED EXAMPLE 14 Standard deviation and mean from frequency table

Find, correct to 2 decimal places, the mean and standard deviation of the following scores.

Score	Frequency
8	7
9	3
10	6
11	8
12	2
13	4
14	1
15	8

Steps	Working
1 Enter the data in the STAT mode of your calculator.	
2 Then find the mean, $\bar{x}$ and s_x.	$\bar{x} = 11.307\ldots$ $s_x = 2.472\ldots$
3 Round to 2 decimal places.	$\bar{x} \approx 11.31$ $s_x \approx 2.47$

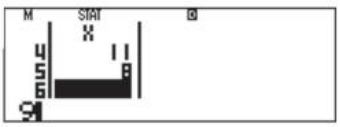
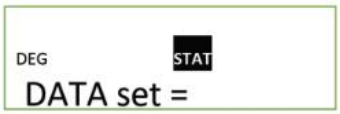
WORKED EXAMPLE 15 Interpreting standard deviation

The shoe sizes of men's shoes sold one Friday in 2 different stores are shown.

City store: 8, 9, 10, 11, 8, 9, 10, 13, 10, 12, 10, 9, 8, 6, 8, 9, 10, 10

Suburban store: 6, 8, 10, 12, 10, 8, 9, 9, 9, 6, 8, 10, 12, 11

Which store has the greater concentration of sizes sold? **Justify** your answer.

Steps	Working
1 Enter the data for each store into the STAT mode of your calculator.	 
2 Find the mean, $\bar{x}$ and s_x.	City: $\bar{x} \approx 9.44$, $s_x \approx 1.62$ Suburban: $\bar{x} \approx 9.14$, $s_x \approx 1.88$
3 Define 'concentration' and make a decision.	More concentrated means that the data would be more tightly grouped together. This means that the standard deviation would be lower.
4 Write the answer.	The sizes sold in the city were more concentrated as the standard deviation was lower.

Exam hack

Look at the practical situation to see if being more spread out or closer together is what you are looking for.

EXERCISE 10.6 The standard deviation

ANSWERS p. 421

Recap

1 Calculate the interquartile range of the data in this stem-and-leaf plot.

A 10

B 15

C 21

D 52

Stem	Leaf
36	7 8 9
37	4 4 6 8
38	0 0 1 3 5 9
39	1 1 5 7
40	3 5 5 5
41	9

Key: 36|7 means 367

2 **a** Calculate the range and interquartile range of the following data.

14.2, 12.4, 15.1, 14.3, 12.7, 15.6, 15.2, 18.1, 25.4, 12.4, 14.1, 15.8, 14.1

b Explain why there is such a big difference in the 2 measures of spread. Which would be the more appropriate one here?

Mastery

Use the STAT mode of your calculator for this exercise, rounding your answers to 2 decimal places where necessary.

3 WORKED EXAMPLE 13 Find the mean and standard deviation of each set of data.

a 11, 6, 6, 11, 7, 8, 9, 10, 7, 12

b 8, 6, 3, 4, 7, 7, 6, 3, 0, 10, 7, 5

c 16, 16, 13, 18, 14, 12, 12, 14, 16, 17, 12, 12, 17, 16, 16, 13

4 Find the mean and standard deviation of each set of data.

a 12, 12, 9, 8, 15, 8, 11, 12, 10, 12, 8, 15

b 10, 7, 6, 13, 6, 10, 5, 13, 6, 16, 7, 14, 8, 14, 6

c 20.8, 25.4, 23.1, 24.4, 18.9, 20.3, 18.5, 23.8, 20.6, 22.6

5 WORKED EXAMPLE 14 Find the mean and standard deviation of

a

Score	Frequency
8	7
9	3
10	6
11	8
12	2
13	4
14	1
15	8

b

Score	Frequency
3.9	1
4.0	2
4.1	4
4.2	6
4.3	3
4.4	0
4.5	0
4.6	2

6 WORKED EXAMPLE 15 Determine which set of data is more consistent.

A: 5, 9, 13, 9, 13, 13, 0, 10, 2, 7

B: 9, 9, 4, 7, 6, 11, 9, 14, 13, 7, 11, 2

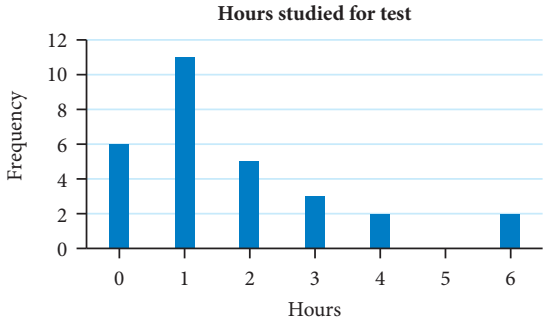
7 Determine which set of data is more varied.

A: 13, 47, 27, 23, 24, 28, 26, 13, 11, 23

B: 15, 26, 21, 25, 25, 22, 22, 27, 26, 28, 23, 25

Exam practice

8 A teacher asked a sample of students how long they had studied for their final exam.



The standard deviation, correct to 2 decimal places, is

A 1.62 **B** 1.64 **C** 1.65 **D** 1.72

9 (6 marks) The data shows the number of errors that machine operators had made in a week.

5	2	0	4	3	9	4	0	2	1
1	3	4	8	3	0	0	1	0	8
7	0	0	4	5	3	9	7	6	2

Calculate the

a measures of centre – mean, median and mode. *[3 marks]*

b measures of spread – range, interquartile range and standard deviation. *[3 marks]*

10 (3 marks) CF Two brands of light globe are tested to see how long they will last (in hours).

Brand X:	1100	900	995	1050	1000	1100	960	875	900	1425
Brand Y:	975	1000	1050	1100	950	1020	990	975	1050	990

Which of the 2 brands is more reliable? Justify your answer.

11 (5 marks) CU Sam is studying the house sale statistics in his suburb. The sales in the last month are shown.

$845 000	$1 250 000	$955 000	$1 130 000
$1 050 000	$980 000	$1 100 000	$970 000
$3 600 000	$1 080 000	$845 000	$975 000
$880 000	$900 000	$1 150 000	$980 000

Find the measures of centre and spread that best describe this data. Justify your decisions.

Videos
The shape of a frequency distribution

Outliers

Worksheet
Shapes of distributions

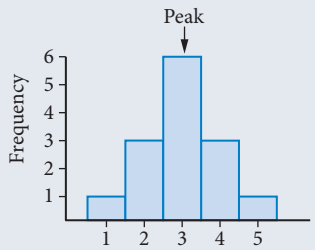
10.7 The shape of a distribution

The way data is spread is referred to as the shape of the data **distribution**. The shape of a distribution can be **symmetrical** or **skewed** and can also have added features such as being **multimodal** or having **outliers**. Looking at the visual displays of the data allows the shape of the distribution to be seen.

The shape of a distribution

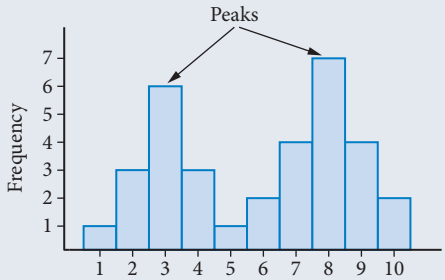
Symmetrical

- A **symmetrical distribution** has one mode.
- It is roughly the same on either side of the mode.



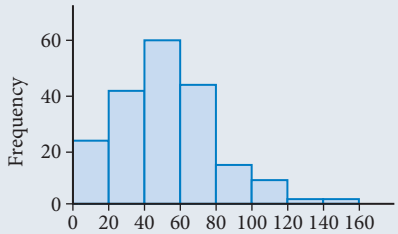
Multimodal

- A **multimodal distribution** has more than one distinct peak.
- The peaks do not have to be of the same height.
- A bimodal distribution (2 peaks) is the most common.



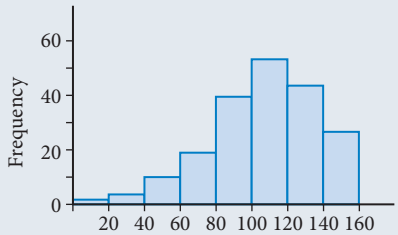
Positively skewed

- A **positively skewed** distribution has a long tail on the **right-hand** (positive) side.
- It has the data grouped together near the start.



Negatively skewed

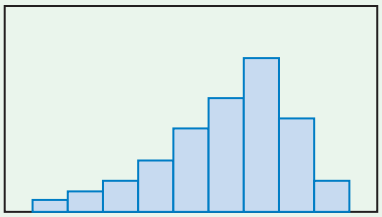
- A **negatively skewed** distribution has a long tail on the **left-hand** (negative) side.
- It has more data grouped together at the higher end.



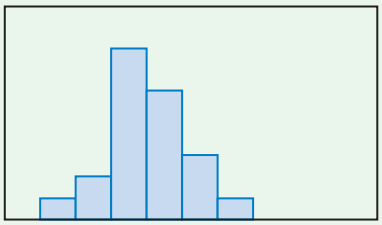
WORKED EXAMPLE 16 Describing shapes of distributions

Describe the shape of each distribution shown.

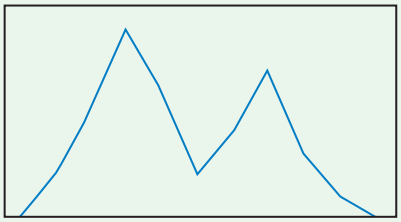
a



b



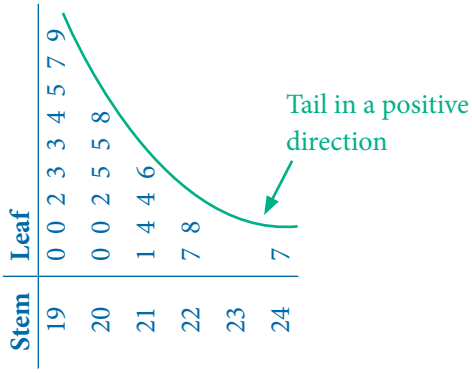
c



d

Stem	Leaf
19	0 0 2 3 3 4 5 7 9
20	0 0 2 5 5 8
21	1 4 4 6
22	7 8
23	
24	7

Key: 19|2 means 19.2

Steps	Working
a The graph has a long tail towards the negative part of the number line.	The distribution is negatively skewed.
b The graph has one peak and is close to balanced on each side.	The distribution is approximately symmetrical.
c The graph has 2 distinct peaks.	It is a bimodal distribution.
d 1 If you look at this side-on, you can see the shape. The data is grouped at the lower end: there is a tail in the positive direction.	
2 Answer the question.	It is a positively skewed distribution.

In some numerical datasets, a single value or a group of values may be quite distant from the rest of the values. If these extreme values are accurate and correct and are not due to error, then they are **outliers**.

Outliers

- An extreme value that stands out from other data values could be an outlier.
- It is important to test if a value is an outlier – it could be legitimate, or there could be errors in data collection.
- One test uses the quartiles and the interquartile range. Boundary values, known as the **lower fence** and **upper fence**, are calculated. If the data value lies outside the fence, then it is an outlier.
- The lower fence is calculated using $Q_1 - 1.5 \times \text{IQR}$.
- The upper fence is calculated using $Q_3 + 1.5 \times \text{IQR}$.

WORKED EXAMPLE 17 Identifying outliers

Determine if there are any possible outliers in each set of data.

a 26, 21, 19, 23, 18, 8, 16, 20, 20, 25, 23, 20

b

Stem	Leaf
19	0 0 2 3 3 4 5 7 9
20	0 0 2 5 5 8
21	1 4 4 6
22	7 8
23	
24	7

Key: 19|2 means 19.2

You say '*possible* outlier' because you can never be 100% certain whether the extreme value was measured correctly or with error.

Steps	Working
a 1 Arrange the data in order.	
Break data into quartiles.	
12	8, 16, 18, | 19, 20, 20, | 20, 21, 23, | 23, 25, 26
6 | 6	
3 | 3 | 3 | 3	
2 State the quartiles.	$Q_1 = \frac{18+19}{2} = 18.5$ $\quad Q_3 = 23$
3 Find the interquartile range.	IQR = 23 – 18.5 = 4.5
4 Find the 'fences'.	$Q_1 - 1.5 \times \text{IQR} = 18.5 - 1.5 \times 4.5 = 11.75$ $Q_3 + 1.5 \times \text{IQR} = 23 + 1.5 \times 5.5 = 29.75$
5 Mark the fences on the data and see if any values lie outside of the 'fences'.	8, | 16, 18, | 19, 20, 20, | 20, 21, 23, | 23, 25, 26 | lower fence upper fence
6 State the answer.	The value 8 is outside the fence, so it is a possible outlier.
b 1 The data is already in order.	
2 Break data into quartiles	
22	
11 | 11	
5 | 5 | 5 | 5	

Stem	Leaf
19	0 0 2 3 3 4 5 7 9
20	0 0 | 2 5 5 8
21	1 4 4 6
22	7 8
23	
24	7

Key: 19|2 means 19.2

3 State the quartiles.	$Q_1 = 19.4$ $\quad Q_3 = 21.4$
4 Find the interquartile range.	IQR = 21.4 – 19.4 = 2
5 Find the fences.	$Q_1 - 1.5 \times \text{IQR} = 19.4 - 1.5 \times 2 = 16.4$ $Q_3 + 1.5 \times \text{IQR} = 21.4 + 1.5 \times 2 = 24.4$

6 Mark the fences on the data. Check if any values are outside the 'fences'.

lower fence (16.4)

Stem	Leaf
19	0 0 2 3 3 4 5 7 9
20	0 0 | 2 5 5 8
21	1 4 4 6
22	7 8
23	
24	7

upper fence (24.4)

Key: 19|2 means 19.2

7 State the answer.

The value 24.7 is outside the upper fence, so it is a possible outlier.

Analysing data

When you analyse a set of data, it is important that you do the following:

- Organise the data into a frequency table if necessary.
- Display the data in an appropriate visual display.
- Calculate **summary statistics** of the data, such as measures of centre (mean, median or mode) and measures of spread (range, interquartile range or standard deviation).
- Look at the shape of the data.
- Look for any unusual values. Test to see if they are **outliers**.

WORKED EXAMPLE 18 Analysing a data set

The heights, in cm, of a group of 24 students were recorded in the stem-and-leaf plot.

Stem	Leaf
14	1
15	5 6 6 8 9 9
16	0 0 1 2 5 5 6 7 8 8 8
17	0 0 1 1 1
18	5

Key: 15|6 means 156

a Find the

i mode **ii** mean **iii** median

iv range **v** interquartile range **vi** standard deviation

b Is there a possible outlier? **Justify** your answer.

c What is the shape of the distribution?

d Which measure of centre best represents the data?

e Which measure of spread best represents the data?

Steps	Working
a i There are three 168s and 171s.	Data has 2 modes: 168 cm and 171 cm.
ii Use the STAT function on your calculator.	$\bar{x} = 163.833\,33 \approx 163.8$ cm
iii 1 Find the middle of the data. 24; 12 | 12	Stem-and-leaf: 14: 1; 15: 5 6 6 8 9 9; 16: 0 0 1 2 5 | 5 6 7 8 8 8; 17: 0 0 1 1 1; 18: 5
2 Median is the average of 165 and 165.	Median is 165 cm.
iv Find maximum and minimum values to calculate the range.	range = max − min = 185 − 141 = 44 cm

v 1 Break data into quartiles

24
12 | 12
6 | 6 | 6 | 6

Stem	Leaf
14	1
15	5 6 6 8 9 | 9
16	0 0 1 2 5 | 5 6 7 8 8 8 |
17	0 0 1 1 1
18	5

2 State the quartiles.

$Q_1 = 159$ $\quad Q_3 = \dfrac{168 + 170}{2}$

$= 169$

3 Find the interquartile range.

$\text{IQR} = Q_3 - Q_1$

$= 169 - 159$

$= 10\,\text{cm}$

vi Use STAT function on your calculator.

$s_x = 8.317\ldots \approx 8.3$

b 1 Find the fences.

$Q_1 - 1.5 \times \text{IQR} = 159 - 1.5 \times 10 = 144$

$Q_3 + 1.5 \times \text{IQR} = 169 + 1.5 \times 10 = 184$

2 Mark the lower fence and upper fence on the stem plot and see if any values are outside of the fence.

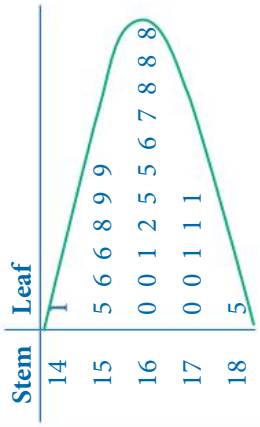
Stem	Leaf
14	1 |
15	5 6 6 8 9 9
16	0 0 1 2 5 5 6 7 8 8 8
17	0 0 1 1 1
18	| 5

3 State the answer.

The values 141 cm and 185 cm are outside the fences, so they are both possible outliers.

c Look at the shape of the distribution (by turning the plot sideways).

symmetrical



d Outliers: median not affected

The mode does not represent this data well.
The mean and the median are very close, BUT the median is the best measure of spread as it is not affected by the 2 outliers.

e Outliers: IQR not affected

The range and standard deviation are both affected by the outliers. Therefore, the interquartile range is the best measure of spread as it is not affected by the 2 outliers.

EXERCISE 10.7 The shape of a distribution

ANSWERS p. 422

10.7

Recap

1 Which of the following data sets will have the lowest standard deviation?

A

Stem	Leaf
14	0
15	1
16	9 9 9
17	1 2 3 4 4 4 5 7 8
18	1 2 3 4 4 5 5 6 8 9

B

Stem	Leaf
14	1
15	5 6 6 8 9 9
16	0 0 1 2 5 5 6 7 8 8 8
17	0 0 1 1 1
18	5

C

Stem	Leaf
14	1 2 3 4
15	0 3 5 7 9
16	0 1 3 4 6 7
17	0 0 1 1 1
18	0 1 2 3

D

Stem	Leaf
14	1 2 2 3 5 6 6 7 7 8
15	0 3
16	
17	0 4 5 8
18	0 1 2 3 5 7 8 9

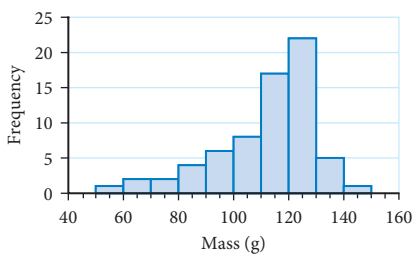
2 Calculate the mean and standard deviation of the following IQ scores for a class of students. Answer correct to one decimal place.

107 100 94 99 103 115 89 102 108 91 72 97 121 113 75 108 115 106 109 110

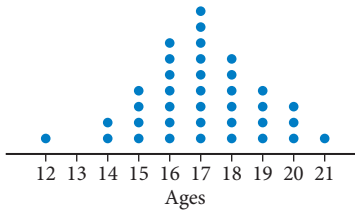
Mastery

3 WORKED EXAMPLE 16 Describe the shape of each distribution shown.

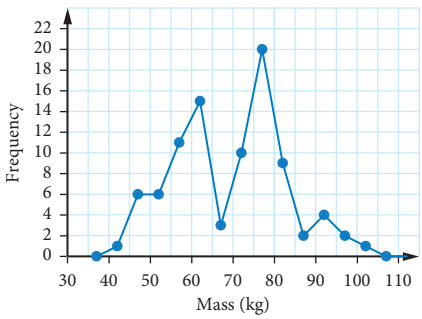
a



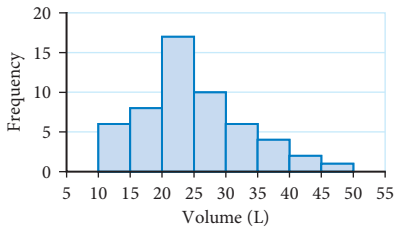
b



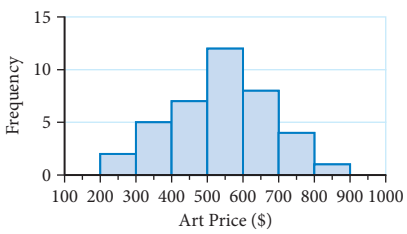
c



d



e



f

Stem	Leaf
14	0
15	1
16	1 3
17	1 2 3 4 4 4 5 7 8
18	1 2 3 4 4 5 5 6 8 9

Key: 15|6 means 156

4 WORKED EXAMPLE 17 Following from question **2**, the IQ scores for a class of students are given:

107 100 94 99 103 115 89 102 108 91 72 97 121 113 75 108 115 106 109 110

a Calculate the interquartile range.

b Calculate the value of the fences.

c Does the data contain any possible outliers? Justify your results.

5 Use the test to find possible outliers in each data set.

a 34.4, 48.1, 43.4, 42.6, 42.5, 6.1, 61.3, 31.6, 39.5, 56.1, 43.7, 37.2, 39.5

b

Stem	Leaf
14	0
15	1
16	1 3
17	1 2 3 4 4 5 7 8
18	1 2 3 4 4 5 5 6 8 9

Key: 15|1 means 151

6 WORKED EXAMPLE 18 The masses, in kg, of 40 dogs at a kennel were recorded.

3	15	8	24	70	25	8	4	37	21
18	34	28	9	11	7	21	15	19	11
27	26	24	8	4	16	12	21	15	38
16	5	18	24	31	23	24	13	9	34

a What type of data is the mass of a dog? Explain.

b Organise this data into a stem-and-leaf plot.

c Comment on the distribution of the data.

d Are there any possible outliers? Justify your decision.

e Calculate a measure of centre that you think is appropriate. Justify your decision.

f Calculate a measure of spread that you think is appropriate. Justify your decision.

Exam practice

7 Which statement below is correct?

A There are no possible outliers.

B $86 is a possible outlier.

C $148 is a possible outlier.

D Both $86 and $148 are possible outliers.

Stem	Leaf
8	6
9	
10	0 0 5 8 8
11	0 0 4 4 4 7
12	1 2 4 6
13	
14	8

Key: 8|6 means $86

8 (7 marks) The speeds of motorists were checked by police radar. The results are shown.

98	115	102	91	104	98	95	89	100	95
100	102	95	102	97	95	86	98	105	97

a Draw a display and comment on the shape of the distribution. *[3 marks]*

b Calculate the interquartile range. *[1 mark]*

c Calculate the lower and upper fence. *[1 mark]*

d Are there any outliers? Justify your decision. *[2 marks]*

9 (3 marks) CF The following data represents maximum temperatures in degrees over a 2-week period.

27 29 26 28 30 28 29 32 30 29 27 30 22 32

Justify why the temperature of 22 is an outlier.

10 (6 marks) CU A student recorded the ages of residents in an aged care facility.

81	92	78	93	84	98	79	90	79	98	81	94
83	89	72	93	76	55	93	81	103	82	89	78
90	72	93	80	87	76	86	72	88	75	91	80
92	79	85	100	80	99	77	95	78	84	99	87

Comment on any unusual features.

Video
Exam question analysis: Analysing data

EXAM QUESTION ANALYSIS

Exam-style question (7 marks)

The response time in seconds to calls to emergency telephone operators are shown in the stem-and-leaf plot below.

Stem	Leaf
2	2 3
3	1 4 8
4	0 1 4 4 5 5 5 7 7
5	0 0 1 2 2 3 3 4 5 8 9
6	0 2 3
7	9

Key: 3|1 means 31 seconds

a Calculate the interquartile range. *[2 marks]*

b Determine whether there are any **outliers**. Justify your answer. *[3 marks]*

c Determine the best **measure of centre** for representing this dataset. Justify why this is the most appropriate one. *[2 marks]*

Reading the question

- Key concepts you need to be clear on are the interquartile range, how to test for outliers and measures of centre.
- **Justify** means you need to show the maths to back up your decision.

Thinking about the question

- Interquartile range is 2 marks. So, one for finding Q_1 and Q_3. The second mark is for the IQR.
- Remember the formula for the outlier fences is on the formula sheet.
- Measure of centre: choose from mode, mean or median. Need to think of when each is best.

Worked solution (✓ = 1 mark)

a

29

14 | 14

7 | 7 | 7 | 7

Q_1 ↑ Q_3

median

Stem	Leaf
2	2 3
3	1 4 8
4	0 1 | 4 4 5 5 5 7 7
5	0 0 1 2 2 3 3 4 | 5 8 9
6	0 2 3
7	9

Key: 3|4 = 34

$Q_1 = \frac{41 + 44}{2} = 42.5$ $\quad Q_3 = \frac{54 + 55}{2} = 54.5$ ✓ for both

IQR = **54.5 − 42.5 = 12** ✓

The interquartile range is 12 seconds.

Stem	Leaf
2	2 3 | lower fence = 24.5
3	1 4 8
4	0 1 |4 4 5 5 5 7 7
5	0 0 1 2 2 3 3 4 |5 8 9
6	0 2 3
7	|9

upper fence = 72.5

b lower fence: $Q_1 - 1.5 \times IQR = 42.5 - 1.5 \times 12 = 24.5$ ✓

upper fence: $Q_3 + 1.5 \times IQR = 54.5 + 1.5 \times 12 = 72.5$ ✓

∴ there are 3 possible outliers: **22, 23 and 79 seconds**. ✓

c The mean is affected by the outliers. The median is not. The mode is not affected BUT also not a great measure of centre. ✓

The median is 50. This is the best measure of centre as it is not affected by the 3 possible outliers. ✓

10 Chapter summary

Types of data

Categorical data are categories.

- **Nominal data** has no order.
- **Ordinal data** has an order; e.g. A, B, C or 1st, 2nd, 3rd and 4th preference.

Numerical data are numbers.

- **discrete data** has only particular values; e.g. goals scored or number of people
- **continuous data** can have any values.

Frequency table

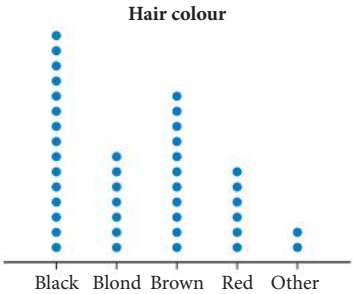
Data

Hair colour	Tally	Frequency
Black	𝍸 𝍸 𝍸	15
Blond	𝍸 ||	7
Brown	𝍸 𝍸 |	11
Red	𝍸 |	6
Other	||	2
Total		**41**

There were 41 answers given.

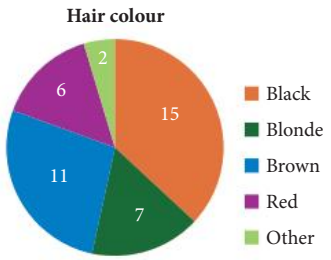
Dot plots

- Useful as long as the data is not too spread out or the data set is not too large.
- Can be used for categorical and discrete data.



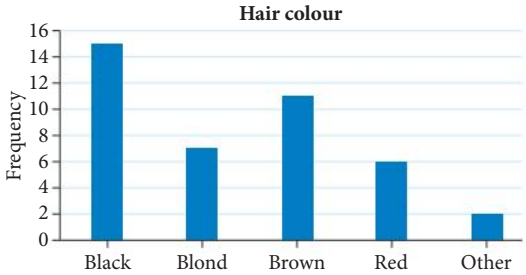
Pie charts

- Only suitable for categorical data.
- The angle for each value $= \dfrac{\text{frequency}}{\text{total}} \times 360°$



Bar/column charts

- Can be used for categorical and discrete data.



Grouped frequency tables

- When numerical data has a large range of values, the data is grouped into class intervals.
- The intervals must all be the same size.
- The class centre is found by averaging the lowest and highest value of the interval; e.g. $\dfrac{35 + 40}{2} = 37.5$

Ages	Class centre	Tally	Frequency
35–<40	37.5	𝍸	5
40–<45	42.5	𝍸 ||	7
45–<50	47.5	𝍸	5
50–<55	52.5	||||	4
55–<60	57.5	||	2
Total			**23**

Stem-and-leaf plots

- The 'stem' is the leading digits and the 'leaf' is the last digit.
- There is always a title and a key.
- If there is a lot of data, the stem may be split in half.

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4
4	6 7 8 9 9
5	1 2 2 3
5	5 7

This is 40 to 44.

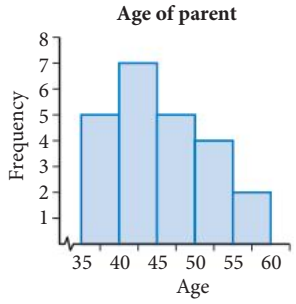
This is 45 to 49.

Key: 3|5 means 35

Puzzle
Data and statistics crossword

Histograms

- They are like column charts but do *not* have spaces between the columns.
- They have a half-column gap at the beginning.



Measures of centre

- It's an advantage that the mean uses all data, BUT it can be affected by outliers.
- Median looks at the middle value and is not affected by **outliers**.
- Mode is not as useful – depends on the situation.

Mean ($\bar{x}$)

- The mean is the average of the data values.

$$\text{mean} = \frac{\text{sum of all the values}}{\text{number of values}} = \frac{\sum x}{n}$$

- When the data is represented in a frequency table:

$$\text{mean} = \frac{\text{total of } f \times x \text{ column}}{\text{total of the } f \text{ column}} = \frac{\sum fx}{\sum f}$$

- As the mean of grouped data is calculated using the class centre, it only gives an estimate of the mean.

Median

- The median is the middle value when all data are written in order from smallest to largest.
- If there is an even number of data values, then it is the average of the middle 2 values.
- In general, the median is the $\left(\frac{n+1}{2}\right)$th value.
- If there is a large number of data values in a frequency table, it may be necessary to look at the cumulative frequency to find the median.

Mode

- The mode is the most common data value or category, with the highest frequency.
- There can be more than one mode.
- For grouped data, the modal class is the group that has the highest frequency.

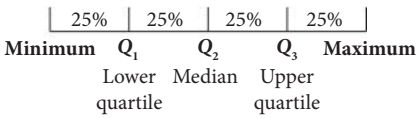
Measures of spread

- The more spread out the data, the higher the measure of spread.
- The more bunched together the data, the lower the measure of spread, meaning it is more consistent.

Range

- range = maximum – minimum
- It takes all values into account so can be affected by outliers.

Interquartile range (IQR)



- Data is separated into quarters.
- If a quartile is between 2 values, take their average like for the median.
- The IQR is the range of the middle 50% of the data.

$$\text{IQR} = Q_3 - Q_1$$

- Because of this, the IQR is not affected by outliers.

Standard deviation, s_x

- Uses all the data values.
- Use the STAT function on your calculator.

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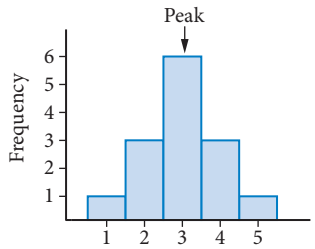
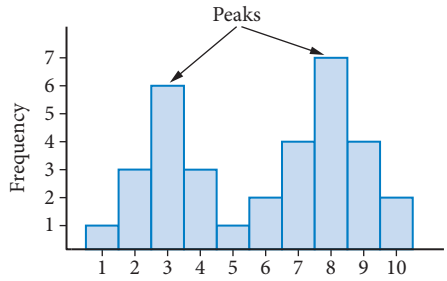
The shape of a distribution

Symmetrical



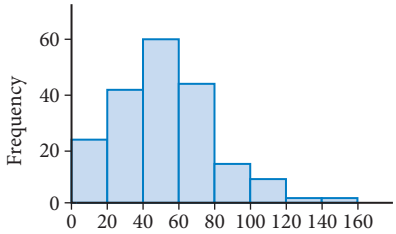
- A **symmetrical** distribution has one mode.

Multimodal



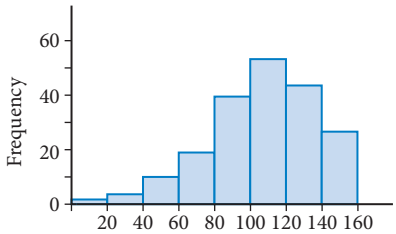
- A multimodal distribution has more than one distinct peak.
- The peaks do not have to be of the same height.
- A bimodal (2 peaks) is the most common.

Positively skewed



- A **positively skewed** distribution has the data grouped together near the start.
- It has a long tail on the right-hand (positive) side.

Negatively skewed



- A **negatively skewed** distribution has more data grouped together at the higher end.
- It has a long tail on the left-hand (negative) side.

Outliers

- It is important to see if a value is an outlier – it could be legitimate, or there could be errors in data collection.
- The outlier test uses the quartiles and the **interquartile range**. Boundary values, known as the **lower fence** and **upper fence**, are calculated. If the data value lies outside the fence, then it is an outlier.
- Lower fence = $Q_1 - 1.5 \times \text{IQR}$
- Upper fence = $Q_3 + 1.5 \times \text{IQR}$

Analysing data

- Organise the data into a frequency table if necessary.
- Display the data in an appropriate visual display.
- Calculate measures of centre (mean, median or mode) and measures of spread (range, interquartile range or standard deviation).
- Look at the shape of the data.
- Look for any unusual values. If there are any, test to see if they are possible outliers.

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 5 short response questions

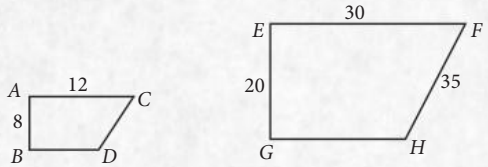
Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 The 2 shapes are similar.

The length of CD is



A 14 **B** 17

C 20 **D** 23

2 Researchers conducted a survey of 156 retail workers to determine if their happiness in their job was related to their gender. Their happiness was rated from: 1 (unhappy) to 5 (very happy) and their gender was: male, female, non-binary, unknown.

In this survey, the variables happiness and gender are

A both categorical variables.

B both numerical variables.

C numerical and categorical variables respectively.

D neither numerical nor categorical variables.

3 In 10 different soccer games, Harmony scored the following number of goals:

0 2 0 0 4 2 1 2 0 0

Her median score was

A 0 **B** 0.5 **C** 1.1 **D** 2.5

4 A score of 37 was added to this data. Which of the following measures will change?

Score	Frequency
33	2
34	4
35	6
36	1
37	1

A range **B** median **C** mode **D** mean

5 Simplify $5 - 3(x + 1)$.

A $2x + 1$ **B** $2x + 2$ **C** $2 - 3x$ **D** $8 - 3x$

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Section 2 5 short response questions

13 marks

6 (4 marks) Harley surveyed the people in his class on their favourite type of pet. The results are shown.

cat	dog	snake	snake	cat	dog	dog	dog	dog	horse
dog	horse	dog	cat	rat	horse	snake	rat	cat	fish
horse	bird	fish	fish	cat	dog	fish	cat	bird	dog

a What type of data is this? *[1 mark]*

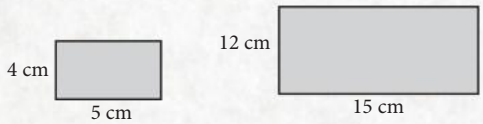
b Construct a frequency table to represent this data. *[2 marks]*

c What is the mode? *[1 mark]*

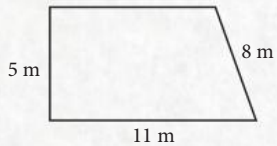
7 (2 marks) Draw a histogram of this frequency table.

Height	Class centre	Tally	Frequency
150-<155	152.5	||	2
155-<160	157.5	|||	3
160–<165	162.5	~~||||~~ ||	7
165–<170	167.5	~~||||~~	5
170–<175	172.5	||	2
175–<180	177.5	|	1
Total			**20**

8 (2 marks) Calculate the area scale factor for these similar rectangles.



9 (3 marks) Joe is building a fence around his chicken coop. Calculate the length of fencing he will need by finding the perimeter of the coop shown. Answer correct to one decimal place.



10 (2 marks) Xi installed a cylindrical water tank in her backyard. Calculate the amount of water that it holds, to the nearest litre, if the tank is 3 metres tall and has a diameter of 2.4 metres.

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions

12 marks

1 (3 marks) Aiden performed the following matrix multiplication:

$$\begin{bmatrix} 4 & 2 \\ 2 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \text{His answer looked like} \begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}.$$

Without even looking at the numbers, his teacher knew he was incorrect.

His teacher told him to go back and look at the **sizes of the matrices** and then return with an explanation of how he should have known it was incorrect.

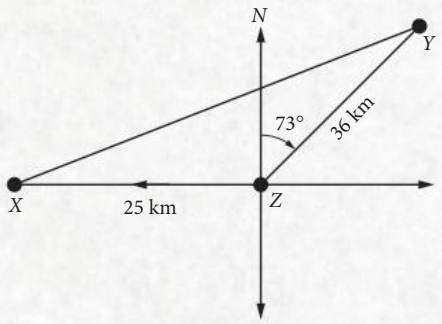
Write an explanation that Aidan could have used for his teacher.

2 (5 marks) The shoe sizes of a group of Year 11 students are shown.

8	9	5	6	8	7	6	8	12	9	8	7	6
10	8	6	7	5	8	11	6	8	9	10	8	8

What is the typical shoe size of this group? Justify your answer.

3 (4 marks) The diagram shows the locations of 3 towns: X, Y and Z.
Town X is due west of Z, and the bearing of Y from Z is 073°.



Calculate the distance directly from X to Y.

9780170484749

CHAPTER

11 THE SINE AND COSINE RULES

Syllabus coverage
Nelson MindTap chapter resources
Terminology
11.1 The sine rule
11.2 The sine rule for angles
11.3 The cosine rule
11.4 The cosine rule for angles
11.5 Triangle area formulas
11.6 Applications of trigonometry
Exam question analysis
Chapter summary
Cumulative examination 1
Cumulative examination 2

Prior learning
The sine and cosine rules

Syllabus coverage

UNIT 2, TOPIC 2: APPLICATIONS OF TRIGONOMETRY

Applications of trigonometry

- Calculate the area of a non-right-angled triangle and solve related practical problems.
  - area $= \frac{1}{2}bc\sin A$, given two sides, b and c, and an included angle, A
  - Heron's rule: area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$, given three sides, a, b and c
- Solve two-dimensional problems involving a non-right-angled triangle, $\triangle ABC$, with sides, a, b and c, and corresponding angles, A, B and C.
  - sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (ambiguous case excluded)
  - cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$
- Solve two-dimensional practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings.

Modified from General Mathematics 2025 v1.1 General senior syllabus 2025. https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/general-mathematics © State of Queensland (QCAA) 2025 https://www.qcaa.qld.edu.au/ copyright 2024, licensed under CC 4.0 https://creativecommons.org/licenses/by/4.0/

Videos (6):

11.1, 11.2 The sine rule
11.3 The cosine rule
11.4 The cosine rule for angles 1
11.5 The sine area formula
11.6 The sine and cosine rules
Exam question analysis The sine and cosine rules

Prior learning (1):

11.1 The sine and cosine rules

Worksheets (9):

11.1 Trigonometric calculations
• The sine rule – Finding lengths of sides
11.2 The sine rule – Finding angles
11.3 Finding an unknown side
11.4 The cosine rule – Angles and sides
• Finding an unknown angle
11.6 Sine rule problems • Cosine rule problems
Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

cosine rule	degree mode	Heron's rule	included angle
inverse trigonometric function	non-right-angled triangle	semi-perimeter	sine rule

11.1 The sine rule

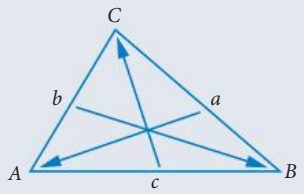
In Chapter 8 you used trigonometry to solve problems involving right-angled triangles.
For **non-right-angled triangles**, you can use the **sine rule** or **cosine rule**.

The sine rule

For any triangle $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b, c are the sides opposite the respective angles A, B, C.
You use the sine rule in problems involving 2 sides of a triangle and the angles *opposite* them.



To solve problems using the sine rule:

1. Draw a diagram, if not given.
2. Label the angles A, B, C and sides a, b, c.
3. Write the sine rule.
4. Substitute in the given values.
5. Solve for the unknown.
6. Answer the question.

Make sure your calculator is in **degree mode**.

Drawing arrows will help to check you have matching pairs.

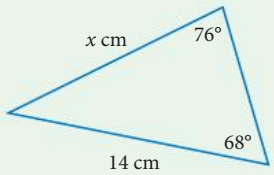
Video
The sine rule

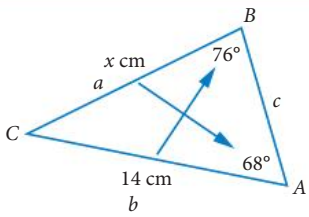
Worksheets
Trigonometric calculations

The sine rule – Finding lengths of sides

WORKED EXAMPLE 1 Finding a side with the sine rule

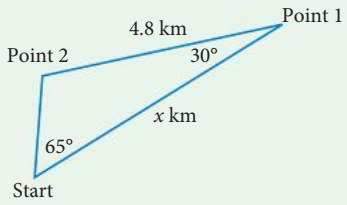
Calculate the value of x, correct to one decimal place.



Steps	Working
1 Look at the angles and sides being used. Draw arrows to match up the 2 sides to their opposite angle.	
2 Label the angles A, B, and the sides a, b.	
Put a on the unknown side and b on the given side.	
3 Write the sine rule for the angles being used.	$\frac{a}{\sin A} = \frac{b}{\sin B}$
4 Substitute in the given values.	$\frac{x}{\sin 68°} = \frac{14}{\sin 76°}$
When solving problems using the sine rule, side c and angle C are not required.	
5 Solve for the unknown.	$x = \frac{14}{\sin 76°} \times \sin 68°$
Multiply both sides by $\sin 68°$.	$x = 13.377\ldots$
Round correct to one decimal place.	$x \approx 13.4$

WORKED EXAMPLE 2 Sine rule problems

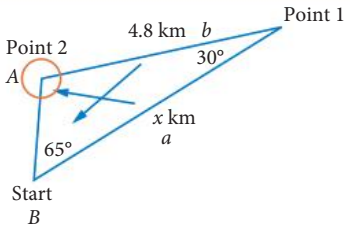
A cross-country track was set out as shown in the diagram.



Exam hack

If the angle needed is not known, use the angle sum of a triangle = 180° to work it out.

Calculate the length of the first leg from the start to Point 1.
Answer correct to one decimal place.

Steps	Working
1 Look at the 2 sides being used. Draw the arrows to match up the 2 sides to their opposite angle. A missing angle (circled) is opposite x, but we can use angle sum of a triangle to find the angle.	 The angle at Point 2 = 180° − (65° + 30°) = 85°.
2 Write the sine rule.	$\frac{a}{\sin A} = \frac{b}{\sin B}$
3 Substitute in the given values.	$\frac{x}{\sin 85°} = \frac{4.8}{\sin 65°}$
4 Solve for the unknown. Multiply both sides by $\sin 85°$. Round to one decimal place.	$x = \frac{4.8}{\sin 65°} \times \sin 85°$ $x = 5.276\ldots$ $x \approx 5.3$
5 Answer the question. Make sure units are included.	The length of the first leg from the start to Point 1 is 5.3 km.

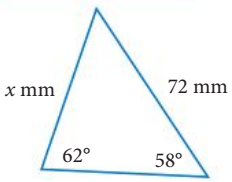
EXERCISE 11.1 The sine rule

ANSWERS p. 423

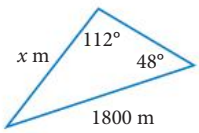
Mastery

1 WORKED EXAMPLE 1 Calculate the value of x, correct to one decimal place.

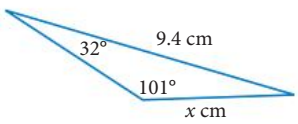
a



b

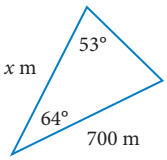


c

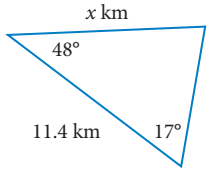


2 WORKED EXAMPLE 2 Calculate the value of x, correct to 2 decimal places.

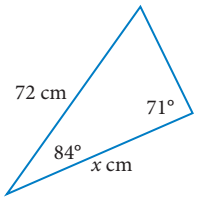
a



b

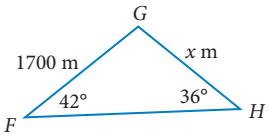


c

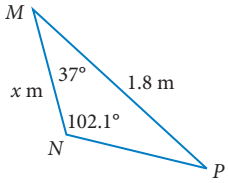


3 Calculate the value of the unknown, correct to the nearest whole number.

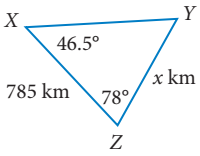
a



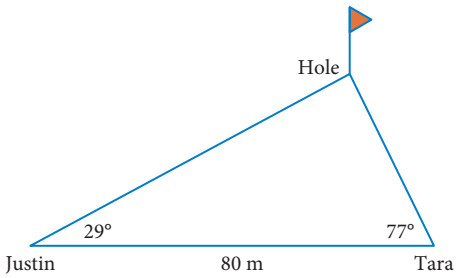
b



c

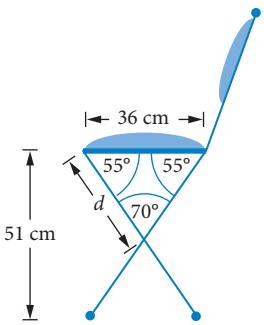


4 Tara and Justin are on the fairway of a golf course standing 80 m apart. The angles of each golfer to the hole are shown on the diagram.



What distance will each golfer have to hit the ball in order to reach the hole?
Answer correct to 2 decimal places.

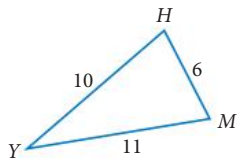
5 The folding chair shown has a seat that is 36 cm deep. The legs of the chair are inclined at an angle of 55° to the seat and join at an angle of 70°. The seat is 51 cm above the floor.



Find the distance from the seat to where the legs are joined, d. Answer correct to one decimal place.

Exam practice

6 Which rule is true for $\triangle HMY$?



A $\dfrac{6}{\sin M} = \dfrac{10}{\sin Y}$

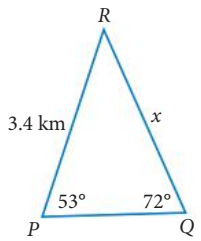
B $\dfrac{6}{\sin Y} = \dfrac{10}{\sin M}$

C $\dfrac{6}{\sin Y} = \dfrac{11}{\sin M}$

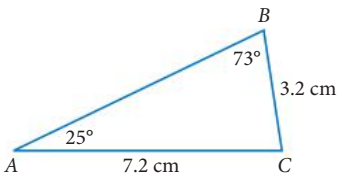
D $\dfrac{11}{\sin Y} = \dfrac{10}{\sin M}$

7 (2 marks) Chris maps out his 3 favourite fishing spots P, Q and R, as shown.

Calculate the distance between spots Q and R. Answer to one decimal place.

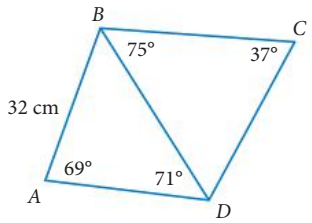


8 (2 marks) Calculate the length of AB, correct to one decimal place.



9 (3 marks)

a Show that the length of BD is 31.6 cm. *[1 mark]*



b Calculate the length of BC, correct to the nearest whole number. *[2 marks]*

10 (3 marks) CF A cable car runs from a point in a straight line and finishes on top of the mountain. Cath was at the beginning of a cable car ride and measured the angle of elevation to the top of the ride to be 27°. She walked along level ground 100 m closer to the top of the ride. The angle of elevation to the top of the ride was now 31°.

Use a diagram to help find the length of the cable car ride, correct to the nearest metre.

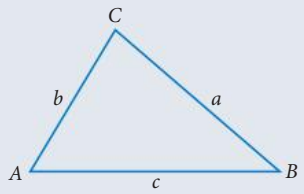
11 (3 marks) CU Atharva walked for 7.3 km on a bearing of 142°, then turned and walked on a bearing of 210° until he was due south of his starting point. Determine the distance Atharva is from his starting point. Answer correct to one decimal place.

11.2 The sine rule for angles

To find an unknown angle using the sine rule, rewrite the sine rule upside-down.

The sine rule for angles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Exam hack

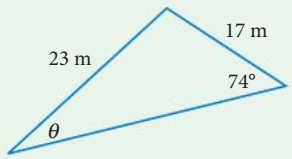
This form of the sine rule is not on the formula sheet. You have to *know* to write it this way to find an angle.

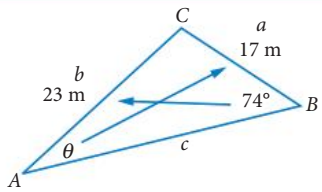
Video The sine rule

Worksheet The sine rule – Finding angles

WORKED EXAMPLE 3 Finding an angle with the sine rule

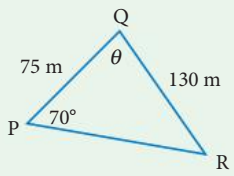
Find the value of θ, correct to the nearest degree.

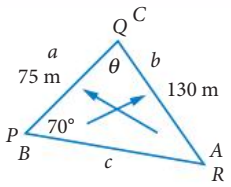


Steps	Working
1 Look at the 2 angles being used. Draw the arrows to match up the 2 angles to their opposite side.	
2 Label the angles A, B, C and the sides a, b, c. Remember a is opposite A, b is opposite B and c is opposite C.	
3 Write the sine rule for angles.	$\frac{\sin A}{a} = \frac{\sin B}{b}$ C and c are not required.
4 Substitute in the given values.	$\frac{\sin\theta}{17} = \frac{\sin 74°}{23}$
5 Solve for the unknown: – Multiply both sides by 17. – Use inverse sine to find θ.	$\sin\theta = \frac{\sin 74°}{23} \times 17$ $\theta = \sin^{-1}\left(\frac{\sin 74°}{23} \times 17\right)$ $\theta = 45.27\ldots$
Round to the nearest degree.	$\theta \approx 45°$

WORKED EXAMPLE 4 Application using sine rule

Calculate the value of θ, correct to the nearest degree.



Steps	Working
1 Draw the arrows to match up the 2 angles to the opposite sides. Don't know the side opposite θ. So, find $\angle R$ and use the angle sum of a triangle.	
2 Label the angles A, B, C and the sides a, b, c.	
3 Write the sine rule.	$\frac{\sin A}{a} = \frac{\sin B}{b}$
4 Substitute in the given values.	$\frac{\sin R}{75} = \frac{\sin 70°}{130}$
5 Solve for the unknown: – multiply both sides by 75.	$\sin R = \frac{\sin 70°}{130} \times 75$
– take the inverse to find R.	$R = \sin^{-1}\left(\frac{\sin 70°}{130} \times 75\right)$ $R = 32.82\ldots$
Round to the nearest degree.	$R \approx 33°$
To find θ, use the angle sum of a triangle is 180°.	$\therefore \theta = 180° - (33° + 70°) = 77°$

Exam hack

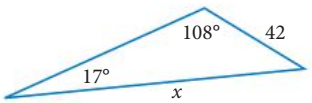
If you don't have the side opposite the angle you want, find the other angle. Then use the angle sum of a triangle is 180°.

EXERCISE 11.2 The sine rule for angles

ANSWERS p. 423

Recap

1 The expression that gives the value of x is

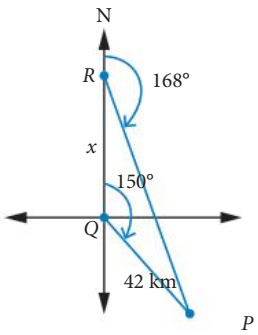


A $\frac{17}{\sin 42°} \times \sin 108°$ **B** $\frac{42}{\sin 17°} \times \sin 108°$ **C** $\frac{42}{\sin 17°} \times 108$ **D** $\frac{42}{\sin 108°} \times \sin 17°$

2 The bearing of P from Q is 150° and the distance from P to Q is 42 km.

R is directly north of Q and the bearing of P from R is 168°.

Calculate the distance from Q to R, correct to one decimal place.



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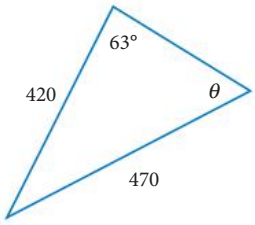
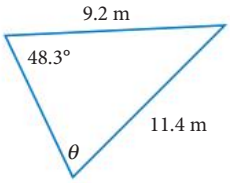
Mastery

3 WORKED EXAMPLE 3 Calculate the value of θ, correct to one decimal place.

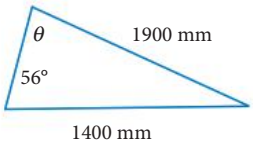
a



b

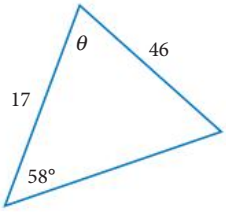


c

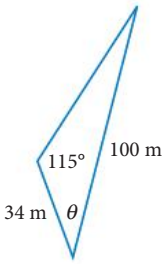


4 WORKED EXAMPLE 4 Calculate the value of θ, correct to nearest degree.

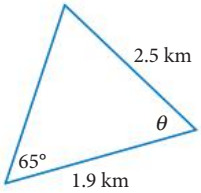
a



b

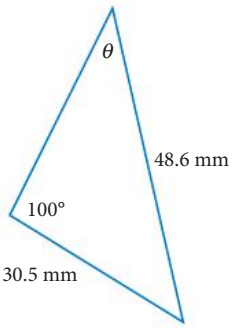


c

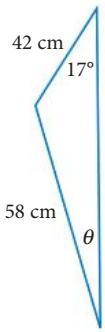


5 Calculate the value of θ, correct to one decimal place.

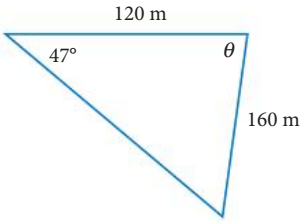
a



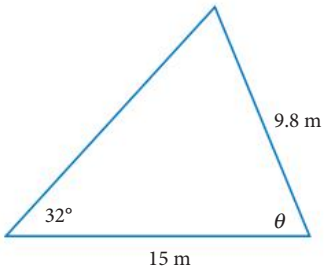
b



c



6 The cross-section of a patio roof is shown.



Calculate the value of θ to the nearest degree.

Exam practice

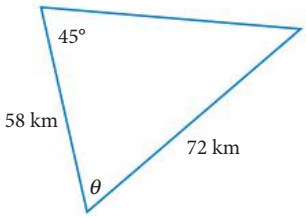
7 The value of θ is closest to

A 35°

B 48°

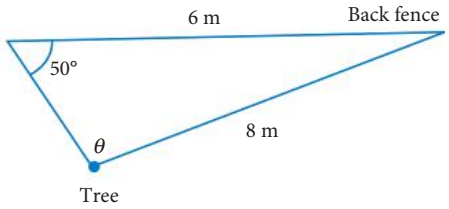
C 87°

D 100°

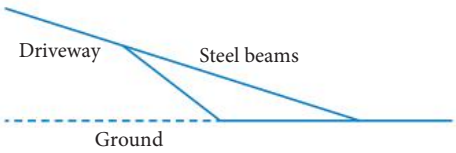


8 (3 marks) Chen made a triangular vegetable garden that was built along the back fence and came out to a tree, as shown.

Calculate the angle that the garden makes at the tree.



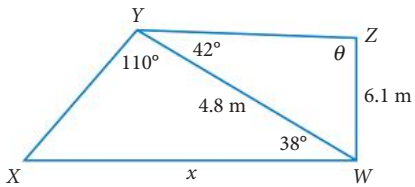
9 (4 marks) Fatima has had issues with the steepness of her driveway and had steel beams put in to resolve the issue. The steel beams are 5.4 m long and reach 3.1 m up the driveway. The driveway makes an angle of 27° with the ground.



a Copy the diagram and mark on it all of the given information. *[1 mark]*

b Calculate the angle that the steel beam makes with the ground, correct to the nearest degree. *[3 marks]*

10 (4 marks)

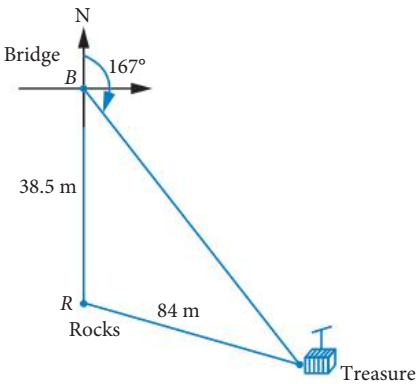


a Calculate the value of θ, correct to one decimal place. *[2 marks]*

b Find the length of WX, correct to one decimal place. *[2 marks]*

11 (3 marks) Find the size of the largest angle of $\triangle ABC$ when $\angle B = 28°$, $c = 6.6$ cm and $AC = 12$ cm.

12 (3 marks) CF Xi drew a map to show the location of his buried treasure, T, in relation to 2 landmarks. The bridge, B, is due north of the rocks, R, and the distance between them is 38.5 m. The bearing of the treasure from the bridge is 167°. The distance from the rocks to the treasure chest is 84 m. Calculate the bearing of the treasure from the rocks. Answer correct to nearest degree.



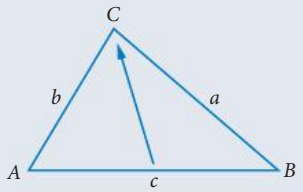
13 (3 marks) CU A green boat, G, sights the top of a cliff, T, at an angle of elevation of 24°. A red boat, R, is 200 metres further out to sea. The red boat is 1100 metres from the top of the cliff in a direct line. Determine the angle of elevation of the top of the cliff from the red boat. Answer correct to the nearest degree.

11.3 The cosine rule

The **cosine rule** is another way to find unknown sides and angles in non-right-angled triangles.

The cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$



Use the cosine rule in problems involving 3 sides and one angle of a triangle.

Video The cosine rule

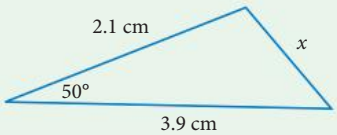
Worksheet Finding an unknown side

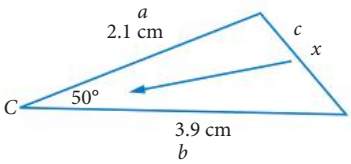
To solve problems using the cosine rule:

1. Draw a diagram, if not given.
2. Label the angles A, B, C and sides a, b, c.
3. Write the cosine rule.
4. Substitute in the given values.
5. Solve for the unknown.
6. Answer the question.

WORKED EXAMPLE 5 Finding a side with cosine rule

Calculate the value of x correct to 2 decimal places in the triangle shown.



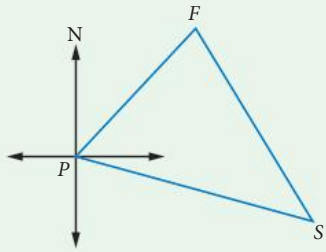
Steps	Working
1 Label the given angle C. Label the side opposite the angle c (an arrow will help with this). Label the other 2 sides a and b (it doesn't matter which is a and which is b).	
2 Write the cosine rule.	$c^2 = a^2 + b^2 - 2ab\cos C$
3 Substitute the given values.	$x^2 = 2.1^2 + 3.9^2 - 2 \times 2.1 \times 3.9 \times \cos 50°$
4 Simplify and solve for the unknown.	$x^2 = 9.0911\ldots$ $x = \sqrt{9.0911\ldots}$ $x = 3.015\ldots$
Round correct to 2 decimal places.	$x \approx 3.02$

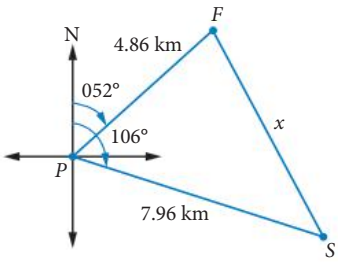
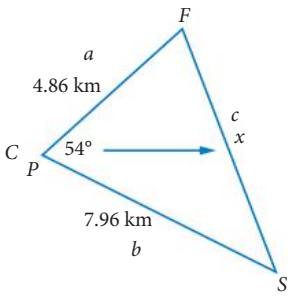
Exam hack

In any triangle, the longest side is always opposite the largest angle. The smallest side is opposite the smallest angle.

WORKED EXAMPLE 6 Applying the cosine rule

From a point P a tower, F, is 4.86 km on a bearing of 052°. There is a second tower, S, which is 7.96 km from P on a bearing of 106°. Find, correct to 2 decimal places, the distance between the towers.



Steps	Working
1 Copy the diagram and mark on it the given information.	
2 In $\triangle PFS$, $\angle FPS = 106° - 52° = 54°$ Draw an arrow from the given angle, 54°, to the side opposite. Label the angle C and the side opposite c. Label the other 2 sides a and b.	
3 Write the cosine rule.	$c^2 = a^2 + b^2 - 2ab\cos C$
4 Substitute in the given values.	$x^2 = 4.86^2 + 7.96^2 - 2 \times 4.86 \times 7.96 \times \cos 54°$
5 Solve for the unknown.	$x^2 = 41.503\ldots$ $x = \sqrt{41.503\ldots}$ $x = 6.442\ldots$
Round correct to 2 decimal places.	$x \approx 6.44$
6 Answer the question.	The distance between the towers is 6.44 km.

9780170484749

EXERCISE 11.3 The cosine rule

ANSWERS p. 423

Recap

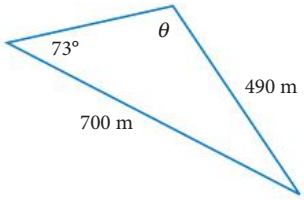
1 Which equation is correct for the triangle shown?

A $\dfrac{\theta}{700} = \dfrac{\sin 73°}{490}$

B $\theta = \dfrac{\sin 73°}{700} \times 490$

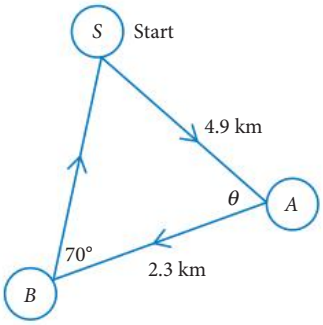
C $\dfrac{\sin\theta}{700} = \dfrac{\sin 73°}{490}$

D $\sin\theta = \dfrac{\sin 73°}{700} \times 490$



2 A school fun run course was set out as shown.

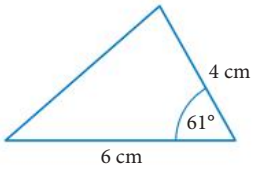
Calculate the value of θ, correct to one decimal place.



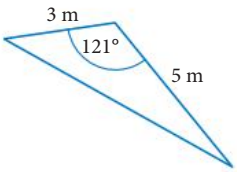
Mastery

3 WORKED EXAMPLE 5 Calculate the unknown side length in each triangle. Answer to one decimal place.

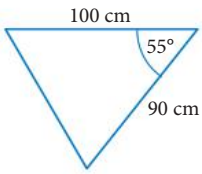
a



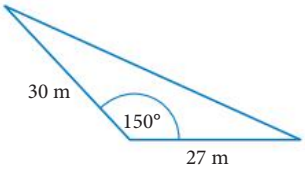
b



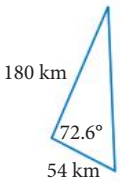
c



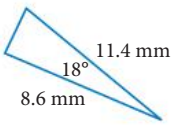
d



e



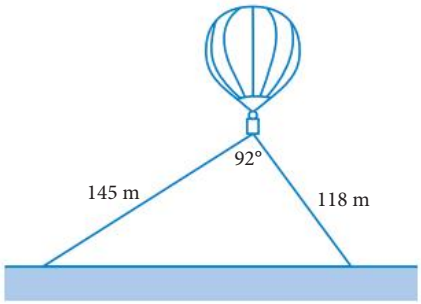
f



4 WORKED EXAMPLE 6 A hot air balloon is fixed to the ground by 2 ropes that are 145 m and 118 m long.

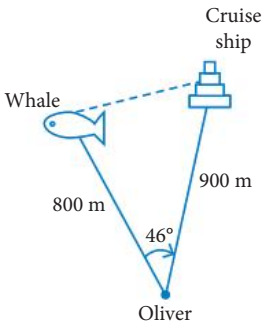
The angle between the ropes is 92°.

How far apart are the ropes fixed to the ground?



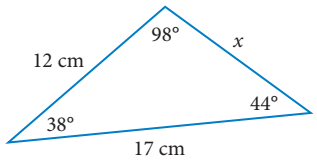
5 Oliver is at a beach lookout when he observes a whale at a distance of 800 m and a cruise ship at a distance of 900 m. The angle between the 2 lines of sight is 46°.

What is the distance between the whale and the cruise ship? Answer correct to the nearest metre.



Exam practice

6 Which rule is true for the triangle shown?



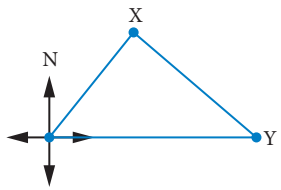
A $x^2 = 12^2 + 17^2 - 2 \times 12 \times 17 \times \cos 44°$

B $17^2 = 12^2 + x^2 - 24x \times \cos 98°$

C $x^2 = 12^2 + 17^2 - 2 \times 12 \times 17 \times \cos 98°$

D $12^2 = x^2 + 17^2 - 34x \times \cos 38°$

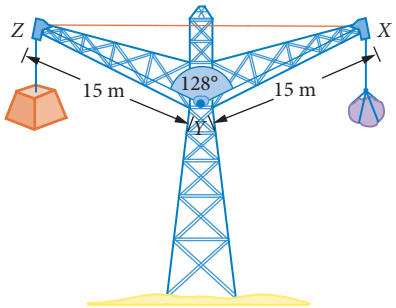
7 (3 marks) Tony hikes from his campsite on a bearing of 042° for 8 km to X. He then hikes to Y, which is due east of his starting point. Y is 11.5 km from his campsite.



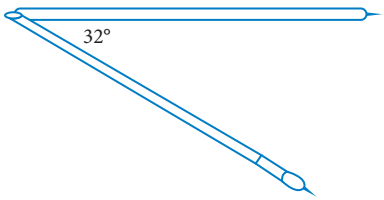
a Copy the diagram and mark on it the given information. *[1 mark]*

b How much further did Tony hike than if he had travelled directly from his starting point to Y? Answer correct to one decimal place. *[2 marks]*

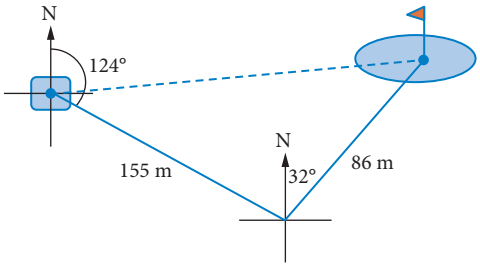
8 (3 marks) The diagram shows a crane with 2 arms, each 15 m long. The arms of the crane are separated by an angle of 128°. The load is attached at point *X* and a counterweight of equal mass is attached at point *Z*. Find *XZ*, correct to one decimal place.



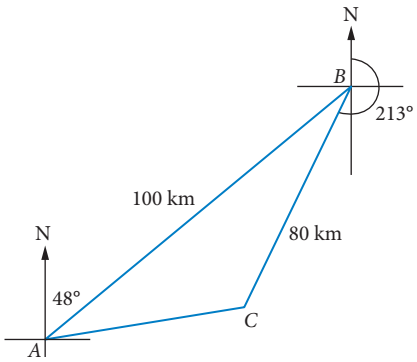
9 (3 marks) A pair of compasses is used to draw circles. The pair shown here has been opened so that the angle between the arms is set at 32°. The arms of the compass are each 12 cm long. Find the diameter of the circle that could be drawn with the arms in this position.



10 (3 marks) CF A golfer plays a wayward shot from the tee. She hits the ball 155 m away from the tee at a bearing of 124°. The golfer's recovery shot is much better. She hits the ball 86 m on a bearing of 032° and it ends up in the hole! How much further was the path taken by the golfer compared with the direct distance between the tee and the hole?



11 (3 marks) CU The location of 3 towns, *A*, *B* and *C* is shown in the diagram. The bearing from *A* to *B* is 048°, and the bearing from *B* to *C* is 213°. The towns at *A* and *B* are 100 km apart; the towns at *B* and *C* are separated by 80 km. How far apart are the towns *A* and *C*?

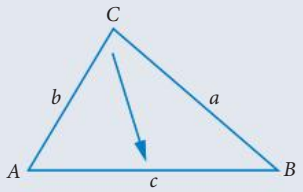


11.4 The cosine rule for angles

To find an unknown angle using the cosine rule, transpose the formula to make cos C the subject.

The cosine rule for angles

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

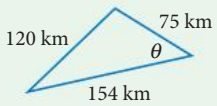


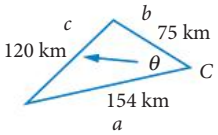
Exam hack

This form of the cosine rule rule is not on the formula sheet, so you have to remember the transposed formula or transpose it each time.

WORKED EXAMPLE 7 Finding an angle with the cosine rule

Calculate the value of θ, correct to one decimal place.



Steps	Working
1 Draw an arrow from θ to the side opposite. Label the angle C and the side opposite c. Label the other 2 sides a and b.	
2 Write the cosine rule.	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
3 Substitute in the given values.	$= \frac{154^2 + 75^2 - 120^2}{2 \times 154 \times 75}$
4 Solve for the unknown.	$\cos C = 0.6467\ldots$
Use the $\cos^{-1}$ function on the unrounded value on your calculator.	$C = \cos^{-1}(0.6467\ldots)$ $C = 49.699\ldots$
Round correct to one decimal place.	$C \approx 49.7$
5 Answer the question.	The value of θ is 49.7°.

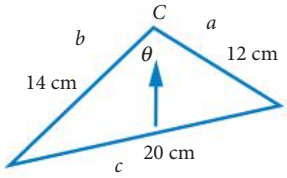
Video The cosine rule for angles 1

Worksheets The cosine rule – Angles and sides

Finding an unknown angle

WORKED EXAMPLE 8 Application of the cosine rule

Calculate correct to the nearest degree the largest angle in the triangle with sides 12 cm, 14 cm and 20 cm.

Steps	Working
1 Draw a triangle, labelling the sides 12 cm, 14 cm and 20 cm.	
2 Draw an arrow from the largest side to the angle opposite. Label this angle C and the side opposite c. Label the other 2 sides a and b.	
3 Write the cosine rule.	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
4 Substitute the given values.	$= \dfrac{12^2 + 14^2 - 20^2}{2 \times 12 \times 14}$
5 Solve for the unknown. Use the $\cos^{-1}$ function on your calculator. Round correct to nearest degree.	$\cos C = -0.17857\ldots$ $C = \cos^{-1}(-0.17857\ldots)$ $C = 100.2\ldots°$ $C \approx 100°$
6 Answer the question.	The largest angle is 100°.

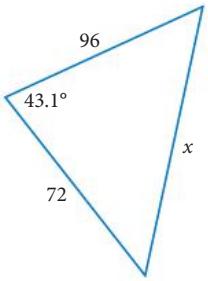
EXERCISE 11.4 The cosine rule for angles

ANSWERS p. 423

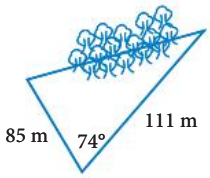
Recap

1 The value of x is closest to

A 63 **B** 66 **C** 120 **D** 4306



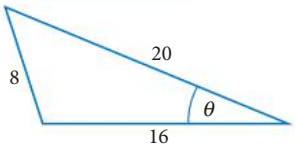
2 Kiara surveyed her block of land. The block is triangular and has sides of 85 m and 111 m, and the angle between these sides is 74°.
One part of her property line goes through a forest and she could not measure it directly. Calculate the perimeter of her block of land. Answer correct to the nearest metre.



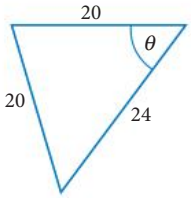
Mastery

3 WORKED EXAMPLE 7 Find θ, correct to the nearest degree.

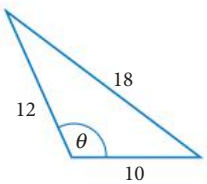
a



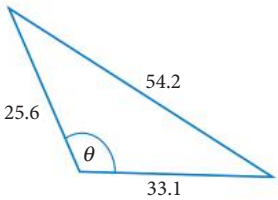
b



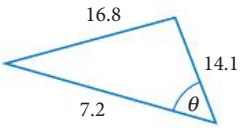
c



d



e



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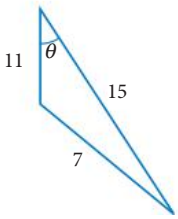
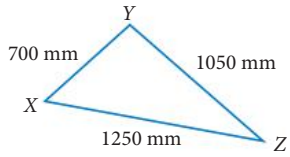
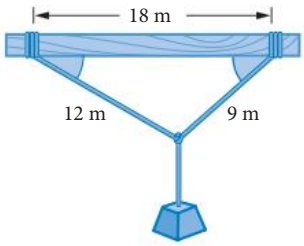


4 WORKED EXAMPLE 8 Calculate the size of the smallest angle.



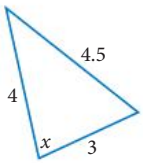
5 In $\triangle ABC$, $a = 22$, $b = 11.4$ and $c = 16.5$.
Calculate the size of the largest angle.

6 A weight is supported by cables attached to a horizontal beam, as shown. Find the angle that each cable makes with the beam.



Exam practice

7 The value of x in this triangle is closest to



A 85°

B 79°

C 63°

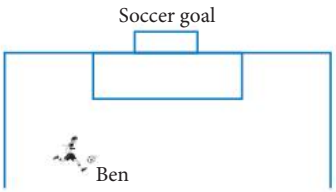
D 72°

8 (3 marks) A ladder has 2 legs 1.8 m long and when it is standing on the ground, the legs are 0.8 m apart.

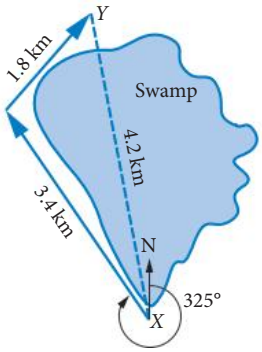
a Draw a diagram and mark on it the given information. *[1 mark]*

b Calculate the angle between the legs, correct to the nearest degree. *[2 marks]*

9 (3 marks) A soccer goal is 7.3 m wide. When Ben shoots for goal, he is 9 m from one goalpost and 6.5 m from the other.
What angle will the shot need to be within if Ben is to score a goal?
Answer to the nearest degree.



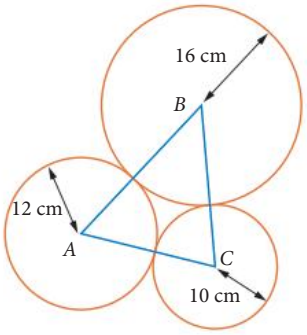
10 (3 marks) CF A hiker encounters a swamp at point X, as shown. To travel around the swamp, the hiker walks 3.4 km from X on a bearing of 325° and then walks 1.8 km to point Y on the other side of the swamp. Find the bearing of X from Y.



11 (3 marks) CU Three circles of radii 10 cm, 12 cm and 16 cm are arranged so that they just touch one another, as shown in the diagram.

The centres of the circles form the vertices of the triangle $\triangle ABC$.

Find, correct to one decimal place, the largest angle in the triangle.



Video
The sine area formula

11.5 Triangle area formulas

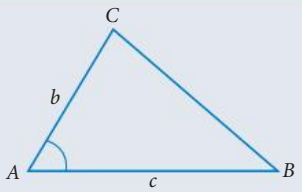
We can find the area of a triangle using $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height.

The area of a triangle can also be found using trigonometry and **Heron's rule**.

Triangle area using trigonometry

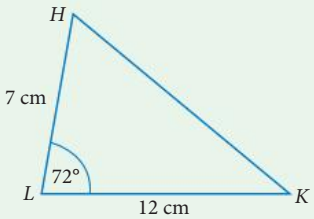
For 2 sides b and c and the **included angle** A:

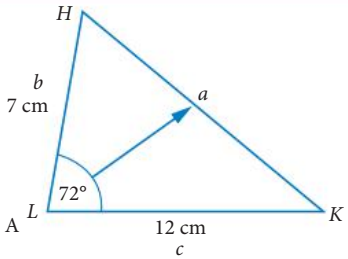
$$\text{Area} = \frac{1}{2}bc\sin A$$



WORKED EXAMPLE 9 Area of a triangle using trigonometry

Calculate the area of this triangle, correct to one decimal place.



Steps	Working
1 Draw an arrow from the angle to the side opposite. Label the angle A and the side opposite a. Label the other 2 sides b and c.	
2 Write the formula.	$\text{Area} = \frac{1}{2}bc\sin A$
3 Substitute in the given values.	$\text{Area} = \frac{1}{2} \times 7 \times 12 \times \sin 72°$
4 Solve using a calculator.	$= 39.94\ldots$
Round correct to one decimal place.	$= 39.9$
5 Answer the question.	The area is 39.9 cm^2.

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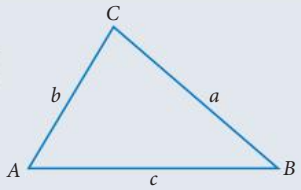
Heron's rule

For 3 sides of a triangle a, b and c:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

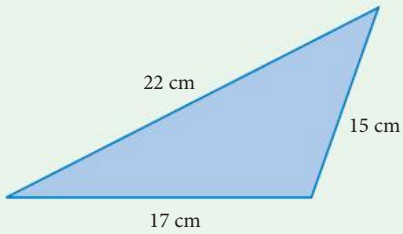
where $s = \dfrac{a+b+c}{2}$.

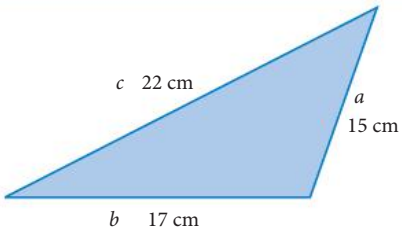


s is called the **semi-perimeter** because it is half of the perimeter of the triangle.

WORKED EXAMPLE 10 Heron's rule

Calculate the area, correct to one decimal place.



Steps	Working
1 Label the sides a, b, c.	
2 Calculate s first. Write the formula. Evaluate.	$s = \dfrac{a+b+c}{2}$ $= \dfrac{15+17+22}{2}$ $= 27$
3 Calculate the area. Write the formula. Evaluate, correct to one decimal place.	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{27(27-15)(27-17)(27-22)}$ $= 127.27\ldots$ $= 127.3\,\text{cm}^2$.

Exam hack

Your calculator recognises $\sqrt{27(27-15)(27-17)(27-22)}$ as $\sqrt{27 \times (27-15) \times (27-17) \times (27-22)}$.

EXERCISE 11.5 Triangle area formulas

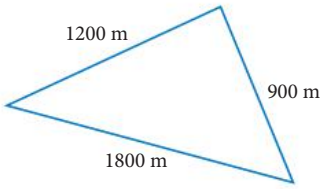
ANSWERS p. 423

Recap

1 The size of the largest angle is closest to

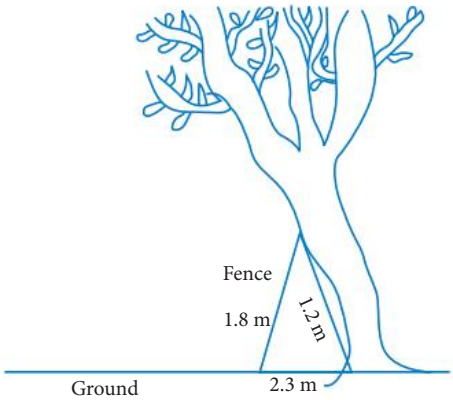
A 117° **B** 26°

C 36° **D** 90°



2 A fence 1.8 m tall starts to fall over and is leaning on a tree, as shown.

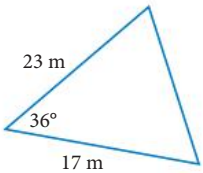
What is the angle the fence is making with the ground? Answer to the nearest degree.



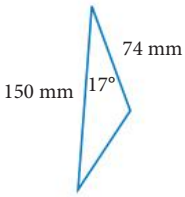
Mastery

3 WORKED EXAMPLE 9 Calculate the area of each triangle, correct to one decimal place, using trigonometry.

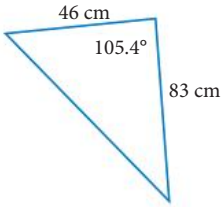
a



b

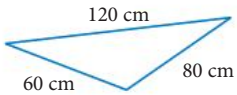


c

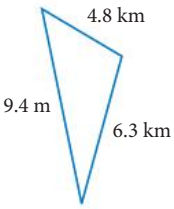


4 WORKED EXAMPLE 10 Calculate the area of each triangle using Heron's rule, correct to nearest whole number.

a



b



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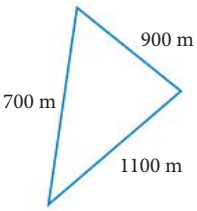
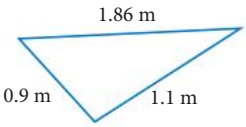


5 Calculate the area of each triangle using the appropriate method. Answer correct to one decimal place.

a



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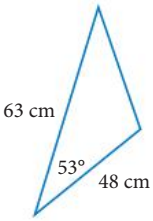

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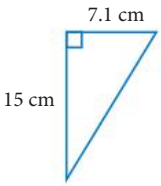
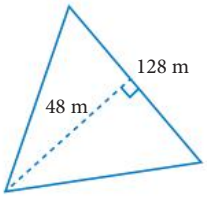


d



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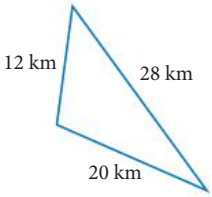

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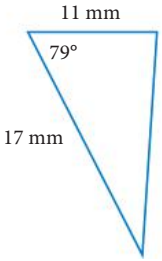

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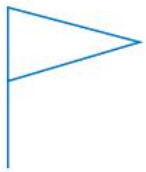
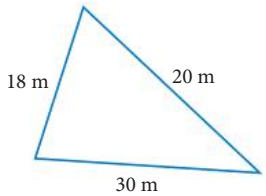
6 A triangular block of land has edges of 156 m, 172 m and 90 m. What is the area of the block of land, correct to the nearest square metre?

7 A flag has 2 sides that are 45 cm long and have an angle of 21° between them. What is the area of material needed to make the flag?



Exam practice

8 The expression for the area of this triangle is



A $\sqrt{34(34-9)(34-10)(34-15)}$

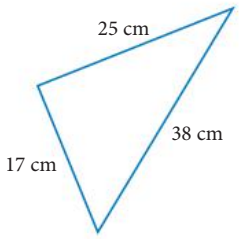
B $\sqrt{34(34-18)(34-20)(34-30)}$

C $\sqrt{68(68-18)(68-20)(68-30)}$

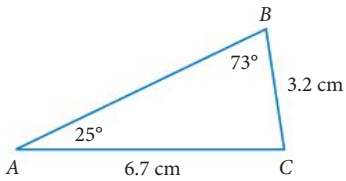
D $\sqrt{68(34-18)(34-20)(34-30)}$

9 (3 marks) Mary needs to work out the cost of manufacturing a metal plate with the dimensions shown.

Calculate the cost of manufacturing 1500 metal plates if they cost 0.5 cents per square centimetre.

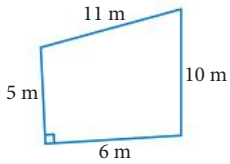


10 (3 marks) Calculate the area of this triangle, correct to the nearest cm^2.



11 (3 marks) CF A shade sail is to be installed over a playground. The shade sail has the dimensions shown.

Calculate the area that the shade sail covers.



12 (3 marks) CF Two sides of a triangle are 4.8 m and 7 m and the area is 8.3 m^2. Find the angle included by the 2 sides.

13 (4 marks) CU Davina has built a new home with a flat roof. She intends to waterproof the roof so that she can build gardens on it to help her home blend in with the natural surrounds. The plans show that the roof consists of 2 shapes. The first is a parallelogram that has side lengths of 20 m and 36 m and has an angle of 64° between them. The second is an equilateral triangle with side length of 30 m.

Davina has been quoted \$65.40 per square metre to waterproof her roof. She has allowed a budget of \$60 000 for the waterproofing. Does she have enough? Justify your answer.

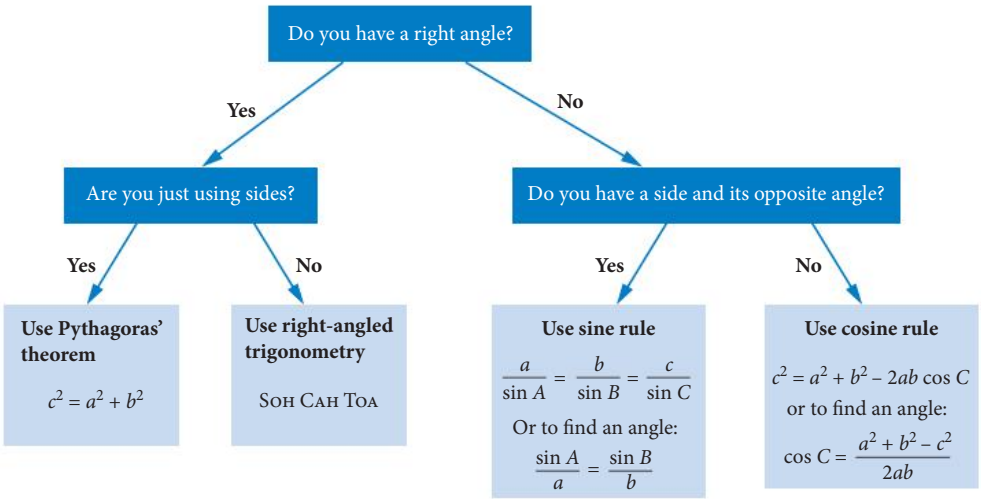
Video
The sine and cosine rules

Worksheets
Sine rule problems

Cosine rule problems

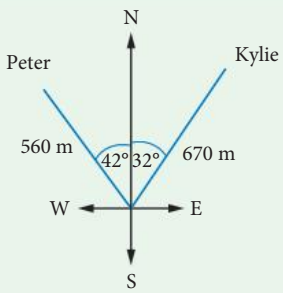
11.6 Applications of trigonometry

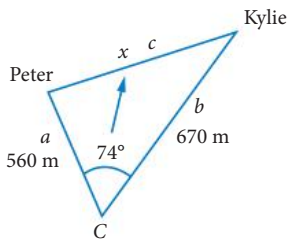
You can use this flowchart to choose the method for a practical application problem.



WORKED EXAMPLE 11 Application on the ground

Peter and Kylie leave a Toursit Information Centre and walk in different directions. Peter walks 560 m in the direction 318°, while Kylie walks 670 m in the direction 032°. Find their distance apart.



Steps	Working
1 Is there a right angle?	It is a non-right-angled triangle.
Don't have side and angle opposite.	Use the cosine rule.
2 Draw a diagram showing the triangle and label the unknown.	angle = 42° + 32° = 74°
Draw an arrow for matching angle and side. Label C, and the side opposite c. Label the other sides a, b.	
3 Write the cosine rule.	$c^2 = a^2 + b^2 - 2ab\cos C$
4 Substitute in the given values.	$x^2 = 560^2 + 670^2 - 2 \times 560 \times 670 \times \cos 74°$
5 Solve using a calculator.	$x^2 = 555\,661.72\ldots$ $x = \sqrt{555\,661.72\ldots}$ $x = 745.42\ldots$
As the question does not state a given accuracy, the nearest metre is appropriate.	$x \approx 745$
6 Answer the question.	The distance between Kylie and Peter is approximately 745 metres.

Exam hack

Look to see if your answer makes sense: 745 m looks reasonable.

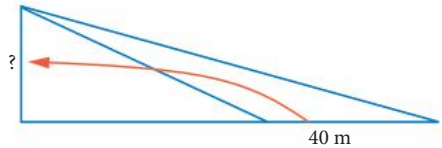
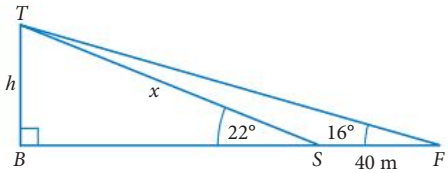
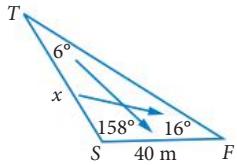
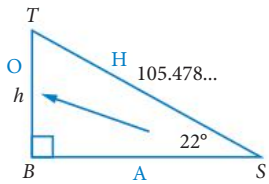
9780170484749

When the unknown is in a triangle connected to another triangle, you may need to find a common side. Try drawing a curved arrow from a known side in one triangle to the unknown side. It could show you what to do first.

WORKED EXAMPLE 12 Connected triangles

Jermaine sees that the top of a sand dune at the end of a beach is at an elevation angle of 16°. She walks 40 m closer and finds that the angle of elevation increases to 22°. What is the height of the sand dune, correct to one decimal place?

Witteart/Dreamstime LLC

Steps	Working
1 Draw a diagram and mark the known and unknown sides. Draw a curved arrow from the known side to the unknown side. It goes through the side you need to find first.	
Mark all the information. Call the unknown side h and the intermediate side x. x and the known side (40 m) are in $\triangle FST$.	
2 Draw $\triangle FST$ and mark in known information. Then find the other 2 angles. Draw an arrow from the unknown side, a, to the opposite angle, A. Do the same for known side, b, and angle, B.	$\angle FST = 180° - 22°$ (angles on a straight line) $= 158°$ $\angle STF = 180° - (16° + 158°)$ (angle sum of a triangle) $= 6°$ 
Have side and opposite angle, so use the sine rule. Write the formula.	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$
3 Substitute in the given values.	$\dfrac{x}{\sin 16°} = \dfrac{40}{\sin 6°}$
4 Solve for the unknown. Multiply both sides by $\sin 16°$. Do not round the answer as you will need it in the next step.	$x = \dfrac{40}{\sin 6°} \times \sin 16°$ $x = 105.478\ldots$
5 Now draw $\triangle BTS$ and mark in known information. Label sides O, A, H.	

6	Right-angled. Use SOH.	$\sin\theta = \frac{O}{H}$
7	Substitute in values.	$\sin 22° = \frac{h}{105.478\ldots}$
8	Solve by multiplying.	$h = \sin 22° \times 105.478\ldots$
		$h = 39.512\ldots$
	Round to one decimal place.	$h \approx 39.5$
9	Answer the question.	The height of the sand dune is 39.5 m.

Exam hack

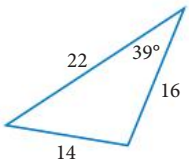
Do not use a rounded value during calculations.
Use your **Ans** button to use the unrounded answer.

EXERCISE 11.6 Applications of trigonometry

ANSWERS p. 423

Recap

1 The correct expression for the area of this triangle is



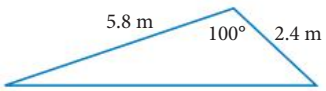
A $\frac{1}{2} \times 14 \times 16$

B $\frac{1}{2} \times 14 \times 16 \times \sin 39°$

C $\sqrt{52(52-14)(52-16)(52-22)}$

D $\sqrt{26(26-14)(26-16)(26-22)}$

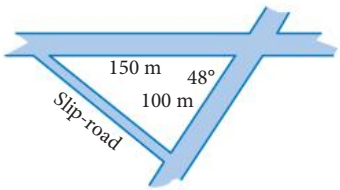
2 Calculate the area of the garden patch shown. Answer correct to the nearest square metre.



Mastery

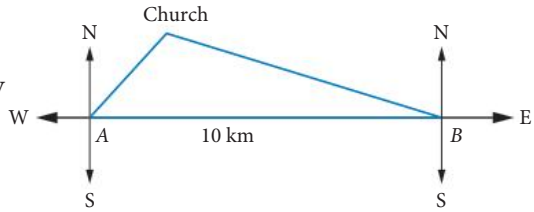
3 WORKED EXAMPLE 11 A fence is to be built on the perimeter of a triangular island where 2 roads and a slip-road meet. The angle between the 2 roads is 48°, and the distances along each road to the slip-road are 100 m and 150 m.

Find the length of the fence.

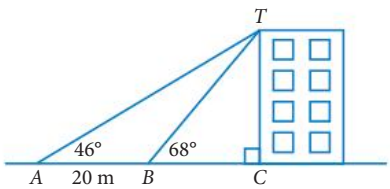


4 Dafina is walking on a straight road running east–west between 2 towns, A and B, 10 km apart. On her walk, she sees a church some distance away in a northerly direction. The bearing from A to the church is 048° and from B, the church is at a bearing of 293°.

Find the distance from A to the church.



5 WORKED EXAMPLE 12 To calculate the height of a building, Madison measures the angle of elevation to the top as 46°. She then walks 20 m closer to the building and measures the angle of elevation as 68°.



a Calculate $\angle ABT$.

b Use $\triangle ABT$ to find the length of BT.

c Now, use $\triangle BCT$ to find the height of the building. Answer correct to one decimal place.

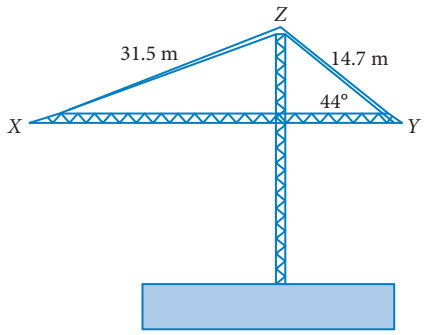
6 To calculate the height of an Aboriginal boundary tree, Eva measures the angle of elevation to the top as 38°. She then walks 21 m closer to the tree and measures the angle of elevation as 52°.

a Draw a diagram and show the given information.

b How tall is the tree? Answer correct to one decimal place.

Exam practice

7 A large crane is being used on a construction site. The crossbeam (XY) is supported by 2 metal stays, XZ and ZY, which are 31.5 m and 14.7 m long respectively. ZY makes an angle of 44° with the crossbeam.



What would you use to find the size of $\angle ZXY$?

A tan

B Pythagoras' theorem

C sine rule

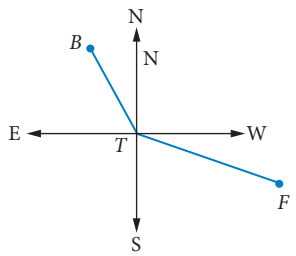
D cosine rule

8 (3 marks) Olivia is bushwalking. She sees 2 mountain peaks. On her map, the distances to the peaks are 8 km and 12.5 km. She estimates that the angle between her lines of sight to the peaks is 105°.

a Draw a diagram and mark on it the given information. *[1 mark]*

b How far apart, as the crow flies, are the peaks? *[2 marks]*

9 (4 marks) A national park ranger is located in an observation tower, T. The ranger sees a small bushfire, B, at a bearing of 329° at a distance of 12 km. The fire response team, F, is currently located at a bearing of 106° and 15 km from the tower.



a Copy the diagram and mark on it the given information. *[1 mark]*

b Show that $\angle BTF = 137°$. *[1 mark]*

c Determine the distance from the fire response team to the bushfire? *[2 marks]*

10 (4 marks) Sophie and Will set out from point A at the same time and both travel for 4 hours. Sophie travels at 30 km/h along a straight road in the direction 042°. Will travels at 24 km/h along another straight road in the direction 142°.

a How far did Sophie and Will each travel? *[1 mark]*

b Draw a diagram and mark on it the given information. *[1 mark]*

c Find their distance apart at the end. *[2 marks]*

11 (4 marks) CF To an observer on top of a cliff that is 120 m above sea level, a boat appears to be at an angle of depression of 17°. After the boat sails directly towards the cliff, the angle of depression increases to 37°. How far did the boat sail, to the nearest metre?

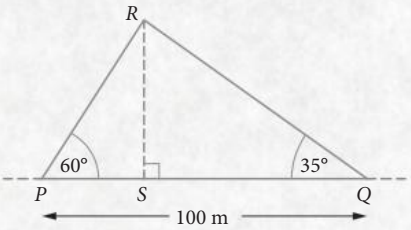
12 (4 marks) CU Toby is flying from Brisbane to Dalby at a speed of 150 km/h. He thinks his altimeter, which shows a height of 15 000 feet (4600 m), is faulty. Looking ahead, he sees the top of a hill marked as 405 m on his map at an angle of depression of 13°. Two minutes later, the angle of depression has increased to 20.6°. Use Toby's altitude to state what the radio operator should tell Toby about his altimeter. Justify your answer.

Video Exam question analysis: The sine and cosine rules

EXAM QUESTION ANALYSIS

Exam-style question (5 marks)

Two points, P and Q, are 100 m apart on level ground. A vertical tower stands somewhere between P and Q, as shown in the diagram.



The angles of elevation to the top of the tower from P and Q are 60° and 35° respectively. Determine the height of the tower.

Reading the question

- Identify the topic that you are working with – words like angles of elevation tell you that the solution method involves trigonometry.
- The words 'level' and 'vertical' indicate there is a right angle in there.

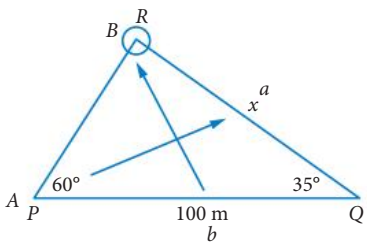
Thinking about the question

- The only given side (100 m) is in $\triangle PQR$ → which does *not* contain RS.
- Plan what you need to do. You'll need to use $\triangle PQR$ to find a side that you can use to then find the height of the tower (RS).
- Notice that RS is in the right-angled triangle QRS. So, if RQ (or RP) is found then it could be used to find RS.

Worked solution (✓ = 1 mark)

$\angle PRQ = 180° - (60° + 35°)$

$= 85°$ ✓

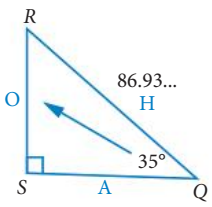


$$\frac{x}{\sin 60°} = \frac{100}{\sin 85°} \checkmark$$

$$x = \frac{100}{\sin 85°} \times \sin 60°$$

$$x = 86.93\ldots \checkmark$$

Now, for $\triangle QRS$:



$$\sin 35° = \frac{RS}{86.93\ldots} \checkmark$$

$RS = \sin 35° \times 86.93\ldots$

$RS = 49.86\ldots$

$RS \approx 50$

The tower is 50 m high. ✓

Exam hack

Don't round your answer at this point – it may give you a rounding error! You need to use it in the next part. You can press your **Ans** button to enter the full value.

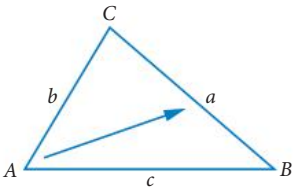
Exam hack

Make sure you go back and reread the question. This is a good opportunity to make sure you have answered the question and done everything required.

11 Chapter summary

Labelling triangles

- Label the angles of a triangle using A, B, C, and label the sides opposite the respective angle using a, b, c.
- Arrows are useful to match sides and angles.



Sine rule

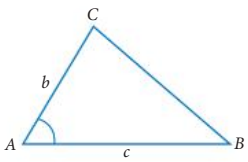
- Use the **sine rule** when you are using 2 sides and the opposite 2 angles.
- On formula sheet: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- To find an unknown angle, write the sine rule upside down: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$.
- NOTE: When solving, we are looking for A or B, so use the inverse sine function ($\sin^{-1}$).
- The angle sum of a triangle = 180° is used if you need to know the third angle.

Cosine rule

- Use the **cosine rule** when using an angle and the 3 sides.
- On formula sheet: $c^2 = a^2 + b^2 - 2ab\cos C$
- When solving, we are looking for c → so $\sqrt{\ }$.
- In any triangle, the longest side is *always* opposite the largest angle and the smallest side is *always* opposite the shortest side.
- To find an unknown angle, use $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

  Remember, we are looking for C, so use the inverse cosine function ($\cos^{-1}$).
- Either remember the formula or solve the other formula each time.

Area of a triangle



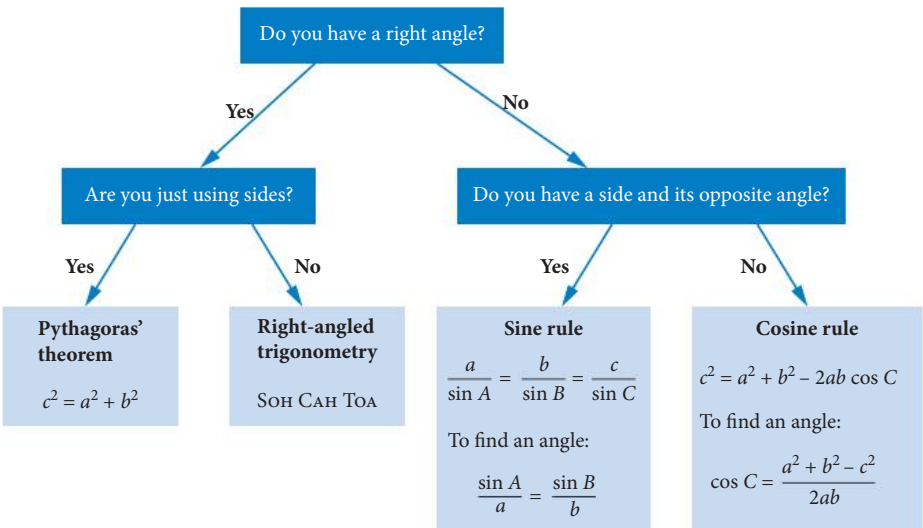
Area $= \dfrac{1}{2}bc\sin A$

Heron's rule:

Area $= \sqrt{(s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

Selecting what to use



Worksheet
General Maths Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

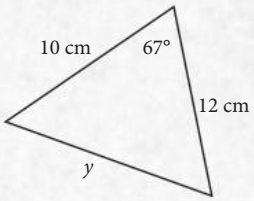
Section 2 (13 marks): 6 short response questions

Total: 18 marks

Section 1 5 multiple choice questions

5 marks

1 The value of y in this triangle is closest to



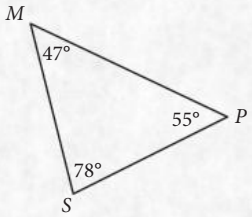
A 6.6 cm **B** 12.3 cm

C 15.6 cm **D** 55.2 cm

2 Charlie and Samara are salespeople who earn commission. As supervisor, Charlie earns 17% commission on his sales, as well as 3% commission on Samara's sales.
Calculate Charlie's total commission if his sales were \$5800 and Samara's were \$3500.

A \$1091 **B** \$1860 **C** \$1581 **D** \$4743

3 Which rule is true for $\triangle MPS$?

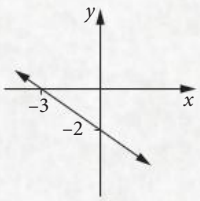


A $\dfrac{p}{\sin 55°} = \dfrac{s}{\sin 47°}$ **B** $\dfrac{m}{\sin 47°} = \dfrac{p}{\sin 78°}$

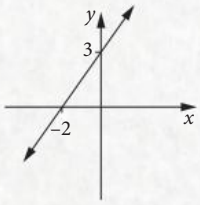
C $\dfrac{s}{\sin 78°} = \dfrac{m}{\sin 47°}$ **D** $\dfrac{m}{\sin 55°} = \dfrac{s}{\sin 78°}$

4 Which is the graph of $y = -2x + 3$?

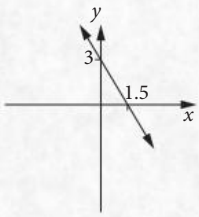
A



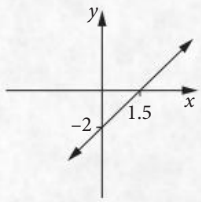
B



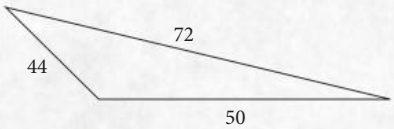
C



D



5 The expression that gives the area of this triangle is



A $\sqrt{166(166-44)(166-50)(166-72)}$

B $\sqrt{166(166-22)(166-25)(166-36)}$

C $\sqrt{83(83-44)(83-50)(83-72)}$

D $\sqrt{83(83-22)(83-25)(83-36)}$

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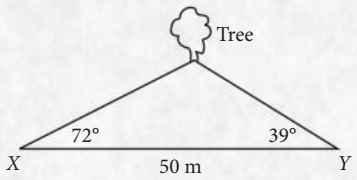
Section 2 6 short response questions

13 marks

6 (2 marks) A playground swing is 2.5 m long. When swinging forward, it makes an angle of 68°. Calculate the horizontal distance it travels when swinging forward.

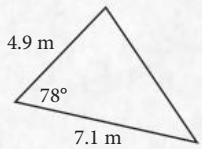
7 (3 marks) Gustavo wanted to work out the distance to a tree across the river. He measured the angle to the tree from point X and then from point Y which was 50 metres away from point X. He recorded them on the diagram shown.

Determine the distance of the tree from point X. Answer to the nearest metre.



8 (1 mark) Sam is retiling his triangular patio. The dimensions are shown.

Determine the area of his patio, correct to one decimal place.

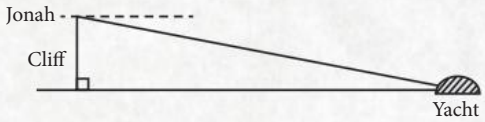


9 (2 marks) Find the product of these 2 matrices. If it is not possible, explain why.

$$\begin{bmatrix} 4 & 2 \\ -5 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 4 & 0 & -1 \end{bmatrix}$$

10 (2 marks) Pedro invests \$7500 at 4.2% p.a. simple interest. How much will he have in total at the end of 2 years?

11 (3 marks) From the top of a cliff 100 m above sea level, Jonah saw a capsized yacht. He estimated the angle of depression to be 8°.



a Copy the diagram and mark on it the given information. *[1 mark]*

b How far was the yacht from the base of the cliff to the nearest metre? *[2 marks]*

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

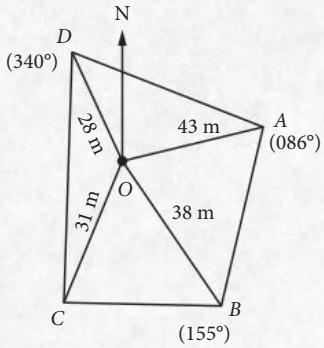
Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions

12 marks

1 (6 marks) Radial surveys are used to measure a piece of land. The bearings of the corners are taken from a central point, O. The following diagram shows a radial survey of a field $ABCD$.



a The farmer wants to put a fence around $\triangle AOD$. Find the length of fencing required. Answer correct to the nearest metre. *[3 marks]*

b The area of $\triangle BOC$ is $470\,\text{m}^2$. Use this to work out the bearing of C from O. *[3 marks]*

2 (3 marks) The total cost for 2 adults and 3 children to see a theatre production was \$123. It cost \$53 for one adult and a child to go.
Use equations to find the cost of an adult ticket.

3 (3 marks) Felicity is flying a kite and had extended 250 m of string when an updraft made the kite rise suddenly and she had to let out another 60 m of string. If the angle of elevation to the kite was 38° before the updraft, how far has the kite been driven vertically?

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CHAPTER

12 COMPARING DATA

Syllabus coverage
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UNIT 2, TOPIC 5: UNIVARIATE DATA ANALYSIS 2

Comparing data for a single numerical variable across two or more groups

- Construct and use parallel box plots, including identifying possible outliers, to compare datasets in terms of median, spread (range and IQR) and outliers to interpret and communicate the differences observed in the context of the data.
  - outliers (identifying): $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$ where Q_1 is lower quartile and Q_3 is upper quartile
- Compare datasets in terms of mean, median, range, IQR and standard deviation, interpret the differences observed in the context of the data, and report the findings in a systematic and concise manner.

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Videos (6):

12.1 Five-number summaries from frequency tables • Outliers
12.2 Box-and-whisker plots • Box plots and outliers
12.3 Double boxplots
Exam question analysis Comparing data

Prior learning (1):

12.1 Comparing data

Worksheets (13):

12.1 Five-number summaries 1 • Five-number summaries 2
12.2 Box plots • Box plots 1 • Box plots 2 • Box plots: graphics calculator
12.3 Box-and-whisker plots
12.5 Comparing city temperatures • Comparing word lengths • Comparing sports scores • Comparing group measures • Investigating young drivers
Cumulative exams General Maths Year 11 formula sheet

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

Terminology

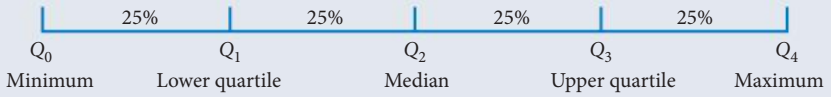
box plot	distribution of data	five-number summary	interquartile range (IQR)
lower fence	lower quartile (Q_1)	mean ($\bar{x}$)	measure of centre
measure of spread	median	mode	negatively skewed
outlier	parallel box plot	range	standard deviation (s_x)
summary statistics	symmetrical distribution	upper fence	upper quartile (Q_3)

12.1 The five-number summary

Five-number summary

A **five-number summary** of a set of data consists of:

- the minimum value
- the **lower quartile** (Q_1)
- the **median**
- the **upper quartile** (Q_3)
- the maximum value.



Worksheets
Five-number summaries 1
Five-number summaries 2

WORKED EXAMPLE 1 Calculating a five-number summary

Find the five-number summary for the data 5, 8, 9, 5, 7, 3, 6, 8, 6, 5, 5, 2, 9, 5, 6, 7, 4.

Steps	Working
1 Arrange the data in order.	2, 3, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7, 8, 8, 9, 9
2 Count the number of data values. Identify the positions of the quartiles.	17 8 | 8 4 | 4 | 4 | 4 Q_1 median Q_3
3 Mark these on the data. Median is the 9th value. Q_1 is between the 4th and 5th value. Q_3 is between the 13th and 14th value.	2, 3, 4, 5 | 5, 5, 5, 5, 6, 6, 6, 7, 7 | 8, 8, 9, 9 Q_1 median Q_3
4 Determine the values.	$Q_1 = 5$ median = 6 $Q_3 = \frac{7+8}{2} = 7.5$
5 Mark the minimum and maximum.	2, 3, 4, 5 | 5, 5, 5, 5, 6, 6, 6, 7, 7 | 8, 8, 9, 9 min Q_1 median Q_3 max
6 State the answer.	The five-number summary is 2, 5, 6, 7.5, 9.

Video
Five-number summaries from frequency tables

WORKED EXAMPLE 2 Five-number summaries from frequency tables

Determine the five-number summary for this data.

Score	Frequency
17	2
18	2
19	5
20	6
21	7
22	5
23	2
Total	**29**

Steps	Working
1 Count the number of data values.	29
2 Identify the positions of the quartiles.	14 | 14 7 | 7 | 7 | 7 Q_1 median Q_3

3 Add a cumulative frequency column to the table. Use it to find the quartiles: see next step.

Score	Frequency	Cumulative frequency	
17	2	2	first 2 values
18	2	4	values 3, 4
19	5	9	values 5, 6, 7, 8, 9
20	6	15	values 10 to 15
21	7	22	values 16 to 22
22	5	27	values 23 to 27
23	2	29	values 28, 29
Total	**29**		

Q_1

Median

Q_3

Max and min

4 Median is the 15th value.	median = 20
Q_1 is between the 7th and 8th value.	Both the 7th and 8th values are 19. $Q_1 = 19$
Q_3 is between the 22nd and 23rd value.	The 22nd value is 21 and the 23rd value is 22. $Q_3 = \frac{21+22}{2} = 21.5$
5 Write the extremes.	min = 17, max = 23
6 Answer the question.	The five-number summary is 17, 19, 20, 21.5, 23.

When you examine data, **always** check for possible **outliers**.

Calculate the **upper fence** and **lower fence** to test for outliers (refer to Chapter 10 *Analysing data*, on page 319).

Video
Outliers

Testing for possible outliers

A data value is a possible outlier if it is:

- less than the lower fence: $Q_1 - 1.5 \times \text{IQR}$

or

- more than the upper fence: $Q_3 + 1.5 \times \text{IQR}$.

Remember, you say '*possible* outlier' because you can never be 100% certain whether the extreme value was measured correctly or with error.

Exam hack

On the formula sheet, the outlier test is written like this:

outliers (identifying) $Q_1 - 1.5 \times \text{IQR} \le x \le Q_3 + 1.5 \times \text{IQR}$

lower fence upper fence

If the value is not inside the fences, it's a possible outlier.

WORKED EXAMPLE 3 Checking for outliers

Determine the five-number summary for this data:

28, 51, 36, 54, 41, 75, 46, 52, 39, 51, 47, 53, 45, 51, 40, 51, 40, 56

Identify possible outliers.

Steps	Working
1 Arrange the data in order.	28, 36, 39, 40, 40, 41, 45, 46, 47, 51, 51, 51, 51, 52, 53, 54, 56, 75
2 Find the total number of data values. Break the data into quartiles.	$n = 18$ 9 | 9 4 | 4 | 4 | 4 Q_1 median Q_3
3 Mark these on the data. Median is between the 9th and 10th values. Q_1 is the 5th value. Q_3 is the 14th value.	28, 36, 39, 40, 40, 41, 45, 46, 47, | 51, 51, 51, 51, 52, 53, 54, 56, 75 Q_1 median Q_3
4 Calculate the values.	$Q_1 = 40$ $\text{median} = \frac{47+51}{2} = 49$ $Q_3 = 52$
5 State the minimum and maximum.	min = 28, max = 75
6 Calculate the interquartile range.	$\text{IQR} = 52 - 40 = 12$
7 Calculate the lower and upper fences.	lower fence $= Q_1 - 1.5 \times \text{IQR}$ $= 40 - 1.5 \times 12$ $= 22$ upper fence $= Q_3 + 1.5 \times \text{IQR}$ $= 52 + 1.5 \times 12$ $= 70$
8 Mark fences on data.	lower fence (22) upper fence (70) | 28, 36, 39, 40, 40, 41, 45, 46, 47, | 51, 51, 51, 51, 52, 53, 54, 56, | 75 min Q_1 median Q_3 max
9 Identify any possible outliers.	75 is a possible outlier. So, change max to 56.
10 Answer the question.	The five-number summary is 28, 40, 49, 52, 56 with a possible outlier of 75.

EXERCISE 12.1 The five-number summary

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Mastery

1 WORKED EXAMPLE 1 Find the five-number summary for each dataset.

a 26, 27, 23, 24, 27, 25, 23, 19, 25, 21, 22, 20, 27, 21

b 2, 6, 5, 9, 2, 6, 6, 9, 7, 5, 4, 2, 3, 7, 8, 6, 6, 7, 4, 2, 6

2 WORKED EXAMPLE 2 Find the five-number summary for each dataset.

a

Score	Frequency
3	2
4	4
5	5
6	5
7	3
8	10
9	4
10	3
11	3
12	2
Total	**41**

b

Score	Frequency
22	3
23	7
24	4
25	11
26	13
27	10
28	6
29	5
30	3
31	2
32	2
Total	**66**

c

Stem	Leaf
4	1
5	0 3 6
6	1 3 8 8 9
7	0 1 1
8	3 4 6 7
9	8

Key: 4|1 means 4.1

d

Score	104	105	106	107	108	109	110	111	112
Frequency	7	9	11	17	21	25	16	9	5

3 WORKED EXAMPLE 3 Find the five-number summary for each dataset, checking for outliers.

a 12, 19, 14, 27, 14, 23, 14, 15, 14, 21, 15, 23, 13, 40, 14, 16, 15, 23, 14

b 10, 16, 12, 12, 7, 13, 10, 13, 8, 14, 9, 13, 11, 13, 10, 13, 12, 13, 3, 14, 10, 13

4 For the following data:

34 39 42 16 31 44 38 42 56 36 42 31

a find the five-number summary.

b use the data to show that 25% of the data is less than the lower quartile, Q_1.

c use the data to show that 50% of the data is above the median.

5 For the following data:

7.3 8.3 2.9 8.1 3.2 4.5 6.1 7.5

a find the five-number summary.

b use the data to show that 25% of the data is greater than the upper quartile, Q_3.

c use the data to show that 50% of the data is in between the lower quartile, Q_1, and the upper quartile, Q_3.

Exam practice

Use the following information for Questions 6 and 7.

The weights, in kg, of 120 six-week-old puppies were recorded.

The five-number summary is

0.4, 1.2, 1.5, 3.7, 5.1.

6 Which statement is **not** true?

A Half of the weights were below 1.5 kg.

B 25% of the weights were less than 1.2 kg.

C 25 of the puppies weighed more than 3.7 kg.

D 90 of the puppies weighed less than 3.7 kg.

7 Which statement is correct for calculating the upper fence?

A $3.7 + 1.5 \times 4.7$ **B** $3.7 - 1.5 \times 2.5$ **C** $3.7 + 1.5 \times 2.5$ **D** $3.7 - 1.5 \times 4.7$

8 (4 marks) The number of devices owned by 15 people is shown.

1 0 2 1 2 2 4 0 1 3 2 2 3 2 1

Calculate the five-number summary.

9 (6 marks) Nirmalan calculates the five-number summary for the heights, in cm, of students in his class to be

150 165 169 173 180

He also notes that there is an outlier of 186 cm.

a What percentage of students are

i below 150 cm? *[1 mark]*

ii above 173 cm? *[1 mark]*

iii between 165 and 173 cm? *[1 mark]*

b Nirmalan is 170 cm tall and says he is taller than most of the students in his class. Is he correct? Justify your answer. *[1 mark]*

c Calculate the upper fence and show that Nirmalan is correct when he says that 186 cm is an outlier. *[2 marks]*

10 (4 marks) CF The stem-and-leaf plot represents the number of goals scored by a netball team's 2 shooters in the last 16-week season.

Stem	Leaf
1	3 4 4
1*	6 7 8 9 9
2	0 1 2 2 3 3 3 4
2*	5 5 6 6 6 6 8 8 8
3	0 2 4
3*	6 7 9
4	3

Key: 1|3 means 13

1*|8 means 18

Abby said that there are no outliers in this data. Is she correct? Justify your answer.

11 (4 marks) CU Arjun wrote down the exam results for 10 students and worked out the five-number summary. When he went back and looked at his notes, he could not read one of the results. What he had was:

Results: 83 95 91 70 74 84 88 80 93

Five-number summary: 70, 80, 86, 93, 95

He remembered that there were no results that were the same. What must the missing result have been? Justify your answer.

Video
Box-and-whisker plots

Worksheets
Box plots
Box plots 1
Box plots 2
Box plots: Graphics calculator

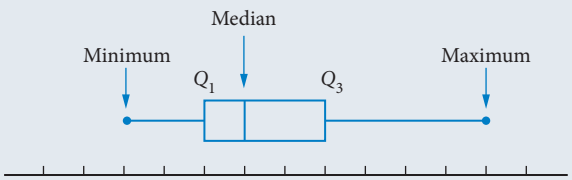
12.2 Box plots

Constructing a box plot

To construct a box plot:

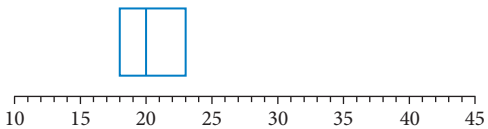
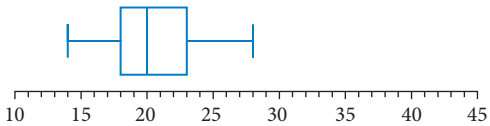
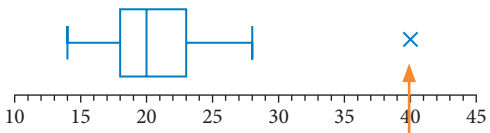
1. Draw an axis that covers all the data.
2. Above the scale, draw a box from Q_1 to Q_3.
3. Draw a vertical line through the box at the median value.
4. Draw horizontal lines ('**whiskers**') from the ends of the rectangle out to the minimum and maximum values.
5. Once you have shown a possible outlier, it is marked as a dot (or cross) and the end of the whisker is moved to the previous value.

A **box plot**, also known as a **box-and-whisker plot**, is a visual representation of the five-number summary.



WORKED EXAMPLE 4 Box plots

Draw a box plot for the five-number summary 14, 18, 20, 23, 28 with an excluded outlier of 40.

Steps	Working
1 Draw the scale from 10 to 45.	
2 Draw the box from 18 to 23 and draw in the median at 20.	
3 Draw the whiskers out to 14 and 28.	
4 Add the outlier.	

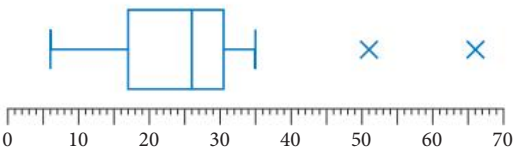
When you draw by hand, a cross is easier to see than a dot.

9780170484749

WORKED EXAMPLE 5 Construct a box plot and check for outliers

Draw a box plot for this dataset, showing possible outliers as appropriate.

18, 51, 35, 66, 28, 6, 11, 20, 28, 15, 30, 30, 29, 16, 34, 28, 31, 18, 12, 24, 11, 20, 19, 33

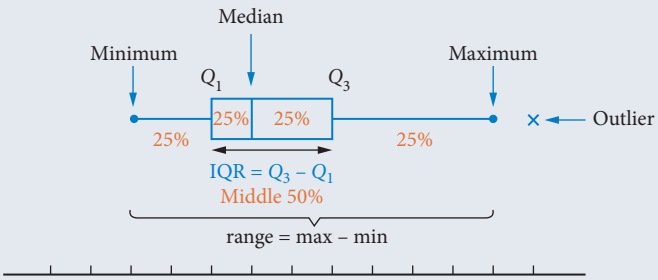
Steps	Working
1 Arrange in order.	6, 11, 11, 12, 15, 16, 18, 18, 19, 20, 20, 24, 28, 28, 28, 29, 30, 30, 31, 33, 34, 35, 51, 66
2 Find the quartiles and interquartile range. There are 24 data values, $n = 24$.	24 12 | 12 6 | 6 | 6 | 6 6, 11, 11, 12, 15, 16, | 18, 18, 19, 20, 20, 24, | 28, 28, 28, 29, 30, 30, | 31, 33, 34, 35, 51, 66 $Q_1 = \frac{16+18}{2} = 17$ $\text{median} = \frac{24+28}{2} = 26$ $Q_3 = \frac{30+31}{2} = 30.5$
3 To look for outliers, find the IQR.	$\text{IQR} = Q_3 - Q_1$ $= 30.5 - 17$ $= 13.5$
4 Find the fences.	$\text{lower fence} = Q_1 - 1.5 \times \text{IQR}$ $= 17 - 1.5 \times 13.5$ $= -3.25$ $\text{upper fence} = Q_3 + 1.5 \times \text{IQR}$ $= 30.5 + 1.5 \times 13.5$ $= 50.75$
5 Any value below −3.25 and above 50.75 could be a possible outlier.	lower fence (−3.25) | 6, 11, 11, 12, 15, 16, | 18, 18, 19, 20, 20, 24, | 28, 28, 28, 29, 30, 30, | 31, 33, 34, 35, | 51, 66 upper fence (50.75)
6 State possible outliers.	51 and 66 are possible outliers.
7 Write the five-number summary, excluding possible outliers.	Five-number summary is 6, 17, 26, 30.5, 35 with possible outliers at 51 and 66.
8 Draw the box plot, showing possible outliers.	

Video
Box plots and outliers

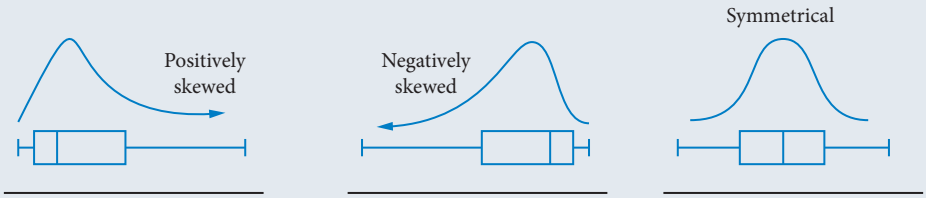
Reading and interpreting a box plot

From a box plot you can:

- calculate the range and interquartile range

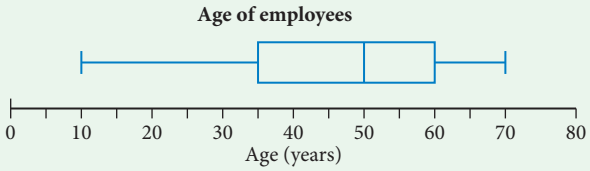


- classify the shape of the data
  If the width of a 25% interval is small, then the data is grouped together. If it is wider, then the data is spread out.



WORKED EXAMPLE 6 Read and interpret a box plot

The following box plot shows the distribution of the ages of 72 employees at a company.



a What is the age of the oldest employee?

b What percentage of employees are aged more than 60?

c What percentage of employees are aged between 35 and 60?

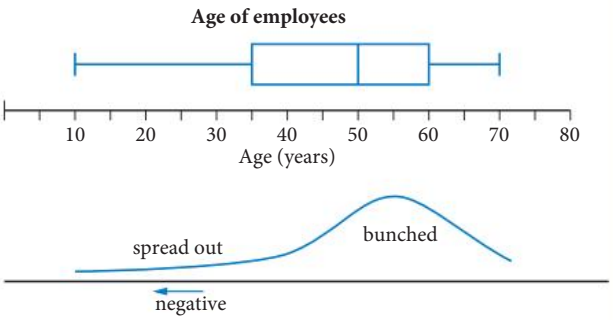
d How many employees are aged less than 35?

e Write the five-number summary.

f What age at the lower end would be considered outliers?

g What would the shape of the distribution of the data be?

Steps	Working
a The oldest age is at the end of the whisker.	The oldest employee is 70 years old.
b 1 60 is the top end of the box. This is the upper quartile.	More than 60 means above the upper quartile.
2 Answer the question.	25% of employees are aged above 60.
c 1 35 is the lower quartile and 60 is the upper quartile.	75% – 25% = 50%
2 Answer the question.	50% of employees are aged between 35 and 60.

d **1** 35 is the lower quartile.	Less than 35 = 25% 25% of 72 = 0.25 × 72 = 18
2 Answer the question.	18 employees are aged less than 35.
e Reading from the box plot.	The five-number summary is: min = 10, Q_1 = 35, median = 50, Q_3 = 60, max = 70
f **1** Need the lower fence, so find the IQR.	IQR = $Q_3 - Q_1$ = 60 − 35 = 25
2 Calculate the lower fence.	lower fence = $Q_1 - 1.5 \times$ IQR = 35 − 1.5 × 25 = −2.5
3 Answer the question.	Any age below −2.5 would be considered an outlier. Hence, there could be no outliers at the lower end.
g The width of the 25% intervals is larger at the lower ages. This means that the data is more spread out there. At the higher ages it is more bunched up.	The data is slightly negatively skewed. 

Exam hack

As it isn't a big difference, the term 'slightly' is used.

EXERCISE 12.2 Box plots

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Recap

1 Hunter had gathered all his test results for the year. He calculated the five-number summary for his data:

$$\text{min} = 72,\ Q_1 = 74,\ \text{median} = 77,\ Q_3 = 81,\ \text{max} = 97$$

Which of the following statements are true?

A His most frequent result was 77.

B The range is 7.

C Half of his results were between 74 and 81.

D The interquartile range is 25.

2 **a** Calculate the five-number summary for the following data.

Score	Frequency
10.6	4
10.7	6
10.8	1
10.9	7
11.0	4
11.1	0
11.2	2

Exam hack

Note that if the question tells you the total number of data values (scores), this lets you check by counting when you are listing them in order.

b Are there any possible outliers? Justify your answer.

Mastery

3 WORKED EXAMPLE 4 Draw a box plot for each five-number summary:

a min = 14, Q_1 = 18, median = 19, Q_3 = 24.5, max = 29

b {24, 36.5, 40.5, 45, 60}

4 WORKED EXAMPLE 5 Draw a box plot for each five-number summary.

a min = 3, Q_1 = 5, median = 6, Q_3 = 8.5, max = 11, with an outlier at 18

b 9, 17, 21, 23, 27 with outliers of 2 and 38

5 The data below shows the ages of 17 people at a dinner party.

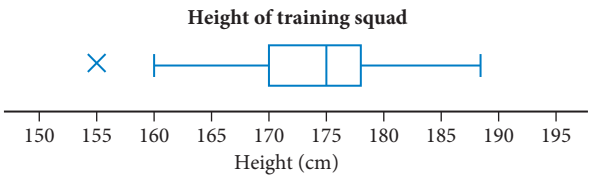
21 23 21 23 21 23 22 25 19 22 9 24 21 22 19 24 20

a Identify if there are any outliers. Justify your answer.

b Write the five-number summary for the ages.

c Draw a box plot for the ages, showing possible outliers as appropriate.

6 WORKED EXAMPLE 6 The box plot shows the heights of rugby players in a training squad of 24 players.



a Find the five-number summary.

b Calculate the percentage of players over 178 cm.

c Calculate the percentage of players less than 170 cm.

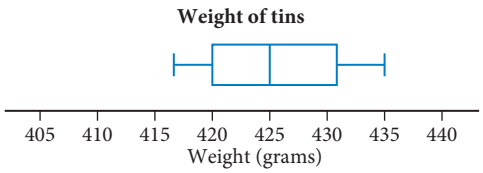
d Count the number of players more than 175 cm.

e Calculate the interquartile range.

f Find the value of the lower fence and, hence, show that 155 is a possible outlier.

g Comment on the shape of the data.

7 Sam was doing quality control on the weights of tins coming off the production line. The results of a sample of 1000 tins are shown.



a Find the five-number summary, including checking for possible outliers.

b Calculate the percentage of cans under 420 g.

c Determine the number of cans under 420 g.

d The tins are advertised as 420 g. What would Sam say in terms of the weight of tins in her quality control report? Justify your answer using the box plot.

8 Comment on the shape of each data distribution.

a

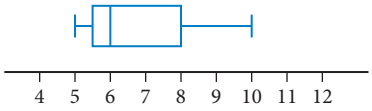
b

c

d

Exam practice

Use the following box plot to answer Questions 9 and 10.



9 The shape of the distribution can best be described as

A symmetrical but bunched at the centre.

B symmetrical.

C negatively skewed.

D positively skewed.

10 What is the interquartile range?

A 0.5 **B** 2 **C** 2.5 **D** 5

11 (7 marks) The number of errors in a page of typing for 28 business college students were recorded as follows.

4 1 0 1 2 9 4 5 7 2 0 1 9 5 32 9 1 0 2 1 4 7 6 3 5 4 8 0

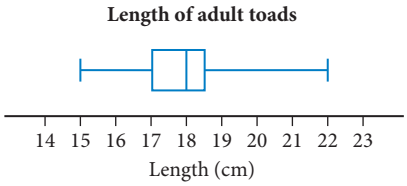
a Calculate the five-number summary, and state if there are any outliers. *[3 marks]*

b Draw a box plot of the data. *[2 marks]*

c State the typical number of errors. Justify your answer. *[2 marks]*

12 (3 marks) CF The length of a sample of 37 adult cane toads is shown in the table. The box plot that was drawn for this data is also shown.

Length	Frequency
15	1
16	4
17	8
18	15
19	4
20	4
21	0
22	1



Is the box plot correct? Justify your answer.

13 (5 marks) CF The masses, in grams, of a hoard of 47 denarius coins found in Gaul (modern France) from Roman times were checked and found to be, in order, as follows:

1.89 2.34 2.34 2.4 2.4 2.49 2.5 2.51 2.55 2.55 2.57 2.58 2.6 2.6 2.63 2.64

2.64 2.65 2.66 2.66 2.66 2.66 2.67 2.67 2.68 2.7 2.71 2.71 2.71 2.71 2.71 2.72

2.72 2.73 2.76 2.78 2.78 2.78 2.78 2.79 2.81 2.86 2.88 2.91 2.93 2.96 3.03

Cade states that the typical mass of the coins is 2.71 g.

Is Cade correct? Justify your answer.

Video Double boxplots

Worksheet Box-and-whisker plots

12.3 Parallel box plots

Box plots allow you to examine the relative spread of data easily. When you compare 2 or more sets of data, you can use **parallel box plots** on the same scale to make immediate comparisons. They are particularly useful for small sets of data.

Parallel box plots

- 2 or more box plots are drawn on the same axes.
- They allow both the centre and spread of the distributions to be found and compared visually.
- Comparison statements should comment on:
  - shape
  - centre (the median)
  - spread (the IQR)
  - possible outliers.

WORKED EXAMPLE 7 Constructing and comparing parallel box plots

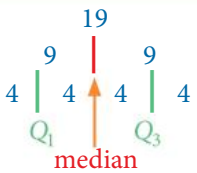
The ages of people in 2 cinemas are listed below.

Cinema A: 33 34 39 41 43 43 43 44 47 49 50 50 58 58 58 60 62 70 70

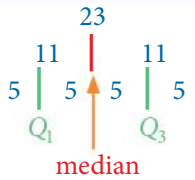
Cinema B: 28 33 39 42 44 46 49 49 49 50 53 54 60 61 62 63 64 64 64 70 70 85 87

a **Construct** parallel box plots of these datasets.

b **Compare** the ages of the people in the 2 cinemas.

Steps	Working
a 1 Find the five-number summary for cinema A by breaking the data into quartiles.	 33 34 39 41 43 43 43 44 47 49 50 50 58 58 58 60 62 70 70 The five-number summary for cinema A is 33, 43, 49, 58, 70.

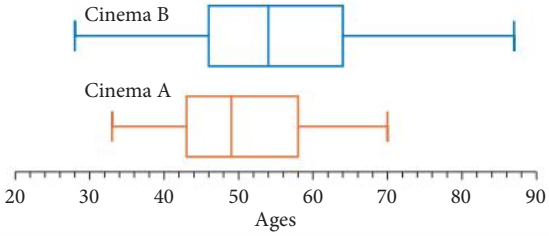
2 Find the five-number summary for cinema B by breaking the data into quartiles.



28 33 39 42 44 46 49 49 49 50 53 54 60 61 62
63 64 64 64 70 70 85 87

The five-number summary for cinema B is 28, 46, 54, 64, 87.

3 Draw the box plots on the same scale.



b 1 Compare by looking at:

– shape — Both datasets are approximately symmetrical.

– centre — The median and quartiles are higher for cinema B than for cinema A. It can be seen that the box for B is positioned higher than for A. This means that the people at cinema B are generally older than those at cinema A.

(BUT cinema B also has the youngest person.)

– spread — Cinema B has a slightly higher IQR of 18 compared to 15 for cinema A, but a much greater range of 59 compared to 37 for cinema A. This means the ages of people at cinema B are more spread out.

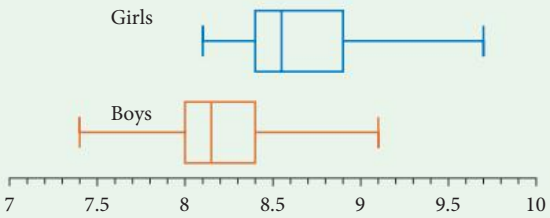
– outliers — There are no outliers in the box plots.

2 Write a general comment to sum up the answer. Put it in practical terms for the question.

Although cinema B contains the youngest person, the people in cinema B are generally older than those in cinema A. The ages of people in cinema B are also more spread out than for cinema A.

WORKED EXAMPLE 8 Comparing data using parallel box plots

The data shows the times (in seconds) taken by a group of 14-year-old students to run 50 metres.



For the time taken for the boys and girls, **compare** the

a **i** shape of the data **ii** centre of the data **iii** spread of the data **iv** outliers.

b Write a statement **comparing** the time taken by the girls and the boys.

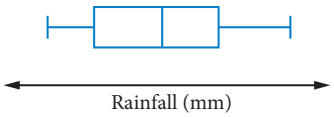
Steps	Working
a Interpret the situation, so you know what you are looking at:	Looking at racing times: the smaller the time, the faster/better the runner.
i the shape	The boys look approximately symmetrical. The girls look positively skewed.
ii measures of centre	The median and quartiles for the girls are higher than for the boys. This can be seen visually as the box for the girls is positioned higher than it is for the boys. The girls, in general, take longer than the boys.
iii measures of spread	The data for the girls has a slightly higher IQR of 0.5 compared to 0.4 for the boys. The range shows the same. Data for the girls is slightly more spread out than for the boys.
iv outliers	There are no outliers in the box plots.
b Write a statement comparing the time taken for the girls and the boys. Put it in practical terms for the question.	The boys were generally faster (had lower times) than the girls, although there were some girls who were faster than many of the boys.

EXERCISE 12.3 Parallel box plots

ANSWERS p. 425

Recap

1 The rainfall, in mm, each day in a month was recorded in the following box plot:



The shape of the data can best be described as

A symmetrical **B** bimodal **C** negatively skewed **D** positively skewed

2 Construct a box plot for the data represented by this five-number summary:

$$\text{min} = 4.5,\ Q_1 = 7,\ \text{median} = 12,\ Q_3 = 20,\ \text{max} = 25, \text{ and an outlier of } 42$$

Mastery

3 WORKED EXAMPLE 7 Construct parallel box plots of these datasets.

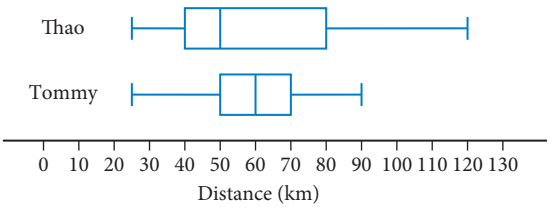
a Set A: 10 13 14 14 14 19 22 25 27 33 33 33 34 36 37 38 38 39 40 46 47 57

Set B: 7 12 13 20 21 25 26 28 28 29 30 31 33 34 37 38 40 43 55

b Set A: 15 32 13 28 14 23 20 29 15 26 18 26 11 28 19 24 15 30

Set B: 15 25 11 24 17 22 16 19 11 23 15 17 16 21 17 19 9 19 14 20 16

4 WORKED EXAMPLE 8 The following box plots show the distance, in km, travelled by 2 sea turtles, Thao and Tommy, each day for a month.



For the distance travelled by each sea turtle, calculate the

a **i** shape of the data **ii** centre of the data **iii** spread of the data **iv** outliers

b Write a statement comparing the distance travelled by the sea turtles.

5 The age of participants at 2 health retreats are given:

Retreat A:	50	76	49	58	39	69	53	33	46	64	36	55
	49	80	48	75	40	60	67	69	58	79	56	
Retreat B:	40	69	50	58	79	65	67	54	37			
	56	60	65	78	57	50	46	71	57			

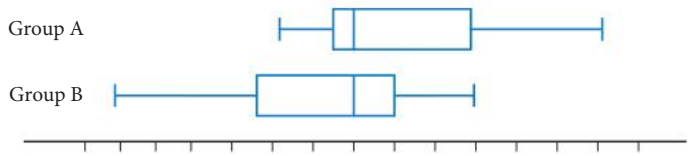
a Draw a parallel box plot of the data.

b Compare the data in terms of shape, centre, spread and outliers.

c Write a statement comparing the ages of participants at the health retreats.

Exam practice

6 For the following parallel box plots, which statement is **not** correct?



A The median for the two groups is the same.

B The IQR is the same for both sets of data.

C 25% of data for group A is greater than all the data for group B.

D Group A data is negatively skewed and group B data is positively skewed.

7 (5 marks) The relative humidity, as a %, shows the amount of water vapour in the air. The higher the relative humidity, the greater the chance of rain.

The relative humidity in 2 towns at 3 pm each day is recorded over 2 weeks.

Town A: 33, 35, 67, 45, 48, 67, 84, 56, 58, 57, 45, 48, 68, 56

Town B: 45, 48, 67, 78, 79, 84, 65, 58, 43, 59, 69, 89, 78, 69

a Draw parallel box plots. *[3 marks]*

b Payal said that these results show that it is more likely that there will be storms in town A. Is she correct? Justify your answer. *[2 marks]*

8 (4 marks) CF Golf balls have a nominal diameter of 4.3 cm. The diameters, in cm, of a sample of 20 balls from 2 makers were recorded. The following five-number summaries were found:

Beaut Ball Company:

min = 4.27, Q_1 = 4.285, median = 4.295, Q_3 = 4.30, max = 4.34

E-Zee Ball Company:

min = 4.26, Q_1 = 4.295, median = 4.305, Q_3 = 4.345, max = 4.39

Decide which company has the better production processes. Justify your answer.

9 (6 marks) CU 25 smokers who entered a new program to stop smoking were surveyed on the number of cigarettes that each smoked per day. Two months after completing the program, the same people were surveyed again.

Cigarettes per day before program:

9 30 8 32 10 36 18 20 10 27 17 24 18 11 12 24 17 21 15 23 15 22 18 13 22

Cigarettes per day after program:

8 34 9 21 11 23 8 23 12 9 24 19 25 17 23 16 22 17 27 16 24 12 10 21 9

Use parallel box plots to determine the success of the program and justify your answer.

12.4 Using summary statistics

Using summary statistics

The analysis of data should include looking at:

- measures of centre
  - mean: average of all data values, good for large sets of data; advantage is that it uses all data values
  - median: gives the middle value, so not affected by outliers
  - mode: the most frequent value; depends on the situation whether this is useful
- measures of spread
  - range = max – min: not as useful as it is affected by outliers
  - IQR = $Q_3 - Q_1$: looks at middle 50% of data, so not affected by outliers
  - standard deviation: good for large sets of data, advantage is that it uses all data values
- shape:
  - positively skewed
  - negatively skewed
  - symmetrical
- possible outliers

Measure of spread

- Larger = more spread out/varied.
- Smaller = more consistent.

WORKED EXAMPLE 9 Comparing data using mean and standard deviation

The age at which a sample of 100 people in 2 different countries got their drivers' licence is shown.

Country A	
Age	Frequency
16	40
17	29
18	17
19	10
20	4
Total	**100**

Country B	
Age	Frequency
16	0
17	7
18	62
19	25
20	6
Total	**100**

Use mean and standard deviation to **compare** the age at which drivers got their licence in each country.

Steps	Working
1 Use the STAT function on the calculator to find the mean and standard deviation of each country.	Country A: $\bar{x} = 17.09$, $s_x = 1.1555$ Country B: $\bar{x} = 18.3$, $s_x = 0.6890$
2 Compare the means.	Country A has a lower mean.
3 Compare the standard deviations.	Country A has a higher standard deviation.
4 Write a comparison statement in practical terms.	In general, Country A's drivers got their licence at a younger age. Although there was more variation in the age, they got their licence younger compared to Country B's drivers.

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When using summary statistics, write comparisons in a real-life context.

Considering the real-life context will tell you what is best in that situation, such as if you want a higher or lower measure of centre.

WORKED EXAMPLE 10 Using summary statistics to compare data

Compare these results of 15 students who study both Art and Music.

Art: 38, 28, 56, 35, 40, 17, 53, 29, 80, 60, 40, 32, 53, 41, 47

Music: 63, 52, 64, 58, 48, 56, 34, 60, 48, 55, 61, 46, 49, 53, 43

a Are there any outliers in either dataset? **Justify** your answer.

b **Calculate** the mean, median and mode.

c **Calculate** the range, IQR and standard deviation.

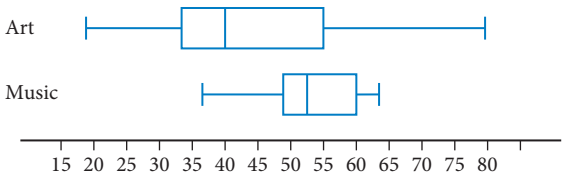
d **Comment** on the shape of the distributions.

e Which measure of centre is best for this data? **Explain**.

f Which measure of spread is best for this data? **Explain**.

g Using the summary statistics, **compare** the results.

Steps	Working
a **1** For Art:	five-number summary: 17, 32, 40, 53, 80
Order the data, break into quartiles, write the five-number summary. Calculate fences to look for outliers.	IQR $= 53 - 32 = 21$ lower fence $= 40 - 1.5 \times 21 = 8.5 \rightarrow$ no outlier upper fence $= 53 + 1.5 \times 21 = 84.5 \rightarrow$ no outlier
2 For Music:	five-number summary: 34, 48, 53, 60, 64
Order the data, break into quartiles, write the five-number summary. Calculate fences to look for outliers.	IQR $= 60 - 48 = 12$ lower fence $= 48 - 1.5 \times 12 = 30 \rightarrow$ no outlier upper fence $= 53 + 1.5 \times 12 = 71 \rightarrow$ no outlier
3 Answer the question.	There are no outliers in either set of data.
b Art:	
mean – use STAT function	$\bar{x} = 43.3$
mode – most frequent	mode = 53
median – from five-number summary	median = 40
Music:	
mean – use STAT function	$\bar{x} = 52.66\ldots$
mode – most frequent	mode = 48
median – from five-number summary	median = 53
c **1** Art:	
range = max − min	range $= 80 - 17 = 63$
IQR from five-number summary	IQR = 21
std deviation – use STAT function	std deviation = 15.517
2 Music:	
range = max − min	range $= 64 - 34 = 30$
IQR from five-number summary	IQR = 12
std deviation – use STAT function	std deviation = 8.225…

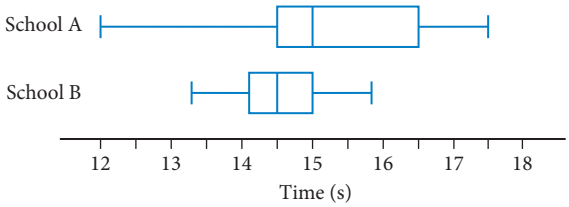
d **1** Look at the box plots.	
2 Comment on the shape.	Art: slightly positively skewed Music: slightly negatively skewed
e Answer the question with justification.	There are no outliers. The mean uses every piece of data. The mean is the best measure of centre.
f Answer the question with justification.	There are no outliers. The standard deviation uses every piece of data. The standard deviation is the best measure of spread.
g Answer the question with justification.	The students did comparatively better in Music; this is shown by the higher mean. The Music results were also more consistent; this is shown from the lower standard deviation.

EXERCISE 12.4 Using summary statistics

ANSWERS p. 425

Recap

1 The box plots below show the time taken, in seconds, for students from 2 different schools to run a 100 m race.



Which of the following statements is true?

A School A had the larger box, so they had the fastest runner.

B School A had the fastest runner but school B had a lower median and were more consistent.

C The median for school B was lower than for school A, so they had the fastest runner.

D School A had the best time as they had a runner who finished in 18 seconds.

2 Construct a parallel box plot to show the results of 18 students in the 2 subjects.

Science: 74 62 82 83 60 73 67 82 85 62 56 59 56 82 56 64 68 75

English: 94 67 84 91 40 65 56 83 58 45 89 65 89 46 42 59 43 87

Mastery

3 WORKED EXAMPLE 9 The lengths of words in typical definitions in 2 biology textbooks were counted.

Text 1: 5, 8, 10, 6, 9, 6, 10, 8, 5, 6, 8, 10, 8, 7, 9, 7, 8, 10

Text 2: 12, 8, 5, 24, 7, 9, 5, 5, 22, 5, 13, 9, 10, 9, 5, 17, 14, 8, 14, 7

a Calculate the five-number summary.

b Are there any possible outliers? Justify your answer.

c Compare the difficulty of the texts, assuming that longer words make a text harder to read.

4 A company has 2 factories that produce the same products. Salomi knows that the machine in Factory X is meeting quality control checks for producing the correct weight in cans. She obtains the weights, in grams, of cans produced in both Factory X and Factory Y in a day. The results are below.

Factory X	
Weight	Frequency
297	1
298	3
299	6
300	89
301	325
302	516
303	1522
304	402
305	203
306	78
307	5
Total	**3150**

Factory Y	
Weight	Frequency
299	2
300	8
301	189
302	431
303	1245
304	840
305	387
306	24
307	15
308	7
309	2
Total	**3150**

a Find the mean and standard deviation for both factories.

b Compare the machines in the different factories and let Salomi know if there is a problem. Justify your answer.

5 WORKED EXAMPLE 10 The following times, in seconds, to complete a task were obtained by 18 students from 2 different classes.

Class A: 17 21 15 18 17 14 19 17 15 18 15 16 24 16 18 17 16 19

Class B: 17 15 16 21 14 16 17 19 15 14 18 15 14 16 17 22 14 18

a Are there any outliers in either dataset? Justify your answer.

b Calculate the measures of centre – mean, median and mode.

c Calculate the measures of spread – range, IQR and standard deviation.

d Comment on the shape of the distributions.

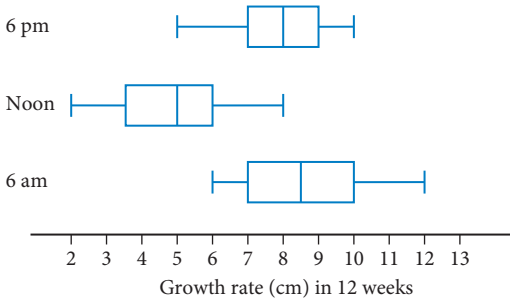
e Which measure of centre is best for this data? Explain.

f Which measure of spread is best for this data? Explain.

g Using the summary statistics, compare the time taken for each class to complete the task.

Exam practice

6 A nursery was looking into the effect the time of the day of watering had on growth rate, if any. Three samples of 50 plants each were set up. One sample was watered at 6 am, one sample was watered at noon and the third sample was watered at 6 pm. The parallel box plots show the growth rate, in cm, of these samples over a 12-week period.



From the box plots, it can be concluded that

A noon is the best time as it had the highest growth rate.

B the growth rate is more variable for noon than for 6 am.

C 6 am is the best time as it has the highest median and similar variability to the other times.

D 6 pm has the less variable growth rate and so it is the best time to water the plants.

7 (4 marks) A golf club compares 2 players to assess which one should be selected for their competition team. They looked at the last 10 times the 2 players had played. They are looking at the number of shots the players took over 18 holes. Their results are:

Lachie:	62	72	74	70	68	76	72	73	68	78
Cohen:	72	74	71	73	69	70	72	71	69	72

Which golfer would you select for the team? Justify your decision.

8 (4 marks) Mia asked 15 people who each attended the Taylor Swift and Pink concerts how far they had travelled, in kilometres, to attend. The results are shown below.

Taylor Swift:	21	18	36	38	470	4	8	17	24	21	9	51	24	15	20
Pink :	7	15	25	32	33	50	3	30	21	3	28	30	48	40	27

What can Mia say about the comparison between the distances these fans travelled to see these stars perform? Justify your answer.

9 (6 marks) A large catering company buys its vegetables direct from producers. It has samples of brown onions from 2 farmers, with the masses of the onions (in grams) shown below.

Farmer Dan:	201	212	263	253	73	292	190	198	200	289
	321	122	348	374	129	143	96	208	187	206
Farmer Joe:	189	269	162	165	152	218	161	270	122	192
	199	190	127	204	195	140	156	205	133	198

a Calculate an appropriate measure of centre. Justify your choice. *[2 marks]*

b Calculate an appropriate measure of spread. Justify your choice. *[2 marks]*

c Which producer should the company choose? Carefully justify your choice. *[2 marks]*

10 (4 marks) CF A local cricket club has narrowed the choice of the last batsman for a match down to 2 candidates. Their scores in their last 12 matches are as follows.

'Basher' Smith: 6, 27, 34, 22, 31, 21, 0, 32, 21, 21, 37, 21

'Slogger' Jones: 18, 25, 0, 17, 27, 12, 24, 25, 42, 25, 28, 27

Which batsman should the club choose, and why?

11 (4 marks) CU The pH of the water in a swimming pool should be close to 7.6. The pH level is set with an automatic dosing system and can be adjusted.

The pH in 2 pools with different automatic dosing systems was set at 7.6. The pH was measured at 3 pm each day for a month with the following results.

Easy as ABC	
pH	Frequency
7.1	2
7.2	5
7.3	4
7.4	4
7.5	10
7.6	5
7.7	0
Total	**30**

Setnforget	
pH	Frequency
7.1	1
7.2	1
7.3	4
7.4	2
7.5	12
7.6	6
7.7	4
Total	**30**

Which dosing system would you recommend? Explain your answer.

Worksheets
Comparing city temperatures

Comparing word lengths

Comparing sports scores

Comparing group measures

Investigating young drivers

12.5 Comparing datasets

Comparing datasets

When you compare data, it is important to choose the best method of comparison.

- Parallel box plots give a visual representation; easy to see shape, centre (median) and spread (IQR), and presence of outliers.
- If there are no outliers, mean and standard deviation are better measures of centre and spread.
- Range can be used as a supporting value, but it is affected greatly by outliers, so is not that useful on its own.
- Mode is only useful if you are looking for the most popular.

WORKED EXAMPLE 11 Comparing data

The ages of Year 11 students' mothers and fathers are shown below.

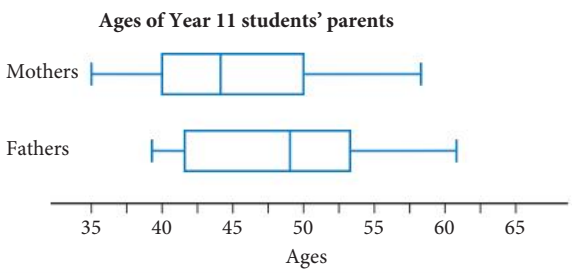
Mothers' ages:

38	43	35	52	55	57	47	49	39	44
46	43	44	48	44	40	51	53	36	42
49	52	39	44	40					

Fathers' ages:

41	46	38	55	58	57	48	48	40	49
45	49	48	52	48	43	57	61	38	42
51	54	41	44	49					

Compare the ages of the mothers and fathers, including a visual display to show shape.

Steps	Working
For each dataset:	
1 Order data, break into quartiles, write five-number summary.	Mothers' ages: 35, 40, 44, 50, 57 Fathers' ages: 38, 42.5, 48, 53, 61
2 Calculate fences and look for outliers.	Mothers' ages: IQR = 50 − 40 = 10 lower fence = $40 - 1.5 \times 10 = 25$ upper fence = $50 + 1.5 \times 10 = 65$
No values below 25 or above 65.	There are no outliers.
	Fathers' ages: IQR = 53 − 42.5 = 10.5 lower fence = $42.5 - 1.5 \times 10.5 = 26.75$ upper fence = $53 + 1.5 \times 10.5 = 68.75$
No values below 26.75 or above 68.75.	There are no outliers.
3 Visual display – small dataset so draw a parallel box plot.	

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4 No outliers, so calculate the mean and standard deviation.	Mothers' ages: $\bar{x} = 45.2,\ s_x = 5.99$ Fathers' ages: $\bar{x} = 48.08,\ s_x = 6.44$
5 Comment on the shape of the box plots.	Both plots are close to symmetrical (although mothers' ages might be very slightly positively skewed).
6 Write a statement comparing the data.	The fathers of Year 11 students are in general a little older than the mothers, and there is more variation in the ages of the fathers.

EXERCISE 12.5 Comparing datasets

ANSWERS p. 426

Recap

1 Two golfers are comparing their results in a number of tournaments. The following calculations were made:

Josef: $\bar{x} = 68$ and $s_x = 0.46$

Gustavo: $\bar{x} = 65$ and $s_x = 0.67$

Which statement is **not** true?

A Gustavo did better overall but Josef is more consistent.

B Josef did better overall and is more consistent.

C Gustavo did better overall and is more consistent.

D Josef did better overall but Gustavo is more consistent.

Exam hack

In practical situations, always think whether you want highest or lowest.

So, in golf → low score,
races → low times,
test mark → high score.

2 The age at which a sample of 100 people in 2 different countries got their drivers' licences is shown.

Country A	
Age	Frequency
16	40
17	29
18	17
19	10
20	4
Total	**100**

Country B	
Age	Frequency
16	0
17	7
18	62
19	25
20	6
Total	**100**

Use mean and standard deviation to compare the age that drivers got their licence in each country.

Mastery

3 WORKED EXAMPLE 11 Compare the ages of students working part-time at a take-away food outlet and a large hardware store.

Take-away: 15 19 16 16 15 19 15 17 16 16 15 16 16 17

Hardware: 18 21 17 19 19 20 17 20 17 20 17 22 19 20

4 Estelle and Matthew collected information about capybaras, the South American rodent. They recorded the masses, in kilograms, of a sample of 13 male and female adult capybaras.

Males: 59 48 35 41 58 44 61 53 49 52 56 48 42

Females: 58 43 64 66 52 59 56 53 35 60 56 59 45

Look at the data and write a statement about the masses of the male capybaras compared to the female capybaras. Justify your comments with calculations and a diagram.

5 The annual earnings of a sample of 24 plumbers and teachers are shown below.

Plumbers

Stem	Leaf
8	4 8
9	3 4 6 7 7 8 9
10	1 3 4 5
11	3 3 3 6 6 8
12	0 1 9
13	7 8

Key: 9|3 means $93 000

Teachers

Stem	Leaf
7	1 2 6 8
8	4 4 4 4 6 6 8 9
9	2 7 8 9 9
10	2 4 7 8
11	3 3
12	
13	9

Key: 9|2 means $92 000

a Are there any possible outliers? Justify your decision.

b Calculate relevant statistics for each dataset. Justify why they are appropriate.

c Describe the distribution of both datasets.

d Comment on the comparison between the annual earnings of this sample of plumbers and teachers.

Exam practice

6 The ages of players in 2 professional netball teams have the following summary statistics:

Nettie Allstars

mean: 23.5 and standard deviation: 1.8

Centre Flyers

mean: 25.7 and standard deviation: 3.2

In comparing the age distribution of the 2 teams, which of the following statements is not true?

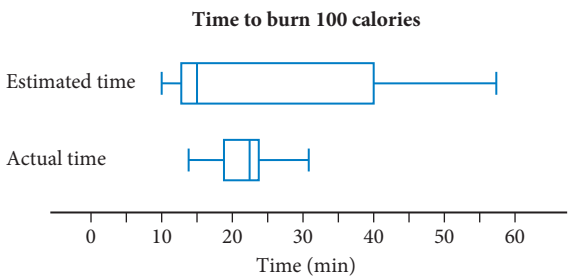
A The Centre Flyers have the oldest player, and the ages are more spread out.

B The Nettie Allstars have a younger team age, and their ages are more consistent than Centre Flyers.

C The Nettie Allstars have the youngest player, and the ages are more spread out.

D The Centre Flyers have an older team age, and their ages are more consistent than Nettie Allstars.

7 (3 marks) Idris believes that everyone underestimates the time it takes to burn calories. He asked a class to estimate how long they thought it would take to burn 100 calories by walking on a treadmill. He then recorded the actual time taken using the treadmill tracker. The results are shown in the parallel box plots.



Do Idris's results support his theory? Justify your answer by referring to medians.

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8 (5 marks) CF The frequency tables below show the maximum daily temperatures (°C) recorded in July for Mackay and Emerald over 75 years.

Mackay	
Maximum daily temperature	Frequency
18	1
19	5
20	9
21	13
22	18
23	15
24	11
25	2
26	1
27	0
Total	**75**

Emerald	
Maximum daily temperature	Frequency
18	0
19	7
20	29
21	15
22	13
23	5
24	4
25	0
26	1
27	1
Total	**75**

Use a statistical method, including a graph, to compare the maximum daily temperature in July and provide a detailed discussion of the distribution.

9 (5 marks) CU A **back-to-back stem-and-leaf plot** allows an easy comparison for 2 sets of data. The stem-and-leaf plots share a stem and have 2 sets of leaves, one to the right of the stem and one to the left.

The numbers of bookings each week for 2 restaurants were represented in the following back-to-back stem-and-leaf plot.

Swift's Steakhouse		**Brodie's Brasserie**
Leaf	Stem	Leaf
1	0	
	1	5
	2	
7 7 6 6 6	3	
9 5 4 4 2 1 1 1	4	5 9
9 9 8 7 7 6 6 5 5 5 3 3 2 1 0 0 0 0 0	5	0 0 6
7 5 5 4 4 3 3 2 1 1	6	1 3 6 7 9
9 5 3 2 1	7	1 2 2 3 5 5 6 7 7 8 8 9
6	8	0 1 1 1 3 3 4 5 6 7 7 8 8 8 9 9
	9	2 3 5 6 7 7 8 8
4	10	2 2
	11	
	12	
	13	1 5
Key: 6|3 means 36		Key: 10|2 means 102

Compare the restaurants. State any conclusions you can make about the restaurants and include another statistical diagram in your solution. Justify your conclusions.

Video Exam question analysis: Comparing data

EXAM QUESTION ANALYSIS

Exam-style question (6 marks)

Josie calculated the mean test mark for a group of 8 students to be 6.5. The median was 6 and the lower and upper quartiles were 4.5 and 7.5 respectively. The minimum was 4. The marks were all whole numbers. Comment on the shape of the distribution of the test marks.

Reading the question

- Need to comment on the distribution.
- Most of the five-number summary is given: 4 out of 5
- What could the 8 marks be?

Thinking about the question

- Determine the maximum mark, missing from the five-number summary.
- The 8 marks will be grouped in 2s for the median and quartiles.
- The mean is the total divided by the number of students.
- or 4, 4, 5, #, #, 6, 9, #

Worked solution (✓ = 1 mark)

As the minimum mark is 4, the median is 6 and the lower and upper quartiles are 4.5 and 7.5 respectively, the data must be arranged in order as

4, 4, 5, #, #, #, #, # ✓

↓ ↓ ↓

4.5 6 7.5

The 2 #s in the centre could be 6, 6 or 5, 7 to make the **median 6**. ✓

The 2 #s around 7.5 could be 7, 8 or 6, 9 to make $Q_3 = 7.5$.

There are 3 possibilities.

Case 1: 4, 4, 5, 6, 6, 7, 8, x,

Case 2: 4, 4, 5, 6, 6, 6, 9, x

or Case 3: 4, 4, 5, 5, 7, 7, 8, x

In all 3 cases, sum of marks $\boldsymbol{\Sigma x = 40 + x}$ ✓

But the mean is 6.5, so $6.5 = \dfrac{\Sigma x}{8}$.

$\Sigma x = 8 \times 6.5 = 52$.

$\therefore\ 40 + x = 52$

$x = 12$

The maximum mark is 12. ✓

12 could be a possible outlier, so test for it.

IQR = 7.5 – 4.5

= 3

upper fence = 7.5 + 1.5 × 3

= 12

12 is not greater than 12, so it is not an outlier. ✓

In all 3 cases, the distribution is positively skewed, but only by one mark that is very much higher than the others. ✓

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12 Chapter summary

Five-number summary

- A **five-number summary** consists of:
  min, Q_1, median, Q_3, max

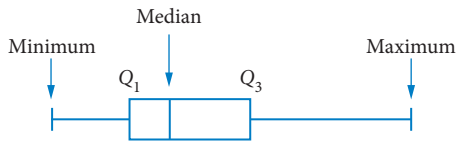
Testing for possible outliers

- A data value is a possible outlier if it is:
  – less than the **lower fence**: $Q_1 - 1.5 \times \text{IQR}$
  or
  – more than the **upper fence**: $Q_3 + 1.5 \times \text{IQR}$.
- If the value is outside the fences, it's an outlier.

Box plots

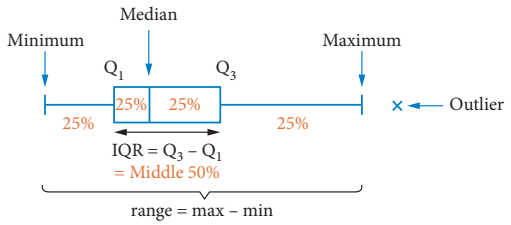
Box plots are:

- also known as **box-and-whisker plots**.
- a visual representation of the five-number summary.

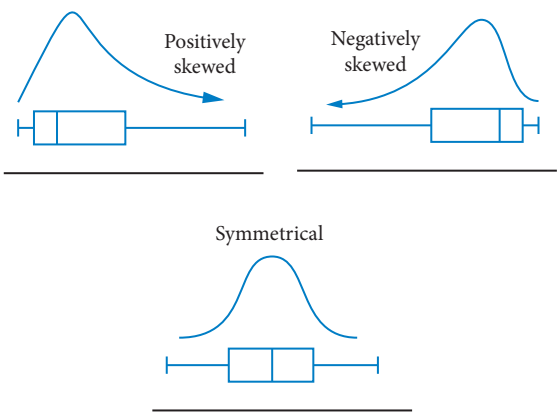


Reading and interpreting a box plot

- From box plots you can calculate the range and interquartile range, as well as questions based on the fact that 25% of data values are in each section.



- The shape of the data can also be seen. If the 25% interval is narrow, the data is grouped together. If it is wider, the data is more spread out.



Parallel box plots

- Allow both the centre and spread of the distributions to be found and compared visually.
- Comparison statements can be made to describe
  – shape
  – centre (the median)
  – spread (the IQR)
  – possible outliers.

Using summary statistics

The analysis of data should include looking at:

- measures of centre
  – mean: average of all data values; good for large sets of data, advantage is that it uses all data values
  – median: gives the middle value, so not affected by outliers
  – mode: the most frequent value; depends on the situation if this is useful
- measures of spread
  – range: max – min; not as useful as it is affected by outliers
  – IQR: $Q_3 - Q_1$, looks at middle 50% of data, so not affected by outliers
  – standard deviation: good for large sets of data, advantage that it uses all data values
- shape
  – positively skewed
  – negatively skewed
  – symmetrical
  – possible outliers

Comparing datasets

When comparing datasets, it is important that you choose the best method of comparison.

- Parallel box plots give a visual representation. Easy to see shape, centre (median) and spread (IQR), as well as the presence of outliers.
- If there are no outliers, then mean and standard deviation are better measures of centre and spread.
- Range can be used as a supporting value but as it is affected greatly by outliers, it is not that useful on its own.
- Mode is not as useful, but if you are looking for most popular – then it has its place.

Worksheet
General Maths Year 11 formula sheet

Cumulative examination 1

Simple familiar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

Section 1 (5 marks): 5 multiple choice questions

Section 2 (13 marks): 5 short response questions

Total: 18 marks

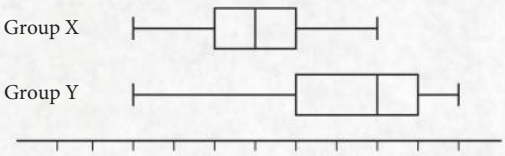
Section 1 5 multiple choice questions

5 marks

1 If $x = \begin{bmatrix} 4 & -2 \\ 3 & 8 \end{bmatrix}$ and $y = \begin{bmatrix} 5 & 0 \\ -1 & 2 \end{bmatrix}$, which matrix shows the calculation of $3x - 2y$?

A $\begin{bmatrix} -1 & -2 \\ 4 & 6 \end{bmatrix}$ **B** $\begin{bmatrix} 2 & -6 \\ 7 & 20 \end{bmatrix}$ **C** $\begin{bmatrix} 22 & -6 \\ 7 & 28 \end{bmatrix}$ **D** $\begin{bmatrix} 2 & -6 \\ 11 & 20 \end{bmatrix}$

2 For the following parallel box plots, which statement is **not** correct?



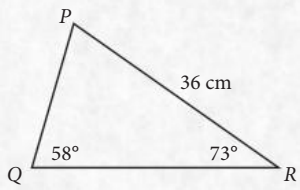
A The minimum value for the 2 groups is the same.

B The IQR is the same for both sets of data.

C 50% of the data for group Y is greater than all of the data for group X.

D Group Y is negatively skewed and group X is symmetrical.

3 The length of PQ is closest to



A 31.9 cm **B** 34.4 cm

C 40.6 cm **D** 42.5 cm

4 A group of 8 students did a topic test and received these marks:

47 54 36 45 68 32 51 40

After a revision session they were tested again. Each student's mark had increased by 10. Compared with the first result, the second set had

A a larger mean and a larger standard deviation.

B a larger mean and a smaller standard deviation.

C the same mean and a larger standard deviation.

D a larger mean and the same standard deviation.

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5 For the following data, calculate the value of the fences used to identify outliers.

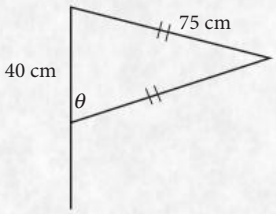
74 32 45 52 45 29 60 56 72 45 37 60 51

A $37.25 \le x \le 52.25$ **B** $12.5 \le x \le 88.5$ **C** $29 \le x \le 74$ **D** $41 \le x \le 60$

Section 2 5 short response questions

13 marks

6 (2 marks) A school designed a new pennant flag for their sporting teams. The flag has the dimensions as shown.

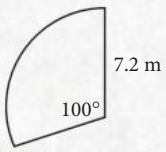


Calculate the angle that the flag makes with the pole, θ.

7 (3 marks) Find the five-number summary for this dataset.

38	41	32	52	37	44	31	42	31	43	31	48	37	45
38	42	34	40	32	44	34	43	33	44	38	41	33	

8 (2 marks) Calculate the area of this sector, correct to one decimal place.



9 (3 marks) Draw a box plot for the five-number summary 20, 25, 33, 35, 42, with the excluded outliers 5, 57, 62.

10 (3 marks) Solve $3(x - 2) = 7x + 1$.

Worksheet
General Maths
Year 11 formula sheet

Cumulative examination 2

Complex familiar and unfamiliar

Perusal time: 2 minutes Working time: 30 minutes

Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.

3 short response questions (12 marks)

1 (2 marks) Calculate the original price of a computer that sold for \$807.50 after it was reduced by 15%.

2 (4 marks) The angle of elevation of the top of a 70 m high cliff from a boat is 36°. The boat then sails 200 metres further out to sea to point *D*. What is the angle of elevation to the top of the cliff from *D*? Answer to the nearest degree.

3 (6 marks) The daily maximum temperatures (°C) in two Queensland towns over a fortnight were:

Town A: 22, 24, 28, 32, 35, 27, 24, 29, 28, 29, 27, 31, 23, 30

Town B: 25, 26, 26, 24, 26, 28, 28, 26, 29, 30, 27, 24, 29, 31

Which town is more likely to be near the sea? Justify your answer.

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Answers

CHAPTER 1

EXERCISE 1.1

1 **a** 47.5% **b** 312.5% **c** 292% **d** 79.7%

2 **a** 0.14, $\frac{7}{50}$ **b** 2.3275, $2\frac{131}{400}$ **c** 0.0056, $\frac{7}{1250}$ **d** 0.23, $\frac{23}{100}$

3 **a** $\frac{16}{25} = 64\%$, $\frac{25}{35} \approx 71.43\%$, so 25 out of 35 is greater.

b $\frac{32}{45} \approx 71.11\%$, $\frac{38}{55} \approx 69.09$, so 32 out of 45 is greater.

4 **a** \$179.20 **b** about \$185.79 **c** 13.392 g **d** about \$220.21

5 **a** 16.67% **b** 56.25% **c** 56.52% **d** 11.29% **e** 14.81% **f** 7.94%

6 **a** 16.47% **b** 12.28% **c** 95.83% **d** 10.53% **e** 192.68% **f** 7.78%

7 \$5440 **8** \$645 **9** \$392

10 about 488.37 **11** about 10.5 kg

12 6270 g (6.27 kg) **13** D **14** B

15 C **16** 350 mL

17 Suppose the original price is x. The percentage increase is $\frac{1600}{x}\%$. The percentage decrease is $\frac{1600}{x+16}\%$.

18 percentage increase $= \frac{3000}{x-30}\%$

EXERCISE 1.2

1 B **2** 22.03%

3 **a** \$17/kg **b** 4.33 points/game **c** 19.2 km/h **d** 12.5 m^2/L **e** 36 km/h **f** 100 000 kg/m^3

4 **a** 10 m/s **b** \$460/kg **c** 0.033 cm/s **d** 0.865 c/g

5 \$16.44/h

6 The 65-hectare paddock has a higher yield of 11.6 bags/hectare.

7 Nathan scored more points (158 to 144)

8 352 km **9** B **10** A

11 Yes, it will grow to 1.07 m in 21.4 months.

12 The heartbeat rates were 60 beats/min, 90 beats per min and 180 beats/min, so it increases with more exercise.

EXERCISE 1.3

1 C **2** 360 L

3 **a** \$2431 **b** \$4862 **c** \$10 533

4 \$3639.38 **5** \$47 621.60 **6** \$54 893.28

7 **a** \$73 000 per year **b** \$2104.40/week **c** \$44.01/h **d** \$42.90/h

8 \$1086.75

9 B **10** C **11** B **12** C

13 **a** \$387.24 **b** 5.5 hours

14 Shaye

15 Kristy's calculated salary = \$90 720. Her actual salary = \$98 280. Kristy's answer is incorrect as her calculated answer equates to 48 weeks rather than 52.

EXERCISE 1.4

1 D **2** \$39.27/h **3** \$1747.20

4 \$498.40 **5** \$53.91 **6** \$1415

7 \$430.56 **8** \$1825.70 **9** \$1351.44

10 C **11** C **12** \$1099.14

13 \$1329.75 **14** \$2111.83

EXERCISE 1.5

1 D **2** \$5690.25

3 **a** \$353.13 **b** \$44.14 **c** No, there would probably be fewer customers on weekdays.

4 \$9181.50 **5** \$40.63/h; \$9750

6 **a** \$23 603 **b** \$2632.09 **c** \$2083.05

7 He takes \$900, making \$365 at \$22.81/h.

8 **a** \$1138.60, \$21.15/h **b** \$4571.43

9 **a** \$278.80 **b** \$46.57 **c** \$232.23 after paying petrol and \$38.71/h, but the real cost of running his car is more than the petrol, so his real earnings is less.

10 A **11** D **12** B

13 \$129 **14** at least \$11 333 **15** \$29.95/h

EXERCISE 1.6

1 \$1120 **2** D

3 \$445.70/fortnight **4** \$639/fortnight

5 \$872.60/fortnight; \$35 687.60

6 \$892.60/fortnight; \$35 167.60

7 C **8** B **9** D

10 Danielle got \$455.20/fortnight, Sam got \$449.70/fortnight. Their Youth Allowance may have changed because their combined earnings may be used in their assessment.

EXERCISE 1.7

1 $395.30 **2** $806.00/fortnight = $20 956 in a year

3 Paid weekly, so budget weekly.

Income		Expenses	
Wage	$510	Flat	$102.50
		Food, etc.	$95.00
		Bus	$24.00
		Mobile	$50.00
		Entertainment	$105.00
		Clothes	$38.00
Total income	$510	Total expenses	$414.50

Savings: $95.50/week, $4966 a year. Ayla could save more by reducing her mobile account and entertainment expenses.

4 Paid fortnightly, so budget fortnightly.

Income		Expenses	
Wage	$1380	Flat	$450
		Power	$30
		Groceries	$190
		Entertainment	$300
		Bike payments	$90
		Running costs	$30
		Clothes	$56
		Mobile	$50
Total income	$1380	Total expenses	$1196

Savings: $184/fortnight, $4784 a year.

Justin could save more by spending less on takeaways and parties.

5

Expenses	
Air fares	$898
Airport transfers	$330
Accommodation	$1995
Meals	$910
Other	$350
Total expenses	$4483

6 Paid fortnightly, so budget fortnightly.

Income		Expenses	
Wage	$1360	House + electricity	$290
		Food, etc.	$180
		Mobile	$90
		Entertainment	$160
		Bus	$64
		Clothes	$84
Total income	$1360	Total expenses	$868

Savings: $492/fortnight, $12 300 a year.

7 Monthly savings = $2257

8 a

Expenses	
Girls cabins	$900
Boys cabins	$700
Teachers cabin	$100
Catering	$11 712
Buses	$1928
Total expenses	$15 340

b Cost per pupil = $170

c $130 for contingencies is probably enough.

9

Expenses	
Sausages	$600
Bread rolls	$550
Tomato sauce	$54
Total expenses	$1204

income: $3800

profit: $2596

CUMULATIVE EXAMINATION 1

1 C **2** D **3** A

4 C **5** B **6** $996.40

7 $167.70; $27.95/h **8** $329.38

9 a $30 **b** $240

c Samar needs to reallocate $250 over two weeks. He could not buy clothes for two weeks ($80 × 2 = $160) and reduce his entertainment to $25/wk ($45 × 2 = $90)

CUMULATIVE EXAMINATION 2

1 Charlie should choose Method 1; using the average sales total of $16 550: Method 1: 26% commission = $4303/month, Method 2: 9% commission + retainer of $550/week = $3872.83/month. Method 1 earns more.

2 Mia will need to work an additional 19 hours during the week.

3 No, Phong will be $872 short. It will take an additional 10 weeks (1 year and 4 weeks) to have enough for the deposit.

CHAPTER 2

EXERCISE 2.1

1 a $a = 150$ m **b** $b \approx 141.8$ mm **c** $c \approx 1.54$ cm

d $d \approx 8630$ mm **e** $e = 0.7$ m **f** $f \approx 19.4$ mm

2 a $a = 17$ m **b** $b \approx 19.2$ mm **c** $c \approx 4.7$ cm

d $d = 35$ km **e** $e \approx 10$ m **f** $f \approx 19.1$ cm

3 a no **b** no **c** yes

4 a 547 cm **b** 15.97 mm

5 Area = 8 cm^2 **6** 712 m **7** 15.2 m

8 16.12 cm

9 a 8.5 cm **b** 10.39 cm

10 D **11** 4082 mm **12** 275 m

13 6.8 m **14** about 20.7 m **15** 32.29 m

EXERCISE 2.2

1 B 2 C
3 a 25 m b 240 mm^2 c 0.0058 km^2
d 2100 000 mm e 45.721 m f 21 m^2
g 36 000 m^2 h 0.568 cm^2 i 6.72 m^2
4 a i 88.4 cm ii 400.05 cm^2 ≈ 400 cm^2
b i 39.84 m ii 126.3 m^2
c i 16.7 mm ii 63.9 mm^2
d i 95 mm ii 420 mm^2
e i 31 cm ii 27 cm^2
f i 648 mm ii 14 580 mm^2
g i 89.2 mm ii 497.29 mm^2
h i 98 m ii 270 m^2
i i 4712mm ii 1 767 145 mm^2
j i 80 m ii 380 m^2
k i 7.3 m ii 1.75 m^2
l i 127 mm ii 575 mm^2
5 a 0.6 m^2 b 1963.5 mm^2 c 150 cm^2
d 12 m^2 e 113.1 cm^2 f 7.1 m^2
6 a i 39.3 cm ii 64.3 cm iii 245.4 cm^2
b i 0.8 km ii 3.8 km iii 0.6 km^2
c i 60.8 mm ii 94.8 mm iii 517.0 mm^2
7 15.0 cm 8 10.5 cm by 10.5 cm
9 44.8 cm 10 20.0 m^2
11 C 12 about 2243 kg 13 140.76 cm^2
14 $3308 (for 58 posts, 111.5 m of rails, 892 full palings, gate)

EXERCISE 2.3

1 A 2 C
3 a i 94 cm ii 500 cm^2
b i about 5.95 m ii 2.13 m^2
c i 12 m ii 7.9 m^2
d i 75.4 cm ii 370.2 cm^2
e i 5390.4 m ii 1 878 500 m^2 (1.8785 km^2)
f i 52.84 cm ii 179.0 cm^2
g i 37 m ii 65 m^2
h i 20.02 cm ii 25.33 cm^2
i i 726.35 mm ii 32 202 mm^2
4 a 372.2 cm^2 b 38.8 m^2 c 4 426 820 mm^2
d 98.17 m^2 e 28.9 km^2 f 21.46 cm^2
5 10.28 m ≈ 10.3 m
6 a $2577.33 b $597.08 (including gate)
7 a 85 posts, 423 m of rail
b 1208 palings c $5208
8 $152 744.23
9 a 26.24 m^2 b 194.34 m^2 c 63.99 m^2
10 160.2 cm^2
11 a Court 30.48 m × 15.24 m; centre circle 0.9144 m diameter; goal circles 9.7536 m diameter.
b goal attack and goal defence 310 m^2, 66.7%; goal shooter and goal keeper 155 m^2, 33.3%; wing attack and wing defence 272 m^2, 58.6%; centre 390 m^2, 83.9%
12 $10 644.96
13 a 228 tiles b 5.76 L, so 6 L
c 3 bags d $1467.90
14 A 15 1714 cm^2 16 36.6%
17 4.27 L are required; need to buy a 4 L tin and a 1 L tin.

EXERCISE 2.4

1 D 2 B 3 B 4 B
5 a 2706 cm^2 b 288 cm^2 c 825 cm^2
d 4632 cm^2 e 64.45 m^2 f 24 m^2
g 3076 mm^2 h 3358 mm^2 i 9000 mm^2
6 a 32.5 m^2 b 1731.8 mm^2 c 1306.9 cm^2
d 3267.3 cm^2 e 3758.6 mm^2 f 1445.13 mm^2
g 1134.1 cm^2 h 763.4 mm^2 i 39.3 m^2
7 a 70.48 m^2 b $684.60
8 989 m^2 9 22.6 m^2
10 a 179 m^2 b $2431.50
11 a 22.9 m^2 b 2.5 L
12 49.3 m^2 13 C
14 $SA = \pi r^2 + 2\pi rh + \pi rs$
15 $1939.08
16 about $1370
17 382 labels (with some wastage at the end of the roll)

EXERCISE 2.5

1 D 2 C 3 B
4 a cm^3 (mL) b cm^3 (mL)
c m^3 (kL) d ML
e cm^3 (mL) f L
g cm^3 (mL) h L
i m^3 j cm^3 (mL)
5 a 26 000 000 cm^3 b 0.32 m^3 c 280 ML
d 0.8 L e 56 m^3
6 a 2.4 L b 3200 cm^3 c 7500 mL
d 0.73 L e 41 mL f 8200 L
g 432 cm^3 h 0.003 24 m^3 i 2.32 L
7 a 19 974.2 cm^3 b 4712.4 mm^3
c 8.6 m^3 d 1125.4 cm^3
8 a 82 m^3 b 619.7 cm^3 c 3.927 m^3
d 2205.4 cm^3 e 3234 mm^3
9 a 1480 cm^3 b 18 200 cm^3 c 285 cm^3
d 1.17 m^3 e 1.59 m^3 f 248 cm^3
g 2.744 m^3 h 22.6 cm^3 i 27 m^3
10 a 1 922 800 mm^3 b 1922.8 cm^3
11 2.083 m
12 a about 105 h b about 117 h

EXERCISE 2.6

1 C 2 A
3 a 1 243 547 mm^3 b 1437 cm^3 c 8379 mm^3
d 1206.4 cm^3 e 37 875 cm^3 f 18 m^3
g 867 415.1 cm^3 h 147 436.1 cm^3 i 1030 cm^3
4 1.624 L
5 a 2400 m^3 b 2400 kL c 148 hours
6 a 44 m^3 b about 21 min

7 **a** 193.8 m^3 **b** about 11.8% **c** 183.7 m^2
8 \$5140.80
9 72 cm^2
10 137 m
11 195 281 mm^2 (1952.81 cm^2)
12 26.3 m

CUMULATIVE EXAMINATION 1

1 B **2** C **3** A
4 D **5** B
6 **a**

b 68 m^2
7 29.81 cm **8** \$24.75/h **9** \$26 250

CUMULATIVE EXAMINATION 2

1 \$498.75 **2** 4464 **3** \$440.66

CHAPTER 3

EXERCISE 3.1

1 **a** **i** \$0.04/g, \$0.02/g **ii** 110 g
b **i** \$3.72/kg, 2.64/kg
ii As long as you use it in reasonable time, 1 kg.
c **i** \$0.52/100 mL, \$0.55/100 mL **ii** 300 mL bottle
d **i** \$2.37/pack, \$2.30/pack **ii** 12-pack
2 The leftover from a sachet is difficult to keep, so it is really 90 g for \$2. If she can keep the bottle in the fridge for at least 4 weeks after opening, it is cheapest. Otherwise, the tubs are best.
3 If he made 2 meals, the chuck steak is best, otherwise the gravy beef.
4 A
5 Pastell Drömmar (Pastel Dreams), Vintermagi (Winter Magic), Glittrande Gåvor (Glittering Gifts)

EXERCISE 3.2

1 C **2** D
3

a	Table	\$800	75%	\$1400
b	Software bundle	\$300	90%	\$570
c	Bottle of soft drink	\$3.00	50%	\$4.50

4 **a** 81.2% **b** 125.8%
5 **a** \$1.14 **b** \$2.21
6 **a** \$148.75 **b** \$280
7 **a** \$53; \$19 **b** 26.4%
8 **a** \$19.80 **b** \$217.80 **c** \$10.91
9 6.25% **10** \$455
11 Fishing reel: 33.3% profit, video camera: 31.8% profit, jewellery box: 28.6% loss, filing cabinet: 62.5% profit, printer: 25% loss
12 \$300; \$471
13 Because 90% × 80% = 72%, not 70%.
14 \$240

EXERCISE 3.3

1 A **2** B
3 **a** \$180 **b** \$320 **c** \$1440
4 **a** 7.5% **b** 6.5% **c** 6.25%
5 **a** \$2375.63 **b** \$8925 **c** \$14 004
6 15% **7** \$3112.50; \$81.64
8 \$145.03 **9** \$162 500
10 \$1 248 000 **11** D **12** 21.3%
13 \$7440, \$7448; the second investment option is better.

EXERCISE 3.4

1 C **2** A
3 **a** US\$521.84 **b** US\$952.36 **c** A\$5244.52
d A\$843.17
4 **a** NZ\$458.90 **b** 270 550 rupee **c** A\$1750.12
d A\$5019.24
5 A\$4902.60 **6** \$412.97
7 \$1548.89 **8** ¥74 244.98

EXERCISE 3.5

1 A **2** £506.17
3 **a** 0.8% **b** \$40 000
4 **a** 9000 **b** \$450 000 **c** 1000 shares
5 \$20
6 **a** \$210 **b** 5.07%
7 5.08% **8** \$1929.24
9 **a** Another Chance Pty Ltd = 6.31
Because I Can Pty Ltd = 11.2
b Because I Can Pty Ltd has been the better investment.
10 \$150

CUMULATIVE EXAMINATION 1

1 C **2** B **3** A
4 B **5** D **6** \$771.40
7 **a** \$2260 **b** 4.5%
8 0.47 cm
9 **a** 1800 cm^3 **b** 1609 cm^3 **c** 10.6%

CUMULATIVE EXAMINATION 2

1 16.6 m **2** Product 2 is better, \$11 260 instead of \$11 236.
3 A\$265.81

CHAPTER 4

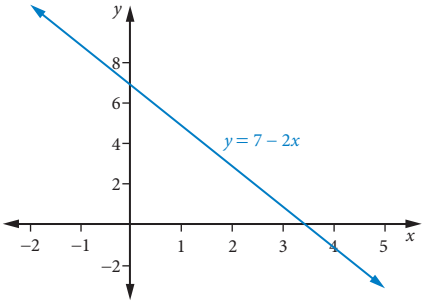
EXERCISE 4.1

1 **a** $2g + 2h + 10$ **b** $6m - 4 + 5n$
c $4p - 19$ **d** $3 - 5x^2 - 8y + 3xy$
e $5q^2 + 3pq - 4$ **f** $6a - 5 + 10b - 3ab$
g $2m^2 + 14m$ **h** $-y - 17$
2 **a** $8ab^2c$ **b** $-15d^3g^2$ **c** $\frac{3y}{2x}$
d $42y^4x^5$ **e** $\frac{17x^2y}{14z}$ **f** $48p^5q^5$
g $\frac{-3b}{a^2}$ **h** $\frac{19h^3j^2}{21k}$

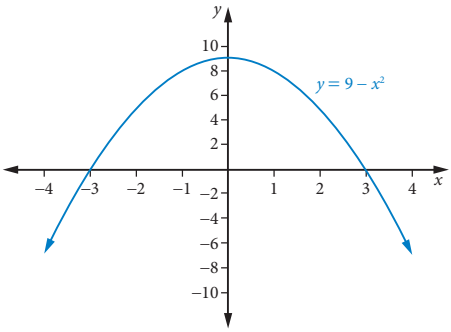
3 linear: **a**, **b**, **d** and **f**; non-linear: **c, e, g, h, i**

4 **a**, **c**, **d**, **f**, **h** and **i** are non-linear.

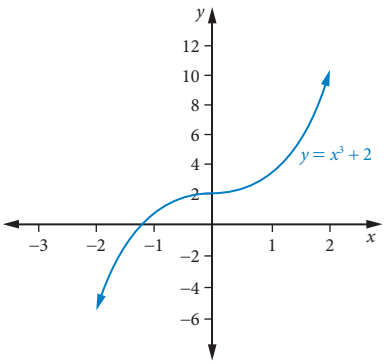
5 **a** linear



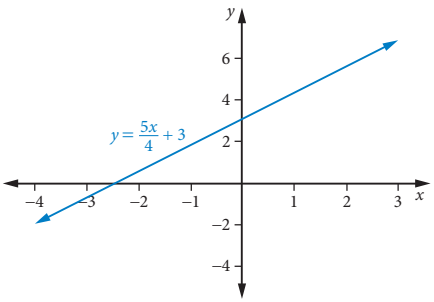
b non-linear



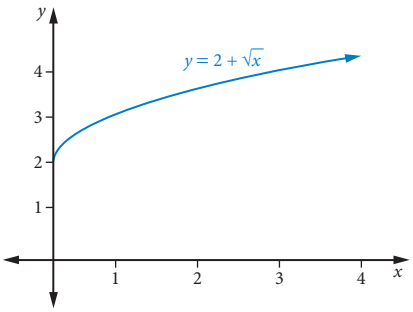
c non-linear



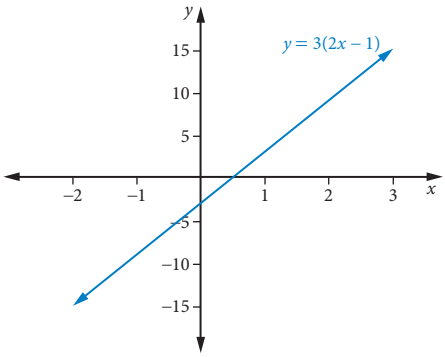
d linear



e non-linear

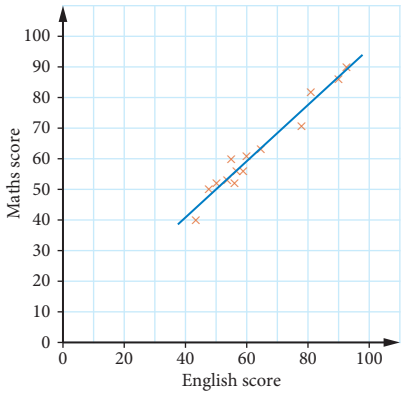


f linear



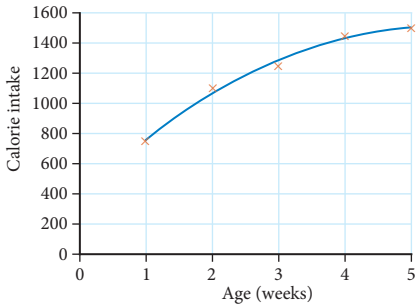
6 **a**, **d**, **g** and **h** are linear; **b**, **c**, **e** and **f** are non-linear.

7 **a** There is an approximately linear relationship.



b about 68

8 non-linear



9 D

10 non-linear; The graph of $A = \pi r^2$ is not linear. The highest power on a variable is 2. Could also use a table of values to show that there is not a common difference.

11 linear

EXERCISE 4.2

1 C **2** C

3 **a** $y = -11$ **b** $y = 10$ **c** $y = 14$

4 **a** $A = 17$ **b** $A = -5$ **c** $A = 27$
d $A = 49$ **e** $A = -5$ **f** $A = -21$

5 **a** $Y = 7.1$ **b** $Y = 10.08$ **c** $Y = 18.01$
d $Y = -25.23$ **e** $Y = -15.5$ **f** $Y = -4$

6 **a**

x	0	1	2
y	−3	2	7

b

C	−1	4	5
D	9	−1	−3

7 315 km

8 **a** 9 **b** 21.41

9 \$3180

10 **a** \$8442.60 **b** \$18 741.77 **c** \$11 934.07

11 **a** 339.29 cm^2 **b** 904.78 cm^2

12 1.2 amperes **13** 1.33

14 250 cm^3

15 25 and 26 **16** $k_1 : k_2 = 4 : 3$

EXERCISE 4.3

1 D **2** B

3 **a** $t = \frac{2u - v}{3au}$ **b** $c = \frac{Ik - G}{kt}$

c $b = m(c - a) + a$

d $u = sa - at$ or $a(s - t)$

e $d = \frac{ac}{b}$ **f** $k = \frac{d - h}{n}$

4 **a** $R = a \pm \sqrt{d^2 - c}$

b $a = cv^2 + b$

c $b = \sqrt{\frac{y}{4ac^2}}$ **d** $c = \frac{(d + 5)^2}{b}$

e $c = \frac{b}{a - bm}$ **f** $v = \frac{fu}{f - u}$

g $r = \sqrt[3]{\frac{3V}{4\pi}}$ **h** $y = 1 \pm \sqrt{9 - (x - 2)^2}$

5 **a** $x = \frac{1}{2}(y + 7)$ **b** $x = \frac{1}{3}(y + 13)$

c $x = -2y - 1$ **d** $x = \frac{3}{5}y + 7$

e $x = \frac{1}{11}(7y + 18)$ **f** $x = \frac{1}{51}(15y - 8)$

6 **a** $y = \frac{x + 6x^2 - 8}{3}$ **b** $y = \pm\sqrt{5x + 12}$

c $y = \frac{1}{2}(5x - 3)^2$ **d** $y = \frac{7}{3x}$

e $y = -\frac{3}{5 - 2x}$ **f** $y = -\frac{49x^2 + 28x + 1}{2}$

7 **a** $A = z - x$; $A = -2$ **b** $A = \frac{15x}{z^2}$; $A = 1.8$

c $A = \frac{-\sqrt{6xy} + z}{3}$; $A = -\frac{1}{3}$

8 **a** $u = 6$ **b** $m = -4$ **c** $E = 17.5$

d $h = 45$ **e** $t = 3.75$ **f** $r = 6.5$

9 6000 m **10** A

11 5.03 m

12 6 tries

13 $V = \frac{\pi}{3}(R^2 + r^2 + Rr)h$

EXERCISE 4.4

1 B **2** $h = \sqrt[2]{\frac{m}{B}}$

3

r (cm)	0.5	1	1.5	2
C (cm)	3.14	6.28	9.42	12.56

4

C	−5	10	25	30
F	23	50	77	86

5

G		*K*			
		1	2	3	4
H	6	11	5.5	$3\frac{2}{3}$	2.75
	12	17	8.5	$5\frac{2}{3}$	4.25
	18	23	11.5	$7\frac{2}{3}$	5.75

6

F		*a*			
		5	10	15	20
m	20	100	200	300	400
	30	150	300	450	600
	50	250	500	750	1000

7 **a** =20*A2+15 **b** \$115

8 **a**

	A	B	C
1	*r*	*h*	Curved surface area
2	1	10	62.8318
3	2	10	125.6637
4	3	10	188.4956
5	4	10	251.3274
6	5	10	314.1593
7	6	10	376.9911
8	7	10	439.823
9	8	10	502.6548
10	9	10	565.4867
11	10	10	628.3185
12	11	10	691.1504
13	12	10	753.9822
14	13	10	816.8141
15	14	10	879.6459
16	15	10	942.4778

b

	A	B	C
1	*r*	*h*	Curved surface area
2	10	10	628.3185
3	10	11	691.1504
4	10	12	753.9822
5	10	13	816.8141
6	10	14	879.6459
7	10	15	942.4778
8	10	16	1005.31
9	10	17	1068.142
10	10	18	1130.973
11	10	19	1193.805
12	10	20	1256.637

c about 747.7 cm^2

9 a

	A	B
1	Time (t)	Interest (I)
2	1	62
3	2	124
4	3	186
5	4	248
6	5	310
7	6	372
8	7	434
9	8	496
10	9	558
11	10	620

b 17 years

10

	A	B	C	D
1	a	d	n	l
2	5	6	17	101
3	111	−9	10	30
4	24	8	25	216

a 101 **b** 30 **c** 216

11 a

A	B	C	D	E	F	G	H
s				s			
		1	1.1	1.2	1.3	1.4	1.5
r	1	6.28	6.60	6.91	7.23	7.54	7.85
	1.1	7.26	7.60	7.95	8.29	8.64	8.98
	1.2	8.29	8.67	9.05	9.42	9.80	10.18
	1.3	9.39	9.80	10.21	10.62	11.03	11.44
	1.4	10.56	11.00	11.44	11.88	12.32	12.75
	1.5	11.78	12.25	12.72	13.19	13.67	14.14
	1.6	13.07	13.57	14.07	14.58	15.08	15.58
	1.7	14.42	14.95	15.49	16.02	16.56	17.09
	1.8	15.83	16.40	16.96	17.53	18.10	18.66
	1.9	17.31	17.91	18.50	19.10	19.70	20.29
	2	18.85	19.48	20.11	20.73	21.36	21.99

b $s = 2.0$ m

12

t	1	2	3	4
d	5.6	11.2	16.8	22.4

13 14.3 cm

14 The cheapest combination that produces at least 300 mL is $A = 5$ and $B = 15$ at a cost of \$1.30.

CUMULATIVE EXAMINATION 1

1 B **2** C **3** D

4 A **5** C

6 a

C	5	15	30
S	175	125	50

b Yes, highest power 1 or there is a common difference.

c \$20

d It is too expensive and there would be no stalls rented.

7 a $r = \sqrt{\frac{SA}{4\pi}}$ **b** radius = 9 cm

8 a \$4979

b 23%

9 a Devilishly Delicious: \$1.61 vs \$2.83 per 100 g

b \$4.84

CUMULATIVE EXAMINATION 2

1 Line 2: $8y + 12 - 2y - 2 = x$

2 \$1750, 14.29

3

BMI		h (m)		
		1.64	1.85	1.93
m (kg)	56	20.8	16.4	15.0
	79	29.4	23.1	21.2
	91	33.8	26.6	24.4

55.6% are considered unhealthy.

CHAPTER 5

EXERCISE 5.1

1 B **2** D

3 a 0.5 **b** 2.5 **c** 1.8

d 0.5

4 a i $\triangle KLM \parallel\!| \triangle ZYW$ **ii** scale factor $= \frac{1}{3}$

b i $LCFGH \parallel\!| ZTWVS$ **ii** scale factor = 3.5

c i $\triangle RBP \parallel\!| \triangle GHE$ **ii** scale factor = 2.5

d i $GFEH \parallel\!| TRUS$ **ii** scale factor = 1.6

e i $\triangle PLO \parallel\!| \triangle YKO$ **ii** scale factor = 2.2

f i $MDFS \parallel\!| QTYE$ **ii** scale factor = 1.5

5 a 9.5 m **b** 125 cm **c** 4 cm

d 63 mm **e** 165 cm **f** 13 mm

6 a T **b** T **c** F

d T **e** T **f** F

g F **h** T **i** F

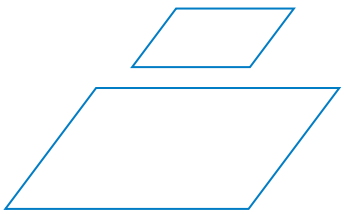
j F **k** F **l** T

m T

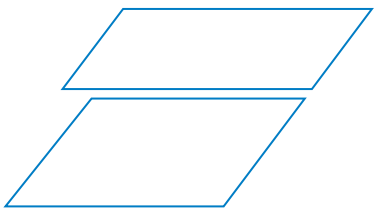
7 35 cm

8 a The parallelograms are not necessarily similar as the ratios of corresponding sides may not be equal.

b Many answers are possible.



c Many answers are possible.



9 $BA \parallel\!| ED$, $AC \parallel\!| DC$, $BC \parallel\!| EC$ $\angle A = \angle D$, $\angle B = \angle E$, $\angle C$ common

10 $x = 3$ **11** $x = 12$

12 $x = 25$, $y = 4$ **13** $x = 9$

EXERCISE 5.2

1 B
2 scale factor $= \frac{1}{2}$; $x = 2$
3 C (SAS) 4 A (SAS)
5 a, b Teacher to check.
c Yes, $\triangle DAF \parallel\!| \triangle KUT$ (AAA)
6 a $\triangle ABC \parallel\!| \triangle XYZ$ (AAA)
b $\triangle MSQ \parallel\!| \triangle AFD$ (AAA)
c No test can be applied.
d $\triangle KLD \parallel\!| \triangle DLP \parallel\!| \triangle KDP$ (AAA)
e $\triangle MCN \parallel\!| \triangle CDN$ (SSS)
f $\triangle XYW \parallel\!| \triangle ZUX$ (RHS)
g $\triangle PQS \parallel\!| \triangle QSR$ (SSS)
7 a $\triangle CKQ \parallel\!| \triangle MWL$ (AAA)
b $\triangle BIZ$ and $\triangle NPV$ are not necessarily similar.
c $\triangle BRT$ and $\triangle EAN$ are not necessarily similar.
d $\triangle APS \parallel\!| \triangle KRM$ (SSS)
e $\triangle CPW \parallel\!| \triangle MLE$ (SAS)
f $\triangle AMP \parallel\!| \triangle QSD$ (AAA)
g $\triangle PRZ$ and $\triangle DKL$ are not similar.
h $\triangle AMW \parallel\!| \triangle ESL$ (RHS)
i $\triangle WVR \parallel\!| \triangle PEN$ (SSS)
8 a $w = 50°$, $z = 45°$, $x = y = 85°$
b $x = 12$ c $n = 12$, $m = 25.2$
d $b = 15$ e $x = 12$
f $x = 20$
9 A 10 D
11 $\angle GFH = \angle GEI$ ($\angle\!\!-\!\!\angle$ rule)
$\angle GHF = \angle GIE$ ($\angle\!\!-\!\!\angle$ rule)
$\angle FGH = \angle EGI$ (Common)
$\therefore \triangle FGH \parallel\!| \triangle FGH$ (AAA)
12 Consider $\triangle WXZ$ and $\triangle XYZ$.
$\angle WXZ = 90° - \angle XWZ$
$\angle XYZ = 90° - \angle XWZ$
$\angle WZX = \angle XZY = 90°$
$\triangle WXZ \parallel\!| \triangle XYZ$ (AAA)
Use a similar approach to show that $\triangle WXZ \parallel\!| \triangle WXY$.
Hence, $\triangle WXZ \parallel\!| \triangle XYZ \parallel\!| \triangle WXY$.
13 5.6 cm

EXERCISE 5.3

1 A 2 C 3 6.25 m
4 143.3 – 8 = 135.3 m 5 34.3 m
6 9 m 7 24.5 m 8 174 cm
9 18.75 m

EXERCISE 5.4

1 Yes, scale factor = 2
2 $AB = DE$ (scale factor = 2)
$BC = EF$ (scale factor = 2)
$AC = DE$ (scale factor = 2)
$\therefore \triangle ACB \parallel\!| \triangle DFE$ (SSS)
3 D 4 C 5 B
6 a 1 : 100 b 1 : 100 000 c 1 : 1000
d 1 : 10 000 e 1 : 50 f 1 : 200 000
7 a 1 m b 1 m c 5 m
d 100 m e 6.5 km f 50 m
8 a 1 : 5000 b 1 : 10 000 000
c 1 : 1000 d 1 : 50
e 1 : 400 000 f 1 : 5000
9 a 15 m b 2.1 m c 1.0 m
d 4.0 m e 1.3 m, 0.8 m
10 a 2000 mm b 900 mm c 800 mm
d 1200 mm e 1600 mm f 500 mm
11 a 4.1 m b 1.7 m c 53 cm
d 12 cm
12 \$1021.80; reasoning will differ.
13 a 7 m by 7 m b \$4346.30
14 \$5775
15 a 12 cm = 18 km b ≈1.47 cm
16 16 cm 17 6.875 km
18 56 sheets

EXERCISE 5.5

1 Shape and angles are the same, scale factor = 3.5
2 20 m 3 D 4 B
5 a i 1.75 ii 3.0625 iii about 18.4 cm^2
b i 1.4 ii 1.96 iii 784 mm^2
c i 1.8 ii 3.24 iii about 3.03 m^2
d i 0.55 ii 0.3025 iii about 257.1 cm^2
6 a 62 500 b 153.1 m^2
7 a i 36 ii 2772 cm^2
b i 16 ii 201.0625 m^2
8 C
9 a i 2 ii 8 iii 14.4 cm^3
b i 1.5 ii 3.375 iii 6075 mm^3
c i 1.25 ii 1.953 125 iii ≈ 97.7 m^3
d i 0.4 ii 0.064 iii ≈ 2.1 L
10 D 11 B 12 17.6 cm^3
13 ≈ 1.28 14 ≈ 300 cm^2

CUMULATIVE EXAMINATION 1

1 D 2 D 3 A
4 B 5 C 6 similar (AAA)
7 \$1642.50 8 $x = 3.6$ cm 9 339.29 cm^2
10 a ¥115 200 b A\$900 c A\$306.38

CUMULATIVE EXAMINATION 2

1 height = 5.46 m
2 \$93.75 3 \$51.10
4 height = 0.245 m

CHAPTER 6

EXERCISE 6.1

1 $x = 12$ is a solution for **a**, **b**, **c** and **e**.

2 **a** $x = 34$ **b** $a = 24$ **c** $u = 9$
d $d = 84$ **e** $f = 41$ **f** $k = -29$
g $e = -14$ **h** $y = 143$ **i** $h = -11$

3 **a** $k = 0$ **b** $x = -2.1$ **c** $t = 6$
d $g = 3$ **e** $p = -4$ **f** $u = 23$
g $y = -3$ **h** $n = -14$ **i** $a = 1.3$

4 **a** $z = 7$ **b** $x = -3$ **c** $y = 4$
d $a = 5$ **e** $b = 3$ **f** $c = -1$
g $p = 7$ **h** $u = -6$ **i** $q = -40$
j $m = 12$ **k** $t = 3$ **l** $x = 15$

5 D **6** B

7 **a** $y = 8$ **b** $x = 675$ **c** $d = -21$

8 $x = 3$; side lengths: 11 m, 11 m, 13 m, therefore it is an isosceles triangle.

9 width = 70 m

EXERCISE 6.2

1 yes

2 **a** $x = -13$ **b** $m = 7$

3 **a** C **b** A

4 **a** $g = -3$ **b** $x = 2$ **c** $s = -5$
d $u = -3$ **e** $w = -12$ **f** $q = 2$
g $x = -3$ **h** $y = 4$ **i** $r = 5$

5 **a** $b = \frac{5}{8}$ **b** $x = -1\frac{3}{7}$ **c** $w = 1\frac{3}{8}$
d $a = 5$ **e** $y = 19$ **f** $n = 1$
g $h = -2\frac{3}{5}$ **h** $u = -\frac{5}{7}$ **i** $d = 3\frac{1}{2}$

6 **a** $b = 8.5$ **b** $y = -4.5$ **c** $m = -12$
d $r = -9\frac{1}{3}$ **e** $z = 4\frac{4}{5}$ **f** $u = 1\frac{4}{9}$
g $x = 1\frac{1}{5}$ **h** $d = \frac{4}{17}$ **i** $m = -12$

7 **a** $e = 14$ **b** $y = 13$ **c** $p = 3$
d $x = -1\frac{5}{17}$ **e** $n = -3\frac{3}{4}$ **f** $k = 3$
g $w = -3$ **h** $q = -20$ **i** $t = -1\frac{2}{9}$

8 **a** true **b** false **c** false
d true

9 Line 2, $3x - 7x = -4x$, not $4x$.

10 Line 1, $-3(x - 2) = -3x + 6$, not $-3x - 6$.

EXERCISE 6.3

1 no

2 **a** $x = 3$ **b** $n = 5$

3 **a** D **b** A
c B **d** D

4 **a** $10(12 + n) = 180$ **b** $\frac{14}{n - 7} = 2$
c $\frac{(n + 15)}{2} - 5 = 3$
d $2(4n + 17) = -6$
e $3 + 7(n + 2) = 11$ **f** $n + \frac{1}{3}n - 14 = 9$
g $2(10n - 7) = -12$ **h** $\frac{2}{3}n - 7n = 10$

5 **a** $P = 6w$ **b** $P = (4w + 8)$ cm
c $P = (4w + 6)$ m **d** $P = 8w$
e $P = (6w + 8)$ mm **f** $P = (8w - 10)$ cm
g $P = (10w + 56)$ m **h** $P = (4w + 24)$ mm

6 **a** **i** $y + 7 = 4$ **ii** $y = -3$
b **i** $m + 9 = 2$ **ii** $m = -7$
c **i** $a + 31 = 13$ **ii** $a = -18$
d **i** $x + 12 = 8$ **ii** $x = -4$
e **i** $d + 3 = -13$ **ii** $d = -16$
f **i** $26 - y = 39$ **ii** $y = -13$
g **i** $-12 + g = 4$ **ii** $g = 16$
h **i** $7 - w = -4$ **ii** $w = 11$

7 60 marks **8** 10 years old

9 15 and 16 **10** 15 cm, 15 cm, 24 cm

11 **a** $\frac{x}{5} = 30$ **b** $10 + 4x = 26$ **c** $3(3x - 6) = 18$

12 age = 78 years **13** 15 **14** \$296

EXERCISE 6.4

1 $k = -4$

2 The number is 6.

3 **a** Q3 **b** Q1 **c** Q4
d Q1 **e** Q2 **f** Q3

4 **a** $y = -3x + 2$

x	0	2	5
y	2	−4	−13

b $y = 4x + 1$

x	−3	0	3
y	−11	1	13

c $y = 5x - 12$

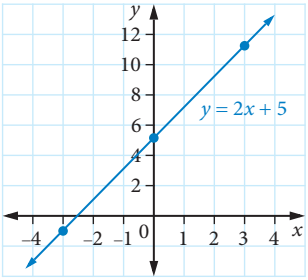
x	−2	0	2
y	−22	−12	−2

d $y = 7 - 3x$

x	−1	0	3
y	10	7	−2

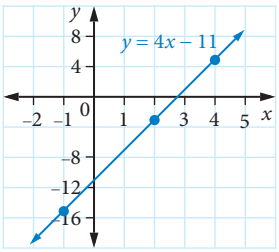
5 **a** $y = 2x + 5$

x	−3	0	3
y	−1	5	11



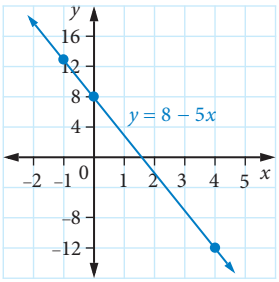
b $y = 4x - 11$

x	−1	2	4
y	−15	−3	5



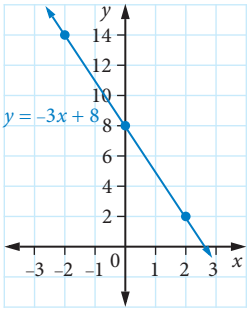
c $y = 8 - 5x$

x	−1	0	4
y	13	8	−12

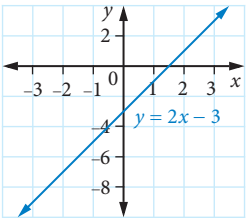


d $y = -3x + 8$

x	−2	0	2
y	14	8	2



6 **a**



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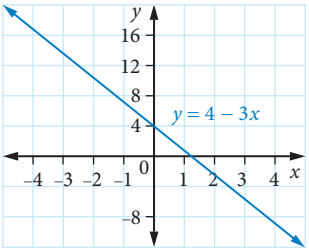

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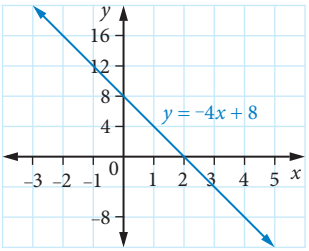
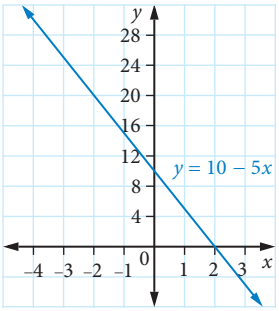
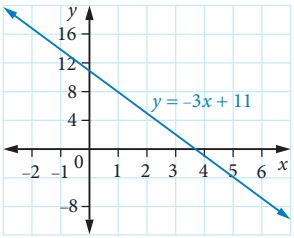


d



e



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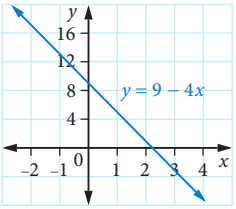
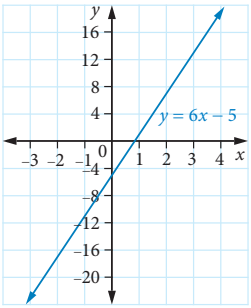
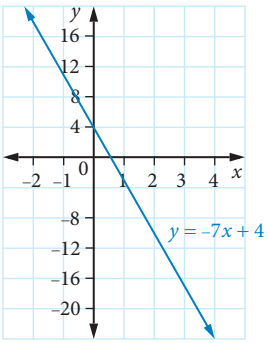


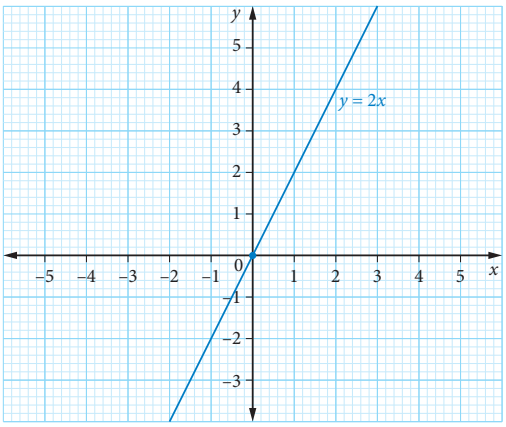
g



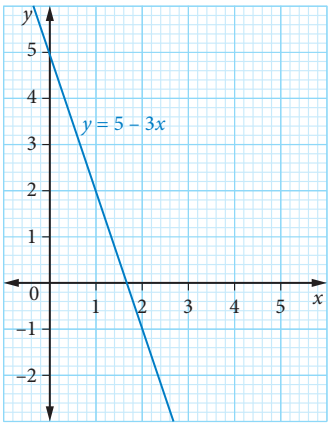
h



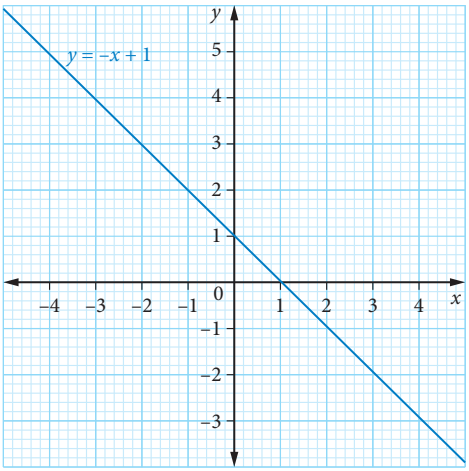
7 a $m = 2, c = 0$



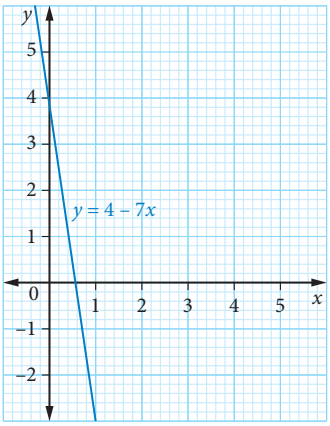
b $m = -3, c = 5$



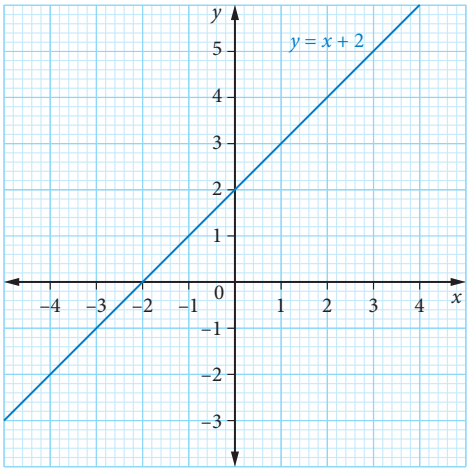
c $m = -1, c = 1$



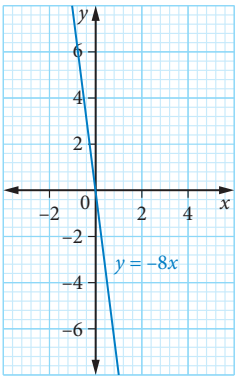
d $m = -7, c = 4$



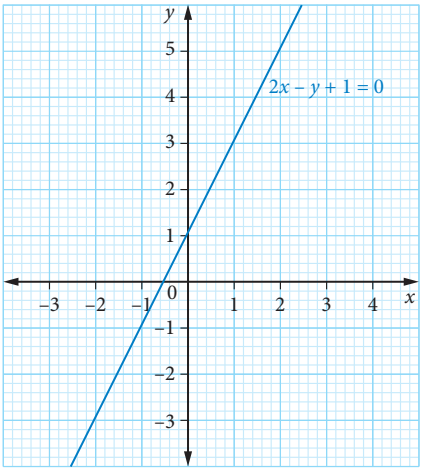
e $m = 1, c = 2$



f $m = -8, c = 0$

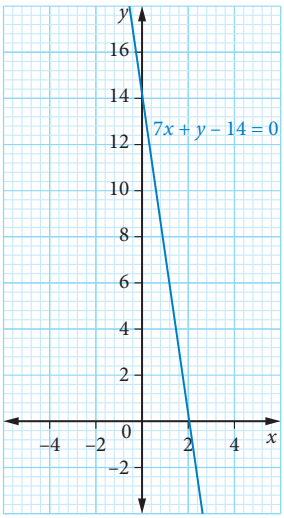


g $m = 2, c = 1$

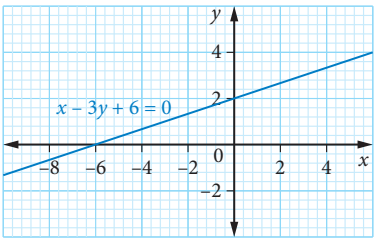


ANSWERS

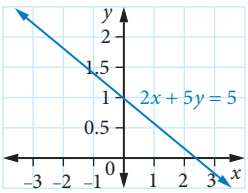
h $m = -7$, $c = 14$



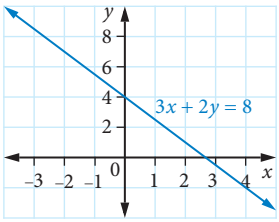
i $m = \frac{1}{3}$, $c = 2$



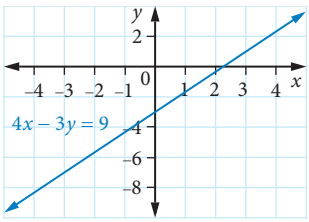
8 a $(2\frac{1}{2}, 0)$, $(0, 1)$



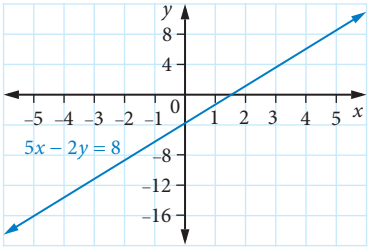
b $(2\frac{2}{3}, 0)$, $(0, 4)$



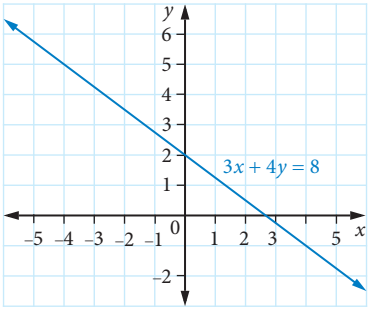
c $(2\frac{1}{4}, 0)$, $(0, -3)$



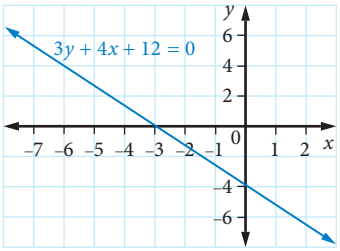
d $(1\frac{3}{5}, 0)$, $(0, -4)$



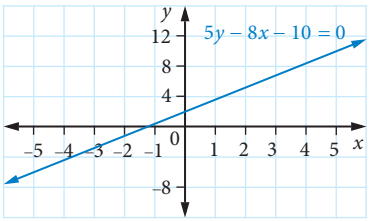
e $(2\frac{2}{3}, 0)$, $(0, 2)$



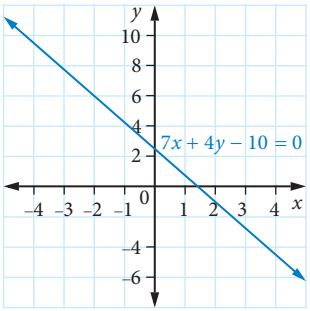
f $(-3, 0)$, $(0, -4)$



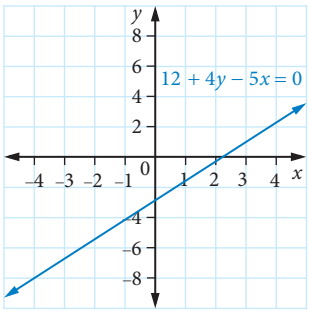
g $(-1\frac{1}{4}, 0)$, $(0, 2)$



h $(1\frac{3}{7}, 0)$, $(0, 2\frac{1}{2})$



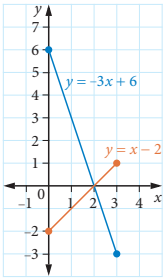
i $(2\frac{2}{5}, 0), (0, -3)$



9 **a** $(-4, 2)$ and $(2, -7)$ lie on the line.
b $(1, -2)$, $(3, 0)$ and $(-3, -6)$ lie on the line.
c $(-2, 2)$ and $(3, 12)$ lie on the line.
d $(3, -7)$ and $(-4, 14)$ lie on the line.
e $(2, 0)$ and $(-4, -18)$ lie on the line.

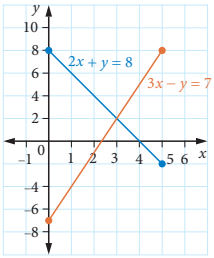
10 D **11** A **12** B

13 **a**



b $(2, 0)$ **c** $(2, 0)$ satisfies $y = x - 2$.
d $(2, 0)$ satisfies $y = -3x + 6$.

14 **a**



b $(3, 2)$ satisfies both functions.

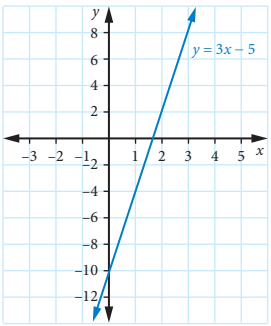
15 9 units

16 **a** linear **b** non-linear
c non-linear **d** linear

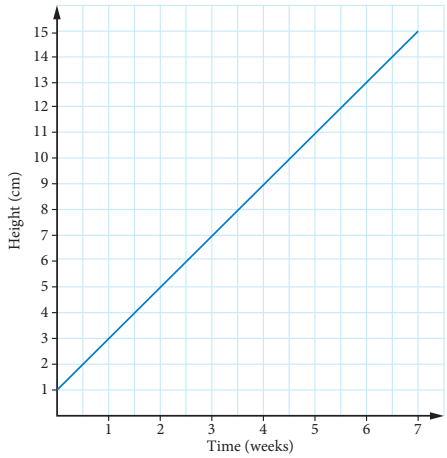
17 $y = 3x - 4$

18 Yes, the point $(1, 6)$ is on the line of $y = 2x + 4$.

19 **a** $y = 3x - 5$
b



20



The grass is likely to reach 15 cm after 7 weeks.

EXERCISE 6.5

1 **a** $y = 2x + 4$ **b** $y = -3x + 1$
c $y = 5x - 6$ **d** $y = 3x - 2$

2 x-intercept 8, y-intercept 20

3 **a**, **d**, **f** and **g** are positive; **b**, **c**, **h** and **i** are negative; **e** and **j** are neither.

4 **a** $-\frac{2}{7}$ **b** $-\frac{3}{2}$ **c** 2
d not defined **e** $-\frac{1}{2}$ **f** $-\frac{1}{2}$
g $\frac{3}{4}$ **h** 0 **i** $\frac{1}{5}$ **j** $\frac{1}{2}$

5 **a** 3 **b** −2 **c** 2
d 7 **e** $\frac{1}{8}$ **f** −2

6 **a** $y = 3x + 4$ **b** $y = -x - 3$ **c** $y = \frac{2}{3}x + 1$
d $y = 4x + 6$ **e** $y = -2x + 10$ **f** $y = -x + 6$

7 **a** $y = 4x + 12$ **b** $y = -2x - 5$ **c** $y = 3x$
d $y = \frac{2}{3}x + 1$ **e** $y = -6x + 15$ **f** $y = \frac{1}{2}x - 8$

8 Many answers are possible, for example, $(2, -4)$ and $(-1, 2)$.

9 $y = 4x - 1$

10 **a** The solution is not correct.
b Second line should read $2 - (-1)$ for the change in x, therefore the denominator should be 3. The gradient should be $m = -\frac{2}{3}$ for the answer.

11 D **12** B **13** $y = 5x - 3$

14 $y = -3x$ **15** $C = 12t + 10$

16 $y = -2x + 6$, x-intercept is 3.

EXERCISE 6.6

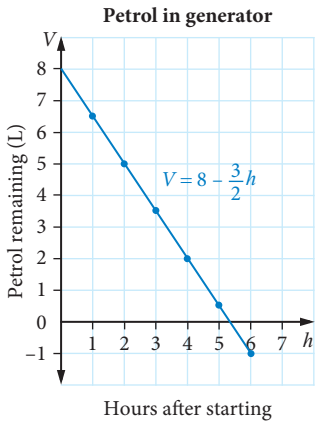
1 −2.5 **2** $y = \frac{15x}{2} - 11$

3 y-intercept $c = 25$ **4** B

5 **a**

h	0	1	2	3	4	5	6
V	8	6.5	5	3.5	2	0.5	−1

b

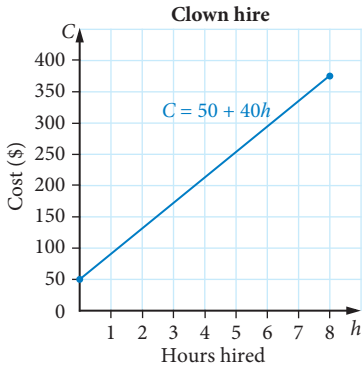


c linear function **d** after 3.3 hours **e** 5 L

f The volume of petrol in the generator before it started.

g The time in hours after it starts at which there is no petrol in the generator.

6 a



b linear function **c** \$170 **d** 3.75 hours

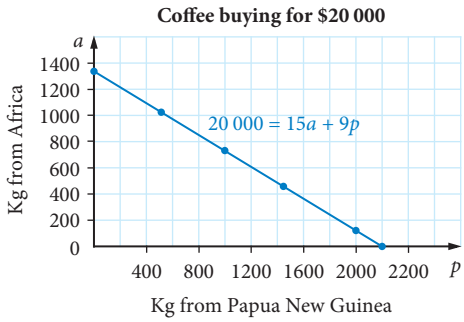
e The cost of hiring the clown regardless of how many hours they work. It is sometimes called a service fee or a call-out fee.

f The hourly rate of pay for the clown.

7 a $20\,000 = 15a + 9p$

b

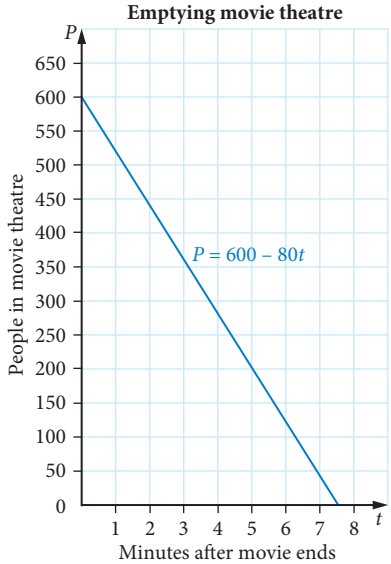
p	0	500	1000	1500	2000
a	1333	1033	733	433	133



c About 850 kg of African coffee.

8 a $P = 600 - 80t$

b

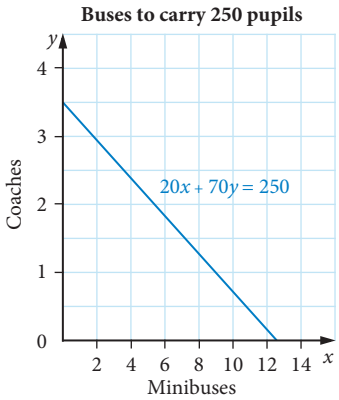


c about 3.8 minutes

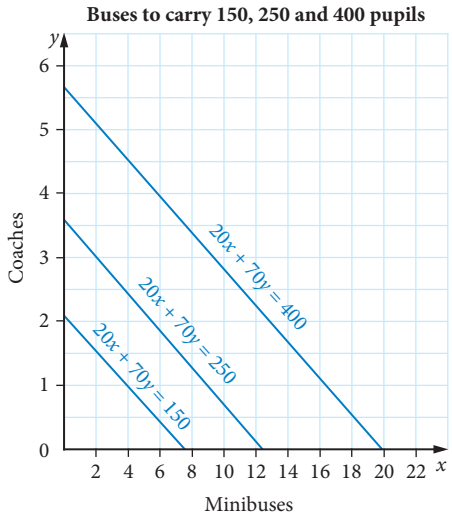
9 a $20x + 70y$ students

b $20x + 70y = 250$

c



d



e The graphs are parallel.

10 $h = 1000 - 80t$; 12.5 minutes

11 Josh is cheaper at \$270 for 4 hours. This is \$10 less than Nadine.

CUMULATIVE EXAMINATION 1

1 D **2** A **3** A

4 B **5** D

6 x-intercept $(12, 0)$, y-intercept $(0, 30)$

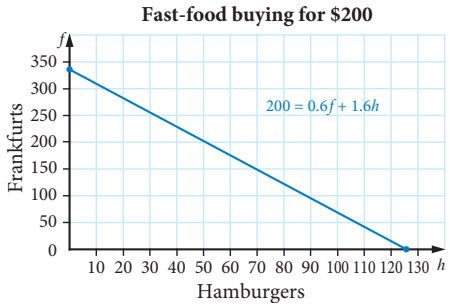
7 **a** \$3.12 **b** \$1092

8 The triangle has the larger area, $A = 60\,\text{cm}^2$.

9 **a** $200 = 0.6f + 1.6h$

b

h	0	25	50	75	100	125
f	333	267	200	133	67	0

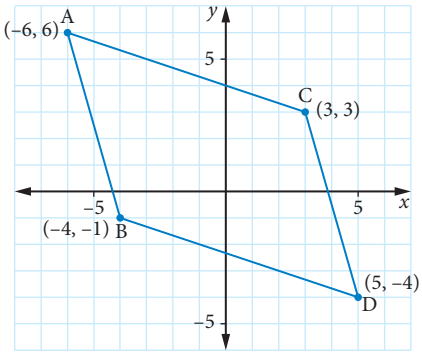


c 147 frankfurts

CUMULATIVE EXAMINATION 2

1 $600 - 12t$, 96 L **2** $x = 1.6$

3



Opposite lines have the same gradients.

AC $m = \dfrac{-1}{3}$ = BD $m = \dfrac{-1}{3}$

AB $m = \dfrac{-7}{2}$ = CD $m = \dfrac{-7}{2}$

4 $S = 2213.25\,\text{cm}^2$

CHAPTER 7

EXERCISE 7.1

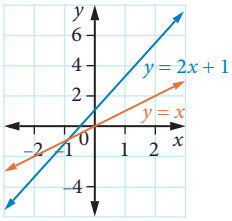
1 **a** $(2, 4)$ **b** $(1, 4)$ **c** $(1, 6)$
d $(-2, 5)$ **e** $(1, 5)$ **f** $(-2, 4)$

2 **a** $(-1, 5)$ **b** $(1, 1)$ **c** $(3, 7)$
d $(0, -2)$

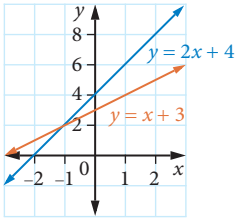
3 D **4** B

5 **a, b, d, e, g, h** and **j** are simultaneous solutions.

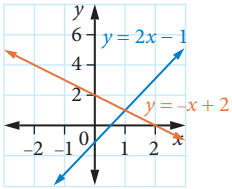
6 **a** $x = -1$, $y = -1$ or $(-1, -1)$



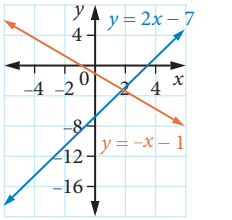
b $x = -1$, $y = 2$ or $(-1, 2)$



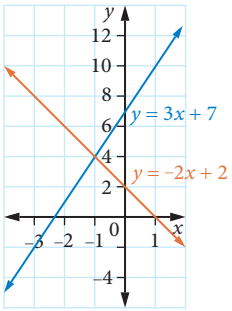
c $x = 1$, $y = 1$ or $(1, 1)$



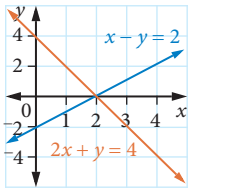
d $x = 2$, $y = -3$ or $(2, -3)$



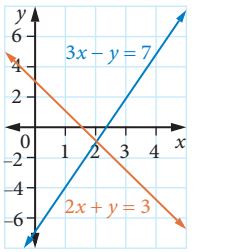
e $x = -1$, $y = 4$ or $(-1, 4)$



7 **a** $x = 2$, $y = 0$ or $(2, 0)$

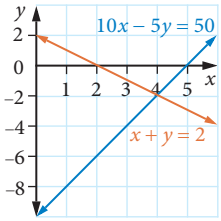


b $x = 2$, $y = -1$ or $(2, -1)$

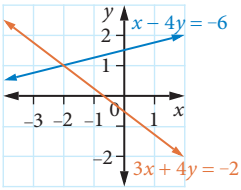


ANSWERS

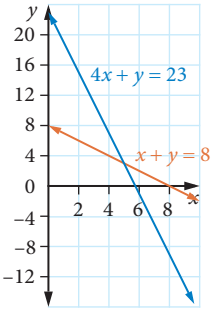
c $x = 4, y = -2$ or $(4, -2)$



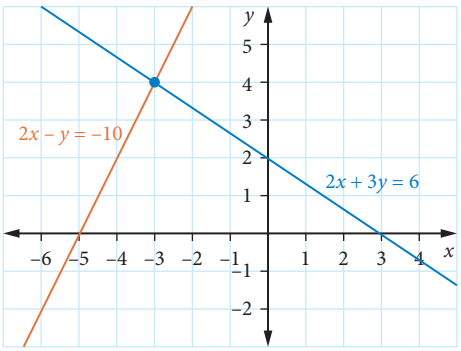
d $x = -2, y = 1$ or $(-2, 1)$



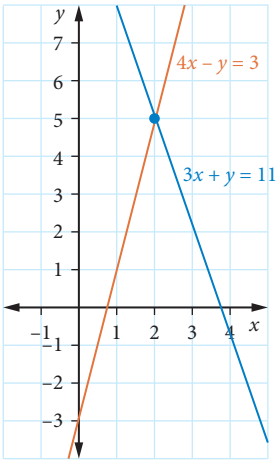
e $x = 5, y = 3$ or $(5, 3)$



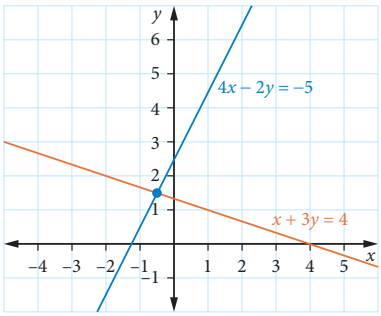
8 **a** $x = -3, y = 4$ or $(-3, 4)$



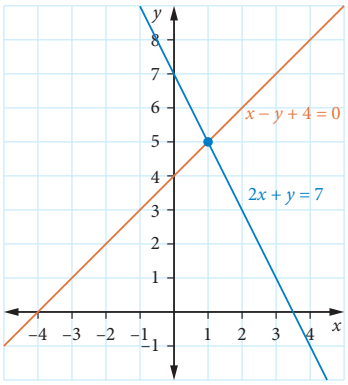
b $x = 2, y = 5$ or $(2, 5)$



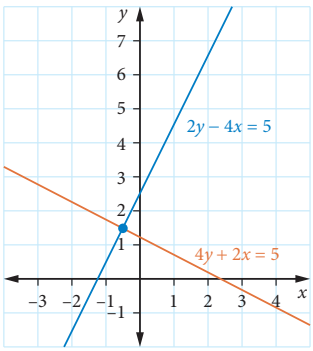
c $x = -\frac{1}{2}, y = 1\frac{1}{2}$ or $\left(-\frac{1}{2}, 1\frac{1}{2}\right)$



d $x = 1, y = 5$ or $(1, 5)$



e $x = -\frac{1}{2}, y = 1\frac{1}{2}$ or $\left(-\frac{1}{2}, 1\frac{1}{2}\right)$



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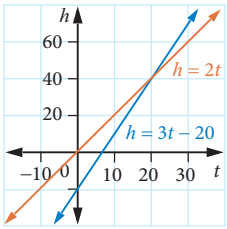
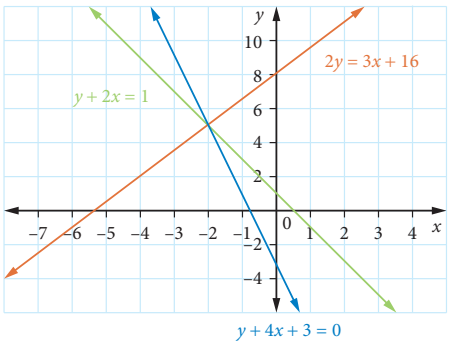
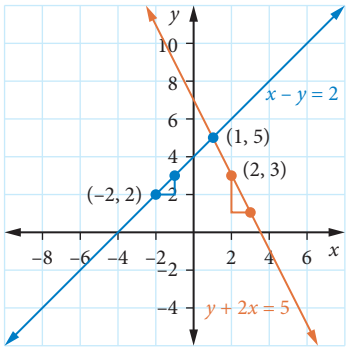


Same height after 20 seconds.

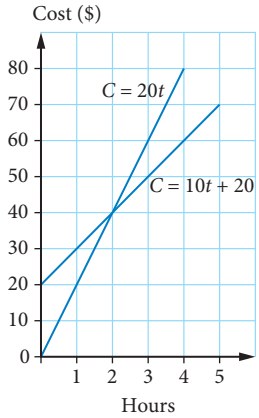
10 Simultaneous solution is $x = -2$ and $y = 5$ or $(-2, 5)$.



11 Simultaneous solution is $x = 1$ and $y = 5$ or $(1, 5)$.



12



The two business will both cost \$40 after 2 hours, $(x = 2, y = 40)$.

EXERCISE 7.2

1 $y = 3x + 4$ **2** B **3** D **4** B

5 **a** $p = 5q - 8$ **b** $p = 7, q = 3$

6 **a** $x = 5, y = 2$ **b** $x = 6, y = 18$ **c** $a = 2, b = 1$
d $c = 8, d = 3$ **e** $c = 4, d = 1\frac{1}{2}$ **f** $m = -2, n = 1$

7 **a** $x = 3, y = 2$ **b** $x = 5, y = 2$ **c** $x = 0, y = 3$
d $a = 5, b = -7$ **e** $c = 5, d = 4$ **f** $x = 2, y = -1$

8 **a** $m = 3, n = 1$ **b** $a = 3, b = 2$ **c** $m = 1, n = -2$
d $x = 3, y = -3$ **e** $x = 4, y = -3$ **f** $a = -2, b = 1$

9 Power bank \$20, Earphones \$26

10 **a** $l = w + 2$ and $2(l + w) = 68$
b $l = 18, w = 16$

11 $x = \frac{b}{2a}, y = \frac{b}{2}$ **12** 375 tools

EXERCISE 7.3

1 Yes, $(2, 4)$ is the simultaneous solution.

2 $(3.5, 3)$

3 **a** subtract **b** add **c** add
d subtract **e** add **f** add

4 **a** $3x + 3y = -6$ **b** $-8x - 4y = 20$
c $10x - 4y = 22$ **d** $30x + 6y = 24$

5 **a** $(4, 2)$ **b** $(6, -2)$ **c** $(2, 4)$
d $(2, 1)$ **e** $(2, 1)$ **f** $(9, 3)$

6 **a** $(13, -8)$ **b** $(4, 2)$ **c** $(4, -3)$
d $(5, 0)$ **e** $(3, 2)$ **f** $(-2, 1)$

7 **a** $(3, 4)$ **b** $(-2.2, 2.2)$ **c** $(1, 5)$
d $(3, -1)$ **e** $(-4.65, 1.41)$ **f** $(4, -1)$

8 The two numbers are 255 and 167.

9 $(4, -3)$

10 The two numbers are 8 and 19.

11 450 adults attended.

12 calculator = 80 g, notepad = 200 g

EXERCISE 7.4

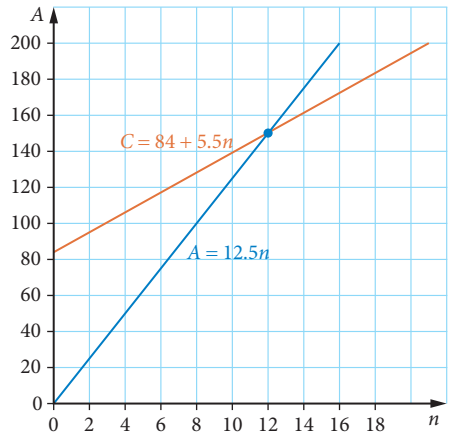
1 $y = 2$

2 x-intercept = $(4, 0)$, y-intercept = $(0, 2)$

3 **a** Flynn is 3 km ahead of Aiden.
b Aiden is 2 km ahead of Flynn.
c This is the time when they are the same distance from the sports ground (or the same distance from Aiden's house).

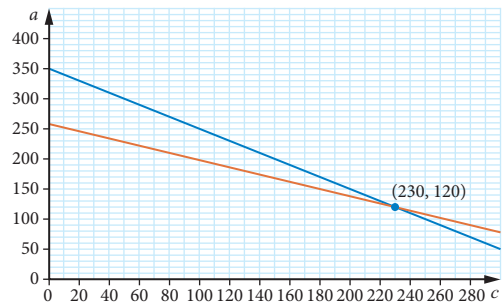
4 D **5** B

6 **a**



b Break-even point is when $n = 12$.
c The point at which sales revenue equals production costs.
d profit of \$28

7 **a**



b 230 children, 120 adults

8 **a** $4b + 3c = 21$ and $2b + 4c = 19.8$
b A bagel costs \$2.46 and a croissant costs \$3.72.

9 $a = 40$ cm, $b = 10$ cm

10 Sam 50 bricks, Peta 200 bricks

11 strawberries \$4/tray, cherries \$6/tray

12 length = 27 cm and width = 21 cm

13 adult's ticket \$7, child's ticket \$2

14 132 serves of hot chips, 44 meat pies and 33 Chiko rolls.

15 2 hats

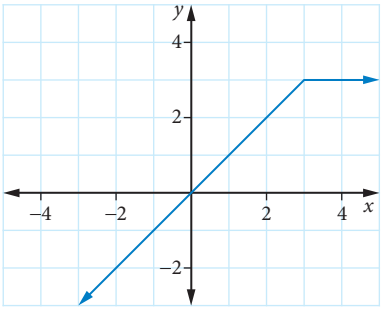
EXERCISE 7.5

1 **a** $(2, -3)$ **b** $y = x - 5$

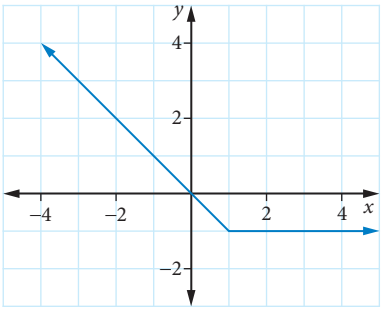
2 C **3** D **4** B **5** A **6** C

ANSWERS

7 **a**



b



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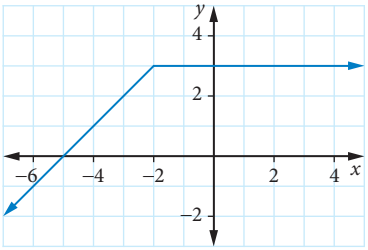

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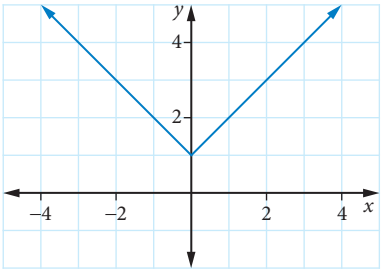

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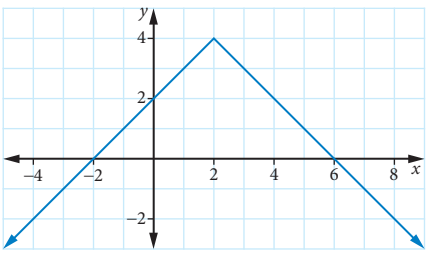

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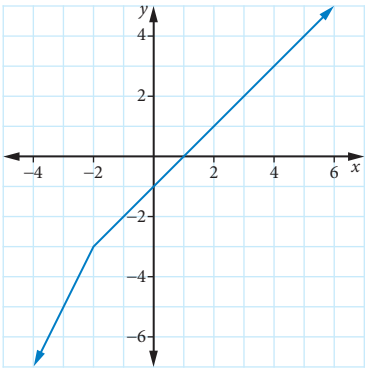
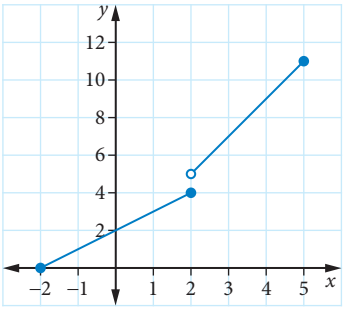


8 **a**



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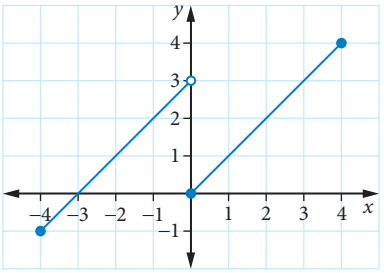

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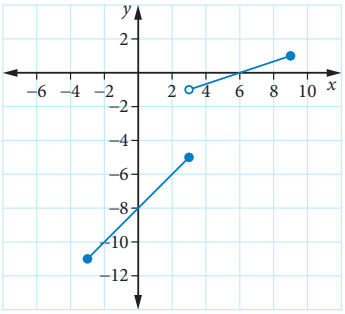
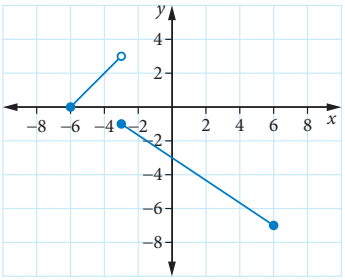


d



9 **a** The graph is made up of 4 line segments. The gradient of each line segment increases as the taxable income increases.

b The highest rate of taxation is paid for taxable incomes greater than \$80 000.

c The lowest rate of taxation is paid on taxable income less than \$18 000.

d **i** \$0 **ii** about \$19 500

iii About \$11 000 **iv** about \$7000

e 0.31

10 **a** $-3 \le x \le 1$ and $1 \le x \le 5$

b $m = 2$ and $c = 1$

c $y = 2x + 1$

d $y = -x + 4$

e $y = \begin{cases} 2x + 1 & \text{if} \quad -3 \le x \le 1 \\ -x + 4 & \text{if} \quad 1 \le x \le 5 \end{cases}$

(Note: One of the '≤' next to 1 can also be replaced by '<')

11 **a** Edin's home, the starting point

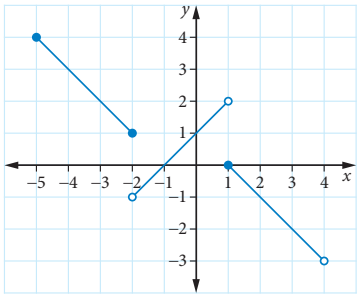
b 7 km

c Edin stopped for 1 hour.

d hour 6–7

e 17 km

12



EXERCISE 7.6

1 $x = 2, y = 5$ **2** $(5, 0)$

3 The 2nd and 3rd segments cannot overlap. This means that the segments for $-2 < x \leq 1$ and $0 < x \leq 2$ have not been drawn correctly.

4 **a** **i** \$20 **ii** \$20 **iii** \$15 **iv** \$10

b \$895

c The cost of the t-shirt decreases (in steps) as more are ordered. The set up and fixed production costs are distributed over a greater number of t-shirts and so the cost per t-shirt is less.

d \$5

5 **a** \$5/h **b** \$10/h **c** 4 hours **d** \$80

6 **a** \$9 **b** \$12 **c** \$10.50

d \$10.50 **e** \$100

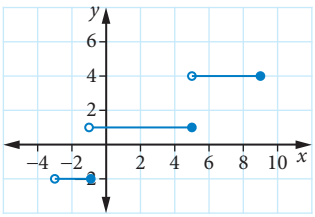
7 **a** **i** 10 cents **ii** 15 cents

iii 5 cents **iv** 7.5 cents

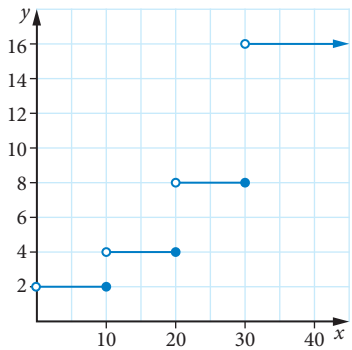
b \$23.50

c The cost per copy decreases (in steps) as more are printed.

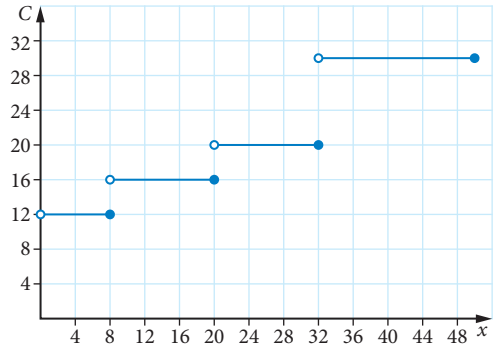
8 **a**



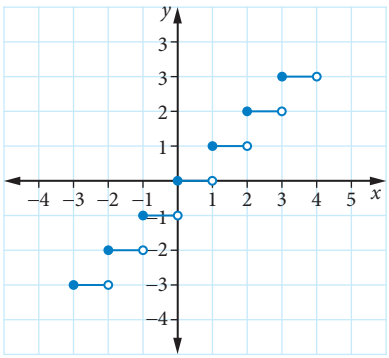
b



c



d

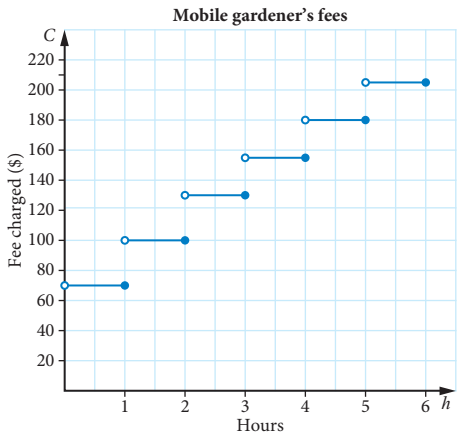


9 **a** \$3.50 **b** \$4.75 **c** \$7

10 $C = \begin{cases} \$7.50 & \text{if } 0 < t \leq 10 \\ \$10 & \text{if } 10 < t \leq 15 \\ \$15 & \text{if } 15 < t \leq 25 \\ \$20 & \text{if } t > 25 \end{cases}$

11

hours (h)	0	0.5	1	1.5	2	2.5	3	3.5	4	5	6
Cost (C)	0	70	70	100	100	130	130	155	155	180	205



CUMULATIVE EXAMINATION 1

1 B **2** D **3** B

4 A **5** D **6** $x = 3, y = 2$

7 **a** \$40 **b** \$440 **c** \$24

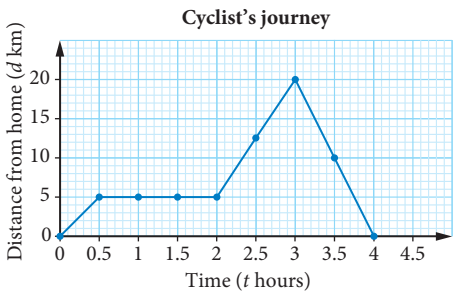
8 **a** \$2750 **b** \$2475

9 **a** \$0.75/share **b** 6.2

c As the P/E ratio is low, an investor would see this favourably as it indicates less money would need to be invested for each dollar of profit gained.

CUMULATIVE EXAMINATION 2

1



2 mother = 30, daughter = 10

3 3.6 m

4 11 5c coins and 16 20c coins

CHAPTER 8

EXERCISE 8.1

1

	Opp	Adj	Hyp
a	*ST*	*SU*	*TU*
b	*j*	*k*	*i*
c	*CD*	*CE*	*DE*
d	*a*	*c*	*b*
e	*VX*	*VW*	*WX*
f	*VW*	*VX*	*WX*

2 a $\sin\theta = \frac{85}{128}$; $\cos\theta = \frac{96}{128}$; $\tan\theta = \frac{85}{96}$

b $\sin\theta = \frac{10.96}{11.62}$; $\cos\theta = \frac{3.85}{11.62}$; $\tan\theta = \frac{10.96}{3.85}$

c $\sin\theta = \frac{52}{63}$; $\cos\theta = \frac{36}{63}$; $\tan\theta = \frac{52}{36}$

3 a 0.8480 **b** 0.9736 **c** 3.1166
d 6.2981 **e** 24.1322 **f** 203.9410

4 a 34.85 **b** 58.81 **c** 77.74
d 31.79 **e** 30.00 **f** 18.31

5 A **6** 0.6 **7** 0.643 **8** $\frac{3}{5}$

EXERCISE 8.2

1 C

2 a 6.16 **b** 860.65 **c** 43.34

3 a 2.7 m **b** 141.3 km **c** 3.5 cm

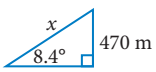
4 a 133.5 m **b** 466.9 mm **c** 108.1 cm

5 a 10.97 m **b** 36.84 cm **c** 461.36 mm
d 43.68 km **e** 1.73 m **f** 6.55 m

6 2.9 m **7** 4.97 m **8** 5.4 m

9 685 m **10** D **11** 22.3 m

12 a



b 3.2 km

13 30.5 m **14** 7.18 m **15** 16.0 cm

EXERCISE 8.3

1 A **2** 10.55 m

3 a 50° **b** 24° **c** 13°
d 31° **e** 71° **f** 27°

4 52°; 38° **5** 20.6° **6** 48°

7 D **8** 31.6° **9** C

10 34°

11 $\theta = 4.654\ldots°$. This is less than the allowed maximum. So, it meets the standard.

12 16.4°

EXERCISE 8.4

1 D **2** 26.8° **3** 108 m

4 37 113 m **5** 228 cm **6** 20°

7 58.5 m **8** D **9** C

10 a Teacher to check diagram
b 15.10 m

11 a 44.5 m **b** 71.18 m

12 37.3 m

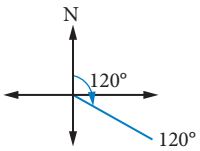
13 51.2 km/h

EXERCISE 8.5

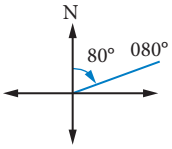
1 B **2** 21 m

3 a 070° **b** 280° **c** 115°
d 248° **e** 207° **f** 143°

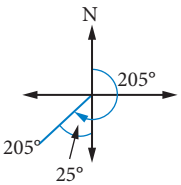
4 a



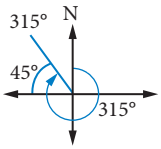
b



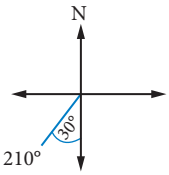
c



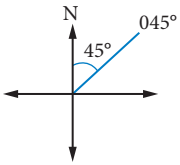
d



e

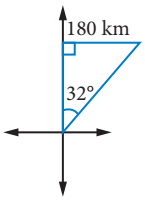


f



5 a 393 m **b** 338°

6 a



b 288.1 km

7 104.2 km **8** D

9 a 25.1 km **b** 209°

10 389 km **11** 135° **12** 28.3 km

CUMULATIVE EXAMINATION 1

1 B **2** C **3** C
4 D **5** B
6 **a** \$997.50 **b** 10.4%
7 $a = \sqrt{c^2 - b^2}$
8 \$10 **9** 31.4°
10 $\cos A = \frac{9}{41} (= 0.219\ldots)$, 77°
11 19.88 m

CUMULATIVE EXAMINATION 2

1 942.5 m^2
2 16 466 m
3 3.0 m
4 \$785

CHAPTER 9

EXERCISE 9.1

1 **a**, **c**, **e** and **f** represent matrices.
2 **a** 3×4
b **i** 4 **ii** 9 **iii** 5 **iv** 0
v 2 **vi** There is no a_{43}.

3 **a** $M = \begin{bmatrix} 3 & 2 & 4 & 1 & 5 & 0 & 3 & 6 & 1 \\ 1 & 8 & 1 & 7 & 1 & 8 & 0 & 5 & 8 \\ 4 & 0 & 5 & 6 & 4 & 5 & 9 & 1 & 7 \\ 7 & 9 & 2 & 0 & 1 & 1 & 10 & 9 & 1 \end{bmatrix}$

b **i** 0 **ii** 1 **iii** 0
4 **a** Because there is no entry in one cell.
b Place a very high cost in that cell to indicate secondary sourcing—say \$1000.

c $T = \begin{bmatrix} 4.20 & 4.30 & 1000 & 4.40 \\ 9.10 & 8.80 & 8.70 & 9.00 \\ 6.20 & 6.40 & 6.50 & 6.10 \\ 3.80 & 3.90 & 3.60 & 3.70 \end{bmatrix}$

d 9.00

5 **a** $B = \begin{bmatrix} 4 & 9 & 20 & 40 \\ 1 & 2 & 4 & 9 \\ 0.2 & 0.3 & 0.5 & 1 \end{bmatrix}$

b 4 **c** 0.3
d It is the amount of epoxy resin needed to make a shower base.

6 **a** $P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 0 & 6 & 4 & 0 \end{bmatrix}$

b Mosquito larvae do not eat the other animals.
c It is 0 because water beetles do not eat each other.
7 D
8 **a** order = 2×3
b **i** 7 **ii** 8
9 **a** 0.7324 USD

b $E = \begin{bmatrix} 1 & 0.7324 & 0.5874 & 86.24 \\ 1.3654 & 1 & 0.8020 & 117.75 \\ 1.7024 & 1.2469 & 1 & 146.82 \\ 0.01159 & 0.00893 & 0.006811 & 1 \end{bmatrix}$

c 0.8020
d 1.2469

EXERCISE 9.2

1 2×3 **2** -2
3 **a** A row matrix of size 5.
b A 3×4 matrix.
c A column matrix of size 4.
d A square matrix of order 3.
e The 3×4 zero matrix.
f The identity matrix I_3.
g A 2×5 matrix where every element is 1.
h The zero row matrix of size 6.
i A square matrix of order 2.

4 $I_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

5 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

6 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

7 Teacher to check.
8 **a** $\begin{bmatrix} 35 & 45 \\ 30 & 40 \end{bmatrix}$

b order = 2
c square matrix
9 **a** false **b** false **c** true **d** true
10 C

11 **a** $T = \begin{array}{c} \\ \text{Bus} \\ \text{Train} \\ \text{Car} \\ \text{Bike} \\ \text{Scooter} \end{array} \begin{array}{c} \begin{array}{cc} \text{Sat} & \text{Sun} \end{array} \\ \begin{bmatrix} 8950 & 3890 \\ 6500 & 3950 \\ 25\,700 & 34\,500 \\ 1590 & 2340 \\ 850 & 720 \end{bmatrix} \end{array}$

b $\begin{array}{c} \begin{array}{ccccc} \text{Bus} & \text{Train} & \text{Car} & \text{Bike} & \text{Scooter} \end{array} \\ \begin{bmatrix} 12\,840 & 10\,450 & 60\,200 & 3930 & 1570 \end{bmatrix} \end{array}$

The car is used the most over the weekend.

EXERCISE 9.3

1 A
2 **a** 200 **b** 175 **c** 120
3 **a** $\begin{bmatrix} 5 & 6 \\ 8 & -5 \end{bmatrix}$ **b** $\begin{bmatrix} -3 & 10 \\ 17 & -13 \end{bmatrix}$

c $\begin{bmatrix} 8 & -1 \\ 6 & -2 \end{bmatrix}$ **d** $\begin{bmatrix} -1 & -3 \\ 5 & -9 \end{bmatrix}$

e $\begin{bmatrix} -3 & -6 \\ -9 & 3 \end{bmatrix}$

f $\begin{bmatrix} -8 & 4 \\ 9 & -8 \end{bmatrix}$

g $\begin{bmatrix} -2 & 13 \\ 12 & -4 \end{bmatrix}$

h $\begin{bmatrix} -9 & -2 \\ -1 & -7 \end{bmatrix}$

i $\begin{bmatrix} 6 & 0 \\ -3 & -6 \end{bmatrix}$

j $\begin{bmatrix} 6 & 0 \\ -3 & -6 \end{bmatrix}$

k $\begin{bmatrix} 15 & 30 \\ 45 & -15 \end{bmatrix}$

l $\begin{bmatrix} -24 & 16 \\ 32 & -40 \end{bmatrix}$

m $\begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix}$

n $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

o $\begin{bmatrix} 1.2 & 2.4 \\ 3.6 & -1.2 \end{bmatrix}$

p $\begin{bmatrix} 7 & -6 \\ -14 & -7 \end{bmatrix}$

4 a $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 7 \\ 9 & 2 & 3 \\ -4 & 2 & -4 \end{bmatrix}$

b $\begin{bmatrix} 5 & 8 & 9 \\ 0 & 5 & 10 \\ 5 & 6 & 1 \\ 7 & 5 & -2 \end{bmatrix}$

c $\begin{bmatrix} 0 & 1 & 5 \\ 2 & -1 & 5 \\ 4 & 8 & 6 \\ 3 & -3 & -2 \end{bmatrix}$

d $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 7 \\ 9 & 2 & 3 \\ -4 & 2 & -4 \end{bmatrix}$

e $\begin{bmatrix} 1 & 4 & 3 \\ -2 & 3 & -1 \\ -4 & -2 & -4 \\ 4 & 3 & 2 \end{bmatrix}$

f $\begin{bmatrix} -3 & 2 & 7 \\ 0 & -1 & -2 \\ -5 & 6 & 3 \\ 7 & -5 & 2 \end{bmatrix}$

g $\begin{bmatrix} 3 & -2 & -7 \\ 0 & 1 & 2 \\ 5 & -6 & -3 \\ -7 & 5 & -2 \end{bmatrix}$

h $\begin{bmatrix} -2 & -9 & -11 \\ 2 & -5 & -3 \\ 4 & -4 & 2 \\ -11 & -3 & -2 \end{bmatrix}$

i $\begin{bmatrix} -5 & -7 & -4 \\ 2 & -6 & -5 \\ -1 & 2 & 5 \\ -4 & -8 & 0 \end{bmatrix}$

j $\begin{bmatrix} 5 & 7 & 4 \\ -2 & 6 & 5 \\ 1 & -2 & -5 \\ 4 & 8 & 0 \end{bmatrix}$

k $\begin{bmatrix} 16 & 12 & 4 \\ 0 & 12 & 24 \\ 20 & 0 & -4 \\ 0 & 20 & -8 \end{bmatrix}$

l $\begin{bmatrix} -3 & -12 & -9 \\ 6 & -9 & 3 \\ 12 & 6 & 12 \\ -12 & -9 & -6 \end{bmatrix}$

m $\begin{bmatrix} -2 & -10 & -16 \\ 0 & -4 & -8 \\ 0 & -12 & -4 \\ -14 & 0 & 0 \end{bmatrix}$

n $\begin{bmatrix} 2 & 1\frac{1}{2} & \frac{1}{2} \\ 0 & 1\frac{1}{2} & 3 \\ 2\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 2\frac{1}{2} & -1 \end{bmatrix}$

o $\begin{bmatrix} \frac{3}{4} & 3 & 2\frac{1}{4} \\ -1\frac{1}{2} & 2\frac{1}{4} & -\frac{3}{4} \\ -3 & -1\frac{1}{2} & -3 \\ 3 & 2\frac{1}{4} & 1\frac{1}{2} \end{bmatrix}$

p $\begin{bmatrix} 0 & 17 & 31 \\ 0 & 5 & 10 \\ -5 & 24 & 9 \\ 28 & -5 & 2 \end{bmatrix}$

5 a $\begin{bmatrix} 4 & 4 & 2 & -3 \\ 2 & -2 & 2 & -5 \\ -1 & 7 & -2 & 7 \end{bmatrix}$

b $\begin{bmatrix} 7 & -1 & 7 & -8 \\ -4 & 10 & -10 & -4 \\ 4 & 12 & -4 & 8 \end{bmatrix}$

c $\begin{bmatrix} 2 & 8 & 6 & -10 \\ -6 & 4 & -2 & -12 \\ 0 & 10 & -4 & 6 \end{bmatrix}$

d $\begin{bmatrix} 2 & 8 & 6 & -10 \\ -6 & 4 & -2 & -12 \\ 0 & 10 & -4 & 6 \end{bmatrix}$

e $\begin{bmatrix} -6 & 5 & -4 & 3 \\ 1 & -8 & 9 & -2 \\ -4 & -7 & 2 & -5 \end{bmatrix}$

f $\begin{bmatrix} 2 & -4 & -4 & 7 \\ 8 & -6 & 4 & 7 \\ -1 & -3 & 2 & 1 \end{bmatrix}$

g $\begin{bmatrix} 2 & -4 & -4 & 7 \\ 8 & -6 & 4 & 7 \\ -1 & -3 & 2 & 1 \end{bmatrix}$

h $\begin{bmatrix} 5 & -9 & 1 & 2 \\ 2 & 6 & -8 & 8 \\ 4 & 2 & 0 & 2 \end{bmatrix}$

i $\begin{bmatrix} 15 & 0 & -5 & 10 \\ 25 & -20 & 15 & 5 \\ -5 & 10 & 0 & 20 \end{bmatrix}$

j $\begin{bmatrix} -18 & 15 & -12 & 9 \\ 3 & -24 & 27 & -6 \\ -12 & -21 & 6 & -15 \end{bmatrix}$

k $\begin{bmatrix} 4 & 16 & 12 & -20 \\ -12 & 8 & -4 & -24 \\ 0 & 20 & -8 & 12 \end{bmatrix}$

l $\begin{bmatrix} -15 & 0 & 5 & -10 \\ -25 & 20 & -15 & -5 \\ 5 & -10 & 0 & -20 \end{bmatrix}$

m $\begin{bmatrix} 3 & -2\frac{1}{2} & 2 & -1\frac{1}{2} \\ -\frac{1}{2} & 4 & -4\frac{1}{2} & 1 \\ 2 & 3\frac{1}{2} & -1 & 2\frac{1}{2} \end{bmatrix}$

n $\begin{bmatrix} 0.1 & 0.4 & 0.3 & -0.5 \\ -0.3 & 0.2 & -0.1 & -0.6 \\ 0 & 0.5 & -0.2 & 0.3 \end{bmatrix}$

o $\begin{bmatrix} 2 & -1\frac{2}{3} & 1\frac{1}{3} & -1 \\ -\frac{1}{3} & 2\frac{2}{3} & -3 & \frac{2}{3} \\ 1\frac{1}{3} & 2\frac{1}{3} & -\frac{2}{3} & 1\frac{2}{3} \end{bmatrix}$

9780170484749

p $\begin{bmatrix} -6 & 15 & -16 & 17 \\ 23 & -40 & 39 & -2 \\ -16 & -13 & 6 & 1 \end{bmatrix}$

6 **a** $x = -5$ **b** $x = -1$ **c** $x = 2$

7 $A = \begin{bmatrix} 3 & 2 \\ 3 & 0 \end{bmatrix}$ **8** $M = \begin{bmatrix} -8 & 5 \\ -8 & -5 \end{bmatrix}$ **9** B

10 **a** $\begin{bmatrix} 32 & -2 \\ 2 & 8 \end{bmatrix}$ **b** $\begin{bmatrix} 12 & 8 \\ 22 & 6 \\ 0 & 36 \end{bmatrix}$

11 **a** $\begin{bmatrix} 300 & 400 & 200 & 150 \\ 400 & 220 & 300 & 200 \\ 250 & 150 & 400 & 150 \end{bmatrix}$

b $\begin{bmatrix} 30 & 70 & 20 & 20 \\ 20 & 30 & 40 & 30 \\ 40 & 10 & 50 & 10 \end{bmatrix}$

c $\begin{bmatrix} 50 & 100 & 10 & 15 \\ 15 & 50 & 30 & 10 \\ 30 & 25 & 20 & 40 \end{bmatrix}$

d $A - B + C = \begin{bmatrix} 320 & 430 & 190 & 145 \\ 395 & 240 & 290 & 180 \\ 240 & 165 & 370 & 180 \end{bmatrix}$

12 **a** $\begin{bmatrix} 4.50 & 9.15 & 15.70 \\ 7.85 & 14.80 & 22.05 \\ 7.10 & 15.70 & 20.55 \\ 11.05 & 22.05 & 26.95 \end{bmatrix}$

Customer prices are rounded to the nearest 5 cents.

EXERCISE 9.4

1 **a** column matrix size 2×1
b square matrix order 2

2 $\begin{bmatrix} 4 & 6 & 4 \\ -3 & 11 & -1 \end{bmatrix}$

3 **a** yes **b** yes **c** no
d yes **e** no **f** yes

4 **a** $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **b** $\begin{bmatrix} -7 & 4 \end{bmatrix}$

c $\begin{bmatrix} 6 & -2 & 4 \\ -9 & 3 & -6 \\ 12 & -4 & 8 \end{bmatrix}$ **d** $\begin{bmatrix} -2 & 1 \\ 6 & 3 \end{bmatrix}$

e $\begin{bmatrix} -17 & 9 \\ 8 & -16 \end{bmatrix}$ **f** $\begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}$

g $\begin{bmatrix} 6 & -15 \\ -2 & -14 \end{bmatrix}$ **h** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

i $\begin{bmatrix} -12 & 6 \\ -8 & 4 \end{bmatrix}$ **j** $\begin{bmatrix} -1 & 5 \\ -4 & 7\frac{1}{2} \end{bmatrix}$

k $\begin{bmatrix} 0 & 3 \\ -3\frac{2}{3} & -2 \end{bmatrix}$ **l** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5 $\begin{bmatrix} 33 & 21 \\ 35 & 40 \end{bmatrix}$ and $\begin{bmatrix} 33 & 21 \\ 35 & 40 \end{bmatrix}$; they are the same.

6 $\begin{bmatrix} -11 & 7 & -11 \\ 7 & 13 & 7 \\ -11 & 7 & -11 \end{bmatrix}$ and $\begin{bmatrix} -11 & 7 & -11 \\ 7 & 13 & 7 \\ -11 & 7 & -11 \end{bmatrix}$; they are the same.

7 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. The product is a zero matrix.

8 The number of columns in the first is not the same as the number of rows in the second, so the product of the matrices cannot be found.

9 $\begin{bmatrix} 7 & 1 \\ 6 & 6 \end{bmatrix}$ The answer is the same as the original matrix as it was multiplied by an identity matrix.

10 $\begin{bmatrix} 0 & 2 & 5 & 1 \\ -3 & 1 & 2 & 1 \end{bmatrix}$ The answer is the same as the original matrix as it was multiplied by an identity matrix.

11 **a** yes **b** no **c** no **d** yes

12 **a** $\begin{bmatrix} 60 & 37 \\ 18 & 16 \end{bmatrix}$ **b** $\begin{bmatrix} 20 & 12 \\ -4 & 6 \\ 0 & 2 \end{bmatrix}$

13 **a** $\begin{bmatrix} 12 & 10 & 8 \\ 15 & 8 & 4 \\ 10 & 10 & 10 \\ 12 & 12 & 4 \\ 16 & 6 & 12 \end{bmatrix}$ **b** $\begin{bmatrix} 117.80 \\ 155 \\ 190 \end{bmatrix}$

c Brisbane \$4483.60, Logan \$3767.00, Chermside \$4628.00, Cairns \$4033.60, Gold Coast \$5094.80.

EXERCISE 9.5

1 **a** yes **b** yes **c** yes **d** no
2 **a** identity **b** row **c** square **d** zero

3 **a** $\begin{bmatrix} 3 & 3 \\ 6 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 12 & 4 \\ 12 & 4 \end{bmatrix}$ **c** $\begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$

d $\begin{bmatrix} 22 & -18 \\ -12 & 10 \end{bmatrix}$ **e** $\begin{bmatrix} 25 & 0 \\ 0 & 4 \end{bmatrix}$

4 **a** $\begin{bmatrix} 27 & 0 \\ 0 & 64 \end{bmatrix}$ **b** $\begin{bmatrix} -28 & -36 \\ 27 & -1 \end{bmatrix}$

c $\begin{bmatrix} -39 & 114 \\ 19 & 94 \end{bmatrix}$ **d** $\begin{bmatrix} -68 & 76 \\ 57 & -49 \end{bmatrix}$

e $\begin{bmatrix} 49 & 38 \\ -114 & -84 \end{bmatrix}$

5 **a** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

c $\begin{bmatrix} 5000 & 5000 \\ 5000 & 5000 \end{bmatrix}$ **d** $\begin{bmatrix} -107 & 84 \\ -252 & 61 \end{bmatrix}$

e $\begin{bmatrix} 7281 & -6160 \\ -4400 & 3761 \end{bmatrix}$

6 **a** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

All powers of A are the 2×2 zero matrix.

7 **a** $\begin{bmatrix} 3 & 0 & -21 \\ 25 & 4 & -15 \\ 14 & 0 & 10 \end{bmatrix}$ **b** $\begin{bmatrix} 63 & -45 & 0 \\ 90 & 18 & 0 \\ -27 & -63 & 0 \end{bmatrix}$

c $\begin{bmatrix} 6 & -4 & 2 \\ 6 & -4 & 2 \\ 6 & -4 & 2 \end{bmatrix}$ **d** $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

8 **a** $\begin{bmatrix} 1 & 75 & -15 \\ 0 & 1 & 0 \\ 0 & -15 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$

c $\begin{bmatrix} 13 & 33 & 11 \\ 22 & 13 & 11 \\ 33 & 22 & 13 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

9 $\begin{bmatrix} 13 & 6 \\ 8 & 13 \end{bmatrix}$

10 As the matrix is an identity matrix, squaring, cubing or any additional powers will always result in the matrix remaining exactly the same.

11 All equal $\begin{bmatrix} 9 & -4 \\ 18 & -8 \end{bmatrix}$

They are all the same as the original matrix.
$M = M^2 = M^3 = \ldots$

EXERCISE 9.6

1 Defining a matrix product means checking that the matrices can be multiplied.
So, checking if the number of columns of the first equals the number of rows of the second.

2 $\begin{bmatrix} 14 & 12 \\ 60 & 86 \end{bmatrix}$

3 C

4 **a** $\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ **b** $\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$

c $\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$ **d** $\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$

5 **a** $C = \begin{array}{c c} & \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$

b 2

6 **a** 1 **b** none **c** 1

7 **a** $\begin{bmatrix} 10.60 & 25.00 \\ 8.80 & 17.20 \\ 5.40 & 12.80 \\ 10.00 & 16.40 \end{bmatrix}$ **b** $\begin{bmatrix} 18.55 & 43.75 \\ 15.40 & 30.10 \\ 9.45 & 22.40 \\ 17.50 & 28.70 \end{bmatrix}$

c $\begin{bmatrix} 7.95 & 18.75 \\ 6.60 & 12.90 \\ 4.05 & 9.60 \\ 7.50 & 12.30 \end{bmatrix}$

8 \$1207

9 $\begin{array}{c c} & \begin{array}{cc} \text{Wheels} & \text{Seats} \end{array} \\ \begin{array}{c} \text{Junior} \\ \text{Senior} \end{array} & \begin{bmatrix} 56 & 20 \\ 98 & 32 \end{bmatrix} \end{array}$

10 **a** $\begin{bmatrix} 7160 \\ 9640 \\ 7140 \\ 7200 \\ 11\,580 \\ 9180 \end{bmatrix}$ **b** $\begin{bmatrix} 2506 \\ 3374 \\ 2499 \\ 2520 \\ 4053 \\ 3213 \end{bmatrix}$

c $\begin{bmatrix} 9666 \\ 13\,014 \\ 9639 \\ 9720 \\ 15\,633 \\ 12\,393 \end{bmatrix}$

11 A receives \$27 265, B \$42 329, C \$49 532.

12 George \$147.40, Frank \$201.30, Maria \$168.40

CUMULATIVE EXAMINATION 1

1 D **2** B **3** C

4 A **5** D

6 **a** -2 **b** 4 **c** 6

7 $\begin{bmatrix} -3 & -2 \\ -1 & 14 \end{bmatrix}$

8 $x = -18$, $y = -8$ or $(-18, -8)$

9 height = 10.9 m

CUMULATIVE EXAMINATION 2

1 $x = -1$, $y = \frac{1}{2}$

2 35 km

3 $m = \frac{1}{4}$, $n = \frac{1}{4}$

4 No, the operation cannot be evaluated as not all terms will have the correct orders.
$CB + A$ (2×2) cannot be added to DB (1×2).

CHAPTER 10

EXERCISE 10.1

1 a ordinal, categorical
  b continuous, numerical
  c continuous, numerical
  d discrete, numerical
  e nominal, categorical
  f discrete, numerical
  g nominal, categorical
  h nominal, categorical
  i continuous, numerical
  j discrete, numerical

2 yes; it is numerical but can't be 'measured', so discrete, numerical.

3 B

4 disagree; Peter has just averaged the 3 prices without taking into account the amount of each grade of tomato sold. If most of the tomatoes sold are the cheap ones, the average will be closer to that price.

5 Jess is correct. It may be written discretely (rounded) but it is still a distance, which is continuous.

6 a Age is a measurement, so numerical, continuous.
  b There can only be an exact number of students, so numerical, discrete.

7 The data is categorical, so it doesn't make sense to average the responses, even when they are coded as numbers.

EXERCISE 10.2

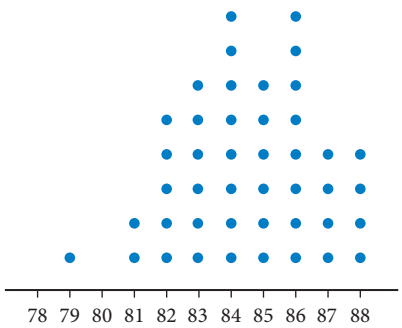
1 A

2 Can't be measured, only exact values, so numerical, discrete.

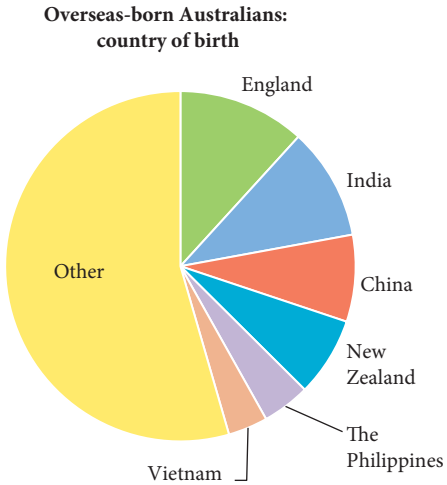
3

Vehicle	Frequency
Cars	93
Motorbikes	9
Trucks	30
Taxis	10
Other	42
Total	**184**

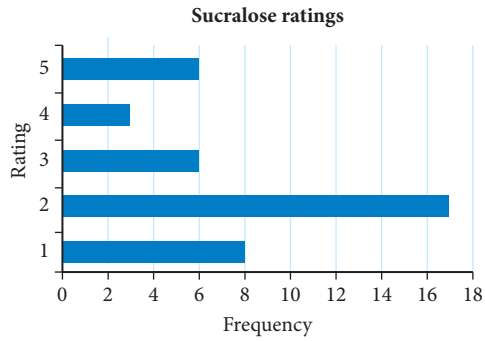
4



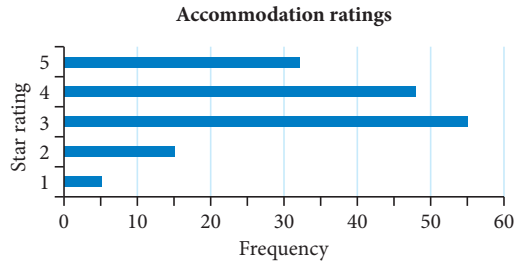
5



6



7



8 a 70 b 150 c Action
  d 840 e 32.1%

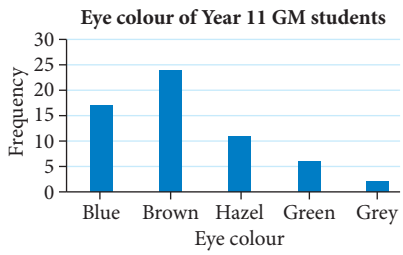
9 B

10 a

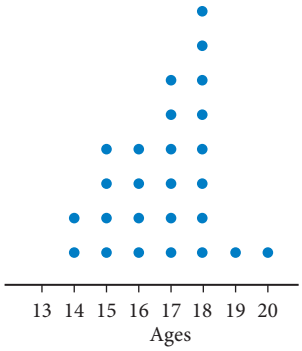
Colour	Tally	Frequency
Blue	𝍸 𝍸 𝍸 ||	17
Brown	𝍸 𝍸 𝍸 𝍸 ||||	24
Hazel	𝍸 𝍸 |	11
Green	𝍸 |	6
Grey	||	2
Total		**60**

  b 28.3%

  c



11 a



b 18 **c** 61.5%

12 a incorrect; 20% of 360° = 72° not 20°.

b 284 **c** 2080

EXERCISE 10.3

1 A

2

Number siblings	Tally	Frequency
0	𝍸 ||	7
1	𝍸 |||	8
2	𝍸 |||	8
3	||||	4
4	||	2
5		0
6	|	1
Total		**30**

3

Score	Tally	Frequency
0–<10	|	1
10–<20	|	1
20–<30	||||	4
30–<40	||||	4
40–<50	𝍸 |	6
Total		**16**

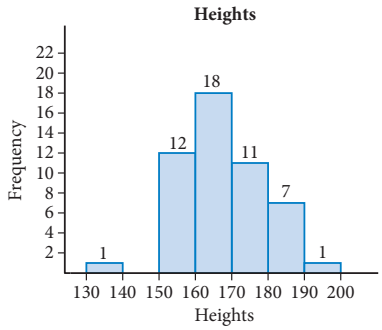
4 a 34 and 96

b

Stem	Leaf
3	4 6
4	0 5 5 6
5	2 5 6 8
6	0 4 7 8
7	1 4 5 8 8 8 8
8	0 0 2 8
9	0 2 4 6

Key: 4|5 means 45

5

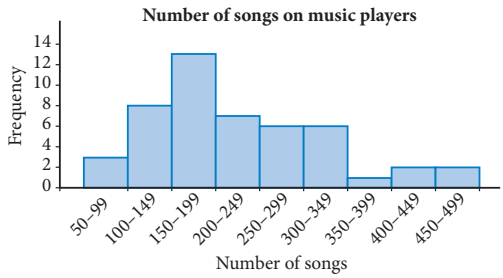


6 a Use class intervals of 50; as the data goes from 53 to 497. So, if it goes by 50s, this gives ~10 groups.

b

Number of songs	Tally	Frequency
50–99	|||	3
100–149	𝍸 |||	8
150–199	𝍸 𝍸 |||	13
200–249	𝍸 ||	7
250–299	𝍸 |	6
300–349	𝍸 |	6
350–399	|	1
400–449	||	2
450–499	||	2
Total		**48**

c



d discrete, numerical

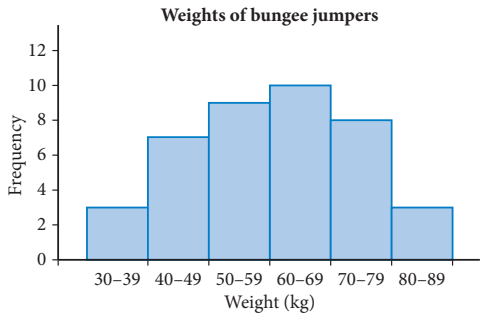
e There was a wide spread of numbers of songs, but most had between 100 and 350.

7 B **8** D

9 a

Weight (kg)	Frequency
30–39	3
40–49	7
50–59	9
60–69	10
70–79	8
80–89	3
Total	**40**

b



c 55

10 **a** continuous, numerical as it is a measurement

b from 35 years to less than 39 years, but not including 39 years

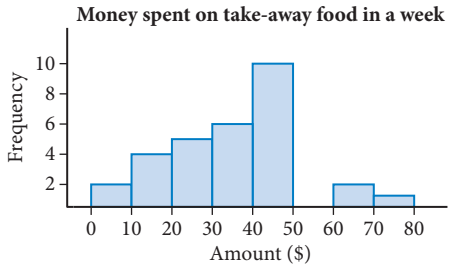
c 47–<51 **d** 45 **e** 41

f

Fathers' ages	Frequency
35–<39	2
39–<43	2
43–<47	3
47–<51	16
51–<55	9
55–<59	3
59–<63	6
Total	**41**

11 **a** discrete; money is exact, it is not a measurement.

b



c don't know exactly; just know it was between $70 and $80.

d

Stem	Leaf
0	0 6
1	2 5 5 8
2	2 5 5 8 8
3	0 0 1 5 6 7
4	0 0 0 1 2 2 5 8 8 9
5	
6	5 9
7	4

Key: 2 | 5 means 25

e It can tell you the actual values. So, can see maximum spent on take-away food was $74.

12 The stem-and-leaf plot as it shows you the actual values but still shows the shape of the distribution.

EXERCISE 10.4

1 B

2 Dog weighs from 5 kg to less than 10 kg, but not 10 kg.

		Mean	Median	Mode
3	**a**	27.5	27	28
	b	5.9	6	6, 8
4	**a**	13.3	14	14
	b	214.1	213.5	212, 217
5	**a**	7.4	7	7
	b	18.4	19	19
6	**a**	4.1	3.5–<4	3–<3.5
	b	169.9	170–<175	160–<165, 170–<175
7	**a**	49.7	46.5	46
	b	136.2	130	129

8 A **9** B

10 **a** mode = 1, median = 2, mean = 3.3

b median, as it is not affected by the very high values of 10 but still shows the middle value

11 **a** mean ≈ $4433, median class = $4000–$4499, modal class = $3500–$3999

b $4000–$4499 (median class), because the mean is affected by the few very high incomes.

12 78 songs

13 All measures of centre are approx. the same. So, 18 cars is typical.

EXERCISE 10.5

1 B

2 **a** mean = 166.1 cm, median class = 165–<170, modal class = 165–<170 and 170–<175

b Although there is a wide range of heights in the office, most are around 165–<170 cm.

		Range	IQR
3	**a**	7	5
	b	7	3.5
	c	20	12
4	**a**	16	6.5
	b	7	4
	c	5	2
5	**a**	9	4
	b	8	3
6	**a**	56	23.5
	b	5	1.5

7 C

8 **a** 8 **b** 2

c IQR as it best represents the data. It is not affected by the unusually high age of 18.

9 middle half → IQR = 7. So, the middle half would be finished within 7 seconds.

10 Tena is more consistent as she has a lower range and IQR.

EXERCISE 10.6

1 C

2 **a** range = 13, IQR = 2.3

b Range includes even the extreme value, whereas the IQR only looks at middle 50% of the data. So, IQR is more appropriate here.

		Mean ($\bar{x}$)	Std deviation (s_x)
3	**a**	8.7	2.21
	b	5.5	2.68
	c	14.63	2.09
4	**a**	11	2.49
	b	9.4	3.70
	c	21.84	2.36
5	**a**	11.31	2.47
	b	4.2	0.18

6 set B **7** set A **8** C

9 **a** mean = 3.37, median = 3, mode = 0

b range = 9, IQR = 4, standard deviation = 2.93

10 Brand Y is more reliable as it has a lower standard deviation. This means it is more consistent.

11 Best measure of centre is median and best measure of spread is IQR, as neither are affected by the very large value of $3 600 000.

EXERCISE 10.7

1 B

2 mean = 101.7, std. dev. = 12.70

3 **a** negatively skewed **b** symmetrical

c bimodal **d** positively skewed

e symmetrical **f** negatively skewed

4 **a** 14

b 74.5 and 130.5

c yes; 72 as it is a value outside the fences.

5 **a** 6.1 and 61.3

b 140 and 151

6 **a** It's a measurement, so continuous, numerical.

b

Stem	Leaf
0	3 4 4
0*	5 7 8 8 8 9 9
1	1 1 2 3
1*	5 5 5 6 6 8 8 9
2	1 1 1 3 4 4 4 4
2*	5 6 7 8
3	1 4 4
3*	7 8
4	
4*	
5	
5*	
6	
6*	
7	0

Key: 1 | 3 means 13 and 1* | 5 means 15

c mostly symmetric apart from 70

d 70 is a possible outlier

e As there is an outlier, use median = 18.

f As there is an outlier, use IQR = 14.5.

7 C

8 **a** Answers will vary. Sample answer:

Stem	Leaf
8*	6 9
9	1
9*	5 5 5 5 7 7 8 8 8
10	0 0 2 2 2 4
10*	5
11	
11*	5

Key: 10 | 2 means 102 years and 10* | 5 means 105 years

b IQR = 7

c 84.5 and 112.5

d 115 is a possible outlier

9 22 is an outlier as it is below the lower fence of 22.5.

10

Stem	Leaf
5	
5*	5
6	
6*	
7	2 2 2
7*	5 6 6 7 8 8 8 9 9 9
8	0 0 0 1 1 1 2 3 4 4
8*	5 6 7 7 8 9 9
9	0 0 1 2 2 3 3 3 3 4
9*	5 8 8 9 9
10	0 3

Key: 7 | 2 means 72 and 7* | 6 means 76

The residents have a median age of 84.5 and the middle 50% of the data is within 13.5 years (IQR = 13.5). These are the best measure of centre and spread as they will not be affected by any unusual values, like 55. The ages are bimodal, possibly because women live longer, and the 55-year-old is an outlier. They would be in an aged care facility possibly because nothing else suitable is available.

CUMULATIVE EXAMINATION 1

1 A **2** A **3** B

4 D **5** C

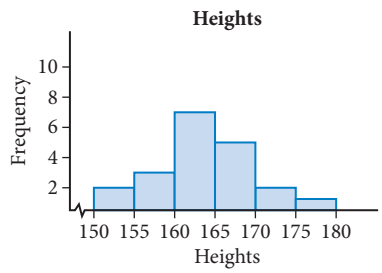
6 **a** categorical, nominal

b

Pet	Tally	Frequency
Cat	~~IIII~~ I	6
Dog	~~IIII~~ IIII	9
Horse	IIII	4
Bird	II	2
Snake	III	3
Fish	IIII	4
Rat	II	2
Total		**30**

c dog

7



8 9

9 28.8 m

10 13 572 L

CUMULATIVE EXAMINATION 2

1 a When you multiply a (3×2) matrix with a (2×1) matrix, you should get a (3×1) matrix, not a (3×2) matrix.

2 mean = 7.81, median = 8, mode = 8
Typical shoe size is 8 as all 3 measures are close to the same value.

3 60.35 km

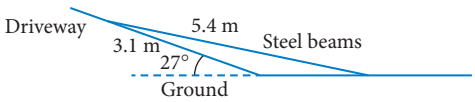
CHAPTER 11

EXERCISE 11.1

1 a 69.2 mm **b** 1442.7 m **c** 5.1 cm
2 a 780.96 m **b** 3.68 km **c** 32.18 cm
3 a 1935 m **b** 1 m **c** 691 km
4 Justin 81.09 m; Tara 40.35 m
5 31.4 cm **6** B **7** 2.9 km
8 7.5 cm
9 a $BD = \dfrac{32}{\sin 71°} \times \sin 69° \approx 31.6$ cm
b 49 cm
10 738 m **11** 13.5 km

EXERCISE 11.2

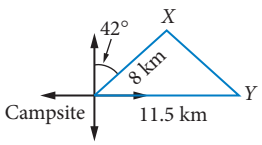
1 B **2** 62.4 km
3 a 52.8° **b** 37.1° **c** 37.7°
4 a 104° **b** 47° **c** 71°
5 a 38.2° **b** 12.2° **c** 99.7°
6 94° **7** D **8** 35°
9 a



b 15°
10 a 31.8° **b** 8.5 m
11 137° **12** 161° **13** 20°

EXERCISE 11.3

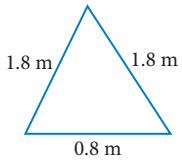
1 C **2** 83.8°
3 a 5.4 cm **b** 7.0 m **c** 88.2 cm
d 55.1 m **e** 171.8 km **f** 4.2 mm
4 190.1 m **5** 671 m **6** B
7 a



b 5.1 km
8 27.0 m **9** 13.2 cm
10 66.4 m **11** 30.7 km

EXERCISE 11.4

1 B **2** 316 m
3 a 22° **b** 53° **c** 109°
d 134° **e** 99° **f** 26°
4 34° **5** 103° **6** 36°, 26° **7** B
8 a



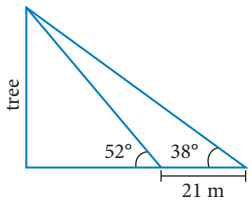
b 26°
9 53° **10** 170° **11** 70.8°

EXERCISE 11.5

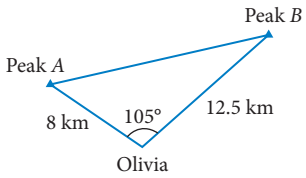
1 A **2** 31°
3 a 114.9 m^2 **b** 1622.7 mm^2 **c** 1840.5 cm^2
4 a 2133 cm^2 **b** 14 km^2 **c** 314 195 m^2
5 a 0.3 m^2 **b** 1207.5 cm^2 **c** 53.3 cm^2
d 3072.0 m^2 **e** 103.9 km^2 **f** 91.8 mm^2
6 6984 m^2 **7** 363 cm^2
8 B **9** \$1246
10 11 cm^2 **11** 52.7 m^2
12 29.6°
13 not enough; needs \$7810 more.

EXERCISE 11.6

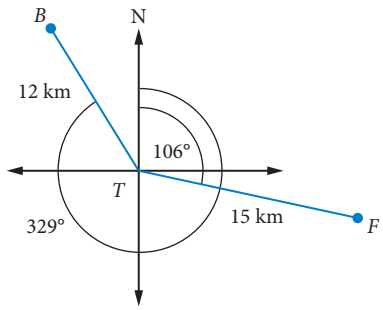
1 D **2** 7 m^2
3 361.5 m **4** 4.31 km
5 a 112° **b** 38.4 m **c** 35.6 m
6 a



b 42.1 m
7 C
8 a



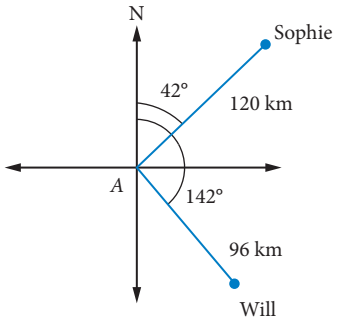
b 16.5 km
9 a



b $\angle BTF = 31° + 106° = 137°$
c 25.1 km

10 **a** Sophie 120 km, Will 96 km

b



c 166.2 km

11 233 m

12 2992 m above hill, height = 3397 m
Altimeter is incorrect. It is reading higher than it is.

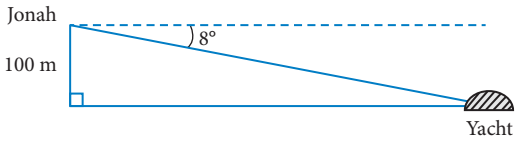
CUMULATIVE EXAMINATION 1

1 B **2** A **3** C
4 C **5** C **6** 2.8 m
7 34 m **8** 17.0 m^2

9 $\begin{bmatrix} 20 & 4 & 6 \\ -11 & -5 & -11 \\ 28 & 0 & -7 \end{bmatrix}$

10 \$8130

11 **a**



b 712 m

CUMULATIVE EXAMINATION 2

1 **a** 128 m **b** 208°
2 \$36
3 85 m

CHAPTER 12

EXERCISE 12.1

1 **a** {19, 21, 23.5, 26, 27}
b {2, 3.5, 6, 7, 9}

2 **a** {3, 5, 8, 9, 12}
b {22, 25, 26, 28, 32}
c {4.1, 5.85, 6.9, 8.35, 9.8}
d {104, 107, 108, 109.5, 112}

3 **a** {12, 14, 15, 23, 27} with possible outlier at 40
b {7, 10, 12, 13, 16} with possible outlier at 3

4 **a** {31, 32.5, 38.5, 42, 56} with a possible outlier at 16
b 3 of the 12 are less than 32.5. So, $\frac{3}{12} \times 100 = 25\%$.
c 6 of the 12 are above the median 38.5.
So, $\frac{6}{12} \times 100 = 50\%$.

5 **a** {2.9, 3.85, 6.7, 7.8, 8.3}
b 2 of the 8 are greater than 7.8. So, $\frac{2}{8} \times 100 = 25\%$.
c 4 of the 8 are between 3.85 and 7.8.
So, $\frac{4}{8} \times 100 = 50\%$.

6 C **7** C

8 {0, 1, 2, 2, 3} with possible outlier at 4.

9 **a** **i** 0%
ii 25%
iii 50%
b no; only taller than just over half – not **most**.
c Nirmalan is correct as 186 is above the fence.

10 Abby is incorrect. 43 is a possible outlier.

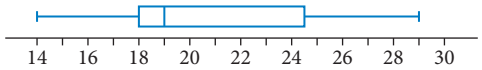
11 94; teacher to check justification

EXERCISE 12.2

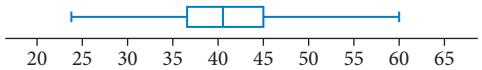
1 C

2 **a** {10.6, 10.7, 10.9, 10.95, 11.2}
b no; no values outside fences

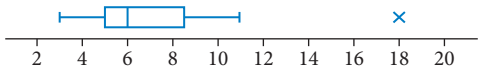
3 **a**



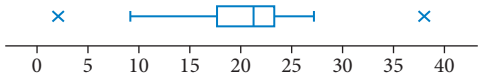
b



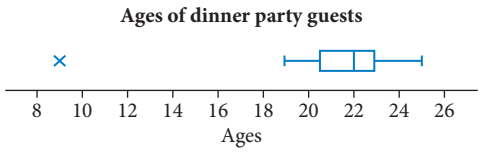
4 **a**



b



5 **a** yes; 9 is below the lower fence.
b {19, 20.5, 22, 23, 25}
c



6 **a** {160, 170, 175, 178, 188} and outlier at 155.
b 25%
c 25%
d 12
e 8
f 158. So, 155 is possible outlier as lower than fence.
g Distribution of data is approx. symmetrical (although bunched in the centre).

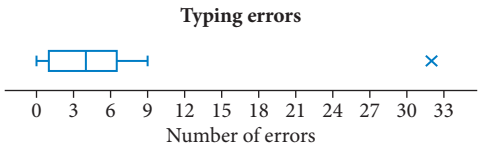
7 **a** {417, 420, 425, 431, 435}. No outliers.
b 25%
c 250 tins
d A large percentage are under 420 g. The data is quite spread out. Sam should recommend looking at the machine to give a more consistent result. A quarter of customers not getting enough is not good enough.

8 **a** symmetrical
b positively skewed
c symmetrical but bunched in the middle
d negatively skewed

9 D **10** C

11 **a** {0, 1, 4, 6.5, 9} and possible outlier of 32

b



c Typical error of 4 errors is appropriate. This is the median as there was an outlier)

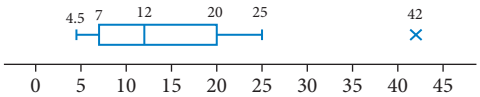
12 no; 22 should be an outlier which makes the new max to be 20.

13 Cade is incorrect. There is only 1 outlier and the median is 2.67 g. So, typical value would be represented by 2.67 g.

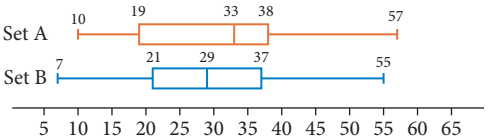
EXERCISE 12.3

1 A

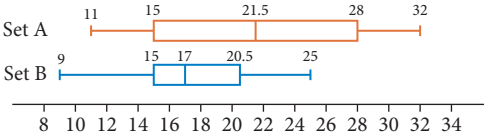
2



3 **a**



b

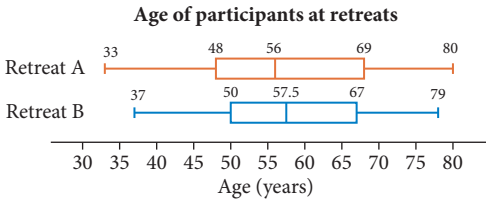


4

		Thao	Tommy
a	**i**	positively skewed	symmetrical
	ii	50	60
	iii	IQR = 40	IQR = 20
		range = 95	range = 65
	iv	no outliers	no outliers

b Thao's distances travelled were more varied than Tommy's, shown by larger IQR and range, but Tommy typically travelled further, shown by larger median.

5 **a**

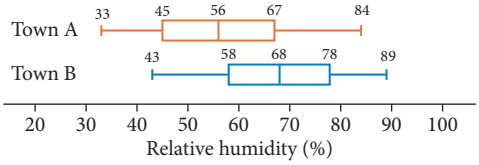


b Both close to symmetrical. Retreat A medium slightly lower, so typically lower age. Retreat A had a wider age range than B. Neither had outliers.

c Although very similar, participants at retreat A had a slightly lower age, and their ages were more varied than for participants at retreat B.

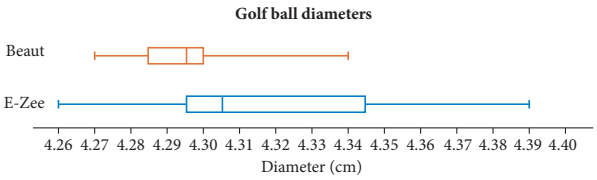
6 D

7 **a**



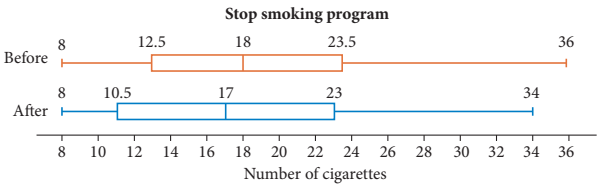
b Payal is incorrect. Looking at the box plots, it is clear that town B has higher relative humidity and hence more likely to have storms.

8



Beaut Ball Company has better production processes. Both companies have median diameters close to 4.30 cm but Beaut Ball Company's results were more consistent.

9

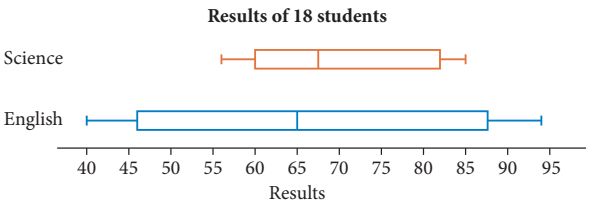


Slightly fewer cigarettes smoked per day after the program, shown by the box moving down slightly but overall, I would not determine the program a success. The reduction was too minimal.

EXERCISE 12.4

1 B

2



3 **a** Text 1 = {5, 6, 8, 9, 10}, Text 2 = {5, 6, 9, 13.5, 24}

b No outliers. No values outside of fences.

c Text 1 would be easier to read as the words are consistently shorter than in Text 2.

4 **a** Factory X: $\bar{x} = 302.9$, $s_x = 1.23$; Factory Y: $\bar{x} = 303.3$, $s_x = 1.14$

b Since Salomi knows that factory X is fine, she can be assured that factory Y is fine. It has a very similar mean and is actually more consistent than Factory X, shown by the lower standard deviation.

5 **a** Yes. Class A has a possible outlier at 24.

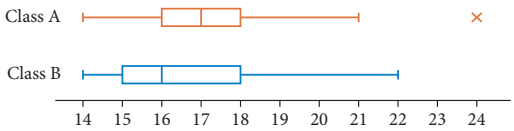
b

	A	B
mean	17.3	16.6
median	17	16
mode	17	14

c

range	10	8
IQR	2	3
std dev	2.40	2.36

d By drawing parallel box plots:



Class A is symmetrical, although grouped around centre except for the possible outlier. Class B is positively skewed.

e Class A has an outlier, so median best.

f Class A has an outlier, so IQR best.

g Class A had a higher median and IQR, So, class A had less consistent results and took longer than class B.

6 C

7 Cohen would be the best selection. The 2 players have the same mean but Cohen is more consistent, as shown by the lower standard deviation.

8 Pink fans consistently travelled a little further to see her, although there was one Taylor Swift fan who travelled a very long way (470 km).

9 **a** No outliers in either data set, so use mean. Farmer Dan = 215.25; Farmer Joe = 182.35

b No outliers in either data set, so use standard deviation. Farmer Dan = 81.81; Farmer Joe = 41.07

c Answers will vary. One possible solution: Dan's onions are larger (215 g mean vs 185 g) and more varied in size (80 g vs 47 g), so they are more suitable for a variety of uses than Joe's. Dan's should be chosen unless the caterer wants consistently small onions.

10 Slogger had the higher median and the IQRs were the same. So, the club should choose Slogger Jones as he gets more runs and has the same consistency as Basher Smith.

11 The Setnforget is better than the Easy as ABC because it is more consistent. They both need to have their level adjusted slightly.

EXERCISE 12.5

1 A

2 Drivers in country A got their licences at a younger age than in country B, but the age at which people got their licence was more consistent in country B.

3 The students working at the hardware store are generally older: mean of 19 compared to 16.3.

4 Female capybaras typically weigh more than males, but the masses of the males were more consistent.

5 **a** yes; \$139 000 is an outlier for teachers.

b There is an outlier, so use median and IQR.

Plumbers: median = \$104 500, IQR = \$20 000

Teachers: median = \$90 500, IQR = \$19 000

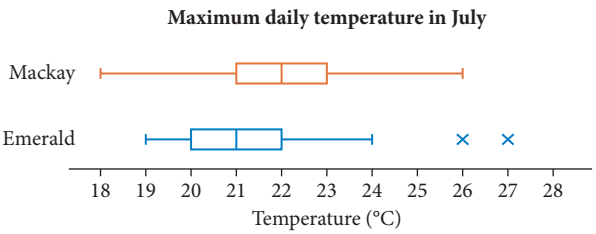
c Plumbers: bimodal (not skew); teachers: positively skewed.

d Plumber's income are higher and more spread out than teachers.

6 B

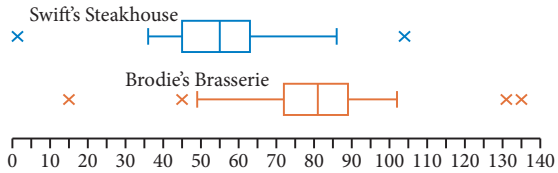
7 There was a lot of inconsistency in the estimated times to burn 100 calories, shown by the much larger IQR for estimated times. So, although some over-estimated the time, the lower median for estimated time compared to actual time supports his theory that people 'underestimate' the time it takes to burn 100 calories.

8



The results for Mackay and Emerald have the same consistency, as indicated by the same IQR. Emerald has a lower maximum temperature in July, as shown by the median, although there are 2 possible outliers with very high values in Emerald.

9



Brodie's Brasserie generally had more bookings per week than Swift's Steakhouse but the 2 had a similar consistency in their bookings. Both had some weeks with unusually high and low bookings.

CUMULATIVE EXAMINATION 1

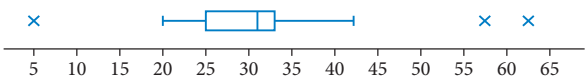
1 D **2** B **3** C

4 D **5** B **6** $74.5° \approx 75°$

7 {31, 33, 38, 43, 52}

8 45.2 m^2

9



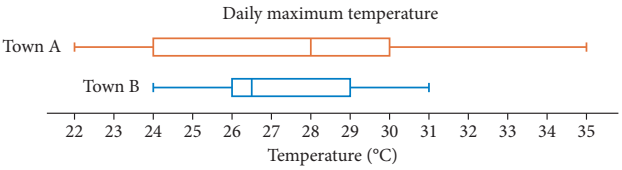
10 $x = -1\frac{3}{4}$

CUMULATIVE EXAMINATION 2

1 \$950

2 13°

3 Town B is more likely to be near the sea as it has less variation in maximum temperature.



Town B is more likely to be near the sea due to its lower temperatures (lower mean and median).

Glossary and index

ABSTUDY Government allowance paid to eligible Aboriginal apprentices and students. (p. 19)

adjacency matrix A matrix that shows direct connections between points in a network. (p. 278)

adjacent side The side that is next to a given angle in a right-angled triangle, which, with the hypotenuse, forms the arms of the angle. (p. 235)

Age Pension A pension paid to eligible Australians who are aged over 67 and not working. (p. 20)

algebraic expression (or **expression**) Variables and numbers connected by operations (p. 110)

angle of depression The angle formed by looking down from a horizontal line. (p. 245)

angle of elevation The angle formed by looking up from a horizontal line. (p. 245)

annual Each year; in a year. (p. 9)

annulus The shape that is the area between two circles with the same centre. (p. 51)

apex The vertex opposite the base of a cone or a pyramid. (p. 59)

arc length The length of a part of a curve, measured along the curve. (p. 46)

area The size of a plane or curved surface, measured in square units. (p. 43)

assets test An assessment of the value of a person's possessions (for example, property, car), to work out the amount of an allowance or pension the person will be paid. (p. 20) *See also* **income test**.

Austudy A government allowance paid to eligible apprentices and students. (p. 19)

axis (of a Cartesian plane) A fixed line used for reference on a graph. (p. 178) *See also* **Cartesian plane, *x*-axis, *y*-axis**.

back-to-back stem-and-leaf plot Stem-and-leaf plots to the left and right of the same stem for 2 related sets of data, such as the ages of mothers and fathers of a group of students. (p. 389)

balance (of equation) The values of both sides of an equation are equal. You do the same thing to both sides to maintain the balance. (p. 118) *See also* **inverse operation**.

bar chart A rectangular graph divided into vertical or horizontal bars with lengths or heights proportional to values. (p. 291) *See also* **column chart** and **pie chart**.

base The flat end of a prism or pyramid. (p. 59)

bearing A compass direction stated as either a 3-digit angle clockwise from north, or as an angle east or west of north or south. (p. 249) *See also* **true bearing**.

box-and-whisker plot, **box plot** A display of the five-number summary of a set of data; the quartiles are shown by the left and right ends of a rectangle ('box'), divided by the position of the median; lines ('whiskers') extend from the ends of the rectangle to show the positions of the minimum and maximum values. (p. 370)

budget Lists of income and expenses. A *balanced* budget has equal totals of income and expenses. (p. 22)

buying price *See* **cost price**.

capacity Another name for volume, used for the volumes of containers for liquids. (p. 66)

Cartesian plane or **number plane** A grid system referenced by intersecting horizontal and vertical lines called the x-axis and y-axis respectively. The axes have scales starting from 0 at the intersection and a point on the plane is described by 2 coordinates from the x- and y-axes. (p. 178)

casual worker An employee whose hours of work may vary from day to day or week to week. (p. 12)

categorical data A statistical variable with data that are categories (names), not numbers. (p. 289) *See also* **numerical variable, nominal variable, ordinal variable**.

chart A graph or display of data. (p. 291) *See also* **pie graph, dot plot, stem-and-leaf plot, bar chart, column chart, histogram**.

circumference The perimeter or boundary of a circle. (p. 44)

class centre The middle of a class. (p. 297)

class interval, **class** A range of data values in a frequency table of grouped data, such as 5–9 or 9.5–19.5. (p. 297)

coefficient The number that precedes a variable in an algebraic term. For example, the coefficient of $5x$ is 5. (p. 110)

column chart A graph with vertical bars with heights proportional to values. (p. 291) *See also* **bar chart**.

column matrix A matrix that has only one column. (p. 264)

commission Payment to workers such as salespeople as a proportion of the value of sales or services. (p. 15) *See also* **retainer**.

communications network A network that shows direct communication links. (p. 278)

compass rose The directions N, E, S and W shown by a cross with an arrow pointing upwards to the north. (p. 249)

cone A circular pyramid; a 3D shape with a base that is a circle with the side rising to an apex (point). (p. 59)

congruent figures Figures with the same size and shape. (p. 137)

constant (term) A term in an algebraic expression or equation that is a number. It has a fixed value. (p. 110)

continuous data A numerical variable such as height that can take on any possible value on a scale. (p. 289) *See also* **discrete variable**.

coordinates An ordered pair of numbers used to identify a position. Cartesian coordinates have a horizontal x-coordinate and a vertical y-coordinate obtained from the x- and y-axes respectively. (p. 178)

cosine The trigonometric ratio given by

$\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ in a right-angled triangle. (p. 235)

cosine rule A rule relating the sides and angles of a triangle. For $\triangle ABC$, $a^2 = b^2 + c^2 - 2bc \cos(A)$ or $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$. (pp. 335, 343)

cost price, buying price The amount of money paid for an item by a shop. (p. 92)

cross-section A shape made by 'cutting' a plane through a 3D shape. (p. 59)

currency The money of a country, particularly one unit; for example: dollar, euro, yen. (p. 98)

curved surface area The area of the curved part of the surface of a solid shape such as a cylinder, cone or sphere. (p. 59)

cylinder A can-shape with a uniform circular cross-section. (p. 59)

degree mode The mode of a calculator so that it calculates trigonometric ratios expressed in degrees. (p. 335)

dependent variable The variable that is calculated from the value of other variables in a function. (pp. 111, 189) *See also* **independent variable**.

Disability Support Pension A pension paid to eligible Australians who cannot work because of a disability or impairment that affects their ability to work. (p. 21)

discount The amount taken off the marked price of an item. (p. 88)

discrete data A numerical variable, such as number of children, can only take on specific, distinct values, in this case, whole numbers. (p. 289) *See also* **continuous variable**.

discretionary expenses Expenses for something that is not essential, could be reduced or removed from a budget. The opposite of **fixed expenses**. (p. 22)

distribution (of data) The way that data is spread. (p. 318) *See also* **negatively skewed, outlier, positively skewed, symmetrical**.

dividend The amount paid to shareholders for each share from company earnings. (p. 100)

dividend yield The dividend as a percentage of the market value of a share. (p. 100)

dot plot A graph with each discrete value shown by a dot above a horizontal scale. (p. 291)

double time Double the normal rate of pay for overtime or as a penalty rate. (p. 12)

element (of a matrix) One of the numbers in a matrix. (p. 261)

elimination method An algebraic method for solving simultaneous equations. Multiples of the equations are added or subtracted to eliminate one variable. The solution for the remaining variable is found, then used to find the other variable. (p. 207) *See also* **substitution method**.

equation A mathematical sentence stating that 2 algebraic expressions are equal. (pp. 110, 169)

exchange rate The rate at which one currency is exchanged for another (for example, US dollars for Australian dollars). (p. 98)

expression *See* **algebraic expression**.

face Any flat surface of a 3D (solid) object. (p. 59)

face value The original price at which a share in a company was sold. (p. 100)

first quartile *See* **lower quartile**.

five-number summary The minimum, lower quartile, median, upper quartile and maximum of a set of data. (p. 365)

fixed expenses Essential expenses that must be incurred, cannot be changed or removed. The opposite of **discretionary expenses**. (p. 22)

flat-rate interest *See* **simple interest**.

formula A mathematical equation that shows the relationship between different variables. (p. 116)

frequency The number of times a data item occurs. (p. 291) *See also* **frequency table**.

frequency table A table showing data and its frequencies. (p. 291)

full-time (worker) A person who is employed for at least 38 hours a week, employed on at least a weekly basis. (p. 12) *See also* **part-time worker**.

goods and services tax (GST) A tax paid to the Australian government when goods or services are bought; currently 10% of the sale price. (p. 91)

gradient *See* **slope**.

gradient–intercept form *See* **slope–intercept form**. (p. 178)

graph A visual representation of a function or data. (p. 291) *See also* **chart, bar graph, column graph, linear function, pie chart**.

grouped data *See* **class interval**.

hemisphere Half a sphere. (p. 59)

Heron's rule A formula used to calculate the area of a triangle using the lengths of the three sides. Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$. (p. 350)

histogram A column graph of the frequency distribution of a continuous variable. There are no gaps between the columns. (p. 297)

hypotenuse The longest side in a right-angled triangle; the side opposite the right angle. (pp. 35, 235)

identity matrix A square matrix I with every element 0 except for 1s in the diagonal $i_{11}, i_{22}, i_{33}, \ldots$ like $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. (p. 264)

image An object that has undergone a transformation. (p. 137) *See also* **object**.

included angle The angle between 2 particular sides of a triangle. (pp. 142, 350)

income test An assessment of a person's regular income, to work out the amount of an allowance or pension a person will be paid. (p. 19) *See also* **assets test**.

independent variable The variable that does not depend on the value of other variables in a function. (pp. 111, 189) *See also* **dependent variable**.

intercept The point where a function cuts an axis (x-axis or y-axis). The value on the axis at that point. (p. 178) *See also* **x-intercept**, **y-intercept**.

interest rate, **nominal interest rate** The cost of a loan or investment as a percentage of the loan/investment, usually stated for one year. (p. 95)

interest The cost of a loan or investment. (p. 95)

interquartile range (IQR) A measure of spread that is the difference between the third and first quartiles. IQR = $Q_3 - Q_1$ (p. 309)

inverse operation The operation that reverses the effect of another operation; for example, + and – are inverse operations of each other. (pp. 118, 169)

inverse trigonometric function The functions that give an angle for values of $\sin(x)$, $\cos(x)$ and $\tan(x)$, written as $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. (p. 236)

like terms Algebraic terms in an expression with identical variable parts; for example, $3x$ and $9x$. (p. 110)

linear equation An algebraic equation in which the highest power of the variable is 1, such as $y = 2x + 7$ or $4x - 3 = 9$. The graph of a linear equation is a straight line. (p. 110)

linear function or **linear equation** The equation of a straight line. (p. 178)

linear expression An algebraic expression in which the highest power of the variable is 1, such as $3x - 4$. (p. 110)

linear model A model that involves a linear function (equation). (p. 189)

loss The amount lost by a business when items are sold for less than they cost; the cost price minus the selling price. (p. 92) *See also* **profit**.

lower fence The value $Q_1 - 1.5 \times \text{IQR}$ used to determine if a low data value is a possible outlier. (pp. 319, 366) *See also* **IQR, outlier, upper fence**.

lower quartile (Q_1) The value below which a quarter of the values of a statistical distribution lie. (pp. 309, 365)

marked price *See* **selling price**.

market price The current price paid for a share on the stock exchange. (p. 100)

mark-up The amount added to the cost of an item for sale in a shop. (p. 88)

matrix (plural: **matrices**) A table of numbers in parentheses ( ) or brackets [ ], like $A = \begin{bmatrix} 1 & 0.5 & 4 & 3 \\ 0 & -2 & 7 & 3 \end{bmatrix}$. (p. 261)

matrix product The product of an $n \times m$ matrix A and an $m \times p$ matrix B is an $n \times p$ matrix C whose elements c_{ij} are the sums of the product of the ith row of A and the jth column of B. (p. 271)

mean A measure of centre; the average of the values of a data set; the sum of all the values divided by the number of values. (p. 303)

measure of centre A measure of the middle of statistical data or distributions, such as the **mean**, **median**, **mode**. (p. 303)

measure of spread A measure of the spread of statistical data or distributions, such as the **range**, **interquartile range** and **standard deviation**. (p. 309)

median, **2nd quartile (Q_2)** A measure of centre; the value below which half of the values of a statistical distribution lie. (pp. 303, 365)

modal class The class with the highest frequency in a statistical distribution. (p. 303)

mode The data value or category in a statistical distribution that has the highest frequency. (p. 303)

multimodal distribution A frequency distribution with two or more distinct peaks, two or more values of high frequency. (p. 318)

multiplication (of a matrix) by a scalar *See* **scalar multiple (of a matrix)**.

negatively skewed, **skewed to the left** Describes a statistical distribution in which the left tail is longer; most of the values are concentrated on the right of the figure. (p. 318)

net (of a 3D shape) The 2D shape that can be folded to make a solid shape. (p. 59)

nominal data A categorical variable with data that cannot be ordered, such as the names of continents. (p. 289) *See also* **ordinal variable**.

non-linear equation An equation that is not a linear equation. (p. 110)

non-linear expression An expression that is not a linear expression. (p. 110)

non-right-angled triangle A triangle that does not have a right (90°) angle. (p. 335)

numerical data A statistical variable with data that is a number, not a category. (p. 289) *See also* **categorical variable**, **discrete variable**, **continuous variable**.

object The original figure prior to undergoing a transformation. (p. 137) *See also* **image**.

operation +, –, ×, and so on that do something to numbers and/or variables. (p. 118)

opposite side The side opposite the given angle in a right-angled triangle. (p. 235)

order (of a square matrix) The number of rows and columns of a square matrix. (p. 261) *See also* **size (of a matrix)**.

ordered pair Two numbers placed in order inside parentheses, as x- and y-coordinates. (p. 178)

ordinal data A categorical variable that has data with a natural order, such as finishing position in a race. (p. 289) *See also* **nominal variable**.

origin The point (0,0) on the Cartesian plane where the x- and y-axes intersect, usually labelled with the letter O. (p. 178)

outlier A value that is well outside the range of the rest of the values in a set of data. (pp. 309, 319, 366) *See also* **lower fence, upper fence**.

overtime Work outside the normal or agreed hours of work. (p. 12)

parallel lines Lines with the same slope or gradient; lines that extend in the same direction and never meet. (p. 179)

parallel box plot Box plots of 2 data sets arranged side by side on a common scale in order to compare them. (p. 376)

part-time (worker) A person who is employed for regular hours less than 38 hours a week, on at least a weekly basis. (p. 12)

penalty rate A higher than normal pay rate for work on a weekend, Sunday or public holiday. (p. 12)

per annum (p.a.) Each year; the normal time for which interest is quoted. (p. 95)

percent (%) Out of 100. (p. 3)

percentage A fraction with a denominator of 100, written as %. (p. 3)

perimeter The length around the outside or boundary of a two-dimensional shape. (p. 43)

pie chart A graph that has sectors of a circle proportional in area and central angle to the values of a variable. Also called a **sector graph**. (p. 291)

piecewise linear function A linear function that is defined by 2 or more sub-functions, each applied to a particular interval of the domain of the function. (p. 216)

piecewise linear graph The graph of a piecewise function. (p. 216)

piecework Payment for each item made or task completed. (p. 16)

plane shape A two-dimensional shape that can be drawn on a flat surface. (p. 42)

point of intersection The point at which two or more graphs intersect. (p. 199)

positively skewed, skewed to the right Describes a statistical distribution in which the right tail is longer; most of the values are concentrated on the left of the figure. (p. 318)

power of a matrix A matrix multiplied by itself repeatedly; A^n is the matrix product of n factors A. (p. 275)

price-to-earnings (P/E) ratio The market price of a share divided by the earnings per share. (p. 100)

principal The initial amount of a loan or investment. (p. 95)

prism A three-dimensional (solid) shape with a uniform cross-section. (p. 59)

profit The amount made by a business when items are sold for more than they cost; the selling price minus the cost price. (p. 92) *See also* **loss**.

pronumeral *See* **variable**.

pyramid A three-dimensional shape with flat faces whose base is a polygon and whose sides are triangles meeting at a common vertex (apex). (p. 59)

Pythagoras' theorem A rule used for calculating the sides of right-angled triangles. For $\triangle ABC$ with hypotenuse c, $c^2 = a^2 + b^2$. (p. 35)

quadrant One of the quarters of the coordinate plane formed by the axes. (p. 178)

quartile 3 values that separate a data set into 4 quarters; the **lower quartile** (Q_1), the **median** (Q_2) and the **upper quartile** (Q_3). (p. 309)

range (of a set of data) The difference between the highest and lowest values of a statistical variable. (p. 309)

rate A comparison of 2 different quantities, such as money and time in \$/hour. (p. 6)

retainer A fixed amount per work period in addition to a commission and does not depend on sales or services provided. (p. 15) *See also* **commission**.

right-angled triangle A triangle with a 90° angle. (p. 35)

rise The change in the y values of a function measured from left to right. (p. 179)

row matrix A matrix that has only one row. (p. 264)

royalty Money paid for the use of intellectual property. (p. 16)

run The change in the x values of a function measured from left to right. (p. 179)

salary A fixed payment to an employee stated as an annual or monthly amount. (p. 9)

scalar multiple (of a matrix), scalar multiplication The product of a number and a matrix; every element of the matrix is multiplied by the number. (p. 267)

scale drawing A drawing that shows a real object with accurate sizes reduced or enlarged by a certain factor (called the scale). (p. 152)

scale factor, scale The fraction or ratio that relates the size of a drawing to the size of the actual object it represents. (pp. 137, 152)

sector A part of a circle that lies between two radii and the circumference. (p. 46)

selling price, marked price The price for which a shop sells items. (p. 92)

semi-perimeter Half the perimeter of a shape. (p. 351)

share Part of the ownership of a company. (p. 100)

shareholder The owner of some shares in a company. (p. 100)

similar figures Shapes that have the same shape but not necessarily the same size. One is an enlargement of the other, with corresponding sides proportional and corresponding angles equal. (p. 137)

simple interest, flat-rate interest Interest calculated on the original principal (only) of a loan or investment. (p. 95)

simultaneous equations Two or more equations (with several variables) that are considered together. (p. 199)

simultaneous solution The values that make all the equations in a system of simultaneous equations true. (p. 199)

sine The trigonometric ratio given by $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$ in a right-angled triangle. (p. 235)

sine rule A rule relating the sides and angles of a triangle. For $\triangle ABC$, $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$ (p. 335)

size (of a matrix) The dimensions of a matrix; an $n \times m$ matrix has n rows and m columns. (p. 261) *See also* **order (of a square matrix)**.

skewed to the left *See* **negatively skewed**.

skewed to the right *See* **positively skewed**.

slope (or **gradient**) The mathematical measure of the steepness of a line, calculated using the formula

$m = \dfrac{\text{rise}}{\text{run}}$ (p. 178)

slope–intercept form (or **gradient-intercept form**) The way of writing the equation of a line so that the gradient and y-intercept are immediately apparent; that is, $y = mx + c$, where m = gradient and $c = y$-intercept. (p. 178)

solution The value(s) of the variable(s) that makes an equation true, solves the equation. (p. 169)

sphere A ball shape. (p. 59)

square matrix A matrix that has an equal number of rows and columns. (p. 264)

standard deviation A measure of spread that is the square root of the mean of the squares of deviation of the values from the mean. (p. 314)

stem-and-leaf plot, **stem plot** A display of data in which the highest place values such as tens are shown in a column (the stem) and the lower place values such as units are shown horizontally across from their stems (the leaves). (pp. 297, 389)

step graph The graph of a step function, consisting of a series of line segments that look like steps. (p. 223)

subject (of a formula) A single variable on the left-hand side of a formula (equation). (p. 116)

substitution method An algebraic method for solving simultaneous equations. One of the equations is written as a formula for one of the variables. This is substituted into the second equation to find the other variable. (p. 203)
See also **elimination method**.

summary statistics Statistical measures calculated from data such as mean, median, mode, range, interquartile range and standard deviation. (pp. 321, 380)

surface area The areas of the faces of a three-dimensional (solid) shape. (p. 59)

symmetrical distribution A statistical distribution that is roughly symmetrical on both side of the central peak. (p. 318)

tangent The trigonometric ratio given by $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ in a right-angled triangle. (p. 235)

term Part of an algebraic expression separated from others by addition or subtraction. (p. 110)

third quartile *See* **upper quartile**.

three-dimensional (3D) An object or solid that has three dimensions: length, width and height (or depth). (p. 38)

time-and-a-half 1.5 times the normal rate of pay for overtime or as a penalty rate. (p. 12)

transpose (a formula) To move a given term or variable from one side of a formula (or equation) to the other. (p. 118)

trigonometry The study of relationships between the sides and angles of triangles. (p. 235)

trigonometric ratio, **trigonometric function** The ratios between the sides of a right-angled triangle. (p. 235)
See also **sine**, **cosine**, **tangent**.

true bearing A compass direction such as 084° measured as an angle clockwise from north, from 000° to 360°. (p. 249)

two-dimensional (2D) A shape that only has 2 dimensions, length and width, and no thickness (or height or depth). (p. 36)

undefined Not having a mathematical meaning. (p. 179)

unit cost (or **unit price**) The price of 1 item, 1 kg, 1 litre, etc. of goods, used to compare prices sold in different-sized packages. (p. 86)

univariate data Statistical data involving only one variable. (p. 289)

upper fence The value $Q_3 + 1.5 \times \text{IQR}$ used to determine if a high data value is a possible outlier. (pp. 319, 366)
See also **IQR, lower fence, outlier**.

upper quartile (Q_3) The value below which three-quarters of the values of a statistical distribution lie. (pp. 309, 365)

variable, **pronumeral** A symbol standing for a number. A *statistical* variable is often the quantity under discussion. (p. 110)

vertex (pl. **vertices**) A corner or a point where lines or edges meet on a shape or network. (p. 35)

volume A measure of the size of a three-dimensional (solid) shape using cubic units. (p. 66)

wage A payment for work stated as an hourly rate. (p. 9)

x-axis The horizontal axis on the Cartesian plane. (p. 178)

x-coordinate The distance that a point is to the right (positive) or left (negative) of the origin. (p. 178)

x-intercept The value where the graph of the function cuts the x-axis. (p. 179)

y-axis The vertical axis on the Cartesian plane. (p. 178)

y-coordinate The distance that a point is above (positive) or below (negative) the origin. (p. 178)

y-intercept The value where the graph of the function cuts the y-axis. (p. 178)

Youth Allowance Government allowance paid to eligible apprentices and students aged under 25 years of age. (p. 18)

zero matrix A matrix with every element 0. (p. 264)

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